

e  
Book

# **BASIC MATH FORMULAS**

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## Logarithms

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## Symbol Meaning

N	Set of Natural numbers
Z	Setot Integers numbers
Q	Set of Rational numbers
R	Set of Real number
C	Set of Complex numbers
U	The union of
∩	Intersected with
⊂	ls a subset of
⊆	is a subset of or equal to
⊄	Is not a subset of
⊃	Is superset of
⊇	Is a superset of or equal to
⊅	is not a superset of
\	Set Difference
∈	Is an element of
∉	Is not an element of
[a, b]	Closed interval
]a,b[	Open interval
{a, b, c}	Set of Elements
Ø or {}	Empty Set
( )	Group (do first)
+	Addition (plus, add)
-	Subtraction (minus, subtract)

$\div$	Division
$\times$	Multiplication (times)
$+$	Plus-minus
$<$	Is less than
$\leq$	Is less than or equal to
$>$	Is greater than
$\geq$	Is greater than or equal to
$\Leftrightarrow$	Equivalence
$\Rightarrow$	Implication (implies)
$=$	Equality (is equal to)
$\neq$	Inequality (is not equal to)
$\approx$	Approximately (equal to)
$\equiv$	Congruence
$\Sigma$	Summation
$\Pi$	Product
$\nabla$	Gradient
$\wedge$	And (propositional logic)
$\vee$	Or (propositional logic)
$\exists$	Existential quantification (there is)
$\nexists$	There is not
$\forall$	Universal quantification (for all)
$\neg$ or $\sim$	Negation
$\#$	Cardinality
$:$	Such that
$\therefore$	Therefore

$\therefore$	Because
QED	End of proof (quod erat demonstrandum)
GCD	Greatest Common Divisor
LCM	Lowest common Multiple
$\sqrt{\phantom{x}}$	Square Root
$\sqrt[3]{\phantom{x}}$	Cube Root
!	Factorial
%	Percent
%o	Per mille
°F	Degrees Fahrenheit
°C	Degrees Celsius
$\alpha - A$	Alpha
$\beta - B$	beta
$\gamma - \Gamma$	Gamma
$\delta - \Delta$	Delta
$\varepsilon - E$	Epsilon
$\zeta - Z$	Zeta
$\eta - H$	Eta
$\theta - \Theta$	Theta
$\iota - I$	Iota
$\kappa - K$	Kappa
$\lambda - \Lambda$	Lambda
$\mu - M$	Mu
$\nu - N$	Nu
$\xi - \Xi$	Xi
$\omicron - O$	Omicron

$\pi$	Pi
$\rho$	Rho
$\sigma$	Sigma
$\tau$	Tau
$\nu$	Upsilon
$\varphi$	Phi
$\chi$	Chi
$\psi$	Psi
$\omega$	Omega
$\angle$	Angle
$\Delta$	Angle Measure
$^\circ$	Degrees
$'$	Minutes
$''$	Seconds
$\perp$	Is perpendicular to
$\parallel$	Is parallel to
$\sin()$	Sine
$\cos()$	Cosine
$\tan()$	Tangent
$\cot()$	Cotangent
$\square$	
$v$	Vector
$\  v \ $	Norm of
$ x $	Absolute value (modulus)
$\bar{x}$	Mean
$\tilde{x}$	Median

$\log_a()$	Logarithm with base a
$\ln()$	Natural Logarithm (with base e)
$\log()$	Common Logarithm (with base 10)
$f(x)$	Function
$f'(x)$	Derivative of Function
$\int$	Integral (differential calculus)
$\text{Dom}(f)$	Domain of the function f
$\text{Ran}(f)$	Range of the function f
$f^{-1}$	Inverse Function
$f \circ g$	Function Composition
$\lim()$	Limit
$x \rightarrow a$	x approaches a
$\infty$	Infinity
$\pi$	Pi, $\pi = 3,14159265359\dots$
$e$	Euler's constant, $e = 2,7182 \dots$
$\Phi$	Golden Ratio, $\Phi = 1,6180$
$i$	Imaginary number, $i^2 = -1$
$R(z)$	The real part of a complex number
$I(z)$	The imaginary part of a complex

# Areas

## Square

$$A = l^2$$

$l$  = length of side

## Rectangle

$$A = w \times h$$

$w$ : width,  $h$  : height

## Triangle

$$A = \frac{b \times h}{2}$$

$b$  : base,  $h$  : height

## Rhombus

$$A = \frac{D \times d}{2}$$

$D$  = large diagonal,  $d$  : small diagonal

## Trapezoid

$$A = \frac{B \times b}{2} \times h$$

$B$  : large side,  $b$  : small side,  $h$  : height

## Regular polygon

$$A = \frac{P}{2} \times a$$

$P$  : perimeter

$a$  : apothem

## Circle

$$A = \pi r^2$$

$$P = 2\pi r$$

$r$  : radius

$P$  : perimeter

### Cone (lateral surface)

$$A = \pi r \times s$$

$r$  : radius

$s$  : slant height

### Sphere (surface area)

$$A = 4\pi r^2$$

$r$  : radius

## Volumes

### Cube

$$V = s^3$$

$s$  : side

### Parallelepiped

$$V = l \times w \times h$$

$l$  : length,  $w$  : width,  $h$  : height

### Regular prism

$$V = b \times h$$

$b$  : base,  $h$  : height

### Cylinder

$$V = \pi r^2 \times h$$

$r$  : radius,  $h$  : height

### Cone (or pyramid)

$$V = \frac{1}{3} b \times h$$

$b$  : base,  $h$  : height

## Sphere

$$V = \frac{4}{3}\pi r^3$$

$r$  : radius

## Functions and Equations

### Directly Proportional

$$y = kx \quad k = \frac{y}{x}$$

### Inversely Proportional

$$y = \frac{k}{x} \quad k = yx$$

$k$  : Constant of Proportionality

### Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Concavity

$$ax^2 + bx + c = 0$$

Concave up :  $a > 0$

Concave Down :  $a < 0$

### Discriminant

$$\Delta = b^2 - 4ac$$

### Vertex of the parabola

$$V\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$$

## Concavity

$$y = a(x - h)^2 + k$$

concave up :  $a > 0$

concave down:  $a < 0$

Vertex of the parabola  $V(h, k)$

## Zero-product property

$$A \times B = 0 \iff A = 0 \vee B = 0$$

$$\text{ex: } (x + 2) \times (x - 1) = 0 \iff$$

$$x + 2 = 0 \vee x - 1 = 0 \iff x = -2 \vee x = 1$$

## Difference of two square

$$(a - b)(a + b) = a^2 - b^2$$

$$\text{ex: } (x - 2)(x + 2) = x^2 - 2^2 = x^2 - 4$$

## Perfect square trinomial

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$\text{ex: } (2x + 3)^2 = (2x)^2 + 2 \cdot 2x \cdot 3 + 3^2 = 4x^2 + 12x + 9$$

## Binomial theorem

$$(x + y)^n = \sum_{k=0}^n {}^n C_k k^{n-k} y^k$$

# Exponents

## Product

$$a^m \times a^n = a^{m+n}$$

$$\text{ex: } 3^5 \times 3^2 = 3^{5+2} = 3^7$$

$$a^m \times b^m = (a \times b)^m$$

$$\text{ex: } 3^5 \times 2^5 = (3 \times 2)^5 = 6^5$$

## Quotient

$$a^m \div a^n = a^{m-n}$$

$$\text{ex} : 3^7 \div 3^2 = 3^{7-2} = 3^5$$

$$a^m \div b^m = (a \div b)^m$$

$$\text{ex} : 6^5 \div 2^5 = (6 \div 2)^5 = 3^5$$

$$\text{ex} : 5^3 \div 2^3 = \left(\frac{5}{2}\right)^3$$

## Power of Power

$$(a^m)^p = a^{m \times p}$$

$$\text{ex} : (5^2)^3 = 5^{2 \times 3} = 5^6$$

## Zero Exponents

$$a^0 = 1$$

$$\text{ex} : 8^0 = 1$$

## Negative Exponents

$$a^{-n} = \left(\frac{1}{a}\right)^n$$

$$\text{ex} : 3^{-2} = \left(\frac{1}{3}\right)^2$$

$$\text{ex} : \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$$

## Fractional Exponents

$$a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

$$\text{ex} : a^{\frac{4}{3}} = \sqrt[3]{2^4}$$

## Radicals

### Multiplication

$$\sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{x \times y}$$

$$\text{ex: } \sqrt[3]{2} \times \sqrt[2]{5} = \sqrt[2]{2 \times 5} \Rightarrow \sqrt[3]{10}$$

## Division

$$\sqrt[n]{x} \div \sqrt[n]{y} = \sqrt[n]{\frac{x}{y}}$$

$$\text{ex: } \sqrt[4]{8} \div \sqrt[4]{3} = \sqrt[4]{\frac{8}{3}}$$

## Addition

$$a \sqrt[n]{x} \pm b \sqrt[n]{x} = (a \pm b) \sqrt[n]{x}$$

$$\text{ex: } 4 \sqrt[3]{5} - 2 \sqrt[3]{5} = (4 - 2) \sqrt[3]{5} \Rightarrow 2 \sqrt[3]{5}$$

## Exponents

$$(\sqrt[n]{x})^p = \sqrt[n]{x^p}$$

$$\text{ex: } (\sqrt{2})^3 = \sqrt{2^3} \Rightarrow \sqrt{8}$$

## Radicals

$$\sqrt[n]{\sqrt[p]{x}} = \sqrt[n \cdot p]{x}$$

$$\text{ex: } \sqrt[3]{\sqrt[2]{5}} = \sqrt[3 \times 2]{5} \Rightarrow \sqrt[6]{5}$$

## Exponentiation

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\text{ex: } \sqrt[3]{4^5} = 4^{\frac{5}{3}}$$

$$(\sqrt[n]{a})^n = a$$

$$\text{ex} : (\sqrt{3})^2 = 3$$

### Simplifying Radicals

$$(\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

$$\text{ex} : (\sqrt{4})^5 = \sqrt{4^5}$$

## Trigonometry

### Trigonometry Ratios

$$\sin \alpha = \frac{\text{opp.}}{\text{hyp.}}$$

opp. opposite

hyp. : hypotenuse

$$\cos \alpha = \frac{\text{adj.}}{\text{hyp.}}$$

adj. : adjacent, hyp. : hypotenuse

$$\tan \alpha = \frac{\text{opp.}}{\text{adj.}}$$

opp. : opposite, adj.: adjacent

### Fundamental identities

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha}$$

### Law of sine (aka sine rule)

$$\frac{\sin A}{a} - \frac{\sin B}{b} = \frac{\sin C}{c}$$

### Law of Cosines (aka cosine rule)

$$a^2 = b^2 + c^2 - 2bc\cos A$$

### Heron's formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

### Exact Values

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

$$\sin\left(\frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\tan\left(\frac{\pi}{6}\right) = \sqrt{3}$$

### Angle Relationships

$$\sin(-\alpha) = -\sin\alpha$$

$$\cos(-\alpha) = \cos\alpha$$

$$\tan(-\alpha) = -\tan\alpha$$

$$\sin(\pi - \alpha) = \sin\alpha$$

$$\cos(\pi - \alpha) = -\cos\alpha$$

$$\tan(\pi - \alpha) = -\tan\alpha$$

$$\sin(\pi + \alpha) = -\sin\alpha$$

$$\cos(\pi + \alpha) = -\cos\alpha$$

$$\tan(\pi + \alpha) = \tan\alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \frac{1}{\tan \alpha}$$

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos \alpha$$

$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin \alpha$$

$$\tan\left(\frac{\pi}{2} + \alpha\right) = -\frac{1}{\tan \alpha}$$

$$\sin\left(\frac{3\pi}{2} - \alpha\right) = -\cos \alpha$$

$$\cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin \alpha$$

$$\tan\left(\frac{3\pi}{2} - \alpha\right) = \frac{1}{\tan \alpha}$$

$$\sin\left(\frac{3\pi}{2} + \alpha\right) = -\cos \alpha$$

$$\cos\left(\frac{3\pi}{2} + \alpha\right) = \sin \alpha$$

$$\tan\left(\frac{3\pi}{2} + \alpha\right) = -\frac{1}{\tan \alpha}$$

## Trigonometric Equations

$$\cos x = \cos \alpha \iff x = \alpha + 2k\pi \vee x = -\alpha + 2k\pi, k \in \mathbb{Z}$$

$$\tan x = \tan \alpha \iff x = \alpha + k\pi, k \in \mathbb{Z}$$

### Sum Formulas

$$\sin(a + b) = \sin a \times \cos b + \sin b \times \cos a$$

$$\cos(a + b) = \cos a \times \cos b - \sin a \times \sin b$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \times \tan b}$$

### Difference Formulas

$$\sin(a - b) = \sin a \times \cos b - \sin b \times \cos a$$

$$\cos(a - b) = \cos a \times \cos b + \sin a \times \sin b$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \times \tan b}$$

### Double Angle Formulas

$$\sin(2a) = 2 \times \sin a \times \cos a$$

$$\cos(2a) = \cos^2 a - \sin^2 a$$

$$\tan(2a) = \frac{2 \times \tan a}{1 - \tan^2 a}$$

## Geometry

### Euler's Polyhedral Formula

$$F + V = E + 2$$

F : Face, V: Vertex, E : Edge

Sum of interior angles of a regular polygon

$$S_i = (n - 2) \times 180^\circ$$

$n$ : Number of sides

### Pythagorean theorem

$$H^2 = C_1^2 + C_2^2$$

Hypotenuse : $H$

Leg:  $C_1$  e  $C_2$

### Distance between two points

$$\overline{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

ex : A(8, 2) e B(4, -1)

$$\begin{aligned}\overline{AB} &= \sqrt{(8 - 4)^2 + (2 + 1)^2} \iff \\ \overline{AB} &= \sqrt{16 + 9} \iff \overline{AB} = 5\end{aligned}$$

### Midpoints

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

ex : A(2, 6) e B(4, -2)

$$M\left(\frac{2 + 4}{2}, \frac{6 + (-2)}{2}\right) \iff M(3, 2)$$

### Slope-intercept form

slop :  $m$ , Y intercept :  $b$

$$y = mx + b$$

### Vector Form

Direction vector  $\vec{u} \square (u_1, u_2, u_3)$

Point  $(x_0, y_0, z_0)$

$$(x, y, z) = (x_0, y_0, z_0) + k(u_1, u_2, u_3), k \in \mathbb{R}$$

### Cartesian Form

Direction vector  $\vec{u} (u_1, u_2, u_3)$

Point  $(x_0, y_0, z_0)$

$$\frac{x - x_0}{u_1} = \frac{y - y_0}{u_2} = \frac{z - z_0}{u_3}$$

### Parametric Form

Direction vector  $\vec{u} (u_1, u_2, u_3)$

Point  $(x_0, y_0, z_0)$

$$\begin{cases} x = x_0 + Ku_1 \\ y = y_0 + Ku_2, k \in \mathbb{R} \\ z = z_0 + Ku_3 \end{cases}$$

### Equation of a plane

#### Cartesian Form

Normal vector  $\vec{n} (n_1, n_2, n_3)$

Point  $(x_0, y_0, z_0)$

$$n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$$

#### Scalar Form

Normal vector  $\vec{n} (n_1, n_2, n_3)$

$$n_1x + n_2y + n_3z + d = 0$$

## **Equation of a circle**

Center( $x_0, y_0$ ) and radius r

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

## **Equation of a Sphere**

Center( $x_0, y_0, z_0$ ) and radius r

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

## **Equation of an ellipse**

Center(h, k) Axis a and b

$$\left(\frac{x - h}{a}\right)^2 + \left(\frac{y - k}{b}\right)^2 = 1$$

## **Logic**

### **Conjunction**

p	q	$p \wedge q$
V	V	V
V	F	F
F	V	F
F	F	F

### **Conjunction**

p	q	$p \vee q$
V	V	V
V	F	V
F	V	V

F F F

### Implication

p q p → q

V V V

V F F

F V V

F F V

### Law of non contradiction

F ∧¬ p ⇒ F

### Law of the excluded middle

p ∨¬ p ⇒ V

### Double Negation

¬(¬ p) ⇒ p

### Commutativity

Conjunction p ∧ p ⇒ q ∧ p

Disjunction p ∨ p ⇒ q ∨ p

### Associativity

Conjunction (p ∧ p) ∧ r ⇒ p ∧ (q ∧ r)

Disjunction (p ∨ q) ∨ r ⇒ p ∨ (q ∨ r)

### Neutral Element

Conjunction p ∧ V ⇒ p

Disjunction p ∨ F ⇒ p

### Absorbing Element

Conjunction  $p \wedge F \iff F$

Disjunction  $p \vee V \iff V$

### Idempotence

Conjunction  $p \wedge p \iff p$

Disjunction  $p \vee p \iff p$

### Distributive Property

Conjunction  $p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$

Disjunction  $p \vee (p \wedge r) \iff (p \vee p) \wedge (p \vee r)$

### Properties of Implication

Transitive  $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

Implication and Disjunction  $(p \Rightarrow q) \iff \neg p \vee q$

Negation  $\neg(p \Rightarrow q) \iff p \wedge \neg q$

Contrapositive of an implication  $(p \Rightarrow q) \iff (\neg q \Rightarrow \neg p)$

### Properties of Equivalence

Double implication  $(p \iff q) \iff [(q \Rightarrow q) \wedge (p \Rightarrow q)]$

Transitive  $[(p \iff q) \wedge (q \iff r)] \Rightarrow (p \iff r)$

Negation  $\neg(p \iff q) \iff [(p \wedge \neg q) \vee (q \wedge \neg p)]$

### De Morgan's laws

Negation of a Conjunction  $\neg(p \wedge q) \iff \neg p \vee \neg q$

Negation of a Disjunction  $\neg(p \vee q) \iff \neg p \wedge \neg q$

### Negation of Universal Quantifier

$\neg(\forall x, p(x)) \iff \exists x : \neg p(x)$

### Negation of Existential Quantifier

$$\neg (\exists x : \neg p(x)) \iff \forall x, \neg p(x)$$

## Vector

### Notation

$$\boxed{AB} = B - A = (b_1 - a_1, b_2 - a_2)$$

### Magnitude

$$\| \boxed{u} \| = \sqrt{(u_1)^2 + (u_2)^2}$$

### Square of magnitude of a vector

$$(u)^2 = \| \boxed{u} \|^2$$

### Calculations

$$A + \boxed{u} = (a_1 + u_1, a_2 + u_2)$$

$$\boxed{u} + \boxed{v} = (u_1 + v_1, u_2 + v_2)$$

$$k \times \boxed{u} = (k \times u_1, k \times u_2)$$

### The Scalar or Dot Product

$$\boxed{u} \cdot \boxed{v} = u_1 \times v_1 + u_2 \times v_2$$

$$\boxed{u} \cdot \boxed{v} = \| \boxed{u} \| \times \| \boxed{v} \| \times \cos(\boxed{u} \wedge \boxed{v})$$

### Angle between two lines

Direction vector of lines :  $\boxed{u}$  e  $\boxed{v}$

angle :  $\alpha$

$$\cos \alpha = \frac{|u \cdot v|}{\|u\| \|v\|}$$

## Statistic

### Summation Rules and Properties

$$\sum_{i=p}^n \lambda = (n - p + 1)\lambda$$

$$\sum_{i=1}^n \lambda x_i = \lambda \sum_{i=1}^n x_i$$

$$\sum_{i=1}^n (x_i + y_i) = \sum_{i=1}^n x_i + \sum_{i=1}^n y_i$$

$$\sum_{i=1}^n x_i = \sum_{i=1}^n x_i + \sum_{i=p+1}^n x_i$$

### Used Symbols

#### Statistical sample

$$x = (x_1, x_2, x_3, \dots, x_n)$$

#### Sample Size

$$N$$

#### Absolute Frequency

$$n_i$$

#### Relative Frequency

$$f_i = \frac{n_i}{N}$$

### Cumulative (Absolute) Frequency

$$N_i$$

### Cumulative Relative Frequency

$$F_i$$

### Ungrouped Data

$$\bar{x} = \frac{\sum_{i=1}^k x_i}{N}$$

### Grouped Data

$$\bar{x} = \frac{\sum_{i=1}^k n_i x_i}{N}$$

$$\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{N}$$

### Median

$$\text{If } N \text{ is odd } M_e = x_k, k = \frac{N+1}{2}$$

$$\text{If } N \text{ is even } M_e = \frac{x_k + x_{k+1}}{2}, k = \frac{N}{2}$$

### Sum of Deviations from the mean

$$\sum_{i=1}^k d_i = \sum_{i=1}^k (x_i - \bar{x}) = 0$$

## **Sum of Squared Deviations from the Mean**

Ungrouped Data

$$SS_x = \sum_{i=1}^k (x_i - \bar{x})^2$$

$$SS_x = \sum_{i=1}^k x_i^2 - k\bar{x}^2$$

Grouped Data

$$SS_x = \sum_{i=1}^k (x_i - \bar{x})^2 n_i$$

## **Sample Variance**

$$S_x^2 = \frac{SS_x}{N - 1}$$

## **Sample Standard Deviation**

$$S_x = \sqrt{\frac{SS_x}{N - 1}}$$

## **Sequences**

### **Arithmetic sequences**

Common difference  $r = u_{n+1} - u_n$

Expression for the nth term  $u_n = u_1 + (n - 1)r$

### **Monotonicity**

Increasing if  $r > 0$

Decreasing if  $r < 0$

$$\text{Sum of the first } n \text{ terms } S_n = \frac{u_1 + u_n}{2} \times n$$

## Geometric Sequences

$$\text{Common ratio } r = \frac{u_{n+1}}{u_n}$$

$$\text{Expression for the } n\text{th term } u_n = u_1 \times r^{n-1}$$

## Monotonicity

Increasing if  $u_1 > 0 \wedge r > 1$

Decreasing if  $u_1 < 0 \wedge r > 1$

Not Monotonic if  $r < 0$

$$\text{Sum of the first } n \text{ terms } S_n = u_1 \times \frac{1 - r^n}{1 - r}$$

## Simple Interest

$$FV = P \times (1 + r \times t)$$

## Compound Interest

$$FV = P \times (1 + r)^t$$

## Derivatives

### Average rate of change between two points

$$\text{Slope of the Secant Line } [a, b] \text{ SSL} = \frac{f(b) - f(a)}{b - a}$$

### Rate of change at a point

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

### Constant

$$a' = 0$$

### Multiplication by constant

$$(mx)' = m$$

### Power Rule

$$(u^n)' = n \times u^{n-1} \times u'$$

### Root

$$(\sqrt[n]{u})' = \frac{u'}{n \times \sqrt[n]{u^{n-1}}}$$

### Exponential

$$(a^u)' = u' \times a^u \times \ln a$$

### Exponential base

$$(e^u)' = u' \times e^u$$

### Sum Rule

$$(u + v)' = u' + v'$$

### Product Rule

$$(u \cdot v)' = u'v + uv'$$

## Quotient Rule

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

## Chain Rule

$$(gof)' = g'(f) + f'$$

## Sine

$$(\sin u)' = u' \times \cos u$$

## Cosine

$$(\cos u)' = -u' \times \sin u$$

## Tangent

$$(\tan u)' = \frac{u'}{\cos^2 u}$$

## Logarithms

$$(\log_a u)' = \frac{u'}{u \times \ln a}$$

## Natural logarithm

$$(\ln u)' = \frac{u'}{u}$$

## Probability and Sets

### Commutative

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

### **Associative**

$$A \cup (B \cup C) = A \cup (B \cup C)$$

$$A \cap (B \cap C) = A \cap (B \cap C)$$

### **Neutral element**

$$A \cap \emptyset = A$$

$$A \cap E = A$$

### **Absorbing Element**

$$A \cup E = E$$

$$A \cap \emptyset = \emptyset$$

### **Distributive**

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cup (B \cup C) = (A \cap B) \cup (A \cap C)$$

### **De Morgan's laws**

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

### **Laplace laws**

$$P(A) = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

### **Complement of an Event**

$$P(\overline{A}) = 1 - P(A)$$

### **Union of Events**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

## Conditional Probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

## Independent Events

$$P(A | B) = P(A)$$

$$P(A \cap B) = P(A) \times P(B)$$

## Permutation

$$P_n = n! = n \times (n - 1) \times \dots \times 2 \times 1$$

## Permutations without repetition

$${}^n A_p = n^p$$

## Combination

$${}^n C_p = \frac{{}^n A_p}{p!} = \frac{n!}{(n - p)! \times p!}$$

## Probability Distribution

### Average value

$$\mu = x_1 p_1 + x_2 p_2 + \dots + x_k p_k$$

### Standard deviation

$$\sigma = \sqrt{\sum_{i=1}^k p_i (x_i - \mu)^2}$$

## Binomial distribution

$$P(X = k) = {}^n C_k \cdot p^k \cdot (1 - p)^{n - k}$$

# Logarithms

## Definition

$$\log_a b = x \iff b = a^x$$

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log_a a^b = b$$

## Product

$$\log_a(u \times v) = \log_a u + \log_a v$$

## Quotient

$$\log_a \left(\frac{u}{v}\right) = \log_a u - \log_a v$$

## Exponential

$$\log_a u^v = v \times \log_a u$$

## Change of Base

$$\log_a u^v = \frac{\log_b u^v}{\log_b a}$$

# Special Limits

$$\lim_{x \rightarrow +\infty} \frac{a^x}{x^p} = +\infty \quad (a, p \in \mathbb{R})$$

$$\lim_{x \rightarrow +\infty} \frac{\log_a x}{x} = 0 \quad (a > 1, a \in \mathbb{R})$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\ln(x) - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow +\infty} \frac{\sin x}{x} = 0$$

$$\lim_{u_n \rightarrow +\infty} \left(1 + \frac{k}{u_n}\right)^{u_n} = e^k$$

$$\log\left(1 + \frac{1}{n}\right)^n = e \quad (n \in \mathbb{N})$$

## Integrals and primitives

### Common primitives

$$\int 1 \, dx = x + c, \quad c \in \mathbb{R}$$

$$\int (u(x))^\alpha \cdot u'(x) \, dx = \frac{(u(x))^{\alpha+1}}{\alpha+1} + c, \quad \alpha \in \mathbb{R} \setminus \{-1\}, \quad c \in \mathbb{R}$$

$$\int \frac{u'(x)}{u(x)} \, dx = \ln(|u(x)|) + c, \quad c \in \mathbb{R}$$

$$\int e^u(x) \cdot u'(x) \, dx = e^u(x) + c, \quad c \in \mathbb{R}$$

$$\int \sin(u(x)) \cdot u'(x) \, dx = -\cos(u(x)) + c, \quad c \in \mathbb{R}$$

$$\int \cos(u(x)) \cdot u'(x) dx = \sin(u(x)) + c, c \in \mathbb{R}$$

### Linearity rules of Integration

$$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

$$\int k \cdot f(x) dx = k \int f(x) dx$$

### Properties of Definite Integrals

$$\int_b^a f(x) dx = - \int_a^b f(x) dx$$

$$\int_b^a f(x) dx = 0$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$\int_a^b (f(x) + g(x)) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

$$\int_a^b k \cdot f(x) dx = k \int_a^b f(x) dx$$

### Barrow's rule

$$\int_a^b f(x) dx = F(b) - F(a).$$

where  $F$  is primitive from  $f$  in the interval  $[a, b]$

## Complex Numbers

### Complex Number

$$z = a + bi$$

### Conjugate

$$\bar{z} = a + bi$$

## Symmetry

$$- z = - a - bi$$

## Equality

$$a + bi = c + di \iff a = c \wedge b = d$$

## Addition

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

## Subtraction

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

## Multiplication

$$(a + bi) \times (c + di) = (ac - bd) + (ad + bc)i$$

## Division

$$\frac{a + bi}{c + di} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} i$$

## Inverse

$$z^{-1} = \frac{1}{z}$$

$$z^{-1} = \frac{1}{|z|^2} \cdot \bar{z}$$

## Properties

$$\bar{\bar{z}} = z$$

$$|z| = |\bar{z}|$$

$$|z|^2 = z \bar{z}$$

$$\operatorname{Re}(z) = \frac{z + \bar{z}}{2}$$

$$\operatorname{Im}(z) = \frac{z - \bar{z}}{2i}$$

### Exponential to Algebraic form conversion

#### Angle

$$\arg(z) = \theta$$

$$\theta = \tan^{-1} \left( \frac{b}{a} \right)$$

#### Distance

$$|z|$$

$$|z| = \sqrt{a^2 + b^2}$$

### Exponential form

#### Complex Number

$$z = |z|e^{i\theta}$$

$$z = |z|(\cos \theta + i \sin \theta)$$

#### Conjugate

$$\bar{z} = |z|e^{i(-\theta)}$$

#### Symmetry

$$-z = |z|e^{i(\theta + \pi)}$$

#### Multiplication

$$z_1 = |z_1|e^{i\theta_1}$$

$$z_2 = |z_2| \cdot e^{i\theta_2}$$

$$z_1 \times z_2 = |z_1| |z_2| \cdot e^{i(\theta_1 + \theta_2)}$$

### Division

$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} \cdot e^{i(\theta_1 - \theta_2)}$$

### Exponentiation

$$z^n = |z|^n \cdot e^{in\theta}$$

### Radicals

$$\sqrt[n]{|z|^n \cdot e^{i\theta}} = \sqrt[n]{|z|} \cdot e^{i\left(\frac{\theta + 2k\pi}{n}\right)}, \quad k \in \{0, \dots, n-1\}, \quad n \in \mathbb{N}$$

## Basic math formulas



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