ASTRONOMISCHE NACHRICHTEN.
Band 212. Nr. 5085. 21.

New Theory of the Aether. By *T. J. J. See.*

(Fourth Paper.) (With 3 Plates.)

By way of introduction, we remark at the outset that this Fourth Paper is occupied chiefly with the foundations of the wave-theory of light. The subject is presented from a new point of view, in harmony with the electrodynamic wave-theory of magnetism, to which I have been led by the researches on electrodynamic action and universal gravitation outlined in the preceding papers.

As will be remembered by those familiar with the historical development of the wave-theory of light, *Newton*, *Huyghens* and *Euler* had not considered the modern theory of vibrations confined to the plane of the wave-surface, normal to the direction of propagation. Indeed these great founders of the physical sciences did not discriminate between the nature of the molecular oscillations which produce sound and those which produce light. But about 1817 Dr. *Thomas Young*, in England, and *Fresnel* and *Arago*, in France, were led to assume that in light the molecular motions of the aether are normal to the direction of the ray, like the lateral vibrations of a stretched cord. This view seemed like a very startling hypothesis, and thus for a time it encountered great opposition.

At a somewhat earlier period both *Poisson* and *Cauchy* had been occupied with profound researches in the mathematical theory of wave-motion, and each of these eminent geometers presented a number of brilliant memoirs to the Paris Academy of Sciences, chiefly between the years 1810 and 1840. When the first of these researches were presented to the Academy the venerable *Lagrange*, who died in 1813, was still numbered among its most honored members; and *Laplace* continued to take a deep interest in the wave-theory till his death in 1827.

It thus appears that *Lagrange* died before *Young* and *Fresnel* brought forward the theory of transverse vibrations (1817) for explaining the interference and polarization of light; but *Laplace* lived to witness this development for ten years; and, with his pupil *Poisson*, always held to the historical views of wave movement handed down by *Newton*, *Huyghens* and *Euler*, that at a great distance from the source the vibrations of the particles of the aether are largely in the direction of the radius drawn from the center of disturbance, as in the theory of sound.

From these circumstances, and the new physical experiments of *Young*, *Fresnel* and *Arago*, on polarization and interference, there arose a celebrated controversy on the wave-theory of light, which occupies a prominent place in the Memoirs of the Paris Academy, 1819–1839. A brief but lucid review of these papers as they successively appeared is given by *Lloyd* in his contemporary Report on the Progress and Present State of Physical Optics, made to the British Association for the Advancement of Science, 1834, and reprinted in *Lloyd*'s Miscellaneous Papers connected with Physical Science, London, 1877, pp. 19–148. It will be remembered that *Lloyd* had experimentally confirmed *Hamilton*'s theoretical prediction of conical refraction, and therefore speaks with authority.

After the appearance of *Poisson*'s memoir of 1819, the French academicians were divided into two groups: the geometers, led by *Laplace*, *Poisson*, *Lamé*, contending that at great distances from the source of disturbance the vibrations of the particles are in the direction of the radius, as held by *Newton*, *Huyghens*, *Euler*, and *Lagrange*; and the physical group, led by *Fresnel*, *Arago*, and *Cauchy*, claiming that in light the vibrations are transverse to the direction of propagation, and thus exactly opposite to those recognized in the theory of sound.

This celebrated philosophical controversy extended over some twenty years, but never led to any satisfactory conclusion. The mathematical genius of *Cauchy* came to the rescue of *Fresnel*'s experiments, by showing the possibility of a medium transmitting transverse waves. Yet neither *Cauchy* nor *Fresnel* showed how such transverse waves could arise; and after the death of *Poisson*, in 1839, there was a gradual acquiescence in the doctrine, without any theoretical explanation of the origin, of the transverse waves in light. Since 1840 there has been no change in the theory, though it often has appeared far from satisfactory to eminent investigators who expect unbroken continuity for the whole body of wave-phenomena in nature.

In his lucid article on Light, Encyclopedia Americana, 1904, Prof. *Chas. S. Hastings*, of Yale University, states the crucial difficulty more recently encountered by the wave-theory of light as follows:

»This great work of *Fresnel* was looked upon, as indeed it well deserves to be, as one of the greatest monuments to the human understanding — comparable to *Newton*'s doctrine of universal gravitation — and it long remained of almost unquestioned authority. Ultimately, however, one of its fundamental postulates, namely, that the vibrations are always at right angles to the direction of the motion of the light, began to give rise to difficulties. The fact also that the theory could not determine specifically whether the direction of vibration of plane-polarized light is in the plane of polarization or perpendicular to it was not only a manifest incompleteness, but it was a constant stimulus to a critical inspection of its premises. The more these points were studied the more insoluble the difficulties appeared, until there came to be a tolerably widespread belief that the theory was not only incomplete, but that in some way it must be essentially in error.«

From the development given below it appears that after considerable uncertainty, extending over a full century, the New Theory of the Aether now makes it possible to reconcile the difficulties which so perplexed the illustrious geometers and physicists of the Paris Academy of Sciences. This greatly simplifies our view of the wave-theory of light, without introducing any arbitrary hypotheses. And as the new wave-theory connects the theory of light directly with the theory of sound, according to the views of *Poisson*, 1830, it must be considered not the least fortunate solution of a problem which greatly bewildered some of the most illustrious academicians of France.

1. As the Aether is a Gas, and thus Isotropic in all Directions for Ordinary Terrestrial Distances, it is a Fundamental Error in the Wave-Theory of Light to deny Radial Motion, in Order to hold to the Doctrine of Vibrations almost wholly transverse to the Direction of a Ray.

It is fully realized that the modern wave-theory of light is so vast a subject that any treatment, even of the foundations merely, necessarily is much more incomplete than those given in standard treatises[1]) on light. Yet even a partial discussion of the foundation principles, provided it unfolds a new aspect of the theory of light, may be welcome to investigators who seek the laws of nature.

Thus I deem it worth while to present the results at which I arrived. Under no other principles have I been able to bring the varied phenomena of light into harmony with those of electricity, magnetism, and gravitation.

And since Prof. *Majorana*, of Rome, in the Philosophical Magazine, vol. 39, May, 1920, pp. 488–504, has been able to confirm experimentally the conclusion respecting gravitation to which I was led in 1917, (Electrod. Wave-Theory of Phys. Forc., vol. 1, p. 155) — that the amount of matter within the heavenly bodies is much greater than we heretofore have believed, actually making the sun's true mass three times that accepted by astronomers, — we see evidence of a coming transformation of doctrine in physical science, greater than any which has occurred since the age of *Kepler, Galilei*, and *Newton*. The new theory of the lunar fluctuations, motion of Mercury's perihelion, and of the problems of the aether treated of in AN 5044, 5048, seems to have triumphed incontestably.

Under the circumstances it will not do to shut our eyes to new conceptions just because they have not been handed down by traditions. When so many difficulties have arisen in the wave-theory of light, which can not be overcome on the old theory, it seems to be a sign of error in the assumed foundations of the theory itself; and the need for a modification of the theory is therefore urgent, not only in the hope of winning new truth, but also of attaining harmony and simplicity.

If by following the principles of the new theory of the aether already unfolded we have been able to confirm the work of 1917, — as by *Majorana*'s experiment of 1919, — and also obtain a much simpler view of magnetism, electrodynamic action and universal gravitation, — there is plain indication that we should attempt to harmonize the wave-theory of light with this theory of the aether.

In venturing upon this new line of thought, in accordance with the views of *Poisson*, 1830, it is of course understood that investigators should welcome suggestions for improvements which have not yet been made, owing to difficulties in the old point of view, as handed down by tradition from the days of *Young, Fresnel, Arago*, and *Cauchy*.

In preparing the third paper we discovered a new method for determining the absolute density of the aether, and developed a process by which we were enabled to calculate this density at the surfaces of the sun and planets of the solar system. This new method was found to be applicable to any stellar or sidereal system, where the force of gravity is known by observation, and thus may be extended throughout the immensity of space.

The method has proved to be of great importance in confirming and definitely establishing the small density of the aether, in accordance with the views of *Newton, Herschel, Kelvin*, and *Maxwell*. This not only does away with the strange claim put forward by electronists that the aether may have an immense density (estimated to be 2000 million times that of lead!), but also definitely establishes the compressibility of the aether when powerful forces act quickly, as in the explosions of dynamite, which was successfully employed by Prof. *Francis E. Nipher* of St. Louis, to disturb the quiescence or the medium.

Since the aether therefore is a gas, with properties which make this medium approximately isotropic for ordinary distances at the surface of the earth, though aeolotropic in respect to the heavenly bodies, as distant centres of wave-agitation, we perceive that the doctrine of the wave-theory of light, that the vibrations are wholly transverse to the direction of the ray, rests on a fundamental error, and a correction is required to take account of the gaseous character of the aether, and its equal compressibility in all directions. Thus, contrary to the assumptions of *Green*, and others, who get rid of the longitudinal component by arbitrarily making that component of the velocity infinite; there is a longitudinal component in light, as in sound; but it is very small, because it depends on the ratio of the amplitude to the wave-length $A/\lambda = 10^{-5}$, due to the very slight compressibility of the aether. The longitudinal component thus becomes $\varLambda = (A/\lambda)\cdot\varrho$, where ϱ is the spherical projection factor, about $^1/_{40}$, deduced from Fig. 1, Plate 7; so that the longitudinal component probably does not exceed 1/4000000[2]). According to the very accurate experiments described by Prof. *Hastings*, in section 5 below, *Huyghens*' construction for the extra-

[1]) Among the great standard treatises on light, that by Sir *John Herschel*, Encyclopedia Metropolitana, 1849, is to be especially commended for its comprehensiveness, and because it reflects the state of the subject just after the epoch of *Young, Fresnel* and *Arago*. *Drude*'s Theory of Optics, translated by *Mann* and *Millikan*, (Longmans, Green & Co., London and New York, 1917) is the best recent treatise with which I am familiar. Lord *Rayleigh*'s article Wave-Theory, Encyclopedia Britannica, 9th ed., 1887, presents a masterly survey of the subject, based on great personal experience, and may be unreservedly recommended.

[2]) Compare the later calculation in the notes of Sept. 12 in section 4, and in section 8, below, which indicate that this component is about $1 : (66420\cdot 10^6)$.

ordinary wave surface certainly is accurate to $1:10^6$, which therefore lends a remarkable support to the new theory of transverse waves in light.

Finally, it remains to point out that although in our new theory of the aether we usually speak of the waves as resembling the waves on the surface of still water, — which convey to the mind the image of particles revolving in circular or elliptical paths, while the wave form moves on, — yet, as in the theory of sound, it is allowable, in many phenomena, to conceive the oscillations of the particles to take place in such narrow ellipses as to be practically rectilinear, in the normal to the wave front, according to *Poisson*'s theory of 1830. Such approximate rectilinear motion always is referable to simple harmonic motion, according to the ordinary theory of uniform motion in the circle of reference. Thus our theory is not restricted in any way, but is applicable to any possible elliptical oscillation of the particle, from a circle on the one hand, to a straight-line ellipse on the other, as in the displacements referred to simple harmonic motion in the theory of sound.

In the third paper on the new theory of the aether (AN 5079), near the end of section 8, equations (86) to (88) and beyond, we have carefully cited the reasoning of *Poisson*, who devoted over 25 years to the mathematical theory of waves, and in his last papers (1819–1839) maintained that at a great distance from the source of disturbance the motion of the molecules always is sensibly normal to the wave front, as in the theory of sound.

Thus *Poisson* never concurred in the views of *Fresnel*, *Arago*, and *Cauchy*, which were gradually adopted in the traditional wave-theory of light. And it must be plainly pointed out that *Fresnel*'s doctrine of purely transverse waves was an assumption pure and simple, which offered a needed explanation of the interference of polarized light.

It is a matter of authentic record that at first *Fresnel* and *Arago* hesitated to take such a radical departure as to postulate transverse waves (cf. *Arago*'s Eulogy on *Fresnel*, English translation, Boston, 1849, pp. 212–213).

In regard to the reluctance of the early investigators to admit a lateral vibration in light, it may be pointed out that *Huyghens*, *Newton*, and *Euler* had held to the view of oscillations chiefly in the line of the rays, though *Euler*'s equations involve no necessary restrictions as to the direction of vibration, being of the same general form as in the theory of sound,

$$d^2u/dt^2 = -c\cdot\partial^2u/\partial x^2 \quad u = a\sin[2\pi/\lambda\cdot(Vt-x)]\,. \quad (1)$$

But at length, *Young* began to entertain the idea that the molecules of the aether might oscillate in parallel directions transverse to the direction of the ray, though he thought that longitudinal vibrations might exist also. *Fresnel* independently reached the idea of transverse vibrations, but like *Young* he could not account for it dynamically.

In his History of the Inductive Sciences, vol. II, 3rd ed., 1857, pp. 332–333, Dr. *Whewell* quotes the remarks of *Fresnel*: »M. *Young*, more bold in his conjectures and less confiding in the views of geometers, published it before me, though perhaps he thought of it after me.« And from personal information of the progress of the theory of transverse waves, Dr. *Whewell* adds:

»And M. *Arago* was afterwards wont to relate, (I take the liberty of stating this from personal knowledge) that when he and *Fresnel* had obtained their joint experimental results, of the non-interference of oppositely-polarized pencils, and when *Fresnel* pointed out that transverse vibrations were the only possible translation of this fact into the undulatory theory, he himself protested that he had not the courage to publish such a conception; and accordingly, the second part of the memoir was published in *Fresnel*'s name alone. What renders this more remarkable is, that it occurred when M. *Arago* had in his possession the very letter of *Young* (Jan. 12, 1817), in which he proposed the same suggestion.«

From the circumstances here reported it will be seen that *Fresnel* and *Arago* did not feel very secure[1]) in their position, under the criticisms of *Laplace*, *Poisson* and their followers. Accordingly *Fresnel* and *Arago* were more than glad to have the mathematical support of *Cauchy*, in favor of the possibility of transmitting transverse waves, if once they existed. But that was all that *Cauchy*'s analysis proved. It did not indicate how such transverse waves would arise in nature, nor did *Fresnel* and his followers throw any light on this difficult problem.

Accordingly it appears that the origin of the transverse vibrations in light has never been explained on a satisfactory basis; and for that reason it is hoped that the simple theory in section 4 below may commend itself to geometers and natural philosophers.

Another difficulty of quite fundamental character in the wave-theory of light has been before me for many years. We commonly have offered to us for illustration of transverse waves the vibrations of a single stretched cord: this looks obvious and convincing, when we deal only with a single cord free to vibrate in empty space.

But in the theory of light we should have to imagine all space, in the sphere $V = {}^4/_3\pi r^3$, $r = 0$, $r = r$, about the source of light, filled entirely full of such cords, which would thus mutually crowd each other on every side; so that no one of them would have the assumed freedom of the single cord used in our class-room illustrations. The surface of the sphere has the area $S = 4\pi r^2$, and for a spherical shell of thickness dr, the volume is $4\pi r^2 dr$, and the integral of volume is $V = 4\pi\int r^2 dr$.

Now by no possibility can the sphere surface $S = 4\pi r^2$ be increased. Accordingly no one cord can be moved sidewise, in transverse vibration, without crowding all the other cords extending outward from the centre, unless we assume simultaneous motion of all the cords in the same direction

[1]) In another place, Hist. of the Induct. Sciences, vol. II, p. 350, Dr. *Whewell* explains the embarrassment of *Arago* as follows: M. *Arago* would perhaps have at once adopted the conception of transverse vibrations, when it was suggested by his fellow-labourer, *Fresnel*, if it had not been that he was a member of the Institute, and had to bear the brunt of the war in the frequent discussions of the undulatory theory, to which theory *Laplace*, and other leading members, were so vehemently opposed, that they would not even listen with toleration to the arguments in its favour. I do not know how far influences of this kind might operate in producing the delays which took place in the publication of *Fresnel*'s papers.«

for the spherical shell $4\pi r^2 dr$. The chances are infinity to one against this occurring.

These considerations alone show that the old wave-theory of light is inadmissible. The same difficulty does not arise in *Poisson*'s theory of 1830, which makes the vibrations normal to the wave front, as in sound, and thus allows vibrational increase of space equal to $dV = 4\pi r^2 dr$, where dr is the amplitude of the oscillations. With the new theory as to why the waves are mainly transverse, more fully set forth in section 4, below, it is believed that the last outstanding difficulty in the wave-theory of light has been removed. But before quitting this subject, we may state the expansive difficulty pointed out above with somewhat greater mathematical rigor. If Φ be the velocity-potential, we have the usual differential expression

$$d\Phi = u dx + v dy + w dz\,. \qquad (2)$$

Now it is well known that the line integral of the tangential component velocity around any closed curve of a moving (incompressible) fluid remains constant throughout all time; so that when $d\Phi$ is a complete differential, the circulation $\int d\Phi$ is zero, just as in the obvious case when the fluid is at rest:

$$\delta \int d\Phi = \delta \int (u dx + v dy + w dz) = 0\,. \qquad (3)$$

When the fluid is incompressible this integral round a closed circuit is evanescent, and the momentum, like the circulation, is zero; but for a compressible fluid, the existence of a velocity-potential Φ does not imply evanescence of the integral momentum round a closed circuit (cf. Lord *Rayleigh*, Theory of Sound, 2nd ed., 1896, vol. 2, pp. 8–9).

In the case of the aether, however, the fluid is so nearly absolutely incompressible that the above theorems will hold, and we may take $d\Phi$ to be essentially an exact differential; so that the velocity in any direction is expressed by the corresponding rate of change of Φ, and therefore

$$du/dx + dv/dy + dw/dz = \partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2\,. \qquad (4)$$

Let us now consider any closed surface, such as that of the sphere already spoken of, $S = 4\pi r^2$. Then the rate of flow of the fluid outward, across the element dS, becomes:

$$dS \cdot d\Phi/dn\,.$$

And when the density is constant, the total loss of fluid in time dt is given by the double integral:

$$(\delta/\delta t)(^4/_3 \pi \sigma r^3) = \iint d\Phi/dn \cdot dS\, dt \qquad (5)$$

where the integration is to be extended over the entire surface $S = 4\pi r^2$.

Now when the sphere surface S is full both at the beginning and at the end of dt, the loss of fluid vanishes, so that

$$(\delta/\delta t)(^4/_3 \pi \sigma r^3) = \iint d\Phi/dn \cdot dS\, dt = 0\,. \qquad (6)$$

The equation of continuity, for an incompressible fluid deduced from the spacial element $dx\,dy\,dz$, under this condition of no loss of fluid across the boundary, is

$$\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2 = 0$$

or briefly $\nabla^2\Phi = 0\,. \qquad (7)$

And as *Poisson*'s equation of wave motion is

$$\partial^2\Phi/\partial t^2 = a^2 \nabla^2 \Phi \qquad (8)$$

we see that $\nabla^2\Phi = 0$, excludes the existence of waves, if this condition held rigorously for the time dt.

Wherefore we conclude that in traversing the surface S, the condition in (6) will hold for the wave from the centre at the beginning and also at the end of the time dt, corresponding to the propagation of a wave through all its phases, over the wave-length λ, which represents a complete oscillation of the fluid.

But for shorter intervals, the equation (6) will not hold rigorously; so that temporarily, over an interval less than the wave frequency, $\tau = 2\pi/v = \lambda/V$, there is both slight compressibility and a flow of the fluid across the boundary $S = 4\pi r^2$; and, for $\delta t < \tau$ we have:

$$(\delta/\delta t)(^4/_3 \pi \sigma r^3) = \iint d\Phi/dn \cdot dS\, dt = \pm dm \qquad (9)$$

where dm is the total fluid temporarily lost, an infinitesimal mass positive or negative.

Accordingly, in the wave motion of the aether, there is slight compressibility, and a minute temporary radial motion of the fluid does take place. Hence we cannot have purely transverse motion, as assumed in the traditional form of the wave-theory of light due to *Fresnel* and *Cauchy*.

During the last half century these problems have been discussed by many eminent natural philosophers — Lord *Kelvin*, *Maxwell*, Lord *Rayleigh*, *Larmor*, *Glazebrook*, etc., — but whilst they give up *Green*'s views, they do not reach satisfactory accord in their views of the aether. A useful summary of their reasoning is given in *Daniell*'s Principles of Physics, 3rd edition, 1895, p. 510. Under the circumstances we have felt that the older views must be entirely abandoned, and the waves in the aether treated as in *Poisson*'s Theory of 1830. There is no experimental evidence of different velocities for compressional and distortional waves, and no such assumptions are authorized by the existing state of our knowledge.

2. *Maxwell*'s Electromagnetic Theory of Light rests on Vibrations wholly transverse to the Direction of a Ray, and thus in View of the above Considerations the Electromagnetic Theory also must be rejected as not based strictly on the Laws of Nature.

We have just outlined the geometrical and physical difficulty encountered by *Fresnel*'s classical conception of vibrations wholly transverse to the direction in which light is propagated; and have shown how waves flat in the equators of the atoms, under haphazard arrangement of the atomic planes, would be equivalent to the uniform spherical distribution of the elliptical vibration paths exhibited to the eye in Fig. 1, Plate 7. This new principle in the wave-theory of light gives two remarkable results:

1. From any spherical source of light, or luminous mass, where the number of atoms is large, it would lead to vibrations so nearly transverse, that the longitudinal component probably would not exceed the value $1/(4 \cdot 10^6)$, and thus be insensible[1]) to observation in optical experiments.

[1]) A much smaller value $1/(66420 \cdot 10^6)$, is reached in section 4 below, Sept. 12, 1920.

2. It makes the molecules oscillate primarily in the direction of the normal to the wave-front, as held by *Huyghens, Newton, Euler, Lagrange, Laplace* and *Poisson*, prior to the theory of lateral vibrations of the stretched cord introduced by *Young, Fresnel* and *Cauchy*. Thus we have at once a vindication of the profound wave-theory of *Poisson*, 1830, without need for recourse to the artificial and dynamically inadmissible theory of *Fresnel*, that the vibrations are wholly transverse.

The above citations from *Whewell* show that *Young, Fresnel* and *Arago* were loth to entertain the theory of purely lateral vibrations, which they could not account for dynamically, as contrary to the views of geometers since the age of *Newton*. Apparently it never occurred to *Young* and *Fresnel* that a theory of projection for *Poisson*'s normal elliptical paths, such as is shown in Fig. 1, Plate 7, multiplied by the small ratio A/λ, would give mean vibrations almost normal to the ray, without the strained and unnatural theory of lateral motion appropriate to a stretched cord.

The theory of lateral vibrations, drawn from the example of the stretched cord, is approximately correct, as respects the smallness of the longitudinal component, but it is wholly lacking in physical basis, as shown above in section 1. Moreover it introduces an unfortunate and unnecessary conflict between the doctrines of experimental physics and geometry. The eminent experimenters, *Fresnel* and *Arago*, and the great analyst *Cauchy*, were thus arrayed against *Laplace, Poisson*, and *Lamé*; yet apparently it was not possible for these illustrious academicians to settle the controversy which thus arose, because the premises in their reasoning departed from the order of nature.

If the theory above traced be admissible, it follows that the claims of geometers since the days of *Newton* and *Euler*, as put forth by *Laplace* and *Poisson*, certainly were correct, that at a great distance from the source of the disturbance the molecular oscillations are normal to the wave front. On the other hand, the average vibration in light is nearly normal to the ray, owing to the effect of the spherical projection from the variously tilted elliptical paths at the source of the light, and the smallness of A/λ. Accordingly we are impressed with the necessity of the most crucial test of the premises underlying our reasoning in natural philosophy.

In order to outline this defect clearly, we shall now treat of the difficulty of the electromagnetic theory of *Maxwell*, which will also show the unwarranted assumptions underlying the *Fresnel-Cauchy* wave-theory.

»If I knew,« says Lord *Kelvin*, (Baltimore Lectures, 1904, p. 9) »what the electromagnetic theory of light is, I might be able to think of it in relation to the fundamental principles of the wave theory of light. But it seems to me that it is rather a backward step from an absolutely definite mechanical motion that is put before us by *Fresnel* and his followers to take up the so-called electromagnetic theory of light in the way it has been taken up by several writers of late. In passing, I may say that the one thing about it that seems intelligible to me, I do not think is admissible. What I mean is, that there should be an electric displacement perpendicular to the line of propagation and a magnetic disturbance perpendicular to both. It seems to me that when we have an electromagnetic theory of light, we shall see electric displacement as in the direction of propagation, and simple vibrations as described by *Fresnel* with lines of vibration perpendicular to the line of propagation, for the motion actually constituting light.«

If Lord *Kelvin* had such difficulty in understanding the electromagnetic theory of light, it undoubtedly is very allowable for the present writer to attempt to put the theory of light on a simpler basis.

The figure from *Maxwell*'s Treatise on Electricity and Magnetism, vol. II, p. 439, cited below, will put before our minds the electric and magnetic vibrations, conceived to be in planes at right angles to each other, and thus calling forth the above severe criticism by Lord *Kelvin*, who was long an associate and friend of *Maxwell*. It seems to be certain that Lord *Kelvin* was very much bewildered by the unnatural complications of the electro-magnetic theory, and thus it proved of little or no value to him.

In his Électricité et Optique, 1901, p. 73, *Poincaré* has pointed out the difficulties and contradictions he found in following *Maxwell*'s processes. »Il ne faut pas attribuer à cette contradiction trop d'importance. J'ai exposé plus haut en effect les raisons qui me font penser que *Maxwell* ne regardait la théorie du déplacement électrique ou du fluide inducteur que comme provisoire, et que ce fluide inducteur auquel il conservait le nom d'électricité, n'avait pas à ses yeux plus de réalité objective que les deux fluides de *Coulomb*.«

The importance of having a perfectly clear understanding of *Maxwell*'s electromagnetic theory is so great that we quote his reasoning in full. It is not very long, and the deductions will justify it (pag. 438–39–40).

»790. Let us now confine our attention to plane waves, the fronts of which we shall suppose normal to the axis of z. All the quantities, the variation of which constitutes such waves, are functions of z and t only, and are independent of x and y. Hence the equations of magnetic induction, (A), Art. 591, are reduced to

$$a = -\mathrm{d}G/\mathrm{d}z \qquad b = \mathrm{d}F/\mathrm{d}z \qquad c = 0 \qquad [13]\ (10)$$

or the magnetic disturbance is in the plane of the wave. This agrees with what we know of that disturbance which constitutes light.«

»Putting $\mu\alpha$, $\mu\beta$ and $\mu\gamma$ for a, b and c respectively, the equations of electric currents, Art. 607, become

$$\begin{aligned} 4\pi\mu u &= -\mathrm{d}b/\mathrm{d}z = -\mathrm{d}^2F/\mathrm{d}z^2 \\ 4\pi\mu v &= \mathrm{d}a/\mathrm{d}z = -\mathrm{d}^2G/\mathrm{d}z^2 \qquad [14]\ (11) \\ 4\pi\mu w &= 0\,. \end{aligned}$$

»Hence the electric disturbance is also in the plane of the wave, and if the magnetic disturbance is confined to one direction, say that of x, the electric disturbance is confined to the perpendicular direction, or that of y.«

»But we may calculate the electric disturbance in another way, for if f, g, h are the components of electric displacement in a non-conducting medium,

$$u = \mathrm{d}f/\mathrm{d}t \quad v = \mathrm{d}g/\mathrm{d}t \quad w = \mathrm{d}h/\mathrm{d}t\,. \qquad [15]\ (12)$$

»If P, Q, R are the components of the electromotive intensity,

$$f = K/4\pi \cdot P \quad g = K/4\pi \cdot Q \quad h = K/4\pi \cdot R \qquad [16]\ (13)$$

and since there is no motion of the medium, equations (B), Art. 598, become

$$P = -\mathrm{d}F/\mathrm{d}t \quad Q = -\mathrm{d}G/\mathrm{d}t \quad R = -\mathrm{d}H/\mathrm{d}t. \qquad [17]\ (14)$$

Hence

$$u = -K/4\pi \cdot \mathrm{d}^2F/\mathrm{d}t^2$$
$$v = -K/4\pi \cdot \mathrm{d}^2G/\mathrm{d}t^2 \quad w = -K/4\pi \cdot \mathrm{d}^2H/\mathrm{d}t^2. \qquad [18]\ (15)$$

Comparing these values with those given in equation [14], we find

$$\mathrm{d}^2F/\mathrm{d}z^2 = K\mu \cdot \mathrm{d}^2F/\mathrm{d}t^2$$
$$\mathrm{d}^2G/\mathrm{d}z^2 = K\mu \cdot \mathrm{d}^2G/\mathrm{d}t^2 \quad 0 = K\mu \cdot \mathrm{d}^2H/\mathrm{d}t^2. \text{«} \qquad [19]\ (16)$$

»The first and second of these equations are the equations of propagation of a plane wave, and their solution is of the well known form

$$F = f_1(z - Vt) + f_2(z + Vt)$$
$$G = f_3(z - Vt) + f_4(z + Vt). \qquad [20]\ (17)$$

The solution of the third equation is

$$H = A + Bt \qquad [21]\ (18)$$

where A and B are functions of z. H is therefore either constant or varies directly with the time. In neither case can it take part in the propagation of waves.«

»791. It appears from this that the directions, both of the magnetic and the electric disturbances, lie in the plane of the wave. The mathematical form of the disturbance therefore agrees with that of the disturbance which constitutes light, being transverse to the direction of propagation.«

»If we suppose $G = 0$, the disturbance will correspond to a plane-polarized ray of light.«

»The magnetic force is in this case parallel to the axis of y and equal to $1/\mu \cdot \mathrm{d}F/\mathrm{d}z$, and the electromotive intensity is parallel to the axis of x and equal to $-\mathrm{d}F/\mathrm{d}t$. The magnetic force is therefore in a plane perpendicular to that which contains the electric intensity.«

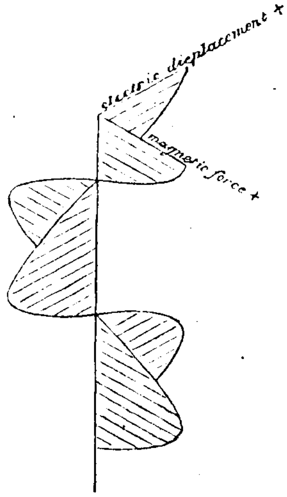

Fig. 2 = *Maxwell*'s Fig. 67.

»The values of the magnetic force and of the electromotive intensity at a given instant at different points of the ray are represented in Fig. 67, (cf. Fig. 2), for the case of a simple harmonic disturbance in one plane. This corresponds to a ray of plane-polarized light, but whether the plane of polarization corresponds to the plane of the magnetic disturbance, or to the plane of the electric disturbance, remains to be seen.«

Critical Analysis of *Maxwell*'s Processes.

1. *Maxwell* conceived the vibrations to be entirely in the wave-front, normal to the axis of z, and thus wholly dependent on x and y. This is a pure assumption, in accordance with the orthodox theory, but indefensible, as is more fully shown hereafter.

2. It appears that *Maxwell* did not regard the electric or magnetic vibrations as having any kind of vortical rotation as the wave form moves on, because he expressly states, near the close of section 791, that »this corresponds to a ray of plane-polarized light,« which in the orthodox classical theory of *Fresnel* is conceived to be direct linear vibrations, at right angles to the direction of the ray, as shown in *Maxwell*'s figure.

3. After much investigation, we have reached the conclusion that such suppositions are pure hypotheses, not justified by anything in nature. For we cannot hold the aether to be a superfine gas, the aetherons having all the degrees of freedom appropriate to *Poisson*'s equation

$$\partial^2\Phi/\partial t^2 = a^2(\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2) \qquad (19)$$

and fail to admit three component motions depending on x, y and z.

4. There was a celebrated controversy on this point between *Poisson* and *Fresnel* and their followers, in the Institute of France, (1819–1839), but to the end *Poisson* held to the conclusion that in general the vibrations are not normal to the direction of the ray. *Fresnel* himself held such views, in virtue of the necessity of explaining polarization, interference, etc.; and *Cauchy*'s mathematical researches seemed to indicate that if vibrations existed normal to the ray, they could be propagated in the aether.

5. There is no doubt that any kind of vibrations, once established in the aether, may be propagated in that medium; but this does not show that the actual vibrations in polarized light are of this type. Here is a fundamental error in the wave-theory of light, which the wave-theory of magnetism has enabled us to correct.

6. We hold that light must have a longitudinal component depending on the ratio of the amplitude to the wave length, which is small but finite. In the Philosophical Magazine for Sept., 1896, *Fitzgerald* has a thoughtful and useful paper on this subject, beginning as follows:

»In most investigations on the propagation of light, attention has been concentrated on the transverse nature of the vibration. Longitudinal motions have been relegated to the case of pressural waves, and investigators have devoted themselves to separating the two as much as possible. In Sir *George Stokes*'s classical paper on Diffraction, and in Lord *Kelvin*'s Baltimore Lectures, the existence of a longitudinal component is mentioned; but it is mentioned only to show that it is very small and that the motion is mostly transverse. Now the longitudinal component is no doubt generally small, except in the immediate neighbourhood of a source; but it by no means follows that, as a consequence, the actual direction of motion is transverse at all points in a wave. In every complicated wave there are points and often lines along which the transverse component vanishes, and at all these places the small longitudinal component may be, and often is, of great relative importance, so that the actual motion is largely in the direction of wave-propagation at these places.« (cf. *Fitzgerald*'s Scientific Writings, 1902, p. 418.)

7. The principle of the dependence of the longitudinal component in light on the ratio of the amplitude to the

wave lenght, $A = A/\lambda \cdot \varrho$, will enable us on the one hand to reconcile the views of *Poisson*, on wave propagation, with those of *Fresnel* and *Cauchy*; and on the other hand to correct a fundamental defect in the wave-theory of light, which has stood for nearly a century.

8. Thus it will be seen that *Maxwell*'s figure above given has handed down the defect of lack of rotation of the wave elements, whatever be the amplitude, and therefore does not represent nature. No wonder that Lord *Kelvin* and others have failed to understand the electromagnetic theory. As given by *Maxwell* it is contrary to the profound and conscientious researches of *Poisson*, which were critically examined by *Laplace* and *Fourier*, and not at all authorized by the researches of *Cauchy*. With *Poincaré*, therefore, we dismiss *Maxwell*'s electromagnetic theory as ‚provisoire', not deduced from the laws of nature, but from certain arbitrary assumptions, and therefore fundamentally defective.

3. The *Cauchy-Fresnel* Theory of wholly Transverse Vibrations dynamically Inadmissible for a Gaseous Medium of High Elasticity and practically Incompressible, whether Isotropic or Aeolotropic.

In his celebrated article on the Wave-Theory, Encyclopedia Britannica, 9[th] ed., the late Lord *Rayleigh* often points out the weakness of the wave-theory of light, and shows that although we may adopt it as a working hypothesis, we are not to trust the theory as a representation of nature. Thus on pp. 422–445–446, he points out *Green*'s assumption that the longitudinal component has infinite velocity, in order to get rid of this difficulty; but it is evident that Lord *Rayleigh* regarded this procedure as a somewhat violent hypothesis, scarcely justified by any known phenomenon. *Rayleigh* says:

»The idea of transverse vibrations was admitted with reluctance, even by *Young* and *Fresnel* themselves. A perfect fluid, such as the ethereal medium was then supposed to be, is essentially incapable of transverse vibrations. But there seems to be no reason a priori for preferring one kind of vibration to another; and the phenomena of polarization prove conclusively that, if luminous vibrations are analogous to those of a material medium, it is to solids, and not to fluids, that we must look. An isotropic solid is capable of propagating two distinct kinds of waves, — the first dependent upon rigidity, or the force by which shear is resisted, and the second analogous to waves of sound and dependent upon compressibility. In the former the vibrations are transverse to the direction of propagation, that is, they may take place in any direction parallel to the wave front, and they are thus suitable representatives of the vibrations of light. In this theory the luminiferous ether is distinctly assimilated to an elastic solid, and the velocity of light depends upon the rigidity and density assigned to the medium.«

»The possibility of longitudinal waves, in which the displacement is perpendicular to the wave-front, is an objection to the elastic-solid theory of light, for there is nothing known in optics corresponding thereto. If, however, we suppose with *Green* that the medium is incompressible, the velocity of longitudinal waves becomes infinite, and the objection is in great degree obviated.«

On page 422 *Rayleigh* had already indicated the limitations of the elastic-solid theory:

»For these and other reasons, especially the awkwardness with which it lends itself to the explanation of dispersion, the elastic-solid theory, valuable as a piece of purely dynamical reasoning, and probably not without mathematical analogy to the truth, can in optics be regarded only as an illustration.«

In order to set forth this difficulty somewhat more clearly we shall outline the mathematical theory of plane waves in homogeneous elastic solids. The new theory of magnetism, in relation to light, recently developed, requires for comparison a definite outline of the theory of plane waves in a homogeneous elastic solid. It is only in this way that we can decide whether the waves from a magnet are similar to those of a solid, or are of a somewhat different nature.

The following very brief outline is founded on Lord *Kelvin*'s article Elasticity, Ency. Brit. 9[th] ed., p. 824–5; but is in accord with the researches of *Cauchy, Rankine, Green,* Lord *Rayleigh, Love,* and many other eminent authorities.

(i) Definitions. Let the rectangular axes OX, OY, OZ be so oriented that OX is perpendicular to the wave front, and OY, OZ in the plane of the wave front. Then if α, β, γ be the displacements of a particle of the solid, whose undisturbed coordinates are (x, y, z) we have for any time the disturbed coordinates $x+\alpha$, $y+\beta$, $z+\gamma$. Accordingly the displacements α, β, γ are functions of x and t, and this is the definition of wave motion.

There is therefore a simple longitudinal strain ξ in the direction of OX, and two differential slips, η parallel to OY, and ζ parallel to OZ, which are simple distortions, in the shear of planes of the material one over the other.

The values are

$$\xi = \mathrm{d}\alpha/\mathrm{d}x \qquad \eta = \sqrt{2}\cdot \mathrm{d}\beta/\mathrm{d}x \qquad \zeta = \sqrt{2}\cdot \mathrm{d}\gamma/\mathrm{d}x\,. \tag{20}$$

(ii) Calculation of the work done to produce strain.

If W denote the work per unit volume required to produce this strain, the stress quadric becomes:

$$W = {}^1\!/_2\,(A\xi^2 + B\eta^2 + C\zeta^2 + 2D\eta\zeta + 2E\zeta\xi + 2F\xi\eta) \tag{21}$$

which is an ellipsoidal surface, A, B, C, D, E, F being moduluses of elasticity of the solid.

If p, q, r be the three components of the traction per unit area of the wave front, we shall have the linear equations connecting the strain and slips with the moduluses of elasticity:

$$\begin{aligned} p &= A\xi + F\eta + E\zeta \\ q\sqrt{{}^1\!/_2} &= F\xi + B\eta + D\zeta \\ r\sqrt{{}^1\!/_2} &= E\xi + D\eta + C\zeta\,. \end{aligned} \tag{22}$$

Now let it be further assumed ξ, η, ζ fulfill linear relations, with the moduluses of elasticity in the three directions:

$$\begin{aligned} M\xi &= A\xi + F\eta + E\zeta \\ M\eta &= F\xi + B\eta + D\zeta \\ M\zeta &= E\xi + D\eta + C\zeta\,. \end{aligned} \tag{23}$$

The resulting determinantal cubic gives three real positive values for M, which define the ways in which the solid may be strained. If we substitute any one of these values in (23), we may derive the ratios $\xi:\eta:\zeta$; and the components of the traction yield

$$p = M\cdot d\alpha/dx \quad q = M\cdot d\beta/dx \quad r = M\cdot d\gamma/dx\,. \tag{24}$$

The three components of the whole force due to the tractions of the sides of an infinitesimal parallelopiped $\delta x\,\delta y\,\delta z$ of the solid obviously are:

$$dp/dx\cdot\delta x\,\delta y\,\delta z \quad dq/dx\cdot\delta x\,\delta y\,\delta z \quad dr/dx\cdot\delta x\,\delta y\,\delta z\,. \tag{25}$$

Now these component forces are in equilibrium with the mass ϱ in the same element of space; and hence we have the resulting equations:

$$\begin{aligned} d^2\alpha/dt^2\cdot\varrho\,\delta x\,\delta y\,\delta z &= dp/dx\cdot\delta x\,\delta y\,\delta z \\ d^2\beta/dt^2\cdot\varrho\,\delta x\,\delta y\,\delta z &= dq/dx\cdot\delta x\,\delta y\,\delta z \\ d^2\gamma/dt^2\cdot\varrho\,\delta x\,\delta y\,\delta z &= dr/dx\cdot\delta x\,\delta y\,\delta z\,. \end{aligned} \tag{26}$$

(iii) Equations of motion for waves in an elastic solid.

Without regard to the space of the element, therefore, the equations of motion are:

$$\begin{aligned} dp/dx = \varrho\cdot d^2\alpha/dt^2 \qquad dq/dx &= \varrho\cdot d^2\beta/dt^2 \\ dr/dx &= \varrho\cdot d^2\gamma/dt^2\,. \end{aligned} \tag{27}$$

Substituting the values of ξ, η, ζ from (20), in (23) and integrating in respect to x, we get

$$\begin{aligned} A\alpha+(F\beta+E\gamma)\sqrt{2} &= M\alpha \\ F\alpha+(B\beta+D\gamma)\sqrt{2} &= M\beta\sqrt{2} \\ E\alpha+(D\beta+C\gamma)\sqrt{2} &= M\gamma\sqrt{2}\,. \end{aligned} \tag{28}$$

The three roots of his determinantal cubic may be called M_1, M_2, M_3; and the corresponding values of the ratios β/α, γ/α, determined by (28), may be denoted by b_1, c_1, b_2, c_2, b_3, c_3.

Accordingly the complete solution of (27), subject to (28), becomes of the form:

$$\begin{aligned} \alpha &= \alpha_1+\alpha_2+\alpha_3 \\ \beta &= b_1\alpha_1+b_2\alpha_2+b_3\alpha_3 \\ \gamma &= c_1\alpha_1+c_2\alpha_2+c_3\alpha_3 \\ \alpha_1 &= f_1[x+t\sqrt{(M_1/\varrho)}]+F_1[x-t\sqrt{(M_1/\varrho)}] \\ \alpha_2 &= f_2[x+t\sqrt{(M_2/\varrho)}]+F_2[x-t\sqrt{(M_2/\varrho)}] \\ \alpha_3 &= f_3[x+t\sqrt{(M_3/\varrho)}]+F_3[x-t\sqrt{(M_3/\varrho)}]\,. \end{aligned} \tag{29}$$

(iv) Three different wave velocities inferred.

In the above equations $f_1, f_2, f_3, F_1, F_2, F_3$ are arbitrary functions. Owing to the form of these expressions it is therefore inferred that there are three different wave velocities, namely:

$$V_1 = \sqrt{(M_1/\varrho)} \quad V_2 = \sqrt{(M_2/\varrho)} \quad V_3 = \sqrt{(M_3/\varrho)} \tag{30}$$

and three different kinds of waves, determined by (28), and depending on the aeolotropic character of the solid. The waves are therefore very complex, but are much simplified in an isotropic medium.

Simple case of waves in an isotropic solid.

Let the solid be isotropic, and then the moduluses of elasticity reduce to the Form:

$$\begin{aligned} B = C \qquad D &= E = F = 0 \\ M_1 = A \qquad M_2 &= M_3 = B\,. \end{aligned} \tag{31}$$

Accordingly, the above three different kinds of waves with three different velocities now reduce to just two: Compressional or Longitudinal.

1. A compressional wave, like that of sound in air, or other elastic fluid, with the motion normal to the wave front. This corresponds to the conclusion reached by *Poisson* in his celebrated memoir of 1830, and holds for any elastic medium.

2. A transverse wave, with the motion parallel to the wave front. This wave depends on the assumed properties of an elastic solid, which resists shearing motion, as when one layer slides over another.

(v) The simplest case of waves in an incompressible solid, aeolotropic or isotropic.

When the solid is incompressible *Green* has shown from equation (21) above, that the modulus of elasticity $A=\infty$; and hence the displacement along the x-axis vanishes, or $\alpha = 0$, $\xi = 0$. Therefore (21) becomes simply

$$W = B\eta^2+C\zeta^2+2D\eta\zeta\,. \tag{32}$$

And the first of (23) vanishes, leaving merely:

$$B\eta+D\zeta = M\eta \qquad D\eta+C\zeta = M\zeta\,. \tag{33}$$

This restriction of the oscillations to the plane of $\eta\,\zeta$, gives a determinantal quadratic instead of cubic, yielding two wave velocities and two wave modes. The velocity along the axis of x is thereby taken to be infinite and α disappears; leaving the two velocities:

$$V_2 = \sqrt{(M_2/\varrho)} \qquad V_3 = \sqrt{(M_3/\varrho)}\,. \tag{34}$$

And in the case of isotropy, $V_2 = V_3$, as in (31), and M_2 and M_3 are principal moduluses, each equal to the modulus of rigidity.

As Lord *Kelvin* points out, M_1 is a mixed modulus of compressibility and rigidity — not a principal modulus generally, because the distortions by differential motions of planes of particles parallel to the wave front give rise to tangential stresses orthogonal to them, which do not influence the wave motion.

(vi) Conclusion applicable to the elastic medium of the aether gas.

This outline of the theory of plane waves in homogeneous elastic solids enables us to form a fair idea of the possible types of motions of waves in the aether. When the motion of the aether wave is not through ponderable bodies, it is free of most restrictions, and follows rectilinear paths: if through ponderable masses, the action always follows *Fermat*'s minimum path, defined by *Hamilton*'s stationary condition, $\delta\int ds = 0$.

Accordingly we learn from the above analysis that most any kind of motion may be transmitted by the waves of an elastic solid: and the question to be discussed is therefore not the type of waves which may be transmitted, but rather the type of waves which actually exist in nature, and have therefore to be transmitted by the aetherial medium.

This is mainly an observational question, and the observations should therefore be extended to the phenomena of magnetism and gravitation as well as to those of light and heat.

1. Since the aether is a gas, and therefore compressible, by extremely powerful quick-acting forces, it follows from the kinetic theory, that even if the propagation of waves

by means of vibrations wholly transverse to the direction of a ray of light be a geometrical possibility, and *Cauchy* showed, and *Airy* and *Herschel* confirmed by independent researches, it is physically inadmissible to assume transverse displacements, and deny corresponding longitudinal displacements, such as was implied in the theory of *Poisson*, 1830, and suggested by *Fitzgerald*'s paper on the Longitudinal Component in Light, 1896.

2. For such an arbitrary restriction would give the aether gas anisotropic properties, — symmetrical as respects the xy-plane, but unsymmetrical in respect to the z-axis, along which the light is propagated, — for no assignable physical reason, except that the light is propagated along the z-axis.

3. And this unsymmetrical anisotropy would change its direction in space with the change in the direction of the ray of light, or the mere rotation of the axis about the origin of coordinates; and hence we see that the hypothesis is physically inadmissible. Such a physical doctrine that the property of the aether changes with the direction of the ray can no more hold a place in natural philosophy than can an established reductio ad absurdum in geometry.

4. If we view the aether in free space, as homogeneous and isotropic, except as rendered heterogeneous and aeolotropic at great distances, as of the celestial bodies, — as shown in the first paper on the New Theory of the Aether, AN 5044, — we cannot admit that its vibratory motion is different in different directions, and changing with the direction in which the light is allowed to travel.

5. Therefore if we admit a series of transverse displacements of the aether particles for making waves of the type imagined by *Fresnel, Cauchy,* Sir *John Herschel, Airy, Kelvin* and *Maxwell,* we must admit also corresponding longitudinal displacements of the aether in the direction of wave propagation — thus giving rise to rotations about mean positions, or true waves of the type imagined by *Poisson.*

6. Instead of the special polarized waves imagined by *Maxwell* of the type described in section 2 above, and implying merely a rectilinear side oscillation of the particles, like that of a stretched cord, we should therefore imagine waves of the *Poisson* type, referable to simple harmonic motion as illustrated by the modified figure of *Airy* for the surface of still water.

The geometrical conditions are fixed by the equations:

$$\begin{aligned} u &= a\cos(2\pi\, t/\tau+p) \\ v &= b\cos(2\pi\, t/\tau+q) \\ w &= c\cos(2\pi\, t/\tau+r) \end{aligned} \qquad \begin{aligned} &(u/a)^2+(v/b)^2+(w/c)^2 = 1 \\ &s = \sqrt{(u^2+v^2+w^2)}\,. \end{aligned} \tag{35}$$

7. It is therefore evident that in adopting *Cauchy*'s ideas of vibrations similar to that of a stretched cord, *Herschel* was misled, and he in turn misled *Airy* and others — substituting a mere geometrical abstraction, and practically a physical impossibility, for the valid physical theory of *Poisson,* which makes the vibrations of the aether similar to those of sound, but A/λ very small.

8. The result has been a traditional false teaching in the wave-theory of light, as hinted at by *Fitzgerald* in the memoir »On the Longitudinal Component of Light,« (The Scientific Writings of *Fitzgerald*, p. 418), and by Professor *Chas. S. Hastings*, Encyclopedia Americana, 1904, article Light, quoted in section 1 above, where it is pointed out that the conviction has grown that the wave-theory is in some way wrong.

9. It is obvious that waves of the types imagined by *Cauchy* and *Fresnel* could be transmitted by the perfectly elastic aether if they existed — as is correctly held by *Herschel* and *Airy* — but the question of fact remains: Do they in general exist?

10. This important question must be answered in the negative. For in magnetism we recognize, from *Faraday*'s rotation of a beam of polarized light, 1845, the rotations of the elements of the aether, the atoms having their equators lying in parallel planes. In common luminous bodies, on the other hand, no such parallelism in the atomic planes can be assumed: indeed this parallelism must be emphatically denied.

11. And as we cannot have luminous bodies, with the atomic planes all parallel, as in magnetism; so also we can not imagine these atoms so tilted as to send rays to us only from their combined poles. Hence the wave-theory of light as heretofore taught is physically inadmissible.

12. We must hold that the waves of light in general are flat in the planes of the equators of the atoms, and these planes tilted at all possible angles, as explained below in Section 4. If the axis of z be in the plane of the equator of the vibrating atom, the oscillation will be of the plane wave type commonly shown. If the axis of z lies in the northern hemisphere of the atom, the approaching waves, as we look at them, will seem to rotate left handed, in the form of a left handed helix. If the z-axis lies in the southern hemisphere of the atom, the waves received will seem to rotate right handed, like the coils of a right handed helix.

4. Geometrical Reasons why the Vibrations of Ordinary Light are mainly Transverse.

If we contemplate the hemisphere presented to our view by a luminous spherical source of light, such as the sun, it is evident that the waves propagated towards the observer will cover a surface of area

$$A = 2\pi r^2\,. \tag{36}$$

And in orthogonal projection this area will be reduced by one half, and become merely the area of a single great circle of the sphere

$$A' = \pi r^2\,. \tag{37}$$

The sphere surface seen by us in projection is enormously fore-shortened and contracted in area at the border, while at the centre no decrease in apparent area takes place. If therefore the atoms emit waves which are flat in the planes of their equators, and a haphazard arrangement of the atomic planes holds true, as should occur in a non-magnetic sphere, it follows that the beam of light emitted by the sun should have its vibrations so largely peripheral that, with A/λ very small, it will present practically the appearance of transverse vibrations, — as long taught in the wave-theory of light.

In order to examine into this subject somewhat more critically we may proceed as follows. Let Fig. 3, Plate 7, represent an orthogonal projection of the sun's hemisphere, with the centre at O, and the coordinate axes OX and OY as shown in the diagram. Then, if we subdivide the quadrant of the circle into 20 parts, corresponding at the centre to an angular distance of $4\overset{\circ}{.}5$ between the small circles about that point O as a pole, we may plat a curve along the radius OX which will represent a section of the visible surface of the hemisphere, as if the area were not decreased by the orthogonal projection. The equal distances along the radius OX will represent equal values of the sine of the polar distance, θ, or equal values of the cosine of the latitude reckoned from the base of the hemisphere here represented by the lower circle.

The curve may be drawn from a table of natural sines or cosines by taking y proportional to this function, so that the change will make a curve of the kind indicated in the Fig. 3, Plate 7, which is repeated on both sides of O, in order to show to the eye the enormous condensation of surface near the circumference of the projected hemisphere. In fact the double curve on both sides of O is a semicircle, drawn about Y as a centre, and thus exceedingly simple.

The coordinates of the curve, to four places of decimals, and the surface integral I for the component of *Poisson*'s radial wave motion in line of sight, equation (38), are:

Angle θ_i	$X_i = \sin\theta_i$	$Y_i = 1 - \cos\theta_i$	I
0°	0.0000	0.0000	0.00308
4.5	0.0785	0.0031	0.00915
9.0	0.1564	0.0123	0.01502
13.5	0.2334	0.0277	0.02052
18.0	0.3090	0.0489	0.02551
22.5	0.3826	0.0762	0.02985
27.0	0.4540	0.1090	0.03342
31.5	0.5224	0.1474	0.03634
36.0	0.5878	0.1910	0.03814
40.5	0.6494	0.2397	0.03916
45.0	0.7071	0.2929	0.03906
49.5	0.7603	0.3506	0.03824
54.0	0.8090	0.4122	0.03625
58.5	0.8526	0.4776	0.03350
63.0	0.8910	0.5460	0.02978
67.5	0.9238	0.6174	0.02761
72.0	0.9511	0.6910	0.02050
76.5	0.9723	0.7666	0.01501
81.0	0.9877	0.8436	0.00914
85.5	0.9969	0.9215	0.00310
90.0	1.0000	1.0000	$\sum_{i=1}^{i=20} = 0.50238$

$$I = (\sin\theta_i - \sin\theta_{i-1}) \sin {}^1/_2(\theta_i + \theta_{i-1}) \qquad I_m = 0.025119 = {}^1/_{40}, \text{ nearly.}$$

$$I_m = {}^1/_{40} \qquad \varDelta = A/\lambda \cdot I_m . \tag{38}$$

From these considerations it is evident that if we imagine the atoms in the sphere to have their equatorial planes directed radially, which will be the average position in a large mass, under haphazard atomic arrangement, the effect will be to give us an enormous preponderance of transverse vibrations near the periphery of our luminous globe, or in a ray of ordinary light from a globe like the sun or a star. This reasoning applies to any luminous body or flame, such as that from a Bunsen-burner in our laboratories, which have haphazard arrangement of the atomic planes, all atoms vibrating so rapidly that from any single atom several hundred waves of the same type will reach the eye of the observer before the translatory motions of the luminous atoms will produce appreciable change.

In his Undulatory Theory of Optics, 1866, pp. 155–156, *Airy* says:

»Common light consists of successive series of elliptical vibrations (including in this term plane and circular vibrations), all the vibrations of each series being similar to each other, but the vibrations of one series having no relation to those of another. The number of vibrations in each series must amount to at least several hundreds; but the series must be so short that several hundred series enter the eye in every second of time.«

This criterion of *Airy* obviously is fulfilled by the light from any luminous source, since even in a very small mass the atoms are numbered by the trillion, and no change in their average orientation occurs with the lapse of time, though individual atoms in their mutual interactions will slowly shift their individual equatorial planes to new positions, as the millions of millions of vibrations are emitted.

The centre of the yellow light of the spectrum has a frequency of 517 500 000 000 000 vibrations per second; and thus with such an enormous flow of waves, they might be subdivided into ten thousand million successive series and still leave a flood of 51750 groups of waves beating upon the eye in a second. Accordingly, *Airy*'s criterion is perfectly consistent with the motions of the individual atoms, in mutual collisions at the rate of say 10 000 000 000 per second (cf. AN 5044, p. 66), which is about the average for terrestrial gases under laboratory conditions.

Returning now to our figure for illustrating the enormous preponderance of transverse rays in a beam of ordinary light, we easily find by calculation that 62 percent of the light comes from the zone $\theta = 90°$ to $\theta = 51°45'27''$, near the periphery of the orthogonally projected sphere surface. We may even extend this zone inward to $\theta = 44°25'30''$ and still not approach the centre of the circle more than 0.30 of the radius; yet this outer zone to θ' includes 71.4 percent of the luminous sphere surface. Thus we see from the corresponding small circles drawn in the figure about the pole O, why in ordinary light it may be described as practically transverse — since a great preponderance of the light from the atoms acts as if the vibratory motion were in the plane of the wave surface.

The great hollowing out of the curve of light near the centre of the figure, from which alone indications of a longitudinal component could be expected to come, and the smallness of the factor A/λ, shows why there is such a feeble indication of this longitudinal component in our actual experiments. It is not surprising therefore that in his Undulatory Theory of Optics, 1866, p. 91, Sir *George Airy* says:

»The reader who has possessed himself fully of this hypothesis, will see at once the connection between all the experiments given above.«

»For the general explanation of these experiments, and for the accurate investigation of most of the phenomena to be hereafter described, it is indifferent whether we suppose the vibrations constituting polarized light to take place parallel to the plane of polarization, or perpendicular to it. There are reasons, however, connected with the most profound investigations into the nature of crystalline separation and into the nature of reflection from glass, etc., and confirming each other in a remarkable degree, that incline us to choose the latter: and thus:«

»When we say that light is polarized in a particular plane, we mean that the vibration of every particle is perpendicular to that plane.«

»Thus, in the undulation constituting the ordinary ray of Iceland spar, the vibration of every particle is perpendicular to the principal plane of the crystal: in that constituting the extraordinary ray, the vibration of every particle is parallel to the principal plane. When light falls upon unsilvered glass at the polarizing angle, the reflected wave is formed entirely by vibrations perpendicular to the plane of incidence: the transmitted wave is formed by some vibrations perpendicular to the plane of incidence, with an excess of vibrations parallel to the plane of incidence.«

»The reader will perceive that it is absolutely necessary to suppose, either that there are no vibrations in the direction of the wave's motion, or that they make no impression on the eye. For if there were such, there ought in the experiment of (98) to be visible fringes of interferences: of such however there is not the smallest trace.«

If we examine the figure, we find from the integral in the plane xy, that the total light emitted is given by the expression

$$L = \int_0^x y \, dx \tag{39}$$

To derive a corresponding expression for the *Poisson* waves emitted radially from the sphere surface, we put

$$x = \sin\theta \qquad y = 1 - \cos\theta \qquad dx = \cos\theta \, d\theta \,. \tag{40}$$

And we integrate for θ between the limits 0 and $\frac{1}{2}\pi$, and, for the surface generated by revolving the axis of x, we use ω between the limits 0 and 2π. Thus we have as the surface integral of the hemisphere

$$L' = \int_0^{\frac{1}{2}\pi} \int_0^{2\pi} (1 - \cos\theta) \cos\theta \, d\theta \, d\omega = 2\pi \,. \tag{41}$$

To find the light in a beam we calculate the reduction of area by orthogonal projection.

If now we integrate for the light distributed over a more limited surface $S = f(\theta, \omega)$, we shall find the value of the integral so trifling, that till $\theta = 44° 25' 30''$, $x = 0.7$, only 28.6 percent of the light will be included in the central canopy. Moreover the average factor for the part of the *Poisson* radial wave motion in the line of sight is only $\frac{1}{40}$, and the ratio $A/\lambda = 10^{-5}$, making[1])

$$\Lambda = A/\lambda \cdot \varrho = 1/(4 \cdot 10^6) \,. \tag{42}$$

Accordingly one would expect experimenters to reach the very conclusion announced by *Airy*, in the above passage, that there is not the smallest trace of visible fringes of interference due to the longitudinal component, which of course has to come from the light near the centre of the canopy. *Airy* personally repeated the experiments which he described and reduced to mathematical expression: so that his conclusions have been widely accepted by natural philosophers.

It is by virtue of *Airy*'s careful experimentation and analysis of the wave-theory of light, following the independent and profound analysis of Sir *John Herschel*, in the great treatise on Light, Encycl. Metropol., 1849, that we adopt *Airy*'s presentation of the subject as authoritative. Our conclusions therefore are as follows:

1. About 71.4 percent of the sphere surface is included within the elevation of $45° 34' 30''$ from the base of the hemisphere. This part of the sphere is a zone so near the circumference as to appear to the observer to be essentially peripheral. Hence the origin of the belief, in view of the smallness of the ratio A/λ, that the vibrations are actually transverse, and the integral for the longitudinal component insensible to the experimenter.

2. Light vibrations coming from this periphery would appear essentially as transverse waves; and by proper optical appliances could be polarized into right handed, left handed, circularly polarized or elliptically polarized light, as seen in that transmitted through crystals.

3. As only 28.6 percent of the sphere surface remains in the larger zone, near the pole, and a considerable part of the vibrations on that polar surface could be resolved likewise into circularly or elliptically polarized light, we see that in ordinary light, the average vibration is described as made up of elliptical vibrations (*Airy*, Undulatory Theory of Optics, 1866, p. 156).

4. In discussing experiments leading up to *Lloyd*'s observations on conical refraction, *Airy* notes, in regard to polarization of light, that »if common light be incident, (which not improbably consists of successive series of waves polarized in every conceivable plane) rays will be formed directed to every point of the [Newton's]ring, each ray having the polarization proper to its point of the ring; and a conical sheet of light will be formed within the crystal« (Undulatory Theory of Optics, p. 106). Again, summarizing the description of ordinary polarization, *Airy* draws three conclusions: (1) »If from common light we produce, by any known contrivance, light that is polarized in one plane, there is always produced at the same time light more or less polarized in the plane perpendicular to the former» (p. 89).

5. On this first conclusion *Airy* comments as follows: »The first leads at once to the presumption that polarization is not a modification or change of common light, but a resolution of it into two parts equally related to planes at right angles to each other; and that the exhibition of a beam of polarized light requires the action of some peculiar forces (either those employed in producing ordinary reflection and refraction or those which produce crystalline double refrac-

[1]) The calculations made Sept. 12, 1920, as given in the note to section 8, below, make $A/\lambda = 1 : 1660508000$, which would make $\Lambda = 1 : (66420 \cdot 10^6)$ — a value hopelessly beyond the range of observation. — Note added, Sept. 12, 1920.

tion) which will enable the eye to perceive one of these parts without mixture of the other. This presumption is strongly supported by the phenomena of partially polarized light. If light falls upon a plate of glass inclined to the ray, the transmitted light, as we have seen, is partially polarized. If now a second plate of glass be placed in the path of the transmitted light, inclined at the same angle as the former plate, but with its plane of reflection at right angles to that of the former plate, the light which emerges from it has lost every trace of polarization; whether it be examined only with the analyzing plate B, or by the interposition of a plate of crystal in the manner to be explained hereafter (145). This seems explicable only on the supposition that the effect of the first plate of glass was to diminish that part of the light which has respect to one plane (without totally removing it), and that the effect of the second plate is to diminish in the same proportion that part of the light which has respect to the other plane; and therefore that, after emergence from the second plate, the two portions of light have the same proportion as before. On considering this presumption in conjunction with the second and third conclusion, we easily arrive at this simple hypothesis explaining the whole«:

»Common light consists of undulations in which the vibrations of each particle are in the plane perpendicular to the direction of the wave's motion. The polarization of light is the resolution of the vibrations of each particle into two, one parallel to a given plane passing through the direction of the wave's motion, and the other perpendicular to that plane; which (from causes that we shall not allude to at present), become in certain cases the origin of waves that travel in different directions. When we are able to separate one of these from the other, we say that the light of each is polarized. When the resolved vibration parallel to the plane is preserved unaltered, and that perpendicular to the plane is diminished in a given ratio (or vice versa), and not separated from it, we say that the light is partially polarized.«

6. In view of the considerations here deduced by *Airy*, we see why the spherical distribution of waves from atoms in every conceivable plane will give rays directed to every point of the circumference of the end of a beam of light; just as in *Airy*'s discussion of the polarization in Newton's-rings, it is held that the waves »are polarized in every conceivable plane«, and »rays will be formed directed to every point of the ring, each ray having the polarization proper to its point of the ring.«

7. To view this reasoning graphically, imagine a series of planes drawn through the centre of the sphere and fixed at equal intervals normal to a meridian of the circumference having its pole in the observer's eye. Then imagine the whole set of fixed planes rotated about the pole through the observer's eye, and stopped at successive intervals of the circumference equal to those between the fixed planes. The equatorial portions of the hemisphere will thereby be divided into rectangular compartments with areas equal to $r^2 \cos\lambda\, d\lambda\, d\omega$, were ω is the angle about the pole, and λ is the latitude. To get compartments of equal areas in higher latitudes, the revolving system must stop at intervals equal to $d\omega/\cos\lambda = (2\pi/n)\sec\lambda$. From these considerations we perceive that in higher latitudes the number of rectangular compartments decreases rapidly; and if the number of flat wavelets of light are proportional to the rectangular areas on the sphere, the wave disturbance in light will be almost wholly peripheral, or transverse.

8. Small as is the amount of light depending on the vibrations in or near the line of vision, our sphere shows that the central great circles distributed in haphazard fashion, do not lie in the line of vision, but pass around it on all sides; and hence we perceive that the disturbance necessarily is rotational in character, and nearly transverse to the direction of propagation.

9. From considerations based on polarization, — tending to show that in the ordinary ray of Iceland spar the vibration of every particle is perpendicular to the principal plane of the crystal, while in that constituting the extraordinary ray, the vibration of every particle is parallel to the principal plane — the polarized light in both cases being already systematically resolved by the action of the crystal — *Airy* concludes in article 101 of his Undulatory Theory of Optics, that there is not the smallest trace of visible fringes of interferences.

10. If the considerations on the spherical distribution of the planes of the flat wavelets above deduced be valid, *Airy*'s results could be true, and yet give us an unlimited number of component flat wavelets not originally normal to the direction of the wave propagation, but inclined to it by the angle ε, as in the electrodynamic formula of *Franz Neumann*, 1845:

$$M = II'\iint 1/r \cdot \cos\varepsilon\, ds\, ds' = -\chi\chi'\iint (1/r)(dx/ds \cdot dx'/ds' + dy/ds \cdot dy'/ds' + dz/ds \cdot dz'/ds')\, ds\, ds' \qquad (43)$$

$$\cos\varepsilon = \cos\alpha \cos\alpha' + \cos\beta \cos\beta' + \cos\gamma \cos\gamma' = \cos(I, I') \qquad (44)$$

and yielding the general formula for electrodynamic action in universal gravitation, or *Ampère*'s theory of elementary electric currents about the atoms:

$$\Omega = \iiiint\!\!\iint II' \cos(I, I') [(x-x')^2 + (y-y')^2 + (z-z')^2]^{-1/2}\, dx\, dy\, dz\, dx'\, dy'\, dz'. \qquad (45)$$

Thus the difficulties of reconciling the wave-theory of light with the electrodynamic theory disappear. The resolved waves in polarized light are largely normal to the direction of propagation, but their original component flat wavelets were not, being in atomic planes inclined at all angles.

5. Other Fundamental Objections to *Fresnel*'s Theory that Light Waves are purely Linear Transverse Motions.

(i) Certain circumstances favorable to the old wave-theory of light permitted it to progress but did not establish it on a permanent basis.

1. In his memoir of 1830 *Poisson* showed that in elastic media waves propagated from a centre are essentially like sound waves, and at great distances the molecules move mainly in the direction of the normal to the wave front. But *Poisson* died in 1839, while *Cauchy* lived on till 1857; and moreover the deceptive argument drawn from the vibrations of an elastic cord misled *Herschel* and *Airy*, who failed to perceive that the underlying premise implies anisotropy in the medium.

2. As *Poisson* never concurred in the theory of vibrations normal to the direction of the ray, *Fresnel* and *Arago* sought comfort in the analytical results of *Cauchy*. And because such waves are theoretically possible when once they are generated, it was inferred that light has such motion as is observed in the vibrating cord.

3. *Cauchy*'s analysis seems to have proved that if waves normal to the direction of propagation be started, they could be propagated by such transverse motion; yet he did not explain how they would arise, or would be started normal to the direction of propagation. Nor did his associates see the anachronism implied in a medium with anisotropic properties along x, y, z, $-z$ being in the direction of the ray, whatever that may be.

4. After a visit from *Arago*, 1816, *Young* began to form a theory of waves with motions normal to the direction of propagation. They were held to be similar to undulations carried along a stretched cord, as stated in a letter April 29, 1818, (cf. *Whittaker*'s History of the Aether, p. 122). This example of the vibrating cord gave a physical analogy which was afterwards adopted by *Fresnel, Herschel, Airy* and others, but it was really an anachronism; for it implied a »stringy« condition in the aether, in any direction the wave might travel, but not in other directions. The z-component of the vibration along the ray vanished, which made $\zeta = 0$, and therefore $s = \sqrt{(\xi^2+\eta^2)}$ becomes confined wholly to the wave surface.

5. As we have seen above, *Green* took the velocity of the longitudinal component of the waves to be infinite; which left the finite motion wholly in the wave surface. In the case of a gaseous aether of course there is no authority for this procedure; and thus it simply begged the question, by offering an arbitrary hypothesis.

6. *Hamilton*'s prediction of conical refraction (conf. *Whittaker*'s History of the Aether, p. 131) only showed *Fresnel*'s ideas of the theory to be correct in general, but was not an accurate test in all details. The theory above outlined will explain conical refraction equally well. Accordingly in the absence of definite objections, the old wave-theory triumphed by default, at least temporarily; yet the assumptions made to get rid of the longitudinal component never were satisfactory, and could not be justified, because based on an arbitrary hypothesis.

7. The physical inadmissibility of *Green*'s postulate that the longitudinal component has infinite velocity (*Green*'s Collected Papers, p. 246) is easily shown by the following considerations:

a) In his work on Sound, Chap. V, *Tyndall* shows that when the bow of a violin is given a stroke along the violin string a shrill sound arises, owing to the rapidity of the wave along the string, — giving high pitch to the sound. Owing to its higher elasticity, waves travel say ten times more rapidly along the string than through the air.

b) Now it is easy to see that this is analogous to *Green*'s unauthorized procedure, which amounts to assuming a »stringy« condition of the aether in any direction in which light is sent. And the chance that the assumed longitudinal component would not manifest itself in some way is very slight, since the aether, with excessively small density, is naturally taken to be a gas, and the velocity of the aetheron $\bar{v} = 471000$ kms.

8. Again, in his work on Sound, (p. 73), *Tyndall* shows that a sharpness of shock, or rapidity of vibration, is necessary for producing sonorous waves in air. »It is still more necessary in hydrogen, because the greater mobility of this light gas tends to prevent the formation of condensations and rarefactions.«

Therefore the aether should present enormous difficulties in the generation of waves therein, and such is observed to be a fact. By way of experiment Prof. *Nipher* alone has generated disturbances in the aether; and to produce them he had to use dynamite, which gives intense forces quickly exerted. Observation thus verifies the high velocity of the aetheron, and will not permit us to assume different velocities of the aether wave in different directions.

(ii) Purely transverse vibrations in light would imply only transverse undulations in magnetism and electrodynamic action, which is contrary to observation.

The theory of transverse waves was first admitted somewhat reluctantly by *Young* and *Fresnel* in the early part of the 19th century, (1802–1829). But under the celebrated experiments on diffraction, double refraction, polarization and interference conducted by *Fresnel* and *Arago*, the theory became a new means of discovery. This apparent experimental triumph of the undulatory theory aroused such interest that a long series of brilliant mathematical researches were entered upon by the eminent natural philosophers then resident at Paris — *Navier, Poisson, Cauchy* and *Lamé*.

It is true that these mathematicians were by no means agreed among themselves as to the details, yet their work was mathematically so impressive that it created great interest in other countries, more especially in England, and was adopted by *Airy, Hamilton* and *Herschel*, and subsequently by *Green, Thomson* (Lord *Kelvin*), *Stokes, Maxwell*, and *Rayleigh*. In this way the undulatory theory as now taught came into wide use; and yet it was always suspected to be somewhat defective, and we shall now point out some additional reasons why the traditional view can not be valid.

1. The theory of purely transverse waves in light is directly inconsistent with the rotations actually known in magnetism, and with the electrodynamic action of a current on a magnetic needle, in such phenomena as *Oersted*'s experiment of 1819.

2. For if the motion of the aether is linear and transverse in light, it would be logical to conclude that it must be of the same type in the waves by which electrodynamic action is propagated across space. Indeed, experiment proves

that both actions have the same velocity, and take place in the same medium. And we have no grounds for assuming a difference of wave type.

3. Yet we know by actual observation that in *Oersted*'s experiment of 1819 the magnetic needle not only is directed in a definite way, depending on the direction of the current, but also attracted to the conductor by the action of electrodynamic waves propagated from the wire, as first pointed out by the present writer in 1914.

4. Now the electrodynamic waves discovered in 1914 can not be wholly transverse, as held by *Fresnel* and his followers in the wave-theory of light; for in that case there could be no actual attraction on the needle. On the contrary, *Maxwell* held (Treatise on Electricity and Magnetism, 3[d] ed., § 793) that such transverse waves exert a slight repulsion, and on the premise employed it is difficult to refute his conclusion.

5. In order to exert the observed attraction, the electrodynamic waves must have rotations somewhat like those observed in water waves; and the needle must so orient itself that the elementary Ampère-currents of electricity about the atoms coincide in direction with those in the electrodynamic waves propagated from the wire.

6. The observed attraction of the magnetic needle to the wire therefore is inconsistent with *Fresnel*'s doctrine of purely transverse waves, as taught in the theory of light and adopted by *Maxwell* in his electromagnetic theory. Now magnets themselves have circulation of currents about their atoms, as first shown by *Ampère*'s experiments with currents in 1822; and these currents about the atoms give rise to the rotations about the Faraday-lines of force, thus forming the waves propagated outward from magnets. It is only in this way that we can imagine how magnets presenting unlike poles attract; and, when like poles are presented, repel, by a mechanism at last disclosed to our vision.

7. Therefore the magnetic needle is attracted to a conducting wire by the electrodynamic waves propagated outwardly from it; and magnets themselves also attract by sending out waves defined by the well known rotations about the Faraday-lines of force. Accordingly it follows that all such waves must necessarily involve rotations in the aether to make up the waves; and the waves incontestably are not wholly transverse, but only transverse in somewhat the same way that water waves are transverse.

8. The *Fresnel* theory of purely transverse light waves thus again is definitely disproved, and we may reconcile the varied mathematical researches of *Poisson*, *Cauchy*, and *Lamé*. It should be noticed, however, that *Cauchy*'s reasoning had no physical basis, to control the legitimacy of the hypotheses underlying it, except the artificial analogy with the vibrating cord. *Poisson* and *Lamé* on the other hand never were fully convinced that the motion in light is wholly transverse. The theory outlined in section 4 above probably had never occurred to them.

9. Accordingly there are real weaknesses in the traditional wave-theory of light; and the difficulties noticed by the earlier investigators have never been satisfactorily overcome. The objections here pointed out appear to be new, and absolutely fatal to the theory of wholly transverse waves as held by *Fresnel*. He was essentially a specialist in light, rather than a mathematician and all around natural philosopher, like *Poisson*, who never did believe that in nature the aetherial vibrations could be as *Fresnel* imagined. The temporary scepticism of the illustrious *Poisson* is now verified from a new point of view, after the lapse of nearly a century.

10. It is remarkable that such a palpable perversion as the theory of wholly transverse vibrations gained currency in science through the misdirected reasoning of the followers of *Cauchy*. They seem to have been misled by beautiful general formulae, valid enough as applied to wave motion in crystalline media, but utterly deceptive as applied to the simple case of the aether itself, viewed in free space as a uniform isotropic medium, which furnishes the general basis for the undulatory theory of light. This outcome is the more remarkable and unfortunate, since *Poisson* was a greater and more sagacious physical philosopher than *Cauchy*, who was chiefly a pure mathematician.

(iii) Difficulties in the wave theory of light as outlined by Prof. *Chas. S. Hastings*.

In a letter to the present writer, dated Aug. 17, 1916, Prof. *Hastings* speaks as follows:

»That light vibrations necessarily are transverse only is proved in many ways — perhaps most obviously by the fact that complete polarization is possible.«

»If light waves fall normally on a refracting surface, any free element of volume in the first medium is sustained in permanent transverse vibrations of definite period, but if it is attached to an element of the second medium as at the interface, the second medium having either a greater density or a greater rigidity, it will not (although necessarily retaining the same period) move so far from its position of equilibrium. Just at this region, therefore, so far as the first medium is concerned, we must add a system of waves of opposite phase and of an amplitude easily calculable from the ratio of light velocities in the two media. — this constitutes the reflected light.«

»Now consider the refracted light. The element of volume just below the interface has the same period and amplitude as the attached element above; it is therefore a portion of a system of waves propagated in the same direction as the incident waves but with a velocity determined very simply by the density and rigidity.«

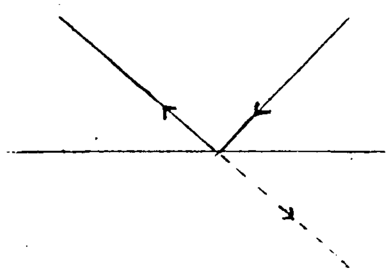

Fig. 4. Professor *Hasting*'s diagram of the path of light at the interface.

»It is when we consider oblique incidences that we get into difficulties. *Fresnel* assumed that the same condition held in these cases also, but, as you can readily see from the diagram, there could exist a stable state of vibrations at the surface only when there is a system of compressional waves also proceeding from the interface in a direction and of an intensity easily calculable if the ratio of volume-elasticity to rigidity is known. Now no such system of longitudinal waves exists under any circumstances, because the energy carried by the reflected and refracted wave systems taken together always equals the energy carried to the refracting surface by the incident wave system. (This, by the way, is the direct answer to your principal question. I might stop here but the fixed habit of an old teacher leads me to add: —) In order to get rid of the obvious difficulty *Fresnel* assumed that the volume-elasticity of the ether, both free and associated with matter, is infinitely great, in which case the velocity of the longitudinal wave system would be infinite and it would carry no energy with it. Aside from the fact that absolute incompressibility is difficult to conceive there are other serious difficulties in the theory connected with the phenomena of double refraction.«

»*Stokes* is said to have invented an elastic-solid theory, which, however, carried with it as a necessary consequence the proof that *Huyghens*' construction of the extraordinary wave surface in Iceland spar is slightly erroneous, say in the fourth decimal. *Fitzgerald* attempted to test this by accurate measurements (the first made since *Huyghens*!) but failed in attaining adequate precision. Finally I demonstrated (Amer. Jour. Sci. somewhere) that *Huyghens*' construction is certainly accurate to $1 \cdot 10^{-6}$.«

»More recently *Kelvin*, who was especially desirous of getting a defensible elastic-solid theory of light, proposed a zero volume-elasticity, or a collapsible ether. This gives zero velocity for compressional waves and hence no energy is carried away from the interface. *Kelvin* apparently left his readers to imagine an outer boundary condition which would prevent the ether-universe from collapsing.«

6. Outline of the General Theory of the Waves from any Body, whether due to Light, Magnetism, Electrodynamic Action or Universal Gravitation.

(i) Results of *Poisson*'s analysis for wave motion.

As we have seen, in the third paper, *Poisson* reduces (Memoir of 1830, p. 556) the sextuple integral for the propagation of waves to the double integral:

$$\begin{aligned}\Phi = (1/4\pi)\int_0^{\pi}\int_0^{2\pi} F(x+at\cos\theta,\ y+at\sin\theta\sin\omega,\ z+at\sin\theta\cos\omega)\, t\sin\theta\, d\theta\, d\omega \\ +(1/4\pi)(d/dt)\int_0^{\pi}\int_0^{2\pi}\Pi(x+at\cos\theta,\ y+at\sin\theta\sin\omega,\ z+at\sin\theta\cos\omega)\, t\sin\theta\, d\theta\, d\omega.\end{aligned} \tag{46}$$

Now the equation

$$lx+my+nz=0 \tag{47}$$

represents a plane through the origin. And

$$lx+my+nz-(at+s)=0 \tag{48}$$

represents a plane with perpendicular $p=(at+s)$ from the origin.

If plane waves proceed from the equator of an atom, the radius of the spherical wave surface about the atom will increase with at; and the disturbance, in the plane of the flat wave, in the equator of the atom, will travel away with the velocity at, and remain parallel to the original in all parallel planes. Thus $lx+my+nz-(at+s)=0$ represents the disturbance in the equatorial plane of the flat waves from any atom, propagated in every direction parallel thereto.

Our integration should include the disturbances along these planes in which the waves are flat. Accordingly, for the waves from any atom we have

$$\begin{aligned}\Phi = (1/4\pi)\int_0^{\pi}\int_0^{2\pi} F\{l(x+at\cos\theta)+m(y+at\sin\theta\sin\omega)+n(z+at\sin\theta\cos\omega)-(at+s)\}\, t\sin\theta\, d\theta\, d\omega \\ +(1/4\pi)(d/dt)\int_0^{\pi}\int_0^{2\pi}\Pi\{l(x+at\cos\theta)+m(y+at\sin\theta\sin\omega)+n(z+at\sin\theta\cos\omega)-(at+s)\}\, t\sin\theta\, d\theta\, d\omega.\end{aligned} \tag{49}$$

And if we integrate this expression for the waves from all the atoms of a body, we shall have

$$\begin{aligned}\Phi = \int_0^{r}\int_0^{\pi}\int_0^{2\pi}(\sigma/4\pi)\int_0^{\pi}\int_0^{2\pi} F\{l(x+at\cos\theta)+m(y+at\sin\theta\sin\omega)+n(z+at\sin\theta\cos\omega)-(at+s)\}\, r^2\sin\theta\, dr\, d\theta\, d\omega\cdot t\sin\theta\, d\theta\, d\omega \\ +\int_0^{r}\int_0^{\pi}\int_0^{2\pi}(\sigma/4\pi)(d/dt)\int_0^{\pi}\int_0^{2\pi}\Pi\{l(x+at\cos\theta)+m(y+at\sin\theta\sin\omega)+n(z+at\sin\theta\cos\omega)-(at+s)\}\, r^2\sin\theta\, dr\, d\theta\, d\omega\cdot t\sin\theta\, d\theta\, d\omega.\end{aligned} \tag{50}$$

This equation may be simplified somewhat by a transformation employed by *Poisson* in his Memoir of 1819, p. 127. In this we put:

$$la = p\cos\theta' \qquad ma = p\sin\theta'\sin\omega' \qquad na = p\sin\theta'\cos\omega' \tag{51}$$

and then the second terms under the integral signs become of the form

$$t\{\cos\theta'\cos\theta+\cos(\omega-\omega')\sin\theta'\sin\theta\} = t\cos\psi \qquad d\Omega = \sin\theta\, d\theta\, d\omega \tag{52}$$

and therefore

$$\begin{aligned}\Phi = \int_0^{r}\int_0^{\pi}\int_0^{2\pi}(\sigma/4\pi)\int_0^{\pi}\int_0^{2\pi} F\{lx+my+nz-(at+s)+t\cos\psi\}\, r^2\sin\theta\, dr\, d\theta\, d\omega\cdot t\sin\theta\, d\theta\, d\omega \\ +\int_0^{r}\int_0^{\pi}\int_0^{2\pi}(\sigma/4\pi)(d/dt)\int_0^{\pi}\int_0^{2\pi}\Pi\{lx+my+nz-(at+s)+t\cos\psi\}\, r^2\sin\theta\, dr\, d\theta\, d\omega\cdot t\sin\theta\, d\theta\, d\omega.\end{aligned} \tag{53}$$

(ii) Simplified expressions for all the elements of a spherical surface with motion making any angle with the radius. Accordingly, when we have equations of the type found in *Poisson*'s expression (Memoir of 1819, p. 127):

$$P = \int_0^\pi \int_0^{2\pi} f(g\cos u + h\sin u\sin v + k\sin u\cos v)\sin u\,\mathrm{d}u\,\mathrm{d}v \tag{54}$$

we may put

$$g = p\cos u' \qquad h = p\sin u'\sin v' \qquad k = p\sin u'\cos v' \tag{55}$$

and thus obtain:

$$P = \int_0^\pi \int_0^{2\pi} f\{p[\cos u'\cos u + \cos(v-v')\sin u'\sin u]\}\sin u\,\mathrm{d}u\,\mathrm{d}v \tag{56}$$

By using the simplifying formulae:

$$\cos\psi = \cos u'\cos u + \cos(v-v')\sin u'\sin u \qquad \mathrm{d}\omega = \sin u\,\mathrm{d}u\,\mathrm{d}v \tag{57}$$

this reduces to

$$P = \int_0^\pi \int_0^{2\pi} f(p\cos\psi)\,\mathrm{d}\omega\,. \tag{58}$$

»Thus this quantity P represents the sum of all the elements of the spherical surface, multiplied each by a given function of the cosine of the angle comprised between its radius and a radius determined in position.«

A wave flat in the equator of the atom is defined by

$$lx + my + nz - (at + s) = 0\,. \tag{48}$$

The coordinates for the spherical propagation of the wave are

$$x + at\cos\theta \qquad y + at\sin\theta\sin\omega \qquad z + at\sin\theta\cos\omega\,. \tag{59}$$

Hence

$$\begin{aligned}\Phi = {} & (1/4\pi)\int_0^\pi \int_0^{2\pi} F\{l(x+at\cos\theta)+m(y+at\sin\theta\sin\omega)+n(z+at\sin\theta\cos\omega)-(at+s)\}\,t\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\omega \\ & +(1/4\pi)(\mathrm{d}/\mathrm{d}t)\int_0^\pi \int_0^{2\pi} \Pi\{l(x+at\cos\theta)+m(y+at\sin\theta\sin\omega)+n(z+at\sin\theta\cos\omega)-(at+s)\}\,t\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\omega\,.\end{aligned} \tag{60}$$

These solutions are general for wave motion in light, magnetism or similar natural phenomema; and thus it remains to examine certain expressions in *Gauss*' Theory of Terrestrial Magnetism, to ascertain if these phenomena are consistent with the wave-theory. But before entering upon magnetic phenomena, we summarize the hypotheses underlying *Poisson*'s analysis as briefly as possible.

(iii) The equations for waves propagated spherically in an elastic medium.

Consider a system of waves propagated spherically, from any point, whose coordinates are x, y, z, t. Then the coordinates of the disturbed molecules at any time t, will be found in a sphere surface: $(at)^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$

$$\begin{aligned} x - x' &= r\cos\theta &&= at\cos\theta \\ y - y' &= r\sin\theta\sin\omega &&= at\sin\theta\sin\omega \\ z - z' &= r\sin\theta\cos\omega &&= at\sin\theta\cos\omega\,. \end{aligned} \tag{61}$$

Accordingly at the time t the coordinates of the disturbed molecules will be:

$$x + at\cos\theta \qquad y + at\sin\theta\sin\omega \qquad z + at\sin\theta\cos\omega\,. \tag{62}$$

And *Poisson*'s solution yields the integral over the sphere surface (at, θ, ω):

$$\begin{aligned}\Phi = {} & (1/4\pi)\int_0^\pi \int_0^{2\pi} F\{x+at\cos\theta,\ y+at\sin\theta\sin\omega,\ z+at\sin\theta\cos\omega\}\,t\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\omega \\ & +(1/4\pi)(\mathrm{d}/\mathrm{d}t)\int_0^\pi \int_0^{2\pi} \Pi\{x+at\cos\theta,\ y+at\sin\theta\sin\omega,\ z+at\sin\theta\cos\omega\}\,t\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\omega\,.\end{aligned} \tag{63}$$

And the equation of wave motion is:

$$\partial^2\Phi/\partial t^2 = a^2(\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2)\,. \tag{64}$$

The fundamental equations

$$\begin{aligned} &\mathrm{d}u/\mathrm{d}t = a^2\,\mathrm{d}s/\mathrm{d}x \quad \mathrm{d}v/\mathrm{d}t = a^2\,\mathrm{d}s/\mathrm{d}y \quad \mathrm{d}w/\mathrm{d}t = a^2\,\mathrm{d}s/\mathrm{d}z \\ &\mathrm{d}s/\mathrm{d}t = \mathrm{d}u/\mathrm{d}x + \mathrm{d}v/\mathrm{d}y + \mathrm{d}w/\mathrm{d}z \quad s = (1/a^2)\,\mathrm{d}\Phi/\mathrm{d}t \end{aligned} \tag{65}$$

lead to the components of the velocity of any molecule

$$u = \mathrm{d}\Phi/\mathrm{d}x + U \quad v = \mathrm{d}\Phi/\mathrm{d}y + V \quad w = \mathrm{d}\Phi/\mathrm{d}z + W \tag{66}$$

where U, V, W are arbitrary functions of x, y, z, in accordance with the conditions laid down by *Lagrange* in the Mécanique Analytique.

(iv) *Gauss*' theorem that the sum total of positive and negative magnetic fluid in any magnet is zero confirms the wave-theory of magnetism.

In his Allgemeine Theorie des Erdmagnetismus, 1838, p. 21, *Gauss* has shown that the sum total of positive and negative fluid in the entire earth is zero, so that

$$\int \mathrm{d}\mu = 0\,. \tag{67}$$

The expression for the potential, due to the magnetic mass μ, is

$$V = -\int 1/\varrho\cdot\mathrm{d}\mu \tag{68}$$

where the integral to be extended over the whole magnet, and ϱ denotes the distance of the element of magnetic mass $\mathrm{d}\mu$ from the point acted on (x', y', z').

In rectangular coordinates we have:

$$\varrho = \sqrt{[(x-x')^2+(y-y')^2+(z-z')^2]} \qquad (69)$$

and, in the spherical coordinates used by *Gauss*,

$$\varrho = \sqrt{\{r^2+r_0{}^2-2rr_0[\cos u \cos u_0 + \sin u \sin u_0 \cos(\lambda-\lambda_0)]\}} \quad (70)$$

u and u_0 being polar distances, r and r_0 being radii of the earth, λ_0 a fixed longitude, and λ a variable longitude, to be used in extending the integration throughout the mass.

Thus

$$V = -\int 1/\varrho \cdot d\mu = -\iiint(\sigma/\varrho)\, dx\, dy\, dz$$
$$= -\iiint(\sigma/\varrho)\, dr \cdot r\, du \cdot r \sin u\, d\lambda . \qquad (71)$$

Accordingly when we extend *Gauss'* theorem to the entire terrestrial globe we have the expression for the potential:

$$V = -\int_0^{2\pi} d\lambda \int_0^{\pi} \sin u\, du \int_0^{r} (\sigma/\varrho)\, r^2\, dr . \qquad (72)$$

This will give the potential at any point (r_0, u_0, λ_0), which may be outside the earth, as in the moon or sun.

(v) Extension of this theorem to the electrodynamic action between two spheres, as the earth and sun.

Imagine electric currents to circulate around the atoms of two globes: it is required to consider the resulting electrodynamic action. We have the sextuple integral

$$P = \iiiiiint(1/\varrho)\, i\, i' \cos(i, i')\, \sigma\, dx\, dy\, dz\, \sigma'\, dx'\, dy'\, dz' . \qquad (73)$$

The two masses may be called M and m, the latter being the sun.

1. The coordinates of the sun (r_0, u_0, λ_0) may be taken as fixed, while the integration is being extended over the earth.

2. In the same way the coordinates of the earth as seen from the centre of the sun may be regarded as fixed while the integration is being extended over the sun's mass.

3. The two masses as respects each other are thus reduced to weighted points of mass M and m. The action of the sun on the earth's atoms is equivalent to the action of the earth on the sun's atoms:

$$P = m\int_0^{2\pi} d\lambda \int_0^{\pi} \sin u\, du \int_0^{r} (\sigma/\varrho)\, r^2\, dr \qquad (74)$$

$$= M\int_0^{2\pi} d\lambda \int_0^{\pi} \sin u\, du \int_0^{r} (\sigma/\varrho)\, r^2\, dr . \qquad (75)$$

And both of these expressions are zero, in accordance with (67) and (71); for if in the case of the earth's magnetism involving $^1/_{1375}{}^{\text{th}}$ part of the atoms, $\int d\mu = 0$, which means $\iiint(1/\varrho)\, \sigma\, dx\, dy\, dz\, i\, i' \cos(i, i') = 0$; so also in the case of electrodynamic action depending on all the atoms, it follows that

$$\int d\mu/\varrho = \iiint(1/\varrho)\, i\, i' \cos(i, i')\, \sigma\, dx\, dy\, dz = 0 \quad (76)$$

if the integration is rigorously restricted to the limits specified in (74) and (75).

Now it happens that the actions between two globes M and m are not restricted to their centres as seen from each other; but the globes subtend measurable angles 2ω, $2\omega'$, and the atoms are correspondingly dispersed. When the mass is concentrated at the centre, suppose it restricted to a minute measurable area of unit size; then the actual expanded bodies will be larger than this minute area in proportions of ν and ν' times. If the action on unit mass in the minute area be one unit, the action of all the mass in M will be $\nu\sigma$ times that powerful; and that of all the mass in m will be $\nu'\sigma'$. Hence the necessity of integration over every area however small and however minute the density.

(vi) The wave action positive as in the observed case of gravitation.

If the concentration of the action of the distant body in the centres M and m be indicated by integration with rigidly fixed limits, 2π in the case of λ, and π in the case of u, — which restricts the mutual action to a single minute area — we may write two integrals for the whole action: one with no spacial distribution, and the other variable throughout the solid angles 2ω, $2\omega'$:

$$\begin{aligned} P &= \iiiiiint(1/\varrho)\, i\, i' \cos(i, i')\, \sigma\, dx\, dy\, dz\, \sigma'\, dx'\, dy'\, dz' \\ &= \int_0^{2\pi}\int_0^{\pi}\int_0^{r}\int_0^{2\pi}\int_0^{\pi}\int_0^{r}(1/\varrho)\, i\, i' \cos(i, i')\, \sigma r^2\, dr\, d\lambda \sin u\, du\, \sigma' r_0{}^2\, dr_0\, d\lambda_0 \sin u_0\, du_0 \qquad = 0 \\ &+\int_0^{2\omega}\int_0^{2\omega}\int_0^{r}\int_0^{2\omega'}\int_0^{2\omega'}\int_0^{r_0}(1/\varrho)\, i\, i' \cos(i, i')\, \sigma r^2\, dr\, d\lambda \sin u\, du\, \sigma' r_0{}^2\, dr_0\, d\lambda_0 \sin u_0\, du_0 \qquad = A . \end{aligned} \qquad (77)$$

The latter expression A is positive, because all the factors depending on the cosine, $i\, i' \cos(i, i')$ are positive — the total angles of integration being in excess of a whole or semicircumference by the amounts 2ω, $2\omega'$. This last expression (77) explains why gravitation always appears as a positive force, though the electrodynamic action on a point vanishes, — because also it emits no waves. Both bodies fill measurable space, and the angular overlap is 2ω, $2\omega'$ when the action of all the atoms in both bodies is considered.

7. Why Reflected Light is Polarized in a Plane at Right Angles to the Plane of Incidence and Reflection: Confirmation of *Fresnel's* Views.

(i) Mechanical analogies are convincing.

1. We have found the aether to be enormously elastic, so that when any pencil of the medium is filled with a beam of light, which consists of waves tilted at all angles and flowing on in almost infinitely rapid succession, the pencil may be viewed as maintaining its figure by the elasticity of the medium and the rapid succession of the waves. If the pencil of light strikes a solid or liquid surface, the speed of the wave motion is suddenly checked, and reaction on the equilibrium of the pencil at the boundary takes place: so that the vibrations in certain directions are altered by the contact with the solid or liquid surface.

2. To judge by a tangible and familiar experiment, as to what may happen to a pencil of light, we may compare it to the stream of water flowing from the nozzle of a garden hose. The cross-section of the stream of water is assumed to be circular, and we recognize that the forces which keep it so, are chiefly the forward motion and surface tension, — the attraction of the water for itself. In the case of the pencil of light, the equilibrating forces depend on the elastic power within the aether, and thus are different; but the effect produced is very similar, for any slender cylinder filled with a flow of waves.

3. Now we know by daily observation, that when a round stream of water is thrown by a hose against a solid wall, the cross-section of the stream ceases to be circular, and becomes highly flattened, so that the new cross-section of the stream becomes an ellipse, having its longer axis normal to the plane of incidence and reflection. The flattening of the reflected stream of water is easily seen by the most careless of observers: and thus analogy leads us to expect a similar flattening of the vibrations in a beam of reflected light. It is true that the flattened stream of water is not vibrating like the aether, yet the reflected stream is flattened, and tends to retain that figure, with elliptical cross-section.

4. It has been proved by flash-light photography that when liquid drops are forming and falling, the detached spherules oscillate about a mean figure, — being alternately prolate, then spherical, and finally oblate. In the case of drops therefore the particles of the fluid oscillate about a mean position, under the influence of surface tension. The figure of the drop is drawn out of shape at the instant of detachment, and in falling the action of surface tension restores the normal figure, and carries it beyond, so that the globules oscillate about their mean form, which is spherical.

5. Now in the same way, when a pencil of light is reflected from a solid or liquid surface, the act of reflection brings into play, for an infinitesimal time dt, certain forces which tend to flatten the beam, as reflected, in a plane normal to the plane of incidence and reflection. Considering merely the relative motion of the beam in respect to the solid or liquid, we may regard the circular pencil as struck by the solid or liquid in the act of reflection. Owing to its elasticity, each element of the pencil rebounds like a rubber ball — flattened in the plane normal to that of polarization, as we see in the actual behavior of rubber balls in collision. Since each element of the pencil is elastic, there is incessant recovery from the flattening effect — so that the pencil continues to vibrate, but by relative crowding of the vibrating aetherons it has lost its circularity of cross-section, and become elliptical, owing to the restricted freedom imposed in the process of reflection.

6. In Fig. 6, Plate 8, we may imagine equal amplitudes of vibration, in all directions from the centre of the incident beam as shown above; but after reflection the resistance thus encountered forces the circle into the ellipse, as shown. The mutual crowding towards the centre, owing to restricted freedom at the instant of reflection, forces the pencil as a whole out at the sides, and thus it takes on a very elliptical form for the cross-section. In spite of a notable flattening of the pencil of light the aether particles describe ellipses — not straight lines, as often stated, in the theory of polarization.

7. It has been shown by the recent researches of *Heiberg* that *Archimedes* used mechanical means of proving his theorems, at least in the first instance, and then made them rigorous by improving the geometrical demonstration. Accordingly, in dealing with polarization, we are justified in adopting similar methods. And the only question is one of devising a valid model which affords a true analogy. To this end we rely upon the evidence of experiments, in phenomena easily understood and admitting of but one interpretation.

8. The model of a reflected stream of water above outlined certainly is mechanically valid. And it may be confirmed and extended by considering the instantaneous forced form of a series of rubber balls, in such close succession as to be united into a solid tube, like the stream of water, yet not actually touching prior to reflection. At reflection each ball would be flattened by the resistance of the reflector, so that the vibrations of the aether in the pencil take the same form, as observed in polarized light.

9. When liquid drops are formed, in the breaking of a jet, flash-light photography shows them in rapid vibration, owing to surface tension. They form, and vibrate up and down, under gravity; but the waves of the aether pencil would vibrate normal to the plane of polarization, when they are reflected. The vibrations in a plane at right angles to the plane of polarization thus necessarily results from the reflection of waves in an elastic medium.

10. Accordingly, on the basis of actual experience, in well defined phenomena, it is impossible to imagine any kind of vibrations in reflected light other than that at right angles to the plane of polarization.

If we adopted the *Maccullagh-Neumann* theory of vibrations, in the plane of polarization, we should have to expect mechanically a similar effect when a circular stream of water is reflected by a smooth solid wall. No such effect is observed. And as reflection is equivalent to a blow against the round moving stream, renewed at every instant, at infinitesimal intervals dt, we see clearly that the distortion of the vibrations should take place, with the longer axis of the new elliptical motion at right angles to the plane of polarization. No other result is mechanically possible.

(ii) Analysis of light vibrations.

Let the three components of the revolving light vector be:

$$\begin{aligned} u &= a\cos(2\pi\, t/\tau+\alpha) \\ v &= b\cos(2\pi\, t/\tau+\beta) \\ w &= c\cos(2\pi\, t/\tau+\gamma) \end{aligned} \qquad \begin{aligned} (u/a)^2+(v/b)^2+(w/c)^2 &= 1 \\ s &= \sqrt{(u^2+v^2+w^2)}\,. \end{aligned} \tag{78}$$

The fourth of these equations indicates that the path described by the end of the light vector is an ellipse; and the fifth equation gives the displacement relatively to the equilibrium position of the aether particle at any time.

By altering the angles through $\frac{1}{2}\pi-\theta$, we are enabled to use sines in the place of cosines:

$$\begin{aligned} \sin(2\pi\, t/\tau+p) &= u/a = \sin(2\pi\, t/\tau)\cos p+\cos(2\pi\, t/\tau)\sin p \\ \sin(2\pi\, t/\tau+q) &= v/b = \sin(2\pi\, t/\tau)\cos q+\cos(2\pi\, t/\tau)\sin q \\ \sin(2\pi\, t/\tau+r) &= w/c = \sin(2\pi\, t/\tau)\cos r+\cos(2\pi\, t/\tau)\sin r\,. \end{aligned} \tag{79}$$

The quantities u, v, w, represent the rectangular coordinates of the end of the revolving light vector; and the equation for the path, quite independently of the time, may be obtained by eliminating t from equation (79), by the following process. If we multiply the expanded form of these equations by $\sin(q-r)$, $\sin(r-p)$, and $\sin(p-q)$ respectively, and add them, the right hand members will be found to vanish, and we get:

$$(u/a)\sin(q-r)+(v/b)\sin(r-p)+(w/c)\sin(p-q)=0. \quad (80)$$

This linear equation connects u, v, w, which are the rectangular coordinates of the end of the light vector; and hence we see that the path described by it lies in a plane passing through the origin. The path of the vibration therefore is a plane curve.

To get the path as projected on the coordinate planes, we use two of the equations (79). Thus from the first two of these equations we obtain:

$$\begin{aligned}\sin(2\pi t/\tau)(\cos p\sin q-\cos q\sin p)&=\ \ (u/a)\sin q-(v/b)\sin p\\ \cos(2\pi t/\tau)(\cos p\sin q-\cos q\sin p)&=-(u/a)\cos q+(v/b)\cos p.\end{aligned} \quad (81)$$

If we square and add these two expressions, we get

$$\sin^2(p-q)=u^2/a^2+v^2/b^2-2(u/a)(v/b)\cos(p-q). \quad (82)$$

And we see that this equation is that of an ellipse whose principle axes coincide with the coordinate axes when $p-q={}^1/_2\pi$, so that only the first terms of the right member remain, and the left member is unity:

$$1=u^2/a^2+v^2/b^2. \quad (83)$$

This represents elliptically polarized light, in which a and b may have any ratio.

If we put $w=0$, $a=b$, $p-q=\pm{}^1/_2\pi$, we have the conditions for circularly polarized light:

$$\begin{aligned}u&=a\sin(2\pi t/\tau) & v&=\ \ a\cos(2\pi t/\tau)\ \text{(right handed)}\\ u&=a\sin(2\pi t/\tau) & v&=-a\cos(2\pi t/\tau)\ \text{(left handed)}.\end{aligned} \quad (84)$$

When the vibration ellipse reduces to a straight line, or in practice approximately so, $w=0$, $p-q=0$, or $p-q=\pi$, we have by taking the square root of (82):

$$u/a\pm v/b=0. \quad (85)$$

In wave motion, the intensity of the action, or the energy of the disturbance, is proportional to the square of the amplitude. Hence we add, for the geometrical sum, the squares of the component amplitudes A, B, C, and thus obtain:

$$\mathcal{J}=A^2+B^2+C^2. \quad (86)$$

We shall now apply this composition theorem to polarized light. It is well known that such light is free from interference, when polarized in planes mutually inclined at right angles, but always gives an intensity equal to the sum of the intensities of the separate rays.

(iii) Analysis of the composition of polarized light compared with the evidence of observations.

Let us superpose upon the ray defined by equations (79) and traveling along the z-axis, a ray of equal intensity, but polarized at right angles to it. If the components of this new ray be u', v', w', and the phase difference be δ; then we shall have

$$\begin{aligned}u'&=B\sin(2\pi t/\tau+q+\delta) \quad v'=-A\sin(2\pi t/\tau+p+\delta)\\ w'&=C\sin(2\pi t/\tau+r+\delta)\end{aligned} \quad (87)$$

Except for the phase difference δ, these components become identical with those in (79), by rotating the coordinate system through 90°, about the z-axis. Accordingly, we have by taking the sums of the components, thus geometrically compounded:

$$\begin{aligned}u+u'&=A'^2=A^2+B^2+2AB\cos(\delta+q-p)\\ v+v'&=B'^2=A^2+B^2-2AB\cos(\delta+p-q)\\ w+w'&=C'^2=2C^2(1+\cos\delta).\end{aligned} \quad (88)$$

By simple addition we have from (86) the following geometrical composition of the components of the light vectors:

$$\mathcal{J}'=A'^2+B'^2+C'^2 \quad (89)$$

which is equivalent to

$$\mathcal{J}'=2\mathcal{J}+2C^2\cos\delta-4AB\sin\delta\sin(q-p). \quad (90)$$

But it is found by experiment that we have sensibly $\mathcal{J}'=2\mathcal{J}$, or the intensity of the compound ray is equal to the sum of the intensities of the separate rays, and independent of the phase difference δ. Hence it follows that the second and third terms in (90) are so small as to be insensible to observation. Therefore we conclude that within the limits of observation:

$$C=0 \qquad p-q=0. \quad (91)$$

That is, in polarized light the radius vector is sensibly perpendicular to the direction of propagation of the ray, and the motion therefore sensibly transverse. Also from equations (82) or (85) it follows that the particles vibrate sensibly in a straight line.

From this analysis, it follows that rays which have suffered double refraction or reflection at the polarizing angle are plane-polarized, and thus consist of vibrations which are sensibly transverse. We use the term sensibly transverse, instead of absolutely transverse, in order to reconcile other facts of observation with mathematical theory.

It is shown by experimental research that when plane polarized light is reflected from metals, the effect is to convert it into elliptical polarization, — the degree of the ellipticity depending on the direction of the incident ray, and on its plane of polarization, as well as on the nature of the reflecting metal (cf. *Ganot*'s Physics, 14th ed., 1893, § 672, p. 656).

When the plane-polarized light is reflected from silver, the resulting polarization is almost circular — probably because silver is so perfect a conductor of electric or aether wave motion, that the normal tendency to elliptical motion is largely restored. But if the plane-polarized light be reflected from galena, a lead ore of low electric conductivity, the resulting polarization remains almost plane.

Now since elliptically polarized light never vanishes, when examined in a Nicol prism, though at alternate positions it becomes fainter, such elliptical motion in light must be considered the general type of vibration of the aether particle. If therefore plane-polarized light, by reflection from metallic surfaces, is rendered decidedly more elliptical in its motion, it would seem to follow that in plane-polarized light the motion is never strictly rectilinear; on the contrary such light always has in its motion a slight elliptical element, which permits of notable restoration, by reflection from silver and other high conducting metals.

It is for these reasons that, in our discussion of the above equations, (82) to (91), we admit the plane-polarized vibrations to be only sensibly transverse, not rigorously transverse, in rectilinear paths.

This conclusion from the combination of experimental research with mathematical analysis fully sustains our view that there necessarily is a longitudinal component in light. Any other view than that here set forth is contradicted by well established facts of observation, which appear to admit of but one interpretation.

8. The Undulatory Explanation of the Phenomenon of Interference in Polarized Light conforms to *Poisson*'s Theory of the Elliptical Vibrations of the Aether Particles mainly in the Direction of the Normal to the Wave Surface.

(i) Explanation of interference when the particles describe ellipses.

We have shown in section 1 and 4 above that the traditional theory of the transverse vibrations in light is not strictly rigorous, but requires rational revision, to take account of the geometrical conditions specified by *Poisson*, and the related electrodynamic waves from each atom, which underlies the theory of magnetism. Thus it is advisable to reexamine the bearing of these results on the theory of interference of polarized light.

1. The ordinary explanation of interference handed down from the days of *Young* and *Fresnel* is based upon an assumed analogy with the side vibrations of an elastic cord. This theory allows disturbances given the cord to travel along it, while the particles of the cord have only a transverse motion. But we have seen that this explanation begs the question, in that it practically assumes a »stringy« condition of the aether, whereas *Poisson*'s theory of elliptical vibrations, with their major axes in the direction of the normal to wave surface, gives an almost identical result, without physical premises involving the anisotropy of the medium, or geometrical postulates of purely lateral motion which cannot be admitted.

2. Accordingly, the analogy of the waves conveyed along a twisted cord seemed so plausible to those who did not study the problem deeply, that it came into general use, and still holds its place today. Yet a more mischievious doctrine seldom has been introduced into science, because although plausible, it is dynamically and geometrically unsound in principle.

For why is the aether, in the traditional form of the wave-theory, assumed to be capable of a transverse motion of appreciable dimensions, but incapable of an equally large longitudinal motion? The chief reason for this hypothesis — for it is merely a convenient hypothesis — is the problem of explaining interference, and polarization. It is known from modern research that diffraction only requires that the length of the waves shall be small compared to the dimension of the aperture.

3. Fig. 8, Plate 9, shows how a split beam of plane-polarized light may produce interference fringes when they differ in phase by $^1/_2\lambda$.

For reasons of simplicity in construction the oscillations of the particles in the figure are taken to be circular, yet similar reasoning will hold true for elliptical paths of any kind, and hence the results here shown are true for every kind of vibrations in polarized light.

It will be noticed that the rays consist of plane waves with amplitude A and wave length λ, and the ratio A/λ is comparatively small, but here drawn on a scale large enough to enable us to see the rotation of the elements of the wave at every point. The waves are imagined to be flat in the plane of the paper, and hence they have a longitudinal component depending on the amplitude A.

4. The adjacent diagram of light and dark bands shows the interference effects, and is seen to have strips of darkness and of light, where the motions of the split rays are such as to destroy the rotation, or augment it by the superposition of the separate effects. If, for example, the wave travel along the x-axis, the displacement of the particle parallel to the y-axis is η, and ξ parallel to the x-axis:

$$\begin{aligned} \eta &= a\sin(2\pi x/\lambda+\alpha) = \mathrm{d}y \\ \xi &= a\cos(2\pi x/\lambda+\alpha) = \mathrm{d}x \qquad (92) \\ s &= \sqrt{(\eta^2+\xi^2)} = a, \text{ in circular motion.} \end{aligned}$$

Now a detailed treatment of the leading phenomena of interference is beyond the scope of this paper; yet we may sketch briefly the wave-theory of this subject, in order to show that in spite of the defect above pointed out in the form of the wave-theory of light handed down by *Young*, *Fresnel*, *Arago*, and *Cauchy*, this defect does not invalidate the explanation of interference.

(ii) Theory of the light and dark bands.

An adequate treatment of diffraction phenomena would require a mathematical discussion of *Fresnel*'s integrals (*Drude*'s Theory of Optics, pp. 188–196), which have the form:

$$\xi = \int_0^v \cos {}^1/_2\pi v^2\,\mathrm{d}v \qquad \eta = \int_0^v \sin {}^1/_2\pi v^2\,\mathrm{d}v. \qquad (93)$$

These functions may be thought of as the rectangular coordinates of a point in the light plane $\xi\eta$. Accordingly, from (93) we have at once:

$$\mathrm{d}\xi = \mathrm{d}v\cos {}^1/_2\pi v^2 \qquad \mathrm{d}\eta = \mathrm{d}v\sin {}^1/_2\pi v^2 \qquad (94)$$

$$\mathrm{d}s = \sqrt{(\mathrm{d}\xi^2+\mathrm{d}\eta^2)} = \mathrm{d}v. \qquad (95)$$

And when the spacial length s is measured from the origin, we have

$$s = v. \qquad (96)$$

The functions ξ, η are illustrated by the following fig. 9 (*Drude*'s Theory of Optics, p. 192), which has been calculated by the method originally due to *Cornu* (Jour. de Phys., 3, 1874).

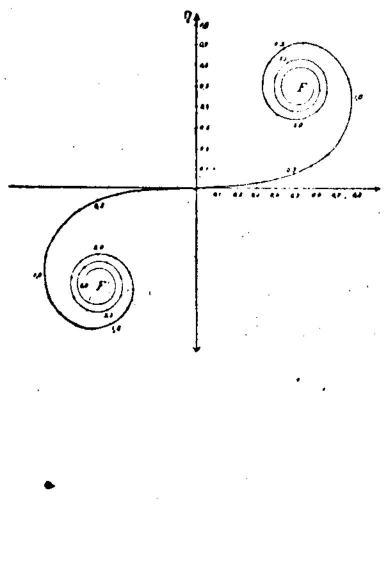

Fig. 9.

Diagram of the double spiral of *Fresnel*'s integrals, for the diffraction of light. The curve coils about the two asymptotic points F and F'. where $v = +\infty$, and $v = -\infty$. The maxima and minima of intensity lie approximately at the intersections of the line FF' with the spiral curve. If the free intensity be 1, the maxima are $\mathcal{J}_1 = 1.34$, $\mathcal{J}_2 = 1.20$, $\mathcal{J}_3 = 1.16$; the minima $\mathcal{J}_1 = 0.78$, $\mathcal{J}_2 = 0.84$, $\mathcal{J}_3 = 0.87$ (cf. *Fresnel* Oevres Complètes, 1, p. 322).

It is shown by *Cornu*'s method that for the asymptotic points F and F' we have

$$\xi_F = \int_0^\infty \cos {}^1/_2 \pi v^2 \, dv \qquad \eta_F = \int_0^\infty \sin {}^1/_2 \pi v^2 \, dv \,. \tag{97}$$

These integrals may be evaluated by putting x, y as the rectangular coordinates of a point P, $x^2+y^2=r^2$, where r is the distance from the origin. If, therefore, we put:

$$\int_0^\infty e^{-x^2} dx = M \qquad \int_0^\infty e^{-y^2} dy = M \tag{98}$$

we get for their product the double integral:

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy = M^2 . \tag{99}$$

Accordingly $dx\,dy$ may be looked upon as a geometrical surface element in the xy plane; and the problem is to evaluate *Fresnel*'s integrals for the diffraction. It is shown that the asymptotic point F has the coordinates

$$\xi_F = \int_0^\infty \cos {}^1/_2 \pi v^2 \, dv = {}^1/_2 \qquad \eta_F = \int_0^\infty \sin {}^1/_2 \pi v^2 \, dv = {}^1/_2 \tag{100}$$

with corresponding integrals for the point F', whose coordinates are negative.

In the more general problem of diffraction we have the two integrals:

$$C = \int \cos [f(x, y)] \, d\sigma \qquad S = \int \sin [f(x, y)] \, d\sigma . \tag{101}$$

Here the function

$$f(x, y) = (\pi/\lambda)(1/\varrho_1 + 1/\varrho_0)[x^2 \cos^2 \varphi + y^2] \tag{102}$$

and σ is a small opening of any form in a screen of infinite extent, while the radii vectores

$$\begin{gathered} \varrho_1 = \sqrt{(x_1{}^2+y_1{}^2+z_1{}^2)} \quad \varrho_0 = \sqrt{(x_0{}^2+y_0{}^2+z_0{}^2)} \\ \varphi = \text{angle of } z\text{-axis with } \varrho_1 \quad \cos(n\varrho_0) = -\cos(n\varrho_1) = \cos\varphi . \end{gathered} \tag{103}$$

Near a straight edge [1]) these functions C and S become:

$$\begin{aligned} C &= \int_{-\infty}^{x'} \int_{-\infty}^{+\infty} dx\, dy \cos \{(\pi/\lambda)(1/\varrho_1 + 1/\varrho_0)[x^2 \cos^2 \varphi + y^2]\} \\ S &= \int_{-\infty}^{x'} \int_{-\infty}^{+\infty} dx\, dy \sin \{(\pi/\lambda)(1/\varrho_1 + 1/\varrho_0)[x^2 \cos^2 \varphi + y^2]\} . \end{aligned} \tag{104}$$

(iii) Application of the theory to the formation of diffraction patterns.

It is found in the application of the above functions, that the equations give the central fringe intensely white, with adjacent blackest area, near the centre of the pattern; here the double integral totally vanishes, but on either side of the centre there remains some illumination. When a space has been traversed along the x-axis equal to a certain length, the light reappears as it should do by the above equations depending on sine and cosine, with corresponding periodicities. The sharpness of the boundary is an essential condition of a well defined diffraction pattern; and without the fulfillment of this geometrical condition, a satisfactory exhibition of the theory can hardly be realized.

In practical experience therefore the values of the double integrals often are somewhat approximate, — the formulae being rigorously true in the centre of the dark and light bands, when the screen effect is mathematically sharp, but at other places only partly true, — and thus we have interference bands, shading away gradually and attaining maxima at intervals, where the contrast reaches a maximum, as shown in the diagram.

In general the researches of experienced physical investigators have shown that the theory of transverse waves accounts for the diffraction pattern with great accuracy. In section 5 above Prof. *Hastings* states that he found *Huyghens*' construction accurate to $1:10^6$, which is a remarkable degree of precision, and equally valid as applied to diffraction phenomena.

Now in our slight correction of the foundations of the wave theory of light, given in sections 1 and 4, we found that an accuracy of $1:(4 \cdot 10^6)$ might be attained before any sensible outstanding phenomena would be likely to arise [2]). And as this is below the limits of our perception in modern experiments, we may dismiss the question as beyond the limits of detection in the present state of physical science.

But to show that a real longitudinal component should exist in light waves, though it is excessively small, we may recall an actual measurement of the smallness of the longitudinal component in a well determined experiment with sound. The late Lord *Rayleigh* observed the musical note f^{iv} due to a pipe of an organ which could be heard at a distance of 820 metres; and found by measurement that the

[1]) One of the great historical difficulties in the wave-theory of light was the problem of explaining with geometrical rigor the propagation in straight lines, since on *Huyghens*' principle each particle of the aether in the wave front becomes a centre of disturbance. The above integrals, as worked out by modern geometers, have their limits so fixed as to include the whole region of disturbance, yielding appropriate fringes due to interference, but otherwise giving rectilinear propagation.

The celebrated geometer *Poisson*, as we learn from the careful note appended to his posthumous memoir of 1839, pp. 151–152, was much occupied with the problem of the rectilinear propagation of light during his last illness:

»Quand le mal moins avancé lui permettait encore de causer science avec ses amis, il a dit qu'il avait trouvé comment il pouvait se faire, qu'un ébranlement ne se propageât dans un milieu élastique que suivant une seule direction; le mouvement propagé suivant les directions latérales étant insensible aussitôt que l'angle de ces directions avec celle de la propagation était appréciable. Il arrivait ainsi à la propagation de la lumière en ligne droite. Plus tard, cédant au mal, et se décidant enfin à interrompre l'impression de son mémoire: c'était pourtant, a-t-il dit, la partie originale, c'était décisif pour la lumière; et cherchant avec peine le mot pour exprimer son idée, il a répété plusieurs fois: c'était un filet de lumière. Puissent ces paroles, religieusement conservées par les amis de M. *Poisson*, les dernières paroles de science qui soient sorties de sa bouche, mettre les savants sur la trace de pensée, et inspirer un achèvement de son oeuvre digne du commencement.«

It is unfortunate that *Poisson*'s memoirs have become very scarce, and thus little known to modern readers. It has long been recognized that there is great need for the reprinting of *Poisson*'s Collected Works. But for my good fortune in obtaining a set of *Poisson*'s celebrated memoirs on waves, formerly belonging to the library of Sir *John Herschel*, the results brought out in this paper would not have been possible.

[2]) In the note dated Sept. 12, below, it is shown that the longitudinal component is $\Lambda = (A/\lambda)\rho = 1:(66420 \cdot 10^6)$, very much smaller than first estimated.

amplitude of the oscillation in these waves could not be greater than 0.06 of a millimetre.

Now in case of f^{iv} there are 2784 vibrations per second, and the length of the wave, under a velocity of 332000 mm per second, is therefore 120 mm. If the amplitude be 0.06 mm, as found by Lord *Rayleigh*, it follows that the wave length is 2000 times the amplitude. As a concrete example of the molecular oscillations which produce musical sound, this result is quite remarkable.

In the case of light we can determine the wave length very accurately, but we cannot measure the amplitude of the aether waves by any direct process[1]). Yet if the length of typical musical waves be some 2000 times their amplitude, it will follow, from the nature of the similar cause involved, that for so elastic a medium as the aether the waves should also be enormously longer than their amplitudes — much greater than 100 times, as assumed by *Kelvin, Maxwell,* and *Larmor*. This value of $A/\lambda = 10^{-2}$ is a relatively small amplitude, but it gives a longitudinal component 20 times larger than that noted in the sound wave above cited.

Accordingly there is reason to believe that in the case of so highly elastic a medium as the aether the amplitude A is less than 1 : 100000th of the wave-length, or at least 1000 times smaller than *Kelvin, Maxwell,* and *Larmor* assumed. This would make the ratio in the case of the very elastic aether fifty times smaller than was observed by Lord *Rayleigh* for typical musical sound in our air. Such a value as $1 : 10^5$ certainly is not too large, but it may be that the ratio should be considerably smaller yet.

The following figure 11 illustrates the interference phenomena observed when light passes through a glass wedge, with the sides mutually inclined at a small angle.

This too represents interference, much like that of polarized light shown in the preceding discussion, but it exhibits the phenomena more in detail; and the phenomena exhibited are consistent with rotating elements in the waves like those in the first diagram. The wedge of glass explains why the waves interfere in bands at right angles to the height of the wedge. In the first diagram the direction of the height of the wedge, for separating the phase of the wave by $\frac{1}{2}\lambda$, would have to be imagined horizontal, and the light returned along a path parallel to its emission.

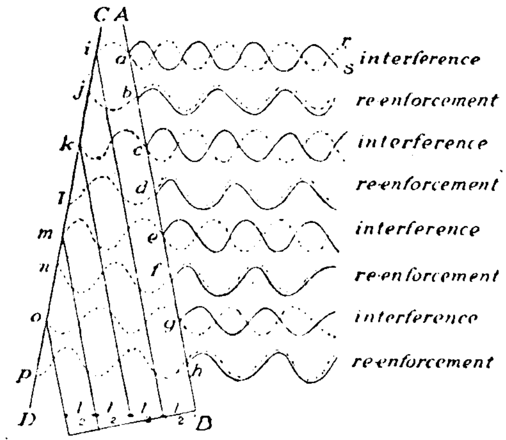

Fig 11. Familiar illustration of interference and reenforcement, when the light of a candle falls upon a glass wedge (*Millikan* and *Gale*). This gives bright and dark bands, parallel to the edge of the wedge, exactly as in the case of Newton-rings about the centre, in the case of a lens.

(iv) Integral expressions for the kinetic and potential energy of the medium when filled with waves.

Let ξ, η, ζ be the rectangular coordinates of a particle at the time t, then the differentials $d\xi$, $d\eta$, $d\zeta$ will denote the component velocities of a particle in the medium which is propagating the waves. The particle is oscillating periodically about a mean position, at any time t, and thus has both a velocity of which the components are $d\xi$, $d\eta$, $d\zeta$, and a distortion from its mean position, or displacement. It is well known that in wave motion half the energy is kinetic, half potential: therefore the kinetic energy due to the component velocities of the particles becomes:

$$T = \tfrac{1}{8}(\mu/\pi)\iiint[(d\xi/dt)^2+(d\eta/dt)^2+(d\zeta/dt)^2]\,dx\,dy\,dz\,. \qquad (105)$$

For the potential energy due to the distortion of the elements of the medium we have:

$$W = (1/4\pi K)\iiint[(d\zeta/dy-d\eta/dz)^2+(d\xi/dz-d\zeta/dx)^2+(d\eta/dx-d\xi/dy)^2]\,dx\,dy\,dz\,. \qquad (106)$$

In these equations the component velocities of the wave disturbances are $d\xi/dt$, $d\eta/dt$, $d\zeta/dt$, and the distortions of the form of the elements $\delta_1 = d\zeta/dy - d\eta/dz$, $\delta_2 = d\xi/dz - d\zeta/dx$, $\delta_3 = d\eta/dx - d\xi/dy$ give the displacements of the elements of the medium along the coordinate axes.

The total energy in the medium at any point is the sum of these two energies: $T + W = \Omega$, or

$$\Omega = (1/4\pi)\iiint\{\tfrac{1}{2}\mu[(d\xi/dt)^2+(d\eta/dt)^2+(d\zeta/dt)^2]+1/K\cdot[(d\zeta/dy-d\eta/dz)^2+(d\xi/dz-d\zeta/dx)^2+(d\eta/dx-d\xi/dy)^2]\}dx\,dy\,dz \qquad (107)$$

which illustrates the agitation of a medium filled with waves.

[1]) Since writing the above paragraph, it has occurred to me that we may calculate the theoretical ratio of the amplitude to the wave-length of the aether by the following process. We have proved that the aether is $\varepsilon = 689321600000$ times more elastic than air in proportion to its density. And it is this elasticity which gives the aether waves their enormous velocity; and, as compared to air, the amplitude should be smaller in proportion to the square root of this number. For when a wave in the aether begins to be generated it speeds away so rapidly, under the enormous elasticity, that the amplitude is small in the same proportion that the velocity v is great. Now from the above value we find that $\sqrt{\varepsilon} = 830254$; and as the ratio in air furnished by Lord *Rayleigh*'s experiments is 1 : 2000, we have for the aether the relative ratio: $1/830254 \cdot 1/2000 = 1/1660508000$, or 830254 times smaller than the ratio of the amplitude to the wave-length in the musical sound investigated of Lord *Rayleigh*. The true ratio thus appear to be 16605 times smaller than that indicated above, and should be $A/\lambda = 1 : (16605 \cdot 10^5) = 1 : (1.660508 \cdot 10^9)$, which makes $\Lambda = (A/\lambda)\rho = 1 : (66420 \cdot 10^6)$. — Note added Sep. 12, 1920.

The wave-theory indicated by all the phenomena of nature.

In concluding this discussion we draw attention to the indications of nature from the widest survey of physical phenomena:

1. In the whole domain of mathematical physics, modern investigations lay great stress on boundary problems. Now boundary conditions naturally would have great importance if natural forces are due to the action of waves; because at the boundary of solid or liquid bodies the velocity of propagation is changed very suddenly by the resistance, and the tendency to refraction and dispersion.

2. In his celebrated article on light, Encycl. Metrop., 1849, section 561, Sir *John Herschel* shows that the forces producing refraction are such as »may be termed infinite«. It is now recognized that these powerful actions appear in dispersion and diffraction, as well as in refraction, and give rise to the molecular forces, which in a future paper will be referred to wave motions, thus confirming the great importance of the wave-theory for all the phenomena of nature.

3. Now quite aside from the physical considerations of particular phenomena, we have general mathematical methods for treating partial differential equations, invented by *Fourier*, *Poisson*, *Cauchy* and other geometers about a century ago. Thus in our time practically all the equations of mathematical physics turn on the treatment of partial differential equations, as in sound, heat, light, electrodynamic action, magnetism, etc. And these general mathematical methods, so largely devised by *Fourier* and *Poisson*, point to waves in the aether as the underlying cause of physical forces.

4. Accordingly, the importance of boundary conditions, in problems of the transmission of energy through matter undergoing sudden transition of property, by virtue of fixed domains of discontinuity, and thus requiring the methods of partial differential equations for their exact treatment, seemed to be so remarkable an argument for the wave-theory that it should engage the attention of geometers and natural philosophers who aim at extending the researches of *Fourier* and *Poisson*.

9. Theory of the Propagation of Wave Energy, under *Poisson*'s Equation $\partial^2\Phi/\partial t^2 = a^2\nabla^2\Phi$, in a Continuous Elastic Solid, with an Analysis which shows Waves traveling in Different Directions.

In the New Theory of the Aether (AN 5044, 5048) we have calculated the mean molecular velocity of the aetheron to be $\bar{v} = {}^1/_2\pi V = 471239$ kms per second, and shown that the aether obeys certain laws of density and rigidity not heretofore suspected. The length of the mean free path is about 573000 kms, and in free space less than one collision per second occurs between the free aetherons, under normal motion. Owing to the decrease of density and rigidity towards large bodies like the sun and earth, all our old analogies with the traditional elastic solid have to be carefully revised, and adapted to the new theory with extreme caution.

After very careful consideration of these problems, in the light of the data contained in the first, second, third papers on the New Theory of the Aether, I believe we may safely conclude that, notwithstanding the very extraordinary physical properties of the aether, in a certain sense it behaves as an elastic solid for quick acting forces: namely, that the aether will faithfully transmit any kind of vibration communicated to it, whether it involve dilatation of volume or mere change of form of any element $dx\,dy\,dz$.

Unless we grant this extraordinary power of transmission of wave motion, we can scarcely reconcile the new theory, including the extreme velocity of the aetheron $\bar{v} = {}^1/_2\pi V = 471239$ kms per second, with the known extreme elasticity of the aether, which is $\varepsilon = 689321600000$ times greater than that of air in proportion to its density. It is evident that the aether is so different from air, in respect to the high velocity of the aetheron, and the enormous elasticity of the medium, that no movement of any kind can occur in it without the most perfect response to whatever waves arise.

In this sense I regard the aether as an infinite aeolotropic elastic solid; but I do not assume that all the physical restrictions of the ordinary elastic solid, which we can subject to experiment in our laboratories, necessarily hold for the aether. Some of these physical restrictions, which we ascribe to molecular forces in solids, may be and probably are missing in the aether, — owing to the absence of the complex molecular structure known in solids, and to the enormously greater rapidity of the motion of the aetherons.

Our conclusions therefore are as follows:

1. Any movement whatever given to the aether will be faithfully transmitted, — owing to the extremely high velocities of the aetherons, which gives the medium both extremely great elasticity and high rigidity, — yet the medium is not like ordinary solids, in that it has an extraordinarily small density.

2. The aether, therefore, has most of the wave transmitting properties of an elastic solid — will transmit any kind of wave; yet always with one velocity only, a uniform velocity $V = 3\cdot 10^{10}$ cms, which is somewhat different from what is attributed to ordinary elastic solids, with two different velocities, of the following kind, namely:

(C) A compressional wave in an extended mass, say of steel, depending on both the compressibility k and the rigidity n:

$$V_c = \sqrt{(k+{}^4/_3 n)} = 655000 \text{ cms per second} \qquad n = 0.95\cdot 10^{12} \qquad k = 1.84\cdot 10^{12}. \tag{108}$$

(T) A purely transverse distortional wave (without change of volume) expressed by the simpler formula:

$$V_d = \sqrt{(n/\sigma)} = 348000 \text{ cms per second} \tag{109}$$

in the case of an extended mass of steel, σ = density = 7.85.

Thus for steel the former value is nearly twice the latter, which renders the theory doubtful, in view of the non-separation of the earthquake waves of these two classes.

3. In certain respects the aether is more like a gas than a solid, and up to this time it is probable that experiment has not fully established the two velocities theoretically predicted for an elastic solid by *Poisson*, *Cauchy* and other mathematicians. In his Tides and Kindred Phenomena

of the Solar System, 1899, pp. 261–2, Sir *George Darwin* remarks in regard to earthquake phenomena:

»The vibrations which are transmitted through the earth are of two kinds. The first sort of wave is one in which the matter through which it passes is alternately compressed and dilated; it may be described as a wave of compression. In the second sort the shape of each minute portion of the solid is distorted, but the volume remains unchanged, and it may be called a wave of distortion. These two vibrations travel at different speeds, and the compressional wave outpaces the distortional one. Now the first sign of a distant earthquake is that the instrumental record shows a succession of minute tremors. These are supposed to be due to waves of compression, and they are succeeded by a much more strongly marked disturbance, which, however, lasts only a short time. This second phase in the instrumental record is supposed to be due to the wave of distortion.«

»If the natures of these two disturbances are correctly ascribed to their respective sources, it is certain that the matter through which the vibrations have passed was solid. For, although a compressional wave might be transmitted without much loss of intensity, from a solid to a liquid and back again to a solid, as would have to be the case if the interior of the earth is molten, yet this cannot be true of the distortional wave. It has been supposed that vibrations due to earthquakes pass in a straight line through the earth; if then this could be proved, we should know with certainty that the earth is solid, at least far down towards its center.«

This reasoning implies that this eminent natural philosopher was in doubt as to the validity of the two-velocity theory, in practice, with actual masses like the earth.

In studying earthquake seismographic records and discussions I find the disturbance to rise very gradually and die down equally gradually. Thus I have not been able to verify the assumption of two distinct types of waves: we merely find that at a great distance from the source of disturbance the earthquake waves are spread out like a spectrum. This spreading out might be due to varying resistance to waves of one type, but of different length, as in optics.

On purely physical grounds it seems difficult to imagine the distortional wave being actually separated from the compressional wave. That actual nature would effect this ideal separation seems very doubtful. And so far it is not supported by earthquake phenomena admitting of verification by observation on the propagation of waves through our globe.

5. Accordingly, it appears that the actual propagation of waves in solids deserves further study. Our premises so frequently are false that the actual facts, in regard to solids both homogeneous and heterogeneous, deserve more statistical inquiry, in cases where a definite decision may be attained. In his article on Light, Encycl. Brit., 9th ed., § 19, p. 446, the late Lord *Rayleigh* says that in such bodies as jelly the velocity of the longitudinal vibrations is a large multiple of the velocity of the corresponding transverse vibrations. No doubt there is some assumed evidence for such a statement, besides the calculations above given, but as no authorities for conclusive experiments of this type are known to me, I think a result of such delicacy should be received with great caution.

6. A few cases, however, even if true, are not enough to establish general conclusions; and in view of the difficulty of conceiving how the two classes of waves can be actually separated in nature — one set of waves inevitably tending to run into the other — the only safe course is to appeal to a variety of experiments, under conditions which may lead to an experimentum crucis.

Notwithstanding this uncertainty as to the true order of nature — the theory being not certainly verified by experiment, — it seems best to examine briefly the chief mathematical conditions imposed by the propagation of waves in an elastic solid. In an elastic solid, the equation of *Poisson*

$$\partial^2\Phi/\partial t^2 = a^2\nabla^2\Phi \qquad (110)$$

is satisfied by the dilatation and three components of rotation as follows:

$$\delta = \partial\alpha/\partial x + \partial\beta/\partial y + \partial\gamma/\partial z \qquad (111)$$

$$\omega_1 = {}^1/_2(\partial\gamma/\partial y - \partial z/\partial\beta) \quad \omega_2 = {}^1/_2(\partial\alpha/\partial z - \partial x/\partial\gamma) \quad \omega_3 = {}^1/_2(\partial\beta/\partial x - \partial y/\partial\alpha) \qquad (112)$$

α, β, γ being the displacements at any point $p\ (x, y, z)$.

In the elastic solid solutions, the components of rotation ω_1, ω_2, ω_3 are connected by the well known relation:

$$\partial\omega_1/\partial x + \partial\omega_2/\partial y + \partial\omega_3/\partial z = 0 \qquad (113)$$

and only two of the three sets of solutions are independent. Combining these with the solution for δ, we have, in all, three sets of independent solutions.

Take a rectangular volume of the elastic substance $x = 0$, $x = \alpha$, $y = 0$, $y = \beta$, $z = 0$, $z = \gamma$. Then at any time $t = 0$, $\Phi = \Phi_0$; and by *Fourier*'s theorem the value of Φ_0 for any point within $\alpha\ \beta\ \gamma$ may be expressed by the following triple summations, which include all positive integral values of l, m, n from 0 to ∞:

$$\Phi_0 = \sum_{l=0}^{l=\infty}\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty} A_{lmn}\cos(l\pi x/\alpha)\cos(m\pi y/\beta)\cos(n\pi z/\gamma) + \sum_{l=0}^{l=\infty}\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty} B_{lmn}\sin(l\pi x/\alpha)\cos(m\pi y/\beta)\cos(n\pi z/\gamma) + \cdots \qquad (114)$$

(cf. Lord *Rayleigh*, Theory of Sound, 2nd ed., 1896, p. 70).

The full set of eight coefficients, for all possible arrangements of sines and cosines, are given by the integral expressions:

$$A_{lmn} = (8/\alpha\beta\gamma)\iiint\Phi_0\cos(l\pi x/\alpha)\cos(m\pi y/\beta)\cos(n\pi z/\gamma)\cdot dx\,dy\,dz \qquad (115)$$

$$B_{lmn} = (8/\alpha\beta\gamma)\iiint\Phi_0\sin((l\pi x/\alpha)\cos(m\pi y/\beta)\cos(n\pi z/\gamma)\cdot dx\,dy\,dz \qquad (116)$$

$$C_{lmn} = (8/\alpha\beta\gamma)\iiint \Phi_0 \cos(l\pi x/\alpha)\sin(m\pi y/\beta)\cos(n\pi z/\gamma)\cdot dx\,dy\,dz \tag{117}$$
$$D_{lmn} = (8/\alpha\beta\gamma)\iiint \Phi_0 \sin(l\pi x/\alpha)\sin(m\pi y/\beta)\cos(n\pi z/\gamma)\cdot dx\,dy\,dz \tag{118}$$
$$E_{lmn} = (8/\alpha\beta\gamma)\iiint \Phi_0 \cos(l\pi x/\alpha)\cos(m\pi y/\beta)\sin(n\pi z/\gamma)\cdot dx\,dy\,dz \tag{119}$$
$$F_{lmn} = (8/\alpha\beta\gamma)\iiint \Phi_0 \sin(l\pi x/\alpha)\cos(m\pi y/\beta)\sin(n\pi z/\gamma)\cdot dx\,dy\,dz \tag{120}$$
$$G_{lmn} = (8/\alpha\beta\gamma)\iiint \Phi_0 \cos(l\pi x/\alpha)\sin(m\pi y/\beta)\sin(n\pi z/\gamma)\cdot dx\,dy\,dz \tag{121}$$
$$H_{lmn} = (8/\alpha\beta\gamma)\iiint \Phi_0 \sin(l\pi x/\alpha)\sin(m\pi y/\beta)\sin(n\pi z/\gamma)\cdot dx\,dy\,dz\,. \tag{122}$$

As Φ is a scalar quantity, we may suppose the rate of increase at any time $t = 0$, to be denoted by $\partial\Phi_0/\partial t$, which may be expanded in series similar to that in (114), but with accented coefficients, A'_{lmn}, B'_{lmn}, C'_{lmn}, etc.

Knowing the initial values of Φ and $\partial\Phi/\partial t$, we may at once write down the complete solution of (110), which is easily seen to be:

$$\begin{aligned}\Phi = &\sum_{l=0}^{l=\infty}\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}\cos(l\pi x/\alpha)\cos(m\pi y/\beta)\cos(n\pi z/\gamma)\,(A_{lmn}\cos pt + A'_{lmn}\sin pt)\\ &+\sum_{l=0}^{l=\infty}\sum_{m=0}^{m=\infty}\sum_{n=0}^{n=\infty}\sin(l\pi x/\alpha)\cos(m\pi y/\beta)\cos(n\pi z/\gamma)\,(B_{lmn}\cos pt + B'_{lmn}\sin pt) + \cdots\end{aligned} \tag{123}$$

In order to satisfy (110) we must have:

$$p^2 = a^2\pi^2(l^2/\alpha^2 + m^2/\beta^2 + n^2/\gamma^2). \tag{124}$$

We may now combine terms which have the same values of l, m, n in (123), and thus we find that Φ can be expressed as a sum of terms of the form:

$$\Phi = \sum K\cos(pt \pm l\pi x/\alpha \pm m\pi y/\beta \pm n\pi z/\gamma - \varepsilon) \tag{125}$$

where the summation is to be extended to all values of $\pm l$, $\pm m$, $\pm n$, and the constants K and ε are of course different for each set of values.

Put in this form, it is clear, as *Jeans* remarks (Dynamical Theory of Gases, 2nd ed., p. 383, 1916), that the solution represents sets of plane waves traveling in different directions. But from (124) it follows that all the waves are propagated with the same velocity a, as in the luminiferous aether.

If the elastic solid has continuous character, its particles have dynamically all the degrees of freedom appropriate to the aether, which is an absolute continuum, the finest molecular or atomic structure in the universe. A medium so constituted has the capacity to transmit waves from any direction. And in case the medium is the ultimate medium underlying the physical universe, no energy can be lost in the movement of the waves, which move incessantly from one body to another, and in free space travel with the velocity of light.

When the velocity of the waves is retarded, energy is expended, and pressure developed by the retardation of the wave front. Forces of a more intricate kind arise when refraction, dispersion, diffraction, etc., develop, as in the encounter with particles or bodies in which the velocity is suddenly changed, and the wave-field redistributed, so that the density and local internal pressure of the aether is altered. But we can only treat of this topic when we come to deal with molecular forces, which heretofore have defied explanation, owing to lack of a kinetic theory of the aether and the undeveloped state of the wave-theory.

Usually it is assumed that in an elastic solid both compressional and distortional waves co-exist, though propagated with different speeds. The two equations of *Poisson* thus become:

$$\begin{aligned} a_1 &= \sqrt{(k + {}^4/_3 n)} \qquad \partial^2\Phi/\partial t^2 = a_1\nabla^2\Phi \\ &\text{for the compressional wave;} \\ a_2 &= \sqrt{(n/\sigma)} \qquad \partial^2\Phi/\partial t^2 = a_2\nabla^2\Phi \\ &\text{for the distortional wave.}\end{aligned} \tag{126}$$

With most solids the latter velocity a_2 is considerably smaller than a_1, the velocity of the compressional wave. In the numerical example of steel above cited, a_1 is nearly twice as large as a_2, but it still is uncertain to what extent a real separation of the two kinds of waves takes place. In other words, the two kinds of waves are distinct and should be separated, in theory; but it is quite uncertain whether this occurs in actual practice, owing to the limitations of freedom of movement in such material bodies as we find in nature. There is only one velocity of waves in the aether.

In the case of earthquake waves, there is no evidence of separation of the two kinds of waves, — all the seismographic records being explicable by the unequal velocities incident to mere wave-length, and thus having different speeds of propagation.

It is true that the earth's crust is a very complex structure, and the movement incident to an earthquake involves release of strain, and thus consists of a series of adjustments of the quasi-solid lava beneath faulted and mutually crowding blocks of granite some 20 miles thick. Perhaps we could not expect distinct separation in such a mass of tremors, partly direct and partly reflected, by the faulted blocks of the crust.

Yet if the two classes of waves actually separated in practice, we ought to perceive two distinct shocks from earth waves incident to explosions, as of powder magazines, masses of T.N.T., and other high explosives, which are powerful enough to be felt at a great distance, but do not involve complex direct and reflex actions in the crust, as in the lava adjustments due to earthquakes.

So far as I have been able to ascertain there is no well established record of double waves from such explosions above ground; and thus the experimental evidence would seem to point to a merging of the two classes of waves into one.

In the case of the aether it is certain that only one class of waves is observed, which in free space travel with uniform velocity, as in the case of sound in gases. Accordingly the aether certainly behaves as a gas, yet its elasticity is so great that waves of any kind may be transmitted, as in an elastic solid, but apparently the velocity is uniform, whether the waves involve a rigidity, with sliding of one layer over another, or compression, as in gases.

10. Geometrical Theory of the Transmission of Light and other Physical Forces along *Fermat*'s Minimum Path, $\delta\tau = \delta\int 1/v \cdot ds = 0$.

(i) The problem of refraction in the minimum path.

For any path in space, with radius of curvature ϱ, and curvature $1:\varrho$, we have for the length of the curved path s and the curvature:

$$s = \varrho\chi \qquad 1/\varrho = d\chi/ds \tag{127}$$

where χ is the angle between the osculating tangent planes, and ds is the element of the curve, and ϱ the radius of curvature, for the osculating circle passing through three consecutive points.

The curvature for any path is

$$1/\varrho = \sqrt{[(d^2x/ds^2)^2+(d^2y/ds^2)^2+(d^2z/ds^2)^2]}\,. \tag{128}$$

And the direction cosines of the radius of curvature

$$\gamma_1 = \varrho\, d^2x/ds^2 \qquad \gamma_2 = \varrho\, d^2y/ds^2 \qquad \gamma_3 = \varrho\, d^2z/ds^2. \tag{129}$$

Now in refraction, the path must be consistent with the principle of least time, and also conform to the principle of least action. The principle of least time was recognized by the Greek geometers at Alexandria, about 300 B.C., in the constructions of *Euklid*, (cf. Electrod. Wave-Theory of Phys. Forc., vol. I, 1917, pp. 63–66), but the principle of the minimum path, in simple refraction, was discovered by *Fermat* (1601–1665), who found the actual path to conform to the law:

$$\tau = l_1 v_1 + l_2 v_2 \tag{130}$$

where the second member is made up of the sum of two terms, each a product of the length of path, l, by the velocity in that path, v.

In gradual refraction, such as that of light in the atmosphere, the direction of the ray changes at every point, chiefly because of the varying density. And thus if τ be the time of passage, we have the integral

$$\tau = \int 1/v \cdot ds\,. \tag{131}$$

And *Fermat*'s condition of the minimum path becomes:

$$\delta\tau = \delta\int 1/v \cdot ds = 0\,. \tag{132}$$

To bring out the geometrical conditions of the theory of the minimum path, we have to develop the subject somewhat as outlined in the author's work of 1917.

By the method of the Calculus of Variations, equation (131) yields

$$\delta\tau = \int 1/v \cdot d\delta s - \int 1/v^2 \cdot ds\,\delta v\,. \tag{133}$$

If λ be the wave-length, it is obvious that the velocity would be defined by the functional relation

$$v = f(\lambda, x, y, z) \tag{134}$$

the form of the function f depending on the arrangement of the parts of the medium.

Making use of this value of v in (133) we obtain

$$\delta\tau = \int (1/v)\,(dx\, d\delta x + dy\, d\delta y + dz\, d\delta z)/ds$$
$$-\int (1/v^2)\, ds\,(dv/d\lambda\cdot\delta\lambda + dv/dx\cdot\delta x + dv/dy\cdot\delta y + dv/dz\cdot\delta z) \tag{135}$$

or
$$\delta\tau = [(1/v)\,(dx/ds\cdot\delta x + dy/ds\cdot\delta y + dz/ds\cdot\delta z)]$$
$$-\delta\lambda\int(1/v^2)\, dv/d\lambda\cdot ds - \delta x\int(1/v^2)\, dv/dx\cdot ds$$
$$-\delta y\int(1/v^2)\, dv/dy\cdot ds - \delta z\int(1/v^2)\, dv/dz\cdot ds\,. \tag{136}$$

The last three integrals of (135), under *Hamilton*'s stationary condition, vanish, because the fixed terminal points make δx, δy, δz each equal to zero. The rest of the expression depends on the terminal points of the path, and on the wave-length only.

These conditions therefore lead to four equations

$$\begin{aligned} \delta\tau/\delta x &= (1/v)\, dx/ds & \delta\tau/\delta y &= (1/v)\, dy/ds \\ \delta\tau/\delta z &= (1/v)\, dz/ds & \delta\tau/\delta\lambda &= -\int(1/v^2)\, dv/d\lambda\cdot ds\,. \end{aligned} \tag{137}$$

Now the tangent to the curved path ds is defined by the three differential direction cosines, fulfilling the condition

$$(dx/ds)^2+(dy/ds)^2+(dz/ds)^2 = 1\,. \tag{138}$$

And therefore if we square and add the first three equations of (137) we shall obtain

$$(\delta\tau/\delta x)^2+(\delta\tau/\delta y)^2+(\delta\tau/\delta z)^2 = 1/v^2\,. \tag{139}$$

(ii) Geometrical conditions fulfilled by *Hamilton*'s characteristic function.

In 1823, when only eighteen years of age, *Hamilton* obtained insight into his method, and gradually introduced the consideration of a characteristic function A defined by the following differential equation for a single particle of unit mass,

$$\delta A = [dx/dt\cdot\delta x + dy/dt\cdot\delta y + dz/dt\cdot\delta z]$$
$$-(dx_0/dt\cdot\delta x_0 + dy_0/dt\cdot\delta y_0 + dz_0/dt\cdot\delta z_0) + t\,\delta H \tag{140}$$

where H is the constant of the total energy $H = T + V$.

If the moving particle be entirely free, the seven variables in the right member of (140) are independent of one another; and thus the characteristic function A fulfills the following remarkable differential equations:

$$\begin{aligned} \partial A/\partial x &= dx/dt & \partial A/\partial x_0 &= -dx_0/dt \\ \partial A/\partial y &= dy/dt & \partial A/\partial y_0 &= -dy_0/dt \\ \partial A/\partial z &= dz/dt & \partial A/\partial z_0 &= -dz_0/dt \\ & & \partial A/\partial H &= t\,. \end{aligned} \tag{141}$$

Therefore we have at once

$$(\partial A/\partial x)^2+(\partial A/\partial y)^2+(\partial A/\partial z)^2 =$$
$$= (dx/dt)^2+(dy/dt)^2+(dz/dt)^2 = v^2 = 2(H-V) \tag{142}$$
$$(\partial A/\partial x_0)^2+(\partial A/\partial y_0)^2+(\partial A/\partial z_0)^2 =$$
$$= (dx_0/dt)^2+(dy_0/dt)^2+(dz_0/dt)^2 = v_0{}^2 = 2(H-V_0) \tag{143}$$

Now it is obvious that if physical forces be due to wave-action, these forces also will conform to the remarkable geometrical properties of *Hamilton*'s characteristic function, and his analysis will be applicable alike to the propagation of light, electrodynamic action and universal gravitation.

Since the characteristic function A satisfies the partial differential equation:

$$(\partial A/\partial x)^2+(\partial A/\partial y)^2+(\partial A/\partial z)^2 = v^2 = 2(H-V) \quad (144)$$

it follows that the partial differential coefficients with respect to the coordinates represent the components of the velocity in a motion possible under the forces whose potential is V. And as V is the potential energy of the system, this result is very remarkable; for it assimilates the propagation of wave disturbances, such as light, and electrodynamic action, to the action of universal gravitation, which also fulfills the same condition.

By partial differentiation of (144) with respect to the co-ordinates we have

$$\left.\begin{aligned}
&\partial A/\partial x\cdot\partial^2 A/\partial x^2+\partial A/\partial y\cdot\partial^2 A/\partial x\,\partial y+\partial A/\partial z\cdot\partial^2 A/\partial x\,\partial z = \\
&\quad = -\partial V/\partial x = X = d^2x/dt^2 = (d/dt)(dx/dt) \\
&\partial A/\partial x\cdot\partial^2 A/\partial x\,\partial y+\partial A/\partial y\cdot\partial^2 A/\partial y^2+\partial A/\partial z\cdot\partial^2 A/\partial y\,\partial z = \\
&\quad = -\partial V/\partial y = Y = d^2y/dt^2 = (d/dt)(dy/dt) \\
&\partial A/\partial x\cdot\partial^2 A/\partial x\,\partial z+\partial A/\partial y\cdot\partial^2 A/\partial y\,\partial z+\partial A/\partial z\cdot\partial^2 A/\partial z^2 = \\
&\quad = -\partial V/\partial z = Z = d^2z/dt^2 = (d/dt)(dz/dt).
\end{aligned}\right\} \quad (145)$$

Also, differentiating in respect to t, we have

$$\left.\begin{aligned}
&dx/dt\cdot\partial^2 A/\partial x^2+dy/dt\cdot\partial^2 A/\partial x\,\partial y+dz/dt\cdot\partial^2 A/\partial x\,\partial z = \\
&\quad = (d/dt)(\partial A/\partial x) \\
&dx/dt\cdot\partial^2 A/\partial x\,\partial y+dy/dt\cdot\partial^2 A/\partial y^2+dz/dt\cdot\partial^2 A/\partial y\,\partial z = \\
&\quad = (d/dt)(\partial A/\partial y) \\
&dx/dt\cdot\partial^2 A/\partial x\,\partial z+dy/dt\cdot\partial^2 A/\partial y\,\partial z+dz/dt\cdot\partial^2 A/\partial z^2 = \\
&\quad = (d/dt)(\partial A/\partial z).
\end{aligned}\right\} \quad (146)$$

On comparing equations (145) and (146), we find that

$$dx/dt = \partial A/\partial x \quad dy/dt = \partial A/\partial y \quad dz/dt = \partial A/\partial z \quad (147)$$

satisfy simultaneously the two sets of equations.

If now we take α, β to be constants which may combine with H to give the complete integral of (144), it follows that the corresponding path and the time of its description are given by the equations:

$$\partial A/\partial\alpha = \alpha_1 \quad \partial A/\partial\beta = \beta_1 \quad \partial A/\partial H = t+\varepsilon \quad (148)$$

where α_1, β_1, ε are three additional arbitrary constants.

By complete differentiation of (148) with respect to t, through the three coordinates x, y, z, we have at once:

$$\left.\begin{aligned}
&\partial^2 A/\partial x\,\partial\alpha\cdot dx/dt+\partial^2 A/\partial y\,\partial\alpha\cdot dy/dt+ \\
&\qquad +\partial^2 A/\partial z\,\partial\alpha\cdot dz/dt = 0 \\
&\partial^2 A/\partial x\,\partial\beta\cdot dx/dt+\partial^2 A/\partial y\,\partial\beta\cdot dy/dt+ \\
&\qquad +\partial^2 A/\partial z\,\partial\beta\cdot dz/dt = 0 \\
&\partial^2 A/\partial x\,\partial H\cdot dx/dt+\partial^2 A/\partial y\,\partial H\cdot dy/dt+ \\
&\qquad +\partial^2 A/\partial z\,\partial H\cdot dz/dt = 0.
\end{aligned}\right\} \quad (149)$$

Similar differentiation in respect to α, β, H, respectively, gives:

$$\left.\begin{aligned}
&\partial^2 A/\partial\alpha\,\partial x\cdot\partial A/\partial x+\partial^2 A/\partial\alpha\,\partial y\cdot\partial A/\partial y+ \\
&\qquad +\partial^2 A/\partial\alpha\,\partial z\cdot\partial A/\partial z = 0 \\
&\partial^2 A/\partial\beta\,\partial x\cdot\partial A/\partial x+\partial^2 A/\partial\beta\,\partial y\cdot\partial A/\partial y+ \\
&\qquad +\partial^2 A/\partial\beta\,\partial z\cdot\partial A/\partial z = 0 \\
&\partial^2 A/\partial H\,\partial x\cdot\partial A/\partial x+\partial^2 A/\partial H\,\partial y\cdot\partial A/\partial y+ \\
&\qquad +\partial^2 A/\partial H\,\partial z\cdot\partial A/\partial z = 1.
\end{aligned}\right\} \quad (150)$$

On comparing these two sets of equations, we find

$$dx/dt = \partial A/\partial x \quad dy/dt = \partial A/\partial y \quad dz/dt = \partial A/\partial z. \quad (151)$$

And as the first members of these equations represent the components of the velocity of the moving particle, it follows that the second members also represent the same thing. Accordingly the proposition stated after equation (144) above is established, and obviously applies equally to light, electrodynamic action and gravitation.

(iii) The physical interpretation of *Hamilton*'s analysis points to wave-action.

We have now to consider the physical interpretation of *Hamilton*'s analysis, and we note first that the celebrated function A was invented by *Hamilton* for the treatment of light. Yet if all physical forces depend on waves due to vibrations in atoms, — with equatorial planes lying haphazard, or mutually inclined at various angles, — it will apply also to magnetism, gravitation, and all kinds of electrodynamic action. *Hamilton*'s characteristic function A is therefore above all a wave-function, equally applicable to all the forces of the universe.

To interpret the above analysis, for the path of light, through a physical medium like the luminiferous aether, we resume the equation

$$(\delta\tau/\delta x)^2+(\delta\tau/\delta y)^2+(\delta\tau/\delta z)^2 = 1/v^2. \quad (152)$$

And we see that if we can obtain a complete integral of this equation, containing therefore two arbitrary constants α, β, in the form

$$\tau = F(x, y, z, \lambda, \alpha, \beta) \quad (153)$$

then the derived equations

$$\left.\begin{aligned}
\partial\tau/\partial\alpha &= \partial F(x, y, z, \lambda, \alpha, \beta)/\partial\alpha = \alpha' \\
\partial\tau/\partial\beta &= \partial F(x, y, z, \lambda, \alpha, \beta)/\partial\beta = \beta'
\end{aligned}\right\} \quad (154)$$

will represent two series of surfaces, whose intersections give the path of the light in the medium.

As α' and β' are also arbitrary constants, the four constants α, β, α', β' are necessary and sufficient for the purpose of making the two intersecting surfaces each pass through any two given points $p_0(x_0, y_0, z_0)$ and $p(x, y, z)$.

These *Hamilton*ian considerations, on simple refraction in non-homogeneous media, show, as was originally found by *Fermat*, that the actual path is that of least time, as well as that of least action.

Now in the case of light the physical cause of such action is known to be waves in the highly elastic aether, and propagated with unequal velocities, in different media, according to density, effective elasticity, and wave-length. Increase of density, due to the presence of ponderable matter, hinders the progress of the wave of given length, while increase of elasticity under thinning out of the matter accelerates it. And in general decreasing the wave-length increases the retardation in velocity.

Equiactional surfaces, orthogonal to the path of light, are so distributed that the distances between them, for geometrical reasons, are always inversely as the velocity in the corresponding path.

Now [it is clearly shown in the third paper on the New Theory of the Aether (AN 5079), that electrodynamic action is conveyed by waves, traveling in free aether with the velocity of light, and therefore these waves will follow the same general laws as the waves of light. Such a physical cause necessarily takes the path of least time and of least action, which is also that of least resistance to the disturbances of the medium. And as the motions of the planets conform to these principles, the question may properly be asked whether any other cause than electrodynamic wave-action could be imagined to produce the attractions of the heavenly bodies.

This question has been dealt with at some length in the second paper (AN 5048), and from the additional discussion included in section 12 below it would seem to follow incontestably that no cause other than wave-action could explain the phenomena of universal gravitation.

11. The New Wave-Theory of Light accounts for all Known Optical Phenomena — Refraction, Dispersion, Anomalous Dispersion, Diffraction, Interference, and the Aberration of Light from the Fixed Stars.

(i) The problem of refraction.

It now remains to survey briefly the leading optical phenomena, to see if the new wave-theory of light will explain the observed phenomena as well or better than the old wave-theory, which assumes vibrations entirely normal to the direction of the ray, as in the motion of a stretched cord, but does not assume vibrations flat in the planes of the equators of the atoms.

And, first, the phenomenon of simple refraction presents no difficulty. For the bending of the light always is due to the unequal resistance offered to the two sides of the wave front, — the one which is more resisted being held back in its advance and the other therefore propagated more rapidly, and thus turning the direction of the ray of light towards the denser medium. This reasoning holds for refraction in water, a prism of glass, or such a slightly heterogeneous medium as the earth's atmosphere, where the air is nearly homogeneous for small distances, yet in the larger problems of the globe arranged in concentric layers, with increased density and refractive power towards the earth's surface.

On the old wave-theory of light this explanation has always been considered satisfactory; and on the new wave-theory it is equally valid, because we consider a beam of light to be made up of an infinite number of independent waves from the separate vibrating atoms. And as each wave is transmitted independently of the rest by the superfine medium of the aether, — just as on a telephone or telegraph wire large numbers of independent messages may be sent at the same instant — it follows that in transmitting the infinitely complex waves of common light, each atomic wave will be refracted exactly as if the others did not exist, and the integral effect after traversing a distance ds will be that all the waves will be refracted in the same direction, owing to the greater resistance on the same side of their common wave front.

Accordingly, the explanation of refraction remains unchanged, while that for dispersion is improved, as shown below.

(ii) The phenomena of dispersion, including anomalous dispersion.

In ordinary refraction, as we have seen, all the rays depending on the waves emitted by the individual atoms, are bent in the same direction; and thus it is evident that if waves be of unequal length, they will encounter unequal resistance, — the shorter waves, owing to their more rapid oscillation, being relatively more resisted than the longer ones. The result of this unequal resistance is that the waves are dispersed, as in the spectrum, the longer waves being least refracted, while the shorter waves, in normal dispersion, suffer maximum refraction, thereby producing the spectrum effect of dispersion, as in a grating.

Now however, many separate waves enter a refracting medium, the refractive action on each vibration occurs as if the other vibrations did not exist: thus we have not merely refraction but also dispersion. In fact dispersion, depending on difference of wave length, seems to imply that the separate atoms, or same atoms, are emitting not only their own distinct waves, but in most cases each atom gives quite a variety of these waves, as we see by comparing the table of wave-lengths for the different elements, as sodium, calcium, hydrogen, iron, titanium, etc.

The observed phenomenon of dispersion is therefore favorable to the new wave-theory; for we realize from the known phenomena of the spectral lines that each atom has its own several periods of vibration; and thus dispersion, or unequal refraction depending on wave length, ought to occur.

As for anomalous dispersion the problem is more complex, because the substances giving this phenomenon exhibit extremely variable effects. But as each atom of a given substance emits its own characteristic waves, there is no reason why the effect of a given refractive medium should affect atoms of the different substances in the same way. The proportion of energy absorbed changes with each substance, and the resistance to each color is a function of the wave-length, but not the same for all wave-lengths, owing to the variable molecular reaction on the passing light waves.

Accordingly just as refraction depends on the wave-length, for homogeneous waves of one color, so also anomalous dispersion must depend on different resistances for different colors or wave-lengths, — due either to the absorptive effects of the substance, by which different wave lengths are unequally affected, with the thinning out of particular wave-lengths, or to the increased resistance of the substance to certain waves, thus causing them to crowd over into an adjacent part of the spectrum.

In the well known case of fuchsine, with the abnormal deviation of the violet rays, by which this color is less deviated than the longer red rays, we may suppose the fuchsine to have an inherent attraction for the violet rays great enough to offset its shorter wave length as compared to the red.

Kundt's careful observations on anomalous dispersion showed that it was common in bodies having surface color — or a different shade by reflected light from that given by transmitted light. Now since in reflection we perceive the colors which are not absorbed, it follows that bodies presenting surface color, different from that shown by light transmitted through them, must absorb the colors which they do not transmit. And therefore in transmission the spectrum is deficient, — certain waves being absorbed or taken up by the vibrating molecules, — so as to make possible the observed deviation of the remaining waves from their arrangement in the normal spectrum shown by a grating.

It would appear from these considerations that the phenomenon of anomalous dispersion is highly favorable to the wave-theory. Unless all molecules emitted and absorbed waves appropriate to their own molecular structure, according to *Kirchhoff*'s law, it does not seem possible to account for the actual results of observation. The theory that each molecule or atom vibrates in its own period, so as to absorb certain waves in transmission, but reflect others from the surface of a body so constituted, seems to harmonize all known facts in a simple way.

(iii) The problem of diffraction, interference, stellar aberration.

The phenomenon of diffraction consists in the bending of the waves through small apertures and at sharp corners, by which light is spread around and gathered into fringes which become distinct. The wave-theory accounts for the phenomenon, under the hypothesis that the waves are very short, which is fully verified by actual measurements. In fact for a given width of slit, different colored light gives an appreciable change in the position of the fringes, depending on the length of the waves in the light used: which obviously confirms the wave-theory, not only as heretofore taught, but also as now modified to take account of waves flat in the planes of the equators of the atoms. The theory of the waves from the individual atoms therefore does not add to the difficulty of the problem of diffraction in any way.

In the matter of interference, the conclusion is similar, as we have already found in section 8 above. This is natural, since the waves from each atom are by hypothesis independent of those from the other atoms; and whatever the positions of the equators, each wave is transmitted by the aether independently of the waves from the other atoms. Interference thus takes place in the modified theory just as in the older theory, except for the detailed changes already described.

In AN 5048, p. 183, we have given a new and simple explanation of the problem of stellar aberration. It is so very direct and simple as to be remarkable. In view of the difficulty felt since *Bradley*'s discovery in 1727, which has been increased rather than decreased by the investigations of the last half century, it is surprising that this simple analysis of the problem of stellar aberration has not been developed before. It presents no difficulty from the old or the new point of view of the wave-theory, but rests wholly on the motion of the earth relatively to the independent motion of the rays of the star, in the moving wave-field carried along with the earth in its orbital motion about the sun.

All that we need consider is the independent motion of the rays of light relatively to the moving earth. We therefore give the parallel rays of light a common backward motion exactly equal and opposite to the forward motion of the earth in its orbit. The diagonal of the parallelogram gives the true motion of light relatively to the moving earth; and by drawing this diagonal of the parallelogram we have a direct and perfectly satisfactory explanation of the stellar aberration.

(iv) *Stokes*' investigation of 1845 harmonizes with the new theory of stellar aberration.

In the Phil. Mag., 1845, 27.9, Sir *Gabriel Stokes* attempted to examine the theory of aberration so as to find out what distribution of velocity may be imparted to the aether about the earth, without changing the path of the rays of light in space. As the new kinetic theory of the aether (AN 5044) was not yet developed, *Stokes* was unwilling to accept the view that the earth could pass freely through the aether whithout setting it in motion; and he tried to find the conditions which would leave the observed aberration unchanged.

If c be the velocity of light in the stagnant aether, in a direction whose direction-cosines relative to axes fixed in space are l, m, n, and the components of the supposed velocity of the aether at any point are u, v, w; then prior to the development of the new kinetic theory, with $\bar{v} = {}^1/_2\pi V$, the velocity of the ray in space at the point in question would be
$$V = c + lu + mv + nw\,. \tag{155}$$

Fermat's minimum path and *Hamilton*'s principle of stationary time, as applied by *Stokes*, would lead to the geometrical condition
$$\delta\tau = \delta\int ds/(c + lu + mv + nw) = 0\,. \tag{156}$$

To quantities of the first order in $(u, v, w)/c$, this is equivalent to
$$\delta\tau = \delta\int ds/c - \delta\int (1/c^2)\cdot(u\,dx + v\,dy + w\,dz) = 0\,. \tag{157}$$

If the medium fulfills hydrodynamically irrotational conditions, without whirling motion of the parts en mass, so that $d\Phi = u\,dx + v\,dy + w\,dz = 0$ is a perfect differential, the second integral will depend on the values of u, v, w at the terminal points, and thus will be independent of the motion in the aether about the earth. When this hydrodynamical condition is satisfied, the path of the ray of light, between two points whose velocities ave given, is determined wholly by the values of these velocities and does not depend on the motion of the aether between these points in the path of the light.

If the terminal points be x_0, y_0, z_0, and x_1, y_1, z_1, — and the intervening medium be filled with a uniform stream of aether flowing with a uniform velocity whose components are u, v, w, — then we shall have
$$\delta\tau = \delta\int_{x_0 y_0 z_0}^{x_1 y_1 z_1} 1/c\cdot ds - \delta\int_{x_0 y_0 z_0}^{x_1 y_1 z_1} 1/c^2\cdot(u\,dx + v\,dy + w\,dz) = 0 \tag{158}$$
$$= \delta\int_{x_0 y_0 z_0}^{x_1 y_1 z_1} ds - \delta\left[1/c\cdot\{u(x_1 - x_0) + v(y_1 - y_0) + w(z_1 - z_0)\}\right]. \tag{159}$$

But by hypothesis the second term of the right member of this last equation is zero, and therefore we have
$$\delta\tau = \delta\int_{x_0 y_0 z_0}^{x_1 y_1 z_1} ds = 0\,; \tag{160}$$

Accordingly the path s obviously is a straight line, in the free aether, from (x_0, y_0, z_0) to (x_1, y_1, z_1), which are the terminal points of the path. *Stokes* found that the differentially irrotational condition would be fulfilled if the aether behaves like a perfect fluid for the slow motion of material bodies through it.

Now in our new theory of the aether (AN 5044, 5048) we have shown that the aether particles fulfill the law of mean velocity $\bar{v} = \frac{1}{2}\pi V = 471239$ kms/sec.

Accordingly, the earth's motion is only 1 : 15708th part of the mean velocity of the particles. And since the velocity of the earth is very small and nearly uniform, owing to the circularity of the orbit, it follows that our planet experiences no secular resistance from the aether.

Moreover, the earth carries its aether wave-field with it, all arranged in perfect kinetic equilibrium, with law of density and wave amplitude

$$\sigma = \nu r \qquad A = k/r \tag{161}$$

extending away from it indefinitely. Thus a ray of light from a fixed star enters the earth's aether wave-field as if this medium were absolutely stagnant. And under the relative motion of the rays of light and the moving earth, the stellar aberration discovered by *Bradley*, 1727, really takes place, just as in the emission theory of light.

For the ray of light from the star pursues a straight line in the earth's wave-field, and the identical component of the earth's motion forward, but directed backward, may be transferred to the moving rays of light before they reach our globe. Thus, relatively to the moving earth, the rays of light really come from the direction in which the stars appear, and ds is a straight line.

This explanation of stellar aberration is therefore geometrically rigorous and perfectly satisfactory. And since in the new wave-theory of light, no change is made in a ray of light as respects velocity and direction, but only as regards the internal tilting of the planes of the vibrations from the individual atoms, we perceive that the explanation of aberration leaves nothing to be desired.

Accordingly it follows that in respect to aberration not the smallest difficulty is encountered in the confirmation of the new wave-theory of light. Such entire agreement, in such diversified optical phenomena, can have no other meaning than that the new wave-theory of light accords with the order of nature.

Other phenomena examined under the new wave-theory of light.

In addition to the above general phenomena there are many special phenomena which might be used to investigate the nature of light. With this object in view I have looked into a variety of observed data to ascertain if any contradiction of the new wave-theory could be established, or even rendered probable. No such result could be brought out, though I have gone over the principal phenomena in optics and electro-optics.

1. Polarization in crystals, which presents complex and intricate interference phenomena, and would be likely to offer a contradiction if any existed in nature.

2. *Brewster*'s law, $n = \operatorname{tg}\varphi$, where n is the index of refraction, and φ the angle of polarization by reflection. The partial failure of this law discovered by *Jamin* and others, when φ differs from $55°\,35'\,30''$, seems to point to the new theory rather than the old. It appears that the outstanding residuary phenomena, not in conformity with this law, but yielding maximum polarizing effect when $n = \operatorname{tg}\varphi$, is not easily explained on the old conceptions of waves wholly transverse to the direction of propagation.

3. The external conical refraction mathematically predicted by Sir *W. R. Hamilton* about 1832, and soon afterwards experimentally verified by *Lloyd* for aragonite was found to be definite and decisive. Yet in examining the cusp-ray refraction *Lloyd* found that the »boundaries were no longer rectilinear, but swelled out in the form of an oval curve« — showing a very gradual diffusion, due to appreciable scattering of light (cf. *Lloyd*'s Miscellaneous Papers Connected with Physical Science, London, 1877, p. 14, figures i and k).

4. Nearly all the very exact measurements on polarized light by Lord *Rayleigh*, *Drude*, *Jamin*, and others bring out residuary phenomena which show a sensible departure from the classic undulatory theory (cf. *Glazebrook*, Physical Optics, London, 1914, pp. 355–387).

5. In the domain of electro-optics, the *Kerr* phenomenon directly points to the wave-theory, including the rotation of the plane of polarization by magnetism; and all this is even more consistent with the new wave-theory than with the old. If the poles of an electro-magnet are polished, and plane polarized light is reflected therefrom, it is found that when no current passes the plane of polarization is not rotated. If then the current flows in one direction, there is a corresponding rotation of the plane of polarization; and the moment the current flows in the opposite direction, and thus changes the pole to opposite polarity, the plane of polarization is rotated in the opposite direction. This is very definite proof of the wave-theory, both for optics and magnetism, for the *Kerr* and *Zeeman* phenomena.

6. The production of elliptically polarized light by letting a polarized beam fall upon a transparent insulator, such as glass, liquids or gases, under strong electric stress, — the region being filled by electric waves rotating in definite direction, as in a magnetic field — was first discovered by *Kerr*, and confirmed by *Becquerel*, *Kundt*, *Röntgen*, *Quincke*, *Lippich*, *Du Bois*, and others. When the medium is connected with the poles of an electric machine, the waves constituting the discharge make it possible to produce double refraction, as in a crystal, and in *Zeeman*'s phenomena, where the spectral lines are doubled. All these phenomena are found to harmonize with the new wave-theory, quite as well as or better than with the classic theory of *Fresnel*.

12. The Wave-Theory of Gravitation towards a Single Body extended to the Case of Waves from Two Equal Bodies by means of the Geometrical Theory of Confocal Conics, in Conformity with the Observed Motions of Planets and Comets under the Newtonian Law.

(i) Why the aether remains heterogeneous and presses towards a single body like the sun.

1. In our theory of the emission of light and heat waves from the sun, (AN 5044), we have shown that under the spherical expansion of the wave surface in free space, the amplitude of the waves follows the law

$$A = k/r \tag{162}$$

and the force towards the centre due to the receding waves is therefore as the square of the amplitude:

$$f = A^2 = k^2/r^2 \qquad (163)$$

which has the form of the law of gravitation observed in nature.

2. The mere existence of waves, as of light and heat, — which certainly radiate from the sun with tremendous energy, — thus necessarily operates to make the aether heterogeneous, according to the law $\sigma = \nu r$. There is no doubt of this law holding for light and heat waves; and if gravitational and magnetic waves exist, they too will follow this same law. It appears that ‚Magnetic Storms' and ‚Magnetic Tides' are referable only to waves, as shown in my work on Physical Forces, 1917; and aside from the connection of electrodynamics with gravitation previously shown to exist, it is fair to ask the broad question:

What is the probability that the force $f = A^2 = k^2/r^2$ would give an appropriate wave amplitude $A = k/r$, unless gravitational waves also exist? No such coincidence could occur by mere chance! In fact the chances against such a coincidence occurring for all the atoms of a body in the potential

$$V = \iiint[(x-x')^2+(y-y')^2+(z-z')^2]^{-1/2}\cdot\sigma\,dx\,dy\,dz \qquad (164)$$

is at least infinity of the third order (∞^3) to 1.

Moreover, since electrodynamic action certainly is due to waves, and these exert a mechanical action like magnetism and gravitation, what is the chance that there is a sudden break in the continuity of natural forces at the boundary which is assumed to divide electrical action from universal gravitation? Evidently the probability is zero. For we find by experiments on all the forces of nature that the doctrines of the correlation of forces and the conservation of energy are valid. Thus it is impossible to separate gravitation from the other forces of nature, whose electrical character is so well established.

3. The aether is thus thinned out by wave agitation, towards all single masses; and as the aetherons have a velocity of 471239 kms per second, we perceive that the elasticity, $\epsilon = 689321600000$ times greater than that of our air in proportion to its density, would secure an instant homogeneity of the aether everywhere but for the incessant action of the receding waves. Accordingly the world is filled with waves, constantly received and constantly emitted from all atoms. The waves are in some way due to the motions of the aetherons, which collide with and are reflected by the reactions of the atoms.

4. Thus on the one hand, the receding waves would give by reaction the central pressure of gravitation; and on the other, the resulting heterogeneity of such an elastic kinetic medium also implies the same central pressure. Owing to the enormously rapid motions of the aetherons they tend powerfully to become equally distributed, and thus make the medium homogeneous, but as they are reflected violently from the atoms, — the collisions and reflections keeping up the waves incessantly — the medium remains heterogeneous, with the energy of the central inrush of the aetherons just balancing the loss of energy by the waves receding away.

5. In *Drude*'s Theory of Optics, 1917, pp. 179–180, (English translation by *Mann* and *Millikan*), a very remarkable theorem is drawn from the rigorous formulation of *Huyghens*' principle, as follows:

»When the origin lies within the surface S,

$$4\pi s_0 = \int\{\partial[s(t-r/V)/r]/\partial r\cdot\cos(nr) - (1/r)\partial s(t-r/V)/\partial n\}\,dS. \quad [35] \qquad (165)$$

»This equation may be interpreted in the following way: The light disturbance s_0 at any point P_0 (which has been taken as origin) may be looked upon as the superposition of disturbances which are propagated with a velocity V toward P_0 from the surface elements dS of any closed surface which includes the point P_0. For, since the elements of the surface integral [35] are functions of the argument $t-r/V$, any given phase of the elementary disturbance will exist at P_0, r/V seconds after it has existed at dS.«

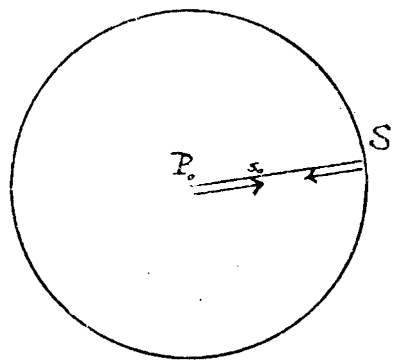

Fig. 12. Diagram of the disturbance s_0 reflected from the surface S about the point P_0, and thus maintained in perpetuity.

6. It thus appears that disturbances emanating from P_0 towards dS in a conical solid angle $d\omega$, may be ascribed to disturbances from the element dS of the same conical solid angle $d\omega$, from any closed surface about P_0. Just as the integral of the outflowing waves gives $4\pi s_0$, equation (165), so also the integral of the energy of the inflowing disturbances are equal, and oppositely directed, which proves the proposition.

7. The integral of the vibrations in the separate sources of the inflowing disturbances dS has to be taken over the whole closed surface, and thus the calculation is complicated, involving a surface integral at the interval dt over the solid angle $\omega = 4\pi$ about the point P_0. And in order to maintain the action the integral has to be renewed at infinitely short intervals, dt, corresponding to surface thickness

$$dr, \quad dV = 4\pi r^2\int_0^{dr} dr. \qquad (166)$$

But as these renewed integrals have the same value for the interval dt, we may take dV constant,

$$dV = 4\pi r^2\int_0^{dr} dr = c \qquad (167)$$

owing to the uniformity of the propagation of light.

Accordingly, if the aetherons were once heterogeneous, in spacial distribution, they would always rush inward, and perhaps generate waves even if they did not already exist. But once existing, and emitted as light, heat or other like radiation, the heterogeneous density of the aetherons will always exist. Hence the wave-field about a body like the sun depends on the kinetic exchange of the rapidly moving aetherons, under the steady outflow of waves, and is therefore eternal like matter itself.

8. This explains rigorously the central pressure of gravitation. If waves exist, the density thereby becomes $\sigma = \nu r$; and since the waves of light and heat fulfill this law, the waves of magnetism and of gravitation also necessarily conform to it.

The moon's fluctuations show that gravitational waves really exist, and are long enough to traverse the earth's mass, just as similar gravitational waves traverse the bodies of Jupiter, Saturn, etc. It also follows that the aether is excessively fine grained, otherwise these refractive phenomena would not be distinctly realized, so as to become sensible to observation in the effects they produce on the moon's motion.

9. The above mathematical theorem, relative to the inward propagation of the disturbances from a closed concentric surface S, with velocity V, equal to the velocity of the waves traveling outward from the centre P_0, will be fulfilled by the energy flow conveyed through the aether by the individual aetherons from any spherical surface $S = 4\pi r^2$. It is not necessary that the disturbances s_0 from the elements of the enclosing surface dS be waves; they may be stresses due to the energy of the individual aetherons produced by the heterogeneity incident to the receding waves, and thus converging to the centre whence the waves come.

Accordingly, the above integral (165) rigorously fulfills the geometrical condition for a heterogeneous aether: it is kept to the law of density $\sigma = \nu r$ by the receding waves, and the aetherons always pressing inward, by virtue of this very heterogeneity, and the enormous elasticity $\varepsilon = 689321600000$ times greater than that of our air in proportion to its density.

(ii) Physical illustration of the effects of waves from the two foci of an ellipsoid, corresponding to a double star with equal components.

The accompanying wave plate Fig. 13 (*Guillemin*, Les Phénomènes de la Physique, 1869, p. 182) represents a faint system of confocal conics due to waves receding from two equal centres, such as a double star of equal components:

(a) The confocal hyperbolas represent the reacting pressures at the ellipsoidal boundary, if reflection were to take place there, or the inwardly directed stresses fulfilling the above equation for $4\pi s_0$, under *Huyghens'* principle for this more complex system of two bodies, instead of the one central mass already considered.

(b) Each wave from any centre as it reaches the hypothetical ellipsoidal boundary is met there by a wave from the other centre; and in reflection the reaction from the assumed bounding surface is in the direction of the hyperbolas, as shown in the figure. The reflection is perpendicular to the surface of the bounding ellipsoid; and, whether reflected or not, the stresses are along the hyperbolas shown.

(c) If one of the bodies be nearly insensible in mass, it is obvious that the other will emit practically all the waves, and the reaction or reflection would be central, as in the case of a spherical body like the sun. When there is a single centre of waves, a comet may be made to move about it in a conic section, by giving it an initial velocity equivalent to the integrated effect of the two bodies from infinity, (the smaller being now removed from the simplified problem). Accordingly if the influence from the other focus be cut off, at the instant of starting, yet its integrated effect be included in the initial velocity, we have the motion in conic sections for a single body as laid down by *Newton*. There are infinite systems of hyperbolas, parabolas, ellipses, which may be described, depending on the initial conditions, as more fully set forth below.

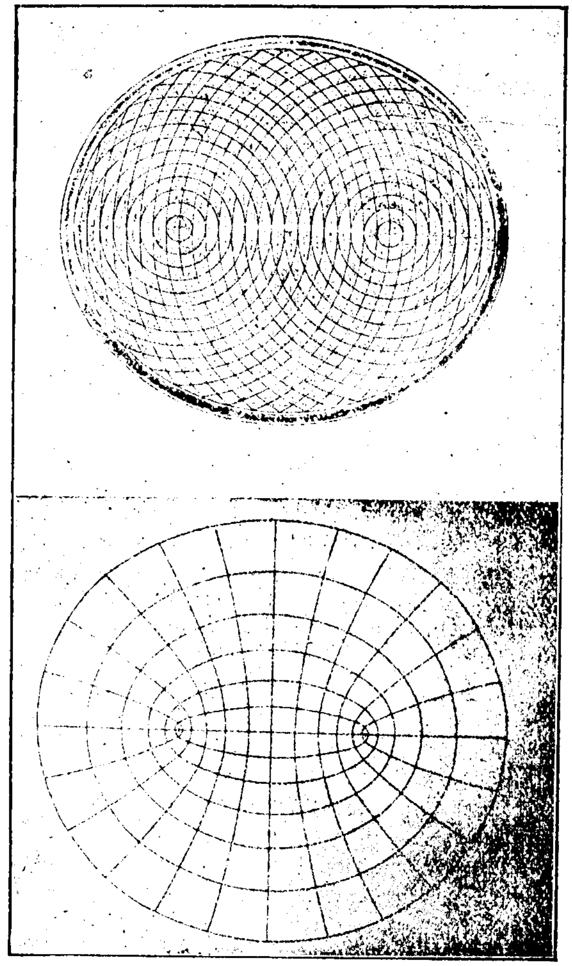

Fig. 13. The upper figure is a diagram of the waves propagated from two equal foci. As reflected from the enclosing ellipsoidal surface, they produce the confocal hyperbolas normal thereto. The entire system of confocal conics is made more distinct in the lower figure.

(d) These novel considerations throw a new light on dynamical problems, and bring the laws of celestial mechanics into harmony with the wave-theory. They are therefore of deepest interest in the theories of the motions of bodies. Every possible motion in a system of two bodies is accounted for, by the effects of perfectly simple waves, and the resulting stresses in the aether, towards central masses. Celestial mechanics thus acquires a hydrodynamical basis, the aether being always subjected to stresses, owing to the waves receding from the stars and other bodies of the physical universe.

A very remarkable comparison may now be made between the waves from two foci reflected from an enclosing ellipsoidal surface, and that above given for waves reflected from a spherical surface enclosing a single centre.

1. We have seen that if the waves emanating from a single centre be reflected from the enclosing spherical surface $S = 4\pi r^2$, we have the equation (165).

2. From this equation it follows that if we imagine a wave-field established, in kinetic equilibrium, about a radiating star, and suddenly enclose that star by a perfectly reflecting surface, $S = 4\pi r^2$, the energy near the centre will flow outward, till reflected at the enclosing boundary, while that near the boundary will as steadily flow inward, to restore the energy lost by the central spherical shells,

$$4\pi\int_0^r r^2\,\mathrm{d}r.$$

3. And as the velocity of propagation V is constant, we have

$$4\pi\int_r^{r+\mathrm{d}r} r^2\,\mathrm{d}r = 4\pi\int_R^{R-\mathrm{d}R} r^2\,\mathrm{d}r. \tag{168}$$

Accordingly, the loss of wave energy from the centre and its perfect restoration goes on without ceasing, and the motion of the waves thus confined is eternal.

4. Now in the same way, let us imagine waves emanating from two equal foci, as in the case of a double star with equal components, and suppose both foci suddenly enclosed by a perfectly reflecting, confocal, concentric, ellipsoidal surface:

$$x^2/(a^2+\lambda)+y^2/(b^2+\lambda)+z^2/(c^2+\lambda) = 1. \tag{169}$$

Then the waves from either focus will return to the other in an interval of time $\mathrm{d}t$, corresponding to the distance $2a$, traveled before and after reflection, in any plane section of the ellipsoidal surface; and thus the wave-field about either focus will be perpetual. And just as the wave-field reflected for restoration is perpetual, so also the inward stress, from the aether outside the surface, is equal to the radiant energy constantly reflected, and thus also eternal. This is the foundation of celestial dynamics, resulting from the new theory of the aether.

5. The inwardly directed system of confocal hyperbolas indicate the direction of the wave stresses sustained by the ellipsoidal reflecting surface. And since if we remove the surface, the waves will proceed into infinite space, we recognize that a wave-field about the two radiating foci must [redacted] inward stress of the [redacted] normal to [redacted] aether thus exerts its stress along the tangents to the systems of confocal hyperbolas.

6. This geometrical description conveys to our minds a very clear dynamical illustration of the behavior of the aether about a system of **two equal stars**. The inward stress is no longer directed **to each centre separately**, but the total effects for the two centres are combined as shown by the system of confocal hyperbolas. The system of confocal conics shown in the accompanying illustrations is thus of the highest dynamical interest.

(iii) The wave-theory rigorously extended to a system of two bodies, by means of the geometrical theory of confocal conics.

We have just investigated the physical theory of waves propagated from the two foci of an ellipsoid, and shown that very remarkable phenomena may thus arise. As the theory thus outlined may have great dynamical importance, it is necessary to examine the problem somewhat more critically from the point of view of geometrical rigor.

Perhaps it is not immediately obvious what all the physical phenomena would be in a wave-field about two equal stars. Yet there obviously is ample assurance that should the wave-theory triumph for a pair of equal binary stars, it would necessarily hold for triple and quadruple stars, and sidereal systems of higher order such as we find in the globular clusters. These splendid sidereal systems are so crowded with stars in their inner spherical shells as to attain a perfect blaze of starlight towards the centre, and thus the glory of globular clusters, like M.13 in Hercules, ω Centauri, and 47 Toucani, is unrivaled by any other objects in the starry heavens.

Accordingly we recall briefly the geometry of confocal conics, in the hope of illuminating the wave-fields in sidereal systems of high order, so much studied by the elder *Herschel*.

The equation of a system of confocal conics in the xy-plane is

$$x^2/(a^2+\lambda)+y^2/(b^2+\lambda) = 1. \tag{170}$$

And for the more general system of confocal conics, in tri-dimensional space, the corresponding equation is

$$x^2/(a^2+\lambda)+y^2/(b^2+\lambda)+z^2/(c^2+\lambda) = 1. \tag{169}$$

From the forms of these equations, we perceive that, what applies to the plane of xy, will apply also to the system of confocal conoids in xyz. Thus for the sake of simplicity we shall consider the system of confocal conics chiefly in the plane xy, as sufficiently general for the requirements of our present problem in tri-dimensional space.

If λ is positive in the equation, the resulting curve is an ellipse; but if λ is negative the curve becomes an hyperbola. The transition from the ellipse to the hyperbola is explained as follows.

From the form of (170) we perceive that the principal axes of the curve will increase as λ increases, and their ratio will tend more and more to equality as λ increases. Accordingly a circle of infinite radius, ($a = b = \infty$), gives the *limiting form of the elliptical confocals.*

On the other hand, when λ is negative, the principal axes will decrease as λ increases; and the ratio

$$\varrho = (b^2+\lambda)/(a^2+\lambda) \tag{171}$$

will also decrease as λ increases. The ellipse thus becomes flatter and flatter, until λ is equal to $-b^2$, when the minor axis vanishes, $b^2+\lambda = 0$; and the major axis is equal to the distance between the foci. The curve thus narrows down to the line-ellipse joining the foci, which is a limiting form of one of the confocals.

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If the process continue till $b^2+\lambda = -\eta$, a small negative quantity, the transverse axis of the hyperbola is very nearly equal to the distance between the foci; and the complement of the line joining the foci is a limiting form of the hyperbola. This limiting form of the hyperbola is the narrow hyperbola shown in the figure at the right and left respectively. When $b^2+\lambda = -\eta$ is a larger negative quantity, the hyperbola spreads its branches more widely and the vertex becomes more distant from the foci on the horizontal axis, as shown in the central part of the figure. As λ becomes greater and greater, the angle between the asymptotes of the hyperbola increases, and in the limit both branches coincide with the axis of y.

Accordingly, we perceive that by making λ approach $-b^2$, we narrow up the confocal ellipses into a straight line joining the foci. And when the change continues still further, $b^2+\lambda = -\eta$, a very small negative quantity, the curve passes from the straight line joining the foci into another straight line running from either focus to infinity, which give the line-hyperbola, corresponding to the internal line-ellipse. The point describing the line-ellipse thus ceases to move between the foci, and returns to the other focus through infinity, when the limiting elliptical confocal passes into the limiting hyperbolic confocal. When λ is negative and numerically greater than a^2, the curve is imaginary.

Let us now return to the above figures, and imagine two equal wave centres, as from a double star of equal components, like γ Virginis; then obviously we have two equal wave-fields, one about each focus, with the double system of confocal hyperbolas, as shown in the above figure. The entire solid angle about the centre of the confocal ellipses is $\Omega = 4\pi$.

But we may split the system of confocal hyperbolas into two equal parts, on either side of the median plane, each equal to $\frac{1}{2}\Omega = 2\pi$.

It will be evident on reflection that all the hyperbolas about the lower focus will curve about the right star f, just as in the case of comets revolving about our sun: and all about the upper focus will curve about the left star f'. And these infinite systems of hyperbolas will include curves of all possible eccentricity, with a perihelion distance less than a, half the distance between the two foci.

The waves propagated from two equal stars by generating a doubly infinite system of confocal conics — the ellipses cutting the corresponding hyperbolas at right angles — fix the paths of infinite varieties of comets about either focus, as will be more fully discussed below.

(iv) Geometrical properties of confocal conics.

(a) Two conics of a confocal system pass through any given point — one an ellipse, the other an hyperbola. After the above outline this is almost obvious, without further discussion, for if the equation of the original conic be

$$x^2/a^2+y^2/b^2 = 1 \tag{172}$$

the equation of the confocal conic is

$$x^2/(a^2+\lambda)+y^2/(b^2+\lambda) = 1\,.$$

And it is obvious that this curve will pass through the given point $(x'y')$, if

$$x'^2/(a^2+\lambda)+y'^2/(b^2+\lambda) = 1\,. \tag{173}$$

To find the solution for this condition, we remember that $b^2 = a^2-a^2e^2$, and put $b^2+\lambda = \eta' = a^2-a^2e^2+\lambda$, and thus obtain from (173)

$$x'^2\eta'+y'^2(\eta'+a^2e^2)-\eta'(\eta'+a^2e^2) = 0$$

or

$$\eta'^2-\eta'(x'^2+y'^2-a^2e^2)-a^2e^2y'^2 = 0\,. \tag{174}$$

This is a quadratic with two roots, both real, but of opposite signs, and thus there are two conics, $b^2+\lambda = +\eta'$ being the ellipse, and $b^2+\lambda = -\eta'$ being the corresponding hyperbola.

(b) One conic of a confocal system and only one will touch a given straight line.

From the equations

$$lx+my-1 = 0 \qquad x^2/(a^2+\lambda)+y^2/(b^2+\lambda) = 1 \tag{175}$$

we find for tangency:

$$(a^2+\lambda)l^2+(b^2+\lambda)m^2 = 1 \tag{176}$$

which is linear in λ, and yields one value of λ, corresponding to one confocal conic, and only one, bounding the given straight line. This might be tangent to the ellipse, or to the hyperbola, but not to both at the same point, because the hyperbolas always are at right angles to the ellipses at their intersections.

By subtraction we have from the two equations

$$x^2/a^2+y^2/b^2 = 1 \qquad x^2/(a^2+\lambda)+y^2/(b^2+\lambda) = 1$$

$$x'^2/[a^2(a^2+\lambda)]+y'^2/[b^2(b^2+\lambda)] = 0\,. \tag{177}$$

And as the condition of tangency is

$$xx'/a^2+yy'/b^2 = 1 \qquad xx'/(a^2+\lambda)+yy'/(b^2+\lambda) = 1 \tag{178}$$

we see that (178) shows the rectangularity of the curves at their intersections.

(v) Application of the theory of confocal conics to the motions of comets, as under the wave-theory of physical forces.

Referring to the figure given above for the waves from two equal stars, we notice that the boundary there represented is one of the confocal ellipses; others of greater oblateness are shown nearer the centre of the figure, but the approximations to the line-ellipses very near the centre are omitted, for reasons of clearness.

It will be found that the spherical waves propagated from these two centres give the confocal ellipses, and also the confocal hyperbolas, as clearly outlined in this figure. The independent circles about the two foci are at distances at_1, at_2, at_3, $\ldots at_n$.

At the boundary the waves from the two foci are reflected, with reaction in the direction of the perpendicular to the surface. Hence we see that the normals at these points of reflection give the confocal hyperbolas. Accordingly, if waves were traveling with uniform velocity from both foci, and reflected at the confocal elliptical boundary, there would thereby result stresses in the aether directed along the confocal hyperbolas at the intersections of these two systems. This result of the intersecting system of confocals is very remarkable, since it will hold for every point of infinite space, and thus for ellipses and hyperbolas of every possible form, mutually intersecting at right angles, as shown in the figure.

It was established by the researches of Prof. *Strömgren*, of the Royal Observatory, Copenhagen, about 1910–11, that all the comets heretofore observed describe ellipses about the sun in one focus. It had previously been supposed that the orbits of certain comets were hyperbolic, yet greater refinement of research proved the elliptical character of all these orbits; so that they return to our sun, and thus are relics of our primordial solar nebula, as set forth in my Researches on the Evolution of the Stellar Systems, vol. II, 1910.

If the comets had greater than the parabolic velocity of movement relatively to our sun, $v > k\sqrt{(1+m)} \cdot \sqrt{(2/r)}$, the paths would be hyperbolas; such orbits, however, are not yet of record. It is obvious that we can now interpret the physical significance of the system of confocal conics, in conformity with the observed laws of celestial mechanics, and the indications of the Wave-Theory of Physical Forces.

For example, if a comet with zero velocity were to cross the boundary to enter the field about the two foci, in the above wave-figure, the instantaneous stresses to the foci, on the line of the reflected waves, would cause the comet to pursue the indicated hyperbola, passing through the point (x, y). Under slightly modified conditions this reasoning might be greatly extended, but we shall not enter upon it here.

In conclusion, it only remains to add that in the fifth and sixth papers I hope to throw some light on the obscure physical cause underlying molecular and atomic forces. The calculation of the wave-stresses at the boundary of a liquid globule, such as a rain drop or a drop of dew, will lead us to the cause of surface tension, constantly acting for the generation of minimal surfaces throughout nature.

It is not by chance that all liquid drops take the spherical form! The geometer may discover therein a great secret of the physical universe!

If so, this advance will illuminate also the difficult problem of capillarity, which has already engaged the attention of so many eminent geometers. Whence we hope to attack the subject of cohesion and adhesion, and even of explosive forces, which heretofore have appeared even more bewildering.

Mr. *W. S. Trankle* has laid me under lasting obligations by facilitating the completion of this fourth paper. And Mrs. *See*'s sympathetic interest in these researches has lent a support which often proved so invaluable as to be beyond all praise.

Starlight on Loutre, Montgomery City, Mo., 1920 Sept. 6. *T. J. J. See.*

Der Veränderliche RS Virginis 142205. Von *M. Esch*, S.J.

Diesen Veränderlichen habe ich nur in den Jahren 1900, 1902, 1918 und 1919 häufiger beobachtet. Meine Vergleichsterne waren:

	ASV	HA	BD	Gr.	Autorität
C		b	$+5°2889$	$6^{m}82$	HA 57
B		b^1	$+6°2878$	7.28	HA 74
D		b^2	$+6°2891$	7.36	HA 74
β	1	a	$+5°2879$	7.76	HA 57
A	2		$+4°2859$	8.00	HA 70
ϑ	4	c	$+5°2880$	8.13	HA 57
c			$+4°2862$	8.3	BD
a	5		$+4°2866$	8.45	HA 74
ϑ_1		d	$+6°2881$	8.85	HA 57
α		c^1	$+5°2878$	9.45	HA 57
d	6		$+5°2884$	9.9	
b	7	f	$+4°2863$	10.07	HA 57
p	8	g	$+5°2883$	10.13	HA 57
γ	11			$11^{m}0$	
ζ	13	m		11.24	HA 57
m	16	n		11.92	HA 57
δ	22	q		12.83	HA 57
q	21			12.8	
k	25			13.1	
y		q^1		13.26	HA 57

Die 1. Spalte gibt meine Bezeichnung, die 2. Spalte die Nummer im Atlas Stellarum Variabilium VI 5174, die 3. Spalte die Bezeichnung in HA 57.244, die 4. Spalte die Nummer in der B.D., die 5. Spalte die angenommene Größe, die 6. Spalte die Autorität für diese Größe. Die paar Sterne, für welche photometrische Größen nicht vorliegen, sind selten benützt; ihre Größen wurden in Anlehnung an meine Schätzungen und die des ASV näherungsweise festgelegt; c behielt die BD-Größe. Meine sämtlichen Beobachtungen folgen hier:

Jul. Dat. 241····	Schätzung	Größe	Jul. Dat. 241····	Schätzung	Größe	Jul. Dat. 241····	Schätzung	Größe
5132.4	a 15 v 4 b L3	$9^{m}73$	5232.4	v o $k\pm$	$13^{m}1$	5801.7	β 6–7 v 1–2 ϑ L2–3 ☾	$8^{m}06$
5145.5	v o b	10.07	5372.7	ϑ 15 v 1–2 α unbeständ.	9.36	5806.7	ϑ o v L2 ☾	8.13
5150.6	v o p L2–3	10.13	5544.4	v 3 m neblig	11.7	5812.5	ϑ 2 v o c unbeständig	8.3
5159.6	b 5 v 5 ζ[1]) ☾	10.66	5574.4	δ 1–2 v 2–3 k	12.9	5822.7	ϑ 3 v 10 b schlechte Bilder	8.58
5166.5	b 10 v 2 ζ, v o γ L3	11.04	5735.8	β 11 v 2 a	8.34	5843.4	$a >> v > b$ neblig	9.5
5173.4	ζ 2 v 6 m ☾	11.41	5754.8	β o v ☾ unbeständig	7.76	5852.4	a 10 v 1–2 b L2	9.86
5180.5	γ 2–3 v 5 m L3 ☾	11.3	5758.7	β o v unbeständig	7.76	5868.4	b 5 v 10 ζ L2	10.46
5185.4	ζ 5 v 2 m L3	11.73	5773.8	β o v	7.76	5895.4	$\zeta > v$, ζ o v Wolken	11.3
5193.5	m o $v\pm$ L2	11.9	5779.7	β o–1 v 8 ϑ	7.78	7242.7	a 9–10 v 1 b L2 ☾	9.92
5205.5	m 6 v 3 δ L2	12.53	5790.7	β 3 v 7 ϑ	7.87	7276.7	m o v	11.92
5220.4	v o δ unsicher	12.8	5793.7	β 5 v 2 ϑ dunstig	8.02	7297.4	m 3–4 v 1 δ	12.6

[1]) Im Tagebuch steht ε (= ASV 24) für ζ, was offenbar eine Verwechslung ist.