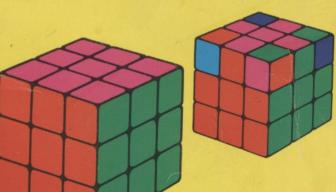


# CONQUER THAT CUBE





CZES KOŚNIOWSKI

Cambridge University Press

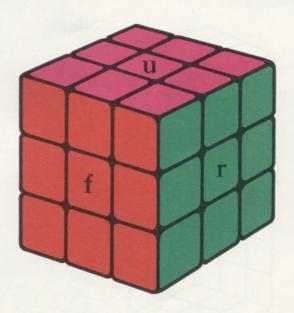
#### CONQUER THAT CUBE

Dr. Czes Kośniowski School of Mathematics, University of Newcastle upon Tyne

The Cube was invented by a Hungarian Professor of Architecture, Ernö Rubik, of the School of Commercial Artists in Budapest. It has become the craze of the eighties — once you lay your hands on one it becomes compulsive.

To solve the mysteries of the cube is not easy, you may hit lucky and get a solution in a few minutes or you may struggle for many frustrating months.

Conquer that Cube gives an easy, step by step solution to the cube. With this book the cube will no longer remain a mystery to you. As well as the basic cube solution the book shows you how to solve cubes with pictures, the barrel puzzle and other multi-sided puzzles. In addition it shows you how to achieve some pretty patterns on your cube.



Conquer that Cube is the third edition of Master the Rubik Cube Easily © C. Kośniowski 1981, first published 1981 by Kornworthy Publications.

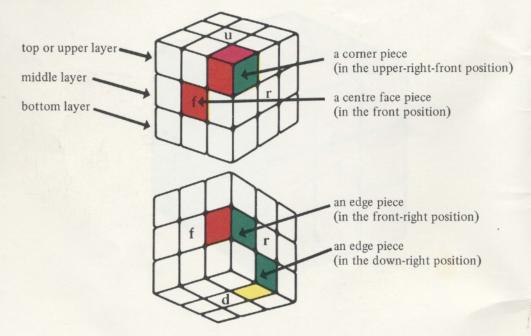
#### THE PIECES

The six sides of the cube will be referred to as

- f front
- b back
- r right
- 1 left
- u upper
- d down

There are three types of pieces in the cube — centre face pieces, edge pieces and corner pieces. Centre face pieces have one colour on them, edge pieces have two colours and the corner pieces have three colours — see the diagrams below.

The position that a piece occupies is referred to by the face or faces that it lies in - see the diagrams for examples.



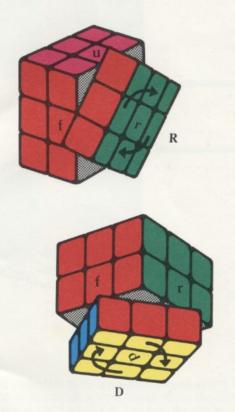
#### DISMANTLING THE CUBE

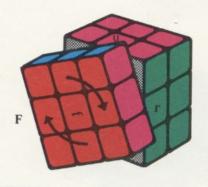
The cube may be dismantled and reassembled quite easily. Turn the top face about 45° and with a screw driver gently prise up an upper edge piece which should easily pop out. It is then possible to remove all other pieces (except the centre face pieces) — no force is necessary. The cube can be easily reassembled leaving an upper edge piece to be replaced last, again the top face should be turned about 45° to pop the last piece in. Warning: Reassemble the cube with all pieces correctly in place — unless of course you are playing tricks on a friend.

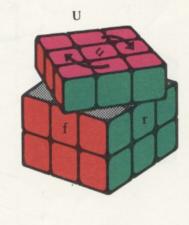
#### THE MOVES

Capital letters denote a clockwise quarter-turn of the corresponding face (clockwise as viewed by an observer looking at the face).

- F front
- B back
- R right
- L left
- U upper
- D down







R clockwise quarter-turn of right face (i.e. turn right face clockwise through 90°) etc.

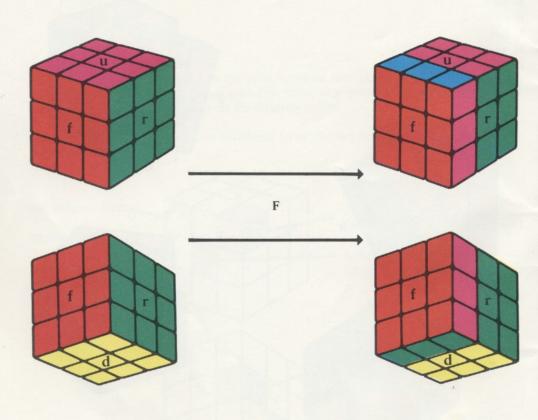
 $\mathbb{R}^2$ 

two clockwise quarter-turns (or two anticlockwise quarter-turns), etc. anticlockwise quarter-turn (read R<sup>-1</sup> as "R inverse" or "right inverse").  $R^{-1}$ 

Note.  $R^{-1} = R^3$ , etc.

## A MOVE

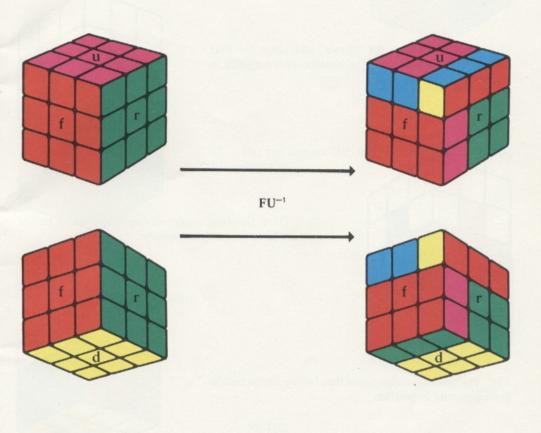
We record the effect that a move has on a cube by diagrams. The diagrams often show two views of the cube. For example: The move  $\mathbf{F}$ .



To undo the effect of the move F perform  $F^{-1}$  ("F inverse") the inverse of F. Note that the inverse of  $F^{-1}$  is F.

#### A SEQUENCE OF MOVES

A number of moves performed one after another is called a sequence of moves, for example  $FU^{-1}$  (this means F followed by  $U^{-1}$ ). The result of  $FU^{-1}$  is shown below.



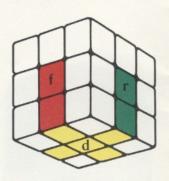
To undo the effect of a sequence of moves use the inverse of that sequence. This means do the sequence in reverse order using the inverse of each move. For example the inverse of  $FU^{-1}$  is  $UF^{-1}$  (thus  $UF^{-1}$  undoes the effect of  $FU^{-1}$ ).

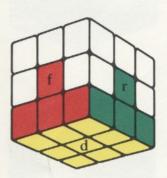
In a long sequence of moves brackets are sometimes used, for example  $F(URU^{-1}R^{-1})F^{-1}$ , these brackets are included as an aid to memory. The inverse of this sequence is  $F(RUR^{-1}U^{-1})F^{-1}$ .

#### **OUTLINE OF SOLUTION**

The solution to the cube given in this book is in seven stages and begins on page 8. Each stage requires several sequences of moves. Briefly the seven stages are as follows.

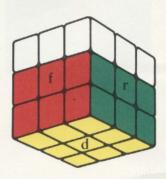
1) Choose a colour for "down" and place the four edge pieces that belong to the bottom layer correctly in position.

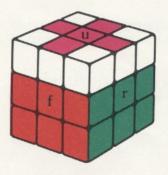




2) Place the four corner pieces that belong to the bottom layer correctly in position.

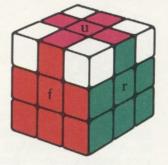
3) Place the four edge pieces that belong to the middle layer correctly in position.

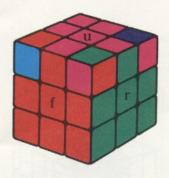




4) "Flip" the upper edge pieces to their correct colour, making a "cross".

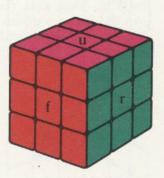
5) Put the top edge pieces in correct position.





6) Put top corners into their position, possibly facing the wrong way.

7) Put top corners in correctly.



#### **NOTES**

Sometimes it is not necessary to go through all seven stages.

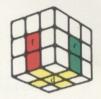
With a little practise each stage becomes easier and more automatic.

In the diagrams not all pieces are shown coloured. We only colour those necessary to show what is happening.

Note that the colours on your cube may be different to those in the diagrams of this book.

Warning. Whenever you start a sequence of moves do not be distracted until you have finished — otherwise you may have to go right back to the beginning!

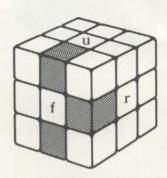




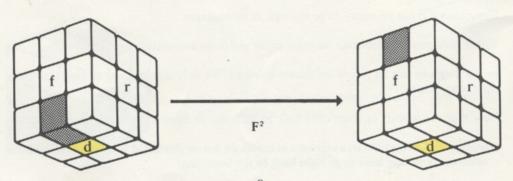
Choose a colour for "down" (d) — in this book yellow will be the down colour and the yellow centre piece will always remain down. The opposite centre piece will always remain up. Any of the other four centre pieces can be "front" (f) simply by rotating the whole cube (keeping "down" down). Although the up and down centre pieces remain fixed, the "front", "right", "left" and "back" ones are not fixed except temporarily when making a sequence of moves.

Having chosen a colour for down the first stage is to place the four edge pieces that belong in the bottom layer correctly in position — so that their colours match the centre piece colours. Do the following for each of the bottom edge pieces that are not correctly in their position.

Select an edge piece that belongs to the bottom layer but which is not correctly in position. Locate it and rotate the whole cube (keeping "down" down) until your selected piece appears in one of the three positions front-down, front-right or front-upper as in the diagram, then go on to †, †† or ††† respectively.

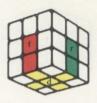


 $\dagger$  Selected piece incorrectly in bottom layer. With the selected piece in the front-down position use  $F^2$  to move it to the top layer and turn over to  $\dagger\dagger$ .

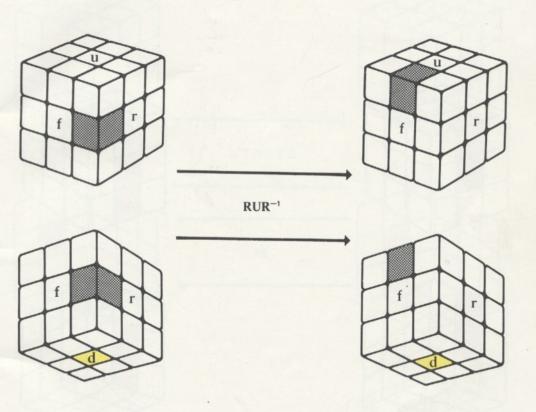




# STAGE 1 (continued)



 $\dagger\dagger$  Selected piece in middle layer. Hold the cube so that the selected piece is in the front-right position and apply  $RUR^{-1}$  — this move is designed so that any down edge piece already correctly in position is not accidentally moved out. Then go on to  $\dagger\dagger\dagger$  over the page.

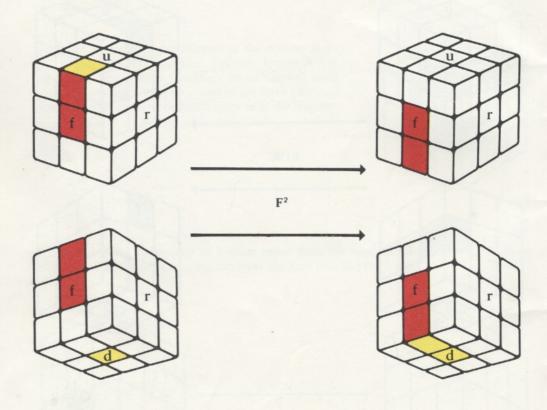




## STAGE 1 (continued)



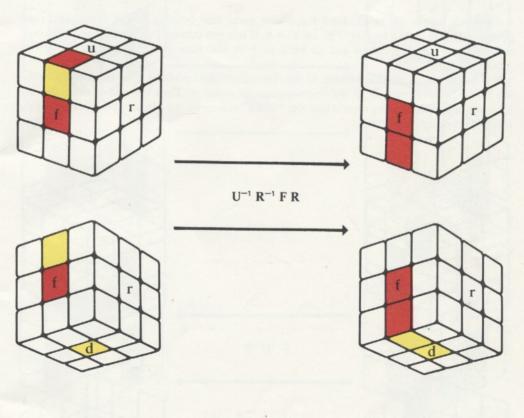
††† Selected piece in upper layer. If the selected piece is in the upper layer rotate the whole cube until the position that it belongs to is in the front-down position. Use U as many times as necessary so that the selected piece is in the front-upper position. To get the selected piece correctly into position use  $F^2$  or  $U^{-1}R^{-1}FR$  — see the diagrams to tell you which to use.



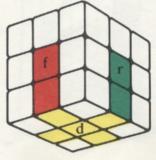


# STAGE 1 (continued)





The process described should be repeated until all four down edge pieces are correctly in position. On completion of stage 1 your cube should resemble the following.

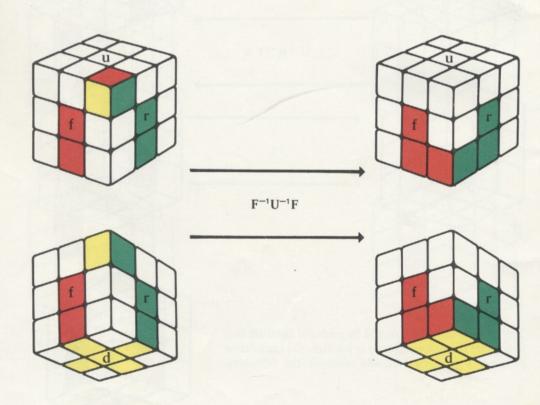


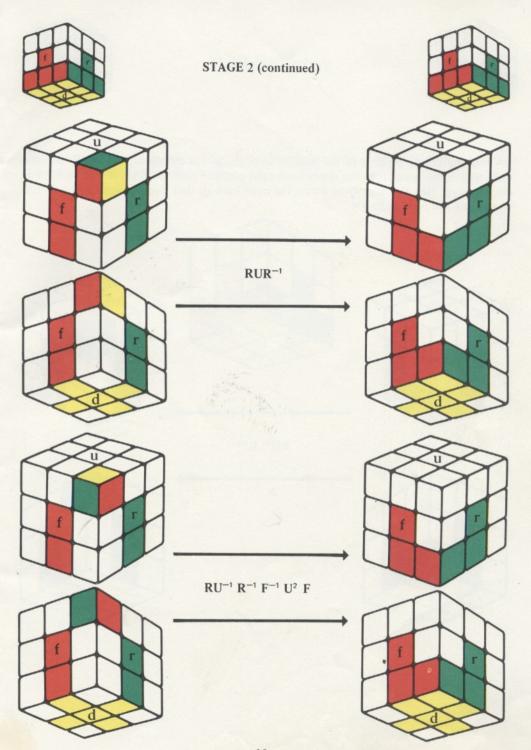




The second stage is to put the four corner pieces that belong to the bottom layer correctly into position. Do the following for each of the four pieces that are not correctly positioned.

- ★ Choose a colour for front. Find the corner piece that belongs to the down-front-right position. If it is in the top layer go on to  $\star\star$ . If it is not correctly in the bottom layer then either choose another colour and go back to  $\star$  or else turn over the next page to  $\star\star\star$ .
- $\star$  If the corner piece that belongs to the down-front-right position is on the top layer then use  $U, U^2, \ldots$  to move it to the front-upper-right position. Then use one of the following three sequences depending on whether the "down" colour on the piece to be moved is facing front, right or up respectively.



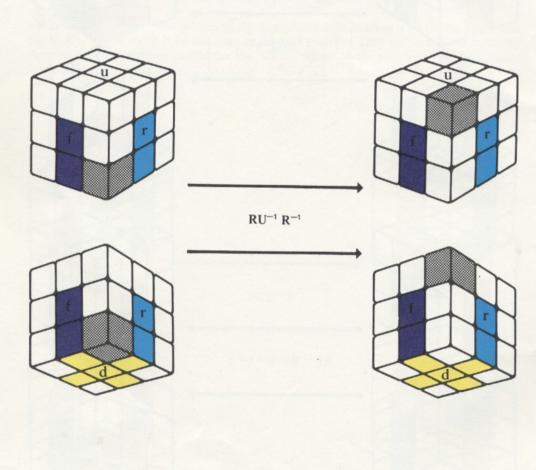




## STAGE 2 (continued)



\*\*\* If the corner piece is in the bottom layer (in an incorrect position) rotate the cube until that corner piece is in the down-front-right position then use  $RU^{-1}R^{-1}$  to move it to the top layer (see diagram). Now rotate the cube back so that your chosen front colour is front again and go back to \*\*.

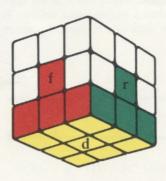


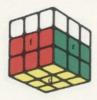


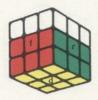
# STAGE 2 (continued)



At the end of Stage 2 all four bottom corner pieces should be correctly in position and your cube should resemble the following.

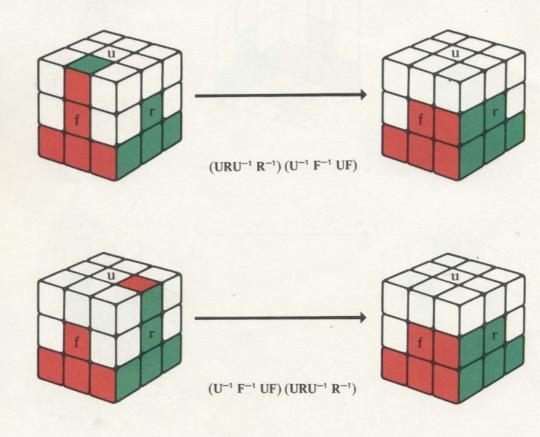






Stage 3 is to place the four edge pieces that belong to the middle layer correctly in position. Such an edge piece is either in the top layer or the middle layer (possibly incorrectly).

Edge piece in top layer. If the middle edge piece is in the upper layer then use  $U, U^2, \ldots$  to ensure that its side colour matches a centre piece colour. Then use one of the following sequences to get that middle edge piece into its correct position.

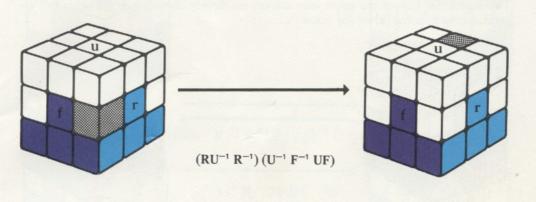




## STAGE 3 (continued)

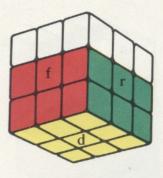


Edge piece incorrectly in middle layer. If the middle edge piece is incorrectly in the middle layer then use the following sequence to move it to the top layer and proceed as before.



Note that apart from doing U first, this sequence is the same as the first sequence on the opposite page.

The above will enable you to get all middle edge pieces into their correct position. At the end of stage 3 your cube should look like the following.



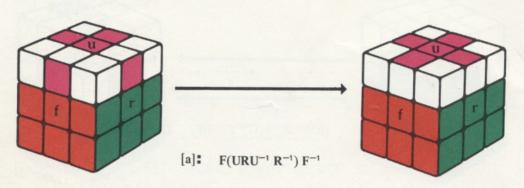




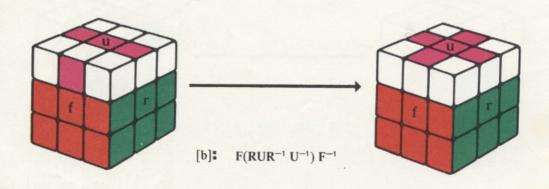
Stage 4 is to "flip" the upper edge pieces to their correct colour, making a "cross", (don't worry about the side colours of the top layer at this stage). The number of upper edge pieces that are correct is either 4, 2 or 0.

Four correct. If all four are correct go to stage 5.

Two correct. If two of the upper edge colours are already correct then the other two are made correct by using [a] or [b] below.



Sequence [a] flips upper-front and upper-right.



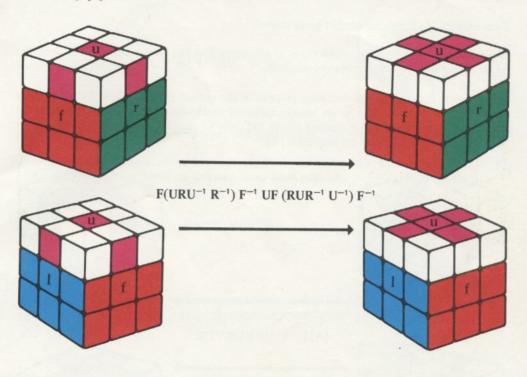
Sequence [b] flips upper-front and upper back.



# STAGE 4 (continued)

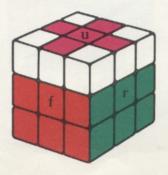


None correct. If none of the upper edge colours are correct then use [a] followed by [b] — this is illustrated below.



Sequence [a] U [b] flips all upper edges.

At the end of stage 4 your cube should look like the following.





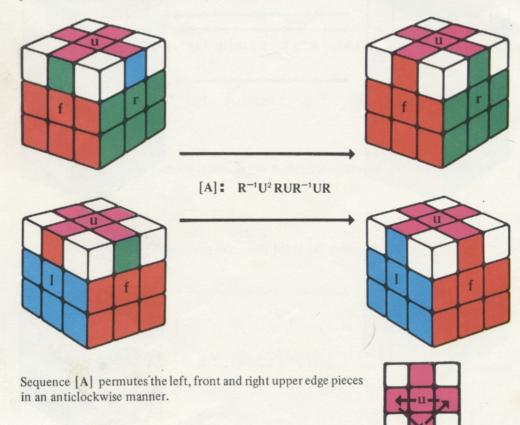


Stage 5 is to put the top edge pieces into their correct position. Either 0, 1, 2 or 4 will be in their correct position.

Four correct. If all four are correct go to stage 6.

None correct. By using U as many times as necessary you can ensure that 1, 2 or 4 of the upper edge pieces are in their correct position.

One correct. If precisely one top edge piece is in the correct position then the other three have to be permuted either in an anticlockwise or clockwise manner. If they have to be permuted in an anticlockwise manner rotate the cube so that the piece that is in its correct position is at the back, then use sequence [A].





## STAGE 5 (continued)



If the three pieces have to be permuted in a clockwise manner apply  $U^2$ , rotate the whole cube until the upper edge is at the back, then apply [A]. Alternatively use [C], the inverse of [A].

[C]:  $R^{-1}U^{-1}RU^{-1}R^{-1}U^{2}R$ 



Sequence [C] permutes the left, front and right upper edge pieces in a clockwise manner.

Two correct. If two adjacent edge pieces are in the correct position use U followed by [A] which will interchange the upper-left and upper-back edge pieces.

U[A]: UR-1U2RUR-1UR

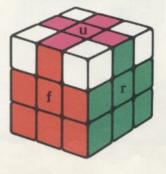


If two opposite edge pieces are in the correct position then use [A] followed by  $U^{-1}$  followed by [A] again which will interchange the upper-front and upper-back edge pieces.

[A]  $U^{-1}$  [A]:  $(R^{-1}U^2RUR^{-1}UR)U^{-1}(R^{-1}U^2RUR^{-1}UR)$ 



At the end of stage 5 your cube should resemble the following.

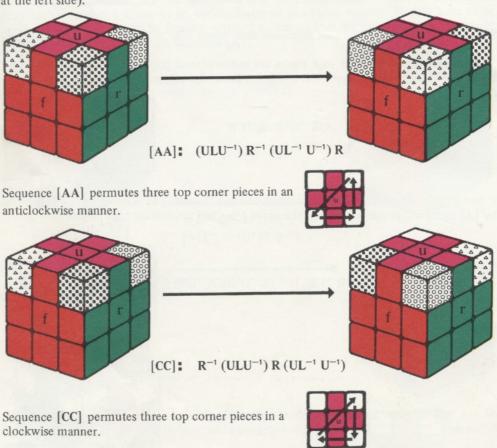






Stage 6 is to place the four top corner pieces in their right place although possibly facing the wrong way (stage 7 will twist the pieces correctly into place). Either 4, 1 or 0 will be in the right place. If all four are in their right place, go to stage 7.

One right. If exactly one piece is in its right position arrange the cube so that this piece is in the left-back-upper position (shown unshaded in the diagrams). Then permute the other three pieces in an anticlockwise or clockwise manner by using [AA] or [CC] respectively. (Recall that L moves the left side of the cube in a clockwise direction as seen when looking at the left side).



Remark: [CC] is the inverse of [AA], also using [AA] twice has the same effect as using [CC] once.



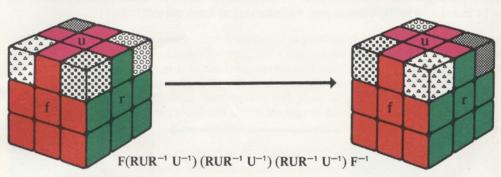
## STAGE 6 (continued)



None right. If none of the four corner pieces is in the right place then using [AA] ensures that precisely one is in the right place. Then use [AA] or [CC] to finish off.

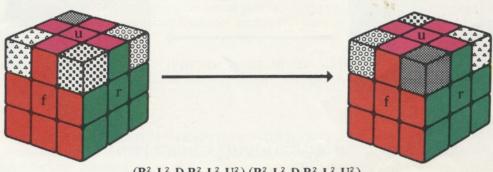
Alternatively, the following interchanges two pairs of adjacent top corners.





On the other hand the following interchanges opposite top corners. Recall that D moves the down (bottom) side of the cube in a clockwise direction as seen when looking at the down side.



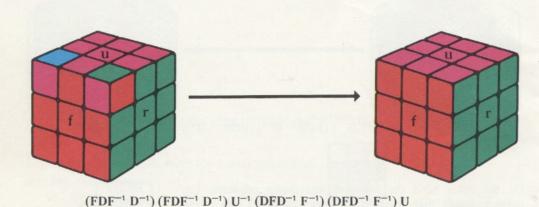


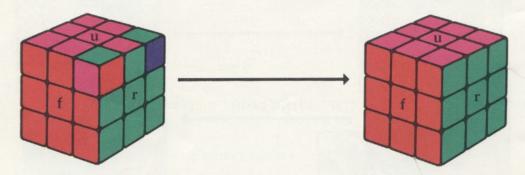




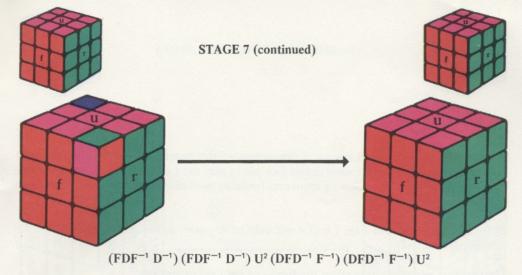
Stage 7, the final stage, is to twist the top corner pieces so that they are correctly in position. Either 4, 2, 1 or 0 are correctly in position. If all 4 are then you are finished.

Two correct. If exactly two of the upper corner pieces are correctly in position then one of the incorrectly positioned ones requires a clockwise twist while the other requires an anti-clockwise twist. Place the cube so that the corner piece that requires a clockwise twist is in the front-upper-right position. Then use one of the following three sequences depending upon where the other incorrect piece is. Recall that D moves the down (bottom) side of the cube in a clockwise direction as seen when looking at the down side.





 $(FDF^{-1} D^{-1}) (FDF^{-1} D^{-1}) U (DFD^{-1} F^{-1}) (DFD^{-1} F^{-1}) U^{-1}$ 



None correct. If no upper corner piece is correctly in position then two require a clockwise twist and two require an anticlockwise twist. Place the cube so that one of the corner pieces that requires a clockwise twist is in the front-upper-right position. Use one of the sequences in the "two correct" section making sure that at the end two of the upper corner pieces are correct. A further sequence from that section will place all pieces correctly in position. Alternatively use the following sequences which are shorter but harder to remember.

The sequence

twists the upper-front-right and upper-back-left pieces clockwise, the two other upper corner pieces being twisted anticlockwise. (Recall that B moves the back side of the cube in a clockwise direction as seen when looking at the back side.)

The sequence

twists the two front upper corner pieces clockwise and the two back ones anticlockwise.

One correct. If one upper corner piece is correctly in position then the other three either all require a clockwise twist or all require an anticlockwise twist. By using one of the sequences in the "two correct" section you can ensure that two of the upper corner pieces are correctly in position. A further sequence from that section will place all pieces correctly in position.

Alternatively

(FDF<sup>-1</sup>D<sup>-1</sup>) (FDF<sup>-1</sup>D<sup>-1</sup>) U<sup>-1</sup> (FDF<sup>-1</sup>D<sup>-1</sup>) (FDF<sup>-1</sup>D<sup>-1</sup>) U<sup>-1</sup> (FDF<sup>-1</sup>D<sup>-1</sup>) (FDF<sup>-1</sup>D<sup>-1</sup>) U<sup>-1</sup> twists each of the corner pieces front-upper-right, left-upper-front and back-upper-left in a clockwise direction.

While

UU (DFD<sup>-1</sup>F<sup>-1</sup>) (DFD<sup>-1</sup>F<sup>-1</sup>) U (DFD<sup>-1</sup>F<sup>-1</sup>) (DFD<sup>-1</sup>F<sup>-1</sup>) U (DFD<sup>-1</sup>F<sup>-1</sup>) twists each of the corner pieces front-upper-right, left-upper front and back-upper-left in an anticlockwise direction.



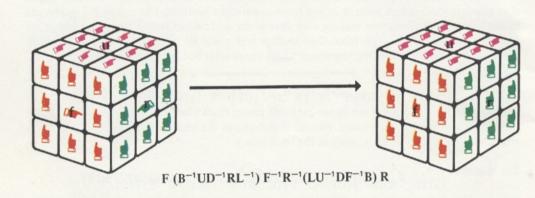
#### **CUBES WITH PICTURES**

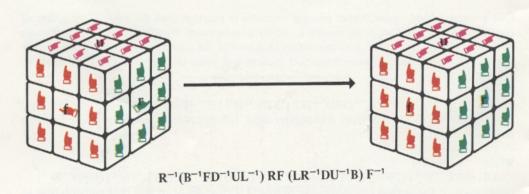


If your cube has extra markings on it (for example fruit, numbers, portraits etc.) then after you have completed stage 7 some centre face pieces may need a turn in relation to the other pieces on that face. The following sequences (possibly used more than once) will enable you to complete your cube.

Note: By being careful in stage 1 to 7 it will only be necessary to use at most the third of the following sequences.

The first two sequences only affect the front and right centre pieces.



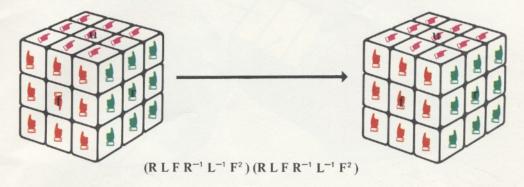




## **CUBES WITH PICTURES (continued)**



The third sequence only affects the front centre piece.

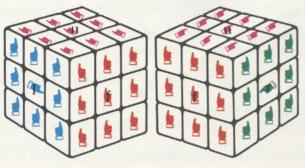


Other cases may be dealt with by using the above sequences a few times. Some examples are given below.



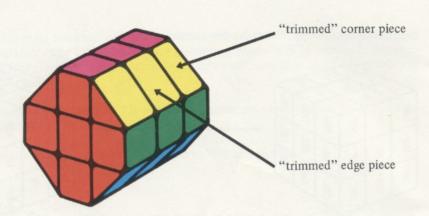
In this case use the first sequence followed by the third.

For this case use the first sequence, then rotate the whole cube so that I becomes front and use the first sequence again.



#### THE BARREL OR OCTAGONAL PRISM PUZZLE

This puzzle is just the Cube with four edges trimmed so that two of the faces are octagonal (eight-sided) in shape. There are ten different colours on the Barrel, two colours appear nine times and the other eight each appear three times.



The solution to this puzzle is much the same as the solution to the cube.

Stage 1. Choose a colour for "down" that is on a centre face piece and appears three times. You can easily put the two edge pieces with that colour on them correctly into position. To complete stage 1 you will have to choose any two of the "trimmed" edge pieces and put them into the bottom layer.

Stages 2 and 3. These stages are the same as for the cube but perhaps a bit more confusing.

Stage 4. Make sure that both edge pieces having the upper colour are flipped.

Stage 5. Make sure that both edge pieces having the upper colour are correctly in position.

Stage 6. You may come across a situation not described on pages 22, 23 in stage 6 for the cube. If so, use the following sequence:

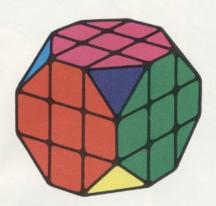
# $(R^{-1}U^2 RUR^{-1}UR) U^{-1} (R^{-1}U^2 RUR^{-1}UR)$

(This is just the sequence [A]  $U^{-1}$  [A] on page 21.) Then use  $U^2$  if necessary and you will have a situation described on pages 22, 23 after which you will be able to complete stage 6.

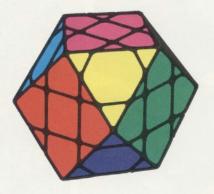
Stage 7. No problems arise here and you will be able to complete the octagonal prism puzzle.

## OTHER MULTI-SIDED PUZZLES

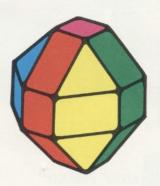
There are many other multi-sided puzzles which are constructed by trimming the cube in various ways. Some examples are given here. The solutions to these puzzles are as for the cube or the barrel.



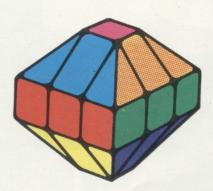
**Truncated Cube** 



Cuboctahedron



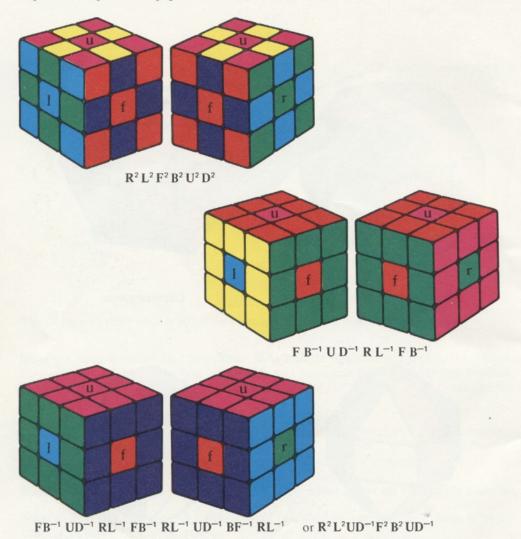
Spherical Cube or Rhombicuboctahedron



**Cushion or Diamond** 

#### PRETTY PATTERNS

To obtain the pretty patterns start with a completed cube and perform the sequence. Remember what is front and up, then to get back to your completed cube do the inverse sequence as explained on page 5.

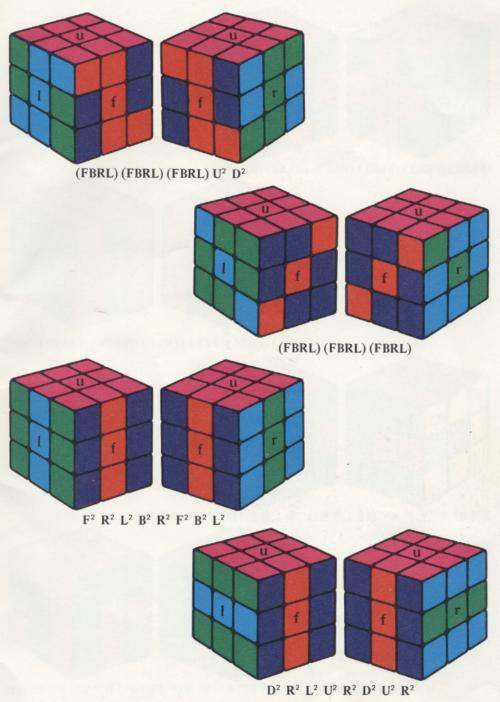


Moves of the type  $(RL^{-1})$ ,  $(FB^{-1})$ ,  $(UD^{-1})$  etc. are called slice moves. Using sequences of slice moves often results in pretty patterns, some are shown here — can you see which? (Note that  $R^2L^2 = (RL^{-1})(RL^{-1})$  is a slice move).

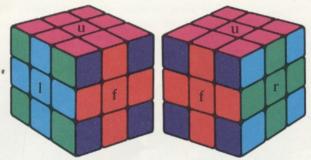
Moves of the form (RL), (RB), (UD) etc. are called antislice moves. Again, sequences of antislice moves often lead to pretty patterns, some are shown here.

Try also (FB) (UD) (R<sup>-1</sup> L<sup>-1</sup>) (FB).

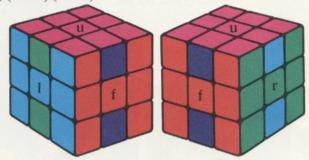
#### PRETTY PATTERNS



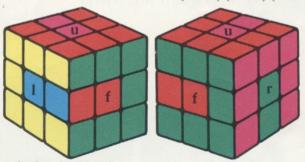
#### PRETTY PATTERNS



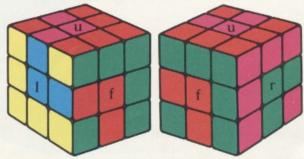
(FBRL) (FBRL) (FBRL) (RLFB) (RLFB)



(FBRL) (FBRL) (FBRL) (RLFB) (RLFB) (RLFB) U<sup>2</sup> D<sup>2</sup>



 $L^{-1} \; R^2 \; F^{-1} \; L^{\; -1} \; B^{-1} \; U \; B \; L \; F \; R \; U^{-1} \; R \; L \; R \; L^{-1} F \; B^{-1} \; U \; D^{-1} \; R \; L^{\; -1}$ 

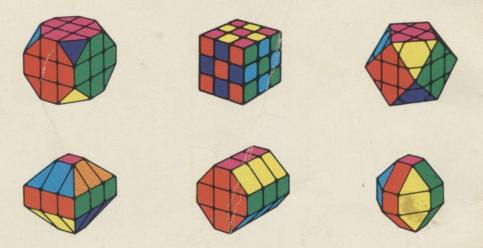


 $RUF^2\ D^{-1}\ RL^{-1}\ FB^{-1}\ D^{-1}\ F^{-1}\ R^{-1}\ F^2\ RU^2\ FR^2\ F^{-1}\ R^{-1}\ U^{-1}\ F^{-1}\ U^2\ FR$ 

The Cube was invented by the Hungarian Ernö Rubik as a teaching aid for his architecture students. It helped them to appreciate shape and space. Since the Cube first appeared in Europe in 1978 many millions have been sold. In this little book Czes Kośniowski presents an easy-to-follow solution to the Cube. It has the following unique features:

- Every page is illustrated in full colour with every possible configuration accounted for and instantly recognisable. So you can't go wrong or get lost!
- Each stage of the solution consists of a short sequence of clearly explained and easily memorised moves. It is easy to take stock of your progress before going on.
- ★ Once you have solved the Cube, Czes explains how you can master the illustrated Cube, the Barrel puzzle and other multi-sided puzzles.
- For extra enjoyment Czes shows you how to make pretty patterns from your Cube.

If you have been baffled, fascinated or frustrated by the Cube, this is the book you need to help you enjoy its variety, meet its challenge and discover its potential.



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