MATHEMATICS FORMULAS HANDBOOK

The Ultimate Handbook for Success

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Dedicated to aspiring mathematicians, may this handbook be your guiding light through the world of formulas.

Happy exploring!

From the Author,
Abu Muniru.

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INTRODUCTION

I welcome you all to the "Mathematics Formulas Handbook: The Ultimate Handbook for Success." I understand that many people may not enjoy mathematics due to its formulas, logical equations, and challenging questions that require significant thought.

But don't worry any further, because this book is here to help. We conducted research and surveys to identify commonchallenges in mathematics. Based on our findings, we've decided to start the book with "Number System," the first chapter, as many learners struggle with understanding this fundamental concept.

Some may not realize its importance, but the revolution in mathematics begins with the number system. Without grasping the basics, progress in any subject is challenging.

This well-structured book aims to make navigation easy for you, ensuring a smooth and enjoyable learning experience with formulas.

Happy learning!

CHAPTER 1: NUMBER SYSTEM

A number system in mathematics is a way of expressing numbers using digits or symbols, typically organized in a systematic pattern. It provides a consistent method for representing and manipulating numerical quantities.

Natural Numbers

Natural numbers are a set of positive integers that begin from 1 and continue indefinitely. They are the counting numbers used for basic counting and ordering. These numbers do not include zero or any negative values.

Example: 1, 2, 3, 4, 5, 10, 20, 30, 100, 2000 etc.

Whole Numbers

Whole numbers consist of zero and all positive integers without any fractional or decimal parts.

Example: 0,1,2,3,4, 10, 30, 99, 39, 300, etc.

Integers

Integers are whole numbers that include positive, negative, and zero.

Positive Integers: Whole numbers greater than zero.

Examples: 1, 7, 123.

Negative Integers: Whole numbers less than zero.

Examples: -2, -45, -789.

Zero: The integer that represents the absence of quantity.

Example: 0.

Note: Arithmetic operations (addition, subtraction, multiplication, division) are applicable to integers.

Rational Numbers

Rational numbers are numbers that can be expressed as the quotient or fraction $\frac{b}{a}$, where a and b are integers and b is not equal to zero. In other words, they can be written in the form $\frac{q}{p}$, where p and q are integers.

Examples: $\frac{1}{2}, \frac{3}{5}, \frac{7}{17}, \frac{5}{6}, \frac{7}{8}\ etc.$

Irrational Numbers

Irrational numbers are real numbers that cannot be expressed as fractions of two integers. Unlike rational numbers, irrational numbers have non-repeating, non-terminating decimal expansions.

Example: $\sqrt{2}, \pi$ etc.

Real Numbers

Real numbers include all rational numbers (which can be expressed as fractions) and irrational numbers (which cannot be expressed as fractions).

Complex Numbers

A complex number is comprised of a real part (a) and an imaginary part (b). The real and imaginary parts are denoted as $Re(Z)$ and $Im(Z)$ respectively, where $Z = a + bi$.

Note: $i^2 = -1$

Standard Form/Scientific Notation

Standard form, also known as scientific notation, is a way of expressing very large or very small numbers in a concise and standardized format.

Example

Positive number	Standard form
10	10^1

100	10^2
1,000	10^3
10,000	10^4
100,000	10^5
1,000,000	10^6
10,000,000	10^7
100,000,000	10^8
1,000,000,000	10^9
1,000,000,000,000	10^{12}
1,000,000,000,000,000	10^{15}

Decimal number	Standard form
0.1	10^{-1}
0.01	10^{-2}
0.001	10^{-3}
0.0001	10^{-4}
0.00001	10^{-5}
0.000001	10^{-6}
0.0000001	10^{-7}
0.00000001	10^{-8}
0.000000001	10^{-9}
0.000000000001	10^{-12}
0.000000000000001	10^{-15}

Decimal Numbers

Decimal numbers are a way of representing real numbers using a decimal point to separate the whole part from the fractional part. Each digit's position to the right of the decimal point represents a power of 10.

A decimal number can be expressed as $a.bcd$, where a is the whole part, and bcd is the fractional part.

Examples:

i. 25.37: Here, 25 is the whole part, and .37 is the fractional part.
ii. 0.005: In this case, the whole part is 0, and .005 is the fractional part.

Place Value:

- The digit to the left of the decimal point is in the "ones" place.
- The digit to the right of the decimal point is in the "tenths" place.
- Subsequent digits move to "hundredths," "thousandths," and so on.

Example:

Number	Decimal value
10.1	One decimal points
10.10	Two decimal points
10.001	Three decimal points
123.1024	Four decimal points
234.11934	Five decimal points
12.26754378	Eight decimal points

Significant Figures

Significant figures are digits in a numerical value that contribute to its precision. They include all certain digits and the first uncertain digit. Understanding and using significant figures is crucial in scientific and mathematical contexts to convey the precision of measurements and calculations.

Rules for Identifying Significant Figures:

- All nonzero digits are considered significant. (e.g., 456 has three significant figures.)
- Any zeros between significant digits are also significant. (e.g., 7003 has four significant figures.)
- Leading zeros (zeros to the left of the first nonzero digit) are not significant. (e.g., 0.008 has one significant figure.)
- Trailing zeros in a decimal number are significant. (e.g., 120.00 has five significant figures.)

Binary Numbers

Binary numbers are a fundamental part of the digital world, representing information using only two digits: 0 and 1. In contrast to the decimal system (base-10), which utilizes 10 digits (0-9), the binary system relies on powers of 2.

Note:

- Each digit in a binary number is called a "bit" (binary digit).
- The rightmost bit has a value of 2^0, the next 2^1, then 2^2 and so on.

Conversion from Decimal to Binary:

i. Divide the decimal number by 2.
ii. Record the remainder as the least significant bit (LSB).
iii. Continue dividing the quotient until the quotient is 0, recording remainders along the way.
iv. The binary equivalent is the sequence of remainders read in reverse.

Octal Numbers

Octal numbers are a base-8 numeral system, meaning they use eight digits: 0, 1, 2, 3, 4, 5, 6, and 7. Each digit's position in an octal number represents a power of 8.

Hexadecimal Numbers

Hexadecimal numbers are a base-16 numeral system widely used in computing and digital electronics. In this system, each digit represents four binary digits, providing a concise way to express large binary numbers.

Hexadecimal uses 16 symbols: 0-9 and A-F, where A stands for 10, B for 11, and so on, up to F for 15.

Rounding Numbers

Rounding numbers is a common practice used to simplify numerical values while maintaining a certain level of precision. It involves adjusting a number to a specified place value.

Rounding to a Specific Decimal Place

Example: Rounding 15.678 to the nearest tenth.

Solution:

15.678 rounded to the nearest tenth is 15.7.

Rounding to the Nearest Whole Number

Example: Rounding 23.89 to the nearest whole number.

Solution:

23.89 rounded to the nearest whole number is 24.

Rounding with Decimal Places

Example: Rounding 3.14159 to three decimal places.

3.14159 rounded to three decimal places is 3.142.

Rounding Strategies

Example: Rounding 47.492 using different strategies (round up, round down, or round to the nearest even number).

Solution:

47.492 rounded up is 47.5, rounded down is 47.4, and rounded to the nearest even number is 47.4.

Prime Numbers

A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

Prime numbers have exactly two distinct positive divisors: 1 and the number itself.

They cannot be formed by multiplying two smaller natural numbers.

Examples:

- 2: The smallest prime number, as it only has divisors 1 and 2.
- 3: Another basic prime number, divisible only by 1 and 3.
- 5: A prime number with no divisors other than 1 and 5.

Prime Factor

Prime factorization is the process of expressing a composite number as the product of its prime factors. Prime factors are the prime numbers that multiply together to give the original number.

Example: Prime Factorization of 24:

Step 1: Identify the prime factors of 24.
$24=2\times2\times2\times3$

Step 2: Express as the product of prime factors.

$24=2^3$ x 3.

Even and Odd Numbers

- Even Numbers: Integers divisible by 2 without leaving a remainder.

Even Numbers: 2, 4, 6, 8, 10, ...

- Odd Numbers: Integers not divisible by 2 without leaving a remainder.

Odd Numbers: 1, 3, 5, 7, 9, ...

Ordinal Numbers

Ordinal numbers represent the position or order of elements in a set. They convey the idea of rank or sequence.

Example: 1st, 2nd, 3rd, etc.

CHAPTER 2: SETS

A set is defined by its elements. If an object belongs to the set, it is denoted as an element of that set.

Representation of set

Sets can be represented using curly braces. For example, if A is a set containing elements 1, 2, and 3, it is written as

$A = \{1, 2, 3\}$.

SET IDENTITIES

Identity Element of Union

- The union of set A with the empty set $(\emptyset)$ is the set itself.

  $\therefore A \cup \emptyset = A\emptyset$

Identity Element of Intersection

- The intersection of a set A with the universal set U is the set itself.

  $\therefore A \cap U = A$

Domination Laws

- The union of a set A with the universal set U is the universal set.

  $A \cup U = U$

- The intersection of set A with the empty set $(\emptyset)$ is the empty set.

  $A \cap \emptyset = \emptyset$

Complement Laws

- The union of a set A with its complement (A') is the universal set.

  $A \cup A' = U$

- The intersection of a set A with the empty set $(\emptyset)$ is the empty set.

  $A \cap \emptyset = \emptyset$

Idempotent Laws:

- The union of a set A with itself is the set itself

  $A \cup A = A$

- The intersection of a set A with itself is the set itself

  $A \cap A = A$

Commutative Laws

- The union of sets A and B is equal to the union of sets B and A

  $A \cup B = B \cup A$

- The intersection of sets A and B is equal to the intersection of sets B and A

  $A \cap B = B \cap A$

Associative Laws:

- The union of sets A, B and C is associative

  $(A \cup B) \cup C = A \cup (B \cup C)$

- The intersection of sets A, B and C is associative

  $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive Laws

- Union distributes over intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- Intersection distributes over union

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Set symbols

{a, b, c, d} = Set

$\cup$ = Union of sets

$\cap$ = Intersection of sets

$\in$ = element of

Δ = Symmetric difference

A' = Complement of a set

$\subseteq$ = Subset

$\supseteq$ = Superset

U = Universal Set

Ø = Empty Set or void set

|A| = Cardinality of a set

$P(A)$ = Power set

$A * B$ = Cartesian product

CHAPTER 3: ALGEBRA

Algebra is a branch of mathematics that deals with mathematical symbols and the rules for manipulating these symbols.

Algebraic Rules

Associative property

$(a + b) + c = a + (b + c)$

Commutative property

$a + b = b + a$

Distributive property

$a * (b + c) = a * b + a * c$

Identity property

- $a + 0 = a$
- $a * 1 = a$

Inverse property

$a + (- a) = 0$

Exponential rules

- $a^m * a^n = a^{m + n}$
- $\frac{a^m}{a^n} = a^{m - n}$

Zero exponent rule

$a^0 = 1$

Negative exponent

$a^{-n} = \frac{1}{a^n}$

Multiplying polynomial

$(a + b)(c + d) = ac + ad + bc + bd$

Factoring Formulas

- $a^2 - b^2 = (a + b)(a - b)$
- $a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$
- $a^4 - b^4 = (a - b)(a + b)\left(a^2 + b^2\right)$
- $a^5 - b^5 = (a - b)\left(a^4 + a^3b + a^2b^2 + ab^3 + b^4\right)$

Product formulas

- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$
- $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Binomial Formulas

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

Or

$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1} + {}^nC_2a^{n-2} + \ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_na^n$, where ${}^nC_n = \frac{n!}{k!(n-k)!}$ are the binomial coefficients.

Pascal's Triangle

$(x+y)^0 = 1$

$(x+y)^1 = 1 \quad 1$

$(x+y)^2 = 1 \quad 2 \quad 1$

$(x+y)^3 = 1 \quad 3 \quad 3 \quad 1$

$(x+y)^4 = 1 \quad 4 \quad 6 \quad 4 \quad 1$

$(x+y)^5 = 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

$(x+y)^6 = 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$

$(x+y)^7 = 1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$

$(x+y)^8 = 1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$

$(x+y)^9 = 1 \quad 9 \quad 36 \quad 84 \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1$

$(x+y)^{10} = 1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1$

Indices

a) $a^{m} \times a^{n} = a^{m+n}$

b) $\frac{a^m}{a^n} = a^{m-n}$

c) $(a^m)^n = a^{mn}$

d) $a^0 = 1$

e) $a^{-n} = \frac{1}{a^n}$

f) $\sqrt[n]{a^m} = a^{m/n}$

g) $(ab)^m = a^m \, x \, b^m$

h) $(\frac{a}{b})^m = \frac{a^m}{b^m}$

i) If $a^m = b^m$ $(m \neq 0)$, then $a = b$.

j) If $a^m = a^n$ then $m = n$.

Logarithm

a) If $a^x = M$ then $\log_a M = X$

b) $\log_a 1 = 0$

c) $\log_a a = 1$

d) $a^{\log_a m} = M$

e) $\log_a MN = \log_a M + \log_a N$

f) $\log_a (\frac{M}{N}) = \log_a M - \log_a N$

g) $\log_a M^n = n\log_a M$

Roots

a) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

b) $\sqrt[n]{a}\,\sqrt[m]{b} = \sqrt[nm]{a^m b^n}$

c) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$

d) $\frac{\sqrt[n]{a}}{\sqrt[m]{b}} = \sqrt[nm]{\frac{a^m}{b^n}}, b \neq 0$

e) $(\sqrt[n]{a})^n = a$

f) $\sqrt[n]{a^m} = a^{\frac{n}{m}}$

g) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

EQUATIONS

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

$$D = b^2 - 4ac$$

Quadratic equation roots

a) $\alpha + \beta = -\frac{b}{a}$

b) $\alpha\beta = \frac{c}{a}$

c) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

d) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

e) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

f) $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

g) $\alpha^3 + \beta^3 = (\alpha + \beta)\left((\alpha + \beta)^2\right) - 3\alpha\beta$

h) $\alpha^4 + \beta^4 = \left[(\alpha + \beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$

i) $\alpha^2\beta + \beta^2\alpha = (\alpha + \beta)(\alpha\beta)$

j) $\frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha + \beta)\left((\alpha + \beta)^2\right) - 3\alpha\beta)}{\alpha\beta}$

k) $\sqrt{\alpha} + \sqrt{\beta} = \sqrt{(\alpha + \beta) + 2\sqrt{\alpha + \beta}}$

Completing the square method

- Move the actual constant to one side of the equation $ax^2 + bx = -c$.
- Divide both side by the coefficient of x^2 or by the coefficient of the first term. $\frac{ax^2}{a} + \frac{bx}{a} = \frac{-c}{a}$

- After dividing, take half of the coefficient of the second term, square it and sum it to the both sides of the equations. $\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4ac}$
- Simplify the equation to get the solution.

Slope Formula

Slope = $\frac{(y_2 - y_1)}{(x_2 - x_1)}$

Linear equation

$ax + b = 0$

Quadratic Equation

$ax^2 + bx + c = 0$

Graph Equations

- $y = mx + c$
- $y = -mx + c$
- $y = mx - c$
- $y = mx$
- $y = -mx$

Slope Intercept

$y = mx + b$

Midpoint Formula

$\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}$

Distance Formula

d= $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Percentage error

Percentage error = $\frac{\text{error}}{\text{measurement}}$ x 100%

If The True Value is known

Percentage error = $\frac{\text{error}}{\text{true value}}$ x 100%

Arithmetic progression (A.P)

$T_n = a + (n - 1)d$

Sum of an A.P

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric Progression (G.P)

$T_n = ar^{n-1}$

Sum of G.P

- $S_n = \frac{a(1 - r^n)}{1 - r}$ $\quad r < 1$
- $S_n = \frac{a(r^n - 1)}{r - 1}$ $\quad r \geq 1$

CHAPTER 4: GEOMETRY

Geometry is a branch of mathematics that deals with the properties, measurements, and relationships of points, lines, angles, surfaces, and solids.

Points, Lines, and Planes

- **Point:** A location in space with no size. Denoted by a dot.
- **Line:** A straight path with no thickness extending infinitely in both directions.
- **Plane:** A flat surface that extends infinitely in all directions.

Example: In a coordinate system, the point (2, 3), the line AB, and the plane P.

Right angle Triangle

Right angle triangle is $90°$

Pythagorean Theorem

$a^2 + b^2 = c^2$

Square formula

Area = L^2

Perimeter = 4S

Diagonal of a square = $S\sqrt{2}$

Triangle Formula

Area = ½bh

Perimeter = $a + b + c$

Semi-perimeter = $\frac{a+b+c}{2}$

Law of Sines = $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Law of Cosines = $c^2 = a^2 + b^2 - 2ab\cos(C)$

Rectangle formula

Area = $L * B$

Perimeter = $2(L + B)$

Diagonal = $\sqrt{L^2 + B^2}$

Circle formula

- Area of a sector = $\frac{\theta}{360°}$ x $\pi r^2\ cm^2$
- Diameter of a circle = $D = 2r$
- Radius of a circle = $\frac{D}{2}$
- Area of circle = πr^2
- Chord length = $2\sqrt{r^2 - d^2}$
- Circumference of a circle = $2\pi r$
- Perimeter of a sector = 2r + $\theta/360°$ × 2πr
- Perimeter of a sector = 2r + L

- Area of a segment = Area of a sector – area of a triangle = $\frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

Kite formula

Area = $\frac{1}{2} * Product\ of\ Diagonals$

Perimeter = $2 * (Length\ of\ longer\ diagonal + Lenght\ of\ shorter\ diagonal)$

Sphere formula

- The formula for the volume of a sphere(V) = $\frac{4}{3}\pi r^3$
- Surface Area of a Sphere(A) = $4\pi r^2$
- Diameter of a sphere(D) = 2r
- Circumference of a sphere(C) = $2\pi r$

Cylinder formula

- Curved surface area = $2\pi rh$
- Curved surface area with one end closed = $2\pi rh + \pi r^2$
- Curved surface area with two ends closed =$2\pi r(h + r)$
- Volume of a cylinder = $\pi r^2 h$

Cone formula

- Curved surface area = πrl

- Total surface area = $\pi rl + \pi r^2$
- Volume of a cone = $\frac{1}{3}\pi r^2 h$

Cube formula

- Volume of a cube(V) = S^3
- Surface area(A) = $6S^2$
- Diagonal of a cube = $s\sqrt{3}$
- Length of the cube's edge = $\sqrt[3]{V}$

Cuboid formula

- Volume of a cuboid(V) = lwh
- Surface area of a cuboid(A) = $2(lw + lh + wh)$
- Diagonal of a cuboid(d) = $\sqrt{l^2 + w^2 + h^2}$

Trapezium formula

- Area of a trapezium(A) = $\frac{1}{2} * (a + b) * h$
- Perimeter of a trapezium(P) = $a + b + c + d$

Pyramid formula

- Volume of a pyramid(V) = $\frac{1}{3}\text{Base area} \times \text{height}$
- Surface area of a pyramid(A) =

  $\frac{1}{2} * perimeter\ of\ base * Slant\ height + Base\ area$

Rhombus formula

- Perimeter of a rhombus = $4S$
- Area of a Rhombus = $\frac{1}{2} * d_1 * d_2$

Parallelogram formula

Area = Base * Height

Perimeter = $2 * (Length\ of\ base + length\ of\ adjacent\ side)$

CHAPTER 5: TRIGONOMETRY

Trigonometric Functions

i. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

ii. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

iii. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

iv. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

v. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

vi. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

Pythagorean identities

- Fundamental Pythagorean identity

$cos^2\theta\ + sin^2\theta\ = 1$

- Tangent Pythagorean identity

$tan^2\theta + 1 =\ sec^2\theta$

- Cotangent Pythagorean identity

$$1 + cot^2\theta = cosec^2\theta$$

Reciprocal identities

i. $$cosec\theta = \frac{1}{sin\theta}$$

ii. $$sec\theta = \frac{1}{cos\theta}$$

iii. $$cot\theta = \frac{1}{tan\theta}$$

complementary identities

i. $$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

ii. $$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

iii. $$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

iv. $$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

v. $$\sec\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec}\theta$$

vi. $$\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

Sum and difference identities

i. $\sin(a \pm b) = \text{sinacosb} \pm cosasinb$

ii. $\cos(a + b) = \text{cosacosb} - \text{sinasinb}$

iii. $\cos(a - b) = \text{cosacosb} + \text{sinasinb}$

iv. $\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

v. $\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

Sum to product identities

i. $\text{sina} + \text{sinb} = 2\sin\left(\frac{a + b}{2}\right)\cos\left(\frac{a - b}{2}\right)$

ii. $\text{sina} - \text{sinb} = 2\cos\left(\frac{a + b}{2}\right)\sin\left(\frac{a - b}{2}\right)$

iii. $\text{cosa} + \text{cosb} = 2\cos\left(\frac{a + b}{2}\right)\cos\left(\frac{a - b}{2}\right)$

iv. $\text{cosa} - \text{cosb} = -2\sin\left(\frac{a + b}{2}\right)\sin\left(\frac{a - b}{2}\right)$

Product to sum identities

i. $2\text{SinaCosb} = \sin(a + b) + \sin(a - b)$

ii. $2\text{CosaSinb} = \sin(a + b) - \sin(a - b)$

iii. $2CosaCosb = \cos(a + b) + \cos(a - b)$

iv. $2SinaSinb = \cos(a - b) - \cos(a + b)$

Double angle identities

- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- $\cos 2\theta = 2\cos^2\theta - 1$
- $\cos 2\theta = 1 - 2\sin^2\theta$
- $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Special angles identities

Special angle identities at 30°

i. $\sin 30° = \frac{1}{2}$

ii. $\cos 30° = \frac{\sqrt{3}}{2}$

iii. $\tan 30° = \frac{1}{\sqrt{3}}$

iv. $\cot 30° = \sqrt{3}$

v. $\sec 30° = \frac{\sqrt{3}}{2}$

vi. $\csc 30° = 2$

Special angle identities at 45°

i. $\sin 45° = \frac{\sqrt{2}}{2}$

ii. $\cos 45° = \frac{\sqrt{2}}{2}$

iii. $\tan 45° = 1$

iv. $\cot 45° = 1$

v.

$\sec 45° = \sqrt{2}$

vi. $\csc 45° = \sqrt{2}$

Special angle identities at 60°

i. $\sin 60° = \frac{\sqrt{3}}{2}$

ii. $\cos 60° = \frac{1}{2}$

iii. $\tan 60° = \sqrt{3}$

iv. $\cot 60° = \frac{1}{\sqrt{3}}$

v. $\sec 60° = 2$

vi. $\csc 60° = \frac{2}{\sqrt{3}}$

CHAPTER 6: DIFFERENTIAL CALCULUS

Derivative Formulas for Elementary Functions

i. $\frac{d}{dx}(K) = 0$ (constant rule)

ii. $\frac{d}{dx}(Ku) = K\frac{du}{dx}$

iii. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$ (sum and difference rule)

iv. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dv}$ (product rule)

v. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (quotient rule)

vi. $\frac{dy}{du}.\frac{du}{dy} = 1$ (chain rule)

vii. $\frac{d}{dx}(x^n) = nx^{n-1}$ (power rule)

viii. $\frac{d}{dx}(e^x) = e^x$ (exponential function)

ix. $\frac{d}{dx}(a^x) = a^x \log a$

x. $\frac{d}{dx}(\log x) = \frac{1}{x}$

xi. $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$

Differentiation of trigonometric functions

i. $\frac{d}{dx}(\sin x) = \cos x$

ii. $\frac{d}{dx}(\cos x) = -\sin x$

iii. $\frac{d}{dx}(\tan x) = sec^2 x$

iv. $\frac{d}{dx}(\cot x) = -cosec^2 x$

v. $\frac{d}{dx}(\sec x) = \sec x \tan x$

vi. $\frac{d}{dx}(\mathrm{cosec}\, x) = -cosecx\ cotx$

Operation rule on trig functions

vii. $\frac{d}{dx}(\sin u) = \cos u \frac{du}{x}$

viii. $\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$

ix.
$$\frac{d}{dx}(\tan u) = sec^2 u\frac{du}{dx}$$

x. $$\frac{d}{dx}(\cot u) = -cosec^2 u\frac{du}{dx}$$

xi. $$\frac{d}{dx}(\sec u) = \sec u \tan u\frac{du}{dx}$$

xii. $$\frac{d}{dx}(cosecu) = -cosecu\ cot\ u\frac{du}{dx}$$

Differentiation of inverse trig functions

i. $$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

ii. $$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

iii. $$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

iv. $$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

v. $$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

vi. $$\frac{d}{dx}\left(cosec^{-1}x\right) = -\frac{1}{x\sqrt{x^2-1}}$$

Differentiation of hyperbolic functions

i. $$\frac{d}{dx}(\sinh x) = \cosh x$$

ii. $$\frac{d}{dx}(\cosh x) = \sinh x$$

iii. $$\frac{d}{dx}(\tanh x) = sec^2x$$

iv. $$\frac{d}{dx}(\coth x) = -cosec^2x$$

v. $$\frac{d}{dx}(\text{sech}\, x) = -sechx\ tanhx$$

vi. $$\frac{d}{dx}(cosech\ x) = -cosechx\ cothx$$

Differentiation of inverse hyperbolic functions

i. $$\frac{d}{dx}\left(\sinh^{-1}x\right) = \frac{1}{\sqrt{x^2+1}}$$

ii. $$\frac{d}{dx}\left(\cosh^{-1}x\right) = \frac{1}{\sqrt{x^2-1}}$$

iii. $$\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2}$$

iv. $$\frac{d}{dx}\left(\coth^{-1} x\right) = -\frac{1}{1-x^2}$$

v. $$\frac{d}{dx}\left(\operatorname{sech}^{-1} x\right) = -\frac{1}{x\sqrt{1-x^2}}$$

vi. $$\frac{d}{dx}\left(\operatorname{cosech}^{-1} x\right) = -\frac{1}{x\sqrt{1+x^2}}$$

CHAPTER 7: INTEGRAL CALCULUS.

integration Formulas of Elementary Functions

i.
$\int a dx = ax + c$ (constant rule)

ii.
$\int x dx = \frac{x^2}{2} + c$ (variable rule)

iii.
$\int \frac{1}{x} dx = \ln x + c$ (reciprocal rule)

iv.
$\int e^x dx = e^x + c$ (exponential rule)

v.
$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ (power rule)

vi.
$\int c f(x) dx = c \int f(x) dx$ (multiplication by constant)

vii.
$\int (u \pm v) dx = \int u dx \pm \int v dx$ (sum and difference rule)

Integral of trigonometric functions

i. $\int \sin x dx = \; -\cos x + c$

ii. $\int \cos x dx = \; \sin x + c$

iii. $\int \tan x dx = \; ln|secx| + c$

iv. $\int secx \, dx = \; ln|tanx + secx| + c$

v. $sin^2 xdx = \; \frac{1}{2}(x - sinx\, cosx) + c$

vi. $\int cos^2 xdx = \frac{1}{2}(x + sinx\, cosx) + c$

vii. $\int tan^2 xdx = \; tanx - x + c$

viii. $\int sec^2 xdx = \; tanx + c$

ix. $\int cosec^2 xdx = \; - cot^2 x + c$

x. $\int \frac{1}{\sqrt{1 - x^2}} dx = \; \sin^{-1} x + c$

xi. $\int \frac{1}{\sqrt{1 \; + x^2}} dx = \; \tan^{-1} x + c$

Integration by substitution

$\frac{dx}{du} \int f(u) du$

Integration by part

$$\int u dv = uv - \int v du$$

CHAPTER 8: BUSINESS MATHEMATICS

Profit formulas

- Profit = S.P – C.P (where; S.P is selling price and C.P is cost price)
- Profit % = $\frac{Profit}{C.P} x \frac{100}{1}$

Loss formulas

- Loss = C.P – S.P
- Loss % = $\frac{loss}{C.P} x \frac{100}{1}$

Selling and Cost price formulas

- Selling price = $\frac{\text{C.P}\,(100\ +\ \text{profit\%})}{100}$
- Selling price = $\frac{\text{C.P}\,(100\ +\ \text{loss\%})}{100}$
- Selling price = market price – discount
- Selling price = price after discount + value added tax.
- Cost price = $\frac{100\ \times\ \text{S.P}}{100\ +\ \text{profit\%}}$
- Cost price = $\frac{100\ \times\ \text{S.P}}{100 - \text{loss\%}}$

Simple interest

- I = $\frac{PRT}{100}$ (where; p = principal, R = rate, T = time and I or S.I = simple intrest)
- Amount (A) = $P + I$

Compound interest

- C.L = $P\left[\left(\frac{1+R}{100}\right)n-1\right]$
- Amount (A) = $P\left(I + \frac{R}{100}\right)^n$

Future Value (FV) of a Single Sum

- $FV = PV(1+r)^t$

Present value (PV) of a single sum

- $PV = \frac{FV}{(1+r)^t}$