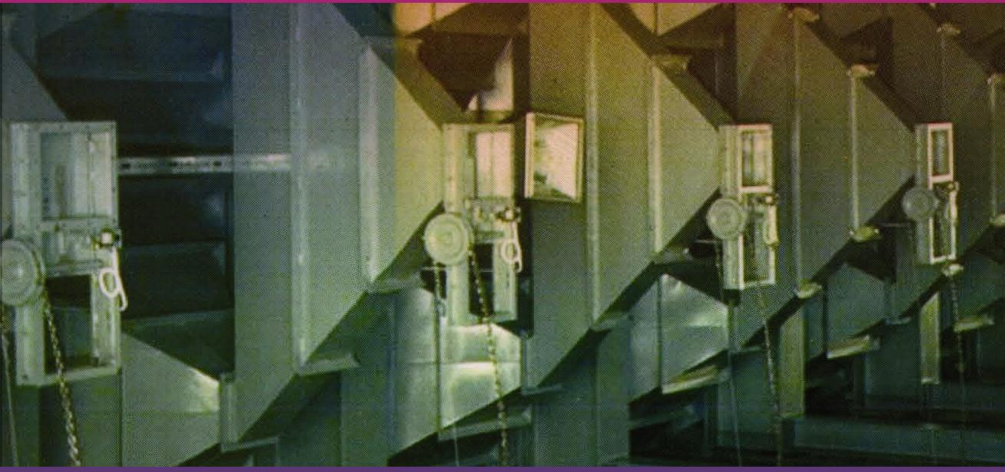
MODERN TEXTILE TECHNOLOGY



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One
Technological Analysis of Spinning

In the beginning of the history of spinning, progress in spinning technique was mainly made by accumulating empirical facts; that is to say, by repeating a set of procedures such as setting a spinning, condition and measuring the resultant properties and structures of the spun fibers. In melt spinning, we can predict the diameter and temperature and the tension in a running filament if the spinning conditions and the rheological properties of a polymer used in the spinning process are given; the predicted values are, of course, in good agreement with the experimental results. Such a prediction, however, can be made only when no significant crystallisation occurs during the spinning process.

In the theory of Kase and Matsuo, mean values of temperature and of stress over the whole area of a transverse section of the filament were used. In the process of high speed spinning, however, the variables such as temperature, stress, orientation and crystallinity must be expressed as functions of the radial distance from the central axis of the filament as well as the distance from the spinneret. For example, consideration of the radial distribution of these variables is inevitable in discussing inhomogenous structures such as skin-core structure. In order to understand the spinning process, it is indispensable for us to know how structure will be formed during the process as well as to carry out the detailed technological analysis of the process.

For the technological analysis of spinning, three fundamental equations are derived from the conversion of energy, the conversion of momentum and the conversion of matter, respectively. Here, for simplification, the two differential operators are defined as follows:

$$\nabla \cdot = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \quad (1)$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y} + V_z\frac{\partial}{\partial z}$$

where t is the time, (x,y,z) the spatial co-ordinates, $V(V_x, V_y, V_z)$ the velocity of polymer, and i, j and k are the unit vectors in the x, y, and z directions, respectively.

Fundamental equations

Conversion of energy

Consider an arbitrary small-volume element dv fixed in space along the spinning path and an enthalpy change in the element. The fundamental equation for conservation of energy can then be derived.

Inflow of heat through conduction

We assume that the thermal conductivity of the polymer K_c is independent of temperature T. The net inflow of heat in unit time conducted through the xy-plane into the element, the centre of which is located at (x,y,z), is expressed as

$$\left(K_c \frac{\partial T}{\partial z}\bigg|_{z+dz/2} - K_c \frac{\partial T}{\partial z}\bigg|_{z-dz/2} \right) dxdy = K_c \frac{\partial^2 T}{\partial x^2} dxdydz \quad (3)$$

Accordingly, by summing such quantities for all the three directions, heat transferred into the element in unit time is

$$Kc\nabla^2 T dv \; (\nabla^2\text{: Laplacian}) \qquad (4)$$

Inflow of heat accompanying transfer of matter

For the polymer, the specific heat at constant pressure, the heat of crystallisation, the crystallinity and the density are expressed as C_p, ΔH, X and ρ respectively. We assume that C_p, and ΔH are independent of T and that the enthalpy per unit mass (H) is a function of T and X.

Accordingly, by summation of such quantities for all the three directions,

$$-\nabla.(\rho \; HV)dv \qquad (5)$$

Using the equation of continuity $\partial R/\partial t = -\nabla.(\rho V)$ (modified equation 4),

$$-\nabla \cdot (\rho \; HV)dv = H\frac{\partial \rho}{\partial t} - \rho V \cdot \nabla H \qquad (6)$$

Conservation of enthalpy

Here, we direct our attention to the conservation of enthalpy within dv

$$\frac{\partial(\rho H)}{\partial t} = K_c \nabla^2 T + H\frac{\partial \rho}{\partial t} - \rho V \cdot \nabla H \qquad (7)$$

$$\text{Since } \partial H/\partial T = C_p \text{and } \partial H/\partial X = \text{-}\nabla H, \qquad (8)$$

$$\nabla H = C_p \nabla T\text{-}\nabla H.\nabla X$$

Substitution of this relation into equation 7 gives

$$\rho\frac{\partial H}{\partial t} = K_c \nabla^2 T - C_p \rho V \cdot \nabla T - \rho \Delta H V \cdot \nabla X \qquad (9)$$

Conservation of matter

The following equation is derived on the basis of the balance of the matter which flows into the volume element.

$$\frac{D\rho}{Dt} = -\rho\nabla \cdot V \qquad (10)$$

Assuming that the polymer is not a compressible material,

$$\Delta.V = 0 \qquad (11)$$

Conservation of momentum

Based on the balance of the momentum which flows into the volume element the following equation is obtained

$$\rho\frac{DV}{Dt} = -\nabla P - [\nabla \cdot p] + \rho g \qquad (12)$$

where P is the pressure, in a normal sense, of isotropic fluid, p the excess stress tensor and g the acceleration of gravity. $[\nabla.p]$ is a vector, and, for example, one of its components is expressed as

$$[\nabla \cdot p]_x = \frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} + \frac{\partial p_{xz}}{\partial z} \qquad (13)$$

Under the assumption of non-compressibility, P has no physical meaning, but will be determined by the boundary condition. When the spinning direction is chosen as the z-axis, the quantity, $P_{zz} - P_{xx}$, corresponds to the tensile stress. In addition to the three equations corresponding to conservation of energy, matter and momentum, we must take account of constitutive equations (rheological equations), equations of crystallisation kinetics, the equation concerning molecular orientation (birefringence Dn) and the thermodynamic equation of state. Before turning to the discussion of each of these equations, we shall direct our attention to the number of unknown variables and of equations. The important equations governing the boundary conditions are the equation of thermal conduction on the surface of filament and that of air resistance.

Constitutive equations

Various complicated formulae have been proposed so far as constitutive equations which relate the excess stress tensor p to the thermal and deformation history. The simplest example is Newton's equation of viscosity.

Equation of crystallisation kinetics

The nucleation rate of polymers at a constant temperature is greatly accelerated by molecular orientation. For the present, however, there is no general formula expressing the quantitative relation between the crystallisation rate and molecular orientation. Furthermore, the crystallisation under molecular orientation may be different from ordinary unoriented crystallisation, which is expressed in terms of the nucleation rate, the growth rate of the nucleus and the mode of geometrical growth. At any rate, the process of structural change in oriented crystallisation has never been clarified.

This will be discussed in a subsequent part of this chapter. If crystallisation kinetics are described in the form of the Avrami equation, $X = 1 - \exp(-K_A t^n)$, with increasing molecular orientation the rate constant K_A increases rapidly and the Avrami index n decreases to unity or even below. In reality, the Avrami equation applies only in the early stages of crystallisation. In addition, it should be noted that secondary crystallisation becomes prominent in the advanced stages of crystallisation. Adopting the birefringence Δn (or the tensile stress σ) as a parameter relating to molecular orientation, we can tentatively express the rate constant of crystallisation K_A at a constant temperature as a function of T and Δn.

Nevertheless, there still remains a question of how to utilize the corresponding data for describing non-isothermal crystallisation. Though we have no approved answers to this question, the following expression for the crystallinity X can be adopted without any gross errors.

$$X = 1 - \exp \{-\int_0^t K(T, \Delta n)dt')^n\} \qquad (14)$$

where the equation for kinetics of isothermal crystallisation is expanded into the case of non-isothermal crystallisation and the relation, $K(T, \Delta n) = \{K_A (T, \Delta n)\}^{1/n}$ is assumed. Equation 16 is an integral equation which describes X using the history of T and Δn (namely, σ).

Relation between tensile stress and birefringence

An experimental linear relation between the birefringence A n and the tensile stress a during spinning has been reported for small $\sigma(\sigma<3\times10^7$ dyne/cm^2 for polyethylene terephthalate [PET]). This is easily understood because the theory of rubber elasticity has definitely proved that σ/KT (K is Boltzmann's constant) is directly proportional to Δn for small a (Gaussian chain approximation).

In the case of high speed spinning, however, we must take account of the relation between Δn and σ for large a because a readily reaches up to 10^8 -10^9 dyne/cm^2. While a can approach infinity, Δn has its maximum value defined by the intrinsic birefringence Δn_m. Hence, with increasing σ, the rate of increase of Δn decreases and Δn itself approaches a constant value. Ziabicki and Jarecki obtained a numerical relation between the value of Δn in the steady state (Δn_{st}) and σ/KT for Langevin chains. The above applies to steady-state flow but the spinning process is in the non-steady state in terms of molecular orientation. Thus, Δn is always smaller than Δn_{st}, which is the value of Δn in the steady state for the present a. Assuming a single delay time τ,

$$\mathrm{D}\Delta n/\mathrm{D}t = (\Delta n_{st} - \Delta n)/\tau \tag{15}$$

then through σ/KT, Δn_{st} is a function (and through T, τ is a function) of position and time.

Solution spinning

In the process of dry spinning, the fiber structure is formed by forcing the polymer solution out through a fine nozzle and then evaporating the solvent. Accordingly, dry spinning can be treated as a problem of structure formation in a two-component system of the polymer and its solvent.

Technological analysis of the dry spinning is, therefore, much more difficult than that of melt spinning which is treated as a problem of structural formation in a one-component system. An equation of diffusion for a two-component system is needed to describe structure formation within the filament, and an equation of evaporation rate of the solvent at the boundary between two phases is also needed on the surface of the filament. Moreover, some equations used in the mathematical treatment of melt spinning should be modified to apply them to dry spinning. In the process of wet spinning, where a solution of the polymer is forced out through a nozzle into a non-solvent for the polymer, mass transfer for both the solvent and non-solvent must be considered.

Technological analysis of wet spinning is, accordingly, much more complicated than that of dry spinning. In a spinning process accompanied by chemical reactions, quantitative analysis is almost impossible. As for the spinning process without any chemical reactions, the phase diagram using triangular co-ordinates for a three-component system of polymer (P), solvent (S) and non-solvent (N) can be drawn. In this case, it is the ratio F_N/F_S of the flux of solvent from the filament into a spinning bath (F_S) to that of non-solvent from the bath into a filament (F_N) that determines which path is followed on the phase diagram in the process of fiber formation.

Structural formation during spinning

The structural changes in the spinning process, crystallisation, gelation, and phase separation, are discussed here. Crystallisation arising from the state of molecular orientation in a polymer solution, melt or amorphous solid is termed 'oriented crystallisation'. Crystallisation during spinning is a typical example of oriented crystallisation.

Structure formation in oriented crystallisation is of great interest because it is a phenomenon reflecting the nature of the macromolecule. Since the 1960s, studies on oriented crystallisation have been carried out extensively and the publications so far are too many to mention. Of all these studies, that of flow-induced crystallisation of polyethylene in particular attracted researchers' attention. When polyethylene is crystallised from solution by stirring the solution, what is called the shish-kebab structure is formed. The following are general features of oriented crystallisation:

— The morphology of the crystallized materials changes according to the degree of molecular orientation.

— With increasing degree of molecular orientation, the temperature which gives the maximum rate of crystallisation goes up and, in some cases, the maximum rate itself also increases by several orders of magnitude.

— The mechanism of oriented crystallisation may be very different from that of non-oriented crystallisation.

Oriented crystallisation from melts

The knowledge that we have so far acquired of structure formation from oriented melts of flexible polymers is summarized as follows:

— In advance of crystallisation, a spatial non-uniformity of density occurs. A domain with higher density has a rod-like shape elongated along the direction of molecular orientation. The diameter of the domain depends strongly on the degree of molecular orientation and decreases with increasing degree of orientation. The domain, generally speaking, is of a size which can be seen with a light microscope.

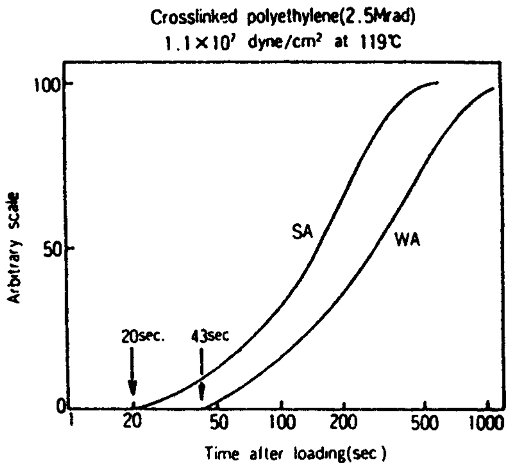

Fig. 1 Oriented crystallisation of cross-linked polythelene

— Within such a domain, density fluctuations on a scale of several tens of mn occur within a stacked lamella-like structure in which constituent 'lamallae' are developed perpendicularly to the direction of molecular orientation. The fluctuation can be detected by meridional small-angle X-ray scattering. At this stage of structure formation, wide-angle X-ray diffraction from the crystalline state is still not observed. In conclusion, there exists a mesomorphic state during transformation from the amorphous state to the crystalline.

— In the shish-kebab structure and the row structure (a structure showing an appearance like a Japanese mat, 'tatami'), crystalline lamellae are stacked in the direction of molecular orientation. Such morphology was interpreted by Keller and co-workers as a structure consisting of folded-chain lamellar crystals grown epitaxially on a long and slender nucleus composed of extended chains. Row structure, however, can readily emerge even from very weakly-oriented melts, and in a thin film of the polymer we can observe many nuclei which are aligned in a line in the rod-like domain. It is, therefore, presumed that the concept of a nucleus composed of extended chains would be a product of too extreme modelling. We should rather venture to say that the characteristic of structure formation in oriented crystallisation is that oriented nuclei are apt to align themselves in a line. In thin films crystallized under molecular orientation with a rather high degree of orientation, however, a crystalline domain of about 200 nm in length and about 15 nm in width was identified by high-resolution electron microscopy as a domain in which lattice fringes are observable. Accordingly, of course, we cannot deny the existence of extended-chain crystals. Hence, in order to gain a better understanding of the structure formation during crystallisation under flow (oriented crystallisation), the overall conformation of specific molecular chains in the material should be determined, probably by neutron scattering.

Non-uniformity of density generated in a polymer system which was already deformed or is now being deformed mechanically has been noted recently and termed stress-induced phase separation. In this regard, an interesting result of simulation has been reported. In this paper, a two dimensional gel with a triangular network at its equilibrium state is assumed.

A state with different strains which depend on the position is given as an initial condition and the process of strain relaxation with time is simulated on the basis of dissipative molecular dynamics. As a result of such simulation, the existence of a metastable state is predicted, in which expanding and contracting phases coexist. If we regard the polymer melt as a gel in which the points of entanglement appear and disappear temporarily, we can understand the fact that in the relaxation process of a system deformed by elongational flow, there exists a metastable state which has spatial non-uniformity of density.

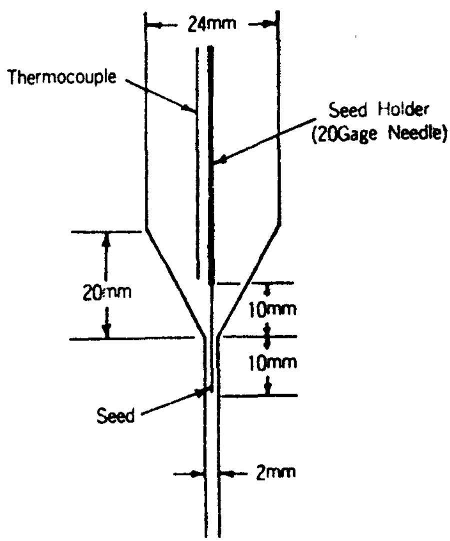

Fig. 2. Apparatus for oriented crystallisation of polyethelene from solution

Oriented crystallisation from solution

The previous subsection introduced the fibrous precipitates grown from a solution of high density polyethylene, the so-called shish-kebab structure. It should be noted that the formation process of this structure was not a matter of common knowledge until comparatively recently.

A 0.01wt% xylene solution of ultra-high molecular weight polyethylene ($M_w = 3 \times 10^6$) was flowed through a funnel-shaped pipe as illustrated in Fig. 2. A polarized light microscope was used to observe the formation process of the fibril at the tip of the seed crystal which was suspended in the pipe. The results indicated that a gel-like amorphous fibril, which had a higher concentration of the polymer and was non-oriented, was formed first as a precursor and then crystallisation synchronised with the emergence of birefringence which was induced by tensile stress exerted on the fibril through the fluid. From the time dependence of the change of the measured birefringence, the crystallisation was expressed by the Avrami equation with an Avrami index of 2. McHugh *et al.*, therefore, concluded that the crystallisation in question consisted of one-dimensional growth (growth in the flow direction) initiated by homogeneous nucleation. The formation of a precursor structure before crystallisation bears a resemblance to structure formation in oriented crystallisation from a melt and is therefore of great interest.

Gelation and phase separation

Theoretical and experimental studies on gelation and phase separation of polymer systems have flourished. According to experiments on a polyviny-lalcohol (PVA)-water system by Komatsu *et al.* the solgel transition curve intersects the SD (spinodal decomposition) curve.

i) Area corresponding to a homogeneous sol state.

ii) Area in which liquid-liquid separation takes place due to SD but gelation does not occur.

iii) Area in which gelation takes place due to SD.

iv) Area in which gelation takes place without any liquid-liquid separation.

The molecular orientation induced by the flow of such a solution, needless to say, shifts these curves. In dry spinning of the system in question, the path which the system follows on the phase diagram changes according to the spinning conditions and consequently the mode of structure formation and the structure itself in the spun filament will vary.

Fiber structure

For flexible polymers, uniaxial orientation is realised by the unfolding of their folded chains by stretching. Such orientation is also given to a rigid polymer by elongational flow of a liquid crystalline phase of the polymer; uniaxial orientation is a structural characteristic of fibers. The fiber structure in commercial fibers has been investigated by transmission electron microscopy (TEM), scanning electron microscopy (SEM), X-ray diffraction, and so on.

Poly (aryl-ether-ether-ketone) (PEEK)

PEEK is fairly resistant to electron bombardment and, thus, is a suitable material to take lattice images by high-resolut ion TEM. Several drops of a hot solution of PEEK in a-chloronaphthatene were sandwiched between two glass slides at 300°C. Just after evaporation of the solvent, a thin molten film of PEEK between the slides was oriented and crystallized by displacing one of the two slides. A fine crystallite connecting adjacent lamellae in the direction of the fiber axis is clearly seen; the authors have called this a 'tie-crystallite'.

The authors are satisfied by the explanation that the correlation in orientation between crystallites which belong to a single microfibril is maintained by a tie-crystallite. Such tie-crystallites may pass through several lamellae. Tie-crystallites have also been recognised in the lattice images of uniaxially oriented thin films of PE and poly (4-methyl-1-pentene), whose images were taken at 4.2K using a cryogenic transmission electron microscope equipped with a superconducting objective lens.

Fiber structure of rigid chain polymers

Fibers of rigid chain polymers are commonly produced by solidifying their liquid crystalline domains, in which extended chains are aligned parallel to each other, with all chains being oriented uniaxially owing to elongational flow working on the domains.

Poly (p-phenytene terephthalamide) (PPTA)

A Kevlar fiber was annealed at 400°C under the condition of constant length, and then fibrillar fragments were obtained by tearing them off from the fiber. The fragments thus prepared were used as specimens for TEM. Such a banded texture was also observed in longitudinal thin section and in the fibrillar fragments of PPTA. Careful inspection of the figure, however, shows that in some areas the crystallites are aligned along the microfibrils. The size of the crystallites is of the order of 10-20 nm both in width and in length, and is much smaller than that expected from the nature of the rigid polymer chain and its chain length. This inconsistency seems to arise from twisting of microfibrils around their own chain axes.

Simulation of high-speed spinning

The process of high-speed melt spinning has been proposed and developed as an attempt to produce highly oriented filaments by a single-stage process without any drawing after spinning. Since a commercial winder with a take-up speed of about 6000 m/min became available in the latter half of the 1970s, studies on high-speed spinning have been greatly advanced. The features of high-speed spinning are as follows:

— Polyethylene terephthalate (PET) does not crystallize appreciably in normal melt spinning, but does at a spinning speed of about 4000 m/min or more.

— An abrupt necking-like change of diameter of the filaments during high-speed spinning is recognized and closely related with crystallisation.

— The resultant spun filaments have a skin-core structure: the skin region is highly oriented and crystallized, but the core region has weak orientation and low crystallinity.

— The spun filaments have rather high extensibility, and accordingly are utilized only for special purposes.

— A spinning speed of 7000 m/min or more lowers the quality of spun filaments.

In these experiments, it was confirmed that the necking point moves up and down within certain limits, and that crystallisation hardly occurs at all before necking, but increases very suddenly at the onset of necking. The real cause of this necking phenomenon is not yet known. One idea of the cause is based on the time dependence of elongational viscosity.

A plot of the elongational viscosity β (t ε) in a non-steady state vs time in a situation where a melt of low density PE was elongated at a constant strain rate ε. $\beta(t\ \varepsilon)$ increases with time and attains a maximum. This indicates that the structural change due to elongational flow has a complex influence on the viscosity, such that the viscosity depends on the strain itself as well as on the strain rate. Hence, it might be deduced that the viscosity reaches its maximum during the spinning process and this triggers the neck formation.

On the other hand, another view that such a maximum might be due to experimental error and cannot exist has also been presented. A second idea as to the cause of necking is the following. It was experimentally reported that when the ratio of élongational viscosity to shear viscosity exceeds necking occurs. According to this result, we can say that necking takes place when elongational strain attains a certain magnitude during spinning.

A third idea comes from attention to the structural change which occurs before oriented crystallisation. That is to say, the system itself is transformed into another state in which necking can readily take place. For the moment, we cannot determine which of the above three ideas is true. As a model polymer, PET with a molecular weight of about 23,000 was taken, and mathematical relations found in the literature were utilized. As for unknown numerical relations, appropriate assumptions were made, and a working hypothesis was also introduced for the mechanism of structural formation during spinning. In this simulation, the same spinning conditions as above were used, and a cooling condition was fixed at horizontal cooling air velocity of 40 cm/sec. Though the magnitude of initial tension must be given as an initial condition of calculation, this was roughly estimated by an established procedure in which crystallisation is neglected. Since the take-up speed is to be determined in principle from the calculated result, it cannot be introduced as a spinning condition.The change of diameter of the filament during spinning.

The upper curve in the same figure shows the experimental necking stress for an unoriented PET film annealed for 3 hrs at 120°C. It is deduced that the real necking stress during spinning should be rather smaller than the experimental necking stress mentioned above. Intersect each other at a point corresponding to about 150°C definitely predicts the appearance of necking around this point during high-speed melt spinning.

Two
Applications of Textile Mechanics

A somnolent drunkard, breathing stertorously in the gutter on a Saturday night, is in a blissful state of equilibrium. A similar argument holds true when an object is undergoing uniform motion, either in linear or in circular fashion. All the visible evidence indicates that the uniform motion will continue. As far as we can tell by direct observation, the moon will carry on revolving around the earth in a perfectly satisfactory manner, to the delight of lovers and poets, for the foreseeable future, and the dire predictions of astronomers can be regarded as irrelevant nonsense in the context of one night, one year or even one lifetime.

In both these cases, then, the object is in equilibrium with its surroundings. This equilibrium may be maintained is by the complete absence of any force acting on the body. We have already seen that a body remains at rest, or continues to move uniformly in a straight line, so long as no external force acts on it. In practice, however, it is virtually impossible to visualize any object existing in complete isolation from any force whatsoever unless one enters the realms of metaphysics or leaves the physical universe so far behind that gravitational attraction no longer exists even to the most minute degree.

Nevertheless, equilibrium can easily be achieved in the everyday experience of mere mortals, so we must seek a more realistic way of defining its existence. Consider a particular single fibre, in a roving can, awaiting spinning. Let us assume that, from its rest position, it is pulled uniformly into the drawframe and, after passing through one drafting zone, becomes part of the fly, drifting gently down until it settles on the ground. From the mechanical point of view, the fibre is initially at rest, is suddenly accelerated to a specific speed and moves at this speed towards the frame. It is then subjected to an acceleration in the drafting process and, before it can achieve uniform motion again, is thrown or blown, with a sideways' acceleration, out of the fibre bundle. It now undergoes an acceleration vertically downwards under the influence of gravity until it finally returns to the rest state once again.

We can examine, in a somewhat simplified way, the forces acting on the fibre throughout the changes. In rest positions, the weight of the fibre, the force produced by the effect of gravity on its mass, is acting vertically downwards and an equal but opposite force, the normal reaction at the surface (or other fibrous material) on which it rests, is acting on the fibre to prevent it from falling under the gravitational attraction. The same two forces exist when the fibre is moving at uniform speed in the roving.

In addition, the frictional forces which maintain the roving in a coherent form must be present but, since the fibre is not accelerating, we deduce that any force of this nature must be balanced by an equal but opposite force in some way. This fact leads us to a preliminary definition of equilibrium as a state in which there is no net force acting on the body; that is, all the external forces acting on the body must be able to give a resultant of zero. It is then easy to see why, in the remaining parts of its motion, our fibre is not in equilibrium. By the very nature of the drawing operation, a fibre must be accelerated forwards by forces originating with the pull of the roller nip and transmitted gradually by inter-fibre frictional contact throughout the drafting zone.

In free fall, the fibre accelerates downwards under the influence of gravitational force and, once again, can not be in equilibrium. In the case just considered, we have assumed that each force was exactly balanced by an equal but opposite one. In addition, because the fibre is so small, we can make the additional assumption that all forces act at the same point. In general, however, this assumption may not be valid if we are considering a large body.

Suppose, for example, we wish to change the position of the builder motion on a spinning frame. We do not grab the shaft in both hands and attempt to turn it manually; we use a crank handle and apply the force at a distance from the shaft, as shown in Fig. 1, despite the fact that the resistance to motion is at the shaft itself. Not only does this keep the hands free from grease, but it also enables us to turn the shaft more easily for reasons which will emerge later in this chapter. As a matter of simple observation, however, we can note that, the longer the throw of the crank (i.e. the greater the distance between the handle and the shaft) the easier becomes the task of turning the mechanism. Two useful points emerge from this simple observation. In the first case, a force can be made more effective by moving its point of application away from the point at which its effect must be felt, a fact which, as anyone who has served time with hard labour will readily admit, makes the lever a useful tool.

In the second case, the act of applying a force at a point other than its point of effect achieves a rotational rather than linear movement. It is obvious, then, that the combination of a force and the distance from its point of application to its point of effect can have specific properties and we define the product of the two quantities as the moment of the force. The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

The moment of a force also represents the tendency of the force to cause turning and is usually expressed as either clockwise or anticlockwise in a diagram illustrating the mechanism.

Centre of gravity

In all our discussion in the earlier chapters of this work, there has always been the assumption that a body could be represented by a single point through which all forces pass and at which all effect is felt. It is now time to examine the differences between this ideal state and the one pertaining in practice. The simplest way to begin is to consider the effect of gravity on the body at rest. In theory, any object can be subdivided into smaller and smaller units until each of the units achieved can be regarded as a single point. Let us avoid the temptation to exchange philosophical thoughts with the Ancient Greeks about the nature of the atom, or to argue with medieval theologians about the number of angels who can dance on the point of a pin, by declining to define the precise magnitude of such a point.

Each of these minute units, left undisturbed in its original position would experience individually a force directed towards the centre of the earth, which would represent the weight of that individual unit. The force which we call the weight of the body is merely the resultant of very large numbers of these individual forces. It is obvious that, in a body of appreciable size, these forces cannot all act through the same point, so their effect when summed at a single resultant must be one of rotation. It is equally obvious, however, that a body at rest cannot be rotating, so the net effect of the forces acting to rotate the body clockwise must be exactly balanced by those acting in the opposite sense. For any selected position of the body, then, it is possible to combine the individual small forces into a single resultant with a particular line of action such that no net rotational effect is present. If this process is repeated for all possible positions of the body, it will be observed that all these resultant lines of action will pass through a single point and this point is defined as the centre of gravity. The centre of gravity of a body is that point, fixed with respect to the body, through which the line of action of the body weight always passes, no matter what the position of the body. As a result, the body can be regarded for most purposes as having all its mass concentrated at the one point.

As long as we take into account all the movements undergone by the body in translational or rotational modes, we can treat the body in our calculations as a point at its centre of gravity containing the whole mass. Thus the force of the body's weight can always be assumed to act at its centre of gravity, a fact which simplifies calculations considerably. Already seen that, when two equal but opposite forces act at a point, their resultant is zero. Let us now take the case where the two forces do not act at a point, but act at two different points on the same body and yet are parallel to one another. The system is illustrated in Fig. 1.

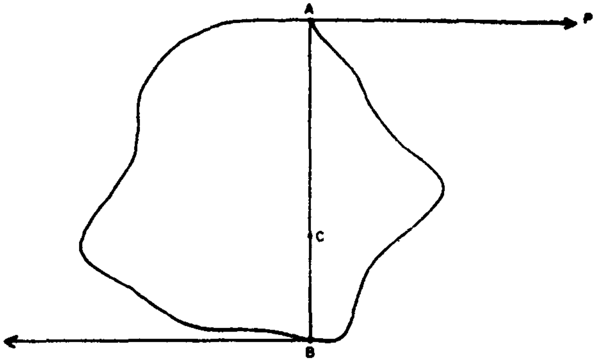

Fig. 1. Parallel forces acting at two different points

The presence of this rotation implies that some force is being exerted on the body, but it is obviously impossible to characterise this force as a single resultant of P and P' since they are parallel. Such a pair of equal parallel forces, acting in opposite directions but not in the same line of action, is called a couple; the definition might also fit the partners of many a modern marriage!

Let us choose any two points A and B such that each point is on the line of action of one force and the line AB is perpendicular to the lines of action. On the line AB, let us now select any point C and take moments about that point.

The moment P is P × AC, in a clockwise direction. Similarly, the moment of P is P' × BC, also in a clockwise direction.

The sum of the moments about C is thus:

$$P \times \mathrm{AC}\ P' \times \mathrm{BC} = P(\mathrm{AC} + \mathrm{BC}), \text{ since } P = P'$$

$$= P \times \mathrm{AB}, \text{ since AC} + \mathrm{BC} = \mathrm{AB}$$

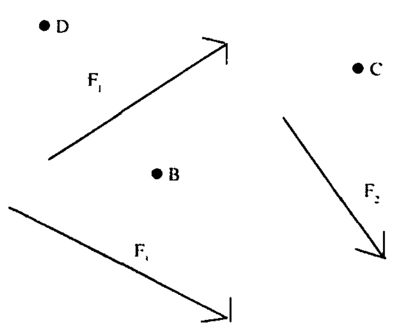

Fig. 2. Forces acting in a plane

This product is known as the moment of the couple and is a constant for any given couple, since no conditions were laid down in the selection of C except that was on a line at right angles to the direction of the forces. The distance AB is known as the arm of the couple. If the point C is fixed and the lines of application of the forces are moved about in any way, but with the distance between them remaining constant, the moment of the couple about C (and hence about any point in the plane) will remain the same and the rotational tendency of the couple on the body will, therefore, be unchanged. It follows logically, as the discerning student will no doubt be able to prove for himself, that to counteract the action of a couple, it is necessary to apply another couple of opposite rotational effect and of the same moment. The properties of a couple may thus be summarized as follows:

— A couple produces, or tends to produce, only rotation with no translational movement, and whenever a body rotates or tends to rotate a couple must be acting.
— The moment of a couple about any point in the plane in which it acts is constant and is measured as the product of one force and the arm of the couple.
— A couple can never be replaced by a single force, but can be replaced by another couple of the same moment and the same direction of rotation.
— The resultant of a set of two or more couples is a single couple, of moment equal to the algebraic sum of the moments of all the individual couples.
— A couple can never be balanced by a single force, but can be balanced by any other couple of equal moment but opposite rotation.

These properties lead us to an extension of the definition of equilibrium, since a body in which two equal and opposite forces are acting may not necessarily be in equilibrium if the two forces do not have any common point of action. In general, then, a body is only in equilibrium if there is no resultant force acting on it and if the resultant moment about any and all points on the body is zero.

Sign conventions

So far we have had to resort to the rather tedious device of specifying clockwise or anticlockwise rotation to describe a direction of movement, a procedure which can lead to problems if you happen to tell the time by a digital watch or by a sun dial in the Northern hemisphere. In order to eliminate the need for repetition, the idea of assigning as algebraic sign to describe direction of rotation is an obvious step which is usually taken. It is immaterial whether you define clockwise rotation to be positive or negative, as long as you are constant in your definition.

The usual convention, which will be followed in this book, arbitrarily defines this clockwise movement as a negative rotation; no doubt some of the more activist students will find just cause for an uprising against such blatant discrimination. It is necessary of course, to specify also the direction of observation in the case of any rotation, occurring in three-dimensional space, but this book is happily not concerned with such nit-picking trifles and it is assumed that the system has already been committed firmly and securely to the two-dimensional framework of the piece of paper on which it is depicted before we are called upon to define the sign of the movement.

If A is taken as the point at which rotation is important, all three forces tend to cause clockwise rotation, so their moments are all negative. At B, however, only F_1, and F_2 tend to cause clockwise rotation, while F_3 tends to cause anticlockwise movement. The moment of F_3, therefore, is positive at B. In similar manner it is easy to see that both F_2 and F_3 have a positive moment about C, while all three forces have positive moments about D. The sign of the rotational movement then, can be changed by adopting a different point of measurement of the effect even though no change in magnitude, direction, or line of application of a force takes place.

Principle of moments

It is now possible to define more clearly exactly what we mean by the term principle of moments or, as it is sometimes called, moments condition. This states that, for a body in equilibrium under the influence of any number of forces, the algebraic sum of the moments of all the forces, about any point in the plane, is zero.

The critical part of this definition, of course, is the fact that the point used the origin for taking moments is immaterial. At any point whatsoever in the plane of the forces the sum of the clockwise or negative moments is exactly equal to the sum of the anticlockwise, or positive, moments.

Example 1

A rod AB, of length 30 cm, pivoted 10 cm from A, is used by a rather naughty student for suspending glassware to dry after a Chemistry practical laboratory period. One piece of apparatus, with a mass of 500 g, is hung from A, a second piece, of mass 100 g, is hung from B and a third piece, of mass 400 g, is hung 7.5 cm from the pivot on the side nearest B. Show that, as long as a slamming door does not disrupt the situation, the student will unfortunately escape his just deserts.

The system is illustrated in Fig. 5, the pivot point being shown as O. Only the item at A gives rise to an anticlockwise moment.

Thus, Moments anticlockwise are given by:

$$0.5 \times 9.81 \times 0.1 = +\ 0.490 \text{ N m}.$$

in Clockwise moments are given by:

$$0.1 \times 9.81 \times 0.2 + 0.4 \times 9.81 \times 0.075 = -\ (0.196 + 0.294) \text{ Nm},$$
$$= -\ 0.490 \text{ N m}.$$

Thus, clockwise and anticlockwise moments are equal, the system is in equilibrium, and the student survives to torment other lecturers. Further examples illustrating the truth of this statement will appear in due course at a later part of this chapter; it is one of the most important and widely used principles in mechanics.

Equilibrium

Examined superficially the state of equilibrium and have defined the condition as one in which no net force acts on a body. It is now time to consider a little more carefully the exact nature of this situation. You may remember the old story of the Greek philosopher who challenged his students to balance an egg on one end. When they finally abandoned the attempt several days, he nonchalantly tapped one end of the egg very gently so as to flatten the apex of the shell without actually breaking it, upon which the egg would, of course, stand up steadily with no problem.

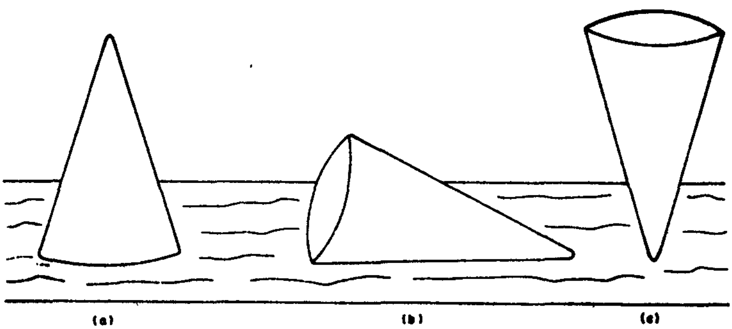

Fig. 3. Stable, metastable, and unstable equilibrium

The story is not included merely to fill up space, but to illustrate an important point about equilibrium, though propose that we transfer our attention from an egg to a spinning top to reduce the mess and to take the stress out of the chicken's existence. There are, in theory, three ways in which we can place a spinning top (or an egg with a flattened base) on a flat horizontal surface. We can stand it on its plane area, we can lay it on its side, or we can balance it on the point with the flat base in the air in the position usually adopted during spinning.

The latter mode is, of course, the most difficult and usually considered impossible, though we can cheat a little as in the case of the egg and flatten the point with a file; after all, who are we to question the honesty of a Greek philosopher?

Suppose, for example, that we apply to the cone a gentle push, of sufficient force to move it slightly from the equilibrium position. In the first sideways, rests briefly on a point on the rim circumference furthest from the application point of the force, then returns to its original position as soon as the force is released. In the second case, the cone will roll along its curved surface, in some kind of circular path, as long as the force is present and will stop, as soon as the latter is removed, at whatever position it has then reached.

For the cone balanced precariously on its smaller end, however, the situation is much more delicate. Even a slight force, applied for the briefest possible time, is sufficient to cause it to topple, with a suitably impressive crash, onto its side so that it ends up in the same state as the second case. The three situations are obviously different and are distinguished by name. The first one is an example of stable equilibrium, to which the body returns automatically, because the state is more stable than any other nearby one, after a small displacement has occurred as a result of a force acting for a limited time. The second state is one of *metastable* equilibrium, in which application of force causes a change in position but no increase or decrease in stability of the body.

The third position is, fairly obviously, one of unstable equilibrium, in which the body remains only as long as no additional force is exerted on it and only because there is a precarious, though uneasy, situation in which no net force is present on the body for the time being as all forces present are delicately balancing one another. Any movement from this position destroys the equilibrium and the body immediately reverts to the nearest state of stable or metastable equilibrium. It should be particularly noted that the body cannot revert to a true unstable-equilibrium state since the tendency of the body nearly, but not quite, in such a state is to move away from, rather than towards, the unstable position.

Equilibrium under two forces

When the egg, or the top, or the cone is in one of the three possible equilibrium states, there is really only a single external force, that of gravitational attraction, acting on the body. If external forces are applied to a body, however, there are two conditions which must be met if the latter is to remain in equilibrium. In the first case, the resultant of the forces must be zero, since otherwise an acceleration would be imposed on the body and it would begin to move (or change its rate of movement if it were already in motion). Secondly, the resultant moment of all the forces about any axis must be zero, since there would otherwise be a rotational tendency which would nullify the equilibrium state.

There are only two forces are applied, these conditions impose rather severe restrictions on the situation. If equilibrium is to be maintained, the two must be equal and opposite, since they would otherwise have a resultant and the first condition would no longer exist. In addition, they must lie on the same line of action (but in opposite sense, naturally, to maintain the first condition) since otherwise they would constitute a couple and the second condition would no longer hold true. Perhaps the most important application of this situation in practice is the one where an object is held at rest by an upward force resisting that of gravity. Let us suppose that we apply such a force, U, by means of a suspending string. Since the force of gravity acts vertically downwards, and since we seem helpless to change this fact, it follows that *U* must act vertically upwards. It is obvious, by considering the two foregoing conditions, that *U* must be equal to the weight of the object, the force *W* exerted by gravity on its mass, and that it must pass through the centre of gravity, G, since the mass of the object acts there also.

This may be achieved in two ways; the suspending string can be attached vertically above G, at A, or vertically below G, at B. A slight displacement from each of these two equilibrium states serves to indicate that the former is a stable one, whereas the latter is unstable, as indicated in the diagram. A further example of the situation may be seen when an object such as a vehicle, or a bale of fibre, or it piece of equipment is rocked sideways; as long as the equilibrium is stable the object will return to its original position but as soon as the boundary of stability is passed it will topple.

This occurs whenever the direction of action of the mass, assumed to be vertically downwards through the centre of gravity, falls outside the area of the base on which the object rests, so that the upthrust and the weight together form a couple tending to topple the body.

Example 2

A bale of cotton fibre, of height 1.80 meters has a square base of side 1.20 meters. Calculate the height to which one edge of the base can be lifted before the bale topples over. The weight *W* of the bale acts vertically downwards through A, the edge at which the upthrust, *U*, is exerted and the edge B, at a distance h above the ground, is at its maximum height before toppling occurs.

By trigonometry:

$$\sin \alpha \; h/1.20$$

But sin α is also equal to 1.20/AC, where AC is given, by

Pythagoras's theorem, in:

$$AC^2 = 1.80^2 + 1.20^2,$$

i.e.:

$$AC = 4.68 \text{ m}.$$

Thus

$$h/1.20/4.68,$$

or:

$$h = 0.31 \text{ m},$$

i.e.: the edge of the base can be lifted to a height of 31 cm before toppling occurs. Thus, if the bale edge is lifted to a height of less than 31 cm and then released, it will fall back to its original position, but if this height is exceeded the couple consisting of weight and upthrust will cause it to topple to the stable position of equilibrium in which it rests on one long side.

Equilibrium under three forces

The addition of a third external force to the system introduces a slightly larger element of freedom into the situation. If two of the forces are represented by the vectors P and Q they can be replaced by their resultant R so that this resultant, together with the third force, S, can be regarded as belonging to a two-force system. Thus, for equilibrium, R must be equal and opposite to S and must act along the same line. It follows that the lines of action of all three forces must lie in the same plane, since otherwise R and S could not be collinear.

There are thus two ways in which the forces may be distributed; either they all three act at the same point, or they are all three parallel to one another, since no other arrangement permits all these conditions to be satisfied. Where three parallel forces are present, equilibrium exists when the resultant force is zero and this can only be the case if the algebraic sum of the forces is zero and if the algebraic sum of the moments of all the forces about any point in the plane is zero. Then, the sum of the magnitudes of P and Q must equal the magnitude of S and the moment of P about 0 must equal the moment of Q about O.

Example 3

A Simple lever arrangement for applying pressure to the rollers of a padder one such lever being used at each side of the padder to achieve even pressure. If the mass of the roller is 25 kg, the mass of the loading bob is 40 kg, the pressure point is 15 cm from the fulcrum and the centre of gravity of the loading bob is 60 cm from the fulcrum, calculate the force exerted on the fabric passing between the rollers and the force exerted by the lever on the fulcrum. Each lever is kept in equilibrium by three forces, the bob weight, the upward supporting pressure, P, of the roller bush and a force F at the fulcrum.

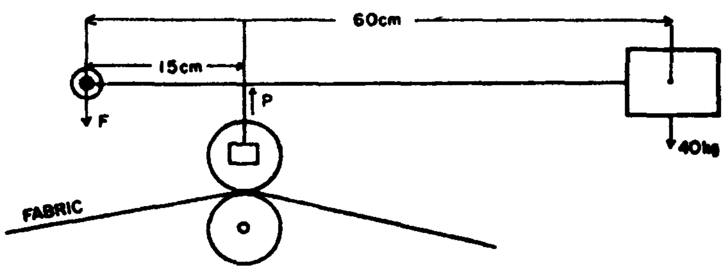

Fig. 4. Padder pressure system

Applying the principle of moments, and taking moments about the fulcrum to eliminate the unknown force there, we have:

$$P \times 0.15 = 40 \times 9.81 \times 0.60,$$

i.e.:

$$P = 1569.6 \text{ N}.$$

This must also be the downward force exerted on each end of the roller by a bush, so that the total force exerted is double this quantity. In addition, the mass of the roller itself exerts a force of 25×9.81 N on the fabric.

Thus, total force on fabric = $(1569.6 \times 2 + 25 \times 9.81)$ N,

i.e. total force on fabric = 3384-4 N.

Since the three forces on a lever are in equilibrium, there must be no net force in any direction.

Thus, the force at the fulcrum must act vertically. In addition, we know that the upward pressure of 1569.6 N and the downward force of 40×9.81 or 392.4 N, must be exactly balanced by *F*. Thus *F* acts vertically downwards and has a magnitude of (1569.6 – 392.4) N.

Hence force exerted on fulcrum is 1177.2 N. Let us now consider the case where the three forces are not parallel and must, therefore, act at a single point. A vector diagram by completing the parallelogram with sides *P* and *Q* and diagonal *R*. In addition, *S* is equal in magnitude, but opposite in direction, to *R*, so can be represented by a vector -*R* on the diagram. Thus, we can use two adjacent sides and a diagonal of the parallelogram, its shown, to represent the three forces in magnitude and direction.

It is also evident that the three are arranged in sequence around the triangle and that either triangle of the original parallelogram may be used. This is the rule of the triangle of forces; three non-parallel forces in equilibrium may be represented in magnitude and direction by the three sides of a triangle taken in sequence. It is important to note that the triangle does not necessarily enable the point of application to be established. In dealing with practical situations, then, care must be taken to ascertain which object is in equilibrium, what are the three forces acting and where they are applied. It is useful, in the latter context, to remember that, if two of the forces can be shown to pass through a single point, the third one must also act there, if this is not obvious from a diagram illustrating the problem.

Example 4

If the pressures applied are found to be 15 N at the top roller, A, and 50 N at the bottom roller, B, find the magnitude and direction of the force on the fulcrum. Consider the equilibrium of the rollers. Since the pressures at *A* and *B* are parallel, there must be a third force, *T*, in the link with a magnitude equal to the sum of the pıessures.

Thus:

$$T = 65 \text{ N}.$$

Now consider the equilibrium of the weighting lever. Three forces are acting on it, these being exerted by the mass of the weighting bob, vertically downards, the tension in the link at an upwards direction of $\pi/3$ to the horizontal and the force *F* at the fulcrum, unknown in magnitude and direction. It is possible to determine *F* by calculating its horizontal and vertical components and then combining them, by trigonometry, to find.

The magnitude and direction of the resultant. Alternatively, we can extend the fine of action of the tension and the mass. Until they meet in a point, and then construct a line through this point and the fulcrum to give -the line of action of the force acting there. The usual tedious and dreary calculation then gives, eventually, a value for the magnitude of *F*.

General conditions of equilibrium

If we now extend our interest to the situation where more than three forces are exerted on a body at equilibrium, an even greater degree of latitude in the magnitude and direction of the forces arises. In the first case, the lines of action need not necessarily all lie in the same plane, since the resultant of any two does not have to balance a single force, but merely the resultant of a number of other forces. In addition, if the forces are not all parallel, the lines of action need not necessarily pass through a single point, by a similar line of reasoning. At this stage the mathematical juggling becomes unnecessarily boring for the poor textile technologist, who will probably never need to consider such complex situations anyway, but it can be shown by an extension of the simple considerations just met that there are, in general, two conditions which should hold if equilibrium exists.

The first of these is that the resultant force on the body is zero and the second requirement is that the algebraic sum of the moments of all the forces acting must be zero no matter which point is taken as reference.In order to test for equilibrium, it is necessary to establish a three-dimensional set of co-ordinate axes, usually indicated as x, y, and z axes, and to resolve each force into its components in the three directions. For the first condition to be met, the resultant net force in each of the three directions must be zero.

The second condition is met if the algebraic sum of the moments of all the components in any one direction, about any point in the plane of the other two axes, is zero. (If, for example, the x-direction is being investigated, the net moment of all the forces in this direction about any point in the y-z plane must be zero, and the same condition must pertain in both of the other two comparable situations representing the other two similar combinations). All of this becomes rather complicated, as may be appreciated, and the student is advised to approach each such problem with care.

The preferred, method of dealing with this type of situation is to pass the calculation on to someone else junior to yourself and to criticize when it is carried out incorrectly. For the office boy who finally has to tackle it, however, no such ploy is available and an alternative technique is to substitute each force by an equivalent one in such a way that all the forces can be treated as though they are all parallel, all passing through a single point, or all lying in the same plane.

In any one of these cases, it is then an easy matter, by obtaining resultants for two or more forces, to simplify the problem to one which can be handled readily by applying the methods given earlier in this chapter. This technique of replacing a force by an equivalent will occur again later in the chapter, but is relatively simple to understand.

Example 5

Show that the machine is not in equilibrium. The forces acting on the body include the four tensions acting horizontally, its weight acting vertically downwards at the centre of gravity, G, and the reaction between the body and the surface on which it stands, which, in the absence of friction is a purely vertical force acting upwards.

Since the two vertical forces cancel each other, we need only consider the four tensions. It appears that the tensions may all meet in a point, *X*, so that the absence of equilibrium cannot be deduced by inspection. It is, of course, perfectly possible to calculate the components of each force resolved in any two directions at right angles, but the alternative method of transposition will be used here. Each force is replaced by a parallel force of equal magnitude, acting at G.

The resultants of *AB* (*E*) and of *CD* (*F*) are then obtained by means of parallelograms of forces, so that the situation reduces to one of two forces acting at a point. It is clear from the diagram that, if E and *F* are combined into a single force, *K*, this is zero. Thus, the machine is not in equilibrium, since a net horizontal force is exerted on it.

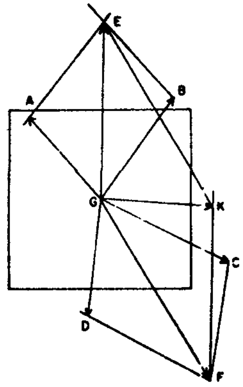

Fig. 5. Transposition of forces

Centre of gravity

Definition of centre of gravity, that the line of action of the weight of a body always passes through this point no matter which position is adopted by the body. In certain well-defined cases, the location of the centre of gravity can be predicted accurately by inspection, so long as certain conditions are met. For such prediction to be effective, the material of which the body is made must be homogeneous. In particular, the density of the material must remain constant throughout its entire volume, with no local variations.

In the second case, the body must be symmetrical about one or more axes. (In special instances, the first condition may be modified by symmetry; thus, it is possible to predict the centre of gravity of a flywheel in which holes have been cut, with precision, to reduce the mass, as long as the casing is of uniform density in the solid regions and as long as the holes are exactly symmetrical about the same axis as is the rest of the wheel). Logical consideration can carry us a stage further. Suppose that we have a composite body consisting of two portions, each of which has a predictable centre of gravity. If we label the two such positions *E* and *F*. Then we can partially predict the position of *G*, the centre of gravity of the entire body. It is obvious that it must lie somewhere on the, line joining *E* and *F*, since it is only on this line that the moments of both the individual parts of the body will be zero, a condition which is necessary for the location of G and which is inherent in the definition of the centre of gravity.

The precise location of G along the line cannot, in general, be discovered by inspection, but can usually be calculated even in quite complicated cases as long as the condition of symmetry is present.

Location by calculation

In determining the centre of gravity by calculation, the principle of moments is extremely useful. As we have already seen, the body can be regarded as having its mass concentrated at this point, so can be replaced for the purpose of calculations as a force acting at that point. We can assume that two forces, proportional to the masses of the two portions, act at E and F, respectively. Location of G then consists in determining the point at which the moments of these two forces are exactly in balance.

Example 6

A cylindrical steel shaft of length 25 cm and of diameter 2 cm is inserted tightly into a cylindrical bushing of diameter 5 cm made from the same material. If the bushing is 10 cm in length and 2 cm of the shaft protrudes through the bushing, calculate the centre of gravity of the bushed shaft.

The centres of gravity of bushing and shaft, by symmetry, are *E* and *F*, respectively. Thus, G lies on the line *EF* and we assume that it is at a distance *x* from one end, *A*. *E* and *F* are located at 7 cm and 12.5 cm, respectively, from *A*. There are two ways of proceeding and the one to choose depends on the situation.

We can either take moments about any point in the system and say that the moment of the combined weights acting at G is the sum of the moments of all other forces or we can take moments about G and say that there must be no net moment resulting there from other forces combined. In the first method taking moments about A, we derive the equation:

$$m_b \times 7 + m_s\ 12.5 = \mathrm{m} \times x,$$

where m_b is the mass of the bushing, m_s the mass of the shaft and m the combined mass. Since the materials are the same, they must have the same density, so that mass is directly proportional to volume

Thus:

$$7v_b + 12.5v_s = (v_b + v_s)x,$$

where v_b and v_s. are the volumes of bushing and shaft, respectively.

But:

$$v_s = 2r_s^2 l_s,$$

where r_s and l_s are the radius and length of the shaft, respectively.

We know that $r_s = 1$ cm and $l_s = 25$ cm, hence:

$$v_s = 2 \times 1^2 \times 25 = 50\ \mathrm{cm}^3.$$

Similarly:

$$\begin{aligned} v_b &= 2(r_b - r_s)^2 l_b \\ &= 2 \times 1.5^2 \times 10 \\ &= 45\ \mathrm{cm}^3. \end{aligned}$$

So:

$$7 \times 45 + 12.5 \times 50 = 95 \times x,$$

$$x = \frac{315 + 625}{95} = \frac{940}{95} = 9.89\ \mathrm{cm}.$$

Thus: the centre of gravity of the shaft is at a point 9.9 cm from the end with the bushing. In the alternative method of calculation, we take moments about G, to give the equation:

$$m_b (x - 7) = m_s (12.5 - x).$$

The same substitution gives us:

$$v_b (x - 7) = v_s (12.5 - x),$$

or

$$45 (x - 7) = 50 (12.5 - x).$$

Thus:

$$45x - 315 = 625 - 50x$$

or:

$$95x = 940,$$

which gives the same result as above.

The situation can become a little more complicated if there is a change in uniformity of the material used in making the composite body. A change may arise either because the shape is varying or because materials with two different densities are used. In such cases the geometry and/or the densities of the component parts must be taken into account in determining the mass of each one before the final stage of the calculation is carried out.

Example 7

The spindle of a ring-spinning frame consists of a steel shaft 40 cm in length fitted with a bronze bushing, 4 cm in length, and of diameter 6 cm. The shaft protrudes 2 cm through the bushing and the shank has a diameter of 3 cm at this end. From a point 2 cm above the bushing, the shaft is uniformly tapered until, at its other end, its diameter is reduced to 1 cm. Calculate the location of its Centre of gravity, if the specific gravities of steel and brass are 7.7 and 8.9, respectively. The spindle may be regarded as a having three components, these being:

i) tapes steel section AB, of length 32 cm,
ii) uniform steel section BE, of length 8 cm, and
iii) the bronze bushing

The latter two components have a common centre of gravity, *F*, since they are symmetrical about an axis perpendicular to that of the spindle. Let us assume that the centre of gravity of the entire spindle, G, and that of the tapered component, H, lie at distance *a* and *b*, respectively, from A.

We know the mass of metal between A and H must equal that between H and B; since the same metal is used for this entire component, AH and HB must also be equal in volume. It can readily be shown (or so the mathematicians claim) that the volume of a truncated cone with radii of *R* and *r* for the larger and smaller ends, and with a height *h*, is given by:

$$V = 1/3 \times h\ (R^2 + Rr + r^2)\pi.$$

If the radius of the spindle cross-section at H is r_H, then:

$$V_{AH} = 1/3 \times b\ (r_H^2 + 0.5r_H + 0.25)\pi.$$

and:

$$V_{HB} = 1/3 \times (32 - b)\ (r_H^2 + 1.5r_H + 2.25)\pi.$$

Since *H* is the centre of gravity of this section, the two volumes must be equal. As the student may verify, attempts to carry out a mathematical solution are rather complex, to put it mildly, so a more realistic approach may be a graphical one. It can be readily shown, by geometry, that:

$$r_H = \frac{b + 16}{32}$$

so that, if various values of are assumed, r_H (and hence the two volumes) can easily be calculated. This calculation has been carried out in Table 1 and the relationship between the volumes and b have been plotted. As can be seen here, the two volumes are equal when b is about 22.5 cm, so we assume that H is this distance from A. (An alternative way of proceeding would be to assume the existence of two complete cones, one from B to the hypothetical apex and one from A to the same point.

Table 1

Variation in volumes of sections of spindles with distance from end

b (cm)	V_{AH} (cm^3)	V_{HB} (cm^3)
0	0	108.9
8	4.7	99.0
16	29.3	79.6
24	61.3	47.6
32	108.9	0

For the remainder of the problem we merely have two components, the tapered one with centre of gravity at H, a volume of 108.9 cm^3 and a specific gravity of 7.7, and the bushed cylindrical shank. It is now necessary to calculate the masses of these two components. The mass of the tapered section is simply derived, as the product of volume and density, and has a value of 838.53 g . For the bushed shank, we must calculate separately the volumes of shank and bushing, then calculate the respective masses from the two densities.

Now:

$$\begin{aligned} \text{volume of shank} &= \pi r^2 l \\ &= \pi \times (1.5)^2\ 8\ \text{cm}^3 \\ &= 56.6\ \text{cm}^3, \\ \text{volume of bushing} &= \pi\ (R - r)2l \\ &= \pi \times (3 - 1.5)^2 \times 4 \\ &= 28.3\ \text{cm}^3. \end{aligned}$$

$$\text{mass of shank} = 56.6. \times 7.7$$
$$= 435.82 \text{ g},$$
$$\text{mass of bushing} = 28.3 \times 8.9$$
$$= 251.87 \text{ g},$$

i.e. total mass of bushed shank = 687.69 g.

This mass is exerted at F, 36 cm from A.

The mass of the tapered section is 838.53 g and is exerted 22.5 cm from A. Thus, by the principle of moments, we can equate the moment of the total mass acting at G to the sum of the two component masses, using A as our reference point,

i. e.:

$$(687.69 + 838.53)a = 687.69 \times 36 + 838.53 \times 22.5$$

$$a = 28.58.$$

Thus: the centre of gravity of the spindle is at a point 28.6 cm from the upper end. Although this method can be applicable to almost any object, one with a complicated shape can lead to very tedious calculations. An alternative approach which is often of value uses a difference calculation. In this case, we assume the body is completely symmetrical, then make an allowance for the missing portions which destroy the symmetry by subtracting the effect which the missing material has on the position of the centre of gravity. It is frequently possible to make what would normally be a rather complex calculation into a simple one by this means.

Example 8

Find the centre of gravity of a slotted disc, shown in Fig. 6, used as part of a reciprocating guide-bar mechanism.

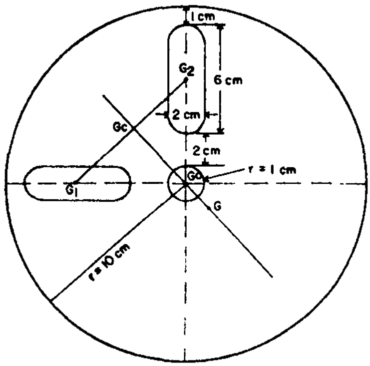

Fig. 6. Centre of gravity of a slotted disc

The centre of gravity, G_0 of the original disc including the central hole must be at the centre of the disc, by symmetry. Similarly, the centres of gravity G_1 and G_2 of the pieces of metal cut away to form the two identical slots must be at the geometric centre of each slot. Obviously, then, the centre of gravity, G_c of the missing metal from the slots must be at the centre of the line joining G_1 and G_2. Thus, the centre of gravity, G, of the slotted disc must lie somewhere in the line passing through G_c and G_0, and will be on the opposite side of G_0 from G_c. If the disc is of uniform thickness and material, the masses of the various parts are proportional to their areas.

Now:

area of entire disc $= \pi r^2$
$= 314.16$ cm^2,
area of central hole $= \pi \times 1_2$
$= 3.14$ cm^2,

i.e.:

total area of disc before slots are cut = 311.02 cm^2.

The area of the slots cut away is calculated for each one as the area of a rectangle 4 cm by 2 cm plus the area of a circle of radius 1 cm,

$$\text{total area of slots} = 2\ (\pi \times 1^2 + 2 \times 4)$$
$$= 2\ (\pi + 8)$$
$$= 22.28\ \text{cm}^2.$$
$$\text{area of metal remaining} = (311.02 - 22.28)$$
$$= 288.74\ \text{cm}^2.$$

By calculation or measurement, the distance G_0G_c can be shown to be 3 cm. Thus, the missing metal has a mass proportional to 22.28 cm^2 acting at a distance 3 cm from G_0, while the remainder has a mass proportional to 288.74 cm^2 acting at an unknown distance x (from G to G_0) from G_0. These two masses must have equal moments about G_0, since it is the effect of removing the smaller one which has caused the displacement in the centre of gravity.

Thus:

$$22.28\ 3 = 288.74x$$

or:

$$x = 0.23\ \text{cm}.$$

Thus: the centre of gravity of the disc is 0.23 cm from the centre, on the axis of symmetry of the two slots. With even more complicated shapes, of course, this type of calculation also becomes more tedious and, if a highly irregular object is to be dealt with, calculation becomes impossible. At this stage, the only sensible procedure is to throw away all the mathematical paraphernalia, abandon attempts to appear intelligent and adopt the idyllic life of primitive man or of a castaway on a desert island. It is tempting to think that this involves forgetting the entire chapter, but there is one last resort which can almost invariably be made to locate the centre of gravity, and that is the use of direct measurement.

Location by measurement

The definition tells us that the line of action of the body weight always passes through this point, so we merely need to suspend the object from points around its periphery and find the location of the unique point which is always vertically below the point of suspension. The lever is first suspended from point A and a line AD is drawn vertically downwards from A, then the process is repeated with suspension at B to derive the line BE.

The two intersect at G, the centre of gravity, and a third suspension from C shows that CF also passes through the same point. Derivation of this point brings to attention a situation where difficulty may be experienced. If the body is very thick, however, and particularly if it does not have a uniform dimension in any direction, practical problems arise. The suspension process applied to such an object. A similar difficulty arises for a body with its centre of gravity located outside its boundaries, as shown in the second part of the same figure.

In both these cases, an exact identification of the centre of gravity is difficult, and the student's only consolation must be that no one else can check the accuracy of his work. One is tempted to recommend an intelligent guess in such circumstances; though not, of course, in the presence of witnesses. Frequently, the exact location of a centre of gravity as an actual point is an unnecessary refinement; in many cases, it is merely the axis on which it lies which is needed. For example, the exact location of G is unimportant as long as we know the vertical axis through which the mass of the balance arm acts.

$$mgx = Fy,$$

where m is the mass of the arm, g is the gravitational attraction and F is the force recorded on the spring balance.

If the lever cannot be removed, its mass may easily be unknown and, in these circumstances, it is only possible to calculate the moment of the mass, mgx, about the fulcrum without knowing precisely where the mass acts.

Practical applications

The principle of moments abound in the everyday experiences of human beings. Every time a force is exerted at some point other than where its effect is felt, the existence of a moment is immediately evident. It is instructive to examine a little more closely some of these situations. Perhaps, in this mechanized age, it is appropriate to consider first of all the steering wheel on which we have become so utterly dependent.

Every time you turn the car, boat, tractor, aircraft, or whatever other vehicle gives you your kicks, you apply a couple to the wheel as you push with one hand and pull with the other, as in Fig. 6; for the cycle or motor-cycle fiends the same is true for handlebars.

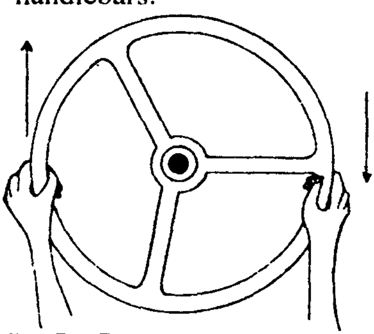

Fig. 7. Forces on a steering wheel

For those stalwarts who shun the mechanical life, we can perhaps mention the sailing dinghy as an example of an application of the principle of moments. The wind strikes the mainsail and the effect, felt at the centre of pressure, would tip the boat or cause it to move sideways if it were not for the centre-board, or keel. The latter, potentially forced towards the windward direction by the couple exerted, meets water resistance in the process and experiences a reaction.

This reaction, together with the forward component of wind pressure, brings about the seemingly miraculous movement of the boat in a direction which apparently defies all logic. If you happen to be the athletic type but prefer to remain on terra firma (and the firmer it is the less the terror), you are no doubt skilled in the art of kicking a football in such a manner that it slices off unaccountably to the right, missing the goal by a good ten yards in the process.

It may be of little consolation now, but there is a rational explanation, in addition to the previous night's celebration party, to account for the mystery. When you kicked the football, your foot struck it in such a way that the line of action of the blow passed to the left of the centre of gravity. The forward impulse, P, in conjunction with the frictional force resisting movement, F, formed a couple which imparted a clockwise spin to the ball. This spin, when combined with the effect of air resistance, caused the ball to veer to the right, with the disastrous effect which leaves you off next week's team.

Perhaps if you'd known that sooner you'd have dallied longer at the bar with the blonde and not even turned up for the game. You could have pleaded a mild touch of bubonic plague and so missed the disgrace of that one disastrous kick, thus ensuring your place in the cup team on a subsequent day.

Applications in textiles

Textile applications of the principle of moments are so widespread that the list is virtually endless. Almost every single drive component, whether it be a pulley, a gear wheel, a chain drive, a ratchet mechanism, a lever, a connecting rod system, a cam, or any other of the many devices used in the industry, involves in some way a use of the mechanical principles discussed in this chapter. Application of a couple may bring about a rotation or may merely balance another couple, and many types of calculation using the principle of moments may be envisaged. Sample ones are given in the following worked examples.

Example 9

Find the tension on the specimen when the angle a is (i) $\pi/6$ rad and (ii) $\pi/3$ rad. Although the pendulum and pointer are slowly moving as the left-hand grip moves, we can regard the system as being instantaneously in equilibrium at any given position, as indeed is the case if the traverse is stopped at that position. Taking moments about the fulcrum of the lever, we have:

$$T \times 0.04 = 40 \times 9.81 \times 0.40 \cos \alpha.$$

Thus, when $\alpha = \pi/6$ rad:

$$T = 25 \times 40 \times 9.81 \times 0.866$$
$$= 8495.5 \text{ N},$$

and, when $\alpha = \pi/3$ rad:

$$T = 25 \times 40 \times 9.81 \times 0.5$$
$$T = 4905.0 \text{ N}.$$

The tension on the specimen in the two cases is, respectively, 8495 N and 4905 N.

Three
Liquid Crystal Polymers

Melt spinning is the most rational fibre-formation process for the polymer. Liquid-crystal polymers include the lyotropic liquid-crystal type and the thermotropic liquid-crystal type. Pioneering studies on the latter were carried out by Economy of Carborundum Co., Jackson of Eastman Kodak and Calundann of Hoechst-Celanese, together with their collaborators, and some of the polymers are now commercially available as resins. In the fibre field, however, although numerous studies have been proceeding since 1970, 'VECTRAN' is the only fibre that is currently commercially available. Continuing research and development work may yet achieve super-fibres having higher performance and characteristics at lower cost.

Thermotropic liquid-crystal polymers

Desirable examples for this purpose are aromatic polymers comprising a highly symmetrical and rigid rodlike structure. With wholly aromatic polyesters, the basic structures include a self-condensation polyester of (a) *p*-hydroxybenzoic acid (HBA) and (b) a polyester comprising units from terephthalic acid (TA) and units from hydroquinone (HQ).

The above polymers, however, have melting points ((a): 610°C, (b): 600°C) higher than their decomposition temperatures (400-450°C) due to their molecules being too rigid, and therefore are unprocessable in the thermotropic liquid crystal form. To overcome the problem, attempts have been made by various methods to decrease the melting point to below the decomposition temperature at the expense, to some extent, of the rigidity and crystallinity of the polymers. The methods include:

1. The introduction of units from a flexible alkyl group into the main chain.
2. Introduction of a substituent into each of the aromatic rings in the main chain.
3. Co-polymerization of two rigid molecules.
4. Introduction of a rodlike non-linear structure.

These methods have been able to provide a series of thermotropic liquid-crystal polymers that are injection-moldable or melt-spinnable, which will be described later. In practice, thermotropic liquid-crystal polymers are obtained by polymerization of an aromatic diol acetate and an aromatic dicarboxylic acid or self-condensation of an aromatic acetoxycarboxylic acid.

Spinning of thermotropic liquid-crystal polymers

Liquid crystals have a flowable, optically anisotropic structure above a certain temperature and lack, in this temperature region, sufficient energy for their individual molecules to rotate freely as in a liquid. Consequently, many intermolecular interactions occur in the liquid and there form domains comprised of parallel-arranged molecules. Nematic, liquid crystals generally form, with an orientation direction parallel to the long axes of the molecules present. In terms of processability, thermotropic liquid-crystal polymers generally allow the molecules to be oriented when subjected to shear stress and also exhibit long orientation relaxation times. These are the main features that distinguish them from conventional flexible polymers. In a transition flow study of thermotropic liquid-crystal polymers, relaxation of stress can last seconds, whereas the relaxation of orientation can be a matter of several minutes. When a thermotropic liquid-crystal polymer is extruded through an orifice under a high shear stress, its molecules orient to a high degree and the oriented structure is almost completely maintained during solidification by cooling due to the long relaxation time. From a technical viewpoint, the following three processes are available for spinning thermotropic liquid-crystal polymers:

1. Spinning of Polymers having a comparatively low molecular weight (low η_{inh}).
2. Spinning of high molecular weight polymers at a temperature higher than their melting point.
3. Spinning of high molecular weight polymers at a temperature lower than their melting point (supercooled spinning).

While operation of process 1 is rather easy, the as-spun fibre obtained has a low strength and hence requires a protracted heat treatment for achieving high performance. With process 2, the molecular weight is increased to the maximum level that still assures spinnability and the spinning is conducted at a high temperature and high shear rate to decrease melt viscosity.

The as-spun fibre obtained has a strength of 5-20 g/d which can be further improved by a relatively short heat treatment. The thermal stability of the polymer used and spinning skill are the key points in this process. Process 3 comprises extruding a high molecular weight liquid-crystal polymer having a ΔH_c of not more than 10 J/g at a temperature not more than the melting point and not less than the freezing point and taking up the as-spun fibre at a relatively low speed. This process is of interest, because the polymer thermally decomposes only slightly and the as-spun fibre obtained has a high strength and requires only short heat-treatment time.

Heat treatment

As-spun fibre obtained by melt spinning a thermotropic liquid-crystal polymer already has a high strength and modulus. To improve the performance still further, the as-spun fibre is subjected to heat treatment, which improves the strength and modulus, and also the thermal resistance. The as-spun liquid-crystal polymer fibre is generally heat treated in a reduced-pressure atmosphere of an inert gas, or air, with the by-products that form being continuously removed. The heat treatment is conducted at a temperature not more than the melting point, to avoid the filaments sticking. Since the melting point itself increases during heat treatment, the treatment temperature can eventually be higher than the original melting point of the polymer.

As will be described later, it has been confirmed that heat treatment causes solid-phase polymerization, thereby increasing the molecular weight. The reaction mechanism is, however, complex and there may possibly occur cross-linking reactions and also elongation of molecular chains.

Fibre formation

Fibres of flexible group

A copolyester obtained by polymerising 60 mol% of HBA and 40 mol% of polyethylene terephthalate (hereafter referred to as PET/HBA:40/60) is a raw material of commercial interest from the viewpoint of production cost. This Polymer however has not yielded particularly good results as a material for fibre.

With PET/HBA:20/80, Zachariades and Logan report that this copolymer was spun at a high temperature of 330°C to give a fibre having a strength of 225 MPa and a modulus of 40 GPa. VA-tile PET/ HBA:40/60 is a random copolymer, 20/80 is considered to contain blocks of HBA. According to a patent, irregular particles formed upon block-wise polymerization of HBA were removed and the copolymer thus homogenized was spun at 310° C to give a fibre having a strength of 10. 1 g/d, which was then heat treated at 170-265°C for 4 hours, to show an improved strength of 25.2 g/d. Jackson used, instead of PET, a polymer from 3,4 -carboxy-benzene-propionic acid (CPA) and HQ.

A copolyester of CPA-HQ/HBA:20/80 was spun at a temperature of 335°C and at a take-up speed of 1070 m/min, to obtain an as-spun fibre having a strength of 6.9 g/d and a modulus of 470 g/d, which they further heat-treated at 325°C for 0.5 hour to obtain a fibre having a strength of 20.3 g/ d and a modulus of 960 g/d.

Fibres of rodlike molecule

Fibre formation from a copolymer of HBA and 2,6 hydroxynaphthoic acid (HNA), which is commercially employed for VECTRAN, is explained here. X-ray analysis by Gutieffez et al. has revealed that the polymer of HBA/ HNA is a random copolymer. Calundann and Jaffe 3 spun a polymer of HBA/HNA:75/25 having anη_{inh} (inherent viscosity as determined in pentafluorophenol) of 5.7 dl/g and a melting point of 302°C, to obtain an as-spun fibre having a strength of 12.1 g/d, elongation of 2.8% and modulus of 541 g/d.

The polymer had a comparatively high molecular weight and was spun at a temperature well above the melting point. Upon melting, hydrolysis and thermal decompo-sition of the polymer occur, degrading the polymer and generating gases that markedly decrease the spinnability. To minimize the decrease in η_{inh} during spinning, it becomes necessary to lower the moisture content of pellets and to decrease the spinning temperature as far as possible. Figure 1 shows the relationship between the rate of shear T and the melt Viscosity Flexible polymers such as PET generally maintain their viscosity in low shear regions. On the other hand, with thermotropic liquid-crystal polymers, the viscosity falls nearly linearly as shear rate rises, thus showing marked non-Newtonian behaviour, and is influenced to a large extent by temperature changes.

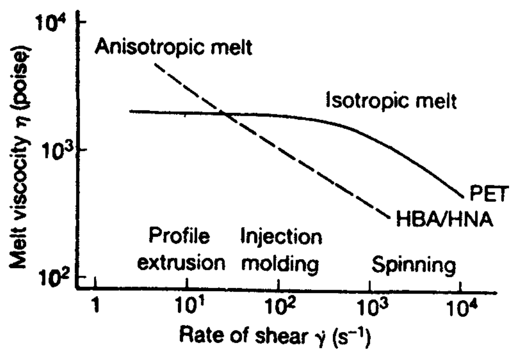

Fig. 1. Relationship between shear rate and melt viscosity

Fibre formation from thermotropic liquid-crystal polymers includes no drawing process, and so the fineness (denier) of the as-spun fibre becomes that of the finished fibre. To obtain a fine-denier fibre, it is necessary either to use a small-diameter orifice or to increase the spinning draft. However, as seen from Fig. 1, when a thermotropic liquid-crystal polymer passes through an orifice, leaving behind a low τ region, molecular orientation occurs to a high degree, thereby rendering it impossible to employ a large spinning draft. Consequently, to obtain a fine-denier fibre, the orifice diameter must be made small, and as a result τ increases. A larger τ in turn causes the melt viscosity to decrease, which is favourable for spinning high η_{inh} polymers.

On the other hand, HBA/HNA attenuates rapidly just below the orifice and the attenuation is nearly complete at a distance of 10 cm from the orifice. Calundann and Jaffe also state that solidification of extruded streams occurs rapidly and that orientation and structure formation are complete within 10 cm from the orifice. Under the usual spinning conditions, no 'die-swell' phenomenon is observed. Krigbaum et al. carried out spinning of HBA/HNA:75/25 and reported that the die swell is a function of shear stress and is not observed above a shear stress of about 8 × 10^4 dyn/cm. Calundann and Jaffe for the relationship between η_{inh} and strength of as-spun fibre. The strength reaches a maximum of nearly 15 g/d at an η_{inh} of about 7 dl/g, but then decreases because stable spinning becomes impossible dile to high viscosity. Good spinnability is obtained with a slightly lower η_{inh}, which gives an as-spun fibre having a strength, elongation and modulus of about 12 g/d, 2% and 600 g/d, respectively. Heat treatment is conducted at a temperature below the melting point to further improve the properties.

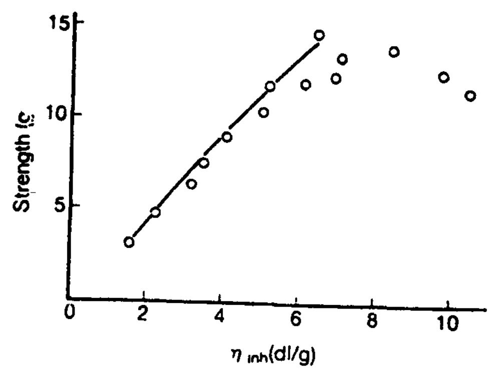

Fig. 2. Relationship between η_{inh} and strength of as-spun fibre

The endothermic melting peak shifts toward higher temperature and has a smaller area with increasing treating temperature. A fibre -treated until the endothermic peak temperature exceeds about 300°C is no longer meltable under atmospheric pressure and, when heated at higher temperatures, gradually decomposes while keeping its fibre shape. The heat treatment time required to give sufficient improvement in properties and the strength achieved may vary depending on the manufacturing conditions of the polymer and the way in which the heat treatment is conducted.

Yoon reported that a polymer of HBA/HNA:73/27 was spun and the as-spun fibre was heat treated at 270°C for 30 minutes, to give a fibre having a strength of about 17 3.5 GPa, which eventually reached about 3.71 GPa (29.8 g/d). The strength of as-spun fibres comprising HBA/HNA markedly increases upon heat treatment, but the modulus increases only slightly. An as-spun fibre spun from a copolymer obtained by copolymerization of HBA/HNA with 4,4-biphenol (BP) and terephthalic acid (TA), with reduced amount of HNA, i.e. HBA/HNA/BP/TA: 60/5/17.5/17.5 improves, on heat treatment at 300 to 310°C, not only in strength but also in modulus from 634 up to 1110 g/d.

Fibre formation from EKONOL (i.e. poly HBA)-related liquid-crystal polymers was studied by Economy et al. and these products have since been promoted by Sumitomo Chemical Co. up to the stage of development under the name of 'EKONOL FIBRF'. Sumitomo spun a copolymer having a non-linear (rigid-angular) component, isophthalic acid (IP) of composition HBA/IP/TA/BP:60/5/15/20, at a temperature of 350°C and at a take-up speed of 160 m/min to obtain an as-spun fibre having a strength, elongation and modulus of 5.5 g/d, 1.4% and 410 g/d, respectively. When heat treated the as-spun fibre achieved a strength and modulus of 30.8 g/d and 1420 g/d, respectively.

Fibres of aromatic ring-substitution type

The homopolymer of phenylhydroquinone (PhHQ) and TA has a melting point of 341°C and is melt-spinnable. Jackson et al. heat treated an as spun fibre of this polymer having a fineness, strength and modulus of 1.2 d, 4.8 g/d and 500 g/d, respectively, at 340°C for 1 hour to obtain a fibre having a strength and modulus of 32 g/d and 910 g/d, respectively. Copolymers have lower melting points and hence even those having high molecular weight (high η_{inh}) are spinnable.

It is worth noting that a polymer of PhHQ-TA/BP-TA/HBA:80/10/10 having a high η_{inh} of 9.3 dl/g has a melting point of 288°C and gives, upon spinning at 324°C, an as-spun fibre having a remarkably high strength of 20.1 g/d. While draw resonance often occurs in the spinning of polymers having low heats of fusion, ΔH_f, of not more than 10 J/g, spinning them at a temperature below their melting point and above their freezing point, i.e. supercooled spinning, can eliminate draw resonance, thereby rendering it possible to conduct stable spinning. For example, a polymer of PhHQ-TA/HNA:96/4 having an 71inh of 6.9 dl/g has a melting point of 348°C and a ΔH_c, of 2.1 J/g.

Spinning this polymer at 3500°C causes serious draw resonance so that the extruded streams are difficult to take up. Spinning at 324°C, however, assures stable take-up, to obtain an as-spun fibre having a strength of 15.6 g/d. Further heat treatment gives, for example, from a copolymer of PhHQ-TA/HBA:83.8/16.2, a fibre having a strength of 34.8 g/d and from a copolymer using chlorohydroquinone, i.e. CIHQ-TA/CIHQ-NA:70/30, a fibre of 30.4 g/d.

Fibre structure

Structure of thermotropic liquid-crystal polymer fibres varies depending on the raw material polymer and fibre manufacturing conditions but generally is a highly oriented fibrillar structure. With HBA/HNA fibre, macrofibrils having a size of about 5μm, fibrils having a size of about 0.5μm and microfibrils having a size of about 0.05μm, together with a very thin skin layer having a thickness of about 1μm have been observed. Donald et al observed a banded structure in polarized light, with the striations lying perpendicular to the fibre axis. This structure closely resembles those previously reported in lyotropic polymers.

The principal dispersion temperature increases, the degree of orientation shows almost no change and the crystallinity obtained from specific weight or by X-ray analysis increases only a little. On the other hand, the number average molecular weight increases by about 3 times. While the arrangement of HBA and HNA in the molecular chain does not change, the a-axis and b-axis X-ray data do change, which suggests that sequences rich in HBA may have aggregated. Yang and Krigbaum identified block formation from HNA caused by melt interchange. Erdemir et al. reported the results of an X-ray study, based on changing the composition of a liquid-crystal polymer comprising HBA/HQ-IP, of the structural changes between unoriented polymer, as-spun fibre and heat-treated fibre.

As-spun fibre from the polymer having a composition of 501 50 has a slightly oriented para-crystalline structure, but there forms, upon heat treatment, a new crystal structure which is different from those of the corresponding homopolymers. There is a fibre type which, upon heat treatment, shows a marked increase in melting point and strength but shows little change in the modulus and another type which shows a marked increase, not only in melting point and strength, but also in modulus.

Four
Textile Machines

Several definitions are needed before we can examine either aspect in detail. In this context, a machine is defined as any instrument, system or device that does useful work. The effort is the force required at the driving, or input, end of the machine, and the load is the resistance that must be overcome at the place where the useful work is being done.

The principle of work states that, in theory:

work done by effort = work done in overcoming load,

but, in practice, the presence of friction complicates the situation and a better equation would be;

work done by effort = work done on load + work lost in overcoming friction,

the latter term not, of course, being regarded as useful work. There are, in addition, other ways in which work can be wasted, and the fact that work is actually done on the load does not necessarily mean that the work is strictly useful. When a heavy load of cargo is placed in the hold of a ship, for example, the net displacement is, perhaps, a lateral one of 10 m. In order to achieve that displacement, however, it may be necessary for a crane to lift the cargo by a distance of, say, 20 m and, after a sideways displacement, lower it by the same amount.

The work done is different in the two cases, but the 'useless' work of lifting and lowering is forced on the stevedores by the circumstances of the task.

A hole in the ship's side would simplify the procedure no end but may introduce undesirable effects on the ship's functional ability once it emerged onto the high seas and encountered a storm.

The decision of whether work is useful or not, then, may not be unequivocal in practice. From the point of view of the machine, however, the distinction is a simple one. If the work is done directly in bringing about a movement of the load (or in some equivalent way, such as exertion of pressure on the load), it is said to be useful, no matter where it causes the load to go.

If, on the other hand, the work is expended in overcoming friction, in generating noise, in moving the point of application or line of action of some component within the machine itself, or in any other way that is not applied directly to the load, it is not regarded as useful. Obviously, the effectiveness of any machine is improved by increasing the useful proportion of the work that must be expended by the effort in order to bring about the desired change at the load. We can thus define efficiency of a machine as the ratio of the useful work available at the output to the work that must be put in to the device, so that, if friction were absent, the machine would have an efficiency of 1.0, or 100%.

Friction is omnipresent and efficiency must always be less than unity (or 100%), but the machine designer attempts to reduce friction as far as possible to maintain a high efficiency. The use of a bearing, in which sliding friction, present between two surfaces, is replaced by rolling friction of much lower magnitude, is a familiar gambit, as also is the use of a lubricant to take advantage of the low shear-resistance of a liquid. The problem of lubricant viscosity, however, deserves some mention. In order to operate satisfactorily, a machine must be lubricated to reduce metal-to-metal friction. With insufficient lubrication (i.e., if lubricant viscosity is too low), such friction would reduce efficiency (and machine life) drastically.

On the other hand, the use of too high a viscosity in the lubricant would tend to hamper the ease with which components move, and hence again reduce efficiency. Selection of the correct lubricant, then, reflects a compromise situation and requires careful design considerations. On the assumption that the designer knows what he is doing, the wise operator of a machine will follow the manufacturer's lubrication recommendations reasonably faithfully.

Efficiency can also be reduced if some of the work potentially available from a machine is diverted to some other non-useful function. The use of energy in overcoming the resistance to motion of a tight component, or in producing noise, is an obvious example of such diversion. As teeth enter into mesh, contact occurs across the width of a tooth. The curved profile of each tooth permits a rolling contact to take place as the teeth move more closely into mesh, so that frictional resistance is reduced, when meshing is correct, and the tooth can roll into and out of mesh without the need for metal surfaces to slide while they are in contact with one another. If meshing is too tight, however, this facility no longer exists.

A tooth penetrates more deeply than it should do into the gap intended to receive it, so that one of two possible problems occurs. On the one hand, the penetration may be so deep that the teeth contact at a position where the profile is so thick that there is not room to accommodate a tooth in a gap. As can be seen, the thickness of a tooth increases gradually from tip to base, so that too deep a mesh can easily cause this problem to occur. As a result, jamming of the gear train takes place and movement stops. An attempt to use force to restore the rotation will cause undue wear in mirror cases of jamming or can even cause breakage of one or more teeth in a case of serious tightness. Even if the gear profile is made with such a small taper that this type of jamming cannot occur, however, another problem can arise if meshing is too tight.

In normal mesh, a space remains between the outer edge of a tooth and the base of the gap into which it fits throughout the entire meshing process. If meshing is too close, though, the outer edge of a tooth may come into contact with the bottom surface of the gap, which again causes problems. Excessive wear will take place in mild cases of overmeshing of this type, while jamming or tooth breakage can also occur as before if serious tightness exists.

Yet another possibility in this case is the presence of excess thrust on the shafts carrying the sprockets, which may cause bowing of the shafts, increased friction at the bearings, misalignment of some other components in the mechanism, or, if use under such conditions is prolonged, breakage of some part of the system where stress has bee concentrated. A different situation is created when meshing is too loose. Operation in such circumstances means that the tip of a tooth does not penetrate sufficiently deeply into the gap awaiting it, and the tooth shape (designed, of course to be used it) correct mesh) means that there is a considerable gap between the non-contacting faces of adjacent teeth as driving occurs.

There is thus a time delay after one tooth has moved out of mesh before the tooth coining into mesh makes contact, a fact that creates three undesirable effects. In the first case, the driven cog wheel tends to slow or to stop momentarily because the driving force is not maintained. In the second case, the entire process of energy transfer depends on the thrust given by a single tooth, the one in full mesh, rather than having the support of the two adjacent teeth as in correct mesh, for a brief instant of time during each movement of one tooth distance. Finally, when contact does occur, an impact takes place rather than a smooth mating because of the slight difference in speed between the two rotating components. The net result of these three effects, as can readily be envisaged, is a jerky motion in the driven shaft and an increase in wear on the mating teeth.

A moment's thought will show the reader that the problem is self perpetuating, since wear will reduce the amount of metal on the teeth, which thus reduces tooth thickness and hence compounds the looseness of mesh. Even if meshing is correctly arranged, however, problems of backlash and wear are still encountered and must be considered in the design of a gear train. The gear teeth are so cut that the size of a tooth is slightly less than that of the gap into which it must fit. As a result of slight variations in tooth size, it may be that an oversize tooth comes into juxtaposition with an undersize hole, and there must be sufficient clearance for this situation to be tolerated without jamming.

In addition, slight movements in a gear shaft, or in the bearing carrying it, can change the meshing characteristics slightly, an eventuality that must again be accepted and for which allowance must be made. Finally, when the rolling contact of tooth against tooth occurs at the mesh point, minute vibration, or surface roughness, or particles of foreign matter can again alter slightly the effective size of a tooth. In consequence, the minute difference in size between tooth and gap in a gear wheel is intentionally incorporated at the design stage, and it is the resulting gap, at the position of closest contact, that is defined as the backlash. Naturally, a large amount of backlash leads to lower efficiency as a result of the intermittent nature of the contact that call be experienced, so the magnitude of the backlash is reduced as the precision (and cost) of the gear components are increased.

The wear that occurs in a component can arise from many sources. Particles of metal can be dragged out of a surface by excess friction or call be chipped off by repeated violent contact. If satisfactory component design is assumed, these two sources of wear may usually be attributed to incorrect meshing and are self-propagating. Tight fit means scraped surfaces, which lead to rough surfaces or stray metal particles and hence to a tighter fit. Too loose a fit causes chipping of particles, which thus increases the gap between contacting surfaces and hence loosens the fit still further.

Wear can also occur if excess grit or metal particles are allowed to accumulate because of insufficient frequency of lubricant changes, or from the use of incorrect lubricant. Some wear in the normal life of a machine is, of course, inevitable, but the responsibilities of the user in minimizing the amount of wear are, one hopes, obvious. One final way in which efficiency can be reduced is of particular interest at present, when manufacturers are being made more and more aware of their responsibility to employees.

The production of noise diverts some work into a non-useful expenditure, and, even though the production of acoustic energy does not represent the highest single waste source of loss of power in a plant, it is important because it is so pervasive, so difficult to eliminate, and responsible for such tremendous problems among personnel. Noise arises when the air molecules adjacent to a machine are set into vibration, and this vibration must obviously draw the energy required to initiate it from the power source of the machine.

It is, perhaps, rather an understatement to say that textile machines in general are not quiet, and the cacophony that assaults the ears in the traditional textile mill must be produced by considerable expenditure of energy over a long period of time. Unfortunately, waste energy is paid for at the same rate as useful energy, so the sound of a textile machine in operation is not music in the ears of the accountant any more than in those of the operative. The effect of prolonged noise on the operative has only recently been recognized fully. It has always been accepted that deafness is a likely, and permanent, legacy of long employment in the weaving shed, the spinning room, or the twisting area.

Recent work has indicated, however, that many other disorders can be blamed on noise exposure. Psychological problems, such as stress, frustration, or tension, are believed to be increased in such situations. As a result, employees may become careless, inattentive, or even wilfully destructive, and the manufacturer is, once again, paying for the noise his machinery produces, this time in a less direct way.

In this respect, efficiency is decreased in terms of human factors as well as in mechanical terms, a distinction to be mentioned in more detail in due course. Practical factors are important in the application of drive mechanisms to enable a piece of machinery to run successfully and at maximum efficiency. Unless the designer, the operator, the mechanic, and the employer are aware of the pitfalls and work its a team to avoid them, the best of machines cannot function satisfactorily, even if it is kicked with religious regularity. If a malfunction of any kind remains uncorrected, problems will arise. In the limit, when the machine jams, all the input work is being diverted to the task of attempting to free the obstruction, and efficiency drops to zero. Persistent use under faulty conditions can lead only to disaster, and naughty words from the firm's accountant may subsequently be expected.

Theoretical factors

Assuming the practical conditions are maintained at as favourable a state as possible, we can now turn our attention to the theoretical aspects governing the use of machines. Two further definitions are needed. The *velocity ratio* (V.R.) is defined as the ratio of the rates of movement of effort and load, i.e.:

$$\text{V. R.} = \frac{\text{rate of movement of effort}}{\text{rate of movement of load}}$$

and the *mechanical advantage* (M.A.) is defined as the ratio of load to effort, i.e.:

M.A. = load/effort.

By definition, V.R. = x/y, and M.A. = W/P. By the principle of work, with friction ignored, the work done by the load and that done by the effort are identical.

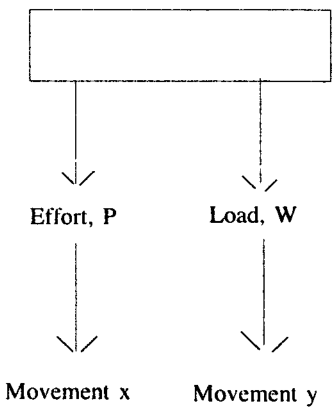

Fig. 1. Theory of machine operation

Thus

$$Px = Wy$$

i.e

$$W/P = x/y$$

or

$$\text{M.A} = \text{V.R}$$

In a theoretically perfect machine with no friction, then, the mechanical advantage and velocity ratio are equal. In practice, however, the presence of friction causes a deviation from this equality. The efficiency, η, of a machine is defined as:

$$\eta = \frac{\text{useful work done by machine on load}}{\text{total work done on machine by effort}}$$

Work done is calculated as the product of force and distance moved, so that in our example:

$$\eta = \frac{Wy}{Px}$$

This can be rearranged as:

$$\eta = \frac{W}{P} \div \frac{x}{y}$$

But, by definition:

$$\frac{W}{p} = \text{M.A.,}$$

and

$$\frac{x}{y} = \text{V.R.,}$$

i.e.,

$$\eta = \frac{\text{M.A}}{\text{V.R}}$$

In all machines, friction is present and, as already mentioned, there are other factors, such as noise and internal movements within the machine itself, that use up some of the available energy. By the conservation of energy, then, we can stipulate as the condition of operation:

W.D. by effort = W.D. on load + W.D. on friction +

W.D. for internal operation, where W D. denotes work done.

The work done on the load is thus always less than the work done by the effort, so that efficiency must always be less than unity. The efficiency of a machine is often estimated by calculating the theoretical mechanical advantage (i.e., on the assumption that all the work done on the machine is converted to useful work, or efficiency is 100%) and using it to predict the load that this perfect machine should be capable of overcoming. A comparison with actual performance then gives a direct measure of efficiency under actual operating conditions.

Types of machine

All mechanisms, no matter how complex, are made up from simple machines, of which there are only two basic types. The first of these is the lever, a category that includes disguised levers, such as pulleys, gear wheels, cranks, and wheel-and-axle machines, together with other similar variations. In the lever, a rigid rod or other body is free to rotate about a given point or axis, and the effort applied to one point on the lever is used to rotate it about the axis. As a result, a different point, on the lever also moves and constrains the load to move with it. In all the examples, common features may be discerned, There is a fixed point the fulcrum, around which free rotation can occur.

In the basic lever, the rotating body is a rigid, straight rod, though the rod may become shaped, or may even be replaced by a wheel configuration, in more complex instances. At some point on the rigid system, a load is situated, and the whole purpose of the machine is to move this load. Movement is achieved by application of an effort at some other point, the effect of this application being transmitted through the rigid system to the location of the load, which is thereby moved under the influence of the resulting force acting on it. The operation of all levers is governed by a simple mathematical relation, which is easily derived.

The length AO is known as the *effort arm* of the system and BO as the *load arm*. The effect of a rotational force increases as the distance from the point of rotation increases, a fact that can easily be observed qualitatively by noting the relative ease of steering a bicycle by the handlebars or moving a wheelbarrow by lifting the handles. An attempt to steer the bicycle by grasping the stem, or to move the wheelbarrow by lifting the walls, is likely to end in generally unpleasant consequences for the intrepid experimenter.

The effect of this force can be measured by a factor known as its moment, defined as the product of the force and the perpendicular distance from the point of rotation to the line of action. In any situation where a load is maintained in position, or moved at a uniform rate, by an effort in a lever system, it is obvious that the effect of the two forces at the fulcrum must be balanced. If displacement from the horizontal position is negligibly small, the two forces act at right angles to the rod. Thus, the perpendicular distance between the fulcrum and either force is the respective arm of the lever, i. e.:

$$\text{effort} \times \text{effort arm} = \text{load} \times \text{load arm}.$$

This is a simple illustration of the *principle of moments*, to be discussed in further detail at a later stage in this work.

Example 1

The weighting lever for a loom let-off is 50 cm in length, pivoted at one end. and supported by a chain placed 10 cm from that end. If the mass of the lever is 2 kg and masses of

8 kg and 12 kg are placed 40 cm and 45 cm, respectively, from the fulcrum, find the tension in the chain. The mass of the bar may be regarded as being located at its midpoint, i.e., at 25 cm from the fulcrum.

There are thus four forces maintaining equilibrium in the bar, these being:

a) the tension, T, in the chain;
b) the force due to the lever's mass, (2×9.81) N at a distance of 0.25 m from the fulcrum;
c) a force of (8×9.81) N at 0.40 m from the fulcrum; and
d) a force of (12×9.81) N at 0.45 m from the fulcrum.

The tension acts anticlockwise and the other three forces act clockwise around the fulcrum.

Taking moments about the fulcrum:

$$\text{anticlockwise moment} = T \times 0.1$$

$$\begin{aligned}\text{clockwise moment} &= (2 \times 9.81 \times 0.25) + (8 \times 9.81 \times 0.40) \\ &\quad + (12 \times 9.81 \times 0.45) \\ &= 4.905 + 31.392 + 52.974 = 89.271.\end{aligned}$$

Thus, by the principle of moments:

$$0.1T = 89.271,$$

i.e.:

$$T = 892.71,$$

i.e., tension in chain = 892.7 N.

The velocity ratio of a lever system is also derived very simply. A movement of magnitude x in the effort applied at A is needed to cause a movement of magnitude y in the load at B. By definition, the velocity ratio is the ratio of the rates of movement of effort and load, and, since the movements at A and B occur simultaneously, this ratio must therefore be the same as that of the actual displacements. Thus, the velocity ratio is equal to the ratio of x to y, which, by similar triangles, is equal to the ratio of AO to BO. In other words:

$$\text{velocity ratio} = \frac{\text{effort arm}}{\text{load arm}}$$

a result that is generally applicable to all types of lever. One type of lever. But not yet mentioned, is the wheel and axle. In this machine, two cylinders of unequal diameter are mounted on a common shaft, and the load is attached by means of a rope to the smaller one. The effort is applied to the larger, and, because it is applied at a greater distance from the fulcrum (the axis of rotation in this case), a given load can be lifted by using an effort of lower magnitude.

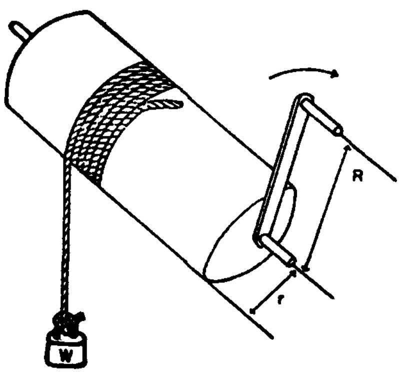

Fig. 2. A simple winch

The rope is wound onto an axle of radius a and simultaneously removed from one of radius b. In each case, the velocity ratio can be calculated as the ratio of distances moved, and a value of R/r is obtained. In one revolution of the differential winch, the effort moves a distance $2\pi R$ around the circle, a length $2\pi a$ of the load-carrying loop is wound onto the larger axle, and a length $2\pi b$ is unwound from the smaller one. Shortening of this loop by a net amount of $2\pi(a - b)$ thus occurs, and, since this decrease is shared between the two sides of the loop, net movement of the load is half the amount, Or $\pi(a - b)$. The velocity ratio is therefore $2R/(a - b)$ in this case.

Example 2

The radii of the two axles are 2 cm and 5 cm, respectively, and the crank radius is 40 cm. It is found that a mass of 1.2 kg suspended at the end of the crank in its horizontal position is just able to lift a load of 18 kg. Calculate the efficiency of the machine.

We have:

$$\text{V. R} = 2\text{R}/(\text{a} - \text{b})$$

$$= \frac{2 \times 0.40}{0.03}$$

$$= 26.67$$

By defenition

$$\text{M.A} = \text{load/effort}$$

$$= 18/1.2$$

(note that we have omitted g, since its value appears in both numerator and denominator), i.e.:

$$\text{M.A} = 15\text{-}0.$$

We also have:

$$\text{efficienct} = \frac{\text{M.A.}}{\text{V.R.}}$$

$$= 15.0/26.67$$

Hence efficiency = 56.2%

Gear pairs can also be regarded as lever systems. One revolution of the effort handle turns the axle carrying the load through n/N revolutions, where n and N are the number of teeth in the effort and load gears, respectively. The force produced at the circumference of the effort cog is transmitted, via the meshing, to the load cog, which then operates as a wheel-and-axle machine to lift the load. The velocity ratio is thus equal to NR/nr.

Example 3

A pinion with 128 teeth is attached to an axle of radius 8 cm. It meshes with a second pinion with 75 teeth, turned by a crank of radius 30 cm. If the efficiency of the system is 72% crank, to calculate the mass of a machine part that requires a force of 112 N, applied to the lift it.

We have:

$$\text{V.R.} = NR/nr$$
$$= 128 \times 30/(75 \times 8)$$
$$= 6.40$$

We also have:

$$\text{efficiency} = 72\%$$

i.e.:

$$\frac{\text{M.A}}{\text{V.R}} = 0.72$$

Thus:

$$\text{M.A} = 6.40 \times 0.72$$
$$= 4.61$$

But:

$$\text{M.A} = \text{load/effect}$$

i.e.:

$$\text{force exerted by machine part on system} = 4.61 \times 11\text{N} = 516.3 \text{ N},$$

i.e.:

$$\text{mass} = 516.3/9.81 \text{ kg}.$$

Hence mass of machine part = 52.6 kg.

In the case of a single pulley, the load and effort are both applied at the same distance from the fulcrum, so that the velocity ratio is unity. In the sheaved pulley block. However, the four pulleys are acting as a system of levers. The effort E, as it moves through unit distance, causes a shortening of the loop holding the bottom pulley, which, in turn, causes a shortening of the loop holding the second-lowest one. In each case, the movement of the rope is shared between two sides, so that any movement in a rope causes a movement of half that amount in the pulley around which the rope passes.

Thus, since there are two supported pulleys, the load moves one-quarter the distance of the effort, and the velocity ratio is equal to 4. The final example of this type of lever is the Weston differential, often used to lift heavy loads, such as a bale of cloth, to a loading bay higher in a building. The machine consists of three pulleys, two cast as a single upper wheel and one as the load pulley beneath the first ones. Over the larger effort pulley, under the load pulley, over the smaller effort pulley, and then, via the slack side, back to the starting point.

Both the wheels of the upper pulley are indented, in a way resembling chain sprockets, so that the chain cannot slip. If the numbers of such indentations are N and n in the larger and smaller wheel, respectively, then one revolution of the composite wheel will be associated with a movement of N

links upwards in the left-hand side of the load loop and of n links downwards in the right-hand one. The net shortening of this loop is thus $(N-n)$ links, and, since each side shortens by half this amount, the load must rise by $\frac{1}{2}(N-n)$ links. Simple calculation then gives the velocity ratio as $2N/(N-n)$.

The machine is a useful one in that it usually has an efficiency less than 0.5. This means that more then half the effort is used in overcoming friction, so that the maximum frictional force must exceed the load. Thus, when the effort is removed, frictional resistance is high enough to prevent the device from running backwards, so that the load remains supported by this frictional force.

Example 4

A Weston differential is used to lift a fibre bale of mass 800 kg from a delivery lorry. If the two wheels in the machine have radii of 15 cm and 12 cm, respectively, and the efficiency is 44%, find how many men (each capable of exerting a force of 500 N) would have to haul on the lifting chain together to unload the bale.

We have:

$$\begin{aligned} \text{V.R.} &= 2N/(N-n) \\ &= 2 \times 15/(15-12) \\ &= 10 \end{aligned}$$

and

$$\begin{aligned} \text{M.A} &= \text{efficiency} \times \text{V.R} \\ &= 0.44 \times 10 \\ &= 4.4 \end{aligned}$$

Then, since

$$\text{force exerted by load} = 800 \times 9.81 \text{ N}$$

$$= 7848 \text{ N},$$

$$\text{force exerted by effort} = 7848/4.4 \text{ N}$$

$$= 1783.6 \text{ N},$$

i.e., four men are needed to operate the lifting device.

Inclined planes

In an inclined plane, the effort is applied to the plane and causes it to move. The resulting movement applies a force to the load and constrains it to move in turn, as shown in Fig. 2.

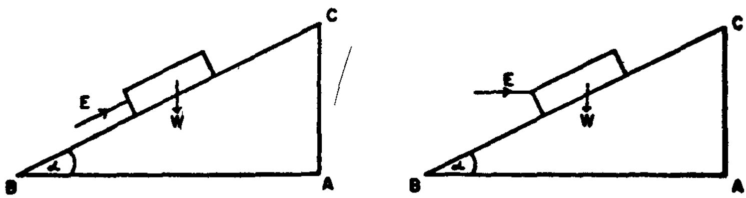

Fig. 3. The inclined plane is as a machine

It is apparent from the diagram that, if the effort is applied up the slope, the velocity ratio is given by BC/AC, or 1/sin α (i.e., cosec α), whereas, for horizontal application of effort, the velocity ratio is AB/AC, or 1/tan α (i.e., cot α). Thus, the velocity ratio is determined only by the slope of the inclined plane and must therefore be constant, no matter where the point of application of either load or effort is situated.

Example 5

A motor-driven winch is used to haul a fibre bale of mass 560 kg up a ramp with a height of 1m and a slope length of 5 m. The bale moves up the ramp in 20 and the 440V motor draws a constant current of 2.1A throughout this time. If the coefficient of friction m, between bale and ramp is 0.22, calculate the efficiency of the motor and which combined.

Angle of slope of plane, = 0.21 rad, since sin $\alpha = 0.200$.

Hence:

$$\cos \alpha = 0.98$$

Since

force exerted vertically downwards by bale

$$= 560 \times 9.81\text{N}$$

$$= 5493.6 \text{ N}$$

the reaction normal to the palne is given by

$$R = 5493.6 \cos \alpha$$

$$= 5493.6 \times 0.98$$

$$= 5383.7\text{N}$$

and if is the coefficient of friction:

$$\text{frictional force} = \mu R$$

$$= 0.22 \times 5383.7$$

$$= 1184.4\text{N}$$

We also have:

$$\text{components of loaddown slope} = 54936 \sin\alpha$$

$$= 1098.7 \text{ N}$$

Thus:

$$\text{total force required to move load} = (1098.7 + 1184.4)\text{N}$$

$$= 2283.1 \text{ N}$$

Now:

Thus: $$\frac{1}{\sin\alpha}$$

since this force is applied up the slope, i.e

$$V.R = 5$$

and, by definition:

$$\begin{aligned} M.A. &= \text{load/effort} \\ &= 5493.6/2283.1 \\ &= 2.41. \end{aligned}$$

Thus

$$\begin{aligned} \text{efficiency} &= M.A./V.R. \\ &= 2.41/5, \end{aligned}$$

i.e. efficiency of inclined plane = 48%

Now

work done in moving bale up slope

$$\begin{aligned} &= \text{force} \times \text{ distance moved} \\ &= 2283.1 \times 5 \text{ J} \\ &= 11415.5 \text{ J}. \end{aligned}$$

This work is done in 20 seconds, i.e.:

rate of doing work = 11415.5/20 J/s.

thus

power used by 440-V motor in drawing 2.1 A

$$= 440 \times 2.1 \text{ W}$$

But

power expended = 570.8 W.

$$= 924.0 \text{ W},$$

and thus:

$$\text{efficiency} = \text{useful power/power put in}$$
$$= 570.8/924.0,$$

i.e., efficiency of motor and winch = 62%.

As was the case for the lever, the inclined plane can again masquerade in various disguises such as a wedge, a screw, a worm gear, or a cam. A screw, which carries a helical thread of constant pitch working in a similarly threaded nut or hole, moves forward by one pitch for each revolution of the shaft. If the effort is applied at the end of a lever of arm length r.

Example 6

A screw of pitch 2.5 cm is worked by a lever 2.2 m in length. If a force of 15 N applied at the end of the lever is just sufficient to raise a mass of 450 kg, calculate:

a) the velocity ratio;
b) the efficiency;
c) the work done by the effort in raising the mass 10 cm.

We have:

$$\text{V.R} = 2\pi r/p$$
$$= 2\pi \times 2.2/0.025$$

Hence velocity ratio = 552.9.

We also have:

$$\text{force exerted by load} = 450 \times 9.81 \text{ N}$$
$$= 4414.5 \text{ N},$$

and thus:

M.A. = load/effort

= 4414.5/15

= 294.3,

so that:

efficiency = M.A.N.R.

= 294.3/552.9.

Hence efficiency = 53%.

Since work done (W.D.) = force × distance moved:

W.D. on load = 4414.5 × 0.1 = 441.4 J.

Since efficiency is 53%, this work must be 53% of the work done by the effort, i.e.:

W.D, by effort = 441.4 × 100/53 J.

Hence W.D. by effort = 832.8 J.

The meshing is in the form of an inclined plane and the load is subsequently lifted by lever action of a wheel and axle type. Calculation again is straightforward and gives a value of *Nr*/*r*' for velocity ratio. In more complicated cases, however, it is usually safer to work from first principles.

Example 7

In the drive mechanism for the flat chain of a carding engine, the driving rope turns a pulley of diameter 33 cm compounded with a single worm. This worm operates a cog wheel with 32 teeth, compounded again with a single worm. The latter drives a cog wheel with 56 teeth, compounded with the chain wheel, which has 16 teeth on a 3.8 cm pitch. Calculate the velocity ratio. If the efficiency is 10% and a force of 36 N is needed at the rim of the driving rope pulley to move the flats, what resistance do they offer? The diagram of the mechanism is given in Fig. 4.

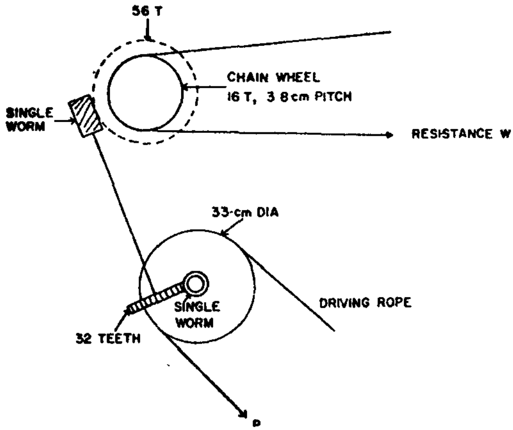

Fig. 4. The chain drive for a carding-flat system

For one revolution of the driving-rope pulley, the rope moves 33π cm, i.e.:

$$\text{rope movement} = 1.04 \text{ m},$$

and

$$\text{the movement produced in the flat chain} = (1/32)\ (1/56) \times 16 \times 3.8 \text{ cm},$$

$$\text{chain movement} = 0.0339 \text{ cm, or } 0.000339 \text{ m}.$$

Thus:

$$\text{V.R.} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= 1.04/0.000339,$$

i.e., velocity ratio = 3067.8

Now:

$$\text{M,A.} = \text{efficiency} \times \text{V.R.}$$
$$= 306.8,$$

and we have:

force exerted at drive pulley = 36 N,

thus, since

$$\text{M.A} = \frac{\text{force applied to load}}{\text{force exerted by effort}}$$

force applied to load = $36 \times 306^{\cdot 9}$ N.

This force is used in overcoming resistance to movement.

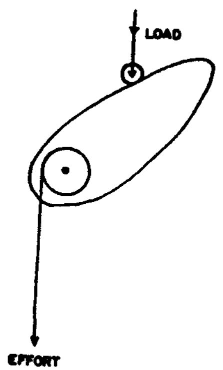

Fig. 5. The cam as an inclined plane

Hence resistance to movement = 11044.8 N. It should be noted that this resistance is itse!f a frictional one, but the work done in overcoming friction in this case is regarded as useful work, since the object of the mechanism is to keep the flat chain moving at the correct speed against any resistance it offers to movement. Of the 36 N effort required at the driving-pulley rim only 3.6 N used in overcoming chain friction, since efficiency is 10%. The remaining 32.4 N are used to overcome friction within the mechanism, particularly at the two worm gears. A further point that should be noted is the fact that the machine would not reverse on removal of the effort even if efficiency were above 50%, since the frictional load is unable to apply any force back through the system. The velocity ratio cannot usually be calculated theoretically because of the complexity of cam shapes and must be determined directly by measurement of the distances moved by load and effort.

Comparison of levers and inclined planes

The major difference immediately noticed between the lever and the inclined plan e is one of mobility. In the lever, there is a fixed point (the fulcrum) about which a rotation takes place, and this point enables the work done by the effort to be transferred along the lever and to act on the load. Apart from this rotation, there is no movement, so that the points of application of the effort and the load remain fixed relative to each other and to the lever.

In the inclined plane, on the other hand, movement is essential to the operation of the machine. The effort is applied to the plane and causes it to move, and this movement is transmitted to the load, which thus produces movement there in turn. The point of application of the effort is usually fixed, but the point of application of the load must vary along the plane or no work can be done by the machine.

This fundamental difference between the two machine types leads in turn to a practical difference. By means of bearings and lubricants, the rotation in a lever can be made to occur very easily, with few or no significant frictional effects. In the inclined plane, however, where movement must take place, frictional resistance tends to become more noticeable and work must be done to overcome it. This work is wasted, as far as the load is concerned, so that the efficiency of the inclined-plane type of machine tends, in general, to be considerably lower than that of the lever type.

Example 8

A machine of mass 500 kg must be raised a vertical distance of 7.5 cm to allow insertion of vibration-reducing mountings. Two types of jack are available, a lever type and a screw type. In the lever type, the effort arm is 150 cm in length, the load arm is 15 cm, and the fulcrum pin, with a diameter of 4 cm, has a coefficient of friction of 0.2 in its housing.

The screw jack is turned by means of a lever 30 cm in length and has a screw, of diameter 2 cm and pitch 0.65 cm, with a coefficient of friction of 0.15 in its housing. Compare the mechanical behaviour and general usefulness of each type in carrying out the operation.

In both cases:

$$\text{force exerted by machine on jack} = 500 \times 9.81 \text{ N}$$
$$= 4905 \text{ N},$$

and since

$$\text{movement} = 7.5 \text{ cm},$$

then

$$\text{useful W.D.} = 4905 \times 7.5/100 \text{ J}$$
$$= 367.9 \text{ J}.$$

For the lever type

$$\text{V.R.} = \frac{\text{length of effort arm}}{\text{length of load arm}}$$
$$= 150/15$$
$$= 10.$$

Thus, if 100%, efficiency is assumed, effort would be 490.5 N, and hence:

total force on fulcrum pin = (4905 + 490.5) N

= 5395.5 N,

so that:

approximate frictional resistance = 0.2 × 5395.5

= 1079.1 N.

For a lift of 7.5 cm and a load-arm length of 15 cm, the angle of movement of the fulcrum, α, is given by:

$$\sin \alpha = 0.5 \text{ approximately}$$

i.e.:

$$\alpha = 0.52 \text{ rad.}$$

since

circumference of fulcrum pin = $\pi \times 4$ cm,

its surface movement = $4\pi \times \frac{0.46}{2\pi}$

= 0.92 cm,

and

frictional W.D. at fulcrum = 1079.1 × 0.92/100 J

= 9.9 J.

Thus

total W.D. = 367.9 + 9.9 1

= 377.8 J

and hence

efficiency = 367.9/3778 × 100%

$$= 97.4\%.$$

With an efficiency as high as this, the approximation used in determining frictional resistance does not require a correction.

Now, substituting in:

$$\text{M.A.} = \text{efficiency} \times \text{V.R.}$$

$$\text{M.A.} = 0.974 \times 10$$

$$= 9.7 \text{gives:}$$

For the screw type:

$$\text{V. R.} = 2\pi r/p$$

$$= \frac{2\pi \times 30}{0.65}$$

$$= 290.0.$$

Since number of revolutions of screw for a 7.5cm lift is equal to distance moved/pitch:

$$\text{number of revolutions} = 7.5/0.65$$

$$= 11.5.$$

The total length of inclined plane is thus 11.5 times the screw circumference, i.e.:

$$\text{length of plane} = 11.5\pi \times 4 \text{ cm}$$

$$= 1.45 \text{ cm}$$

Now:

$$\text{frictional force at the screw} = \mu R$$

$$= 0.15 \times 4905 \text{ N}$$

(approximately, since the direction in which the load acts is not quite exactly at right angles to the plane)

$$= 735.8 \text{ N,}$$

i. e.:

$$\text{W.D. against friction} = 735.8 \times 1.45 \text{ J}$$
$$= 1066.9 \text{ J.}$$

Thus:

$$\text{total W.D.} = 1066.9 + 367.9 \text{ J}$$
$$= 1434.8 \text{ J.}$$

and hence

$$\text{efficiency} = 367.9/1434.8 \times 100\%$$
$$= 25.6\%.$$

Now, substituting in:

$$\text{M.A.} = \text{efficiency} \times \text{V.R.}$$

gives

$$\text{M.A.} = 25.6 \times 290$$
$$= 7424.0.$$

For the two types, a table of mechanical properties can be drawn up for comparison as in Table 1.

Table.1

	Lever	Screw
V. R.	10	290
M.A.	9.7	7424
Efficiency	97%	26%

The values of velocity ratio show that the lever is much faster than the screw, but the screw, with a much higher mechanical advantage, requires a much smaller effort. On the other hand, the low efficiency of the screw type means that more work must be done in lifting the load. There are, however, other factors of importance apart from these obvious mechanical ones. In the first case, the screw jack, with an efficiency below 50%, cannot reverse, whereas the lever can. Thus, if the lever type is used, the effort must be maintained throughout the operation, if the load is not propped up, or the machine will fall on someone's toes to that person's noticeable distress.

The screw jack has to have effort applied, in the reverse direction, in order to lower the load. In the second case, the larger size of the screw jack means that it requires appreciable clearance beneath the machine before it can be inserted for the lifting operation, and this space may not be available. In practice, it may be advisable to use the ' lever jack, easily inserted, to raise the machine to a height sufficient to allow insertion of the screw jack, which is then used for the subsequent lifting operation in the interests of safety. Notice also that, in this simple example, we have assumed the machine to be lifted bodily, whereas in practice it might be more convenient to lift one leg, or one side, at a time and insert the mountings in turn.

Law of the machine

If the load on which a machine is operating is increased, it is obvious that an increase in effort is required to enable the operation of the machine to continue. In an ideal machine, with negligible friction, we have already seen that the mechanical advantage, the ratio of load to effort, is equal to the velocity ratio, a constant determined only by the relative rates of movement of load and effort. For such a machine, then, the relation between load and effort is a linear one and the effort P is proportional to the load W.

In the practical case, this is no longer true. Some of the effort P is used in overcoming the frictional resistance present within the machine or associated with the movement of load and effort. In consequence, the actual effort, Q, that must be applied to the machine to move the load W is greater than the theoretical amount, P, by a quantity depending on the nonproductive work that must be done in bringing about operation of the machine itself. We can distinguish between theoretical and practical conditions by defining two sets of constants. Thus, the theoretical mechanical advantage, already defined as:

$$\text{M.A.} = W/P$$

can have an analogous practical complement defined as W/Q. The efficiency, defined as the ratio of mechanical advantage to velocity ratio, can then be shown to be given by:

$$\text{efficiency} = W/Q - W/P$$

$$= P/Q.$$

When the load W is increased, the direct proportionality between P and W does not extend to the relation between Q and W. Even if W is zero, a finite amount of effort must be exerted in practice merely in order to set the machine into operation. This amount will always be needed, whatever the magnitude of W, and so its presence must be accepted under all working conditions. In addition, an increase in the load will increase to a slight extent the nonproductive work of the machine and will thus increase the practical effort Q at a rate slightly higher than that of the theoretical effort, P.

As W increases from zero, P increases proportionately, also from zero. The effort Q, on the other hand, has a finite value even at zero load and then increases in a linear manner but at a slightly higher rate than P. Equations can be fitted to the two lines and are, in the example given:

$$P = 0.1 \ W$$

and:

$$Q = 0.12 \ W + 13.3.$$

The latter equation, relating actual effort to load, is known as the law of the machine for the particular system under consideration. In the case chosen, the taw of the machine is a linear one, but it is possible in many cases to have a quadratic, or even more complex, equation expressing this relation. The method of establishing the equation is, however, the same in all cases.

The effort required to operate a machine, Q, is determined for different values of load, W, and the graph-relating the two quantities plotted, the equation being derived by subsequent measurement. Or calculation it is not possible, of course, to derive values of P in practice of this factor can only be estimated approximately, practice, and the magnitude from a knowledge of the frictional behaviour of the machine, in simple cases.

Example 9

The load and effort of a sheaved pulley a system with a velocity ratio of 4 were measured at various values of load and found to be as set in table 2.

Table 2

Load (N)	0	50	100	150	200	250	300
Effort(N)	14.2	28.9	42.3	57.8	71.2	86.7	100.1

Derive the law of the machine and find the resistance to motion of the moving parts.

$$\text{slope} = 0.29.$$

The effort, Q, and the load, W, are then related by the equation:

$$Q = 0.29\ W + 14.2,$$

and this is the law of tile Machine.The resistance to motion, W_0, is the load that must he overcome before the effort begins to do useful work and is equal to the negative intercept on the load axis, calculated when $Q = 0$. Thus:

$$0.29\ W_0 = 14.2$$

i.e.:

$$W_0 = -48.97.$$

Hence resistance to motion = 49.0 N.

The change of efficiency with load may be deduced in a similar manner. When there is no load (i.e., $W = 0$), the efficiency must be zero too, since P is zero. As the load increases from zero, the useful work initially increases very rapidly in comparison with the lost work used in driving the machine.

The efficiency therefore rises rapidly as light loading is placed progressively on the machine. As the loading increases further, the lost work constitutes a smaller proportion of the total work, and the change in useful work becomes less noticeable. As a result, the efficiency increases more and more slowly until, eventually, an increase in load causes no further increase in efficiency and the machine is operating at its maximum efficiency. The changes taking place may be seen if a graph is plotted to show the variation of efficiency with load, and a typical curve.

Example 10

In the sheaved pulley block of the previous example, plot graphs showing the dependence on load of (a) mechanical advantage and (b) efficiency. By using the expressions:

$$\text{M.A.} = \text{load/effort}$$

and:

$$\text{efficiency} = \text{M.A./V.R.},$$

the values of M.A. and efficiency for the last reading may be determined as:

$$M.A. = 300/100.1 = 2\text{-}997,$$

and:

$$\text{efficiency} = 2.997/4 = 0\text{-}749.$$

The other values may be calculated similarly, and the results tabulated as in Table 3.

Table. 3

Load (N)	*Effort (N)*	*M.A.*	*Efficiency*
0	14.2	0	0
50	28.9	1.73	0.43
100	42.3	2.36	0.59
150	57.8	2.60	0.65
200	71.2	2.81	0.70
250	86.7	2.88	0.72
300	100.1	3.00	0.75

Since there is a constant ratio between M.A. and efficiency, it is obvious that the same curve can be fitted to both parameters so long as the ordinate is scaled correctly in each case.

Use of the law of a machine

Most practical machines consist of a combination of several or many simple ones, since they are made up from levers, pulleys, screws, and so on. It is possible to show that the overall efficiency of such a machine is equal to the product of the efficiencies of all the component parts, but a practical measurement would bypass these values and be used to determine only the overall efficiency of the entire machine.

Example 11

A weaving shed uses electrical power from an external 120-kV supply line. The efficiencies of the separate parts of the power train are specified independently as transformer (97%), cables (97%), motors (82%), and gear drive, coupling a motor to a loom (97%). The driving shaft of a loom operates at 180 rev/min, the driving pulley has a diameter of 40 cm, and the average effective force from the belt to the pulley rim during operation is 103.2 N. Calculate the current flowing along the 120-kV line into the transformer if there are four looms running simultaneously.

We have:

$$\text{angular speed of driving shaft} = 180 \text{ rev/min} = 3 \text{ rev/s},$$

and hence:

$$\text{surface speed of pulley} = 2 \times 0.20 \times 3 = 3.78 \text{ m/s}.$$

Thus:

$$\text{W.D. on pulley per second} = 103.2 \times 3.78 \text{ J},$$

i.e.:

$$\text{power dissipated per machine} = 390.1 \text{ W},$$

and hence:

$$\text{total power dissipated} = 1560.4 \text{ W, with four machines running.}$$

Now:

$$\text{overall efficiency of supply} = 0.97 \times 0.97 \times 0.82 \times 0.97 = 0.75,$$

$$\text{power supplied to transformer} = 1560.4 \times 100/75 \text{ W} = 2080.5 \text{ W}.$$

current drain = 2080.5/120000 A

= 0.01734 A.

Hence current flowing along 120-kV line = 17.3 m A.

It is easy to visualize this type of calculation in simply measured parameters like electrical ones, but the identification of efficiency for each individual component part of a complex machine is a time-consuming job and scarcely worth the effort except in certain esoteric design applications. The normal procedure in setting such a machine into operation, or in establishing its potential usefulness in prototype form, is to test for efficiency at different loads and then decide the optimum operating conditions with regard to the work expected from it.

Textile-machine efficiency

In considering a textile machine, it is a little more difficult to establish the optimum operating conditions. The function of such a machine is to carry out a given operation on a textile material, rather than solely to perform useful mechanical work. A textile is not a rigid structure and can thus not be moved by the usual mechanical method of giving it a good push.

Textiles, like donkeys, have to be coaxed indirectly to behave in the desired way, and the judicious use of a metaphorical carrot may not necessarily be achieved in a mechanically efficient manner. The most complicated piece of textile machinery is, however, merely a complex combination of many simple machines, each behaving in accordance with' the laws already discussed. Even though the most important function of the machine is to produce textile goods of an acceptable quality and in satisfactory quantity, the designer will try to ensure that, within these major constraints, his equipment will work as efficiently as possible to reduce power consumption and wear to a minimum. At this point, it is important to distinguish clearly between textile-machinery efficiency from the mechanical and the production points of view.

Mechanically, machine efficiency is determined exactly as for any other machine; the load is progressively increased, and the resulting increase in effort, or in power consumption, is monitored. There may be practical difficulties in arriving at an estimate of the load because of the nature of textile materials, but there is usually some device of substitution to enable a reasonably accurate value to be obtained. For example, a roller that normally moves a fibre assembly can be fitted instead with a belt and scale pan, and the effort required to raise the pan with different loads can, be measured.

Example 12

A continuous-filament yarn extruded at 0.5 m/s is cold-drawn at different draw ratios by a variable-speed roller of diameter 12 cm and offers a force of resistance, which may be regarded as constant, of 2.5 N to the drawing operation.

With the yarn absent, the variable-speed drive was operated at different angular speeds, and, at each speed, measurement was made of the load that could just be lifted, by means of a light inextensible cord, by the roller. Plot a curve relating efficiency and draw ratio, on the assumption that filament slip is negligible.

At the fastest speed:

surface speed of drawn filament = 2 × 0.06 × 500/60 m/s

= 3.14 m/s.

Since slip is negligible:

draw ratio = V.R. = 3.14/0-5

$$= 6.28.$$

Now, at this speed

$$\text{force exerted by test loading} = 0.048 \times 9.81 \text{ N}$$
$$= 0.471 \text{ N},$$

i.e.:

$$\text{effort} = 0.471 \text{ N}.$$

Now:

$$\text{M.A.} = \text{load/effort}$$
$$= 2.5/0.471$$
$$= 5.31,$$
$$\text{V.R.} = 6.28.$$

A similar calculation may be carried out for each speed, the results obtained being given in Table 4.

Table 4

Roller Speed rev/min	*Surface Speed m/s*	*V. R. Draw Ratio*	*M.A.*	*Efficiency*
0	0	0	0	0
100	0.63	1.26	0.50	0.40
200,	1.26	2.52	1.40	0.56
300	1.88	3.76	2.63	0.70
400	2.51	5.02	4.03	0.80

The graph relating efficiency to draw ratio from which the shape of the curve may be seen to be complex and non-linear, though smoothly curved. This smoothness is characteristic of machines of a continuously variable type as in the example, but a more discontinuous type of curve is often obtained for machines where the load varies in an erratic fashion. An obvious instance of this kind of behaviour would be found if, for example, the variation of efficiency with displacement of a loom crankshaft could be plotted over a cycle of operation.

Load variation in this case changes suddenly and significantly at various stages of the cycle of operation. Production efficiency is, however, a completely different problem. It is obtained by a comparison of the actual production of a given machine with the estimated production of which the machine is capable. Reduction of efficiency from the theoretically perfect value, 100%, occurs in this case because the machine experiences idle time from causes such as setting up, maintenance, shift-changing, doffing, material breakage, and so on.

It has no direct connection with the mechanical efficiency of the machine, and, unless mechanical efficiency is so poor that it interferes with the successful operation of the machine, there is usually not even an indirect connection between the two. Although this kind of efficiency is of vital importance to the manufacturer, it is not at all predictable in mechanical terms. All that can be done by the textile technologist is to ensure that each machine is operating at its highest mechanical level, receives regular maintenance or service, and is carrying out its work on material with which it is compatible.

Five
Circular Motion

The choice of which analytical method to use is governed by the application in question, some rotational motions being treated most easily by the first method and some by the second. In the case of the moon rotating around the earth, or in the study of electron acceleration in a cyclotron, the motion of a particle is obviously of more importance, and the same is true of an analysis of the movement of a traveller around a ring in a spinning frame.

In the case of an aircraft propeller or of the sweep in a radar scanner tube, however, the movement of the radius vector is of greater importance. In textile appli-cations, the rotation of the seed yarn in an open-end spinning operation is similarly more closely related to the radius, rather then the particle, treatment.

Equations of circular motion

The simplest example of circular movement, uniform circular motion, has already been met and occurs when an object travels at uniform speed round a fixed central point, O, as shown in Fig. 1. If the object moves from A to B, so that the radius OA moves through an angle θ we say that its angular velocity, ω about O is the change in the angle θ per unit time. If the time taken in moving from A to B is t seconds, the angular velocity is therefore given by:

$$\omega = \theta/t \text{ rad/s.}$$

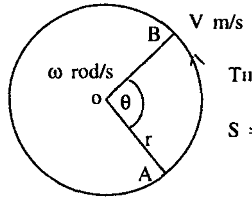

Fig. 1. Uniform circular motion

If we consider to find out the angular velocity of the seconds hand of a clock. In one revolution, the hand rotates through a full circle, or 2π rad. Time taken for a revolution is 1 min or 60 s, i.e.:

$$\omega = \theta/t \text{ rad/s.}$$

Example 1

A hydroextractor is revolving at 15.7 rad/s; calculate its period of revolution and its rotational speed in rev/min.

We have:

$$\text{speed of rotation} \quad \omega = 15.7 \text{ rad/s.}$$

Thus:

$$\text{period } T = 2\pi/\omega$$

$$= 2\times 3.14/15.7 \text{ s,}$$

i.e., period of rotation = 0.4 s.

Thus, in 60 s, the hydroextractor makes 60/0.4 revolutions, i.e., rotational speed = 150 rev/min.

If the length of the are AB in Fig. 1 is given by S and the length of the radius b r, then the angle θ, by definition, is given by:

$$\theta = S/r$$

which may be rearranged as:

$$S = r\theta$$

Division by t, the time taken in moving through the distance S, gives:

$$S/t = r\theta/t$$

But S/t is the speed, v, of the rotating object and θ/t is ω the angular velocity.

Thus:

$$v = r\omega.$$

Example 2

Find the surface speed of the cage of the hydroextractor used in Example 1 if the cage has a diameter of 2.55 m.

We have:

diameter of cage = 2.55 m,

i.e.:

radius of rotation = 1.275 m,

and:

angular velocity = 15.7 rad/s.

Hence:

$$v = r\omega$$
$$= 1.275 \times 15.7$$
$$= 20.02 \text{ m/s},$$

i.e., surface speed of cage is about 20 m/s.

If an object attached to a string is rotated at constant speed in a circle, it is evident that a force is present to maintain the motion. The existence of this force can, indeed, make itself known in a tangible manner if the string breaks and the object flies off to contact a pane of glass or the head of a passer-by! The presence of this force, the centripetal force, implies that an acceleration must exist to produce it, and, since the object continues to rotate around a central point (barring accidents), the towards the centre of the circle. This can be shown most simply by a vector analysis. Suppose that a particle moves at constant speed v, in a circular path of radius r, through two equal small angles α in its motion from A to B.

The velocity v can be resolved at A into a vertical component, $+v_v$ and a horizontal component, $-v_h$, on the assumption that velocities become more positive in the upward direction and to the right, for the vertical and horizontal components, respectively, as in the usual convention. Similarly, the velocity at B may be resolved into components $+v_v$ and $+ v_h$, so that the change in the particle's velocity is $(+ v_v) - (+ v_v)$ in the vertical direction and $(+v_h) - (-v_h)$ in the horizontal one. The net change in velocity is thus $2v_h$ in the horizontal direction from left to right, so that the acceleration must take place in the same direction, that is, towards the centre of rotation. Thus:

$$\frac{v_h}{v} = \frac{CD}{BC}$$

$$\frac{BC}{OB} = \text{ (from similar triangles).}$$

If the triangles become very small, this becomes equivalent to x/r where x is the distance from either A or B to C, their mid-point.

The time taken for the movement from A to B is:

$$t = \frac{2x}{v}$$

and, since acceleration, f, is defined as the change of velocity in unit time:

$$f = 2v_h + \frac{2x}{v} = \frac{vv_h}{x}$$

Substituting from the equivalence

$$\frac{v_h}{v} = \frac{x}{r}$$

gives:

$$f = \frac{v}{x} \cdot \frac{vx}{r} . = \frac{v^2}{r} .$$

We have already seen that v = rω, so that the acce-leration may be written as

$$f = r\omega^2.$$

Thus, an object moving in a circle of radius r with a constant speed v has a constant acceleration of v^2/r or $r\omega^2$ towards the centre.

If the speed and angular velocity are constant, the two relations

$$S = vt$$

and

$$\theta = \omega t$$

hold good for the particle and radius treatments, respectively.

It is sometimes convenient, particularly with regard to calculations involving rotation in electrical devices such as motors, to operate in revolutions per second, rather than radians per second. For this purpose, the transformation equation

$$\pi \text{ rad} = 180°$$

is of importance. By its use, we can see that, in terms of n, the number of revolutions per second, or of T, the period of rotation, equations expressing angular velocity, ω angular displacement, θ and linear velocity, v, may easily be derived. Thus:

$$\omega = 2\pi n = \frac{2\pi}{T};$$

$$\theta = 2\pi nt = 2\pi \frac{t}{T};$$

$$v = r\omega = 2\pi nr$$

$$= \frac{2\pi r}{T}.$$

Example 3

Cloth emerging from a tenter is being collected on a take-up roller and, at a time when the diameter of the cloth on the roller is 60 cm, the period of rotation of the roller is found to be exactly 10 s. Calculate the speed at which cloth is moving through the tenter.

We have:

diameter of cloth on roller = 60 cm,

i.e.:

radius of rotation = 0.30 m,

and:

period of rotation = 10 s.

The surface speed of rotation, v, is given by:

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi \times 0.30}{10} = 0.1884.$$

Hence cloth speed through tenter = 0.19 m/s.

An alternative method of doing the calculation would be to find the angular velocity and to derive the surface speed from that, as follows:

$$\text{period of rotation} = 10\text{s},$$

i.e.:

$$\text{angular velocity} = 2\pi$$

$$= 0.628 \text{ rad/s}.$$

Thus, surface speed,

$$v = r\omega$$

$$= 0.628 \times 0.30$$

$$= 0.1884$$

As before, cloth speed through tenter = 0.19 m/s.

The rotation is taking place at constant speed. If the speed is changing but the acceleration is uniform, it is possible to derive equations of motion analogous to those pertaining in linear movement. The uniform acceleration, f, of the particle velocity is matched by a corresponding uniform angular acceleration, α. If the initial particle speed and angular velocity are u and ω_0 respectively, then transformation (or derivation from first principles) yields equivalences as follows:

$$v = u + ft \text{ becomes } \omega = \omega_0 \ \alpha t;$$

$$S = ut + \tfrac{1}{2}ft^2 \text{ becomes } \theta = \omega_0 t + \tfrac{1}{2}\alpha r^2;$$

and

$$v^2 = u^2 + 2fS \text{ becomes } \omega^2 = \omega_0^2 + 2\alpha\theta.$$

Example 4

A carding engine is started from rest to its full speed of 180 rev/min with a uniform acceleration of 36 rad/s². Find the time taken to reach full speed and the angle through which the driving shaft moves during this time.

The final angular velocity is:

$$\omega = 180 \text{ rev/min}$$
$$= 3 \text{ rev/s}$$
$$= 18.85 \text{ rad/s.}$$

Thus:

$$\omega_0 = 0$$
$$\omega = 18.85 \text{ rad/s}$$

and

$$\alpha = 36 \text{ rad/s}^2$$

Substituting these values in:

$$\omega = \omega_0 + \alpha t$$

gives:

$$18.85 = 0 + 36t$$
$$t = 18.85/36.$$

Hence time to reach running speed = 0.524 s.

Furthermore, since:

$$\omega_0 = 0$$
$$\alpha = 36 \text{ rad/s}^2$$

and

$$t = 0.524 \text{ s,}$$
$$\theta = \omega_0 t + ½ t^2,$$
$$= 0 + ½.36\ (0.524)^2$$

Hence angle moved by shaft = 4.94 rad.

Maintaining uniform circular motion

When a particle moves in a circle with uniform speed of v m/s in a circle of radius r m, it has an acceleration of v^2/r, or $r\omega^2$,

m/s^2 directed towards the centre of the circle. We have also seen that a centripetal force acts on the particle, again towards the centre, and the origin of this force can now be sought. Since force is the product of mass and acceleration, the magnitude of the centripetal force can be calculated as mv^2/r, or $mr\omega^2$, newtons for a particle of mass m kg. This force is obviously not inherent in the particle itself and, unless one assumes the presence of a poltergeist, must have an origin external to the particle.

The nature of the rotation governs the source of the accelerating force; for a string swung in a circle, a slight flick of the hand in synchronism with the period of oscillation imparts a force at intervals sufficiently close to maintain movement. The inertia of the rotating mass is sufficient to smooth out the jerkiness of the application and create an illusion of uniform external force. In a rotating planet, however, the external force is gravitational and is applied continuously, a rather fortunate fact since an Earth with hiccoughs would be an uncommonly inconvenient place on which to live. The crucial fact in both cases, then, is the presence of a continuous, or continuing, force applied from some external source. Without this force, the rotation would cease, and its presence is necessary for maintaining the circular motion, but an apparent difficulty arises.

If we look more carefully at the string that restrains the stone rotating in a circle, we find that the force directed towards the centre of rotation manifests itself as a tension in the string. At first sight, this tension should cause the string to shorten, or crumple, but this obviously does not take place. The solution to the problem is found in the presence of an equal but opposite force in the string, directed outwards from the centre of rotation.

The centripetal force T_1 is just balanced by the force T_2 so that the net force in the stretched string is zero, and the string neither stretches further nor loses its tension once equilibrium has been reached. The tensions T_1 and T_2 are, of course, of equal magnitude, and T_2 is known as the centrifugal force. The student should avoid the common tendency to confuse the two; centrifugal force does not act on the rotating body, as is often stated, but is merely set up within the string as an equal but opposite force of reaction to maintain equilibrium when the centripetal force acting on the rotating body is transferred to the string.

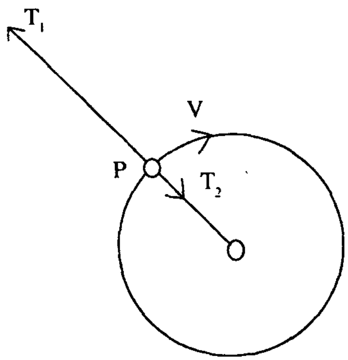

Fig. 2. Centripetal and centrifugal forces

The proof of this point lies in observation of what happens when the string breaks. If the centrifugal force T_2 were acting on the body, the latter would be flung outwards in the radial direction at the instant of break, and this does not happen. What does take place, of course, is a cessation of circular motion because the force maintaining it, which was directed towards the centre of rotation, ceases to act, and the body flies off in a tangential direction.

At the same time, the centrifugal force also ceases to act because it is no longer needed to balance the centripetal force, so the broken end of the string is no longer under the two tensions. It thus crumples initially and then falls to take up a vertical position from the point of suspension under the influence of gravity. Part of the confusion surrounding the concept of centrifugal force arises from a lack of awareness of its true nature. Suppose, for example, that a passenger is standing in a bus as it goes rapidly round a corner. He is thrown radially outwards in the process and naturally attributes his undignified landing in a lady's lap to a force exerted on him, in a direction radially outwards from the centre of rotation, which he calls centrifugal force.

Whether the lady accepts his explanation or not is immaterial, but it is certainly not based on sound scientific principles. He was not struck by any object, so no force was transmitted from an external source to him. The bus did not fly outwards, so there was no centrifugal force applied to the whole system. It follows, therefore, that the reason for his sudden lapse of manners (or plunge into romance, depending on the lady's reaction) must have been inherent in his motion before the incident. What happens, of course, is a problem in inertia, not in centrifugal force. Before the turn, the bus with all its contents, including passengers, is moving forwards in a straight line at uniform speed.

In order to turn (that is, to move into a circular motion), the presence of an external force is required. For the bus, this is achieved by the thrust of the wheels against the road surface. For the seated passengers, it is achieved by the thrust of the fixed superstructure of the bus against various portions of their anatomy. Our poor victim, however, has little such assistance. His feet, it is true, are carried inwards by movement of the floor beneath them, but his upper body receives no such force and thus continues to move in a straight line directly along the original line of motion. Since this is no longer the line of motion of the rest of the bus, he falls headlong into the delightful (or dreadful) predicament already defined. His movement, then, is caused not by the presence of a centrifugal force, but by the absence of a centripetal one. His inability to distinguish between the two is caused by the fact that he is carrying his frame of reference with him. To him, the bus appears fixed and he seems to be standing still in it, so that he is the only moving object when he loses his balance. To an external observer standing outside the bus, however, the motion appears quite different. From his point of view, the bus swerves suddenly and the man continues to move in a straight line, so that the lady literally turns into him.

The external fixed observer is also moving without being aware of the fact, of course, since he is standing on the surface of a planet that is rotating about its own axis, revolving about the sun, rushing outwards from some point in our galaxy, and, for all we know, undergoing an infinite number of other motions when defined from more and more remote frames of reference. The mechanical treatments of these successive complications are somewhat difficult and must be deferred (with regrets, naturally) to a more opportune time.

Forces acting during circular motion

We have seen that, for a body rotating at the end of a piece of string, a centripetal force acts on the body and is directed along the string towards the centre of rotation. Such a force in isolation, of course, would not represent a stable arrangement. In the first case, the string in which this force is present remains taut, so that an equal tension, acting outwards, must be present in the string. If these were the only forces acting, the object would be in a situation of unstable equilibrium and would presumably revert to the common stable situation in which it hung vertically at rest under the influence of gravitational attraction.

This situation is a stable one because there is a force, the effect of gravity, present to maintain the tension in the string, and the same must also be true when rotation in an equilibrium state is occurring. In such a case, the force is provided as an inertia resistance, R, acting radially outwards on the rotating object and represents the resistance of the object to the inward force causing it to deviate from the tangential path that it would follow if the retaining string were suddenly to be broken.

From the equation expressing force as the product of mass and acceleration, i.e.:

$$F = mf,$$

it is possible to derive an expression for the centripetal force C (and hence for the tension or inertia resistance, since all three have the same magnitude) in the form:

$$C = mv^2/r$$

or:

$$C = mr\omega^2$$

The centripetal force acting inwards must be balanced (since the walls do not collapse) by an equal but opposite force acting outwards, which is, in fact, the inertia resistance of the contents to motion. In the centrifuge, where separation of a precipitate from a liquid takes place, the heavier mass of the solid particles causes a larger force to act upon them than upon the 'particles' of liquid, so that they have a larger inertia resistance. They are thus held more firmly against the extreme part of the rotating tube, which allows separation to be effected easily. In hydroextraction, both fabric and water experience inertial thrust, as before, which causes them to move towards the inner surface of the rotating cylinder.

The walls of this cylinder are, however, in the form of a perforated cage, which acts as a barrier only to the cloth. The inertia resistance of the water droplets allows them to be forced through the holes, at which point they suddenly cease to be acted upon by the centripetal force. They are flung off the cage tangentially, hit the outer wall of the machine, lose their kinetic energy on impact, and then run down the side of the wall under the influence of gravity until they reach the drain in the floor. The physical size of the cloth prevents it from passing through the holes, so that it must remain inside the cage and rapidly loses water until a certain moisture content remains in the fabric.

This stage is reached when the resistance to movement of the residual traces of water in the fabric, arising from molecular or capillary attraction, can no longer be overcome by the force applied externally on the water molecules in a manner tending to make them move outwards.

Example 5

A hydroextractor consists of a perforated shell of diameter 80 cm, mounted on a vertical spindle and rotating at 300 rev/ min. With what force will 1 kg of yam press against the inside of the dryer?

We assume that the entire mass is located at the surface of the dryer once equilibrium is reached.

The speed of rotation is:

$$\begin{aligned} \omega &= 300 \text{ rev/min} \\ &= 5 \text{ rev/s} \\ &= 10\pi \text{ rad/s} \end{aligned}$$

We then have:

$$\begin{aligned} \text{centripetal force } C &= mr\omega^2 \\ &= 1\times 0.40\ (10\pi)^2 \\ &= 394.8 \text{ N.} \end{aligned}$$

This must be balanced by inertia resistance of yarn. Hence force exerted by yarn on inner wall = 394.8N.

Practical applications

Conical pendulum

If a mass is attached by a string to a fixed point and moves around horizontally in a circular motion, the volume swept out by the locus of the string is in the form of a cone. When the object rotates, the inward centripetal force, C, is derived from the horizontal component of the tension, T, in the string, while the vertical component of T balances the force of gravity. If the speed of rotation is increased, the inward force, C, must increase to maintain circular motion, and the only way in which this can occur is for the horizontal component of T to increase. This increase will obviously take place if T is directed more nearly towards the horizontal, that is, if the radius of the circle of rotation increases. By vector analysis of the forces acting, it is a simple matter to derive an expression:

$$h = rmg/C,$$

in which r is the radius of rotation, m the mass of the object, g the gravitational acceleration, C the centripetal force, and h the height of the cone of revolution. The centripetal force may, however, also be expressed as:

$$C = mr\omega^2$$

so that:

$$h = rmg/mr\omega^2$$

or

$$h = g/\omega^2$$

Thus the height of the cone of revolution depends only on the acceleration due to gravity and the angular velocity; in particular, the length of the string and the weight of the rotating object do not affect it. A heavy bob, B, suspended by a string of length 50 cm, is rotated from a fixed point P at a rate of 50 rev/min. Find the diameter of the circle, centre O, that is the locus of the movement of the bob.

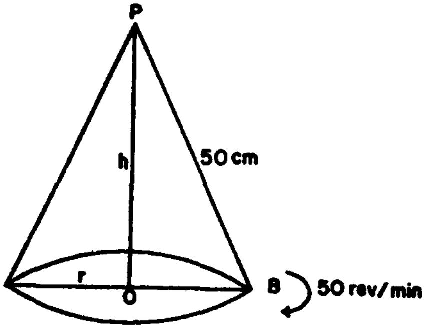

Fig. 3. Rotating-bob system

We have:

angular velocity $= 50$ rev/min

$= 5/6$ rev/s

$= 10\pi/6$ rad/s

$= 5.24$ rad/s

The conical height, h, is given by:

$$h = g/\omega^2$$

$$= 9.81/5.24^2 \text{ m}$$

$$= 0.357 \text{ m}$$

$$= 35.7 \text{ cm.}$$

By Pythagoras:

$$r^2 = l^2 - h^2,$$

where l is the string length, i.e.:

$$r^2 = 2500 - 1274.49$$

$$= 1225.51,$$

from which:

$$r = 35.01 \text{ cm.}$$

i.e., diameter of locus of bob = 70 cm.

A vertical shaft, driven from the engine, has two arms pivoted on it, each of them carrying a heavy steel ball. These balls are connected by arms to a sleeve, *S*, which operates as a metering device for the steam, gas, compressed air, or other power source energizing the mechanism. If the engine speed increases, the balls move further from the shaft, which thus lifts the sleeve to reduce the supply of fuel. Conversely, if the engine speed falls, the balls move in, the sleeve is lowered, and the fuel supply increases to restore once again the preset operating conditions.

Example 6

A governor mechanism has upper and lower arms of equal length. Ignoring frictional effects, how much will the sleeve rise if the speed changes from 50 to 52 rev/min? By geometry, the sleeve will rise twice as far as the vertical movement of the balls.

The initial angular velocity is:

$$\omega_1 = 50 \times 2\pi/60 \text{ rad/s}$$
$$= 5.24 \text{ rad/s}.$$

Hence the initial cone height, h_0, is given by:

$$h_0 = g/\omega_1^2$$

$$= \frac{9.81}{(5.24)^2} \text{ m} = 0.357 \text{ m}.$$

The final angular velocity is:

$$\omega_2 = 52 \times 2\pi/60 \text{ rad/s}$$

Hence the final cone height, h_1, is given by:

$$h_1 = \frac{9.81}{(5.24)^2} \text{ m} = 0.330 \text{ m}.$$

Thus the change in height of the balls is given by:

$$(h_1 - h_0) = (0.357 - 0.330)\text{ m}$$
$$= 2.7\text{ cm},$$

and hence the sleeve will rise 5.4 cm.

Tension in a rotating ring, belt, or thread

Textile processing involves many situations in which a flexible band is required to pass around a pulley, guide, or other circular object. In addition, there are cases in which a circular motion is inflicted on a yarn in a particular operation; the rotation of a yarn on the surface of a package, or during ballooning, springs to mind very easily. Consider a short length, x, of yarn subtending a small angle $\delta\theta$ at the centre of its circle of rotation. We assume the yarn to be moving at a speed of v in a circle of radius r. The centripetal force C, which keeps it in its circular path, must be derived from the tensions T at each end of the length x.

$$C/T = x/r.$$

If the mass per unit length is w, then the mass of the element x is wx and:

$$C = \frac{wxv^2}{r}.$$

But:

$$C = \frac{Tx}{r}.$$

Hence:

$$T = wv^2.$$

What is the tension, due to its motion in a circle, in a yarn of 30 tex on the surface of a package revolving at a surface speed of 20 m/s?

Since the mass per unit length $w = 0.00003$ kg/m and the surface speed $v = 20$ m/s, the tension in the yarn, is given by:

$$T = wv^2$$

$$= 3.10^{-5} \times 20 \times 20 \text{ mN}$$
$$= 12.0 \text{ mN}$$

This figure is obviously much lower than the tensile strength of the yarn, so that it is unlikely that yarn breakage would take place during winding as a result of the centripetal force. It should be remembered, however, that there will also be a winding tension, and the additional stress placed on the yarn by centripetal force if much higher winding speeds are used may be sufficient to cause problems in fine yarns. A metal wheel or rim, with a much higher mass per unit length, may suffer a large enough force to burst the wheel if rotation at too high a speed is attempted.

Example 7

A rope flywheel on a mill engine has a s a rim speed of 30 m/s. If the density of the metal of the rim is 7200 kg/m^3 and it has a cross-sectional area of 5 cm^2, calculate the tension in the rope caused by centripetal force in the rim. If the mass per unit length of the rope is 1.34 kg/m, calculate the additional tension caused by centripetal force in the rope.

We have:

volume of a 1-m length of rim = 5×10^{-4} m^3

Hence mass per unit length of rim is:

$$w = 7200 \times 5\ 10^{-4}\text{kg} = 3.6 \text{ kg}$$

and

tension $= wv^2$

$$= 3.60 \times 30^2\text{N}$$
$$= 3240 \text{ N}$$

This is not a dangerous stress for cast iron, but tent a fourfold increase in tension. For this relationship, so that a doubling of speed brings about reason, flywheel surface speeds rarely exceed this 30 M/S value by any great amount.

For the rope, $w = 1.34$ kg/m, and hence:

$$\text{additional tension} = 1.34 \times 30^2 \text{ N}$$
$$= 1206 \text{ N}.$$

In addition, of course, there must be extra tension in the rope for carrying out the work of driving the machinery, but the total tension must not exceed the tensile strength of the rope. For a rope of diameter 4 cm, a safe working tension might be of the order of 2500 N, but only 1294 N of this, in the example given, would be available for doing work. The remaining 1206 N would be required to change the direction of movement of the rope and hence would not be usable in pressing the rope against the pulley grooves to derive frictional grip for power transmission.

Non-circular Movements

There are two types of motion, linear and circular, occur frequently in textile operations and can be described mathematically. A third important type, which has some connexion with both of the other two, is simple harmonic motion, or S.H.M. The movement of a pendulum bob or of a tuning fork can be cited in illustration of S.H.M. In arriving at a definition of S.H.M., it is convenient to illustrate the motion by means of an example.

The disc D rotates about a pivot 0, and a peg Q, fixed near the circumference, engages a slot S, which is rigidly fastened to the arm of position *P*. The latter can move in only a vertical direction as a result of the constraining effect of the walls of the fixed cylinder C. As the wheel rotates, the vertical motion of Q causes a movement of P in the cylinder, the horizontal motion of Q merely causing it to move sideways in the slot. At the top and bottom of the circle, Q is moving instantaneously in a horizontal direction, so that the piston is stationary, but, at all other positions of Q, some vertical component of motion is present, and the piston is thus driven in an oscillating, or reciprocating, manner. This motion is the simple harmonic motion of P and is thus the projection in the vertical plane YY' of the circular motion of Q. Examples of this type of conversion from circular to reciprocating motion include the rotating crank and sliding crank pin encountered in many textile mechanisms.

The movement of a thread guide bar in reeling, o: of the shedding griffe in a Jacquard loom, are typical instances. A crank AB, driven by rotation of the bottom shaft of the loom, is connected by a flexible coupling to a long arm BC, which is flexibly coupled at its other end to the cross lever CD. At D, a third flexible coupling attaches the griffe to the cross-arm, which pivots about its centre E. As the bottom shaft rotates, C moves up and down and, since E is located at a fixed point, the griffe moves in the same manner but in reverse phase.

The movement of CD about E is actually a rotation, so that the ends each follow a curved path, but, if the length of the cross lever is considerably greater than the distance moved by its ends, this effect is small enough to be ignored. The reverse kind of conversion, where an S.H.M. is converted into circular motion, is best envisaged in the internal combustion engine. Ignition of the fuel-vapour/air mixture causes the piston to move downwards in its cylinder, the connecting rod being forced down as a result. This linear movement acts on the crank, thus forcing the crankshaft to turn in a circular manner, and this rotation is transmitted to the drive of the vehicle. The force of the initial thrust is such that rotation of the crankshaft continues after the piston has reached its lowest point, thus causing it to rise back up the cylinder to complete the cycle in readiness for the next combustion thrust.

A consideration of the factors involved in transmitting motion is deferred until a later chapter, but it is evident that the movement in each case is predictable, and fixed by the geometry of the system constraining the travel paths of the components.

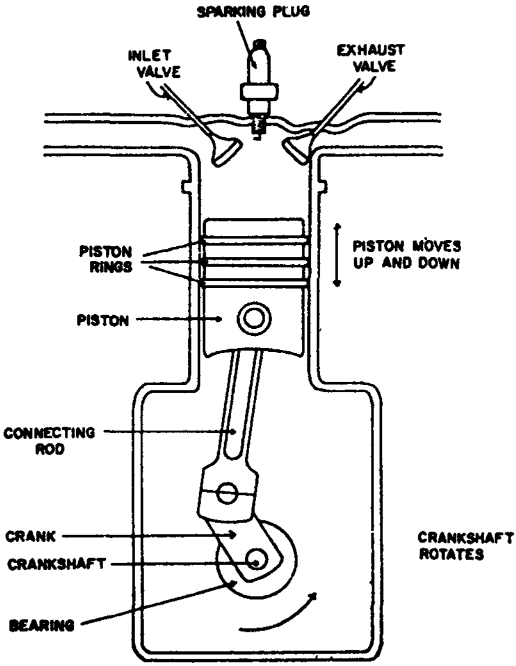

Fig. 4. The internal-combustion engine

Angular S.H.M.

The obvious example of angular S.H.M. is the vibration of a simple pendulum, a heavy weight swinging in one vertical plane at the end of a light string or rod, as illustrated in Fig. 4. If the pendulum, pivoted at 0, is released when the bob B is at the highest point, A, of its path, it will have a tendency to fall vertically downwards under the influence of gravity. Because it is restrained by the pendulum arm, however, this is impossible.

The horizontal component of the tension in the arm causes a tendency for sideways movement to take place, and so the initial motion at A is along the tangent to the are described by the radius of the arm. There is then an immediate adjustment of the tension components in the arm as a result of the consequent slight change in position, and this adjustment is a continuous one, so that the pendulum bob moves with steadily increasing speed until it reaches the lowest point, C, of its travel at a position vertically below O. It is then moving horizontally and would, if free to do so, travel ballistically forwards and downwards in a trajectory akin to that of a projectile.

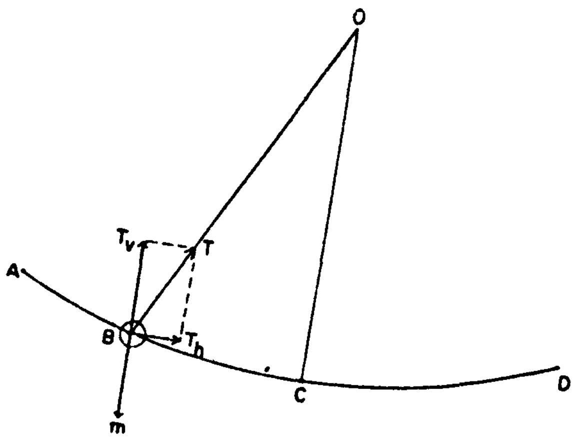

Fig. 5. The simple pendulam

Once again, however, the restraining effect of the arm prevents such a course of action. Instead, the bob continues to follow the radius are, but as a result there is now an upward component to its movement in opposition to the effect of gravity.

Thus its speed steadily decreases until it finally comes to rest at D, symmetrically opposite to A on the are and, theoretically, at the same height as A, if such factors as friction and air resistance are ignored. Oscillation in the reverse direction, in a precisely similar sequence, then takes place until the bob again comes to rest at A, when the entire cycle recommences.

If a particle moves at a constant speed around the circle, on which an arbitrary diameter AB has been drawn it is obvious that, at A and B, there is no component of velocity along the diameter while, at all other points, the instantaneous velocity vector may be resolved into components parallel and perpendicular, respectively, to AB. Examples of this type of S.H.M., in addition to the many applications of a pendulum, include the balance wheel of a clock or watch, the vibration of a magnet in a compass as it settles down to indicate the north direction, and a variety of reciprocating movements in textiles. In many of these cases, the angle moved through is Email, and the motion can be regarded as approximately linear.

A typical example of such a case is the sley motion of the power loom, which oscillates only through an angle of about 0.2 rad or less. Although the top of the sley moves in an are, the circular component of the motion can be neglected in simple loom calculations and the beating-up process can be regarded as an example of approximately linear S.H.M. back and forth into the fell of the cloth. When angular movement is appreciable, however, it is necessary to take this circular component into account in calculations or in machine design and the problem must be regarded as one in angular S.H.M.

An obvious example of this type of movement is the oscillatory motion of a doffer comb at the end of a carding sequence. The comb consists of a row of long teeth carried on a single shaft, which is in turn carried by two arms. These arms are connected to a second shaft which is driven in a rocking motion, by means of an eccentric coupling, from the card drive mechanism. This rocking motion brings about a vibration back and forth of the two arms, and hence of the shaft carrying the teeth, so that the comb vibrates rapidly near the surface of the card doffer roller, removing fibres from the latter in the process.

Damped and forced vibrations

In the oscillation of a simple pendulum just discussed, the extreme positions of vibration were assumed to be of equal height above the horizontal, and to be regained on each swing. These assumptions would only be true if no resistance movement, arising from friction at the pivot or impedance by air molecules, were present.

In practice, each oscillation would decrease slightly in amplitude from the energy imparted by moving it to its initial position of displacement had all been expended in overcoming these forces of resistance. Such a movement is known as a absence of resistance to movement. A weight W, suspended from a spiral spring S, is pulled downwards to cause the system to oscillate in a vertical vibration. The amplitude of oscillation is measured as a function of time by means of a pen, attached to the spring, and a chart moving horizontally at the side.

The same experiment is then carried out with the weight submerged in liquids, such as water, oil and glycerine in turn. As may be seen in the figure, the approximately free oscillation in air is damped more and more heavily as the viscosity of the damping liquid is increased. In all cases, the change of amplitude with time is an example of an exponential decay, but the time of decay varies from extremely long in the case of air to extremely short in the case of glycerine.

In many instances this damping effect would be a serious handicap in using a vibrating device. In order to overcome it, it is sometimes possible to impart a small pulse of kinetic energy, at the crucial instant in the cycle, to maintain the amplitude of oscillation undiminished. The resulting motion is known as a forced vibration, the most familiar example probably being the pulse imparted to the pendulum of a clock or the balance-wheel of a watch by the escapement mechanism driven by the weight or the spring, respectively.

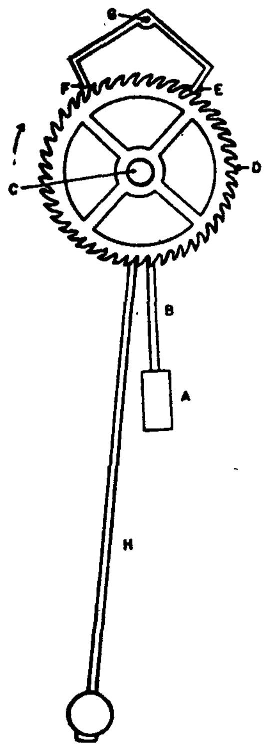

Fig. 6. The escapement mechanism of a clock

A heavy mass, A, is suspended by a flexible wire B from the drum C on the escape-wheel D. The wire makes several turns around the drum so that the mass could-drop an appreciable distance without becoming disconnected if it were free to do so. Rotation of the escape-wheel is restricted, however, by the pallets, E and F, of the anchor, G, to which the pendulum H is rigidly fastened. As the pendulum swings, it causes a rocking motion to take place in the anchor, so that the two pallets move in and out of mesh with the teeth in turn and the escape-wheel can rotate by one tooth for each swing as E is released and before F is engaged.

At the same time, the pressure exerted by the escape wheel on the pallet, arising from the load exerted on the wheel by the gravitational pull of the mass, transmits a small impulse to the pallet. This impulse is fed to the pendulum in such a phase that it tends to impart an additional push in the direction of movement, so that the oscillation is maintained undiminished and pendulum movement does not cease.

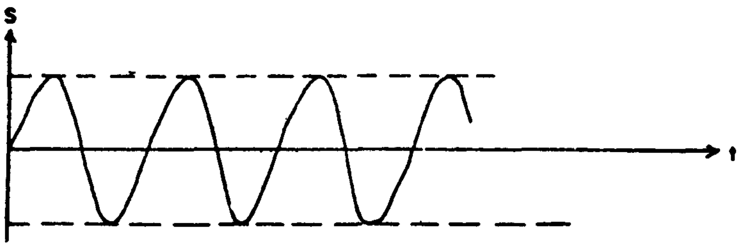

Fig. 7. The displacement-time curve for S.H.M.

In modern clocks, the tension is applied by the mainspring, rather than by a falling mass, and the oscillation of the balance-wheel replaces that of the pendulum, but the principle remains the same. In addition to S.H.M., whether free, damped, or forced, there are a large number of non-uniform, non-harmonic movements that are, nevertheless, repetitive.

The needles in a knitting frame, the ring rail of a spinning frame, the healds of a loom and the yarn guide in a winder all move in a cyclic manner, but their movement is neither linear nor simple-harmonic. It is this type of motion which is classed together as 'non-harmonic periodic vibrations', and, as will later be shown, it can, in fact, be analysed in terms of linear or harmonic components of motion. It is sufficient for the moment, however, to examine a typical such movement qualitatively by means of a displacement-time diagram.

Equations for this motion will be presented later in this chapter, but the general shape of the displacement-time curve for S.H.M. is as shown in Fig. 4, derived by applying a time abscissa to the to-and-fro movement up and down the diameter of the circle, displacement then becoming the ordinate. This kind of diagram can be plotted, as will later be shown, for any type of S.H.M. and is always characterised, so long as the motion is simple harmonic, by the sinusoidal waveform shown. In the case of a non-harmonic periodic vibration, however, the curve is no longer sinusoidal. The trumpet guide, through which the yarn passes, is moved from side to side across the width of the rotating package by means of an endless screw thread, cut in the form of a groove, around the circumference of a rotating roller on which the guide is carried.

This nut is so shaped that it can pass around the curvature of the screw groove at the ends where reversal occurs, but can not be turned so far that it can jump into the wrong groove by accident at the crossover points along the length of the grooved roller. As a result, the trumpet repeatedly moves backwards and forwards between the two ends of the roller, but travels at a uniform speed during any given traverse. The method of analysing such a motion mathematically is to be examined in due course.

Linear S.H.M.

The qualitative method of arriving at a sinusoidal shape for the displacement-time curve of a typical S.H.M. can now be examined a little more closely. Suppose that a particle moves around a circle of radius r and Centre O with a uniform angular velocity w, as in Fig. 5. As the particle moves once round the circle from A in an anticlockwise direction back to A, the foot of the perpendicular from the particle's position to the diameter BOC moves from O to B, back through O to C, then finally back once more to O. This oscillating motion of the perpendicular along the diameter BOC is S.H.M. Suppose now that, at a given instant, the particle is at D, where angle AOD is q and the foot of the perpendicular from D to OB is E.

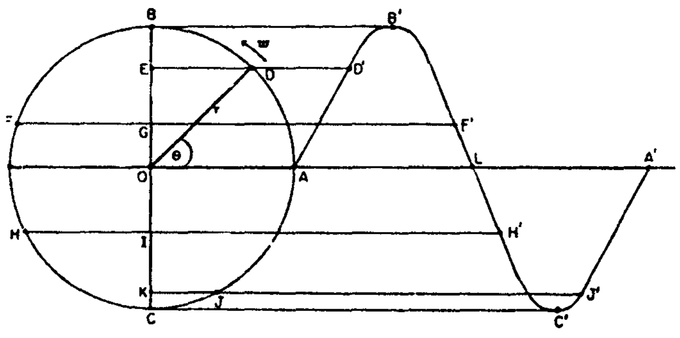

Fig. 8. The origins of S.H.M.

In a similar way, when the particle is at points F, H, and J, the foot of the perpendicular is at G, I, and K. This projection may be carried out for any number of points and, if the distance of the foot of the projection from is plotted as a function of time, it is obvious that a regular curve, alternating between the extreme values when the particle is at B and C, will be obtained.

A definition of simple harmonic motion can be obtained from a consideration of Hooke's Law, to be discussed more fully in a later volume of this work. The law states, in simple terms, that the restoring force, in any situation where a particle is held by strain away from its equilibrium position, is proportional to extension.

Thus, when a body is displaced from its equilibrium position by some constraint, then, as long as the elasticity of the system is not exceeded, the restoring force is proportional to the displacement. In consequence, the acceleration of the body is also proportional to the displacement and is directed towards the rest position; this is the fundamental relation defining S.H.M. and may be expressed briefly as:

$$f = -\phi S,$$

where f is the acceleration, S the displacement, and ϕ a constant of proportionality. The negative sign indicates that the acceleration is directed towards the Centre and that a retardation begins to act on the body immediately after it passes through the equilibrium position. Omission of the negative sign would imply that acceleration increased with distance from the equilibrium point, so that the body could never reverse its motion and return to the equilibrium position again.

The definition of simple harmonic motion derived from this treatment may thus be summarised as the motion of a particle with an acceleration that is always directed towards a fixed point and directly proportional to the distance of the particle from that point. The manner in which simple harmonic motion can be derived from circular motion may also be illustrated by the technique of producing displacement diagrams for motion of this kind. Let us suppose that a reciprocating motion in a bar is to be derived from the rotation of a crank by means of a flexible coupling. As the crank rotates, the bar moves backwards and forwards with S.H.M. and is made to move only in a straight line because the guides will not allow it to deviate from this path. The crank circle is now divided into any number of equal parts, sixteen being used in this case for convenience of construction. Each point sixteen being used in this case for convenience of construction. Each point obtained is now numbered consecutively in the direction of rotation, starting from a reference point established when the bar is at its position of nearest approach to the crankshaft. At this point, of course, the crankshaft, connecting rod, and bar are in the same straight line.

A base line is now established tangential to the crank circle at point O and thus perpendicular to the direction of movement of the reciprocating bar. Along this base line, at any convenient position, a scale is constructed, of any convenient length but with sixteen equal points numbered to correspond with the points established on the circle. Between each pair of points in the same horizontal plane a line is now drawn, parallel to the base line and extended at least to the length of the scale; point 8, of course, has a tangential line to itself because it is the only point at that distance vertically from point O. An ordinate is now constructed by drawing a line through point O on the base scale and perpendicular to the base.

The displacement diagram can now be plotted very simply by establishing the point on each horizontal line at which the position on the circle corresponds with the position on the base scale. The series of points so obtained is then joined to make a smooth continuous curve with the usual sinusoidal wave form. The horizontal axis may be calibrated in terms of time, since the distance between points Q and 16 must represent one revolution; a rotational speed of 6 rev/min has been assumed in Fig. 9.

The diagram has now become a displacement-time curve for the movement of the bar, with zero time and zero displacement being established at the instant when the bar is just about to leave the position of nearest approach to the crank circle.

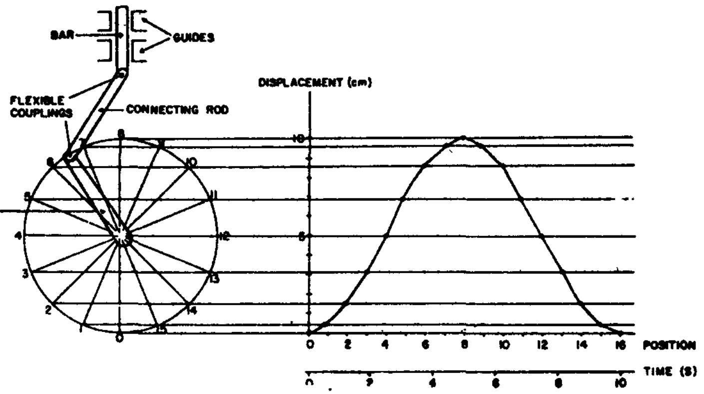

Fig. 9. Reciprocating motion derived from a crank

In practice, an error is introduced by the fact that the projection of the connecting rod on the axis line of the bar varies as a result of the rotation; this error can only be ignored if the connecting rod is very long, so that its projection along the direction of the axis does not change significantly with crank position. In certain cases, however, it is not possible to use a long connecting rod, and allowance must be made for errors of motion in machinery design by constructing an angular-displace-ment diagram.

The sleeve pinion drives the crank wheel, and the latter carries a crank that oscillates the quadrant, which in turn oscillates the quadrant pinion. Both the sleeve pinion and the quadrant pinion are mounted loosely on the driving shaft of the machine. Between the two is a clutch, fitted tightly to the shaft, which can be made to engage with either of the two pinions by means of a sliding key. When the clutch is engaged with the sleeve pinion, as in the diagram, the driving shaft rotates at a constant speed in one direction and the quadrant pinion oscillates idly on the shaft. When the clutch is moved to the left to engage with the quadrant pinion, the drive is transmitted through the crank, quadrant, and quadrant pinion, so that the driving shaft is now made to oscillate backwards and forwards through almost one complete revolution in each direction in turn.

The quadrant gear must turn through the same angle as the quadrant lever, since the two are rigidly joined, and the angle through which the quadrant pinion turns must be greater than the angular movement of the quadrant in the proportion of the radius of the quadrant gear to the radius of the quadrant pinion. It is therefore only necessary to work out the displacement diagram for the quadrant lever, and the method of doing so is one that can generally be applied to find the movement of a machine part.

The positions of the crank and quadrant centres, C and Q, are first fixed to scale, and the crank circle is drawn with subdivision into equal parts as before. With O as centre, an are is drawn to represent the path of the Centre of the pin joining the quadrant lever and connecting rod. Taking each point 0, 1, 2, etc., on the crank circle in turn, a series of points 0', 1', 2' etc., on this are is cut off in such a way that each distance 0-0, 1-1, 2-2', etc., is equal to the length of the connecting rod.

The angular displacement of the quadrant lever from its zero position Q0' can now be measured, for each position, by means of a protractor and the results plotted on a base line, as before, to give the angular displacement-time diagram shown. The deviation from the S.H.M., resulting from the shortness of the connecting rod and the curved path of the quadrant gear, can now be seen, since the quadrant displacement deviates slightly from the theoretical curve. For example, in true S.H.M., equal angular displacement would take place in each of the four quarters of a revolution, but this is obviously not so in this case, mainly because of the connecting rod size. The effect of the curved path is seen in the displacements at positions 2 and 6, which differ slightly; with a straight path they would coincide. For comparison, a true S.H.M. curve is generated as before, by using a semi-circle at the side of the displace-ment-time curve, and is shown as a dotted line. It is often useful to check in this way whether a motion that appears to be S.H.M. actually is such a movement, so that any deviation can be taken into account if necessary in machine design or use.

Oscillation in a spring

The energy stored in the stretched spring tends to make it contract and, when contract occurs, the spring overshoots the equilibrium rest point and thus becomes compressed. The energy stored in this compressed state then causes the spring to expand, and inertia again causes an overshoot of the equilibrium position, so that stretching takes place once more and the cycle can recommence. If damping is not high, the oscillation can continue for a considerable time and an analysis of the motion may be carried out. The simplest parameter to measure, possibly, is the time taken for one cycle of the oscillation, from one extreme position to the other and back again to the first; the came given to this time is the period of the oscillation, T.

The other obvious measure is the distance from either extreme position to the equilibrium rest point; in an undamped or free oscillation this distance is constant and is called the amplitude of the oscillation. The period is the time taken for the particle to make one complete revolution of the circle and is thus measured by the time between corresponding points on adjacent cycles of the oscillating curve plotted at the right. The times represented by LM, PQ and RS are thus all equal to the period of the oscillation. The amplitude is the distance from the rest point to an extreme position, so may be given as OB, OC, or the maximum height of either the positive or the negative wave above the baseline of the oscillating curve. It is possible to establish some quantitative relationship for these and other parameters of

S.H.M. by a further examination of the motion, together with some principles which have, hopefully, been learned from earlier chapters of this work.

Equations of linear S.H.M.

In the period of oscillation, a particle travels exactly once around the circle. Since the angle swept out is a full circle or 2π radians, and the angular velocity is ω rad/s, it follows that the period is given by:

$$T = \frac{2\pi}{\omega} \text{ s.}$$

The amplitude of the motion is equal to r, the radius of the circle, and we must now determine how the distance from 0 of the foot of the perpendicular varies with time. Suppose that, when the particle is at point D, the distance of E (the foot of the perpendicular from D on to BC) is y. By geometry:

$$y = r \sin \theta.$$

But, if the angular velocity is ω then the angle θ swept through in a time t, is given by:

$$\theta = \omega t$$

Thus, combining these two formulae, we have an equation for y:

$$y = r \sin \omega t$$

The oscillating motion is thus a sinusoidal one with a characteristic shape. As discussed, the period of oscillation is the time required for one cycle, the motion from a point on one particular part of one wave to the corresponding point on the next wave, to take place. The number of such cycles occurring in one second is the frequency of the oscillation in hertz, not to be confused with a certain organisation dealing with transportation.

A well-known frequency is that of standard electrical mains power, which is delivered at 50 Hz in Great Britain, at 60 Hz in North America, and at an apparently random frequency, if at all, in certain parts of the world, which shall be unnamed. It is now possible to calculate the acceleration and velocity of an object undergoing S.H.M. At the point D, the acceleration of the particle rotating in a circle is to $\omega^2 r$ and is directed along the radius DO towards O.

Hence, the acceleration at the foot of the perpendicular, E, towards 0 is given by geometry as $\omega^2 r \sin \theta$. But $r \sin \theta$ has already been defined as y. Thus the acceleration, f, at E towards O is given by:

$$f = -\omega^2 y.$$

But ω^2 is a constant, since the circular motion is uniform. Thus, the acceleration at E towards 0 is directly proportional to the distance of E from O. In order to express the fact that the acceleration is always directed towards O, we use a negative sign. The correct mathematical expression for the acceleration of a particle undergoing S.H.M. is, then:

$$f = -\omega^2 y.$$

Example 1

A thread guide-bar in a reeling machine is made to reciprocate by the method. As the crank revolves at constant speed, the crank pin slides in the slot at the end of the bar and forces it to move backwards and forwards. The movement is S.H.M. and is linear as a result of the constraining effect of the guides. If the diameter of the crank handle is 6 cm and the maximum acceleration of the guide bar is 6.5 m/s^2, calculate the speed of rotation of the crankshaft.

We know from the equation

$$f = -\omega^2 y$$

that maximum acceleration must occur when y is a maximum, since o) is a constant. In addition, the maximum value of y must obviously be the amplitude of' the S. H. M. It is apparent. That the limits of movement of the guide bar occur when the bar and the crank are in the same line, the two extreme situations occurring when the crank is on the same and the opposite sides as the bar, respectively, of the crankshaft. Thus, the total movement of the bar is 6 cm and the amplitude of the motion is 3 cm.

Then:

$$y_{max} = 3 \text{ cm}$$
$$= 0.03 \text{ m}$$

and, since:

$$f_{max} = -\omega^2 y_{max}$$

then:

$$-6.5 = -\omega^2 \times 0.03$$

i.e.:

$$\omega^2 = \frac{6.5}{0.03}$$

and:

$$\omega = 14.7 \text{ rad/s.}$$

Hence angular velocity of crankshaft = 14.7 rad/s.

By geometry, then, the velocity v of E along the diameter BOC can be obtained by vector resolution and is given by:

$$v = r\omega \cos \theta.$$

By the well-known trigonometrical relation:

$$\sin^2 \theta + \cos^2 \theta = 1$$

we can rewrite the equation of velocity as:

$$v = r\omega \sqrt{(1 - \sin^2 \theta)}.$$

However, we have seen that $y = r \sin \theta$ and thus:

$$v = r\omega \sqrt{\left(\frac{r^2 - y^2}{r^2}\right)}$$

$$= r\omega \sqrt{\left(1 - \frac{v^2}{r^2}\right)}$$

Thus

$$v = \omega\sqrt{(r^2 - y^2)}.$$

The maximum velocity, v_{max}, occurs as the particle executing S.H.M. passes through the rest point, and is thus present when y is zero.

Hence, $v_{max} = \omega r$

Example 2

In a loom producing plain-weave fabric, one heald shaft is lifted 12 cm, and the other one simultaneously lowered the same distance, in a quarter of a second. If the crankshaft turns through exactly half a revolution in the process, calculate the relative velocity of the warp threads controlled by the shafts, as they pass one another at the centre of the shed, assuming exact S.H.M. is taking place.

The crankshaft makes half a revolution in 0.25 s, i.e.:

angular velocity = 2 rev/s

= 12.57 rad/s.

The heald-shaft movement = 12 cm, i.e.:

radius of generating circle = 6 cm

= 0.06 m.

We now have:

$$v_{max} = \omega r$$
$$= 12.57 \times 0.06$$
$$\doteq 0.75 \text{ m/s}$$

Each set of warp threads has this velocity at the mid-point of' the S.H.M., which must occur at the centre of the shed.

Hence relative velocity of warp threads as they cross is 1.5 m/s.

One further useful piece of information is the fact that the maximum and average speeds are related in any S.H.M. The generating point, moving in a circular path of radius r, moves a distance of $2\pi r$ units while the point moving with S.H.M. moves a distance of $4r$, since r is also the amplitude of the motion.

Thus:

$$\frac{v_{max}}{v_{av}} = \frac{2\pi r}{4r}$$

$$= \frac{\pi}{2},$$

i.e. the ratio of maximum speed to average speed in any linear S.H.M. is equal to $\pi/2$. We now have formulae for calculating the instantaneous displacement, velocity and acceleration of a particle undergoing linear S.H.M. and for calculating the amplitude, period, frequency and maximum velocity of its motion.

One further important parameter, encountered reasonably frequently in textiles, is the energy present at a given displacement. In an oscillating spring, this property is particularly important, and this example is a convenient one to use in our calculations.

Suppose that a mass m is suspended by a spring, and reaches an equilibrium position at A where the force of gravity on m is just balanced by the upward force applied by the stretched spring. The stiffness of the spring, expressing the tension that is produced in the spring by extension of unit distance, is a N/m and the extension at the equilibrium position is d metres. At this equilibrium position, the tension in the spring must just balance the gravitational force acting on the mass, so that:

$$\sigma d = mg$$

Suppose now that the mass is displaced a further distance x metres to a new position B and then released, so that S.H.M. about A occurs between the extreme positions B and C. At B, there is an increase of σx in the tension, while at C there is a corresponding decrease of σx, since AB and AC are equal if damping is ignored. At either extreme position, therefore, there is a force of σx, acting towards the rest position, available for acceleration.

Thus the accelerating force acting on the mass is directly proportional to the displacement from the equilibrium position and, in consequence, the same must be true of the acceleration, a fact which confirms that the movement must indeed be S.H.M. As the mass rises and falls, a continuous exchange of energy is taking place, though the total energy present remains constant at all times since no external work is done by or on the system. If we take B, the lowest position of the mass, as our datum reference level, all the energy present may then be regarded as strain energy in the spring.

In the top position, the strain energy in the spring has been reduced, but has been replaced in the system by a corresponding amount of potential energy as a consequence of the elevation of the mass above the datum reference level. As the mass passes the equilibrium position, it has both potential and kinetic energy, while the spring has an amount of strain energy intermediate between the energies it possesses at the upper and lower positions.

At B, the strain energy in the spring is equal to the work done in stretching it by a distance of $(d + x)$ metres. This has already been defined as the total energy, E, of the system, which is thus given by:

$$E = \tfrac{1}{2}\ \sigma(d + x)^2$$
$$= \tfrac{1}{2}\ \sigma d^2 + \tfrac{1}{2}\sigma x^2 + \sigma dx$$

At C, when the mass has risen to its highest position, its potential energy P_c is given by:

$$P_c = mg \times 2x$$
$$= \sigma d \text{ x } 2x.$$

The strain energy in the spring, E_c is given by:

$$E_c = \tfrac{1}{2}\sigma\ (d - x)^2$$
$$= \tfrac{1}{2}\ \sigma d^2 + \tfrac{1}{2}\ \sigma x^2 - \sigma dx.$$

Thus, total energy E in system, given by $E = P_c + E_c$, is:

$$E = \sigma d \times 2x + \tfrac{1}{2}\sigma d^2 + \tfrac{1}{2}\sigma x^2 - \sigma dx$$
$$= \tfrac{1}{2}\ \sigma d^2 + \tfrac{1}{2}x^2 + \sigma dx,$$

the same value as in the bottom position.

At the equilibrium position, A, the mass is in motion. The potential energy, P_A, is given by:

$$P_A = mgx$$
$$= \sigma \text{dx, as shown earlier in this section.}$$

The strain energy, E_A, is given by:

$$E_A = \tfrac{1}{2}\sigma x^2$$

But the total energy, E, must be the sum of potential, strain and kinetic energies, and must be equal to its former value.

Thus kinetic energy, K_A, must be:

$$K_A = \frac{1}{2}\sigma x^2$$

If the velocity of the mass as it passes through A is v m/s, an alternative expression for kinetic energy is:

$$K_A = \frac{1}{2}mv^2$$

Thus:

$$\frac{1}{2} mv^2 = \frac{1}{2} \sigma x^2$$

or:

$$v^2 = \frac{\sigma x^2}{m}$$

This occurs when the velocity has its maximum value, v_{max} since the equilibrium point is at the centre of the S.H.M. Thus:

$$v_{max} = x \sqrt{(\sigma/m)}.$$

We have already shown, however, that:

$$v_{max} = \omega r,$$

where ω is the angular velocity and r is the radius of the circle from which the S.H.M. is projected. In this case, r is equal to x, the extreme displacement from equilibrium, so that the equivalence:

$$x \sqrt{(\sigma/m)} = \omega r$$

becomes:

$$x \sqrt{(\sigma/m)} = \omega x$$

or:

$$\omega = \sqrt{(\sigma/m)}.$$

The period T is the time taken for one revolution of the generating circle, so that:

$$T = 2\pi/\omega.$$

Thus:

$$T = 2\pi\sqrt{(m/\sigma)}.$$

It is clear that the period of oscillation depends only on the stiffness and the mass, and is completely independent of the amplitude of the oscillation so long as the motion remains simple harmonic in nature.

Example 3

A mass of 5 kg, suspended from a spring, extends it 25 cm. If the mass is pulled down a further 2 cm and released, find the periodic time of the motion and the highest speed of the mass.

We have:

$$\text{tension in spring} = 5 \times 9.81 \text{ N} = 49.05 \text{ N}.$$

i.e.:

$$\text{stiffness, } \sigma = 49.05 \text{ N/25 cm} = 196.2 \text{ N/m}.$$

Thus:

$$\text{period, } T = 2\pi\sqrt{(m/\sigma)} = 2\pi\sqrt{(5/196.)} = 1.00 \text{ s}.$$

Hence period of oscillation = 1 second.

Substituting in:

$$T = 2\pi/\omega$$

gives

$$\omega = 2\pi \text{ rad/s (or 1 rev/s)}.$$

The radius of the theoretical generating circle for the motion is equal to its amplitude, i.e.:

$$r = 2 \text{ cm}$$

$$= 0.02 \text{ m.}$$

Hence maximum speed (passing through equilibrium position) is 0.13 m/s.

Linear S.H.M. in circular motion

S.H.M. in a line may be represented as the projection of circular motion and it is possible to show that circular motion can be described completely, by selection of conditions, in terms of two linear S.H.M. at right angles. Suppose that a particle describes a circle of radius r m with a uniform angular velocity ω rad/s. If our reference point of zero time is taken as the particle passes point A then, after a further time t, the angle θ is equal to ωt rad. OA and OB are taken as co-ordinate axes, the opposite ends A' and B' of the perpendicular diameters representing negative values. The co-ordinates of the rotating particle P are then given by:

$$x = r \cos \theta$$
$$= r \cos (\omega/t),$$

and:

$$y = r \sin \theta$$
$$= r \sin \theta$$
$$= r \cos (\omega t - \pi/2)$$

Thus, point M moves along the x-axis AOA' with linear S.H.M. and point N moves along the y-axis BOB' with linear S.H.M. of the same amplitude and period, but differing in phase by $\pi/2$ radians (i.e., a quarter of a period). The problem may be approached from another aspect, and it is instructive to do so in order to prove the existence of the two S.H.M. by a different method. The velocity of the particle at time t is ωr m/s, at an angle ($\pi/2$ -θ) to OE and θ OB.

The acceleration of P at the same time is $\omega^2 r$ towards O, at an angle θ to OA and ($\pi/2$ -θ) to OB.

We can now resolve the vectors of displacement, velocity, and acceleration in the two directions OA and OB.

In direction OA:

$$\text{displacement } x = \text{OM}$$
$$= r \cos \theta$$
$$= x = r \cos (\omega t);$$
$$\text{velocity } v_x = -\omega r \sin (\omega t)$$
$$= -\sqrt{\{\omega^2 (r^2 - x^2)\}};$$
$$\text{acceleration } f_x = -\omega^2 r \cos \theta$$
$$= -\omega^2 x.$$

In direction OB, similarly:

$$\text{displacement } y = r \sin (\omega t);$$
$$\text{velocity } v_y = \sqrt{\{\omega^2 (r^2 - x^2)\}};$$
$$\text{acceleration } f_y = -\omega^2 y.$$

In both cases, the acceleration is proportional to the displacement from the point O and is directed towards that point; this is the fundamental feature of S.H.M. In addition, the values for displacement and velocity correspond to those already derived so that we can either regard this as further evidence of the presence of S.H.M. in each case or use it as an alternative proof of the equations of linear S.H.M. already derived.

Angular S.H.M.

The resemblances between angular and linear S.H.M. have already been mentioned, and it is now time to compare the two types of motion in more detail. The major difference, of course, is the path followed by the motion in each case, the linear S.H.M. being restricted to a straight line, and the angular one being constrained to a circular are. The similarities between the two, however, are much more noticeable than are the differences. If the upper end of the rod is pivoted at the point P and the mass is displaced to one side and then released, the system will begin to oscillate in simple harmonic motion.

Suppose that, at a given instant, the rod is inclined at an angle θ to the vertical position of equilibrium, as in the figure. The forces acting on the mass are the tension, T, in the rod and the gravitational attraction vertically downwards. If we resolve forces along the rod, we find:

$$T = mg \cos \theta.$$

At right angles to the rod, there is only the weight component, $mg \sin \theta$, in this direction, so that the system cannot be in equilibrium. This force must therefore produce an acceleration, f, towards the vertical position and, in the tangential direction, we find:

$$-mg \sin \theta = mf,$$

the negative sign being used to indicate that the force is directed towards O whereas displacement is measured positively from O and is therefore in the opposite direction. In order to demonstrate that S.H.M. is actually occurring, it is now necessary to make the assumption that the maximum displacement is small, and hence that the angle θ is small at all times in the oscillation. Under these circumstances, sin θ becomes equal to θ, in radians, and is also equal to y/l where y is the displacement of the mass from its rest position. With this assumption:

$$mf = -mg\theta = -mgy/l$$

i.e.

$$f = -(g/l)y.$$

Since g and l are both constant, it follows that the acceleration

f is proportional to the distance y from the rest position, and the oscillation is thus S.H.M. under the specified conditions.

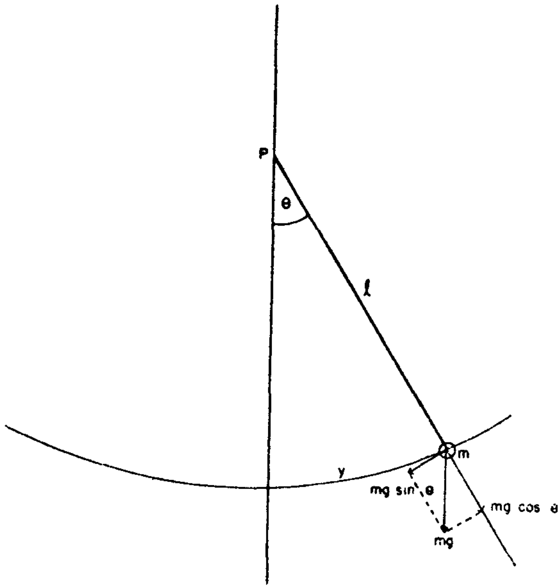

Fig. 10. Angular S.H.M. in a simple pendulam

The expression can be compared with the corresponding one in linear S.H.M., where

$$f = -\omega^2 y.$$

Combination of the two expressions gives:

$$\omega = g/l$$

and, since the period of oscillation, T, is equal to $2\pi/\omega$, we find:

$$T = 2\pi\sqrt{(l/g)}.$$

Now, we are trying to find out the length of a seconds pendulum for a grandfather clock. The seconds pendulum must swing from one side to the other in exactly one second.

Hence the period is 2 s for a complete oscillation, and substituting in:

$$T = 2\pi\sqrt{(l/g)}.$$

gives:

$$2 = 2\pi\sqrt{(l/9.81)}.$$

i. e.:

$$l = 9.81/\pi^2$$

$$= 0.994 \text{ m}.$$

Hence the seconds pendulum has a length of 99.4 cm.

For any given point on the earth or in space, g is constant, so that the period of oscillation depends only on the length of the pendulum and is, in particular, independent of the mass of the bob. This fact has been used to determine the value of g, the acceleration due to gravity, by means of timing the oscillations of a pendulum. The equation:

$$T = 2\pi\sqrt{(l/g)}.$$

may be rearranged as:

$$T^2 = \frac{4\pi^2 l}{g}$$

or:

$$g = \frac{4\pi^2 l}{T^2}$$

If the period, T, is measured for a pendulum at several different values of l a graph with T_2 as abscissa and l as ordinate can then be plotted and will have a slope proportional to g. For maximum accuracy, as wide a range of lengths as possible should be used, the average period should be calculated from the time taken for many oscillations (say, 50–100) to occur, as small an angle of oscillation as possible (say, less than 0.2 rad) should be used, and the length of the pendulum should be calculated as the distance from the point of suspension of the fine wire to the centre of the small spherical bob used as the mass.

Values of g ranging from 9.832 m/s^2 at the pole to 9.780 m/s^2 at the equator are currently accepted as accurate, and the value of 9.81 m/s^2 used in this book is the generally accepted value. The resemblance between linear and angular S.H.M. can be further seen by developing equations for the angular motion, as was done for the linear one. The formula for velocity, for example:

$$v = \omega\sqrt{(r_2 - y_2)}$$

may be rewritten, since

$$v = \pm \sqrt{\{(g/l)\ (a_2 - l_2 \sin^2 \theta)\}},$$

where a is the amplitude, l the length of the pendulum, and θ the instantaneous angular displacement.

We know that velocity is zero when the displacement is a maximum, that is, when:

$$l \sin \theta = a$$

i.e., when:

$$\theta = \sin^{-1} \{\sqrt{(a/l)}.$$

Similarly, we know that maximum velocity occurs at the mid-point of oscillation, that is, when:

$$\sin \theta = 0,$$

i.e.:

$$v_{max} = \pm a\sqrt{(g/l)}$$

the ± sign indicating that maximum velocity is attained during both directions of travel.

The kinetic energy, K, equal to ½ mv^2, is given by:

$$K = \frac{mg}{2l}(a^2 - l^2 \sin^2 \theta).$$

The total energy of the oscillating system must be experienced when the pendulum bob is at its lowest position, that is, when $\theta = 0$ and v has its maximum value.

Then total energy E is given by:

$$E = \frac{mga^2}{2l}$$

and, since total energy E is the sum of kinetic and potential energies, the potential energy P is given by

$$P = \frac{mga^2}{2l} - \frac{mg}{2l}(a^2 - l^2 \sin^2 \theta)$$

$$= mgl \sin^2 \theta.$$

Other equivalences between linear and angular S.H.M. may be derived in similar manner by the student as needed.

Exact and approximate angular S.H.M

In deriving formulae for S.H.M.. we made the assumptions that the mass was small, that the rod suspending it was light, and that the angular displacement θ was always so small that the equivalence $\theta = \sin \theta$ was an exact one. In practice, none of these three conditions normally holds true. The mass usually has an appreciable size and occupies a considerable volume, so that its effect can no longer be assumed to be concentrated at a point.

The imprecision is further compounded by the fact that the mass of the suspending rod is usually large enough to be significant, and these two factors must be taken into account. The system forms a compound pendulum, to be dealt with in a later chapter, and is handled mathematically by assuming the existence of an equivalent simple pendulum having the same period. The magnitude of the displacement, however, falls within a different category. Even in the compound pendulum, the oscillation must be small for the derived formulae to be applicable.

Any large displacement (greater than, say, about 0.2 rad) introduces into the equations an error large enough to produce inaccuracies in using them to carry out calculations. As a result, it is necessary either to restrict movement to very small oscillations or to accept the fact that all derived results are approximations only. An experiment intended to measure the value of g was carried out with a simple pendulum exactly $2m$ in length, but the oscillation was not restricted to small angles. Derive a curve showing the percentage error in the calculated value of g as a function of θ. We can assume that the correct value of g is the one obtained for the smallest oscillation.

The relationship:

$$T = 2\pi\sqrt{(l/g)}$$

may be written:

$$g = 4\pi^2 l/T^2.$$

When θ 0.05 rad, $T = 2.84$ s,

i.e.:

$$g = 4\pi^2 \times 2/2.84^2$$
$$= 9.879 \text{ m/s}^2.$$

When $\theta = 1.00$ rad, $T = 3.09$ s,

i.e.:

$$g = 4\pi^2 \times 2/3.09^2$$
$$= 8.269 \text{ m/s}^2.$$

Thus:

error in calculating g = (9.789 - 8.269)

= 1.520

and:

percentage error $\frac{1.520}{9.789}$ = × 100% = 15.33%

An exactly similar calculation may then be carried out for each value of g.

Damped and forced vibrations

We have assumed for our simple pendulum, swinging in air, that the amplitude of ,he movement, like the period, is constant. In practice, this is not so, even if all the other requirements of the simple pendulum are met. A simple pendulum, set into motion and then left to swing freely, will execute S.H.M. with an amplitude that decreases gradually until it eventually comes to a rest at the vertical equilibrium position. Since this position does not change, it is obvious that the loss in kinetic energy represented by the diminishing movement is not arising as a result of translation into potential energy.

The only logical alternative, barring the mysterious forces so beloved of -science-fiction writers, is the hypothesis that the energy is being dissipated by doing work. As a corollary to this, since we postulate an isolated system, there must be some resistance to motion on which the work is done and two sites of such resistance may readily be identified. The first of these is at the point of suspension of the pendulum. No matter how carefully the suspension of the pendulum has been designed, the fact that there is movement there means that friction is automatically present. In the worst case, this frictional component may be high, bringing the pendulum to rest quickly, and in the best case, where the finest bearings are used, a coefficient of friction of as low as perhaps 0.01 may be achieved, so that an extended time of observation is needed to notice any effect.

In either case, however, the resistance to motion is there and absorbs some of the energy contained in the oscillating system, bringing it eventually to rest. The second source of resistance is virtually omnipresent and approximately constant for a given pendulum. It is the resistance of the air to movement of the pendulum through it and, short of placing the pendulum in a vacuum, cannot be varied except by changing the shape of the moving components. Air is a mixture of gases and, as the pendulum swings through it, gas molecules are forced aside. Even though the energy required to move a molecule is so microscopically small, the large number of gas molecules that must be pushed out of the way need, in total, an appreciable amount of work, supplied from the energy of oscillation, to move them all.

As a result of the combination of these two factors, then, the pendulum swings slowly diminish if the device is left swinging freely, the time taken depending to some extent on such factors as lubrication at the suspension bearing, or aerodynamic design of the streamlining on the bob and rod. The same principle may be extended to all forms of S.H.M., whether they are linear or angular. The oscillation of it spring, or of a cylinder bobbing up and down in it liquid, of the waves on the surface of a pond where it stone hits dropped or in an electronic circuit where excitation hits occurred; all these and other examples of S.H.M. have in common with the pendulum the fact that oscillation dies away once the source of energy is removed. The phenomenon is a general one known as damping and is worth examining more closely.

Degrees of damping

Damping can be categorised, in general, into such groups as light (or underdamped), critical, and very heavy (or overdamped). For the sake of reference, these three categories are defined fairly rigorously by common agreement. Slight damping includes all the examples just quoted. In this category, oscillations are maintained, but they gradually die away at a rate dependent on external factors, as discussed.

The period of oscillation is slightly greater than would be the case for the given system if damping were not present. The diminishing amplitudes a_1, a_2, a_3, a_4, . . . at the extreme positions of successive oscillations form a geometric series such that

$$a_1/a_2 = a_2/a_3 = a_3/a_4 \ldots \delta,$$

where δ is called the decrement of the system. Critical damping occurs when the resistance to motion is just sufficient to prevent oscillation, but not so great that the return to the rest position takes an inordinately long time. One-quarter of a vibration takes place, and the period is considerably longer than would be the case in the undamped state.

In overdamping, the resistance is so great that the return to rest is delayed extensively and the period is vastly greater than would be expected in the absence of damping. As a general principle natural resistance leads to underdamping, whereas critical damping and overdamping are the result of an external application of additional resistance deliberately designed to bring about the damped state of affairs.

It should be noted that critical damping brings the motion to rest in the shortest possible time, so that instruments that involve an oscillation followed by a reading are usually designed to operate at, or near, the critically-damped, or dead-beat, point. An analytical balance, or a moving-coil electrical instrument such as a galvanometer, may be quoted as examples of devices incorporating this aspect of design.

Forced vibrations

In order to keep a system with a degree of damping in continuous oscillation, it is necessary to apply an external force in a particular way. It is obviously no use if the force is a continuous one, because there will then be either no effect or no net effect. The latter case arises where the force is in the same direction as the oscillation. If the force is continuous, the desirable acceleration which it produces in one direction will be exactly counter balanced by the undesirable deceleration produced when the force opposes the instantaneous direction of movement.

What is needed is a force that is present when it reinforces the motion and absent at a time when it would oppose movement. An attractive alternative is a force that oscillates in such a way as to bring about this state of affairs and, where such a periodic device is used, it is said to vibrate at the forcing frequency. When a system vibrates freely, it does so at a constant rate and with a fixed period of oscillation.

We have just seen, however, that one side-effect of damping is to increase the period of vibration, so that, to design a forced system operating under optimum conditions, we need to know the period that would pertain in the absence of damping. This is known as the mean free period and can be determined either by a particularly nasty calculation or by an experiment in which the damping is progressively increased, the resulting effect on the period is noted, and a graphical extrapolation to zero damping is carried out in order to obtain the period that would exist under these (theoretical) conditions. From this, it is a simple matter to calculate the natural frequency of the vibrating system and, if the same frequency is to be used, to adjust the forcing frequency to this value.

Types of forced oscillation

A system may be forced into oscillation by a periodic driving force of any frequency and will then execute vibrations with the same (or nearly the same) frequency as that of the driving force. The driving force may operate, in general, at a frequency below, equal to, or greater than, that of the driven system, and the three cases have distinctive properties. Two pendulums, A and B, are suspended by cords from a light rod PQ, which is itself also suspended by cords.

If A is the driver and is set swinging, its motion is transmitted via the coupling PQ to B, which begins to oscillate, at first, with its natural frequency. After a short time, however, B is forced to vibrate with the same frequency as A; the time needed to bring about this steady state is short if A has a much greater mass than B, and becomes longer as the difference in mass between the two is made smaller.

The three types of operating conditions, as differentiated above, are achieved by changing the relative lengths of the cords suspending the two masses. If the cord at A is longer than that at B, A is the longer pendulum and thus has the longer period (since $T = 2\pi\sqrt{(l/g)}$). In this case, the frequency of the driver is less than that of the driven. When this happens, the driven pendulum vibrates in phase (or approximately so) with the driver, with a relatively small amplitude. If the two cords are of the same length, the driver and the driven pendulums have the same frequency.

Under these conditions, the amplitude of the forced vibration is very great and the situation is known as a *resonance*. The driven pendulum vibrates a quarter of a period (i.e., $\pi/2$ in phase difference) behind the driver. Finally, if the cord at A is shorter than that at B, A has the shorter period and the driver vibrates at a higher frequency than the driven. In this case, the driven pendulum vibrates with a relatively small amplitude (or has a phase difference of

π) behind the driver. The changes in behaviour are best summarized by means of two diagrams. The increased rate of change of the dependent variable around the resonance point can be clearly seen in both diagrams when the system is lightly damped. It is, of course, possible to introduce damping into a situation of forced oscillation, and the effect of doing so is shown in both diagrams. Damping reduces the amplitude of the forced vibration at all frequencies.

The amplitude attained under steady-state conditions of forced vibration represents an equilibrium in which dissipation of energy by the driven and supply of energy from the driver are balanced. It should be noted, in particular, that amplitude reduction is greatest at resonance, so that the sharpness of this phenomenon is reduced. In addition, the phase difference changes more slowly from its resonance value of $\pi/2$ as the frequency difference increases either way, so that the net effect of damping is to produce a smaller response by the driven pendulum over a wide range of frequencies, rather than it noticeably large response over a relatively narrow frequency band. This general statement is applicable to all kinds of forced vibration, whether referring to molecular bonds, electronic circuits, or simple mechanical systems such as the one we used to explain the phenomena occurring in forced oscillations.

'Non-harmonic' periodic vibrations

A non-harmonic periodic vibration is one that repeats at a regular interval but cannot be described by an equation of S.H.M. An oscillograph of the noise emanating from the rollers of a drafting machine; the vibration must be repeated, or the oscilloscope would not yield a clear image, but the curve is obviously not sinusoidal in nature and therefore cannot be classed as S.H.M. It is somewhat startling to realize that, despite the attention paid to then., vibrations of true

S.H.M. are almost non-existent it) nature. Almost without exception, each oscillation nominally described as S.H.M. suffers some defect that distorts it to such an extent that it is no longer a pure sinusoidal one. Indeed, the difficulty of obtaining a pure sine wave can be appreciated merely by examining the specification sheet for an electronic instrument intended to generate tones.

Despite unbelievably high cost, there is always a residual error in the purity of the sine wave generated by the equipment, this error naturally decreasing as the cost soars towards astronomical figures. As a distorted sine wave is strictly classed as a non-harmonic periodic vibration, it will be realized that such vibrations are, in fact, omnipresent.

The single note from the virtuoso playing on the world's finest specimen of the musical instrument with the purest tone is non-harmonic, even on a good day. All other noises, of course, fall far behind this one in quality, so that every sound we hear and every vibration we see or feel must of necessity be non-harmonic. The expression is, however, something of a misnomer, since it is possible by mathematical means to introduce a harmonic nature to all vibrations, even the ones that, like grandfather's snoring, seem to have nothing remotely harmonic about them.

Resolution of non-harmonic vibrations,

Let us consider for a moment the case of a body executing two S.H.M. vibrations simultaneously in the same straight line. The body may be a pendulum with two forced vibrations impressed on it, a vibrating string in a musical instrument, or merely the medium in which a disturbance is produced by electronic or acoustic energy. Whatever the case, we can describe each of the vibrations mathematically and picture them as sine waves. For simplicity in the next step, let us assume that one oscillation has exactly three times the wavelength of the other, that the two are exactly in phase at the reference time chosen as zero on our time scale, and that they have the same amplitude. At any instant in time, then, the body is subjected to both vibrations simultaneously.

The easiest way to determine the net effect is to note the amplitude that is imparted by each at a given instant, and to add the two amplitudes together in order to find the net amplitude of the overall vibration at that instant. Obviously, account has to be taken of the algebraic sign of an amplitude so that, if the two waves are out of phase at the instant of measurement.

The converse process can, of course be carried out on this wave, in order to separate it into two distinct, individual, S.H.M. components. By a mathematical treatment, which need not concern us in this book, this harmonic analysis can be carried out for a waveform of any degree of complexity, so long as it is periodic, and it can thereby be separated into individual sinusoidal components. Thus, a more descriptive term than the conventional one for non-har-monic periodic functions would be multiple-harmonic periodic functions; traditionally, however, the former nomenclature is the one used. The process's of resolution is useful in identifying, for example, noisy components in a textile mechanism.

The sound emitted by the mechanism is received by a microphone, amplified, and passed to a wave analyser, an electronic instrument that carries out harmonic analysis and measures the amplitude of each specified narrow band of frequencies present in the noise source. By setting the instrument to carry out a continuous sweep across the entire audio-frequency spectrum, and by careful selection of filters to ensure that a very small bandwidth is used, it is possible to identify very quickly those frequencies at which troublesome noise is emitted. Simple calculations, based on a knowledge of shaft speeds, gearing and so on, then enable the components giving rise to the noise at any specific frequency to be identified.

Equations for periodic vibrations

We have already seen in Section 8.2.3 that an equation for S.H.M. can be written:

$$y = r \sin \omega t$$

where y is the distance from the rest point at time t, r is the maximum displacement, and ω is the angular velocity of the circular motion theoretically generating the S.H.M. It can be shown mathematically, by a proof known as Fourier's theorem, that any periodic vibration that repeats regularly can be represented by a series of simple-harmonic terms with frequencies that are integral multiples of that of the original function.

This means that, if a vibration is repeated at a frequency of n times per second, we can write it down as the resultant of a set of S.H.M. vibrations of frequencies 0, n, $2n$, $3n$, $4n$, . . . where the angular velocity ω is equal to $2\pi n$. If we take the zero-frequency (i.e. constant) term as a reference, we can assign a phase lag ε_1, ε_2, ε_3. . . to each subsequent term so that, if the beginning of the zero-frequency signal is taken as zero on the time scale, each of the following terms begins at a time ε_1, ε_2, ε_3. . . etc. later. Thus, the equation of the first harmonic term is then

$$y = r_1 \sin (2\pi nt + \varepsilon_1).$$

and, in subsequent terms, n increases by an integer and the phase angle changes. The complete general equation for any non-harmonic periodic vibration can then be written as:

$$y = r_0$$

$$+ r_1 \sin(2\pi nt + \varepsilon_1)$$
$$+ r_2 \sin(2\pi nt + \varepsilon_2)$$
$$+ r_3 \sin(2\pi nt + \varepsilon_3)$$
$$+ r_4 \sin(2\pi nt + \varepsilon_4)$$
$$+ \ldots\ldots\ldots\ldots$$

A standard mathematical transformation enables us to express each of the phase angles c as a function of the frequency n, so that the expression may be rewritten as:

$$y = r_0$$
$$+ r_1 \sin(2\pi.nt) + b_1 \cos(2\pi.nt)$$
$$+ r_2 \sin(2\pi.nt) + b_2 \cos(2\pi.nt)$$
$$+ r_3 \sin(2\pi.nt) + b_3 \cos(2\pi.nt)$$
$$+ r_4 \sin(2\pi.nt) + b_4 \cos(2\pi.nt)$$
$$+ \ldots\ldots\ldots$$

and this equation is applicable to all periodic vibrations.

The series is an infinite one and it is naturally not necessary for all terms to be included for a specific waveform. If the oscillation occurs symmetrically about the x-axis (i.e., $y = 0$ at the rest position), for example, then the coefficient r_0 becomes zero. If in addition the wave is sinusoidal, then all coefficients apart from r_1 wid b_1 become zero and the equation simplifies to:

$$y = r_1 \sin 2\pi nt,$$

the cosine term usually being omitted because we assume the oscillation starts at zero time. There is no phase difference, so that all the b coefficients become zero. The frequency of one wave is three times that of the other, so that only the terms with n and $3n$ remain. Finally, the two components have the same amplitude, and y is zero at the rest point, so that the equation reduces to:

$$y = + r_1 \{\sin (2\pi nt) + \sin (2\pi.3nt)$$

There are several types of oscillation that are worth noting because they occur regularly in various applications. Typical of these are the square wave, which represents a periodic reversal or on-off switching, the 'saw-tooth' wave, in which a slow uniform change is succeeded by a rapid recovery to the beginning of the slow uniform change, and the triangular *wave*, in which a slow uniform change in one direction is succeeded by a change at the same rate but in the reverse direction. The square wave, associated with all switching operations, contains all the term of odd values of n, exactly in phase, r being proportional to 1/n. The equation is therefore:

$$y = r \{\sin (2\pi.nt) + 1/3 \sin (2\pi.3nt) + 1/5 \sin (2\pi.5nt + ...\}.$$

The 'saw-tooth' wave, used in television applications, contains all the sine terms, again with no phase difference and again with r proportional to 1/n. It is thus written:

$$y = r \{\sin (2\pi.nt) + \tfrac{1}{2} \sin (2\pi.2nt) + 1/3 \sin (2\pi.3nt + ...\}.$$

The triangular wave, of importance in the theory of vibrating strings, is represented by the series of all the odd cosine terms with r proportional to $1/n^2$ and is expressed as:

$$y = r \{\cos (2\pi.nt) + 1/3^2 \cos (2\pi.3nt) + 1/5^2 \cos (2\pi.5nt + ...\}.$$

Thus, no matter how complex a wave, it is always possible to analyse it into its harmonic content and to assign to it an equation containing only terms representing oscillations of a pure sinusoidal waveform.

Six
Transmission of Motion

The essential features of a belt drive include first the properties of the belt itself. It must be continuous, flexible, and, if possible, seamless, so that no disruption or jerkiness in driving occurs as the seam passes over any part of the driven machinery. Strength is also another vital feature; the ability of a belt to transmit power depends on the existence of tension in the belt, and it must therefore be strong enough to with stand the resulting forces imposed on it. The transmission of power occurs by friction, so that suitable surface characteristics are necessary in the belt to ensure, as far as possible, that slip does not occur too readily.

Finally, the fact that the drive is an endless one means that changes in direction must occur at two or more places in the drive. These changes are brought about by the use of pulleys or guides, and these in turn are subject to certain requirements. The pulley should be free to rotate easily, usually with the aid of bearings having very low frictional resistance to rotation, as will be described later in this chapter.

The material of which the pulley is made, usually a metal or plastics, should be chosen to ensure long life, with no mechanical or thermal damage in use, and should have surface features compatible with the aim of high frictional contact. A belt guide, on the other hand, is usually required to be stationary, either to bring about a change of direction or to prevent unwanted sideways movement of the belt during operation. The forked stick used in transferring a belt between loose and driven pulleys is an obvious instance of such an application. At the point of contact between belt and guide, friction is likely to be present, with a resulting increase in wear, power loss, and heating in the system.

The guide material, and particularly the surface, must be selected with a view to minimizing these frictional effects. The notable advantages of a belt drive are simplicity, cheapness, and versatility. A wide range of belt lengths and types is available, so that a drive system can normally be devised easily for any machine. The fact that precision components are not needed, as in other drives, also reduces costs considerably. The use of different sizes of pulleys, belts, and guides means, of course, that an endless variety of combinations can be achieved and ensures that a drive can be obtained with any desired shaft separation, which thus gives greater freedom to machine designers. The major disadvantages include slip, wear, stretch, and the problems of non-planar operation.

Slip can reduce drastically the power that a belt drive transmits, and wear or stretch in the belt changes its length, and hence the tension and power-transfer capability. In order to combat this change, some means of adjusting the length of the belt path, typically by the use of adjustable pulley-positioning is usually incorporated in a drive. A belt system is also essentially two-dimensional in nature. Whereas a slight amount of pulley offset can be tolerated, possibly with the aid of belt guides, a large amount of sideways displacement is not permissible, and an additional belt drive, with a common shaft between the two drives, is necessary where a lateral transfer must be used.

The line shaft, turned by a drive motor at some convenient point, is a case in point. Each machine driven from it requires a separate pulley, fitted to the shaft in such a way that the drive belt does not have to suffer sideways distortion. Two drives that are not operating in planes parallel to one another cannot be interconnected by the use of belt-and-pulley systems alone.

In carrying out calculations involving belt drives, a simple formula, if belt thickness and slip are ignored, may readily be derived. If a pulley of diameter d_1 m is rotating at a speed of n_1 rad/s. A second pulley, of diameter d_2 in and speed n_2 rad/s, will similarly have a surface speed of $\frac{1}{2}d_2n_2$ m/s.

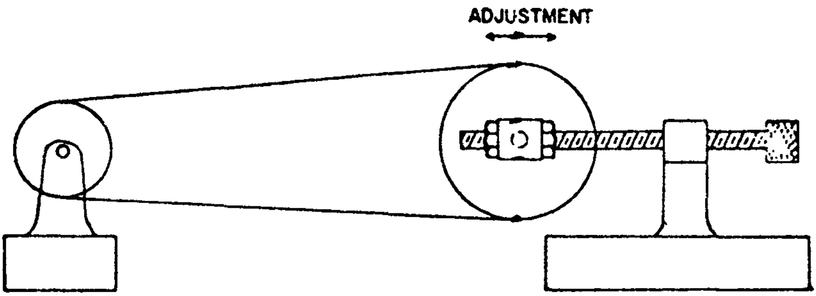

Fig. 1. Belt-drive operating conditions

If the two pulleys are connected in a belt drive, then their surface speed must be equal, so that:

$$d_1n_1 = d_2n_2$$

from which any one of the four quantities can be calculated if the other three are known, and from which it is also obvious that the angular speed of a pulley in a belt drive is inversely proportional to its diameter.

Example 1

A machine is driven by a belt from an overhead line shaft rotating at 140 rev/min. The line-shaft and machine pulleys have diameters of 40 and 30 cm, respectively. We can find out the angular speed of the machine pulley in rad/s.

We have:

$$\text{line-shaft speed} = 140 \text{ rev/min}$$
$$= \frac{140}{60} 2\pi \text{ rsd/s}$$
$$= 14.66 \text{ rad/s;}$$

$$\text{diameter of line-shaft pulley} = 40 \text{ cm}$$
$$= 0.40 \text{ m;}$$

$$\text{diameter of machine pulley} = 30 \text{ cm}$$
$$= 0.30 \text{ m.}$$

Since:

$$d_1 n_1 = d_2 n_2$$
$$0.40 \times 14.66 = 0.30 \times n_2,$$

from which:

$$n_2 = \frac{0.4 \times 14.66}{0.3}$$
$$= 19.55.$$

Hence machine-pulley rotational speed = 19.6 rad/s.

there is no need to convert diameters into metres, as is done in this example. The student is advised, however, to acquire the habit of using SI units regularly so that errors of magnitude arising from mistakes in using multiplying prefixes with units do not occur. When the belt bends round the pulley, the outer surface, AB, stretches, while the inner one, CD, contracts by a similar extent. The centre layer, shown as a dotted path, remains unchanged in length, so that it represents the true length of belt that should be used in calculations. Thus, the true diameter of are is the pulley diameter plus one belt thickness, and this quantity should, strictly speaking, be used in calculations.

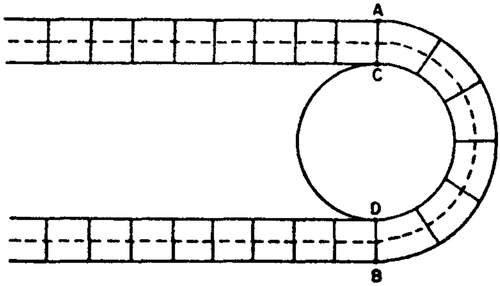

Fig. 2. Effect of thickness in belt drives

For large pulleys and relatively thin belts, however, the difference is not significant, especially when it is realized that a belt drive is dependent on friction and that all calculations are therefore necessarily approximate. Belt slip occurs even in the best drive systems and can range between about 1% and 5%, depending on such factors as load, surface conditions, and belt tension, before it becomes too large to be tolerated. One obvious last point is the fact that a belt drive is completely reversible; the functions of driven and driving pulleys are inter-changeable, and either direction of rotation may be used. Modifications of the single belt drive, as already discussed, include multiple-belt and V-belt systems.

Rope drives, sometimes used in place of belt ones, also operate in a similar manner, but it is often more difficult to estimate the effective pulley diameter. For circular ropes, the effective pulley diameter must be measured as the distance from the centre of the shaft to the centre of the rope, whereas for V-section ropes the part of the section that is neither compressed nor stretched should be taken as the reference point.

The compressible nature of rope often makes these estimates difficult, so that, if possible, the effective pulley circumference should be found expe-rimentally by measuring the distance moved by a point on the rope, along the straight path between pulleys, for one revolution of each of the pulleys. This measurement is simplified if a marker of some kind is fastened to the rope and a pointer is used to note the initial position of this marker.

Example 2

A loom is driven by a V-rope drive from a motor running at 960 rev/min. The effective pulley diameters when the rope is new are 8 cm and 40 cm for motor and loom, respectively. After wear, however, the rope sinks in by 1.5 mm, and slip (which was not present intially) becomes 5%. We can calculate the percentage change in running speed of the loom-drive shaft.

We have:

angular speed of motor pulley = 960 rev/min

= 16 rev/s

= 100.5 rad/s.

Hence:

initial driven speed of loom shaft $= 100.5 \times \frac{0.08}{0.40}$

= 20.1 rad/s

After wear, the rope sinks in by 1.5 min at each side, i.e., the pulley diameter decreases by 3 min in each case.

Thus, new diameters are 0.077 in and 0.397 in, respectively.

In addition, only 95% of the power is transmitted, since 5% is lost in slip.

Thus:

$$\text{loom speed} = 100.5 \times \frac{0.077}{0.397} \times \frac{95}{100}$$
$$= 18.5 \text{ rad/s.}$$

Hence:

$$\text{loss in running speed} = 1.6 \text{ rad/s}$$

i.e.:

$$\text{change in speed} = \frac{1.6}{20.1} \times 100\%$$

Hence running speed decreases by 8.0%.

Chain drives

The other major type of simple drive commonly compared with the belt-type system is the chain drive. In this, the endless 'belt' consists of a large number of chain links joined continuously together, and the pulleys are replaced by sprocket wheels with pointed teeth, designed to engage the links of the chain around their circumference. The bicycle chain drive is the obvious example of such a system, and its limited flexibility is obvious to anyone who has ever wrestled with a bicycle chain after removing it from the machine.

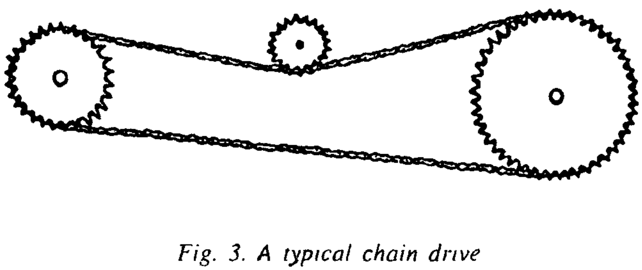

Fig. 3. A typical chain drive

In correct operation, however, flexibility is adequate to permit a change of direction unless an attempt is made to use too small a sprocket diameter for the gauge of chain chosen. The major difference between belt and chain drives lies in the mechanism of power transfer. Frictional transfer is no longer needed, since the tension in the chain is maintained by the force applied between the link and the tooth in contact it. As a result, tension can be lower, the only requirement being that the chain must not become so loose that it slips from the sprockets. Apart from this abnormal eventuality, slip cannot occur. The simplicity and versatility of belt drives are also present in this case, because a variety of chain and sprocket types or sizes is readily available.

There is, however, a significantly increased cost associated with chain drives. Not only does the cutting of a sprocket wheel require more time than the casting of a pulley, but a reasonable precision of workmanship is also needed to ensure that the teeth and links mesh smoothly. The drive is again completely reversible and suffers once more from the disadvantage of being usable only in a virtually planar configuration. It has, however, one important additional property not present in a belt drive.

If the teeth are cut suitably, the sprocket can be made to mesh with a cog wheel ill a gear train, to be discussed in the next section, so that an additional factor in drive design is available to the engineer planning a machine. Calculations involving chain drives are possibly somewhat simpler than is the case with a belt drive. Sprockets must engage with chain links means that each link must have precisely the same length,' each sprocket tooth must be of the same size as all others, and there must be an exact whole number of teeth around every sprocket. As a result, the speed at which the chain moves is controlled more precisely, and, since the tooth-spacing is the same for both sprockets of a chain, the number of teeth on a given sprocket is exactly proportional to its diameter.

Example 3

One part of the mechanism for driving the drafting rollers in a spinning frame contains a chain drive operated by a pulley of 25.0 cm diameter rotating at 100 rev/min. The mean link K length of the chain is 8.45 mm, and the output shaft is required to rotate at 13 rad/s. How many teeth must the sprocket on this shaft have?

We have:

pulley diameter = 0.250 m,

so that:

circumference = 0.786 m.

Since tooth-spacing must be equal to mean link length:

tooth-spacing = 0.00845 m,

and hence:

$$\text{number of teeth} = \frac{0.786}{0.00845}$$

= 93, to nearest integer.

Now:

input speed = 100 rev/min

= 10.47 rad/s.

Substituting in:

$$n_1 d_1 = n_2 d_2$$

and noting that the diameter is proportional to the number of teeth gives:

$$10.47 \times 93 = 13 \times d_2,$$

i.e.:

$$d_2 = \frac{10.47 \times 93}{13}$$

$$= 74.90.$$

Hence driven sprocket must have 75 teeth.

Meshing drives

Gears

In meshing drives, as long as operating conditions are satisfactory, the drive is always positive; that is, as long as the driving component has sufficient force available to move, the driven component must also move even if an attempt to prevent movement is made. Resistance to movement of the driven component can only result in either complete stoppage of the entire motion or breakdown of some part of the drive.

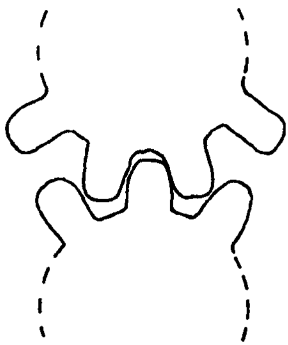

Fig. 4. A typical gear drive

Each cog wheel is cut in a toothed pattern around its circumference, teeth and gaps being cut exactly to the correct size so that each tooth of one pinion fits precisely into a gap of the other. A tooth and a gap are not exactly equal in size; the gap is a slight but significant amount larger so that the meshing can take place without jamming, as will be discussed in connexion with backlash during a subsequent section of this chapter. The reason for using the rounded shape for a tooth section will also become clear. At present, it is sufficient to understand that any rotational movement of one wheel produces a corresponding and predictable movement in the other, so that power applied to the shaft of one wheel is automatically transferred via the meshing to the shaft of the other.

The principle of operation of a gear train is summarized by this basic fact. As one pinion rotates, one of its teeth moves into a gap of the other and exerts a sideways pressure on the adjacent tooth. Rotation occurs, and the design of the system is such that a second tooth of the first pinion begins to mesh with a gap of the second one before contact at the initial position has been broken.

The materials used in gear construction have traditionally been of metal, case-hardened steel or cast iron, depending on the application, being most commonly employed. The noise problem associated with these gears, to be discussed in due course, has recently led to the introduction of new alloy or plastics components in gear trains. The performance of these materials compares favourably with that of the ferrous metals in such matters as power-handling capability and wear life, so that their use may be expected to increase in the future. An obvious advantage of a gear train lies in the positive nature of the drive.

There is no possibility of frictional slip or of chain breakage, as in a simple drive, so that energy transfer is regulated in a predictable and inevitable manner. The absence of stretching, too, means that, once correct meshing conditions have been established, no further adjustment to the system is required throughout the useful life of the gear pinions. Against these advantages, however, some disadvantages have to be accepted.

The first of these lies in the establishment of correct meshing. As will later be recounted, the use of too tight or too loose a fit between meshing components leads to various operating problems, so that the correct placement of pinions is quite critical. This in turn leads to a requirement for high precision in manufacturing the gears and such associated components as shafts or bearings, a factor naturally reflected in higher costs. Problems of wear, friction, and noise are also prevalent; again, their origins will be discussed in due course. To alleviate some of these problems, several variations of the basic gear wheel have been designed, the most important one being the spiral gear.

When meshing between normal cogs takes place, the entire width of one tooth comes simultaneously into contact at all parts, in theory, with the entire width of the mating tooth in the second pinion. In a spiral-gear system, however, the tooth orientation is at an angle to the axis of the wheel. A particular feature of the spiral gear is its ability to change the direction of drive. The most frequent, arrangement has the two shafts at right angles again, but other orientations are possible by suitable tooth-cutting.

If the wheels are of the same size, they are known as mitre wheels, and, if the shaft centre lines are in different planes, as is often the case in spindle drives, the wheels are known as skew bevels. A combination of spiral and bevel gears, going by the name of spiral bevel, is also used and, like the spiral gears, is much quieter in operation as a result of the rolling action during contact meshing prevailing with normal gears.

As with the chain drive, the diameter of a gear wheel is proportional to the number of teeth around its circumference in any particular drive, since identical tooth-spacing is needed on all meshing cogs in the system for successful transmission of power. When two shafts are connected by a gear train, the speeds are in inverse proportion to the numbers of teeth in the two wheels connected respectively to them. Where a more complex arrangement is needed, each meshing pair can be considered in turn, and a general expression can be derived as:

$$\omega_2 = \frac{N_1}{N_2}\omega_1,$$

where ω_1 and ω_2 are the angular speeds of the driver and driven shaft, respectively, and N_1 and N_2 are the products of the numbers of teeth in the driver and driven wheels respectively. The input to the gear train is at cog A, which is operating at ω_A rad/s.

The cog C of the compound carrier, composed of two cogs C and D on a common shaft, is driven through the intermediate sprocket B, and D in turn drives the output shaft attached to cog F through the intermediate gear E. Thus, A and D are drivers, B and E are intermediate carriers, and C and F are driven.

For the mesh between A and B:

$$\omega_B = \frac{N_D}{N_F}\omega_A$$

Similarly:

$$\omega_C = \frac{N_B}{N_C}\omega_{B.}$$

i.e.:

$$\omega_C = \frac{N_A \times N_B}{N_B \times N_C}$$

$$= \frac{N_A}{N_C}$$

In like manner, we can show that:

$$\omega_F = \omega_D.$$

But $\omega_C = \omega_D$, since they are on a common shaft, and hence:

$$\omega_F = \frac{N_A N_B}{N_C N_F} \omega_A,$$

a result that follows the general rule given above.

Example 4

A typical arrangement of the gearing between the crankshaft and the tappetshaft of a loom. One pick is put into the cloth for each revolution that the crankshaft makes, so that the number of picks per pattern repeat is given by the ratio of the speed of the crankshaft to that of the tappetshaft. For two picks per repeat, the tappets are placed directly on the bottom shaft, but for three, four, and five picks per repeat, gear pairs AB, CD, and EF, respectively, are engaged. If sprocket A has 28 teeth, we can find out the number of teeth required for each of the other five cogs.

we have:

speed of crankshaft = 180 rev/min

so that:

$$\text{speed of bottom shaft, } \omega_1 = 180 \times \frac{30}{60} \text{ rev/min}$$
$$= 90 \text{ rev/min}$$

For three picks per repeat, *A* and *B* are in mesh and:

$$\text{speed of tappetshaft, } \omega_2 = \frac{180}{3}$$
$$= 60 \text{ rev/min.}$$

Substituting in:

$$\omega_2 = \frac{N_1}{N_2} \times \omega_1$$

gives:

$$60 = \frac{28}{N_2} \times 90$$

i.e.,

$$N_2 = \frac{28 \times 90}{60} = 42.$$

Hence cog B must have 42 teeth.

For four picks per repeat, C and D are in mesh and:

$$\text{speed of tappetshaft} = \frac{180}{4}$$

$$= 45 \text{ rev/min.}$$

Thus, ratio of gears C and D must be 45:90, or 1:2.

But the distance between shaft centres is fixed, so the sum of the diameters of each pair of wheels must be the same.

If we assume that all gear wheels are cut to the same tooth pattern, as is usual, it follows that, since the number of teeth is proportional to the diameter, the sum of the number of teeth in each pair must also be the same. Thus, since:

$$N_A + N_B = 28 + 42$$

$$= 70$$

then:

$$N_C + N_D = 70,$$

but:

$$N_D = 2N_C$$

and hence:

$$3N_C = 70$$

Thus:

NC = 23, to the nearest integer,

and:

$$N_D = 46.$$

$N_C + N_D = 69$ rather than 70 will give a slightly loose mesh, but the difference will not be critical and is within the tolerance of normal drive conditions. Thus, cogs C and D must have 23 and 46 teeth, respectively.

For five picks per repeat, E and F are in, mesh and:

$$\text{speed of tappetshaft} = \frac{180}{5}$$

$$= 36 \text{ rev/min}$$

Thus:

$$N_E : N_F = 36{:}90 \text{ or } 2{:}5.$$

As before:

$$N_E + N_F = 70.$$

and, since:

$$2N_F = 5N_E,$$

then:

$$7N_E = 140,$$

i.e.:

$$N_E = 20,$$

and:

$$N_F = 50.$$

Thus, cogs E and F must have 20 and 50 teeth, respectively. The speed of a given part must often be changed in order to produce some difference in the material being processed. Obvious examples are the change wheels used for altering draft or twist in a spinning frame, for varying take-up in a loom, or for changing the number of coils per traverse of yarn guide in a winding machine. In such cases, it is customary to calculate a gearing constant in order to simplify the selection of the correct sprocket for any particular set of operating conditions. This constant takes into account all meshing other than the one in which the change pinion is involved, so that a repetitive calculation is avoided as far as possible. Try to can find out the draft constant and the range of drafts available on the machine, expressed as front, intermediate, back, and total drafts. For any pair of rollers, the draft is given by the ratio of the surface speeds. III the front zone, diameter of front roller = 2.54 cm, i.e.:

$$\text{circumference} = 2.54\pi \text{ cm.}$$

Similarly, circumference of second roller = 2.46π cm.

The ratio of surface speeds can be calculated from circumference and gear teeth. Thus:

$$\text{front draft} = \frac{2.54\pi \times 66 \times 66 \times 60 \times 40}{2.46\pi \times 24 \times 24 \times N_A \times 25}$$

$$= \frac{749.6}{N_A}$$

In the intermediate zone, similarly:

$$\text{intermediate draft} = \frac{2.46\pi \times 36 \times 41}{2.54\pi \times 34 \times 36} = 1.17$$

In the back drafting zone:

$$\text{back draft} = \frac{2.54\pi \times 70 \times 60}{2.78\pi \times 20 \times N_C} = \frac{191.9}{N_C}.$$

The total draft is given by the Product of the three zone drafts. Thus:

$$\text{total draft} = \frac{749.6 \times 1.17 \times 191.9}{N_A N_C}$$

$$= \frac{168302.4}{N_A N_C}$$

In each case, the maximum draft is obtained when the cog with the fewest teeth is used, and the minimum draft is obtained when the one with the most teeth is used. Thus:

i.e.: range of front draft is from $\frac{749.6}{93}$ to $\frac{749.6}{25}$

from 8.06 to 29.98;

i.e.: range of back draft is from $\frac{191.9}{64}$ to $\frac{191.9}{21}$

from 3.00 to 9.14;

range of total draft is from $\frac{168302.4}{93 \times 94}$ to $\frac{168302.4}{21 \times 25}$

i.e.:

28:28 to 320.58

Worms

This is essentially a screw, cut in a helical pattern along the length of a shaft. The ridge of the screw is cut in such a way that the cross-section of the thread is capable of meshing with the teeth of a suitable gear wheel. The drive is unidirectional, in that the worm can rotate to drive the wheel but the wheel cannot be made to drive the worm because the thrust of rotation is almost at right angles to the direction of movement of the ridges of the worm.

The worm is prevented from moving sideways by collars on the driving shaft, and, as it rotates, the pressure of the inclined plane that constitutes the worm acts sideways on a t ooth of the gear wheel to cause the latter to turn slowly. In one revolution of the worm in the direction shown, tooth A must remain in the groove until, at the end of the revolution, it will have moved to position B.

Thus, one revolution of the worm moves the worm wheel one tooth, and, if the wheel has 100 teeth, 100 revolutions of the worm are required to produce one revolution of the wheel. In calculations, this worm drive may therefore be regarded as a driving wheel with one tooth. Worms are sometimes cut with a double or triple screw thread, an example of a double worm being. In this case, it is apparent that a tooth moves from A to B during one revolution, so that the double worn moves two teeth per revolution and may thus be regarded in calculations as a driving wheel with two teeth.

A treble worm may be regarded as a driving wheel with three teeth. The easiest way to distinguish between single, double, and treble worms is to examine the end of the worm, where the beginning of each thread car easily be seen. In operation, worm drives offer several advantages in compensation for the disadvantage of being non-reversing. In the first case, it is easy to achieve very low gear ratios in a relatively small space.

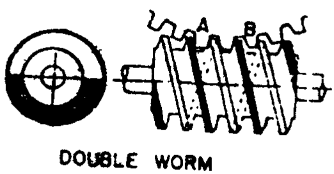

Fig. 5. Double worm

The ratio of 1 : 100 present in our example, for instance, would require a cog with 600 teeth if a driving sprocket with only six teeth, probably the smallest size that could give a smooth operation, were available. Alternatively, an extra shaft giving a reduction drive could be incorporated, but either of these two arrangements would occupy considerably more space than the worm drive. As a second advantage, the fact that motion is unidirectional serves a useful purpose. Since pressure is always applied between the same contacting surfaces, there need never be a sudden separation as happens in other types of drive. Thus, backlash can be made much smaller, almost to the point of being nonexistent, and meshing is smoother, so that noise, friction, and wear are all reduced. In consequence, worm drives tend to be preferred to pinion drives whenever a low-gear mechanism with no requirement for reversing is needed in a machine.

Example 6

A double worm at 40 rev/min drives a sprocket of diameter 15 cm. If the worn pitch is 131.6 threads/m, calcualte the angular velocity in rad/s of the driven shaft.

We have:

$$\text{worm pitch} = 131.6 \text{ threads/m}$$

i.e.:

$$\text{pitch-spacing} = \frac{100}{131.6} \text{ cm} = 0.760 \text{ cm.}$$

Since worm must mesh with sprocket, pitch of sprocket must also be 0.760 cm. Now:

$$\text{sprocket diameter} = 15 \text{ cm,}$$

i.e.:

$$\text{circumference} = 15\pi\text{cm}$$

$$= 47.124 \text{ cm,}$$

and hence:

$$\text{number of teeth} = \frac{47.124}{0.760} = 62.005.$$

Thus the sprocket has 62 teeth.

Since the worm is a double one, it can be regarded as equivalent to a sprocket with two teeth.

Substituting in:

$$\omega_2 = \frac{N_1}{N_2}\,\omega_1$$

gives:

$$\omega_2 = \frac{2}{62} \times 40 = 1.29 \text{ rev/min}$$

$$= \frac{1.29 \times 2\pi}{60} \text{ rad/s}$$

$$= 0.135 \text{ rad/s.}$$

Thus the driven shaft operates at 0.14 rad/s.

Ratchets

A ratchet system is normally used to convert a reciprocating, or oscillating, movement into an intermittent rotary one. The wheel receives its rotatory force from the driving pawl, which has a to-and-fro movement as shown. On its driving movement (i.e., from left to right in the diagram), the pawl pushes the wheel round in the direction of the arrow, usually (but not always) by just one tooth. On its backward movement (i.e., to the left), the pawl slips back over one tooth (or more, if a motion involving several teeth has been designed), ready for its next forward movement.

During this backward movement of the driving pawl, a second pawl (the retaining, or holding, pawl) prevents any backward movement of the wheel. It is fixed freely on a fulcrum so that wheel movement in the forward direction is not restricted at all, but it is weighted to drop behind each ratchet tooth and so prevent any backward movement from taking place. If the ratchet serrations are relatively large, the retaining pawl is a compound one, made up from several individual pawls of slightly different lengths mounted on the same pivot.

The backward movement of the Wheel that can then take place is reduced because one or other of the compound pawl elements will drop into the locking position no matter where the positive movement delivered by the driving pawl allows the wheel to stop. Once again, the motion is obviously unidirectional and, equally obviously, is intermittent.

There is a regular, predetermined sequence repeated discontinuously, and several sources of friction, noise, and wear may easily be identified. The movements of each pawl as it slides over the top of one ratchet, drops back to the bottom of the adjacent one, and then slides forward to apply a blow to the stationary wheel to reactivate its movement are all obvious contributors to these three problems.

Example 7

A ratchet wheel, R, receives one tooth movement (usually) for each reciprocation of the sley (i.e., for each weft pick inserted in the cloth). The wheel R is compounded (i.e., on a common shaft) with sprocket A, which drives the beam wheel D via the compound carrier BC. The wheel D is fixed to the take-up roller E, which drives the cloth roller and pulls the cloth forward, by frictional contact in both cases. The wheel A is a change wheel, used to alter the pick-spacing in the cloth. If R has 50 teeth, B 75 teeth, C 15 teeth, D 120 tee ˙ h, and E a circumference of 40 cm, calculate the change constant and the pick-spacing when a sprocket with 50 teeth is used at A.

Suppose R has gone through one complete revolution, i.e., 50 picks have been inserted.

During this time:

distance moved by cloth = distance moved by take-up roller.

In one revolution of R, there is one revolution of A, and thus:

$$\text{number of revolutions of B} = 1\times \frac{N_A}{N_B}$$

Since B and C are compounded, C also makes NA/NB revolutions, and hence:

$$\text{number of revolutions of D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D}$$

Thus E also makes

$$\frac{N_A N_C}{N_B N_D} \text{ revolutions.}$$

In one revolution, surface of E moves 40 cm. Thus, in $N_A N_C / N_B N_D$ revolutions, surface of E moves:

$$\frac{40 \times N_A N_C}{N_B N_D} \text{ cm} = \frac{40 \times 15 \times N_A}{75 \times 120}$$

$$= \frac{N_A}{15} \text{ cm.}$$

i. e.:

$$\text{distance moved by cloth} = \frac{N_A}{15} \text{ cm.}$$

Thus, 50 picks are inserted in $N_A/15$ cm, and hence:

$$\text{pick-spacing} = 50 \div \frac{N_A}{15}$$

$$= \frac{750}{N_A} \text{ picks/cm}$$

i.e., change constant = 750.

If A has 50 teeth, then:

$$\text{pick-spacing} = \frac{750}{50} \text{ picks/cm,}$$

$$= 15 \text{ picks/cm.}$$

Hence weft-spacing = 15 picks/cm.

This is the spacing calculated at loom tension, and it is customary to add 1½% to allow for relaxation after removal from the loom. In this example, the practical value of change constant would become 750 + (750 × 1.5)/100, or about 762.

The number 762 is often called the dividend of the motion, since it is the number that, when divided by the number of teeth in the change wheel, gives the weft-spacing in picks/cm. If the change wheel is a driven one, the constant must be multiplied by the number of teeth to give the weft-spacing, and it is common in such cases to choose cogs so that, after allowing for contraction, the value of the constant is 1.

In theory, the number of picks per centimetre is then equal to the number of teeth on the change wheel, but tension-change uncertainty and the possibility of cloth slip make it necessary to accept considerable uncertainty in this approximation.

Hunting-cog motions

When gear wheels mesh, it is obvious that, if the number of teeth in one cog is equal to, or an exact multiple of, the number of teeth in the other, then the same teeth will mesh together at regular intervals. As far as possible, this should be avoided, because it leads to uneven tooth wear, but it is often impossible to escape such a situation because of the requirements of the relative shaft speeds in a gear train.

In the gears connecting the crankshaft and the bottom shaft of a loom, for example, successful operation of the machine demands that the crankshaft wheel must have exactly half the number of teeth of the bottom-shaft wheel. Heavy wear takes place on the teeth that mesh when the large force required for shuttle propulsion is transmitted, since this always occurs at the same position of the crankshaft. In order to overcome this problem, it is customary to insert into the gear train an idler wheel, which is merely an additional cog wheel inserted between the other two sprockets and which does not affect the speed of rotation of either of them. This idler, or carrier, wheel is often made with one tooth more (or less) than the number in one of the wheels with which it meshes, which thus ensures even distribution of high force transfer and hence even tooth wear. The odd tooth is then referred to as a hunting-cog; a similar principle is also used, in a slightly different manner, for stop-motions on various pieces of equipment in a manner.

Two wheels, with different numbers of teeth, mesh together and each one has a projecting piece of metal, shaped like a tooth, fixed to the side. In one wheel, the projection P_1 coincides with a tooth, whereas, in the other, the projection P_1 coincides with a gap between two adjacent teeth. As a result, the gears can mesh normally except when P and P, coincide, at which time the wheels are forced apart, one of them being Mounted in such a way that this can occur without damage.

The time elapsing between consecutive conjunctions of P and P_1 will depend on the numbers of teeth in the two wheels and both must make ail exact number of revolutions in the process. When they do coincide, however, the forcing apart of the wheels prevents meshing, and it stop in the motion takes place. This can be used to stop the drive mechanism, to actuate a counting device for length or speed measurement, or to carry out any similar auxiliary operation automatically at regular, predetermined intervals.

Example 8

A hunting-cog measuring motion on a cotton scutcher. One cog wheel, A, is fixed on a calendar roller, of diameter 12 cm, around which the cotton passes. The other one, B, is on a stop-motion lever that stops the roller every time the hunting-cogs come together. If A has 21 teeth and B has 71 teeth, find the length of cotton that passes round the roller between stoppages. The lowest integral proportion between the numbers of teeth is 21:71, since the two do not have a common factor.

Thus, the cogs will meet every 71 revolutions of A (i.e., every 21 revolutions of B). The length of cotton passing during this time given by

$$l = 71 \times \frac{12\pi}{100} \text{ m}$$

$$= 26.77 \text{ m.}$$

Hence length of cotton passing between stoppages = 26.8 m.

Geneva mechanisms

The Geneva mechanism, sometimes known as the pin and star wheel, is a device for obtaining an intermittent movement, in a very precise manner, from a constant-speed shaft. The driver is an ordinary sprocket with a pin fitted, at a point near the circumference, in a direction parallel to the constant-speed shaft on which this sprocket rotates.

The driven wheel is shaped as shown and has radial slots cut at regular intervals around its circumference. As the pin rotates, it enters one of these slots, moves the driven wheel through a predetermined angle, and then leaves the slot so that the rotation of the driven wheel ceases. The number of slots around the wheel, the position of the pin, and the shaft-spacing are arranged to give a rotation of any desired fraction of a complete revolution of the driven wheel for each single revolution of the driver.

There is usually a ridge, or a large hub, on the driver to keep the star wheel locked in position when it is not turning, and additional pins can be positioned around the driver if desired to introduce additional complication into the intermittent motion. It should be observed that a pin wheel with only one pin must rotate at twice the speed of a pin wheel with two evenly spaced pins to give the same number of movements of the star wheel in a given time.

When the star wheel is making its movement, however, it will move twice as fast in the former case as in the latter. For various reasons, the slower movement is preferred, so it is quite common to see two or more pins fitted to the pin wheel in such mechanisms.

Mangle wheels

The driving pinion, mounted on this shaft, is very small, usually with six or eight teeth, and these engage with pins fixed, in the same way as for the Geneva mechanism, around the circumference of the driven wheel. When the pinion has turned the wheel to the end of the pin rack, it rolls around the end pin and drives on the inside of the wheel, which thus reverses the direction of rotation. On reaching the other end of the pin rack, the pinion again rolls around the end pin to the outside position, and rotation in the initial direction recommences.

The shaft carrying the pinion must be able to make a slight sideways movement to allow rolling around the end pins, and the motion produced is a slow, constant-speed rotation of nearly one revolution followed by a similar rotation in the reverse direction. The mechanism is used frequently in such applications as moving a thread guide along a slow traverse in a winding machine.

Auxiliary components

Shafts and bearings are almost invariably used in combination and are therefore best treated together for discussion. They are jointly responsible for carrying virtually all transmission components and for maintaining precise alignment of these components to ensure successful operation of a mechanism.

Bearings are responsible for reducing friction, as described in an earlier volume, and hence for increasing efficiency of operation. It is perhaps this latter function that is most familiar to the student, since maintenance of a machine frequently involves the provision of lubricant to bearing surfaces to ensure continued, trouble-free running. There are, however, more fundamental requirements that the machine designer must bear in mind in selecting components. In the first case, the needs for correct meshing must be met.

All components must be properly aligned, in all three dimensions. The distance between centres of all gear wheels must be exactly correct. The proper changes in direction, speed, and other such parameters must be achieved without interfering with this meshing in any way, and there must be no unwanted play, or excess backlash, throughout the entire mechanism. These specifications are met by careful attention to shaft and bearing design.

There must be no distortion in the shafts, which must thus be perfectly circular in cross-section and not subject to any bowing or other deviation from a true cylindrical shape. A perfect fit between shaft and bearing, between bearing and housing, and between components within the bearing must be ensured, with no grit or other surface roughness present to ruin this fit. Shafts and bearings must be exactly aligned so that the shafts are parallel to one another. Finally, the quality of the components must be such that this ideal state is maintained over long periods of use. Strength, ability to with stand distortion, and resistance to wear must all be taken into account in selecting materials, type, and size for the components at the design stage.

Obviously such attention to detail exacts a price. In any range of components it is usual to find a selection of tolerances, the components built to higher precision naturally costing more than those produced to a slightly less critical specification. This is a general provision, similar ranges of tolerance being available for all types of components and for a diversity of sizes from wrist-watch to large machinery-scale equipment.

Cams

A cam is a device that uses the principle of the inclined plane to convert rotary motion into an oscillating movement, frequently of a complex kind. Cam mechanisms are of many varieties, and it is somewhat difficult to survey the entire range in detail, so that only a few of the more important types pertaining to textile technology will be mentioned.

They may loosely be classified into revolving cams (sometimes referred to as tappets) and direct-action cams, which are stationary inclined planes used to give a reciprocating movement to any needles or similar components passing across their region of influence. Revolving cams may be further subdivided into positive or negative ones. A positive cam controls the movement of the part that it activates at all times of the cycle, whereas a negative cam controls the movement in only one direction and must rely on gravity, or a device such as a spring, to return the controlled part to its original position for completion of the cycle.

In one case, the cam is a positive one, controlling the needle movement by means of a groove in which the needle butt moves. In the other, the needles are merely lifted by the cams and must rely on gravitational force to drop back into a lower position when cam action ceases. The whole topic of knitting-machine cams is a very complicated one, and the reader is referred, with a sigh of relief, to specialist textbooks on this subject for further information.

In many instances, the nature of the movement given by the cam must be known in order to achieve successful design, or operation, of a machine. In theory, this knowledge is acquired by measuring the movement of the reciprocating part as the cam (or any component geared to it) turns through successive equal angular movements, and then constructing a displacement-time or displacement-angular-rotation diagram.

An alternative technique, simpler in practice but less capable of high accuracy, is to obtain a rubbing of the cam shape by means of a paper and a charcoal, wax, or pencil marker in a manner akin to the method used to obtain an impression of a coin or a church brass. From this rubbing, a displacement diagram for the movement of the reciprocating part may then be constructed. An example will, it is hoped, make this procedure a little less incomprehensible. A rubbing of the cam shape is taken either by wrapping the paper around the cam (in the case of a 'face' cam) or by pressing the paper against the cam to obtain a trace of the edge or groove (in the case of a 'plate' cam).

A face cam is one in which the cam follower is lifted by the contouring of the face of a surface, whereas a plate cam is one in which the edge of the cam actuates the follower. Once the rubbing has been achieved, compass-and-ruler constructions are all that are needed to obtain a clear indication of the motion induced by cam action. Consider. As the camshaft rotates, the lever controlling the sideways movement of the thread guide-bar is pushed laterally until the cam has reached its maximum radius position. As further rotation occurs, reversal of the bar's motion takes place, as a result of the effect of the weight, until the cam's position of minimum radius is achieved, when the motion starts once again.

The rubbing obtained from the cam, together with the construction used to derive an indication of the motion. Contact between cam and lever takes place an anti-friction bowl, and it is the movement of the centre of this bowl that is to be determined. Strictly speaking, this movement takes place in a circular arc, but it is customary to ignore this fact, in view of the slight curvature involved, and to assume that the locus of bowl-centre movement is a straight line. This line is first constructed by connecting the centre of the camshaft, as located on the rubbing at A, with the centre of the anti-friction roller, B, on the assumption that A and B are at their nearest point of approach at the starting time.

With centre A and radius AB, a circle is now drawn around the cam, and this circle is divided into any number of equal parts, a larger number of parts giving greater accuracy. A radius from A to each of the circle-dividing points is then drawn. As the camshaft revolves at uniform speed, these radii are brought to the horizontal position, occupied originally by AB, at regular time intervals.

The centre point of the roller, at each position in turn, is found by constructing the circle of diameter equal to that of the roller, with centre on a radial line, which just touches the cam outline. For each centre so constructed, an arc of a circle, with centre A and radius equal to the distance between A and the centre of the particular circle in question, is drawn to cut the extended portion of the original line AB.

The distance. BC, between the nearest and furthest approaches of the roller centre to A, is known as the lift of the cam, and cam displacement can now be plotted as a function of cam position by reading off the proportion of lift that takes place as each radial position reaches the base line. This line is drawn as abscissa for a graph, for which the ordinate is given as bowl displacement from the innermost position. All displacements, for one complete revolution of the cam, are plotted, and the movement of the thread guide-bar at any cam position can then be deduced by taking into account the constant magnification factor relating movement of the bar to roller-centre movement. The procedure can he reversed, if desired, in designing a cam shape intended to give a particular movement. A similar mode of operation enables one to analyse or design any cam movement, and the student is encouraged to attempt the use of the technique in a practical situation..

Example 9

A shedding tappet is to be designed for a three-shaft weave, the shaft being down for one pick and up for two. The treadle lever is connected directly to the shaft, and the 'down' position of the bowl corresponds to the 'down' position of the shaft, the lifting operation being carried out by means of a spring. Particulars of the movement are as follows:

nearest distance between bowl and tappet centres = 9cm.;

lift of tappet = 5 cm;

diameter of anti-friction bowl = 7.5 cm;

duration of dwell = 1/3 pick.

If the shaft movement is S.H.M. and the lift line is assumed to be straight and passing through the tappetshaft centre, as before, design the tappet by geometric construction. A diagrammatic sketch of the motion. Which also shows details of the constructions used in the tappet-design procedure. Since the weave is three-shaft, the loom crankshaft will make three revolutions for each revolution of the tappet. From a common point A, taken to be the tappet centre, three straight lines are drawn such that they trisect any circle of centre A.

The angle between any two of these lines represents the angular rotation of the tappet during one revolution of the crankshaft. We are told that dwell time is 1/3 pick, so that, for one-third of each revolution of the crankshaft, no heald-shaft movement takes place. Each of our three angles is therefore trisected, as shown, so that any circle of centre A is subdivided into angles of $2\pi 9$ and $4\pi 9$, repeated three times in sequence around the circle. Each angle of $2\pi/9$ then represents a dwell period and each angle of $4\pi/9$ a change period, as shown. With centre A, circles of radius 9 and 14 cm, respectively, are then drawn. At some position on these circles, the minimum and maximum separations, respectively, of tappet and bowl centres will occur. Each of the six $2\pi/9$ segments present in one or other change period is then trisected again, so that each change-period angle of $4\pi/9$ has been divided into six equal angles of $2\pi/27$.

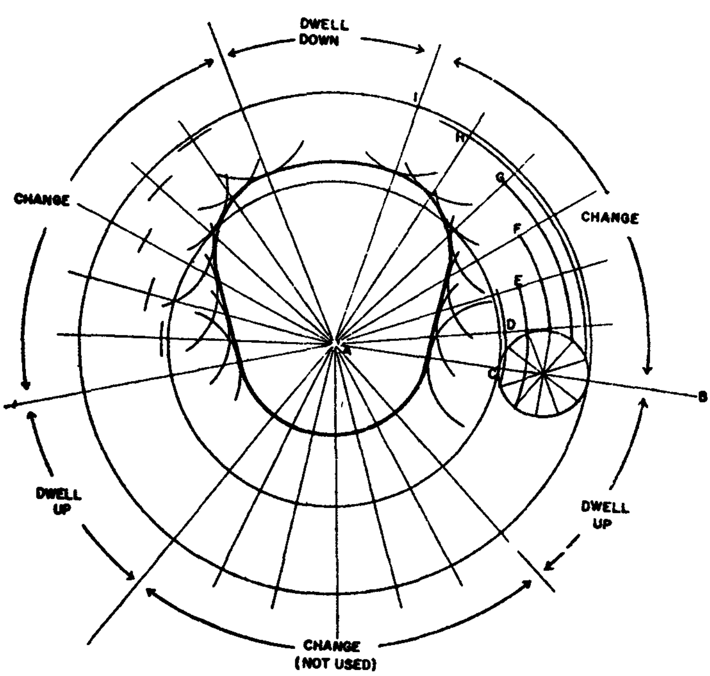

Fig. 6. Shedding-tappet design

Select as a reference line the line AB, which separates a dwell period from a change period. On this line construct a circle, of diameter equal to the lift, located at the lift portion of the line (i.e., just touching the two circles already drawn). Divide this circle into twelve equal segments, six on each side of the reference line. With centre A, draw a series of arcs such that each arc cuts this circle at the point where a segment line meets it, as shown, and extend each arc to meet one of the lines marking the 2n/27 segments of circles with centre A. By this construction, we obtain a series of points C, D, E, F, G, H, 1, and, since their distance from the start of lift has been derived by projection of a circle on a straight line, their distance from the start of lift must divide the lift in proportions equivalent to S.H.M.

From point C, a circle of diameter 7-5 cm (i.e., equal to that of the anti-friction bowl) is then drawn, and a similar circle is drawn from each of the other six points D-1. The outline of the tappet required at this particular change period can then be obtained merely by drawing the line touching these circles. For one of the other two change periods, the entire operation must be repeated in the relevant area. For the third one, since no change takes place when the heald is up, the tappet outline consists of a circle on which the bowl rests at its distance of minimum separation from the tappet shaft.

Calculation shows this to be a circle of radius 5.25 cm, obtained by subtracting the radius of the bowl from that of the circle at lift start. Similarly, at the dwell zone between the two active change periods, the tappet outline is a circle that just allows the bowl to rest at its maximum distance from the tappetshaft centre and thus has a radius of (14 - 3.75) cm, or 10.25 cm.

Cranks and eccentrics

The title of this section provides an almost irresistibly strong temptation to deviate from a strict consideration of textile matters to explore the foibles of the poor humans who must earn their meagre living in the shadow of the discipline.

Every textile technologist worth his salt has at his fingertip a fund of stories recounting the abnormalities of character of an incredibly large number of people connected with textiles, but the urge to succumb to this weakness must be resolutely cast aside in the interests of economy and of the Editor's sanity. For the purpose of the section, then, cranks and eccentrics are mere mechanical devices with a practical function in machine operation. A crank is a linkage designed to convert a rotational movement into a linear-oscillatory one, the reverse operation also being feasible if desired. The most obvious illustration of the latter application is the bicycle pedal; as the rider's leg moves up and down, its reciprocating motion is converted to a rotational one by means of the crank connecting the pedal to the chain sprocket wheel.

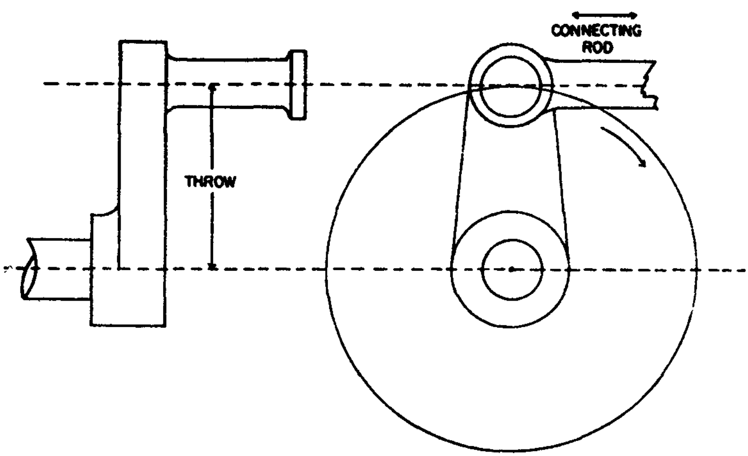

Fig. 7. A crank drive

Essentially, the crank is a rigid rod fixed at one end to the centre of rotary movement and with a joint at the other end to provide a flexible connexion to the oscillating component.

An eccentric performs a similar function but with a slightly different mode of operation. It consists of a rotating disc, but with the axis of rotation displaced from the centre of the disc so that the latter 'wobbles' markedly as it revolves.

It is there obvious that the rim of the disc moves in an oscillatory manner as rotation takes place, so that a reciprocating motion may be obtained by suitably harnessing this rim movement. The resulting movement is identical to that which could be derived by the use of a crank with a throw equal to the distance separating the geometric centre from the axis of rotation of the eccentric, as shown. An example may again help in understanding the use of the device.

A common method of deriving the reciprocating motion of the shedding griffe of a jacquard loom. As the bottom shaft rotates, the crank attached to it moves the long shaft connected to the lever AB, pivoted at P, which controls the vertical movement of the griffe. Since P is at the midpoint of AB, griffe movement is equal in magnitude, but opposite in direction, to the movement of A. We assume that the crankpin C is initially at its highest position (i.e., that the griffe is in the fully down position) and divide the crank circle into twelve equal segments, as before. As the crankpin moves through the first segment, A moves downwards vertically, and, if the connecting arm is long, the distance of movement can be assumed equal to the vertical height of the crankpin below its starting position.

This height, however, is equal to the projection of the angular rotation of the crankpin on the vertical diameter of the crank circle. In a similar fashion, each segment through which the crankpin passes can be used to derive a projection point on the vertical diameter of the crank circle. On completion of one revolution, the displacement-time diagram may be plotted, as shown, by the usual method, and it becomes apparent that the reciprocating movement so derived is an example of S. H. M.

Idler wheels

An idler wheel-was used in order to change continuously the mating points of a pair of gear wheels to prevent uneven wear. As a device to reverse the direction of rotation. In the first coupling, gear A turns clockwise and the mating gear, B, must therefore turn anticlockwise. The insertion of an intermediate gear, C, between the two, however, changes this state of affairs. The fact that A turns clockwise means that C turns anticlockwise, so that B must now turn clockwise. The number of teeth in the idler wheel does not affect the relative speeds of rotation of A and B, since a movement of one-tooth width on A produces a movement of one tooth width on C and hence the same movement oil B. In the first case, of course, a movement of one tooth width on A must automatically cause the same movement on B if correct

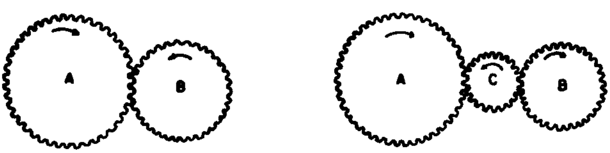

Fig. 8. Idler-wheel operation

meshing exists between the two. In the diagram given, the shaft separation between A and B has been increased to accommodate C but, if shaft separation must remain fixed, the use of smaller gears at A and B (keeping the ratio between them fixed, naturally, to maintain the same relative speeds) enables C to be incorporated easily. Where A and B are closely situated, C can be inserted to one side of the line AB without affecting performance as long as correct meshing at the two points of contact is provided.

Seven
Ultra-fine Fibers

Ultra-fine fibres were produced by melt-blow spinning and flash spinning in the late 1950s. These fibres were not of the continuous filament type but were fine staple fibres of random length which found no application except for being processed into nonwoven sheets immediately after spinning. Ultra-fine fibres of a continuous-filament type have a relatively recent history. A petal-shaped conjugate fibre described in a Du Pont patent was probably the first example of a potential ultra-fine filament. This patent was issued in 1961 as one of several patents for the production of fibres with a triangular cross-section. It was aimed at producing a fibre with a sharp edge by utilizing the boundary of two components *A* and *B*.

Another patent, issued simultaneously from Du Pont, described splitting two-component conjugate fibres of non-circular cross-section into the two separate components after weaving. No attention was paid at that time to combining these technologies to produce ultra-fine fibres, since the fibre with a sharp edge was the primary concern. Okamoto *et al.* of Toray developed conjugate spinning technology for the production of ultra-fine filaments in the mid 1960s by increasing the splitting number of *A* and *B* components.

Here two components, *A* and *B*, are arranged alternately and extruded to yield a conjugate filament which is split into ultra-fine fibres after processing. This was the first attempt to produce an ultra-fine fibre intentionally.

Outline of ultra-fine fibre products

The definition of ultra-fine fibre has varied according to the convention employed. For instance, A micro-denier fibre is a fibre finer than 1.2 dtex for polyester and finer than 1.0 dtex for polyamide. Although a rather thick (1 denier or more) fibre is sometimes claimed as an ultra-fine fibre commercially, an ultra-fine fibre should preferably be specified as a fibre of less than 0.5 d. This chapter therefore deals with filaments of approximately 0.05-0.5 d. A fibre of less than 0.1 d is sometimes referred to as a super ultra-fine fibre. Ultra-fine fibres are classified into two types: (i) a continuous-filament type and (ii) a random (staple) type.

Manufacturing processes for ultra-fine fibres

Continuous-filament type

Ultra-fine fibre of the continuous-filament type is now produced by a variety of methods including:

- Direct spinning (conventional extrusion).
- Conjugate spinning (extrusion of polymer components arranged alternately):
    - a) islands-in-a-sea type;
    - b) separation type or splitting type;
    - c) multi-layer type.

Random (staple) type

Ultra-fine fibres of the random type are produced by:

- Melt-blowing or jet spinning.
- Flash-spinning.
- Polymer-blend spinning.

— Centrifugal spinning.
— Fibrillation or violent flexing.
— Turbulent flow-moulding
— Bursting.
— Other methods.

Characteristics of ultra-fine fibres

Different ultra-fine fibres are designed to provide the following characteristics:

— Softness, flexibility and smoothness.
— Fine textile structure.
— Micro-pockets in fabrics.
— High filament density at the textile surface.
— Large surface area per unit weight, and a characteristic interfacial property.
— Small radius of curvature (resulting in luster and characteristic colour)
— Large aspect ratio (the ratio of length to diameter) and easy entanglement.
— Good interpenetrating capacity in other materials.
— Quick stress relief.
— Low resistance to bending.
— Bio-singularity relative to living tissues and fluids.
— Fine, sharp edges.

Since the cross-sectional area A, the second moment of the cross-section M and the torsion I_p are given by $A = (\pi/4)D^2$, $M = (\pi/64)D^4$, and $I_p = (\pi/32)D^4$, with D being the diameter of the cross-section of a filament, these values decrease exponentially as the diameter D decreases. Thus the flexibility (item 1 in the above list) of ultra-fine fibres is the result of a small cross-sectional diameter.

Spinning of the continuous-filament type

The direct spinning method is an extension of conventional spinning, where the spinning conditions are optimised so as to be suitable for the production of ultra-fine fibres. In the application of conventional melt spinning, the following problems can be foreseen:

— Fibre break-down (dripping).
— Variation of filament thickness.
— Spinneret clogging.
— Denier variability among filaments in a single yam.

The following precautions are taken in order to avoid these problems:

— Optimization of polymer viscosity (i.e. a higher spinning temperature to reduce viscosity).
— Optimization of the spinneret design (i.e. the spinning holes arranged so as to ensure homogeneous cooling).
— Optimization of the ambient temperature underneath the spinneret (i.e. quenching, cooling, rate control).
— Optimization of filament assembly (i.e. assembly nearer to the spinneret).
— Optimization of spinning draft (i.e. spinning tension control).
— Lower rates of extrusion (i.e. stable polymer transmission)
— Purification of spinning polymer (i.e. high-efficiency filtration).

The conditions for production of ultra-fine fibres of less than 0.3 denier are that the polymer melt viscosity should be adjusted to be less than 950 poise, the cross-sectional area per spinneret hole should be less than 3.5×10^{-4} cm^2, the ambient, temperature at 1-3 cm underneath the spinneret should be kept below 200°C, and the extruded filaments should be assembled at 10-200 cm underneath the spinneret.

Asahi Chemical Industry Co. has succeeded in producing ultra-fine polyester fibre of less than 0.15 denier by extruding polyester of melt viscosity less than 480 poise through a spinneret with over 300 holes, of less than 1×10^{-4} cm^2 cross-sectional area per hole, arranged concentrically.

However, the extruded polymer tends to form droplets and e xhibits no drawability unless the thermal environment immediately below the spinneret is suitably controlled. The ambient temperature at cm underneath the spinneret holes must be kept below 150°C by blowing cold air from the circumference of the spinning threadline to enable the polymer to be drawn into filaments.

These concentrically arranged filaments should all be cooled at the same rate. Then the filaments are assembled at 20-70 cm underneath the spinneret holes, and wound up as undrawn fibre. This undrawn fibre can be drawn conventionally to yield ultra-fine fibre of less than 0.15 denier. The Teijin Co. has investigated the influence of air friction on the high-speed spinning of ultra-fine fibre. Although the PET filament is drawn only 4-6-fold in a conventional process, it can be drawn 10-20-fold in a particular condition called superdraw.

Du Pont has proposed a method for producing ultra-fine fibres by super-draw, but no industrial application has been implemented so far because of the unstable and restricted conditions required. Ultra-fine fibres can be produced by wet spinning as shown below. The following points should be observed in the technical application:

— The solvent should be chosen so as not to form a porous structure in the resulting fibre, and the solution concentration should be adjusted to the optimum level for ultra-fine fibre spinning.

— The spinning solution should be filtered through a fine filter to eliminate particles larger than 1/3 of the nozzle diameter.

— The nozzle diameter should be less than 30 gm and a multi-hole system is desirable for industrial purposes.

— Drawing can be performed by a conventional technique.

In the case of acrylic fibre spinning using a polyacrylonitrile/ vinyl acetate/sodium methacryl-sulphonate (92.5/7.0/0.5) copolymer, an ultrafine acrylic fibre of 0.06-0.4 denier with a tenacity of 3.0 g/denier and 26% extension is produced under the following spinning conditions .

— Specific viscosity: 0.17-0.19.

— Solvent: dimethylacetamide or dimethylformamide.

— Concentration: 16-19% solute.

— *Filter*: sintered metal filter of less than 10 m filtration cut-off.

— Nozzle diameter: 20-30 mm.

— Number of nozzle holes: 40 000-80 000.

A single-component ultra-fine fibre is obtained by direct spinning, and the later processes require no complicated processing such as splitting into two components or removing a second component. However, filament breakage and fluff formation are often observed in direct-spinning of ultra-fine fibres, and a handle of high quality cannot be expected.

Conjugate spinning with alternately arranged polymers

The technical problems in direct spinning can be solved by conjugate spinning, which yields homogeneous ultra-fine fibres. The idea of conjugate spinning predates direct spinning for the production of ultrafine fibres. Okamoto *et al* (Toray) and Matsui *et al* (Kanebo) investigated the extrusion of conjugate fibres with a cross-section consisting of highly-dispersed conjugate components by modifying the spinneret structure. Okamoto suggested that the resulting conjugate fibres should be termed 'alternately-arranged-polymer fibres' in order to express correctly both the morphology in the longitudinal direction and the islands-in-a-sea structure in the lateral direction. Alternately-arranged polymer spinning is classified into two types from the technical viewpoint:

— The islands-in-a-sea type, where the sea component is removed by dissolving it in a solvent, and

— The separation type or splitting type, where Breen of DuPont's patent has been applied to ultra-fine fibreization. In either case, no drawing is required at spinning and the micro-fibreization is performed in the form of fabrics. No special technical problems arise at later processing compared with conventional spinning. Here, two-component polymer flows of a conjugate type or a sheath-and-core type are assembled into a single flow.

The number of two-component polymer flows in a bundle determines the thickness of the resultant filaments. Although it depends on the assembling process of two-component polymer flow, the sheath-and-core type has an advantage with respect to obtaining the desired number of split filaments since the components in the splitting type may mingle during the assembling process. Polyester, nylon, polypropylene, polyethylene and polyphenylene sulfide are among the polymers employed as island components.

A sea-component, such as polystyrene, a 2-ethylhexyl acrylate copolymer or a copolymer of ethylene terephthalate and sodium sulfo-isophthalate, is removed by dissolving it in a solvent after conventional processing into woven, knitted or nonwoven fabrics. Thus there is no fundamental difference in spinning and further processing from conventional melt spinning. For example, the extrusion temperature is

275-300°C when PET is employed as an island component. The micro-fibreization takes place after the macro-filaments have been processed conventionally into fabrics. This technology has provided a means of industrial production of suede-type artificial leather, silk-like fabrics, wiping cloths and fine filters made of ultra-fine fibres.

The number of islands in the ultra-fine multifilament yam is specified by the design of the spinneret. The ratio of the island component to the sea component is determined by the extrusion rate of each component. Here, the island component is sheathed in the sea component. The spun and drawn fibres are processed into fabrics and micro-fibreized by removing the sea component Processing and finishing of islands-in-a-sea fibres in non-woven sheet. Three-component spinning can be carried out with two island-components by designing a three-component spinneret assembly. The sea-component can be reduced to 2-10% of the total components from the purely technological point of view, but the space between the ultra-fine filaments is also reduced and this may lead to poorer handle of the products.

When the sea-component is small in amount and not miscible with the island-component, the splitting can be carried out mechanically. Since the ultra-fine filaments are sheathed by the sea-component in the islands-in-a-sea type. This technology is capable of producing a PET filament of 0.00009 denier. Only 4.16 g of such a filament would be enough to stretch from the earth to the moon.

The technology has been further extended to spin a multi-component conjugate fibre, and a suede-type artificial leather of high dyeability has been developed with a three-component conjugate extrusion where the two-component (polyester and nylon 6) ultra-fine fibre had a sheath-and-core structure. Many variations will be found in islands-in-a-sea spinning, such as extremely-many-islands-component spinning for wiping cloths, non-circular cross-sectional or non-even lateral surface island-omponent spinning, and blended island component spinning.

Splitting type spinning

This type of spinning aims to utilize the second component in the final product as well as the first by splitting the two components mechanically instead of removing the second component by dissolving. The fundamental principles of this technology will be found in the inventions by Breen and Tanner of Du Pont, although these inventors had no intention of applying their idea to the production of ultra-fine fibres. Since the commercial success of the artificial leather made from the alternately-arranged two-component fine fibre, this alternative spinning technology has been refined by Kanebo, Teijin and Toray to produce ultra-fine fibres.

Their ultra-fine spinning technology consists of the combination and separation of PET and nylon, the use of benzyl alcohol for effective shrinkage of the nylon component, and the establishment of dyeing technology for dyeing PET and nylon of different dyeing characteristics simultaneously with high colour fastness. These ultra-fine fibres have been developed mainly into moisture-permeable and water-repellent fabrics, along with suede-type artificial leather, silk-like fabrics and wiping cloths.

The ultra-fine fibreization is performed by a mechanical or chemical process in the splitting and separation types of spinning. Here the point is how many divisions of two components can be achieved. The commercial ultra-fine fibres are now produced by a specially designed spinneret. Toray and Kanebo employ a spinneret with a *-shaped cross-section, which is a modified version of that described in Breen's patent. The fibre, with a + -shaped cross-section, divides the second component into four wedges in this process.

Kanebo has added another wedge component to each of the four wedges in order to increase the splitting probability in later processes. Its cross-section looks like a shaped conjugate fibre. The basic design of the spinneret is similar to those of Du Pont and Toray for the petal-shaped conjugate fibre.

The hollow was introduced to avoid flattening and sharp edges in the resulting filaments even when the splitting number is increased. A good handle will be achieved when the spacing between ultra-fine filaments is large in the fabric. The hollow portion preserves the spacing between filaments after splitting, although the split filaments tend to arrange with the original cross-section before splitting because of the spatial restrictions in the fabric. A proper choice of the spinneret and component polymers is vital in ultra-fine fibre spinning by this technique. PET and nylon 6 form the most popular combination found in the commercial products, since both components have a similar temperature range for extrusion and drawing.

Fluffing during the drawing or weaving process can be avoided by improving the adhesion of two components, for example, by using the copolymer of PET and sodium sulfo-isophthalate as the PET component. Splitting during the spinning process can be reduced by increasing the spinning speed so that the PET and nylon exhibit similar shrinking behaviour.

Multi-layer type spinning

Liquids can be multi-layered by static mixers, which have been applied to multi-layer type spinning. Kanebo, Kuraray and Toray investigated this multi-layer type spinning technology. Here the two components, polyester and nylon 6, are spun into a conjugate fibre of multi-layered structure with an oval-shaped cross-section, which is micro-fibreized into filaments of 0.2-0.3 denier during the dyeing process.

Random-type spinning

The polymer melt is blown apart immediately after extrusion by an air-jet stream in this method, so it is sometimes termed 'jet spinning'. Thus, this method is an application of spraying technology rather than true spinning. The melt viscosity is lower than that of the conventional polypropylene used for melt spinning, and an appropriate range of melt viscosity should be chosen to achieve the required result. The spinneret has a sharp edge and the jet stream blows off the extruded polymer melt into ultra-fine fibres that are collected as nonwovens.

New technology has been developed for the production of electrets on these polypropylene nonwovens to meet the requirements for a clean environment.

Flash spinning

Flash-spun fibre is counted as an ultra-fine fibre. Figure. 16 is a photograph of a fibre network obtained by spreading a single stream of fibre spun from one spinneret hole. The filament thickness varies from 0.01 denier to 10 denier, and the average denier per filament is approximately 0.14.15 denier.

The filament cross-section is non-circular, and some filaments contain micro-bubbles. This technology was found by Du Pont accidentally while examining the explosion behaviour of organic solvents for safety research. As an example of the process, polyethylene is dissolved in hydrocarbon or methylene chloride, heated under pressure, and jetted from a nozzle into a micro-networked fibre termed a 'plexifilament'. Ethylene chloride and fluorocarbon have been used as solvents, but they are now being replaced with other solvents which will not destroy the ozone layer.

This technology was first aimed at producing pulp for synthetic paper, but has been converted to produce wrapping materials including wrappings for domestic use and envelopes. The spinning speed is too high to reel in the product in the form of fibre, so the product is made into a sheet. Polymer is dissolved in liquid gas and made into transparent (homogeneous) solutions at high temperature under high pressure.

The spinning solution is then liquid-liquid phase-separated to make a turbid solution, which is jetted out of a nozzle into air to form a fibrous network. The homogeneous polymer solution (prepared by extrusion or in autoclave, and kept at a high temperature under high pressure) is extruded through point a and the orifice in the pressure-reducing chamber *b*.

Since the pressure at point a is reduced in chamber *b*, the polymer solution separates into two phases, and is then jetted into air through the nozzle. In the process from b to c, the solvent is gasified instantly and produces a jet stream. The temperature drops to room temperature, and the dissolved polymer is solidified and drawn to form a fibre of high strength. Since the micro-phases of solvent are dispersed in the polymer gel which expands explosively at the time of jetting, the resulting fibre is composed of an extremely thin network of non-circular cross-section fibrils like a cobweb.

Polymer blend spinning

In this method the conjugate fibre is produced by extruding and drawing a blended polymer melt of two components. The arrangement of the dispersed and non-dispersed (matrix) components is determined by the mixing ratio of the components and their melt viscosities. UCC and others found that discontinuous ultra-fine fibres could be obtained by removing the matrix component while investigating gut extrusion.

Fukushima *et al.* of Kuraray successfully applied this method in the production of artificial leather. A conventional spinning facility can be easily converted for this type of polymer blend spinning by adding a mixer-extruder. Here, the fibre fineness cannot be controlled and the fibre often breaks during spinning, although the spinning stability is strongly dependent on the combination of polymers.

Eight
Optical Fibers

Optical fibers are classified into three groups according to the types of core material, quartz, multi-component and plastic optical fibers (POF). The quartz optical fiber is used for long-distance optical communication including public trunk lines because its transmission loss is as low as 1 dB/km or under.

The multi-component optical fiber is used for middle-distance communication of 1-2 km including local area networks (LAN) in plants and fiberscopes. The glass optical fiber has a shortcoming in that its processing requires great skill because it is expensive, brittle and hard to process. Meanwhile, the plastic optical fiber has advantages in that its inexpensive, flexible, light and easy to process, though the transmission loss is as high as 120-130 dB/km.

It is, therefore, widely used for short-distance wave guides including optical sensors, optical transmission in equipment, mobile optical communication and display. This chapter outlines the process for manufacturing plastic optical fibers, the technique for improving the transmission loss and the latest product with a multiple optical element.

Structure of plastic optical fiber

The structure of optical fiber is classified by the transmission methods into the step index (SI) and graded index (GI) types.

Commercial plastic optical fiber is of the SI type. The GI type has not been commercialized yet. It is interesting that the transmission loss and transmission bandwidth of the GI type plastic optical fiber are 134 dB/km and 360 MHz. km, respectively. Recently, a single mode plastic optical fiber has been proposed and manufactured on an experi-mental basis and the types are being rapidly varied.

The structural dimensions of the plastic optical fiber using PMMA as the core material are being standardized in such a way that the core diameter and sheath thickness of a plastic optical fiber 1000 mm in outer diameter are 980 and 10 mm, respectively. The area ratio of the core is so large that it is characterized easily by connecting an optical fiber to a light source and another optical fiber in alignment.

Material of plastic optical fibre

The materials of both the core and sheath of the plastic optical fiber require high transparency. Although commercially available PMMA and polycarbonate (PC) are generally used as the core material, silicone and thermosetting resins, MMA-methacrylic anhydride copolymer, MMA maleic anhydride copolymer, MMA-methacrylicimide copolymer and ionomers are being investigated. Since the refractive index of the sheath material should be lower than that of the core material, fluoroplastics including poly (vinylidene fluoride), Teflon FEP, Teflon AF, fluorinated methacrylate and fluroinated polycar-bonate are used as the sheath material.

Polymethyl-pentene is used as the sheath material for a plastic optical , fiber with a relatively high refractive index of 1.59 using fluorinated polycarbonate as the core material.

Causes of transmission loss of plastic optical fiber

The causes of the transmission loss of the plastic optical fiber are classified into the loss inherent in the material itself and the external loss from the manufacturing technology. Kaino investigated the internal losses theoretically including the losses on absorption and scattering for a plastic optical fiber using PMMA as the core material and estimated that the limit of the internal loss was 106.2 dB/km at a wavelength of 650 nm in the visible LED.

The typical transmission losses as a function of wavelength of Eska Extra (produced by Mitsubishi Rayon) which has the lowest transmission loss among the current commercially available plastic optical fibers. Although the transmission loss at a wavelength of 568 nm is 60-70 dB/km which is 30-40 dB/km smaller than the limiting loss, it is considerably lower than the value for the product manufactured on the industrial scale. This transmission loss is 120 dB/km at a wavelength in the vicinity of 650 nm in the visible LED used for communication which is close to 106.2 dB/km, the limiting loss obtained by Kaino.

Manufacturing plastic optical fiber using PMMA as core material

The schematic flow diagram of the process for manufacturing plastic optical fiber using PMMA as the core material. Although suspension and bulk polymerizations are applicable to the process, the latter lowers the transmission loss because the usages of the polymerization initiator and the molecular weight modifier are substantially less. The transmission loss is, however increased by pellestising the core material made by the bulk polymerization and by melt spinning to produce the plastic optical fiber. Mitsubishi Rayon started to produce high-purity PMMA for the first time by an industrial continuous bulk polymerization of MMA in 1970.

Based on the actual results obtained by the operation of the process, Mitsubishi Rayon put Eska Extra, the plastic optical fiber for communication, on the market in 1983 by combining the continuous polymerization technology of MMA with the melt spinning technology of polyester and polypropylene fibers. The transmission loss of this product was half that of conventional plastic optical fibers and the transmission distance was increased to as long as 150 m.

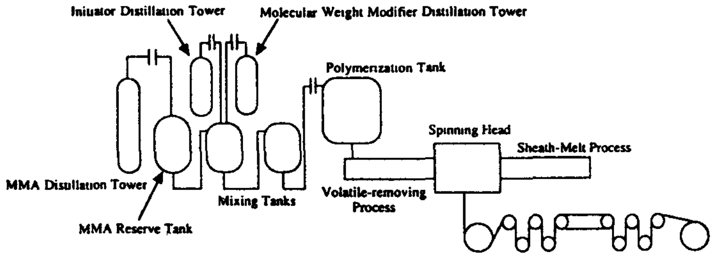

Fig. 1. Flow chart of POF production

MMA contains a polymerization inhibitor, which should be removed by distillation. When a hydroquinone derivative, generally used as the polymerization inhibitor, remains in the final product, it absorbs visible wavelengths. The dissolved oxygen in the monomer reacts with the monomer or molecular weight modifier in the polymerization process and forms a peroxide which easily colours PMMA.

MMA peroxides contained in the monomer should also be removed. These peroxides can be removed by pretreatment with a reducing agent followed by distillation. Various methods for removing compounds contained in the monomer affecting the visible wavelengths have been described. After removing the polymerization inhibitor and molecular weight modifier from the monomer by distillation and also thoroughly removing impurities by vapour and liquid-phase filters, the monomer is ready for polymerization.

A vapour phase filter using hollow yams with an aperture of 700A is effective. Various other purification processes for the monomer have also been proposed. The bulk polymerization process is most suitable for the manufacture of plastic optical fibers because polymerization additives, other than the polymerization inhibitor and molecular weight modifier, are not used. Mitsubishi Rayon has developed a process for manufacturing PMMA involving continuous bulk polymerization of MMA followed by a process for removing volatile matter such as unreacted monomer.

PMMA is manufactured in this newly-developed process in such a way that MMA is polymerized using tens of ppm of the polymerization initiator and molecular weight modifier at 130-180°C for several hours at a yield of approximately 60 wt% and the volatile matter, including unreacted monomer and molecular weight modifier, is then removed. This polymerization process is an integral part of the manufacturing process of Eska Extra. It is important for stable polymerization to produce a uniformly mixed blend of monomer and polymer.

Since the termination reaction is retarded by the increase of the viscosity in the polymerization reaction, the polymerization sometimes proceeds rapidly by the gel effect into a runaway reaction. It is, therefore, important to remove the heat of polymerization effectively by reducing the viscosity to below a specified value and uniformly stirring the polymerization product. Since variations in the degree of polymerization and the viscosity of the blend cause variations in the blend transportation and volatile-removing process, the transmission loss of the plastic optical fiber can vary.

The polymerization temperature is preferably kept low so that there is no gel effect. Oligomers including MMA dimer are produced as by products by increasing the polymerization temperature.

Since the oligomers are so thermally unstable that they are easily coloured, the absorption of the plastic optical fiber is increased. Important features of the volatile-removing process are to remove the volatile matter including monomer, at a temperature as low as possible, to avoid locally heating the polymer by high shearing, to prevent any dead space from forming in the equipment and to inhibit the generation of impurities from the equipment. The special features of these two processes will now be described.

Extruder with degassing hole

In a conventional extruder with a degassing vent, an upward degassing vent is connected to the evacuation line ahead of the blend-feeding hole and directly connected to the extruder cylinder. In the newly-developed extruder, however, the degassing vent is fixed behind the blend-feeding hole so that part of the blend is sent to the rear of the shaft of the screw through an inverse pitch screw channel, so that the volatile matter is steadily discharged together with the blend from the system without mixing the impurities generated by rubbing at the shaft seal with the polymerization product.

Since the direction of the degassing hole is horizontal, the polymer discharged from it together with the monomer does not return to the screw channel. Impurity-free polymer is, therefore, continuously extruded from the end of the extruder.

Cylindrical vertical purifier

The blend is fed from a small hole at the end of the feeding pipe against the upper inside wall of the cylinder of the volatile-removing equipment. The temperature of the inside wall is controlled in advance at a temperature such that the viscosity of the polymer is kept at 5000 poise or less. The syrup fed against the heated wall flows down, spreading on the wall in the form of film. Since the cylinder is evacuated from the degassing hole, volatile matter contained in the blend is effectively removed.

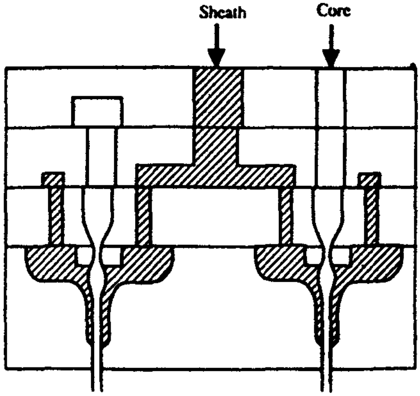

Fig. 2. Spinning nozzle for conjugate spinning

The plolymer free from the volatile matter is fed to a polymer tank located in the deaeration zone while maintaining the free level at a constant height in the lower part of the cylinder. The volatile matter is removed by gently stirring the syrup to release trapped bubbles under the free level.

The polymer is then transported downward under pressure into a screw and discharged from its outlet. The polymer is prevented from contamination with the discharge from the under shaft seal by discharging it from a separate outlet. Since this purifier is provided with a vertical rotating shaft, there is little vibration and the polymer is never contaminated by abrasion.

Spinning technique for plastic optical fiber using PMMA as core material

The core material melted in the volatile-removing process is supplied to the core-sheath composite spinning nozzle through a constant-volume gear pump. Attention has to be paid to the following matters in the spinning of plastic optical fiber:

1. Avoiding long residues of the polymer in piping and equipment

2. Preventing extraneous matter being generated from the equipment
3. Designing a high-level spinning nozzle.
4. Optimizing the viscosities of molten core and sheath materials.

It is necessary to shorten the length of the piping after the Polymerization process as much as possible and to electropolish the inside surface of the pipe to reduce the surface resistance. It is also necessary to increase the bending radius of the pipe as much as possible. It is important to conduct dead-end polymerization to prevent the polymer from staying in the piping end of the equipment. Dead-end polymerization is a process in which polymerization does not proceed after a specified time has passed.

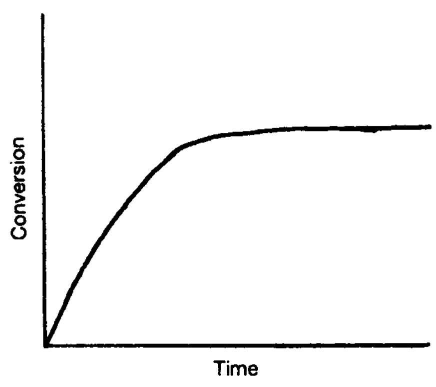

Fig. 3. Dead-end polymerisation: convension with time

Even if any dead reactant space remains present in dead-end polymerization, the reactant is not polymerised there in practice. For preventing the generation of extraneous matter from the equipment, the volatile-removing equipment has a mechanism for discharging contaminant polymer from the shearing part and degassing hole. It also has a constant-volume gear pump which steadily discharges the contaminant produced from the rotating-shearing part. The transmission loss caused by imperfect fibrous structure, including irregular interface and unequal core diameters, is mainly derived from the spinning process.

Optimum design of the spinning nozzle and adaption of the fluidity of polymer to the spinning conditions are required for reducing the loss. The transmission loss of the plastic optical fiber is increased by the colour developed by heating the core material, it should be spun at as low a temperature as possible. The melt viscosity of the core material is, however, raised to tens of thousands of poise at such a low temperature.

The operational conditions including molecular weight, temperature, inner diameter of nozzle and output should be optimized by keeping the shear stress in the nozzle at 10^6 dyne/cm^2 or less to prevent the development of melt fracture. Although composite melt spinning is effectively used for a composite fiber and multilayer film, the instability of the shape of the interface is interesting. A low viscosity material generally encapsulates a high viscosity material during passage through a circular die because the low viscosity material migrates to the high shear region at the die surface. Since the smoothness of the interface between the core and the sheath is important, the melt viscosities of core and sheath should be optimized.

The interface between the materials at the melt viscosity ratio of 1 is uneven, while that at the melt viscosity ratio of 2 is smooth. The relationship between the melt viscosity ratio of core to sheath and the transmission loss of the plastic optical fiber. Although the transmission loss is decreased by increasing the melt viscosity ratio up to 10 or higher, the variability of the outer diameter of the plastic optical fiber is increased by decreasing the melt viscosity of the sheath too much. The conceptual relationship between the melt viscosities or core and sheath and the sectional shape of plastic optical fiber.

Research into reducing the transmission loss of plastic optical fiber has been energetically pursued and values for commercially available plastic optical fiber products are as low as 120 dB/km at a wavelength of 650 nm. Since the value for the plastic optical fiber using PMMA as the core material should still be capable of reduction by a further 20 dB/km, improvements in the process and material are expected. For further decreases in the transmission loss of the plastic optical fiber, the development of transparent materials including floropolymers is expected. When these are developed, the transmission loss may theoretically be reduced to 5 dB/km.

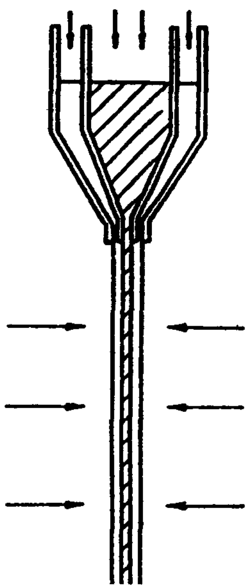

Fig. 4. Process for manufacturing organic-silicone core POF.

Other processes for manufacturing plastic optical fiber

Process for manufacturing plastic optical fiber using PC as core material

The plastic optical fiber using PC as the core material has a heat resistance of 120°C or higher which is 30°C or so higher than that of the plastic optical fiber using PMMA as the core material. After commercialization of this plastic optical fiber by Mitsubishi Rayon in 1986, Fujitsu, Teijin Kasei, Idemitsu Petrochemical and Asahi Kasei have also successfully commercialized it. PC polymer is dissolved in an organic solvent containing methylene chloride, and unreacted substances and by-products are removed from the solvent solution by washing.

The polymer is recovered by removing the solvent using a spray drying method. This recovered polymer is usually pelletized by melt extrusion at a temperature higher than the crystalline melting point of 245°C, but the polymer thermally degrades at 280-320°C. Even though pellets of such a PC polymer containing thermal decomposition products can be used for the melt spinning of plastic optical fiber, high-transmissibility product cannot be obtained.

It is, therefore, necessary to directly carry out the melt spinning of polymer recovered from the polymerization process without pelletizing it to produce a plastic optical fiber with high transmissibility. It is also necessary to inhibit crystallization of polymer during the melt spinning. Since the crystallization point of PC reaches its maximum at approximately 190°C, the molding temperature should be 210°C or higher.

Process for manufacturing plastic Optical fiber using organic silicone as core material

The plastic optical fiber using organic silicone as core material is characterized by flexibility and resistance to heat and chemicals. Sumitomo Denko has commercialized such a fiber composed of silicone rubber as the core and Teflon FEP as the sheath. Three manufacturing processes are as follows:

— A mixture of vinyl alkyl siloxane and a platinum catalyst is filtered and the filtrate is injected into a hollow tube of a tetrafluoroethylene/ hexafluoro-propylene copolymer (FEP) under vacuum followed by thermal polymerization.

— Using a mixture of liquid siloxane polymer and a hydrogen chloroplatinate as the core material, and liquid siloxane polymer with lower refractive index that of the core materials as the sheath material, both materials are fed at the same time through nozzles 2 mm and 4 mm in, diameter, respectively to a thermal cross-linking stage in a heater.

— A fiber of the core material is manufactured in the same way as in 2 and the sheath material is coated by dip coating.

Process for manufacturing plastic optical fiber using thermosetting resin as core material

Since the plastic optical fiber using thermosetting resign as core material has good retention of shape at high temperature, Hitachi Densen has placed it on the market. It is composed of a thermosetting design as the core material and Teflon FEP as the material. The monomer is fed through a pump into a stainless steel tube lined with Teflon. It is polymerized in a hot water tank so that the viscosity of the polymer reaches 10^4 poise. A fibrous resin is extruded from the nozzle and heated by an infrared heater to form the core. The resin is then coated with a cladding material.

Multi-picture element plastic optical fiber

Diverse applications of plastic optical fibers including light transmitters, optical sensors, optical branching fibers and image guides are expected to be developed. Sheet and block-type Multi-picture element plastic optical fibers have been developed. Since these plastic optical fibers are presently manufactured by accurately arranging a filament, this process requires much time. An integral melt spinning process is being developed. The processes and their special features will now be described.

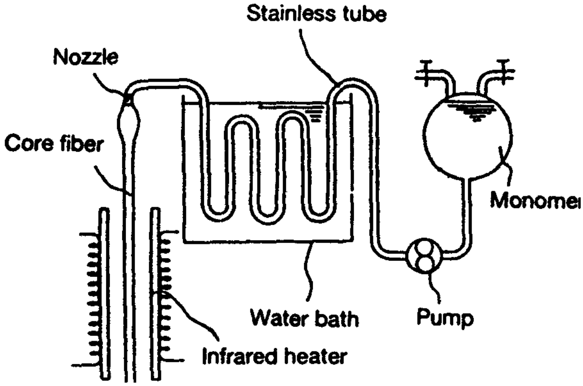

Fig. 5. Process for manufacturing thermosetting resign core POF.

Sheet-type plastic optical fiber

A sheet-type plastic optical fiber is used as an optical line sensor and image guide for reading drawings and detecting defects. It is manufactured as follows. The core and sheath materials are spun through a composite spinning nozzle in the same way as the single filament plastic optical fiber except that the spinning nozzle has many circularly arranged orifices. Many fibers are brought near to each other along the spin-guide and are stuck together in the form of a circular arc on a sticking guide located slightly apart from, and directly below, the spin-guide. They are then passed through an arranging guide located below the sticking guide and the fibers arranged in a straight line are drawn off by the nip rolls. Since the transit distances of the fibers from the spinning nozzle to the spin-guide are equal, uniform sheet type optical transmitters can be manufactured without deformation by processing.

An image fiber composed of a bundle of many ultra-fine optical fibers has been used for endoscopes and the like.

Since multi-component glass and quartz are brittle, further development of plastic image fibers is expected. In 1988 Mitsubishi Rayon developed and commercialized a plastic image fiber composed of a bundle of approximately 1500 stepindex type plastic optical fibers 10-20 µm in diameter based on their precise composite melt spinning technique. Following further programs for thinning the fiber and improving the resolving power, a plastic image fiber composed of a bundle of approximately 3000 ultrafine optical fibers 10 µn or less in diameter was developed in April 1991. It reveals that the circular plastic image fiber with a diameter of approximately 0.5 mm has approximately 3000 plastic optical fibers very closely packed.

Each fiber corresponds to one picture element and is approximately 9 mm in diameter. The diameter of core and the thickness of sheath are approximately 7 mm and 1 mm, respectively. The plastic image fiber is characterized by higher flexibility, greater flexing resistance and resolving power and a brighter image than those of a glass image fiber. The modulus of elasticity of the plastic image fiber is 1/10 or less that of a quartz image fiber and it is very flexible. The brightness of image is expressed by:

$$E = FK_e \qquad [1]$$

where E = brightness of image, F = performance index of the plastic optical fiber constituting a plastic image fiber and K, = ratio of core area to the total cross-sectional area of fiber. Using fibers with the same K, , E is proportional to F. F is expressed by the following formula:

$$F = (NA)2\ 10(-Od/10) \qquad [2]$$

where NA = number of apertures, a = transmission loss and I = length of fiber. Although the plastic image fiber has a number of apertures as large as 0.5 and the transmission loss is as high as 600 dB/km, the plastic image fibers that are only a few metres long have larger F values than those of image fibers composed of other materials. The molten core material fed to the nozzle is divided into core-forming plates and the sheath and intervening materials are fed to the circumference of the core inside the nozzle.

A fiber composed of concentrically arranged core-sheath intervening components is discharged from the outlet and integrated into a uniformly arranged plastic, image fiber at the integrating nozzle. Since organic optical fibers have special features for short-distance optical transmission media including low transmission loss and high heat resistance, various applications are expected.

Nine
Textile Impacts of Impulsive Forces

The impulse of a force is defined as the product of that force and the time for which it is acting. The importance of the time of action of a force is best illustrated by means of an example. Suppose that a man with a mass of 80 kg jumps from a wall 5 m in height and lands on the ground with a velocity that simple calculation puts at about 10 m/s.

Just before impact, momentum is given by:

$$\text{momentum} = mv = 80 \times 10 \text{ m kg/s.}$$

Momentum after impact is zero, so that:

$$\text{momentum change} = 800 \text{ m kg/s}$$

i.e.:

$$\text{impulse} = 800 \text{ m kg/s} = 800 \text{ Ns.}$$

If the man's legs are kept stiff, he is brought to a halt very quickly, whereas he is brought to rest more slowly if he flexes his knees during the landing. Let us assume that the motion is arrested in 0.1 s in the former case and 1 s in the latter.

Impulse is the product of force and time of action, so that the forces in the two cases are given by:

$$800 = F_1 \times 0.1 \text{ for stiff knees;}$$

$$800 = F_2 \times 1.0 \text{ for flexed knees.}$$

Thus, with stiff knees, the force of impact is 8000 N, whereas flexing reduces the impact force to 800 N. To anyone who has jumped from a high wall with stiff legs this explanation will save the trouble of asking the hospital staff what happened. It is obvious from the previous example that, for a given change of momentum in a body, the magnitude of the force necessary depends on the time for which it is allowed to act.

The shorter the time of action, the greater is the force required to produce the impulse, and, for a blow of very short duration, an extremely high force is required. Such a force is known as an impulsive force, but it is impossible to specify exactly what is meant by 'very short duration'. An impulsive force is commonly defined, in rather vague terms, as a force applied for such a short time that the resulting change in momentum takes place practically instantaneously.

Obvious examples are the blow from a hammer, the jerk on the coupling as a train starts, or, in textiles, the blow given to a loom shuttle by the picker stick. It is normally impossible to measure directly the time of action and the magnitude of the blow, so that an impulsive force is usually described in terms of the momentum change that it produces.

The occurrence is known as an impact, and this is usually defined as the entire process from just before the contact takes place to just after the bodies have separated (or, if they do not separate, to the time at which their subsequent movement is uniform). Thus, the effects of acceleration before contact or of friction and air-resistance afterwards are not normally regarded as a part of the phenomenon of impact.

True impact conditions are seldom found in textiles; the blow causing shuttle-picking is one of the few obvious ones. In other apparent cases of impact, there are usually other effects present, such as the action of the swell in stopping the shuttle or of the check band in stopping a mule carriage. As a result, problems involving short-lived forces in textiles are frequently very difficult to solve, and considerable oversimplifications have to be made in an elementary book of this kind.

The time of application of the force on A must be equal to the time of application of the force on B. Thus, the magnitude of the impulse on A must be equal but opposite to the magnitude of the impulse on B, and, since impulse is equal to change in momentum, there can be no change in momentum, as a result solely of the collision, because the net impulse is zero for that instant only. This is an example of the principle of conservation of momentum, which states that, if no external forces act on a system of colliding objects, the total momentum of the objects remains constant. At the instant of collision, the force acting between the two bodies causes deformation, and, during this stage, the centres of gravity approach one another until the balls have the same velocity, V.

The work needed to bring about this deformation must come from somewhere and is, in fact, derived from the kinetic energy of the moving bodies. The same source supplies the energy required to produce any noise or heat generated by the collision so that, even though momentum remains constant, there is a slight but significant loss in the kinetic energy of the system. In practice, however, we can normally ignore this loss for calculation purposes. In the example given, we can therefore use the principle of conservation of momentum to relate the velocities before, during, and after collision. Thus:

initial momentum $= m_1u_1 + m_2v_1$;
momentum during collision $= (m_1 + m_2)V$;
momentum after collision $= m_1u_2 + m_2v_2$.

Each of these three terms can be regarded as equal if, and only if, the collision is a perfectly elastic one, a condition that is impossible to achieve in practice. It is time now to examine what happens in normal cases.

Coefficient of restitution

In a perfectly elastic collision, separation of the two bodies begins to take place immediately they acquire their common speed V. The strain energy that has been used in producing the contact deformation is released as the correct shapes are restored and is converted back to kinetic energy. In such a case, then, there is no loss of energy, but this ideal situation never occurs in practice. With most imperfectly elastic bodies, there is a smaller change in velocity (and hence in momentum) than would be the case if perfect elasticity existed between the colliding objects. The deviation from perfect elasticity is measured by means of the *coefficient of restitution, e.*

This constant is defined, in any given collision, as the ratio of the change in velocity or momentum of either body to the change that would take place if elastic conditions prevailed. Suppose, for example, that three balls are dropped from a height of 1 m onto the ground The first one, which is perfectly elastic, will rebound to a height of 1 m. The polymeric compound known as 'silly putty' is an example of an almost perfectly elastic material and has caused either delight or anguish, depending on whether it soars into the air or cannons through a window, to many thousands of children.

The second ball (made, perhaps, from clay) is perfectly inelastic and does not rebound at all from the ground but merely lies there after dropping with a dull thud. The third one represents normal behaviour in the vast majority of such cases.

Suppose that the coefficient of restitution, e, is 0.8. Since the ball is released from a height of 1 m with no velocity and falls freely, we can use the equation:

$$v_2 = u_2 + 2fS$$

to calculate for the impact velocity, v_1, a value:

$$v_1 = \sqrt{(2g)},$$

where g *is* the acceleration due to gravity.

The rebound velocity after impact, v_2, is given by:

$$v_2 = ev,$$
$$= eV(2g).$$

The ball now rises at a decreasing velocity, because of gravitational deceleration, until its velocity becomes zero again, and the height that it reaches in the process is found, by using the same equation of motion:

$$v_2 \quad u_2 + 2fS,$$

to be given by:

$$S = v_2^2/2g$$
$$= e\sqrt{/(2g))^2/2g}$$
$$= e^2.$$

Since e is 0.8 in this case, then, the ball must rise to a height of 0.64 m on rebound.

Where the velocity of B (with mass m_1) changes from u_1 to u_2 and the velocity of A (with mass m_2) changes from v_1 to v_2, we can assume a transfer of momentum M from A to B, during the initial collision, to cause both balls to move at their common velocity V.

The initial momentum of A, m_2v_1, loses a quantity of momentum M in the process, so that:

$$m^2V = m_2v_1 - M.$$

Similarly, B gains this momentum, so that:

$$m_1 V = m_1 u_1 + M.$$

Adding the two equations gives

$$(m_1 + m_2)V = m_1 u_1 + m_2 v_1.$$

If the collision is perfectly elastic, the transfer of momentum during separation is equal to that during approach, so that:

$$m_2 V = m_2 v_1 - M.$$

Similarly, B gains this momentum, so that:

$$m_1 V = m_1 u_1 + M$$

Adding the two equations gives

$$(m_1 + m_2)V = m_1 u_1 + m_2 v_1$$

If the collision is perfectly elastic, the transfer of momentum during separation is equal to that during approach, so that:

$$m_2 v_2 = m_2 v_1 - 2M;$$

$$m_1 u_2 = m_1 u_1 + 2M.$$

Thus, by adding, we obtain:

$$m_2 v_2 + m_1 u_2 = m_2 v_1 + m_1 u_1.$$

If the collision is perfectly inelastic, there is n(transfer of momentum after impact, so that the two balls continue to move together at velocity V.

If a coefficient of restitution e exists for the collision, then simple calculation shows that:

$$m_2 v_2 = m_2 v_1 - M(1 + e);$$

$$m_1 u_2 = \mathrm{m}_1 u_1 + M(1 + e).$$

Again, by adding, we obtain:

$$m_2 v_2 + m_1 u_2 = m_2 v_1 + m_1 u_1.$$

Thus, in all three cases, momentum is conserved in accordance with the principle which says that it should be. The kinetic energy of the balls, however, is not necessarily conserved, as may be seen in a calculation.

Example 1

A sphere of mass 20 kg, moving at 8 m/s, overtakes and collides with a second one of mass 40 kg, travelling at 5 m/s along the same straight line. Find the velocities of the two after impact and the kinetic-energy loss in each case:

a) if collision is perfectly inelastic;
b) if collision is perfectly elastic;
c) if the coefficient of restitution is 0.5.

If V is the common velocity when the spheres are moving together and M is the momentum transferred, then:

$$20V = 20 \times 8 - M.$$

and

$$40V = 40 \times 5 + M.$$

i.e.:

$$60V = 360$$

and:

$$V = 6 \text{ m/s}.$$

Substitution in either of the original equations gives:

$$M = 40 \text{ Ns}.$$

Total K.E. before impact is given by:

$$K_, = \frac{1}{2} \times 20 \times 8^2 + \frac{1}{2} \times 40 \times 5^2 = 1140 \text{ J}.$$

a) For a perfectly inelastic collision, the two spheres continue to move together at a velocity of 6 n^{-1}/s. Total K.E. after impact is given by:

$$K_2 = ½ (40 + 20) \times 6^2$$

$$= 1080 \text{ J},$$

i.e., velocity of both spheres after impact *is* 6 m/s, and loss in kinetic energy = 60 J.

(b) For a perfectly elastic collision, suppose that the velocities of the spheres after impact are V^2 (for the 20-kg one) and u^2 (for the 40-kg one). Then:

$$20\ v_2 = 20 \times 8 - 2M$$

$$40\ u_2 = 40 \times 5 + 2M.$$

Substituting $M = 40$ in these two equations gives:

$$v_2 = 4 \text{ m/s},$$

and

$$u_2 = 7 \text{ m/s},$$

i.e., the smaller sphere moves at 4 m/s and the larger one at 7 m/s after impact.

Total K.E. after impact *is* given by:

$$K_2 = ½ \times 20 \times 4^2 + ½ \times 40 \times 7^2 = 1140 \text{ J},$$

i.e., there is no loss in K.E. on impact.

c) If the coefficient of restitution *is* 0.5, then:

and

$$40\ u^2 = 40 \times 5 + M(1 + 0.5).$$

Again, substituting $M = 40$ gives:

$$v^2 = 5 \text{ m/s}$$

and

$$u_2 = 6.5 \text{ m/s},$$

i.e., the smaller sphere moves at 5 m/s and the larger one at 6.5 m/s after impact.

Total K.E. after impact is given by:

$$K_2 = \tfrac{1}{2} \times 20 \times 5^2 + \tfrac{1}{2} \times 40 \times (6.5)^2$$
$$= 1095 \text{ J},$$

i.e., loss of K.E. on impact = 45 J.

Impulses and impacts in textiles

Powered impacts

It might be thought that the only impulse of value to the textile technologist is the one to kick the recalcitrant machine or the stupid foreman; let us hope that such uncivilized behaviour is under control and examine some other impulses. Even though conditions of impact are not common in textile processing, the few that do exist are of some importance, and some of them are worth a closer look.

In general, they can be classed as powered impacts, where the contacting components are thrown together by an externally applied force; gravitational impacts, where gravitational attraction causes one of the components to drop onto the other, and textile impacts, where at least one of the components is a textile structure, and the impact is likely to affect its performance. The first two types of impact often involve textile structures too, but an attempt to distinguish between the three types will be made. The most obvious case of powered impact is, ot course, in the behaviour of the shuttle.

The blow imparted to it by the picking stick at the beginning of its travel is visible (and audible!) evidence of an impulsive force. At the end of its travel, the shuttle undergoes a second impact as it strikes the picker, though the effect is modified somewhat in this case by the effect of swell friction in absorbing some of the energy of motion. One result of these two collisions, taking place as they do in opposed directions, is to waste an appreciably large quantity of energy.

The high energy that must be imparted to the shuttle to ensure its correct flight is almost completely retained at the end of the travel, since little is dissipated in overcoming the frictional resistance of the air to movement of the streamlined shuttle. At the end of the flight, the shuttle comes to rest, so that all the energy remaining must there be dissipated, partly by the impact with the picker, partly by overcoming swell friction, and partly by the energy absorbing strain of the check strap. It is possible, by ignoring the two last-mentioned factors at the moment of impact, to estimate the energy loss on initial collision.

Example 2

A shuttle of mass 0.5 kg, moving at a velocity of 10 m/s, strikes a picker of mass 0.15 kg, connected by means of a check strap of mass 0.4 kg to the identical picker at the other end of the sley. Assuming shuttle and pickers to move together after the impact, and ignoring the effect of swell friction or check-strap extensibility, find the velocity with which shuttle and picker strike the box end. How much energy is lost in the impact of shuttle and picker?

We have:

$$\text{momentum before impact} = 0.5 \times 10$$

$$5 \text{ N s;}$$

$$\text{total mass after impact} = 0.5 + C.4 + 2 \times 0.15 \text{ kg}$$

$$= 1.2 \text{ kg;}$$

and

$$\text{momentum after impact} = 1.2v.$$

where v is the common velocity of shuttle and picker.

By the principle of conservation of momentum:

$$5 = 1.2v,$$

$$v = 4.17 \text{ M/s}.$$

Thus velocity at which box end is struck *is* 4.17 m/s.

K.E. before impact is given by:

$$K_1 = ½ \times 0.5 \times 10^2 = 25 \text{ J}.$$

K.E. after impact is given by:

$$K_2 = ½ \times 1.2 \times (4.17)2$$

$$= 10^{-43} \text{ J},$$

i.e., loss of K.E. on impact = 14-57 *J*.

Other examples of powered-impact situations can be observed in weaving. During the beating-up operation, the loom sley moves rapidly towards and away from the fell of the cloth, and, although the rocking motion is positively controlled by the rotation of the crank, the speed with which the reed is driven against the weft is usually high enough to justify the assumption that impact conditions are present to some extent. The same assumption can be made in connexion with the shedding process.

Again, heald movement is controlled by the tappets, but the speed of lifting and failing of the healds ensures that some degree of impact occurs in the mechanism. It is difficult to establish in a general way exactly where this impact takes place for a particular loom design, but the familiar crashing sound that accompanies heald movement indicates that the manufacturer's attempts to reduce impulsive forces to a minimum are not entirely successful. Similar instances may be found in other processes of textile manufacture. The high speed of operation of modern knitting machines ensures that needles and cams come into contact forcefully, despite the effort to reduce the severity of impact by careful cam design.

As a result, chatter and wear are often encountered in this part of the machine.

The demise of the mule in woollen spinning, mainly as a consequence of its inefficiency, was assisted to some extent by the waste of energy incurred when the carriage strikes the stop in a noticeable impact. The student will no doubt be able to identify other impact phenomena in the mill; to plagiarize an old cliche he merely has to follow his ears!

Gravitational impacts

This type of impact occurs when an object, previously lifted to a given height above its rest position, is subsequently allowed to fall freely, under the influence of gravity, until it strikes a restraining surface. Naturally, the greater the length of free-fall path, the higher is the impact velocity and the greater the impulsive force; the mass of the object, too, will influence the magnitude of this force.

In this machine, the hammer is lifted slowly by the block on the rotating drum, but, when it reaches the top of its travel and slides from the block, it is free to crash down onto the fabric in the bowl and does so with a satisfying thud. The time of action of the force is increased to some extent by the fact that soft, compressible wet fabric is being struck, but the impact proportion of the resulting blow is still appreciable, as may be judged by carelessly leaving a thumb in the wrong place during operation of the hammer.

Once more, other examples may be identified in the manufacturing train. The return of needles or healds to their rest position, in knitting or weaving machines where these movements are achieved by gravity, is a common example, particularly on older machinery. The faller bars drop out of the drafting zone in a gill-box, and the near-empty shuttle drops from the magazine in an automatic changing mechanism. In these examples, of course, the magnitude of any impulsive force is minimized as far as possible, in contrast with the situation in fulling, and the impact is a waste, rather than a use, of energy, but the principle of gravitational impact can, in fact, be usefully applied in a somewhat different way.

It is frequently desirable to know the impact conditions in a piece of operating machinery where it is impossible to carry out any direct measurements. In such cases, it is possible to simulate the impact occurring when two surfaces collide at a given relative velocity by isolating the two surfaces and allowing one of them to fall, under gravitational attraction , into contact with the other one. The contact force, the rebound distance, or other such parameters may then be measured precisely and the effects of impact calculated. The advantage of the method, as the astute reader will immediately grasp, is the fact that the gravitational force at any particular location is sufficiently constant to ensure virtually perfect predictability and repeatability of experimental conditions.

Example 3

In order to simulate operating conditions, a shuttle is dropped from a height of 5 in and allowed to strike a block of material similar to that used for making pickers. If the shuttle rebounds 15 cm, what is the coefficient of restitution?

In the initial fall:

$$u = 0,$$

$$S = 5 \ m, \text{ and}$$

$$f = 9.81 \text{ m/s}^2.$$

Thus:

$$v^2 = u^2 + 2fS$$

$$= 0 + 2 \times 9.81 \times 5,$$

i.e.

$$v = 9.90 \text{ m/s}.$$

After rebound:

$$S = 15 \text{ cm} = 0.15 \; m,$$

$$f = -9.81 \; m/s^2,$$

$$v = 0.$$

Substituting in:

$$v^2 = u^2 + 2fS$$

gives:

$$0 = u^2 - 2 \times 9.81 \times 0.15,$$

i.e.:

$$u = 1.72 \text{ m/s}.$$

But:

$$e = \frac{\text{velocity after rebound}}{\text{velocity before rebound}}$$

$$= \frac{1.72}{9.90} = 0.174.$$

Hence coefficient of restitution = 0.17.

The student is again left to the devices of his own powers of observation for more examples of gravitational impact. In this kind of impact, the textile material plays a significant part, rather than just being present in a passive role to be hit as, for instance, in the fulling process just discussed. Apart from the use of impact in fulling, one or two cases where impact is necessary in processing may be noted. It should be pointed out at this stage that, in addition to the impulsive forces existing where two bodies collide together, there are other types of sudden stress that may be classed under the general heading of impact.

Compressive stress, where two equal, large forces are suddenly applied simultaneously to the two ends of a rigid body, can produce impact symptoms within the body. Similarly, tensile stress, where a rigid or flexible object is suddenly pulled simultaneously at both ends by-two equal forces of large magnitude, can induce certain characteristics of impact phenomena.

A detailed treatment of these two types of impact is, however, outside the scope of this book and the reader is referred, with a clear conscience and a sigh of relief on the part of the author, to one of the more advanced textbooks dealing with the subject. It is possible, nevertheless, to note these two situations in a qualitative way and to see how they affect textile structures. In the conversion of continuous-filament tow to staple-fibre top, successful breaking of the continuous strand is achieved by chopping, stretching, or abrading, and, in the first two cases, some degree of impact may be observed.

The sudden blow as the chopper blade strikes the tow, or as the tensile stress is rapidly applied, is sufficient to overcome the shear, or tensile, strength, respectively, of the filament bundle, and division into staple fibres thereby ensues. In certain textile test methods, for determining tensile, tearing, and bursting strengths, some degree of impact is present. Perhaps the most obvious example of this kind is the ballistic type of testing machine, in which a pendulum is held in a horizontal position and suddenly released.

The heavy bob is attached to a cloth specimen in such a way that a high force is exerted on the latter as the pendulum reaches its vertical position. The pendulum continues its swing and breaks the specimen, but the impact taking place checks its movement to some extent, and the work-to-break for the cloth sample can be obtained by measuring the maximum travel of the pendulum past the vertical position.

Even in other testing machines, however, where the load may be applied relatively slowly.

Perhaps less esoteric examples of impact make themselves known in cases where there is an undesirable risk of damage to the textile material. In the opening stage, the beaters live up to their name and thump the fibres mercilessly. At the card, the taker-in pushes the fibres quickly into the nip of the roller, and the doffer comb, at the other end, strikes the web rapidly as it vibrates. As the shuttle flies across the loom, the yarn that it carries is subjected to sudden stress, as also is the warp yarn as it is rapidly jerked upwards or downwards by the eye of the moving heald.

In the loop-forming process during knitting, a sinker is frequently used and is forced very quickly against the yarn. The tufting operation, in which the needles oscillate vertically at a high frequency, puts stresses repeatedly on the yarn. The small child, grabbing a shirt or blouse to pull it off in a hurry during a temper tantrum, tears a hole in it and is suitably chastised as well as being sent to bed early, the latter punishment having caused the tantrum in the first place.

Ten
Solution Spinning

Solution spinning are divided into two parts: Dry spinning is where polymer solution is solidified through evaporation of polymer solvent. Wet spinning can be divided further into three methods based on three different physicochemical principles: the liquid-crystal method, the gel method and the phase-separation method. In the liquid-crystal method, a liquid-crystalline solution of a lyotropic polymer is solidified through the formation of a solid crystalline region in the solution. In the gel method, polymer solution is solidified through the formation of intermolecular bonds in the solution. This phenomenon is called gelation. Gelation is caused by a temperature or concentration change in the solution.

In the case of phase separation, two different phases appear in the solution, one polymer-rich, the other polymer-lean. The phase diagram of a ternary system consisting of polymer, solvent and precipitating agent (non-solvent). The curve represents the boundary between a homogeneous, one-phase system (above the curve), and a heterogeneous, two-phase system (under the curve). In dry spinning, the spinning dope (SD) increases in polymer concentration along the line from SD to pure polymer.

In the region above the line A, the spinning dope decreases in polymer concentration and solidification never occurs. To the right of the line B, the polymer concentration increases and the spinning solution solidifies through the formation of a gel (or oriented crystals in the case of lyotropic polymer). In the region between the lines A and B, phase separation occurs, yielding polymer-rich and polymer-lean phases. The gel method and the liquid crystal method result in the formation of a solid homogeneous phase as the initial structure of the fiber, but in the case of phase separation, a heterogeneous two-phase system appears as the initial structure. This difference in structure significantly affects the choice of fiber-forming method.

Spinning technology of an acrylic filament yarn

Significant developments have been made in the field of phase equilibrium and phase separation of polymer solutions, which have led to a theoretical understanding of the spinning technology of acrylic fibers. The two component solution of polymer and solvent. If the temperature of a homo-geneous polymer solution at point E is lowered to that at point F, the solution becomes thermodynamically unstable, which leads to the generation of 'concentration fluctuation'.

The solution becomes visibly cloudy, so the point F is called the 'cloud point'. The curve which links cloud points is called the 'cloud point curve'. A further drop of solution temperature beyond the point F leads to separation of the solution into two phases, a polymer-lean phase (A) and a polymer-rich phase (B). It has been proved by Kamide and Manabe that this phase separation occurs according to the process called 'micro-phase separation'. In the polymer solution, critical nuclei are formed because of concentration fluctuation if the temperature of the solution is lowered to a temperature below the cloud point.

The critical nuclei grow to larger particles called 'primary particles'. If the polymer concentration of the solution (V^{o}_{x}) is lower than the concentration at the critical solution point (V^c_x), the polymer-rich phase is a dispersed phase. On the other hand, if V^o_x is higher than V^c_x the polymer-lean phase becomes the dispersed phase. Afterwards these primary particles collide and amalgamate with each other and grow to larger particles called 'secondary particles', which further amalgamate and fuse to create the fiber structure. Such micro-phase separation phenomena can be seen only in the case of a three component solution of polymer, solvent and nonsolvent. A phase diagram of such a three component system comprising poly (acrylonitrile/ methyl acrylate) copolymer, solvent (strong aqueous nitric acid solution) and nonsolvent (water).

If a dilute aqueous nitric acid solution (coagulant) E is added to a spinning solution F, the composition of a mixture changes along the line connecting F and E, crossing the cloud point curve at tile point marked by a white circle. As the polymer concentration at this point is lower than that at the critical solution point, marked by a dotted white circle, the polymer-rich phase forms a dispersed phase. On the other hand, if a more dilute aqueous nitric acid solution (coagulant) D is added to the solution F, the concentration of the resulting mixture changes along the line connecting F and D, crossing the cloud point curve at the point marked by an open square.

As the polymer concentration at this point is higher than that of the critical solution point, in this case the polymer-lean phase forms a dispersed phase and the polymer-rich phase forms a continuous phase. When the concentration of coagulant increases, at first dr_{max} decreases gradually and then it increases rapidly after reaching a minimum. This minimum concentration is called the critical concentration. Generally, the method of spinning with coagulant concentration below this point is called 'low concen-tration spinning' while that with coagulant concentration above this point is called 'high concentration spinning'. When spinning is carried out in the low concentration region, a continuous layer (skin) appears on the surface of the spun dope, then the nonsolvent in the coagulant diffuses through this skin into the spun dope inducing coagulation inside it and a change in volume. As the skin is rather rigid, it cannot deform in proportion to the shrinkage of the inside volume caused by the amalgamation of polymer particles. This is the reason macro-voids are formed inside the fiber. The more the concentration of the coagulant decreases, the greater the number and the larger the voids that are formed inside the fiber. These voids often induce opacity in a fiber. If the elongation and drying of a fiber are properly carried out, a transparent fiber without voids can be obtained. On the other hand, if spinning is carried out in the high concentration region, the coagulated fiber does not have a skin but has a structure formed uniformly by aggregation of polymer particles.

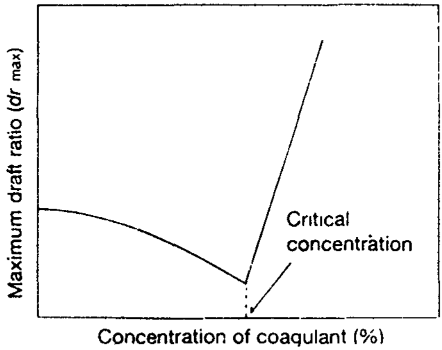

Fig. 1. Dependence of maximum draft ratio on concentration of coagulant

In this case, coagulation does not proceed as rapidly as in the case of low concentration spinning, but because of the lack of skin, both solvent and nonsolvent diffuse smoothly between the inside of the fiber and the external coagulant, which makes the fiber structure uniform. As the phase separation occurs in the coagulant at a higher concentration of solvent than in the case of low concentration spinning, the polymer particles contain a larger amount of solvent and are more easily elongated and fused with each other under extensional forces in the spinning process than in low concentration spinning.

However, high concentration spinning has some defects. Firstly, a fiber of this type has very low tenacity in the coagulation bath and is easily broken by extensional forces. Secondly, a larger amount of Polymer comes out into the coagulant than in low concentration spinning. In the history of the development of acrylic fibers, the manufacturing method for staple fiber was developed first, then that for filament yarn on the basis of the manufacturing technology for staple fiber. The manufacturing technology for staple fiber is that of the manufacture of fibers in the form of a tow which has a total denier of several tens of thousands, so the spinning velocity is generally less than a few hundred meters per minute. On the other hand, much effort has gone into development of highspeed spinning of continuous-filament yarn, which will now be discussed.

Spinning technology of an acrylic filament yarn

There are two methods of spinning an acrylic fiber. One is an immersed-jet method where spinning dope is extruded through a spinneret which is immersed in a coagulation bath. The other is a dry-jet wet spinning method (or air-gap method) where a dope is extruded through a gaseous atmosphere from a spinneret which is set above a coagulant bath. In the case of the immersion method, the spinning dope starts coagulating immediately after it is extruded from a spinneret, so it is difficult to draft filaments in the coagulation bath at a high rate, especially in the case of low concentration spinning.

On the contrary, it is easy to draft extruded dope at a high rate while it is in the air in the case of the air-gap method. In addition, filaments made by this process have a smooth and glossy surface compared with those made by the immersion process. The manufacturing technology for acrylic filament yarn has been developed on the basis of these two spinning technologies.

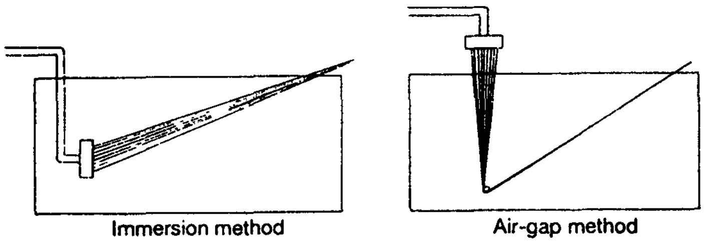

Fig. 2. Spinning methods for acrylic fiber

Immersion method

A spinning dope made of polyacry-lonitrile or an acrylonitrile copolymer (hereafter referred to as PAN) dissolved in *ca* 70% nitric acid solution is extruded into a coagulant bath containing ca 30% aqueous nitric acid solution at -3°C. After coagulation, the fiber is drawn out from the coagulant bath by means of a set of rollers (14) rotating at 5-10 m/min. After complete removal of solvent in the washing bath (6), the fiber is passed by means of a set of rollers (15) to an elongation bath (7) which is filled with hot liquid or steam. The fiber is elongated at least fourfold between rollers (15) and (16). The fiber thus oriented is then dried, first in a dryer (9) after treatment with oil or size, then in a second dryer after being treated with finishing agent, and finally wound up on a winder (13).

The reason for drying the fiber in two stages is to obtain good mechanical properties. In the first dryer, the fiber shrinks by 5-10%; in the second dryer it is elongated by 0-10%. By controlling the shrinkage and elongation rate, good mechanical properties can be obtained. The characteristics of the fiber obtained by this process are an excellent lustre and a handle like that of silk. This silky handle is derived from micro-stripes on the fiber surface. This fiber has high tensile strength and low tensile elongation.

Air-gap spinning method

A spinning apparatus utilizing the air-gap spinning method developed by the Monsanto company Spinning dope made from a 26% PAN solution in N,N"-dimethylacetainide (DMAc) is extruded from the spinning nozzle (13) into the air. After running for 1.3 cm in the air, it enters a coagulant bath containing 50% aqueous DMAc solution. The fiber is elongated in the coagulant bath to some extent by the tension applied by a roller (22), then removed from the coagulation bath at a velocity of 9.3 m/min.

Then it is elongated 6.1 times in an elongation bath containing hot water at 100°C. The fiber is cleared of solvent with hot water at 50-80°C while it runs 30-40 times over rollers (28 and 30). The fiber is dried on drum dryers (46 and 47) after being treated with oil in a bath (45). The fiber velocity on the drums is about 46 m/min. The characteristics of this process are air-gap spinning and drying without tension after shrinking the elongated fiber in hot water. By this process, a fiber with a smooth glossy surface and high toughness can be obtained.

Highspeed spinning of an acrylic filament yam

Technological requirements

The following requirements should be fulfilled in order to achieve highspeed spinning in the manufacture of an acrylic filament yarn:

1. *Rapid coagulation.* In acrylic fiber manufacture the fiber is elongated by up to several tens of times, after

coagulation, to obtain appropriate mechanical properties through molecular orientation and densification. The fiber velocity in the coagulation bath is not so high as that for a regenerated cellulose, which is not substantially elongated after coagulation if the same winding velocity is used, but much higher speed is necessary compared with a conventional acrylic spinning method.

The following are measures required to achieve high speed coagulation:

a) In the case of an immersion method:
   i) High velocity of spinning dope at extrusion when spinning in a low-concentration spinning region.
   ii) High velocity of spinning dope at extrusion and a large draft ratio when spinning in a high-concentration spinning region.

b) In an air-gap spinning method, high velocity of spinning dope at extrusion and a large draft ratio.

   Resulting from these measures, the maximum velocity of a fiber in a coagulation bath is only a few tens of m/min in the case of low-concentration spinning utilizing an immersion method due to the increase of pressure behind the spinneret, an increase of the fiber tension in the coagulation bath, and low dr_{max}. Consequently, the other two methods have been employed in the development of high-speed spinning.

2. Rapid removal of the solvent. As mentioned before, elongation of the fiber after coagulation is indispensible in acrylic fiber spinning in contrast to regenerated cellulose-fiber spinning. This permits relatively low fiber speeds in the coagulation bath for acrylic fiber spinning but makes it difficult to utilize a net conveyer system after removing the solvent.

   The following are the measures for rapid removal of solvent using highspeed spinning:

a) Low resistance of washing liquid and high efficiency of washing in a washing process such as the Hoffman type (straight type), or

b) sufficient retention time of fiber on Nelson-type rollers.

The former method is advantageous from the viewpoint of saving space and cost of equipment, but it is difficult to balance the washing effect with the resistance of the washing liquid. The latter method is advantageous from the viewpoint of balance but requires larger and more expensive equipment. From the viewpoint of mechanical properties it is desirable to dry the fiber at low tension. The following measures decrease fiber tension while drying:

1. Low applied tension while drying on a drum-type dryer and during any subsequent relaxation of the fiber to reduce residual tension.
2. Drying the fiber on Nelson rollers, if necessary with tapering rollers.
3. Drying the fiber without tension on a net conveyor.

The highspeed spinning apparatus for spinning in the high concentration region using an immersion-spinning method of the Monsanto Company. The following is an example of spinning conditions used in this process. A spinning dope made from a 25% PAN solution in DMAc is extruded into a coagulant bath containing 83% aqueous DMAc solution at 30°C. and is drawn out from the bath by means of the rollers R-1 and R-2 in an air atmosphere. The solvent is removed from the fiber on the rollers R-2 and R-3 with deionized water at 70°C.

The fiber is elongated with steam at 105°C (8) between washing rollers. The fiber is then elongated again (8'), which is heated electrically at 250°C, treated with a finishing agent, dried on the rollers R-4, and wound up on a bobbin. In this process the tension of the running fiber is controlled at an appropriate level, especially during washing and drying when it is kept at a low level. If the fiber is elongated in the tube 8', a fiber with high strength and low elongation is obtained, but if the fiber is shrunk in this tube a fiber with low strength, high elongation and low shrinkage in boiling water is obtained. Additionally, the shrinkage in boiling water is lowered if the fiber on the bobbin is treated with steam. Spinning speeds up to 1000m/min are available using this process.

Highspeed spinning with an air-gap method

The spinning dope, a PAN solution in DMAc, is extruded downwards from the spinneret, falls 1-20 mm in air and enters a coagulant bath containing aqueous DMAc solution (for example, 70% at 40°C). After coagulation the fiber is washed with hot water, elongated 1.8-4.5 times in boiling water, and then elongated 6-12 times with steam at 2-4 kg/cm^2. After the elongation, the fiber is dried and then shrunk using a hot roller, a hot plate or high pressure steam. The first elongation, with boiling water, has a great influence on the second elongation, with high pressure steam.

Unless the fiber is elongated to the appropriate extent in the first stage, a high elongation rate cannot be obtained in the second stage. If the fiber is elongated 2-4 times in the first stage, a high rate of elongation is possible in the second stage and a maximum velocity of 2160 m/min can be obtained. By employing an air-gap method, the velocity of the spinning dope before coagulation can be increased, but the high resistance of the coagulant liquid has to be overcome.

If the resistance is too high, the fiber breaks in the coagulant bath, so high speed cannot be obtained. In this apparatus the coagulant liquid, introduced through an inlet (7), flows down through a spinning funnel (3). In this funnel, excess coagulant liquid overflows into an outer funnel (6) and flows out through an outlet (8), maintaining the liquid head at a desired level. The velocity of the coagulant liquid in the straight part of the funnel can be controlled by changing the inner diameter of the funnel. This downward flow of the coagulant liquid decreases its resistance to the fiber compared with a static coagulant bath, so the fiber is elongated and coagulated smoothly. By employing this spinning funnel and a net (mesh) conveyer for drying, a spinning velocity of 1000-2000 m/min can be obtained. Table 1 shows the mechanical properties of a conventional acrylic filament manufactured by an immersion process in the low-concentration region and that of a filament manufactured by the highspeed spinning process with an air-gap method.

The fiber obtained from the highspeed process has higher tensile elongation, knot strength and abrasion strength and lower shrinkage in boiling water than the conventional fiber. These advantages of the fiber from the highspeed process are the result of employing an air-gap method and drying without tension on the net conveyor.

Table 1

Properties of fiber from the conventional spinning method and the highspeed spinning method

	Conventional	*Highspeed spinning*
Tensile strength (g/d)	5.1	3.0
Tensile elongation (%)	13.9	27.8
Knot strength (g/d)	2.6	10
Knot elongation (%)	7.5	25.2
Maximum twist number (t/m)	1092	1928
Shrinkage in boiling water (%)	6.8	1.5
Fibrillation grade	2nd	5th

The micro-stripes are thought to be formed through depression of voids in the fiber which are common in fibers made in the low-concentration region when the fiber is elongated in the coagulant bath. On the other hand, the fiber made in the highspeed spinning process is elongated mainly in air just after extrusion from the spinneret and only slightly in the coagulant bath. This is thought to be the reason why this fiber has a smooth surface.

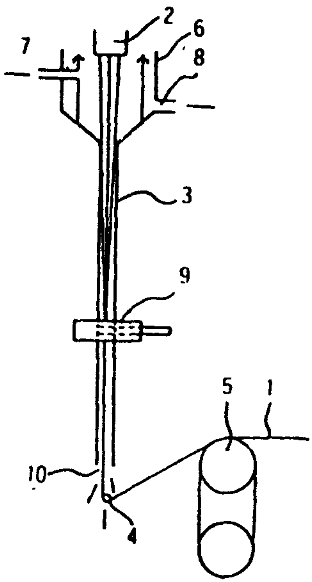

Fig. 3. Spinning funnel for the air-gap method

Acrylic filament yam has been used rarely in the high fashion field. However, as the highspeed process has realized large improvements both in quality and cost, it will in future be widely used in general textile and industrial fields, like regenerated cellulose filament yarn which is already used in such fields.

Spinning technology of Bemberg rayon

The industrial technology for manufacturing Bemberg rayon (ISO term cupro, FTC term cupra, i.e. cuprammonium-regenerated cellulose; hereafter referred to as Bemberg rayon) was developed in 1918 by J P Bemberg in Germany. Thereafter, many companies worldwide introduced this technology and started the production of Bemberg rayon. Nowadays only a few companies, including Asahi Chemical Industry in Japan and Bemberg SpA in Italy, continue to produce Bemberg rayon. This is due to its lack of competitiveness against viscose rayon arising from the use of expensive materials such as copper and ammonia to dissolve the cellulose. In manufacturing Bemberg rayon, cotton linters are used as the raw material.

Cotton linters are short fibers which are rubbed off cotton seeds after cotton fiber is cut from them. Refined linters are obtained by boiling raw cotton linters with alkali solution and then bleaching them. The refined linters are characterized by a narrow distribution of degree of polymerization, chemical purity and low content of oxidized groups. Copper hydroxide is dissolved by aqueous ammonia, forming a complex salt (tetra-ammonium copper hydroxide). The refined linters are added to copper ammonium solution which contains copper hydroxide as a precipitate. Cellulose forms a complex with tetra-ammonium copper hydroxide which dissolves in the solution.

Meyerio proposed the following chemical formula for the complexed product. The spinning technology of Bemberg rayon is based on the method developed by Thiele in 1901 in Germany. The spinning dope extruded from a spinneret flows down with hot water (coagulant) through a spinning funnel in the shape of a cone. In the funnel the spinning dope is coagulated gradually while being elongated several hundredfold and thus the fiber is formed.

Review of spinning methods

Hank-spinning method

The hank-spinning method is one of the oldest Bemberg spinning methods and is a batch process which uses a winding frame called a Hank in Gemany. The fiber on the Hank is taken off it in the shape of a ring and then regenerated and dried. The spinning speed is 40-80 m/min. The cellulose cuprammonium solution is extruded from a spinneret with a diameter of 0.6-1.0 mm into hot water (the spinning water) in a spinning funnel.

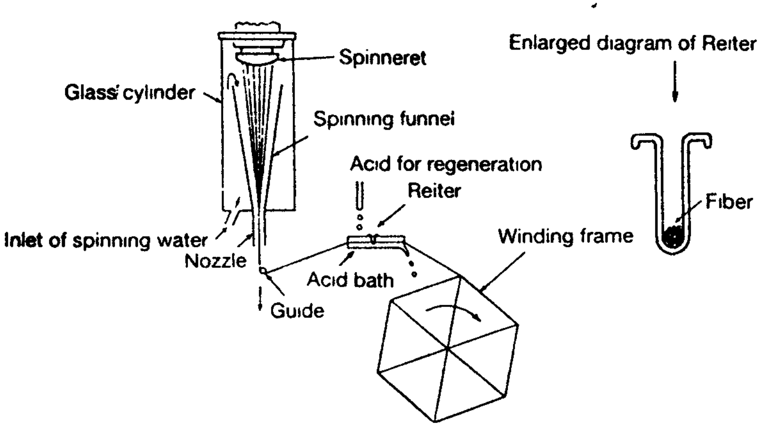

Fig. 4. Hank-spinning method

The extruded spinning dope flows down in the funnel, being coagulated and elongated at the same time. Generally, the degree of coagulation depends on the ratio of the ammonia and copper which is removed from the fiber to the amount of ammonia and copper initially present in the fiber. The changes in the velocity of the spinning water and the fiber in the funnel. In the upper part of the funnel, spinning dope extruded from the spinneret flows down under gravity, gradually growing thinner by elongation. During this time, ammonia diffuses into the hot water through the surface of the fiber and so coagulation starts from the surface. The elongation at this stage causes cracking of the surface layer, diffusion of ammonia from the new surface layer ensues and coagulation proceeds. Through repetition of these processes, coagulation and elongation continue. When the coagulation rate reaches a certain point, the fiber changes to an insoluble complex called a blue yarn. The viscosity of the fiber rises rapidly and it losses its fluidity.

In the middle section of the funnel, cellulose molecules, which still form a complex with copper, are oriented longitudinally by means of a winding force transmitted from the lower part of the funnel. In the lower part of the funnel, the fiber is elongated only slightly by the winding force. After the fiber leaves the funnel outlet, it is regenerated with acid and dried in later processes without substantial elongation.

The fundamental structure of the fiber, including the orientation of molecules and the degree of crystallization which control the fiber properties, is considered to be determined at the stage of the formation of the blue yarn. So the extent of elongation in the upper part of the funnel and the tension imposed on the blue yarn from its formation to the stage of regeneration have a decisive influence on the fiber properties. Bozza and Elsasser thoroughly researched this spinning process. Bozza expressed the differential increase of fiber velocity dvf by the following equation using fiber velocity, vf, at a distance x from a spinneret, tension imposed on the fiber at that point f, cross section area of the fiber q and viscosity μ.

$$\mathrm{d}vf = (1/3\mu)\ (f/q)\ dx$$

Elsasser determined the tension f and viscosity μ of the fiber at a series of distances from the spinneret by inserting $\mathrm{d}vf/\mathrm{d}x$ into this equation calculated from the diameter of the fiber in the funnel, which was measured directly. From these results he introduced the idea of 'optimum coagulation state' and concluded that the tension imposed on the fiber at the optimum coagulation state determines the strength of the fiber. According to his calculation, the fiber at this state has a viscosity of 56500 poise which equals that of rather soft asphalt.

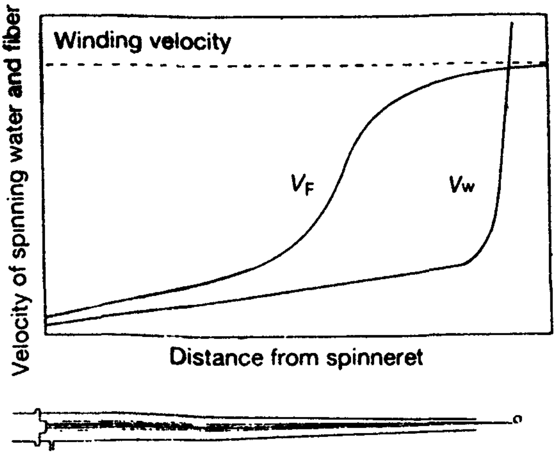

Fig. 5. Change of velocity of spinning water and fiber

Therefore, it is considered that the fundamental structure of the fiber is formed at an early stage of coagulation in this spinning method. The blue yarn which comes out from the bottom of the funnel with the spinning water changes its direction and is separated from the water by means of a guide set under the funnel, and is wound up on a frame after being treated with 6% aqueous sulphuric acid solution for final regeneration. This solution is poured on to the fiber on the frame during winding to remove ammonia and copper. The Hank yarn is manufactured without twist and is woven or knitted either without twist or after being twisted. There is a unique feature of the Hank process called Reiter colligation that makes it possible to handle Hank yarn without twist in the spinning process and the later processes.

The Reiter is a small instrument set just before the apparatus for sulphuric acid treatment. When the blue yarn comes into contact with the sulphuric acid solution, rapid regeneration occurs with generation of active OH groups and elimination of water from the yam. If filaments in the yarn are in close contact with each other, they bond to each other by means of chemical bonds based on OH groups. The Reiter with an appropriate curvature at its bottom for the passage of the yarn is set at the point where regeneration of the yarn takes place most intensively in order to get the filaments into close contact with each other and bind them with OH bonds.

These OH bonds are formed intentionally so that they ensure that yarns can be handled easily in the manufacturing processes during spinning, knitting or weaving. At the same time they also ensure that filaments can move freely enough apart in the fabric to give it soft touch and good uniform appearance. For this reason, the Reiter colligation should be reversible. The extent of colligation is controlled by changing the position and the curvature of the Reiter.

The yarn wound on the Hank is then conveyed to a regeneration apparatus where it is further regenerated, washed thoroughly for several hours, treated with finishing oil and then dried in a dryer for several hours. In the Hank process, the spinning speed is restricted by two factors. One is the use of a Hank frame for winding. If the spinning speed is increased, the increased tension of the yarn tightens the yarn wound on to the frame and binds the filaments together both within the yarn and between the yarns, which leads to insufficient regeneration and drying. The. other restrictive factor is the spinning condition in the funnel. These resrictions are overcome by the following continuous spinning method.

Continuous-spinning method

Two types of continuous-spinning apparatus, the Hoffman type and the Duretta type, are known thus far. The main differences between them are in the regeneration and drying processes. Only the Hoffman process will be described here. In the Hoffman apparatus, the fiber, which comes out of the funnel, runs straight through the regeneration stage and the drying apparatus and is wound up continuously on a winder. The spinning speed in this process is 100-150 m/min. One of the technological improvements which make a higher speed possible is the employment of the double spinning-funnel method. 15 In the single spinning-funnel method of the Hank process, an increase of spinning speed requires:

— Improvement of funnel dimensions.
— Changes in the amount and temperature of the spinning water.

However, if these conditions are changed to facilitate elongation at high speed, the fiber comes out of the funnel uncoagulated. This problem is solved in the double spinning-funnel method by separating the role of spinning water into coagulation and elongation. In this method, the temperature of the 'first' spinning water used for the upper funnel is lower than that in the Hank method, to facilitate elongation, but the temperature of the second spinning water used for the lower funnel is higher to ensure sufficient coagulation.

Additionally, in the lower funnel, turbulent flow is generated by mixing the first water and the second water; this accelerates coagulation. The change in velocity of the spinning water and the fiber and also the degree of coagulation in the funnel as a function of the distance from the spinneret. The degree of coagulation is very low up to the outlet of the upper funnel but rises rapidly after the fiber reaches the second water. If the temperature of the first water is too high, coagulation proceeds too far and copper hydroxide from the fiber sticks to the outlet of the upper funnel. This causes problems such as a change in velocity of the water and breakage of the fiber.

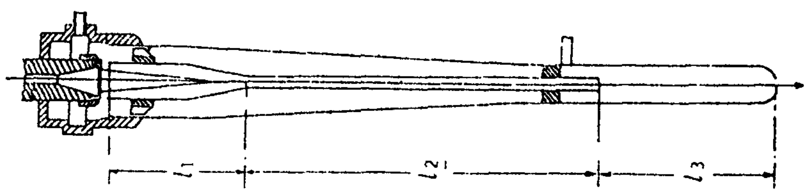

Fig. 6. Double spinning funnel

On the other hand, if the temperature is too low, insufficient coagulation causes problems such as breakage of the fiber and deterioration of fiber properties. The other technological improvement required for a continuous-spinning process at high speed is an acceleration of regeneration. In the Hank-spinning method, it takes many hours to regenerate and dry fibers as they are tied up closely in a bundle.

On the other hand, in the Hoffman process, the regeneration is finished in a few seconds because the fiber runs straight through a regeneration bath which, in any case, should not be too long because of the cost of the machine.

In order to accelerate regeneration, replacement of water around the fiber is important. It is also essential to keep the fiber structure in a state which facilitates diffusion of water and of ions such as sulphuric acid and copper. The sulphuric acid solution flows in the regeneration bath against the flow of the fiber. The water layer which covers the yarn is removed by suppressing and supporting dams. A most important point is not to keep the filaments in the yarn bound together but to keep them apart from each other in the bath.

The water layer can then be taken off smoothly and replaced with fresh water which has lower ammonium and copper ion concentration. This condition can be attained by ensuring that the coagulation level is sufficiently high before the fiber reaches the regeneration bath. In addition to these factors, it is important to keep the fiber structure, especially that of the surface, rather porous in order to accelerate diffusion of ions in the regeneration bath. The dependence of the degree of gloss, swelling and dye absorbancy on the concentration of sulphuric acid which the yarn encounters first in the process.

The dependence of residual copper concentration in the fiber on the concentration of sulphuric acid in the first acid bath. As the acid concentration increases, the copper concentration of the fiber just after the first acid bath decreases but that of the final product increases. This means that the acid concentration of the first acid bath should be kept low, for example lower than 0.5%, in order to attain a low level of copper concentration in the final product. The Hoffman-type spinning method is superior to the Hank-spinning method because it is continuous and permits higher spinning speed. However, it still has the following drawbacks:

- Low spinning speed: almost twice as high as that of the Hank process, but much lower than that of synthetic fibers. Higher speed brings about deterioration of fiber properties and leads to fiber breakage in the process.
- Frequent contact of the fiber with parts of the apparatus

causes filament breaks (generation of fluffs) and even yarn breaks.

— Problems with residual copper: even slight changes of spinning condition tend to cause increased residual copper which has a large influence on the dyeabllity of the fiber.

— Due to the regeneration and drying under high tension, the tensile elongation is small and shrinkage at the boil is large. These properties are sometimes useful during handling but restrict the fiber's range of use.

In order to overcome the drawbacks of this process and realize a much higher spinning speed, the following technological requirements should be fulfilled:

— Coagulation and elongation at much higher speed.

— Regeneration and drying under no tension or low tension.

— Continuous process from spinning to winding.

These requirements are satisfied by the following highspeed spinning method.

NP-type spinning method

Highspeed spinning technology in the NP method

A spinning speed of about 400 m/min is realized in the NP method. In the double funnel method of the Hoffman process, the following conditions lead to the highest spinning speeds:

— Lower temperature and larger amount of the first spinning water.

— Higher temperature and larger amount of the second spinning water.

However, as the spinning speed rises, the resistance of the spinning water to the fiber in the second funnel increases, the fiber properties deteriorate and many fluffs are generated. The limit of Hoffman double funnel is about 200 m/min.

the NP-type spinning apparatus. This apparatus comprises an upper funnel, an instrument called a CJ (short for 'coagulation jet') and a lower funnel. The upper part of the upper funnel contains a mesh for smoothing the flow of the spinning water in order not to entangle filaments just extruded from the spinneret and not to enhance coagulation. The shape of the upper funnel is designed so that the filaments can be highly elongated smoothly without enhancing coagulation.

The apertures, with diameters of several mm, open in a circle, the centre of which coincides with the centre of the running fiber on the undersurface of the CJ. The second spinning water is supplied from these apertures. No further spinning water is supplied to the lower funnel, but the first and second waters flow into it and mix there. A controlled head of water is maintained in this funnel. The change in velocity of the spinning water and the fiber and the degree of coagulation.

Regeneration, drying and winding technology in the NP method

It is not realistic to apply the regeneration and drying apparatus of the Hoffman process to highspeed spinning at several hundreds to 1000 m/min because of the high tension which will produce frequent fiber breaks and high boil shrinkage, and also increase investment cost. In the NP process a net (mesh) conveyer is employed to regenerate and dry fiber. After being treated with sulphuric acid the fiber is drawn by a DKR (double kick roller) down on to a reversing roller. The fiber is oscillated transversely to its direction by an instrument with an amplitude of several cm just before the DKR, so that the fiber on the roller forms an endless narrow belt with a width of several cm.

The fiber belt is transferred from the roller to the net conveyer upside down so that the fiber can be withdrawn from the fiber belt in an orderly fashion. The fiber belt on the main net is treated with sulphuric acid and finishing oil, then dried and finally humidified to a water content of 11%. The fiber belt is covered with a thin net while it runs through these processes to maintain its order.

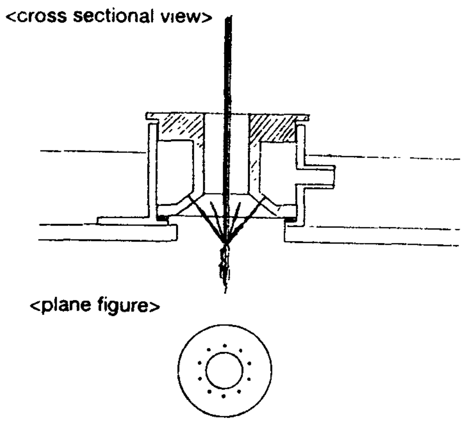

Fig. 7. Structure of CJ water-supply device

The withdrawing apparatus has two functions:

— To withdraw fiber from the fiber belt and let it run forward.

— To control the position of the end of the belt where the fiber is drawn out.

Sometimes the fiber is withdrawn from the fiber belt with a short section not undone. This can lead to three effects:

— Undoing of the belt to a straight fiber.

— Forming of a knot (or loop).

— Fiber breakage.

Effects 2 and 3 cause quality problems and reduce throughput. Therefore, preventing these imperfections is an important matter. The positioning of the end of the belt is controlled as follows. The endpoint of the belt is detected by means of a light sensor system, where a light is set above the main net and a photo cell is set underneath it. If the withdrawal speed is too low, the belt cuts the light beam. The winding speed is raised to a higher level until the photo cell receives the light; once the photo cell receives the light, the winding speed is reduced to a lower level. There are many reasons for changes in the end point of the belt, such as differences in length and denier of the fiber and difficulty in withdrawing the fiber from the belt.

The merits of employing a net conveyer for the high speed spinning process are:

1 The much lower speed of the net conveyer compared with the spinning speed provides sufficient time for regeneration, drying and humidifying of the fiber.
2 Fiber with high tensile strength and low boil shrinkage can be manufactured since it is treated in the spinning process without tension.
3 No fiber breaks occur on the net. Even if the fiber is broken at winding, an operator can withdraw the fiber from the fiber belt on the net and re-start winding.
4 The investment cost and maintenance charge are reduced because the net speed is low and the number of machine parts is small.
5 It is theoretically possible, using a net conveyer, to realize highspeed manufacturing at several thousand m/min, if suitable highspeed spinning technology can be developed.

Fiber structure and properties of NP fiber

The NP and Hank fibers have high tensile elongation and low boil shrinkage compared with the fiber from the continuous process, because of the treatment under low tension in the regeneration, drying and humidifying processes. The fiber from the continuous process has the smallest number of fluffs per unit fiber length, the second best is the NP fiber. The neps found in the NP fiber stem from loops. From the viewpoint of ease of feeding, fiber properties and fiber quality, the NP fiber is considered to have a good aptitude for the weft of woven fabrics. Recently, the NP fiber has attracted notice because of its high aptitude for the air-jet loom process, introduced mainly for the weaving of regenerated cellulose fiber.

There are drawbacks in the NP fiber such as low abrasion resistance after resin treatment and a tendency to fibrillate. These faults become more apparent as the spinning speed increases, probably due to structural defects in the surface layer as a result of highspeed spinning.

UNP-type manufacturing method

The UNP method, an improvement of the NP method, was developed to attain higher productivity and to overcome the drawbacks of the NP method and has the capacity to spin at a speed of 1000 m/min. It employs the same technology as that of the NP method, especially for the processes after spinning. However, in order to attain higher spinning speed and better properties such as abrasion resistance after resin treatment, the following requirements should be fulfilled:

— Increase of coagulation ability of the spinning funnel.

— Achievement of high elongation.

— Optimization of elongation and coagulation.

Requirements 1 and 2 are necessary for high spinning speed and 3 is for improvement of fiber properties.

The dependence of the abrasion resistance on the ratio of the final spinning speed to the maximum elongation speed due to spinning water, V_F/V_L. Here, the final speed V_F is the speed of the fiber drawn by a DKR and the maximum elongation speed at spinning *VL* is the speed of the fiber which is produced only by spinning water free from the force exerted by a DKR, or the speed of the fiber at the point where the elongation by the drawing force of the DKR begins.

It is clear that the abrasion resistance rises as $V_F V_L$ increases and, in order to improve it, it is necessary to elongate the fiber at an early stage of coagulation. A straight funnel with smaller diameter gives fibers with less fluffs because the higher elongation in the early stage of coagulation improves the balance of elongation to coagulation. However, it has been shown that such a straight funnel causes turbulence of the spinning water and therefore the fiber path was disturbed and even broken.

A new type of spinning funnel was introduced in order to solve this problem. This new type of funnel has a tapered end with the smallest diameter at the foot. The aim of this funnel is to prevent breakage of the fiber by turbulent water and at the same time to attain high elongation by increasing the speed of the spinning water. In addition to this, it was shown that the second funnel, which was important to achieve the necessary coagulation level in the NP method, produced a large resistance against the fiber in the UNP method.

Therefore, a second CJ was introduced instead of the second funnel to decrease the resistance from the water and to increase the coagulation ability. In addition to this, the freefall length was increased. The change in velocity of the spinning water and the fiber, and the change in degree of coagulation. The employment of this type of spinning apparatus has made it possible to manufacture fibers with low fluff level, good abrasion resistance and low fibrillation tendency.

Other characteristics of the UNP process

The frequency of loops in the UNP process is one tenth of the NP process. This is considered to be because entanglements in the fiber belts occur less often using DKR drawing and it becomes easier to withdraw the fiber from the fiber belts at high speeds. The effect is considered to stem from the fact that fiber drawn by the DKR reaches the net conveyer before the crimp produced by the blades of the DKR disappears.

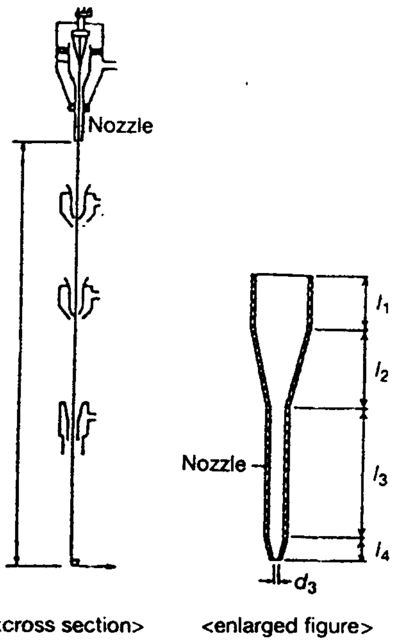

Fig. 8. UNP-type spinning funnel

Spinning technology of spandex fibers

In the 1940s, the IG company developed Perlon U, which however was a non-elastic polyurethane fiber rather like nylon. The elastic polyurethane fiber 'Lycra' was developed in 1959 by Du Pont, and thereafter many companies started developing elastic polyurethane fibers. The segmented polyurethane used for elastic polyurethane fibers is a copolymer of a soft segment and a hard segment.

Rubber-like elasticity is given by combining a soft segment which gives polymer elon-gation and a hard segment which gives polymer strength. In the polymerization process, a high molecular-weight diol is converted to a prepolymer which has an isocya-nate group at each end by combining it with two molar equivalents of disocyanate. Polytetramethyleneglycol (PTMG), a polyadipate, or polycaprolactone are used as the high molecular weight diol and diphenylmethane 4,4' -disocyanate (MDI) or toluene-2,4-diisocyanate (TDI) is used as the capping agent.

This prepolymer is then converted to high molecular weight polyurethane by combining it with a chain propagation agent such as a diamine or some other bifunctional active hydrogen compound such as a diol. The urea bonds (in the case of diamine) or urethane bonds (in the case of diol), which form in this reaction, produce hard segments. Hydrazine and ethylene diamine are the most commonly used diamines. As there is a large amount of free disocyanate in the pre-polymer, the polymer produced has the following structure.

—IAIAIAI—I-I-I—IAIAIAIAI—I-I-I—IAIAIAI—

Here I is diisocyanàte, A is diamine and —is high molecular weight diol. The sequence IAIAIAJAI corresponds to a hard segment and I—I—I— corresponds to a soft segment. Generally, a dry- or wet-spinning method is used for manufacturing spandex fiber but the melt-spinning method is also applied. There are two methods of wet spinning, one using a solution of the final polymer and the other a prepolymer.

In the former method, the polymer solution is extruded into a coagulation bath through a spinneret and after coagulation, combination and fusion between filaments take place, the fiber is wound up. When using prepolymer, the solution of prepolymer is extruded into diamine solution where chain propagation takes place. Therefore this spinning method is often called reaction spinning. In the case of dry spinning the heated polymer solution is extruded from a spinneret into a hot gaseous atmosphere in a spinning tube, where the solvent is removed. The filaments are combined to form a yarn, fused with each other and wound up. This is now the most usual spinning method employed for spandex manufacture.

Dry spinning of spandex fibers

The process of structural formation of the fiber in dry spinning is more complex than in melt spinning, since the former is a two-component system, while the latter is a one-component system. However, assuming that there is no change of phase due to vaporization of solvent and that the fiber can be treated as a continuous body, the spinning process can be analysed theoretically like the melt-spinning processes.

The region surrounded by two horizontal planes which are separated by a vertical distance Δz and a cylindrical surface of the fiber is considered. For this region, four equations define the balance of materials (polymer and solvent), momentum and energy. By transforming these equations, four independent equations can be introduced that define the weight ratio of solvent, sectional area of fiber, spinning tension and fiber temperature. and express the spinning conditions.

The results of calculations using these equations. The dotted lines denote the case (a) that solvent vaporization is restricted by a boundary layer and the continuous lines denote the case (b) that it is restricted by diffusion. The solvent concentration in the region near the spinneret is higher for case (a) than for case (b) but the concentration decreases rapidly in the next region, the concentration for case (a) becomes lower than that of case (b) and the time until the solvent is completely removed is longer for case (a). The concentration dependence of the diffusion coefficient is not taken into consideration.

In fact, the diffusion coefficient is considered to decrease as the solvent concentration decreases, so it takes much longer to remove the solvent completely. The temperature of the fiber drops at first, then remains constant for a while and gradually rises to the temperature of the circumferential gas. The first drop is the result of the vaporization of the solvent which takes latent heat from the fiber. Then the temperature reaches wet-bulb temperature and remains there because the latent heat equals the heat transferred from the circumferential gas to the fiber. Thereafter, the temperature starts to rise as the amount being vaporized decreases.

The cross sectional area drops rapidly at first because of elongation and then decreases more gradually because of the vaporization of the solvent. Thereafter, as the amount of solvent vaporization decreases and the fiber becomes solid, the change of cross-sectional area diminishes. The tension of the fiber, which is very small at first, rapidly increases as the solvent vaporizes, but soon reaches a plateau.These four values shift by a factor of two along the horizontal axis if the amount of spinning dope increases twofold. This means that if the amount of spinning dope is increased in order to increase the spinning speed, the length of the spinning tube should be extended proportionately to the increase in the amount of dope.

Practical dry spinning of spandex

The conceptual diagram of a spinning apparatus for dry spinning of spandex. The polymer solution, which is already de-aerated, filtered and heated up to the required temperature, is extruded from a spinneret into a spinning tube. From the top of the tube, inert gas such as Kempf gas is introduced into the tube.

The solvent is removed from the fiber, which becomes thinner as it passes down the tube. After emerging from the tube, the fiber is twisted. The twist travels upstream along the fiber to a point in the upper part of the tube where the filaments comprising the fiber fuse together and form an aggregate of filaments. This fusion is indispensable for subsequent processing of the fiber if it is not to be damaged. Various twisting devices such as air jet, liquid flow or roller can be used.

The twist imparted is removed by the time the fiber reaches the first roller by means of the tension of the fiber itself. The fiber then receives finishing oil by contact with the finishing roller and is wound up by way of the second roller. Various technological improvements have been made to obtain stable spinning and diminish denier fluctuation. A technological improvement concerning the flow control of the heated gas. The heated gas flows down through a ring-shaped passage formed between the spinning tube and a cylindrical body set under the spinning nozzle and is removed from the tube at a point sufficiently far from the nozzle.

This flow control prevents the oscillation of filaments and minimizes denier fluctuation. Generally, in dry spinning the heated gas is supplied in parallel with the direction of the fiber either co-currently or counter-currently, a method where the heated gas is supplied perpendicularly to the fiber as in melt spinning. The velocity of the gas is 1-30 cm/sec. This method prevents gas turbulence and heat transfer fluctuations and therefore decreases the entanglement of fibers and denier fluctuations. Moreover, this method accelerates the vaporization of solvent and facilitates higher spinning speed.

Highspeed spinning of spandex

The spinning speed of spandex is typically about 500 m/min; much slower than that of synthetic fibers such as PET made by a melt spinning method. In the spinning of spandex, an increase of the fiber tension causes undesirable changes such as an increase of modulus and a decrease of tensile elongation, Therefore, lowering the fiber tension is the main theme of the technological development in order to attain highspeed spinning. It is also necessary to increase the rate of solvent vaporization.

The developments required are:

— A method to supply the heat energy needed to evaporate the solvent. The amount of solvent to be evaporated increases proportionally to the increase of spinning speed, yet it is not desirable to lengthen the spinning tube proportionately from the viewpoint of economy and fiber tension. Therefore, the residence time of the fiber in the tube must become shorter. An increase of the gas supply rate causes turbulent flow. An increase in temperature of the gas is limited by the melting point of the fiber.

— Technology for diminishing fiber tension. As mentioned before, an increase in the fiber tension causes a rise of modulus and a decrease of tensile elongation. The velocity of the gas becomes lower than that of the fiber. The resistance of the gas to the fiber increases as the fiber velocity increases.

— Highspeed twisting technology. As mentioned before, the fiber is twisted after it comes out of the tube by means of an air jet, liquid flow or rotating roller. The rate of revolution of the twister must be increased to attain higher spinning speed.

— Highspeed winding technology. It is difficult to wind up spandex yarn as it has low modulus and high tensile elongation. The technological difficulties increase proportionately to the increase of spinning speed.

Research and development is advancing on each of these themes in order to improve fiber properties.