PRACTICAL GEODESY

COMPRISING

CHAIN SURVEYING AND THE USE OF SURVEYING INSTRUMENTS

LEVELLING AND TRACING OF CONTOURS

TOGETHER WITH

SANITARY SURVEYS OF TOWNS

TRIGONOMETRICAL COLONIAL MINING AND MARITIME SURVEYING;

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PREFACE TO THE FIRST EDITION.

I HAVE employed the term 'Geodesy' as one well suited by its comprehensive meaning to embrace the various subjects treated of in the present work; and, by adding the word 'Practical,' I have designed to indicate, that actual practice in the field is the especial object I have had in view. This has not been lost sight of throughout the work, and the subject has been so arranged as to enable the student, at the outset, to commence a simple survey, minute practical directions being given to guide him in each new process, or to prepare him to surmount, by his own judgment, difficulties which may arise under an unforeseen aspect.

Proceeding through the elementary processes, the student is brought, by regular gradations, to understand the principles, and to appreciate the bearings of the more perfect methods which the demands of an advanced state of civilisation render it necessary to employ, in order to ensure that degree of accuracy which is now deemed essential.

In the Chapter on Trigonometrical Surveying, I have taken the opportunity of making the student acquainted, by a description of some of the interesting trigonometrical operations of this and other countries, with the highly scientific processes required for the successful performance of an extensive survey. It is true, that in Great Britain the trigonometrical operations are nearly completed; but we find that, as every year increases the civilisation and adds to the wealth of our older Colonies, similar operations, conducted with an approach to equal accuracy, are called for in those portions of the British Empire.

It has been deemed also that, with the tide of emigration flowing so freely towards the New Colonies, a treatise on Surveying would have been incomplete without a short description of the method of surveying especially applicable to New Colonies.

An exposition of the insufficiency of the instruments in general use for mining surveys has been deemed of importance, for it appears strange that mining surveys should, to this day, be performed with instruments which have long been laid aside as too inaccurate for surveys above ground, although the latter seldom present difficulties of so serious a character as those met with in subterraneous operations.

In the Chapter on Maritime Surveying, particular care has been taken to convey practical information on those details, which, although they may be familiar to the mariner, are novel to the engineer and land-surveyor, on whom the duty not unfrequently devolves of conducting surveys of harbours and of coasts connected with interior triangulation.

I have in the course of the work acknowledged, by references at the foot of the page, the sources from whence I have derived any portion of the contents. But I am desirous especially to point out the valuable assistance which I have received in the Chapter on Latitude and Longitude from Sir J. F.W. Herschel's *Treatise on Astronomy*.

Finally, I would express a hope that the present volume, by affording frequent illustrations of the application of mathematical knowledge to purposes eminently practical, may be instrumental in encouraging, amid the varied occupations of an active life, the pursuit of that science, which, if cultivated with judgment, leads to most beneficial results, and is capable of promoting the highest intellectual enjoyments.

London, April, 1842.

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PRACTICAL GEODESY.

CHAPTER I.

SURVEYING WITH THE CHAIN.

TO CONSTRUCT a map of a district of country consists in making on paper, on a smaller scale, a figure similar to the district, and representing thereon the prominent objects on its surface.

A *topographical* map or plan differs from a *geographical* map in this particular, that the first is intended to represent all or nearly all the details that appear on the surface of the country; the second embraces a comparatively greater extent of country, and aims only at determining the relative position of the principal points and most important objects.

LAND SURVEYING consists in measuring, with a view to subsequent delineation on a map or plan, the boundaries and forms of natural and artificial objects on the surface of the ground, ascertaining and fixing their relative position, and finding the superficial content or area of each part, and of the whole.

Land is surveyed either by means of a chain only, or by combining with it a theodolite or other angular instrument. I commence by describing the first of these methods, as being the most elementary; and well adapted for the measurement of a surface of small extent.

The Surveying Chain.

The chain is a linear measure constructed of any given arbitrary length, divided by links into a stated number of units of extension. A chain of 100 feet, divided into 100 links, is perhaps the most convenient for general use, being equally applicable to the purposes of the engineer and land surveyor, and combining readily horizontal with vertical distances, the latter being uniformly measured in feet, whatever unit may otherwise be adopted for the former. When the sole object is to obtain the acreage, *Gunter's chain*, equal in length to 66 feet, is the most convenient, because of the facility it affords for computation: for, as 10 square chains are equal to 1 acre, and the chain is divided into 100 links, the contents, expressed in chains and links, are converted into acres and decimals of an acre, by simply dividing by 10. (See *Computation of Areas*, page 30.)

To guide the eye in counting the number of feet or links, brass marks are fastened at every tenth foot or link, and distinguished from each other by notches, varying in number according to their position with respect to the extremity of the chain, so that the surveyor can, by simple inspection, readily read any required length.

Accompanying the chain are 10 arrows, which are used in succession to mark the chain-lengths in measuring a line.

Method of Measuring with the Chain.

The chain is used by two persons, one of whom is called the leader, the other the follower. The point from which the measurement is to commence, as also the direction of the line to be measured, being determined, the leader, who has been supplied with the 10 arrows, draws out the chain in the required direction, as indicated to him by the follower, who holds the other end at the starting point. An arrow is then thrust perpendicularly into the ground by the leader at the point where the chain terminates; he then proceeds onward, drawing the chain after him, and repeats the same operation throughout the length of the line, the arrow last put down serving always as the mark to which the follower is to bring his end of the chain as a new station of departure.

The arrows are taken up by the follower as he advances, and when the 10 arrows have thus changed hands, they are all returned to the leader to be used again. In this manner the arrows are changed from one to the other at every 10 chains' length, till the whole length of the line is measured, care being taken to enter every such change in the field-book. At the end of the line, the number of changes, added to the number of arrows in the follower's hand, and to the number of links extending from the last arrow put down to the extremity of the measured distance, gives the entire length of the line.

When long distances are to be measured, a very convenient check on the operation of counting the chains, is obtained by using 10 supplementary arrows, distinguished from the common ones by brass or other marks, these arrows being put down in succession at every tenth chain. In this case, the requisite number of arrows indicating 1 chain each, is reduced to 9; and as each one of the supplementary arrows represents the measure of 10 chains' length, when the 10 supplementary arrows shall have passed into the hands of the follower, a distance of 100 chains' length will have been measured. Another advantage resulting from this arrangement is that it avoids the inconvenience occasioned by the follower having no arrow to measure from, after he has given his 10 arrows to the leader.

It will be found useful in practice to fasten to the heads of the arrows pieces of red cloth or other distinguishing mark, to cause them to be more readily found when placed in long grass, or brushwood, etc.

In the operation just described, care is necessary, first, to ensure the line gone over being a straight line; secondly, that the ends of the chain are made to coincide as accurately as possible with the arrows placed to mark each extremity. A deviation from a straight line causes the apparent length measured to be greater than the true length: this follows from Euclid's 20th proposition of the first book, which proves that any two sides of a triangle are together greater than the third. Also, the frequent repetition of errors in the coincidence of the extremities of the chain with the arrows, may render deviations in the aggregate important, which, viewed singly, would be inconsiderable.

I would therefore recommend a young surveyor, when employing as chainmen labourers who have been unused to the work, to cause them to measure a certain distance, say half a mile or a mile, on a level road, several times, until they shall have learned, by a careful attention to the directions above given, to obtain very nearly the same result at each measurement. This preliminary caution will save the surveyor much subsequent loss of time; first, by training his assistants to their work; and secondly (which is of more importance), by impressing them with the necessity of a very careful admeasurement as an element in the successful performance of the survey.

Chain to be compared with a 'Standard.'

The length of the chain should, from time to time, be tested by a careful comparison with a standard chain kept for the purpose. To facilitate and expedite this comparison, two strong pickets may be driven firmly on a level portion of ground, at a distance from each other corresponding to the length of the standard chain, notches being cut, or nails driven, to mark its exact length. The coping of a horizontal wall is also well suited to receive such marks. They are thus rendered permanent; and every day a comparison of the working chain may be made with the standard length without loss of time. This precaution is indispensable; for the chain, being composed of numerous pliable links joined together by three small unwelded rings which give it flexibility, is, from its construction, constantly liable on the one hand to expand at the joints, and on the other to have the links bent when dragged over rough surfaces. Of such importance is this examination held by scientific and practical men, that the Commissioners for the restoration of the Standards of Weight and Measure recommend in their report, dated December 21, 1841, that 'no person shall be admitted to give evidence in any court of justice, of having measured land, after the passing of the contemplated Act, with any other than a stamped (standard) measure, or a measure which has been compared, on each day on which any part of the measurement has been made, with a stamped (standard) measure.' The chain used for measuring may be left unaltered, if it be about half an inch longer than the standard, as it can never be stretched perfectly straight, but adapts itself by its weight to the small inequalities on the surface. The chain also, when used in wet weather, becomes shorter, in consequence of the insertion of dirt between the rings. If the excess be great, the length must be diminished by the removal from each extremity of one or more of the rings. The same correction, whatever it be, should be made equally at both ends, in order that the middle point the chain may be in its true position, in which case the error, subdivided among the remaining parts, will be trifling, and not worth being taken into account. If, on the contrary, the working chain be found too short, this will arise from the links being bent; consequently the correction will be effected by straightening them.

Surface to be divided into Geometrical Figures.

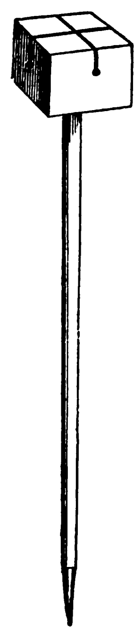
In measuring an estate, a parish, or any comparatively small portion of land, the surface may be supposed to be divided into a system of arbitrary geometrical figures, bounded by right lines, either inscribed within or circumscribing the area to be measured; but so disposed that they shall pass near all objects included in the survey, and serve to determine their positions and forms. The object of this imaginary division is to facilitate the measurement of all irregular boundaries, which, if they were traced independently of these auxiliary lines, into all their windings, would lead to a great consumption of time in the operation, and to inaccuracy in the results, owing to the number of mutually dependant angles to be measured. After having divided the surface into a number of geometrical figures bounded by right lines, the sides of these figures are used as bases from which the irregular boundaries and other objects are measured by means of shorter lines at right angles, termed offsets.

Offsets.

These offsets, when short, are measured with an offset-staff, 10 feet in length; and with a second chain, or, in preference, a measuring tape, when the offsets are too long for the staff to measure them conveniently. The limit to the length of these offsets is fixed in a great measure by the degree of accuracy aimed at in the survey, as well as by the scale to which the plan is to be drawn: in general it is not advisable to make use of offsets more than about 100 links in length.

Offsets, with few exceptions, are measured at right angles to the main line; the length of the offset being determined, the position of the object referred to is fixed with reference to the main line. These right angles must of course be set off correctly. When the length of the offset is only a few links, the right angle is generally set off by the eye. For long offsets, it is advisable to use an instrument. That formerly employed for setting out perpendicular lines was the cross-staff.

The Cross-Staff or Surveying Cross.

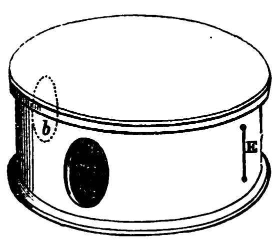


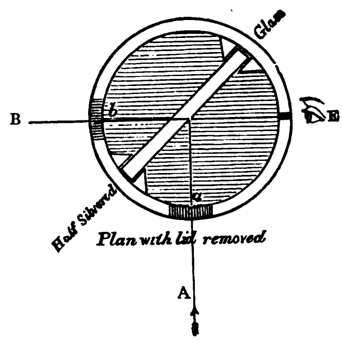
The simplest form of the cross-staff consists in a cubical top formed of hard and well-seasoned wood, fixed on a pointed staff nearly equal in height to the observer's eye, to prevent stooping. Two vertical cuts are made by a tenon-saw, exactly at right angles to each other, of such a depth as to leave enough wood to maintain the parts firmly in their places. The saw-cuts form the "sights." But, to extend the field of vision laterally, two centre-bit holes should be made at the bottom of the cuts, to be used in the first instance in finding the objects.

The staff being thrust into the ground, with two of the sights placed in the direction of the main line, points out the perpendicular required, by means of the other two sights. This instrument presents the advantage of simplicity, but it cannot be used on hard roads or in towns; and in soft or swampy ground it is not to be relied on. Hence it has been generally superseded by the *optical square, or reflecting surveying cross*, a small circular box, of about 2 to 3 inches in diameter, which marks a right angle with accuracy and precision.

Optical Square.

This box contains a strip of looking glass, from the upper half of which the silvering is removed so as to admit





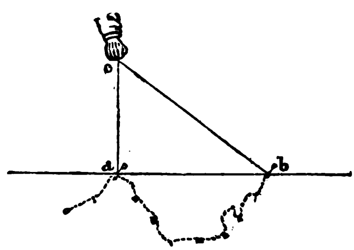
of direct vision through it, while the lower half acts as a reflector. The eye placed at E, will observe the distant object B by direct vision through the hole *b* and upper unsilvered half of the reflector; while if another object A is placed at right angles to the line E B, it will be seen through the hole *a*, and reflected by the lower silvered half of the mirror to the eye at E, and thus appear to coincide with the object B. The direction of the half-silvered glass is adjusted so as to form half a right angle, or 45°, with the line E *b*.

With this instrument, therefore, a line may be laid out at right angles to another, from any point in it, by simply standing over the given point, and looking through the eye-slit along the line, having an assistant to go with a mark or ranging rod in the direction in which the perpendicular is required, and signing to him to move to the right or to the left, until his rod is seen by reflection to coincide with a staff fixed on the line along which the observer is looking. When the coincidence takes place, the rod is fixed in the ground.

If, instead of erecting a perpendicular from a given line at a given point, it be required to find on a line the point of intersection of a perpendiculur from a fixed object, as a house, a tree, etc., the observer himself must move along the line until the image of the object appears, as before, in the direction of the line, and the place where he then stands marks the spot where the perpendicular would fall.

Method of raising a Perpendicular with the Chain only.

The following is a correct method of setting out a perpendicular with the chain only, but it is evidently too laborious for offsets. Let *a b* be the line on which it is required to erect a perpendicular from the point *a*. Fix an arrow in the ground at *a*, through the ring of the chain denoting 20 links, and measure 40 links on *a b*. At *b* fix the *extreme* end of the chain; then holding the brass ring denoting the 50 links, or the centre of the chain, draw the chain tightly in the direction *c*, the sides of the triangle will then be in the proportion of 30, 40, 50, and consequently *b a c* will be a right angle; ($30^2 + 40^2 = 50^2$), Euc. I. 47.



SURFACE TO BE SURVEYED SHOULD BE DIVIDED INTO TRIANGLES.

In making a survey with the chain only, we are confined to the simplest geometrical figure — namely, the triangle; for, of all plane geometrical figures, it is the only one of which the form cannot be altered, if the sides remain constant. That the triangle possesses this property is evident from the theorem (Euclid, I. 7) which proves that, 'Upon the same base, and on the same side of it, there cannot be *two* triangles that have their sides which are terminated at one extremity of the base equal to one another, and likewise those which are terminated in the other extremity, equal to one another.'

Comprehensive Triangles.

The surface to be measured is therefore divided into a series of imaginary triangles; and, in this division, it must be borne in mind that the triangles are to be as large, with reference to the whole surface to be measured, as is consistent with the nature of the ground (see diagram, page 14); for, by such an arrangement, we are acting on this important principle in all surveying operations, that it is well generally to work from a whole to the parts, and rarely from parts to the whole. By the first method errors are subdivided, and time and labour economised; by the second, the errors inseparable from all operations that do not deal with abstract quantities are increased as each step in the work advances.

Proof or Tie-Lines.

The sides of these triangles are first measured; and, as a necessary check on this first part of the work, a straight line is, in addition, measured from one of the vertices of each triangle to a point in or near the middle of the opposite side. This fourth line is called a tie-line or proof-line, and is an efficient means of detecting errors, if any have been committed, in the measurement of the sides of the triangle. This fourth measurement is made in accordance with a maxim which ought invariably to be acted upon in surveying operations, whether limited or extensive, simple or complex; namely, that where accuracy is aimed at, the dimensions of the main lines, and the positions of the most important objects, should be ascertained or tested by at least two processes, independent of each other.

Subsidiary Triangles.

Within the larger triangles, as many tie-lines and smaller triangles (see diagram, page 14) are to be measured as may be necessary to determine the position of all the objects embraced in the survey. The directions of the lines forming the sides of these secondary triangles are so selected or disposed, that they shall connect and pass close to as many objects as possible, in order that the offsets to be measured from them may be as short, and as few in number, as practicable.

If the sides of these secondary triangles be in any case so distant from the objects whose positions are to be determined as to require a length of offset greater than the proposed limit of about 100 links, it then becomes advisable to construct, either on the whole or a portion of the side of the triangle as a base, a smaller offset triangle with its sides so disposed that they shall either embrace, or pass very near to, the objects to be measured by their intervention. (See triangle $f\,i\,k$, diagram, page 14).

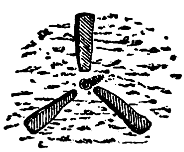
Preliminary Reconnaissance.

The disposition and general combination of these triangles demanding care and judgment, it is customary, previous to commencing any measurement, to walk over the ground for the purpose of obtaining a general knowledge of the surface, and all the relative positions of the most conspicuous objects. The acquisition of this knowledge, depending on the *coup d'œil*, is much assisted by an eye-sketch drawn with rapidity, and showing some of the principal roads, streams, churches, etc. This hand-sketch is not to be drawn to any scale; and its object is attained if it simply bear a general resemblance to a plan of the ground, as it will thereby assist the memory in the distribution of the surface into triangles.

The sides of the larger triangles are to pass as close as possible to the external boundaries to be surveyed; the triangles should, moreover, be made to approach, as nearly as practicable, to the equilateral form, avoiding with care very acute or obtuse angles, because the farther the form of the triangle is removed from the equilateral, the greater will be the alteration in the form of the figure and in its area, should any error be committed in the measurement of any one of the sides.

Station-Points.

The triangles having thus been disposed to the greatest advantage, pickets are placed in the ground at each vertex



of the triangles, and cuts are made in the sod or ground, as shewn in the annexed figure. Their general form and position is then noted in the hand-sketch previously made, and distinctive letters are written on the diagram at each point of intersection (see diagram, page 14). This arrangement admits of easy reference in the field-book, or, on the ground, to any triangle or part of a triangle.

Station-Lines.

The points of intersection of all straight lines, as well as the vertices of the triangles, are always points measured to or from: they are called station-points or stations, and the lines connecting them station-lines, thereby distinguishing them from the simple offset lines.

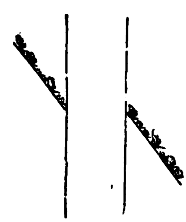
FIELD-WORK.

Field-Book.

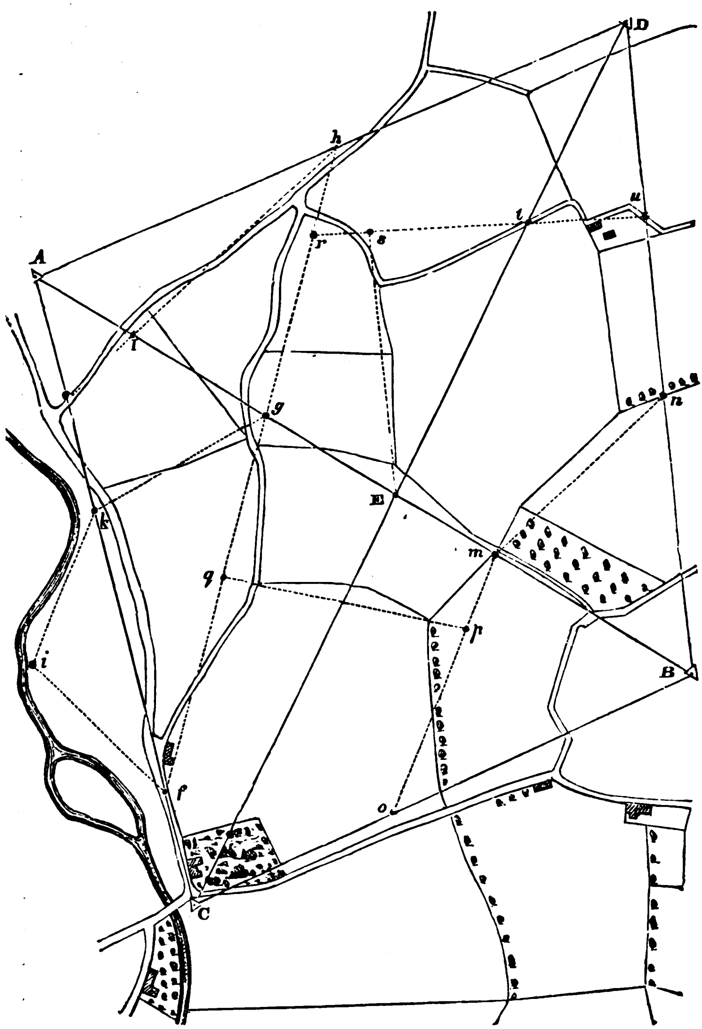
The hand-sketch, or rough diagram, is usually made in the field-book, that is, a book in which every minute step of the operations gone through in the survey is to be entered with precision at the time. The entries in the field-book should be made with an indelible pencil, or written in ink, in the field; but the first is to be preferred, as it is difficult to make legible entries in ink in wet weather. The field-book should be paged for convenience of reference.

The field-book is ruled into three columns (see diagram, p. 15); in the middle one are set down the distances on the station-line, at which any mark, offset, or other observation is made) and in the right or left-hand column are entered the offsets and observations made on the right or left-hand respectively of the station-line.

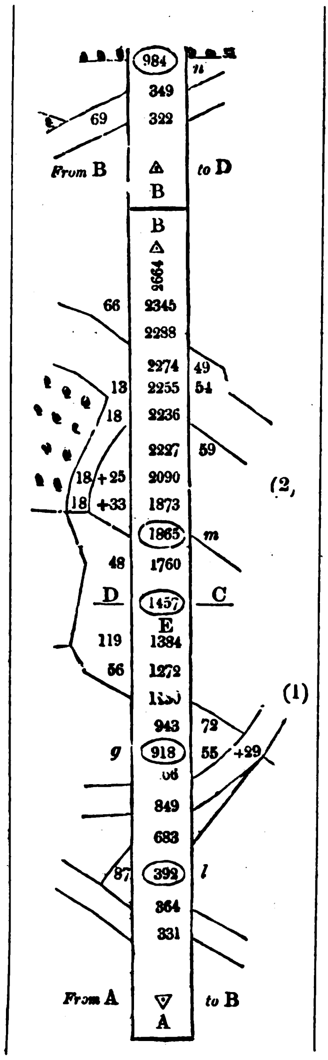
It must be borne in mind, that the middle column in the field-book represents the position of, or, rather, the station-line itself. If the station-line, therefore, should be crossed on the ground by a fence, or any boundary, meeting it obliquely, its representation or symbol in the field-book must not be made to pass obliquely across the middle column, but must arrive at one side of the column and leave it on the other, at points precisely opposite, as it would do were the middle column merely of the thickness of a line. Inattention in this particular causes much confusion in the relative position of offsets.



It is a universal rule, and of advantage for the sake of perspicuity, to begin the entries in the field-book at the bottom of the leaf, which, under ordinary circumstances, would be considered the last in the book; the reason is evident, inasmuch as it places the field-book in the same position as the station-line with respect to the surveyor, who keeps his face directed towards the distant station. The crossings of the fences, roads, streams, etc., and the corners of fields, and other remarkable turns in the boundaries, to which offsets are taken, are to be shown by joining lines in a manner somewhat similar to the form which they assume on the ground.



It is a too common error with the inexperienced surveyor to neglect this approximation to the real forms of the objects, as also to make his entries faintly, and with a careless hand. It cannot be too strongly impressed on the surveyor, that the work which he is called upon to perform depends for its accuracy in a very great measure on the order, system, and neatness bestowed on all the steps, whether of delineation or of measurement. Proper attention in keeping the field-book saves much time in plotting, and guards against the errors unavoidably arising from reference to a confused field-book. Moreover, care bestowed in the first essays will amply reward the surveyor, by giving accuracy of eye, freedom, and steadiness of hand, qualities indispensable to his success.



Form for Field-Book, having reference to the Plan in p. 14.

Field-Work.

On commencing the measurement of a station-line, the letter corresponding to the starting point in the rough diagram, or hand-sketch, is entered first at the bottom of the middle column, and on each side are written the letters marking the extremities of the line, thus: from A to B, A being the starting point. In the same manner, on arriving at the end of the line corresponding to the point B, the letter B is written in the middle column above the closing distance, and above the letter a line is drawn across the middle column to denote that the line terminates in that point. In the form of field-book given in the preceding page, are entered the observations for the line A B, on the diagram in page 14. The numbers placed within a vinculum on the right hand side of the page, indicate the changes of the 10 arrows. If the line be a short one, and a staff at either extremity be seen from every part of it, it may be ranged by the eye. If the line be so long, or on such uneven ground that the staff, fixed at its extremity, is occasionally lost sight of, the line must be ranged either by the eye or with a telescope, from a commanding position; pickets or ranging rods being placed at convenient distances to mark the right line. Advancing along it with the chain, the distance of the crossing of every fence or natural boundary, as also from all points from which offsets are taken, is noted in the field-book, together with the length of these offsets measured to the right or left. At every station, that is, any of the points determined by the intersection of the sides of the triangles, or of tie-lines with those sides, a picket is driven, so that the precise spot may be readily found again in subsequent parts of the operation. The number of links written in the field-book as corresponding to the distance of such station, is encircled by a line to distinguish it from other points, and by the side of the circle the letter corresponding to the point in the diagram is written. This process is continued until the measurement has extended to the extremity of the line, of which the entire length is written down in larger characters than the rest; and, as an additional distinction, written in a line parallel with the vertical sides of the field-book; for example, '2664,' in the form of field-book given in page 15. When it is not convenient to measure exactly *from* any of the marks left for the tie-lines, the place measured from may be described as being so many links from one station towards another; and where a station mark is not measured *to* precisely, the exact place at which the measurement stops is shown by writing 'turn to the right (or left)' so many feet towards such a station, it being always understood that these auxiliary distances are measured along the station-line.

In this manner, the sides of all the triangles are measured in succession, and their dimensions, with the additional assistance of offsets, give the means of ascertaining all boundaries, external and internal, positions of houses, etc., and of finding the area of the whole and of every part, by direct computation from the field-book. But to obtain the contents of each inclosure by computation, would be a process very laborious and generally unnecessary: the contents of the whole should be ascertained by computation from the sides of the large circumscribing triangles; the areas of the inclosures may be afterwards obtained by measurement from the plan, their accuracy being tested by a comparison of the sum of the areas of the inclosures, with the area comprised within the exterior boundary, as obtained from direct computation.

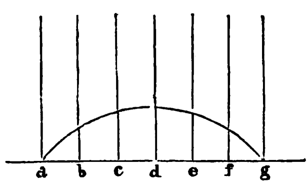
Reduction of Lines to a Horizontal Base.

I now come, in connexion with this question of areas or superficial contents, to the consideration of an important principle, namely, the reduction, of the lines measured over steep slopes, to the horizontal plane.

Having to lay down on a plan or flat surface, boundaries and lines at different inclinations, in order to avoid distortion in the outline, and to bring all the details duly within the triangular framework, it is absolutely necessary that we refer to, or project all lines and points upon a plane. The plane adopted to receive this common projection, is the horizontal plane. It is not, therefore, the actual surface that we have to protract, but the diminished quantity that would result, had the whole been reduced to a horizontal base.

This distinction, which is indispensable for the purpose of laying down a plan of the surface, is supported, when the question is viewed under its social aspect, by the obvious principle that since plants shoot up vertically, the vegetable produce (with the exception of grasses, and a few other objects of culture), on a rounded eminence, does not in general exceed in quantity what would have grown upon its base.

A diagram makes this proposition evident; for let the vertical lines *a*, *b*, *c*, etc., represent the position of plants growing as closely as possible, or as is judged advantageous, from the horizontal surface *a*, *g*: it is manifest that if a curved line be drawn resting on the base *a*, *g*, and representing the inclined surface of the soil, the same number of plants only can grow in the vertical direction which plants tend to assume. The arrangement, therefore, as a matter of graphic necessity, is not inconsistent with the order followed by nature in respect of many varieties of produce.



Reduction of Lines to a Horizontal Base may be effected by Calculation.

All sloping or hypothenusal distances are, consequently, reduced to their horizontal lengths. When the lines are long, and the slopes much varied and considerably inclined, this reduction may be made by calculation, or by reference to tables of reduction usually engraved on the vertical arcs of angular instruments, which, while they shew on one side the angle of elevation or depression, give on the other the number of units per hundred that have to be deducted to reduce the hypothenuse to its corresponding horizontal length. This subject I do not at present wish to discuss, especially as in small surveys, performed with the chain only, an allowance or reduction is generally made in the field by construction or estimation, as the measurement proceeds.

The Reduction may be effected by Construction.

If the slope be not very steep, the reduction is accomplished by holding the lower end of the chain above the ground, as nearly horizontal as can be judged by the eye, allowing a pointed plummet to hang from the hand that holds the chain, to indicate where the arrow shall be placed. If the slope be steep, one half or one quarter of the chain is raised, as being more easily brought to a horizontal position; and on precipitous banks, the offset-staff or measuring tape is substituted, as giving more correct results, with greater expedition. It may be observed, that when the chain is thus held suspended, it cannot be straightened, its links describing the catenary curve; but as a compensation for the shortening of the chain, caused by the bend, it is found that the pull at each end of the chain to diminish the curvature caused by its weight, tends to open the unwelded elastic rings, and thus to add very sensibly to the length which it would have when laid upon the ground. Nevertheless, the bending of the chain is an element of inaccuracy in this process, which is further made erroneous by the difficulty of ascertaining exactly, without lateral or longitudinal error, the point vertically beneath the elevated extremity of the chain; and by the unavoidable deviation from the horizontal line, when it has to be estimated by the eye.

The Reduction may be effected by Estimation.

The surveyor who does not use an angular instrument for the purpose of ascertaining the required reduction, learns by habit to estimate and make at each chain's length on the ground, an approximate reduction. In such an operation, he will be much assisted by the subjoined table, which may be copied on the first leaf of the field book, and the principal elements of which are easily learned after a few references and practical applications. The inclination in such cases is, of course, estimated solely by the eye; and I describe this method, not to recommend it, but because it will aid the surveyor in approaching to accuracy, in exceptional instances, where he may not have the assistance of an angular instrument. The reductions are purposely made approximate in the table, in order not to distract the attention by fractional quantities in the application of a process, in itself only an approximation.

REDUCTION in links upon each length of 100 links for the following Angles of Inclination, or for the following Rates of Inclination, expressed in the terms of the Horizontal Base and Vertical Height.

ANGLE.		Corresponding Rate of Inclination (Approximative).		Reduction in Links for every 100 links (Approximative).	
Degrees.				Links.	Decimals.
4	...	1 in 15	...	0	·25
6	...	1 „ 9½	...	0	·50
7	...	1 „ 8	...	0	·75
8	...	1 „ 7	...	1	·00
10	...	1 „ 6	...	1	·5
11½	...	1 „ 5½	...	2	·00
14	...	1 „ 4½	...	3	·00
16¼	...	1 „ 3¾	...	4	·00
18¼	...	1 „ 3	...	5	·00
20	...	1 „ 2¾	...	6	·00

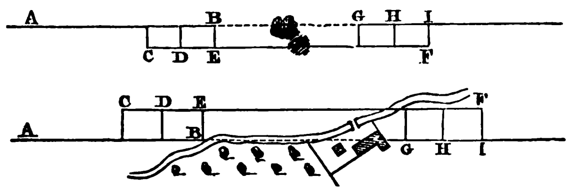
The necessary reduction, as estimated on the ground, is effected, as the measurement proceeds, by putting the chain forward the exact number of links denoted by the table as due to the angle, or the rate of inclination. This mechanical method possesses this advantage, that the crossing of the fences or natural boundaries, and the position of the offsets, are at once entered in the field-book, with the required reduction. Practised surveyors obtain by this simple means, results much more accurate than would have been expected; but I repeat, when perfect accuracy is sought, and when the survey is extensive, the angles of inclination should be observed, and the proper deduction obtained by computation, and allowed when the work is being plotted. This part of the subject is fully explained in treating of levelling, and the interior filling-in of a trigonometrical survey.

Practical Methods of Measuring Inaccessible Distances, and of avoiding Obstacles in running Lines.

Cases of obstruction in the measurement of a line offered by the intervention of trees, buildings, rivers, lakes, etc., are readily overcome by practical geometry, even without the aid of an angular instrument. But when the difficulty cannot be surmounted with ease by the chain, it is always better to make use of some angular instrument, which, with the aid of plane trigonometry, will enable the surveyor to solve all difficulties.

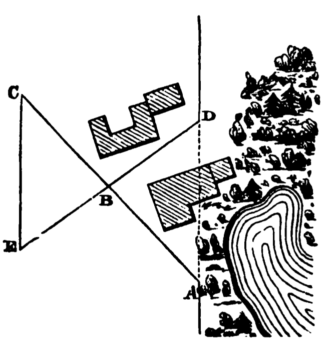
In ranging his lines, the surveyor should be careful to dispose them so that they shall, if possible, pass clear of trees, houses, and other impediments. If, in spite of all his efforts to the contrary, he finds it impracticable to avoid them altogether, he may proceed thus for passing them.

The measured line A B being obstructed by a tree, a brook, or a house, etc., staves are set up at C, D, and E, at equal distances from the measured line, and far enough from it to enable the new line C D E F, to pass clear of the obstacle. This new line parallel to A B, is then measured till the obstruction is passed, when, by setting other staves, G, H, I, at distances from the second line equal to those first set out, a return is made to the direction of the original line, which is pursued as before.

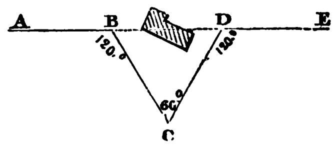


When the obstacle thus avoided is a tree, it is called a 'sight tree,' and for the purpose of facilitating future reference, it is marked in a particular manner by an arrow-head or otherwise, cut at or near the points where the direction of the line meets the tree, both in front and rear.

Another method of passing such obstacles, is by the construction of equal triangles: thus, let A D be the direction of the line under measurement, the further progress of which is interrupted at A. From A measure A C in any direction; and leave a central mark B; from D, an accessible point on A D, measure a line D B E, making B E equal to B D, then C E will be equal to A D, the distance required. This method is inapplicable if the line has not been previously ranged and determined by signals fixed beyond D, and visible from it.

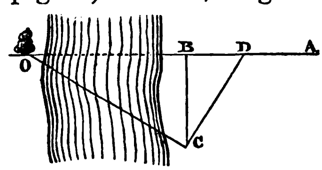


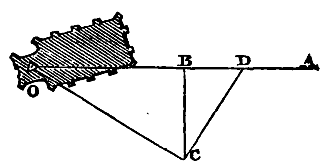
To avoid a similar obstacle, using an angular instrument. The measured line A B is interrupted at B; make the angle A B C equal to two-thirds of two right angles, and proceed along B C far enough to clear the obstacle. At C, measure the angle B C D equal to one-third of two right angles, and proceed along C D to a point D, making C D equal to B C; at D, measure the angle C D E equal to two-thirds of two right angles.



The triangle B C D, is by construction equilateral, each angle being equal to one-third of two right angles; hence, B D is equal to B C or C D; and D E is continued in the direction of the original line.

Required along the line A B produced, the distance B O, inaccessible to direct measurement with the chain. At B, raise the perpendicular B C of any convenient length, by making with the chain a triangle whose sides are in the ratio of 3, 4, and 5 (see page 9). At C, range in the same manner the line C D perpendicular to C O, and produce it to meet A B in D; measure B D. Then because B C D and B C O are equiangular (Euc. VI. 4.)



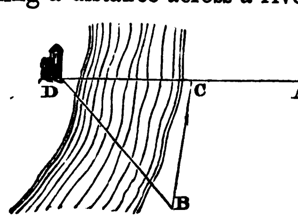


B D : B C : : B C : B O,

and (Euc. VI. 16.).

$$\text{B O, the length required} = \frac{\text{B C}^2}{\text{B D}}.$$

A ready method of determining a distance across a river is offered by that property of the triangle which consists in the external angle being equal to the two interior and opposite angles. Thus, on the line A D, the distance C D is required.



At C, measure any angle A C B (it should not be less than 90° for the sake of accuracy); and setting the instrument to half that angle, proceed along the line C B, until the object D subtends with C the angle set; then C D B is an isosceles triangle, having the side C B equal to the side C D, the distance required.

Many other methods, more or less simple, may be readily contrived to solve these questions, and others of a like nature;—they all resolve themselves into the principle of constructing, on the accessible surface, triangles either equal or similar, to others resting on the inaccessible distances.

PLOTTING.

Of the Scales generally used for Plans.

The scale to which a survey is to be plotted, must first be determined: as, for instance, 1 inch to a mile, 6 inches to a mile, $26\frac{2}{3}$ inches to a mile, or 3 chains to an inch, or any other dimension suitable to the object of the survey.

The following are among the scales adopted in this country for the special objects referred to:—

Three chains to an inch, $\frac{1}{2376}$ of the actual linear measurement of the ground to be mapped, $26\frac{2}{3}$ inches to a mile, is well adapted to the plotting of surveys of parishes or estates. It is the scale adopted by the Tithe Commissioners for their first-class maps.

Two chains to an inch, $\frac{1}{1584}$ of the linear measurement of the ground, or 40 inches to a mile, is used for plans of building-grounds or valuable property, on which it is required to measure minute portions of land, but it is too small for the survey of large towns.

Scales of 100 feet and 200 feet to an inch, are convenient for engineering purposes, owing to the facility they afford for decimal calculations.

For extensive surveys, smaller scales have been generally adopted. The Ordnance Survey of Ireland is plotted and engraved in outline to a scale of 6 inches to a mile, $\frac{1}{10560}$ of the actual size: the same scale was adopted on the Ordnance Survey for the Northern Counties of England, the southern part having been published on a scale of one inch to a mile, or $\frac{1}{63360}$ of the actual size, on which scale the general map of the whole of England and Wales is to be completed and published.

In accordance with the recommendation of the Statistical Congress held at Brussels, in September, 1853, and with the majority of the highest scientific and professional opinions forwarded to the Government in favour of the adoption, for national surveys, of 'Scales bearing a definite numerical proportion to the linear measurement of the ground to be mapped,' the Lords of the Treasury have ordered,* in reference to the Ordnance Survey of Scotland, that the surveys of Ayrshire, Dumfrieshire, and other districts, should be laid down and drawn on paper to the scale of $\frac{1}{2500}$ of the linear measurement of the ground, being equal to $25\frac{1}{3}$ inches to a mile nearly, and the prevailing scale for Cadastral surveys on the Continent.

Plans and sections for projected lines of inland communication, or generally for public works, requiring the sanction of the Legislature, are required by the 'Standing Orders' to be drawn to scales, not less than 4 inches to the mile, $\frac{1}{15840}$ for the plan, and 100 feet to the inch, $\frac{1}{1200}$ for the section.

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* 'Correspondence respecting the Scale for the Ordnance Survey, and upon Contouring, and Hill Delineation,' presented to Parliament, 1854, page 344.

This Parliamentary Return will be found full of matter of the greatest interest to the Engineering Student, and I would particularly invite his attention to the admirable reports which it contains on these important subjects from Lieut.-Colonel Dawson, R.E., of the Tithe and Enclosure Commission.

Universal Notation for Scales.

To assist in giving more precise ideas of the relative proportions of the scales used, and described in such various ways, the proportion they bear to the actual size, or, more correctly, to the linear measurement of the ground to be mapped, should invariably be given in fractions as used above, and thus expressed in a language as universal as numerical notation. The practice is general on the Continent.

Contraction or Expansion of Paper.

To receive the plotting of the survey, strong drawing-paper is stretched on a board, or mounted on linen. The scale adopted for the plotting of the survey is then ruled on the plan, to serve for future reference, because the paper is liable to alter its dimensions, through hygrometrical changes of the atmosphere, after it has been cut off from the board. The experience obtained by the examination of the great number of Tithe maps inspected by Lieut.-Colonel Dawson, shows generally a contraction in the length of the lines as first laid down; the average of which, on the scale of 3 chains to an inch, or $\frac{1}{2376}$, is from one-fourth to one-half per cent., and requires, therefore, an allowance varying from one-half to one perch per acre.

Triangles first Plotted.

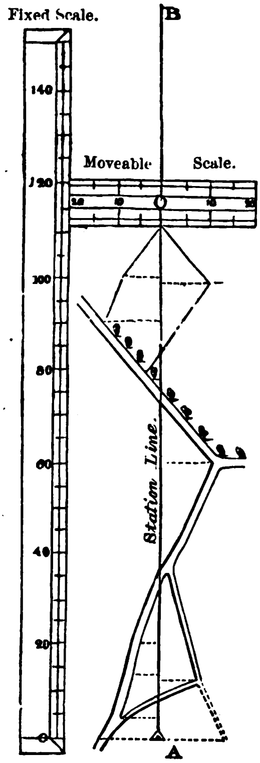
The principal triangles are first laid down in pencil by the intersections of their sides. The lengths of these may be taken from a diagonal scale, and laid down with a pair of compasses; but if their lengths exceed the span of common compasses, they should be laid down by means of a beam-compass graduated to inches, and having a vernier which reads to one-hundredth of an inch. The points of intersection of the sides of the smaller triangles, and of all proof-lines, are then pricked off, and marked in pencil with the corresponding letters, taken from the rough diagram in the field-book.

Plotting of the Details.

The detail is afterwards plotted, proceeding on the paper in the same order as that which was followed in the field. A long scale with bevelled edges, divided into inches and such parts of inches as the scale used for the plan may require, is kept, by means of weights, at a fixed distance from, and parallel to, the main line—the zero of the scale being opposite the starting point of the line. A shorter scale, divided on the edges in the same manner as the first, with its zero point in the middle, is made to slide along the fixed scale at right angles, whilst its central point keeps in coincidence with the station-line. The scales in general use for this purpose have been made of ivory; but, as they are very liable to irregular contraction and expansion, I would recommend well-seasoned box-wood scales in preference.

The lengths entered in the middle column of the field-book are measured on the first scale, and the perpendicular offsets on the offset scale: each point thus determined by two ordinates, is marked on the plan with a fine-pointed pencil, or pricked with a fine needle. These points are, from distance to distance, joined by pencil lines.

Proceeding in this manner from side to side of each triangle according to the order followed in the field-book, the whole of the survey is laid down in pencil. The annexed diagram shews the position of the scales in plotting.



Example of Plotting in Progress.

Drawing of the Plot.

The principal triangles are then ruled in with red lines to be preserved and exhibited as constituting the basis of the whole work. The various boundaries and objects are drawn in Indian ink with the steel drawing-pen: straight lines, rectangular boundaries, such as those of buildings, etc., are ruled; irregular boundaries, such as streams and winding fences, are drawn freely by the hand, the steel-pen being held upright, with its broad edge parallel to the direction of the line to be drawn.

Conventional Signs.

Dwelling-houses are tinted with light flat tints of carmine; out-buildings with Indian ink; streams, rivers, lakes, etc., are tinted with light Prussian blue, laid in flat tints, a second tint being passed along the edges next to the upper boundary of the plan, to make them somewhat darker than the rest; trees are etched in Indian ink with the pen, or washed in with the brush; houses, roads, canals, bridges, etc., are drawn as shown in the plate of conventional signs.* (See page 1).

Plans are usually drawn with their tops to the north. As I suppose the surveys to have been, up to this time, performed without angular instruments, I have given (see note, on setting out *meridian line,* Chapter III.) the means of setting out, on the ground, an approximate meridian line, by a simple mechanical process. The line thus set out in the field, from the apex of one of the triangles, can be connected by one or more tie-lines with one of the sides; and the survey may then be plotted with the north upwards.

Lastly the writing required on the plan is to be disposed in parallel directions from east to west, with the exception of the names of rivers, canals, chains of mountains, etc., which are to be adapted to their natural sinuosities. The disposition of the writing requires taste; and it is always to be borne in mind, that the utility of a plan depends very much on that facility of reference which is obtained by the relative keeping of the names. The size of the letters must in some degree depend on the situations in which they are placed; but, as a general rule, the names should be so written as to be legible at distances proportionate to the importance of the objects to which they refer. Thus, the names of hundreds should be legible at greater distances than those of parishes, villages than single houses, gentlemen's houses than cottages, etc.†

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* Through the kindness of Lieut.-Col. Dawson, of the Tithe Commission Office, I am enabled to give the accompanying plate of conventional signs, which are admirably adapted for detailed plans.

† Colonel Colby's *Instructions for the Interior Survey of Ireland.*

ON THE COMPUTING OF AREAS.

The superficial content of land is generally expressed in statute acres, roods, and perches. The acre is equal to 10 square chains of 66 feet or 22 yards in length, which chain, called Gunter's chain, is divided into 100 links: an acre, therefore, is equal to $(66)^2$ ft. $\times$ $10 = 43{,}560$ square feet, or $(22)^2$ yds. $\times$ $10 = 4{,}840$ square yards, or equal to $(100)^2$ links $\times$ $10 = 100{,}000$ square links; the rood is one-fourth of an acre, and the perch one-fortieth of a rood. When the computation of acreage, therefore, is the object, the linear distances should be measured or expressed in links, the area thence obtained being in square links, is reduced to acres by a simple inspection, by cutting off with a decimal point the last five figures, the remaining figures to the left representing the acres: the decimal fraction, multiplied by 4, gives the roods; and the decimal part of this last product, multiplied by 40, gives the poles or perches.

Example.—Required the acreage of a Field containing 530,500 square links:—

$$
\begin{array}{ll}
5)30500 & \ldots\ 5 \text{ Acres} \\
\quad\ \ 4 & \\
\hline
1)22000 & \ldots\ 1 \text{ Rood} \\
\quad\ 40 & \\
\hline
8)80000 & \ldots\ 8\tfrac{8}{10} \text{ Perches}
\end{array}
$$

Answer . . . . 5A. 1R. 8·8P.

The Table annexed gives, by simple inspection, the roods and perches answering to the decimals.

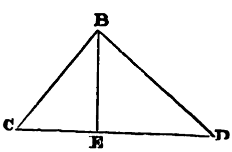
A DECIMAL TABLE

For the Use of LAND SURVEYORS, showing the Decimals answering to every Rood and Perch in the Acre.

Perch.	Dec.	1 Rood.	2 Roods.	3 Roods.	Perch.	Dec.	1 Rood.	2 Roods.	3 Roods.
+ 0	·000	·250	·500	·750	21	·131	·381	·631	·881
+ 1	·006	·256	·506	·756	+22	·137	·387	·637	·887
+ 2	·012	·262	·512	·762	+23	·144	·394	·644	·894
+ 3	·019	·269	·519	·769	+24	·150	·400	·650	·900
+ 4	·025	·275	·525	·775	+25	·156	·406	·656	·906
+ 5	·031	·281	·531	·781	+26	·162	·412	·662	·912
+ 6	·037	·287	·537	·787	+27	·169	·419	·669	·919
+ 7	·044	·294	·544	·794	+28	·175	·425	·675	·925
+ 8	·050	·300	·550	·800	+29	·181	·431	·681	·931
+ 9	·056	·306	·556	·806	+30	·187	·437	·687	·937
+10	·062	·312	·562	·812	+31	·194	·444	·694	·944
+11	·069	·319	·569	·819	+32	·200	·450	·700	·950
+12	·075	·325	·575	·825	+33	·206	·456	·706	·956
+13	·081	·331	·581	·831	+34	·212	·462	·712	·962
+14	·087	·337	·587	·837	+35	·219	·469	·719	·969
+15	·094	·344	·594	·844	+36	·225	·475	·725	·975
+16	·100	·350	·600	·850	+37	·231	·481	·731	·981
+17	·106	·356	·606	·856	+38	·237	·487	·737	·987
+18	·112	·362	·612	·862	+39	·244	·494	·744	·994
+19	·119	·369	·619	·869	+40	·250	·500	·759	
+20	·125	·375	·625	·875					

The area of the principal triangles in the survey should, in all cases, be computed from the length of their sides (reduced to their horizontal bases), as obtained from the field-book. The operation (see investigation of theorem in note*) is simple in all cases, but rapidly performed with logarithmic tables. The contents of the fields and other inclosures could be similarly obtained by computation without measurement on the plan, but the process would

* Formulæ for Areas of Triangles.



Let A = area of triangle B C D,
h = B E, the height of the triangle,
B C = d, B D = c, C D = b,

then $A = \frac{b \cdot h}{2}$, but $h = d \sin. C$,

therefore,

$$A = \tfrac{1}{2}\, b \cdot d \sin. C. \quad (1)$$

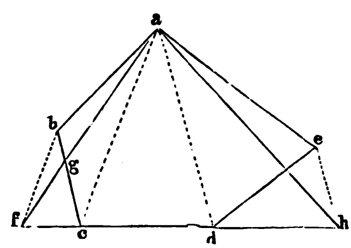
Again, sin. B : sin. D : : b : d, whence

$d = \frac{b \sin. D}{\sin. B}$; by substitution, we have be extremely laborious. The area is therefore obtained from measurement on the plan, and a comparison of the sum of the areas of each inclosure with the area of the whole, as deduced by direct computation from the field-book, guards against the introduction of any casual errors of importance.

Determination of Areas by Geometrical Construction.

The area of fields, or inclosures, might be calculated from the plan, by reducing the irregular polygons, constituting the inclosures, to triangles equivalent in area, as in the following example.

In the annexed irregular polygon *a b c d e*, draw a pencil line from *a* to *c*, and, with a parallel ruler, draw the pencil line *b f* parallel to *a c*, cutting *c d* produced in *f*; join *a f*, then the area of the quadrilateral *a f d e* is equal to the area of the original figure. For, the triangles



$$A = \tfrac{1}{2}\,\frac{b^2 \sin. C \sin. D}{\sin. B} = \frac{b^2 \sin. C \sin. D}{2 \sin. (C + D)} \quad (2)$$

To obtain the area in terms of the sides; substituting in equation (1) the value of $\sin. C = 2 \sin. \tfrac{1}{2} C \cos. \tfrac{1}{2} C$ (trig.) we obtain

$A = b\,d \sin. \tfrac{1}{2} C \cos. \tfrac{1}{2} C$; but, making $s = \dfrac{d + b + c}{2}$

(trig.) $\sin. \tfrac{1}{2} C = \sqrt{\dfrac{(s - b)(s - d)}{d\,b}}$, and

(trig.) $\cos. \tfrac{1}{2} C = \sqrt{\dfrac{s(s - c)}{d\,b}}$; substituting, we have

$$A = b\,d \sqrt{\frac{s(s - b)(s - c)(s - d)}{b^2 d^2}}$$

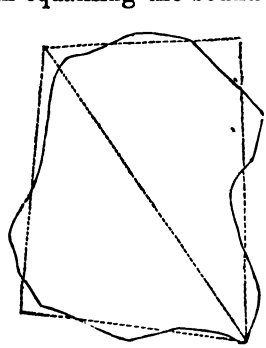
$A = \sqrt{s(s - b)(s - c)(s - d)}$ = area of triangle.

a b f, *c b f*, being on the same base, *b f*, and between the same parallels, are equal; from each take away the common triangle, *b g f*, the remainders, *a b g*, *c f g*, are equal. Bnt in the alteration of the figure, *c f g* has been substituted for *a b g*, therefore the area of the quadrilateral, *a f d e*, is equal to the area of the original figure. In the same manner, by drawing *e h* parallel to *a d*, intersecting *c d*, produced in *h*, and joining *a h*, the area of the triangle *a f h* is made equal to the area of the quadrilateral *a f d e*, and consequently equal to the original figure, the contents of which are obtained by multiplying the base, *f h*, by half the height of the triangle.

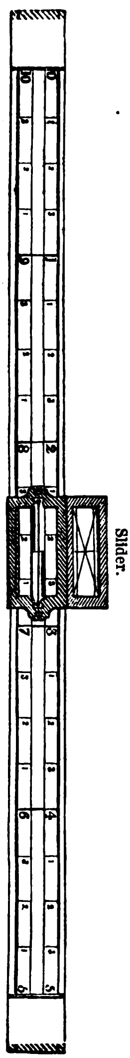
It is evident that, whatever may be the number of sides of the polygon, a similar process will reduce it to an equivalent triangle; but it is no less manifest, that the method would be very tedious if the boundary of the inclosure or figure were very crooked, presenting the additional objection of inaccuracy, owing to the number of intersections to be obtained through the mechanical assistance of a parallel ruler.

Determination of Areas by 'Equalisation' of Boundaries.

A more rapid method consists in equalising the boundaries by the eye, by applying the straight edge of a transparent piece of horn, or of gum paper, in such a manner that the small parts cut off by it, from the crooked figure, should be equal to those taken in. Pencil lines (represented by the dotted lines in the sketch) are then ruled along the edge of the horn, and the figure is afterwards reduced to triangles, the contents of which are easily computed.



Various other means of obtaining contents from the direct measurement of plans have been adopted; I shall limit my description to the following method, by which the necessity for calculation is avoided; and which, from its great simplicity and accuracy, has now generally superseded all others. Its first general application was made in the Tithe Commission Office, about the year 1840.



Description of the 'Computing Scale.'

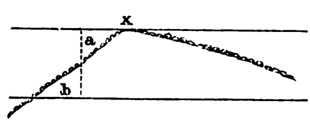
A scale, represented in the annexed diagram, is made equal in length to 50 chains, as plotted on the plan: it is divided by transverse lines into 20 parts; each part, being 2½ chains in extent, would therefore, between parallels 1 chain in width, represent a space equal to 1 rood; two of the parts, being 5 chains in extent, between the same parallels, would represent a space equal to 2 roods; and so on for the whole length of the scale, which itself would, between parallels 1 chain asunder, represent a space equal to 20 roods, or 5 acres.* A slider is attached to the scale, admitting of motion in the direction of its length; it bears a vernier scale,† with spaces equal in length to 1 part on the primary scale (or to 2½ chains), divided into 40 parts, each of which spaces would, therefore, with a height of 1 chain, represent $\frac{1}{40}$ of a rood, or 1 perch. The slider carries, in a projecting frame, a fine hair, or wire, drawn across its centre, at right angles to the direction of the length of the scale.

* See Computation of Areas, p. 30, where it is stated that
$1 \text{ acre} = 1 \text{ chain}^2 \times 10 \text{ chains}$.

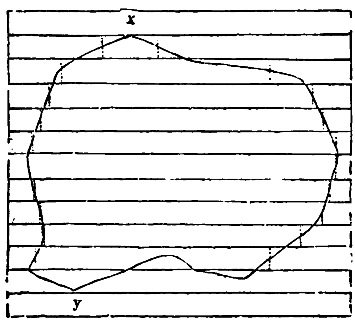
† For a description of the "Vernier," see p. 40.

Mode of using the 'Computing Scale.'

To apply this scale to the measurement of areas, a piece of strong transparent paper (called horn-paper) with parallel lines ruled, 1 chain apart, is laid over the field the area of which is required, and kept fixed by a weight. The ruled paper must be so applied that two opposite salient points of the field, such as *x y*, in the figure page 36, may touch some two of the parallel lines. The scale is then placed upon the ruled paper in a direction parallel to the lines; and the vernier in the sliding frame, being set opposite to the zero point on the scale, is brought also to coincide with, or intersect, that portion of the left-hand boundary fence which is inclosed between the upper pair of parallel lines. If the wire and boundary fence do not exactly coincide, the scale must be moved bodily to the right or left until the spaces, *a*, *b*, intersected between the parallels by the wire and fence, are judged by the eye to be equal to, and to compensate, one another. The scale is then kept fixed by the pressure of the hand, while the sliding frame is moved to the right-hand boundary of the field, and the cross-wire adjusted in the same manner as before: so far, the number of divisions on the scale passed over by the cross-line, in its movement from left to right, indicates the number of roods and parts of roods included in the field between the upper parallel lines. The *slider* remaining fixed in position, the *scale* is moved downwards 1 chain, that is, to the next lower division of the parallel lines, and the vernier is brought, as before, by the movement of the *scale*, to coincide with, or to compensate, the left-hand boundary. The coincidence, or compensation, being established,



the sliding *frame* is moved to bring the cross-line to coincide with, or to compensate, the right-hand boundary; the division opposite the cross-line then indicates the number of acres, roods, and perches comprised within the two upper parallel divisions. When the slider has, by a repetition of the process, passed over the whole length of the scale, 5 acres have been measured. The operation is then continued by moving the slider from *right* to *left* (the equalization commencing on the right-hand side), until the slider having reached the starting point or zero of the scale, 10 acres have been measured, the quantity measured being indicated by the scale itself. The mechanical operation is continued and repeated in a similar manner, until the whole field has been measured.



The student is recommended to take the 'computing scale' in hand, and follow, by measurement on a plan, each step of the operation as above described. He will then easily understand the process, and appreciate the rapidity and accuracy of the results.

On Copying Plans.

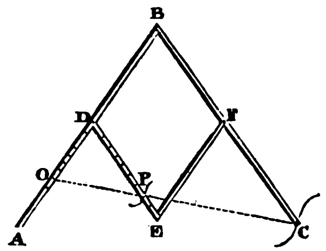
It rarely happens that one copy only of a plan is required. When the lines and boundaries are regular, duplicates may be made by laying the plan upon the sheet of paper or vellum on which the copy is to be drawn, and pricking, with a fine needle, through all the angular points necessary to define the figures: the punctures being then connected by pencil lines, the plan is finished by drawing these in Indian ink.

This method is not suitable to the transfer of irregular or curved boundaries. An accurate and rapid way of copying these, or plans of small extent, is by means of the instrument called a copying glass. It consists of a large piece of plate glass, set in a frame of wood, which can be inclined to any angle, in the same manner as a reading or music desk. On this glass the original plan and the fair sheet of paper are laid, and the frame being raised to a suitable angle, a strong light is thrown, by means of tin reflectors or otherwise, on the under side of the glass, whereby every line in the original plan is seen distinctly through the fair sheet. The copy is at once made in ink, and finished while being traced.

Plans of greater extent cannot be conveniently copied by means of the 'copying glass.' Moreover, being generally mounted on linen, or other material, which renders them opaque, they do not admit of the operation just described. In such cases, the plan is first traced in Indian ink on transparent tracing paper. The first copy is then carefully laid over the fair sheet; black-leaded or transfer paper being first placed under the tracing. All is steadied by numerous weights laid along the edges, or by drawing-pins fixed into the drawing-board or table; a fine and smooth point is then passed over each boundary or mark on the tracing, with a pressure of the hand sufficient to cause a clear pencilled mark to be left on the fair sheet by the black-leaded or 'transfer paper.' The whole outline is thus obtained, and afterwards drawn in Indian ink in the usual way.

On Reducing Plans.

Plans may be reduced by means of the pentagraph, to any proportion wanted. The instrument consists of a jointed rhombus, B D E F, made of brass, and having the two sides, B D, B F, extended to double their length; the side D E, and the branch D A, are marked from D with successive divisions, D O being made to B O always in the ratio of D P to B C. Small sliding boxes for carrying a pencil, or a tracing point,



are placed at P and C, and secured in their positions by screws; the point O is made the centre of motion, and rests on a fulcrum or support of lead; and the tracer is fixed at C, while the pencil is lodged in P. From the property of similar triangles, the three points, O, P, and C, must range in the same straight line, which is divided at P in the ratio required. While the point C, therefore, is carried along the boundaries of any figure, the intermediate point, P, will trace out a similar figure, reduced in the proportion of O C to O P, or of O B to O D, the proportion required.*

By changing the relative positions of the tracer and pencil, the figure would be enlarged in the same proportion; but errors are so much increased by enlarging plans with the pentagraph, that it ought not to be resorted to where accuracy is required, except in cases where the field-books are either lost, or do not afford the data required to plot on a larger scale, and when there are no means of repeating the survey.

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* Leslie's *Geometry*, page 431.

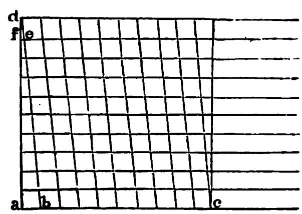
CHAPTER II.

SURVEYING INSTRUMENTS.

The Diagonal Scale.

WHEN a line is to be divided into equal parts, so numerous and minute, that they would be indistinct, some method, not embodying direct subdivision, must be adopted in order to estimate those minute fractional parts. The diagonal scale, an ingenious application of the property of similar triangles, described in Euc. VI. 4, was among the earliest methods resorted to for this purpose.

In the annexed figure, the line ac is divided into any number of equal parts, or into n times ab, and a space equal to each of these parts is indirectly subdivided into n secondary parts, by means of diagonal lines, of any arbitrary length, raised from the points marking the primary divisions. These diagonals decline, in their entire lengths



from the perpendicular by intervals equal to one of the primary parts, and they are cut transversely into n equal parts by equidistant lines parallel to ac. In the triangles, adb, fde, we have

$$bd : ab :: de : ef,$$

but de is, by construction, equal to the n^{th} part of bd, therefore ef is equal to the n^{th} part of ab, or $ef = \frac{1}{n} \times ab$.

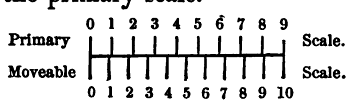
But, by construction, $ab = \frac{1}{n} \times ac$, hence, by substitution,

$$ef = \frac{1}{n} \times \frac{1}{n} \times ac = \frac{ac}{n^2}.$$

The Vernier Scale.

This subdivision by diagonals is still in general use, and constituted the first improvement in dividing astronomical and geodesic instruments.* In this part of its application it has been superseded by the vernier scale, the simplest and most ingenious of the methods hitherto invented for the minute subdivision of lines. It obtains this object by measuring the *differences* between the divisions of two approximating scales; one of which is fixed, and called the primary scale; the other moveable, and called the vernier.

If a space, on the primary scale, be divided into a given number of parts, equal to $n-1$ (in the figure equal to 9), and a space, equal in length to the first, be divided on the moveable scale into a number of parts equal to n (in the figure equal to 10), these latter parts will each be smaller than the first by the n^{th} part (in the figure the 10^{th} part) of a division on the primary scale.



For, let $a=$ the length of a division on the primary scale,
$b=$ the length of a division on the moveable scale; then, by hypothesis,

$$(n-1)\,a=n\,b, \text{ or}$$
$$n\,a-a=n\,b, \text{ and}$$
$$a-\frac{a}{n}=b\,; \text{ or } a=b+\frac{a}{n}\,;$$

that is, b, a division on the moveable scale, is smaller than a, a division on the fixed scale, by the n^{th} part of a.

The edges of the two scales being applied to each other, so that the extreme end of the vernier, which is marked ♀ and is called the index, coincides with a division on the

* LESLIE'S *Geometry*.

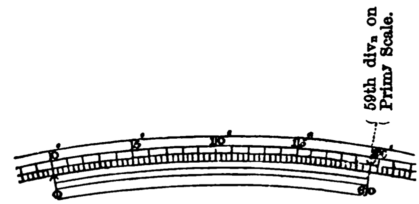
fixed scale, then the quantity of aberration at the first division of the vernier will be $\frac{a}{n}$ (in the figure $\frac{1}{10}$ of a), at the second, $\frac{2\,a}{n}$, and so on to the n^{th} division, when the aberration becomes equal to $\frac{n\,a}{n}$ or a; and therefore a coincidence always obtains simultaneously at the first and last division of the vernier. In moving the vernier forward, the quantity of aberration at each successive division will diminish by the extent of the space moved over, until a new coincidence takes place at the first division, which coincidence shows the amount of displacement to have been $\frac{a}{n}$. In the same manner, the amount of displacement may be made equal successively to $\frac{2\,a}{n}$, $\frac{3\,a}{n}$ ..... $\frac{(n-1)\,a}{n}$, and, finally, $\frac{n\,a}{n}$ or a, when the space moved over becomes equal to one of the divisions on the primary scale.

If 1 inch, for example, be divided on the primary scale into 10 equal parts, and a space on the vernier equal to 9 of these parts be itself divided into 10 equal parts, the difference between a division on the primary scale and a division on the vernier will be equal to $\frac{1}{10}$ of the first, and therefore equal to $\frac{1}{100}$ of an inch.

A general rule for reading with a vernier may be expressed thus: observe the number of parts on the primary scale that is equal in length to the same number *increased by one* on the vernier; this last number is the denominator of a fraction whose numerator is unity, and which expresses the subdivision of the *parts* on the primary scale.

Application of the Vernier Scale to the Theodolite.

For example, on the common 5-inch theodolite, the circle forming the primary scale is divided into 360 degrees, each degree being subdivided by shorter lines into 20 minutes. Now a space, equal in length to 59 of these subdivisions, is adopted for the length of the vernier, and divided into 60 parts; each of these parts on the vernier is,



Vernier, divided into 60 *parts.*

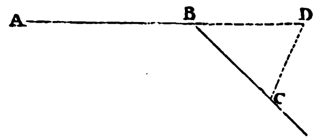
therefore, smaller than those on the circle by $\frac{1}{60}$, or by 20 seconds. In reading with the instrument, an account is first taken of the degrees, and fractional parts of a degree, as shown by the index of the vernier; then the eye is passed along the vernier until some one of its lines, coinciding with any line on the primary scale, is found. The quantity to be added to the first approximate measure is then obtained by counting the number of divisions included between the zero or index of the vernier, and the coincident line, each division corresponding, as before explained, to 20 seconds. To simplify the reading, every third line on the vernier is made longer than the rest, and represents minutes, while the shorter intervening lines represent one-third of a minute, or 20 seconds.

THE THEODOLITE.

The method of surveying by the chain alone is applicable only to surveys of comparatively small extent, and simple in their outlines; for, even in small surveys, the intervention of towns, villages, high inclosures, or other obstacles, may be found to render the measurement of right lines by the chain extremely difficult; and, by isolating different portions of the work, cause inaccuracies that may be avoided by the use of an angular instrument.

Measurement of Angles by Chainwork only.

Angles, it is true, may be determined by the chain alone, by measuring the sides of small triangles disposed for the purpose, thus: let A B represent a line measured to a station B, from whence a second line B C, forming an angle with A B, is to be measured. To determine the angle A B C, prolong A B

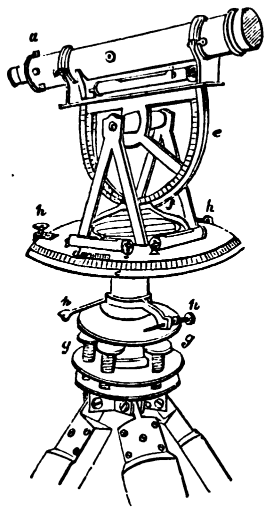


to D, make B C equal to B D (in order to construct a well-conditioned triangle), and measure the chord D C; the three sides of the triangle B D C being known, the angle D B C, or its supplement A B C, is determined. This is a method which is, however, rarely resorted to; for no time is gained by its adoption, and the chances of error are considerably multiplied, owing to the numerous additional lines to be measured. Moreover, it is to be observed, that angles can in general be measured in the field more correctly with an instrument than the length of lines with the chain, especially over uneven ground, or in an inclosed country.

The instrument in general use, for the purpose of measuring angles in surveying, is the theodolite, of which there are several constructions, differing slightly in the arrangement and adjustment of the parts. A detailed and clear description of these varieties, as also of nearly all the instruments used in surveying, is given by Mr. Simms, in his valuable *Treatise on Mathematical Instruments.* The following is a description of the theodolite in most general use.

Description of the Theodolite.

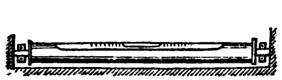
The theodolite consists of two circular plates, the upper turning freely on the lower, and both having a horizontal motion by means of a vertical axis. This axis, with the view to render the motion of the circular plates independent one of the other, is made of two parts, external and internal; the former secured to the lower plate, and the latter to the upper plate. The circumference of the lower plate is divided into 360 degrees, and parts of a degree, as described when treating of the vernier: and at the extremities of a diameter of the upper plate are fixed two verniers.*



a Diaphragm with adjusting screws.
b Spirit-level.
c Lower circular plate.
d Vernier to lower circular plate.
e Vertical arc.
f Vernier to vertical arc.
g Parallel-plate screws.
h Clamp and tangent screws.

These circular plates are intended to measure horizonta angles, *i.e.*, angles in a plane parallel to the horizon,—they must therefore be adjusted in the horizontal plane. This adjustment is effected by means of four screws (called parallel-plate screws), set in pairs opposite to each other. Two spirit-levels, placed at right angles to each other, on the upper plate, serve to guide the moving of the screws.

A spirit-level is a glass tube nearly filled with a liquid, generally spirit of wine, and hermetically sealed. The tube has a slight and regular curvature; it is placed with the convex side upwards, and the bubble in that position occupies the higher central part. Several divisions marked on the

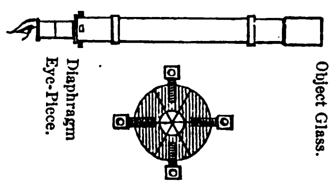


* Simms, *on Mathematical Instruments.*

tube at equal distances on each side of the bubble, serve to indicate very slight deviations from horizontality; that level being most sensible whose upper side most nearly approaches to a mathematically straight surface.

The upper and lower horizontal plates are retained in any required position, either conjointly or separately, by distinct sets of clamp-screws; and tangent-screws afford the means of fixing them with more precision than can be attained by the hand alone. A frame, resting on the upper plate, supports the axis of the telescope in angular recesses called Y's, from their resemblance to that letter. A horizontal motion, therefore, given to the telescope for the purpose of observing an object, may, by the above arrangement of the double vertical axis, and double clamps, be communicated to either or both of the circular plates.

In the focus of the eye-piece and object glass, and at right angles to the length of the telescope, are placed three lines formed of fine wires or spider's web; one horizontal,

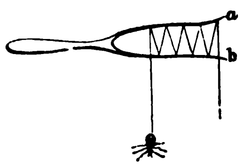


Front view of Diaphragm.

the other crossing its middle point, so as to form an acute angle with each other. These wires serve to point the axis of the telescope with certainty to any object or part of an object.

The best wires in use are generally made of platinum; they are prepared exceedingly fine by rolling platinum with silver, and drawing the combined metals to a very fine wire. The silver is then dissolved in nitric acid, which, having no influence on the platinum, leaves the wire of this metal much finer than it could be made by merely mechanical means.

When these wires are broken, an accident to which their fineness renders them liable, they are easily replaced by cobwebs in the following manner:—'A piece of wire is bent into a shape something like a fork, the opening *a b* being rather larger than the diameter of the diaphragm. A cobweb being selected, from the extremity of which a spider is suspended, it is wound round the fork in the manner represented in the sketch, the weight of the insect keeping it constantly tight. The web is thus stretched ready for use; and when it is required to fix on a new thread, it is merely necessary to put a little gum or varnish on the diaphragm, and adjust one of the threads to its proper position, as indicated by faint notches on the metal.'*



From the lower part of the telescope is suspended a spirit-level, which, being more sensible than the levels fixed on the vernier plate, is used in the final adjustment of the circular plates to the horizontal position, previous to taking observations in which great accuracy is desired.

To the under part of the telescope is attached a vertical semicircular arc, for the purpose of observing altitudes and depressions. The axis of this vertical arc rests, at equal heights above the vernier plate, on two points in the frames supported by it; consequently, when the upper plate is horizontal, the semicircular arc attached to the telescope is in a vertical plane. The angle of inclination of the telescope is indicated by a fixed index and vernier attached to the upper plate in the locus of a perpendicular, let fall from the centre of the axis of the vertical arc. The vertical arc is adjusted and retained at any required angle of inclination, by means of a clamp and tangent-screw. One side of the

* LIEUT. FROME'S *Trigonometrical Surveying*, p. 21.

arc is graduated into degrees and parts of a degree; the other side shows the difference between a hypothenuse of 100 units and the base in right-angled triangles, calculated to degrees of inclination of the hypotenuse from 0° to 45°. By means of this table of differences, the required reduction can be accurately made when actually measuring over an inclined plane, in order to reduce it to its horizontal base.

The above constitute the essential characteristics of the instrument, but a compass-box is usually attached to the upper circular plate. The compass is sometimes used for noting the bearings of different stations with the meridian, as a check on the measured angles. Also, in setting out a long straight line, the extremities of which are invisible from intermediate points, the bearing of the line may, with advantage, be taken at its extremities, and at intermediate points, in order to serve as a check, but as a check only, on the straightness of the line.

The theodolite is fixed, by means of a screw, on the staff-head of the three legs which form its stand. Beneath the centre of the staff-head, a hook is attached for the purpose of suspending a plummet to guide the observer in placing the instrument exactly over the station at which the observations are to be made.

ADJUSTMENTS OF THE THEODOLITE.

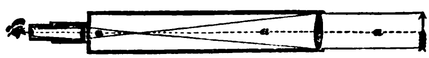
Correction of Parallax.

Before the theodolite can be advantageously used, it must be examined and proved to be in perfect adjustment. The first adjustment to be attended to, is the correction of parallax. For this purpose, draw out the tube of the eye-piece till the cross wires appear clearly defined; then place the eye opposite the middle of the aperture of the eye-glass, and point the telescope to any distinct distant object. Next move the eye slightly to the right and left, and if any apparent motion of the object be thereby occasioned, such motion is said to arise from parallax in the telescope, and the correction of it consists in drawing out the tube of the eye-piece a little more or less, until the required stability of the image takes place.

As an additional precaution against error from parallax, it is well always to observe, as nearly as possible, through the middle of the aperture of the eye-glass.

The Line of Collimation and its Adjustment.

The next adjustment is that of the line of collimation, *i.e.*, a line (*a a* in diagram) passing through the point



of intersection of the cross wires, fixed in the focus of the object and eye-glasses, and the centres of those glasses. The adjustment consists in making this line of collimation to coincide with the axis of the cylindrical rings in which the telescope turns. This is necessary, because the observations are made and registered under the supposition that such a coincidence exists, as it is only by reference to the cylindrical rings or external tube of the telescope, that we estimate the direction of the optical axis, whether in azimuth or altitude: or, in other words, either horizontally or vertically. This adjustment is effected as follows:—

Make the intersection of the cross wires to coincide with some well-defined part of a distant object; then turn the telescope half round in its Y's, till the level lies above it; if the same point be not then covered by the centre of the wires, move this centre one-half of the amount of deviation by means of the diaphragm screws, and correct the other half by elevating or depressing the telescope. If the coincidence of the wires and the object then remain perfect in both positions of the telescope, the line of collimation in altitude or depression is correct; but if not, the operation must be repeated until the adjustment is satisfactory. A similar process will adjust the line of collimation in the vertical plane.

Adjustment of the Level.

The next adjustment is that which fixes the level, attached to the telescope, in a position parallel to the rectified line of collimation, or longitudinal axis of the telescope. The necessity for this adjustment is evident, as the only means of judging of the horizontal position of the optical axis, or of the amount of deviation from it, is by reference to the spirit-level, which is assumed to be parallel to the said axis. To effect this adjustment, the clips that retain the telescope in its place being open, and the vertical arc clamped at or near zero, bring the air-bubble of the level to the centre of the tube by turning the tangent-screw, which moves the vertical arc; then reverse the telescope in its Y's, end for end. If the bubble do not return to the middle of the tube, bring it back one-half by the capstan-headed screw placed at one end of the tube (to elevate or depress that end of the level), and the other half by the tangent-screw that acts on the vertical arc. This process is to be repeated until the adjustment is perfect.

Adjustment of the Circular Plates.

Another adjustment is that by which the circular plates are placed horizontally. This is also necessary to insure accuracy in the angles of elevation and depression, and also because any deviation from the horizontal plane would introduce a proportional error in the measurement of what are assumed to be horizontal angles.

Place the instrument as nearly level as can be done by the eye, fasten the lower horizontal plate by its clamp, leaving the upper plate free, and move the latter so as to place the telescope over two of the parallel-plate screws; then bring the bubble of the level under the telescope to the middle of the tube by the tangent screw of the vertical arc; next turn the upper plate 180° from its former position; if the bubble do not return to the middle, half the difference is to be corrected by the parallel-plate screws, and half by elevating or depressing the telescope by means of the tangent-screw. The same operation is to be repeated over the other pair of parallel-plate screws, so that the air-bubble of the spirit-level attached to the telescope shall remain constantly in the centre of the tube, in whatever position it is placed. The two small levels on the vernier-plate are then to be adjusted, by means of their capstan-headed screws; and, when so adjusted, these levels will serve to place the circular-plates in a horizontal plane much more quickly than by the process before described.

'Index Error' in Vertical Arc.

The vernier of the vertical arc is next to be attended to: it is correct if it point to zero when the foregoing adjustments have been made, and any deviation may be rectified by the attached screws; or, if the deviation be small, note the quantity of deviation as an index error, and apply it, + or −, to each vertical angle observed. This deviation is best determined by repeating the observation of an altitude or depression in the reversed positions, both of the telescope and vernier-plate; the two readings will have equal and opposite errors, and the half of their difference will be the index error.

On Measuring Angles with the Theodolite.

To measure Horizontal Angles.

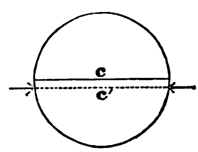
The adjustments before described having been carefully examined and rectified, place the theodolite exactly over the station from whence the angles are to be taken, by means of the plumb-line suspended from its centre. Then level the circular-plates with the parallel-plate screws, by bringing the telescope, with the vertical arc clamped at zero, over each pair alternately. Clamp the lower horizontal limb in any position, and direct the telescope to one of the objects to be observed, moving it till the object and cross wires coincide; then clamp the upper limb, and by its tangent-screw make the intersection of the wires exactly bisect the object; now read off the two verniers, which are respectively marked A and B, noting the degrees, minutes, and seconds, of A, and the minutes and seconds of B, and take the mean of the two readings.

Next, release or unclamp the upper plate, and move it round until the telescope is directed to the second object, whose angular distance from the first is required, and by the clamp and tangent screws make the cross wires to bisect the object. Again read off the verniers, and the difference between their mean, and the mean of the first readings, will be the angle required; thus:

	Vernier A.	Vernier B.	Mean.
1st reading . . .	142° 36′ 20″ . .	36′ 40″ . .	142° 36′ 30″
2nd reading . .	228° 43′ 20″ . .	43′ 0″ . .	228° 43′ 10″
		Difference . . .	86° 6′ 40″

Errors of 'Eccentricity' in the Circular Plates.

The object of reading off the two verniers placed diametrically opposite to each other, is to counteract the errors in the construction. For whatever be the amount of these errors, their effects are diminished by taking the mean of the two readings as above described. By the double reading the effects of eccentricity in the two circular-plates and their axes are neutralised; for the effect of eccentricity is always to increase one such reading by exactly the same quantity by which it diminishes the other.* This correction is one of essential importance, because the principle of the instrument requires that the circles should be concentric with the axes on which they are made to turn, and with each other. However, in cases where great accuracy is required, the observer should not rest satisfied with one measurement of the angle, even thus corrected from the error due to eccentricity: for the errors due to graduation, and those resulting from the observations themselves, are also diminished by repetition.

Vernier A.  Vernier B.

Errors of Graduation.

To whatsoever degree of perfection the construction of astronomical or geodesic instruments may be brought, it constitutes only an approximation to geometrical accuracy; and among the varied operations of this high branch of mechanical art, none presents greater difficulties than the accurate division of the circumference of a circle turned in

* 'The geometrical property of the circle whereby two lines, intersecting one another in any point within a circle, cut off opposite arches of the circumference, the sum of which is the same as if they intersected one another in the centre, contributes to the perfection of the instrument. For it follows that, in a circular instrument, whether the centre about which the index turns be the true centre or not, the mean of the two opposite arcs is the exact measure of the angle to be found.'—(PLAYFAIR'S *Works*, vol. iv. page 503.)

metal into 360 equal parts, and these again into smaller subdivisions. 'The attainment of perfect accuracy in this work has hitherto baffled the utmost stretch of human skill and industry; nor, if executed, could it endure. The ever-varying fluctuations of heat and cold have a tendency to produce, not merely temporary and transient, but permanent, uncompensated changes of form in all considerable masses of those metals which alone are applicable to such uses; and their own weight, however symmetrically formed, must always be unequally sustained, since it is impossible to apply the sustaining power to every part separately: even could this be done, at all events force must be used to move and fix them, which can never be done without producing temporary, and risking permanent, change of form.'*

Errors of Observation.

The next class of errors, called errors of observation, arises from inexpertness, defective vision, atmospheric indistinctness, momentary instrumental derangement due to the want of a firm basis to support the instrument, slips in clamping, looseness of screws, etc.

Means of Counteracting the Errors of Graduation and of Observation.

To obviate, in a great degree, these errors, the 'principle of repetition,' an invention generally attributed to Borda, is thus applied.† After making the second bisection, as above described, leave the upper plate clamped to the lower plate, and release the clamp fixing

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* SIR J. F. W. HERSCHELL'S *Astronomy*, p. 66.

† 'The first idea of this excellent contrivance occurred to Tobias Mayer, of Gottingen, whose name is so well known in the history of Astronomy.'—(PLAYFAIR'S *Works*, vol. iv. page 503.)

the latter to the external axis; now move the horizontal limb with the telescope to point to the first object, till the cross wires are in coincidence with it. Fixing it thus by the clamp which acts on the external axis, release the upper plate, and turn the telescope towards the second object, and again bisect it by means of the clamp and tangent-screw of the *upper* plate. 'Let this process be repeated as often as is deemed advisable (suppose ten times for very accurate geodesic operations); then will the final arc read off on the circular plate be ten times the required angle, affected by the joint errors of all the ten observations, but only by the same constant error of graduation, which depends on the initial and final readings alone. Now, the errors of observation, when numerous, tend to balance and destroy one another; so that, if sufficiently multiplied, their influence will disappear from the mean result. There remains, then, only the constant error of graduation, which comes to be divided in the final result by the number of observations, and is, therefore, diminished in its influence to one-tenth of its possible amount, or to less if need be. The abstract beauty and advantage of this principle seems to be counterbalanced in practice by some unknown cause, which, probably, must be sought for in imperfect clamping,' and the straining of the parts consequent on the action of the tangent-screws.

Test of Accuracy in Horizontal Angles.

A proof of accuracy of a number of horizontal angles, if they surround the station, is obtained by adding them all together, and their sum, if correct, will be 360°. Also, if they be taken at several stations (near enough to each other to make the spherical excess inappreciable), the sum of all the interior angles of the polygon formed by joining the stations by straight lines will be equal to twice as many right angles as the polygon has sides, wanting 4 right angles (Euc. I. Cor. 32.). Thus, if the figure have 3 sides, the sum of the interior angles will be equal to 180°; if 4 sides, the sum will be equal to 360°.

To Measure Angles of Elevation or Depression.

Unclamp the vertical arc, and direct the intersection of the cross wires of the telescope to the object. Note the reading of the vertical arc, and repeat the operation with the telescope turned half round in its Y's; that is, with the level uppermost: the mean of the two readings will neutralise the effect of any error that may exist in the line of collimation. When very great accuracy is aimed at, or when it is desired to ascertain the index error, two more readings may be taken, in a similar manner, with the upper or vernier plate and telescope reversed longitudinally. If there be any index error, it will be equal to half the difference between the mean of both readings, which, if no error existed, would be equal. The mean of the four readings will thus be free from the effects of any error that might exist in the adjustment of the line of collimation, as well as from the index error of the vertical arc.

Magnetic Bearings.

The magnetic bearing of an object is taken by simply reading the angle pointed out by the compass needle, when the object is bisected. With the common compass attached to the theodolite (usually from 4 to 5 inches in diameter), the angle cannot be obtained with certainty nearer than one degree to the truth.

In conclusion, it may be observed that the telescope generally used (see diagram, page 48), shows the objects inverted; the reason for the inversions is, that fewer glasses being required, objects are seen more clearly. Practice quickly renders the inversion of the image immaterial to the observer. However, a second eye-piece is usually provided, which shows objects in their natural position, and may be substituted at the eye-end of the telescope.

The Spirit-Level.

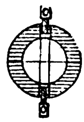
The knowledge of the principal parts of the theodolite and its adjustments, will enable the student to understand, by a simple inspection, the use and adjustments of the Spirit-Level.

Description of the Spirit-Level.

The instrument consists of a telescope precisely similar to that of the theodolite, and requiring analogous correction for parallax and adjustment of the line of collimation. To the telescope is attached a spirit-level, the longitudinal axis of which should be likewise, and for the same reason as in the theodolite, parallel to the line of collimation. Finally, the object of the instrument being to obtain a constantly horizontal axis of vision, the telescope or line of collimation is brought into a horizontal plane by means of the parallel-plate screws.

Parallax. The correction for parallax is made by bringing the tube carrying the eye-glass into such a position that the point of intersection of a distant object shall not be altered by a slight vertical or lateral movement of the eye.

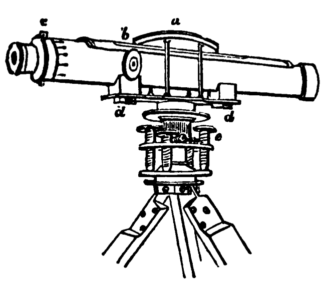
Collimation. In the construction of the instrument known as the Y spirit-level, the telescope rests on supports similar to those which retain the telescope of the theodolite:—for this construction, the line of collimation is rectified, as in the theodolite, by turning the telescope half round in its Y's after the first observation has been made, and noting whether the horizontal cross wire then intersects the same point. The cross wires in the diaphragm of the level, are arranged as shown in the diagram; the horizontal wire marks the intersection of the horizontal visual ray with the staff; the two vertical wires serve to direct the telescope, so that the staff shall be seen between them, and thus be in the axis of the lenses, and likewise in a vertical direction.



In the Y level, the spirit-level is suspended beneath the telescope, as in the theodolite, and requires a similar adjustment to bring its longitudinal axis parallel to the line of collimation, which is performed by reversing the telescope end for end in its Y's, and altering the screws as described in the adjustments for the theodolite.

But in those modifications of the instrument known as 'Troughton's Improved Level,' and 'Gravatt's Level,' and which, from superior compactness, greater stability of the adjustments, and increased optical power, are more generally used, the adjustment of the line of collimation and of the spirit level, are made in a different manner.

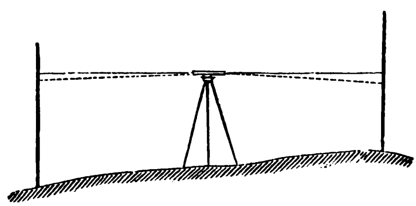
By a reference to the annexed diagram, which represents 'Troughton's Improved Level,' it will be seen that



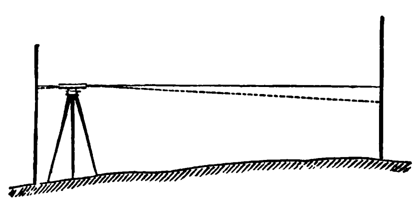
a Compass-box.
b Spirit-level.
c Diaphragm with adjusting screws.
d Screws to adjust spirit-level.
e Parallel-plate screws.

the telescope is attached to the bearing-frame, and the spirit-level firmly fixed to the tube of the telescope. The spirit-level is adjusted perpendicularly to the vertical axis of the instrument, by correcting half of the observed deviation by the screws *d,* and the other half by the parallel-plate screws *e*, until the telescope can be moved round in any direction without any material change taking place in the position of the bubble.

The spirit-bubble being thus adjusted, the instrument will fulfil the object of giving a line in any direction parallel to the plane of the horizon, provided that the line of collimation be itself parallel to the longitudinal axis of the spirit-level. This is examined and corrected as follows:—



Let two staves be held upright at a distance of 400 or 500 feet from each other, and the instrument set up exactly midway between them. The line of collimation (whether it be correct or not, as appears from the figure), will intersect the two staves on the same level; the difference of reading on the staves being noted, the instrument



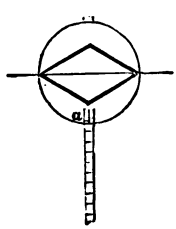
is removed to a point near one of the staves, and if in this new position the difference of reading on the staves be the same as before, the line of collimation is correct; if not, the error is corrected by raising or depressing the diaphragm (as the case may be), until the difference of reading on the staves shall be, in this second position of the instrument, the same as it was in the first.

The application and mode of action of the parallel-plate screws needs no explanation.

LEVELLING STAVES.

The levelling staff simply consists of a rectangular rod or rods divided into feet, and hundredths of a foot, by black lines appearing on a white ground. The hundredth part of a foot can be distinctly read at a distance of 400 or 500 feet with 'Troughton's Improved' or 'Gravatt's' Level. Two levelling staves are required with each instrument.

In certain operations of levelling (as explained under that head), it is required to determine the intersection of the horizontal axis of vision on the staff at very great distances. For this purpose, a sliding vane with a rhombus, marked by thick black lines on a white ground, is attached to the staff, on which it forms a conspicuous object. The angular points of the rhombus can be intersected with great precision by the horizontal wire, at a considerable distance. By means of signs, the staff-holder is directed to raise or lower the vane until the horizontal wire bisects the rhombus; and the reading, as shown by the index *a*, is noted



and registered by the assistant. When this index is placed, as in the figure (which is the usual construction), below the centre of the rhombus, the divisions, instead of commencing with zero at the bottom of the staff, are graduated at a distance above zero, equal to the distance from the index to the centre of the rhombus.

CHAPTER III.

TRIGONOMETRICAL SURVEYING.

THE characteristic difference between a trigonometrical survey, and a survey as described in the first chapter, consists in this:—that, whereas in the latter, the relative positions of the principal and secondary stations are ascertained by direct *linear measurement;* in the former, they are ascertained by trigonometrical *calculations*, based upon the direct linear measurement of a single line only (called therefore a base line), combined wilh the observation of angles as hereafter described. This trigonometrical operation is indispensable to obtain accuracy in the performance of any extensive survey. The first step consists in the measurement of the base line.

MEASUREMENT OF BASE LINE.

The measurement of a base line, from which the sides of the triangles of an extensive series are to be calculated, is a most difficult operation,* and one in which every refinement which mechanical ingenuity can devise, has been of late adopted, with a view to obtain almost mathematical accuracy.

The length of the base is made to depend in general on the proposed length of the sides of the triangles which are to be deduced from it; but circumstances seldom allow it to exceed from seven to eight miles in extent, as its position has to be selected on an even plane, as nearly as possible horizontal, and otherwise conveniently adapted for purposes of measurement.

* SIR J. F. W. HERSCHELL'S *Astronomy;* Trigonometrical Survey of England and Wales; DELAMBRE, *Base du Système Métrique.*

Standard Measures.

A necessary precaution at the outset of the operation consists in carefully comparing with a well-known standard the particular instrument or unit to be employed as the medium of measurement. 'By the word standard, or by a standard yard or standard metre, is meant a certain extent of space in one direction, in the abstract, without any reference to wooden rods or metallic bars. But as it is impossible to measure a line without some material standard, we are compelled to adopt as the practical definition of a yard, metre, etc., the length of a certain bar (usually of metal) called a standard. But as from a change of temperature, the length of the bar (as compared with the length of others not subjected to the same trial) is found to change, then we must specify the degree of temperature under which this certain bar must be placed in order to present the exact length required. The length of a base, measured with a standard at a higher temperature than that specified, must be increased; and, if measured at a lower temperature, must be diminished, according to the ratio of increase or diminution in the length of the bar ascertained by experiment as due to one or more degrees of temperature.'*

The temperature to which English standards are referred is 62° Fahrenheit. In the measurement of the arc to serve as the basis of the metre, the French geometricians adopted the temperature of 13° Réaumur, or 61¼° Fahrenheit.

There have been in England, to within a late period, several standard measures of authority, differing all slightly one from the other; but at the recommendation of the Commissioners of Weights and Measures appointed in

* AIRY'S *Figure of the Earth.*

1818, the parliamentary standard, prepared by Bird in 1760, was adopted as the foundation of all legal weights and measures, and declared the 'unit or only standard measure of extension of the United Kingdom.'

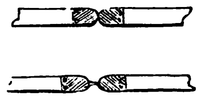
Four other standards were prepared by Captain Kater, of the same length as this imperial standard yard, and one was deposited at the Exchequer, Westminster; one at Guildhall, London; one in Edinburgh; and one in Dublin. But these, being perfect copies, were to be referred to only upon extraordinary occasions for scientific purposes; four other standards were therefore made at the same time, with great precision (although not with the extreme care bestowed on those first described), for the purpose of being deposited in the same places, to be used as the standard yards, by reference to which those employed in commerce were to be sized or adjusted.

Instruments used in measuring Base Lines for extensive Trigonometrical Operations.

Different instruments have been employed at different times as the means of measuring base lines. Deal rods, which had been originally used in England and on the Continent, were soon laid aside in exact operations, as experience demonstrated that they were liable to sudden and irregular changes from dryness, humidity, or other causes.* General Roy found that deal rods, not varnished, were lengthened about half an inch in 300 feet by exposure for one night to moisture. However, for ordinary surveying operations, deal rods saturated with boiling oil, and afterwards covered with a thick coat of varnish, will be found sufficiently exact, as this process tends to protect them in a great measure from the effects of hygro-

* *Trigonometrical Survey of England and Wales.*

metric changes in the atmosphere.* When deal rods are employed, their ends should be protected by metal caps, which prevent their wearing, and ensure a more perfect contact.



In the measurement of a base on Hounslow Heath, in 1784, glass rods, which expand or contract less than steel, iron, or brass, were substituted instead of deal. Their extremities were furnished with caps of bell-metal, connected with the rods by springs; the caps were brought in each operation to a certain mark on the rods, in order that unequal compression of the rods (tending of course more or less to affect their length by bending them) might be avoided. The change was crowned with success, as was proved by a subsequent measurement of a distance of 1000 feet with the same glass rods on the one hand, and with a steel chain of perfect workmanship on the other; from the result of which it appeared that the difference would have been little more than half an inch upon the whole base of 27,404 feet, had it been measured with each respectively. The same experiments further shewed, that hollow glass tubes were less liable to sudden expansion and contraction than solid glass rods.

As the above test had also proved the steel chain to be as accurate as the glass rods, and as it was more convenient to use, it was subsequently employed in the measurement of bases of verification in different parts of the kingdom. In using the steel chain, a drawing post and a weight post are required, a given weight being always applied to one end of the chain, while the other end is fixed to the drawing post. The chain is made to rest in deal coffers, in order to obtain a perfectly level surface: thermometers are laid at different distances along the chain, and the coin-

* PUISSANT'S *Géodésie.*

cidence at the end of the chain made only when the thermometers read alike. Thermometers have to be used where the standard measure is a chain of this character, because it is affected in the direction of its length by changes of temperature; and certain minute reductions have to be made for the purpose of obtaining the true length of the base for a fixed point of the thermometer.

In the measurement of the Irish base of between 7 and 8 miles on the plain of Magilligan, 'and in which the greatest possible error is supposed not to exceed 2 inches,'* a beautiful and novel apparatus was devised by Col. Colby, in which by using compensating expansions, he obtained an unalterable linear measure, and therefore obviated the necessity of noting the temperature, as well as all subsequent reductions.

The indication, to be found in special treatises on the subject, of the many precautions adopted by eminent scientific and practical men in this kingdom and other countries, will show the great importance that attaches to the careful admeasurement of the base, on which the accuracy of the remainder of the work entirely depends. For ordinary engineering, or territorial surveys (not partaking of a national character, such as those for which the careful admeasurements above referred to were conducted), operations less minute and precise would be sufficient. Let the surveyor, however, not construe this into an opinion that the measurement of his base may be made hastily. The tendency of accumulation of error is so great, even in trigonometrical operations of moderate extent, that he will certainly find it most advantageous to bestow, in the first instance, especial care on that leading operation. The amount of care, and labour, and time to be devoted to it, must, however, be determined by the peculiar circum-

* FROME'S *Trigonometrical Surveying*, page 3.

stances of each survey, as regards its objects, its extent, the degree of accuracy sought for, and the scale to which it is to be plotted.

The unit of measure having then been selected according to the degree of care to be bestowed on the work, the base line should be ranged with a theodolite or transit instrument,* and traced by means of pickets driven into the ground, at convenient intervals, in the same vertical plane.

Ranging a Straight Line.

The necessity for ranging straight lines with great accuracy frequently occurs in practice. For this purpose, the theodolite or transit instrument having been fixed very firmly, the axis of the vertical arc, or of the pivots of the telescope, must be adjusted to a truly horizontal position with great care. Marks are then fixed in the ground, at different distances, in a continuous vertical plane, as far as the power of the telescope will permit; the instrument is then taken forward to within three or four marks or pickets of the extremity of the line ranged, and fixed correctly over one of them, first, by means of the plummet, secondly, by the intersection of the cross wires of the telescope directed to the back and forward pickets successively. Boards 12 or 15 inches square, with concentric black and white rings painted on both sides of the board, make good ranging marks. The mark is made to move in horizontal grooves cut into two posts driven firmly into the ground; and when the centre of the mark has been brought into the line, it is fastened by wedges to the posts. A picket

* The *transit instrument* is one resembling a theodolite in those parts designed for moving the telescope in a vertical plane; and its especial object is to offer steadiness and accuracy of vertical movement and measurement.

is next driven into the ground, its position being determined by the plummet, and a notch cut in it under the centre of the mark to secure the line. By this method, Messrs. Dixon and Mason measured with accuracy a continuous straight line of 100 miles in length in the provinces of Maryland and Pennsylvania.*

Double Measurement of Base Line.

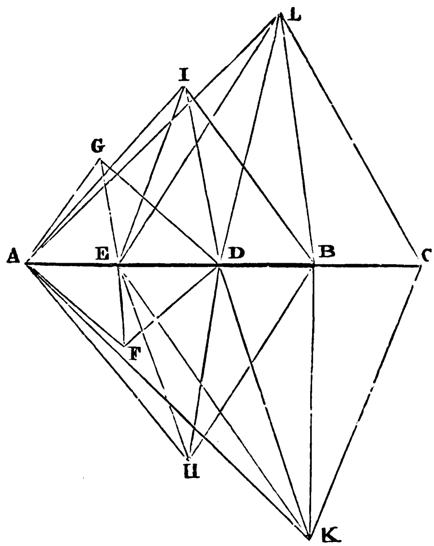
When the base line has been accurately ranged, it is measured; and, as a necessary precaution in even ordinary operations, it is measured at least twice, in opposite directions. When the base has been measured in a direction inclined to the horizon, a section of the line is carefully taken with the spirit-level or theodolite, as hereafter explained, for the purpose of reducing the line to its horizontal base.

To Verify and Prolong a Base by Triangulation.

'Besides the marks at the extremities of a base-line, which, if the base is to form a ground-work of a survey of considerable extent, should be constructed so as to be permanent, as well as minute; intermediate points should be carefully determined and marked during the progress of the measurement, by driving strong pickets or making some clearly-defined mark. These marks serve for testing the accuracy of the different portions, and reciprocally comparing them with each other, thus:—let A B represent the portion of the base actually measured, and B C that to be added by calculation, for the purpose of extending the base to C, in order to obtain a more eligible termination. The points, E, D, have been marked during the measurement. The stations, F, G, are selected, so that

* *Phil. Trans.*, 1768. Memoir by Messrs. Dixon and Mason.

the angles at E may be nearly right angles, the points themselves nearly equidistant from the line, and G E, E F, about equal to A E. Similar conditions determine the positions of H, I, K, and L. At A, as well as at every point previously marked on the base, and selected on each side of it, angles are observed to every other point. With these data,



G E and E F are determined; from each of these E D is obtained by calculation; and from A E and E D, and A D as bases, I D and D H are obtained; and lastly, by similar processes, B L and B K are found, as the mean results of many operations, all tending to check each other. B C is finally obtained from B L and B K independently, used as bases in the triangles B L C, B K C.'*

In this manner, the Irish base, on the plain of Magilligan, was prolonged about two miles, the termination of the north end of the base being ill-adapted to serve as a station for general observation of the angles.

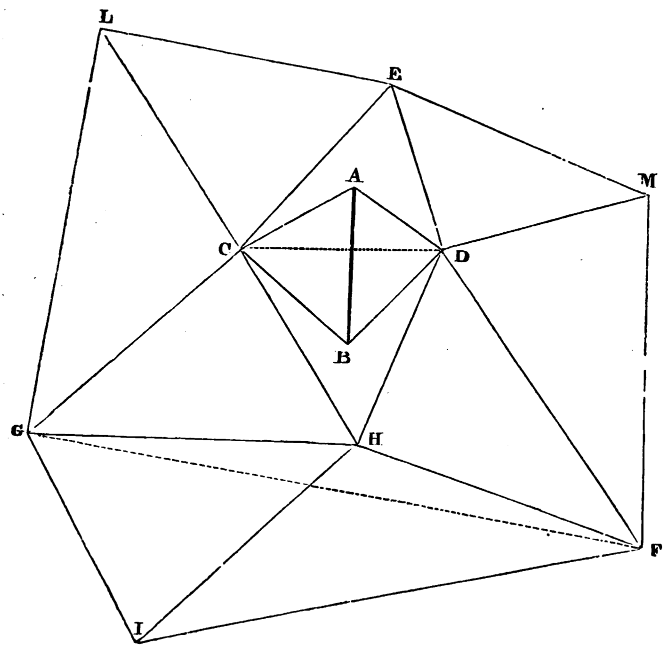
TRIANGULATION.

A base having been measured with every precaution demanded by the nature of the work of which it forms

* FROME'S *Trigonometrical Surveying*, page 9.

a part, the next step is the triangulation, or the division, of the country to be surveyed, into a series of great triangles, the angles of which are placed at stations clearly visible from each other. The angles are generally measured by means of the theodolite.

The following figure represents such a series of triangles. A B is the base, C, D, etc., are stations visible



from both its extremities, and E, F, G, H, etc., other stations, on commanding points in the country, by the connexion of which the whole surface may be divided, as it were, into a net-work of triangles. Now, in the triangle A B C, the angles A, B, C being observed, and one of the sides, A B, measured, the other two sides may be calculated by the rules of trigonometry; and thus each of the sides, A C, B C, becomes, in its turn, a base capable of being employed as the known side of other triangles. All the stations may, in this manner, be accurately determined and laid down; and as this process may be carried on to any extent, a map of the whole country may be thus constructed, and filled to any degree of detail required. The triangles ought not, however, to be laid down until their accuracy has been tested by the actual measurement of one or more of the distant sides, which are therefore called bases of verification. Care should be taken, during the progress of the work, to calculate many of the sides of the triangles from several independent data, in order to prove the identity of the results.*

Of 'Well-conditioned' Triangles.

In this process, it is necessary to be careful in the selection of the stations, so as to form triangles free from any *very* great inequality in their angles. For instance, the triangle H G F, in the last figure, would be an improper one to determine the situation of F from observations made at H and G, because the angle F being very acute, a small error in the angle G or H would produce a great one in the position of F upon the line H F.

In general no angle less than 30° should be used, unless the nature of the localities should render its adoption necessary. If this condition be attended to, the accuracy of the determination of the calculated sides, in a series of triangles, will not fall much short of that which would be attained by actual measurement, were it practicable. For experience tends to prove, that when all the triangles of a series are *well-conditioned*,† the errors in the measure of the angles do not cause the consequent errors in the

* SIR J. F. W. HERSCHEL'S *Astronomy*, chap. iii.

† See PUISSANT'S *Géodésie*, vol. i. p. 136; and Notes to LESLIE'S *Trigonometry*, page 471 et seq.

sides to accumulate through each successive step in the operation, but that they, on the whole, tend to compensate each other.

These remarks, as to the best practical form to be given to the triangles, apply equally to the surveying operations with the chain only, as described in the first chapter, and should be borne in mind in the arrangements for such a survey.

Mode of Increasing Triangles without making them 'Ill-conditioned.'

But, in an extensive triangulation, it is necessary that the sides of the primary triangles should be much longer than the original measured base;—the sides of the triangles must therefore be increased without admitting any ill-conditioned triangles. This is accomplished with rapidity, as follows:—

A B is supposed to be the measured base, and C and D the nearest trigonometrical points.* All the angles being observed, the distances of C and D from the extremities of the base are obtained by calculation. In each of the triangles, A D C, B D C, we then have two sides, and the contained angle, to find D C, one calculation acting as a check upon the other. This line D C is again made the base from which the distances, from D and C, of the trigonometrical stations, E and F, are computed; and the length of E F is afterwards obtained in the two triangles, D E F and C E F. In like manner, the relative positions of H, G, K, etc., are obtained, and therefore, as we recede from the base, it will speedily become practicable to use, as bases, the sides of triangles much larger, and embracing

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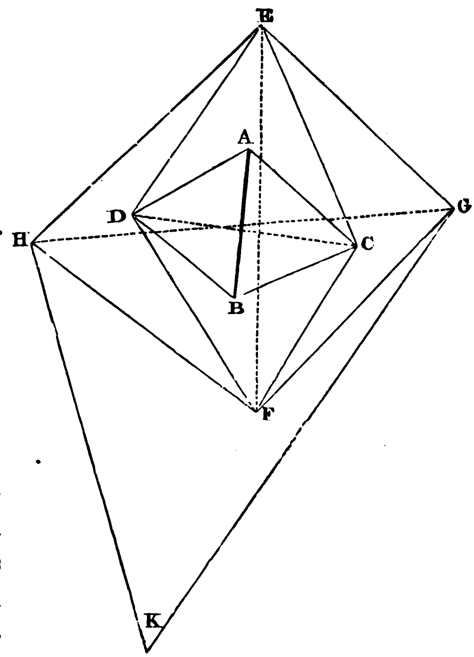
* See FROME'S *Trigonometrical Surveying*, p. 14, and SIR J. F. W. HERSCHEL'S *Astronomy*, p. 148.

much greater intervals. 'Thus it becomes easy to divide the whole face of a country into great triangles, from 30 to 40, or even 100 miles in their sides, according to the nature of the country. The vertices of these great triangles being once well determined, the country is afterwards, by a second series of subordinate operations divided into smaller or secondary triangles, and these again into others of a still minuter order, till the final filling in is brought within the limits of personal survey and draftsmanship, and till a map is constructed with any required degree of detail.'



In the late trigonometrical operations constituting the framework of the Irish survey, it was found advantageous to introduce triangles with sides from 70 to 90 miles in length:—in the triangulation carried on in the southern parts of England by General Mudge and Mr. Dalby, they deemed triangles whose sides were from 12 to 18 miles in length, preferable, for the general purposes of the survey, to triangles of greater dimensions.

If, for any cause, it has been found advisable to commence the triangulation before thc base has been measured, the sides of the triangles may be calculated from an assumed base, and afterwards corrected for the difference between this imaginary quantity and the real length of the base line. Or, as was found to be the case with one of the Indian bases,* if the length of the base has not originally been, from the want of access to correct standards, reduced with the most rigid accuracy, the triangulation may be easily rectified. The property of similar angles readily points out the method to be employed.

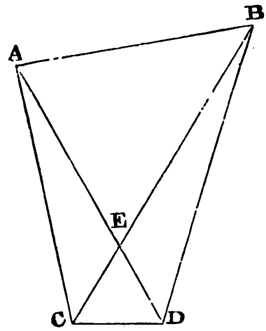
Reduction of Angles to the Centre of the Station.

In extending a series of triangles in populous neighbourhoods, wooded districts, or occasionally under other circumstances, instead of planting moveable signals at each point of observation, it will be found more convenient and more economical to select permanent well-defined objects, such as steeples, towers, windmills, etc., for the principal stations in the triangulation. But when a choice is made of such objects, the theodolite or circular instrument can seldom be placed in the centre or axis of the station. The observer, in such cases, approaches as near to the centre as he can with advantage, and calculates the quantity of error which the minute displacement may occasion. But, as a general rule, it will be found expedient to take pains to select objects, such as towers, etc., that will admit of the theodolite or instrument being placed in the centre of the station, the corrections for eccentricity being so troublesome and so complex, as to consume much time in additional observations, and present additional chances of error in the aggregate. I subjoin in a note examples of some of the precautions and corrections used in great surveys.

NOTE.—Suppose it be required to determine the angle A C B, which the remote objects A and B subtend at C, the centre of the

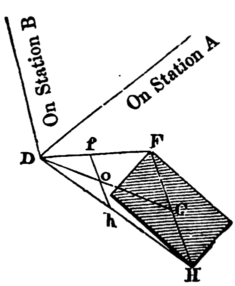
* *Asiatic Researches*, vol. xiii., and *Philosophical Transactions*. 1853.

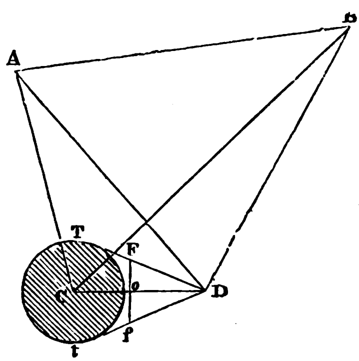
permanent station; the instrument is placed in the immediate vicinity at the point D, and the distance D C with the angle A D C noted, while the principal angle A D B is observed; the central angle A C B, may then be computed from the rules of trigonometry.*



When the instrument is placed at the base of a tower or high permanent object, the centre of which is the true vertex of the triangle, it sometimes happens that the centre of the station cannot itself be seen, but the direction of that centre from the axis of the instrument is required for the purpose of measuring the angle A D C. The direction of the centre is found as follows:—

First, supposing the base of the station or signal to be rectangular; from the extremities of the diagonal H F, draw the lines D F, D H, and measure their lengths; then on D F, take any point



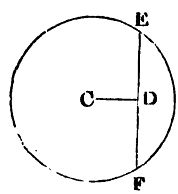


* See Notes to LESLIE'S *Trigonometry*, page 469, and PUISSANT'S *Géodésie*, page 182, et seq.

f, and from D H, cut off D *h*, so that D F : D H : : D *f* : D *h*, then *f h* is parallel to F H, one of the diagonals. Bisect *f h* in *o*, join D *o*; D *o* will be in the direction D C required.

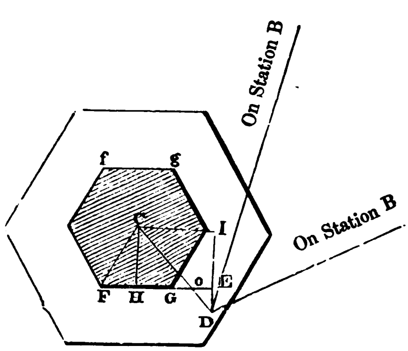
Supposing the base of the tower or signal to be circular; draw the tangents D T, D *t*, by sweeping the telescope of the theodolite round, until the visual ray describes a line touching the circumference of the tower. From D T, D *t*, cut off equal lines D F, D *f*, join F *f*, and bisect it in *o*; the line D *o* joining D and *o* will be in the direction of the centre C. The points F and *f* should be chosen as near as possible to the tower, in order that F *f* may be as long as possible.

If the instrument be placed within the circumference of the circle; through D, the axis of the instrument, draw the chord E F, bisected in D. On D erect the perpendicular D C; D C will be the direction required.



If the position of the point D be such that both extremities of a diameter of the base cannot be seen from it, or if the base of the station be a regular polygon of any number of sides, the direction D C is obtained as follows:—

From D draw D E perpendicularly to F G or F G produced, measure D E. Bisect F G in H, and measure E H. Obtain the value of C H by measuring half the distance between the opposite sides F G, *f g*, or by the following proportion, in which the angle F C H is known, and depends on the number of sides of the regular polygon:—



$$C\,H : F\,H :: \cos. F\,C\,H : \sin. F\,C\,H$$

$$\text{hence } C\,H = F\,H \frac{\cos. F\,C\,H}{\sin. F\,C\,H}.$$

Then in the triangles D I C, D E o, we have

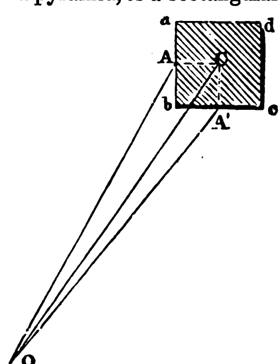
$$D\,I : I\,C :: D\,E : E\,o, \text{ or}$$

$$E\,o = \frac{D\,E\,.\,I\,C}{D\,I},$$ from which equation the point of intersection of D C with F E, and consequently the direction D C, is obtained.

Form of Objects under Observation.

Attention is also to be paid to the form of objects or signals under observation. Those which do not terminate in a point,

whether presenting the form of a truncated pyramid, or a rectangular top, may lead to errors, when they are illuminated obliquely by the sun, causing thereby the observer to direct his telescope, not to the centre of the signal, but to the centre of the face exposed to the light. For example, let $a\ b\ c\ d$ be the base of the signal observed from O. If, on account of the distance, the illuminated face $a\ b$ can alone be seen, the telescope will be directed to the point A, a middle point in $a\ b$ instead of the point C, the centre of the signal,



the amount of error being equal to the angle A O C. The value for this error is

$$A\,O\,C = \frac{A\,C \sin. A\,C\,O}{A\,O},$$

and the shorter the distance A O, provided it be great enough to prevent the sides in the shade from being seen, the greater will be the error. In very accurate trigonometrical operations, this correction is not to be neglected, as the error due to this cause has been known in cases of truncated pyramids with broad bases to amount to 15″ or 20″. *

Sketch of Permanent objects used as Stations.

When observations are made to churches, towers, or other permanent objects, it is desirable to make a slight sketch of their general form and appearance from the point whence they are observed; and if the signal be irregular in its outline, a mark should be made showing the part of the object intersected, in order to avoid errors when the same object has to be viewed from another distant station, or if it should be necessary to re-observe the angles at a future time.

It is also desirable that a plan of the station or observation be made, marking the position of the axis of the instrument, with written dimensions of its distance, by ordinates from some marked and fixed points. These data are useful to identify the station for future observations, and serve also when reducing angles to the centre.

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* DELAMBRE, *Base du Système Métrique*, vol. i., page 221.

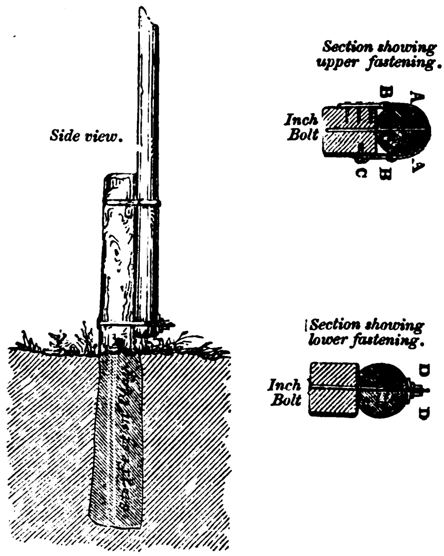
Practical Directions on the Construction of Signals.

Permanent objects are, as before observed, to be chosen in preference for signals: their advantages are solidity, and consequent steadiness and durability; they also economize time and money otherwise expended in the special erection of signals. But such permanent objects do not always exist in localities best suited for stations, and they perhaps would form ill-conditioned triangles. They are, in those cases, to be determined in position by intersections, but special signals must be used for the summits of the triangles. Even when permanent objects are applicable, they do not always dispense from the necessity or expediency of placing temporary signals on their summits, in order to render their intersection by the cross wires more precise.

To the variety of form and character of signals there is, of course, no limit; they depend on the nature of the country, its capabilities, the distances between the stations, the importance of the work, the outlay contemplated, etc. A few details, however, on signals adapted to different circumstances, may be given at this stage of our course.

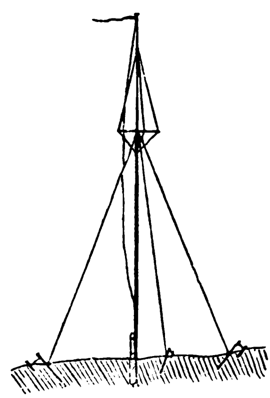
In common surveys, embracing from several parishes to a whole country, as also in surveys for railroads, canals, and similar works, sides of triangles or connected bases have to be measured from 2 or 3, to 6 or 8 miles in length. For the purpose of ranging such lines, signals may be made by firmly fastening straight poles in the tops of high isolated trees, if their position gives the direction required.

An economical signal, suitable for lines of 8 or 10 miles in length, is represented in the annexed sketch, in which a long pole or mast, forming the signal, is held in a vertical position by a strong post, the lower part of which is firmly fixed in the ground to a depth of 6 or 8 feet. A collar, towards the upper part of the post, confines the



A *Iron band*, 2in. × $\frac{3}{8}$in.
B *Hinges.*
C *Staple,*
D *Nut and washers.*

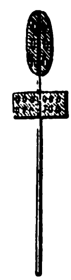
mast in its place, its lower end being fastened by a pin, round which it freely revolves as on a centre in a vertical plane, when the collar is unclapsed or unlocked. If the pole be very high, and made of two pieces, or placed in an exposed position, it is strengthened and kept upright by means of guys and stays. Such a signal admits of the instrument being placed in the axis of the signal, which is for that purpose let down temporarily by being made to revolve on the pin that supports its base.



All posts or masts erected for signals are usually made to bear bunting flags; these should be at least of two colours,—red and white, when they are to be seen projected against trees or dark ground,—green and red, when they are to be relieved against the sky.

But as flags are useless in calm weather, a state of the atmosphere selected by preference for observations, they may be advantageously replaced by a small cone or barrel fastened towards the top of the pole. This may be painted red, if relieved against trees or ground; black, if relieved against the sky.

A good and easily recognised signal is also made by fixing at the top of the pole or mast a circular disk of sheet iron 2 or 3 feet in diameter, and a rectangular plate 4 or 5 feet long, by 1½ broad; they are placed at right angles, one above the other. On their faces circular openings are cut, to diminish the surface of resistance offered to the air. The diagram represents this form of signal viewed obliquely to both planes. It is much used on railways, and found well adapted to be seen at considerable distances.

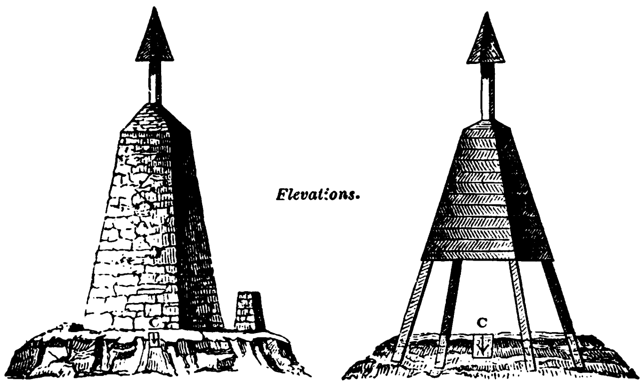


In mountainous districts, the summits of the mountains are selected as the sites of the stations; and the signals may be made of pyramidal heaps of stone (or 'cairns'), raised over a permanent mark fixed in the ground to denote the axis of the station, and over the centre of which a pole, similar to those previously described, is kept steady and upright by the stones surrounding its base.

In very elevated situations, where the signals are exposed, not merely to the influence of storms, but also to the destroying effects of alternating snows and frosts, it may be desirable in national surveys to give such signals greater stability. The annexed figures represent the signals used in a triangulation carried across the Alps,* in which some

* *Opérations Géodésiques exécutées en Piémont et en Savoie*: Atlas, Milan, 1827.

of the stations were at a greater elevation than the glaciers, and on steeps accessible only to men of firm nerves.*



Elevations.

FORMS OF ENTRY FOR HORIZONTAL ANGLES.

When no repetition is made in the reading of the angles, they may be entered in the field-book according to the annexed form.

OBSERVATIONS at East end of Base on PUTNEY HEATH. June, 1841.

Name of Station.	Vernier A.	Vern. B.	Mean Reading.	Right or left of first object.	Remarks.
	° ′ ″	′ ″	° ′ ″		
Chimney, Heathfield-house	70 19 0	18 40	70 18 50	. .	Cloudy, and light wind.
Wimbledon Windmill . .	103 32 20	32 40	103 32 30	R.	
Combe Wood Telegraph .	115 14 20	14 50	115 14 35	R.	Sketches of Stations in page 4 of field-book A.
West end of Base. . . .	159 30 20	30 20	159 30 20	R.	
Finger-post, Cross-roads .	194 57 0	57 10	194 57 5	R.	
Putney Heath Telegraph .	224 9 40	10 0	224 9 50	R.	
Chimney, Heathfield-house	70 19 0	18 40	70 18 50	R.	

The entries in the fifth column, denoting whether the signal is read to the right or left of that immediately pre-

* One of the stations, Roche Melon, was 11,650 feet above the sea.

ceding, facilitate the recognition of the objects at any future time.

When multiples of the angles are taken by the method of repetition explained in page 53, and when great care is otherwise bestowed on the observations, they are conveniently entered according to the following form, which was that adopted in the Piedmontese triangulation,* and in which the state of the barometer and thermometer is also registered, as the state of the atmosphere influences the amount of refraction, a subject of which we shall treat in the next chapter.

ANGLES between the SIGNALS, AVEBURY (left), LONG KNOLL (right).
June 26, 1823, 10 A.M. First Series.

No. of repetitions.	Multiple arcs read from Vernier A.	Single.	Vernier.	First and last reading.	Means.	Remarks.
	° ′ ″	′ ″		° ′ ″	° ′ ″	
1	76 40 20	40 20·0	A.	0 0 0	46 43 20	Barometer 29·276.
2	153 20 40	„ 20·0		46 43 20		
3	230 1 0	„ 20·0	B.	90 0 10	„ 43 20	Attached Thermometer 61°.
4	306 41 20	„ 20·0		43 30		
5	23 21 40	„ 20·0	C.	180 0 10	„ 43 20	Detached Thermometer 57°.
6	100 2 0	„ 20·0		43 30		
7	176 42 20	„ 20·0	D.	270 0 0	„ 43 10	Calm and clear.
8	233 22 40	„ 20·0		43 10	4) 70	
9	330 3 0	„ 20·0		Mean . . .	46 43 17·5	
10	46 43 20	„ 20·0				

Twice the circumference+46° 43′ 17·5″=766° 43′ 17·5″

Mean angle . . . . . . = 76° 40′ 19·7″

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* *Opérations Géodésiques exécutées en Piémont et en Savoie*: Milan, 1825.

TRIANGLES, NOT PLANE, BUT SPHERICAL.

With respect to the angles thus observed, and the triangles combined from them, they are not, rigorously speaking, *plane* but *spherical*, existing on the surface of a sphere, or rather, to speak correctly, of a spheroid.* In small triangles, of six or seven miles in the sides, an extent which the ordinary surveyor or engineer will rarely require to exceed, this consideration may be altogether neglected, as the difference is imperceptible.

In more extensive geodesical operations, the spherical excess of the sum of the 3 angles of a triangle must be taken into consideration. It is, indeed, absolutely necessary to do so with great triangles, because, by the observations themselves, the sum of the 3 angles in any of them is always found to be rather more than 180°.† Ramsden's large theodolite, three feet in diameter, was the first instrument by which this excess was observed. It is always a minute quantity, seldom exceeding 4″ to 5″, in the triangles used in geodesical operations, and may therefore be altogether neglected in ordinary surveying operations.

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* 'Were the earth's surface a plane, the sum of the three angles would be exactly 180°; and the excess above 180° is so far from being a proof of incorrectness in the work, that it is essential to its accuracy, while it offers at the same time another palpable proof of the earth's sphericity.

'The true way then of conceiving the subject of a trigonometrical survey, when the spherical form of the earth is taken into consideration, is to regard the triangles into which the country is divided, as the bases of an assemblage of pyramids converging to the centre of the earth. The theodolite gives us the true measures of the angles included by the planes of those pyramids; and the surface of an imaginary sphere, at the level of the sea, intersects them in an assemblage of spherical triangles, above whose angles, in the radii prolonged, the real stations of observation are raised by the superficial inequalities of mountain and valley.'

† SIR J. F. W. HERSCHEL'S *Astronomy*, p. 149.

*Exercises for Calculation.**

One side and adjacent angles given, hence

$$S = \tfrac{1}{2}a^2 \frac{\sin. B. \sin. C}{\sin. (B. + C)},$$

in which $a = 32608{\cdot}64$ feet.

Vertices of the Triangles.	Observed Angles.	Spherical excess.	Error.	Corrected Angles for calculation.	Distances in feet.
A. Gootydroog Station	65° 18′ 41″·19	..	..	65° 18′ 41″·10	
B. North end of base .	87 27 16·45	..	..	87 27 16·36	
C. South end of base .	27 14 2·64	..	..	27 14 2·54	
	180 0 0·28	0·13″	+0·15	180 0 0	
Gootydroog Station from North end of base .					16423·9
Gootydroog Station from South end of base .					35853·8
B. North end of base .	35° 4′ 2″·45	..	..	35° 4′ 2″·09	
C. South end of base .	105 3 6·43	..	..	105 3 6·07	
D. Paumdy-hill Stat. .	39 52 52·21	..	..	39 52 51·84	
	180 0 1·09	0·22	+0·87	180 0 0	
Paumdy-hill Station from North end of base .					49111·3
Paumdy-hill Station from South end of base .					29218·8
Dean Hill	62° 22′ 48″·75	..	..	62° 22′ 47″	
Butser Hill	48 28 41·5	..	..	48 28 40	
Highclere	69 8 35·	..	..	69 8 33	
	180 0 5·25	4·07″	+1·18	180 0 0	
Butser Hill to Highclere					148031
Dean Hill to Butser Hill					156122
Dean Hill to Highclere . . . . ,					125085

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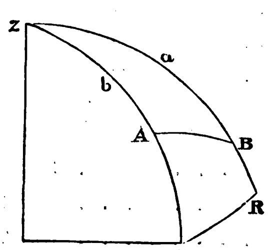
* Colonel Lambton's Account of the Indian Survey, *Asiatic Researches*, vol. xiii. p. 33.

Reduction, to a Horizontal Angle, of an Angle observed between two Objects situated in a Plane oblique to the Horizon.

Supposing the angular distance between the objects A and B (which objects are at different elevations) to be observed with the theodolite, it will, by the construction and adjustment of the instrument, be reduced to the horizontal angular distance between them. And this result presents one of the great advantages of the theodolite; for, as with measured lines, which, it was shown (Chapter I.), must be reduced to their horizontal value before they can be plotted, so is it with angles. They must similarly be reduced to their horizontal values before they can be protracted, or before the triangles to which they belong can be laid down. But when, instead of the theodolite, the sextant or repeating circle is used to measure the angular distance between two objects, the horizontal angle, or angular distance of the projections of those points on the plane of the horizon, must be deduced by calculation. It may be obtained as below:



Let A and B be the objects observed, and H R the projections of A and B on the plane of the horizon, required the arc H R. Observe the angular elevations A H, B R, as explained in the description of the Sextant.

H R is the same as the vertical angle Z in the spherical triangle Z A B, of which the measured angle A B is the base, and the complements of the elevations are the sides. The vertical angle is therefore obtained by the formula

$$\text{Sin. } \tfrac{1}{2}\, Z = \sqrt{\frac{\sin.\ (s-b)\ \sin.\ (s-a)}{\sin.\ b\ \sin.\ a}}$$

in which s is equal to half the sum of the sides.

On the Continent, where Borda's repeating circle, although gradually giving place to the theodolite, is yet very generally used, the angles to be observed between terrestrial objects have frequently to be reduced to the horizon: a work of great labour if a simpler formula than the above had not been found. Considering the triangle as one composed of two sides differing little from a quadrant, a formula less complex is obtained, and from it tables have been calculated which give very rapidly the reduction required. The investigation of the formula will be found in PUISSANT'S *Géodésie,* vol.i., page 109.

BASES OF VERIFICATION TO BE MEASURED.

The angles of a series of triangles having been observed, and the sides calculated from independent data to prove their accuracy, an additional test is adopted by the actual measurement of one or more of the distant sides to serve as bases of verification; these sides being, of course, measured with the same care as it was deemed expedient to bestow on the measurement of the original base. The accuracy of the work having been thus ascertained, the next step consists in plotting or protracting the triangles to the scale determined upon for the survey.

Surveys to be Plotted with the North upwards.

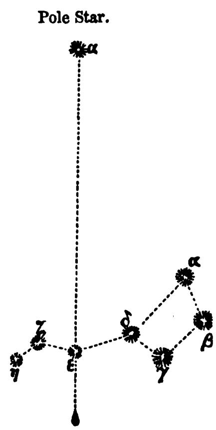
It is customary generally to plot surveys with the north upwards, and invariably so all surveys of counties or of kingdoms. For this purpose, and under any circumstances, the direction of the meridian with reference to the triangulation is to be laid down. It is necessary, therefore, to observe the direction of the meridian with respect to the original base, or some one of the sides of the principal triangles, from which the azimuthal distance of each part is given. When treating on the subject of longitude, we shall give an account of the methods that may be adopted to ascertain with precision the direction of the meridian at any one station; we shall here describe some ready methods by means of which its direction can be obtained with approximate accuracy.

Meridian Line.

Fix the theodolite at one of the stations used in the triangulation, and some hours before mid-day direct the telescope so that the cross wires shall touch the upper or lower limb of the sun in the east; note the horizontal and vertical readings of the arcs;—repeat the operation at short intervals, taking care to direct the intersection of the cross wires to the same limb of the sun that was before observed, and note all the readings in their regular succession.

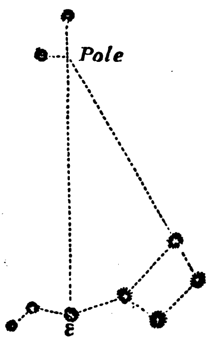
Again, in the afternoon, when the sun descends westwards, clamp the vertical arc to the last reading, and note the horizontal angle at the time of the sun's limb touching the intersection of the cross wires. The vertical arc being clamped in succession in the descending series of the vertical angles, all the horizontal readings at the time of each successive intersection are entered. The point on the horizontal limb half way between all the readings will give the angle to which the vernier is to be placed, in order that the telescope may point to the position occupied by the sun at noon. A picket, driven into the ground in that direction, serves to mark the meridian line, and the angle, formed between it and any side of the triangles having the selected point for a vertex, being taken, the azimuthal direction of each and all the sides of the triangles is obtained.*

The same method, however, is applicable without correction to the observation of a fixed star; and the pole star, from the facility with which it is identified, is frequently selected for the purpose, being observed at the time of its greatest apparent eastern and western elongation. But, if the telescope of the theodolite be not powerful enough to observe the star under these conditions (as one of the observations must generally be made by daylight), a very close approximation may be had by remembering that the pole star very nearly reaches the true meridian, when it is in the same vertical plane with the ϵ or Alioth, or the star in the tail of the Great Bear, which is nearest to the quadrilateral. The vertical position can be ascertained by means of a plummet. To see the cross wires in the field of the telescope at the same time with the star, a faint light should be placed near the object glass. When the pole star has been brought correctly into the central part of the angle formed by the intersection of the cross wires, the horizontal limb is firmly clamped, and the telescope brought down to the horizon; and a light, seen through a small aperture in a board, and held at some distance by an assistant, is moved according to signals, until it is bisected by the wires. A picket



* This method would be quite correct, if the sun moved constantly in the same parallel, but the change in his declination between the time occupied by the observations renders necessary a minute correction, which in ordinary surveying operations may be neglected altogether.

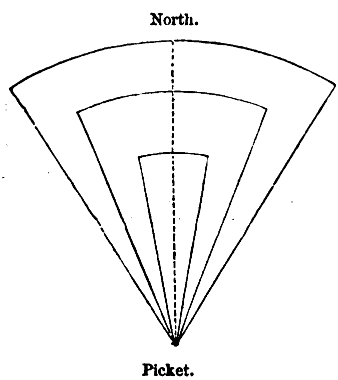
driven into the ground underneath the light serves to mark the meridian line for reference by day, when the angle, formed between it and the side of the triangle, may be measured.



The true situation of the North Pole may also be nearly ascertained by the following indications of the stars near it. In the first place, as shown above, a straight line drawn from the pole star to the star Alioth, or ε, in the Great Bear, passes through the poles, and a perpendicular to this line at the pole passes through the small star nearest the pole. Finally, the stars called the 'Pointers,' in the above-named constellation, point almost directly towards the pole. The pole star is distant from the pole about $2\frac{3}{4}$ degrees.

When no angular instrument is at hand, an approximate meridian line may be set out as follows:—

Drive a thin staff or picket vertically on a level piece of ground, a gravel walk for instance. Several hours before noon measure the length of shadow thrown by the picket, and from the base of the staff as a centre, with the length of the shadow as radius, describe an arc of a circle from west to east. About the same interval of time after mid-day, observe the point where the extremity of the shadow again coincides with the arc; a line drawn from the centre of the staff to the middle point of the arc thus intersected, will be nearly in the



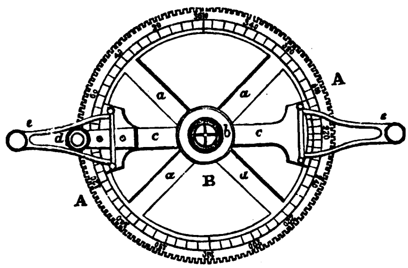
direction of the meridian. It will be better to describe three or four such arcs, at different elevations of the sun, and to make use of the mean of their central points to trace the meridian line.

Of Protracting the Triangulation.

In protracting an extended triangulation, which has been conducted with the minute precautions indicated in this chapter as necessary to ensure great accuracy, it will be better to lay down the triangles from the lengths of their sides, than by measuring the angles; because measures of length can be taken from a scale, and transferred to the plan with more exactness than angles can be pricked off from a protractor. If, however, the triangulation is to be laid down on paper, which is subject to changes from difference in the state of the atmosphere, having a greater effect on the accuracy of the plotted work than the difference between the degrees of exactness of triangles protracted by the length of their sides, or by measurement of the angles, it is not essential to adopt the course here indicated. Hence, for ordinary surveys, the triangulation is more frequently plotted by means of the angles, using, for the sake of expedition, a circular protractor. There are various kinds of protractors, among which I select the following, which is found expeditious and accurate.

The Circular Protractor.

'It consists of an entire circle, A A, connected with its centre by four radial bars, *a a*, etc. The centre of the metal is removed, and a circular disk of glass fixed in its place, on which are drawn two lines crossing each other at right angles, and dividing the small circle into four quadrants, the intersection of the lines denoting the centre of the protractor. When the instrument is used for laying down an angle, the protractor must be so placed on the paper that its centre exactly coincides with or covers the angular point, which may easily be done, as the paper can be seen through the glass centre-piece.



'Round the centre, and concentric with the circle, is fitted a collar, *b*, carrying two arms, *c c*, one of which has a vernier at its extremity, adapted to the divided circle, and the other a milled head, *d*, which turns a pinion, working in a toothed rack round the exterior circle of the instrument; sometimes a third arm is applied, at right angles to the other two, to which the pinion is attached, and a vernier can then (if required) be applied to each of the other two, and it also prevents the observer disturbing that part of the instrument with his hand when moving the pinion. The rack and pinion give motion to the arms, which can be thus turned quite round the circle for setting the vernier to any angle that may be required. Upon a joint near the extremity of the two arms (which form a diameter to the circle) turns a branch, *e e*, which, for packing, may be folded over the face of the instrument, but when in use, must be placed in the position shown in the figure; these branches carry, near each of their extremities, a fine steel pricker, the two points of which, and the centre of the protractor, must (for the instrument to be correct) be in the same straight line. The points are prevented from scratching the paper as the arms are moved round, by steel springs, which lift the branches a small quantity, so that, after setting the centre of the protractor over the angular point, and the vernier in its required position, a slight downward pressure must be given to the branches, and each of the points will make a fine puncture in the paper; a line drawn through one of these punctures and the angular point will be the line required to form the angle.

'Any inaccuracy in placing the centre of the protractor over the angular point may easily be discovered, for, if incorrectly done, a straight line drawn through the two punctures in the paper will not pass through the angular point; which it will do, if all be correct.

'The face of the glass centre-piece on which the lines are drawn is placed as nearly even with the under surface of the instrument as possible, that no parallax may be occasioned by a space between the lines and the surface of the paper.

'By help of the vernier, the protractor is graduated to single minutes, which, taking into consideration the numerous sources of inaccuracy of this kind of proceeding, is the smallest angular quantity that we can *pretend* to lay down with certainty.'*

If, however, for the sake of greater accuracy, it be preferred to lay down the triangles by means of their sides, beam compasses with vernier scales attached should be used in this operation. The meridian must then also be plotted by means of measures of length. A ready way offers itself, by calculating the lengths of the sides in a right-angled triangle, having for one of its angles the azimuthal distance of the observed side, and the said side for the hypothenuse.

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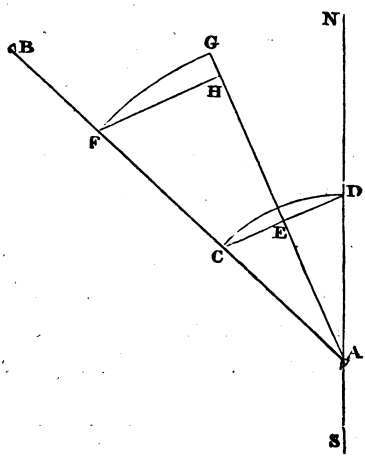
* SIMMS on *Mathematical Instruments.*

But the following method will be found more convenient.



Let A B be the side of the triangle, the azimuthal angle of which has been ascertained with reference to N S, the meridian line. Take from an accurately divided diagonal scale, exactly five inches as a radius, and from A, as a centre, describe an arc C D; now the chord of an arc being equal to twice the sine of half the arc, the chord C D is equal to twice C E, the sine of half the angle C A D. Take a radius A F equal to twice A C, and describe the arc F G intersecting the radius A B in F, draw the sine F H, then by similar triangles:—

$$A F : A C :: F H : C E, \text{ but}$$
$$A F = 2 A C \text{ by construction, therefore}$$
$$F H = 2 C E = C D;$$

that is, the chord of a given arc is equal to the sine of half the arc with double the radius.

The radius of the tables of natural sines is equal to 1 or 10; and, having taken the half of 10 or 5 inches for the radius A C, the natural sine of half the given angle taken from the tables, will correspond to F H, the sine of half the given angle, with double the radius; but F H was proved equal to C D; the natural sine, therefore, of half the given angle to a radius 10, will be equal to the chord of the whole angle to a radius 5. Having taken that distance from the same scale of inches as the radius, place one foot in the point C, and with the other, mark the point D on the arc C D, then through D and A draw the line N S, which will be the direction of the meridian.

When the operations of a Trigonometrical Survey are extended, in eastern or western directions, beyond spaces of about 60 miles from a fixed meridian, it is expedient to observe new meridians, in order to avoid errors which would otherwise take place as the result of computations made on the supposition of the earth's surface being a plane. Within a limit of about 60 miles, such a supposition produces no sensible error.*

Interior detail of a Trigonometrical Survey.

The triangulation for a survey being accomplished, the filling in of the interior detail, such as roads, streams, legal and ecclesiastical boundaries, towns, villages, houses, woods, etc., presents little difficulty. The larger triangles being subdivided into others of a smaller size, the sides of these are measured with the chain, and the field-book is kept according to the form given in the first chapter, the surveyor entering into such detail as the object of the work may demand, even to the minute tracing of all fields and enclosures. His object, however, may not always be to make detailed property plans, but simply to lay down the roads, rivers, boundaries of woods, and other great lines of artificial or natural demarcation. In this case, the survey of the roads, rivers, woods, etc., is made with the chain and theodolite, according to a process to which the term 'traversing' is applied, and for the description of which the reader is referred to chapter VI. While measuring the sides of the triangles or station lines, the surveyor takes the angles of elevation and depression, both for the purpose of

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* *Trigonometrical Survey*, vol. ii., page 4.

reducing the inclined lines more correctly to the horizontal plane, as also to obtain as many altitudes as possible over the surface of the district surveyed; and, as these levelling operations form an essential part of the trigonometrical survey, the subject of levelling must next be considered before detailing more fully the practical operations connected therewith.

NOTE.—Frequent reference has been made in the course of this chapter to the work describing the operations of the 'Trigonometrical Survey for England and Wales.' The reader who is desirous to study this branch of the subject more fully, is recommended to consult that work.

CHAPTER IV.

ON LEVELLING AND REFRACTION.

Definition of 'Levelling.'

LEVELLING is the art of finding a line parallel to the horizon at one or more stations, in order to assign the difference of altitude between one place and another. 'Two or more places are on the same level, when they are equally distant from the centre of the earth. Also, one place is higher than another, or above the level of it, when it is further from the centre of the earth, and a line, equally distant from that centre in all its parts, is called a line of true level. Hence, because the earth is round, that line must be a curve, and make part of the earth's circumference, or at least be parallel to it.' *

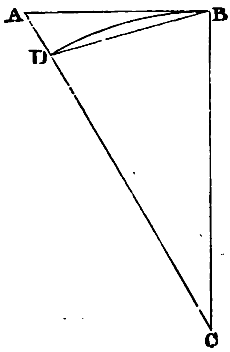
Difference between the apparent and the true Level.

But, as the lines of sight which determine relative levels cannot evidently trace a curve parallel to the earth's surface, a horizontal line can be traced only by a series of right lines, tangent to the earth's surface, approximating more nearly to a line of true level the shorter the sides of the circumscribing polygon are chosen.

Let the arc B D be a portion of the earth's surface with the centre C; and let the tangent A B, horizontal at B, meet the vertical line C D in A. The line B A

* HUTTON'S *Mathematical Dictionary.*

will be the apparent line of level, and the arc B D the true

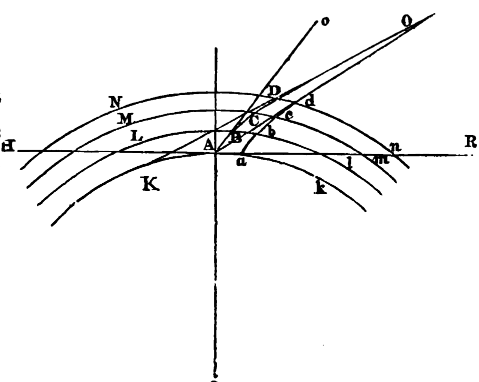


line of level from the point B, and at any point D; A D is the height of the apparent above the true level. This difference, it is evident, is always equal to the excess of the secant of the arc B D, above the radius of the earth.

The quantity of depression, A D, is easily computed; for $A\,B^2 = (2\,B\,C + A\,D)\,A\,D$ (Euc. III. 36), or very nearly $= 2\,B\,C\,.\,A\,D$, hence $A\,D = \frac{A\,B^2}{2\,B\,C}$. As 2 B C, the diameter of the earth, may be assumed as a constant quantity, the depression is proportioned to the square of the distance. In the space of one mile, this depression will amount to $\frac{5280}{7916}$ parts of a foot,—and from this we derive an easily remembered formula for the approximate correction for curvature, which may be expressed in feet by two-thirds of the square of the distance in miles.

Of Refraction.

But this effect, due to the earth's curvature, is modified by another cause arising from optical deception. Experience has shown that rays of light, in passing obliquely from a medium of a given density into another of greater density, change their direction, and approach more nearly to that of a perpendicular, raised to the common surface, at the point where they enter the denser medium. Now, the atmosphere increasing gradually in density from its external limits to the surface of the earth, may be supposed to consist of successively superposed minute layers, each concentric with the general surface of the sea, and each of which is more rarefied, or specifically lighter, than that immediately beneath it; and denser or specifically heavier than that immediately above it. A ray of light, therefore, passing obliquely through the atmosphere, for example from a higher to a lower level, to the eye of an observer, passes from the rarer to the denser strata; and following the above law of optics, it will be diverted from its original course, and made to approach more and more nearly to a perpendicular to the horizon. It will thus describe a curve concave to the earth's surface; but it is a law in optics that an object is seen in the direction which the visual ray has on arriving at the eye, without regard to what may otherwise have been its course between the object and the eye: the object appears, therefore, in the direction of the tangent to this curve. This optical effect or apparent displacement of the object is called *refraction*. Every difference of level, accompanied, as it must be, with a difference of density in the strata of the atmosphere, will have, corresponding to it, a certain amount of refraction; and as the curve described by each ray of light is concave next the earth, the tangent to the curve will lie above it, and consequently the object will appear more elevated above the horizon than if there were no atmosphere.



'Suppose a spectator placed at A, any point of the earth's surface K A *k*; and let L *l*, M *m*, N *n*, etc., represent successive strata of decreasing density, into which we may conceive the atmosphere to be divided, and which are spherical surfaces concentric with K *k*, the earth's surface. Let O represent the object under observation, whether terrestrial or a heavenly body, within or without the utmost limit of the atmosphere; then, if the air were away, the spectator would see it in the direction of the straight line A O. But in reality, when the ray A O passes from a rarer into a denser stratum, suppose at *d*, it will by the laws of optics begin to bend *downwards*. But as it advances downwards, the strata continually increasing in density, it will continually undergo greater and greater refraction in the same direction; and thus, instead of pursuing the straight line O *d* A, it will describe a curve O *d c b a*, continually more and more concave downwards, and will reach the earth not at A, but at a certain point *a* nearer to O. This ray, consequently, will not reach the observer's eye. The ray by which he will see the object O is, therefore, not O *d* A, but another ray, which, had there been no atmosphere, would have reached the earth at K, a point behind the observer; but which, being bent by the air into the curve O D C B A, actually arrives at A. Hence, the object O will be seen, not in the direction O A, but in that of A *o*, a tangent to the curve O D C B A at A. But because the curve described by the refracted ray is concave downwards, the tangent A *o* will lie above A O, the unrefracted ray; consequently, the object O will appear more elevated above the horizon H R, than it would appear were there no such atmosphere. Since, however, the disposition of the strata is the same, or assumed as being the same, in all directions around A, the visual ray will not be made to deviate laterally, but will remain constantly in the same vertical plane O A E, passing through the eye, the object, and the earth's centre.' *

Exceptions to this rule have been observed, and lateral

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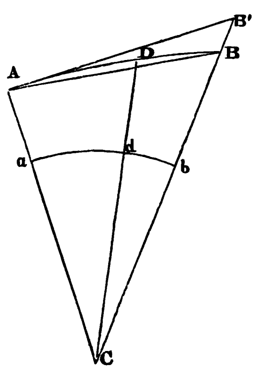
* HERSCHELL'S *Astronomy*, p. 27.

deflection has been the consequence of a supposed subversion of equilibrium in the same concentric ring. Under certain states of the atmosphere, denser strata have also been supposed to be temporarily incumbent on rarer strata, the curve or path of the refracted ray becoming in such a case convex downwards, whereby a double curvature is produced, the effects of which there are as yet no means of estimating, and consequently correcting:—such cases fortunately are of rare occurrence.

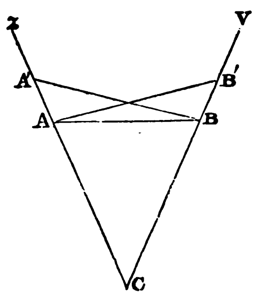
Of the Measurement of 'Refraction.'

I now proceed to the investigation of a formula for measuring this refraction, supposing it to occur only in a vertical direction, and thus tending to raise the apparent position of the object.

Let C be the centre of the earth, and *a d b* its surface; if from a station, A, a distant object, B, be observed, the visual ray, from B, will describe the curve, B D A, and the object will appear situated at B′, in the direction of the tangent to the curve at A. The angle B A B′ therefore is the measure of the displacement caused by refraction.



The nature of the curve, B D A, is unknown; but, as in all geodesical operations the distance, A B, is always comparatively small, the curve, B D A, may be assumed circular, as being an arc of the osculating circle to the curve. Under this hypothesis, the angle, B A B′, is equal to half the arc, A B. (Euc. III. 20 and 32.) With an object, D, the refraction would be measured by half the arc, A D, hence the refraction is proportional to the arcs, A D, A B. But the arcs, A B, A D, may be considered as proportional to the arcs, *ad*, *ab*, on the earth's surface; hence the amount of atmospheric refraction varies as the angle formed by vertical lines drawn from the extremities of the curve of refraction; or making R = refraction, and C = angle at earth's centre, then $R = n\,C$, *n** being a coefficient deduced from experiments, and which remains constant in the same state of the atmosphere. The following is the method adopted to obtain the value of this coefficient, which is found by experiment to vary according to the elevation of the object above the horizon of the observer.



Let C, as before, be the centre of the earth, and A and B two stations, of which the zeniths will be Z and V, respectively. The station B, observed from A, will be seen in B′, owing to the effect of refraction; and the station A, observed from B, will appear in A′, for the same reason. The angles between each object and the zenith (*i.e.*, the sum of the angle of depression and 90°, or the difference between the angle of elevation and 90°), will, when observed, be too small, being diminished by the measure of the angle of refraction; *i.e.*, the observed zenithal angle at A will be Z A B′, and at B, V B A′.

The exterior angle of a triangle being equal to the two interior and opposite, we have

$$ZAB = ACB + ABC,$$

$$VBA = ACB + BAC, \text{ and}$$

$$ZAB + VBA = 2\,ACB + ABC + BAC = 180^\circ + ACB.$$

* The coefficient, *n*, varies because the curve, A D B, varies.

From this equation, we find that the sum of the *true* zenithal angles of the two stations is equal to two right angles + the contained arc; hence the excess of 180^0 + the contained arc above the sum of the *observed* zenithal distances, will give the measure of the sum of aberration due to refraction in both observations. If the observations at A and B have been made precisely at the same moment, and when the state of the atmosphere, therefore, would have had the same effect on both observations, the amount of error divided by 2 will give with precision the amount of correction to be made at each angle. It is extremely difficult, however, with great distances (those in which the correction is most wanted), to make simultaneous and reciprocal observations of this kind; but a series of observations should be taken at each station under the most favourable circumstances, about noon of a cloudy calm day, when, in our climate, the tremulous motion in the air is commonly the least; the mean of the results may then be assumed as the value of R in the equation, $R = n\,C$, whence the coefficient, n, can readily be found.

Let D and d represent the difference obtained by subtracting 90 from each zenithal distance; then, without refraction, we should have $D + d = A\,C\,B$: and if the sum of the refractions $= 2\,R$, then $2\,R = A\,C\,B - (D + d)$, care being taken to give the proper signs to D and d, which become negative when the zenithal distance is less than 90^0:—the mean refraction at each station is, therefore,

$$R = \frac{A\,C\,B - (D + d)}{2}.$$

It is customary to describe the amount of refraction in terms of the distance between the stations, expressed in degrees or parts of a degree. This expression for the distance, which is then called the 'contained arc,' is obtained by the following proportion: —

$$365{,}110 \text{ feet} : \left\{ \begin{matrix} \text{distance between} \\ \text{the stations} \end{matrix} \right\} :: 1^0 : \text{contained arc},$$

the length of 1^0 at the earth's circumference being, at a mean valuation, equal to 365,110 feet, or 69·15 miles.*

* When the angle of elevation exceeds 8° or 10°, as in astronomical observations, the amount of refraction has been ascertained with precision by the comparison of the results of a great number of observations, made as follows. A circumpolar star which passes the zenith, and another which grazes the horizon, are followed with an altitude and azimuth circle (an instrument constructed on the same principles and with the same movements as the theodolite) through their own diurnal course; and the exact apparent forms of their diurnal orbits, or the ovals into which they are distorted by refraction, are traced. Their deviation from circles, which is deduced from the recorded reading of the vertical arc at every noment, gives the measure of the refraction due to all degrees of elevation; which is found to decrease rapidly from the horizon, where it is greatest, to the zenith, where it becomes nothing. Accurate tables of the mean astronomical refractions are prepared, by means of which the angles of elevation of celestial bodies are to be corrected.

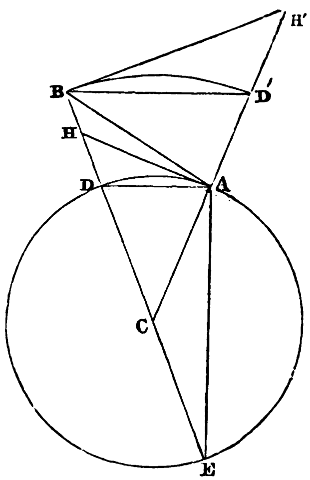
When the object observed is nearer to the horizon than 8° or 10°, the refraction, then termed terrestrial refraction, has been found to vary in a very irregular manner, changing materially with all changes in the state of the atmosphere. Different values for this coefficient have, therefore, been adopted by different observers. General Roy, in the operations of the Trigonometrical Survey, assumed it at $\frac{1}{10}$th or $\frac{1}{14}$th of the contained arc in cases where it had not been ascertained by actual observation; but, in examining the correction for refraction obtained from actual observations in that survey, we see it varying from $\frac{1}{7}$th to $\frac{1}{34}$th of the contained arc; the greater number of these corrections, however, oscillating between $\frac{1}{10}$th and $\frac{1}{16}$th.† It is evident, therefore, that, for terrestrial refraction, it is impossible to generalise a formula in the present state of knowledge; and extreme cases of extraordinary refraction have been mentioned which no previous calculations could have prepared the observer to guard against. When tracing out the base on Hounslow Heath, General Roy had directed the telescope fixed at King's Arbour towards

† In the account published of COLONEL LAMBTON'S *Indian Survey*, the refraction is stated as having varied from $\frac{1}{7}$th to $\frac{1}{30}$th of the contained arc.

Levelling with the Theodolite by means of Trigonometrical Calculation.

Vertical angles, in the mensuration of heights with the theodolite, being estimated from the direction of the spirit-level or plummet, they are consequently measured by reference to a tangent to the surface of the earth at the point of observation.

Let the points, A and B, represent two remote objects, and C the centre of the earth. With the radius C A describe a circle; draw A H, tangent to the circle at A, and C B, cutting the circumference in D and E, and join E A and A D. Then B H will be the apparent, and B D the true, elevation of B as compared with A. Through B, with



Hampton Poor House, 'at which end of the base a flag-staff had been erected: this, for a long time, he endeavoured in vain to discover, till at last, very unexpectedly, it suddenly started up into view, and so high it seemed to be lifted, that the surface of the ground where it stood became visible. This will appear the more extraordinary, when it is considered that a right line, drawn from the eye of the observer at King's Arbour to the other end of the base, would pass 8 or 9 feet below the surface of the intermediate ground.

'On the same base line, 30 pickets had been driven, 100 feet from each other, so that their heads appeared through the telescope to be in a right line; this was done in the afternoon. The following morning proved uncommonly dewy, and the sun shone bright; when, having occasion to replace the telescope, it was remarked that the heads of the pickets exhibited a curve concave upwards: in the afternoon, when the ebullition in the air subsided, the curve appearance was lost.'—*Trig. Survey*, vol. i. p. 175.

the radius C B, describe the arc B D′, and draw B H′ tangent to the arc B D′ at B. Then if B is observed from A, the angle of elevation is too small by the angle D A H; if A, on the contrary, is observed from B, A H′ will be the apparent, and A D′ the true, depression of A below B, the angle of depression being too large by the angle D′ B H′. The value for H D, the difference between the true and apparent elevation, has been given (page 96) in terms of the distances A B and A C; it may be given in terms of the angle at C, being $= \frac{1}{2}$ C, for D A H = angle A E D in the alternate segment of the circle $= \frac{1}{2}$ A C D (Euc. III. 20).

Hence the true vertical angle at any station will be found by adding to the angle observed with the theodolite when it is an angle of elevation, and when it is an angle of depression, deducting from it half the measure of the contained or intercepted arc.

This measure, depending upon the curvature of the earth, which is neither uniform nor regular, should (mathematically speaking) be deduced, for each particular place, from the length of the corresponding degree of latitude. Such nicety, however, is very seldom required. It will be sufficiently accurate in practice to assume the mean quantities, and to consider the earth as a globe. Assuming its mean diameter at 7,916 miles, the arc of a minute on a great circle would be equal to 6,085 feet nearly, and the correction to be added to the observed vertical angle will amount to one second nearly for every 101 feet contained in the intervening distance.

The vertical angle being hence obtained, and the horizontal distance between the two stations computed from previous data of the triangulation, the height B D, or the difference of elevation between A and B, is obtained by a single computation from the triangle B A D, in which the side A D is given, the angle B A D = observed angle $+ \frac{1}{2}$ A C D, and the angle D B A $= 90° -$ (observed angle + ½ A C D); for A D B may be considered a right angle.

Example.—Taking the distance between Maker Heights and Kit Hill stations, in the eastern part of Cornwall, at the computed length of 67,822 feet, and the angle of elevation at Maker Heights, corrected from the errors due to refraction equal to 28′ 7″; required the height of Kit Hill, Maker Heights having been found, by levelling with the spirit-level, to be 402 feet above the sea at low water.

The angle of elevation at Maker Heights, corrected from the effect of refraction, is equal to . . . . 28′ 7″

to which, adding half the contained arc, or . . $\frac{11' \ 9''}{2}$

the true angle of elevation becomes . . . . . 33′ 41·5″;

then because,

Rad. : tan. angle of elev. : : horiz. dist. : diff. of elevation, we have,

Log. rad.	10·0000000
: log. tan. 33′ 41″ . .	7·9911551
: : log. 67822 feet . .	4·8313706
: log. 664·6	2·8225257

Maker Heights elevated above the sea	402 feet;
To which add difference of elevation .	664·6 „
Kit Hill elevated above the sea . .	1066·6 „

Altitudes thus obtained not always to be relied upon with certainty.

It is to be observed, that unless reciprocal angles of elevation and depression have been taken in the same state of the atmosphere, at each station, the levels obtained

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* *Trigonometrical Survey*, 1795.

cannot be securely depended upon, owing to the constantly varying condition of the atmosphere, and the consequent difficulty of ascertaining the true coefficient of correction. As an instance of the great discrepancies that refraction has been known to cause, we may compare* the height of St. Ann's Hill, as obtained from observations taken by General Roy in 1787, at the station near Hampton Poorhouse, with that deduced from observations made at the same station in 1792, when (the axis of the instrument being at the same height above the ground) the angle of elevation in the first case was 17′ 39″, and in the second 8′ 11″. The height deduced from the first observation was 321 feet, and that deduced from the second, 240 feet; presenting the very great difference of 81 feet, or upwards of 30 per cent.

As to the relative heights obtained by the observation of the reciprocal angles, they cannot, unless the reciprocal observations have been made exactly at the same moment, be depended upon as approaching nearer the truth than about 10 feet,† with triangles whose sides are from 7 to 10 miles, and so on in proportion. The best time for making such observations is on a cloudy day, when the tremulous motion in the air is commonly the least.

The elevations of points, therefore, obtained thus trigonometrically, are not sufficiently exact for many delicate engineering operations. They are well adapted for general reconnaissances, and for high mountainous districts; but for surveys that are to form the basis of engineering works, more exact processes must be adopted. The accurate results required are produced by means of the spirit-level, and the operations for these results will be found described in chapter VI.

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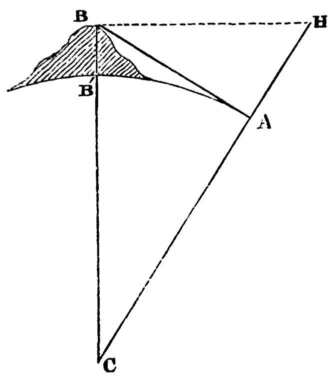
* *Trig. Survey.*, vol. i., p. 172. † *Trig Survey.*, vol. i., p. 173.

Determination of the Altitude of a Point near the Sea by a single Observation.

When the horizon of the sea is visible from an elevated station, the altitude of the observer above the level of the sea can be ascertained by a simple observation, with tolerable accuracy, as follows:—*



Let B be the station of the observer, from whence the line B A is drawn tangent to the surface of the sea. The angle of depression H B A is equal to the observed angle, plus the correction for refraction, which must be assumed at a mean value.

In the right-angled triangle A B C, the elevation of the point B, or

$$B\,B' = C\,B - A\,C, \text{ and } C\,B = \frac{A\,C}{\cos. C}, \text{ therefore,}$$

$$B\,B' = \frac{A\,C}{\cos. C} - A\,C = \frac{A\,C - A\,C \cos. C}{\cos. C}$$

$$= A\,C. \frac{(1 - \cos. C)}{\cos. C} = A\,C. \frac{(1 - \cos. C) \sin. C}{\sin. C \;\cos. C};$$

but because (trig.)

$$\frac{\sin. C}{\cos. C} = \tan. C, \text{ and } \frac{1 - \cos. C}{\sin. C} = \tan. \tfrac{1}{2}\, C,$$

we have by substiution,

$$B\,B' = A\,C \tan. \tfrac{1}{2}\, C \tan. C$$

Whatever may be the height of the station of observation B on the earth's surface, the angle of depression, which is equal to the angle at C, will always be very small. In

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* PUISSANT'S *Géodésie*, vol. i., p. 355.

such a condition of the triangle we may asssume (trig.)

$$\tan. \tfrac{1}{2} C = \tfrac{1}{2} \tan. C; \text{ then, by substitution}$$

$$B B' = \frac{A C}{2} \tan.^2 C,$$

making the *observed* angle of depression $= D$, and assuming the correction for refraction in this climate at the mean value of $\frac{1}{10}$th of the contained arc,

$$C = D + \tfrac{1}{10} C, \text{ or } D = C - \tfrac{1}{10} C = C\,(1 - \tfrac{1}{10}), \text{ whence}$$

$$C = \frac{D}{(1 - \frac{1}{10})} = \frac{1}{(1 - \frac{1}{10})} D. \text{ By substitution,}$$

$$B B' = \tfrac{1}{2} A C \tan.^2 \left(\frac{1}{1 - \frac{1}{10}} . D\right), \text{ and because the}$$

angle D is very small,

$$B B' = \tfrac{1}{2} A C \frac{1}{(1 - \frac{1}{10})^2} \tan.^2 D,$$

$B B' = \frac{1}{2} A C (1 + \frac{1}{9})^2 \tan.^2 D$, from which the required altitude $B B'$ is obtained nearly.

Example.—Required the height of a station from which the horizon of the sea was seen depressed 22′ 36″, taking the correction for refraction at $\frac{1}{10}$th of the contained arc, and the radius of the earth = 20,888,000 feet.

Log. $\frac{1}{2}$	$\bar{1}$·6989700
Log. 20,888,000 feet . .	7·3198969
Log. $(1 + \frac{1}{9})^2$	0·0915140
Log. tan.^{2} (22′ 36″) . .	5·6356816
Log. 557·3 feet	2·7460625

Height of station above the level of the sea = 557 feet.

Altitude of each Station determined Trigonometrically during the progress of a Trigonometrical Survey.

Having now explained* the method of levelling with the theodolite, and described the corrections requisite in

* Throughout the preceding remarks no account has been taken of the spheroidal shape of the earth, and in all levelling operations

the operation, it remains only to observe that in the progress of the triangulation the angles of elevation or depression are taken of each station, for the purpose of obtaining the relative elevation of each, and their absolute altitude above the level of the sea. These angles of inclination are entered in an additional column in the field-book; or, if the work be extensive, it is better to enter these angles in a separate field levelling-book, in the following form.

Form of Trigonometrical Levelling Field-Book.

From	To	Horizontal Reading or Index Error.	Apparent Elevation or Depression.	Reduction of Hypot. to Horiz. Base per 100 feet.	Remarks.
		′ ″	° ′ ″		
A	B	0 20	Depr. 3 0 0	0·14	
A	C	0 20	Elev. 6 30 0	0·65	

In the description of the adjustments of the theodolite it was remarked, that when the adjustments of the optical axis, parallelism of the level, and horizontal position of the circular limb were perfect, the index of the vertical arc should point to zero; but as some alterations unavoidably take place in these delicate adjustments by the carriage of the instrument, the third column is designed to receive the entry of the index error. No entry is, of course, made here, if the method of compensating for this error described in page 48 be adopted. The fourth column contains the apparent elevation or depression of the object; in the fifth are entered the number of feet to be subtracted per 100, as

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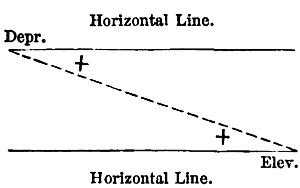
of this character, even those connected with the most delicate geodesical operations, it may be neglected; for the effects of refraction and the errors always inseparable from angular observations are more considerable than errors produced in geodesical operations by neglecting the extremely minute influence of the shape of the earth as differing from that of a sphere.

shown on the vertical arc; and the sixth is left for remarks. Among these remarks, a few horizontal angles to surrounding objects should occasionally be entered.

To 'Observe' so as to avoid necessity for Correction in the Vertical Angles.

As before remarked, if from any station the telescope be directed towards the ground, or the top of a signal at the next station, a correction is required in the observed vertical angle. If, for example, the ground be the point observed, the correction is additive if the vertical angle be one of elevation, and negative if the observed angle be one of depression. To avoid the necessity for this correction, the instrument should be set up (as nearly as possible) at a constant height above the ground, and the staff used for the observations should have a cross bar or vane fixed at the same height above the ground as the axis of the telescope, which bar or vane is to be bisected by the cross wires in observing. This precaution cannot always be used in the primary triangulation, and is, of course, inadmissible when the observed objects are permanent structures, such as church steeples, towers, etc.; but it should be universally adopted when levelling for the interior detail of a survey. In this latter case, the reciprocal angles of elevation and depression should be taken in order to ensure accuracy, and when the distances are so short that the effects of curvature and refraction are not sensible, the reciprocal angles, if observed correctly, ought to be equal to one another, the horizontal lines at each station being to our senses parallel.



Tabular Form for Reduction to Horizontal Base, and for Calculation of Altitudes.

In cases where the distances are short, and the relative altitudes are not required, the reduction of the lines to the horizontal plane, previously to their being used in plotting, may be made by reference to the column for the reduction of the hypothenuse to the horizontal base, as entered in the levelling field-book, by the reading off one side of the vertical arc. But when the distances are long, or the relative altitudes required, logarithmic computation should be used. In an extensive survey, time will be saved, and errors guarded against in this operation, if the entries for the calculation are made according to the following form, thus described in COLONEL COLBY'S *Instructions for the Interior Survey of Ireland.*

Plan and Plot.	Measured distances.	Elevation or depression.	Calculations of reductions to the horizon. — Hyp. × cos. of angle of inclination.	Horizontal distances.	Calculation of Vertical distances. — Hyp. × sin. of angle of inclination.	Relative altitude.	Altitude above low-water mark,	Remarks.
							A 21·50	Obtained by levelling with the spirit-level.
B	A 1942 B	elev. 2° 29′ 30″	9·9996351 3·2882492 — 3·2878843	1940 3	8·6382280 3·2882492 — 1·9264772	84·43	105·93	
	B 2156 C	depr. 1 21 20	9·9998804 3·3336488 — 3·3335292	2155·4	8·3739542 3·3336488 — 1·7076030	51·00	54·93	
	C 859 D	elev. 4 27 0	9·9986888 2·9339932 — 2·9326820	856·4	8·8898007 2·9339932 — 1·8237939	66·65	121·58	

'In the first column of this register, the designations of the plans and plots in which the points or lines are contained are entered. The second column shows the measured length in feet of the station line, which length is to be written between the letters marking its extremities, thus, A 1942 B. The third column shows the mean or elevation depression of the second object deduced from the reciprocal angles in the levelling field-book, after applying the corrections indicated in the third column of that book, and those for curvature and refraction when very long distances render their effect sensible. The fourth column contains the logarithmic cosine of the angle in the preceding column, and the logarithm of the distance; the natural number answering to the sum of these logarithms is entered in the fifth column. The sixth column contains the logarithmic sine of the angle, and the logarithm of the distance; the number answering to the sum of these two logarithms is entered in the seventh column. The eighth column contains absolute altitudes above the low-water mark. The altitudes in this column are to be proved by always commencing at some point whose altitude is known, either from the trigonometrical survey, or by levelling with the spirit-level, and proceeding in a regular series of additions or subtractions to some other point of which the altitude is also known in like manner. In connexion with these levelling operations, observations should be made for the purpose of ascertaining the heights of the rise and fall of the tide, both at springs and neaps, at various places on the coast, etc., the altitude above low-water (spring tides) of some conspicuous part of each of the points which has been trigonometrically determined; and of a sufficient number of other points, found by levelling, etc., to prevent the accumulation of error in the altitudes given in the register.

'The survey thus performed will furnish a great number of accurate heights, at short distances from one another, over the district surveyed; it will be easy to render this part of the work complete, and subservient to future local improvements, without devoting much additional time to this object. Not only the heights of hills, but also those of the lowest parts of the necks which connect them should be given; also the heights and depths of lakes, and the altitudes of rivers and streams in various parts of their courses. As churches are usually very prominent objects, the heights of the ground on which their towers or belfries are erected should be given as points for future reference; and a knowledge of the altitudes of mines and mineral deposits, and of manufactories, towns, and villages, will tend to facilitate internal improvements. The heights of canals should be given at all the locks, and the heights of the summit levels of roads; and also, when it can be conveniently done, the height over which a new canal or road must unavoidably pass to connect a valuable mineral deposit, or principal market or manufactory, with some adjacent harbour, navigable river, or existing canal.'

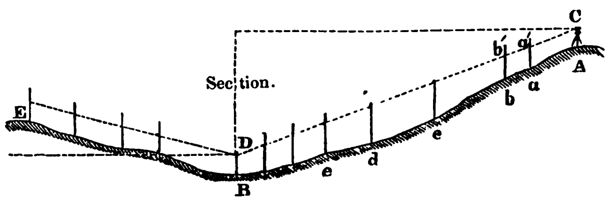
CHAPTER V.

LEVELLING FOR SECTIONS.

Levelling for Sections with the Theodolite.

WHEN the theodolite is used in levelling for sections along a continued straight line, much time would be lost by placing the instrument at every change of level which it is desired to mark on the section. The following method is adopted to obtain the section required.

The section line having been ranged, and pickets driven at all great changes of inclination of the ground, the theodolite is set up at one extremity of the line, and the intersection of the cross wires made to bisect the vane on a staff erected at the site of the first picket, the vane being as nearly as possible of the same height as the axis of the theodolite. The angle of elevation or depression having been noted, a levelling staff with a sliding vane is taken by an assistant to each irregularity of the ground offering itself in succession between the observer and the second station, and the vane is raised or depressed on the staff according to signs made by the observer, until the centre of the vane is intersected by the cross wires. When the vane is thus fixed, the reading on the staff is noted by the assistant; this mode of levelling being only resorted to when the distances are too great to enable the observer himself to read the divisions on the staff. For example: Let A be the position of the theodolite at the first station, and B that of the staff fixed at the second station. Between A and B, the intermediate positions *a*, *b*, *c*, *d*, etc., for holding the levelling staff, are determined by the irregularities of the ground. The angle of depression to B is observed, and the vertical arc being clamped in that position, the assistant places the levelling staff successively at *a*, *b*, *c*, etc., the centre of the vane being brought into the line of sight, C D, and the heights, *a a'*, *b b'*, etc., noted.



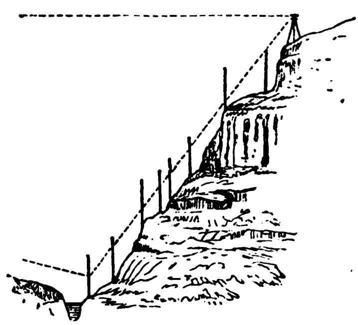
The instrument is afterwards brought to B, from whence the reciprocal angle of elevation to C is observed as a check on the work, and the telescope afterwards directed upon the staff fixed at the third station, E, in the line of section. The theodolite being clamped in that position, the same operation is repeated, to note the irregularities between B and E. In laying the section down upon paper, a horizontal line being drawn, the angles of elevation and depression can be protracted, and the distances laid down on the inclined lines as they were measured by the chain at the time of the observations being taken. The respective heights of the vane of the staff being then laid off from these points in vertical direction, will give the points, *a*, *b*, *c*, etc., marking the outline of the ground.

Levelling with the Theodolite not recommended for Sections.

As regards this method of levelling for sections, it may be observed, that in all cases it is inferior in point of accuracy to levelling with the spirit-level, and that it seldom saves much in point of time. A serious objection to it is the necessity for entrusting the reading of the staff to an assistant, and for making a previous careful inspection of the line to determine the site of each station of the theodolite, in order that the staff, when held on the intervening irregularities, may be long enough to be intersected by the line of sight.

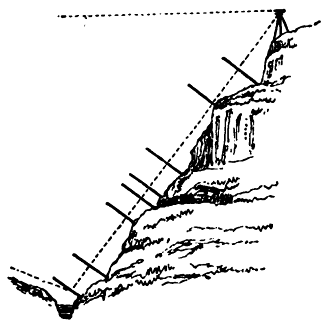
May be used in Exceptional Cases with advantage.

This mode of levelling for sections should not, therefore, be adopted, except in cases where the line of country, of which a section is required, is intersected by deep and precipitous ravines, or sea cliffs, to cross which much time is consumed when levelling with the spirit-level, owing to the difficulty in fixing the instrument in places suited to the proper reading of the staff, and owing also to the great number of readings required in a short horizontal distance. With the theodolite, on the contrary, it is sufficient to place the instrument at the top or bottom of the ravine, and take the differences of level as above described.



The leveller should, in such an operation, supply the staff-holder with a plummet, and take means of ensuring that the staff is held exactly upright, otherwise his work would be incorrect.

If he hesitate thus to expose the result of his work to the chance of the staff-holder not holding the staff upright, he may cause the staff to be held in directions removed from the vertical, so as to be at right angles to the line of sight. To effect this, the staff-holder should slowly incline the staff backwards and forwards (that is, to and from the observer). The lowest reading which can then be obtained from the staff will be when it stands perpendicular to the line of sight, because the perpendicular is the shortest line that can be drawn from a fixed point to a straight line.



Corrections for Curvature and Refraction may be generally neglected in Levelling for Sections.

In thus levelling with the theodolite when the distances from station to station are long enough to make the effects of curvature and refraction sensible and of practical importance, they should be taken into account. It rarely happens, however, that such a correction is required, because, within such distances as are adopted in practice, the corrections due to these causes are more minute than the errors caused by the difference of elevation between the axis of the theodolite and that of the vane above the ground, and other disturbing causes. For example, the correction for curvature and refraction combined is,

at ¼	mile, only	0·0357	foot
½	„	0·1430	„
¾	„	0·3216	„
1	„	0·5717	„

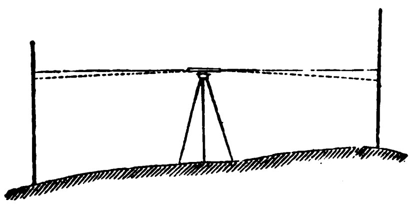
and, as the vane is to be raised or depressed according to signs made by the observer at the instrument, a greater distance than half a mile between station and station is but rarely adopted.

Levelling with the Spirit-Level.

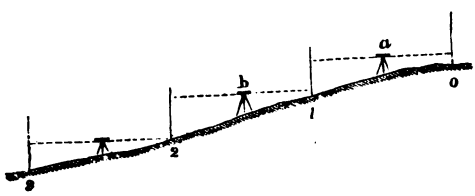
The spirit-level, with its present improved construction, presents the most accurate means of obtaining a section along a continuous line, or of ascertaining the difference of level between isolated stations.

When practicable, place the Spirit-Level midway between the back and fore sight.

The spirit-level serves to trace a series of lines tangent to a great circle passing through the axis of the instrument, the centre of which circle is the centre of the earth; and if the instrument be placed *in the middle* of each of these straight lines successively, the difference of level between

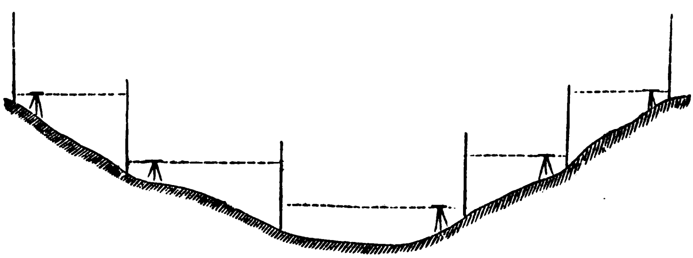


the extremities of each line will be obtained without any error arising from curvature or refraction, or from imperfect adjustment of the line of collimation, as already explained.

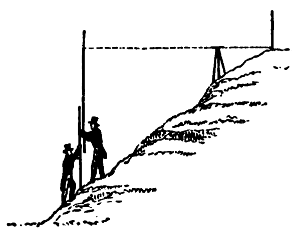


The mode of proceeding is thus: the level is fixed at *a*, and adjusted by the parallel plate-screws; the difference of reading between the first or 'back' station, *o*, and the second or 'forward' station, 1, is registered in the field book (see form page 124) opposite the distance between the stations; the staff at the station 1 is kept unmoved, while the instrument is taken forward and fixed at *b*. From *b*, the reading of the staff at 1, which then becomes a 'back' station, is registered, as also the reading of the next 'forward' station, 2; from these data the difference of elevation between not only the stations 1 and 2 is obtained, but also, by combination, the difference between the extremes, 0 and 2, is given. The same process is continued for any required distance, giving the elevation of each intermediate point, as well as the relative height of the extremes.

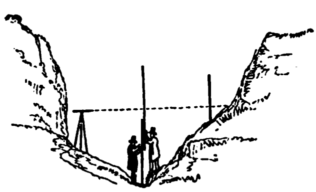
In common levelling operations, corrections for curvature and refraction may be neglected, even when the instrument is not placed half-way between the staves, because the distance at which the staves can be read is so small as to render their effects inappreciable; at a distance of 500 feet, for instance, the correction due to both causes is only 0·00513 of a foot. Therefore, when the line of collimation is itself properly adjusted, the instrument need not necessarily be placed midway between the back and forward stations, when a section of an inclined surface of ground is being taken; but its position may be so chosen that observations can be made each way, with the staves at a considerable distance from each other. When the ground is nearly level, it is better to fix the instrument midway between the staves; but, when crossing a valley, the instrument, if properly adjusted, should be placed, for the sake of expedition, nearer the back stations in going down the inclination, and nearer the forward stations in



rising on the opposite side; this arrangement avoids the necessity of taking the sights inconveniently close to one another. The alternation also tends, by a compensation of errors, to correct in the final result the effect of any error that might be due to imperfect adjustment in the line of collimation.

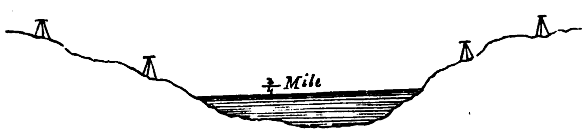


When levelling over a steep slope, or when the level has been fixed so that the staff falls below the range of the horizontal ray, much time may be saved by having the second staff held at a definite distance up the first, so that the horizontal line of the telescope may intersect the second staff, although it may not the first.



By this means, the surveyor saves not only time, but also avoids a repetition of adjustment, and therefore probable errors. This would be very useful in crossing ravines for the purpose of reading the lowest point of the valley, without necessarily going near the bottom.

In crossing wide rivers or estuáries, if the weather be calm, and there be no strong currents, the levels may be connected with great expedition by assuming the surface



of the water to be level, and reading, by two separate observations, the right and left bank elevation at the same instant of time.

'Bench-Marks.'

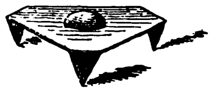
In levelling for a long section, bench marks or fixed stations that can be again found, ought to be chosen at certain distances, rarely more than half a mile asunder, and their elevation ascertained and registered on the section. These bench-marks need not be, and in practice rarely are, on the line of the section itself. Their use is to give greater facilities for checking the accuracy of the work as a whole, and for correcting errors that may have been made, by merely repeating the levels between those marks in the distance, between which the error is detected, without its being necessary to retrace the entire work a second time. Bench-marks should therefore consist of permanent objects, so defined by a slight sketch and description in the column for remarks, that they may be easily found again. Gate-posts, mile-stones, notches cut on stumps of trees, and similar points of reference, readily present themselves: for the prosecution of works in progress of execution, it is customary to drive short hard-wood piles in convenient places to serve as bench-marks.

Chain-men and Staff-holders.

In levelling for a section, two chain-men are required by the surveyor to measure the distances, as also two staff-holders, who, as they place the staff at any particular distance along the chain, are to give or call out the distance to the surveyor, who stands by his level.

Tripod to support the Staff.

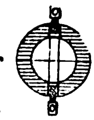
Each staff-holder should be provided with an iron tripod, *i. e.*, a triangular piece of plate-iron, with its corners turned down to act as cramps, whereby it may be steadily fixed in the ground; and with a hemispherical projection on the middle of the upper surface on which the base of the staff is to be placed. The tripod being firmly driven into the ground, by pressure of the foot, serves as a fixed point on which the staff may, when upright, be turned round without the slightest change taking place in its elevation; whereas, if no tripod, or other similar contrivance for obtaining a firm basis, be used, the staff, if resting on grass, clay, or gravel, etc., is liable to undergo a very sensible change in its elevation when turned round from one side to the other. Frequent repetitions of such alterations in its elevation, would, in the course of a long section, introduce serious errors, which may be altogether avoided by the use of the tripod.



Some levelling staves have been constructed with a tripod, or a contrivance designed for the same end, fastened to the base of the staff by a pivot that admits of the staff being turned freely round. A decided objection to this construction, is the liability which there is of the staff-holder unintentionally, especially in windy weather, or through carelessness, displacing the tripod by moving the staff; when, on the contrary, the tripod is separate from the staff, it is not liable, when once firmly fixed in the ground, to be moved, until taken up expressly after the observation has been made. Also in taking the level of bench-marks, it is inconvenient to have the tripod fastened to the staff. The tripod can easily be made by any village blacksmith.

Staff held upright by means of a Plummet.

By means of a small plummet, usually introduced in the side of the staff, the staff-holder is enabled to hold it upright in a vertical plane at right angles to a plane passing from the staff to the instrument. The vertical wires placed in the diaphragm of the telescope, serve to detect any deviation in the staff from a true vertical position in the latter plane; and the surveyor, before registering the reading, takes care that the staff appears properly between these vertical wires, and parallel to them. That the staff, when observed, should be held in a truly vertical position, is obvious; or if it be inclined, the intersection of the visual ray with the staff, will give a number too great by the amount of difference between the leg of the right-angled triangle and the hypothenuse, which, in the deviation from the vertical position, forms one of the acute angles of the triangle. In windy weather, when it is very difficult for the staff-holder to keep the staff upright, he should be directed to wave it backwards and forwards to and from the leveller, who, by noting the lowest or smallest reading, thus obtains the observation when the staff is at right angles to the horizontal ray of vision.



The following is a good form for keeping the field-levelling book. The entries shown below in Roman figures are the only ones which it is necessary to make in the field; those in Italic are entries and reductions which may be made afterwards, and form part of the office work.

Rise.	Back Sight.	Fore Sight.	all.	Reduced heights.	Distance in feet.	Remarks.
				21·34	....	B. M., Top of key-
	B.M.3·86	4·88	*1·02*	*20.32*	0	stone, canal bridge.
1·53	6·43	4·90		*21·85*	100	
2·46	4·90	2·44		*24·31*	140	
·74	7·96	7·22		*25·05*	146	
	7·22	9·31	*2·09*	*22·96*	158	Fence.
2·04	9·29	7·25		*25·00*	167	
·93	7·25	6·32		*25·93*	200	
·79	6·32	5·53		*26·72*	240	
	4·53	4·71	*·18*	*26·54*	264	
1·46	4·71	3·25		*28·00*	300	
1·32	3·25	1·93		*29·32*	317	
	1·93	5·46	*3·53*	*25·79*	341	
1·11	8·24	7·13		*26·90*	352	Double Fence.
	7·13	9·01	*1·88*	*25·02*	360	
2·51	9·01	6·50		*27·53*	365	
	6·50	7·91	*1·41*	*26·12*	400	
	7·91	8·65	*·74*	*25·38*	500	
	6·38	6·76	*·38*	*25·00*	600	
	6·76	6·76		*25·00*	700	
·96	6·76	5·80		*25·96*	800	
·28	5·80	5·52		*26·24*	880	
1·56	5·52	3·96		*27·80*	887	
	3·96	6·75	*2·79*	*25·01*	900	
1·62	7·69	6·07		*26·63*	903	Occupation Road.
·04	6·07	6·03		*26·67*	921	
	6·03	7·58	*1·55*	*25·12*	930	
2·92	7·58	4·66B.M.		*28·04*	944	B. M., Plinth of bridge.
22·27	*168·99*	*162·29*	*15·57*			*21·34+6·70=28·04*
15·57	*162·29*					
6·70	*6·70*					

Method of Entries in Field Book.

The only entries registered in the field are those of the second, third, and sixth column, with those made under the head of remarks. In the second column are entered the back sights, in the third column the fore sights. And here it may be observed, that as each station becomes a forward and a back station alternately, the terms back and fore relate only to the respective position of two stations, and not to their position as being back or forward with reference to the position of the instrument or observer. When the level of any two points is taken, that point or station on which the first observation is made is a 'back' station, that on which the second observation is made is a 'forward' station, although both may be behind the instrument with reference to the direction in which the line is being levelled. There may be several sets of back and forward sights taken without the instrument itself being moved; in such a case, the *reading of the fore sight of one set becomes the reading of the back sight of the next succeeding set*, and is therefore repeated: an example of this is seen in the second line, and other subsequent entries in the form of field-book given in the adjoining page; 4·90, the fore sight of the second line, is entered as the back sight in the third. This must be the case so long as the instrument remains unmoved, but when the instrument itself is carried forward, then the back sight ceases to be necessarily the same as the preceding fore sight, because the visual ray does not, after the displacement of the level (except in some chance instances), intersect the staff at the same elevation as it had done previous to its displacement. The distance corresponding to each fore sight is entered in the column for distances, in a line with the fore sight to which it refers.

Reduction of the Levels in the Field-Book.

The filling-in of the remaining columns of the field-book, or the *reducing* of the levels, is performed as follows:—the difference between each back and fore sight is taken; when the fore sight is less than the back sight, it is thereby shown that the staff must have been in a higher position for the forward than for the back reading, a rise is therefore denoted, and the difference between the two readings is entered in the first column, under the head of *rise*. If, on the contrary, the fore sight be greater than the back sight, it is thereby shown that the staff must have been in a lower position for the forward reading than for the back reading; a fall is therefore denoted, and the difference between the two readings is entered in the fourth column, under the head of *fall*. The whole page being thus reduced, as a check on the arithmetical operations, the sum of the first four columns is taken, and if the additions and the subtractions be correct, the difference between the *sums of the rises and falls* will be the same as the difference between the *sums of the back and fore sights*. In the example, page 124,

$$22{\cdot}27 - 15{\cdot}57 = 168{\cdot}99 - 162{\cdot}29 = 6{\cdot}70.$$

If the sum of the back sights exceed the sum of the fore sights, or the sum of the rises exceed the sum of the falls, a total rise is denoted; in the example a total rise of 6·70 is indicated; if the contrary holds, a total fall is denoted.

The fifth column, or that of reduced heights, remains to be filled. For that purpose, either an arbitrary elevation above a base, called a *datum line*, is assumed as that of the first or starting point; or the level of the starting point, as obtained from previous observations, is entered at the top of the column. In the example, 21·34 feet was the height of the bench mark from which the work was commenced. The elevation of each succeeding point is then obtained by adding or subtracting the corresponding quantity taken from the column of rise or fall, as the case may be; and this process of addition or subtraction being repeated throughout the entire column, the reduced height corresponding to the last station will be higher or lower, as the case may be, than the height corresponding to the first station, precisely by the difference between the sums of the back and fore sights, or of the rises and falls. In the example, 21·34 (assumed as the elevation of the starting point, a bench-mark, above the datum line) and 6·70, the difference referred to, indicating a rise, are added together, and give

$$21{\cdot}34 + 6{\cdot}70 = 28{\cdot}04;$$

28·04 is therefore the height of the last point above the assumed datum line, and all the intervening numbers indicate the relative elevation of the corresponding point above the same datum.

The Arithmetical Check in no case to be neglected in reducing the Levels.

It is important not to neglect in any instance the means here indicated of checking the accuracy of the additions or subtractions; for these being in a great measure mechanical operations, there is a liability to clerical or other errors when many pages of the field-book, or long sections, are to be reduced rapidly.

To select such a Datum as to avoid Negative Quantities in the Reduced Heights.

The elevation of the starting point is assumed, as I have said, at an arbitrary quantity. When this starting point is not at a lower level than all the succeeding points, it is desirable, in most cases, to choose a number high enough to avoid the necessity of using negative quantities for the reduced heights that follow, as they would to some extent tend to confuse the work, and an accidental omission of the sign minus would lead to serious errors in the reduction and plotting.

Levels should always be Checked.

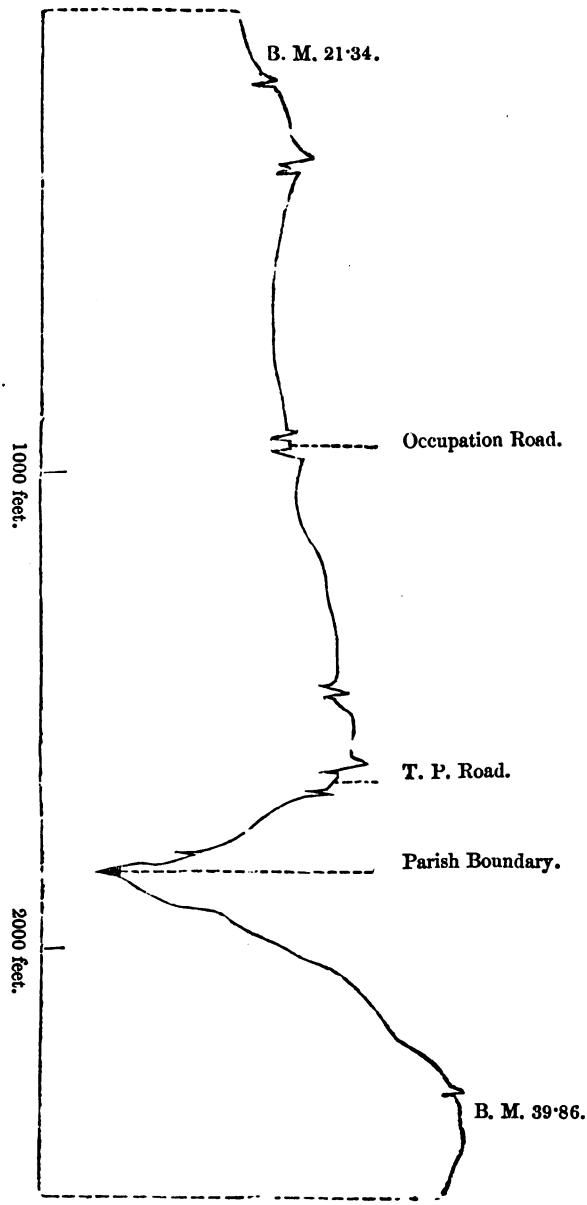
After a section line has been levelled, a second series of levels should *always* be taken from bench-mark to bench-mark, to check the previous work. In this case it is, of course, unnecessary to chain the distances; all that is required is to ascertain the relative elevations of the bench-marks; if these be found the same in the first and second operations, the work is correct: if any sensible difference exist, that part of the section line included between the bench-marks that do not correspond in altitude must be levelled over a third time, in order that the error may be detected and corrected.

Of Plotting the Section.

The plotting of the section is performed in a manner similar to the plotting of a plan: the elevation and position of each point that has been levelled are determined by means of two ordinates at right angles to one another, the horizontal distance being measured along one ordinate, the vertical distance or height along the other. The points having been marked with a fine-pointed pencil, they are all joined by straight lines as the plotting proceeds. The accompanying section is plotted from the data given in the form of the field-levelling book, page 124.

Horizontal and Vertical Scales usually Different.

Unless it be in a few exceptional cases, in which sections are plotted on an exceeding large scale, it is the



Datum Line, Summit Level of Grand Junction Canal.

Horizontal Scale, 400 *feet to* 1 *inch*. Vertical Scale, 20 *feet to* 1 *inch*.

custom to exaggerate the vertical scale or height, in order to render more prominent to the eye those particular dimensions, forms, or irregularities of the ground, which the section is especially intended to exhibit. Sections over the general surface of cultivated country in lowland districts, if plotted to a true scale, vertically and horizontally, of four inches to the mile (the horizontal scale required for sections of roads, railways, or canals to be deposited with the Houses of Parliament), would frequently appear almost as straight lines, and certainly fail to indicate in a striking manner heavy cuttings or embankments that might be requisite in such lowland countries to construct projected lines of communication. The foregoing section is plotted to a horizontal scale of 400 feet to the inch, and to a vertical scale of 20 feet to the inch; the vertical height is therefore exaggerated 20 times.

Section Paper.

For the purpose of receiving the plotting of sections, a paper is prepared, on which are engraved faint lines, dividing its dimensions horizontally and vertically into twentieths of an inch, i.e., with lines ruled parallel and at right angles to each other, at the distance of $\frac{1}{20}$th of an inch. Much time is saved by the use of this section paper, as no scale is required for the plotting; it has also the advantage of facilitating correct and rapid measurements of any particular part, the measurement being made by simply counting the engraved lines or divisions, instead of applying a scale. Any regular contraction or expansion of the paper, moreover, does not affect the accuracy of such measurements, the scale being itself embodied on the paper.

Levelling with the Spirit-Level more accurate than by Angles of Elevation, or Depression with Theodolite.

Before concluding the subject of levelling with the spirit-level, I would observe that the late improvements made in the construction of the instrument, and in the mode of reading the levelling staves (improvements due, in a great measure, to the special attention paid to levelling for railway sections), may be said to leave nothing to be desired in point of accuracy and expedition. And I would repeat, that for the purpose of taking sections, the theodolite, or other angular instrument used for levelling by means of angles of elevation or depression, is inferior to the spirit-level.

Theodolite may be used as a Spirit-Level, but the practice not recommended.

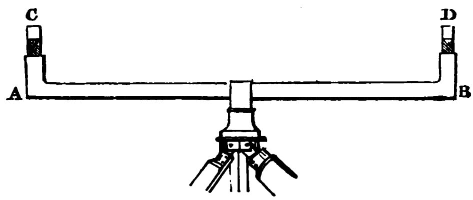
The theodolite may be used as a substitute for the spirit-level, that is, by giving its telescope only a horizontal movement; but it is not advisable so to employ it. In the first place, the greater weight of the instrument would prove a hindrance to rapid operations; secondly, the number of clamps and adjustments required to obtain a truly horizontal motion, renders the time in which the instrument remains so adjusted, much shorter than with the spirit-level. It should be thus used, therefore, as a substitute only, by way of expediency, but not, as a general practice, to supersede the spirit-level.

*The Water-Level.**

Circumstances may occur in which the surveyor or engineer would find it necessary to take a section without

* MALORTIE'S *Topography*, vol. ii., p. 59.

having a good instrument at hand. He might, in such a case, use the water-level, an instrument that may be readily constructed in a very short time. It consists simply of a cylindrical tube A B (usually made of tin), the extremities of which are bent at right angles to the length of the tube,



and support two cylinders, C D, of very transparent glass, open at both ends. When the instrument is to be used, water, slightly coloured in order to define its surface more clearly, is poured into one of the cylinders, and immediately communicates with the other by means of the tube A B. When the water is in a state of rest, the surfaces in each glass tube are on the same level, and the instrument requires, therefore, no adjustment. In making an observation, the surveyor places his eye in the line of the two surfaces of the water, and the intersection, with the staff, of this visual line, gives the reading required. A staff with a sliding vane must be used for this operation, because the observer being unassisted by a telescope, would be unable to read the divisions on the staff, except when placed very near to the instrument. This level, when used in calm and clear weather, is capable of giving results with surprising accuracy.

CHAPTER VI.

SPECIAL APPLICATION TO 'PARISH OR ESTATE SURVEYS,' 'ROAD, RAILWAY, OR CANAL SURVEYS,' 'TOWN SURVEYS,' &c.

THROUGHOUT the preceding chapters, while the student's attention was being directed to the various kinds of surveying or levelling operations that he may be called upon professionally to undertake, it was necessary to keep the theoretical investigations, side by side, with the description of the processes to which they became subservient. For, as it has been previously stated, no practical rules can be given universally applicable in all details to any one surveying operation:—the processes must all be modified according to the difficulties to be overcome; and a surveyor, ignorant of the principles on which all the processes, however varied, must be based, would frequently be at a loss how to proceed, or use methods so ill chosen as to cause waste of time, and produce errors from their want of simplicity.

Now, however, it will be assumed that the student is master of the principles previously set forth; that he understands the uses and adjustments of the instruments described; and, in short, that he is prepared to commence to work professionally, in a manner lucrative to himself, and satisfactory to his employers.

This chapter is then to be devoted to the explanation of field and other working operations to be undertaken; first, with a view to obtain correct results for practical application; and, secondly, to obtain these results in a manner that shall consume the least amount of time and labour consistent with the indispensable requisite of accuracy.

'Parish or Estate Surveying.'

For surveys of parishes, estates, or other divisions of property partaking of the same character as to general size, and ordinary compactness, the operations are of a similar character. In the following description, therefore, although for the sake of brevity, the survey of a parish will be alone referred to, it is to be understood that the same processes will be equally applicable to surveys of estates, townships, hundreds, etc.

Previous Maps or Plans.

The surveyor, on arriving at the scene of his labour, will ascertain if any map or plan of the parish is extant. For if he have access to such a plan, no matter how imperfect it may be, it will still be of some assistance to him (proportionate to its accuracy), in enabling him to dispose the conduct of his work with greater expedition.

Local Information as to Boundaries, etc.

He will then procure the assistance of intelligent persons (not fewer than two, if possible, in order that the information given by one may be tested by the other), well acquainted with the boundary, and every object in the parish, to accompany him over it, so as to acquire a knowledge of its magnitude, general form, and bearings, and to ascertain if it contain elevated spots, either natural or artificial, from which commanding views of large portions of the parish may be obtained.

Practical Hints for the Advantageous Disposal of the Work.

'In cultivated countries, let the roads and lanes, or footpaths, be examined, to see if they can be made subservient to the purposes of the survey; because, if so, they ought to be used on account of their offering no impediments to the use of the chain or other instruments; while woods, hedges, deep ditches, and other obstacles, frequently occasion delay. To overcome some of the difficulties, the surveyor should be provided with a small hatchet, as it is frequently necessary to cut a chain or sight-way through underwood, as well as to cut and drive marking stakes. One of his men should also be provided with a wallet or strong bag slung over his shoulder for carrying refreshments and necessary implements, such as a hand-saw, small spade, pickets, etc. If the surveyor has to plot or draw his own plans from the measurement taken, the author recommends, from his own experience, the following distribution of time. To spend the first day in the field, taking measurements, and to draw or plot the work so taken early the following morning; that done, to resume the field-work till dark, and on the following morning to plot the second day's work, and so on. The reason of this is obvious. Field measuring is laborious and fatiguing work, and after having spent a long day upon it, the surveyor is in general in no condition for fine drawing or scale measuring; his hand is unsteady from exertion, and the light of evening is unfavourable to his operations. But after a night of refreshing sleep, he will be well prepared for drawing on the following morning, when the light is good, and he retains a perfect recollection of the positions and particulars of the places he has been over on the previous day, and may even be able to supply small omissions, if such have been made in his field-book, and they do not relate to measurements. If errors or omissions occur, he detects them, and has an opportunity of correcting them by revisiting the spot before another day's work is commenced. And as a skilful draughtsman will have no difficulty in plotting as much work in two hours as can be measured upon the ground in ten or twelve, it will be seen that no delay is occasioned by this arrangement. The drawing work may all be finished before an early breakfast, after which the surveyor proceeds to the ground, and will generally find himself so fatigued after six or eight hours' work in the field (for he should carry his dinner with him to avoid delay), that he will have little inclination to do more.'*

Each day's work should be dated in the field-book—such a memorandum frequently proving of use for subsequent reference.

Details required for the Plan.

The plan will be required to represent accurately, and in their true relative positions, the several objects which occupy the surface of the ground; such as roads, rivers, lakes, ponds, canals, streams, drains, parks, woods, fences, houses, and other buildings, bridges, etc.; also the boundaries of the parish and its various sub-divisions; such as townships, estates, unions, etc. These may all be represented according to the Table of Conventional Signs, referred to in page 29, Chapter I.

Contents of the Fields and Inclosures.

The gross contents of the parish, as well as the acreage of each field or inclosure, must also be obtained. The

* MILLINGTON'S *Engineering*, page 134.

latter will be determined by admeasurements on the plans, the former by some means which will make the correctness of that area independent of the result obtained by summing up the contents of each inclosure, minute errors in many of which would escape observation, if not checked by comparison with the correctly ascertained whole. 'It is essential, in fact, to arrive at the total area of the parish by direct admeasurement of the space included within its external boundary; and the simplest and cheapest means by which a survey and plan may be made for effecting this object appears to be as follows:—

Sketch of Leading Operations.

'1stly. To measure two straight lines through the entire length and breadth of the parish.

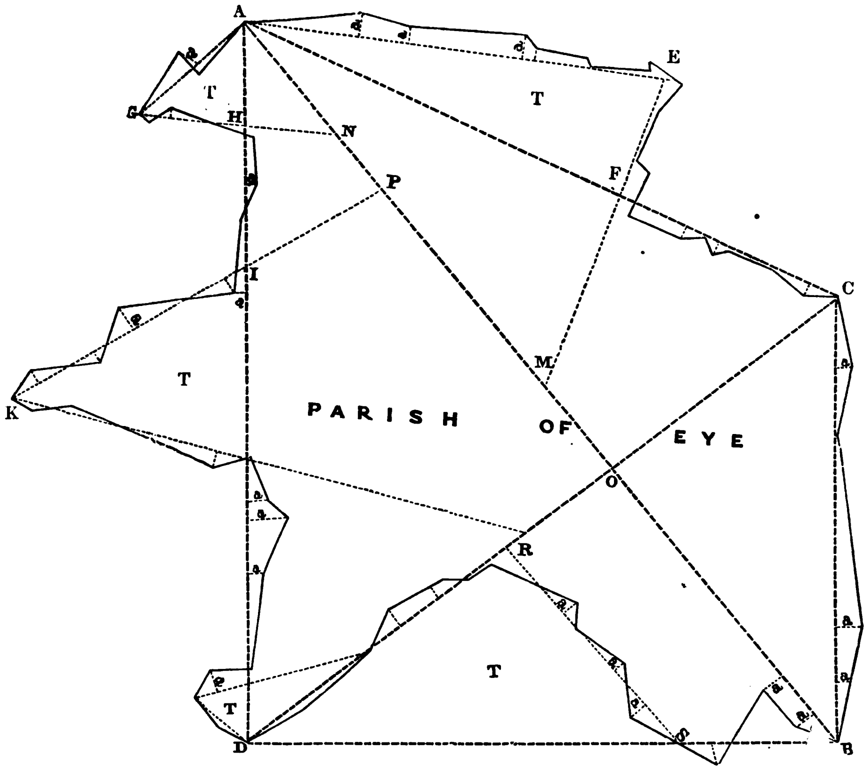
'2ndly. To connect the ends of these lines by means of other measured lines; and,

'3rdly. From these connecting lines (by measured triangles and offsets) to determine the entire parish boundary.

'The true area of the parish may then be obtained by calculation from the measured distances, and by admeasurement of the included space on the plan.

'Lines of the description herein proposed to be measured are ordinarily used by surveyors in the construction of their plans, but are not always shown on the finished map; I propose to retain them permanently, for purposes which will presently appear.

'The object and application of these lines will be better seen by reference to the accompanying diagram, representing the parish of Eye.



'The two main-lines which I should recommend to be measured through it are marked A B, and C D.

'A C, C B, B D, D A, are the connecting-lines.

'T, T, T, T, are the triangles constructed upon the connecting-lines.

'*a*, *a*, *a*, *a*, are the offsets, or perpendicular distances of the several angular points of the parish boundary from the measured lines.

'Now, if the main-lines, A B, and C D, be measured accurately, and their true lengths, from the point (O), at which they cross one another, be laid down upon the plan, it will be seen that the connecting-lines, A C, C B, etc., will form efficient checks on the general direction of the two main-lines with reference to one another.

'A satisfactory check on the *lengths* of the several lines will, by the same means, be afforded; for, as the points, A, C, B, D, are in each case determined by the intersections of three lines, an error in any one of these lines must immediately be discovered.

'Thus the true relative position of four extreme points (A, C, B, D,) in the parish boundary will be obtained, and such portions of the boundary as fall within the ordinary range of offset distances from the connecting-lines (A C, C B, etc.,) will also be determined, and may be laid down in their true positions.

'The more remote parts of the parish boundary may be determined by means of the triangles (T, T, T), the sides of which (E F, G H, K I, etc.,) being prolonged on the ground to intersect the main-lines, A B, C D (as these do at M, N, P, etc.), may be laid down correctly in position and direction upon the plan.

'By this simple process the whole boundary will be determined, and the total area may then be ascertained.

'Among the objects to be particularly attended to in practice, is that of reducing the lines, measured over steep slopes in hilly districts, to the horizontal plane.

'This demands special mention, because some inattention to it is not very unusual, though the necessity for such reduction is well known to practised surveyors, and all should be alive to the importance of using a theodolite, spirit-level, or other assured means, in the measurement of lines over hilly ground, for determining the exact allowance to be made.

'Without this reduction of the lines, they cannot be laid down in plan upon a flat surface, and distortion of the outline must inevitably result.

'Care must be taken in all cases to measure the lines straight to the points desired, and this will require more particular care in a mountainous, rocky, marshy, wooded, or thickly-inhabited country.

'The expedients in use among practised surveyors will of course be resorted to for overcoming any difficulties which may attend the measurement of these main-lines, and the theodolite offers a never-failing resource in all cases where a departure from the direct line is inevitable.'*

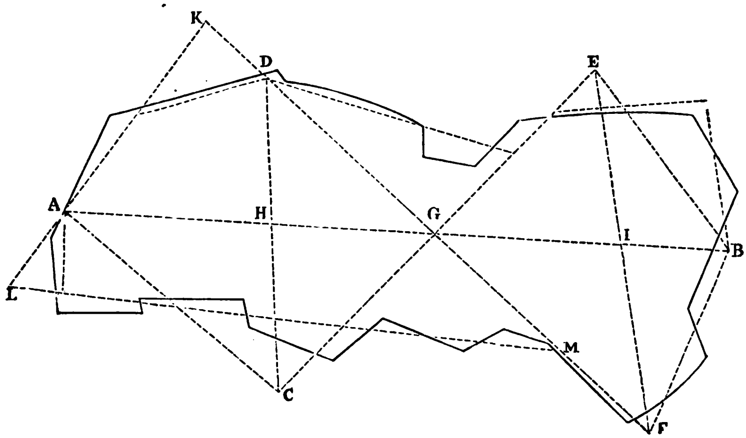
Operations based on the same general principle to be modified according to circumstances.

It is not of course to be understood that the system of lines here recommended is to be universally applied. It is undoubtedly the best arrangement in all cases in which the parishes are of a compact form. In cases of marked deviations from such a form, modifications of the system (dependent, nevertheless, on the same fundamental principles), will be adopted, as exemplified in the lines recommended for the survey of a parish in the following diagram.

Comparing it with the figure in page 138, it will be seen that there are still retained the main lines, C E and D F, traversing the parish nearly at right angles, but that instead of forming a single trapezium, as in the example of the parish of Eye, by joining their extremities, two trapeziums are formed, so as to adapt the lines more closely to the boundary of the parish than the single form could be made to do. Still, the same principle guides the operation which is thus explained in detail. 'A B is the base line, E F and D C the transverse main lines. The tie line, F G, forming one side of the trapezium, E B F G, it will be perceived, is extended to D and K, forming a side of

* *Report to the Tithe Commissioners*, by Captain Dawson, R.E., 1837.

the other trapezium, G C A K; and, in like manner, another side, E G, of one trapezium, is extended and forms a side, G C, of the other trapezium; then the measurement of the main lines, A B, E F, and C D, will most



effectually test the accuracy of position of the stations, C, D, E, F; and a further check will be afforded by the lines, K A, A C, E B, B F, and L M.'* Without pursuing this part of the subject further into details, it is sufficient to refer to the diagram, which shows—

1st. Two straight lines to be measured through the length and breadth of the parish.

2nd. The ends of these straight lines to be connected by measured lines.

3rd. Measured triangles and offsets resting on the main or connecting lines, to determine the boundary.

Interior Detail.

For completing the interior detail of the parish, other lines must be measured; and those which may be most

* BRUFF'S *Engineering Field-Work*, page 57.

advantageously selected for the purpose appear to be the following, viz.—

1st. Lines connecting any remarkable objects in the parish, such as churches, windmills, towers, obelisks, high rocks, dove-cots, summer-houses, etc.

2nd. Lines measured from such objects within the parish in the direction of similar objects in the adjacent parishes, noting particularly the points at which they cut the previously measured lines and the boundary of the parish. It will be well to continue the measurement of the lines to the objects themselves, when they are within a moderate distance of the parish boundary.

Lastly. Such other lines to be measured among the preceding lines, and connecting them with one another as may be requisite for determining the positions of fences and other objects not previously obtained.*

Ranging of long Station Lines.

From the great length of the lines required in this description of survey, their extremities can very rarely be seen from every point along the lines; they must be ranged, therefore, by intermediate points placed sufficiently near to have always some of them in sight to guide the chain-men. When the direction of a station line, therefore, has been determined, set up at least two ranging rods, and before losing sight of or passing the first of these, set up a third; and, in like manner, before the second is passed, or becomes invisible, set up a fourth, and so on. To ensure the continuity of the straight line, by having all the ranging rods successively placed in the same vertical plane, the surveyor should be provided with a plumb-line and plummet, which, when held over a point in the line, so as to cover

* *Report of the Tithe Commissioners*, by CAPTAIN DAWSON, R.E 1837.

or hide from view the first rod, should, at the same time, cover all those placed beyond the first and this, whether the country be level or undulating.

Of the actual Measurement.

The line being traced out, the measurement is commenced from a fixed mark. The chain used will be Gunter's (66 feet divided into 100 links). Many persons, for the sake of having a light chain to carry, purchase those that are made of thin wire; but they cannot be so well depended on. The strength of the wire should be such as to permit the chain to be stretched without great liability to the links opening and expanding, an inconvenience to which even the strongest chains are liable.

The surveyor will also bear it in mind to test daily the accuracy of his chain by reference to a standard as explained in page 4, Chapter I. The field-book is to be kept in the manner explained in the first chapter; the crossing of all fences and divisions of property being carefully noted.

It is the usual practice to make or fix secondary station-marks at such crossings, but some surveyors adopt the method of leaving marks or pickets at stated intervals on the line, a system which has the recommendation of great convenience for reference, and for the subsequent filling in of the detail by cross measurements from one station to another. 'It is scarcely possible to tell where the 'close' will be; and as it may frequently happen to be in the middle of a field, a great loss of time will take place, and some uncertainty arise, if, to determine the intersection, it becomes necessary to refer back to the station at the next hedge, and re-measure the main line from the station to the point of intersection. But if, as recommended, stations be left on the main lines at stated intervals, say 10 chains apart, it is evident no difficulty will arise in determining the exact point of intersection, as well as the chainage from one of those stations. The method of procedure by this plan is to drive stakes at the determined distances, say at 10 chains apart, and inscribe the chainage rudely thereon in Roman character; thus, if the stakes are left at 10 chains apart, call 10 chains I., 20 chains II., and so on.'* 'When the first main line is measured (a side of one of the great triangles for example), at any spot supposed to be a good termination for the first line, set up a mark, and, if sufficiently acquainted with the locality, a new line may be immediately commenced therefrom, either by taking a forward or backward mark in the determined direction. But, if the surveyor should not possess the requisite knowledge of the locality, it will be necessary for him to explore the direction in which he contemplates running his line previous to setting it out, and in doing so it will be found advantageous roughly to range out his track, but which will occasion little loss of time, as the marks employed, from being properly distributed over the ground, can be quickly placed in correct position when the direction is finally determined on. It will probably be the better way to commence ranging out the direction of this line from its expected termination, towards its commencement, not troubling about the point of intersection, so long as it is sufficiently near to the extremity of the previous line, and free from any positive objection.' The measurement of the second line is then carried through precisely in the same manner as the first. For the determination of the third line, the assistance of the theodolite in measuring the angle between the first and second will be frequently useful, as, from the data then obtained, the angle may be set out for the direction of the third line so as to insure that it will intersect the first at no great distance from the original starting point.

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* BRUFF'S *Engineering Field Work*, page 63.

No delay to take place in plotting the Work.

The main lines, and the field-work generally, should, as explained, be plotted as soon as possible after the measurements are made.

The plan should be made on good drawing paper, previously mounted on linen.

Scales for the Plans.

The scales commonly used for content surveys of parishes and estates are those of 3 chains and 4 chains to an inch. The scale of 3 chains to an inch is in most common use throughout the country at the present time; and it is the scale on which the surveys made under the Inclosure, Tithe, and Poor Law Assessment Acts are generally laid down. The 4 chain scale is occasionally used for estate surveys in districts where the inclosures are large and of regular form. But it appears to be generally admitted by experienced surveyors, that the use of this scale should be confined to such districts, and to skilful hands; and that the scale of 3 chains to an inch is the smallest that can with safety be used, in all cases, for plans from which the contents are to be computed.

For the plans of towns a larger scale is required.

Station Lines to be plotted and their lengths written.

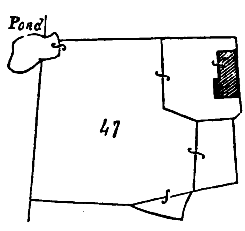
The scale having been determined, the main lines are laid down with straight edges and beam compasses.

The chained lines are then drawn with a fine pointed pencil, to be afterwards, in all cases, traced in red ink, and marked with a reference to the number of the line in the field-books, or to the page of the book in which the notes of the measurement are entered; thus, L 1, L 2, L 3, etc., if referring to the lines, or P 1, P 2, P 3, etc., when referring to the pages. And when several field-books are used, each book should have a distinctive letter assigned to it, which may be added to the reference upon the plan after the number referring to the line or page; thus, L 1, A; P 1, B, etc.

When the lines are very long, inconvenience may occasionally arise from the want of a straight edge of sufficient length to reach from one extremity to the other. When such is the case, a good substitute will be found in a silk thread blackened with black lead, and snapped when correctly over the points.

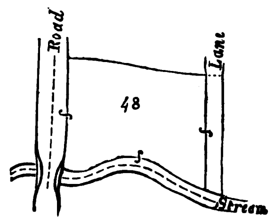
Numbering of the Fields or Parcels of Land.

'Each separate parcel of land is to be numbered upon the plan, the numbers following in succession from No. 1 to the highest number required. These numbers are to correspond with those given in the reference book, in which must be specified the name and description of each field or enclosure, with its true quantity or contents in statute measure, the names and description of the owners and occupiers thereof, the state of cultivation of the several lands, whether as arable, meadow, pasture, wood, coppice or common land, garden, orchard, hop-ground; or howsoever otherwise.



'When the quantity of two or more *contiguous* parcels of land is given in one sum, the reference number must be repeated on each parcel, or the parcels must be connected by a brace, as in the annexed sketch, where it is shown by the brace (∽), that four parcels of land, the house, and pond, are all included in the quantity assigned to No. 47.

'Where the whole breadth or any portion of a road, stream, etc., is included with the adjoining field, the brace must also be used; and where only part of the road, stream, etc., is included, the exact limit to which the

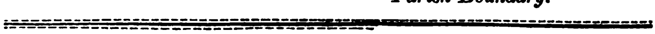


quantity applies must be marked with a dotted line on the plan.—See No. 48 in the annexed sketch; of which the quantity is thus shown to include the whole breadth of the lane, half the road, and half the stream.'

Parish or other Boundaries.

The parish boundary should be shown in all cases by a dotted line; and when it passes along the middle of a fence, the dots should be drawn on both sides of the fence, thus—

Parish Boundary.



Fence.

'When a road forms part of the boundary of a parish, both fences of the road should be shown, and it will be desirable also to mark the abutments of other fences upon the outer fence of the road.

'The same remark will apply to rivers generally, and, in Lincolnshire and other fen districts, to droves, and the drains by which they are bounded, etc.

'When a parish boundary passes through a field or other inclosure without being defined by a fence, the whole of such field or inclosure should be shown on the plan, with the parish boundary (marked by a dotted line) passing through it. The area of the included portion only, of such field or inclosure, will appear in the schedule; but the area of the excluded portion may with propriety be given on the plan, and be marked as belonging to the adjoining parish.'*

Wherever disputes exist respecting the boundaries, the boundaries claimed by both parties should be shown upon the plan.

In all cases of fences, the actual boundary line of the adjacent properties should be marked upon the plan, whether it be the central line or the side of a hedge, ditch, wall, bank, etc.; and when the fence belongs entirely to one property or the other, that should be indicated by the proper mark.

Generally, the centre of the ditch, when the separation of a property is formed by a bank and ditch, is the boundary: with a wall, the middle of a wall, etc. But there are, regulating these important points, various local customs, with which the surveyor should at the outset make himself acquainted.

'The ordinary usage in other respects is to be observed, of placing the north towards the top of the plan, writing the name of the parish and county as a title, the name and address of the surveyor, the date of performance, the scale, and the total contents. And when a parish is mapped in two or most distinct parts, a notice to that effect must be added to the title.

'When alterations are made in the field-books, an explanation of the cause of the alteration is to be entered at the same time; and erasures in the field-books should on no account be made.'

The quantities are not to be written upon the plan, but in the reference book only.

The quantities in the reference book are to be arranged

* *Report* by Captain Dawson, R.E., to the Tithe Commissioners.

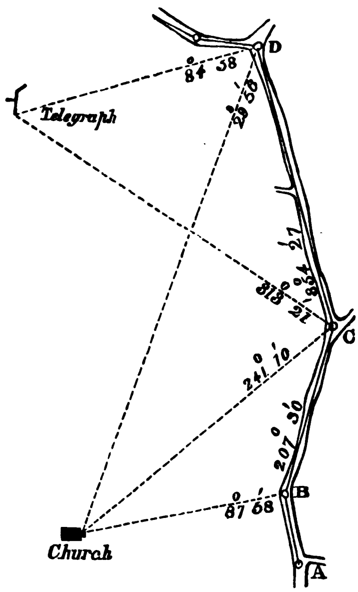
in the consecutive order of their reference numbers; and figures are invariably to be used for the reference, the use of letters being less clear in practice.

It is essential that the plan, when completed, should be such as that the admeasurement on the plan of the inclosures therein represented should correspond with the quantities assigned to them in the book of reference. The original plain working plan is always deemed the most valuable document, with all the lines of construction, names, and reference figures shown upon them.

'TRAVERSING.'

In the description of the parish survey, provision has been made for the determination of the true position of every object on the surface. It frequently happens that a surveyor may be called upon to make merely a survey of a road, canal, river, or boundary, without its being required to survey any of the objects on either side. This operation is performed by a process called 'Traversing,' by which, with the assistance of the theodolite or other angular instrument, the surveyor is enabled with great expedition to perform the work required.

Let A, B, C, D, etc., in the diagram, represent a road to be surveyed: at the starting point (A), selected at the side of the road so that the traffic on the road may not interfere with the work, a staff, with a cross bar fixed at the usual height of the axis of the theodolite above the ground, is erected; and from A a straight line is measured in the direction in which the road is to be surveyed to a point B, selected at or near the first turn or bend, and at the side of the road, offsets being taken to the right and left, as may be required, to terminate the width and boundary of the road: these measurements are entered as usual in the field-book. At B the instrument is adjusted and set to zero, the telescope having first been directed to the staff at A: the upper plate being then unclamped, the readings of one or more conspicuous objects in the neighbourhood are taken; and, lastly, the telescope being directed upon a staff fixed by an assistant at the forward station C, chosen at the next bend in the road, the upper plate is firmly clamped, and the *forward* angle read off. Proceeding from B to C, the line B C is measured



with accompanying offsets as before, and the theodolite adjusted over the station C. The ranging staff brought forward from A and fixed at B, serves as the object on which the telescope is to be directed for the *back* angle, by means of the movement of the *lower* limb, the upper plate still remaining clamped to the last forward reading. The staff at B being properly intersected by the cross wires, the lower plate is then firmly clamped, and the upper plate having been unclamped, the same conspicuous objects are intersected as before, and the telescope is afterwards directed to the next forward station D, when the same operation is repeated, to be afterwards continued throughout the entire length of the road to be traversed.

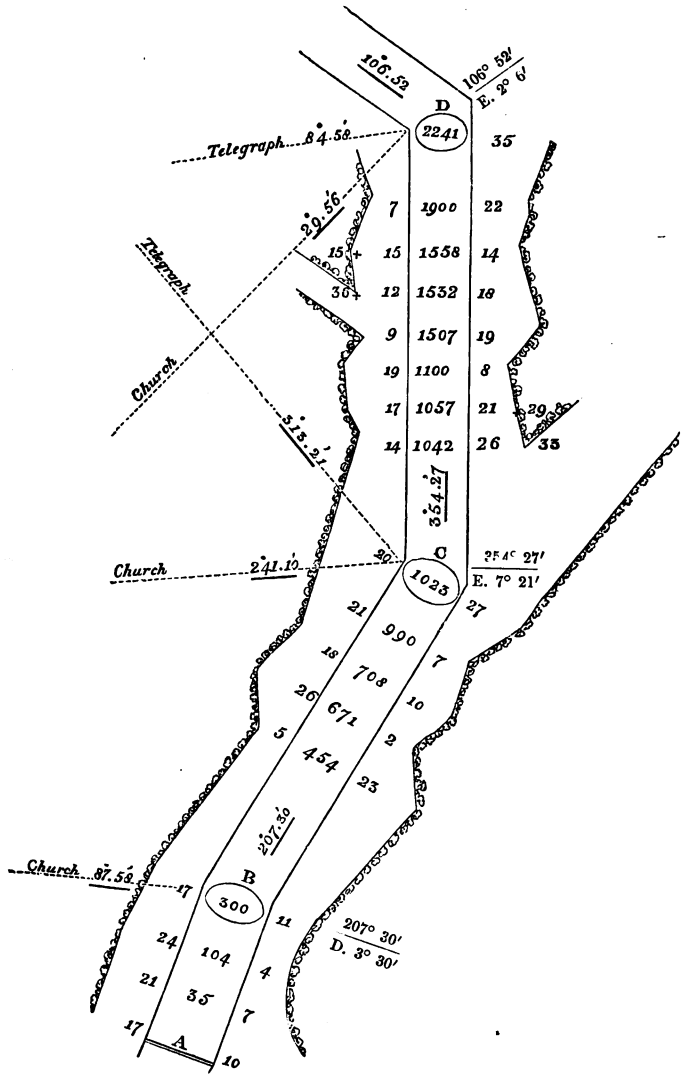
It will be observed that, by leaving the upper plate clamped for the *back* reading at the same angle as the preceding *forward* reading, the readings of the horizontal limb, for the angles forward, indicate the direction of each station with reference to the first line on which the telescope was set, which may therefore be called the first meridian. This method of reading saves, in the subsequent plotting, the trouble of changing the position of the protractor at every angle.

The angles read at each station may be entered, at the corresponding station in the field-book, at the end of the measurements relating to the station line immediately preceding, as in the form (page 152). If the 'forward' angles be, as in the diagram, written at right angles to the direction of the 'forward' lines, the angles of elevation or depression may be noted under the forward angles, fractionwise.

In the example, the line A B is the first meridian to which all the angles are referred; but during the progress of the work other lines are selected as meridians, in order to facilitate the plotting. To constitute any one of the lines a meridian line, it is only necessary to fix the vernier at zero for the back angle, instead of retaining the preceding forward angle.

Pickets are generally driven at each station to mark the precise spot, in case it should be necessary to refer to it again; and at the close of the day's work, if the survey is incomplete, angles should in addition be taken to several fixed points near the last station, in order that it may be identified with ease when the work is resumed.

The reason for taking angles to surrounding conspicuous objects (if they be not so distant as to make the angles of intersection very acute, and thereby form ill-conditioned triangles) is, that they may serve as a check on the work; for the several bearings on the same object should, in the plotting, meet at a common point of intersection. However, from the dependence of each new station line for its



direction upon those that precede it, errors will, except under favourable circumstances, be introduced into the work. The different parts of a traverse should therefore be referred to certain accurately fixed points, and any errors thus discovered may, if small, be corrected and allowed for in the plotting.

The surveyor, in plotting, marks off, around the first station, the bearing of the angles referred to the first meridian; and, taking from the field-book, in succession, the length of each station line connected with that first meridian, he transfers the direction from the central point by means of a parallel ruler. The same operation is repeated around each station at which a new meridian has been assumed. The station lines, when checked and found correct as to length and direction, are marked in faint red lines, and the road itself is plotted from the offsets given in the field-book.

Highly-finished circular protractors are made in metal with verniers attached, capable of reading to minutes and fractions of minutes; but these protractors are liable to be altered in position by the movement given to the arms bearing the verniers. For general use, circular pasteboard protractors, with the interior circle cut out, will be found more convenient;—these can be made by describing a circle with a radius of (say) five inches, and dividing the circumference to one-half or one-fourth of a degree, by measuring, from an accurately-divided diagonal scale, the lengths of the chords as given in the table of natural sines, and applied in the manner explained in page 89. Quantities less than one-half or one-fourth of a degree may be estimated by the eye.

In road or other traversing, it would scarcely be necessary to read to seconds, if the direction of the lines were to be plotted solely by reference to the protractor. But independently of the fact, that, after a little practice, no more time is required to read the seconds than the single minutes, it may be observed here, that when great accuracy is sought, the direction of the lines is calculated by the resolution of right-angled triangles, in which the angle registered forms one of the data. It is especially when traversing for the surveying of mines, that calculation may be thus resorted to with advantage; we have therefore reserved for the chapter on Mining Surveys the necessary explanations on this subject.

Surveying for Railways.

These surveys, though extensive in length, are rarely so in breadth, being usually confined to a narrow strip of the country through which the road, railway, or canal is to pass. The ordinary method of triangulation cannot, therefore, be applied, and the mode of proceeding resembles more nearly in character that described for road traversing, with this distinction, that the station lines, instead of being a few hundred or thousand feet in length, may be made to extend from five to ten miles, according to the general direction of the proposed work, or nature of the country. These great station lines are connected with each other by repeated and most careful observations of the angles formed by their intersection, as also by angles taken to conspicuous objects. These station lines being determined as to direction, and carefully ranged and measured, serve as great bases, on each side of which the fields and other details to be embraced in the plot must be next surveyed.

Detail required on Plans for Public Works to be executed with the sanction of the Legislature.

Before describing how this detail may be most readily obtained, it will be useful to consider the kind of information which it is necessary to provide. The greater number of such surveys as are now being described are made for the purpose of obtaining the sanction of the Legislature for the execution of contemplated works.

For plans that are to be thus prepared for Parliament, certain regulations defined under the Standing Orders of the Houses of Parliament must be complied with, and those regulations the surveyor should thoroughly understand, in order to ensure that his work shall comply with them.

Scale and extent of the Plan.

The plan must be upon a scale of at least four inches to a mile, and must describe the line or situation of the whole of the proposed work, and the lands in or through which the same will be made; and also any communication intended to be made with the proposed work.

The General Act gives powers for the diversion of the line of railway, or road, etc., 100 yards on either side of the line shown on the deposited plans; excepting where the line passes through towns, or continuous houses, and then to the extent of 10 yards only; the limits of such deviation on each side of the line of railway must be defined upon the plan, and the lands included within them must be shown.

It is not essential that the limit of deviation should always extend to 100 yards where circumstances (*e.g.*, the existence of a farmstead, or turnpike road, or a park,) render it advisable to restrict the power to deviate within narrow limits. Upon this latter supposition, the line of deviation will be drawn so as to exclude the excepted property, and the lands beyond that line, although within 100 yards, need not be described or numbered.

If the plan is on a scale of less than a quarter of an inch to every 100 feet, there must be an additional plan upon that scale (¼ inch to 100 feet) of any building, yard, court-yard, or land, within the curtilage of any building, and of any ground, cultivated as a garden, included within the limits of deviation.

These enlarged plans are so frequent a source of error, as to render it usually expedient to draw the whole plan to the enlarged scale.

Details to be introduced on the Plan.

The plan is to exhibit thereon the distances, in miles and furlongs, from one of the termini; if the plan is not lithographed, the distances will be best marked in figures of red ink, to distinguish them from the figures of the fields, which will be in black ink.

A memorandum of the radius of every curve, not exceeding one mile in length, must be noted on the plan in chains where the curve occurs. Where a tunnel is intended to be constructed, it must be marked by a dotted line on the plan.

Numbering of each distinct Property, Road, Path, River, Stream, etc.

Each distinct property, divided by any visible boundary from another property, should have a separate number, with this exception, that any collection of buildings and grounds within the curtilage of a building belonging to one person, and in one occupation, may be described under one number—thus, 'Farm house, yards, garden, barn, and sheds.'

When it is necessary to interpose a number, a duplicate number should be added, thus, 8_a, 4_a.

The numbering should recommence in every parish. All lands included within the limits of deviation shown by lines drawn on the plan, and all lands which those lines touch, must be numbered and described; if the limits of deviation are not defined, all the property shown on the plan must be numbered and described.

Public roads should have a separate number, in each parish, where they appear on the plan. Private or occupation roads should have a separate number, if fenced off from adjoining land; so also, footpaths, if repaired by the parish, or if fenced off, should be numbered. Navigable streams, and mill streams, must be separately numbered.

Field Operations for the Survey.

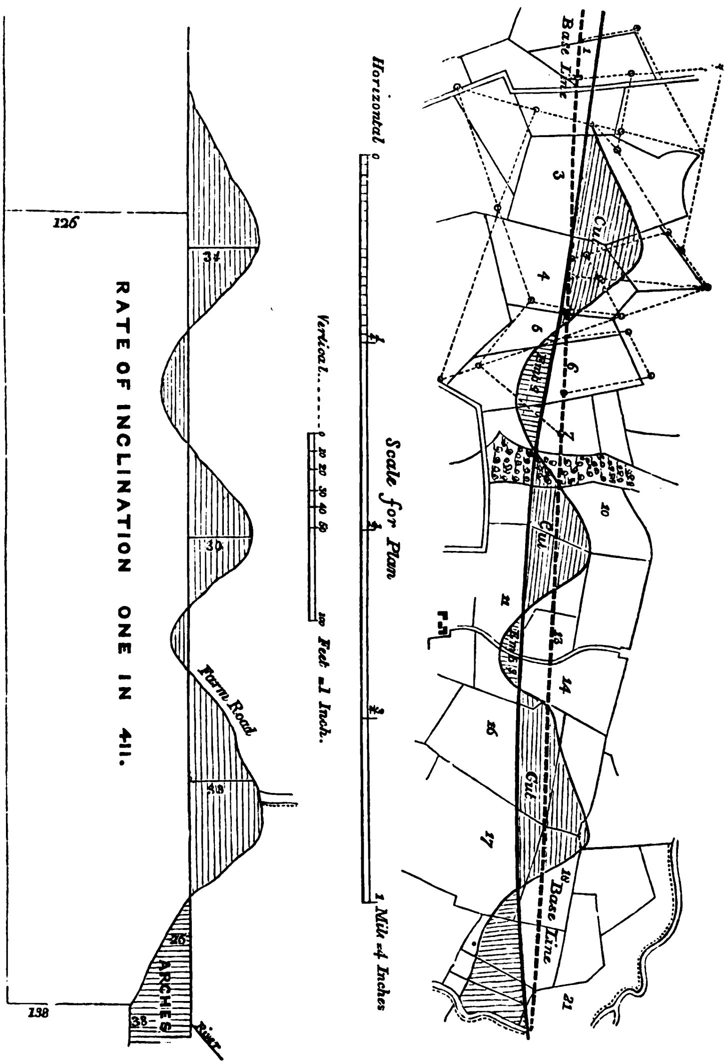
Understanding the amount of information required, the surveyor will proceed with his survey, either by the chain only, raising offset triangles upon various portions of each base, so disposed as to embrace the detail with the least amount of measurement, or by combining with the chain an angular instrument.

'The first plan requires the surveyor to close as often as possible on the base, intersecting it with cross lines wherever it can be done, as by such means, when a point on one side of the base is fixed, a line extended therefrom beyond the base on the other side, to any part where the chainage is noted also, becomes likewise a fixed point.' The dotted lines on the upper part of the annexed sketch represent such lines of construction. And when, as in the example, the enclosures are small, and the fences vary frequently in direction, this plan of triangulating with the chain will be found as rapid as any other.

In a more open country, however, where the fences are fewer and more regular, the use of the pocket sextant will be found greatly to expedite the work, by enabling the surveyor to obtain the details with much fewer measurements.

Longitudinal and Cross Sections of the Surface.

The survey being completed, and the line of railway marked approximately by the engineer, one or more longitudinal sections and numerous cross sections are taken, to



furnish data for the final selection of the preliminary or Parliamentary line. When the latter has been finally marked on the plan, its section is taken, carefully tested, and it is then drawn to the same horizontal scale as the plan, and to a vertical scale of not less than one inch to every 100 feet; it must show the surface of the ground marked upon the plan, and traversed by the proposed line of railway.*

Level of Line of Rails.

It must exhibit by a line the intended level of the upper surface of the rails, and it must show a uniform horizontal datum line throughout the whole length of the work and its branches. This datum line must refer to a fixed point near one of the termini, and the point of reference must be stated in writing on the section.

Selection of 'Datum.'

This fixed point should be some marked unvarying object, easily accessible to the public, *e.g.*, a tide-mark, chiselled in a dock gate, the plinth of a pillar, or other public building.

The surface water level of a canal at a particular spot, would not be good, because that level varies; the surface of a road, without stating the particular point, would also be too vague.

* The plan in the figure, page 158, is prepared from the published plate appended to the 'Standing Orders' as formerly published, and embodies the principle of 'Sectio-planography.' The intention of this mode of representation, was to enable proprietors and others, not familiar with the use of plans, to judge more readily of the probable effect and appearance of a projected railway, considered in reference to the properties to be passed over. It forms no longer, however, part of the 'Standing Orders' (1845).

Horizontal Distances to be marked on the Section.

The distances from one of the termini must be marked along the datum line in miles and furlongs, to correspond with the distances marked on the plan.

Height of Line to be written at every change of the gradient.

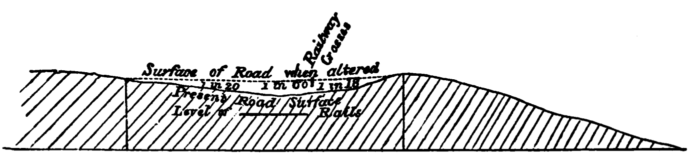
A vertical measure from the datum line to the line of railway, must be marked in feet and inches at each change of the gradient or inclination, and the proportion or rate of inclination between each such change must also be marked.

Wherever the railway is intended to cross any turnpike road, public carriage road, navigable river, canal, or railway, or is intended to form a junction with a railway, the distance of the surface of those objects from the upper surface of the rails, must be marked in figures upon the section, at the point of crossing or contact; and even if the levels coincide, that fact should be stated upon the section.

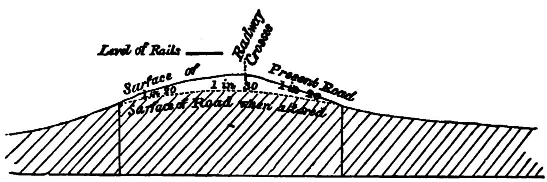
Where a railway crosses a road or navigation, or is crossed by a bridge or viaduct, the height and span of each arch must be marked.

The extreme height of the surface of the railway over the surface of the ground, must be marked in figures in the case of every embankment, however trifling; and likewise in every cutting the extreme depth of the railway, below the surface of the ground, must also be marked in figures.

If any alteration is intended to be made in the present level or rate of inclination of any turnpike road, carriage road, or railway, that alteration must be stated on the section; the road or railway must be numbered, and there must be a cross section (with a reference to that number), on a horizontal scale of 1 inch to every 330 feet, and on a vertical scale of 1 inch to every 40 feet.



Public Road No. 1.



Turnpike Road No. 2

The cross section must show in figures, as well as by measurement, the present level and the intended level of the road.

Intended tunnels and viaducts must be marked on the section, and special care must be taken that the lengths of the tunnels on the plan and section agree.

Observations.

A horizontal and vertical scale must be given on the plan and section.

The scale and datum line, as well as all the other foregoing requisites, must be shown upon each plan for deposit with the parish clerks.

It must also be borne in mind, that, where the level of any road is altered, the ascent of any turnpike road must not be more than 1 foot in 30 feet; and of any other public carriage road, not more than 1 foot in 20 feet.*

* It must be understood that alterations are occasionally made in the 'Standing Orders' as regards Surveys.

Survey after an Act has been obtained.

The preceding details refer to surveys for parliamentary deposit. When the Act for a public work of this kind has been obtained, it is customary to make a more accurate survey on a larger scale, about three chains to the inch, of which the accuracy must be perfect, inasmuch as it is to form the basis of contracts for the sale and transfer of property, measurement of earth-work, and generally for the preparation of estimates.

The line is also then re-levelled, and such deviations made in its course, within the parliamentary limit of 100 yards, as may appear desirable upon the possession of more exact details.

The general directions herein given for surveys of railways, will be found applicable also for the survey of roads, canals, etc.

TOWN SURVEYS.

Scale for the Plans.

For rural districts, a scale of 3 chains to 1 inch, or $26\frac{2}{3}$ inches per mile, has been described as of a sufficient size to represent the fields or other subdivisions of the surface. But where groups of houses and small tenements are to be shown, the quantities of which amount only to a few perches, a map on a much larger scale is indispensable for the accurate representation of such minute subdivisions.

Maps of towns have of late been frequently made on the scale of 1 inch to 1 or 2 chains, or from 40 to 80 inches to a mile, and some of the London parishes have even been made to a scale of 1 inch to 30 feet. Scales of 5 feet (60 inches) to 1 mile, or 1 inch to 88 feet, and also of 10 feet (120 inches) to 1 mile, or 1 inch to 44 feet, have been adopted for maps of some of the towns surveyed by the Ordnance. Experience has shewn the larger scale to be well adapted to the many purposes for which surveys of towns are required. It is applicable for sanitary purposes, admitting of the correct shewing of the lines of house and street drainage, the distribution of water-pipes, the stations of fire-plugs and fountains; upon such a map, the lines and directions of gas-pipes, and the station of gas-lamps and other objects may also be displayed. Characteristic marks have been inserted on the premises to render the map useful for the purpose of rating and valuations. With an engraved copy of a map so marked, it is suggested that any public officer making valuations may go over a whole district and check, in a general way, any account read from a rate-book, and note the omission of any sort of premises, such as stabling or out-houses, which may appear not to have been rated. In like manner, with such a map, the collection of the rates may be checked.

If building improvements and new lines of road be under consideration, it is deemed that the marks characteristic of the different descriptions of premises and their occupancy will give a new and additional value to such survey, in showing at once the general nature of the property that would have to be taken down, or that would be affected by the proposed undertaking.

For railway purposes, the parts of towns adjoining to the proposed works are usually plotted to a scale of 40 or 50 feet to the inch. The engineer and the proprietors of the land to be encroached upon by the works are thereby enabled to judge of the extent of property interfered with to within about a foot or two in length; and the engineer has the means of devising such works as may appear desirable, to avoid, as much as possible, interference with valuable property.

Town Surveys performed usually by 'Traversing.'

It is evident, therefore, that surveys which are to be plotted on such large scales must be made with the utmost care. They are usually performed by a process similar to that described under the head 'Traversing,' the theodolite being used to take the bearings of all the streets.

Town Surveys performed more accurately and expeditiously by the aid of Triangulation and Angular Intersections.

The work will be much expedited, and rendered more accurate, by taking angles from spires or commanding heights to remarkable points presented by the buildings, public and private, such as observatories, remarkable roofs, projections, chimneys, etc., noting each object in the field-book, at the time of observation, by slight sketches of each object observed, sufficiently exact to serve as a means whereby it may easily be recognised. Such intersections taken from three points, themselves fixed in position, will give the site of the principal buildings; and when these are laid down, 'traverses,' run in the intermediate streets and roads, will give the details with accuracy, the angular intersections serving as checks on the directions of the traverse lines.

Offsets and Measurement of Detail taken with the Measuring Tape or Staff.

In town surveys, the measuring tape is much used for the offsets, and especially for the measurements of the rears of houses, yards and small enclosures, through which the chain cannot be taken, to follow a station line.

Principal Lines of Construction only to be shewn on the Plan.

In rural districts, the surveyor is recommended to retain on the plan (showing them in red ink) all the lines of construction. For the plans of towns, this practice would lead to confusion, owing to the multiplicity of lines required. Those only, therefore, which mark the directions of leading streets, and form (as it were) great bases on which the remainder of the detail depend, should be marked.

'SKETCHING IN' DETAILS FOR MILITARY AND OTHER SURVEYS.

The completion of surveys, in which many principal points are determined, and of which a portion of the outline has been measured and plotted, and which are not designed to be accurate in the details, such as travelling maps, etc., may be expedited by taking the plan itself to the ground, and measuring with the box-sextant the angles between fixed points, or taking their bearings with the meridian by the prismatic compass; the directions of the lines and the measured distances being at once protracted on the plan, no field-book is kept. A protractor scale is required to set off the angles or bearings; one of its edges being divided as a plotting scale suited to the scale on which the plan of the survey is to be drawn. When plans are thus required to be taken to the field, they should be traced on bank-post paper, which admits of being folded without injury, so as to expose any portion of the plan, and only that portion which may be required at one time. In order to enable the surveyor to draw or plot more neatly, the paper may be folded over a rectangular piece of board, fitting on a field sketch-book or portfolio, and retained in place by leather bands at each corner.

It is not unfrequently of advantage (to the military surveyor especially) to be able to construct a plan, or to fill in the interior detail between fixed stations, with tolerable accuracy, without instruments. By pacing, distances may be measured; bearings or directions taken with the aid of a straight walking-stick; and the distances plotted at once on the field sketch-book by the aid of a plotting scale:—the intention of such reconnoissances, and their chief recommendation being, that the plan may be constructed at the same time that the surveyor walks over the ground. The greatest difficulty consists in setting out correctly on the field-sketch the first few objects; their relative distances should therefore be paced, and their bearings taken with care. They may serve, when determined, in giving the position of other points or objects, by the intersection of two or more lines passing through these objects fixed in position. The expeditious method of measuring distances approximatively by pacing, is not so liable to error as might be supposed, if due precaution has been taken to ascertain the length or value in feet or yards of a given number of steps made at the *common walking pace.* An attempt to make each step equal to one yard would produce greater inaccuracies, because of the difficulty of preserving, for any length of time, the steps of an unusual length. It is better to ascertain their length by actual trial, by pacing several times a known measured distance at the usual rate and pace. A very general measure of the length of steps is 24 steps to a chain, or to 22 yards.

Independently of pacing, practice in sketching outline plans trains the eye to judge short distances with tolerable accuracy. A knowledge of the laws of perspective, both linear and aërial, is also of some assistance; for, as perspective teaches us to transform actual into apparent forms, so it may teach to deduce actual from apparent forms, and thence obtain their true dimensions.

CHAPTER VII.

ON SURVEYING AS APPLICABLE TO THE COLONIES.

COLONIAL surveying is distinguished from the usual land surveying, previously described, by a marked difference in its objects. In cultivated countries in which every portion of the land is claimed by a proprietor and an occupier, and the surface of which is divided into estates with known boundaries, or separated into legal and ecclesiastical divisions, the business of the surveyor consists in making, on a plan, a faithful representation of the existing demarcations and artificial objects, as well as of the natural features, and in collecting and arranging all data which may contribute to convey a knowledge of the physical aspect of the country. In new colonies, on the contrary, the first purpose of the surveyor, instead of being directed to the measurement of existing lines or boundaries, consists in actually setting out on the ground the limits of stated quantities of land or 'sections,' previously to their being conveyed to the purchasers.

Bearing in mind this difference, we proceed to describe a suitable mode of conducting such a survey.

When treating of trigonometrical surveying, it was explained, that by it only could perfect accuracy be attained in the survey of an extensive district; but, at the same time, the description of the mode of operation made it manifest that it necessarily involves, in its prosecution, both considerable expenditure of money, and great consumption of time. In a new country, probably covered with timber or dense and tangled underwood, intersected by impassable rivers, and inaccessible marshes, and presenting other serious physical obstacles, the consumption of time and money must be proportionably increased.

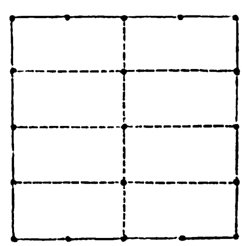
The resources of a new colony are evidently unable, on the one hand, to bear the burden of the expenditure necessary to obtain perfect accuracy; and, on the other hand, its thriving condition must be injured by the delay necessarily consequent on the operation.

Perfect accuracy must, therefore, in the first instance, be sacrificed to economy, and a method adopted capable of providing for the immediate division of the surface into suitable allotments, with an approximate accuracy sufficient for the exigencies of the moment, and attained at the least possible cost.

The square, or rectangle, admitted to be the figure best adapted for the subdivision of lands, is found to lend itself the most readily to such objects. The size of these rectangular divisions or sections must depend on the means of the settler, and the agricultural capabilities of the soil. In Lower Canada, the minimum size of the sections has been fixed at 200 acres; that adopted in South Australia contains 80 acres; and the greater number of the purchases in South Australia having been made of sections of the smallest size admitted by the regulations, it may be inferred, that the average means of settlers would be more readily met by the smaller sections. Whatever may be the dimensions adopted, the mode of operation will be similar: it will be the first object of the survey to provide for setting out on the ground the limits of such sections, in the district selected as the most suitable for immediate settlement.

The lines forming the boundaries of the rectangular sections are generally ranged in the direction of the cardinal lines; this ranging should be performed with the theodolite, the surveyor bestowing special care on the reading of the right angles, and never omitting to test the correctness of his work, by measuring the angles formed at the intersections of the boundary lines. The meridian line may be obtained, as described in page 88, by observations of the sun: or, as the surveyor has, in such operations, frequently to pass his nights in the field, he may check the direction of his meridian lines by observations of the polar-star.

The directions of these lines, or boundaries of sections, are ranged by the surveyor himself, while labourers are employed in clearing the lines, by cutting the brush-wood or under-wood to a width of about 3 or 4 feet. When large trees impede the progress of the line, they are passed, as described in page 21. As the clearing proceeds, the boundaries of the section are marked by strong pickets, driven into the ground at short distances of a few hundred feet, and projecting above the surface not less than from 2 to 3 feet: the bark should also be taken off, in order to render them more easily recognised. At the angular points of the sections, *i.e.*, at the intersections of the boundary lines, three or more pickets should be driven, in order to distinguish especially those points in the boundaries. If the minimum size of the sections adopted be 80 acres, these pickets would be driven at every quarter of a mile, or at every twentieth chain of 66 feet. If the cardinal lines we have described have been ranged at a distance of one mile from each other, they will, by their intersections, divide the surface into rectangles containing one square mile, or 640 acres. These larger rectangles would afterwards be subdivided into the assumed sections of 80 acres each, by ranging lines corresponding to the dotted lines in the figure, and driving interior section pickets at their points of intersection.



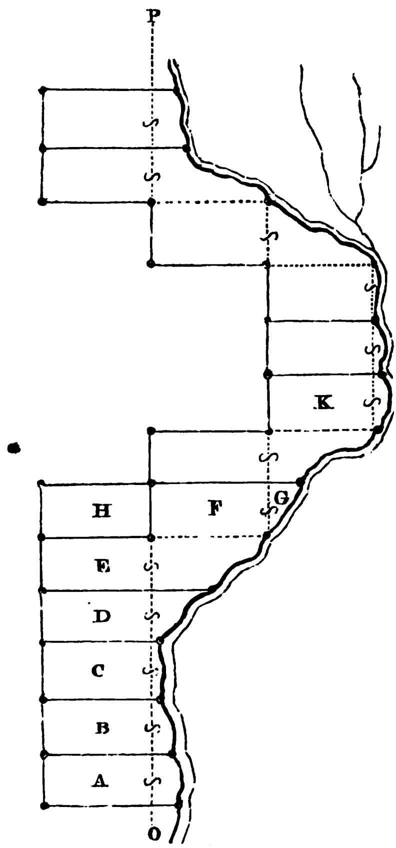
Varying circumstances of locality and physical aspect will necessarily occur to modify the uniformity of this system of survey, demanding occasional departure, both from the direction of the cardinal lines, and from the average size of the allotments, in order that they may be adapted to the natural features of the country. In this adaptation, the element of greatest influence, is the direction of the coasts, or of rivers and streams: for, looking to the great advantage which water frontage affords, the object must be to distribute that advantage as equally and generally as possible throughout the settlement. 'It becomes, therefore, a question of importance to determine the proportion which should be observed, in water sections, as to frontage and depth. The proportion in the land sections, as above described, is that of 2 to 1, the sides being half a mile, and a quarter of a mile, respectively.

'In the water-sections, the proportion may be increased, say, as 3 to 1, or as 4 to 1.'

It may be desirable, under certain circumstances, to vary the size of the water sections, in the manner indicated on the sketch, and subject to proper regulations:—The first line should be run from the point O, not more than 10 chains from the river, and the broken spaces between the line and the river should be added to the sections immediately behind them.

If the directions of the lines be fixed by means of the divided limb of the theodolite, and preserved by careful ranging, approximate accuracy may be attained to within certain limits. And if an allowance be made to the purchaser, ample enough to cover casual errors, the possession of the full quantity of land he has paid for will be thereby secured to him; and the trigonometrical operation might consequently be deferred until ulterior objects should call for greater accuracy, and the colony be better able to pay for it.

The allowance would involve a present sacrifice of land, a loss of little moment as compared with the immediate



saving of the great expenditure involved in the execution of a trigonometrical survey. As to the amount of this allowance for errors, '3 per cent. would be ample'; and this, added to a further liberal allowance of 2 per cent. on account of public roads, would reduce the purchase-money 5 per cent. on the total amount of land specified in the conveyance.

The map representing the sections, would not be held as giving a faithful copy of their exact form as set out on the ground, but would represent them as true rectangles, whatever might be the deviation from that form in the sections actually set out on the ground. On the sides of the rectangles, the place of the intersections of such rivers, streams, and other natural features as have been crossed in the regular progress of the survey would be marked, but their course in the intermediate areas would be for the time left unmeasured; and thus a skeleton map only would be formed. The allowance made to the purchasers, by securing them from loss, would render the probable distortion of the sections unimportant at first; and when future exigencies should demand accuracy, their true form and position could, by means of an ulterior trigonometrical survey, be determined, and projected on a map, with sufficient accuracy to constitute it a just record, and conclusive evidence of the boundaries of property.

In the prosecution of this work, the surveyor would require the assistance of 2 chainmen, 2 woodmen, and 2 or more labourers, according to the local difficulties of the district: with proper assistance, he could, in dense underwood or brushwood, set out a distance of about a mile per day. The Gunter's chain of 66 feet would be the most convenient for such operations, because 80 such chains constituting a linear mile, it lends itself readily to the binary division.

The necessity for economy and rapidity in the performance of surveys in all new colonies has been so strongly felt, as to lead sometimes to too wide a departure from accuracy, in order to obtain the objects in view. Time has been economized, first, by making the sections much larger than those we have described; secondly, by using less precise instruments, and bestowing less care in the ranging and measurement of the lines. By adopting the larger allotments, fewer linear miles have to be measured per square mile set out, and time is economised; but, independently of the consequent exclusion of settlers whose means are unequal to the purchase of large sections, this course diminishes the number of checks on the work, and by causing the accumulation of errors to be distributed over a more extensive space, renders their detection and correction more laborious and intricate, and thereby tends to induce their being passed over unregarded and unremedied.

Under the second head, the use of the compass as the sole means of obtaining the direction of the lines, necessarily introduces serious errors, which are again increased when the lines are not properly ranged by staves fixed in a straight line with a transit instrument or a theodolite. Such has been the practice in North America, where the proposed direction is determined by means of the compass. Moreover, the lines are there set out without a telescope, simply by the aid of sights raised on the instrument, and instead of being ranged by staves or rods, they are ranged by the bearing of large trees which stand in the line of sight.

It is obvious that great want of precision must result in ranging a line, when the only object to determine its direction is a tree of considerable diameter. Yet the errors due to this cause are not so considerable as those consequent on variations of the compass, which have been known to exceed half a degree in the course of a day.*

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* I am indebted for the information embodied in this chapter to the able report *On Surveying as applicable to the Colonies,* made by Captain Dawson, R.E., to the Secretary of State for the Colonies, 1840. It is important to notice, that since the date of the report, this system of survey has been followed in Australia, New Zealand, and other British Colonies, and that it has in all cases proved admirably adapted to the purposes in view.

The anticipations as to cost and rapidity of execution, as well as accuracy, have been fully realised.

CHAPTER VIII.

'HILL DRAWING' AND 'CONTOURS.'

ALL objects marked in position on an outline map, are projected according to the laws of the orthographic projection; in other words, their respective distances are reduced to a common horizontal plane. But as surveying aims at making a faithful image of the surface of the country, the map remains incomplete if the rise and fall of the hills, the undulations of the surface, and its whole relief be left unrepresented. It would evidently be an imperfection, that on the same plan or map the highest mountain ground and the lowest land should all appear as if on the same level. There are several methods adopted for the purpose of conveying the necessary knowledge on this point. The first and the most general is that called Hill Drawing or Hill Sketching.

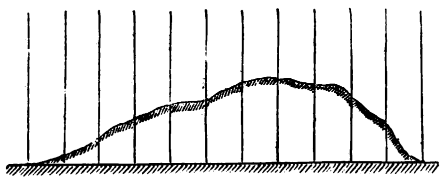
Representations of Hills by Light and Shade.

In the same manner that a certain disposition of light and shadow can convey to the mind an idea of the familiar objects around us, so can a physical representation of a hilly surface, by means of light and shade, pórtray the form and height of the hills, the depth and direction of the valleys, and the massive external characteristics of the natural stratification of the country. The only difficulty arises from this circumstance, that the representation of the natural features must be made as if seen from above, a point of sight to which the eye is not accustomed; for in the limited view which it can embrace of objects around, the effects of perspective always present the *apparent* as different from the *real* forms. Nevertheless, by attending to the principles and effects of light and shade, plans may be so shaded and finished that an inspection of them would make the observer acquainted with the relief of the ground, as it would appear if a reduced model of it were placed under his eye.*

Steepness of the Slope, an element to regulate the Shade.

The side or slope of a hill, being inclined to the horizon, receives in proportion to its surface a smaller quantity of vertical light than the summit, or than a horizontal plane at its base. A horizontal surface receives an equal portion of light with the inclined surface resting upon it,

Vertical Rays of Light.



and as the inclined surface is of greater extent, it will be darker than the horizontal, in proportion to the degrees of inclination and consequent increase of the extent of surface.

* The Ordnance Map of England (which is published on a scale of 1 inch to the mile), with a representation of the natural inequalities of the ground by means of light and shade, has proved of incalculable value to the engineers and surveyors employed to design and lay out improved lines of communication throughout the country. By a simple inspection of the map, a knowledge is obtained of the prominent ridges and of the most practicable passes through them, and the engineer can at once direct the surveyor to proceed to the minute examinations of those lines which are likely to prove available for the proposed work. Without access to such maps, numerous trials, surveys, and sections, would have to be made at considerable expense of time and labour in order to combine as much topographical information as is conveyed on the Ordnance Maps.

A sheet of white paper, bent into the form of a ridge, and placed under a vertical light, affords a simple illustration of this effect, the shade becoming darker as the inclination is increased.* From this is derived directly the principle, that in the representation of the varied forms of ground, the shade applied to the side of hills should be proportionate to the steepness of the slope. The draughtsman thus gives a physical representation of the hills, generally intelligible to all, and enabling the engineer or professional man to estimate the relative steepness of the hills from a comparison of the relative intensity of the shades.

Height also to be an element to regulate the Shade.

Height, as well as steepness, is a characteristic to be attended to in the representation of hills. On looking at a reduced model of ground, the highest point attracts the eye in the first instance; in the same manner, it should retain its prominence in its representation on the plan, and all differences of altitude should have their relative importance in the drawing. This is effected by combining height with steepness, as the two elements to regulate the depth of shade. This combination produces a correct physical representation, by imitating the effects of aërial perspective. The higher points of the country, being nearest the eye, are supposed to present a greater intensity of light and shadow, both of which diminish in degree as the surface becomes of less altitude, because of the supposed intervention, between the surface and the eye, of a greater body of the atmosphere. Through this cause the valleys are slightly shaded or tinted; thereby they cannot be confounded with the tops of the highest mountains, which alone are to be left quite white. Were

* Mitchell's *Outlines of a System of Surveying*, page 69.

this principle unattended to, the flat parts of valleys, and the level summits of mountains, being both left unshaded, could not be distinguished from one another, and the true appearance of relief would be lost.

Relative importance of features to be preserved.

In the representation of ground, all features, whether of primary or subordinate importance, whether high mountains or small ridges, should be represented as fully as the scale will admit; but each feature, as in nature itself, or in a model, should be in proper keeping, the smaller features being kept subordinate to those of greater magnitude. The importance of preserving each feature or object in its proper 'keeping' is well understood in drawing or painting when cultivated as a branch of the fine arts, and it is one of the elements most conducive to beauty and truth of representation. When unattended to, the drawing presents a harsh effect of spottiness. Until of late years it was altogether neglected in topographical drawing; high mountains, howsoever extensive their base might be, or however varied their ramifications into other subordinate forms, were represented much in the same manner, and with the same tone, as the inferior hills of the low country.

Mr. Dawson (for more than half a century connected with the Topographical Department of the Ordnance Survey of England and Wales) was the first to correct this defect, by establishing the following general principles.

1. That a plan is to be considered as a full-face portrait of a country.

2. That mountains, hills, and hollows, are to be considered as features varying the general face of the ground.

3. That every feature must be conceived and expressed

as a whole object; that is, according to its effect on the eye as a whole form.

4. That features must be drawn according to their assembled effect as a general whole.

And as to Mr. Dawson was entrusted, for many years, the instruction in hill-drawing of the officers of engineers on their leaving the Royal Military Academy at Woolwich, the conventional representation of hills, as so many isolated features, all partaking of the same character, was by degrees abandoned, and the improved system, which aims at copying nature both in the great masses and the minute details, was generally introduced.

Various Methods of Shading Hills.

Different methods of shading, and various dispositions of lines are used to give the physical effect of relief to a map.

'Vertical' Light.

Of these methods, that which supposes the light to fall vertically on the ground is well suited to give the physical representation required, and, by causing the intensity of the shade to depend on the height of the hills and the relative steepness of their sides, it is best adapted to offer a precise and uniform method of ascertaining these important characteristics.

'Side' Light.

There are numerous advocates for the representation of hills, under a supposed 'side light,' the light in that case descending obliquely at a fixed angle from one side of the map. The sides of the hills next the light receive it more or less brilliantly, according as they are inclined more or less nearly at right angles with its rays; and the shade on the sides removed from the light increases in intensity as the slopes increase in steepness. This style may be rendered most expressive by a skilful draughtsman, especially when the character and strike of the hills is favourable to the direction of the light; this is the case when the strike of the hills is at right angles or nearly so to the direction of the light, and when the steepest sides of the hills are uniformly on the shaded side. Such a disposition of the forms of ground can only obtain over a very limited space, and when the map comprehends an extensive district, the steepest sides of the hills will in some cases be opposed to the light, in others range in directions parallel to it, and in both circumstances the less inclined slopes will receive the darkest shading, and may mislead as to the character of the country. With this style of representation, the hills are generally made to partake more or less of the same character, appearing almost uniformly steepest on the sides removed from the light. It is a disadvantage inherent to this method, that by it much more scope is left to the taste of the draughtsman, and the topographical language thereby loses some part of its universality; also if, on looking at a map so drawn, the light whereby it is examined should happen to fall in a direction contrary to that in which it is assumed to proceed, the effect of relief is, to some extent, diminished; and the writing and outline necessarily introduced on the plan contribute also to mar its effect. Most beautiful drawings of the mountainous districts of Wales have been executed by Mr. Dawson for the Ordnance Survey in this style; and they could not probably be surpassed in truth of execution, or in pictorial effect and breadth of expression. There can be little doubt, however, that artists equally skilful could give as rich an effect by means of the vertical light, and if so, the latter style should seem to claim the preference; for maps so drawn may be considered as furnishing more precise data, the relative height and steepness of the hills being represented by corresponding depths of shade, and the writing and outline interfering less with the general effect.

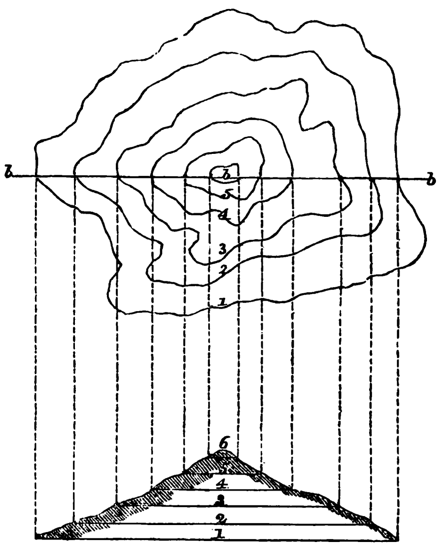
Contour Lines.

The physical effect of shading can be produced by etched lines; when this method is adopted, the lines may be drawn on the plan in the relative position that would be occupied by lines traced on the ground parallel to the horizon. Such lines are called *horizontal contours*.

Principle of Contour Lines.

If we imagine horizontal lines vertically equidistant, that is, each line separated from the adjoining lines above and below by a given constant altitude, they will, when projected orthographically on the plan, necessarily approach or recede from each other, according as the slopes are more or less steep; on ground of slight inclination they will be distant from each other and produce light shades, whereas on steep slopes they will approach closer to each other, and consequently produce a depth of shade proportionate to the steepness of the slope. If a series of such horizontal lines, all vertically equidistant, be traced from the base to the summit of a hill, they will present a correct geometrical and physical representation of the hill; and the vertical distance which separates the contours being known, a profile or section can be drawn in any direction whatever.

'To give an idea of the principle of contour lines, we may suppose a hill, or any elevation of land covered with water, and that we want to trace the course of all the levels at every 4 feet of vertical height; suppose the water to subside 4 feet at a time, and that at each subsidence the line of the water's edge is marked on the hill; when all the water is withdrawn, supposing the hill to be 24 feet high, it will be marked with a set of 6 lines, denoting the contours of each of the levels, exactly 4 feet above each



other. The marks thus supposed to be left on the hill being surveyed and drawn on the plan, would supply information more exact than a model in relief.'*

Origin of Contour Lines.

The idea of employing horizontal lines for the representation of forms suggested itself, as early as 1742, to Philippe Buache,† when observing the horizontal marks left on the land by the gradual subsidence of the waters of an inundation. But he only aimed at tracing them as lines of equal soundings on hydrographical charts. Thirty-

* *Evidence* of J. Butler Williams, C.E., *before Health of Towns Commission*, p. 448, vol. ii., 8vo., 1844.

† Girard, *Sur les Eaux Publiques de Paris*, page 141.

three years later, Ducarla proposed to adapt imaginary lines, following the same law, to the representation of the features of ground; and, in 1782, Dupain-Triel, in arranging the method into a system, recommended that the lines should generally be vertically equidistant.* However, owing to the want of precise data, and accurate outline maps, the principle was not fully carried out until of late years, when these vertically-equidistant horizontal lines, which we shall call *normal contours*, came to be determined in the field by means of the theodolite, spirit-level, or other instrument. These normal contours have been adopted in the French cadastre since 1818,† and are now also adopted in the English and Irish Survey, in which latter work their introduction dates from the year 1838.

The normal horizontal lines may be traced on the ground by means of the theodolite or the spirit-level; but before these contours can be traced, an accurate outline map of the ground must be constructed.

Practical Method of Tracing 'Contours.'

The contours may be traced thus. Ranging rods should be fixed at certain distances along the ridge or water-shed lines, and along the valleys or water-courses, and such other natural lines as best define the undulations of the surface; and the positions of these rods must be marked on the plan. A short picket being driven between two of the ranging rods at the proposed level of one of the contours to be traced, the theodolite, or, in preference, the spirit-level, is placed at a convenient distance from the

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* PUISSANT'S *Figure du Terrain*, page 81.

† *Ibid.*, page 67; also, *Instructions données en* 1818 *aux Ingénieurs Géographes par l'Adminstration du Dépôt de la Guerre.*

picket, and at such an elevation that, when adjusted horizontally, the line of sight shall intersect a levelling-staff placed on the picket. For this purpose a staff with a sliding vane must be used, as it admits of being observed at greater distances than the improved levelling-staves. The centre of the vane is, by attention to signals, brought by the staff-holder to the point of intersection of the visual ray, and fixed in that position; the staff is then taken to a point nearly on the same level between the two next ranging rods, and moved up or down the slope until the centre of the vane again coincides with the horizontal line of sight. A picket is driven at this spot, and the operation is continued round the valley or brow of the hill as far as the observer can see the staff distinctly; the position of the spirit-level is then changed, and the same horizontal line continued as far as required. The next contour, either above or below, is traced in exactly the same manner at the required vertical distance from the first. While sweeping the telescope of the spirit-level from point to point, the observer takes care to note where the visual ray intersects any object, such as the base of a house, gate, corner of a field, etc., which is marked on the plan, and he thereby obtains the elevation of an additional number of points. The positions of the others that have been observed, as marked on the ground by pickets, are fixed by measuring with the chain their distances from the ranging rods whose positions are known. Finally, the normal contours are traced by joining (by lines nearly straight, but partaking of the general roundness of the ground) the various points indicating the same elevation. It is almost needless to observe, that fixed points must be carefully levelled at different distances to serve as bench-marks for the detection of errors that would unavoidably accumulate, were contour lines to be carried great distances without checks from independent data.

Vertical Equidistance of 'Contours.'

These contours, for the sake of clearness, and to avoid unnecessary labour, are kept at a convenient vertical distance from each other. No general rule can be given for the distance to be adopted; it must vary according to the scale of the plan, and the nature of the country to be represented. If the scale be small, the vertical distance on the ground must be increased, and if the country be very mountainous and rugged, with steeper slopes to the hills than are met with in the lower cultivated grounds, the time and labour required for tracing horizontal lines, separated by small vertical distances, would be incommensurate with their practical application; such a country being usually uncultivated and barren. In such localities, therefore, the contours may be traced at wider intervals. For low cultivated lands they should be much closer.

The vertical distance adopted on the Irish Survey for the scale of 6 inches to a mile, or $\frac{1}{10560}$ of the actual linear measure, is 50 feet for the cultivated part of the country, and 100 feet for the mountainous districts. On the French Survey, the plans of which are drawn to a scale of 400 centimètres to a mètre, or $\frac{1}{20000}$ of the actual size, the vertical distance is 10 mètres. On the special plan made of Paris and its environs, to a scale of $\frac{1}{5000}$ of the actual size, the vertical distance between the contours is only 2 mètres.

The contours should run wherever the nature of the country may require, or a short portion of a contour may be interpolated. It is not necessary that they should be at equal or fixed distances above one another; the general selection of an equal distance for the same district being recommended simply because thereby greater facilities are offered for reference and for using the contours, and by the adoption of a system, errors are less likely to be introduced into the work. 'For towns built on regular natural surfaces with steep falls, I believe that contour lines, with a vertical equidistance of 4 feet, would suffice; but for flat ground, or ground very irregular in its form, or for ground liable to be flooded, 2 feet would be a good vertical equidistance. Such contours projected on a plan plotted to a scale of either 10 feet or 5 feet to a mile (the Ordnance scales for towns), would convey nearly all the information required for sanitary and structural improvements. Contour lines drawn with a vertical equidistance of 1 mètre, were found by M. Girard, engineer to the water-works of Paris, sufficient for all the purposes of the water-works; and it may be observed here, that the plan adopted for watering the streets of Paris by fountains (*bornes-fontaines*) demanded a very exact knowledge of the levels; for the site of these fountains had to be so selected, that, by placing each in a culminating point of a street, it might be made to water the greatest possible extent of surface.

'A distance of 2 feet between each contour would be sufficient to guide builders in their private drainage. But to assist them more effectually, and in general for the purpose of reference by all designers of physical and structural improvements, it would be highly desirable that, in addition to the careful registry of the altitudes on maps accessible to the public, exact indications of the levels should be inserted at convenient places on public permanent objects. The heights, all referred to the same datum, should be inscribed in feet on the public buildings, and at the corners of streets in towns; and, in the environs of towns, on mile-stones, gate-ways, or any convenient fixed object.

'Contours, thus regulated, would supply the necessary information for the most advantageous distribution of the sewerage. They would likewise suffice to guide in the improvements in the levels of streets, when any repairs should offer opportunities of diminishing irregularities in the gradients at a small cost, and they would as a matter of course, be referred to in laying down the water-pipes. Contours representing by a *combined* view all the points of elevation, would be of great value in the designing of plans, and forming estimates for new districts of towns.

For country districts, embracing an even surface or gradual swelling features, contour lines with a vertical equidistance of 4 feet would be sufficient, not only for all purposes of road, railway, or canal projects, but also for the very delicate operations of drainage and irrigation. In districts presenting swelling and rounded forms, without abruptness, but with more marked falls, contour lines separated by a vertical equidistance of 8 feet would, I believe, be sufficient: and in still more hilly districts, the contour lines might be traced even wider apart.'*

The normal contours, traced by means of an instrument, being too much separated to produce a shade capable of giving a forcible representation of the hills in relief, other lines, parallel to the normal ones, are introduced in order to constitute a shade. This part of the work may be made mathematically correct, by assuming that a given number of lines of the same thickness shall always be introduced between the normal contours, which lines by being brought close together on steep slopes, will there produce dark shades, and lighter shades on gentler slopes, being separated in the ratio of the variation of inclination. It would be extremely difficult, and it is unnecessary, to follow that law with geometrical precision. The normal contour will guide the eye and hand in giving the proper depth of shade, by means of intermediate etched lines, freely drawn parallel or nearly so to the normal contours; which contours will be sufficiently close to give a correct section of the ground,

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* Evidence of J. BUTLER WILLIAMS, C.E., *Health of Towns Commission Report*, vol. ii.

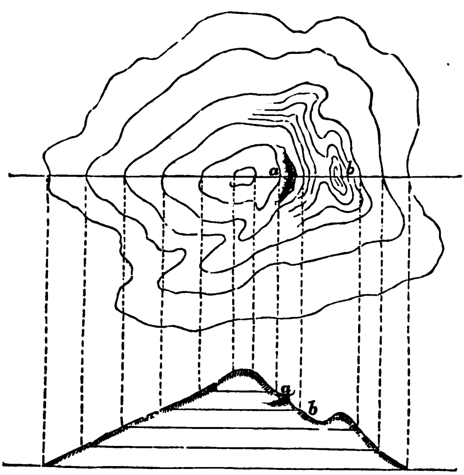
by combining the measured and *varying* horizontal distances between the contours, with the *constant* vertical height which separates them.

On the French cadastre, those who trace the normal contours in the field have instructions not to introduce any intervening horizontal lines on the plan. The physical relief is afterwards produced in the office, by using the normal contours as guides for the intensity of shade.

On the Irish Survey, the same preliminary process is followed, but with this important ulterior improvement, that the maps, with the normal contours traced upon them, are afterwards placed in the hands of field-parties, whose duty consists in filling in the details to produce physical relief, and give expression of form and character to the drawings.

To obtain accuracy in details, this last process is preferable; for the vertically equidistant contours frequently omit important intervening features, as is shown in the adjoining plan and section, wherein the rocks at *a*, and the

PLAN.

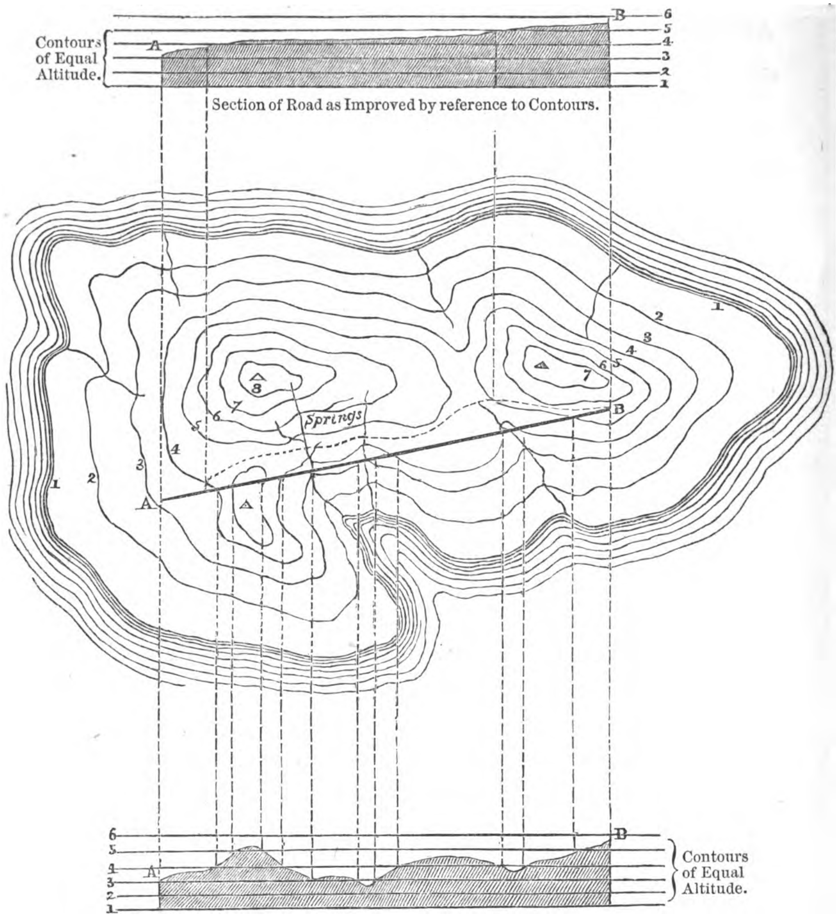


SECTION.

back fall at *b*, are left unrecorded, simply because they present themselves between the normal contours.

Use of 'Contour Lines' illustrated.

'To illustrate the use of contour lines, let a road surveyor be requested to lay out a line of road from A to B (see annexed sketch), he could, by reference to contour



Section of Straight or Unimproved Road.

lines drawn within sufficiently close limits, select such a direction as would give the most favourable *gradients,* without taking any trial levels, or even visiting the locality. He could, for example, obtain from the plan, the section or varying gradients of a road, in the direction represented on the sketch by the straight line A B, and compare its length and its gradients with those of a line of road made to deviate in such improved directions as would be indicated by the contour lines generally (as shown by the dotted line in the above sketch), increasing in a slight degree the total length of the road, but avoiding back-falls, and reducing the gradient to a minimum amount of steepness by spreading the total rise or fall almost evenly throughout the whole line of road. The saving in time, by reference to such a map, is self-evident, inasmuch as no preliminary levelling would be required, and the saving in expense could not be less than the cost for levelling from four to five times the whole length of the line of road; for it is but rarely that even a common road of considerable length could be set out in the most advantageous manner without at least half a dozen trial-sections. Such trial-sections, after they have served the immediate purpose required, become useless for any other work; whereas, levels taken for contour lines, are always ready to be referred to for any project of internal improvement. But the use of contour lines would become even more important to an engineer engaged in setting out a railway or canal. The necessity for easy gradients in the first description of work, and of level lines in the second, render deep excavations and high embankments unavoidable. The proportion of saving in the cost of these great works, that may be the result of even slight deviations, can scarcely, in a broken and varied surface of country, be over estimated. Indeed, many of the existing lines of railway in the kingdom present striking examples of unnecessary expenditure in cuttings or embankments that might have been avoided or greatly diminished, had the engineer had access to trustworthy documents, presenting, under one connected view, the relative elevations of adjoining hills and valleys.

A map with such contours accurately traced upon it, is of much greater value to the engineer than if left simply in outline, inasmuch as he can trace sections in any direction, by measuring the horizontal distances along the line of section between each pair of contours, the vertical distances being a known constant quantity.

Horizontal contours can be traced by the eye with considerable accuracy, especially when the surveyor is assisted by the altitudes obtained in the trigonometrical operations serving for the construction of the outline map. The process before described is, from its mechanical nature, slow; that which I now proceed to describe is rapid in execution, and tolerably correct for a small scale (say 1 inch or 2 inches to a mile), where experience has trained the eye to accuracy. It is well adapted for reconnoissances of a country, and it is much used by military engineers. The civil engineer would, however, frequently find the same advantage in using it in his preliminary examinations of countries for the purpose of selecting general lines of communication.

In the field, when the eye is alone depended upon, the horizontal lines are traced in pencil, by close parallel hatchings; and when the whole drawing is finished, the normal contours are traced at the required vertical distances apart, by following the general direction of the pencil lines, and checking their truth by means of the trigonometrical elevations or other heights marked on the map. The contours, when a complete circuit is made, must return to the point of departure, and if it were attempted by the eye alone to trace normal contours which are isolated from each other, no degree of previous experience would suffice for the attainment of the object.

'Vertical' Style of Hill Drawing.

The practice has long prevailed of transferring to the fair drawing or to the copper plate for engraving, not the horizontal lines as sketched in the field, but vertical lines at right angles to them, which represent the course that would be followed by water in its descent down the slope. This style of hill drawing is called the 'vertical style,' in contradistinction to the first described, which is called the 'horizontal style.' The vertical style is rarely used for the field-work; but whatever style may be adopted in the field, the maps, when engraved, have hitherto been etched in the vertical style.

It seems difficult, on first consideration, to account for this substitution of vertical lines for the horizontal lines, as originally drawn in the field, especially when it is considered that any change must diminish in the copy the value due to the original document. Also, for engineering purposes, the change is inconvenient, as the horizontal lines must again be restored by tracing them at right angles to the vertical ones; and such a change cannot be made without introducing errors. The chief cause of the alteration, seems to be that the vertical lines can be etched by the draughtsman, and more especially by the engraver, with greater facility and rapidity than the horizontal lines: it is believed, however, that, to a certain extent, the greater facility of engraving in the vertical style is due to the artists having exclusively practised in that style.

For a small scale, such as that of 1 or 2 inches to a mile, when it is intended to introduce into the plan all the details that the scale admits of, etching with the crowquill for the fair plan is to be preferred. But for plans aiming at less exactness of detail, or prepared on a larger scale (from 3 inches to a mile and upwards), washing in the tints with Indian ink is much preferable, as it admits of greater rapidity of execution. The field-work is, nevertheless, finished as before described in pencil, and normal contours are, if possible, traced and drawn with the pen; the tints of Indian ink are then laid on according to the rule of the depth of tint being proportioned to the height of the hills, and the steepness of the slopes. To soften the tints, when it is required to represent a rounded form, two hair pencils are used, one as the colour brush, the other as the water brush. The shades are laid on with the colour brush, and softened by passing the water brush rapidly along their edges. The water brush should not have much water, as it would in that case lighten the shadow to a greater extent than is intended, and leave a ragged harsh edge. Tints may be rounded without softening the edges with the water brush, by using very light colour, and applying one tint over another, with the boundary of the upper tint not reaching to the extreme limit of the tint below it;—a beautiful effect of clearness and transparency is by this means given to the drawing. Also, when depth of shade is required, it is best produced by the application of several light tints in succession; for when the full depth is given by a single wash, its effect is rough and opaque. No tint is to be laid over another until the first is dry; and a little indigo mixed with the Indian ink improves its colour and adds to the richness of effect.

The roads must be left either untinted or lighter than the adjacent shade, in order that they may be easily seen on the map. When woods have to be represented, the shading used for the trees, instead of interfering with the shadows due to the slopes, may be made to harmonize with them, and to contribute to the general effect by presenting greater or less depth, according to the position of the woods on the sides or summit of the hills.

CHAPTER IX.

MINING SURVEYS.

A MINING plan is the chronicle wherein are recorded all geological and mineralogical data connected with the mining district. It serves also as a guide by which new workings may be directed; and if made with strict geometrical accuracy, it saves one heavy item of expense in all subterraneous works, namely, that arising from lost labour in driving false headings. It guards also against the occasional destruction of human life by unexpectedly opening into a gallery, or cutting into a protecting dyke, supposed by the indications of an inaccurate plan to have been at a greater distance.

The want of subterranean plans, constructed with as great a degree of geometrical precision as those representing the surface of the ground, has been long felt by scientific and practical men. With a very few exceptions, no correct or trustworthy record of subterranean works are preserved in any of the important mining districts of Great Britain; and so strong is the impression of too general neglect in this particular, that the highest authorities in mining engineering have recommended, 'that in future leases of mines, the proprietors should introduce a clause to require the adventurers to keep sections and plans of all their workings.' But, manifestly injurious as the general inexactness of mining plans is acknowledged to be, the erroneous impression continues to prevail, that mining surveys can be performed with instruments incomparably less accurate, and with precautions much less stringent than those which are now deemed indispensable to the perfect success of surveys on the surface. It is further to be observed, that surveys on the surface seldom present obstacles equal to those of subterraneous works, in many of which, the difficulty of access, and the great irregularity and varied ramifications of the levels, demand all the care and skill which the experienced surveyor can command.

It may be said to be the universal practice, at present, to perform mining surveys by means of the mariner's or miner's compass, for the observation of horizontal angles. No strictly accurate surveys can be performed by such means, and the following are among the causes of error inherent in the present system.

First. Angles cannot be measured nearer to the truth than from a $\frac{1}{4}$ to $\frac{1}{8}$ a degree with the miner's compass, the diameter of which is rarely large enough to admit of a more minute subdivision of the circumference,—but especially because the mode of action of the needle does not admit of the application of a vernier to the circumference.

Secondly. The horizontal or azimuthal angles, indicated by the magnetic needle, cannot be read with precision, because the needle is seldom parallel to the plane of the instrument, but has either its north or south pole raised some distance from the plane. Unless then, in viewing the needle, the eye be kept exactly in the vertical plane, passing through its longitudinal axis, the index will not be seen projected upon its proper place on the circle; and as we evidently cannot, in reading the angle, always be certain that the eye is placed in the required position, we are frequently liable to refer the index to the wrong division, or incorrectly to appreciate its apparent or parallactic distance from the right division.

Thirdly. The diurnal variations of the needle are too important to be neglected, as they sometimes amount to $\frac{1}{8}$ a degree in an interval of twelve hours. The use of the compass necessitates, therefore, an inquiry into, and a record to be made of, the variations of the needle at different periods of each day's work, as well as on consecutive days. For in the very same locality, variations of 1° have been observed in an interval of fourteen days, and of 1¾° in an interval of six months.*

Lastly. The use of the common miner's compass is evidently inapplicable to mines of such minerals as are possessed of undoubted magnetic properties,—and independently even of these, the needle may be made to deflect through local attractions, among which might perhaps be included the electro-magnetic currents to which Mr. R. W. Fox is inclined to trace the existence of metallic veins. Above all, the powerful influence of an iron railroad, a most common requisite in all mines, has long been ascertained; so great is its effect, that the needle, when held at the joint between two rails, will immediately place itself in a direction parallel to their length.† This last cause of error may probably be destroyed by using an improved compass needle, a late invention of Mr. James Ramsden, intended to obviate the derangement of the nativity of the needle, caused by the presence of the iron rails, and various metallic substances. The improved needle is thus described in No.293 of the *Mining Journal*, for 1841. 'It consists of three parts, viz., two bars of steel, each 1½ inch long, and each having a north and south pole, divided in the middle by a brass bar, which separates the north pole of one bar from the south pole of the other by about 1½ inch. The affinity of one of these for the other is so strong that the polarity of the needle as a whole is maintained.'

The above remarks amply testify the insufficiency of the miner's compass for the performance of accurate underground surveys, and to its inherent imperfections are added

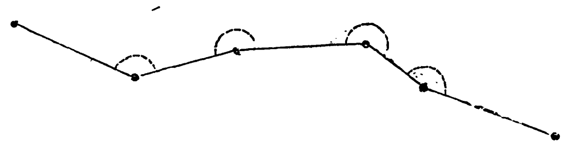
* *Annales des Mines*, tome ix., 1835, page 88.

† *Ibid.*, tome ix., 1836, page 99.

those of the instruments for measuring the angles of inclination.

The quadrantal or semicircular instruments used for this purpose are applied as follows:—A fine silken cord or brass wire is tightly stretched in the direction of the line to be measured: to this wire the diameter or straight side of the instrument is applied, and the angle of inclination is denoted on the divided arc by a plummet depending from its centre. If the distance be so long as to cause a sensible amount of deflection or curvature in the wire, the instrument is applied at two or three intermediate stations, in order to counteract the effect of the curvature by combining the several readings. The instrument thus acting without the aid of a telescope directed to a well-defined fixed point, can give only an approximation to the truth.

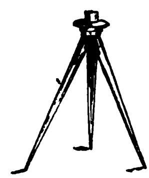
Finally, the common method of protracting the work on the plan, by causing each succeeding bearing to embrace



and confirm all the antecedent errors of observation and plotting, can lay no claim to accuracy.

This last description of errors is not, it must be observed, inherent in the principle of surveying with the compass; for the method of plotting by reference to three normal co-ordinates, as we shall presently explain, is equally applicable to the system of observation just described as to that which it is proposed to substitute for it, namely observation with the theodolite. The substitution of the theodolite is recommended on the grounds of incomparably greater accuracy, and also of greater rapidity of execution, inasmuch as it avoids the cumbrous method of taking the angles of inclination as above described.

The common theodolite, in order to adapt it to mining observations, requires a slight modification in the construction of the stand, whereby it may be easily disengaged from the staff for the purpose of placing it on any support or bearing afforded by a locality too confined to admit of the common stand being used. The inferior parallel plate should have in its middle part a hollow socket of several inches in depth, and slightly conical. This is intended to receive a brass projection attached to the stand which serves the same purpose as the screw of the staff head in the ordinary construction. To proceed with the survey of a mine, if its adits or galleries be not too steep and irregular, 3 tripod stands are provided similar to those used in common field-work, but shorter, and having, instead of the common staff head, a flat broad head (with 2 small spirit levels at right angles to each other), on which can be screwed, when required, a brass pivot made slightly conical, so as to fit into the hollow socket of the lower parallel plate, which is made fast to it by a friction screw.



If the galleries be too steep and rough to admit of the steady adjustment of such tripods, wooden supports may be substituted of the annexed form, armed with spikes or

Side view.

Plan.

cramps to fasten them to the polling boards or shores on the sides of the galleries.

Lamps are used as the objects to be observed: they are made so that the focus of the light shall be at the same elevation, when placed on one of the tripods or rests, as the axis of the theodolite when fixed in the same place. A hollow socket beneath the lamp, provided also with a friction screw, fits on the projecting brass stand of the tripods or rests.

To carry forward a survey with due correctness and expedition, the surveyor must have thc assistance of two chainmen, and two labourers to carry the instruments and fix the forward lamp. A lamp is placed at the starting point from which the chaining commences, and the surveyor goes forward with the theodolite, in the direction of the line to be surveyed, to the farthest point from which he can see the light of the lamp back. One of the tripod stands is then placed at that point to receive the theodolite, and the telescope is directed towards the light, the vernier being fixed at zero; the angle of inclination is also noted. During the time occupied in this operation, one of the assistants has moved forward with a lamp and a tripod stand, which he fixes at the farthest point at which his light can be seen from the instrument. The forward angle is then read on the horizontal limb, and the angle of inclination having also been noted, the instrument is taken forward to the stand on which the second light was placed, and the lamp is itself removed to serve for a second forward station, while the first light is brought forward to the first position of the theodolite, and the observations are continued as before. The chainmen measure the distances at the same time that the instrument is moved forward by the surveyor, and he enters all the observations in the field-book according to the following form.

No. of hypo-thenuse.	Measured distances in feet.	Angles of inclination.	Readings of horizontal limb. Back sight.	Readings of horizontal limb. Fore sight.	Horizontal angles.	Remarks.
		° " '	° '	° '	° '	
1	133	depr. 2 30 0 elev. 4 29 30	0 0	189 16	189 16	
2	150	depr. 4 29 30 elev. 7 32 0	189 16	357 29	168 13	
3	302	depr. 7 31 30 elev. 15 20 0	357 29	178 53	181 24	
4	220	depr. 15 20 0 elev. 0 30 0	178 53	357 41	178 48	

The operation above described will be found to be similar to that practised for road or town surveys, and known by the name of 'traversing.' However, the operation as conducted in subterraneous surveying, presents certain advantages not possessed by that usually carried on on the surface. First, the mode of observing with three fixed supports so contrived that the observed object and the axis of the instrument always occupy in succession the same position, prevents all inaccuracies from eccentricity of the instrument or the object staves. This eccentricity, in the common method of traversing on the surface, forms a serious and practically unavoidable element of error, for the axis of the theodolite being fixed in position by means of a plummet suspended over a picket of a certain diameter, cannot with precision be placed truly in the axis marking the centre of the object-staff just removed. This defect would be entirely obviated by using, as recommended for subterraneous surveys, three tripod stands.

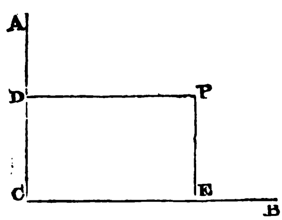
Secondly, the superior distinctness and precision afforded by a small bright light used as a point of sight in mining surveys, is well appreciated by all who have had the opportunity of comparing it with the intersection of the centre of a fixed vane placed across the staff, used as the object of sight in surveys on the surface.

Lastly, in subterraneous surveying, the surveyor is protected from all unsteadiness in the instrument, consequent on the agitation of the atmosphere.

From these remarks it may be inferred, that, as 'traversing' is a process mathematically correct in theory, although to a certain extent erroneous in practice, the mining surveyor has the means of approaching more nearly to geometrical precision, by following the practical directions given above.

The work may be plotted at once from the field-book, by means of a circular protractor, as described under the head of 'traversing,' page 150. That method, by referring all the angles to a common meridian, presents the advantage, that a trifling error in the *direction* of one of the lines does not affect the *direction* of the lines succeeding. But it counteracts the tendency to accumulation of errors only as regards the *direction* of the lines, and not as regards their length, errors in which may continue to accumulate through each step of the plotting. By carrying out the principle to its full extent, and referring the lengths of the lines as well as their directions to common meridians or co-ordinates, this remaining liability to error in the plotting is obviated.

The position of a point in a *plane* can be determined by the length of two straight lines drawn from it parallel to, and terminated by, two lines given in position, and perpendicular to each other.

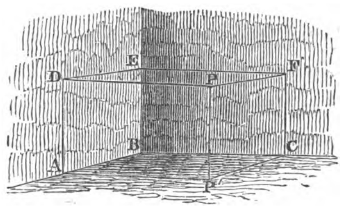


Thus, if the two straight lines, A C, C B, perpendicular to each other, and intersecting in C, be given in position, and P be any point in the same plane from which P D and P E are drawn parallel to B C and A C, if P D or C E, and P E be given, it is evident that the position of P would be determined. It is in this manner that offsets from a main or station-line determine the position of side objects in a survey, or offsets from a datum line determine the section of a line of country in plotting the levels.

Again, the position of a point in *space* can be determined by the length of three straight lines, formed by the mutual intersection of three planes at right angles to each other.

Thus, to determine the position in space of the point

P, it is first referred to a horizontal plane A B C given in position, by a perpendicular P P′ drawn to that plane. The point P′ is then determined, as shown above, by two perpendiculars, P′ A, P′ C, drawn to the lines A B, B C, given in position. This



is evidently the same operation as referring the point P to the three planes, A B C, A B E, C B E, perpendicular to each other, for the lines P′ A, P′ C, are evidently equal to the lines P D and P F respectively. The lines drawn from the point P to the three planes fixed in position are called co-ordinates. These three rectangular planes, by a reference to which the position of any point can be determined, may be compared to the floor and two of the side walls of a room.* Perpendiculars drawn from the point to the walls and floor would intersect them in points called its projections. This is the method universally employed by engineers and architects to convey a representation of the different parts of a proposed edifice, —the projections on the three planes being denominated *plans*, *elevations*, and *sections*. The plan is the projection of the parts of the edifice on the horizontal plane, the elevation and section are the projections of the parts of the edifice on two vertical planes at right angles to each other.

The terms of altitude, latitude, and longitude, are analogous to the three lines or co-ordinates drawn to three planes fixed in position.

It is proposed, then, to apply this universal method of determining the position of points in space to the plotting of mining surveys. For that purpose the three planes are

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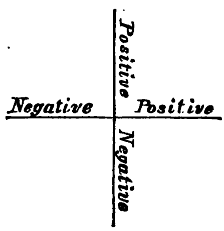
* Lacroix, *Complément des Elémens de Géométrie.*

supposed to intersect in the starting point of the survey, one of them being horizontal, the others vertical and perpendicular to each other. The measured lengths, together with the angles of inclination, determine the position of each point with respect to the horizontal plane; in other words, its altitude. This co-ordinate altitude is said to be positive when it lies above the horizontal plane; negative when it extends below it.

The measured lengths with the horizontal angles determine the position of each point with reference to the vertical planes, one of which may be called the plane of the meridian, the other the parallel of the latitude. When a co-ordinate longitude lies to the right of the meridian it is positive, when it lies to the left it is negative. When a co-ordinate latitude lies above, or, as it were, to the north of the normal parallel of latitude, it is positive; when below it, or, as it were, to the south of it, it is negative.



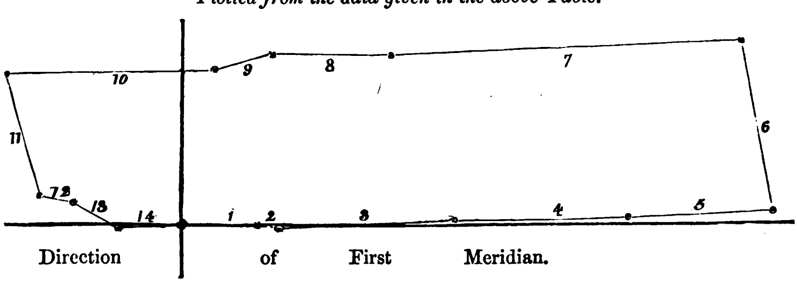
The solution of right-angled triangles — a process quickly performed by means of logarithmic tables — determines the position of all the points from the data given in the field-book. The steps in the process are best disposed according to the form given in pages 204, 205, in which the distances of each station from the three planes are registered independently of each other, and can be plotted in the same manner as a section. The additional time occupied in the calculation is, in some measure, saved in the plotting, which is much more expeditious when performed by means of two rectangular co-ordinates than by the use of the protractor.

When new galleries have to be conducted in connexion with, or having a reference to, old workings, it will be more accurate to set out their directions from data obtained from the table of registered lengths than to measure them from the plan.

Underground surveys require to be referred to, and connected with, surveys on the surface. When the connexion can be made by adits, it is effected by traversing out of the adit to the surface: when it must be accomplished through a narrow shaft, the bearing of the first line, used in the underground survey, with the magnetic meridian is ascertained with the greatest care, and in order to detect the existence of any local cause of attraction, many observations should be made under different circumstances, and at different parts of the line of which the bearing is required. As soon as this has been satisfactorily determined, a line with the same bearing is set out at the mouth of the shaft, the surveyor using the same precautions to guard against local or accidental deflexion of the magnetic needle. The direction of the first line being thus marked on the surface, the other lines which have been traversed underground are set out, and marked by pickets on the surface, from the data given in the field-book.

No. of Hypothenuse.	Angles of inclination. Elevation.	Angles of inclination. Depression.	Measured distances.	Horizontal angles.	Angles of direction with reference to plane of the meridian. +	Angles of direction with reference to plane of the meridian. —	Logarithms of measured distances.	Logarithms of cosines of angles of inclination.	Logarithms of sines of angles of inclination.	Logarithms of sines of angles of direction.	Logarithms of cosines of angles of direction.
	°	° ′		° ′	° ′	° ′					
1		0 38	15·60		0 0		1·1931246	9·9999735	8·0435009	0·	10·
2		5 9	5·55	189 16	9 16		0·7442930	9·9982433	8·9530996	9·2069059	9·9942950
3		4 16	36·70	168 13		2 31	1·5646661	9·9987947	8·8715646	8·6425634	9·9995809
4		1 33	36·70	181 24		1 7	1·5646661	9·9998411	8·4321561	8·2897734	9·9999175
5	0 5		30·60	178 48		2 19	1·4857214	9·9999994	7·1626960	8·6066226	9·9996449
6	0 30		36·50	82 34		99 45 / 80 15 sup.	1·5622929	9·9999835	7·9408419	9·9936813	9·2287839
7	0 16		73·60	97 21	177 36 / 2 24 sup.		1·8668778	9·9999953	7·6678445	8·6219616	9·9996189
8	0 38		25·45	182 39		179 45 / 0 15 sup.	1·4056878	9·9999735	8·0435009	7.6398160	9·9999959
9	6 22		14·00	165 25	165 40 / 14 20 sup.		1·1461280	9·9973132	9·0448954	9·3936852	9·9862663
10	4 36		34·05	192 58	178 38 / 1 22 sup.		1·5321171	9·9985988	8·9041685	8·3774988	9·9998764
11	0 38		25·80	96 21	94 59 / 85 1 sup.		1·4116197	9·9999735	8·0435009	9·9983553	8·9388496
12	0 37		6·95	94 32	9 31		0·8419848	9·9999748	8·0319195	9·2183635	9.9939815
13		3 26	10·10	203 1	32 32		1·0043214	9·9992198	9·7773334	9·7306129	9·9258681
14		1 59	14·70	143 45		3 43	1·1673173	9·9997396	8·5391863	8·8117264	9·9990856

Plotted from the data given in the above Table.



Logarithms of measured distances reduced to horizontal base.	Logarithms of the distances of each station from three planes passing through each station.			Distances obtained from Logarithms in preceding Columns.						Distances of each Station from principal Planes passing through First Station.					
	Log. distances from			Distances from						Distances from					
Hypothenuse × cosine of angle of inclination.	Horizontal plane = hyp. × sin. of angle of inclination.	Meridian plane = horiz. dist. × sine of angle of direction.	Latitude plane = horiz. dist. × cosine of angle of direction.	Horizontal Plane.		Plane of Meridian.		Plane of Latitude.		Horizontal Plane.		Plane of Meridian.		Plane of Parallel.	
				+	−	+	−	+	−	+	−	+	−	+	−
1·1930981	$\bar{1}$·9494422	0·	1·1930981		0·17			15·60			0·17			15·60	
0·7425363	$\bar{1}$·6973926	$\bar{1}$·9494422	0·7368313		0·50	0·89		5·45			0·67	0·89		21·05	
1·5634608	0·4362307	0·2060242	1·5630417		2·73		1·61	35·56			3·40		0·72	57·61	
1·5645072	$\bar{1}$·9968222	$\bar{1}$·8542806	1·5644247		0·99		0·71	36·68			4·39		1·43	94·29	
1·4857209	$\bar{2}$·6484174	0·0923435	1·4853658	0·04			1·24	30·57			4·35		2·67	124·86	
1·5622764	$\bar{1}$·5031348	1·5559577	0·7910603	0·32			35·97		6·18		4·03		38·64	118·68	
1·8668731	$\bar{1}$·5347223	0·4888347	1·8664920	0·34		3·08			73·70		3·69		35·56	44·98	
1·4056613	$\bar{1}$·4491887	$\bar{1}$·0454773	1·4056572	0·28			0·11		25·45		3·41		35·67	19·53	
1.1434412	0·1910234	0·5371264	1·1297075	1·55		3·44			13·48		1·86		32·23	6·05	
1·5307159	0·4362856	$\bar{1}$·9082147	1·5305923	2.73		0·81			33·93	0·87			31·41		37·88
1·4115932	$\bar{1}$·4551206	1·4999485	0·3504428	0·29		25·70			2·24	1·16			5·72		30·12
0·8419596	$\bar{2}$·87390[illegible]	[illegible]3231	0·8359411	0·07		1·15		6·85		1·23			4·57		23·27
1·0035412	[illegible]	[illegible]541	0·9294093		0·60	5·42		8·50		0·63		0·85			14·77
1·1670571	$\bar{1}$·[illegible]	[illegible]35	1·1661427		0·51		0·95	14·66		0·12			0·10		0·11

[illegible] this series of observations, a polygon has been desc[illegible]d, the last measurement closing the work by return[illegible]g to the station of departure. This example is taken from an excellent Essay on Subterraneous Surveying, published by M. Combes, in the ninth volume of the *Annales des Mines,* third series; in which the application of this method of referring the data to three co-ordinates, which was first recommended by M. D'Aubuisson, is strongly advocated.

CHAPTER X.

OF LATITUDE AND LONGITUDE.

The Sextant.

WHEN it is required to measure the angular distance between two objects, the observer being himself on an unstable basis (at sea for example), the theodolite cannot be used for the purpose, because the principle of its construction, and the nature of its adjustments, demand that it should rest on a firm support. If, moreover, it be required to measure with perfect accuracy, at any specified moment of time, the angular distance between two objects in motion, this cannot be accomplished by the theodolite, because the telescope must be directed in succession from one object to the other, and a certain amount of time must necessarily elapse between the first and second sight.

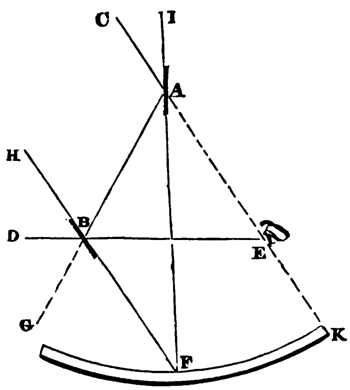
The sextant (a modification of Halley's quadrant, so called from its reputed inventor, though the merit of the invention seems due to Newton*), obviates these difficulties, as it may be used when held simply in the hand, and gives, by a single sight or observation, the angular distance between two objects. Its principle and construction are as follows:—

Principle of the Sextant. Let A and B be two mirrors, moveable on axes parallel to each other, the second mirror, B, being half silvered to admit of the passage of rays of light through half its area.

Let a ray of light, proceeding from the object, C, be

* Sir J. F. W. Herschell's *Astronomy*, and Hutton's *Mathematical Dictionary*.

reflected from the mirror, A, and after a second reflexion from the half-silvered glass, B, enter the eye at E; also let a second object, D, be seen by direct vision through the half-silvered glass, B, required the angle subtended at E by the objects, C and D.



Produce the plane of the mirrors until they shall intersect in F, the angle A F B = ½ angle C E D. For, producing A B to G, we have

$$GBE = BAE + AEB, \text{ (Euc. I. 16)}.$$

But, because the angle of incidence is equal to the angle of reflexion,

$$HBA = FBE = GBF, \text{ (Euc. I. 15)}.$$

$$\text{therefore } GBE = 2\,GBF = BAE + AEB;$$

$$\text{but } 2\,GBF = 2\,BAF + 2\,AFB \text{ (Euc. I. 16)},$$

$$\text{therefore } BAE + AEB = 2\,BAF + 2\,AFB.$$

But, because

$$CAI = BAF = FAE,$$

$$BAE = 2\,BAF,$$

taking equals from equals, we have

$$AEB \text{ (or } CED) = 2\,AFB.^*$$

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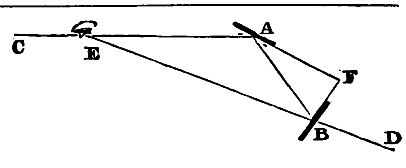
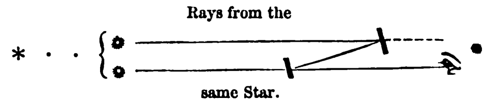
* If the mirrors were placed at such an angle that the image of the object, C, would be reflected to the eye at E, then we have



$$CEB = EBA + EAB, \text{ but}$$
$$EBA = 180^\circ - 2\,ABF, \text{ and}$$
$$EAB = 180^\circ - 2\,BAF, \text{ therefore}$$
$$CEB = 360^\circ - (2\,ABF + 2\,BAF), \text{ but}$$
$$2\,AFB = 360^\circ - (2\,ABF + 2\,BAF), \text{ therefore}$$
$$CEB = 2\,AFB.$$

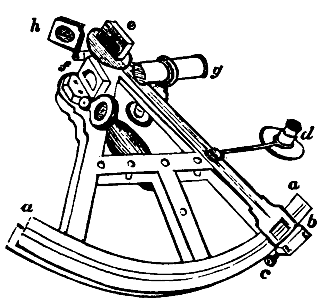
If the two mirrors be parallel to each other, and the angle formed by their planes equal therefore to zero, then



a distant object seen by direct vision, and its reflected image will appear to coincide; the parallax of the two mirrors, or their separation, not being sufficiently great to render the deviation from parallelism between the direct and reflected rays sensible.

If to the mirror A (see fig. page 207), moveable round an axis perpendicular to the plane A F B G, an index, A F, be attached, passing along a graduated arc, F K, and if the mirror B be fixed, with its plane parallel to C K and to the axis of the mirror A, then the angle F A K = angle A F B = ½ angle C E D. If the arc, D K, therefore, being equal to one-sixth of the circumference or 60°, be divided as if it were equal to 120°, the number of degrees marked, as included in the arc, F K, will be equal to the angle C E D. This theorem regulates the dividing of the arc of the Sextant.

Construction of the Sextant. The annexed figure represents a sextant so framed as not to be liable to bend. The

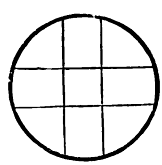


a Graduated arc.
b Vernier.
c Tangent-screw.
d Microscope.
e Mirror, or Index-glass.
f Half-silvered glass.
g Eye-tube.
h Coloured glasses.

arc is generally graduated to 10′ of a degree, which are subdivided by the vernier into 10″, a space equal to 59 divisions on the arc being subdivided on the vernier into

60 parts. An arm carrying the vernier or index moves round an axis placed at the centre of the circle, of which the graduated limb forms an arc. Over this axis or centre of motion, and attached to the arm, a mirror is fixed perpendicular to the plane of the instrument, so that a movement given to the index is communicated to the mirror. The index is clamped and adjusted by the usual clamp and tangent screws. A second glass, half silvered to admit of direct and reflected vision, is attached to the frame, nearly opposite the first mirror, and with its plane perpendicular to the plane of the instrument. The zero of graduation on the limb is placed so that the vernier shall indicate zero when the two mirrors are parallel to each other.

Objects are observed with this instrument, either through a plain tube, or through a telescope. It is better for the learner to use the plain tube, owing to the increased difficulty of bringing the objects into the field of view when the telescope is employed. The field of the telescope contains wires, as shown in the annexed diagram.



Dark glasses, of different depths of shade and colour, are attached to the instrument, to be used when the sun is observed, so as to moderate the intensity of the light.

ADJUSTMENTS OF THE SEXTANT.*

'The requisite adjustments are the following:—

'1st. The index and horizon-glasses must be perpendicular to the plane of the instrument;

'2nd. The planes must be parallel to each other when the index division of the vernier is at 0° on the arc;

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* SIMMS on *Mathematical Instruments.*

'3rd. The optical axis of the telescope (when used) must be parallel to the plane of the instrument.'

To Adjust the Index-glass. Move the index forward to about the middle of the limb, then, holding the instrument horizontally, with the divided limb from the observer, and the index-glass to the eye, look obliquely down the glass, so as to see the circular arc, by direct view and by reflection, in the glass at the same time; if they appear as one continued arc of a circle, the index-glass is in adjustment. If it requires correcting, the arc will appear broken where the reflected and direct parts of the limb meet. This, in a well-made instrument, is seldom the case, unless the sextant has been exposed to rough treatment. As the glass is in the first instance set right by the maker, and fixed in its place, its position is not liable to alter, therefore no direct means are supplied for its adjustment in the best instruments.

'*To examine the half-silvered glass, and set it perpendicular to the Plane of the Sextant.* The position of this glass is known to be right, when, by a sweep of the index, the reflected image of an object passes exactly over or covers its image as seen directly; and any error is easily rectified by turning the small screw at the lower end of the frame of the glass.

'*To examine the Parallelism of the Planes of the two Glasses, when the Index is set to Zero.* This is easily ascertained; for, after setting the zero on the index to zero on the limb, if you direct your view to some distant object, the sun for instance, you will see, by giving a slight movement to the instrument, that the two images (one seen by direct vision through the unsilvered part of the horizon-glass, and the other reflected from the silvered part) coincide or appear as one, if the glasses are correctly parallel to each other; but if the two images do not coincide, the quantity of their deviation constitutes what is called the index error. Supposing the index to point to the left of the zero, when the two images coincide exactly, the angle read will be too great by the distance between the index and the zero of the arc; for at the point where the index is when the two images coincide, ought to commence the graduation. For the same reason the angle read will be too small if the index be outside the arc when the images coincide. The effect of this error on an angle measured by the instrument is exactly equal to the error itself: therefore, in modern instruments, there are seldom any means applied for its correction, it being considered preferable to determine its amount previous to observing or immediately after, and apply the correction with its proper sign to each observation. The amount of the index error may be found in the following manner: clamp the index at about 30′ to the left of zero, and looking towards the sun, the two images will appear either nearly in contact or overlapping each other; then perfect the contact, by moving the tangent-screw, and call the minutes and seconds denoted by the vernier, the reading *on* or *within* the arc, that is the reading within the graduation. Next place the index about the same quantity to the right of zero, or on the arc of excess, and make the contact of the two images perfect as before, and call the minutes and seconds on the arc of excess the reading *off* or *without* the arc; and half the difference of these numbers is the index error; additive when the reading on the arc of excess is greater than that on the limb, and subtractive when the contrary is the case.

Example.—Reading on the arc . .	31′	56″
,, off the arc . .	31	22
Difference	0	34
Index error . . = –	0	17

In this case the reading on the arc being greater than that on the arc of excess, the index error, = − 17″, must be subtracted from all observations taken with the instrument, until it be found, by a similar process, that the index error has altered.'

'*To make the Line of Collimation of the Telescope parallel to the Plane of the Sextant.* This is known to be correct, when the sun and moon, having a distance of 90° or more, are brought into contact just at the wire of the telescope which is nearest the plane of the sextant (see diagram, page 209), fixing the index, and altering the position of the instrument to make the objects appear on the other wire; if the contact still remains perfect, the axis of the telescope is in proper adjustment; if not, it must be altered by moving the two screws which fasten, to the up-and-down piece, the collar into which the telescope screws. This adjustment is not very liable to be deranged.'

Of the Use of the Sextant.

The large sextant is rarely required for observations (on land) of terrestrial or of celestial bodies; but for purposes of navigation or for maritime surveying it is of essential importance. Instruments, in fact, constructed on this principle are the only instruments capable of being used on ship-board for determining altitudes, or measuring the angular distances of objects.

Mode of Holding the Instrument.

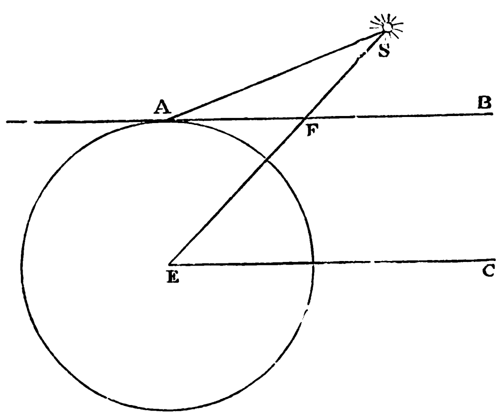
The instrument is held lightly in the right hand, and moved until its face is in the plane passing through the eye and the two objects of which the angular distance is required. When altitudes, therefore, are to be observed, the instrument is held in a vertical plane; when horizontal or oblique angles are to be measured, it is held in a horizontal or oblique plane.

Mode of Observing with the Sextant.

When the altitude of an object (the sun, for instance) is to be observed at sea, where no level or artificial horizon can be used, the observer having turned down one or more of the dark glasses, according to the brilliancy of the object, holds the instrument in a vertical plane passing through the sun, having the sea horizon before him. Directing his sight to that part of the horizon immediately beneath the sun, he then with the left hand lightly slides the index forward, until the image of the sun, reflected from the index-glass, appears in contact with the horizon, seen through the unsilvered part of the horizon-glass. He then clamps the index firm, and turns the tangent-screw carefully and lightly, to make the contact of the upper or lower limb of the sun and the horizon perfect, when it will appear a tangent to the circular disc. The angle read off, corrected from the index error, if any, is the observed altitude. If, instead, the angle between two terrestrial objects be required, the observer holds the instrument steadily in a plane passing through the objects and the eye, and directing his sight to the object on his left, he slides, with the left hand, the index forward until the two objects are brought nearly to coincide. He then clamps the index as before, and by the tangent screw renders the contact perfect. When the angular distance is thus observed, between terrestrial objects, for surveying or similar purposes, the only correction required when the objects are at the same level is that for the index error, if any. When the objects are at different elevations, it may be requisite further to apply the reduction to the horizontal angle as explained in page 84. When the altitude of the sun, however, has been observed, there are certain other corrections required to obtain the true altitude before the observation can be made available for calculation or other purposes. These corrections are as follows:

To or from the angle read add or subtract the sun's semidiameter, as given in the *Nautical Almanac*,* according as the lower or upper limb is observed, to obtain the apparent altitude of the sun's centre. Before we can use this observation for determining the time, the latitude, etc., it must be further corrected for refraction and for parallax caused by the distance of the observer from the earth's centre, to obtain the true altitude, subtracting the former and adding the latter; and when the sea horizon is employed, a quantity must also be subtracted for the dip, which is unnecessary when the altitude is taken by means of an artificial horizon.

The correction for parallax is always additive. Let E be the earth's centre, A the place of observation on the



earth's surface, S the sun, A B the artificial horizon, E C the true horizon, then the parallax is the difference be-

* The *Nautical Almanac* is published annually, by order of the Lords Commissioners of the Admiralty, generally four years forward; in it are entered the sun's longitude, right ascension, declination; the planets' longitudes, latitudes, times of passing the meridian; the times of solar and lunar eclipses, together with those of Jupiter's satellites; the distance of the moon from the sun and certain fixed stars at the beginning of every third hour; and in general the times when any remarkable appearances take place, being all computed for Greenwich time.

tween the angles S A B and S E C. The latter is the angle required, and is greater than the observed angle, for F E C = E F A, A B, and E C being parallel,

But E F A = F A S + F S A, therefore

S E C = E F A > S A B.

Consequently the correction for parallax, or for reduction of the observations to the earth's centre, must be additive.

'Tables for obtaining the above corrections may be found in MR. BAILEY'S *Astronomical Tables*, etc., in the ***Requisite Tables***, or in any modern work on navigation.

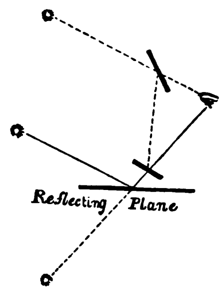
'*Example* (*from an Observation on Shipboard*).

Obs. alt. of the sun's lower limb (July)		=	61°	13′	5″
Index error		=	−		17
Apparent altitude		=	61	12	48
Sun's semidiameter (*see Table, p.* 288).		=	+	15	46
Sun's parallax (*see Table, p.* 288) . .		=	+	0	4
Refraction (*see Table, p.* 287)	32″		61	28	38
Dip of the horizon for an elevation of 18 feet .	−4 3·0	=	−	4	35
True altitude of the sun's centre .		=	61	24	3

When a 'lunar distance,' i.e., the distance between the sun and moon, or between the moon and a fixed star or planet, is required, the instrument is held in the plane passing through the eye and the two objects, the fainter object being observed by direct, the brighter by reflected vision.

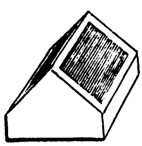
The inconvenience of measuring with the sextant angles between terrestrial objects, whose horizontal distance is that which is generally required, is, that the angles, being measured in planes parallel to the plane in which the eye and the two objects are situated, have to be reduced to their horizontal value, as explained in page 84.

When the altitude of a celestial object is to be taken on land with the sextant, an artificial horizon is used. Of these there are various constructions, all of which aim at presenting a reflecting plane parallel to the natural horizon, from which the rays of the celestial object may be reflected to an eye placed in the direction of the rays of reflexion. The angle measured in such a case is double the altitude of the object above the true horizon.



Fluid Artificial Horizon.

Among the various fluids used for the purpose of presenting such a reflecting surface, mercury has been found most useful. But during the observations its surface must be protected from agitation by the external air; for this purpose a roof-shaped cover is placed over the trough in which the mercury is contained, two plates of glass being fixed in the sides. These plates of glass should have perfectly parallel faces to avoid irregular refraction; but as this cannot be ensured exactly, two observations ought always to be made with the roof in reversed positions, in order to correct any error occasioned by undue refraction.

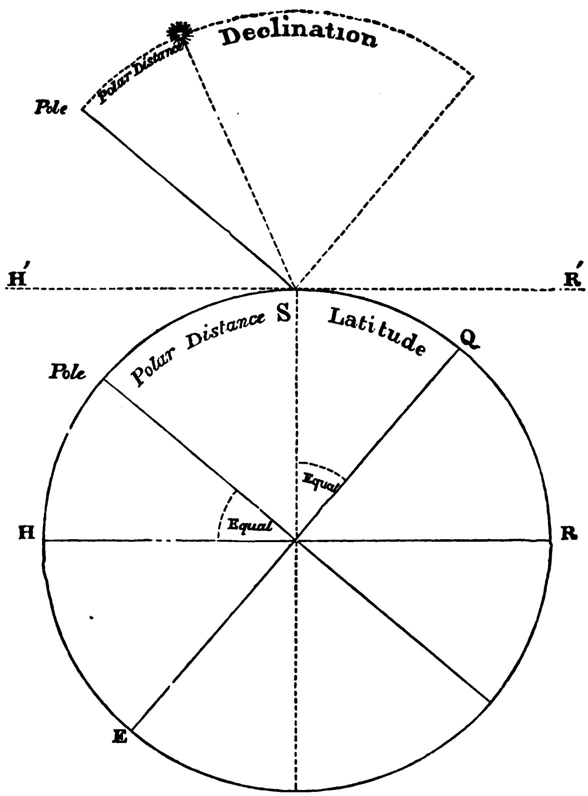


Mirror for an Artificial Horizon.

The mercurial reflector thus obtained presents an accurately level surface, but is frequently inconvenient and necessarily slow in its application. A more portable artificial horizon is one formed of a well-polished reflector, supported on three adjusting screws, the motion of which, guided by a very sensitive spirit-level tube that may be laid on the surface of the reflector, enables the observer to make the latter perfectly level. This horizon will be found for ordinary purposes convenient and quickly adjusted.

OF LATITUDE AND LONGITUDE.

The latitude of a place on the earth's surface is its angular distance from the equator, and it is equal to the



H R represents the true horizon.
H′ R′ „ apparent horizon.
E Q „ equator.
S „ the position of the observer.

altitude of the elevated pole above the horizon. This angular distance or latitude can be determined by observing the altitude of the pole, or the greatest altitude of a celestial body whose declination at the time of the observation is known: the declination of a celestial body being its distance from the equinoctial, or the complement of its distance from the pole. The declination of a celestial body is, in fact, the same as the latitude of a place on the earth's surface, that is, the complement of the polar distance.

Deeming it of great importance, that the principles upon which the solution of the various problems connected with longitude should be understood, I am induced to insert the following eminently clear exposition of those principles from Sir J. F. W. Herschel's *Astronomy*, pages 133, *et seq*.

'To determine the latitude of a station is easy. It is otherwise with its longitude, whose exact determination is a matter of more difficulty. The reason is this: we are obliged in both cases to resort for their determination to marks external to the earth, i. e., to the heavenly bodies; but to observers situated to stations on the same *meridian*, i. e., differing in latitude, the heavens present different aspects at *all* moments; to observers situated on the same *parallel*, i. e., differing only in longitude, the heavens present the same aspects. In the former case there *is*, in the latter there is *not*, anything in the appearance of the heavens, watched through a whole diurnal rotation, which indicates a difference of locality in the observer.

'But no two observers, at different points of the earth's surface, can have at the same instant the same celestial hemisphere visible. Suppose, to fix our ideas, an observer situated at a given point of the equator, and that at the moment when he noticed some bright star to be in his zenith, and therefore on his meridian, he should be suddenly transported, in an instant of time, round one quarter of the globe in a westerly direction, it is evident that he will no longer have the same star vertically above him: it will now appear to him to be just rising, and he will have to wait six hours before it again comes to his zenith, *i.e.*, before the earth's rotation from west to east carries him back again to the line joining the star and the earth's centre, from which he set out.

'The difference of the cases, then, may be thus stated, so as to afford a key to the astronomical solution of the problem of the longitude. In the cases of stations differing only in latitude, the same star comes to the meridian at the same *time,* but at different *altitudes.* In that of stations differing only in longitude, it comes to the meridian at the same *altitude*, but at different *times.* Supposing, then, that an observer is in possession of any means by which he can certainly ascertain the *time* of a known star's transit across his meridian, he knows his longitude; or, if he knows the difference between its time of transit across his meridian and across that of any other station, he knows their difference of longitude. For instance, if the same star pass the meridian of a place, A, at a certain moment, and that of B exactly one hour of sidereal time, or one twenty-fourth part of the earth's diurnal period, later, then the difference of longitudes between A and B is one hour of time or 15°, and B is so much west of A.

'In order to a perfectly clear understanding of the principle on which the problem of finding the longitude by astronomical observations is resolved, the reader must learn to distinguish between time, in the abstract, as common to the whole universe, and therefore reckoned from an epoch independent of local situation, and *local time*, which reckons, at each particular place, from an epoch or initial instant, determined by local convenience. Of local reckoning we have instances in every sidereal clock in an observatory, and in every town clock for common use. The sidereal clock is regulated by observing the meridian passages of the more conspicuous and well-known stars, and this operation is called getting the *local time*.

'Suppose, now, two observers, at distant stations, A and B, each independently of the other, to set and regulate his clock or chronometer to the true sidereal time of his station. It is evident that, if one of these chronometers could be taken up without deranging its going, and set down by the side of the other, they would be found, on comparison, to differ by the exact difference of their local epochs; that is, by the time occupied by any star, in passing from the meridian of A to that of B; or, in other words, by their difference of longitude, expressed in sidereal hours, minutes, and seconds.

'Were chronometers perfect, nothing more complete and convenient than this mode of ascertaining differences of longitude could be desired. An observer, provided with such an instrument, and with a portable transit, or some equivalent method of determining the local time at any given station, might, by journeying from place to place, and observing the meridian passage of stars at each (taking care not to alter his chronometer, or let it run down), ascertain their difference of longitude with any required precision.

'The chronometer, however, though greatly improved by the skill of modern artists, is not yet sufficiently perfect to be relied on implicitly. Observers have, therefore, sought to resort to other means of communicating from one station to another a knowledge of its local time. The simplest and most accurate method by which this can be accomplished, is by telegraph or other signals, such as the flash of gunpowder, the explosion of a rocket, the sudden extinction of a bright light, or any other which admits of no mistake, and can be seen from one station to the other. The moment of the signal being made is noted by each observer by his respective watch set to local time, consequently, when the observers communicate their observations of the signal to each other, the difference of their local time, and therefore of their longitudes, become known. But circumstances seldom admit of the use of these artificial signals; natural ones have therefore been employed as their substitute; and the eclipses of Jupiter's satellites being visible at once over a whole terrestrial hemisphere, afford, in addition to their universality, the great advantage that the time of their happening at any fixed station, such as Greenwich, can be predicted with great certainty. An observer, therefore, at any other station wherever, who shall have observed one or more of these eclipses, and ascertained his local time, instead of waiting for a communication with Greenwich, to inform him at what moment the eclipse took place there, may use the *predicted Greenwich time* instead, and thence, at once, and on the spot, determine his longitude. The predicted Greenwich time is always published five years in advance, in the *Nautical Almanac*. The nature of this observation is, however, such that it cannot be made at sea, so that, however useful to the geographer, it is of no advantage to navigation. Moreover, the returns of the eclipses are of only occasional occurrence; and in their intervals, and when cut off from all communication with any fixed station, it is indispensable to possess some means of determining longitudes on which the geographer and navigator can implicitly rely for a knowledge of their positions. Such a method is afforded by

'*Lunar Observations.*

'If there were in the heavens a clock furnished with a dial-plate and hands, which always marked Greenwich time, the longitude of any station would be at once determined, so soon as the *local* time was known. Now the offices of the dial-plate and hands of a clock are these:—the former carries a set of marks upon it, whose position is known; the latter, by passing over and among these marks, informs us, by the place it holds with respect to them, what it is o'clock, or what time has elapsed since a certain moment when it stood at one particular spot.

'In a clock, the marks on the dial-plate are uniformly distributed all around the circumference of a circle, whose centre is that on which the hands revolve with a uniform motion. But it is clear that we should, with equal certainty, though with much more trouble, tell what o'clock it was, if the marks on the dial-plate were *un*equally distributed—if the hands were eccentric, and their motion not uniform—provided we knew, 1st, the exact intervals round the circle at which the hour and minute marks were placed; which would be the case if we had them all registered in a table, from the results of previous careful measurement;—2ndly, if we knew the exact amount and direction of eccentricity of the centre of motion of the hands;—and, 3rdly, if we were fully acquainted with all the mechanism which put the hands in motion, so as to be able to say at every instant what was their velocity of movement, and so as to be able to calculate, without fear of error, *how much time* should correspond *to so much angular* movement.

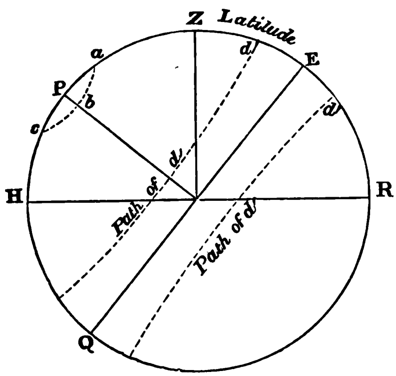
'The visible surface of the starry heavens is the dial-plate of our clock, the stars are the fixed marks distributed around its circuit, the moon is the moveable hand, whose position among them can at any moment, when it is visible, be exactly measured by the help of a sextant, just as we might measure the place of our clock-hand among the marks on its dial-plate with a pair of compasses, and thence, from the known and calculated laws of its motion, deduce the time.

'Such a clock would, no doubt, be considered a very bad one; but, if it were our *only* one, and if incalculable interests were at stake on a perfect knowledge of time, we should justly regard it as most precious, and think no pains ill bestowed in studying the laws of its movements, or in facilitating the means of reading it correctly. Such, in the parallel we are drawing, is the lunar theory, whose object is to reduce to regularity the indications of this strangely-irregular going clock, to enable us to predict, long beforehand, and with absolute certainty, whereabouts among the stars, at every hour, minute, and second, in every day of every year, in Greenwich local time, the moon would be seen from the earth's centre, and *will* be seen from every accessible point of its surface; and such is the *lunar method* of longitudes. The moon's apparent angular distance from all those principal and conspicuous stars which lie in its course, as seen from the earth's centre, are computed and tabulated with the utmost care and precision in the *Nautical Almanac.* No sooner does an observer, in any part of the globe, at sea or on land, measure its actual distance from any one of those *standard stars* (whose places in the heavens have been ascertained for the purpose with the most anxious solicitude), than he has, in fact, performed that comparison of his local time with the local times of every observatory in the world, which enables him to ascertain his difference of longitude from one or all of them.'

Having prepared the reader by the above simple explanations to understand the general principles on which the determination of latitude and longitude depends, I now proceed to illustrate their practical application.

To determine the Latitude of a Place.

First, by the mean altitude of a circumpolar star, observed at the time of its upper and lower culmination. This method is the simplest, as it requires no correction for the declination of the star observed—the observation is made with a transit instrument or theodolite, carefully adjusted in the plane of the meridian, the readings of the vertical arc being noted at the time of the upper and lower transits of the star. These readings are then corrected for atmospheric refraction (see Table, page 229), and the mean reading gives the latitude of the place.



For if *a b c* be the path of the star about the pole, P, Z the zenith, and H R the horizon, then is *a* H the altitude of the star upon the meridian when above the pole, and *c* H the same when below the pole; hence, because *a* P = *c* P when both are corrected for refraction, therefore $\frac{aH + cH}{2} = HP = EZ$; hence the height of the pole, H P, is equal to E Z, the latitude of Z, *i. e.*, its angular distance from the equator.

A second method is by a single observation of the meridian altitude of the sun, or a star whose declination is known. Thus the altitude, R *d* or R *d'*, of the celestial body, *d* or *d'*, being observed, its zenith distance or co-altitude, is known. Then, to the zenith distance, add the declination *d* E (as given in the *Nautical Almanac*), when the star and place of observation are on the same side of the equator, or subtract the declination, d' E, when they are on different sides; and the sum or difference will be the latitude, E Z, required. In this case, as in the former, the observed altitudes must be corrected for refraction; for parallax, caused by the distance of the observer from the earth's centre; and for the semi-diameter of the sun when it is the object observed.

Example 1. On August 11, 1841, the double meridional altitude* of the sun's lower limb was observed with a sextant to be 104° 27′ 45″; required the latitude of the place of observation, the observer being north of the sun:—

Double altitude		2)	104°	27′	45″
Apparent altitude			52	13	52
Add sun's semi-diameter, the lower limb having been observed (p. 230) . .	= + 15′ 49″				
Refraction (p. 229)	= − 0 45	+		15	10
Parallax . .	= + 0 6				
True altitude			52	29	2
Co-altitude, or zenith distance .			37°	30′	58″
August 11, 1851, declination north +			15	18	0
Latitude			52	48	58

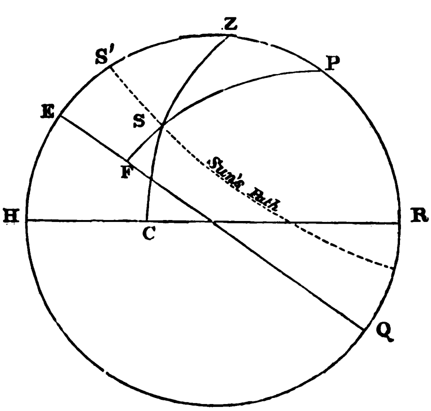
When the natural horizon is used instead of an artificial horizon for an observation made with the sextant, an additional correction for the dip of the horizon is required: this correction is also obtained from tables prepared for the purpose, and which are published in BAILEY'S *Astronomical Tables*, or in any modern work on navigation.

* That is, the altitude having been taken with an artificial horizon (see page 216), twice the height was obtained.

Example 2. On September 21, 1829, in longitude 60° E., the meridian altitude of the sun's lower limb was 56° 26′, the observer being south of the sun, and the height of his eye 26 feet above the surface of the sea—required the latitude of the place of observation:—

Altitude of ☉		56°	26′	00″
☉'s semi-diameter . . .		+ 0	16	00
		56	42	00
Dip of the horizon . . .		− 0	4	52
Apparent altitude ☉ . . .		56	37	08
Refraction − 38·4″ } Parallax + 4·4 } correction .		− 0	0	34
True altitude ☉ . . .		56	36	34
Co-altitude, or zenith distance .		33	23	26
Sept. 21, 1829, decl. north	− 0° 43′			
Correction for longitude	− 4			
		− 0	47	00
Latitude		32	36	26

To find the Local Time, or to Regulate Time-pieces by Observations of the Sun.



Determine the latitude of the place according to the preceding problem, observe the altitude of the sun at the time indicated by the time-piece, take the sun's declination for the day and time of observation from the *Nautical Almanac*, with the above data, the time-piece may be corrected as follows:

In the triangle Z P S, we have S, the position of the sun at the time of observation,

Z P, the complement of the latitude,

Z S, the zenith distance, or the complement of the observed altitude,

And P S, the polar distance, which is the complement of the declination at the time of the observed altitude.

The three sides of the triangle being thus given, the angle Z P S, usually called the '*hour angle*,' that is, the difference between the times of the sun's being at S and at S′ on the meridian, is obtained.

The angles of a spherical triangle, in which the sides are given, may be obtained by the following formula (THOMPSON'S *Trigonometry*, page 33):

$$\text{Sin. } \tfrac{1}{2}\,A = \sqrt{\frac{\sin.(s-b)\sin.(s-c)\,r^2}{\sin. b \sin. c}};$$

or as it may be expressed,

$$\text{Sin. } \tfrac{1}{2}\,A = \sqrt{\frac{r}{\sin. b} \cdot \frac{r}{\sin. c} \cdot \sin.(s-b)\sin.(s-c)}.$$

By taking the logarithms of both mumbers of the equation, we find

$$\log.\sin.\tfrac{1}{2}\,A = \tfrac{1}{2}\left\{\begin{array}{l} 10 - \log.\sin. b + 10 - \log.\sin. c + \\ \quad \log.\sin.(s-b) + \log.\sin.(s-c). \end{array}\right.$$

In a similar manner we find

$$\log.\cos.\tfrac{1}{2}\,A = \tfrac{1}{2}\left\{\begin{array}{l} 10 - \log\ \sin. b + 10 - \log.\sin. c + \\ \qquad \log.\sin. s + \log.\sin\ (s-a), \end{array}\right.$$

$$\text{and } \log.\tan.\tfrac{1}{2}\,A = \tfrac{1}{2}\left\{\begin{array}{l} 10 - \log.\sin. s + 10 - \log.\sin.(s-a) \\ \quad + \log.\sin.(s-b) + \log.\sin.(s-c). \end{array}\right.$$

This last formula is, perhaps, the most convenient for practice.

Example.—Given the apparent altitude of the sun's lower limb in lat. $54^0\ 36'$, April 4, 1823, equal to $29^0\ 24'$, at ten minutes past nine in the morning, by a clock, to find the error of the clock.

	°	′	″
SC, or the altitude, corrected for semi-diameter, parallax, and refraction . . . . . =	29	38	25
	90	0	0
(See fig. p. 226) Co-altitude SZ =	60	21	35
ZE =	54	36	
ZP =	35	24	

SP = 90 − decl. = 84° 33′ 6″

In triangle, ZSP

	°	′	″		
SZ =	60	21	35		
ZP =	35	24	0		
SP =	84	33	6		
2 s =	180	18	41		
s =	90	9	20	+ 10 − log. sin. s	0·0000016
s − SZ =	29	47	45	+ 10 − log. sin. (s − a)	0·3037217
s − ZP =	54	45	20	+ log. sin. (s − b)	9·9120612
s − SP =	5	36	14	+ log. sin. (s − c)	8·9897025
					2)19·2054870
½ ZPS =	21	49	57	log. tan. ½ ZPS =	9·6027435
			2		
ZPS =	43	39	54.		

To reduce this to time, we have (one hour being = 15°),

15° : 43° 39′ 54″ :: 1 : 2h. 54m. 39s., to be subtracted from 12.

h.	m.	s.	
12	0	0	
2	54	39	
9	5	21	apparent time, because deduced from an observation.
+	3	15	equation of time.
9	8	36	true time or mean time.
9	10	0	time by clock.
Error	1	24	too fast.

TABLE OF REFRACTIONS.

Apparent Altitude. °	′	Refraction. ′	″
0	0	33	51
	5	32	53
	10	31	58
	15	31	5
	20	30	13
	25	29	24
	30	28	37
	35	27	51
	40	27	6
	45	26	24
	50	25	43
	55	25	3
1	0	24	25
	5	23	48
	10	23	13
	15	22	40
	20	22	8
	25	21	37
	30	21	7
	35	20	38
	40	20	10
	45	19	43
	50	19	17
	55	18	52
2	0	18	29
	5	18	5
	10	17	43
	15	17	21
	20	17	0
	25	16	40
	30	16	21
	35	16	2
	40	15	43
	45	15	25
	50	15	8
	55	14	53
3	0	14	35
	5	14	19
	10	14	4
	15	13	50
	20	13	35
	25	13	21
	30	13	7
	35	12	53
	40	12	41
	45	12	28
	50	12	16
	55	12	3

Apparent Altitude. °	′	Refraction. ′	″
4	0	11	52
	10	11	30
	20	11	10
	30	10	50
	40	10	32
	50	10	15
5	0	9	58
	10	9	42
	20	9	27
	30	9	11
	40	8	58
	50	8	45
6	0	8	32
	10	8	20
	20	8	9
	30	7	58
	40	7	47
	50	7	37
7	0	7	27
	10	7	17
	20	7	8
	30	6	59
	40	6	51
	50	6	43
8	0	6	35
	10	6	28
	20	6	21
	30	6	14
	40	6	7
	50	6	0
9	0	5	54
	10	5	47
	20	5	41
	30	5	36
	40	5	30
	50	5	25
10	0	5	20
	10	5	15
	20	5	10
	30	5	5
	40	5	0
	50	4	56
11	0	4	51
	10	4	47
	20	4	43
	30	4	39
	40	4	35
	50	4	31

Apparent Altitude. °	′	Refraction. ′	″
12	0	4	28·1
	10	4	24·4
	20	4	20·1
	30	4	17·3
	40	4	13·9
	50	4	10·7
13	0	4	7·5
	10	4	4·4
	20	4	1·4
	30	3	58·4
	40	3	55·5
	50	3	52·6
14	0	3	49·9
	10	3	47·1
	20	3	44·4
	30	3	41·8
	40	3	39·2
	50	3	36·7
15	0	3	34·3
	30	3	27·3
16	0	3	20·6
	30	3	14·4
17	0	3	8·5
	30	3	2·9
18	0	2	57·6
19	0	2	47·7
20	0	2	38·7
21	0	2	30·5
22	0	2	23·2
23	0	2	16·5
24	0	2	10·1
25	0	2	4·2
26	0	1	58·8
27	0	1	53·8
28	0	1	49 1
29	0	1	44·7
30	0	1	40·5
31	0	1	36·6
32	0	1	33·0
33	0	1	29·5
34	0	1	26·1
35	0	1	23·0
36	0	1	20·0
37	0	1	17·1
38	0	1	14·4
39	0	1	11·8
40	0	1	9·3
41	0	1	6·9

App. Alt. °	Refraction. ′	″
42	1	4·6
43	1	2·4
44	1	0·3
45	0	58·1
46	0	56·1
47	0	54·2
48	0	52·3
49	0	50·5
50	0	48·8
51	0	47·1
52	0	45·4
53	0	43·8
54	0	42·2
55	0	40·8
56	0	39·3
57	0	37·8
58	0	36·4
59	0	35·0
60	0	33·6
61	0	32·3
62	0	31·0
63	0	29·7
64	0	28·4
65	0	27·2
66	0	25·9
67	0	24·7
68	0	23·5
69	0	22·4
76	0	21·2
71	0	19·9
72	0	18·8
73	0	17·7
74	0	16·6
75	0	15·5
76	0	14·4
77	0	13·4
78	0	12·3
79	0	11·2
80	0	10·2
81	0	9·2
82	0	8·2
83	0	7·1
84	0	6·1
85	0	5 1
86	0	4·1
87	0	3·1
88	0	2·0
89	0	1·0

SEMI-DIAMETERS OF THE SUN FOR THE DIFFERENT MONTHS THROUGHOUT THE YEAR.

Days.	Jan.	Feb.	March.	April.	May.	June.
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
1	16 18	16 15	16 10	16 1	15 53	15 48
11	16 17	16 13	16 7	15 58	15 51	15 46
21	16 17	16 11	16 4	15 55	15 49	15 46
	July.	August.	Sept.	Oct.	Nov.	Decem.
1	15 46	15 47	15 ~~53~~	16 1	16 9	16 16
11	15 46	15 49	15 56	16 3	16 12	16 17
21	15 46	15 51	15 58	16 7	16 14	16 18

15

PARALLAX OF THE SUN ON THE FIRST DAY OF EACH MONTH, THE MEAN HORIZONTAL PARALLAX BEING 8·60″ 8″.74

Altitude.	Jan.	Feb. Dec.	March. Nov.	April. Oct.	May. Sept.	June. Aug,	July.
″	″	″	″	″	″	″	″
90	0·00	0·00	0·00	0·00	0·00	0·00	0·00
85	0·79	3·76	0·76	0·75	0·74	0·74	0·74
80	1·52	1·52	1·51	1·49	1·48	1·47	1·47
75	2·26	2·26	2·25	2·23	2·21	2·19	2·19
70	2·99	2·91	2·97	2·24	2·92	2·90	2·89
65	3·70	3·69	3·67	3·63	3·60	3·58	3·57
60	4·37	4·36	4·34	4·30	4·26	4·24	4·23
55	5·02	5·01	4·98	4·93	4·89	4·86	4·85
50	5·62	5·61	5·58	5·53	5·48	5·45	5·44
45	6·19	6·17	6·13	6·08	6·03	5·99	5·98
40	6·70	6·68	6·64	6·59	6·53	6·49	6·48
35	7·17	7·15	7·11	7·04	6·99	6·94	6·93
30	7·58	7·56	7·51	7·41	7·39	7·34	7·33
25	7·93	7·91	7·86	7·79	7·73	7·68	7·67
20	8·22	8·20	8·15	8·08	8·01	7·97	7·95
15	8·45	8·43	8·38	8·30	8·24	8·19	8·17
10	8·62	8·59	8·54	8·47	8·40	8·35	8·33
5	8·73	8·69	8·64	8·56	8·50	8·44	8·42
0	8·75	8·73	8·67	8·60	8·53	8·48	8·46

AUGMENTATION OF MOON'S SEMIDIAMETER ACCORDING TO HER INCREASE IN ALTITUDE.

The Moon's horizontal semidameter is found in page 3 of each month in the *Nautical Almanac,* for every day at mean noon and midnight at Greenwich; and the Sun's in page 2 for every mean noon.

	Horizontal Semidiameter.					
Moon's app. Altitude.	14′ 30″	15′ 0″	15′ 30″	16′ 0″	16′ 30″	17′ 0″
°	″	″	″	″	″	″
0	0·00	0·00	0·00	0·00	0·00	0·00
3	0·71	0·75	0·80	0·86	0·92	0·97
6	1·41	1·50	1·60	1·71	1·83	1·94
9	2·11	2·25	2·40	2·56	2·73	2·90
12	2·81	3·00	3·20	3·41	3·63	3·86
15	3·50	3·74	3·99	4·25	4·52	4·80
18	4·17	4·46	4·76	5·07	5·39	5·73
21	4·84	5·18	5·52	5·89	6·26	6·65
24	5·49	5·88	6·27	6·68	7·11	7·54
27	6·13	6·56	7·00	7·46	7·93	8·42
30	6·75	7·23	7·71	8·22	8·74	9·28
33	7·35	7·88	8·40	8·96	9·52	10·12
36	7·93	8·50	9·07	9·67	10·28	10·92
39	8·49	9·10	9·72	10·36	11·02	11·66
42	9·03	9·68	10·34	11·02	11·72	12·44
45	9·55	10·23	10·93	11·65	12·39	13·15
48	10·05	10·76	11·49	12·25	13·03	13·83
51	10·52	11·26	12·02	12·81	13·63	14·46
54	10·95	11·72	12·52	13·34	14·19	15·06
57	11·35	12·15	12·98	13·83	14·72	15·62
60	11·72	12.55	13·40	14·29	15·20	16·13
63	12·06	12·91	13·79	14·70	15·64	16·60
66	12·37	13·24	14·14	15·08	16·04	17·03
69	12·64	13·53	14·46	15·41	16·39	17·40
72	12·88	13·79	14·73	15·70	16·70	17·73
75	13·08	14·01	14·96	15·95	16·96	18·01
78	13·24	14·18	15·15	16·15	17·18	18·24
81	13·37	14·32	15·30	16·31	17·35	18.42
84	13·46	14·42	15·41	16·42	17·47	18·55
87	13·52	14·48	15·47	16·49	17·54	18·62
90	13·54	14·50	15·49	16·51	17·57	18·65

To determine the Longitude of a Place.

Of the various methods which astronomers have devised for determining the longitude of a place, I shall only describe the method of determining it by 'lunar observation,' a mode which possesses the advantage of being easily applied at sea, and does not involve complex calculations. For additional information on this subject I would refer to PEARSON'S *Practical Astronomy*, RIDDLE'S *Navigation*, NORIE'S *Navigation*, etc.

To find the true angular distance of the moon from a star or the sun, it is necessary that the altitudes of the moon and that of the other object, whether a star or the sun, be measured in order to correct the observed angular distance from the effects of parallax and refraction. For the moon is always seen lower than her true place, because, owing to her vicinity to the earth, the apparent depression caused by parallax is a greater quantity than the apparent elevation caused by refraction: the sun is always seen higher than his true place, his great distance rendering parallax of less effect than refraction. These apparent changes from the true positions cause the true distance to be almost always greater or less than the observed distance, and obtaining the true from the observed distance is called technically, 'clearing the lunar distance,' that is, correcting it from the effects of parallax and refraction.

For greater accuracy the three observations, i. e., the angular distance and the two altitudes, should be taken simultaneously; but if they are taken in succession by one observer, he is to bestow especial care in measuring the lunar distance, taking the altitudes as rapidly as possible. He proceeds in the following order, taking first, the two altitudes with the time of each observation; secondly, the lunar distance repeated several times with the time of each observation, from whence a mean of the times and distances is deduced; lastly, the altitudes in reverse order. The altitudes are then reduced to the mean time of the lunar distance by the following proportion. As the difference of times between the observations is to the difference of the corresponding altitudes, so is the difference between the time at which the first altitude was taken and the mean time, to a fourth number which, added or subtracted from the first altitude, according as it is increasing or diminishing, will give the altitude reduced to the mean time.*

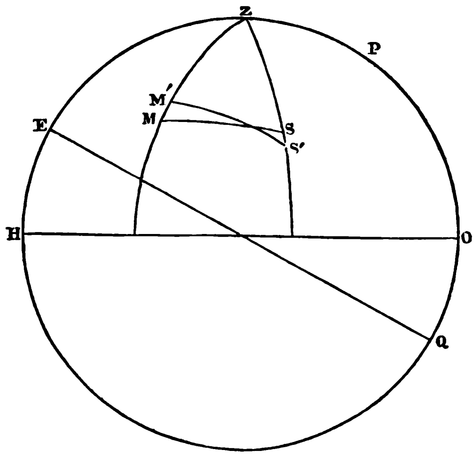
The obtaining of the true distance, called 'clearing the lunar distance,' is a problem in spherical trigonometry. Of such vital importance at sea is its correct solution, that the most eminent astronomers have turned their attention to the subject with the view to simplify it. Tables, the results of their labours, are given in all works on navigation, with directions for their use; by their means, an operation otherwise laborious is much expedited, and placed within the reach of all who are moderately versed in simple trigonometrical operations.

The principle is as follows:

In the following figure (p. 234) Z represents the zenith, P the pole, M the observed place of the moon, and S that of the sun or star. The data given are M S, the measured angular distance; and Z M and Z S, the two zenith distances (or co-altitudes) from whence the angle M Z S is found, the value of which is evidently not affected by refraction or parallax, which act on vertical lines. The true place of the moon is *elevated above* its apparent place, and that of the sun or star is *depressed below* its apparent place. Let M′ and S′ represent the corrected places of these bodies, we have then Z M′ and Z S′—the zenith distances *corrected for refraction and parallax*—and the angle

* NORIE'S *Navigation*, page 224.

Z, before found, to find the true linear distance, M′ S′, in the triangle Z M′ S′.



*Example.**—On May 4th, 1838, at $10^h\ 38^m\ 12^s$ by chronometer, the following observations were taken, in latitude 51° 23·40′ north, to find the longitude.

Double altitude ☽̄ 74° 42′ 13″, taken with a sextant.

Double altitude Spica Virginis 56° 30′ 44″ taken with a sextant.

Distance * ☽ 32° 25′ 55″.

	°	′	″
Observed double altitude of moon's upper limb (☽̄) . . . . 2)	74	42	13
Apparent altitude of moon's upper limb .	37	21	6·5
Moon's semi-diameter horiz. at $10^h\ 7^m$ (*Nautical Almanac*) . 14′ 45·3″; Augmentation for 37·21″ altitude (*Table*, p. 231) . 8·5″	0	14	53·8
Apparent altitude ☽	37	6	12·7
	90	0	0
ZM, apparent co-altitude, or zenith distance	52	53	47·3

* FROME'S *Trigonometrical Surveying.*

Observed altitude Spica Virginis .	2)56° 30′ 44″
Apparent altitude	28 15 22
	90 0 0
ZS, apparent co-altitude or zenith dist. .	61 44 38
Distance * ☽	31 25 55
To be diminished by ☽'s corrected semi-diameter	0 14 53·8
MS, or apparent lunar distance . .	31 11 1·2

Then in the triangle, ZMS, we have the three sides, to find the angle, MZS (see formula, page 227).

$$\begin{aligned} MS &= 31^\circ\ 11'\ 1''{\cdot}2 \\ ZS &= 61\ \ 44\ \ 38 \\ ZM &= 52\ \ 53\ \ 47{\cdot}3 \\ \text{the sum or } 2S &= 145\ \ 49\ \ 26{\cdot}5 \end{aligned}$$

$\frac{2S}{2} = S = 72^\circ\ 54'\ 43''{\cdot}25 + 10 -$	log. sin.	0·0200380
$S - MS = 41\ \ 43\ \ 42{\cdot}05 + 10 -$	log. sin.	0·1767871
$S - ZS = 11\ \ 10\ \ \ 5{\cdot}25 +$	log. sin.	9·2871013
$S - ZM = 20\ \ \ 0\ \ 55{\cdot}95 +$	log. sin.	9·5343639
		2)19·0182903
$\frac{1}{2} Z = 17\ \ 53\ \ 53$	log. tan. $\frac{1}{2}$ Z =	9·5061451
2		
Angle Z 35 47 46		

Then, to correct the zenith distances, ZM and ZS, for refraction and parallax:

☽'s apparent zenith distance, ZM . =	52° 53′ 47″·3
Refraction . . . . . . +	0 1 14·1
	52 55 1·4

	°	′	″
Parallax (from p. 3, *Nautical Almanac*) –	0	43	7·4
Z M′, corrected zenith distance . .	52	11	54
Z S, Spica Virginis' apparent zenith dist.	61	44	38
Refraction . . . . . +		1	45
Z S′, corrected zenith distance . .	61	46	23

Then, in the triangle, Z M′ S′, to find the side, M′ S′, we have

$$Z M' = 52^\circ\ 11'\ 54''$$
$$Z S' = 61\ \ 46\ \ 23$$
$$\text{Angle Z (page 235)} = 35\ \ 47\ \ 46, \text{ required M}'\text{S}'.$$

To obtain the third side of the triangle, we may use the following formulæ (THOMSON'S *Trig.*, page 35):

Log. tan. ϕ = log. tan. Z M′ + log. cos. Z – 10,

Log. cos. M′ S′ = log. cos. Z M′ + log. cos. (Z S′ – ϕ) – log. cos. ϕ,

in which ϕ is an auxiliary arc, introduced to simplify the computation.

Log. tan. Z M′ (52° 11′ 54″)	+	10·1102916
Log. cos. Z (35° 47′ 46″)	+	9·9092156
	–	10·
Log. tan. ϕ (46° 17′ 11″)	.	10·0195072
Log. cos. Z M′ (52° 11′ 54″)	+	9·7877009
Log. cos. Z S′ – ϕ (15° 29′ 12″)	+	9·9839525
		19·7716534
Log. cos ϕ (46° 17′ 11″) .	–	9·8395606
Log. cos. M′ S′ (31° 13′ 6″)	.	9·9320929

M′ S′, the corrected or 'cleared' lunar distance, is 31° 13′ 6″.

By the *Nautical Almanac*, it appears that the Greenwich mean time answering to this distance must be between 9 P.M. and midnight. For,

	°	′	″
Lunar dist. at 9 P.M., Greenwich .	32	2	35
Ditto at midnight, ditto . .	30	33	43
Difference for 3 hours of time . .	1	28	52
Lunar dist. at 9 P.M., Greenwich .	32	2	35
Corrected lunar distance . . .	31	13	6
Difference for $1^h\ 40^m\ 13^s$ . . .	0	49	29

deduced from the following proportion:

$$1^\circ\ 28'\ 52'' : 49'\ 29'' :: 3 : 1^h\ 40^m\ 13^s.$$

Greenwich mean time for same distance,

$$=9^h+1^h\ 40^m\ 13^s=10^h\ 40^m\ 13^s$$

Mean time at place of observation $=10^h\ 38^m\ 12^s$

Longitude (West) . . . $2^m\ 1^s$

or, in space, 8″.

Meridian Line.

The method of obtaining a meridian line by observations of the sun, given in page 88, would be perfectly accurate if the sun moved constantly in the same parallel. However, his advance in the ecliptic, or the change in his declination during the interval elapsed between the first and last observations, requires a correction on the mean results of the observations, varying according to the season of the year. The required correction is greatest near the time of the equinoxes, as the change in the sun's declination is then the most rapid. The middle point of the horizontal arcs, as obtained by equal altitudes, is to the west of the true meridian when the sun is advancing towards the elevated pole, and to the east of the true meridian when he is receding from the elevated pole. From mid-winter to mid-summer the sun gradually approaches the north pole, and from mid-summer to mid-winter gradnally recedes from it.

To apply the correction to two observations of equal altitude, the time of each observation must be noted:

Let T = time of first observation

T′ = time of second observation

D = sun's declination at the time of the first observation } as obtained from the *Nautical Almanac.*

D′ = do. second observation }

then the

$$\text{Correction} = \tfrac{1}{2}\,(D-D')\ \text{sec. lat. cosec.}\ \frac{(T-T')}{2}$$

Practical Rule. To the log. of half the change of declination during the interval between the observations, add the log. secant of the latitude, and the log. cosecant of half the interval of time between the observations, converted into space; the sum - 20 will be the log. of the correction in seconds of space.

*Example.** 'The readings of the horizontal limb at equal altitudes of the sun were 130° 10′ 15″ and 32° 36′ 15″, therefore the middle point or reading of the approximate meridian was 81° 23′ 15″. The interval of time between the observations was 5 hours, the half of which converted into space = 37° 30′. The sun's hourly change of declination = 56·77″, therefore the change for half the interval = 141·92″ (approaching the north pole). The latitude of the place = 51° 28′ 39″, required the correction to be applied to the middle point to obtain the direction of the true meridian.

* SIMMS on *Mathematical Instruments*, page 97.

$\frac{1}{2}(D - D') = 141 \cdot 92''$	. . . log.	2·1520436
Lat. $= 51° \; 28' \; 39''$	. . . . sec.	0·2056388
$\frac{1}{2}(T - T') = 37° \; 30''$	. . cosec.	0·2155529
Correction $374 \cdot 31''$	. . . . log.	2·5732353
$= 6' \; 14 \cdot 31''$		
Reading of the middle point	. .	81° 23′ 15″
Correction	. . . . . .	0 6 14
Reading of the instrument when set to the true meridian	. . .	81 17 1

When the meridian is deduced from equal altitudes of a circumpolar star no correction is required.

Most instructive and interesting details on the determination of meridian lines, as also on the mode of conducting the measurement of arcs of a meridian, and of determining geodesically the latitudes and longitudes of stations, will be found in the second volume of the *Trigonometrical Survey of England and Wales.*

CHAPTER XI.

ON MARITIME SURVEYING.

MARITIME Surveying has for its object the determining (for the purpose of representation on hydrographical plans or charts) of coasts, harbours, inlets, rocks, shallows, soundings, and whatever particulars may serve to direct the mariner on his voyage, or point out the dangers to be avoided.

Practical Directions. Observations for the construction of a chart are made with reference to fixed points on shore. The relative positions of those points are ascertained, either with great precision, or with a degree of approximate accuracy proportionate to the extent of detailed information to be given on the chart. When perfect accuracy is aimed at, many stations on shore are in the first instance fixed in position by means of a trigonometrical survey, executed according to the methods previously described, and in which the accuracy of the work is to be tested as usual by the measurement of one or more bases of verification. The stations in the triangulation being selected with reference to the ultimate end in view, will be chosen so as to determine the position of remarkable headlands, beacons, lighthouses, and other objects of primary importance to the mariner. With these data, whatever extent of coast may be embraced by the projected hydrographical work, each series of operations at sea will be confined to spaces comparatively limited, and the whole will consist of numerous detailed charts correctly joined and harmonized by means of the triangulation on shore. A description, therefore, of the mode of operation adopted for the marine survey of a single harbour or limited sea-reach, will apply equally to the system adopted in the performance of a continuous survey embracing an extensive line of coast.

The triangulation on shore is generally performed in the first instance, but it may proceed simultaneously with the maritime survey, taking care that it shall be kept somewhat in advance of the latter.

Tide Gauges. The triangulation being supposed completed, the first step for the maritime operations consists in the selection of localities suited to the erection of tide-gauges, divided into feet and tenths, to be fixed in a vertical position. The zero point of each gauge is to be referred to a fixed permanent bench mark by means of the spirit-level, in order that the gauge may be refitted in its original position, should it be displaced by the violence of the sea or any other cause.

After a series of observations, these gauges serve, in the first instance, to give the lowest point of the lowest tide at full and change of the moon,—and to the level of this lowest point the depths of all soundings are referred. The gauges, in addition, give for every day and portion of each day, on which soundings are made, the amount of rise and fall of the tide,—and, by means of these latter observations, all registered soundings are reduced to the lowest level. The necessity for this is obvious, inasmuch as it would be impossible to take the soundings even of a limited area at the precise time of low water.

By taking advantage of a quay or other local circumstance, the observations relating to the higher stages of the tide may frequently be made from the shore, though they, perhaps, cannot be carried down to low water. A second gauge must then be provided and placed further out to seaward, so that when the tide shall be on the point of leaving the first, the observer may proceed to the second.

In such a case, the relative altitude of the zero division of the first gauge, as compared with some division of the second, must be carefully ascertained with the spirit-level, and, each gauge being denoted by a distinctive letter, proper entries must be made in the field-book to record the time of changing from one gauge to the other. In most cases, however, a careful selection will enable the observer to find a suitable locality in which the base of the tide-gauge will not be left dry by the retreating tide.

A trustworthy assistant, provided with a well-regulated watch, is to be stationed at each tide-gauge for the purpose of registering the height of the tide at regular intervals of time, usually every quarter of an hour. At each tide-gauge station, a meridian line should be marked (see pages 87 and 88), in order that the tide-registrar may regulate his watch by the course of the sun. If at any time it be required to alter the time of the watch, both the date and the amount of such alteration are to be entered in the field-book.

Time of High Water at Full and Change. It is important to take advantage of the opportunity offered by the tide-gauge to note with great precision the time of high-water at full and change of the moon, as this information is always required on the chart. While on this subject, I cannot do better than to add the following 'suggestions on the observations of the heights and times of the tides,' as given by Professor Whewell.

First, as to time: 'The time used in tide observations may be *mean** or *apparent* time,† but it should always be noted which is employed, and by what means obtained.'

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* *Mean* time, or *equal* time, is that which is measured by an equable notion, e.g., the time given by a watch or clock. It is distinguished from apparent time arising from the unequal motion of the earth.

† *Apparent* or *relative* time, is the sensible measure of any duration by means of motion, e.g., the time shewn by a sun dial, the sun's time.

'The *establishment* of any place, or the time of high-water at *full* and *change** of the moon, should always be noted in the field-book, not as is commonly done,' at such an hour of the day, 'but as being so many hours after the moon's transit, the time of which is easily known *from the tables.*'† 'If the tide be observed according to *mean time*, and the time of the moon's transit be determined according to *apparent time*, it will be necessary to apply the *equation of time*‡ to the interval.'

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* *Full* and *change* given in common almanacs, usually marked thus ● for new moon or change, and thus ○ for full moon.

† The moon's transit, or meridian passage at Greenwich is given in the *Nautical Almanac*, in page 4 of each month. To find the mean time of transit of the moon under any other meridian (see page 579, *Nautical Almanac*, 1843).

Suppose the meridian to be the west of Greenwich 45° or 3^h, required the mean time of transit on Jan. 25, 1843.

'The meridian being to the west of Greenwich, the transit will take place after the Greenwich time of transit on the 25th, therefore, take the difference between the meridian passage on the 25th and 26th, from the *Nautical Almanac.*

		h.	m.
Meridian passage on 25th	.	20	44·5
Do. do. 26th	.	21	40·1
Difference	.	0	55·6

Then $\overset{h.}{24} : \overset{h.}{0}\ \overset{m.}{55{\cdot}6} :: \overset{h.}{3} : \overset{m.}{7}$ which, added to the Greenwich mean time of transit, gives

	h.	m.
	20	44·5
+	0	7
	20	51·5

for the mean time of transit at the given meridian.

Had the assumed meridian been 3^h to the east of Greenwich, the transit would have taken place before the transit at Greenwich, and the proportional part of the difference between the 24th and 25th must, in this case, have been subtracted. The times thus deduced are only approximate; but they are sufficiently accurate for the purposes usually required.

‡ The *equation of time* denotes the difference between the mean and apparent time. The greatest difference occurs about the 1st of November, amounting to 16′ 14″.

Secondly, as to time of high-water; 'The instant to be registered is, of course, that when the surface of the water is highest; but, if the water be perfectly still, it changes very slowly when near the highest point, and appears to be stationary for some moments. To avoid the difficulty produced by this circumstance, some observers have registered, not the time when the water is highest, but two instants of equal height before and after the greatest; and the time of greatest height is supposed to bisect this interval.

To obviate the effect of waves in rendering the surface uncertain, the following apparatus may be used. Let a pipe be fixed upright by the side of the gauge, in such a situation that at low-tide time the water shall reach its lower part. The bottom of the pipe must be stopped, and a number of small holes, about a ¼ inch in diameter, be made in or near the bottom. A float, nearly filling the pipe, is to be placed in it, and to carry a light upright rod, divided into feet and decimals, which are to be read off by means of an index or mark fastened to the top of the tube. The apertures in the bottom of the tube will allow the float to rise and fall with the general surface, without any sensible loss of time; while the smallness of those apertures will prevent the oscillations of the waves from affecting the inside of the tube.

No precise directions can be given as to the proper number of stations for gauges; this must be determined from the information obtained from pilots or fishermen on the coast, as to the variations in the amount of rise and fall of the tide at different spots. As a general rule, it may be observed, that in all narrow channels, and especially in rivers where obstacles cause greater accumulations of water, and consequent exceptional irregularities in the change of the level of the tides, a greater number of gauges are to be used than on an open seaboard.

The annexed table may be used as a form for the registry of the tide-tables required for surveying operations.

Form of Tide-Table.

At Station No. 1. *Entrance into*____________ *River*
July , 18 .

Month. Day.	Moon's transit.			Moon's *age* at noon.*	Remarks.
	h. m.	h. m.	feet.	days.	
21		11 15 A.M.	7·7		Watch set at apparent time by meridian observation of the sun.
		30	8·1		
		45	8·4		
		12 0 noon	8·9	26·4	Watch 1 minute slow.
		15	9·2		
		30	9·4		Light wind, S.S.E.
		45	9·5		Results.
	9 11	1 0	9·6		feet. in.
	mean	15	9·5		Greatest depth . .
	time.	30	9 4		Least . . . . .
		45	9·1		Whole rise . . .
		2 0	8·7		
		15	8·2		h. m.
					Began to rise at . .
		30	7·9		High water at . .

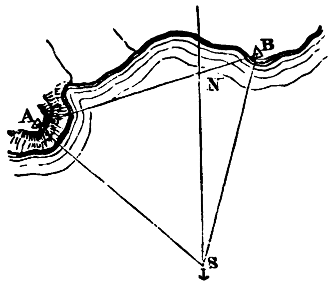
Operations at Sea.

If there be not on the shore permanent well-defined stations, such as churches, towers, light-houses, or other beacons, fixed by triangulation, the surveyor erects the necessary signals at the vertices of the triangles. Those which are, when viewed from the sea, projected on the ground behind them, should be painted white; those which are projected against the sky, or on a sandy beach, should be painted black or red. These preliminaries arranged, the observer is prepared to commence his operations at sea, having secured the assistance of an able pilot, and of men skilful in the use of the lead.

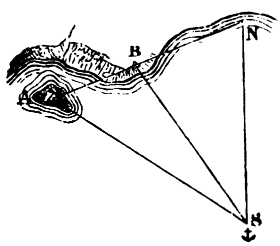
* The moon's age is given in common Almanacs.

Three methods may be used for determining, by reference to fixed points on shore, the locality of any station at sea, such as a rock, shoal, reef, etc.

The first consists in observing, by means of an azimuthal compass, the bearing of two or more points on shore, whereby the position of the observer is determined when the position and bearing of the points on shore are given, with respect to each other. For, let A and B be two objects on shore, fixed in position with reference to one another and to their bearing with the meridian; and let S be the position of the observer, from which the angles, A S N and B S N, formed by the objects, A and B, with the meridian, are observed.

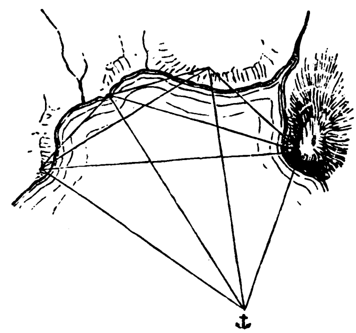


In the triangle A S N, the angles at S and N are given, consequently the angle at A is known; in the same manner the angles at B, in the triangle B S N, is known. Then, in the triangle A B S, the side A B, and the adjacent angles, A and B, being given, the point S may be found.



This method gives at best but a loose approximation, because the angles cannot be determined by the compass nearer than within about 1 or 2 degrees of the truth: the only recommendation in its favour is its great rapidity and facility of execution. When resorted to, it is advisable to employ two compasses, one at a height of 2 or 3 feet above the other, and use the arithmetical mean of the two readings.

The second method consists in observing at the same time, by means of preconcerted signals, from two or more stations on shore, the bearing of the observer at sea with some fixed objects. Theoretically, this method is the most accurate, but practically it is found that even well-concerted signals cannot always ensure simultaneous observation. As the times of observation must, moreover, be registered, at all the stations as well as at sea, a single error in the series, arising from unseen signals, leads to constant misapprehension, and can scarcely be rectified by a subsequent comparison of the different field-books, if the series embrace many observations. Independently of these objections, others present themselves in the form of a greater consumption of time, and a necessity for an increased number of experienced observers.

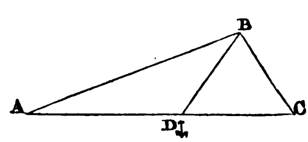


The third method consists in measuring from the boat, or vessel, or rock, by means of the sextant, the angles subtended by three or more objects on shore, the positions of which are given—from these data the position of the observer is determined.

*Theorem.**

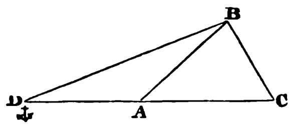
The mutual distance of three remote objects being given, with the angles which they subtend at a station in the same plane, to find the relative place of that station.

Let the three points, A, B, and C, and the angles, A D B and B D C, which they form at a fourth point, D, be given; to determine the position of D.



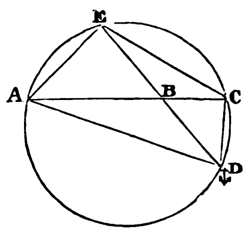
* LESLIE'S *Trigonometry*, page 385, *et seq.*

First. Suppose the station, D, to be situate in the direction of two of the objects, A and C.

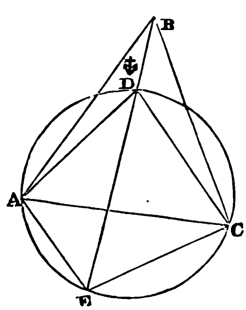


All the sides, A B, A C, and B C, of the triangle, A B C, being given, the angle B A C is found, and in the triangle A B D, the side A B, with the angles at A and D, being given, the side A D is found, and consequently the position of the point D is determined.

Secondly. Suppose the three objects, A, B, C, to lie in the same direction.



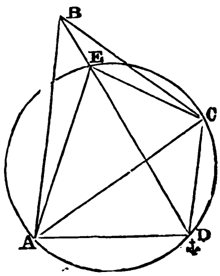
Describe a circle about the extreme objects, A, C, and the station, D; join D A, D B, and D C; produce D B, to meet the circumference in E, and join A E and C E. In the triangle A E C, the side A C is given, and the angles E A C and E C A, being (Euclid III. 21) equal to C D E and A D E, are consequently given; wherefore the side A E is found. The triangle, A E B, having thus the sides A E and A B, and their contained angle EAB or BDC given, the angle A B E, and its supplement A B D, are found. Lastly, in the triangle A B D, the angles A B D and A D B, with the side A B, are given, whence B D is found. But, since the angle A B D, and the distance B D, are assigned, the position of the station D is evidently determined.



Thirdly. Let the three objects form a triangle, and the station D be either within or without it.

Through D, and the points A and C, describe a circle: draw B D, cutting the circumference in E, and join E A, and C E.

1. In the triangle A E C, the side A C, and the angles A C E and C A E, which are (Euclid III. 21) equal to A D B or its supplement, and to B D C or its supplement, being given, the side A E is found.



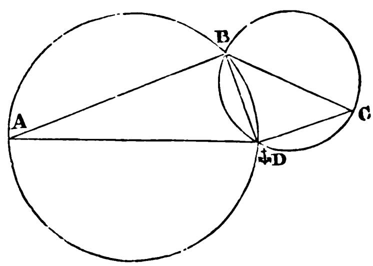
2. All the sides of the triangle A B C being given, the angle C A B is found.

3. In the triangle B A E, the sides A B and A E are given, and the contained angle E A B (being either the difference or the sum of C A E and C A B), is also given, whence the angle A B E or A B D is found.

4. In the triangle D A B, the side A B, and the angles A B D and A D B, being given, the side A D or B D is found, and consequently the position of the point D, with respect to A and B, is determined. By a like process the relative position of D and C is deduced; or C D may be calculated from the sides A C, A D, and the angle A D C, which are given in the triangle C A D.

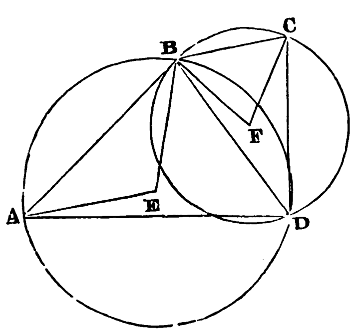
It is obvious that the calculation will fail, if the points B and E should happen to coincide. In fact, the circle then passing through B, any point D whatever in the opposite arc A D C will answer the conditions required, since the angles A D B and D B C, being now in the same segment, must remain unaltered.

This third case, in which the three objects form a triangle, involves the conditions under which the problem has in general to be solved; the first case, in which two of the objects, and the second, in which the three objects are in a line, occurring but rarely. The reader will, however, doubtless have been impressed with the extremely laborious nature of the solution which this case involves, demanding no less than four separate trigonometrical calculations before the required answer is obtained. It would evidently therefore, be a most tedious process, and one but little suited to practical purposes; other means have therefore been devised of solving the problem which are better suited to practice.



The first of these consists in a geometrical construction. Let A, B, and C, be the three stations, and D the position of the observer, at which the angles A D B, B D C, have been measured.

On A B (Euclid III. 33) describe a segment containing an angle equal to that subtended by the objects A and B, and on B C describe another segment, B D C, containing an angle equal to that subtended by the objects B and C; the point D, where the two circumferences intersect, will evidently mark the station required. Should the two circles have the same centre, their circumferencs must obviously coincide, and therefore every point in the containing arc will answer the conditions required, in which case the problem becomes indeterminate.

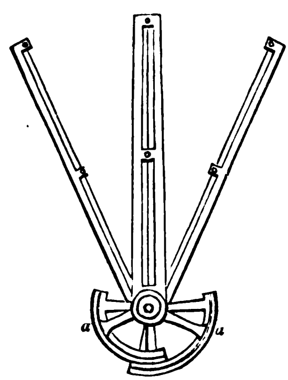


Example. Let the three objects on shore, A, B, C, be fixed in position; and let the angle subtended at D by A B be equal to 50°, and the angle subtended by B C be equal to 40°; to find the point D by construction. Subtract double the angle A B D from 180°, and take half the remainder, equal 40°. Lay off this angle at A and B, the two lines forming the angles with A B will meet in E, the centre of a circle passing through A, B, D (Euc. III. 20). Again, subtract double the angle B D C from 180°, and take half the remainder, equal to 50°. Lay off this angle at B and C; the two lines forming the angles with B C will meet in F, the centre of a circle passing through B, C, and D. The point D, where the two circles intersect, marks the station required.

But this process, although much simpler in point of construction than that previously explained, would yet be exceedingly tedious where a great number of stations had to be determined. To simplify the construction, an instrument, called the station-pointer, has been invented: it affords means of laying down the work with great rapidity, and with sufficient accuracy for all practical purposes. The following is a description of the instrument.

The Station Pointer

Is formed by three limbs or rulers, A, B, C, which revolve round a common centre, in such a manner that B and C may be set to form any angles with A. 'The middle ruler is double, and has a fine wire stretched along its opening; the other rulers have likewise a fine wire stretched from end to end, and so adjusted by the little projecting pieces which carry them, that all the three wires tend to the centre of the instrument, where they would meet if produced.

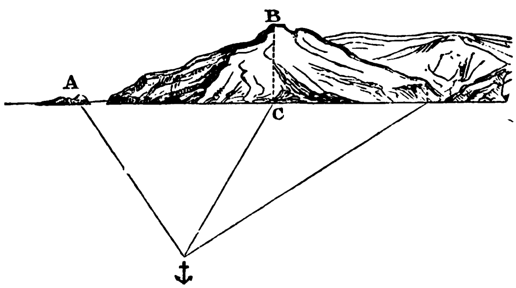


Through the centre is an opening sufficiently large to admit a steel pricker.' The middle limb carries at the extremities of two arms, the verniers *a* and *a′*. Two arcs, of about 100° each, are connected with the limbs B and C, in such a position that, when the instrument is closed, the verniers *a* and *a′* mark zero; and when the limbs are opened, the angles they respectively form with A are marked by the verniers on their corresponding arcs. The angles subtended by three stations on shore, at the place of the observer at sea, are the measures of the readings to which the verniers are to be set; and, when properly fixed, the instrument is laid on the plan, and moved till the three wires pass through the three stations: the centre of the instrument then occupies the relative place of the observer, and a dot marked by the steel pricker determines the point D on the plan.

It is evident, from this description, that, in the absence of the station-pointer, a graduated circle marked on any transparent paper, or on a plate of glass, may be used instead, by drawing on the upper surface of such a circle lines diverging from the centre at the given angles; the circle being moved until these radii pass through the stations, the centre of the circle will give the point required.

The demonstration given above (page 251) proves that, with the exception of the case in which the observer is in the circumference of the circle passing through the three stations, the measure of two angles is sufficient to determine his position. In practice, however, as many angles as possible ought to be observed to the surrounding stations, in order to obtain greater accuracy; and in no case ought the observer to rest satisfied with the measure of two angles only, unless necessity compels him to adopt this extreme limit.

The angles are observed with the sextant or reflecting circle, and are consequently measured in the plane of the objects. If this plane be inclined to the horizon, and a result rigorously accurate be sought, the angles of elevation of each station above the horizon should at the same time be observed, to afford data for reducing the hypothenusal to the horizontal angle. But this reduction may be neglected in all cases where the difference of elevation between the objects does not exceed 2^0 or 3^0, and when the observed angle is larger than 20^0 or 25^0; for the reduction to the horizon would, in such cases, deal with quantities more minute than the amount of error to which the measures of all angles observed at an unstable station are liable.



When the difference of elevation between the objects is considerable, an ideal vertical line (see sketch) may be drawn from the higher object downwards to an elevation corresponding to that of the lower object, and this, with some experience and correctness of eye, will give results sufficiently near to the truth.

With the sextant no telescope should be used, because the objects are more quickly brought into the field of the mirrors by the unassisted eye, and rapidity of execution is most important in the observations.

Metallic reflectors should be used instead of the glass mirrors, as sea water, to the effect of which the sextants are constantly exposed in such observations, rapidly destroys the silvering. Another reason for preferring the metallic reflector is, on account of the indistinctness caused by the objects being reflected from both surfaces of the transparent mirror, which indistinctness is very much increased when a telescope is used.

When there is only one observer engaged in the operation, he should, when measuring the angles subtended by objects on shore, have at least three sextants ready at his hand, so as to measure with different instruments the separate angles in rapid succession, without losing the time that would otherwise be necessary to enter the reading of the sextant after each observation. For, on unstable stations, such as boats or vessels, the angles should be taken as nearly as possible at the same instant.

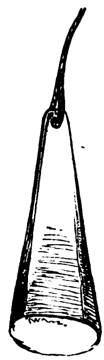
The field-books should be kept with an indelible pencil, the mark of which is not liable to be effaced by the washing of sea-water. The form of field-book to be used in registering the angles is given below. In the first column is entered the index error of the sextant; in the second, the precise moment of commencing the observations, with remarks as to the 'status' of the boat or vessel, whether at anchor or otherwise; in the third column are entered the angles. The single letter opposite the bracket marks the first station on shore or station of departure, and a note is made, stating whether the other stations are to the right or left of the first.

July, 1841. *Bay of* ________.

Index Error. Sextant.			Observed Angles.	Remarks.
			To the right.	
′ ″	h. m.		° ′	
Sextant 1 .. 0 10	8 10	A	I 26 18	
„ 2 .. 0 20	boat at anchor.		B 68 54	
„ 3 .. 1 0			C 89 42	
			To the left.	
Errors as before.	8 25	C	A 58 28	
	steadied by the oars.		B 24 32	
			E 82 4	This angle doubtful.

Sounding Lines.

Sounding lines should be made of strong pliable cord, known as 'lead line,' divided into feet by different coloured rags, or other marks. The lead, fastened at one extremity, is shaped like the frustum of a cone, with the base hollowed out to hold some grease, to which the sand or mud at the bottom of the water may adhere, serving thus to show the probable nature of the anchorage. Lines are used different in length and strength, and leads differing in weight, according to the depth of the water in which the casts are made. The lines, especially when new, must be occasionally compared during a day's work with a standard measure always at hand, as they are liable to great and sudden changes. It is almost needless to observe that, in open waters, an experienced leadsman must be employed, whose reading of the depths should, nevertheless, be frequently checked. When the soundings are deep, the boat's way must be stopped at each cast, in order that the depth may be measured in a vertical direction.



Sounding Lead.

On shallows or reefs near the surface, and generally in all anchoring grounds of small depth, where accuracy is consequently of the utmost importance, sounding rods, divided into feet, and weighted at the extremities, may with advantage be substituted to obtain greater correctness.

The grease let into the hollow base of the lead, or the 'arming,' shows the nature of the surface of the bottom; but before we pronounce upon the quality of an anchorage, we should likewise know, if possible, the nature of the material for some depth under the immediate surface. This object is accomplished by means of a lance-shaped pike, of a length and weight proportionate to the depth to which it is desired to penetrate beneath the surface. The part

Lead weight.

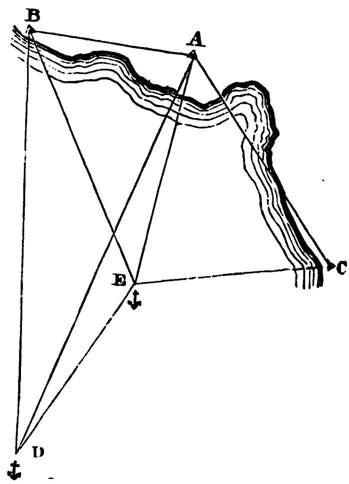
below the lead is indented in the same manner as a rasp, and the indentations on its surface bring up specimens of the deposit or formation traversed by the instrument, thus indicating in some degree its nature. If the pike be impeded in its progress by rock, its bent or broken point, when brought up to the surface, gives evidence of the fact. Additional value may, by such means, be given to the soundings; and in the field-book, at the entries of the nature of the bottom, a mark should be made to distinguish the data obtained by this instrument from those obtained by the common lead. (See form of field-book, page 260.)

Survey of Shallows, Reefs, Sunken Rocks, etc.

As it is of importance that the position of shallows, reefs, sunken rocks, etc., should be determined with the utmost possible accuracy, the observer should, whenever practicable without actual danger, cast anchor while making his observations, in order to insure greater accuracy in the measurement of his angles, choosing for his stations on shore those which will subtend the largest angles at the place of the boat. Several remarkable points bring thus determined at anchor, others may be fixed by means of two angles taken with rapidity while the boat is steadied by the oars. Sails ought rarely to be used when observations are made in these cases, as it is impossible under such circumstances to steady the boat. Moreover, the sails obstruct the sight in the measurement of the angles.

When it is required to determine a shoal or reef, etc., so far out to sea that only two objects on shore are visible, an assistant boat is moored temporarily between the distant observer and the coast, in such a place that one additional station can be seen from it. At a given signal, angles are observed from the assistant boat to the three objects on shore and to the distant boat; and from the latter, angles are at the same signal measured to the two stations on shore and to the assistant boat.

Thus, in the annexed figure, let D be a station on the distant reef from which the two elevated stations, A and B, can be seen, let E be the position of the assistant boat from which the three stations, A, B, and C, can be seen; then at the same moment of time, the angles CEA, AEB, BED, are observed from E, and the angles EDA and ADB from D,—and by their means the position of D is determined. For the point E is fixed in position by means of the observed angles CEA, AEB—and it becomes, therefore, a fixed station with reference to D, from whence two angles are observed to three stations fixed in position.*

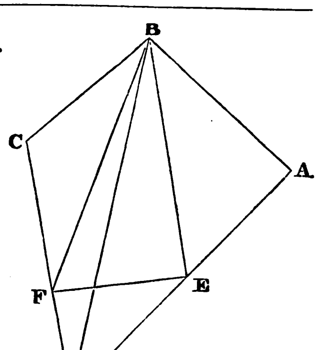


If breakers or currents denoting danger are observed in a certain state of the tide, and it be impossible at the

* This question becomes somewhat more difficult of solution if only one angle can be observed from E. It is enunciated thus by Leslie:

'The mutual distances of three remote objects, two of which only are seen at once from the same station, being given, with the angles observed at two stations in the same plane, and the intermediate direction of these stations being also given—to find their relative places.

'Suppose the three points, A, B, and C, are given, with the angles AEB and BFC, and likewise the angles AEF and EFC; to find the relative situation of the stations E and F.



time to anchor over them, or to fix a buoy to mark their locality, their position should be determined approximately by intersections of prominent objects on shore, so disposed as to guide the observer to the spot in a more favourable state of the tide, when a perfect calm may leave no trace whereby the dangerous pass can be recognised.

A sketch or profile of the coast should be made before each series of important observations; and the stations should be defined and referred to on the sketch. These profiles are useful, not only in assisting the observer to recognise the coast when constructing his chart, but also in presenting to mariners the appearance of headlands and other striking points.

'Produce AE and CF to meet in D, and join BD; the angle EDF, being equal to AEF + CFE − 180° (Euc. I. 13, and Cor. 32), is given. Now, in the triangle EBF,

$$\text{Sin. BFE} : \text{sin. EBF} :: \text{EB} : \text{EF},$$

and in the triangle EDF,

$$\text{Sin. EDF} : \text{sin. DFE} :: \text{EF} : \text{ED};$$

wherefore (LESLIE, V. 23, *Elements of Geometry.*)

$$\text{Sin. BFE} \times \text{sin. EDF} : \text{sin. EBF} \times \text{sin. DFE} :: \text{EB} : \text{ED},$$

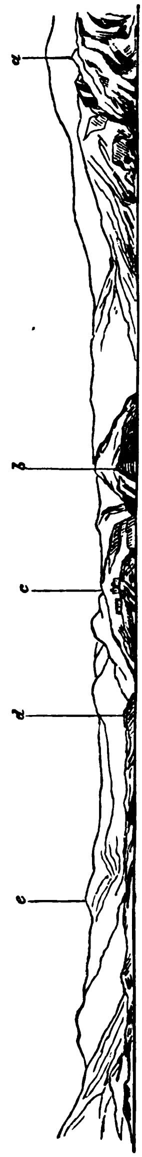
and, consequently, the ratio of EB to ED is found. Again, the angle BED, being the supplement of AEB, is given, and

$$\text{Sin. BFE} \times \text{sin. EDF} : \text{sin. EBF} \times \text{sin. DFE} :: \text{EB} : \text{ED}, \\ :: \text{R} : \text{tan. } b,$$

and $\text{R} : \text{tan.}(45^\circ - b) :: \text{cot.} \tfrac{1}{2} \text{BED} : -\text{cot.}(\tfrac{1}{2} \text{BED} + \text{EBD})$ or $\text{cot.}(180^\circ - \tfrac{1}{2} \text{BED} - \text{EBD})$, whence the angle EDB is given. The angles which all the three objects, A, B, C, subtend at the point D are therefore all given, and hence the position of D is determined by the preceding proposition. But BD being found, the several distances BE, ED, and BF, FD, are thence obtained, and consequently the position of each of the stations E and F is determined.'

As this operation is somewhat too laborious for practice, in such exceptional cases as this, when only two stations could be seen, the azimuth compass might be used, as by its means the position of a station is determined with only two fixed objects in sight.—LESLIE'S *Trigonometry*, page 389.

On the chart directions are given for sailing or working into harbour, such as—'Lighthouses in one S.E. ½ S., lead over the bar, and up the channel to within two cables of the buoys.' Such and similar information is to be entered on the authority of trustworthy pilots and others well acquainted with the locality.



While standing off and on to detect shoals or changes of level, a certain fixed direction must be followed and entered in the field-book, and whenever the direction changes, the point thus formed becomes a station from which angles are to be taken to the fixed objects on shore. Soundings are to be taken at each of these stations, and also in passing from one station to another. The soundings taken at the stations are entered in a column opposite the observed angles, and any intermediate soundings are to be entered between them. When constructing the chart, the intermediate soundings are to be distributed at equal distances along the line between each pair of stations, the time of taking the soundings being noted only at the stations where angles are observed.

The annexed form of field-book will illustrate the mode of operation.

Index error. Sextant.	Hours and Minutes.	Angles observed to Objects on Shore.	Soundings. Feet.	Remarks, July, 1841.
20″				The watch being compared with that of the tide-registrar, the first sounding is taken near station A.
	8 10		3; 5, rock 7; 9; 12, rock; 14, sand.	Proceeding in a line from A to F.
	8 19 steadied by the oars.	To the right, ° ′ A { I.. 26 18 B.. 68 54 C.. 89 42	15, sand.	
			19; 24; 29.	Continuing in same direction.
	8 25	To the right, H past C { I.. 58 28 B.. 24 32 A.. 82 4	31, sand and mud.	
			40; 45; 49; 50.	Ditto.
	8 35	To the left. I { A.. 22 32 B.. 28 48 E.. 28 48	52, mud.	
			48; 39; 36, mud; 32, mud.	Direction changed towards C.
	8 50 at anchor.	To the left, C { B.... 38 36 I.... 63 24 A.... 107 30 To the right, D.... 26 18 E.... 67 36	29, sand:— rock by the sounding pike.	
	9 0 weigh anchor.		25; 20, sand; 19; 20, sand and shells.	Continuing in same direction.
	9 10	To the right, C { D.. 30 4 E.. 84 16 F.. 95 20	29, shells and mud.	
			39; 30, sand; 21; 17, sand; 8, sand.	Change direction, and proceed S. by compass, in order to cross the bar which appears to connect the rock E with the shore.
	9 25	To the left, E { D.. 21 13 C.. 49 1 F.. 77 55	6, sand.	
			8; 18, sand; 21; 27; 35, sand.	Continuing in same direction.

Reduction of Soundings.

The reduction of soundings consists in deducting from the depths, as registered in the field-book, proportionate quantities varying with the time, in order that all the depth may be referred to the lowest level of the tide. These quantities are obtained from the data supplied by the tide-registrar, and are arranged according to the annexed form for each day on which soundings and observations have been made.

Name of tide registrar.	Date, 1841.	Time.		Deductions.	Time.		Deductions.
		h. m.	h. m.	feet.	h. m.	h. m.	feet.
A. B.	July.	8 0 to	8 15	5	3 25 to	3 55	9
		8 15	8 35	4	3 55	4 30	10
		8 35	9 5	3	4 30	5 0	11
		9 5	11 0	2	5 0	5 25	12
		11 0	11 35	1	5 25	5 45	12
		11 35	0 10	2	etc.	etc.	etc.
		0 10	0 30	3			
		0 30	1 10	4			
		1 10	1 40	5			
		1 40	2 10	6			
		2 10	2 45	7			
		2 35	3 25	8			

For example, if the sounding over the highest part of a sand-bar gave a depth of 27 feet at the time that the annexed table or the tide-gauge indicated an elevation of 11 feet above the lowest tide, the sounding marked on the chart would be $27 - 11 = 16$ feet. If the depth over the shoal were registered at only 6 feet at the same time that the annexed table or tide-gauge indicated an elevation of

11 feet above low-water, then the true level of that part of the shoal at full and change of the moon would be $6 - 11$, indicating that it would be left high and dry at an elevation of 5 feet above the lowest tide.

The final entry in ink of the soundings is made in fathoms and quarter fathoms on all English charts; no decimals ought to be used, because the non-observance of the decimal point or its accidental omission by the engraver, a case of no rare occurrence, would lead to most disastrous consequences, which would not necessarily follow from a mistake made in the entry of a fraction.

Maritime Surveying without the aid of Triangulation on Shore.

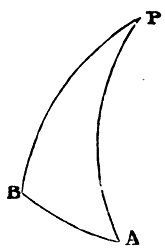
I now proceed to describe a method of constructing charts embracing a great extent of coast, second in point of accuracy to that which has for its basis a regular system of triangulation. This triangulation demands a considerable expenditure of time and money, and, of course, requires a free access to the country. It cannot, therefore, be put in practice where the country is in the occupation of an enemy, nor in cases where a considerable outlay of money is inexpedient. The following method, free from those drawbacks, is capable of giving results remarkable for their comparative accuracy: its principle or characteristic is this, that instead of commencing, as in triangulation, with the measurement of a short base, from which a net-work of triangles is to be spread over a great extent of country, the first step in this process consists in using, for the basis of the operations, a very long base, say of 40 or 50 miles, and filling in the intermediate details by working from the whole to part, thus subdividing, at each intermediate step, any error that may exist in the original base, instead of accumulating error upon error, as must be the consequence were the contrary system to be adopted, without the checks which a trigonometrical operation affords. It may be observed here, that, in triangulation on shore, the accumulation of errors is prevented by the extreme care and precaution which we have described as necessary when treating on that subject; and that errors, if any have crept in, are detected by the measurement of the basis of verification.

A judicious choice of the primary stations, or the stations determining the extremities of the great base, is important; but of course no precise directions can be given which shall be applicable to all circumstances of locality and climate. In general it may be observed, that the stations should be elevated peaks or headlands easily recognised; they must be at a considerable distance from each other (say 40 or 50 miles); but still the distance must not be so great as to prevent several intermediate points from being seen from both stations by the use of good instruments. Such intermediate stations are to be selected along the whole line of coast to be surveyed.

The locality of the first station being selected, its latitude and longitude are to be obtained by careful astronomical observations made from it (see pages 224 and 232); and in addition, if the observations be made by day, the azimuthal bearings of as many important points as can be seen between the first and second primary stations are to be taken. But should any hostile feeling of the inhabitants prevent the observations being made by day, these last objects cannot, of course, be observed, but greater time may be bestowed on the astronomical observations; the latitude being obtained by observations of the pole-star, reduced by means of the tables given in the *Nautical Almanac;* and the longitude by observation of some of the stars usually selected for that purpose.

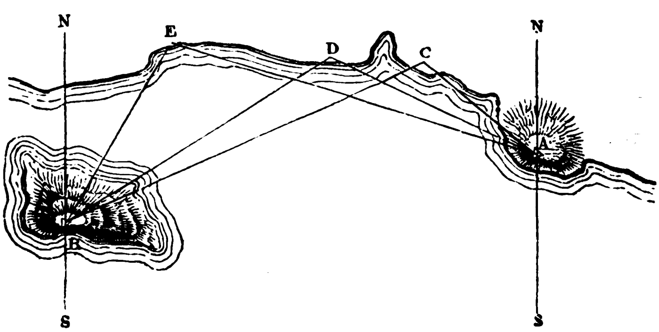
Without stopping at any intermediate points, similar observations are to be made consecutively at all the *primary* stations along the whole line of coast to be embraced in the survey. This continuity in the astronomical observations affords the important advantage, that the relative positions of the stations are determined while the rate of the chronometer remains unchanged: and in sailing back to the starting point to commence the work in detail, a thorough acquaintance with the leading features of the coast is obtained, and an occasional check may be made on the previous observations.

The distance between any two of the primary stations, whose latitudes and longitudes are thus determined, is obtained as follows:—



In the annexed figure, let P represent the pole of the earth, and A and B the two stations. The longitudes being known, the angle P, or their difference of longitude, is given. The sides, P A and P B are also given, being the respective co-latitudes of the two stations. We have, therefore, in the spherical triangle, ABP, two sides, and the contained angle, from which we get the length of the opposite side, AB.

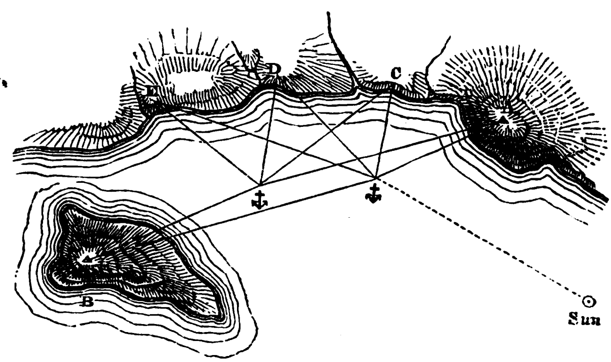
It is evident, that the line joining any two of these stations is the arc of a great circle on the earth's surface; it must, therefore, be reduced to its chord, which is equal to



twice the sine of half the arc (see page 92). This reduction having been attended to, several intermediate points will also have been determined, if the observations at the primary stations have been made by day, as shown in the last figure, where A and B are the primary stations, from which the azimuthal bearings of the stations C, D, E, etc., have been observed.

But if the observations have been made at night, when no intersections could be obtained, intermediate points are to be fixed in position by forming two or more temporary stations of the vessel anchored at convenient distances from A and B, and about 8 or 10 miles from the shore.

The watches having been regulated with great care during the observations made at night, the position of the vessel is determined by observing the angles subtended between the sun and the primary stations, and notifying the exact time of the observations.



The time gives the sun's azimuth, and from it is deduced the azimuth of the two primary stations from the vessel. The intermediate stations, C, D, E, etc., are obtained by the intersection of their lines of direction, as observed at two or more of such stations of the vessel. Then a secondary series of points on shore, nearer to each other, is determined from the vessel, at a distance of 2 or 3 miles from the coast. Lastly, when these are protracted, the diagram is ready to receive the topographical details of the coast, and the soundings, the first being marked by sketching, the second according to the method already described. This process is repeated between the several primary stations, and the entire chart being then joined, represents an extensive district, the details of which have been obtained by proceeding from the whole to part, as before recommended.

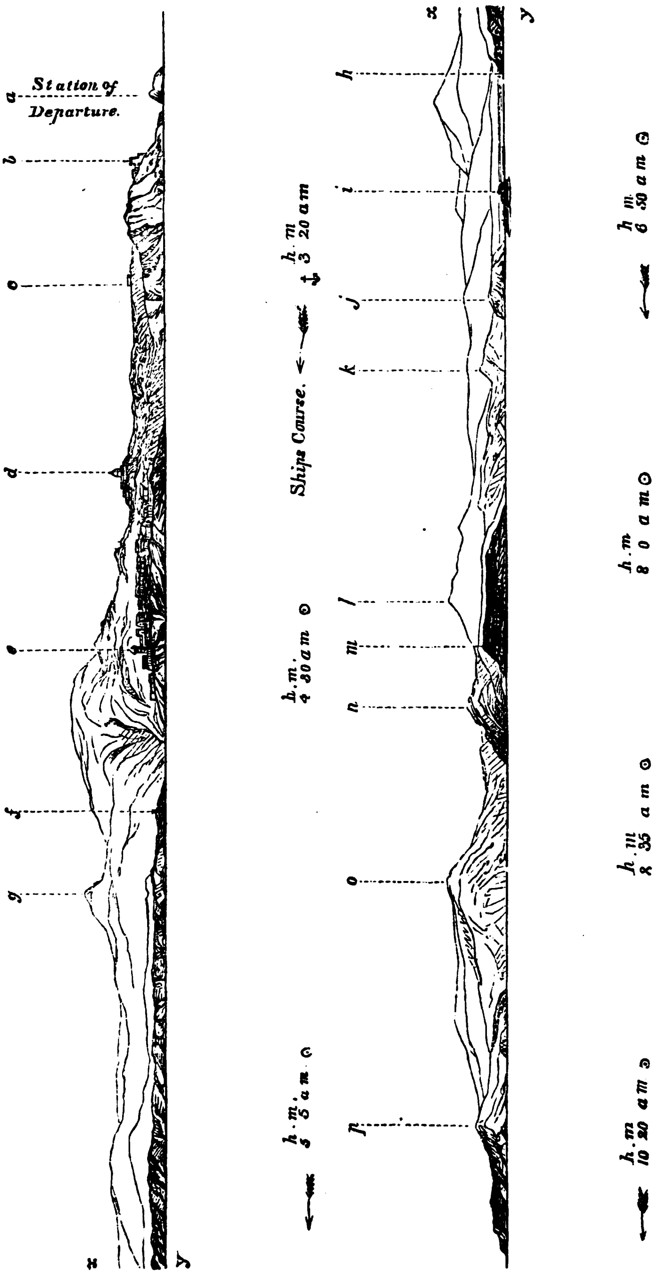
Maritime Surveying under Sail.

Third, in point of accuracy, but nevertheless highly useful, and sufficiently correct to be of great assistance to the mariner, is the survey made of a coast while sailing along it. It is, in fact, in this manner, that nearly all original maritime surveys of new colonies, or newly-explored lands, have been made. This process differs in some of its details from those we have before described; but in common with them the information is acquired by means of the angular distances between remarkable points on land, as observed from the vessel, which should, if possible, be brought to anchor, or steadily hove to, while the observations are being made. But, as the angles measured are subtended by objects on shore which are not and cannot be visited, arbitrary definitions must be employed to denote and recognise those objects, and sketches or profiles of the coast should be made from each point of observation. Indeed, under the present circumstances, such profiles may be said to be absolutely indispensable. These profiles save the necessity for written description, and assist in detecting angles that may have been entered glaringly wrong through haste or any other cause: they are also of great assistance in protracting the work, by bringing back a vivid recollection of the appearance of the points observed, their situation to the right or left of the station of departure, and other circumstances attending the observation; finally, they are advantageously referred to when sketching in the ground-plan or contour of the coast. It is customary also to mark, on these profiles, each point observed by a distinctive letter, and to write opposite to it the angle which it forms with the station of departure, in addition, of course, to the regular entry in the field-book. As a general rule, the result of each day's observation should be protracted in the evening, when every occurrence is fresh in the observer's memory.

At each station of the vessel, astronomical observations are made to determine its position, at the same time that the angular distances between objects on shore are measured; two observers should therefore be employed, in order that the observations of the celestial bodies, and of the objects on the coast, may be made at the same moment of time. The angle formed between the sun and the first point of departure on shore should likewise be observed, especially when the sun is just appearing, or is not much elevated above the horizon, as in this case the reduction to the horizontal angle may be omitted. The azimuthal bearing also of the first point from the vessel is to be taken by one or more azimuthal compasses, and the mean azimuthal angles serve to confirm its direction as obtained from the observed position of the sun, and to give it independently, if the sun has not been observed.

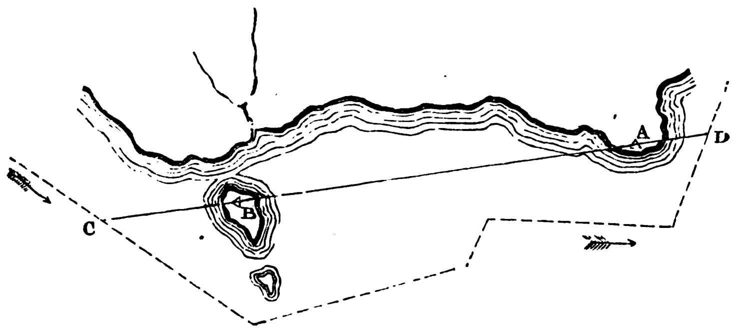
Azimuthal angles, thus taken, even under favourable circumstances, cannot be relied upon nearer than to two degrees or more, owing to the movement of the vessel, and the constant change in the variation of the compass. In a small boat, as the motion is much greater, the errors may be expected to increase also.

Similar observations, bearing on the same and on new points that open in sight, are repeated at various distances


Station of
Departure.
a
b
c
d
e
f
g
x
y
Ships Course.
h. m.
3 20 a m
h. m.
4 30 a m
h. m.
5 5 a m
h
i
j
k
l
m
n
o
p
x
y
h m
6 30 a m
h. m
8 0 a m
h. m
8 35 a m
h. m
10 20 a m

as the vessel proceeds. In sailing from one vessel's station to another, especial care must be taken not to lose sight for a moment of the points to which angles have been observed, as their continually changing aspects would otherwise make it difficult to recognise them. Inattention to this point would infallibly lead to numerous errors, and to delay and confusion in the construction of the charts. To assist the eye, a prominent and easily recognised object should always be chosen as the point of departure.

Also while sailing from one ship's station to another, a reckoning of the rate of going is to be kept carefully by the log-line; this should seldom be relied upon to determine



the ship's course, as its inaccuracy is notorious; but it may serve as a collateral check on the distance of the vessel's stations as obtained from astronomical observations, and must sometimes, perforce, be used when angles are measured to objects on shore under conditions of the atmosphere that do not admit of astronomical observations.

When standing on and off the coast, especial care must be had to take advantage of the appearance, in the same straight line with the ship, of any two of the observed points, the time of such transits being entered in the field-book. These bearings are useful when laying down the points and ship's course on the chart. It must be observed, however, that when one of these stations, thus appearing in the line, is very distant, there is no certainty of its extreme projection or lowest point being seen. This additional element of error may be avoided by having an observer on the look out at the mast-head, to give notice of the exact instant when he sees both the points in a line.

The angles measured are entered in the field-book, precisely in the same manner as those we have before described; the soundings and the reckoning, and other remarks, are entered in the field-book according to the form annexed.

Time.	Knots.	Fathoms.	Courses.	Winds.	Soundings.	Remarks.
A.M.					Fathoms.	
2	..	..	At Anchor.	W.	15	Weighed, and stood to A.
3	1	6	S.S.W.	W.	14¼	
4	2	..	S.	.. ..	20	
5	2	..	S. by W.	.. ..	23½	
6	3	4	S.S.W.	.. ..	20	
7	2	..	S.W.	N.W.	18½	
8	$\frac{1}{1}$	..	E. by N.	Tacked.	22¾	
9	1	4	E.	W.	27	
10	1	6	S.W.	Tacked.	21	
11	3	..	.. ..	S.W.	19	
12	3	..	.. ..	S.W.	16½	

Measuring Distances by Sound.

Sound, as a means of ascertaining great distances approximately, is not to be neglected. Its velocity, as ascertained by the latest experiments,* may be assumed at 1089, or in round numbers 1090 feet per second of time; a watch, therefore, by which an observer can measure with accuracy fractions of seconds, will enable him to determine

* *Phil. Trans.*, 1824. Dr. Moll's Account of the Experiments on the Velocity of Sound.

a distance of several miles within about 100 or 200 yards. Observations of this nature should not be undertaken, without being provided with a good instrument for measuring fractions of a second with precision. A stop-watch, known by the name of chronograph,* answers this condition. It is so constructed, that one of its hands, which performs a revolution in a second, can be made to touch, with its extremity, the dial-plate at any instant of time, by the sudden pressure of a lever, and, leaving there a black dot, proceed without more than this momentary stoppage of its rotation.

To estimate distances by this method, two boats or vessels are moored at some distance from each other, and guns are fired alternately from each vessel, whilst the time elapsed between the flash and the report is noted by means of the stop-watch. The time occupied by the passage of the light is equal to zero, and a simple proportion gives the distance between the two vessels. Angles being observed from each vessel to objects on shore previously agreed upon, the positions of these objects are determined with relation to the base or distance between the vessels.

Distances may also be obtained, by approximation, by means of the instrument known as Dr. Brewster's micrometer telescope, and described in BREWSTER'S *Philosophical Instruments*.

---

* The chronograph is of Bréguet's invention.

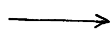
'Il (Bréguet) a construit des montres, dont l'aiguille marque subitement et à volonté un point très-visible sur le cadran, sans que l'impulsion donnée cause la moindre interruption dans la marche de l'instrument. On peut mesurer ainsi, avec une éxactitude rigoureuse, la durée des effets observés, ce qui est l'objet d'un grand nombre de recherches physiques. Nous devons ajouter qu'un artiste Français, M. Rieusacq, a employé le premier un procédé de ce genre pour des usages civils. M. Bréguet a changé le caractère de l'instrument, et lui a donné un nouveau degré de précision.'—*Eloge Historique de M. Bréguet*, par M. LE BARON FOURIER, Mem. de l'Acad., tom. vii., 1827.

Charts.

Charts are protracted with the true meridian pointing towards the top. At convenient places a mariner's compass is drawn, and the variation is shown by a small fleur-dé-lis, which terminates the magnetic north and south line, drawn at the proper angle through the centre of the compass. Along the coast, lines of 1 fathom, 2 fathoms, 3 fathoms, etc., are marked by dotted lines, thus—

. . . .
.. .. } 1, 2, 3, fathom lines.
... ...

The run of the flood tide is marked thus: 

That of the ebb tide thus . . . . . . 

Buoys are marked thus . . . . . 

Good anchoring places . . . . . 

Stopping places . . . . 

Between high and low water:—

Rocks . . . . . . 

Sand . . . . . . 

Mud . . . . . . 

Under the soundings, letters are added, denoting the nature of the bottom, thus:—

s, for sand; *m*, for mud; *r*, for rock; etc.

The measure of length employed is that of nautical miles (60 to a degree), each mile being divided into 10 cables.

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