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New Theory of the Aether. By *T. J. J. See.*

(Second Paper.)

1. Gravitational Action propagated with the Velocity of Light.

In the first paper on the New Theory of the Aether, AN 5044, we have shown that the existence of this medium is a necessary condition for conveying physical action from one body to another across the celestial spaces, and have given the elements of the kinetic theory of the aether-gas as the subtile vehicle of energy. *Maxwell* had a very clear conception of this medium 47 years ago, when he pointed out, in the closing paragraph of the celebrated Treatise on Electricity and Magnetism, 1873, vol. II, p. 403, that »whenever energy is transmitted from one body to another in time there must be a medium or substance in which the energy exists after it leaves one body and before it reaches the other«.

No better description can be given of the aether, as the vehicle of energy, than that just quoted. And since *Maxwell* says that the energy must exist in the medium, after it has left one body, but before it has reached the other, owing to the propagation in time, we see that this energy obviously must be conveyed through the agency of waves travelling with the velocity of light, just as radiant heat from the sun and electrodynamic action travel with the same velocity, 300000 kms per second.

From the celebrated letter of *Gauss* to *Weber*, March 19, 1845, (*Gauss*, Werke 5.629) we learn that as early as 1835 *Gauss* looked upon physical action across space as conveyed in time, and was trying to formulate a law of this action, but put it aside temporarily, and only recurred to it when *Weber* had formulated his fundamental electrodynamical law, published in 1846:

$$f = (mm'/r^2)\{1-(1/c^2)(dr/dt)^2+(2r/c^2)\,d^2r/dt^2\}. \qquad (1)$$

The first term of this formula is *Newton*'s law of gravitation, 1686, whilst the other terms take account of the effects of induction in the relative motion of the two bodies m and m'. The minor terms thus give the energy effects of the velocity and acceleration or change of velocity, under wave action, in the direction of the radius vector, as required by the present author's Electrodynamic Wave-Theory of Phys. Forc., vol. 1, 1917.

In the work here cited (p. 143–149) I have calculated the effects of *Weber*'s law upon the progressive movement of the perihelia, periplaneta, and periastra of the best known bodies of the solar system and of the sidereal universe. The tabulated $\delta\varpi$ is the progression of the orbital perihelia in a Julian century, owing to the propagation of gravitation with the velocity of light.

Progression of Perihelia in a Julian Century, *Weber*'s Law.

Planets	$\delta\varpi$
Mercury	14."511
Venus	2.9125
The Earth	1.2964
Mars	0.45619
Jupiter	0.02104
Saturn	0.004613
Uranus	0.00080395
Neptune	0.0002615
Satellites	
The Moon (Earth)	0.00637
Phobos (Mars)	0.02651
Deimos »	0.011098
Ariel (Uranus)	0.18439
Umbriel »	0.13235
Titania »	0.080504
Oberon »	0.060339

Satellites		$\delta\varpi$
Jupiter:	V	4."233655
	I	1.8212385
	II	1.143450
	III	0.715544
	IV	0.40508
	VI	0.068685
	VII	0.064658
	VIII	0.034681
	IX	0.034128
Saturn:	Mimas	1.2403
	Enceladus	0.966394
	Tethys	0.78066
	Dione	0.60811
	Rhea	0.43644
	Titan	0.188423
	Hyperion	0.157842
	Iapetus	0.102376
	Phoebe	0.018751

Comets	$\delta\varpi$
Encke	0."62198
Tempel$_2$	0.11461
Brorsen	0.223072
Tempel-L. Swift	0.120988
Winnecke	0.140366
De Vico-E. Swift	0.078878
Tempel$_1$	0.067616
Finlay	0.118332
D'Arrest	0.089622
Biela (I)	0.12570
Wolf	0.075939
Holmes	0.062773
Brooks	0.063389
Faye	0.066373
Tuttle	0.051076
Olbers	0.0077034
Halley	0.012637
Newton, 1680	0.0121184
1843 I	0.219474
1882 II	0.1081733

Binary Stars	$\delta\varpi$
η Cassiopeiae	0."00066902
β Persei	3291.927
40 o^2 Eridani	0.000316413
α Aurigae	65.7045
α Canis maj.	0.0069185
α Geminorum	0.00051013
α Canis min.	0.0066018
γ Virginis	0.00308893
α Centauri	0.0018777
70 Ophiuchi	0.001068103
δ Equulei	0.1524535
85 Pegasi	0.0432729

This table shows that the difference between the effects of *Newton*'s law, with fixed perihelia, and of *Weber*'s fundamental electrodynamic law, with progressing perihelia, is always small. The chief interest centers around the motion of Mercury's perihelion, which *Leverrier* in 1859 found to have a progression larger than known gravitational theory would explain.

As the unexplained motion of Mercury's perihelion found by *Newcomb* in 1881 is about 43" per century, the above effect of *Weber*'s law removes 14″5 of the total amount [1]), leaving outstanding about 28″5 instead of the 43″ assumed in *Einstein*'s Theory of Relativity. The outstanding 28″5 can be explained by the transformation and absorption of wave energy from the atoms on the opposite side of the sun, yielding a law of attraction of the very form approved by *Newton* in the Principia, 1687:

$$f = mm'/r^{2.0000001046}. \qquad (2)$$

This explanation of the motion of Mercury's perihelion is more fully discussed below. Such a result was long ago anticipated by *Newton*, and in 1894 carefully examined and proposed by *Hall*, and subsequently used by *Newcomb* and *Seeliger*. It therefore has the sanction of the most eminent astronomers, and as it rests upon a known physical cause, it involves no vague and chimerical reasoning such as underlies *Einstein*'s mystical Theory of Relativity.

Towards the end of this paper, we develop a new view of the experiments of *Michelson* and *Morley*, 1887, and of Sir *Oliver Lodge*, 1891–97, which results from the kinetic theory of the aether, originally outlined by *Newton*, 1721, approved by *Maxwell* and *S. Tolver Preston*, 1877, and recently developed by the present writer, as shown in the first paper. This new view of the chief physical experiments on which the theory of relativity so largely rests may well claim the attention of natural philosophers. As bearing on the same question we treat carefully of the outstanding motions of the perihelion of Mercury and of the lunar perigee; and show that neither phenomena lends the slightest support to non-Newtonian mechanics.

In fact, although the theory of relativity has occupied much space in scientific literature, and many treatises, memoirs, and other papers have appeared on the subject, it is impossible for a careful observer to escape the conviction that the whole development heretofore brought out is false and misleading, — a veritable foundation laid on quicksand — and that some day philosophers will wonder that such an improvised absurdity ever became current among men. Among the most pernicious of these temporary doctrines is *FitzGerald*'s hypothesis, which under the kinetic theory of the aether is wholly untenable.

A considerable number of persons are much impressed with the admissibility of any doctrine which becomes current among contemporaries, yet the study of the history of science shows that truth is neither dependent upon popularity, nor discovered by majorities, but by the few individuals who think carefully and frequently in complete isolation, and who thus attain superior vision into the deeper mysteries of nature.

In promulgating his new System of the World, 1543, *Copernicus* describes his reasoning in daring to depart from the opinion of the majority:

»Though I know«, he says, »that the thoughts of a philosopher do not depend on the judgment of the many, his study being to seek out truth in all things as far as that is permitted by God to human reason: yet when I considered«, he adds, »how absurd my doctrine would appear, I long hesitated whether I should publish my book, or whether it were not better to follow the example of the Pythagoreans and others, who delivered their doctrines only by tradition and to friends«.

2. The Effect of Resistance is to break up Long Waves into Shorter Ones and actually to increase the Amplitude of the Principal Component, as noticed in Breakers at the Sea Shore.

In his celebrated work on Tides and Waves, Encyclopedia Metropolitana, 1845, Sir *George Airy* obtained one of the most comprehensive and useful theories of wave motion ever developed. *Airy*'s theory has the advantage of being intensely practical, because it applies to wave motion in a canal, water being the chief liquid found upon the earth, and nearly incompressible. The formula for the periodic time of the waves is

$$\tau^2 = (2\pi\lambda/g)\,(e^{4\pi k/\lambda}+1)/(e^{4\pi k/\lambda}-1)\,. \qquad (3)$$

It may be shown analytically that when the wave length is shortened, as by resistance to the movement of the fluid, the exponential expression $e^{4\pi k\lambda}$ increases, and thus the amplitude increases [2]). This change has been much discussed in various treatises and memoirs, and we shall not attempt to add to it here, except in the practical application of the result to physical problems.

Now *Airy* finds (art. 201–210) the following theoretical curves for the breaking up of water waves in rivers, considered as straight canals, with smooth banks. After explaining his analysis of these theoretical waves in water, *Airy* interprets the results as follows:

»(201). To represent to the eye the form of the wave produced by the combination of the two terms, we have constructed the curve in figure 9. The horizontal line represents the level line of the mean height of water: the elevation or depression of the curve represents (on an enormously exaggerated scale) the elevation or depression above the mean height, given by the expression above. The value of x' is supposed to increase from the left to the right: on which supposition the quantity $mvt-mx'$, representing the phase of the wave, diminishes from the left to the right (mvt being constant).«

»(202.) To exhibit to the eye the law of the ascent and descent of the surface of the water at different points of the canal, the figures 10, 11, 12, and 13 are constructed. The first of these is intended for the point where the canal communicates with the sea: the others for points successively more and more distant from the sea. The horizontal line is used as a measure of time, or rather of phase $mvt-mx'$: in which, for each station, x' is constant: the elevation or depression of the corresponding point of the curve represents the corresponding elevation or depression of the water above its mean height, as given by the expression above.«

»An inspection of these diagrams will suggest the following remarks:

[1]) In the Monthly Notices for April, 1917, p. 504, Dr. *Silberstein* treats at some length of the *Einstein* calculations, based on *Gerber*'s formula (Zeitschr. Math. Phys. 43.93–104, 1898) in which for the Newtonian potential M/r is put $M/r(1-1/c\cdot dr/dt)^2$, and concludes: »As far as I can understand from *Jeffrey*'s investigation, (MN 77.112–118), it would rather alleviate the astronomer's difficulties if the sun by itself gave only a part of these 43 seconds.« Accordingly this is all the more reason for adopting *Weber*'s law, though I reached it from a different point of view.

[2]) This increase of amplitude will prove of high importance in the new theory of molecular forces, to be dealt with in a future paper.

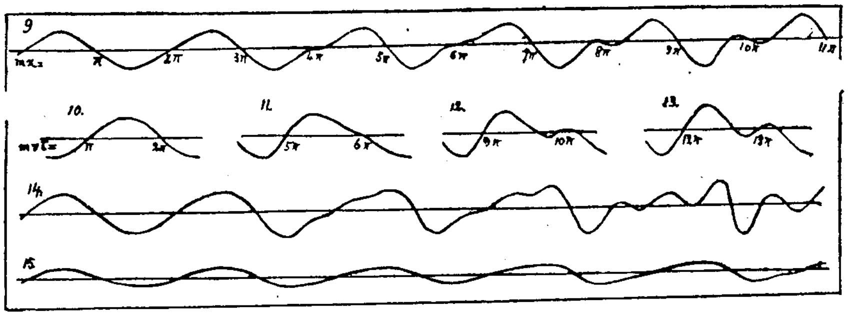

Fig. 1. *Airy*'s graphical illustration of the breaking up of waves under resistance. The canals considered are connected with the sea and of uniform width.

9. Theoretical form of tide-wave in a shallow river, to second approximation. $mx = 0$, first station, the sea; $mx = 4\pi$ = second station, $mx = 8\pi$ = third station.
10–13. Theoretical tidal curves for different stations on the river. 10 = first station at mouth of river, 11 = second, 12 = third, 13 = fourth station.
14. Theoretical form of tide-wave in a shallow river to third approximation with large tide.
15. The same with small tide.

»(203.) When the wave leaves the open sea, its front slope and its rear slope are equal in length, and similar in form. But as it advances in the canal, its front slope becomes short and steep, and its rear slope becomes long and gentle. In advancing still further, this remarkable change takes place in the rear slope: it is not so steep in the middle as in the upper and the lower parts: at length it becomes horizontal at the middle: and, finally, slopes the opposite way, forming in fact two waves (figure 9).«

»(204.) At the station near the sea (see figure 10), the time occupied by the rise of the water is equal to the time occupied by the descent: at a station more removed from the sea (figure 11) the rise occupies a shorter time than the descent: the rise is steady and rapid throughout, but the descent begins rapid, then becomes more gentle, then becomes rapid again: at stations still farther from the sea (figures 12 and 13) the descent, after having begun rapid, is absolutely checked, or is even changed for a rise, to which another rapid descent succeeds: in this case there will be at that station two unequal tides corresponding to one tide at the mouth of the canal.«

This numerical and practical discussion by *Airy*, with curves for illustrating the results is more satisfactory than any purely theoretical analysis of the effects of resistance, and thus all we need to do is to point out, that, just as water waves in canals degenerate and break up into partial waves, under the action of a variable resistance, depending on the depth of the water, and its distance up the river from the sea: so also in the aether, the long waves encounter resistance which progressively is more and more disintegrating on their existence, kinetic stability, and continuity. Accordingly we may be sure that long waves in the aether will undergo corresponding changes by disintegration into shorter waves, and that the chief component will have increased amplitude.

There are various physical illustrations of this effect which may be cited, as when the sun's radiation impinges on the earth, and the longer invisible infra-red rays, so much studied by *Langley*, pass into heat waves of shorter wave length.

Again, in our electric stoves and heaters, the electric current, made up of very long waves, first develops heat, so that the resisting wire acquires a dull glow, then a red heat, and finally becomes incandescent, with light of shorter and shorter wave length the longer the action continues. The transition here sketched is therefore known to be a reality in dealing with the transformation of electric energy into heat and light, under conditions observed daily in every part of the world.

The analogies here cited are so obvious and familiar to us in the changes noticed when waves pass into breakers at the sea shore, that it seems impossible to deny the validity of the conclusion above drawn from every day experience, and fortified by the profound researches on tides and waves produced by one of the greatest mathematicians and natural philosophers of the past age.

To those who hesitate at the contrast between water and aether, we point out that it is true that water is heavy and inert, and sluggish in its movements, whereas the aether is excessively rare, with density at the earth's mean distance equal to 438×10^{-18}, and having an enormous elastic power, 689321600000 times greater than that of our air in proportion to its density. Thus the light and electric waves in the aether travel 902000 times faster than sound waves in the air, and about 200000 times faster than sound in water at 30° C., which travels 4.54 times faster than in air, owing to the high incompressibility of the water.

There is thus much difference between the speed of waves in the aether and in water, even if the dense water, like the rare aether, be highly incompressible. But notwithstanding this difference, due chiefly to the extreme rarity of the aether, water being in comparison with aether 228×10^{13} times denser, there is a substantial physical basis for comparison of the actions in the two media.

Our reasoning therefore is not speculative or hypothetical, but purely practical, since it rests upon facts definitely determined by experience, and verified by careful observations of recognized phenomena of the physical universe.

In order to bring out the practical bearing of the wave-theory upon the motion of the perihelion of Mercury, and the lunar fluctuations, discussed below, we notice that as long ago as 1901, Professor *Planck* of Berlin supposed that in all matter there were a great number of »resonators« of every possible period (Ann. d. Physik, 4.556, 1901). Thus matter would receive and emit vibrations of all possible periods, as postulated in the Electr. Wave-Theory of Phys. Forc. 1.85–88, 1917. The lunar fluctuations occur where the sun's gravitational waves have to traverse the solid mass of the earth, and thus the action on the moon is decreased near the time of lunar eclipses; and the moon partially released from the sun's control, thus tends to fly the tangent. This gives rise to disturbances in the mean motion which *Newcomb* declared to be the most enigmatical phenomenon presented by the celestial motions.

Now the lunar theorists were unable to find the periodicities required to explain the lunar fluctuations, until I discovered the obstructing cause at work, near the shadow of the earth, to modify the sun's gravitative action on the moon.

If this explanation of the fluctuations of the moon be conceded, a similar cause will have to be admitted to act on the planet Mercury, which renders our sun gravitationally unsymmetrical or lopsided, as if a small part of the matter on the opposite side of the sun were removed, or ineffective, owing to the interposition of the sun's huge globe in the path of gravitational action. In other words, owing to refraction, dispersion, absorption, large masses of matter exercise a slight screening effect.

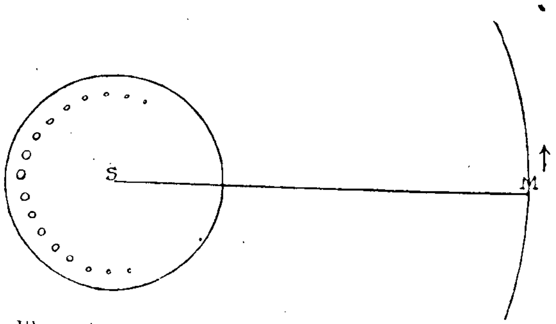

Fig. 2. Illustrating the absorption and circular refraction of some of the waves from part of the matter in the side of the sun opposite to Mercury, as if parts of the Sun's mass had been removed, and the globe thus rendered slightly lopsided. Compare also Fig. 3, in section 5 below.

Mercury therefore is less attracted than if the strict law of inverse squares established by *Newton* held, and thus we have the feebler law of force explained below:

$$f = mm'/r^{2.0000001046} = (mm'/r^2)\cdot(1/r^{0.0000001046}) \qquad (4)$$

whence arises the hitherto unexplained progression of Mercury's perihelion, by 28".44 per century, which has proved so bewildering to geometers and astronomers ever since *Leverrier* discovered the difference in 1859.

This explanation is very much simpler than any heretofore offered, and as it harmonizes the motion of Mercury with the motion of the moon, under well established physical laws, without introducing any vague and chimerical hypotheses, it would seem difficult to deny its essential physical truth.

3. Explanation of the outstanding Motion of the Perihelion of Mercury, based on the Electrodynamic Wave-Theory of Physical Forces.

Aside from the investigation of the amount of the outstanding motion of Mercury's perihelion, by *Leverrier*, 1859, and by *Newcomb*, 1881, duly noted below, we cite the following researches as offering various explanations of the phenomenon:

1. Untersuchungen über die Bewegung des Planeten Merkur, and other notices of researches by Dr. *J. Bauschinger*, AN 109.32.

2. Über die Bewegung des Merkurperihels, by *P. Harzer*, AN 127.81, 1891. *Harzer* investigates the effects of unequal moments of inertia of the sun about polar and equatorial axes, and of the matter in the corona, and finds these hypotheses admissible.

3. A Suggestion in the Theory of Mercury, by *A. Hall*, AJ 14.49, 1894. *Hall* adopts the suggestion of *Newton* that the law is not exactly that of the inverse squares, and puts

$$f = mm'/r^{2.00000016}. \qquad (5)$$

4. Hypothesis that gravitation towards the sun is not exactly as the inverse square of the distance, Astronomical Constants, p. 118, by *S. Newcomb*, 1895. *Newcomb* adopts *Hall*'s hypothesis, with very slight modification:

$$f = mm'/r^{2.0000001574}. \qquad (6)$$

5. Über die empirischen Glieder in der Theorie der Bewegung der Planeten Merkur, Venus, Erde und Mars. VJS 41.234–240, by *H. Seeliger*.

Das Zodiakallicht und die empirischen Glieder in der Bewegung der inneren Planeten. Sitz.-Ber. d. Kgl. Akad. d. Wiss. zu München, 36.595–622, by *H. Seeliger*.

Seeliger assumes the matter of the zodiacal light to be distributed in two ellipsoids, an outer one and an inner one, which will effect Mercury's perihelion, as observed, without disturbing the other planets. He gets a very perfect agreement with observations, fully as good as that supplied by *Einstein*'s theory, without the vagueness of relativity. *Seeliger*'s chief results are:

		Newcomb	*Seeliger*'s Ellipsoids	Outstanding Residuals
Mercury	$e\,d\pi =$	+8".64	+8".49	+0".15
	$\sin i\,d\Omega =$	+0.61	+0.62	−0.01
	$di =$	+0.38	+0.49	−0.11

Seeliger's theory applies equally well to Venus, the Earth and Mars.

6. A Memoir on the outstanding anomalies of the celestial motions, by Professor *E. W. Brown*, Amer. Journ. of Science, 29, in which various hypotheses, including the effects of the magnetic fields of the earth, sun and moon, are examined and rejected. See also Report of British Association for 1914, for Prof. *Brown*'s Address to Section A, p. 311–321.

7. *Einstein*'s General Theory of Relativity, 1916, in which this author uses the value $\delta\varpi = +43''$, and deduces the term of *Gerber*'s formula:

$$V = (M/r)(1 - 1/c\cdot dr/dt)^{-2} \qquad (7)$$

required to be added to the law of gravitation to make this difference between theory and observation disappear. By using the value $\delta\varpi = +43''$ per century, and deducing a very exact agreement based on this difference, instead of the difference 28."44, which results from *Weber*'s law, *Einstein* adds to the improbability of his theory.

It has long been remarked that among the outstanding motions of the solar system recognized by astronomers during the past sixty years, and of which geometers have sought a valid explanation, none is more justly celebrated than the excessive progression of the perihelion of Mercury, announced by *Leverrier* to the Paris Academy of Sciences, Sept. 12, 1859, (CR 49.379).

Leverrier's announcement of an outstanding motion of 38" per century in Mercury's perihelion seemed to find almost immediate confirmation in Dr. *Lescarbault*'s supposed observation of an intra-mercurial planet named Vulcan; and this anomaly therefore was made the basis for the provisional elements assigned to the new planet.

If on the one hand later observational researches, during many total solar eclipses, have shown no signs of an intra-mercurial planet, it may be noticed, on the other, that the fullest confirmation of *Leverrier*'s analysis of the planetary motions, 1859, has been obtained by later investigators, especially by *Newcomb*, who used all the observations of the transit of Mercury from 1677 to 1881, and deduced an outstanding motion in excess of that found by *Leverrier*, namely about 43" per century. (Astron. Pap. of the Amer. Ephem., 1.367–484, 1881.)

Accordingly, *Leverrier* spoke conservatively in the original announcement of his discovery, when he said:

»The necessity of an increase in the secular motion of the perihelion of Mercury results exclusively from the transits of the planet over the disc of the sun. The exactitude of these observations is beyond doubt.«

The anomalous motion of Mercury's perihelion thus established by *Leverrier* and *Newcomb*, has been widely discussed in natural philosophy, and in fact combined with the *Michelson-Morley* experiment of 1887, for laying the foundation of a Theory of Relativity, on which already many treatises have appeared, without, however, thus constituting a simple and consistent physical doctrine which commands universal assent.

There are, I think, grave reasons for doubting the whole Theory of Relativity, as now developed, on grounds which will be more fully outlined in treating of the *Michelson-Morley* experiment. For the present it must suffice to allude to the unsatisfactory theory resulting from *Leverrier*'s discovery of an outstanding motion in Mercury's perihelion, and the growth in natural philosophy of a doctrine which many regard as both vague and chimerical.

In 1894, Prof. *Asaph Hall* of Washington outlined a new view of the anomalous motion of Mercury's perihelion (AJ 14.49), based on the hypothesis that for some unknown reason the Newtonian law of the inverse squares might not be strictly correct.

Already in 1686, while preparing the Principia, (Lib. I, sect. IX), Sir *Isaac Newton* had considered such a possible modification of the law of attraction; and even included some computations, in which he assumes that the central force departs a little from the inverse square of the distances.

Newton found that the perihelia would move forward under such a modification of the law of attraction (Lib. I, sect. IX, Prop. XLV, Prop. XXXI, cor. I), but considered the observed approximate fixity of the planetary perihelia a strong proof of the accuracy of the law of the inverse squares. His final view evidently is expressed in the General Scholium to the Principia, 1713, where he says that in receding from the sun gravitation »decreases accurately in the duplicate proportion of the distances as far as the orb of Saturn, as evidently appears from the quiescence of the aphelia of the planets; nay, and even to the remotest aphelia of the comets, if these aphelia also are quiescent«.

In the Mécanique Céleste, 1799, *Laplace* likewise concluded that the law of gravitation holds accurately for the satellites as well as for the planets, (Liv. II, ch. I, § 6). In Liv. XVI, chap. IV, however, *Laplace* investigated more fully the effect on certain terms of the moon's motion of some assumed changes in the Newtonian law of attraction, but from his remarks it is evident that he did not consider it probable that there is a departure from the strict law of the inverse squares.

Thus, up to the time of *Leverrier*'s researches on the motion of Mercury, 1859, there were no well established deviations from the Newtonian law which might be made the basis of observational inquiry, so as to serve as a crucial test of the accuracy of that law.

In his paper of 1894, however, Professor *Asaph Hall* sagaciously remarks:

»If the Newtonian law of attraction is not a rigorous law of nature, or if it is modified slightly under certain conditions, probably this lack of rigor would become apparent first among the swiftly moving bodies of our solar system, such as our moon and the planet Mercury« (AJ 14.49).

Our moon indeed does not move so swiftly, but owing to its great proximity to the earth and the eclipse records extending over nearly 3000 years, the motion is very accurately known, — both by observation and by theoretical research and calculation, — so that the smallest disturbances may become sensible to observation (cf. Electr. Wave-Theory of Phys. Forc., 1.113, 1917), which doubtless is the chief point Prof. *Hall* had in view.

That *Leverrier*'s researches on the motion of Mercury, 1859, set in motion several new lines of inquiry of great theoretical importance is shown by two investigations developed within the next fifteen years.

1. The researches of *Tisserand* on the motion of a planet under *Weber*'s electrodynamic law, communicated to the Paris Academy of Sciences, Sept. 30, 1872, by the eminent geometer *Bertrand*, who had inspired these investigations.

2. The problem proposed in 1873 by *Bertrand* to the Paris Academy of Sciences, (CR 84), to find the closed curve described by a planet when the forces have the form of an unknown function $R = \Phi(x, y)$ of two independent variables x and y, and the differential equations of motion are

$$m \cdot d^2x/dt^2 = -R \cdot x/r \qquad m \cdot d^2y/dt^2 = -R \cdot y/r \qquad (8)$$

it being required to find the function R whatever be the initial values of the coordinates x_0, y_0, and of the components of the velocity

$$x_0' = (dx/dt)_0 \qquad y_0' = (dy/dt)_0 \,. \qquad (9)$$

The solution of this problem showed that this function always takes the form $R = m r^n$, where m is the mass of the planet, and r the radius vector.

It was *Bertrand*'s theoretical improvement in the treatment of *Newton*'s problem of a moving perihelion which led to *Hall*'s hypothesis of 1894, for explaining the excess in the motion of the perihelion of Mercury. Since *Hall*'s hypothesis has been further developed by the writer's recent researches in the Electr. Wave-Theory of Phys. Forc., it is necessary to treat of these successive steps for attaining an Electrodynamic Theory of the motion of Mercury's perihelion.

(1). *Bertrand*'s solution of *Newton*'s problem of finding the central force for a moving perihelion. As proposed to the Academy of Sciences, in 1873, *Bertrand*'s problem reads (CR 77):

»We consider a planet attracted by the sun under a force of which the intensity depends only on the distance. We suppose known this one fact: that the planet describes a closed curve, whatever be the magnitude and direction of its velocity. We have to find the law of attraction from this single datum.«

Bertrand remarks that as the force is central, the motion takes place in a plane through the centre of the sun, and *Kepler*'s law of equal areas in equal times holds true. If the force have the form

$$R = m r^n \qquad (10)$$

it is found that there result just two formulae:

$$R_2 = m/r^2 \qquad (11)$$

$$R_1 = m r\,. \qquad (12)$$

And these are the only two laws of attraction which permit a planet to describe a closed curve, whatever be the initial data (the velocity being nevertheless below a certain limit). And if we suppose the attraction zero at an infinite distance, there remains only one formula (11), or the law of *Newton*, which could thus be deduced from the sole fact of observation: that any planet whatever describes a closed curve, without our being able to know the nature of this curve (cf. *Tisserand*'s Mécanique Céleste, 1.48, 1889).

Resuming *Newton*'s problem of a moving perihelion, *Bertrand* derives a perfectly general formula for the arc Θ swept over by the planetary radius vector between the minimum value (r_1) and the maximum value (r_2)

$$\Theta = [\pi/\sqrt{(n+3)}]\times \{1+{}^1/_{24}(n-1)(n+2)[(r_2-r_1)/(r_2+r_1)]^2+\cdots\}. \qquad (13)$$

He remarks that when r_2-r_1 tends towards zero, we have in the limit the Theorem of *Newton*, 1686:

$$\operatorname{Lim}\Theta = \pi/\sqrt{(n+3)} \qquad (14)$$

which applies to an orbit almost circular described by a planet under the influence of a central force proportional to a power of the distance.

If for the motion of a planet around the sun, we take with *Newton*, $n = -2$, $R = m/r^2$, the relation (14) gives $\Theta = \pi$, which is rigorous. Thus it only remains to find what will happen when we modify slightly the exponent -2 in the Newtonian law of gravitation.

If, for example, we supposed $n = -2.001$, it follows that we should have:

$$\lim\Theta = \pi/\sqrt{(1-0.001)} = \pi\,(1+{}^1/_2 0.001+\cdots) = 180^\circ\,5'\,24'' \qquad (15)$$

or a progression of the apsis line at each revolution of 10′48″, which is so large a quantity as to be totally inconsistent with observation. Without further examination of the effects of changing the exponent in *Newton*'s law (cf. Principia, Lib. I, Prop. XLV), we recognize that the change in the exponent must be extremely small. This case has been considered by Prof. *Asaph Hall*, who has applied the hypothesis to the motions of the planets and of our moon.

(2) *Hall*'s hypothesis of 1894, that the law of attraction may be $f = mm'/r^{2+\nu}$, where $\nu = 0.00000016$. In A. J. No. 319, June 3, 1894, Prof. *Asaph Hall* remarks that on applying *Bertrand*'s formula to the case of Mercury — with *Newcomb*'s value of the outstanding motion of the perihelion, or 43″ per century — he finds that the perihelion would move as the observations indicate by taking

$$n = -2.00000016 \qquad (16)$$

the difference of the exponent from the law of *Newton* being $\nu = 0.00000016$.

The change in the law of attraction required for producing this progression of the line of apsides is therefore very minute. If we use *Weber*'s law, as in the author's Elect. Wave-Theory of Phys. Forc. and *Newcomb*'s value of the outstanding motion of Mercury's perihelion (Astr. Pap. of the Amer. Ephem. 1.473): namely, $\delta\varpi = 42''.95$, we shall obtain an outstanding motion of 28″.44 per century, which is to be accounted for by modification of the exponent in the law of attraction.

(3) Law of attraction indicated by the outstanding motion of Mercury's perihelion. As the motion of Mercury's perihelion offers the principal difficulty in modern celestial mechanics, we take the law of attraction to have the form:

$$f = mm'/r^{2+\nu} \qquad [\delta\varpi]_{00} = +28''.44 \qquad (17)$$

and determine ν by the condition that the outstanding centennial motion of the perihelion shall be $+28''.44$.

If the perihelion shifts 28″.44 in 100 years, it will shift 0″.2844 in one year; and as there are 4.1521 revolutions of this planet in a year, the shift will be 0″.0684956 in a single revolution, and therefore, 0″.0342478 in a half revolution of Mercury.

By *Bertrand*'s formula (13) above, we notice that when the orbit is considerably eccentric, as in the case of the planet Mercury, the term depending on $[(r_2-r_1)/(r_2+r_1)]^2 = e^2$ becomes sensible. In fact Θ in this formula depends on the products of two series as follows:

$$\begin{aligned}\Theta &= \pi/\sqrt{(n+3)}\times \{1+{}^1/_{24}(n-1)(n+2)[(r_2-r_1)/(r_2+r_1)]^2+\cdots\}\\ &= \pi/\sqrt{(1-\nu)}\cdot\{1+{}^1/_{24}(3+\nu)\,\nu\cdot e^2+\cdots\}\\ &= \pi\,\{1+{}^1/_2\nu+{}^3/_8\nu^2+\cdots\}\{1+{}^1/_8\nu e^2+{}^1/_{24}\nu^2 e^2\}\end{aligned} \qquad (18)$$

$$\begin{aligned}\Theta &= \pi\,\{1+{}^1/_2\nu+{}^1/_8\nu e^2+{}^5/_{48}\nu^2 e^2+{}^{15}/_{192}\nu^3 e^2+\cdots\}\\ &= \pi\,\{1+\nu({}^1/_2+{}^1/_8 e^2)+\cdots\}\end{aligned} \qquad (19)$$

Accordingly, our equation of condition is:

$$\begin{aligned}\Theta &= \pi\,\{1+\nu({}^1/_2+{}^1/_8 e^2)+\cdots\}\\ &= 180^\circ\,0'\,0''.0342478 = 648000''.0342478\,.\end{aligned} \qquad (20)$$

As the coefficient of the term involving ν in the case of Mercury becomes $({}^1/_2+{}^1/_8 e^2) = 0.5052839$, we find from (20) by calculation that

$$\nu = 0.0000001045977 \qquad (21)$$

And the modified Newtonian law becomes:

$$f = mm'/r^{2.0000001046}. \quad (22)$$

Applying this law of attraction (22) to the eight principal planets of the solar system we have the following table of centennial progressions for their perihelia:

	$[\delta\varpi]_{00}$		$[\delta\varpi]_{00}$
Mercury	28″44	Jupiter	0″577448
Venus	11.1341	Saturn	0.2325307
The Earth	6.8496	Uranus	0.0815288
Mars	3.6418	Neptune	0.0415681

The progression of the perihelia here calculated from the modified Newtonian law are not contradicted by any known phenomena. The exact position of the perihelion of Venus is not well defined by observations, owing to the great circularity of the orbit; and some slight uncertainty also attaches to the position of the perihelia of the earth and of Mars.

It will be seen that the change made in the Newtonian law is exceedingly minute. For the change in the exponent the ratio is

$$\tfrac{1}{2}\nu = 1046/20000000000 = 1/19120459 \quad (23)$$

a little less than one nineteen-millionth of the whole. Such an infinitesimal alteration in the resulting attractive force would give no sensible effect in a single revolution, but as the change $\delta\varpi$ accumulates with the lapse of time, it finally becomes very sensible, and we are obliged to take account of the secular progression of the perihelion.

This cumulative effect is very similar to the alteration in the moon's mean longitude which results from the secular acceleration of the moon's mean motion, first explained by *Laplace* in 1787, under forces which are insensible for short intervals, but by continuing for long ages in the same direction, finally become sensible and have to be calculated in the formation of tables of the moon designed for use over many centuries.

4. The Modification of the Newtonian Law indicated by the outstanding Difference between the observed and calculated Motions of the Lunar Perigee.

Just as the motion of Mercury's perihelion is the chief means for throwing light on the form of the law of attraction for the planets of the solar system, so also the motion of the lunar perigee affords the best criterion for the form of the law of attraction operating on the motion of the satellites. As the subject has been but little discussed heretofore, we shall briefly outline the results of astronomical research on this interesting problem.

In the Monthly Notices of the Royal Astronomical Society 74.396, 1914, Prof. *E. W. Brown* gives the annual motion of the lunar perigee depending on the ellipticity of the earth as follows:

$$(\partial\varpi/\partial t)_\epsilon = +6''41, \text{ for an oblateness of } 1:296.3. \quad (24)$$

He adds that for an oblateness of 1 : 297, the value would be reduced by the factor

$$(1/297-0.001734):(1/296.3-0.001734) \quad (25)$$

and become:

$$(\partial\varpi/\partial t)_\epsilon = +6''38. \quad (26)$$

From these data it follows that the annual motion of the lunar perigee for an oblateness of 1 : 298.3 would be

$$(\partial\varpi/\partial t)_\epsilon = +6''32. \quad (27)$$

The above values by *Brown*, as thus reduced to an oblateness of 1 : 298.3, are confirmed by the part of the motion of the lunar perigee depending on the ellipticity of the earth's figure calculated by Dr. *Hill*, in his supplement to *Delaunay*'s Theory of the Moon's Motion, Astron. Pap. 3.334, namely:

$$(\partial\varpi/\partial t)_\epsilon = +6''82. \quad (28)$$

This value, however, refers to *Hill*'s oblateness of 1 : 287.71, and must be reduced to correspond to the oblateness of 1 : 298.3; which leads to a result differing only 0″01 from that found by *Brown* and cited above. *Hill*'s value for this reduced ellipticity of the earth therefore is:

$$(\partial\varpi/\partial t)_\epsilon = +6''33. \quad (29)$$

Hence we conclude that this value of the annual perturbation of the lunar perigee depending on the ellipticity of the figure of the earth is very accurately known. The difference in these two authorities would be only 0″0124 per annum, or 1″24 in a century, which is below the limit of determination in the present state of science.

Prof. *E. W. Brown* also gives data to show (MN 75.514), that when the theoretical secular acceleration of the perigee is determined with the highest accuracy, it is 16″ per century smaller than the observed centennial motion of the perigee. This is for an ellipticity of the earth of 1 : 297. By changing the ellipticity to 1 : 294 *Brown* reduces this value from 16″ to 3″; and by taking an ellipticity of 1 : 293.7, the outstanding difference entirely vanishes.

Such a large value of the oblateness, however, seems to be quite inadmissible; and thus on calculating the excess in the actual motion of the perigee over the theoretical motion, for an oblateness of 1 : 298.3, I find it to be 21″9, or say 22″ per century. If we admit this ellipticity of the earth[1]), — which is decisively indicated by the four best methods —, namely:

1) Pendulum observations of gravity, as discussed by *Helmert* and the U. S. Coast Survey,
2) Geodetic measurements of arcs on the earth's surface,
3) The lunar inequality in latitude,
4) The fluid-theory of the earth, isostasy and *Laplace*'s law of density;

then it will follow incontestibly that the moon has an outstanding motion of its perigee of about 22″ per century, almost exactly one half the outstanding motion observed in the perihelion of Mercury.

To form a better idea of the accuracy heretofore attained in these calculations, of the centennial motions of the lunar perigee, we recall the results of *Hansen* and *Brown*:

	Observed	Calculated	Diff. O−C	Authority
$[d\varpi/dt]_{00}$ =	14643560″	14643404″	+156″	*Hansen*, Darlegung, 1864, p. 348
$[d\varpi/dt]_{00}$ =	14643520″	14643504″	+ 16″	*Brown*, MN 75, 1915.

[1]) In the writer's »Determination of the oblateness of the terrestrial spheroid«, begun in 1904, but not yet published, this question has been carefully examined, and the value 1 : 298.3 shown to be the most probable of the various values heretofore proposed.

As above pointed out, the difference of 16″ per century here indicated by *Brown*'s calculation of the theoretical motion of the perigee becomes 22″ when the ellipticity of the earth is reduced to 1 : 298.3.

It is also to be noticed that the observed centennial motion of the lunar perigee used by *Hansen* is 40″ larger than that used by *Brown*. It would seem that very little doubt could attach to the increased accuracy of *Brown*'s observed motion, though owing to the fluctuations in the mean longitude the value 14643520″ for the observed centennial motion of the perigee may yet admit of some improvement, if any of the observational equations should prove to be vitiated by this troublesome cause.

Indeed, it is a little difficult to understand why so considerable a difference as 40″ per century should exist in the observed centennial motion of the perigee used by two such very modern authorities as *Hansen* and *Brown*. For the position of the perigee is given with considerable accuracy from the eclipse records of the Greeks, and the calculations of *Hipparchus* and *Ptolemy*; and as about 226 revolutions of the perigee would occur in 2000 years, the motion of the perigee ought to be quite accurately fixed by the eclipse records of the Greek astronomers. The above difference of 40″ per century, increasing as the square of the time, in 20 centuries would accumulate to 16000″, nearly four and a half degrees, or about nine times the diameter of the moon.

The difference of 100″ between the above calculated centennial motions of the perigee is less striking than it otherwise would appear, but such differences warn us not to overrate the accuracy attained.

It seems remarkable that the eclipse records of the Greeks would leave the position of the perigee open to so much uncertainty. Besides, in the modern observations of the moon since 1750, which are quite accurate, an uncertainty of even 20″ per century, or an accumulated difference of 57.″8, in the interval of 170 years, ought not to exist. Still more intolerable is the difference of 115.″6, based on the difference of 40″ per century! But *Hansen* was unaware of the fluctuations in the moon's mean longitude; and as the fluctuations affect the node as well as the longitude, it may also have vitiated sensibly his calculation of the observed centennial motion of the perigee.

It is worthy of notice that *Hansen*'s outstanding difference between the observed and calculated centennial motion of the lunar perigee is $O-C = +156''$; while *Brown*'s values make this difference $O-C = +22''$. The mean of these two values is $O-C = +89''$.

Now, in default of definite knowledge it is not quite safe to assume that *Hansen*'s values are wholly wrong, and *Brown*'s entirely right, notwithstanding the preeminence of the latter's exhaustive researches in the lunar theory. Both investigators may be somewhat in error, for one reason or another, or for several reasons combined. Thus, apparently the safest thing is to assume that the truth lies between $+156''$, as found by *Hansen*, and $+22''$, which results from *Brown*'s calculations. And as we do not know what weights should be assigned to these extreme values, we can only take the simple mean of the two outstanding motions of the perigee, and thus we have: $[(\partial\varpi/\partial t)_\epsilon]_{00} = +89''$. (30)

It is to be observed also that in our researches on the outstanding motions of Mercury's perihelion, we found the exponent of *Newton*'s law should be modified from 2 to $2+\nu$, where $\nu = 0.0000001046$.

To calculate the resulting outstanding motion for the lunar perigee we notice, in the first place, that the effect of the time of propagation of gravitation by *Weber*'s law, as shown in the table of section 1 above, is almost insensible, $\delta\varpi = 0.''00637$ per century. Thus we need consider only the effect of the exponential change for a body having a mean motion 3.219763 times greater than that of Mercury. And since the unexplained motion of Mercury's perihelion is 28.″44, we get for the corresponding motion of the lunar perigee

$$[(\partial\varpi/\partial t)_\epsilon]_{00} = +28.''44 \times 3.219763 = +91.''57. \quad (O-C) \quad (31)$$

This calculated value is so very near the mean of the values found by *Hansen* and *Brown* as to appear worthy of attention. If for example, *Hansen*'s value $O-C = +156''$ were 65″ too large, leaving $O-C = +91''$, while *Brown*'s were as much too small, yielding $O-C = 22''+65'' = +87''$, the two values would be quite reconciled. And since *Hansen* and *Brown* disagree as to the value of the observed centennial motion of the perigee to the astonishing extent of 40″, the possibility of such unknown errors in their several results is not to be wholly excluded.

Accordingly, for some hitherto unsuspected reason, *Hansen*'s value of the observed centennial motion of the perigee may be substantially correct, namely:

$$[(\partial\varpi/\partial t)_\epsilon]_{00} = +14643560''. \quad (32)$$

In this case, it would suffice to assume an error of 18″ per century in *Brown*'s calculated motion of the perigee.

Unfortunately Prof. *Brown* even proposed to adopt an oblateness of the earth of 1 : 293.7, as if to avoid a modification of the form of the Newtonian law [1]); and hence it seems not wholly improbable that an error of 18″ per century in the calculated centennial motion of the perigee may have been introduced, through some step based upon the tacit assumption of the strict rigor of the Newtonian law.

Under the circumstances, since *Hansen*'s value of the outstanding residual in the centennial motion of the perigee apparently was obtained without prejudice, it should not be rejected, till *Brown*'s values are independently tested and found to be not only the more accurate, but also wholly free from possible prejudice due to assumed rigor in the Newtonian law, or other systematic cause which might thus unexpectedly creep in.

Under the present circumstances, it follows that if the outstanding residual in the centennial motion of the perigee be $[(\partial\varpi/\partial t)_\epsilon]_{00} = +91.''57$ the exponent of the law of attraction for the moon would be the same as that for the planet Mercury, namely: $f = mm'/r^{2.0000001046}$. (33)

[1]) In his address to the British Association in Australia, 1914, p. 316, *Brown* estimates that the exponent in the Newtonian law does not differ from 2 by a fraction greater than 1 : 400000000 = 0.0000000025; but the present discussion shows that this prediction probably overrates the accuracy we are justified in claiming, from 10 to 42 times.

In conclusion, it would appear from this investigation that the change in the exponent for the law of attraction may be the same for the moon and for Mercury. But if future researches should develop a smaller difference in the observed and calculated centennial motions of the lunar perigee, such as 22″ per century, which seems to be the minimum value now admissible; then there would be a smaller value of ν in the exponent of the modified law of *Newton*. The value 22″ per century leads to a value about one-fourth of that found for the planet Mercury, as may be seen from the following considerations.

The moon makes 1336.85126 revolutions in a century, and therefore: $11''/1336.85126 = 0''.0082283$ is the amount of this secular progression of the perigee in half a lunation. The equation of condition:

$$\theta = \pi\{1 + {}^1/_2\nu + \cdots\} = \pi\{1 + 0.0082283/648000\} \quad (34)$$

therefore gives $\nu = 0.000000025396$.

But although there can be no assurance that this modification of the exponent for the earth would be the same as for the sun — the earth being so different in density, size, and physical constitution from the sun — yet at present apparently we are not justified in using this smaller value, because in the existing state of our knowledge there are no definite grounds to authorize it.

Accordingly for the sake of simplicity and uniformity the value of ν applied to the motion of the perihelion of Mercury is preferable also for the motion of the lunar perigee.

5. Outline of the Cause of the Fluctuations of the Moon's Mean Motion.

In the Electrodynamic Wave-Theory of Physical Forces, vol. 1, 1917, it is shown that the previously unexplained fluctuations of the moon's mean motion, discovered by *Newcomb* in 1909, after a study of the moon's motion extending over more than forty years, (1867–1909), is due to the refraction, dispersion, and perhaps absorption of the sun's gravitational waves in passing through the solid globe of the earth. The result is a slight decrease in the sun's gravitative action upon the moon when near the shadow of our globe in space, by which, near the time of Lunar eclipses, the moon is slightly released from the sun's control, and in the tendency to »fly the tangent«, has certain long period disturbances introduced into its mean motion.

An attempt to find such disturbances in the motion of the moon depending on the 18-year period, had been made by Dr. *K. F. Bottlinger*, in a crowned prize Inaugural Dissertation, at the University of Munich, Die Gravitationstheorie und die Bewegung des Mondes, (Freiburg i. B., 1912). *Bottlinger* deduced some evidence of an 18-year period, but in the case of the longer disturbances (61.7006 years, and 277.59 years respectively) he was not able to find the slightest indications of the required periods; so that in his address on the moon's motion at the meeting of the British Association in Australia, 1914, p. 319, Prof. *E. W. Brown* spoke as follows:

»The shading of gravitation by interposing matter, e. g. at the time of eclipses, has been examined by *Bottlinger*. For one reason alone, I believe this is very doubtful. It is difficult to see how new periodicities can be produced; the periods should be combinations of those already present in the moon's motion. The sixty to seventy years fluctuation stands out in this respect, because its period is not anywhere near any period present in the moon's motion or any probable combination of the moon's periods. Indeed Dr. *Bottlinger*'s curve shows this: there is no trace of the fluctuation«.

From this citation it is evident that *Bottlinger* not only had not convinced *Brown* of the reality of the fluctuations depending on the interposition of our globe in the path of the sun's gravitative action, but also that *Brown* felt that an explanation of the 60-year and 275-year periods in the observed fluctuation could not be based on the theory of gravitational disturbances depending on the known cycles of the moon's motion, in relation to the eclipse periodicities.

Notwithstanding this confidence of Professor *Brown*, resulting from his great experience in the lunar theory, I was fortunate enough to discover such long period inequalities in the moon's motion, bearing the closest analogy to the forces acting in the great inequality of Jupiter and Saturn, of which the physical cause was discovered by *Laplace* in 1785, — after *Euler* and *Lagrange* had searched in vain for the mystery underlying the celebrated 900-year inequality of these great planets.

Without attempting to give a detailed account of these researches in the lunar theory, we shall endeavor to outline briefly the leading points, because this advance of 1917 bears very directly on the wave-theory, above applied to the motion of the perihelion of Mercury and of the lunar perigee.

It is shown from an extension of *Maxwell*'s theory of circular refraction in the eye of a fish (Cambridge and Dublin Math. Journal, vol. XI), that a similar circular refraction of gravitational waves occurs when the path of these waves is through the solid mass of the earth. For in the earth, as in the eye of the fish, the concentric shells are each of uniform density, but with the density increasing from layer to layer towards the centre. Thus a circular refraction of the sun's gravitational waves will occur in propagation through the globe of the earth, and also of the moon's gravitational waves in passing through the same globe, owing to the concentric layers of which it is made up. The accompanying figure 3 (pag. 156) illustrates the refraction of the sun's waves in passing through the earth.

By virtue of this circular refraction of the gravitational waves in passing through the globe of the earth, it follows that the mutual interpenetration of the waves from the sun and moon are not the same when the earth interposes its solid mass in their path of action.

The result is a weakening of the sun's gravitative action on the moon; and, when our satellite is thus partially released from the sun's control, it tends to »fly the tangent«, as near the time of lunar eclipses. The outcome is a series of disturbances in the moon's mean longitude depending on the motions of the perigee and node of the lunar orbit, with respect to the Saros or eclipse cycle.

The principal eclipse cycles, incessantly repeated in the theory of the moon's motion, are the following [1]):

[1]) Cf. Electrod. Wave-Theory of Phys. Forc., I.101–102.

1. The Saros, made up of 223 synodic months $= 6585.32$ days, discovered by the Chaldeans and used at Babylon for predicting the return of eclipses, in conjunction with the eclipse year of 346.62 days.

2. The eclipse year of 346.62 days, the average time of the sun in passing around the heavens from the moon's node and returning to the same node again as it retrogrades under the sun's disturbing action in 18.6 years. Nineteen of these eclipse years make 6585.78 days, almost exactly equal to the cycle of the Saros given above, which is 6585.32 days.

The difference in these two periods is only 0.46 of a day, and therefore after 18 Julian years 10.82 days (0^d46 less than 19 eclipse years) the Saros of eclipses is very nearly repeated, except that the location on the terrestrial globe is $0^d32 = 7^h40^m48^s$ further west in longitude.

3. The nodical or draconitic month made up of 27^d21222: and thus $242 \times 27^d21222 = 6585^d357$. This again is of almost the same length as the 223 synodic months and 19 eclipse years defined in paragraphs 1 and 2 above.

4. The anomalistic month made up of 27^d55460; and thus $239 \times 27^d55460 = 6585^d549$. Accordingly, after 223 months the moon not only returns very closely to its original position in respect to the sun and node, but also in respect to the line of apsides of the moon's orbit; so that the perturbations near perigee, during the interval of the difference in these two cycles, $6585^d549 - 6585^d32 = 0^d229 = 5^h29^m8$ are so small as to modify but very slightly the return of the cycle of eclipses composing the Saros.

Accordingly, these four fundamental lunar cycles recur in the following periods:

1. The Saros ꝏ 223 synodic months $= 6585^d32$
2. 19 eclipse years of 326^d62 each $= 6585.78$
3. 242 nodical or draconitic months of 27^d21222 each $= 6585.357$
4. 239 anomalistic months of 27^d55460 each $= 6585.549$

Now the Saros $= 6585^d32 =$ 18 Julian years 10.82 days, or 18.0293 sidereal years of $365^d2563582$ (*Hansen*). And according to *Neison* the period of the circulation of the lunar perigee is 8.855 years. In the 10th edition of his Outlines of Astronomy, 1869, p. 472, Sir *John Herschel* uses the period $3232^d575343 = 8.85031$ Julian years, which is only slightly different from the value cited above.

Accordingly, the forward motion of the perigee will carry it twice around the heavens in 17.71 years, while the node revolves in the retrograde direction in 18.6 years. Thus if we call ☊ the yearly motion of the node, and ϖ the corresponding motion of the perigee, we have

$$\Omega = -19^\circ.35484 = 360^\circ/18.6$$
$$\varpi = +40^\circ.6550 = 360^\circ/8.855. \qquad (35)$$

From the above data, it follows that the node will retrograde through 360° in 18.6 years, but in the same time the lunar perigee will progress through an angle of $756^\circ.183 = 720^\circ + 36^\circ.183$; so that after an interval of 18.6 years the perigee is displaced forward by $36^\circ.183$ in respect to the restored node.

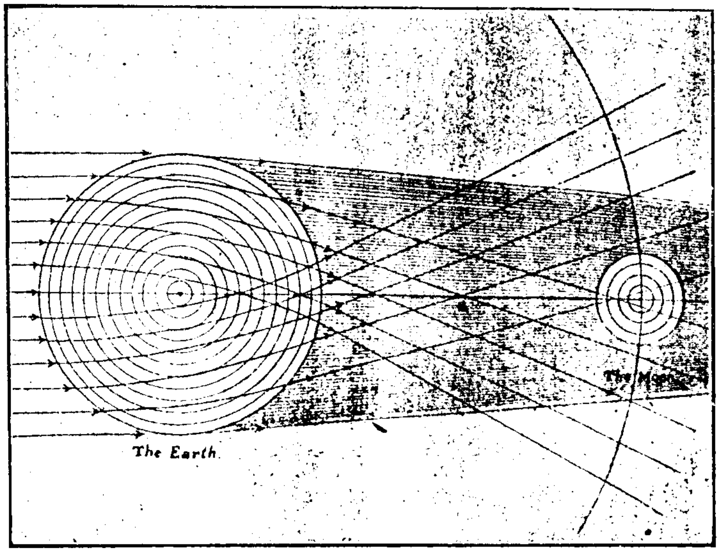

Fig. 3. Refraction of the sun's gravitational waves in passing through the earth's mass, by which the moon is slightly released from the sun's control near the time of lunar eclipses.

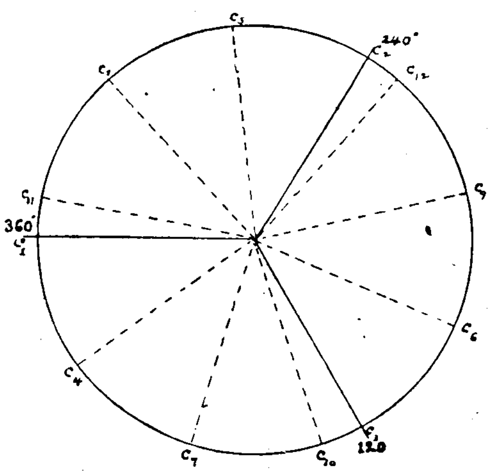

Fig. 4. Illustration of the progress of the moon's perigee in respect to the node, in the 61.7-year fluctuation.

(1) Determination of the period of the 60-year fluctuation.

It is very easily shown that owing to the relative magnitudes of these direct and retrograde revolutions, the angular conjunctions will tend to recur in the regions of 360°, 240°, 120°, like the actual conjunctions of the planets Jupiter and Saturn in the theory of the celebrated 900-year inequality which was first theoretically explained by *Laplace* in the year 1785. Here, too, as in the theory of Jupiter and Saturn, the conjunction lines move forward. The amount of the displacement is $36^\circ.183$ in 18.6 years; and in 3.31648 such periods, 3.31648×18.6 years $= 61.7006$ years, the angular conjunction which started out at the angle 360° will revolve forward through 120°, and the cycle of angular conjunctions at all three points will begin over again, exactly as in the great inequality of Jupiter and Saturn. This leads at once to the second long inequality in the moon's mean motion, which, without suspecting the cause, *Newcomb* estimated at »60 years, more or less«. His judgment of the period was surprisingly accurate; and as he concluded that the coefficient might be about 3″0, here again his value could be adopted.

(2) Determination of the period of the great fluctuation in 277.590 years.

In the case of the great fluctuation in the moon's mean motion, of which *Newcomb* estimated the period at about 275 years, the calculation of the period is somewhat similar to that just cited, but also somewhat different. It is physically obvious that the modification of the sun's gravitation in passing through the body of the earth will depend on the relative shifting of the line of angular conjunctions node-perigee.

Now it is easily found by calculation that the angles of the node-perigee are in angular conjunction, on a line 11°670 in advance of the original conjunction, after an interval of 17.9971 years. For in this time the perigee progresses over an arc of $4\pi + 11°670$, and the node retrogrades over an arc of $2\pi - 11°670$, and meet exactly at the conjunction line specified.

The problem thus arises to find the interval in which this secular displacement of the angular conjunction line will complete the cycle in the moon's motion due to the reduction of gravitation near the shadow of the earth. In each period of 17.9971 years, the node retrogrades through the angle 2π in respect to the shifting mean position of the perigee, and in the same interval the perigee progresses through the double of this angle, 4π, in respect to the retrograding mean node; so that on the average their opposite motions amount to 6π in 17.9971 years.

As the physical effect of the reduction of gravity near the shadow of the earth is the same whether the shifting conjunction line node-perigee refer to ascending or descending node, we perceive that this advancing conjunction line need only sweep over the angle π to give the required interval for completing the cycle due to the changes of gravitation near the shadow of the earth.

Now $180°/11°670 = 15.422$, and therefore in an interval of 15.422×17.9971 years $= 277.590$ years, the cycle of the changes of gravitation near the shadow of the earth will be complete.

This is the period of the great fluctuation in the moon's mean longitude which *Newcomb* estimated at 275 years, from the modern observations, and used in calculating the secular acceleration from the eclipse records extending over 2600 years since the era of the Babylonians.

The diagram in Fig. 5 presents to the eye a continuous representation of the changes in node (outside circle) and perigee (inside circle) during 18 years. At the end of 18 years they both are in conjunction at 1, near the original line of conjunction, 360°, but 11°670 further forward. In each of these periods of 18 years the nodes turn to every part of the heavens, so that eclipses occur all around the earth's orbit, with the earth and moon at all possible distances from the sun. In this interval the lunar perigee revolves twice, and the node once; so that the effect of the progression of the perigee goes through symmetrical phases in respect to the earth's orbit in 18 years, as shown by the above diagram.

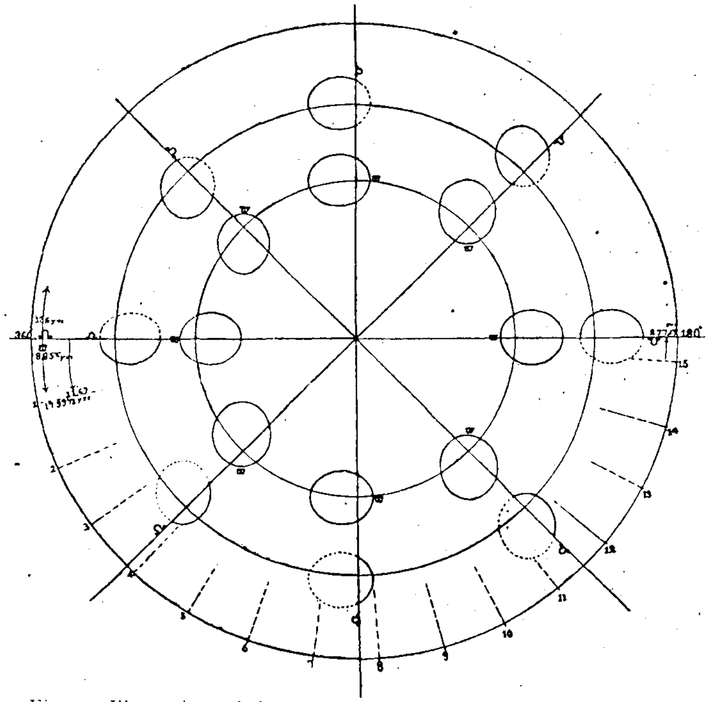

Fig. 5. Illustration of the progress of both node and perigee for producing the moon's great fluctuation in 277.59 years.

This diagram also illustrates the secular progress of the line node-perigee, the restoration to parallelism in this conjunction line, advancing by 11°670 every 17.9971 years, and requiring 277.590 years for completing the full cycle of a semi-circumference.

We may express this result also by observing that physically the decrease of gravitation near the shadow of the earth will take place with equal effect whether the eclipse be near the ascending or the descending node; and this decrease will always correspondingly affect the moon's mean longitude. Therefore, the 18-year movement of node-perigee conjunction line over the arcs $1, 2, 3 \cdots n$, where $n = 15.422$ at 180°, will comprise all possible combinations of the conjunction line node-perigee for modification of the sun's gravity on the moon when near the shadow of the earth.

(3) Determination of the 18-year period of the Saros cycle.

The Saros cycle is so well known that we need scarcely add that a minor disturbance in the moon's mean longitude will recur in this period of 6585.32 days $= 18.0293$ years. In this period the symmetrical eclipse cycle of 223 lunations is complete and the eclipses begin to repeat themselves, with the moon very near the same relative position with respect to the sun and node, and also with respect to the line of apsides or perigee. This Saros cycle of the Chaldeans gives rise to a minor disturbance in the moon's mean longitude, with period of 18.0293 years, and a coefficient of about 1″0. It is the smallest of the moon's sensible fluctuations, yet indicated by the researches of *Newcomb* and *Bottlinger*, and illustrated graphically by the accompanying Fig. 6 (p. 159).

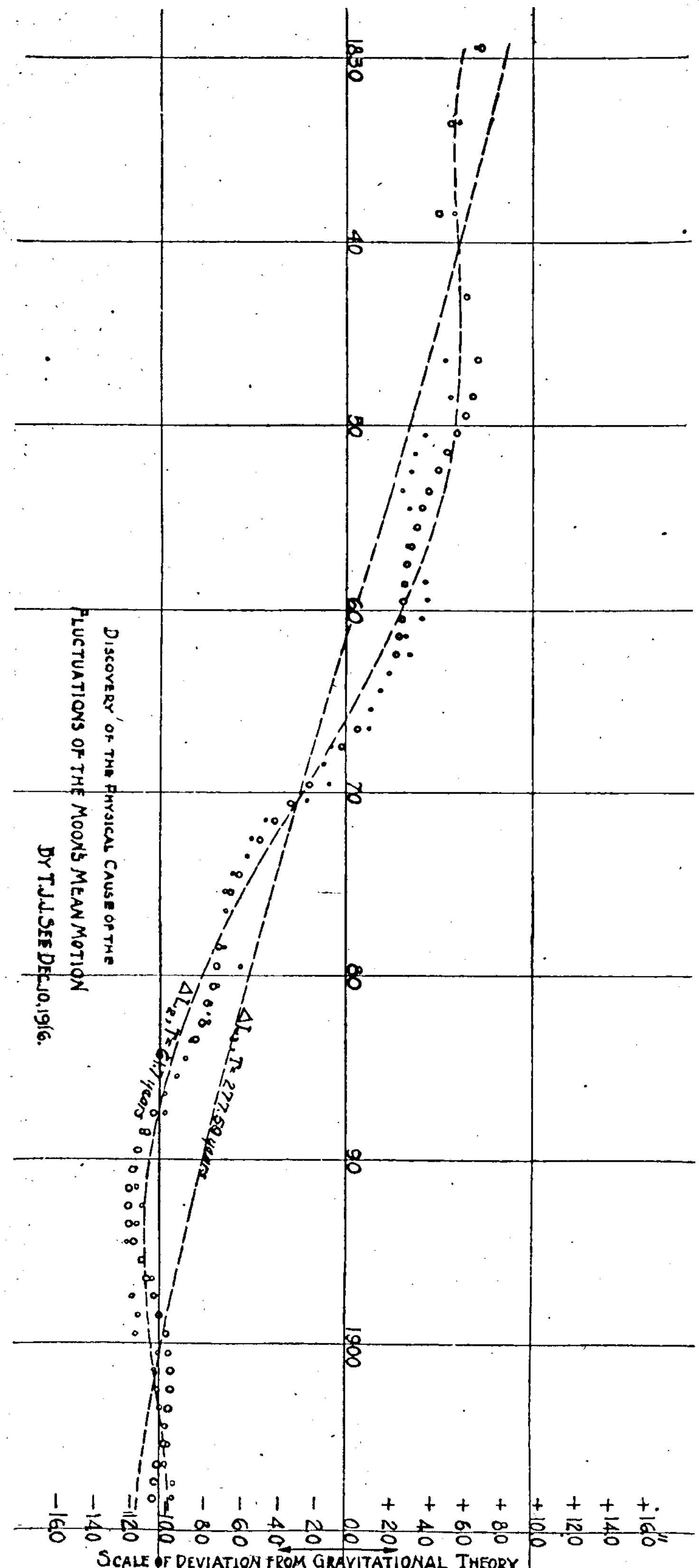

Fig. 6. Graphical illustration of the three chief fluctuations in the moon's mean motion. The small dots represent the observed, the small circles the calculated, places of the moon.

The close analogy of this explanation of the 61.7-year period in the mean motion of the moon, which is the most powerful of these fluctuations, with the celebrated 900-year inequality of Jupiter and Saturn is shown by the following figure 7 for illustrating *Laplace*'s discovery, 1785, involving a forward shift of the conjunction lines of these two great planets through 120°.

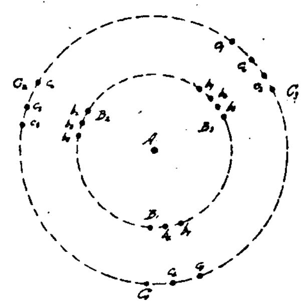

Fig. 7. Diagram of the shifting conjunctions of Jupiter and Saturn, for illustrating *Laplace*'s discovery of the cause of the great inequality.

In the case of *Laplace*'s discovery the conjunction lines of the planets revolve forward, whereas in the case of the lunar fluctuations, it is the angular movement of the conjunction lines node-perigee which has to be considered. On this relative angular movement depend the small uncompensated forces for the release of the moon from the sun's gravitational control when near the shadow of the earth, whence arises the long period inequalities in the moon's mean longitude.

I arrived at the cause of the lunar fluctuations from the study of the close analogy with the great inequality of Jupiter and Saturn, which I had suspected, and called to the attention of Prof. *E. W. Brown*, in 1914, after reading his address to the British Association in Australia. It appears that neither *Brown* nor *Bottlinger* had been encouraged by results of the researches they had made; yet *Bottlinger*'s investigation of 1912 proved very suggestive to me, and the analogy with *Laplace*'s discovery of 1785 was so close that it finally enabled the cause of the lunar fluctuations to be made out.

In the Observatory for May, 1918, Mr. *Harold Jeffreys* has a review of my researches on the lunar fluctuations. After recounting the method employed, and admitting the force of the results brought to light, he finally holds (p. 219) that the angular progressions in the two fluctuations, namely that in 61.7006 years and 277.59 years respectively, should stand in the ratio of exactly 9 : 2, and thus that my constants are not quite exact. If the shorter period of 61.7006 years be exact, *Jeffreys*' argument would make the longer period 277.652 years, instead of 277.59 years found by me.

This difference is very trifling, and of no practical importance, but the relation 9 : 2 may eventually be of value to the future investigators of these movements when the course of centuries shall make known the constants of the lunar movement with increasing precision. At present I think the final value of these periods can scarcely be attained, because each fluctuation involves very slightly the period of the other; so that we scarcely know which period may be chosen, or how the two may be adjusted and compensated to the exact ratio of 9 : 2.

In closing this brief discussion of the lunar fluctuations, which *Newcomb* pronounced the most enigmatical phenomenon presented by the celestial motions, it is scarcely necessary to add that the result attained is a very notable triumph for the wave-theory.

We have seen that the motion of the perihelion of Mercury admits of a half a dozen different explanations, in addition to the mystical one offered by *Einstein*, which is devoid of physical basis; and finally the natural and simple explanation based on the wave-theory, and outlined above in section 3.

On the other hand, the lunar fluctuations, which are vastly more complicated than the motion of Mercury's perihelion, admit of but a single known explanation, namely, that discovered by the present writer in 1916. It is therefore with some reason that the most experienced physical mathematician at Cambridge wrote me, Jan. 28, 1917:

»I wish the perihelion of Mercury could be resolved similarly (to the new work on the lunar fluctuations). Otherwise we have an unlimited number of ingenious kinds of relativity on our hands; which will be remarkable for self-contradiction of the principle that everything is relative«.

It is just such confusion as this that I have labored to get rid of, and now my theory of the motion of Mercury's perihelion is found to conform to the wave-theory, and to correspond to the ideas of *Newton*, 1686, that the law of gravitation in certain cases differs a little from the exact law of the inverse squares — the difference being explained by the wave-theory, and the nature of the aether.

6. Gravitational Action is propagated by Stresses due to Waves in the Aether, but *Maxwell's* conception that the Stress is based on Pressure in the Direction of the Line of Force and on an equal Tension in all directions at right angles thereto is not admissible.

From the electrod. wave-theory of gravitation, outlined in the writer's work of 1917, it follows that gravitation is propagated by stresses in the aether due to the interpenetration of waves, and the action across space therefore travels with the velocity of light. This mode of action is already outlined also in the first paper on the new theory of the aether, AN 5044. Forty-seven years ago in the celebrated Treatise on Electricity and Magnetism, 1873, vol. 1, Chap. V, §§ 103–116, *Maxwell* gave a remarkable theorem for the stresses between two electrically charged material systems, as producible by a distribution of stress over closed surfaces about these systems.

He takes two electrical systems, namely, E_1, with volume density ϱ_1, of the element whose coordinates are x_1, y_1, z_1; and similarly for the other system, $E_2, \varrho_2, x_2, y_2, z_2$. Then the x-component of the force acting on the element of E_1, owing to the repulsion of the element of E_2, will be:

$$\begin{aligned} dX &= \varrho_1\,\varrho_2\,(x_1-x_2)/r^3\cdot dx_1\,dy_1\,dz_1\,dx_2\,dy_2\,dz_2 \\ r^2 &= (x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2 \\ X &= \iiiiiint (x_1-x_2)/r^3\cdot\varrho_1\,\varrho_2\,dx_1\,dy_1\,dz_1\,dx_2\,dy_2\,dz_2 \end{aligned} \tag{36}$$

This is as in the theory of action at a distance, and the integrals will not be altered by extending the limits from $-\infty$ to $+\infty$.

Maxwell then proceeds to remark (§ 105) that if the action of E_2 on E_1 is effected, not by direct action at a distance, but by means of a distribution of stress in a medium extending continuously from E_2 to E_1, it is manifest that if we knew the stress at every point of any closed surface s which completely separates E_1 from E_2, we shall be able to determine completely the mechanical action of E_2 on E_1. Accordingly, he concludes that if it is possible to account for the action of E_2 on E_1 by means of a distribution of stress in the intervening medium, it must be possible to express this action in the form of surface integrals extending over the surface s, which completely separates one system from the other.

Maxwell then develops the solution at some length, and after obtaining the required mathematical expressions, (§§ 105–110), remarks (§ 111): »I have not been able to make the next step, namely to account by mechanical considerations for these stresses in the dielectric. I therefore leave the theory at this point.«

It can be shown that the action of waves, flat in planes normal to the lines of force will explain the mechanical difficulties here noted by *Maxwell*. For in his work on Matter, Aether and Motion, Boston, 1894, Prof. *A. E. Dolbear* describes an experiment of the following kind:

»If a dozen disks five or six inches in diameter are set loosely an inch apart upon a spindle a foot long, so that they may be rotated fast, yet left free to move longitudinally upon the spindle, they will all crowd up close together, as the pressure is less between them than outside. If one can imagine the spindle to be flexible and the ends brought opposite each other while rotating, it will be seen that the ends would exhibit an apparent attraction for each other, and if free to approach, would close up, thus making a vortex ring with the sections of disks. If the axis of the disks were shrinkable, the whole thing would contract to a minimum size that would be determined by the rapidity of the rotary movement, in which case not only would it be plain why the ring form was maintained, but why the diameter of the ring as a whole should shrink. So long as it rotated it would keep up a stress in the air about it. So far as the experimental evidence goes, it appears that a vortex ring in the air exhibits the phenomenon in question.«

The behavior of the flexible spindle in this experiment is analogous to that of the lines of force, which *Faraday* long ago observed had a notable tendency to shorten themselves. The gaseous medium of the air between the disks is thinned out, by the effect of the centrifugal force, just as the aether itself is near a magnet, owing to the rotations[1]) of the wave elements about the lines of force. Hence the lines of force tend to shorten themselves, as *Faraday* observed in his experiments with magnets and electric currents.

In view of this experiment it is not remarkable therefore that the lapse of time has confirmed *Maxwell's* stresses

[1]) We hold the lines of force to be filaments of the aethereal vortices, due to rotations of the wave elements, as the waves recede from a magnet. If dm be the element of aethereal mass in rotation, and the z-axis coincide with the axis of the magnet, the angular momentum of an element in the plane of the magnetic equator will be: $A = \Sigma\,dm\,(y\cdot dx/dt - x\cdot dy/dt)$. This momentum of masses of aether $\Sigma\,dm$, about the axis of the line of force, tends to beat back the aether in the equatorial plane, and causes it to press in on the two ends, parallel to the z-axis. Hence we see the inevitable tendency of the lines of force to shorten themselves. Cf. *Maxwell*, On Physical Lines of Force, 1862, Scientific Papers, Vol. 1, p. 508.

for electrical action, yet shown on the other hand that the stresses conceived by him for gravitation are invalid, because in this latter case he conceived the pressure to be in the direction of the lines of force.

Maxwell's conclusion as to gravitation is announced in the article Attraction (Scientific papers, vol. 2, p. 489): »To account for such a force (gravitation) by means of stress in an intervening medium, on the plan adopted for electric and magnetic forces, we must assume a stress of an opposite kind from that already mentioned. We must suppose that there is a pressure in the direction of the lines of force, combined with a tension in all directions at right angles to the lines of force. Such a state of stress would, no doubt, account for the observed effects of gravitation. We have not, however, been able hitherto to imagine any physical cause for such a state of stress.«

It seems remarkable that *Maxwell* himself should not have seen the error underlying this reasoning. When we whirl a stone by a string, it is the tension of the cord which holds the stone in its circular path, thus overcoming the centrifugal force. If the string breaks, the stone goes flying away, along the tangent to the instantaneous path at the moment when the tension of the string is released.

Innumerable examples of this central tension or pulling, necessary to overcome centrifugal force, should have occurred to *Maxwell*, as perfectly analogous to the forces which hold the planets in their orbits.

It was seven years after the death of *Maxwell* (1879) before the mathematical test required to overthrow the validity of his gravitational stresses was given by Prof. *George M. Minchin* in his Treatise on Statics, Oxford, 1886, Vol. II, pp. 448–455. *Minchin* calculates the Maxwellian gravitational stress intensities at any point P and finds the components to be:

$$A = -R^2/8\pi\gamma \quad B = R^2/8\pi\gamma \quad C = R^2/8\pi\gamma \qquad (37)$$

where R is the resultant force intensity, and γ the gravitation constant. These expressions show that the three principal stresses are equal. The component A along the line of force, is, by *Maxwell*'s hypothesis, a pressure, and the other two components are tensions.

Apparently Prof. *Minchin* never seriously suspected the fallacy underlying *Maxwell*'s assumption, that pressure in the medium along the radius vector of a planet could make its orbit curve about the sun, where in fact a tension, corresponding to the full breaking strength of stupendous cables of steel, is required to be exerted for holding a planet in its elliptical path. The nature of the curvature of the elliptic orbit was established by *Kepler* from the observations of *Tycho*, 1609, and first explained by *Newton* from the law of gravitation, 1687.

After a very learned discussion, Prof. *Minchin* only reaches the conclusion that since on trial, the mathematical conditions specified by the stress analysis are not fulfilled, — »either gravitation is not propagated by the Maxwellian stress, or the aether is not of the nature of a solid body.«

This is a good historical example of a false premise, on which much ingenious mathematical effort was spent, without detecting the physical error underlying the hypothesis. It will forcibly remind natural philosophers of *Einstein*'s bizarre proposal to do away with the aether, without substituting any medium or substance in the planetary spaces which might exert contractile power for holding the planets and stars in their orbits.

It is scarcely necessary to add that if the signs of *Maxwell*'s stresses given above be changed, so as to give a component of tension in the line of force, and two equal pressures at right angles thereto, thus:

$$A = +R^2/8\pi\gamma \quad B = -R^2/8\pi\gamma \quad C = -R^2/8\pi\gamma \qquad (38)$$

gravitational phenomena would be explained.

In the Electrod. wave-theory of Phys. Forces, 1917, pp. 131–133, will be found an explanation of why the aether tends to contract between any two bodies, as the sun and earth. This may be made a little more obvious by the following diagram, in which each body is shown surrounded by a wave-field, the aether near either body being so agitated by the waves from its own atoms as to be of less density towards either centre than in the remoter spaces between the masses.

We are to conceive the waves from either centre, by interpenetrating with those from the other centre, undoing the wave stress, depending on the other mass, and thus causing a constant tendency of the aether to collapse, which results in-pulling with maximum tension along the right line connecting the two bodies.

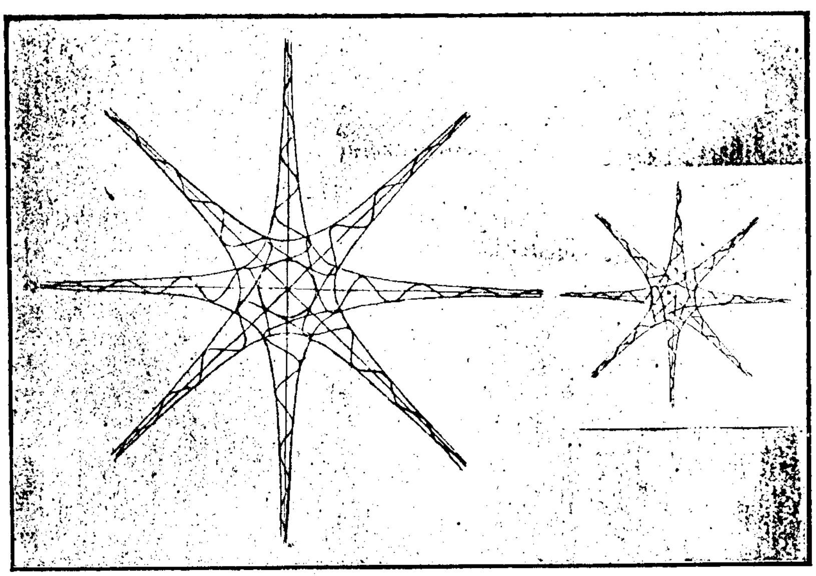

Fig. 8. Illustration of the development of stress between the sun and earth, owing to the interpenetration of the waves, rotating in opposite directions, from these two independent wave-fields, thus causing a tendency to collapse, in the medium between the two bodies, which furnishes the tension required to hold the planets in their orbits.

This gives us a very simple and direct grasp of the mechanism underlying the planetary forces, which is not very different from those operative in electricity and magnetism, except for the essentially haphazard arrangement of the planes of the atoms in the heavenly bodies. These bodies are only slightly magnetic, — this power depending on the lining up of a small fraction of their atoms, in planes which are mutually parallel, as in common magnets; while the great mass of the atoms are tilted haphazard. The resulting action yields the central force called gravity, instead of the duality of powers noted by *Airy* (Treatise on Magnetism, 1870, p. 10) for the magnetic attraction directed towards two poles.

7. Sextuple Integration, under *Fourier*'s Theorem, for solving *Poisson*'s Partial Differential Equation $\partial^2\Phi/\partial t^2 = a^2\nabla^2\Phi$ for the velocity-potential, in a medium like the aether, capable of freely propagating waves.

We consider the partial differential equation for the velocity-potential Φ in wave motion:

$$\partial^2\Phi/\partial t^2 = a^2\nabla^2\Phi \qquad \Phi = \Omega(x, y, z, t)$$
$$d\Phi = \partial\Phi/\partial x\cdot dx + \partial\Phi/\partial y\cdot dy + \partial\Phi/\partial z\cdot dz \qquad (39)$$

$d\Phi$ being an exact differential, to which *Poisson* (Traité de Mécanique, 1833, Tome II, p. 697) and *Cauchy* have given so much attention, in the period immediately preceding and following the development of *Fourier*'s analysis, (1807–1821). This method finally appeared in the celebrated Théorie Analytique de la Chaleur, 1821. Besides the above reference to *Poisson*'s Mechanics, we cite the important memoirs indicated below [1]).

Poisson usually treats his differential equation in the form:

$$\partial^2\Phi/\partial t^2 - a^2(\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2) = 0.$$

Thus Φ is any solution of the equation (39), which involves three variable coordinates, x, y, z, and the time, t.

By a well known form of *Fourier*'s theorem we have:

$$\Omega(x) = 1/2\pi\cdot\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{(\xi - x)\lambda\sqrt{(-1)}}\cdot\Omega(\xi)\cdot d\xi\, d\lambda. \quad (40)$$

And as this will apply to the several variables, we get by three successive integrations between the limits $-\infty$ and $+\infty$:

$$\Phi = \Omega(x, y, z, t) = (1/8\pi^3)\iiiiii e^{A\sqrt{(-1)}}\cdot\Omega(\xi, \eta, \zeta, t)\cdot d\xi\, d\eta\, d\zeta\, d\lambda\, d\mu\, d\nu. \quad A = (\xi - x)\lambda + (\eta - y)\mu + (\zeta - z)\nu. \quad (41)$$

If now we substitute the derivatives of this result in (39), observing by the form of A, in (41), that we have upon actual derivation:

$$(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)\, e^{A\sqrt{(-1)}} = e^{A\sqrt{(-1)}}(-\lambda^2 - \mu^2 - \nu^2) \quad (42)$$

we have for the solution of the original equation involving the four variables:

$$\partial^2\Phi/\partial t^2 - a^2(\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2) = 0$$

$$\Phi = \Omega(x, y, z, t) = (1/8\pi^3)\iiiiii e^{A\sqrt{(-1)}}\cdot\{\partial^2/\partial t^2 + a^2(\lambda^2 + \mu^2 + \nu^2)\}\,\Omega(\xi, \eta, \zeta, t)\cdot d\xi\, d\eta\, d\zeta\, d\lambda\, d\mu\, d\nu = 0 \quad (43)$$

(limits of integration $-\infty$ and $+\infty$). This equation will be satisfied, if $\Omega(\xi, \eta, \zeta, t)$ is determined so as to satisfy the equation:

$$\partial^2\Omega(\xi, \eta, \zeta, t)/\partial t^2 + a^2(\lambda^2 + \mu^2 + \nu^2)\,\Omega(\xi, \eta, \zeta, t) = 0. \quad (44)$$

We therefore integrate this differential equation, and in place of arbitrary constants, we introduce arbitrary functions Ψ_1 and ψ_1 of ξ, η, ζ. Accordingly our solutions yield the following particular integrals:

$$\Omega(\xi, \eta, \zeta, t) = e^{Bht\sqrt{(-1)}}\,\Psi_1(\xi, \eta, \zeta) \qquad \Omega(\xi, \eta, \zeta, t) = e^{-Bht\sqrt{(-1)}}\,\psi_1(\xi, \eta, \zeta) \qquad B = (\lambda^2 + \mu^2 + \nu^2)^{1/2}. \quad (45)$$

If now we substitute the first of these in (41), and include the integration factor $1/8\pi^3$ in the arbitrary function, we have (limits of integration $-\infty$ and $+\infty$):

$$\Phi = \Omega(x, y, z, t) = \iiiiii e^{(A + Bht)\sqrt{(-1)}}\,\Psi_1(\xi, \eta, \zeta)\cdot d\xi\, d\eta\, d\zeta\, d\lambda\, d\mu\, d\nu. \quad (46)$$

This is a particular integral of equation (41), and the second value in (45) would lead to an identical result, as may be proved by actual substitution. Thus it only remains to complete the solution from such particular solutions.

Let $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2 = \Delta \qquad t = e^{\theta}$ (47)

so as to reduce the given equation to the symbolical form: $\Phi - [\Delta/D(D-1)]\,e^{2\theta}\cdot\Phi = 0$ (48)

where $\partial/\partial\theta = D$. Then the transformation: $\Phi = e^{-\theta}\cdot\partial\chi/\partial\theta = \partial\chi/\partial t$ (49)

will give: $\chi - [\Delta/D(D-1)]\,e^{2\theta}\cdot\chi$ (50)

which is of the same form as the equation for Φ in (48).

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[1]) 1. *Fourier.* Oeuvres de *Fourier*, Tomes I et II, publiées suos les auspices du Ministère de l'Instruction Publique par les soins de *Gaston Darboux*, Paris, 1888.

2. *Poisson*: a) Mémoire sur la Théorie des Ondes, Déc. 18, 1815; Mém. de l'Acad., T. I.
b) Mémoire sur l'Intégration de quelques équations linéaires aux différences partielles, et particulièrement de l'équation générale du mouvement des fluides élastiques. Juill. 19, 1819, Mém. de l'Acad., T. III.
c) Mémoire sur le Mouvement de Deux Fluides Élastiques Superposés. Mars 24, 1823, Mém. de l'Acad., T. X.
d) Mémoire sur l'Équilibre et le Mouvement des Corps Élastiques. Avril 14, 1828, Mém. de l'Acad., T. VIII.
e) Mémoire sur l'Équilibre des Fluides. Nov. 24, 1828, Mém. de l'Acad., T. IX.
f) Mémoire sur la Propagation du Mouvement dans les Milieux Élastiques. Oct. 11, 1830, Mém. de l'Acad., T. X.
g) Mémoire sur l'Équilibre et le Mouvement des Corps Crystallisés. Oct. 28, 1839, Mém. de l'Acad., T. XVIII.

3. *Cauchy*: a) Théorie de la Propagation des Ondes à la surface d'un Fluid Pesant d'une Profondeur Indéfinie, 1815.
b) Sur l'Intégration d'Équations Linéaires. Exercises d'Analyse et de Physique Mathématique, T. I, p. 53.
c) Sur la Transformation et la Réduction des Intégrales Générales d'un Système d'Équations Linéaires aux différences partielles, ibid. p. 178.

It thus follows that χ admits of expression in the form (46), and therefore by merely changing the arbitrary function, we have (limits of integration $-\infty$ and $+\infty$):

$$\chi = \Omega'(x,y,z,t) = \partial/\partial t \iiiint\!\!\iint e^{(A+Bht)\sqrt{(-1)}} \cdot \Psi_2(\xi,\eta,\zeta) \cdot d\xi\, d\eta\, d\zeta\, d\lambda\, d\mu\, d\nu\,. \tag{51}$$

To get the complete integral from these independent particular integrals (46) and (51), we add the two solutions multiplied by arbitrary constants, (cf. *Hattendorff*'s edition of *Riemann*'s Partielle Differentialgleichungen, 1882, p. 100), which may be included under the sextuple integral signs (limits of integration $-\infty$ and $+\infty$):

$$\Phi_1 = c_1\,\Phi + c_2\,\chi \tag{52}$$

$$= \iiiint\!\!\iint e^{(A+Bht)\sqrt{(-1)}} \cdot \Psi_1(\xi,\eta,\zeta) \cdot d\xi\, d\eta\, d\zeta\, d\lambda\, d\mu\, d\nu$$

$$+ \partial/\partial t \iiiint\!\!\iint e^{(A+Bht)\sqrt{(-1)}} \cdot \Psi_2(\xi,\eta,\zeta) \cdot d\xi\, d\eta\, d\zeta\, d\lambda\, d\mu\, d\nu\,. \tag{53}$$

These sextuple integrals admit of reduction to double integrals leading to a form of solution originally obtained by *Poisson*; but *Cauchy* has made this reduction by means of a trigonometrical transformation. The only essential precaution to be taken is to avoid processes by which the functions to be integrated become infinite within the limits.

The above equation belongs to the general form

$$\partial^2 \Phi/\partial t^2 = \Delta\,\Phi \tag{54}$$

where Δ is a function of the derivatives with respect to the coordinates $\partial/\partial x$, $\partial/\partial y$, $\partial/\partial z$. For all such equations the method above outlined furnishes directly a solution expressed by sextuple integrals, which are reducible to the *Poisson-Cauchy* double integrals, if Δ is homogeneous and of the second degree, as in the case of a sphere surface, with radius increasing uniformly with the time:

$$x^2+y^2+z^2 = c^2 t^2 \tag{55}$$

where c is the parameter representing the velocity of light.

As was long ago pointed out by *Fourier*, *Poisson* and *Cauchy*, integrals of this type are peculiarly appropriate for the expression of those disturbances involving the transmission of energy in a medium, as in the steady flow of waves, whether of sound, light, heat or electrodynamic action. These wave disturbances are propagated through the medium in question with a finite velocity, and unless the waves are regularly renewed the original disturbance leaves no trace behind when it has passed by; so that the upkeep of the energy flow involves periodic renewal of disturbances for maintaining the steady flow of waves. In his Théorie Analytique de la Chaleur, 1821, *Fourier* continually emphasizes the incessant movement of heat.

Solution of *Poisson*'s equation for the velocity-potential Φ in wave motion from n bodies.

Let there be n bodies emitting waves: m_1 with coordinates (x_1, y_1, z_1, t_1) surrounded at the instant t_1 by an infinite series of wave surfaces, which for simplicity we may suppose to be spherical:

$$x_1^2+y_1^2+z_1^2-c_1^2 t_1^2 = 0$$
$$x_2^2+y_2^2+z_2^2-c_2^2 t_1^2 = 0$$
$$\cdots\cdots\cdots$$
$$x_I^2+y_I^2+z_I^2-c_I^2 t_1^2 = 0\,.$$

Accordingly, at the time t_1 there are $\sum_{I=1}^{I=\infty} I = \infty$ of these concentric wave surfaces, all moving with the velocity c, which is the velocity of light. But the time t also flows on, and if there be i intervals, the summation $\sum_{i=1}^{i=\infty} i = \infty$ will yield for the double integration of intervals and waves:

$$\sum_{i=1}^{i=\infty}\sum_{I=1}^{I=\infty} i\,I = \infty^2$$

which corresponds to all the points in an infinite plane.

Imagine another system of coordinates (ξ_I, η_I, ζ_I), with its origin at the centre of gravity of m_1 $(\xi_h, \eta_h, \zeta_h, t_1)$, to which the moving waves are referred at i times, so that for the n bodies we have:

For the Bodies.	For the Waves emitted.	
$m_1\,(x_1, y_1, z_1, t_1)$	$m_1\,(\xi_{hi}, \eta_{hi}, \zeta_{hi}, t_i)$	
$m_2\,(x_2, y_2, z_2, t_1)$	$m_2\,(\xi_{Ii}-\xi_{hi}, \eta_{Ii}-\eta_{hi}, \zeta_{Ii}-\zeta_{hi}, t_i)$	
$m_3\,(x_3, y_3, z_3, t_1)$	$m_3\,(\xi_{Ii}-\xi_{hi}, \eta_{Ii}-\eta_{hi}, \zeta_{Ii}-\zeta_{hi}, t_i)$	(56)
. . . .	. . . .	
$m_n\,(x_n, y_n, z_n, t_1)$	$m_n\,(\xi_{I_n i}-\xi_{hi}, \eta_{I_n i}-\eta_{hi}, \zeta_{I_n i}-\zeta_{hi}, t_i)$.	

Then, from the preceding investigation it will follow that the solution of *Poisson*'s equation $\partial^2\Phi/\partial t^2 = a^2\nabla^2\Phi$ for the velocity-potential Φ and transmission of energy of wave motion, in the case of n bodies will be similar to that already found for a single wave centre, except that as the waves from the several bodies are everywhere superposed, the velocity-potentials Φ_1, Φ_2, $\Phi_3 \cdots \Phi_n$ from the several centres must be added together to get the total effect, $\Phi_1+\Phi_2+\Phi_3+\cdots+\Phi_n = \Phi$, when the waves from the n bodies mutually interpenetrate, giving maximum tension in the right lines which connect the bodies in pairs, and maximum pressure in the prolongation of these lines beyond the masses.

Accordingly, if we introduce the amplitude of the waves from each mass, $A_{Ii} = k_{Ii}/\sqrt{(\xi_{Ii}^2+\eta_{Ii}^2+\zeta_{Ii}^2)}$ and retain the amplitude $e^{(A+Bht)\sqrt{(-1)}}$ for deteriorating wave changes, under resistance, we shall find for the general solution the expression (all integrations between the limits $-\infty$ and $+\infty$):

$$\Phi = \Phi_1, (m_1) + \Phi_2, (m_2) + \Phi_3, (m_3) + \cdots \Phi_n, (m_n) \tag{57}$$

$$\Phi_n, (m_n) = \iiiint\!\!\iint k_{I_n i}/\sqrt{[(\xi_{I_n i}-\xi_{hi})^2+(\eta_{I_n i}-\eta_{hi})^2+(\zeta_{I_n i}-\zeta_{hi})^2]} \cdot e^{(A_n+B_n h_n t_1)\sqrt{(-1)}} \cdot \Psi_n(\xi_n,\eta_n,\zeta_n) \cdot d\xi_n\, d\eta_n\, d\zeta_n\, d\lambda_n\, d\mu_n\, d\nu_n$$

$$+ \partial/\partial t \iiiint\!\!\iint k_{I_n i}/\sqrt{[(\xi_{I_n i}-\xi_{hi})^2+(\eta_{I_n i}-\eta_{hi})^2+(\zeta_{I_n i}-\zeta_{hi})^2]} \cdot e^{(A_n+B_n h_n t_1)\sqrt{(-1)}} \cdot \psi_n(\xi_n,\eta_n,\zeta_n) \cdot d\xi_n\, d\eta_n\, d\zeta_n\, d\lambda_n\, d\mu_n\, d\nu_n\,.$$

This solution of *Poisson's* equation for the velocity-potential Φ is well calculated to show the complexity of the problem of explaining the forces which govern the operations of the physical universe. The velocity-potential is essentially a function of the elasticity in a gas, condensation alternating with rarefaction, by which wave motion once generated is maintained at all points of space, and at velocities suitable to the elasticity and density of the medium at these points. Thus wherever waves penetrate the velocity-potential must also exist.

And we see not only that the domain penetrated by the waves includes all space, from minus infinity to plus infinity, in a sextuple integration, which corresponds to an integration connecting every point of space with every other point; but also that it must be continuous, that is, repeated for every pair of points two and two.

The waves from the individual atoms are infinitely more complex still, and in fact cannot be given except by an integral like the foregoing, infinitely extended. This infinite integral could be written out analytically, yet its contemplation would aid us but little in grasping the infinitely complex phenomena of nature.

In practice it suffices to remember that from every body an infinitely complex system of waves goes forth, to interpenetrate and combine with the like infinitely complex wave systems going forth from all other bodies. The summation of all these disturbances is an infinite integral of the effects of small commotions, the final result of which is the system of forces operating throughout the physical universe.

In the Principia (Lib. III, Props. VI–VIII and Prop. XXIV) Sir *Isaac Newton* points out how the gravitative force due to one body may penetrate into the regions occupied by any other body or system, just as if the other body or system did not exist; so that each body or system acts independently of the others, yet the final effect is a combination of the separate effects. Gravitation, therefore, is an interpenetrating power — just such an influence as would arise from waves propagated from the several centres, and extending throughout all parts of the system of the world.

8. Geometrical Conditions fulfilled by the Velocity-potential Φ, expressions for the molecular velocity and condensation at any distance from the source of disturbance, with an indication of the energy due to the waves of various lengths observed in nature.

The solution of the problem of vibrating cords runs back to *Daniel Bernoulli* and *D'Alembert*, but the method of analysis was generalized by *Lagrange*, and *Poisson* has greatly improved the theory for application to all classes of waves. The energy in the wave function depends on three coordinates, x, y, z, and the time t, because when a disturbance originates in a medium it spreads in all directions, sometimes at rates depending on the wave conductivity along certain axes, but always at a rate defined by the time t.

If the medium be gaseous, as in the kinetic theory of the aether, Φ must be the velocity-potential [1]). Accordingly, we outline the equations of such a medium:

$$\begin{aligned} d\Phi &= \partial\Phi/\partial x\cdot dx+\partial\Phi/\partial y\cdot dy+\partial\Phi/\partial z\cdot dz \\ &= u\,dx+v\,dy+w\,dz \\ u = \partial\Phi/\partial x \quad & v = \partial\Phi/\partial y \quad w = \partial\Phi/\partial z \end{aligned} \tag{58}$$

where u, v, w are the component velocities.

The general equation of equilibrium is:

$$dV = X\,dx+Y\,dy+Z\,dz$$

$$\text{whence}\quad X = \partial V/\partial x \quad Y = \partial V/\partial y \quad Z = \partial V/\partial z\,. \tag{59}$$

Now put $\int(1/\varrho)\,dp = P$; and we have the well known relations:

$$(1/\varrho)\,\partial p/\partial x = \partial P/\partial x \quad (1/\varrho)\,\partial p/\partial y = \partial P/\partial y \quad (1/\varrho)\,\partial p/\partial z = \partial P/\partial z \tag{60}$$

$$V-P = \partial\Phi/\partial t+{}^1/_2[(\partial\Phi/\partial x)^2+(\partial\Phi/\partial y)^2+(\partial\Phi/\partial z)^2]\,. \tag{61}$$

And the equation of continuity:

$$\partial\varrho/\partial t+\partial/\partial x(\varrho\cdot\partial\Phi/\partial x)+\partial/\partial y(\varrho\cdot\partial\Phi/\partial y)+\partial/\partial z(\varrho\cdot\partial\Phi/\partial z)=0\,. \tag{62}$$

For an incompressible fluid the second expression in (62) vanishes:

$$\partial^2\Phi/\partial x^2+\partial^2\Phi/\partial y^2+\partial^2\Phi/\partial z^2 = 0\,. \tag{63}$$

But the aether is not incompressible, and this equation therefore does not apply to any gaseous medium.

In general the exact form of the wave surface cannot be defined, owing to changes in the density and elasticity of the bodies penetrated by the advance of the wave front. If the medium be symmetrical in respect to three axes at right angles, as in the case of certain crystals, then the wave surface, from a disturbance at the centre of such a mass, will pass from the spherical form:

$$x^2+y^2+z^2-c^2t^2 = 0 \tag{64}$$

and take the form of an ellipsoid of three unequal axes:

$$x^2/\alpha^2+y^2/\beta^2+z^2/\gamma^2-c^2t^2 = 0 \tag{65}$$

where the axes α, β, γ denote the conductivities along the axes of the ellipsoid, and $ct = 1$, at any stage of the progress with the wave surface in the form of the ellipsoid:

$$x^2/\alpha^2+y^2/\beta^2+z^2/\gamma^2 = 1\,. \tag{66}$$

It follows therefore that the problem of wave motion involves the solution of *Poisson's* equation:

$$\partial^2\Phi/\partial t^2 = a^2(\partial^2\Phi/\partial x^2+\partial^2\Phi/\partial y^2+\partial^2\Phi/\partial z^2)\,, \tag{67}$$

where a is the velocity of the wave propagation (cf. *Poisson*, Traité de Mécanique, 1833, tome II, p. 663–720; or Lord *Rayleigh's* Theory of Sound, vol. II, chapter XIII).

Let u, v, w be the component velocities parallel to the axes Ox, Oy, Oz of an element of mass dm, at the instant t, so that,

$$x-x' = \int u\,dt \quad y-y' = \int v\,dt \quad z-z' = \int w\,dt\,. \tag{68}$$

[1]) If for any part of an elastic fluid mass $d\Phi = u dx+v dy+w dz = 0$ be a perfect differential at one moment, it will remain so for all subsequent time. When Φ is single valued, the integral round any closed circuit vanishes, $\int d\Phi = 0$. This is the irrotational condition of hydrodynamics. Hence, with condensations and rarefactions alternating, and of equal intensity, in wave motion, the above condition $\int d\Phi = 0$ is met by the plane wave $\Phi = A\cos[2\pi/\lambda\cdot(x-at)]$, which is typical of the velocity-potential in general.

If we neglect the squares of the velocities $\partial\Phi/\partial x$, $\partial\Phi/\partial y$, $\partial\Phi/\partial z$, and put $v = 0$, $w = 0$, Φ will become a function of x and t only:

$$\partial^2\Phi/\partial t^2 = a^2\,\partial^2\Phi/\partial x^2$$
$$\Phi = \Omega\,(x, t) = A\cos[2\pi/\lambda\cdot(x - at)]\ ^{1}). \tag{69}$$

The solution obviously is an undulation of flat wavelets parallel to the axis of x, traveling with velocity a.

Let ζ be the velocity in the direction of the radius vector, so that the resultant

$$\zeta = \sqrt{(u^2+v^2+w^2)} \tag{70}$$

then since for spherical disturbances

$$x^2+y^2+z^2 = r^2 \qquad x\,dx+y\,dy+z\,dz = r\,dr$$
$$u = \zeta\,x/r \qquad v = \zeta\,y/r \qquad w = \zeta\,z/r \tag{71}$$

we get $\quad u\,dx+v\,dy+w\,dz = \zeta\,dr \qquad \zeta = \partial\Phi/\partial r \qquad (72)$

$$\partial\Phi/\partial x = \partial\Phi/\partial r\cdot x/r \qquad \partial\Phi/\partial y = \partial\Phi/\partial r\cdot y/r$$
$$\partial\Phi/\partial z = \partial\Phi/\partial r\cdot z/r. \tag{73}$$

Differentiating a second time, we have

$$\partial^2\Phi/\partial x^2 = \partial^2\Phi/\partial r^2\cdot x^2/r^2+\partial\Phi/\partial r\cdot(y^2+z^2)/r^3$$
$$\partial^2\Phi/\partial y^2 = \partial^2\Phi/\partial r^2\cdot y^2/r^2+\partial\Phi/\partial r\cdot(z^2+x^2)/r^3 \tag{74}$$
$$\partial^2\Phi/\partial z^2 = \partial^2\Phi/\partial r^2\cdot z^2/r^2+\partial\Phi/\partial r\cdot(x^2+y^2)/r^3.$$

By means of these values, *Poisson*'s equation,

$$\partial^2\Phi/\partial t^2 = a^2\,(\partial^2\Phi/\partial x^2+\partial^2\Phi/\partial y^2+\partial^2\Phi/\partial z^2)$$

becomes $\quad \partial^2\Phi/\partial t^2 = a^2\,(\partial^2\Phi/\partial r^2+2/r\cdot\partial\Phi/\partial r). \qquad (75)$

This is the same as

$$\partial^2 r\Phi/\partial t^2 = a^2\,(\partial^2 r\Phi/\partial r^2) \tag{76}$$

the complete integral of which is

$$r\Phi = f(r+at)+F(r-at) \tag{77}$$

where f and F are two arbitrary functions.

By extending his analysis (Traité de Mécanique, 1833, vol. II, p. 706) *Poisson* shows that since $\zeta = \partial\Phi/\partial r$, we have

$$\zeta = 1/r\cdot f'(at-r)+1/r^2\cdot f(at-r)$$
$$s = 1/ar\cdot f'(at-r). \tag{78}$$

Accordingly, *Poisson* concludes that at a great distance from the centre of this disturbance we may neglect the second terms of the values of ζ, which are divided by r^2, in comparison with the first, which are divided by r. Thus for the whole duration of the movement we get for the condensation or dilatation $s \qquad s = \zeta/a. \qquad (79)$

By equation (78), therefore, the velocity of the molecules in a gaseous medium decreases inversely as r, just as in the amplitudes of the waves postulated in the kinetic theory of the aether. The condensation or dilatation s varies as the velocity in the direction of the radius vector, which itself varies inversely as r; and also inversely as a, the velocity of wave propagation. Accordingly, for a highly elastic medium, s is small, and decreases very rapidly; which confirms our view that the amplitudes of the aether waves are very minute, and decrease inversely as r in receding from the sun.

In finishing this paper, Febr. 19, 1920, I am surprised to notice *Poisson*'s sagacious remark (p. 706): »La vitesse propre des molécules d'air décroîtra alors en raison inverse de r«: which affords an unexpected verification of the writer's formula for the amplitudes of the aether waves, $A = k/r$, also derived from the kinetic theory, but by a different process. It thus appears that *Poisson* had such a result for the waves of sound 87 years ago, and its neglect for nearly a century is remarkable.

As Lord *Rayleigh* points out in his Theory of Sound, 2nd edition, 1896, vol. II, p. 16: the rate at which energy is transmitted across unit area of a plane parallel to the front of a progressive wave may be regarded as the mechanical measure of the intensity of the radiation. This is the basis of Lord *Kelvin*'s celebrated paper of 1854, »On the possible density of the luminiferous medium, and on the mechanical value of a cubic mile of sunlight«, (Trans. Roy. Soc., Edinburgh, 1854), which we have used, in our first paper on the new theory of the aether, for calculating the density of this medium. The energy transmitted, in the direction of the three coordinate axes, Φ being taken successively as a function of x (and t), y (and t), z (and t) only, is given by the approximate equations:

$$\partial^2\Phi/\partial t^2 = a^2\cdot\partial^2\Phi/\partial x^2 \qquad \partial^2\Phi/\partial t^2 = a^2\cdot\partial^2\Phi/\partial y^2$$
$$\partial^2\Phi/\partial t^2 = a^2\cdot\partial^2\Phi/\partial z^2 \tag{80}$$

which are expressed in (75) above.

In case the gravitational wave transmission occurs within a mass of density ϱ, we have *Poisson*'s equation for the potential:

$$\partial^2 V/\partial x^2+\partial^2 V/\partial y^2+\partial^2 V/\partial z^2+4\pi\varrho = 0 \tag{81}$$

instead of the equation of *Laplace*:

$$\partial^2 V/\partial x^2+\partial^2 V/\partial y^2+\partial^2 V/\partial z^2 = 0. \tag{82}$$

And thus within an elastic solid the equation (80) would become:

$$\partial^2\Phi/\partial t^2 = a^2\cdot(\partial^2\Phi/\partial x^2+\partial^2\Phi/\partial y^2+\partial^2\Phi/\partial z^2+4\pi\varrho) \tag{83}$$

which is of the form adopted by *Riemann*, for the induction of electric currents, in the memoir presented to the Royal Society of Göttingen in 1858, but subsequently withdrawn, and after the death of the author, published in Poggendorff's Annalen 131.237–263, 1867.

This investigation of *Riemann* was examined by *Clausius* (Poggendorff's Annalen 135.612) who doubts the validity of the mathematical processes for the phenomenon of electric induction, chiefly on the ground that the hypothesis that potential is propagated like light, does not lead either to the law of *Weber* or to the other laws of electrodynamics.

In our Electrod. Wave-Theory of Phys. Forc., however, it is not held that potential is propagated like light; on the contrary that the potential is a function $V = f(x, y, z, \varrho)$, is fixed in space, yet depends on the total accumulated stress due to wave amplitudes of all the matter involved. Hence this criticism is not valid against the wave-theory here dealt with.

Moreover, we use *Poisson*'s equation for the potential, $\nabla^2 V+4\pi\varrho = 0$, only within solid masses, *Laplace*'s equation $\nabla^2 V = 0$ applying to all free space. Thus we adopt a transition between these two equations at the boundary of any mass of matter, as long recognized by geometers and natural philosophers.

The physical meaning of the transition is the sharp difference in velocity of propagation for all aether waves at

[1]) Lord *Rayleigh*, Theory of Sound, vol. II, p. 15–16, 2nd edition, 1896.

the boundary of a mass of matter; and moreover the decrease in total accumulated stress due to the aether waves from all the atoms, as the moving point $p(x, y, z)$ enters the body of density ϱ, and leaves behind a part of the mass, — the aether waves coming from the atoms of this shell from all directions just balancing in a homogeneous sphere. But whatever the law of density or form of the body, there is a change in the sum of the second differentials of the potential at the boundary of the body, from *Laplace*'s to *Poisson*'s equation:

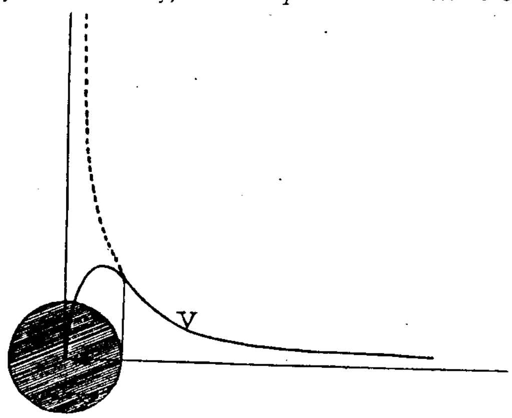

Fig. 9. Curve of the potential function V, showing its asymptotic decrease with the distance, and the tendency to an asymptotic increase towards the centre; but owing to finite dimensions of the mass, a gradual decline to zero.

This difference between *Laplace*'s equation of the potential for free space, and *Poisson*'s corresponding equation for space filled with matter of density ϱ, owing to the intervention of boundary conditions, is distinctly favorable to the wave-theory of physical forces. We therefore presented the treatment of the wave equation of *Poisson* $\partial^2 \Phi/\partial t^2 = a^2 \nabla^2 \Phi$ for free space, by the general method of integration based on *Fourier*'s theorem.

This solution will hold for waves of any initial wave length, propagated with the velocity of light, from n bodies, in all parts of space, and everywhere mutually interpenetrating so as to generate maximum tension in the right lines connecting the n bodies in pairs, in accordance with the observed phenomena of universal gravitation.

If the solution will hold for separate bodies, from which spherical waves are emitted, it obviously will hold also for separate vibrating particles, within a single body; but here the mathematical difficulty is increased, by virtue of the unequal conductivity which heterogeneous solid bodies offer to wave propagation; so that the expression of the effects of the waves from the atoms would be infinitely complex. Yet the above equation (57) gives the approximate representation of the propagation of wave energy from atoms, which may be useful in certain problems of molecular physics.

The solution in (57) already involves an infinitely complex integration, repeated n-times for the n bodies of the universe. To include the initial waves of all possible lengths, we should have to integrate this complex expression for Φ between the limits $\lambda = 0$, $\lambda = \infty$, involving all possible periodicities, the number of which is: $n = [V/\lambda]_{\lambda=0}^{\lambda=\infty}$.

Now, according to the researches of Prof. *Planck* on thermodynamic radiation, the energy E of wave length λ is given by the rather complex expression

$$E_\lambda \, d\lambda = (k/RT\lambda)/(e^{k/RT\lambda} - 1) \cdot 8\pi RT\lambda^{-4} \, d\lambda \qquad (84)$$

which admits of integration within certain limits.

In this formula, R and T are the gas-constant and absolute temperature, $k = hV$, V being the velocity of light, and h is *Planck*'s new constant, $h = 6.55 \times 10^{-27}$ ergs secs, so that if the wave frequency be ν, $\lambda = V/\nu$ and

$$x = k/RT\lambda = h\nu/RT. \qquad (85)$$

And *Planck*'s fundamental equation for the quantum of energy of ν frequency is

$$\varepsilon = h\nu. \qquad (86)$$

By the use of *Planck*'s formula therefore

$$E_\lambda \, d\lambda = 8\pi RT\lambda^{-4} [x/(e^x - 1)] \, d\lambda. \qquad (87)$$

This integration, to take account of the various wave lengths, could be carried out, but the subject is in too primitive a condition to be undertaken at present.

9. A Definite Criterion for deciding between the Great and Small Densities claimed for the Aether.

In Section 1 of the first paper on the new theory of the aether, we have cited the claim put forward by certain electronists, that, on the hypothesis of incompressibility, the aether has a density 2000 million times that of lead. In his Aether of Space, 1909, p. 91–105, Sir *Oliver Lodge* finds from electrical theory that the density of the aether is 10^{12}, a million million times that of water.

It is only fair to point out that as the aether transmits waves, as in light, heat, magnetism, electrodynamic action, and radio telegraphy, of the most varied length, and of various amplitudes, it is not conceivable that it should be incompressible, so that the dilatation is zero in the equation:

$$\delta = \partial\alpha/\partial x + \partial\beta/\partial y + \partial\gamma/\partial z = 0 \qquad (88)$$

where α, β, γ, are the displacements, and

$$\partial^2\Phi/\partial t^2 = a^2 (\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2) = 0$$

by equation (63). For this would make the wave velocity infinite, which is contrary to observation. Accordingly, whilst the aether is highly incompressible, owing to the enormous velocity of the aetherons, and the resulting kinetic elasticity, this medium certainly is not incompressible.

In the article Aether, Encyclopedia Britannica, 11th ed., 1911, Prof. Sir *Joseph Larmor* is more poised and cautious than the writers previously cited, but his faith in the older theories is so shaken, that he intimates that the ratio of the amplitude of the waves to the wave length, taken by *Maxwell* and *Kelvin* at about 10^{-2}, may be enormously overestimated. *Larmor* adds: »It is not impossible that the coefficient of ultimate inertia of the aether is greater than the coefficient of inertia (of a different kind) of any existing substance«; which shows his tendency to an abandonment of the older theory, under the teachings of the electron theorists.

It thus appears that the excessively small density, found by *Kelvin* and *Maxwell*, namely, about 10^{-18}, or my own value at the earth's mean distance 438×10^{-18}, is opposed by the modern teaching in favor of an enormous density, about 10^{12}, as stated by Sir *Oliver Lodge*. The difference between the two results presents an enormous contrast, namely the almost unlimited factor:

$$F = 10^{30}, \text{ with the value of } Kelvin \text{ and } Maxwell; \quad = 0.0023 \times 10^{30}, \text{ with } See\text{'s value.} \quad (89)$$

Accordingly, progress is nearly impossible with this irreconcilable difference of opinion among the learned. *Brooks* and *Poyser*, as representatives of the opinion of the electronists, state: »There is no intrinsic difficulty in either view, but at present (1912) no method is known by which we may hope to discriminate between them.«

The present writer has therefore labored to develop a criterion for the rejection of one of these competing values, which would leave the other in possession of the field. Besides the above criticism, that the finite velocity of wave propagation excludes the incompressibility of the medium, I have given in the Observatory, Nov. 1918, p. 411–412, a brief discussion of the consequence of the intolerable disagreement in the values of the aether density.

A simple calculation has enabled me to exclude *Lodge*'s density as wholly inadmissible, because if true the energy of the waves from the sun falling upon a single square centimetre of the earth's surface would be able to vaporize the entire terrestrial globe in less than one minute of time, when we use *Bigelow*'s value of the constant of solar radiation, and *Kelvin* and *Maxwell*'s density.

The mass of the earth is 5956292000000000000000 metric tons. If we take the average specific heat of the globe at 0.2, and the vaporizing point of its average matter at 3000° C., the total amount of heat required to reduce it to vapour — the interior being assumed to be without heat — would be

$$H = 5.956292 \times 10^{21} (0.2 \times 3000) \times 1000000 \text{ calories}, \quad = 6 \times 5.956292 \times 10^{29} = 3.6 \times 10^{30}, \text{ nearly.} \quad (90)$$

Now *Bigelow*'s value of the solar constant is 3.98 cal. per minute, or 0.0663 cal. per second; and, as *Lodge*'s value of the density of the aether is about 10^{30} that above cited from *Kelvin* and *Maxwell*, and 0.0023×10^{30} times my own value, we have for the effect of such an increase in density the raising of the solar radiation by the factor 10^{30}:

$$H = 0.0663 \times 10^{30} = 6.63 \times 10^{28}, \ Kelvin \text{ and } Maxwell \quad \text{or } H = (0.0663 \times 0.0023) \times 10^{30}, \text{ with } See\text{'s value.} \quad (91)$$

The first of these values would vaporize the earth in 54 seconds of time, the second in 0.277 of a day. But in nature this vaporization does not occur, and thus we conclude that the density of the aether stands at a value near that fixed by *Kelvin* and *Maxwell* many years ago, but slightly improved in the writer's new theory of the aether.

In the Observatory, for Dec., 1918, p. 446, Sir *Oliver Lodge* has attempted to reply to my criticism by pointing out that the energy of the solar radiation depends on the amplitude of the wave, compared to the wave length, which with *Kelvin* and *Maxwell* I took at 10^{-2}, a value pronounced by Sir *Joseph Larmor* (in the article Aether, p. 292) »a very safe limit«. *Lodge* also adds: »many facts have suggested that the amplitude of the most brilliant light is exceedingly small compared with its wave length«.

Now if any good ground can be adduced for decreasing the ratio of the amplitude to the wave-length, I am willing to consider such a modification in the belief of the most eminent physicists, — such as *Kelvin, Maxwell, Larmor* — but it should be pointed out that to make the reconciliation of the extreme values complete, the ratio of the amplitude to the wave length will have to be lowered by the enormous factor

$$F = 10^{-30} \quad (92)$$

so that A/λ now taken at 10^{-2}, would become

$$A'/\lambda = 10^{-32}. \quad (93)$$

The difficulty of this extreme step is so great that I dismiss it as quite inadmissible. Until new evidence, resting on ground more secure than mere assumption, is available it must be held that Sir *Oliver Lodge*'s attempt to reply to this criticism completely breaks down. For even if we took

$$A/\lambda = 10^{-5}, \text{ or } A/\lambda = 10^{-6} \quad (94)$$

— which are values 1000 or 10000 times more extreme than appealed to the experienced judgements of Lord *Kelvin, Maxwell* and *Larmor*, — the required factor would scarcely be reduced in a sensible degree; and practical experience in physical science certainly would not justify us in exceeding the limit of 10^{-6}.

As a final argument against the electrical theory, assigning the aether a density of 2000 million times that of lead (namely: 11.352 × 2000000000 = 22704000000000 times that of water!), we may recall the familiar experience of a man swimming in water. Here the swimmer is immersed in an inert liquid of about the same density as his body; yet to move about a strong exertion is required of the most powerful muscles, completely under the control of the will.

If the liquid had the density of quicksilver, the swimmer would scarcely sink down to his boot-tops, and his muscles would be altogether too feeble to displace such an inert and heavy liquid, if he were required to move through it: yet he could walk over such a magma, by great effort, analogous to that required when we walk in very yielding volcanic ashes.

Now the density of mercury (13.6) is a little greater than that of lead (11.352), but the moment we consider an aether 2000000000 times denser than lead, we perceive the culmination of absurdity! Even if it penetrated all bodies quite perfectly, and gave equal pressure on all sides, still some displacement of the particles would be required when we move about in it, as in the case of water displaced by a swimmer. Obviously no living physical body would be capable of displacing such a dense medium; and we see that even the strongest stars, planets and comets would be dispersed to atoms under the changing resistance such a medium would interpose to their variously accelerated motions. The electrical theory assigning the aether a density 22704000000000 greater than that of water is therefore the best possible illustration of a physical Reductio ad Absurdum, and we know that either some premise or some link in the chain of reasoning eventually will not bear investigation [1]).

In the article Aether, Encyclopedia Britannica, 11th ed., 1911, Prof. Sir *Joseph Larmor* concludes that we must treat the aether as a plenum. Under the influence of electrical theory, he even speaks as if the aether were not molecular. In discussing the transparency of the celestial spaces, — to which much attention was given by *Cheseux* and *Olbers*, *W. Herschel* and *W. Struve* — (cf. Etudes d'Astron. stell., St. Pétersbourg, 1847) — *Larmor* first recalls the well known transparency of space shown by astronomical research, and then adds:

»If the aether were itself constituted of discrete molecules, on the model of material bodies, such transparency would not be conceivable. We must be content to treat the aether as a plenum, which places it in a class by itself; and we thus recognize that it may behave very differently from matter, though in some manner consistent with itself, — a remark which is fundamental in the modern theory.«

The first part of this reasoning apparently implies that the aether is not molecular, at least »on the model of material bodies«. This may be correct in part, because no one would suppose the aether to be made up of complex molecules, underlaid by a finer medium, such as the aether is to the more complex masses of common matter. On the other hand there is not the smallest objection to an aethereal medium made up of spherical perfectly elastic monatomic elements, so called aetherons, having a diameter of 1:4005th of a hydrogen molecule, and a mass of 15.56 millionths of a millionth of such a molecule, such as we show do really exist.

As no finer medium would underly such a monatomic aether, it could not dissipate the energy of wave motion, »on the model of material bodies«, and thus it would fulfill *Larmor's* condition of a plenum. This would give such an excessively fine monatomic molecular structure that the medium would penetrate all material bodies, but waves in such an aether would be very noticeably retarded in solid or liquid bodies, and much less so in gases, in accordance with physical experience.

That the aether must necessarily be molecular follows at once from our every day experience with such granular bodies as fine gravel, grains of corn, sand, shot or mustard seed. If we fill a glass vessel with such coarse granular masses, and insert the fingers or any solid body, such as a rod, into the granules, we perceive that they are thrust aside to make way for the hand or solid rod. If we fill the vessel with water, oil, alcohol, ether, or any similar liquid, our experience in such displacement is the same. The liquid is visibly thrust aside and this holds even when the molecular structure is relatively so fine that a drop of water might be magnified to the dimensions of the earth without exhibiting the molecules of larger size than footballs, — as shown by Lord *Kelvin* in his well known researches on the size of atoms.

But it will be said that the aether penetrates all bodies, and thus we cannot sensibly displace it, as we can water, oil, alcohol or ether. We reply that it is perfectly true that the aether penetrates freely all bodies, even the dense and highly elastic or rigid masses of the earth, sun and stars, almost as if their molecular structure were absent: yet we learn from the phenomena of refraction and diffraction in our laboratories, that light waves in the aether are very perceptibly retarded in their motions through transparent bodies; and in our investigation of celestial phenomena, we find from the investigation of the motion of the moon that the sun's gravitational waves, though of such length as to pass through the earth, are yet sensibly refracted, and perhaps dispersed or partially absorbed at the time of total eclipses of the moon, — whence arises the fluctuations of the moon's mean motion established by *Newcomb* in 1909, and explained by the present writer in 1916, (cf. Electrod. Wave-Theory of Phys. Forces, vol. 1).

From these considerations it appears that we have both terrestrial and celestial evidence that the aether is molecular, but of such excessively fine grained structure that no finer medium whatever underlies it: thus it penetrates all bodies freely, under an elastic power, or expansive tendency, 689321600000 times greater than our atmosphere exhibits in proportion to its density, as more fully shown in the first paper, sect. 4.

10. The Kinetic Theory of the Aether accords with the Views of *Newton*, 1721, and of *Maxwell*, 1877.

In order to further illuminate the above discussion we may recall the earlier though little known views of *Newton* and *Maxwell*, on the physical constitution of the aether.

a) Views of Sir *Isaac Newton*, Treatise on Optics, 3rd ed., 1721, p. 325 et seq. [2])

»Qu. 20. Doth not this Aethereal Medium in passing out of Water, Crystal, and other compact and dense Bodies, into empty Spaces, grow denser and denser by degrees, and by that means refract the Rays of Light not in a point, but by bending them gradually in curve lines? And doth not the gradual condensation of this Medium extend to some distance from the Bodies, and thereby cause the Inflexions of the Rays of Light, which pass by the edges of dense Bodies, at some distance from the Bodies?«

»Qu. 21. Is not this Medium much rarer within the dense Bodies of the Sun, Stars, Planets and Comets, than in the empty celestial Spaces between them? And in passing from them to great distances, doth it not grow denser and denser

[1]) In the Optics, 1721, pp. 342–3, *Newton* discusses the very problem here treated of in the following manner: »The resistance of water arises principally and almost entirely from the vis inertiae of its matter; and by consequence, if the heavens were as dense as water, they would not have much less resistance than water; if as dense as quick-silver, they would not have much less resistance than quick-silver; if absolutely dense, or full of matter without any vacuum, let the matter be never so subtile and fluid, they would have a greater resistance than quick-silver. A solid globe in such a medium would lose above half its motion in moving three times the length of its diameter, and a globe not solid (such as are the planets) would be retarded sooner. And therefore to make way for the regular and lasting motions of the planets and comets, it's necessary to empty the heavens of all matter, except perhaps some very thin vapours, steams or effluvia, arising from the atmospheres of the earth, planets and comets, and from such an exceedingly rare aethereal medium as we described above. A dense fluid can be of no use for explaining the phaenomena of nature, the motions of the planets and comets being better explain'd without it.«

[2]) Quoted at length, because this edition is very inaccessible to the modern reader.

perpetually, and thereby cause the gravity of those great Bodies toward one another, and of their parts towards the Bodies; every Body endeavouring to go from the denser parts of the Medium towards the rarer? For if this Medium be rarer within the Sun's Body than at its surface, and rarer there than at the hundredth part of an inch from its Body and rarer there than at the fiftieth part of an inch from its Body, and rarer there than at the Orb of Saturn; I see no reason why the increase of density should stop anywhere, and not rather be continued through all distances from the Sun to Saturn, and beyond. And though this Increase of density may at great distances be exceeding slow, yet if the elastick force of this medium be exceeding great, it may suffice to impel Bodies from the denser parts of the Medium towards the rarer, with all that power which we call Gravity. And that the elastick force of this Medium is exceeding great, may be gathered from the swiftness of its Vibrations. Sounds move about 1140 English feet in a Second Minute of Time, and in seven or eight Minutes of Time they move about one hundred English Miles. Light moves from the Sun to us in about seven or eight Minutes of Time, which distance is about 70000000 English Miles, supposing the horizontal Parallax of the Sun to be about 12″. And the Vibrations or Pulses of this Medium that they may cause the alternate Fits of easy Transmission and easy Reflexion, must be swifter than Light, and by consequence above 700000 times swifter than Sounds. And therefore the elastick force of this Medium, in proportion to its density, must be above 700000 times 700000 (that is above 490000000000) times greater than the elastic force of the Air in proportion to its density. For the Velocities of the Pulses of elastic Mediums are in a sub-duplicate Ratio of the Elasticities and the Rarities of the Mediums taken together.«

»As Attraction is stronger in small Magnets than in great ones in proportion to their bulk, and Gravity is greater in the surfaces of small Planets than in those of great ones in proportion to their bulk, and small Bodies are agitated much more by electric attraction than great ones; so the smallness of the Rays of Light may contribute very much to the power of the Agent by which they are refracted. And so if any one should suppose that Aether (like our Air) may contain Particles which endeavour to recede from one another (for I do not know what this Aether is) and that its Particles are exceedingly smaller than those of Air, or even than those of Light: The exceeding smallness of its Particles may contribute to the greatness of the force by which those Particles may recede from one another, and thereby make that Medium exceedingly more rare and elastick than Air, and by consequence exceedingly less able to resist the motions of Projectiles, and exceedingly more able to press upon gross Bodies, by endeavouring to expand itself.«

»Qu. 22. May not Planets and Comets, and all gross Bodies, perform their Motions more freely, and with less resistance in the Aethereal Medium than in any Fluid, which fills all Space adequately without leaving any Pores, and by consequence is much denser than Quick-silver or Gold? For instance; If this Aether (for so I will call it) should be supposed 700000 times more elastic than our Air, and 700000 times more rare; its resistance would be above 600000000 times less than that of Water. And so small a resistance would scarce make any sensible alteration in the Motions of the Planets in ten thousand years.«

In *Newton*'s views above quoted, Qu. 20, dating from 1721, it will be noticed that he not only held the aether to be a superfine gas, of enormous elasticity, but also calculated this elastic power to be $\varpi = 490000000000$ times greater than that of air in proportion to its density. By the most careful calculations that can be made today, we find this relative elastic power to be $\varpi = 689321600000$; which shows that the value found by *Newton* two centuries ago was 71 percent correct, — a wonderfully accurate result, even for so incomparable a geometer as *Newton*!

His remarks in Qu. 22 have been misconstructed by Sir *Oliver Lodge* (Introduction to his »Aether of Space«, 1909), in an effort to make it appear that *Newton* held the aether to have a large density, but the context shows the misconstruction involved in this claim. When *Newton* says that there is »less resistance (to the planets) in the aethereal medium than in any fluid which fills all space adequately without leaving any pores, and by consequence is much denser than quick-silver or gold?«, he means that the aether is very fine grained, more so than any material fluid like quick-silver or gold, which has pores. He thus held the aether to be so fine grained that it could truly act as a plenum, yet assigned this medium excessively small density. »May not its resistance be so small as to be inconsiderable? For instance: If this aether (for so I will call it) should be supposed 700000 times more elastic than our air, and above 700000 times more rare« — which shows clearly that *Newton*'s value of the density of the aether is:

$$\sigma = {}^1/_7\, 0.001293 \times 10^{-5} = 0.000000001849 \qquad (95)$$

that of water = 1, or $\sigma = 1/700000$, that of air = 1.

b) Views of *Maxwell*, 1877.

In the article Aether, Encyclopedia Britannica, 9th ed., p. 572, 1878, *Maxwell* speaks as follows regarding the molecular constitution of the aether: »Mr. *S. Tolver Preston* (Phil. Mag., Sept. and Nov., 1877) has supposed that the aether is like a gas whose molecules very rarely interfere with each other, so that their mean path is far greater than any planetary distances. He has not investigated the properties of such a medium with any degree of completeness, but it is easy to see that we might form a theory in which the molecules never interfere with each other's motion of translation, but travel in all directions with the velocity of light; and if we further suppose that vibrating bodies have the power of impressing on these molecules some vector property (such as rotation about an axis) which does not interfere with their motion of translation, and which is then carried along by the molecules, and if the alternation of the average value of this vector for all the molecules within an element of volume be the process which we call light, then the equations which express this average will be of the same form as that which expresses the displacement in the ordinary theory.«

Accordingly it will be seen that the present paper is a development of the reasoning sketched by *Newton*, 1721, and again briefly outlined by *Maxwell* in 1877.

The vector property, such as rotation about an axis, which *Maxwell* supposes might be impressed on the aether molecules, will be furnished by the wave motion in the aether, when the waves are taken to be flat in the planes of the equators of ordinary atoms. This is shown in the theory of magnetism outlined in the first paper, and will be treated of more fully in the third paper, in connection with a correction to the fundamental conceptions of the wave-theory of light.

11. Under the Kinetic Theory of the Aether *Michelson*'s celebrated Experiment of 1887 should yield a Negative Result. New Theory of Stellar Aberration based on the Motion of Light relatively to the moving Earth.

In the Philosophical Magazine for 1887, Prof. *Michelson* describes the famous experiment which he devised to detect the effect of a supposed aether drift past the earth, due to an assumed effect of the earth's orbital motion. In this experiment a beam of light, from a terrestrial source, is split into two parts, one of which is sent to and fro across the line of the supposed aether drift, while the other is sent along the line of the aether drift.

A semi-transparent mirror set at a 45° angle is employed to split the beam, and a pair of normal and ordinary mirrors set perpendicular to the two half beams, are employed to return the half beams whence they came, thus enabling them to enter the observer's eye through a telescope.

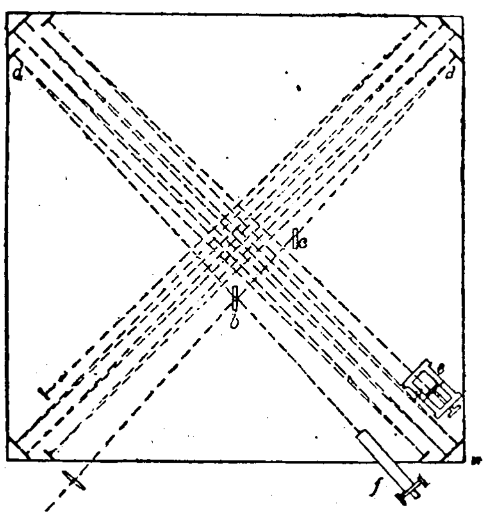

Fig. 10. Illustration of the paths of the split beam of light in *Michelson*'s experiment of 1887, one part traveling along the direction of the earth's orbital motion, the other at right angles thereto.

The apparatus was mounted on a stone support about 4 feet square, and one foot thick, and this stone in turn mounted on a circular disk of wood which floated in a tank of mercury. The resistance to rotation of the floating disk is very small, and a slight pressure on the circumference enables the observer to turn it around in say five minutes, with practically no oscillation.

The path of the light, from a terrestrial source, is thus made parallel and perpendicular to the direction of the earth's orbital motion; and the two half beams mutually interchanged for observation of the relative displacement of the interference fringes.

In his work on Light Waves and their Uses, 1903, p. 158, *Michelson* sums up his experience thus:

»It was found that there was no displacement of the interference fringes, so that the result of the experiment was negative and would, therefore, show that there still is a difficulty in the theory itself; and this difficulty, I may say, has not been satisfactorily explained«.

By the reasoning given below, in describing *Fitzgerald*'s hypothesis, sect. 12, it is shown that the effect sought is very small, depending on the square of $v/c = 1/10000$, the ratio of the velocity of the earth in its orbit to the velocity of light, and thus of the order of 1 : 100000000. But *Michelson* estimates that by his improved apparatus he could see fringe displacements of 1 part in 4000000000 if they existed; and thus the precision of the apparatus exceeded the magnitude of the fringe displacement sought by forty fold.

On repeated trial, under favorable conditions, everything behaved exactly as if the aether were stagnant. *Michelson* therefore suspected the difficulty to be in the theory itself; and we shall now examine into this question, to see if any ground for this impression can be found.

Owing to the translatory motion of the earth, we may change the fixed Newtonian coordinates to correspond to uniform motion in the direction of the x-axis:

$$x' = x - vt \quad y' = y \quad z' = z \quad t' = t. \tag{96}$$

At the initial epoch $t = 0$, we may equate these coordinates to zero, and our transformations, owing to the motion of the earth, become:

$$x' = a_1(x - vt) \quad y' = b_1 y_1 \quad z' = c_1 z_1. \tag{97}$$

Since the velocity of light is the same in reference to the fixed and moveable systems of coordinates, at the instant $t = t' = 0$, we get for identities of the spherical wave surfaces propagated from the moving source of light:

$$x^2+y^2+z^2 = c^2 t^2 \quad x'^2+y'^2+z'^2 = c^2 t'^2 \tag{98}$$

where c is the velocity of light.

Under the kinetic theory any heavenly body carries an electrodynamic wave-field about its centre of figure, in perfect kinetic equilibrium. The amplitude of the waves and therefore the density of the aether is arranged as shown in the accompanying diagram (p. 183), where the two stars may have the independent motions indicated by the vectors. The motion of either star automatically carries with it that star's own wavefield, and each field is independent of the other, just as the field of light waves emitted by any star is independent of that propagated from any other star. Hence owing to the earth's orbital motion we have the phenomenon of stellar aberration, as if the aether were really stagnant, because the wave-field has no motion relatively to the earth, though the earth itself moves, and thus generates the aberration, as follows:

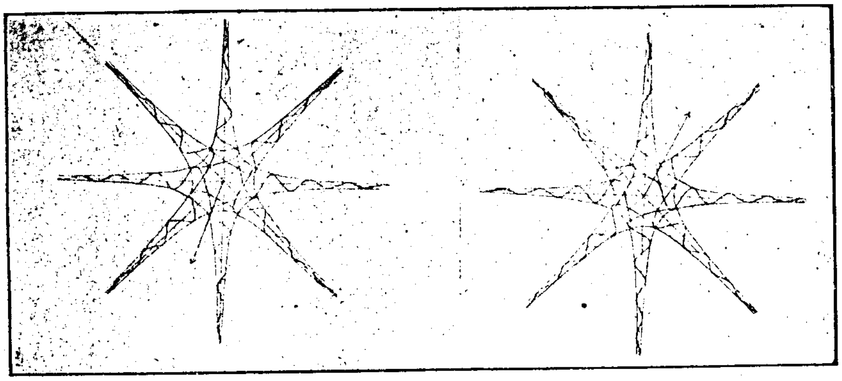

Fig. 11. Illustration of the proper motion of two stars which carry with them concentric wave-fields in perfect kinetic equilibrium, just as they carry their spheres of gravitational influence due to these waves. There is thus no such thing as a motion of the aether past the earth, in the sense imagined by *Young*, 1803, who compared the aether, supposed to be streaming through the earth, to the wind blowing through the tops of trees.

The light from a distant star travels independently of the motion of the earth and of its moving aether wave-field. Hence to take account of the earth's forward motion, in respect to space, we may imagine the parallel rays of light from the star to be given a backward motion *Sb* identical with the forward motion of the earth, *Ef*. This is the true motion of the light relatively to the moving earth, and by this simple device, stellar aberration is perfectly explained. The light actually comes from the direction *ES*, and a refractive medium in the path will have no effect whatever.

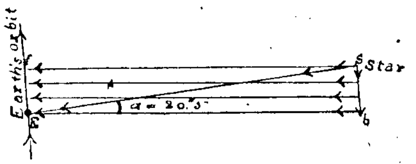

Fig. 12. A direct and simple explanation of the phenomenon of stellar aberration, based on the motion of light relatively to the moving earth.

The reasoning of *Klinkerfues*, about the refractive index of the medium in which the light penetrates, does not deal with the motion of the light relatively to the moving earth, and thus has no bearing on the subject. And likewise *Airy*'s observational experiment, with the zenith telescope tube 36 inches long, filled with water (Greenwich Observations, 1871, p. 1–16), is misapplied ingenuity[1]). The negative results obtained by these authorities is proof of the correctness of the simple view here set forth.

Accordingly, just as each star carries its own wave-field with it, so also, each particle of vibrating matter in the earth, sends out its system of spherical waves, and the whole wave-field in kinetic equilibrium, moves with the earth, and the gravitational potential depends on the integration of all these wavelets between the limits $-\infty$ to $+\infty$.

Thus the triple integral for the potential corresponds to a trebly infinite system of wavelets due to stresses decreasing with the distance, yet superposed at all points of space, but the potential for any body itself is finite, as in the theory of action at a distance.

$$v = \iiint \sigma / \sqrt{[(x-x')^2 + (y-y')^2 + (z-z')^2]} \times \times dx\, dy\, dz\,. \qquad (99)$$

Some of the individual wave surfaces from any one particle become:

$$\begin{aligned} x^2 + y^2 + z^2 &= c^2 t^2 \\ x'^2 + y'^2 + z'^2 &= c^2 t'^2 \\ x''^2 + y''^2 + z''^2 &= c^2 t''^2 \qquad (100) \\ \cdots\cdots\cdots & \cdots\cdots \\ x^{\nu 2} + y^{\nu 2} + z^{\nu 2} &= c^2 t^{\nu 2}\,. \end{aligned}$$

The individual wave surfaces have a common and parallel displacement in space, $v = ds/dt = 30$ kms, owing to the orbital motion of the earth.

Yet the stress of the aether, in kinetic equilibrium, is determined by the compounding of the effects of the waves emanating from the earth. This fixes the density and rigidity of the aether, which is arranged symmetrically about the vibrating particles of the globe. Accordingly, under the kinetic theory, the aether is stagnant in respect to the moving earth, precisely as found by *Michelson* in his celebrated experiment of 1887.

Hence no theory but the kinetic theory, with the particles moving 1.57 times faster than light, can be admitted. This follows at once from our investigation of the enormous elasticity of the aether, which gives the physical cause of the observed velocity of 300000 kms per second, for the wave motions constituting light and electricity.

Thus it only remains to state clearly the kinetic hypothesis underlying the wave-theory of physical forces, namely: We conceive all atoms of matter to receive and to emit waves, without regard to the motion of these atoms relatively to other atoms, just as we know the stars emit their typical spectral lines in spite of their proper motions in space.

Accordingly, as the Aether corpuscles have the enormous velocity of 471000 kms per second, this medium is taken to be in kinetic equilibrium about the moving earth, which will secure the law of density $\sigma = \nu r$, and of wave amplitude $A = k/r$. For the aether has an elasticity 689321600000 times greater than that of our air in proportion to its density, and if any lack of perfect kinetic equilibrium existed, it would disappear from the aethereal envelope of the earth

[1]) Though I have examined many authorities I can find no satisfactory explanation of the aberration. They are all confused by some such reasoning as the following, from *Michelson*'s Light Waves and their Uses, 1903, p. 151: »The objection to this explanation (*Bradley*'s) was, however, raised that if this angle (20″.5) were the ratio of the velocity of the earth in its orbit to the velocity of light, and if we filled a telescope with water, in which the velocity of light is known to be only three-fourths of what it is in air, it would take one and one-third times as long for the light to pass from the center of the objective to the cross-wires, and hence we ought to observe, not the actual angle of aberration, but one which should be one-third greater. The experiment was actually tried. A telescope was filled with water, and observations on various stars were continued throughout the greater part of the year, with the result that almost exactly the same value was found for the angle of aberration.«

in an infinitely small fraction of a second, owing to the mean velocity of the aetherons being 471000 kms per second.

12. Sir *Oliver Lodge*'s Experiments for detecting the Viscosity of the Aether, 1891–97, and *Fitzgerald*'s Hypothesis of a contraction of the dimensions of bodies in the direction of their motion.

In the Philosophical Transactions, 1893–97, Sir *Oliver Lodge* describes elaborate experiments with revolving steel disks, about a meter in diameter, which he had spun with the highest possible speed, in close proximity to a split beam of light, arranged as in *Michelson*'s experiment of 1887, in the hope of discovering a relative displacement of the fringes, due to viscosity of the aether. The experiment was well conceived, and executed with great skill, but it failed to give the smallest indication of a displacement such as viscosity of the aether would be supposed to yield. The results were entirely negative, and *Lodge*, like *Michelson*, could only conclude that the aether behaves as if it were absolutely stagnant.

Let us now consider why the negative results of *Michelson* and *Lodge* follow, if the aether be a kinetic medium such as *Newton*, *Maxwell* and Dr. *S. Tolver Preston* conceived it to be, and such as we have found it to be by exact calculations.

If the aether be corpuscular, the particles having a velocity 1.57 times that of light, it is obvious that it will adjust itself instantly to any state of steady motion, and that this kinetic equilibrium will be obtained more rapidly than even the propagation of light. And when Sir *Oliver Lodge*'s moving disk is revolving steadily, the aether will act as if it were absolutely stagnant.

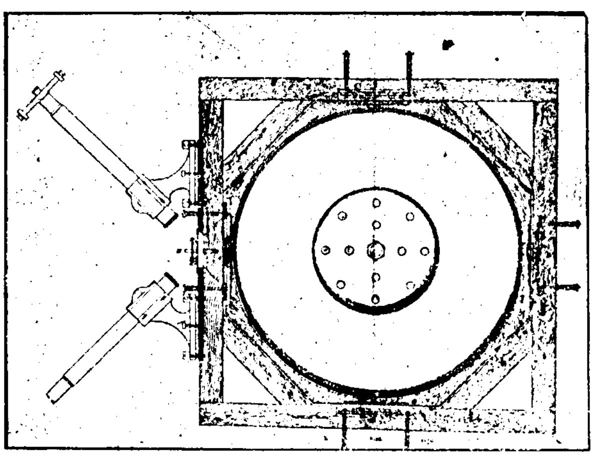

Fig. 13. Illustration of Sir *Oliver Lodge*'s apparatus for effecting a displacement of the aether, owing to viscosity, by the rapid rotation of disks of steel, near which a split beam of light is passed.

Hence the conclusion reached by Sir *Oliver Lodge* (Aether of Space, p. 82), as to the revolving disk experiments, was natural enough and quite justified in the premises, when he declared: »I do not believe the ether moves. It does not move at a five-hundredth part of the speed of the steel disks. Further experience confirms and strengthens this estimate, and my conclusion is that such things as circular-saws, flywheels, railway trains, and all ordinary masses of matter do not appreciably carry the ether with them. Their motion does not seem to disturb it in the least.«

»The presumption is that the same is true for the earth; but the earth is a big body — it is conceivable that so great a mass may be able to act when a small mass would fail. I would not like to be too sure about the earth — at least, not on a strictly experimental basis. What I do feel sure of is that if moving matter disturbs ether in its neighborhood at all, it does so by some minute action, comparable in amount perhaps to gravitation, and possibly by means of the same property as that to which gravitation is due — not by anything that can fairly be likened to etherial viscosity. So far as experiment has gone, our conclusion is that the viscosity or fluid friction of the ether is zero. And that is an entirely reasonable conclusion.«

In view of our theory of a kinetic medium, we may now go further than *Fresnel*, *Michelson* and Sir *Oliver Lodge*, and declare that as the corpuscular aether readjusts itself instantly to any state of steady motion, it follows that the motion of the earth can in no way disturb it. There is planetary induction indeed, from the wave-effect due to the relative motion of the sun and earth, but this is observable only by magnetic instruments, and not by means of other apparatus used in physical experiments.

If, as is definitely proved by calculation, the aether has an elasticity 689321600000 times greater than that of our air in proportion to its density, it is obvious that it not only penetrates all bodies, but even the electrodynamic waves in the aether may traverse the body of the terrestrial globe with only a small resistance, giving merely refraction, dispersion, and perhaps absorption of part of the energy, as we have shown in the theory of the lunar fluctuations (Electrod. Wave-Theory of Phys. Forc., vol. I, 1917). It not only follows that this adjustment of the aether to any state of steady motion will occur, but also that no power in the universe could prevent such a kinetic adjustment, in the all-pervading medium, under the above stupendous elastic power which it exerts against itself. It is thereby rendered almost incompressible, the waves traveling with a velocity of 300000 kms per second.

The physical meaning of such rapid propagation of waves is this: When a wave begins to be generated, the disturbance speeds away very rapidly, so that the movement is not cyclicly complete until a wave length λ has been traversed. As the amplitude a is very small, compared to λ, — as Lord *Kelvin*, *Maxwell* and *Larmor* have shown, — it follows that the aether is nearly incompressible, though the density at the sun's surface is only

$$\sigma = 2.0 \times 10^{-18}.$$

These last considerations also show why we cannot disturb the aether by revolving disk experiments. Accordingly it is not remarkable that Prof. *F. E. Nipher*, of St. Louis, has succeeded in disturbing the aether only by means of explosions of dynamite, an explosive of enormous power and excessively quick action. This not only shows the futility of viscosity experiments, with comparatively slow, steady motions, as when the revolving disks, a meter in diameter, make 66 rotations in a second [1]), but also confirms the

[1]) This is only 1 : 2356195 of the velocity of the aetheron, 471239000 m per second.

extremely rapid readjustment of the aether when disturbed. Therefore it follows that our theory of a kinetic medium, with the particles traveling 1.57 times faster than light, is in accordance with all the established facts of observation.

After giving a summary of all the known effects (Aether of Space, p. 62-63), *Lodge* concludes that the aether behaves under experiment as if it were stagnant with respect to the earth. »Well then, perhaps it is stagnant. The experiments I have quoted do not prove that it is so. They are equally consistent with its perfect freedom and with its absolute stagnation, though they are not consistent with any intermediate position. Certainly, if the aether were stagnant nothing could be simpler than their explanation.«

The new theory of the aether as a kinetic medium gives the stagnant quality sought by *Michelson* and *Lodge*, yet it preserves the »perfect freedom« with which the experiments are consistent.

Accordingly, the aether being a perfectly elastic corpuscular medium, always adjusting its internal stresses with at least the velocity of light — the individual particles having a velocity of 1.57 times greater yet, — it follows that around a body moving with uniform velocity there could be exerted no sustained forces, impressed or acting upon the atoms, to alter the linear dimensions of the uniformly moving body; and hence we reject *Fitzgerald*'s hypothesis as altogether misleading.

Fitzgerald's Hypothesis, that the linear dimensions of bodies are altered by motion relative to the aether, superfluous and misleading.

In Nature for June 16, 1892, Sir *Oliver Lodge* mentions a conversation with the late Prof. *Geo. F. Fitzgerald*, (cf. also *Lodge*'s Aether of Space, 1909, p. 68) to the effect that the dimensions of material bodies are slightly altered when they are in motion relative to the aether. The negative result of the *Michelson-Morley* experiment of 1887 was the occasion which called forth *Fitzgerald*'s hypothesis.

If V be the velocity of the earth's orbital motion, c the velocity of light, l the length of path traversed by the beam of light divided in *Michelson*'s experiment; then, one of the two portions of a split beam of light should make its journey in less time than the other by the interval $V^2 l/c^2$, if the aether itself be motionless, as *Michelson* supposed. This difference, however, would be compensated if the arm of the apparatus pointed in the direction of the earth's motion were shorter than the other by an amount $^1/_2 V^2 l/c^2$, which would follow if the linear dimensions of moving bodies are contracted in the direction of their motion in the ratio of $(1-{}^1/_2 V^2/c^2)$ to 1.

Now for the earth the ratio in question is:

$$V/c = 30\,\text{km/sec} : 300000\,\text{km/sec} = 1/10000 \qquad (101)$$

and the square

$$V^2/c^2 = 1/100000000 \qquad (102)$$

which shows that the alteration in dimensions — namely

$$\delta l = ({}^1/_2 V^2/c^2)\, l \qquad (103)$$

is only one two hundredth millionth. The minuteness of this hypothetical observed effect would make detection by experiment extremely difficult, even if a valid method could be devised. But let us consider, on other grounds, whether such an alteration in dimensions is consistent with sound physical laws.

By this hypothesis of *Fitzgerald*, the end-on-dimensions of a moving body is shortened

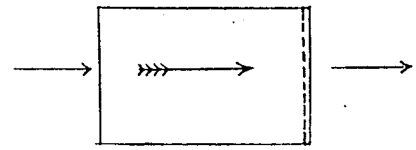

Fig. 14. Illustration of *Fitzgerald*'s hypothesis that the dimensions of a body, moving freely, uniformly, and without constraint, is decreased in the direction of the motion.

as shown in the figure. This hypothetical change is not postulated for the starting of a body in motion — where its figure might be changed in overcoming inertia, when the forward velocity is being developed — but for a body already in uniform rectilinear motion, and thus so far as is known subjected to no strain of its linear dimensions.

Newton's first law of motion (Principia, Lib. I, Axioms) is: »Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.«

»Projectiles persevere in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity. A top, whose parts by their cohesion are perpetually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the planets and comets, meeting with less resistance in more free spaces, preserve their motions both progressive and circular for a much longer time.«

If these axioms were obvious to Sir *Isaac Newton*, it will no doubt be equally obvious to us that a body may have its dimensions altered in acquiring a velocity, — as when a ball is struck by a bat — yet the elasticity of the body will immediately assert itself, so that the figure will oscillate about its mean or undisturbed form; and after a certain time the original figure will become restored. And thereafter there will be no permanent change of figure. This is a fact of universal experience, and may be verified experimentally in our laboratories by all manner of actual measurements.

The most careful physical experiments show that bodies placed under constraint, tend very rapidly to restore their figures of equilibrium. Accordingly it follows that bodies having uniform motion of long duration in any direction, could not undergo changes of figure, in virtue of uniform motion, without physical constraint, which in turn would call forth the power of restitution, at the instant of release. Hence in uniform unrestrained motion no alteration in the figure of equilibrium appropriate to a state of rest would be possible, and *Fitzgerald*'s contraction hypothesis is contrary to the order of Nature.

In concluding this second paper, it is scarcely necessary to point out that prior to the development of the kinetic theory of the aether, experiments like those made by *Michelson* and *Morley* and Sir *Oliver Lodge* led to the idea of a stagnant aether. There are indeed profound reasons why the aether should act as if it were absolutely stagnant, whereas the particles really move 1.57 times faster than light, and thus the medium instantly adjusts itself to any state of steady motion, whatever it may be; because the motion of the aetheron is 10000 fold faster than that of our swiftest planets, and over two millions of times faster than any steady artificial motions which we can make experimentally, as in the researches of Sir *Oliver Lodge* with rapidly revolving disks of steel.

On the old hypotheses the *Michelson-Morley* experiment of 1887 was admirably adapted to detect the effect of the earth's motion through the aether. Little did these eminent experimenters dream that the earth carried its wave-field of aether with it, — all infinitely extended and adjusted in perfect kinetic equilibrium. This wave-field has decreased density towards the centre, in virtue of the increased amplitudes of the waves emanating from the atoms, and thus is truly stagnant about the moving earth in respect of waves of light from distant stars, in the phenomenon of aberration.

Accordingly, whether the components of the split beam of light, from a terrestrial source, as used by *Michelson*, travel in the direction of the earth's orbital motion, or at right angles thereto, no shift of the fringes is theoretically possible, because of the perfect kinetic equilibrium of the wave-field of the aether about the earth and extending away from it indefinitely.

For similar reasons *Fitzgerald*'s hypothesis rests on a false premise, and only beclouds the reasoning in this difficult subject. The fundamental condition required for real progress is a valid kinetic theory of the aether, such as *Newton* first outlined two hundred years ago, and *Maxwell* approved in 1877, but left very incomplete, owing to the premature death of this great mathematician.

Since the difficulties connected with the motion of the perihelion of Mercury and of the lunar perigee, as well as the lunar fluctuations, which *Newcomb* pronounced the most enigmatical phenomena presented by the celestial motions, are fully overcome, without any mystical doctrine such as *Einstein* introduces, it is evident that the whole theory of relativity, as heretofore developed, is shaken to its foundations, and will no longer deserve the serious consideration of natural philosophers.

For several years experienced investigators in all parts of the world have wondered at the strange sight presented by British men of science in unjustifiably abandoning the established natural philosophy of *Newton*, and hastily embracing the untenable speculations of *Einstein* when the facts of observation themselves are insecurely established.

And as for the overdrawn statement of Prof. Sir *J. J. Thomson*, President of the Royal Society, that the supposed larger value of the solar deflection of light indicated by the eclipse observations of May 29, 1919, »is the most important result obtained in connection with the theory of gravitation since *Newton*'s day, and it is fitting that it should be announced at a meeting of the society so closely connected with him«, it suffices to call attention to the unfortunate impression thus conveyed to investigators, who remember on the one hand the historical fact that the Royal Society in 1686 refused to publish[1]) *Newton*'s Principia, and thus it had to be issued at the private expense of Dr. *Edmund Halley* (cf. *Brewster*'s Life of *Newton*, 2 vols., 1855), and on the other hand the vast development and perfection of the theory of gravitation since made by *Euler, Clairault, Lagrange, Laplace, Poisson, Bessel, Gauss, Hansen, Leverrier, Airy, Delaunay, Adams, Tisserand, Gyldén, Hill, Newcomb, Poincaré, Darwin,* and several eminent geometers still living.

In contradistinction to the singular spectacle thus presented in the Royal Society, it is a relief to find a much more cautious attitude in the Monthly Notices for Nov., 1919, p. 23, where Prof. *Newall* gives good reasons for rejecting *Einstein*'s theory of the deflection of light in the sun's field, in favor of optical refraction.

In the Nineteenth Century Magazine, for Dec., 1919, Sir *Oliver Lodge* likewise is skeptical; for he reasons that if we accept *Einstein*'s theory in its entirety, »the death knell of the aether will seem to have been sounded, strangely efficient properties will be attributed to emptiness, and theories of light and of gravitation will have come into being unintelligible on ordinary dynamical principles«. Such protests would indicate that the Newtonian philosophy still has some supporters in England, but apparently they are not aware of the real strength of their cause, as now brought to light in the New Theory of the Aether.

Accordingly, in view of the comprehensive results already reached in the New Theory of the Aether, the defenders of the Newtonian mechanics could hardly wish for a more complete triumph. And it is gratifying to realize that it is based upon the original conceptions of Sir *Isaac Newton* himself, after the simple and elegant theory of this great philosopher had been almost completely abandoned by his countrymen.

I am indebted to my young friend Mr. *E. L. Middleton*, for valuable assistance in the completion of this investigation.

Starlight on Loutre, Montgomery City, Missouri, 1920 Febr. 19. *T. J. J. See.*

[1]) The well known delay of 14 years (1807–1821) in the publication of *Fourier*'s mathematical researches on the theory of heat seems to place the Paris Academy of Sciences in an equally unfortunate light. In the Éloge Historique of *Fourier* delivered by *Arago*, blame is placed on the commissioners of the Academy — *Lagrange, Laplace* and *Legendre* — for poisoning the pleasure of *Fourier*'s triumph, which Lord *Kelvin* has also criticized. As no commissioners could be more competent than the three geometers just cited, history often is witness to the weaknesses of the highest academies of sciences; and hence, in his very original Researches in the Lunar Theory, 1877, Dr. *G. W. Hill* had recourse to private publication, which probably was better than the fate accorded to *Newton* and *Fourier*.