During the present year, observational research in reference to this object is likely to prove of an unusually interesting character, as there are several features which deserve (and will doubtless receive) special attention. During 1901, the red spot exhibited a singular acceleration of motion when compared with its rate in previous years, for it maintained a longitude of 45°, consistently with a rotation period of 9h. 55m. 40s.·6, upon which system II. of Mr. Crommelin's ephemerides is based. It will therefore be most important to trace the position of the spot in ensuing months, as it seems probable that this curiously durable marking, after a constantly increasing retardation between about 1878 and 1900 which augmented its rotation period from 9h. 55m. 34s.·5 to 9h. 55m. 41s.·8, will now travel with greater celerity and give a period becoming shorter with the time until the minimum is reached. If these oscillations in velocity are developed at regular intervals, it will soon be possible to determine the length of the cycle for observations of the red spot or of the hollow in the great southern belt date from 1831. There is some significant evidence in support of the conjecture that the motion of certain markings as well as the apparition of particular spots in various latitudes are recurrent on the planet, and some of the reappearances appear to take place at periods not differing materially from the time of Jupiter's revolution round the sun. The red spot may be looked for near mid-transit on March 13 at 18h. 7m., on March 25 at 18h. 4m., on March 30 at 17h. 13m., on April 6 at 18h. 0m., and on April 11 at 17h. 9m.

Observers should now endeavour to redetect the rapidly moving dark spots which appeared in the north temperate belt of Jupiter in 1880 and 1891. There were a number of spots visible in this latitude in 1901, but the writer found their mean rate of rotation 9h. 55m. 50s. This is nearly 8 minutes in excess of the rotation period found for certain irregular markings in approximately the same latitude in the autumn of 1880 which gave a rate of 9h. 48m. It is a very singular circumstance that in a similar latitude of Jupiter spots are developed showing respectively the shortest and longest rotation periods of any which have ever been observed.

In 1901, a large dark spot was often seen in the south or tropical[1] zone of the planet, and this may prove a repetition of the object observed in the same latitude in 1889–91. This spot exhibits a rotation period of 9h. 55m. 18s.·5, and its more rapid movement will enable it to overtake the red spot in about June, 1902, should both the markings remain visible until that month. The longitude of the south temperate or tropical spot will be as follows during the next three months, and it will follow the red spot at the time-intervals stated if the latter object retains the same longitude (=45°) as in 1901 :—

1902	Longitude	Follows Red spot. h.	m.
March 17	110·5	1	48
April 17	94·1	1	21
May 17	77·7	0	54

The writer obtained an observation of this marking on February 27, when it appeared to be central at 18h. 40m., which would make its longitude 123°·8, but it was very imperfectly seen. The instrument used was a 4-inch Cooke refractor, power 175.

Another important feature for reobservation in 1902 will be the white and dark spots plentifully grouped along the equatorial region of the planet. In the three years 1898, 1899 and 1900, the rotation period of the equatorial current differed very little, the mean value from a large number of spots being 9h. 50m. 24s., or 5m. 17s.·7 less than the rate of the red spot. But in 1901 the mean rotation period of 28 equatorial spots observed at Bristol was 9h. 50m. 29s., or 5m. 12s. less than that of the red spot.

When further observations of these variations have been pursued during many oppositions, the outcome may be both interesting and important as affording a good clue to the physical condition and phenomena of the planet. That great atmospheric changes are in progress on the disc is evident, and it is the facility with which they may be observed and compared which renders this object a singularly attractive one to the possessors of telescopes.

W. F. DENNING.

[1] This interesting marking exhibited a motion coinciding with that of objects placed in the planet's south temperate zone, though its position encroached on the south tropical as well as the south temperate region.

MAGIC SQUARES AND OTHER PROBLEMS UPON A CHESS-BOARD[1]

THE construction of magic squares is an amusement of great antiquity; we hear of them being constructed in India and in China before the Christian era, whilst they appear to have been introduced into Europe by Moschopulus, who flourished at Constantinople early in the fifteenth century. On the diagram you see a simple example of a magic square, one celebrated as being drawn by Albert Dürer in his picture of "Melancholy," painted about the year 1500 (Fig. 1). It is one of the fourth order, involving 16 compartments or cells. In describing such squares, the horizontal lines of cells are called "rows," the vertical lines "columns," and the oblique lines going from corner to corner

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

FIG. 1.

"diagonals." In the 16 compartments are placed the first 16 numbers, 1, 2, 3, . . . 16, and the magic property consists in this, that the numbers are placed in such wise that the sum of the numbers in every row, column and diagonal is the same, viz., in this case, 34.

It is probable that magic squares were so called because the properties they possessed seemed to be extraordinary and wonderful; they were, indeed, regarded with superstitious reverence and employed as talismans. Cornelius Agrippa constructed magic squares of orders 3, 4, 5, 6, 7, 8, 9, and associated them with the seven heavenly bodies, Saturn, Jupiter, Mars, the Sun, Venus, Mercury and the Moon. A magic square engraved on a silver plate was regarded as a charm against the plague, and to this day such charms are worn in the east.

However, what was at first merely a practice of magicians and talisman makers has now for a long time become a serious

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

FIG. 2.

study for mathematicians. Not that they have imagined that it would lead them to anything of solid advantage, but because the theory of such squares was seen to be fraught with difficulty, and it was considered possible that some new properties of numbers might be discovered which mathematicians could turn to account. This has, in fact, proved to be the case; for from a certain point of view the subject has been found to be algebraical rather than arithmetical, and to be intimately connected with great departments of science, such as the "infinitesimal calculus," "the calculus of operations" and the "theory of groups."

In the next diagram (Fig. 2) I show you a magic square of order 5, the sum of the numbers in each row, column and

[1] A discourse delivered at the Royal Institution on Friday evening, February 14, by Major P. A. MacMahon, F.R.S.

diagonal being 65. This number 65 is obtained by multiplying 25, the number of cells, by the next higher number, 26, and then dividing by twice the order of the square, viz., 10. A similar rule applies in the case of a magic square of any order. The formation of these squares has a fascination for many persons, and, as a consequence, a large amount of ingenuity has been expended in forming particular examples and in discovering general principles of formation. As an example of the amount of labour that some have expended on this matter, it may be mentioned that in 1693 Frénicle, a Frenchman, published a work of more than 500 pages upon magic squares. In this work he showed that 880 magic squares of the fourth order could be constructed, and in an appendix he gave the actual diagrams of the whole of them. The number of magic squares of the order 5 has not been exactly determined, but it has been shown that the number certainly exceeds 60,000.

As a consequence it is not very difficult to compose particular specimens, and, for the most part, the fascinated individuals, to whom I have alluded, have devoted their energies to the discovery of principles of formation. Of such principles I will give a few, remarking that the cases of squares of uneven order 1, 3, 5 . . . are more simple than those of even order 4, 6, . . . and that no magic square of order 2 exists at all. The simplest of all methods for an uneven order is shown in the diagram (Fig. 3), where certain additional cells are added to the square, the numbers written as shown in natural order diagonally, and then the numbers which are outside the square

				5				
			4		10			
		3	16	9	22	15		
	2	20	8	21	14	2	20	
1		7	25	13	1	19		25
	6	24	12	5	18	6	24	
		11	4	17	10	23		
			16		22			
				21				

FIG. 3.

projected into the empty compartments according to an easily understood law. The second method is associated with the name of De la Loubère, though it is stated that he learnt it during a visit to Siam in 1687. The number 1 (see Fig. 2) is placed in the middle cell of the top row, and the successive numbers placed in their natural order in a diagonal line sloping upwards to the right subject to the laws:—

(1) When the top row is reached, the next number is written at the bottom of the next column.

(2) When the right-hand column is reached, the next number is written on the left of the row above.

(3) When it is impossible to proceed according to the above rules, the number is placed in the cell immediately below the last number written.

If we commence by writing the number 1 in any cell except that above indicated, a square is reached which is magic in regard to rows and columns, but not in regard to diagonals.

Subsequent writers have shown that starting with the left-hand bottom cell and using the move of the knight instead of that of the bishop, the general principle of De la Loubère will also lead to a magic square (Fig. 4). The next method is that of De la Hire, and dates from 1705. Two subsidiary squares are constructed as shown, the one involving five numbers 1, 2, 3, 4, 5, and the other five numbers 0, 5, 10, 15, 20. When these squares are properly formed and a third square constructed by adding together the numbers in corresponding cells, this third square is magic (Fig. 5). Time does not permit me to enter into the exact method of forming the subsidiary squares, and I will merely mention that each of them possesses a particular property, viz., only five different numbers are involved, and all five appear in each column and in each row; in other words, no row and no column contains two numbers of the same kind, but no diagonal property is necessarily involved. Such squares are of a great scientific importance, and have been termed by Euler and subsequent writers "Latin squares," for a reason that will presently appear. From a scientific point of view, the chief interest of all arrangements such as I consider this evening lies, not in their actual formation, but in the enumeration of all possible ways of forming them, and in this respect very little has been hitherto

7	20	3	11	24
13	21	9	17	5
19	2	15	23	6
25	8	16	4	12
1	14	22	10	18

FIG. 4.

achieved by mathematicians. No person living knows in how many ways it is possible to form a magic square of any order exceeding 4. The fact is, that before we can attempt to enumerate magic squares we must see our way to solve problems of a far more simple character. For example, before we can enumerate the squares that can be formed by De la Hire's method we must take a first step by finding out how many Latin squares can be formed of the different orders. For the order 5 the question is, "In how many ways can five different objects be placed in the cells so that each column and each

3	4	1	5	2
2	3	4	1	5
5	2	3	4	1
1	5	2	3	4
4	1	5	2	3

15	0	20	5	10
0	20	5	10	15
20	5	10	15	0
5	10	15	0	20
10	15	0	20	5

18	4	21	10	12
2	23	9	11	20
25	7	13	19	1
6	15	17	3	24
14	16	5	22	8

FIG. 5.

row contains each object?" It may occur to some here this evening that such a discussion might be interesting or curious, but could not possibly be of any scientific value. But such is not the case. A department of mathematics that is universally acknowledged to be of fundamental importance is the "theory of groups." Operations of this theory and those connected with logical and other algebras possess what is termed a "multiplication table," which denotes the laws to which the operations are subject. In Fig. 6 you see such a table of order 6 slightly modified from Burnside's "Treatise on the Theory of Groups"; it is, as you see, a Latin square, and the chief problem that awaits solution is the enumeration of such tables; the questions are not parallel because *all* Latin squares do not give rise to tables in the theory of groups; but still, we must walk before we can run, and a step in the right direction is the enumeration of *all* Latin squares. When I call to mind that the theory of groups has an important bearing upon many branches of physical science, notably upon dynamics, I consider that I have made good my point.

I now concentrate attention on these Latin squares, and observe that the theory of the enumeration has nothing to do with the particular numbers that occupy the compartments; the only essential is that the numbers shall be different one from another. My attention was first called to the subject of the Latin square by a work of the renowned mathematician Euler, written in 1782, entitled "Recherches sur une nouvelle espèce de Quarrés Magiques." I may say that Euler seems to have been the first to grasp the necessity of considering squares possessing what may be termed a magical property of a far less recondite character than that possessed by the magic squares of the ancients, and, as we shall see presently, he might have gone a

I	A	B	C	D	E
A	B	I	D	E	C
B	I	A	E	C	D
C	E	D	I	B	A
D	C	E	A	I	B
E	D	C	B	A	I

FIG. 6.

step further in the same direction with advantage and have commenced with arrangements of a more simple character than that of the Latin square, with arrangements, in fact, which present no difficulties of enumeration, but which supply the key to the unlocking of the secrets of which we are in search. He commences by remarking that a curious problem had been exercising the wits of many persons. He describes it as follows:—There

$a\alpha$	$a\beta$	$a\gamma$	$a\delta$	$a\epsilon$	$a\theta$
$b\alpha$	$b\beta$	$b\gamma$	$b\delta$	$b\epsilon$	$b\theta$
$c\alpha$	$c\beta$	$c\gamma$	$c\delta$	$c\epsilon$	$c\theta$
$d\alpha$	$d\beta$	$d\gamma$	$d\delta$	$d\epsilon$	$d\theta$
$e\alpha$	$e\beta$	$e\gamma$	$e\delta$	$e\epsilon$	$e\theta$
$f\alpha$	$f\beta$	$f\gamma$	$f\delta$	$f\epsilon$	$f\theta$

FIG. 7.

are 36 officers of six different ranks drawn from six different regiments, and the problem is to arrange them in a square of order 6, one officer in each compartment, in such wise that in each row, as well as in each column, there appears an officer of each rank and also an officer of each regiment. Of a single regiment we have, suppose, a colonel, lieutenant-colonel, major, captain, first lieutenant and second lieutenant, and similarly for five other regiments, so that there are in all 36 officers who must be so placed that in each row and in each column each rank is represented, and also each regiment. Euler denotes the six regiments by the Latin letters *a*, *b*, *c*, *d*, *e*, *f*, and the six ranks by the Greek letters α, β, γ, δ, ϵ, θ, and observes that the character of an officer is determined by a combination of two letters, the one Latin and the other Greek; there are 36 such combinations, and the problem consists in placing these combinations in the 36 compartments in such wise that every row and every column contains the 6 Latin letters and also the 6 Greek letters (Fig. 7). Euler found no solution of this problem in the case of a square of order 6, and since Euler's time no one has succeeded either in finding a solution or in proving that no solution exists. Anyone interested has, therefore, this question before him at the present moment, and I recommend it to any-

$a\alpha$	$b\gamma$	$c\beta$
$b\beta$	$c\alpha$	$a\gamma$
$c\gamma$	$a\beta$	$b\alpha$

FIG. 8.

one present who desires an exercise of his wits and a trial of his patience and ingenuity. It is easy to prove that when the square is of order 2, viz. the case of 4 officers of two different ranks drawn from two different regiments, there is no solution; Euler gave his opinion to the effect that no solution is possible whenever the order of the square is two greater than a multiple of four. In other simple cases he obtained solutions; for example, for the order 3, the problem of 9 officers of three different ranks drawn from three different regiments, it is easy to discover the solution shown in the diagram (Fig. 8), and, as demonstrated by Euler, whenever one solution has been constructed there is a simple process by which a certain number of others can be derived from it. Now if you look at that diagram and suppose the Greek letters obliterated, you will see that the Latin letters are arranged so that each of the letters occurs in each row and in each column, the magical property mentioned above, and for this reason Euler termed such arrangements Latin squares and stated that the first step in the solution of the problem is to enumerate the Latin squares of a given order. As showing the intimate connection between the Græco-Latin square of Euler and ordinary magic squares, it should be noticed that the method of De la Hire, by employing Latin and Greek letters for the elements in his two subsidiary Latin squares, gives rise immediately to the Græco-Latin square of Euler. Euler says in regard to the problem of the Latin square, "The complete enumeration of the Latin squares of a given order is a very important question, but seems to me of extreme difficulty, the more so as all known methods of the doctrine of combinations appear to give us no help," and again, "the enumeration appears to be beyond the bounds of possibility when the order exceeds 5." Moreover, Cayley, in 1890, that is 108 years later, gave a *résumé* of what had been done in the matter, but did not see his way to a solution of the question. Under these circumstances, you will see how futile it is to expect a solution of the magic-square problem when the far simpler question of the Latin square has for so long proved such a tough nut to crack. The problem of the Latin square has eventually been completely solved, and in order to lead you up gradually to an understanding of the method that has proved successful, I ask you to look at the Latin square of order 5 that you see in the diagram (Fig. 9). The first row of letters can be written in any order, but not so the second row, for each column when the second row is written must contain two different

a	b	c	d	e
b	d	e	a	c
c	e	d	b	a
d	c	a	e	b
e	a	b	c	d

FIG. 9.

letters. We must, therefore, be able to solve the comparatively simple question of the number of possible arrangements of the first two rows. For a given order of the letters in the first row, in how many ways can we write the letters in the second row so that each column contains a pair of different letters? This is a famous question, of which the solution is well known; it is known to mathematicians as the "problème des rencontres." It may be stated in a variety of ways; one of the most interesting is as follows:—A person writes a number of letters and addresses the corresponding envelopes; if he now put the letters at random into the envelopes, what is the probability that not a single letter is in the right envelope?

Passing on to the problem of determining the number of ways of arranging the first three rows so that each column contains three different letters, it may be stated that up to 1898 no solution of it had been given; while it is obvious that as the number of the rows is increased the resulting problems will be of enhanced difficulty. A particular case of the three-row problem had, however, been considered under the title "problème des ménages" and a solution obtained. It may be stated as follows:—

A given number of married ladies take their seats at a round table in given positions; in how many ways can their husbands be seated so that each is between two ladies, but not next to his own wife? For order 5, that is 5 ladies, the question comes to this:—Write down 5 letters and underneath them the same letters shifted one place to the left; in how many ways can the third row be written so that each column contains three different letters? This particular case of the three-row problem for any order presents no real difficulty. The results are that in the cases of 3, 4, 5, 6 . . . married couples there are 1, 2, 13, 80, &c., ways.

Since the year 1890, the problem of the Latin square has been completely solved by an entirely new method, which has also proved successful in solving similar questions of a far more recondite character, and I am here this evening to attempt to give you some notion of the method and some account of the series of problems to which that method has been found to be applicable.

There is, as viewed mathematically, a fundamental difference between arithmetic and algebra; the former may be regarded as an algebra in which the numerical magnitudes under consideration are restricted to be integers; the two branches contemplate discontinuous and continuous magnitude respectively. Similarly, in geometry we have the continuous theory, which contemplates figures generated by points moving from one place to another and in doing so passing over an infinite succession of points, tracing a line in a plane or in space, and also a discontinuous theory, in which the position of a point varies suddenly, *per saltum*, and we are not concerned with any continuously varying motion or position. The present problems are concerned sometimes with this discontinuous geometry and sometimes with an additional discontinuity in regard to numerical magnitude, and the object is to count and not to measure. Far removed as these questions are, apparently, from the subject-matter of a calculus of infinitely small quantities and the variation of quantities by infinitesimal increments, my purpose is to show that they are intimately connected with them and that success is a necessary consequence of the relationship. I must first take you to a much simpler problem than that of the Latin square, to one which in a variety of ways is very easy of solution, but which happens to be perhaps the simplest illustration of the method. In the game of chess a castle can move either horizontally or vertically, and it is easy to place 8 castles on the board so that no piece can be taken by any other piece. One such arrangement is shown in Fig. 10. The condition is simply that one castle must be in each row and also in each column. Every such arrangement is a diagrammatic representation of a certain mathematical process performed upon a certain algebraical function. For consider the process of differentiating x^8; it may be performed as follows:—Write down x^8 as the product of 8 x's,

$$x\ x\ x\ x\ x\ x\ x\ x,$$

and now substitute unity for x in all possible ways and add the results; the substitution can take place in eight different ways, and the addition results in $8x^7$, which will be recognised as the differential coefficient. Observe that the process of differentiation is thus broken up into eight minor processes, each of which may be diagrammatically represented on the first row of the chess-board by a unit placed in the compartment corresponding to the particular x for which unity has been substituted. If we now perform differentiation a second time, we may take the results of the above minor processes and in each of them again substitute unity for x in all possible ways; since in each the substitution can take place in seven different ways, it is seen that we can regard the process of differentiating twice as composed of $8 \times 7 = 56$ minor processes, each of which can be diagrammatically represented by two units, one in each of the first two rows of the chess-board, in positions corresponding to the substitutions of unity for x that have been carried out. Proceeding in this manner in regular order up to the eighth differentiation, we find that the whole process of differentiating x^8 eight times in succession can be decomposed into $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40{,}320$ minor processes, each of which is denoted by a diagram which slight reflection shows is a solution of the castle problem (Fig. 11). There are, in fact, no more solutions, and the whole series of 40,320 diagrams

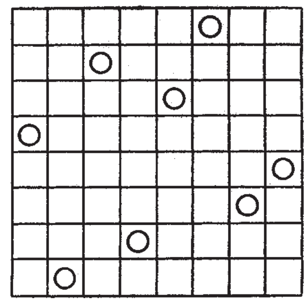


FIG. 10.

constitutes a picture in detail of the differentiations. Simple differentiations of integral powers thus yield the enumerative solutions of the castle problem on chess-boards of any size.

We have here a clue to a method for the investigation of these chess-board problems; it is the grain of mustard seed which has grown up into a tree of vigorous growth, throwing out branches and roots in all sorts of unexpected directions. The above illustrations of differentiation gave birth to the idea that it might be possible to design pairs of mathematical processes and functions which would yield the solution of chess-board problems of a more difficult character. Two plans of operation present themselves. In the first place we may take up a particular question, the Latin square for instance, and attempt to design, on the one hand, a process, and, on the

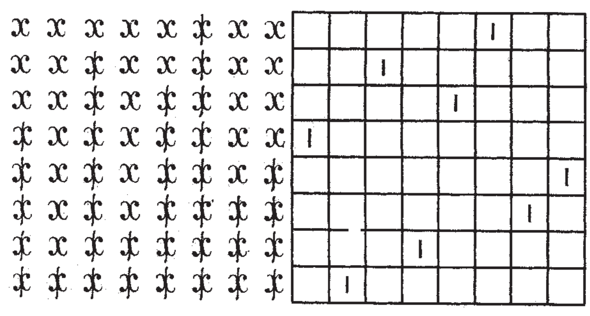


FIG. 11.

other hand, a function the combination of which will lead to the series of diagrams. In the second place, we may have no particular problem in view, but simply start by designing a process and a function, and examine the properties of the series of diagrams to which the combination leads. The first of these plans is the more difficult, but was actually accomplished in the case of the Latin square and some other questions; but the second plan, which is the proper method of investigation, met with great success, and the Latin square was one of its first victims, a solution of a more elegant nature being obtained than that which had resulted from the first plan of operations. There is such an extensive choice of processes and functions that many solutions are obtainable of any particular problem. I will now give you an idea of a solution of the Latin square, which is not the most elegant that has been found, but which is the most suitable to explain to an audience. Suppose we have five collections of objects, each collection containing the same five different objects, *a, b, c, d, e* (Fig. 12). I suppose the objects distributed amongst five different persons in the following manner:—The first person takes one object from each collection, so as to obtain each of the five objects; he can do this in 120 different ways; we will suppose that he takes *a* from the first, *b* from the second, *c* from the third, *d* from the fourth, *e* from the fifth; the collections then become as you see in Fig. 12, second row. Now suppose the second man to advance with the intention of taking one object from each collection and obtaining each of the five objects, he has not the same liberty of choice as had the first, because he cannot take *a* from the first collection or *b* from the second, &c. However, he has a good choice in his selection, and we will suppose him to take *b* from the first collection, *d* from the second, *e* from the third,

(*abcde*)	(*abcde*)	(*abcde*)	(*abcde*)	(*abcde*)
(.*bcde*)	(*a*.*cde*)	(*ab*.*de*)	(*abc*.*e*)	(*abcd*.)
(..*cde*)	(*a*.*c*.*e*)	(*ab*.*d*.)	(.*bc*.*e*)	(*ab*.*d*.)
(...*de*)	(*a*.*c*..)	(*ab*...)	(..*c*.*e*)	(.*b*.*d*.)
(....*e*)	(*a*....)	(.*b*...)	(..*c*..)	(...*d*.)

FIG. 12.

a from the fourth, *c* from the fifth. The collections then become as you see in the third row. The third man who has the same task finds his choice more restricted, but he elects to take *c* from the first, *e* from the second, *d* from the third, *b* from the fourth and *a* from the fifth. The fourth man finds he can take *d, c, a, e, b*, and this leaves *e, a, b, c, d* for the last man. If we plot the selections that have been made by the five men, we find the Latin square shown in Fig. 9.

Every division of the objects that can be made on this plan gives rise to a Latin square, and all possible distributions give rise to the whole of the Latin squares. Now it happens that a mathematical process exists (connected with algebraical symmetric functions) that acts towards a function representing the five collections in exactly the same way as I have supposed the men to act, and when the process is performed five times in succession, an integer results which denotes exactly the number of Latin squares of order 5 that can be constructed. Moreover, *en route* the "problème des rencontres" and the problems connected with any definite number of rows of the space are also solved.

I will now mention some questions of a more difficult character that are readily solved by the method. In the

a a b c	*a a b b*	*a a a b*
a b c a	*a b a b*	*a a b a*
b c a a	*b b a a*	*a b a a*
c a a b	*b a b a*	*b a a a*

FIG. 13.

"problème des ménages" you will recollect that the condition was that no man must sit next to his wife. If the condition be that there must be at least four (or any even number) persons between him and his wife, the question is just as easily solved. Latin squares where the letters are not all different in each row and column are easily counted. Illustrations of these are shown in Fig. 13. One of these extended to order 8 gives the solution of the problem of placing 16 castles on a chess-board, 8 black and 8 white, so that no castle can take another of its own colour.

Theoretically, the Græco-Latin squares of Euler can be counted, but I am bound to say that the most laborious calculations are necessary to arrive at a numerical result or even to establish that in certain cases the number sought is zero.

Next consider a square of any size and any number of different letters, each of which must appear in each row and in each column, while there is no restriction as to the number that may appear in any one compartment. In this case the result is very simple; suppose the square of order 4 (Fig. 14), and that there are seven different letters that must appear in each row and column; the number of arrangements is $(4\,!)^7$, viz., 4, the order of the square, must be multiplied by each lower number and the number thus reached multiplied seven times by itself.

Finally, if there be given for each row and for each column a different assemblage of letters and no restriction be placed upon the contents of any compartment, the number of squares in which all these conditions are satisfied can be counted. This, of course, is a far more recondite question than that of the Latin square, and cannot be attacked at all by any other method.

I now pass to certain purely numerical problems. Suppose we have a square lattice of any size and are told that numbers are to be placed in the compartments in such wise that the sums of the numbers in the different rows and columns are to have any given values the same or different. This very general question, hitherto regarded as unassailable, is solved quite easily. The solution is not more difficult when the lattice is rectangular instead of square and when any desired limitation is imposed upon the magnitude of the numbers.

Up to this point, the solutions obtained depend upon processes of the differential calculus. A whole series of other problems, similar in general character, but in one respect essentially different, arises from the processes of the calculus of finite differences. Into these time does not permit me to enter. In the case of magic squares as generally understood, the method brought forward marks a distinct advance in connection with De la Hire's method of formation by means of a pair of Latin squares, but apart from this a great difficulty is involved in the condition

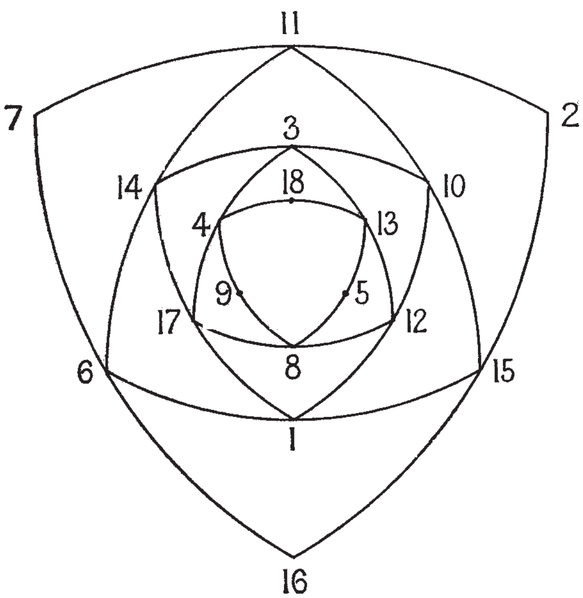
abcd		ef	g
e	abc	dg	f
f	deg	ab	c
g	f	c	abde

FIG. 14.

that no two numbers must be the same. Still, a statement can be made as to a succession of mathematical processes which result in a number which enumerates the magic squares of a given order. In any cases except those of the first few orders, the processes involve an absolutely prohibitive amount of labour, so that it cannot yet be said that a practical solution of the question has been obtained.

Scientifically speaking, it is the assignment of the processes and not the actual performance of them that is interesting; it is the method involved rather than the results flowing from the method that is attractive; it is the connecting link between two, to all appearance, widely separated departments of mathematics that it has been fascinating to forge and to strengthen. Of all the subjects that for hundreds of years past have from time to time engaged the attention of mathematicians, perhaps the most isolated has been the subject of these chess-board arrangements. This isolation does not, I believe, any longer exist. The whole series of diagrams formed according to any given laws must be regarded as a pictorial representation, in greatest detail, of the manner in which a certain process is performed. We have to exercise our wits to discover what this process is. To say and to establish that problems of the general nature of the magic square are intimately connected with the infinitesimal calculus and the calculus of finite differences is to sum the matter up. Much, however, remains to be done. The present method is not able to deal with diagonal properties, or with arrangements which depend upon the knight's move. The subject is only in its infancy at present. More workers are required who will, without doubt, introduce new ideas and obtain results far transcending those we are in possession of now. The latest work has shown that the method is applicable to boards of triangular and trapezoidal shapes, and also to solid boards in three dimensions, so that the remote ground occupied by magic and Nasik cubes will soon be invaded.

In conclusion, I bring before you an interesting example of magic arrangement that I found whilst engaged in rummaging amongst the books and documents of the old Mathematical Society of Spitalfields (1717–1845) for the purpose of extracting something which might interest or amuse, if it might not instruct, the audience I addressed in Section A of the British Association for the Advancement of Science at Glasgow last autumn. It is an arrangement of the first eighteen numbers on five connected triangles; the magical property consists in the circumstance that the numbers 19, 38 and 57 appear as sums in a variety of ways. The number 19 appears nine times, 38 twelve times and 57 fourteen times (Fig. 15).



$$\begin{aligned}
9 &= 7+12=14+5=4+15 \\
&= 6+13=17+2=9+10 \\
&=16+3=1+18=8+11 \\
38 &= 7+11+14+6=11+2+15+10=15+16+6+1 \\
&=11+10+3+14=10+15+1+12=1+6+14+17 \\
&=14+3+4+17=3+10+12+13=12+1+17+8 \\
&= 3+13+18+4=13+12+8+5=8+17+4+9 \\
57 &= 7+14+4+5+12+15=6+17+9+13+10+2=16+1+8+18+3+11 \\
&= 7+11+2+15+16+6=11+10+15+1+6+14=14+3+10+12+1+17 \\
&= 3+13+12+8+17+4=4+18+13+5+8+9 \\
&= 9+4+3+10+15+16=18+13+12+1+6+7=5+8+17+14+11+2 \\
&= 9+8+12+10+11+7=18+4+17+1+15+2=5+13+3+14+6+16
\end{aligned}$$

FIG. 15.

I should say that I feel conscious that I have not been able to introduce the subject of my lecture without occasional and, perhaps, in the circumstances, unavoidable obscurity. For the rest, I have felt somewhat doubtful as to the interest I might arouse in these problems, but the managers honoured me by inviting me to display to you some of the chips from a pure mathematician's workshop, and I felt no hesitation in accepting.

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