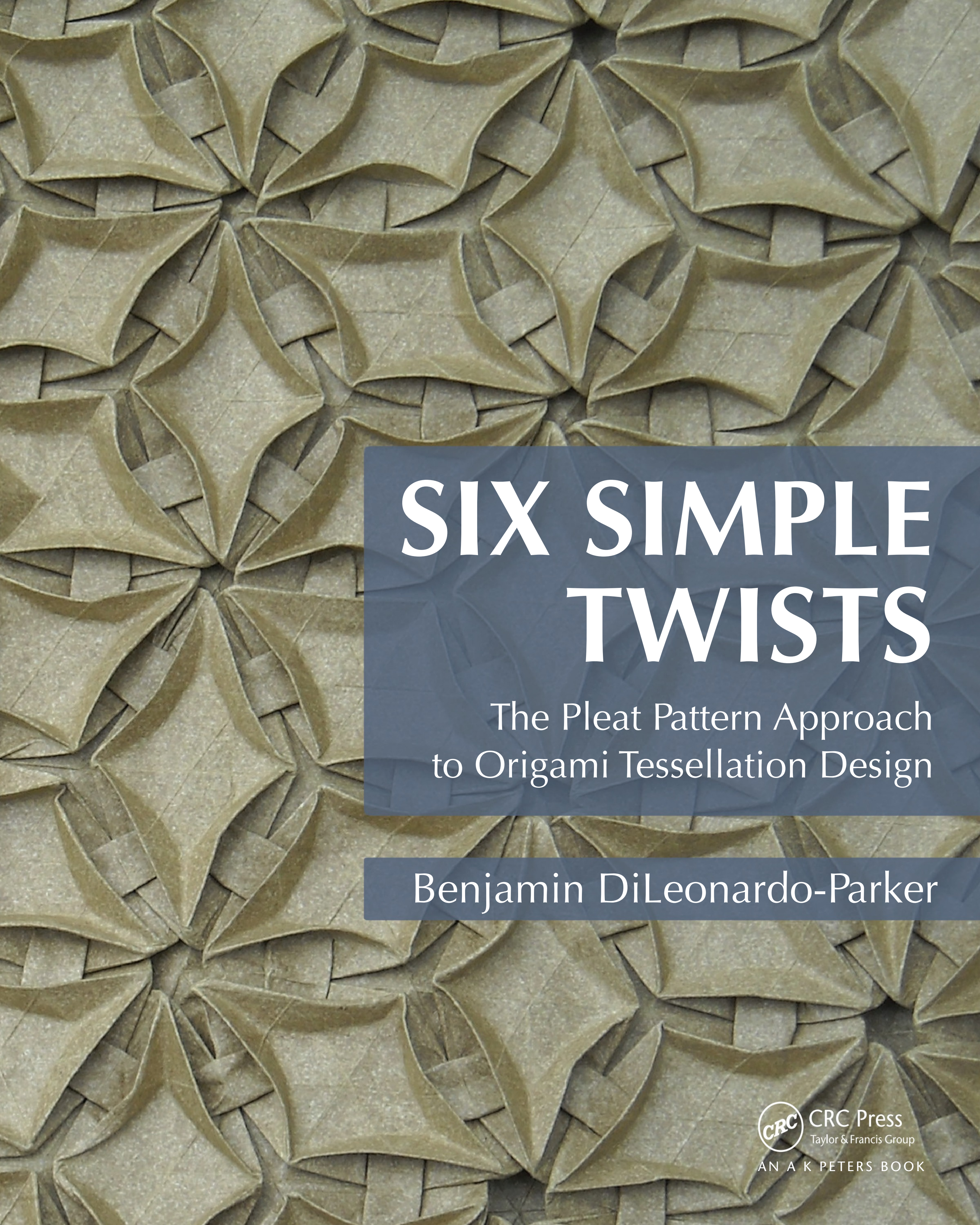

SIX SIMPLE TWISTS

The Pleat Pattern Approach to Origami Tessellation Design

Benjamin DiLeonardo-Parker

CRC Press
Taylor & Francis Group
AN A K PETERS BOOK

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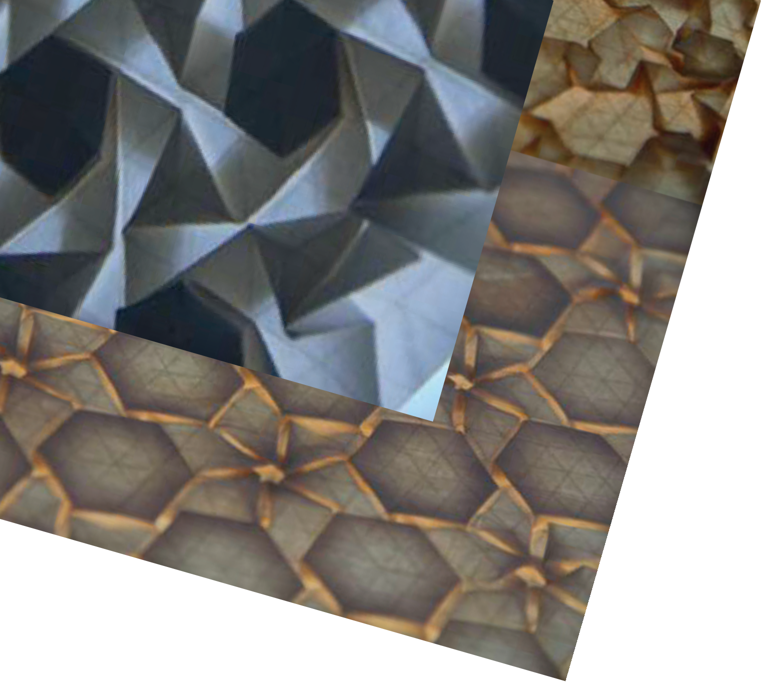

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Preface

What are origami tessellations? Why does this book exist?

Origami has risen in popularity around the world in the past century. It started as a fun and educational pastime for schoolchildren (which is how most people encounter the craft) and grew into an exciting study for artists, mathematicians, engineers, teachers, and anyone else with an inquisitive mind. These people meld their varied backgrounds to explore how the paper interacts with itself. This is often done through simple play. Origami artists certainly know about paper play. They enjoy thinking, “If this is 1×1, let’s see what a 2×1 looks like!” “I wonder what it would look like if this angle were 30° larger,” or simply “Let’s make it bigger!” There are many subfields of study, and more units and models are being discovered and published daily.

The study of origami tessellating is one branch of this medium. Tessellations are patterns of polygons that tile regularly on a plane. Origami tessellations accomplish this using one sheet of paper without cuts, generally using pleats and twists, or mountainous corrugations. Oftentimes the paper is folded into a grid, especially when a folder is learning. Despite the rise in popularity of this genre over the past decade, the study of tessellations has been difficult to access for many folders. I believe that this is due to lack of literature on the subject, the tedium of folding a grid (a valuable teaching and experimentation tool), and the many different types of tessellations from which to choose.

When I started attending origami conventions in 2007, there were few people folding tessellations, and far fewer classes on the subject. The classes were either based around a procedure for folding a specific tessellation or on the mathematical concept behind a family of tessellations. These represent two extremes. The first is an extension of traditional origami teaching methods in which there is a design and a method of folding, and the student learns that method, step by step. Some folders go on to modify the design, but many memorize and repeat. The latter extreme allows for a myriad of designs to be created through mathematical progression; however, all but the most ambitious folders may become paralyzed by the many options or intimidated by the mathematics. Neither is superior to the other, but in my experience, a mixture of the two makes the activity accessible to more folders.

So we look at those who have successfully designed a tessellation that was either never folded before or was otherwise independently discovered. How did they do it? Did they stumble upon it through experimentation? Or did they use complex algorithms to figure it out? The truth is most likely a bit of both.

An Internet image search for origami tessellations will yield varying results, from flagstone tilings to waterbomb-based tessellations, to corrugations, and other styles within the field. If one were to try to study how all of these are done using one unifying technique, the task would be enormous and daunting, since they are each distinct styles and the approach is different for each. I restrict my studies to similar techniques and draw conclusions from those progressions before figuring out how to spill over the concepts into other styles. I began studying waterbomb-based tessellations, which are mountainous single-layer patterns. After a time, I decided to switch and study something completely different: what I call *pleat patterns*. After years of design, it is time to write my thoughts down and describe how I view what is happening when a tessellation is folded. No one questions whether it is possible to fold these patterns, because origami tessellations can be seen at art galleries and origami conventions. The questions are, “How is it done?” and “What are the further possibilities?” This book provides some of those answers.

Acknowledgments

I owe many people acknowledgments for their help and inspiration in writing *Six Simple Twists.*

Thank you to Ilan Garibi, Tom Crain, Mary MacDonnell, Margo Lynn, and my parents, Catherine DiLeonardo and Andrew Parker, all of whom helped me with proofreading and content editing.

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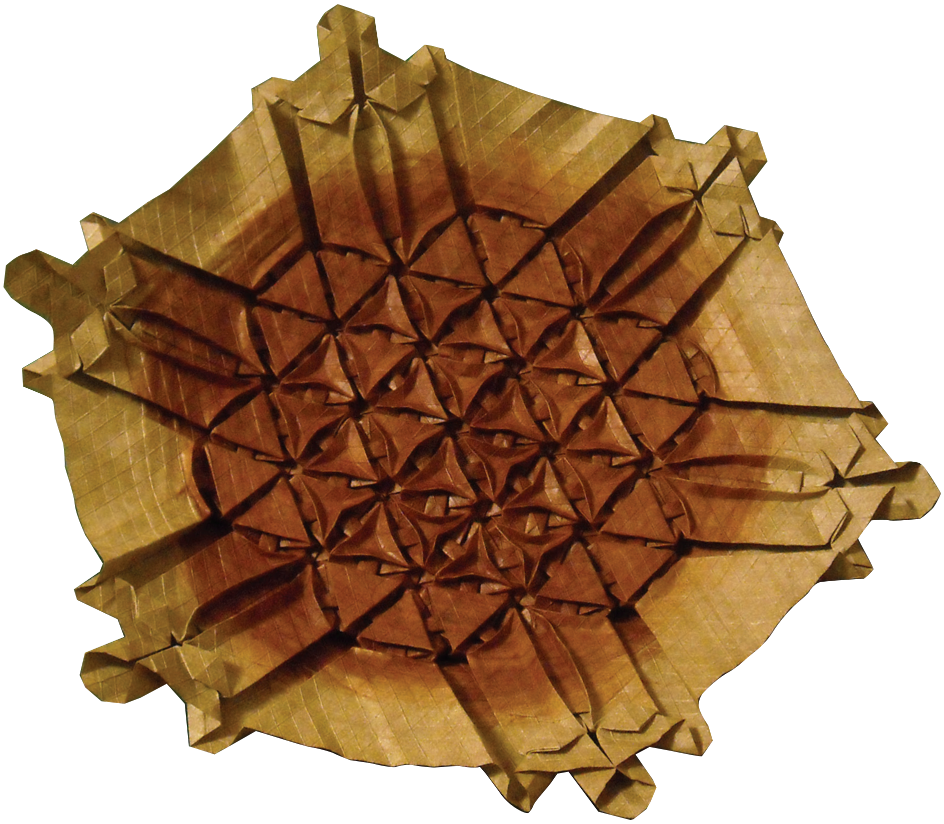
Thank you to Chris Huestis, for the generous use of his studio; Jeff Holcombe, for the conversation and use of his camera; Christine Dalenta, for building a successful artistic collaboration with me; and Jordan Smith, for the use of his eyes and the tangential knowledge on any subject.

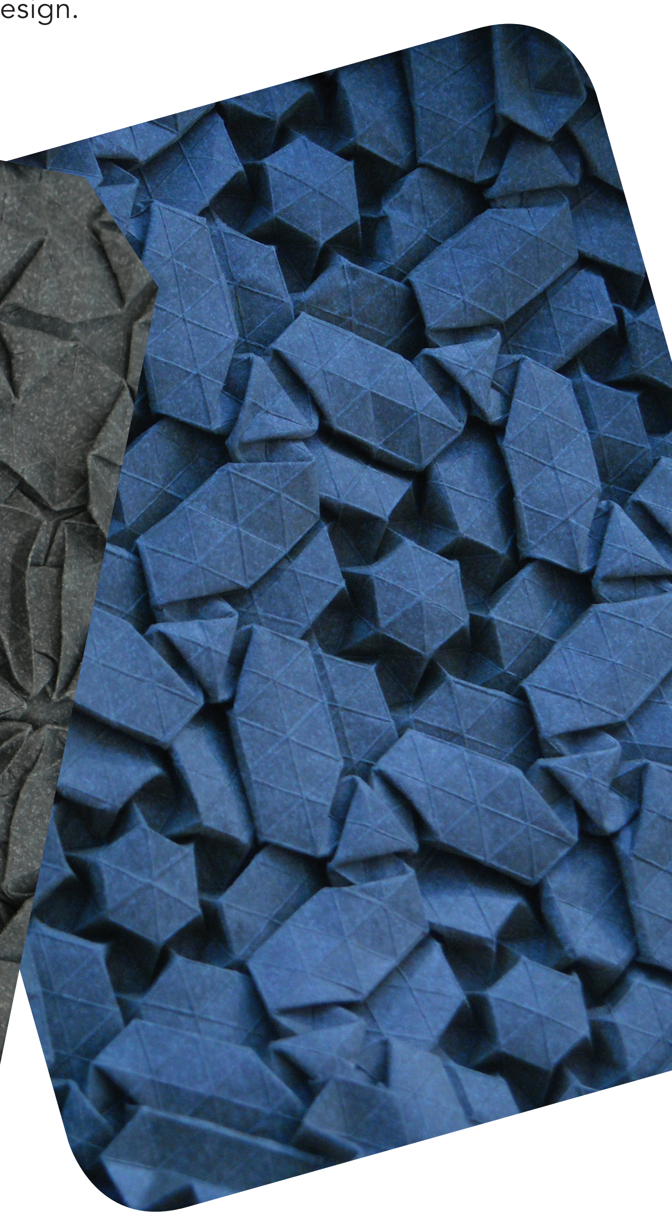
Thank you to my many origami friends throughout the world, with whom I have shared the love of this art for many years, including Eric Gjerde, Christiane Bettens, Philip Chapman-Bell, Joel Cooper, Ray Schamp, Tom Crain, Joseph Wu, Beth Johnson, Michael Assis, and countless others.

Thank you for your inspiration and support.



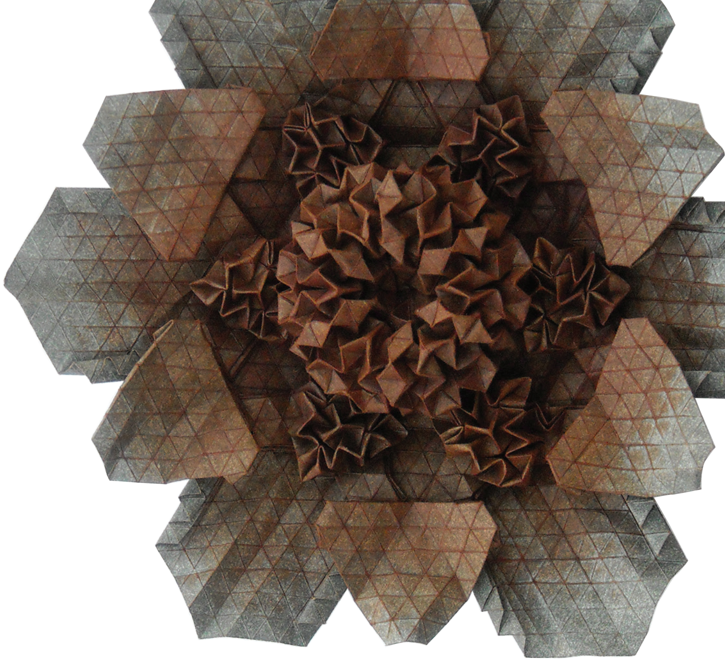
About the Author

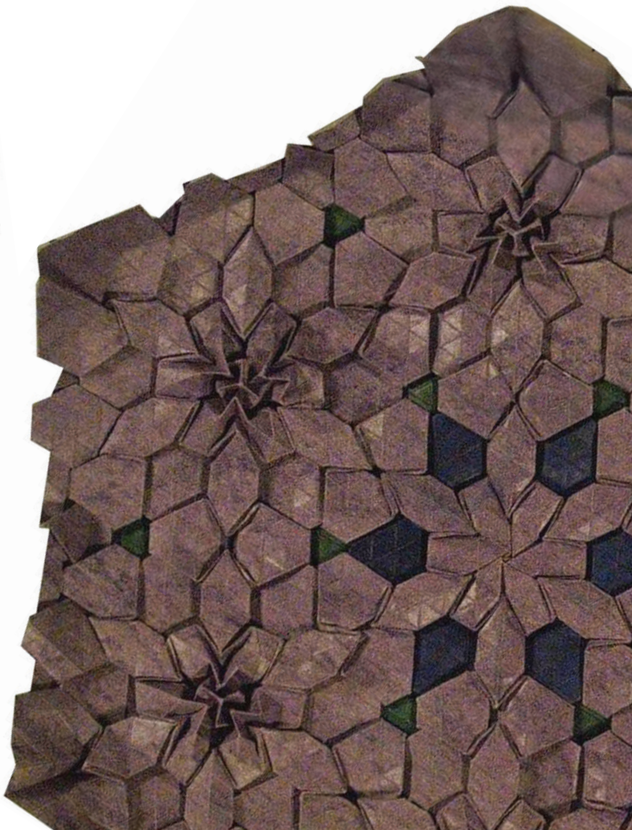
Benjamin DiLeonardo-Parker, a working origami pattern artist since 2009, studied French and Italian Language and Literature at the University of Pittsburgh. He uses his art career to explore other activities, including electronics, woodworking, and alternative photography. He has taught and exhibited at origami conventions and art shows throughout the United States and La Escula-Museo Origami de Zaragosa, an origami museum in Spain. Parker is currently collaborating with the alternative photographer Christine Dalenta to fold light-sensitive paper, which Dalenta exposes while folded, resulting in the impression of the folds modulating the material. Parker wishes to expand his and others' understanding of what is possible with a sheet of paper and how it can be applied with other media to inspire creativity and design.



Chapter 1

The Six Simple Twists





1.1 The Basics and Preparation

While the study of origami tessellating is different from the study of traditional origami, at its core, the field uses many of the same concepts. Many of the same terms are used, and it is necessary to understand these before delving too far. In school, we are exposed to origami diagrams with lines and arrows showing step by step how to fold an origami model. This is the most common way to document instructions, since it is comparatively easy to understand and replicate, depending on the clarity of the diagram. An Internet search for "origami diagram" will yield thousands of results.

However, as a folder progresses in skill level, there is an appeal to using a different type of documentation: the *crease pattern*. A crease pattern is a blueprint of folds with lines that mark the creases that compose an origami design. You can create your own crease pattern by folding a model out of a sheet of paper, unfolding it, and then drawing the creases that you see. Many designers use crease patterns to facilitate the design process and to teach others how the design was created.

Origami uses two primary types of folds: *mountain* and *valley*. A mountain fold is a fold that points toward the viewer. A valley fold is the opposite: a fold that points away from the viewer. Whether a fold is mountain or valley depends on the user's point of view. If you are looking at one type of fold, flip the paper to see the other type. Crease patterns describe mountain and valley fold allocation with respect to the other folds on that side of the paper. If you fold a sheet of paper away from you, it makes a valley fold. If you pinch a sheet in the middle, it makes a mountain fold.

In my crease patterns, I include an additional pair of figures, called *mountain points* and *valley points*. A mountain point is an intersection of creases that rise toward the viewer. A valley point is an intersection of creases that descend away from the viewer. While not a necessary thing to note, I find that marking mountain and valley points at certain intersections can make a crease pattern neater and can clarify some ambiguities in deciphering a crease pattern. You may notice as you go through the exercises that I do not include mountain and valley points at every intersection, only in those that I feel will help the folder understand the structure.

1.2 How Tessellations Differ from Traditional Folding

Traditional origami designs are those that have been in existence for so long that we do not know their origin. These folds use bases as starting points to get the paper in the correct spot and then decoration to fill in. Well-known designs such as the crane and the waterbomb fit this category.

Tessellation folders are largely unconcerned with the paper resembling an object or animal. They simply enjoy the pattern and the process. The goal is not a base, but rather constraints in the form of polygonal twists that create the structure of a tessellation. If you understand the structure, you can substitute different polygonal twists for others that share the correct properties with the original connection. This creates a multitude of possibilities, which can be overwhelming to a beginner. Hence, this book is primarily concerned with only six twists that are useful for getting started. After you understand how to wield these effectively, you can learn how to create your own.

1.3 Fold Bias and Manipulation

Take a sheet of paper and fold it edge to edge. It does not matter precisely where the fold is; for this exercise, we are only concerned with the crease line. Remove your hands, but do not actively unfold the paper. What did the paper do? Depending on the material you used and how hard you creased, the paper unfolded at least a little by its own power, but not entirely. You will probably see a valley fold with the paper opened a little. Different material will yield different results.

Why didn't it open entirely? The paper has a memory of the fold in it, and this is necessary to acknowledge if you wish to manipulate it. This crease has a very strong valley bias from the viewer's perspective. The *bias* of a fold describes a crease that has been unfolded and the strength of the memory of the original fold. As with a mountain or valley fold, whether a crease has a mountain or valley bias depends on the point of view of the describer.

In tessellations, there are often setup steps that involve refolding specific creases to increase the mountain or valley bias. This can make it easier to fold more than one crease simultaneously and to fit more layers of paper into place at once. In gridding, it is necessary to understand crease bias, how to strengthen it by refolding, and how to neutralize it through backcreasing. *Backcreasing* a crease involves folding a crease in the direction opposite of its bias to weaken it. Backcreasing can turn a strong valley bias into a weaker mountain bias or a strong mountain bias into a weaker valley bias. One goal of backcreasing is to keep the crease biases predictable and as neutral as possible throughout the material.

1.4 How to Read the Exercises

To diagram these tessellations, I employ photographs and text, the same as traditional diagramming. I also include crease patterns at each step, for two reasons. The crease pattern of a particular step may offer additional clues about how a fold is done that text alone lacks. Also, this allows the folder to become accustomed to interpreting crease patterns for tessellations. The last numbered step of a diagram shows the full crease pattern for the item being diagrammed, as does the page immediately after the diagram. As you are learning, you should scan this full-sized crease pattern, print it, and fold it. Crease patterns in steps leading to the final step are partial, and should be interpreted as such. I use arrows and other movement markers where necessary, but much of the goal of the diagrams is to encourage you to read the crease patterns. Diagramming a twist or full tessellation is very inefficient, both in terms of space and time taken to document. If your understanding of twist construction can transcend the standard diagram format, you will become a stronger, more innovative designer.

There are conventions when drawing crease patterns, and books that include these conventions define them clearly. The diagrams in this book are more complicated than most, since they reflect the subtleties of the crease bias. (See Figure 1.1).

1.5 Obtaining a Hexagon

Many folders prefer to begin with a rectangle or square. Since this book is exclusively concerned with triangular and hexagonal symmetry, I find it more straightforward to fold directly from a hexagonal sheet of paper. Once you understand how to fold from

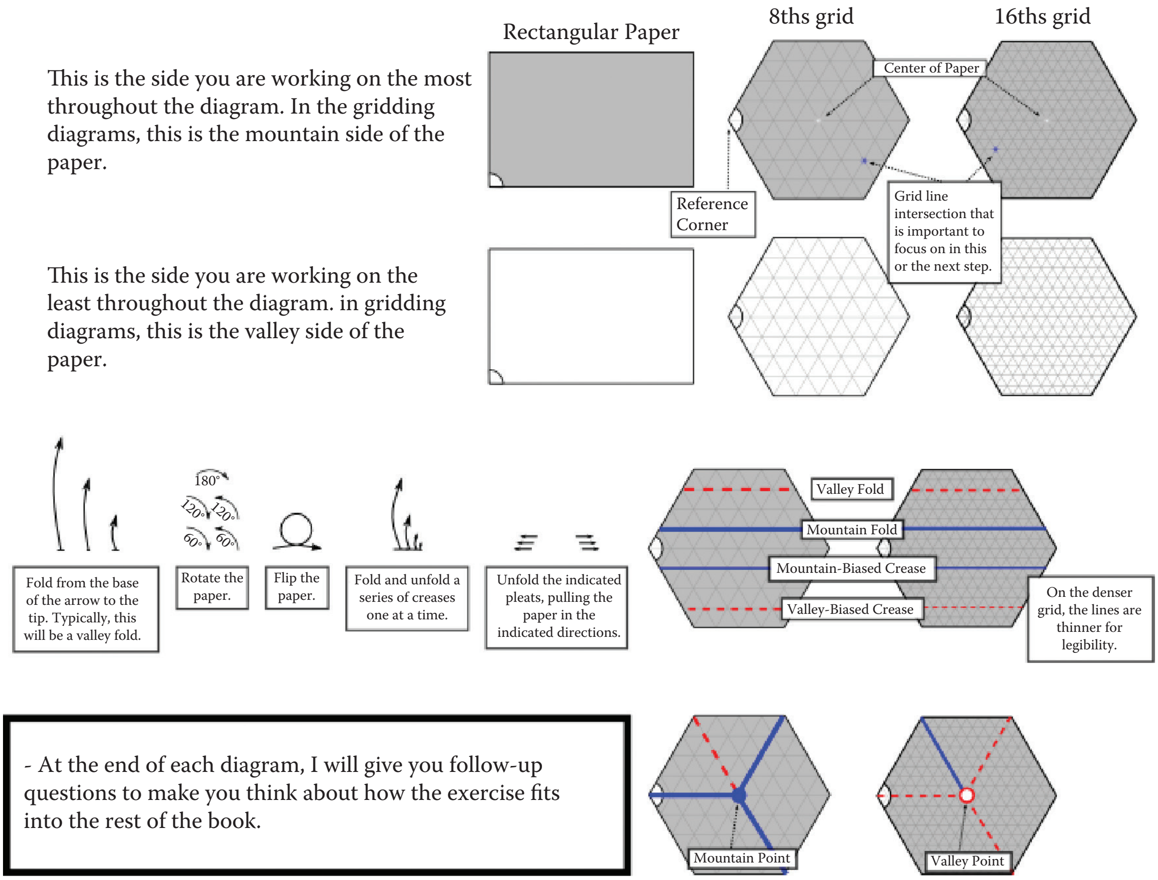


Figure 1.1

a hexagon, the concepts are transmutable to any shape of paper.

There are several methods of cutting a hexagon from a square or rectangle. Starting out, I recommend folding standard printer paper. Printer paper is comparatively sturdy, with convenient dimensions, and it is abundant enough that you can experiment with it without a large investment. You can use the square origami paper (kami) that you find in craft stores, but the folding process for obtaining a hexagon from a square is slightly different and more complicated. This is demonstrated in Figure 1.2.

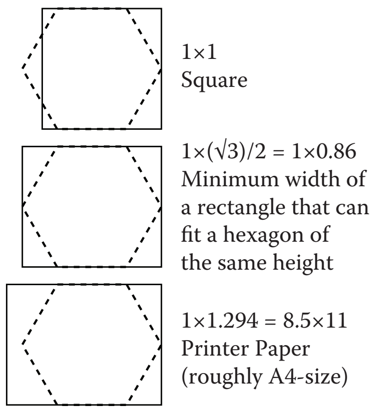
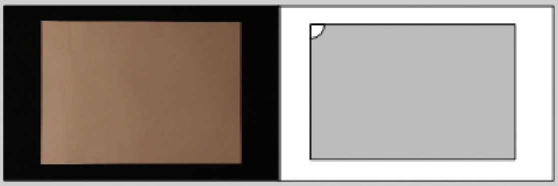


Figure 1.2

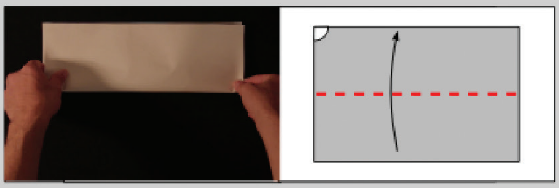
Constructing a Hexagon from a Rectangle

The most simple way I've found to fold a hexagon from a rectangle is to fold the paper in half and construct a line 60° off of the center line. From this, you can derive the rest of the form.

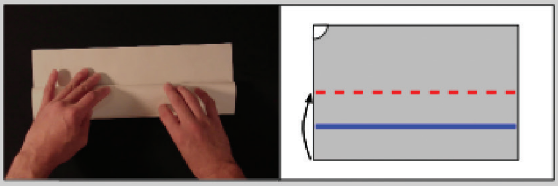
There are many methods for folding a hexagon, but I find this one to be relatively easy to teach and the excess can be used for other experimentation.



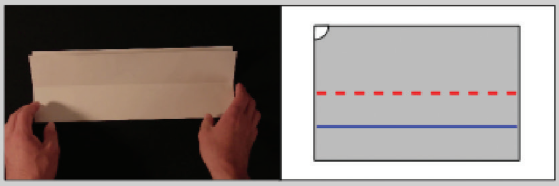
1. Begin with a rectangular sheet of paper.



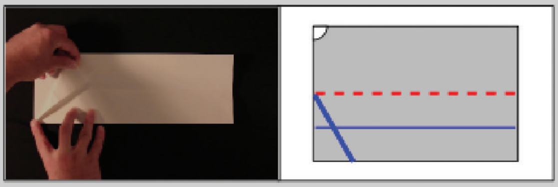
2. Valley fold in half along the length.



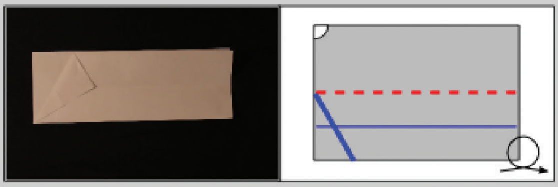
3. With the raw edges away from you, fold the top layer toward you to meet the closest edge (this is the center of the paper). Then, unfold this crease.



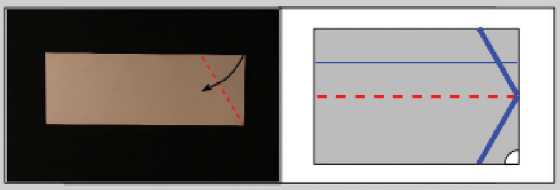
4. This shows finished step 3.



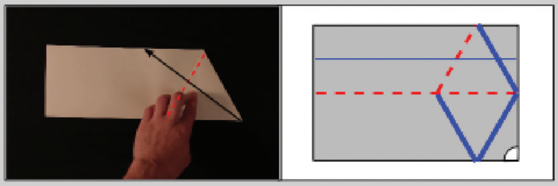
5. Bring the top-left corner to the crease formed in step 3. Have the resultant crease go through the lower-left corner.



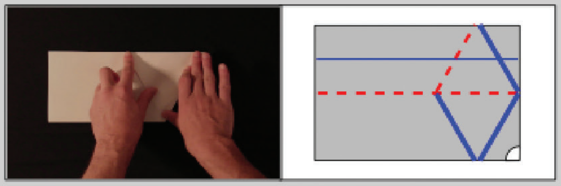
6. This is what step 5 looks like when finished. The lower-left corner angle is now 60°, and we can construct the rest of the hexagon from this. Flip the paper laterally.



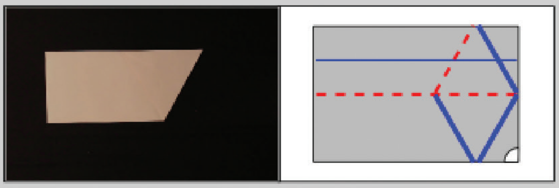
7. Now we will duplicate the 60° to the other side. Using the first 60° angle as a guide, fold the top-right corner down to match.



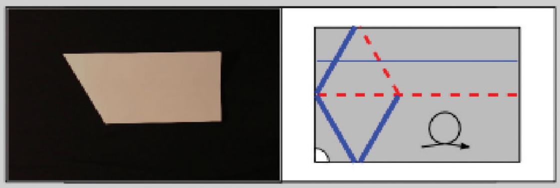
8. Bisect the top-right corner with a valley fold. This will replicate the 60° angles through what will be the center of the hexagon.



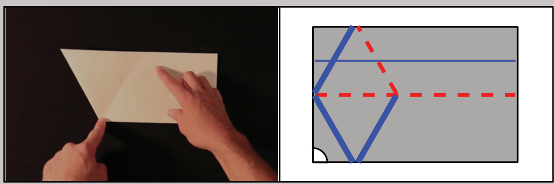
9. The right edge of the paper should meet the top edge.



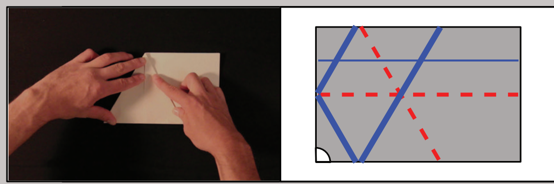
10. This shows finished step 8.



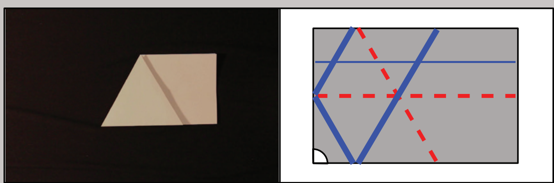
11. Flip the paper laterally.



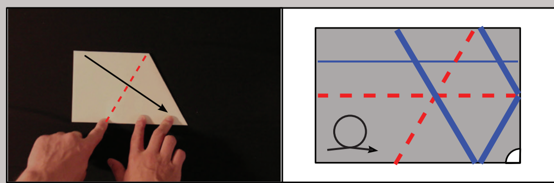
12. Replicate the 60° angles again through the paper by bisecting the lower-left corner and folding the top-left corner to meet the bottom.



13. This shows step 12 in progress.

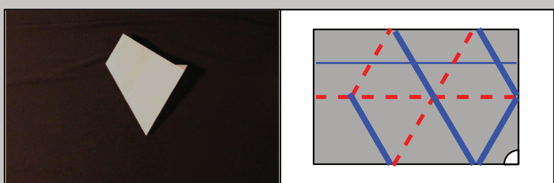


14. This shows finished step 12.

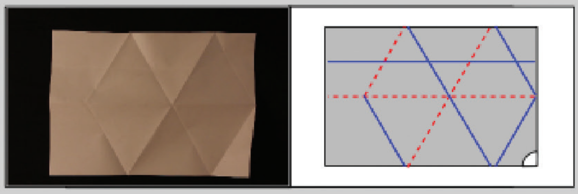


15. Flip the paper laterally. Now, we will complete the hexagon with the final 60° replication. Bisect the top-right corner with another valley fold, folding the top edge of the paper to meet the right edge. The left side will pass the bottom edge.

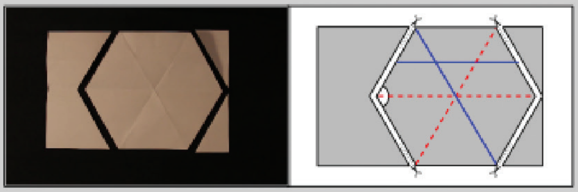
The valley fold will pass through where my left index finger is placed.



16. This shows finished step 15.



17. Unfold the paper, and you will see the outline of the hexagon to cut out.



18. This shows the finished hexagon.

- As this book progresses, you will notice that it is important to acknowledge the bias of a fold, whether it is mountain or valley, and how strong that bias is.

- Can you identify the crease biases in the finished hexagon using this method? How would you go about making the crease biases become more neutral again?

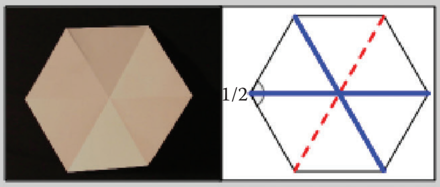
- The solution for how to neutralize the crease biases will be in the first few steps of the Basic Gridding diagram, but attempt to figure it out for yourself first.

1.6 How and Why to Fold a Grid

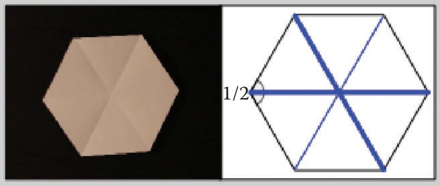
One of the primary hurdles with teaching or learning tessellations is grid folding. As you are learning the basics, a grid is a critical element to keeping the folds in line. But not everyone knows how to fold a grid, and of those who know, not everyone knows how to do it efficiently or accurately. One of the most common phrases I hear when showing people my work for the first time is "I don't have the patience for that." The main requirement is not patience, but rather confidence. Without the confidence that the effort one puts into a grid will lead somewhere, there is no impetus to experiment.

The first step to overcoming this lack of confidence is becoming comfortable with the paper. If the grid does not look even, or if you don't want to turn the grid into a work of art, turn it into an experimental grid. Or challenge yourself to work on an uneven grid and see how it differs from a "perfect" one. There is a lesson in each sheet of paper, and you will become quicker and more accurate with practice.

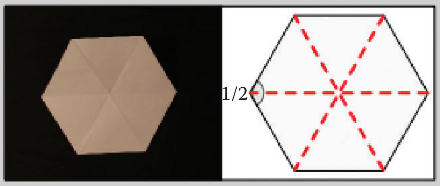
Basic Gridding



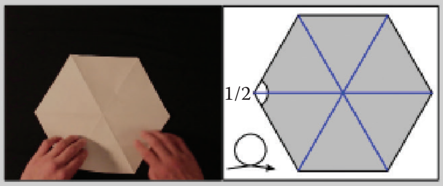
1. When you unfold the hexagon learned in the last section, there will be a strong mountain bias on two diagonals and a valley bias on the third. If the opposite is true, flip the paper.



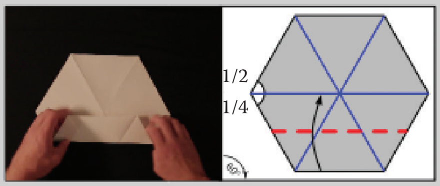
2. Reverse the bias of the valley fold diagonal so that all three have mountain fold biases. You will have to flip the paper to do this and then flip it back.



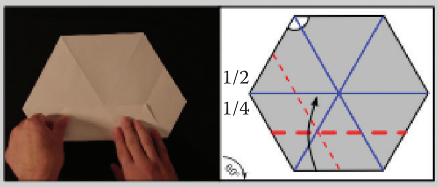
3. One at a time, reverse each crease and then unfold. This will result in all three diagonals having a very weak valley bias. This is important during gridding for uniformity. We will call this the valley side of the paper.



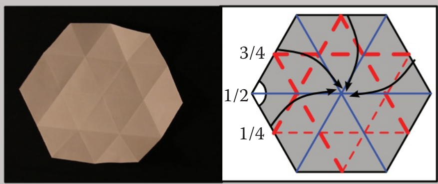
4. Flip the paper. You should be able to feel the difference between the folds on this side and folds on the other. We will call this the mountain side of the paper, and it is the primary side on which we will work.



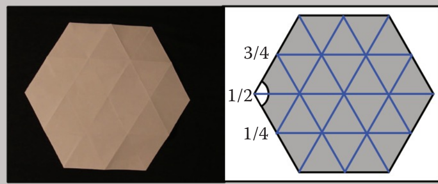
5. Starting with the edge that is closest to you, fold from the edge of the hexagon to the center line. Unfold.



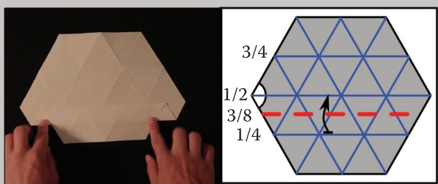
6. Rotate the paper 60° clockwise, and fold the next edge into the center line. Unfold.



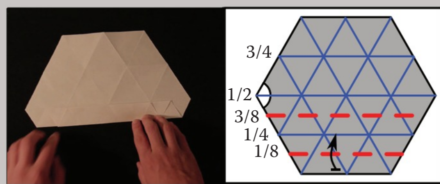
7. Repeat step 5 with each edge, one after the other, until all six edges have been folded into the center line and unfolded.



8. Now we have valley folds encroaching on our mountain side. One at a time, reverse each valley fold so it becomes a mountain fold and unfold. You will have to flip the paper back and forth to do this. After you are finished, all crease biases should be uniform.



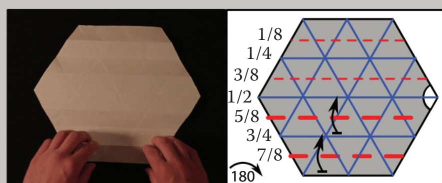
9. You now have a grid divided into 4ths, which is the starting point for more complex grids. Find the mountain crease adjacent to the center line. Fold that crease into the center. Do not unfold.



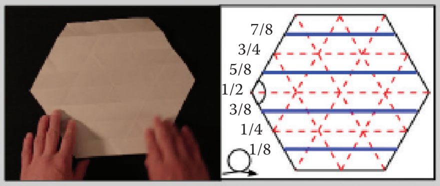
10. Fold the edge of the paper onto the stack, like an accordion. Unfold entirely. You will find valley fold biased creases on the mountain side of the paper.

To go from 4ths to 8ths, we need to detail the grid, or add a crease between each set of adjacent creases already in the paper. Detailing is a two-step process. The first step is subdividing an axis, or a set of parallel lines. The second step is backcreasing. Then, replicate the process with the remaining two axes.

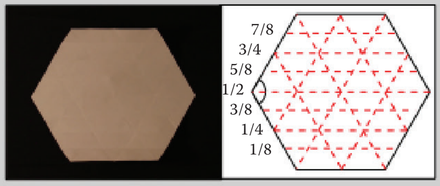
To subdivide an axis, make the creases equidistant between each pair of adjacent parallel creases. To backcrease, reverse the direction of the new valley folds made by subdividing. When you are finished with these steps for all three axes, you will have a grid with double the complexity that you started with.



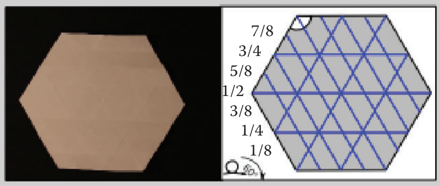
11. Rotate the paper 180° and repeat steps 9 and 10 on the other half of the paper. You have now detailed on axis.



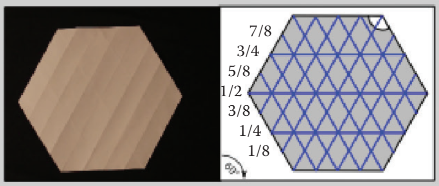
12. Flip the paper onto the valley side. On this side, every other crease is a mountain fold. One at a time, starting with the mountain fold closest to you, backcrease and unfold each mountain crease so that they become valley biased.



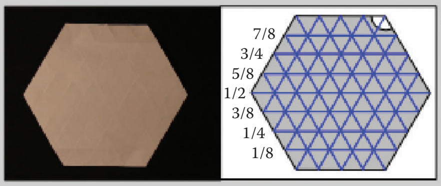
13. You have now backcreased this axis.



14. Return the paper to the mountain fold side and rotate the paper so the axis you just worked on is not parallel to you. Repeat steps 9–13 on this axis.



15. Now you have subdivided and backcreased two axes. Rotate 60° and repeat steps 9–13 with the final axis.



16. You have now made an 8ths grid. Practice this extensively, as this is vital to being able to experiment with twists.

- It is acceptable to have some of the grid lines not intersect at a point for the moment. Practice and you will become more accurate over time.

- Can you easily identify the mountain side and the valley side of the paper? If there is one or more stray crease bias on the wrong side of the paper, backcrease it.

- This method uses binary subdivision to create more complex grids. The next steps will be 16ths, then 32nds, etc. Can you figure out how to divide a hexagon into thirds instead of halves? This can be a base to folding it into 6ths, 12ths, 24ths, etc. Keep in mind the possibilities as you go through this book.

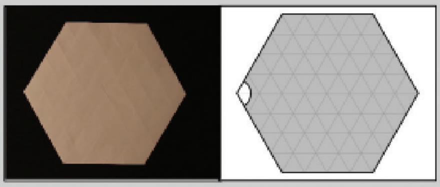
1.7 Basic Pleat Design

Many tessellations, including the ones illustrated in this book, are composed of *pleats*, which are overlaps on an otherwise flat material. Pleats are commonly found in clothing and window curtains. If a portion of the material overlaps another portion, it has a pleat.

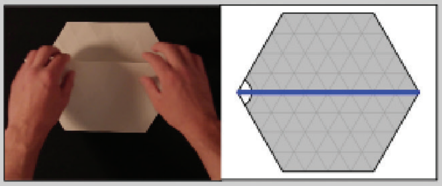
Let us examine this broad definition of a pleat. If there is an overlap, there must be creases or bends in the plane. In the case of the paper, these are often hard creases, and those creases are often straight lines, rather than curves. Since nonparallel lines will eventually intersect, it is more common to use parallel pleats, which are easier to control and study mathematically. A *parallel pleat* is a pleat where all of the creases are parallel. A parallel pleat must be composed of an equal number of mountain and valley folds and travel indefinitely in each direction, only stopping when it crosses another feature, such as another pleat or the edge of the material. Let's fold a pleat through the middle of the grid that you made in Section 1.6.

How to Fold an MV Pleat

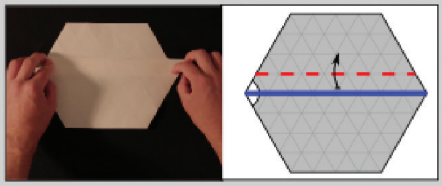
Depending on orientation, this is also called a VM pleat. This is the simplest pleat you will learn and is involved in each of the six simple twists. The M stands for mountain fold, and the V stands for valley fold. Whichever is closer to the viewer is the first letter, so the naming depends on orientation. Many of the twists you will start with will be composed of VM or MV pleats. Understanding these pleats and how to manipulate them will go a long way toward understanding how they interact with other elements on the paper.



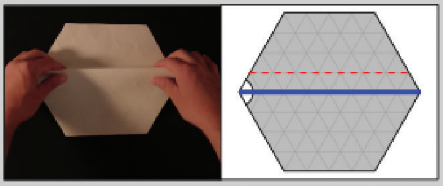
1. Begin with a hexagonal sheet of paper, gridded to 8ths.



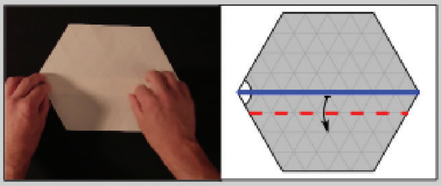
2. Pinch the line that you wish to be the mountain fold of the pleat. In this case, we will use the center line.



3. Push the pinched crease away from you so that it lays flat. The valley fold that results from it should be one grid line away from the mountain fold. This is an MV pleat.



4. Now, we will change the orientation of the pleat. Lift the center line from the table again.



5. Pull the mountain fold toward you, and the paper will form a valley fold next to it. You've changed the orientation of the pleat from MV to VM.

The pleat pattern method revolves around how to make pleats in the paper to create a limitless number of designs. Master pleat manipulation, and the twists will come more easily.

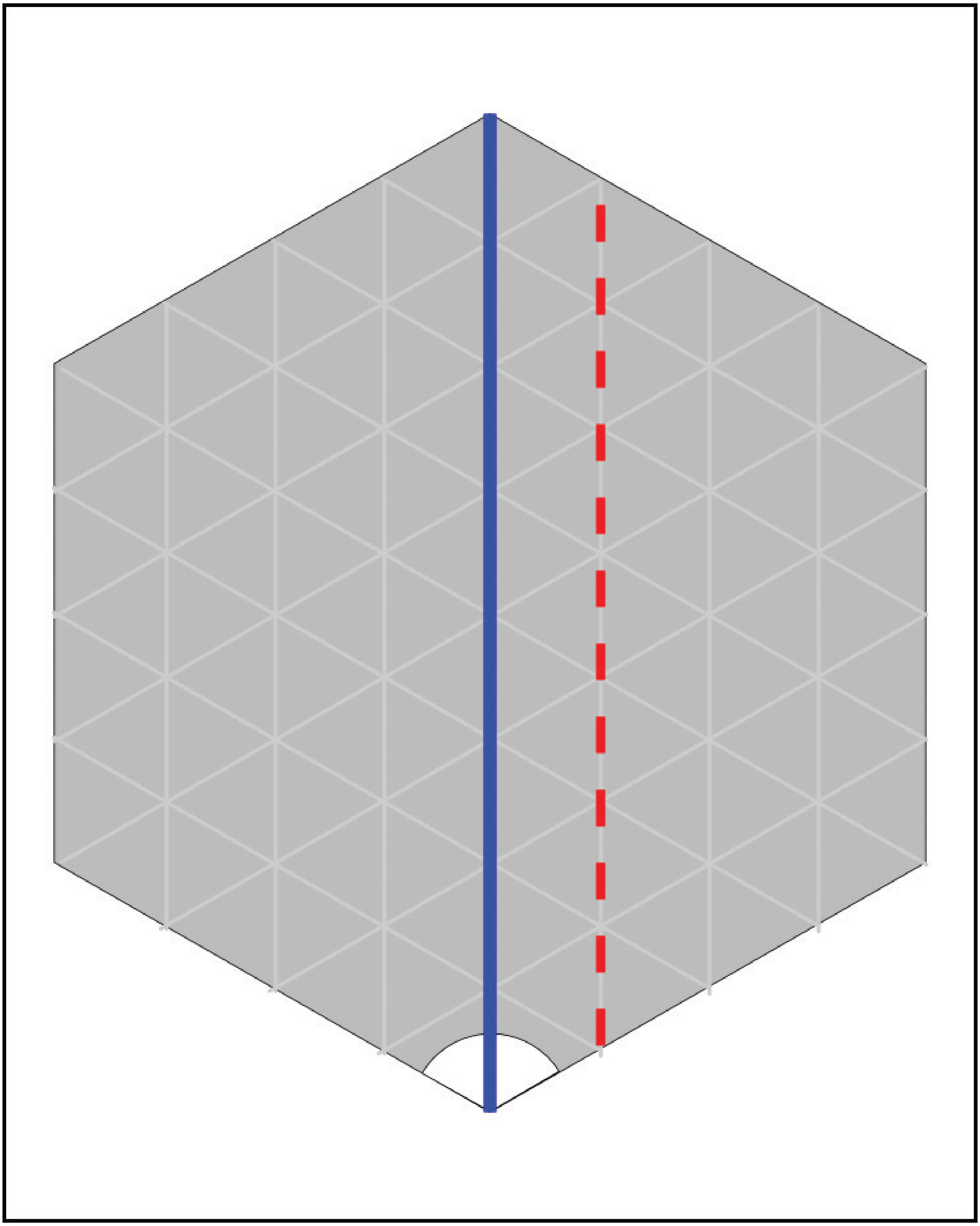
- The way the pleat moves across the paper is called the *direction of travel*. In twists, pleats will travel across the paper and cross other pleats, and they interact. In the exercise you just completed, the direction of our pleat was from one corner of the hexagon to the other.

- The way the pleat is flattened is called the *orientation* of the pleat. In step 3, it was oriented away from the folder. In step 4, it was oriented off the table. In step 5, it was oriented toward the folder. The factor here is in which direction the mountain fold pinch is brought.

- Can you neutralize the crease biases so that we are back to a relatively neutral bias grid? If you can, you will have an easier time refolding this paper for another exercise.

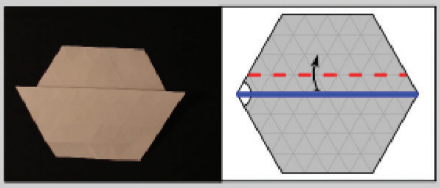
- Can you fold a pleat where the mountain fold is one grid line off of the center line?

- What happens when you fold an MV pleat, unfold it, and then fold another MV pleat along a different diagonal? Then, What happens when you try to refold them both at the same time?

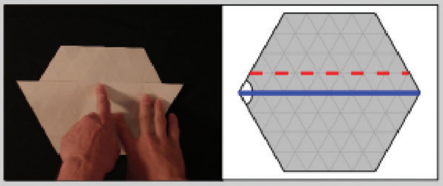


How to Split a Pleat

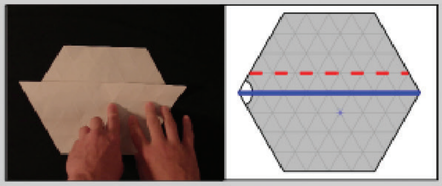
A single pleat is not so interesting to study. Things get interesting when you start to look into collisions of pleats, called *pleat intersections*. Pleats travel from one direction to the other, and when they intersect another element, they interact with it. Folders can force this by splitting a pleat at a certain point, which is a skill that this exercise will teach.



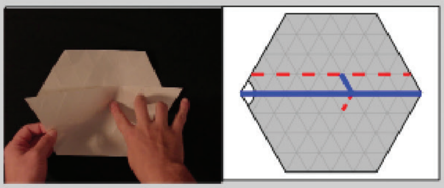
1. Begin with an MV pleat with the mountain fold along the center line.



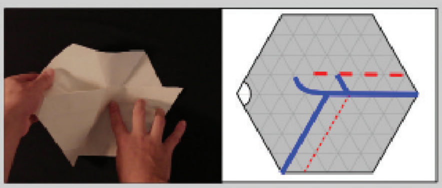
2. Identify the center of the paper in the folded form with your finger.



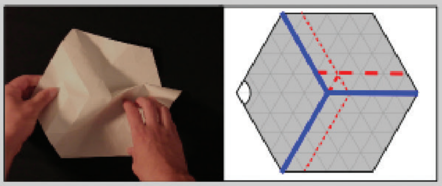
3. Now move your finger toward yourself and to the right one triangle. This will be the point where you split the pleat. Mark this point.



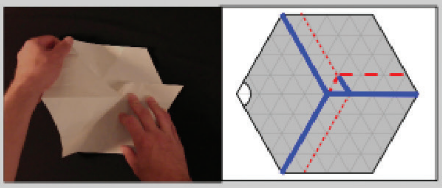
4. Lift the left side of the paper from the table, and keep the right side flat.



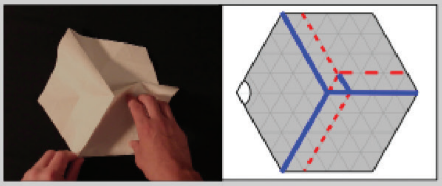
5. On the underside of the left area of the paper, use your fingers to open the pleat.



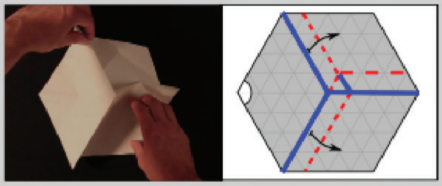
6. The left side is opened now.



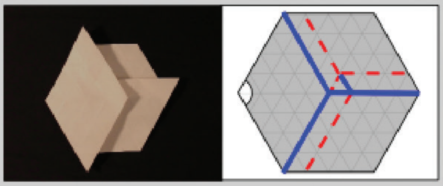
7. Sculpt the grid line running from the top left to the center so that it is a mountain fold.



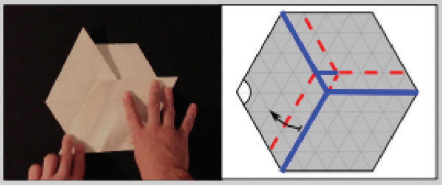
8. Do the same with the grid line that runs from the lower-left corner to the middle.



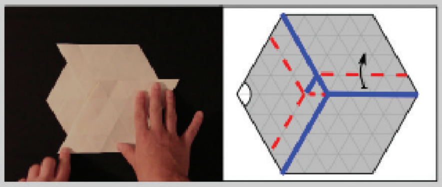
9. Flatten the mountain folds.



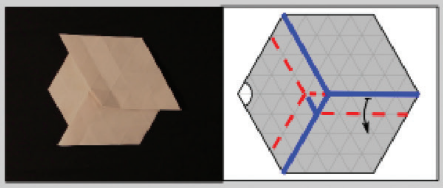
10. You have now split the pleat and have created a pleat interaction. Note that this is not a twist (we will get to that in the next section). Play with the orientation of the pleats and see what combinations you can get.



Reorienting the pleats



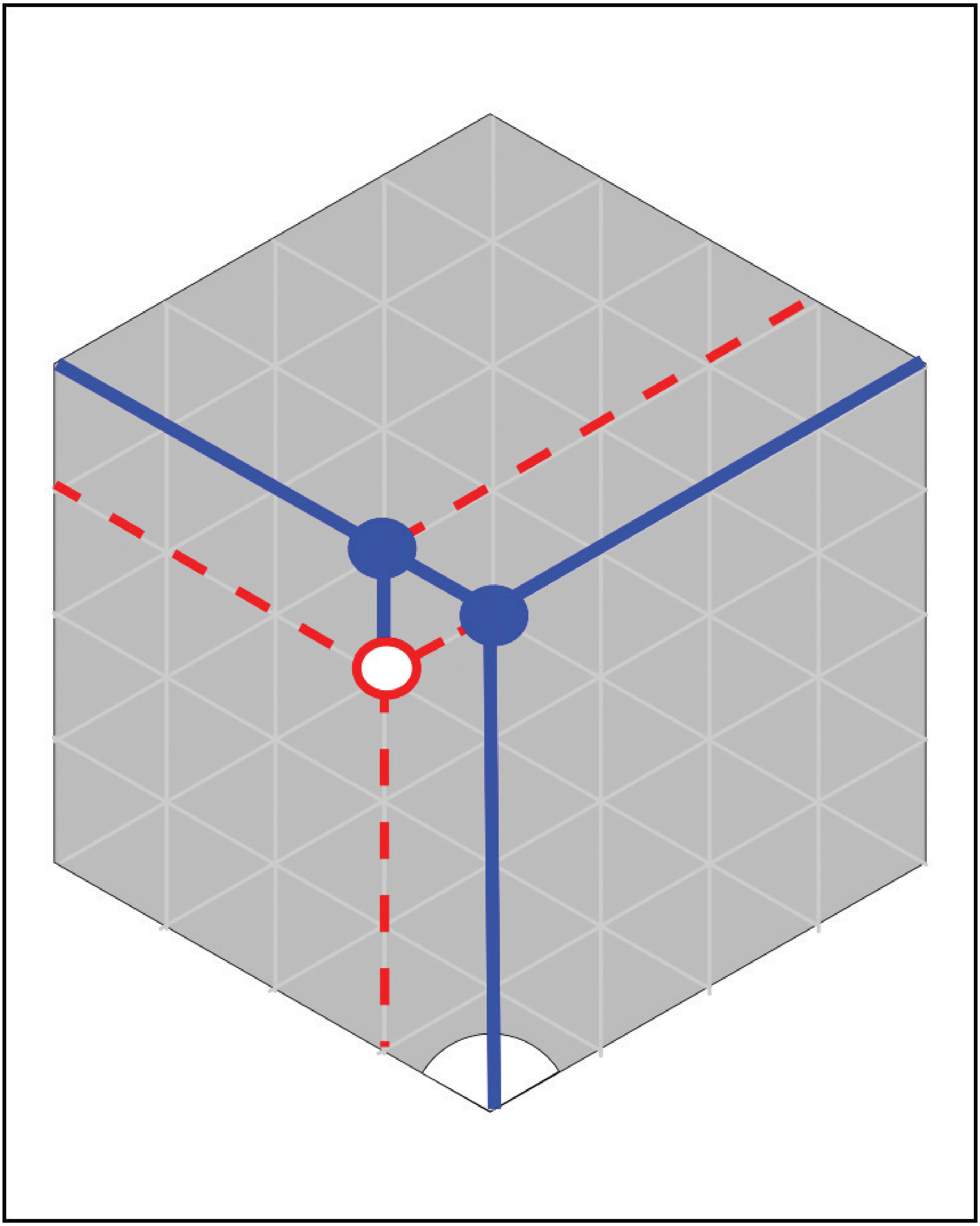
Reorienting the pleats



Reorienting the pleats

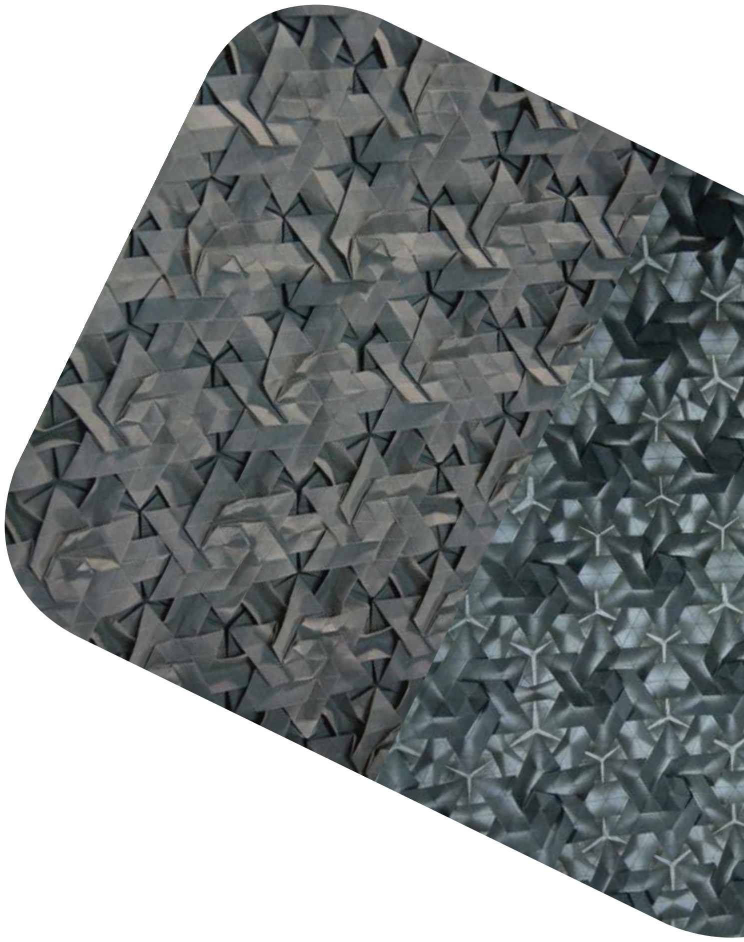
- What happens when you split at a different point than the center?

- After the split, can you figure out how to split one of the resultant pleats again? How many splits can you fit on an 8ths grid?



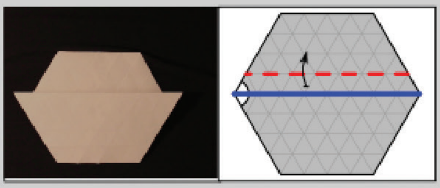
1.8 Diagrams of the Six Simple Twists

This section explains how to fold the six simple twists. The name is somewhat deceptive, as most of these twists involve complex maneuvers. There are often several moving parts coming together simultaneously. However, they are the core upon which this book is based, so they must be learned if they are to be applied to full-scale tessellations.

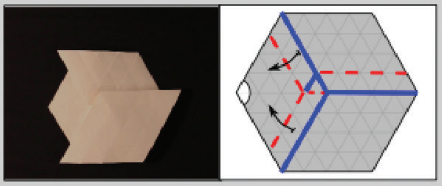


Triangle Twist

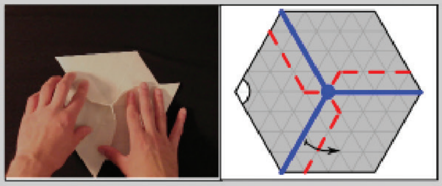
The triangle twist is an extension of pleat splitting. It is composed of three intersecting pleats and is the most basic twist to learn using a triangle grid.



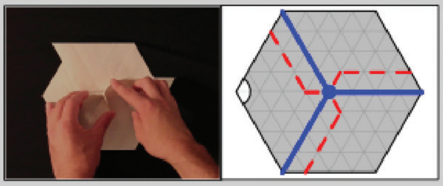
1. Begin with an MV pleat on the center line of a hexagon gridded to 8ths.



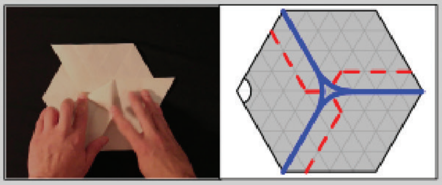
2. Split the pleat in the center. Make a note of which pleats are oriented clockwise around the center and which ones are oriented counterclockwise around the center.



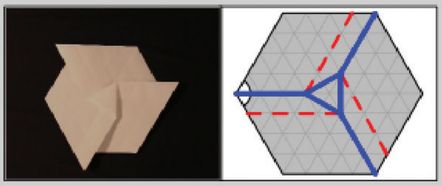
3. Reorient the pleats so that they all go counterclockwise around the center of the hexagon. The center will rise from the table forming a peak.



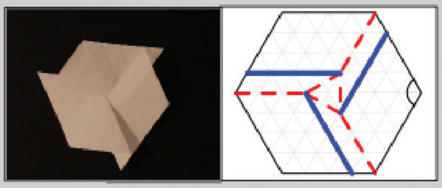
4. Push counterclockwise on all three points of the base of the peak.



5. As you press, the center will begin to spread apart, forming new creases off of the grid lines.



6. This is the finished triangle twist.

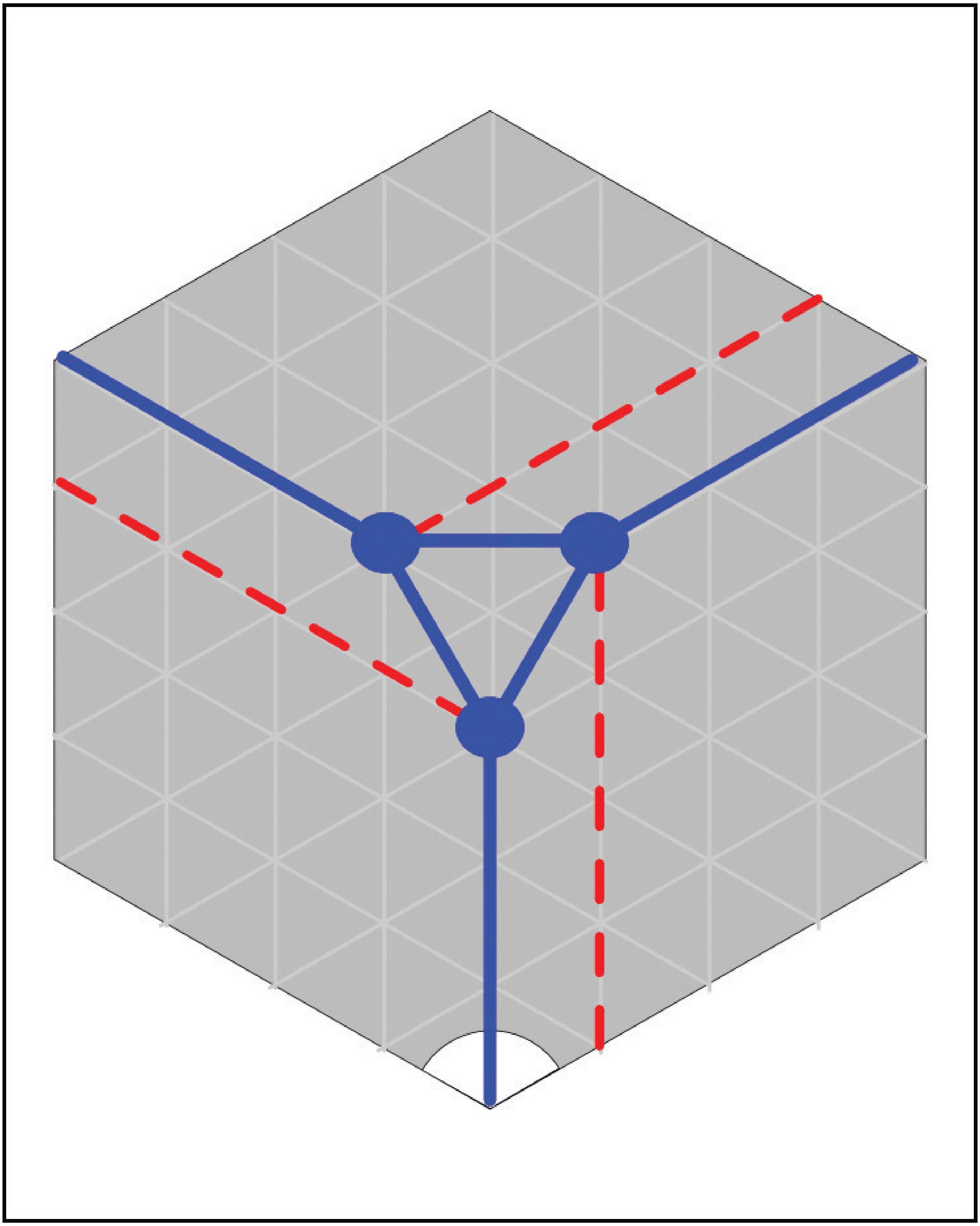


Reverse

- If the center is not a perfect triangle, there are generally two reasons for it: one or more of the pleats were a different width from the others, or one of more of the pleats were nonparallel. Make sure the valley folds of the pleats are on the grid lines.

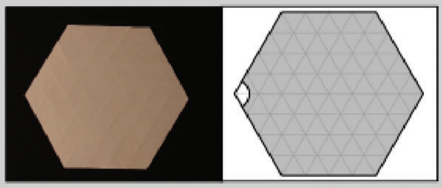
- Can you fold the twist going clockwise instead of counterclockwise?

- If you had split the pleat at a different point than the center in step 2 and folded the twist from there, how would it differ from the result we got?

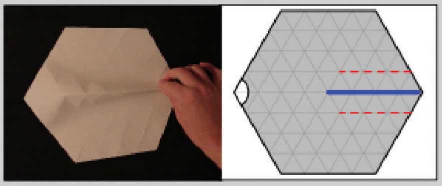


Triangle Spread Twist

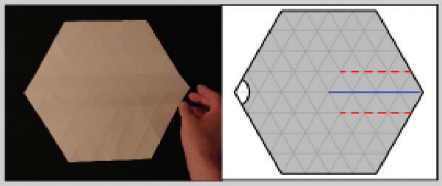
In the triangle twist, there are three intersecting pleats, and the mountain folds of the pleats intersect at the same point, the center of the paper. Pleats can be offset from the intersection, creating new twists, generally called spread twists, as in the triangle spread. Two of the three pleats will hit the corner of the hexagon, whereas one will be off by one grid line.



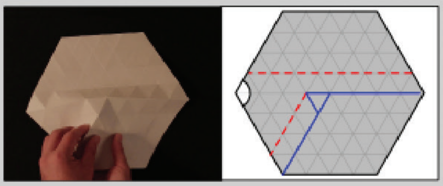
1. Begin with a hexagonal sheet of paper, gridded to 8ths.



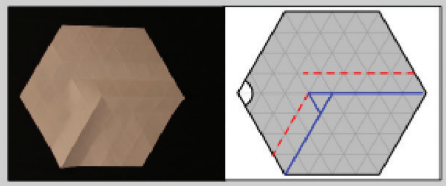
2. Pinch the right side of the center line from the corner into the middle. Do not crease the entire grid line.



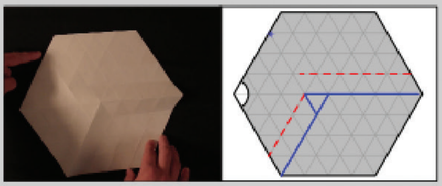
3. When you release, there should be a memory of the pinch embedded into the paper. You have just increased the mountain bias of that section of the center line.



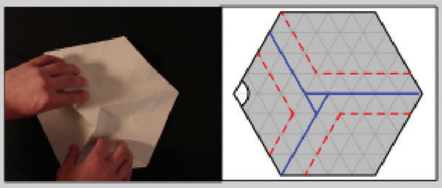
4. Next, make the same pinch emanating from the lower-left corner toward the center. Again, do not pinch the entire length of the grid line.



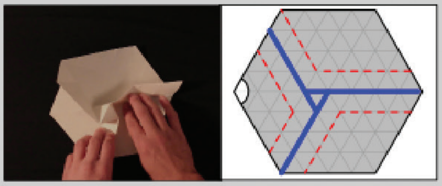
5. Release this pinch as well. If you start to have difficulty seeing the creases, draw the mountain folds of the pinches.



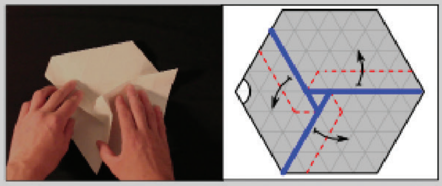
6. Identify the top-left corner, and then go one grid line left of that. Mark the line on the left edge of the paper.



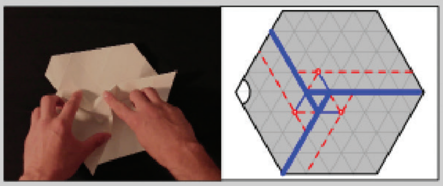
7. Make the same pinching formation you made in steps 3 and 4 toward the center, but not the entire grid line.



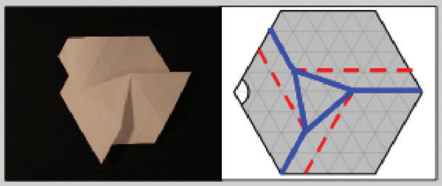
8. Refold all three of the pinches, and you will find the paper clusters around and raises one of the triangles in the center of the paper.



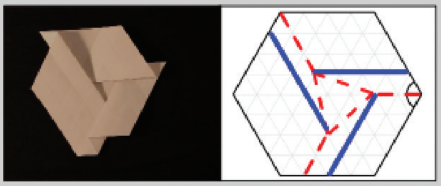
9. Fold each of the creating pleats so they lay flat.



10. As you flatten the pleats more, the center will spread apart, forming new creases off of the grid lines. Push on the corners of the center triangle to guide it into place.

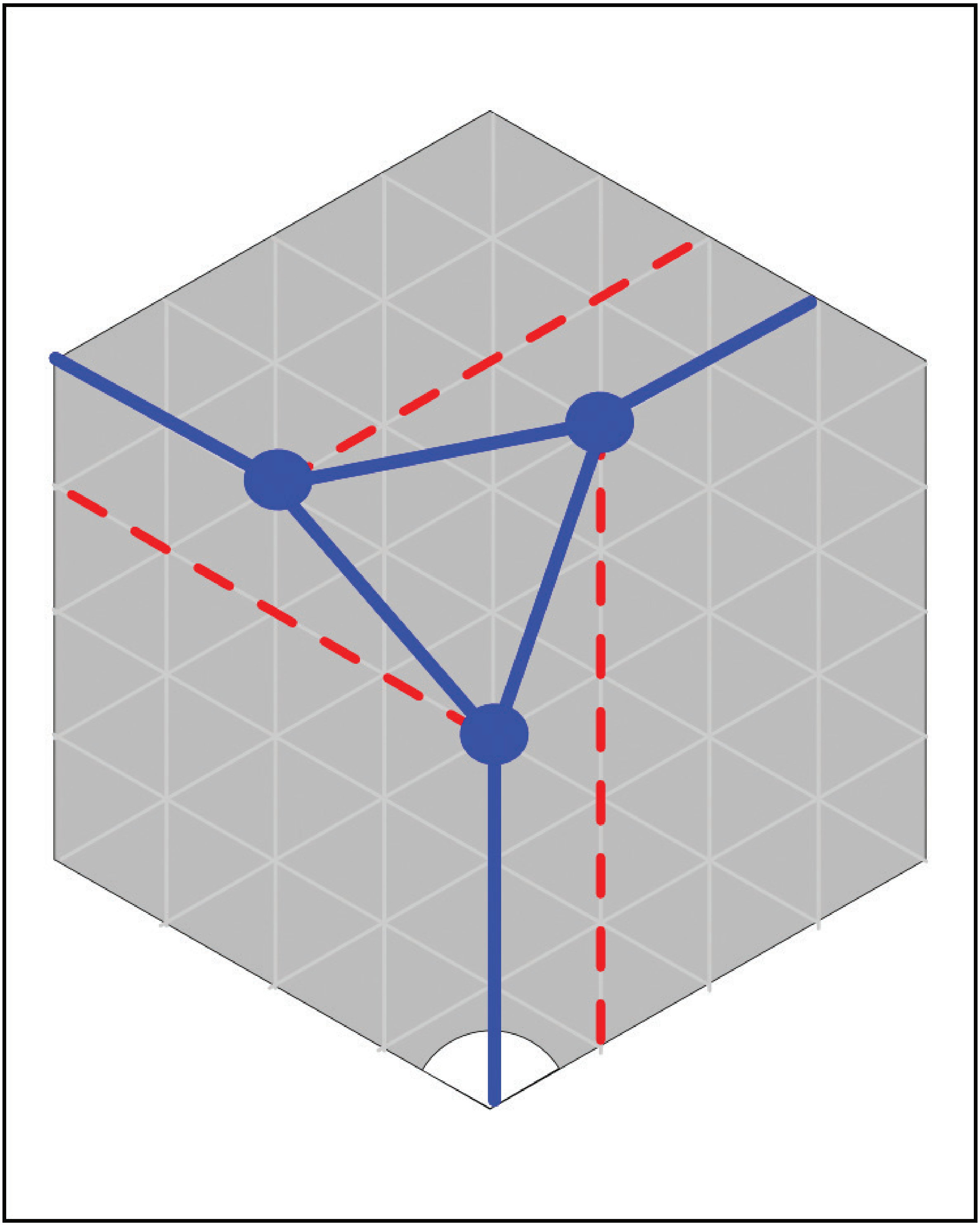


11. This shows the finished triangle spread twist.



Reverse

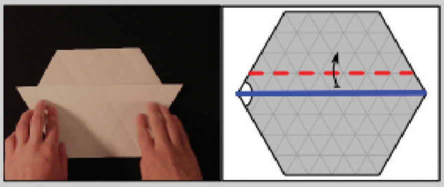
- Can you fold the twist rotating clockwise instead of counterclockwise? This may not be as easy as with the triangle twist. (Hint: the pleats in place in this diagram will not work if you wish to reverse the direction of the twist.)



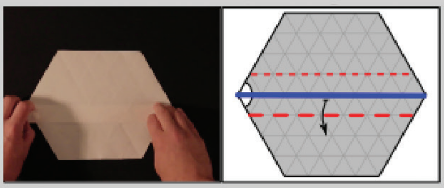
Hex Twist

A triangle twist is the composition of three intersecting pleats. As an extension, a hex twist is the composition of six intersecting pleats. It serves many of the same tiling functions as the triangle twist, which we will see in Chapter 2, and is one of the most useful twists to know how to fold.

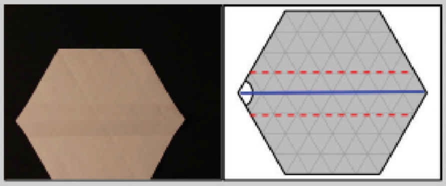
Unfortunately, I consider it to be somewhat harder to teach than the other simple twists. This is largely due to the degree of rotation the center hex has to make. If you have difficulty with this, look to the last step's crease pattern and follow it carefully ... and practice!



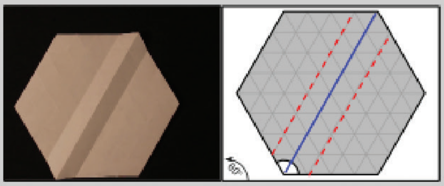
1. Start with the gridded hexagon and a pleat through the center line.



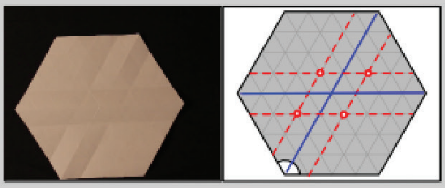
2. Change the orientation of the pleat.



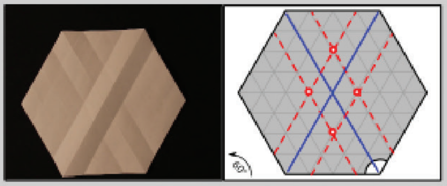
3. Unfold. There should be significant bias in the folds. This is the first of three pleats that we will make. First, we need to prepare the pleats.



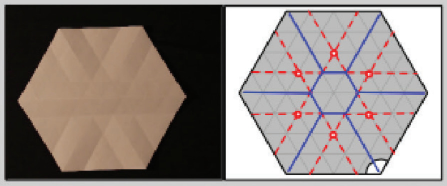
4. Turn counterclockwise 60°. Now, we're going to work on the next pleat.



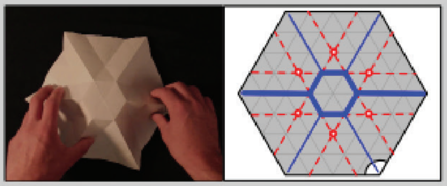
5. Repeat steps 1–3 on this diagonal. If it helps, flatten the paper entirely before starting this step.



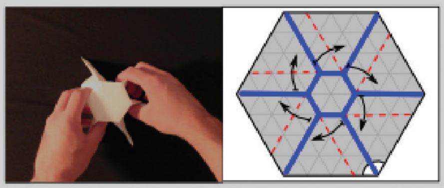
6. Turn the paper counterclockwise another 60°.



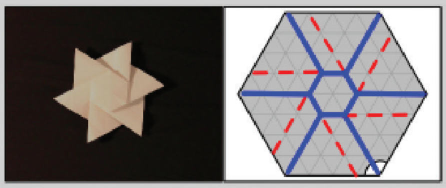
7. Repeat steps 1–3 with the third pleat. There should be a hexagon shape in the middle pushing toward you. If it does not push toward you, push from the back of the paper so that all of the biases around the outline of the hexagon are mountains.



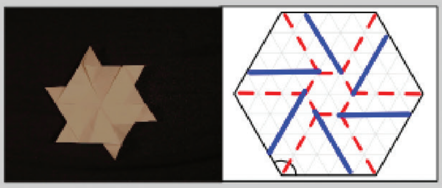
8. Now we will work on getting the pleats folded. Pinch opposite pleats and walk your hands into the middle.



9. When you can't left the hexagon any farther off of the table, twist the center hexagon clockwise. This motion will set the pleats.



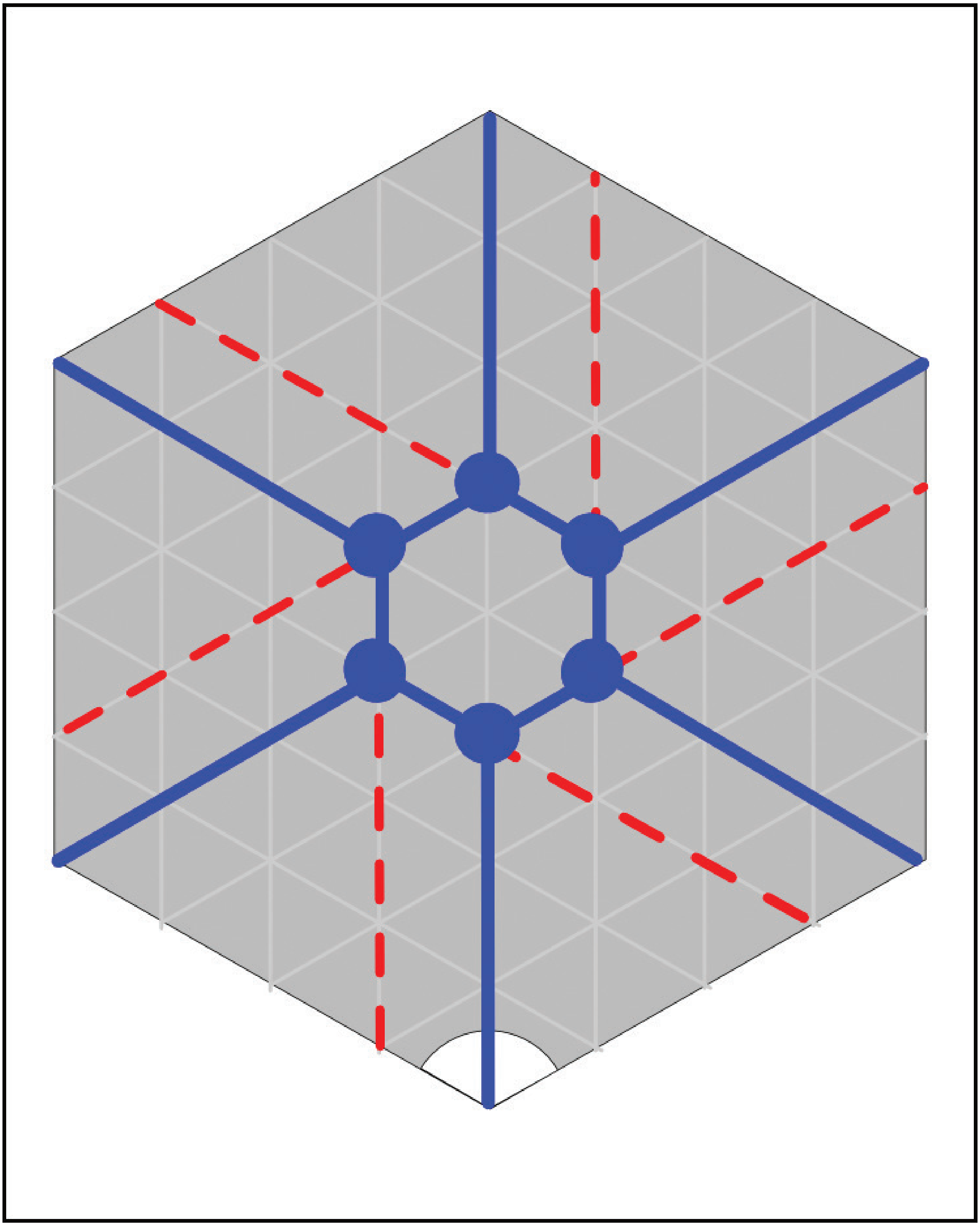
10. This is the result when the entire hexagon is flattened. Step 9 is a complex maneuver and should be practiced thoroughly.



Reverse

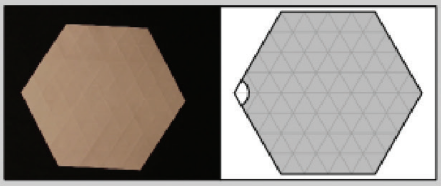
- Can you fold the twist going counter-clockwise instead of clockwise? This is doable without changing where the pleats are.

- What happens when you change the orientation of one pleat but not the others?

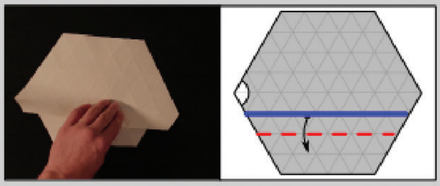


Hex Spread Twist

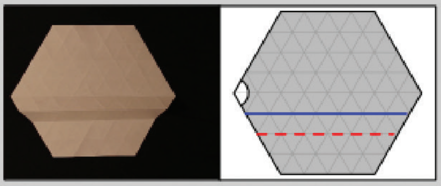
Like the triangle spread, the hex spread is a composition of six pleats that do not intersect at a point. This in combination with the triangle spread creates a very intricate weave pattern.



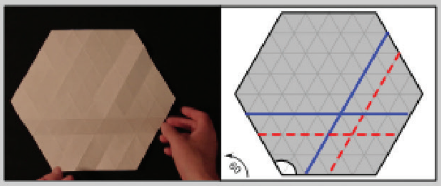
1. Begin with a hexagonal sheet of paper, gridded to 8ths.



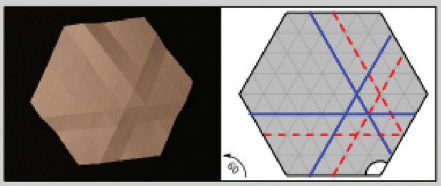
2. Create a VM pleat with the mountain fold one grid line away from the center line oriented toward you.



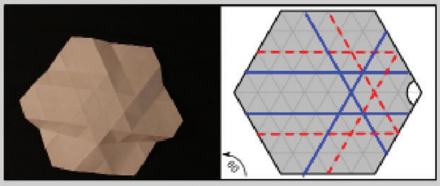
3. Unfold. There will still be the memory of step 2 in the paper.



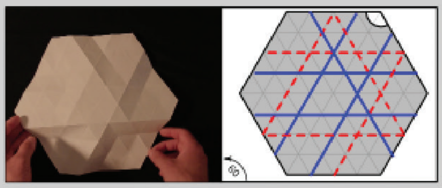
4. Turn the paper 60° counterclockwise and repeat with the second pleat, again one grid line removed from the center line. Then unfold.



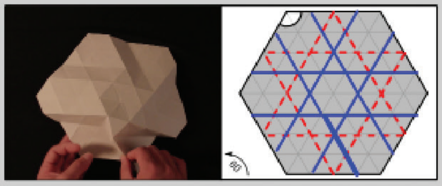
5. Turn again, fold the third pleat, and unfold.



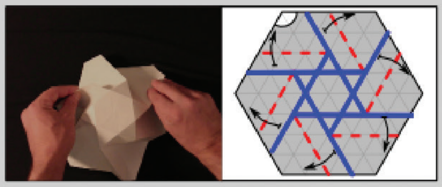
6. Continue to turn and fold and until all six pleats are folded and unfolded. This image shows four of the six pleats complete.



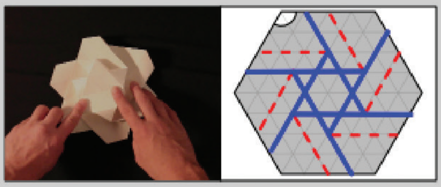
7. This shows five of the six pleats complete.



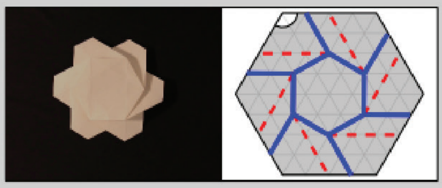
8. This shows all six pleats finished. Choose a pleat, and pinch it from the corner until it hits another mountain-biased crease.



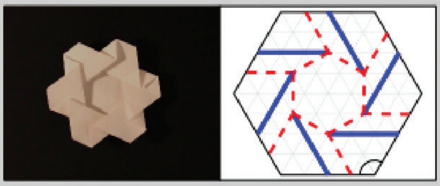
9. Repeat with all six pleats. You can do them all at the same time if you feel adventurous. As you pinch more pleats, start to lay them flat in a clockwise orientation.



10. When you are finished, there will be a star puff in the center.



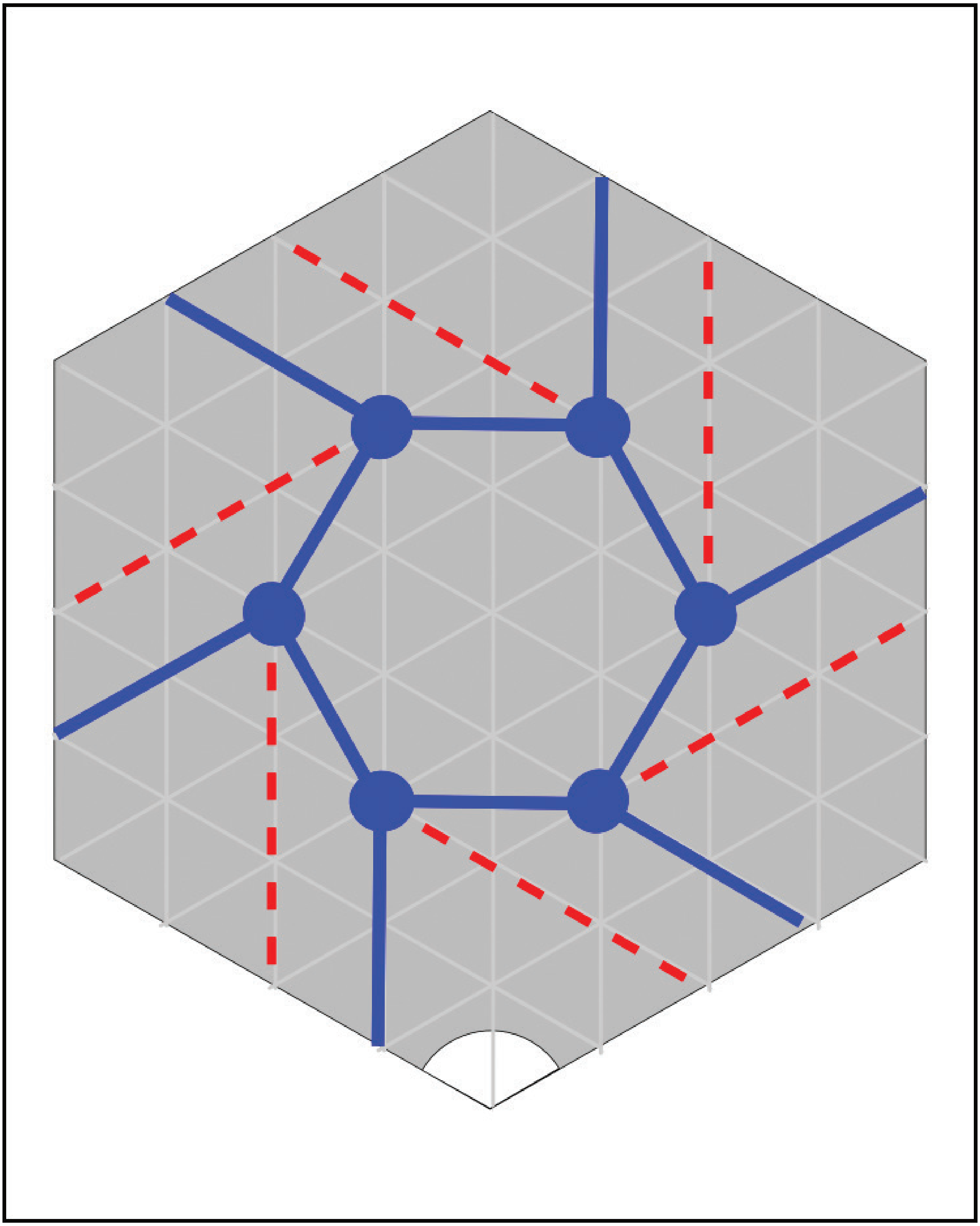
11. Push down on the interior corners of the star, and the hexagon will flatten out, creating new creases and a hex spread twist.



Reverse

- Can you fold the twist going counter-clockwise instead of clockwise? You will have to change where the pleats lie on the paper.

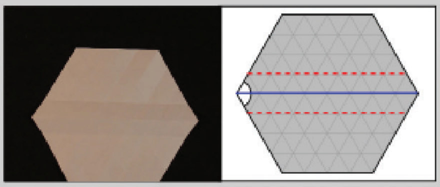
- What happens when you change the orientation of one pleat, but not the others? What about when you change all six?



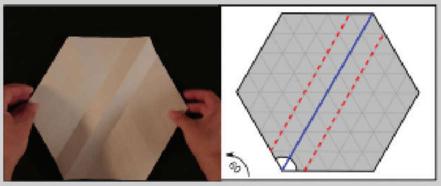
Rhombic Twist

Up to this point, we've learned two twists with three-pleat intersections and two with six-pleat intersections. These have all been equiangular collisions, meaning each pleat is either 120° or 60° from its neighbor.

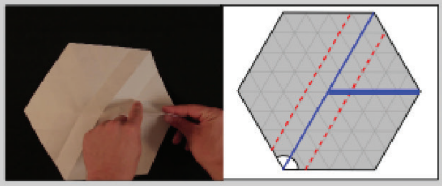
As you study tessellations, you will find pleats colliding at many different angles, not always radially symmetric. The rhombic twist is one such intersection.



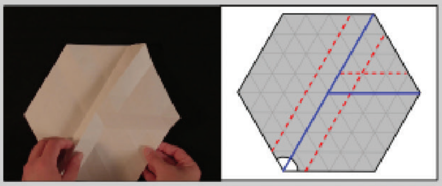
1. Start with a hexagonal sheet gridded to 8ths and place a pleat through the middle. Flatten the pleat in both orientations, and then unfold.



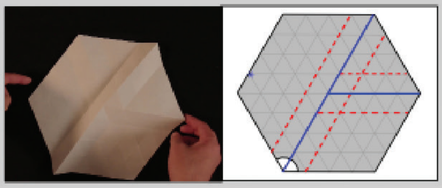
2. Turn the paper counterclockwise 60°.



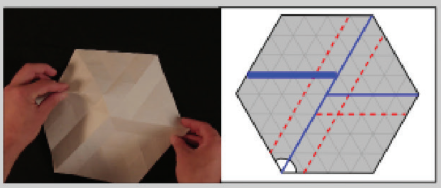
3. Pinch the diagonal from the right corner toward the center.



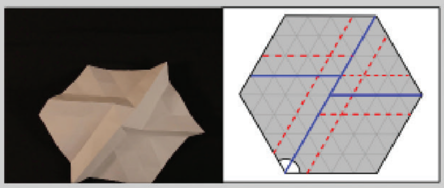
4. Unfold. Notice that the mountain fold of the second pleat intersects the first pleat at the center of the hexagon.



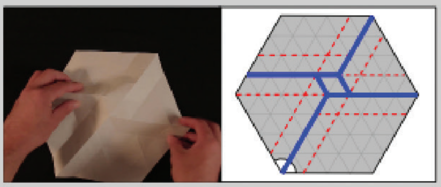
5. Find the grid line one away from the diagonal line away from you. Mark that point.



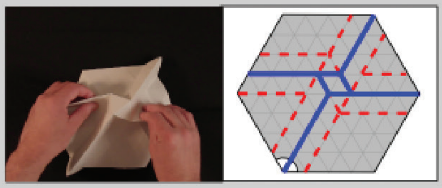
6. Pinch that grid line from the left edge to where it intersects the first pleat.



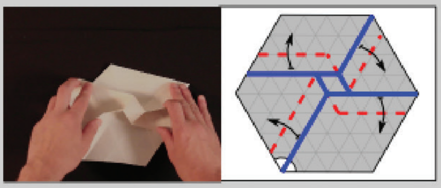
7. When you release, you should see a strong bias of the four pleats and a rhombus plateau in the middle of them.



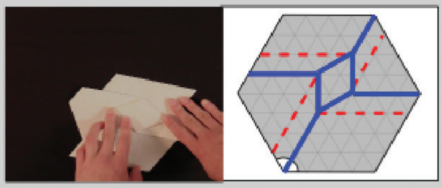
8. Pinch the pleats formed in steps 4 and 6 simultaneously.



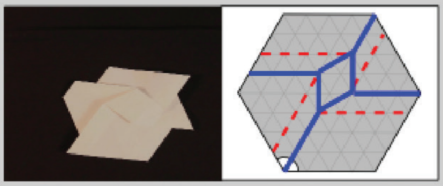
9. Push inward, and the rhombus plateau will rise off of the table.



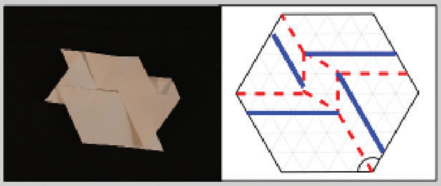
10. Lay the pleats flat in a clockwise orientation.



11. Flatten the rhombus. This will create new creases and a larger rhombus around it.

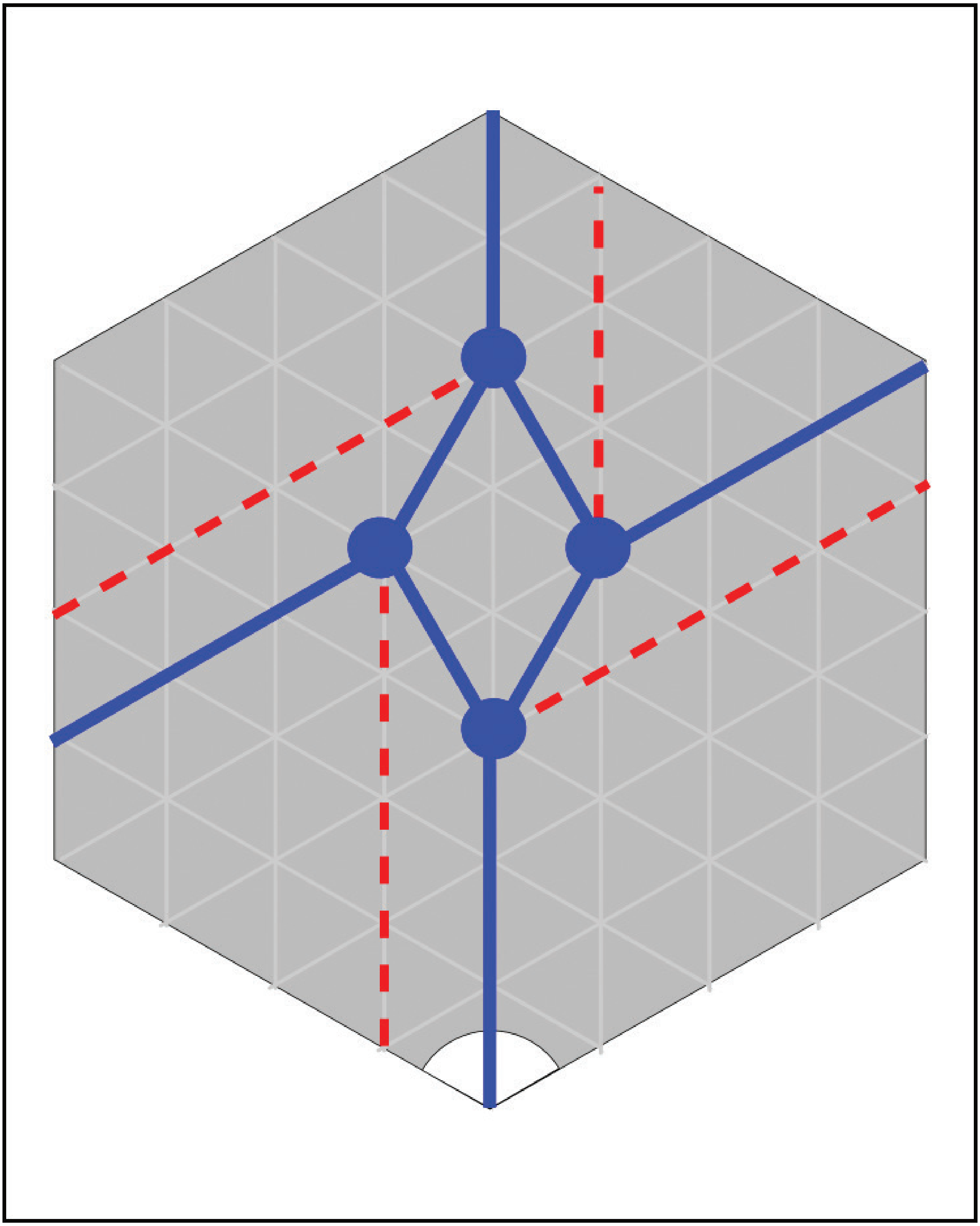


12. This is the finished rhombic twist.



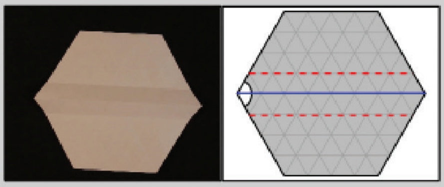
Reverse

- What happens when you change the orientation of one or more of the pleats?
- Can you change the direction of the twist? You will have to change where the pleats are on the paper.

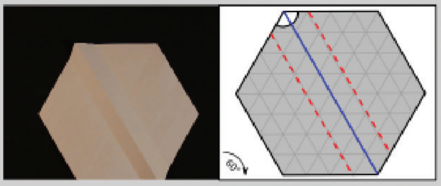


Arrow Twist

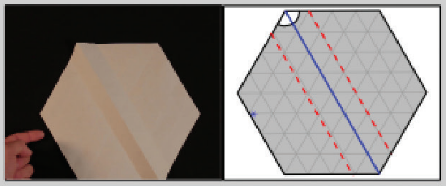
Other artists may have different names for this twist, but this is my name for it. The surface looks almost the same as the rhombic twist, but the pleat formation is different. This is the first twist that combines different pleats. Instead of four same-width pleats, this has three pleats of one triangle width and one of two triangle width.



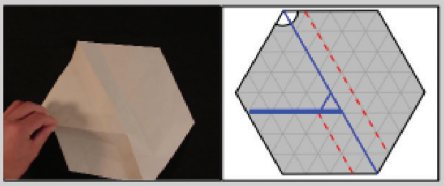
1. Start with a hexagonal sheet gridded to 8ths, and place a pleat through the middle. Flatten in both orientations and then unfold.



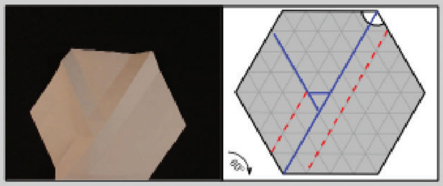
2. Rotate clockwise 60°.



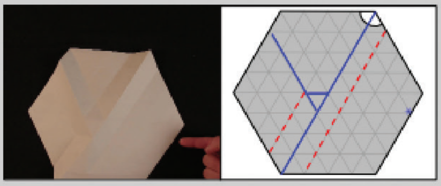
3. Identify the grid line one off the diagonal toward you on the left edge, and mark it.



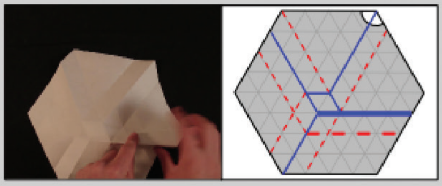
4. Pinch that grid line from the edge to where it intersects with the mountain fold of the first pleat. You will see a triangular plateau in the paper. Unfold.



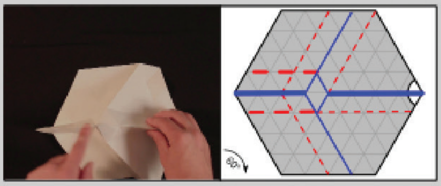
5. Rotate clockwise 60°.



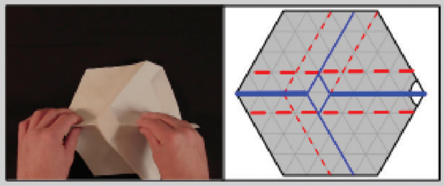
6. As with step 3, identify the grid line one off of the diagonal toward you on the right edge, and mark it.



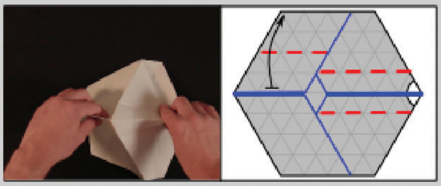
7. Pinch that grid line from the edge to where it intersects the first pleat as in step 4. The plateau will now be a rhombus, rather than just a triangle.



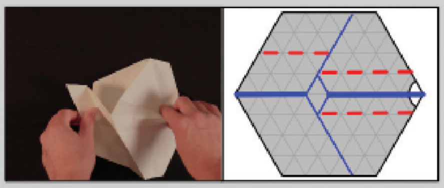
8. Rotate the paper 60° clockwise. Pinch the mountain folds of the pleats that run to the corners of the paper. The rhombus plateau will rise from the table.



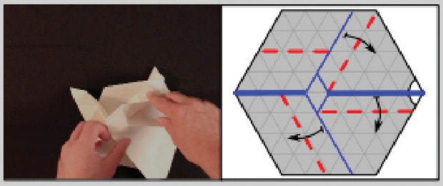
9. The pleats are not set up fully yet. The left pleat needs to be double the width it currently is.



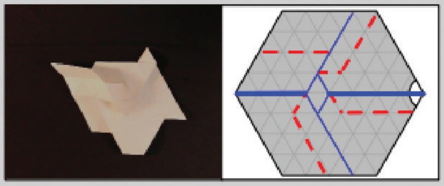
10. Bring the left pleat away from you and lay it flat. The mountain fold of the left pleat will line up with the farthest edge of the hexagon from you.



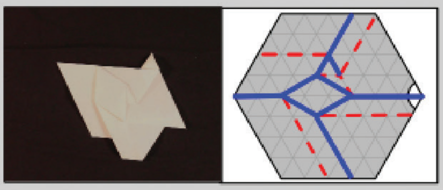
11. Now we can work on the other pleats.



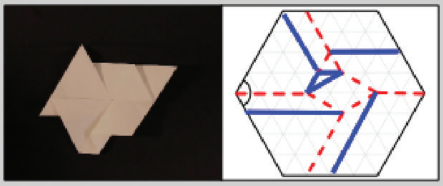
12. Fold the remaining pleats in a clockwise orientation. The rhombus plateau will twist with them.



13. This is what the twist looks like with the pleats flattened. Push down on the corners of the rhombus plateau, and you will get a larger rhombus.

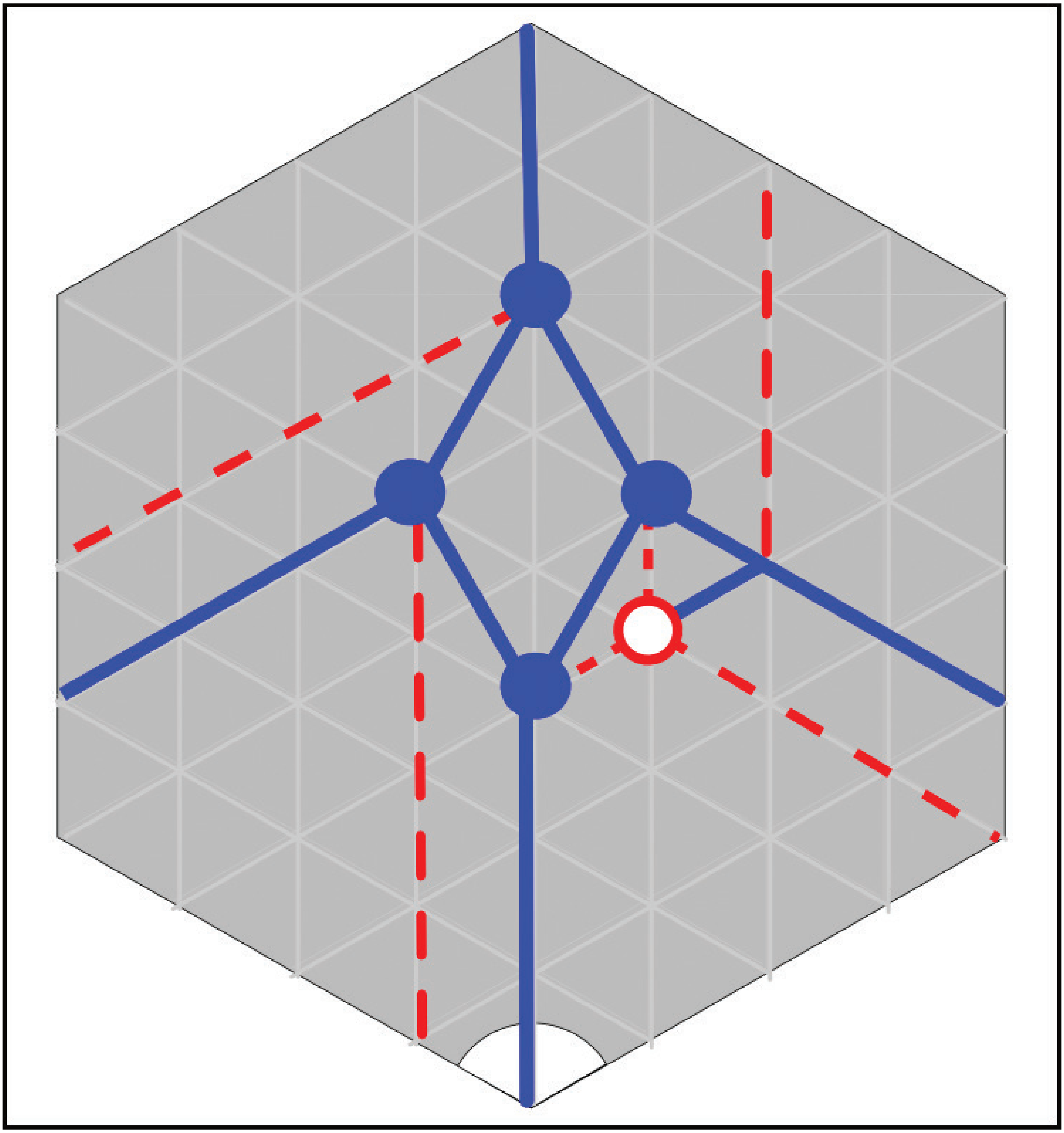


14. This shows the finished arrow twist.



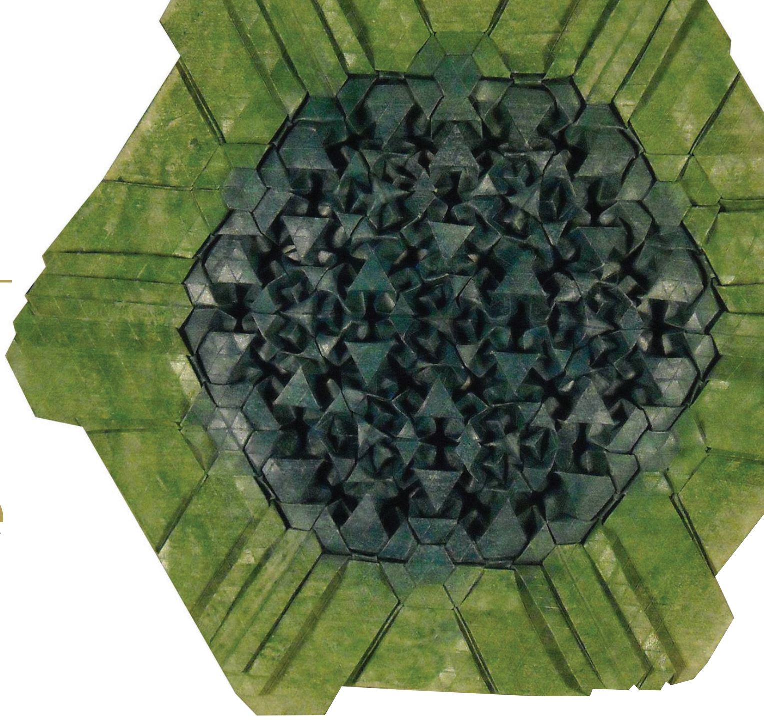
Reverse

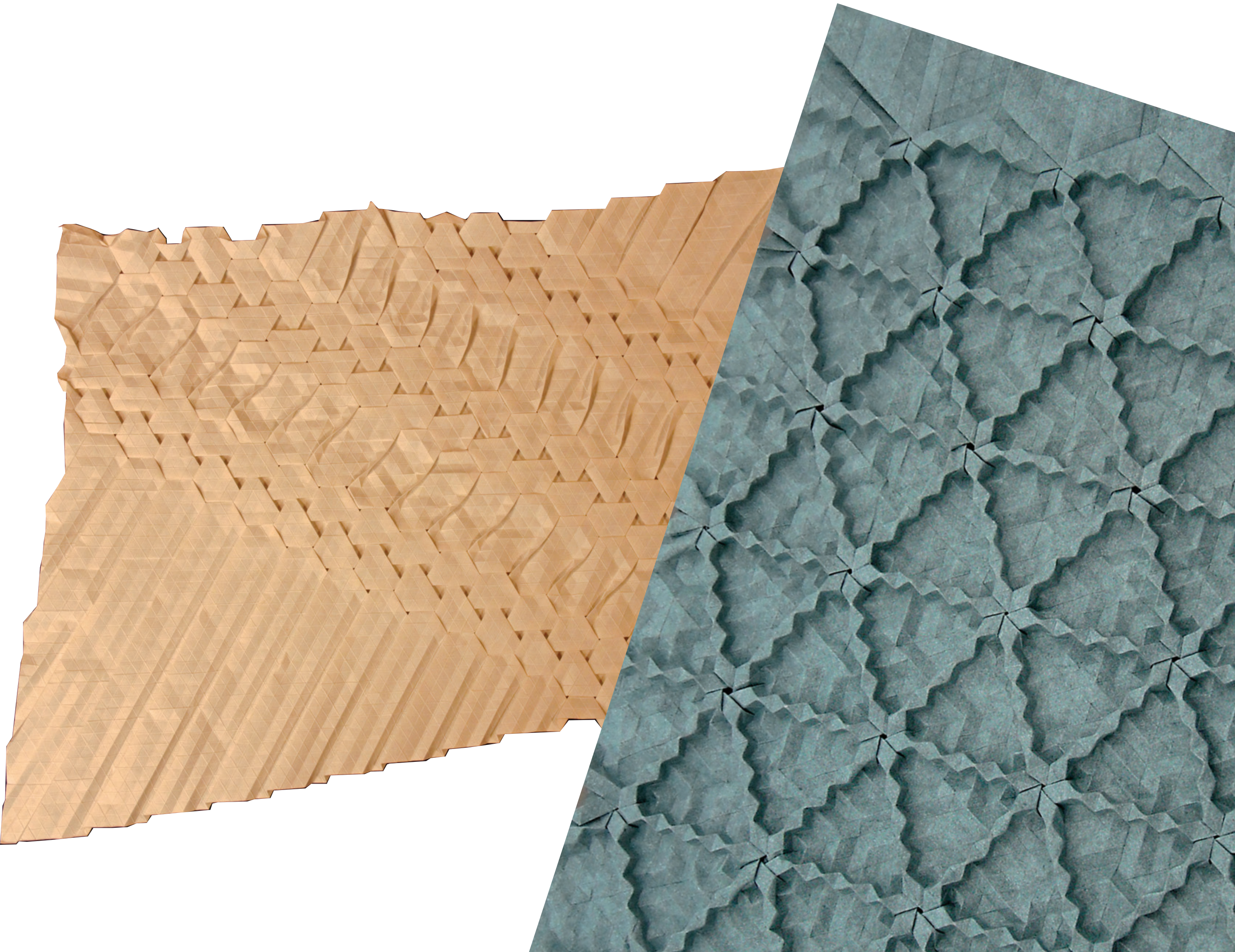
- You might have an easier time twisting the arrow counterclockwise than the rhombic twist. You do not have to change where the pleats are on the paper.



Chapter 2

How to Use the Six Simple Twists





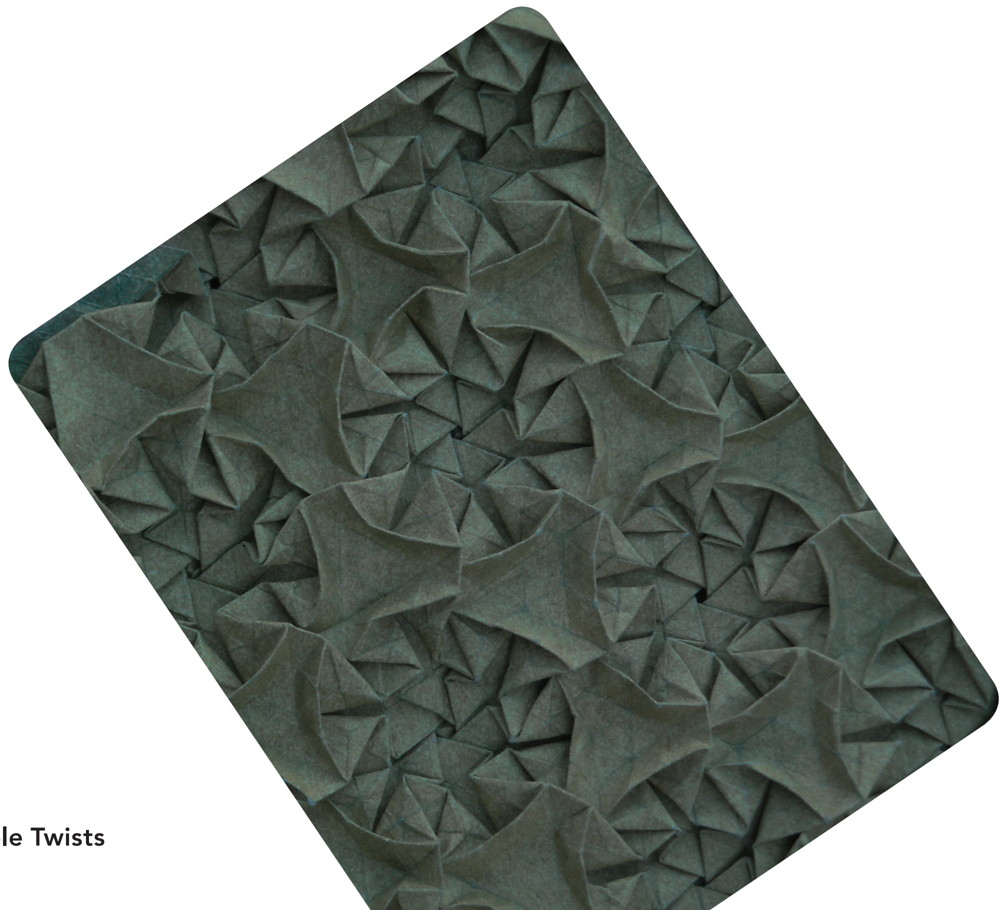
2.1 Intermediate Gridding

The purpose of this book is not to teach processes, but rather to convey understanding of design. For that, there are basic components that need to be mastered. This book presents those as the six simple twists. Once you have an understanding with which to continue, this book will help you use those tools to create amazingly beautiful patterns. There will be variants and progressions, abstraction, mathematics, and openings to create your own types of tessellations and implement them in other media. If you master the basics, the possibilities are vast.

Now that we are encountering more complex folding, we need a denser grid. The 8ths grid has enough area to use single twists with very little unfolded paper remaining. But if you want to tile the twists, this is insufficient. Each twist consumes a certain amount of the grid, and the more concentrated the grid, the greater number of twists it can accommodate.

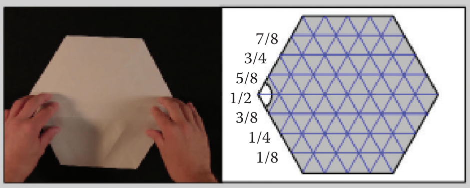
Origami is primarily a binary process: "fold in half," "bisect this angle," "repeat with both sides," are common steps in any type of folding. Gridding is binary as well. Unless you fold a grid and then remove some of the edges, you will fold the paper into $1/2^x$ parallel grid lines per axis for each of the three axes, which becomes 8ths, 16ths, 32nds, 64ths, and so forth. For now, we will detail the 8ths grid into 16ths and then 32nds, which will be sufficient to create many patterns, but you can use the process to make more complex grids when you wish to venture beyond the scope of these exercises.

Due to the rhythmic quality of gridding, I recommend that you enjoy music in the background while you grid. While listening to music, you will be surprised at how much progress you can achieve.

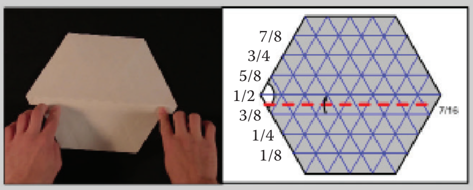


Intermediate Gridding

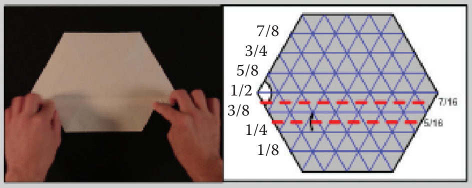
You already have the concept for gridding the 8ths diagram. To detail an 8ths into a 16ths grid, follow the same process of subdivision, backcreasing, and replication. In order to do this, it is often easier to create a stack of folds in an accordion fashion throughout a full axis from edge to opposite edge and then unfold in preparation for the backcreasing.



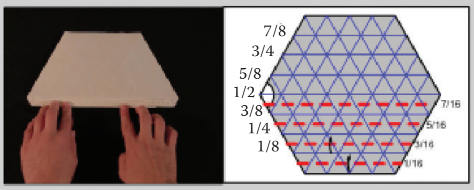
1. Begin with a hexagon gridded to 8ths with the mountain side up.



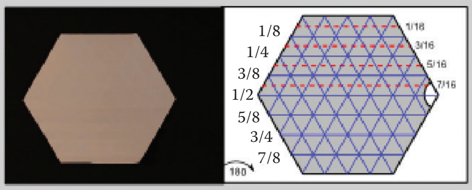
2. Locate the closest crease to the diagonal, and fold that into the center of the paper. Do not unfold.



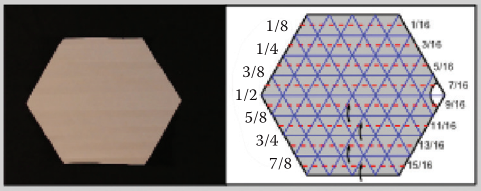
3. Locate the next mountain-biased crease closest to you, and fold that onto the previous fold.



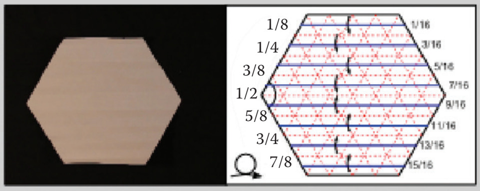
4. Two folds are on the stack now. Continue folding more layers until the entire side is folded like an accordion. Here you can see the full stack finished. Unfold.



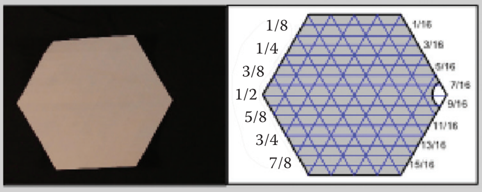
5. Rotate the paper 180°. We have now subdivided half of one axis.



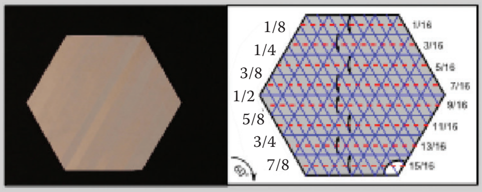
6. Repeat steps 2–4 on this side of the paper, and then unfold.



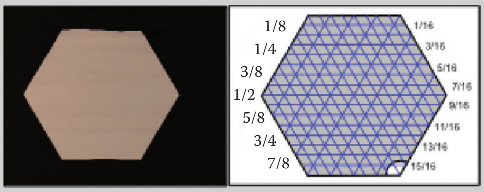
7. Flip the paper. You will notice that every other crease on this axis is a mountain fold, but you are on the valley side of the paper. Starting with the closest mountain crease to you working away from you, backcrease each mountain-biased crease one at a time, and then unfold until all of the crease biases are neutral.



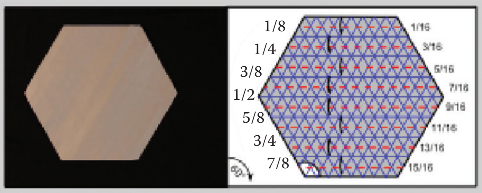
8. Flip the paper back to the mountain side. All of the creases should have uniform bias.



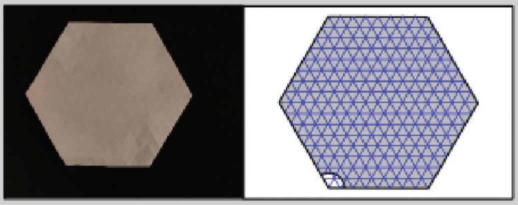
9. Rotate the paper 60° clockwise, and repeat the process of subdivision and backcreasing on this side.



10. This is the finished second axis.



11. Rotate 60° clockwise again, subdivide, and backcrease the third axis.



12. You have now finished a 16ths grid. This will help you achieve more complicated and interesting patterns.

2.2 Tessellation Basics, without Folding

There are many ways to tile polygons. Most tessellations we see in our lives are on wallpaper or floors, and they include either squares and octagons or triangles and hexagons. Occasionally, you see Penrose tilings, which involve pentagons and complementary shapes. The essential point is that you must use combinations of shapes whose interior angles total 360°. With a triangle grid, the most common shapes involve triangles, hexagons, and rhombi, with 60° and 120° angles throughout. When simply drawing shapes freehand, it is possible to tile the plane using only equilateral triangles and hexagons in three main ways (shown in Figures 2.1–2.3).

The first two are regular tiling: they only use a single polygon, the triangle (Figure 2.1) and the hexagon (Figure 2.2), respectively. The third is a semiregular tiling: there are two different shapes used (Figure 2.3). However, each vertex is identical: each one has a triangle, hexagon, triangle, and hexagon. When naming tessellations with regular polygons, the convention is to use the number of sides in the shapes around each vertex. So, the triangle tessellation is the 3.3.3.3.3.3. or 3^6, since there are six shapes around each vertex, all triangles (which have three sides). The honeycomb tiling is called a 6.6.6. or 6^3, and the triangles and hexagon tiling is called a 3.6.3.6. An irregular tessellation that appears often is the rhombile tiling, which is shown in Figure 2.4.

Notice that the rhombi in the last tiling are not regular shapes because they are equilateral but not equiangular. Also note that there are two different types of vertices: one with six 60° angles around it, and one with three 120° angles around it.

Knowledge of these four patterns helps with the understanding of how to tile twists.

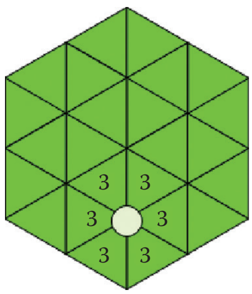


Figure 2.1

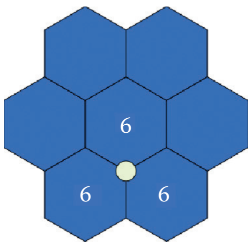


Figure 2.2

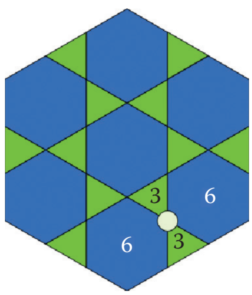


Figure 2.3

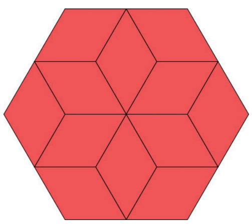


Figure 2.4

There are many more patterns (3.4.6.4., 4.4.4.4., and 3.3.3.4.4.). We will deal primarily with the 3^6, 6^3, 3.6.3.6., and the rhombile tessellations. For our purposes, most of the twists will be triangular or hexagonal, meaning they will be in clusters of six or three.

2.3 Applying Tessellation Knowledge to the Paper

The above section combined drawing shapes with matching the edges to create tilings. The pleats can be used to create tessellations by aligning with the lines that are drawn into the paper. This translates to two different possible methods of studying the patterns: viewing twists as the vertices and the fields between them as the corresponding polygons that tile, or viewing the twists as the polygons and the fields between them as the corresponding vertices. The pleat pattern method presented in this book focuses more on the twists and pleats than the fields between them.

However, an origami twist is not a polygon. It's the polygon and the lines that emanate from it. When designing, I am often thinking about how I wish to tile the paper during folding. Do I want three hexagons in a cluster, like a hexagonal tiling (6^3)? Or do I think this pattern is better suited by having a triangle in the middle of the hexagons, like a 3.6.3.6. tiling? Will the hex twist you started with in the center be surrounded by other hex twists, or triangle twists, or rhombic twists? How will this affect the pleats emanating from the center? How can you use the pleats again to make the subsequent twists? Practice helps, but a solid understanding of tessellations helps more.

2.4 Your First Tessellation

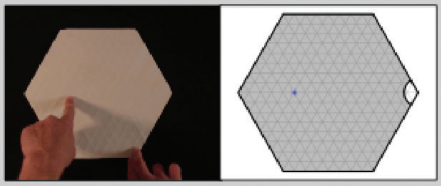
Now that we know how to make some twists and we understand some rules that govern a tiling, let's make a full tessellation. The simplest tiling is a cluster of six triangle twists. This tessellation employs the pleat-splitting technique extensively.

Triangle Twist Cluster

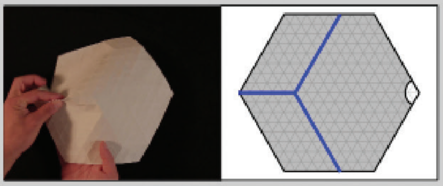
This could alternatively be called a 3.3.3.3.3.3. or 3^6 tessellation. The notation means that there will be clusters of six connected triangles. As your skills improve, you can get more iterations as you get denser grids. There are two primary skills that are useful in this diagram: *distance counting* and *pleat locking*.

When counting distance between triangle twists, I count from the center of the twist (where the three mountain folds intersect) to the center of its neighbor. We will have a distance of three between our twists.

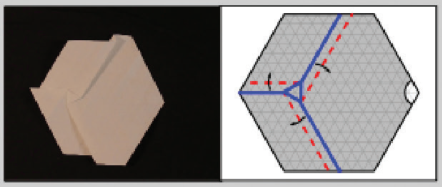
After the fourth twist in the cluster, two pleats will clash, and you will have to lock one pleat into another one. This technique can be useful when working on multiple clusters at once. The lock is easy to form and undo, and it holds the paper together while you are working on another part. Always remember, do not be afraid of unfolding the paper. Anything you unfold, you can refold.



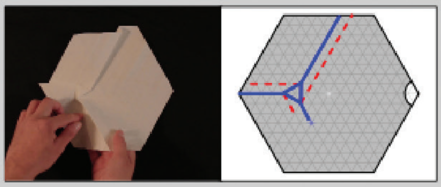
1. Begin with a hexagon gridded to 16ths. Find the point on the grid three triangles away from the center along one of the diagonals. This will be the center of the first twist. Mark that point.



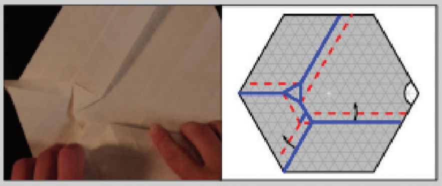
2. Pinch from the marked point to the closest corner to create a pleat. We will construct the first twist off of this pleat. Go through the process of pleating that pinch and splitting it to the top and bottom edges of the hexagon.



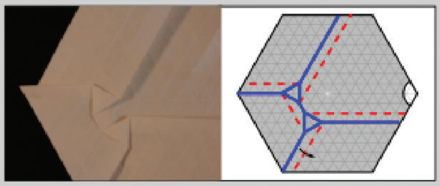
3. The first triangle twist is complete. Notice that the twist is not centered on the paper. Also take note of where the mountain folds of each pleat reach the edges. The first one reaches the corner directly, but the others are three grid lines away from their closest corners. This is because we started at a point other than the center of the hexagon.



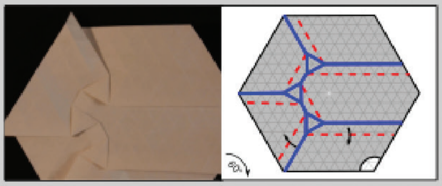
4. Take the pleat closest to you, and find the point along the mountain fold that is three grid lines away from the center of the first twist. You may have to unfold the first twist to find it and then refold. Mark this new point.



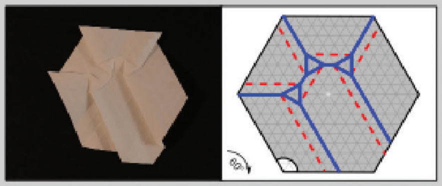
5. This is where we will split the mountain fold to form the second triangle twist. Split the pleat at that point.



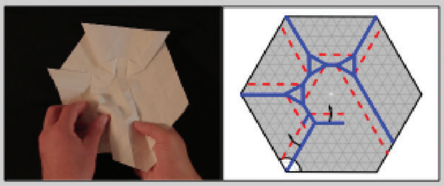
6. Twist the split. Note that the first twist was clockwise and the second was counterclockwise. We will return to why this is in a later chapter.



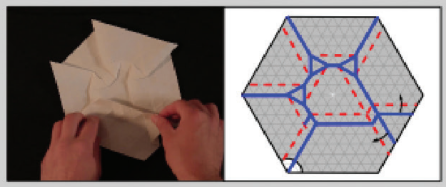
7. Rotate clockwise 60°, and create the third triangle twist three grid lines away from the second. This one will rotate clockwise.



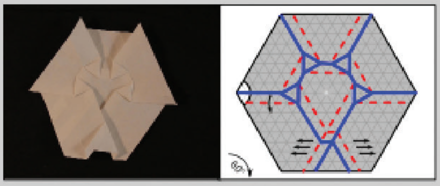
8. Turn 60° clockwise again. Then, try to visualize where the pleats go from here. Three units away from the third twist, there will be another split. The pleat that travels to the right will clash with one of the pleats from the first twist. What do you do then?



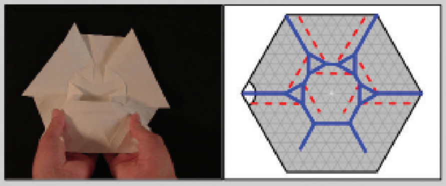
9. The most effective way I have found to deal with clashing pleats is to lock one in place with the other. As you are splitting the pleat emanating from the fourth twist, ignore the pleat emanating from the first and have the new one run through it.



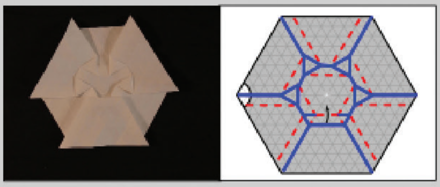
10. This shows the finished lock.



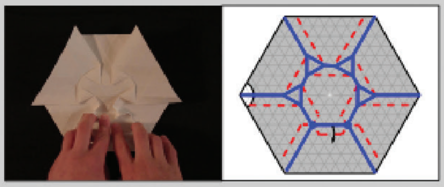
11. Finish the fourth twist, and rotate the paper 60° clockwise. Then in one swift movement, pull the pleats apart just enough for the lock to unfold. The goal is to undo the lock while keeping the twists that have already formed together.



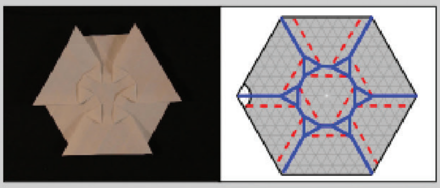
12. Now you can form the splits for the remaining two twists. Identify the split locations three triangle widths away from the neighboring twists centers. The pleats that do not go toward another twist will go directly into the corner of the hexagon.



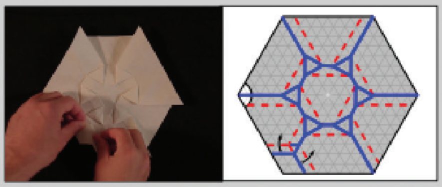
13. This is the completed split. You've actually set up the final two twists' pleats in a single motion.



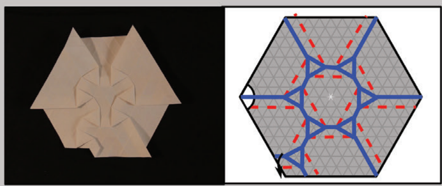
14. Reorient the middle pleat of the last twists so that you can form the remaining triangles.



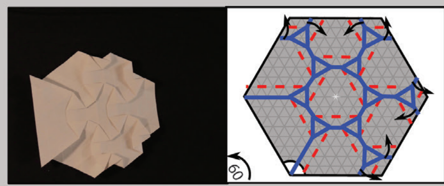
15. This is the finished triangle twist cluster.



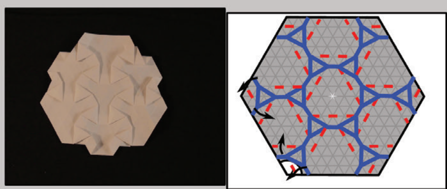
16. If you wish to continue the pattern, there is just enough room for another iteration of triangles. Choose a pleat that goes from a triangle to the corner, and split it.



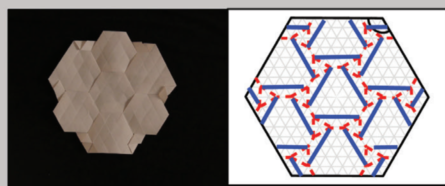
17. Make these pleats rotate counterclockwise, and you will have the first twist of the next iteration.



18. Rotate as necessary, filling in the next iteration of twists.



19. This is the finished extended triangle twist cluster.

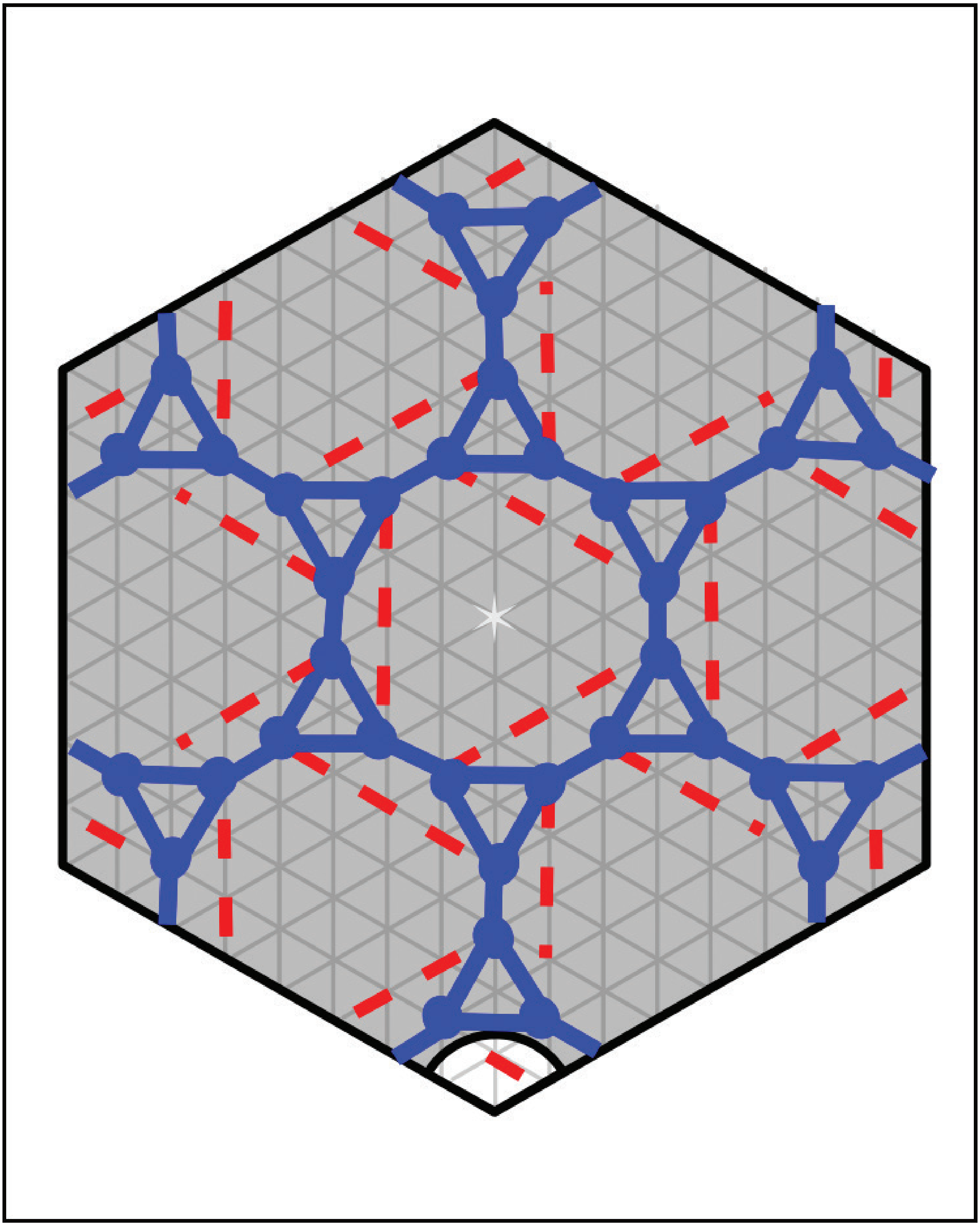


Reverse

- In step 3, I asked you to make note of where the mountain folds of the pleats reach the edges of the paper. This is a valuable tactic in keeping track of where the next twist should be.

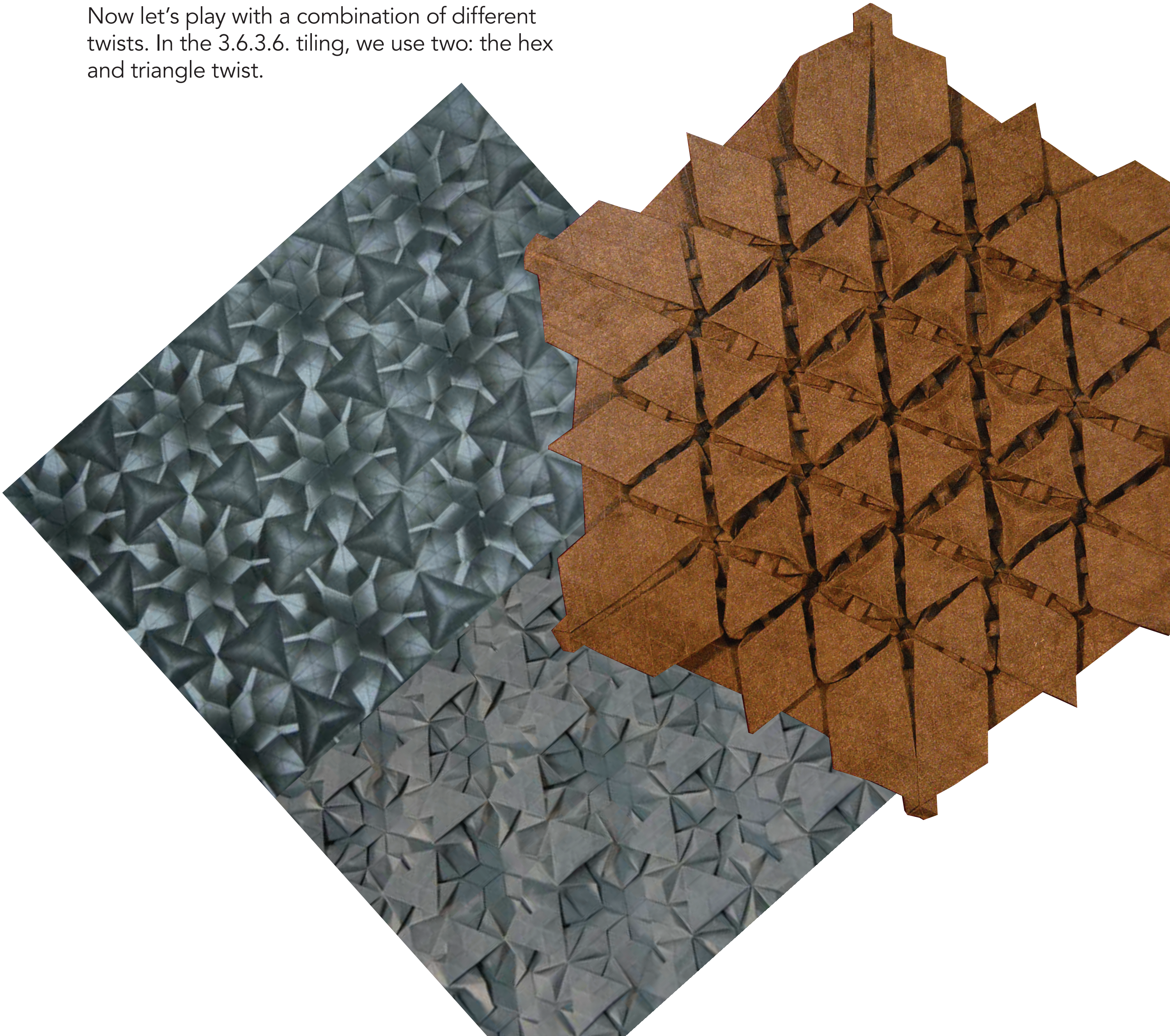
- How would this tessellation look if you had a triangle grid that was 17 triangle widths across the diagonal? Or 18? More paper means more real estate to use for the tiling. What if you had folded this on the 8ths grid instead?

- Can you fold the 3^6 tessellation where the triangles are tow grid lines away instead of three? How about four?



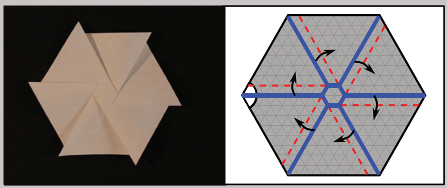
2.5 Your Second and Third Tessellations

Now let's play with a combination of different twists. In the 3.6.3.6. tiling, we use two: the hex and triangle twist.

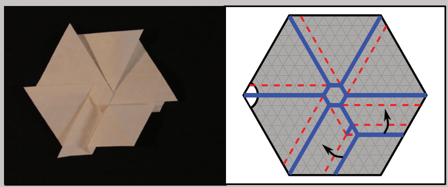


3.6.3.6. Twist Cluster

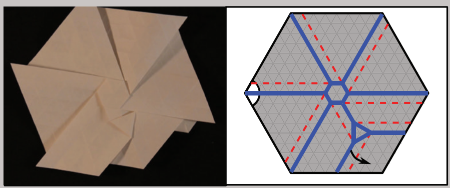
3.6.3.6. means that there will be clusters of connected polygons consisting of a triangle, a hexagon, another triangle, and another hexagon. Now we are combining different twists in the same pattern. This exercise starts with a hex twist in the center of the paper, and each pleat of the hex twist flows into a triangle twist. Pleats from two adjacent triangle twists intersect to create a second hex twist. This can be repeated as many times as the folder wishes (or for which there is room on the paper). Always pay attention to where the mountain folds of the pleats are and where they intersect the edges of the paper.



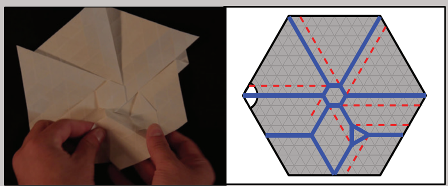
1. Begin with a hex twist in the center of the paper with a clockwise rotation.



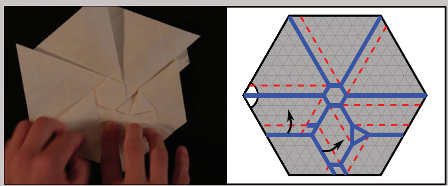
2. Identify the point four triangle widths away from the center of the hexagon. You will have to unfold the hexagon to identify it and then refold. Mark that point. Split the pleat at that point.



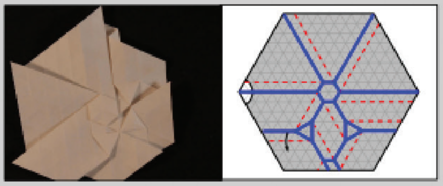
3. Flatten the triangle twist by orienting the last pleat counterclockwise.



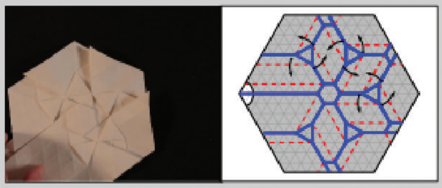
4. Choose another pleat emanating from the hex twist, and repeat the split.



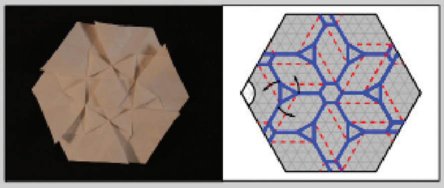
5. Flatten the split. Make the clashing pleat of the second twist run through the obstructing pleat of the first twist.



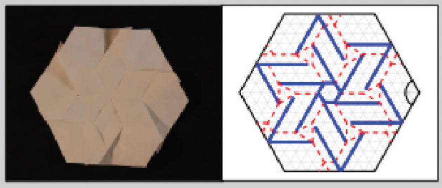
6. Turn the second split into a triangle twist.



7. Continue this process around the hexagon, rotating as necessary.



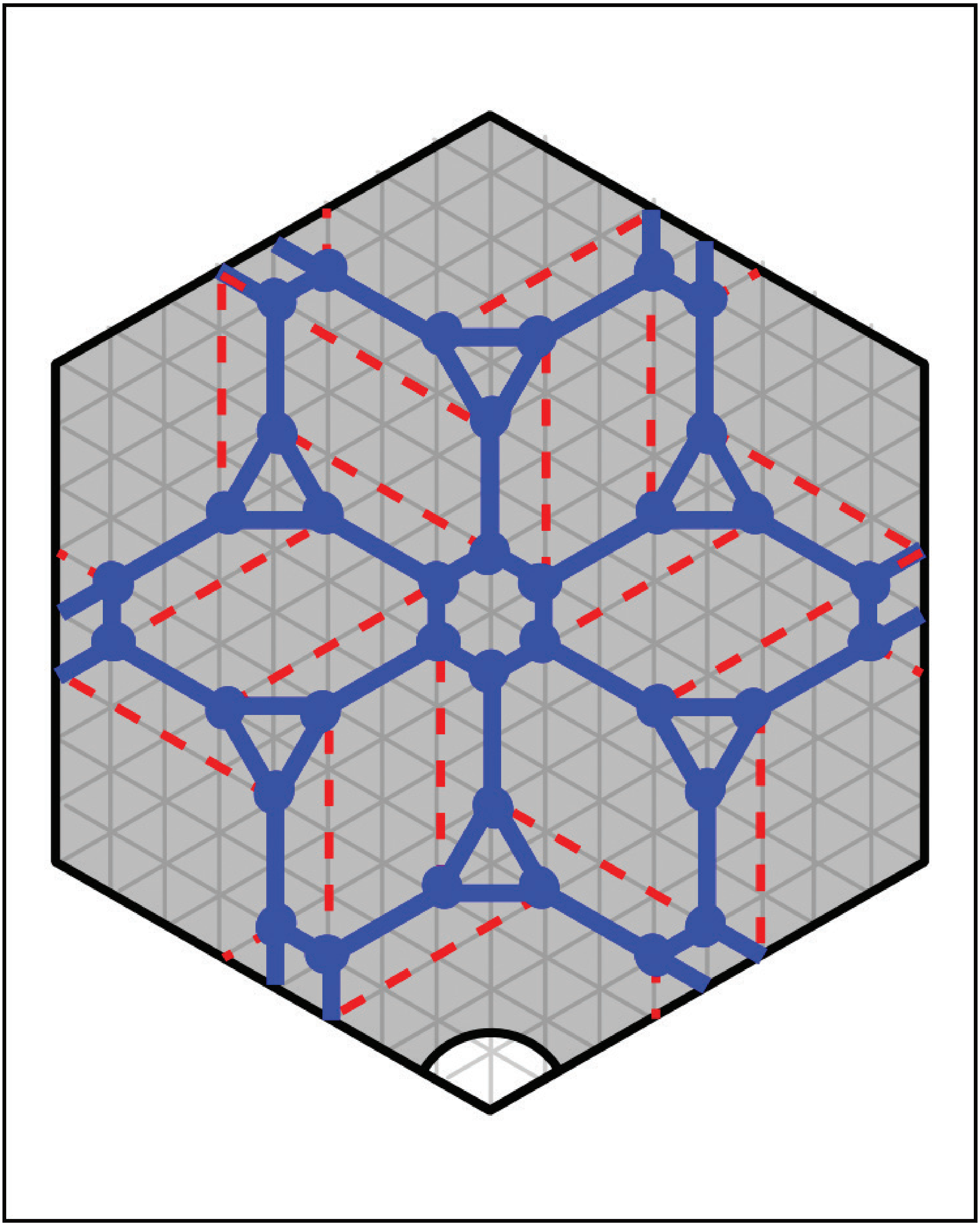
8. This is the completed 3.6.3.6. pattern.



Reverse

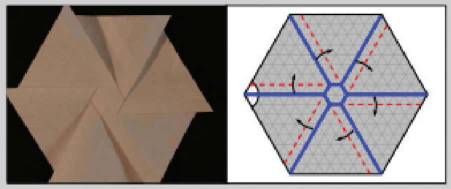
- Notice that there is only one hexagon in this pattern. Why is that? When you learn how to fold 32nds grids in the Advanced Gridding diagram, return to this page and fold this on a denser grid. See how many iterations of the 3.6.3.6 pattern you can fit onto it.

- Why do all of the triangles rotate counterclockwise but the hexagon rotates clockwise?

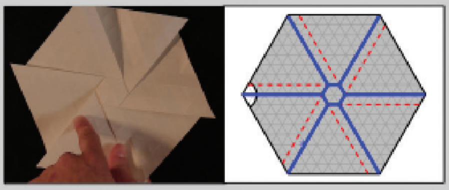


Hexagonal Failed Cluster

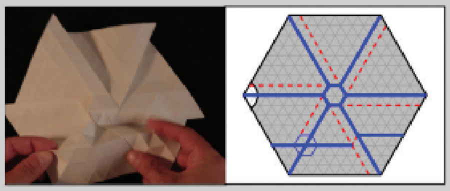
This can also be notated as the 6.6.6. or 6^3 tiling, meaning that there should be clusters of three connected hexagons. At least, this would make sense, but sometimes you want to fold something, and it simply will not work the way that you want it to. It's up to the designer to figure out why, but when explaining why or how a concept doesn't work, it is informative to have you fold it and see for youself. Fold and try to figure out what the difficulties are. The answer is revealed in the next section.



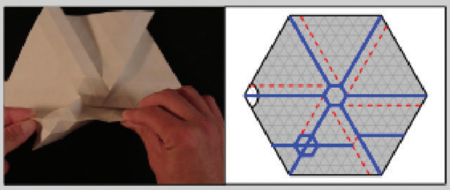
1. Begin with a hexagonal twist in the center of the paper rotated clockwise.



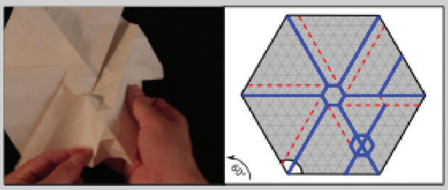
2. Find the point five grid lines away from the center of the paper along the mountain fold of one of the pleats, and mark it. You will have to unfold the paper to do this. Then refold.



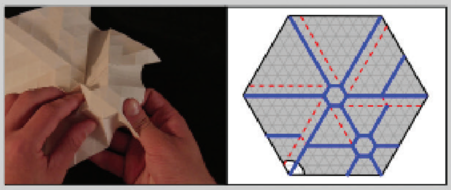
3. This point will be the center of the second hexagonal twist. Open the pleat and pinch a mountain fold laterally that intersects the mountain fold of the first pleat at the marked point.



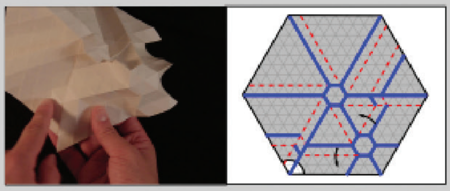
4. As you pinch, you should see a warped hexagon at the intersection. This will be your hexagonal plateau that you will twist flat once all of the pleats are finished.



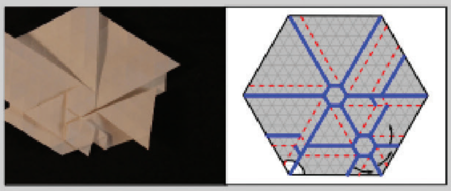
5. Rotate the paper as needed.



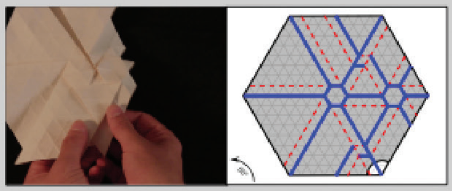
6. Next, pinch the third pair of pleats running through the center of the hexagon. You should clearly see the hexagonal plateau.



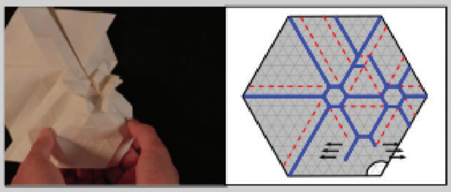
7. Where the new pleats clash with older pleats, lock them in place. Twist the hexagon counterclockwise, and flatten the pleats.



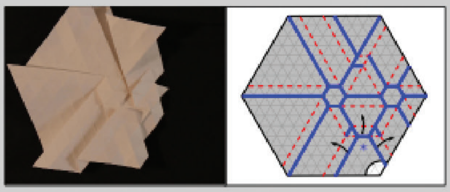
8. This is the finished second hex twist.



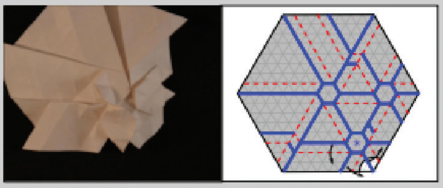
9. Rotate as needed.



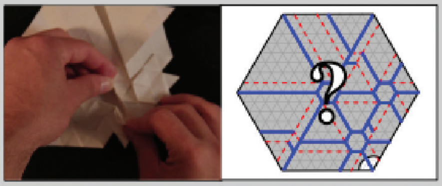
10. As with the lock in the other tessellations, pull apart and open the pleats so that you can fold the third hex twist.



11. Lay the paper flat in a split. This will be half of the hex twist with two of the six pleats in place already formed from the other twists. We can form the rest of the pleats around, one at a time. Mark the center of this new twist.



12. Form the remaining four pleats, and try to get the hex twist to lay flat in either a clockwise or counterclockwise formation. The paper will resist.



13. Actually, the paper will resist a lot. Should you fold it clockwise or counterclockwise?

- The 3^6 tiling lays flat without much issue, as does the 3.6.3.6 tiling. What happens here? This will be answered in the next section, but it has to do with pleat orientation and twist rotation. Fiddle with the last step, and see if you can flatten the final hexagon without continuing. If you get discouraged, continue to the next section to find one solution.

2.6 Chirality

Over the past few sections, you have learned several factors in folding a tessellation: the tiling, the twist(s) you are using, pleat distance, and the pleats you are using. Now we'll add one more factor: *chirality*. If the pleats that compose a twist are asymmetric in their assortment, the intersection will be chiral, and this must be factored into the design. VM and MV pleats are asymmetric, so twists composed of all VM or all MV pleats are achiral. Chiral twists naturally want to rotate in the opposite direction from their neighbors, in the same way that interlocking gears must rotate in opposite directions.

Consider Figures 2.5 and 2.6.

Notice in Figure 2.6 how the connection between the clockwise (CW) hexagonal twists clashes. Since one of the pleat connections in the 6^3 tiling does not line up with its neighbor, we have to figure out a way to work around this misalignment.

2.7 Back-Twisting and Releasing Pleats

In the twists we have learned so far, each pleat of the twists rotates either clockwise or counterclockwise around the center, the intersection of the pleats. If the twist rotates clockwise, so do each of the pleats, and vice-versa. We can release the pleats from this constraint by back-twisting. *Back-twisting* is the process of forcing a twist to rotate in the opposite direction around the center from the pleats. Generally, this releases the pleats so that they can move in either orientation.

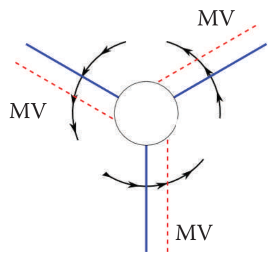


Figure 2.5

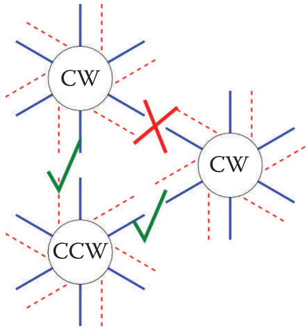
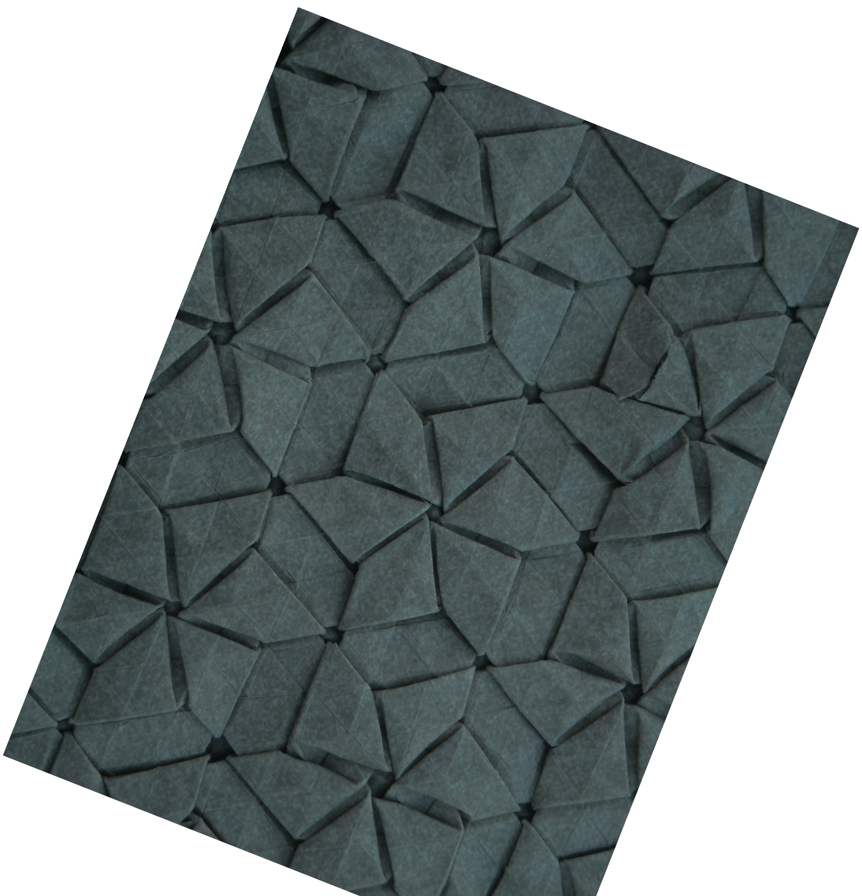


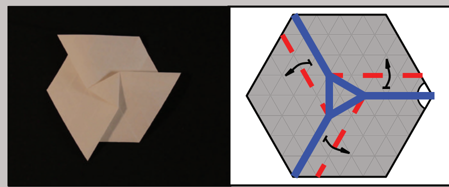
Figure 2.6



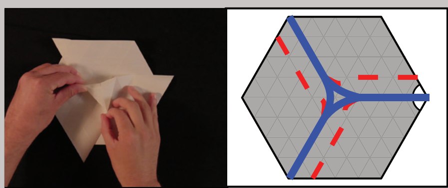
Triangular Back-Twist

To this point, all of our twists have included several intersecting pleats and the actual twist that forms at the intersection. Each of these elements can be studied separately, but the twists we have gone over have acted as a unit, meaning if the pleats are oriented clockwise around the intersection point, the twist will also go clockwise.

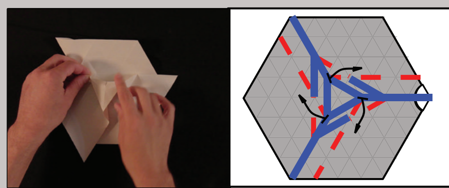
This does not have to be the case. One way to unlock the pleats and orient them opposite of the twist is to back-twist the center. Let's revert to our 8ths grid because we're only working with a single intersection now.



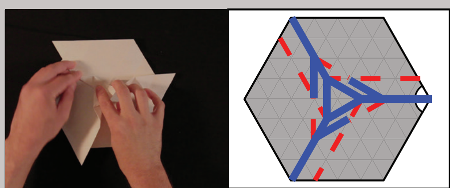
1. Begin with a triangle twist on an 8ths grid with counterclockwise rotation.



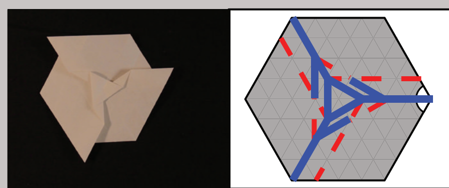
2. Pinch two corners of the triangle.



3. Use the corners of the triangle to twist the center clockwise. Do not unfold or reorient the pleats for this maneuver.



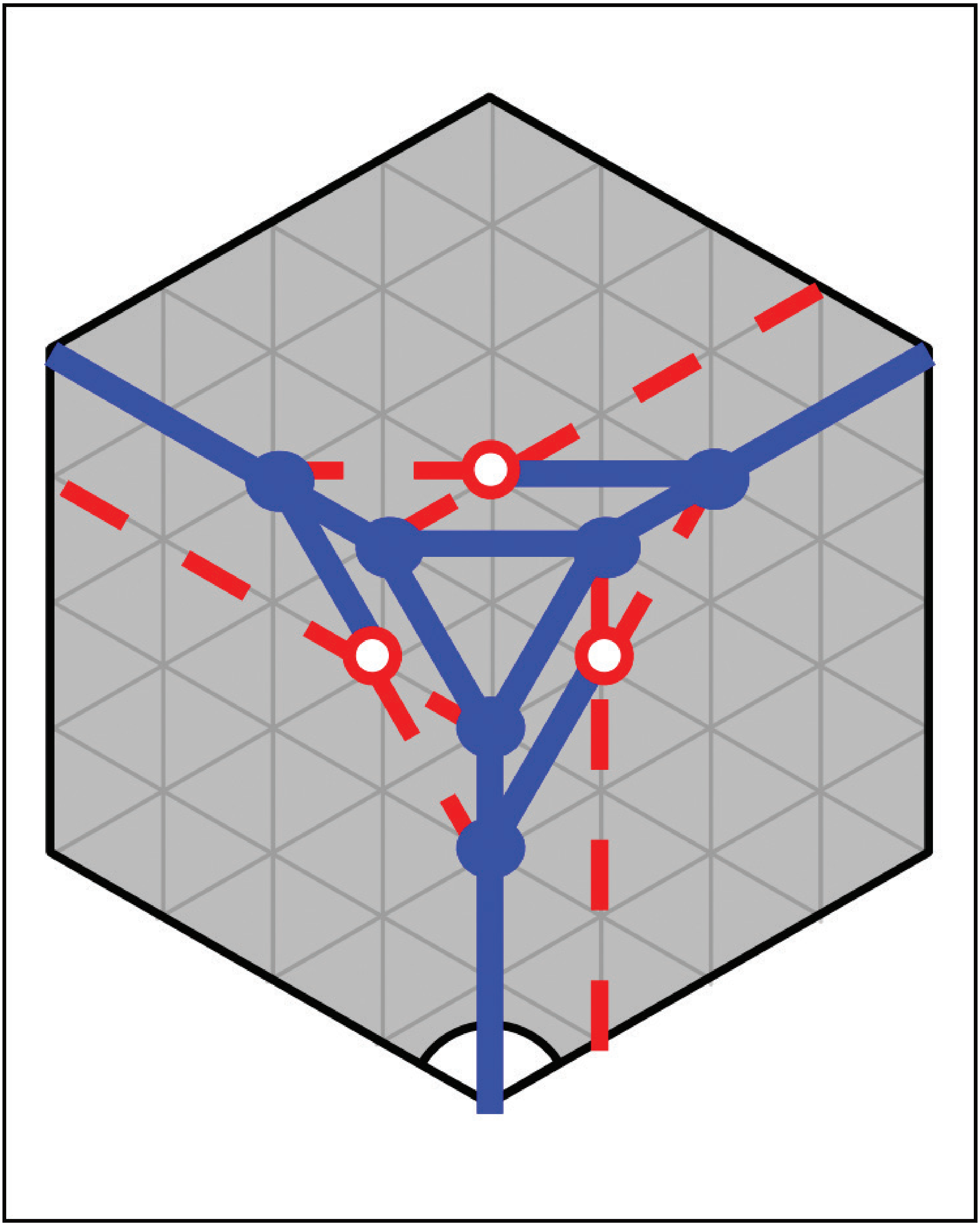
4. Once the triangle has been rotated so that the corners of the triangle point toward the corners of the hexagon, flatten the twist down again.



5. This is the finished back-twist.

- Work on back-twisting a hex twist. The goal is to make the center twist rotate in the opposite direction from the pleats around the center. Once back-twisted, the pleats are free to orient themselves in either direction, clockwise or counterclockwise.

- Can you incorporate a hexagonal back-twist into a hex twist cluster? If we remove the restriction that all of the twists have to go in the opposite direction, this should allow you to fold the pattern more easily.



The triangle twist can be back-twisted. In the same way, the hex and arrow twists may also be back-twisted. The others offer resistance to the back-twisting process, and we will learn why that is in a later section.

Consider the triangle, hex, and arrow twists and where the corners of the plateau of each twist flow into the connecting pleat (shown in Figure 2.7).

Now, regard the back-twist version of each of these (shown in Figure 2.8).

The corners of the center twist have moved opposite of the pleats that composed them. Additionally, the pleats are free to become oriented in whichever direction they choose.

Now, we can create a 6^3 tiling by folding a hex back-twist and having the pleat flow into a second hex back-twist. That will flow into subsequent hex back-twists and create the tiling. The entire tessellation will lay flat. The orientation of the pleats in the result does not strictly matter. The pleats will not be symmetric, but it is a solution for avoiding the chirality issue the 6^3 tiling often presents.

2.8 Pleat Flattening

Another way to avoid the chirality issue is to simply have an *achiral intersection*. An achiral intersection is one where each of the pleats is symmetric, meaning the mountain and valley folds of each pleat are in a symmetric pattern. A common example is the VM2MV pleat, which involves a valley fold, a mountain fold, another mountain fold, and a valley fold. Figure 2.9 shows the crease pattern for the VM2MV pleat.

A twist composed of VM2MV pleats connects with another similar twist in the manner shown in Figure 2.10.

This is how achiral intersections make tiling easier. If you can create a hex twist composed of VM2MV pleats, you can create a 6^3 tessellation using this method of avoiding chirality issues.

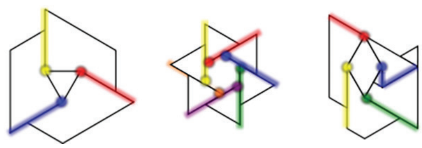


Figure 2.7

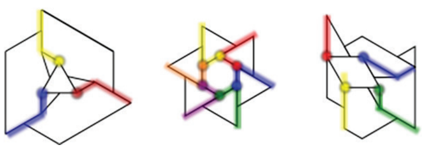


Figure 2.8

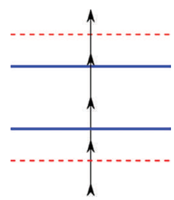


Figure 2.9

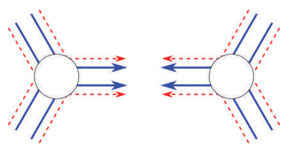
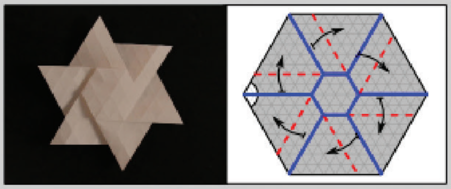


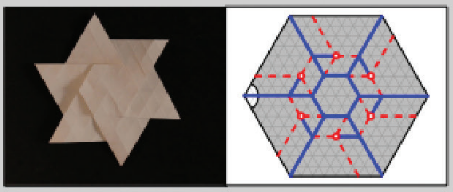
Figure 2.10

Hex Twist with Flattened Pleats

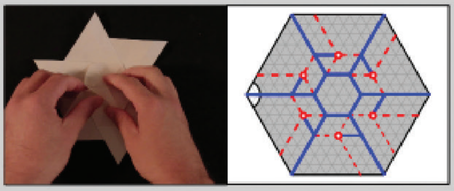
Flattening the pleat is a method of avoiding chirality issues. If two pleats connect and the creases line up with each other, the tiling can continue unabated. To flatten a pleat into a VM2MV, the easiest way is to take a VM pleat that is two triangle widths wide (V2M) and separate it. For this, we need to begin with a hex twist composed of pleats that are two triangle widths wide. Folded from a 16ths grid, this will look identical to a hex twist folded from an 8ths grid in terms of size, but there will be a denser underlay of grid on the paper.



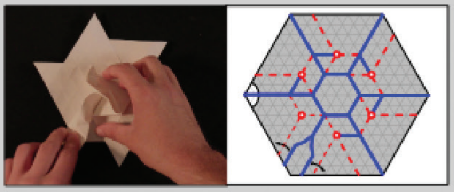
1. Begin with a hexagonal sheet of paper, gridded to 16ths, with a hex twist in the center composed of pleats two triangle widths wide.



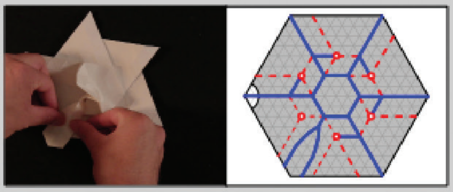
2. Back-twist the center hex. The center should rotate counterclockwise, while the pleats should rotate clockwise around center.



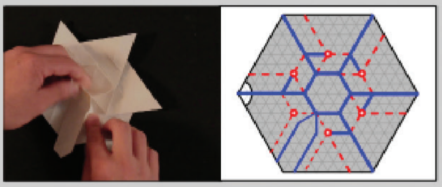
3. Lift a corner of the twist.



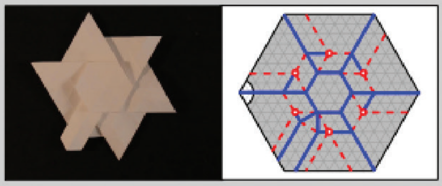
4. Lift the pleat that emanates from the chosen corner and starting from the corner of the paper toward the center, flatten the pleat.



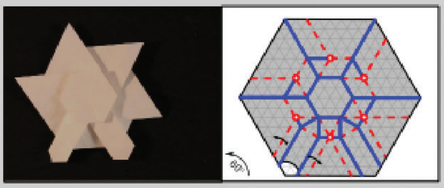
5. The form will not lay flat until you are finished flattening completely.



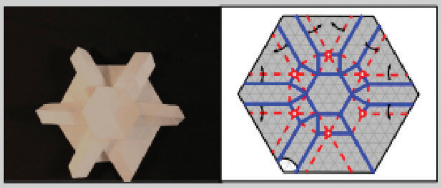
6. You may have to move an adjacent pleat out of the way to flatten. Fortunately, the pleats are loose, so you can do that.



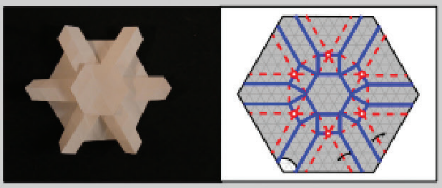
7. This is the first pleat flattening finished.



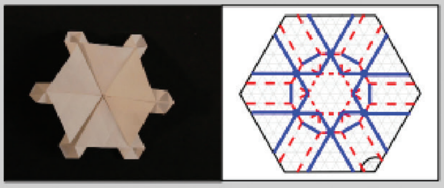
8. Rotate counterclockwise 60°, and flatten the second pleat. The pleats will clash in the middle, so you have to choose one to be on top of the other.



9. Continue to rotate and flatten pleats until all six have been flattened.



10. This is the finished hex twist with flattened pleats.



Reverse

- Can you fold a hex twist with single-width pleats and flatten the pleats? This is a little trickier because the flattening has to go between the grid lines, rather than using the creases already in the grid.

- If you can fold the hex twist with flattened pleats from a hexagon composed of single-width pleats, can you figure out how to fold the 6^3 tiling with these twists?

2.9 Different Types of Pleat Formats

Parallel pleats can be described using a method I have touched upon in previous sections. By designating the mountain folds of the pleats as M, the valley folds as V, and the space separating them by a numbered unit relative to the rest of the pattern, we can accurately describe any parallel pleat. When the distance is one unit, the distance number may be omitted.

For example, the VM pleat is a valley fold one unit away from a parallel mountain fold. The MV pleat is a mountain fold one unit away from a parallel valley fold. MV and VM can be used to describe the same two folds seen from different points of view. In the hex twist with flattened pleats, we used VM2MV pleats, which were composed of a valley fold, a parallel mountain fold one unit away, another parallel mountain fold two units away from the previous fold, and finally a valley fold one unit away from the previous fold. You may use this nomenclature as you wish for any style of pleat, such as in Figures 2.11–2.13.

I mentioned earlier that the arrow twist had two types of pleats, three that were single-width and one that was double-width. The single-width pleats are VM or MV pleats. The double-width pleat is a V2M or M2V, in relation to the rest of the twist.

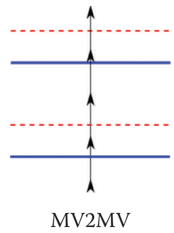


Figure 2.11

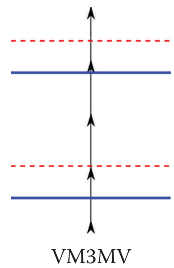


Figure 2.12

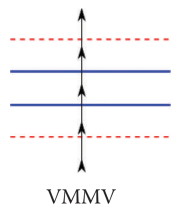


Figure 2.13

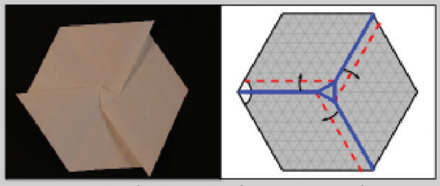
2.10 The Referential Method of Tessellation Formation

As you begin to study more complex pleats, it becomes difficult to create new twists by simply splitting the pleats as we have been.

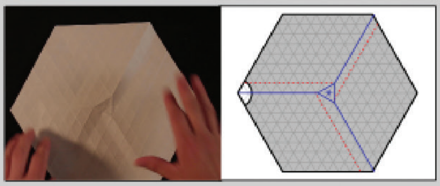
The referential method of tessellation formation is a powerful way to integrate more complicated concepts. The referential method involves folding a single twist, unfolding it, and then determining where the next twist should be. The folder then makes the second twist as if the first had not been folded and then unfolds the second one. The folder next completes the pattern folding and unfolding each twist separately, and finally refolds all of the twist using the resulting bias in the paper.

Referential Method of Tessellation Formation

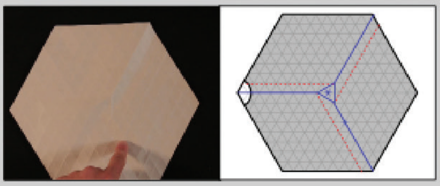
So far, we've constructed a twist using pleats emanating from previous twists, generally from a split. The referential method expands on the idea that sometimes you have to fold and unfold to get a twist in the correct spot, and then refold everything. This is a different method of connecting twists that is useful for more complex twist clusters. I will use a triangle twist cluster as an example, but it is a tactic that can be useful for any tessellation construction.



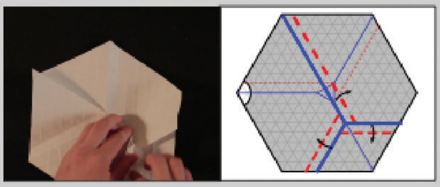
1. Begin with a triangle twist in the center of the hexagon. Unfold.



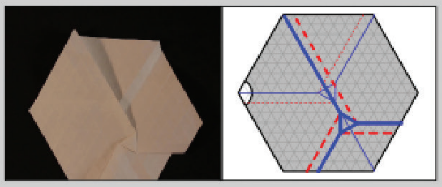
2. With the paper unfolded, you should still see the original creases from the triangle twist, the center, the pleats, whether a crease was a mountain or valley, etc. Mark the center of the twist.



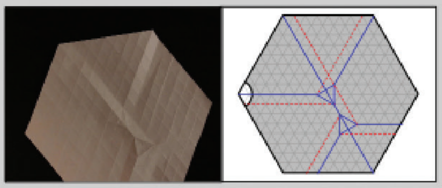
3. Using this subtle image of the last twist, find the point three triangles away from the center of the last twist. Mark this point.



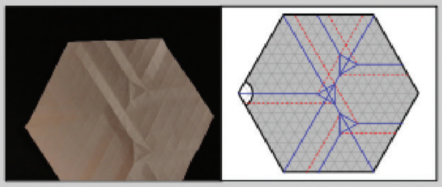
4. Construct a pleat from the corner through this point, and then split that pleat at the point.



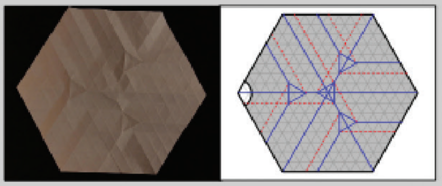
5. Reorient the pleats clockwise to make a second twist.



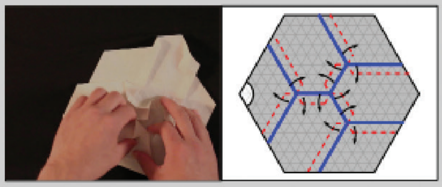
6. Unfold.



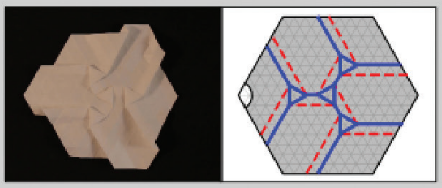
7. Repeat the process of folding and unfolding until you have a cluster of four triangle twists, one in the center surrounded by three.



8. This step shows the finished cluster.



9. Now, refold the pleats one at a time. Because of chirality, the center twist will rotate counterclockwise while the others will rotate clockwise.



10. Once the pleats are in place, you can flatten the twists as you would a normal twist cluster. This step shows the finished cluster.

2.11 Flagstone Tessellations and Offsetting the Pleats

So far, we have created tessellations that contain a twist, from which emanates pleats, and each pleat is used to flow into another twist. Consider Figure 2.14.

Here we see two pleats that will collide and become one. This is because they have the same crease pattern, a valley and a mountain in the same orientation. Suppose we offset one of them so that the mountain folds miss each other but are still parallel, as in Figure 2.15.

The mountain fold of the pleats do not line up at all. However, through experimentation, folders have learned that the paper self-corrects and flattens to become a different type of connection between twists. This style of design has been a called a *flagstone tessellation* because of the resulting resemblance to flagstone sidewalk tiles.

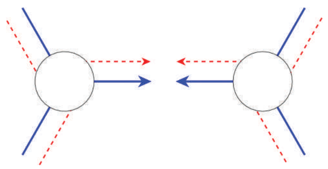


Figure 2.14

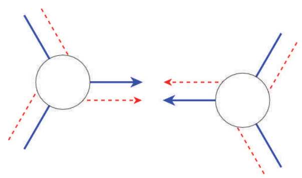
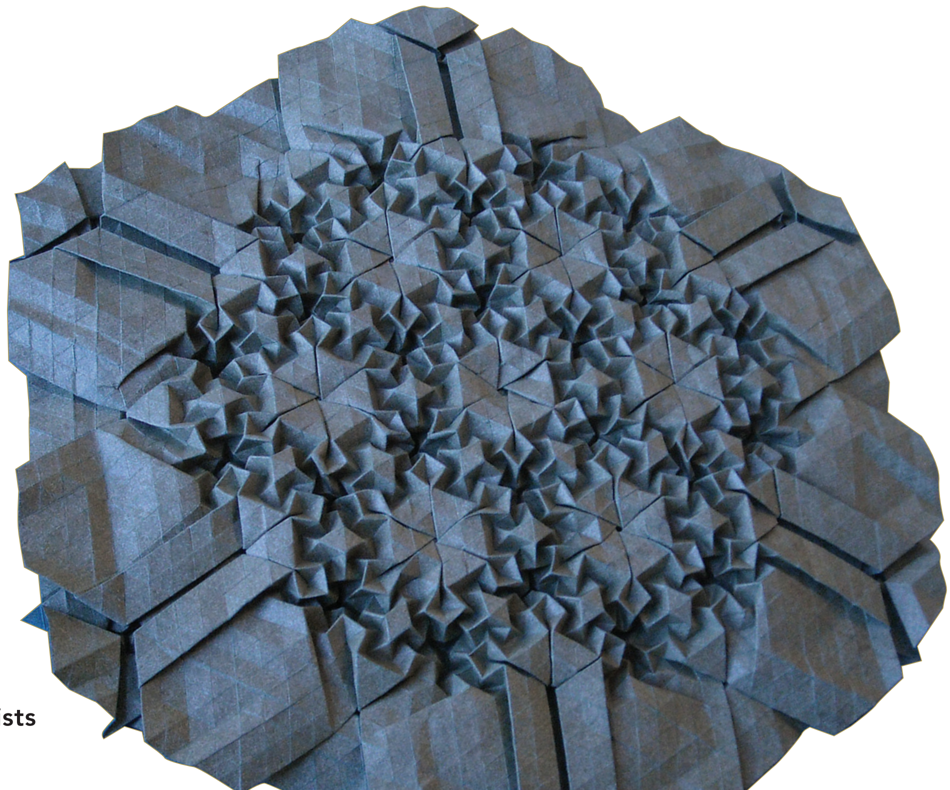


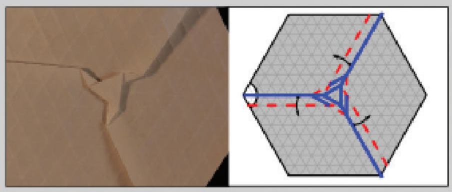
Figure 2.15



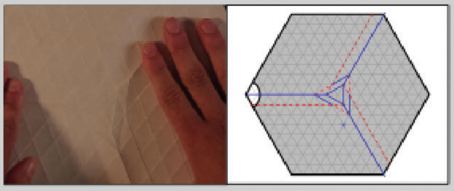
3^6 Flagstone Tessellation

Up to now, the diagrams have all connected twists directly through the pleats. There is a twist with pleats emanating from the twist; a pleat travels across the paper and then connects directly with another pleat, which flows into a new twist.

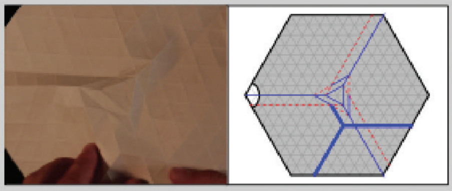
Pleats connecting twists do not have to line up directly. Two twists can be offset from one another, creating another method of varying the design. Tessellations composed of twists offset from one another are often called *flagstone tessellations*. I find the referential method particularly effective for folding flagstone tessellations cleanly.



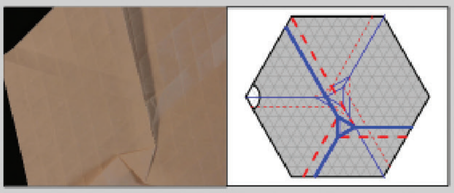
1. Begin with a triangle back-twist in the center rotated in either direction.



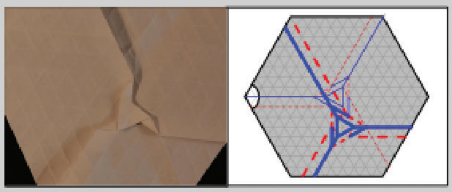
2. Unfold. You should see the center of the original twist, as well as other features from the memory in the paper. Identify the point two triangles away from the center on the lower-right pleat and then one triangle offset counterclockwise. Mark this point.



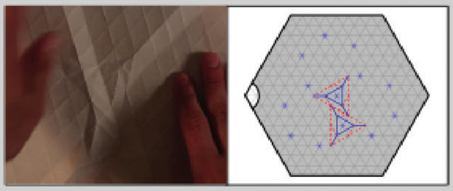
3. Construct pleats intersecting at this point with one pleat heading toward the first twist.



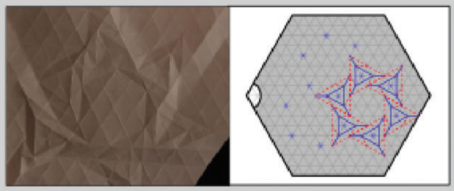
4. Use the pleats to form a triangle twist. Run the pleat directly through the first triangle.



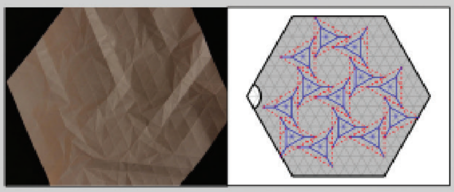
5. Back-twist the second triangle twist.



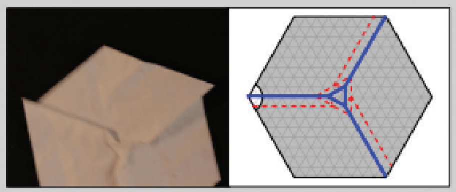
6. Unfold. The impressions of the two twists should be right next to each other. Continue the pattern of finding the point two grid lines away and one offset counterclockwise from the previous triangle center. Mark these points throughout the hexagon before continuing.



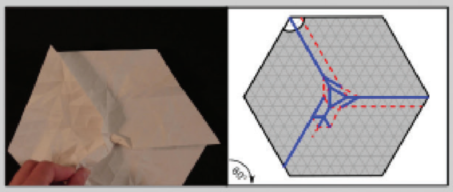
7. Fold the next triangle twist, back-twist it, and then unfold. Make sure to note exactly where the center of the next twist is before committing to flattering the pleats. Continue this pattern throughout the paper.



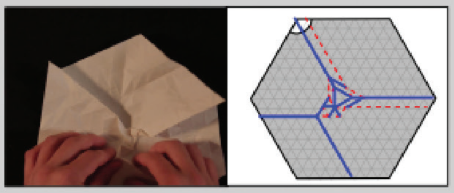
The finished cluster reference creases



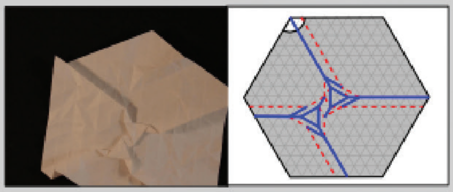
8. Refold the center back-twist.



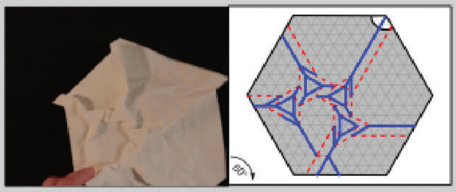
9. Rotate 60° clockwise to get a better view. Push left on the mountain fold of the pleat with your thumb. This will make the pleat travel one grid line to the left.



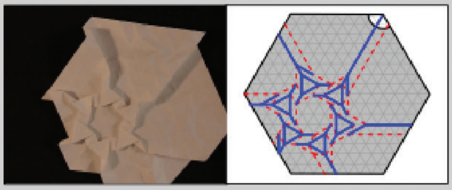
10. Find the center point of the second triangle twist. Split this new pleat at that point.



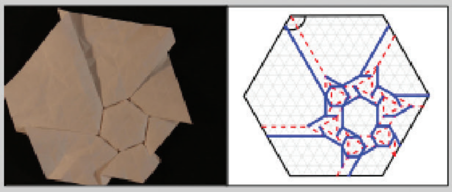
11. Flatten this new twist. The creases should already be in the paper. The pleat that runs toward the center triangle will fold flat. The other pleats will extend to the edge of the paper.



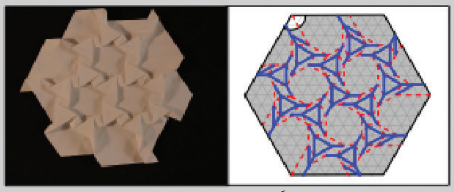
12. Rotate as necessary. Continue the process of pushing the one pleat to the left, splitting that pleat, and forming new twists. Keep in mind that once you get to four twists in a circle, there will be a clashing pleat, which you can lock with another pleat.



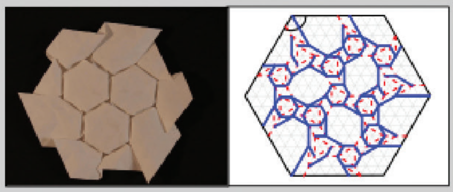
13. This is the finished folded cluster.



Reverse of one cluster



14. This is the finished 3^6 flagstone tessellation.

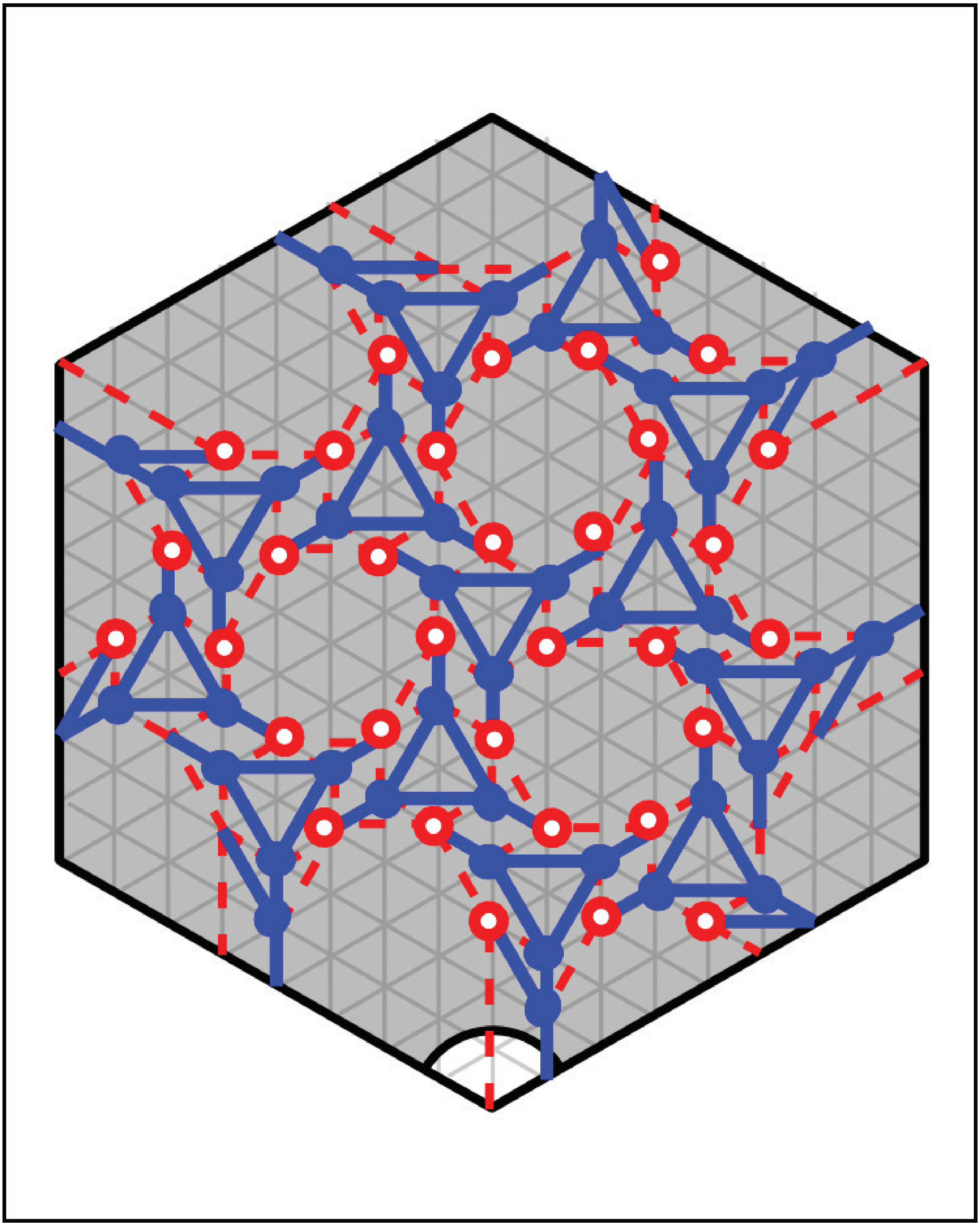


Full reverse

- As you are experimenting with the offsetting technique, play with spacing. The triangle twists that surround the center twists were two grid lines away and one grid line offset counterclockwise. What if they were three grid lines away and one offset? Or two lines offset? How does that affect the design? What if they were offset one grid line clockwise instead of counterclockwise?

- Can you start with a back-twisted hexagon in the center and have an offset pleat go into a new triangle twist? Can you tile this to make a 3.6.3.6. flagstone tessellation?

- Once the full tessellation is folded, you may notice something strange about this assortment. All of the twists are rotating counterclockwise! Because of that, there is no rule in flagstoning that says that adjacent twists have to go in opposite directions. Using this information, can you fold a 6^3 flagstone tessellation?



2.12 Twist Decorations and Progressions

Since the patterns we make are geometric progressions, so is the design process. Each of the six simple twists—along with each of the countless other twists documented and yet to be documented—has many different variations. Many of those follow specific geometric progressions. A designer selects which twists to use (and through them the pleats emanating from them), the spin of the twists (and how to deal with any relating chirality issue that may arise), and decoration that is to be given to the pattern. There is a myriad of options. That is what has held my fascination with folding these items for so long.

We have already discussed two of the more basic progressions: pleat length and offsetting. When calculating pleat length, what you are really doing is drawing a line between the centers of two twists. We talked a bit about what happens if the twist centers are not connected by a grid line, and then you get offset pleats and a flagstone quality on the reverse of the paper. There are many different ways of modifying a single aspect of known functions with the paper.

Another progression we have discussed is the twist's *spread*, which describes the manner in which paper sections are brought together by the twists' pleats. Consider the six simple twists and the sections of paper they bring together (see Figures 2.16–2.21).

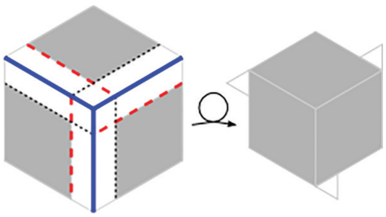


Figure 2.16

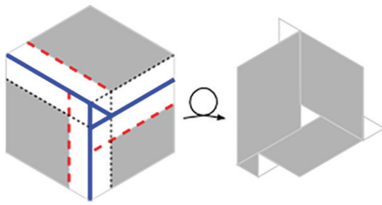


Figure 2.17

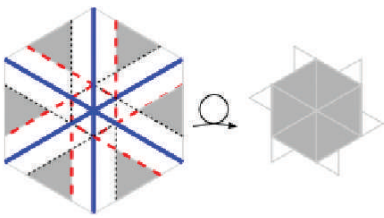


Figure 2.18

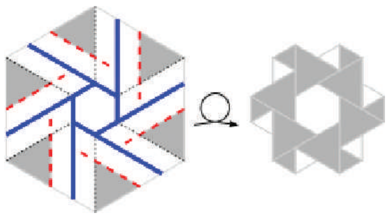


Figure 2.19

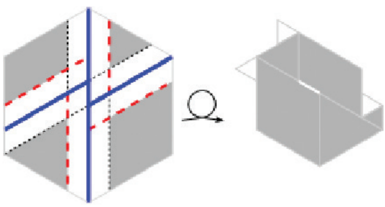


Figure 2.20

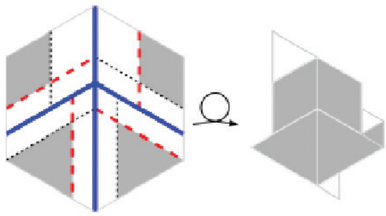


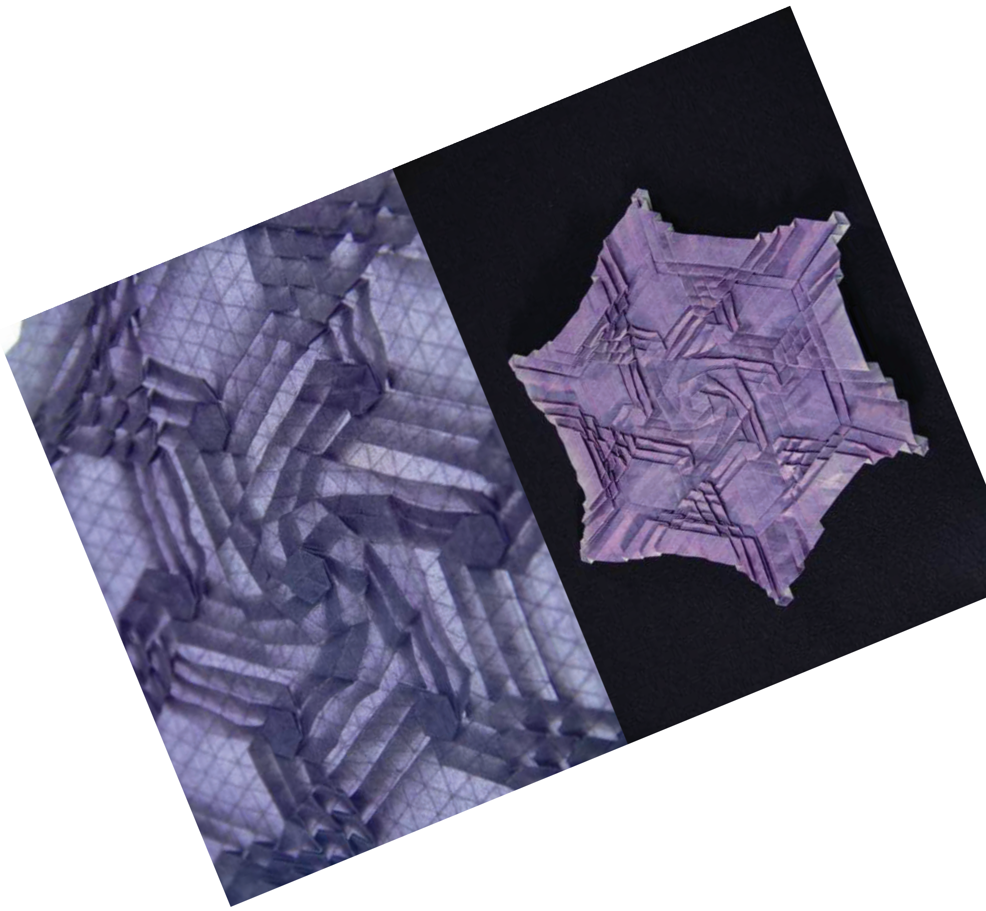
Figure 2.21

You will notice that the triangle, hex, and arrow twists (Figures 2.16, 2.18, and 2.20, respectively) all come to a point. The rhombic twist (Figure 2.21) creates a line equal to one grid triangle's side. The other two have openings corresponding to the twist types themselves: a triangle (Figure 2.17) and a hexagon (Figure 2.20).

If there is no negative space between the twists, the twist has no spread. In this event, a twist is usually able to be back-twisted. If it is spread over two points (as in the rhombic twist), it has a spread across a line. The spread triangle and hex twists both have spreads of polygons, specifically an equilateral triangle and equilateral hexagon, respectively.

2.13 Twist Expansion

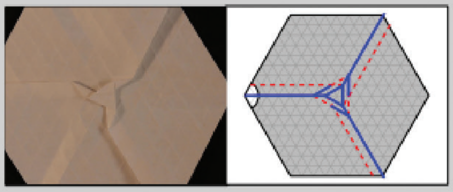
Another progression that is frequently used is twist expansion. *Twist expansion* is a method of increasing the area a twist encompasses without increasing the width of the pleats that form it.



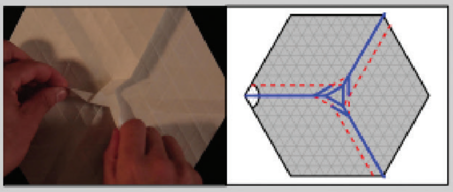
Triangle Twist Expansion

The triangle twist is composed of single-width pleats that intersect at a point. The triangle spread is composed of single-width pleats whose intersection is offset one or more grid lines from a point. It is possible to fold a triangle twist with pleat widths of two triangles or more. These are all progressions from a common ancestor, the triangle twist. The hex twist has its own parallel progressions and resultant modified twist, as do the rhombic and arrow twists.

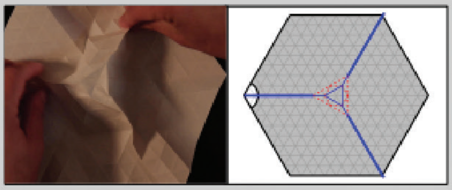
Expansion is a way of making the center of the twist encompass more area without enlarging the width of the pleats. This can lead to yet another modification to the triangle twist.



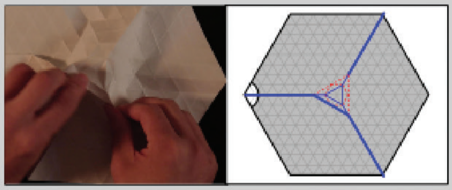
1. Begin with a triangle back-twist in the center of the paper.



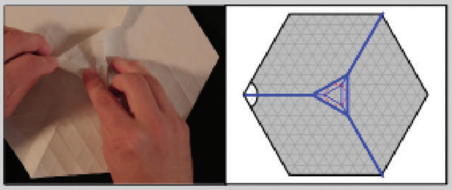
2. Pinch two of the pleats.



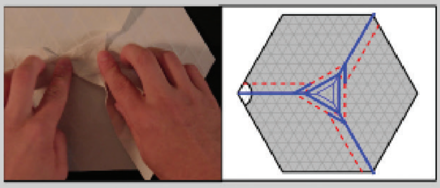
3. In one shift motion, pull the twist apart, but keep the pleats folded. This should result in a plateau of paper coming off the table toward you, surrounded by valley folds. These creases are not on the grid.



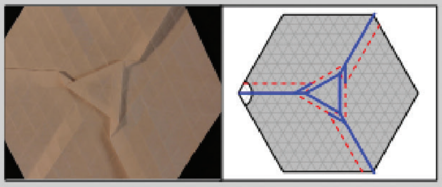
4. One at a time, turn these valley fold outline creases into mountain folds.



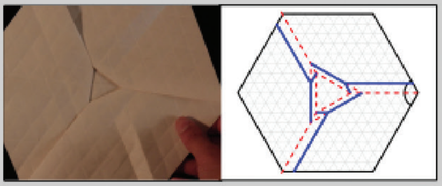
5. This step shows more of the triangle being formed.



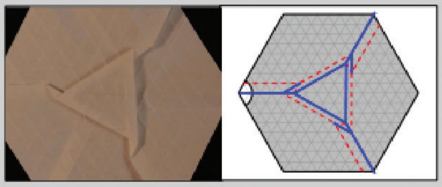
6. When all three mountain folds have been formed, press down as you would for a triangle back-twist.



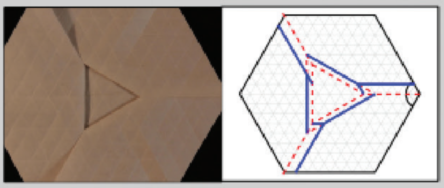
7. This is the finished triangle twist expansion.



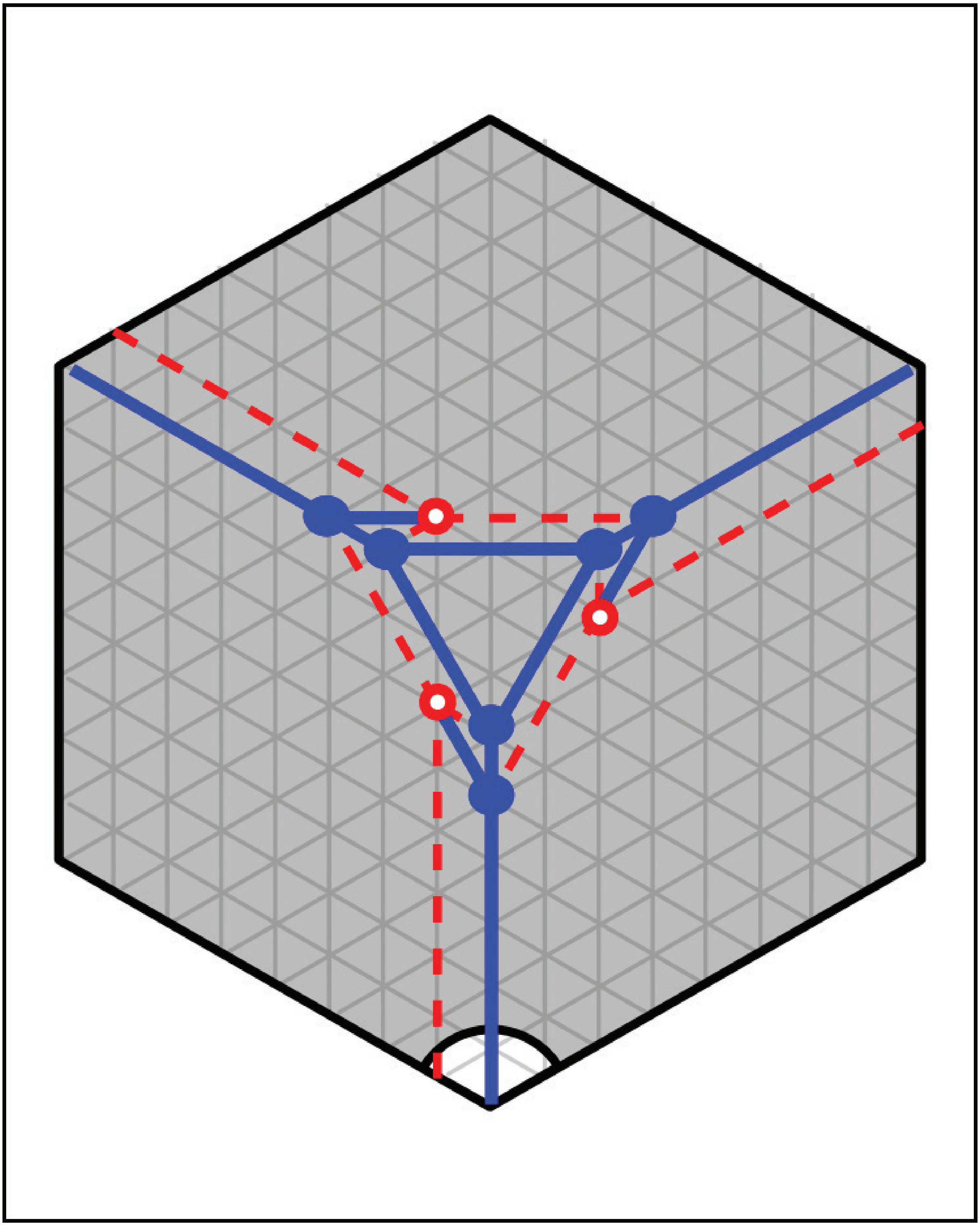
Reverse



8. Using the same process of opening the center while keeping the pleats folded, expanding the twist, forming the outlines of the next row, and reflattening, you can continue the progression to make even larger expansions. This shows the second step of the progression.

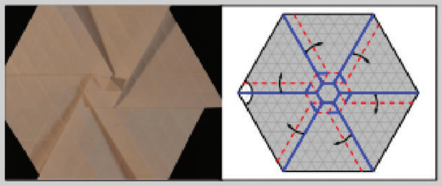


Reverse

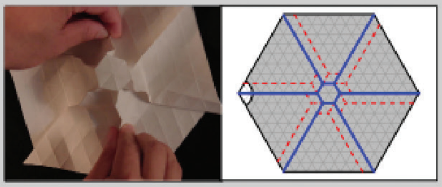


Hex Twist Expansion

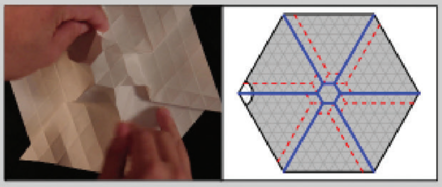
As with the triangle twist expansion, the hex twist expansion involves pinching opposite pleats in the twist, pulling the twist apart, forming a plateau of a large hexagon around it, and flattening.



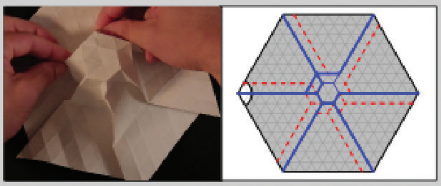
1. Begin with a hexagonal back-twist in the center of the paper.



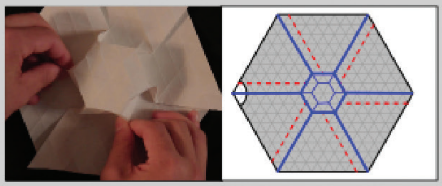
2. Pinch two opposite pleats close to the twist, and pull the center apart. Keep the pleats folded.



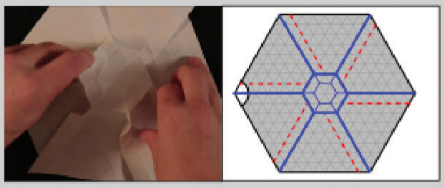
3. The center hex will unfold, revealing another large hexagon around it. Pinch in the sides of that hexagon so that they become mountain-biased creases.



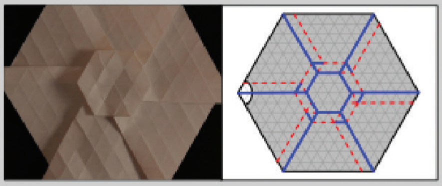
4. This shows two sides of the outer hexagon completed.



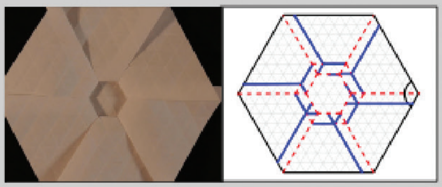
5. All six sides of the hexagon are completed. This will make a larger plateau in the center.



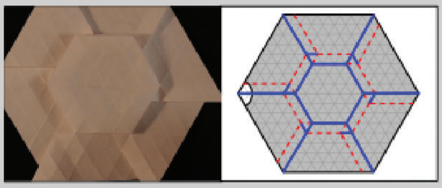
6. Press down on the plateau. This will create valley folds on the outside of the larger hex.



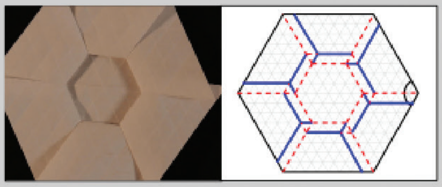
7. This is the finished hexagon twist expansion.



Reverse



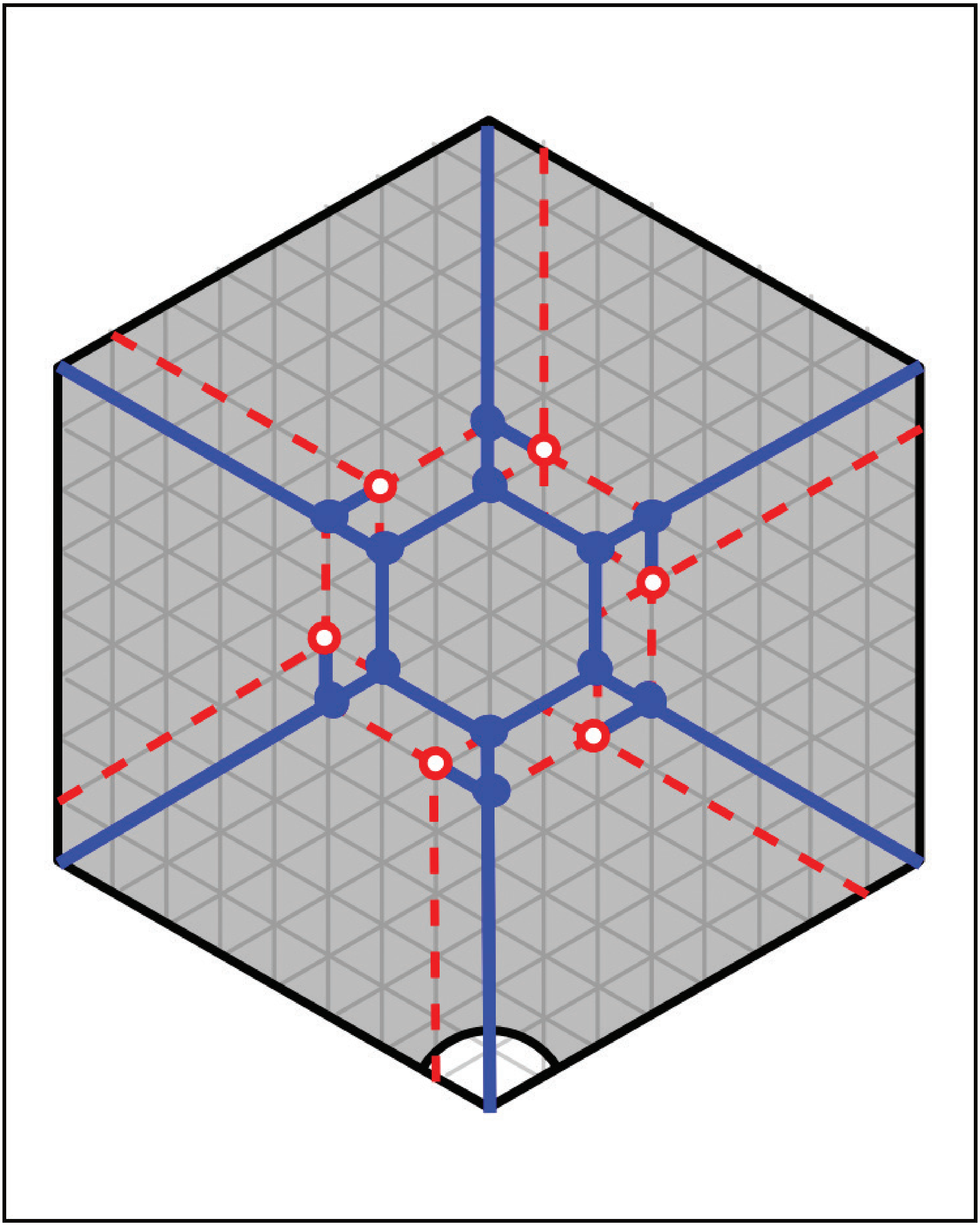
8. This is the next progression of the expansion.



Reverse

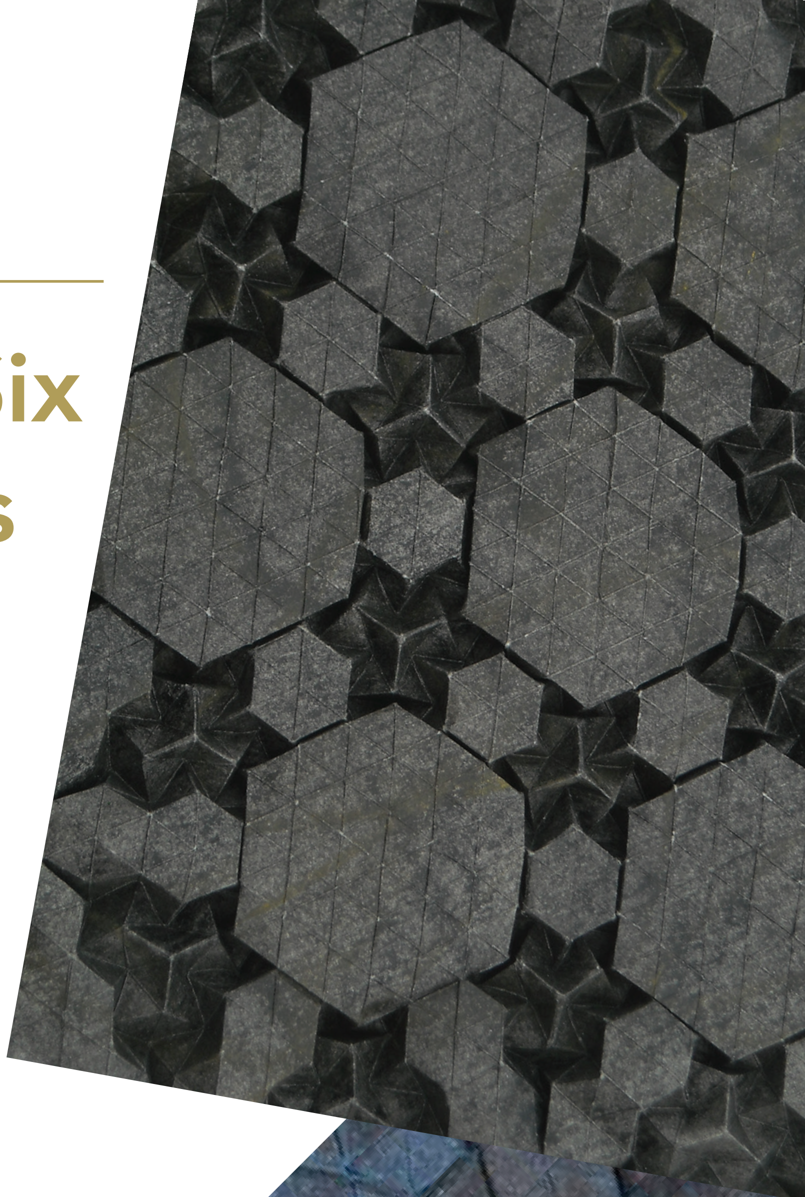
- Now that you know how to expand the triangle and the hexagonal twists, try to expand the arrow twist as well.

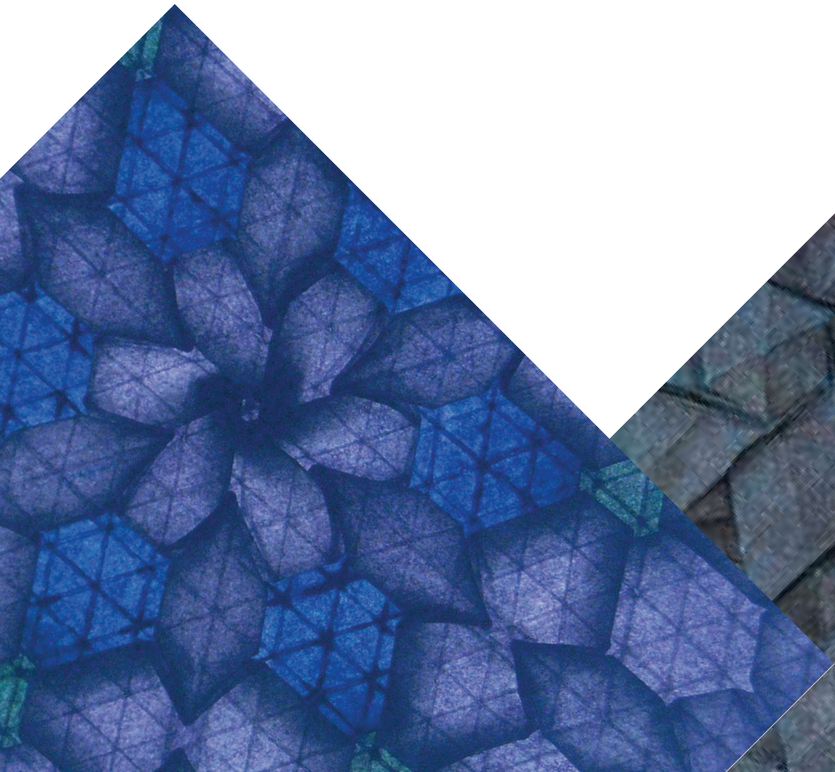
- Try to incorporate this modification into a tessellation. You may need to increase the distance between twists to make it fit.

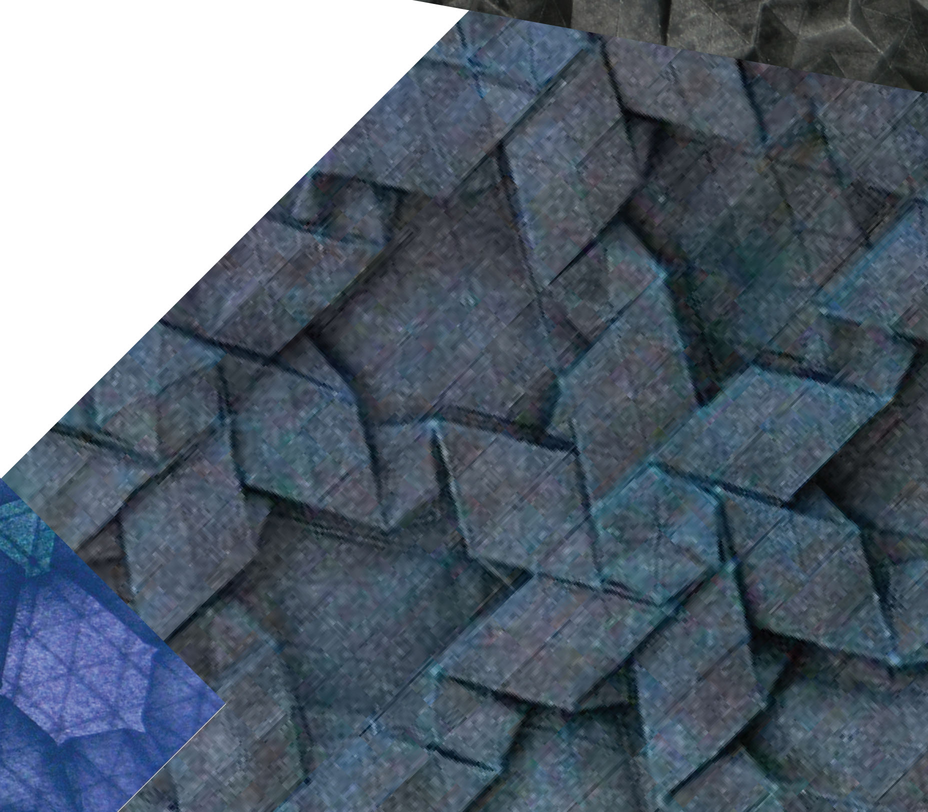


Chapter 3

Beyond the Six Simple Twists



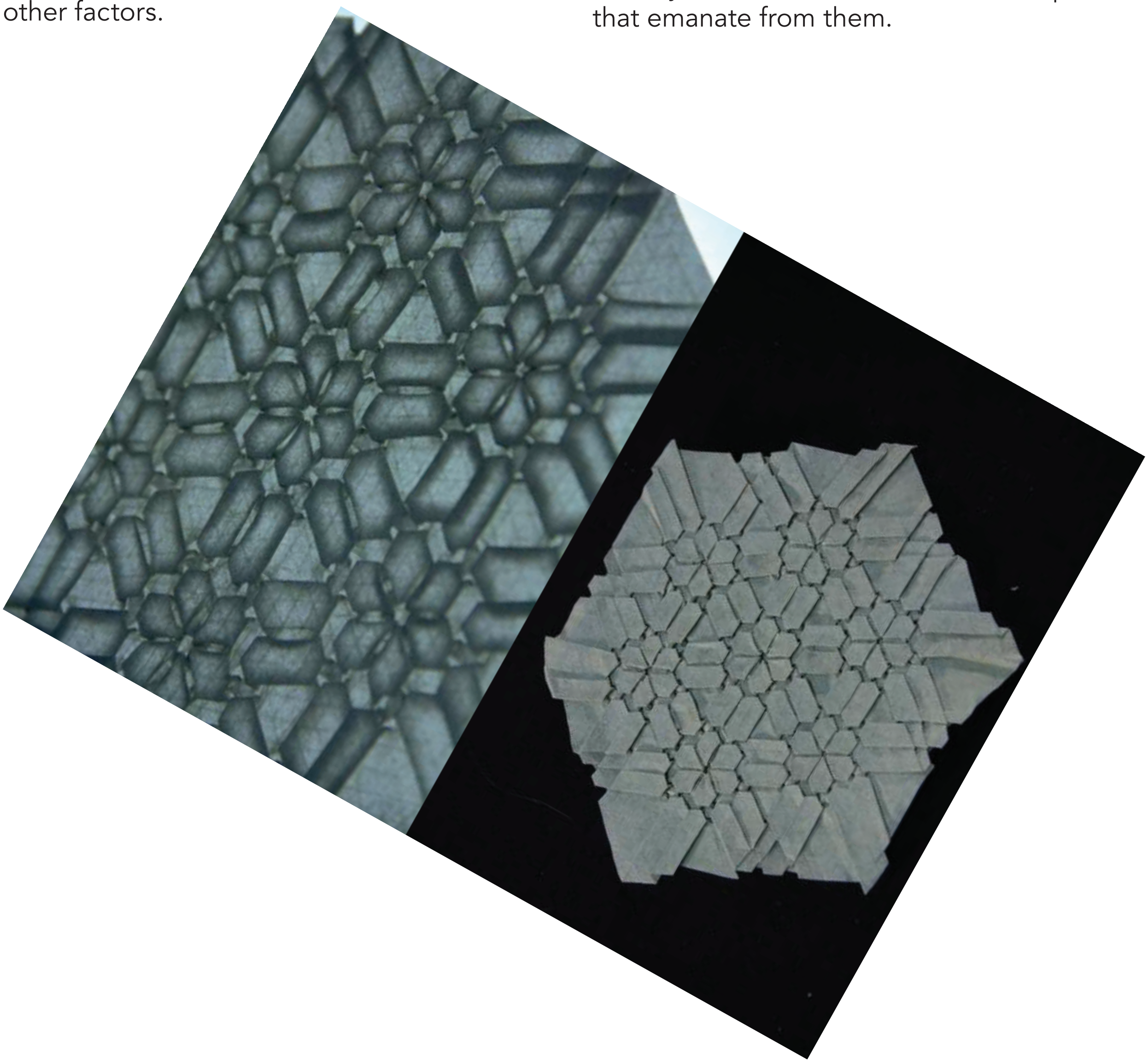




3.1 The Next Step

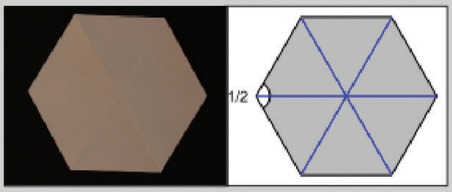
Throughout the various exercises and diagrams so far, we have explored six twists, a few methods of tiling them, and a few modifications. We have identified progressions, a variety of pleat structures, and rotational restrictions. Using these six twists, we have laid the groundwork for you to fold hundreds of tessellations by modifying the twists, the pleat offsetting, the spacing, complexity, and other factors.

There are far more than six twists. The number of possibilities for paper tiling is enormous indeed. Over the years I and other tessellation folders have discovered many different twist designs. Many are modifications of the twists you have learned already, but many are unique. Some are clusters of twists, while some are just different pleat allocations. Some are flat-foldable, while others are not. All are useable in some fashion in a tessellation if you understand how to use the pleats that emanate from them.

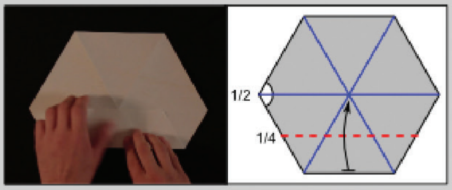


Advanced Gridding Techniques

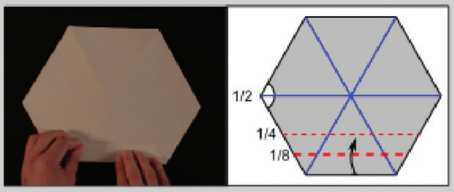
An issue arises with the process of detailing as you get denser grids. It is possible to go as small as your fingers will allow, but the physicality of the paper offers its own constraints. One of the dangers is that as the grid gets denser, the creases that are nearest to the edges of the paper get closer. If you are not careful with backcreasing, the edges can tear. To circumvent this, you can prepare the grid with what I call the grid skeleton. I find that starting a grid with a skeleton allows the folder to focus on the edges at the beginning and speed up after the skeleton is created.



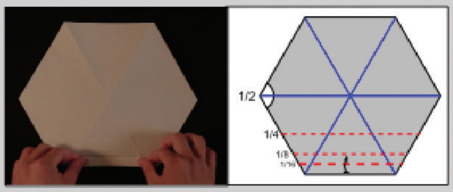
1. Begin with a hexagon with all of the diagonals folded with mountain-biased creases.



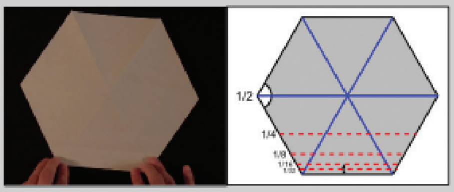
2. Valley fold from the closest edge to the center line and unfold to make the 1/4 line.



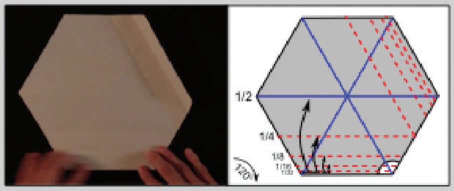
3. Valley fold from the closest edge to the crease formed in step 2 and unfold to mark the 1/8 line.



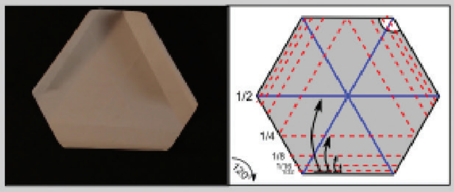
4. Valley fold from the closest edge to the creased formed in step 3 and unfold to mark the 1/16 line.



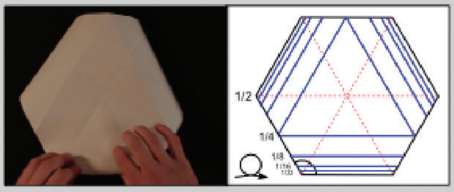
5. Valley fold from the closest edge to the crease formed in step 4 and unfold to mark the 1/32 line.



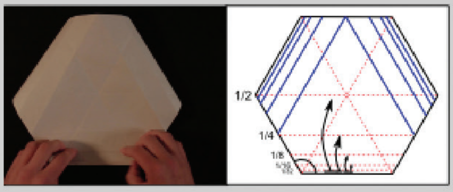
6. Rotate 120° counterclockwise and repeat steps 2–5 on this edge.



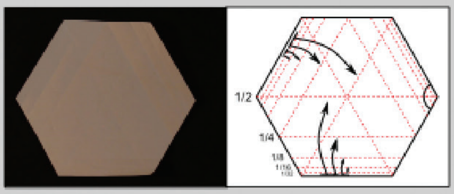
7. Rotate 120° counterclockwise and repeat steps 2–5 on this edge as well. Notice only three of six sides are completed, but none of the creases cross each other. This helps with accuracy. Before we fold the remaining three edges, we'll backcrease these.



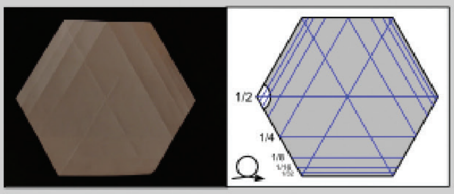
8. Flip the paper to the valley side. You should see mountain folds on this side. Backcrease them to return the bias to uniformity.



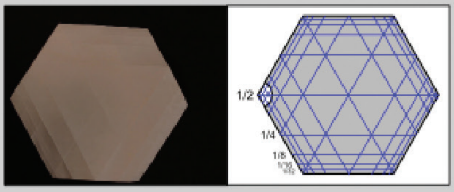
Backcreasing of the first edge



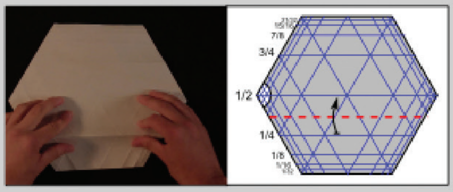
9. Backcrease the remaining two edges, rotating as needed.



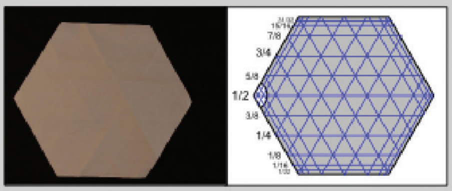
10. Flip the paper to the mountain side.



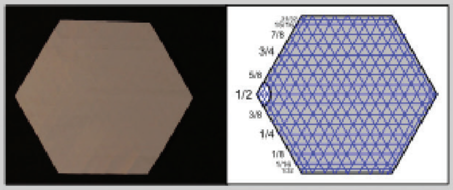
11. Repeat steps 2–9 on the remaining three edges. Once you have done all six edges, you have a finished grid skeleton.



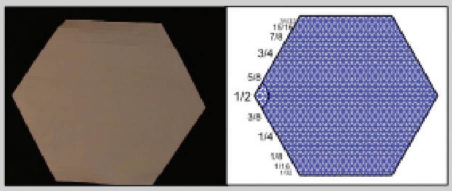
12. To detail, pinch the grid line one away from the center line and valley fold into the center line. Construct the rest of the 8ths grid as we did in the first gridding diagram.



13. Rotate as needed, and fold the 8ths grid with backcreasing.



14. Rotate as needed, and fold the 16ths grid with backcreasing.



15. Rotate as needed, and fold the 32nds grid with backcreasing.

- Did the skeleton make it easier to fold the denser grid? If not, keep experimenting until you find a method that suits you better.

- Using the 32nds grid, fold some of the other tessellations that we saw earlier in the book. You should be able to see the full patterns more clearly because you can get more iterations.

- Are you adventurous enough to fold a 64ths grid using the same method? Challenge yourself and find what you are comfortable with. Try it with a larger sheet of paper if you want, and see how far you can go.

3.2 Designing a New Tessellation

I am often asked whether I plan the pattern out before I fold, and generally the answer is no. Sometimes I have a particular twist I want to use and figure out the tiling as the pattern goes on. There are three levels I consider during the design process: the format level, the twist level, and the decoration level.

The *format level* of a tessellation refers to the tiling and density of the pattern. Will you use a hex twist with triangular twists surrounding it, as in a 3.6.3.6. tiling? How far apart will the twists be from one another? Will a pleat emanating from one twist connect directly with a pleat emanating from its neighbor, or will the twists be offset from one another as in a flagstone tessellation?

The *twist level* of a tessellation refers to which twists and pleats you will use. A tessellation composed of spread triangle twists has the same format as a tessellation composed of regular triangle twists. When designing, ask yourself if you use triangle spreads for the places where three pleats intersect. If you have a format that uses a hexagon in the center, will you use VM pleats or something more complicated, such as VM2MV pleats?

The *decoration level* happens after the pleats have been set, when the folder decides whether to modify the twists or modify the space between the twists. An example of this is the twist expansion progression.

Any of these questions can be answered prior to or during the folding process.

3.3 Describing Twists

As you progress with tessellation experimentation, you will discover more twists. Each twist will have a specific pleat assignment and a specific way the pleats collide in the middle. Using decoration, you may find that two twists can have the same pleat assortment but entirely different center creases, as in the triangle twist and the iterations of the triangle twist expansion progression. This leads to a method of cataloguing twists. If a folder knows which pleats create the twist, the angle at which the pleats intersect, and how much the pleats are offset from one another, it answers most of the questions necessary to fold the twist. On a triangle grid, the pleats will generally intersect in one of the formations in Figures 3.1–3.5 with variants to the pleat structure and point(s) of intersection.

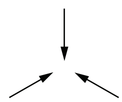


Figure 3.1

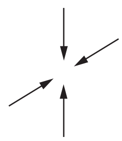


Figure 3.2

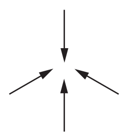


Figure 3.3

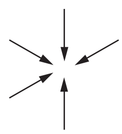


Figure 3.4

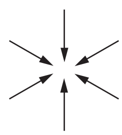


Figure 3.5

However, once you transmute the concepts in this book to other grid types, or even remove the grid and simply calculate what you want on the open paper, many more intersection formations are attainable.

So, to create a nomenclature for a twist, we need these factors: pleat formation, angle of intersection, and offset. I call this naming scheme a *pleat pattern* of a twist. This differs from a crease pattern of a twist since it is not a full schematic of the creases in a twist. Yet, it offers enough information about the twist to convey how to fold it.

To draw the pleat pattern of a twist, perform the following steps:

Step 1: Determine the *driving* crease of each pleat in the intersection. The driving pleat will default to the mountain fold. If there are multiple mountain folds, it will be the first crease that is crossed rotating counterclockwise around the intersection.

Step 2: Draw only the driving crease of each pleat in a bold line starting from the same distance away from the intersection and have them approach the center of the intersection at the same rate.

Step 3: When one driving crease hits another, it stops. Continue all such creases until they hit another.

Step 4: If the intersections of all of the driving creases are not connected, connect them with bold lines. These are called *connecting lines*. Connecting lines are not folded, but are used in the pleat intersection description once the pleat pattern is drawn. There may be several ways to draw the connecting lines. It will not affect the way the intersection is folded, only the way it is described below.

Step 5: Using the driving creases as a base, construct the rest of the pleat, beginning each crease the same distance away from the center of the intersection and drawing them at the same rate toward the center. Stop a nondriving crease of a pleat when it hits the driving crease of any pleat or one of the lines made in Step 4.

Now, how do we describe this schematic? To name a twist using its pleat pattern:

Step 1: Determine the crease allocation of the top pleat of the intersection. If there is not one at the top, rotate counterclockwise until you find one. Write the pleat's crease allocation using M's for mountains, V's for valleys, and a numerical unit to denote the relative separation between the creases. If the distance is one unit, you may omit the number.

Step 2: Rotate counterclockwise until you find the driving crease of another pleat or a connecting line. Mark the angle difference in superscript next to the first pleat's crease allocation. If the angle is greater than 180°, rotate clockwise instead and write the angle as negative.

Step 3: If the next item counterclockwise is a pleat, determine the crease allocation of the second pleat. Write the pleat's crease allocation after the superscript angle.

If the next item counterclockwise is a connecting line (made in Step 4 of the previous section), continue to Step 4.

Step 4: If the driving crease or connecting line you just described connects to the next angle without extending beyond or retracting from the previous angle of rotation, continue to Step 5.

Step 4a: If the driving crease or connecting line extends beyond the previous angle of rotation, mark the distance of extension with +X, where X is the number of units of the extension.

Step 4b: If the driving crease or connecting line retracts from the previous angle of rotation, mark the distance of retraction with −X, where X is the number of units of the extension.

Step 5: Rotate again counterclockwise until you hit the driving crease of the next pleat or the next connecting line counterclockwise. Mark the angle difference in superscript next to the first pleat's crease allocation. If that angle is greater than 180°, rotate clockwise instead and write the angle as negative.

Step 6: Repeat Steps 1–4 counterclockwise for each pleat and connecting line until each pleat, connecting line, and angle have been marked. The angles should sum to 360°.

Step 7: If the pleats and connecting lines are divisible into 360° symmetrically, you may shorten the description to the number of divisions multiplied by the pattern of each division. In this case, the multiplier times the superscript angle will sum to 360°.

Let's use the pleat pattern and schematic description steps to describe the intersection of the triangle twist: Figure 3.6 shows the naming process based on two pleat intersections.

The first intersection could be used to create a triangle twist or triangle twist expansion. The second is more complicated and has different twists that can be created from it, as shown in Figure 3.7.

See if you can describe the intersections in Figure 3.8 using this method.

Finally, Figures 3.9–3.14 show the pleat patterns for the six simple twists.

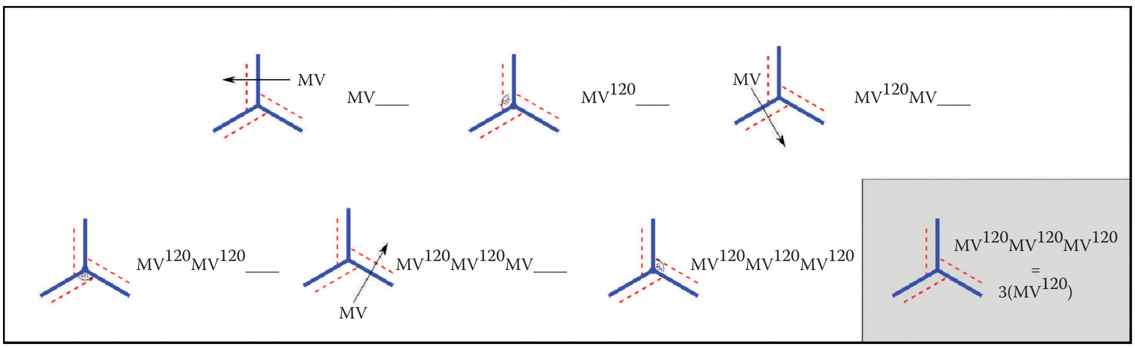


Figure 3.6

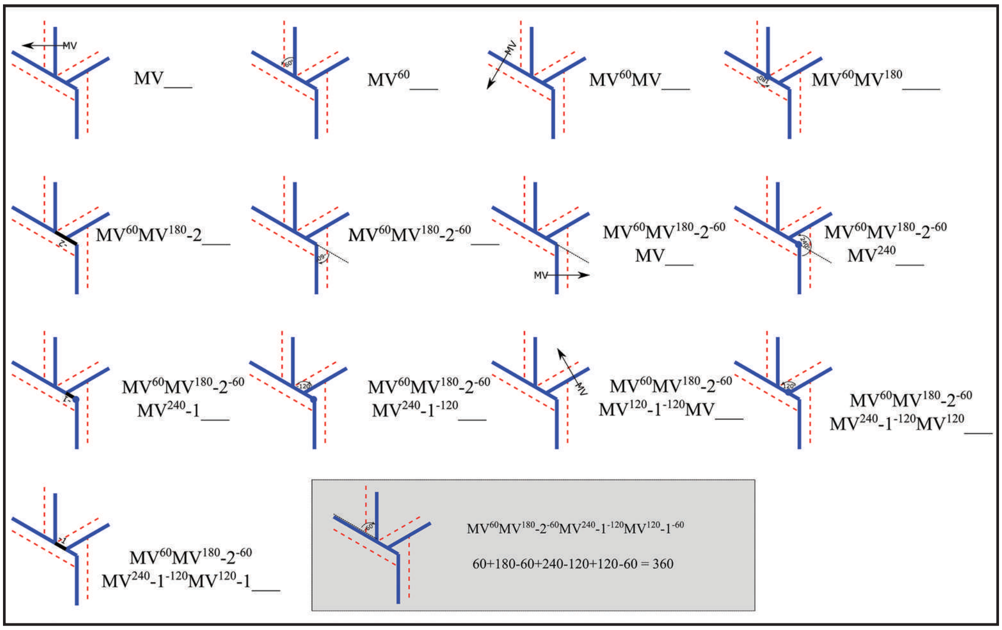


Figure 3.7

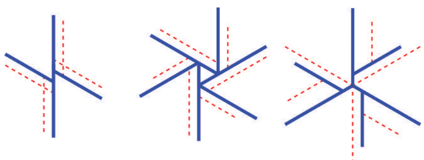


Figure 3.8

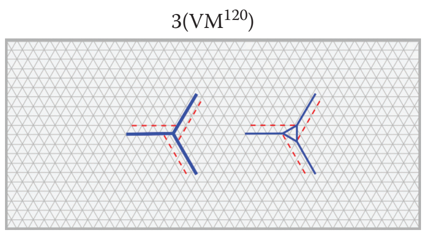


Figure 3.9

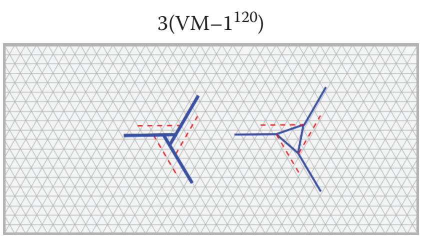


Figure 3.10

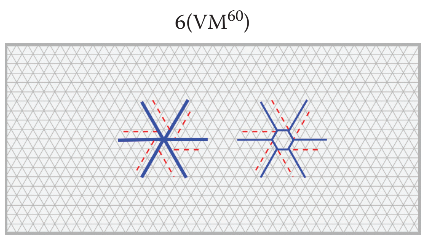


Figure 3.11

$6(VM\text{–}1^{60})$

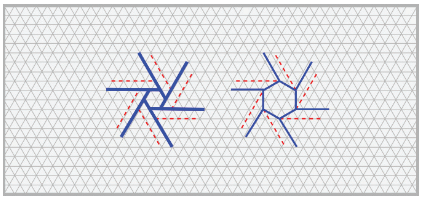


Figure 3.12

$2(VM^{60}\text{–}1VM^{120})$

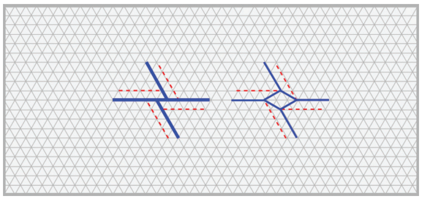


Figure 3.13

$V2M^{120}VM^{60}VM^{60}VM^{120}$

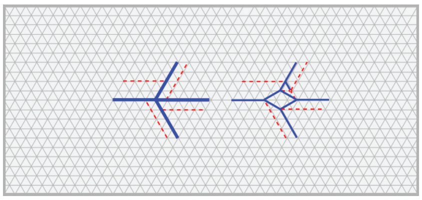


Figure 3.14

3.4 We Don't Care about the Middle

At least, we don't care about the middle for the sake of the name. This naming scheme merely provides a structure that will give a direction to build a twist from. Several different twists can be folded from the same pleat pattern.

The power of such a system is the third method of documentation beyond the traditional diagrams and the exacting crease patterns. Pleat patterns and crease patterns often work in conjunction. Coupled with a photograph, an experienced folder can figure out how the twist is formed from the pleat allocation alone.

3.5 Tips for Tricky Twists

There is no universal rule for understanding the process for folding a twist. Sometimes it is easier to precrease everything that's not on the grid (or reinforce grid lines' biases) to make a twist fold flat; this is the principle behind the referential method. Other times it's easier to use the tension in the paper to guide certain other sections of the paper to their destination; this is seen in the hex twist, where folding the surrounding paper codes the paper to fall into the right place. There is, however, a powerful rule of thumb that I use when folding: the Mountainous Rule. The *Mountainous Rule* says that you have more control over the paper that comes toward you than the paper that goes away from you.

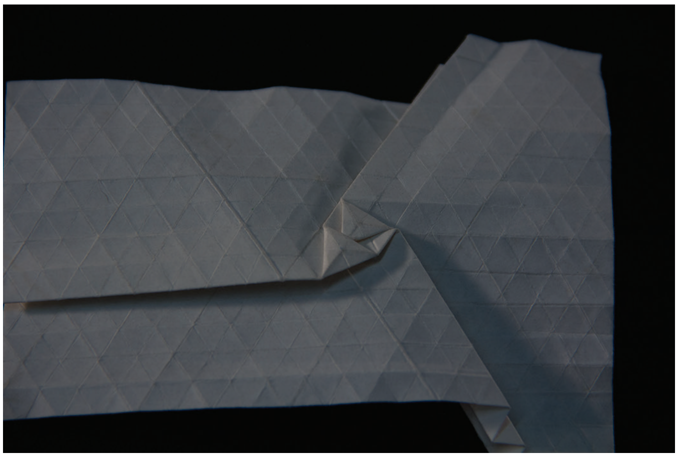
Imagine you are standing on a sheet of paper that extends for miles in all directions, such that you cannot see the edges of the paper. However, there is enough slack in the paper that you can make a pleat. How do you fold this if you cannot reach to the reverse of the paper? In tessellations, you are often folding in the center of a sheet without regard for the edges of the paper. In order to make a pleat, you need to pinch a mountain fold and then fold it over (which will create a valley fold as well). If you don't flip the paper (and you can't in the above scenario), then the only way to make a valley fold is with a pleat, and the only way to make a mountain fold is with a pinch.

Likewise, when pleats intersect, an area between where they intersect is pushed off of the plane, either toward the folder (mountain) or away (valley). Twists are easier to manipulate if the material is rising toward the folder than away. This is because if the material is going away from the folder, static sections of the paper block the folder's vision of what the paper is doing. Additionally, you do not want to have to reach to the other side of the paper until the twist is complete. Then, if you wish to place the twist on the other side of the paper, it is easier to flip the paper and again make the twist area come toward you.

This notion may seem intuitive after practice, but it is worth acknowledging. If you are experiencing difficulty setting a twist, push the paper you want to make the twist with toward you and set the pleats all the way to the edge of the paper (or as far as they can go). This will give the twist structure that you can fill in afterward. If one of the pleats is off, move it in the appropriate direction. If there are other features in the way, it may be helpful to unfold them. You can refold them after the twist you are working on is set.

3.6 The Database

This is a good start, but hardly an exhaustive list, of a set of twists and twist clusters that I enjoy folding and feel are a good base to study. I've included the crease patterns, pleat patterns, and photos. Any of these can be used in your tessellation to drastically increase the variety in your designs. Use them in your own patterns, or modify and document the modifications to create your own!



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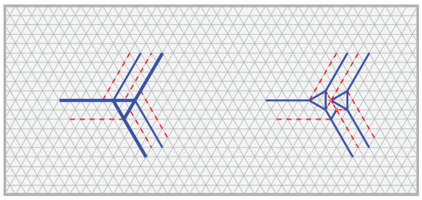
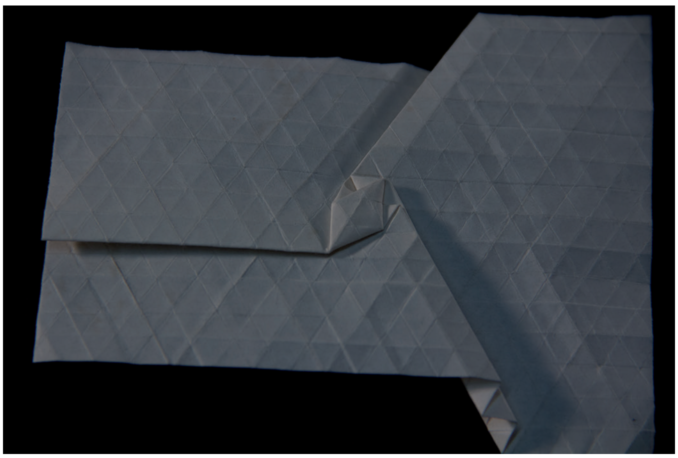
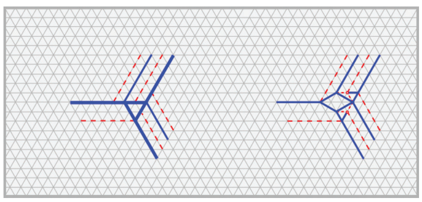
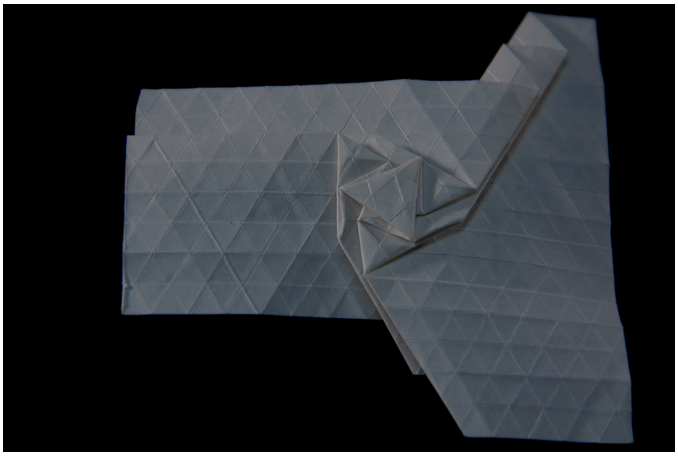


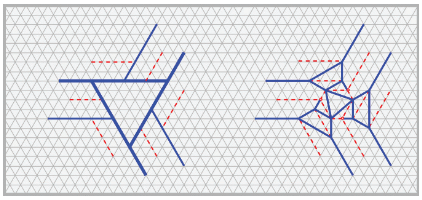


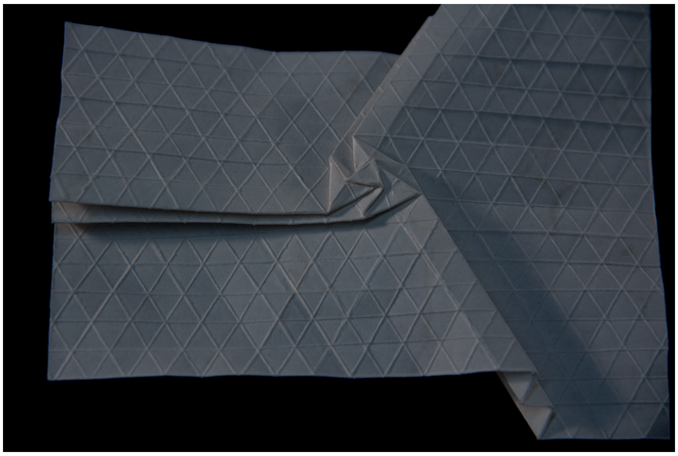
T002
$M2V{-}2^{120}MVMV{-}2^{120}MVMV{-}2^{120}$



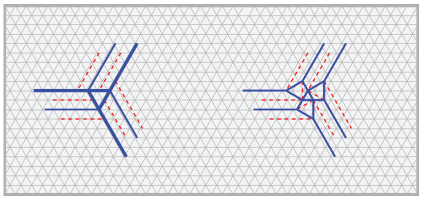


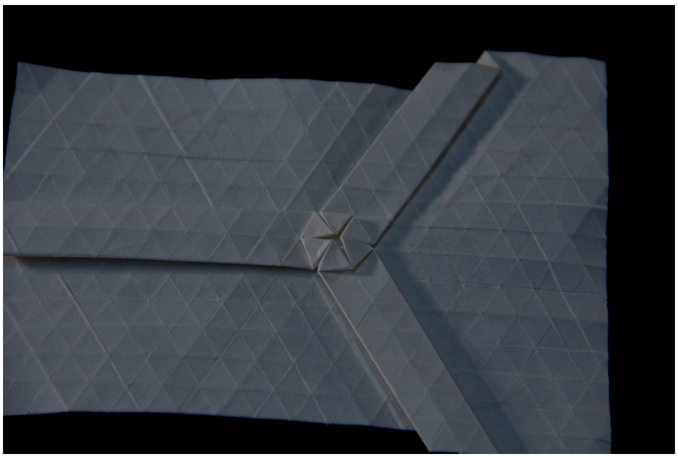
T003
$3(\text{VMVM}^{120}-7)$



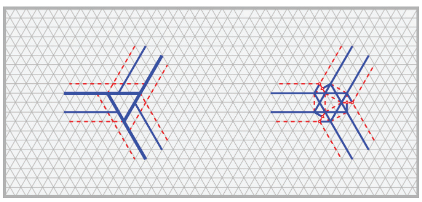


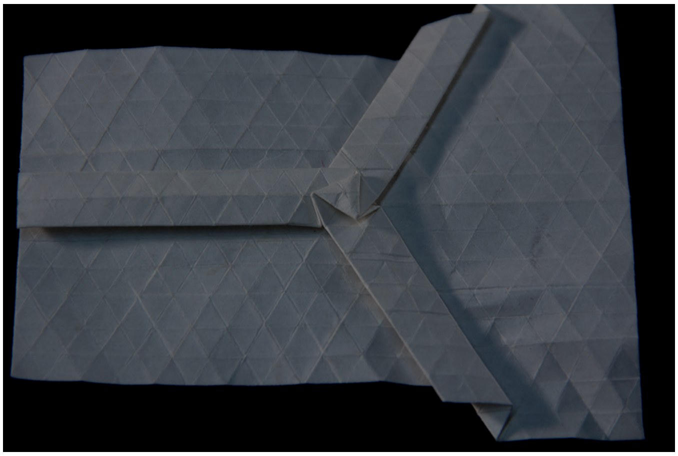
T004
$3(\text{MVMV}-2^{120})$



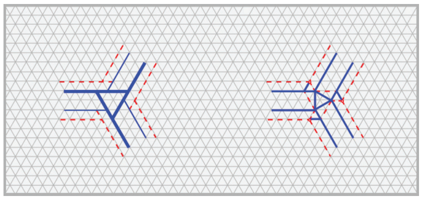


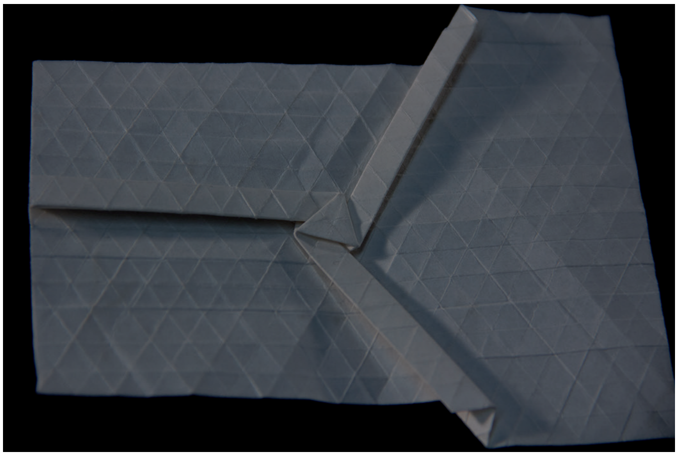
T005
$3(\text{VM2MV}^{120}-3)$



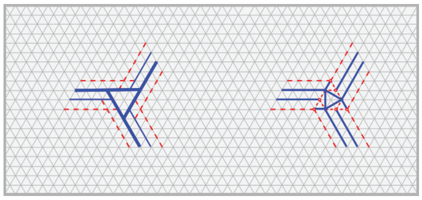


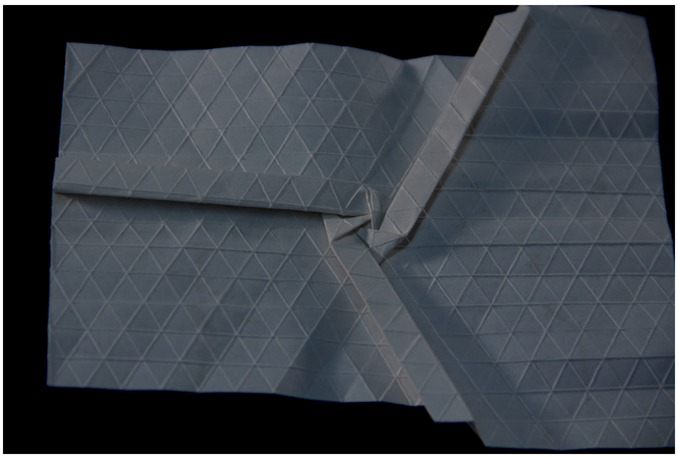
T006
3(VM2MV–3^{120})



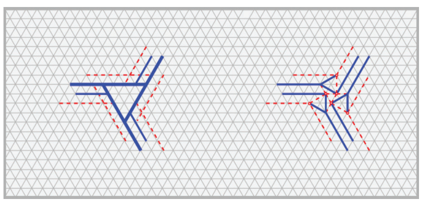


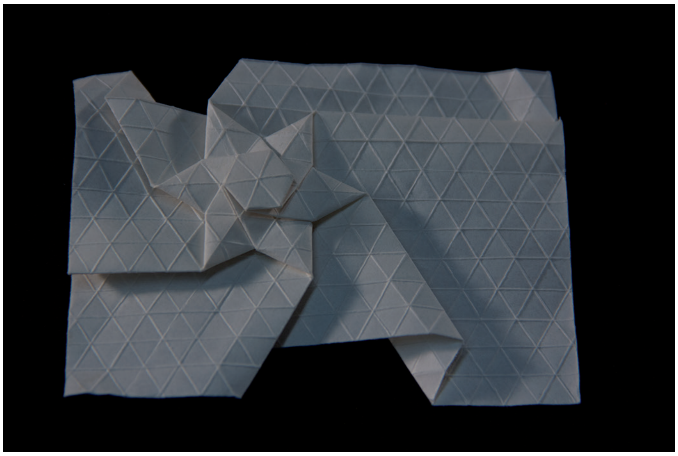
T007
3(VMMV–3^{120})



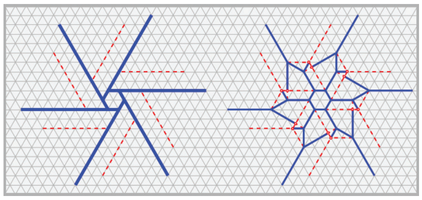


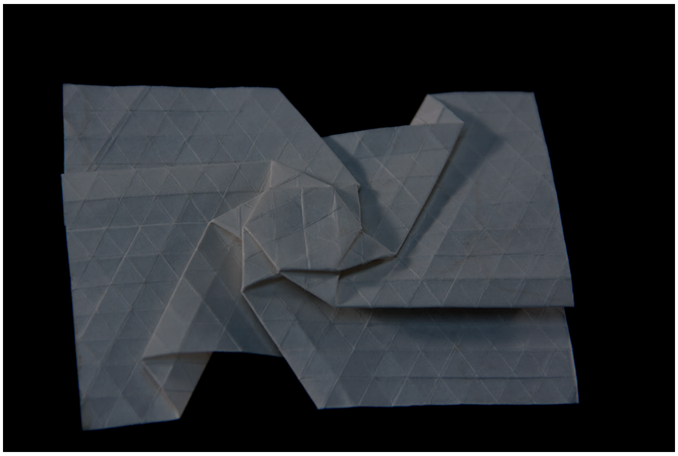
T008
3(VMM–4^{120})



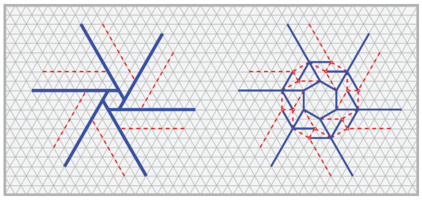


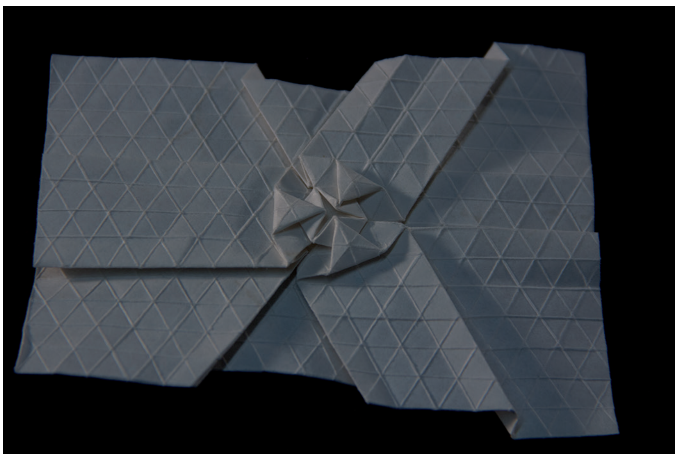
H001
$6(M2V+1^{60})$



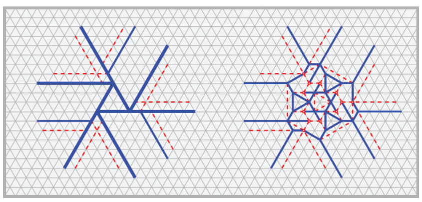


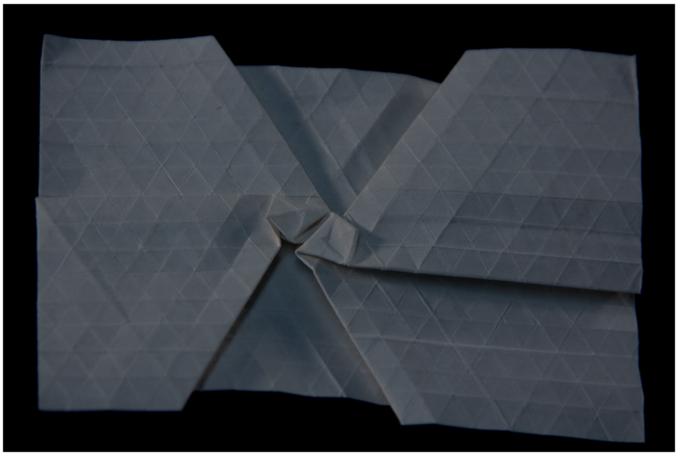
H002
$6(V2M^{60}-1)$



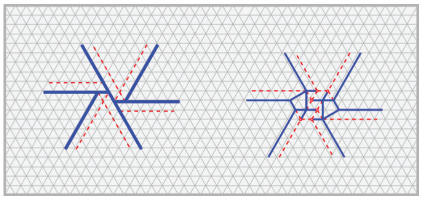


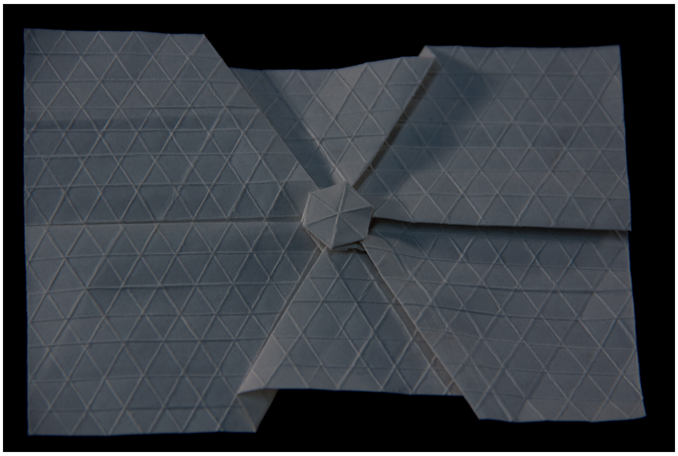
H003
$3(MV^{60}VM4MV^{60}-3)$



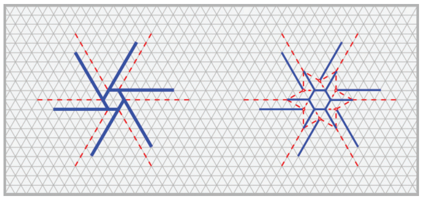


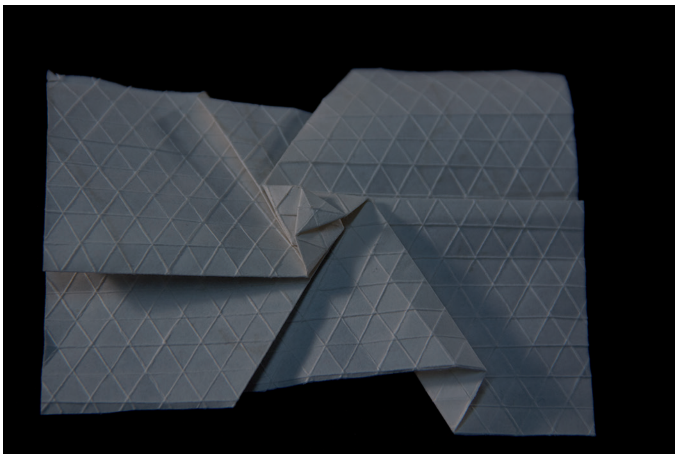
H004
$2(\mathrm{VM}{-}1^{60}\mathrm{VM}{-}1^{60}\mathrm{MV}^{120}{-}1^{-60})$



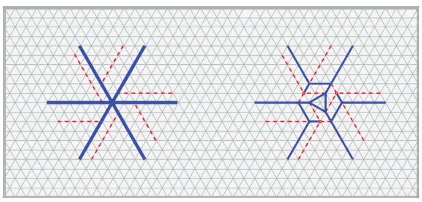


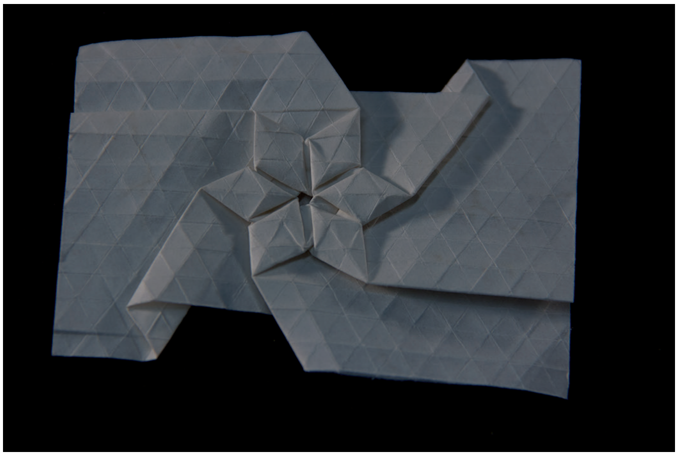
H005
$6(\mathrm{VM}{+}1^{60})$



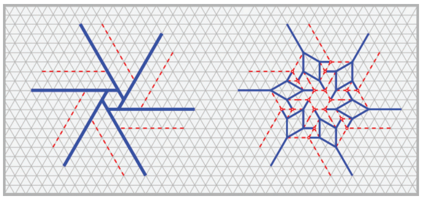


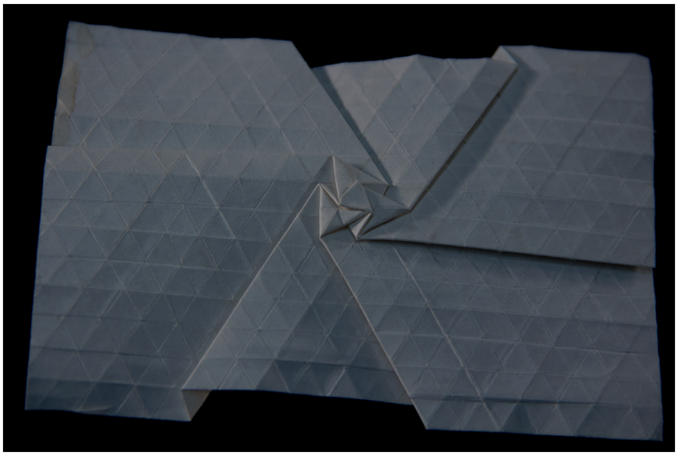
H006
$3(\mathrm{MV}^{60}\mathrm{M}2\mathrm{V}^{60})$



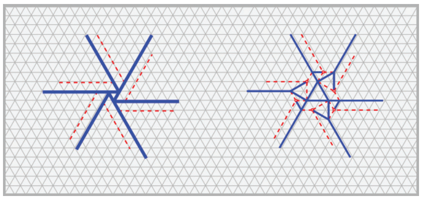


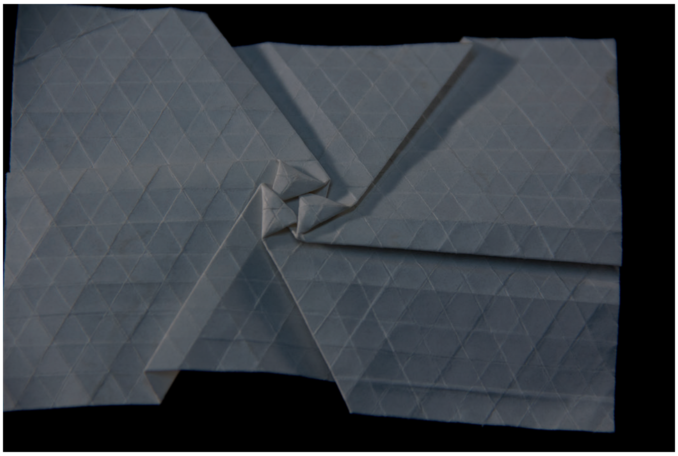
H007
6(V2M–1^{60})



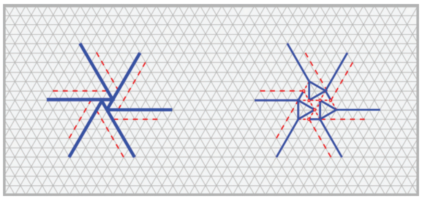


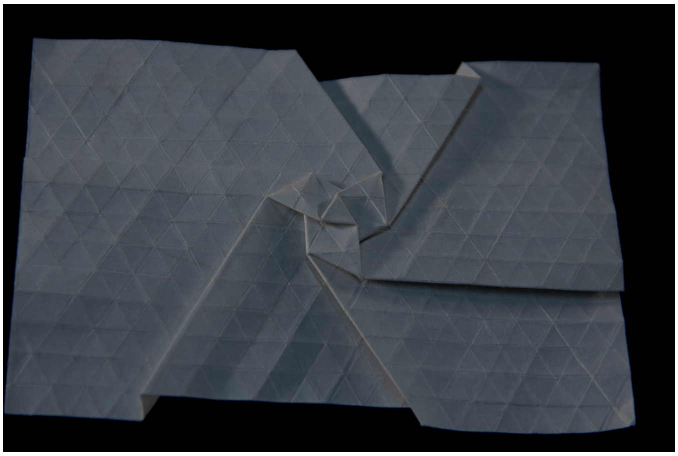
H008
3(VM^{60}VM–1^{60})



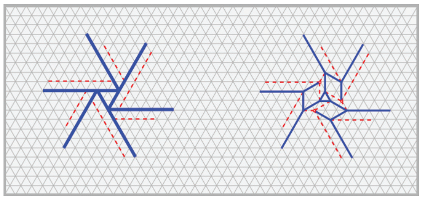


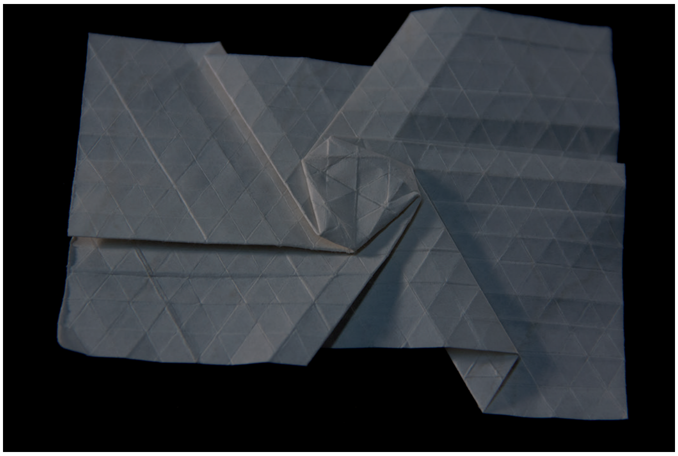
H009
3(VM^{60}VM–1^{60})



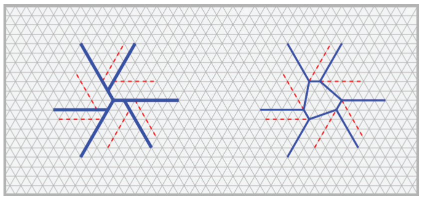


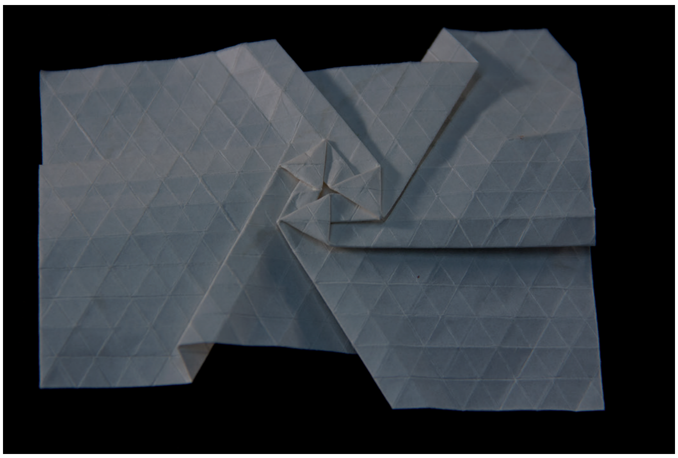
H010
$3(VM^{60}VM{-}2^{60})$



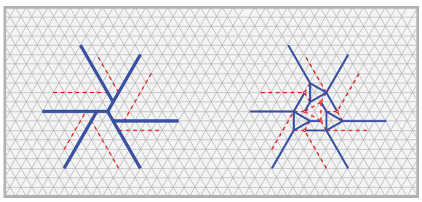


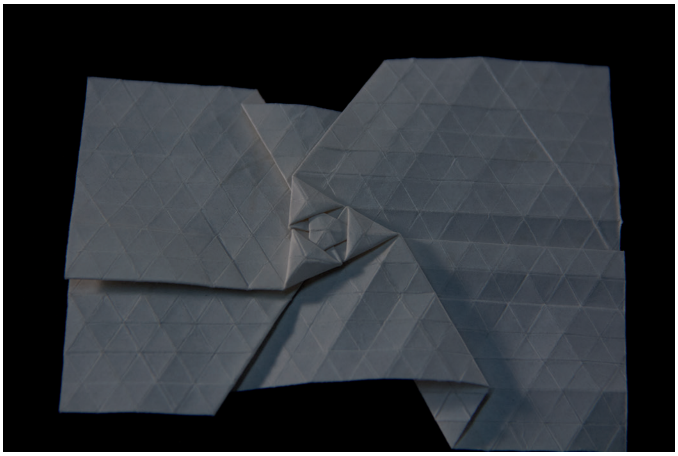
H011
$3(M2V{+}1^{120}{-}1^{-60}MV^{60})$



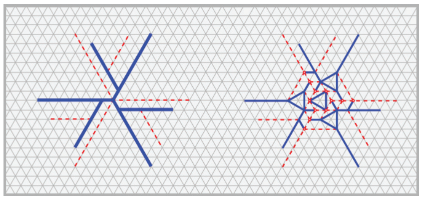


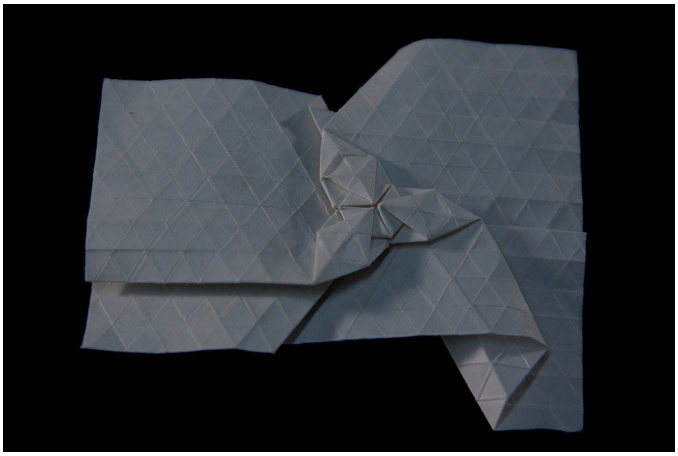
H012
$3(VM^{120}{-}1^{-60}V2M{-}1^{60})$



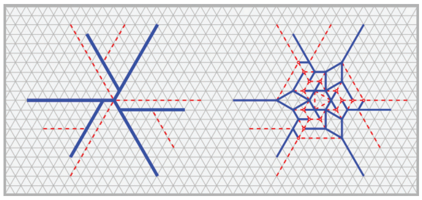


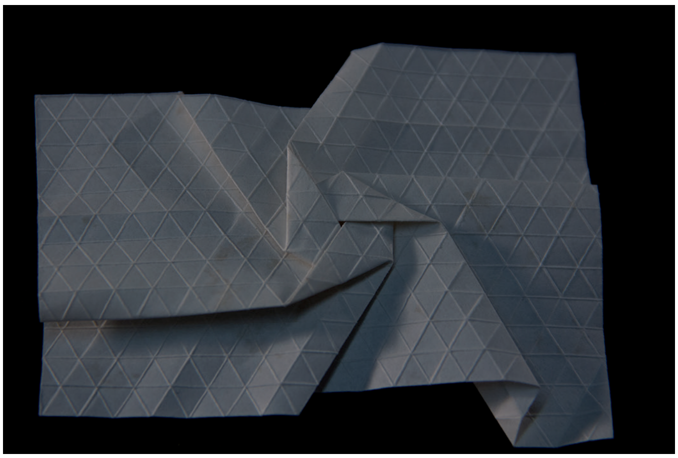
H013
$3(MV^{120}{-}1^{-60}M2V{-}1^{60})$



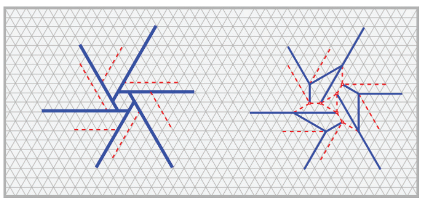


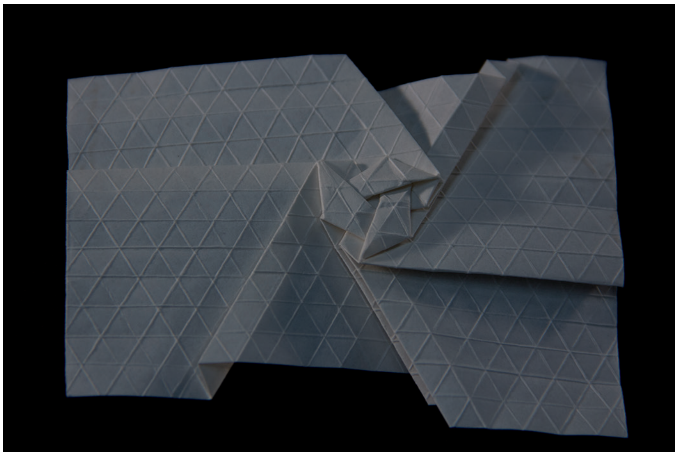
H014
$3(MV^{120}{-}1^{-60}{-}1M3V^{60})$



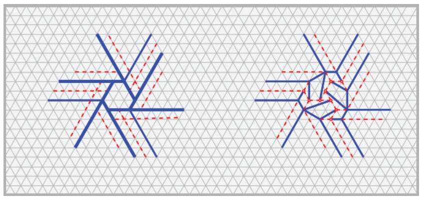


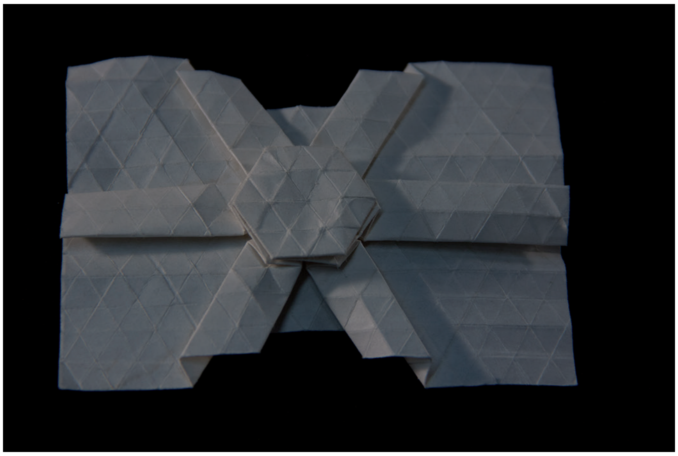
H015
$3(MV{+}1^{60}M2V{+}1^{60})$



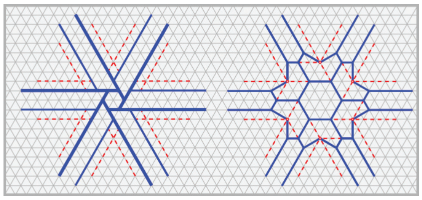


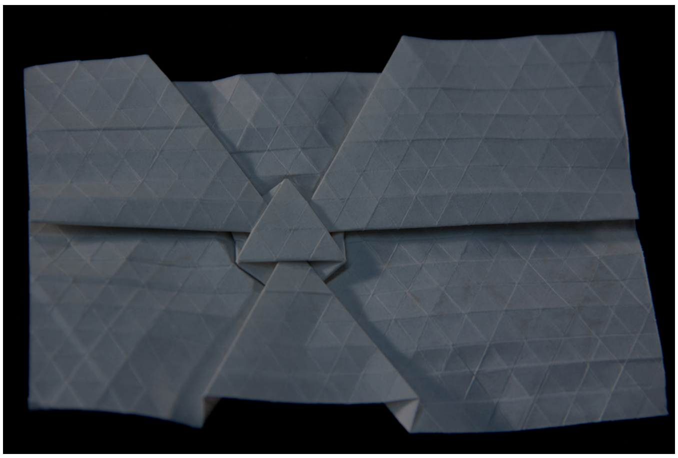
H016
$3(VM{-}2^{60}VMVM{-}1^{60})$



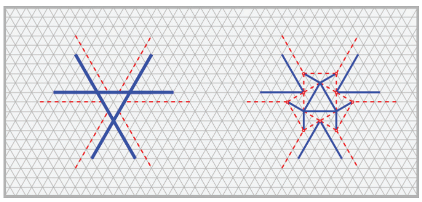


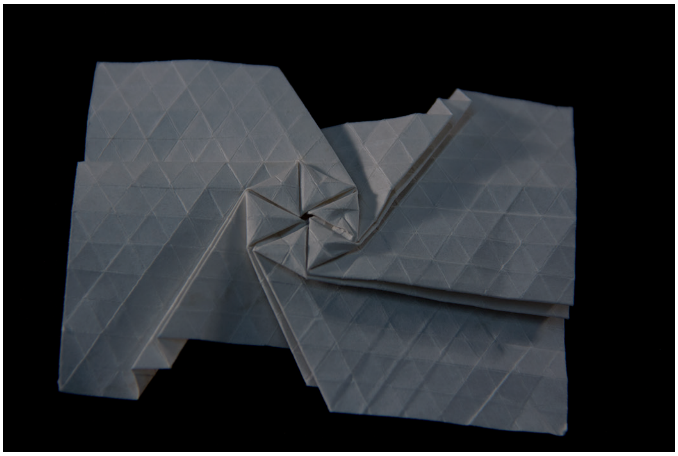
H017
$6(VM2MV{-}1^{60})$



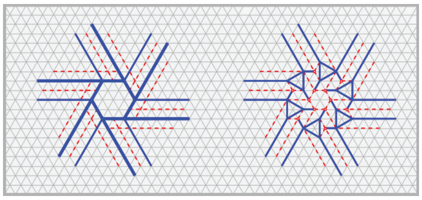


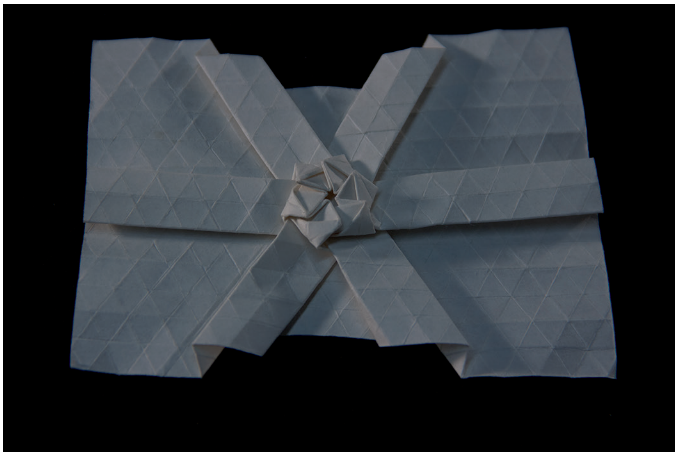
H018
$3(VM^{60}MV^{120}{-}3^{-60})$



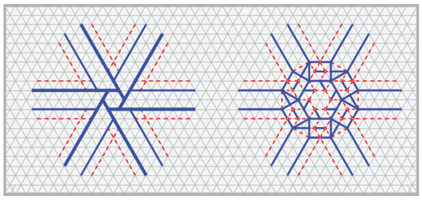


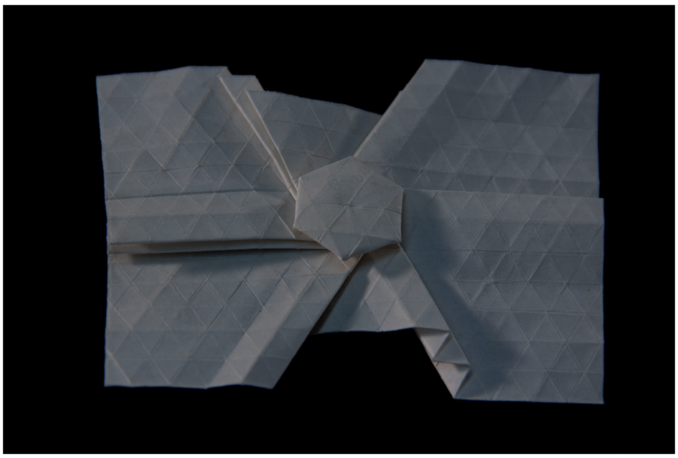
H019
$6(\text{VMVM}-2^{60})$



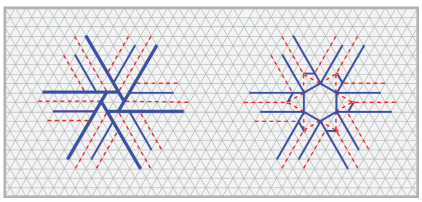


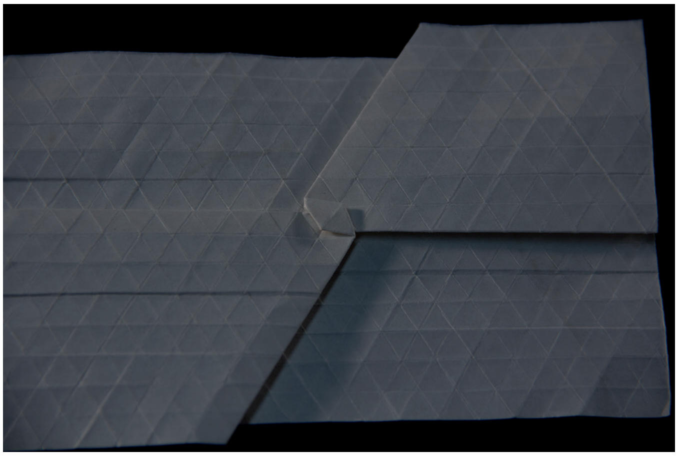
H020
$6(\text{VM2MV}-1^{60})$



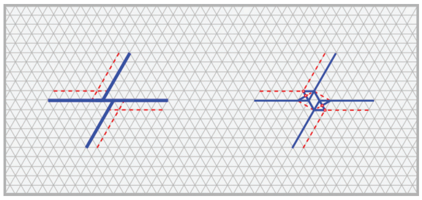


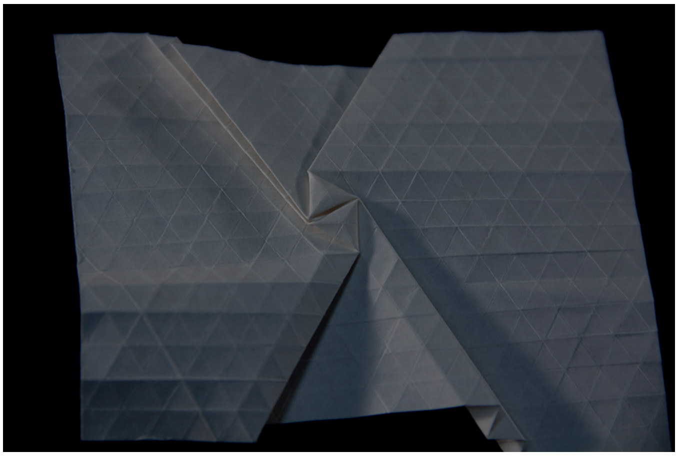
H021
$6(\text{VMVM}-1^{60})$



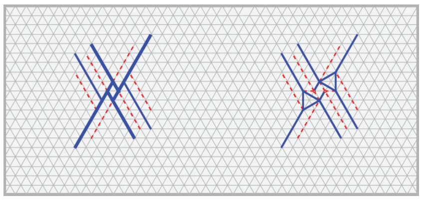


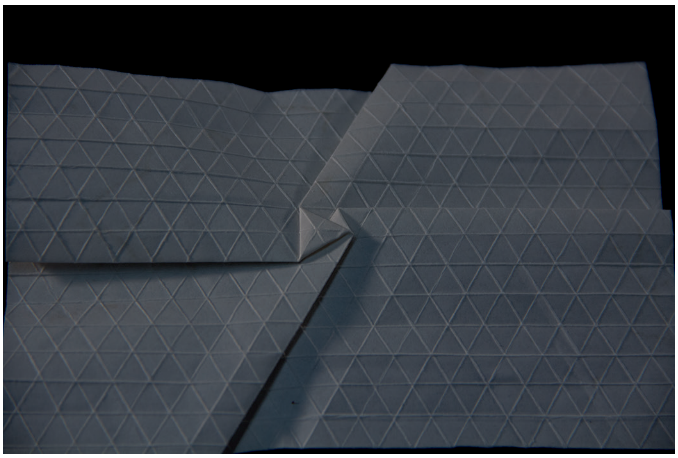
R001
$2(VM+1^{60}MV^{120})$



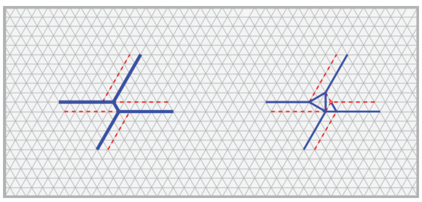


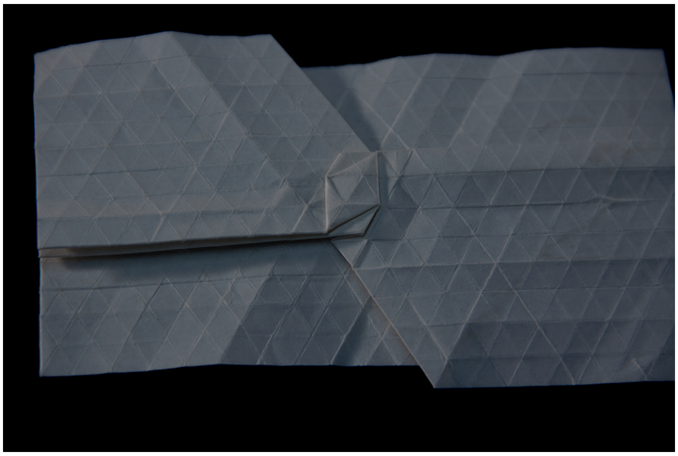
R002
$2(MVMV-1^{120}MV-1^{60})$



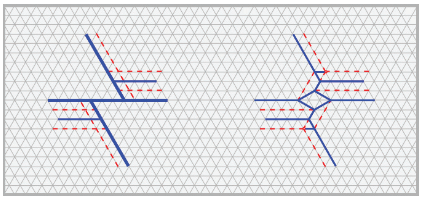


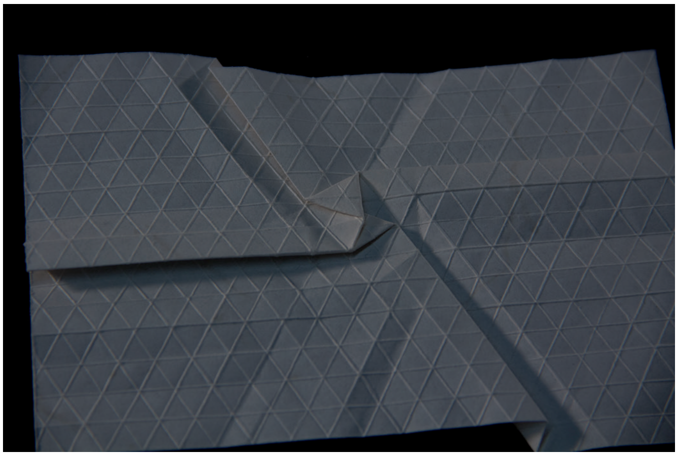
R003
$2(MV^{120}-1^{-60}MV^{120})$



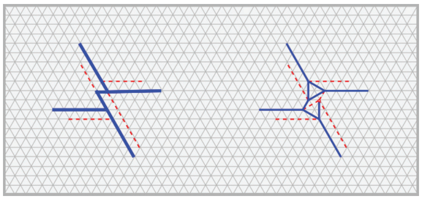


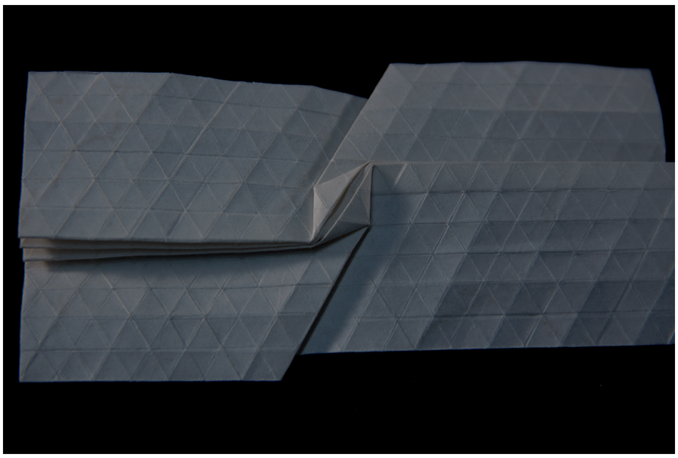
R004
$2(VM^{60}-3MVMV^{120})$



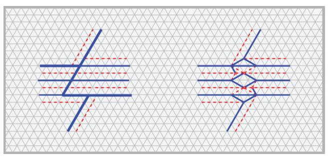


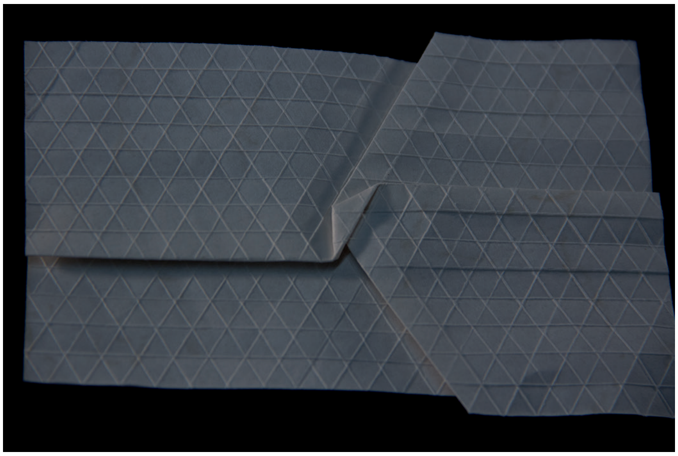
R005
$MV^{60}-1^{120}-2^{120}MV^{120}MV+2^{60}MV-1^{120}$





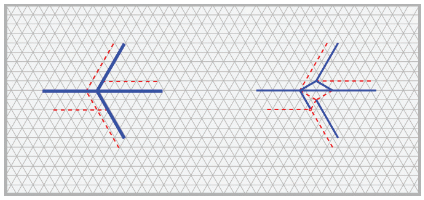
R006
$MVMVMV^{60}\text{-}4^{120}\text{-}3^{-120}MV^{120}MVMVMV+3^{60}\text{-}4MV^{120}$

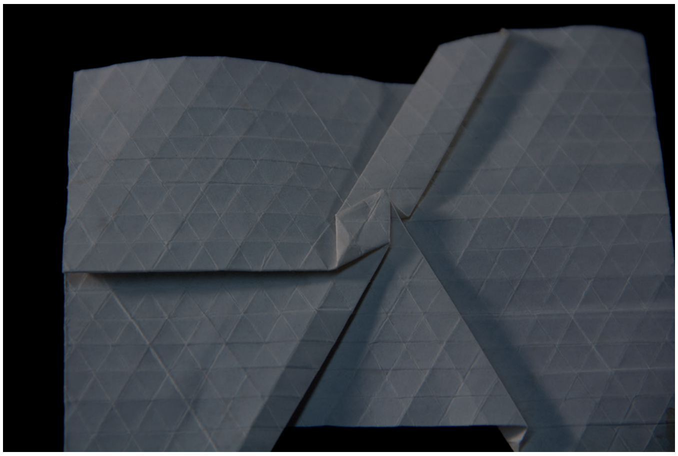




A001

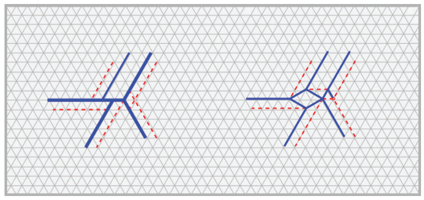
$M2V^{120}VM^{60}MV^{60}MV^{120}$

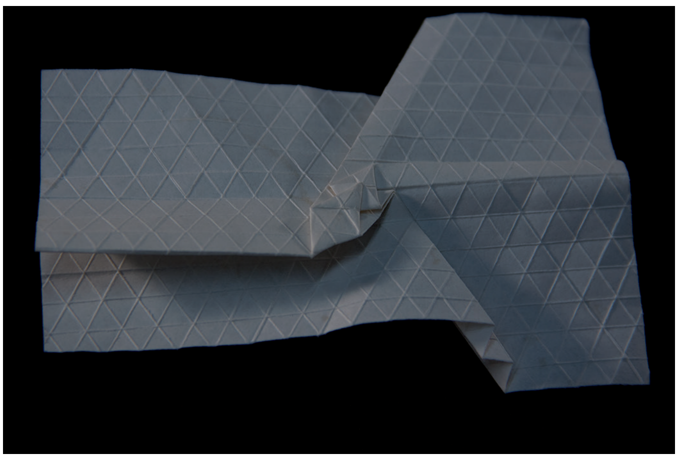




A002

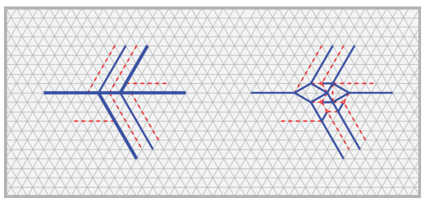
$MV^{60}MV^{120}-1^{-60}MV^{120}VM2MV^{120}$

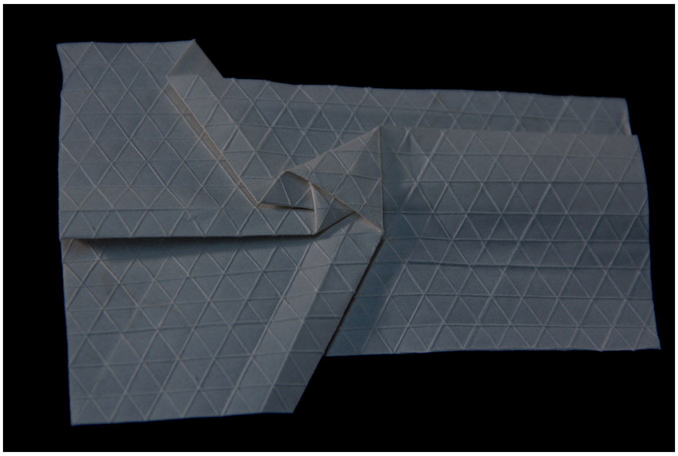




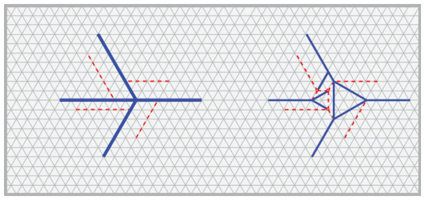
A003

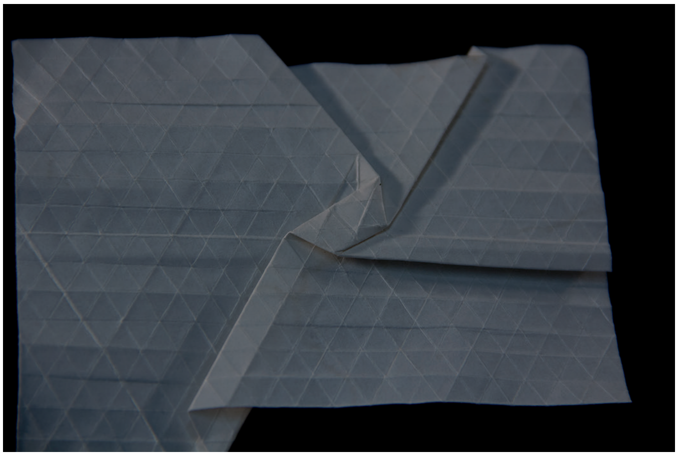
$M3V-2^{120}MVMV^{60}MV-2^{60}MVMV^{120}$



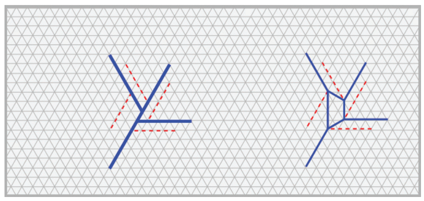


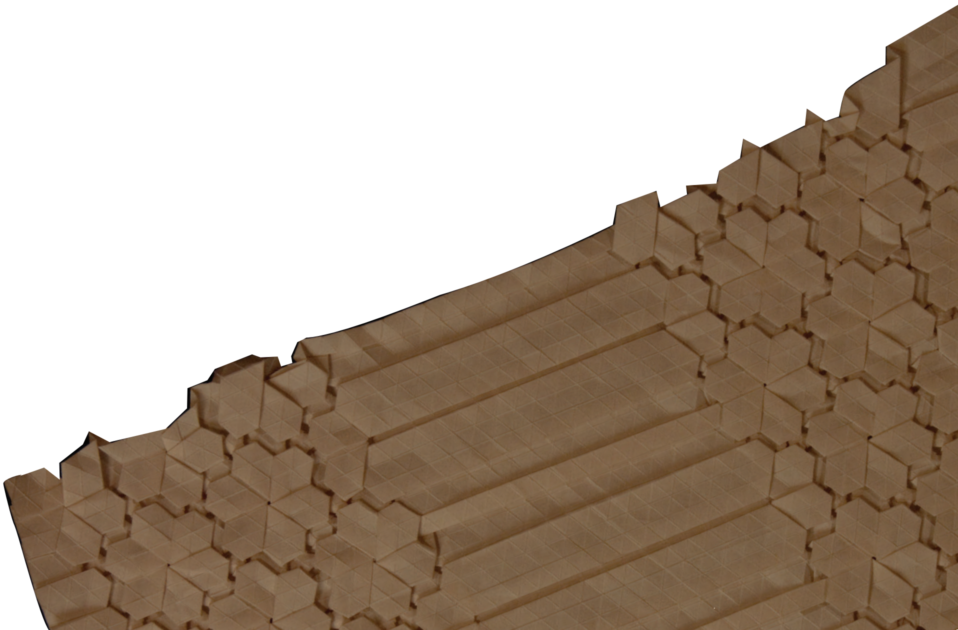
A004
$M2V^{60}MV^{60}M2V^{120}M2V^{120}$





A005
$VM^{120}V2M{-}1^{120}VM^{60}VM{-}1^{60}$





Final Thoughts

I consider these the basics of origami tessellation design using the pleat pattern approach. There is a great deal more to study, including different progressions and documentation of an enormous number of twists.

On a personal note, one of the main purposes for writing this book is to solidify a core method of folding and understanding these designs. Writing this book has given me a good foundation, and now I can advance with my own artwork. Although I have worked with tessellations and symmetric tilings for a long time, I am beginning to branch out toward the asymmetric, creating compositional works expanding upon these basics. Other artists are doing the same to great effect. It is an exciting era for practitioners of this medium.

I truly believe that the study of origami will lead folders to unlock certain secrets about the way the world works. I have used origami as a medium to teaching many other skills, including electronics to work with backlighting pieces in a frame, woodworking to make my own frames, artistic composition, and art marketing. In addition to that, I believe that once a folder is beyond the basics, he or she can create incredibly intricate masterpieces that represent their view of the world. Many artists are already doing this.

The practice of origami tessellation has given me many hours of enjoyment, and I sincerely hope that it will provide the same for you as well. Thank you for your interest in this growing art form.

Benjamin Parker

For Further Study

I hope that this book encourages you to explore the possibilities of paper design further. But where should you go from here? There is significant other literature on origami and a few books in English on origami tessellations.

Lang, Robert. 2011. *Origami Design Secrets: Mathematical Methods for an Ancient Art*, Second Edition. A K Peters/CRC Press.

Many tessellation folders, myself included, started studying this field because of Robert Lang's first edition of this book. In it, there is a diagram of koi with a tessellation of scales down its back. If you wish to understand one of the pivotal approaches to modern origami design, this is an essential read.

Lang, R. 2016 (forthcoming). *Twists, Tilings, and Tessellations*. CRC Press.

Robert Lang is currently writing his own tessellation book, publishing date to be determined. While *Six Simple Twists* focuses on mainly the pleats that create twists, he will go into the mathematics that governs the twists themselves. *Twists, Tilings, and Tessellations* explores the three topics of its title and much more in addition, describing the underlying mathematics and techniques for designing, constructing, and folding a wide range of geometric origami.

Lister, D. *The Lister List*, British Origami Society. http://www.britishorigami.info/academic/lister/ (accessed January 1, 2015).

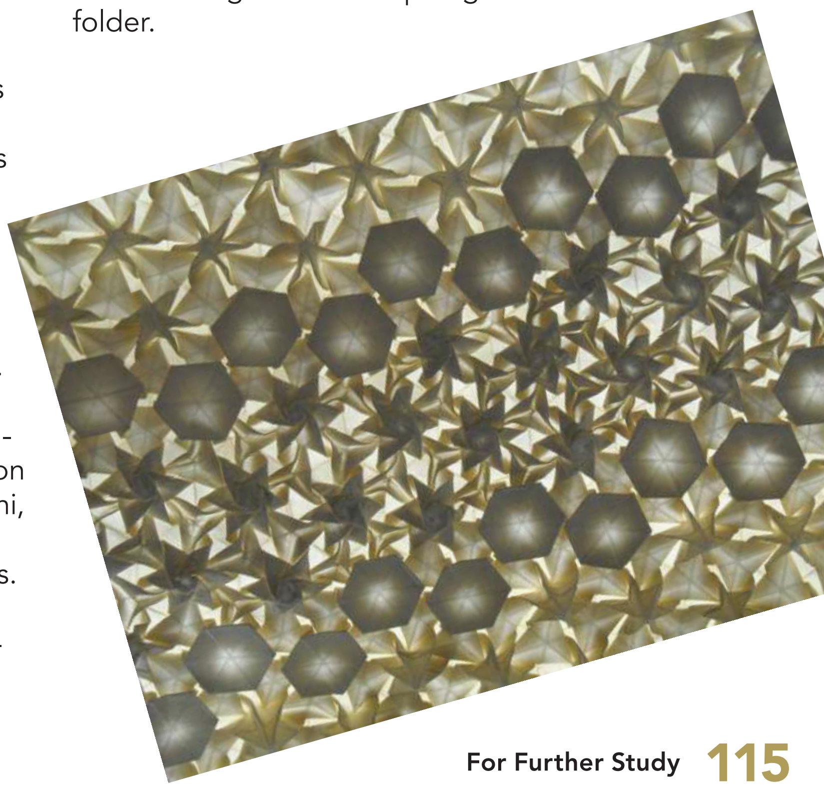
Hosted on the British Origami Society's website, David Lister has many valuable articles on the history of origami. His tessellation history article discusses Yoshihide Momotani, Shuzo Fujimoto, and some of the founding models that inspired the tessellation studies. There are also many more articles that you may wish to explore regarding other paper-folding topics.

Gjerde, E. 2008. *Origami Tessellations: Awe-Inspiring Geometric Designs*. A K Peters/CRC Press.

Eric Gjerde presents a very popular and attractive introduction to origami tessellations with many excellent examples of patterns. His perspective is different from my own. I highly recommend his book for any origami artist, but especially if you have had difficulty with the exercises in *Six Simple Twists* or simply wish to have another perspective on how twists are formed.

Rutzky, J. and Palmer, C. 2011. *Shadowfolds: Surprisingly Easy-to-Make Geometric Designs in Fabric*. Kodansha USA.

Palmer has pioneered the technique of using the tessellation concepts to fold fabrics, rather than paper. Rutzky and Palmer present several projects in a beautiful format that offers another perspective on how the twists can be formed. The properties of the fabrics being different from the paper, this can offer new challenges for the aspiring tessellation folder.



Recreational Math

"**Six Simple Twists** is a gentle introduction to the vast, wide world of origami twist tessellations that brings the reader through the successive stages of folding, understanding, and, ultimately, designing origami geometric patterns based on the powerful hexagon grid system. The six twists of the title—interacting configurations of paper pleats—may be simple in isolation, but Parker shows how they can be combined in uncountable variety. Parker presents more than a bare plan; he gives tips and techniques, guidelines and hints using standard origami notation where it is useful, but also introducing new notations that aid both understanding and folding. … photographs of the author's own beautiful tessellation creations … illustrate the concepts within the book and inspire the reader to her/his own inventions."
—Dr. Robert J. Lang, Origami Artist and Consultant, LangOrigami.com

"… unlike more traditional forms of origami, [origami tessellation] does not have a consistent system of diagramming or even a vocabulary of its own. In this book, Ben Parker addresses this problem. The interested novice will find important information to get started folding tessellations … The advanced folder will find a thoughtful approach to defining and describing the geometry of origami as revealed in tessellation. Whatever level of experience readers bring to this book, they will find something to challenge them. … Ben brings artistry and an analytic perspective to this subject, and provides much useful information and inspiration."
—Joel Cooper, Origami Artist

Benjamin DiLeonardo-Parker is an origami pattern artist. He has taught and exhibited at origami conventions and art shows throughout the United States and at La Escula-Museo Origami de Zaragosa, an origami museum in Spain.

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