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PARABOLIC REFLECTORS FORMED BY INFLATION

R. N. BRACEWELL and K. M. PRICE†
Department of Electrical Engineering, Stanford University, Stanford, CA 94305, U.S.A.

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Abstract—Paraboloids of revolution over 1 meter in diameter have been formed from flat sheets by inflation without the use of a mold or template. Quality is adequate for microwave use and for most high-concentration applications for focusing solar energy. The process is suitable for making reflectors in ones or twos to special focal lengths and diameters, but could also be automated for production runs.

INTRODUCTION

Parabolic reflectors a few meters in diameter are required for a number of purposes such as microwave relay antennas, solar concentrators, searchlight mirrors and astronomical telescopes. The cost is extremely high for telescope mirrors where surface accuracy of a fraction of an optical wavelength is required, but even where surface tolerance may be relaxed to 1 mm or so, as with centimeter-wave antennas, the cost is not low. For example, the cost will generally be orders of magnitude higher than the material cost. The reason for this lies with the mode of manufacture. Several techniques depend upon the construction of a steel mold or dies, as with vacuum forming and pressing (sheet metal), a graphite mold (glass slumping) or a mold made from plaster or other soft material (fiberglass). The initial cost of the mold is substantial and has to be spread over the number of reflectors manufactured. An unattractive feature of a valuable mold is that the focal length is frozen so that no flexibility is gained that might allow the mold to be adapted to other focal lengths later. Templates are cheaper than molds and have been used for spinning microwave antennas from sheet metal.

Composite paraboloids up to 5 m in diameter have been built from identical spherical segments in cases where one or two optical reflectors have been needed, and even machining from solid metal has been the economical way to go in some cases. Even so, when only a few reflectors of special dimensions are needed, there has been no really economical process available. In some fields, such as solar energy, economic considerations are paramount.

We have therefore looked into a moldless clamp-and-inflate fabrication method for sheet metal reflectors. First we describe the method, then we study the design parameters and report on tests that have been carried out.

BASIC METHOD

In Fig. 1, we see two pieces of sheet metal M clamped between two circular steel rings R. Fluid under pressure is introduced between the plates through a valve V, forcing the sheets apart. The stress in the sheet metal rises beyond the elastic limit and plastic flow sets in as the assembly inflates like a balloon. When the desired amount of dishing is reached, the valve is closed, and the pressure is released. This procedure is more direct than inflation of collapsible molds[1], inflation of membranes to which solidifiable substances can be applied[2], or pumping up of a membrane that shapes epoxy resin to which hardener may be added[3].

DESIGN PARAMETERS

At first sight it might seem that rather high pressures might be needed to plastically deform a substantial volume of sheet metal, but in fact readily available and relatively safe tire-inflation pressures are found to suffice.

Let D = reflector diameter; F = focal length; β = rim slope angle = arctan $(4F/D)$; α = semiangle subtended at focus = arcsin $\{F/D\ [0.125 + 2(F/D)^2]\}$; and δ = depth of parabola = $F - D/(2\tan\alpha)$. The foregoing quantities specify the parabolic geometry (Fig. 2). In the example worked out we consider a 1.12 m diameter reflector with a 1.12 m focal length. For this case, $\beta = 76°$, $\alpha = 28°$ and $\delta = 7$ cm.

Further quantities are as follows: t = sheet metal thickness; p = fluid pressure; f_y = yield stress of sheet metal; L = axial load; and T_1 = tension in sheet metal per unit length at rim. The pressure p acting on a circular area of diameter D produces an axial load L given by

$$L = (\pi/4)D^2p. \tag{1}$$

In equilibrium this must also equal the axial component $T_1 \sin\beta$ acting on the perimeter πD, so

$$\pi D T_1 \cos\beta = L, \tag{2}$$

and, if the yield stress is just reached,

$$f_y = T_1/t. \tag{3}$$

Combining these equations we find that the pressure required to take the metal into plastic deformation is

$$p = 4f_y(t/D)\cos\beta. \tag{4}$$

If we use 1100-O aluminum, whose yield stress is 35 MPa

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†ISES member.

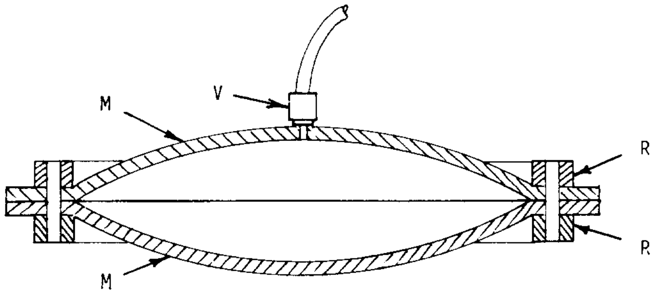

Fig. 1. Fluid admitted under pressure through valve V inflates sheet metal M held between clamping rings R.

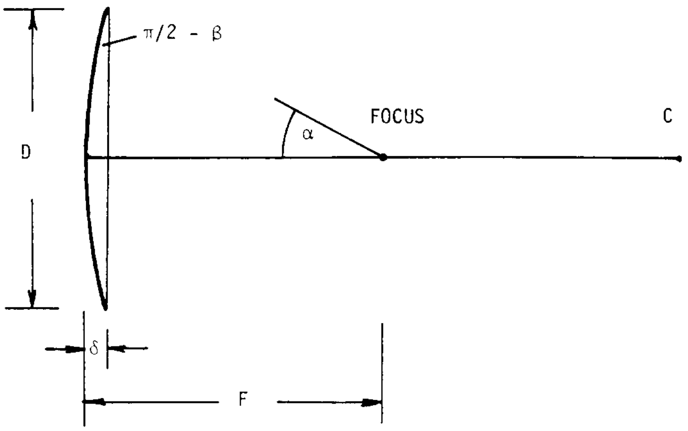

Fig. 2. Geometry of a paraboloid. The center of curvature of the sphere touching the paraboloid at its vertex is at C.

and a thickness of 3 mm, we find that the pressure p needed is 100 kPa (15 psi). On the other hand if we use 1.5 mm cold-rolled steel sheet with a yield stress of 262 MPa, then we need a pressure of 360 kPa (52 psi). On the basis of these modest calculated pressures we were encouraged to make experimental trials.

FABRICATION TESTS

Two tests will be described in detail. The first used a pair of 0.9 m (35-in.) diameter steel clamping rings made of 12.7 mm square bar bent into rings by passing through rollers and welding. Material was 6061-0 aluminum 2 mm (0.080 in.) thick. The rings were held together with 44 high strength 4.83 mm (10–32) steel screws. The two sheets were each inflated to a height of 5 cm. Overpressure is convenient to speed up the inflation forming, and gasket cement was found necessary in order to counter leakage. Two satisfactory dishes resulted which were very stiff and could be walked on without damage. Thus the possibility of fabricating reflectors with such modest tooling as a pair of clamp rings was confirmed.

In order to investigate the method itself in more detail a more elaborate fixture was constructed in the form of a 1.2 m (48-in.) diameter circular table 22.4 mm thick with 36 bolt holes for 19 mm (3/4 in.) bolts and a valve. A clamping ring 5 cm wide by 1.9 cm thick and 1.2 m outside diameter was provided. This new installation bypasses effects due to flexibility of the clamping system itself and to the insertion of the valve in one sheet. Several satisfactory 1.12 m (44-in.) reflectors were formed with a height of 7.5 cm from 6061-O aluminum 2 mm thick and 1100-O aluminum 3 mm thick.

SHAPE MEASUREMENTS

Very smooth shapes were produced which in the radial cross sections examined showed root-mean-square departures of about 0.5 mm from the paraboloid of best fit. At a wavelength of 30 mm, rms errors of about 3 mm are tolerable, so for microwave antenna applications the shape is essentially perfect. Some astigmatism resulted from the fact that the yield stress was higher in the direction in which the aluminum sheet was rolled in the course of manufacture, and with hardened alloys this effect was even more noticeable. A paraboloid is soft against deflection of the rim into an ellipse and for this reason needs a stiff mounting ring at or near the rim. The astigmatism is easily taken out as the reflector is fixed to its mount.

There is no reason to think that plastic deformation under fixed pressure will lead to a paraboloidal shape, in fact a spherical shape would be the equilibrium axisymmetric surface for homogeneous isotropic material in the absence of flexural rigidity. However, there is not much difference between a sphere and a paraboloid in our shape range as may be verified by comparing the parabola $y^2 = 4.48x$ with the circle $(x - 2.24)^2 + y^2 = (2.24)^2$. For example, at the rim $y = 0.56$ m, we have $x = 7.00$ cm for the parabola and $x = 7.12$ cm for

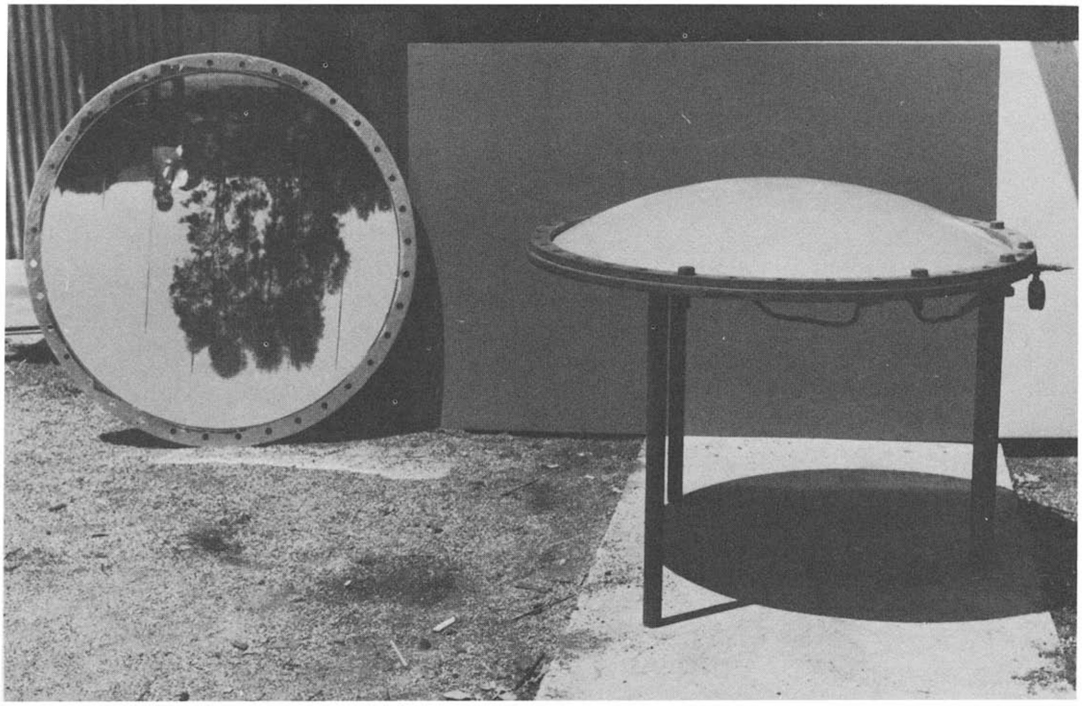

Fig. 3. A finished reflector (left) and a reflector about to be released from its clamping ring (right).

the circle, a discrepancy of only 1.12 mm. The parabola of *best fit* to the given circle would agree even more closely with an average departure of only about 0.3 mm. Although we have made our surface shape measurements to within 0.3 mm, it is clearly a matter of some refinement to ascertain the exact shape achieved by clamp-inflate forming.

OPTICAL PERFORMANCE

The 1.12 m reflector of 1100-O aluminum 3 mm thick was covered with Scotchcal chrome film 0.13 mm (0.005 in.) thick. The very smooth surface achieved is apparent in Fig. 3. Exposure to incident sunlight revealed a hot focal area 1 cm in diameter and essentially all light was received on a 2 cm circle. Ideal theoretical area concentration† for a paraboloid with $F/D = 0.9$ would be 9000.

CONCLUSION

Clamp-inflate forming of parabolic reflectors has been demonstrated as a feasible and economical method of fabrication. The quality achieved is immediately adequate for microwave antennas and some solar energy applications. The highest possible concentrations as may be required for thermophotovoltaic conversion of sunlight to electricity [5–9] may also be reachable. More tests under controlled conditions with accurate surface shape measurements will be necessary to establish the full potentiality for high precision. For small numbers, ring clamps are suggested, but for production runs of a certain length the convenience of a table and one ring would pay. Fittings such as vice grips could be used instead of screws to speed things up. For long production runs an automatic ring clamping press and guillotine can be imagined together with automatic inflation and a height measuring microswitch to halt inflation.

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†Area concentration is defined as the ratio of the concentrator aperture area to the receiver area. At high concentration, area concentration may differ substantially from the commonly quoted "flux concentration", the ratio of the flux density at a point in the receiver plane to the flux density in the aperture plane [4].