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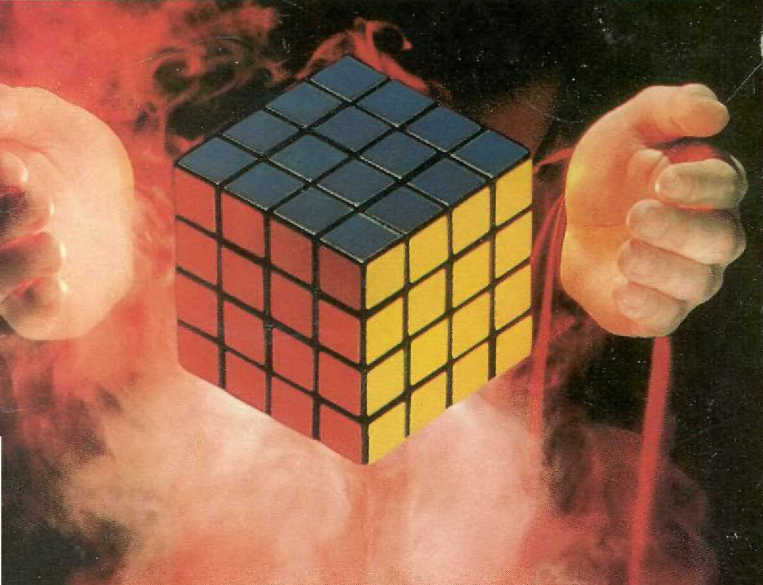
HOW TO SOLVE Rubik's Revenge™*

FULL-COLOR STEP-BY-STEP INSTRUCTIONS TO SOLVE THE SUPERCUBE



JEFFREY ADAMS

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JEFFREY ADAMS

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Introduction

Mastering the Cube was just the beginning — now there's the Supercube, alias Rubik's Revenge.™ It's a lot harder, but don't despair. Here is a complete, step-by-step solution to the 4x4x4 Supercube.

This book will help you solve the Supercube, and more. You will understand the Supercube, because before you get bogged down in the step-by-step solution you learn the Basic Moves which *alone* are enough to solve it! These are the *only* moves you need to solve this mind-boggling puzzle.

The nine Basic Moves are listed for convenience on a fold-out flap on the back cover so you can refer to them easily as you follow the solution. You will soon be

able to dispense with the step-by-step solution, and remember only these moves. To make things even simpler, the Basic Moves (with one exception) are all based on a single Basic Basic Move, called a commutator (see pg. 14). It is one of the marvels of the Supercube that one simple procedure allows you to solve this fantastically complex puzzle. There is very little memorization needed! And if you are familiar with the original cube, you will soon see that solving the Supercube isn't much harder.

Don't worry about speed at first. This solution is *conceptual* and *easy to follow* rather than fast, and requires very little memorization. Anyhow, no matter how fast you can solve it there's bound to be someone half your age who can do it twice as fast! The beauty of the puzzle lies in its extraordinary complexity yet ultimate simplicity, in its symmetry and in its patterns. With an understanding of the Supercube you can begin to explore the fascinating world of Supercube patterns.

For those of you set on breaking records (and wrists) there are shortcuts to help you speed up the solution. We have included a sketch of an alternative solution in which you solve the centers first. You may be interested in some more advanced processes, and a sample of possible Supercube patterns. And for the experts there is a brief section on Supercube-theory.

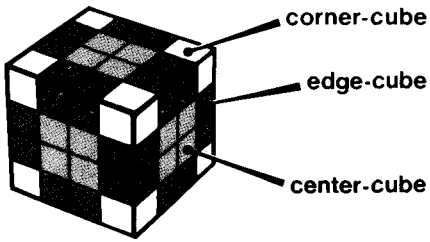
Happy Cubing!

Guide to the Solution

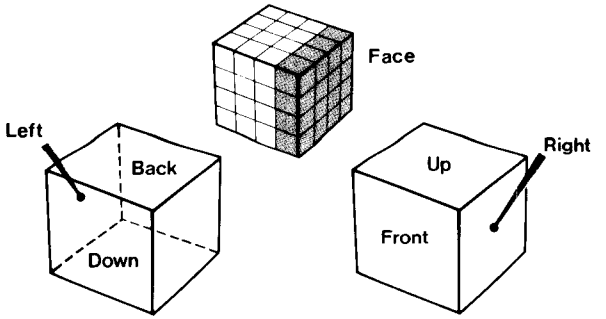
We need a convenient notation to denote moves, parts of the cube, and so forth. We use a simple extension to the Supercube of the notation developed by David Singmaster in Notes on Rubik's Magic Cube, (somewhat of a standard in the industry). If you are familiar with his notation you may wish to skim this section quickly.

Parts of the cube

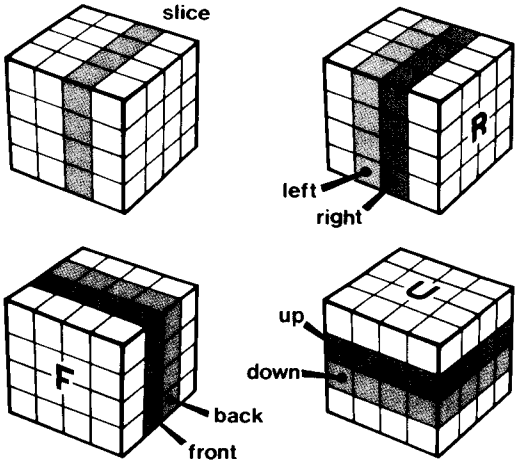
There are 56 sub-cubes in the cube:
24 center-cubes, 24 edge-cubes
and 8 corner-cubes.



There are 6 faces consisting of 16 visible sub-cubes. These are labelled as follows: Up, Down, Left, Right, Front and Back; or U, D, L, R, F and B for short.

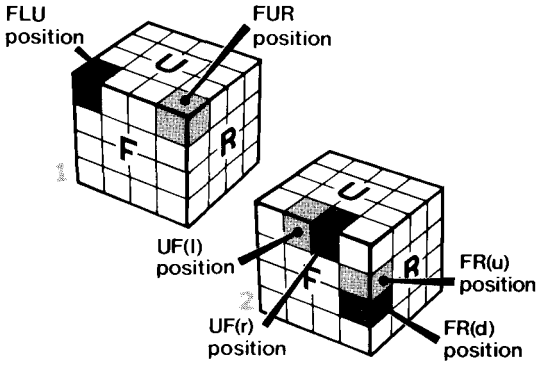


There are also 6 slices consisting of 12 visible sub-cubes. These are labelled as follows: up, down, left, right, front and back; or u, d, l, r, f and b for short.



We label positions within the cube as follows: the position at the Front-Up-Right corner of the cube is called the Front-Up-Right position,[1] or FUR-position. The Front-Left-Up position is called the FLU-position, and so on.

It is not enough to refer to the UF-position for an edge position since there are two such; we distinguish them by using UF(r) and UF(l).[2] The same holds for FR(u) and FR(d),[3] UR(f) and UR(b), and so on. We will not bother labelling centers.

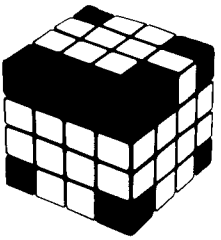


In a 3x3x3 cube the centers are fixed, so you know where every sub-cube must go. So if a corner-cube must go to the FUR-position, we call it the FUR corner-cube. Similarly for edges.

Unfortunately there are no fixed centers in a Super-cube. However, at each step some sub-cubes will be considered fixed, so we label the other sub-cubes in reference to them. For example after Step I the corners are all solved and must remain fixed. Then if your cube looks like this,[3] the UF(r)-edge and the UF(l)-edge are the two with blue and red sides. The FR(u)-edge and FR(d)-edge are the ones with blue and green sides. So in this example, an FR(u or d) edge is in the

UR(f)-position. (The FR(u) and FR(d) edges have the same colors, and are essentially indistinguishable, see pg. 34).

As another example in Step I you fix the FLU-corner.[4] This determines the rest of the corners. For example, if your cube is like this one[4] the FUR corner-cube is the one with the red, blue and green sides. The FRD corner-cube has blue, green and yellow sides.



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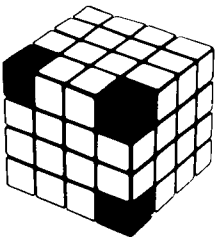

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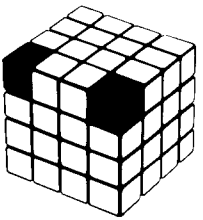
We say a corner sub-cube is correctly oriented if (it is in the correct position and) its colors are aligned correctly (with respect to whatever is considered fixed). Otherwise it is incorrectly oriented. For example, in diagrams[5] and,[6] the FLU-corner is considered fixed, and the FUR-corner is incorrectly or correctly oriented respectively.

We use the same terminology for edges even though strictly speaking an edge which looks incorrectly oriented is really in the wrong position (see pg. 34).



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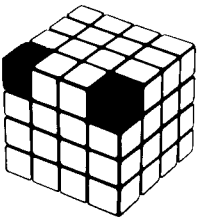

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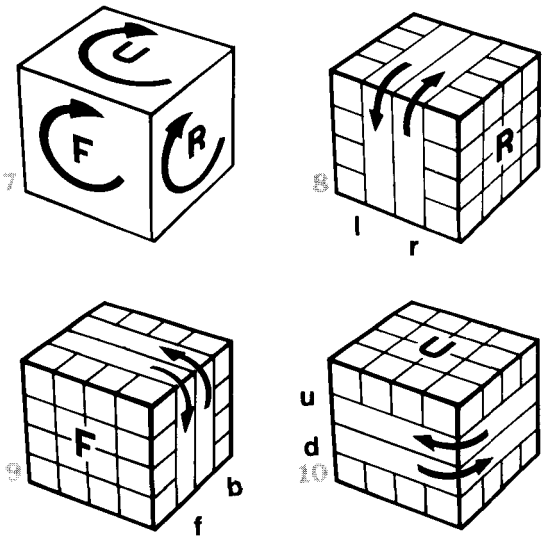
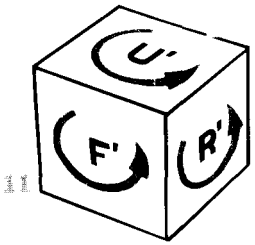
Moves

R means: turn the Right face one turn (90 degrees) clockwise.[7] The clockwise move is determined by looking directly at the face. Similarly **L**, **F**, **B**, **U** and **D**. **r** means: turn the right slice one turn (90 degrees) clockwise.[8] Clockwise for the right slice is determined by looking at the Right face. **l**, **f**, **b**, **u** and **d** are similar.[9,10] Note that **r** and **l** appear to go in opposite directions.



R′ ("R-prime") means: turn the Right face one turn counter-clockwise; also **L′**, **F′**, **B′**, **U′**, and **D′**; and **r′**, **l′**, **f′**, **b′**, **u′** and **d′**.



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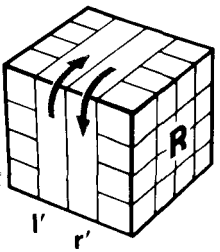

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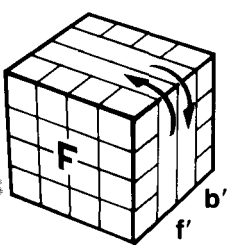

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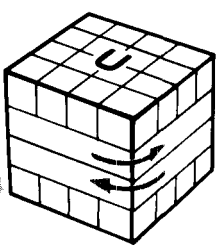


14

A sequence of moves is given by a string: **R′URU′** means apply **R′**, then **U**, then **R**, and finally **U′**.

Parentheses or brackets simply clarify the moves: **L(F′UFU′)L** = **LF′UFU′L**. Brackets [ ] will be used to indicate that the process involves conjugation (see pg. 14). Superscript ² means done twice: **R²** = **RR**, **(R′URU′)²** = **R′URU′R′URU′**.

Note that **RR′** does not do anything; neither does **(FR′lU)(U′l′RF′)**. That is, to undo a sequence of moves, reverse their order, change all primes to non-primes, and vice-versa. This move which undoes a given move is called its inverse, and is denoted by a ′: **(FR′lU)′** = **U′l′RF′**.

Basic Moves

The catalogue of Basic Moves contains all the moves you need to solve the Supercube. We suggest you familiarize yourself with these moves, especially Basic Move 5, before going on to the step-by-step solution.

It is time we explained the unifying principle behind these moves. Look at Basic Move 1: R'URU'. It moves four corners and 6 edges (in three pairs) in a very simple way. A $3 \times 3 \times 3$ cube may be completely solved using only this move and a few variants of it! This move is the simplest example of a commutator. Any move of the form (move A) (move B) (inverse of move A) (inverse of move B) is called a commutator. *Commutators are the key to the cube and to the Supercube.*

COMMUTATOR =
(A B)(-A-B) = AB·A'B'

They are the building blocks of the solution. It is easy to break up any process you like into smaller ones achievable with commutators. Although this does not always give the fastest way to do something, remember, what we are after first is clarity, not speed. Speed will come later.

All the Basic Moves but 7 and 8 are commutators, and Basic Move 8 has a commutator in it: Sometimes, as in Basic Moves 5 and 6, we build commutators on top of commutators and voilà: a ten-move sequence that only affects three edges.

Another essential part of this solution (any solution, for that matter) is a similar process — *conjugation*. This allows you to increase the usefulness of any process. For example Basic Move 3 very conveniently orients two adjacent corners. But what if (as happens in Step I.C., pg. 30) you want to orient two opposite corners? No need to find a whole new process; simply move the two corners into adjacent positions, apply the given move (Basic Move 3) and move them back the way they came. That is in this case, one applies L [Basic Move 3]L'.

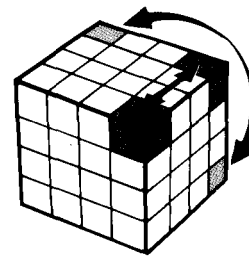
Any move of the form (move A) [move B] (inverse of move A) is called a conjugate of move B. It has the same effect as move B, on different parts of the cube. Brackets [ ] are used to point these moves out — they are used extensively in Step II.

CONJUGATE =

A • B • −A

Basic Move 1: (Steps I-A and B)

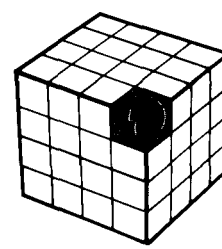
R'URU'



This moves four corners, just like on a $3 \times 3 \times 3$ cube. It also moves three pairs of edges. Its inverse is
UR'U'R.

Also

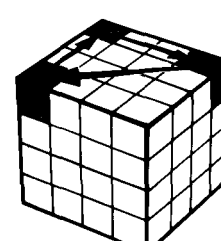
$(R'URU')^2$



orients the FUR corner. It also orients three corners on the Back face, and moves three pairs of edges. Its inverse is
$(UR'U'R)^2$.

Basic Move 2: (Step I-B)

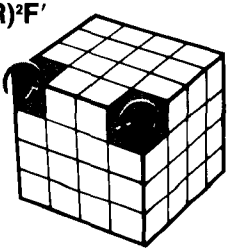
L'(URU')L(UR'U')



This moves three corners clockwise. Basic Moves 1 and 2 are all that you need to get all corners in place. Its inverse is
(URU')L'(UR'U')L.

Basic Move 3: (Step I-C)

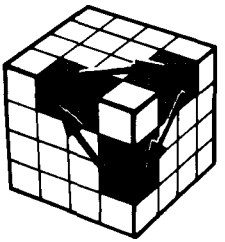
move W = $(R'URU')^2F(UR'U'R)^2F'$



This orients two corners, and is all you need to orient the corners. Its inverse is **move W′ = $F(R'URU')^2F'(UR'U'R)^2$.**

Basic Move 4: (Steps II-A, B and C)

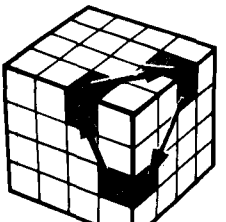
(RBLF)U(F'L'B'R')U'



This moves three pairs of adjacent edges as if they are glued together. It does not affect corners or centers. Its inverse is **U(RBLF)U'(F'L'B'R').**

Basic Move 5: (Steps II-B and C)

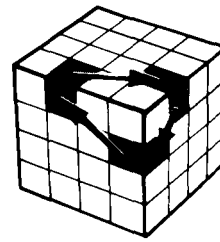
move X = (F'rFr')U(rF'r'F)U'



This moves three edges without affecting corners or centers. It is the workhorse of this solution. It and Basic Move 6 form two halves of Basic Move 4. Its inverse is **move X′ = U(F'rFr')U'(rF'r'F).**

Basic Move 6: (Steps II-B and C)

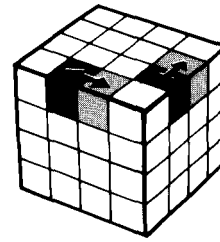
move Y = (F'I'FI)U(I'F'IF)U'



This also moves three edges; see Basic Move 5. This is included for the sake of completeness. In the step-by-step solution we use only Basic Move 5. We use this move in the shortcuts; see Section 6. Its inverse is **move Y' = U(F'I'FI)U'(I'F'IF).**

Basic Move 7: (Steps II-A, B and C)

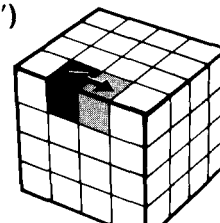
R'U²R²UR'U'R'U²•LFRF'L'



This exchanges two pairs of edges, essentially orienting them. It does not affect corners or centers. It comes from the same move on a $3 \times 3 \times 3$ cube which flips two edges in place. Its inverse is itself.

Basic Move 8: (Step II-C)

I²D²[uF'u'F]D²I²•(LUL')u'(LU'L')

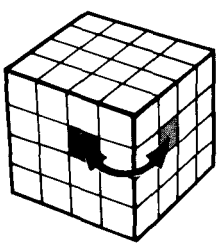


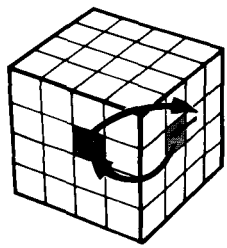
This switches just two edges (which is impossible on a $3 \times 3 \times 3$ cube). It also moves some centers. For a move which does not affect centers see Section 8. This move is only used once in the solution. Its inverse is (essentially) itself.

See p. 60

Basic Move 9: (Steps III-A and B)

R'(fr'f')R(frf')





This essentially switches two centers on adjacent faces without affecting corners or edges. That is, it moves three centers as shown, but if you ignore centers shuffling around a single face you can think of it as above. This is all you need to solve the centers. Its inverse is **(fr'f')R'(frf')R.**

Summary of Notation

Faces:

F - Front	L - Left	U - Up
B - Back	R - Right	D - Down

Slices:

f - front	l - left	u - up
b - back	r - right	d - down

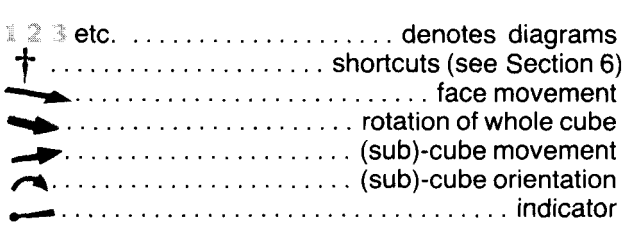
Sub-cubes and Positions:

FUR: Front-Up-Right *corner* (cube or position)
UF(r): Up-Front-(right slice) *edge* (cube or position)

Moves:

R: turn the Right face one turn clockwise
R′: turn the Right face one turn counter-clockwise
r: turn the right slice one turn clockwise
r′: turn the right slice one turn counter-clockwise
RB′lF : R, then **B′**, then **l**, then **F**
(RB′) (lF): RB′lF
(RB′lF)′: RB′lF "Inverse" = **F′l′BR′**
$(RB'lF)^2$: = RB′lFRB′lF
R[UD]R′: = RUDR′ (same as parentheses but only used in conjugation)
†:see shortcuts (Section 6)

Color Key



Remember:

★ Follow all formulas exactly.

★ Watch for the use of Basic Moves, and refer to them on the fold-out.

★ Do not skip steps. You may do some steps your own way as long as you make sure to do exactly the right thing without messing up what has already been done. In particular you may want to do all of Step I (corners) by yourself. You may also find it easier to get the first few edges in place in Step II by yourself.

★ Don't lose your place.

★ There are no fixed centers!

★ Do not move the cube as a whole unless instructed to do so — this changes the meaning of U, F, etc.

★ Don't worry about sub-cubes you will solve later until you get there. Step III is the easiest one, you may wish to try it first for practice.

★ Have fun!

Step-by-Step Solution

OUTLINE OF THE SOLUTION

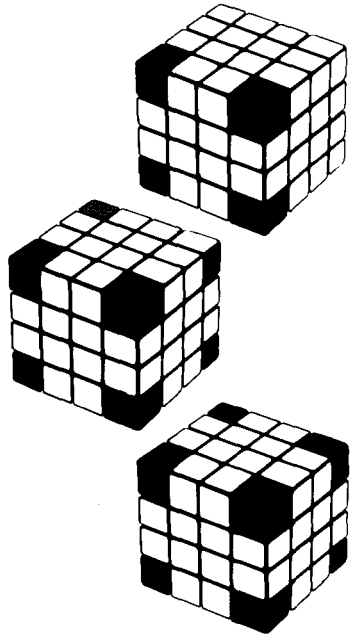
STEP I

SOLVE THE CORNERS

A. Solve the Front-corners

B. Put remaining corners in place, not oriented

C. Orient remaining corners

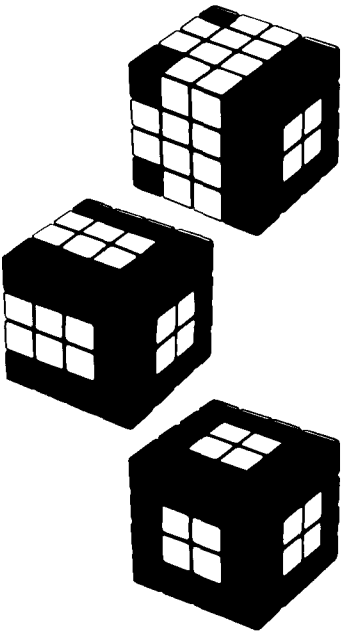


STEP II
SOLVE THE EDGES

A. Solve the Right-edges

B. Solve the left and right slice-edges

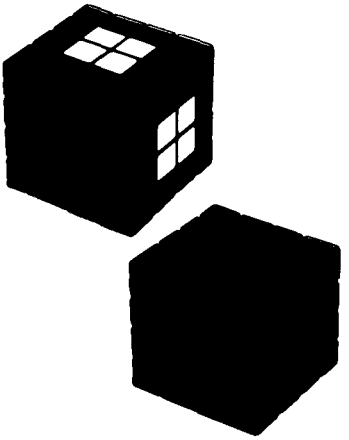
C. Solve the remaining edges



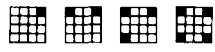
STEP III
SOLVE THE CENTERS

A. Solve the Front-centers

B. Solve the remaining centers, one face at a time

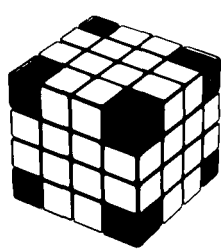


STEP I



SOLVE THE CORNERS

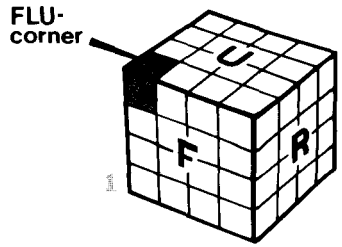
In this step, you put all of the corners in their correct positions, correctly oriented. As explained in the introduction, since at this point there are no fixed centers to refer to, what this means is that the corners should be lined up like this:



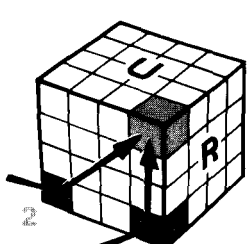
If you wish, you may ignore all edges and centers and pretend you are solving a mini $(2 \times 2 \times 2)$-cube. You probably know how to do this yourself. If you do, then go ahead and solve the corners, and go right to Step II (pg. 32). If not, follow the bouncing ball:

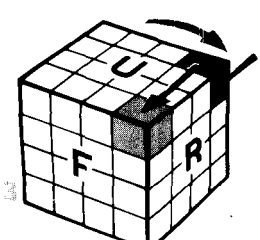
A. Solve the four corners in the Front face.
   1. Look at the corner-cube in the FLU-position; (somewhat arbitrarily) consider this corner-

cube solved, i.e. in its correct position and orientation. This completely determines what the final position of every corner-cube must be.

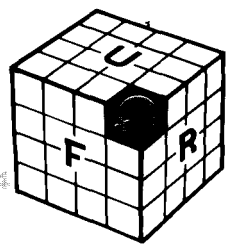


2. Solve the FUR-corner.
   The corner-cube which belongs here is the one having two sides the same colors as the Front and Up sides of the FLU-corner.
   a. Find the FUR corner-cube.
      There are three possibilities:
      i. *The FUR-cube is already in the FUR-position.* Go to step b.
      ii. The FUR-cube is in the Front face, but at the FDR position or the FDL-position. Apply **R** or **DR** respectively to put it at the FUR-position. Go to step b.
      iii. The FUR-cube is in the Back face. Rotate the Back face to put it at UBR. Apply **D'R'D** to put it at FUR. Go to step b.





b. The FUR-cube is at the FUR-position, possibly incorrectly oriented. If it is correctly oriented, i.e. the Front and Up sides of it match the Front and Up sides of the FLU-corner, go to step 3. If not, and it wants to turn counter-clockwise, apply **(R'URU')²**;[4] if it wants to turn clockwise apply **(UR'U'R)²**.[5] Go to step 3.



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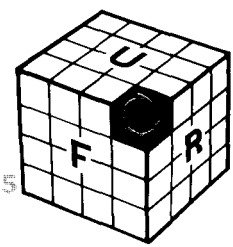
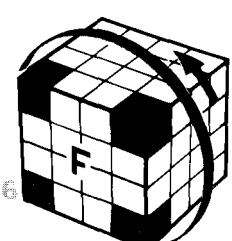


5

3. Rotate the whole cube counter-clockwise one turn, keeping the Front face Front. Repeat step 2 twice more, to solve the remaining two Front corners.[6] The four Front corners are now solved.[7] Go to step B.



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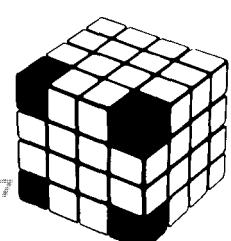
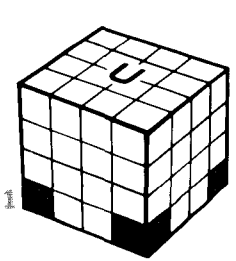

7

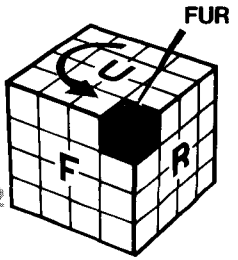
B. Put remaining corners in place, not oriented.

1. Move the whole cube to put the Front face with its solved corners Down,[1] so you are now solving the four Up-corners.

Rotate the Up face to put the FUR corner-cube in its correct position.[2]



1

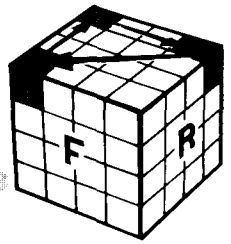


2

Examine the three remaining Up-corners.
There are three possibilities:

i. The other three corners are also already in the correct position. Go to step C.

ii. The other three Up corners are all in the wrong position. If they want to move clockwise,[3] apply **L'(URU')L(UR'U')**. If they want to move counter-clockwise,[4] apply **(URU')L'(UR'U')L**. All Up-corners are now in the correct position. Go to step C.



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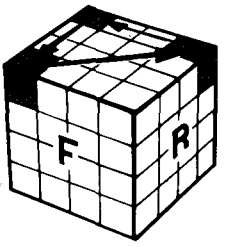
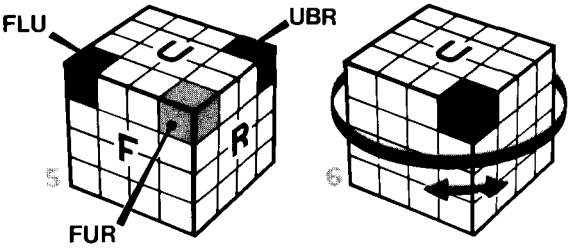


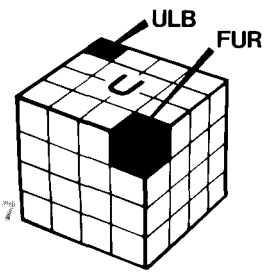
4

iii. There are two Up-corners in the correct position, and two wrong.

If the other correct Up-corner is adjacent to the FUR-corner (i.e. at FLU or UBR) apply **U**.[5] There is now one correct Up-corner (UBR or ULB) and three wrong; rotate the whole cube keeping the Up face Up, to put the correct one at FUR.[6] You are now in the situation of step ii so go to step ii.



If, on the other hand, the other correct corner is opposite the FUR-corner (i.e. at ULB),[7] apply **U'•L'D'[R'URU'] DL** . All Up-corners are now in the correct position. Go to step C.

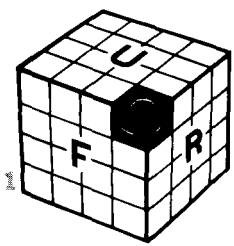


C. Orient the Up-corners.
All corners are now in the correct position, with the Up-corners possibly not oriented. It is easy to use Basic Move 3 and its inverse:
move W = (R'URU')²F(UR'U'R)²F'
move W' = F(R'URU')²F'(UR'U'R)²

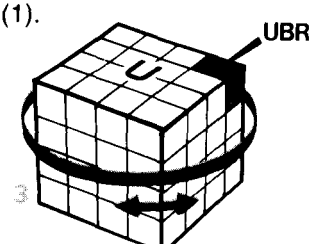
to orient the Up-corners one or two at a time. This is a spot where it is easier to do this yourself than describe it explicitly, but nontheless:
There are four possibilities:

i. All Up-corners are already correctly oriented. Go to step II.

ii. All four Up-corners are oriented wrong. If the FUR corner-cube wants to be turned counter-clockwise, [1] apply **move W**; if it wants to be turned clockwise, [2] apply **move W′**. There are now two or three corners oriented incorrectly. Go to step iv or iii.



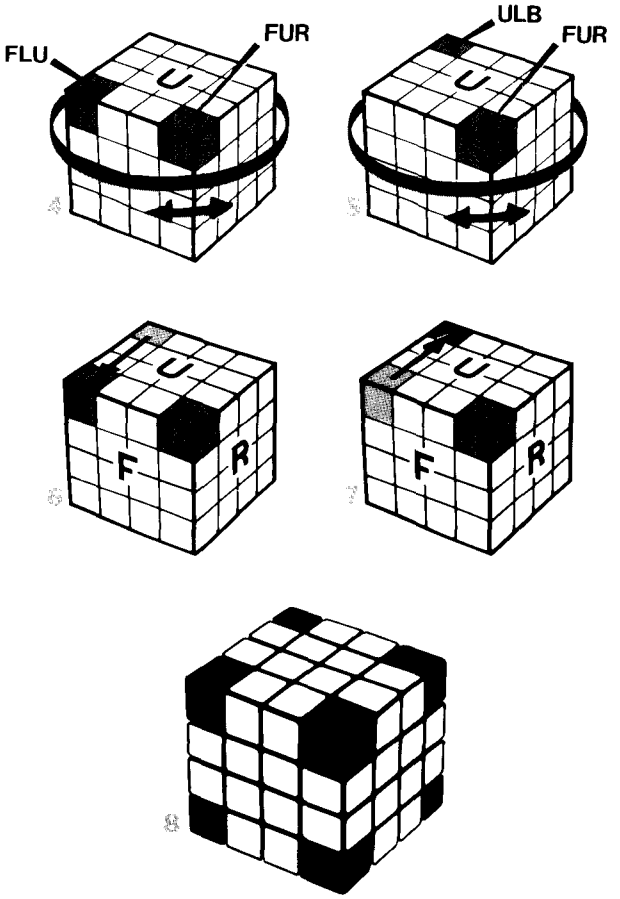
iii. Three corners are oriented incorrectly, and one correctly. Rotate the whole cube, keeping the Up face Up, to put the correct corner at UBR. [3] If the FUR-corner wants to be turned counter-clockwise, apply **move W**; if it wants to be turned clockwise, apply **move W′**. There are now two adjacent corners wrong. Go to step iv.(1).



iv. There are two corners wrong, and two correct. There are two possibilities:

(1). The two incorrect corners are adjacent. Rotate the whole cube, keeping the Up face Up, to put the incorrect corners at FLU and FUR.[4] If the FUR-corner wants to be turned counter-clockwise apply **move W**; if it wants to be turned clockwise, apply **move W′**. All corners are now completely solved.[8] Go to step II.

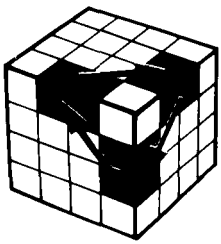
(2). The two incorrect corners are opposite. Rotate the cube, keeping the Up face Up, to put the incorrect corners at FUR and ULB.[5] Apply **L** to put these at FLU and FUR as in (1).[6] If the FUR-corner wants to be turned counter-clockwise, apply **move W**; if it wants to be turned clockwise, apply **move W′**. Apply **L′** to return the cube to its previous position.[7] That is, apply **L[move W** or **move W′]L′**. All corners are now completely solved.[8] Go to step II.


FLU
FUR
U
ULB
FUR
U
U
F
R
U
F
R

STEP II

SOLVE THE EDGES

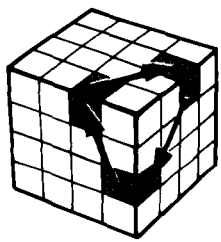
All corners are now solved and you have come to the only difficult step: solving the edges. Here this is done in essentially the same manner as on a $3 \times 3 \times 3$ cube. On an ordinary cube you only need one process (a commutator): **F'UFU'**, which moves three edges. On the Supercube **F'UFU'** looks the same, with pairs of edges glued together.



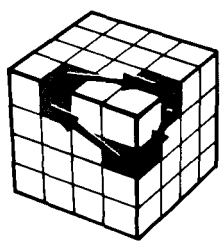
First of all, when you solve the edges on a $3 \times 3 \times 3$ cube using **F'UFU'** you leave the corners till last so it doesn't matter that this moves some corners. But on the Supercube you have already solved the corners and you don't want to mess them up. So instead use the following commutator (Basic Move 4) which has the same effect on edges but does not move corners: **(RBLF)U(F'L'B'R')U'**.

Secondly, you need to accomplish this on single edges (not pairs). So on a Supercube you use:

move X =
(F'rFr')U(rF'r'F)U'



move Y =
(F'l'Fl)U(l'F'lF)U'



Thus X and Y are two halves of Basic Move 4: **(move X) (move Y) = (RBLF)U(F'L'B'R')U'**. It is useful to remember that X moves one "inner" and two "outer" edges (meaning adjacent to the pivotal corner FUR or not), and Y moves one "outer" and two "inner" edges.

Note that moves X and Y are commutators. For example move X is a commutator of a commutator **(F'rFr')** and a face move **(U)**.

Incidentally, moves X and Y do not affect corners or centers!

In principle (and in fact!) moves X and Y may be used to solve the edges much like on a $3 \times 3 \times 3$ cube. Of course inverses and conjugates are used repeatedly (see pp. 13 and 14).

As indicated in the Basic Moves, we actually only use move X in the step-by-step solution. We eliminate the need for move Y by the use of move X in conjunction with Basic Move 7. The solution is less complicated this way, with fewer cases to analyze; but possibly takes a few more moves to execute since Basic Move 7 may be used a few extra times. Places where move Y may be used are indicated by a † and given as shortcuts in Section 6.

Generally when the first of a pair of adjacent edges is placed, you may be sloppy and use Basic Move 4. However, when doing the second one you must be more careful and use move X.

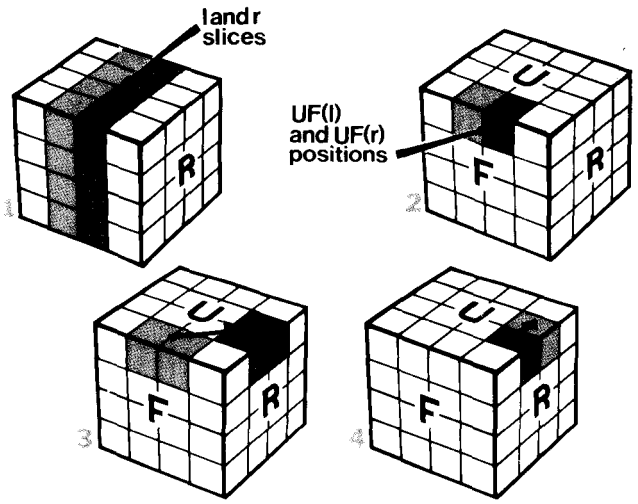
One last comment before getting down to business. Unlike on a $3 \times 3 \times 3$ cube, it is impossible to flip edges in place. What happens is, if the FR(d) edge is in the correct (FR(d)) position, it is necessarily correctly oriented. But remember the FR(u) edge looks identical to the FR(d) edge — if *it* is put in the FR(d) position, it will necessarily be oriented wrong. So if an edge looks oriented wrong, it's not, it's in the wrong position! We will still use the term "incorrectly oriented" to mean an edge which looks like it wants to be flipped in place (but actually wants to change places with its neighbor).

So without further ado:

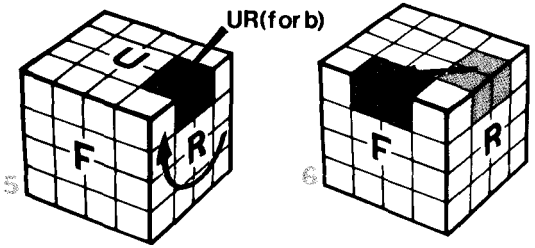
A. Solve the Right-edges.

① Solve the UR(f) and UR(b) edges.

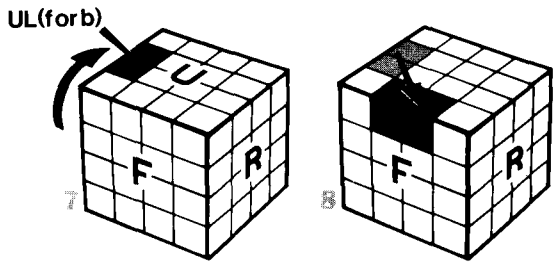
The general plan is: put the UR(f) and UR(b) edges into the l and/or r slices (steps a and b), put them at UF(l) and UF(r) (step c), and then into UR(f) and UR(b), (step d), ignoring orientation. Then orient them if necessary (step e).



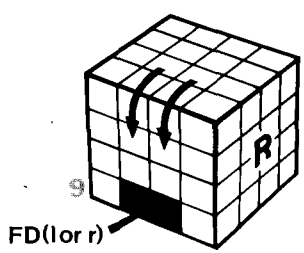
a. Put one UR(f or b) edge in the l or r slice.
Find a UR(f or b) edge. If it is already in the l or r slice, go to step b.
Otherwise, there are two possibilities:
   i. It is in the Right face. Rotate the Right face enough times to put it at UR(f or b), apply **F[(RBLF)U(F'L'B'R')U']F'** to put it in the l or r slice, and rotate the Right face back to where it was (thus fixing the corners).
   Go to step b.



ii. It is in the Left face. Rotate the Left face enough times to put it at UL(f or b),[7] apply **(FRBL)U(L'B'R'F')U'** to put it in the l or r slice,[8] and rotate the Left face back to where it was (thus fixing the corners). Go to step b.

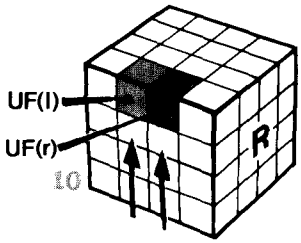


b. One UR(f or b) edge is now in either the l or r slice. Rotate this slice to put this edge in the FD(l or r) position.[9] Now repeat step a once to put the second UR(f or b) edge into the l or r slice. Go to step c.

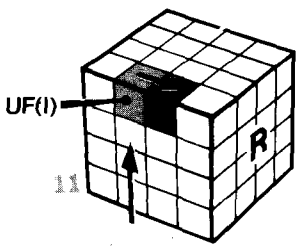


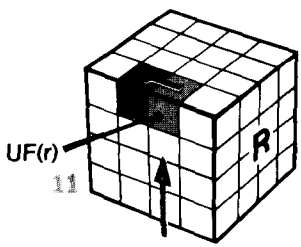
c. Both UR(f or b) edges are in the l and/or r slices. Now you put the UR(f or b) edges at UF(l) and UF(r). There are two possibilities.

i. If the UR(f or b) edges are in different slices (one in l and one in r) rotate these slices to put them at UF(l) and UF(r).[10] Go to step d.

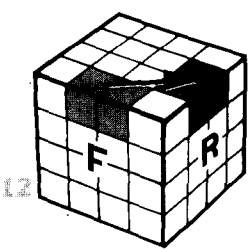


ii. If the UR(f or b) edges are in the same slice, rotate that slice to put one of them at UF(l or r),[11] and apply Basic Move 7: **R'U²R²UR'U'R'U²LFRF'L'** *to move this edge to* the other slice.[11] Go to step i.

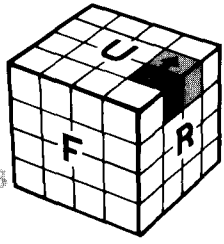




d.† The UR(f or b) edges are now at UF(l) and UF(r). Apply **F[U(RBLF)U'(F'L'B'R')]F'** to put them at UR(f) and UR(b).[12] Go to step e.

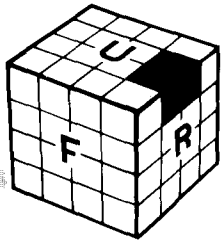


e. If both UR(f or b) edges are correctly oriented, go to step 2. If they are both incorrectly oriented,[13] apply Basic Move 7: **R'U²R²UR'U'R'U²LFRF'L'**, and go to step 2.



13

(2) Both UR(f or b) edges are now solved.[14] Rotate the whole cube, keeping the Right face to the Right.[15] Repeat step 1. After four repetitions the Right edges will all be solved.[16]



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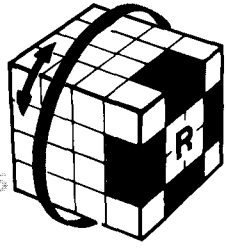

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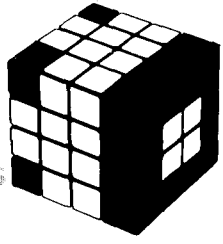
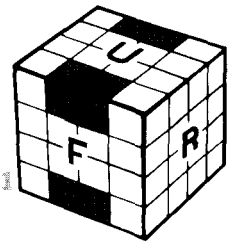
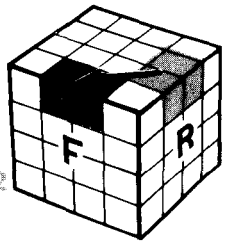

16

B. Solve the l and r slice edges.
All the edges on one face are now solved. Move the whole cube to make this the Left face. You now solve the l and r slice edges.[1]



1. Solve the UF(l) and UF(r) edges.
This is sort of opposite to step A: put the desired edge into the Right face, and then into the UF(l or r) position.[2] Do this for each edge (steps a and b), ignoring orientation. Then orient them if necessary (step c).[3]



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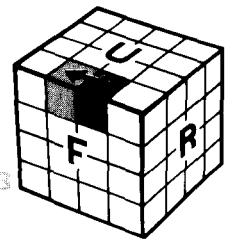
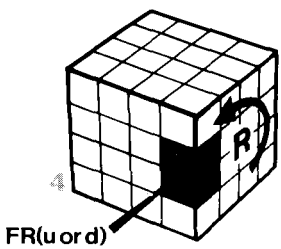


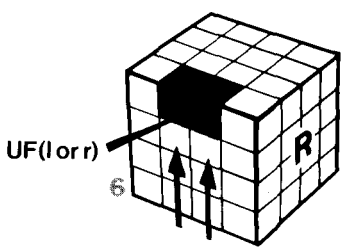
3

a. Put one UF(l or r) edge in the UF(l or r) position. Find a UF(l or r) edge. If it is already at UF(l or r), go to step b. Otherwise, there are two possibilities:

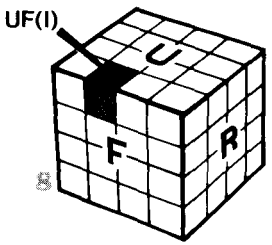
i.† It is in the Right face. Rotate the Right face enough times to put it at FR(u or d).[4] Apply **(RBLF)U(F'L'B'R')U'** to put it at UF(l or r),[5] and rotate the Right face back to where it was. Go to step b.



ii. It is in the l or r slice. Rotate this slice enough times to put it at UF(l or r),[6] and apply **(RBLF)U(F'L'B'R')U'** to put it in the Right face,[7] and rotate the l or r slice back to where it was. Go to step i.



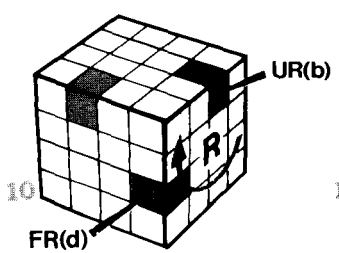
b.†One UF(l or r) edge is now in the UF(l or r) position, possibly incorrectly oriented. To minimize the number of decisions you have to make, it is best to have this at UF(l).[8] If it is at UF(l), fine; if it is at UF(r),[9] apply Basic Move 7: **R'U²R²UR'U'R'U²LFRF'L'** to put it there.



You now put the other UF(l or r) edge at UF(r).
Find this edge.
There are two possibilities:

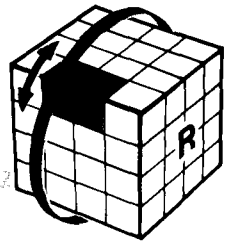
i. It is in the Right face. You can rotate the Right face to put it at either UR(b) or FR(d),[10] but not both, so rotate the Right face to either put it at
   * UR(b), and apply **move X'**,[11]
   * FR(d), and apply **move X**,

   to put it at UF(r); and rotate the Right face back to where it was. Go to step c.



ii. It is in the l or r slice. (If it is already at UF(r), go to step c). Rotate the whole cube keeping the Right face to the Right to put it at UF(l or r)[12] (and to put the edge already at UF(l) out of the

way). Apply **(RBLF)U(F'L'B'R')U'** to put it in the Right face, and rotate the cube back to where it was. Since the UF(l or r) edge in question is now in the Right face, go to step i.



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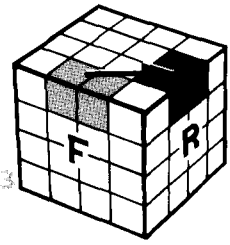
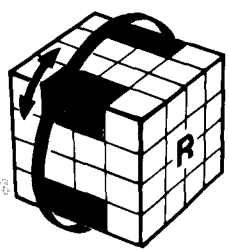


13

c. Both UF(l or r) edges are now at UF(l or r). If they are both correctly oriented, go to step 2. If they are both incorrectly oriented, apply Basic Move 7: **R'U²R²UR'U'R'U²LFRF'L'**, and go to step 2.

(2.) Both UF(l or r) edges are now solved. Rotate the whole cube, keeping the Right face to the Right. Lather, Rinse, Repeat. After four repetitions all of the edges in the l and r slices are solved.



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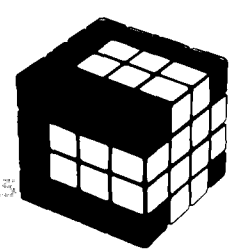
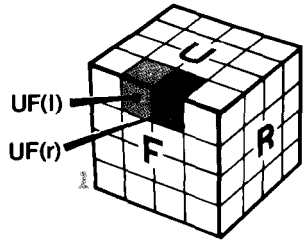


15

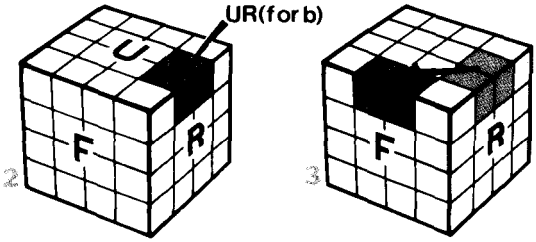
C. Solve the remaining edges.

All of the edges are now solved except those on a single face. Move the whole cube to make this face the Up face.

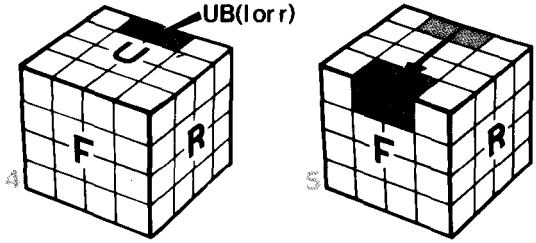
(1.) Solve the UF(l) and UF(r) edges.



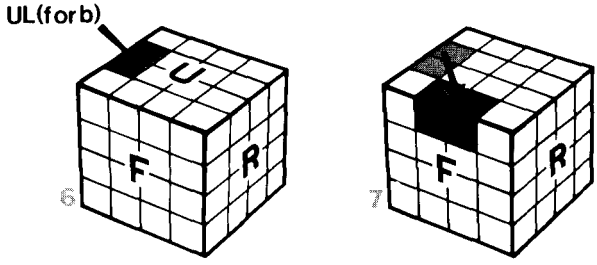
a. Find a UF(l or r) edge. If it is already at UF(l or r), go to step b. Otherwise, there are three possibilities:
   i. It is at UR(f or b).
      Apply **B'R'[(RBLF)U(F'L'B'R')U']RB**
      to put it at UF(l or r).
      Go to step b.



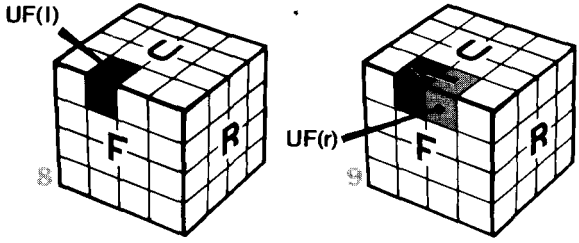
   ii. It is at UB (l or r).
      Apply **B'R'[U(RBLF)U'(F'L'B'R')]RB**
      to put it at UF(l or r).
      Go to step b.



iii. It is at UL(f or b).[6]
Apply **R'U'][(RBLF)U(F'L'B'R')U']UR** to put it at UF(l or r).[7]
Go to step b.



b.† One UF(l or r) edge is now in the UF(l or r) position, possibly incorrectly oriented. To minimize the number of decisions you have to make, it is best to have this at UF(l).[8] If it is at UF(l), fine; if it is at UF(r),[9] apply Basic Move 7: **R'U²R²UR'U'R'U²LFRF'L'** to put it there.

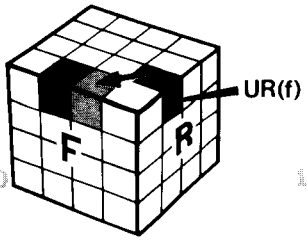


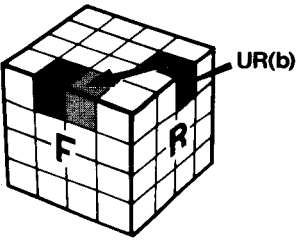
You now put the other UF(l or r) edge in the UF(r) position.
Find this other edge.
There are three possibilities:

i. It is at UR(f or b). If it is at

* UR(f), apply **B'R'[move X]RB**,[10]
* UR(b), apply **FU[move X']U'F'**,[11]

and go to step c.

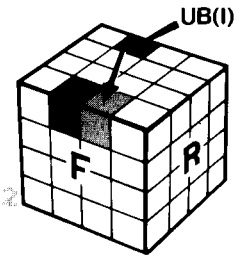


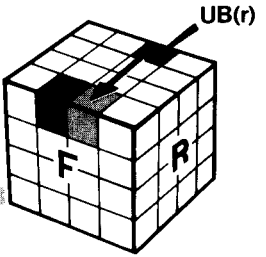


ii. It is at UB(l or r). If it is at

* UB(l), apply **FU[move X]U'F'**,[12]
* UB(r), apply **B'R'[move X']RB**,[13]

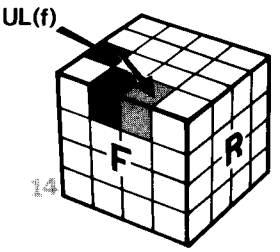
and go to step c.

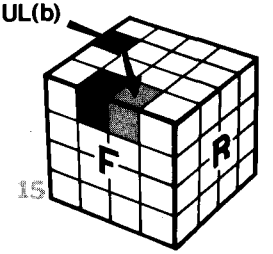




iii. It is at UL(f or b). If it is at
* UL(f), apply **R'U'[move X]UR**,[14]
* UL(b), apply **LF[move X']F'L'**,[15]

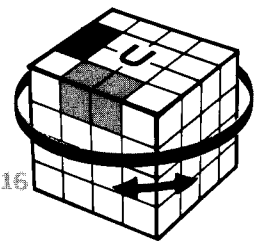
and go to step c.



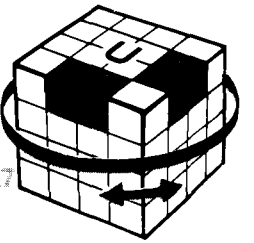


c. The UF(I or r) edges are now at UF(I) and UF(r). If they are both correctly oriented, go to step 2. If they are both incorrectly oriented, apply Basic Move 7: **R'U²R²UR'U'R'U²LFRF'L'**, and go to step 2.

2. Both UF(I or r) edges are now solved. Rotate the whole cube, keeping the Up face Up, to put these at UL(f) and UL(b).[16] You now solve the UF(I) and UF(r) edges. To do this, repeat step 1 once. After this the only edges not solved are at UR(f or b) and UB(I or r). Go to step 3.



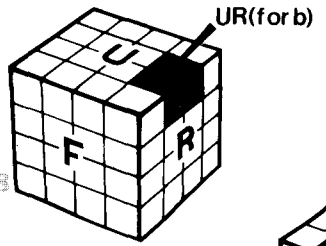
③ The UL(f), UL(b) and UF(l), UF(r) edges are now solved. Rotate the whole cube, keeping the Up face Up to put the *unsolved* edges at UF(l or r) and UR(f or b).[17]

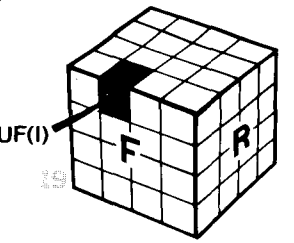


a.† Find both the UF(l) and UF(r) edges.
There are three possibilities:

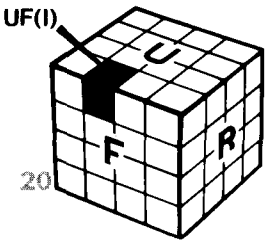
i. They are both already at UF(l or r). Go to step c.

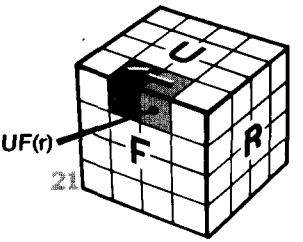
ii. They are both at UR(f or b).[18] Apply **lU'F'U[move X]U'FUl'**. There is now a UF(l or r) edge at UF(l).[19] Go to step b.





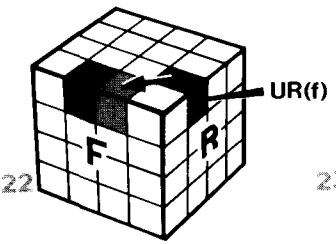
iii. One is at UF(l or r), and one at UR(f or b).
- * If the one at UF(l or r) is at UF(l), go to step b.
- * If the one at UF(l or r) is at UF(r), apply Basic Move 7: **R'U²R²UR'U'R'U²LFRF'L'**, and go to step b.

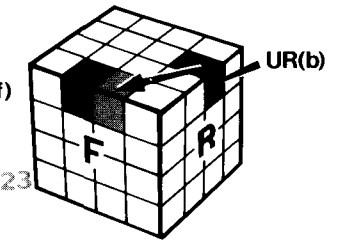




(b) There is now a UF(l or r) edge at UF(l) and the other one is at UR(f or b).
If the other one is at:
- * UR(f), apply **bR'b'[move X]bRb'**,
- * UR(b), apply **bR'b'[move X']bRb'**,

and go to step c.





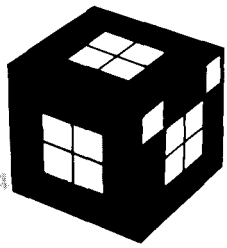
c. The UF(l or r) edges are now both at UF(l or r), and the UR(f or b) edges are both at UR(f or b). You now orient them.

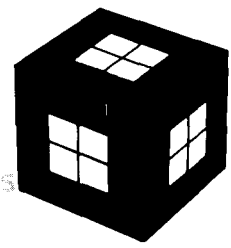
There are three possibilities:

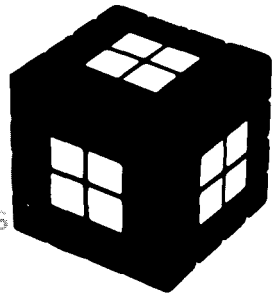
i. All four of the UF(l or r) and UR(f or b) edges are incorrectly oriented.[24] Apply Basic Move 7: **R'U²R²UR'U'R'U²LFRF'L'** to remedy this, and go to Step III.

ii. Exactly two (adjacent) edges are incorrectly oriented. Thus you want to switch these two edges. (This cannot be done on a $3 \times 3 \times 3$ cube, but this is a Supercube!) Move the whole cube to put these at UF(l) and UF(r).[25] Apply (carefully) **D²[uF'u'F]D²[l²•(LUL')u'(LU'L')]**. Go to Step III. BASIC MOVE 8

iii. All edges are oriented correctly.[26] Go to Step III.





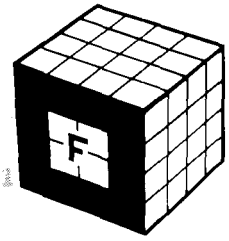


STEP III

SOLVE THE CENTERS

The worst is over. Solving the centers is a piece of cake. Solving the centers on each of the six faces is identical; the procedure below is simply repeated six times. You only use Basic Move 9.

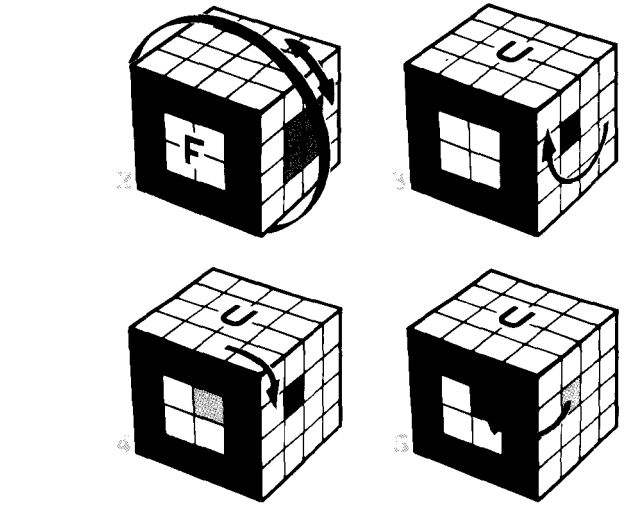
A. Solve the Front-centers.
Since solving the centers on each of the faces is identical, we illustrate this for a single color, say blue. So move the whole cube to put the blue face Front.



1. Move a single blue center onto the Front face (from some other face). Find a blue center-cube on some face other than the Front face.

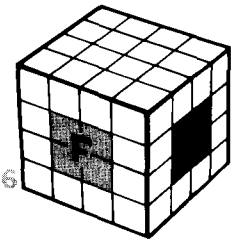
There are two possibilities:

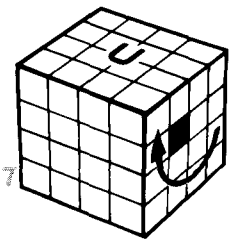
i. It is on the U, R, D or L face. Rotate the whole cube, keeping the Front face to the Front, to put this blue center-cube on the Right face. Rotate the Right face to put this blue center-cube in the upper left-hand slot on the Right face. Rotate the Front face to put any non-blue center-cube in the upper right-hand slot of the Front face. (Remember how you did these two things, you need to undo them in a moment.) Apply **R'(fr'f')R(frf')** to bring the desired blue center-cube to the Front face. Rotate first the Front face and then the Right face to undo what you did a moment ago, thus restoring all corners and edges to their solved positions. It is easy to figure out how to do this by looking at the corners and edges, and by remembering to move the Front face first. Go to step 2.

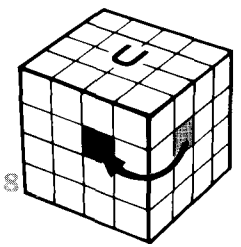


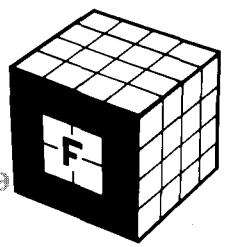
ii.† The desired blue center-cube is in the Back face. You apply the same procedure as in step i to move this blue to another face adjacent to the Front face. You can then apply step i to move it to the Front (blue) face. Move the whole cube to put the Back face, the one with the blue center you want, to the Right,[6] and also some other face whose centers are not yet solved to the Front.[6] Rotate the Right face to put the desired blue center-cube in the upper left-hand slot of the Right face.[7] Apply **R′(fr′f′)R(frf′)** to bring this blue center-cube to the Front face,[8] and rotate the Right face back to where it was.

Move the whole cube to return the blue face to the Front.[9] Go to step i.

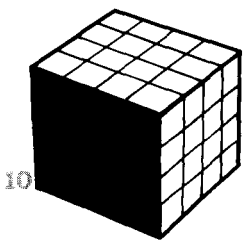








2. Repeat step 1 up to four times to get each of the blue center-cubes onto the Front (blue) face one-by-one. After four times through, the Front face is completely solved.[11]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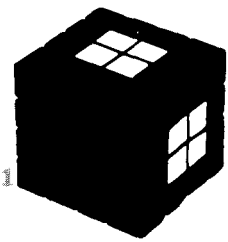

11

(B) The centers on one face (the blue face) are solved. Now you solve the center-cubes on the remaining faces, one face at a time.

Choose any other face to do. Repeat step A exactly, with the color of this face in place of "blue." Thus you solve the center-cubes on another face without disturbing the ones you have already done. Repeat step A four more times, doing the faces one-by-one in any order with one restriction: the only thing to watch out for is that you do not leave two opposite faces till last. For example, do the faces in this order: F, U, R, B, L, D.

Do this five times. The Supercube is now solved!!!

FIRST SOLVED
SUNDAY, 3:45 PM
9-26-82

Congratulations!

†Shortcuts

Here are a few ways to trim the number of moves in the solution. You should only consider these when you feel comfortable with the solution as it stands. Then depending on your mental and physical agility you can try to cut down your time.

Try to find your own shortcuts — there are almost as many shortcuts as there are steps.

II.A.1.d. (pg. 37) Put the UR(f) and UR(b) edges in place:
If the Up sides of these edges are
* Up, apply **F[U(RBLF)U'(F'L'B'R')]F'**,
* Front, apply **FR'[(F'R'F'R'F')(RFRFR)]RF'**,

and go to step 2 (step e is no longer necessary).

II.B.1.a.i. (pg. 40) Put one UF(l or r) edge in the UF(l or r) position:
If the Front side of this edge is
* Front, apply **(RBLF)U(F'L'B'R')U'**,
* Right, apply **R[U(RBLF)U'(F'L'B'R')]R'**,

and go to step b.

II.C.1. (pg. 42) solving the first four Up-edges is shortened similarly.

II.B.1.b. (pg. 40) Putting the second UF(l or r) edge at UF(l or r):
If a UF(l or r) edge is at UF(r), find the other UF(l or r) edge. If it is in the Right face, you can rotate the Right face to put it at UR(f) or FR(u), so rotate the Right face to put it at
* UR(f), and apply **move Y'**,
* FR(u), and apply **move Y**,

to put it at UF(l), and rotate the Right face back to where it was. Go to step c.

II.C.1.b. (pg. 44) Solving the Up-edges:
If the UF(l or r) edge is at UF(r),
find the other UF(l or r) edge.
There are three possibilities:

i. It is at UR(f or b); apply **FU[move Y']U'F'** or **B'R'[move Y]RB** respectively, and go to step c.
ii. It is at UB(l or r); apply **B'R'[move Y']RB** or **FU[move Y]U'F'** respectively, and go to step c.
iii. It is at UL(f or b); apply **LF[move Y']F'L'** or **R'U'[move Y]UR** respectively, and go to step c.

II.C.3.a. (pg. 47) Solving the final four edges:
This is similar to the preceding two shortcuts. At possibility iii, if there is a UF(l or r) edge at UF(r), do the following. The other one is at UR(f) or UR(b); apply **f'R'f[move Y'** or **move Y]f'Rf** respectively, and go to step c.

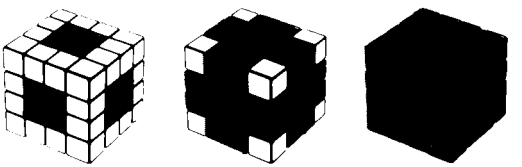
Solving the Up-edges may be considerably speeded up by the use of moves 6.a-d, pg. 61, which give four different ways of moving three Up-edges in only eight or ten moves.

III.A.1.ii. (pg. 52) Solving the centers; the desired blue center-cube is in the Back face:
The blue center-cube may be brought directly to the Front face as follows. Rotate the Back face to put the blue center-cube in the upper left-hand slot (as viewed from the Back) of the Back face. Rotate the Front face to put any non-blue center-cube in the lower left-hand slot of the Front face. Apply **$(Fu^2F'd^2)^2$**, and rotate the Front and Back faces back to where they were.

Alternative Solution

Below is another method for solving the Supercube. Its stages are:

I. Solve the centers
II. Solve the edges
   A. Solve the Right-edges
   B. Solve the l and r slice-edges
   C. Solve the Left-edges
III. Solve the corners
   A. Solve the Front-corners
   B. Put the remaining corners in place, not oriented
   C. Orient the remaining corners



The advantages of doing things in this order are:

1. Centers, being first, go very quickly.
2. Edges are a little easier than before. You don't have to worry about corners, and except for Basic Move 8 and some slice moves, the moves used before do not affect centers anyway. You need to modify your use of slice moves, and find a replacement for Basic Move 8.
3. Corners are just like on a $3 \times 3 \times 3$ cube with which you are probably familiar.

The disadvantages are:

1. You must make sure to solve the centers in the correct arrangement of colors. This is not so much a problem to do as it is cumbersome to describe in words.
2. You don't have centers to kick around anymore. So you may need to exchange two edges without affecting any centers (Basic Move 8 affects some centers). You may need to apply Basic Move 8~ once (see below).
3. You may need to exchange two corners without affecting anything else. This is not so bad; see Basic Move 10.~

As Basic Moves you use:

Basic Move 1~ (Steps III~-A and B)

In place of **R'URU'**, use **(R'URU')**^{3}, so as not to affect edges or centers.

Basic Move 2~ (Step III~-B)

Same as Basic Move 2.

Basic Move 3~ (Step III~-C)

Same as Basic Move 3.

Basic Move 4~ (Steps II~-A, B and C)

In place of **(RBLF)U(F'L'B'R')U'**, simply use **F'UFU'**, since you don't need to worry about affecting corners.

Basic Move 5~ (Steps II~-A, B and C)

Same as Basic Move 5.

Basic Move 6~ (Steps II~-A, B and C)

Same as Basic Move 6.

Basic Move 7 ˜ (Steps II˜-A, B and C)

Same as Basic Move 7.

Basic Move 8 ˜ (Step II˜-C)

Now you need **u²B²[(f'l'fb'r'f')d(frbf'lf)•B'L'U² [move X']U²LB]B²u²** which exchanges the FR(u) and FR(d) edges but does not affect centers.

Basic Move 9 ˜ (Step I˜)

In place of **R'(fr'f')R(frf')** you may simply use **F'uFu'** which also essentially switches two centers. (It actually moves five centers). It moves some edges but now you don't care about this.

Basic Move 10˜ (Step III˜-B)

(F²r²F²)(U²r²)³(F²r²F²)•U'•F²B²D[R²F²B²L²F²B²]D'B²F²

This moves four corners, and nothing else. Following it by **(URU')L'(UR'U')L** (i.e., inverse of Basic Move 2), exchanges the FUR and UBR corners.

Briefly, you solve the Supercube as follows:

I. ˜ Solve the Centers

This is just like before, except that Basic Move 9˜ (essentially) switches the upper right-hand center-cube of the Right face and the upper left-hand center-cube of the Front face.

As a shortcut to this step (see pg. 55), **F'u²Fu²** essentially switches the upper left-hand corner of the Front face with the upper right-hand corner of the Back face.

Since you don't have edges and corners to guide you, you must decide beforehand which centers

belong where, that is what the final arrangement of colors must be. You can do this by remembering what the solved cube looks like, or by studying the corners and one way or another imagining them solved and hence telling you where the centers belong.

II.˜ Solve the Edges

This is just like before with a few changes. Throughout, replace **(RBLF)U(F'L'B'R')U'** with **F'UFU'**.

A. Solve the Right-edges
   This is just like before, except at the end of step 1 you return the l and r slices to where they started to restore the centers.

B. No change is necessary.

C. Again this is the same, except that in step 3.c.ii. replace Basic Move 8 by Basic Move 8.˜

If you don't like Basic Move 8,˜ you may use a hybrid: use Basic Move 8, thus moving some centers, and then solve them again following the original solution Step III.

III.˜ Solve the Corners

This is exactly the same as on a $3 \times 3 \times 3$ cube with one amusing addition: you may have to exchange two corners.

For example, you may roughly follow Step I of the main solution, replacing **R'URU'** with **(R'URU')**^{3}. Of course you must be more careful in getting the first few corners in place.

If you wind up with exactly two corners in the wrong position and the rest correct, you cannot simply apply **U** (as you could in Step B.iii) since you are concerned about the edges. In effect, Basic Move 10˜does just apply **U** and then fixes the edges. As noted, following it with the inverse of Basic Move 2 achieves the exchange of two corners.

More Useful Moves

Now that you have mastered the Basic Moves and know how to use them to solve the Supercube, you may want to learn some more processes. Some may be used to speed up solving the Supercube. Others are more specialized, and are useful for making patterns. Now you are free to really *play* with the Supercube — experiment, try these moves in different combinations, create your own patterns!

Any process you know on a $3 \times 3 \times 3$ cube will be useful on a Supercube. Because there are so many of them to be found in books on the ordinary cube we have only included a few of these.

Some of these moves are certainly not the shortest ones possible. See if you can improve on them. As always we prefer moves which may be a little longer but have some method behind their madness: keep your eyes peeled and you will see commutators and conjugates lurking everywhere.

1. $(Fu^2F'd^2)^2 = (Fu^2F')d^2(Fu^2F')d^2$

Essentially exchanges the lower left-hand center on the Front face with the upper left-hand one on the Back face. See pg. 55.

2. F′uFu′

Essentially exchanges the upper left-hand center of the Front face with the upper right-hand center of the Right face, and also moves five edges. See Basic Move 9¯, pg. 58.

3. b′u′bu

This commutator cycles a pair of triplets of centers for a total of six, one on each face. Useful conjugates of it are:

3a. **RU²LB²[b′u′bu]B²L′U²R′**

3b. **R′F²L′D²[b′u′bu]D²LF²R**

4. $(U^2r^2)^3 = U^2r^2U^2r^2U^2r^2$

Exchanges two opposite pairs of Up-edges and opposite Up-corners, and also four centers. So:

4a. **$(F^2r^2F^2)(U^2r^2)^3(F^2r^2F^2)\cdot U^2$**

Exchanges just two opposite pairs of Up-edges.

4b. **$(F^2r^2F^2)(U^2r^2)^3(F^2r^2F^2)\cdot U'\cdot$**
$F^2B^2D[R^2F^2\,B^2L^2F^2B^2]D'B^2F^2$

Moves only four corners, following it by **(URU′)L′(UR′U′)L** exchanges just two corners. See pg. 58.

5. (R′F′R′F′R′)(FRFRF)

Moves six edges.

5a. **R′[(R′F′R′F′R′)(FRFRF)]R**

Moves six edges just like Basic Move 4 but with different orientations.

6. a. **(LF′L′)f′(LFL′)f**

b. **f(R′FR)f′(R′F′R)**

c. **b′(U′RUR′)b(RU′R′U)**

d. **(FU′F′U)b(U′FUF′)b′**

Moves three edges in the Up face. See pg. 55.

7. (r′uru′)R(ur′u′r)R′·F(ur′u′r)F′(r′uru′)

Switches two centers on the Right face, and two on the Front face.

Patterns

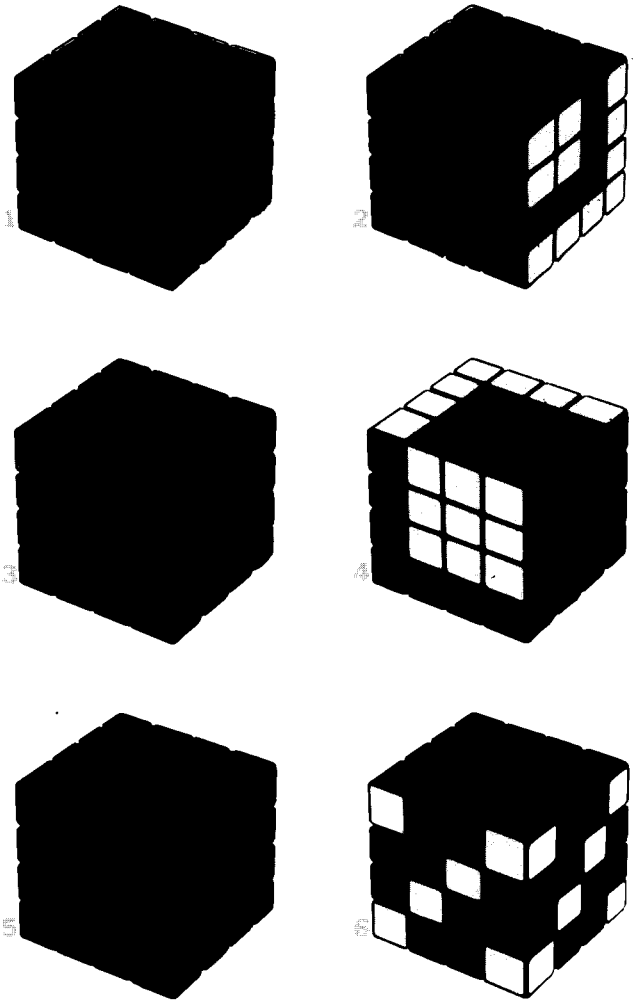
The fun really begins once you have learned to solve the Supercube and can start playing with patterns. Your understanding of the Basic Moves opens up a vast territory to be explored. The Supercube has 177,628,724,197,557,644,876,978,255,387,965,784,064,000,000,000 positions. If this is not vast I don't know what is! The moves in Section 8 are also useful for patterns.

Here are a few patterns to get you going. There are literally billions of patterns on the Supercube where the colors on the F, U and R faces intermingle, as do those on the L, B and D faces. There are numerous "rings" (no. 5), "paths" (no. 2), "mesons and quarks" and so forth (nos. 3 and 4), and stripes and diagonals (nos. 1 and 6). These are but a tip of the proverbial iceberg. Incidentally, you can make a ring around a single corner without a ring around the opposite corner.

You can find a lot of patterns by imagining the Supercube to be an ordinary cube or even a mini $(2 \times 2 \times 2)$-cube. If you only move faces, never slices, the edges stay glued in pairs and move like the edges in an ordinary cube. Note that unlike on an ordinary cube you can exchange just two (pairs of) edges, or two corners. This is done in pattern 1. On the other hand,if you always move each slice together with its face (i.e. l and L, f and F, etc.),the Supercube behaves like a $2 \times 2 \times 2$ cube (whose sub-cubes are $2 \times 2 \times 2$ cubes!).

Try combining patterns, especially patterns 3, 4, 5 and even 2.

Let your imagination go!


1
2
3
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6

Afterword — for the Expert

The Supercube (to be precise the Supercube with a picture on each face) is a splendid example of what mathematicians call a non-commutative group. Many abstract concepts of group theory become clear when applied in this concrete situation. Here we discuss a few of the simpler aspects of this particular iceberg.

Remember the commutative law, $ab = ba$? This does not hold on a Supercube. If it did, a commutator $aba'b'$ would not do anything, but of course it does. Commutators do so much that you can do almost anything with them. More precisely, the commutator subgroup is of index 4 (on an ordinary cube it is of index 2). That is, there are essentially only three things you cannot do with commutators, which is another way of saying how very non-commutative the group is or, in layman's terms, this is a damn hard puzzle!

Where did that number on page 62 come from? Well, ignoring the fact that there are 4 identical centers of each color, there are $(8!24!24!/2^{12})\cdot(3^8 2^{24})$ ways to disassemble the Supercube and reassemble it. Not all of these can be obtained by legal moves: since an edge in a given position can only have one orientation the $2^{24}/2^{12} = 2^{12}$ is spurious, and as on an ordinary cube the 3^8 should only be 3^7. You cannot switch just two centers, so divide by 2, to get $8!24!24!3^7/2$. Finally because of identical centers the answer is $(8!24!24!3^7/2) / (4!^6/2) = 8!24!24!3^7/4!^6$. Multiply this out and you get the 48 digit number on pg. 62 (exercise).

With color illustrations throughout

The complexities will baffle even those millions who mastered the cube! But now you can solve Rubik's Revenge™ with Jeffrey Adams's full-color step-by-step instructions. Here are the secrets to this brilliant new solution:

- **The simplest, most direct solution available**
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- **Easy-to-follow basic steps lead you through the entire process**
- **Shortcuts speed the sequences to the solution**
- **An alternate solution will intrigue cubemasters**
- **A full appendix offers further fun supercube possibilities**
- **Special fold-out flap summarizes nine basic solution-steps**

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