How To Solve Every SUDOKU Puzzle - Volume 2

				8		9		
4		2	3			6		
9		1	5					3
			6	7	8			
						1		
					6	2		7
		9						5
		3		4				

Sudoku - No more a Puzzle

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Table of contents

If you've mastered the easier, yet the surer way of solving Sudoku using the Possibility Matrix Approach, it is possible that you may still want to learn the other, more frequently used method (the Conventional Method), though it may be more complex and more difficult to learn.

This eBook presumes that you have already learnt and understood the Possibility Matrix Method. Why? Because that's the easiest way to learn to solve Sudoku. Our own studies have shown that those who have learnt the Possibility Matrix Method find it far easier to learn this method than first-time learners of this Method.

If you would like me to hand-hold you, I would recommend that you to read the eBook on the Possibility Matrix Method first. This eBook will not do the hand-holding that the other one does. However, this doesn't mean that you can't read this and understand this method all by itself. Just that I would not recommend this.

Those of us who don't want to learn the Conventional Method, or find it not very interesting, or difficult or unnecessarily cerebral, don't need to learn it; because we have already learnt to solve Sudoku using a smart approach that is less taxing.

Solving Sudoku is partly an art; particularly when you follow this method. Whereas the Possibility Matrix Method was more scientific.

Using the Conventional Method, you will be able to solve the more difficult Sudokus only from experience. Till then do more Sudokus. When you get stuck, perhaps you can switch over to the Possibility Matrix Method.

Here's wishing you a long and happy Sudoku experience!

Why would you want to learn this method since you already know one method? Your reason could be:

(i) If you learn the method that most people use anyway, you may find it easier to talk to them in their language when you get to discuss Sudoku in social conversations.

(ii) You have more time to learn one more method, and you feel challenged too, and so you want to learn it.

(iii) Knowing 2 methods helps you solve the puzzles quicker.

(iv) There are those who would claim this to be the smarter method; so why not have a go at this method as well?

(v) Or simply because, it's there!

You may have any other of your own reasons as well.

But let's be clear: You don't necessarily have to learn this method to be able to solve Sudokus (unless, of course, YOU WANT to learn), as we've already seen. Possibility Matrix Method is good, and guarantees solutions to every Sudoku.

But just in case YOU decide to learn it, let's face it - this one is not going to be as easy as the other one. You should have the patience to go thru this somewhat cerebral method, to learn it completely and to put it to practice. Some, no doubt, learn it spontaneously; but many would need to put in an extra effort.

Luckily for you, since you've already learnt the Possibility Matrix method, you'll find it much easier to learn it than others would, despite this being a more difficult method, per se`. For this reason, my recommendation is that you may try to learn this method ONLY after you've solved quite a few puzzles using the Possibility Matrix Method.

Let's now try to understand what makes this method more difficult to learn and apply.

Sudoku involves solving Cells based on the values in Rows, Columns and Major Squares. When we need to solve a certain Cell by just referring between a Row and a Column, or between a Column and a Major Square, or between a Row and a Major Square, the problem is in 2-D (2 Dimensions).

But when we need to solve for each Cell across all the 3 Dimensions (3-D), Rows, Columns and Major Squares (which is the case with the not-too-easy Puzzles), that's when it gets a little tricky. It requires what is called the 'Spatial Ability'. Some of us are inherently good at it, and the rest of us need to put in an extra effort.

While the Conventional Method tries to take the bull by its horns and attempts to solve the method in 3-D (after solving in 1-D and 2-D to the extent possible), the Possibility Matrix Method tries to find a bypass approach that solves it using a simpler method.

The Possibility Matrix Method is a mathematical approach while the Conventional Method is more of an intuitive approach. Not that there is no mathematics or logic to the Conventional Method; there are, in fact, a set of definite procedures (we call them Approaches) that have emerged over time, and they are getting fine-tuned by Sudoku enthusiasts. So, newer approaches and variations of the old ones are appearing every day.

But to solve a given Sudoku, you keep these approaches at the back of your mind, and take an 'on the spot' intuitive decision on which of these approaches you would apply, for which cells, and in which order.

Though you may make use of the Possibility Matrix even in the Conventional Method, you don't depend on the Possibility Matrix as the tool. You try to avoid using it as much as you can, and you try to assign values to Cells as far as possible. It's strictly not essential, but that's what most people do.

Of course, you can choose the approach that best suits your temperament.

As explained already, the difference between the 2 methods is that, while the Possibility Matrix Method attempts to find the values for every Cell, the Conventional method, by and large, attempts to find the right Cells for every value. Also, some of the approaches used in the Conventional Method presume that a given puzzle has one and only one solution. Which is most often the case with puzzles published in reputed magazines. Whereas the Possibility Matrix Method makes no such assumption. And therefore, it can help you find all the solutions (and also the fact of 'No Solution' wherever it is the case).

And interestingly, if you know both the approaches, you could combine them as you like. You could use some of the techniques of one Method to solve parts of the puzzle and use some of the techniques of the other Method to solve the other parts. And you could go back and forth.

So, if you wish to use the Conventional Method, but can't understand (or remember and apply) some of the techniques of the Conventional Method, don't tax yourself. Sudoku is a game, not a school lesson. Learn as much as you can. And when you don't know how to proceed, fall back on the Possibility Matrix Method. You can, in fact, go back and for the between the two methods.

I advocate learning the Possibility Matrix Method first, which guarantees solutions to Sudoku puzzles of any complexity.

Let's now learn the Conventional Method to solving Sudoku.

Since we've already learnt the Possibility Matrix method fully and systematically, we will not do many exercises at every stage, but we'd rather have some exercises at the end of the eBook.

This method consists of unlimited number of approaches. I said unlimited number because, if you become a member of any one of the many Sudoku communities, you'll find on-going discussions on the available approaches, and research on a few new approaches.

And with all these approaches, you're still not guaranteed a solution. Unless, of course, you include the 'Trial & Error' approach (which is strictly not a part of the Conventional Method), to your armory. This is part of the reason why I advocate learning the Possibility Matrix Method first, which guarantees a solution to every Sudoku puzzle of any complexity.

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Of the unlimited number of approaches, we'll learn the most popular

12, which should normally be sufficient to solve most of the puzzles that appear in newspapers and magazines. You may discover that some of these Approaches overlap with the steps we'd learnt in the Possibility Matrix Method, but nevertheless, we'll see these approaches also as part of the Conventional Method for the sake of completeness.

Before you proceed to learn this method, a few words about the exercises - while part of puzzles would be used to highlight and illustrate the 12 Approaches, complete (and semi-complete) exercises are reserved for the very end, since real-life puzzles are combinations of many of the Approaches.

Understand the Approaches, but then memorize the 'Pattern to look for' below. It is these patterns that you should look for in every given puzzle. And solve as learnt in the Approach. The more quickly and easily you're able to recognize these patterns, the more you're on your way to mastering this Method.

Note: When you interact with others who use the Conventional Method, you'll find that many of them refer to 'Major Square' as 'grid'.

Approach 1

Let's start with the obvious. If there's a Cell that can obviously take one and only one value, we could fill it with that value.

							3	
							7	
						2		8
						9		
1		5		2			?	
							5	
							4	

What values can Cell (6,8), marked '?' in black take? Obviously nothing other than '6'. So you can fill it with the value '6'. This is called the 'Singles' or 'Naked Singles' Approach.

Pattern to look for: Intersection of well-populated "Row - Column - Major Square".

Approach 2

If a certain value is required in a Row/ Column/ Major Square, but except for one Cell the other Cells in that Row or Column or Major Square can either not take the value because of the occurrence of the value in the respective 'other Rows/ Columns/ Major Squares' or if the other prospective Cells are already filled with other values, we could fill that one Cell with that value.

7								
			?	8	9			
								7

Rows 4 and 6 already have '7's. Row 5 doesn't yet have one. The '7' in Row 5 can't be in the Mid Left or Mid Right Major Squares as there are '7's in them already. This leaves us with only 3 candidate squares, viz., Cells (5, 4), (5, 5) and (5, 6). However, (5, 5) and (5, 6) are already filled with other values. So, the value '7' for Row can only be placed in Cell (5, 4) marked '?' in black. This is called the 'Hidden Singles'

Approach.

Together, both the Approaches (1 & 2) are also called 'Unique Value' Approaches.

Interchanging (and slightly modifying) Rows & Columns, you may have the pattern as below: Let's see this with the help of an example.

			7					
				?				
				8				
				9				
					7			

			7					
					?			
7								
							7	
				7				

Or, even as:

			2	3	?			
7								
			4	5	6			

The above 2 Approaches are fairly simple, and chances of detection of such instances in a puzzle are high.

After repeatedly applying the above 2 Approaches, if you're unable to solve a puzzle, you may have to start applying the more complex Approaches that follow.

Pattern to look for : Occurrence of the a value in 1 or 2 Major Square(s) and its non-occurrence in another Major Square that doesn't have that value yet, all these Major Squares being placed vertically/ horizontally in one set of Rows/ Columns.

Approach 3

Sometimes, even if a certain value is not yet there in a Row (or Column), you can still eliminate that value as a possibility from a set of Cells in that Row (or Column). This is because we may not yet have reached the stage where we can fix that value to any specific Cell in that Row (or Column), but yet, from the interactions with a Major Square, and from the occupied values in other Cells in the Row (or Column), it is clear that soon a Cell in that Row (or Column) would definitely take that value. So, you can rule out the chances of other Cells in that Row (or Column) taking the same value.

Let's learn this thru an example:

2	4							
3	5							
			?	?	?	?	?	?
		6						

Cells (3,4), (3,5), (3,6), (3,7), (3,8) and (3,9), marked '?' in red, can't take the value '6'.

Why? Because, the Top Major Square still needs a '6'; and leaving out the already occupied Cells and the Cells in column 3 (because Column 3 already has a '6') in the Top Major Square, the only 2 possible positions for the value '6' are Cells (3, 1) and (3, 2). In either case, no other Cell in Row 3 can take the value '6'.

Interchanging (and slightly modifying) Rows & Columns, you may have the pattern as below:

2		4						
				6				
3		5						
	?							
	?							
	?							
	?							
	?							
	?							

This is called 'Direct Interaction' or 'Row/Column-Major Square Interaction' or 'Row/Column-Block Interaction' Approach.

Pattern to look for: A Rectangular pattern of Cells filled up in a Major Square; a particular value not yet filled up in the above Major Square being found on one side of it horizontally or vertically, in the Row/ Column that has Cells to be filled in.

Approach 4

Let's see a situation that's similar to the above, but this time, it is across 2 Major Squares instead of Row/Column-Major Square. Here, though a certain value is not yet there in a Major Square, you can still eliminate that value from a set of Cells in the adjoining Major Squares (i.e., in the same Rows/ Columns). This is because we may not yet have reached the stage where we can fix that value to any specific Cell in that Major Square, but yet, from the interactions with the other Major Squares, and from the occupied values in the Major Square, it is clear that soon a Cell in that Major Square would definitely take the value. So, you can rule out the chances of other Cells in that Row (or Column) taking the same value.

Let's see this thru an example:

						2	9	8
		5						
						1	?	?
						?	?	?
						?	?	?
						?	?	?
						7	8	?
						4	?	3
5								

Here, both the Top Right and Bottom Right Major Squares still need a '5'. Given the puzzle above, the only candidate Cells for '5' in the Top Right Major Square are Cells (3, 8) and (3, 9). And the only candidate Cells for '5' in the Bottom Right Major Square are Cells (7, 9) and (8, 8). Whichever of these Cells takes the value '5' in these 2 Major Squares, it is clear that there's no place for another '5' in these 2 Columns in the Mid Right Major Square. So, we can eliminate the possibility of value '5' from being allotted to Cells (4, 8), (4, 9), (5, 8), (5, 9), (6, 8), and (6, 9), all marked '?' in red. And '5' in this Major Square can only appear in one of the 3 Cells (4,7), (5,7) and (6,7), marked '?' in black.

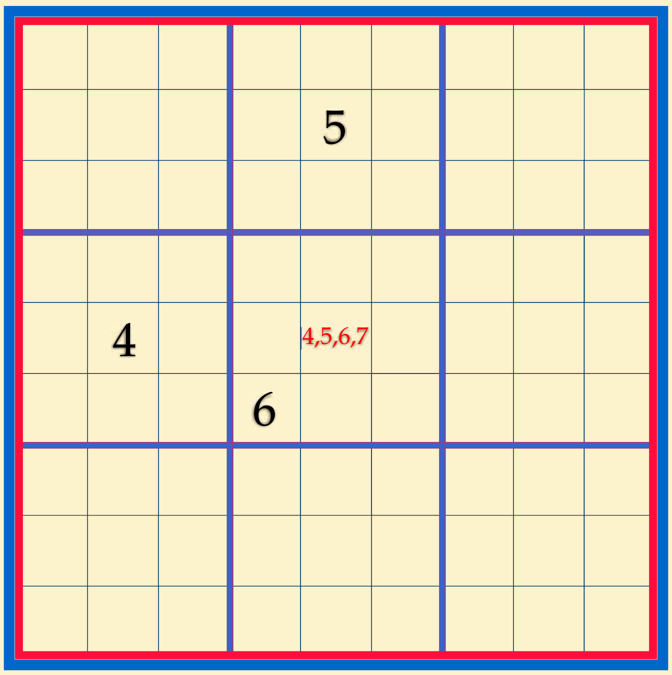
This is called 'Indirect Interaction' or 'Major Square-Major Square Interaction' or 'Block-Block Interaction' Approach.

Pattern to look for: There are 3 Major Squares, all at the Top/ Middle/ Bottom/ Left/ Centre/ Right. All these 3 Major Squares still need a certain value, which is actually present in a Major Square perpendicular to (i.e., by the side of) two of them.

Approach 5

If, among the possible set of values that a Cell can take, one or more values have been taken by one or more Cells in its Row/ Column/ Major Square, then you can rule out these values for the Cell. This is just the same as the Reduction Approach that we've seen in the Possibility Matrix Method.

Though we've learnt this in the Possibility Matrix Method, for the sake of completeness, let's see this thru an example:



Let's say that through other inferences, we know that Cell (5,5) can only take the values {4,5,6,7}, and the values '4, '5', '6' were obtained successfully only in the last step.

Now, Cell (5,5) can take only the value '7', as below.

Approach 5

				5				
	4			7				
			6					

This is called the 'Reduction' Approach.

Pattern to look for: When any values have been arrived at just now, all the unfilled Cells in the Row, the Column and the Major Square that the newly arrived at value is in are potential candidates for Reduction..

If, among the possible set of values that certain Cells can take, a certain subset of values will necessarily have to be shared only among certain Cells, then, you can rule out these values from the other Cells.

Let's see this thru an example:

In a Row, there are 4 Cells yet to be filled in. And the possible values in these Cells are:

{1,2}, {1,2}, {1,2,3,4}, and {1,2,3,4}.

Whereas the Cells with the possible values {1,2,3,4}can take any one of the 4 values, there are 2 Cells that can only take one of 2 values '1' or '2'. Either of the Cells can take the value '1' and the other can take the value '2'. This means that, in either case, the other 2 Cells cannot take the values '1' and '2'. So, you can eliminate these values from the possibilities for these Cells and so, the set of possible values reduces to:

{1,2}, {1,2}, {3,4}, and {3,4}.

In a slightly more complex form, a similar situation could be as below:

{1,2,3}, {1,2,3,4,5}, {1,2,3}, {1,2,3,6,7}, {1,2,3,4,6}, {1,2,3,5,7}, and {1,2,3}.

And this will reduce to:

{1,2,3}, {4,5}, {1,2,3}, {6,7}, {4,6}, {5,7}, and {1,2,3}.

This is called 'Naked Groups' or 'Naked Subsets' Approach. This approach can be applied to Rows, Columns as well as Major Squares.

Pattern to look for: There are 3 or more unfilled Cells in a Row/ Column/ Major Square, and they share 3 or more possible values among themselves. Of these, 2 or more Cells can take exactly the same set of values.

This is very similar to the previous Approach. Here, among the possible values that certain set of Cells can take, the subset of values that will necessarily have to be shared only among certain Cells is not apparent but has to be deduced. Here again, you can rule out these values from the other Cells.

Let's slightly modify the example above and see how and when to apply this:

Let's say 7 Cells can take the following possible values:

{1,3}, {1,2,3,4,5}, {2,3}, {1,2,3,6,7}, {1,2,3,4,6}, {1,2,3,5,7}, and {1,2}.

We can see that still, the red-colored Cell-values means that these 3 Cells can take no value other than '1', '2' and '3' among them. This means that these values are not available to the other Cells. So, you can reduce this also to:

{1,3}, {4,5}, {2,3}, {6,7}, {4,6}, {5,7}, and {1,2}.

This is called 'Hidden Groups' or 'Hidden Subsets' Approach. This approach can also be applied to Rows, Columns as well as Major Squares.

Pattern to look for: There are 4 or more unfilled Cells in a Row/Column/ Major Square, and they share 3 or more possible values among themselves, though in the form of subsets.

Approach 8: If, among the possible set of values that certain Cells can take in a Row (or Column), a certain value will necessarily have to be taken between a couple of pairs in the same Row (or Column), then, you can rule out these values from the other Cells in their Columns (or Rows).

We'll see this thru an example.

								M,4
	A,4							
	X,4							Y,4
	B,4							
								N,4
	P,4							Q,4

Where 'x', 'y' (here '4'), and 'p', 'q' are single values, and 'A', 'B', 'M' and 'N' can be any set of one or more values. And let's say that '4' is not a possible value in any other Cell in these Rows (3 and 9). (That is, if you were to construct a Possibility Matrix, '4' can't figure in any other Cell in these 2 Rows.)

Let's see this with some values in the place of x, y, etc.

								1,2,4
	4,7,8							
	2,4	1,2,6				1,2,6		4,6
	4,7,9							
								1,2,4,6
	3,4					2,3,6	2,3,6	4,6

In this case, if Cell (3, 2) takes the value '4', then, Cell (3, 9) can't take the value '4'. And Cell (9, 2) can't take the value '4' either. This leaves us with Cell (9,9) taking the value '4'.

Likewise, if Cell (3,9) takes the value '4', then, Cell (9,2) will also take the value '4'.

So, either Cell (3, 2) and (9, 9) take the value '4', or Cell (3, 9) and (9, 2) take the value '4'.

In either case, no other Cell in these 2 Columns can take the value '4'.

You can eliminate the value '4' from the possible set of values {4,7,8}, {4,7,9} in Column 2 and {1,2,4} and {1,2,4,6} in Column 9. This reduces them to {7,8}, {7,9} in Column 2 and {1,2} and {1,2,6} in Column 9. (i.e., More generically, {A, 4}, {B, 4}, {M, 4} and {N, 4} reduce to: ({A}, {B}, {M} and {N}).

This is called 'X-Wing' Approach. You can have the Rows and Columns interchanged here, and this approach is still applicable.

Pattern to look for: A certain value that can be taken by only one of 2 Cells in certain Row, and the same value can be taken by only one of 2 Cells in the same Columns in another Row. That is, four Cells falling in 2 identical Rows and Columns share a prospective value. (This is also true of Four Cells falling in 2 identical Rows and Major Squares, OR, in 2 identical Columns and Major Squares).

Approach 9

This is similar to the previous approach, but in 3 Rows (or Columns). Here, there are 2 candidates for a value in each Row (or Column). And these candidates fall in the same set of 3 Columns (or Rows). Then, you can rule out this value from the other Cells in their Columns (or Rows).

We'll see this thru an example.

								M,4
	A, 4							
	x, 4	z, 4						
		r, 4						y, 4
	B, 4							
								N, 4
		L, 4						
	p, 4							q, 4

Where 'x', 'y', 'z', 'p” 'q', and 'r' are any values (and 'A', 'B', 'M', 'N', and 'L') are any set of values, and '4' is not a possible value in any other Cell in these Rows (3, 4 and 9), you can eliminate the value '4' from the possible set of values ({A,4}, {B,4}, {M,4}, {N,4}, and{L,4}) in all other cells in these Columns.

Let's see what the given situation means. This means that either Cell (3,2) or Cell (3,3) must be a '4'; and, either Cell (4,3) or Cell (4,9) must be a '4'; and, either Cell (9,2) or Cell (9,9) must be a '4'.

However, if Cell (3,2) is a '4', Cell (3,3) can't be a '4' and Cell (9,2) can't be a '4' either. The only Cell in R9 that can be a '4' is (Cell (9,9); so, Cell (4,9) can't be a '4'. Since R4 still needs a '4', Cell (4,3) must be a '4'.

Alternatively, if Cell (3,3) is a '4', Cell (4,3) can't be a '4', and Cell (3,2) can't be a '4' either; Since R4 needs a '4', Cell (4,9) must be a '4', which means Cell (9,9) can't be a '4'. Since R9 needs a '4', Cell (9,2) must be a '4'.

Let's now see what this boils down to Column-wise: Either Cell (3,2) is a '4' or Cell (9,2); and, either Cell (3,3) is a '4' or Cell (4,3); and, either Cell (3,9) is a '4' or Cell (9,9).

That is, one of the Cells in each of these Columns is a '4'. So, no other Cell in these Columns can be a '4'.

So, ({A,4}, {B,4}, {M,4}, {N,4}, and{L,4}) reduce to ({A}, {B}, {M}, {N}, and{L}).

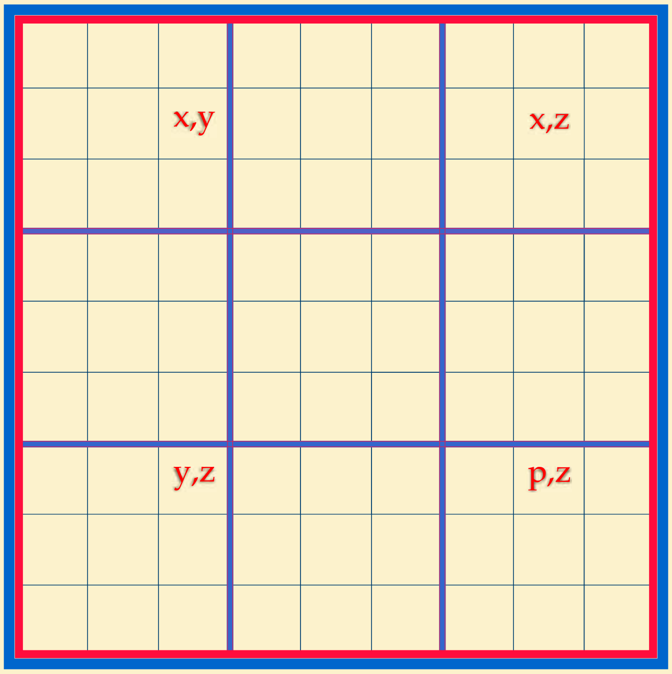
This is called 'Sword-Fish' Approach. You can have the Rows and Columns interchanged here, and this approach is still applicable.

> Pattern to look for: Six Cells falling in 3 identical Rows and Columns share a prospective value.

Approach 10

This is also somewhat similar to the previous ones in the sense that it helps eliminate certain possibilities based on 'not so apparent' logic.

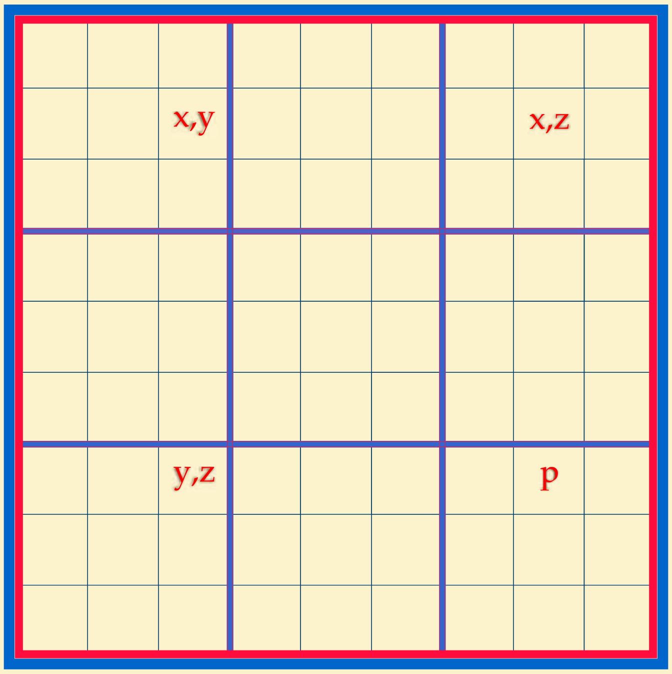
The logic here is that, if you have a row-column, (or) row-major square (or) column-major square intersection where 4 (or more) cells have a x-y, x-z, y-z formation, you can rule the possibility of 'z' in the fourth Cell. Here, x, y and z represent a specific value between '1' and '9'.



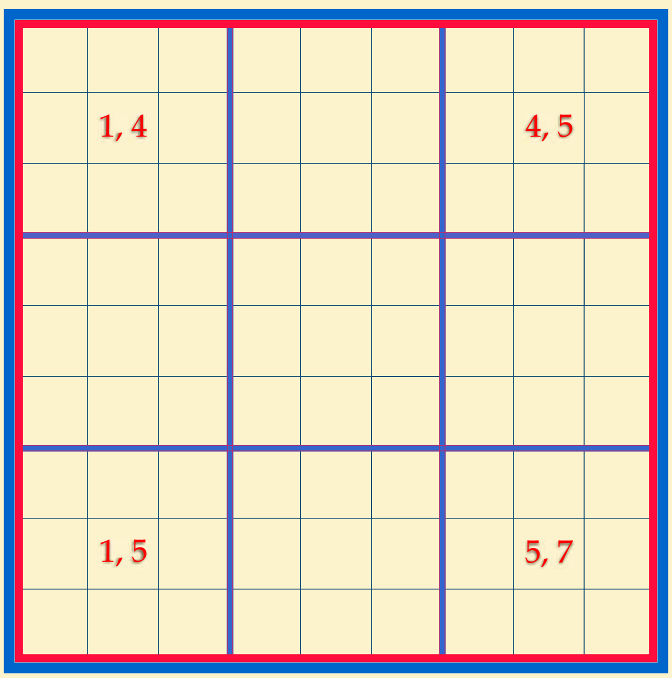
Cell (2, 3) can be only 'x' or 'y'. Let's say it is 'x'. Then, Cell (2, 8) has to be 'z'. Then, Cell (7, 8) can't be 'Z'.

On the other hand, let's say Cell (2, 3) is 'y'. Then, Cell (7,3) has to be 'z', and so, Cell (7, 8) can't be 'Z'.

So, in any case, Cell (7, 8) can't be 'z', whenever we have a formation like this. So, we can see that we can eliminate 'z' from Cell (7, 8), as below:



Example:



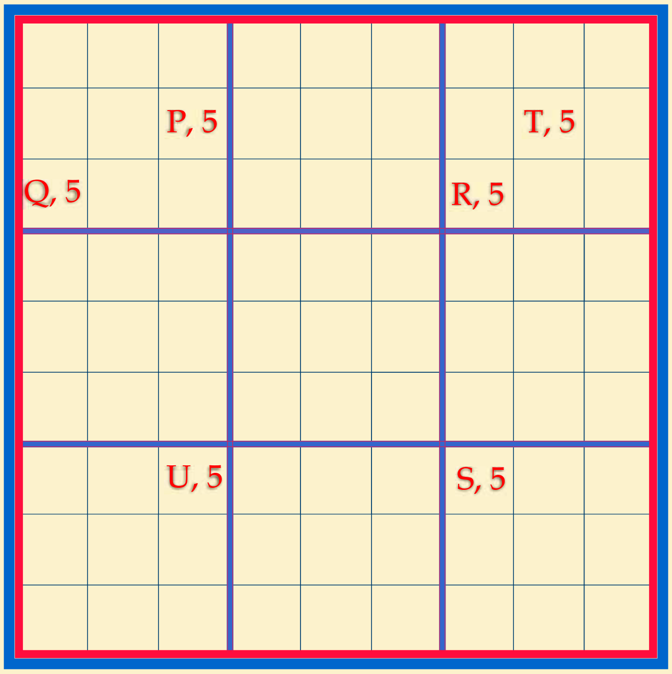
Reduces to

	1, 4						4, 5	
	1, 5						7	

This is called "XY-Wing' Approach. We have seen only the case of row-column interaction. The same obviously holds true for row-major square interaction and column-major square interaction.

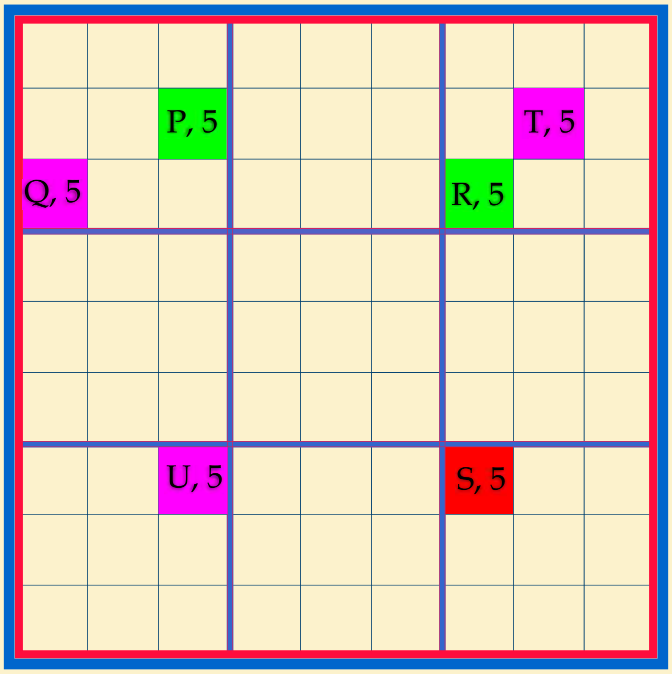
Pattern to look for: Four Cells falling in 2 identical Rows and Columns need 3 prospective values (x, y and z), with the possibilities being {x,y}, {x,z}, {y,z}, and {p,z},. (This is also true of Four Cells falling in 2 identical Rows and Major Squares, OR, in 2 identical Columns and Major Squares).

In certain situations, the linking of Cells across Rows, Columns and Major Squares is not apparent, particularly when they are not in the same Rows or Columns, but if we use color pencils and shade them, we may be able to understand the linking better and eliminate certain possibilities. Let's see this thru an example.



Let's say only Cells (2, 3) or (3, 1) can hold the value '5' in the Top Left Major Square. And let's also say only Cells 2, 8) or (3, 7) can have the value '5'. And let's also say that we need a '5' for Row 7.

When one of the Cells in the Top Left and Top Right Major Squares takes the value '5', the other cannot. Such Cells are 'Alternatives' to each other. Let's Color Alternatives differently, and see if it helps us resolve some Cells.



Here, we can see that if Cell (2, 3) takes the value '5', Cell (3, 1) can't; but Cell (3, 7) can, and Cell (2, 8) can't; and Cell (7, 3) can't. And we have a conflict in Cell (7, 7); while it is an alternative to Cell (3, 7) and so it should take the color different from it, it should also take the color alternative to Cell (3, 7) and so it should take the color different from it too. So, we see that Row 7 cannot have a '5' while we need one. So, this cannot be the solution.

Let's look at the alternative solution. If Cell (3, 1) takes the value '5', Cell (2, 8) and Cell (7, 3) can take the value '5' too, and our requirements are completely met.

So, we can see how coloring has helped us resolve this conflict. Sometimes, we may not be able to completely resolve such conflicts with this Approach, but we may only be able to eliminate the possibility of some values in certain Cells as we have seen with the previous approaches.

This is called 'Coloring' Approach.

> Pattern to look for: A single value yet to be found following a chain of possibilities across the Table, going thru Rows, Columns and Major Squares in any order, such that some of these possibilities would become impossible in case some of the other possibilities are true.

Let's see one more complex example:

	4,7,9						4,5,7	5,7,9
	7,9							7,9
	4,9					4,6,8,9	4,6,8	8,9
	2,6,8	4,8				1,4,6,8		
	2,3,6,8, 9	8,9				1,6,7,8	5,6,7,8	1,5,7,8
		4,8,9				4,8	4,5,8	
	3,6,7,8	1,7,8		1,6,7,8		1,7,8		
		1,7,8		1,7,8		1,7,8		
	4,6,7,8		6,8	1,6,7,8			7,8	1,7,8,9

Here, we find that the value '7' is required to be filled into 6 rows, 6 columns and 6 Major Squares.

If we were to fill in Cell (9,2) with a '7', lets see what all possibilities of filling in the value '7' remain.

	4,9						4,5,7	5,7,9
	9							7,9
	4,9					4,6,8,9	4,6,8	8,9
	2,6,8	4,8				1,4,6,8		
	2,3,6,8,9	8,9				1,6,7,8	5,6,7,8	1,5,7,8
		4,8,9				4,8	4,5,8	
	3,6,8	1,8		1,6,7,8		1,7,8	2	
		1,8		1,7,8		1,7,8	3	
	7		6,8	1,6,8			8	1,8,9

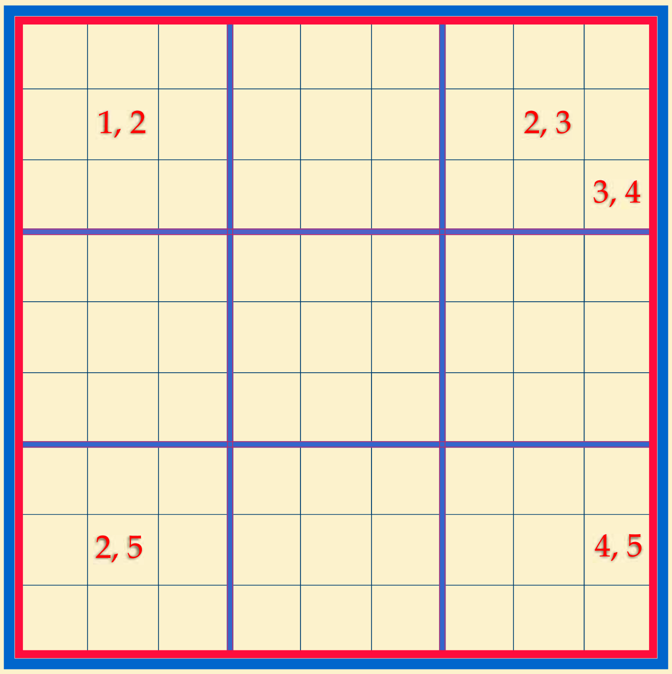
Now, we can fill in '7' only in 4 more columns, whereas we require '7' in 5 more columns. So, we could eliminate the possibility of Cell (9,2) being filled in with a '7'.

So, the Table reduces to:

	4,7,9						4,5,7	5,7,9
	7,9							7,9
	4,9					4,6,8,9	4,6,8	8,9
	2,6,8	4,8				1,4,6,8		
	2,3,6,8,9	8,9				1,6,7,8	5,6,7,8	1,5,7,8
		4,8,9				4,8	4,5,8	
	3,6,7,8	1,7,8		1,6,7,8		1,7,8		
		1,7,8		1,7,8		1,7,8		
	4,6,8		6,8	1,6,7,8			7,8	1,7,8,9

Approach 12

When you can't make a direct deduction, see if a series of deductions (based on the logic: “if this Cell takes this value”) leads to a resolution of the right Cell for a value. This is similar to the Coloring Approach, but is different in the sense that we are not looking for just only one value here, but a series of values. In a manner of speaking, this is similar to the “X-Y Wing Approach”, except that, in this approach, we need not look for a pattern of the position of the Cells like in “X-Y Wing Approach”. And therefore, this is no Axiom, and the deductions must be made on a case-to-case basis. Let's see this Approach thru an example.



If Cell (2, 2) takes the value '1', Cell (2,8) will take '2', Cell (3, 9) will take '3', Cell (8, 9) will take '4' and Cell (8, 2) will take '5'. And there are no conflicts.

However, if Cell (2, 2) takes the value '2', Cell (2,8) will take '3', Cell (3, 9) will take '4', Cell (8, 9) will take '5' and Cell (8, 2) will take '2'. Now there is a conflict. So, this set of values is not right.

So, we are able to deduce that we should go with the assumption that Cell (2, 2) takes the value '1', and fill up the rest of the Cells on this basis.

This is called 'Forcing Chains' Approach.

Pattern to look for: A set of values yet to be found following a chain of possibilities across the Table, going thru Rows, Columns and Major Squares in any order. Typically it is possible to resolve only when we go through chains of Cells with 2 possible values in each Cell, as with more possibilities in some Cells, it will be too complex.

When all the approaches we've learnt fail, use the Tie Breaker Approach we'd learnt in the Possibility Matrix Method (let's call it the 'Trial And Error' Approach here). 'Trial And Error' Approach also helps determine if a puzzle has multiple solutions or any solution at all or not. This Approach is not an inherent part of the Conventional Method, and that's why this is not included formally as part of the Method. Most purists call it the 'Sledge Hammer' Approach, and won't accept it. However, since the Conventional Method doesn't guarantee results for every Sudoku, you may be forced to borrow this 'Tie Breaker' Approach from the Possibility Matrix Method. This Approach also comes in handy when you are unable to proceed (though there may well be a solution without having to resort to this Approach, but you are unable to find it).

In a way, you could say 'Forcing Chains' is also part of the 'Trial And Error' Approach, because you realize whether a chain forces values or not only thru 'Trial And Error'. However, in the case of 'Trial And Error' Approach, you continue with the trial regardless of whether trials result in resolution or not, whereas the 'Forcing Chains' Approach actually helps resolve conflicts successfully without having to try to solve the puzzle completely.

These Exercises are in the increasing order of complexity. Explanations are more detailed initially, and less detailed as we progress (to avoid monotony, though explained sufficiently in detail so that no reader is left in any ambiguity/ doubt at any stage). The first few Exercises have been solved fully, while some of the latter Exercises are semi-solved, to give the reader an opportunity to solve using the previously learnt approaches. However, in the case of semi-solved exercises, the Final answers have been given to help the reader verify their answers.

Solved Exercise: 1

	8		4	1			2	3
5				7	6	9	4	
	3					1		
2	1					8	3	
3			6		2			5
	6	4					7	9
		8					9	
	5	3	1	2				4
9	7			5	4		6	

We will try to find the values for the potential Cells using the Naked Singles Approach. Let us consider Cell (2,2).

Why did we decide on Cell (2,2)? By scanning the puzzle, we realize that Cell (2,2) can take only one value; that's why.

Considering the values in its corresponding row, column and major square, Cell (2,2) can take only one possible value, '2'. Let's assign it first.

	8		4	1			2	3
5	2			7	6	9	4	
	3					1		
2	1					8	3	
3			6		2			5
	6	4					7	9
		8					9	
	5	3	1	2				4
9	7			5	4		6	

Similarly, Cells (2,9), (4,9), (5,7), (5,8), (6,7), (8,1), (8,7) & (8,8) can take the only possible values '8', '6', '4', '1', '2', '6', '7', & '8' respectively. So, let us assign these values to their respective Cells.

	8		4	1			2	3
5	2			7	6	9	4	8
	3					1		
2	1					8	3	6
3			6		2	4	1	5
	6	4				2	7	9
		8					9	
6	5	3	1	2		7	8	4
9	7			5	4		6	

Now, if we consider Cell (3,8), the only possible value for it is '5' (by the Naked Singles Approach considering only Column 8 (let's call it C8 for short). Similarly, in Cells (5,2) & (7,2), the only possible values are '9' & '4' respectively. In Cell (8,6), by the Naked Singles Approach in Row 8 (let's call it R8 for short), the only possible value is '9'. Let us assign These Values To their Respective cells .

	8		4	1			2	3
5	2			7	6	9	4	8
	3					1	5	
2	1					8	3	6
3	9		6		2	4	1	5
	6	4				2	7	9
	4	8					9	
6	5	3	1	2	9	7	8	4
9	7			5	4		6	

Now, if we consider Cell (1,1), the only possible value for Cell (1,1) is '7'. Similarly, Cells (1,6), (1,7), (2,3), (2,4), (3,6), (3,9), (5,3), (5,5), (6,1), (7,1) & (9,7) can take the only possible values '5', '6', '1', '3', '8', '7', '7', '8', '8', '1' & '3' respectively. Let us assign these values to their respective Cells.

7	8		4	1	5	6	2	3
5	2	1	3	7	6	9	4	8
	3				8	1	5	7
2	1					8	3	6
3	9	7	6	8	2	4	1	5
8	6	4				2	7	9
1	4	8					9	
6	5	3	1	2	9	7	8	4
9	7			5	4	3	6	

Now, if we consider the Cell (1, 3), the value should be '9' (by the Naked Singles Approach in R1). Likewise, if we consider Cell (3,5), the only possible value is '9'. For Cell (4,3), the value is '5'. Similarly, for Cells (7,4) & (7,9), the only possible values are '7' & '2' respectively.

7	8	9	4	1	5	6	2	3
5	2	1	3	7	6	9	4	8
	3			9	8	1	5	7
2	1	5				8	3	6
3	9	7	6	8	2	4	1	5
8	6	4				2	7	9
1	4	8	7				9	2
6	5	3	1	2	9	7	8	4
9	7			5	4	3	6	

Now, if we consider Cell (3,1), the value should be '4'. For Cell (3,3), the only possible value is '6', and for Cell (3,4), the only possible value is '2' (by the Naked Singles Approach in the Top Middle Major Square). Similarly, for Cell (9,9), the only possible value is '1' (by the Naked Singles Approach in C1). Likewise, if we consider Cells (4,4), (4,5), (4,6), (6,4), (6,5) & (7,6), the only possible values are '9', '4', '7', '5', '3' & '3' respectively (considering their corresponding rows, columns and major squares). Let us now assign these values to their respective Cells.

7	8	9	4	1	5	6	2	3
5	2	1	3	7	6	9	4	8
4	3	6	2	9	8	1	5	7
2	1	5	9	4	7	8	3	6
3	9	7	6	8	2	4	1	5
8	6	4	5	3		2	7	9
1	4	8	7		3		9	2
6	5	3	1	2	9	7	8	4
9	7			5	4	3	6	1

Finally, we have only five Cells to be filled in. If we consider Cells (6,6) & (7,5), the only possible values are '1' (by the Naked Singles Approach in R6) and '6' (by the Naked Singles Approach in C7) respectively. And, Cell (7,7) has to be '5', (9,3) has to be '2', and (9,4) has to be '8'. Let's fill them in.

7	8	9	4	1	5	6	2	3
5	2	1	3	7	6	9	4	8
4	3	6	2	9	8	1	5	7
2	1	5	9	4	7	8	3	6
3	9	7	6	8	2	4	1	5
8	6	4	5	3	1	2	7	9
1	4	8	7	6	3	5	9	2
6	5	3	1	2	9	7	8	4
9	7	2	8	5	4	3	6	1

Now, we have solved this simple puzzle using only naked singles approach.

Solved Exercise: 2

	6		9			1	7	
7	5			8				
	2	1	3		5			
2	4				8	7		6
6			7		4			2
9		7	2				4	8
			1		6	2	5	
				5			6	4
	3	6			7		9	

In this puzzle, we are not able to find any reductions possible by the Naked Singles Approach. So, let's explore the possibility of Hidden Singles. The values '7' & '6' occur most number of times (6 times). So, let us consider these first.

Consider the value '7' in Cell (2,1) of Row 2 and Cell (1,8) of Row 1. As per the Hidden Singles Approach in the Top Middle Major Square, it is clear that only Cell (3,5) can take the value '7'.

Similarly, considering Columns 7 (C7), Column 8 (C8) & Row 9 (R9), in the Bottom Right Major Square, we find the value of Cell (7,9) as '7'.

Likewise, considering Row 4 (R4), Row 5 (R5) & Row 6 (R6), we arrive at a conclusion that only Cell (6,5) can take '6'. Let us assign these values to their respective Cells.

	6		9			1	7	
7	5			8				
	2	1	3	7	5			
2	4				8	7		6
6			7		4			2
9		7	2	6			4	8
			1		6	2	5	7
				5			6	4
	3	6			7		9	

Now, after assigning the values, if we once again apply the Hidden Singles Approach, we can find the value of the Cell (2,4) as '6' (considering Columns 4, 5 & 6).

Consequently, Cell (3,7) is '6' considering C7, C8 & C9.

Similarly, we find the value of Cell (8,2) as '7' considering Columns C1, C2 & C3 and Rows R7, R8 & R9. Let us assign the values to their respective Cells.

	6		9			1	7	
7	5		6	8				
	2	1	3	7	5	6		
2	4				8	7		6
6			7		4			2
9		7	2	6			4	8
			1		6	2	5	7
	7			5			6	4
	3	6			7		9	

Now, we can find the values of Cells (1,6), (3,8), (4,4), (6,2), (8,4), (9,7) & (9,9) by Naked Singles Approach as '2', '8', '5', '1', '8', '8' & '1' respectively. Let us assign the new found values to their respective Cells.

	6		9		2	1	7	
7	5		6	8				
	2	1	3	7	5	6	8	
2	4		5		8	7		6
6			7		4			2
9	1	7	2	6			4	8
			1		6	2	5	7
	7		8	5			6	4
	3	6			7	8	9	1

Now, after assigning these values, we can easily find the value of Cells (9,4) & (8,7) as '4' & '3' respectively (by Naked Singles Approach). Likewise, the values of Cells (1,5), (3,1), (2,6), (3,9), (4,3), (5,2) & (6,6) can be found using the Naked Singles Approach as '4', '4', '1', '9', '3', '8', & '3' respectively.

	6		9	4	2	1	7	
7	5		6	8	1			
4	2	1	3	7	5	6	8	9
2	4	3	5		8	7		6
6	8		7		4			2
9	1	7	2	6	3		4	8
			1		6	2	5	7
	7		8	5		3	6	4
	3	6	4		7	8	9	1

Now, we can find the values of Cells (5,3), (7,2) & (8,6) by Naked Singles Approach as '5', '9'& '9' respectively. Likewise, we can find the value of Cell (9,5) by Hidden Singles Approach as '2'.

	6		9	4	2	1	7	
7	5		6	8	1			
4	2	1	3	7	5	6	8	9
2	4	3	5		8	7		6
6	8	5	7		4			2
9	1	7	2	6	3		4	8
	9		1		6	2	5	7
	7		8	5	9	3	6	4
	3	6	4	2	7	8	9	1

Now, by Naked Singles Approach, we can find the values for Cells (1,3), (2,7), (2,9), (6,7), (7,1), (8,1) & (9,1) as '8', '4', '3', '5', '8', '1' & '5' respectively. Let us now assign these values to their respective Cells.

	6	8	9	4	2	1	7	
7	5		6	8	1	4		3
4	2	1	3	7	5	6	8	9
2	4	3	5		8	7		6
6	8	5	7		4			2
9	1	7	2	6	3	5	4	8
8	9		1		6	2	5	7
1	7		8	5	9	3	6	4
5	3	6	4	2	7	8	9	1

Now, we can find the values of the Cells (1,1), (1,9), (2,3), (2,8), (4,8), (5,7), (7,3), (7,5) & (8,3) by Naked Singles Approach as '3', '5', '9', '2', '1', '9', '4', '3' & '2' respectively.

3	6	8	9	4	2	1	7	5
7	5	9	6	8	1	4	2	3
4	2	1	3	7	5	6	8	9
2	4	3	5		8	7	1	6
6	8	5	7		4	9		2
9	1	7	2	6	3	5	4	8
8	9	4	1	3	6	2	5	7
1	7	2	8	5	9	3	6	4
5	3	6	4	2	7	8	9	1

Finally, we can fill in the rest of the Cells (4,5), (5,5) & (5,8) by Naked Singles Approach as '9', '1' & '3' respectively.

3	6	8	9	4	2	1	7	5
7	5	9	6	8	1	4	2	3
4	2	1	3	7	5	6	8	9
2	4	3	5	9	8	7	1	6
6	8	5	7	1	4	9	3	2
9	1	7	2	6	3	5	4	8
8	9	4	1	3	6	2	5	7
1	7	2	8	5	9	3	6	4
5	3	6	4	2	7	8	9	1

So, we have been able to solve this simple Sudoku too using only Naked and Hidden Singles Approaches.

2		7			1		8	
8							5	6
5		6				7		
					3			7
			5		7			
9			2					
		2				8		1
6	4							2
	8		4			5		3

We shall try to solve this puzzle using as many of the methods as we can.

Cell (6, 2) takes the value '7' by Hidden Singles Approach, looking at Row 4, Row 5, and Column 3. Now, similarly, Cell (2,3) takes the value '4' too.

2		7			1		8	
8		4					5	6
5		6				7		
					3			7
			5		7			
9	7		2					
		2				8		1
6	4							2
	8		4			5		3

Cells (1,5), (6,9) and (7,8) take the values '5', '5' and '4' respectively by the same method.

2		7		5	1		8	
8		4					5	6
5		6				7		
					3			7
			5		7			
9	7		2					5
		2				8	4	1
6	4							2
	8		4			5		3

Now, Cells (1,4), and (5,9), take the values '6', and '8' respectively (by the Hidden Singles Approach).

2		7	6	5	1		8	
8		4					5	6
5		6				7		
					3			7
			5		7			8
9	7		2					5
		2				8	4	1
6	4							2
	8		4			5		3

Consequently, Cell (8,7) takes the value '9' by Naked Singles Approach.

2		7	6	5	1		8	
8		4					5	6
5		6				7		
					3			7
			5		7			8
9	7		2					5
		2				8	4	1
6	4					9		2
	8		4			5		3

Similarly, Cells (8,8) and (9,8) take the values '7' and '6' respectively.

2		7	6	5	1		8	
8		4					5	6
5		6				7		
					3			7
			5		7			8
9	7		2					5
		2				8	4	1
6	4					9	7	2
	8		4			5	6	3

Solved Exercise: 3

In Top Middle Major Square we need '4'. Cells (3,5) or (3,6) can take the value '4'. Other unfilled Cells in Top Mid Major Square can't take value '4' since they already have a '4' in their respective Columns. Let's now try to identify all the possible values in R3, and then see if we could eliminate some of these possibilities or fix the value '4'.

2		7	6	5	1		8	4,9
8		4					5	6
5	1,3,9	6	3,8,9	2,3,4, 8,9	2,4, 8,9	7	1,2, 3,9	4,9
					3			7
			5		7			8
9	7		2					5
		2				8	4	1
6	4					9		2
	8		4			5		3

By Direct Interaction Approach, we can eliminate possibility of '4' in the other Cells of R3. That is, we get '9' in the Cell (3,9).

2		7	6	5	1		8	4,9
8		4					5	6
5		6				7		9
					3			7
			5		7			8
9	7		2					5
		2				8	4	1
6	4					9	7	2
	8		4			5	6	3

After assigning this value, we get the value '4' in the Cell (1,9) by Naked Singles Approach. Now, by Reduction Approach, we get the values of Cells (1,7) & Cells (1,2) as '3' & '9' respectively.

2	9	7	6	5	1	3	8	4
8	1,3	4					5	6
5	1,3	6				7		9
	1,2, 5,6				3			7
	1,2, 5,6		5		7			8
9	7		2					5
	3,5	2				8	4	1
6	4					9	7	2
	8		4			5	6	3

In Column 2, Cells (2, 2) & (3, 2) have the same unique set of possible values, '1' & '3'. This means, the values '1' and '3' cannot be taken by any other Cell in C2. In such a case, Naked Subsets Approach allows us to eliminate the values '1' & '3' in the other possible Cells of C2. So, we get the value of Cell (7,2) as '5'.

2	9	7	6	5	1	3	8	4
8		4					5	6
5		6				7		9
1,4	2,6	1,5,8			3			7
1,3,4	2,6	1,3	5		7			8
9	7	1,3,8	2					5
	5	2				8	4	1
6	4					9	7	2
	8		4			5	6	3

In Middle Left Major Square, three Cells (4,1), (5,1) & (5,3) can only take one of 3 values '1', '3' or '4'. So, '1', '3' & '4' cannot be the value in any other Cells in the Middle Left Major Square. In such a case, Hidden Subsets Approach allows us to eliminate the values '1', '3' & '4' in the other possible Cells of the Middle Left Major Square. By this deduction, we get value of the Cell (6,3) as '8'.

2	9	7	6	5	1	3	8	4
8		4					5	6
5		6				7		9
					3			7
			5		7			8
9	7	8	2					5
	5	2				8	4	1
6	4					9	7	2
	8		4			5	6	3

By Hidden Singles Approach in C1 & C2, and R5 & R6 we get the value of Cell (4,3) as '5'. Similarly we get value for Cell (8, 6) as '5' and Cell (9, 3) takes the value '9', looking at R8 & C1. Assigning these values to their Cells, and applying the Naked Singles Approach, we get the value '2' for Cell (9,6).

Let's now fill in the possible values in Column 6.

2	9	7	6	5	1	3	8	4
8		4			2,9		5	6
5		6			2,4,8	7		9
		5			3			7
			5		7			8
9	7	8	2		4,6			5
	5	2			6,9	8	4	1
6	4				5	9	7	2
	8	9	4		2	5	6	3

Solved Exercise: 3

By Reduction Approach we get the values of the Cells (2,6), (3,6), (6,6) and (7,6) as '9', '8', '4' and '6' respectively.

We'll now consider the possible values in Top Middle Major Square.

2	9	7	6	5	1	3	8	4
8		4	3,7	2,3,7	9		5	6
5		6	3	2,3,4	8	7		9
		5			3			7
			5		7			8
9	7	8	2		4			5
	5	2			6	8	4	1
6	4				5	9	7	2
	8	9	4		2	5	6	3

By Reduction, we get the values of the Cells (3,4), (2,4), (2,5) and (3,5) as '3', '7', '2' and '4' respectively.

Let's now assign these new-found values in R2 and R3.

2	9	7	6	5	1	3	8	4
8	1,3	4	7	2	9	1,2	5	6
5	1,3	6	3	4	8	7	1,2	9
		5			3	1,2,4,6	1,2,9	7
			5		7	1,2,4,6	1,2,3,9	8
9	7	8	2	1,6	4	1,6	1,3	5
	5	2			6	8	4	1
6	4				5	9	7	2
	8	9	4		2	5	6	3

Solved Exercise: 3

By Reduction Approach, we get the values of the Cells (2,2), (3,2), (2,7) and (3,8) as '3', '1', '1', and '2' as respectively. Let us now consider the possible values select Cells in R6 and R7.

2	9	7	6	5	1	3	8	4
8	3	4	7	2	9	1	5	6
5	1	6	3	4	8	7	2	9
		5			3			7
			5		7			8
9	7	8	2	1,6	4	1,6	1,3	5
	5	2	3,7,9		6	8	4	1
6	4				5	9	7	2
	8	9	4		2	5	6	3

Again, by Reduction, we get values of the Cells (7,4), (6,7) and (6,5) as '9' , '6' and '1' respectively. Consequently, by Reduction, we get the value of the Cell (6,8) as '3'.

Let us now consider the possible values in Middle Centre Major Square.

2	9	7	6	5	1	3	8	4
8	3	4	7	2	9	1	5	6
5	1	6	3	4	8	7	2	9
		5	8,9	6,8,9	3			7
			5	6,9	7			8
9	7	8	2	1	4	6	3	5
3,7	5	2	9		6	8	4	1
6	4	1,3			5	9	7	2
1,7	8	9	4		2	5	6	3

Solved Exercise: 3

By Hidden Singles Approach we get value '1' in the Cell (8,4), looking at C5 and C6. Consequently, we get the values of the Cells (4,4), (8,3), (7,1) and (9,1) as '8', '3', '7' and '1' respectively by Reduction Approach.

2	9	7	6	5	1	3	8	4
8	3	4	7	2	9	1	5	6
5	1	6	3	4	8	7	2	9
1,4		5	8		3			7
1,3,4			5		7			8
9	7	8	2	1	4	6	3	5
7	5	2	9		6	8	4	1
6	4	3	1		5	9	7	2
1	8	9	4		2	5	6	3

By Naked Singles Approach we get value of the Cell (5,3) as 1. By Reduction Approach we get the values of the Cells (4,1) and (5,1) as '4' and '3' respectively. Consequently we get the values of the Cells (9,5), (8,5) and (7,5) as '7 , '8' and '3' respectively by Reduction Approach

2	9	7	6	5	1	3	8	4
8	3	4	7	2	9	1	5	6
5	1	6	3	4	8	7	2	9
4	2,6	5	8	6,9	3	2,4	1,9	7
3	2,6	1	5	6,9	7	2,4	1,9	8
9	7	8	2	1	4	6	3	5
7	5	2	9	3	6	8	4	1
6	4	3	1	8	5	9	7	2
1	8	9	4	7	2	5	6	3

Finally by Reduction Approach, we get the values of the Cells (4,2), (4,5), (4,7), (4,8), (5,2), (5,5), (5,7) and (5,8) as '6' , '9' , '2' , '1' , '2' , '6', '4' and '9' respectively.

2	9	7	6	5	1	3	8	4
8	3	4	7	2	9	1	5	6
5	1	6	3	4	8	7	2	9
4	6	5	8	9	3	2	1	7
3	2	1	5	6	7	4	9	8
9	7	8	2	1	4	6	3	5
7	5	2	9	3	6	8	4	1
6	4	3	1	8	5	9	7	2
1	8	9	4	7	2	5	6	3

So, we have been able to solve this Sudoku puzzle using 6 different Approaches namely, Naked Singles, Hidden Singles, Direct Interaction, Naked Subsets, Hidden Subsets and Reduction Approaches.

Solved Exercise: 4

1		6	2			8		
			5				4	
						7	3	1
		9		5	2	4		
		1	6	8		5		
3	7	8						
	2				5			
		5			3	6		7

Let's try to solve this puzzle also by using as many methods as we can. In this puzzle, we will also try to employ some new method methods that we have not used till now.

Let's try to begin solving the puzzle with the easier Approaches.

Cell (9,2) should take the value '1' by the Hidden Singles Approach considering the '1's in C1 & C3. Cell (8,3) should be '4' by the Naked Singles Approach. Let us assign these values to their respective Cells.

1		6	2			8		
			5				4	
						7	3	1
		9		5	2	4		
		1	6	8		5		
3	7	8						
	2	4			5			
	1	5			3	6		7

Now, we can find the value of Cell (7,9) as '4' by the Hidden Singles Approach considering '4's in C7, C8 & R8.

1		6	2			8		
			5				4	
						7	3	1
		9		5	2	4		
		1	6	8		5		
3	7	8						4
	2	4			5			
	1	5			3	6		7

Similarly, we can find the value of the Cell (7,8) as '5' considering R8, R9 & C7. Let us assign these values to their respective Cells. And by the Naked Singles Approach, we can fill the values of Cell (9,1) and (8,1) as '9' and '6' respectively.

1		6	2			8		
			5				4	
						7	3	1
		9		5	2	4		
		1	6	8		5		
3	7	8					5	4
6	2	4			5			
9	1	5			3	6		7

Now, we can find the value of Cell (1,9) as '5' by the Hidden Singles Approach, considering R2 & C8. By the Naked Singles Approach, we get the value of the Cell (1,8) as '9' and similarly Cell (3,3) takes the value '2'.

1		6	2			8	9	5
			5				4	
		2				7	3	1
		9		5	2	4		
		1	6	8		5		
3	7	8					5	4
6	2	4			5			
9	1	5			3	6		7

After assigning the above values to their respective Cells, we can find the values of the Cells (2,7) & (2,9) as '2' & '6' respectively by the Naked Singles Approach.

Let's now consider the possible values in C2.

1	3,4	6	2			8	9	5
	3,8,9		5			2	4	6
	4,5,8,9	2				7	3	1
	3,6,8	9		5	2	4		
	3,4,5,6,8							
	3,4	1	6	8		5		
3	7	8					5	4
6	2	4			5			
9	1	5			3	6		7

Now, Cells (1,2) & (6,2) of C2 have the same set of possible values viz., '3' & '4'. By the Naked Subsets Approach, we can eliminate '3' & '4' in the possible Cells of C2.

Let's now consider the Bottom Right Major Square for the possible values in its Cells.

1	3,4	6	2			8	9	5
	8,9		5			2	4	6
	5,8,9	2				7	3	1
	6,8	9		5	2	4		
	5,6,8							
	3,4	1	6	8		5		
3	7	8				1,9	5	4
6	2	4			5	1,3,9	1,8	3,8,9
9	1	5			3	6	2,8	7

Solved Exercise: 4

The four Cells (7,7), (8,7), (8,8) & (8,9) can only take '1' or '3'or '8' or '9' as their values. By Hidden Subsets Approach, we can eliminate '1', '3', '8' & '9' in the other possible Cells of that Major Square. So, we get value of Cell (9,8) as '2'.

Let's find the possible values of Cells (9,4) & (9,5).

1	3,4	6	2			8	9	5
	8,9		5			2	4	6
	5,8,9	2				7	3	1
	6,8	9		5	2	4		
	5,6,8							
	3,4	1	6	8		5		
3	7	8	1,9	1,2, 6,9			5	4
6	2	4			5			
9	1	5			3	6	2	7

Now, by Reduction Approach, we get the values of Cells (9,4), (9,5) & (6,8) as '8', '4' & '7' respectively. Let us assign these new values to their respective Cells.

1		6	2			8	9	5
			5			2	4	6
		2				7	3	1
		9		5	2	4		
		1	6	8		5	7	
3	7	8	1,9	1,2, 6,9	1,6,9		5	4
6	2	4	1,7,9	1,7,9	5			
9	1	5	8	4	3	6	2	7

Solved Exercise: 4

In the Bottom Middle Major square, three Cells (7,4), (8,4) & (8,5) can take only '1' or '7' or '9' as their values. By Hidden Subsets Approach, we can remove '1', '7' & '9' in the other possible Cells of that Major Square. So, we get the value of Cell (7,6) as '6'. Consequently, we get the value of Cell (7,5) as '2'.

1		6	2			8	9	5
7,8			5			2	4	6
4,5,8		2				7	3	1
7,8		9		5	2	4		
2,4, 5,7,8								
2,4		1	6	8		5	7	
3	7	8		2	6		5	4
6	2	4			5			
9	1	5	8	4	3	6	2	7

In C1, Cells (2,1) and (4,1) share the same possibilities of values '7' and '8'; so we can eliminate '7' & '8' in the other Cells of C1 by the Naked Subsets Approach.

1		6	2	3,7		8	9	5
7,8		3,7	5	1,3, 7,9	1,7, 8,9	2	4	6
4,5		2		6		7	3	1
7,8		9		5	2	4		
2,4,5				1,3, 7,9				
2,4		1	6	8		5	7	
3	7	8		2	6		5	4
6	2	4		1,7,9	5			
9	1	5	8	4	3	6	2	7

Solved Exercise: 4

In the Top Mid Major Square, we need '3'. The only possibility to get '3' in that Major square are in Cells (1,5) & (2,5). Why? The other unfilled Cells can't take value '3' since C6 and R3 already have '3'. By Direct Interaction Approach, we can eliminate the possibility of '3' in the other Cells of C5.

Similarly in Top Right Major square, the possibility of getting '7' lies in Cells (2,1) & (2,3) of R2 because, C2 & R3 already have '7'. So, we can eliminate '7' from the other possible Cells of R2.

1	3,4	6	2	3,7		8	9	5
7,8	8,9	3,7	5	1,3,9	1,8,9	2	4	6
		2		6		7	3	1
		9		5	2	4		
2,4,5	5,6,8	3,7	1,3,4,7,9	1,7,9	1,4,7,9	1,3,9	1,6,8	2,3,8,9
		1	6	8		5	7	
3	7	8		2	6		5	4
6	2	4		1,7,9	5			
9	1	5	8	4	3	6	2	7

In R5, Cells (5,3), (5,4), (5,5), (5,6) and (5,7) share the possible values of '1', '3', '4', '7', and '9' among themselves. So, by the Hidden Subsets Approaches, we can eliminate '1', '3', '4', '7', '9' from other possible Cells in R5. By this deduction, the Cells (5,1), (5,8) and (5,9) reduce to {2,5}, {6,8}, and {2,8} respectively.

1		6	2			8	9	5
			5			2	4	6
		2		6		7	3	1
7,8	6,8	9		5	2	4		
2,5	5,6,8	3,7	1,3,4,7,9	1,7,9	1,4,7,9	1,3,9	6,8	2,8
2,4	3,4	1	6	8	4,9	5	7	2,3,9
3	7	8		2	6		5	4
6	2	4		1,7,9	5			
9	1	5	8	4	3	6	2	7

By the Coloring Approach, if Cell (6,6) takes the value '4', then Cells (6,1) and (6,2) can't take '4', which means the Mid Left Major Square can't have a '4' in it. We can thus eliminate '4' from the Cell (6,6). By this deduction, '4' can only occupy one of the possible Cells (6,1) or (6,2). So, we can eliminate '4' from the Cell (6,6). Hence, we get the value of the Cell (6,6) as '9'.

1		6	2			8	9	5
			5			2	4	6
		2		6		7	3	1
7,8	6,8	9		5	2	4	1,6,8	3,8
2,5	5,6,8	3,7	1,3,4, 7,	1,7	1,4,7	1,3,9	6,8	2,8
2,4	3,4	1	6	8	9	5	7	2,3
3	7	8		2	6		5	4
6	2	4		1,7,9	5			3,8,9
9	1	5	8	4	3	6	2	7

By the Hidden Subsets Approach, considering the values '2', '3' and '8' in Mid Right Major Square, we find they can only occupy the Cells (4,9), (5,9) and (6,9). So, eliminating these three values from the other Cells in that Major Square and in C9, we get the values of the Cells (5,8) and (8,9) as '6' and '9' respectively. Consequently, by Reduction Approach, we get the values of the Cell (4,8) and (5,7) as '1' and '9' respectively.

1		6	2			8	9	5
			5			2	4	6
		2		6		7	3	1
7,8	6,8	9		5	2	4	1	3,8
2,5	5,8	3,7	1,3, 4,7	1,7	1,4,7	9	6	2,8
2,4	3,4	1	6	8	9	5	7	2,3
3	7	8		2	6	1,9	5	4
6	2	4			5	1,3,9	1,8	9
9	1	5	8	4	3	6	2	7

By Reduction Approach, we get the values of the Cells (7,7), (8,7) and (8,8) as '1' , '3' and '8' respectively.

1		6	2	3,7		8	9	5
			5	1,3,9		2	4	6
		2		6		7	3	1
		9		5	2	4	1	3,8
				1,7		9	6	2,8
		1	6	8	9	5	7	2,3
3	7	8	1,9	2	6	1	5	4
6	2	4		1,7	5	3	8	9
9	1	5	8	4	3	6	2	7

Again, by Reduction Approach, we get value of the Cell (7,4) as '9'.

The values '1' and '7' have to be shared between the Cells (5,5) and (8,5).

So, By the Naked Subsets Approach, we can eliminate the values '1' and '7' from other possible Cells of C5. So, we get value of the Cell (1,5) as '3'

1	4,3	6	2	3		8	9	5
			5	3,9		2	4	6
		2	4,9	6		7	3	1
		9		5	2	4	1	
				1,7		9	6	
		1	6	8	9	5	7	
3	7	8	9	2	6	1	5	4
6	2	4		1,7	5	3	8	9
9	1	5	8	4	3	6	2	7

By the Reduction Approach, we get the values of the Cells (2,5), (3,4) and (1,2) as '9' , '4' and '4' respectively

1	4	6	2	3	7,4	8	9	5
7,8	8,9	3,7	5	9	1,8	2	4	6
4,5	5,8,9	2	4	6	4,8	7	3	1
		9		5	2	4	1	
						9	6	
		1	6	8	9	5	7	
3	7	8	9	2	6	1	5	4
6	2	4			5	3	8	9
9	1	5	8	4	3	6	2	7

By further Reduction, we get the values of the Cells (2,1), (2,2), (2,3), (2,6), (3,1), (3,2) and (3,6) as '7', '8', '3', '1', '5', '9', '2', and '8' respectively. By the Naked Singles Approach in C6, we get value of the Cell (5,6) as '4'.

1	4	6	2	3	7	8	9	5
7	8	3	5	9	1	2	4	6
5	9	2	4	6	8	7	3	1
7,8	6,8	9		5	2	4	1	
2,5	5,6,8	3,7			4	9	6	
2,4	3,4	1	6	8	9	5	7	
3	7	8	9	2	6	1	5	4
6	2	4			5	3	8	9
9	1	5	8	4	3	6	2	7

By further Reduction, we get the values of the Cells (4,1), (4,2), (5,1), (5,2), (5,3), (6,1), (6,2) as '8, , '6' , '2' , '5' , '7', '4' and '3' respectively.

1	4	6	2	3	7	8	9	5
7	8	3	5	9	1	2	4	6
5	9	2	4	6	8	7	3	1
8	6	9		5	2	4	1	3,8
2	5	7	3,7	1,7	4	9	6	2,8
4	3	1	6	8	9	5	7	2,3
3	7	8	9	2	6	1	5	4
6	2	4			5	3	8	9
9	1	5	8	4	3	6	2	7

By Reduction Approach we get the values of the Cells (4,9), (5,4), (5,5), (5,9) and (6,9) as '3' , '3', '1' , '8' and '2' respectively.

1	4	6	2	3	7	8	9	5
7	8	3	5	9	1	2	4	6
5	9	2	4	6	8	7	3	1
8	6	9	3,7	5	2	4	1	3
2	5	7	3	1	4	9	6	8
4	3	1	6	8	9	5	7	2
3	7	8	9	2	6	1	5	4
6	2	4	1,7	1,7	5	3	8	9
9	1	5	8	4	3	6	2	7

Finally, by the Reduction Approach, we get the values of the Cells (4,4), (8,4) and (8,5) as '7' , '1' and '7' respectively.

1	4	6	2	3	7	8	9	5
7	8	3	5	9	1	2	4	6
5	9	2	4	6	8	7	3	1
8	6	9	7	5	2	4	1	3
2	5	7	3	1	4	9	6	8
4	3	1	6	8	9	5	7	2
3	7	8	9	2	6	1	5	4
6	2	4	1	7	5	3	8	9
9	1	5	8	4	3	6	2	7

So, we have been able to solve this Sudoku puzzle using 7 different Approaches namely, Naked Singles, Hidden Singles, Direct Interaction, Naked Subsets, Hidden Subsets, Coloring and Reduction Approaches.

Partly Solved Exercise - 5 (Involving X-Wing Approach)

Here's a puzzle that has been partly solved, and you've reached the position below:

7		1	2	9	8	5	6	
9		5	4					2
2								9
1		7	5	6			2	8
8	2						5	1
5,6	3,6, 5,9	3,6,9	8	1	2	4	7	3,6
3								5
4					3	8		7
5,6	7	6,8	9	4,5,8	1,4, 6,5	2	1,3,4	3,4,6

In Columns 1 & 9, Cells (6,1), (6,9), (9,1) & (9,9) should share the value '6' only among themselves. So, by X-wing Approach we can eliminate '6' from other possible Cells of R6 & R9. Why? If Cell (6,1) takes the value '6', then Cells (6,9) and (9,1) can't take '6'; so, Cell (9,9) should take '6'. Similarly, if Cell (6,9) takes the value '6', then Cell (9,1) should also be '6'. So, we can remove '6' as a possible value from the other Cells of R6 and R9. Hence, we get the value of Cell (9,3) as '8'

7		1	2	9	8	5	6	
9		5	4					2
2								9
1		7	5	6			2	8
8	2						5	1
5,6	3,5,9	3,9	8	1	2	4	7	3,6
3								5
4					3	8		7
5,6	7	8	9	4,5,8	1,4,5	2	1,3,4	3,4,6

Solved Exercise: 5

Solve the puzzle from here, and compare your solution with the answer below.

7	3	1	2	9	8	5	6	4
9	8	5	4	7	6	1	3	2
2	6	4	1	3	5	7	8	9
1	4	7	5	6	9	3	2	8
8	2	6	3	4	7	9	5	1
5	9	3	8	1	2	4	7	6
3	1	2	7	8	4	6	9	5
4	5	9	6	2	3	8	1	7
6	7	8	9	5	1	2	4	3

Partly Solved Exercise - 6 (Involving Sword Fish Approach)

One more puzzle that has been partly solved, and you've reached the position below:

		8	3		1	4		
	4			8			1	3
5	3	1		2			7	8
		4	5		6	8		
8				1				4
		6	2	4	8	7		
	2			9		1	8	6
	8		1				5	
		9	8		2	3	4	7

Cells (1,2), (1,5), (1,9), (6,2), (6,5), (6,9), (9,2), (9,5), (9,9) all are in a network of 3 Rows and 3 Columns, with '5' as a prospective value shared by some of these Cells. We also see Cell (5,2) outside of this network, which can also take the value '5'. In this network of Cells, let's consider Cells which share the value '5', as differently shaded below:

	6,7,9	8	3	5,6,7	1	4		2,5,9
	4			8			1	3
5	3	1		2			7	8
		4	5		6	8		
8	5,7,9			1				4
	1,5,9	6	2	4	8	7		1,5,9
	2			9		1	8	6
	8		1				5	
	1,5,6	9	8	5,6	2	3	4	7

Cells that can take the value '5' are (1,5), (1,9), (5,2), (6,2), (6,9), (9,2) & (9,5). If we apply the Sword Fish Approach, we see that the Cell (5,2) can't take the value '5'. Let us see how.

If cell (1,5) takes the value '5' in C5, then Cells (1,9) & (9,5) can't take '5'; therefore Cell (6,9) takes the value '5' in C9, and Cell (9,2) takes the value '5' in R9. This is possible.

Alternatively, if Cell (1,9) takes the value '5' in C9, then Cell (1,5) & (6,9) can't take the value '5'. Therefore, Cell (9,5) takes the value '5' in C5, and Cell (6,2) takes the value '5' in R6.

And there is no other possibility by which the 3 Rows R1, R6 & R9, as well as the 3 Columns C2, C5 & C9 can have one Cell in each taking the value '5'.

From the above, we see that the possibility of Cell (5, 2) taking the value '5' is eliminated in both the cases. So, we can eliminate the possible value 5 from Cell (5,2), as below:

	6,7,9	8	3	5,6,7	1	4		2,5,9
	4			8			1	3
5	3	1		2			7	8
		4	5		6	8		
8	7,9			1				4
	1,5,9	6	2	4	8	7		1,5,9
	2			9		1	8	6
	8		1				5	
	1,5,6	9	8	5,6	2	3	4	7

Solved Exercise: 6

Solve the puzzle from here, and compare your solution with the answer below.

7	6	8	3	5	1	4	2	9
9	4	2	6	8	7	5	1	3
5	3	1	4	2	9	6	7	8
2	9	4	5	7	6	8	3	1
8	7	5	9	1	3 1	2	6	4
3	1	6	2	4	8	7	9	5
4	2	3	7	9	5	1	8	6
6	8	7	1	3	4	9	5	2
1	5	9	8	6	2	3	4	7

Partly Solved Exercise - 7 (Involving Indirect Interaction Approach)

Here's another partly solved puzzle, and you've reached the position as below:

5	3							
7	1	2	3	5		8		
6	4		2		7			
3	8	5		7				6
1		6	9		8			
9		4	5	6		1		8
	9	3			5		1	7
8	6	1	7	2	9	3	4	5
	5	7					8	

In the Top Left Major Square, there are only 2 Cells (1,3) & (3,3) that can take the value '9'. Likewise, in the Top Mid Major Square, there are only 2 Cells (1,5) & (3,5) that can take the value '9'. The Top Right Major Square also has still to take the value '9'.

5	3	8,9		4,8,9		2,4, 6,7,9	2,6, 7,9	1,2, 4,9
7	1	2	3	5		8	6,9	4,9
6	4	8,9	2	1,4, 8,9	7	5,7,9	3,5, 7,9	1,3,9
3	8	5		7				6
1		6	9		8			
9		4	5	6		1		8
	9	3			5		1	7
8	6	1	7	2	9	3	4	5
	5	7					8	

Solved Exercise: 7

By the Indirect Interaction Approach, we can eliminate '9' as a possible value from the Cells (1,7), (1,8), (1,9), (3,7), (3,8) and (3,9), as below:

5	3	8,9		4,8,9		2,4, 6,7,	2,6,7	1,2,4
7	1	2	3	5		8	6,9	4,9
6	4	8,9	2	1,4, 8,9	7	5,7	3,5,7	1,3
3	8	5		7				6
1		6	9		8			
9		4	5	6		1		8
	9	3			5		1	7
8	6	1	7	2	9	3	4	5
	5	7					8	

Solve the puzzle from here, and compare your solution with the answer below.

5	3	9	8	1	6	7	2	4
7	1	2	3	5	4	8	6	9
6	4	8	2	9	7	5	3	1
3	8	5	1	7	2	4	9	6
1	7	6	9	4	8	2	5	3
9	2	4	5	6	3	1	7	8
2	9	3	4	8	5	6	1	7
8	6	1	7	2	9	3	4	5
4	5	7	6	3	1	9	8	2

Partly Solved Exercise - 8 (Involving X-Y Wing Approach):

Here's another puzzle partly solved, and you've reached the position as below:

9	1	3	6	5	7	4	2	8
8	2	4	3	1	9			
7			8	2	4	1		
6		9	1		8			
3		1	2	4	5	6		
2			9		6			
1			4	9	3			5
4	3		5	8	1	9	7	
5	9	8	7	6	2	3		

We see the pattern of X-Y Wing here, in Cells (5,2), (5,8), (7,2) and (7,8). Here, X is '6', Y is '7', Z is '8', P is '9'.

9	1	3	6	5	7	4	2	8
8	2	4	3	1	9			
7			8	2	4	1		
6		9	1		8			
3	7,8	1	2	4	5	6	8,9	
2			9		6			
1	6,7		4	9	3		6,8	5
4	3		5	8	1	9	7	
5	9	8	7	6	2	3		

If Cell (7,2) takes the value '6', then Cell (7,8) should take the value '8'; consequently, Cell (5,8) can't be '8'. If Cell (7,2) takes the value ' 7', then Cell (5,2) should take the value '8' and hence Cell (5,8) can't take the value '8'. In both cases, Cell (5,8) can't be '8'. So, we can eliminate '8' as a possible value from the Cell (5,8) by the X-Y wing Approach. Hence, we get value of the Cell (5,8) as '9'.

9	1	3	6	5	7	4	2	8
8	2	4	3	1	9			
7			8	2	4	1		
6		9	1		8			
3		1	2	4	5	6	9	
2			9		6			
1			4	9	3			5
4	3		5	8	1	9	7	
5	9	8	7	6	2	3		

Solve the puzzle from here, and compare your solution with the answer below.

9	1	3	6	5	7	4	2	8
8	2	4	3	1	9	7	5	6
7	6	5	8	2	4	1	3	9
6	5	9	1	7	8	2	4	3
3	8	1	2	4	5	6	9	7
2	4	7	9	3	6	5	8	1
1	7	2	4	9	3	8	6	5
4	3	6	5	8	1	9	7	2
5	9	8	7	6	2	3	1	4

Partly Solved Exercise - 9 (Involving Forcing Chains Approach):

This is yet another partly solved puzzle, and you've reached the position as below:

			5				7	1
	7	1		6	9	8	5	3
		5		7	1	4	2	
	1		7	8			4	
		2	1	5	4	3	6	
				9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

Let's first consider C1 for its possible values.

2,3,4,6,8,9			5				7	1
2,4	7	1		6	9	8	5	3
3,6,8,9		5		7	1	4	2	
3,5,6,9	1		7	8			4	
7,8,9		2	1	5	4	3	6	
3,4,5,6,7				9	2	1	8	
1,8	6	4	9		5	7		
7,8	2	3		1		5	9	
1,7,8,9	5							

In C1, Cells (9,1), (8,1), (7,1) and (5,1) share four values such as '1' or '7' or '8' or '9' among themselves. So, by the Hidden Subsets Approach, we can eliminate the values '1', '7', '8', and '9' as possible values from other Cells of Column 1, as below:

2,3,4,6			5				7	1
2,4	7	1		6	9	8	5	3
3,6		5		7	1	4	2	
3,5,6	1		7	8			4	
7,8,9		2	1	5	4	3	6	
3,4,5,6				9	2	1	8	
1,8	6	4	9		5	7		
7,8	2	3		1		5	9	
1,7,8,9	5							

Let's now consider the Top Mid Major Square for the possible values in its Cells.

			5	2,3,4	3,8		7	1
	7	1	2,4	6	9	8	5	3
		5	3,8	7	1	4	2	
	1		7	8			4	
		2	1	5	4	3	6	
				9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

Cells (1,6) & (3,4) share the values '3' or '8' between themselves. So, by the Naked Subsets Approach, we can eliminate the values '3' & '8' from the other possible Cells of that Same Major Square. Now, we have:

			5	2,4	3,8		7	1
	7	1	2,4	6	9	8	5	3
		5	3,8	7	1	4	2	
	1		7	8			4	
		2	1	5	4	3	6	
				9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

Let's now focus on the Left Center Major Square and fill in the possible values in its Cells.

			5				7	1
	7	1		6	9	8	5	3
		5		7	1	4	2	
3,5,6	1	6,9	7	8			4	
7,8,9	8,9	2	1	5	4	3	6	
3,4, 5,6	3,4	6,7		9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

Cells (4,3), (5,1), (5,2) and (6,3) can take only one of the values '6', '7', '8', or '9' among themselves. So, we can eliminate these 4 values from the other Cells in the Left Center Major Square, applying the Hidden Subsets Approach. So, the value '6' gets eliminated from the Cells (4,1) and (6,1). Now, we have:

			5				7	1
	7	1		6	9	8	5	3
		5		7	1	4	2	
3,5	1	6,9	7	8			4	
7,8,9	8,9	2	1	5	4	3	6	
3,4,5	3,4	6,7		9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

Let's now look at the possible Cells for the value '6' in the Top Left and Mid Left Major Squares, as well as Columns 1 and 3. Let's color them all green, initially.

2,3,4,6		6,8,9	5				7	1
	7	1		6	9	8	5	3
3,6		5		7	1	4	2	
	1	6,9	7	8			4	
		2	1	5	4	3	6	
		6,7		9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

We see that, if Cell (1,3) takes value '6', then Cell (1,1), (3,1) , (4,3) and (6,3) can't be 6 which, means that C1 and Mid Left Major square can't have a value '6' at all. So, Cell (1,3) can't have '6'. So, we can eliminate the value '6' from Cell (1,3). We now have:

		8,9	5				7	1
	7	1		6	9	8	5	3
		5		7	1	4	2	
	1		7	8			4	
		2	1	5	4	3	6	
				9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

If we see carefully, we notice that there are a series of 2-possible-value-Cells and all of them are interconnected starting from Cell (1,3) in some way. Let's explore.

Let's see the Cells (1,3) ,(4,3) , (4,6) and (4,1) colored below:

		8,9	5			6,9	7	1
2,4	7	1		6	9	8	5	3
3,6		5		7	1	4	2	6,9
3,5	1	9,6	7	8	6,3		4	
	8,9	2	1	5	4	3	6	
	3,4	6,7	3,6	9	2	1	8	
1,8	6	4	9		5	7		
7,8	2	3		1		5	9	
	5							

Here, if Cell (1,3) takes the value '9', then Cell (4,3) takes the value '6', consequently Cell (4,6) takes the value '3' and Cell (4,1) should, therefore, take the value '5'.

		9,X	5				7	1
	7	1		6	9	8	5	3
		5		7	1	4	2	
5	1	6,X	7	8	3,X		4	
		2	1	5	4	3	6	
				9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

We've added 'X' to the 3 Cells because we're not fixing the other value marked in red to these Cells yet.

What if Cell (1,3) takes the value '8'? Let's see a different set of 2-possible-value-Cells connected.

		9,8	5		8,3	6,9	7	1
	7	1		6	9	8	5	3
6,3		5		7	1	4	2	9,6
3,5	1	6,9	7	8	3,6	9,2	4	
		2	1	5	4	3	6	
				9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5					2,6		

Solved Exercise: 9

Consider the sequence of values in Cells as below:

(1,3) -> (1,6) -> (4,6) -> (4,3) -> (4,7) -> (9,7) -> (1,7) -> (3,9) -> (3,1) -> (4,1)

If Cell (1,3) takes '8' then other Cells values are forced. Cell (1,6) takes '3', Cell (4,6) takes '6' ,Cell (4,3) takes '9', Cell (4,7) takes '2', Cell (9,7) takes '6', Cell (1,7) takes '9', Cell (3,9) takes '6', Cell (3,1) takes '3' and Cell (4,1) takes value '5'.

So, irrespective of whether Cell (1,3) takes the value '9' or '8', Cell (4,1) is forced to take the value '5', as below:

			5				7	1
	7	1		6	9	8	5	3
		5		7	1	4	2	
5	1		7	8			4	
		2	1	5	4	3	6	
				9	2	1	8	
	6	4	9		5	7		
	2	3		1		5	9	
	5							

This is a rather complex case of 'Forcing Chains' Approach. In real life situations, we may or may not be able to notice long chains of 'Forcing Chains'. In fact, there may be many other Forcing Chains involving Cells with more than 2 possible values (sometimes 3 or more values), and we may never be able to notice them due to their complexity. We can only see 'Forcing Chains' of low complexity.

Solve the puzzle from here, and compare your solution with the answer below.

2	3	9	5	4	8	6	7	1
4	7	1	2	6	9	8	5	3
6	8	5	3	7	1	4	2	9
5	1	6	7	8	3	9	4	2
8	9	2	1	5	4	3	6	7
3	4	7	6	9	2	1	8	5
1	6	4	9	2	5	7	3	8
7	2	3	8	1	6	5	9	4
9	5	8	4	3	7	2	1	6