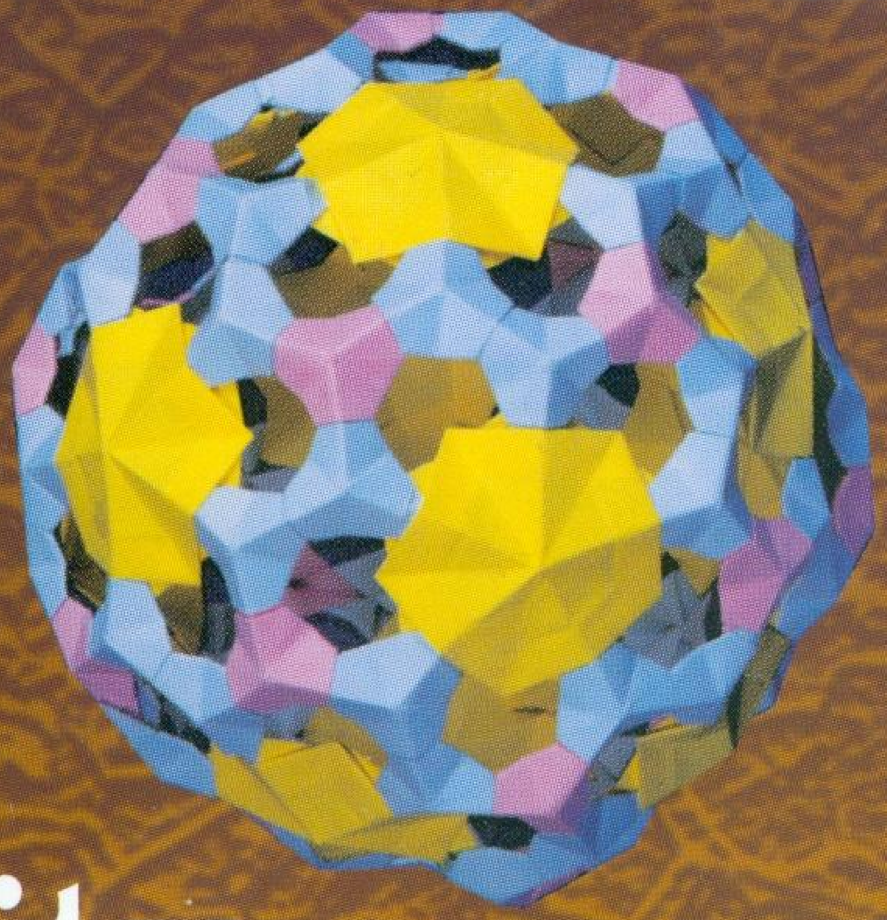
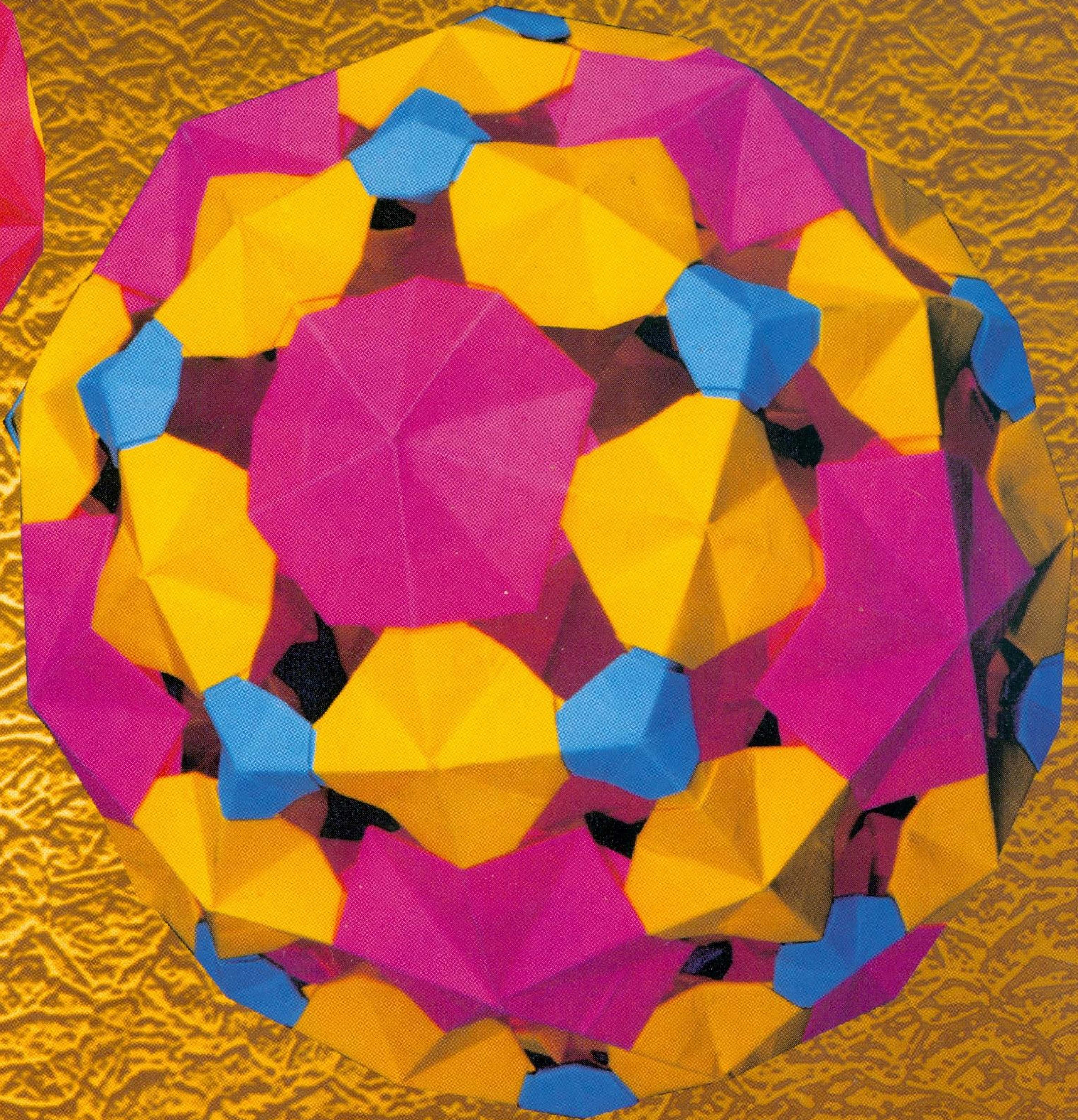
MULTIMODULAR ORIGAMI POLYHEDRA

GREEN EDITION



Archimedeans, Buckyballs, and Duality

Rona Gurkewitz and Bennett Arnstein

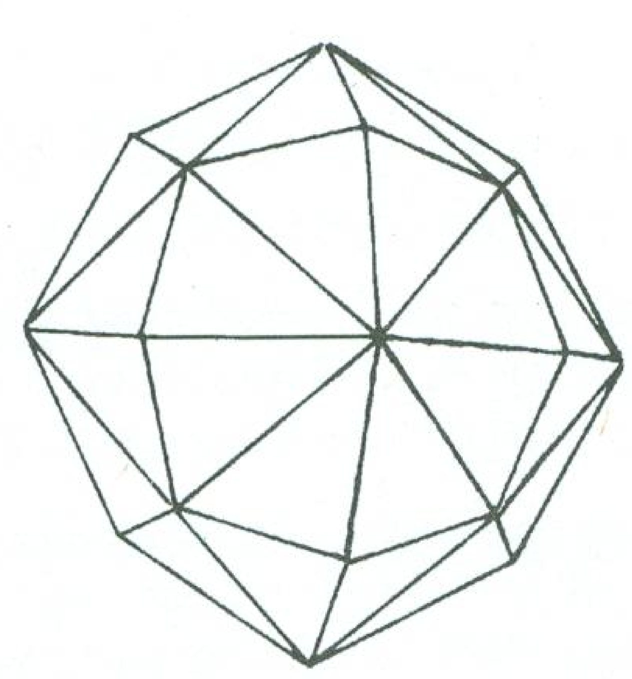


MULTIMODULAR ORIGAMI POLYHEDRA

Archimedeans, Buckyballs, and Duality

RONA GURKEWITZ
AND
BENNETT ARNSTEIN

Photographs by Bill Quinnell



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All models shown in this book were designed by
Rona Gurkewitz and Bennett Arnstein.

INTRODUCTION

This section provides an explanation of the polyhedra created in this book. You don't need it to learn how to fold the models, but if you like modular origami, it will broaden your understanding and help you to create your own models.

Modular origami polyhedra are generally made from a number of identical modules. In this book, in addition to giving instructions for creating models made from identical modules, we give instructions for creating models made from several different but related modules, for which we have coined the term "multimodular origami polyhedra." The modules described in this book together form a system of related folds like those we have previously written about [9, 11], with extensions—that is, the modules are variations of each other. They originate from different polygonal shapes, yet lock together. What they have in common is a tab-and-pocket connection between modules and a relationship to a square- or other polygon-based "waterbomb" base (see below).

A special property of this system is that more than one type of module is used in one model. Finding a system with many models is interesting; extending it to encompass many types of modules that fit together is even more interesting. We do this, first, by basing the modules on different polygons (triangles, squares, pentagons, hexagons, octagons, and decagons) and, second, by sizing them so that the point of a module made from one polygon may be fitted into the pockets of modules made from other polygons, and that the edges of all polygons that are faces of an underlying polyhedron are approximately the same length. (A polyhedron is a three-dimensional shape with flat faces, straight edges where the faces meet, and vertices where the edges meet.)

Another special property of the system is its connection with mathematical duality principles for polyhedra. We start with certain polyhedra whose faces are all regular polygons, and through a process we call "gyroscoping," create models that have the edges of a dual polyhedron of the starting polyhedron as the mountain folds and the vertices of a dual polyhedron of the starting polyhedron as the vertices of the resulting polyhedron. Some of the polyhedra we construct have potential practical applications in molecular science or crystallography.

In this book we give procedures and algorithms for making polyhedra from our modules, but consider only some of the possibilities. The first set of polyhedra we work with is the Archimedean solids and their duals, the less familiar Catalans (p. 9). There are thirteen of each. The Archimedeans have faces that are regular polygons of two or three types, and each vertex in a polyhedron is surrounded by the same sequence of polygonal faces. The Catalans are produced from the Archimedeans by a duality process of polar reciprocation [1, 3, 5, 6, 16] which can be described informally as replacing faces by vertices and vertices by faces, while keeping the same number of edges (which is topological duality) yet being more specific as to which vertices and which faces are used. In this book we show how to construct polyhedra derived from Archimedeans by a process of gyroscoping. These gyroscoped Archimedeans are no longer convex, as the gyroscoping process uses modules based on regular polygon waterbomb bases. (A waterbomb base is formed from a regular polygon by making mountain folds from its center to its vertices and valley folds from its center to the midpoints of its edges. When the vertices are held in a plane, and the center is raised, a waterbomb base is formed. It is not convex because the faces go in and out.) While the gyroscoped Archimedeans are not convex we can observe that their mountain folds and vertices are the edges and vertices of the related Catalans. We can also locate the original Archimedean on the model as well as other polyhedra as subsets of the model's vertices.

The second set of polyhedra we include are what we call buckyballs and hypothetical buckyballs. There are fifteen of these and they are made exclusively from the triangular gyroscope module. A couple of the largest of these models require glue. There are three families of these polyhedra. They are based respectively on pentagons and hexagons, squares and hexagons, and triangles and hexagons. We show how to construct five models from each family. The models get progressively larger, using more and more modules by following a net pattern that can be enlarged and "grown" while keeping their buckyball or hypothetical buckyball properties. The starting polyhedron for the pentagon-hexagon-based models is the truncated icosahedron; for the square-hexagon-based models it is the truncated octahedron, and for the triangle-hexagon-based models it is the truncated tetrahedron. The pentagon-hexagon-based models correspond to natural carbon-based buckyballs. It is left as an open question whether the other hypothetical buckyballs correspond to substances natural or man-made.

The third set of polyhedra we consider are the gyroscoped buckyballs. These can be thought of as being formed from the buckyballs and hypothetical buckyballs using a gyroscope process. We have constructed fifteen of them corresponding to the previously constructed buckyballs and hypothetical buckyballs. They turn out to have very pleasing symmetries, even though they are not convex. Also, their mountain folds and vertices reveal a dual polyhedron.

We have also applied the gyroscope process to the model we call the "egg," a truncated hexadecahedron whose sixteen faces are equilateral triangles. (The hexadecahedron can be thought of as two square-based pyramids attached to a square antiprism.) We came up with this shape, made exclusively from triangle gyroscope modules, through a truncation algorithm, and then gyroscoped it, revealing a dual polyhedron in mountain folds. This model is interesting because it is not spherical, but rather elliptical.

Don't miss the display stand we have designed to hold your models.

We have enjoyed experimenting with different ideas. What is different about the design of the models in the book is that after a first intuition, we set out intentionally to explore the ideas further. This was definitely not doodling. There are other possible models that we have not written about and for which we do not include pictures, namely the Johnson solids and their truncations [2]. (The egg is an approximation of a truncation of a Johnson solid.) Since the Johnson solids have as their faces only regular polygons, they are good candidates for gyroscoping. We hope you will enjoy experimenting with all ninety-two of these models too.

The models in this book are all new except for the truncated icosahedron, the truncated octahedron, the truncated tetrahedron, and the egg. The only family of polygonal modules for polyhedra that we know of with equal edge lengths was published in [10].

I. BACKGROUND MATERIAL

A. Archimedean Solids
B. Duality
C. Gyroscoped Archimedeans
D. Gyroscope Modules
E. Buckyballs
F. The Eggs

BACKGROUND MATERIAL

A. Archimedean Solids

Our first group of models is based on pairs of polyhedra, specifically Archimedean solids and their dual Catalans [2]. The Archimedean solids were known as early as 400 B.C., although they were named at a later date after their rediscovery by Kepler and others. They are also called semiregular solids. They are the only convex polyhedra other than the Platonic solids with regular polygons for faces and with vertices that all have the same sequence of polygons meeting around them. The Platonic solids each have only one type of face; the Archimedean solids each have two or three types of faces.

The Archimedeans are related to the larger set of Johnson solids, which have regular polygons as faces but more than one type of vertex, as well as to the set of uniform polyhedra which have all vertices alike and all faces alike but not necessarily regular, and which are not necessarily convex. The Archimedeans are convex. We consider all thirteen of the Archimedean solids in this book [2].

B. Duality

Other properties of the thirteen Archimedean solids that we found useful for designing models relate to their associated spheres. All vertices of an Archimedean solid lie on a sphere, called the circumsphere, and their edges are tangent to a midsphere at the midpoints of the edges. These properties lead naturally to the standard dual, a polyhedron whose edges are perpendicular to both the Archimedean edges and to the segments from the center of the polyhedron to the midpoints of the Archimedean edges. The set of thirteen such duals is historically designated the Catalan solids. Perhaps the most striking feature of a Catalan solid is that it is composed of just one kind of face that is not a regular polygon. A face of a Catalan solid can be simply constructed by the Dorman-Luke construction [3, 4, 6, 16], which is based on a dualization process known as polar reciprocation [6], with respect to the midsphere. Although topologically equivalent to the Catalan solid, a proportionately different (non-standard) dual occurs when we replace each Archimedean face center with a vertex and each Archimedean vertex with a face. Such an interchange of parts produces a dual with the same number of edges as the Archimedean, and the new faces remain all of one kind.

C. Gyroscoped Archimedeans

Each polyhedron in our first group of models has been produced from an Archimedean through the gyroscoping process and is a melding of that Archimedean and its dual Catalan. We have chosen to designate these new gyroscoped polyhedra with the names of the Archimedeans they are derived from, although they could just as easily have been named after the dual Catalans; they have properties derived from each.

Specifically, these gyroscoped Archimedeans are made up of gyroscope modules that correspond in type to the faces of the Archimedean, namely, triangular, square, pentagonal, hexagonal, octagonal, or decagonal (see modules section). The gyroscope modules function as waterbomb bases with tabs and pockets to lock them together, so the models are not convex. However, amazingly, the moutainfolded edges of the new polyhedra correspond to the edges of the Catalans they are based on: we can see duality happening. Each face of an Archimedean has a vertex placed on it by placing a gyroscope module on it with the mountainfolds of the gyroscope module going from its vertex to the midpoints of the edges of the face of the Archimedean, thereby interchanging the face of the Archimedean with the vertex of the gyroscope module, on the gyroscoped Archimedean.

D. Gyroscope Modules

A gyroscope module is placed on the face of an Archimedean in such a way that it is not apparent at first where the original face is. The mountain folds going from the vertex of the gyroscope module to the tabs and pockets of the module meet the edge of the pocket at the midpoint of an edge of a face of the underlying Archimedean. That is, the edges of the pockets of the gyroscope module lies along the edges of the faces of the Archimedean, but the pockets are shorter than the edges of the faces. Because of this, there are open spaces on some of the models.

As for the faces of the Catalan, they are formed by the mountain folds of the gyroscope module because of certain of their properties. Two gyroscope modules interlock with a tab from one module and a pocket from the other module. The edges of the pockets from both modules meet, so that they are perpendicular to the mountain folds that go through them in a straight line. This property further shows how the models are related to a duality process because the edges of an Archimedean are perpendicular to the edges of its dual. Also, the edges of an Archimedean are bisected by the edges of its dual, and so the mountain folds of the gyroscope modules on an Archimedean bisect the edges of the Archimedean.

E. Buckyballs

The second group of models we have designed includes the buckyballs, otherwise known as the fullerenes, as well as the hypothetical buckyballs. The third group comprises the gyroscoped forms of the buckyballs and the hypothetical buckyballs (pp. 24–38)

Buckminsterfullerene, or C60, is a carbon substance discovered in 1985 by Robert F. Curl, Jr. and Richard E. Smalley of Rice University, and Harold W. Kroto of the University of Sussex, England, who received the Nobel Prize for Chemistry in 1996 for this work. It was named after the designer/philosopher R. Buckminster Fuller because its molecular structure, a truncated icosahedron (like the surface pattern of a soccer ball), resembles the geodesic domes designed by Fuller. Previously, graphite and diamonds were the only known pure carbon substances (and their molecular structure is much simpler geometrically).

At the time of their discovery it was felt that buckyballs held much promise for practical applications, and some progress toward such applications has been made. Possibilities are thought to include drug delivery systems, HIV-blocking drugs, better protective coatings, and improved carbon dating. Recently, other cased structures, of silicon, have been discovered and have raised similar hopes [3].

The fifteen buckyballs and hypothetical buckyballs we construct are made of pentagons and hexagons, squares and hexagons, and triangles and hexagons. These are all regular polygons, so we were able to use the gyroscoping process on them, revealing a dual in the mountain folds, thus creating another group of models. We show a process for constructing each family of buckyballs and hypothetical buckyballs. Only the 3-1-1, 4-1-1, and 5-1-1 are regular polyhedra; the others are approximations.

F. The Eggs

The last models we present are the "egg" and the "gyroscoped egg." The egg is unusual because it has an elliptical rather than spherical shape. The gyroscoped egg truly looks like a decorated holiday egg. The egg is a truncated hexadecahedron. It has forty-eight vertices and so can be made from forty-eight triangle gyroscope modules. It consists of two square rings on its ends, each surrounded by four hexagonal rings, which are themselves surrounded by eight more hexagonal rings and eight pentagonal rings (p. 8). Note that the egg is not a Johnson solid, so it does not have exactly regular faces.

II. GALLERY OF POLYHEDRA

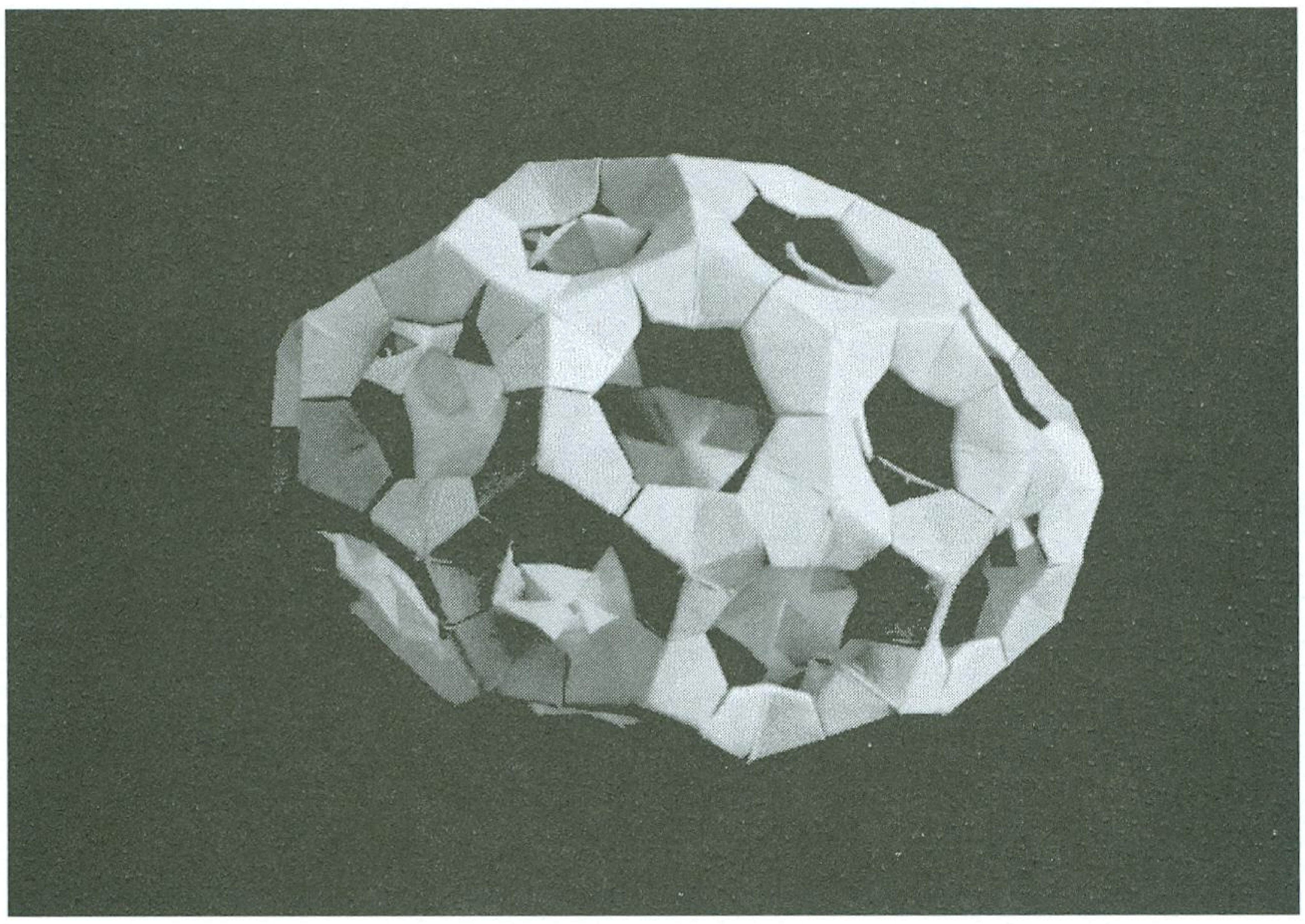
A. The Egg and the Gyroscoped Egg

B. Archimedeans, Catalans, and Gyroscoped Archimedeans

C. Seeds for Growing Buckyballs, Buckyballs, Hypothetical Buckyballs, and Their Gyroscoped Forms

GALLERY OF POLYHEDRA

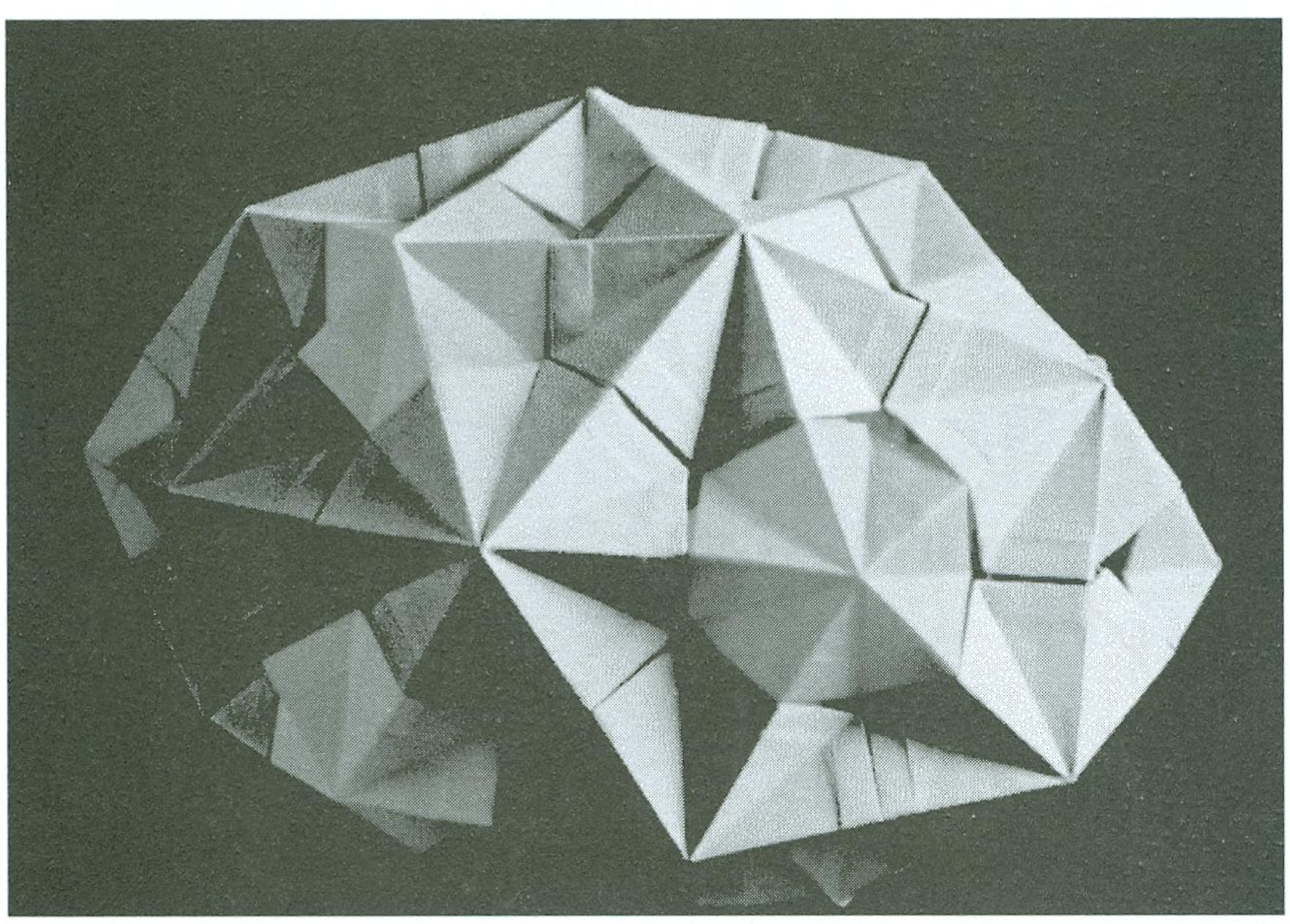
A. The Egg and The Gyroscoped Egg



The Egg
(Truncated
Hexadecahedron)

48 triangle modules
arranged in:

- 2 square rings
- 8 pentagon rings
- 16 hexagon rings



Gyroscoped Egg

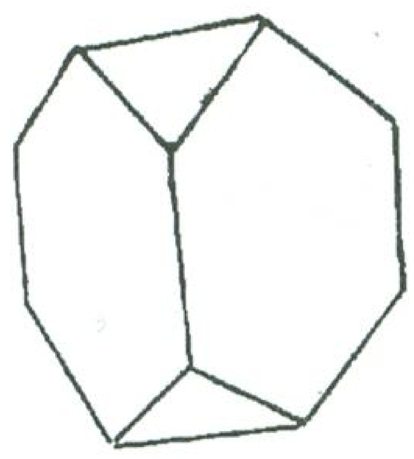
- 2 square gyroscope modules
- 8 pentagon gyroscope modules
- 16 hexagon gyroscope modules

B. Archimedeans, Catalans, and Gyroscoped Archimedeans

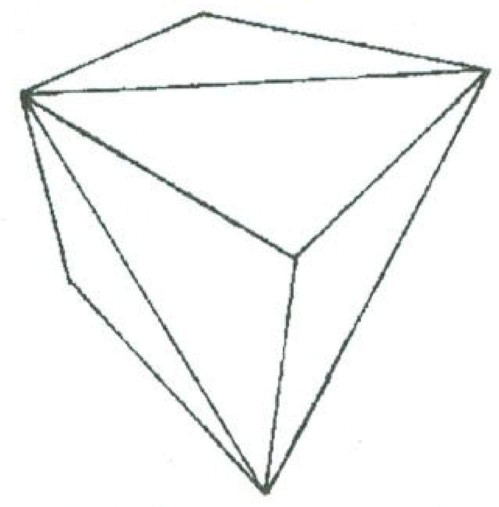
Summary: Archimedeans and their Dual Catalans

This page is a summary of the pairs of polyhedra that are bases for the gyroscoped Archimedeans that follow.

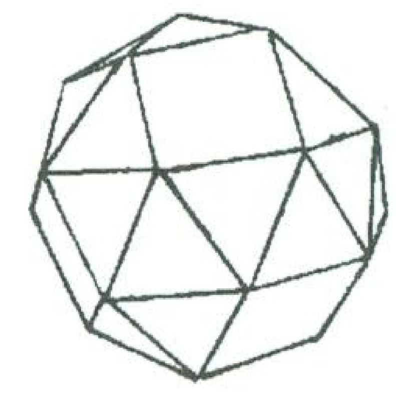
Archimedean	Catalan	Archimedean	Catalan



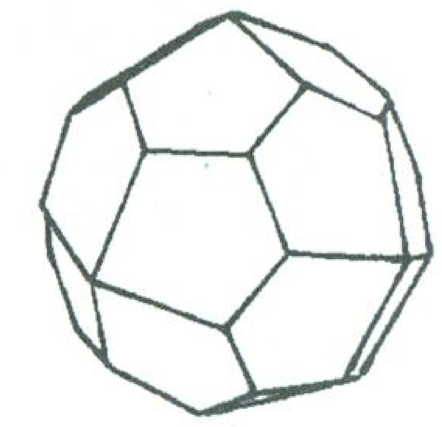
1. Truncated Tetrahedron



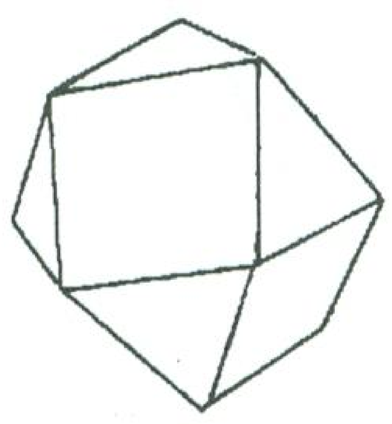
Triakistetrahedron



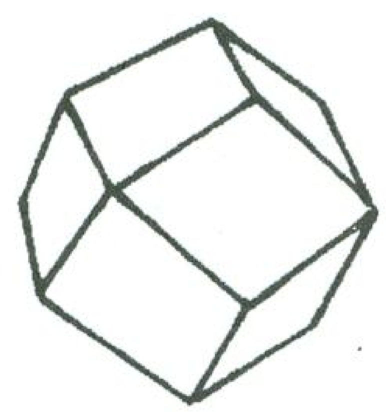
7. Snub Cube



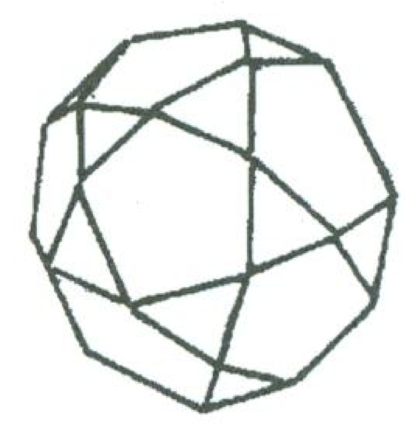
Pentagonal Icositetrahedron



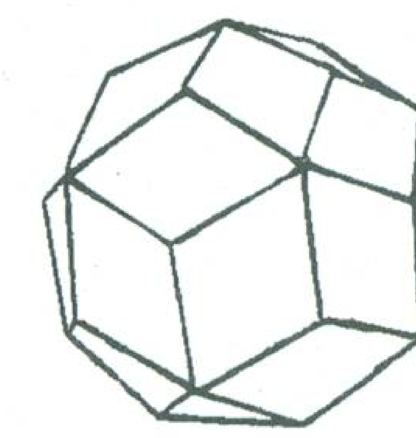
2. Cuboctahedron



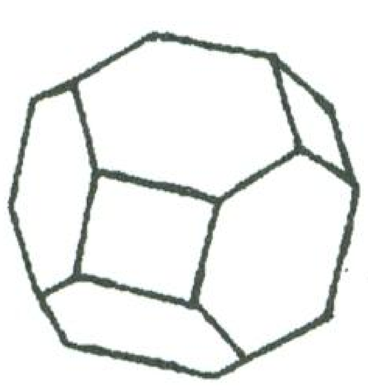
Rhombic Dodecahedron



8. Icosidodecahedron

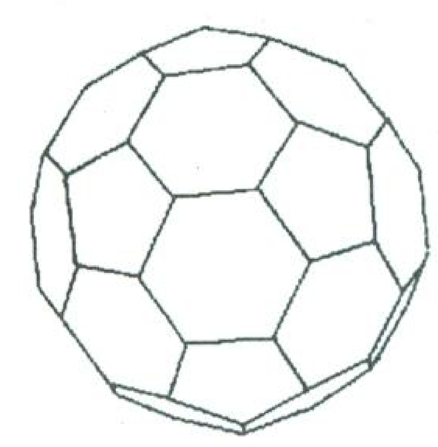


Rhombic Triacontahedron

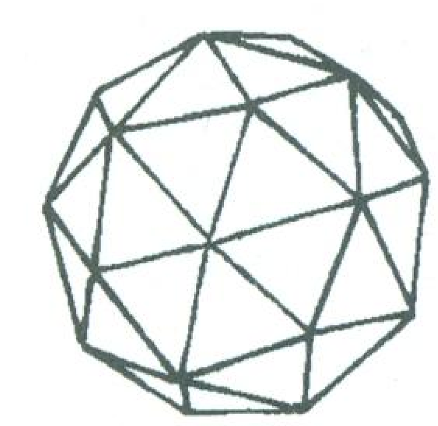


3. Truncated Octahedron

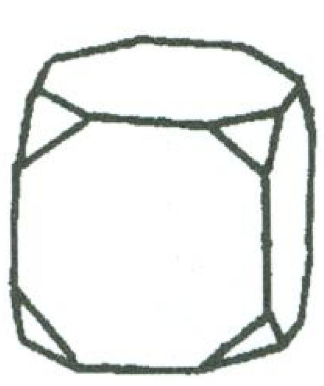
Tetrakishexahedron



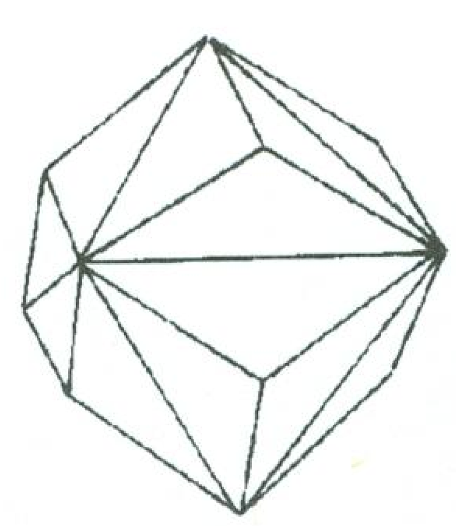
9. Truncated Icosahedron



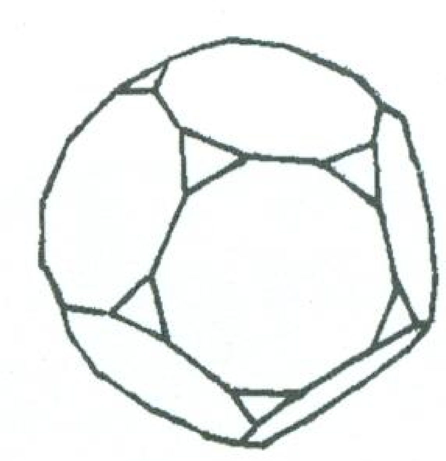
Pentakisdodecahedron



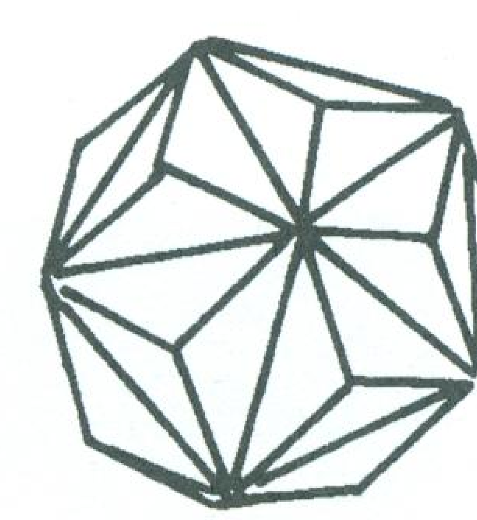
4. Truncated Cube



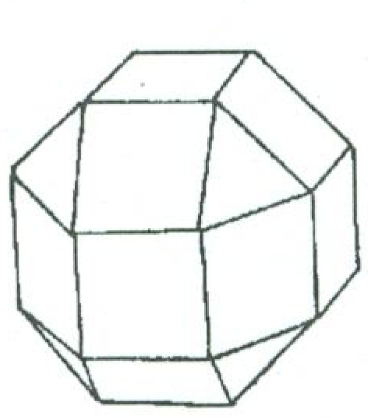
Triakisoctahedron



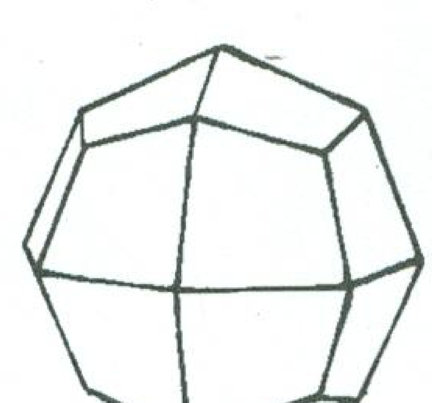
10. Truncated Dodecahedron



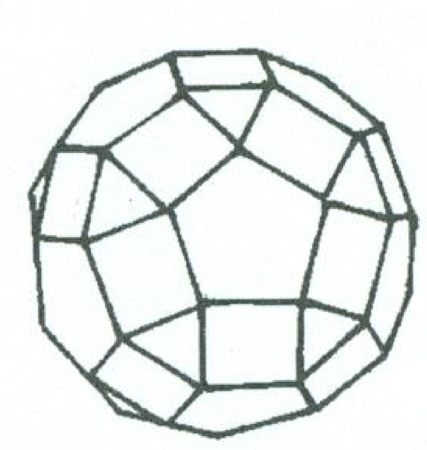
Triakisicosahedron



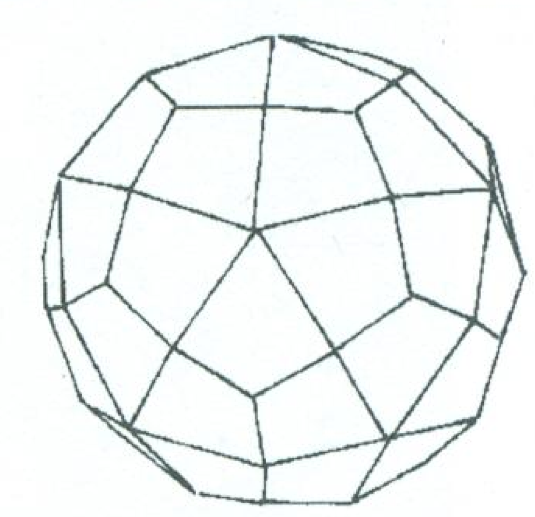
5. Rhombicuboctahedron



Trapezoidal Icositetrahedron



11. Rhombicosidodecahedron

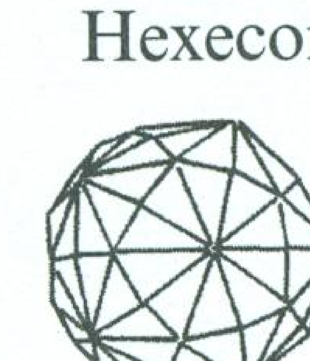


Trapezoidal Hexecontahedron

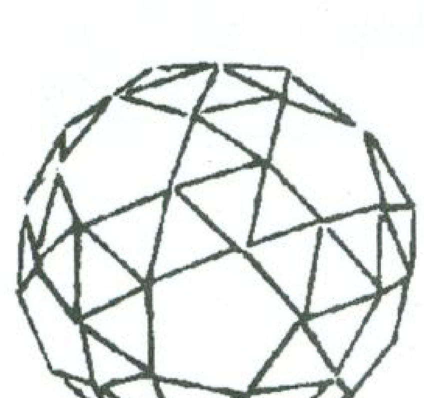
6.Truncated Cuboctahedron

Disdyakis-Dodecahedron

12. Truncated Icosi-dodecahedron



Disdyakis Tria-contahedron

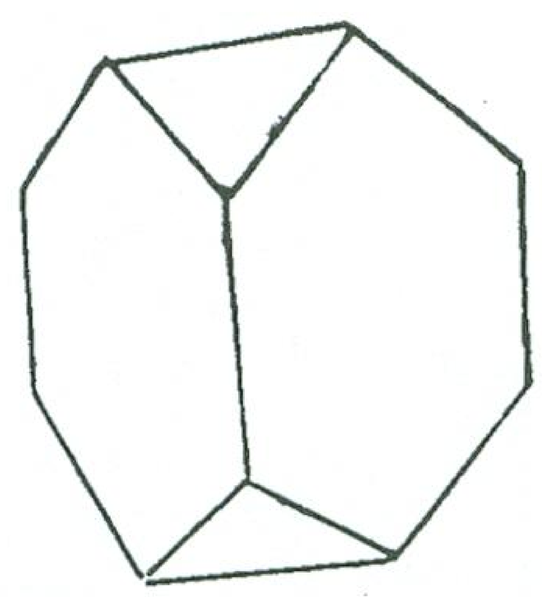


13. Snub Dodecahedron

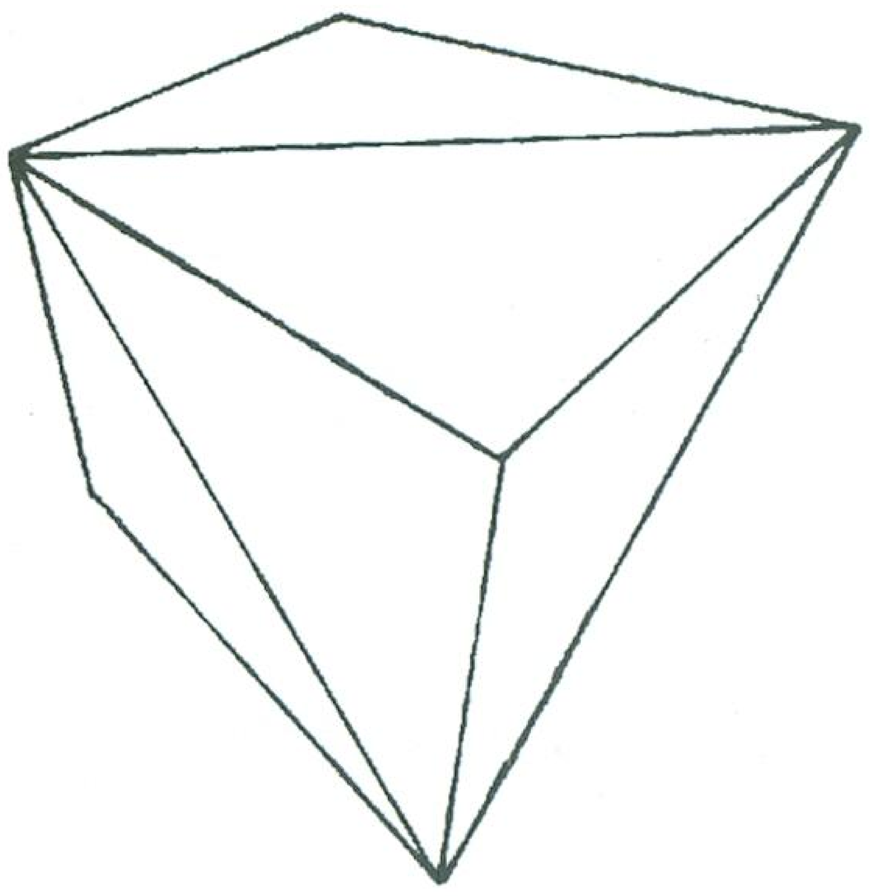
Pentagonal Hexecontahedron

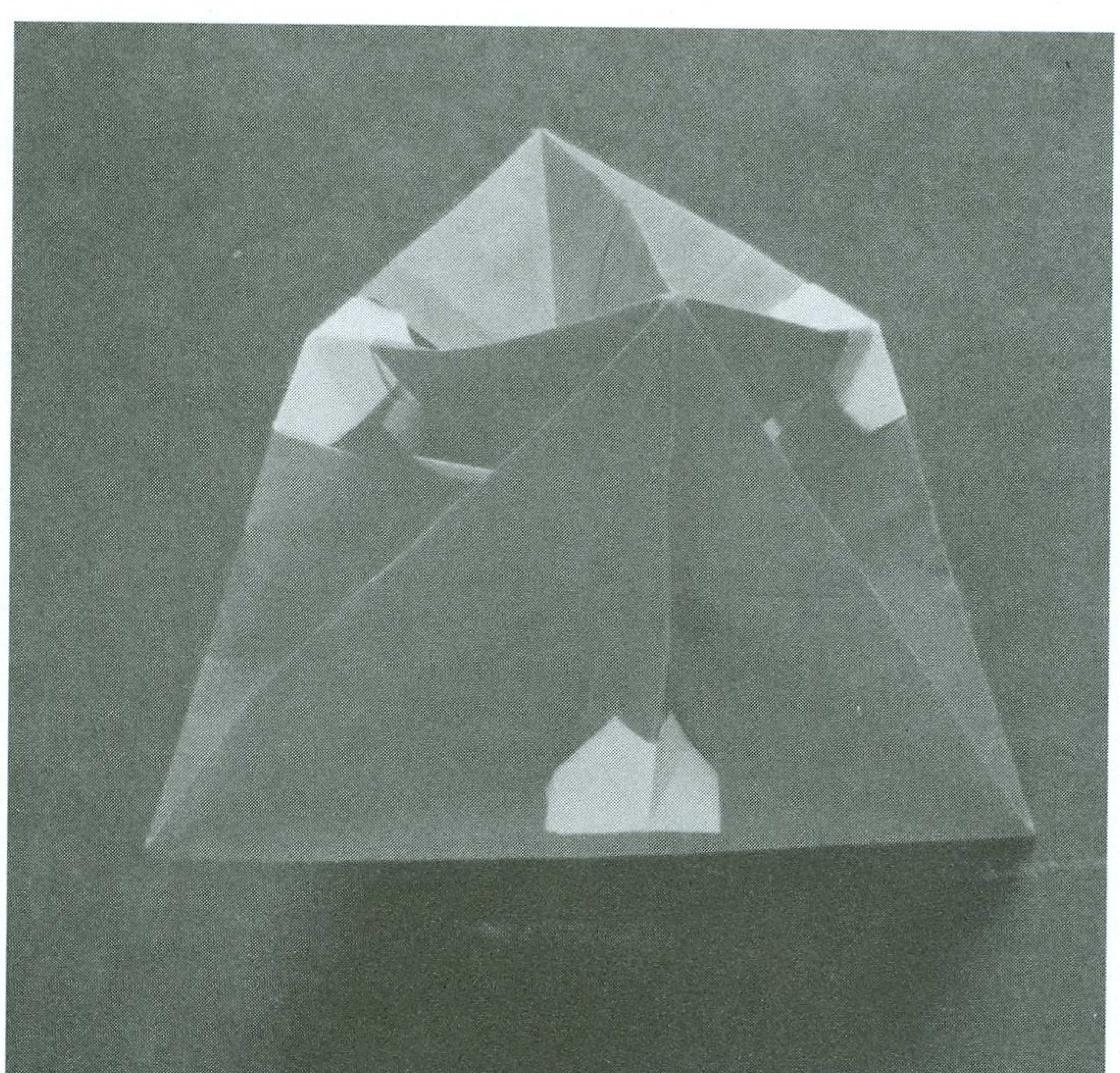
1. Gyroscoped Truncated Tetrahedron

Truncated Tetrahedron



Triakis-tetrahedron

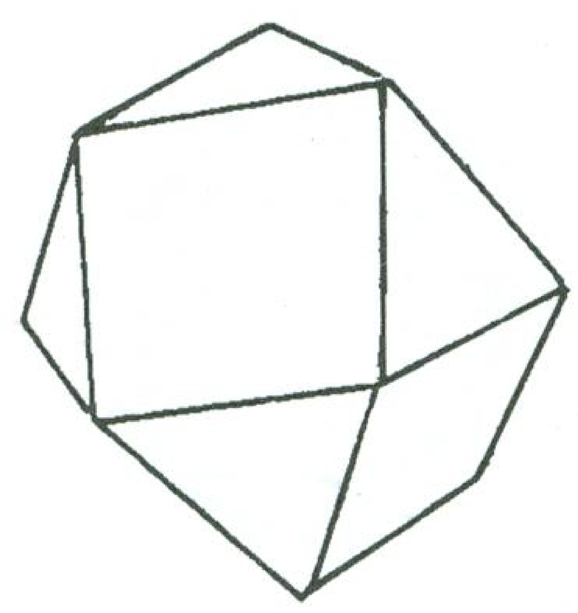




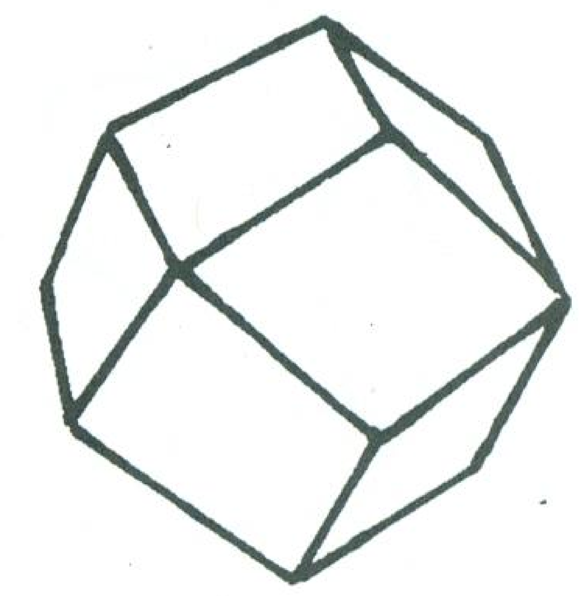
Gyroscope Modules:
4 triangles and
4 hexagons

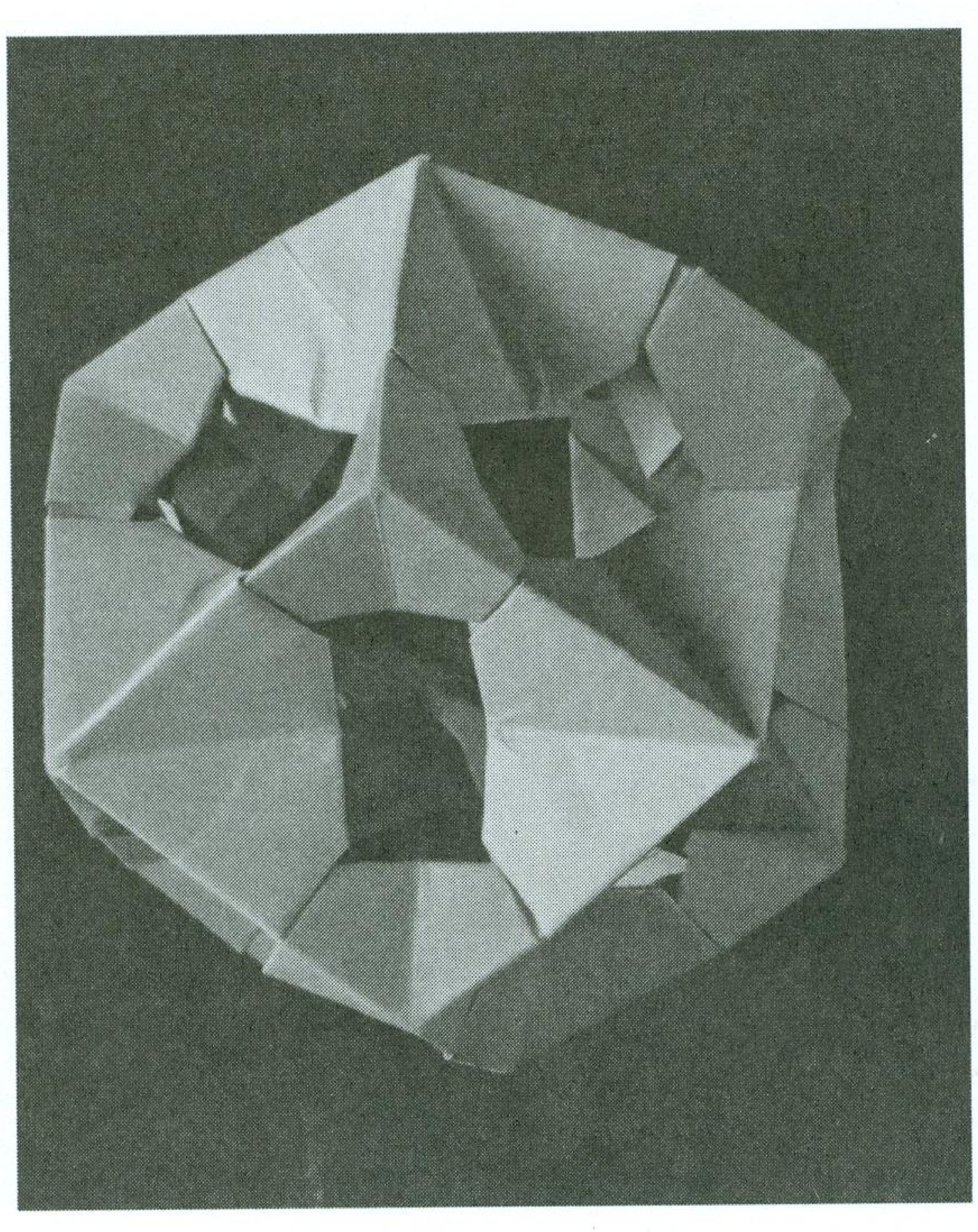
2. Gyroscoped Cuboctahedron

Cuboctahedron



Rhombic Dodecahedron

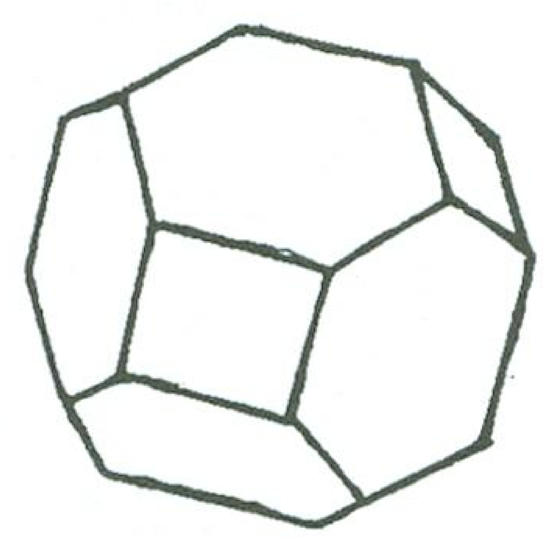




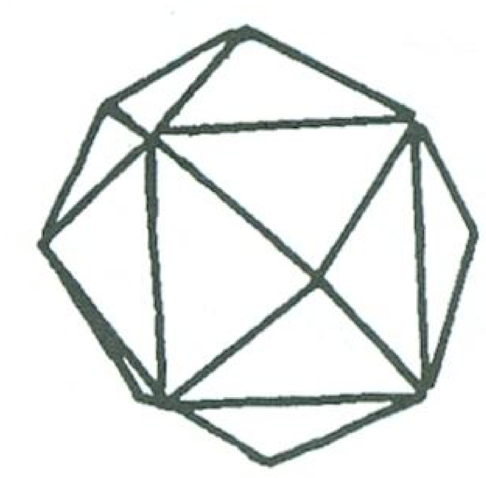
Gyroscope Modules:
8 triangles and
6 squares

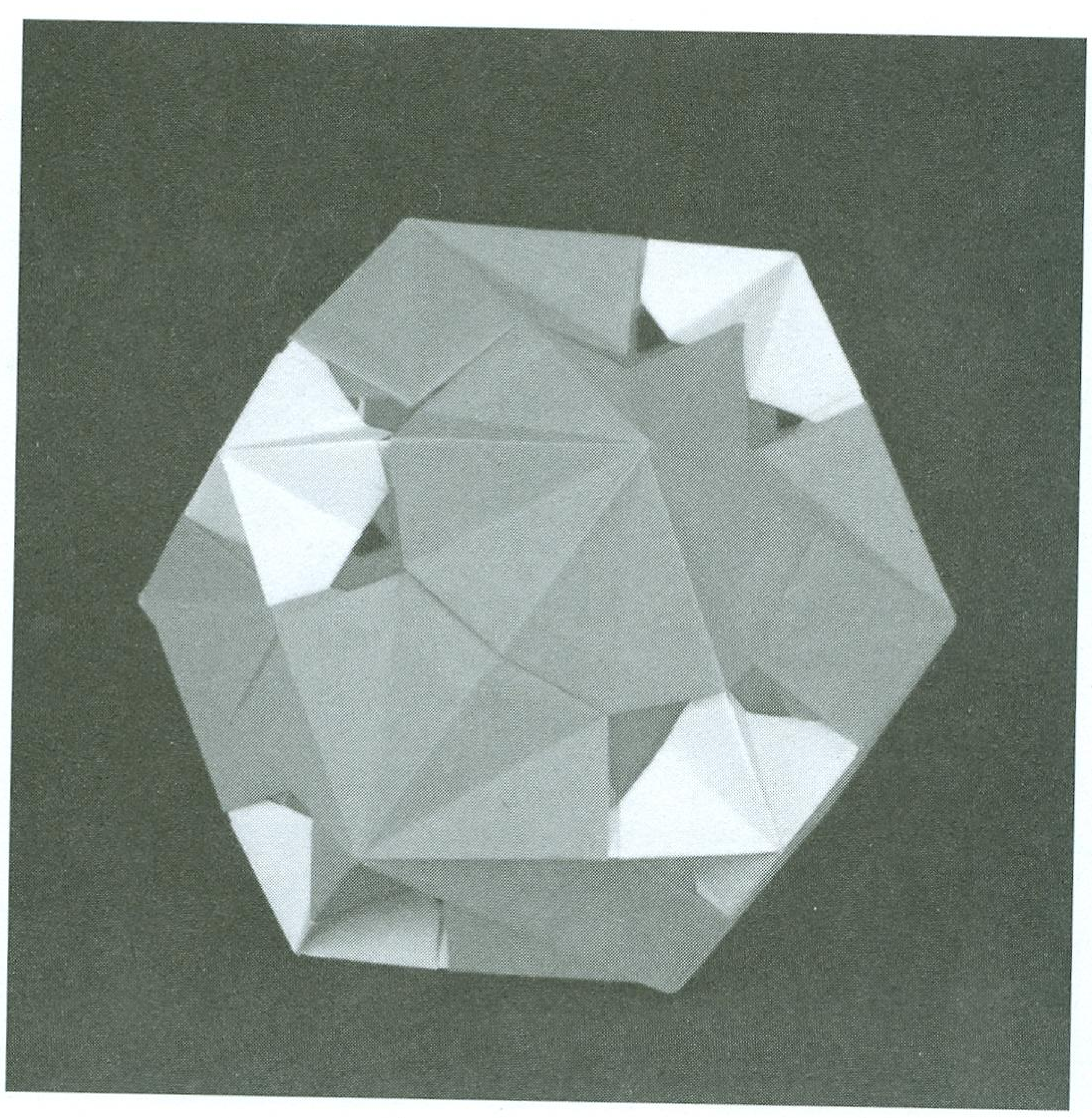
3. Gyroscoped Truncated Octahedron

Truncated Octahedron



Tetrakis hexahedron

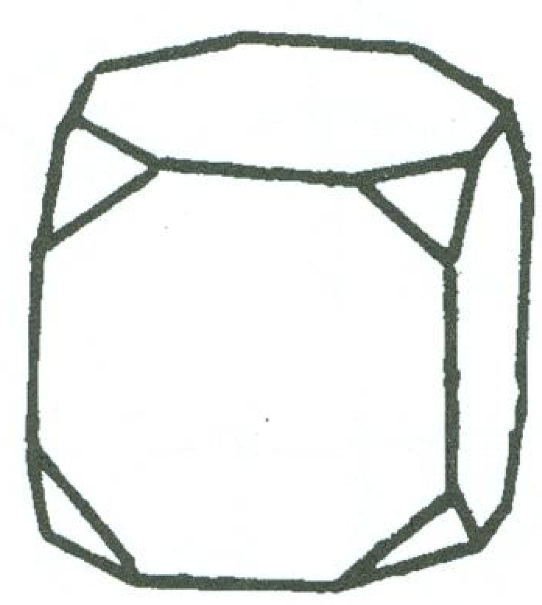




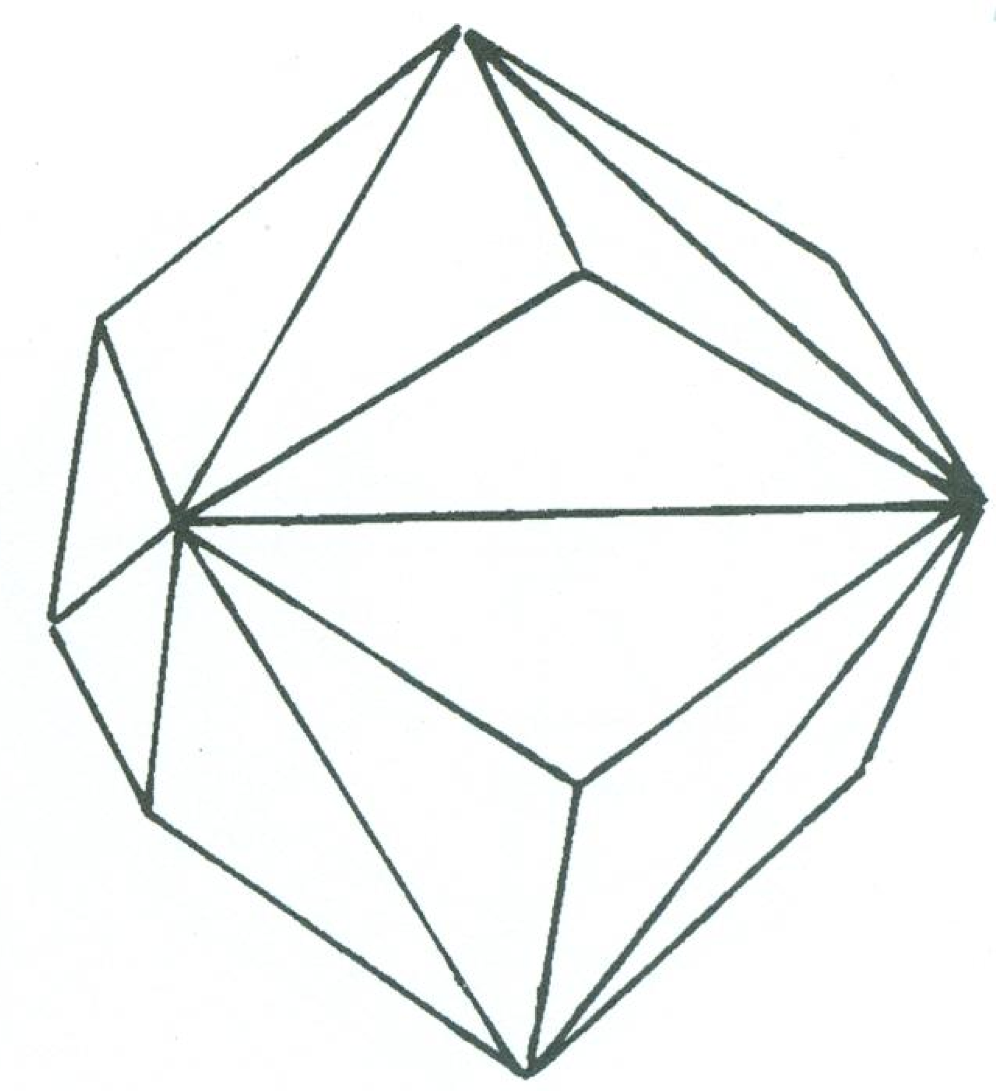
Gyroscope Modules:
6 squares and
8 hexagons

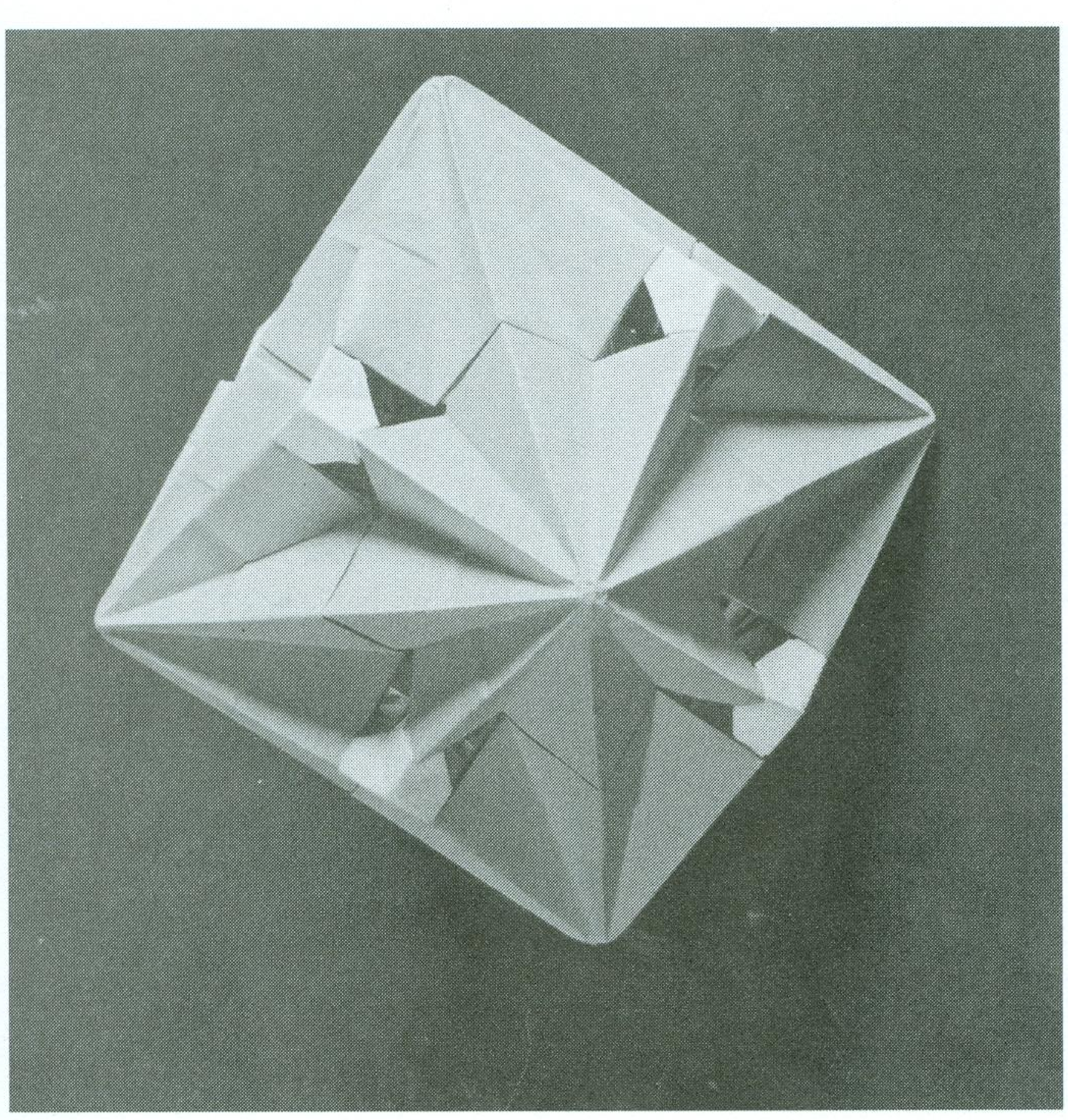
4. Gyroscoped Truncated Cube

Truncated Cube



Triakis-octahedron

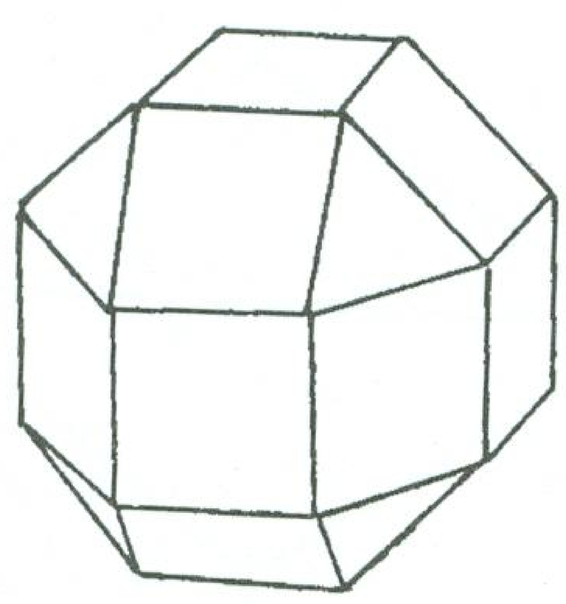




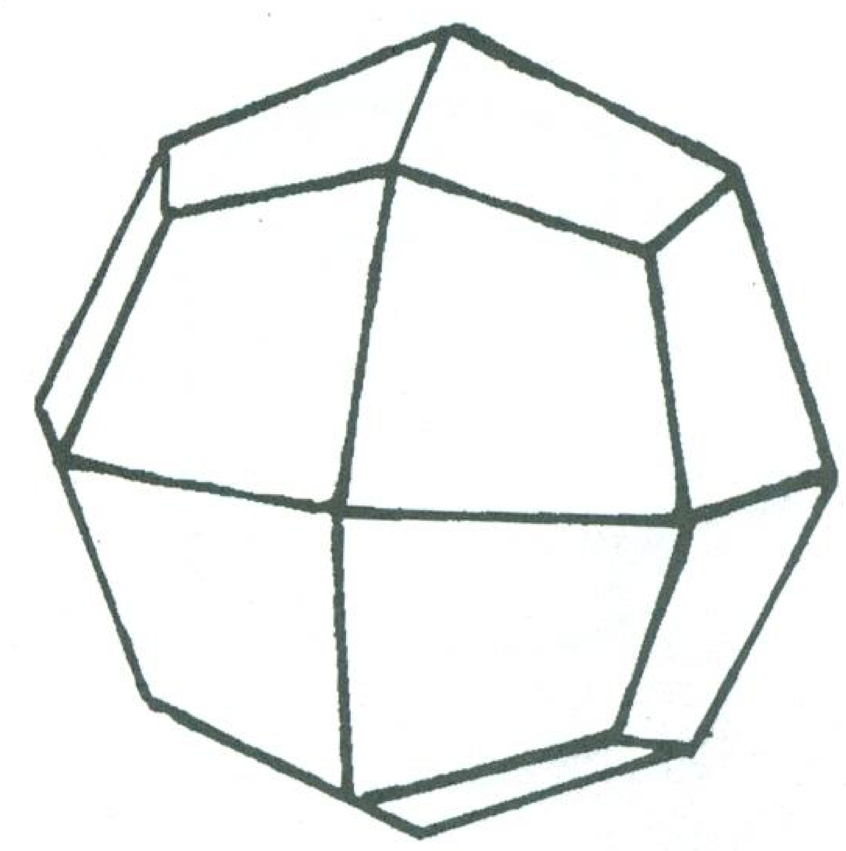
Gyroscope Modules:
8 triangles and
6 octagons

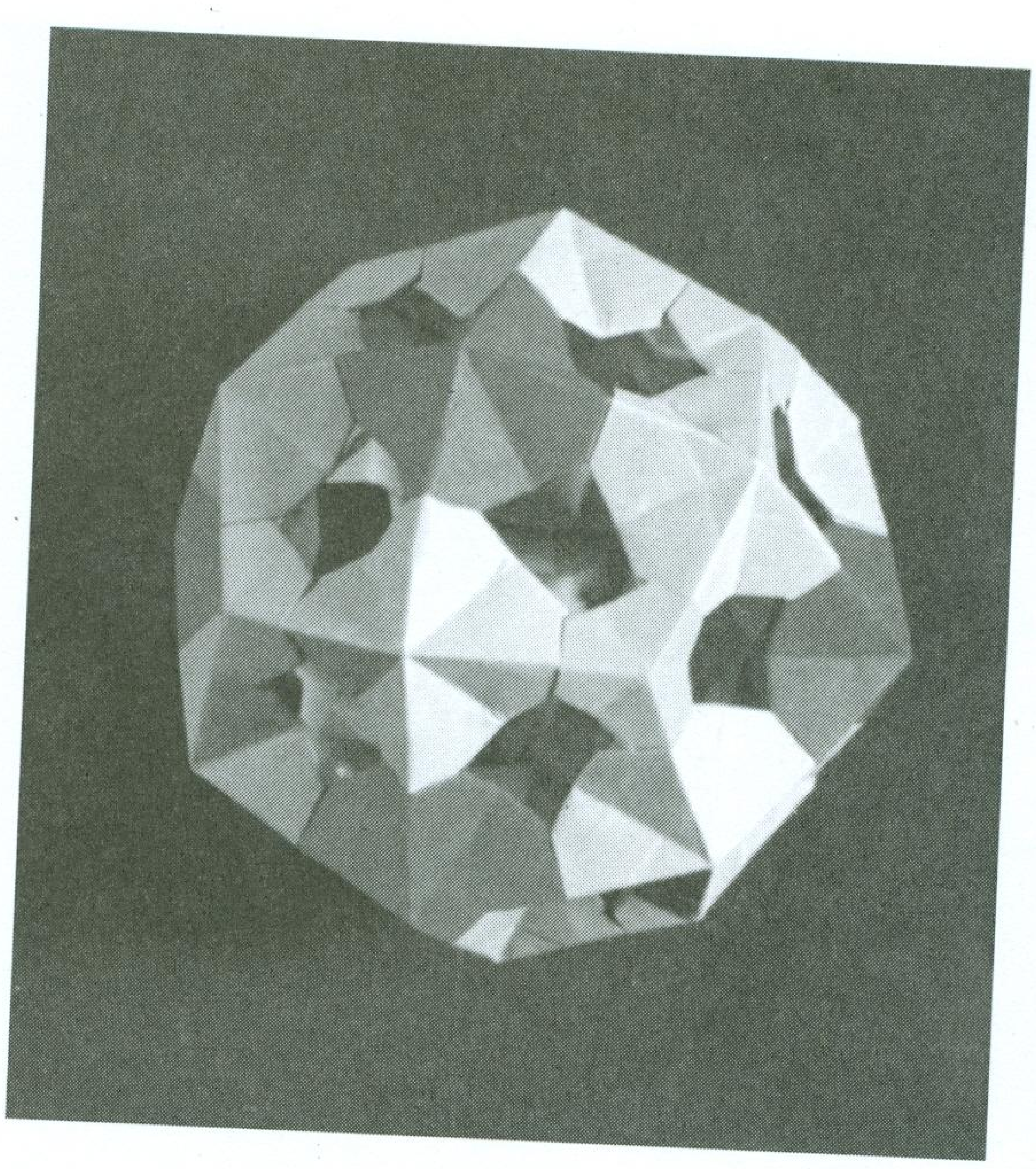
5. Gyroscoped Rhombicuboctahedron

Rhombicub-
octahedron



Trapezoidal
Icositetrahedron

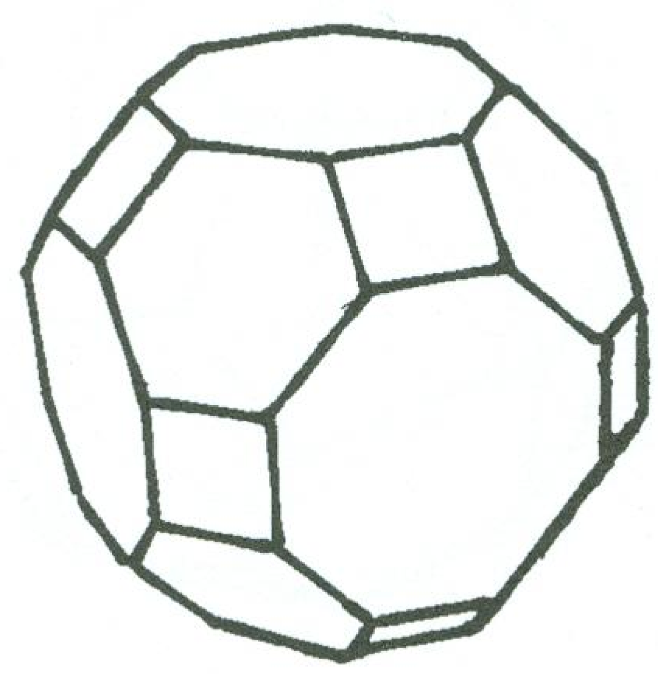




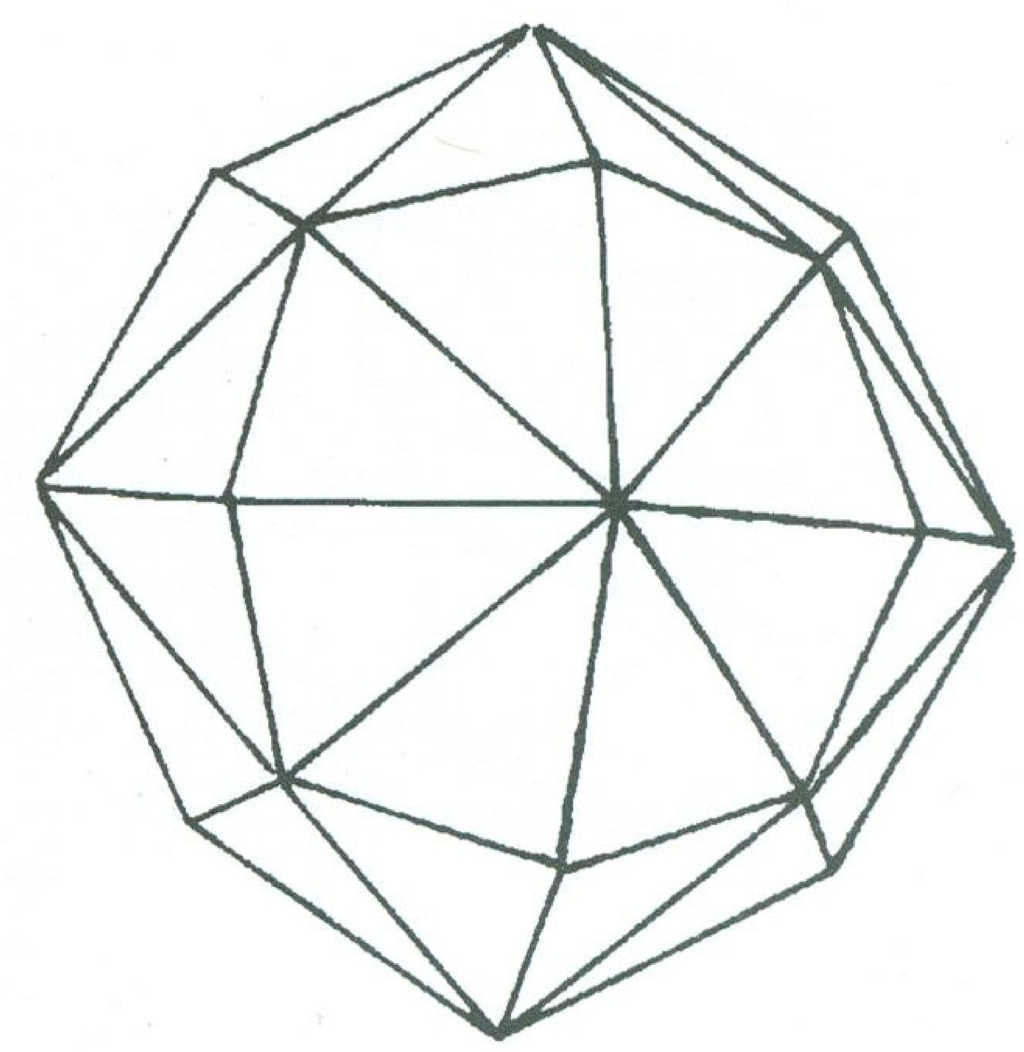
Gyroscope
Modules:
8 triangles and
18 squares

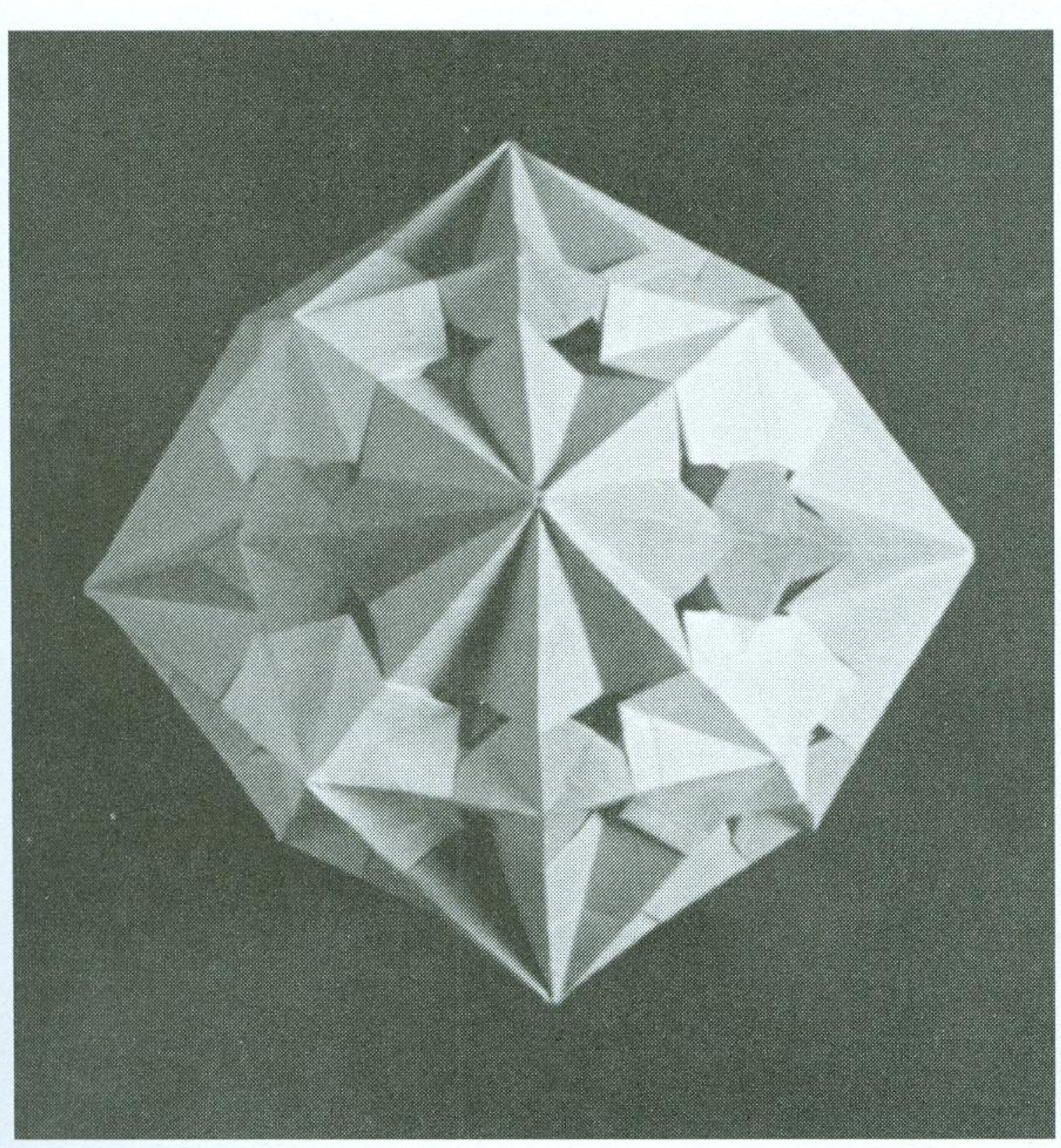
6. Gyroscoped Truncated Cuboctahedron

Truncated Cuboctahedron



Disdyakis Dodecahedron

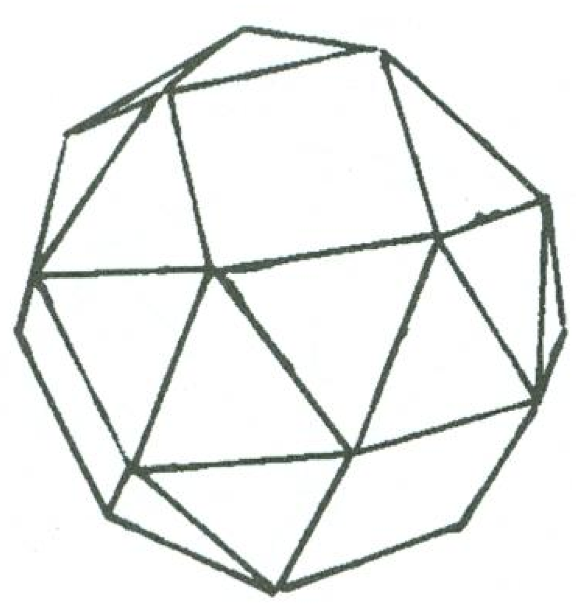




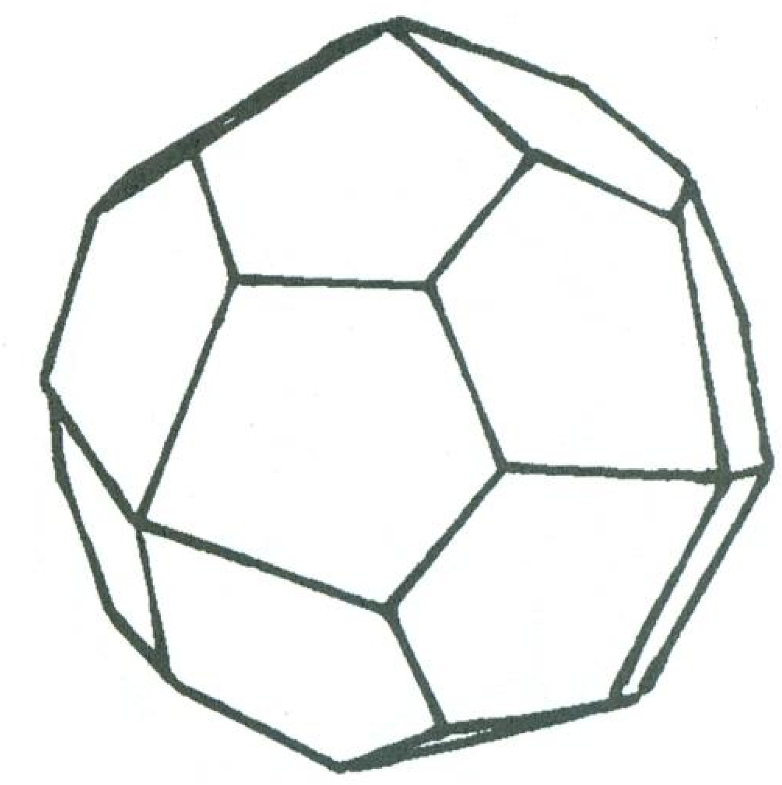
Gyroscope Modules:
12 squares,
8 hexagons, and
6 octagons

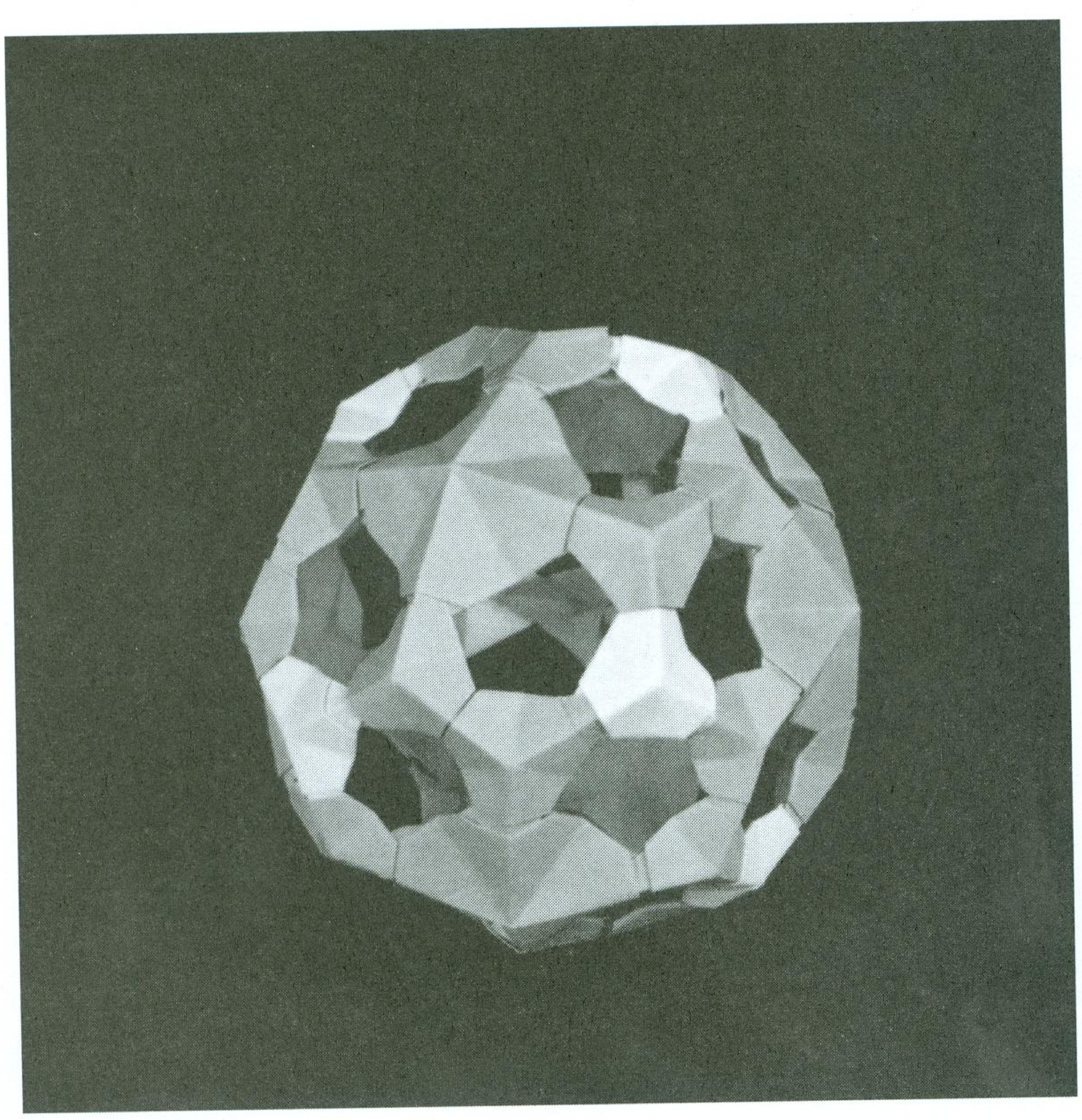
7. Gyroscoped Snub Cube

Snub Cube



Pentagonal Icositetrahedron

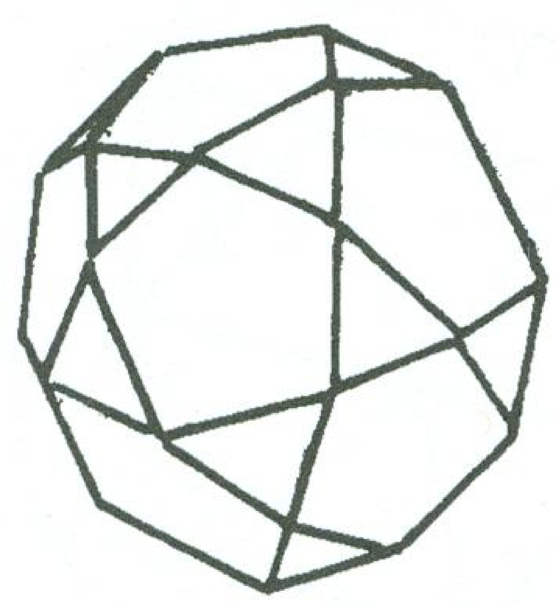




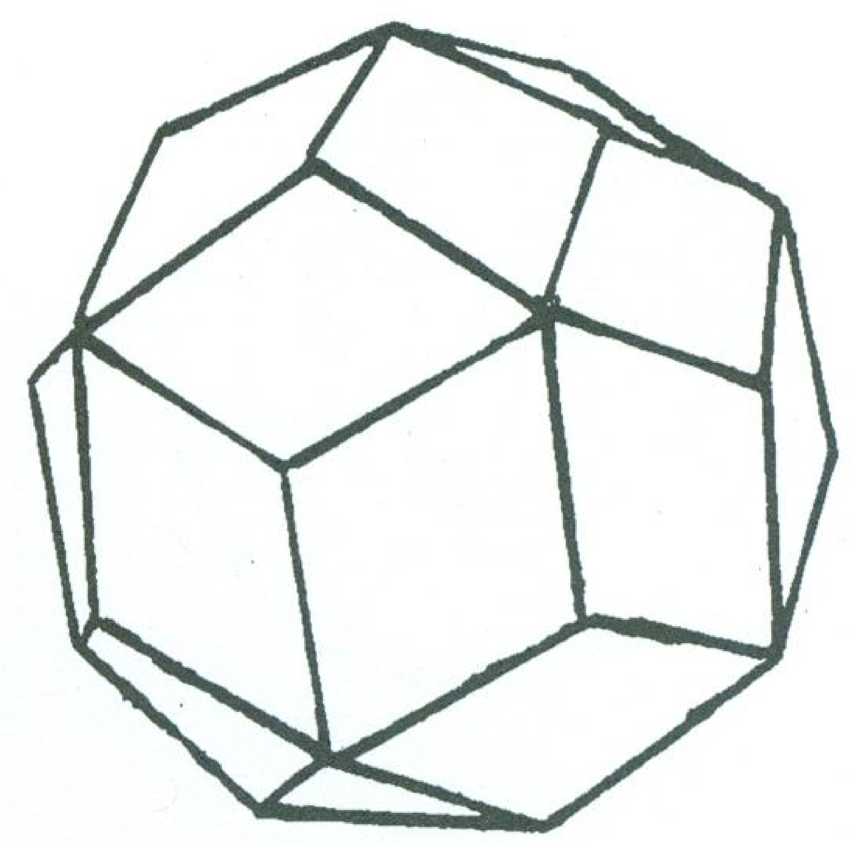
Gyroscope Modules:
32 triangles and
6 squares

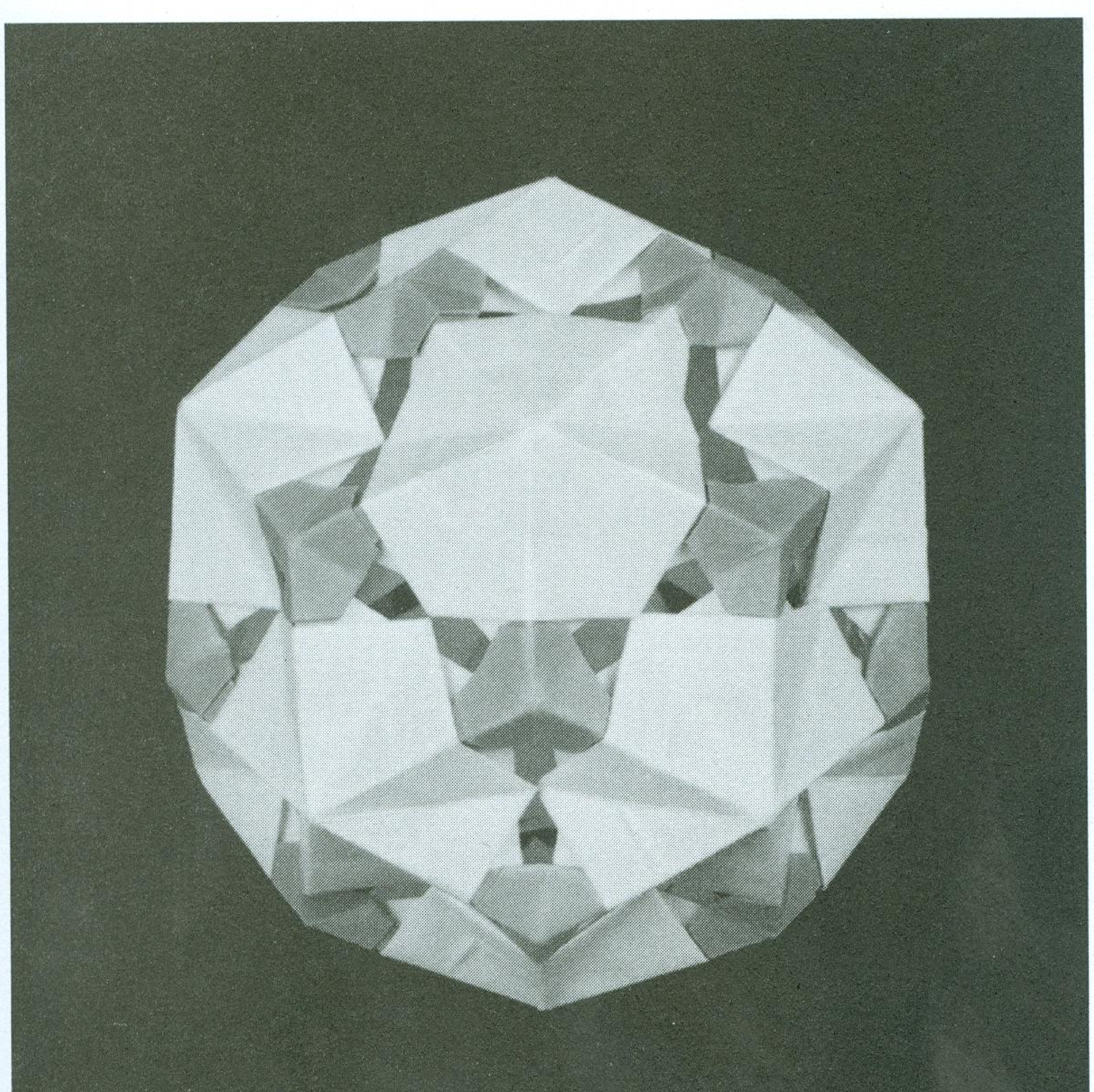
8. Gyroscoped Icosidodecahedron

Icosi-
dodecahedron



Rhombic
Triacontahedron



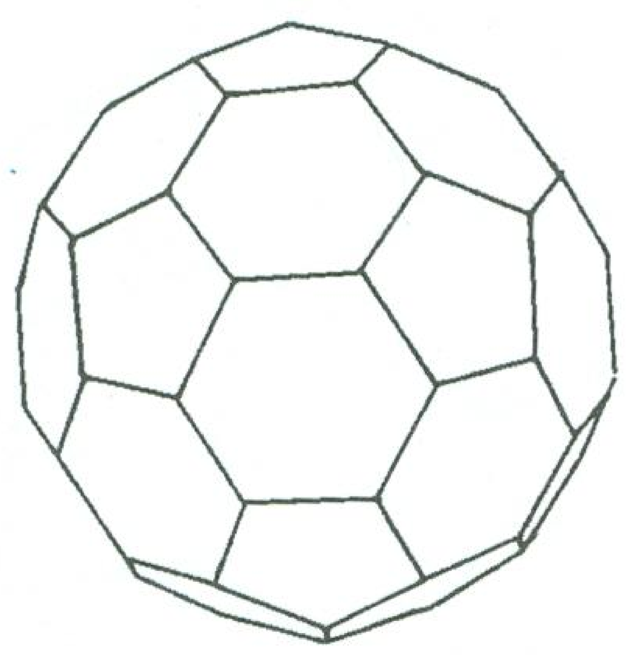


Gyroscope
Modules:
20 triangles

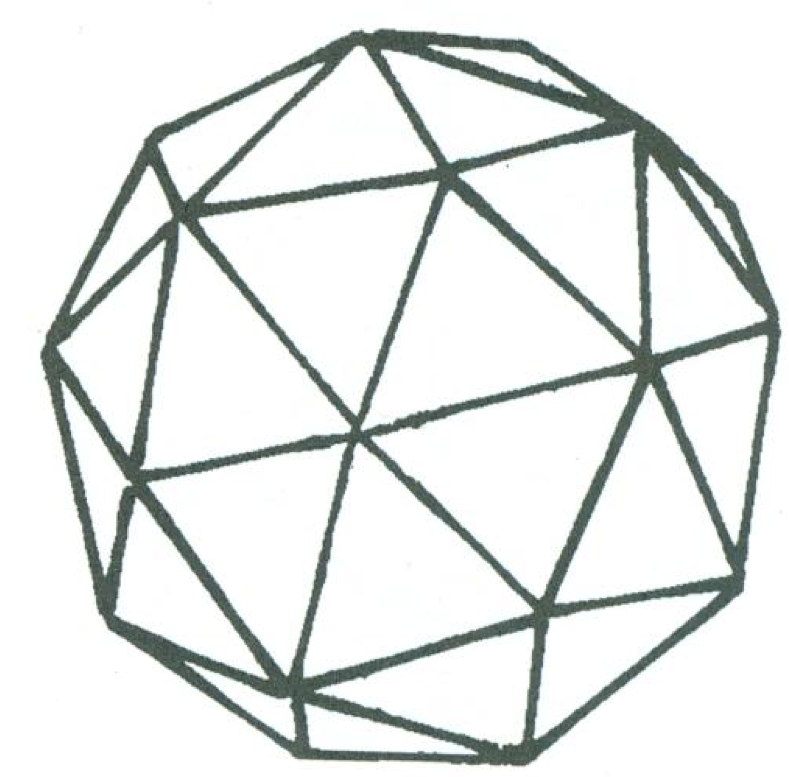
12 pentagons

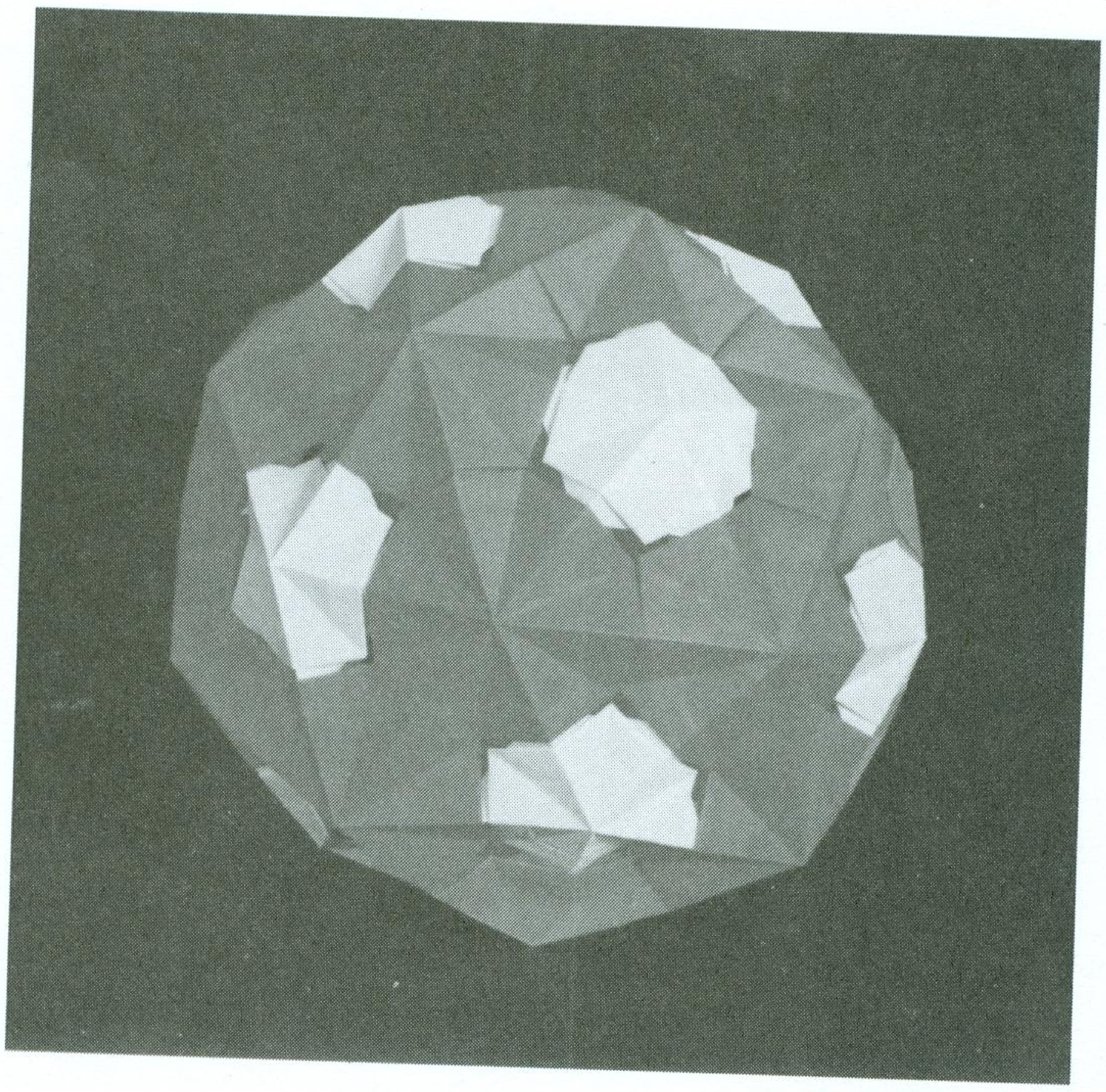
9. Gyroscoped Truncated Icosahedron

Truncated Icosahedron



Pentakis-dodecahedron

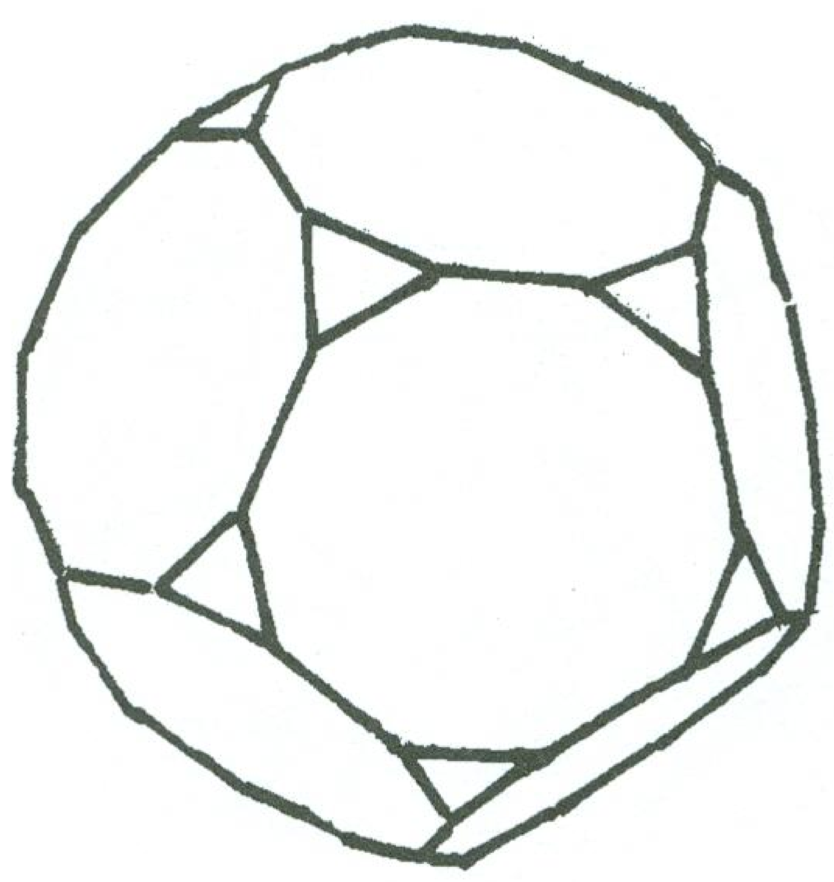




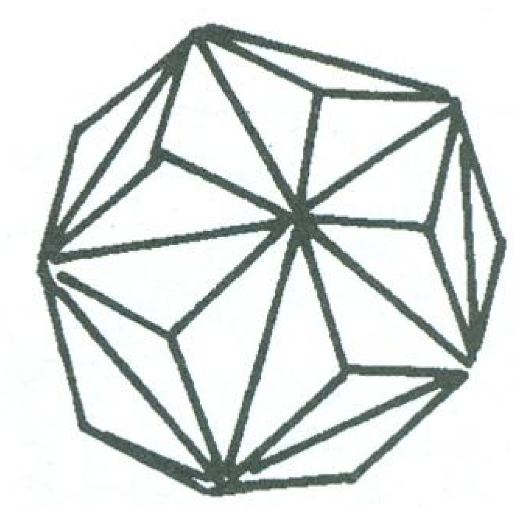
Gyroscope Modules:
12 pentagons and
20 hexagons

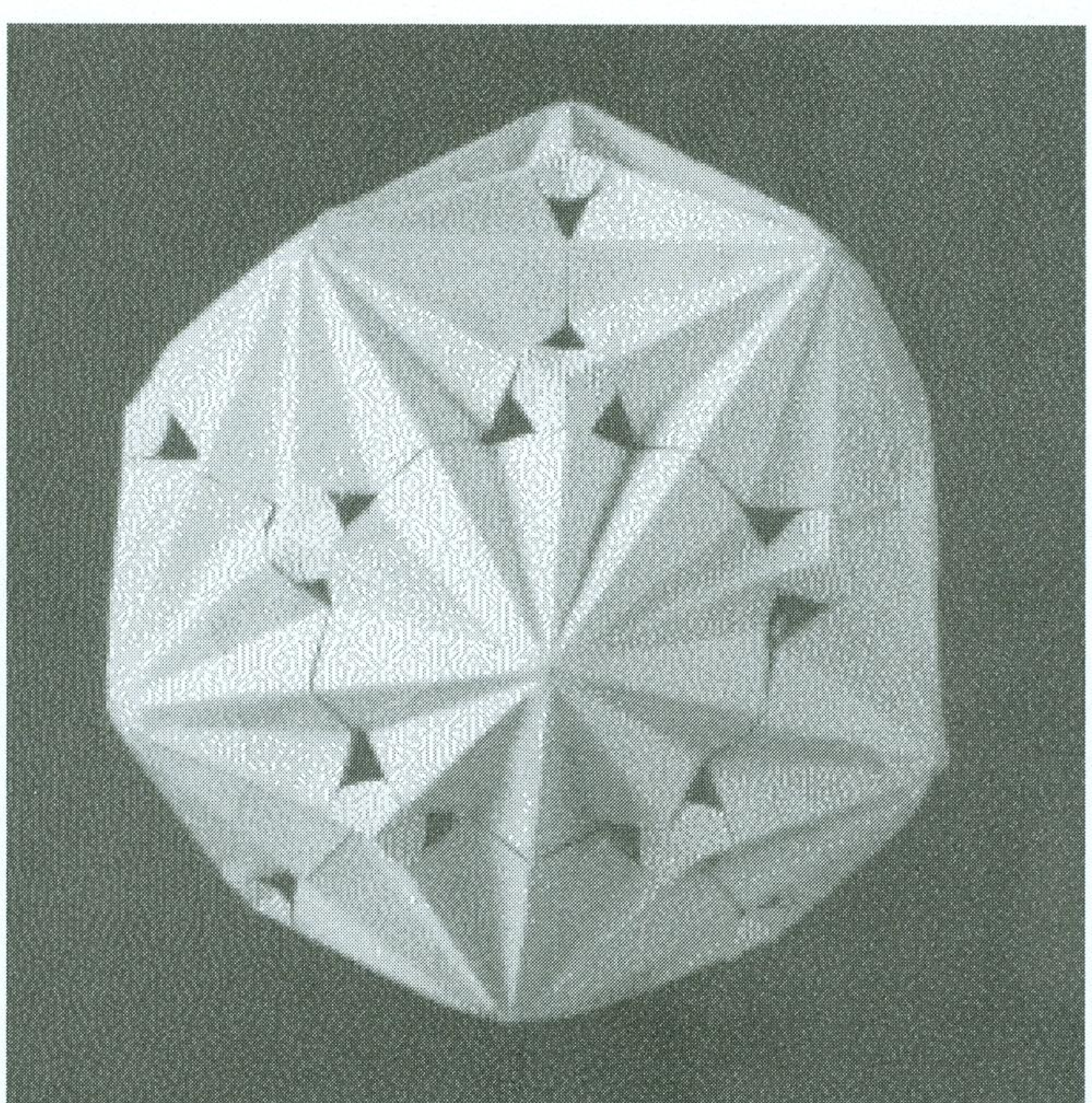
10. Gyroscoped Truncated Dodecahedron

Truncated Dodecahedron



Triakis-Icosahedron

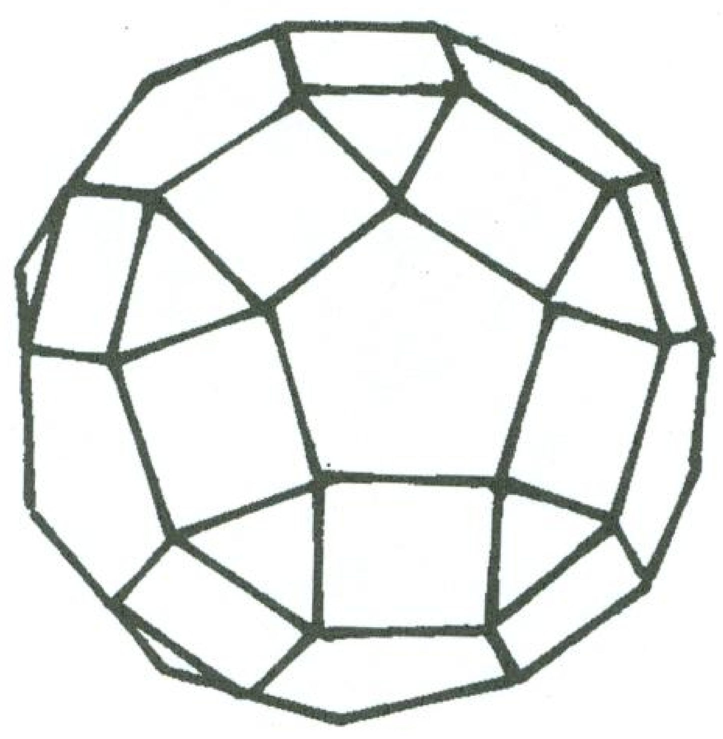




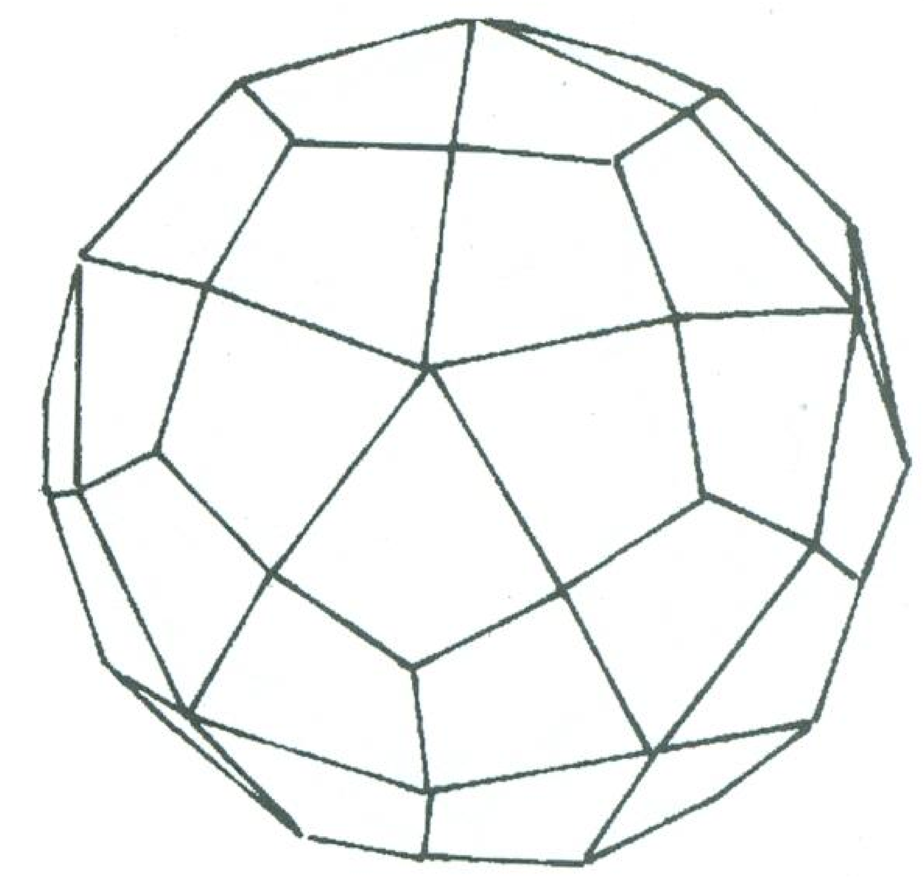
Gyroscope Modules:
20 triangles and
12 decagons

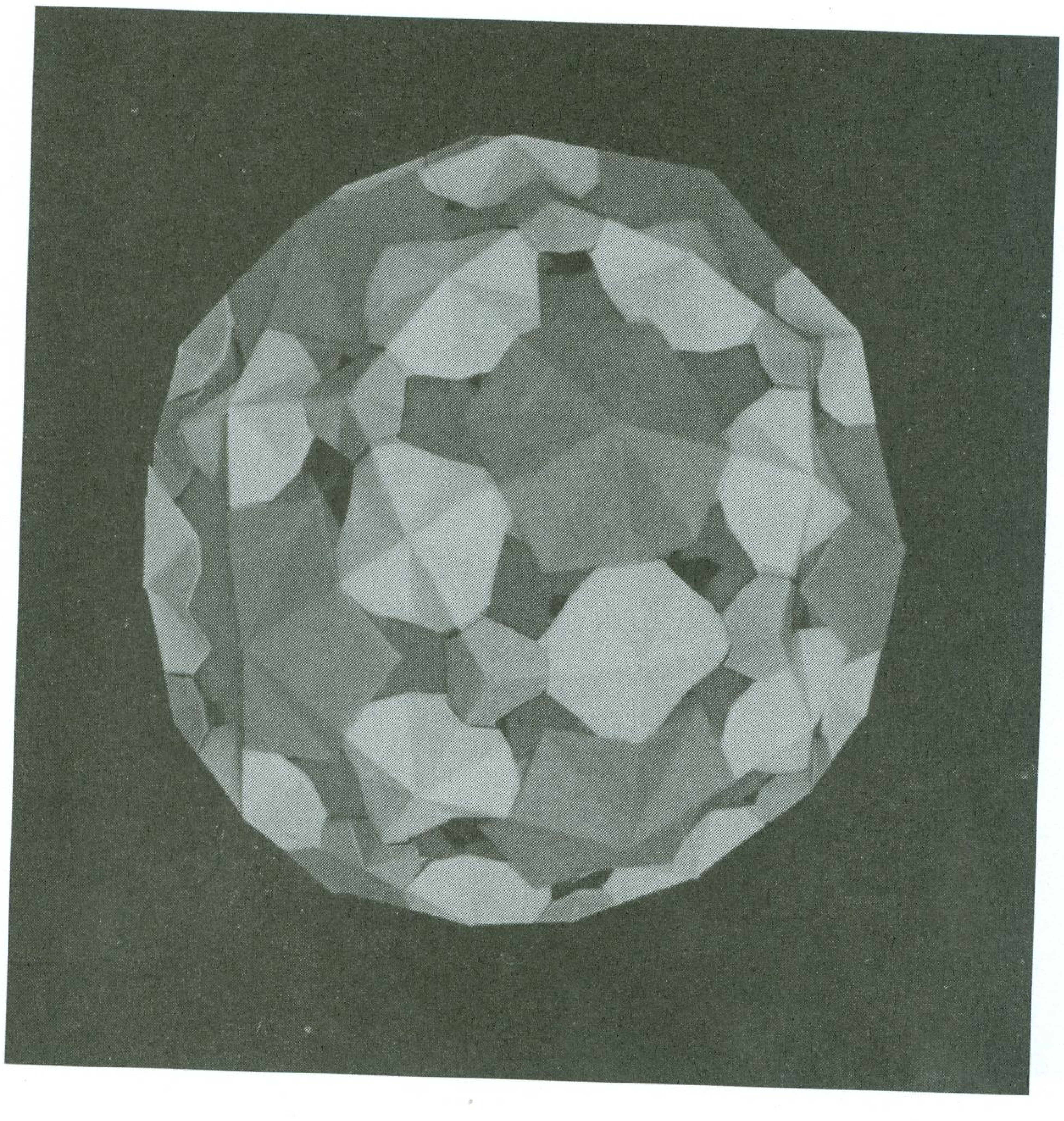
11. Gyroscoped Rhombicosidodecahedron

Rhombicosi-
dodecahedron



Trapezoidal
Hexecontahedron

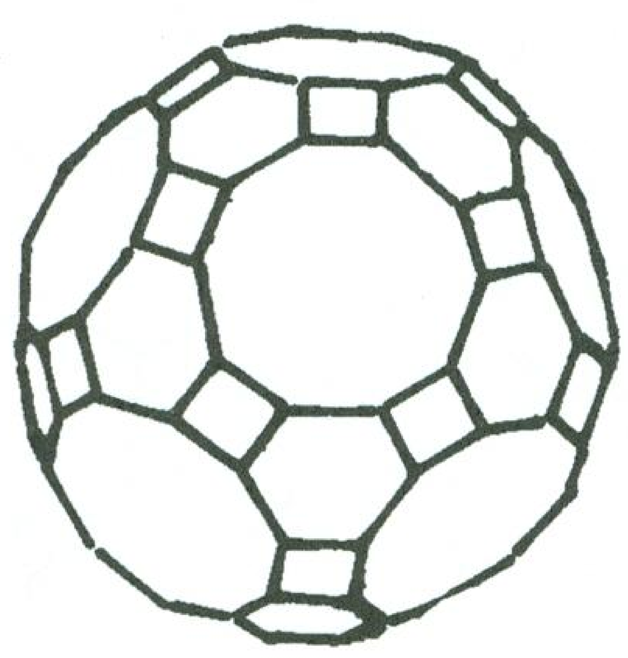




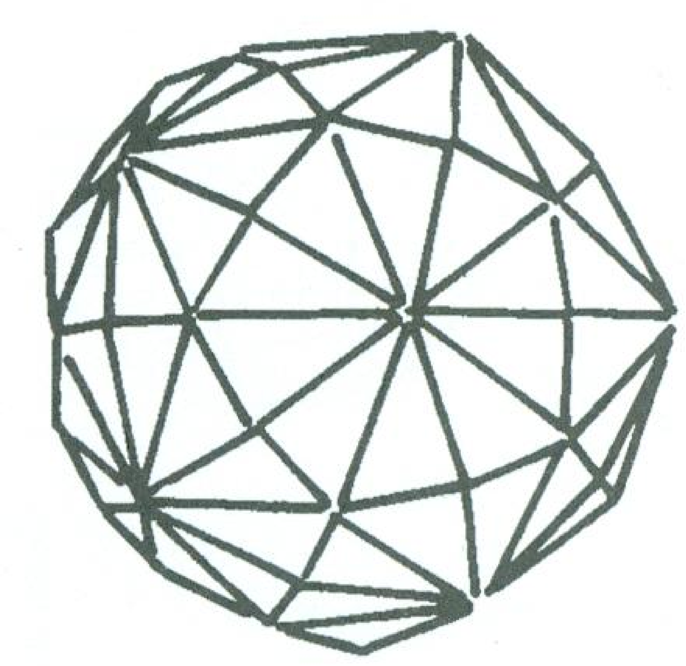
Gyroscope
Modules:
20 triangles,
30 squares, and
12 pentagons

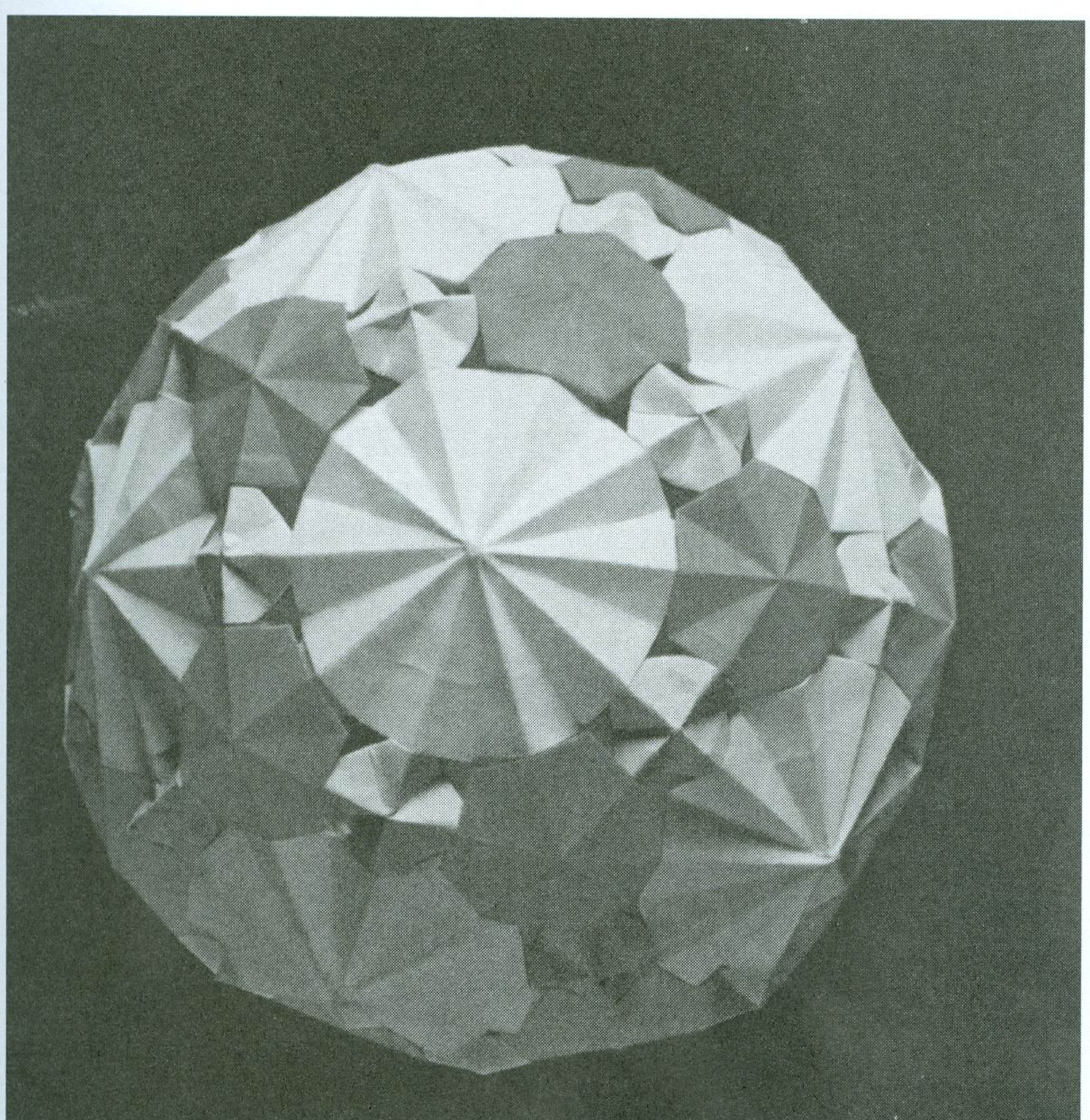
12. Gyroscoped Truncated Icosidodecahedron

Truncated Icosidodecahedron



Disdyakis Triacontahedron

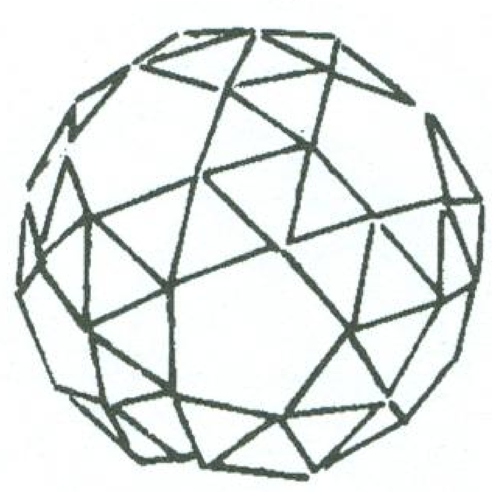




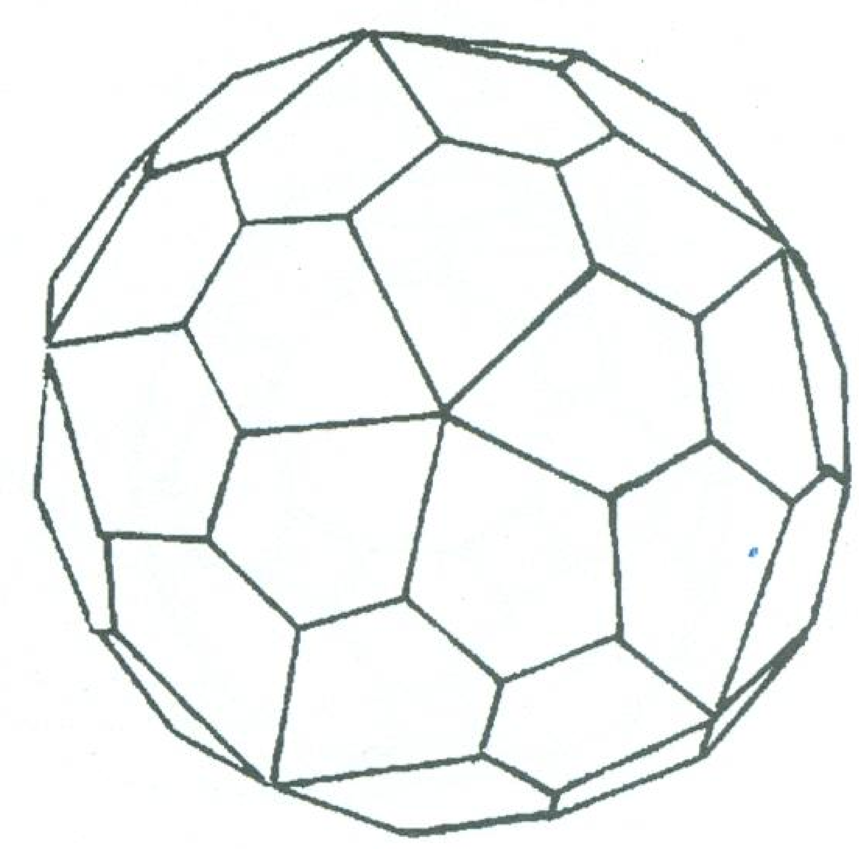
Gyroscope Modules
30 squares,
20 hexagons, and
12 decagons

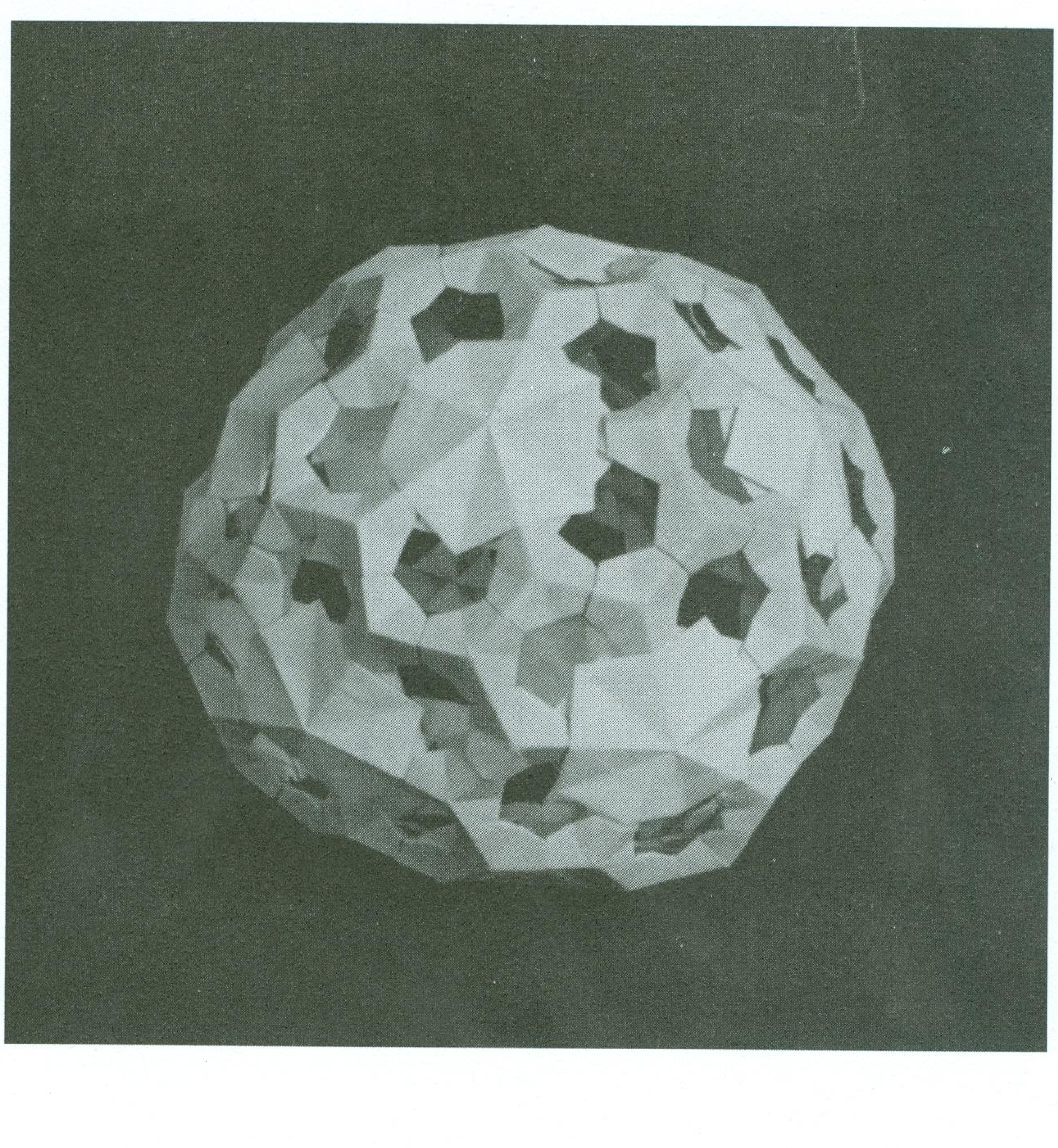
13. Gyroscoped Snub Dodecahedron

Snub Dodecahedron



Pentagonal Hexecontahedron

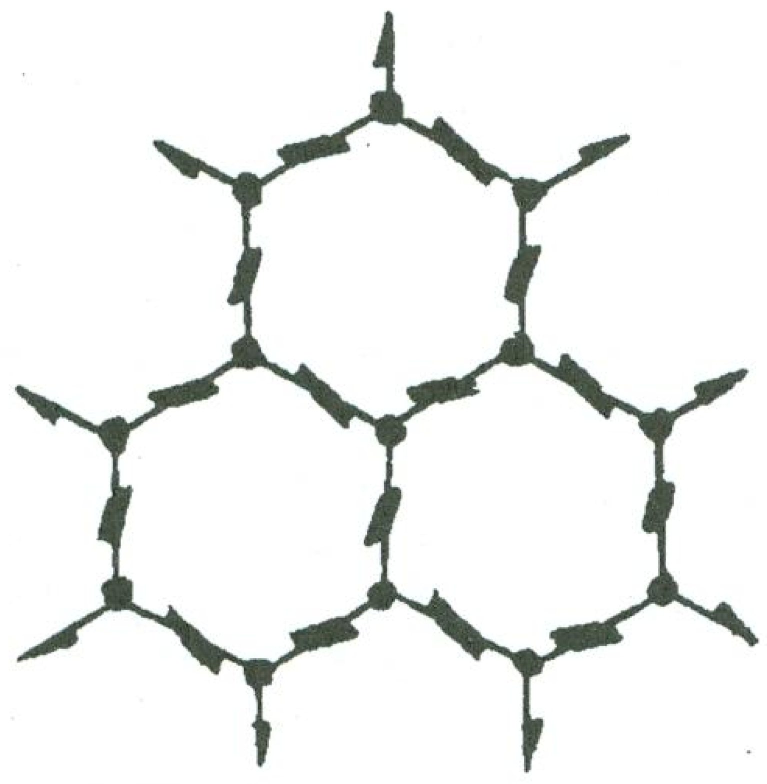




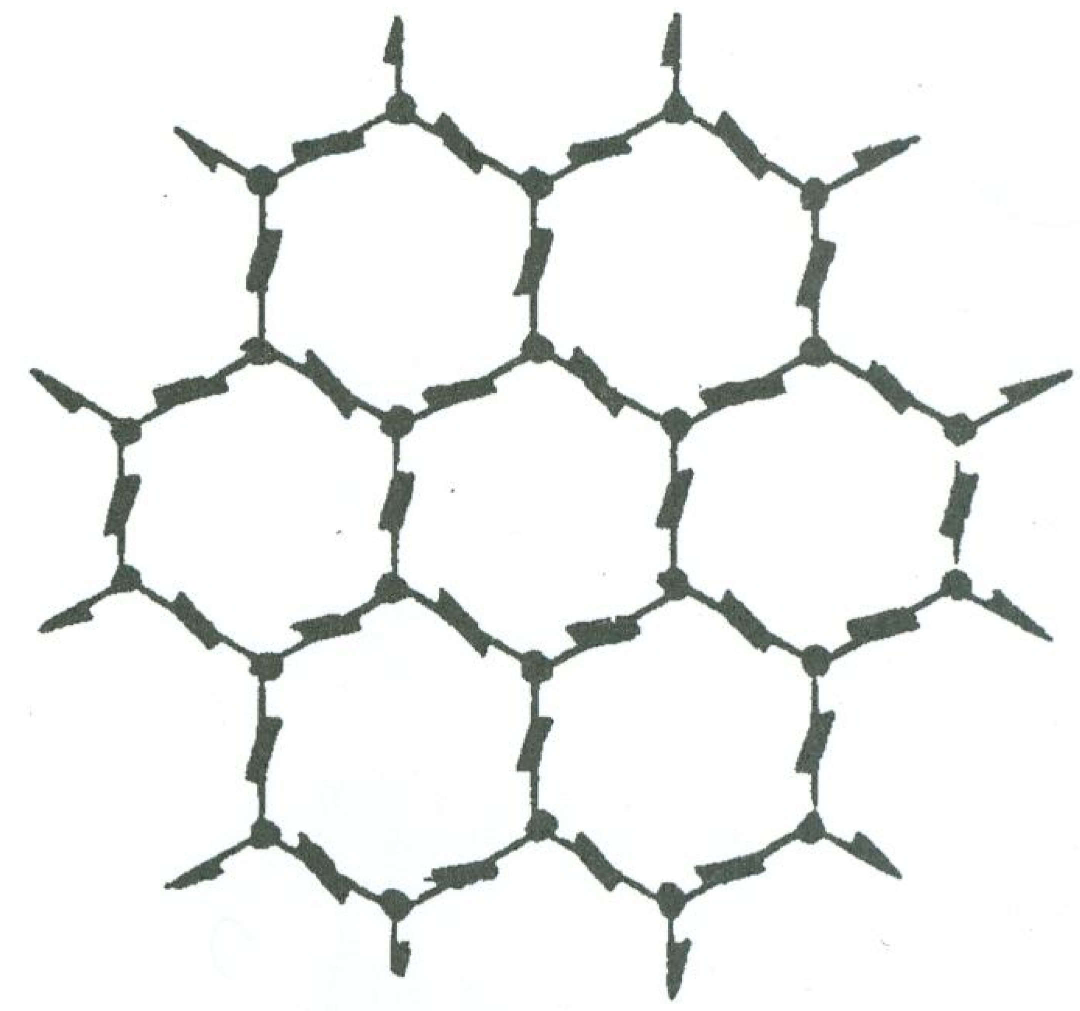
Gyroscope Modules:
80 triangles and
12 pentagons

C. Seeds for Growing Buckyballs, Buckyballs, Hypothetical Buckyballs, and Their Gyroscoped Forms

Triangle Gyroscope Module (one-piece triangle module)



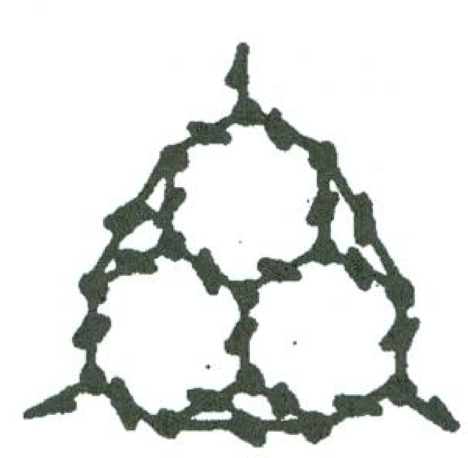
13 Triangle Gyroscopes form 3 hexagons



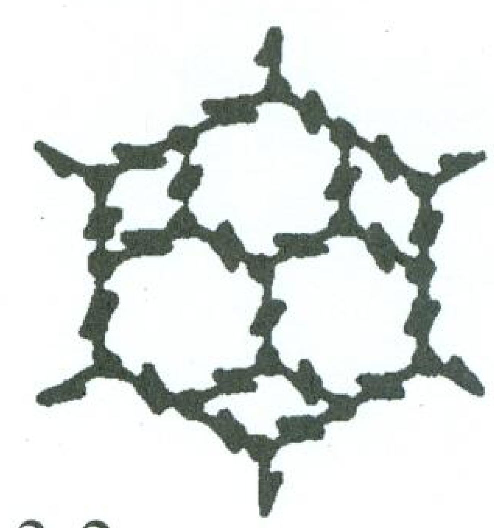
24 Triangle Gyroscopes form a cluster of 7 hexagons

	Tetrahedron Derivative	Cube Derivative	Dodecahedron Derivative [16]

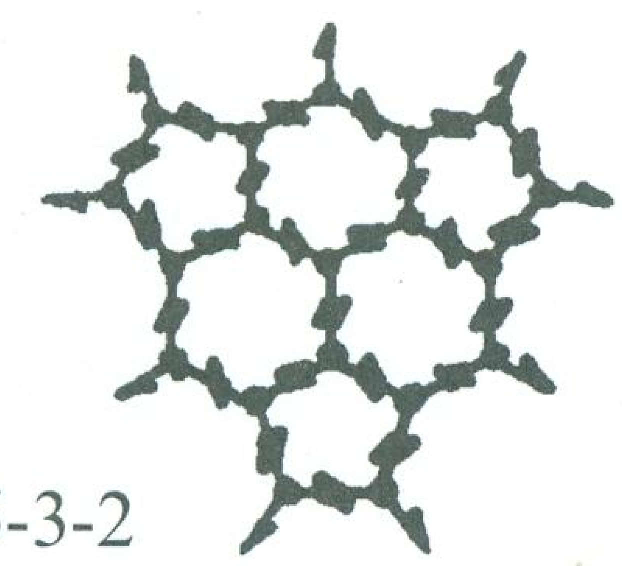
3 Basic Derivative polygons equally spaced around 3 hexagons; each polygon touches 2 hexagons



3-3-2

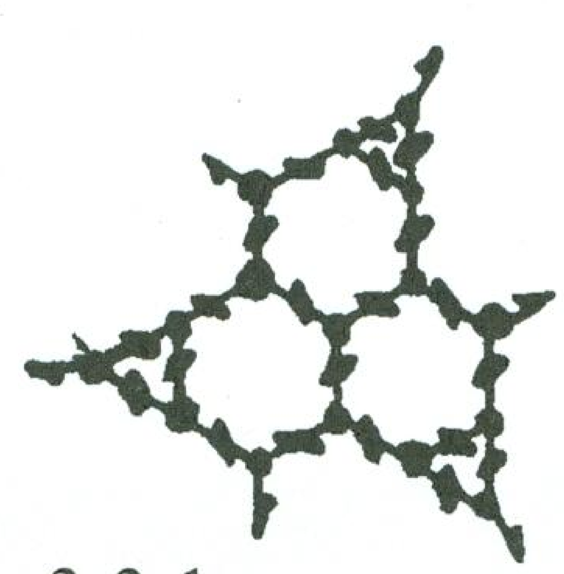


4-3-2

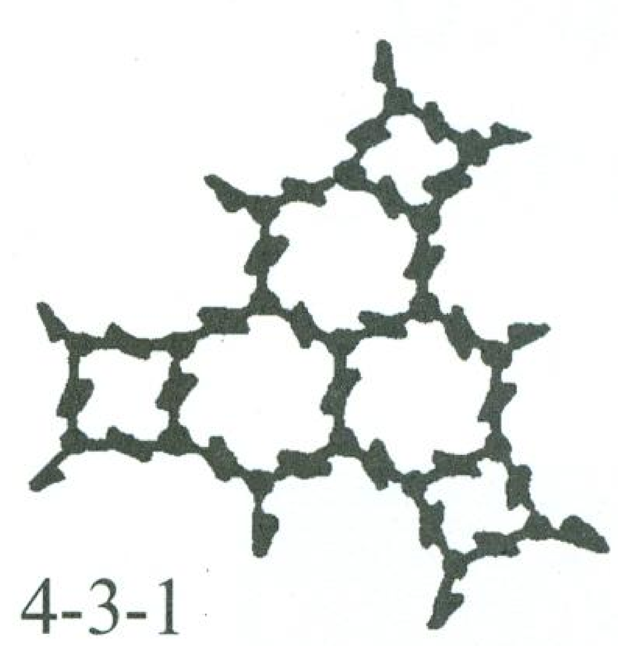


5-3-2

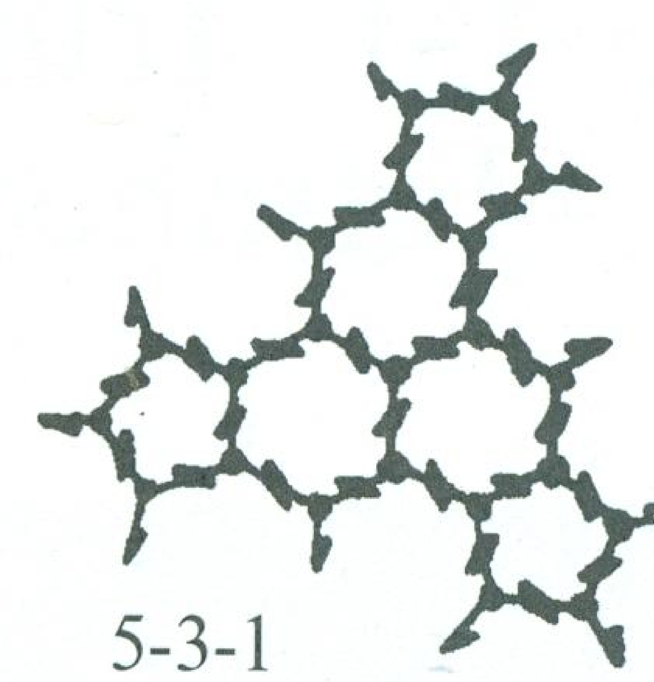
3 Basic Derivative polygons equally spaced around 3 hexagons; each polygon touches 1 hexagon



3-3-1

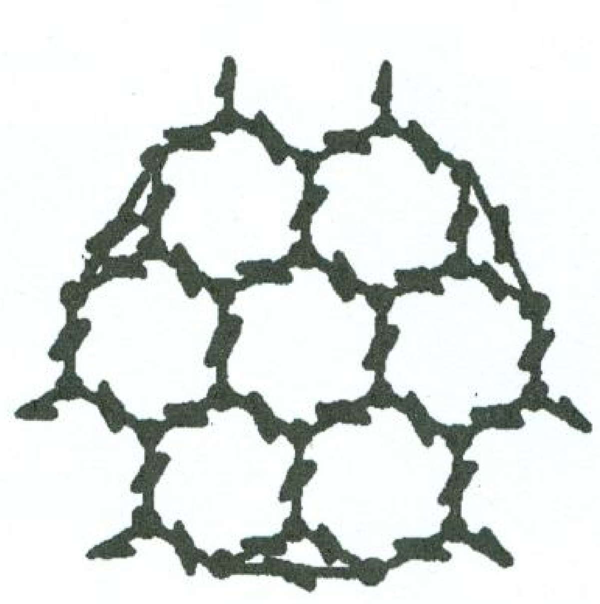


4-3-1

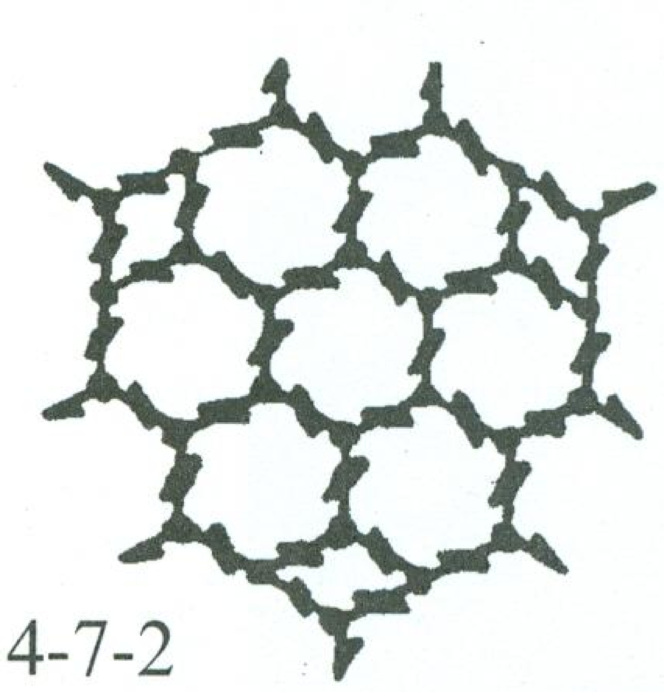


5-3-1

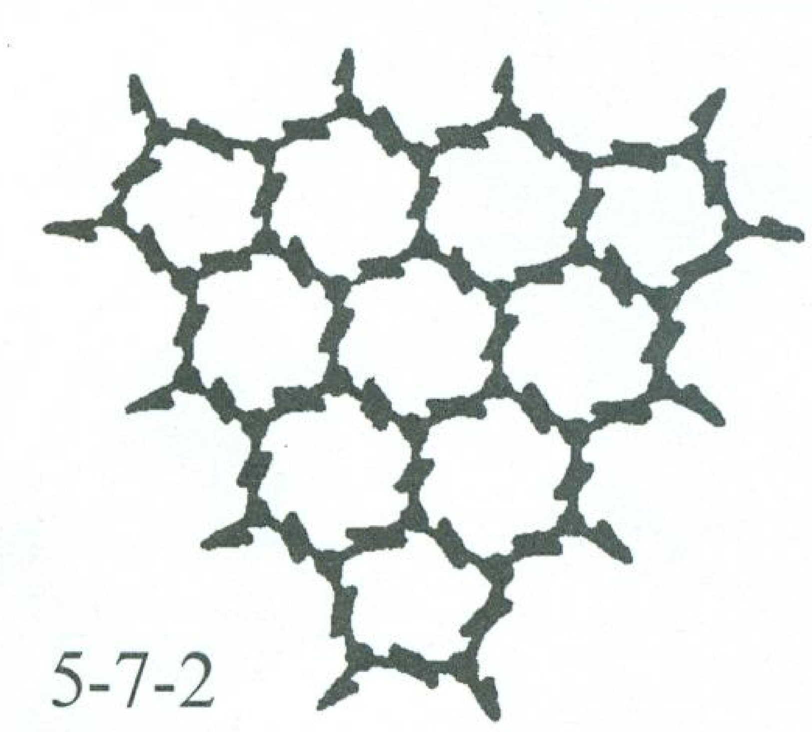
3 Basic Derivative polygons equally spaced around a cluster of 7 hexagons; each polygon touches 2 hexagons



3-7-2

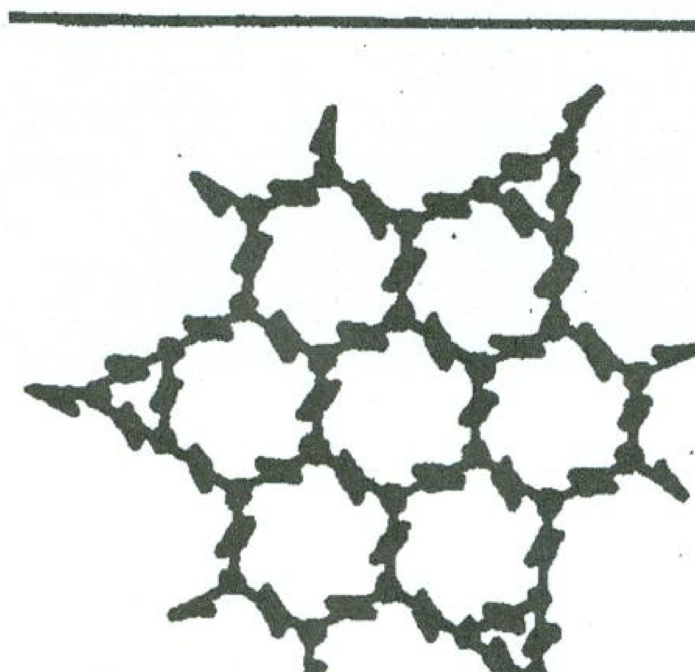


4-7-2

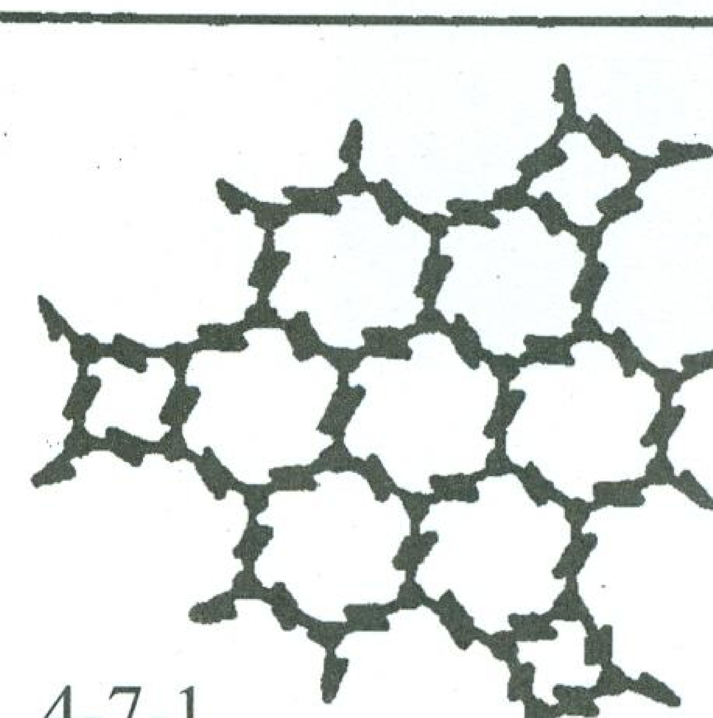


5-7-2

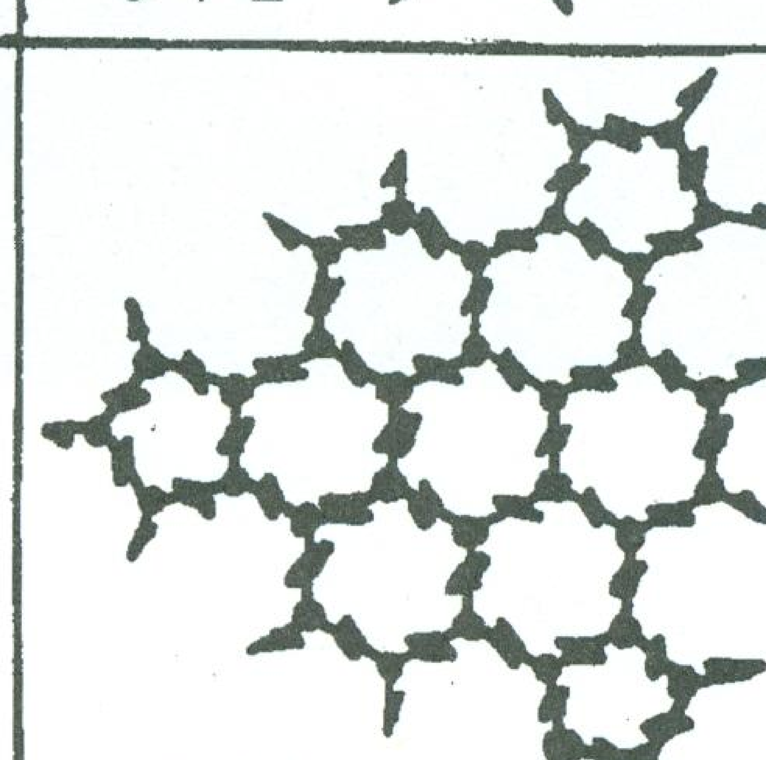
3 Basic Derivative polygons equally spaced around a cluster of 7 hexagons; each polygon touches 1 hexagon



3-7-1



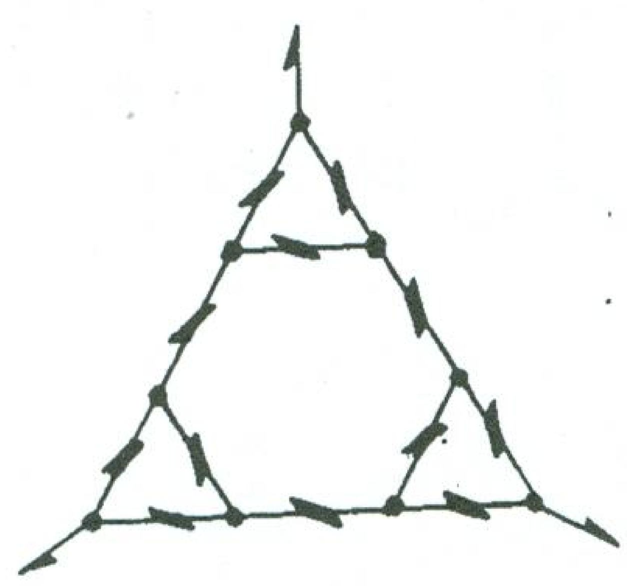
4-7-1



5-7-1

1. 3-1-1

Net



Ring of one hexagon with each triangle touching one edge of hexagon

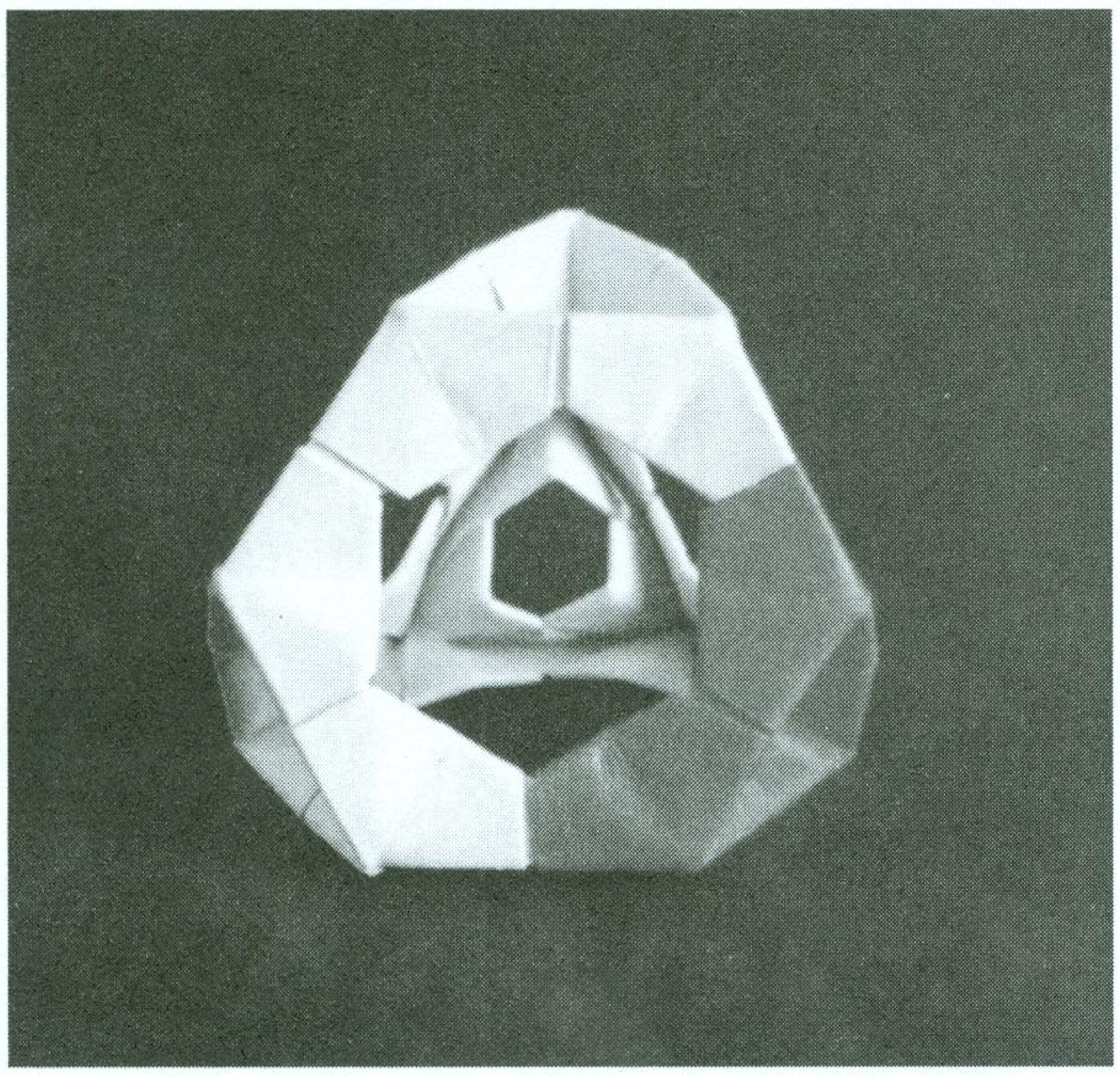
Modules:

12 triangles or

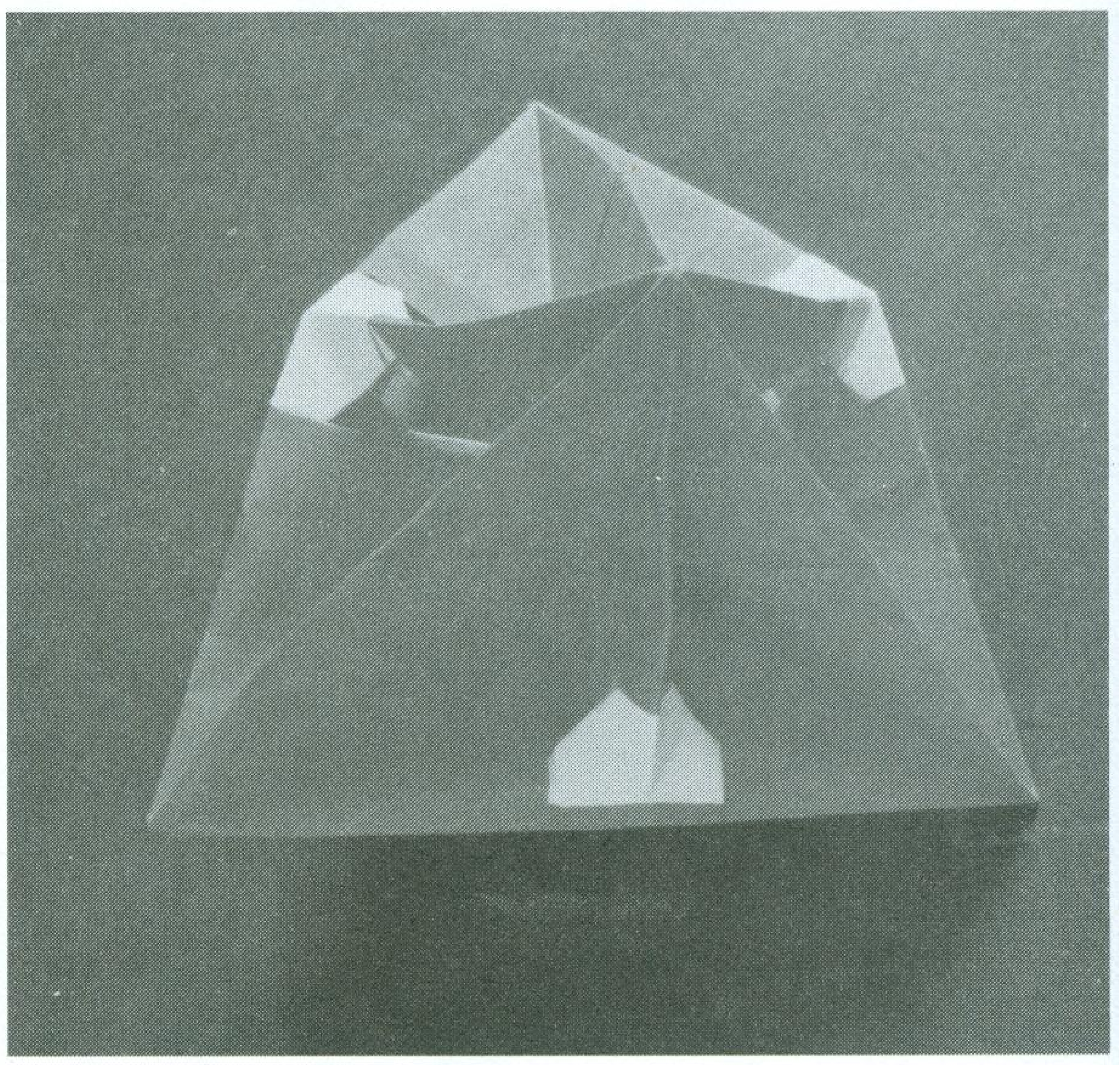
4 triangles and

4 hexagons

3-1-1 Truncated Tetrahedron

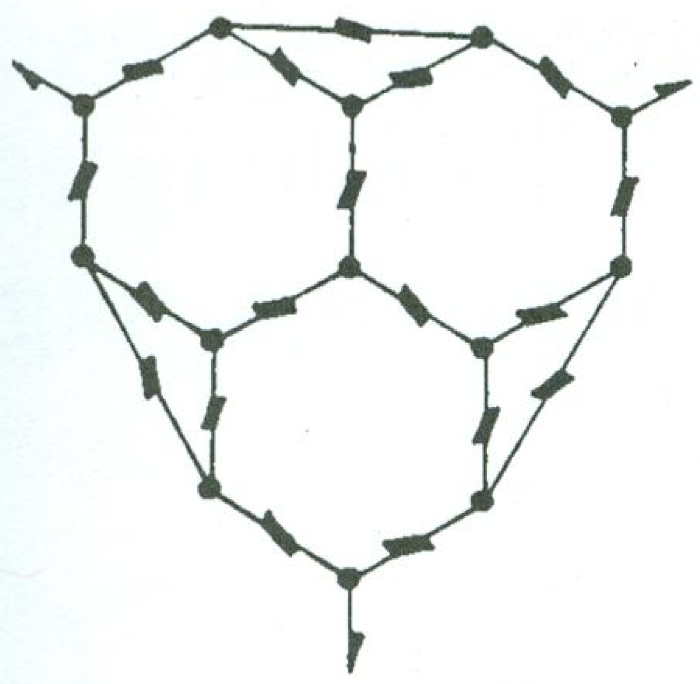


3-1-1 Gyroscoped Truncated Tetrahedron



2. 3-3-2

Net



Ring of three hexagons with triangle touching edge of two hexagons in a ring

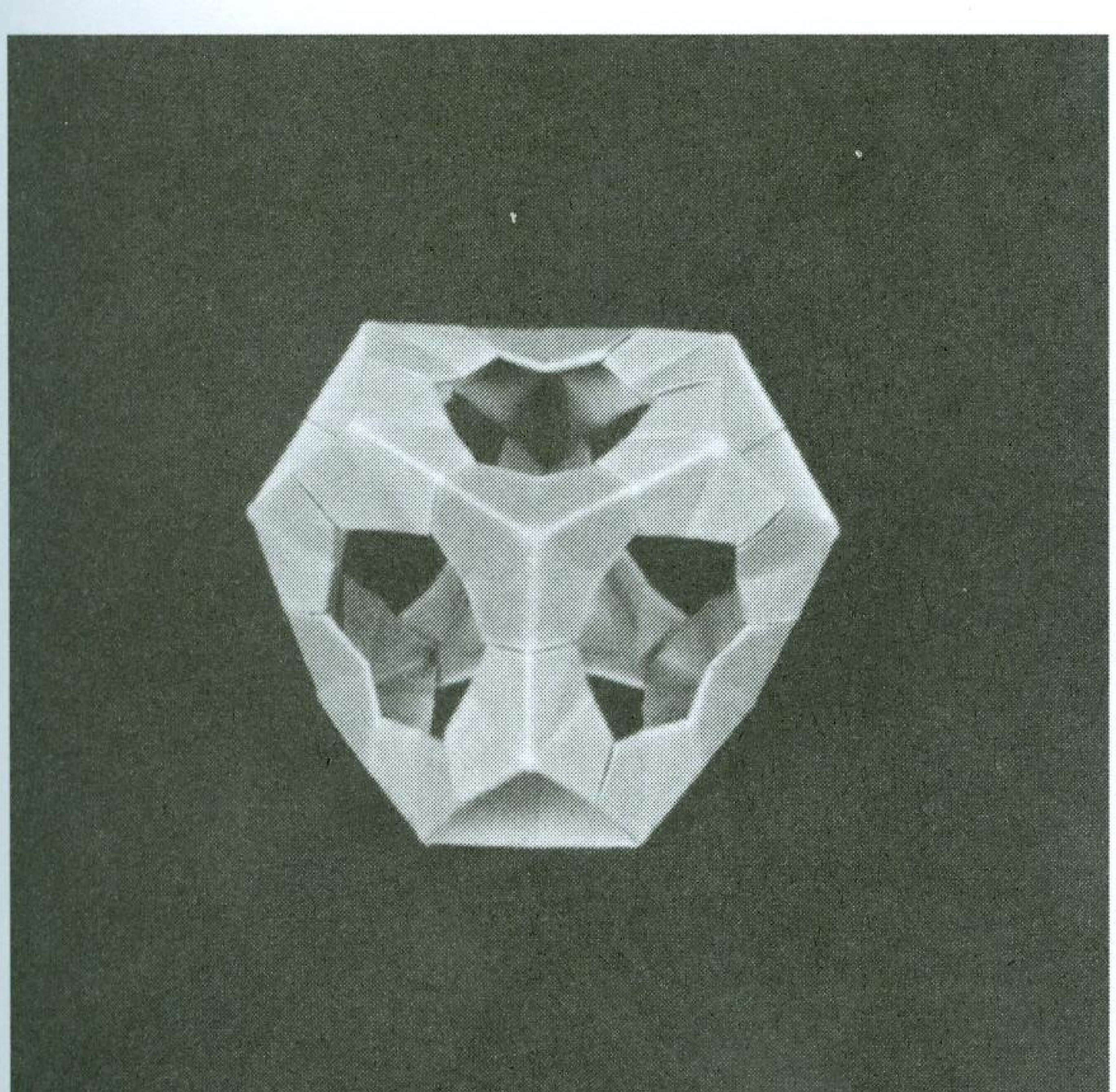
Modules

16 triangles or

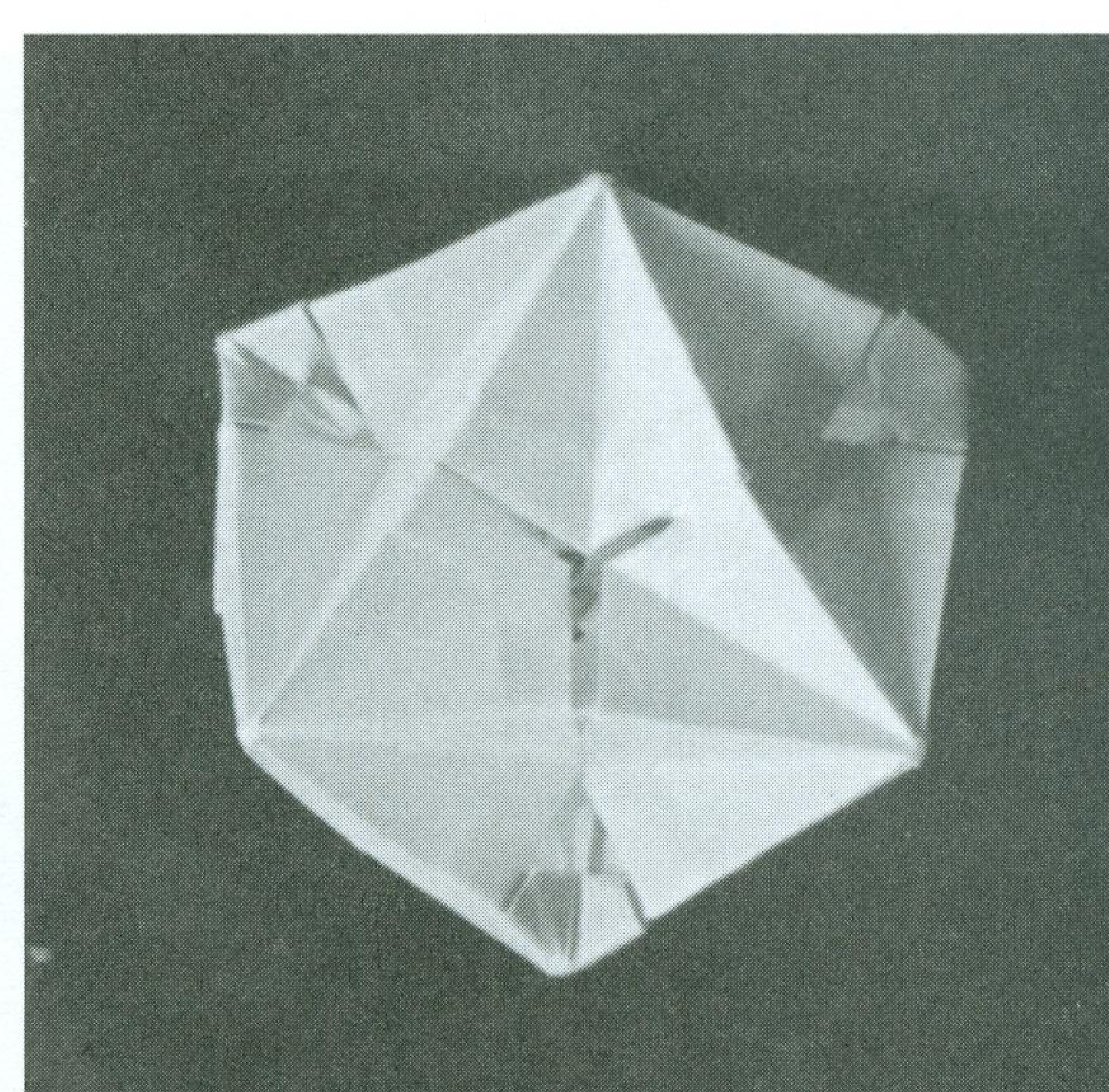
4 triangles and

6 hexagons

3-3-2 Hypothetical Bucky

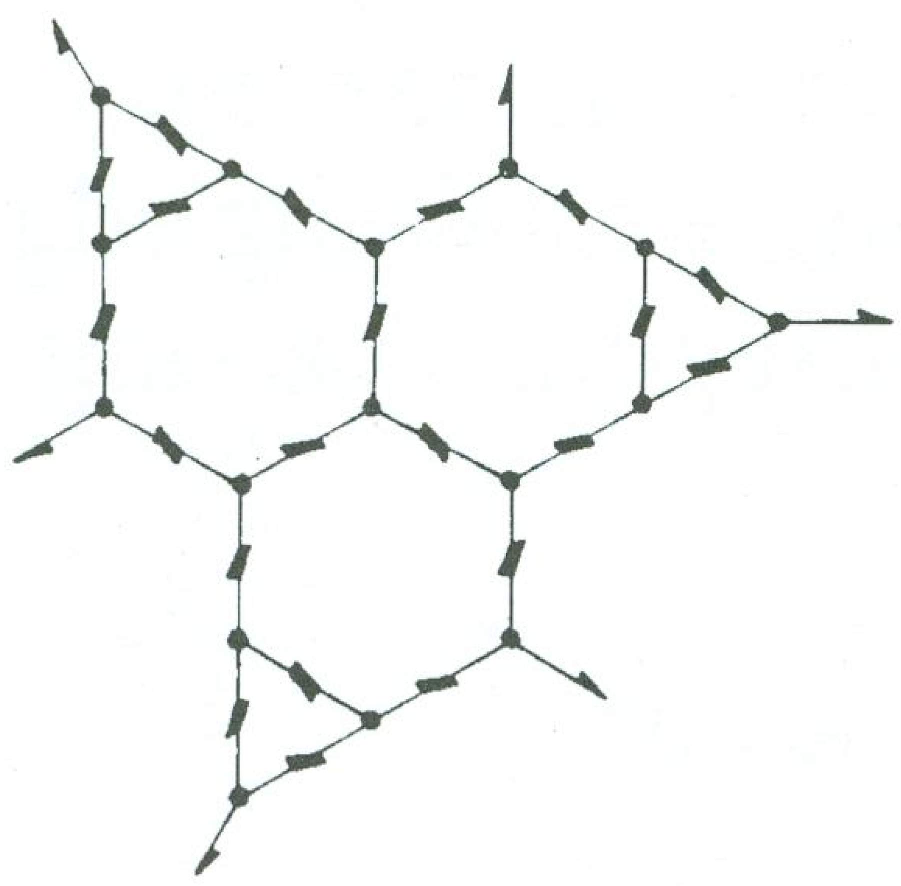


3-3-2 Gyroscoped



3. 3-3-1

Net



Ring of three hexagons with triangle touching edge of one hexagon in a ring

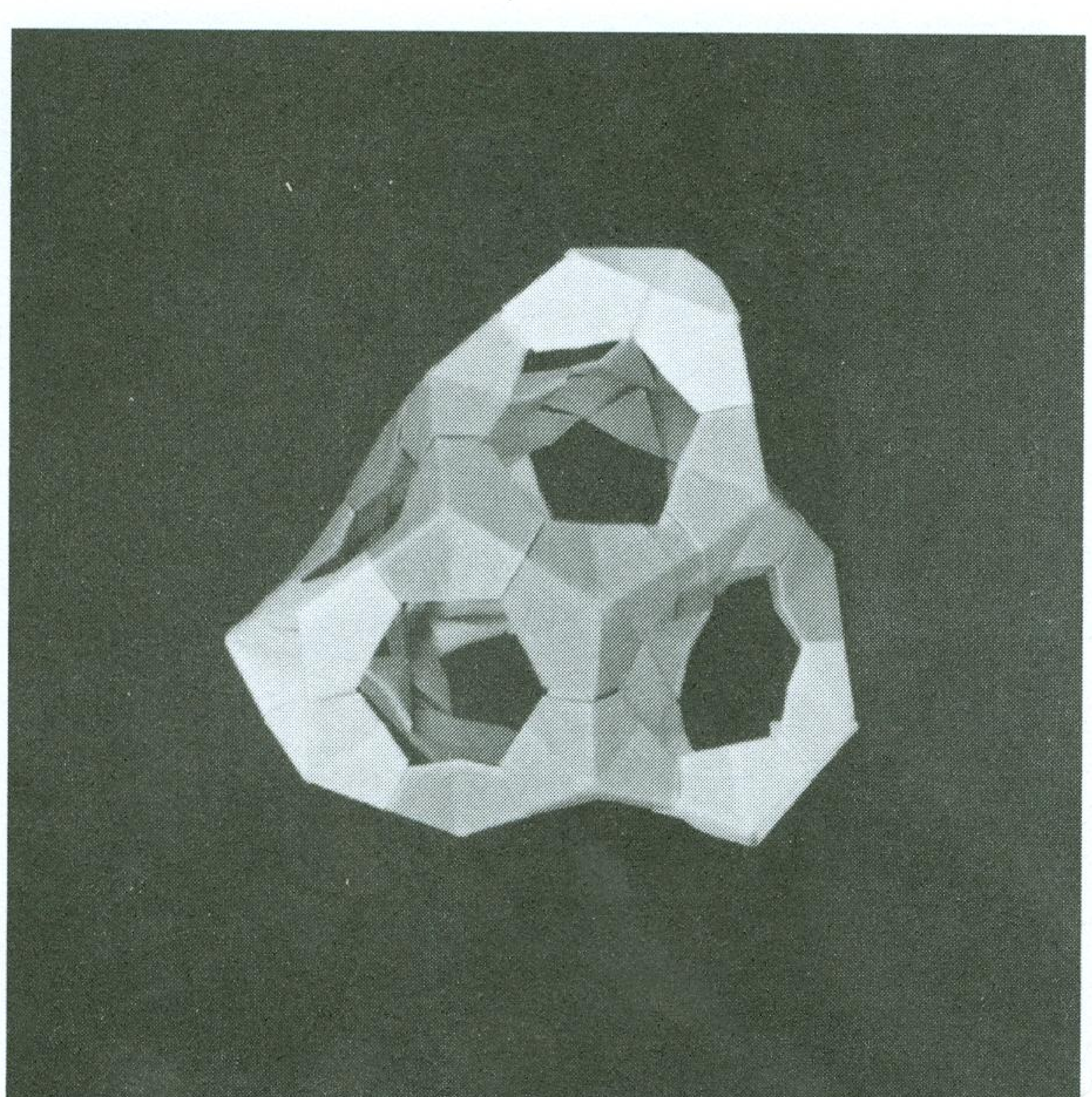
Modules:

28 triangles or

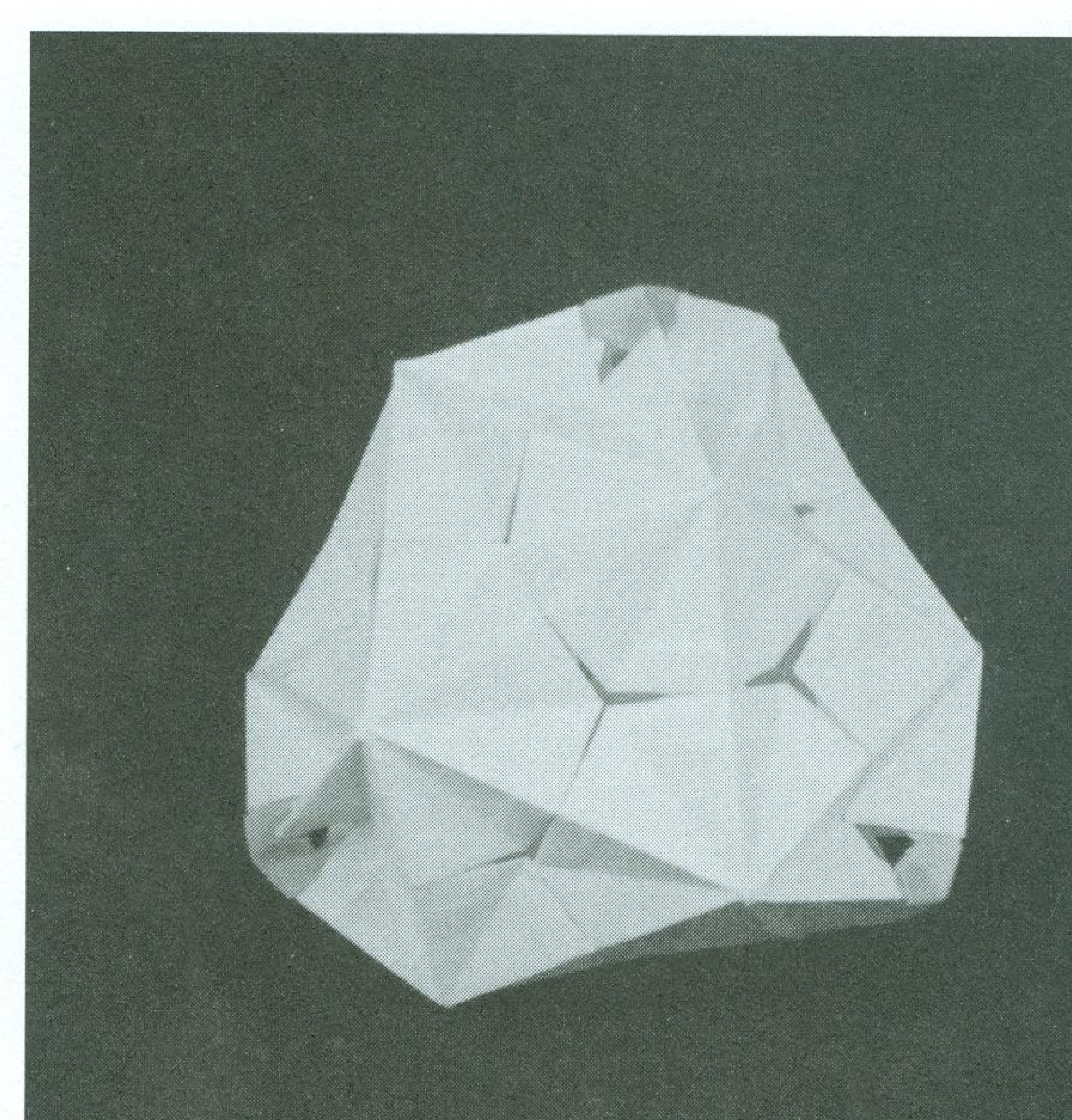
4 triangles and

12 hexagons

3-3-1 Hypothetical Bucky

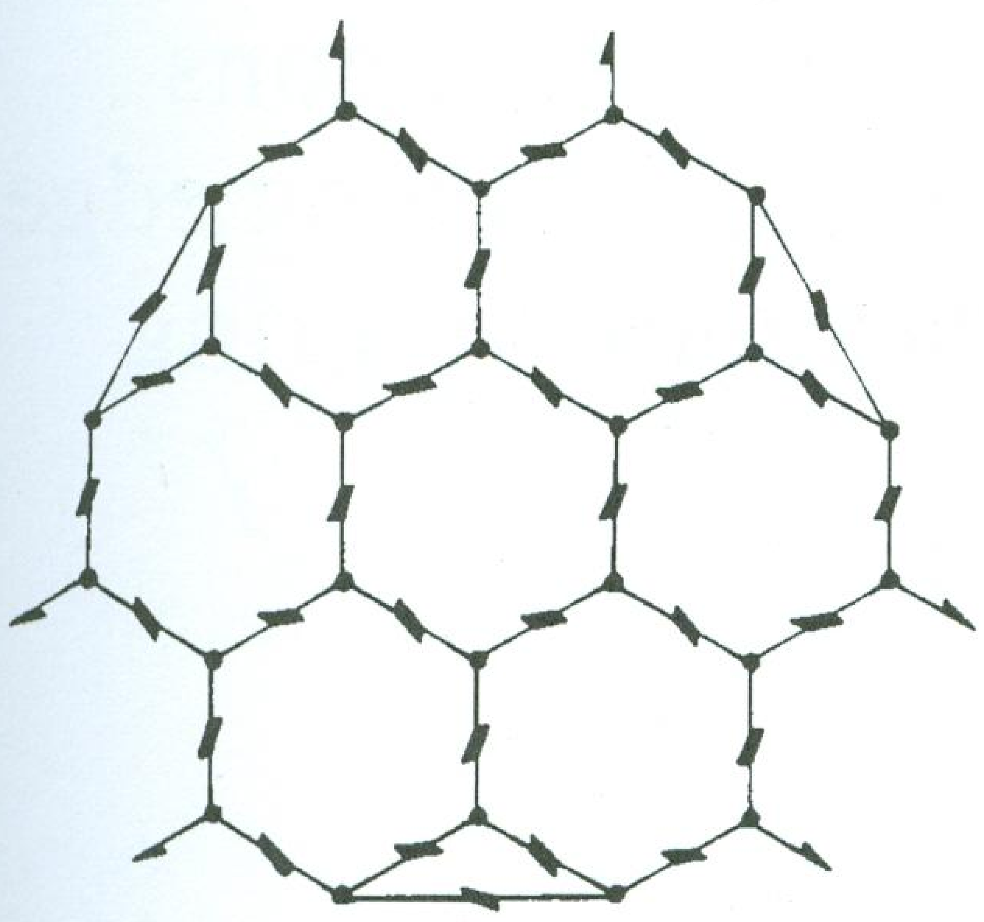


3-3-1 Gyroscoped



4. 3-7-2

Net



Ring of seven hexagons with triangle touching one edge of two hexagons in a ring

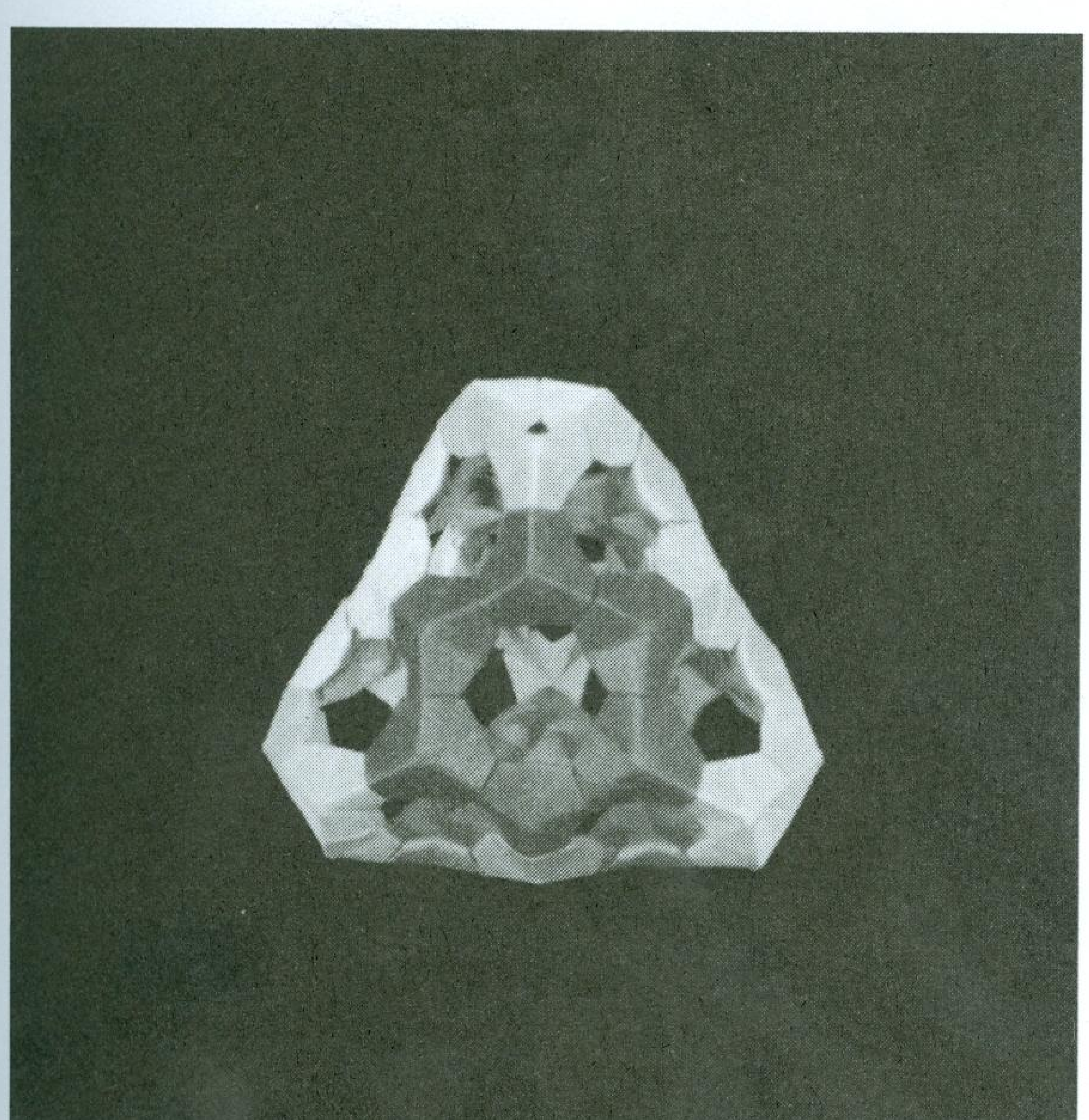
Modules:

36 triangles or

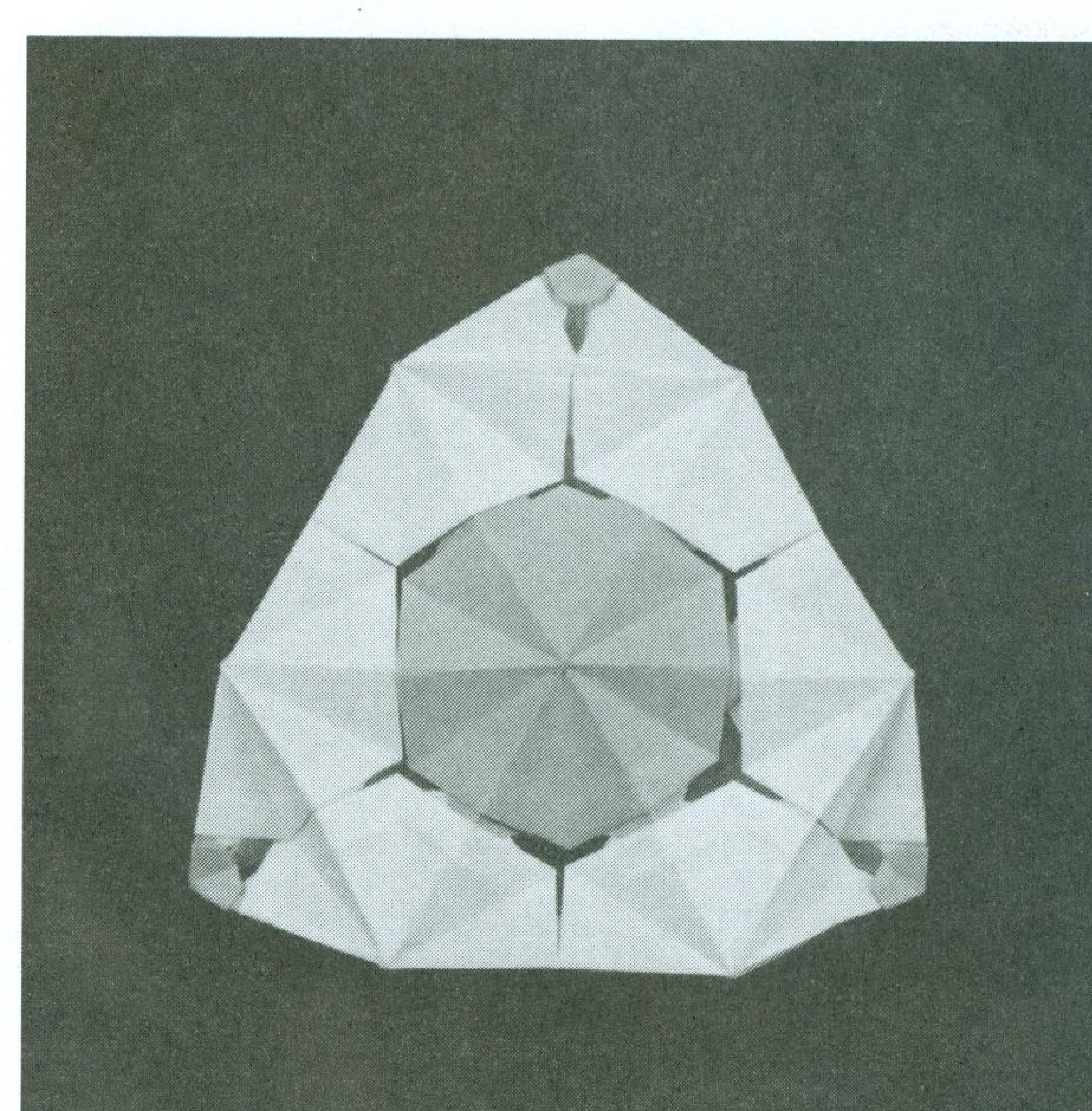
4 triangles and

16 hexagons

3-7-2 Hypothetical Bucky

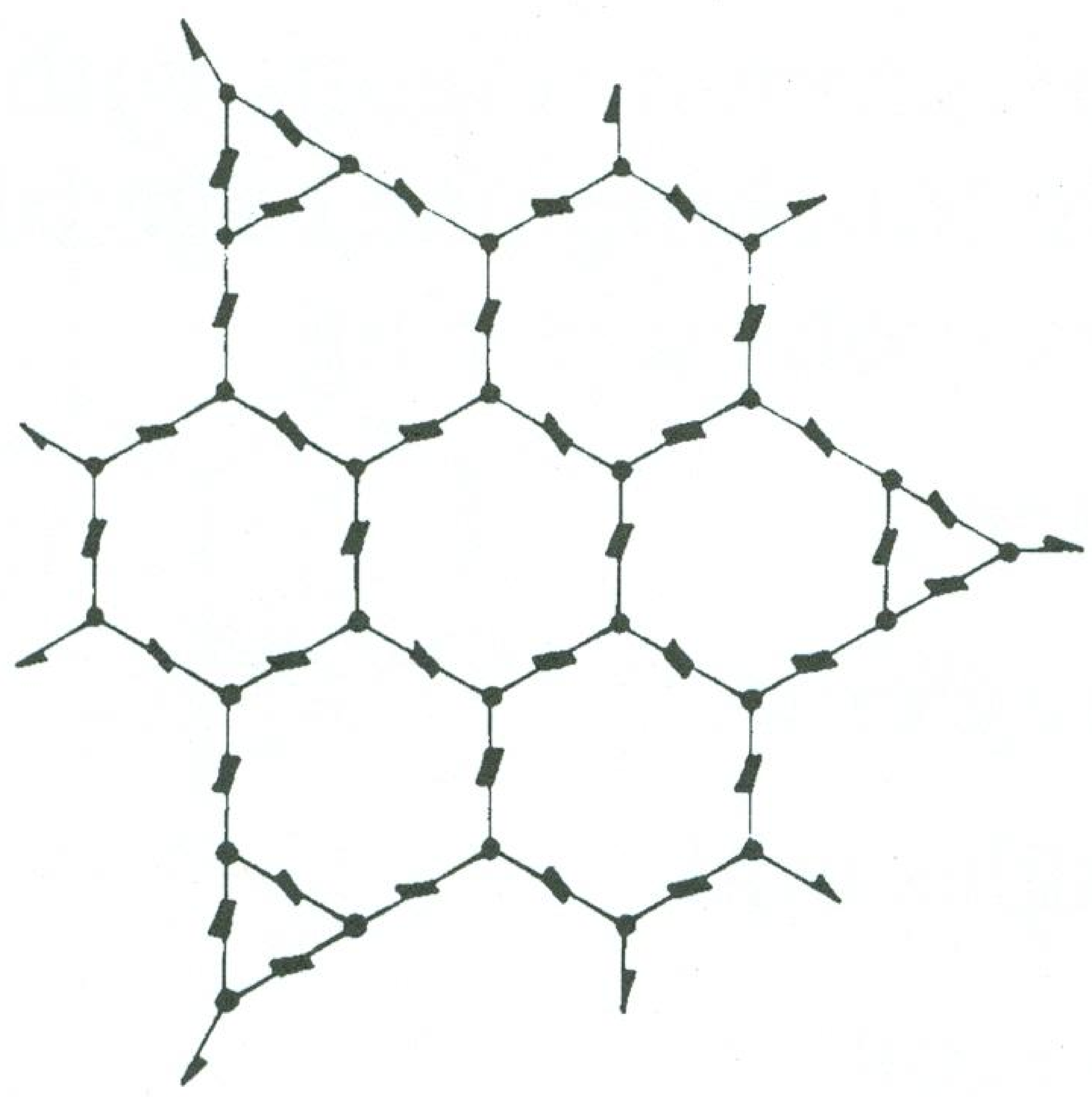


3-7-2 Gyroscoped



5. 3-7-1

Net



Ring of seven hexagons with triangle touching edge of one hexagon in a ring

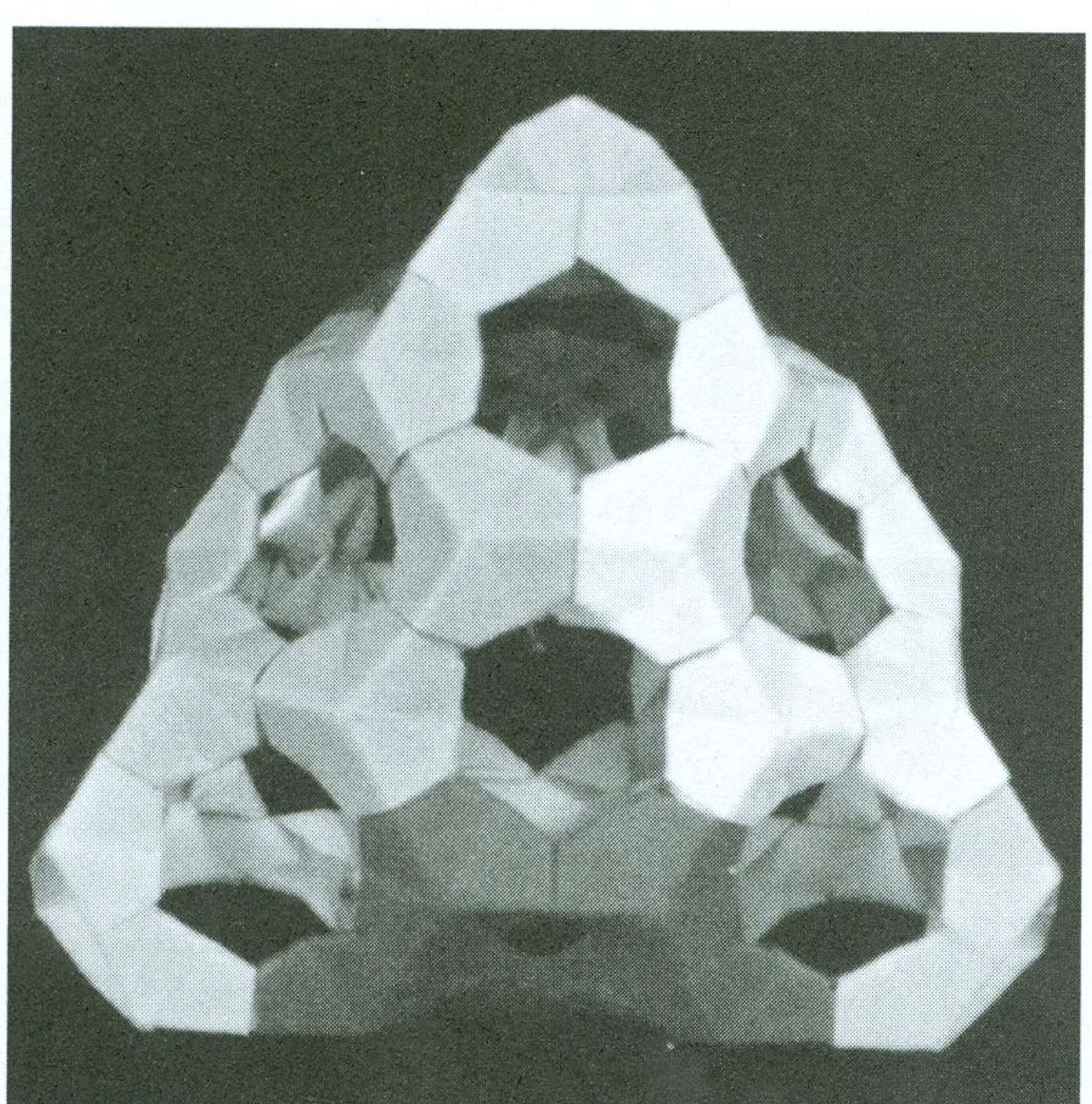
Modules:

48 triangles or

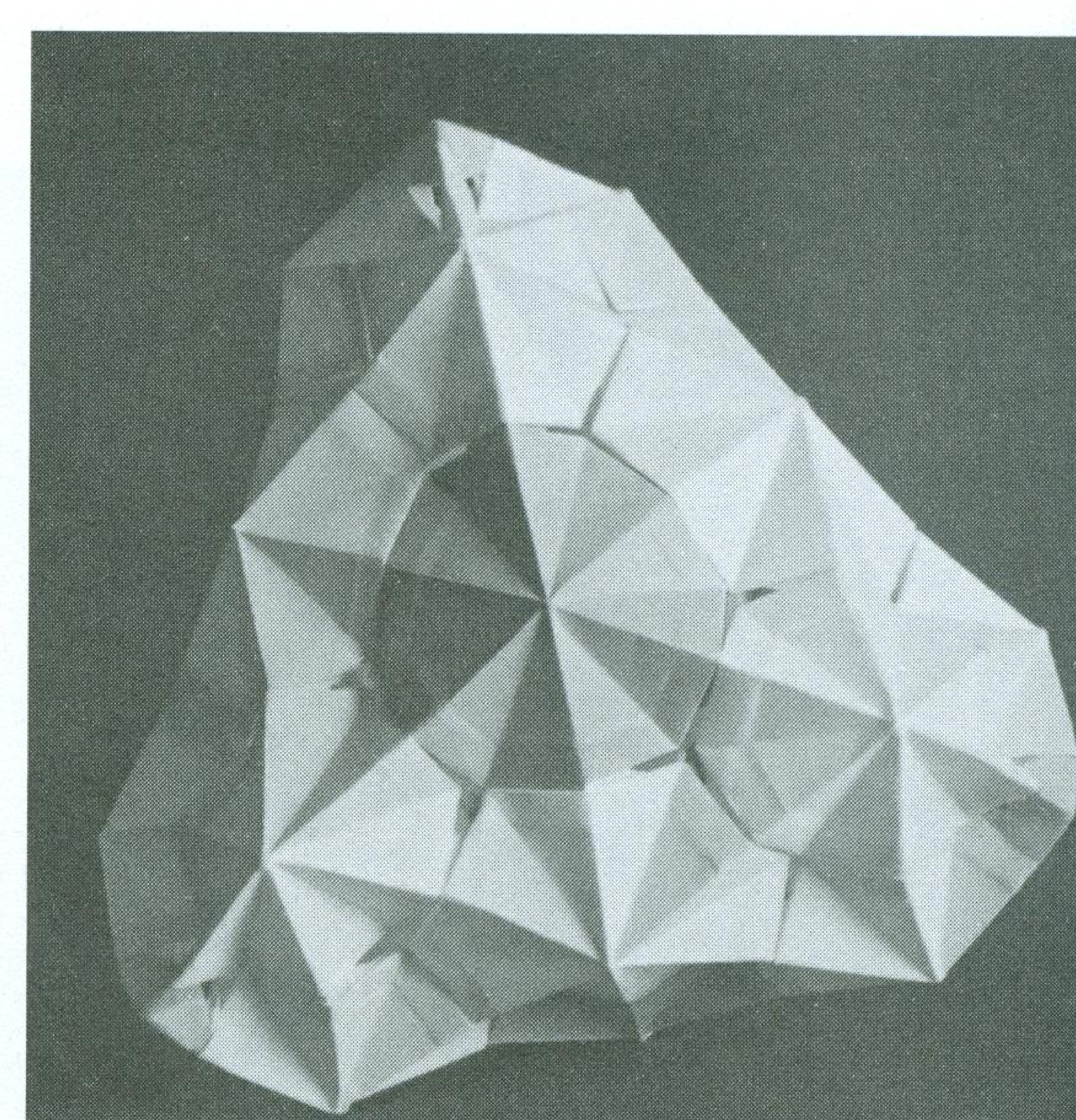
4 triangles

22 hexagons

3-7-1 Hypothetical Bucky

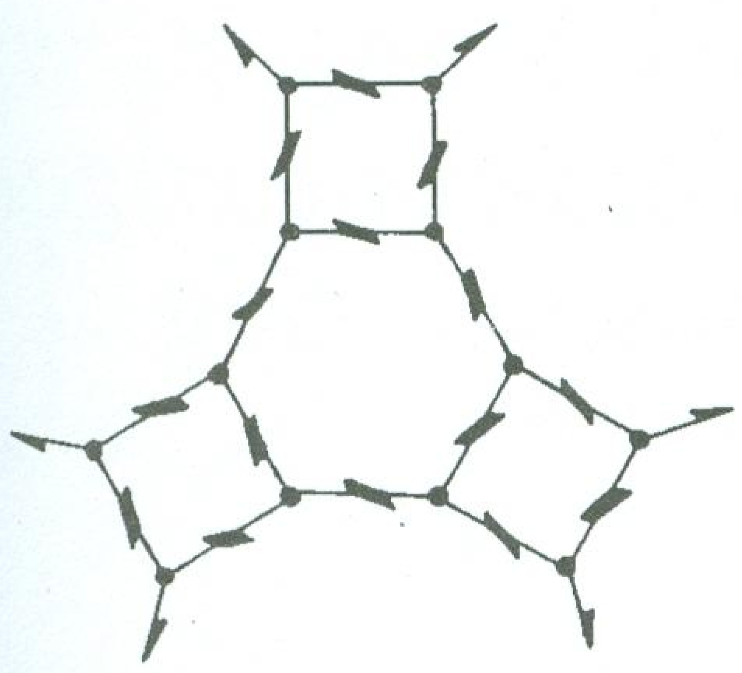


3-7-1 Gyroscoped



6. 4-1-1

Net



Ring of one hexagon with square touching one edge of one hexagon in ring

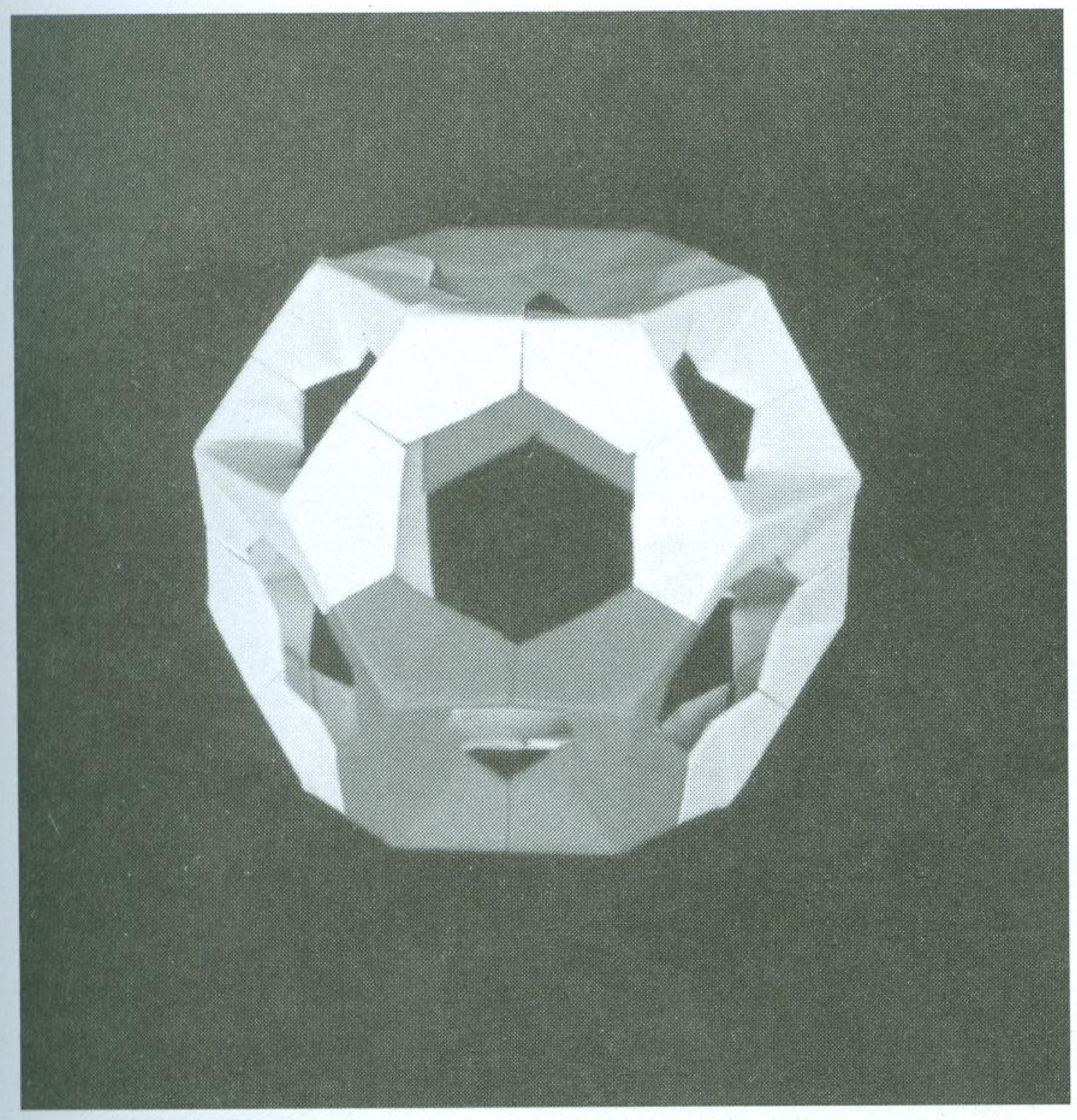
Modules:

24 triangles or

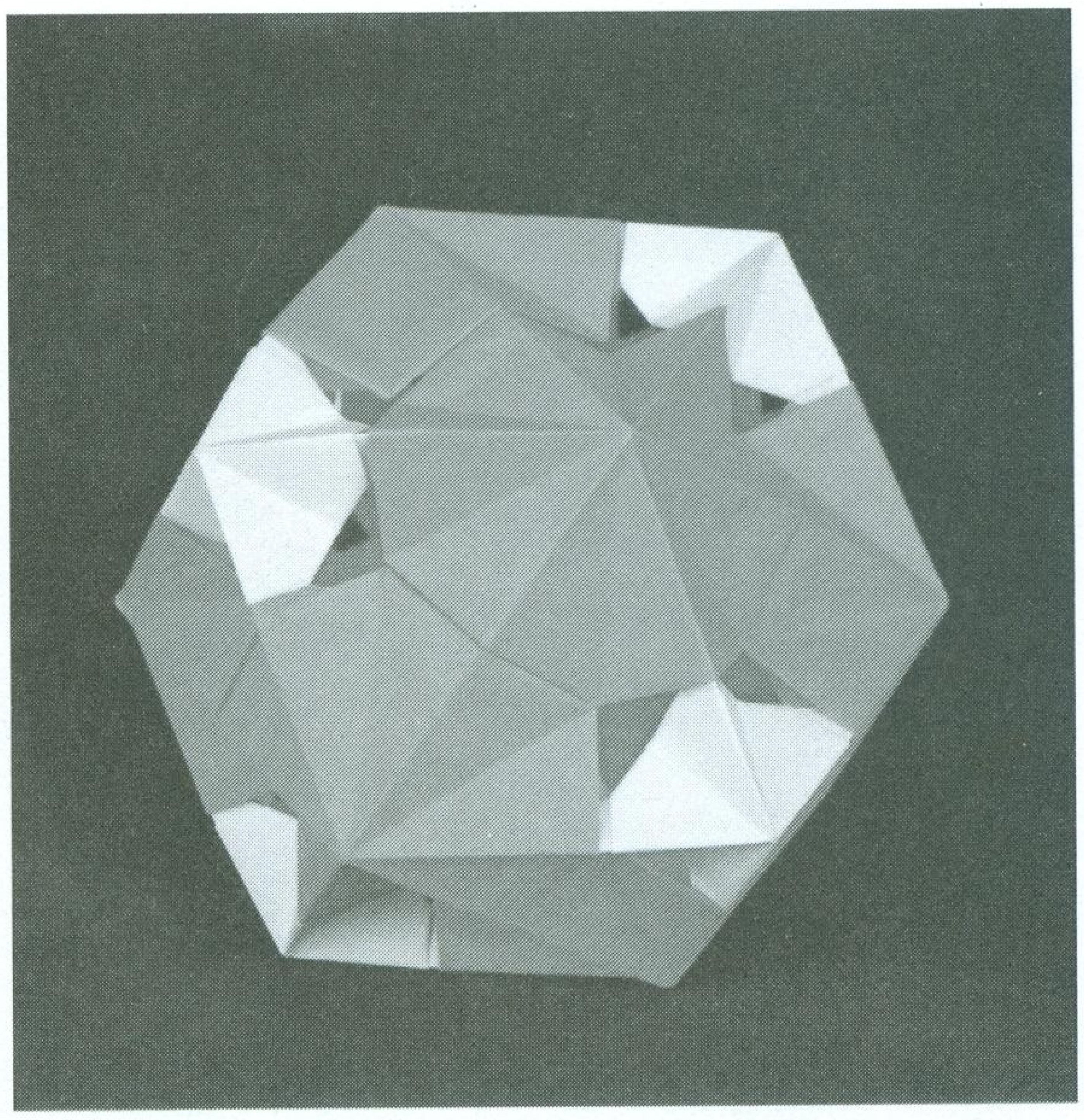
6 squares and

8 hexagons

4-1-1 Truncated Octahedron

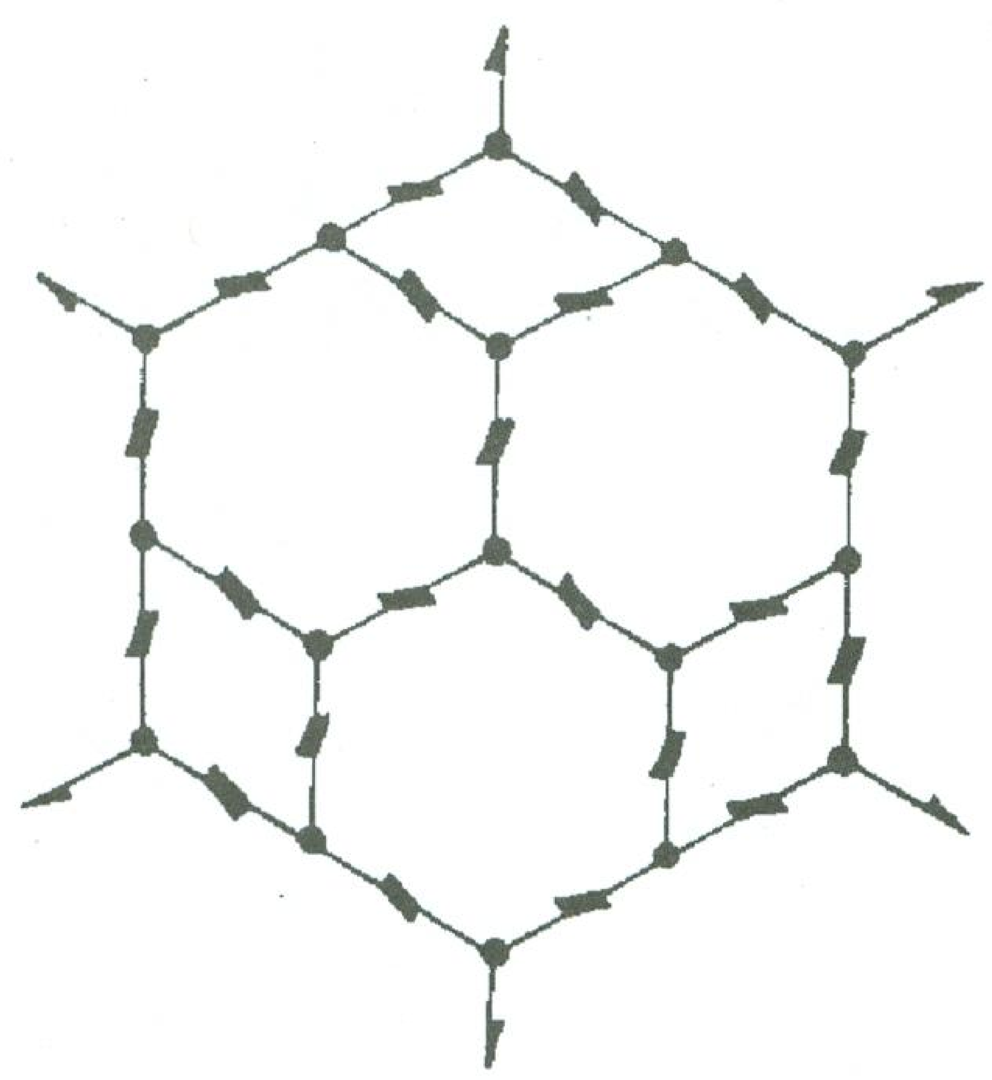


4-1-1 Gyroscoped Truncated Octahedron



7. 4-3-2

Net



Ring of three hexagons with square touching one edge of two hexagons in a ring

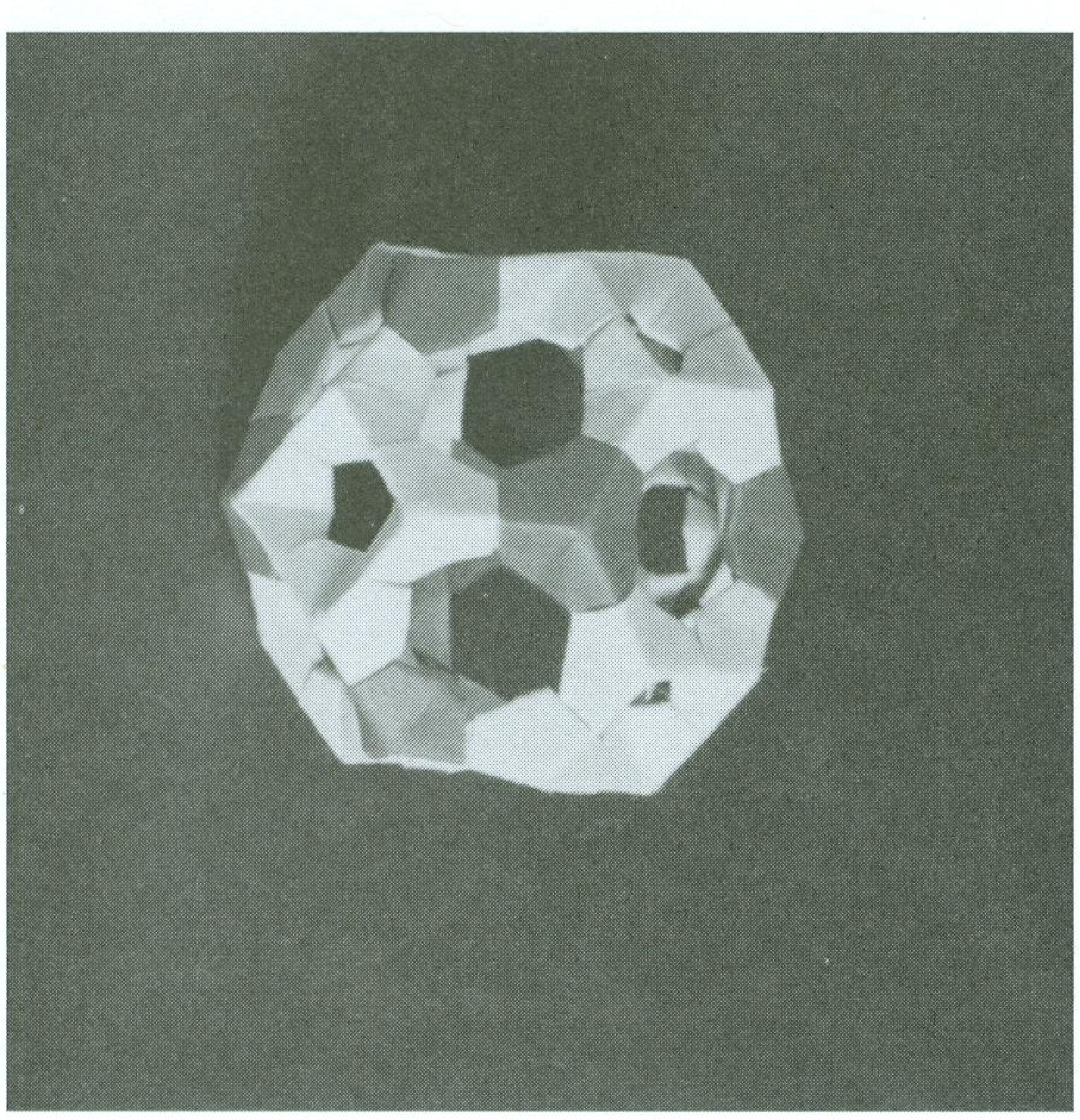
Modules:

32 triangles or

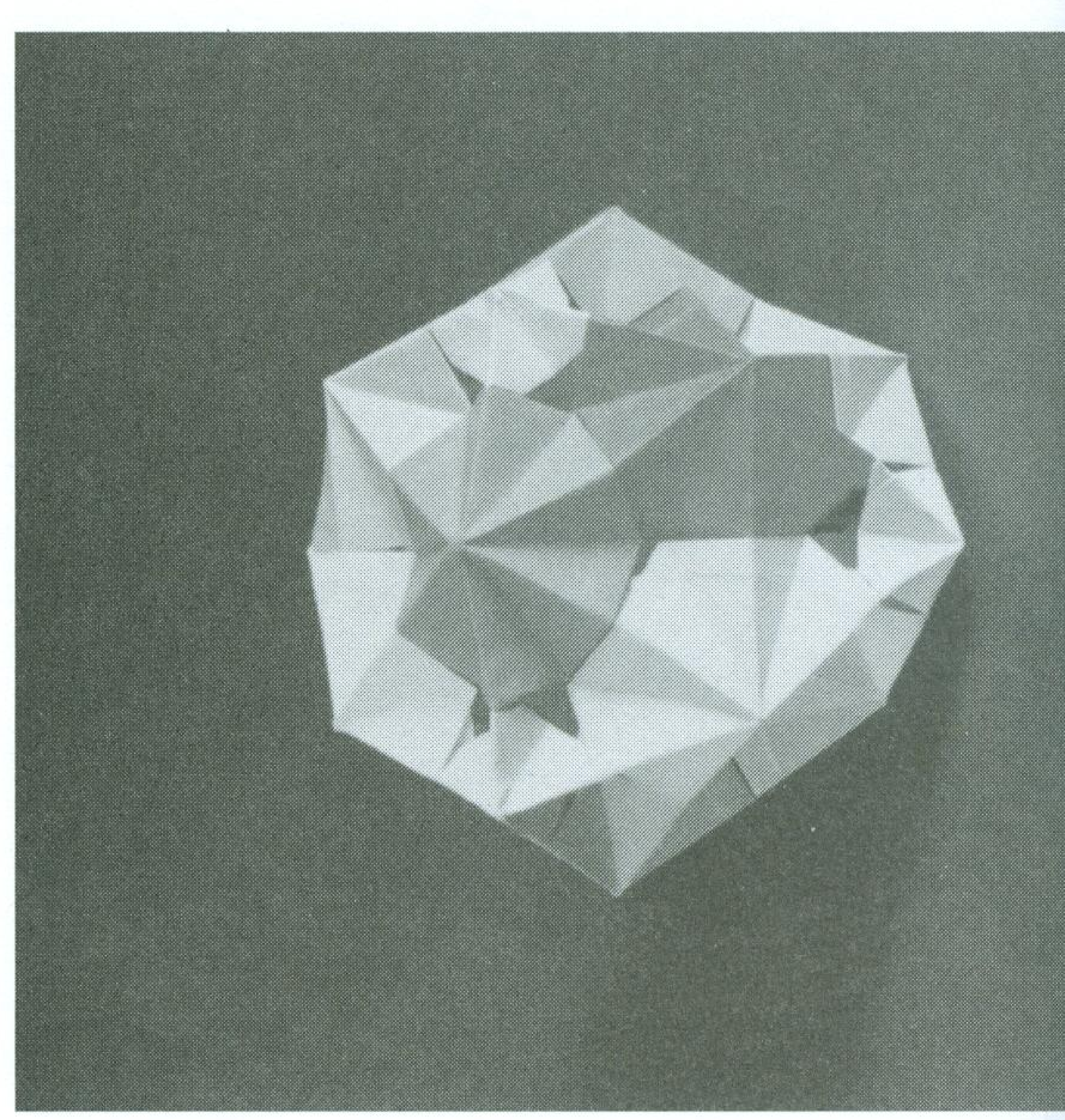
6 squares and

12 hexagons

4-3-2 Hypothetical Bucky

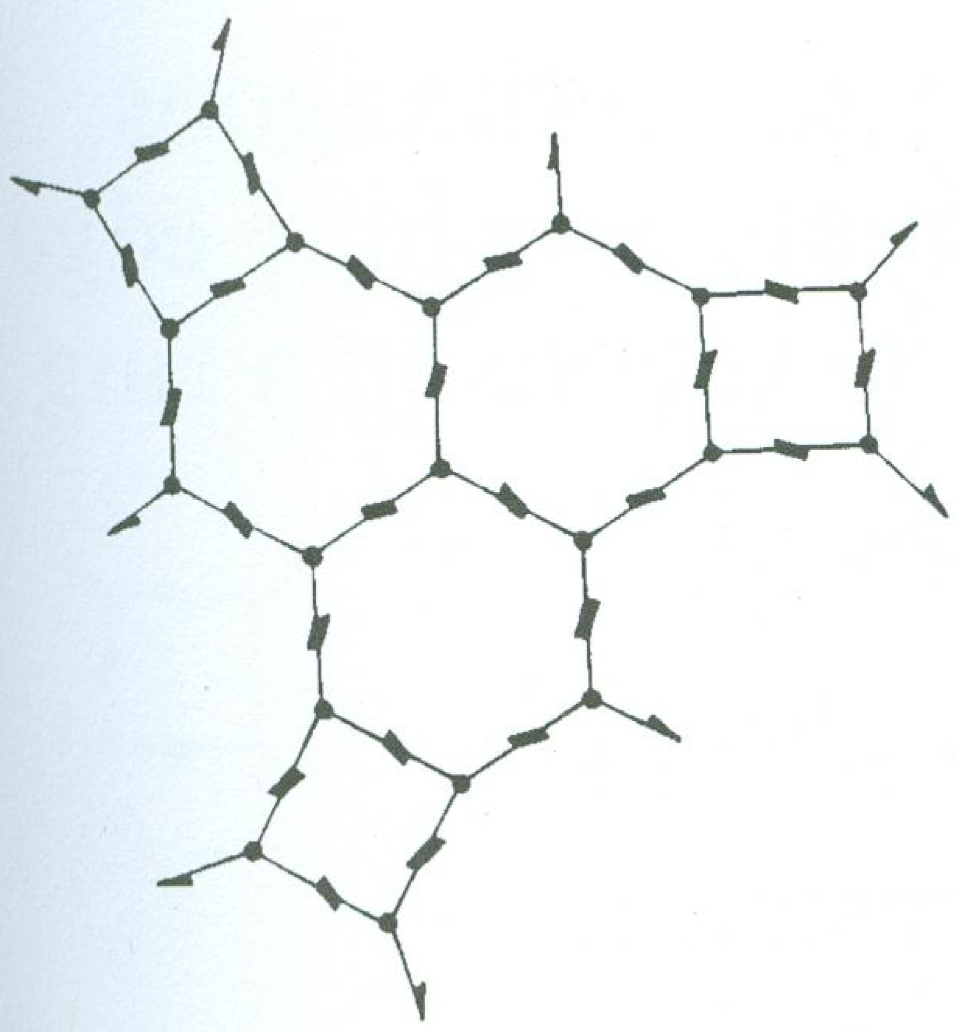


4-3-2 Gyroscoped



8. 4-3-1

Net



Ring of three hexagons
with square touching one
edge of one hexagon in ring

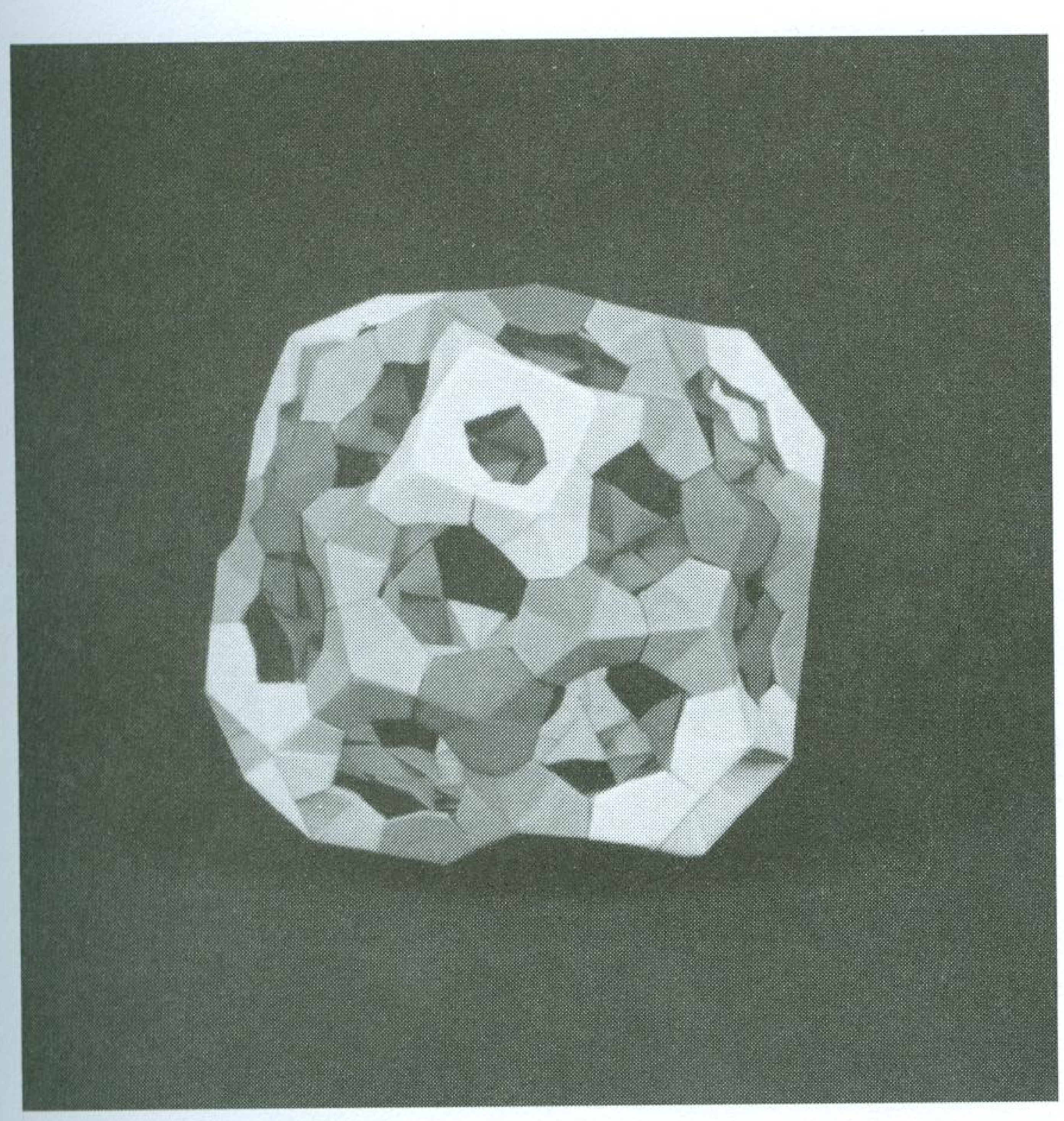
Modules:

56 triangles or

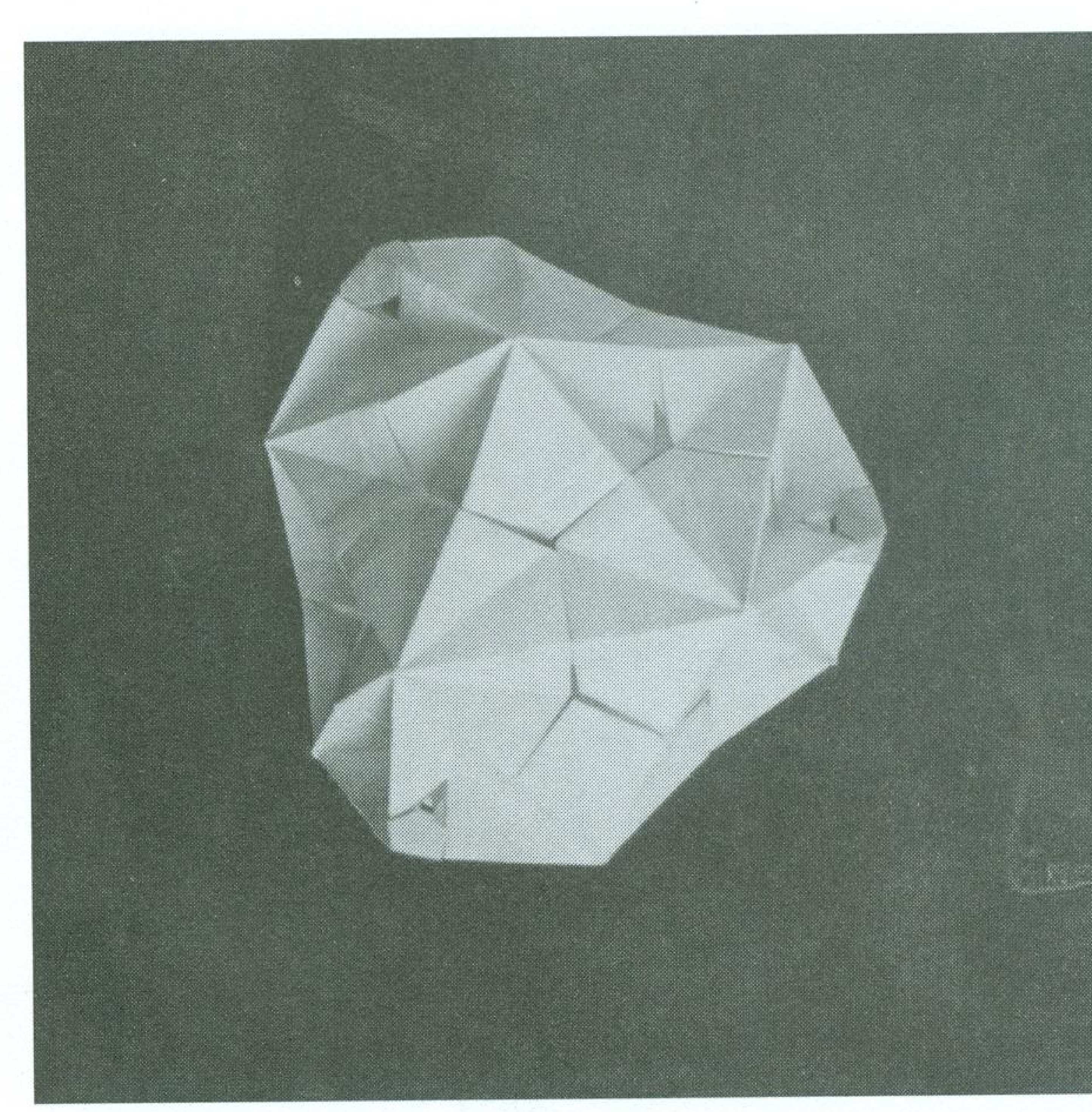
6 squares and

24 hexagons

4-3-1 Hypothetical Bucky

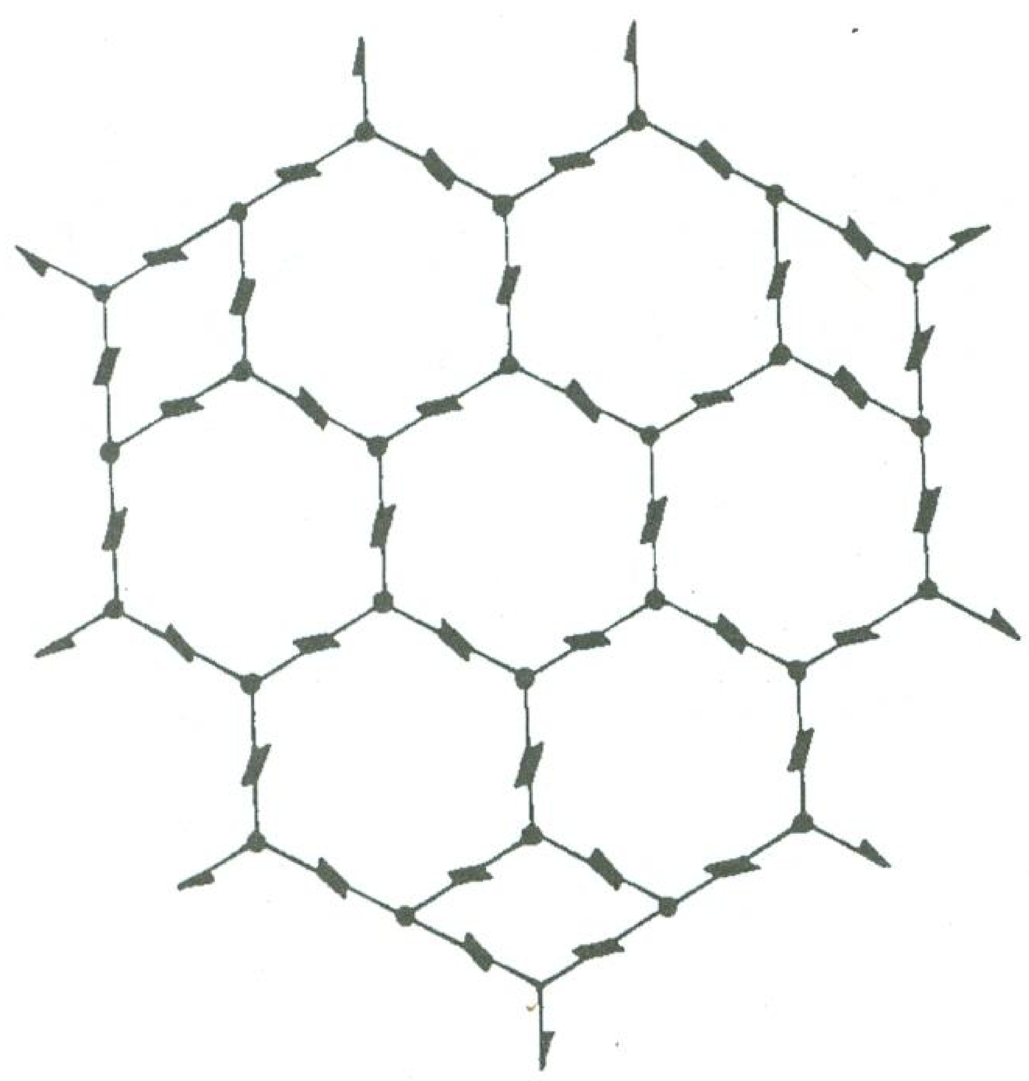


4-3-1 Gyroscoped



9. 4-7-2

Net



Ring of seven hexagons with square touching edge of two hexagons in a ring

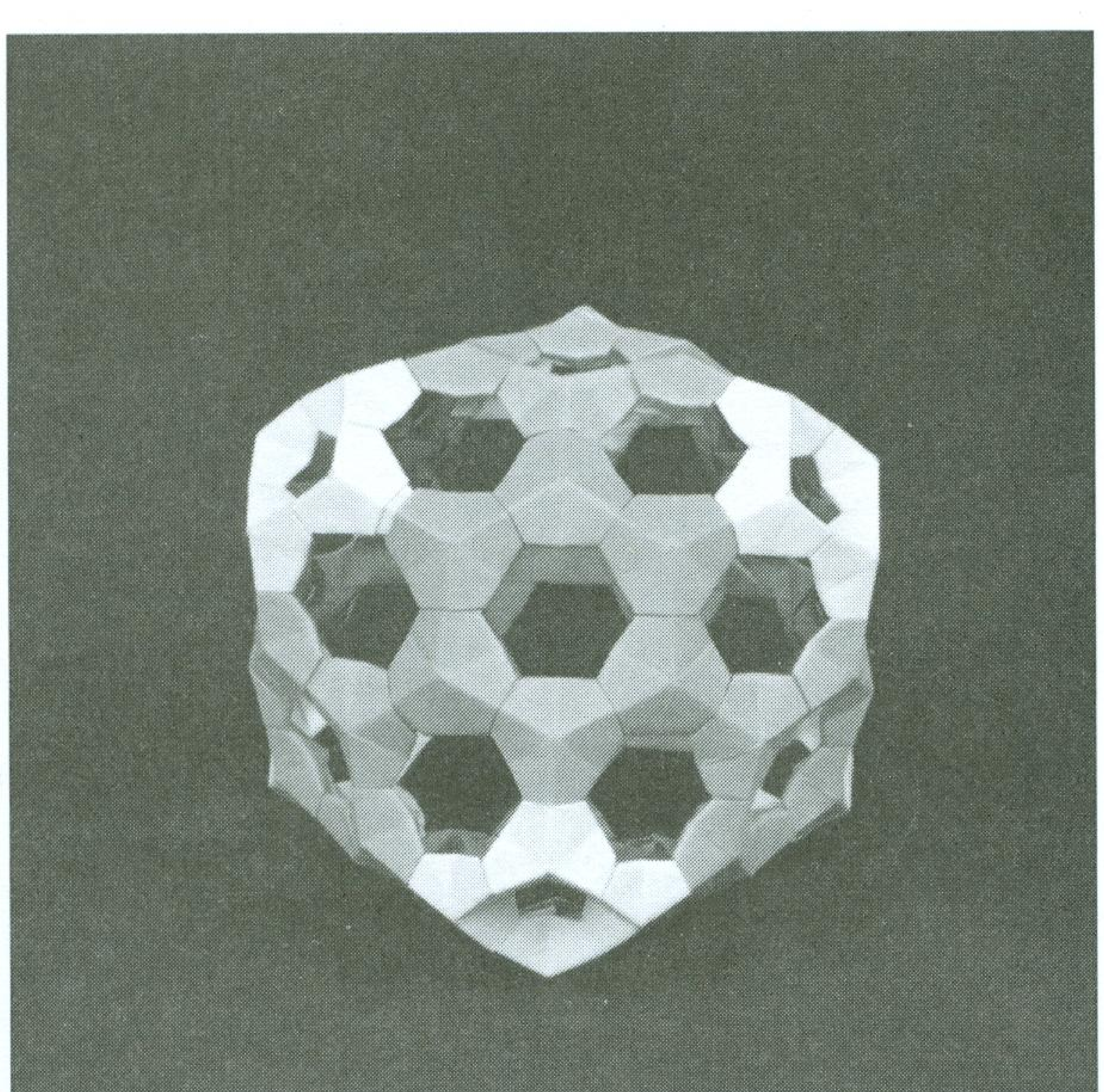
Modules:

72 triangles or

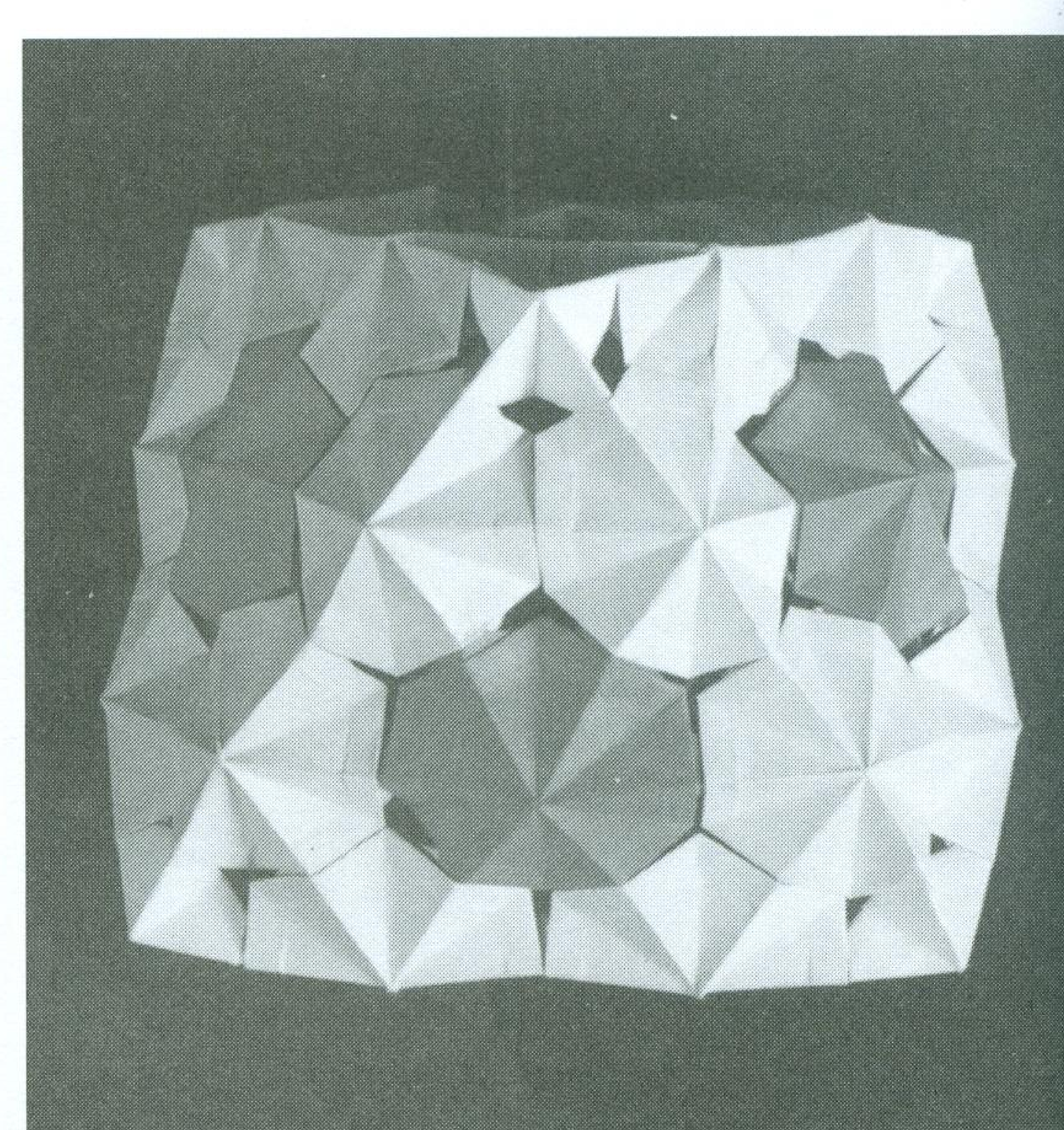
6 squares and

32 hexagons

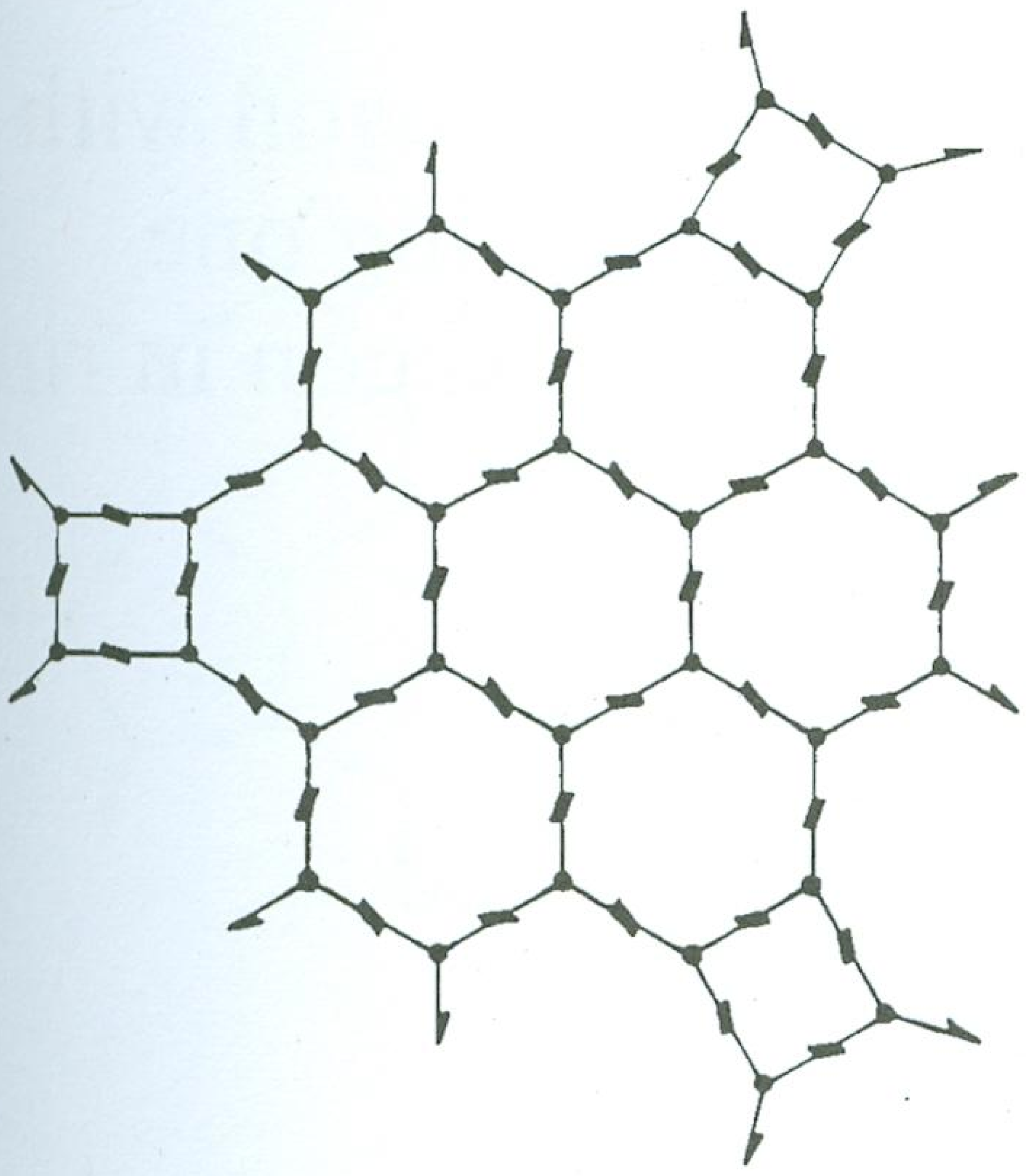
4-7-2 Hypothetical Bucky



4-7-2 Gyroscoped



Net



Ring of seven hexagons with square touching edge of one hexagon in a ring

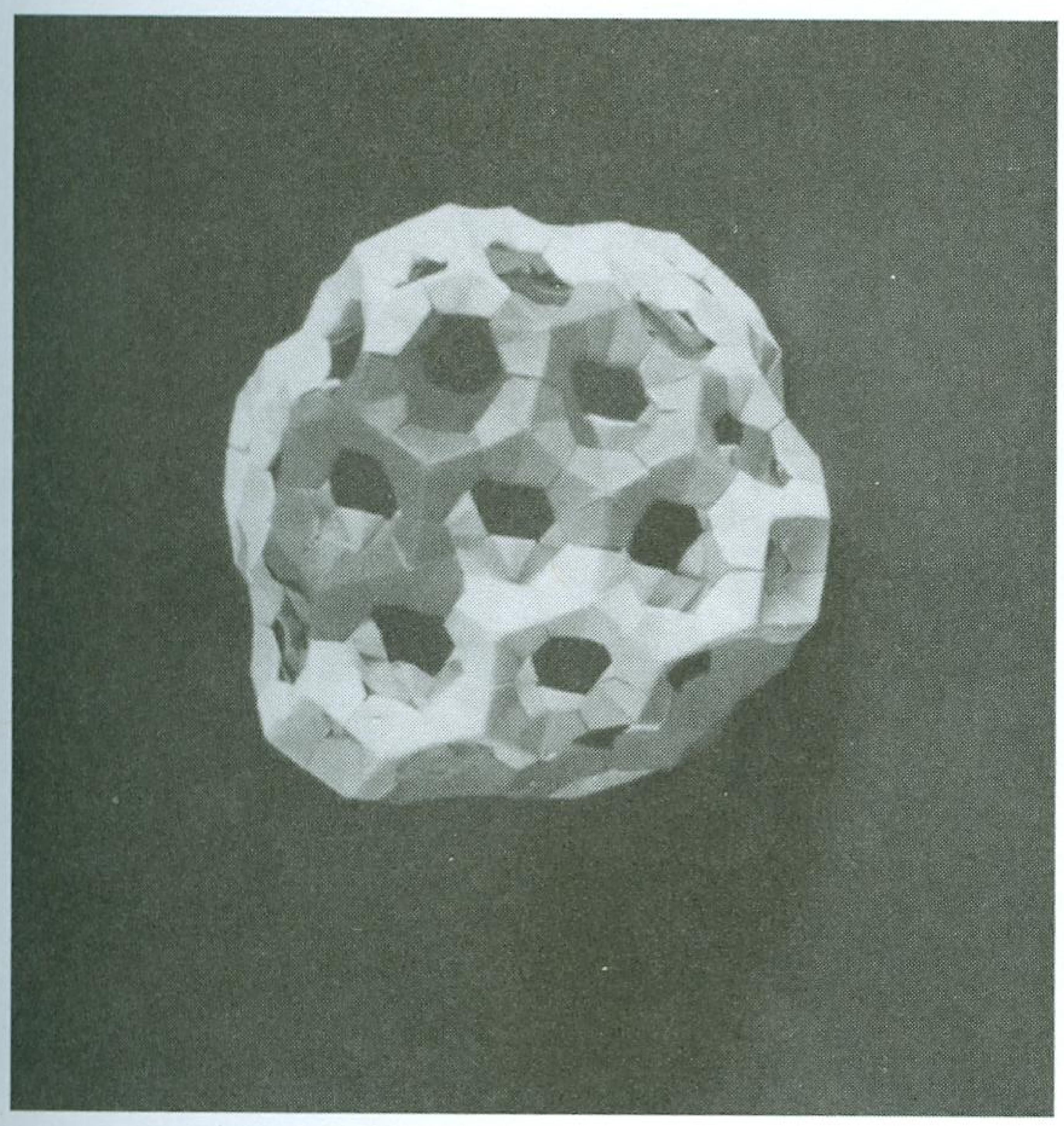
Modules:

96 triangles or

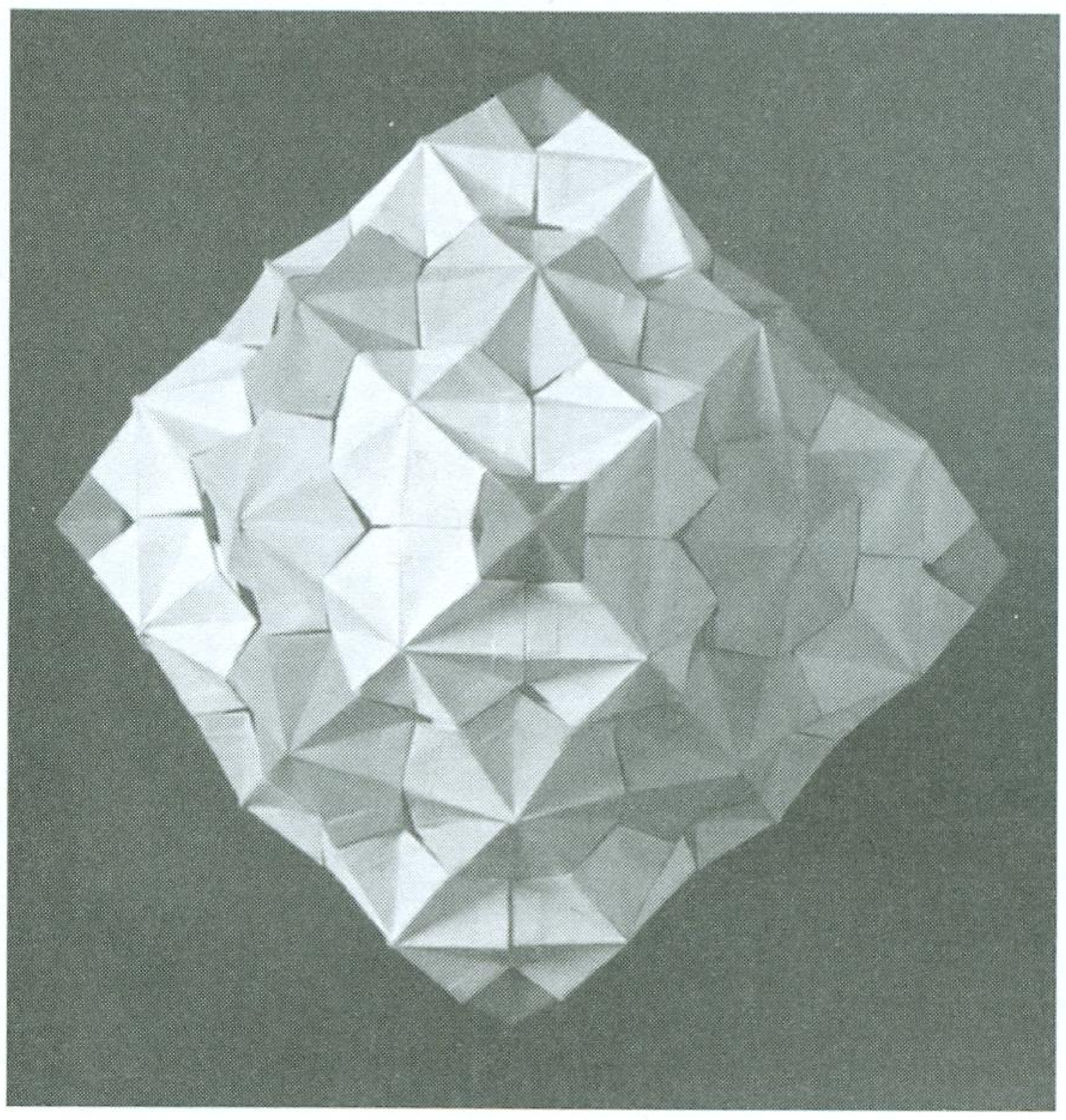
6 squares

44 hexagons

4-7-1 Hypothetical Bucky

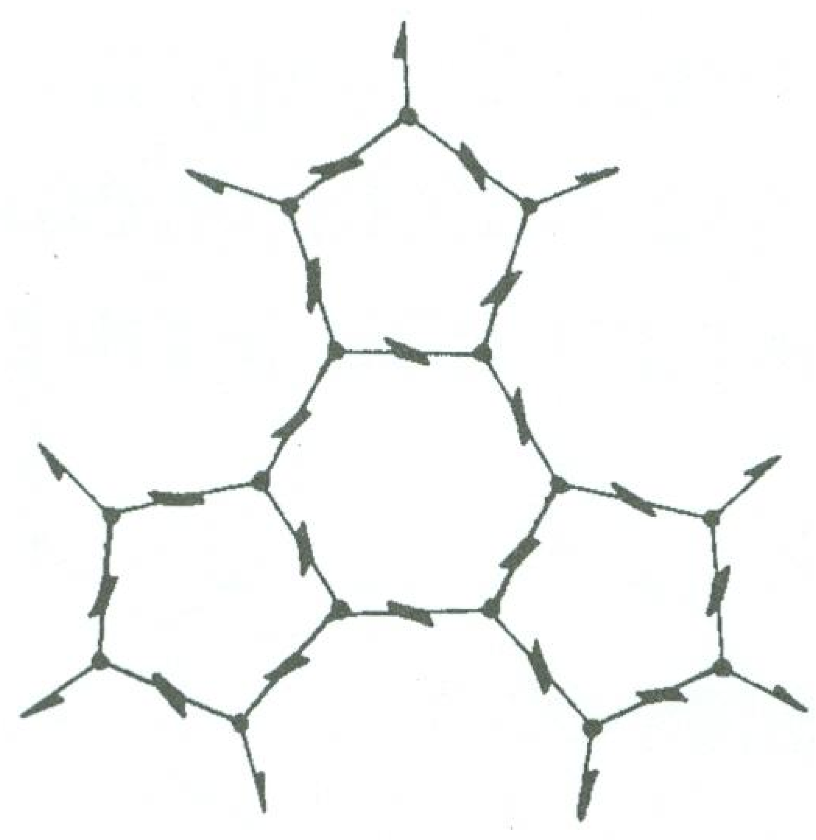


4-7-1 Gyroscoped



11. 5-1-1

Net



Ring of one hexagon with pentagon touching one edge of one hexagon in ring

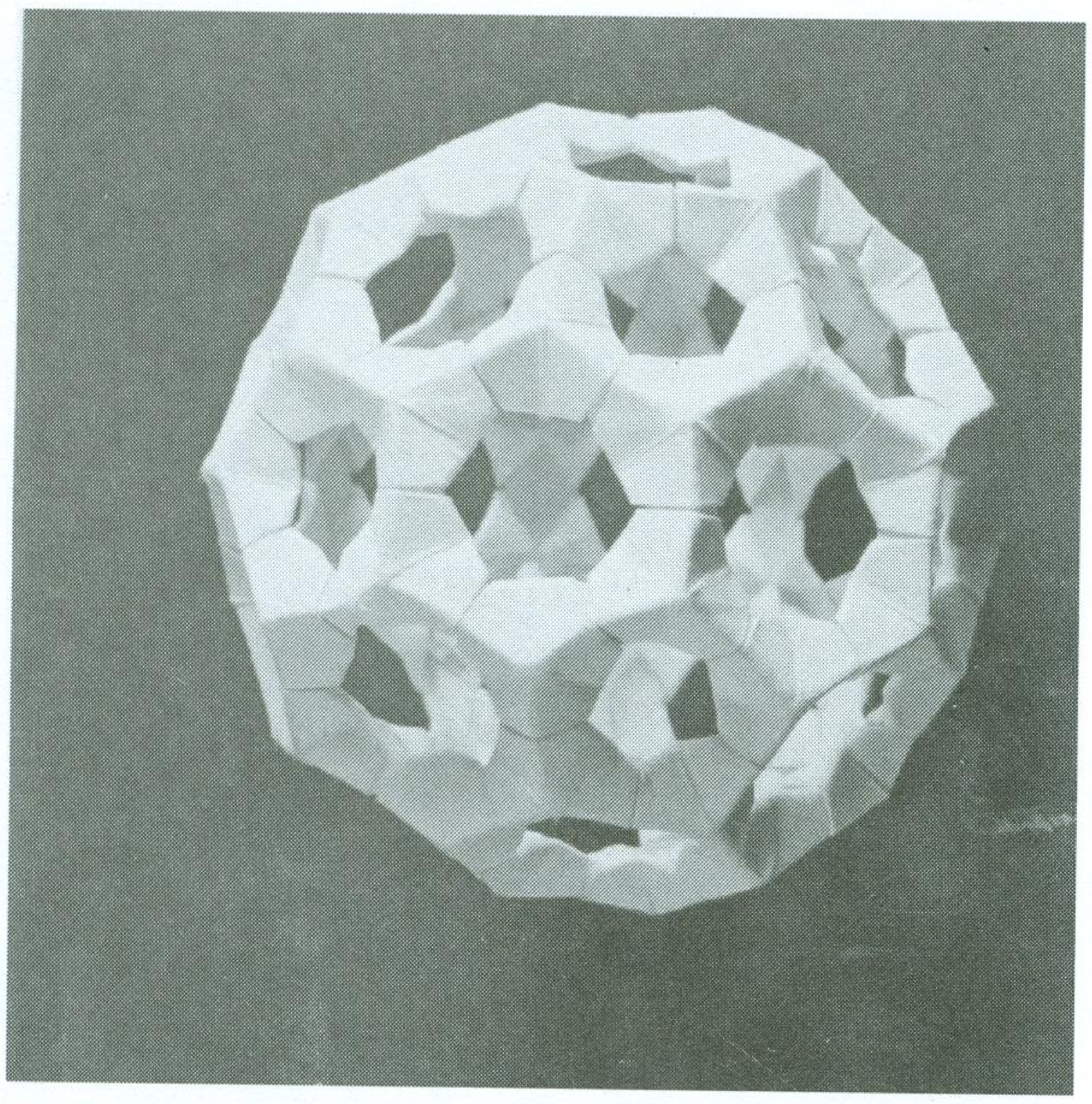
Modules:

60 triangles or

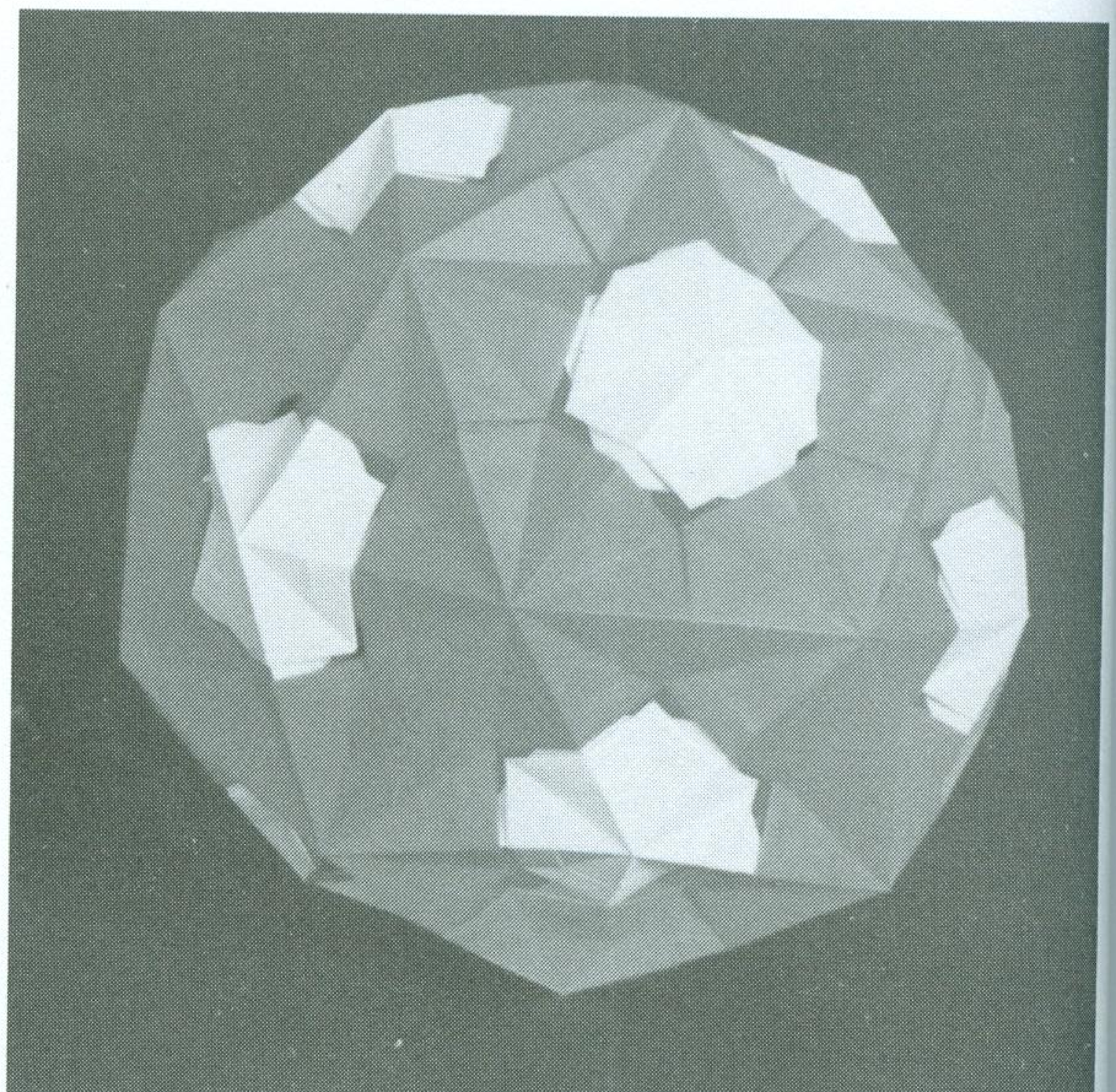
12 pentagons and

20 hexagons

5-1-1 C60 Truncated Icosahedron

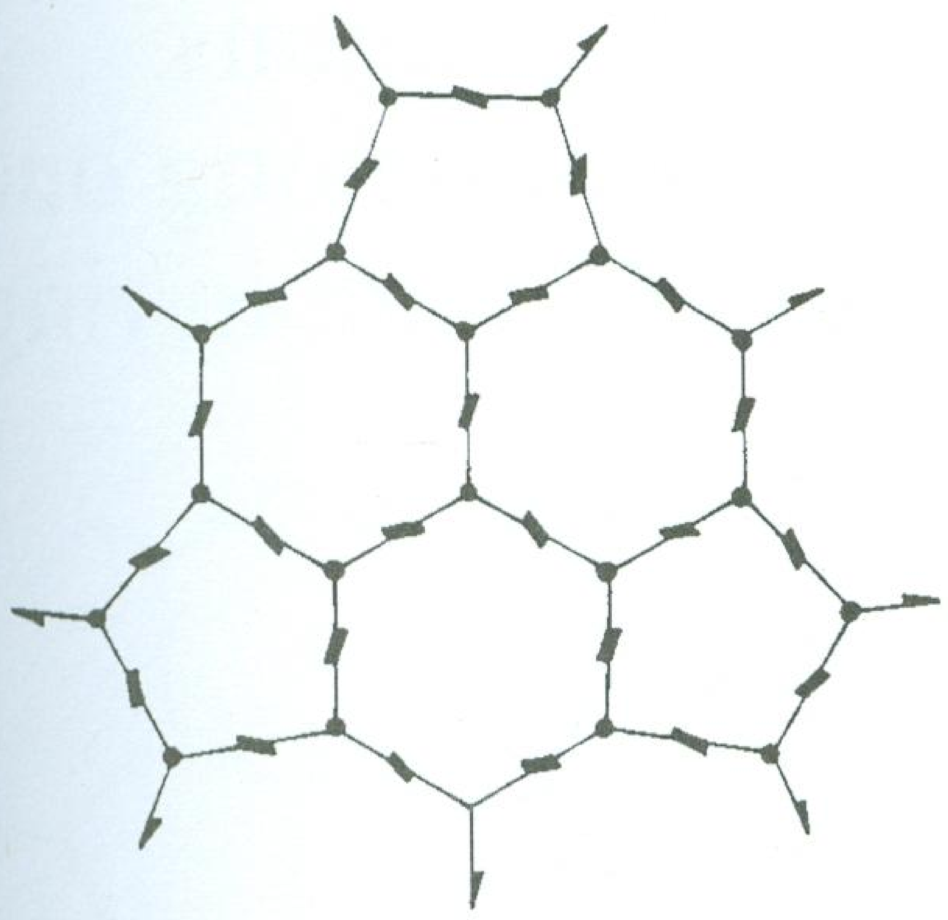


5-1-1 Gyroscoped C60



12. 5-3-2

Net



Ring of three hexagons with pentagon touching two edges of one hexagon in ring

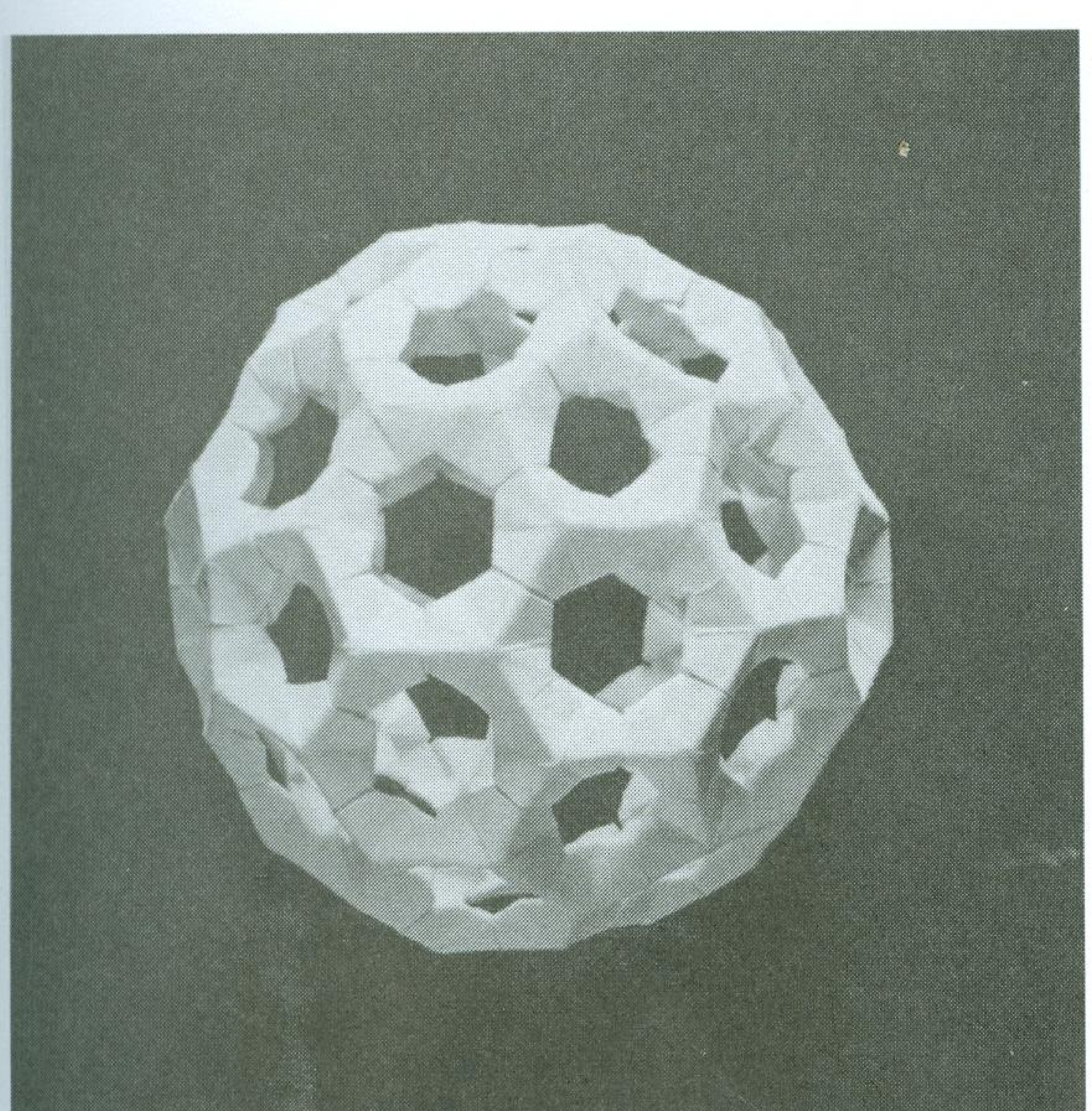
Modules:

80 triangles or

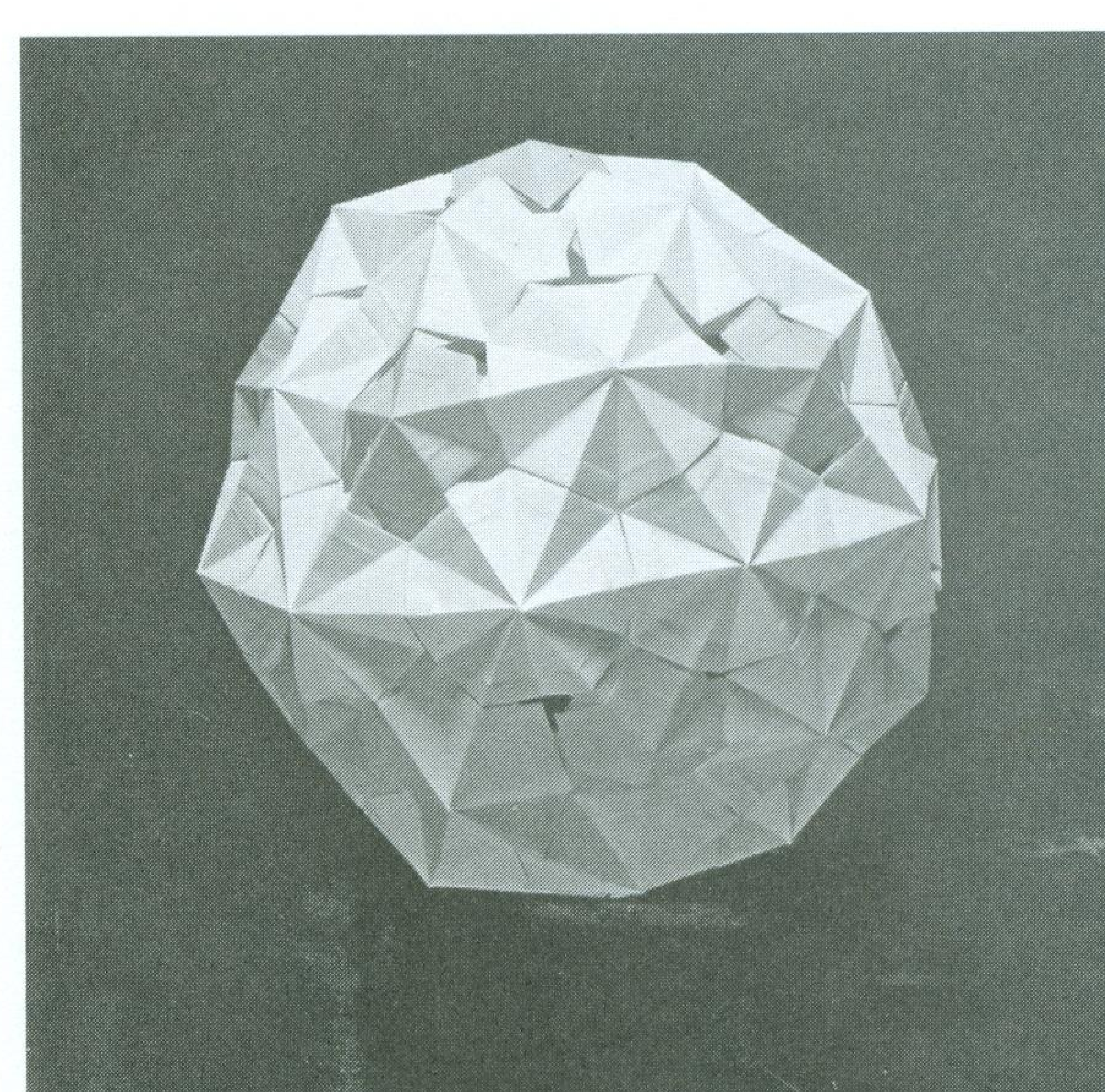
12 pentagons and

30 hexagons

5-3-2 C80 Bucky

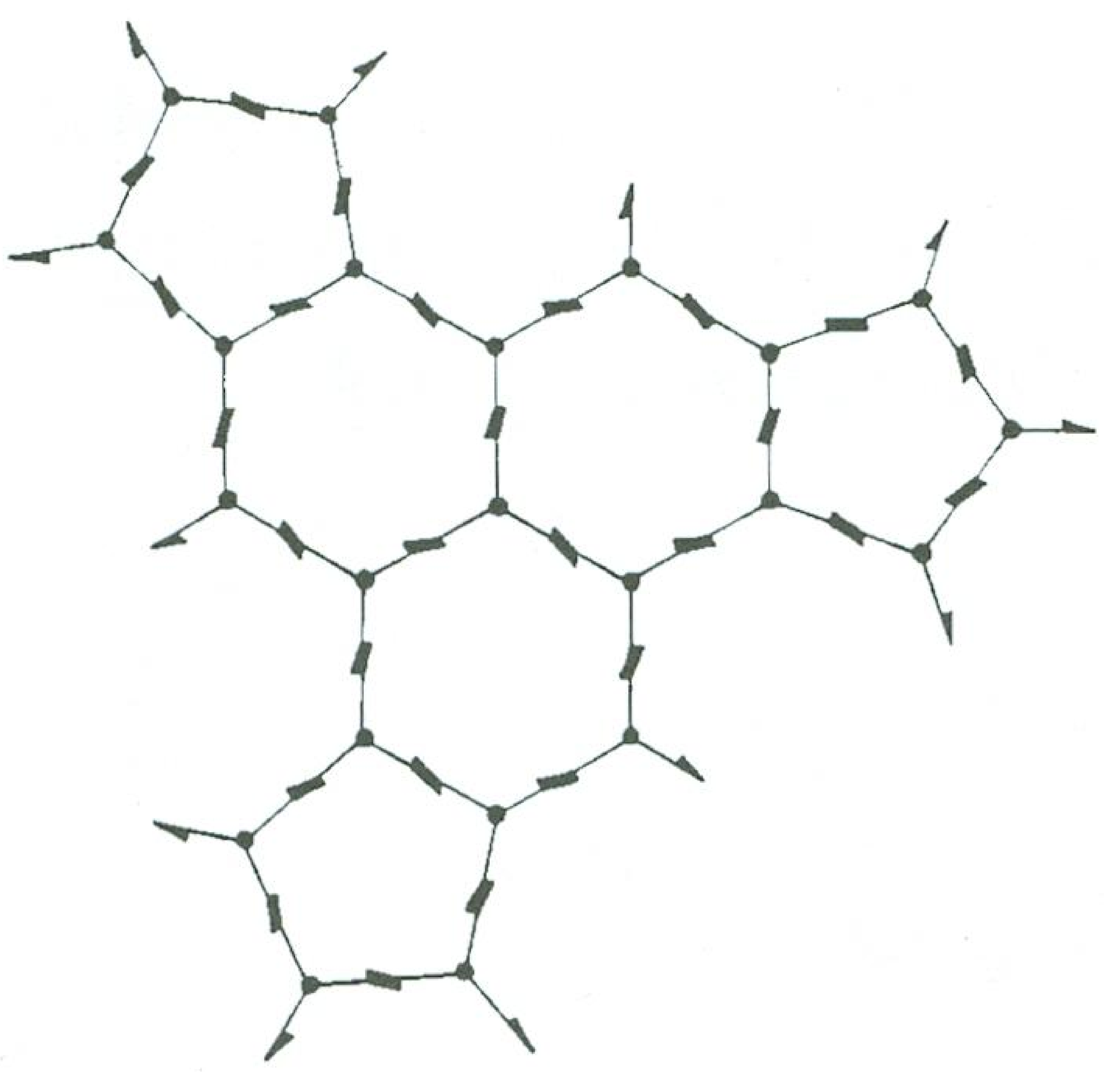


5-3-2 Gyroscoped C80 Bucky



13. 5-3-1

Net



Ring of three hexagons
with pentagon touching one
edge of one hexagon in ring

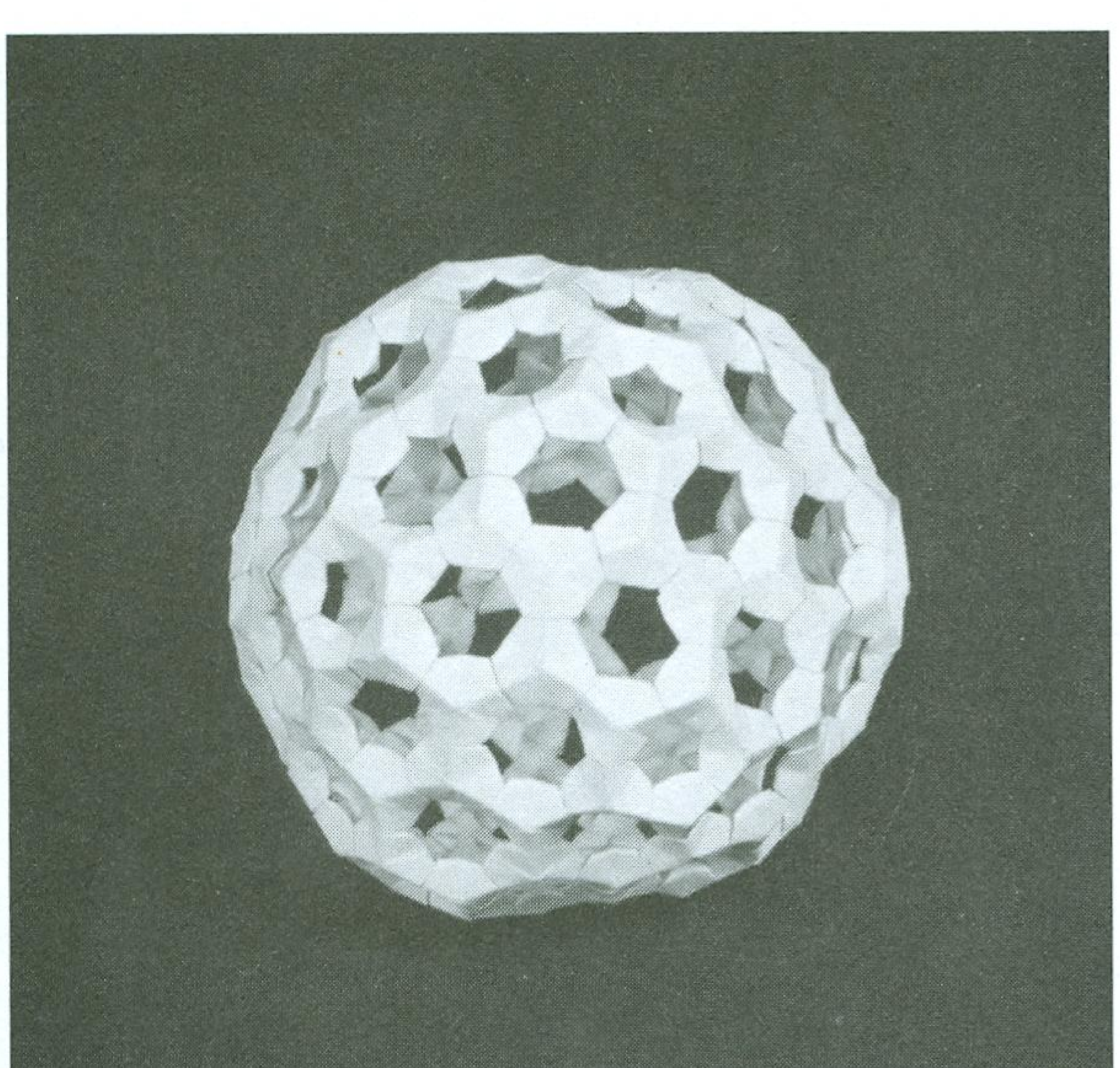
Modules:

140 triangles or

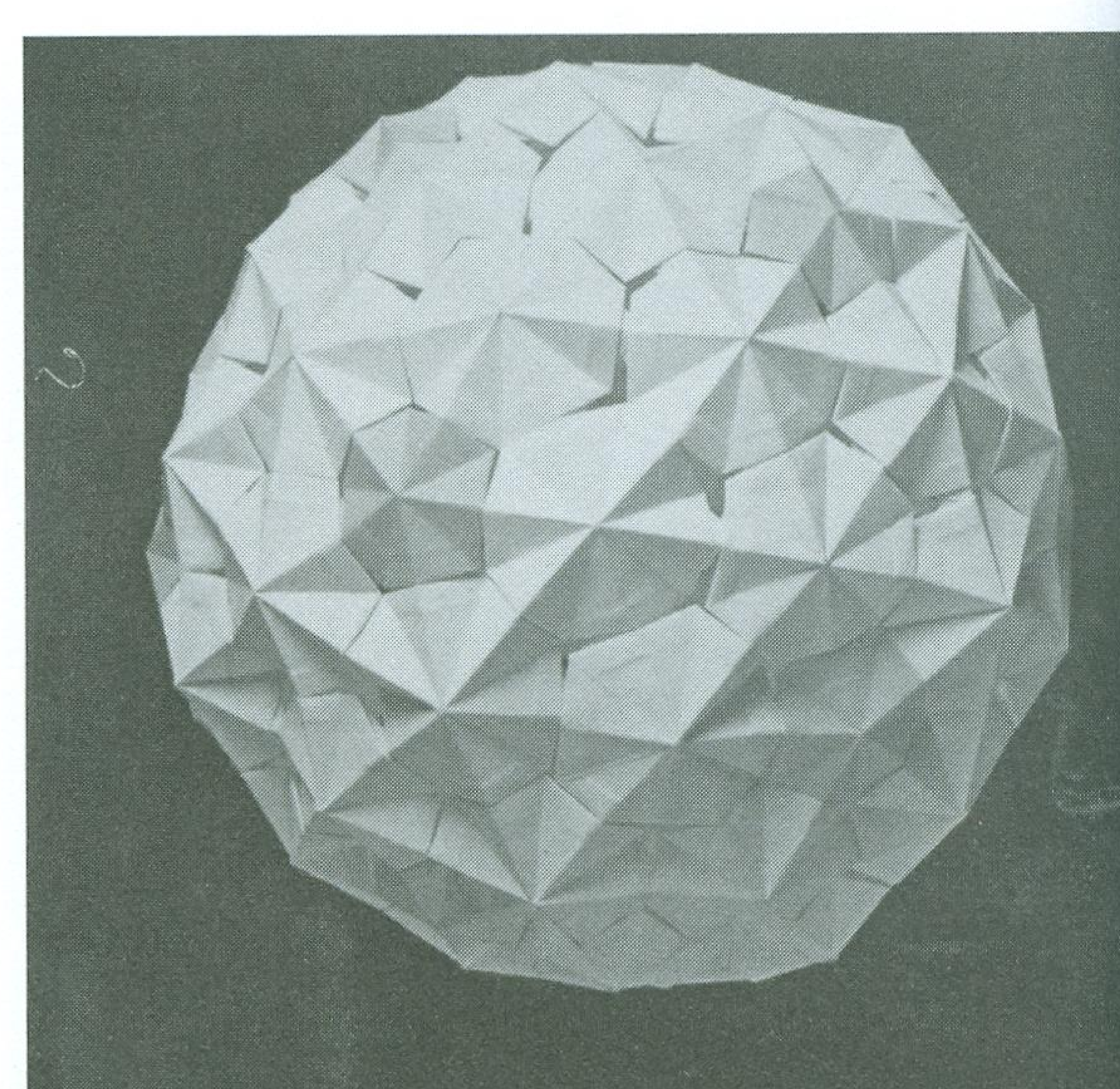
12 pentagons and

60 hexagons

5-3-1 C140 Bucky

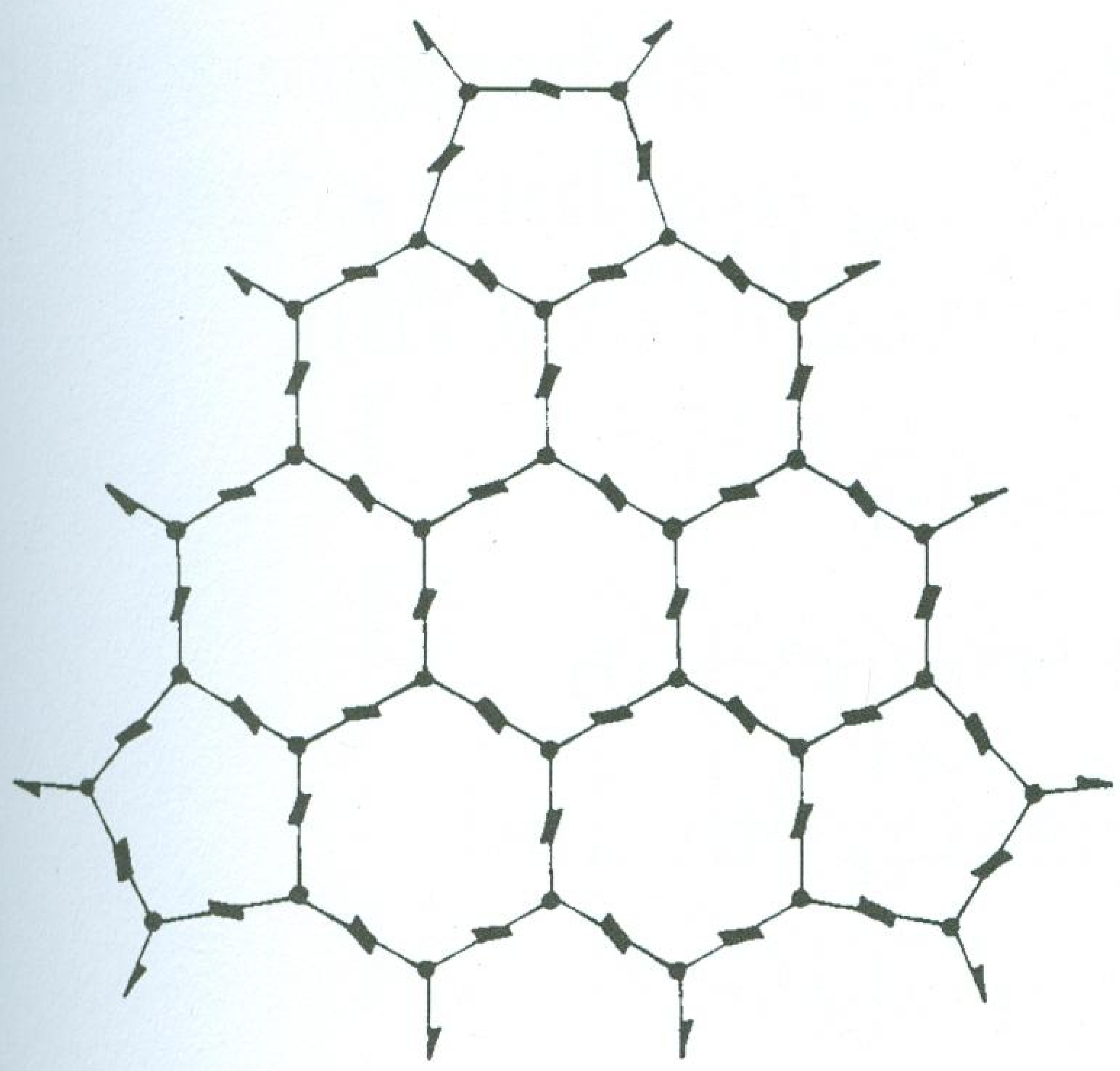


5-3-1 Gyroscoped C140 Bucky



14. 5-7-2

Net



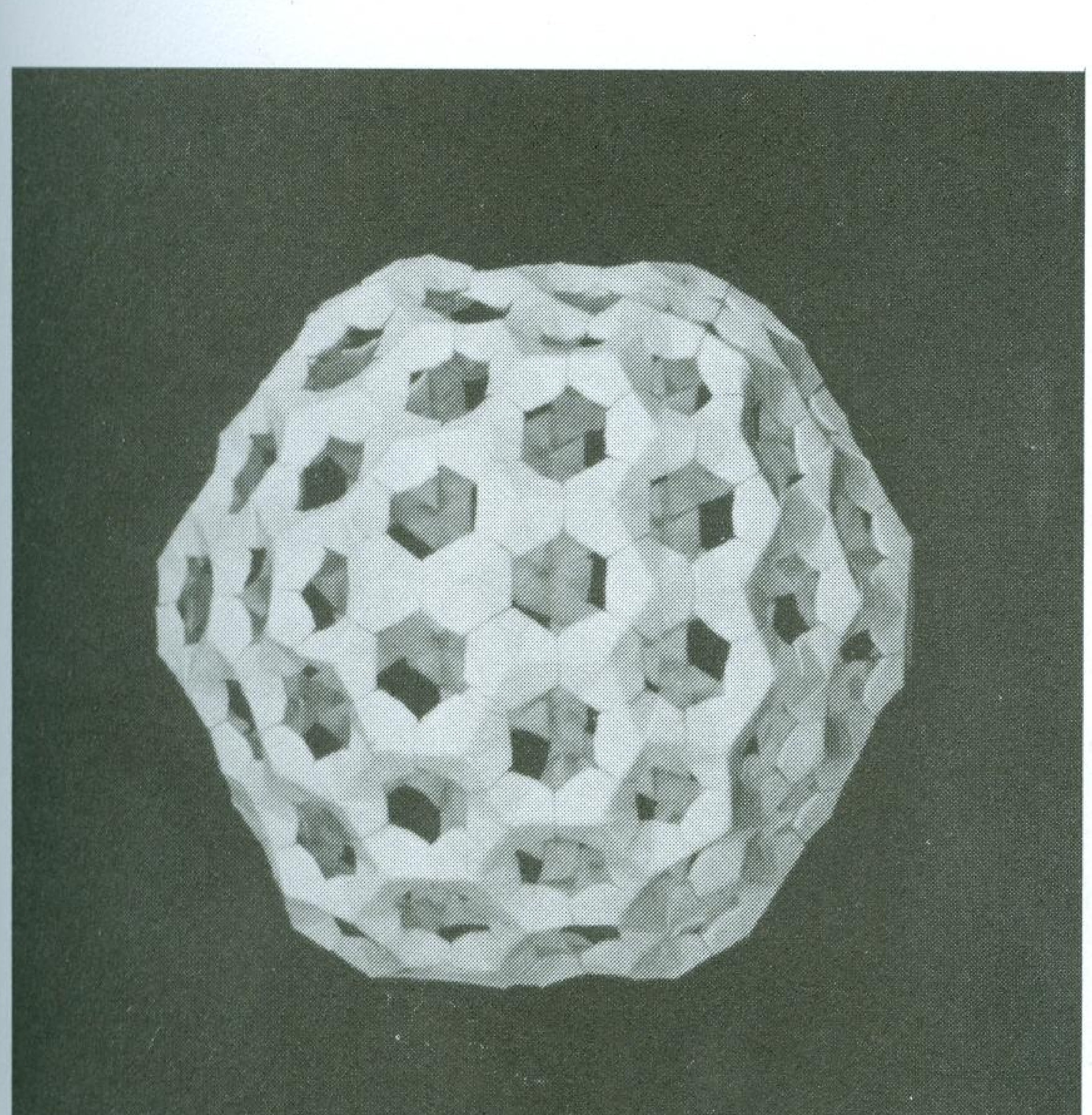
Ring of seven hexagons with pentagon touching edge of two hexagons in a ring

Modules:
180 triangles or

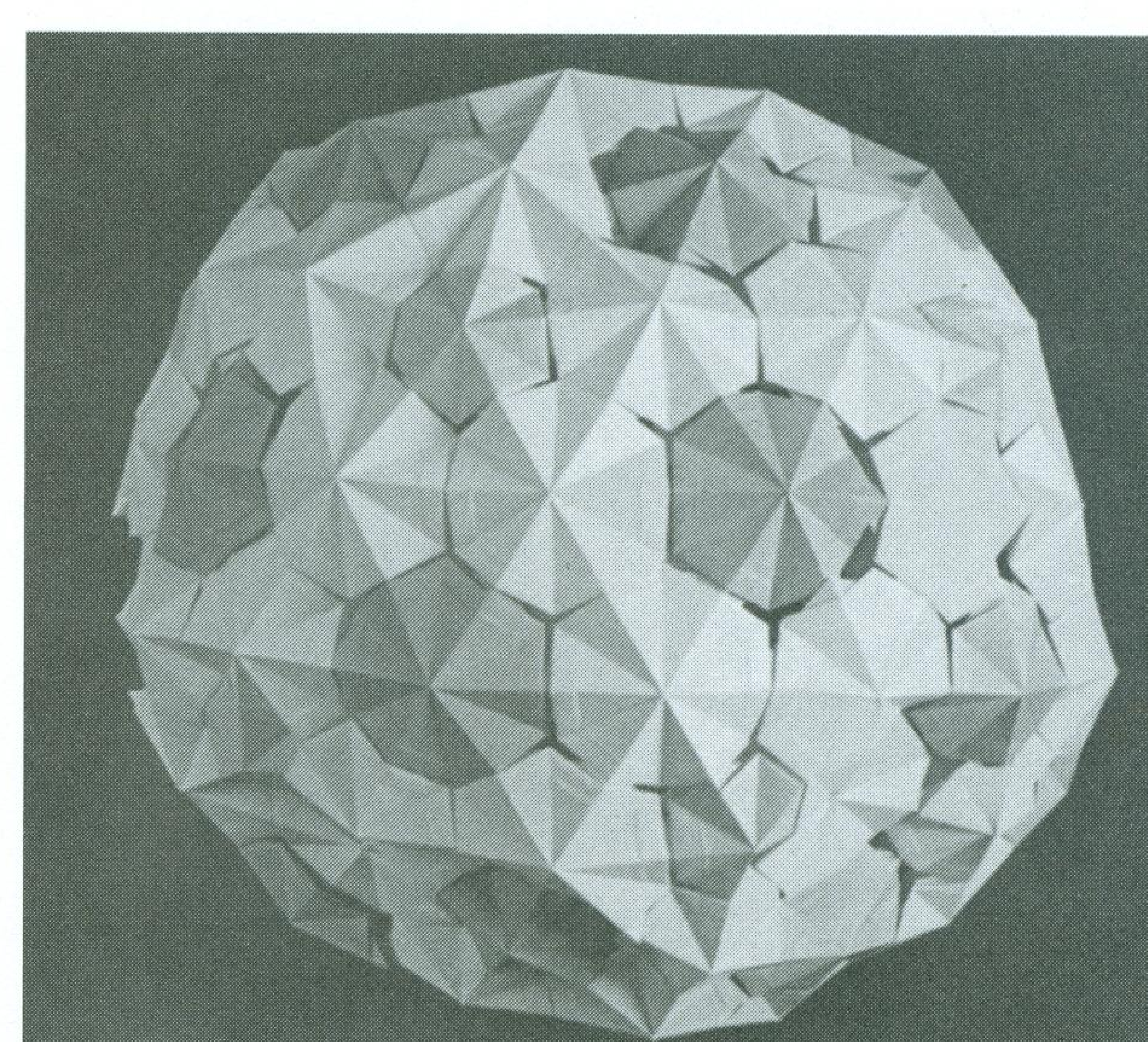
12 pentagons and

80 hexagons

5-7-2 C180 Bucky

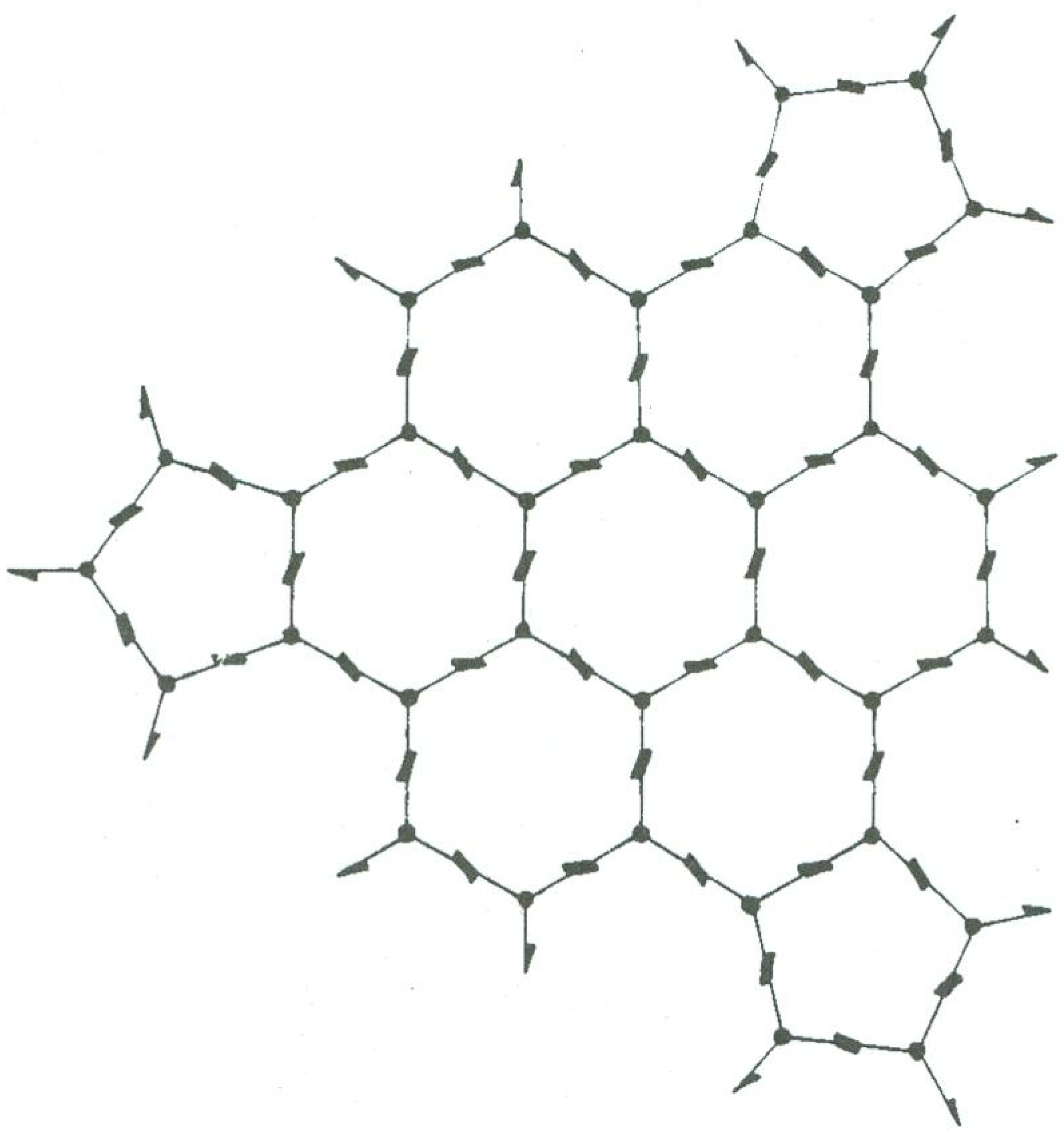


5-7-2 C180
Gyroscoped



15. 5-7-1

Net



Ring of seven hexagons with pentagon touching edge of one hexagon in a ring

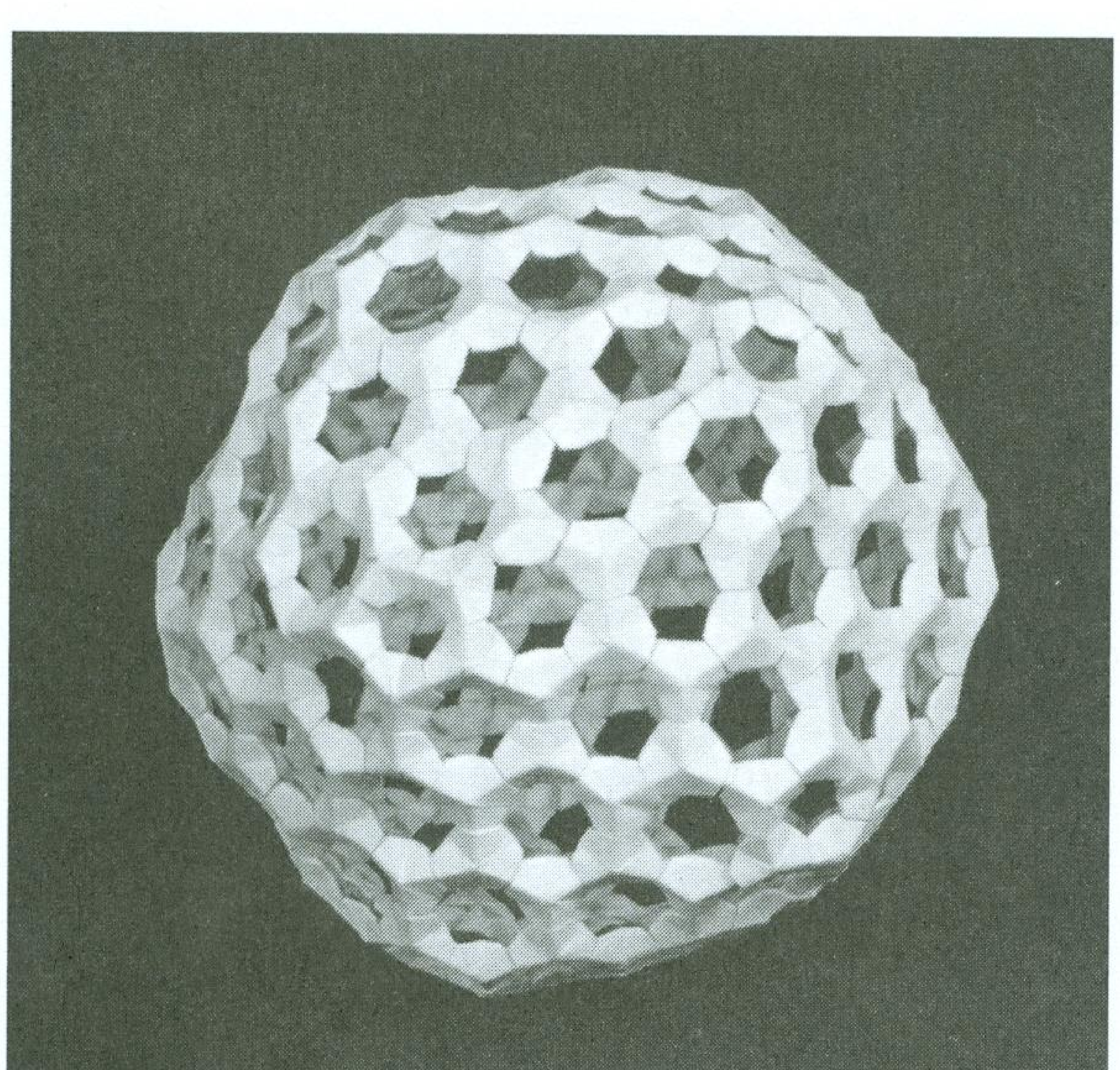
Modules:

240 triangles or

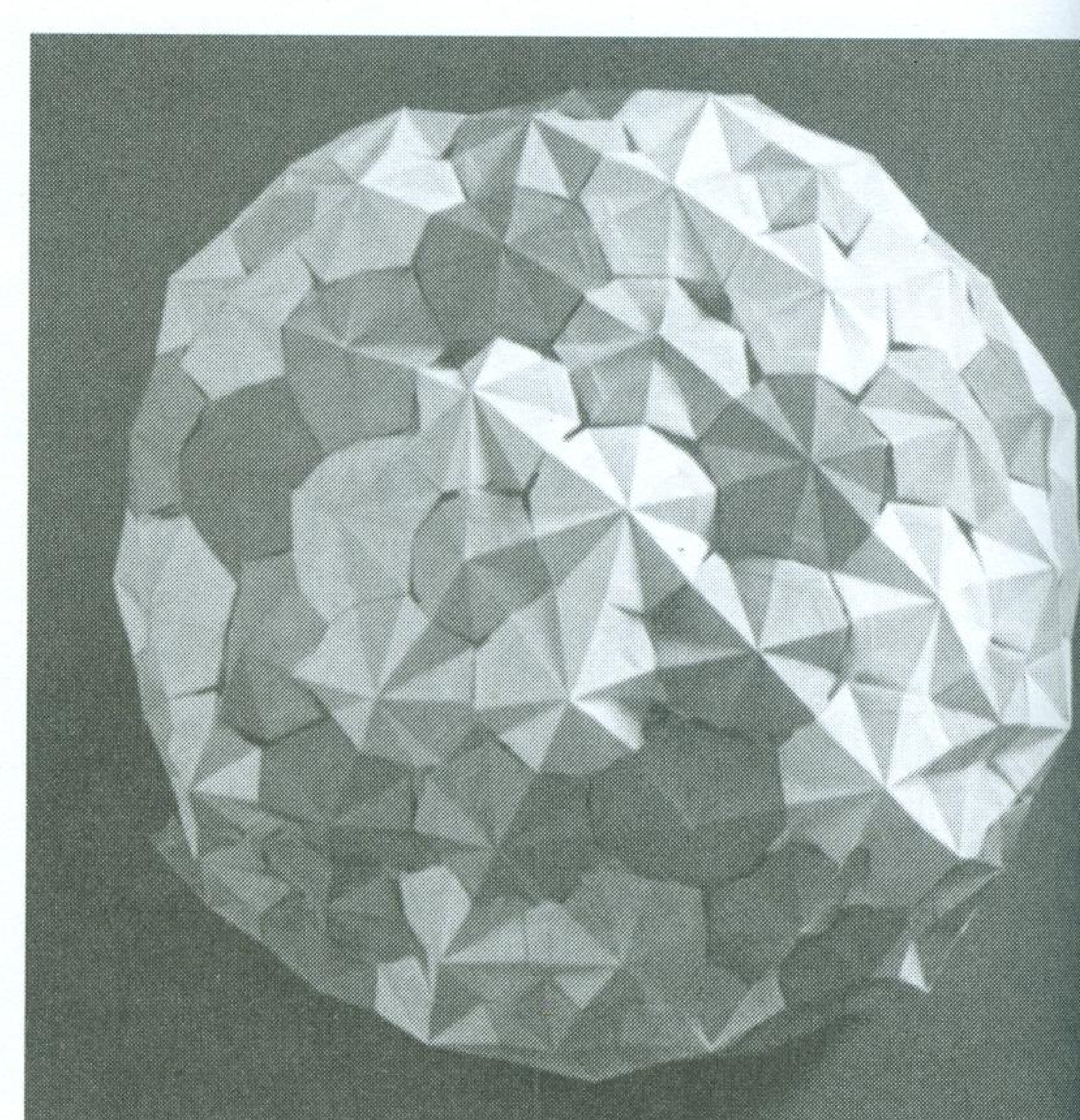
12 pentagons and

110 hexagons

5-7-1 C240 Bucky



5-7-1 Gyroscoped C240 Bucky



III. MODEL CONSTRUCTION

A. Model Selection
B. Module Selection
C. Module Sizing
D. Nets as Guides; Naming System for Polyhedra
E. Model Assembly
F. Four-Sided Display Stand from a Pentagon Waterbomb Base

MODEL CONSTRUCTION

A. Model Selection

When selecting a model to make, you should first consider what is involved. In the Gallery (p. 7) you will find photos of the models, with information on the number and type of modules needed for each. For models made from more than one type of module, you will often need a different size paper for each type of module. This is to allow the modules of different types to fit together. In a following section, we give some sizings for each model's modules. These sizings produce manageably sized models with modules that are not too hard to fold. Some of the larger buckyballs and their gyroscoped forms as well as the gyroscoped Archimedeans that contain octagons and decagons are larger and require some glue.

B. Module Selection

When making models with only pentagons and hexagons, the pentagon and hexagon expanded spike ball modules can be substituted for pentagon and hexagon gyroscope modules. We include them because they have better locks, using connectors. As you might guess, these modules involve more folding and must be made large so that the connector will be a manageable size.

C. Module Sizing

Sizing the modules is the key to making the models. In this section our first goal is to present some sizing sets that will allow you to make models which are not too big and yet use modules that are a convenient size that are not too hard to fold. A sizing is based on a part of the starting polygon or square for the module. Different construction methods for polygons may make certain ways of sizing easier than others. Our determination of the sizing sets is based on an empirical and pragmatic method akin to what scientists did with crystals a long time ago. Basically, we folded modules and partial models and measured them, or the holes where modules need to fit.

First, consider how to make the buckyballs and the egg. These models are made exclusively from triangle gyroscope modules, so you can make the module any size you like and two of them will still lock together. We like to divide an 8.5" x 11" sheet into strips after first dividing the 8.5" edge into halves or fourths, giving triangles with altitudes of 4.25" or 2.125" (p. 52). These are convenient sizes for modules, are not too hard to fold, and produce manageably sized models. It is also convenient to divide the 11" edge into eighths. These modules are harder to fold and the model is more difficult to assemble. You may also start with squares, or make a triangle tessellation (p. 51).

For each of the gyroscoped Archimedeans, gyroscoped buckyballs, gyroscoped hypothetical buckyballs and the gyroscoped egg you will need two or three different modules. The modules for a particular model cannot, in general, all be constructed from squares of the same size; if they are, the modules will not fit together. There are, however, some cases in which you may start with squares of the same size, e.g. if the model is a buckyball of pentagons and hexagons only.

We now discuss our methods for constructing modules that fit together.

We have come up with three ready-to-use module sizing sets. If you want to combine different modules in the same model, use sizes from the same sizing set. More generally, we also give basic sizing relationships.

Sizing set 1: This sizing set was chosen because 3" squares and strips are convenient to use.

1. triangle module from 3" strip (altitude 3")
2. square module from 3" square
3. pentagon module from 4.25" square (diagonal of pentagon 4.25")
4. hexagon module from 4.25" square (diagonal of hexagon 4.25")
5. octagon module from 10.2" square (starting square 10.2")
6. decagon module from 15.25" square (distance between opposite edges 12.8")

Sizing set 2: This sizing set was chosen because 8.5" squares are the largest that can conveniently be made from 8.5" x 11" paper. The largest module is made from one of these squares.

1. triangle module from 1.6875" (1 11/16") strip (altitude 1.6875")
2. square module from 1.6875" (1 11/16") square
3. pentagon module from 2.4" square (diagonal of pentagon 2.4")
4. hexagon module from 2.4" square (diagonal of hexagon 2.4")
5. octagon module from 5.75" square (starting square 5.75")
6. decagon module from 8.5" square (distance between opposite edges 7.14")

Sizing set 3: This sizing set was chosen because 2.125" (2⅛") is one fourth of 8.5" and this is convenient for making strips.

1. triangle module from 2.125" strips (8.5" x 11" sheet divided into four strips)
2. square module from 2.125" square (8.5" divided by 4)
3. pentagon module from 3" square (diagonal of pentagon 3")
4. hexagon module from 3" square (diagonal of hexagon 3")
5. octagon module from 7.25" square (starting square 7.25")
6. decagon module from 10.8" square (distance between opposite edges 9.1")

General sizing factors: Sizing factors for a module based on a polygon are based on one part of the starting polygon or the starting square used to obtain the starting polygon. We use the designations AT, SST, SS, D5, SSH, SSO, and D10 to indicate what is being measured for the different starting polygons. For the triangle module, the altitude of the starting triangle (AT) or the length of the side of the starting square (SST) is used, depending on the construction method. For the square, hexagon, octagon and decagon the side of the starting square (SS, SSH, SSO, SSD) is the measure used. For the pentagon and hexagon, the distance being sized is the diagonal of the finished pentagon (D5) or hexagon (D6). It turns out that D6 = SSH.

Dimensions measured:

SST	the side of the square for a triangle
AT	the altitude of a triangle
SS	the side of the square for a square
D5	the diagonal of a finished pentagon

SSP	the side of the square for a pentagon
D6	the diagonal of a finished hexagon
SSH	the side of the square for a hexagon
SSO	the side of the square for an octagon
SSD	the side of the square for a decagon
D10	the distance between opposite edges of a decagon

Here are the sizing factors:

Modules in one model:

Triangle and hexagon	SST = .81 x SSH
Triangle and square	SST = 1.15 x SS, SS = AT
Triangle and pentagon	SST = .81 x D5
Triangle and octagon	SST = .34 x SSO = 1.15 x AT
Triangle and decagon	SST = .27 x D10
Square and hexagon	SS = SSH / 1.4 = .71 x SSH
Pentagon and hexagon	D5 = SSP = SSH = D6
	(all squares the same size if using template)
Triangle, square and pentagon	SS = .71 x D5 = AT
	SST = SS x 1.15 = .81 x D5
Square, hexagon and octagon	SSO / 3.4 = SS
	SS = .29 x SSO
	SSH = .42 x SSO
Square, hexagon and decagon	SS = D10 / 4.28
	SS = .23 x D10
	SSH = 1.4 x SS = 1.4 x .23 x D10 = .33 x D10
	SSD = 1.19 x D10

D. Nets as Guides; Naming System for the Polyhedra

First, a word about hypothetical buckyballs. Hypothetical buckyballs are our invention. In making new buckyballs, we saw a pattern and extended it to make other polyhedra in a similar way, but replacing the buckyball's twelve pentagons with either six squares or four triangles. We tried it and it worked. It is an open question whether the hypothetical buckyballs correspond to something in nature, as the buckyballs do.

The underlying pair of polyhedra for the buckyballs and hypothetical buckyballs is either the icosahedron-dodecahedron, the octahedron-cube or the tetrahedron-tetrahedron. The polyhedra in a pair are duals, so the number of vertices of the first is equal to the number of faces in the second. If we cut off (truncate) the vertices in the first polyhedron just right we get a polyhedron with faces of the second polyhedron and hexagons. The buckyballs are based on the truncated icosahedron and have twelve pentagons and varying numbers of hexagons as faces. Hypothetical buckyballs are based either on the truncated octahedron or the truncated tetrahedron. Those based on the truncated octahedron have six squares and varying numbers of hexagons as faces. Those based on the truncated tetrahedron have four triangles and varying numbers of hexagons as faces.

The first polyhedron in the pair of polyhedra consists exclusively of equilateral triangles and is an approximation to a sphere. We have constructed nets that correspond to a single equilateral triangle of one of these polyhedra, the icosahedron, octahedron or tetrahedron. This equilateral face is divided into hexagons and connected with either pentagons, squares or triangles to give an overall pattern. The buckyballs have pentagons attached to hexagons. We have extended this pattern to hypothetical buckyballs, in which squares are attached to hexagons or triangles to hexagons. However, it should be noted that the buckyballs, hypothetical buckyballs, and the eggs are made up exclusively of triangle gyroscope modules.

The triangles, squares, pentagons, and hexagons we refer to in the buckyballs or hypothetical buckyballs are rings of triangle gyroscope modules, in which each module corresponds to a vertex of the polyhedron. It is in the gyroscoped polyhedra that we see other types of modules.

After we invented and constructed the hypothetical buckyballs, we developed a system for designating them. In this system, each polyhedron has a three-position numerical designator, based on the pattern of the equilateral triangle that is repeated in all the basic polyhedra. The number in the first position indicates whether the face of the polyhedron is a pentagon (5), square (4) or triangle (3). The number in the second position indicates what type of cluster of hexagons fills up an equilateral triangle face of a basic polyhedron (icosahedron, octahedron, or tetrahedron); we consider polyhedra in which the second number is either 1, 3, or 7, indicating a cluster of one hexagon, three hexagons meeting at a point, or seven hexagons with one in the middle and six surrounding it. The number in the third position indicates the relation between the polygon indicated by the first-position number and the cluster of hexagons indicated by the second-position number. The third-position number is always either 1 or 2. A 1 means that three of the polygons indicated by the first-position number surround the cluster indicated by the second-position number symmetrically and that each of them touches only one edge of one hexagon in the cluster. (For the buckyballs, this corresponds to Tom Hull's "PPO," "pentagons pointing out" [16]). A 2 means that three of the polygons indicated by the first-position number surround the cluster indicated by the second-position number symmetrically and that each of them touches one edge of two hexagons in the cluster. (For the buckyballs this corresponds to Tom Hull's "PPI," "pentagons pointing in" [16]). Note that a model with a third-position number of 2 has fewer modules than a model with a third-position number of 1.

The basic polyhedra, therefore, are 5-1-1, 4-1-1, and 3-1-1—the truncated icosahedron, the truncated octahedron and the truncated tetrahedron. The buckyballs we have constructed are 5-1-1 (C-60), 5-3-2 (C-80), 5-3-1 (C-140), 5-7-2 (C-180), and 5-7-1 (C-240). The hypothetical buckyballs based on the octahedron or cube are the 4-1-1, 4-3-2, 4-3-1, 4-7-2, and 4-7-1, and those based on the tetrahedron are the 3-1-1, 3-3-2, 3-3-1, 3-7-2, and 3-7-1.

E. Model Assembly

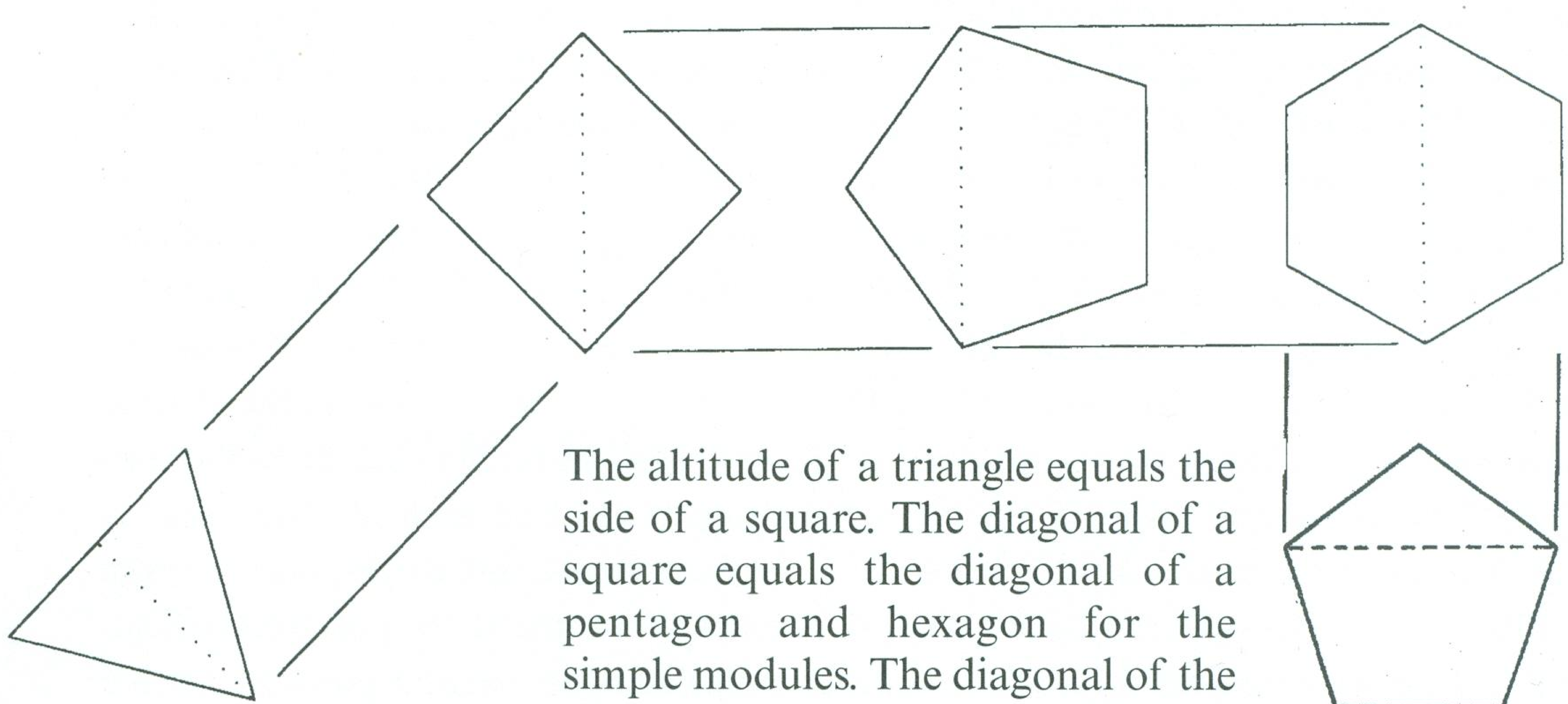
As with all modulars, it is recommended that you start assembling modules and keep adding to the partial assembly, rather than make several partial assemblies and then connect them. When connecting two different modules, the one made from the polygon with the greater number of sides is the receiving module because its pocket and tab are wider. However, the length of the entering tab is usually longer than the depth of the receiving pocket, so the tip of the entering tab must be folded over itself to allow the tab to fit into the pocket so that the edges of the pockets meet. Always first compare the length of the tab to the depth of the pocket to see if the tab needs to be shortened.

Also, while most of the models will stay together on their own, if they are large or will be handled a lot it is a good idea to use glue selectively to hold them together. For example, the crimps in the pentagon and hexagon modules are places where a drop of glue would be useful (see p. 47).

When making large gyroscoped buckyballs, gyroscoped hypothetical buckyballs, and the gyroscoped egg, we recommend that you glue all the hexagons together to make a framework and then pop the triangles, squares, or pentagons into the holes, without glue.

The largest member of each family, whose designation ends in -7-1, is most conveniently made by first making a hexagon framework, then making subassemblies of the basic polygon surrounded by hexagons, then inserting the subassemblies into the openings in the framework. This can be seen from the coloring of the modules in the photographs.

Sizing Starting Polygons



Simple Modules

The altitude of a triangle equals the side of a square. The diagonal of a square equals the diagonal of a pentagon and hexagon for the simple modules. The diagonal of the pentagon equals the distance between opposite sides of a hexagon for Expanded Spike Ball Modules.

Expanded Spike Ball Modules

How to Identify Buckyballs up to C240

C60 (5-1-1) Basic pentagon based Buckyball: Truncated Icosahedron. Each hexagon is surrounded symmetrically by 3 hexagons and 3 pentagons.

C80 (5-3-2) Each hexagon is surrounded symmetrically by 4 hexagons and 2 pentagons.

C140 (5-3-1) Each hexagon is surrounded by 5 hexagons and 1 pentagon.

C180 (5-7-2) Each pentagon is surrounded by 5 hexagons + 1 hexagon at each vertex of a dodecahedron.

C240 (5-7-1) Each pentagon is surrounded by 5 hexagons + 1 hexagon at each vertex of a dodecahedron + 1 hexagon at each edge of a dodecahedron.

Module Quantities Needed

	Gyroscoped Archimedeans Wenninger's Name and Number [7]	tri	sq	pent	hex	oct	dec
6.	Gyroscoped Truncated Tetrahedron	4			4		
7.	Gyroscoped Truncated Octahedron		6		8		
8.	Gyroscoped Truncated Cube	8				6	
9.	Gyroscoped Truncated Icosahedron			12	20		
10.	Gyroscoped Truncated Dodecahedron	20					12
11.	Gyroscoped Cuboctahedron	8	6				
12.	Gyroscoped Icosidodecahedron	20		12			
13.	Gyroscoped Rhombicuboctahedron	8	18				
14.	Gyroscoped Rhombicosidodecahedron	20	30	12			
15.	Gyroscoped Truncated Cuboctahedron		12		8	6	
16.	Gyroscoped Truncated Icosidodecahedron		30		20		12
17.	Gyroscoped Snub Cube	32	6				
18.	Gyroscoped Snub Dodecahedron	80		12			

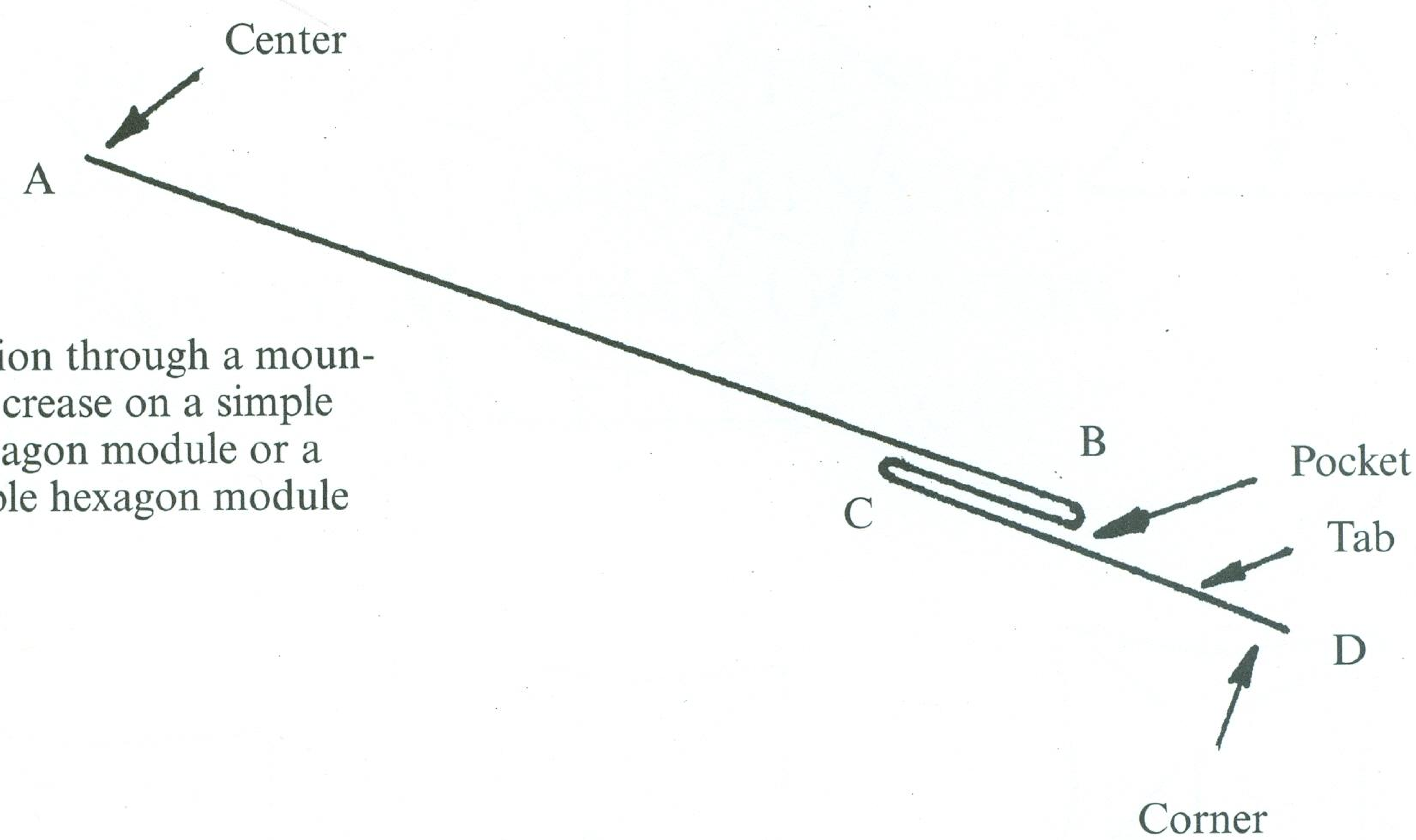
		Buckyball	Gyroscoped Buckyball				
		tri	tri	sq	pent	hex	
1.	3-1-1	12	4			4	truncated tetrahedron
2.	3-3-2	16	4			6	
3.	3-3-1	28	4			12	
4.	3-7-2	36	4			16	
5.	3-7-1	48	4			22	
6.	4-1-1	24		6		8	truncated octahedron
7.	4-3-2	32		6		12	
8.	4-3-1	56		6		24	
9.	4-7-2	72		6		32	
10.	4-7-1	96		6		44	
11.	5-1-1	60			12	20	truncated icosahedron
12.	5-3-2	80			12	30	
13.	5-3-1	140			12	60	
14.	5-7-2	180			12	80	
15.	5-7-1	240			12	110	

Growing Buckyballs: Building and Counting the Modules

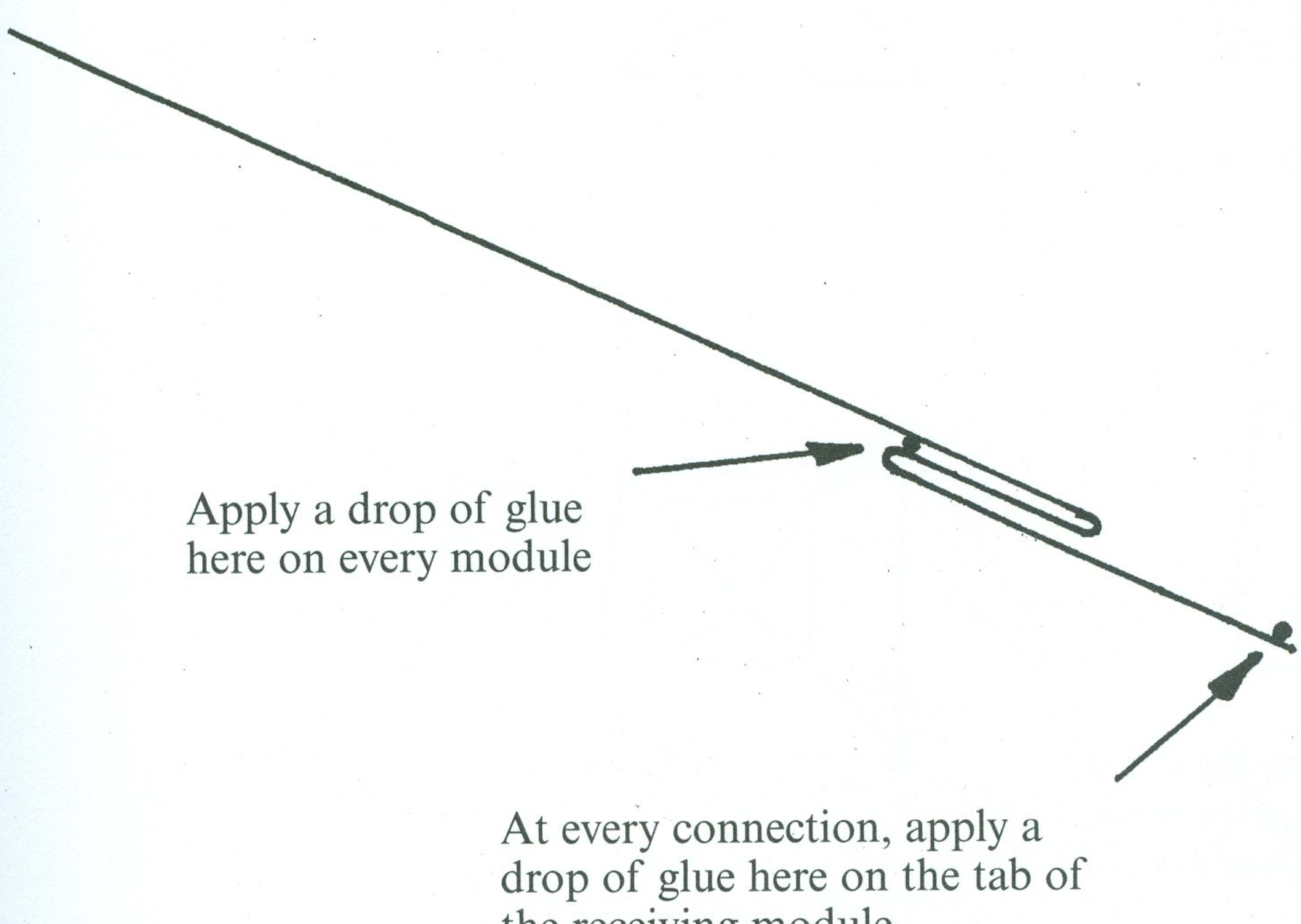
	Tetrahedron Derivative	Cube Derivative	Dodecahedron Derivative
Basic	Truncated Tetrahedron 3-1-1 (4) 3-sided rings 12 modules	Truncated Octahedron 4-1-1 (6) 4-sided rings 24 modules	Truncated Icosahedron 5-1-1 (12) 5-sided rings 60 modules
Chamferred Derivative: slicing off an edge Basic + one module at each corner of the derivative	3-3-2 12 + 4 = 16 modules	4-3-2 24 + 8 = 32 modules	5-3-2 60 + 20 = 80 modules
Snub Derivative: Basic + one module at each corner + 2 modules at each edge of derivative	3-3-1 12 + 4 + 12 = 28 modules	4-3-1 24 + 8 + 24 = 56 modules	5-3-1 60 + 20 + 60 = 140 modules
Basic + 6-sided ring at each corner of derivative	3-7-2 12 + 24 = 36 modules	4-7-2 24 + 48 = 72 modules	5-7-2 60 + 120 = 180 modules
Basic + 6-sided ring at each edge of derivative	3-7-1 12 + 36 = 48 modules	4-7-1 24 + 72 = 96 modules	5-7-1 60 + 180 = 240 modules

How and Where to Apply Glue

When joining two different modules, check to see that the length of the tab, distance B-D, fits inside the depth of the pocket, distance B-C. If the tab is too long, fold over the tip D so the tab fits inside the pocket. When properly connected, the edges B on both modules will touch. D on the entering module slides between B and C on the receiving module.



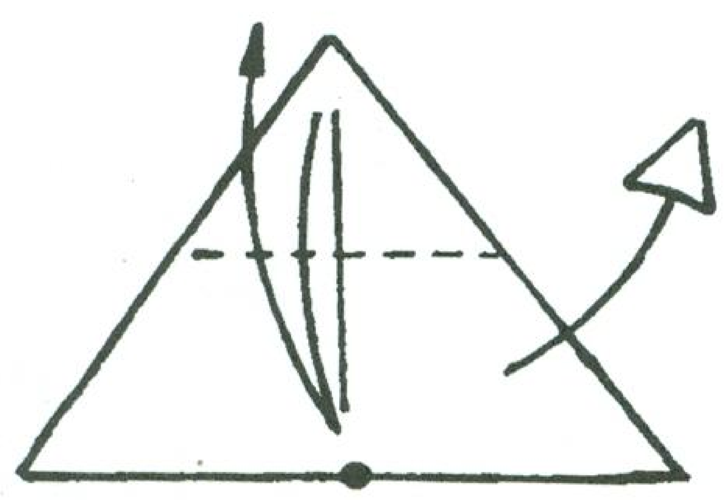
Section through a mountain crease on a simple pentagon module or a simple hexagon module



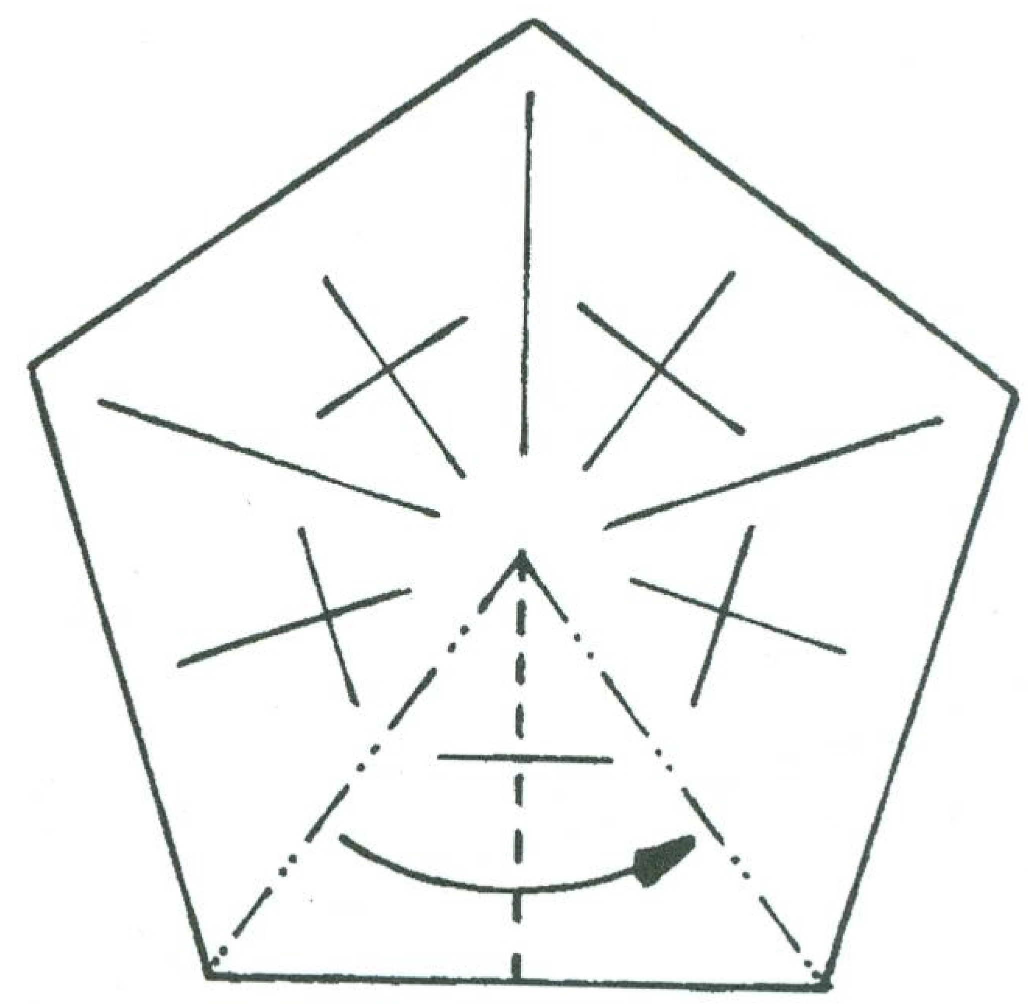
F. Four-sided Display Stand From a Pentagon Waterbomb Base

by Bennett Arnstein

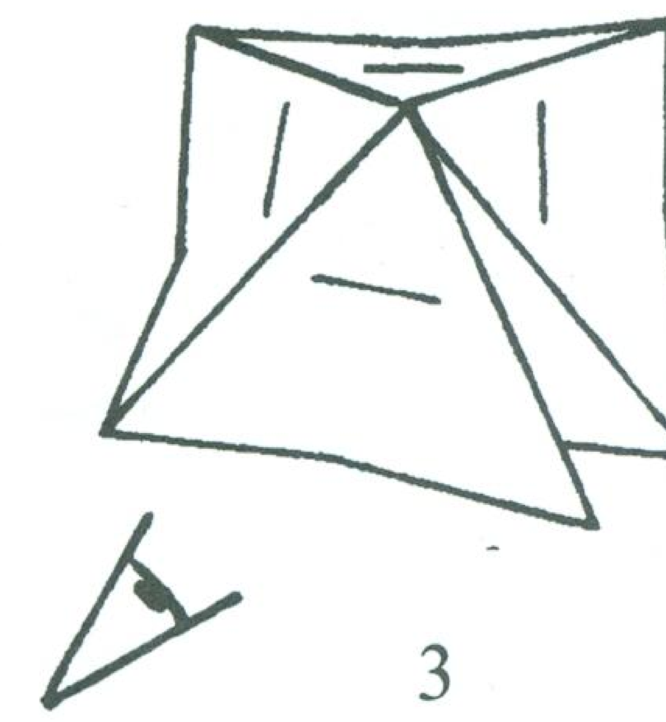
Based on a 3-sided display stand from a square waterbomb base by Douglas Shachnow. Start with figure 3 of the "Expanded Pentagon Spike-Ball Module" on p71.



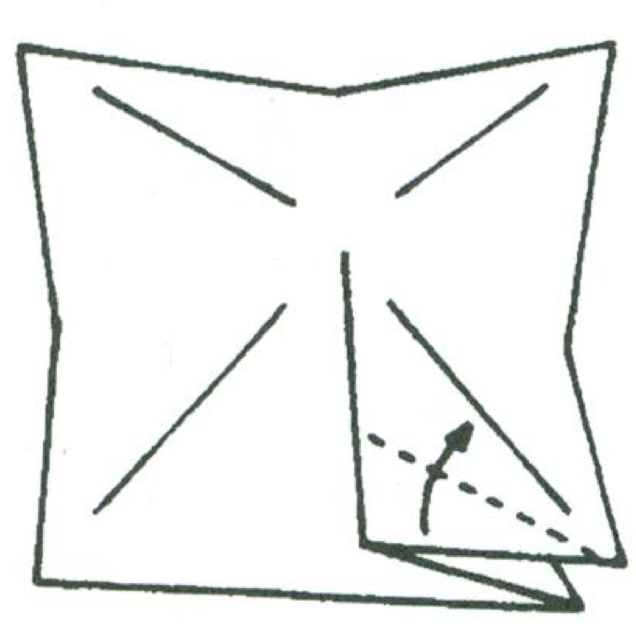
1



2

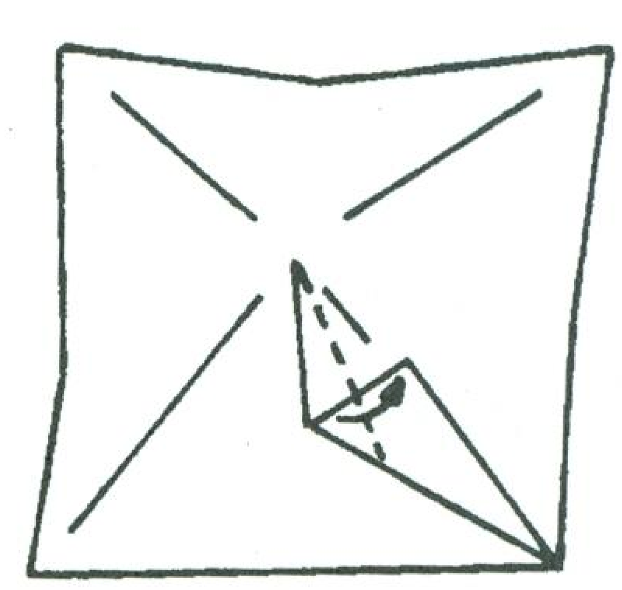


3



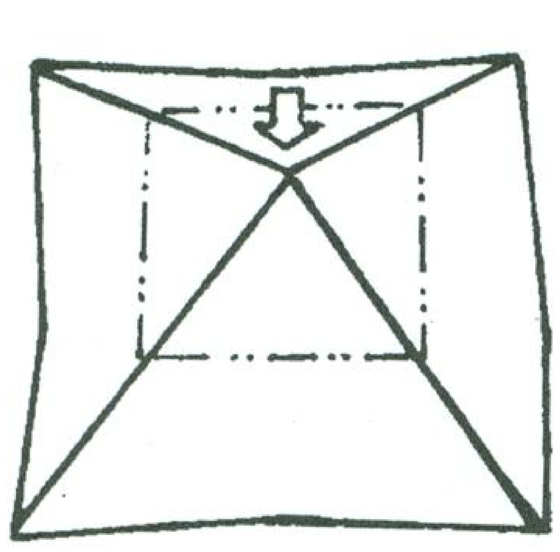
Looking up at the inside

4



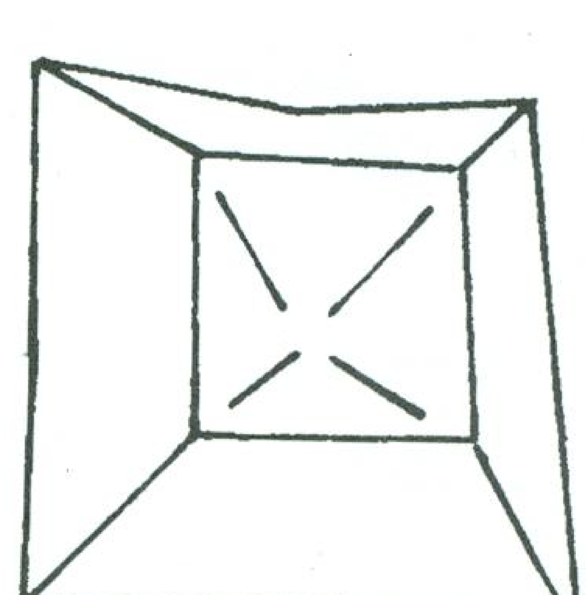
5

6



7

Looking down from the outside. Sink the point using the mountain creases as a boundary.



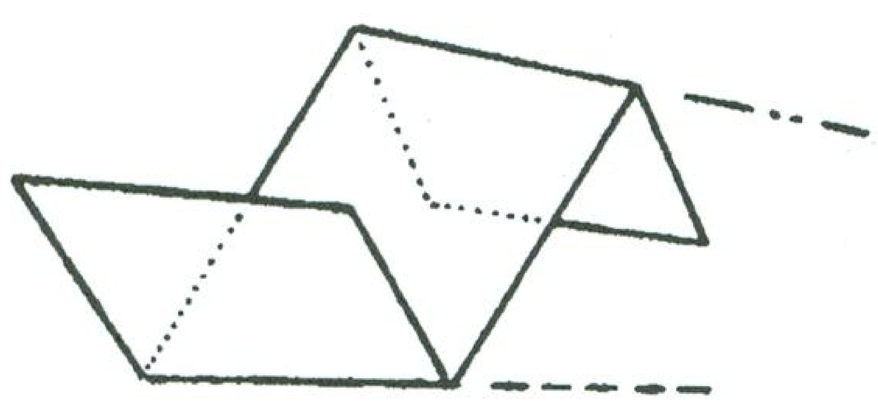
8

IV. POLYGON CONSTRUCTION

A. Symbols
B. How to Make an Equilateral Triangle
C. How to Make a Pentagon
D. How to Make a Hexagon
E. How to Make an Octagon
F. How to Make a Decagon

POLYGON CONSTRUCTION

A. Symbols



Mountain Fold

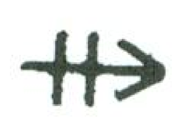
Valley Fold

Fold

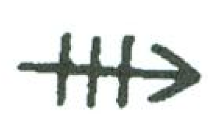
Fold and Unfold

Unfold

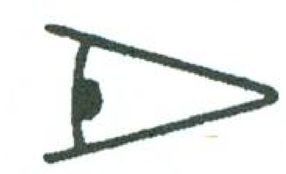
Repeat



Repeat Twice

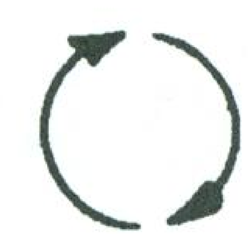


Repeat Three Times

A  B

B is an edge view of A

Turn Over



Rotate Without Turning Over

Fold second crease against first, refold first

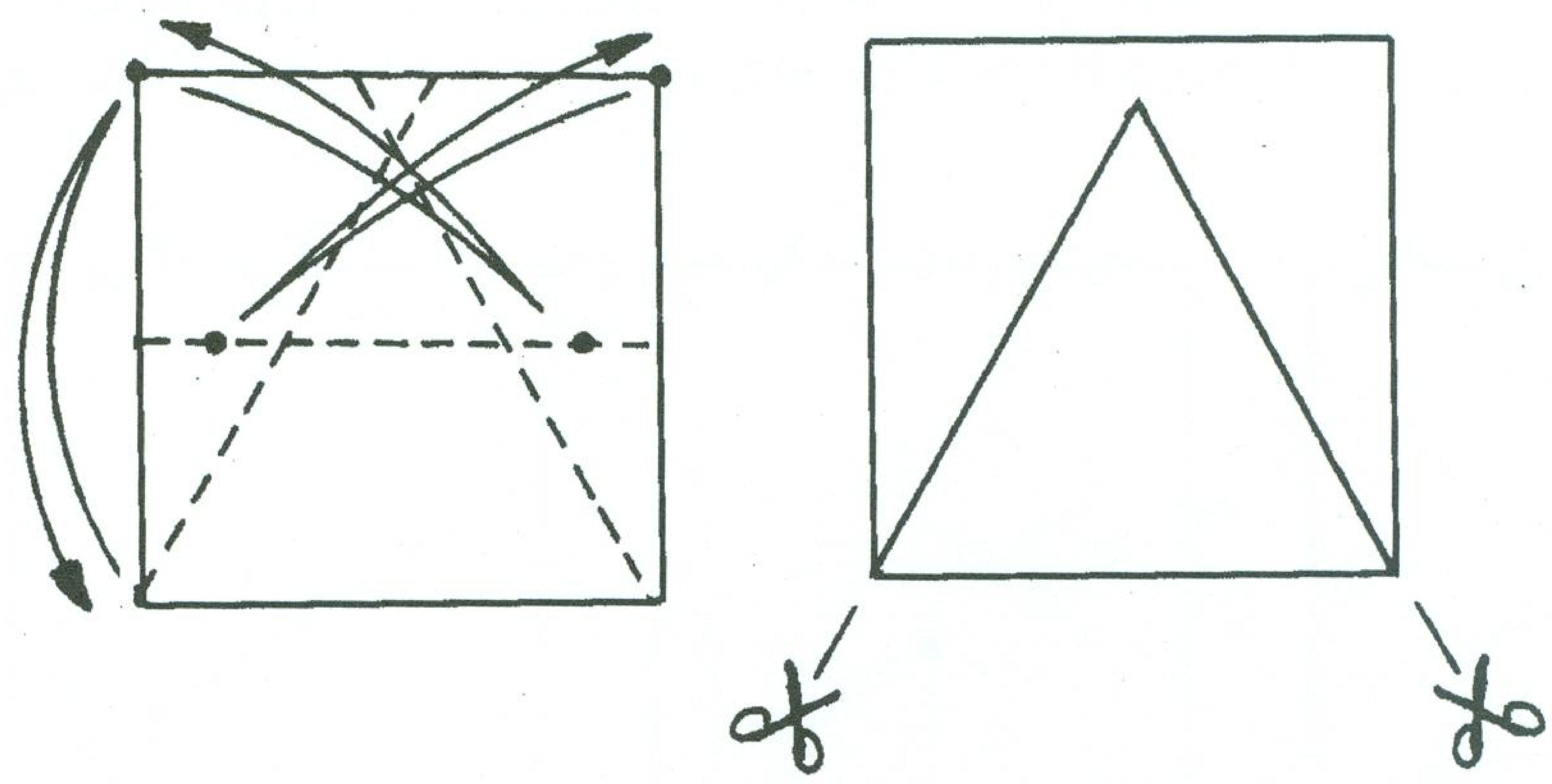
A B

Fold point A to point B

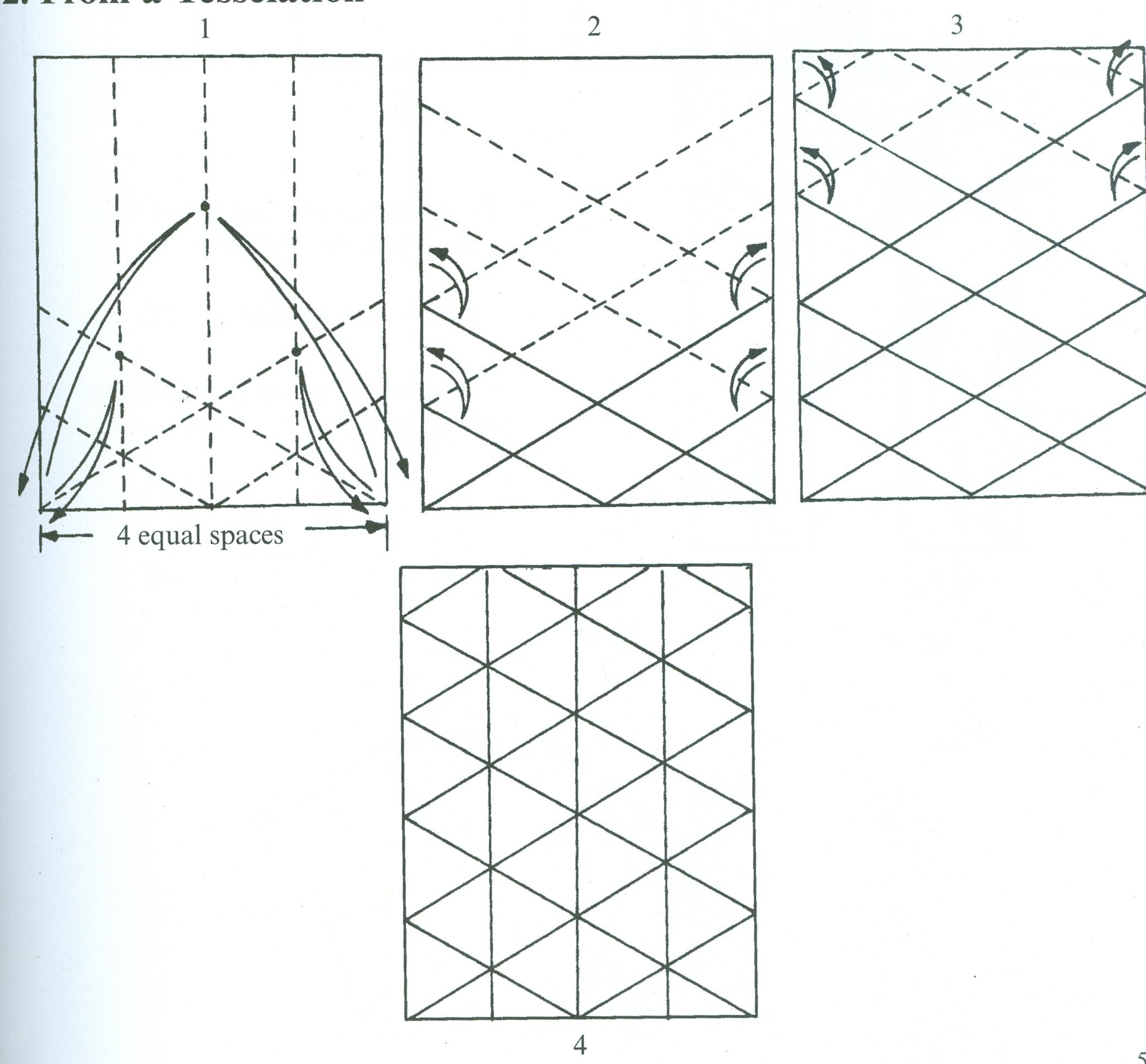
B. How to Make an Equilateral Triangle

by Bennett Arnstein

1. From a Square

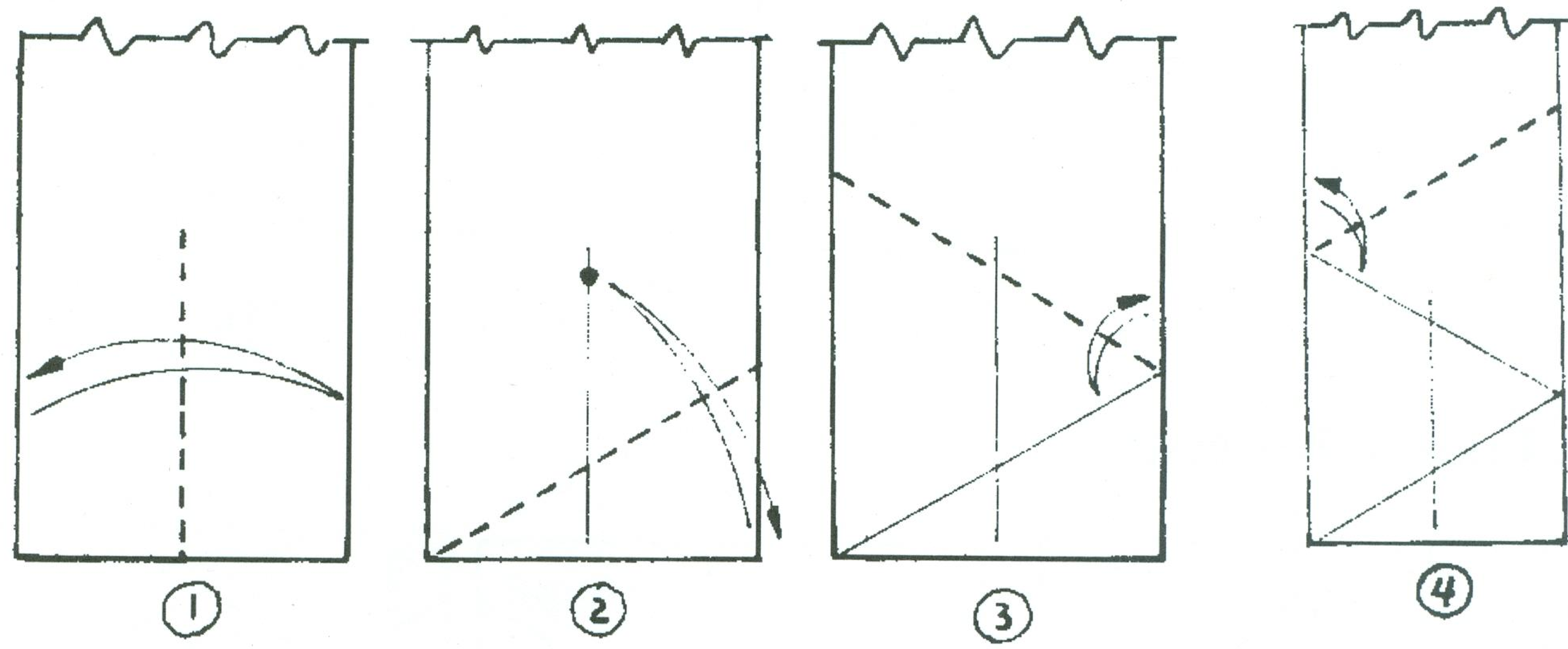


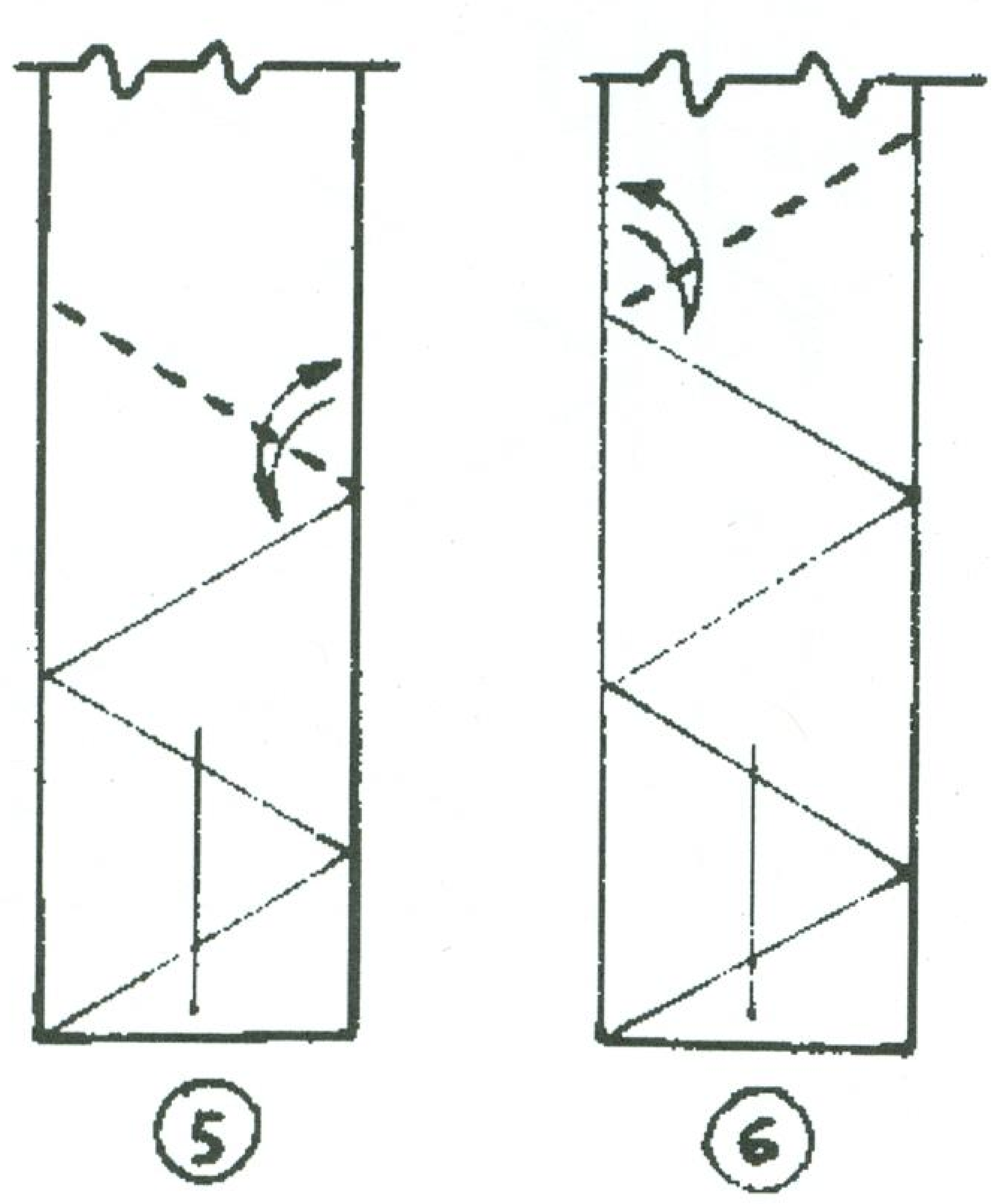
2. From a Tesselation



3. From a Strip

Width of strip = altitude of triangles





To make n triangles, start with a strip whose length=

(n/2 + .5)(altitude x 1.15)

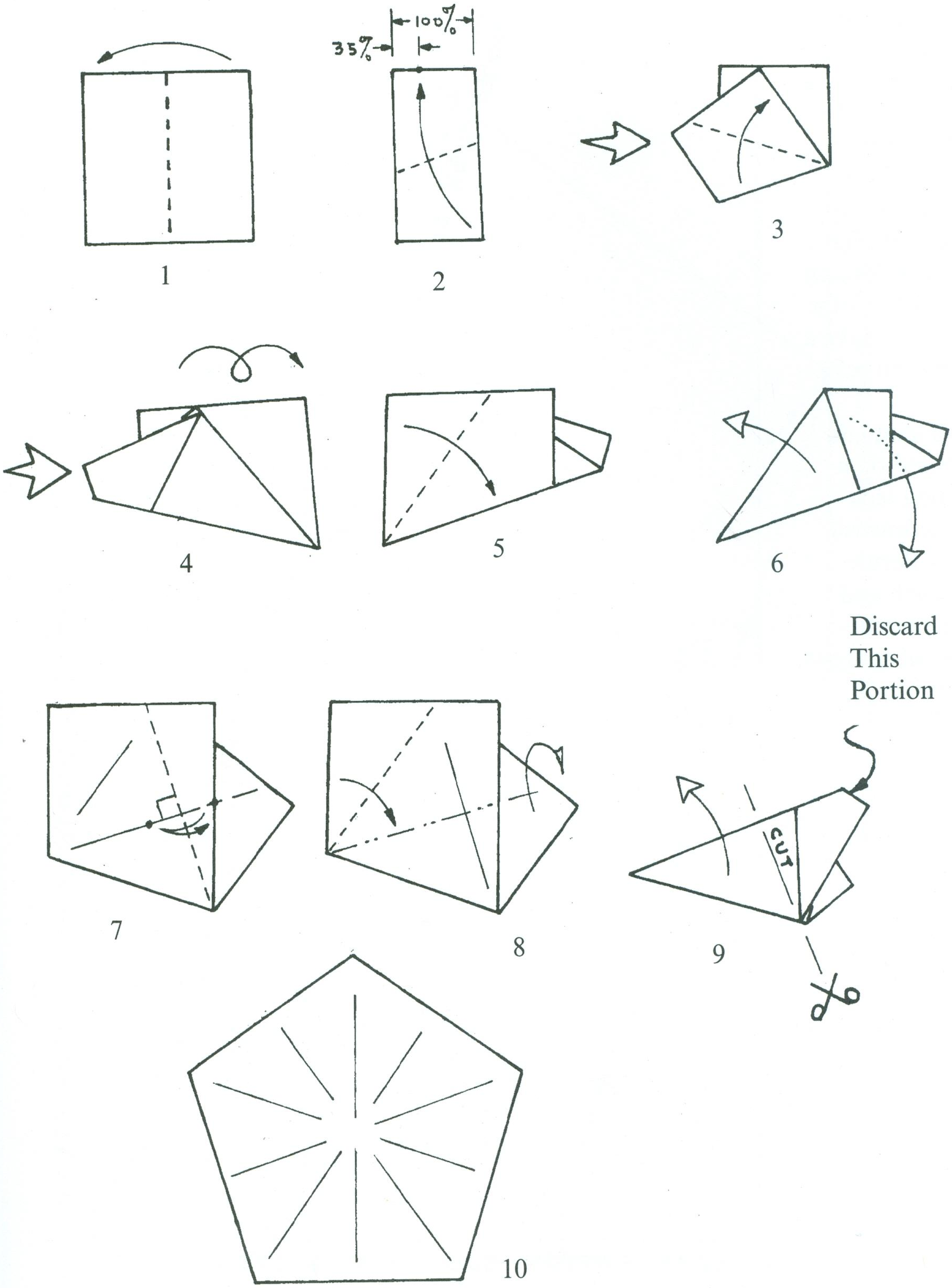
where width of strip = altitude of triangles

C. How to Make a Pentagon

Traditional, USA

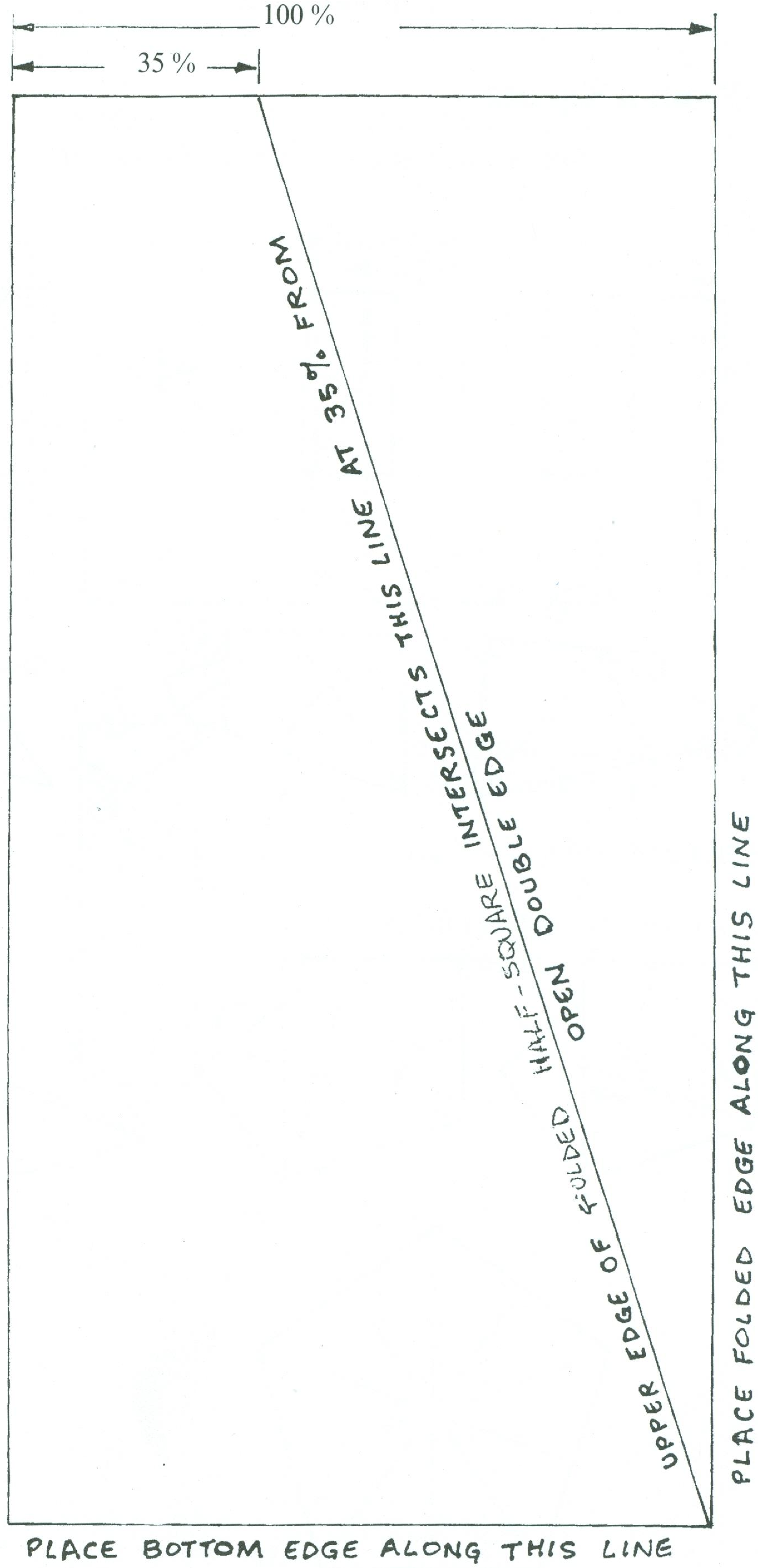
1. From a Square

See full-page enlargement of figure 2 for template.



2. Pentagon Template 1: From a Square

by Bennett Arnstein

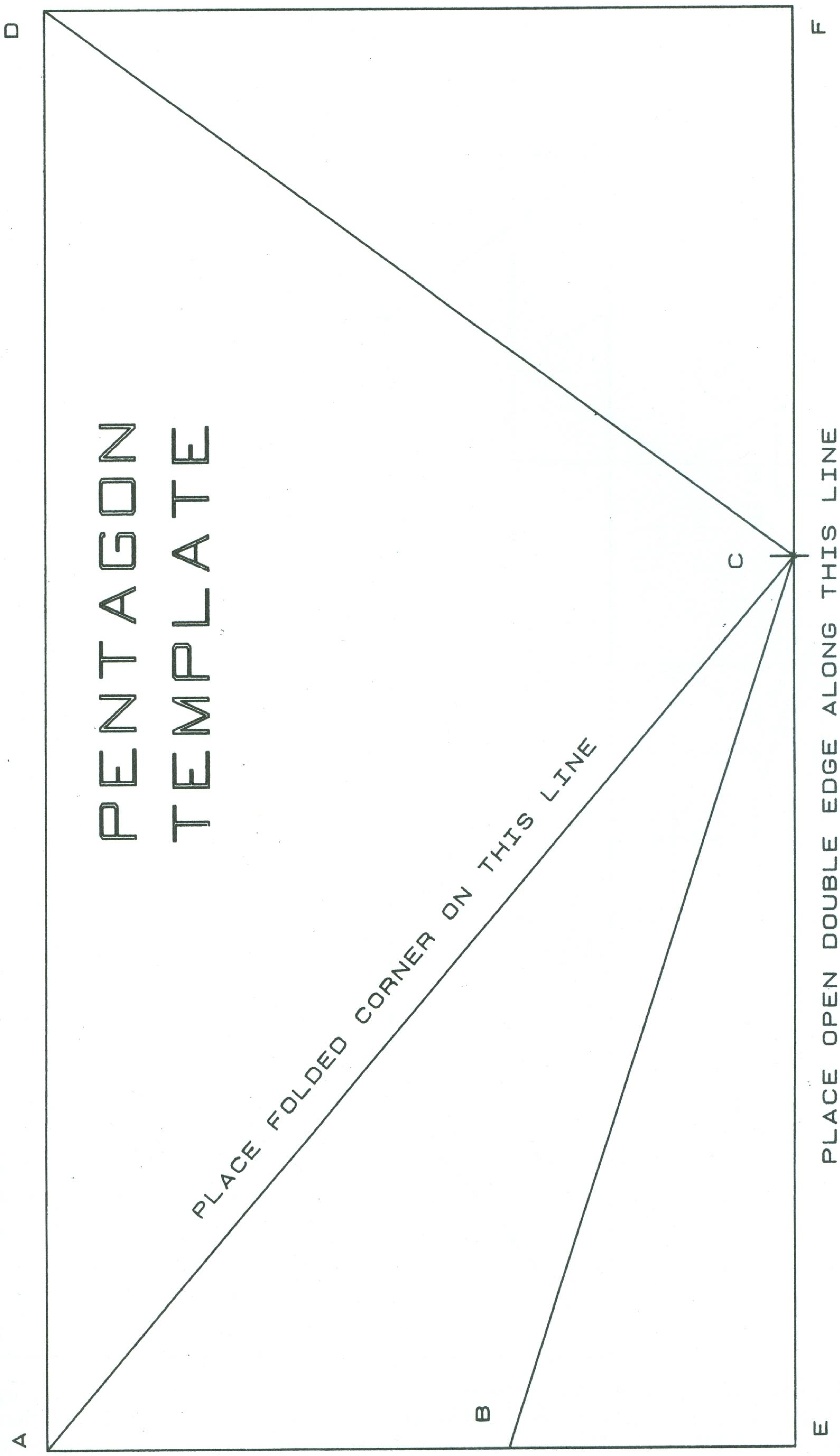


Enlarged view of figure 2 of How to Make a Pentagon p.53. Mark upper edge at 35 % point and proceed from figure 2 of folding instructions for How to Make a Pentagon p.53.

This template is based on a folding technique learned from Bernie Slotnick and some mathematical analysis by Jasper Paulsen.

3. Pentagon Template 2

by Bennett Arnstein



How to Use the Pentagon Template 2

by Bennett Arnstein

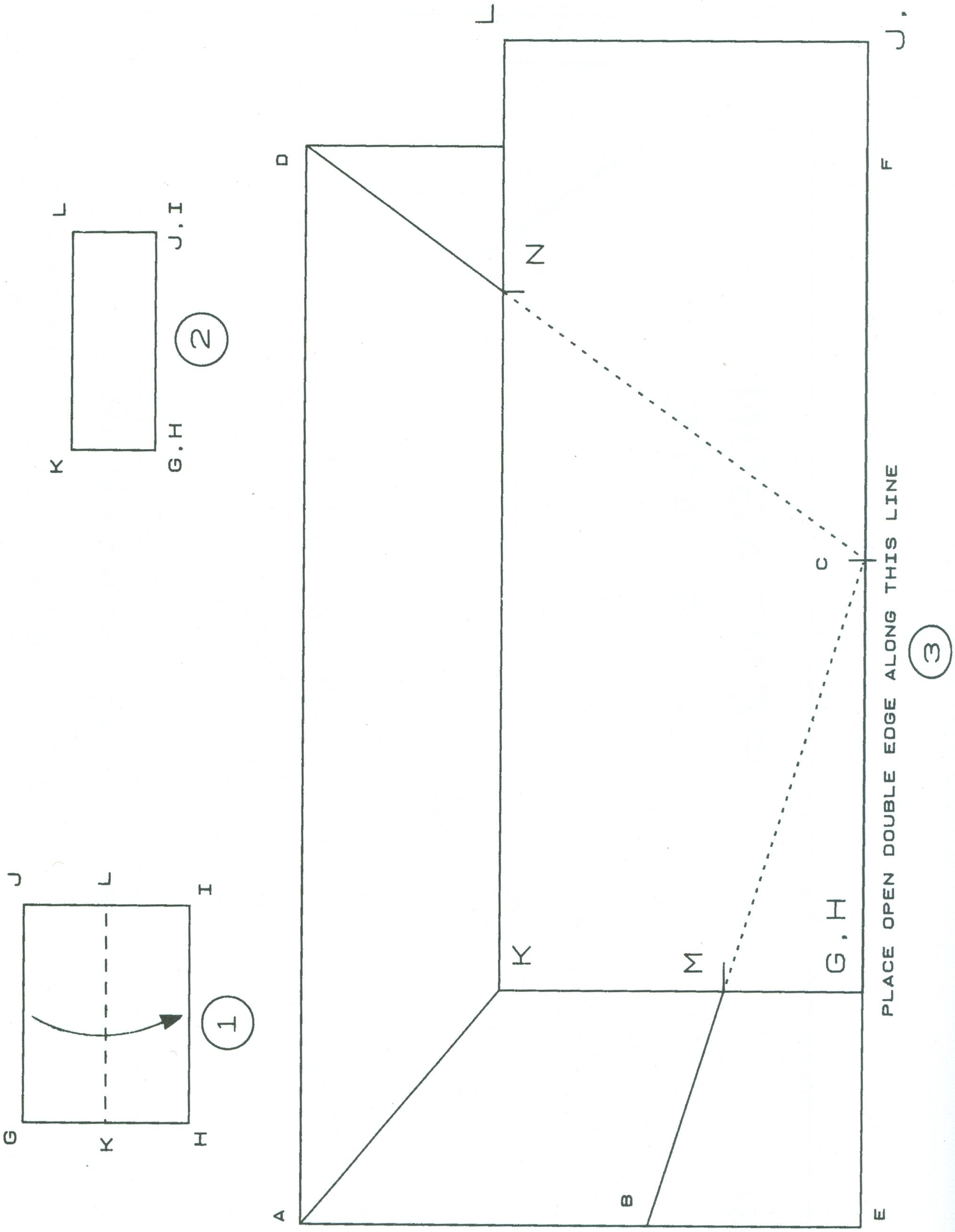
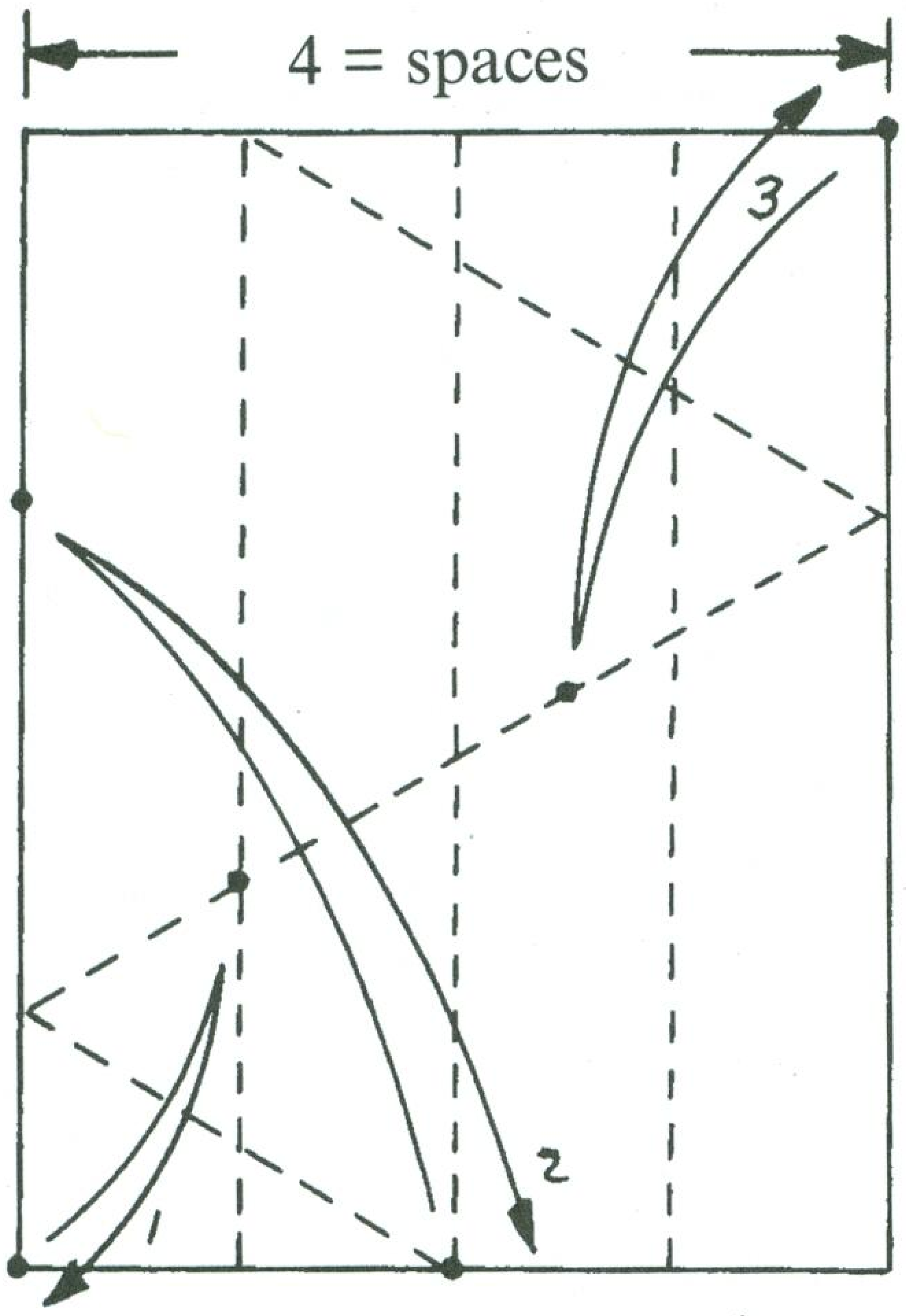


FIG. 1: Fold sheet of paper in half to arrive at FIG. 2. FIG. 3: Slide the folded sheet along line EF on the template until the folded corner K lies in line AC. Mark points M, C, and N on the folded sheet and draw lines MC and CN using a ruler. With a pair of sharp scissors cut lines MC and CN through both layers of the folded sheet. The pentagon, folded in half, is KMCN.

D. How to Make a Hexagon

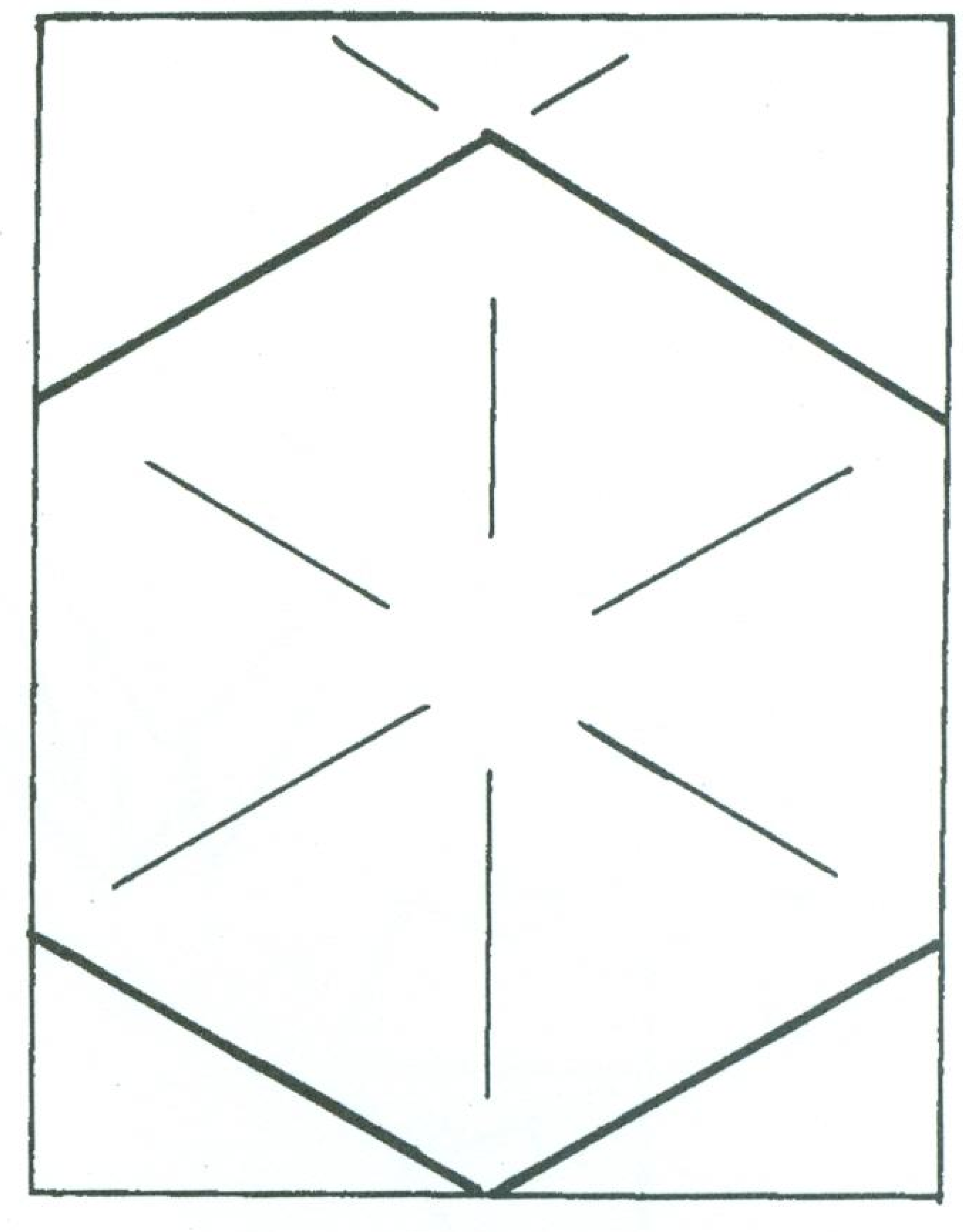
by Bennett Arnstein

From a
Rectangle



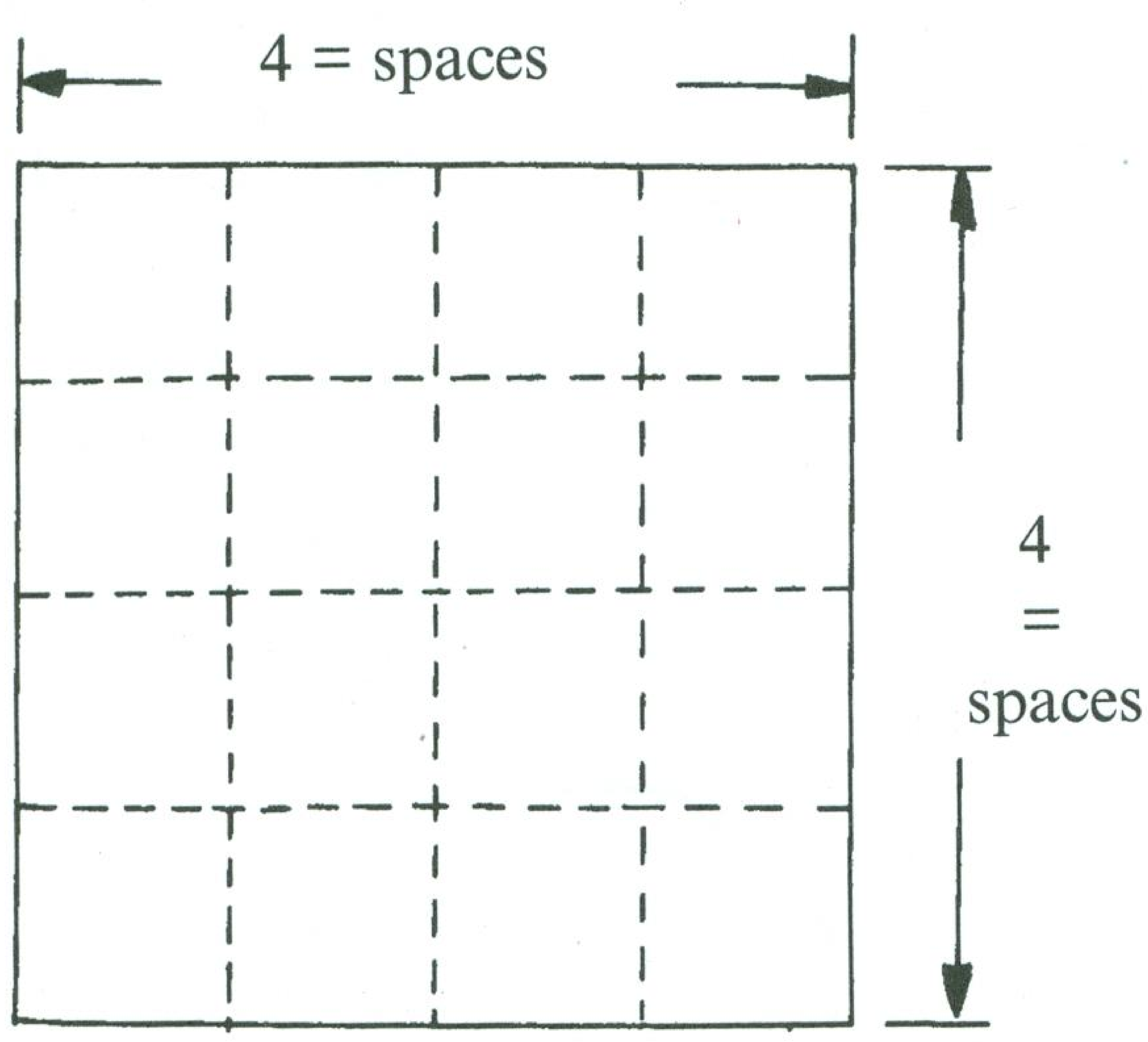
1

Repeat steps on
right side of paper

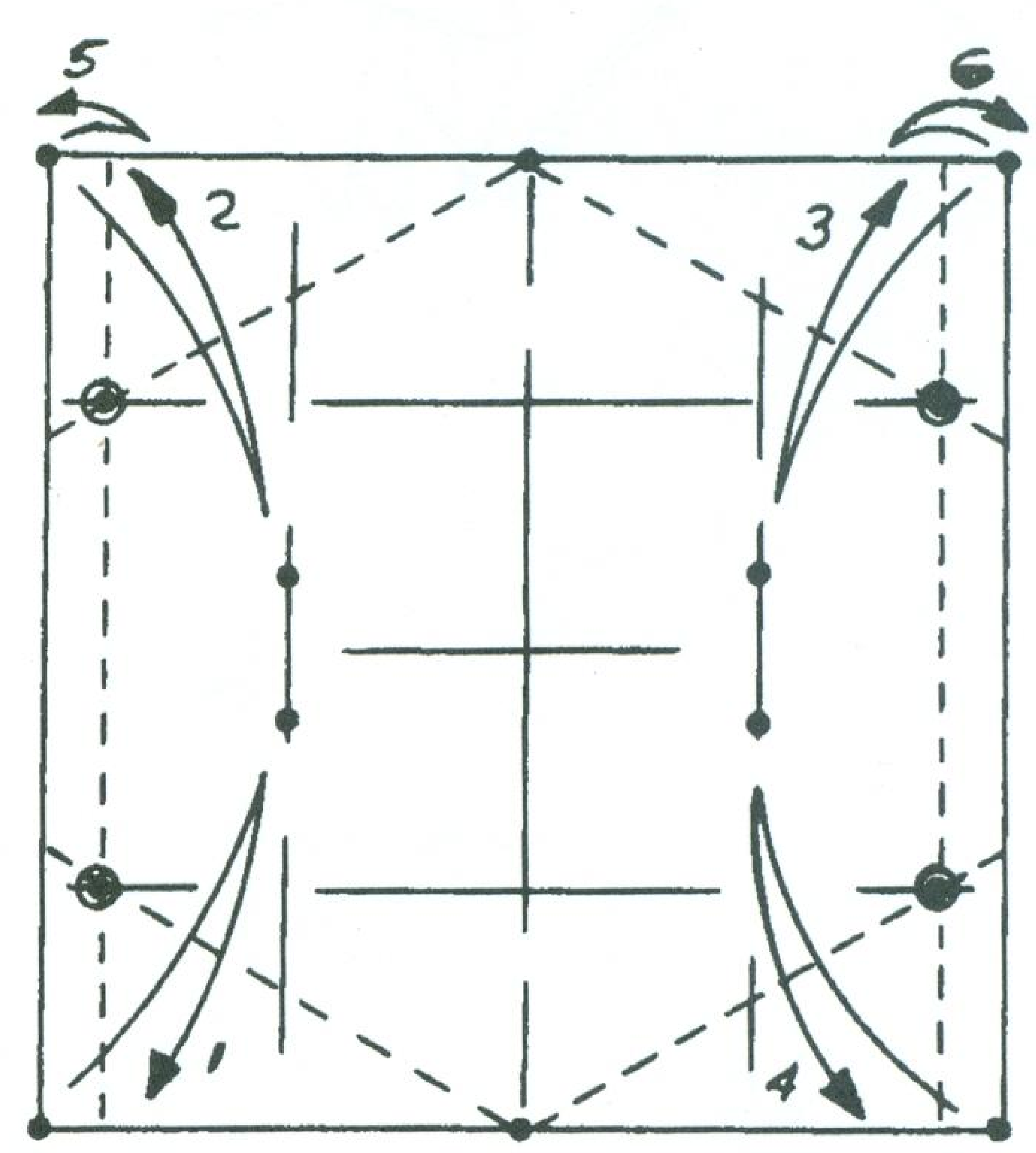


2

From a
Square

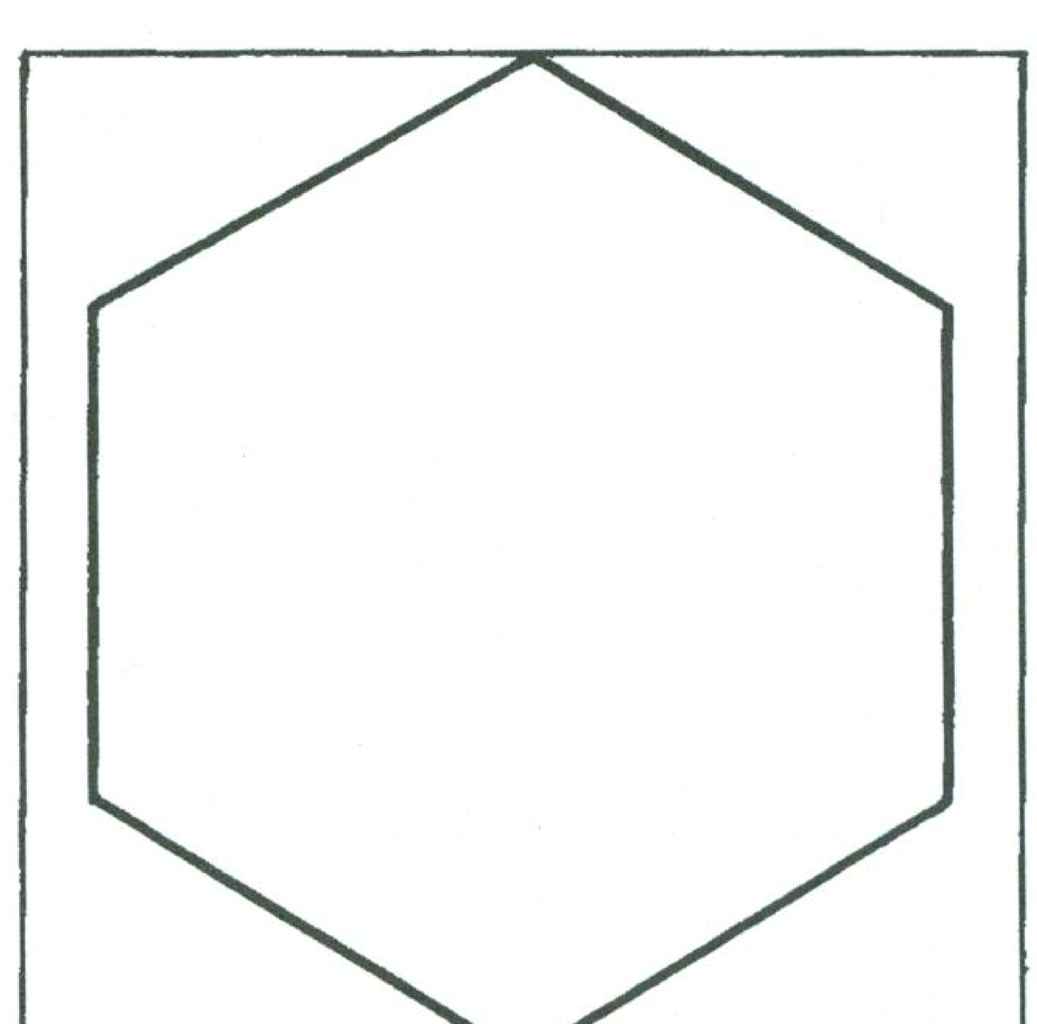


3



4

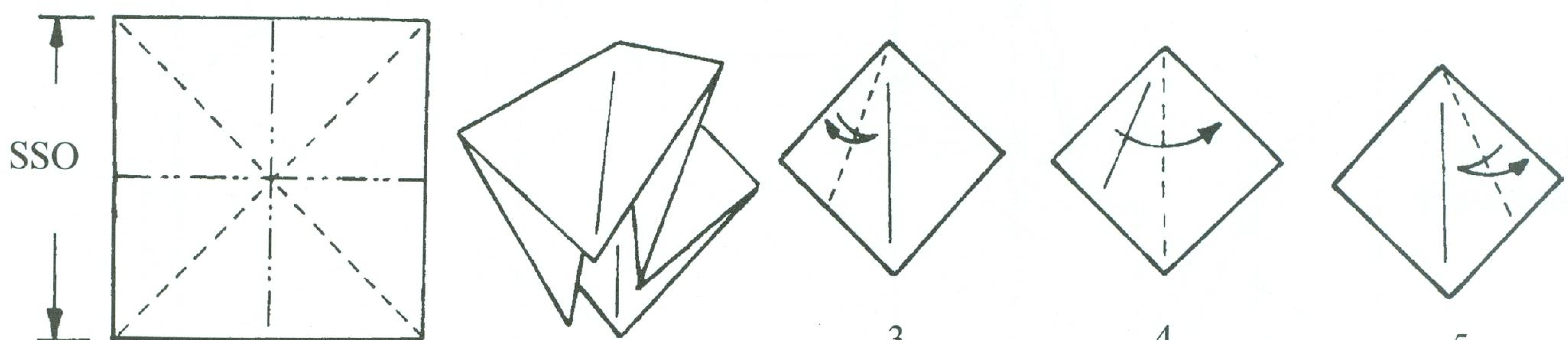
Alternative:
Use a triangle tessella-
tion like the one on p. 52



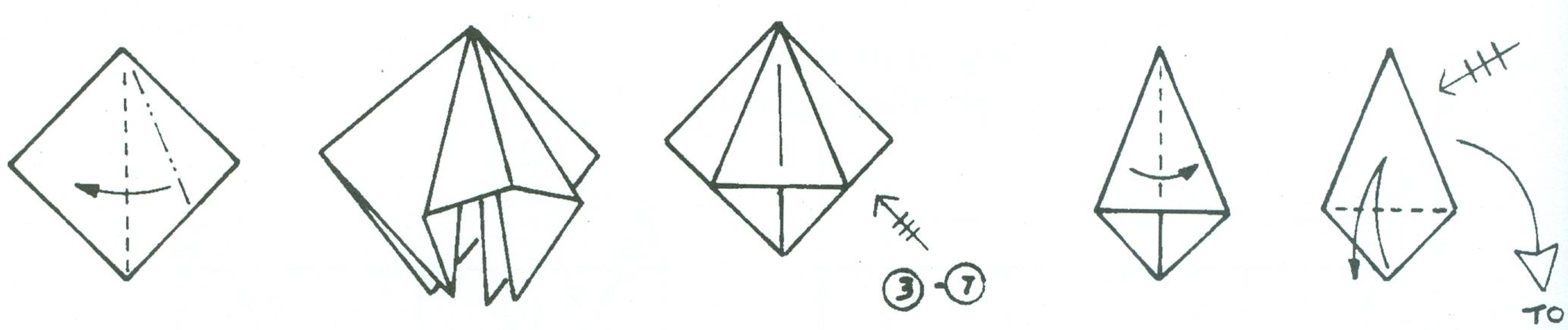
5

E. How to Make an Octagon

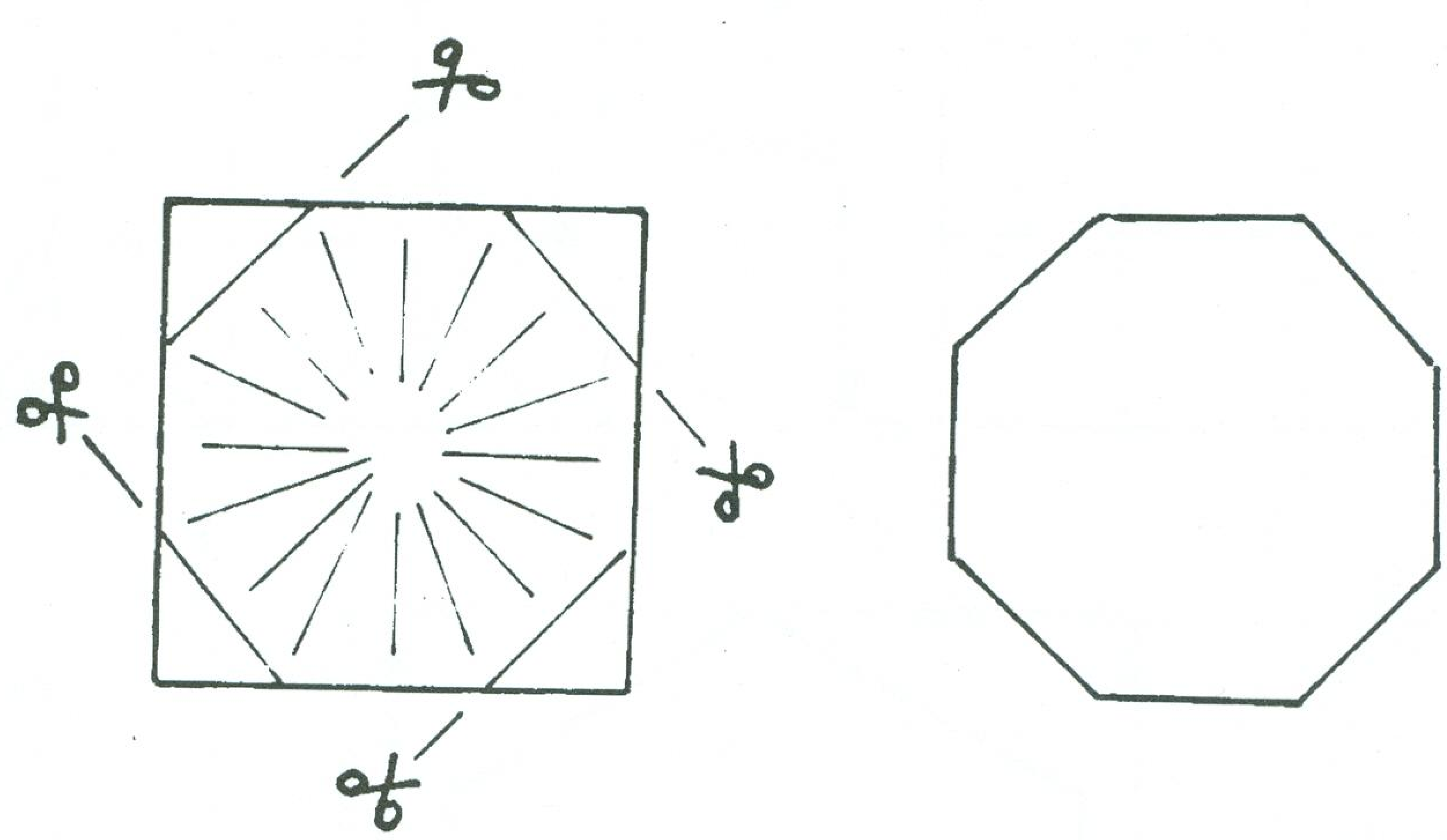
The distance SSO is used in sizing the octagon module so that it mates with other types of modules.



1 2 3 4 5



6 7 8 9 10

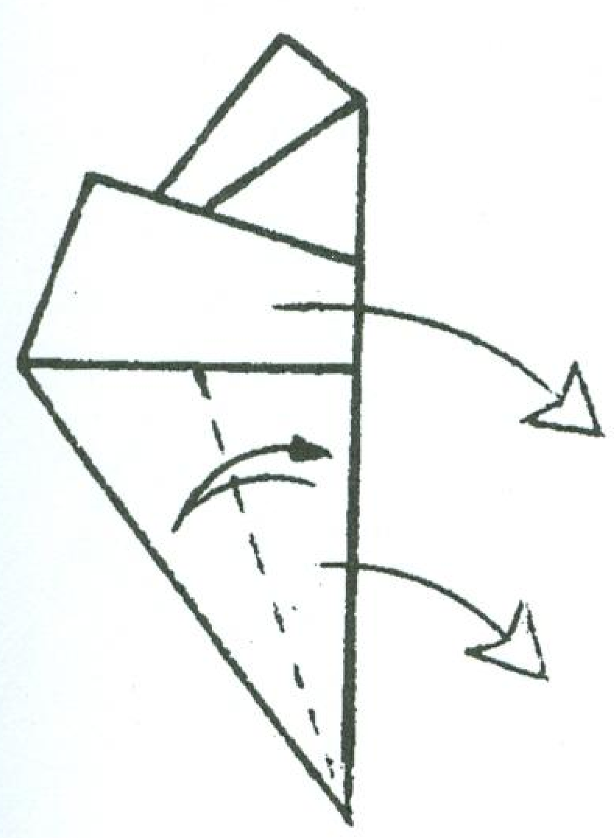


11 12

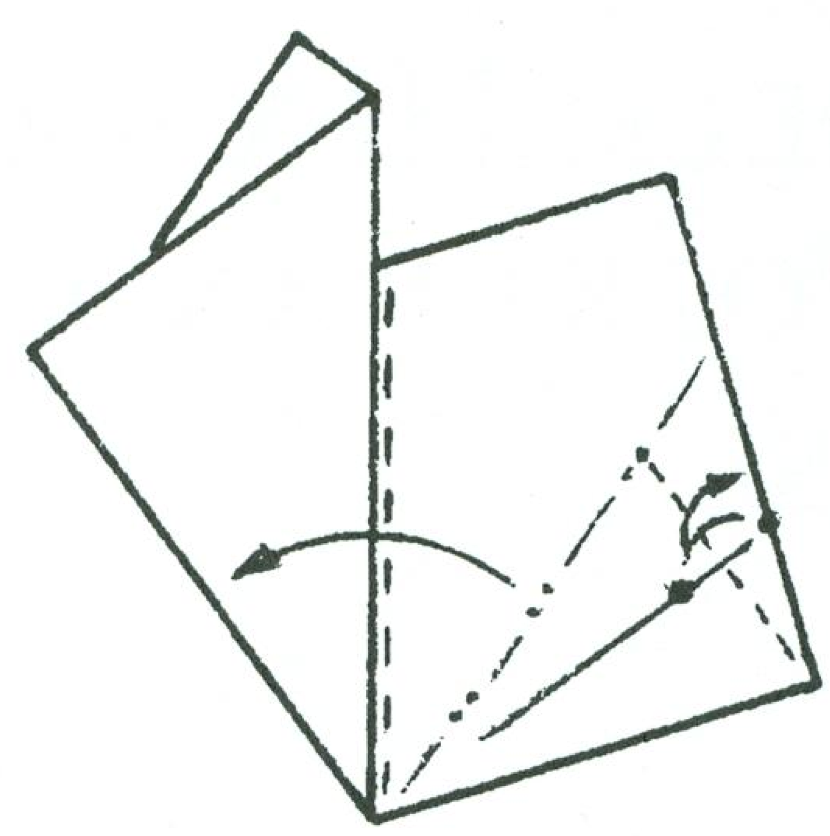
F. How to Make a Decagon

by Bennett Arnstein

Start with figure 6 of How to Make a Pentagon.



1



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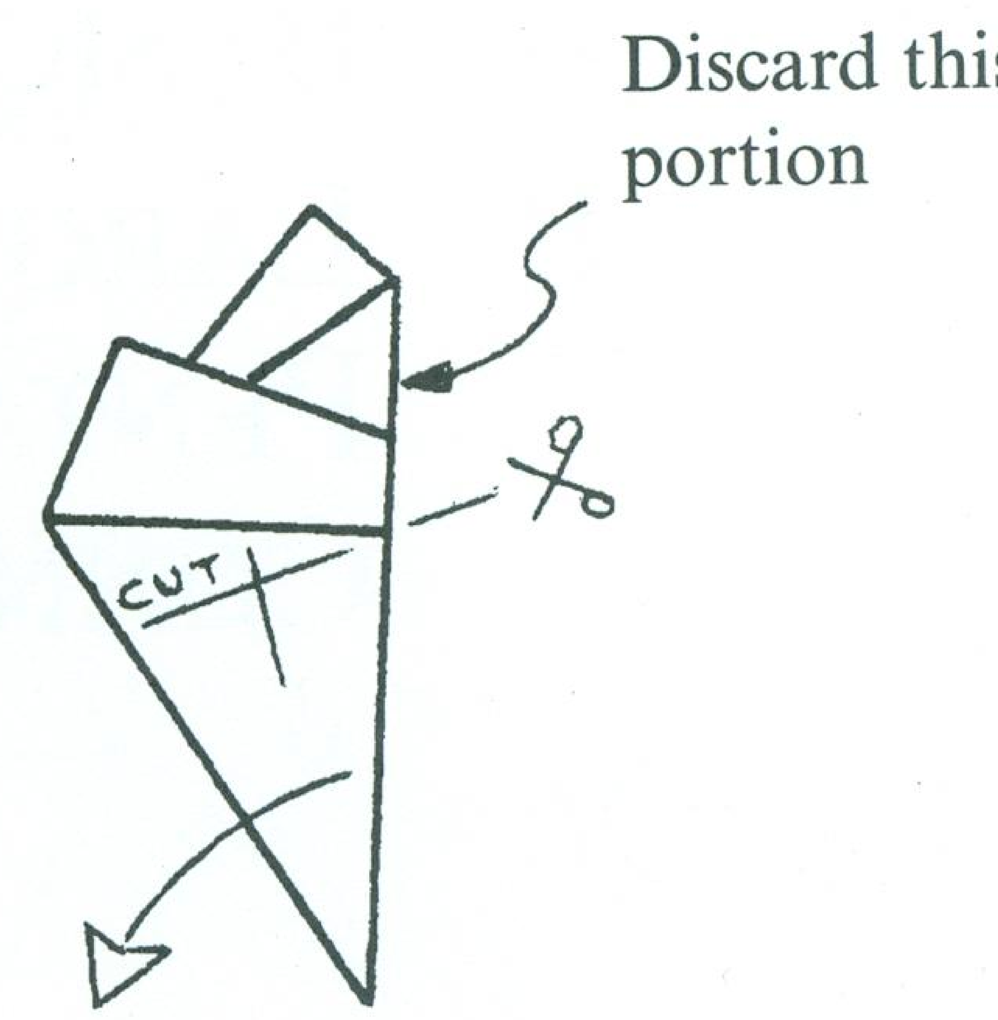

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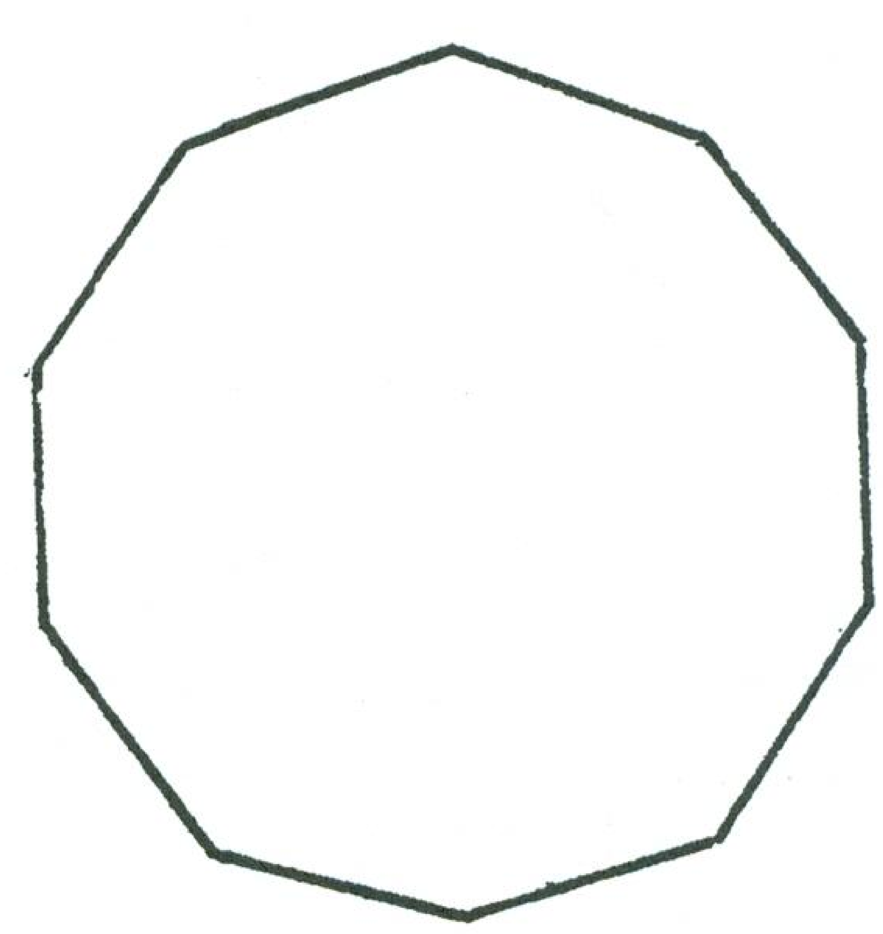


4

V. MODULE CONSTRUCTION FROM POLYGONS

A. Basic Tab and Pocket Lock

B. Large Models: Hexagon/ Pentagon Pocket and Connectors

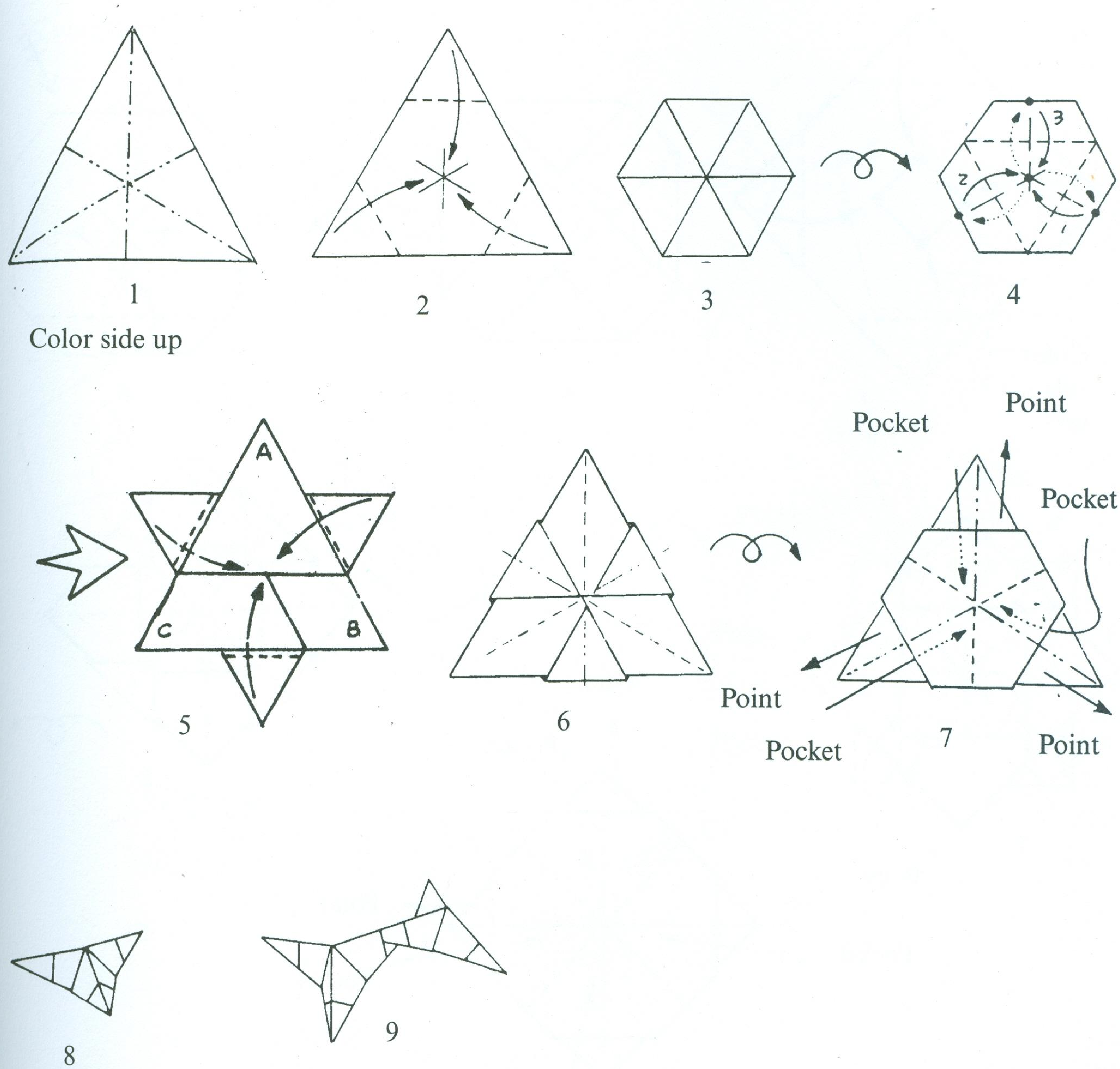
MODULE CONSTRUCTION FROM POLYGONS

A. Basic Tab and Pocket Lock

1. One-Piece Triangle Module

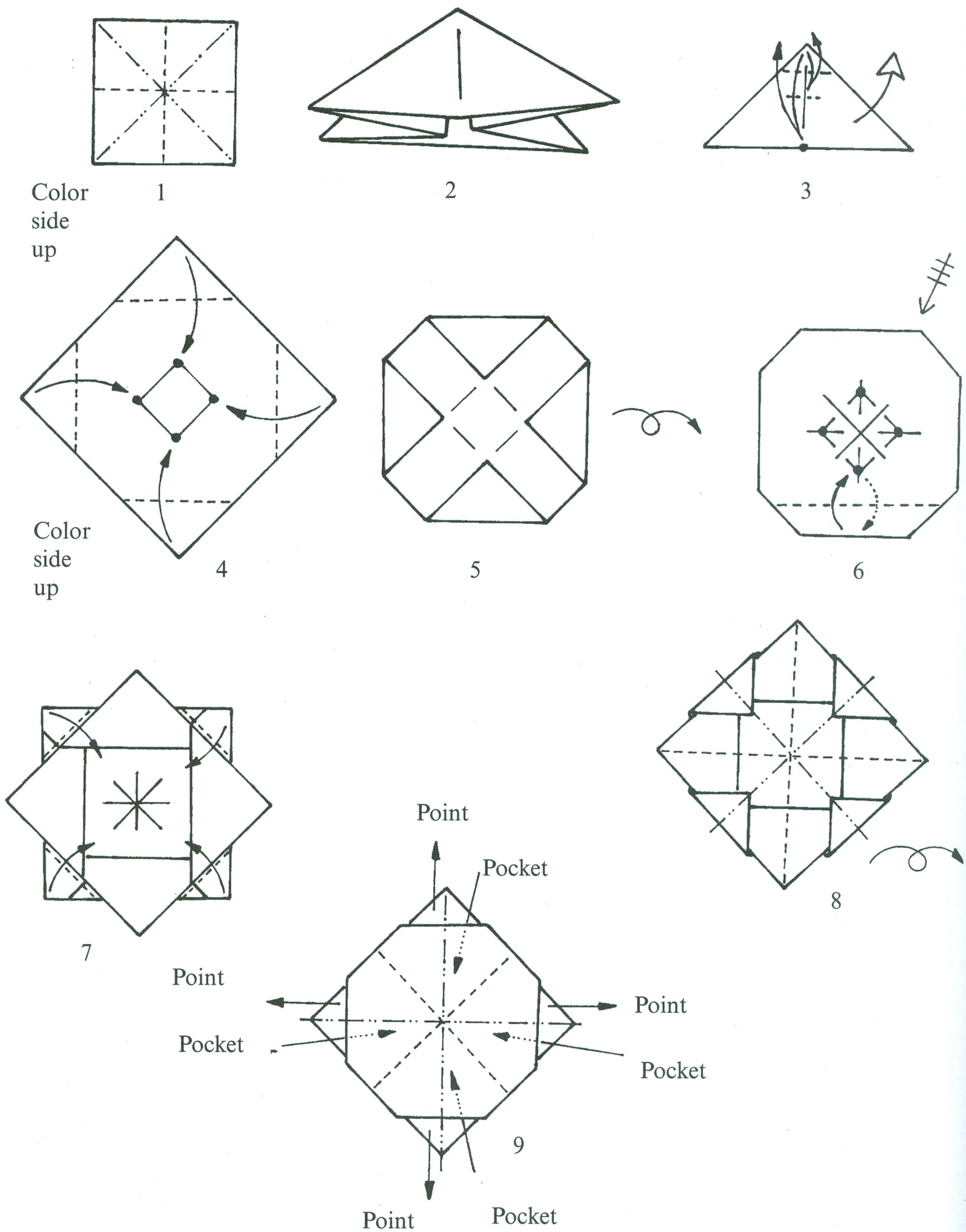
by Bennett Arnstein

This module is the one-piece version. The two-piece version is the gyroscope of [11].

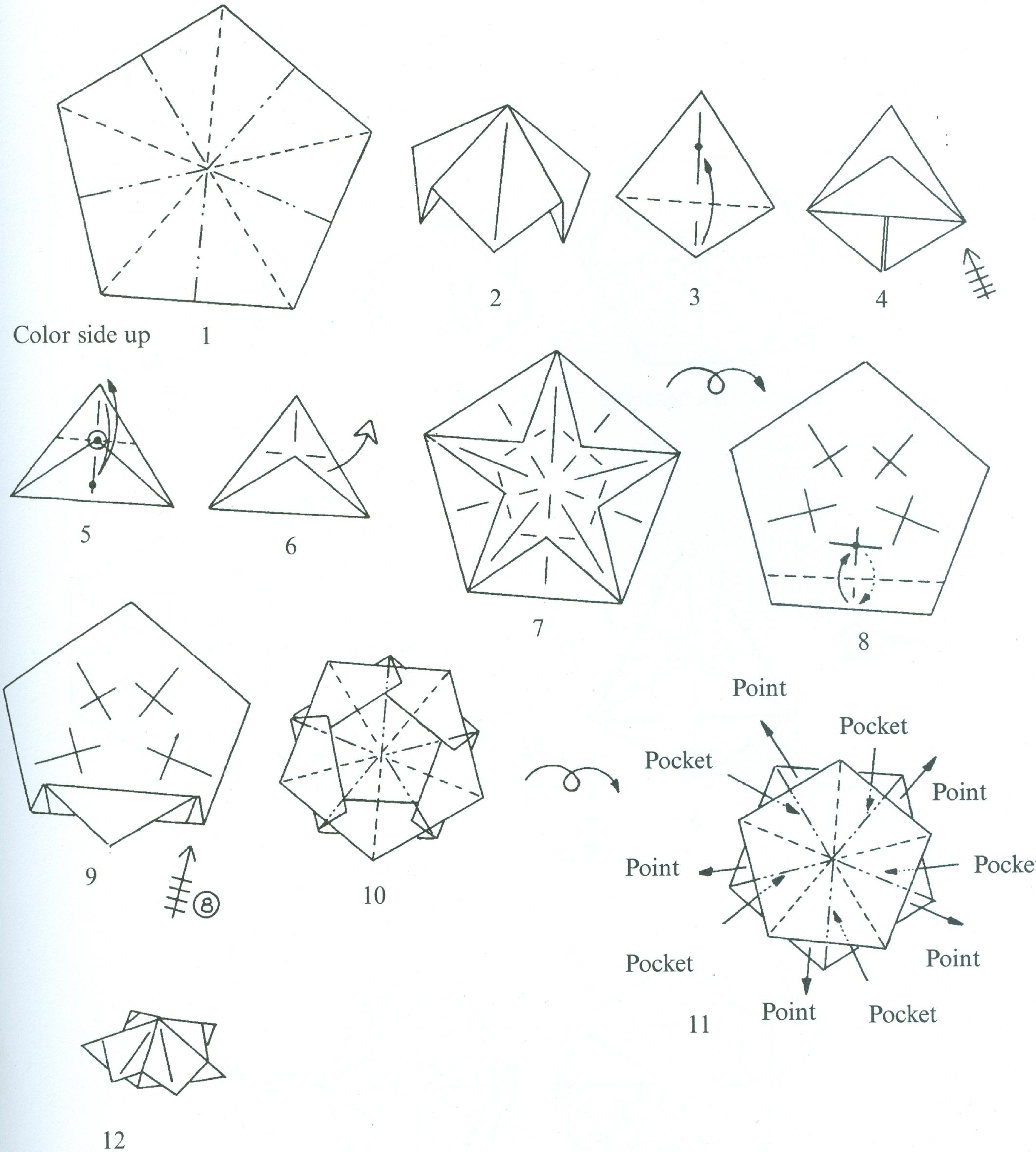


2. One-Piece Square Module

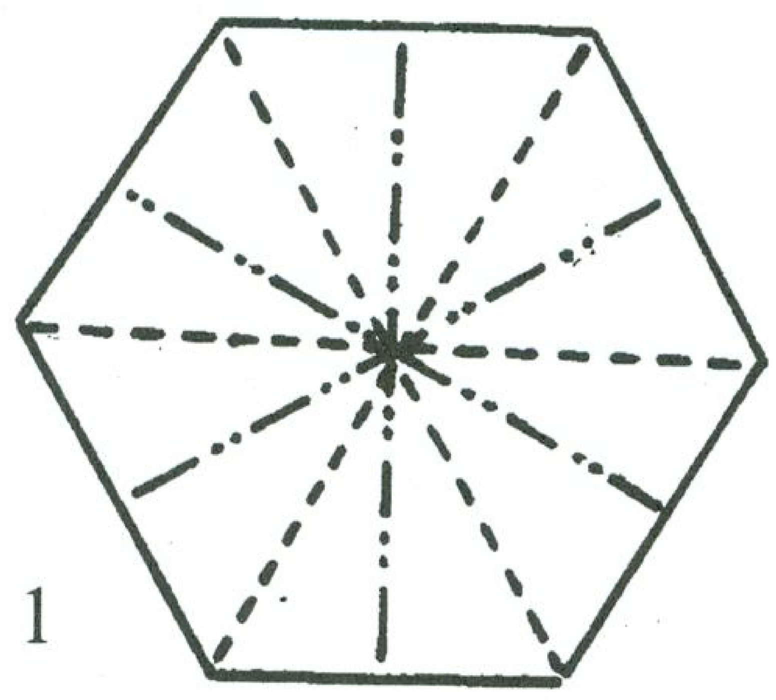
by Bennett Arnstein



3. Simplified Pentagon Module

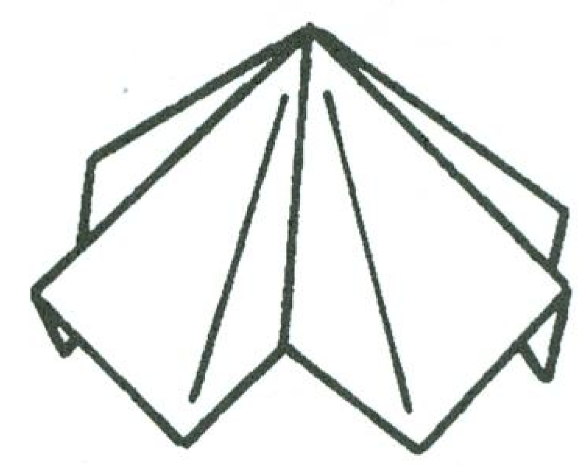


4. Simplified Hexagon Module



1

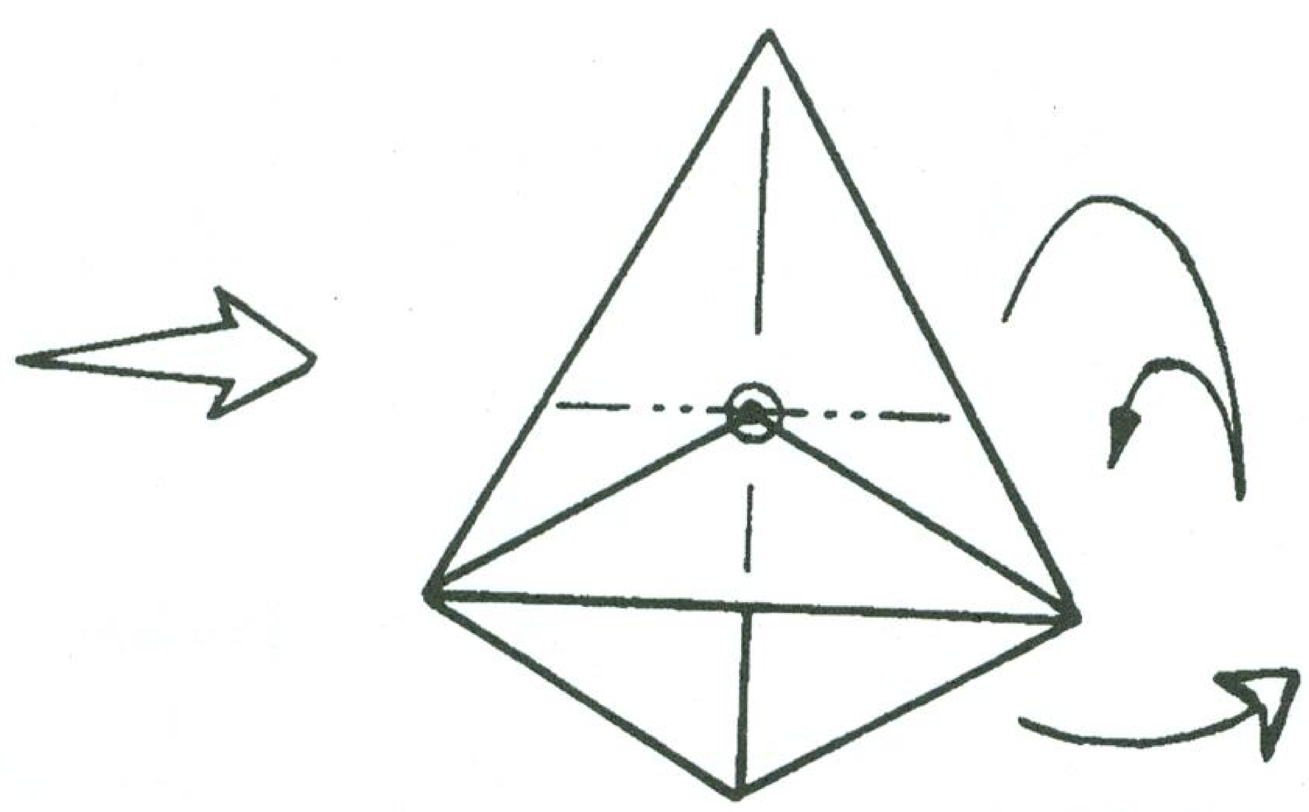
Start with a hexagon



2

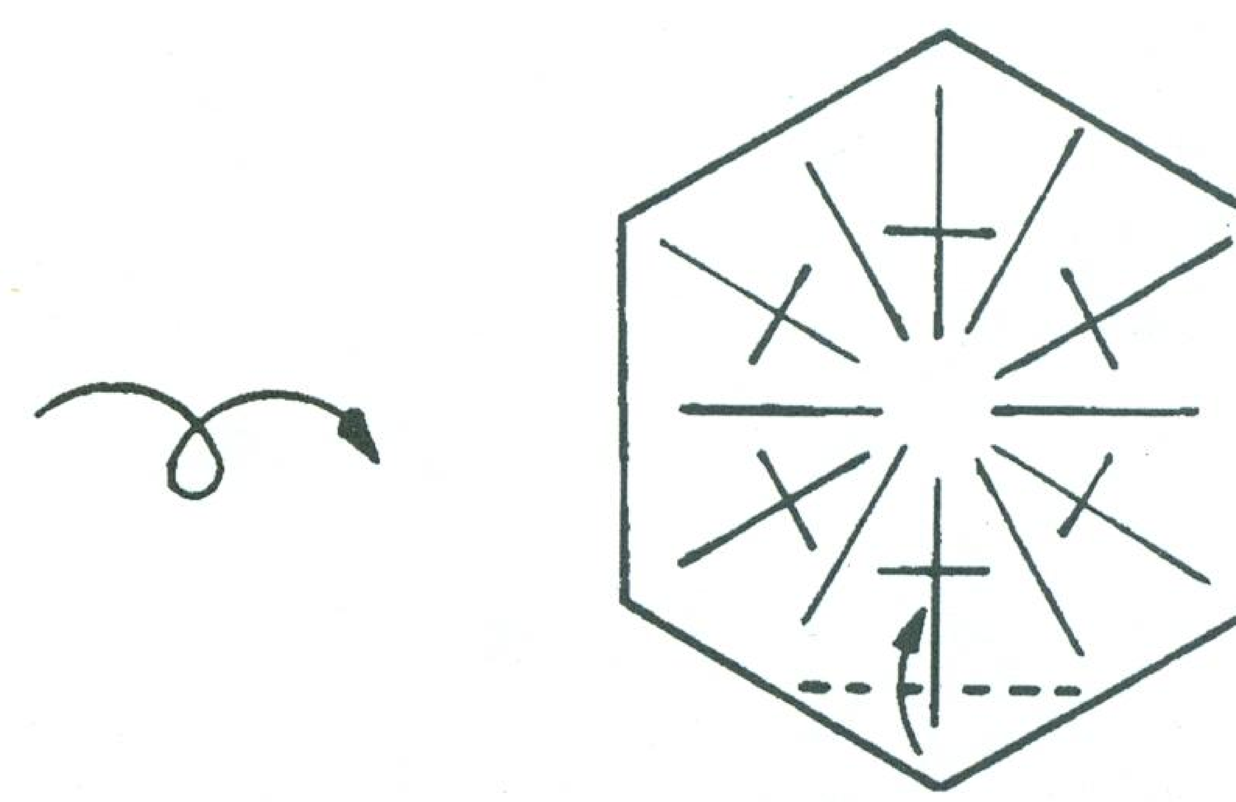
3

Fold top layer only



4

Unfold to figure 1



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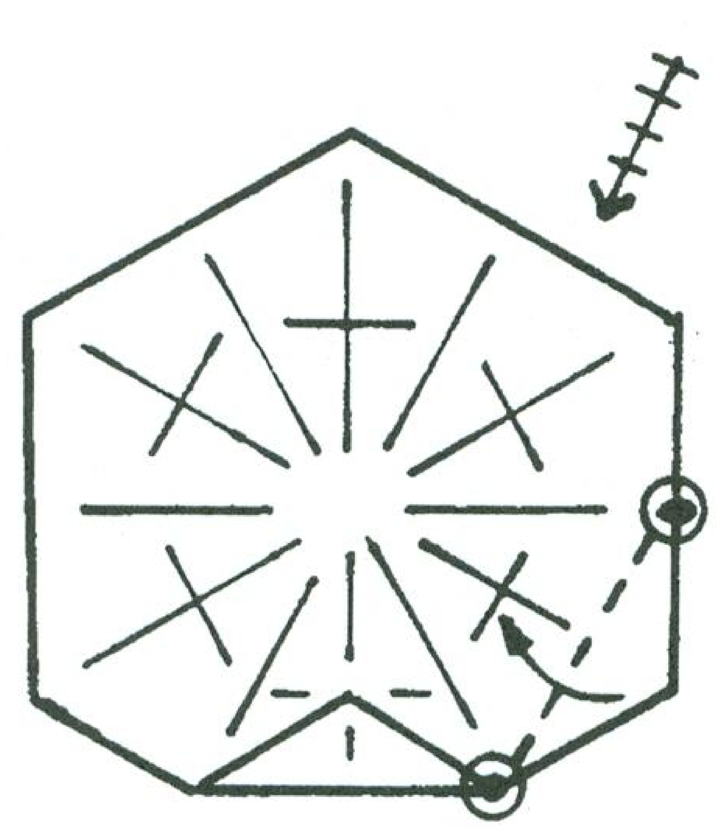

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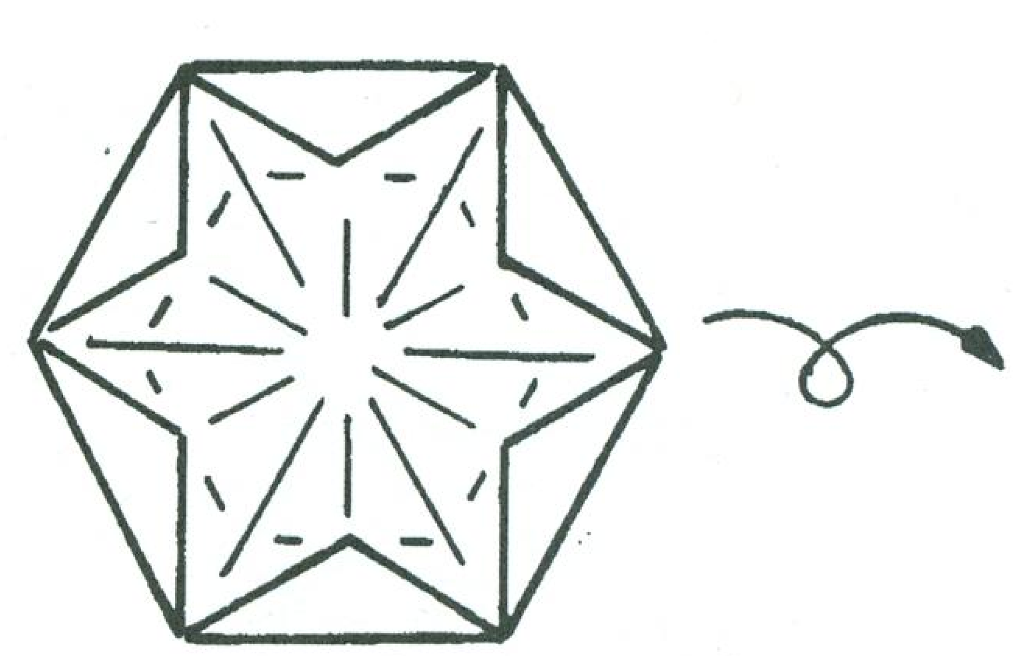

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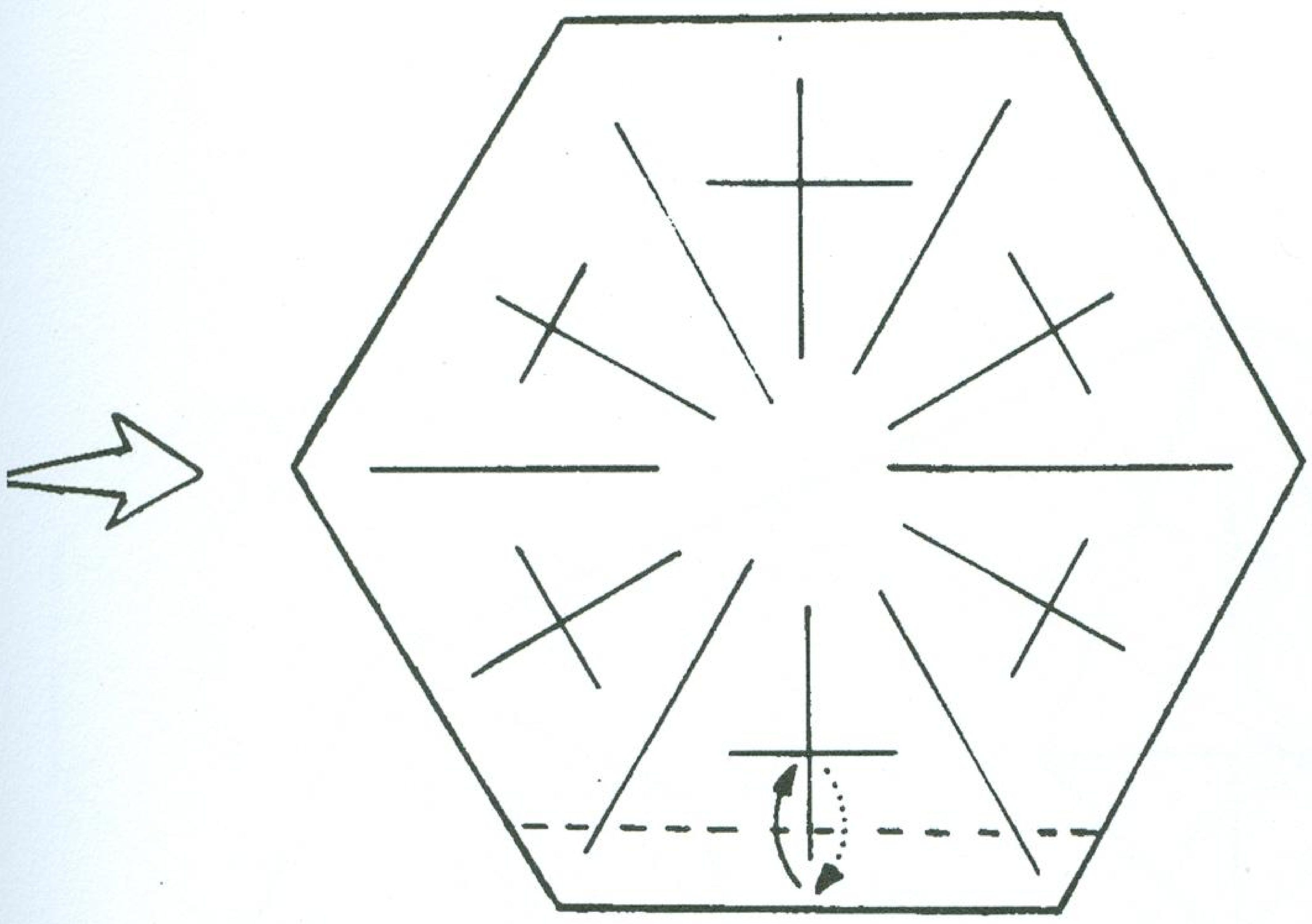

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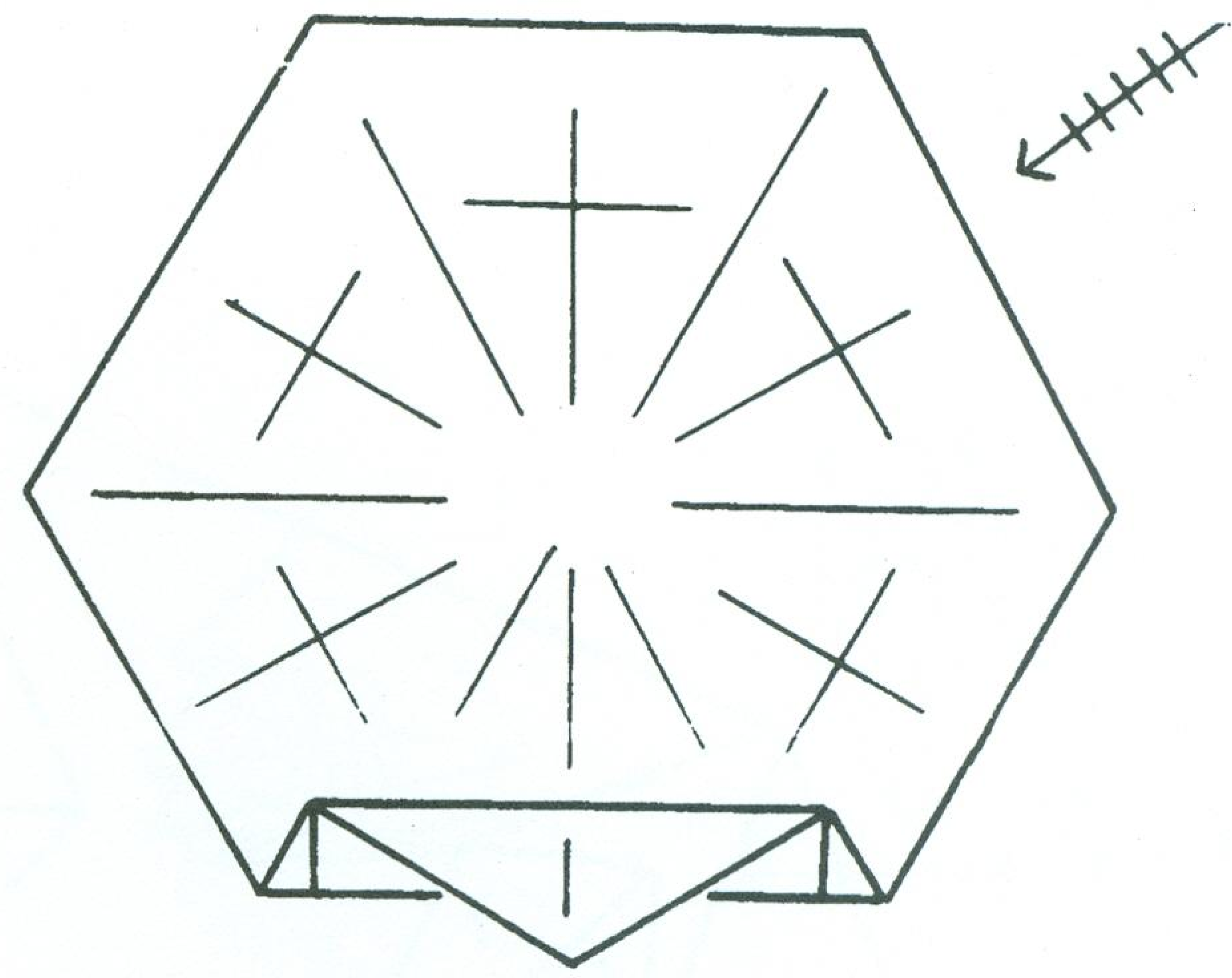

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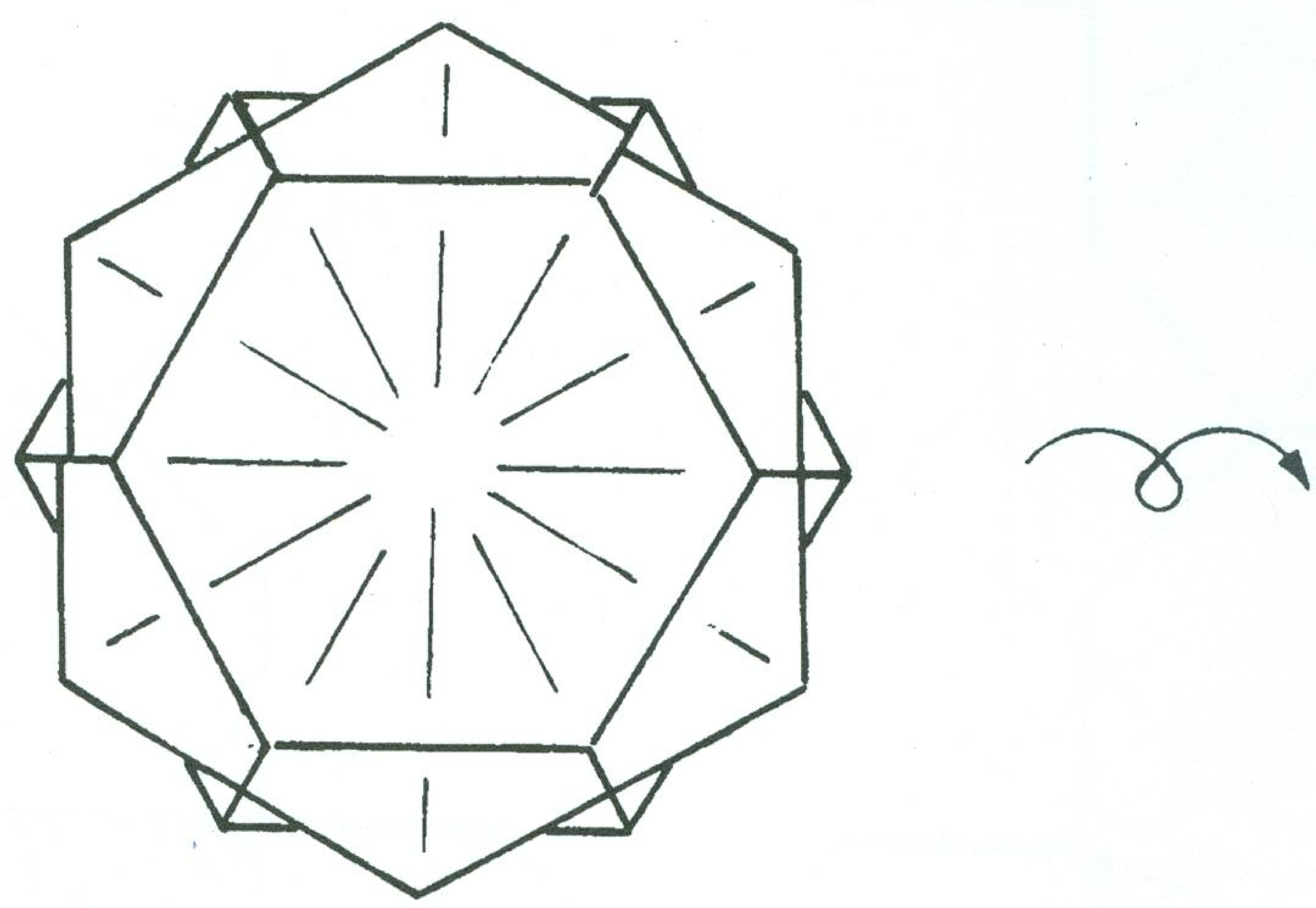

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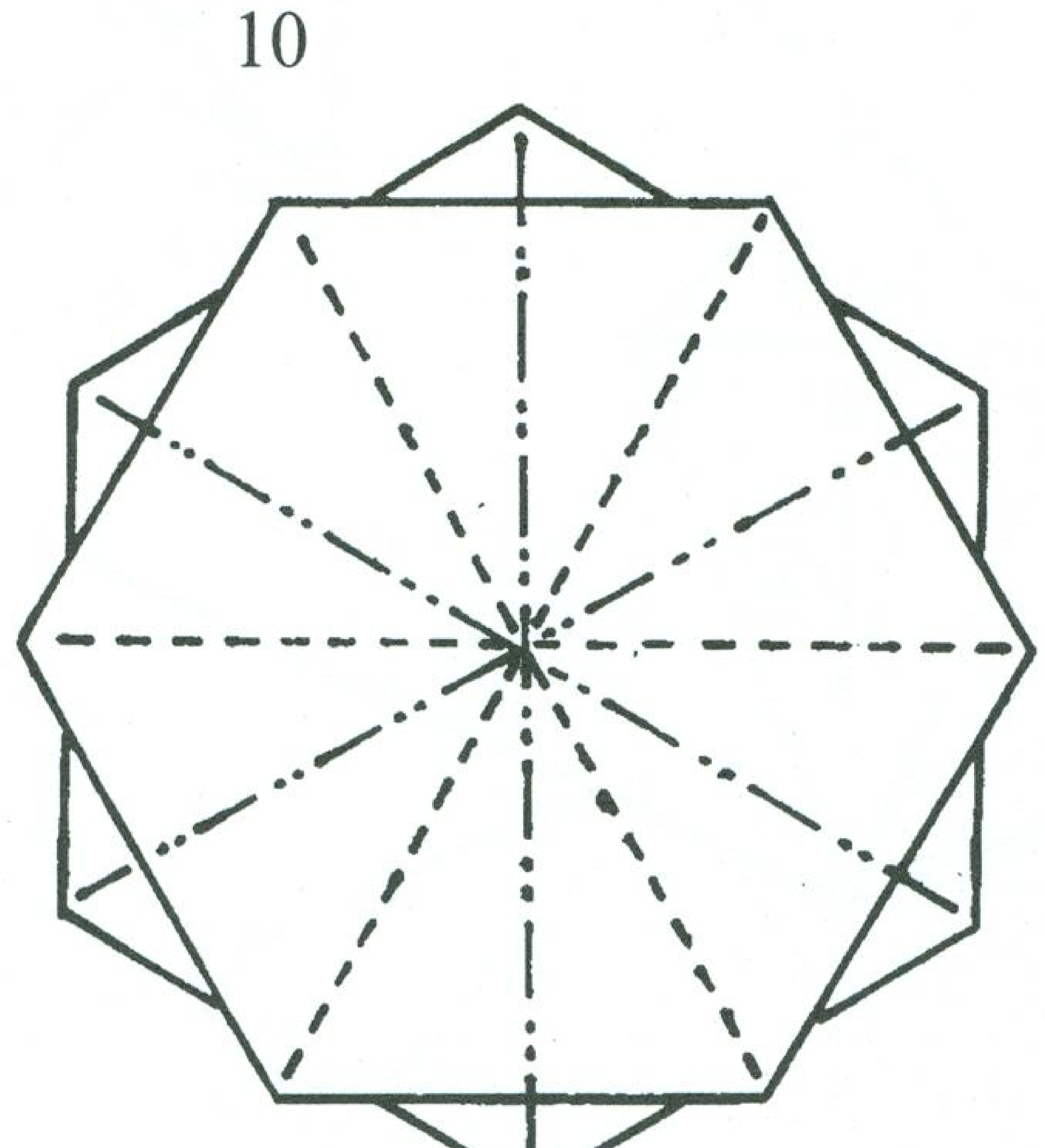

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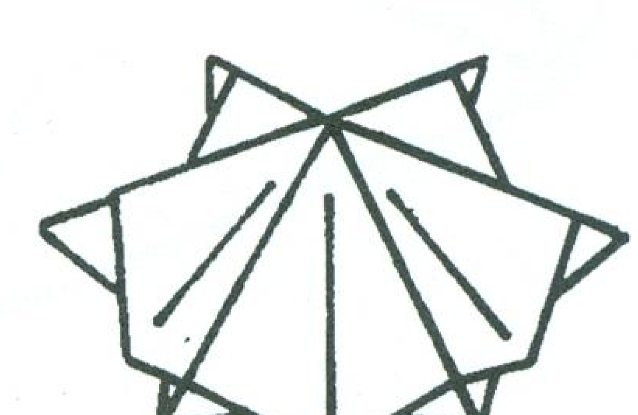
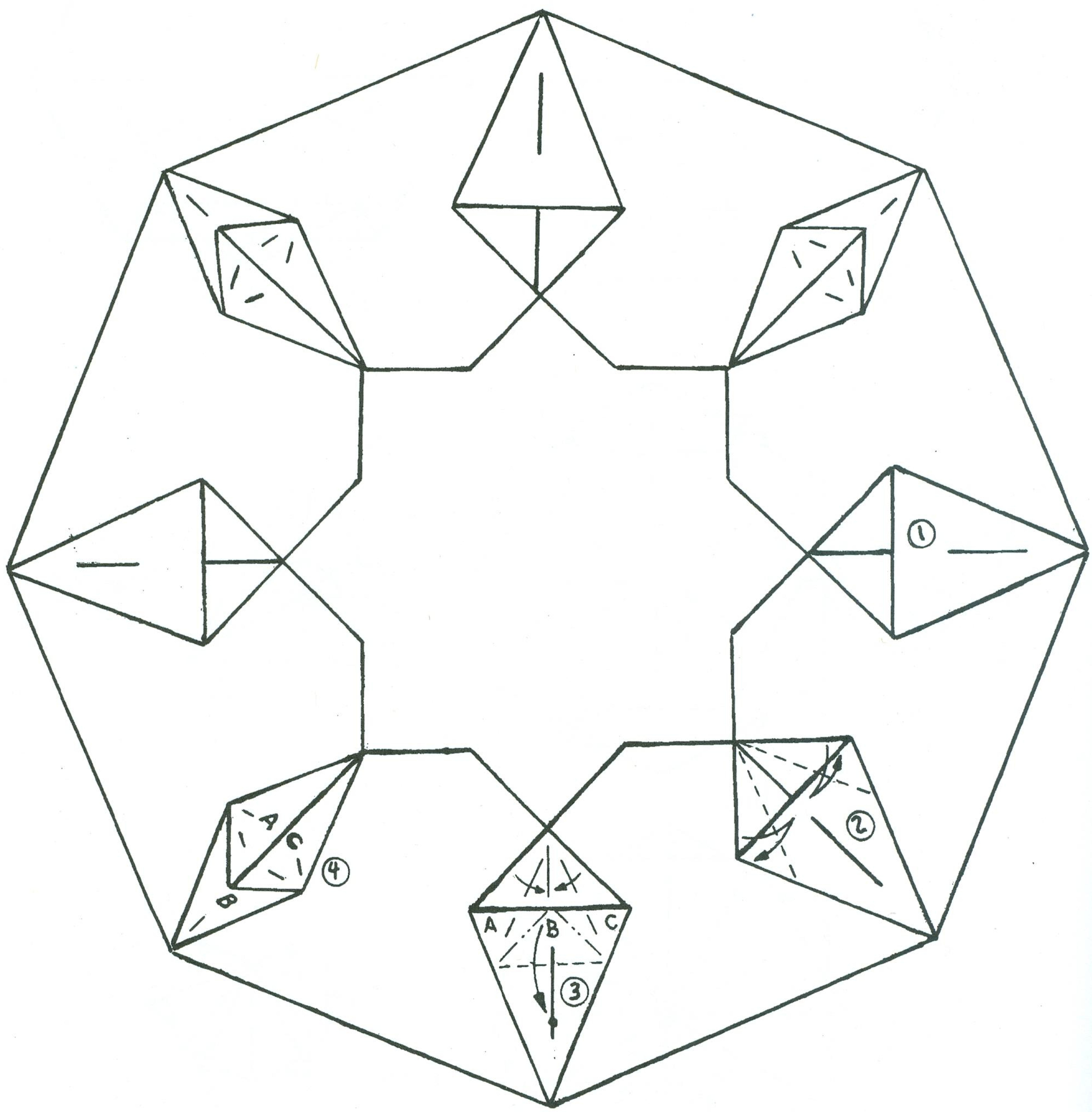


12

5. How to Petal-Fold a Squashed Vane

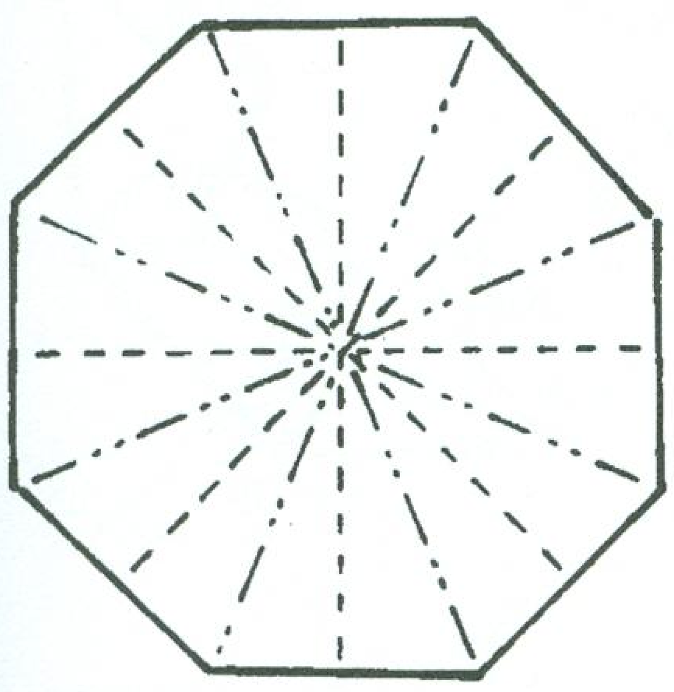
See octagon and decagon modules, pp.67 and 70.



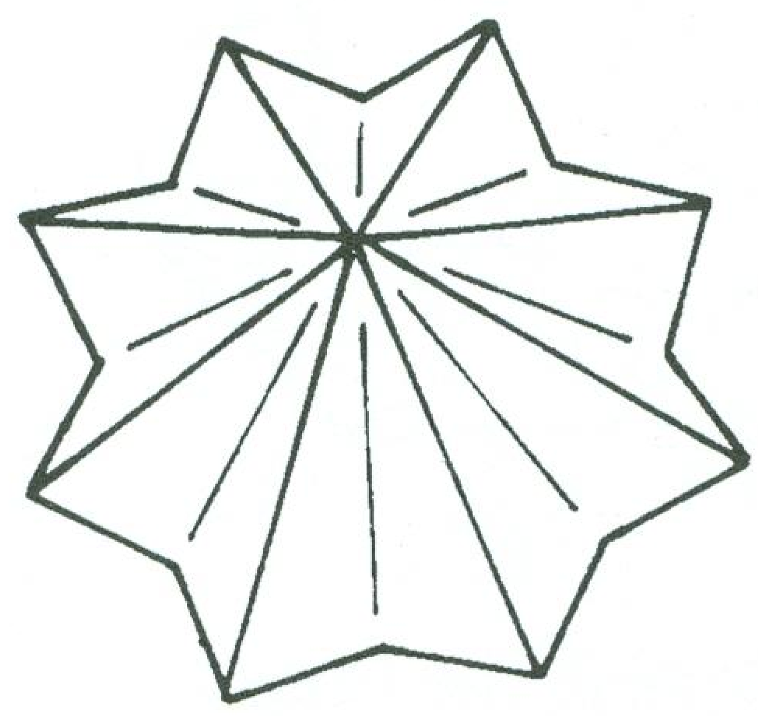
The mountain creases in figure 3 form naturally as point B swings down and points A and C come together.

6. Octagon Module

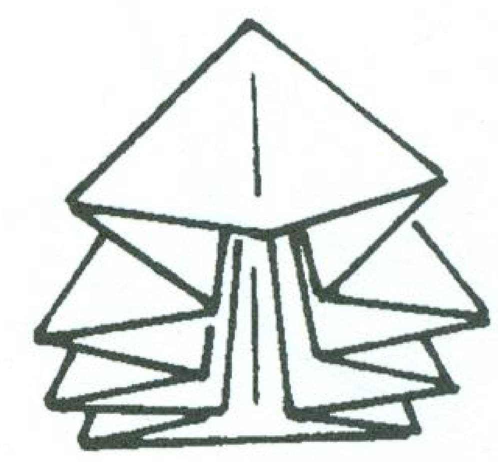
Octagon analog of one-piece square module and pentagon module.



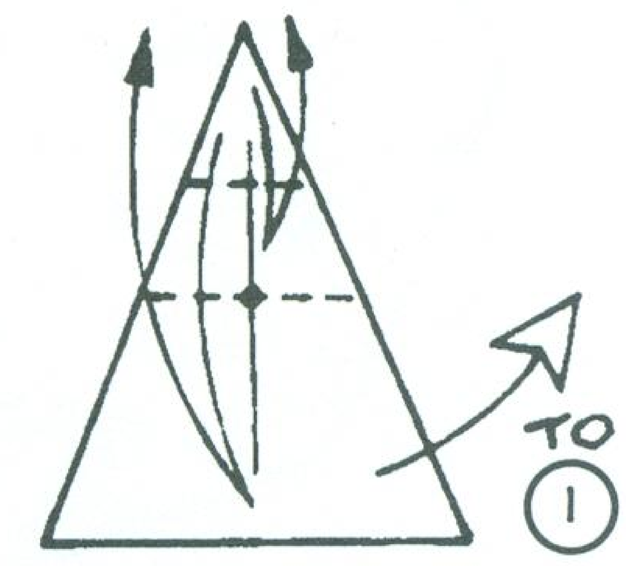
1



2



3



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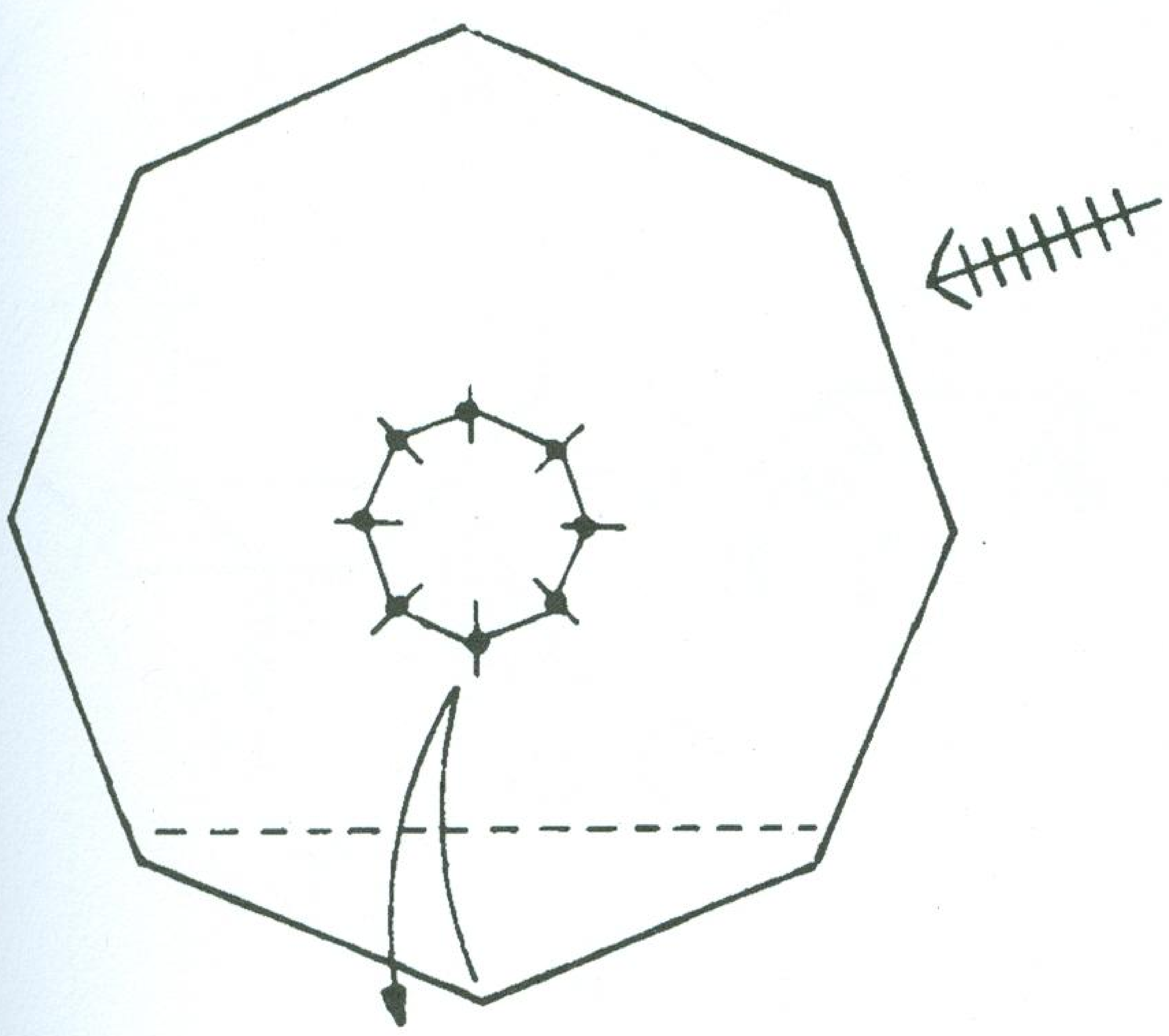

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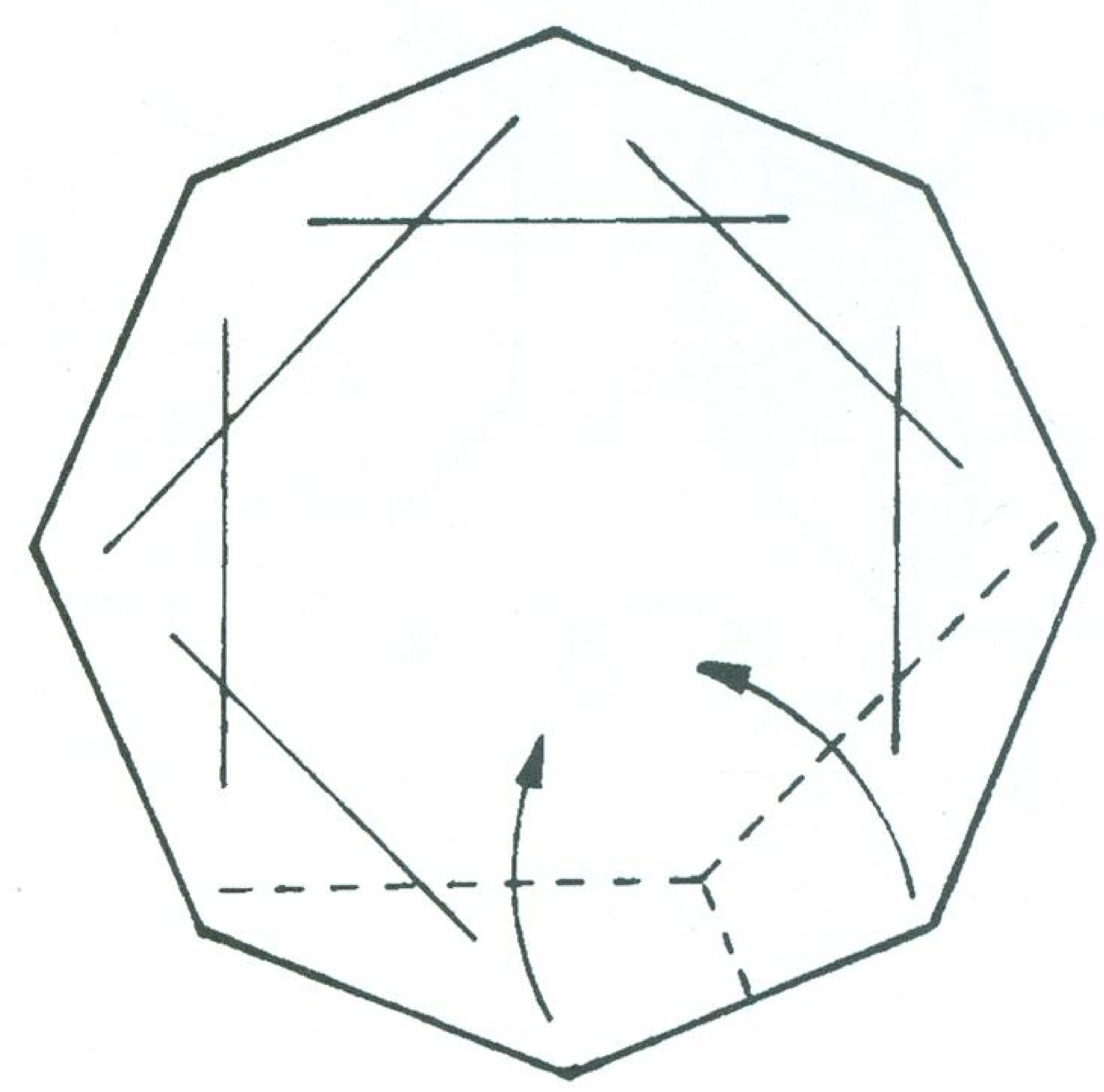

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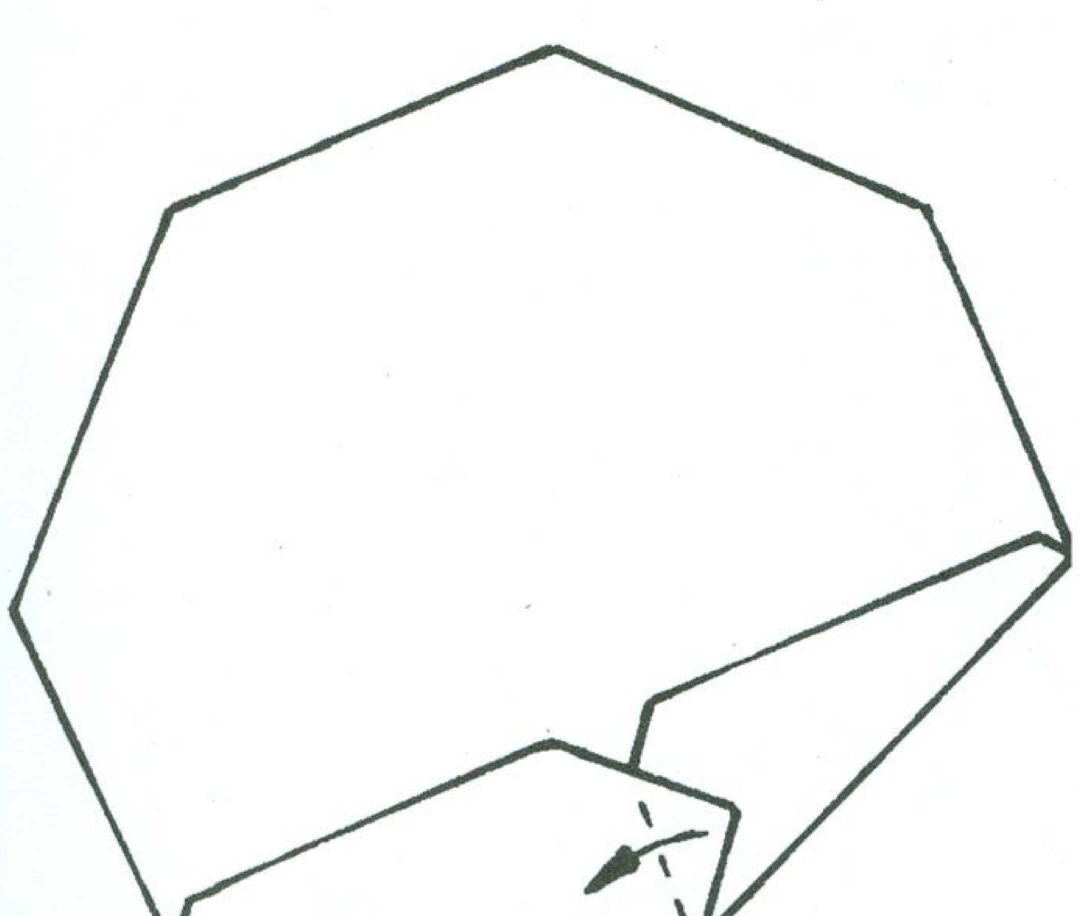

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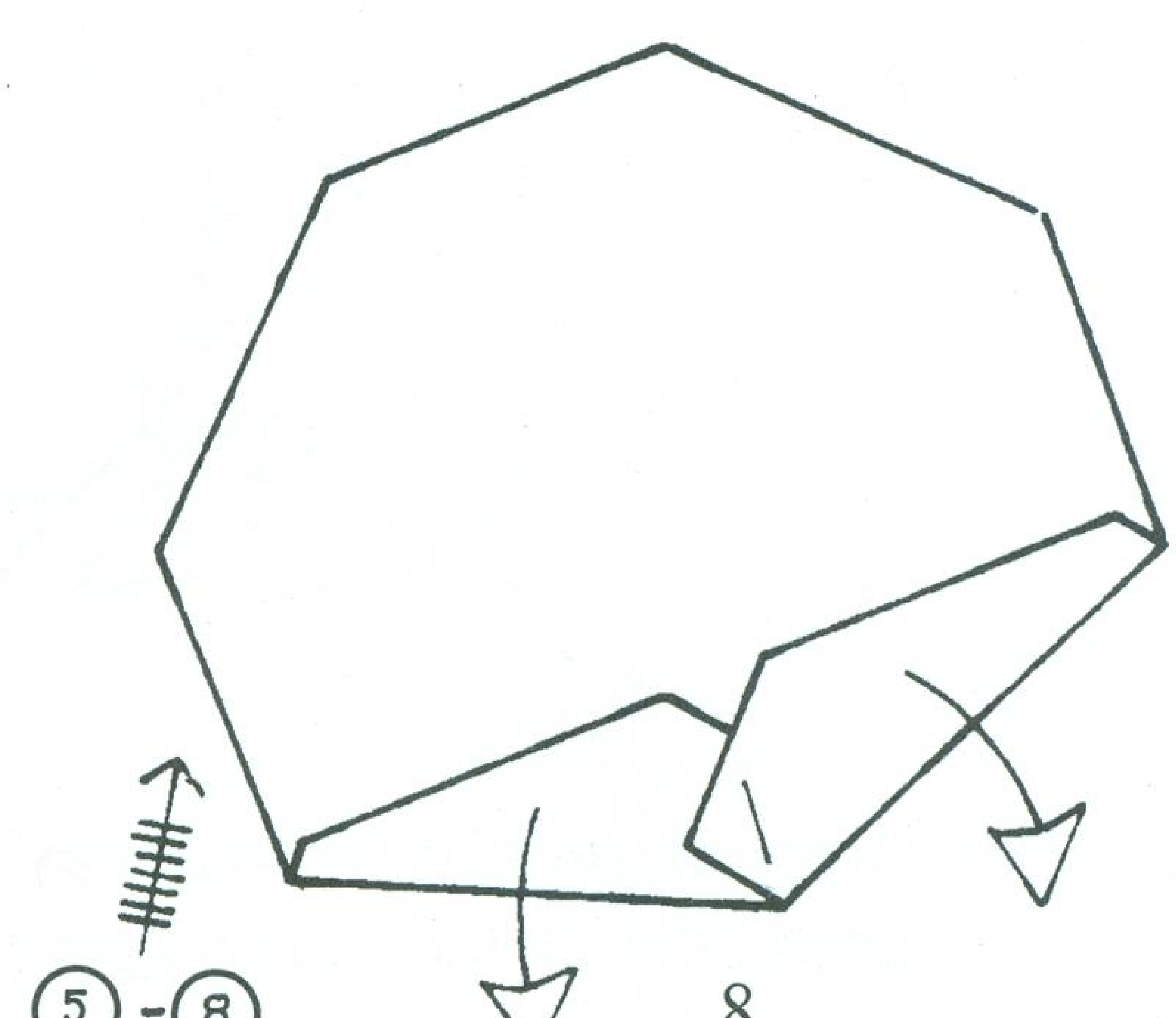

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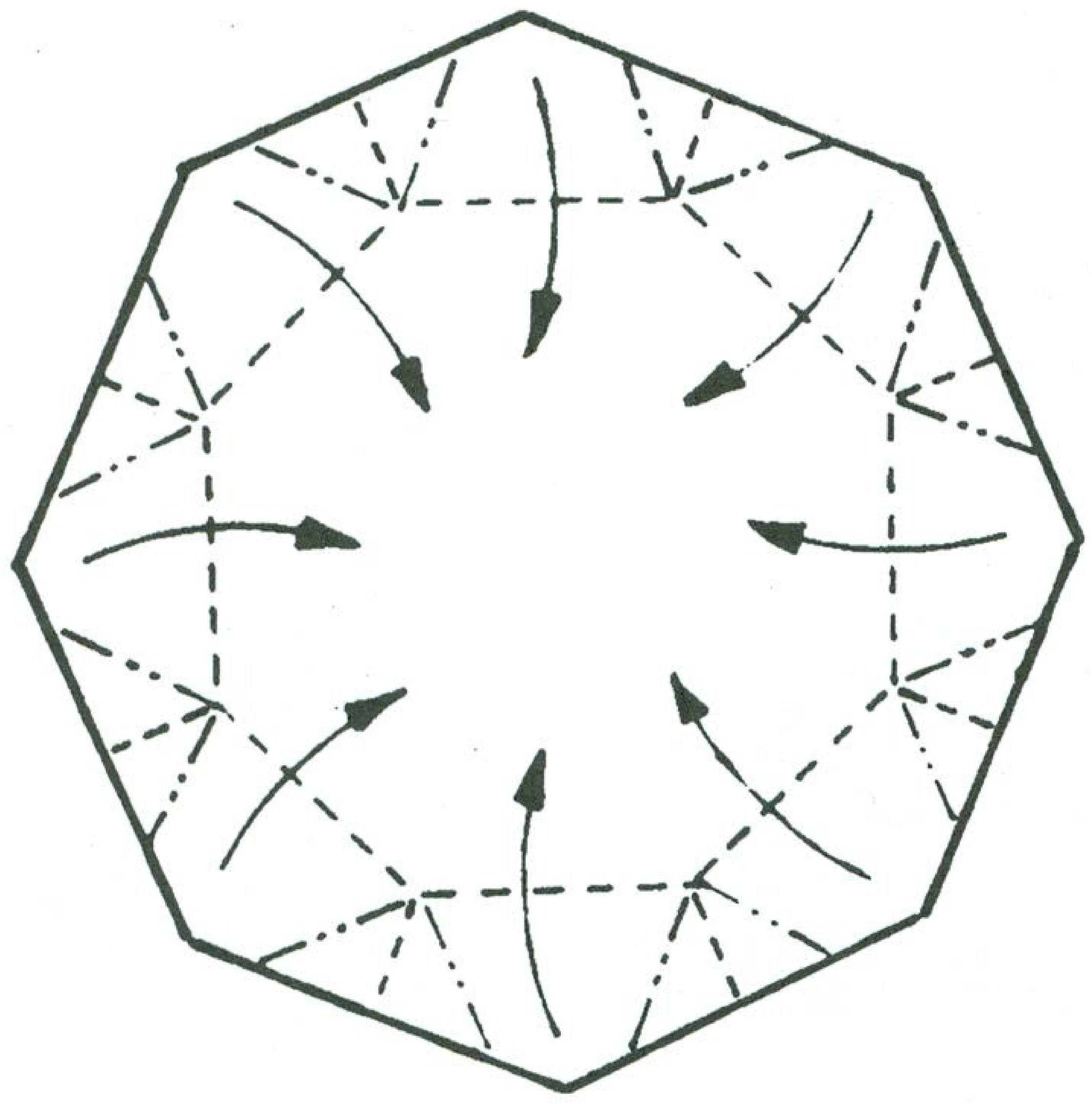

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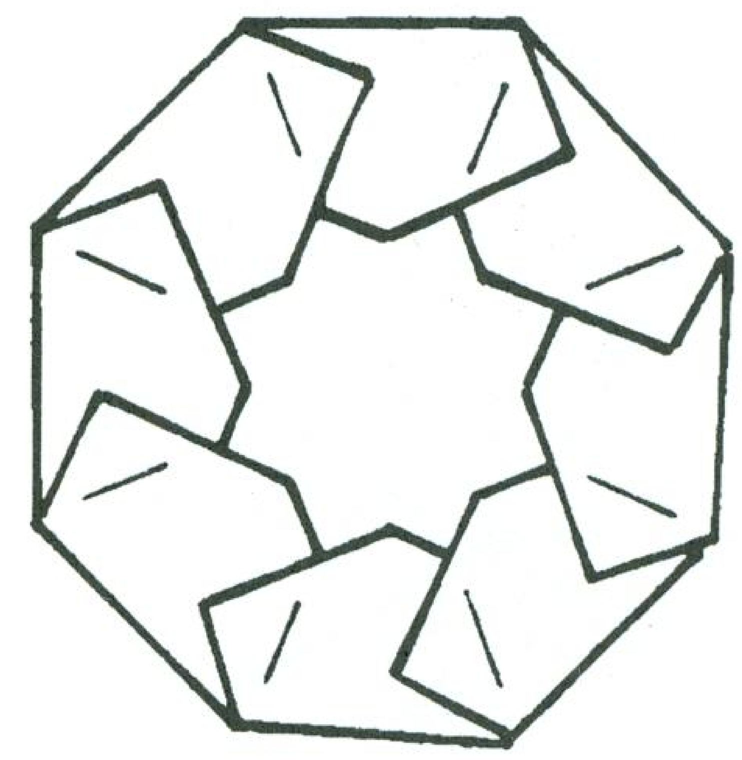
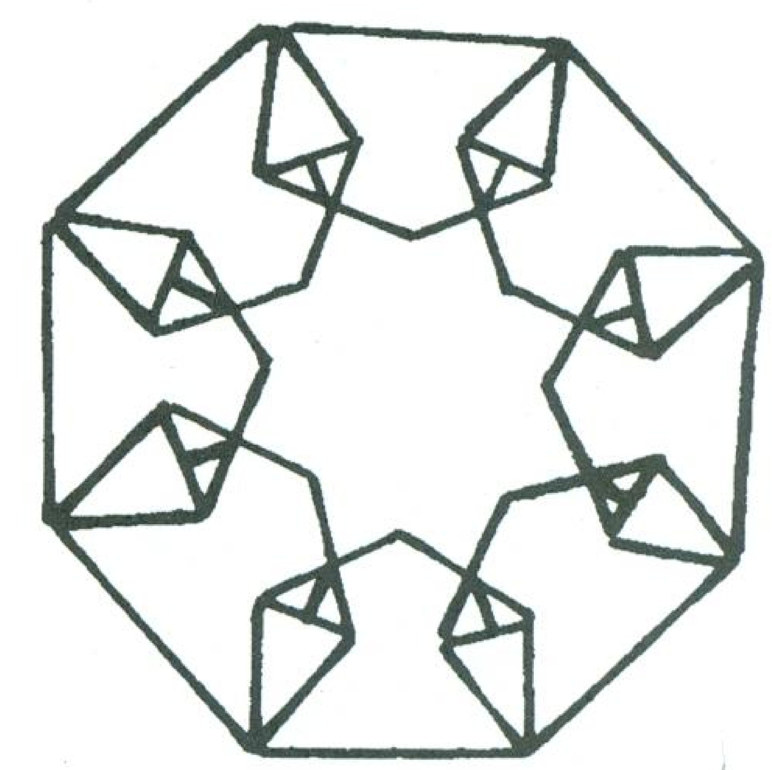


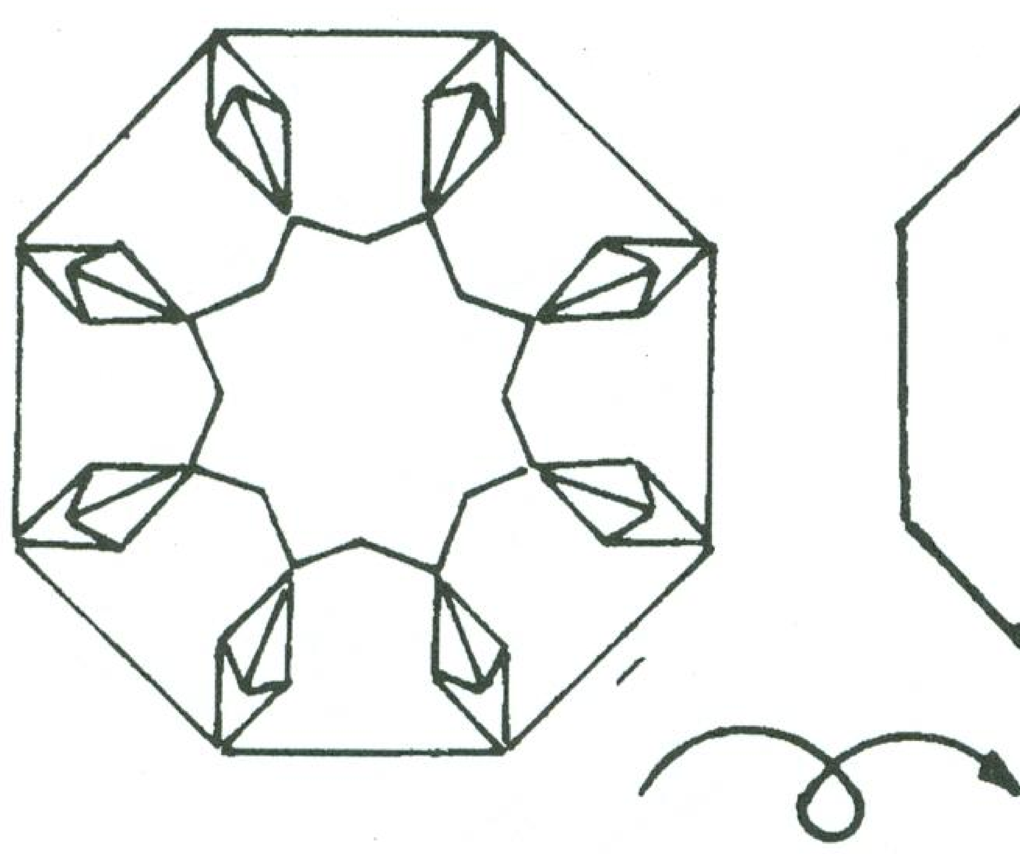
10

Squash the vanes

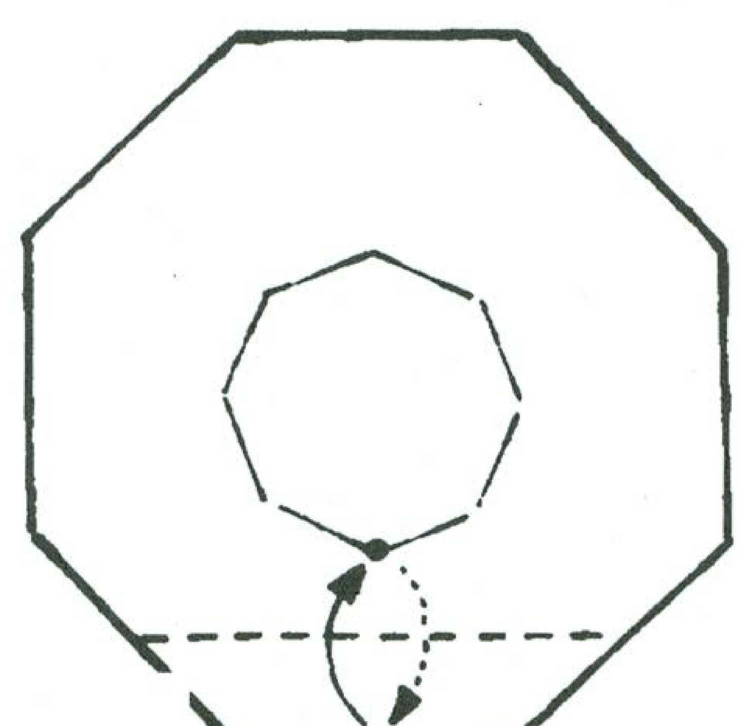


11

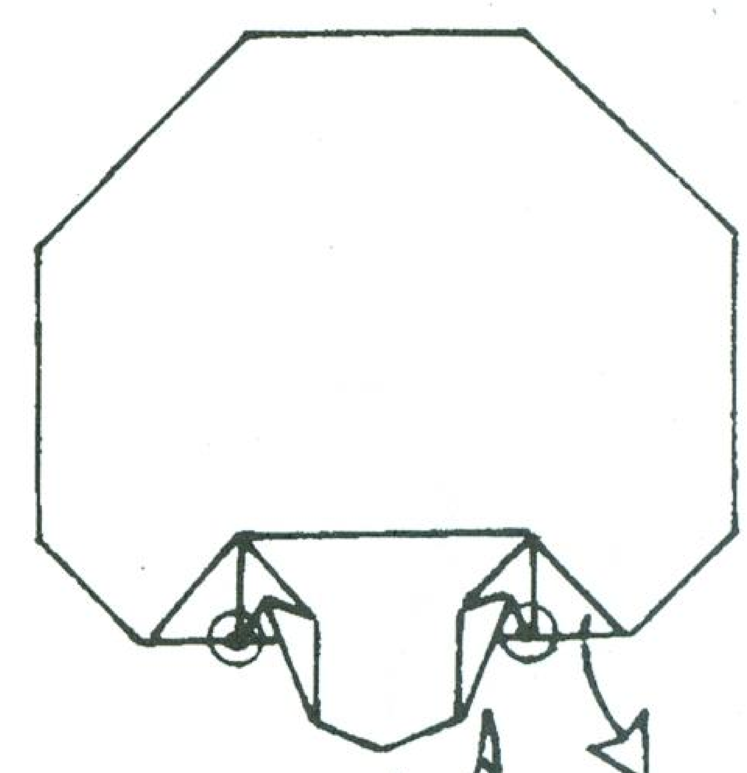
Petal-fold the vanes



12



13



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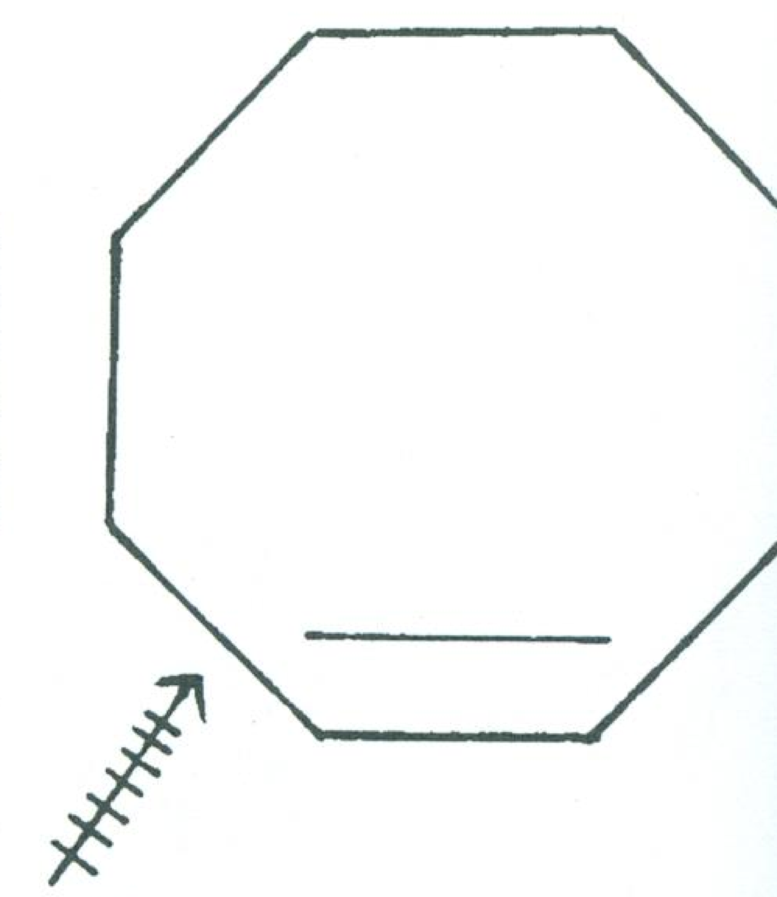
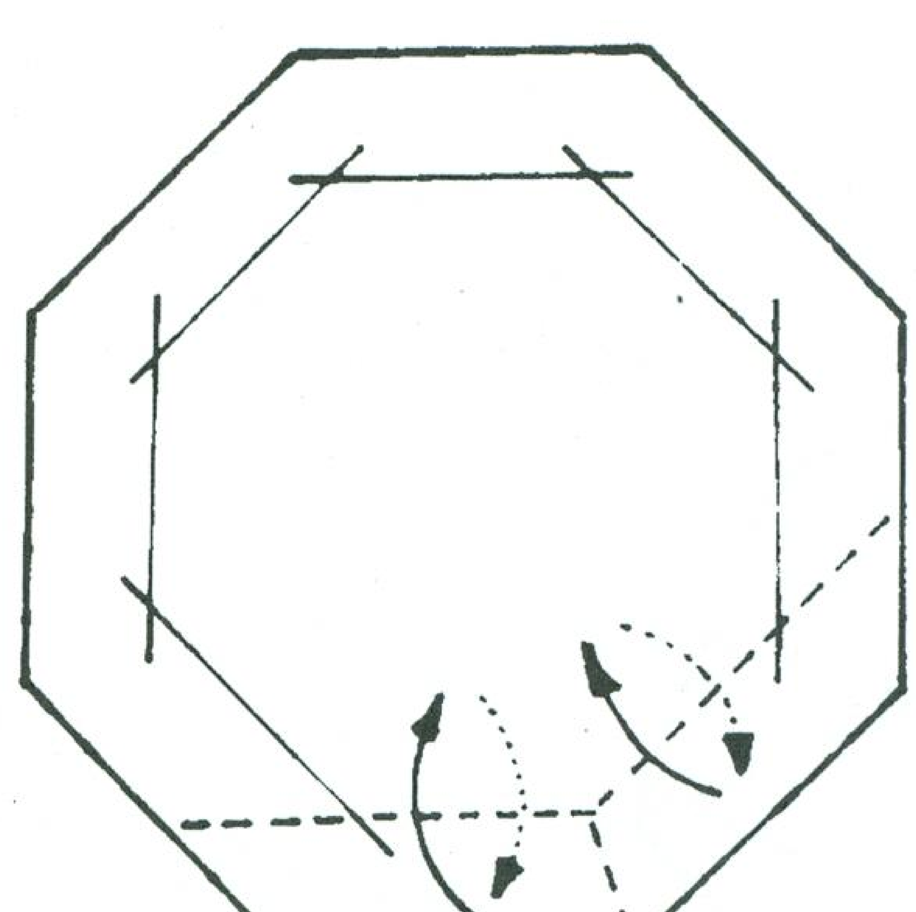
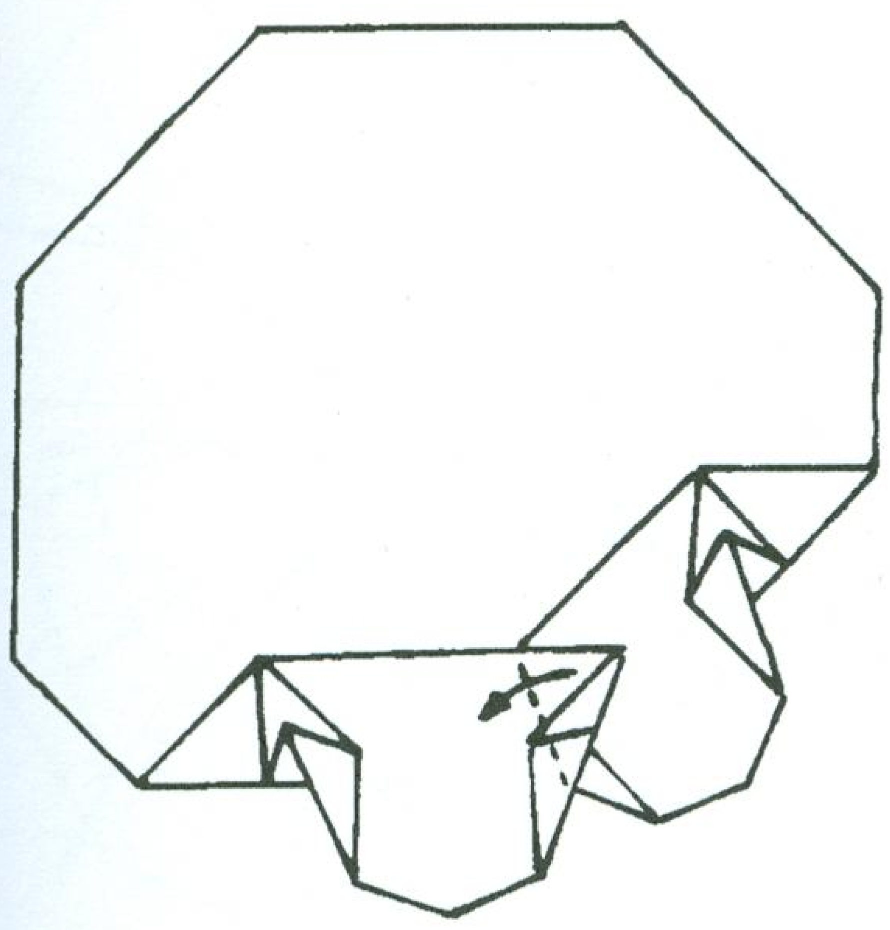


(13) - (14)

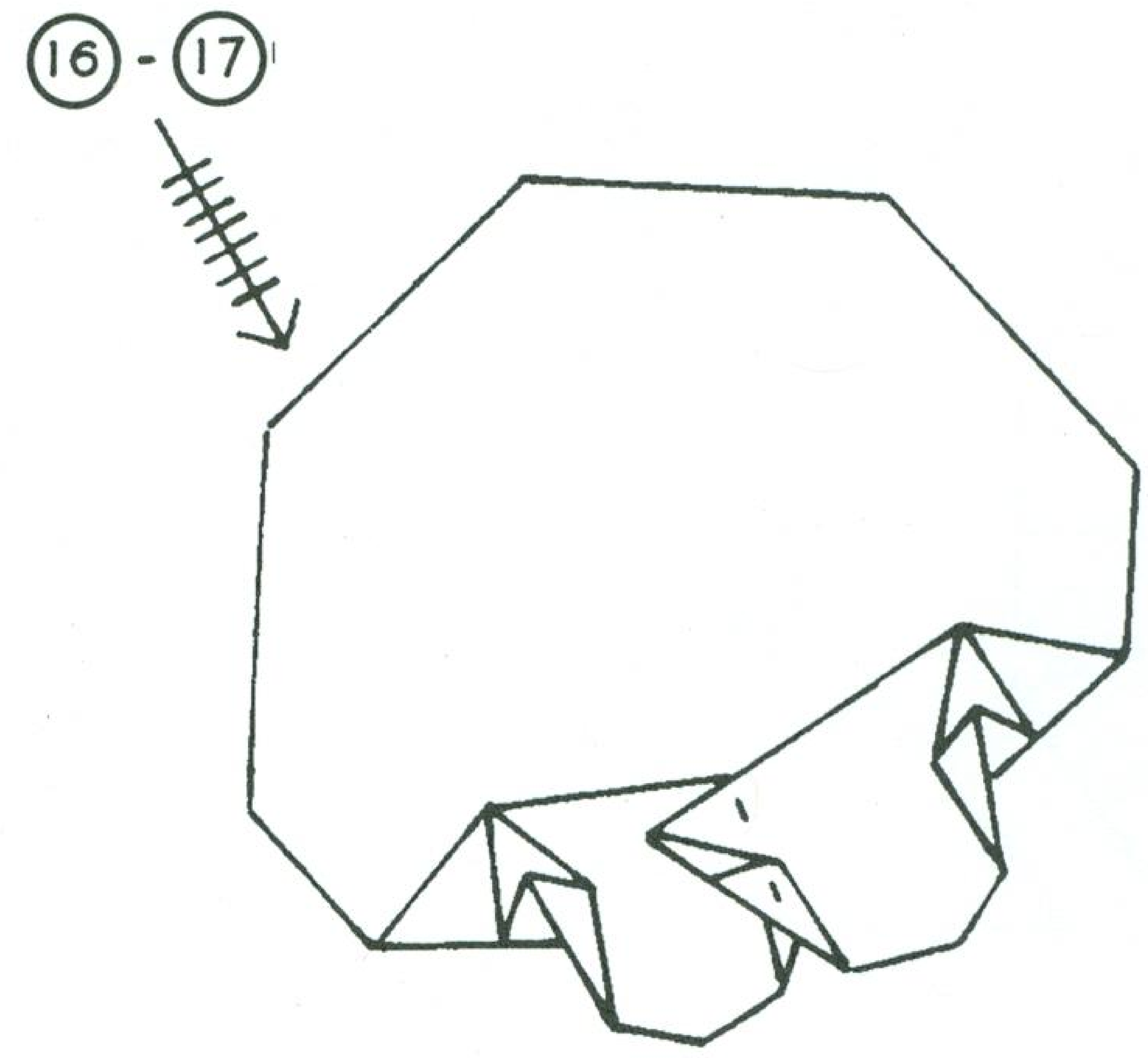
15



16



17



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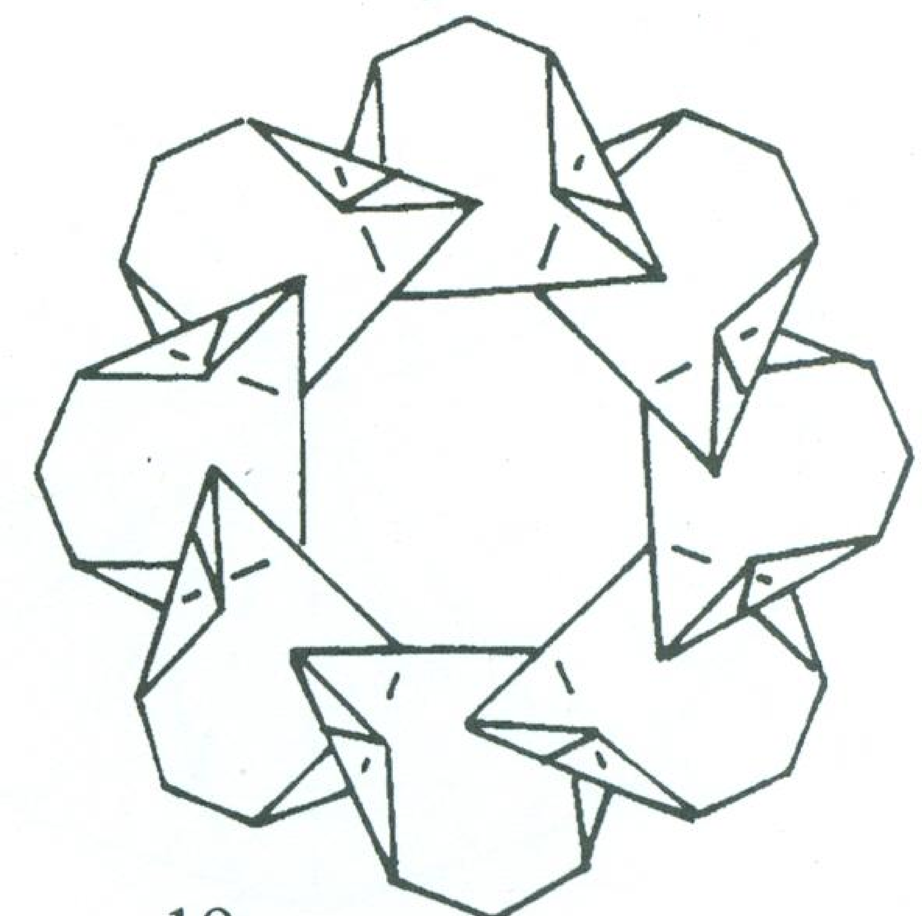
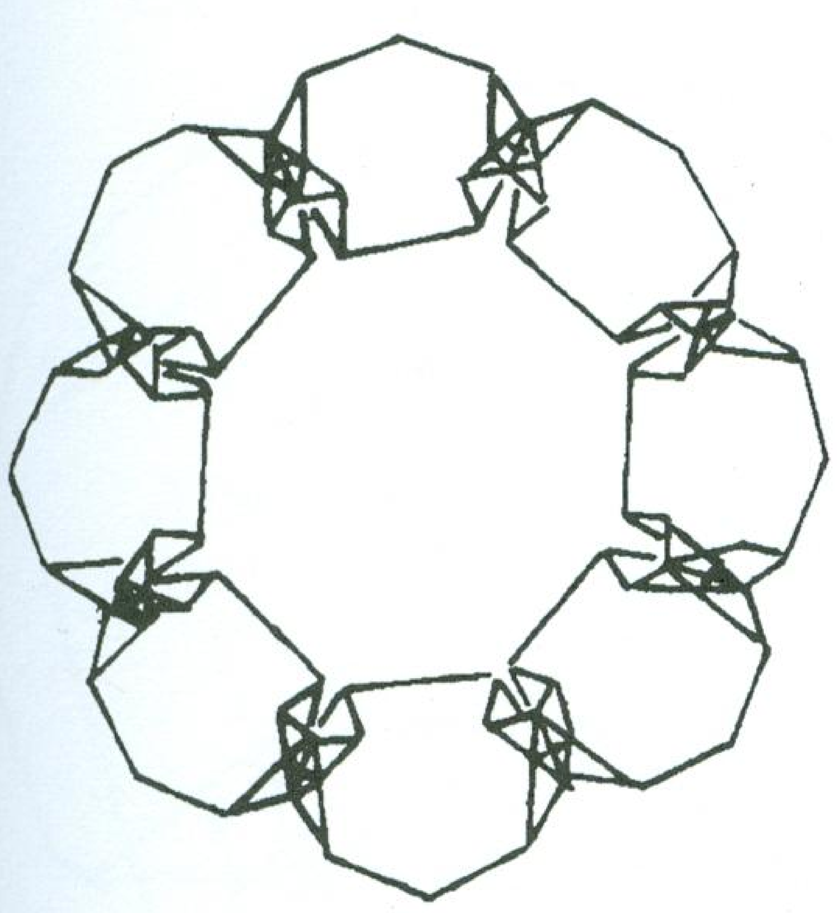


19

Squash the vanes



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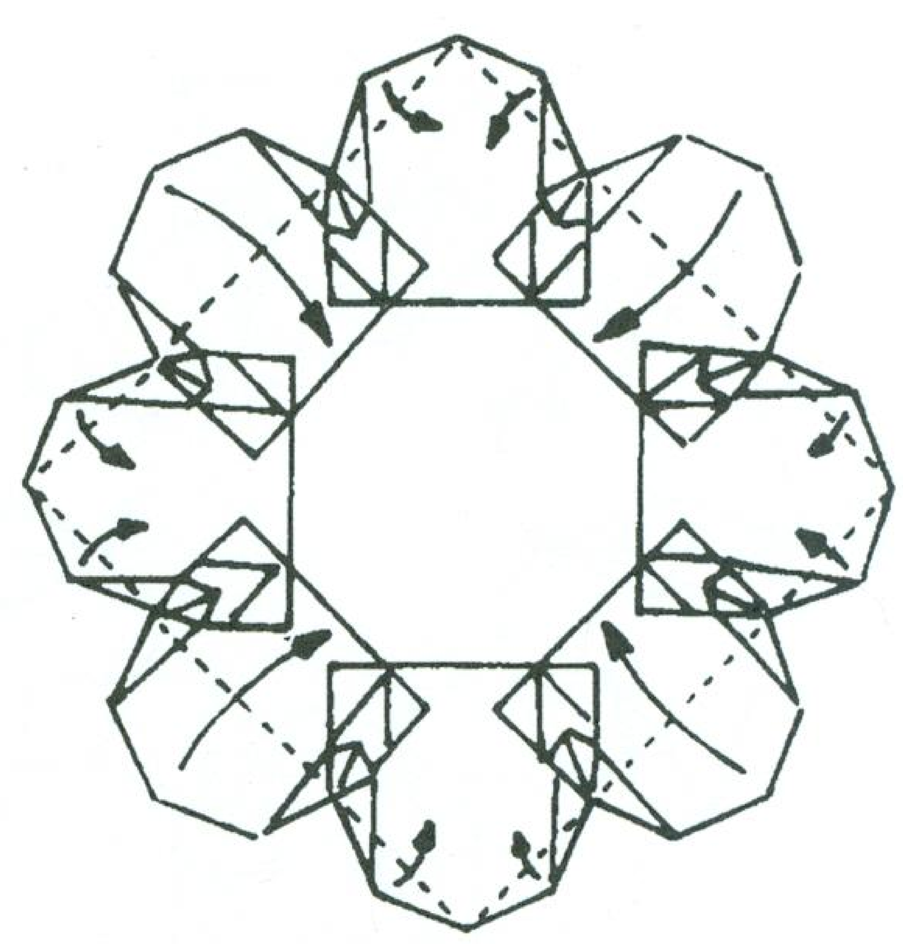

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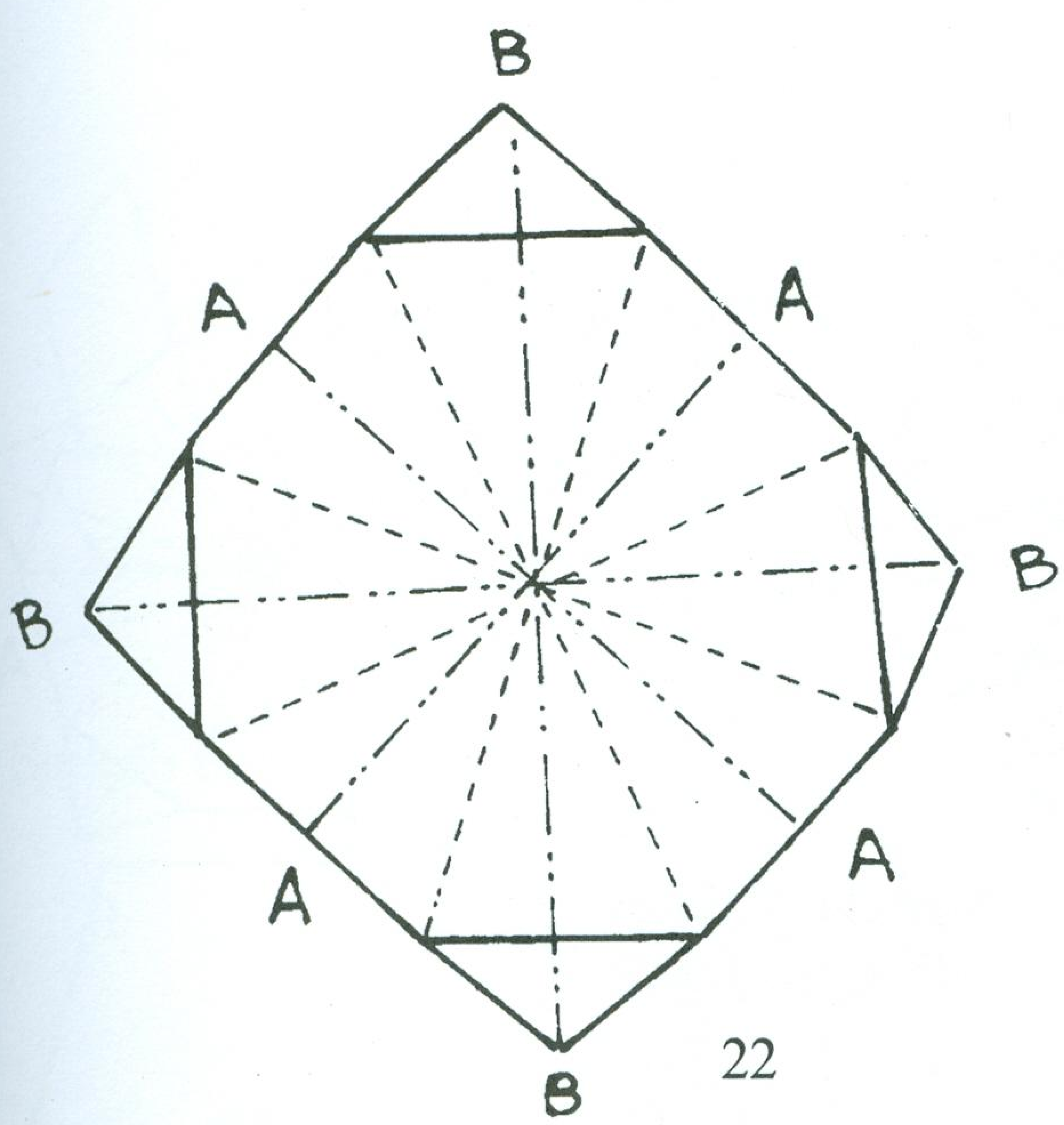

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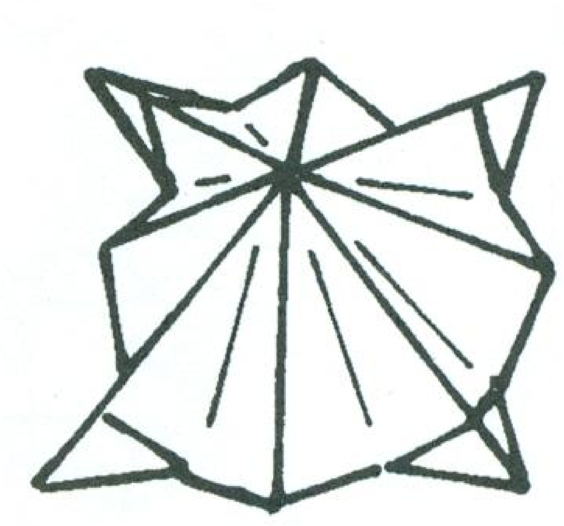
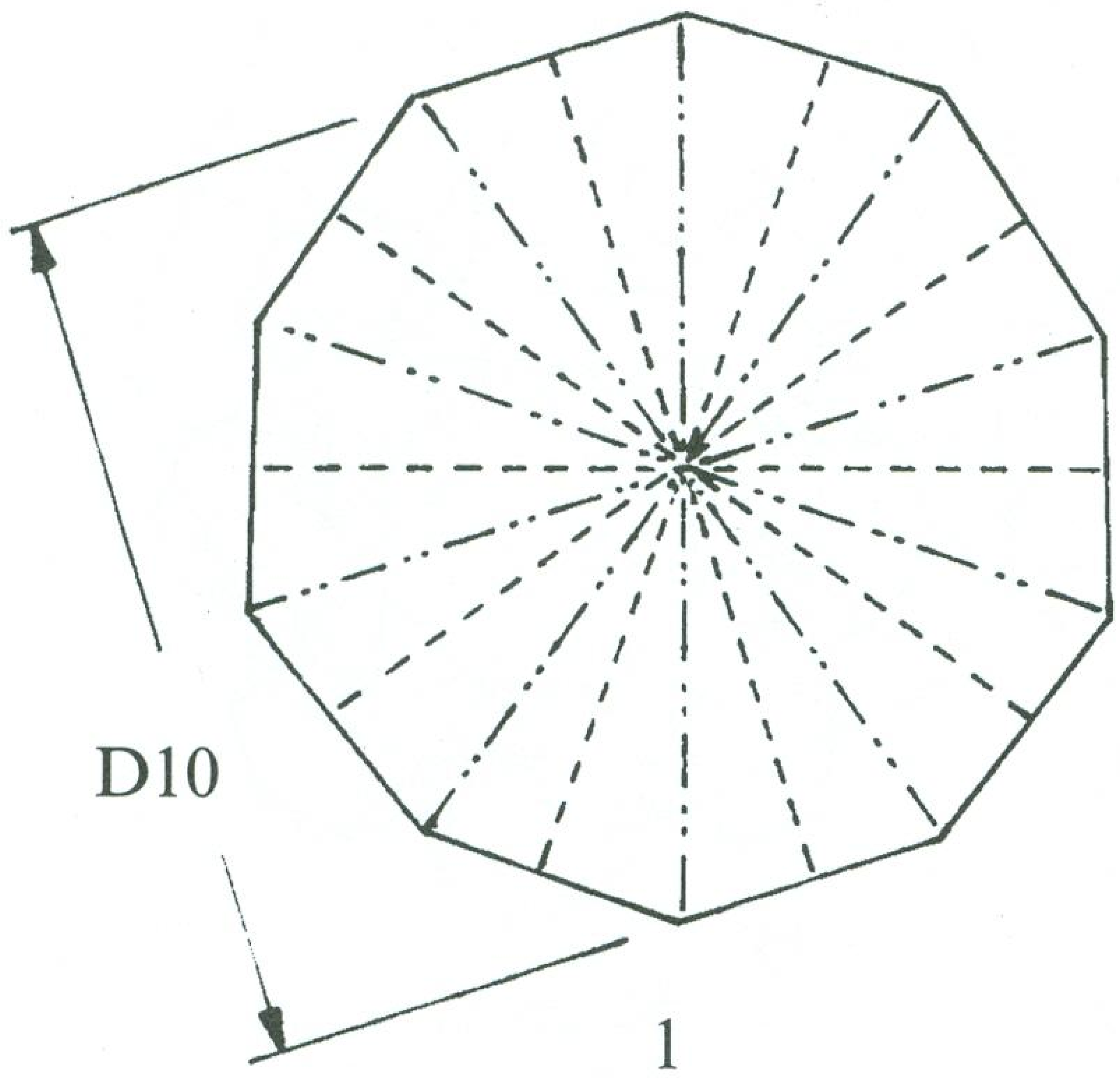


23

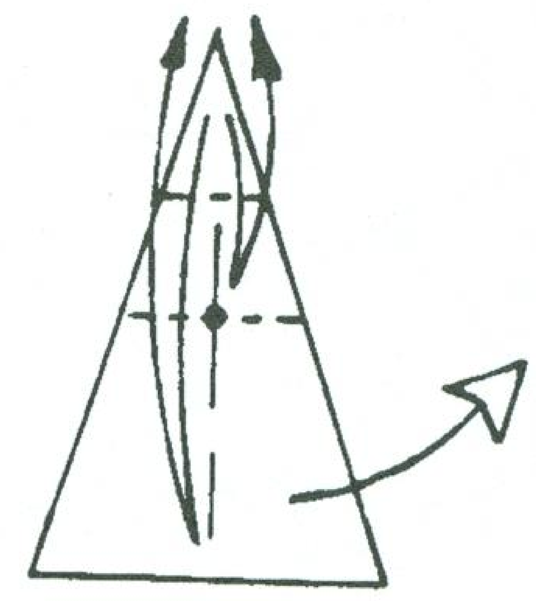
A pockets mate with triangles or squares.
B tabs/pockets mate with hexagons or octagons.

7. Decagon Module: Abbreviated Instructions

The procedure is exactly analogous to that of the octagon module.
Only the critical steps are shown.



1



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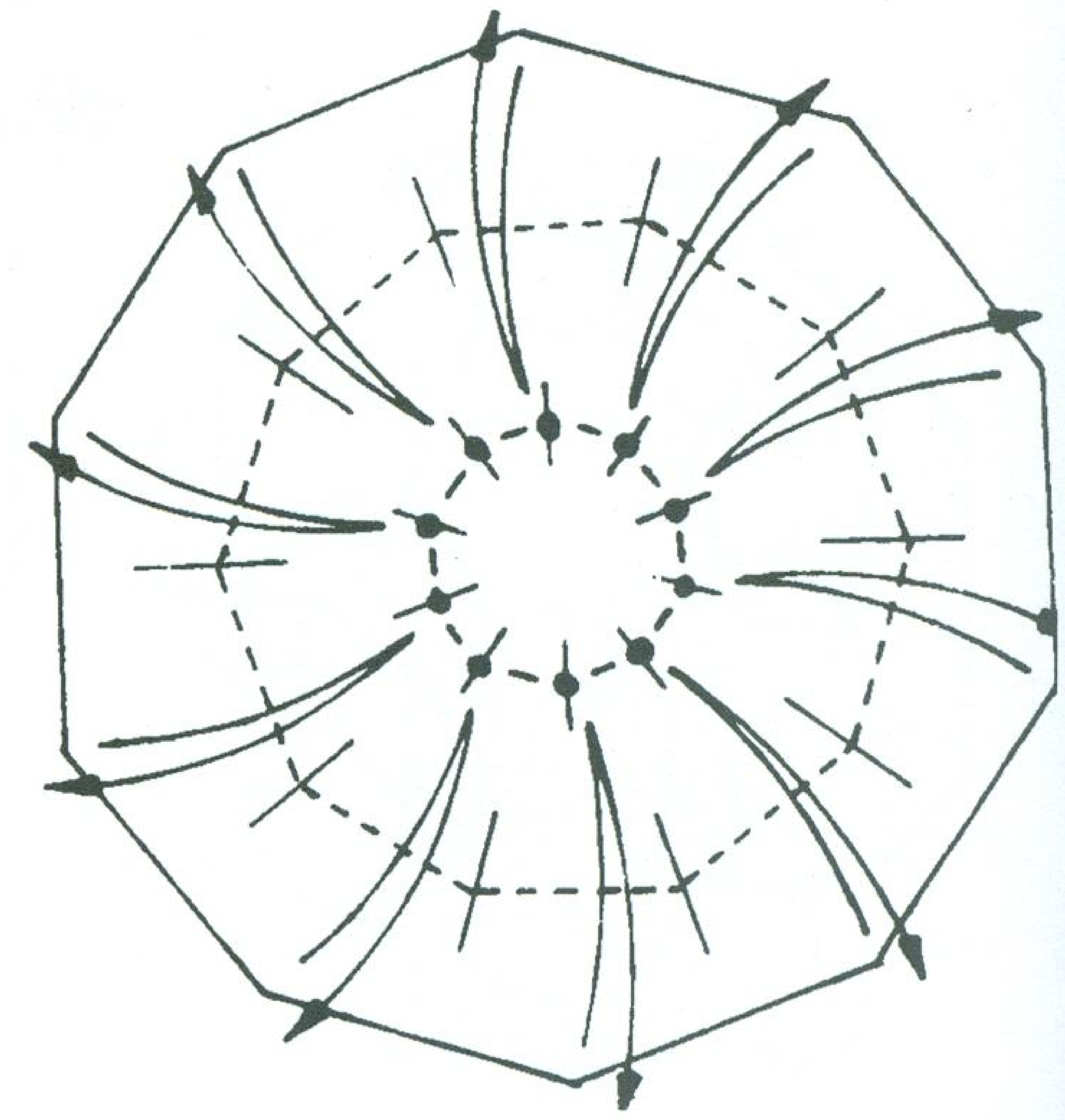

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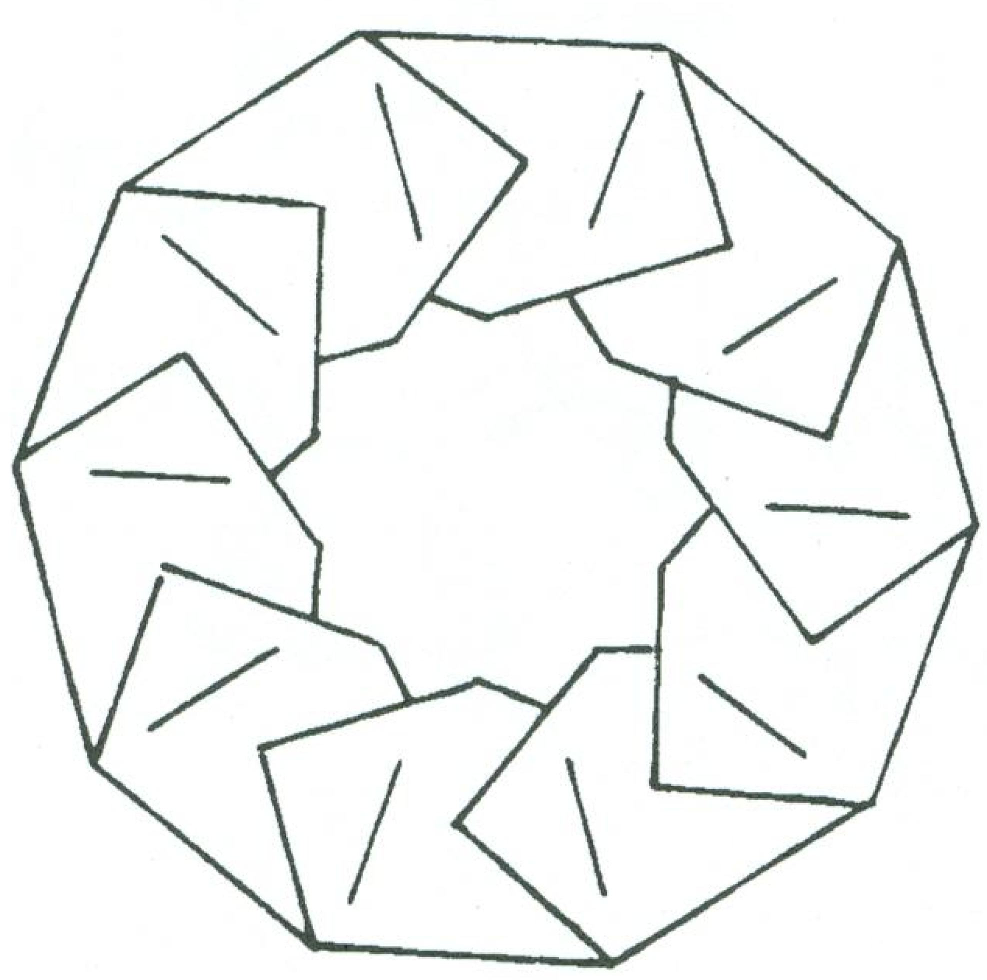

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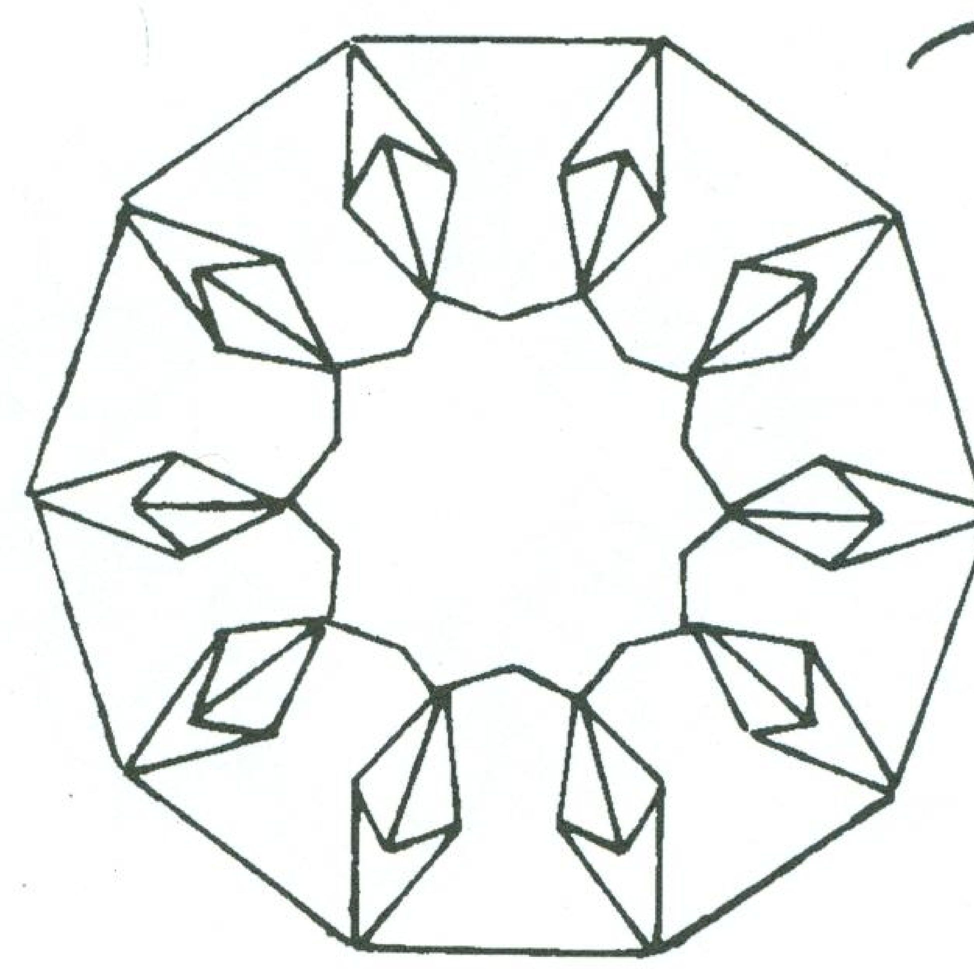

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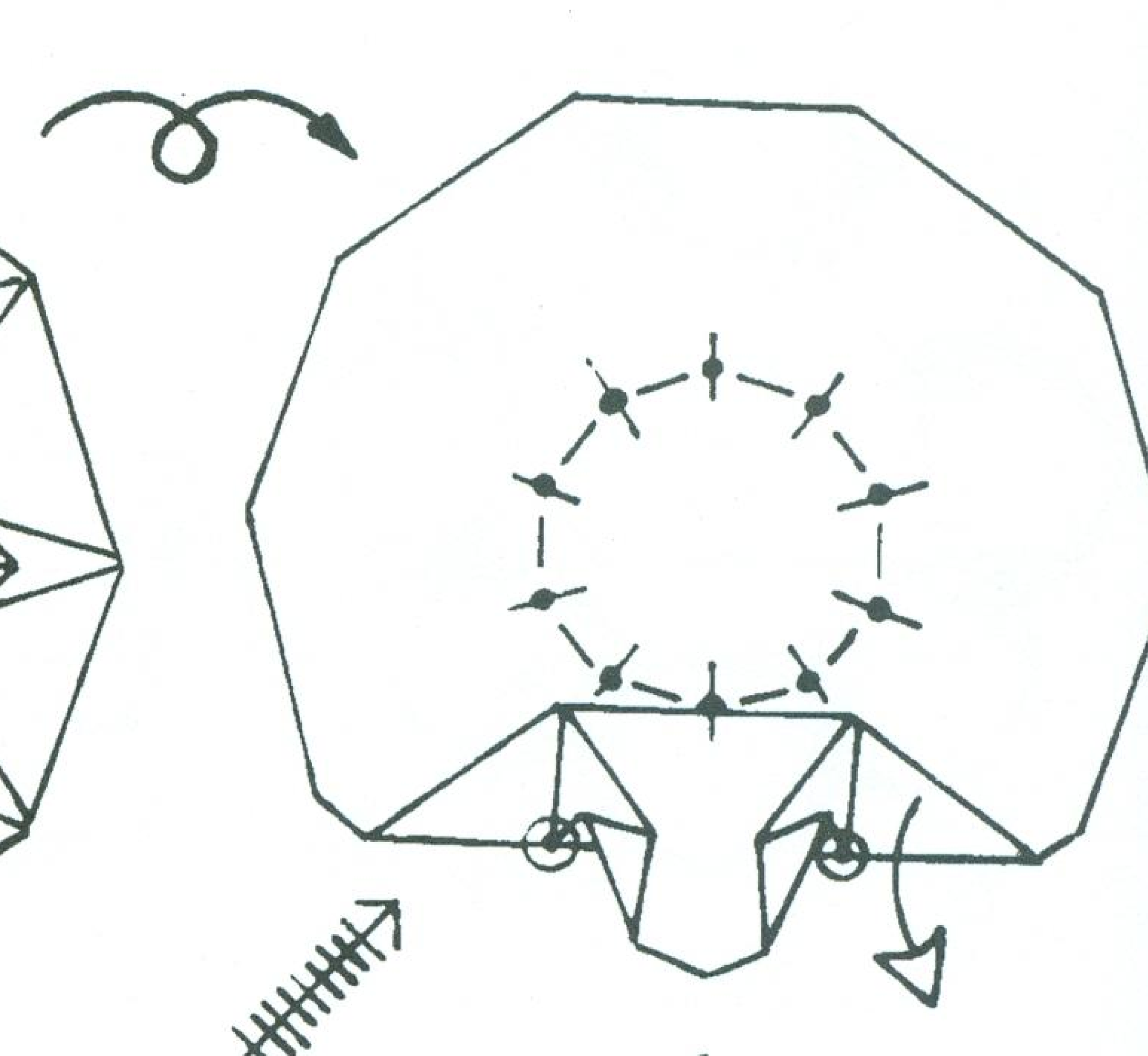

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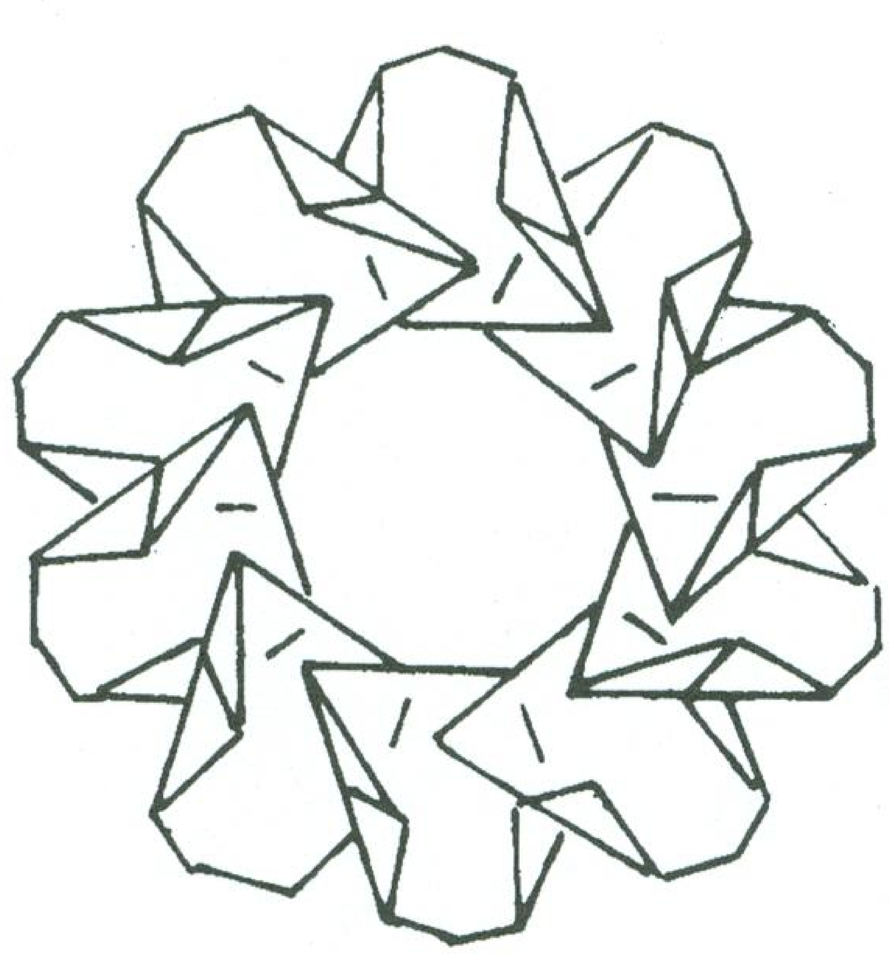

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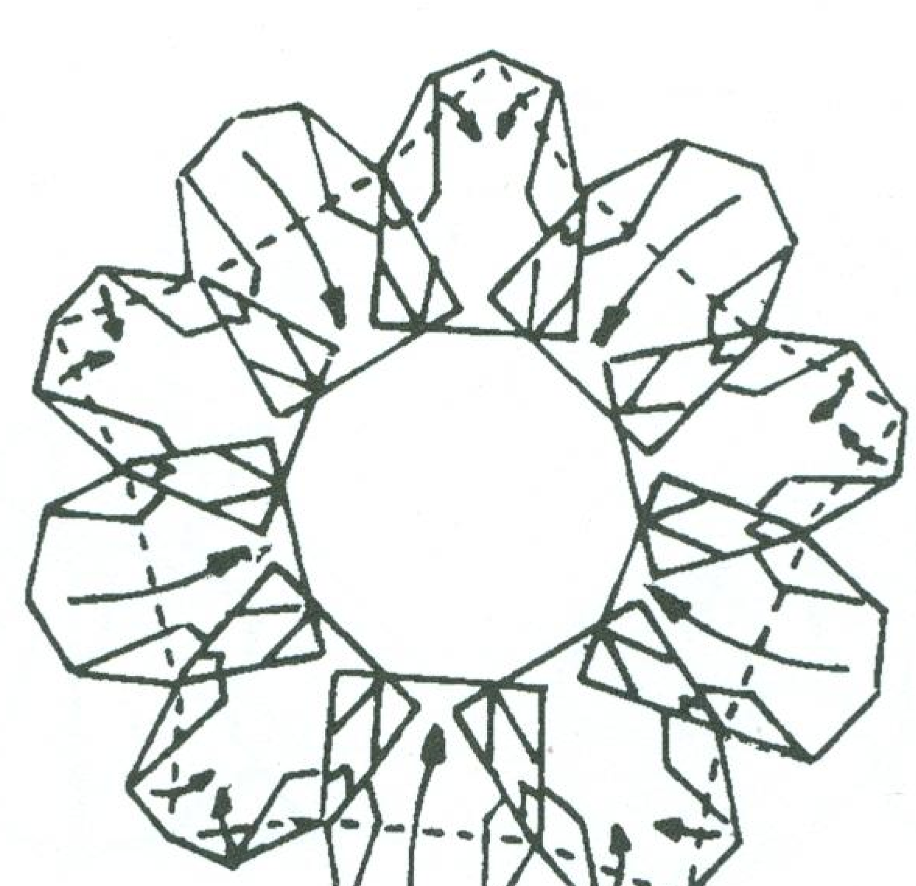

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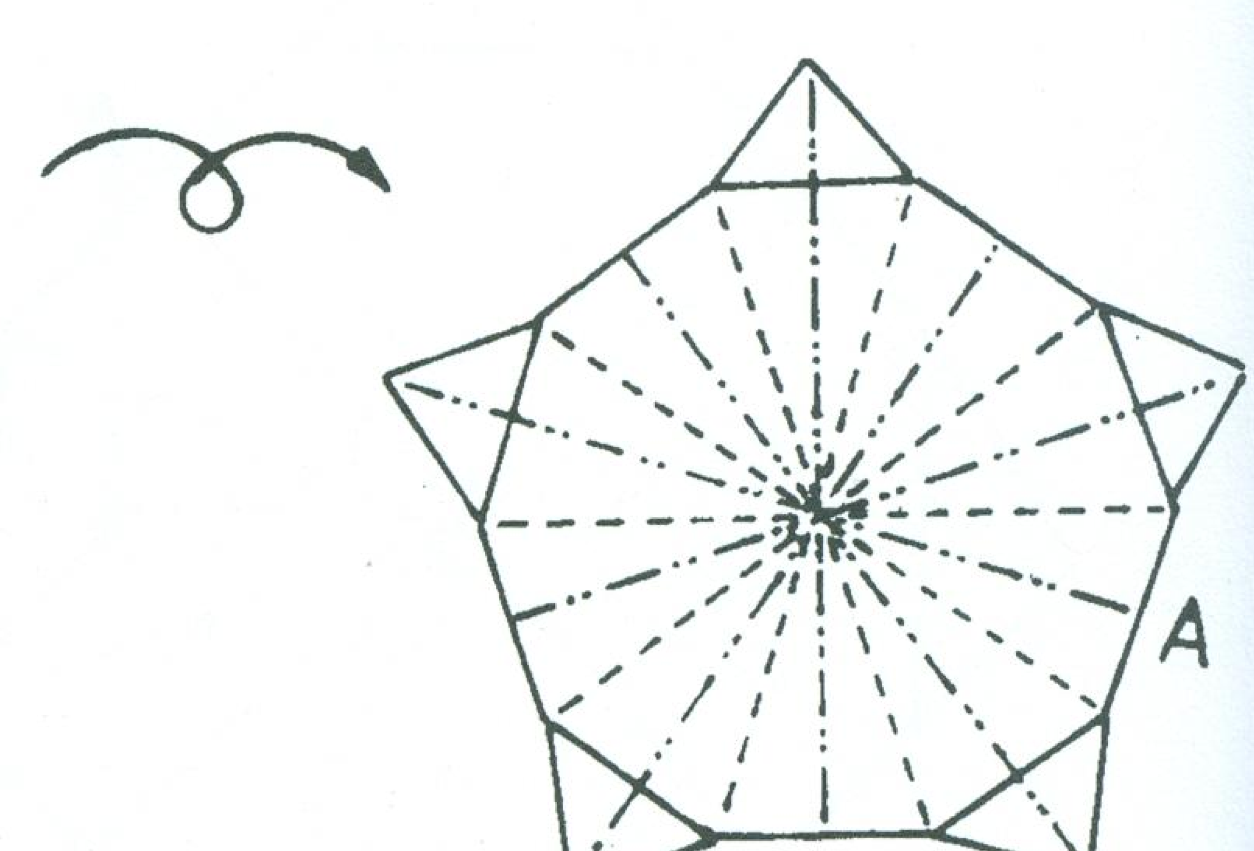


9

B

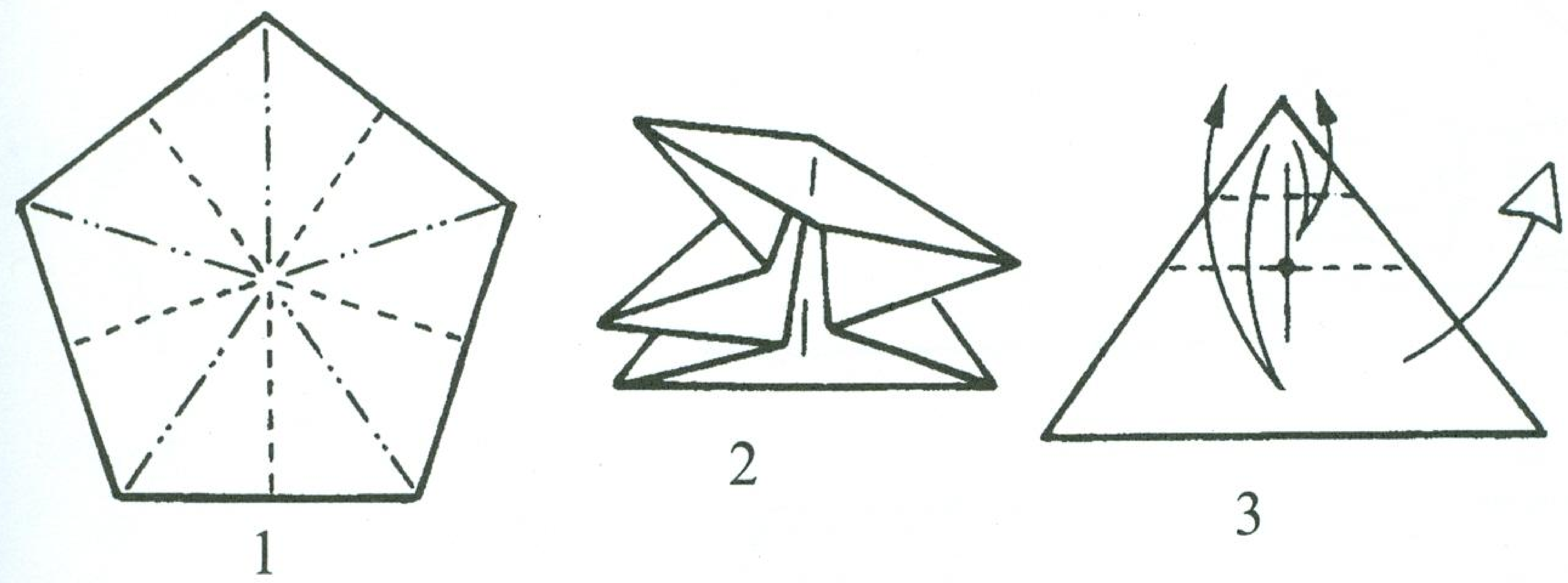
A pockets mate with triangles or squares.

B tabs/pockets mate with hexagons or decagons.

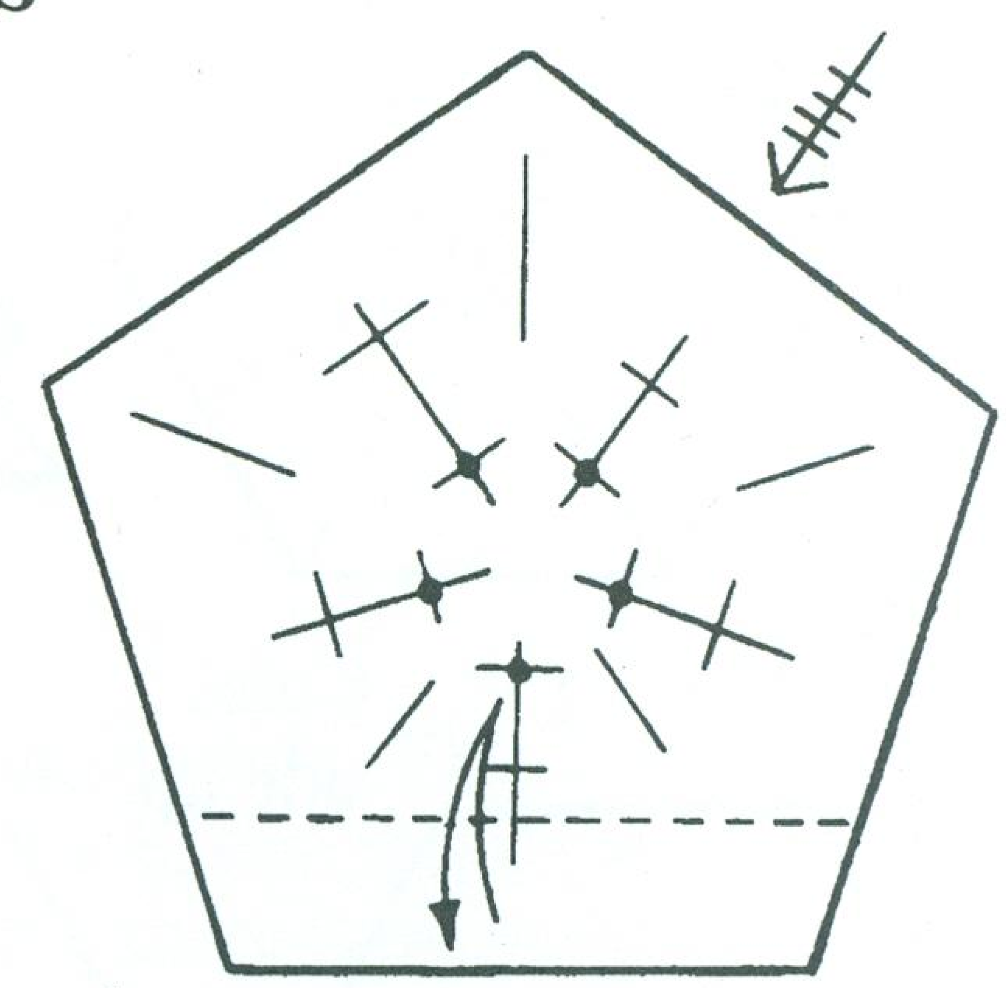
B. Large Models: Hexagon/Pentagon Pocket and Connectors

1. Expanded Pentagon Spike Ball Module: Lockable with Connectors

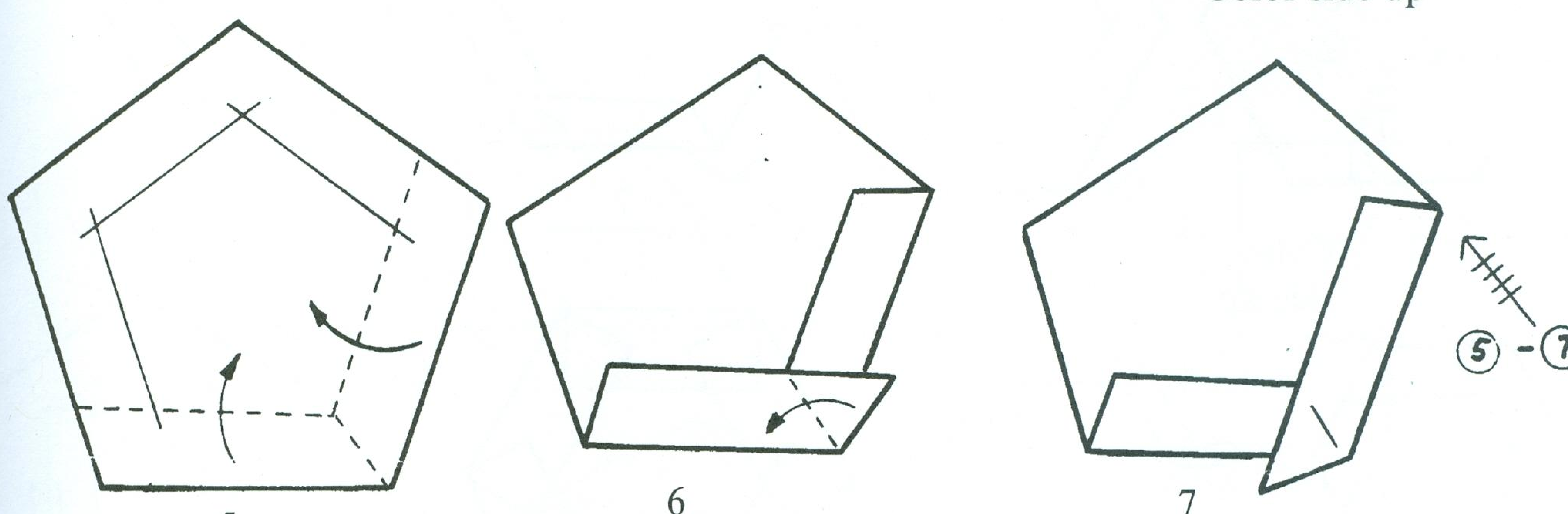
by Bennett Arnstein

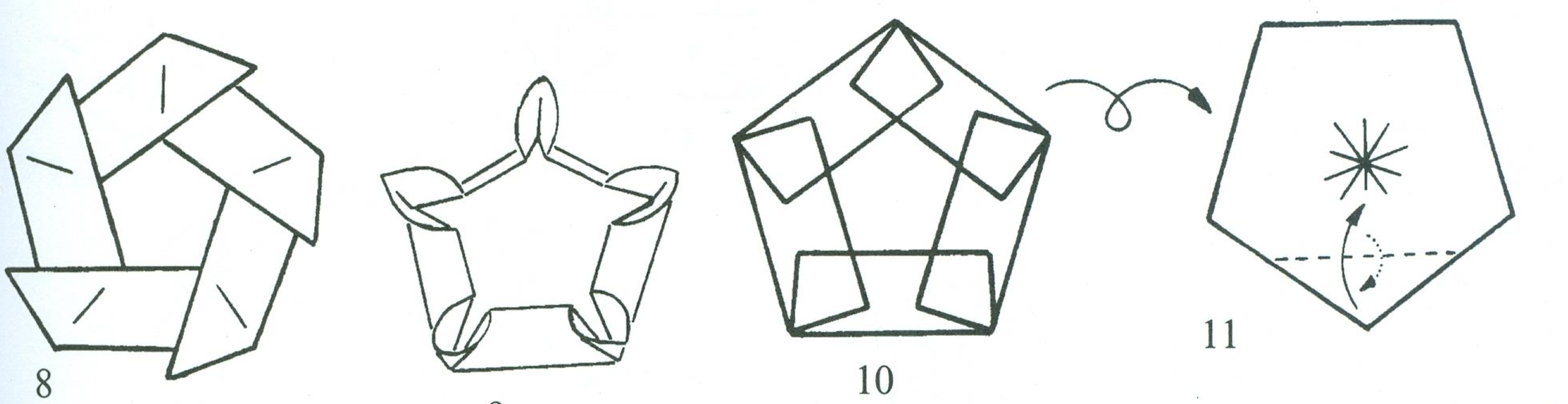


Color side up



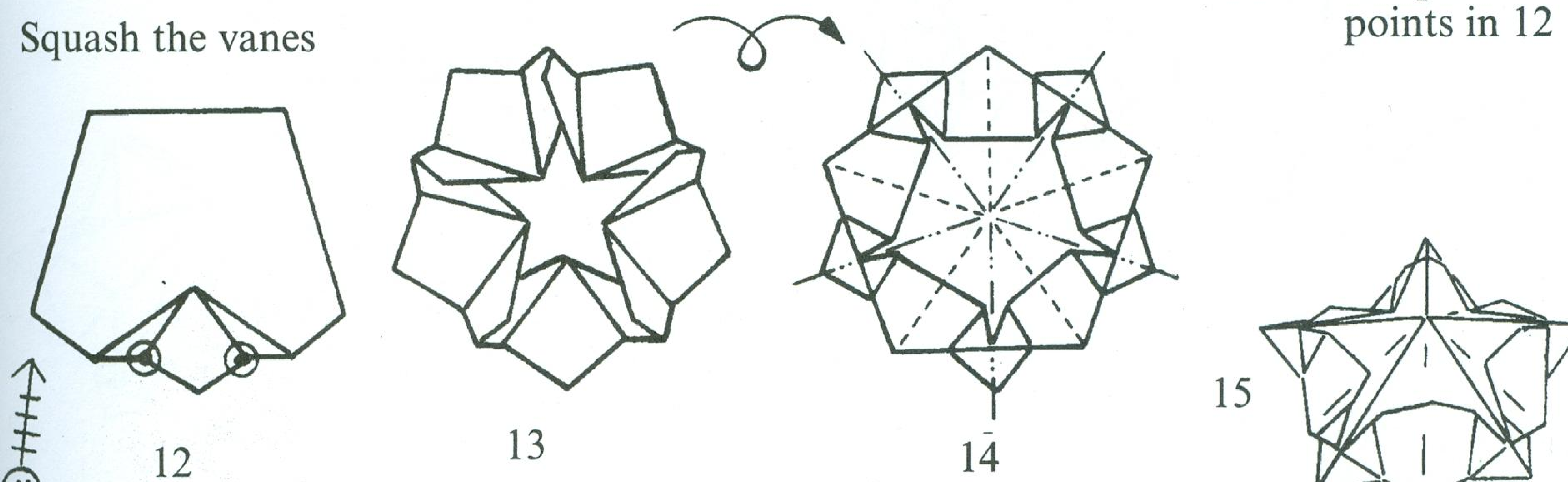
4 Color side up



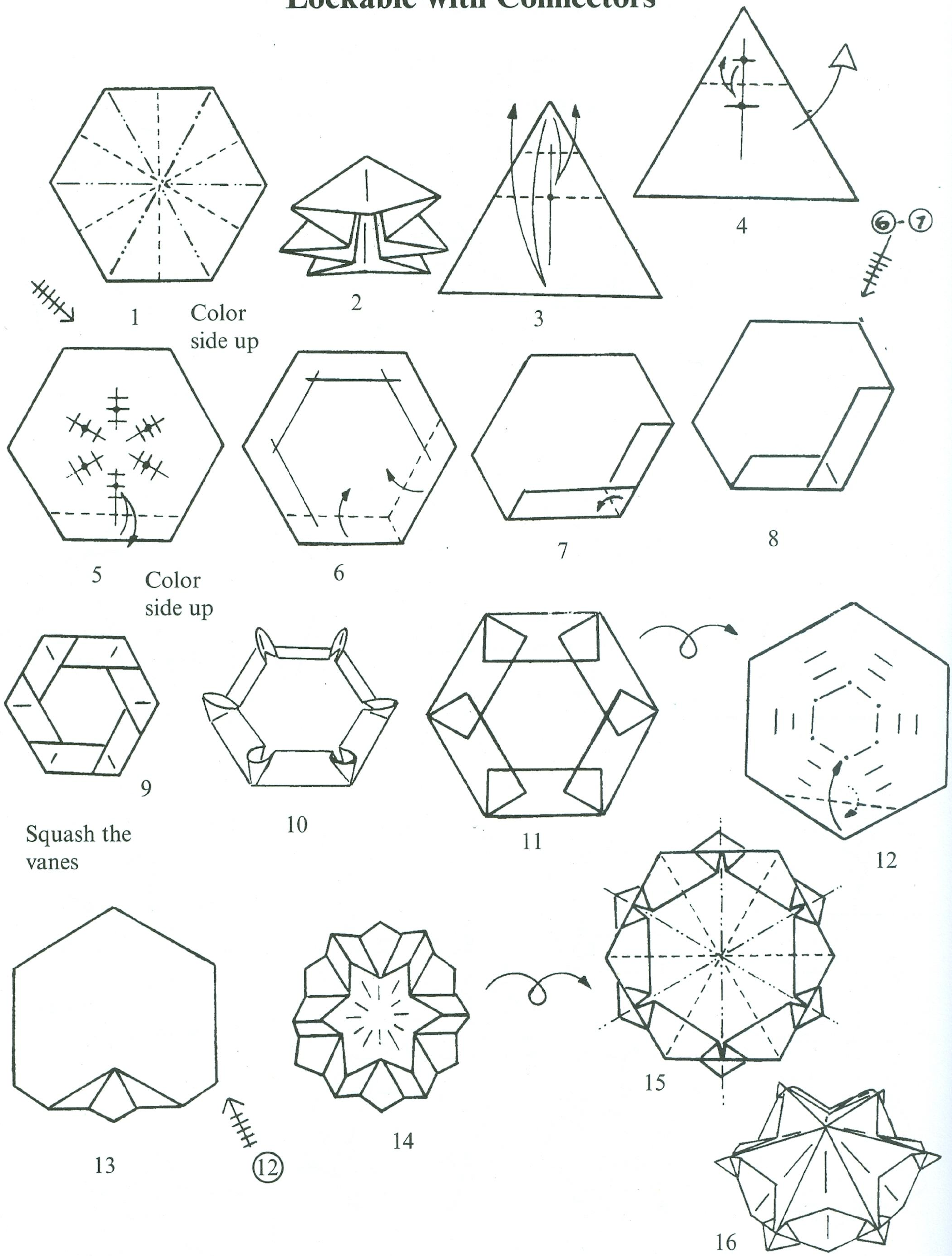


Squash the vanes

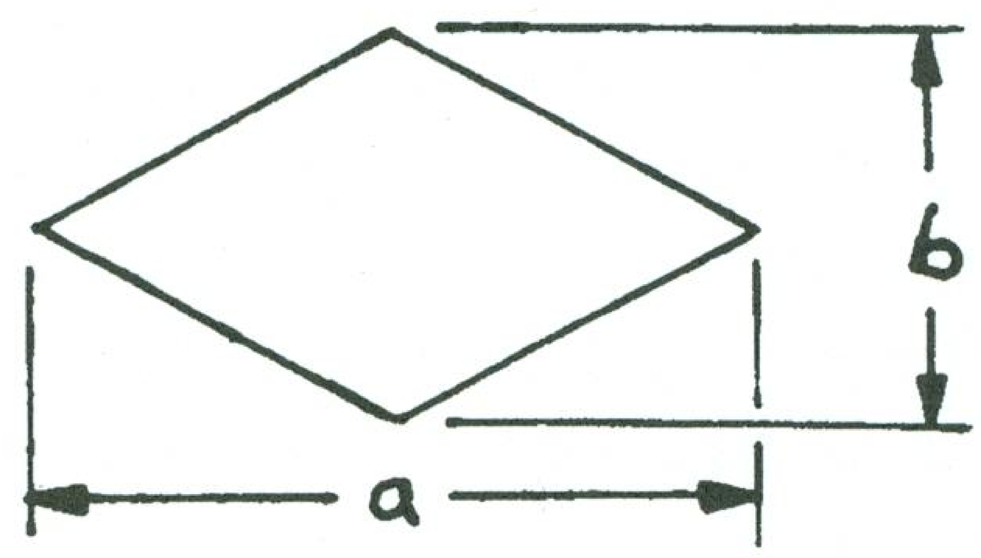
The crease passes through points in 12



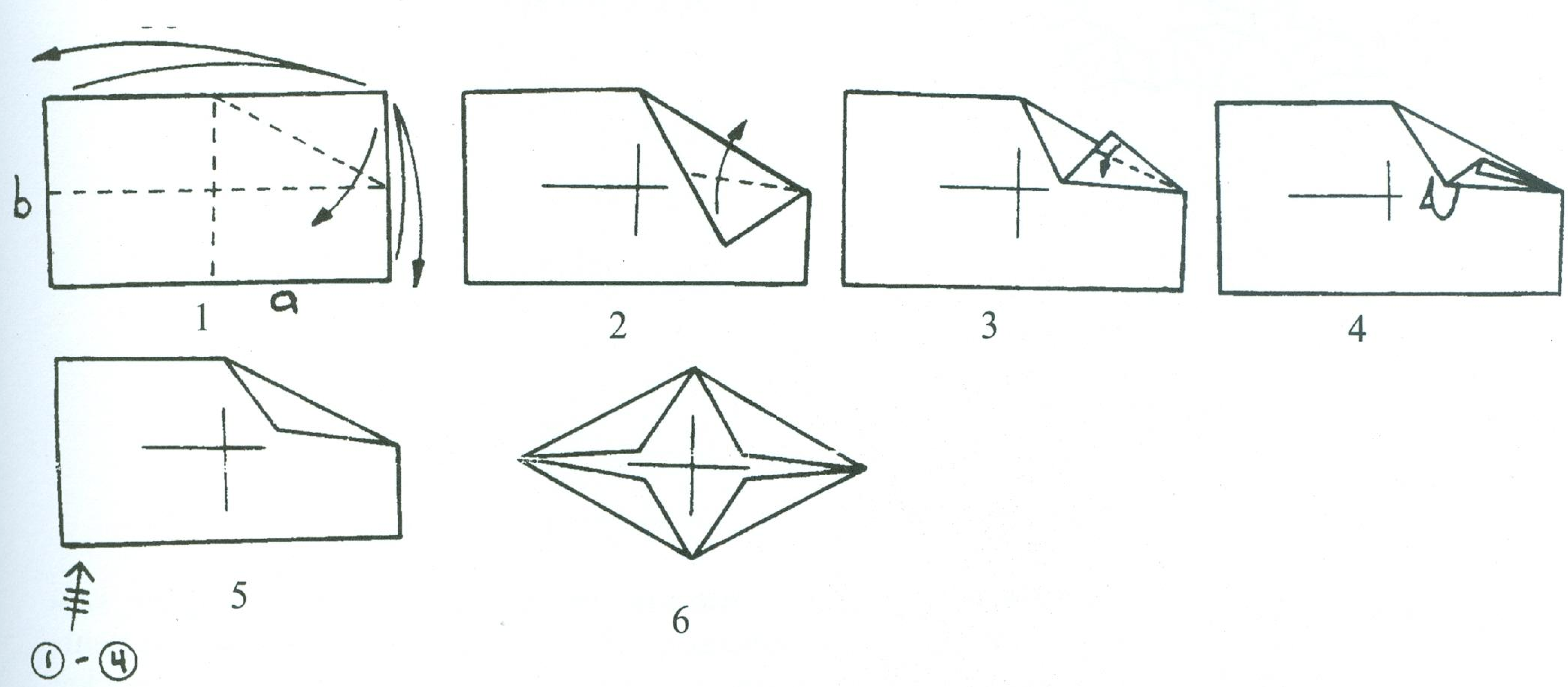
2. Expanded Hexagon Spike Ball Module: Lockable with Connectors



3. Locking Connector for Pentagon and Hexagon Spike Ball Modules

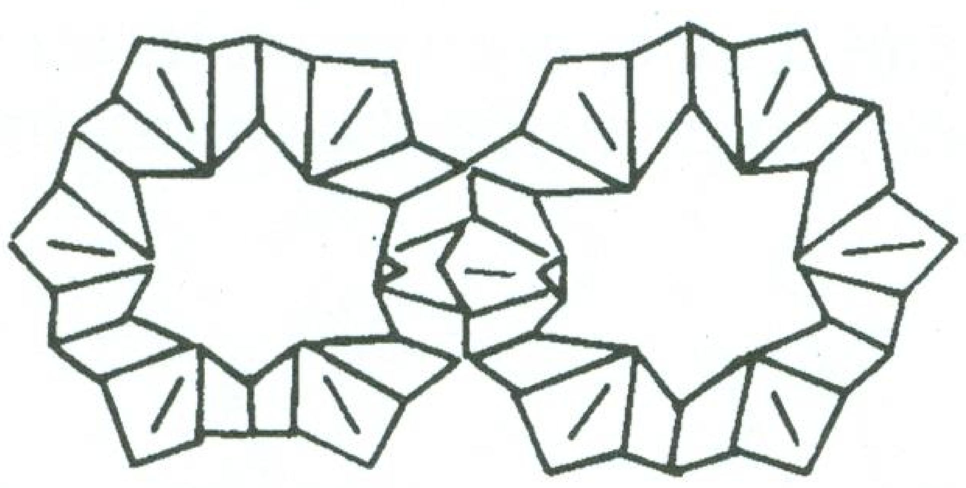


Measure the width of the opening of the pocket on a hexagon module. This is length b. Calculate length a using a = b x 11/6. Cut out rectangles of size a x b, one rectangle for each connector. Follow the directions below. Note that in figures 2 and 3 there is no exact location for the creases. The purpose of these creases is to hide the excess paper inside the connector.

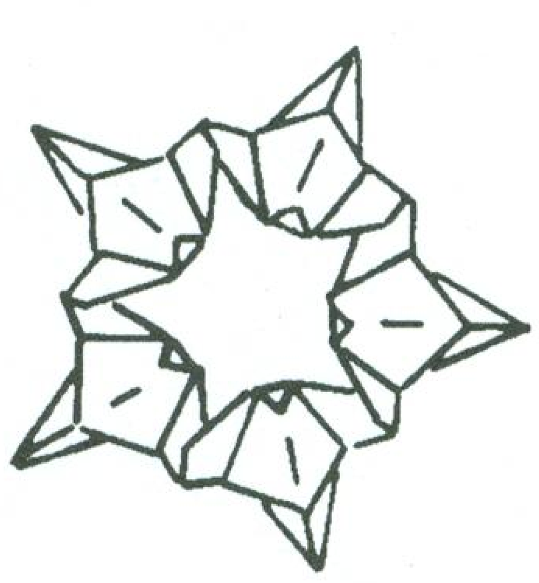


4. Locking Expanded Spike-Ball Modules with Connectors in the Pockets

Hexagon modules are always locked together. Hexagon and pentagon modules are locked together when they are connected at the beginning of construction of a model when there is room to get behind the modules and fold over the tip. To connect a hexagon to a pentagon, first insert the connector into a pocket on the hexagon and lock it by folding over the tip of the pocket and then creasing sharply the radial crease running through the pocket. Then, connect the locked pair to the pentagon by inserting the other end of the connector into a pocket on the pentagon. Lock by first folding the tip of the pocket and then creasing sharply the radial crease running through the pocket.



Two hexagon modules locked together by folding over the tips of the pockets.



When a pentagon module fits inside a five-sided space surrounded by five hexagons, it is not necessary to lock the pentgons to the hexagons. Instead, place a connector in each pocket of the pentagon module and lock the connectors by folding over the tips of the pockets. Next, insert the remaining connector point into a hexagon pocket without locking it.

VI. REFERENCES AND ACKNOWLEDGMENTS

References and Acknowledgments

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CRAFTS & HOBBIES/ORIGAMI

MULTIMODULAR ORIGAMI POLYHEDRA

Archimedeans, Buckyballs, and Duality

Rona Gurkewitz and Bennett Arnstein

Explore the relationship between origami and mathematics with this well-illustrated guide to creating a world of multifaceted wonders. Written by a mathematician and a mechanical engineer, it strengthens origami's link to mathematics and expands its relationship to crystallography.

Through a series of photographs, diagrams, and charts, the authors illustrate the correlation between the origami waterbomb base and the mathematical duality principle of Archimedean solids. They then show how to apply the correlation to models of the buckyball (a carbon-60 molecule resembling a soccer ball, named for R. Buckminster Fuller, designer of the geodesic dome). Using the fascinating process of gyroscope transformation, origamists can transform buckyballs into ever more interesting shapes.

Detailed instructions and clear diagrams offer paper folders a step-by-step path through the intricacies of multimodular origami polyhedra. These remarkable projects will challenge origami devotees in addition to providing perfect adjuncts to classroom demonstrations of geometric principles.