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# MATH BASICS

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## NUMBER SYSTEMS

### NATURAL NUMBERS

The **natural numbers** are the numbers we count with: $1, 2, 3, 4, \ldots$ Zero is not included.

### WHOLE NUMBERS

The **whole numbers** are the numbers we count with and zero: $0, 1, 2, 3, \ldots$

### INTEGERS

The **integers** are the natural numbers, their **negatives,** and zero: $\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$

- The **positive integers** are the natural numbers.
- The **negative integers** are the "minus" natural numbers: $-1, -2, -3, -4, \ldots$

### RATIONAL NUMBERS

The **rational numbers** are all the numbers that can be expressed as **fractions** (positive or negative, proper or improper).

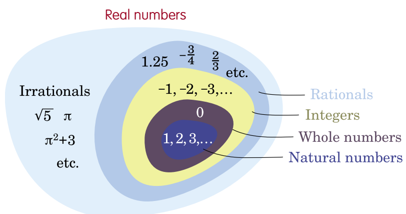
Any rational number can be expressed as $\frac{\text{integer}}{\text{non-zero integer}}$.

- All integers are rational. **Ex:** $4 = \frac{4}{1}$
- All "terminating" decimals are rational. **Ex:** $5.125 = \frac{41}{8}$

### REAL NUMBERS

The **real numbers** are all those that can be represented as points on a number line.

- All rational numbers are real, but the real number line has many, many points that are "between" rational numbers.
  **Ex:** $\sqrt[3]{2},\ \pi,\ \sqrt{3} - 9,\ 0.12112111211112\ldots$




### IMAGINARY NUMBERS

The **imaginary numbers** are square roots of negative numbers. They do not exist on the real number line.

- All of them are some real number multiplied by $i = \sqrt{-1}$.
  **Ex:** $\sqrt{-49}$ is imaginary and is equal to $i\sqrt{49}$ or $7i$.

### COMPLEX NUMBERS

The **complex numbers** are all possible sums of real and imaginary numbers. They are written as $a + bi$, where $a$ and $b$ are real and $i = \sqrt{-1}$ is imaginary.

- All reals are complex numbers (with $b = 0$); all imaginary numbers are complex (with $a = 0$).
- We represent the complex numbers on a 2-dimensional **complex plane,** with the horizontal axis representing the reals and the vertical axis representing the imaginary numbers. The number $a + bi$ is represented by the point $(a, b)$.

## NAMING WHOLE NUMBERS

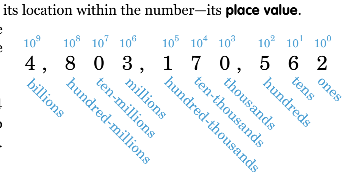
### DIGITS VS. NUMBERS

- **Digits** are symbols. Our number system (the arabic system) uses 10 digits:
  $$0, 1, 2, 3, 4, 5, 6, 7, 8, 9.$$
- **Numbers** are actual values represented by some arrangement of digits. This is an abstract concept.

### PLACE VALUE

How much each digit is worth depends on its location within the number—its **place value.**

- Place values go up by powers of 10. The arrangement of digits in 234 can be reexpressed as $(2 \times 10^2) + (3 \times 10^1) + (4 \times 10^0)$.
- **Commas:** In numbers with more than 4 digits, commas separate off each group of three digits, starting from the left. These groups are read off together.
- **Reading numbers:** The number 4,803,170,562 is read as "four billion, eight hundred and three million, one hundred and seventy thousand, five hundred and sixty-two."




The digit 7 in the ten-thousands' place is worth $7 \times 10{,}000 = 7 \times 10^4$.

### ROUNDING

A **round** number ends with one or several zeroes; how many zeroes depends on the context. **To round** a number is to approximate it with the nearest round number. We specify what place—tens, thousands, etc.—to round to.

- To "round a number to the nearest ... place," look at the place immediately to the right.
  - If the digit there is 5 or larger, **round up**: increase the digit in the place being rounded to by 1 and replace all the digits to the right by zeroes.
  - Otherwise, **round down**: keep the digit and replace all digits to the right by zeroes.

**Ex:** 553,488 rounded to the hundreds' place is 553,500. 77,901 rounded to the tens' place is 77,900.

- When rounding *up* to a place with a 9 in it, change other digits to the left accordingly.
  **Ex:** 5995 rounded to the nearest hundreds' place is 6,000.

## ARITHMETIC

### ADDITION

In the equation $3 + 4 = 7$, the numbers 3 and 4 are the **addends,** and 7 is their **sum.**

### SUBTRACTION

In the equation $15 - 7 = 8$, the number 15 is the **minuend,** 7 is the **subtrahend,** and 8 is their **difference.**

### MULTIPLICATION

In the equation $4 \times 5 = 20$, the number 4 is the **multiplicand,** 5 is the **multiplier,** and 20 is their **product.** Also 4 and 5 are both **factors** of the product 20.

- Multiplication by a natural number is "repeated addition": $3 \times 4$ means "3 added to itself 4 times," or $3 + 3 + 3 + 3$.
- Multiplication is **commutative:** Miraculously, $4 \times 3$ (or 4 added to itself 3 times) gives the same answer: $3 + 3 + 3 + 3 = 4 + 4 + 4 = 12$. The fact that $a \times b = b \times a$ is called the "commutative" property of multiplication (the two numbers can "move past," or commute with, each other).
- Ways to express multiplication:
  - Cross: $4 \times 6 = 24$.
  - Dot: $3 \cdot 9 = 27$.
  - Double pair of parentheses: $(7)(8) = 56$.
  - Single pair of parentheses: $9(6) = 54$.

### DIVISION

In the equation $36 \div 3 = 12$, the number 36 is the **dividend,** 3 is the **divisor,** and 12 is the **quotient.**

- When working with whole numbers, we can **divide with remainder:** $75 \div 8 = 9$, remainder 3.
  - In this example, the number 9 is still called the quotient, and 3 is the **remainder.**
  - The quotient 9 above is sometimes called the **partial quotient** to distinguish from the **total quotient** ($9\frac{3}{8}$ in the equation $75 \div 8 = 9\frac{3}{8}$).
- Dividing by zero is not allowed.
- Ways to express division:
  - Division sign: $72 \div 9 = 8$.
  - Slash: $48/3 = 16$.
  - Fraction: $\frac{28}{7} = 4$.
  - Long division sign: $6\overline{)30} = 5$.
  - Colon (rare): $33 : 11 = 3$.

### ORDER OF OPERATIONS

Arithmetic operations ($+, -, \times, \div$, and raising to powers) are always performed in a specific order. However, expressions enclosed in parentheses are evaluated first—also according to order of operation:

1. **P**arentheses.
2. **E**xponents
3. **M**ultiplication and **D**ivision (left to right).
4. **A**ddition and **S**ubtraction (left to right).

*MNEMONIC:* **PEMDAS**. This is sometimes expanded into the phrase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally." The phrase is somewhat misleading: multiplication and division have equal priority, as do addition and subtraction.

**Ex:** $3 + 2 \times 3^2 - (4 + 5 \times 2)$

1. Parentheses: Evaluate $(4 + 5 \times 2)$, performing multiplication before addition to get $(4 + 10) = 14$. We have $3 + 2 \times 3^2 - 14$.
2. Exponents: Evaluate $3^2 = 9$. We have $3 + 2 \times 9 - 14$.
3. Multiplication and division: $2 \times 9 = 18$. We have $3 + 18 - 14$.
4. Addition and subtraction: $3 + 18 = 21$ and $21 - 14 = 7$, which is the answer.

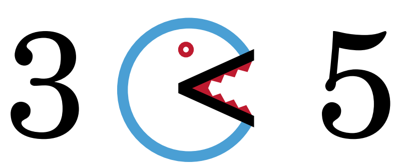
### INEQUALITIES AND SIGNS

**Trichotomy property**: If two numbers are not equal, then one of them is greater than the other one.

| Sign | Meaning | Example |
|---|---|---|
| $<$ | less than | $1 < 2$ and $4 < 56$ and $-29 < -3$ |
| $>$ | greater than | $1 > 0$ and $56 > 4$ and $-3 > -29$ |
| $\leq$ | less than or equal to | $1 \leq 1$ and $1 \leq 2$ |
| $\geq$ | greater than or equal to | $1 \geq 1$ and $3 \geq -29$ |
| $\neq$ | not equal to | $0 \neq 3$ and $-1 \neq 1$ |

The sharp end of an inequality sign points toward the smaller number; the open part toward the larger.

*MNEMONIC:* If you see the sign as a mouth, the mouth wants to eat the larger number.



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# “THE DIFFERENT BRANCHES OF ARITHMETIC—AMBITION, DISTRACTION, UGLIFICATION, AND DERISION.” LEWIS CARROLL

## ARITHMETIC (continued)

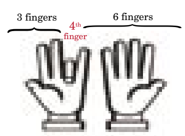
### PROPERTIES OF ADDITION AND MULTIPLICATION

| Property | Addition $(+)$ | Multiplication $(\times$ or $\cdot)$ |
|---|---|---|
| **Commutativity** | $a+b=b+a$ | $a\times b=b\times a$ |
| **Associativity** | $(a+b)+c=a+(b+c)$ | $a\times(b\times c)=(a\times b)\times c$ |
| **Identities exist** | 0 is a number and $a+0=0+a=a$ • 0 is the **additive identity**. | 1 is a number and $a\times 1=1\times a=a$ • 1 is the **multiplicative identity**. |
| **Inverses exist** | $-a$ is a real number and $a+(-a)=(-a)+a=0$ Also $-(-a)=a$. | If $a\neq 0$ then $\frac{1}{a}$ is a real number and $a\times\frac{1}{a}=\frac{1}{a}\times a=1$ Also, $\frac{1}{\frac{1}{a}}=a$. |
| **Closure** | $a+b$ is a real number | $a\times b$ is a real number |
| **Distributive property** (of multiplication over addition): | $a\times(b+c)=a\times b+a\times c$ $(b+c)\times a=b\times a+c\times a$ | |

There are also two (derivative) properties having to do with zero.
—**Multiplication by zero:** $a\times 0=0\times a=0$.
—**Zero product property:** If $a\times b=0$ then $a=0$ or $b=0$ (or both).

### MULTIPLICATION BY NINES

To multiply a number $n$ by 9: Look at your ten fingers, bend down the $n^{\text{th}}$ one from the left, and read off the answer.




In the diagram, bending down the $4^{\text{th}}$ finger leaves 3 fingers on one side, and 6 on the other. So $9\times 4=36$.

### MULTIPLICATION TABLE

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **1** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| **2** | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| **3** | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| **4** | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| **5** | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| **6** | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| **7** | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| **8** | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| **9** | 0 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| **10** | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| **11** | 0 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| **12** | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

- Any number times 1 is itself.
- To multiply a number by 10, attach a zero to the end.
  Ex: $34\times 10=340$
- To multiply a number by 5, you can divide it by 2 and move the decimal point one place to the right.
  Ex: $13\times 5$. Half of 13 is 6.5 so $13\times 5=65$.

## WHOLE NUMBERS: FACTORING

In this section, all numbers are natural numbers.

- If $a\times b=c$, then $a$ and $b$ are **factors** or **divisors** of $c$. You can also say that $a$ and $b$ **go into** $c$ **evenly**. Also, $c$ is a **multiple** of $a$ and of $b$, and **divisible** by $a$ and by $b$.
  Ex: 3 is a factor of 12; 15 is not a multiple of 4; 28 is divisible by 7. The number 1 is a factor of every number.
- If a number is divisible by 2, it is called **even**; if not, it is called **odd**.
  - Even numbers end with even digits: 0, 2, 4, 6, or 8.
  - Odd numbers end with odd digits: 1, 3, 5, 7, or 9.
- A **prime** number has no factors except for itself and 1. The first few primes are worth knowing: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, ...
  As a convention, 1 is usually not considered prime. Every prime except for 2 is odd.

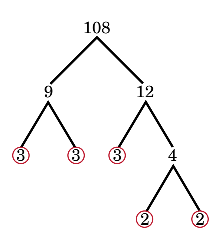
**Fundamental Theorem of Arithmetic:** Every natural number (except for 1) can be written as a product of prime numbers in exactly one way (ignoring rearrangements). Prime numbers are the irreducible building blocks of all natural numbers.

- A **composite** number can be factored in an “interesting” way (neither factor is 1).
  Ex: $12=3\times 4$ is composite, but $7=1\times 7$ is not. Every number (except for 0 and 1) is either prime or composite.
- Determining whether a number is prime is hard. The easiest way to conclude that $n$ is prime is to make sure that every prime number less than $\sqrt{n}$ is **not** a factor of $n$.
  Ex: To check if 589 is prime, test every prime less than $\sqrt{589}\approx 24$. Dividing by 2, 3, 5, 7, 11, 13, 17, 19, 23, we see that 19 is a factor: $589=19\times 31$ and is not prime.

### FACTOR TREES

Every natural number greater than 1 can be factored into a product of primes. If the number is prime, you’re done. If not, factor it, and look at each factor. If they’re both prime, you’re done. If not, factor one or both and repeat...

- This method sometimes is organized in a **factor tree.** There is more than one valid factor tree for the same number, but the prime factors at the end are always the same.
- The end result is called the **prime factorization** of the original number.
- In practice, factoring large numbers is also hard. Encryption software often relies on the difficulty of factoring very large numbers.




$108=2\times 2\times 3\times 3\times 3=2^2\times 3^3$

### DIVISIBILITY RULES

| A number is divisible by... | if... |
|---|---|
| **2** | its last digit is 2, 4, 6, 8, or 0. |
| **3** | the sum of its digits is divisible by 3. |
| **4** | its last two digits (taken together as a two-digit number) are divisible by 4. |
| **5** | it ends in 0 or 5. |
| **6** | it’s even and the sum of its digits is divisible by 3. |
| **7** | **For three-digit numbers:** the quantity $2\times(\text{hundreds' digit})+3\times(\text{tens' digit})+(\text{last digit})$ is divisible by 7. |
| **8** | its last three digits (taken together as a three-digit number) are divisible by 8. |
| **9** | the sum of its digits is divisible by 9. |
| **10** | it ends in 0. |

### GREATEST COMMON FACTOR (GCF)

- A **common factor** of two numbers is any number that is a factor of both.
  Ex: 6 is a common factor of 108 and 126.
- The **greatest common factor** (**GCF**), a.k.a. **greatest common divisor** (**GCD**), of two numbers is the largest of the common factors.
  Ex: The common factors of 108 and 126 are 1, 2, 3, 6, 9, and 18, so their GCF is 18.
- To find the GCF, factor both numbers into their prime factorizations, find all the primes they have in common (counting multiplicity) and multiply them together.
  Ex: $108=2\times 2\times 3\times 3\times 3=2^2\times 3^3$ and $126=2\times 3\times 3\times 7=2\times 3^2\times 7$, so their GCF is $2\times 3\times 3=18$.
- If two numbers have no “interesting” common factors (their GCF is 1), we say that they are **relatively prime**. Ex: 40 and 21 are relatively prime (although both are composite).

### LEAST COMMON MULTIPLE (LCM)

The **least common multiple** (LCM), sometimes known as the **least common denominator** (LCD), of two numbers is the smallest number that is divisible by both.

- To find the LCM, factor both numbers into primes. Each prime factor of either must appear in the LCM at least as many times as it does in each one.
  Ex: 108 has three 3s in its factorization, and 126 has two. So there will be a factor of $3\times 3\times 3$ in their LCM. The LCM of 108 and 126 is $2\times 2\times 3\times 3\times 3\times 7=2^2\times 3^3\times 7=756$.
- If you already know the GCF of two numbers:
  $$(\text{GCF})\times(\text{LCM})=\text{product of the numbers.}$$
  Ex: Since the GCF of 108 and 126 is 18, their LCM $=\frac{108\times 126}{18}=756$.
- The LCM of relatively prime numbers is their product. Ex: The LCM of 21 and 40 is $21\times 40=840$.

# FRACTIONS

A **fraction** is a division in progress; it describes parts of a whole. The top is called the **numerator**, and the bottom is called the **denominator**. $\frac{3}{4}$ is "three fourths" of a whole.

- If a fraction is written with a slash instead of a bar, the numerator comes first.
  **Ex:** $3/4 = \frac{3}{4}$
- The denominator can never be $0$. The expressions $\frac{1}{0}$, $\frac{-4}{0}$, and $\frac{0}{0}$ are all undefined.

## PROPER FRACTIONS, IMPROPER FRACTIONS, AND MIXED NUMBERS

- If its numerator is smaller than its denominator, a fraction is **proper**. A proper fraction denotes a quantity less than $1$. **Ex:** $\frac{3}{4}$ and $\frac{6}{12}$ are proper fractions.
- Otherwise, the fraction is **improper**, and denotes a quantity more than or equal to $1$.
  **Ex:** $\frac{5}{2}$ and $\frac{16}{10}$ are improper fractions.
- A **mixed number** has two parts: a whole number and a fraction. **Ex:** $3\frac{1}{4}$
  The mixed number is effectively the sum of the integer and the fraction: $3 + \frac{1}{4} = 3\frac{1}{4}$. A mixed number denotes a quantity more than $1$.
- **Converting mixed numbers to improper fractions:** Multiply the whole number by the denominator and add the product to the numerator to get the numerator of the improper fraction. The denominator stays the same:
  $$\text{whole} + \frac{\text{numerator}}{\text{denominator}} = \frac{(\text{whole}) \times (\text{denominator}) + (\text{numerator})}{\text{denominator}}.$$
  **Ex:** $2\frac{3}{7} = \frac{2\times7+3}{7} = \frac{17}{7}$
- **Converting improper fractions to mixed numbers:** Divide (with remainder) the numerator by the denominator. The quotient is the whole number; the remainder is the numerator of the fraction; the denominator is unchanged:
  $$\frac{\text{numerator}}{\text{denominator}} = \text{quotient} + \frac{\text{remainder}}{\text{denominator}}.$$
  **Ex:** To convert $\frac{48}{5}$, first divide: $48 \div 5 = 9$, remainder3. So $\frac{48}{5} = 9\frac{3}{5}$.

## EQUIVALENT FRACTIONS

There are many ways to write the same fraction.
**Ex:** $\frac{3}{4}$, $\frac{12}{16}$, and $\frac{-6}{-8}$ are all ways to write the same quantity. They are **equivalent fractions.**

- If a fraction is in **lowest terms,** its numerator and denominator have no common factors.
  **Ex:** $\frac{3}{4}$ is in lowest terms; $\frac{12}{16}$ is not.
- To **reduce** a fraction to lowest terms, divide the numerator and the denominator by their GCF—the largest number that goes into both evenly: $\frac{6}{12} = \frac{6\div6}{12\div6} = \frac{1}{2}$.
- Any whole number $a$ is equivalent the fraction $\frac{a}{1}$.

## COMPARING FRACTIONS

Multiplying or dividing the numerator and the denominator of a fraction by the same number does not change the value of the fraction: $\frac{a}{b} = \frac{ac}{bc}$.

- To determine if two fractions are equivalent, **cross-multiply**: multiply the numerator of one by the denominator of the other and vice versa. If the two products are equal, then the two fractions are equivalent. **Ex:** $\frac{18}{21} = \frac{30}{35}$ because $18 \times 35 = 630$ and $21 \times 30 = 630$. In lowest terms, both fractions become $\frac{6}{7}$.
  $$\frac{18}{21} \times \frac{30}{35}$$
- If cross-multiplying does not give the same product, then the larger fraction is the one that contributes its numerator to the larger numerator-denominator product.
  **Ex:** $\frac{3}{7} > \frac{2}{5}$ because $3 \times 5 = 15 > 14 = 7 \times 2$.

## ADDING FRACTIONS

- **Fractions with the same denominator** are easy to add: add their numerators: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.
  - Sometimes fractions will need to be reduced after addition.
    **Ex:** $\frac{5}{12} + \frac{1}{12}$ Add: $\frac{5+1}{12} = \frac{6}{12}$ Reduce: $\frac{6}{12} = \frac{6\div6}{12\div6} = \frac{1}{2}$
- **Fractions with different denominators** have to be converted to equivalent fractions with the same denominator (**common denominator**) before adding. There are two ways of converting...
  - *Less thinking:* Multiply top and bottom of the first fraction by the denominator of the second fraction; multiply top and bottom of the second fraction by the denominator of the first fraction. The new fractions are equivalent to the original fractions and have a common denominator. Add. Reduce.

**Ex 1:** $\frac{3}{8} + \frac{5}{6}$ 1. Convert: $\frac{3}{8} = \frac{3\times6}{8\times6} = \frac{18}{48}$ and $\frac{5}{6} = \frac{5\times8}{6\times8} = \frac{40}{48}$
2. Add: $\frac{18}{48} + \frac{40}{48} = \frac{58}{48}$ 3. Reduce: $\frac{58\div2}{48\div2} = \frac{29}{24} = 1\frac{5}{24}$

**Ex 2:** $\frac{5}{6} + \frac{5}{12} = \frac{5\times12+5\times6}{6\times12} = \frac{90}{72} = \frac{90\div18}{72\div18} = \frac{5}{4}$

- *Less work:* When the two denominators have common factors, we can use a smaller common denominator than their product—the LCM. Convert both fractions to equivalent fractions whose denominator is the LCM of the original denominators. Add. Reducing and/or convert to a mixed number if necessary. The advantage of this method is that you're working with smaller numbers; less multiplication and division is involved. The advantage is greatest when one of the original denominators is a factor of the other, as in the second example both above and below.
  **Ex 1:** $\frac{3}{8} + \frac{5}{6}$ The LCM of $8$ and $6$ is $24$.
  We convert $\frac{3}{8} = \frac{3\times3}{8\times3} = \frac{9}{24}$ and $\frac{5}{6} = \frac{5\times4}{6\times4} = \frac{20}{24}$. Finally, add: $\frac{9}{24} + \frac{20}{24} = \frac{29}{24} = 1\frac{5}{24}$.
  **Ex 2:** $\frac{5}{6} + \frac{5}{12} = \frac{5\times2}{6\times2} + \frac{5}{12} = \frac{10+5}{12} = \frac{15}{12} = \frac{5}{4}$

## SUBTRACTING FRACTIONS

Subtraction works just like addition.

- **Fractions with the same denominator**: Just subtract the numerators. Reduce as necessary.
  **Ex:** $\frac{13}{18} - \frac{1}{18} = \frac{13-1}{18} = \frac{12}{18} = \frac{12\div6}{18\div6} = \frac{2}{3}$
- **Fractions with different denominators** will need to be converted to equivalent fractions with a common denominator. Convert. Subtract. Reduce if necessary.
  **Ex:** $\frac{11}{4} - \frac{5}{6} = \frac{11\times3}{4\times3} - \frac{5\times2}{6\times2} = \frac{33-10}{12} = \frac{23}{12} = 1\frac{11}{12}$

## MULTIPLYING FRACTIONS

Multiply the numerators. Multiply the denominators. No common denominator necessary.

- *Less thinking:* Multiply first, then reduce. **Ex:** $\frac{3}{4} \times \frac{2}{9} = \frac{3\times2}{4\times9} = \frac{6}{36} = \frac{6\div6}{36\div6} = \frac{1}{6}$
- *Less work:* Cancel any common factors from numerators and denominators, then multiply. No need to reduce. **Ex:**
  $$\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

## DIVIDING FRACTIONS

Multiply by the reciprocal of the divisor. No common denominator necessary!

- Again, cancelling common factors can happen before or after multiplication.
  **Ex 1:** $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{3}{2}$ **Ex 2:** $\frac{3}{4} \div 4 = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$
- You cannot divide by $0$.

## RECIPROCALS

The **reciprocal** or **inverse** of a fraction $\frac{a}{b}$ is the fraction $\frac{b}{a}$. The numerator and the denominator are flipped.

- A fraction whose value is zero—i.e., any fraction $\frac{0}{n}$—has no reciprocal.
- The product of a fraction and its reciprocal is $1$.
- The reciprocal of the whole number $a$ is the fraction $\frac{1}{a}$.

## WORKING WITH MIXED NUMBERS

**Addition and subtraction:**

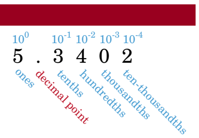
- *Less thinking:* Convert all mixed numbers to improper fractions, add or subtract, then convert back.
  **Ex:** $11\frac{1}{4} - 4\frac{1}{3} = \frac{45}{4} - \frac{13}{3} = \frac{45\times3-13\times4}{12} = \frac{83}{12} = 6\frac{11}{12}$.
- *Less work:* Add or subtract the whole number parts and the fractional parts separately.
  - **Addition:** If the sum of the fractional parts is an improper fraction, convert it to a mixed number and add whole parts together.
    **Ex:** $1\frac{3}{5} + 2\frac{1}{2} = (1+2) + \left(\frac{3}{5} + \frac{1}{2}\right) = 3 + \frac{3\times2+1\times5}{10} = 3 + \frac{11}{10} = 3 + 1\frac{1}{10} = 4\frac{1}{10}$
  - **Subtraction:** If the fractional part of first number is smaller than the fractional part of the second number, "borrow" a $1$ from the whole part by reducing the whole part by $1$ and making the fractional part improper.
    **Ex:** $11\frac{1}{4} - 4\frac{1}{3} = 10 + \left(\frac{4+1}{4}\right) - 4\frac{1}{3} = (10-4) + \left(\frac{5}{4} - \frac{1}{3}\right) = 6 + \frac{5\times3-1\times4}{12} = 6\frac{11}{12}$

**Multiplication and division:** Convert mixed numbers to improper fractions before multiplying or dividing.

# DECIMALS

**Decimals** are another way of expressing parts of a whole—using a place value system.

## INTERPRETING DECIMALS




- The **decimal point** separates the part of a number that is less than $1$ from the part that is greater than $1$. Any whole number has a (usually unwritten) decimal point to its right.
  - If the number has no whole part, the zero before the decimal point may be omitted. **Ex:** $.04 = 0.04$
- Zeroes that occur after the last digit after the decimal point may be dropped or added without changing the value of the number.
  **Ex:** $4.9 = 4.90$; $5 = 5.000$
- **Reading decimals:** The decimal point is marked with "and;" the part after the decimal point is read as a **decimal fraction** (a fraction whose denominator is a power of ten). $5.3402$ is "five and three thousand four hundred and two ten-thousandths."

## COMPARING DECIMALS

- If the whole parts of two numbers differ, then the one with the greater whole part is greater.
  **Ex:** $5.1 > 4.99999$
- To compare the fractional parts, make sure that the numbers have the same number of digits after the decimal point, possibly by padding one of them with zeroes. Compare the padded fractional parts as you would whole numbers. **Ex:** $1.009 < 1.3$ because $1.3 = 1.300$ and $009 < 300$.

## ADDING AND SUBTRACTING DECIMALS

Line up the decimal points, then add or subtract as with whole numbers. Padding with zeroes may make this easier—especially with subtraction.

**Ex 1:** $0.004 + 21.0089 + 7 = 28.0129$
**Ex 2:** $5 - 2.041 = 2.959$

$$\begin{array}{r} 0.0040 \\ 21.0089 \\ +\ 7.0000 \\ \hline 28.0129 \end{array} \qquad \begin{array}{r} {}^{4\ 9\ 9\ 10} \\ 5.000 \\ -2.041 \\ \hline 2.959 \end{array}$$

## MULTIPLYING DECIMALS

Ignoring the decimal points, multiply as you would whole numbers. Place the decimal point so that the answer has as many digits after the decimal point as *both* numbers being multiplied *combined.*

- Watch the end zeroes. They may end up after the decimal point and be insignificant in the answer, but they do count as digits when determining the location of the decimal point.
- Check that the answer makes sense. The product of the whole parts should be of the same **order of magnitude**—in most cases, have the same number of digits before the decimal point—as the answer.
  **Ex:** $3.235 \times 9.22 = 29.8267$

$$\begin{array}{r} 3.235 \\ \times\ \ 9.22 \\ \hline 6470 \\ 6470\phantom{0} \\ 29115\phantom{00} \\ \hline 29.82670 \end{array}$$
3.235 ← 3 digits
9.22 ← 2 digits
29.82670 ← 5 digits

## DIVIDING DECIMALS

- Convert the divisor (the number being divided by) into a whole number by moving the decimal point in both the divisor and the dividend to the right the same number of places. (Exactly the same as multiplying top and bottom of a fraction by the same power of ten.)
  You may need to pad the dividend with zeroes.
- Divide as usual, making sure to line up the digits of the quotient with the corresponding digits of the dividend.
- The decimal point in the dividend indicates the location of the decimal point in the answer.
  **Ex:** $12.6 \div 0.75 = 16.8$

$$\begin{array}{r} 16.8 \\ 75.\overline{)\,1260.0} \\ -75\phantom{0.0} \\ \hline 510\phantom{.0} \\ -450\phantom{.0} \\ \hline 600 \\ -600 \\ \hline 0 \end{array}$$

# CONVERTING BETWEEN FRACTIONS AND DECIMALS

Fractions and decimals are two ways of expressing parts of a whole.

- Decimals (rather than fractions) are often used when the number refers to a physical quantity, such as bushels of rice or square inches.
- Fractions may be more convenient when the numbers involved are small, especially in problems involving a lot of division and multiplication.

## TERMINATING AND REPEATING DECIMALS

The most common type of decimal is a **terminating decimal**: $0.25$ or $1.3$ are both decimals with a finite number of digits after the decimal point. However, there are many real numbers that, written in decimal form, go on forever.

**Ex:** $\pi = 3.14159265\ldots$; $\sqrt{2} = 1.41421356\ldots$

- Some of the decimals that go on forever are **repeating**—after a certain point, a cycle of digits repeats over and over again.
  **Ex:** $0.33333\ldots$; $2.080808\ldots$; $1.8345454545\ldots$.
- The cycle that repeats is usually denoted with an overline bar.
  **Ex:** $0.33333\ldots = 0.\overline{3}$; $1.8454545\ldots = 1.8\overline{45}$
- Numbers that can be expressed as terminating or repeating decimals can be rewritten as fractions—they are **rational.**
- Numbers that are expressed as decimals that go on forever and never repeat—such as $\pi$ or $\sqrt{2}$—are **irrational,** and cannot be expressed as fractions.

## CONVERTING DECIMALS TO FRACTIONS

First convert the decimal to a decimal fraction. The numerator is the part after the decimal point. The denominator is a power of 10—the place value of the *last* digit in the number (the number of zeroes in the denominator should correspond to the number of digits after the decimal point). Next, reduce.

- Reading the decimal aloud should indicate the fraction.
- If the decimal has a whole part, convert to a mixed number first; then convert to an improper fraction if necessary.

**Ex 1:** $0.04 = \frac{4}{100} = \frac{1}{25}$ **Ex 2:** $1.0625 = 1\frac{625}{10000} = 1\frac{1}{16}$

## CONVERTING FRACTIONS TO DECIMALS

Divide, padding the numerator with a decimal point and as many zeroes as necessary.

- If the fraction is in reduced form, and the denominator does not have prime factors except for 2 and 5, then the decimal answer will terminate.
- Otherwise, it will repeat. Either divide until you see the pattern, or stop and get an approximate answer.

**Ex 1:** $\frac{21}{16} = 1.3125$

**Ex 2:** $\frac{6}{11} = 0.\overline{54}$

```
      1.3125            0.5454...
 16 ) 21.0000      11 ) 6.0000...
     –16                55
      50                 50
     –48                 44
       20                 60
      –16                   ...
        40
       –32
         80
        –80
          0
```

# PERCENT

A percentage is another way of expressing parts of a whole, used most often when discussing real-world quantities. The word "percent" literally means "out of 100;" it is abbreviated with the symbol $\%$. **Ex:** $23\% = \frac{23}{100} = 0.23$

- $100\%$ is one whole. More than $100\%$ is more than one whole.

## CONVERTING PERCENT TO FRACTIONS AND DECIMALS

- **Percent to decimals:** Drop the $\%$ sign and move decimal point two places to the left. Pad with zeroes if necessary. **Ex:** $8\% = 0.08$; $1.5\% = 0.015$
- **Decimals to percent:** Move decimal point two places to the right. Add $\%$ sign. Pad with zeroes if necessary. **Ex:** $0.897 = 89.7\%$; $1.9 = 190\%$
- **Percent to fractions:** Drop $\%$ sign and write number over 100. Reduce.
  **Ex:** $37.5\% = \frac{37.5}{100} = \frac{375}{1000} = \frac{3}{8}$; $66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100} = \frac{2}{3}$
- **Fractions to percent:** Convert fraction to decimal; move decimal point two places to the right and attach $\%$ sign.

## WHAT IS __% OF __?

"Of" means multiplication. Convert the percentage to a decimal or fraction and multiply.

**Ex 1:** What is $40\%$ of 56?
Find $0.40 \times 56$ or $\frac{40}{100} \times 56$, which is 22.4.

**Ex 2:** $87.5\%$ of the 16 boys in Nadine's class like to cook. How many boys in Nadine's class like cooking?
$0.875 \times 16 = 14$ cooking enthusiasts.

## WHAT PERCENT OF __ IS __?

## __ IS WHAT PERCENT OF __?

The "main number"—the one representing a whole, or $100\%$—is the one preceded by "of." We will call the other number the "part." In most problems, the main number will be larger. Divide the part by the main number to find the fraction; convert to percent.

**Ex 1:** What percent of 25 is 4?
$\frac{4}{25} = 0.16 = 16\%$.

**Ex 2:** 18 is what percent of 24?
$\frac{18}{24} = \frac{3}{4} = 75\%$.

## __ IS __% OF WHAT NUMBER?

Here, we know the part and seek the whole. Since $(\text{whole}) \times \left(\frac{\text{percent}}{100}\right) = \text{part}$, we have

$$\text{whole} = 100 \times \frac{\text{part}}{\text{percentage}}.$$

**Ex:** 20 is $8\%$ of what number? **Answer:** whole $= 100 \times \frac{20}{8} = 250$.

## PERCENT INCREASE

**Amount increase:**

$$(\text{amount increase}) = (\text{original amount}) \times \left(\frac{\text{percent increase}}{100}\right)$$

**New amount:**

$$(\text{new amount}) = (\text{original amount}) + (\text{amount increase})$$
$$= (\text{original amount}) \times \left(1 + \frac{\text{percent increase}}{100}\right)$$

**Ex:** Phil knew 144 French verbs two months ago. Now he knows $12.5\%$ more verbs. How many verbs does Phil know now? **Answer:** Phil knows $144 \times (1 + \frac{12.5}{100}) = 162$ verbs.

## PERCENT DECREASE

**Amount decrease:**

$$(\text{amount decrease}) = (\text{original amount}) \times \left(\frac{\text{percent decrease}}{100}\right)$$

**New amount:**

$$(\text{new amount}) = (\text{original amount}) - (\text{amount decrease})$$
$$= (\text{original amount}) \times \left(1 - \frac{\text{percent decrease}}{100}\right)$$

**Ex:** Sally the puppy originally cost \$450. How much does Sally cost during the pet store's $34\%$ off going-out-of-business sale? **Answer:** Sally's new price is $\$450 \times (1 - \frac{34}{100}) = \$297$.

## INTEREST

Suppose that dollar amount $P$ is invested at $r\%$ yearly interest for $t$ years.

- **Simple interest:** $I = P\frac{r}{100}t$. Total amount is $P + I = P\left(1 + \frac{rt}{100}\right)$.
- If the length of time is in months, divide by 12 to find out the length of time in years.
  **Ex:** Ivan invests \$2500 at $2.5\%$ simple interest. How much money does he have after 8 months?
  Here, $P = \$2500$, $r = 2.5$ and $t = \frac{8}{12}$. He earns $\$2500 \times \frac{2.5}{100} \times \frac{8}{12} = \$41.67$ in interest and has a total of \$2541.67 at the end.
- **Compound interest:** In practice, most banks calculate how much interest you have earned every day so that during the second day you earn a small amount of interest on the interest that you earned during the first day. The interest is "compounded daily." This makes a difference for large investments.
  - If the interest is compounded $d$ times during the year, then the total amount after $t$ years is
    $$P\left(1 + \frac{r}{100d}\right)^{td}.$$
  **Ex:** If Ivan's interest is compounded monthly, he would have
  $\$2500 \times \left(1 + \frac{2.5}{100 \times 12}\right)^{\frac{8}{12} \times 12} = \$2541.97$—which is a difference of 30 cents.

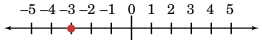
# POSITIVE AND NEGATIVE NUMBERS

## THE NUMBER LINE

The number line is a visual way of keeping track of positive and negative numbers.

- Every point on the line corresponds to some kind of number. The **integers** (whole numbers and their negatives) are evenly spaced and often labelled.
- Positive numbers are to the right of 0, negative numbers are to the left.
- Any given number is larger than every number to its left and smaller than every number to its right, regardless of sign.
- The **absolute value** of a number is its distance from 0. The "absolute value of $n$" is denoted $|n|$. Essentially, $|n|$ is $n$ without the $\pm$ sign.
  - If $n$ is positive, $|n| = n$.
  - If $n$ is negative, $|n| = -n$.

**Ex:** $|3| = 3$; $\left|-\frac{5}{2}\right| = \frac{5}{2}$; $|0| = 0$



The number line and the point -3

## ADDING AND SUBTRACTING SIGNED NUMBERS

*TIP:* Negative signs and subtraction signs are really the same thing.

- **To add numbers with the same sign:** Add the values and keep the sign.
  **Ex:** $-5 + (-1) = -(5 + 1) = -6$
- **To add numbers with different signs:** Subtract the values and take the sign of the "bigger" number (bigger in absolute value).
  **Ex 1:** $-6 + 14 = +(14 - 6) = 8$ **Ex 2:** $7 + (-11) = -(11 - 7) = -4$
- **To subtract any signed number:** Switch its sign and add it.
  **Ex:** $-5 - (-14) = -5 + 14 = 9$
- When subtracting a quantity in parentheses, distribute the negative sign by flipping the sign of *every term* inside the parentheses. **Ex:** $3 - (5 - 1) = 3 + (-5) + 1 = -1$
  Flip only once for a product of two numbers.
  **Ex:** $4 - (3 - 5 \times (-3)) = 4 - 3 + 5 \times (-3) = -14$

On the number line...

- Adding a positive number (or subtracting a negative number) means moving to the right.
- Adding a negative number (or subtracting a positive number) means moving to the left.

## MULTIPLYING AND DIVIDING SIGNED NUMBERS

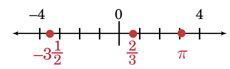
Multiply or divide the numbers disregarding sign. Then choose the sign:

- **Same sign** $(+) \times (+)$ or $(-) \times (-)$: The answer is positive $(+)$.
  **Ex 1:** $(-20) \div (-4) = 5$ **Ex 2:** $(-4) \times \left(-\frac{1}{2}\right) = 2$
- **Different signs** $(-) \times (+)$ or $(+) \times (-)$: The answer is negative $(-)$.
  **Ex:** $4 \div (-3) = -\frac{4}{3}$

## FRACTIONS AND DECIMALS

All of the rules above apply to whole numbers, fractions, decimals—in fact, to all numbers on the number line.

- In signed mixed numbers, the fractional part gets the same sign as the whole part.
  **Ex:** $-3\frac{1}{2} = -\left(3 + \frac{1}{2}\right) = -3 - \frac{1}{2}$



Real numbers on the number line

# POWERS AND ROOTS

Raising to powers, or **exponentiation,** is repeated multiplication—just as multiplication is repeated addition.

## SQUARES AND CUBES

- The **square** of a number $n$, written as $n^2$ and pronounced "$n$ squared," is its product with itself. **Ex:** $3^2 = 3 \times 3 = 9$
  - Any number can be squared (**Ex:** $(1.2)^2 = 1.44$), but squares of integers are called **perfect squares.**
  - The first few perfect squares are $0^2 = 0$, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$, $11^2 = 121$, $12^2 = 144, \ldots$
  - The square of any nonzero number is always positive. (The product of two numbers with the same sign is positive.)
  - Following order of operations, $-4^2 = -(4^2) = -16$, which is not the same as $(-4)^2 = 16$.
- The **cube** of a number $n$, written $n^3$ and pronounced "$n$ cubed," is its product with itself twice. **Ex:** $7^3 = 7 \times 7 \times 7 = 343$. A **perfect cube** is the cube of an integer.
  - The first few positive perfect cubes are $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125, \ldots$

## POWERS AND EXPONENTS

The expression $3^4$ means 3 multiplied by itself 4 times: $\underbrace{3 \times 3 \times 3 \times 3}_{4 \text{ times}}$.

In the expression $3^4$, 3 is the **base**—the number being multiplied—and 4 is the **exponent**—the number of times the multiplication is performed.

- $3^4$ is read as "three to the fourth power." We also say that 81 (which is $3^4$) is a **power** of 3.
- **Natural number exponents:** Any natural number can be an exponent. Any number to the first power is itself. **Ex:** $3^1 = 3$
- **Zero power:** It is convenient to define the zero power of any number to be 1 (you can think of it as the "null" product. *EXCEPTION:* The expression $0^0$ is undefined.

## OPERATIONS ON POWERS

- **Multiplying powers:** If the bases of two powers are the same, then to multiply, add their exponents. **Ex:** $2^3 \times 2^8 = 2^{11}$
- **Dividing powers:** If the bases of two powers are the same, then to divide, subtract their exponents. **Ex:** $3^7 \div 3^4 = 3^{7-4} = 3^3$
- **Raising powers to powers:** To raise a power to a power, multiply the exponents. **Ex:** $(2^3)^2 = 2^6$
  There is no way to combine the sum or difference of two powers into one expression.
  **Ex:** $2^4 - 2^3$ is as "simplified" as it can be before evaluating.
- **Raising a product to a power:** A product raised to a power is equal to the product of powers with different bases. **Ex:** $(3 \times 4)^5 = 3^5 \times 4^5$
- **Raising a quotient to a power:** A power of a quotient is the quotient of the powers.
  **Ex:** $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$
  - This does *not* work for sums and differences: $(3+4)^2 \neq 3^2 + 4^2$.
- **Negative powers:** Raising to a negative power is the same as taking the reciprocal of the positive power. **Ex:** $4^{-2} = \frac{1}{4^2}$

## SQUARE ROOTS

A **square root** of a positive integer $n$, written $\sqrt{n}$, is the positive number whose product with itself is $n$. **Ex:** $\sqrt{25} = 5$. The $\sqrt{\ }$ sign is called the **radical** sign. In $\sqrt{25}$, 25 is the **radicand**.

- It is true that $(-5)^2 = 5^2 = 25$, but by convention, the expression $\sqrt{25}$ means the *positive* square root. To indicate the negative root, write $-\sqrt{25}$.

Perfect squares have whole number square roots. The square roots of all other numbers are on the number line, but they are irrational—their decimal expansions go on forever, never repeating. **Ex:** $\sqrt{2} = 1.41421356\ldots$

- To estimate a square root, sandwich the radicand between perfect squares.
  **Ex:** $\sqrt{57}$ is between 7 and 8 because 57 is between $49 = 7^2$ and $64 = 8^2$.

## SIMPLIFYING SQUARE ROOTS

- **Square root of product:** The root of a product is the product of the roots.
  **Ex:** $\sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$
- **Square root of quotient:** The root of a quotient is the quotient of the roots. **Ex:** $\sqrt{\frac{45}{5}} = \frac{\sqrt{45}}{\sqrt{5}}$
- A square root expression is considered **simplified** if the radicand has no repeated factors. To simplify, factor the radicand and move any factor that appears twice outside the square root sign. **Ex:** $\sqrt{60} = \sqrt{2 \times 2 \times 3 \times 5} = \sqrt{2 \times 2}\sqrt{3 \times 5} = 2\sqrt{15}$

## CUBE ROOTS

A **cube root** of an integer $n$, written $\sqrt[3]{n}$ is the number whose cube is $n$. **Ex:** $\sqrt[3]{512} = 8$

# MEASUREMENT

*For more details, see the Weights and Measures SparkChart.*

## METRIC SYSTEM

Used by most industrialized nations except the United States.

- Basic units: **meter** (m) for length, **liter** (L) for volume, and **kilogram** (kg) for mass.
  - The meter is about $\frac{1}{40{,}000{,}000}$ of the earth's circumference; a liter is the volume of a cube 0.1 m on each side; a kilogram is the mass of a liter of water at 4°C.
- The metric system has principal UNITS (meter, liter, gram) that are made bigger or smaller by different prefixes—which all indicate multiplication or division by some power of ten.
  **Ex:** There are 100 centiUNITS in every UNIT. A kiloUNIT is 1000 UNITS.

| Multiplication factor | Prefix | Symbol | Common examples |
|---|---|---|---|
| $1{,}000{,}000 = 10^6$ | **mega** | M | |
| $1000 = 10^3$ | **kilo** | k | **kilometer** (km), **kilogram** (kg) |
| $1 = 10^0$ | — | — | **meter** (m), **liter** (L), **gram** (g) |
| $0.1 = 10^{-1}$ | **deci** | d | **decimeter** (dm) |
| $0.01 = 10^{-2}$ | **centi** | c | **centimeter** (cm) |
| $0.001 = 10^{-3}$ | **milli** | m | **millimeter** (mm), **milligram** (mg), **milliliter** (mL) |
| $0.000001 = 10^{-6}$ | **micro** | $\mu$ | **micrometer** ($\mu$m) |

## ENGLISH SYSTEM

Used in the United States.

- **Length:** mile (mi), yard (yd), foot (ft), inch (in).
  1 mi = 1760 yds = 5280 ft
  1 yd = 3 ft = 36 in
  1 ft = 12 in
- **Volume:** gallon, quart, pint, fluid ounce (fl. oz).
  1 gallon = 4 quarts = 8 pints = 128 fl. oz
  1 quart = 2 pints = 32 fl. oz
  1 pint = 16 fl. oz
- **Mass:** pound (lb), ounce (oz).
  1 lb = 16 oz

## TIME

1 year = 12 months = 52 weeks = 365 days (366 during a leap year)
1 week = 7 days = 168 hours
1 day = 24 hours = 1440 minutes
1 hour = 60 minutes = 3600 seconds
1 minute = 60 seconds

## SCIENTIFIC NOTATION

Very large and very small numbers—which often come up in chemistry and physics—can be expressed compactly in **scientific notation** as $a \times 10^n$, where $a$ is a decimal between 1 and 10 and $n$ is any integer (possibly negative). Very large numbers have positive $n$; very small numbers have negative $n$.

- **To convert from scientific notation:** In $a \times 10^n$, the exponent $n$ tells how many digits the decimal point must be moved. If $n$ is positive, move the decimal point to the right; if negative, to the left.

**Ex 1:** 0002.343 x $10^{-3}$ = 0.002343

**Ex 2:** 6.59000 x $10^{5}$ = 659000

- **To convert to scientific notation:** The exponent is the number of places that the first nonzero digit of the number must be moved to land into the ones' place.
  - If the number is less than 1, the exponent is negative; if 10 or more, positive; if between 1 and 10, the exponent is zero.

  **Ex 1:** $430 = 4.3 \times 10^2$
  **Ex 2:** $0.109 = 1.09 \times 10^{-1}$
- **Adding** and **subtracting** in scientific notation: The least thought-intensive way to do this is to convert both numbers out of scientific notation, perform the operation, and then convert back.
  *KEY:* You can only add or subtract the coefficients directly if they are multiplied by the same power of 10.
  **Ex:** $1.65 \times 10^5 - 9.0 \times 10^4 = 16.5 \times 10^4 - 9.0 \times 10^4$
  $= (16.5 - 9.0) \times 10^4 = 7.5 \times 10^4$
- **Multiplying** and **dividing** in scientific notation: Multiply (or divide) the coefficients, add (or subtract) the exponents, and convert back to scientific notation.
  **Ex 1:** $\left(3.3 \times 10^{-2}\right)\left(6.20 \times 10^{23}\right) = (3.3 \times 6.20) \times 10^{(-2)+23} = 20.46 \times 10^{21}$
  Convert to get $2.046 \times 10^{22}$.

## SIGNIFICANT DIGITS

Recognizing which digits in a measurement are "significant" helps determine which numbers in a calculation combining several measurements are important.

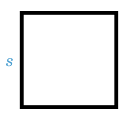
- **Counting significant digits.** In the examples, significant digits are underlined.
  - All nonzero digits are significant. Also, any zero in between nonzeroes is significant.
    **Ex:** $\underline{203}$ cm
  - Zeroes in front of the first nonzero digit are not significant (even if a decimal point is present). **Ex:** $0.000\underline{702}$ g
  - Zeroes after the last nonzero digit are significant only if a decimal point is present.
    **Ex:** $\underline{25}.\underline{00}$ mL, $\underline{3}00$ mm, $0.0\underline{2300}$ sec
- **Significant digits in scientific notation:**
  - All digits written are significant. **Ex:** $\underline{3}.\underline{010} \times 10^{-2}$
  - To indicate that there are two significant digits in 400 kg, write the number in scientific notation as $4.0 \times 10^2$ kg.
- **Multiplication and division:** The number of significant digits in a product or a quotient should be no more than the number of significant digits in any other value involved in the calculation. Round when necessary. **Ex:** $(9.2\,\text{m}) \times (354\,\text{m}) = 3300\,\text{m}^2$
- **Addition and subtraction:** A sum or difference should be as precise as the least precise quantity involved. Again, round.
  - A quantity is **precise** when it is "fine"—when it has significant digits in small places.
    **Ex:** 3,899,900—whose last significant digit is in the hundreds' place—is less precise than 9.9, whose last significant digit is in the tenths' place.

  **Ex 1:** $4.8\,\text{mL} + 5.32\,\text{mL} = 10.1\,\text{mL}$ because the less precise summand (4.8 mL) is precise to the tenths' digit.
  **Ex 2:** $1100\,\text{kg} - 523\,\text{kg} = 600\,\text{kg}$ because the less precise measurement (1100 kg) is precise to the hundreds' digit. However, $1.10 \times 10^3\,\text{kg} - 523\,\text{kg} = 580\,\text{kg}$ because the less precise measurement ($1.10 \times 10^3$ kg) is precise to the tens' digit.

# GEOMETRY AND MEASUREMENT

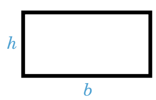
*For more details, see the Geometry SparkChart.*

- The **perimeter** ($P$) of a plane figure is the total distance around its edges. It is measured in units of length: cm, m, km, in, ft. The perimeter of a circle is called the **circumference** ($C$).
- The **area** ($A$) of a plane figure is the number of unit squares that can be fit inside it—a measurement of how much 2-dimensional space it encloses. It is measured in units of length$^2$: cm$^2$, m$^2$, km$^2$, in$^2$, ft$^2$.
- The **volume** ($V$) of a solid is the number of cubic units that can be fit inside it. Measured in units of length$^3$: cm$^3$, m$^3$, km$^3$, in$^3$, ft$^3$.
- $\pi \approx 3.14$



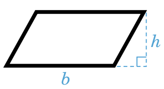

**SQUARE**

**Perimeter:** $P = 4(\text{side length}) = 4s$
**Area:** $A = (\text{side length})^2 = s^2$



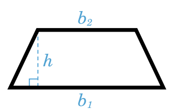

**RECTANGLE**

**Perimeter:** $P = 2(b+h) = 2\big((\text{base}) + (\text{height})\big)$
**Area:** $A = bh = (\text{base}) \times (\text{height})$



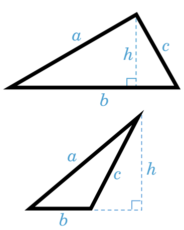

**PARALLELOGRAM**

**Area:** $A = bh = (\text{base}) \times (\text{height})$



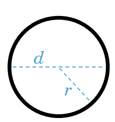

**TRAPEZOID**

**Area:** $A = \frac{1}{2}(b_1 + b_2)h = (\text{average of bases}) \times (\text{height})$



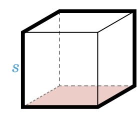

**TRIANGLE**

**Perimeter:** $P = a + b + c = \text{sum of side lengths}$
**Area:** $A = \frac{1}{2}bh = \frac{1}{2}(\text{base}) \times (\text{height})$



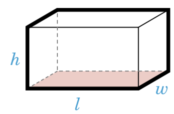

**CIRCLE**

**Circumference:**
$C = 2\pi r = \pi d = 2\pi(\text{radius}) = \pi(\text{diameter})$
**Area:** $A = \pi r^2 = \pi(\text{radius})^2$



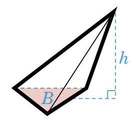

**CUBE**

**Surface area:** $SA = 6s^2 = 6(\text{side length})^2$
**Volume:** $V = s^3 = (\text{side length})^3$



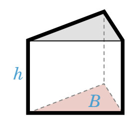

**RECTANGULAR SOLID**

**Surface area:** $SA = 2(lw + lh + hw)$
**Volume:** $V = lwh = (\text{length}) \times (\text{width}) \times (\text{height})$



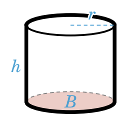

**PYRAMID**

**Volume:** $V = \frac{1}{3}Bh = \frac{1}{3}(\text{Base area}) \times (\text{height})$



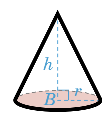

**PRISM**

**Volume:** $V = Bh = (\text{Base area}) \times (\text{height})$



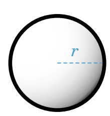

**CYLINDER**

**Volume:** $V = Bh = (\text{Base area}) \times (\text{height})$
$= \pi r^2 h = \pi(\text{radius})^2 \times (\text{height})$




**CONE**

**Volume:** $V = \frac{1}{3}Bh = \frac{1}{3}(\text{Base area}) \times (\text{height})$
$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(\text{radius})^2 \times (\text{height})$




**SPHERE**

**Surface area:** $SA = 4\pi r^2 = 4\pi(\text{radius})^2$
**Volume:** $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\text{radius})^3$

# STATISTICS AND GRAPHS

## STATISTICS

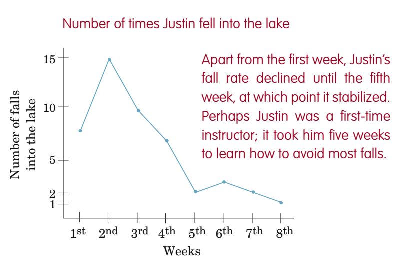
Because lists of data can be cumbersome, we use statistical quantities to typify sets of numbers. Suppose our collection of data is $A = \{2, 2, 1, 8, 9, 3, 6\}$.

- **Mean:** The **mean** or **average** of a collection of numbers is the sum of all the elements divided by the number of elements. The mean of the numbers in set $A$ is $\frac{2+2+1+8+9+3+6}{7} = 4.43$.
- **Median:** The median is the element with the property that there are as many elements smaller than the median as there are elements larger than the median. If you write the elements in order from least to greatest; the median is the middle element. The median of the numbers in $A$ is 3.
  - If there are an even number of elements, the median is the average of the middlemost two.
- **Mode:** The mode is the most frequently occurring value. The mode of the numbers in $A$ is 2.

## GRAPHS

Data is sometimes presented in a **graph**—a pictorial representation of the information. The three most common types of graphs are:

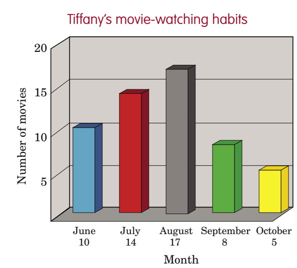
1. **Line graph:** A line graph plots the information on a (usually hidden) grid and connects the data points with lines. The steepness of the connecting lines indicates the rate of increase or decrease from point to point. Stock prices are often presented in line-graph form.



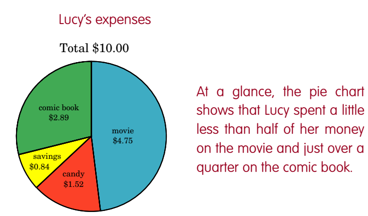

**Ex:** While teaching boating at the Happy Fun summer camp, Justin fell into the water 8 times during the first week, 15 times in the second week, 10 times in the third, 7 times in the fourth, 2 times in the fifth, 3 times in the sixth, 2 times in the seventh, and 1 time in the eighth week.

2. **Bar graph:** A bar graph presents information in a series of bars. Bar graphs show trends, too, but there is more emphasis on sheer amount than in a line graph.

**Ex:** Tiffany saw 10 movies in June, 14 in July, 17 in August, 8 in September, and 5 in October.




3. **Pie chart:** A pie chart (or **pie graph**) presents quantities as parts of a whole; it emphasizes relative amounts.
**Ex:** Budgets are frequently presented in pie-chart form.
**Ex:** Lucy's allowance is \$10. She spent \$1.52 on candy, \$4.75 on a bargain matinee movie, \$2.89 on a comic book, and saved the rest.

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