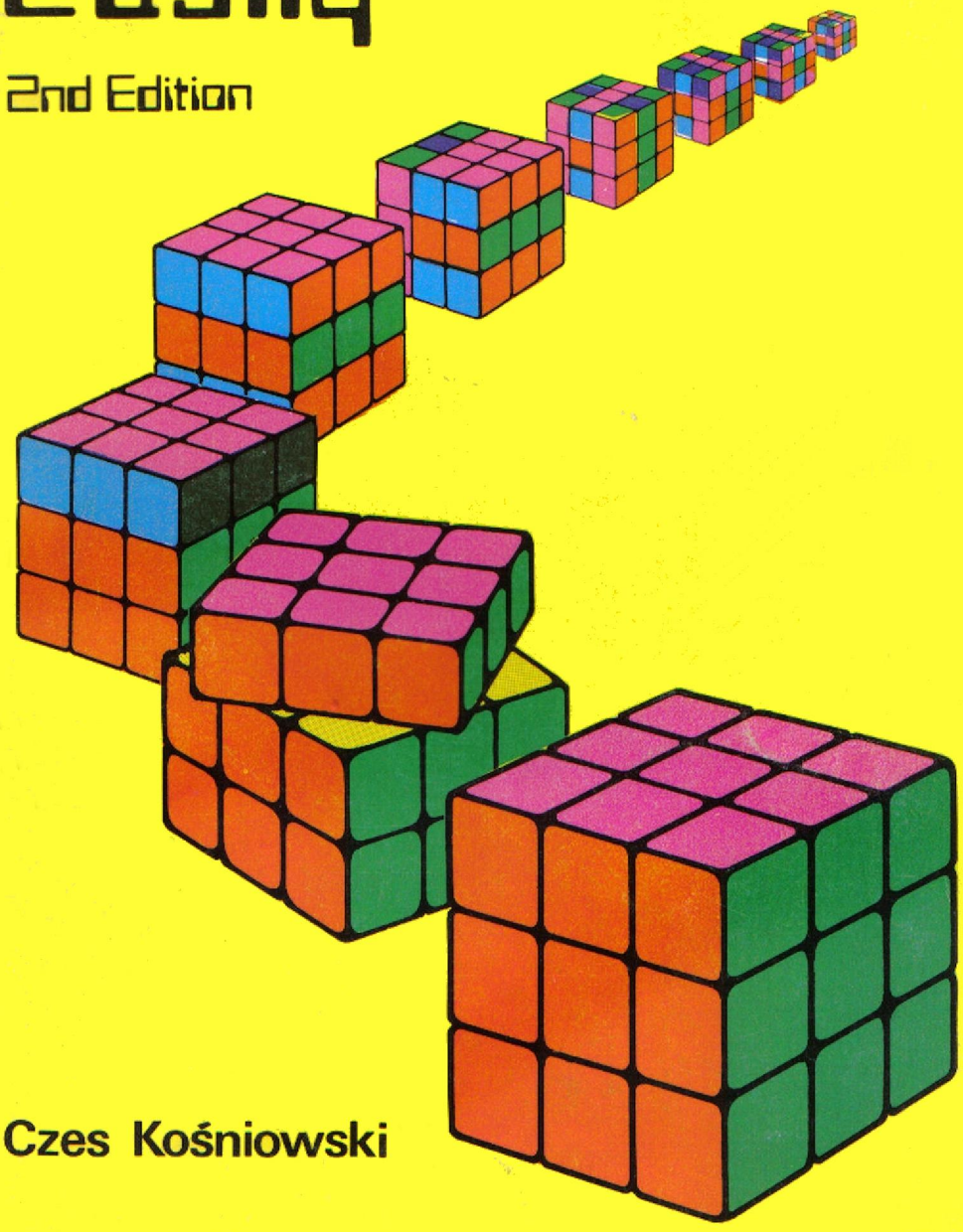
Master the Rubik Cube Easily

2nd Edition



Czes Kośniowski

MASTER THE RUBIK CUBE EASILY

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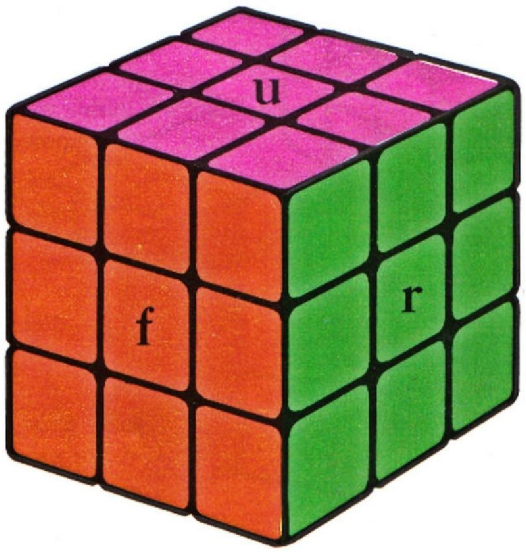
The cube was invented by a Hungarian Professor of Architecture, Ernó Rubik, of the School of Commercial Artists in Budapest.

Rubik's Cube is compulsive. Some hit lucky and get a solution in a few minutes, others struggle for many frustrating months.

This booklet gives an easy, step by step solution to Rubik's Magic Cube. With this booklet the cube no longer remains a mystery to you. Solutions to puzzles related to Rubik's Cube are also given and some pretty patterns are described in the last few pages.

All the moves described in this booklet have been proved mathematically. The solution always works provided you follow the instructions carefully.



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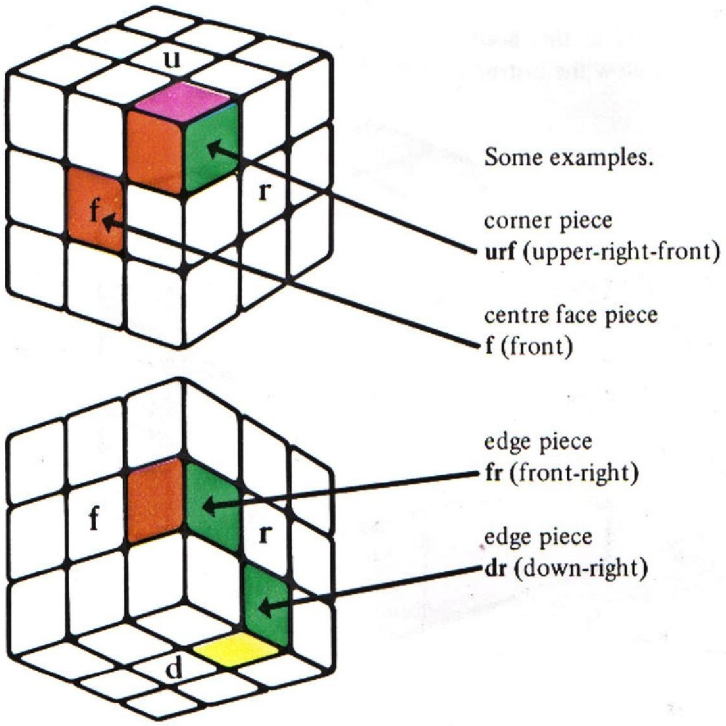
THE PIECES

The six sides of the cube will be referred to as

f front
b back
r right
l left
u upper
d down

Any particular piece of the cube can be designated by the lower case letters naming the face it belongs to. Thus **fr** (or **rf**) stands for the edge piece on the front-right side while **urf** (or **fur** or **fru** or . . .) is the corner piece in the upper-right-front position.



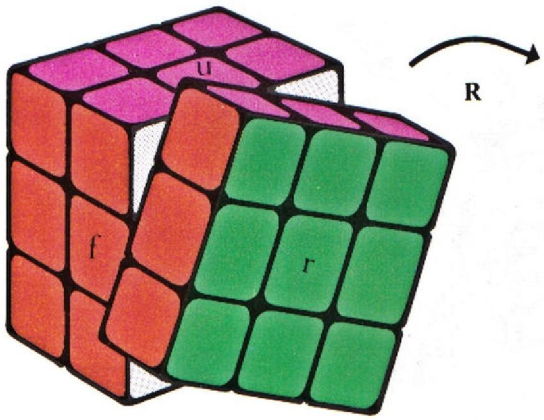
DISMANTLING THE CUBE

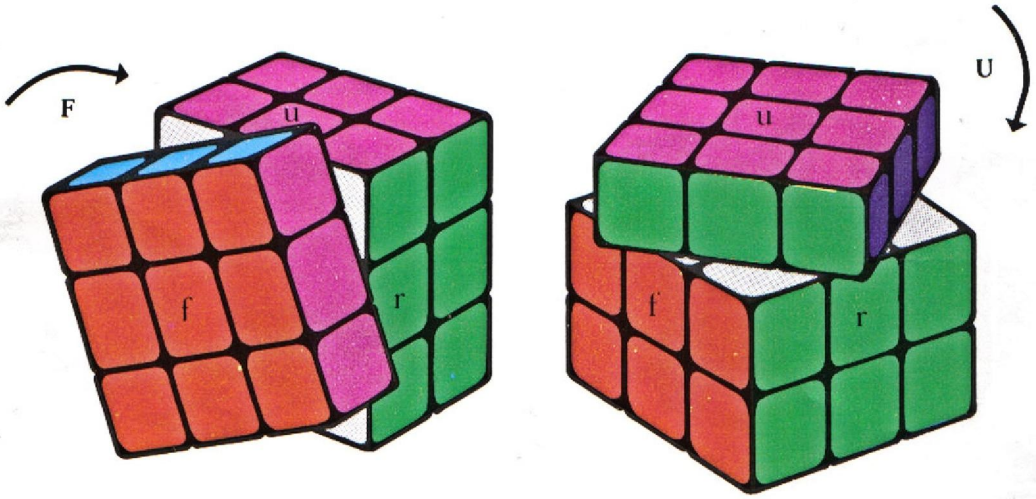
The cube may be dismantled and reassembled quite easily. Turn the top face about 45° and with a screw driver gently prise up an upper edge piece which should easily pop out. It is then possible to remove all other pieces (except the centre face pieces) – no force is necessary. The cube can be easily reassembled leaving an upper edge piece to be replaced last, again the top face should be turned about 45° to pop the last piece in. Warning: Reassemble the cube with all pieces correctly in place – unless of course you are playing tricks on a friend.

THE MOVES

Capital letters will denote a clockwise quarter-turn of the corresponding face (clockwise as viewed by an observer looking at the face).

F front
B back
R right
L left
U upper
D down



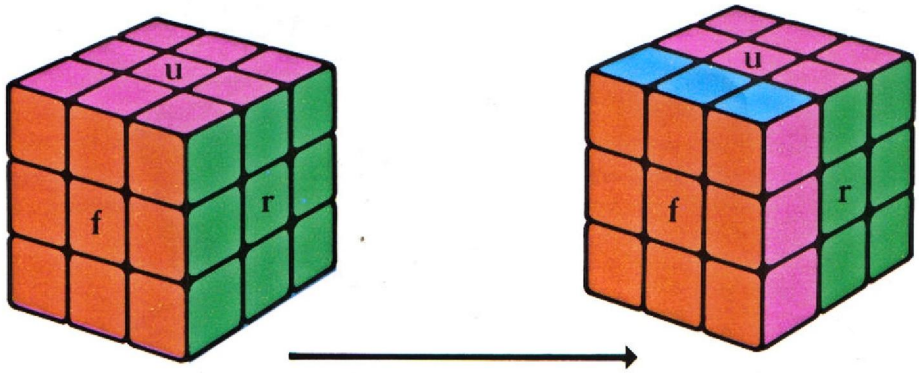


R clockwise quarter-turn of right face (i.e. turn right face clockwise through 90°) etc.
$\mathbf{R}^2$ two clockwise quarter-turns, etc.
$\mathbf{R}^{-1}$ anticlockwise quarter-turn, etc.

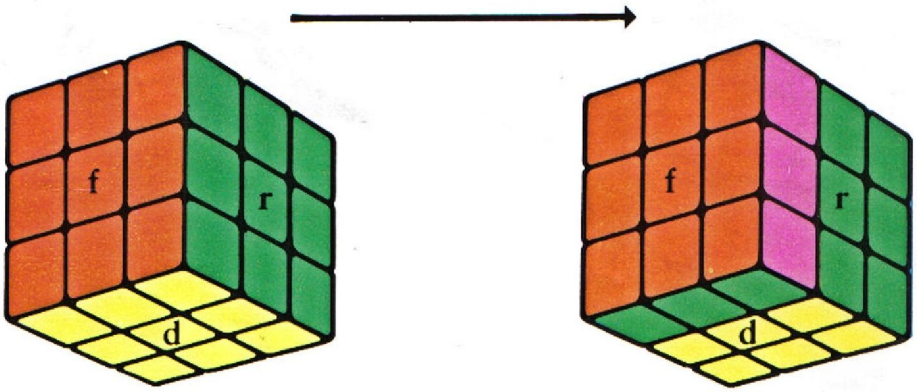
Note. $\mathbf{R}^{-1} = \mathbf{R}^3$, etc.

A MOVE

We record the effect that a move has on a cube by diagrams. The diagrams will often show two views of the cube. For example: The move **F**.



F



We also record the effect of a move by symbols, for example: The move **F**.

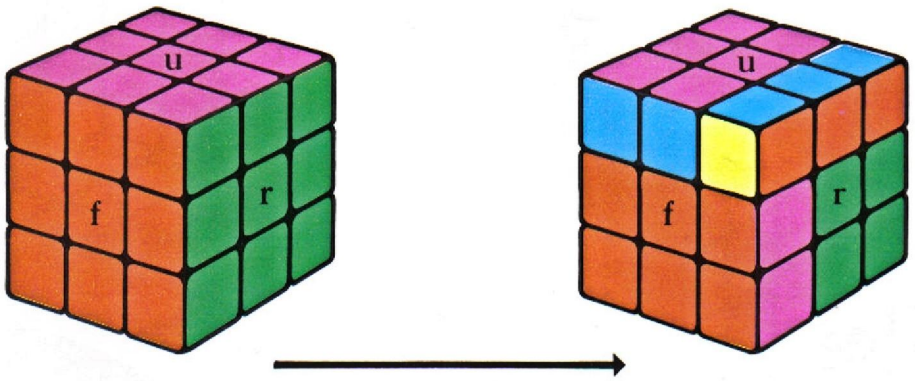
fr → **fd**
fur → **frd**
ur → **ur** etc.

Note that **fr** → **fd** means that the **f** part of **fr** goes to the **f** part of **fd** and the **r** part of **fr** goes to the **d** part of **fd**.

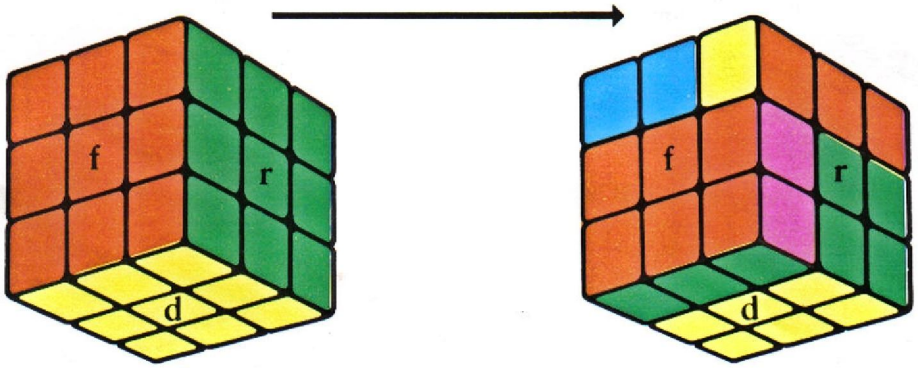
It is not necessary to follow these symbols, they have been included for precision and as a possible aid to the colour-blind.

ANOTHER MOVE

The notation FU^{-1} means **F** followed by U^{-1}. The result of this move is shown below.



FU^{-1}



fur → frd
fdl → rfu
fd → fl

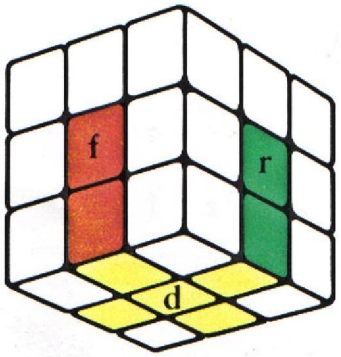
etc.

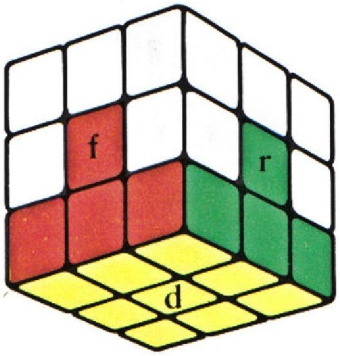
In a long sequence of moves brackets are sometimes used, for example $F(URU^{-1}R^{-1})F^{-1}$, these brackets are included just as an aid to memory.

OUTLINE OF SOLUTION

The solution to the cube given here is in seven stages. Each stage requires several sequences of moves, these are described in the following pages. Briefly the seven stages are as follows.

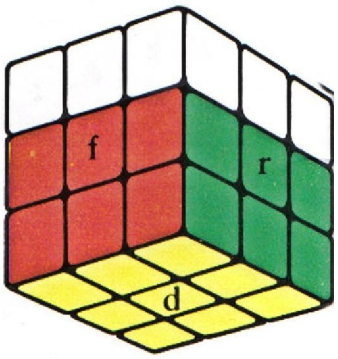
1) Choose a colour for "down" and place the four edge pieces that belong to the bottom layer correctly in position.

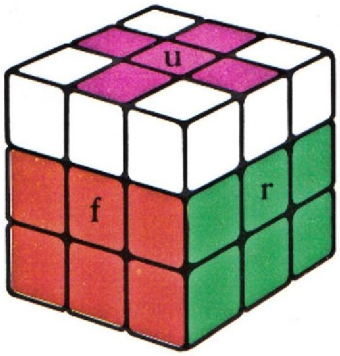




2) Place the four corner pieces that belong to the bottom layer correctly in position.

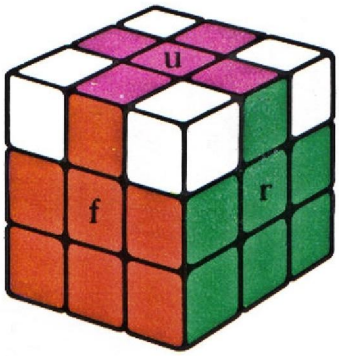
3) Place the four edge pieces that belong to the middle layer correctly in position.

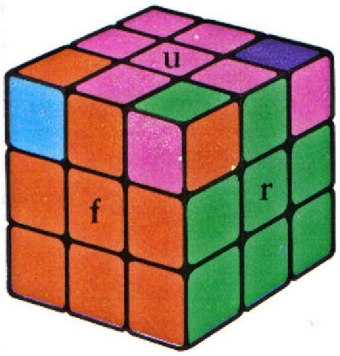




4) "Flip" the upper edge pieces to their correct colour, making a "cross".

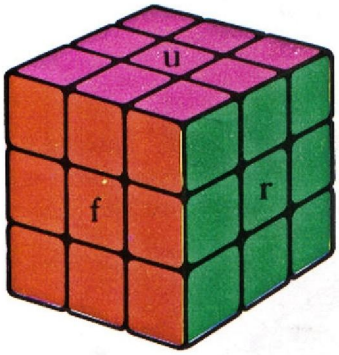
5) Put the top edge pieces in correct position.





6) Put top corners into their position, possibly facing the wrong way.

7) Put top corners in correctly.



NOTES

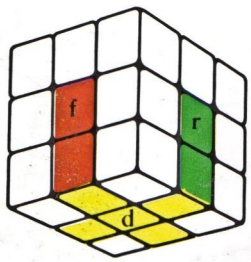
Sometimes it is not necessary to go through all seven stages.

With a little practise each stage becomes easier and more automatic.

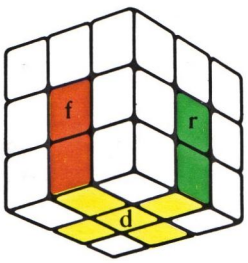
In the diagrams not all pieces are shown coloured. We only colour those necessary to show what is happening.

Note that the colours on your cube may be different to those in the diagrams in this booklet.

Warning. Whenever you start a sequence of moves do not be distracted until you have finished – otherwise you may have to go right back to the beginning!



STAGE 1

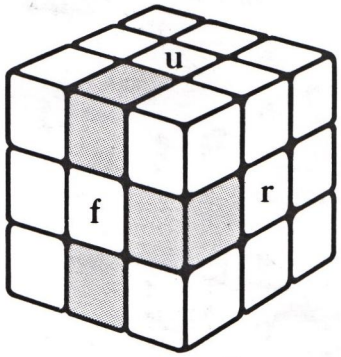


Choose a colour for "down" **(d)** – in these notes yellow will be the down colour and the yellow centre piece will always remain down. The opposite centre piece will always remain up. Any of the other four centre pieces can be "front" **(f)** simply by rotating the whole cube (keeping "down" down). Although the up and down centre pieces remain fixed, the "front", "right", "left" and "back" ones are not fixed except temporarily when making a sequence of moves.

Having chosen a colour for down the first stage is to place the four edge pieces that belong in the bottom layer **(df, dr, dl, db)** into their correct position (i.e. so that their colours match the centre piece colours).

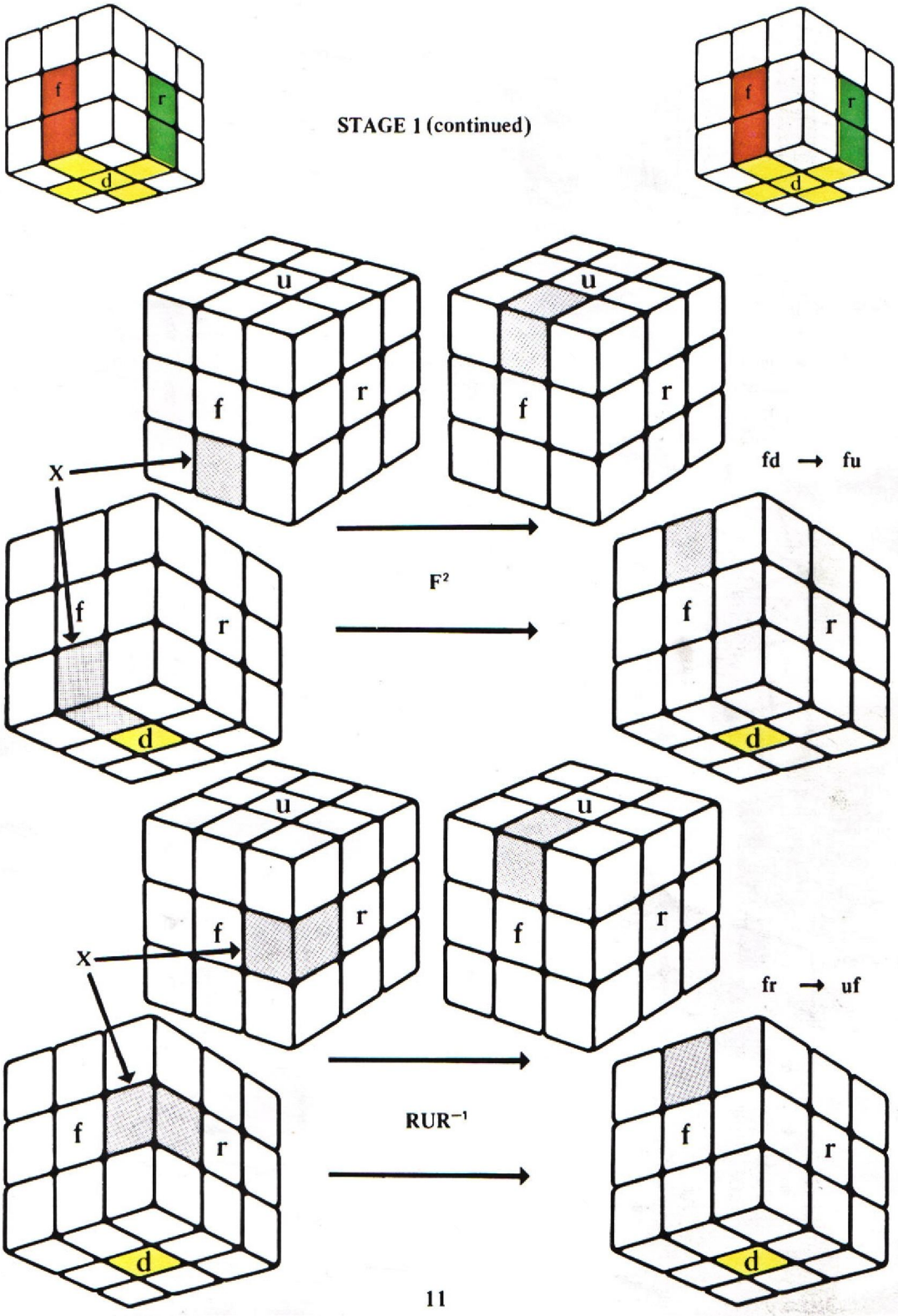
This is achieved in two steps, first by moving the edge piece to the upper layer and then to its correct position in the bottom layer. Proceed as follows:

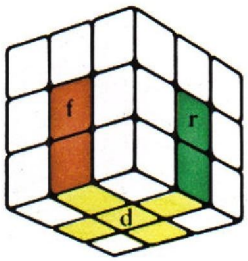
Beginning. Choose a colour for front (eg. red). Call the piece that belongs to the down-front position **X** (for example if **f** is red and **d** is yellow then **X** is the red-yellow edge piece). (If **X** is correctly positioned then choose another colour for front).



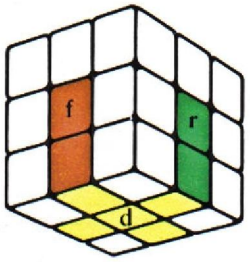
Now rotate the whole cube (keeping "down" down) until you find **X**. By rotating the cube some more if necessary (always keeping "down" down) **X** will be in one of three positions front-down, front-right or front-upper.

By using $\mathbf{F^2}$ or $\mathbf{RUR^{-1}}$ (see diagrams on opposite page) you can ensure that **X** is in the upper layer (i.e. in the front-upper position). These moves are designed so that any down edge piece already in its correct position is not accidentally moved out.

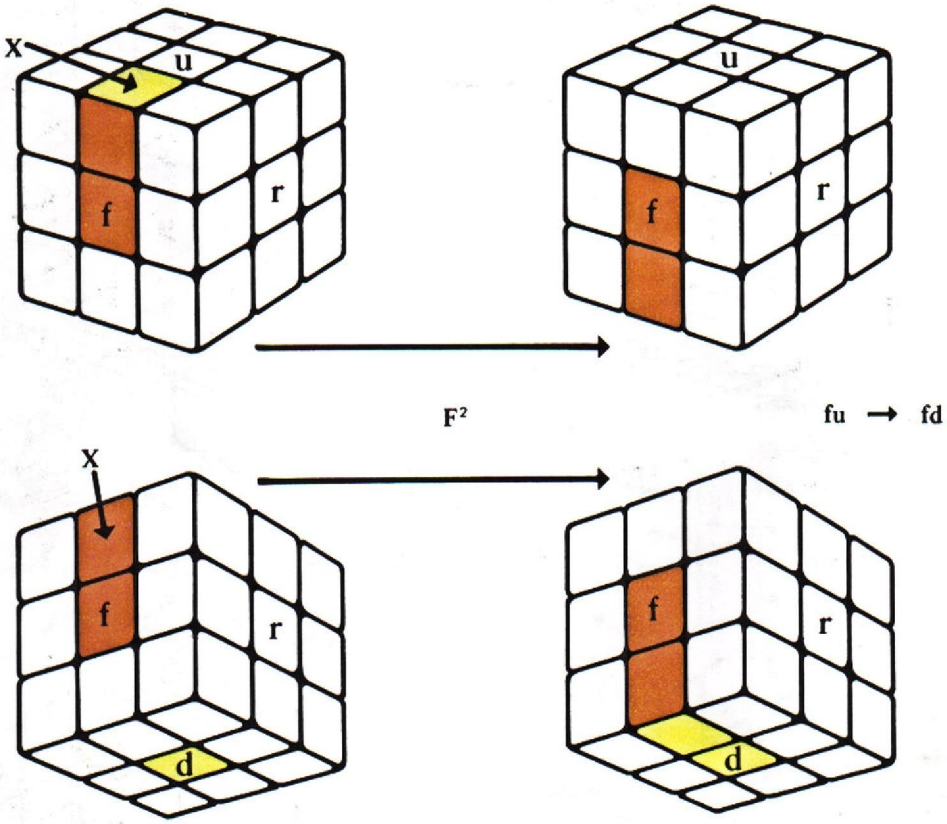

STAGE 1 (continued)
f
r
d
u
f
r
d
u
f
r
X
fd → fu
F²
f
r
d
u
f
r
u
f
r
X
fr → uf
RUR⁻¹
f
r
d

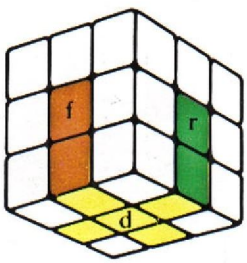


STAGE 1 (continued)

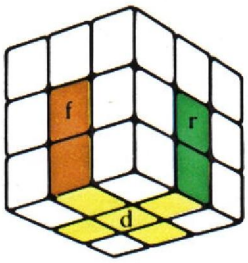


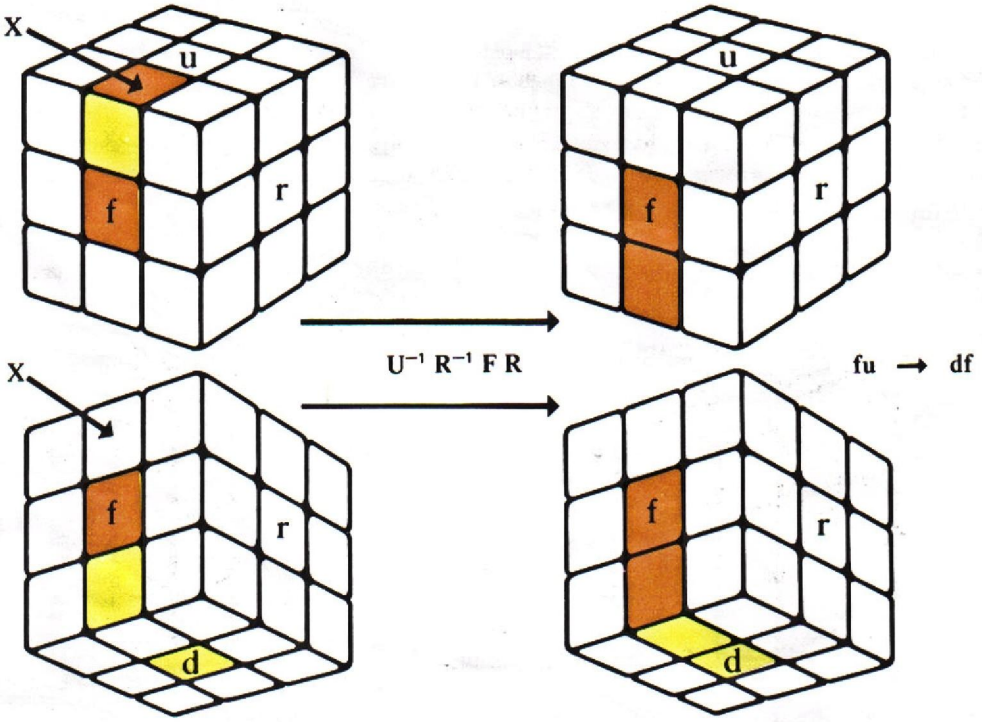
Now rotate the whole cube back so that your chosen colour (red) is in front. By using $\mathbf{U}, \mathbf{U}^2, \ldots$ we can position **X** in the front-upper position. To get **X** into its correct position use $\mathbf{F}^2$ or $\mathbf{U}^{-1}\ \mathbf{R}^{-1}\ \mathbf{F}\ \mathbf{R}$ – see the diagrams. These moves are designed so that any down edge piece already in its correct position is not accidentally moved out.





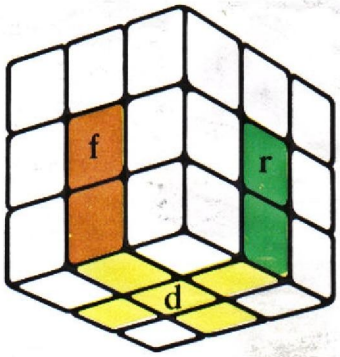
STAGE 1 (continued)

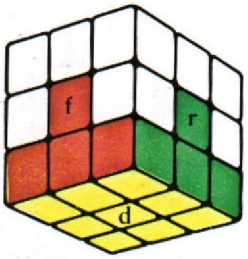




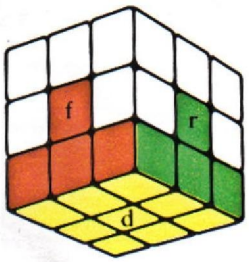
Now go back to the beginning and choose a different colour for front. Repeat the process (always keeping "down" down) until all four down edge pieces are in their correct position.

On completion of stage 1 your cube should resemble the following:





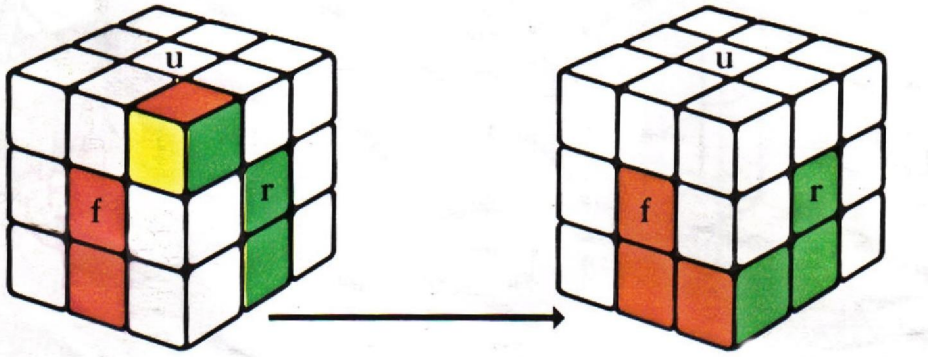
STAGE 2



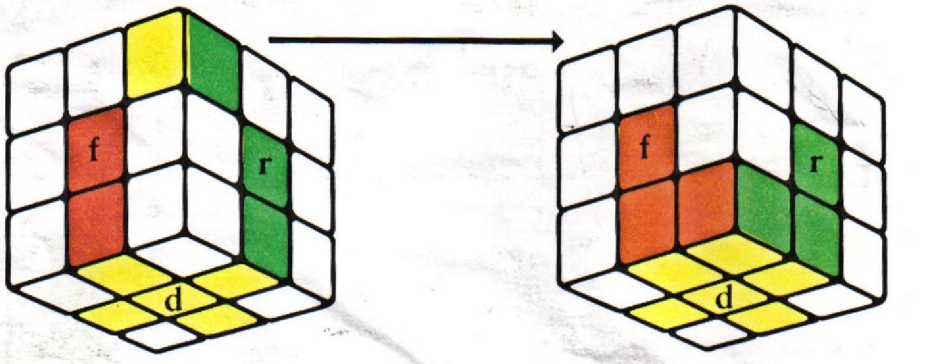
The second stage is to put the four corner pieces that belong to the bottom layer correctly into position. Do the following for each of the four pieces.

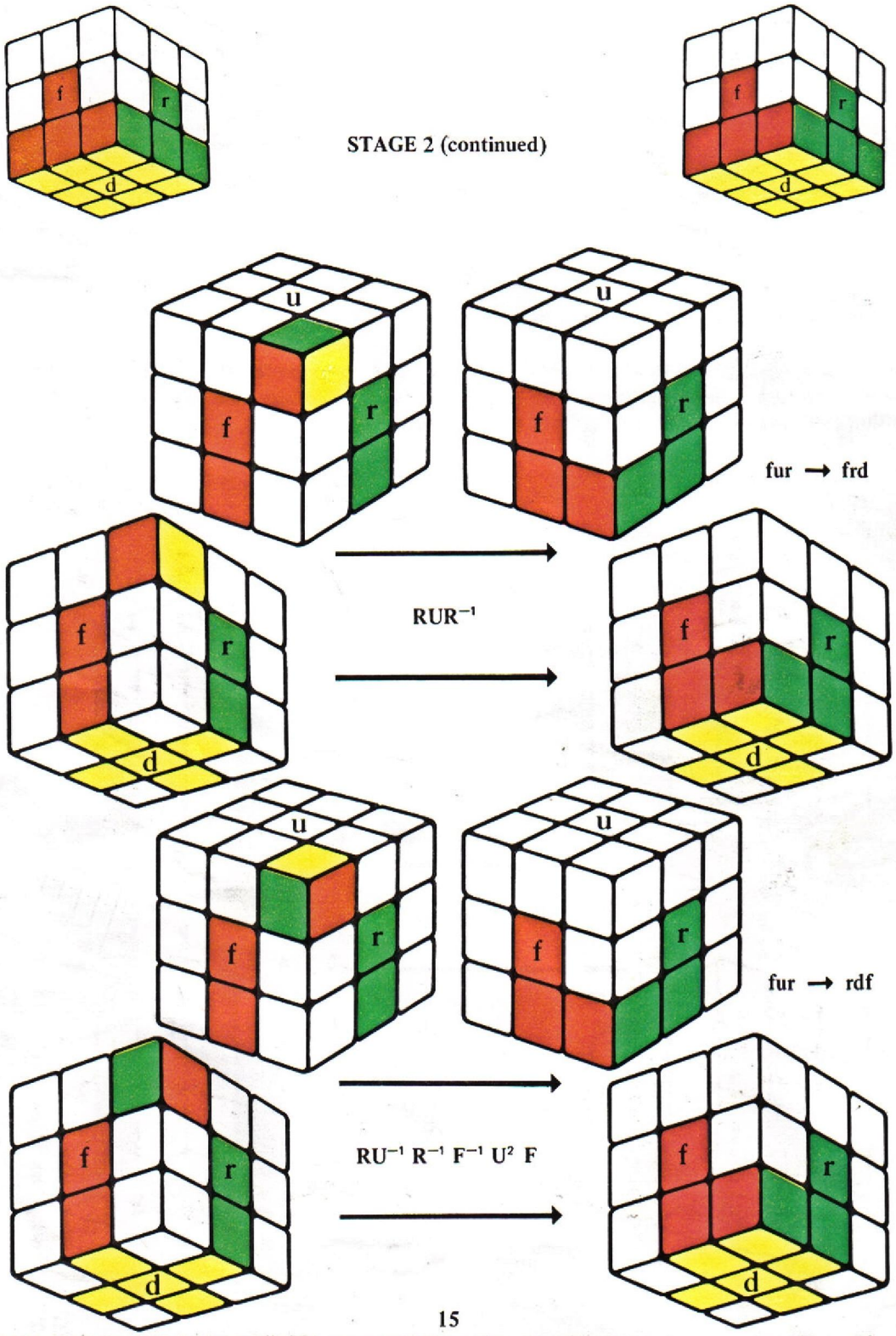
★ Choose a colour for front. Find the corner piece that belongs to the down-front-right position. If it is in incorrectly in the bottom layer then either choose another colour and go back to ★ or else turn over the next page to ★★★.

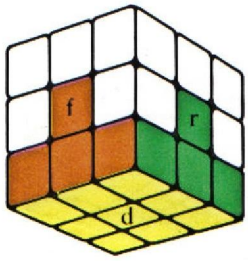
★★ If the corner piece that belongs to the down-front-right position is on the top layer then use **U**, $\mathbf{U}^2$, . . . to move it to the front-upper-right position. Then use one of the following three sequences – note the difference between them.



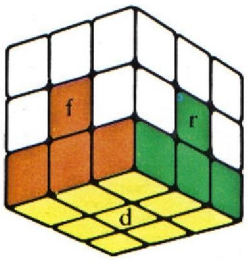
$\mathbf{UR\,U^{-1}\,R^{-1}}$ **fur → dfr**



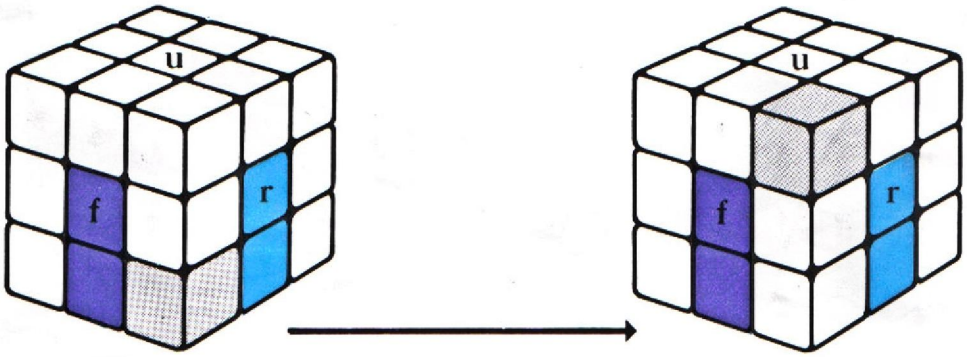

STAGE 2 (continued)
f
r
d
u
fur → frd
RUR⁻¹
fur → rdf
RU⁻¹ R⁻¹ F⁻¹ U² F



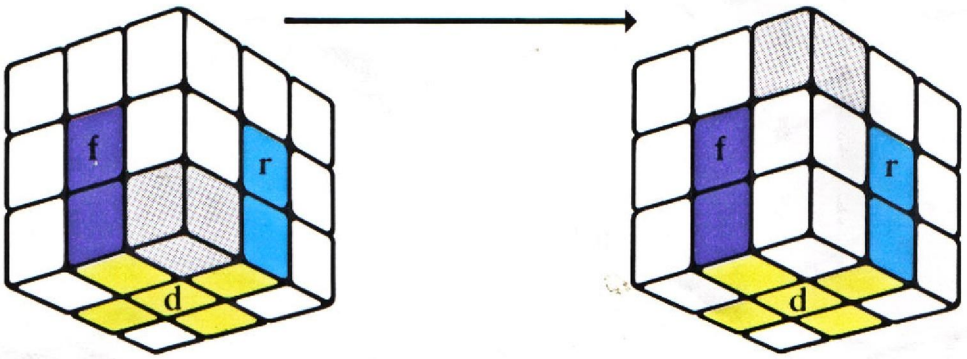
STAGE 2 (continued)

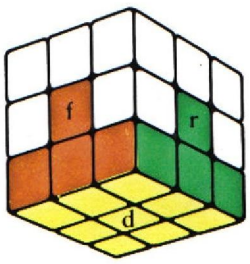


★★★ If the corner piece is in the bottom layer (in an incorrect position) use $RU^{-1}R^{-1}$ to move it to the top layer (see diagram) and go back to ★★.

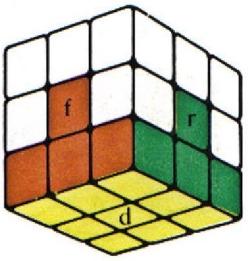


$RU^{-1}R^{-1}$ frd → fur

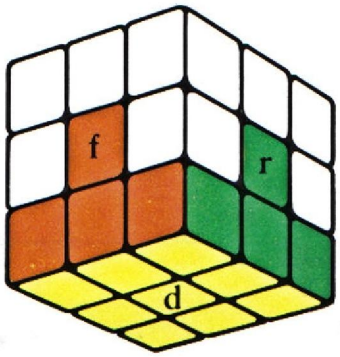


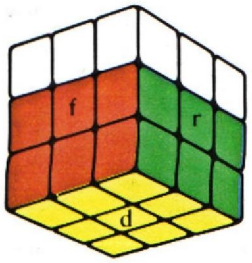


STAGE 2 (continued)

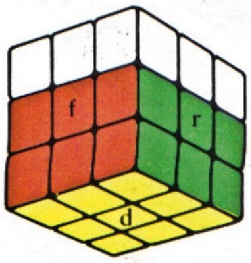


Repeat process with the other corner pieces. At the end of stage 2 your cube should resemble the following:



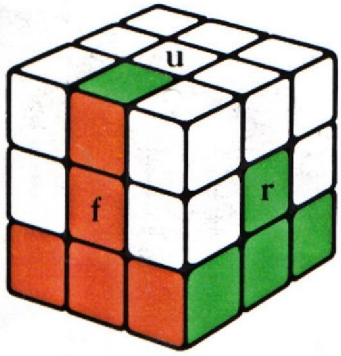


STAGE 3

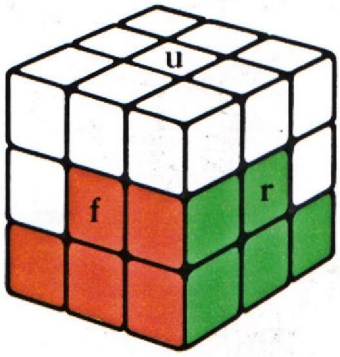


Stage 3 is to place the four edge pieces that belong to the middle layer correctly in position.

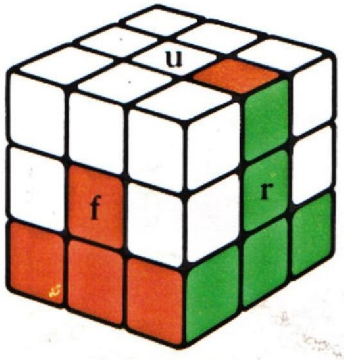
If the middle edge piece is in the upper layer then use $U, U^2, \ldots$ to ensure that one of its colours matches a centre piece colour. Then use one of the following sequences to get that middle edge piece into its correct position.



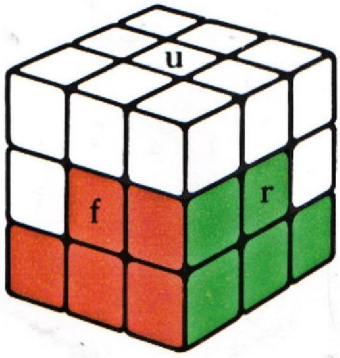
$(URU^{-1}\ R^{-1})\ (U^{-1}\ F^{-1}\ UF)$



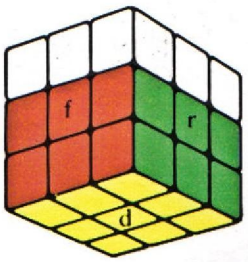
uf → rf



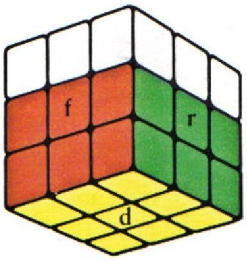
$(U^{-1}\ F^{-1}\ UF)\ (URU^{-1}\ R^{-1})$



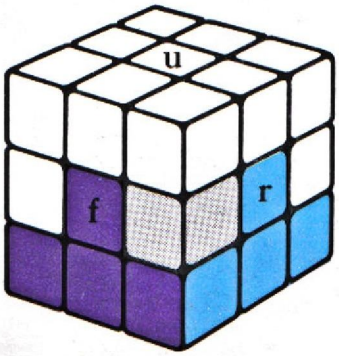
ur → fr



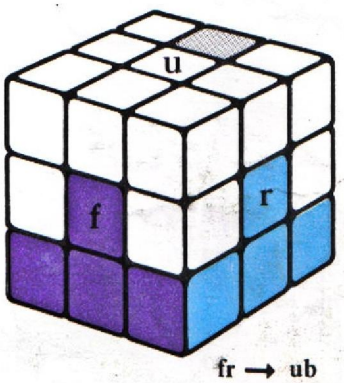
STAGE 3 (continued)



If the middle layer piece is not in the upper but in the middle layer (incorrectly) then use the following sequence to move it to the top layer, then proceed as before.



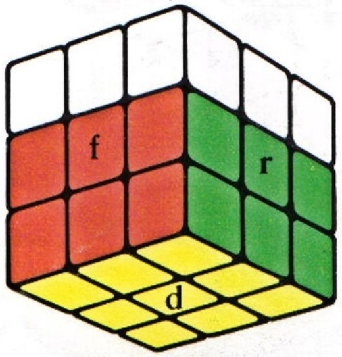
$(RU^{-1}\ R^{-1})\ (U^{-1}\ F^{-1}\ UF)$

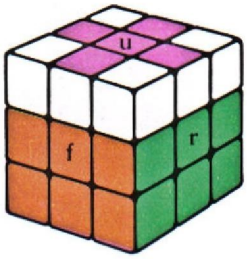


fr → ub

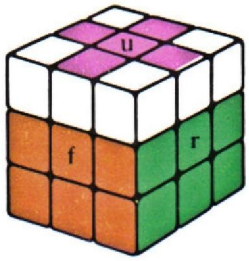
Note that apart from doing **U** first, this sequence is the same as the first sequence on the opposite page.

Repeat until all middle edge pieces are in their correct position. At the end of stage 3 your cube should look like the following.





STAGE 4



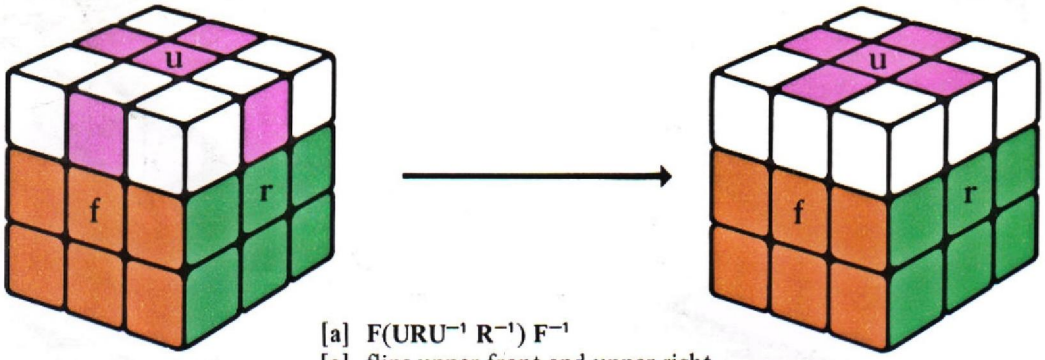
Stage 4 is to "flip" the upper edge pieces to their correct colour, making a "cross", (don't worry about the side colours of the top layer at this stage). The number of upper edge pieces that are correct is either 4, 2 or 0.

Four Correct

If all four are correct go to stage 5.

Two Correct

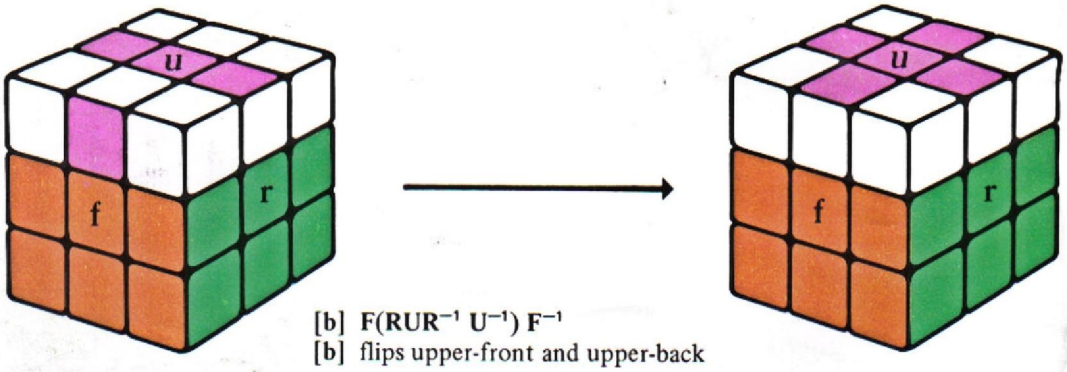
If two of the upper edge colours are already correct then the other two are made correct by using [a] or [b] below.



[a] $F(URU^{-1}R^{-1})F^{-1}$

[a] flips upper-front and upper-right

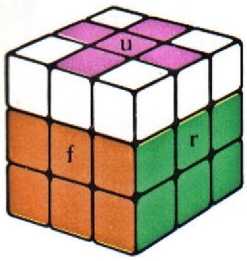
uf → bu → ru



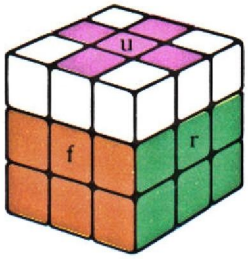
[b] $F(RUR^{-1}U^{-1})F^{-1}$

[b] flips upper-front and upper-back

uf → ru → bu

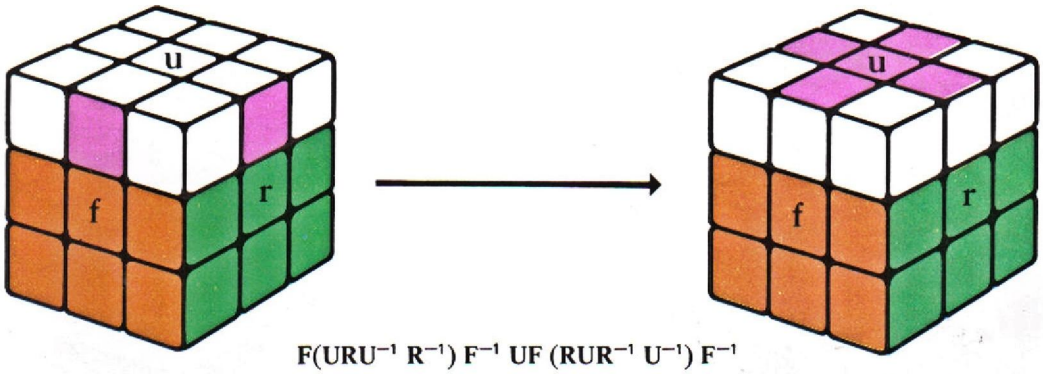


STAGE 4 (continued)

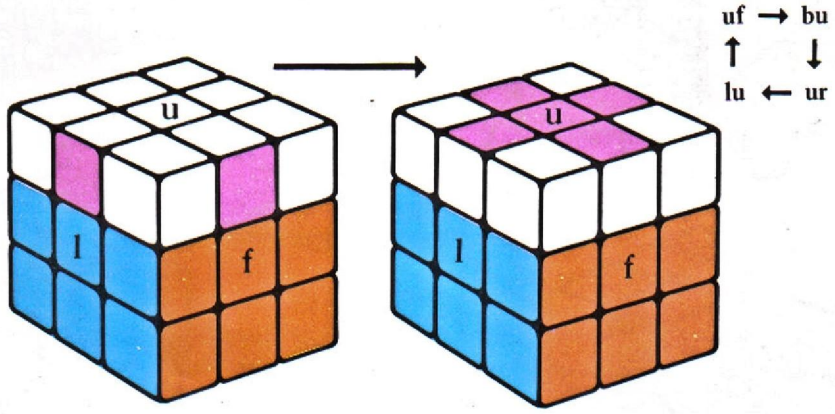


None Correct

If none of the upper edge colours are correct then use **[a]** followed by **U** followed by **[b]** – this is illustrated below.

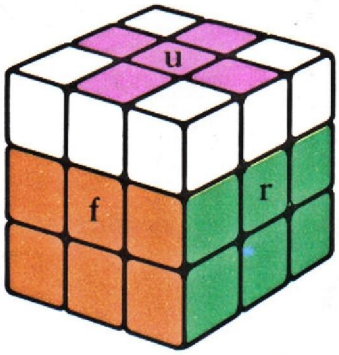


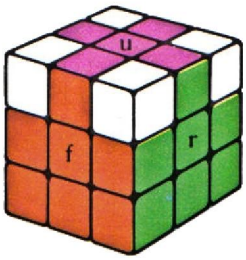
$$F(URU^{-1}R^{-1})F^{-1}UF(RUR^{-1}U^{-1})F^{-1}$$



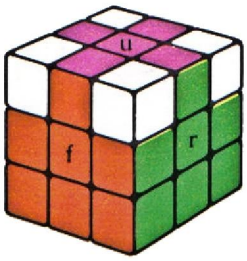
[a] U [b] flips all upper edges

At the end of stage 4 your cube should resemble the following:



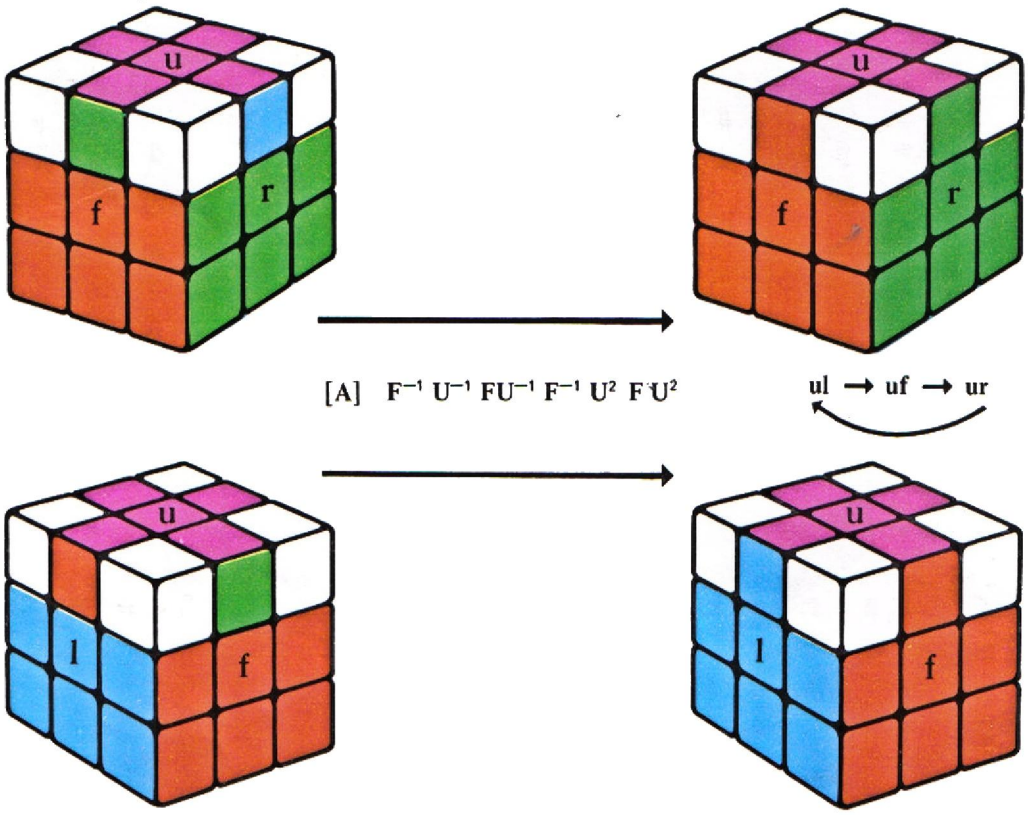


STAGE 5

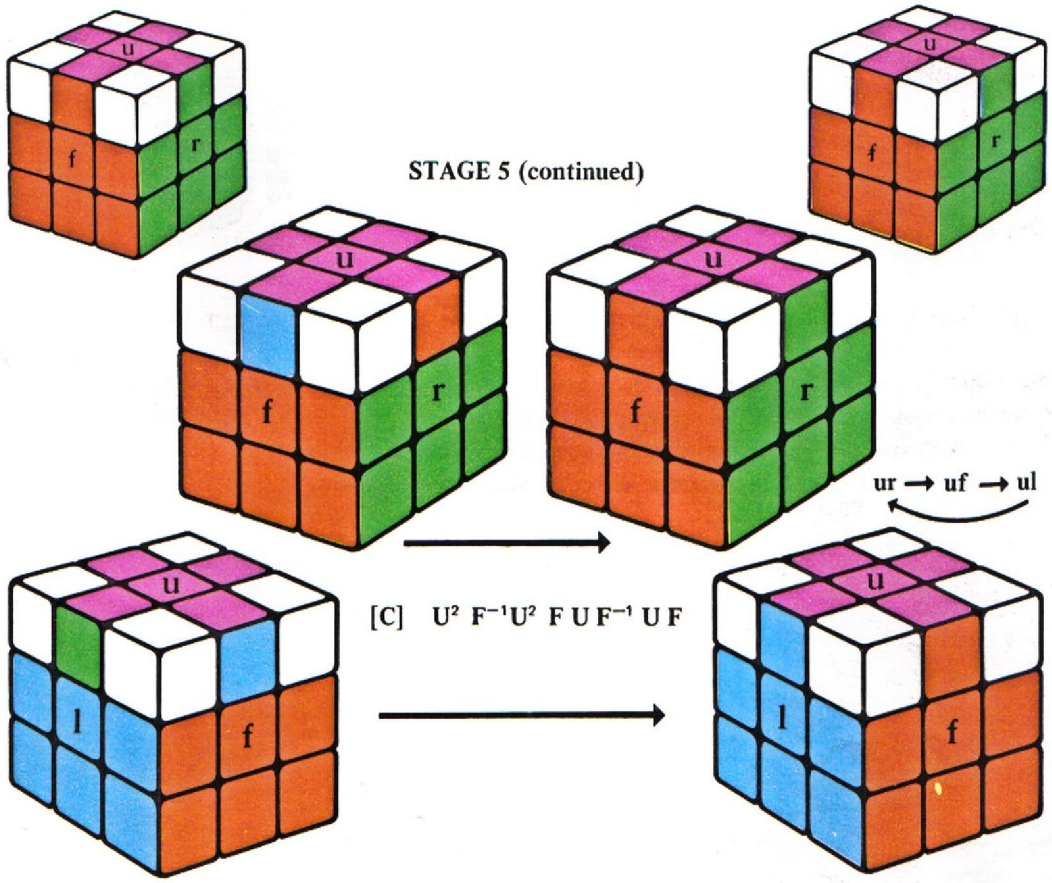


Stage 5 is to put the top edge pieces into their correct position. If all four are correct go to stage 6.

One correct. If precisely one top edge piece is in the correct position then the other three have to be permuted either in a anticlockwise or clockwise manner. This can be achieved by sequence [A] or [C] respectively



[A] permutes the left, front and right upper edge pieces in an anticlockwise manner.



[C] permutes the left, front and right upper edge pieces in a clockwise manner.

Note: It is not necessary to remember [C] because you can either apply [A] twice or else apply U^2, rotate whole cube until the upper edge piece that is in its correct position is at the back, then apply [A].

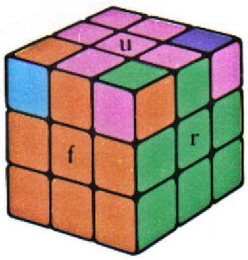
Two adjacent correct. If two adjacent edge pieces are in the correct position then by applying U, U^2 or U^{-1} you can ensure that precisely one is in its correct position. So now use [A] or [C].

Two opposite correct. If two opposite edge pieces are in the correct position then apply the sequence [A] (i.e. $F^{-1}\ U^{-1}\ F\ U^{-1}\ F^{-1}\ U^2\ F\ U^2$). By applying U, U^2, . . you can ensure that exactly one edge piece is in correct position and use [A] or [C] to finish stage 5.

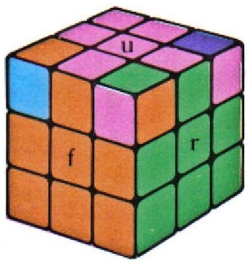
Alternatively use the following (possibly hard to remember) sequence which will interchange the upper-front and upper-back edge pieces.

$(F^2\ D^2\ R^2)\ D\ (L^2\ B^2)\ (L^2\ B^2)\ (L^2\ B^2)\ D^{-1}\ (R^2\ D^2\ F^2)\ U$

(i.e. uf ⟳ ub).

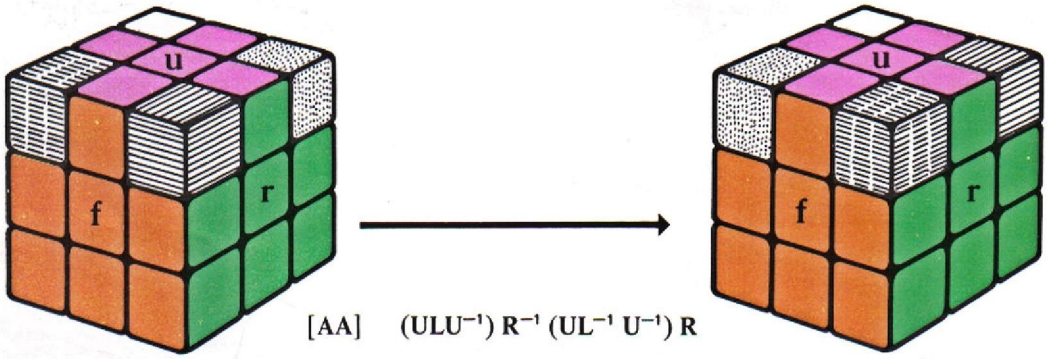


STAGE 6



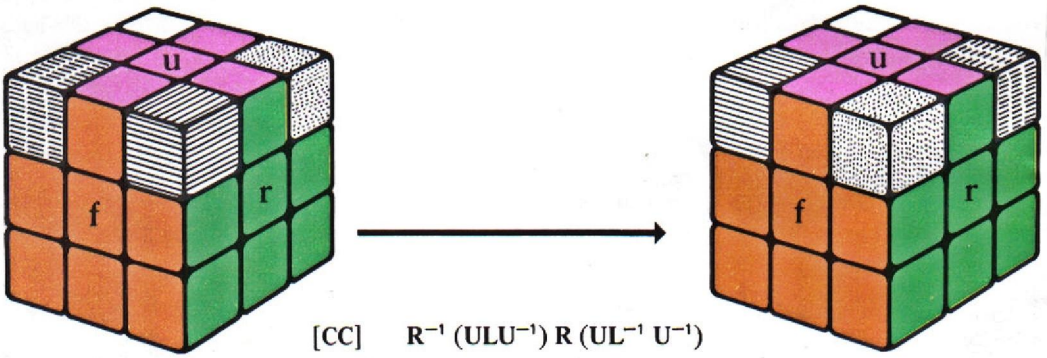
Stage 6 is to place the four top corner pieces in their right place although possibly facing the wrong way (stage 7 will twist the pieces correctly into place).

One right. If exactly one piece is in its right position arrange the cube so that this piece is in the left-back-upper position (shown unshaden in the diagrams). Then permute the other three pieces in an anticlockwise or clockwise manner by using **[AA]** or **[CC]** respectively. (Recall that **L** moves the left side of the cube in a clockwise direction as seen when looking at the left side).



[AA] $(ULU^{-1})\,R^{-1}\,(UL^{-1}\,U^{-1})\,R$

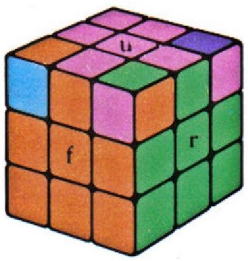
[AA] permutes three top corner pieces in an anticlockwise manner.



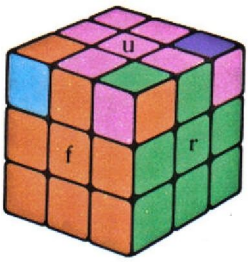
[CC] $R^{-1}\,(ULU^{-1})\,R\,(UL^{-1}\,U^{-1})$

[CC] permutes three top corner pieces in a clockwise manner.

Remark: Using **[AA]** twice has the same effect as using **[CC]** once.

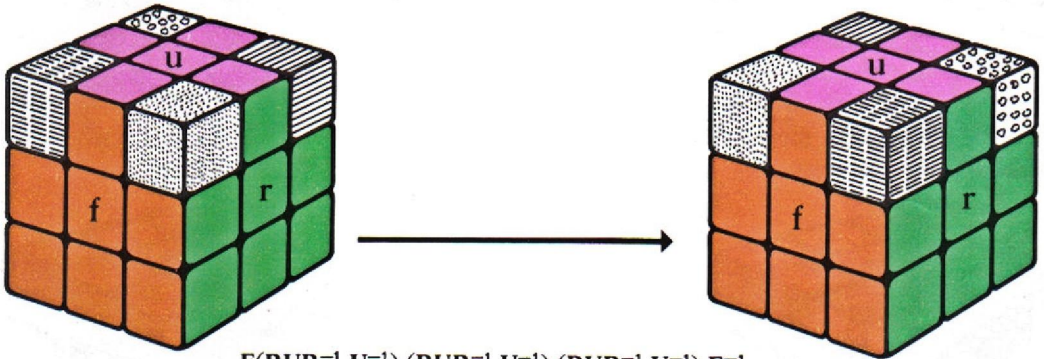


STAGE 6 (continued)



None right. If none of the four corner pieces is in the right place then by using [AA] you will get precisely one in the right place. Then use [AA] or [CC] to finish off.

Alternatively, the following interchanges two pairs of adjacent top corners.



$F(RUR^{-1}\,U^{-1})\,(RUR^{-1}\,U^{-1})\,(RUR^{-1}\,U^{-1})\,F^{-1}$

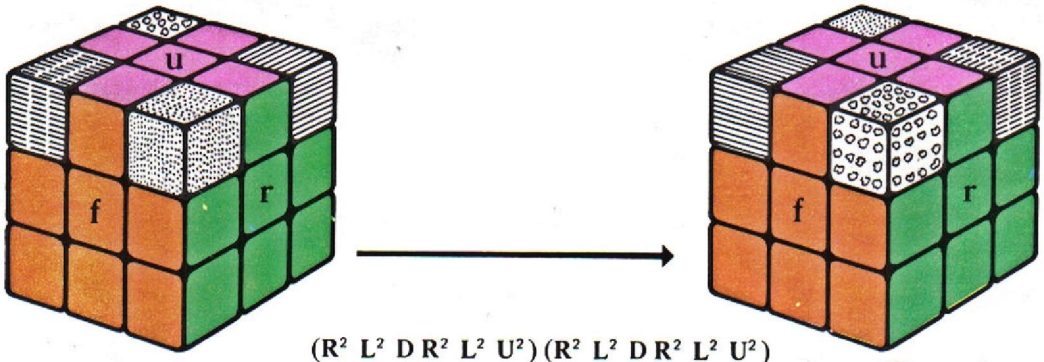
or

$B\,(LUL^{-1}\,U^{-1})\,(LUL^{-1}\,U^{-1})\,(LUL^{-1}\,U^{-1})\,B^{-1}$

ufl ⟳ fur

ulb ⟳ bru

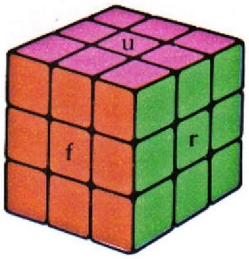
While the following interchanges opposite top corners.



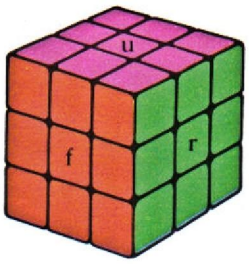
$(R^2\,L^2\,D\,R^2\,L^2\,U^2)\,(R^2\,L^2\,D\,R^2\,L^2\,U^2)$

ubr ⟳ ufl

urf ⟳



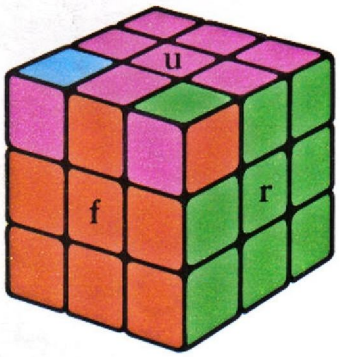
STAGE 7



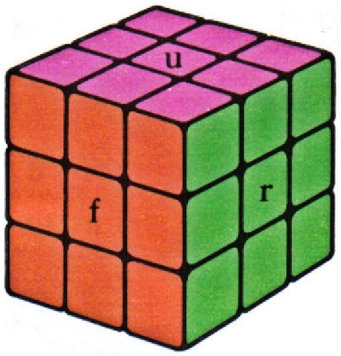
Stage 7, the final stage, is to twist the top corner pieces so that they are correctly in position. Either 4, 2, 1 or 0 are correctly in position. If all 4 are then you are finished.

Two correct. If exactly two of the upper corner pieces are correctly in position then one of the incorrectly positioned ones requires a clockwise twist while the other requires an anticlockwise twist. Place the cube so that the corner piece that requires a clockwise twist is in the front-upper-right position. Then use one of the following three sequences depending upon where the other incorrect piece is.

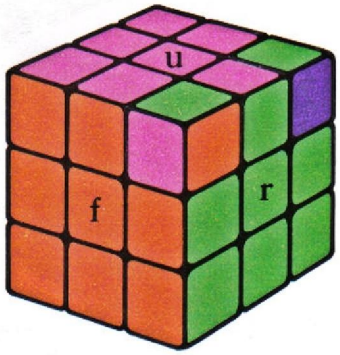
Recall that **D** moves the down (bottom) side of the cube in a clockwise direction as seen when looking at the down side.



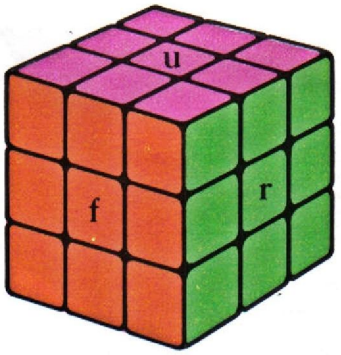
$(FDF^{-1}\ D^{-1})\ (FDF^{-1}\ D^{-1})\ U^{-1}$
$(DFD^{-1}\ F^{-1})\ (DFD^{-1}\ F^{-1})\ U$



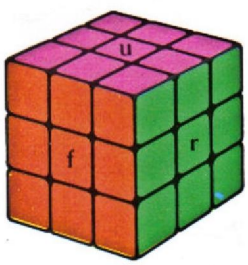
fur → urf
luf → flu



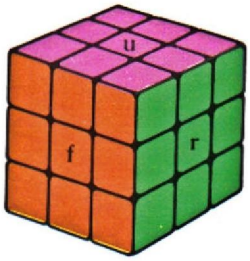
$(FDF^{-1}\ D^{-1})\ (FDF^{-1}\ D^{-1})\ U$
$(DFD^{-1}\ F^{-1})\ (DFD^{-1}\ F^{-1})\ U^{-1}$

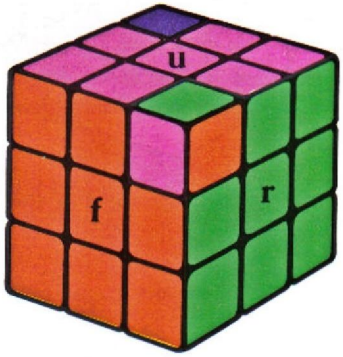


fur → urf
ubr → rub

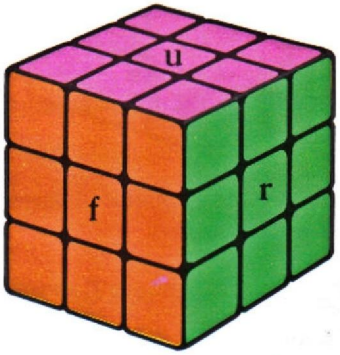


STAGE 7 (continued)





$(FDF^{-1}\ D^{-1})\ (FDF^{-1}\ D^{-1})\ U^2$
$(DFD^{-1}\ F^{-1})\ (DFD^{-1}\ F^{-1})\ U^2$



fur → urf
ulb → bul

None correct. If no upper corner piece is correctly in position then two require a clockwise twist and two require an anticlockwise twist. Place the cube so that one of the corner pieces that requires a clockwise twist is in the front-upper-right position. Use one of the sequences in the "two correct" section making sure that at the end two of the upper corner pieces are correct. A further sequence from that section will place all pieces correctly in position.

Alternatively use the following sequences which are shorter but harder to remember. The sequence $B^2\ (FDF^{-1}\ D^{-1})\ (FDF^{-1}\ D^{-1})\ B^{-1}\ U^{-1}\ (DFD^{-1}\ F^{-1})\ (DFD^{-1}\ F^{-1})\ UB^{-1}$ twists the upper-front-right and upper-back-left pieces clockwise, the two other upper corner pieces being twisted anti-clockwise.

The sequence

$LD\ (U^{-1}\ RUR^{-1})\ (UF^{-1}\ U^{-1}\ F)\ (U^{-1}\ RUR^{-1})\ (UF^{-1}\ U^{-1}\ F)\ D^{-1}\ L^{-1}$ twists the two front upper corner pieces clockwise and the two back ones anticlockwise.

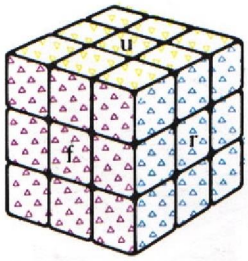
One correct. If one upper corner piece is correctly in position then the other three either all require a clockwise twist or all require an anticlockwise twist. By using one of the sequences in the "two correct" section you can ensure that two of the upper corner pieces is correctly in position. A further sequence from that section will place all pieces correctly in position.

Alternatively

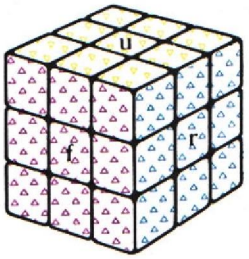
$(FDF^{-1}\ D^{-1})\ (FDF^{-1}\ D^{-1})\ U^{-1}\ (FDF^{-1}\ D^{-1})\ (FDF^{-1}\ D^{-1})\ U^{-1}\ (FDF^{-1}\ D^{-1})\ (FDF^{-1}\ D^{-1})\ U^{-1}\ U^{-1}$ twists each of the corner pieces front-upper-right, left-upper-front and back-upper-left in a clockwise direction (i.e. **fur → urf, luf → ufl, bul → ulb**).

While

$U\ U\ (DFD^{-1}\ F^{-1})\ (DFD^{-1}\ F^{-1})\ U\ (DFD^{-1}\ F^{-1})\ (DFD^{-1}\ F^{-1})\ U\ (DFD^{-1}\ F^{-1})\ (DFD^{-1}\ F^{-1})$ twists each of the corner pieces front-upper-right, left-upper-front and back-upper-left in an anticlockwise direction (i.e. **fur → rfu, luf → flu, bul → lbu**).



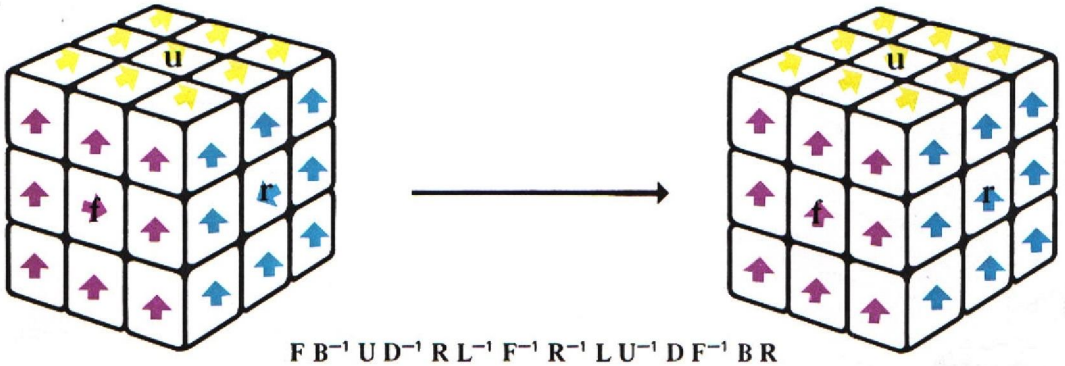
CUBES WITH PICTURES



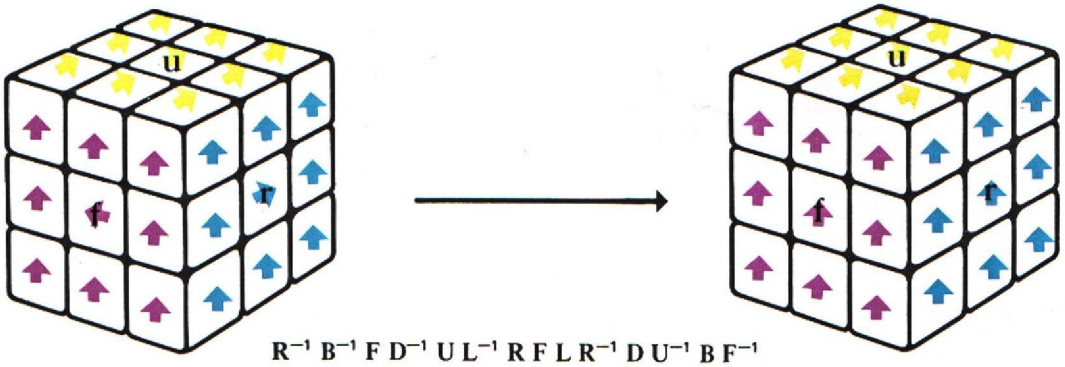
If your cube has extra markings on it (for example fruit, numbers, portraits etc) then after you have completed stage 7 some centre face pieces may need a turn in relation to the other pieces on that face. The following sequences (possibly used more than once) will enable you to complete your cube.

Note: By being careful in stages 1 to 7 it will only be necessary to use at most the third of the following sequences.

The first two sequences only affect the front and right centre pieces.

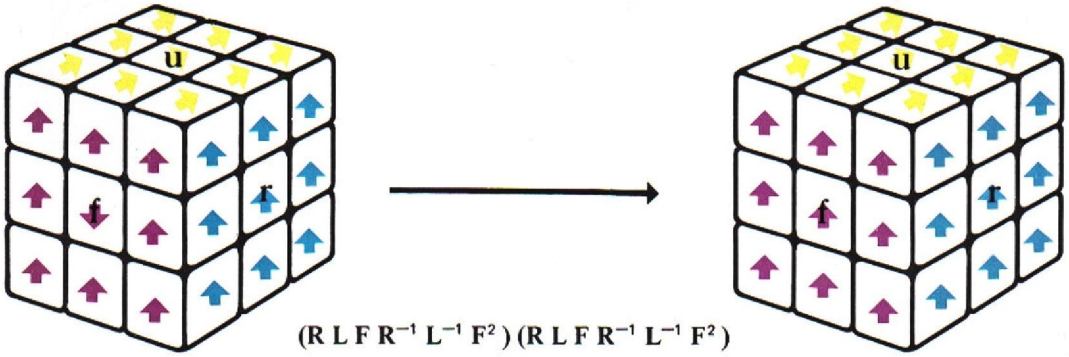


$F\,B^{-1}\,U\,D^{-1}\,R\,L^{-1}\,F^{-1}\,R^{-1}\,L\,U^{-1}\,D\,F^{-1}\,B\,R$



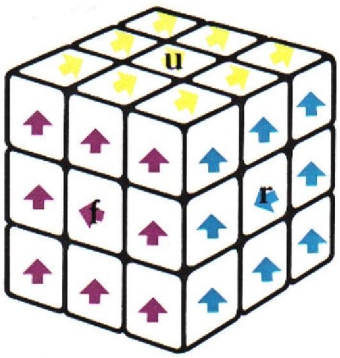
$R^{-1}\,B^{-1}\,F\,D^{-1}\,U\,L^{-1}\,R\,F\,L\,R^{-1}\,D\,U^{-1}\,B\,F^{-1}$

The third sequence only affects the front centre piece.

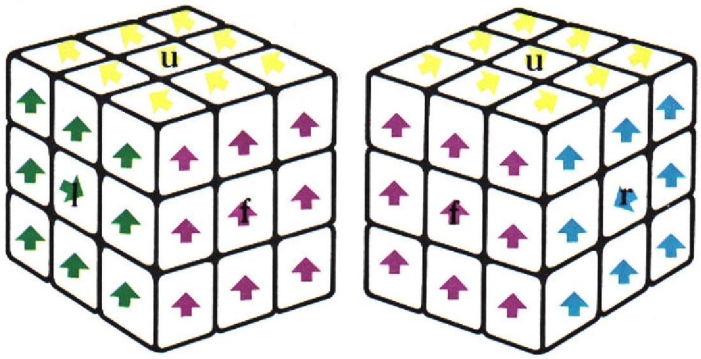


$(R\ L\ F\ R^{-1}\ L^{-1}\ F^2)\ (R\ L\ F\ R^{-1}\ L^{-1}\ F^2)$

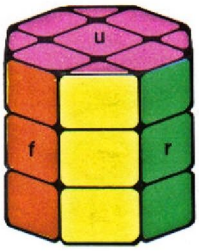
Other cases may be dealt with by using the above sequences a few times. Some examples are given below.



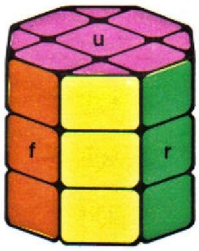
In this case use the first sequence followed by the third.



For this case use the first sequence, then rotate the whole cube so that **l** becomes front and use the first sequence again.

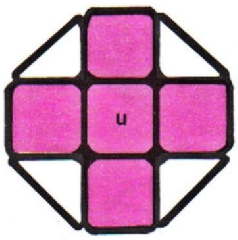


THE BARREL OR OCTAGONAL PRISM PUZZLE



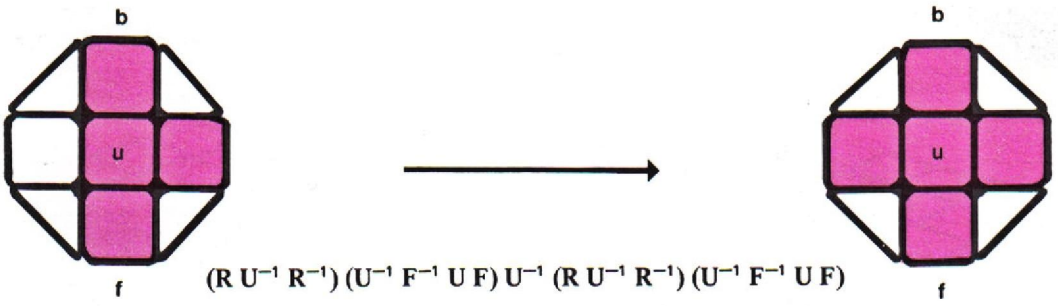
This puzzle is just Rubik's Cube with its vertical edges trimmed so that the top and bottom faces are octagonal (eight-sided). There are ten different colours on the Barrel, two colours appear nine times and the other eight each appear three times. The solution of this puzzle is much the same as the solution to Rubik's Cube. In Stage 1 choose a colour for "down" that appears nine times (the other colour that appears nine times will then be "up"). Stage 2 is easier because you cannot distinguish the four bottom "corner" pieces – so just position them anywhere. In the remainder of the seven stages you may come across two problems, one in Stage 4 and one in Stage 6.

In Stage 4 you need to make a "cross" on the upper face.



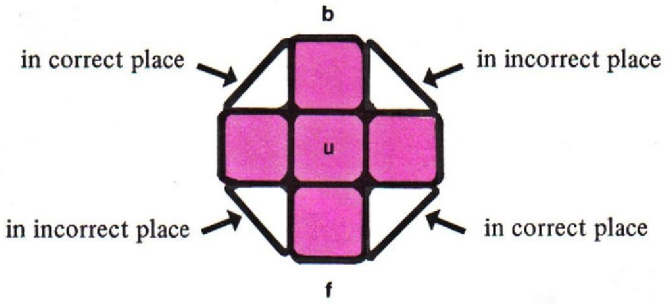
(view from top)

You will either be able to make the cross (in which case go on to Stage 5) or else only three-quarters of the cross. In the case that you can only make three-quarters of the cross use the following sequence and then continue to Stage 5.



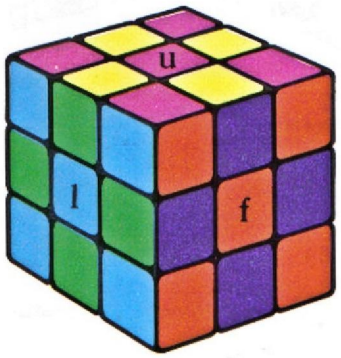
$(R\ U^{-1}\ R^{-1})\ (U^{-1}\ F^{-1}\ U\ F)\ U^{-1}\ (R\ U^{-1}\ R^{-1})\ (U^{-1}\ F^{-1}\ U\ F)$

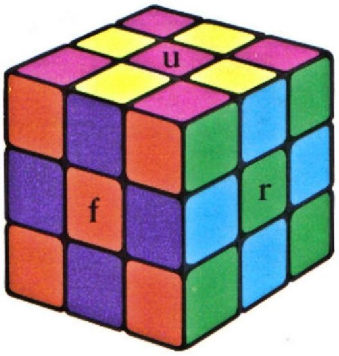
If in Stage 6 you get all the upper "corner" pieces in their correct place go on to Stage 7. Otherwise get exactly two opposite pieces in their correct place as in the diagram below.



Then use $L^{-1}\ R^{-1}\ U^2\ R\ L$. Unfortunately you have to go back to Stage 5 but from now on no problems will arise and you will be able to complete the octagonal prism puzzle.

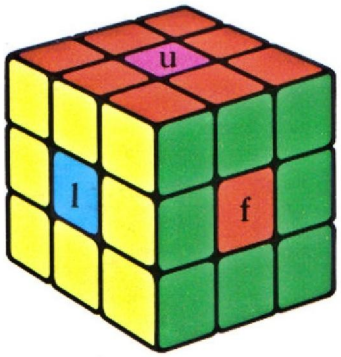
PRETTY PATTERNS

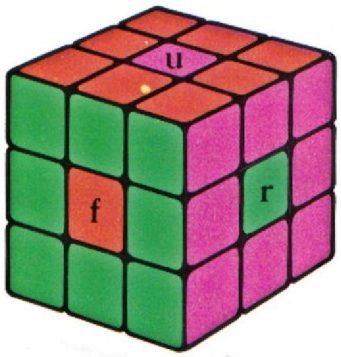


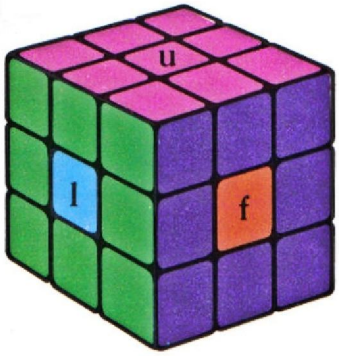


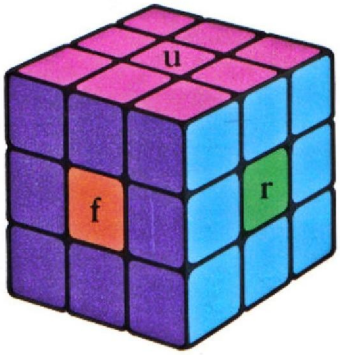
$R^2 L^2 F^2 B^2 U^2 D^2$

$F B^{-1} U D^{-1} R L^{-1} F B^{-1}$







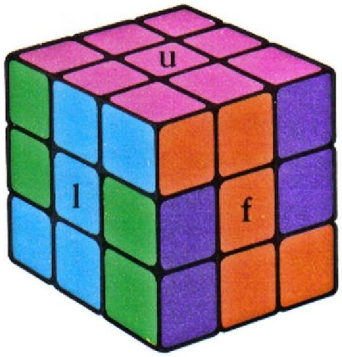


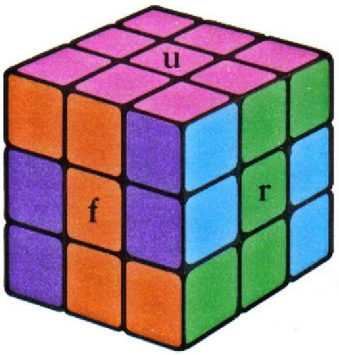
$FB^{-1} UD^{-1} RL^{-1} FB^{-1}$
$RL^{-1} UD^{-1} BF^{-1} RL^{-1}$

Moves of the type (RL^{-1}), (FB^{-1}), (UD^{-1}) etc. are called **slice moves.** Making sequences of slice moves often results in pretty patterns, some are shown here – can you see which? (Note that $R^2 L^2 = (RL^{-1})(RL^{-1})$ is a slice move).

Moves of the form (RL), (FB), (UD) etc. are called **antislice moves.** Again, sequences of antislice moves often lead to pretty patterns, some are shown here. Also try $(FB)(UD)(R^{-1} L^{-1})(FB)$.

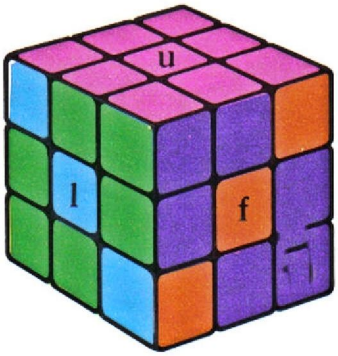
PRETTY PATTERNS

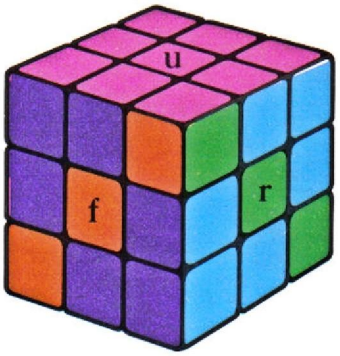


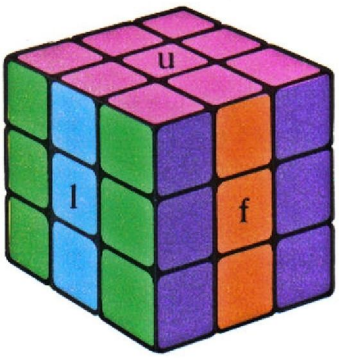


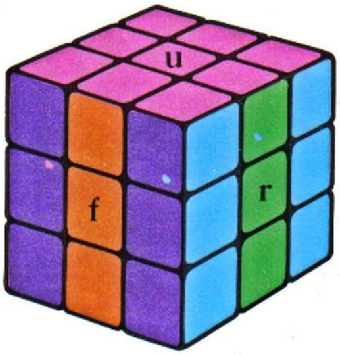
(FBRL) (FBRL) (FBRL) U^2 D^2

(FBRL) (FBRL) (FBRL)



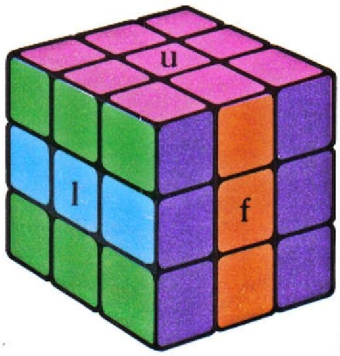


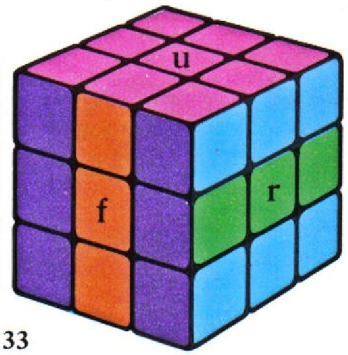




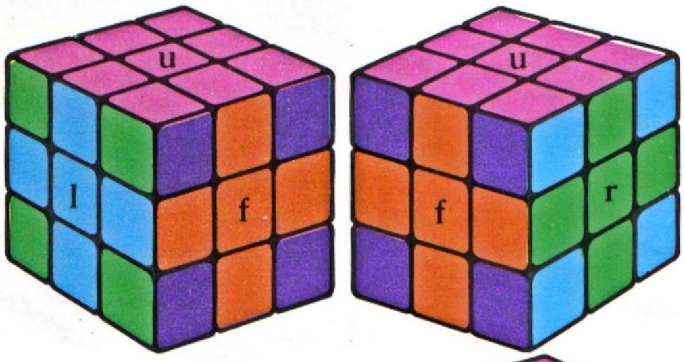
F^2 R^2 L^2 B^2 R^2 F^2 B^2 L^2

D^2 R^2 L^2 U^2 R^2 D^2 U^2 R^2

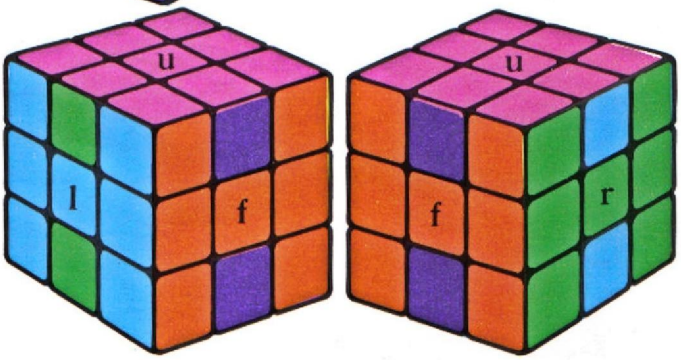




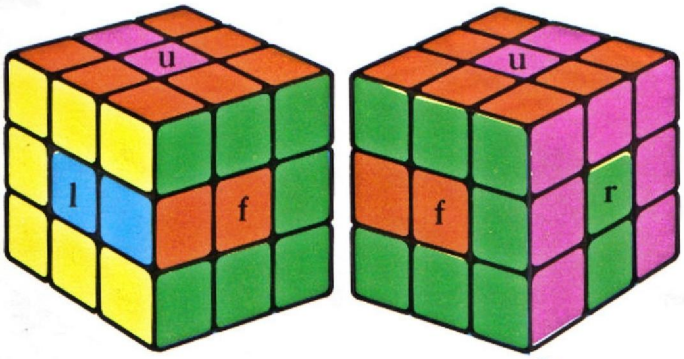
PRETTY PATTERNS



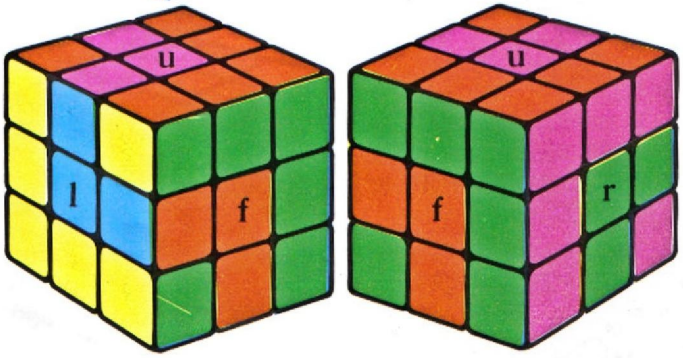
(FBRL) (FBRL) (FBRL)
(RLFB) (RLFB) (RLFB)



(FBRL) (FBRL) (FBRL) (RLFB)
(RLFB) (RLFB) U^2 D^2



L^{-1} R^2 F^{-1} L^{-1} B^{-1} U
B L F R U^{-1} R L R L^{-1}
F B^{-1} U D^{-1} R L^{-1}



RUF^2 D^{-1} RL^{-1} FB^{-1} D^{-1}
F^{-1} R^{-1} F^2 RU^2 FR^2 F^{-1}
R^{-1} U^{-1} F^{-1} U^2 FR