Table of *Gauss*' theory as compared with observations, 1838.

Nr.	Place of Observation	Latitude	East Longitude	Total Magnetic Intensity			Decl.	Incl. or Dip
				I_o	I_c	I_o-I_c	$\delta_o-\delta_c$	$\theta_o-\theta_c$
1	Spitzbergen	+79° 50′	11° 40′	1.562	1.599	−0.037	−1°.32	−0°.83
2	Hammerfest	+70 40	23 46	1.506	1.545	−0.039	−1.55	−0.07
3	Jakutsk	+62 1	129 45	1.697	1.661	+0.036	+5.75	−0.30
4	Porotowsk	+62 1	131 50	1.721	1.658	+0.063	+4.70	−0.45
5	Nochinsk	+61 57	134 57	1.713	1.653	+0.060	+2.23	−0.58
6	Tschernoljes	+61 31	136 23	1.700	1.648	+0.052	+3.50	−0.66
7	Petersburg	+59 56	30 19	1.410	1.469	−0.059	−0.05	+0.63
8	Christiania	+59 54	10 44	1.419	1.456	−0.037	−0.08	+0.05
9	Ochotsk	+59 21	143 11	1.615	1.621	−0.006	+2.60	−0.92
10	Tobolsk	+58 11	68 16	1.557	1.575	−0.018	−3.17	+0.80
11	Tigil River	+58 1	158 15	1.577	1.583	−0.006	+0.23	−1.45
12	Sitka	+57 3	224 35	1.731	1.697	+0.034	+0.43	−0.65
13	Tara	+56 54	74 4	1.575	1.586	−0.011	−1.87	+0.70
14	Catherinenburg	+56 51	68 34	1.523	1.535	−0.012	−0.97	+0.87
15	Tomsk	+56 30	85 9	1.619	1.613	+0.006	−1.22	+0.37
16	Nishny Nowgorod	+56 19	43 57	1.442	1.469	−0.027	−1.62	+1.53
17	Krasnojarsk	+56 1	92 57	1.657	1.638	+0.019	−0.85	+0.60
18	Kasan	+55 48	49 7	1.433	1.477	−0.044	−1.25	+1.20
19	Moscow	+55 46	37 37	1.404	1.446	−0.042	−1.40	+2.20
20	Königsberg	+54 43	20 30	1.365	1.410	−0.045	−0.88	+2.12
21	Barnaul	+53 20	83 56	1.605	1.591	+0.014	−0.42	+0.33
22	Uststretensk	+53 20	121 51	1.656	1.609	+0.047	+2.87	−0.35
23	Gorbizkoi	+53 6	119 9	1.660	1.611	+0.049	+1.82	−0.17
24	Petropaulowsk	+53 0	158 40	1.489	1.521	−0.032	−0.53	−1.68
25	Uriupina	+52 47	120 4	1.667	1.612	+0.055	+2.80	−0.40
26	Berlin	+52 30	13 24	1.367	1.391	−0.024	−1.43	+1.37
27	Pogromnoi	+52 30	111 3	1.640	1.616	+0.024	+0.93	−0.28
28	Irkuzk	+52 17	104 47	1.647	1.616	+0.031	+0.82	−0.05
29	Stretensk	+52 15	117 40	1.649	1.606	+0.043	+1.97	−0.28
30	Stepnoi	+52 10	106 21	1.663	1.615	+0.048	+0.73	−0.03
31	Tschitanskoi	+52 1	113 27	1.668	1.609	+0.059	+1.22	−0.23
32	Nertschinsk City	+51 56	116 31	1.635	1.604	+0.031	+2.18	−0.53
33	Werchneudinsk	+51 50	107 46	1.657	1.612	+0.045	+1.03	+0.18
34	Orenburg	+51 45	55 6	1.432	1.461	−0.029	−0.57	+1.50
35	Argunskoi	+51 23	119 56	1.655	1.595	+0.060	+2.37	−0.27
36	Göttingen	+51 32	9 56	1.357	1.388	−0.031	−1.83	+1.22
37	London	+51 31	359 50	1.372	1.410	−0.038	−1.62	+0.38
38	Nertschinsk Bergw.	+51 19	119 37	1.617	1.593	+0.024	+2.77	−0.43
39	Tschindant	+50 34	115 32	1.650	1.592	+0.058	+1.66	−0.05
40	Charazaiska	+50 29	104 44	1.643	1.599	+0.044	−0.30	+0.18
41	Zuruchaitu	+50 23	119 3	1.626	1.584	+0.042	+1.88	+0.02
42	Troizkosawsk	+50 21	106 45	1.642	1.597	+0.045	+1.37	−0.32
43	Abagaitujewskoi	+49 35	117 50	1.583	1.577	+0.006	+1.77	−0.75
44	Altanskoi	+49 28	111 30	1.619	1.585	+0.034	−1.07	−0.43
45	Mendschinskoi	+49 26	108 55	1.630	1.587	+0.043	+1.13	−0.28
46	Paris	+48 52	2 21	1.348	1.389	−0.041	−2.03	+0.65
47	Chunzal	+48 13	106 27	1.612	1.574	+0.038	+0.40	−0.22
48	Urga	+47 55	106 42	1.583	1.571	+0.012	+0.17	−0.35
49	Astrachan	+46 20	48 0	1.334	1.358	−0.024	−0.47	+2.98
50	Chologur	+46 0	110 34	1.580	1.545	+0.035	+1.15	−0.62
51	Ergi	+45 32	111 25	1.559	1.539	+0.020	+1.22	−0.60
52	Milan	+45 28	9 9	1.294	1.331	−0.037	−2.38	+1.58
53	Sendschi	+44 45	110 26	1.530	1.529	+0.001	+0.83	−0.55
54	Batchay	+44 21	112 55	1.553	1.520	+0.033	+0.72	−0.47
55	Scharabudurguna	+43 13	114 6	1.538	1.502	+0.036	+0.23	−0.48

Nr.	Place of Observation	Latitude	East Longitude	Total Magnetic Intensity I_o	I_c	$I_o - I_c$	Decl. $\delta_o - \delta_c$	Incl. or Dip $\theta_o - \theta_c$
56	Naples	+40° 52′	14° 6′	1.271	1.271	0.000	−3°.55	+2°.45
57	Chalgan	+40 49	114 58	1.459	1.465	−0.006	+0.52	−0.57
58	Pekin	+39 54	116 26	1.453	1.448	+0.005	+0.83	−0.90
59	Terceira	+38 39	332 47	1.457	1.469	−0.012	−0.98	−0.47
60	San Francisco	+37 49	237 35	1.591	1.592	−0.001	+1.45	−1.60
61	Port Praya	+14 54	336 30	1.156	1.168	−0.012	+0.22	+0.20
62	Madras	+13 4	80 17	1.031	1.038	−0.007	—	+2.63
63	Galapagos Islands	− 0 50	270 23	1.069	1.085	−0.016	−0.55	−3.92
64	Ascension	− 7 56	345 36	0.873	0.813	+0.060	−1.12	−3.88
65	Pernambuco	− 8 4	325 9	0.914	0.909	+0.005	−0.07	+0.18
66	Bahia	−12 59	321 30	0.871	0.883	−0.012	+1.10	+1.42
67	St. Helena	−15 55	354 17	0.836	0.811	+0.025	−1.45	−3.15
68	Otaheite	−17 29	210 30	1.094	1.113	−0.019	−2.15	−3.00
69	Mauritius	−20 9	57 31	1.144	1.060	+0.084	+0.15	+0.12
70	Rio de Janeiro	−22 55	316 51	0.878	0.879	−0.001	−0.95	+1.32
71	Valparaiso	−33 2	288 19	1.176	1.094	+0.082	−1.55	−1.18
72	Sydney	−33 51	151 17	1.685	1.667	+0.018	−2.55	−4.63
73	Cape of Good Hope	−34 11	18 26	1.014	0.981	+0.033	+1.10	−1.52
74	Monte Video	−34 53	303 47	1.060	1.022	+0.038	−0.62	−0.10
75	K. George's Sound	−35 2	117 56	1.709	1.658	+0.051	+0.40	−2.03
76	New Zealand	−35 16	174 0	1.591	1.616	−0.025	−2.83	−4.77
77	Concepcion	−36 42	286 50	1.218	1.147	+0.071	−2.08	−1.40
78	Blanco Bay	−38 57	298 1	1.113	1.103	+0.010	−2.05	+0.12
79	Valdivia	−39 53	286 31	1.238	1.145	+0.093	−1.28	−0.57
80	Chiloe	−41 51	286 4	1.313	1.227	+0.086	−1.06	−1.20
81	Hobarttown	−42 53	147 24	1.817	1.894	−0.077	−5.25	−3.63
82	Port Low	−43 48	285 58	1.326	1.257	+0.069	−2.27	−1.27
83	Port Desire	−47 45	294 5	1.359	1.263	+0.096	−3.33	−1.35
84	R. Santa Cruz	−50 7	291 36	1.425	1.321	+0.104	−2.52	−1.45
85	Falkland Islands	−51 32	301 53	1.367	1.276	+0.091	−3.73	−0.65

8. The Mutual Potential Energy and the Mutual Action of Two Magnetic Systems: How the Result is modified by the Magnetic Law of Stress exerted in Curved Lines drawn to the Poles.

(i) The mutual potential energy of two magnetic systems.

1. Imagine a magnet of infinitely small dimensions, like the particles into which a magnet may be broken without losing its physical properties. Suppose the length to be dl, with direction cosines α, β, γ; then if Ω_1 and Ω_2 be the values of the potential Ω at the negative and positive poles respectively, we shall have for the potential energy of this small magnet:

$$d\mu\, dl \cdot d\Omega/dl = m \cdot \partial\Omega/\partial l \qquad (157)$$
$$m = \text{moment of the magnet.}$$

On putting $dv = dx\,dy\,dz$ for the magnetized element we get:

$$m \cdot \partial\Omega/\partial l = I(\alpha \cdot \partial\Omega/\partial x + \beta \cdot \partial\Omega/\partial y + \gamma \cdot \partial\Omega/\partial z)\, dx\, dy\, dz \quad (158)$$

where $I =$ total intensity along the line of magnetization.

By triple integration for all the elements of the system, in a field of potential Ω, on putting $I\alpha = A$, $I\beta = B$, $I\gamma = C$, we get:

$$W = \iiint^{\mu} I(\alpha \cdot \partial/\partial x + \beta \cdot \partial/\partial y + \gamma \cdot \partial/\partial z)(1/D)\, dx\, dy\, dz$$
$$= \iiint^{\mu} (A \cdot \partial\Omega/\partial x + B \cdot \partial\Omega/\partial y + C \cdot \partial\Omega/\partial z)\, dx\, dy\, dz \qquad (159)$$
$$D = [(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2]^{1/2}$$

the distance D corresponding to the point $P(\xi, \eta, \zeta)$.

Here the integration is supposed to be extended throughout the magnetized masses acted upon. Accordingly, if I', α', β', γ', x', y', z' or A', B', C' refer to the acting system, we get for the potential Ω:

$$\Omega = \iiint^{\mu} I'(\alpha' \cdot \partial/\partial x' + \beta' \cdot \partial/\partial y' + \gamma' \cdot \partial/\partial z')(1/D)\, dx'\, dy'\, dz' \qquad (160)$$

where

$$D = [(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2]^{1/2} \qquad (161)$$

as in (159), and thus D is the distance of the point $P(\xi, \eta, \zeta)$.

At any point $P(\xi, \eta, \zeta)$ an infinitely small magnet $A\, dx\, dy\, dz$ situated at $p(x, y, z)$, having its axis parallel to the axis of x, has the component of the potential $A(\xi - x)/D^3$; and as the other components are symmetrical, we get for the potential of the whole magnet, by the integration of the sum of these components, an expression equivalent to (160):

$$\Omega = \iiint^{\mu} \frac{\{A(\xi - x) + B(\eta - y) + C(\zeta - z)\}}{\{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2\}^{3/2}}\, dx\, dy\, dz\,. \qquad (162)$$

Now from (159) and (160) we get for the mutual potential energy of the two magnetic systems:

$$W=\int\int\int^{\mu}\int\int\int^{\mu'}\frac{(\alpha\cdot\partial/\partial x+\beta\cdot\partial/\partial y+\gamma\cdot\partial/\partial z)\,(\alpha'\cdot\partial/\partial x'+\beta'\cdot\partial/\partial y'+\gamma'\cdot\partial/\partial z')\,I\,I'}{[(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2]^{1/2}}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'\,. \qquad (163)$$

The functions A, B, C vanish in free space, where there is no magnetized matter, and therefore we may use these functions in the sextuple integral with the understanding that they are zero everywhere where space is devoid of matter. The limits thus become infinite, and we get:

$$W=\int\int\int_{-\infty}^{+\infty}\int\int\int^{\mu,\mu'}\frac{(A\cdot\partial\Omega/\partial x+B\cdot\partial\Omega/\partial y+C\cdot\partial\Omega/\partial z)\,(A'\cdot\partial\Omega'/\partial x'+B'\cdot\partial\Omega'/\partial y'+C'\cdot\partial\Omega'/\partial z')}{[(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2]^{1/2}}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'\,. \qquad (164)$$

Integrating by parts, and noticing that the surface integral at infinity vanishes, we get simply:

$$W=\int\int\int_{-\infty}^{+\infty\ \mu,\mu'}(A\cdot\partial\Omega/\partial x+B\cdot\partial\Omega/\partial y+C\cdot\partial\Omega/\partial z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,. \qquad (165)$$

$$=-\int\int\int_{-\infty}^{+\infty\ \mu,\mu'}\Omega\,(\partial A/\partial x+\partial B/\partial y+\partial C/\partial z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,. \qquad (166)$$

Here it is assumed that A, B, C vary continuously, but as these functions undergo at the surface such rapid variation as to amount to discontinuity; and thus a finite portion of the integral will arise from an infinitely thin stratum near the surface, as in the *Poisson* formula for surface density μ_0 and volume dentity μ (Mémoires de l'Institut, tome V, 1821):

$$W=\int\int^{\mu_0}\Omega\,\mu_0\,\mathrm{d}S+\int\int\int^{\mu}\Omega\,\mu\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,. \qquad (167)$$

Let therefore Ω' be the potential of the magnet acted upon; then, taking account of the discontinuous change at the surface 4π about the system, we get

$$\partial^2\Omega'/\partial x'^2+\partial^2\Omega'/\partial y'^2+\partial^2\Omega'/\partial z'^2=4\pi\,(\partial A/\partial x+\partial B/\partial y+\partial C/\partial z)\,. \qquad (168)$$

And on multiplying by $\Omega\,\mathrm{d}v=\Omega\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ and integrating, we have by (166):

$$W=-1/4\pi\cdot\int\int\int_{-\infty}^{+\infty\ \mu,\mu'}\Omega\,(\partial^2\Omega'/\partial x^2+\partial^2\Omega'/\partial y^2+\partial^2\Omega'/\partial z^2)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \qquad (169)$$

$$=1/4\pi\cdot\int\int\int_{-\infty}^{+\infty\ \mu,\mu'}(\partial\Omega/\partial x\cdot\partial\Omega'/\partial x+\partial\Omega/\partial y\cdot\partial\Omega'/\partial y+\partial\Omega/\partial z\cdot\partial\Omega'/\partial z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \qquad (170)$$

$$=+1/4\pi\cdot\int\int\int_{-\infty}^{+\infty\ \mu,\mu'}R\,R'\cos\chi\cdot\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \qquad (171)$$

(cf. Sir *W. Thomson*, Reprint of Papers on Electricity and Magnetism, p. 433).

In this last formula R and R' are the resultant forces at any point of space due to the acting and acted upon systems respectively, and χ is the angle between the directions of these resultant forces.

Having now derived very general expressions for the mutual potential and mutual action of two magnets, or magnetic systems, of any form or distribution in space, we are obliged to consider carefully how far the underlying hypotheses conform to the true laws of nature. It has been assumed by *Gauss, Airy, Thomson, Maxwell, Crystall* and other recent investigators:

(I.) The attraction or repulsion according as the poles are unlike or like, between two quantities μ and μ' of magnetism, supposed concentrated in two points at distance r apart is represented by the force:

$$f=\pm\mu\mu'/r^2\,. \qquad (172)$$

(II.) This force in the mutual action of two elements of magnetism is taken to be in the straight line r, joining the elements μ and μ'.

This hypothesis of **rectilinear action** almost presumes that magnetism is similar to gravitation, whereas it is shown in the present paper that magnetic stress is exerted in the curved lines of force typical of magnetism. It is necessary therefore to examine the foundation of the classic theory of magnetism with particular care.

(III.) The classic theory supposes that the unit quantity of magnetism is so chosen that two units of positive magnetism at unit distance apart repel each other with unit force. This definition, underlying the magnetic system of units, gives for the dimensions of a quantity of magnetism, in the magnetic system of units:

$$\mu=L^{3/2}M^{1/2}T^{-1}\,. \qquad (173)$$

(IV.) The strength of a magnetic field, or the resultant magnetic force at a point in the field, is defined to be the force exerted upon a unit of positive magnetism supposed concentrated at the point. So that in general, if R be the resultant magnetic force at the point, the magnetic force exerted on a quantity μ of magnetism concentrated there becomes μR, just as in the case of gravity, where the force is mg.

(ii) How the action is modified by the law of magnetic stress acting tangentially in the curved lines of force directed to the two poles.

We have seen abundant proof that the law:

$$I/g=\eta^2\,(r^2/s^2+r^2/s'^2) \qquad s=\int_0^p\mathrm{d}s \qquad s'=\int_0^{p'}\mathrm{d}s' \qquad (174)$$

represents the connection between magnetism and gravitation.

In every possible situation on the earth's surface we found that in general we have to consider the modification of theory due to the increase in the path of action from

$$r \text{ to } r+\mathrm{d}r\,.$$

To make the reasoning general, and avoid confusion, we shall put: $s=\varrho+\Delta\varrho=\varrho\,(1+\Delta\varrho/\varrho)$ (175)

where ϱ is the rectilinear path chosen by *Gauss* in his Allgemeine Theorie des Erdmagnetismus, 1838. This path ϱ within the earth may be increased by as much as 40 per cent at the equator, so that $\Delta\varrho$ is finite, not infinitely small.

Now expressions for magnetism involve the inverse second power, and therefore we are concerned with:

$$s^2 = \varrho^2 + 2\varrho\,\Delta\varrho + (\Delta\varrho)^2$$

where $\varrho^2 = r^2 - 2rr_0\,[\cos u \cos u_0 + {}$ (176)
$+ \sin u \sin u_0 \cos(\lambda - \lambda_0) + r_0{}^2]$.

Any expression for the force used by *Gauss*, as

$$\Omega = -\int 1/\varrho \cdot d\mu$$
$$= -\int \{r^2 - 2rr_0\,[\cos u \cos u_0 + \sin u \sin u_0 \cos(\lambda - \lambda_0)] + {}$$
$$+ r_0{}^2\}^{-1/2} \cdot d\mu \qquad (177)$$

will therefore be much more complex.

We may put the above expansion in the following form:

$$(\varrho + \Delta\varrho)^2 = \varrho^2 (1 + \Delta\varrho/\varrho)^2 = \varrho^2 [1 + 2\Delta\varrho/\varrho + (\Delta\varrho)^2/\varrho^2]. \qquad (178)$$

And then we have

$$\mu/[\varrho(1 + \Delta\varrho/\varrho)]^2 = (\mu/\varrho^2)[1 + \Delta\varrho/\varrho]^{-2} \qquad (179)$$
$$= (\mu/\varrho^2)\{1 - 2\Delta\varrho/\varrho + 3(\Delta\varrho/\varrho)^2 + {}$$
$$- 4(\Delta\varrho/\varrho)^3 + 5(\Delta\varrho/\varrho)^4 - \cdots\}. \qquad (180)$$

This series converges with sufficient rapidity, except when $\Delta\varrho$ is very large, as in the region of the terrestrial equator. It is evident that beyond the first term, the sum of the terms

$$S = \sum_{\substack{\nu=1 \\ n=-2}}^{\substack{n=n \\ \nu=\nu}} \frac{n(n-1)(n-2)\cdots(n-\nu)}{1\cdot 2\cdot 3\cdot 4\cdots \nu} \cdot \left(\frac{\Delta\varrho}{\varrho}\right)^{\nu} \qquad (181)$$

is negative, because the first term of (179) is smaller than μ/ϱ^2, under the integral (177) used by *Gauss*.

Accordingly as

$$\mu/[\varrho^2(1 + \Delta\varrho/\varrho)^2] < \mu/\varrho^2; \text{ and therefore}$$
$$\int 1/\varrho^2 \cdot d\mu > \int 1/[\varrho^2(1 + \Delta\varrho/\varrho)^2] \cdot d\mu \qquad (182)$$

we may form a table for the earth, using the value $s = \varrho(1 + \Delta\varrho/\varrho)$ in the different latitudes. Thus the table will give the means of integration by quadrature for the average value in the case of our globe. In this way we may find out how much $s > \varrho$, as used by *Gauss*.

Approximate Table for the Increase of the Amount of Magnetism in the Globe, under Curved Line Action compared to the Straight Line Action assumed by *Gauss*:

i = Number of Zone in Quadrant of Globe from the Pole	u_i = Angular Distance of Limit of Zone	s/ϱ = $(1+\Delta\varrho/\varrho)$	s^2/ϱ^2 = $(1+\Delta\varrho/\varrho)^2$	$s^2/\varrho^2 \cdot 2\pi(\cos u_i - \cos u_{i+1})$ = Integral for the Zones of the Sphere by the ratio of the Forces s^2/ϱ^2 acting on them
0	0	1.000	1.000	. . . . .
1	10	1.028	1.056	0.1008
2	20	1.042	1.085	0.3074
3	30	1.050	1.102	0.5103
4	40	1.060	1.123	0.7056
5	50	1.075	1.155	0.8940
6	60	1.096	1.201	1.0801
7	70	1.120	1.254	1.2449
8	80	1.300	1.690	1.7882
9	90	1.414	2.000	2.1815
Mean Value =		1.120	1.266	$\Sigma = 8.8129$

Mean for equal elements of the Solid Angle $\omega = \Sigma/2\pi = 1.4026$.

From the table it appears:

1. The mean ratio of the mere increase of distance s/ϱ is 1.12, while the mean ratio of the squared distances s^2/ϱ^2 required in the divisor of the integral for the forces is 1.266. This means that zone for zone of the conical space about the pole there would be an increase in the calculated amount of the magnetism, — observation, under *Gauss*' defective hypothesis of rectilinear action, yielding only $1/1.266 = 0.80$, nearly. Thus an increase of 25 percent is required, to overcome the effect of the defect in the underlying hypothesis.

2. But instead of taking zone for zone, we should integrate the zones of the sphere by the ratio of the forces s^2/ϱ^2 acting on them, as given in the last column of the above table, which yields a mean for equal elements of all the solid angles, $\omega = 1.4026$. That is, solid angle for solid angle, throughout the hemisphere, the effect of the curved line action rises to about 1.4; so that $1/1.4 = 0.71$ is the calculated part of the magnetism existing in nature, the increase required for defect in hypothesis being about 40 percent, $29/71 = 0.40$.

3. Although the above table is approximate only, — including merely the effect of the changes in s, for one hemisphere, the changes in s' for the other hemisphere being left out of account, — it is evident that the final result will not very greatly exceed the ratio here estimated. It will not, I feel sure, exceed 50 percent; yet this change shows how profoundly all our conceptions of magnetism must be modified, to take account of nature's actions in curved lines compared to the straight lines heretofore employed by physical theorists.

4. The above conclusion that the ratio of increase will not exceed 50 percent rests on a study of the nature of the integrals:

$$I = f + f' = \mu' \iiint^{\mu} \sigma/s^2 \cdot dx\,dy\,dz + \mu' \iiint^{\mu} \sigma/s'^2 \cdot dx\,dy\,dz =$$
$$= \mu\mu'/s^2 + \mu\mu'/s'^2$$

the two terms f and f' being equal at the equator, but each of them falling off rapidly and soon becoming relatively small terms in the opposite hemisphere.

The results of our calculations are given in the accompanying table. It will be seen that the general effect of using s in place of ϱ is to weaken the attractive force everywhere. And as *Gauss*' results follow from the constant use of a path of action which is too short, yet the observed forces were the forces he actually used, we must conclude that the amount of magnetism in the earth very appreciably exceeds that calculated by *Gauss*.

After some examination of this problem, we conclude from the study of the earth's magnetism, that for such a spherical magnet as the earth, the average increase of $s = \varrho(1 + \Delta\varrho/\varrho)$, the second term being the excess of s over rectilinear action, is about:

$$s = \varrho(1 + 0.12/\varrho) \qquad (183)$$

And therefore in the integral

$$\int 1/s \cdot d\mu = \int 1/[\varrho^2(1 + \Delta\varrho/\varrho)^2] \cdot d\mu \qquad (184)$$

the evaluated sum is decreased in the ratio of 1.00 to 1.26. Consequently we find that the amount of magnetism in the globe is about 26 percent greater than observations indicate in virtue of curved line action, zone for zone; and, solid angle for solid angle, at least 40 percent, perhaps 50 percent.

The general theory of curved line action here developed is entirely new in science. It has not been used in any previous work. Thus in *Crystall*'s great article on Magnetism, Encycl. Brit., 9th ed., p. 227, it is stated that the force between two quantities of magnetism is $f = \mu\mu'/r^2$, and »is in the line joining the two points«.

The whole theory of magnetism heretofore in use is based on the theory of rectilinear action, as in *Gauss*' Allgemeine Theorie des Erdmagnetismus, § 3, p. 7, where we read:

»Zur Abmessung der magnetischen Flüssigkeiten legen wir, wie in der Schrift Intensitas vis magneticae etc. diejenige Quantität nördlichen Fluidums als positive Einheit zu Grunde, welche auf eine ebenso große Quantität desselben Fluidums in der zur Einheit angenommenen Entfernung eine bewegende Kraft ausübt, die der zur Einheit angenommenen gleich ist. Wenn wir von der magnetischen Kraft, welche in irgend einem Punkte des Raumes, als Wirkung von anderswo befindlichem magnetischem Fluidum, schlechthin sprechen, so ist darunter immer die bewegende Kraft verstanden, welche daselbst auf die Einheit des positiven magnetischen Fluidums ausgeübt wird. In diesem Sinne übt folglich die in einem Punkt konzentriert gedachte magnetische Flüssigkeit μ in der Entfernung ϱ die magnetische Kraft μ/ϱ^2 aus, und zwar abstoßend oder anziehend in der Richtung der geraden Linie ϱ, je nachdem μ positiv oder negativ ist. Bezeichnet man durch a, b, c die Koordinaten von μ in Beziehung auf drei unter rechten Winkeln einander schneidende Achsen; durch x, y, z die Koordinaten des Punktes, wo die Kraft ausgeübt wird, sodaß

$$\varrho = [(x-a)^2+(y-b)^2+(z-c)^2]^{1/2}$$

und zerlegt die Kraft den Koordinatenachsen parallel, so sind die Komponenten

$$\mu(x-a)/\varrho^3, \quad \mu(y-b)/\varrho^3, \quad \mu(z-c)/\varrho^3,$$

welche, wie man leicht sieht, den partiellen Differentialquotienten von $-\mu/\varrho$ nach x, y und z gleich sind.«

We have cited the foundations of *Gauss*' theory in some detail, not only to show that he takes the force $f = \mu/\varrho^2$ and reckons ϱ rectilinear, »in der Richtung der geraden Linie ϱ«, positive or negative according to the sign of μ; but also to exhibit the *Gauss*ian resolution of the component forces, which is of exactly the same form as in the case of gravitation.

Before coming definitely to the decision that a fundamental error of the kind here described has come down in the classical theory of magnetism, as developed by *Gauss* and other high authorities, I took pains to experiment very carefully as follows:

(a) When iron filings were sprinkled on a plate of glass over the poles of a powerful small magnet, they showed unmistakably the tendency to move towards the poles along the curved lines of force, not straight to the poles.

(b) When soft iron paper fasteners were attached to threads for exploring the field, they showed the same motion — the pulling being in the tangent to the line of force.

(c) Yet not content with such indications, I went to the trouble to test the field carefully, again and again, when the small compass needle was tied to a silk thread for exploring the magnetic field. Upon actual trial it was found that the deflection of the needle against gravity, predicted by the formula for the ponderomotive force:

$$F = mg \operatorname{tg}\chi = \mu\mu'/s^2 - \mu\mu'/s'^2, \text{ (northern hemisphere)} \quad (185)$$

where m = gravitational mass of the smaller magnet, was due visibly and undeniably to a force along the tangent to the line of force. By moving the suspended needle from one position to another, where the lines of force curve rapidly, it could be seen distinctly that the deflection is always in the tangent to the magnetic line of force. The observation is not difficult. The evidence is perfectly unmistakable!

Conclusions.

1. From the experiments here described it follows incontestably that the laws of magnetic attraction and repulsion are not quite so simple as we have supposed. Instead of an attraction or repulsion along the straight line r, we must imagine these stresses exerted along the shortest lines of force

$$s = \varrho + \Delta\varrho = \int_0^p ds, \quad s' = \int_0^{p'} ds'.$$

2. The aether stress along the line of force of minimum length is the maximum tension for the attracted magnetic needle. The resultant is directed to the nearer pole, around the curve, but it changes direction at the equator, and in the other hemisphere is therefore directed to the other pole.

3. The lines of force, as *Faraday* noticed, tend to shorten themselves, and thus are under tension. The tension is greatest nearest the poles, which act as true centres of attraction, yet the stress always acts in curved lines, because there is another pole in the distance to which the other end of the line of force returns.

4. Since the lines of force are vortical filaments of aether in rapid rotation, owing to the wave-action constituting magnetism, — the resultant rotation at any point being in the plane normal to the line of force, — we perceive that the tension in the line of force will always pull the suspended magnetic needle towards the nearer pole of the larger magnet. The chief forces are centred in the opposite ends of the suspended needle, the opposite poles of the needle being pulled to opposite poles of the large magnet.

5. At the equator the action on the two ends of the suspended needle are equal: in other positions the forces are unequal, with the increased attraction towards the nearer pole predominant.

a) There is always a slight bodily deflection of the suspended needle towards the larger magnet — the vortical filament of the aether along the line of force acting like a stretched rope.

b) But near either pole, the stress along the line of force is so much more predominant, that we notice chiefly the attraction towards the pole.

6. As observation confirms the theory of tension along the magnetic lines of force, we perceive that these observations, showing stresses as described above, also confirm and definitely establish wave-action as the cause of magnetism. This explanation is not only sufficient; it is also necessary — the only possible one! Hence in finding the law of magnetic action in curved lines of force, we have discovered the true cause of magnetism!

9. The Feebleness of Gravitation Compared to Electric and Magnetic Forces: Velocity of the Propagation of Universal Gravitation.

(i) Estimates of the feebleness of gravitation.

The force of universal gravitation is so nearly insensible, for small masses, as to lead to the belief that this chief force of nature is a residual effect, in which only a small component of the elastic power of the aether is exerted. By virtue of the stupendous masses of the heavenly bodies, however, this residual component attains gigantic magnitude between the planets and the sun, or between the members of a pair of double stars, as they revolve in their orbits, — the actual stress becoming a maximum and thus often enormous at periastron passage.

a) Thus in the Connexion of the Physical Sciences, 9th ed., 1858, p. 426, Mrs. *Sommerville* remarks:

»Gravitation is a feeble force, vastly inferior to electric action, chemical affinity and cohesion; yet, as far as human knowledge extends, the intensity of gravitation has never varied within the limits of the solar system.«

The latter part of this argument is modified by observations and experiments made within the past twenty years; but the remark on the feebleness of gravitation is eminently appropriate, and today more noteworthy than in former times, when gravitation was less investigated by natural philosophers.

b) In the article Gravitation, Encycl. Brit., 9th ed., 1875, *Ball* points out that on the average the attraction of a magnet is millions of times more powerful than gravitational attraction. This general remark is of such great interest that we propose to test it by actual calculation, as follows. So far as I know this calculation is new, at least I have never met with it in the writings of any modern investigator.

c) It appears by the U. S. Coast Survey observations at the Magnetic Observatory, Cheltenham, Maryland, that in 1906, the horizontal component of the earth's magnetism was $\gamma = 0.00020$ c. g. s., while the inclination or dip was $\theta = 70° 27'$. Now we may take gravity as 981 cm c. g. s., and therefore the horizontal component of the earth's magnetism is to that of gravitational acceleration as:

$$\gamma/g = 2/9810000 = 1/4905000\,. \qquad (186)$$

But $\cos 70° 27' = 0.3347 = 1/3$ very nearly, and thus we have for the ratio of the total intensity at Cheltenham to gravity, 1906, the equation:

$$I/g = \gamma \sec\theta./g = 1/1635000\,. \qquad (187)$$

This is the same result at which we arrived in equation (46) above.

d) It is shown in *Gauss'* Allgemeine Theorie des Erdmagnetismus, 1838, that at the two poles the average intensity of the earth's magnetism is 1.977, or nearly 2, while at the magnetic equator it reduces to 1 approximately. We therefore have the general theorem, that for the region between the magnetic equator and the magnetic poles the intensity of the earth's magnetism lies between the limits given in equation (115):

$$I/g = 1/2000000, \text{ and } I/g = 1/1000000\,. \qquad (188)$$

Accordingly the above value at Cheltenham is typical of the larger part of the globe. The above remarks of Mrs. *Sommerville* and Prof. *Ball* are therefore very appropriate.

e) In 1894, Prof. *C. V. Boys* determined the constant of gravitation or the attraction of a mass of a gram at a distance of a centimetre in a second,

$$\Gamma = 0.0000000666 = 10^{-10} \cdot 666 \text{ c. g. s.} \qquad (189)$$
$$= 1/(1501 \cdot 10^4) \text{ dyne, or practically one-fifteen}$$

millionth of a dyne.

As the dyne is roughly the weight of a milligram, it was remarked by Prof. *G. M. Minchin* of Oxford (Treatise on Statics, 1886, vol. II, p. 251), »how extremely small a magnitude is the constant of gravitation«. This opinion of the feebleness of gravitation has been generally held by investigators since the days of *Newton*; in fact the force was so very small that it was a long time before experimental measurements of the deflections due to gravitation became possible.

In 1774 *Maskelyne* succeeded in detecting the deflection of the vertical by the attraction of Mt. Shehallien in Scotland, from which he deduced a mean density of the earth of 4.71. In 1798 *Cavendish* first used the method of the Torsion Balance, and obtained the value 5.48, which is very near the modern value, 5.50.

In view of the extreme feebleness of gravitation, the accurate determination of the gravitation constant is one of the most difficult experiments in the whole range of physical science.

(ii) Revision of *Maxwell*'s calculation of the stress in the aether incident to gravitation.

In his researches on the stresses in the aether required to produce electric, magnetic or gravitational forces, *Maxwell* derives the following formula (Treatise on Electricity and Magnetism, 1873, sect. 643; *Minchin*'s Treatise on Statics, vol. II, 1886, p. 451):

$$B = R^2/8\pi\Gamma\,. \qquad (190)$$

where Γ is the gravitation constant, $\Gamma = 666 \cdot 10^{-10}$ c. g. s., as above explained, and $R = 981$ cm, is the acceleration of gravity at the earth's surface.

The calculation is as follows:

$\log R = 981 =$	2.9916690	$\log 8$	$= 0.9030900$
$\log(R^2)$	$= 5.9833380$	$\log\pi$	$= 0.4971499$
$\log 10^{10}$	$= 10.0000000$	$\log 666$	$= 2.8234742$
$\log(10^{10}\cdot R^2)$	$= 15.9833380$	$\log(8\pi\, 666)$	$= 4.2237141$
$\log(8\pi\, 666)$	$= 4.2237141$		

$\log(10^{10}\cdot R^2/8\pi\, 666) = 11.7596239$, c. g. s.
$\log(981000) \quad = 5.9916690$
$\log B \quad = 5.7679549$ kg per sq cm
$B \quad = 586077.3$ kg per sq cm. $\qquad$ (191)

In *Maxwell*'s calculation of this stress an error of a decimal place occurs, as is easily shown. He gives the stress as 37000 tons weight per square inch, namely:

$B' = 37000 \cdot 2240 = 82880000$ pounds per sq inch.
$\log(82880000) = 7.9184497$
$\log(6.4516 \cdot 2.2046213) = 1.1530014$
$\log B' = 6.7654483$

$$B' = 5827040 \text{ kg per sq cm.} \qquad (192)$$

But above we found $B = 586077.3$ kg per sq cm, which is almost exactly one tenth of *Maxwell*'s value.

If we recall that *Maxwell* used a slightly different value of Γ from that cited above, it seems certain that he misplaced the decimal point in his reduction.

In pounds per square inch this may be made even more obvious, thus:

$$\begin{aligned} B &= 8335974, \ (\textit{See}) \\ B' &= 82880000, \ (\textit{Maxwell}) \end{aligned} \tag{193}$$

which exhibits very distinctly the misplacement of the decimal point in *Maxwell*'s calculation.

(iii) Table of stresses in the aether at the surfaces of the sun and planets, and at the orbits of the eight principal planets of the solar system.

The following table contains data of much interest, as revealing to us the actual state of the aether at critical points of the solar system.

It will be seen that the aether stress is very great near the surfaces of the larger planets, and especially near the solar surface. This stress varies as the square of the force of gravity, and thus augments rapidly near a large dense mass, where the acceleration of gravity becomes very large. Such a result is in no way remarkable, but on the contrary, to be expected by any one familiar with the wave-process for generating the physical forces pervading nature.

It would be easy to calculate the magnetic stresses by the corresponding formula:

$$B' = R'^2/8\pi\Gamma$$

where R' is the magnetic acceleration expressed in units of some known gravitational acceleration. Thus to compare the stress due to the magnetic field of the earth with gravity at the earth's surface we use the ratio above found, for such a typical station as Cheltenham, namely:

$$R' = I/g = 1/1635000.$$

Table of the aether stresses for the sun and planets.

Name	i	g_i (in cm)	$B_i = g_i^2/8\pi\Gamma$ = Stress (in kg per sq cm)	S_i Stress in units of the tensile strength of steel, 30 tons to the sq in. = 4.7246 metric tons per sq cm	At the Planetary Orbits g_i	Stress = $B_i = g_i^2/8\pi\Gamma$ (in kg per sq cm)
		cm			cm	
Sun	0	27301.6	453934400.0	96085.0	—	—
Mercury	1	188.0	21524.5	4.55	3.947	9.4821
Venus	2	875.37	466660.0	98.77	1.13035	0.77811
The Earth	3	981.0	586077.3	124.05	0.59141	0.21301
Mars	4	377.14	86621.0	18.33	0.25474	0.039519
Jupiter	5	2621.7	4185843.0	885.96	0.021848	0.00029070
Saturn	6	1144.2	797298.0	168.75	0.006499650	0.000025728
Uranus	7	1304.0	822568.3	174.10	0.0016057	0.0000015703
Neptune	8	1464.6	1306336.0	276.50	0.00065404	0.00000026051

And as R'^2 occurs in the above equation, we find that the gravitational stress at the earth's surface, namely 586077 kilograms per square centimetre, must be divided by the enormous number $(1635000)^2 = 2673225000000$, yielding only the utterly insensible stress of 0.00000021924 kilograms per square centimetre. This stress is in the direction of the total magnetic intensity. And as the dip is largely vertical in the chief places of the northern hemisphere, it is not remarkable that the earth's northward pull on a suspended magnetic needle is practically insensible, as found by observers since the days of *Norman*, 1576.

The experimental detection of any modification of weight by magnetization, or even a bodily deflection of a suspended magnetic needle from the vertical, is therefore very difficult in the magnetic field of the earth, where the stresses are so feeble compared to the enormous stress due to gravitation.

(iv) Definite proof that gravitation is propagated with the velocity of light deduced from the connection established between magnetism and gravitation.

1. We have now shown that so far as the magnetism of our globe is regular a definite connection exists between the mean total force of terrestrial magnetism in any latitude and the accelerative force of gravitation at the earth's surface, through the equation:

$$I/g = \eta^2(r^2/s^2 + r^2/s'^2) = (1/1408.12)^2(r^2/s^2 + r^2/s'^2) \tag{14}$$

where r = radius of the earth, and s is expressed in the same unit, the integration for the path $s = \int_o^p ds$ being taken along the line of the magnetic force from the place of observation o to the pole p, and $s' = \int_o^{p'} ds'$ to the other pole p'.

2. It follows from this line of argument:

a) That artificial magnets are produced by the concerted wave-action of electric currents, in coils of wire such as *Ampère* first wound about the bars being magnetized — the atoms having their equators lined up parallel to the equator, or plane perpendicular to the bar's axis at its centre.

b) The magnetic force is due to stress in the aether, under vortical rotation of the aetherons about the lines of force, as the magnetic waves travel outward with the velocity of light. This magnetic force being due to a state of stress in the aether is propagated across space with the same velocity as the electrodynamic waves generating the magnet, which has been found by many careful experimenters to be identical with the speed of light, — 300000 km per second.

3. Now if a real connection exists between terrestrial magnetism and gravitation at the surface at the earth, as shown in this paper, — 1/1408 part of the atoms being

lined up in parallel planes, and giving the attraction directed to the two poles; the rest, 1407/1408, of them lying haphazard, with their planes tilted at all possible angles, and giving only a mean stress towards the centre of the whole mass — then, it will follow that gravitation necessarily is transmitted with the velocity of light. For this velocity certainly is true of the waves generating magnetism, and as magnetism is definitely connected with gravitation, this latter force must of necessity be transmitted across space with the same velocity, namely, $V = 3 \cdot 10^{10}$ cm.

4. The investigation of the connection between the magnetism of the earth and terrestrial gravity now shown to exist, is therefore of the deepest interest, because it furnishes a definite proof that the universal gravitation, which governs the motions of the planets in their orbits, is transmitted with the velocity of light. This is another proof that gravitation is due to wave-action in the aether, because the velocity of transmission deduced from the connection with magnetism corresponds to such waves.

In the Baltimore Lectures, 1904, Appendix F, (reprinted from the Proc. Roy. Soc., vol. 8, June, 1856; Phil. Mag., March, 1857) Lord *Kelvin* treats of *Faraday*'s discovery, 1845, of the rotation of a beam of polarized light when it is passed through heavy glass, carbon disulphide, etc. along the path of a magnetic line of force. This discovery is fundamental in magnetism, and by all competent authorities ranked among the most wonderful of *Faraday*'s discoveries.

Yet although over three fourths of a century have now elapsed since *Faraday*'s discovery, the only investigators who have studied it from the dynamical point of view are *Kelvin*, 1856; *Maxwell*, 1873; and the present writer, 1917. The work of *Kelvin* and *Maxwell* was incomplete, yet extremely suggestive. In fact, since their investigations led to the explanation of the cause of magnetism outlined by me in 1917, and that cause is now definitely demonstrated, it is well to recall the reasoning of both *Kelvin* and *Maxwell*.

Kelvin inferred that »the magnetic influence on light discovered by *Faraday* depends on the direction of motion of moving particles«, and that »*Faraday*'s optical discovery affords a demonstration of the reality of *Ampère*'s explanation of the ultimate nature of magnetism« — corresponding to the wave-theory.

After discussing *Rankine*'s hypothesis of molecular vortices, which he had himself developed at length, *Kelvin* finally concludes: »I think we have good evidence for the opinion that some phenomenon of rotation is going on in the magnetic field, that this rotation is performed by a great number of very small portions of matter, each rotating on its own axis, this axis being parallel to the direction of the magnetic force, and that the rotations of these different vortices are made to depend on one another by means of some kind of mechanism connecting them.« This is *Kelvin*'s early description of what is now the wave-theory.

10. The Degree of Accuracy of the Law of the Inverse Squares for Gravitation and for Magnetism respectively.

(i) The accuracy of the law of the inverse squares for gravitation.

In AN 5048, p. 144–154, we have examined the degree of rigor which may be assigned the *Newton*ian law of the inverse squares, and found that whilst the law certainly is very accurate, yet the exponent is not accurate beyond the ten-millionth, or seventh decimal place.

(a) In the Astronomical Journal, vol. 14, 1894, p. 49, Prof. *A. Hall* considered the admissible change of exponent to account for the motion of Mercury's perihelion. After weighing the evidence carefully *Hall* adopted the modified law:

$$f = mm'/r^{2.00000016}\,. \tag{194}$$

This places the uncertainty in the seventh decimal place of the exponent; and it is impossible for us to deny the admissibility of such a change. For the whole matter was subsequently reviewed by *Newcomb*, (Astronomical Constants, 1895, p. 118) who adopted the same form of law, but carried the development to higher decimal places:

$$f = mm'/r^{2.0000001574}\,. \tag{195}$$

In my own examination of this question, AN 5048, p. 148, I find that when account is taken of *Weber*'s law, with the small terms resulting from the propagation in time, the most probable exponent does not exceed the following:

$$f = mm'/r^{2.0000001046}\,. \tag{196}$$

Since the author prepared the second paper on the New Theory of the Aether, AN 5048, Dr. *Grossmann* of Munich, has very carefully tested the outstanding difference of 43″ in the motion of Mercury's perihelion found by *Newcomb*, 1881, and finds, (AN 5115) that the true outstanding difference very probably lies between 29″ and 38″, but in no case will attain 43″ per century, as so long assumed.

Now as the smaller terms in *Weber*'s law amount to $+14.''5$ for the motion of Mercury, it follows that with *Grossmann*'s results the difference to be accounted for would be between $+14.''5$ and $+23.''5$ per century. This leads to residuals smaller than was used to get my exponent 2.0000001046, which was $[d\varpi]_{00} = +28.''44$ (AN 5048, p. 148). And hence with *Grossmann*'s outstanding motion, and *Weber*'s law (this latter is necessary in any case), the exponent would be less than 2.0000001046, — the axact amount depending on the adopted centennial difference $[d\varpi]_{00}$ taken by me at $+28.''44$.

Making fair allowances for the obvious uncertainty in this outstanding difference, I think it certain that the ten-millionth place of the exponent probably is accurate, namely:

$$f = mm'/r^{2+\nu}, \qquad \nu = 0.0000001 \tag{197}$$

but the hundred-millionth place is in doubt by at least 3 units, so that:

$$\nu = 0.00000010 \pm 0.00000003\,. \tag{198}$$

The uncertainty in this exponent therefore is of the order of three one-hundred-millionths, and it is not easy to see how we can reach a conclusion authorizing a smaller value of this uncertainty.

Now in his address to the British Association in Australia, 1914, p. 316, Prof. *E. W. Brown*, of Yale University, estimates that the exponent in the law of gravitation does not differ from 2 by a fraction greater than 1 : 400000000 = 0.0000000025 — which is at least ten times smaller than the uncertainty indicated above. *Brown*'s premises are open to grave objections, in that he assumed the *Newton*ian law to be quite rigorous, and adopted an oblateness of the earth of about 1 : 294, which is not admissible, (AN 5048, pp. 149-150), because in AN 5103-5104, I have shown that the most probable oblateness of the earth is 1 : 298.3.

The estimated higher accuracy of the law of attraction adopted by *Brown* therefore is not justified by the existing state of our knowledge. In fact the fluctuations of the moon continue to be so troublesome that calculations on the motion of the perigee do not give as exact a criterion for the exponent in the law of attraction as we formerly believed.

All we can do at the present time is to say that the exponent does not differ from 2 by more than about one-twenty-millionth of the whole, or 0.0000001, while the uncertainty is about three in the hundred millionth place, or ± 0.00000003.

It follows from this line of reasoning that the law of gravitation is established with very great accuracy. It is by far the most exact of all the laws of nature, because astronomical observations extend over long ages, and the precision of the observations is very high for about two centuries. In the case of eclipses of the sun and moon the records are fairly complete for 3000 years, and thus they serve to check astronomical calculations back to the time of the Babylonians.

(ii) The accuracy of the law of the inverse squares for magnetism.

In the year 1914, I made a careful estimate of the accuracy assignable to the law of gravitational attraction in *Newton*'s time, and found that it was of the order of one unit in the ten-thousandth place of the exponent, thus, $n = 2.0001 \pm 0.0001$.

It follows from the formulae for the calculation of the motion of the perihelion (cf. *Tisserand*'s Mécanique Céleste, tome 1, p. 50), that the exponent 2.001 would give a displacement of a planet's perihelion amounting to $\delta\varpi = 648''$ in a single revolution. In the same way the exponent 2.0001 would yield a displacement of 64″8, a little over 1′, which is about as high an order of accuracy as was attainable in *Newton*'s time.

If now we turn to the law of magnetic attraction, we find a similar history of progress. In the time of *Newton* the law of magnetic attraction had not been determined, nor even surmised with any degree of probability. Accordingly, *John Michell* of Cambridge, England, in 1750, first showed roughly that the law of the inverse squares probably holds for magnetic attraction. This law was much more rigorously established by the French physicist *Coulomb*, in 1785, by means of delicate experiments with the torsion balance; and it has since been generally received as a true law of nature, so that we have $f = \mu\mu'/r^2$.

The general theory of magnetic attraction was subsequently improved by *Hansteen*, of Copenhagen (Magnetismus der Erde, 1819). Yet it is to *Gauss*, above all others, that we owe the real test of the law of magnetic attraction; for this great mathematician made a successful effort to fix the law of nature with a degree of rigor comparable to that of universal gravitation in the time of *Newton*.

Gauss carried out a series of end-on observations with bar magnets in which the density of the magnetism for either half of the bar was assumed to have the form:

$$f = \lambda x^n. \tag{199}$$

Gauss took the force due to an element of positive magnetism $d\mu$, at the distance D, to have the general form, (Intensitas Vis Magneticae, etc., 1833, § 21):

$$f = d\mu/D^n \tag{200}$$

where n may be any number whatever.

As the result of a long series of delicate experiments, on a magnetized needle, by means of fixed magnets about a foot long and weighing about one pound, he found that while various values of n may be almost equally well adopted for special cases; yet when the distance between the two magnets is sufficiently great, compared to the linear dimensions of either, — ultimately taken as more than four times as great, in the experiments finally devised by *Gauss*, — the best results indicated that the true value is $n = 2$.

If the deflexions in the end-on experiments were denoted by Φ, and in the broadside-on experiments by Φ', the positions of the deflecting magnet are given by the general expressions:

$$\begin{aligned} \operatorname{tg}\Phi &= L_1 r^{-(n+1)} + L_2 r^{-(n+2)} + \dots + L_i r^{-(n+i)} \\ \operatorname{tg}\Phi' &= L'_1 r^{-(n+1)} + L'_2 r^{-(n+2)} + \dots + L'_i r^{-(n+i)} \end{aligned} \tag{201}$$

where $L_1/L'_1 = n$.

His observations, however, required but few terms of the series, and were satisfied by the approximate formulae:

$$\begin{aligned} \operatorname{tg}\Phi &= 0.086870\, r^{-3} - 0.002185\, r^{-5} \\ \operatorname{tg}\Phi' &= 0.043435\, r^{-3} + 0.002449\, r^{-5}. \end{aligned} \tag{202}$$

If r be the distance between the centres of the magnets, measured in metres, and Φ_o, Φ'_o, and Φ_c, Φ'_c be the observed and calculated values, respectively, the experiments of *Gauss*, lead to the following table of results:

r	Φ_o	$\Phi_o - \Phi_c$	Φ'_o	$\Phi'_o - \Phi'_c$
1.1 m			1° 57′ 24″8	+2″8
1.2			1 29 40.5	−6.0
1.3	2° 13′ 51″2	+ 0″8	1 10 19.3	+6.0
1.4	1 47 28.6	+ 4.5	0 55 58.9	+0.2
1.5	1 27 19.1	− 9.6	0 45 14.3	−6.6
1.6	1 12 7.6	− 3.3	0 37 12.2	−3.2
1.7	1 0 9.9	− 5.0	0 30 57.9	−1.2
1.8	0 50 52.5	+ 4.2	0 25 59.5	−3.4
1.9	0 43 21.8	+ 7.8	0 22 9.2	+2.6
2.0	0 37 16.2	+10.6	0 19 1.6	+5.9
2.1	0 32 4.6	+ 0.9	0 16 24.7	+4.9
2.5	0 18 51.9	−10.2	0 9 36.1	−2.5
3.0	0 11 0.7	− 1.1	0 5 33.7	−0.2
3.5	0 6 56.9	− 0.2	0 3 28.9	−1.0
4.0	0 4 35.9	− 3.7	0 2 22.2	+1.7

Those who have studied this subject carefully have found in the above table a double proof of the law of the inverse squares:

1. The fact that $\operatorname{tg}\Phi$ and $\operatorname{tg}\Phi'$ can be so accurately expressed by only the first two terms of the infinite series in (201).

2. The fact that the coefficient of the first term in $\operatorname{tg}\Phi$, namely 0.086870, is exactly double that in $\operatorname{tg}\Phi'$, which is 0.043435.

As a result of the great generality of this analytical theory, together with the novelty of the experimental method, for treating the law of force at great distances, where minor imperfections in the magnets would be minimized, and the refinement of the observations made by *Gauss*, the law of the inverse squares has been regarded as definitely settled by his researches. Accordingly, it seems certain that the exponent in the expression for the law of magnetic attraction is 2, with a probable uncertainty of not more than one unit in the ten-thousandth place:

$$n = 2 \pm \nu \qquad \nu < 0.0001$$

or

$$n = 2.0000 \pm 0.0001 \,. \tag{203}$$

Therefore, it appears probable that the exponent in the law of force in magnetic attraction is known to within about a thousandth part of the accuracy attainable in the refined theories of the heavenly motions developed by the labors of astronomers in the two centuries following the memorable epoch of *Newton*.

The law of magnetic attraction is therefore exceedingly accurate, and with modern apparatus, the refinement doubtless could be carried still further; but it would serve little purpose in the present state of science, as the true law of nature already is plainly indicated.

For it appears from the above analysis of the leading facts that the exponent in the law of magnetic attraction has about the value $n = 2.0001$, essentially identical with the accuracy $n = 2.0001$ attained in the test of the law of gravitation embodied in the Principia by *Newton*, 1687. If this degree of accuracy appeared satisfactory to *Newton*, in the case of gravitation, the accuracy attained by *Gauss* for magnetic attraction leaves very little to be desired.

11. Explanation of the Periodic and Secular Changes in the Earth's Magnetism, including the Cause of the Earth Currents, 'Magnetic Storms', and Aurorae.

In the theory of magnetism a considerable group of errors have been handed down by tradition. Although some of them have been refuted several years ago, they still continue to find place in even the latest treatises. It is therefore necessary to dwell at some length on these errors, in the hope of giving increased currency to valid views on this subject.

(i) Correction of the error in *Lloyd*'s analysis of 1858: Direct magnetic action of the sun and moon established by *Lloyd*'s observations.

In the Philosophical Magazine for March, 1858, Dr. *Humphrey Lloyd*, for many years professor of natural philosophy, and afterwards provost of Trinity College, Dublin, has a learned paper entitled: »On the direct magnetic influence of a distant luminary upon the diurnal variations of the magnetic force at the earth's surface«, which was afterwards reprinted in *Lloyd*'s Treatise on Magnetism, (Longmans, Green & Co., London, 1874, p. 233–239).

We shall review *Lloyd*'s paper briefly in order to point out the error of analysis which vitiates the conclusions drawn from it. These unjustifiable conclusions have been widely circulated in other works, such as the Mathematical Theory of Electricity and Magnetism (3rd ed., 1916) by Dr. *J. H. Jeans*, now secretary of the Royal Society, and are detrimental to the progress of physical science.

Dr. *Lloyd* begins his discussion with the following interesting introductory remarks:

»It has been usual to ascribe the ordinary diurnal variations of the terrestrial magnetic force to solar heat, either operating directly upon the magnetism of the earth, or generating thermo-electric currents in its crust. The credit of these hypotheses has been somewhat weakened by the discovery of a variation which is certainly independent of any such cause, namely, the lunar variation of the three magnetic elements; while at the same time new laws of the solar diurnal change have been established, which are deemed to be incompatible with the supposition of a thermic agency. There has been, accordingly, a tendency of late to recur to the hypothesis that the sun and moon are themselves endued with magnetism, whether inherent or induced; and it is therefore of some importance to determine the effects which such bodies would produce at the earth's surface, and to compare them with those actually observed.«

»I have endeavoured, in what follows, to solve this question, on the assumption that the supposed magnetism of these luminaries is inherent. The result will show the insufficiency of the hypothesis to explain the phenomena; and will therefore bring us one step nearer to their explanation, by the removal of one of their supposed causes.«

He then derives the usual expressions for the total forces X, Y, Z, exerted by a needle delicately suspended about its centre of gravity upon a distant magnetic element m supposed to be in the heavens:

$$\begin{aligned} X &= (Mm/a^3)\,[2\cos\alpha + 3(b/a)\cos\beta + 3(c/a)\cos\gamma] \\ Y &= (Mm/a^3)\,[-\cos\beta + 3(b/a)\cos\alpha] \\ Z &= (Mm/a^3)\,[-\cos\gamma + 3(c/a)\cos\alpha]\,. \end{aligned} \tag{204}$$

Here M is an integral, which we need not explain, a, b, c are the coordinates of the distant magnetic element m, and α, β, γ the angles made by the axis of the suspended terrestrial magnet with the coordinate axes.

Lloyd then proceeds to substitute in these expressions, saying: »If D denote the distance of the centre of the magnet from the centre of the earth, r the earth's radius, λ the latitude of the point (a, b, c) on its surface, and θ the angle contained by the meridian passing through it with that containing the acting magnet,

$$a = D - r\cos\lambda\cos\theta \qquad b = r\cos\lambda\sin\theta \qquad c = r\sin\lambda\,.\text{«} \tag{205}$$

He thus makes the coordinates a, b, c to depend on the hour angle θ, and thus tacitly restricts all changes to the period of the diurnal movement. His analysis is so framed as to exclude the possibility of a semi-diurnal movement of the suspended needle. Yet he proceeds to examine the effect of his analysis, finally adding the following conclusions:

»1. That the effect of a distant magnetic body on each of the three elements of the earth's magnetic force consists of two parts, one of which is constant throughout the day, while the other varies with the hour-angle of the luminary.«

»2. Each of these parts varies inversely as the cube of the distance of the magnetic body.«

»3. The variable part will give rise to a diurnal inequality, having one maximum and one minimum in the day, and subject to the condition:

$$\Delta_\theta + \Delta_{\theta+\pi} = 0\,.$$

»The third of these laws does not hold, with respect either to the solar-diurnal or to the lunar-diurnal variation. Thus, in the solar-diurnal variation of the declination, the changes of position of the magnet throughout the night are comparatively small, and do not correspond, with change of sign only (as required by the foregoing law), to those which take place at the homonymous hours of the day. The phenomena of the lunar-diurnal variation are even more opposed to the foregoing law, the variation having two maxima and two minima of nearly equal magnitude in the twenty-four lunar hours, and its values at homonymous hours having for the most part the same sign. Hence the phenomena of the diurnal variation are not caused by the direct magnetic action of the sun and moon.«

This is one of the most curious specimens of deceptive reasoning which I have ever met with in physical science. If *Lloyd* had used the angle 2θ instead of θ, in the polar expressions for the coordinates a, b, c, so that

$$a = D - r\cos\lambda\cos 2\theta \qquad b = r\cos\lambda\sin 2\theta \qquad c = r\sin\lambda \tag{206}$$

it is evident that the disturbing action resulting would have had a semi-diurnal period, in accordance with magnetic observations, and with gravitational action in the theory of the tides of our seas.

In the theory of our ocean tides, the angle 2θ is used to represent the semi-diurnal forces acting on the sea (cf. *Darwin*'s article Tides, Encycl. Brit.). The reader is also referred to the discussion in AN 5079, pp. 267–270, where *Darwin*'s figure of the semi-diurnal movement of a pendulum will be found useful, in interpreting the following magnetic observations by *Lloyd* himself:

Lunar inequality of the easterly force (η) at Dublin (*Lloyd*, p. 197).

Lunar hours	η = easterly force summer lunations	η = easterly force winter lunations	year
−12	−0.19	−0.09	−0.14
−10	−0.21	−0.08	−0.15
− 8	−0.06	+0.02	−0.02
− 6	+0.09	+0.09	+0.09
− 4	+0.12	+0.13	+0.13
− 2	+0.08	−0.01	+0.03
0	−0.06	−0.09	−0.08
+ 2	−0.04	−0.07	−0.05
+ 4	+0.05	−0.02	+0.01
+ 6	+0.17	+0.08	+0.12
+ 8	+0.06	+0.07	+0.07
+10	0.00	−0.05	−0.03

Lunar inequality of the northerly force (ξ) at Dublin (*Lloyd*, p. 199).

Lunar hours	ξ = northerly force summer lunations	ξ = northerly force winter lunations	year
−12	−0.06	+0.01	−0.03
−10	−0.01	+0.02	+0.01
− 8	+0.08	+0.07	+0.07
− 6	+0.04	+0.05	+0.04
− 4	−0.02	0.00	−0.01
− 2	−0.03	+0.04	+0.01
0	−0.01	+0.02	+0.01
+ 2	0.00	+0.05	+0.08
+ 4	+0.08	−0.03	+0.03
+ 6	−0.01	−0.03	−0.02
+ 8	−0.10	−0.12	−0.11
+10	−0.08	−0.07	−0.08

Lunar inequality of the vertical force (ζ) at Dublin (*Lloyd*, p. 200).

Lunar hours	ζ = vertical force	Lunar hours	ζ = vertical force
−12	+0.05	0	+0.02
−10	+0.00	+ 2	+0.02
− 8	0.00	+ 4	−0.01
− 6	−0.02	+ 6	−0.06
− 4	−0.07	+ 8	−0.01
− 2	+0.10	+10	+0.01

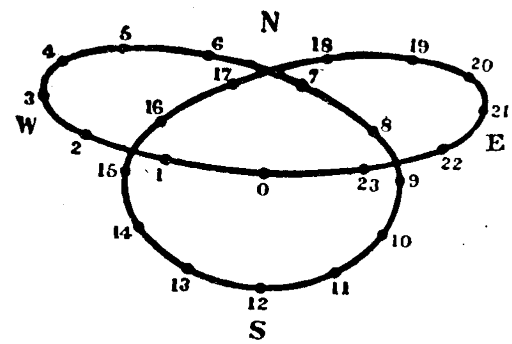

Fig. 6. *Darwin*'s diagram of the semi-diurnal motion of a pendulum, with the hours indicated, latitude 30°.

(ii) Discovery of the magnetic tide by *Kreil* at Prague, 1841, and by *J. A. Broun* independently, 1845: Remarks of *Airy*.

The discovery by *Kreil*, 1841, that there is a true tide in the earth's magnetism, depending on the moon, has long been recognized, and need not be fully discussed here. It suffices to refer to *Stewart*'s elaborate discussion of Terrestrial Magnetism in the article Meteorology, Encycl. Brit., 9th ed.

The existence of this true magnetic tide depending on the moon, was independently discovered by *J. A. Broun*, 1845, and fully confirmed by the elaborate magnetic researches of *Sabine*. The fact of the magnetic tide therefore has been known for about eighty years; yet it has never been explained except by the new theory of magnetism here set forth, and depending on the equations for the ponderomotive forces:

$$F = \mu\mu'/s^2 - \mu\mu'/s'^2, \qquad F' = \mu\mu'/s^2 - \mu\mu'/s^2. \tag{207}$$

When attention was directed to the effect of the changing distance of the moon from the earth, both *Sabine* and *Broun* found the observed changes greater for perigee than for apogee. In fact when *Broun* made an exact analysis of the mean ratio of the apogee effect to the perigee effect, he found it to be as 1 to 1.24 nearly. Examining into the cause of this difference he noticed that the distance of the half orbit near apogee is to that near perigee very nearly as 1.07 to 1.

Broun then adds that tidal forces vary as the cube of the distances, and the cube of 1.07 is 1.23 nearly. As this inference from analytical theory thus agrees very perfectly with observations, he was left in no doubt as to the reality of the suspected tide in the earth's magnetism depending on the moon.

Although a true tide in the earth's magnetism has been known for over 80 years, it is a singular fact that investigators have had great difficulty in interpreting the meaning of these tides. We believe that most of the difficulty has arisen from traditional errors in the theory of magnetism itself, as we shall now proceed to explain.

It appears that among all his eminent contemporaries *Airy* alone — probably as the result of his great work on Tides and Waves, — was able to interpret the magnetic tides of the earth correctly. In his Treatise on Magnetism, 1870, p. 206, *Airy* recognized »a true lunar tide in magnetism, occurring twice in the lunar day, and showing magnetic attraction backward and forward in the line from the Red Sea to Hudson's Bay«.

In view of the cause of magnetism set forth in the present paper it would be difficult to overrate this analysis of *Airy*. First, he assigned to a magnet a »Duality of Powers«, — an attraction towards the two centres known as the poles — and second, he says distinctly that the force of the earth's magnetism directed to the north pole near Hudson's Bay is variable along the line of the magnetic meridian directed through Western Europe to the Red Sea.

In our new theory of the ponderomotive forces of the earth's field, we found the formulae:

$$F = \mu\mu'/s^2 - \mu\mu'/s'^2, \qquad F' = \mu\mu'/s'^2 - \mu\mu'/s^2. \quad (207)$$

Now if under the radiation of the moon or sun any action is exerted upon the earth to change the relative intensity of either pole, the forces F and F' will vary correspondingly. Under these circumstances the ponderomotive forces cannot possibly remain constant; and the result necessarily is a true lunar magnetic tide occurring twice daily, according to which ever pole temporarily is most powerful, thus showing magnetic attraction backward and forward in the line from the Red Sea to the north magnetic pole near Hudson's Bay.

In the case of the sun the changes of the earth's magnetic field due to heat and the magnetic waves in the earth's illuminated hemisphere are so very considerable that the phenomena are somewhat involved. In the case of the moon's action the effects are simpler, though much feebler.

Airy adds that the lunar magnetic forces are considerably less than those which follow the law of solar hours; the mean diurnal solar inequality being about 1/600 of the horizontal force, while the lunar is only 1/12000. Thus the solar influence is approximately 20 times more powerful than that due to the moon. This result need cause no surprise in view of the recently measured high intensity of the sun's magnetism.

(iii) The cause of Earth Currents, 'Magnetic Storms', and the Aurorae.

The explanations of the Aurora Borealis heretofore put forth are artificial, incomplete or unsatisfactory. They all involve something unusual, or out of the ordinary, and therefore must be rejected as inconsistent with the simplicity of the laws of nature.

We propose to develop the magnetic wave-theory of the Aurora, and shall endeavor to show that this phenomenon may be easily understood as soon as we approach the problem from the right point of view.

1. About 1850 it was shown by *Lamont, Wolf, Gautier* and *Sabine* that there is some extraordinary dependence of the amplitude of the magnetic disturbances, or 'Magnetic Storms', of our globe upon sunspots. The amplitudes of the magnetic disturbances follow the same law as the sunspots, in a cycle of about 11 years.

2. But although this periodic magnetic connection has been known now over 70 years, and we might properly have attributed high magnetic power to the sun, yet we could not make out the mode of operation of the magnetic forces, even after the Mount Wilson observers, about 1908, had obtained a better and more direct proof of the intense magnetism of the sun's globe.

3. We pause therefore to unfold a clear view of the disturbance known as an electric current when it is developed in a dynamo of the simplest type. In AN 5079, plate 6, we have explained in some detail the wave field about a wire bearing a current. In the accompanying fig. 7, therefore, we

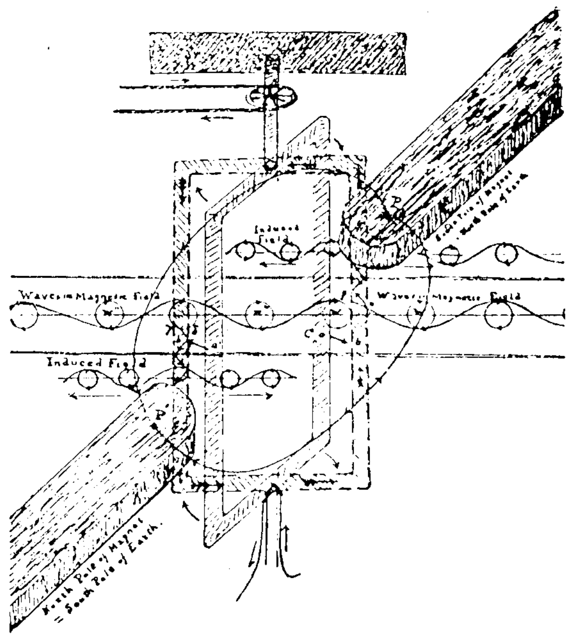

Fig. 7. Generation of electric current by relative motion in a wave-field.

show in the middle section of the figure the nature of the waves of the magnetic field. The waves are to be conceived as traveling in every direction with the velocity of light. What, then, will happen when a loop of wire — which may be viewed as a section of an armature or core — revolves in such a wave field?

4. We know that induction takes place, so that a current tends to be generated and will flow steadily in one direction if the wave-disturbance is properly led off in a wire forming a closed circuit. To get a clear view of this induction process, we need only note that as the wire has both inductance and capacity, any motion of the wire in the wave-field, or any motion of the wave-field relatively to the wire, will produce the induction required to develop the current. By the motion of the wire, or of the wave-field relatively to the wire, the electric equilibrium in the wire is disturbed, as when it moves between the magnetic poles P and P'.

5. Suppose, for example, the copper wire C to move from a to b. At such a point as α the equilibrium of the aether will be disturbed. Under the action of the vortices in the external free aether, those within the wire will tend

to be forced upward, owing to the approach of the wire towards the whirl w. At the point β, on the other hand, the wire recedes from the whirl w', and owing to the enormous elasticity of the aether a suction effect will result, by virtue of which the aether in the wire at β will also tend to be forced upward. Thus, owing to these relative motions, the disturbances at α and β combine to produce a wave disturbance upward in the wire as shown, which is similar to that given in AN 5079, Tafel 6, for a current.

6. But as a current requires a circuit, we must next consider what is the effect of the motion of the wire on the opposite side, $a' b'$. At the point γ the disturbance of the aether due to the vortex w'', will be downward, for reasons similar to those assigned above. At δ, on the other hand, the wire is receding from the vortex w'', and under the high elasticity of the aether, the resulting suction effect will be such as to also direct the disturbance downward. Thus at both γ and δ there is a combination of downward disturbances; and therefore a tendency to develop the electric oscillations or current shown in the diagram, and recognized as a current in AN 5079, Tafel 6.

7. This combination of electric oscillations tending upward on the right, and downward on the left of the wire loop, is the basis of the induced current discovered by *Faraday* in 1831. The rule about »cutting the lines of force« is a very good rule of thumb for artisans and mechanics; but it gives no intelligent view of what actually goes on. Thus we have outlined the process from the point of view of the highly elastic medium the aether is known to be.

8. And we have explained how the relative motion of the wire in respect to the magnetic field filled with vortices must necessarily create about the wire the wave-field called a current. The magnetic vortices, being elements of the magnetic waves, may be conceived as moving with the velocity of light in free space; yet as they are perpetually renewed in situ, they act as if they were stationary relatively to the magnetic poles upon which they depend.

9. As we pointed out above, any motion of the fixed wave-field, or any marked variation of the wave-field, in respect to the wires, will give rise to electric disturbances which might be made the basis of an electric current, if properly led off and directed. We have now to draw attention to the well known fact that the sun's magnetic wave field is quite variable. The magnetic wave disturbances are much worse when sunspots are present than when these spots are absent. As the spots are known to be vortices of highly magnetized matter, we see that the magnetic waves coming from these local areas of the solar surface are the immediate cause of the »Magnetic Storms«. This connection is abundantly established by the researches of astronomers during the past 75 years.

10. In the Phil. Trans., vol. 166, p. 387, *John Allen Broun* found from magnetic phenomena that there was a period of recurrence in the magnetic disturbances of about 26 days, — the period of the solar rotation, — and expressed the belief that certain zones or areas of the solar surface might exert a potent influence on the state of the earth's magnetism during several rotations. In 1904, (MN 65, pp. 2 and 538), *E. W. Maunder* reached similar conclusions, without knowledge of *Broun*'s earlier work. *Maunder* made the period to be 27.28 days, coinciding with the sun's rotation relatively to the observer upon the earth. Mr. *Maunder* regarded his results as demonstrating that the larger magnetic disturbances of the earth originate in the sun; and considered the action as propagated along narrow well defined streams, having their bases in active areas of sunspots, yet possibly preceding and outliving the spots themselves.

11. *Young*'s observations of the violent magnetic tremors near the time of the total solar eclipse in Colorado, 1872, are well known, but as his account is instructive we quote it briefly:

»On August 3, 1872, the chromosphere in the neighborhood of a sun-spot, which was just coming into view around the edge of the sun, was greatly disturbed on several occasions during the forenoon. Jets of luminous matter of intense brilliance were projected, and the dark lines of the spectrum were reversed by hundreds for a few minutes at a time. There were three especially notable paroxysms at $8^h 45^m$, $10^h 30^m$, and $11^h 50^m$ a. m. local time. At dinner the photographer of the party, who was determining the magnetic constants of our station, told me, without knowing anything about my observations, that he had been obliged to give up work, his magnet having swung clear off the scale. Two days later the spot had come around the edge of the limb.«

Young's observations extended over the next two days, and when he afterwards wrote to *Airy* and *Perry* in England, he was surprised to find from their photographs that the needles at Greenwich und Stonyhurst had shown violent tremors, just like that noted in Colorado, and at the same instant of Greenwich mean time, within 10 minutes, — the time in Colorado not having been noted with any great precision.

12. The accompanying Plate 7 of simultaneous magnetic disturbances throughout the world, reduced to Greenwich time, is from a paper by Prof. *W. G. Adams*, in the Phil. Trans., for 1892, A, Plate 8. After examining this record, it is useless to extend our present argument any further.

But whilst the relationship of Sunspots to 'Magnetic Storms', Earth Currents, Aurorae, is long recognized from well defined phenomena of periodicity, the nature of the undoubted connection continues so very obscure that we must carry the examination a little further.

In his Treatise on Magnetism, 1870, p. 204 *Airy* says:

»The periods of great disturbance sometimes occupy a portion of a single day, sometimes several days in succession: they are familiarly known by the name of 'magnetic storms'. They are not connected with thunder-storms or any other known disturbance of the atmosphere; but they are invariably connected with exhibitions of Aurora Borealis, and with spontaneous galvanic currents in the ordinary telegraph wires: and this connection is found to be so certain that, upon remarking the display of one of the three classes of phenomena, we can at once assert that the other two are observable (the Aurora Borealis sometimes not visible here, but certainly visible in a more northern latitude).«

Conclusions as to the Cause of the Earth Currents, 'Magnetic Storms', Aurora Borealis and Aurora Australis.

1. From the above argument it follows that the sun's magnetic wave-field is variable, owing to the changes associated with the sunspot development. Variability in the sun's magnetic wave field leads to variability in the inductive actions of these waves upon the earth; and as our globe has both inductance and capacity, the result is electric disturbances adjusting themselves within the globe, which is a heterogeneous mass three fourths covered by sea, and surrounded by an atmosphere well suited to Geißler tube displays.

2. It is easily shown by experiments on land, especially near the sea, that very considerable differences of electric potential always exists in the ground, and is in constant dissipation and adjustment. The solid heterogeneous earth, sea and sky are under perpetual adjustment of the equilibrium of their respective wave-fields; and as the inductions due to the sun are variable, we see that there will arise a true magnetic tide pulling backward and forward from the Red Sea to Hudson's Bay, whenever the solar inductions take place in such a way as to change the two poles of the globe of the earth or of the sun. If these changes are rapid and violent we frequently have a form of lightning in the upper atmosphere:

$$F = \mu\mu'/s^2 - \mu\mu'/s'^2, \text{ Aurora Borealis.} \qquad (208)$$

$$F' = \mu\mu'/s'^2 - \mu\mu'/s^2, \text{ Aurora Australis.} \qquad (209)$$

3. These simple considerations perfectly explain the earth currents, and the 'magnetic storms', and we may dismiss these two classes of phenomena as now referred to their true cause. For the sun is variable in its action, owing to spot development and rotation, and thus has variable poles, μ und μ, while the earth also rotates steadily; yet its poles μ' and μ' are variable, because the changing light hemispheres are successively subjected to an extremely variable induction by the sun's action. Accordingly the earth currents and 'magnetic storms' immediately follow from the above formulae.

4. As for the aurora, it was remarked by *Halley* in the Phil. Trans., for 1714–1716 (vol. XXIX, no. 341), that this brilliant northern light is a magnetic phenomenon. Perhaps *Halley* was influenced in his conclusions, as more modern investigators have been, by the observed fact that the streamers of the aurora are parallel to the lines of force of the earth's magnetic field. The argument from the parallelism of the streamers to the magnetic lines of force and from the periodicity of the aurora coinciding with that of the 'magnetic storms', earth currents and sunspot development is all that is required to show that the aurora depends upon electric discharges in our atmosphere. The streamers take on faint luminosity along the lines of magnetic force, because the magnetic state of the earth is varying, under the irregular inductive action of the sun, which operates most directly upon the light hemisphere; yet as the night hemisphere of the earth is magnetically a part of the earth's entire system, the dissipation of uncompensated magnetic wave-energy is likely to become visible chiefly by night.

5. In Lowell Observatory Bulletin no. 79, June to November, 1916, Dr. *Slipher* has shown that the aurora usually is present in the sky near the horizon, even by daylight. This is because the air near the horizon is of greatest depth, and thus on the photographic plate the persistence of the auroral rays may become visible in spite of its invisibility to the eye. In high latitudes, especially about the cold poles of our globe, the atmospheric electric potential of the air tends to fall, as *Exner* showed, on the average fifteen times more rapidly than in the equatorial regions of our globe. Thus in cold regions the rapid fall of the electric potential offers an easy release for wave stress in the air when the particles of cirrus and similar clouds are undergoing rapid condensation. Thus electric discharges at great height do not take the form of lightning directed to the earth, but of faintly luminous streamers.

6. Now if we consider the new theory of lightning and of surface tension given in the fifth paper, AN 5130, we perceive that liquid drops, or even globules of ice suspended in a frozen cirrus cloud, under the influence of the earth's magnetic field, would not have the aether stress at the surface exerted with perfect symmetry, but there would be a north and south polarity in the globules, owing to the unsteadiness in the earth's magnetic forces. If this stress yielded to a slight electric rupture, under the wave impulses of the earth's magnetism, the release of energy would generate an agitation yielding luminous streamers in the direction of the magnetic lines of force, as in the aurora. Since the magnetic stress is greatest to the north, the rays often would start there and flash southward, which explains the auroral streamers perfectly, without the introduction of any arbitrary hypothesis. So simple a theory leaves nothing to be desired; without strain it explains all the auroral phenomena of the two terrestrial hemispheres.

7. The folded ribbon bands so often seen in the aurora are perspective shadow effects, produced mainly by alternations of luminous and non-luminous clouds, somewhat analogous to the beams, with truncated columnar aspect, often seen in the sky at sunset or sunrise; yet just as the sources of the auroral light are more hidden than in the case of the sun, so the auroral truncated bands or folded curtains thus are more mysterious than what we see produced by the sun's visible illumination.

8. During the brilliant aurora observed at Mare Island, California, May 14, 1921, I saw the streamers forming very distinctly in several parts of the sky where the clouds were just forming and dissolving. When a faint cloud became visible, yet was not luminous, I found, by watching, that it often would soon exhibit faint streamers running parallel to the earth's magnetic lines.

Thus I am convinced that the light energy of the aurora is wave-energy, which in lower clouds frequently takes the form of lightning directed to the earth; but in the region of the cirrus, where the air is very rare, is released by slight oscillatory changes in the earth's magnetic forces. The aurora is a kind of sheet lightning of the upper atmosphere, and therefore the luminous streamers take the direction of the lines of force in the earth's magnetic field. To this extent *Halley*'s bold conjecture of 1714 is correct. A more detailed account of the brilliant aurora observed May 14, 1921, will be found in AN 5140, p. 81.

(iv) The secular changes in the earth's magnetism.

It only remains to consider the secular changes of the earth's magnetism. Here, unfortunately, we are on uncertain ground, owing to the absence of any well defined criteria to show the cause at work.

1. It was stated by *Airy* that the magnetic north pole probably is revolving around the geographical pole; but after a critical survey of the known data for the south magnetic pole, in section 5 above, we are unable to find definite evidence of any shifting whatever of the earth's magnetic poles. And as the poles appear to be fixed, — if we judge by that in the Antarctic, for which the observed data are most complete — we have been obliged to conclude that the surface secular changes in the earth's magnetism depend mainly on induction effects due chiefly to the sun, as in *Arago*'s rotation experiment of 1825.

2. Since the sun is a strongly magnetized body, there would arise in our globe some eddy currents of considerable intensity; and as the eddy currents for astronomical reasons would recur with approximate regularity in the same direction, for a long time, over considerable areas, it is likely that a very considerable secular change in the magnetic forces near the surface would gradually result. This is the only tangible explanation of the secular changes known to me.

It has the element of simplicity in its favor, and we know that considerable differences of electric potential do really exist in regions very near each other: therefore it seems probable that an adjustment of the planes of the atomic equators would gradually occur so as to shift the direction of the magnetic lines in the same direction; then, later, by mutual reaction upon other masses, the oscillation might go in the other direction.

3. But as the »magnetic domains« of the earth are very unequal in size and intensity, it is probable that the secular changes would be slow and ill-conformed to one uniform rule. The fact is that they are quite confused, some clockwise, others counter-clockwise. Hence this about corresponds to the observed secular changes in the earth's magnetism. Great as these changes become with the lapse of centuries, they evidently do not belong to the poles, which apparently are not shifting. We can only explain the secular changes by the theory of comparatively shallow *Arago* eddy currents. To this hypothesis there is not the slightest objection, yet we must derive the secular changes wholly from observation.

12. Magnetic Attraction depends on a Duality of Powers, and is therefore directed along Curved Lines towards Two Poles, while Gravitational Attraction is directed straight towards a Single Centre of Gravity: Simplicity of the General Laws of Nature.

(i) The connection and the difference between magnetism and gravitation.

It is now clearly shown that magnetism depends on a duality of powers, and therefore the mutual attraction between two magnets is directed along curved lines, according to the formulae for the ponderomotive force:

$$F = \mu\mu'/s^2 - \mu\mu'/s'^2 \qquad F' = \mu\mu'/s'^2 - \mu\mu'/s^2 . \qquad (210)$$

In this respect magnetism is totally unlike the simpler stress of gravitation resulting from the haphazard arrangement of the atomic planes, yet yielding a single force directed in a right line towards the centre of gravity:

1. Gravitational attraction

$$f = mm'/r^2 . \qquad (211)$$

2. Magnetic total intensity, or aether stress

$$I = \mu\mu'/s^2 + \mu\mu'/s'^2 . \qquad (212)$$

It is justly remarked that for a full century investigators have fought shy of searching for the cause of magnetism. The result was so unsatisfactory to physical science that in the later years of his life *Helmholtz* is said to have remarked that our failure to discover the cause of magnetism was the disgrace of the 19th century. Certainly the need for such research has long been felt as a most urgent desideratum of science, and yet little or no progress was made, owing to the confused state of the subject.

Now that a way is opened for referring all the chief forces of nature to wave-action in the aether, we must be careful not to allow attention to be diverted from so fruitful a line of inquiry. There need be, for example, no discussion of the irregularities of the earth's magnetism; we have known of these irregularities all along, but had not perceived the harmonic law connecting the earth's total magnetic force with terrestrial gravitation.

The new method has enabled us to reach the harmonic law:

$$I/g = \eta^2 (r^2/s^2 + r^2/s'^2) \qquad \eta = 1/1408.12 \qquad (213)$$

thus accurately connecting the total intensity of magnetism with gravity throughout the globe. It has enabled us to calculate the total quantity of magnetism in the sun, and to connect the total force of this magnetism at the sun's surface with the known force of solar gravitation by a similar equation

$$I_0/g_0 = \eta_0{}^2 (r_0{}^2/s_0{}^2 + r_0{}^2/s_0'^2) \qquad \eta = 1/157 . \qquad (214)$$

Such a law evidently is in the highest sense a true law of nature. Indeed it has all the generality of the law of gravitation itself. The new law can be applied wherever the intensity of the magnetization is known in units of that observed in the earth or in the sun.

Obviously a similar equation will hold for Jupiter, Saturn or Mars, yet up to this time we have no observed data for fixing the intensity of the magnetization in Jupiter, Saturn or Mars compared to that in the earth or sun. However, just as *Kepler*'s law of planetary motion about the sun:

$$a^3 : a'^3 = t^2 (M+m) : t'^2 (M+m') \qquad M = 1, \text{ usually,} \qquad (215)$$

was generalized for the planets and double stars of mass $M_1 + m_1$, in the form:

$$(M+m) : (M_1+m_1) = a^3/t^2 : a_1{}^3/t_1{}^2 \qquad (216)$$

so also at the surface of any cosmical body the law connecting magnetism with gravitation becomes:

$$I_v/g_v = \eta_v{}^2 (r_v{}^2/s_v{}^2 + r_v{}^2/s_v'^2) . \qquad (217)$$

It seems probable that the magnetization in a body like Jupiter someday may be determined by observation. Even now we may approximate the magnetic component η_J. For since the mass of this great planet lies between that of the sun and earth, we should expect η for Jupiter to lie between 1/1408 and 1/157, thus

$$1/157 > \eta_J > 1/1408 . \qquad (218)$$

And similar reasoning would apply to Saturn, Uranus, and Neptune. In the case of Mars, on the other hand, we should expect the magnetization to be less intense than in the case of the earth; so that we should have

$$1/1408 > \eta_M. \tag{219}$$

And as the mass of Mars is only about 0.1 of the earth's mass, it would not be unexpected if the magnetization of that small planet should prove to be approximately:

$$\eta_M = 1/14080, \text{ or } \eta_M = 1/12000. \tag{220}$$

The magnetization of Venus, by similarity of causes, probably would be comparable with the earth's, and thus very nearly the same as our well determined value $\eta = 1/1408$. Thus the problem of cosmical magnetism takes on new interest in all directions.

There are too many questions as yet unexplored for us to dare to hope to solve all of them in a perfectly simple manner; yet when a way is once opened for a solution, there will eventually appear the investigator who will traverse this unexplored path, just as has happened in extending the dominion of the law of gravitation over the unexpected celestial phenomena discovered since the time of *Newton*.

It is remarkable that at last we have reached a great law of nature connecting magnetism with gravitation, in spite of the remarkable difference in the two forces; and that we are able to refer both of these forces of nature to wave-action in the aether. By this harmonic law geometrically connecting the haphazard wave-action in gravitation, which has the force directed towards a single centre, with the systematic wave-action in magnetism, yielding a duality of powers, or forces directed to two centres, the argument for the cause assigned to either force separately is vastly strengthened.

If for example, the probability be very great in favor of either hypothesis separately, so that the chance of any other result practically vanishes, as happens when the limits α and β are indefinitely near together, in the probability integrals:

$$\int_{\alpha}^{\beta} e^{-h^2x^2}\cdot dx = 1/n, \quad \int_{\alpha'}^{\beta'} e^{-h'^2x'^2}\cdot dx' = 1/n', \quad C\int_{-\infty}^{+\infty} e^{-h^2x^2}\cdot dx = 1 \tag{221}$$

n and n' being very large numbers. And if the two causes at work, as now happens, are connected by the verified harmonic law, then the probability of the compound event, or common cause, for both of the phenomena of magnetism and gravitation being jointly due to wave-action, becomes the reciprocal of the product of these probabilities, when each one has infinitely narrow limits, namely:

$$P = \int_{\alpha}^{\beta} e^{-h^2x^2}\cdot dx \int_{\alpha'}^{\beta'} e^{-h'^2x'^2}\cdot dx' = [1/n \cdot 1/n']_{n=\infty \;\; n'=\infty} = 1/\infty^2. \tag{222}$$

Thus the probability becomes an infinity of the second order, like all the points in an infinite plane to one, that if wave-action underlies either magnetism or gravitation, the same cause underlies both classes of phenomena.

Now the processes involved in gravitation are not directly perceptible to our senses, and thus somewhat obscure. It is not so, however, with the cause of magnetism, — which is profoundly illuminated by *Faraday*'s experiment of 1845, on the rotation of a beam of polarized light, and by a whole train of phenomena in magneto-optics; by the great body of phenomena in electrodynamics; and finally by the vast array of phenomena in wave motion, especially collected together in figure 4 of this paper, and the six preceeding papers on the New Theory of the Aether.

The complete accord of all these magneto-optical phenomena with the undulatory theory in general thus yields another independent probability:

$$p = \int_{\alpha''}^{\beta''} e^{-h''^2x''^2}\cdot dx'' = 1/n''. \tag{223}$$

And the reciprocal of the compound probability of all these causes

$$Pp = \int_{\alpha}^{\beta} e^{-h^2x^2}\cdot dx \int_{\alpha'}^{\beta'} e^{-h'^2x'^2}\cdot dx' \int_{\alpha''}^{\beta''} e^{-h''^2x''^2}\cdot dx'' = [1/n \cdot 1/n' \cdot 1/n'']_{n=\infty,\; n'=\infty,\; n''=\infty} = 1/\infty^3 \tag{224}$$

when the limits of the integrals are narrowed indefinitely, therefore is the chance which supports the wave-theory. Accordingly, the ratio of all the points in infinite space to one represents the probability that magnetism and universal gravitation are due to the common cause of wave-actions in the aether.

(ii) Defects in the theory of magnetism handed down by tradition from the days of *Robert Norman*, 1576; *Gilbert*, 1600; and *Euler*, 1744.

In the Third Paper, AN 5079, p. 244–247, we have given a brief notice of the defective theory adopted by *Euler*, 1744, who regarded the aether as circulating through the axis of a magnet from the south to the north pole, like the blood flowing through the arteries in one direction only, thus returning to the south pole through free space, along the magnetic lines of force. Figure 3, Tafel 4, AN 5079, illustrates *Euler*'s theory according to be original diagrams of his Opuscula, vol. III, Berlin, 1744, plate 1.

The principles thus adopted by *Euler* gave an unfortunate bias to thought in magnetism, and although in later treatises they are somewhat modified so as to conform to modern thought, yet the whole trend of the reasoning in this subject has continued to be on an unsound basis. The usual habit of describing the *Euler*ian circuit from the north to the south pole as »the path of a unit north pole«, is criticised in AN 5079, p. 247 as »unscientific and a very imperfect makeshift«, — the mildest criticism, consistent with truth, which could be made.

But prior to *Euler*'s time certain defects were current which had been handed down from the days of *Robert Norman*, 1576:

1. That magnetization does not change the weight of a body; that by the action of the magnetic forces there is no tendency to a motion of translation.

2. Hence the earth's magnetic action is directive only, being due to a couple.

None of these claims are strictly true, because one magnet exerts upon another the following ponderomotive forces

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1. That magnetization does not change the weight of a body; that by the action of the magnetic forces there is no tendency to a motion of translation.

2. Hence the earth's magnetic action is directive only, being due to a couple.

None of these claims are strictly true, because one magnet exerts upon another the following ponderomotive forces for the northern and southern hemispheres respectively:

$$F = \mu\mu'/s^2 - \mu\mu'/s'^2; \qquad F' = \mu\mu'/s'^2 - \mu\mu'/s^2 \,. \qquad (225)$$

And thus the earth does exert upon a magnet, suspended by a thread through its centre of gravity, a slight deflecting effect; yet it is difficult to observe it experimentally, owing to the force of gravitation exceeding the magnetic force in a ratio of about 1000000 to 2000000 times.

At the earth's magnetic poles, a piece of steel suddenly converted into a magnetic needle would be under two forces: (a) ordinary gravitation, (b) terrestrial magnetism, having a strength of about 1/1000000 of gravitation. As both forces work in the same vertical direction it seems certain that the magnetized steel would weight a little more after magnetization than before, though the change of weight would be small.

We may safely reach this conclusion by observing that if a steel needle or piece of soft iron is suspended by a thread over the pole of a strong steel magnet, the downward pull on the thread is certainly greater in this position above the magnet, than when the strong steel magnet is removed. I have often tried the experiment, and found the effect very noticeable. What is true of a magnetized needle suspended above a magnet therefore is true of a magnetized piece of steel at the earth's magnetic poles. For the downward pull due purely to gravity would be slightly increased by the purely magnetic action of our globe.

The defect in *Norman*'s reasoning, in regard to the earth's magnetic action being directive only, need not be discussed at length, because we have seen that the ponderomotive force does really exist, and vary from place to place; yet the change from place to place is slow, owing to the size of the earth, and the great length of the curved lines s and s'.

It only remains to recall a diagram originally given in *Gilbert*'s work De Magnete, 1600, as follows.

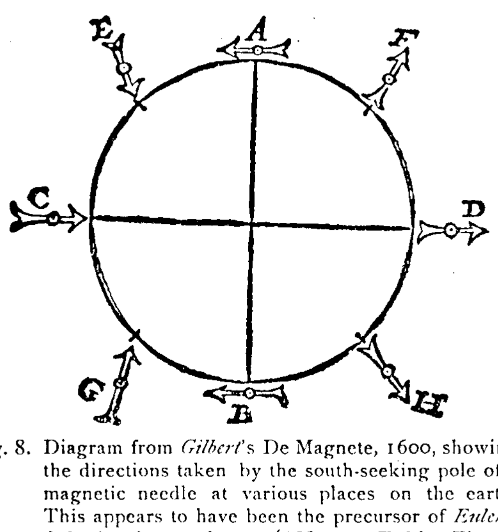

Fig. 8. Diagram from *Gilbert*'s De Magnete, 1600, showing the directions taken by the south-seeking pole of a magnetic needle at various places on the earth. This appears to have been the precursor of *Euler*'s defective theory of 1744, (AN 5079, Tafel 4, Fig. 3.) that the aether flows in at the south pole, and out at the north pole, circulating like the blood in the arteries in one direction only.

This figure is from *Mottelay*'s excellent English translation of *Gilbert*'s work, John Wiley & Sons, New York, 1893, p. 282. The points C and D here correspond with *Euler*'s south pole and north pole, A and B, and thus it may be that *Euler*'s diagram of 1744 had its inspiration in an earlier diagram of *Gilbert*, 1600.

Accordingly the defects in the traditional theory of magnetism doubtless arose originally from the way the south-seeking pole points when brought to the earth's poles. It represents this pointing phenomenon faithfully, but gives us no idea of the correct theory of magnetism, as now expressed by the formulae for the ponderomotive force in the observed mutual action of two magnets.

In AN 5079, p. 247, we have called attention to *Maxwell*'s indistinct reasoning, in his address on Action at a Distance, respecting the push and pull forces between a magnetic needle and a wire bearing a current, as in *Oersted*'s experiment. Thus it only remains to formulate an expression for the ponderomotive force by which the magnetic needle is drawn to the wire, as follows:

$$F = \mu \varkappa i/r + \mu \varkappa i/r \qquad (226)$$

where μ and μ are the strengths of the two poles of the magnetic needle, $\varkappa$ is a constant, i the strength of the current, and r the shortest radial distance of the poles from the axis of the wire.

Biot and *Savart*'s law of 1820 (AN 5079, p. 255), corresponds to the above form of expression; and as the needle acts as if the magnetism were centred in its poles, this extraordinarily simple formula results for the ponderomotive action of a current in a straight wire upon a magnetic needle with poles at the distance r. If the distance of the two poles from the axis of the wire be unequal, r and r', the formula becomes very slightly more complex:

$$F = \mu \varkappa i/r + \mu \varkappa i/r' = \mu \varkappa i\,(1/r + 1/r') \,. \qquad (227)$$

Experiments made by me in 1914, show that these ponderomotive forces are real, and very sensible to observation. If the needle be out of its position of equilibrium, it tends to turn around, as in *Oersted*'s experiment. If reversed from its natural position, there would of course be a change of sign in F, and repulsion would result; but from the point of view of the wave-theory, the case of attraction is far more interesting.

(iii) Results of recent investigations in regard to the cause of magnetism and gravitation.

a) Views of *Airy*: In his Treatise on Magnetism, 1870, *Airy* expresses himself thus:

»In ordinary observation, magnetism is scarcely known except as existing in iron and especially in steel, and as related in some obscure manner to the earth. But there is reason to believe that it is one of the most extensively diffused agents in nature. It can be traced not only in iron but also in every substance into which iron (one of the most widely spread substances in nature) enters in composition. It is found in nickel and other substances, and even in some gases. Wherever a galvanic current exists in nature, whether produced by chemical action or appearing in the thermo-electric form as originating from the effects of heat at the place of union of different substances, magnetic effects can be elicited. On the larger scale it is certain that the whole earth acts as a combination of magnets, and there is reason to think that the sun and the moon also act as magnets«.

Yet after developing a comprehensive treatise written with his usual masterly ability, and making great use and high commendation of the researches of *Gauss*, *Airy* concludes his work (p. 220) with the following somewhat melancholy remark:

»On the whole, we must express our opinion, that the general cause of the earth's magnetism still remains one of the mysteries of cosmical physics.«

b) Views of *Maxwell*: We shall now examine *Maxwell*'s views on magnetism and gravitation. In the celebrated article on Attraction, (Scient. Pap., vol. II, p. 488) *Maxwell* reasons as follows:

»*Faraday* showed that the transmission of electric and magnetic forces is accompanied by phenomena occurring in every part of the intervening medium. He traced the lines of force through the medium; and he ascribed to them a tendency to shorten themselves and to separate from their neighbors, thus introducing the idea of stress in the medium in a different form from that suggested by *Newton*: for, whereas *Newton*'s stress was a hydrostatic pressure in every direction, *Faraday*'s is a tension along the lines of force, combined with a pressure in all normal directions. By showing that the plane of polarisation of a ray of light passing through a transparent medium in the direction of the magnetic force is made to rotate, *Faraday* not only demonstrated the action of magnetism on light, but by using light to reveal the state of magnetisation of the medium he 'illuminated', to use his own phrase, 'the lines of magnetic force'«.

»From this phenomenon *Thomson* afterwards proved, by strict dynamical reasoning, that the transmission of magnetic force is associated with a rotatory motion of the small parts of the medium. He showed, at the same time, how the centrifugal force due to this motion would account for magnetic attraction.«

»A theory of this kind is worked out in greater detail in *Clerk Maxwell*'s Treatise on Electricity and Magnetism. It is there shown that, if we assume that the medium is in a state of stress, consisting of tension along the lines of force, and pressure in all directions at right angles to the lines of force, the tension and the pressure being equal in numerical value and proportional to the square of the intensity of the field at the given point, the observed electrostatic and electromagnetic forces will be completely accounted for.«

»The next step is to account for this state of stress in the medium. In the case of electromagnetic force we avail ourselves of *Thomson*'s deduction from *Faraday*'s discovery stated above. We assume that the small parts of the medium are rotating about axes parallel to the lines of force. The centrifugal force due to this rotation produces the excess of pressure perpendicular to the lines of force. The explanation of electrostatic stress is less satisfactory, but there can be no doubt that a path is now open by which we may trace to the action of a medium all forces which, like the electric and magnetic forces, vary inversely as the square of the distance, and are attractive between bodies of different names, and repulsive between bodies of the same names.«

»The force of gravitation is also inversely as the square of the distance, but it differs from the electric and magnetic forces in this respect, that the bodies between which it acts cannot be divided into two opposite kinds, one positive and the other negative, but are in respect of gravitation all of the same kind, and that the force between them is in every case attractive. To account for such a force by means of stress in an intervening medium on the plan adopted for electric and magnetic forces, we must assume a stress of an opposite kind from that already mentioned. We must suppose that there is a pressure in the direction of the lines of force, combined with a tension in all directions at right angles to the lines of force. Such a state of stress would, no doubt, account for the observed effects of gravitation. We have not, however, been able hitherto to imagine any physical cause for such a state of stress.«

It has been shown in AN 5048, pp. 162–164, that this last view of *Maxwell*, assigning a pressure in the direction of the gravitational force, is erroneous. We treat of his calculations of stress in some detail section 9 (ii) above, and correct a numerical error which has stood about half a century. It thus suffices to add that what *Maxwell* could not solve was even more bewildering to Lord *Kelvin*, *Helmholtz*, and other investigators.

And as light could not be shed on magnetism, it need not surprise us that the mystery of the cause underlying gravitation has remained complete. In a learned paper in the Proc. R. Soc., vol. 54, 1893, p. 457, Prof. *Larmor* concludes by saying that: »The cause of the phenomenon of gravitation has hitherto remained perfectly inscrutable«.

c) Views of Lord *Kelvin*. In the introduction to the English edition of *Hertz*'s Electric Waves, 1893 (pp. xii-xiii), Lord *Kelvin* expressed himself as follows regarding *Faraday*'s researches:

»But before his death, in 1867, he had succeeded in inspiring the rising generation of the scientific world with something approaching to faith that electric force is transmitted by a medium called ether, of which, as had been believed by the whole scientific world for forty years, light and radiant heat are transverse vibrations. *Faraday* himself did not rest with this theory for electricity alone. The very last time I saw him at work in the Royal Institution in an underground cellar, which he had chosen for freedom from disturbance; and he was arranging experiments to test the time of propagation of magnetic force from an electromagnet through a distance of many yards of air to a fine steel needle polished to reflect light; but no results came from those experiments. About the same time or soon afterward, certainly not long before the end of his working time, he was engaged (I believe at the shot tower near Waterloo Bridge on the Surrey side) in efforts to discover relations between gravity and magnetism, which also led to no result.«

»Absolutely nothing has hitherto been done for gravity either by experiment or observation towards deciding between *Newton* and *Bernoulli*, as to the question of its propagation through a medium, and up to the present time we have no light, even so much as to point a way for investigation in that direction. But for electricity and magnetism *Faraday*'s anticipations and *Clerk Maxwell*'s splendidly developed theory have been established on the sure basis of experiment by

Hertz's work, of which his own interesting account is now presented to the English reader by his translator, Prof. *D. E. Jones*. It is interesting to know, as *Hertz* explains in his introduction, and it is very important in respect to the experimental demonstration of magnetic waves to which he was led, that he began his electric researches in a problem happily put before him thirteen years ago by Prof. *v. Helmholtz*, of which the object was to find by experiment some relation between electromagnetic forces and dielectric polarisation of insulators, without, in the first place, any idea of discovering a progressive propagation of this force through space.«

This citation is important for showing the great *Faraday*'s conviction of a connection between magnetism and gravitation, and also for showing the significance of *Hertz*'s researches on forces due to waves propagated through the aether.

Absolute Continuity among the Forces of Nature.

If after two centuries of effort, from the time of *Newton* and *Bernoulli*, such great mystery still hung over this question of the connection between gravitation and electrodynamic action, under electric and magnetic forces, as viewed by the comprehensive and experienced mind of Lord *Kelvin*, still more imperative was it the writer's duty to develop the connection between magnetism and gravitation. Yet this was much more difficult than it might seem at first sight.

1. In Dr. *K. F. Bottlinger*'s Inaugural Dissertation at the university of Munich, 1912, it is stated that *Einstein* had been so unsuccessful in his attempt to connect gravitation with electrodynamic action that he had quite turned away from it. Out of this *Einstein* failure grew the much discussed but false doctrine of Relativity. It was prematurely exploited in England, by the Royal societies and by scientific journals, which have since regretted the record of this ill-advised course.

2. Now that gravitation is directly connected with magnetism, and magnetism itself an electrodynamic phenomenon, it follows incontestably that universal gravitation also is an electrodynamic force. Accordingly it is evident that the present successful breaking down of the complete isolation which so long separated gravitation from the other forces of the universe is a step of no ordinary significance.

3. *Faraday* had the right idea, based on the doctrine of continuity for all the forces of nature, 1866, as above described by Lord *Kelvin*. But in *Faraday*'s day the Royal Society of London would give no heed to his views, any more than they would to the similar views set forth in my Preliminary Paper of 1914, which were regarded favorably, by such an authority as the late Lord *Rayleigh*. Thus, owing to the lack of vision of the referees of the Royal Society, *Faraday*'s correct ideas were smothered for half a century, and never could have triumphed but for my good fortune in getting the work completed and published in these papers on the New Theory of the Aether. In deference to the revered memory of *Newton*, one would have thought that if the referees of the Royal Society could not lead in this work of discovery, they certainly would encourage such effort on the part of others for illuminating the cause of gravitation.

4. But in view of their utter failure, it appears that the same weakness of management exists now, as in 1686, when they evaded the publication of *Newton*'s Principia, and compelled *Halley* to print it at his private expense. Let the simple record of fact — that they discouraged *Newton*, 1686; *Faraday*, 1866; myself, 1914–16; and in 1916–19, actually championed Relativity against the *Newton*ian Philosophy — tell the story of the Royal Society, whether the organization ever is effective in promoting the highest discoveries, such as *Newton* would approve.

The Philosophical Transactions during the last 50 years has perhaps a dozen learned mathematical memoirs on electricity and magnetism, especially on *Maxwell*'s equations for the electromagnetic field, yet not one of the authors discloses the smallest knowledge of the fundamental errors in the theory of magnetism handed down from the days of *Gilbert*, 1600, and *Euler*, 1744, though one or two recent authors discuss the hypothetical Magneton, which has no real existence. Hence as they made no progress towards the cause of magnetism, we have recently had what Dr. *Whewell* calls a stationary period in physical science.

(iv) General considerations on the highest laws of nature.

It is one of the great merits of Sir *Isaac Newton*'s law of universal gravitation:

$$f = mm'/r^2 \qquad (228)$$

that it is extremely simple, and at the same time of the utmost generality. Accordingly 235 years after the publication of the Principia (1687), we find that the *Newton*ian law still accounts perfectly for the observed motions of the celestial bodies, except as modified by the addition of the small terms in *Weber*'s law of 1846, which take account of the induction and change of the induction, under wave-action propagated across space with the velocity of light.

A favorite motto from *Shakespeare*'s King Lear, scene II, was adopted by *Gauss*, in physical science:

»Thou, nature, art my goddess; to thy laws my services are bound;«

and the rule for very perfect productions:

»Pauca sed matura.«

Thus judged by the experience of this great mathematician, the chief object of the explorer of nature is the discovery of general and universal laws which are highly perfect and admit of no exceptions. Accordingly we notice the analytical similarity of the laws of magnetic intensity to the *Newton*ian law of gravitation:

$$I = f + f' = \mu\mu'/s^2 + \mu\mu'/s'^2 \qquad (229)$$

$$g = mm'/r^2 \qquad (228)$$

$$F = \mu\mu'/s^2 - \mu\mu'/s'^2, \qquad F' = \mu\mu'/s'^2 - \mu\mu'/s^2. \qquad (230)$$

The first force is directed to two poles, usually at unequal distances, s and s', along the curved lines of the magnetic force, while the second force is directed to a single centre, the centre of gravity of the whole mass. There is therefore a great similarity in the above laws, yet the difference inevitably resulting from rectilinear attraction towards a single centre, and the duality of powers resulting from stresses directed towards two centres or poles, along curved lines.

As magnetism is characterized by polarity, while gravitation shows an utter lack of polarity, and depends wholly on a mean central action only, one cannot doubt that the above laws of nature are ultimate.

The very simplicity of the law of magnetic attraction, and its close analogy with the *Newton*ian expression for gravitation, shows that the fundamental law of magnetism may be expected to withstand the ravages of time, just as the law of gravitation has done for over two centuries.

Perhaps the above formulae are typical of the highest laws of nature. Magnets exert magnetic attraction only through the mutual action of their poles; the forces are therefore directed to these poles, and follow extremely simple laws, although the action is conveyed along curved lines.

In the case of gravitation, on the other hand, the forces result from the integrations of the actions of the several atoms at their respective distances, yet practically this mean action reduces to a single force directed to the centre of gravity of the mass, at least for spheres and spheroids differing but little from spheres.

We perceive, therefore, as the result of experience, that the great laws of nature are extremely simple, and of the utmost generality in their character. When these qualities are not assured by any announced law, it is to be suspected that what may be called a law of nature is not ultimate, but at best only a first approximation, and sometimes wholly erroneous, without being even approximately true.

The formulae of Relativity[1]) obviously fall in the class of rejected laws. The theory is too complex, vague and chimerical to represent any permanent advance in physical science.

It is justly remarked that it has all the weakness of the Ptolemaic system of astronomy, without the extenuating circumstance, applicable to *Ptolemy*'s complex system, that it arose in a primitive age, when knowledge of the heavens was not yet subjected to the Experimentum Crucis of an exact test. Moreover, Relativity was born of complexity, not of simplicity; it involves so much misapplication of mathematics, without physical basis, — such as the so-called four-dimensional time-space manifolds, curvature of space, geodesic curves, etc. — that an experienced natural philosopher cannot defend it. Thus the doctrine of Relativity has not enlightened, but rather confused the scientific world. Out of all this mass of discussion not one clear truth has emerged, except perhaps the warning to beware of doctrines which pride themselves on their complexity.

The law for the ponderomotive force which two magnets exert on each other, given in equation (230) above, is equally remarkable for its simplicity and its generality. There can be no doubt whatever that this law is ultimate. It is easily verified by experiment, and follows at once from the law of magnetic intensity given above in equation (229).

It frequently is said that the successors of *Kepler* and *Newton* have been so much occupied with the verification of the laws of these great masters, under the complex conditions of the actual universe, that the more recent investigators seldom have been able to add to the laws of nature. Undoubtedly it is a disadvantage to be so much bewildered by a multitude of secondary phenomena that we lose sight of the significant outstanding features of any problem.

No doubt the modern students of magnetism have been bewildered with refinements, and thus were unable to discover the errors in the foundations of the theory of magnetism handed down by traditions dating from 1576, 1600 and 1744. When such fundamental errors of principle are involved in our theories, the mere refinement of measurement will add little to our stock of physical knowledge. Before real progress can be expected the erroneous theory must be unconditionally thrown overboard, and a new start made, on the basis of true laws of nature, which may be recognized, chiefly by their simplicity.

Often we must turn our eyes away from great masses of observations, lest the simplicity of general laws be lost sight of in a hopeless mass of bewildering detail. The investigator often is in the position of the explorer who cannot see the general character of the forest on account of the trees which crowd too close about him. Thus in the present investigation I have depended much less upon the magnetic observations of the past 75 years, than upon the general results already known to *Humboldt* and *Gauss*. As it was the data of their arbitrary scale of intensities, for the earth's magnetism, which enabled me to reach the general laws here given, I have retained their scale in the maps and diagrams of this paper. The multiplier given by *Sabine* for converting these numbers into the absolute scale is 7.57.

And since it was a somewhat hidden diamond accidentally dropped by *Gauss* amidst his profound calculations on the amount of magnetism in the earth, that finally enabled me to work out and verify the connection between the magnetism of the earth and terrestrial gravitation[2]), I gratefully crown the great mathematician with a laurel wreath, by selecting his portrait for the frontispiece to this concluding paper on the New Theory of the Aether.

[1]) Ever since completing the first paper, AN 5044, Jan. 14, 1920, the present writer has recognized clearly the fallacy of the *Einstein* Theory of Relativity. Valid mathematical and physical reasons for the rejection of the whole theory are given in AN 5044, and in AN 5048, the latter paper dated Febr. 19, 1920. It is gratifying to notice that Prof. *Paul Painlévé* and Prof. *Emile Picard,* perpetual secretary, in important communications to the Paris Academy of Sciences, Oct. 24, 1921, have confirmed my conclusions from somewhat different points of view. Both of these eminent mathematicians reject the doctrines of *Einstein*, and support the *Newton*ian mechanics. As they had received copies of the successive papers on the New Theory of the Aether, their announced support of the *Newton*ian doctrines in natural philosophy is not surprising. Under the kinetic theory of the Aether any other position is wholly untenable, but they did well to give public notice of the danger into which many unwary societies and investigators had fallen.

[2]) In an article, written for Popular Astronomy, about 1894, it was stated by Prof. *Asaph Hall* that in his time *Laplace* had done more for astronomy than all the Universities of Europe combined. In the same way it is evident that *Gauss* did more for terrestrial magnetism than all other authorities of every age combined, and henceforth his work takes on vastly increased significance, from the connection now established between magnetism and universal gravitation. Since 235 years have elapsed since the publication of *Newton*'s Principia, and yet no real progress was made towards solving the problem of the cause of gravitation, or showing that gravitation is an electrodynamic phenomenon, we think it very improbable that a solution could be effected without the results given by *Gauss*' method, as further developed in the present paper.

And since *Faraday*'s attempts at experimental proof of a connection between magnetism and gravitation failed, yet the present analytical method of attacking the problem succeeded, by virtue of *Gauss*' theory, I look upon *Gauss*' theory as one of the most precious products of the human intellect. For when every other resource failed, *Gauss*' theory admitted of analytical development which enabled us to solve the greatest outstanding physical problem of the centuries!

As the genius of *Gauss* incidentally brought out the results which gave the fractional part of the earth that is magnetic, and thereby made it possible to connect the magnetism of the globe with terrestrial gravitation, thus breaking down the hitherto inscrutable isolation of the chief force of nature, we may exclaim of *Gauss*, even more appropriately than *Fourier* could of *Laplace*:

»It is the great mathematician whose memory we celebrate.«

By a fortunate circumstance it turns out that *Humboldt* was not only the life-long friend and cooperator with *Gauss* in establishing the study of terrestrial magnetism upon a scientific basis, but also the idol of my youth, the perpetual inspiration to a career of discovery. These early impressions were of the greatest influence during my student days at the university of Berlin, so that I very frequently visited *Humboldt*'s country place at Tegel. Who knows what influence these associations have had in leading me to a rigorous proof of the connection between magnetism and terrestrial gravitation, since these researches were begun in 1914, and such a connection definitely indicated to the Royal Society of London?

Humboldt regarded his discovery of the law of the increase of the earth's total magnetic intensity from the equator towards the poles as the most important result of his American voyage of discovery, 1798–1804. Accordingly, in veneration of the memory of this great man[1]), I cite his own account of the discovery of this great law (Cosmos, *Bohn* Translation, vol. I, pp. 179–181):

»The following is the history of the discovery of the law that the intensity of the force increases (in general) with the magnetic latitude. When I was anxious to attach myself in 1798 to the expedition of captain *Baudin*, who intended to circumnavigate the globe, I was requested by *Borda*, who took a warm interest in the success of my project, to examine the oscillations of a vertical needle in the magnetic meridian in different latitudes in each hemisphere, in order to determine whether the intensity of the force was the same, or whether it varied in different places. During my travels in the tropical regions of America I paid much attention to this subject. I observed that the same needle which in the space of ten minutes made 245 oscillations in Paris, 246 in the Havana, and 242 in Mexico, performed only 216 oscillations during the same period at San Carlos del Rio Negro, (1°53′ north lat. and 80°40′ west long. from Paris), on the magnetic equator i. e. the line in which the inclination = 0, in Peru (7°1′ south lat. and 80°54′ west long. from Paris) only 211; while at Lima (12°2′ south lat.) the number rose to 219. I found in the years intervening between 1799 and 1803, that the whole force, if we assume it at 1.0000 on the magnetic equator in the Peruvian Andes, between Micuipampa and Caxamarca, may be expressed at Paris by 1.3482, in Mexico by 1.3155, in San Carlos del Rio Negro by 1.0480, and in Lima by 1.0773. When I developed this law of the variable intensity of terrestrial magnetic force, and supported it by the numerical value of observations instituted in 104 different places, in a Memoir read before the Paris Institute, on the 26th Frimaire, An XIII, (of which the mathematical portion was contributed by M. *Biot*), the facts were regarded as altogether new. It was only after the reading of the paper, as *Biot* expressly states (*Lamétherie*, Journal de Physique, t. 59, p. 446, note 2), and as I have repeated in the Relation historique, t. 1, p. 262, note 1, that Mr. *de Rossel* communicated to *Biot* his oscillation experiments made six years earlier (between 1791 and 1794) in Van Diemen's Land, in Java, and in Amboina. These experiments gave evidence of the same law of decreasing force in the Indian Archipelago. It must, I think, be supposed that this excellent man, when he wrote his work, was not aware of the regularity of the augmentation and diminution of the intensity, as before the reading of my paper he never mentioned this (certainly not unimportant) physical law to any of our mutual friends, *Laplace*, *Delambre*, *Prony* or *Biot*. It was not till 1808, four years after my return from America, that the observations made by M. *de Rossel* were published in the Voyage de L'Entrecasteaux, t. 2, pp. 287, 291, 321, 480, and 644. Up to the present day it is still usual, in all the tables of magnetic intensity which have been published in Germany (*Hansteen*, Magnet. der Erde, 1819, p. 71; *Gauss*, Beob. des Magnet. Vereins, 1838, p. 36–39; *Erman*, Physikal. Beob., 1841, p. 529–579), in England (*Sabine*, Report on Magnet. Intensity, 1838, p. 43–62; Contributions to Terrestrial Magnetism, 1843), and in France (*Becquerel*, Traité d'Electr. et de Magnét., t. 7, p. 354–367), to reduce the oscillations observed in any part of the earth to the standard of force which I found on the magnetic equator in Northern Peru; so that, according to the unit thus arbitrarily assumed, the intensity of the magnetic force at Paris is put down as 1.348. The observations made by *Lamanon* in the unfortunate expedition of La Pérouse, during the stay at Teneriffe (1785), and on the voyage to Macao (1787), are still older than those of admiral *Rossel*. They were sent to the Academy of Sciences, and it is known that they were in the possession of *Condorcet* in the July of 1787 (*Becquerel*, t. 7, p. 320); but notwithstanding the most careful search, they are not now to be found. From a copy of a very important letter of *Lamanon*, now in the possession of captain *Duperrey*, which was adressed to the then perpetual secretary of the Academy of Sciences, but was omitted in the narrative of the Voyage de La Pérouse, it is stated 'that the attractive force of the magnet is less in the tropics than when we approach the

[1]) *Humboldt*'s extensive and varied researches on the magnetism of the earth extended over more than 60 years. In addition to his own notable series of observations, especially during the travels in America, 1799–1804, and the expedition to Central Asia, 1829, he secured the cooperation of the Russian and British Governments for the establishment of chains of magnetic observatories throughout the northern and southern hemispheres, 1830–1840, and by *Ross* in the Antarctic, 1841. He always cooperated with *Gauss* and *Weber*, and thus aided the observational foundation of *Gauss*' Allgemeine Theorie des Erdmagnetismus, 1838.

After eight years of researches in the wave-theory of physical forces, and the frequent use of *Humboldt*'s Cosmos for exactly forty years, the present author finally was enabled to establish between cosmical magnetism and universal gravitation the remarkable general law of nature: $I/g = \eta^2(r^2/s^2 + r^2/s'^2)$. Hence the addition of his portrait to this concluding paper on the New Theory of the Aether may be dedicated to the revered memory of *Humboldt*.

poles, and that the magnetic intensity deduced from the number of oscillations of the needle of the inclination-compass varies and increases with the latitude'. If the Academicians, while they continued to expect the return of the unfortunate La Pérouse, had felt themselves justified, in the course of 1787, in publishing a truth which had been independently discovered by no less than three different travellers, the theory of terrestrial magnetism would have been extended by the knowledge of a new class of observations, dating eighteen years earlier than they now do. This simple statement of facts may probably justify the observations contained in the third volume of my Relation historique (p. 615): — 'The observations on the variation of terrestrial magnetism, to which I have devoted myself for thirty-two years, by means of instruments, which admit of comparison with one another, in America, Europe and Asia, embrace an area extending over 188 degrees of longitude, from the frontier of Chinese Dzoungarie to the west of the South Sea bathing the coasts of Mexico and Peru, and reaching from 60° north lat. to 12° south lat. I regard the discovery of the law of the decrement of magnetic force from the pole to the equator, as the most important result of my American voyage'. Although not absolutely certain it is very probable that *Condorcet* read *Lamanon*'s letter of July, 1787, at a meeting of the Paris Academy of Sciences; and such a simple reading I regard as a sufficient act of publication. (Annuaire du Bureau des Longitudes, 1842, p. 463). The first recognition of the law belongs, therefore, beyond all question, to the companion of La Pérouse; but long disregarded or forgotten, the knowledge of the law that the intensity of the magnetic force of the earth varied with the latitude, did not, I conceive, acquire an existence in science until the publication of my observations from 1798 to 1804. The object and the length of this note will not be indifferent to those who are familiar with the recent history of magnetism, and the doubts that have been started in connection with it, and who, from their own experience, are aware that we are apt to attach some value to that which has cost us the uninterrupted labour of five years under the pressure of a tropical climate, and of perilous mountain expeditions.«

In conclusion it may be recalled that *Gauss* was much bewildered by the phenomenon of the Northern Light, adding (p. 50): »Die rätselhaften Erscheinungen des Nordlichts, bei welchem allem Anscheine nach Elektrizität in Bewegung eine Hauptrolle spielt« — »the puzzling appearance of the Northern Light, in which according to all appearances electricity in motion plays a leading part«. He adds that it will not do to deny the possibility of such electric currents, but it will be interesting to investigate how such electric currents would arise from the magnetic actions at the surface of the earth.

In the above paper we have not overlooked this recommendation of the great mathematician. On the contrary the new theory of molecular forces (AN 5130) yields so direct and simple an explanation for this harmless discharge of electric energy from the changing globules in the comparatively rare air of the cirrus clouds that the explanation fulfills all known physical conditions by assimilating the aurora to a kind of lightning of the upper atmosphere, which frequently is easily set off in delicate streamers by unusually violent disturbances of the earth's magnetism.

As it has taken eight long years to finish the work recommended by *Gauss*, 84 years ago, I think we may appropriately exclaim with the Poet *Oliver Wendell Holmes*:

»When darkness hid the starry skies
In war's long winter night,
One ray still cheered our straining eyes,
The far-off Northern Light!«
(*Holmes*' Poems, America to Russia, p. 199).

Very grateful acknowledgements are due to two eminent civil engineers, — Captain *Leonard M. Cox*, U. S. N., of Mare Island, and Mr. *Otto von Geldern* of San Francisco; to my friend Mr. *A. E. Axlund*, a very skillful draftsman who has aided in the preparation of the plates; to my associate Mr. *W. S. Trankle*, for contributing greatly to the early completion of the paper; and, above all others, to Mrs. *See*, for an unwavering faith that the hour for the triumph of light over darkness would finally come.

Starlight on Loutre, Montgomery City, Missouri, May 8, 1922.

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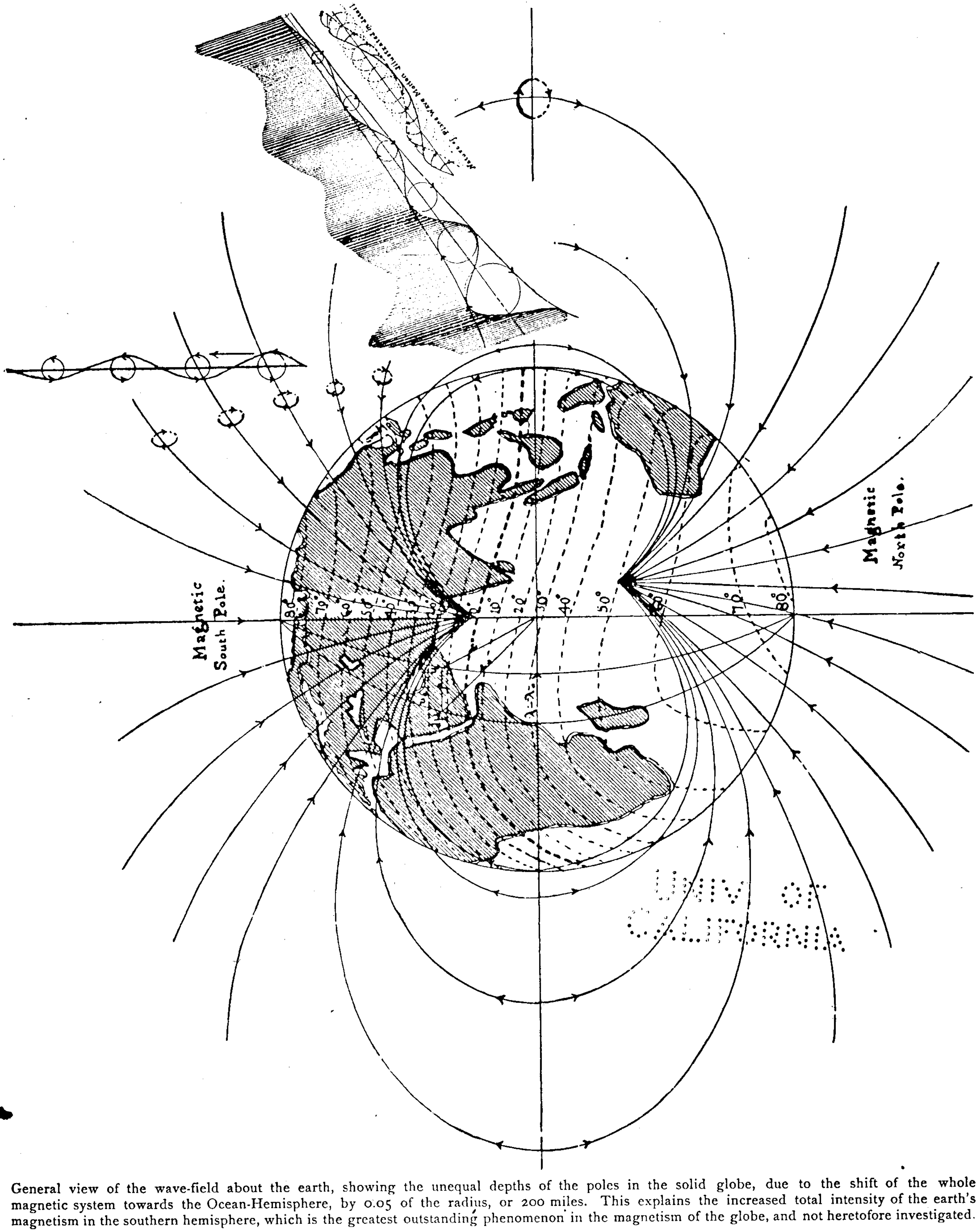
T. J. J. See. New Theory of the Aether.



General view of the wave-field about the earth, showing the unequal depths of the poles in the solid globe, due to the shift of the whole magnetic system towards the Ocean-Hemisphere, by 0.05 of the radius, or 200 miles. This explains the increased total intensity of the earth's magnetism in the southern hemisphere, which is the greatest outstanding phenomenon in the magnetism of the globe, and not heretofore investigated.

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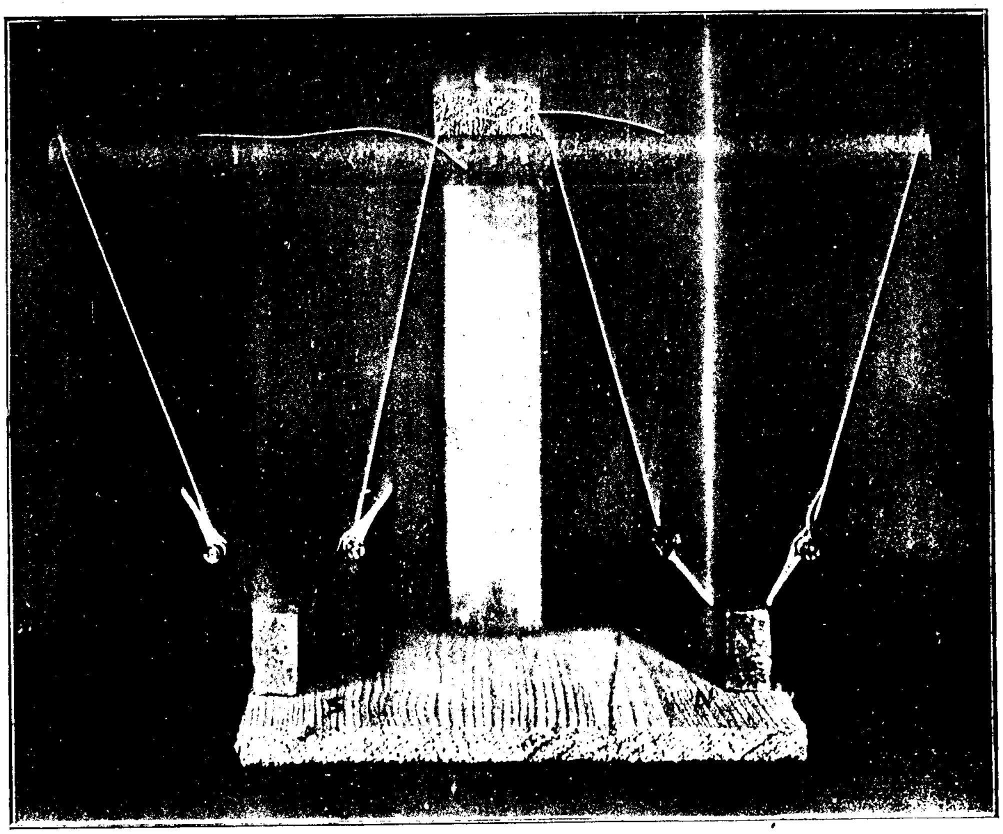

Fig. 1. Photograph of four small magnetic needles, suspended by threads, near a larger strong magnet, which pulls them over bodily to the adjacent poles, and thereby indicates unmistakably the nature of the ponderomotive force. This photograph discloses the true nature of magnetism, and sweeps away the false principles which have come down from the time of *Gilbert*, 1600, and *Euler*, 1744.

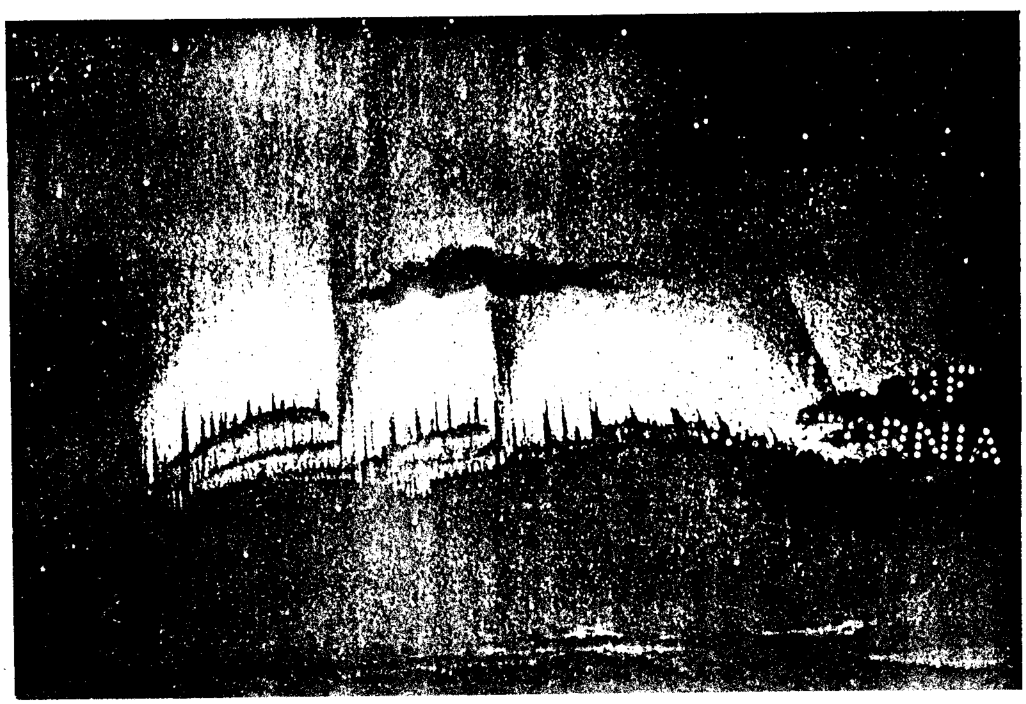

Fig. 2. A typical view of the Aurora Borealis, showing extensive curtain, with ribbon folds at the lower border, and stars visible through the illumination. An aurora of this general aspect was observed by the author at Madison, Wisconsin, Sept. 29, 1895. The brilliant aurora observed at Mare Island, California, May 14, 1921, had less of the well defined curtain, but more of the delicate streamers, and they appeared and disappeared wherever certain thin clouds became visible in the sky. This proves that the aurora is a kind of lightning of the upper atmosphere, the unstable aether stress on the surfaces of the drops escaping as an electric discharge, under the decreased resistance of the rare air, by the mere disturbances of the earth's magnetism, yet this lightning never reaches the surface of the earth like that of the lower atmosphere.

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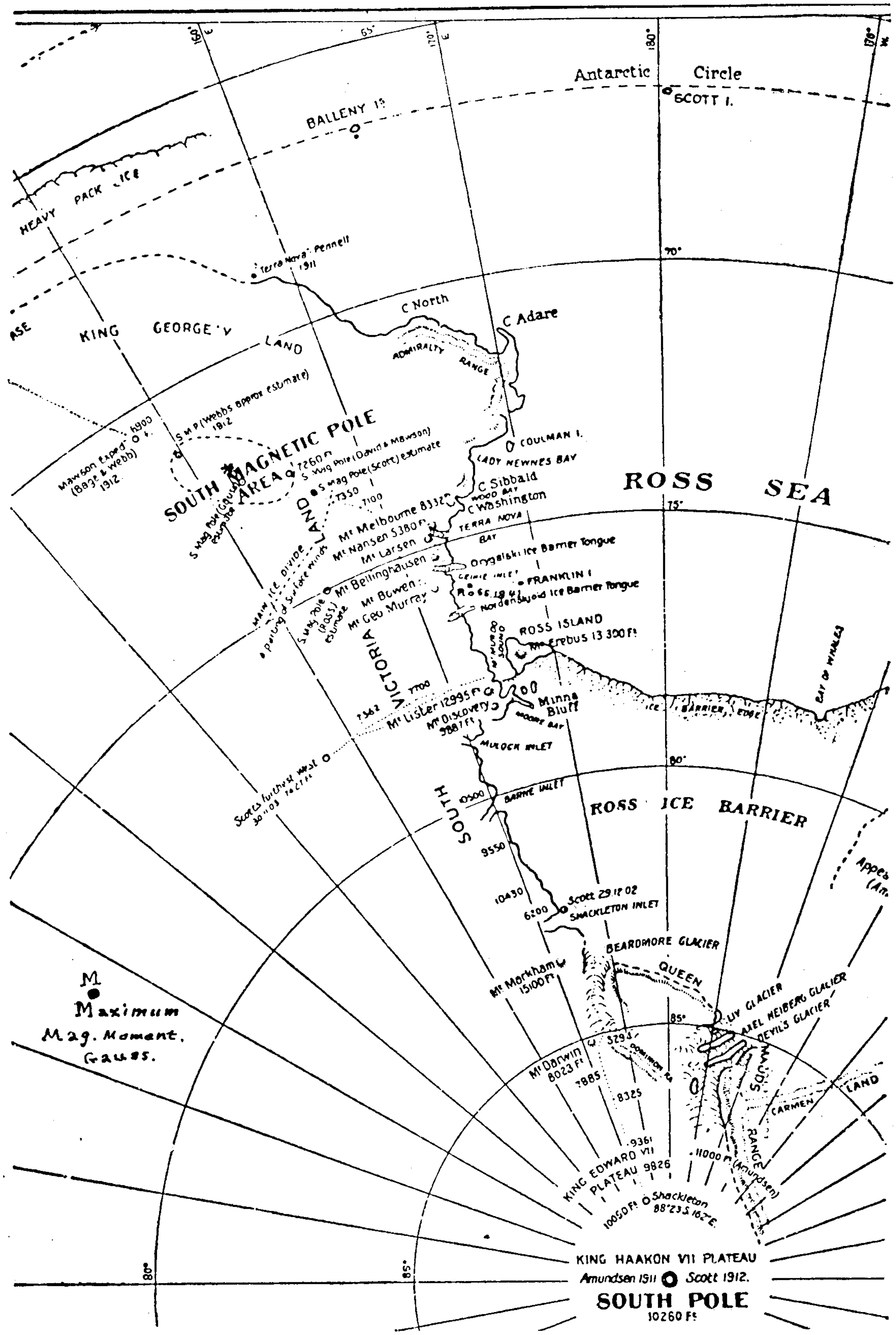

General map of the region of the south magnetic pole, and of the journeys over the ice by *Mawson* from Ross Sea, 1909, and from the other side, 1912. The most probable position of the magnetic pole is shown to be at the star (*), so very near the position assigned by *Gauss'* Theory, 1838, that no certain evidence of motion exists, although 70 years elapsed between *Gauss'* calculations just prior to *Ross'* observations, 1841, and the *Shackleton-Mawson* explorations, 1909–1912.

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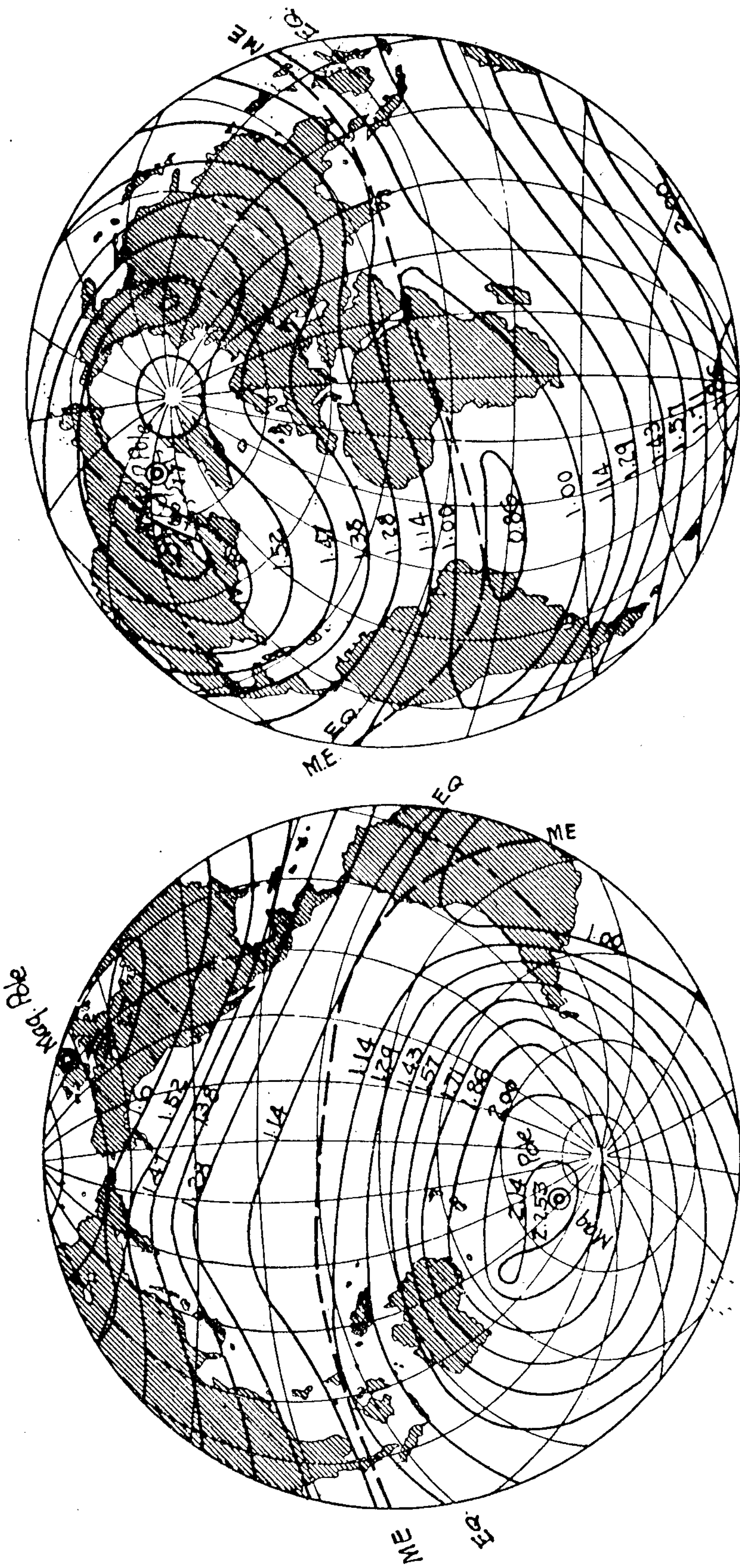

Isodynamic lines of the total intensity of the earth's magnetism, for the equatorial regions. The scale of intensity used is that employed by *Humboldt* and *Gauss*, yet we may pass to the absolute scale by multiplying these numbers by 7·57 (*Sabine*).

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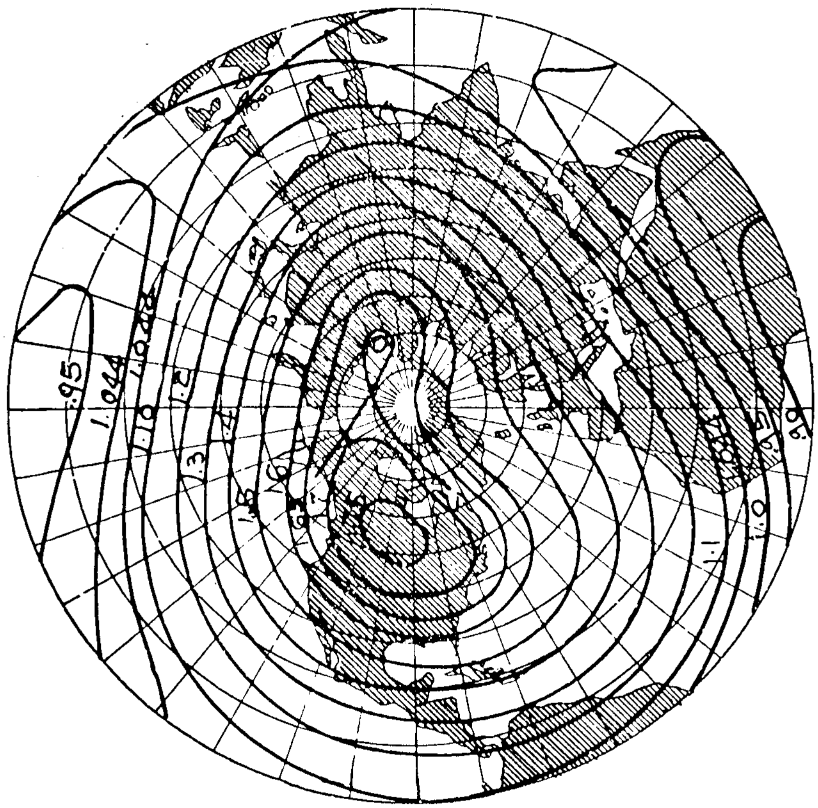

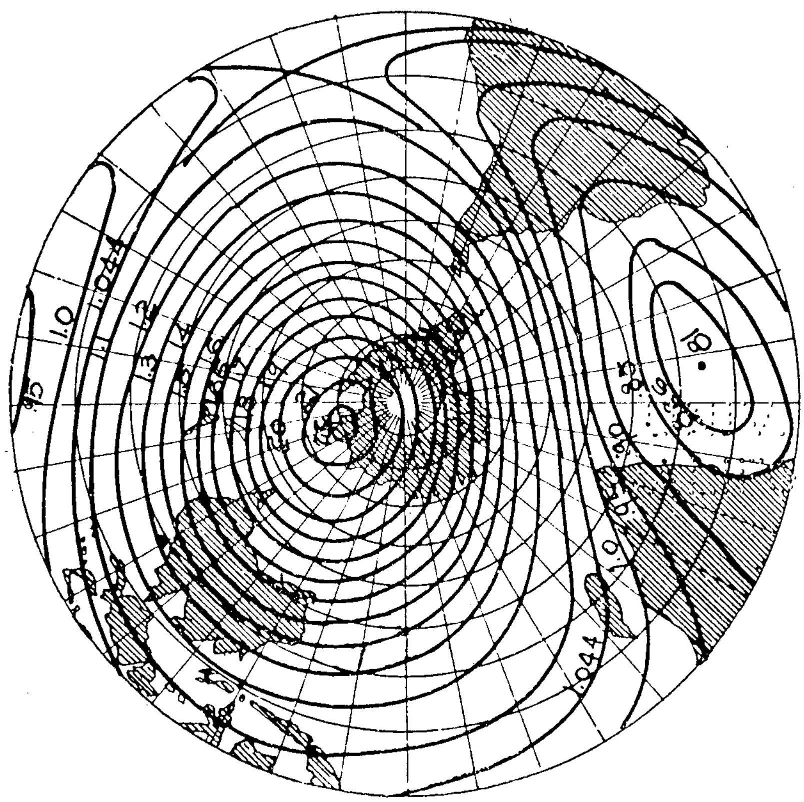

Isodynamic lines for the polar regions of the earth, showing the decreased total intensity in the northern hemisphere which indicates a depth of 0.766 *r* for this pole, while the south pole has a depth of only 0.666 *r*.

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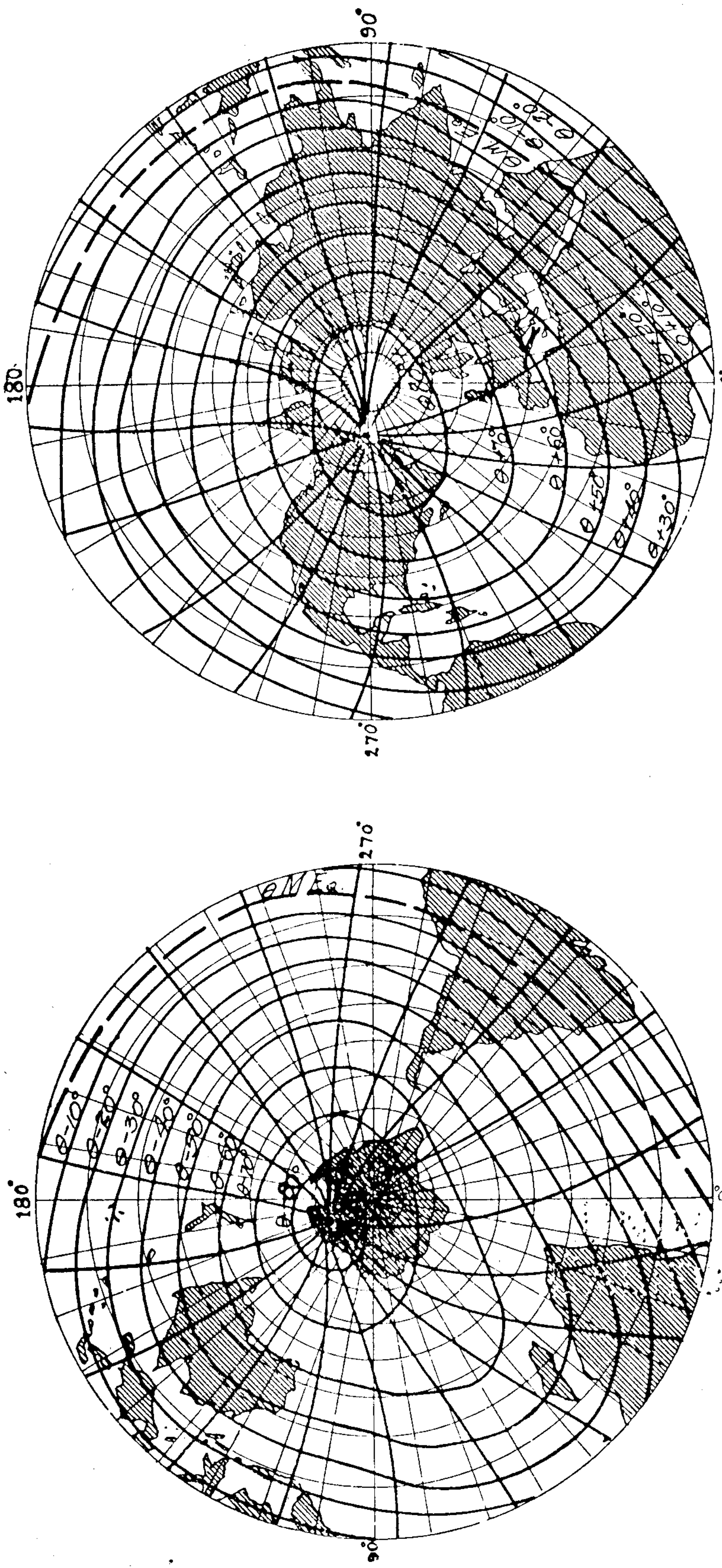

View of the two poles, showing lines of equal dip, and the considerable curvature of the magnetic meridians, which becomes very sensible near the poles.

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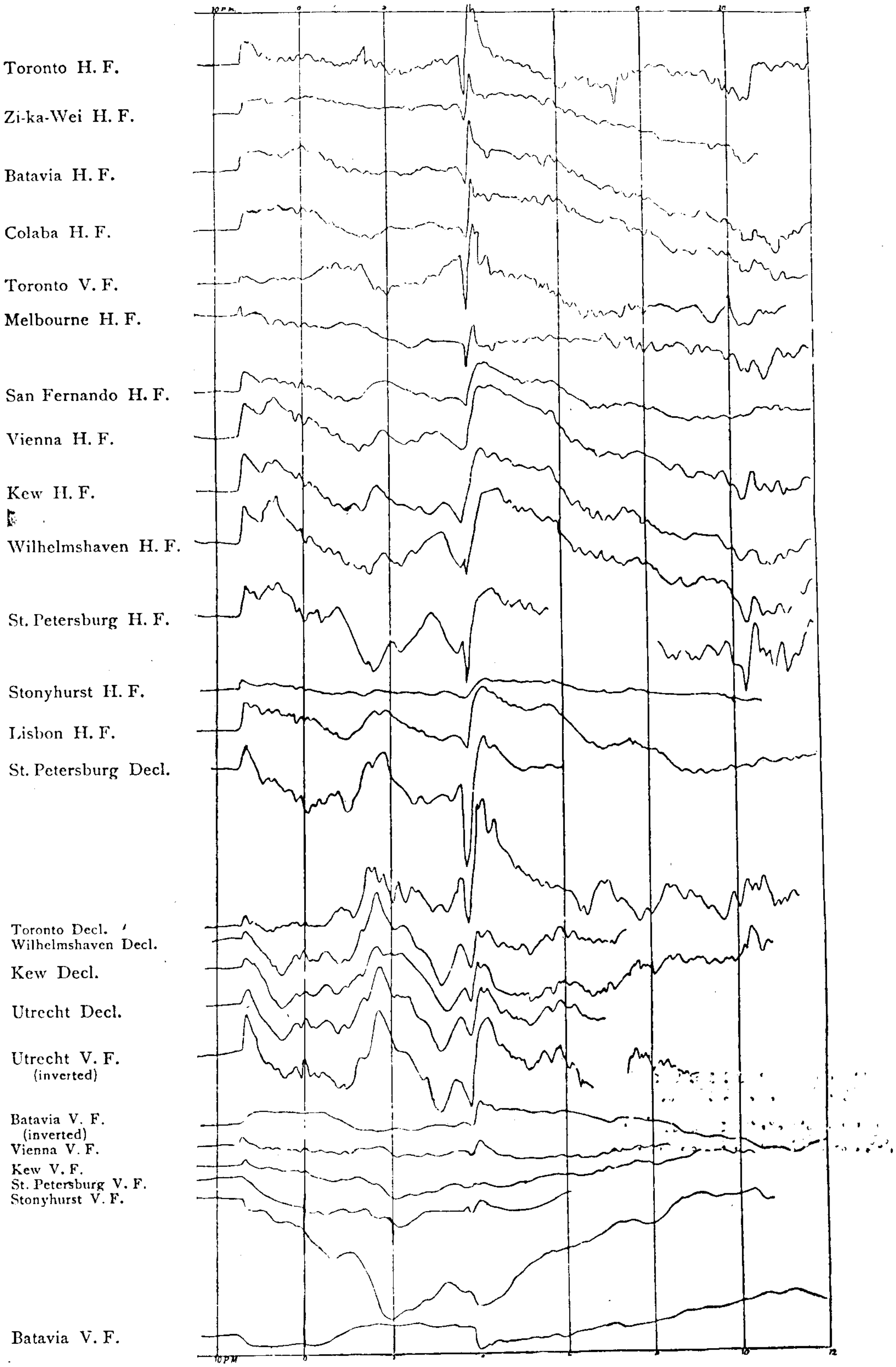

Professor *W. Grylls Adams'* record of simultaneous disturbances of the magnetic needle throughout the world, in the great 'Magnetic Storm' of June 25, 1885.

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