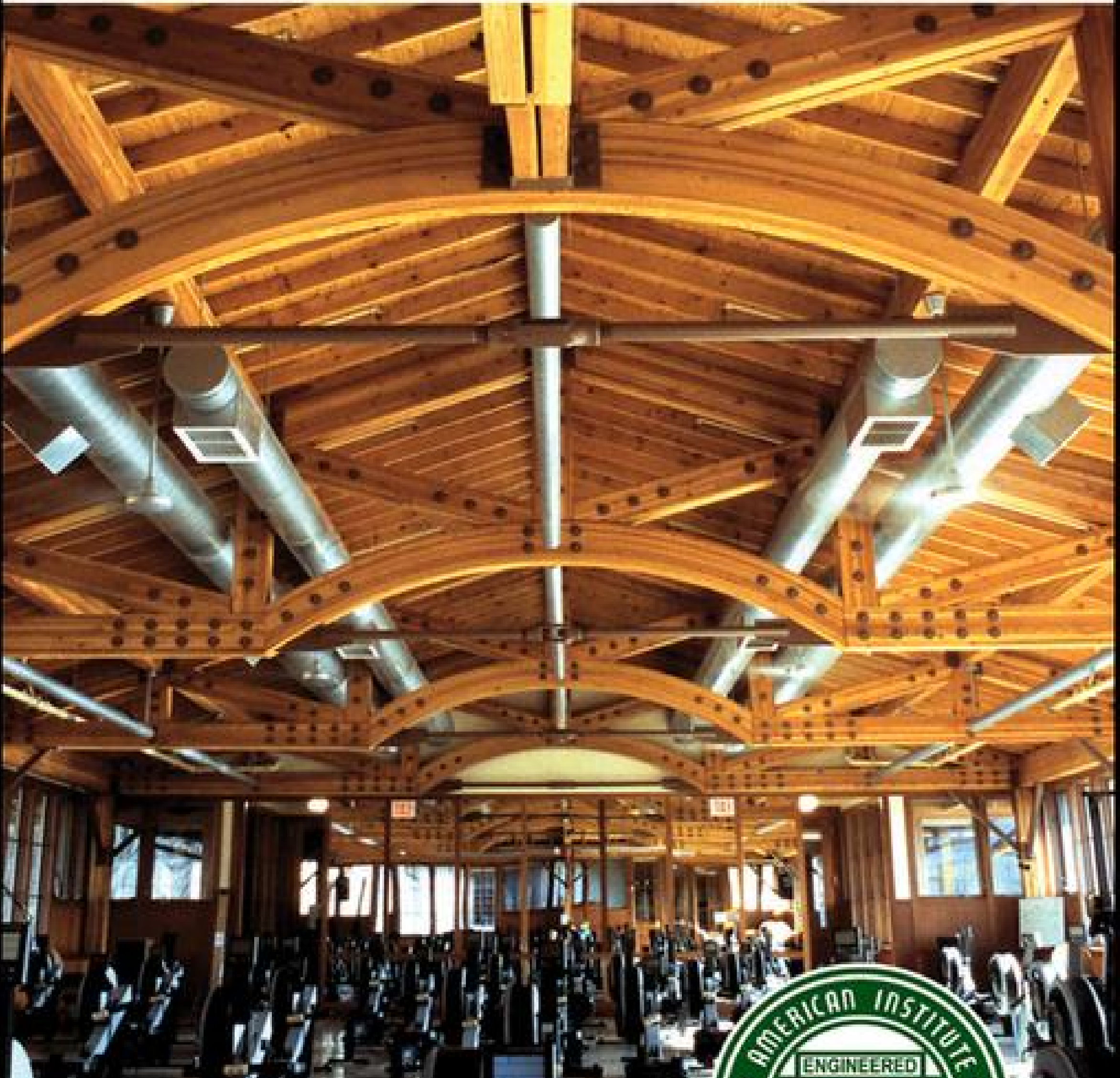
AMERICAN INSTITUTE OF TIMBER CONSTRUCTION



TIMBER CONSTRUCTION MANUAL

Sixth Edition

TIMBER
CONSTRUCTION MANUAL

Timber Construction Manual American Institute of Timber Construction

TIMBER CONSTRUCTION MANUAL

Sixth Edition

AMERICAN INSTITUTE OF TIMBER CONSTRUCTION

WILEY

JOHN WILEY & SONS, INC.

This book is printed on acid-free paper. ♾

Published by John Wiley & Sons, Inc., Hoboken, New Jersey
Published simultaneously in Canada

Library of Congress Cataloging-in-Publication Data:

Timber construction manual / American Institute of Timber Construction.—Sixth edition.
pages cm
Includes index.
ISBN 978-0-470-54509-6 (hardback); 978-1-118-27961-8 (ebk.); 978-1-118-27964-9 (ebk.); 978-1-118-27965-6 (ebk.); 978-1-118-27966-3 (ebk.); 978-1-118-27968-7 (ebk.); 978-1-118-27973-1 (ebk.)
1. Building, Wooden—Handbooks, manuals, etc. I. American Institute of Timber Construction.
TA666.T47 2012
694—dc23

2011051084

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

CONTENTS

PREFACE

The American Institute of Timber Construction (AITC) has developed this *Timber Construction Manual* for convenient reference by architects, engineers, contractors, teachers, the laminating and fabricating industry, and all others having a need for reliable, up-to-date technical data and recommendations on engineered timber construction. The information and the recommendations herein are based on the most reliable technical data available and reflect the commercial practices found to be most practical. Their application results in structurally sound construction.

The American Institute of Timber Construction, established in 1952, is a nonprofit industry association for the structural glued laminated timber industry. Its members design, manufacture, fabricate, assemble, and erect structural timber systems utilizing both sawn and structural glued laminated timber components. These systems are used in homes; schools; churches; commercial and industrial buildings; and for other structures such as bridges, towers, and marine installations. Institute membership also includes engineers, architects, building officials, and associates from other industries related to timber construction.

The first edition of the *Timber Construction Manual* was published in 1966. Changes in the wood products industry, technological advances, and improvements in the structural timber fabricating industry necessitated revisions of the *Manual*. New lumber sizes and revisions in grading requirements for lumber and glued laminated timber were reflected in the second edition published in 1974. The third edition was published in 1985 to reflect new information on timber design methods. The fourth edition of the *Manual* was published in 1994 and contained updated design procedures used for timber construction. The fifth edition (2005) added sections on timber rivet fasteners and load and resistance factor design.

This sixth edition represents a major revision of the format of the *Timber Construction Manual*. Chapters have been completely restructured for more logical and complete presentation of information. Long chapters have been divided into smaller chapters for improved readability and reference.

The sixth edition has also been expanded with completely new chapters on glulam arches, LRFD bridge design, fire safety, and moment splices. More than 30 new design examples have been added, including an appendix entirely composed of design examples.

Preparation of the *Timber Construction Manual* was guided by the AITC Technical Advisory Committee and was carried out by AITC staff, the engineers and technical representatives of AITC member firms, and private consultants. Suggestions for the improvement of this manual will be welcomed and will receive consideration in the preparation of future editions.

Although the information herein has been prepared in accordance with recognized engineering principles and is based on the most accurate and reliable technical data available, it should not be used or relied on for any general or specific application without competent professional examination and verification of their accuracy, suitability, and applicability by a licensed professional engineer, designer, or architect. By the publication of this manual, AITC intends no representation or warranty, expressed or implied, that the information contained herein is suitable for any general or specific use or is free from infringement of any patent or copyright. Any user of this information assumes all risk and liability arising from such use.

CHAPTER 1

TIMBER CONSTRUCTION

1.1 INTRODUCTION

The American Institute of Timber Construction (AITC) has developed this *Timber Construction Manual* to provide up-to-date technical information and recommendations on engineered timber construction. Topics of the first chapter include materials, structural systems, economy, permanence, seasoning, handling, storage, and erection. With an understanding of these topics, the designer can more effectively utilize the advantages of wood construction. Specific design information and recommendations are covered in subsequent chapters, with accompanying design examples. Supplementary information is provided in appendices.

1.2 MATERIALS

This manual applies primarily to two types of wood materials—sawn lumber and structural glued laminated timber (glulam). Sawn lumber is the product of lumber mills and is produced from many species. Glued laminated timbers are produced in laminating plants by adhesively bonding dry lumber, normally of 2-in. or 1-in. nominal thickness, under controlled conditions of temperature and pressure. Members with a wide variety of sizes, shapes, and lengths can be produced having superior strength, stiffness, and appearance. In addition, heavy timber decking, structural panels, and round timbers are also discussed.

1.2.1 Lumber

In its natural state, wood has limited structural usefulness, so it must be converted to a structural form that is compatible with construction needs. The most

Timber Construction Manual American Institute of Timber Construction

common structural wood material is sawn lumber. Sawn lumber is also the primary component of structural glued laminated timber. This section will discuss common growth characteristics of wood and their effects on the properties of structural lumber. It will also discuss common grading systems for lumber.

1.2.1.1 Lumber Grading As it is sawn from a log, lumber is quite variable in its mechanical properties. Individual pieces may differ in strength by as much as several hundred percent. For simplicity and economy in use, pieces of lumber of similar quality are classified into various structural grades. The structural properties of a particular grade depend on the sorting criteria used, species or species group, and other factors.

Rules for determining lumber grades are written by rules writing agencies authorized by the American Lumber Standards Committee (ALSC) [1]. Four such agencies are the Southern Pine Inspection Bureau (SPIB) [2], the West Coast Lumber Inspection Bureau (WCLIB) [3], the Western Wood Products Association (WWPA) [4], and the National Lumber Grades Authority (NLGA) [5]. Lumber grading is also certified by agencies authorized by the American Lumber Standards Committee. Generally, the designer of timber structures is not charged with grading but instead with selecting commercially available grades that meet necessary structural requirements.

Lumber rules writing agencies also establish design values and adjustment factors for each grade. Design values provided by the agencies are published in the *National Design Specification*® (*NDS*®) [6]. These values and factors are generally accepted by model and/or local building codes but are occasionally adopted with amendments particular to the jurisdiction.

Grading is accomplished by sorting pieces according to visually observable characteristics (visual grading) or according to measurable mechanical properties and visual characteristics (mechanical grading). Both grading methods relate key lumber characteristics to expected strength.

1.2.1.2 Characteristics Affecting Structural Lumber Quality Within any given species of wood, several natural growth characteristics observed in structural lumber are important for the determination of quality of the material and the assignment of design values. The main characteristics of concern include: specific gravity, knots, slope of grain, and modulus of elasticity. Other important characteristics include reaction wood, juvenile (pith-associated) wood, and compression breaks. Lumber grading rules regulate these characteristics based on the effect they have on the strength of a piece.

1.2.1.2.1 Specific Gravity Specific gravity is a good index for strength and stiffness of clear wood (free of knots and other strength-reducing characteristics). As the specific gravity of wood increases, so do its mechanical properties (strength and stiffness). The specific gravity of certain species of lumber can be estimated by the amount of latewood in the piece. Because latewood is typically more dense than earlywood, higher proportions of latewood equate to higher specific gravities. This relationship is commonly used in grade rules for structural lumber. Visual grading rules classify lumber according to growth ring measurements as having dense, medium, or coarse grain based on the width of the rings and on the proportion of latewood present. Mechanical grading systems may use weight or calibrated x-ray machines to determine specific gravity.

1.2.1.2.2 Knots A knot is formed by sawing through a portion of the tree trunk that formed around a branch. Knots are considered as defects in structural lumber. The presence of a knot disrupts the longitudinal orientation of the wood fibers as they deviate around the knot. A knot may be intergrown with the surrounding wood or encased by the surrounding wood without intergrown fibers. The latter type of knot is called a *loose knot* and often falls out, leaving a knothole. Both types of knots reduce the capacity and stiffness of a structural member, particularly in tension. Grade rules typically restrict the size, location, and frequency of knots, knot clusters, and knot holes allowed by each grade.

1.2.1.2.3 Slope of Grain It is generally desirable to have the longitudinal axis of the wood cells line up with the longitudinal axis of the structural member. However, irregularities in growth and various methods of sawing employed in the manufacture of structural lumber invariably result in a grain at some angle to the longitudinal axis of the member. Because wood is orthotropic, wood is not as strong to resist loads at angles to the grain as for loads parallel to the grain. Consequently, lumber grading rules typically restrict the general slope of grain allowed in any particular grade. Additionally, high-grade tension laminations used in structural glued laminated timber restrict the amount of localized grain deviations, such as those caused by a knot.

1.2.1.2.4 Modulus of Elasticity In structural lumber, a correlation has been observed between stiffness and other properties. Increases in modulus of elasticity are correlated with increases in specific gravity and strength and with decreases in slope of grain. The correlation between stiffness and strength forms the basis for most common mechanical grading systems, which sort pieces by stiffness.

1.2.1.2.5 Other Characteristics Reaction wood, timber breaks, juvenile wood, and decay each negatively affect the mechanical and physical properties of structural lumber. They are, therefore, limited or excluded by lumber grade rules.

1.2.1.3 Grading Systems Visual grading systems employ trained inspectors to look at each side of a piece of lumber and assign an appropriate grade based on the observed characteristics in the piece. Mechanical grading systems use some sort of device to measure properties not apparently visible, such as density or modulus of elasticity (in addition to visual inspection), to assign grades. Mechanical grading systems may also require random testing of pieces to ensure that the assigned strength and stiffness values are met and maintained over time.

Three main lumber products are produced from mechanical grading systems in the United States: machine stress rated (MSR) lumber, mechanically evaluated lumber (MEL), and E-rated lumber. Pieces graded under the MSR and MEL systems are generally used as individual pieces and are consequently assigned design values. E-rating is used primarily for laminating lumber, and design values for individual pieces are not available. Although mechanical grading is increasing in popularity, visual grading remains the most common grading method employed for structural lumber.

1.2.2 Structural Glued Laminated Timber

The term *structural glued laminated timber* refers to an engineered, stress-rated product of a timber laminating plant comprising assemblies of suitably selected and prepared wood laminations bonded together with adhesives. The grain of all laminations is approximately parallel longitudinally. Individual laminations generally do not exceed 2 in. net thickness. Individual lumber pieces may be joined end-to-end to produce laminated timbers much longer than the laminating stock itself. Pieces may also be placed or glued edge to edge to make timbers wider than the input lumber. As such, glued laminated timbers (glulam) may be made to almost any size; however, shipping considerations generally limit the size of glulam normally produced. Glued laminated timbers may also be manufactured into curved shapes, adding to their appeal for use as architectural elements.

Glulam can be custom manufactured for special applications requiring highly specific members with features such as large sizes, taper, curvature, or special fabrication. The versatility of these custom glulam products allows the designer to maximize creativity. Manufacturers specializing in custom glulam often provide engineering for these applications.

Other manufacturers specialize in producing high volumes of straight (or mildly cambered) glulam stock members in commonly used layups and sizes. Long-length stock members are typically sent to distribution yards, where they are held in inventory and cut to length as needed for immediate availability to builders for use as beams and columns. Manufacturers of these “stock” products generally do not offer engineering services.

The AITC quality mark on a structural glued laminated timber member ensures that the member was manufactured in a facility with strict quality control measures in place. AITC-certified laminators meet the highest quality standards of the industry. AITC-certified glulam is available and accepted throughout the United States. Consequently, engineers and architects may readily specify AITC-certified stock or custom members with high assurance of structural quality and reasonable availability and affordability.

1.2.2.1 Benefits of Glulam Structural glued laminated timber marries the traditional warmth and beauty of wood with modern engineering to create a beautiful material with outstanding structural properties. Some of the benefits of structural glued laminated timber are listed as follows:

- *Environmentally friendly*. Wood is a naturally renewable resource. The wood products industry is committed to sustainable forestry practices. Processing logs to make lumber and glulam uses very little energy, reducing the use of fossil fuels and pollution of our atmosphere. Glulam technology also uses small dimension lumber to make large structural timbers, utilizing logs from second- and third-growth forests and timber plantations. Glulam's efficient use of the highest-quality lumber only where stresses are critical further reduces demand on precious lumber resources.
- *Beautiful*. The natural beauty of wood is unsurpassed. Exposed glulam timbers provide structures with a warmth and beauty unrivaled by other building materials.
- *Strong and stiff.* Glulam's superior strength and stiffness permit larger rooms with fewer columns. Pound for pound, glulam beams are stronger than steel.
- *Dimensionally stable*. Glulam is manufactured from small dimension lumber that is dried prior to laminating. This translates to less checking, warp, and twist than traditional sawn timbers.
- *Durable*. When properly designed to keep the wood dry, glued laminated timbers will last indefinitely. In situations where it is not possible to keep the wood dry, pressure preservative-treated wood or heartwood of a naturally durable species can be used to maximize the service life of the structure. The adhesives used in glulam are waterproof to ensure long life. Wood is also very resistant to most chemicals.
- *Fire resistant*. Structural glued laminated timber has excellent fire performance. Building codes recognize fire ratings of up to one or two hours for properly designed, exposed glulam members. Glued laminated timbers can also be used to meet the Heavy Timber Construction requirements in the building codes.
- *Versatile*. Structural glued laminated timber can be manufactured in a variety of shapes, from straight beams to graceful, curved arches. Sizes of individual members are limited only by shipping capabilities. Components for large assemblies can be fabricated in a plant, transported long distances, and reassembled at the job site.
- *Simple*. Design steps are similar to those for solid sawn lumber and timbers. The structural glued laminated timber industry has adopted a stress classification system to simplify the specification of glued laminated timbers.
- *Cost-effective*. The beauty of glulam framing systems allows structures to be designed and built without costly false ceilings to cover structural components. Installation is fast and easy, reducing costs at the job site. High strength and stiffness permit the use of smaller members for additional cost savings.

- *Dependable*. Glued laminated timbers have been used successfully in the United States for more than 75 years. In Europe, glulam has been used successfully for more than 100 years. AITC's quality program ensures consistent, reliable product performance by inspecting all stages of production for conformance with recognized industry standards.

1.2.2.2 Layup Principles Lumber quality varies significantly within any particular lumber resource. High-quality lumber is more scarce and costly than low-quality lumber. The laminating process offers the manufacturer the unique benefit of placing the best lumber only where stresses are critical and using lower grades of lumber elsewhere. For glulam beams, the highest grades of lumber are typically placed near the top and bottom of the member. This placement makes optimum use of the high strength and stiffness material to resist bending and deflection. Laminated timbers intended primarily to resist uniform axial loads or bending loads about the weak axis are manufactured using the same grade of lumber throughout the cross section. The former type of layup is referred to as an *optimized layup*, while the latter is simply called a *uniform-grade layup*.

Optimized layups are further divided into two categories: *balanced* and *unbalanced* (Figure 1.2.2.2-1). Balanced layups have the same grades of lumber in the top half of the beam as in the bottom half. The two halves are mirror images of each other. Unbalanced beams use higher grades in the bottom half than in the top half. Balanced beams are typically used for continuous-span and cantilevered applications. Unbalanced beams are more efficient for simple spans. Unbalanced beams can also accommodate short cantilevers (up to about 20% of the main span) efficiently.

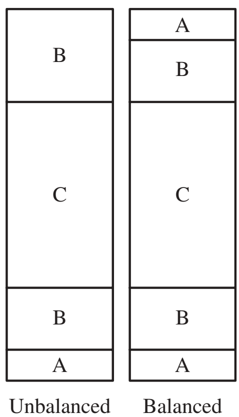


Figure 1.2.2.2-1 Unbalanced layups vs. balanced layups: A, B, and C represent lumber grades, with A representing the highest grade in the lay-up.

1.2.2.3 Combination Symbols Structural glued laminated timber layups are assigned a combination symbol for the purposes of design and specification. The layup requirements for common glulam combinations are provided in AITC 117 [7], Tables B1 and B2. Design values for these combinations are provided in Tables A1-Expanded and A2 of AITC 117 [7] and in Tables 5A-Expanded and 5B of the NDS® Supplement [6].

1.2.2.4 Stress Classes To facilitate selection and specification, similar optimized beam combinations are grouped into stress classes. Designing and specifying with the stress class system simplifies the process for the designer and allows the manufacturer to produce the most efficient combination for his resource that meets the stress class requirements. Design values for the stress classes are shown in Table 5A of AITC 117 [7] and Table 5A of the NDS® Supplement [6]. It is important to note that each layup combination corresponds either to a balanced or unbalanced layup whereas the stress classes do not. Therefore, it is necessary when specifying glulam by stress class to clearly indicate where balanced layups are required.

1.2.2.5 Manufacturing Process Although laminating is a simple concept, the production of modern structural glued laminated timber according to required standards is a complex process. Lumber grades and moisture content are strictly controlled and adhesives must meet rigorous performance requirements. Laminations are machined within precise tolerances and kept clean for bonding. Bonded end joints and face joints are tested daily to ensure that strength and durability requirements are maintained. Code-compliant laminators are required to implement a continuous quality control system with periodic auditing by an accredited inspection agency, such as AITC. The requirements for glulam manufacturing are detailed in ANSI/AITC A190.1 [8] and AITC 200 [9].

1.2.2.5.1 Face Bonds Laminations are stacked and bonded face-to-face to make deep timbers. Adhesive bonds are subject to initial qualification and subsequent daily tests for both durability and strength. Adhesive joints are required to be essentially as strong as the wood they bond. Because of the bond quality and strength, the designer does not need to account for the presence of bond lines during the design process. Consequently, bolts and other fasteners can be placed indiscriminately with regard to bond lines.

1.2.2.5.2 End Joints End joints are used to make individual laminations of essentially unlimited length and to create whole timbers in lengths far exceeding those possible from sawn timbers. The end joint is the most highly engineered and monitored part of the manufacturing process. A manufacturer's end joint strength is established by an initial qualification process, verified through daily quality control tests, and maintained using statistical process control techniques.

These rigorous quality control measures ensure that the strength of the end joint is sufficient to justify the design strength of the glulam beam in which it will be used. Consequently, the designer does not have to account for the presence of end joints when designing structural glued laminated timber members.

1.2.2.6 Appearance Grades Structural glued laminated timber is manufactured to meet any of four standard appearance grades as well as custom appearance options offered by individual laminators. The standard appearance grades are: framing, industrial, architectural, and premium. Framing grade members are surfaced hit-and-miss only to match standard framing lumber dimensions (i.e., 3.5 inches and 5.5 inches wide). This results in a generally poor appearance that is suitable only for concealed applications. Industrial, architectural, and premium grades require more surfacing and are appropriate for applications where the timbers will be exposed to view. The specific requirements for each grade are listed in AITC 110 *Standard Appearance Grades for Structural Glued Laminated Timber* [10].

1.2.2.7 Quality Assurance The International Building Code [11] requires glulam to be manufactured according to ANSI/AITC A190.1 [8], including periodic auditing by an *accredited inspection agency*. The American Institute of Timber Construction (AITC) is accredited to provide inspection and auditing services for structural glued laminated timber and other engineered wood products. The AITC Inspection Bureau visits AITC-certified producers periodically to verify that each plant is meeting the requirements of ANSI/AITC A190.1 and other relevant AITC standards as well as their own internal quality control and procedures manuals. Plants meeting the rigorous requirements of the standard are licensed to use the AITC quality mark on their production (Figure 1.2.2.7-1), signifying compliance with the standard. Building officials recognize the AITC quality mark as evidence that a laminated timber meets the requirements of the code-recognized manufacturing standard. The AITC quality program is summarized in AITC Technical Note 10 [12].

Figure 1.2.2.7-1 AITC Symbol of Quality®

1.2.2.8 Custom Glulam Products In general, the manufacturing process of structural glued laminated timber allows for the efficient use of wood resources, as well as being able to incorporate custom features such as taper, camber, curvature, special appearance, and so on. Such members are well-suited to custom and specialty applications, such as exposed timber trusses, glulam arches, curved beams, and large members. Long lengths, large dimensions, and high design values make glulam desirable for use in situations requiring long spans and/or supporting heavy loads. These applications generally involve engineering calculations provided by licensed professionals, and are typically specific to the individual structures or projects. Indeed, building codes generally require the load-carrying systems of structures to be engineered.

With custom glulam products, the designer has maximum design flexibility. Members can be produced in a wide range of shapes and sizes to fit the particular application. The choice of glulam combinations is practically unlimited. Many custom laminators also provide engineering services, so they can help throughout the design process.

Even though custom glulam products allow for maximum design flexibility, standard sizes should be used where possible. Standard widths for custom southern pine members are 3 in., 5 in., $6\frac{3}{4}$ in., $8\frac{1}{2}$ in., $10\frac{1}{2}$ in., 12 in., and 14 in. Standard widths for custom Alaska cedar and Douglas fir members are $3\frac{1}{8}$ in., $5\frac{1}{8}$ in., $6\frac{3}{4}$ in., $8\frac{3}{4}$ in., $10\frac{3}{4}$ in., $12\frac{1}{4}$ in., and $14\frac{1}{4}$ in. Wider widths and other species may be available upon consultation with the laminator.

1.2.2.9 Stock Glulam Products For many jobs, the engineer (or architect) may specify off-the-shelf products (stock glulam) that are not necessarily project specific. With other considerations equal, the specification of stock products will generally be more economical and time efficient than specialty or custom products.

The prudent design professional is generally cognizant of what particular products are generally available for particular projects or locations. Throughout the United States, 24F-1.8E unbalanced beams in Douglas fir or southern pine are typically stocked by lumber distributors, with the species dependent on the region. Beams with balanced layups may also be available. In addition to beams, it is common for distributors to carry uniform-grade members for use as columns, with grades and species dependent on the region. In some regions, higher strength members of Douglas fir or southern pine up to 30F-2.1E grade beams may be stocked. Depending on the region, Alaska cedar members or pressure-treated southern pine members may be carried in inventory for use where decay resistance is important.

1.2.2.9.1 Widths Stock glulam products are most commonly available in industrial or architectural appearance grades, with finished widths of $3\frac{1}{8}$ in., $5\frac{1}{8}$ in., and $6\frac{3}{4}$ in. in either Douglas fir or southern pine members. In some regions, however, framing appearance grade beams with finished widths of $3\frac{1}{2}$ in., $5\frac{1}{4}$ in., or $5\frac{1}{2}$ in., and 7 in. or $7\frac{1}{4}$ in. may be stocked.

1.2.2.9.2 Depths Depths of stock members are typically multiples of $1\frac{1}{2}$ in. for Douglas fir and $1\frac{3}{8}$ in. for southern pine. Stock beams are typically made in depths of 30 inches or less.

Glulam beams are also commonly manufactured in *I-joist-compatible* (IJC) depths. IJC beams have depths equal to common wood I-joist depths of $9\frac{1}{2}$ in., $11\frac{7}{8}$ in., 14 in., 16 in., 18 in., 20 in., 22 in., or 24 in. for framing within floor spaces. IJC stock beams are typically manufactured in framing or industrial appearance grades.

1.2.2.9.3 Camber Stock members are generally intended for simple-span use and are manufactured with a single radius of curvature. A typical radius for stock beams is 3500 ft; however, different manufacturers may use different standard radius values. In this regard, the design professional seeking to specify stock products will check the suitability of available curvature (camber) values instead of requiring the manufacture of specific cambers for individual members. The present trend of both manufacturers and designers is that of using flat (straight, no camber) or very shallow camber (large radius of curvature) beams. Such members are more easily framed in the field and are have acceptable deflections in service. Beams that span over interior supports or have cantilevered ends are expected to experience significant flexural tension stresses on both top and bottom, and as such, balanced layups are specified. Stock beams with balanced layups are typically manufactured without camber.

1.2.2.9.4 Nonengineered Construction In addition to *engineered* applications, glulam is finding increased use in *nonengineered* construction. Manufacturers typically publish capacity tables and other information (and some provide software) to assist selection of members for simple framing applications.

Nonengineered construction is particularly applicable to residential construction, a great part of which is prescriptive (*conventional light frame construction*). As a framing member, glulam may be substituted (per building official approval) for sawn or built up sawn lumber joists, rafters, headers, and girders. Table 1.2.2.9.4-1 provides 24F-1.8E stock glulam substitutions in both Douglas fir and southern pine for common sawn lumber sizes (single member or multiple-ply) of No. 2 grade Douglas fir-larch or southern pine. The tabulated glulam sizes are also conservative for No. 2 grade hem-fir and spruce-pine-fir. In addition, maximum spans for 24F-1.8E stock glulam beams subject to various loading configurations are included in the *Wood Frame Construction Manual* [13].

1.2.3 Heavy Timber Decking

The term *heavy timber decking* generally refers to lumber sawn with a single or double tongue-and-groove profile on the narrow edges. Heavy timber decking is typically used to form a structural roof in heavy timber systems, and it can also be used for floors. It typically spans 4 ft to 18 ft between timber beams or purlins forming the structural members and serving as the finished ceiling for the

TABLE 1.2.2.9.4-1 24F-1.8E Glulam Beam Sizes to Replace No. 2 Sawn Lumber

Sawn Lumber Nominal Size	24F-1.8E DF Glulam Size to Replace No. 2 DF-L Lumber	24F-1.8E SP Glulam Size to Replace No. 2 SP Lumber
4×8	$3\frac{1}{8} \times 7\frac{1}{2}$	$3\frac{1}{8} \times 8\frac{1}{4}$
4×10	$3\frac{1}{8} \times 10\frac{1}{2}$	$3\frac{1}{8} \times 9\frac{5}{8}$
4×12	$3\frac{1}{8} \times 12$	$3\frac{1}{8} \times 12\frac{3}{8}$
4×14	$3\frac{1}{8} \times 13\frac{1}{2}$	$3\frac{1}{8} \times 13\frac{3}{4}$
4×16	$3\frac{1}{8} \times 16\frac{1}{2}$	$3 \times 16\frac{1}{2}$
6×8	$5\frac{1}{8} \times 7\frac{1}{2}$	$5\frac{1}{8} \times 6\frac{7}{8}$
6×10	$5\frac{1}{8} \times 9$	$5\frac{1}{8} \times 9\frac{5}{8}$
6×12	$5\frac{1}{8} \times 12$	$5\frac{1}{8} \times 11$
6×14	$5\frac{1}{8} \times 13\frac{1}{2}$	$5\frac{1}{8} \times 12\frac{3}{8}$
6×16	$5\frac{1}{8} \times 15$	$5\frac{1}{8} \times 15\frac{1}{8}$
8×10	$6\frac{3}{4} \times 9$	$6\frac{3}{4} \times 9\frac{5}{8}$
8×12	$6\frac{3}{4} \times 12$	$6\frac{3}{4} \times 11$
8×14	$6\frac{3}{4} \times 13\frac{1}{2}$	$6\frac{3}{4} \times 12\frac{3}{8}$
8×16	$6\frac{3}{4} \times 15$	$6\frac{3}{4} \times 15\frac{1}{8}$
(2) 2×8	$3\frac{1}{8} \times 7\frac{1}{2}$	$3\frac{1}{8} \times 8\frac{1}{4}$
(2) 2×10	$3\frac{1}{8} \times 9$	$3\frac{1}{8} \times 9\frac{5}{8}$
(2) 2×12	$3\frac{1}{8} \times 12$	$3\frac{1}{8} \times 11$
(3) 2×8	$5\frac{1}{8} \times 7\frac{1}{2}$	$5\frac{1}{8} \times 6\frac{7}{8}$
(3) 2×10	$5\frac{1}{8} \times 9$	$5\frac{1}{8} \times 9\frac{5}{8}$
(3) 2×12	$5\frac{1}{8} \times 10\frac{1}{2}$	$5\frac{1}{8} \times 11$
(4) 2×8	$6\frac{3}{4} \times 7\frac{1}{2}$	$6\frac{3}{4} \times 6\frac{7}{8}$
(4) 2×10	$6\frac{3}{4} \times 9$	$6\frac{3}{4} \times 9\frac{5}{8}$
(4) 2×12	$6\frac{3}{4} \times 10\frac{1}{2}$	$6\frac{3}{4} \times 11$

space below. Both sawn decking and laminated decking are available. Typical edge joints for decking are shown in Figure 1.2.3-1.

Information on installation and design of two-inch, three-inch, and four-inch nominal thickness tongue-and-groove heavy timber decking may be found in Chapter 10 and in the *International Building Code* [11]. Lumber rules writing agencies publish design values for sawn timber decking.

Glued laminated decking may be manufactured in longer lengths and greater nominal thicknesses and often has higher design values than similar sawn decking. Size, length, and design information for laminated decking should be obtained from the individual manufacturer.

Decking lengths may be specified and ordered to end over supports, where the boards are assumed to act as shallow beams spanning one or more joist

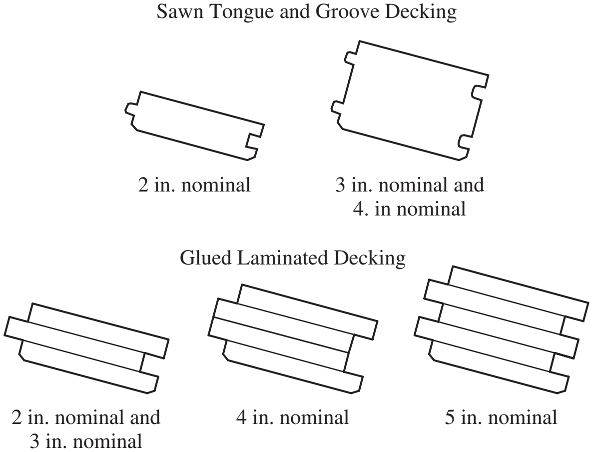


Figure 1.2.3-1 Edge joints for heavy timber decking.

spaces. In many cases, however, it is more economical to specify decking of varied or random lengths. Chapter 10 describes five standard installation patterns for timber decking. Design values for heavy timber decking are established by approved lumber grading agencies. Design values for laminated decking should be obtained from the decking manufacturer.

1.2.4 Structural Panels

Wood structural panels are commonly used for exterior walls and roof surfaces and for floors. Panels with appropriate preservative treatment are sometimes used below grade for wood foundations. Care should be taken to ensure that the selected panels meet the appropriate exposure requirements and satisfy the grade, span rating, and minimum thickness requirements of the local building codes. Structural wood panels should meet the requirements of Voluntary Product Standards PS 1 [14] or PS 2 [15], or those specified by a recognized code evaluation report.

1.2.5 Round Timbers

Modern timber construction most commonly uses members with rectangular cross sections. However, round timber poles and piles continue to be used for some applications. These members are typically left naturally round and tapered, maintaining the shape of the log from which they were cut. Additionally, timbers are occasionally machined to a round or nearly round section.

Poles must conform to ASTM Standard D3200 [16] and piles must conform to ASTM Standard D25 [17] with design values established according to ASTM Standard D2899 [18]. Pole sizes and specifications are per American National Standards Institute (ANSI) Standard O5.1 [19]. Poles and piles must be pressure preservative treated in accordance with the American Wood Protection Association Standards [20] when they are to be used in ground contact or in wet use conditions.

For log construction, grades for round logs and round logs sawn flat on one side are developed in accordance with ASTM Standard D3957 [21]. For members machined to a round cross-section, there is no specific guidance given for grading and assignment of design values. It is recommended that a lumber grading agency be contacted for guidance for assigning grades to members machined to a round cross-section.

1.3 STRUCTURAL SYSTEMS

Structural timber systems take on many forms. Systems discussed in this chapter include: post and beam framing; light frame construction; pole construction, post-frame construction and timber piles; timber trusses; glulam arches; and structural diaphragms.

1.3.1 Post and Beam

Post and beam construction generally consists of roof and floor panels or decking supported by joists or purlins, which are in turn supported by beams or girders, which are supported by columns. Post and beam construction is illustrated in Figure 1.3.1-1. The joists, purlins, beams, and girders are generally framed to carry gravity loads as bending members and the columns resist axial loads. Individual members are designed in accordance with Chapters 4 to 10. Secondary members are typically framed at intervals of 16 in. to 4 ft to accommodate common panel and other material sizes.

Figure 1.3.1-1 Post and beam construction.

Although post and beam framing systems are commonly used for gravity load resistance (roof, snow, and floor loads), they are not inherently suited for lateral load resistance (wind and seismic) without additional elements. For lateral load resistance, bracing is required, and is commonly provided by the addition of cross braces (or "trussing"); let-in bracing, straps, or cables; knee and ankle bracing; and the development of the roof, floor, and wall framing into structural diaphragms and shear walls.

1.3.2 Light Frame Construction

Light frame construction is characterized by repetitive arrangements of small, closely-spaced members installed parallel to each other as wall studs and floor joists. Individual pieces are typically dimension lumber or wood I-joists spaced at intervals of 16–24 inches apart with structural panels spanning across the lumber members forming walls and floors. Light, closely spaced trusses with structural panel sheathing are typically used for roof systems. Larger timber members or built-up members are used as columns and beams where openings are necessary in bearing walls and in floor or roof systems.

Light frame construction is the predominant system used for residential construction in North America. The *Wood Frame Construction Manual* [13] published by the American Wood Council provides design guidance and details for light frame structures including both engineered and prescriptive solutions permitted by the *International Residential Code* [22].

1.3.3 Pole Construction

Pole-type structures generally consist of tapered, round timber poles set in the ground as the main upright supporting members. These poles provide resistance to gravity and lateral loads imposed on the structure.

For resistance of gravity loads, the poles (in some cases referred to as piers) are generally set to bear on undisturbed native soil, engineered fill, or footings. For light gravity loads and/or stronger soils, bearing of a pole on soil may be adequate. For heavier loads and/or weaker soils, the gravity loads must be distributed through spread footings under the poles.

Resistance of lateral loads is achieved by pole bearing laterally on soil or pole embedment in concrete or other fill which bears laterally on surrounding soil. Pole construction relies on the resistance to rotation of the poles provided by pole embedment and backfill. As such, it is critical that the backfill material and placement be suitably specified and its quality assured.

Pole sizes and specifications are per American National Standards Institute (ANSI) Standard O5.1 [19]. Poles must be pressure preservative treated in accordance with the American Wood Protection Association Standards [20]. General considerations applicable to all pole structures include the following:

1. Bracing can be provided at the top of a pole in the form of knee braces or cross bracing in order to reduce bending moments at the base of the pole and to distribute loads; otherwise, the poles must be designed as

vertical cantilevers. The design of buildings supported by poles without bracing requires good knowledge of soil conditions in order to eliminate excessive deflection or side-sway. Where knee braces, cross braces, or other structural elements are attached to the pole and/or roof members, they must be included as integral in the analysis of both vertical and lateral load-resisting systems.

2. Bearing values under butt ends of poles must be checked with regard to the bearing capacity of the supporting soil. Where the bearing capacity of the soil is not sufficient, a structural concrete footing may first be placed under the pole to spread the load. Backfilling the hole with concrete is common practice. With regard to gravity bearing, the concrete backfill can be used to spread the load if a suitable load transfer mechanism is provided from pole to concrete backfill. Friction between pole and concrete should not be relied upon. Studs or dowels may be used; if metal, they must be galvanized or protected against corrosion by some other means. Installation of the stud or dowel in the post must not compromise the pole's resistance to decay. Boring, notching, or other modifications to poles or posts should be done prior to preservative treatment. If boring or notching must be done in the field, the recommendations of the American Wood Protection Association, Standard AWPA M4 [23] must be followed.
3. Where the poles are used to resist lateral loads, it is essential that the backfill material be properly compacted. Sand may be used if placed in shallow, thoroughly compacted (tamped) lifts. Compaction of soil or gravel should be supervised and certified by a geotechnical engineer or other design professional familiar with the site soils conditions and requirements to achieve the needed resistance to lateral movements of the poles. The use of concrete and soil cement backfill will generally result in lesser required embedment depths as the concrete or soil cement provide greater effective pole diameter with regard to lateral bearing.
4. The use of diaphragms and shear walls in pole structures generally results in smaller poles and shallower pole embedment requirements.
5. Pole structures may have excessive deflections for some applications, particularly where there is no bracing, diaphragm, or shear wall action. Deflections of structures with gypsum coverings and glazing should be carefully analyzed.
6. Pole design requires the use of adjustment factors common to wood construction as well as adjustment factors unique to poles. Design values and adjustment factors applicable to timber pole and pile construction are found in the *National Design Specification® for Wood Construction* [6].
7. The intended use of the structure generally determines such features as height, overall length and width, spacing of poles, height at eaves, type of roof framing, and the kind of flooring to be used, as well as any special features such as wide bays, unsymmetrical layouts, or the possible suspending of particular loads from the framing.

1.3.4 Post-Frame Construction

Post-frame construction typically consists of lumber or glulam posts supporting light metal-plate connected trusses and other framing and sheathing or cladding. While the posts in post-frame structures generally resist the gravity loads, lateral forces are typically resisted within the above-ground portion of the structure by a combination of post bending and the diaphragm actions of roof, floors, and walls. Both lateral and vertical forces of the whole superstructure are transferred to the ground via the posts. Posts in post-frame are thus both foundational elements and part of the superstructure.

Post-frame construction has been widely used for agricultural and utility purposes, and has also been used in residential construction. The *Post-Frame Building Design Manual* [24] by the National Frame Builders Association provides guidance for the design of post-frame structures. The primary distinction between pole construction and post-frame construction is in nomenclature, where poles are taken to be tapered with round cross section, and posts are taken to be prismatic and rectangular in section. Post-frame construction generally incorporates the use of metal cladding or structural wood panel sheathing to develop diaphragm action. The stiffness of the diaphragm and posts must be considered together to determine the distribution of loads to posts and end walls. Methods to determine diaphragm action and the distribution of loads to the posts may be found in the *Post-Frame Design Manual* [24] and *Wood Technology in the Design of Structures* [25] with a simplified approach also available [26]. Once the loads on the posts have been determined, the design checks are similar to those described above for pole construction.

1.3.5 Timber Piles

Timber piles are round tapered timber members generally used as foundation elements that are typically driven into the ground. Piles are generally used where soils near the surface are weak or where the structure must be elevated above the surface. Piles may also be used in retaining walls or other structures subject primarily to lateral forces. Piles are typically embedded to greater depths than posts or other types of foundations. Recommendations for the use of timber piles in foundations may be found in *Pile Foundations: Know-How* [27].

Piles may be driven into place or installed in holes prebored by auguring or other means. Tapered timber piles are driven with small end (tip) down. Piles resist vertical forces by a combination of side or skin friction and end bearing. Short piles or piles driven through weak soils until they bear on stronger soils below tend to carry more of the load in end bearing, with the opposite being true of long piles in relatively homogeneous soils.

A number of species are used for piles, with southern pine, Douglas fir, and oak being the most commonly used. Piles must be relatively straight and possess the strength to resist driving stress and support the imposed loads.

1.3.5.1 Pile Driving Equipment used for driving timber piles is of special importance. The energy used to drive the piles must be sufficient to drive the pile, but must not impart excessive forces. Pile butts and tips may be damaged severely by sharp blows. For this reason, it is not desirable to drive timber piles with a drop hammer unless a suitable block is employed to dampen the impact. Air, steam, and diesel hammers are commonly used. Generally, it is desirable to band the butt or driven end of a pile to minimize damage during driving. In some cases where tip damage may occur, a special shoe or fitting is used to protect the tip. Pile driving should be monitored by a quality control professional.

1.3.5.2 Critical Section of Pile Piles may be specified by the circumference of the nominal butt or nominal tip in accordance with ASTM D25 [17]. The ASTM classification allows the designer to specify a pile with adequate dimensions at the critical section. For example, a pile depending on frictional forces along the surface of the pile to support the vertical load will generally have a critical section located away from the tip. On the other hand, an end-bearing pile might have the critical section located at the tip.

1.3.5.3 Preservative Treatment of Piles Preservative treatment of piles should conform to recognized specifications such as American Wood Protection Association (AWPA) Standard U1 Commodity Specification E [20]. Cutoffs at the tops of piles exposing untreated wood should be field treated in accordance with AWPA Standard M4 [23]. Piles used in saltwater are subject to attack by marine borers, and special treatment techniques must be used to minimize degradation.

1.3.5.4 Design of Piles Design of pile foundations and other uses of piles should be based on the recommendations of soils investigations of the sites in question. Design values and adjustment factors are found in the *National Design Specification®* [6]. Such design values typically apply to both wet and dry use. It is generally assumed that piles will be preservative treated and will be used in groups. In cases where no treatment is applied or where piles are used individually, further specific adjustments are applied.

Where the diameter of the pile at the section of critical bending moment exceeds 13.5 in., the bending design value must be multiplied by the size factor given in Equation 3-3, based on the depth of a square section of equivalent cross-sectional area. Except in the case of very slender piles or piles in soils providing little or no lateral support, the column stability factor is not applied. The beam stability factor is not applicable to piles.

1.3.6 Trusses

The subject of timber truss design is quite broad. The discussion in this manual is limited to basic design procedures and the highlighting of features unique to timber truss construction. The design of light metal plate connected wood trusses constructed of dimension lumber is not covered in this manual. Light metal plate connected trusses are generally designed by truss manufacturers utilizing proprietary connection and design techniques.

1.3.6.1 Truss Types Types of timber trusses commonly used are illustrated in Figure 1.3.6.1-1. Architectural considerations generally dictate roof slope and may also dictate truss type. For flat or nearly flat roof trusses, some pitch must be provided for drainage or the trusses must be designed to resist progressive ponding.

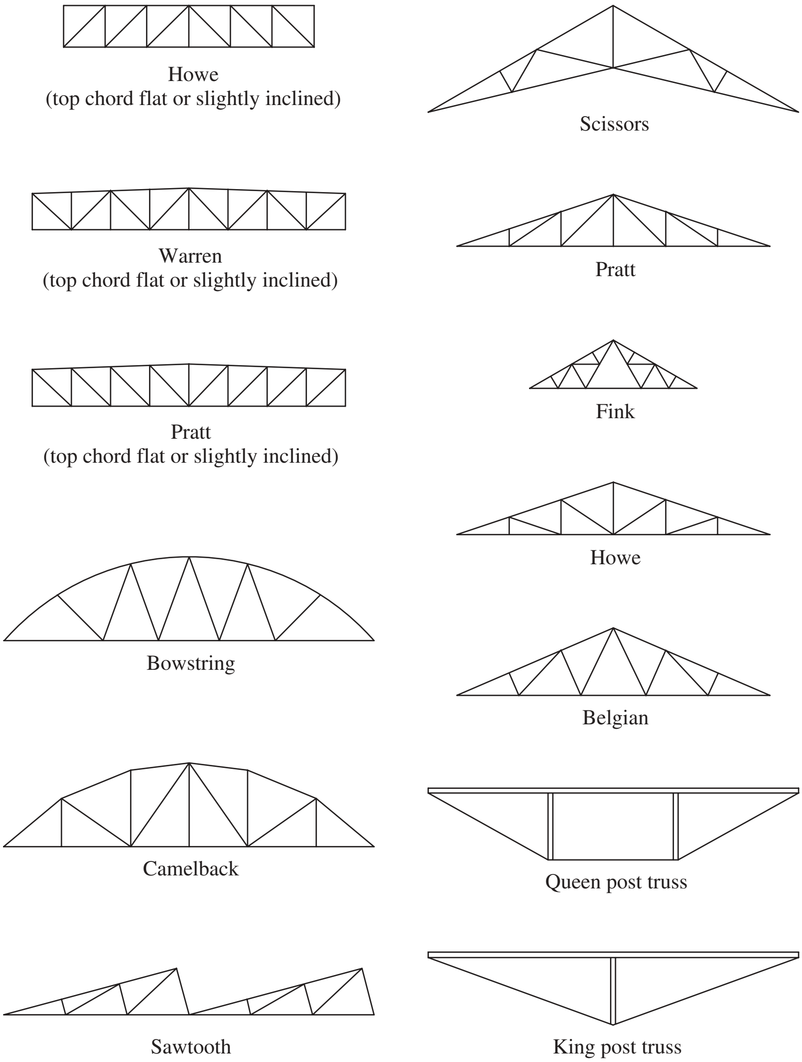


Figure 1.3.6.1-1 Types of timber trusses.

Pratt and Howe trusses are similar except for the orientation of the webs. Pratt trusses have the advantage that for gravity loads, the longer (diagonal) webs are in tension, whereas the shorter (vertical) webs are in compression. These truss names originate from early bridge designers who patented their truss designs. The parallel chord trusses shown in Figure 1.3.6.1-1 were designed for bridge applications with the load applied to the bottom chord, where the triangular trusses of the same names were designed for roof applications with the load applied to the top chord. This accounts for the change in web orientation between the parallel-chord trusses and triangular trusses with the same name.

Bowstring trusses typically have the upper chord shaped to the form of a circular arc with the radius of the top chord equal to the span. Under uniform loading, the chords and heel connections resist the major forces, and the web stresses are small, resulting in light webs and web connections. In addition to normal axial and bending stresses, upper chords of bowstring trusses are subject to moments due to the eccentricity of their curved shape.

Scissor trusses are used to provide more height clearance toward the mid-span. They are commonly used in churches, gymnasiums, various types of assembly halls, and in residential construction. For scissor trusses, horizontal displacement of the truss ends must be considered as part of the design.

Following are recommended span-to-depth ratios for common truss shapes that may be used for determining preliminary truss shape dimensions. Larger span-to-depth ratios may result in excessive member stresses and deflections; smaller ratios may be less economical.

Flat or parallel chord	8–10
Triangular or pitched	6 or less
Bowstring	6–8

1.3.6.2 Truss Members Truss members are designated in three types: top chord, bottom chord, and webs. Webs include all interior vertical or diagonal members between the top and bottom chords. Joints, at which members intersect and connect, are called *panel points.* Chords and webs in all truss types may be constructed as single-leaf, double-leaf, or multi-leaf members. Truss members may be sawn lumber or glued laminated timber. The use of steel rods or other steel shapes for members in timber trusses is acceptable if they fulfill all conditions of design and service.

Glued laminated timber provides many features desirable for truss designs. Glued laminated timber can be made in almost any shape, size, or length, and generally provides higher design values than do sawn timbers. Sawn timbers are limited in maximum length and cross-sectional size and are prone to checking. However, sawn members may provide cost savings. Thus, depending on truss span and loading requirements, trusses can be made of all sawn timber, all glued laminated timber, or a combination of both, and may include some steel tension members. If sawn members are used in conjunction with glued laminated timber, care must be taken to match widths at connections and provide for differential shrinkage.

1.3.6.3 Truss Deflection and Camber The effects of truss deflections must be considered in the design of supporting columns, structural and nonstructural walls, and other fixtures. Timber trusses should be cambered such that total dead load deflection does not produce sag below a straight line between points of support. Additional camber may be appropriate for sustained or heavy live or other loads where sag may impair serviceability or be aesthetically unappealing.

Horizontal deflections of certain trusses must also be computed and accommodated for in the design of the truss support connections. Scissor and crescent trusses may have considerable horizontal deflections. Where horizontal deflection is prevented by the supporting structure, the resulting loads on the truss and support structure must be considered.

1.3.6.4 General Design Procedure For trusses whose members function essentially as pin-connected axial members, member forces may be found by determinate static analysis and deflections by virtual work or other methods. Many trusses, however, incorporate members that are continuous through some connections. For these trusses, static analysis may serve to provide preliminary results, but indeterminate analysis should be used for final member forces and deflections.

Timber truss design typically involves selection of trial member sizes and connection types with iteration until member stress and truss deflection criteria are satisfied. Trial member sizes are generally determined from architectural considerations or engineering judgment, or both. In the absence of other guidance on trial member sizes, preliminary member sizes may be obtained from static analysis considering all joints pinned and the members acting under axial load only. Preliminary deflection information can also be obtained from this idealization.

The availability of proprietary structural analysis and design software enables designers of timber trusses to account for the statically indeterminate features of truss design, unbalanced loads and load reversals, and check combined stresses and deflection quickly and efficiently. Successive iterations of various design features may also result in successive improvements in the truss economy.

For final design, the exact geometry of the truss must be determined—including load points, member sizes, and connection geometry. Where there are knee braces or other structural elements attached to the truss that can lend support or influence load distribution, they must be included as integral with the truss in the overall analysis. If chords are curved, moments resulting from eccentricity of the axial forces must be included in their design.

1.3.6.5 Connections Truss connections should be centric; that is, the centerlines of members at each joint should intersect at a common point so that moments are not induced into the members being connected. Connections should also be detailed in such a way as to allow relative rotation between members (Figures 1.3.6.5-1 and 1.3.6.5-2). If rotation is restrained by the joints, moments in members and connections will result, and the members may split. Single through-bolts connecting the members of multileaf trusses provide true “pin” connections. Slotted or overlapping connecting plates may also be used to allow rotation.

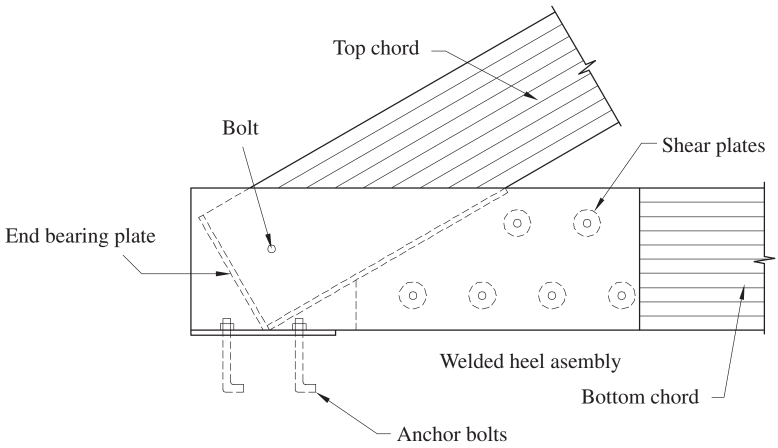


Figure 1.3.6.5-1 Truss heel connection.

Where metal side plates are used to transfer compression loads, buckling of the plates must be considered based on support conditions for the plates. For plates connecting webs to chords, generally the web does not provide lateral stability for the plate but instead the plate is an extension of the web. Therefore, the end of the plate fastened to the web cannot be considered fixed in determining the plate effective length. When the web plates are pinned at the chord, the spacing between bolts in the web should be large enough to provide stability and prevent splitting of the web.

If steel rods are used as tension elements in a truss, written instruction should be given to the truss assembler regarding tightening of the rods. Otherwise, overtightening could occur, which might overstress truss elements when design loads are applied.

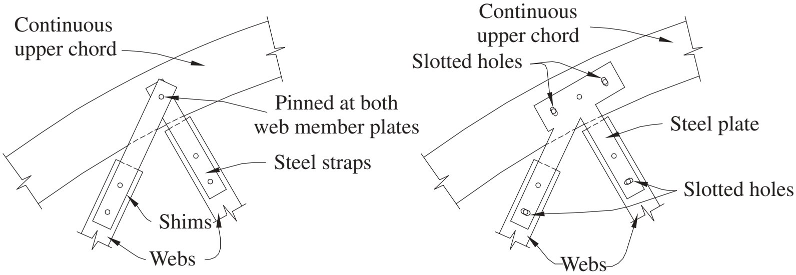


Figure 1.3.6.5-2 Truss chord and web connections.

1.3.6.6 Bracing In structures employing trusses, a system of bracing is required to provide resistance to lateral forces, to hold the trusses true and plumb, and to prevent compression elements from buckling. Both permanent bracing and temporary erection bracing should be designed according to accepted engineering principles to resist all loads that will normally act on the system. Erection bracing is installed during erection to hold the trusses in a safe position until sufficient permanent construction is in place to provide full stability. Permanent bracing forms an integral part of the completed structure. Part or all of the permanent bracing may also act as erection bracing.

1.3.6.6.1 Permanent Bracing Permanent bracing must be provided to resist forces out of the truss plane and provide stability for the truss. Permanent bracing typically consists of a structural diaphragm in the plane of the top chord, horizontal bracing between trusses in the plane of the bottom and top chords, or a combination of both. In addition, individual web members may require permanent bracing to prevent buckling.

A structural diaphragm in the plane of the top chord, acting as a plate girder, transmits forces to end and side walls. This system is the preferred method of providing truss bracing and is usually most economical. It may be necessary to provide additional members acting as the flanges for the diaphragm.

Horizontal bracing between trusses is a positive method of bracing, but it is more costly and should be used only where the strength of a diaphragm described above is insufficient. In effect, such bracing forms an inclined or horizontal truss to resist out of plane loads on the system.

Top chords and web members subject to compressive loads must be adequately braced to satisfy slenderness requirements and resistance to buckling. Similarly, bottom chords and other members subject to compression under wind or other loads may need to be braced. To provide lateral support for truss members subject to compression, the bracing system should be designed to withstand a horizontal force equal to at least 2 percent of the compressive force in the truss chord if the members are aligned.

Even though truss bottom chord forces may be in tension under all loading conditions, there may be circumstances when bottom chord bracing is advised for nonstructural reasons. Truss bottom chords can distort out of plane due to eccentricities introduced by manufacturing variations, unsymmetrical truss loading, and other factors. Distortions out of plane are more likely when chords are non-continuous (spliced) between the support points. The designer should consider including bottom chord bracing when there are sensitive visual sight lines and factors contributing to out of plane distortions.

1.3.6.6.2 Temporary Bracing Temporary or erection bracing is not normally the responsibility of the design engineer. Contract documents should specify who has the responsibility to design and provide temporary bracing systems. Erection truss bracing is installed to hold trusses true and plumb and in a safe condition until permanent truss bracing and other permanent components, such as joists and sheathing contributing to the rigidity of the complete roof structure, are in place. Erection truss bracing may consist of struts, ties, cables, guys, shores, or similar items. Joists, purlins, and other permanent elements may be used as part of the erection bracing.

1.3.7 Structural Glued Laminated Timber Arches

Structural glued laminated timber (glulam) has many advantages over traditional sawn members or other engineered wood products including its ability to be manufactured in a variety of shapes from straight beams and columns to graceful curved members. Glulam can be manufactured with constant cross section along the length or with taper to meet architectural requirements. The glulam arch fully takes advantage of the unique properties of laminated timber construction.

Glulam arches are popular for use in large open structures such as churches and gymnasiums because of their excellent structural performance, inherent fire resistance, and aesthetic appeal. Laminated timber arches are also used for vehicle and pedestrian bridges.

Figure 1.3.7-1 illustrates a number of common arch configurations. Arches may be of either two- or three-hinged design. Tudor arches can generally be used economically for spans of up to 120 feet, and parabolic and radial arches can be used economically for spans of up to 250 feet. The most popular arch configuration in use today is the three-hinged Tudor arch. It provides a vertical wall frame and sloping roof that are commonly used for modern structures. Its appearance is pleasing to most people.

Design of Tudor Arches with Structural Glued Laminated Timber [28] provides detailed guidance for the design of Tudor arches. Manufacturers of arches commonly provide design services for these specialized systems. Chapter 9 of this manual includes a simplified procedure for preliminary design of Tudor arches.

1.3.7.1 Aesthetic Considerations Because structural glued laminated timber arches are typically exposed to view, the aesthetic appeal of the arch is an important design consideration in addition to its structural performance. The building designer and owner should verify that the final geometry meets the aesthetic requirements and any other architectural requirements for the design. Because of the numerous material and geometric parameters involved in arch design, multiple geometries can be utilized to meet the same structural requirements. As with all glued laminated timbers, the specification of the glulam members should clearly include the required appearance grade.

1.3.7.2 Transportation Considerations Arches can pose significant challenges in shipping, particularly with tall wall heights and roofs with low pitch. Critical shipping dimensions of a Tudor arch half are shown in Figure 1.3.7.2-1. To facilitate shipping, it may be necessary to design and manufacture the arch halves with moment splices occurring in the arms (Figure 1.3.7.2-2). These are typically placed at inflection points, or other locations with low bending moments.

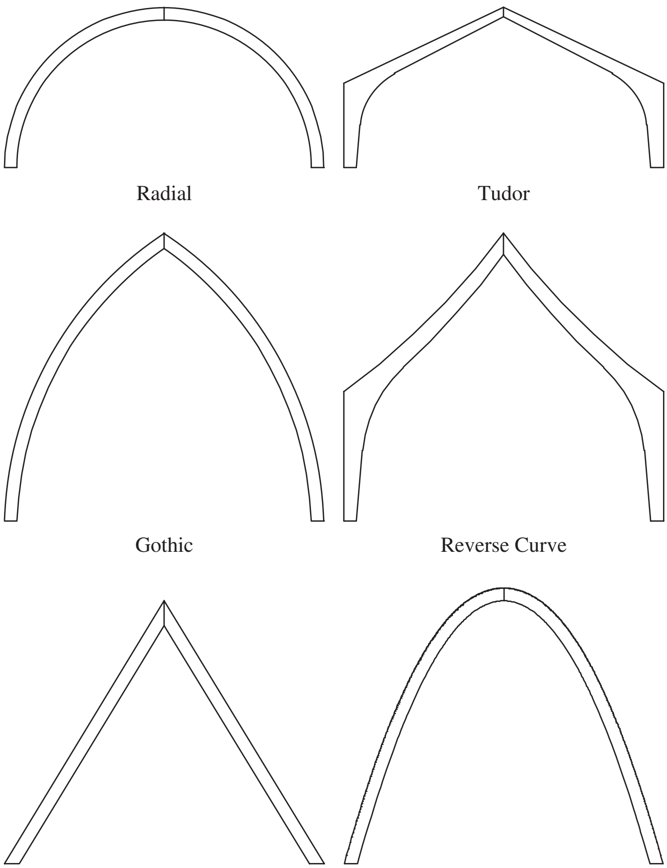


Figure 1.3.7-1 Common arch configurations.

The arch can also be designed and manufactured with a detached haunch to reduce its overall width for shipping (Figure 1.3.7.2-3).

Shipping widths of up to 12 ft can typically be accommodated. Wider loads may be possible depending on the distance from the manufacturing plant to the jobsite and the route the shipment will follow. Glulam manufacturers can provide guidance regarding maximum shipping dimensions for various localities.

For arches with deep haunches, manufacturing constraints may also dictate the use of a detached haunch to facilitate passing the arch through a surface planer. Glulam manufacturers should be contacted early in the design process to determine their maximum recommended haunch depths.

1.3.8 Diaphragms and Shear Walls

Structural diaphragms are relatively thin, usually rectangular, structural systems capable of resisting in-plane shear parallel to their edges. Wood framed roofs,

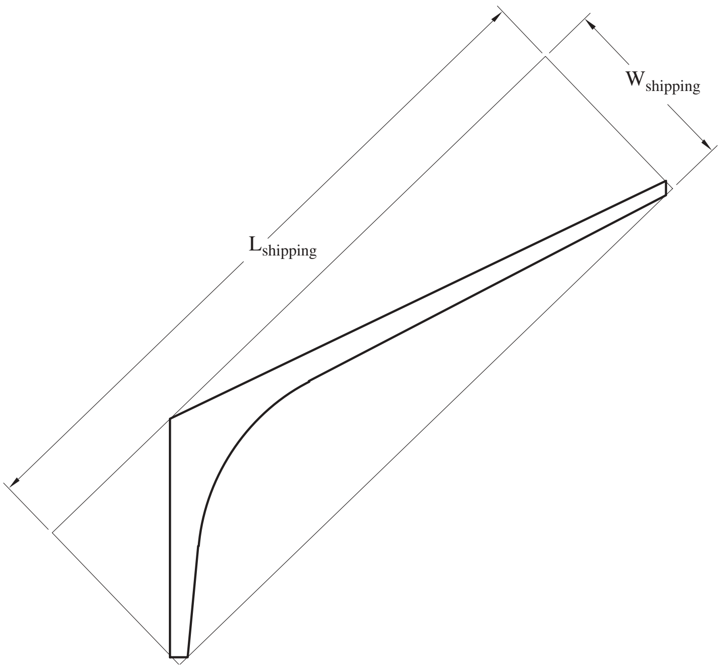


Figure 1.3.7.2-1 Tudor arch critical shipping dimensions.

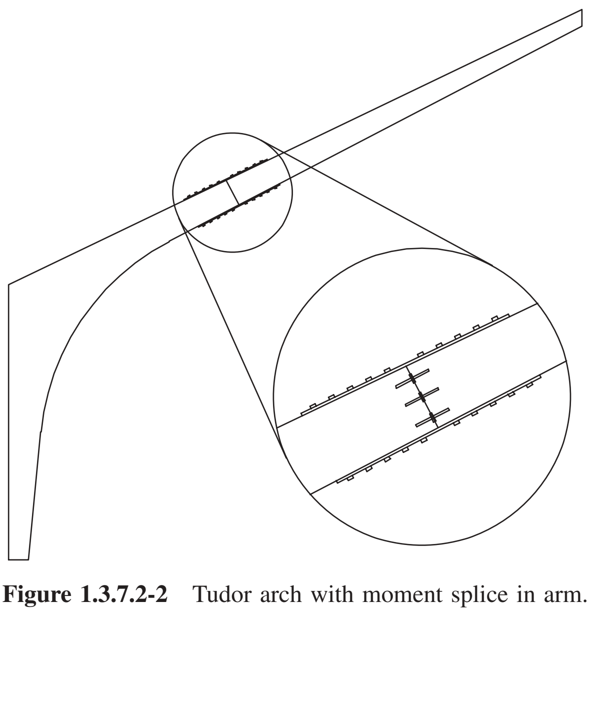


Figure 1.3.7.2-2 Tudor arch with moment splice in arm.

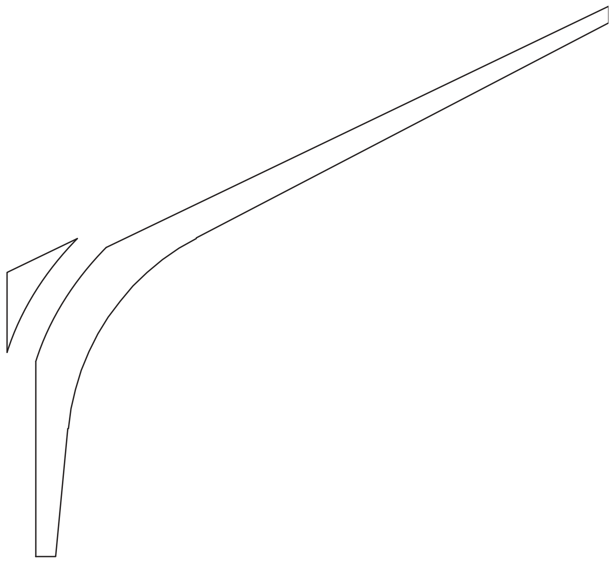


Figure 1.3.7.2-3 Tudor arch with detached haunch.

walls, and floors can generally be made into structural diaphragms with proper detailing and in some cases additional members or connecting hardware. The function of the diaphragm is to brace a structure or parts thereof against lateral forces, such as wind or earthquake loads, and to transmit these forces to the other resisting elements of the structure. Roof and floor systems are often used as horizontal diaphragms. The horizontal diaphragms transfer the lateral forces to vertical diaphragms (shear walls), braced walls of various types, or poles or piles. Common types of diaphragms in wood construction are described below.

1.3.8.1 Wood Structural Panels Structural panel sheathing used to resist out of plane loads (load perpendicular to the panels) can generally be used to also develop diaphragm action by proper detailing of fasteners, boundary members, and attachment to interior support members and blocking. The design of wood structural panel diaphragms is described in *ASD/LRFD Wind & Seismic—Special Design Provisions for Wind and Seismic* [29].

Heavy timber and glued laminated timber decking may be covered with wood structural panel sheathing to develop diaphragm action. Design values for this type of system are considered equivalent to those of blocked structural panel diaphragms of the same sheathing thickness and with the same nailing schedule. The designer must specify and detail the proper panel and boundary nailing conditions for both sheathing and decking.

1.3.8.2 Lumber Sheathing Lumber sheathing, consisting of boards of 1 in. nominal thickness nailed transversely or diagonally at 45° to studs or joists, is sometimes used in wood frame construction. The edges of lumber sheathing may be square, ship-lapped, splined, or tongue-and-groove. When subjected to lateral forces such as wind or earthquakes, lumber sheathing and its supporting framework may act as a diaphragm or shear wall, serving to brace the building against the lateral forces and transmitting these forces to the foundations. Design values for lumber-sheathed diaphragms and shear walls are provided in the *Special Design Provisions for Wind and Seismic* [29], primarily for the evaluation of existing structures. New structures are not typically designed with this type of diaphragm or shear wall.

Lumber sheathing and decking develop only modest amounts of diaphragm action where placed perpendicular to the support members. Diagonal installation of the sheathing lumber a results in considerably greater strength and stiffness than where the sheathing is installed perpendicular across supports. Placing lumber sheathing in two layers of diagonal sheathing, one on top of the other, with the sheathing in one layer perpendicular to the sheathing in the other layer, results in considerably stiffer and stronger diaphragms than single-layer diagonal sheathing.

1.4 ECONOMY

The best economy in timber construction is generally realized when standard-size members can be utilized in a repetitive arrangement with simple connections. However, timber framing, especially glued laminated timber, can be custom fabricated to provide an infinite variety of unique but cost-effective architectural forms and arrangements.

1.4.1 Standard Sizes

The selection of standard sizes and grades in timber construction will result in maximum economy. Standard sizes of glued laminated timber, sawn lumber (boards, dimension lumber, and timbers), are given in Tables 1.4.1-1 and 1.4.1-2. Member length of glued laminated timber is limited, for the most part, only by transportation and handling restrictions. Standard lengths for sawn lumber are generally available in even 2-ft increments, with the maximum length practically limited to 20–30 ft.

1.4.2 Volumetric Measure

The volume of structural timbers is typically measured in terms of board feet (BF). Large volumes of wood are typically measured in thousands of board feet (MBF) and millions of board feet (MMBF). Because price is often stated per thousand board feet, it is important to understand how board footage is measured and to be able to convert into other measures of volume. The wood volume in board feet is calculated based on nominal dimensions; therefore, board foot measure must be converted to actual volume for calculations other than for pricing.

TABLE 1.4.1-1 Standard Sizes for Glued Laminated Timber

Species	Standard Widths for Industrial, Architectural, and Premium Appearance							
Softwoods other than southern pine	$2\frac{1}{8}$	$3\frac{1}{8}$	$5\frac{1}{8}$	$6\frac{3}{4}$	$8\frac{3}{4}$	$10\frac{3}{4}$	$12\frac{1}{4}$[a]	$14\frac{1}{4}$[a]
Southern pine	$2\frac{1}{8}$	3 or $3\frac{1}{8}$	5 or $5\frac{1}{8}$	$6\frac{3}{4}$	$8\frac{1}{2}$	$10\frac{1}{2}$	12[a]	14[a]
	Standard Widths for Framing Appearance[b]							
All softwoods	$2\frac{1}{2}$	$3\frac{1}{2}$	$5\frac{1}{2}$	$7\frac{1}{4}$	—	—	—	—

No. of Laminations	Standard Net Depth of Members, in.		
	Nominal 1 in. Laminations[d]	Nominal 2 in. Laminations	
		$1\frac{1}{2}$ in.[c,d]	$1\frac{3}{8}$ in.[c,d]
4	3	6	$5\frac{1}{2}$
5	$3\frac{1}{4}$	$7\frac{1}{2}$	$6\frac{7}{8}$
6	$4\frac{1}{2}$	9	$8\frac{1}{4}$
7	$5\frac{1}{4}$	$10\frac{1}{2}$	$9\frac{5}{8}$
8	6	12	11
etc.	etc.	etc.	etc.

[a] Laminations wider than 11.25 in. are not generally available. Wider beams are typically manufactured using multiple-piece laminations across the width (Figure 1.4.1-1).
[b] Framing appearance grade members are surfaced "hit or miss" to match conventional framing lumber widths, and are not suitable for applications where appearance is important.
[c] $1\frac{1}{2}$ in. thick laminations are normal for most softwoods; $1\frac{3}{8}$ in. thick laminations are normal for southern pine.
[d] Curved members may use thinner laminations, depending on radius of curvature.

One board foot of sawn lumber or timber is equal to 144 in^3 based on nominal thickness, nominal width, and actual length. For example, a one-foot length of nominal 2 in. × 6 in. or 1 in. × 12 in. lumber measures one board foot. Likewise, a four-inch length of nominal 6 in. × 6 in. timber measures one board foot. The actual volume of a board foot of lumber varies significantly depending on the size of the piece. For example, a board foot of 2 × 6 lumber contains 99 in^3 of wood, while a board foot of 8 × 14 lumber contains 130 in^3 of wood.

Structural glued laminated timber is measured based on the nominal dimensions of the input lumber. For example, a $5\frac{1}{8}$ in. × 12 in. Douglas fir glulam beam is manufactured from eight laminations of nominal 2 in. × 6 in. lumber. Each 2 in. × 6 in. lamination measures one board foot per lineal foot, resulting in a total of 8 board feet per lineal foot for the beam. Similarly, a $5\frac{1}{8}$ in. × $12\frac{3}{8}$ in. Southern pine beam, manufactured from nine laminations of nominal 2 in. × 6 in. lumber measures 9 board feet per lineal foot of beam.

TABLE 1.4.1-2 Standard Sizes for Sawn Lumber

Item	Thickness, in.			Face Widths, in.		
	Nominal	Minimum Dressed		Nominal	Minimum Dressed	
		Dry	Green		Dry	Green
Boards	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{11}{16}$	2	$1\frac{1}{2}$	$1\frac{9}{16}$
	1	$\frac{3}{4}$	$\frac{25}{32}$	3	$2\frac{1}{2}$	$2\frac{9}{16}$
	$1\frac{1}{4}$	1	$1\frac{1}{32}$	4	$3\frac{1}{2}$	$3\frac{9}{16}$
	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{9}{32}$	5	$4\frac{1}{2}$	$4\frac{5}{8}$
				6	$5\frac{1}{2}$	$5\frac{5}{8}$
				7	$6\frac{1}{2}$	$6\frac{5}{8}$
				8	$7\frac{1}{4}$	$7\frac{1}{2}$
				9	$8\frac{1}{4}$	$8\frac{1}{2}$
				10	$9\frac{1}{4}$	$9\frac{1}{2}$
				12	$11\frac{1}{4}$	$11\frac{1}{2}$
				14	$13\frac{1}{4}$	$13\frac{1}{2}$
				16	$15\frac{1}{4}$	$15\frac{1}{2}$
Dimension Lumber	2	$1\frac{9}{16}$	$1\frac{9}{16}$	2	$1\frac{1}{2}$	$1\frac{9}{16}$
	$2\frac{1}{2}$	2	$2\frac{1}{16}$	3	$2\frac{1}{2}$	$2\frac{9}{16}$
	3	$2\frac{1}{2}$	$2\frac{9}{16}$	4	$3\frac{1}{2}$	$3\frac{9}{16}$
	$3\frac{1}{2}$	3	$3\frac{1}{16}$	5	$4\frac{1}{2}$	$4\frac{5}{8}$
	4	$3\frac{1}{2}$	$3\frac{9}{16}$	6	$5\frac{1}{2}$	$5\frac{5}{8}$
	$4\frac{1}{2}$	4	$4\frac{1}{16}$	8	$7\frac{1}{4}$	$7\frac{1}{2}$
				10	$9\frac{1}{4}$	$9\frac{1}{2}$
				12	$11\frac{1}{4}$	$11\frac{1}{2}$
				14	$13\frac{1}{4}$	$13\frac{1}{2}$
				16	$15\frac{1}{4}$	$15\frac{1}{2}$
Timbers	5 and thicker	-	$\frac{1}{2}$ off	5 and wider	-	$\frac{1}{2}$ off

1.4.3 Standard Connection Details

A variety of fasteners and connection hardware is readily available for wood construction. In applications requiring custom design, typical construction details are provided in AITC 104 [30] to assist in the design of safe and durable connections.

1.4.4 Framing Systems

There is great diversity of structural timber framing systems. The relative economy of any one system over another will depend on the particular requirements of a specific job. Consideration of the overall structure, intended use, geographic

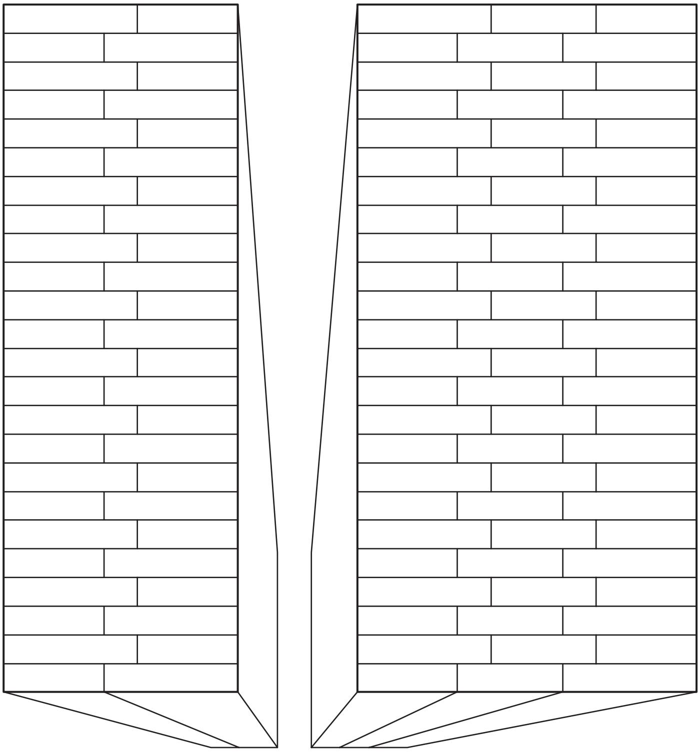


Figure 1.4.1-1 Glulam beams with multiple-piece laminations across the width.

location, required configuration, and other factors play an important part in determining the framing system to be used on a job. Table 1.4.1-1 may be used for preliminary design purposes to determine the economical span ranges for various timber framing systems.

The following additional considerations, when applied to timber framing system design, tend to reduce costs. Connections (joints) should be simple and as few as practically possible. Unnecessary variations in members should be avoided; that is, identical members should be used repetitively where practical, with the number of variations kept to a minimum. Continuous spans and cantilever systems may be used to balance positive and negative moments, reducing required member sizes and costs.

1.4.5 Structural Grades

For projects using sawn lumber and/or "stock" glued laminated members, suppliers should be consulted for the relative availability and economy of various grades and species. For large jobs utilizing numerous glued laminated timbers, and for custom members, better economy may be obtained by specifying the minimum

TABLE 1.4.4-1 Economical Spans for Selected Timber Framing Systems

Primary Framing Systems	Economical Span Range (ft)
Simple Beams	
Sawn Lumber	$\leq$ 32
Glulam	$\leq$ 100
Cantilever Beam Systems	
Sawn Lumber	$\leq$ 24
Glulam	$\leq$ 90
Continuous Beams	
Sawn Lumber	$\leq$ 16
Glulam	$\leq$ 50
Glulam Arches	
Gothic	40–90
Tudor	20–120
A-Frame	20–100
Parabolic	40–250
Radial	40–250
Heavy Trusses	
Flat (parallel chord)	50–150
Triangular (pitched)	50–90
Bowstring	50–200
Light Trusses	
Flat (parallel chord)	20–50
Triangular (pitched)	20–75
Dome	50–500+
Secondary Framing Systems	
1-in. Lumber Sheathing	1–2
2-in. Lumber Decking	4–8
3-in. Lumber Decking	8–14
4 in. Lumber Decking	14–18
Structural Panel Sheathing	1–4
Stressed Skin Panels	8–40
Joists with Sheathing	16–24
Purlins with Sheathing	16–36

required design stresses or stress class for the members, allowing the manufacturer flexibility in choice of laminating combinations.

1.4.6 Appearance Grades for Structural Glued Laminated Timber

The specification of higher appearance grades than needed may significantly increase the cost of the glulam members. It is generally more economical to specify the appearance grade best suited for each job or perhaps different members in a particular job than to require the best appearance grade for all jobs or members.

1.5 PERMANENCE

With proper design details, construction procedures, and usage, wood is a permanent construction material. If proper consideration is given to potential causes of deterioration and their prevention in the design of a project, there will be greater assurance that the structure will be permanent and that maintenance will be minimal. AITC Technical Note 12 [31] provides additional information related to permanence.

1.5.1 Wood Decay

Decay of wood is caused by fungi that grow from microscopic spores. These spores are present wherever wood is used. The fungi use wood substance as their source of nutrition. If deprived of any one of four essentials for life (nutrients, air, moisture, and favorable temperature), decay growth is prevented or stopped and the wood remains sound, retaining its existing strength with no further deterioration. Wood will not be attacked by fungi if it is submerged in water (thereby excluding air), kept continuously below 20% moisture content (excluding sufficient free moisture), or maintained at temperatures below freezing or much above 100°F. Growth can begin or resume whenever conditions are favorable.

The early stages of decay are often accompanied by a discoloration of the wood, which is more evident on freshly exposed surfaces of unseasoned wood than on dry wood. However, many fungi produce early stages of decay that are similar in color to that of normal wood or give the wood a water-soaked appearance.

Later stages of decay produce observable changes in color and texture and in wood volume and density. Decayed wood has reduced strength and fire resistance. In the extreme, the wood appears rotten and crumbly and reduces the member section (Figure 1.5.1-1). *Dry rot* is wood that has decayed in the presence of moisture and has subsequently become dry.

In the design of wood structures, fungal decay is typically prevented by assuring low moisture content or by making the wood toxic to the fungi by pressure preservative treatment (where low moisture content cannot be assured). The *Standard for Preservative Treatment of Structural Glued Laminated Timber*, AITC 109 [32], provides information about preservative treatment of glued laminated timber. The American Wood Protection Association provides information on the treatment of glulam, sawn lumber, and round timbers [20].

1.5.1.1 Detailing to Prevent Decay Proper detailing and construction are important for decay prevention. Rain water and melted snow must be directed away from wood members by adequately sloped framing and appropriate flashing. Roof overhangs with gutters and downspouts are advised. Finish grade around a structure should be sloped to direct runoff away from the structure.

The building envelope should include an appropriate moisture barrier. The American Society of Heating, Refrigeration, and Air-Conditioning Engineers

Figure 1.5.1-1 Advanced decay of unprotected glulam beam.

Fundamentals Handbook [33] can be consulted, as well as the local building authority, for guidance in providing an effective moisture barrier.

Untreated wood should not be installed in direct contact with masonry or concrete. Girder and joist openings in masonry and concrete walls should be large enough to assure that there will be an air space around the sides and ends of these wood members. Where timber members are below the outside soil level, moisture proofing of the outer face of the walls is essential.

Enclosed spaces such as attics and crawl spaces must be adequately ventilated. Moisture from soil floors in crawl spaces can be inhibited from moving into the crawl space by covering the soil with a vapor barrier. Untreated wood must be placed above finish grade (typically a minimum of 6 in.).

1.5.1.2 Use of Preservative Treated or Decay-Resistant Wood Wood has a proven performance of indefinitely long service without special treatments if it is kept below 20% moisture content. However, when wood is exposed to the weather and not properly protected by a roof, eave overhang, or similar covering, or is subjected to other conditions of free water, either preservative treatment is required, or wood that is naturally decay resistant must be used.

Naturally decay-resistant woods include the heartwood of Alaska cedar and redwood. The *Wood Handbook* [34] lists additional domestic woods with heartwood that is naturally resistant to decay.

Special consideration should be taken for structures containing significant sources of moisture such as pools. Special consideration should also be taken for parts of the structure that are outside the building envelope such as decks, porches, and balconies. Prescriptive requirements for wood construction in model building codes generally take into consideration decay potential and reflect many of the above safeguards. Periodic inspection of wood structures is recommended to identify signs of excess moisture, decay, or damage.

1.5.2 Mold and Fungal Stains

Wood may also experience mold and stain. Molds and stains are confined largely to sapwood and are of various colors. Molds generally do not stain the wood but produce surface blemishes varying from white or light colors to black that can often be brushed off. Fungal stains may penetrate the wood and normally cannot be removed by scraping or sanding. The presence of molds and stains are not necessarily signs of decay, as stain-producing fungi do not attack the wood substance appreciably. For most uses in which appearance is not a factor, stains alone are not necessarily unacceptable as wood strength is practically unaffected. Ordinarily, the only effects of stains and mold are confined to those properties that determine shock resistance or toughness. Keeping wood dry is generally sufficient to prevent growth of mold or fungal stains.

1.5.3 Insects

In terms of economic loss, the most destructive insect to attack wood buildings is the subterranean termite. In certain localities, above-ground termites are also very destructive. Other insects attack timber buildings, but, ordinarily, these occurrences are rather rare and their damage is slight. In many cases, these insects can be controlled by the methods used for termites.

1.5.3.1 Subterranean Termites The extent of the occurrence of damage from subterranean and non-subterranean termites is shown in Figure 1.5.3.1-1.

Figure 1.5.3.1-1 Termite damage in the continental United States. A, northern limit of recorded damage done by subterranean termites; B, northern limit of damage done by dry-wood or non-subterranean termites. (From Wood Handbook[34].)

Damage occurrence in general is much greater in southern states where temperature conditions are more favorable. However, damage to individual buildings may be just as great in northern states.

Subterranean termites develop and maintain colonies in the ground from which they build tunnels through the earth and around obstructions to get at the wood they need for food. The worker members of the colony cause the destruction of wood. At certain seasons of the year, male and female winged forms swarm from the colony, fly a short time, lose their wings, mate and, if they succeed in locating suitable places, start new colonies. The occurrence of flying termites (similar in appearance to flying ants) or their shed wings may be an indication of a nearby colony.

Subterranean termites do not establish themselves in buildings by being carried there in lumber, but by entering from the ground nests after the building has been constructed. Termites must have continual access to moisture such as from the soil. Signs of the presence of termites are the earthen tubes, or runways, built by these insects from the ground over the surfaces of foundation walls to reach the wood above. In wood, the termites make galleries that follow the grain, often concealed by a shell of sound wood. Because the galleries seldom show on the wood surface, probing may be necessary to identify termite infestation and damage.

Where subterranean termites are prevalent, the best protection is to prevent their gaining access to the building from the ground. Construction details prescribed by model building codes separating wood from soil and moisture to prevent decay are also useful for preventing termite infestation. In general, foundations must be of preservative treated wood, concrete, or other material through which the termites cannot penetrate. Cement mortar should be used with masonry foundations as termites can work through some other kinds of mortar. Wood that is not impregnated with an effective preservative must be kept away from the ground. Basement floors should preferably be concrete slab on grade. In general, wood floor framing must be treated if within 18 in. of the ground below the floor. Untreated posts must stand off at least 1 in. above concrete slabs unless the slab has been protected from moisture and pest infestation. Moisture condensation on the floor joists and subfloor, which may cause conditions favorable to decay and thus make the wood more attractive to termites, can be avoided by covering the soil with a waterproof membrane. Expansion joint material for slabs must be pest-resistant or treated to resist termites.

All concrete forms, stakes, stumps, and waste wood and other potential sources of infestation must be removed from the building site at the time of construction. Where protection is needed in addition to that obtained by physical methods, the soil adjacent to the foundation walls and piers beneath the building may be thoroughly treated with an appropriate insecticide.

1.5.3.2 Above-Ground Termites Nonsubterranean or dry-wood termites have been found only in a narrow strip of territory extending from central California around the southern edge of the continental United States to Virginia and also in the West Indies and Hawaii (Figure 1.5.3.1-1). Their damage is confined to an area in southern California, to parts of southern Florida, notably Key West, and to Hawaii. The nonsubterranean termites are fewer in number, and their depredations are not rapid, but if they are allowed to work unchecked for a few years, they can occasionally ruin timbers with their tunneling.

In the principal damage areas, careful examination of wood is needed to avoid the occurrence of infestations during the construction of a building. All exterior wood can be protected by placing fine-mesh screen over all holes in the walls or roof of the building. If a building is found to be infested by dry-wood termites, badly damaged wood must be replaced. Further termite activity can be arrested by approved chemical treatments applied under proper supervision that will provide for safety of people, domestic animals, and wildlife. Where practical, fumigation is another method of destroying insects.

1.5.3.3 Other Insects Large wood-boring beetles and wood wasps may infect green wood and complete their development in seasoned wood. The borers can be killed by heating the wood to a center temperature of 130°F for one hour or by fumigation. Once the wood has been cleared of the borers, they will not reinfest seasoned wood. If infested wood is used in a building, the emerging adults may bore $\frac{1}{8}$ in. to $\frac{1}{2}$ in. holes to the surface, penetrating insulation, vapor barriers, siding, or interior surface materials. Infested members may be damaged and require structural or cosmetic repair. If the remaining structure has been built with seasoned wood without initial infestation, infestation of the other members will generally not take place.

Powder-post beetles can infest and reinfest dry wood. The *Lyctus* powder-post beetles, which are encountered most frequently, attack large-pored hardwoods. Their attacks may be recognized by tunnels packed with floury sawdust and numerous emergence holes $\frac{1}{32}$ in. to $\frac{1}{8}$ in. in diameter. Heat or fumigation treatments will kill the beetles but will not prevent reinfestation. Infestation can be prevented by a surface application of an approved insecticide in a light-oil solution. Any finishing material that plugs the surface holes of wood will also protect the wood from *Lyctus* attack. Usually, infestations in buildings result from the use of infested wood, and insecticidal treatment or fumigation may be needed to eliminate them.

Carpenter ants chew nesting galleries in wood. The principal species are large, dark-colored ants, and individuals in the colony may be $\frac{1}{2}$ in. long. They exist throughout the United States. Because the ants require a nearly saturated atmosphere in their nest, an ant infestation may indicate a moisture problem in the wood that could also result in decay damage. Ant infestations can be controlled by insecticides or by keeping the wood dry.

1.5.4 Marine Borers

Fixed or floating wood structures in salt or brackish water are subject to attack by marine borers. Marine borers include shipworms such as *Teredo* and *Bankia*, the pholads *Martesia* and *Zylophaga*, and *Limnoria* and *Sphaeroma*. Almost all marine borers attack wood as free-swimming organisms in the early part of their lives. Shipworms and pholads bore an entrance hole generally at the waterline, attach themselves, and grow in size as they bore tunnels into the wood. *Limnoria* and *Sphaeroma* generally burrow just below the surface of the water.

For areas where shipworm and pholad attack are known or expected and where *Limnoria* attack is not expected, the wood should be pressure treated with a creosote and/or creosote-coal tar solution. For areas where *Limnoria* and pholad attack are known or expected, a dual treatment of waterborne salts and creosote is recommended. Where *Limnoria* attack is known or expected and where pholads are absent, either a dual treatment or waterborne salt preservatives may be used.

1.5.5 Temperature

Wood may be exposed temporarily to temperatures up to 150°F without permanent loss of strength. Where exposed to sustained temperatures in excess of 100°F, wood suffers loss in strength and stiffness as the wood substance is degraded. Strength loss is greater in members simultaneously subject to high moisture content. Design adjustment factors for wood construction take into consideration strength loss at sustained high temperature as well as high temperature and moisture content. Wood at low temperatures tends to have greater strength than at normal temperatures, although this effect is not normally considered in design.

1.5.6 Chemical Environments

Wood is superior to many other common construction materials in its resistance to chemical attack. For this reason, wood is used in storage buildings and for containers for many chemicals and in processing plants in which structural members are subjected to spillage, leakage, or condensation of chemicals. Wooden tanks are commonly employed for the storage of water or chemicals that deteriorate other materials and have the unique feature of being self-sealing due to the expansion of wood where exposed to moisture. Experience has shown that the heartwood of cypress, Douglas fir/larch, southern pine, and redwood is the most suitable for water tanks and that the heartwood of the first three of these species is most suitable for tanks when resistance to chemicals in appreciable concentrations is an important factor. More information on the use of wood in chemical environments is provided in Chapter 2.

1.6 SEASONING

Seasoning, in general, is the drying of wood from its wet or “green” condition when first cut to the end condition in which it is in equilibrium with its surroundings. Since wood shrinks as it dries, it is preferable that wood used during construction be predried to a low moisture content, because the equilibrium moisture content for most end uses in buildings is generally low.

1.6.1 Checking

Wood naturally expands and shrinks with changes in moisture content. In the case of rapid drying, wood at the surface tends to shrink faster than the inner wood, and the surface wood fibers may separate, causing drying "checks." Wood also shrinks at different rates in different directions relative to the growth rings, which can cause checks. Drying checks are common and may be expected in sawn lumber. Drying checks may likewise be expected in glued-laminated members, though to a lesser extent due to the manufacturing process of the members. Glued-laminated members are manufactured using wood with controlled moisture contents generally close to the expected end use conditions of the members.

Proper storage, handling, and final construction details of wood members will, in general, minimize checking. In cases where checking is considered severe, AITC Technical Notes 11 [35] and 18 [36] may be used to evaluate the structural significance of checking in glued-laminated members and ASTM D245 [37] provides some guidance for sawn members.

Where temperature and moisture conditions during construction differ significantly from the building end use (e.g., construction during cold or wet seasons), and where large timber members are exposed to view, it is recommended that the building be brought to end use conditions gradually, to minimize the occurrence of checking.

1.6.2 Shrinkage

Excessive shrinkage of green wood after installation may cause structural and serviceability problems. In its end use, wood used in construction will experience changes in moisture content as surrounding temperature and humidity conditions change. In structures with a good building envelope, these changes tend to be slow, and the wood, fastenings, and properly detailed connections themselves generally accommodate the dimensional changes associated with the changes.

If green wood is used in construction (though not generally recommended), special detailing may be required to accommodate the shrinkage of the members as they dry to equilibrium. This is particularly true if the wood members are large and attached to relatively rigid elements. Equations for estimating wood shrinkage are provided in Chapter 2.

1.7 HANDLING, STORAGE, AND ERECTION

As with any material, care must be taken to protect timber members from damage during the construction process. Proper handling, storage, and erection methods will minimize problems and ensure satisfaction with final product.

1.7.1 Handling

Wood is not a particularly hard material; as such, care must be taken during handling to not damage the members. Forklifts or cranes should be used to lift wood from delivery vehicles to the construction site. Upon delivery, wood members should be tallied and inspected for damage. At the job site, larger members should be handled with fabric slings, because chains and cables tend to mark and damage the wood surfaces. Wood may also be easily marked or scuffed at the site if not protected from soil and traffic.

1.7.2 Storage

Sawn lumber delivered to a job site should be kept off the ground and protected from sunlight and moisture. Wrapped lumber and glued laminated members (generally wrapped) should likewise be stored off the ground with the wrapping intact. While being stored, the underside of the wrapping should be punctured to allow drainage of excess moisture or condensation. For additional protection, the wrapping of glued laminated members should not be removed until the building has been enclosed. Because wood creeps under load, improper storage can result in permanent deformation, so members should not be stored haphazardly or in ways that will introduce undesirable twists or bends.

1.7.3 Assembly

Timber trusses are usually shipped disassembled and are assembled on the ground at the site before erection. Arches, which are generally shipped in halves, may be assembled on the ground or connections may be made after each segment is in position. When trusses and arches are assembled on the ground at the site, they must be assembled on level blocking to permit connections to be fitted properly and tightened securely without damage. The end compression joints should be brought into full bearing and compression plates installed where specified. Field welding of structural connections must be performed in accordance with accepted standards for steel construction and welding [38] [39].

1.7.4 Erection

Before erection, the assembly should be checked for prescribed overall dimensions, prescribed camber, and accuracy of anchorage connections. Erection should be planned and executed in such a way that the close fit and neat appearance of joints and the structure as a whole will not be impaired.

Anchor bolts should be checked prior to the start of erection. Before erection begins, all supports and anchors should be complete, accessible, and free of obstructions. The weights and balance points of the structural timber framing should be determined before lifting begins so that proper equipment and lifting methods may be employed. When long members, spliced members, or timber trusses are raised from a flat to a vertical position preparatory to lifting, stresses entirely different from the normal design stress are introduced. The magnitude and distribution of these stresses will vary, depending on such factors as the weight, dimensions, and type of member. A competent rigger should consider these factors in determining how much stiffening, if any, is required, and where it should be located.

1.7.5 Bracing

All framing must be true and plumb. Temporary erection bracing must be provided to hold the framing in a safe position until sufficient permanent bracing is in place. Erection bracing must accommodate all loads to which the structure may be subjected during erection, including loads from equipment and its operation.

Final tightening of alignment bolts should not be completed until the structure has been properly aligned. Temporary bracing should not be removed until the structure has been properly aligned, diaphragms and permanent bracing have been installed, and connections and fastenings have been finally tightened. The design engineer should be consulted prior to removal of temporary shoring. Retightening of connections prior to final completion or closing in of inaccessible connections is recommended.

1.7.6 Field Cuts

All field cuts in timbers should be coated with an approved moisture sealant if the member was initially coated, unless otherwise specified. If timber framing has been pressure-treated, field cutting after treatment should be avoided where possible. When field cuts in pressure-treated material are unavoidable, additional field-treatment should be provided in accordance with AWPA Standard M4 [23].

1.7.7 Moisture Control

During erection operations, all timber framing, whether sawn or glued laminated timbers, should be protected against moisture absorption. Where practical, fabricated structural materials to be stored for an extended period of time before erection should be assembled into subassemblies for storage purposes. Additional information on handling, storage, and erection of glued laminated timbers is contained in AITC 111 [40].

1.8 CONCLUSION

Timber construction encompasses several different structural systems and materials, giving the designer flexibility to meet his structural and architectural needs. This chapter has presented an overview of timber construction including common wood materials and structural systems. Additional guidance has been given regarding economy, durability, seasoning, handling, storage, and erection of timber materials and systems. Where appropriate, references to other sources of information have been included.

Chapter 2 presents fundamental information regarding the properties of wood. An understanding of these basic wood properties will enable a designer to maximize the benefits of the timber members and systems while minimizing potential weaknesses and problems. Chapters 3 through 15 include detailed information regarding the structural design of timber components and connections using the Allowable Stress Design (ASD) methodology. Chapter 16 provides an overview of Load and Resistance Factor Design (LRFD). Chapters 17–19 discuss the design of timber bridges using both LRFD and ASD provisions. Chapter 20 covers the topic of fire safety in timber construction. Appendices include additional design examples and supplemental information.

CHAPTER 2

WOOD PROPERTIES

2.1 INTRODUCTION

Wood is a cellular organic material made up primarily of cellulose, which comprises the cells and lignin,which bonds the cells together. Wood cells are hollow and vary from about 0.04 to 0.33 in. in length and from 0.0004 to 0.0033 in. in diameter. Most cells are elongated and are oriented vertically in the growing tree, but some, called *rays,* are oriented horizontally and extend from the bark toward the center of the tree.

2.1.1 Hardwoods and Softwoods

Trees are divided into two classes: hardwoods, which have broad leaves, and softwoods or conifers, which have needle-like or scale-like leaves. Hardwoods and softwoods also have distinct anatomies and are readily distinguishable to a trained technician by their gross and microscopic features. Most hardwoods shed their leaves annually at the end of each growing season, and most softwoods shed only damaged or unused needles and are thus termed *evergreens.* Notable conifer exceptions are the tamarack and larch, whose needles turn yellow in the fall and are lost, leaving the tree bare of needles through the winter and early spring.

The terms *hardwood* and *softwood* are somewhat misleading because they do not directly indicate the relative hardness or density of wood. The range of hardness or density significantly overlaps between hardwoods and softwoods. The specific gravity of hardwoods ranges from about 0.15 for balsa to more than 1.0 for some dense species. The specific gravity of softwoods ranges from about 0.25 to 0.65.

2.1.2 Heartwood and Sapwood

The cross section of a tree generally shows several distinct zones: the bark, which protects the tree and typically has a rough surface; a light-colored zone called sapwood; and an inner zone, generally of darker color, called heartwood. Trees grow by adding new layers of cells to the outside of the sapwood. The sapwood functions to conduct sap and to store nutrients, as well as support the tree. As the tree continues to grow, the inner layers of the sapwood stop conducting sap and storing nutrients, becoming heartwood, which acts only to support the tree. In some species, heartwood is significantly more resistant to decay than sapwood. There is no consistent difference between the specific gravity and strength properties of heartwood and sapwood.

2.1.3 Earlywood and Latewood

In climates where temperature limits the growing season of a tree, each annual increment of growth usually is readily distinguishable. Such an increment is known as an annual growth ring, or annual ring. In many wood species, large, thin-walled cells are formed in the spring when growth is fastest, whereas smaller, thicker-walled cells are formed later in the year. The areas of fast growth are called *earlywood* or *springwood* and form the lighter band of the annual ring. The areas of slower growth are called *latewood* or *summerwood* and form the darker band in each ring.

Because latewood typically contains more solid wood substance than does earlywood, it is stronger and denser than earlywood. The proportion of width of latewood to width of annual ring and the number of rings per inch are used as measures of the quality and strength of wood for some species.

2.1.4 Grain and Texture

The terms *grain* and *texture* are used in many ways to describe the characteristics of wood and, in fact, do not have a definite meaning. Terms such as *close-grained* and *coarse-grained* refer to the width of the annual rings, while *straight-grained* and *cross-grained* indicate whether the fibers are parallel or at an angle to the sides of a particular piece. The terms *parallel-to-grain* and *perpendicular-to-grain* are generally structural terms that indicate the direction of load or stress in relation to the longitudinal axis of the wood cells (or fibers). *Texture* generally refers to the fineness of wood structure rather than to the annual rings. When these terms are used in connection with wood, the intended meanings should be clearly defined.

2.1.5 Moisture Content

Water is found in wood as free water in the cell cavities and bound water in the cell walls. *Moisture content* (MC) is generally defined as the weight of water in a piece of wood expressed as a percentage of the "oven-dry" weight of the same piece. The moisture content of wood is important in both design and construction of timber structures.

Wood freshly sawn from living trees is termed *green* and may have a moisture content greater than 100%, because green wood in some species contains more water than wood substance itself. The *Wood Handbook* [1] contains average values for the moisture content of heartwood and sapwood for various species. Once cut, wood generally loses moisture, first from the cell cavities (lumens) and then from the cell walls. The condition in which the free water has left the cell cavities but the cell walls are still saturated is termed the *fiber saturation point*. Below the fiber saturation point, wood shrinks and swells with changes in moisture content. Although the moisture content at fiber saturation varies from species to species, and from piece to piece within a species, a value of 30% moisture content is generally associated with the fiber saturation point.

Wood generally continues to lose moisture, or *season*, until an *equilibrium moisture content* (EMC) is reached between the wood moisture content and the relative humidity and temperature of the surrounding air. Equilibrium moisture content values for wood in the temperature and humidity ranges of practical structural importance are shown in Table 2.1.5-1. The EMC values shown are essentially independent of species. As the surrounding temperature or humidity conditions change, wood takes on or loses moisture to achieve the corresponding new EMC.

When control of the moisture content of the wood is to be achieved or the seasoning process accelerated, which is often the case with structural wood, high temperature kilns are often used to dry the wood to a specified moisture content. Framing lumber is typically kiln dried to a moisture content of 19% or less with an average moisture content of about 15%. Lumber used for laminating is typically kiln dried to 16% or less moisture content, with an average moisture content of about 12%. Wood pieces (particularly larger pieces) are sometimes *surface dried*, in which case allowance is made for additional drying of the interior of the pieces. *Oven-dry* refers to the condition in which wood is dried in a laboratory oven until no further moisture can be driven from the wood.

The rate at which wood reaches equilibrium moisture content varies, depending on three factors: (1) the degree to which the wood has been enclosed or sealed; (2) environmental conditions; and (3) the wood itself. Rapid change in moisture content might cause checking in the wood, as discussed in Chapter 1. Once in use in a structure, the rate of change in moisture content is comparable to seasonal or monthly changes in environmental conditions, and as such, monthly or seasonal values of EMC are generally considered. For structures in many locations, the EMC is significantly lower than the fiber saturation point. The *Wood Handbook* [1] contains 30-year average monthly EMC values for approximately 50 U.S. cities.

2.1.6 Growth Characteristics

Wood contains natural growth characteristics such as knots, slope of grain, compression wood, and shakes, which may adversely affect the strength properties

TABLE 2.1.5-1 Moisture Content of Wood in Equilibrium with Stated Dry-Bulb Temperature and Relative Humidity

Temperature		Relative Humidity																		
°C	°F	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%
−1	30	1.4	2.6	3.7	4.6	5.5	6.3	7.1	7.9	8.7	9.5	10.4	11.3	12.4	13.6	14.9	16.5	18.5	21.0	24.3
4	40	1.4	2.6	3.7	4.6	5.5	6.3	7.1	7.9	8.7	9.5	10.4	11.3	12.4	13.5	14.9	16.5	18.5	21.0	24.4
10	50	1.4	2.6	3.6	4.6	5.5	6.3	7.1	7.9	8.7	9.5	10.3	11.2	12.3	13.4	14.8	16.4	18.4	20.9	24.3
16	60	1.3	2.5	3.6	4.6	5.4	6.3	7.0	7.8	8.6	9.4	10.2	11.1	12.1	13.3	14.6	16.2	18.2	20.7	24.1
21	70	1.3	2.5	3.5	4.5	5.4	6.2	6.9	7.7	8.5	9.2	10.1	11.0	12.0	13.1	14.4	16.0	18.0	20.5	23.9
27	80	1.3	2.4	3.5	4.4	5.3	6.1	6.8	7.6	8.3	9.1	9.9	10.8	11.8	12.9	14.2	15.7	17.7	20.2	23.6
32	90	1.2	2.4	3.4	4.3	5.1	5.9	6.7	7.4	8.1	8.9	9.7	10.6	11.5	12.6	13.9	15.4	17.4	19.9	23.3
38	100	1.2	2.3	3.3	4.2	5.0	5.8	6.5	7.2	7.9	8.7	9.5	10.3	11.2	12.3	13.6	15.1	17.0	19.5	22.9
43	110	1.1	2.2	3.2	4.0	4.9	5.6	6.3	7.0	7.7	8.5	9.2	10.0	11.0	12.0	13.2	14.7	16.6	19.1	22.5
49	120	1.1	2.1	3.0	3.9	4.7	5.4	6.1	6.8	7.5	8.2	8.9	9.8	10.7	11.7	12.9	14.4	16.2	18.6	22.0
54	130	1.0	2.0	2.9	3.7	4.5	5.2	5.9	6.6	7.3	7.9	8.7	9.5	10.3	11.3	12.5	14.0	15.8	18.2	21.5
60	140	0.9	1.9	2.8	3.6	4.3	5.0	5.7	6.3	7.0	7.7	8.4	9.1	10.0	11.0	12.2	13.6	15.4	17.7	21.0
66	150	0.9	1.8	2.6	3.4	4.1	4.8	5.5	6.1	6.7	7.4	8.1	8.8	9.7	10.6	11.8	13.2	14.9	17.2	20.5

Values were calculated using Equation 4-5, *Wood Handbook: Wood as an Engineering Material*, 2010, FPL-GTR-190.

of that member, depending on their size, number, and location within the member. These characteristics are discussed in detail in the *Wood Handbook* [1]. Structural grading rules take into account the effects of these growth characteristics on the strength of wood in establishing design values for lumber and glued laminated timber.

2.1.7 Directional Properties

Wood is anisotropic because of its cellular structure. The structure of wood in any particular log is generally considered to have three axes of symmetry: longitudinal, radial, and tangential (Figure 2.1.7-1). Wood pieces sawn from logs are typically oriented with their long axis (or faces) approximately parallel to the longitudinal axis of the log, but with the other faces indiscriminate with respect to the radial and tangential directions. As such, for practical purposes, directional properties of wood are generally distinguished between *parallel-to-grain* (longitudinal) and *perpendicular-to-grain*. Perpendicular-to-grain values apply to both radial and tangential properties.

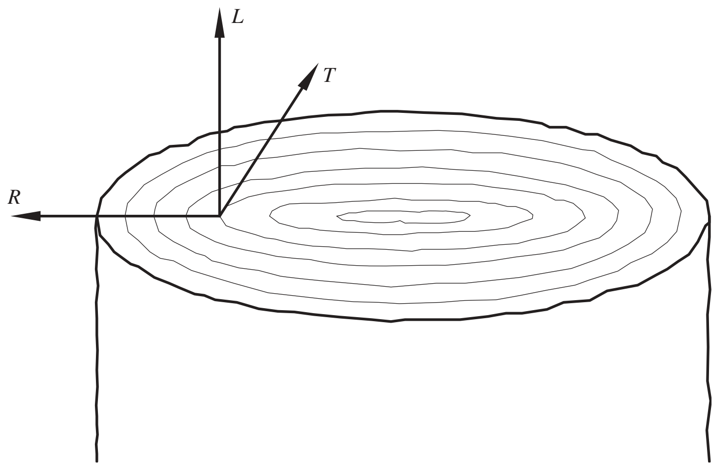


Figure 2.1.7-1 Three principal axes of wood: L, longitudinal (parallel-to-grain); R, radial (perpendicular-to-grain, radial to annual rings); and T, tangential (perpendicular-to-grain, tangential to annual rings).

2.2 SPECIFIC GRAVITY AND SPECIFIC WEIGHT OF COMMERCIAL LUMBER SPECIES

Specific gravity and specific weight values for various lumber species or species combinations are shown in Table 2.2-1. Detailed descriptions of each species and species combinations can be found in the *Wood Handbook* [1] and the *National*

TABLE 2.2-1 Specific Gravity and Specific Weight of Selected Lumber Species

Species or Species Group	Specific Gravity		Specific Weight,
	Oven Dry Volume[a]	12% MC Volume[b]	12% MC (pcf)[c]
Alaska cedar	0.46	0.44	31
Aspen	0.39	0.37	26
Cottonwood	0.41	0.39	27
Douglas fir-larch	0.50	0.47	33
Douglas fir-larch (north)	0.49	0.47	33
Douglas fir (south)	0.46	0.44	31
Eastern spruce	0.41	0.39	27
Eastern white pine	0.36	0.35	24
Hem-fir	0.43	0.41	29
Hem-fir (north)	0.46	0.44	31
Northern white cedar	0.31	0.30	21
Ponderosa pine	0.43	0.41	29
Port Orford cedar	0.46	0.44	31
Red maple	0.58	0.55	38
Red oak	0.67	0.63	44
Red pine	0.44	0.42	29
Redwood, close grain	0.44	0.42	29
Redwood, open grain	0.37	0.36	25
Southern pine	0.55	0.52	36
Spruce–pine–fir	0.42	0.40	28
Spruce–pine–fir (south)	0.36	0.35	24
Western cedars	0.36	0.35	24
Western white pine	0.40	0.38	27
White oak	0.73	0.68	47
Yellow poplar	0.43	0.41	29

[a] Specific gravity from the *NDS*® [2] based on oven-dry weight and oven-dry volume.
[b] Specific gravity based on volume at 12% moisture content and oven-dry weight, calculated using Equation (4-11) of the *Wood Handbook* [1] and specific gravity values from the *NDS*® [2].
[c] Specific weight at 12% moisture content, including the weight of water.

Design Specification® for Wood Construction [2]. Since the weight of a particular piece of lumber may vary with moisture content, specific gravity values used in timber construction are calculated as the oven-dry weight of a piece divided by the volume at a specified moisture content, normalized by the specific weight of water. Typical moisture content values for determining the volume for specific gravity calculations include *green* (above fiber saturation point), 12% MC, and *oven-dry*. Table 2.2-1 shows specific gravity values based on oven-dry weight and volumes for 12% MC and for the oven-dry condition. The values based on the volume in the oven-dry condition are used in the *National Design Specification*® [2] for fastener design.

Specific weight values in Table 2.2-1 include the weight of the water in the wood corresponding to a moisture content of 12%. Specific weight values at 12%

MC are customarily used in design for computing self-weight and dead loads. Specific weight values at other moisture contents up to 30% may be determined with Equations 2.2-1 and 2.2-2.

$$G_m = \frac{G_0}{1 + 0.265\, G_0\, (m/30\%)} \tag{2.2-1}$$

$$\gamma_m = G_m\, (1 + m/100\%)\, \gamma_w \tag{2.2-2}$$

where:

G_m = specific gravity at a specified moisture content, m
G_0 = specific gravity based on oven-dry weight and oven-dry volume
m = moisture content (between 0% and 30%)
γ_m = specific weight of wood at the specified moisture content, m
γ_w = specific weight of water

Occasionally, the weight of wood may need to be calculated at moisture contents above the fiber saturation point. Because the volume and oven-dry weight do not change above the fiber saturation point (30% moisture content), the calculated specific gravity will be equal to that calculated for 30% moisture content. However, the specific weight includes the weight of the water in the wood and will change with moisture content above 30%. Equation 2.2-3 may be used to calculate specific weight for pieces above 30% moisture content.

$$\gamma_m = \gamma_{30} \left(1 + \frac{m - 30\%}{30\%}\right) \tag{2.2-3}$$

where:

γ_{30} = specific weight at 30% moisture content

EXAMPLE 2.2-1 ESTIMATING SHIPPING WEIGHT OF LUMBER

Given: 10 thousand board feet (MBF) of Douglas fir-larch (DF-L) 2 × 6 dimension lumber

Wanted: Estimate the shipping weight at 5% MC, 19%, and 30% MC.

Approach: The specific gravity of DF-L at oven-dry weight and oven-dry volume is obtained from Table 2.2-1. The specific gravity values at 5%, 19%, and 30% are calculated using Equation 2.2-1. Equation 2.2-2 will then be used to calculate specific weights at the three MC levels. Using the definition of board feet from Chapter 1 and the specific weight of water, the total weight of 10 MBF of lumber will be calculated for each of the stated MC levels.

Solution:

From Table 2.2-1, the value of G_0 for DF-L is 0.50.

From Equation 2.2-1, the specific gravity at 5% MC is calculated as:

$$G_5 = \frac{0.50}{1 + 0.265(0.50)\left(5\% / 30\%\right)} = 0.49$$

From Equation 2.2-2, the specific weight at 5% MC is calculated as:

$$\gamma_5 = 0.49\,(1 + 5\% / 100\%)\,62.4\,\text{pcf} = 32\ \text{pcf}$$

From Chapter 1, one board foot of 2 × 6 is equal to one lineal foot of 2 × 6, or, 1.5 in. × 5.5 in. × 12 in. = 99 in^3 of wood; therefore, 10,000 BF of 2 × 6 has a volume of 99,000 in^3 or 573 ft^3.

The weight of 10 MBF at 5% MC is then:

$$W_5 = \left(573\,\text{ft}^3\right)\left(32\ \frac{\text{lb}}{\text{ft}^3}\right) = 18,300\,\text{lb}$$

From Equation 2.2-1, the specific gravity at 19% MC is calculated as:

$$G_{19} = \frac{0.50}{1 + 0.265\,(0.50)\,(19\% / 30\%)} = 0.46$$

From Equation 2.2-2, the specific weight at 19% MC is calculated as:

$$\gamma_{19} = 0.46\,(1 + 19\% / 100\%)\,62.4\,\text{pcf} = 34\,\text{pcf}$$

The weight of 10 MBF at 19% MC is then:

$$W_{19} = \left(573\,\text{ft}^3\right)\left(34\ \frac{\text{lb}}{\text{ft}^3}\right) = 19,500\,\text{lb}$$

From Equation 2.2-1, the specific gravity at 30% MC is calculated as:

$$G_{30} = \frac{0.50}{1 + 0.265\,(0.50)\,(30\% / 30\%)} = 0.44$$

From Equation 2.2-2, the specific weight at 30% MC is calculated as:

$$\gamma_{30} = 0.44\,(1 + 30\% / 100\%)\,62.4\,\text{pcf} = 36\,\text{pcf}$$

The weight of 10 MBF at 30% MC is then:

$$W_{30} = \left(573\,\text{ft}^3\right)\left(36\ \frac{\text{lb}^3}{\text{ft}}\right) = 20,600\,\text{lb}$$

Answer: The weights of 10 MBF of DF-L at 5%, 19%, and 30% MC are 18,300 lb, 19,500 lb, and 20,600 lb, respectively.

Discussion: As shown above, the weight of typical structural wood decreases by about 5 percent from fiber saturation to 19% MC, and another 5 percent from 19% MC to 5% MC.

2.2.1 Effect of Adhesives and Preservative Treatments

Adhesives used in laminating do not have an appreciable effect on specific weight. Some preservative treatments—in particular, oil-borne treatments at high retention levels—may change the specific weight significantly. The American Wood Protection Association *Book of Standards* [3] should be consulted for more detailed information regarding preservative treatment and changes to specific weight.

2.3 DIMENSIONAL CHANGES DUE TO MOISTURE AND TEMPERATURE

Dimensional changes in wood may come about by temperature and moisture content changes in the wood. Wood containing moisture responds to varying temperatures differently than other typical building materials. As wood is heated, it tends to expand due to temperature but also shrinks due to an accompanying loss of moisture. Unless the wood is very dry initially (3% or 4% MC or less), shrinkage due to moisture loss on heating will exceed thermal expansion, resulting in a negative net dimensional change due to heating. In timber design, the dimensional changes due to temperature are considered small in comparison with dimensional changes due to changing moisture content. As such, proper detailing of connections to accommodate wood shrinkage is generally considered adequate to also accommodate dimensional changes due to temperature.

2.3.1 Moisture Content and Shrinkage

Between the fiber saturation point and the oven-dry condition, wood shrinks as it loses moisture and swells as it absorbs moisture. Above the fiber saturation point, there is no dimensional change with variation in moisture content. The amount of shrinkage and swelling differs in the tangential, radial, and longitudinal directions of a piece of wood or lumber. Good engineering design considers shrinkage and swelling in the detailing and use of wood members.

Shrinking and swelling are typically expressed as percentages based on the green dimensions of the wood. Wood shrinks most in a direction tangent to the annual growth rings and somewhat less in the radial direction or across these rings. In general, shrinkage amounts increase with wood density.

Table 2.3.1-1 gives the average tangential, radial, and volumetric shrinkage values for various species drying from the green condition to oven-dry and ranges of values for species groups. Because the faces of pieces of lumber are seldom oriented so that the annual growth rings are exactly tangent and radial to the faces of the piece, it is customary in estimating cross-sectional dimensional changes to use an intermediate or average value between the tangential and radial values. However, for specific cases, where the growth ring orientation is known, it may be appropriate to use either the tangential or radial shrinkage value. The values

TABLE 2.3.1-1 Average Shrinkage Values for Wood Based on Green Dimensions[a]

Species or Species Group	Percent Shrinkage from Green to Oven-Dry		
	Radial	Tangential	Volumetric
Alaska Cedar	3	6	9
Aspen	3–4	7–8	11–12
Cottonwood	3–4	7–9	11–14
Douglas fir-larch	4–5	7–9	11–14
Eastern spruce	4	7–8	11–12
Eastern white pine	2	6	8
Hem-fir	3–5	7–9	10–13
Northern white cedar	2	5	7
Ponderosa pine	4	6	10
Port Orford cedar	5	7	10
Red maple	4	8	13
Red oak	4–5	9–11	14–19
Red pine	4	7	11
Redwood	4	7	11
Southern pine	5	7–8	12
Spruce-pine-fir	3–4	7–8	10–12
Western cedars	2–5	5–7	7–10
Western white pine	4	7	12
White oak	4–7	9–13	13–16
Yellow poplar	5	8	13

[a] Values presented are from the *Wood Handbook* [1] and represent average values for a single species or a a range of average values for a species group. Additional information may be found in the *Wood Handbook.*

in Table 2.3.1-1 can be used to estimate shrinkage using Equation 2.3.1-1.

$$S_m = S_0 \left(\frac{m_i - m_f}{30\%} \right) \tag{2.3.1-1}$$

where:

S_m = shrinkage from initial moisture condition to final moisture content (%)
S_0 = total shrinkage from Table 2.3.1-1 (%)
m_f = final moisture content (at or below 30%)
m_i = initial moisture content (at or below 30%)

Values for longitudinal shrinkage are not tabulated in Table 2.3.1-1, because they are ordinarily negligible. The total longitudinal shrinkage of commonly used species from fiber saturation to oven-dry condition usually ranges from 0.1% to 0.2% of the green dimension. Abnormal longitudinal shrinkage may occur in compression wood, wood with steep slope of grain, and exceptionally lightweight wood of any species.

Because there is considerable variation in shrinkage for individual pieces within any species, it is difficult to predict the exact shrinkage of an individual piece of wood. If the species of wood is known, the values given in Table 2.3.1-1 or the values in the *Wood Handbook* [1] may be used to estimate the average shrinkage of a quantity of pieces, such as a stack of lumber, or the cumulative shrinkage of the wood in a wood structure.

A rule of thumb commonly used to estimate shrinkage perpendicular to the grain in glued laminated timber is to assume that 1% shrinkage occurs for every 5% change in moisture content. This amount of shrinkage corresponds to the average of radial and tangential shrinkage for lumber. Lumber used in laminating tends to be "flat grained" (wide faces more closely parallel with the growth rings); therefore, shrinkage in the direction perpendicular to the glue lines is more comparable to radial shrinkage than tangential shrinkage. As such, the shrinkage of a typical glued laminated beam across the glue lines (decrease in beam depth) tends to be slightly less than the values predicted using the rule of thumb.

EXAMPLE 2.3.1-1 GLULAM BEAM SHRINKAGE

Given: Douglas fir $5\frac{1}{8}$ in. by 24 in. glued laminated beam

Wanted: Determine the decrease in depth (shrinkage) for a change in moisture content from 12% to 5%.

Approach: The decrease in depth will be perpendicular to the wide faces or across the glue lines, which, from the discussion above, will be assumed to be somewhere in between radial and tangential shrinkage, but closer to the radial value.

Solution: From Table 2.3.1-1 a value of 6% is chosen for S_0.

From Equation 2.3.1-1,

$$S_5 = 6.0\% \left(\frac{12\% - 5\%}{30\%} \right) = 1.4\%$$

The shrinkage, therefore, is 1.4% of 24 in., or 0.33 in.

Answer: The expected change in depth of a 24-in. DF glulam beam from 12% MC to 5% MC is 0.33 in. The dimension change of 1.4% agrees with the rule of thumb of 1% change in dimension per 5% change in MC.

Accommodation of shrinkage and swelling is very important in the proper design, detailing, and construction of wood structures and wood structural elements, particularly where constraint introduces stresses perpendicular to the grain. More information on connection detailing is included in Chapter 11.

2.3.2 Dimensional Changes Due to Temperature

The coefficient of linear thermal expansion differs in wood's three structural directions. In the longitudinal direction (parallel-to-grain), the coefficient appears to be independent of specific gravity and species and ranges from about 1.7 $(10)^{-6}$ to 2.5 $(10)^{-6}$ in./in. per °F for oven-dry wood for both hardwoods and softwoods. This value is about one-tenth to one-third of those for other common structural materials. Coefficients of linear thermal expansion across the grain (radial and tangential) are proportional to and range from about 5 to more than 10 times greater than the parallel-to-grain coefficients. Equations relating the coefficient of thermal expansion for oven-dry wood in the across grain (radial and tangential) directions as functions of specific gravity are published in the *Wood Handbook* [1].

2.4 THERMAL INSULATING PROPERTIES

Wood is considered to be a good insulator due to its physical structure, because the cells contain voids that retard heat transfer. The thermal conductivity of wood varies with (1) the direction of grain, (2) specific gravity, (3) moisture content, (4) extractives present in the wood, and (5) growth characteristics such as knots, slope of grain, seasoning checks, and growth rings. Thermal conductivity is approximately the same in radial and tangential directions but is about 2.5 times greater along the grain. Cross-grain thermal conductivity values for various wood species at 12% MC may be calculated using Equation 2.4-1 [1].

$$k = 1.676G + 0.129 \tag{2.4-1}$$

where:

k = thermal conductivity (Btu in./hr ft^2 °F)
G = specific gravity at 12% MC (Table 2.2-1)

Thermal conductivity of individual wood pieces may vary considerably with calculated values. Thermal conductivity also increases with moisture content. Thermal conductivity values at moisture content levels different than 12% may be calculated with equations from the *Wood Handbook* [1].

The thermal resistance, R, for a piece of material with one-dimensional heat flow may be calculated as reciprocal of the thermal conductivity, k, multiplied by the piece thickness, t, or $R = (1/k)t$. Using Equation 2.4-1, the thermal conductivity of Douglas-fir/larch, at 12% MC, is $k = 1.676\ (0.47) + 0.129 =$ 0.92 Btu in./hr ft^2 °F, where $G = 0.47$ from Table 2.2-1. The thermal resistance of a 2 in. (nominal) plank, therefore, is $R = (1/0.92) \times 1.5 = 1.6$ hr ft^2 °F/Btu in. Since wood is commonly used in combination with other building materials for insulating assemblies, consideration must be made for heat flow through the assembly as a whole. The *Fundamentals Handbook* of the American Society of

Heating, Refrigerating, and Air-Conditioning Engineers [4] provides information and example calculations of resistance values for various wood assemblies. Heat capacity and thermal diffusivity values for various wood species at various moisture content levels may be found in the *Wood Handbook* [1].

2.5 WOOD IN CHEMICAL ENVIRONMENTS

As discussed in Chapter 1, wood is naturally resistant to chemical action, making it a favorable building material in chemically adverse environments. Chemical actions of three general types may affect the strength of wood. The first causes swelling and the resultant weakening of the wood. Liquids such as water, alcohols, and some other organic liquids swell wood. This action is almost completely reversible; therefore, if the swelling liquid or solution is removed by evaporation or by extraction followed by evaporation of the solvent, the original dimensions and strength are practically restored. Liquids such as petroleum oils and creosote do not swell wood. The second type of action brings about permanent changes in the wood, such as hydrolysis of the cellulose by acids or acid salts. The third type of action, which is also permanent, involves delignification of the wood and dissolving of hemicelluloses by alkalis.

Experience and available data indicate species and conditions where wood is equal or superior to other materials in resisting the degradative action of chemicals. In general, heartwood of such species as cypress, Douglas fir, larch, southern pine, redwood, maple, and white oaks are quite resistant to attack by dilute mineral and organic acids. Oxidizing acids, such as nitric acid, have a greater degradative action than nonoxidizing acids. Alkaline solutions are more destructive than acidic solutions, and hardwoods tend to be more susceptible to attack by both acids and alkalis than softwoods.

Highly acidic salts tend to hydrolyze wood when present in high concentrations. In hot, arid regions, even relatively low concentrations of such salts have shown signs that the salt might migrate to the surface of railroad ties, which are occasionally wet and dried. This migration, combined with the high concentrations of salt relative to the small amount of water present, causes an acidic condition sufficient to make wood brittle.

Iron salts, which develop at points of contact with plates, bolts, and other fasteners, degrade wood, especially in the presence of moisture. In addition, iron salts may precipitate toxic extractives from the wood and thus lower the natural decay resistance of wood. The softening and discoloration of wood around corroded iron fastenings is a commonly observed phenomenon; it is especially pronounced in acidic woods, such as oak, and in woods such as redwood, which contain considerable tannin and related compounds. The oxide layer formed on iron is transformed through reaction with wood acids into soluble iron salts, which not only degrade the surrounding wood but may catalyze the further corrosion of the metal. The action is accelerated by moisture; oxygen may also play an important role in the process. This effect is not encountered with well-dried wood used in dry locations. Under damp-use conditions, it can be avoided or minimized by using corrosion-resistant fastenings.

Many substances have been employed as impregnants to enhance the natural resistance of wood to chemical degradation. One of the more economical treatments involves pressure impregnation with a viscous coke-oven coal tar to retard liquid penetration. Acid resistance of wood is increased by impregnation with a phenolic resin solutions, followed by appropriate drying and curing. Treatment with furfuryl alcohol has been used to increase resistance to alkaline solutions. Another procedure involves massive impregnation with a monomeric resin, such as methyl methacrylate, followed by polymerization.

2.6 ACOUSTICAL PROPERTIES

The acoustical properties of a material or composite construction are determined by its sound insulation and sound absorption abilities. Sound insulation abilities are measured in terms of the reduction in intensity of sound when it passes through a barrier. Sound absorption refers to the amount of incident sound on a surface that is not reflected by the surface.

Sound insulating values for materials of construction are related to the sound transmission loss for the construction measured in decibels at various frequencies. Like most common construction materials, wood alone does not provide good sound insulation. However, when properly combined with other materials in typical constructions, it will provide a structural unit with satisfactory sound-insulating ability. Sound-absorption values for wood vary with moisture content, direction of grain, and density.

2.7 ELECTRICAL PROPERTIES

The most important electrical properties of wood are its resistance to the passage of an electric current and its dielectric properties. The electrical resistance of wood is utilized in electric moisture meters used to determine moisture content. The dielectric properties of wood are utilized in the high-frequency curing of adhesives in glued laminated members and in the drying of wood.

The electrical resistance of wood varies with moisture content, density, direction of travel of the current with respect to the direction of the grain, and temperature. It varies greatly with moisture content, especially below the fiber saturation point, decreasing with an increase in moisture content. At low moisture content, wood is considered an insulator. Electrical resistance varies inversely with the density of wood, although this effect is slight compared to the variance due to moisture content and is greater across the grain than along it. The electrical resistance of wood approximately doubles for each drop in temperature of 22.5°F, but this relationship varies considerably with the level of moisture content. There is also a variation in electrical resistance between species, which is possibly caused by minerals or electrolytes in the wood itself or dissolved in the water present in the wood.

Metallic salts, such as used in preservative and fire-retardant treatments, may lower the electrical resistance of the wood considerably. Use of wood containing such salts should be avoided for applications where electrical resistance is critical and in processes involving dielectric heating. Moisture meters may give erroneous readings for such wood.

2.8 COEFFICIENT OF FRICTION

Coefficients of static friction for wood in contact with wood and for wood in contact with other materials depend on the moisture content of the wood and surface roughness. The coefficients vary little with species except for those species that contain abundant oily or waxy extractives. Coefficients of static friction for wood have been reported to be approximately 0.7 for dry wood on unpolished steel and 0.4 for green wood on steel. Coefficients of static friction for smooth wood on smooth wood are reported to be 0.6 for dry wood and 0.8 for green wood. It is not usual practice to use friction to resist forces in the design of timber connections.

2.9 CONCLUSION

This chapter has presented an overview of various properties of wood that may be important in design. A basic knowledge of wood-moisture relationships and growth characteristics is fundamental to the use of this material. Other properties discussed in this chapter—such as wood's acoustical, electrical, and insulating properties—have application in industry and in some specialized fields. Further information can be found in the *Wood Handbook* [1].

CHAPTER 3

TIMBER DESIGN

3.1 INTRODUCTION

The design of timber structures requires all components of the structure to be of sufficient size and strength to withstand the forces to which the structure might reasonably be subjected during its useful life. Proper design also requires sufficient stiffness in members and structural systems to prevent deformations associated with reasonable loads from impairing the serviceability of nonstructural systems, or causing the structure to feel or appear unsafe. The selection of the timber elements, connections, and other structural systems to ensure the strength and serviceability described above serves as the context of design for this manual.

Timber design includes the proper evaluation of timber elements, such as individual planks, joists, beams, posts, or columns. Design must also include consideration of the connections of elements, and the overall performance of structural systems. General design provisions are covered in this chapter; subsequent chapters are devoted specifically to the design of structural elements and connections.

This manual includes both allowable stress design (ASD) and load and resistance factor design (LRFD) methodologies. ASD is emphasized through Chapter 15. Chapters 16 and 18 specifically address the LRFD methodology, including design examples. Both methodologies utilize reference design values published in AITC 117 [1] and the *National Design Specification® for Wood Construction* [2]. In the ASD approach, internal stresses are calculated under service (working) load conditions and checked against adjusted allowable design values that take into consideration specific conditions of use. Conditions of use include load duration, moisture, temperature, orientation, and bracing of the member, and others. In the LRFD approach, internal member stresses or forces are calculated at "factored load" levels, and checked against material strengths or member capacities. Strengths or capacities are determined from the reference design values multiplied by the *format conversion factor* and the *resistance factor,* and further adjusted for conditions of use (time effect, moisture, temperature, and so on). For most design applications, the two methodologies produce similar results. The primary difference in the two approaches is in the application of safety factors. ASD design values include implicit safety factors that account for variations in both materials and loads. For LRFD, safety factors are explicitly applied to loads and material properties (resistances).

3.2 LOADS

The loads or forces that act on structures may be divided into two broad categories: dead loads and applied loads. Dead loads generally consist of the weight forces associated with the structure itself and include the weight of all materials permanently attached to the structure. Applied loads generally include the other forces that might reasonably be experienced by the structure during its design life. Applied loads include live loads such as the weights of building occupants and the forces generated by their activities and include the weights of materials stored in or on the structure. Applied loads also include loads or forces brought on by the environment, such as wind, earthquakes, snow, floods, and earth pressures. The design of any structure must include the deliberate determination of all of the appropriate loads and forces.

Building codes typically prescribe minimum loads and forces for design. Building codes also stipulate how the loads and forces are to be resisted, as well as stress or strength values that may be assigned to building materials. Local governing bodies generally develop building codes for their jursidiction by adopting and amending model building codes.

Prior to the design of any structure, the applicable building code requirements should be obtained from the governing building authority. In the absence of a governing building authority, model building codes, such as the *International Building Code* [3] or *International Residential Code* [4], may be used. The design of any structure, regardless of governing codes, must be done in accordance with accepted engineering practice. The methods presented in this manual have historically been considered accepted engineering practice.

The loads and forces acting on timber structures are described in greater detail in the following sections of this chapter. The typical stresses resulting from design loading conditions and corresponding allowable stresses are also described in this chapter. Design methods for individual timber members, connections and fasteners, and building systems used in timber design are covered in following chapters. Wood's strength is dependent on the duration of time over which a particular load is applied. Hence, for timber design, the duration of a load may be as important as its magnitude. Adjustments for load duration are discussed in greater detail in Section 3.4.1.

3.2.1 Dead Loads

Dead loads may be defined as the weights of all permanent components of a building or structure, such as walls, floors, roofs, partitions, stairways, and fixed service equipment. These loads include the weights of materials as a part of the original construction, and should also include anticipated materials to be added at later times. For example, if additional roofing materials are expected to be added at later times (reroofing), the original design of structural framing for the roof should take into consideration the eventual total roof material weights (dead loads), so that the framing system need not be retrofitted or reinforced.

The actual weights of the various materials and systems or assemblies should be used in design if this information is available. Minimum dead load values for common construction materials as recommended by the American Society of Civil Engineers [5] are shown in Table 3.2.1-1. The weight of any structural member under consideration is often referred to as the *self-weight* and is also a dead load. Care should be taken to ensure that self-weights of members are properly considered. Care must also be taken to ensure that material weights on sloped or curved surfaces are properly considered, because the plan area (horizontally projected) weights are greater than the surface area weights.

3.2.2 Live Loads

For the purposes of this manual, live loads are considered to be the loads arising from occupancy or use of a structure and include the weight of occupants, vehicles, and materials placed in or on the structure during normal use, including stored materials and furniture. Other loads or forces, such as those arising from wind, rain, snow, and earthquakes, are considered separately. Minimum live loads recommended by the American Society of Civil Engineers [5] are shown in Table 3.2.2-1 and may be used as a guide in the absence of local code requirements. Conditions of any particular project may require the use of greater loads than those in the tables or provided by local codes. The loads used in design should be clearly shown on the design and construction documents.

3.2.2.1 Floor Live Loads Loads arising from the activities of the occupants on floors of buildings are of two types: distributed loads and concentrated loads. Minimum service loads of both types are listed in Table 3.2.2-1. Floor systems must be designed to carry both types of loads, though not necessarily simultaneously. Where floor support members have multiple spans, the conditions of all-span, alternate-span, and adjacent span loading must be considered. Floor live loads are generally considered to have "normal" load duration of ten years. The live loads listed are treated as static loads but are assumed to accommodate dynamic effects of normal building use and occupancy. Crane loads and cyclic loads associated with machinery are considered separately.

TABLE 3.2.1-1 Minimum Design Dead Loads

Material	Density (lb/ft^3)
Aluminum	170
Bituminous products	
Asphaltum	81
Graphite	135
Paraffin	56
Petroleum, crude	55
Petroleum, refined	50
Petroleum, benzine	46
Petroleum, gasoline	42
Pitch	69
Tar	75
Brass	526
Bronze	552
Cast-stone masonry (cement, stone, sand)	144
Cement, portland, loose	90
Ceramic tile	150
Charcoal	12
Cinder fill	57
Cinders, dry, in bulk	45
Coal	
Anthracite, piled	52
Bituminous, piled	47
Lignite, piled	47
Peat, dry, piled	23
Concrete, plain	
Cinder	108
Expanded-slag aggregate	100
Haydite (burned-clay aggregate)	90
Slag	132
Stone (including gravel)	144
Vermiculite and perlite aggregate, nonload-bearing	25–50
Other light aggregate, load-bearing	70–105
Concrete, reinforced	
Cinder	111
Slag	138
Stone (including gravel)	150
Copper	556
Cork, compressed	14
Earth (not submerged)	
Clay, dry	63
Clay, damp	110
Clay and gravel, dry	100
Silt, moist, loose	78
Silt, moist, packed	96
Silt, flowing	108
Sand and gravel, dry, loose	100
Sand and gravel, dry, packed	110
Sand and gravel, wet	120
Earth (submerged)	
Clay	80
Soil	70
River mud	90
Sand or gravel	60
Sand or gravel and clay	65
Glass	160
Gravel, dry	104
Gypsum, loose	70
Gypsum, wallboard	50
Ice	57
Iron	
Cast	450
Wrought	480
Lead	710
Lime	
Hydrated, loose	32
Hydrated, compacted	45

TABLE 3.2.1-1 (*Continued*)

Material	Density (lb/ft^3)	Material	Density (lb/ft^3)
Masonry, ashlar stone		Machine	96
Granite	165	Sand	52
Limestone, crystalline	165	Slate	172
Limestone, oolitic	135	Steel, cold-drawn	492
Marble	173	Stone, quarried, piled	
Sandstone	144	Basalt, granite, gneiss	96
Masonry, brick		Limestone, marble, quartz	95
Hard (low absorption)	130	Sandstone	82
Medium (medium absorption)	115	Shale	92
Soft (high absorption)	100	Greenstone, hornblende	107
Masonry, concrete[a]		Terra cotta, architectural	
Lightweight units	105	Voids filled	120
Medium weight units	125	Voids unfilled	72
Normal weight units	135	Tin	459
Masonry grout	140	Water	
Masonry, rubble stone		Fresh	62
Granite	153	Sea	64
Limestone, crystalline	147	Wood, seasoned	
Limestone, oolitic	138	Ash, commercial white	41
Marble	156	Cypress, southern	34
Sandstone	137	Fir, Douglas, coast region	34
Mortar, cement or lime	130	Hem fir	28
Particleboard	45	Oak, commercial reds and whites	47
Plywood	36	Pine, southern yellow	37
Riprap (not submerged)		Redwood	28
Limestone	83	Spruce, red, white, and Sitka	29
Sandstone	90	Western hemlock	32
Sand		Zinc, rolled sheet	449
Clean and dry	90		
River, dry	106		
Slag			
Bank	70		
Bank screenings	108		

[a] Tabulated values apply to solid masonry and to the solid portion of hollow masonry.

TABLE 3.2.2-1 Minimum Design Live Loads

Occupancy or Use	Uniform psf (kN/m^2)	Conc. lb (kN)
Apartments (see Residential)		
Access floor systems		
Office use	50 (2.4)	2,000 (8.9)
Computer use	100 (4.79)	2,000 (8.9)
Armories and drill rooms	150 (7.18)[a]	
Assembly areas and theaters		
Fixed seats (fastened to floor)	60 (2.87)[a]	
Lobbies	100 (4.79)[a]	
Movable seats	100 (4.79)[a]	
Platforms (assembly)	100 (4.79)[a]	
Stage floors	150 (7.18)[a]	
Balconies and decks	1.5 times the live load for the occupancy served. Not required to exceed 100 psf (4.79 kN/m^2)	
Catwalks for maintenance access	40 (1.92)	300 (1.33)
Corridors		
First floor	100 (4.79)	
Other floors, same as occupancy served except as indicated		
Dining rooms and restaurants	100 (4.79)[a]	
Dwellings (see Residential)		
Elevator machine room grating (on area of 2 in. by 2 in. (50 mm by 50 mm))		300 (1.33)
Finish light floor plate construction (on area of 1 in. by 1 in. (25 mm by 25 mm))		200 (0.89)
Fire escapes	100 (4.79)	
On single-family dwellings only	40 (1.92)	
Fixed ladders	See Section 4.5	
Garages		
Passenger vehicles only	40 (1.92)[a,b,c]	
Trucks and buses	[c]	
Handrails, guardrails, and grab bars	See Section 4.5	
Helipads	60 (2.87)[d,e] Nonreducible	[e,f,g]

TABLE 3.2.2-1 (*Continued*)

Occupancy or Use	Uniform psf (kN/m^2)	Conc. lb (kN)
Hospitals		
Operating rooms, laboratories	60 (2.87)	1,000 (4.45)
Patient rooms	40 (1.92)	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
Hotels (see Residential)		
Libraries		
Reading rooms	60 (2.87)	1,000 (4.45)
Stack rooms	150 (7.18)[a,h]	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
Manufacturing		
Light	125 (6.00)[a]	2,000 (8.90)
Heavy	250 (11.97)[a]	3,000 (13.40)
Office buildings		
File and computer rooms shall be designed for heavier loads based on anticipated occupancy		
Lobbies and first-floor corridors	100 (4.79)	2,000 (8.90)
Offices	50 (2.40)	2,000 (8.90)
Corridors above first floor	80 (3.83)	2,000 (8.90)
Penal institutions		
Cell blocks	40 (1.92)	
Corridors	100 (4.79)	
Recreational uses		
Bowling alleys, poolrooms, and similar uses	75 (3.59)[a]	
Dance halls and ballrooms	100 (4.79)[a]	
Gymnasiums	100 (4.79)[a]	
Reviewing stands, grandstands, and bleachers	100 (4.79)[a,i]	
Stadiums and arenas with fixed seats (fastened to the floor)	60 (2.87)[a,i]	
Residential		
One- and two-family dwellings		
Uninhabitable attics without storage	10 (0.48)[j]	
Uninhabitable attics with storage	20 (0.96)[k]	
Habitable attics and sleeping areas	30 (1.44)	
All other areas except stairs	40 (1.92)	
All other residential occupancies		
Private rooms and corridors serving them	40 (1.92)	
Public rooms[a] and corridors serving them	100 (4.79)	

(*continues*)

TABLE 3.2.2-1 (*Continued*)

Occupancy or Use	Uniform psf (kN/m^2)	Conc. lb (kN)
Roofs		
Ordinary flat, pitched, and curved roofs	20 (0.96)[l]	
Roofs used for roof gardens	100 (4.79)	
Roofs used for assembly purposes	Same as occupancy served	
Roofs used for other occupancies	[m]	[m]
Awnings and canopies		
Fabric construction supported by a skeleton structure	5 (0.24) nonreducible	300 (1.33) applied to skeleton structure
Screen enclosure support frame	5 (0.24) nonreducible and applied to the roof frame members only, not the screen	200 (0.89) applied to supporting roof frame members only
All other construction	20 (0.96)	
Primary roof members, exposed to a work floor		
Single panel point of lower chord of roof trusses or any point along primary structural members supporting roofs over manufacturing, storage warehouses, and repair garages		2,000 (8.9)
All other primary roof members		300 (1.33)
All roof surfaces subject to maintenance workers		300 (1.33)
Schools		
Classrooms	40 (1.92)	1,000 (4.45)
Corridors above first floor	80 (3.83)	1,000 (4.45)
First-floor corridors	100 (4.79)	1,000 (4.45)
Scuttles, skylight ribs, and accessible ceilings		200 (0.89)
Sidewalks, vehicular driveways, and yards subject to trucking	250 (11.97)[a,n]	8,000 (35.60)[o]
Stairs and exit ways	100 (4.79)	300[p]
One- and two-family dwellings only	40 (1.92)	300[p]
Storage areas above ceilings	20 (0.96)	
Storage warehouses (shall be designed for heavier loads if required for anticipated storage)		
Light	125 (6.00)[a]	
Heavy	250 (11.97)[a]	

TABLE 3.2.2-1 (*Continued*)

Occupancy or Use	Uniform psf (kN/m^2)	Conc. lb (kN)
Stores		
Retail		
First floor	100 (4.79)	1,000 (4.45)
Upper floors	75 (3.59)	1,000 (4.45)
Wholesale, all floors	125 (6.00)[a]	1,000 (4.45)
Vehicle barriers	See Section 4.5	
Walkways and elevated platforms (other than exit ways)	60 (2.87)	
Yards and terraces, pedestrian	100 (4.79)[a]	

[a] Live load reduction for this use is not permitted by Section 4.7 unless specific exceptions apply.

[b] Floors in garages or portions of a building used for the storage of motor vehicles shall be designed for the uniformly distributed live loads of ASCE 7-10, Table 4-1 or the following concentrated load: (1) for garages restricted to passenger vehicles accommodating not more than nine passengers, 3,000 lb (13.35 kN) acting on an area of 4.5 in, by 4.5 in. (114 mm by 114 mm); and (2) for mechanical parking structures without slab or deck that are used for storing passenger vehicles only, 2,250 lb (10 kN) per wheel.

[c] Design for trucks and buses shall be per AASHTO LRFD Bridge Design Specifications; however, provisions for fatigue and dynamic load allowance are not required to be applied.

[d] Uniform load shall be 40 psf (1.92 kN/m^2) where the design basis helicopter has a maximum take-off weight of 3,000 lbs (13.35 kN) or less. This load shall not be reduced.

[e] Labeling of helicopter capacity shall be as required by the authority having jurisdiction.

[f] Two single concentrated loads, 8 ft (2.44 m) apart shall be applied on the landing area (representing the helicopter's two main landing gear, whether skid type or wheeled type), each having a magnitude of 0.75 times the maximum take-off weight of the helicopter and located to produce the maximum load effect on the structural elements under consideration. The concentrated loads shall be applied over an area of 8 in. by 8 in. (200 mm by 200 mm) and shall not be concurrent with other uniform or concentrated live loads.

[g] A single concentrated load of 3,000 lbs (13.35 kN) shall be applied over an area 4.5 in. by 4.5 in. (114 mm by 114 mm), located so as to produce the maximum load effects on the structural elements under consideration. The concentrated load need not be assumed to act concurrently with other uniform or concentrated live loads.

[h] The loading applies to stack room floors that support nonmobile, double-faced library book stacks subject to the following limitations: (1) The nominal book stack unit height shall not exceed 90 in. (2,290 mm); (2) the nominal shelf depth shall not exceed 12 in. (305 mm) for each face; and (3) parallel rows of double-faced book stacks shall be separated by aisles not less than 36 in. (914 mm) wide.

[i] In addition to the vertical live loads, the design shall include horizontal swaying forces applied to each row of the seats as follows: 24 lb per linear ft of seat applied in a direction parallel to each row of seats and 10 lb per linear ft of seat applied in a direction perpendicular to each row of seats. The parallel and perpendicular horizontal swaying forces need not be applied simultaneously.

[j] Uninhabitable attic areas without storage are those where the maximum clear height between the joist and rafter is less than 42 in. (1,067 mm), or where there are not two or more adjacent trusses with web configurations capable of accommodating an assumed rectangle 42 in. (1,067 mm) in height by 24 in. (610 mm) in width, or greater, within the plane of the trusses. This live load need not be assumed to act concurrently with any other live load requirement.

TABLE 3.2.2-1 (*Continued*)

[k] Uninhabitable attic areas with storage are those where the maximum clear height between the joist and rafter is 42 in. (1,067 mm) or greater, or where there are two or more adjacent trusses with web configurations capable of accommodating an assumed rectangle 42 in. (1,067 mm) in height by 24 in. (610 mm) in width, or greater, within the plane of the trusses. At the trusses, the live load need only be applied to those portions of the bottom chords where both of the following conditions are met:

i. The attic area is accessible from an opening not less than 20 in. (508 mm) in width by 30 in. (762 mm) in length that is located where the clear height in the attic is a minimum of 30 in. (762 mm); and
ii. The slope of the truss bottom chord is no greater than 2 units vertical to 12 units horizontal (9.5% slope).

The remaining portions of the bottom chords shall be designed for a uniformly distributed nonconcurrent live load of not less than 10 lb/ft^2 (0.48 kN/m^2).

[l] Where uniform roof live loads are reduced to less than 20 lb/ft^2 (0.96 kN/m^2) in accordance with Section 4.8.1 and are applied to the design of structural members arranged so as to create continuity, the reduced roof live load shall be applied to adjacent spans or to alternate spans, whichever produces the greatest unfavorable load effect.

[m] Roofs used for other occupancies shall be designed for appropriate loads as approved by the authority having jurisdiction.

[n] Other uniform loads in accordance with an approved method, which contains provisions for truck loadings, shall also be considered where appropriate.

[o] The concentrated wheel load shall be applied on an area of 4.5 in. by 4.5 in. (114 mm by 114 mm).

[p] Minimum concentrated load on stair treads (on area of 2 in. by 2 in. [50 mm by 50 mm]) is to be applied nonconcurrent with the uniform load.

3.2.2.2 Roof Live Loads Roof live loads generally arise from construction and reroofing activities and various materials or equipment placed on or supported by the roof. Minimum roof live loads from the *International Building Code* [3] are calculated using Equation 3.2.2.2-1. These loads are assumed to arise from construction and repair activities and are generally assumed to have a cumulative duration of seven days. Roofs to be used for special purposes must be designed for appropriate anticipated loads.

$$L_r = L_0 R_1 R_2 \text{ psf, where } 12 \le L_r \le 20 \text{ psf} \quad (3.2.2.2\text{-}1)$$

where:

L_r = roof live load
L_0 = minimum unreduced uniformly distributed roof live load prescribed by the code
$R_1 = 1$ for $A_f \le 200$ ft^2
$R_1 = 1.2 - 0.001A_f$ for $200 < A_f < 600$ ft^2
$R_1 = 0.6$ for $A_f \ge 600$ ft^2
A_f = tributary area supported by any structural member (ft^2)

$R_2 = 1$ for $F \leq 4$
$R_2 = 1.2 - 0.05F$ for $4 \leq F \leq 12$
$R_2 = 0.6$ for $F \geq 12$

F = inches of rise per foot for a sloped roof or the rise-to-span ratio multiplied by 32 for an arch or dome.

3.2.3 Snow Loads

Snow loads vary widely from region to region. In mountainous areas, snow loads may vary widely in short distances. Factors affecting snow-load accumulation on structures include climatic variables, geographic location, roof exposure, roof slope, roof thermal condition, snow drifting, and sliding snow. Minimum snow loads are generally provided by the building official. In mountainous regions, snow loads may be site-specific, and may be determined as a service by the building official, or it may be required that the snow load be determined by the designer. Structural engineers associations of a number of states publish information helpful for calculating site-specific snow loads. Such information generally consists of ground snow load maps and methods to determine corresponding roof snow loads.

In the absence of specific snow-load information, the amount of snow on the ground at a site for an accepted mean recurrence interval (MRI) must be determined by accepted hydrological methods. The mean recurrence interval typically used is 50 years. Structures containing essential facilities (emergency related facilities) and those containing particularly hazardous materials may be designed to withstand snow (and other) loads associated with greater recurrence intervals.

Snow loads provided by the building official or determined from snow load maps are generally for shallow sloping roofs. Adjustments may be used (if approved by the building official) for reduced snow on steep roofs and on roofs with unobstructed slippery surfaces.

Consideration must also be made for snow accumulations. Examples of snow accumulations to be considered include the following:

1. Snow drifting and accumulating in valleys between parallel and nonparallel ridges
2. Snow drifting against or behind parapets or other roof projections
3. Snow accumulation and ice damming on eaves
4. Snow drifting onto lower roofs (or balconies or decks)
5. Snow sliding onto lower roofs (or other surfaces)
6. Snow drifting from one side of a predominant ridge and accumulating on the other side

The American Society of Civil Engineers [5] and various structural engineers associations provide guidance for the above adjustments. It may be beneficial for a professional charged with designing for snow loads to visit areas with structures loaded with snow to better understand snow accumulations.

Snow loads for mountainous areas, especially in the Western states, may be enormous. Mt. Baker, in western Washington, has received over 90 feet of snow in a single season. Actual snow packs of over 700 psf have been recorded in isolated regions, and some inhabited regions have snow loads ranging up to 300 psf. Roof snow loads are generally prescribed assuming heated structures. Unheated structures, structures with particularly effective insulation, or structures used for cold storage should be given special consideration and the snow loads increased accordingly.

For snow loads, a duration of two months is customarily assumed for design. Although snow may remain on a structure for periods exceeding two months, the amounts of snow for the longer periods of time are generally less than the design load. In cases where near-design level amounts of snow are expected for longer periods of time, a longer load duration should be considered.

3.2.4 Wind Loads

Wind forces on structures are generally calculated from surface pressures. Surface pressures may be positive or negative. Negative (suction) pressures on roofs may produce significant uplift forces in some cases equal to or greater than gravity loads. Both positive and negative pressures exert loads on structural members and act to overturn or slide a structure.

Pressures used for wind design are generally derived from basic wind speeds for a given geographical area. Adjustments are made for height above ground, the conditions of the surrounding terrain (exposure), and for the part of the structure under consideration. Local pressures for individual framing members or pieces of sheathing are higher than the pressures acting over larger surfaces of the structure. Surfaces near discontinuities (roof eaves, rakes and ridges, and wall corners) likewise experience higher pressures than surfaces distant from the discontinuities. Importance factors in wind design are associated with the mean recurrence interval (MRI) of the design wind speeds. An importance factor of 1.0 generally corresponds to a MRI of 50 years; higher importance factors correspond to greater mean recurrence intervals. Structures containing essential facilities (emergency-related) and those containing particularly hazardous materials may be designed to withstand wind loads associated with higher recurrence intervals (higher importance factors).

The design wind speed and exposure condition for a structure should be determined by the designer subject to approval by the local building official and should be indicated on the design and construction documents. Wind pressures and resulting forces on the structure should be calculated per the locally adopted code. In the absence of a locally adopted code, procedures in a model building code, or *ASCE 7–10 Minimum Design Loads for Buildings and Other Structures* [5], should be used. Special consideration should be made for increased wind speeds in valleys, gorges, and mountainous areas. In areas of extremely high wind (e.g., hurricane-prone regions), consideration must also be made for wind-borne debris. For wind loads, a 10-minute load duration is customarily assumed for design.

3.2.5 Earthquake Loads

The ground movements from earthquakes produce significant forces on structures. The Seismic Zone or Seismic Design Category for a proposed structure should be obtained from the building official, as well as the governing building code stipulating seismic design procedures and requirements. In areas of high seismic risk, site location with respect to known faults is important. In some cases, it is necessary to conduct a site-specific study of seismic ground motions. *ASCE 7–10 Minimum Design Loads for Buildings and Other Structures* [5] may be used in the absence of local code requirements, with careful attention to site specific hazards. For earthquake loads, a 10-minute load duration is customarily assumed for design.

With regard to earthquakes, building codes have been developed with the interest of public safety, not for the prevention of all damage to structures. As a minimum, buildings are designed to *not collapse* during the design earthquake event. The degree of damage deemed acceptable during the design earthquake is reflected in the building codes, with the Importance Category or Seismic Design Category of the structure as determined by the building official. The expected performance of a structure during a design earthquake should be clearly agreed upon between designer and owner. The design of a structure to sustain less damage than that allowed by the minimum requirements of the code will generally increase the cost of the structure as well as design costs.

3.2.6 Highway Loads

Timber bridges can be and have been designed to satisfy the requirements of modern vehicle transportation. Load and other design requirements for highway loads should be obtained from the *Standard Specifications for Highway Bridges* [6] or *AASHTO LRFD Bridge Design Specifications* [7], adopted by the American Association of State Highway and Transportation Officials (AASHTO). Due to the increased strength of wood to withstand impact loads, wheel loads used in design for timber bridges are generally treated as live loads with two-month load duration without special consideration for dynamic effects. Dynamic effects must be considered, however, in the metal-to-metal connections and in railing design. Chapters 17–19 provide guidance and examples for timber bridge design.

3.2.7 Railway Loads

Timber railway bridges and trestles have been used extensively, and with the advent of pressure preservative treatments, a long service life may be expected.

For the design of timber bridges and trestles, the recommendations in *Manual of Recommended Practice* [8] of the American Railway Engineering and Maintenance-of-way Association (AREMA) are ordinarily applied. Rail loads are treated as live loads with normal load duration without additional consideration for impact effects for wood members. Dynamic effects must be taken into consideration for the metal-to-metal connections.

3.2.8 Crane Loads

Timber might be chosen for crane beams and girders because of its ability to absorb impact forces. In designing crane beams, wheel loads recommended by crane manufacturers should be used. At a minimum, loads should be taken to be at least equal to the rated capacity of the crane plus the weight of the crane and trolley. In general, crane loads are considered live loads with normal load duration but are increased a certain percentage to account for impact effects. ASCE 7–10 [5] prescribes the minimum impact, lateral, and longitudinal force effects that must be considered in design. Beams or girders supporting cranes should also be designed to prevent undue vertical and lateral deflection in accordance with crane manufacturers' recommendations.

3.2.9 Dynamic Loads

By nature, wood performs well under dynamic loads as previously discussed with regard to bridge and highway loads. As discussed in more detail in Section 3.4.1, certain design stresses may be increased significantly for short duration loads. This is particularly true of stresses induced parallel to the grain, such as bending stresses. Loading or service conditions that produce tension perpendicular to the grain (notches, or fasteners located close to and loaded toward member edges) should be avoided, particularly in repeated load applications.

3.2.9.1 Cyclic Loads Research and experience indicate that wood performs well in situations that might cause fatigue failures in other structural materials. This is true particularly with stresses parallel to the grain, such as in bending. In applications where shear stresses are relatively high and more than one million cycles are expected, the design shear stresses should be adjusted following guidelines in ASCE Paper No. 2470, *Design Considerations for Fatigue in Timber Structures* [9] and/or USDA Forest Products Laboratory Report No. 2236, *Fatigue Resistance of Quarter-Scale Bridge Stringers in Flexure and Shear* [10]. Whereas special consideration for timber in cyclic loading conditions might not be necessary, accompanying mechanical fasteners and connections might require special consideration.

3.2.9.2 Vibration Loads The effects of vibration may generally be neglected in timber structures, except for cases in which the vibration forces are critical, or in which vibrations are objectionable to occupants. Vibrating equipment should be installed with isolation devices and manufacturer recommendations should be followed. In the absence of detailed analysis or information from the manufacturer, it is recommended that the dead load of vibrating equipment be doubled for design of the supporting members. Excess vibration, while not necessarily of structural concern with timber members, may tend to loosen threaded fasteners.

3.2.9.3 Impact Loads Design stresses may generally be increased 100% for impact loads, as shown in greater detail in Section 3.4.1. Capacities for mechanical fasteners and connections, however, are generally increased by a lesser amount, as indicated in the *National Design Specification® for Wood Construction* [2] and/or local building codes.

3.2.9.4 Blast Loads Data on the design of structures resistant to blast loading such as may be caused by weapon explosions are beyond the scope of this manual. Such data may be found in the American Society of Civil Engineers Manual No. 42, *Design of Structures to Resist Nuclear Weapons Effects* [11].

3.2.10 Load Combinations

Design must take into consideration the most severe distribution, concentration, and combination of loads and forces that can be realistically expected on a structure. Table 3.2.10-1 gives load combinations from *ASCE 7–10 Minimum Design Loads for Buildings and Other Structures* [5] for use in allowable stress design. Load combinations generally take into account the full value of any single design load acting individually, as well as fractions of the full loads acting simultaneously. The last two combinations listed require the consideration of reduced dead loads where they offset the effect of other loads.

3.2.11 Design Stresses

Structural members must be selected such that calculated stresses arising from all appropriate load combinations are less than or equal to the corresponding allowable stress values for the load combination and service conditions in question. The design process generally involves determining the critical stress conditions. Stresses due to loads are denoted by a lowercase letter f, with a subscript to indicate the type of stress. Typical stresses analyzed in timber design are listed in Table 3.2.11-1.

TABLE 3.2.10-1 Basic Load Combinations[a] for Allowable Stress Design [5]

D

$D + L$

$D + (L_r \text{ or } S \text{ or } R)$

$D + 0.75(L) + 0.75\ (L_r \text{ or } S \text{ or } R)$

$D + (W \text{ or } 0.7E)$

$D + 0.75(W \text{ or } 0.7E) + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$

$0.6D + W$

$0.6D + 0.7E$

where:

D = Dead load
L = Live load
L_r = Roof live load
S = Snow load
R = Rain load
W = Wind load, and
E = Earthquake load

[a] Fluid (F) and self-straining (T) loads listed in the Basic Combinations of *ASCE 7* [5] have been omitted here for convenience, because they not common in timber design.

TABLE 3.2.11-1 Typical Stresses Analyzed in Timber Design

Stress	Description
f_b	Extreme fiber bending stress
f_v	Shear stress parallel-to-grain
f_c	Compression stress parallel-to-grain
$f_{c\perp}$	Compression stress perpendicular-to-grain
f_t	Tension stress parallel-to-grain
f_{rt}	Radial tension stress in curved members

For any particular structural member, the location of one critical stress may be different from another critical stress, and the two may be examined independently. For example, the critical shear stress in a bending member is at the neutral axis near a support, whereas the location of critical bending stress is at the extreme fiber (top or bottom of the member) near the center of the span for a simply supported member. Other critical stresses may occur at the same location and their combined stress or stress interaction must be considered, for example bending and axial compression or tension.

Where curved members are subject to bending, radial stresses are developed. Bending loads that tend to straighten a curved member produce radial tension stresses, f_{rt}, and are of particular concern. Curved members must be designed so that radial stresses due to applied loads do not exceed the adjusted radial tension stresses. Chapters 4 and Appendix A provide design examples including all of the stresses listed above and appropriate combined stresses.

3.3 DESIGN VALUES

In ASD, structural member sizes and grades are selected such that the stresses due to design loading conditions are not greater than the allowable stresses for the same loading and service conditions (Equation 3.3.1-1). Design values corresponding to each of the stresses in Table 3.2.11-1 are published by rules-writing agencies for sawn lumber and by the American Institute of Timber Construction for structural glued laminated timber. Design values and adjustment factors are also published in the *National Design Specification® for Wood Construction* [2]. Adjustment factors take into consideration the end use of the structural member (wet use versus dry, load duration, etc.).

3.3.1 Strength Design Values

Reference design values for strength properties are generally denoted by the uppercase letter F with a subscript to indicate the type of stress. Adjusted design values are denoted by F'. Design values are discussed in greater detail in this section. Adjustment factors are discussed in detail in the following section. The general design requirement for allowable stress design is:

$$f_{(\ )} \leq F'_{(\ )} \tag{3.3.1-1}$$

where:

$f_{(\ )}$ = the applied or design stress (Table 3.2.11-1)
( ) denotes the type of stress (shear, flexure, compression, etc.)

$F'_{(\ )}$ = the allowable stress determined from the published design value, $F_{(\ )}$, modified by all appropriate adjustment factors

For glued laminated timber, lumber grades commonly vary throughout the beam section, so additional distinction in design values is made for orientation of the member with respect to load. Design values are published for loads causing bending about the x-x axis (i.e., F_{bx}, F_{vx}, etc.) and for loads causing bending about the y-y axis (i.e., F_{by}, F_{vy}, etc.).

Design values for strength properties are typically derived from a characteristic ultimate strength of the wood material for the type of stress specified, except compression perpendicular to the grain, which is based on an accepted deformation limit. Characteristic strength values for timber (with the exception of compression perpendicular-to-grain) are based on an estimated fifth percentile value. For allowable stress design, characteristic values are further reduced to ensure adequate safety. For load and resistance factor design, reference design values (ASD basis) are increased to the appropriate level for LRFD using a format conversion factor. Design values are not published for tension perpendicular-to-grain in timber design, because these stresses should be avoided.

3.3.2 Modulus of Elasticity Design Values

Design values for modulus of elasticity (E and E_{min}) are also provided for deflection and serviceability calculations (E) and for beam and column stability calculations (E_{min}). For optimized combinations of glued laminated timber (members intended to be stressed primarily in bending), E values are provided for each direction of bending (E_x, E_y, $E_{x\,min}$, and $E_{y\,min}$). For uniform-grade glued laminated timbers (members intended to be stressed primarily in axial tension or compression), single values of E and E_{min} are provided to be used for both bending directions. Likewise for dimension lumber and timbers, single pairs of E and E_{min} values are provided.

In general, published E values are intended for use in deflection calculations and include the effect of shear deflection, taken to be an additional 5% for glulam and 3% for lumber.

Published E_{min} values are for use in beam and column stability calculations. These values represent the lower 5% exclusion limit values for the combination (or species group and grade), adjusted to be free of shear effects, and divided by a factor of safety of 1.66. E_{min} values are calculated from the tabulated E values for bending using Equations 3.3.2-1, 3.3.2-2 and 3.3.2-3.

$$E_{min} = \frac{1.05\,E_{05}}{1.66} \quad \text{for glulam} \tag{3.3.2-1}$$

$$E_{min} = \frac{1.03\,E_{05}}{1.66} \quad \text{for lumber} \tag{3.3.2-2}$$

where:

$$E_{05} = E(1 - 1.645\,CoV_E) \tag{3.3.2-3}$$

where:

CoV_E = 0.10 for glulam, 0.25 for visually graded lumber

The 5% exclusion limit value (E_{05}), including the effect of shear, is used specifically to evaluate the effect of ponding, presented in Section 3.7.

3.4 ADJUSTMENT FACTORS

Design values are published for standard conditions and adjusted for end use and loading conditions by the designer. Adjustment factors may also be applied for member size and orientation. Many of the adjustment factors are common to both sawn lumber and glued laminated timber. Others are for use with only one type of wood product. Adjustment factors are published in this manual and the *National Design Specification*® [2]. Some adjustment factors may be prescribed or amended by local building codes; as such, the local building authority should be contacted and all applicable requirements obtained as a part of the design process. Most adjustment factors are cumulative. In general, the design stress is determined by published reference design values multiplied by all appropriate adjustment factors.

The factors covered in this section are applicable to ASD. All but the load duration factor are also applicable to LRFD. Additional factors used in LRFD are covered in Chapter 16.

3.4.1 Load Duration Factor, C_D

Reference design values for timber construction are published for *normal* duration of load, assumed to be ten years. For loads with normal load duration, no adjustment of the design value is necessary for load duration. For loads of shorter or longer duration, the design values for bending, shear parallel-to-grain, compression parallel-to-grain, and tension parallel-to-grain are adjusted in accordance with Table 3.4.1-1. Compression perpendicular-to-grain and modulus of elasticity design values are not adjusted for load duration.

The factors in Table 3.4.1-1 reflect the increased strength of wood for short duration of load. Load duration factors for durations not shown in the Table may be estimated by interpolation. While the load duration factor for dead loads is less than unity (thus decreasing the design value) it seldom governs design except in cases of heavy permanent loads (such as storage uses) or where the dead loads are in excess of 90% of the total load.

TABLE 3.4.1-1 Frequently Used Load Duration Factors[a,b] [2]

Load Duration	C_D	Typical Design Loads
Permanent	0.9	Dead loads
Ten years	1.0	Occupancy live loads
Two months	1.15	Snow loads
Seven days	1.25	Construction loads
Ten minutes	1.6	Wind and earthquake loads
One second	2.0	Impact loads

[a] Load duration factors are not applied to modulus of elasticity, E, nor to compression perpendicular to grain design values, $F_{c\perp}$, based on a deformation limit.

[b] The load duration factor for structural members pressure-treated with water-borne preservatives, or fire retardant chemicals is limited to a maximum of 1.6. The load duration factor for connections is also limited to a maximum of 1.6.

3.4.2 Wet Service Factor, C_M

Reference design values assume in-service moisture contents of 19% or less for dimension lumber and less than 16% for structural glued laminated timber. If wood members are subject to conditions that might cause sustained moisture content values higher than those listed, the wet service factor must be applied. Wet-service factors account for reduced strength of wood at higher moisture content. The wet-service factors account for the immediate effects of moisture on wood properties, but they do not account for degradation due to decay that may be caused by prolonged or repeated exposure to moisture. Where prolonged or repeated wet use is expected, naturally decay-resistant wood or preservative-treated wood may be required. Wet service factors are summarized in Table 3.4.2-1. Southern pine design values for decking and sawn timbers 5 in. × 5 in. and larger require no adjustment for wet service.

3.4.3 Factors Related to Size and Shape

Research has indicated that design values are affected by the size and shape of a member as reflected by its dimensions: thickness, width, and length for sawn

TABLE 3.4.2-1 Wet Service Factor, C_M

Strength Property	F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}	F_{rt}
GLUED LAMINATED TIMBER							
All species, MC ≥16%	0.80	0.80	0.875	0.53	0.73	0.833	0.875
SAWN LUMBER							
Visually graded dimension lumber, MC > 19%	0.85[a]	1.00	0.97	0.67	0.80[b]	0.90	
Visually graded timbers (5 in. × 5 in. and larger) MC > 19%, except southern pine and mixed southern pine[c]	1.00	1.00	1.00	0.67	0.91	1.00	
Visually graded decking MC > 19%, except southern pine[c]	0.85[b]			0.67		0.90	

[a] For visually graded dimension lumber of all species with $(F_b)(C_F) \leq 1150$ psi, $C_M = 1.0$.
[b] Where $(F_c)(C_F)$ of all visually graded species except southern pine ≤ 750 psi, $C_M = 1.0$; when (F_c) for visually graded southern pine ≤ 750 psi, $C_M = 1.0$.
[c] Design values for southern pine and mixed southern pine timbers and decking are used without further adjustment for moisture.

lumber; and width, depth, and length for glued laminated timber. Adjustments for the effects of size and shape are described in greater detail in the next section.

3.4.3.1 Beam Stability Factor, C_L The compression zone of a member subject to bending may become unstable and buckle if not adequately braced (lateral-torsional buckling). Bending members are considered to be adequately braced where the requirements of Table 3.4.3.1-1 are satisfied, and the beam stability factor may be taken to equal 1.0. For all other cases, the bending design

TABLE 3.4.3.1-1 Prescriptive Bracing Rules for Lateral Stability of Bending Members

Member	Rule
Sawn lumber	
Depth/breadth (d/b) (nominal dimensions)	
$d/b \leq 2$	No lateral bracing is required.
$2 < d/b \leq 4$	The ends shall be held in position, as by full-depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.
$4 < d/b \leq 5$	The compression edge shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.
$5 < d/b \leq 6$	Bridging, full-depth solid blocking, or diagonal cross bracing shall be installed at intervals not exceeding 8 feet, the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.
$6 < d/b \leq 7$	Both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.
Combined bending and Axial compression	The depth-to-breadth ratio may be as much as 5 if one edge is firmly held in line. If under all conditions of load the unbraced edge is in tension, the depth-to-breadth ratio may be as much as 6.
Glued Laminated Timber	
Depth/breadth (d/b)	
$d/b \leq 1$	No lateral support is required.
$d/b > 1$	Compression edge supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation.

value must be adjusted by the *beam stability factor*, C_L, to account for potential lateral-torsional buckling. The beam stability factor is not cumulative with the volume factor or flat use factor in glued laminated timbers, but is cumulative with the size factor for sawn lumber.

Five means of preventing lateral rotation of a beam at its end and/or bearing points are as follows:

1. Attachment of the bottom of the beam to a wall, column, or pilaster that prevents movement of the bottom of the beam in a direction perpendicular to the beam axis, and attachment of the top of the beam to the wall or parapet, or to the roof diaphragm, provided the roof is adequately attached to the wall
2. Full-depth blocking between the ends and bearing points of parallel beams
3. Cross bracing between parallel beams at ends and bearing points
4. Attachment of top and bottom of the ends of beams to girts or rim members
5. Suitable and approved connection hardware

In addition, there are five means of providing support against lateral movement or rotation of the beam at intermediate points or continuously along the beam:

1. Attachment of structural roof or floor sheathing or deck material directly to the beam
2. Suitable attachment of roof and floor framing to the beam (girders, joists, trusses)
3. Intermediate full-depth blocking
4. Bridging or bracing at intermediate points
5. Proper attachment of suitable ceiling materials

Where the conditions of Table 3.4.3.1-1 are not satisfied, the beam stability factor must be calculated by Equation 3.4.3.1-1.

$$C_L = \frac{1 + \left(F_{bE}/F_b^*\right)}{1.9} - \sqrt{\left[\frac{1 + \left(F_{bE}/F_b^*\right)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}} \qquad (3.4.3.1\text{-}1)$$

where:

F_{bE} = critical bucking design value for bending members
F_b^* = reference bending design value multiplied by all applicable adjustment factors except C_{fu}, C_L, C_V, and C_I

The critical buckling design value for bending members, F_{bE}, is calculated using Equation 3.4.3.1-2.

$$F_{bE} = \frac{1.20\, E'_{min}}{R_B^2} \tag{3.4.3.1-2}$$

where:

E'_{min} = adjusted modulus of elasticity for beam and column stability calculations
R_B = slenderness ratio of the bending member

Since lateral buckling of the beam is being considered, the modulus of elasticity, E_{min}, for weak axis bending should be used. Design values of E_{min} for lumber are generally the same for both directions of bending, but generally vary for glued laminated timber.

The slenderness ratio, R_B, of the bending member is determined by either of two methods: the *effective length method* or the *equivalent moment method*. Both methods are described in *Technical Report 14 Designing for Lateral-Torsional Stability in Wood Members* [12]. In either case, the slenderness ratio for bending is not permitted to exceed 50.

3.4.3.1.1 Effective Length Method for Determining R_B The effective length method has traditionally been included in the NDS [2] for beam design. Using this method, the slenderness ratio is calculated as a function of effective length with Equation 3.4.3.1.1-1.

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} \tag{3.4.3.1.1-1}$$

where:

ℓ_e = effective length in bending
d = member depth
b = width (breadth)

The effective buckling length, ℓ_e, is a function of the loading conditions, support conditions, the member cross section, and the unbraced length, ℓ_u. The effective length can be determined using Table 3.4.3.1.1-1 for common cases.

The effective-length method has the advantage that it is relatively simple to apply. However, it is not as robust as the equivalent moment method for cases that aren't tabulated.

TABLE 3.4.3.1.1-1 Effective Length, ℓ_e, for Bending Members

Cantilever[a]	where $\ell_u/d < 7$		where $\ell_u/d \geq 7$
Uniformly distributed load	$\ell_e = 1.33\,\ell_u$		$\ell_e = 0.90\,\ell_u + 3d$
Concentrated load at unsupported end	$\ell_e = 1.87\,\ell_u$		$\ell_e = 1.44\,\ell_u + 3d$
Single Span Beam[a,b]	where $\ell_u/d < 7$		where $\ell_u/d \geq 7$
Uniformly distributed load	$\ell_e = 2.06\,\ell_u$		$\ell_e = 1.63\,\ell_u + 3d$
Concentrated load at center with no intermediate lateral support	$\ell_e = 1.80\,\ell_u$		$\ell_e = 1.37\,\ell_u + 3d$
Concentrated load at center with lateral support at center		$\ell_e = 1.11\,\ell_u$	
Two equal concentrated loads at $\frac{1}{3}$ points with lateral support at $\frac{1}{3}$ points		$\ell_e = 1.68\,\ell_u$	
Three equal concentrated loads at $\frac{1}{4}$ points with lateral support at $\frac{1}{4}$ points		$\ell_e = 1.54\,\ell_u$	
Four equal concentrated loads at $\frac{1}{5}$ points with lateral support at $\frac{1}{5}$ points		$\ell_e = 1.68\,\ell_u$	
Five equal concentrated loads at $\frac{1}{6}$ points with lateral support at $\frac{1}{6}$ points		$\ell_e = 1.73\,\ell_u$	
Six equal concentrated loads at $\frac{1}{7}$ points with lateral support at $\frac{1}{7}$ points		$\ell_e = 1.78\,\ell_u$	
Seven or more equal concentrated loads, evenly spaced, with lateral support at points of load application		$\ell_e = 1.84\,\ell_u$	
Equal end moments		$\ell_e = 1.84\,\ell_u$	

[a] For single span or cantilever bending members with loading conditions not specified in Table 3.3.3:

$\ell_e = 2.06\,\ell_u$ where $\ell_u/d < 7$

$\ell_e = 1.63\,\ell_u + 3d$ where $7 \leq \ell_u/d \leq 14.3$

$\ell_e = 1.84\,\ell_u$ where $\ell_u/d > 14.3$

[b] Multiple span applications shall be based on table values or engineering analysis.

3.4.3.1.2 Equivalent Moment Method for Determining R_B The equivalent moment method has been used for the design of steel structures for years, but it has only recently been applied to timber structures. Using this method, the beam slenderness ratio is calculated with Equation 3.4.3.1.2-1.

$$R_B = \sqrt{\frac{1.84\,\ell_u d}{C_b C_e b^2}} \tag{3.4.3.1.2-1}$$

where C_b can conservatively be taken as 1.0 for all cases or can be calculated using Equation 3.4.3.1.2-2 for most cases except for cantilevers.

$$C_b = \frac{12.5M_{\max}}{3M_A + 4M_B + 3M_C + 2.5M_{\max}} \tag{3.4.3.1.2-2}$$

where:

$M_{\max}$ = absolute value of maximum moment in beam segment between points of bracing
M_A = absolute value of moment at $\ell/4$
M_B = absolute value of moment at $\ell/2$
M_C = absolute value of moment at $3\ell/4$
C_e = 1.0 when the beam is braced at the point of load application or when the load is applied to the tension face of the beam or through the neutral axis, otherwise:

$$C_e = \sqrt{\eta^2 + 1} - \eta \tag{3.4.3.1.2-3}$$

where:

$$\eta = \frac{1.3kd}{\ell_u} \tag{3.4.3.1.2-4}$$

The value of k is given in Table 3.4.3.1.2-1 for select cases and can be conservatively taken as 1.72 for all other cases.

For the case of a continuous beam with bracing at the supports and on the top edge only, it is unclear how to apply Equations 3.4.3.1.2-1 and 3.4.3.1.2-2 for calculating a beam stability factor for negative bending. The inflection point has commonly been considered as a brace point; however, Yura [13] indicates that the inflection point cannot be considered a point of bracing unless a brace is actually provided there. He provides Equation 3.4.3.1.2-5 for beam segments with a negative bending moment at either or both ends, rotational bracing at the ends, and lateral bracing along the top edge. The unbraced length is taken as the distance between the points of rotational bracing. (M_1 is taken as 0 in this term if M_1 is negative.)

$$C_b = 3.0 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) - \frac{8}{3}\frac{M_{CL}}{\left(M_0 + M_1\right)} \tag{3.4.3.1.2-5}$$

TABLE 3.4.3.1.2-1 C_b and k Factors for Selected Loading Conditions

Loading Condition	Laterally Braced @ point of loading	Laterally Unbraced @ point of loading	
	C_b	C_b	k
Single Span Beams			
Concentrated load @ center	1.67	1.35	1.72
Two equal conc. loads @ $\frac{1}{3}$ points	1.00	1.14	1.63
Three equal conc. loads @ $\frac{1}{4}$ points	1.11	1.14	1.45
Four equal conc. loads @ $\frac{1}{5}$ points	1.00	1.14	1.51
Five equal conc. loads @ $\frac{1}{6}$ points	1.05	1.14	1.45
Six equal conc. loads @ $\frac{1}{7}$ points	1.00	1.13	1.47
Seven equal conc. loads @ $\frac{1}{8}$ points	1.03	1.13	1.44
Eight or more equal conc. loads @ equal spacings	1.00	1.13	1.46
Uniformly distributed load	1.00[a]	1.13	1.44
Equal end moments (opposite rotation)	1.00	—	—
Equal end moments (same rotation)	2.30	—	—
Cantilever Beams			
Concentrated load @ end	1.67	1.28	1.00
Uniformly distributed load	1.00[a]	2.05	0.90

[a] The unbraced length, ℓ_u, in these cases is zero; therefore, the beam is considered fully-braced with $C_L = 1.0$.

Reprinted with permission from *Designing for Lateral-Torsional Stability in Wood Members* [12].

where:

M_0 = end moment resulting in the larger compression stress on the bottom face
M_1 = other end moment on the unbraced length
M_{CL} = moment at centerline of the unbraced length

Another approach to evaluate stability for negative bending in a continuous beam with top edge bracing is to calculate C_b using Equation 3.4.3.1.2-2, with all positive moments taken as zero. The length between points of rotational bracing is used as the unbraced length in this approach. This approach assumes that the positive moments in the segment do not influence the beam stability for negative bending, because the compression zones in the regions of positive moment are braced to prevent movement.

3.4.3.2 Size Factor, C_F (Sawn Lumber) Design values for lumber vary based on the dimensions of the piece. For most species groups, a single design value is published for a given property and adjusted for size using the size factor,

C_F. The size factor, C_F, is to be applied to the design values for bending, tension parallel-to-grain, and compression parallel-to-grain for various sizes within each grade.

Size factors for use with dimension lumber are tabulated with the design values in the *NDS*® [2]. Design values for southern pine and mixed southern pine dimension lumber are published individually for each size, so the size factor does not apply, except for particular grades with widths greater than 12 in. No size factor adjustment is applied for mechanically graded dimension lumber (MSR and MEL), because size effects are effectively considered by the grading and quality-control processes.

The size factor for 5 in. × 5 in. and larger sawn timbers of depth greater than 12 in. with loads applied to the narrow face is determined by Equation 3.4.3.2-1:

$$C_F = \left(\frac{12\,\text{in}}{d}\right)^{\frac{1}{9}} \tag{3.4.3.2-1}$$

where:

d = member depth

Size factors for 5 in. × 5 in. and larger timbers with loads applied to the wide face are tabulated with the design values in the *NDS*® [2].

Example uses of the size factors are included in design examples in the following chapters. Sawn lumber tends to be limited in width and length, and as such, adjustments for these dimensions are not necessary.

3.4.3.3 Volume Factor, C_V (Glulam) Design values for bending, F_{bx}, in horizontally laminated glued laminated timbers are based on standard beam dimensions of $5\frac{1}{8}$ in. wide × 12 in. deep × 21 ft long. Adjustment for other sizes is made using the volume factor, C_V, Equation 3.4.3.3-1,

$$C_V = \left(\frac{5.125\,\text{in}}{b}\right)^{\frac{1}{x}} \left(\frac{12\,\text{in}}{d}\right)^{\frac{1}{x}} \left(\frac{21\,\text{ft}}{L}\right)^{\frac{1}{x}} \leq 1.0 \tag{3.4.3.3-1}$$

where:

b = member width
d = member depth
L = length of member between points of zero moment
x = 20 for Southern Pine, and
10 for other species

EXAMPLE 3.4.3.3-1 VOLUME FACTOR

Given: $6\frac{3}{4}$ in. × $25\frac{1}{2}$ in. Douglas fir glulam beam spans 30 ft.

Wanted: Determine the volume factor.

Approach: Equation 3.4.3.3-1 will be used with $x = 10$

Solution:

$$C_V = \left(\frac{5.125\,\text{in}}{6.75\,\text{in}}\right)^{\frac{1}{10}} \left(\frac{12\,\text{in}}{25.5\,\text{in}}\right)^{\frac{1}{10}} \left(\frac{21\,\text{ft}}{30\,\text{ft}}\right)^{\frac{1}{10}} = 0.87$$

Answer: The volume factor for the given beam is 0.87.

3.4.3.4 Flat-Use Factor, C_{fu} (Dimension Lumber) Reference design values for bending, F_b, for dimension lumber are based on pieces loaded on edge (transverse loads parallel to the wide face). Where these members are placed *flatwise* and loaded perpendicular to the wide face, the design values for bending may be increased using the flat-use factor, C_{fu}. Flat-use factors are published with the design values [2]. Design values for visually graded decking generally take into consideration flat use with no further adjustment to be taken. The flat-use factor is not applied to visually graded timbers 5 in. × 5 in. and larger.

3.4.3.5 Flat-Use Factor, C_{fu} (Glulam) The flat-use factor, C_{fu}, is applied to the design value for bending for glued laminated timber loaded parallel to the wide faces of the laminations (F_{by}). The flat-use factor for glulam is calculated by Equation 3.4.3.5-1.

$$C_{fu} = \left(\frac{12\,\text{in}}{d}\right)^{\frac{1}{9}} \tag{3.4.3.5-1}$$

Note that the flat-use factor produces an increase of bending stress for depths (widths of the wide faces of the laminations) less than 12 in., and a reduced bending stress for lamination widths greater than 12 in. The term *flat use* is a bit of a misnomer, because the application of this factor depends on the orientation of the laminations with respect to the bending load, not on the relative dimensions of the member.

3.4.3.6 Curvature Factor, C_c (Glulam) The curvature factor, C_c, is applied to the design value in bending, F_b, for curved glued laminated members only. It takes into account the residual stress in laminations bent into curved shapes during the manufacturing process thereby lowering the allowable bending stress for the design loading. The curvature factor is calculated using Equation 3.4.3.6-1.

$$C_c = 1 - 2000\left(\frac{t}{R}\right)^2 \tag{3.4.3.6-1}$$

where:

t = thickness of laminations
R = radius of curvature of inside face of lamination

The curvature factor is significant for glulam timbers manufactured to relatively short curvature radii. For large-radius glued laminated timbers, such as cambered beams (typical radii greater than 1000 ft), the adjustment is not significant and is typically not considered. The reduction of bending strength for a glulam with $1\frac{1}{2}$ in. laminations manufactured to a curvature of radius of 56 ft is only 1%. The curvature factor is not applied to design values in the straight portion of a member regardless of curvature in other portions.

Due to the stresses induced into the laminations during the process of manufacturing curved members, minimum limits on the radii of curvature are recommended for curved structural glued laminated timbers. For curved members manufactured with nominal 2 in. thickness laminations, the minimum radius of curvature (at the inside face) is typically 18 ft for southern pine ($1\frac{3}{8}$ in. laminations) and 27 ft, 6 in. for other softwood species ($1\frac{1}{2}$ in. laminations). For Tudor arches and other tightly curved members manufactured with nominal 1 in. thickness ($\frac{3}{4}$ in. actual thickness) laminations, typical minimum radii of curvature (at the inside face) are 7.0 ft for southern pine and 9.33 ft for other softwood species. The manufacture of curved members with radii shorter than these typically requires standard thickness laminations to be planed to a thinner dimension resulting in more waste and less efficient use of materials. It is recommended that the designer contact the laminator prior to specifying radii shorter than those just described. Curved members also develop radial stresses, which must be considered separately as shown in Chapters 8 and 9.

3.4.3.7 Shear Reduction Factor,* C_{vr} *(Glulam) Reference shear design values for structural glued laminated timber of softwood species are based on large-scale tests of prismatic beams subject to quasi-static loading. They are significantly higher than shear design values traditionally assigned to structural glued laminated timber based on small-scale shear block tests. Based on industry judgment, the increased shear values should not be applied to cases that were not tested. Therefore, the tabulated values must be reduced by multiplication by a shear reduction factor of $C_{vr} = 0.72$ for nonprismatic members and for all members subject to impact loads or repetitive cyclic loads. Prismatic members are defined as straight or cambered members with constant cross-section. Nonprismatic members include, but are not limited to arches, tapered beams, curved beams, and notched members. The shear reduction factor must also be applied for the design of members at connections.

***3.4.3.8 Stress Interaction Factor,* C_I** When the top and bottom of a beam are not parallel to each other, consideration must be given to the interacting effects of bending, compression, tension, and shear parallel-to-grain, and also to compression or tension perpendicular-to-grain. An illustration of the distribution of the shear stresses existing in a tapered beam is shown in Figure 3.4.3.8-1. The specific case shown is that of a tapered beam subject to constant shear along its length, such as a cantilever with a point load or a simple beam with a point load.

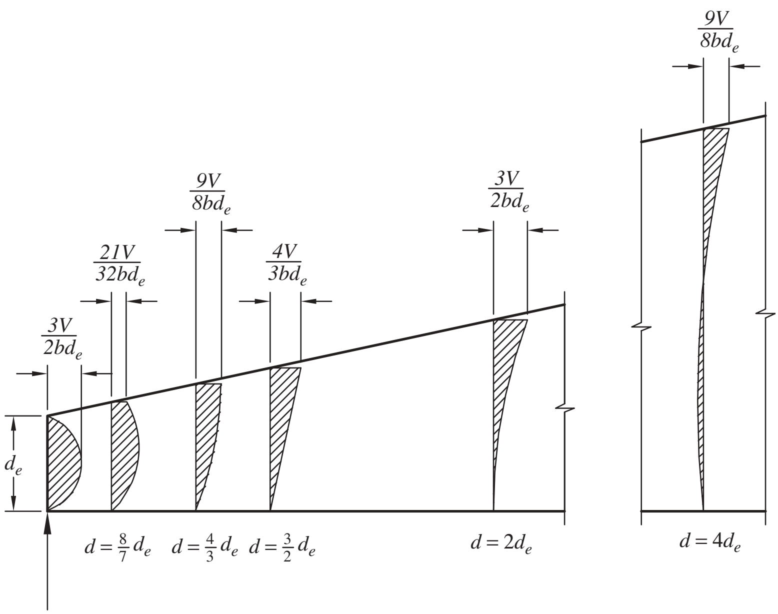


Figure 3.4.3.8-1 Shear stresses in a tapered beam [14].

In a straight prismatic beam, the locations of maximum shear and bending stress do not occur at the same point. However, with a tapered segment as indicated in Figure 3.4.3.8-1, the locations of maximum shear and bending stresses both occur along the tapered face for sections away from the ends. At such locations, the effects of stress interaction must be considered, effectively reducing the amount of load that can be carried by the section. To account for this effect the reference design value in bending is adjusted by the *stress interaction factor* (Equation 3.4.3.8-1).

$$C_I = \frac{1}{\sqrt{1 + \left(\frac{F_b \tan\theta}{F_v C_{vr}}\right)^2 + \left(\frac{F_b \tan^2\theta}{F_{c\perp}}\right)^2}} \qquad (3.4.3.8\text{-}1)$$

where:

θ = taper angle

For a taper on the compression side of a beam, C_I is cumulative with adjustments for lateral stability C_L, but is not cumulative with C_V. Tapering is not recommended on the tension faces of beams, because it causes tension stresses to develop perpendicular to grain. Where tapering on the tension face is unavoidable, the design value for radial tension, F_{rt}, is used in place of $F_{c\perp}$ in the above equation. Although taper on the tension side of beams is not recommended, C_I in such cases is cumulative with C_V, but not with C_L.

3.4.3.9 Column Stability Factor, C_p Short columns fail due to crushing of the wood fibers, while long, slender columns tend to fail by buckling. Columns in the intermediate range fail by a combination of crushing and buckling. The ratio of axial compression stress at failure to the compressive strength of a braced column is described by the column stability factor, C_P. This factor is applied to the design value for compression parallel-to-grain to account for the possibility of buckling reducing the capacity of a column.

Where a column is prevented from buckling in all directions, the column stability factor is 1.0. Otherwise, the column stability factor, C_P, is calculated using Equation 3.4.3.9-1.

$$C_P = \frac{1 + \left(F_{cE}/F_c^*\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_c^*\right)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}} \qquad (3.4.3.9\text{-}1)$$

where:

F_c^* = reference compression design value multiplied by all applicable adjustment factors except C_P
c = 0.8 for sawn lumber
c = 0.85 for round timber poles and piles
c = 0.90 for structural glued laminated timber
F_{cE} = critical column buckling design value (Equation 3.4.3.9-2)

The critical column buckling design value is calculated using Equation 3.4.3.9-2.

$$F_{cE} = \frac{0.822\, E'_{\min}}{\left(\ell_e/d\right)^2} \qquad (3.4.3.9\text{-}2)$$

where:

ℓ_e/d = column slenderness ratio for the direction of buckling being considered
$E'_{\min}$ = adjusted modulus of elasticity for the direction of buckling being considered

The slenderness ratio, ℓ_e/d, must not exceed 75 during construction and must not be greater than 50 in the completed structure.

3.4.3.9.1 Unbraced Column Length, ℓ_u The unbraced length of a compression member is the distance between two points along its length, between which the member is not prevented from buckling. The un-braced length of a column may vary with the buckling direction being considered, as illustrated in Figure 3.4.3.9.1-1. In practice, many columns are supported along their length in one direction (e.g., in the plane of a wall adjoining the column on both sides), and unsupported along their entire length in the other direction (out of the wall plane).

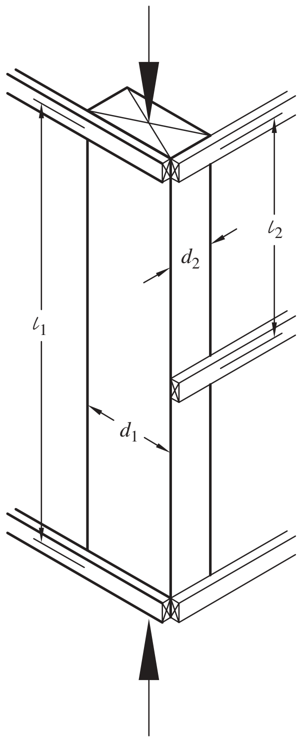


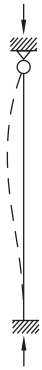
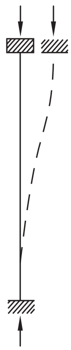
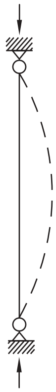
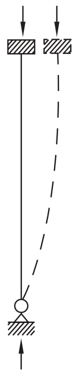
Figure 3.4.3.9.1-1 Simple column lengths and depths.

3.4.3.9.2 Effective Column Length, ℓ_e For a laterally unsupported simple column with pinned ends, the effective length ℓ_e is equal to the total length of the column. Effective lengths for columns with different degrees of fixity at the ends are determined by multiplying the unbraced length by an effective length factor, K_e (Table 3.4.3.9.2-1).

The effective buckling length factors are shown as theoretical values with accompanying recommended values to use for design. The recommended values of K_e for design take into account the lack of perfect fixity in use. Where compression members depend on the rigidity of other in-plane members entering the joint to provide fixity and the combined stiffness of these members is relatively small, K_e can exceed the values shown in Table 3.4.3.9.2-1. Columns with intermediate bracing points may have different theoretical K_e factors along their lengths.

In practice, most columns are square cut and bear on significant areas of connection hardware or other members, or may be attached with significant bolt groups, tending to produce some end fixity. While such bearing conditions may theoretically increase the capacity of the columns due to the increased fixity at

TABLE 3.4.3.9.2-1 Effective Column Length Factors

Buckling modes						
Theoretical K_e value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design K_e when ideal conditions approximated	0.65	0.80	1.2	1.0	2.10	2.4
End condition code		Rotation fixed, translation fixed Rotation free, translation fixed Rotation fixed, translation free Rotation free, translation free				

Reprinted with permission from *National Design Specification® for Wood Construction* [2]. Copyright © 2012. Courtesy American Wood Council, Leesburg, Virginia.

the ends (and corresponding smaller effective lengths), the increased fixity is generally not considered in design calculations; these types of connections are treated as pinned ends for design calculations.

3.4.4 Repetitive Member Factor, C_r

Where three or more parallel members of dimension lumber are spaced 24 in. on center (o.c.) or less, the repetitive member factor $C_r = 1.15$ is permitted to be applied to the design value for bending. The members must be joined to one another mechanically (e.g., built-up beams) or must be joined by floor, roof, or other load-distributing elements capable of supporting the design load. The repetitive member factor takes into consideration the variability of lumber and that stiffer members within a particular grade also tend to be stronger. Where several such members are configured in parallel (as described above), the stiffer member(s) take a greater portion of the applied load. The repetitive member factor is not used with decking because the factor has already been included in establishing the design values for decking. Also, it does not apply to structural glued laminated timber.

3.4.5 Temperature Factor, C_t

Most wood is used under conditions where temperature effects are not significant. However when an elevated temperature of a member is anticipated for an extended period of time or when elevated temperatures are expected to occur simultaneously with maximum design loads, design values must be adjusted by the temperature factors of Table 3.4.5-1.

TABLE 3.4.5-1 Temperature Factor, C_t [2]

Design Values	In Service Moisture Conditions[a]	C_t		
		$T \leq 100°F$	$100°F < T \leq 125°F$	$125°F < T \leq 150°F$
F_t, E	Wet or Dry	1.0	0.90	0.90
F_b, F_v, F_c, and $F_{c\perp}$	Dry	1.0	0.80	0.70
	Wet	1.0	0.70	0.50

[a] Wet service condition for sawn lumber is defined as in service moisture content greater than 19%. Wet service condition for glued laminated timber is defined as in service moisture content of 16% or greater.

3.4.6 Bearing Area Factor, C_b

For bearing lengths less than 6 in. and at least 3 in. from the end of a member, the design values for compression perpendicular to grain, $F_{c\perp}$, may be multiplied by the bearing area factor, C_b, which is obtained from Equation (3.4.6-1).

$$C_b = \frac{\ell_b + 0.375\text{ in}}{\ell_b} \qquad (3.4.6\text{-}1)$$

where:

ℓ_b = bearing length (in.)

The bearing area factor effectively increases the compression perpendicular-to-grain value for bearing conditions away from the ends of a member such as may arise from bearing plates and washers. For round bearing areas, the bearing length is taken to be the diameter of the bearing area.

EXAMPLE 3.4.6-1 BEARING AREA FACTOR

Given: 3 in. × 3 in. timber washer bears against wood perpendicular-to-grain in a connection.

Wanted: Determine the appropriate bearing area factor. Assume that the washer is not near the end of the member.

Solution: Since the bearing condition is away from the end of the member, Equation 3.4.6-1 may be used.

$$C_b = \frac{\ell_b + 0.375\,\text{in}}{\ell_b} = \frac{3\,\text{in} + 0.375\,\text{in}}{3\,\text{in}} = 1.13$$

Answer: The design value for compression perpendicular to grain may be increased by 13%, along with all other appropriate adjustments. The area to be used to determine the bearing stress must be the actual bearing area (net area of the washer).

3.4.7 Adjustments for Treatments

The design values published for sawn dimension lumber and glued laminated timber are applicable to pressure preservatively treated members when treated by an approved process as recommended by the *American Wood Protection Association* [15]. For dimension lumber incised to facilitate preservative treatment, the incising factor, C_i, is applied.

Design Value	C_i
E	0.95
F_b, F_t, F_c and F_v	0.80
$F_{c\perp}$	1.00

For timber treated with waterborne preservative salts or fire retardant chemicals, the load duration factor for impact does not apply. It is further recommended that the effects of fire retardant chemical treatments on strength be considered and that information on these effects be obtained from the company providing the treating and redrying service.

3.4.8 Other Adjustment Factors

The preceding discussion covers adjustment factors usually applicable to timber design. For special cases, such as poles and piles, other adjustment factors may be necessary.

3.4.9 Summary and Application of Adjustment Factors

Adjustment factors for timber design and their application are summarized in Table 3.4.9-1. Where adjustment factors are not applied cumulatively (for example C_L and C_V for glulam), the smaller factor applies.

3.5 DEFLECTION

Serviceability concerns with timber design generally involve the deflections of the framing members. Deflections should be limited to prevent a deflected part from interfering with another part of a structure, to reduce vibration, and to prevent the appearance or feel of an unsafe or sagging structure. Both vertical deflection and horizontal deflection may need to be considered by the designer. Deflection limitations for any particular project must be in compliance with the governing building code. More restrictive deflection limitations may be imposed by the owner or architect. Deflection considerations are also important for the evaluation of ponding as discussed in Section 3.7.

Deflections are generally calculated for spanning members such as joists, beams, and trusses and may also be calculated for columns, frames, shear walls, and diaphragms. Beam deflections are discussed in this section with examples in subsequent chapters. Deflections of trusses and frames are not covered in this manual but can be determined by virtual work or finite element methods. Deflections of shear walls and diaphragms are covered in ANSI/AF&PA SDPWS-2005 *Special Design Provisions for Wind and Seismic* [16].

Wood members experience immediate elastic deformation under load, as well as long-term plastic deformation (or creep) under sustained loads. The ratio of the long-term deflection to the initial deflection of a sustained load is approximated as 1.5 for seasoned wood and 2.0 for green wood members. Where actual deflections are calculated, such as for camber and ponding, both immediate and long-term effects must be considered. Deflection limitation criteria, however, as shown in the following tables and as stipulated in typical building codes, might not require consideration of plastic deflection.

Deflection limitations are generally specified as some fraction of the member span. Two calculation checks are generally performed for beams and other spanning members. The first check is the deflection due to the applied live, wind, or snow load only; the second check is for the deflection due to applied load plus all or some fraction of dead load. Deflection criteria are generally more restrictive for the live, wind, or snow loads only, although either case may govern, depending on the relative magnitudes of the loads. Table 3.5-1 provides recommended deflection limitations for wood members. Where framing members support particularly brittle materials, additional stiffness may be required. In such cases, the designer should obtain deflection limitation criteria from the material manufacturer or other professionals familiar with its use. Where vibrations from equipment are of concern, recommended stiffness requirements should be obtained from the equipment manufacturer.

TABLE 3.4.9-1 Applicability of Adjustment Factors

Member Type and Loading Condition	Design Values and Adjustment Factors
Visually Graded Dimension Lumber	
Bending	$F_b' = F_b C_D C_M C_F C_{fu} C_L C_r C_t C_i$
Tension parallel to grain	$F_t' = F_t C_D C_M C_F C_t C_i$
Compression parallel to grain	$F_c' = F_c C_D C_M C_F C_P C_t C_i$
Compression perpendicular to grain	$F_{c\perp}' = F_{c\perp} C_M C_t C_b$
Shear parallel to grain	$F_v' = F_v C_D C_M C_t C_i$
Modulus of Elasticity for deflection	$E' = E C_M C_t$
Modulus of Elasticity for buckling	$E_{\min}' = E_{\min} C_M C_t$
Mechanically Graded Dimension Lumber	
Bending	$F_b' = F_b C_D C_M C_L C_r C_t C_i$
Tension parallel to grain	$F_t' = F_t C_D C_M C_t C_i$
Compression parallel to grain	$F_c' = F_c C_D C_M C_P C_t C_i$
Compression perpendicular to grain	$F_{c\perp}' = F_{c\perp} C_M C_t C_b$
Shear parallel to grain	$F_v' = F_v C_D C_M C_t C_i$
Modulus of elasticity for deflection	$E' = E C_M C_t$
Modulus of elasticity for buckling	$E_{\min}' = E_{\min} C_M C_t$
Sawn Timbers 5 in. $\times$ 5. In. and Larger	
Bending	$F_b' = F_b C_D C_M C_F C_L C_t$
Tension parallel to grain	$F_t' = F_t C_D C_M C_t C_i$
Compression parallel to grain	$F_c' = F_c C_D C_M C_P C_t$
Compression perpendicular to grain	$F_{c\perp}' = F_{c\perp} C_M C_t C_b$
Shear parallel to grain	$F_v' = F_v C_D C_M C_t$
Modulus of elasticity for deflection	$E' = E C_M C_t$
Modulus of elasticity for buckling	$E_{\min}' = E_{\min} C_M C_t$
Structural Glued Laminated Timber (Glulam)	
Bending	$F_{bx}' = F_b C_D C_M \left(C_V \text{ or } C_L\right) C_c C_I C_t$
Bending	$F_{by}' = F_b C_D C_M C_L C_{fu} C_t$
Tension parallel to grain	$F_t' = F_t C_D C_M C_t$
Compression parallel to grain	$F_c' = F_c C_D C_M C_P C_t$
Compression perpendicular to grain	$F_{c\perp}' = F_{c\perp} C_M C_t C_b$
Shear parallel to grain	$F_v' = F_v C_D C_M C_t C_{vr}$
Modulus of elasticity for deflection	$E' = E C_M C_t$
Modulus of elasticity for buckling	$E_{\min}' = E_{\min} C_M C_t$
Radial tension	$F_{rt}' = F_{rt} C_D C_M C_t$
Decking	
Bending	$F_b' = F_b C_D C_M C_t$
Compression perpendicular to grain	$F_{c\perp}' = F_{c\perp} C_M C_t C_b$
Shear parallel to grain	$F_v' = F_v C_D C_M C_t C_i$
Modulus of elasticity	$E' = E C_M C_t$

TABLE 3.5-1 Recommended Deflection Criteria for Wood Members

CONSTRUCTION[a,d]	LOADS CAUSING DEFLECTION		
	L	S or W	(S or W or L) + $K(D)$[b]
Roof members			
Supporting plaster ceiling	$\ell/360$	$\ell/360$	$\ell/240$
Supporting nonplaster ceiling	$\ell/240$	$\ell/240$	$\ell/180$
Not supporting ceiling	$\ell/180$	$\ell/180$	$\ell/120$
Floor members[c]	$\ell/360$	—	$\ell/240$
Exterior walls and interior partitions			
With brittle finishes	—	$\ell/240$	—
With flexible finishes	—	$\ell/120$	—
Farm buildings	—	—	$\ell/180$
Greenhouses	—	—	$\ell/120$
Vehicular and pedestrian bridges	$\ell/425$	—	—

[a] For cantilever members, ℓ is taken as twice the length of the cantilever.
[b] $K = 1$ except for wood structural members having a moisture content of less than 16 percent at the time of installation and used under dry conditions, where $K = 0.5$ may be used.
[c] The recommended deflection limits for floors are intended for construction in which walking comfort and minimized plaster cracking are the main considerations.
[d] Sources: 2009 International Building Code [3] and AASHTO LRFD Bridge Design Specifications [7].

In cases where the total of the long-term deflection due to dead loads plus the effects of live or other loads acting simultaneously must be considered, the effect of creep (plastic deformation) must be included. Cambering of beams is often used to offset long-term dead load deflection and thus also mitigates the effect of total (dead plus other loads) deflection.

EXAMPLE 3.5-1 DEFLECTION CHECK FOR A FLOOR BEAM

Given: The deflections of a $5\frac{1}{8}$ in. × 12 in. × 20 ft floor beam are calculated to be 0.62 in. under live load and 0.93 in. under live load plus dead load.

Wanted: Determine whether the deflection criteria of Table 3.5-1 are met for ordinary usage.

Approach: The appropriate limits from Table 3.5-1 will be used with the total beam length to calculate allowable deflection for each case. The given deflections will then be compared with the allowable deflections to evaluate suitability with regard to the recommended criteria.

Solution:

$$\delta_L = \frac{\ell}{360} = \frac{20\ \text{ft}\,(12\,\text{in/ft})}{360} = 0.67\,\text{in} > \Delta_L = 0.62\,\text{in} \quad \therefore \text{OK}$$

$$\delta_{D+L} = \frac{\ell}{240} = \frac{20\ \text{ft}\,(12\,\text{in/ft})}{240} = 1.00\,\text{in} > \Delta_{D+L} = 0.93\,\text{in} \quad \therefore \text{OK}$$

Answer: The stated deflections are within the limits specified in Table 3.5-1.

3.6 CAMBER

Camber is built into structural glued laminated timber members by introducing a curvature, either circular or parabolic, opposite to the anticipated long-term deflection. Camber recommendations are provided in Table 3.6-1. Recommendations for camber depend on whether the member is of simple, continuous, or cantilever span. The intent of the camber is typically to provide a straight or nearly straight structural member under the long-term effects of dead load, including plastic deformation or creep. In addition, on long spans, level roof beams may not be desirable because of the optical illusion that the ceiling sags.

Camber may also be specified to ensure adequate roof drainage. To avoid the ponding of water, roof beams should generally have a positive slope or camber equivalent to $\frac{1}{4}$ in. per foot of horizontal distance between the level of the drain and the high point of the roof in addition to the minimum camber to offset deflections due to anticipated loads.

For simple beams, a single radius member is typically selected or specified. For continuous members, either a straight member or custom camber is used. Reverse cambers may be used for cantilever spans and continuous spans under

TABLE 3.6-1 Recommended Camber for Glued Laminated Timber Beams

Beam Usage	Recommended Camber
Roof beams	$1\frac{1}{2}$ times the deflection due to dead load
Floor beams[a]	0 to 1 times the deflection due to dead load plus the sustained portion of live load (such as from storage)
Bridge beams	
Glulam girders	2 times the deflection due to dead load
Stress-laminated bridges	3 times the deflection due to dead load

[a]Floor beams are often specified with little or no camber to prevent interference with other framing components during construction.

some loading conditions but should be specified cautiously as they may also create drainage or ponding problems.

Camber is usually designated in custom members by specifying the amount of camber required in inches and the location along the member. For a simple span beam, the camber is usually specified for the midpoint. The location and camber amount for cantilevered or continuous beams depends on the shape of the deflected structure.

Camber for stock glulam beams is usually designated by the manufacturer as a radius of curvature. Typically, long billets are made with constant radius and cut and sold in shorter pieces. Radius of curvature values of 1600 ft and 2000 ft have been commonly used in the past, with radii of 3500–5000 ft generally preferred in the current construction market to prevent excessive camber interfering with other framing components during construction.

Manufacturing tolerances on camber from ANSI/AITC A190.1 (ref) are shown in Table 3.6-2.

TABLE 3.6-2 Tolerance for Camber

Length of Beam	Tolerance
$L \leq 20$ ft	$\pm\frac{1}{4}$ in.
20 ft $< L \leq$ 40 ft	$\pm\frac{3}{8}$ in.
40 ft $< L \leq$ 60 ft	$\pm\frac{1}{2}$ in.
60 ft $< L \leq$ 80 ft	$\pm\frac{5}{8}$ in.
$L >$ 80 ft	$\pm\frac{3}{4}$ in.

Constant radius shapes have been found to provide suitable camber for large radius (shallow) calculated deflections. Equation 3.6-1 may be used to calculate the camber associated with a constant radius of curvature and beam length.

$$c = R - \frac{\sqrt{4R^2 - L^2}}{2} \tag{3.6-1}$$

where:

c = camber at midspan
R = radius of curvature
L = span

Alternately, Equation 3.6-1 may be cast in the following form to calculate the required radius of curvature for a given camber (Equation 3.6-2).

$$R = \frac{L^2}{8c} + \frac{c}{2} \tag{3.6-2}$$

For cambers typically encountered in industry, the above equation may be approximated by the following (Equation 3.6-3).

$$R = \frac{L^2}{8c} \tag{3.6-3}$$

EXAMPLE 3.6-1 GLULAM BEAM CAMBER

Given: The dead load deflection for a 32 ft, simple span, glued laminated timber is calculated to be 0.49 in. without consideration for creep.

Wanted: Determine the minimum recommended camber and associated radius of curvature for the beam.

Approach: Table 3.6-1 will be used to determine the minimum camber based on deflection and usage (floor beam); Equation 3.6-3 will be used to calculate the associated radius of curvature.

Solution:

$$c = 1.5\Delta = 1.5\,(0.49\,\text{in}) = 0.735\,\text{in} = 0.0613\,\text{ft}$$

$$R = \frac{L^2}{8c} = \frac{(32\ \text{ft})^2}{8\,(0.0613\,\text{ft})} = 2090\,\text{ft}$$

Answer: The minimum recommended camber is 0.74 in. and the associated radius of curvature is 2090 ft.

Discussion: A smaller radius of curvature will produce more camber or a slight crown after the long-term effect of dead load. A more common radius of $R =$ 2000 ft will produce a camber of 0.77 in. Camber should be specified as $\frac{3}{4}$ in. or using a radius of 2000 ft. For the 32 ft beam, the manufacturing tolerance for camber is $\pm\frac{3}{8}$ in. from Table 3.6-2.

EXAMPLE 3.6-2 EVALUATION OF A GIVEN CAMBER

Given: A cambered glued laminated timber with radius of 3500 ft has been recommended for a 24 ft simple-span roof beam. The dead load deflection, including creep, is calculated to be 0.21 in. using design software.

Wanted: Determine whether the 3500 ft radius is suitable for the simple roof beam application.

Approach: Since the calculated deflection already includes the effect of creep, the dead load deflection will not be further modified by the factors of Table 3.6-1, and the use of 0.21 in. will be used directly with Equation (3.6-3) to determine the radius of curvature. The calculated radius of curvature will then be compared with the stated curvature.

Solution:

$$c = 0.21\,\text{in} = 0.0175\,\text{ft}$$

$$R = \frac{(24\,\text{ft})^2}{8\,(0.0175\,\text{ft})} = 4110\,\text{ft}$$

The stated curvature is 3500 ft, which will produce approximately $\frac{1}{4}$ in. of camber, resulting in a slight crown under long-term load.

Answer: The recommended radius of curvature of 3500 ft should be suitable for the stated application.

3.7 PONDING

Ponding of water on roofs or other flat elevated surfaces can cause catastrophic structural collapse if not properly prevented in design. Collapses from ponding are generally the result of insufficient roof stiffness and inadequate roof drainage where the deflection due to water accumulation creates a progressively deeper pond or pool until the supporting structural elements fail. Ponding problems have been observed to be more prevalent on roofs with relatively light design loads, as these systems tend to be more flexible and the weight of ponded water is higher relative to the design loads.

The best design against ponding is to provide adequate roof slope and drainage. A minimum roof surface slope of $\frac{1}{4}$ in. per foot is recommended after consideration of deflection due to design loads. Drains should be properly selected or designed to discharge the design rainfall without creating undue hydraulic head on the roof surface. Consideration must also be made for gravel stops, scuppers, and other appurtenances that may cause water to pond on the roof surface. Consideration should also be made for uneven roof surfacing materials. Drainage systems should be selected to be as free as possible from debris or ice accumulation and should be regularly maintained.

For flat roofs or roofs with less than the recommended minimum slope, the supporting members must be investigated to ensure sufficient stiffness and strength with regard to ponding. ASCE 7 [5] requires that ponding be investigated using the larger of the snow load or rain load, and in the case of the rain load, the primary drainage system for the area of investigation must be considered blocked.

Roof systems in which the secondary framing members, sheathing or decking, and other framing are relatively stiff compared to the primary framing members are regarded as one-way systems for the purposes of investigating ponding. Design procedures for one-way systems are discussed as follows and use a magnification factor approach [17]. For systems where the secondary and other members may themselves experience significant deflection or ponding, the combined effects of the deflections of primary and the other members must be investigated.

The magnification factor approach determines the amount by which any initial roof deflection is magnified by ponding due to the deflected shape. For a very flexible roof member, even a relatively light load (small deflection) might progress to collapse as more and more water ponds over the deflected shape. Less flexible members might theoretically reach a more deflected equilibrium shape, but with increased internal stresses. Failure occurs where such stresses exceed the strengths of the materials. The magnification factor approach is used to determine the increased stresses in the new equilibrium condition. Thus, in roof member design, the magnification factor is calculated and applied to the stresses from the design loads.

The magnification factor for a single-span simply supported beam is given by Equation 3.7-1.

$$MF = \frac{1}{\left(1 - \dfrac{\lambda \gamma s L^4}{\pi^4 E'_{05} I}\right)} \tag{3.7-1}$$

where:

MF = magnification factor to account for ponding

λ = ratio of long-term dead load deflection to immediate dead load deflection (1.5 for glulam and seasoned lumber and 2.0 for unseasoned lumber)

γ = specific weight of the ponding liquid (typically water)

s = tributary width for the member under consideration (or spacing between adjacent similar members)

L = span of member under consideration

E'_{05} = adjusted 5% exclusion limit modulus of elasticity of the member

The E'_{05} value is adjusted by the same factors as for E or E_{min} as discussed in Section 3.3.

Members with insufficient stiffness to prevent ponding instability (collapse) at any initial load level will be manifest algebraically by a negative magnification factor using Equation 3.7-1. A method for determining the critical member stiffness (or spacing) with regard to such collapse may be found in the *Wood Handbook* [18]. An example calculation of the magnification factor follows.

EXAMPLE 3.7-1 MAGNIFICATION FACTOR

Given: A $5\frac{1}{8}$ in. × $13\frac{1}{2}$ in. glulam beam with modulus of elasticity E = 1,800,000 psi is being considered for use in a flat roof framing system where it will span 21 ft. The tributary width for the beam is 8 ft. The beam will be cambered to offset elastic dead load deflection plus creep.

Wanted: Calculate the magnification factor to account for increased stresses due to ponding.

Approach: A value for E_{05} will be calculated using a CoV_E of 10% per Section 3.3.2 that accounts for possible low modulus of elasticity. The roof member is assumed to be in dry service and subject to normal temperatures; thus the adjustments for E_{05} for moisture and temperature will be unity. Equation 3.7-1 will be used to calculate the magnification factor, which will be applied to the stresses caused by the design loads.

Solution:

$$E_{05} = (1{,}800{,}000 \text{ psi})\,[1 - 1.645\,(0.10)] = 1{,}500{,}000 \text{ psi}$$

$$E'_{05} = E_{05}\left(C_M\right)\left(C_t\right) = 1{,}500{,}000 \text{ psi}\,(1)\,(1) = 1{,}500{,}000 \text{ psi}$$

$$\gamma = 62.4\ \frac{\text{lb}}{\text{ft}^3} = 0.0361\ \frac{\text{lb}}{\text{in}^3}$$

$$s = 8 \text{ ft}\,(12 \text{ in/ft}) = 96 \text{ in}$$

$$L = (21 \text{ ft})\,(12 \text{ in/ft}) = 252 \text{ in}$$

$$I = \frac{(5.125 \text{ in})\,(13.5 \text{ in})^3}{12} = 1051 \text{ in}^4$$

$$MF = \frac{1}{\left(1 - \dfrac{1.5\left(0.0361 \text{ lb}/\text{in}^3\right)(96 \text{ in})\,(252 \text{ in})^4}{\pi^4\,(1{,}500{,}000 \text{ psi})\left(1051 \text{ in}^4\right)}\right)} = 1.16$$

Answer: The magnification factor due to ponding for the above beam is 1.16 and will be applied to the stresses in the beam under design (rain plus dead or snow plus dead) loads.

3.8 CONCLUSION

Structural design uses engineering techniques to determine the proper size, strength, and configuration of components in a structure to safely support all design loads. Fundamental considerations for design of timber structures using the allowable stress design (ASD) methodology are discussed in this chapter, including loads, load combinations, and allowable design values. Other design topics discussed in this chapter include deflection, camber, and ponding.

Proper design must consider all loads or forces reasonably expected to occur on a structure. These loads include the weight of the structure (dead loads) and loads applied during expected use of the structure (live loads).

Allowable strength and stiffness values for timber materials are determined by multiplying reference design values by adjustment factors to account for end use, loading conditions, and certain geometric considerations. Reference design values and adjustment factors for common timber materials are published in the *National Design Specification for Wood Construction*® [2].

Deflection, camber, and ponding are important design considerations for timber structures. Deflection must typically be limited in the design of structures to minimize serviceability problems, maintain a good appearance, and provide for occupancy comfort. In some cases, cambered glulam beams can be used to offset downward deflections. Camber can be used to improve appearance or provide positive slope for drainage. In cases where drainage is not adequate, progressive collapse due to ponding must be considered.

CHAPTER 4

TIMBER BEAMS

4.1 INTRODUCTION

Beams are structural components that resist loads by bending. They are typically loaded perpendicular to their longitudinal axis and span across a space between supports. Beams are also commonly called *girders, purlins*, *girts*, *joists, planks,* and *decking*. Large members are commonly called beams and girders, while smaller members are referred to as purlins, girts, and joists, depending on their specific use. Planks and decking are installed flat-wise, providing a roof, wall, or floor surface in addition to supporting loads.

Beams carry loads more efficiently when the direction of the load is parallel to their greater cross-section dimension (Figure 4.1-1, $d > b$). However, as a beam's depth increases relative to its width, the section becomes less stable and requires lateral bracing to prevent buckling or the design capacity must be reduced (through application of the beam stability factor) to account for potential buckling.

This chapter covers the design and analysis of *prismatic* timber beams (straight members without notches or taper). Subsequent chapters will discuss additional considerations for notched, tapered, and curved beams.

4.2 STRUCTURAL EVALUATION OF BEAMS

Timber beams must typically be evaluated for bending stress, shear stress, and deflection. As a rule of thumb, shear parallel-to-grain stress tends to govern the selection of short members, bending stress governs the selection of medium-span members, and deflection considerations govern the selection of long members.

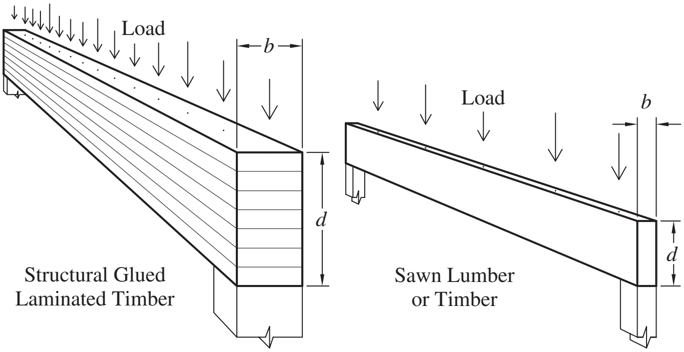


Figure 4.1-1 Typical beams

Bearing stress and other connection conditions must also be checked, although they seldom govern the selection of the beam, itself.

4.2.1 Bending (Flexure)

Bending loads produce flexural tension on one face of the member and flexural compression on the opposite face. Bending stresses are checked for the extreme fiber at the section with greatest bending moment. The applied bending stress is calculated using Equation 4.2.1-1.

$$f_b = \frac{Mc}{I} = \frac{M}{S} \tag{4.2.1-1}$$

where:

f_b = extreme fiber bending stress due to applied loads
M = bending moment due to applied loads
I = area moment of inertia with respect to the neutral axis
c = distance from the neutral axis to the extreme fiber
S = section modulus

For a rectangular section, Equation 4.2.1-1 can be expressed as Equation 4.2.1-2.

$$f_b = \frac{6M}{bd^2} \tag{4.2.1-2}$$

where:

b = beam width
d = beam depth

The design criterion (as with other stresses), is given by Equation 4.2.1-3.

$$f_b \leq F'_b \tag{4.2.1-3}$$

where:

F_b' = adjusted bending stress

4.2.2 Shear Parallel-to-Grain

In wood members subject to bending, shear stresses develop both parallel and perpendicular to the grain. Due to its cellular structure, wood is much stronger to resist shear perpendicular to the grain. Therefore, parallel-to-grain shear stresses (sometimes referred to as *horizontal* shear stresses) always govern shear design. For this reason, shear design values are published only for shear parallel-to-grain. The horizontal shear stress acting on any plane parallel to a beam's neutral axis may be determined by Equation 4.2.2-1.

$$f_v = \frac{VQ}{Ib} \tag{4.2.2-1}$$

where:

f_v = shear parallel-to-grain stress
V = shear force
Q = the first moment about the neutral axis of the area of the section between the plane of interest and outside edge of the cross section on the same side of the neutral axis
b = the width of the plane across which the shear stress is calculated
I = moment of inertia

The shear stress at a section is maximum at the neutral axis. For a rectangular section, the maximum shear stress (at the neutral axis or mid-depth), from Equation 4.2.2-1, becomes Equation 4.2.2-2.

$$f_v = \frac{3V}{2bd} \tag{4.2.2-2}$$

Shear forces are generally greatest near the beam supports. In the computation of shear stress, it is permissible to neglect distributed loads applied within a distance, d, from the edge of the support in cases where the loads are applied on one face and support is provided by bearing on the opposite face (Figure 4.2.2-1). In practice, instead of omitting such loads, the critical shear force is simply taken at a distance, d, from the support. Concentrated loads within the distance, d, from the support are permitted to be multiplied by x/d, where x is the distance between the load and the face of the support.

4.2.3 Deflection

Deflections may be determined using principles of mechanics, energy-based methods, or finite element computer analysis. The appendix contains mechanics-based formulas for shear, bending moment, and deflections for commonly encountered support and loading conditions for beams. Deflection limits

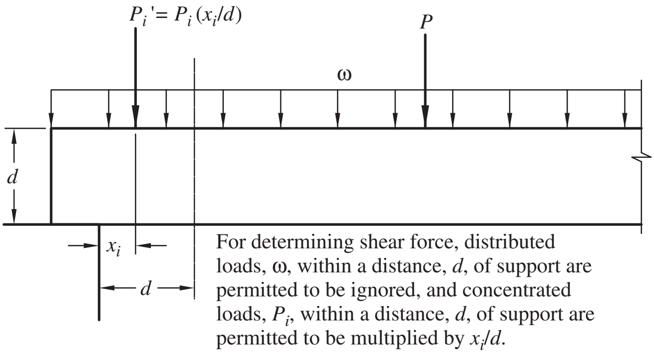


Figure 4.2.2-1 Loads and shear force at ends of beams

are recommended in Chapter 3. In many applications, both the immediate (instantaneous) deflection due to live and other applied loads, as well as the total deflection due to all loads, are considered.

4.3 SIMPLE BEAMS

One of the most common members in wood framing is the *simple beam.* The simple beam is a member that is supported only at its ends and resists bending loads across a single span. The beam ends are idealized as being allowed to freely rotate.

Simple beams are commonly loaded with flexural tension stresses in only the lower (bottom) portion of the member. In such cases, unbalanced glued laminated timber layups are generally more economical and should be specified. As discussed in Chapter 1, unbalanced beams with standard camber are typically stocked in inventory at lumber distribution centers.

4.3.1 Beams with Continuous Lateral Support

Structural sheathing or decking is often attached directly to the compression face of timber beams. These members provide essentially continuous lateral support for the beams, preventing lateral-torsional buckling. For this case, the beam stability factor, C_L, is equal to 1.0.

EXAMPLE 4.3.1-1 SIMPLE BEAM WITH CONTINUOUS LATERAL SUPPORT

Given: A simply supported, structural glued laminated timber is to be used as a beam to support a residential floor load (40 psf). The basic tributary width for the beam is 16 ft and the beam will span 12 ft from the centerline of one support to the centerline of the other. Floor joists at 16 in. on center will span over and bear on top of the beam, and 24F-1.8E DF will be used. A floor dead load of 15 psf is assumed, not including the weight of the beam.

Wanted: Determine an appropriate structural glued laminated timber section.

Approach: Unless the plans specify otherwise, it will be assumed that the floor joists span continuously over the beam. In the condition of two equal spans, the joists will deliver 25% more load over the center support than would be calculated using straight tributary areas alone. The joists will be assumed to provide continuous support of the compression side (top) of the beam. Design values will be obtained from the *National Design Specification®* [1] and deflection criteria will be obtained from Chapter 3. The load duration factor for occupancy live load is 1.0 from Table 3.4.1-1. The beam will be assumed to be in dry conditions of use with normal temperatures, and will be straight or slightly cambered (not curved), and without tapered cuts; therefore, all the adjustment factors will be 1.0 except for C_v and C_L. The beam is assumed to be supported continuously along the compression edge, therefore, $C_L = 1.0$. To determine the required section, the volume factor, C_v, will be estimated as 1.0. Once a trial size is determined, the actual volume factor and self-weight for the beam will be calculated and the beam stresses will be compared to the allowable stresses.

Solution:

Design values:

$$F'_{bx} = F_{bx} C_D C_M \left(C_V \text{ or } C_L\right) C_c C_I C_t = F_{bx} C_V \approx F_{bx} = 2400 \text{ psi}$$

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = F_{vx} = 265 \text{ psi}$$

$$E'_x = E_x C_M C_t = E_x = 1.8 \left(10^6\right) \text{ psi}$$

Floor live load:

$$\omega_L = (40 \text{ psf})\,(16 \text{ ft})\,(1.25) = 800 \text{ plf}$$

Floor dead load:

$$\omega_D = (15 \text{ psf})\,(16 \text{ ft})\,(1.25) = 300 \text{ plf}$$

Bending moment (from applied loads):

$$M = \frac{\left(\omega_L + \omega_D\right) \ell^2}{8}$$

$$M = \left(800 \frac{\text{lb}}{\text{ft}} + 300 \frac{\text{lb}}{\text{ft}}\right) \left(\frac{(12 \text{ ft})^2}{8}\right)$$

$$M = 19{,}800 \text{ lb-ft} = 237{,}600 \text{ lb-in}$$

Estimated required section modulus (based on flexure):

$$S = \frac{M}{F'_b} = \frac{237{,}600 \text{ lb-in}}{2400 \text{ psi}} = 99.0 \text{ in}^3$$

Required depth assuming a width of $b = 5\frac{1}{8}$ in.:

$$d = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6\left(99.0\,\text{in}^3\right)}{5.125\,\text{in}}} = 10.8\,\text{in} \quad \therefore \text{ Try } 5\tfrac{1}{8}\,\text{in} \times 12\,\text{in section}$$

Beam self-weight (Table 2.2-1, DF-L, 12% MC):

$$\omega_{sw} = \gamma bd = \left(33\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{5.125\,\text{in}}{12\,\text{in/ft}}\right)\left(\frac{12\,\text{in}}{12\,\text{in/ft}}\right) = 14\,\text{lb/ft}$$

Total load:

$$\omega_{sw} = \gamma bd = \left(33\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{5.125\,\text{in}}{12\,\text{in/ft}}\right)\left(\frac{12\,\text{in}}{12\,\text{in/ft}}\right) = 14\,\text{lb/ft}$$

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125\,\text{in}}{5.125\,\text{in}}\right)^{\frac{1}{10}}\left(\frac{12\,\text{in}}{12\,\text{in}}\right)^{\frac{1}{10}}\left(\frac{21\,\text{ft}}{12\text{ft}}\right)^{\frac{1}{10}} = 1.06 \le 1.0$$
$$C_V = 1.0$$

Allowable bending stress:

$$F'_{bx} = F_{bx}C_V = 2400\,\text{psi}\,(1.0) = 2400\,\text{psi}$$

Bending moment from total load:

$$M = \frac{\omega_{tl}\ell^2}{8}$$
$$M = \left(1114\frac{\text{lb}}{\text{ft}}\right)\left(\frac{(12\,\text{ft})^2}{8}\right)$$
$$M = 20{,}050\,\text{lb-ft} = 240{,}600\,\text{lb-in}$$

Section modulus:

$$S = \frac{bd^2}{6} = \frac{(5.125\,\text{in})\,(12\,\text{in})^2}{6} = 123\,\text{in}^3$$

Bending stress from total load:

$$f_b = \frac{M}{S} = \frac{240{,}600\,\text{lb-in}}{123\,\text{in}^3} = 1956\,\text{psi} \le F'_b = 2400\,\text{psi} \quad \therefore \text{ OK}$$

End reactions:

$$R = \frac{\omega \ell}{2} = \frac{(1114\,\text{lb/ft})\,(12\,\text{ft})}{2} = 6684\,\text{lb}$$

Shear force from total load:

Since the loads are applied to the top of the beam and the beam supports will be assumed to provide bearing to the bottom, the shear force to be checked may exclude the distributed loads applied within a distance d of the end.

$$V = R - \omega d = 6684\,\text{lb} - (1114\,\text{lb/ft})\,(1\,\text{ft}) = 5570\,\text{lb}$$

Shear stress:

$$f_v = \frac{3V}{2bd} = \frac{3\,(5570\,\text{lb})}{2\,(5.125\,\text{in})\,(12\,\text{in})} = 136\,\text{psi} \le F_v' = 265\,\text{psi} \quad \therefore \text{ OK}$$

Live load deflection limit:

$$\delta_L = \frac{\ell}{360} = \frac{144\,\text{in}}{360} = 0.40\,\text{in}$$

Moment of inertia:

$$I = \frac{bd^3}{12} = \frac{(5.125\,\text{in})\,(12\,\text{in})^3}{12} = 738\,\text{in}^4$$

Live load deflection:

$$\Delta_L = \frac{5\omega_L \ell^4}{384EI} = \frac{5\,(800\,\text{lb/ft})\,(1\,\text{ft/12 in})\,(12\,\text{ft}\,(12\,\text{in/ft}))^4}{384\left(1{,}800{,}000\,\text{lb/in}^2\right)\left(738\,\text{in}^4\right)}$$

$$\Delta_L = 0.28\,\text{in} \le \delta_L = 0.40\,\text{in} \quad \therefore \text{ OK}$$

Dead load deflection:

$$\Delta_D = \frac{5\left(\omega_D + \omega_{SW}\right)\ell^4}{384EI} = \frac{5\,(314\,\text{lb/ft})\,(12\,\text{ft})^4\left(1728\,\text{in}^3/\text{ft}\right)}{384\left(1{,}800{,}000\,\text{lb/in}^2\right)\left(738\,\text{in}^4\right)}$$

$$\Delta_D = 0.11\,\text{in}$$

0.5D + L deflection limit:

$$\delta_{0.5D+L} = \frac{\ell}{240} = \frac{144\,\text{in}}{240} = 0.60\,\text{in}$$

0.5D + L deflection:

$$\Delta_{0.5D+L} = 0.5\Delta_D + \Delta_L = 0.5\,(0.11\,\text{in}) + 0.28\,\text{in} = 0.34\,\text{in}$$

$$\Delta_{0.5D+L} = 0.34\,\text{in} \le \delta_{0.5D+L} = 0.6\,\text{in} \quad \therefore \text{ OK}$$

Answer: For the stated conditions, a $5\frac{1}{8}$ in. × 12 in. Douglas fir (DF) 24F-1.8E beam is satisfactory with regard to bending, shear, and deflection. Since the beam is a simple span beam, flexural tension stresses will be realized only on the bottom of the beam and an *unbalanced* layup should be specified.

Discussion: It is common to express the results of the design checks for bending, shear, and deflection as ratios of actual to allowable values, which in this case are as follows:

Shear: $f_v/F_v' = 136\,\text{psi}/265\,\text{psi} = 0.51 \leq 1.0 \quad \therefore$ OK

Bending: $f_b/F_b' = 1956\,\text{psi}/2400\,\text{psi} = 0.82 \leq 1.0 \quad \therefore$ OK

Deflection (live load): $\Delta_L / \delta_L = 0.28\,\text{in} / 0.40\,\text{in} = 0.70 \leq 1.0 \quad \therefore$ OK

Deflection (total load):
$\Delta_{0.5D+L}/\delta_{0.5D+L} = 0.34\,\text{in}/0.60\,\text{in} = 0.57 \leq 1.0 \quad \therefore$ OK

From the foregoing ratios, it is seen that bending controls the design (giving the ratio closest to, but not exceeding, unity). Since the joists frame over the beam, the beam selected should be checked with regard to clearance. It can be shown that $3\frac{1}{8}$ in. × 15 in. and $6\frac{3}{4}$ in. × $10\frac{1}{2}$ in. 24F-1.8E members would also be satisfactory.

Glued laminated timber is generally priced by cost per board foot. Nominal dimensions of the input lumber are used to calculate the board footage of the glulam beams. The $3\frac{1}{8}$ in. × 15 in. beam is made from (10) 2 × 4s, the $5\frac{1}{8}$ in. × 12 in. beam is made from eight 2 × 6s, and the $6\frac{3}{4}$ in. × $10\frac{1}{2}$ in. beam is made from seven 2 × 8s. From Chapter 1, one board foot (BF) is equal to 144 in^3 based on nominal cross-sectional dimensions and actual length of the input lumber. The board footage calculations (per foot of beam) follow:

For the $3\frac{1}{8}$ in. beam:

$$\frac{\text{BFM}}{\text{ft}} = \frac{(10)\,(2\ \text{in})\,(4\ \text{in})}{\left(144\,\text{in}^3/\text{BF}\right)}\left(\frac{12\,\text{in}}{\text{ft}}\right) = 6.67\,\frac{\text{BF}}{\text{ft}}$$

For the $5 - \frac{1}{8}$ in. beam:

$$\frac{\text{BFM}}{\text{ft}} = \frac{(8)\,(2\,\text{in})\,(6\,\text{in})}{\left(144\,\text{in}^3/\text{BF}\right)}\left(\frac{12\,\text{in}}{\text{ft}}\right) = 8\,\frac{\text{BF}}{\text{ft}}$$

For the $6\frac{3}{4}$ in. beam:

$$\frac{\text{BFM}}{\text{ft}} = \frac{(7)\,(2\,\text{in})\,(8\,\text{in})}{\left(144\,\text{in}^3/\text{BF}\right)}\left(\frac{12\,\text{in}}{\text{ft}}\right) = 9.33\,\frac{\text{BF}}{\text{ft}}$$

The $3\frac{1}{8}$ in. × 15 in. section would likely be more economical, but it would result in lower clearance or headroom.

4.3.2 Upside-Down Installation

Occasionally, unbalanced beams are inadvertently installed upside-down (top-down) (Figure 4.3.2-1), creating a potentially unsafe situation. Unbalanced beams installed upside-down result in the lower-grade compression laminations of the layup being stressed in flexural tension. Flexural tension design values for the compression zone (top) of unbalanced beams are less than those for the tension zone (bottom). As such, beams installed upside-down require reinstallation, replacement, or further analysis to ensure safety and serviceability. For analysis, the bending stress must be compared to the adjusted negative bending design value.

Figure 4.3.2-1 Glulam beam improperly installed upside-down (Photo courtesy of Boise Cascade Corporation, Boise, Idaho.)

For example, the reference design value for a 24F-V4 DF beam subject to negative bending is 1850 psi (Table A1-Expanded of AITC 117 [2]). If this beam were installed upside-down it would have only about three-fourths of its intended capacity. Only if the beam were originally oversized by more than 30% in flexure would it still be acceptable upside-down. The $5\frac{1}{8}$ in. × 12 in. 24F-1.8 DF unbalanced beam selected in Example 4.3.1-1 would *not* be acceptable if installed upside-down. In addition, the beam stability factor for beams installed upside-down may be smaller than the factor calculated for proper installation, further reducing the capacity of the improperly installed beam. The design values for F_{vx} and E_x remain unchanged with regard to upside-down installation.

Furthermore, upside-down installation reverses the camber of a beam. A cambered beam installed upside down will have initial downward deflection (sag) where it was originally expected to have a slight crown. For short spans, the lack of crown may be unnoticeable or of little significance. For longer-span beams, the increased sag produced by the upside-down camber may be unacceptable. For flat roofs, upside-down camber may also cause ponding problems.

4.3.3 Continuous Bracing versus Repetitive Discrete Bracing

Timber beams are often attached to joists at intervals of 24 inches or less, rather than directly attached to sheathing or decking. In this situation, a beam can be designed as a beam loaded by a series of point loads with lateral bracing applied at points of loading; however, it is common practice to consider such loading as uniform with continuous lateral bracing. It can be shown that using concentrated loads from each joist (if the exact joist locations are known) produces shear, bending, and deflection values that vary at most by a few percent from the values obtained from assuming uniformly distributed loads. The following example demonstrates that typical joist (or truss) framing at 24 inches on center or less can also be assumed to provide continuous lateral support.

EXAMPLE 4.3.3-1 SIMPLE BEAM WITH BRACING PROVIDED BY JOISTS

Given: A 3 in. × $13\frac{1}{2}$ in. 24F-1.8E SP glued laminated timber beam supporting snow load on a roof. The beam spans 16 ft.

Wanted: Determine the beam stability factor, C_L, assuming that roof joists spaced at 24 in. o.c. and fastened to the top of the beam provide lateral support.

Approach: The modulus of elasticity for beam and column stability in the $y-y$ direction, $E_{y\min}$, will be used for the beam stability calculations. The effective length method will be used with Table 3.4.3.1.1-1.

Solution:

The unsupported length, ℓ_u, is 24 in., or 2.0 ft. For a span of 16 ft there will be at least seven equally spaced concentrated loads of equal magnitude. From Table 3.4.3.1.1-1, the effective length, ℓ_e is 1.84 × ℓ_u (seven or more equal concentrated loads), or 44.2 in.

Design values:

$$E'_{y\,\min} = E_{y\,\min} C_M C_t = (850{,}000\,\text{psi})\,(1.0)\,(1.0) = 850{,}000\,\text{psi}$$

$$F'_b = F_b C_D C_c C_M \left(C_V \text{ or } C_L\right) C_I C_t$$

$$F'_b = (2400\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\left(C_V \text{ or } C_L\right)(1.0)\,(1.0)$$

$$F'_b = 2760\,\text{psi}\left(C_V \text{ or } C_L\right) = F^*_b\left(C_V \text{ or } C_L\right)$$

Slenderness ratio (Equation 3.4.3.1.1-1):

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(44.2\,\text{in})\,(13.5\,\text{in})}{(3.0\,\text{in})^2}} = 8.14$$

Critical buckling design value (Equation 3.4.3.1-2):

$$F_{bE} = \frac{1.20E'_{y\,min}}{R_B^2} = \frac{1.20\,(850{,}000\,\text{psi})}{(8.14)^2} = 15{,}400\,\text{psi}$$

Beam stability factor (Equation 3.4.3.1-1):

$$C_L = \frac{1+\left(F_{bE}/F_b^*\right)}{1.9} - \sqrt{\left[\frac{1+\left(F_{bE}/F_b^*\right)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$C_L = \frac{1+\left(\frac{15{,}400\text{ psi}}{2760\text{ psi}}\right)}{1.9} - \sqrt{\left[\frac{1+\left(\frac{15{,}400\text{ psi}}{2760\text{ psi}}\right)}{1.9}\right]^2 - \frac{\left(\frac{15{,}400\text{ psi}}{2760\text{ psi}}\right)}{0.95}}$$

$$C_L = 0.99$$

Answer: The beam stability factor, C_L, for the stated beam, assuming the joists provide lateral support every 24 in., is 0.99, nearly unity.

Discussion: This example shows that where joists or other members provide lateral support at relatively short intervals, the supported member can be assumed to be fully supported laterally. In these cases, it is common to also apply the loads from the framing members continuously along the beam (not as concentrated loads at the framing member locations).

4.3.4 Unbraced Beams and Partially Braced Beams

Occasionally, simple beams are installed without full lateral bracing along the top (compression) edge of the beam. In addition, load reversals, such as those caused by wind uplift, may subject the unbraced edge of a beam to compression stresses and potential buckling. For these cases, the design bending capacity of the beam must be reduced through application of the beam stability factor, C_L, described in Chapter 3.

EXAMPLE 4.3.4-1 SIMPLE BEAM UNBRACED ALONG ITS LENGTH

Given: A simply supported, 24F-1.8E SP beam spans 30 ft and is uniformly loaded with 250 lb/ft, resulting in a design moment of 337,500 lb-in. The beam is braced to prevent lateral movement and torsional rotation at the ends only. Assume dry conditions, normal temperatures, and floor live load duration.

Wanted: Determine an appropriate section based on bending.

Approach: A trial beam section will be selected, volume and beam stability factors will be calculated, and the size will be adjusted as necessary such that the adjusted bending stress is not greater than the adjusted design value. The effective length method will be used with Table 3.4.3.1.1-1.

Solution:

Allowable bending stress (estimated):

$$F_b' = F_b C_D C_c C_M \left(C_V \text{ or } C_L\right) C_I C_t$$

$$F_b' \approx F_b C_D C_c C_M C_I C_t = 2400\,\text{psi}\,(1.0)\,(1.0)\,(1.0)\,(1.0)\,(1.0) = 2400\,\text{psi}$$

Required section modulus (estimated):

$$\text{S}_{\text{required}} = \frac{M}{F_b'}$$

$$\text{S}_{\text{required}} \approx \frac{337{,}500\,\text{lb-in}}{2400\,\text{psi}} = 141\,\text{in}^3$$

Required depth based on trial widths of 3 in., 5 in., and $6\frac{3}{4}$ in.:

For $b = 3$ in.:

$$d > \sqrt{\frac{6\text{S}_{\text{required}}}{b}} = \sqrt{\frac{6\left(141\,\text{in}^3\right)}{3\,\text{in}}} = 16.8\,\text{in}$$

For $b = 5$ in.:

$$d > \sqrt{\frac{6S_{\text{required}}}{b}} = \sqrt{\frac{6\left(141\,\text{in}^3\right)}{5\ \text{in}}} = 13.0\,\text{in}$$

For $b = 6.75$ in.:

$$d > \sqrt{\frac{6S_{\text{required}}}{b}} = \sqrt{\frac{6\left(141\,\text{in}^3\right)}{6.75\ \text{in}}} = 11.2\,\text{in}$$

The 3 in. wide beam has a significantly higher depth/width ratio than the other sections, indicating that a larger reduction in capacity can be expected due to the beam stability factor for this unbraced case, so a wider width will be chosen. In anticipation that the volume or beam stability factors will cause some reduction in the bending design value, a slightly larger section (one additional lamination) will be selected, for example, 5 in. × $15\frac{1}{8}$ in.

Section modulus (5 in. × $15\frac{1}{8}$ in. section):

$$S = \frac{bd^2}{6} = \frac{(5\,\text{in})\,(15.125\,\text{in})^2}{6} = 190.6\,\text{in}^3$$

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125\,\text{in}}{5\,\text{in}}\right)^{\frac{1}{10}} \left(\frac{12\,\text{in}}{15.125\,\text{in}}\right)^{\frac{1}{10}} \left(\frac{21\,\text{ft}}{30\,\text{ft}}\right)^{\frac{1}{10}} = 0.945$$

Unbraced length-to-depth ratio:

$$\frac{\ell_u}{d} = \frac{30\,\text{ft}\,(12\,\text{in/ft})}{15.125\,\text{in}} = 23.8$$

Effective length (Table 3.4.3.1.1-1):

$$\ell_e = 1.63\ell_u + 3d = 1.63\,(360\,\text{in}) + 3\,(15.125\,\text{in}) = 632\,\text{in}$$

Slenderness ratio (Equation 3.4.3.1.1-1):

$$R_B = \sqrt{\frac{(632\,\text{in})\,(15.125\,\text{in})}{(5\ \text{in})^2}} = 19.6\ (\leq 50;\ \ \text{good})$$

Critical buckling design value (Equation 3.4.3.1-2):

$$E'_{\min} = E'_{y\,\min} = (850{,}000\,\text{psi})\,C_M C_t = 850{,}000\,\text{psi}$$

$$F_{bE} = \frac{1.20E'_{y\ \min}}{R_B^2} = \frac{1.20\,(850{,}000\,\text{psi})}{(19.6)^2} = 2655\,\text{psi}$$

Partially adjusted bending design value:

$$F_b^* = F_b C_D C_m C_t = 2400\,\text{psi}\,(1.00)\,(1.00)\,(1.00) = 2400\,\text{psi}$$

Beam stability factor (Equation 3.4.3.1-1):

$$C_L = \frac{1 + \left(F_{bE}/F_b^*\right)}{1.9} - \sqrt{\left[\frac{1 + \left(F_{bE}/F_b^*\right)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{2655\ \text{psi}}{2400\ \text{psi}}\right)}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{2655\ \text{psi}}{2400\ \text{psi}}\right)}{1.9}\right]^2 - \frac{\left(\frac{2655\ \text{psi}}{2400\ \text{psi}}\right)}{0.95}}$$

$$C_L = 0.855$$

Adjusted bending design value:

The smaller of C_L and C_V is chosen (C_L= 0.855), because they are not taken cumulatively.

$$F'_b = 2400\,\text{psi}\,(0.855) = 2052\,\text{psi}$$

Bending stress:

$$f_b = \frac{M}{S} = \frac{6M}{bd^2} = \frac{6\,(337{,}500\,\text{lb-in})}{(5\,\text{in})\,(15.125\,\text{in})^2}$$

$$f_b = 1770\,\text{psi} \le F_b' = 2052\,\text{psi} \;\therefore\; \text{OK}$$

Answer: The 5 in. × $15\frac{1}{8}$ in. 24F-1.8E SP glued laminated timber beam is adequate in bending for the stated conditions.

Discussion: The beam stability factor controlled over the volume factor in this example. While deeper beams have greater section modulus, they are also less stable. Complete design of the beam would require consideration of shear and deflection as well. Because the chosen section has approximately 16% excess capacity, a smaller section could be considered; however, it can be shown that a 5 in. × $13\frac{3}{4}$ in. section would be overstressed by about 1%.

EXAMPLE 4.3.4-2 SIMPLE BEAM WITH UNBRACED SEGMENTS

Given: A 60 ft 24F-1.8E DF beam is simply supported and loaded uniformly along its length with 278 lb/ft to develop a maximum moment of 1,500,000 lb-in. The beam is laterally braced only at the ends and at a point 25 ft from one end (Figure 4.3.4-1). Assume snow load duration, dry conditions, and normal temperatures. A trial beam section of 6.75 in × 30 in. was selected based on deflection criteria.

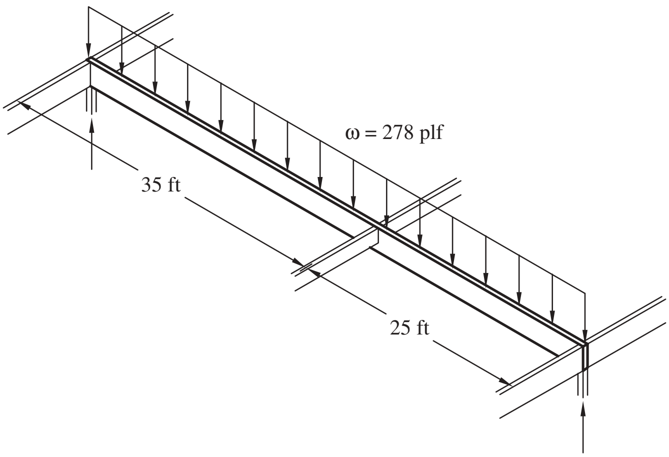


Figure 4.3.4-1 Partially braced beam for Example 4.3.4-2.

Wanted: Check the adequacy of the trial section.

Approach: Volume and beam stability factors will be calculated and the adjusted bending stress will be calculated and compared to the applied bending stress.

The equivalent moment method will be used to determine the slenderness ratio, because the case does not match up with the tabulated common cases.

Solution:

Moments in unbraced segment:

$$M_x = \frac{\omega x}{2}(\ell - x)$$
$$M_A = M_{8.75\,\text{ft}} = 62{,}330\,\text{lb-ft}$$
$$M_B = M_{17.5\,\text{ft}} = 103{,}400\,\text{lb-ft}$$
$$M_C = M_{26.75\,\text{ft}} = 123{,}600\,\text{lb-ft}$$
$$M_{\max} = 1{,}500{,}000\,\text{lb-in} = 125{,}000\ \text{lb-ft}$$

Equivalent moment factor, C_b (Equation 3.4.3.1.2-2):

$$C_b = \frac{12.5M_{\max}}{3M_A + 4M_B + 3M_C + 2.5M_{\max}}$$
$$C_b = \frac{12.5\,(125{,}000)}{3\,(62{,}330) + 4\,(103{,}400) + 3\,(123{,}600) + 2.5\,(125{,}000)} = 1.22$$

Eccentricity factor, C_e (Equations 3.4.3.1.2-3, 3.4.3.1.2-4):

$$k = 1.72\ \text{(for cases not tabulated)}$$
$$\eta = \frac{1.3kd}{\ell_u} = \frac{1.3\,(1.72)\,(30\,\text{in})}{(35\,\text{ft})\,(12\ \text{in/ft})} = 0.160$$
$$C_e = \sqrt{(0.160)^2 + 1} - 0.160 = 0.853$$

Slenderness ratio, R_B (Equation 3.4.3.1.2-1):

$$R_B = \sqrt{\frac{1.84\ell_u d}{C_b C_e b^2}} = \sqrt{\frac{1.84\,(35\,\text{ft})\,(12\ \text{in/ft})\,(30\,\text{in})}{(1.22)\,(0.853)\,(6.75\,\text{in})^2}} = 22.1 \le 50 \qquad \therefore\ \text{OK}$$

Critical buckling design value (Equation 3.4.3.1-2):

$$E'_{\min} = E'_{y\ \min} = (850{,}000\,\text{psi})\,C_M C_t = 850{,}000\,\text{psi}$$
$$F_{bE} = \frac{1.20E'_{y\,\min}}{R_B^2} = \frac{1.20\,(850{,}000\,\text{psi})}{(22.1)^2} = 2090\,\text{psi}$$

Partially adjusted bending stress:

$$F_b^* = F_b C_D C_M C_t C_c C_I = (2400\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\,(1.0)\,(1.0) = 2760\,\text{psi}$$

Beam stability factor (Equation 3.4.3.1-1):

$$C_L = \frac{1 + \left(F_{bE}/F_b^*\right)}{1.9} - \sqrt{\left[\frac{1 + \left(F_{bE}/F_b^*\right)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{2090\text{ psi}}{2760\text{ psi}}\right)}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{2090\text{ psi}}{2760\text{ psi}}\right)}{1.9}\right]^2 - \frac{\left(\frac{2090\text{ psi}}{2760\text{ psi}}\right)}{0.95}}$$

$$C_L = 0.683$$

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125\text{ in}}{6.75\text{ in}}\right)^{\frac{1}{10}} \left(\frac{12\text{ in}}{30\text{ in}}\right)^{\frac{1}{10}} \left(\frac{21\text{ ft}}{60\text{ ft}}\right)^{\frac{1}{10}} = 0.799$$

Adjusted bending design value:

$$F_b' = F_b^* \left(C_V \text{ or } \mathrm{C}_L\right) = 2760\text{ psi}\,(0.683) = 1885\text{ psi}$$

Design stress:

$$f_b = \frac{M}{S} = \frac{6M}{bd^2} = \frac{6\,(1{,}500{,}000\text{ lb-in})}{(6.75\text{ in})\,(30\text{ in})^2} = 1481\text{ psi} \leq F_b' = 1885\text{ psi} \;\therefore\; \text{OK}$$

Answer: The $6\frac{3}{4}$ in. × 30 in. 24F-1.8E DF glulam beam is adequate in bending for the stated conditions.

Discussion: The beam stability factor controlled the volume effect in this example. Although deeper beams have greater section modulus, they are also less stable. Complete design of the beam would require consideration of shear, as well.

4.4 CONTINUOUS MEMBERS

The great majority of wood beams are *simple* or *simply supported.* Simple beams that are horizontal or sloped and that resist primarily gravity loads experience what is commonly referred to as *positive bending* with flexural tension stresses in the bottom portion of the member and flexural compressive stresses in the upper portion. Members that are framed continuously across supports experience flexural tension stresses on the top of the member as well. Bending moments resulting in flexural tension stresses on the top face of a beam are commonly referred to as *negative bending* moments.

Where appropriate, bending moments, bending stresses, and design values are further denoted with the positive (+) and negative (−) signs for distinction and clarity (M^+, M^-, f_b^+, f_b^-, F_{bx}^+, F_{bx}^-, etc.). This is particularly important for glued laminated timbers, because unbalanced members have differing design values based on member orientation relative to loads. It is also important in the evaluation of lateral beam buckling as the compression portion of the beam requires support to resist lateral-torsional movement.

Bending members that are framed continuously across intermediate supports tend to deflect less and, in many cases, may resist greater loads than simply supported members with the same spans. Design of continuous members should include five considerations:

1. Glued laminated timbers should be specified in a balanced combination, because both positive and negative moments are generated in continuous framing.
2. The length, L, used for the volume factor calculation is the distance between points of zero moment.
3. Because the bottom of the member will in some places be in compression, continuous attachment of the top of the member to roof or floor framing, sheathing, or decking will not necessarily prevent buckling of the lower portion of the beam. As such, additional lateral support must be provided or an appropriate beam stability factor calculated for negative bending.
4. Unbalanced glulam members may be used if their reduced ability to resist flexural tension on the upper part of the beam is properly taken into consideration.
5. Beams with stock camber may no longer be appropriate except for very short spans; straight or custom camber beams should be specified.

EXAMPLE 4.4-1 CONTINUOUS BEAM

Given: A glued laminated timber beam will span continuously over two 10 ft openings while supporting 300 lb/ft dead load and 900 lb/ft snow load.

Wanted: Determine the appropriate 24F-1.8E DF sections in both unbalanced and balanced layup based on bending capacity.

Approach: It will be assumed that full lateral support of both top and bottom of the beam may be specified (often accomplished by direct attachment of roof and ceiling framing); therefore, C_L will be 1.0 for both positive and negative bending. The volume factor will be assumed also to be near unity as the loads to be resisted are relatively light and the anticipated section is modest in size.

Solution:
Design values for unbalanced layup:

$$F_{bx}^{+} = 2400\,\text{psi} \quad FF_{bx}^{-} = 1450\,\text{psi}$$
$$F_{vx} = 265\,\text{psi} \quad E_x = 1.8\left(10^6\,\text{psi}\right)$$

Design values for balanced layup:

$$F_{bx}^{+} = 2400\,\text{psi} \quad F_{bx}^{-} = 2400\,\text{psi}$$
$$F_{vx} = 265\,\text{psi} \quad E_x = 1.8\left(10^6\,\text{psi}\right)$$

Maximum positive moment:

$$M^{+} = \frac{9\omega\ell^2}{128} = \frac{9\,(300 + 900)\,\text{lb/ft}\,(10\,\text{ft})^2}{128} = 8438\,\text{lb-ft} = 101{,}250\,\text{lb-in}$$

Maximum negative moment:

$$M^{-} = \frac{\omega\ell^2}{8} = \frac{(300 + 900)\,\text{lb/ft}\,(10\,\text{ft})^2}{8} = 15{,}000\,\text{lb-ft} = 180{,}000\,\text{lb-in}$$

Estimated adjusted negative bending design value for unbalanced beam:

$$F_{bx}^{-\prime} = F_{bx}^{-} C_D \left(C_V \text{ or } C_L\right) \approx 1450\,\text{psi}\,(1.15)\,(1.0) = 1668\,\text{psi}$$

Required section for unbalanced beam:

$$S_{\text{required}} = \frac{M^{-}}{F_{bx}^{\prime -}} = \frac{180{,}000\,\text{lb-in}}{1668\,\text{psi}} = 108\,\text{in}^3$$

Required depth for $b = 3.125$ in.:

$$d \geq \sqrt{\frac{6S_{\text{required}}}{b}} = \sqrt{\frac{6\left(108\,\text{in}^3\right)}{3.125\,\text{in}}} = 14.4\,\text{in} \therefore \text{ Try } 3\tfrac{1}{8}\,\text{in} \times 15\,\text{in}$$

Section modulus ($3\frac{1}{8}$ in. × 15 in.):

$$S = \frac{bd^2}{6} = \frac{(3.125\,\text{in})\,(15.0\,\text{in})^2}{6} = 117\,\text{in}^3$$

Design stress (negative bending):

$$f_b^{-} = \frac{M^{-}}{S} = \frac{180{,}000\,\text{lb-in}}{117\,\text{in}^3} = 1540\,\text{psi}$$

Evaluate volume factor assumption:

The points of zero moments are at the ends and near the center support. The points of zero moment can be located as follows:

$$M = \left(\frac{3}{8}\omega\ell\right)x - \frac{\omega x^2}{2}$$

Setting $M = 0$ and solving for x,

$$M = \left(\frac{3}{8}\omega\ell\right)x - \frac{\omega x^2}{2} = 0$$

gives $x = \frac{3\ell}{4}$ from the end support or $\frac{\ell}{4} = 2.5$ ft from the center support.

For the negative moment, therefore, the length to be used to calculate the volume effect factor is 2.5 ft + 2.5 ft or 5 ft (distance between inflection points on adjacent sides of the center support).

$$C_V = \left(\frac{5.125\text{ in}}{b}\right)^{\frac{1}{10}}\left(\frac{12\text{ in}}{d}\right)^{\frac{1}{10}}\left(\frac{21\text{ ft}}{L}\right)^{\frac{1}{10}} \le 1.0$$

$$C_V = \left(\frac{5.125\text{ in}}{3.125\text{ in}}\right)^{\frac{1}{10}}\left(\frac{12\text{ in}}{15\text{ in}}\right)^{\frac{1}{10}}\left(\frac{21\text{ ft}}{10\text{ ft}}\right)^{\frac{1}{10}} = 1.11 \le 1.0$$

$$C_V = 1.0$$

The volume factor is 1.0, as initially assumed.

Design shear force (at *d* away from support):

$$V = \frac{5\omega\ell}{8} - \omega d = \frac{5\left(1200\frac{\text{lb}}{\text{ft}}\right)(10\text{ ft})}{8} - \left(1200\frac{\text{lb}}{\text{ft}}\right)(15\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right) = 6000\text{ lb}$$

Adjusted shear design value:

$$F'_{vx} = F_{vx}C_D = 265\text{ psi}(1.15) = 305\text{ psi}$$

Design shear stress:

$$f_v = \frac{3V}{2bd} = \frac{3(6000\text{ lb})}{2(3.125\text{ in})(15\text{ in})} = 192\text{ psi} \le F'_{vx} = 305\text{ psi} \qquad \therefore \text{ OK}$$

Estimated adjusted negative bending design value for balanced beam:

$$F_{bx}^{-\prime} = F_{bx}^{-}C_D\left(C_V \text{ or } C_L\right) \approx 2400\text{ psi}(1.15)(1.0) = 2760\text{ psi}$$

Required section for balanced beam:

$$S_{\text{required}} = \frac{M^-}{F_{bx}^{\prime -}} = \frac{180{,}000\text{ lb-in}}{2760\text{ psi}} = 65.2\text{ in}^3$$

Required depth for $b = 3.125$ in:

$$d \geq \sqrt{\frac{6S_{\text{required}}}{b}} = \sqrt{\frac{6\left(65.2\,\text{in}^3\right)}{3.125\,\text{in}}} = 11.2\,\text{in} \qquad \therefore \text{ Try } 3\tfrac{1}{8}\text{ in } \times \text{ 12 in}$$

Section modulus ($3\frac{1}{8}$ in. × 12 in.):

$$S = \frac{bd^2}{6} = \frac{(3.125\,\text{in})\,(12.0\,\text{in})^2}{6} = 75.0\,\text{in}^3$$

Volume factor:

$$C_V = \left(\frac{5.125\,\text{in}}{3.125\,\text{in}}\right)^{\frac{1}{10}}\left(\frac{12\,\text{in}}{12\,\text{in}}\right)^{\frac{1}{10}}\left(\frac{21\,\text{ft}}{10\,\text{ft}}\right)^{\frac{1}{10}} = 1.13 \leq 1.0$$

$$C_V = 1.0$$

Adjusted negative bending design value:

$$F_{bx}^{-\prime} = F_{bx}^{-} C_D \left(C_V \text{ or } C_L\right) = 2400\,\text{psi}\,(1.15)\,(1.00) = 2760\,\text{psi}$$

Design stress (negative bending):

$$f_b^- = \frac{M^-}{S} = \frac{180{,}000\,\text{lb-in}}{75.0\,\text{in}^3} = 2400\,\text{psi} \leq F_{bx}' = 2760\,\text{psi} \qquad \therefore \text{ OK}$$

Design shear force (at d away from support):

$$V = \frac{5\omega\ell}{8} - \omega d = \frac{5\left(1200\,\frac{\text{lb}}{\text{ft}}\right)(10\,\text{ft})}{8} - 1200\frac{\text{lb}}{\text{ft}}\,(15\,\text{in})\left(\frac{1\,\text{ft}}{12\,\text{in}}\right) = 6000\,\text{lb}$$

Adjusted shear design value:

$$F_{vx}' = F_{vx} C_D = 265\,\text{psi}\,(1.15) = 305\,\text{psi}$$

Design shear stress:

$$f_v = \frac{3V}{2bd} = \frac{3\,(6000\,\text{lb})}{2\,(3.125\,\text{in})\,(12\,\text{in})} = 240\,\text{psi} \leq F_{vx}' = 305\,\text{psi} \qquad \therefore \text{ OK}$$

Answer: Suitable size and grade for both unbalanced and balanced layup for the stated conditions are:

Unbalanced: $3\frac{1}{8}$ in. × 15 in. 24F-1.8E DF

Balanced: $3\frac{1}{8}$ in. × 12 in. 24F-1.8E DF

Discussion: A complete analysis would also include deflection checks and possibly camber specifications. In some cases, it is necessary to check both positive and negative bending stresses for continuous beams. It would NOT be appropriate to intentionally design an unbalanced member for upside-down installation, because of potential confusion and improper installation on the jobsite, even though the stronger side of the beam would, in this case, coincide with the greatest moment.

4.5 BIAXIAL BENDING (BENDING ABOUT BOTH AXES)

Beams are commonly intended to be loaded primarily in their strong direction (loaded and supported on edge with load direction parallel to the wide face of the member). Many cases exist, however, where bending members are loaded in both strong and weak directions, or are loaded obliquely. A common example of an obliquely loaded member is a purlin on a sloped roof where the top of the member is framed flush with or parallel to the roof surface (Figure 4.5-1). The components of the loads parallel to the wide and narrow faces must be found; these loads are used to compute flexural stresses with respect to loading in these directions; and the combined effects are evaluated as follows.

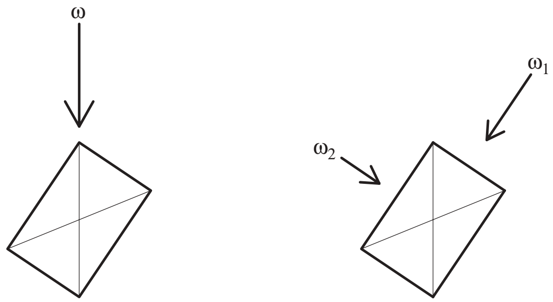


Figure 4.5-1 Obliquely loaded beam

The stress, strength, stiffness, and other properties or conditions of a bending member loaded on edge (in the member's strong orientation) are denoted in the *NDS*® [1] by the subscript 1. The conditions or properties associated with the member being loaded flatwise (weak orientation) are denoted with subscript 2.

Design properties for glued laminated timber are published with respect to x and y orientation, where x is with reference to load direction perpendicular to the wide faces *of the laminations*, and y direction parallel to the wide faces of the laminations. Thus, for horizontally laminated members *with member depth greater than member width*, the 1 and x orientations are coincident; likewise for the 2 and y orientations. However, if the depth of the horizontally laminated beam is less than its width, the 1 orientation corresponds to the y orientation, and the 2 orientation corresponds to the x orientation.

Where members are subject to bending in both directions, Equation 4.5-1 must be satisfied:

$$\frac{f_{b1}}{F'_{b1}} + \frac{f_{b2}}{F'_{b2}[1 - (f_{b1}/F_{bE})^2]} \leq 1.0 \tag{4.5-1}$$

where:

f_{b1} = bending stress about the strong axis
f_{b2} = bending stress about the weak axis
F'_{b1} = adjusted bending design value for strong axis bending
F'_{b2} = adjusted bending design value for weak axis bending
F_{bE} = critical buckling design value for bending (Equation 3.4.3.1-2)

EXAMPLE 4.5-1 BIAXIAL BENDING

Given: A 30 ft long, 24F-1.8E SP glued laminated beam is to be used to support a uniform dead load of 400 plf and a vertical roof load of 800 plf. In addition, the beam is subjected to a horizontal wind load P_W of 4000 lb located at the midpoint of the member, which can act in either direction (Figure 4.5-2). The member is to be used under normal temperature and dry service conditions and is not laterally braced between the end supports. The vertical roof load is assumed to have construction load duration.

Wanted: Determine an acceptable beam section assuming 24F-1.8E SP based on flexure.

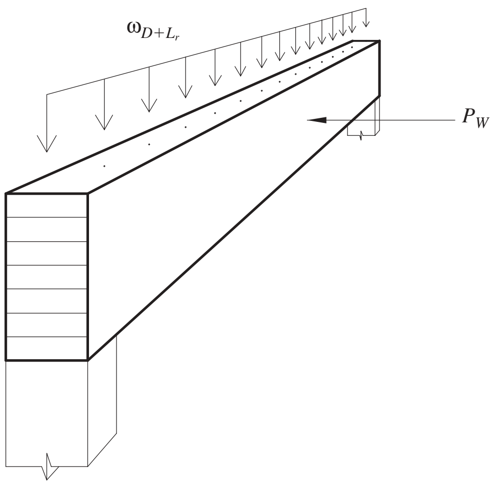


Figure 4.5-2 Biaxially loaded beam—Example 4.5-1.

Approach: The load combinations of Table 3.2.10-1 require that the member be adequate to resist the dead load plus either transient load (L_r or S) as well as the dead load plus 75% of the transient loads acting simultaneously. The load duration factors are obtained from Table 3.4.1-1, as 1.25 for roof load (construction) and 1.60 for wind. A trial beam size will be investigated, and size increased or decreased as appropriate. It will be assumed that the beam is deeper than it is wide.

Unless a very narrow profile is used, it will be assumed that the minimum size will be determined by the dead load plus the full roof live load; therefore, the load duration factor of 1.25 will be used to for the trial size. Also, a factor of 0.85 will be estimated for either C_V or C_L, whichever controls. The applied dead load of 400 plf is assumed to not include the self-weight of the beam.

Solution:

Relevant load combinations:

$$D$$

$$D + L_r$$

$$D + W$$

$$D + 0.75W + 0.75L_r$$

Bending moment (*x*-*x* axis):

$$M_D = \frac{(400\,\text{plf})\,(30\,\text{ft})^2}{8}\frac{12\,\text{in}}{\text{ft}} = 540{,}000\,\text{lb-in}$$

$$M_{L_r} = \frac{(800\,\text{plf})\,(30\,\text{ft})^2}{8}\frac{12\,\text{in}}{\text{ft}} = 1{,}080{,}000\,\text{lb-in}$$

$$M_{D+L_r} = 540{,}000\,\text{lb-in} + 1{,}080{,}000\,\text{lb-in} = 1{,}620{,}000\ \text{lb-in}$$

Reference design values from AITC 117 Table A1 [2]:

$$F_{bx} = 2400\,\text{psi},\ E_x = 1{,}800{,}000\,\text{psi},\ F_{by} = 1450\,\text{psi},\ E_{y\ \min} = 850{,}000\,\text{psi}$$

Allowable bending stress (x-x axis, estimated):

$$F'_{bx} = F_b C_D C_M C_t \left(C_V \text{ or } C_L\right) C_c C_I$$

$$F'_{bx} \approx 2400\,\text{psi}\,(1.25)\,(1.0)\,(1.0)\,(0.85)\,(1.0)\,(1.0) = 2550\,\text{psi}$$

Required section modulus (estimated):

$$S_{x\,(\text{required})} = \frac{M}{F'_{bx}} = \frac{1{,}620{,}000\ \text{lb-in}}{2550\ \text{psi}} = 635\ \text{in}^3$$

Bending moment (*y*-*y* axis):

$$M_y = \frac{P_W L}{4} = \frac{(4000\ \text{lb})(30\,\text{ft})(12\ \text{in/ft})}{4} = 360{,}000\ \text{lb-in}$$

Flat use factor (b = 8.5 in.):

$$C_{fu} = \left(\frac{12\text{ in}}{8.5\text{ in}}\right)^{\frac{1}{9}} = 1.04$$

Allowable bending stress (y-y axis):

$$F'_{by} = F_{by} C_D C_M C_t C_{fu}$$

$$F'_{by} = 1450\text{ psi}\,(1.6)\,(1.0)\,(1.0)\,(1.04) = 2410\text{ psi}$$

Required section modulus (y-y axis):

$$S_{y\,(\text{required})} = \frac{M_y}{F'_{by}} = \frac{360{,}000\text{lb-in}}{2410\text{ psi}} = 149\text{ in}^3$$

Trial southern pine sections:

8.5 in × 22 in ($I_x = 7542\text{ in}^4$; $S_x = 685.7\text{ in}^3$; $I_y = 1126\text{ in}^4$; $S_y = 264.9\text{ in}^3$)

8.5 in × 23.375 in ($I_x = 9047\text{ in}^4$; $S_x = 774.1\text{ in}^3$; $I_y = 1196\text{ in}^4$; $S_y = 281.5\text{ in}^3$)

8.5 in × 24.75 in ($I_x = 10{,}740\text{ in}^4$; $S_x = 867.8\text{ in}^3$; $I_y = 1267\text{ in}^4$ $S_y = 298.0\text{ in}^3$)

Choose trial section of 8.5 in. × 22 in.

Self weight (Table 2.2-1, southern pine, 12% MC):

$$\omega_{sw} = \gamma\, bd = (36\text{ pcf})\left(\frac{8.5\text{ in}}{12\text{ in/ft}}\right)\left(\frac{22\text{ in}}{12\text{ in/ft}}\right)(36\text{ pcf}) = 47\text{ plf}$$

Total dead load:

$$\omega_D = 400\text{ plf} + 47\text{ plf} = 447\text{ plf}$$

Volume factor:

$$C_V = \left(\frac{5.125\text{ in}}{b}\right)^{\frac{1}{x}}\left(\frac{12\text{ in}}{d}\right)^{\frac{1}{x}}\left(\frac{21\text{ ft}}{L}\right)^{\frac{1}{x}}$$

$$C_V = \left(\frac{5.125\text{ in}}{8.5\text{ in}}\right)^{\frac{1}{20}}\left(\frac{12\text{ in}}{22\text{ in}}\right)^{\frac{1}{20}}\left(\frac{21\text{ft}}{30\text{ ft}}\right)^{\frac{1}{20}} = 0.93$$

Ratio of unbraced length to depth:

$$\frac{\ell_u}{d} = \frac{30\text{ ft}\ \ (12\text{ in/ft})}{22\text{ in}} = 16.4$$

Effective length (Table 3.4.3.1.1-1):

$$\ell_e = 1.63\ell_u + 3d = 1.63\,(360\,\text{in}) + 3\,(22\,\text{in}) = 653\ \text{in} = 54.4\,\text{ft}$$

Slenderness ratio (Equation 3.4.3.1.1-1):

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(653\ \text{in})(22\ \text{in})}{(8.5\ \text{in})^2}} = 14.1 \le 50 \qquad \therefore\ \text{OK}$$

Critical buckling design value (Equation 3.4.3.1-2):

$$F_{bE} = \frac{1.20E'_{y\,\min}}{R_B^2} = \frac{1.20(850{,}000\,\text{psi})}{(14.1)^2} = 5131\,\text{psi}$$

Partially adjusted bending stress for $D + L_r$:

$$F^*_{bx} = F_b C_D = 2400\,\text{psi}(1.25) = 3000\,\text{psi}$$

Beam stability factor for $D + L_r$ (Equation 3.4.3.1-1):

$$C_L = \frac{1 + \left(F_{bE}/F^*_{bx}\right)}{1.9} - \sqrt{\left(\frac{1 + \left(F_{bE}/F^*_{bx}\right)}{1.9}\right)^2 - \frac{\left(F_{bE}/F^*_{bx}\right)}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{5131\,\text{psi}}{3000\,\text{psi}}\right)}{1.9} - \sqrt{\left(\frac{1 + \left(\frac{5131\,\text{psi}}{3000\,\text{psi}}\right)}{1.9}\right)^2 - \frac{\frac{5131\,\text{psi}}{3000\,\text{psi}}}{0.95}}$$

$$C_L = 0.94$$

Adjusted design value for $D + L_r$:

$$F'_{bx} = F^*_{bx}\left(C_V\ \text{or}\ C_L\right) = 3000\,\text{psi}\,(0.93) = 2790\,\text{psi}$$

Total load bending moment for $D + L_r$ (x-x axis):

$$M_{D+L_r} = \frac{(447\ \text{plf}\ + 800\,\text{plf})\,(30\,\text{ft})^2}{8}$$

$$M_{D+L_r} = 140{,}300\,\text{lb-ft} = 1{,}683{,}000\,\text{lb-in}$$

Design bending stress for $D + L_r$ (x-x axis):

$$f_{bx} = \frac{M_{D+L_r}}{S_x} = \frac{1{,}683{,}000\ \text{lb-in}}{685.7\ \text{in}^3} = 2455\ \text{psi} \le F'_{bx} = 2787\,\text{psi} \qquad \therefore\ \text{OK}$$

Partially adjusted bending stress for $D + W$ and $D + 0.75W + 0.75L_r$:

$$F_{bx}^{*} = F_b C_D = (2400\,\text{psi})(1.6) = 3840\ \text{psi}$$

Beam stability factor for $D + W$ and $D + 0.75\ W + 0.75\ L_r$:

$$C_L = \frac{1 + (F_{bE}/F_{bx}^{*})}{1.9} - \sqrt{\left(\frac{1 + (F_{bE}/F_{bx}^{*})}{1.9}\right)^2 - \frac{(F_{bE}/F_{bx}^{*})}{0.95}}$$

$$C_L = \frac{1 + \left(\dfrac{5131\,\text{psi}}{3840\,\text{psi}}\right)}{1.9} - \sqrt{\left(\frac{1 + \left(\dfrac{5131\,\text{psi}}{3840\,\text{psi}}\right)}{1.9}\right)^2 - \frac{\left(\dfrac{5131\,\text{psi}}{3840\,\text{psi}}\right)}{0.95}}$$

$$C_L = 0.91$$

Total x-x bending moment for $D + W$ (x-x axis):

$$M_x = \frac{(447\ \text{plf})\ (30\,\text{ft})^2}{8} = 50{,}290\text{lb-ft} = 603{,}000\,\text{lb-in}$$

Adjusted design values for bending for $D + W$ and $D + 0.75\ W + 0.75\ L_r$:

$$F'_{b1} = F'_{bx} = F_{bx}^{*}\ (C_V\ \text{or}\ C_L)\ 3840\,\text{psi}\ (0.91) = 3490\,\text{psi}$$

$$F'_{b2} = F'_{by} = F_{by} C_D C_M C_t C_{fu}$$

$$F'_{b2} = F'_{by} = 1450\,\text{psi}\ (1.6)\ (1.0)\ (1.0)\ (1.04) = 2410\,\text{psi}$$

Design bending stresses for $D + W$:

$$f_{b1} = f_{bx} = \frac{M_D}{S_x} = \frac{603{,}000\ \text{lb-in}}{685.7\ \text{in}^3} = 880\ \text{psi}$$

$$f_{b2} = f_{by} = \frac{M_y}{S_y} = \frac{360{,}000\ \text{lb-in}}{264.9\ \text{in}^3} = 1359\,\text{psi}$$

Design check for $D + W$:

$$\frac{f_{b1}}{F'_{b1}} + \frac{f_{b2}}{F'_{b2}\left[1 - -(f_{b1}/F_{bE})^2\right]} =$$

$$\ldots = \frac{880\,\text{psi}}{3490\,\text{psi}} + \frac{1359\,\text{psi}}{2410\,\text{psi}\left[1 - (880\,\text{psi}/5131\,\text{psi})^2\right]} =$$

$$\ldots = 0.252 + 0.581 = 0.833 \le 1.0\ \therefore\ \text{OK}$$

Design bending stresses for D + 0.75 W + 0.75 L_r}:

$$f_{b1} = f_{bx} = \frac{M_x}{S_x}$$

$$f_{b1} = \frac{\left[(447 \text{ plf} + 0.75\,(800 \text{ plf}))\,(30\,\text{ft})^2/8\right](12 \text{ in/ft})}{685.7 \text{ in}^3}$$

$$f_{b1} = 2061 \text{ psi}$$

$$f_{b2} = f_{by} = \frac{M_y}{S_y} = \frac{0.75\,(360{,}000 \text{ lb-in})}{264.9 \text{ in}^3} = 1019\,\text{psi}$$

Design check for D + 0.75 W + 0.75 L_r:

$$\frac{f_{b1}}{F'_{b1}} + \frac{f_{b2}}{F'_{b2}[1-(f_{b1}/F_{bE})^2]} =$$

$$\ldots = \frac{2061\,\text{psi}}{3490\,\text{psi}} + \frac{1019\,\text{psi}}{2410\,\text{psi}\left[1-(2061\,\text{psi}/5131\,\text{psi})^2\right]} =$$

$$\ldots = 0.59 + 0.50 = 1.09 \;\therefore\; \text{Not Acceptable}$$

The trial size is undersized for the combined roof live load and wind load case. The next size larger (deeper) beam (8.5 in. × 23.375 in.) will weigh slightly more but be stronger in both directions. The 8.5 in. × 23.375 in. 24F-1.8E SP glulam will be evaluated.

Self-weight for 8.5 in. × 23.375 in. (Table 2.2-1, Southern Pine, 12% MC):

$$\omega_{sw} = \gamma_{SP,12\%}bd$$

$$\omega_{sw} = (36 \text{ pcf})\left(\frac{8.5 \text{ in}}{12 \text{ in/ft}}\right)\left(\frac{23.375 \text{ in}}{12 \text{ in/ft}}\right)(36 \text{ pcf})$$

$$\omega_{sw} = 50 \text{ plf}$$

Total dead load:

$$\omega_D = 400 \text{ plf} + 50 \text{ plf} = 450 \text{ plf}$$

Volume factor:

$$C_V = \left(\frac{5.125 \text{ in}}{8.5 \text{ in}}\right)^{\frac{1}{20}}\left(\frac{12 \text{ in}}{23.375 \text{ in}}\right)^{\frac{1}{20}}\left(\frac{21\,\text{ft}}{30\,\text{ft}}\right)^{\frac{1}{20}} = 0.926$$

Ratio of unbraced length to depth:

$$\frac{\ell_u}{d} = \frac{30\,\text{ft}\,(12 \text{ in/ft})}{23.375 \text{ in}} = 15.4$$

Effective length (Table 3.4.3.1.1-1):

$$\ell_e = 1.63\ell_u + 3d = 1.63\,(360\,\text{in}) + 3\,(23.375\,\text{in}) = 657 \text{ in}$$

Slenderness ratio (Equation 3.4.3.1.1-1):

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(657\text{ in})(23.375\text{ in})}{(8.5\text{ in})^2}} = 14.6 \leq 50 \quad \therefore \text{ OK}$$

Critical buckling design value (Equation 3.4.3.1-2):

$$F_{bE} = \frac{1.20E'_{y\,\min}}{R_B^2} = \frac{1.20(850{,}000\,\text{psi})}{(14.6)^2} = 4790\,\text{psi}$$

Partially adjusted bending stress for $D + 0.75\ L_r + 0.75\ W$:

$$F_{bx}^* = F_b C_D = 2400\,\text{psi}(1.6) = 3840\,\text{psi}$$

Beam stability factor for $D + 0.75\ W + 0.75\ L_r$ (Equation 3.4.3.1-1):

$$C_L = \frac{1 + \left(F_{bE}/F_{bx}^*\right)}{1.9} - \sqrt{\left(\frac{1 + \left(F_{bE}/F_{bx}^*\right)}{1.9}\right)^2 - \frac{\left(F_{bE}/F_{bx}^*\right)}{0.95}}$$

$$C_L = \frac{1 + \left(\dfrac{4790\,\text{psi}}{3840\,\text{psi}}\right)}{1.9} - \sqrt{\left(\frac{1 + \left(\dfrac{4790\,\text{psi}}{3840\,\text{psi}}\right)}{1.9}\right)^2 - \frac{\left(\dfrac{4790\text{psi}}{3840\text{psi}}\right)}{0.95}}$$

$$C_L = 0.89$$

Total load bending moment for $D + 0.75W + 0.75L_r$ (x-x axis):

$$M_x = \frac{\left(\omega_D + 0.75\omega_{Lr}\right) l^2}{8} = \frac{(450\text{ plf} + 0.75(800\text{ plf}))\,(30\,\text{ft})^2}{8}$$

$$M_x = 118{,}000\text{ lb-ft} = 1{,}420{,}000\text{ lb-in}$$

Adjusted design values for bending for $D + 0.75W + 0.75L_r$:

$$F'_{b1} = F'_{bx} = F_{bx}^* \left(C_V \text{ or } C_L\right) 3840\,\text{psi}\,(0.89) = 3420\,\text{psi}$$

$$F'_{b2} = F'_{by} = F_{by} C_D C_M C_t C_{fu}$$

$$F'_{b2} = F'_{by} = 1450\,\text{psi}\,(1.6)\,(1.0)\,(1.0)\,(1.04) = 2410\,\text{psi}$$

Design bending stresses for $D + 0.75W + 0.75L_r$:

$$f_{b1} = f_{bx} = \frac{M_\text{x}}{S_\text{x}}$$

$$f_{b1} = \frac{1{,}420{,}000\text{ in-lb}}{774.1\text{ in}^3}$$

$$f_{b1} = 1830\text{ psi}$$

$$f_{b2} = f_{by} = \frac{M_y}{S_y} = \frac{0.75\ (360{,}000\ \text{lb-in})}{281.5\ \text{in}^3} = 958\ \text{psi}$$

Design check for $D + 0.75W + 0.75L_r$:

$$\frac{f_{b1}}{F'_{b1}} + \frac{f_{b2}}{F'_{b2}\left[1 - \left(f_{b1}/F_{bE}\right)^2\right]} =$$

$$\ldots = \frac{1830\ \text{psi}}{3420\ \text{psi}} + \frac{958\ \text{psi}}{2410\ \text{psi}\left[1 - (1830\ \text{psi}/4790\ \text{psi})^2\right]} =$$

$$\ldots = 0.535 + 0.465 = 1.00 \qquad \therefore\ \text{OK}$$

Answer: The 8.5 in. × 23.375 in. 24F-1.8E southern pine beam is adequate for the stated loads. The load combination $D + 0.75W + 0.75L_r$ controls.

Discussion: In the above example, the load combination of D (only) was not shown. This case rarely governs designs; however, in cases of high dead loads, it may govern due to the lower load duration factor.

4.6 TORSION

Torsion stresses are sometimes encountered in timber members. Examples of loading conditions in which torsion may be significant are beams or other members with eccentric transverse loads, bridge stringers resisting guard rail loads, and utility towers. The maximum torsion stress for a rectangular member occurs in the middle of the wide face and is computed using Equation 4.6-1 [3].

$$f_{vt} = \frac{T\ (3a + 1.8b)}{a^2 b^2} \qquad (4.6\text{-}1)$$

where:

f_{vt} = maximum torsion stress under design service load
T = applied internal torque under design service load
a = dimension of the wide face of the member
b = narrow face dimension

For glued laminated timber, the recommended allowable torsional stress is given by Equation 4.6-2 [4].

$$F'_{vt} = F_{vt} C_D C_M C_t = \left(\frac{2}{3} F_{vx} C_{vr}\right) C_D C_M C_t \qquad (4.6\text{-}2)$$

For sawn lumber, the recommended allowable torsional stress is given by Equation 4.6-3 [4].

$$F'_{vt} = F_{vt} C_D C_M C_t C_i = \left(\frac{2}{3} F_v\right) C_D C_M C_t C_i \qquad (4.6\text{-}3)$$

Connections to members subject to torsion must be designed to adequately resist the torsional reactions. Consideration should also be made for the effect of shrinkage at connections and its effect on rigidity. Where torsion stresses may be excessive, design modifications such as additional bracing should be considered to reduce or eliminate torsional effects.

EXAMPLE 4.6-1 TIMBER MEMBER SUBJECT TO TORSIONAL LOADING

Given: The Douglas fir laminated beam in Figure 4.6-1 is subject to load $P = 2100$ lb from a side bracket producing bending about the strong axis and torsion of the beam about its longitudinal axis. The bracket is located 3.5 ft from one end of the 12 ft beam and is such that the load P is applied 3 in. from the wide face of the beam. The size shown has been found satisfactory with regard to strong axis bending without consideration of the torsional effect of the applied load. Normal load duration is to be used. Temperature conditions are to be considered normal and the beam will be dry in service.

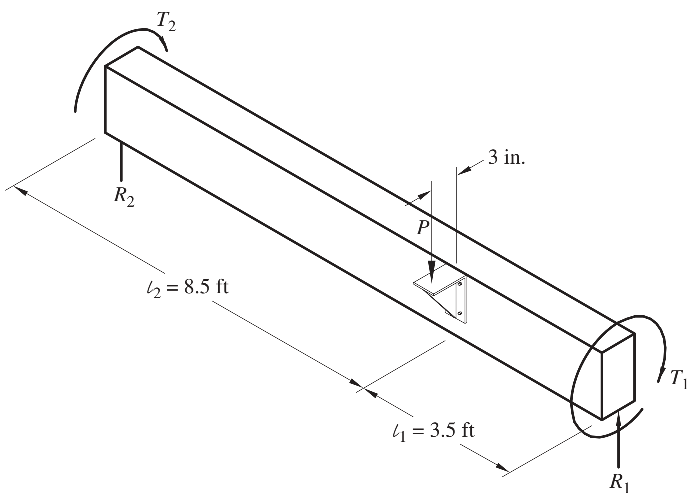


Figure 4.6-1 Beam subject to torsion—Example 4.6-1.

Wanted: Determine the acceptability of the $5\frac{1}{8}$ in. × 9 in. 24F-V4 DF glued laminated timber shown. Recommend an acceptable size if the given size is not suitable.

Approach: A trial size will be investigated with regard to torsion about the longitudinal axis with the torsional stress calculated using Equation 4.6-1. The beam is assumed to be braced adequately at both ends to resist torsion and thus develops torsional reactions at both ends.

Solution:

Torsional load:

$$eP = \left[(5.125\,\text{in})\,/\,2 + 3\,\text{in}\right](2100\,\text{lb})$$
$$eP = (5.563\,\text{in})\,(2100\,\text{lb})$$
$$eP = 11{,}680\,\text{lb-in}$$

Torsional reaction at near end:

$$T = \frac{eP}{1 + \left(\ell_1\,/\,\ell_2\right)} = \frac{11{,}680\,\text{lb-in}}{1 + (3.5\,\text{ft}\,/\,8.5\,\text{ft})} = 8270\,\text{lb-in}$$

Maximum torsional stress:

$$f_{vt} = \frac{T\,(3a + 1.8b)}{a^2 b^2}$$
$$f_{vt} = \frac{8270\ \text{lb-in}\,[3\,(9\,\text{in}) + 1.8\,(5.125\ \text{in})]}{(9\ \text{in})^2\,(5.125\ \text{in})^2}$$
$$f_{vt} = 141\,\text{psi}$$

Allowable torsional stress:

$$F'_{vt} = \left(\frac{2}{3}F_{vx}C_{vr}\right)C_D C_M C_t$$
$$F'_{vt} = \frac{2}{3}\,(265\,\text{psi})\,(0.72)\,(1.0)\,(1.0)\,(1.0)$$
$$F'_{vt} = 127\,\text{psi} < f_{vt} = 141\,\text{psi} \quad \therefore\ \text{Not Acceptable}$$

Try a $6\frac{3}{4}$ in. × 12 in. member.

Eccentricity of load on revised member:

$$e = (6.75\,\text{in})\,/\,2 + 3\text{in} = 6.375\,\text{in}$$

Torsion reaction on revised member:

$$T = 8270\,\text{lb-in}\left(\frac{6.375\ \text{in}}{5.563\ \text{in}}\right) = 9480\,\text{lb-in}$$

Maximum torsional stress on revised member:

$$f_{vt} = \frac{T\,(3a + 1.8b)}{a^2 b^2}$$
$$f_{vt} = \frac{9480\,\text{lb-in}\,[3\,(12\ \text{in}) + 1.8\,(6.75\ \text{in})]}{(12\ \text{in})^2\,(6.75\ \text{in})^2}$$
$$f_{vt} = 70\,\text{psi} \le F'_{vt} = 127\,\text{psi} \quad \therefore\ \text{OK}$$

Answer: The $5\frac{1}{8}$ in. × 9 in. 24F-V4 DF member shown is not sufficient when torsional effects of the given loading condition are considered. A $6\frac{3}{4}$ in. × 12 in. member in the same grade is adequate.

4.7 CONCLUSION

Beams are a common element in structures. They resist loads by bending. Timber beams must typically be evaluated for bending stress, shear stress, and deflection to ensure adequate capacity. Simple beams are supported only at their ends and primarily resist positive bending stresses. As such, unbalanced glulam layups are commonly used. Unbalanced layups are more efficient and economical than balanced layups for simple beams; however, care must be exercised to ensure that the beams are not installed upside-down. For continuous beams, however, both positive and negative bending stresses are developed and balanced layups are recommended.

This chapter presented formulas and examples for the design of *prismatic* timber beams (straight members without notches or taper). Beams subject to torsion stresses were also discussed. Subsequent chapters will cover design of notched, tapered, and curved beams.

CHAPTER 5

TIMBER COLUMNS AND TENSION MEMBERS

5.1 INTRODUCTION

Axially loaded members in timber construction are found in trusses, are used as struts and ties, and are commonly used as columns. These members may resist axial loads alone, or they may support flexural (bending) loads in addition to the axial loads. The flexural component occurs as a result of transverse loads and/or eccentric axial loads.

Compression members (columns) subject to centric axial loads only are the primary subject of this chapter. The term *column* is generally applied to all compression members including truss members, posts, or other structural components stressed in compression.

Tension members are briefly discussed at the end of the chapter and covered by example in Chapters 12, 13, and 14, because they are typically governed by the net section at connections. Members subject to axial loads combined with flexure are covered in Chapter 6.

5.2 COLUMN DESIGN CRITERIA

Centrically loaded columns must be proportioned to resist failure by crushing, buckling, or a combination of both. Very short columns tend to fail by crushing, while long, slender columns tend to fail by buckling. Columns of intermediate length fail due to a combination of crushing and buckling. The column stability factor, C_P, presented in Chapter 3 accounts for each of these possibilities.

Centrically loaded columns must be proportioned so that:

$$\frac{P}{A_g} \leq F_c' = F_c C_D C_M C_t C_P \quad (5.2\text{-}1)$$

and:

$$\frac{P}{A_n} \leq F_c^* = F_c C_D C_M C_t \qquad (5.2\text{-}2)$$

where:

P = concentric axial compression load
A_g = gross cross-sectional area
A_n = net cross-sectional area

5.3 RECTANGULAR COLUMNS

Rectangular columns have the potential to buckle about either of their primary bending axes. Consequently, a critical buckling design value, F_{cE}, must be calculated for buckling about both the *x-x* axis and the *y-y* axis to determine which buckling direction will govern the determination of the column stability factor. Different effective lengths, and differences in the moduli of elasticity (glulam) for bending about the two axes, affect the critical buckling stress and the resulting column stability factor. The smaller critical buckling design value is used in the calculation of the column stability factor.

Columns with rectangular sections typically have a strong direction and a weak direction with regard to buckling. In cases where bracing is provided in only one direction, good column design orients the column so that buckling in the otherwise weak direction is prevented. Where columns are installed within walls and wall sheathing is structurally attached to the column, the effective length for buckling in the plane of the wall approaches zero. In such cases, only buckling perpendicular to the wall plane need be considered. In cases where buckling in both directions is not prevented, the weak direction typically governs, unless the effective lengths substantially differ.

5.3.1 Sawn Lumber Columns

For sawn lumber, only one modulus of elasticity value is typically published. This simplifies column design, allowing the designer to compare slenderness ratios to determine which buckling axis will govern the design, rather than calculating and comparing critical buckling design values. Example 5.3.1-1 illustrates the analysis of a sawn timber column.

EXAMPLE 5.3.1-1 SAWN TIMBER COLUMN

Given: Southern pine, No. 1 dense, 6 × 6 sawn timber column, dry conditions of use.

Wanted: Determine the maximum allowable centric axial load for 5 ft, 10 ft, 15 ft, and 20 ft lengths, assuming pinned-pinned end conditions, no intermediate supports to resist buckling, and snow load duration.

Approach: The capacity of the column will be based on the compression parallel-to-grain design value published for SP No. 1 dense adjusted by the column stability factor for each of the lengths and by setting the adjusted design value equal to the compression stress under load.

Solution:

Design values (from NDS®):

$$F_c' = F_c^* C_P = F_c C_D C_M C_t C_P = 975\text{ psi}(1.15)(1.0)(1.0)C_P = (1121\text{ psi})C_P$$

$$E_{\min}' = E_{\min} C_M C_t = (580{,}000\text{ psi})(1.0)(1.0) = 580{,}000\text{ psi}$$

Effective length:

In this example $\ell_e = \ell = 5$ ft, 10 ft, 15 ft, 20 ft ($K_e = 1.0$ for all 4 cases).

Slenderness ratio:

For the 5 ft column: $\dfrac{\ell_{e1}}{d_1} = \dfrac{\ell_{e2}}{d_2} = \dfrac{5\text{ ft}(12\text{ in/ft})}{5.5\text{ in}} = 10.9 \leq 50 \quad \therefore \text{OK}$

For the 10 ft column: $\dfrac{\ell_{e1}}{d_1} = \dfrac{\ell_{e2}}{d_2} = \dfrac{10\text{ ft}(12\text{ in/ft})}{5.5\text{ in}} = 21.8 \leq 50 \quad \therefore \text{OK}$

For the 15 ft column: $\dfrac{\ell_{e1}}{d_1} = \dfrac{\ell_{e2}}{d_2} = \dfrac{15\text{ ft}(12\text{ in/ft})}{5.5\text{ in}} = 32.7 \leq 50 \quad \therefore \text{OK}$

For the 20 ft column: $\dfrac{\ell_{e1}}{d_1} = \dfrac{\ell_{e2}}{d_2} = \dfrac{20\text{ ft}(12\text{ in/ft})}{5.5\text{ in}} = 43.6 \leq 50 \quad \therefore \text{OK}$

Critical buckling design value:

For the 5 ft column: $F_{cE} = \dfrac{0.822E_{\min}'}{(\ell_e/d^2)} = \dfrac{0.822(580{,}000\text{ psi})}{(10.9)^2} = 4013\text{ psi}$

For the 10 ft column: $F_{cE} = \dfrac{0.822E_{\min}'}{(\ell_e/d)^2} = \dfrac{0.822(580{,}000\text{ psi})}{(21.8)^2} = 1003\text{ psi}$

For the 15 ft column: $F_{cE} = \dfrac{0.822E_{\min}'}{(\ell_e/d)^2} = \dfrac{0.822(580{,}000\text{ psi})}{(32.7)^2} = 445\text{ psi}$

For the 20 ft column: $F_{cE} = \dfrac{0.822E_{\min}'}{(\ell_e/d)^2} = \dfrac{0.822(580{,}000\text{ psi})}{(43.6)^2} = 251\text{ psi}$

Column stability factor, C_P:

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{(F_{cE}/F_c^*)}{c}}$$

For the 5 ft column:

$$C_P = \frac{1 + \left(\frac{4013 \text{ psi}}{1121 \text{ psi}}\right)}{2(0.8)} - \sqrt{\left[\frac{1 + \left(\frac{4013 \text{ psi}}{1121 \text{ psi}}\right)}{2(0.8)}\right]^2 - \frac{\left(\frac{4013 \text{ psi}}{1121 \text{ psi}}\right)}{0.8}}$$

$$C_P = 0.934$$

For the 10 ft column:

$$C_P = \frac{1 + \left(\frac{1003 \text{ psi}}{1121 \text{ psi}}\right)}{2(0.8)} - \sqrt{\left[\frac{1 + \left(\frac{1003 \text{ psi}}{1121 \text{ psi}}\right)}{2(0.8)}\right]^2 - \frac{\left(\frac{1003 \text{ psi}}{1121 \text{ psi}}\right)}{0.8}}$$

$$C_P = 0.651$$

For the 15 ft column:

$$C_P = \frac{1 + \left(\frac{445 \text{ psi}}{1121 \text{ psi}}\right)}{2(0.8)} - \sqrt{\left[\frac{1 + \left(\frac{445 \text{ psi}}{1121 \text{ psi}}\right)}{2(0.8)}\right]^2 - \frac{\left(\frac{445 \text{ psi}}{1121 \text{ psi}}\right)}{0.8}}$$

$$C_P = 0.357$$

For the 20 ft column:

$$C_P = \frac{1 + \left(\frac{251 \text{ psi}}{1121 \text{ psi}}\right)}{2(0.8)} - \sqrt{\left[\frac{1 + \left(\frac{251 \text{ psi}}{1121 \text{ psi}}\right)}{2(0.8)}\right]^2 - \frac{\left(\frac{251 \text{ psi}}{1121 \text{ psi}}\right)}{0.8}}$$

$$C_P = 0.212$$

Adjusted compression stress:

For the 5 ft column: $F'_c = F^*_c(C_P) = (1121 \text{ psi})(0.934) = 1047 \text{ psi}$

For the 10 ft column: $F'_c = F^*_c(C_P) = (1121 \text{ psi})(0.651) = 730 \text{ psi}$

For the 15 ft column: $F'_c = F^*_c(C_P) = (1121 \text{ psi})(0.357) = 400 \text{ psi}$

For the 20 ft column: $F'_c = F^*_c(C_P) = (1121 \text{ psi})(0.212) = 238 \text{ psi}$

Maximum axial load, P:

For the 5 ft column: $P = F_c'bd = (1047 \text{ psi})(5.5 \text{ in})(5.5 \text{ in}) = 31{,}700 \text{ lb}$

For the 10 ft column: $P = F_c'bd = (730 \text{ psi})(5.5 \text{ in})(5.5 \text{ in}) = 22{,}100 \text{ lb}$

For the 15 ft column: $P = F_c'bd = (400 \text{ psi})(5.5 \text{ in})(5.5 \text{ in}) = 12{,}100 \text{ lb}$

For the 20 ft column: $P = F_c'bd = (238 \text{ psi})(5.5 \text{ in})(5.5 \text{ in}) = 7200 \text{ lb}$

Answer: The allowable load for the four different-length SP No. 1 Dense 6×6s are:

$$P = 31{,}700 \text{ lb (5 ft length)}$$

$$P = 22{,}100 \text{ lb (10 ft length)}$$

$$P = 12{,}100 \text{ lb (15 ft length)}$$

$$P = 7200 \text{ lb (20 ft length)}$$

Discussion: As illustrated in this example, the column length has a significant effect on its load-carrying capacity. As the length increases, the column stability factor and column capacity decrease rapidly. Where practical, bracing a column to prevent buckling will significantly increase its capacity.

5.3.2 Glulam Columns

Structural glued laminated timbers manufactured in optimized or uniform-grade lay-ups may be used as columns. Axial design values are published for each. Glued laminated stock beams with camber, however, should generally *not* be used, because the camber induces eccentricity with the axial load. Example 5.3.2-1 illustrates the analysis of a glulam column.

EXAMPLE 5.3.2-1 GLULAM COLUMN

Given: An 18 ft long, $6\frac{3}{4}$ in. × $8\frac{1}{4}$ in., 16F-V2 southern pine glulam timber is to be used as a column. The top and bottom of the column are held to prevent translation, and lateral support is provided at mid-height to resist buckling about the *y-y* axis (weak direction). The column is subject to centric axial loads of 12,000 lb dead and 26,000 lb live and is used in a dry location.

Wanted: Determine the acceptability of the above section for the load combination $D + L$.

Approach: From AITC 117 (1), the 16F-V2 SP layup is normally used as a beam. However, axial design values are provided in AITC 117 Table A1-Expanded [1], and they will be used to evaluate the column. Because the moduli of elasticity are different for the *x-x* and *y-y* directions, critical buckling design values will need to be calculated for both directions.

Solution:

Design values (AITC 117 [1]):

$$F_c' = F_c^* C_P = F_c C_D C_M C_t C_P = (1300\text{ psi})(1.0)(1.0)(1.0) = (1300\text{ psi})C_P$$
$$E_{x\text{ min}}' = E_{x\text{ min}} C_M C_t = 790{,}000\text{ psi}(1.0)(1.0) = 790{,}000\text{ psi}$$
$$E_{y\text{ min}}' = E_{y\text{ min}} C_M C_t = 740{,}000\text{ psi}(1.0)(1.0) = 740{,}000\text{ psi}$$

Effective lengths (Section 3.4.3.9):

$$\ell_{e1} = K_e \ell_1 = (1.0)(18\text{ ft}) = 18\text{ ft} = 216\text{ in}$$
$$\ell_{e2} = K_e \ell_2 = (1.0)(9\text{ ft}) = 9\text{ ft} = 108\text{ in}$$

Slenderness ratios (Section 3.4.3.9):

$$\frac{\ell_{e1}}{d_1} = \frac{216\text{ in}}{8.25\text{ in}} = 26.2 \le 50 \qquad \therefore\ \text{OK}$$
$$\frac{\ell_{e2}}{d_2} = \frac{108\text{ in}}{6.75\text{ in}} = 16.0 \le 50 \qquad \therefore\ \text{OK}$$

Critical buckling design values (Equation 3.4.3.9-2):

$$F_{cE1} = \frac{0.822(790{,}000\text{ psi})}{(26.2)^2} = 946\text{ psi}$$
$$F_{cE2} = \frac{0.822(740{,}000\text{ psi})}{(16.0)^2} = 2376\text{ psi}$$

Column stability factor (Equation 3.4.3.9-1):

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$C_P = \frac{1 + \left(\dfrac{946\text{ psi}}{1300\text{ psi}}\right)}{2(0.9)} - \sqrt{\left[\frac{1 + \left(\dfrac{946\text{ psi}}{1300\text{ psi}}\right)}{2(0.9)}\right]^2 - \frac{\left(\dfrac{946\text{ psi}}{1300\text{ psi}}\right)}{0.9}}$$

$$C_P = 0.624$$

Adjusted compression design value:

$$F'_c = F^*_c C_P = 1300 \text{ psi}(0.624) = 811 \text{ psi}$$

Applied compression stress:

$$f_c = \frac{P}{A} = \frac{12{,}000 \text{ lb} + 26{,}000 \text{ lb}}{(6.75 \text{ in})(8.25 \text{ in})} = 682 \text{ psi} \leq F'_c = 811 \text{ psi} \qquad \therefore \text{ OK}$$

Answer: The 6.75 in. × 8.25 in., 16F-V2 SP timber is acceptable for the stated load condition. In this application, a straight member must be specified, because unbalanced members are often manufactured with camber for beam applications.

5.4 ROUND COLUMNS

Round columns are solid wood members of circular cross section loaded axially, as in the preceding section. Traditionally, the NDS® has recommended that round columns be designed as a square column with equivalent cross-sectional area. This is because only formulae for rectangular sections have traditionally been included in the NDS. The *equivalent square* procedure overestimates the buckling design value for the column by 4.5%, resulting in a somewhat nonconservative design. The correct equation (Equation 5.4-1) for buckling of a round cross section is included herein. Once the buckling design value is determined, the design procedure is identical to that of rectangular columns. The critical buckling design value for a round section is calculated as shown in Equation 5.4-1:

$$F_{cE} = \frac{0.617 E'_{\min}}{(\ell_e/D)^2} \tag{5.4-1}$$

where:

D is the diameter of the cross section.

The slenderness ratio for round columns, ℓ_e/D, should not exceed 43.

5.5 TAPERED COLUMNS

Columns may be tapered from a larger cross section toward a smaller cross section at one end or both ends. Tapered columns are designed similarly to other columns as described in the preceding sections except that calculations are based on representative dimensions for each face, as determined by Equation 5.5-1.

$$d = d_{\text{min}} + (d_{\text{max}} - d_{\text{min}}) \left[a - 0.15 \left(1 - \frac{d_{\text{min}}}{d_{\text{max}}} \right) \right] \tag{5.5-1}$$

where:

d_{min} = minimum dimension for that face of the column
d_{max} = maximum dimension for that face of the column
a = depends on support conditions indicated:
Large end fixed, small end unsupported, or simply supported, $a = 0.70$
Small end fixed, large end unsupported, or simply supported, $a = 0.30$
Both ends simply supported:
Tapered toward one end, $a = 0.50$
Tapered toward both ends, $a = 0.70$

For all other support conditions:

$$d = d_{\text{min}} + \frac{(d_{\text{max}} - d_{\text{min}})}{3} \tag{5.5-2}$$

5.6 SPACED COLUMNS

Spaced columns (Figure 5.6-1) are made of individual column pieces connected to one another by shear plates or split rings through end and spacer blocks. The capacity of the spaced column is the sum of capacities of the longitudinal pieces wherein the fasteners and blocks effectively increase the load carrying capacity of the full length pieces in their weak direction (buckling perpendicular to the wide faces of the longitudinal pieces).

Three limitations for individual longitudinal pieces and block spacing of a spaced column are as follows (Figure 5.6-1):

1. ℓ_1/d_1 must not exceed 80, where ℓ_1 is the distance between lateral supports that provide restraint perpendicular to the wide faces of the individual members,
2. ℓ_2/d_2 must not exceed 50, where ℓ_2 is the distance between lateral supports that provide restraint parallel to the wide faces of the individual members, and
3. ℓ_3/d_1 is limited to 40 where ℓ_3 is the distance between the centroid of connectors in an end block and the center of the spacer block.

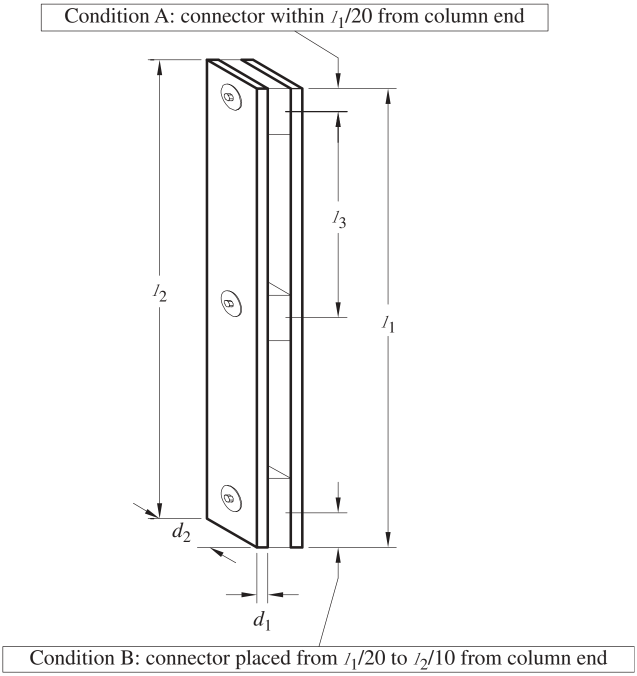


Figure 5.6-1 Spaced column.

There are seven requirements for the spacer and end blocks:

1. For condition A, the centroid of the split ring or shear plate connector, or group of connectors, in the end blocks, must be within $\ell_1/20$ from the end of the column.
2. For condition B, the centroid of the split ring or shear plate connector, or group of connectors, in the end blocks, must be between $\ell_1/10$ and $\ell_1/20$ from the end of the column.
3. Where a single spacer block is located in the middle tenth of the column length, ℓ_1, split ring or shear plate connectors are not required for this block.
4. If there are two or more spacer blocks, split ring or shear plate connectors are required. The distance between any two adjacent blocks must not exceed half the distance between the centers of the split ring or shear plate connectors in the end blocks.
5. For spaced columns used as compression members of a truss, a panel point that is stayed laterally is considered as the end of the spaced column. The portion of the web members, between individual pieces making up a spaced column, is permitted to be considered as the end block.
6. The thickness of spacer and end blocks must not be less than that of the individual longitudinal pieces of the spaced column. The thickness, width,

and length of spacer and end blocks must not be less than required for the split ring or shear plate connectors of size and number capable of carrying the loads required by the following section.

7. The split ring or shear plate connectors in each mutually contacting surface of end block and individual member at each end of a spaced column must be of size and number to provide a load capacity equal to the required cross-sectional area (square inches) of one of the individual members times the end spacer block constant, K_s, from Table 5.6-1.

TABLE 5.6-1 End Spacer Block Constant, K_s [2]

Species Group	Constant
A	$K_s = 9.55(\ell_1/d_1 - 11) \leq 468$
B	$K_s = 8.14(\ell_1/d_1 - 11) \leq 399$
C	$K_s = 6.73(\ell_1/d_1 - 11) \leq 330$
D	$K_s = 5.32(\ell_1/d_1 - 11) \leq 261$

Split ring and shear plate connector design is covered in Chapter 14 of this manual. The species groups listed in Table 5.6-1 are defined in the *National Design Specification*® [2] and are also found in Chapter 14.

The capacity of a spaced column, P', is the sum of the capacities of the individual members. For a spaced column with two identical longitudinal pieces, the capacity is given by Equation 5.6-1.

$$P' = 2F'_c(d_1)(d_2) \tag{5.6-1}$$

where:

P' = column capacity $F'_c = F^*_c C_P$
F^*_c = compression parallel-to-grain design value for the longitudinal pieces multiplied by all applicable adjustment factors except C_P
C_P = column stability factor calculated using Equation 3.4.3.9-1 except that F_{cE} is calculated using Equation 5.6-2:

$$F_{cE} = \frac{0.822 K_x E'_{\min}}{(\ell_e/d)^2} \tag{5.6-2}$$

where:

K_x = 2.5 for fixity condition A
K_x = 3.0 for fixity condition B

The effective length, ℓ_e, of the spaced column must be established by calculation or engineering judgment with the aid of Table 3.4.3.9.2-1, but in no case may it be taken to be less than the actual column length. The allowable compression parallel to grain value F_c' may not exceed the allowable compression-parallel to-grain for the individual pieces treated as simple columns using the slenderness ratio ℓ_2/d_2 (Figure 5.6-1). Where different grades, species, or thicknesses of members are used, the lesser value of F_c' determined for either member is applied to both.

Spaced columns may be loaded eccentrically or loaded to produce combined axial and flexural stresses in the strong direction of the pieces (parallel to d_2). There is no provision, however, to produce combined axial load and flexure or account for eccentric loading in the direction perpendicular to the wide faces of the pieces.

5.7 BUILT-UP COLUMNS

Built-up columns are made of two or more pieces of wood with their wide faces mechanically fastened together, as illustrated in Figure 5.7-1. The capacity of the built-up column with respect to potential buckling in the strong direction of the individual pieces is equal to the sum of the capacities of the individual members. The capacity of the column with respect to potential buckling in the weak direction of the pieces is greater than the sum of the individual pieces, though not as great as a solid piece with the same overall cross-sectional area. The strength of the column with respect to the weak axes of the pieces, where properly fastened, is given as a fraction of the capacity of a similar solid section.

There are five specific requirements of built-up columns:

1. Columns may be built up of from two to five pieces.
2. Each piece is to havea rectangular cross section and must be at least $1\frac{1}{2}$ in. thick (2 in. nominal).
3. All pieces must have the same wide face (depth) dimension.
4. All pieces must be in full contact for the full length of the column.
5. The pieces must be fastened together by nails and bolts according to the requirements of this section.

The effective length for the column may be determined using Table 3.4.3.9.2-1. For built-up columns, a value of C_P for each direction of potential buckling is calculated using Equation 5.7-1, and the smaller of the two is used to calculate F_c'. However, F_c' for built-up columns need not be taken as less than F_c' of the individual laminations designed as individual solid columns.

$$C_P = K_f \left\{ \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{(F_{cE}/F_c^*)}{c}} \right\} \qquad (5.7\text{-}1)$$

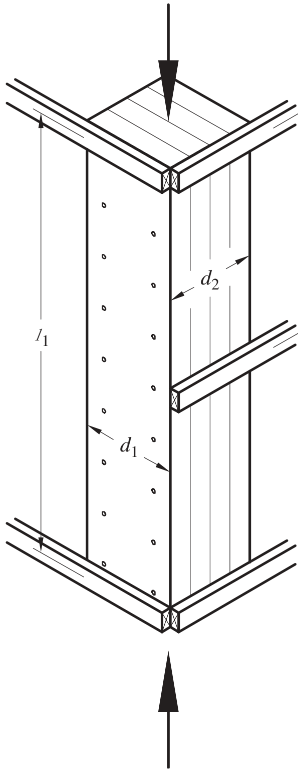


Figure 5.7-1 Built-up column.

where:

F_c^*, F_{cE}, and c are as previously defined for sawn columns,
K_f = 0.6 for built-up columns where ℓ_{e2}/d_2 is used to calculate F_{cE}, and the built-up column is *nailed* according to the rules in Section 5.7.1.
K_f = 0.75 for built-up columns where ℓ_{e2}/d_2 is used to calculate F_{cE} and the built-up column is *bolted* according to the rules in Section 5.7.2.
K_f = 1.0 for built-up columns where ℓ_{e1}/d_1 is used to calculate F_{cE} and the built-up column is either nailed or bolted according to the rules in the following sections.

5.7.1 Nailed Built-up Columns

Nine requirements for mechanically fastening the pieces of built-up columns using nails are as follows:

1. Adjacent nails must be driven from opposite sides of the column.
2. All nails must penetrate all laminations, including at least three-fourths of the thickness of the outermost piece.

3. The end distance for the nail group must be between 15D and 18D, inclusive, where D is the nail diameter.
4. The spacing of nails in a row must be greater than or equal to 20D but not greater than six times the thickness of the thinnest lamination, where the row direction is defined as the longitudinal direction of the column.
5. The spacing between rows of nails is to be between 10D and 20D, inclusive.
6. The edge distance for the group must be between 5D and 20D, inclusive.
7. Two or more rows must be provided where the depth of the individual pieces are greater than three times the thickness of the thinnest piece.
8. Where only one row of nails is required, they must be staggered across the width of the pieces.
9. Where three or more rows of nails are used, nails in adjacent rows must be staggered.

5.7.2 Bolted Built-up Columns

There are seven requirements for mechanically fastening the pieces of built-up columns using bolts:

1. A metal plate or washer must be provided between wood and bolt head and between wood and nut.
2. Nuts must be tightened to ensure that the faces of the adjacent pieces are in contact.
3. For softwood, the end distance for the bolt group must be between 7D and 8.4D, inclusive; and for hardwoods, between 5D and 6D, inclusive, where D is the bolt diameter.
4. The spacing between adjacent bolts in a row must be between 4D and 6 times the thickness of the thinnest lamination, inclusive.
5. The spacing between rows of bolts must be between 1.5D and 10D, inclusive.
6. The edge distance for the group must be between 1.5D and 10D, inclusive.
7. Two or more rows of bolts must be provided where the depth of the pieces is greater than three times the thickness of the thinnest piece.

5.8 COLUMNS WITH FLANGES

Glued laminated and built-up columns are usually square or rectangular, but can also be made with flanges, as shown in Figure 5.8-1. Because of fabrication and handling difficulties, these shapes are not common. The capacities of flanged columns may be limited by the buckling potential of outstanding flange pieces themselves or the difficulty in transferring shear forces from piece to piece, making up the column. The design of a column with flanges should include, at a minimum, the following steps:

1. The allowable compression parallel to grain, F'_c, based on the overall buckling potential of the column and appropriate column stability factor C_P, must be determined. Effective lengths and slenderness ratios must be determined for each direction of potential buckling. The slenderness ratio, ℓ_e/r, must not exceed 175 for either direction, where r is the radius of gyration, $r = \sqrt{I/A}$, and I is the respective area moment of inertia.
2. The allowable compression parallel to grain for the outstanding flange piece(s) should be determined by Equation 5.8-1 and the lesser of this stress and the allowable compressive stress for the whole section used.

$$F'_c \leq 0.030E'_{\min}\left(\frac{t}{b}\right)^2 \tag{5.8-1}$$

   where t and b are defined in Figure 5.8-1.

3. Approved fasteners or adhesives must be used to achieve the necessary shear transfer from piece to piece based on an assumed or known eccentricity or applied flexural load. The designer should require quality assurance in the form of special inspection and/or load capacity verification for such columns.

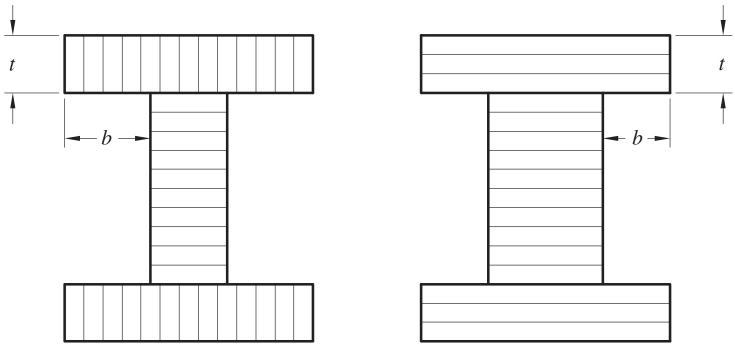


Figure 5.8-1 Columns with flanges.

5.9 TENSION MEMBERS

Tension members in timber construction include webs and chords in trusses, drag struts, and ties. Tension members must be proportioned such that the tension stresses under load do not exceed the allowable tension stresses (Equation 5.9-1). Since tension members are often loaded uniformly (equal tension along the entire length of the member), particular attention must be given to any losses in wood section such as due to notches and holes:

$$f_t = \frac{T}{A_n} \tag{5.9-1}$$

where:

T = tension force
A_n = net or effective area considering section loss due to holes and notches

5.10 CONCLUSION

Axially loaded members are common in timber construction. They are used as columns, struts, ties, and truss members. This chapter has presented design formulas and examples for the design of timber columns subject to centric axial loads. Types of timber columns addressed in the chapter include rectangular columns, round columns, spaced columns, built-up columns, and columns with flanges. A subsequent chapter will discuss timber members subject to combined axial and flexural stresses.

CHAPTER 6

TIMBER BEAM-COLUMNS AND TENSION BEAMS

6.1 INTRODUCTION

Members subject to the combined action of flexural and axial loading are commonly encountered in timber construction. Such members include truss chords, columns subject to side loads, and any axial members loaded eccentrically. Members subject to combined compression and axial loads are commonly referred to as *beam-columns*. In this manual, members subject to combined tension and flexure are referred to as *tension beams*.

6.1.1 Application of Volume Factor for Beam-Columns

Where a member is subject to both bending and axial compression, the combination of the compressive stresses and the flexural tensile stresses results in a smaller compression stress on the flexural tension side and greater compression on the flexural compression side. This affects the manner in which the volume factor, C_V, is applied.

The volume factor, C_V, reflects the effect of member size on the flexural tension strength. If no net tension exists in the section, then the volume effect need not be considered. Since axial compression lessens the net tension on the member, the allowable bending stress (as controlled by flexural tension) is increased by the amount of the compressive stress, f_c. However, the allowable bending stress, F_b', cannot exceed the allowable flexural compression stress, $F_b^* C_L$, on the section. The allowable bending stress for beam-columns is, therefore, determined by Equation 6.1.1-1.

$$F'_{bx} = F^{*}_{bx} C_V + f_c \leq F^{*}_{bx} C_L \qquad (6.1.1\text{-}1)$$

Where $f_c \geq F^{*}_{b}(1 - C_V)$ the flexural compression stress, $F^{*}_{b} C_L$, will control.

6.1.2 End Eccentricity

In actual practice, the loads transmitted to columns from beams may be eccentric, especially when a column supports the end of a beam. The eccentricity of axial loads should be determined from the bearing or other loading conditions and accounted for with the preceding equations. Where the eccentricity is unknown or uncontrolled, a minimum eccentricity of one-sixth of the column dimension parallel to the supported member is recommended. Framing should be detailed to produce centric loading of columns in as much as possible.

6.1.3 Unbalanced or Cambered Glulam Members

Where unbalanced glued laminated timbers are used for members subject to combined axial and flexure loads, the appropriate strong axis bending design value will depend on the direction of bending (positive or negative) relative to the beam layup. Timbers with camber should generally not be used unless the eccentricity introduced by the camber is considered in the design.

6.2 GENERAL EQUATION FOR BEAM-COLUMNS

Equation 6.2-1 can be used for any combination of axial compression and flexural loading, including eccentric axial loading. The first term in the equation represents the compression component of the combined stresses. The second term represents the strong axis bending component. The third term accounts for weak axis bending. For several common cases, the equation can be simplified as shown in subsequent sections.

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1} + f_c(6e_1/d_1)[1 + (0.234 f_c/F_{cE1})]}{F'_{b1}[1 - (f_c/F_{cE1})]} + \frac{f_{b2} + f_c(6e_2/d_2)\left(1 + 0.234(f_c/F_{cE2}) + 0.234\left[\dfrac{f_{b1} + f_c(6e_1/d_1)}{F_{bE}}\right]^2\right)}{F'_{b2}\left(1 - (f_c/F_{cE2}) - \left[\dfrac{f_{b1} + f_c(6e_1/d_1)}{F_{bE}}\right]^2\right)} \leq 1.0 \qquad (6.2\text{-}1)$$

where:

f_c = axial compression stress
f_{b1} = strong axis bending stress
f_{b2} = weak axis bending stress
F'_c = adjusted compression stress
F'_{b1} = strong axis adjusted bending stress
F'_{b2} = weak axis adjusted bending stress
F_{bE} = critical buckling design value for bending
F_{cE1} = critical buckling design value for strong axis buckling
F_{cE2} = critical buckling design value for weak axis buckling
d_1 = wide face dimension
d_2 = narrow face dimension
e_1 = strong axis load eccentricity
e_2 = weak axis load eccentricity

The critical buckling design values for bending and for the strong and weak directions in compression are:

$$F_{cE1} = \frac{0.822E'_{\min}}{(\ell_{e1}/d_1)^2} \tag{6.2-2}$$

$$F_{cE2} = \frac{0.822E'_{\min}}{(\ell_{e2}/d_2)^2} \tag{6.2-3}$$

$$F_{bE} = \frac{1.20E'_{\min}}{R_B^2} \tag{6.2-4}$$

where $E'_{\min}$ in each case is associated with the direction of potential buckling.

In all cases, the compressive axial stress and compressive flexural stress must be less than the corresponding critical buckling design value (Equations 6.2-5, 6.2-6, 6.2-7):

$$f_c < F_{cE1} \tag{6.2-5}$$

$$f_c < F_{cE2} \tag{6.2-6}$$

$$f_{b1} < F_{bE} \tag{6.2-7}$$

Furthermore, the Equation 6.2-8 must be checked to make sure that overstress in weak axis bending is not masked by excess capacity in compression and strong axis bending.

$$\frac{f_c}{F_{cE2}} + \left(\frac{f_{b1} + f_c(6e_1/d_1)}{F_{bE}}\right)^2 \leq 1.0 \tag{6.2-8}$$

EXAMPLE 6.2-1 ECCENTRIC AXIAL COMPRESSION AND SIDE LOAD BENDING

Given: $6\frac{3}{4}$ in. × $10\frac{1}{2}$ in. × 15 ft, pressure-treated, Douglas fir, glulam column is subject to the loading illustrated in Figure 6.2-1. The column is braced to prevent movement in the x and y directions at both the top and bottom. $P =$ 17,000 lb (including 7,000 lb dead load and 10,000 lb snow load); $e_1 = 2$ in.; and $\omega = 200$ lb/ft (wind load only). The column is subject to wet service conditions.

Wanted: Determine the adequacy of Combination 2 DF column with respect to the load combination $D + 0.75S + 0.75W$.

Approach: Design values from AITC 117 will be used with Equation 6.2-1 (with $f_{b2} = 0$ and $e_2 = 0$). Wet service factors will be obtained from Table 3.4.2-1. A load duration factor of $C_D = 1.6$ will be used.

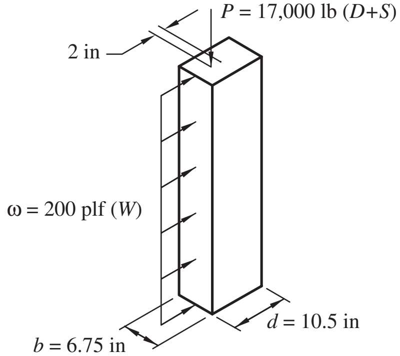


Figure 6.2-1 Beam-column—Example 6.2-1.

Solution:

Design values:

$$F'_c = F^*_c C_P = F_c C_D C_M C_t C_P$$
$$F'_c = 1950 \text{ psi}(1.6)(0.73)(1.0)C_P$$
$$F'_c = (2278 \text{ psi})C_P$$

$$F'_{bx} = F^*_{bx} C_V + f_c \leq F^*_{bx} C_L$$
$$F'_{bx} = F_{bx} C_D C_M C_t C_V + f_c \leq F_{bx} C_D C_M C_t C_L$$
$$F'_{bx} = (1700 \text{ psi})(1.6)(0.8)(1.0)C_V + f_c \leq F_{bx}(1.6)(0.8)(1.0)C_L$$
$$F'_{bx} = (2176 \text{ psi})C_V + f_c \leq (2176 \text{ psi})C_L$$

$$E'_{\min} = E_{\min} C_M C_t = 850{,}000 \text{ psi}(0.833)(1.0) = 708{,}000 \text{ psi}$$

Section properties:

$$A = bd = (6.75\text{ in})(10.5\text{ in}) = 70.88\text{ in}^2$$

$$S_x = \frac{bd^2}{6} = \frac{(6.75\text{ in})(10.5\text{ in})^2}{6} = 124.0\text{ in}^3$$

Bending moment from side load:

$$M_W = 0.75\frac{\omega\ell^2}{8} = 0.75\frac{(200\text{ lb/ft})(15\text{ ft})^2}{8} = 4219\text{ lb-ft} = 50{,}625\text{ lb-in}$$

Bending stress from side load:

$$f_b = \frac{M_W}{S} = \frac{50{,}625\text{ lb-in}}{124\text{ in}^3} = 408\text{ psi}$$

Compression stress:

$$f_c = \frac{P}{A} = \frac{P_D + 0.75P_S}{A} = \frac{7000\text{ lb} + (0.75)10{,}000\text{ lb}}{70.88\text{ in}^2} = 205\text{ psi}$$

Effective length of column ($K_e = 1.0$):

$$\ell_e = K_e\ell = 1.0(15\text{ ft}) = 15\text{ ft} = 180\text{ in}$$

Column slenderness ratios:

$$\frac{\ell_{e2}}{d_2} = \frac{180\text{ in}}{6.75\text{ in}} = 26.7 \leq 50 \qquad \therefore\ \text{OK}$$

$$\frac{\ell_{e1}}{d_1} = \frac{180\text{ in.}}{10.5\text{ in}} = 17.1 \leq 50 \qquad \therefore\ \text{OK}$$

Critical buckling design values for compression (Equation 3.4.3.9-2):

$$F_{cE2} = \frac{0.822E'_{y\text{ min}}}{(\ell_{e2}/d_2)^2} = \frac{0.822(708{,}050\text{ psi})}{(26.7)^2} = 816\text{ psi}$$

$$F_{cE1} = \frac{0.822E'_{x\text{ min}}}{(\ell_{e1}/d_1)^2} = \frac{0.822(708{,}050\text{ psi})}{(17.1)^2} = 1990\text{ psi}$$

The lower critical buckling design value, $F_{cE2} = 816$ psi, is used to determine the column stability factor.

Column stability factor (Equation 3.4.3.9-1):

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{(F_{cE}/F_c^*)}{c}}$$

$$C_P = \frac{1 + \left(\frac{818 \text{ psi}}{2278 \text{ psi}}\right)}{1.8} - \sqrt{\left(\frac{1 + \left(\frac{818 \text{ psi}}{2278 \text{ psi}}\right)}{1.8}\right)^2 - \frac{\left(\frac{818 \text{ psi}}{2278 \text{ psi}}\right)}{0.9}}$$

$$C_P = 0.342$$

Adjusted compression design value:

$$F_c' = F_c^* C_P = 2278 \text{ psi}(0.342) = 778 \text{ psi}$$

Unbraced length for lateral-torsional buckling:

$$\ell_u = \ell = 15 \text{ ft} = 180 \text{ in}$$

Effective length (Table 3.4.3.1.1-1, $\ell_u/d_1 = 17$):

$$\ell_e = 1.63\ell_u + 3d = 1.63(180 \text{ in}) + 3(10.5 \text{ in}) = 325 \text{ in}$$

Beam slenderness ratio (Equation 3.4.3.1.1-1):

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(325 \text{ in})(10.5 \text{ in})}{(6.75 \text{ in})^2}} = 8.65 \le 50 \qquad \therefore \text{ OK}$$

Critical buckling design value for beam (Equation 3.4.3.1-2):

$$F_{bE} = \frac{1.20(708{,}050 \text{ psi})}{(8.65)^2} = 11{,}350 \text{ psi}$$

Beam stability factor (Equation 3.4.3.1-1):

$$C_L = \frac{1 + (F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1 + (F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{(F_{bE}/F_b^*)}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{11{,}350 \text{ psi}}{2176 \text{ psi}}\right)}{1.9} - \sqrt{\left(\frac{1 + \left(\frac{11{,}350 \text{ psi}}{2176 \text{ psi}}\right)}{1.9}\right)^2 - \frac{\left(\frac{11{,}350 \text{ psi}}{2176 \text{ psi}}\right)}{0.95}}$$

$$C_L = 0.988$$

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125 \text{ in}}{b}\right)^{\frac{1}{10}} \left(\frac{12 \text{ in}}{d}\right)^{\frac{1}{10}} \left(\frac{21 \text{ ft}}{L}\right)^{\frac{1}{10}}$$

$$C_V = \left(\frac{5.125 \text{ in}}{6.75 \text{ in}}\right)^{\frac{1}{10}} \left(\frac{12 \text{ in}}{10.5 \text{ in}}\right)^{\frac{1}{10}} \left(\frac{21 \text{ ft}}{15 \text{ ft}}\right)^{\frac{1}{10}}$$

$$C_V = 1.0$$

Adjusted bending design value (Equation 6.1.1-1):

$$F'_{bx} = F^*_{bx} C_V + f_c \leq F^*_{bx} C_L$$

$$F'_{bx} = (2176 \text{ psi}) C_V + f_c \leq (2176 \text{ psi}) C_L$$

$$F'_{bx} = 2176 \text{ psi}(1.0) + 205 \text{ psi} \leq (2176 \text{ psi})(0.988)$$

$$F'_{bx} = (2176 \text{ psi})(0.988)$$

$$F'_{bx} = 2151 \text{ psi}$$

Stresses vs. critical buckling design values (Equations 6.2-5, 6.2-6, 6.2-7):

$$f_c = 205 \text{ psi} < F_{cE1} = 1990 \text{ psi} \qquad \therefore \text{ OK}$$

$$f_c = 205 \text{ psi} < F_{cE2} = 816 \text{ psi} \qquad \therefore \text{ OK}$$

$$f_{b1} = 408 \text{ psi} < F_{bE} = 11{,}348 \text{ psi} \qquad \therefore \text{ OK}$$

Combined stresses (Equation 6.2-1):

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1} + f_c(6e_1/d_1)[1 + (0.234 f_c/F_{cE1})]}{F'_{b1}[1 - (f_c/F_{cE1})]} = \dots$$

$$\dots = \left[\frac{205 \text{ psi}}{778 \text{ psi}}\right]^2 + \dots$$

$$\dots + \frac{408 \text{ psi} + (205 \text{ psi})\left(\frac{6(2 \text{ in})}{10.5 \text{ in}}\right)\left[1 + 0.234\left(\frac{205 \text{ psi}}{1990 \text{ psi}}\right)\right]}{(2151 \text{ psi})\left[1 - \frac{205 \text{ psi}}{1990 \text{ psi}}\right]} = \dots$$

$$\dots = 0.405 \leq 1.0 \qquad \therefore \text{ OK}$$

Answer: The section is adequate for the $D + 0.75S + 0.75W$ load combination.

Discussion: The member should also be checked for adequacy for any other applicable load combinations. Since the combined stress equation resulted in a value significantly less than 1.0, a smaller section could be investigated.

6.3 CENTRIC AXIAL COMPRESSION AND SIDE LOAD BENDING ABOUT BOTH AXES

In the case of centric axial compression plus bending with respect to either direction, Equation 6.2-1 reduces to Equation 6.3-1, and Equation 6.2-8 reduces to Equation 6.3-2.

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}[1 - (f_c/F_{cE1})]} + \frac{f_{b2}}{F'_{b2}[1 - (f_c/F_{cE2}) - (f_{b1}/F_{bE})^2]} \leq 1.0 \quad (6.3\text{-}1)$$

$$\frac{f_c}{F_{cE2}} + \left(\frac{f_{b1}}{F_{bE}}\right)^2 \leq 1.0 \quad (6.3\text{-}2)$$

EXAMPLE 6.3-1 AXIAL COMPRESSION AND SIDE LOAD BENDING ABOUT BOTH AXES

Given: An $8\frac{3}{4}$ in. × 24 in. × 30 ft long, glued laminated timber beam used in a dry location is loaded about the x–x axis with a uniform load consisting of a dead load of 200 lb/ft and snow load of 400 lb/ft. A 2500 lb wind load, P_1, is applied from the side at the midpoint. In addition, the member is loaded in compression with a 30,000 lb load, P, consisting of a 10,000 lb dead load and a 20,000 lb snow load (Figure 6.3-1). The member is supported at the ends to prevent rotation, but is not braced along its length.

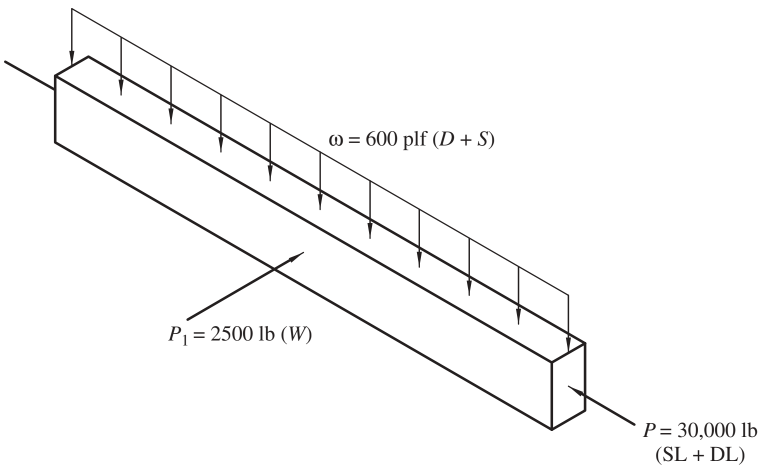


Figure 6.3-1 Beam-column—Example 6.3-1.

Wanted: Determine the adequacy of the 24F-1.8E Douglas Fir glued laminated timber to support the combined loads from $D + 0.75S + 0.75W$.

Approach: Design values for the 24F-1.8E stress class will be used. Equation 6.3-1 will be used to evaluate the section.

Solution:

Design values:

$$F'_{bx} = F^*_{bx} C_V + f_c \le F^*_{bx} C_L$$

$$F'_{bx} = F_{bx} C_D C_m C_t C_V + f_c \le F_{bx} C_D C_m C_t C_L$$

$$F'_{bx} = (2400 \text{ psi})(1.6)(1.0)(1.0) C_V + f_c \le (2400 \text{ psi})(1.6)(1.0)(1.0) C_L$$

$$F'_{bx} = (3840 \text{ psi}) C_V + f_c \le (3840 \text{ psi}) C_L$$

$$F'_{by} = F_{by} C_D C_M C_t C_{fu}$$

$$F'_{by} = (1450 \text{ psi})(1.6)(1.0)(1.0) C_{fu}$$

$$F'_{by} = (2320 \text{ psi}) C_{fu}$$

$$F'_c = F^*_c C_P = F_c C_D C_M C_t C_P$$

$$F'_c = F^*_c C_P = (1600 \text{ psi})(1.6)(1.0)(1.0) C_P$$

$$F'_c = F^*_c C_P = (2560 \text{ psi}) C_P$$

$$E'_{x \text{ min}} = E_{x \text{ min}} C_M C_t$$

$$E'_{x \text{ min}} = (950{,}000 \text{ psi})(1.0)(1.0)$$

$$E'_{x \text{ min}} = 950{,}000 \text{ psi}$$

$$E'_{y \text{ min}} = E_{y \text{ min}} C_M C_t$$

$$E'_{y \text{ min}} = (850{,}000 \text{ psi})(1.0)(1.0)$$

$$E'_{y \text{ min}} = 850{,}000 \text{ psi}$$

Section properties:

$$A = bd = (8.75 \text{ in})(24 \text{ in}) = 210 \text{ in}^2$$

$$S_x = \frac{bd^2}{6} = \frac{(8.75 \text{ in})(24 \text{ in})^2}{6} = 840 \text{ in}^3$$

$$S_y = \frac{db^2}{6} = \frac{(24 \text{ in})(8.75 \text{ in})^2}{6} = 306 \text{ in}^3$$

Compression stress:

$$f_c = \frac{P_D + 0.75P_S}{A}$$

$$f_c = \frac{10{,}000 \text{ lb} + 0.75(20{,}000 \text{ lb})}{210 \text{ in}^2}$$

$$f_c = 119 \text{ psi}$$

Strong axis bending stress:

$$f_{bx} = \frac{M_x}{S_x} = \frac{(\omega_D + 0.75\omega_S)\ell^2}{8S_x}$$

$$f_{bx} = \frac{[200 \text{ plf} + 0.75(400 \text{ plf})](30 \text{ ft})^2(12 \text{ in/ft})}{8(840 \text{ in}^3)}$$

$$f_{bx} = 804 \text{ psi}$$

Weak axis bending stress:

$$f_{by} = \frac{M_y}{S_y} = 0.75\left(\frac{P_1 L}{4S_y}\right)$$

$$f_{by} = 0.75\frac{(2500 \text{ lb})(30 \text{ ft})(12 \text{ in/ft})}{4(306 \text{ in}^3)}$$

$$f_{by} = 551 \text{ psi}$$

Unbraced length for lateral-torsional buckling:

$$\ell_u = 30 \text{ ft} = 360 \text{ in}$$

Effective length for lateral-torsional buckling (Table 3.4.3.1.1-1):

$$\frac{\ell_u}{d} = \frac{360 \text{ in}}{24 \text{ in}} = 15.0$$

$$\ell_e = 1.63\ell_u + 3d = 1.63(360 \text{ in}) + 3(24 \text{ in}) = 659 \text{ in}$$

Beam slenderness ratio (Equation 3.4.3.1.1-1):

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(659 \text{ in})(24 \text{ in})}{(8.75 \text{ in})^2}} = 14.37$$

Critical buckling design value for bending (Equation 3.4.3.1-2):

$$F_{bE} = \frac{1.20(850{,}000 \text{ psi})}{(14.37)^2} = 4939 \text{ psi}$$

Beam stability factor (Equation 3.4.3.1-1):

$$C_L = \frac{1 + (F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1 + (F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{(F_{bE}/F_b^*)}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{4939\text{ psi}}{3840\text{ psi}}\right)}{1.9} - \sqrt{\left(\frac{1 + \left(\frac{4939\text{ psi}}{3840\text{ psi}}\right)}{1.9}\right)^2 - \frac{\left(\frac{4939\text{ psi}}{3840\text{ psi}}\right)}{0.95}}$$

$$C_L = 0.897$$

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125\text{ in}}{b}\right)^{\frac{1}{10}} \left(\frac{12\text{ in}}{d}\right)^{\frac{1}{10}} \left(\frac{21\text{ ft}}{L}\right)^{\frac{1}{10}} \leq 1.0$$

$$C_V = \left(\frac{5.125\text{ in}}{8.75\text{ in}}\right)^{\frac{1}{10}} \left(\frac{12\text{ in}}{24\text{ in}}\right)^{\frac{1}{10}} \left(\frac{21\text{ ft}}{30\text{ ft}}\right)^{\frac{1}{10}} = 0.853 \leq 1.0$$

$$C_V = 0.853$$

Adjusted *x-x* bending design value (Equation 6.1.1-1):

$$F'_{bx} = F^*_{bx}C_V + f_c \leq F^*_{bx}C_L$$

$$F'_{bx} = 3840\text{ psi} + f_c \leq (3840\text{ psi})C_L$$

$$F'_{bx} = 3840\text{ psi}(0.853) + 119\text{ psi} \leq (3840\text{ psi})(0.897)$$

$$F'_{bx} = 3394\text{ psi} \leq 3444\text{ psi}$$

$$F'_{bx} = 3394\text{ psi}$$

Flat use factor (Equation 3.4.3.5-1):

$$C_{fu} = \left(\frac{12\text{ in}}{d}\right)^{\frac{1}{9}} = \left(\frac{12\text{ in}}{8.75\text{ in}}\right)^{\frac{1}{9}} = 1.036$$

Adjusted *y-y* bending design value:

$$F'_{by} = F_{by}(C_D)(C_{fu}) = 1450\text{ psi}(1.60)(1.036) = 2404\text{ psi}$$

Effective column length:

$$\ell_e = K_e\ell_u = 1.0(30\text{ ft}) = 30\text{ ft} = 360\text{ in}$$

Column slenderness ratios:

$$\frac{\ell_e}{d_1} = \frac{360 \text{ in}}{24 \text{ in}} = 15.0 \leq 50 \qquad \therefore \text{OK}$$

$$\frac{\ell_e}{d_2} = \frac{360 \text{ in}}{8.75 \text{ in}} = 41.1 \leq 50 \qquad \therefore \text{OK}$$

Critical buckling design values for column (Equation 3.4.3.9-2):

$$F_{cEx} = \frac{0.822E'_{x\ \min}}{(\ell_e/d_1)^2} = \frac{(0.822)(950{,}000 \text{ psi})}{(15.0)^2} = 3471 \text{ psi}$$

$$F_{cEy} = \frac{0.822E'_{y\ \min}}{(\ell_e/d_2)^2} = \frac{(0.822)(850{,}000 \text{ psi})}{(41.1)^2} = 413 \text{ psi}$$

Buckling about the y-y axis will govern with $F_{cEy} = 413$ psi.

Column stability factor (Equation 3.4.3.9-1):

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{(F_{cE}/F_c^*)}{c}}$$

$$C_p = \frac{1 + \left(\dfrac{413 \text{ psi}}{2560 \text{ psi}}\right)}{2(0.9)} - \sqrt{\left[\frac{1 + \left(\dfrac{413 \text{ psi}}{2560 \text{ psi}}\right)}{2(0.9)}\right]^2 - \frac{\left(\dfrac{413 \text{ psi}}{2560 \text{ psi}}\right)}{0.9}}$$

$$C_p = 0.158$$

Adjusted compression design value:

$$F'_c = F_c^* C_P = (2560 \text{ psi})0.158 = 404 \text{ psi}$$

Stresses vs. critical buckling design values (Equations 6.2-5, 6.2-6, 6.2-7):

$$f_c = 119 \text{ psi} < F_{cE1} = F_{cEx} = 3471 \text{ psi} \qquad \therefore \text{OK}$$

$$f_c = 119 \text{ psi} < F_{cE2} = F_{cEy} = 314 \text{ psi} \qquad \therefore \text{OK}$$

$$f_{b1} = 804 \text{ psi} < F_{bE} = 4939 \text{ psi} \qquad \therefore \text{OK}$$

Combined stresses (Equation 6.3-1):

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}[1 - (f_c/F_{cE1})]} + \frac{f_{b2}}{F'_{b2}[1 - (f_c/F_{cE2}) - (f_{b1}/F_{bE})^2]} = \ldots$$

$$\ldots = \left(\frac{119\text{ psi}}{404\text{ psi}}\right)^2 + \frac{804\text{ psi}}{(3394\text{ psi})\left[1-\left(\frac{119\text{ psi}}{3471\text{ psi}}\right)\right]} + \ldots$$

$$\ldots + \frac{551\text{ psi}}{(2404\text{ psi})\left[1-\left(\frac{119\text{ psi}}{413\text{ psi}}\right)-\left(\frac{804\text{ psi}}{4939\text{ psi}}\right)^2\right]} = \ldots$$

$$\ldots = 0.666 \le 1.0 \qquad \therefore \text{OK}$$

Combined stresses (Equation 6.3-2):

$$\frac{f_c}{F_{cE2}} + \left(\frac{f_{b1}}{F_{bE}}\right)^2 = \frac{119\text{ psi}}{413\text{ psi}} + \left(\frac{804\text{ psi}}{4939\text{ psi}}\right)^2 = 0.314 \le 1.0 \qquad \therefore \text{OK}$$

Answer: The $8\frac{3}{4}$ in × 24 in 24F-1.8E DF member is adequate to carry the stated loads.

6.4 CENTRIC AXIAL COMPRESSION AND SIDE LOAD BENDING ABOUT STRONG AXIS ONLY

In the case of centric axial compression and bending about the strong axis only, Equation 6.2-1 reduces to Equation 6.4-1:

$$\left[\frac{f_c}{F_c'}\right]^2 + \frac{f_{b1}}{F_{b1}'[1-(f_c/F_{cE1})]} \le 1.0 \qquad (6.4\text{-}1)$$

EXAMPLE 6.4-1 AXIAL COMPRESSION AND STRONG AXIS BENDING

Given: A simply supported 5 in. × 12-$\frac{3}{8}$ in. 20F-V3 SP glulam beam-column spans 20 ft and is loaded with a mid-span point load of 2400 lb inducing strong axis bending and a centric axial compressive load of 20,000 lb (Figure 6.4-1). The member is laterally braced at the ends and at the point of load application. The member is subject to normal duration of load and dry conditions of use.

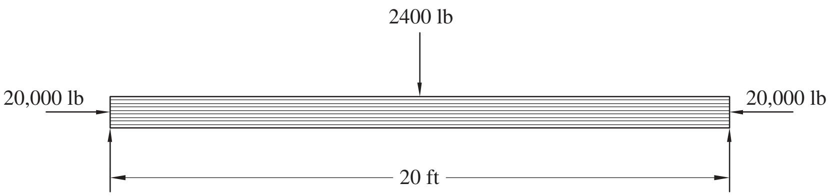


Figure 6.4-1 Beam-column—Example 6.4-1.

Wanted: Determine if the member is adequate to support the given loads.

Solution:

Design values:

$$F'_{bx} = F^*_{bx} C_V + f_c \leq F^*_{bx} C_L$$

$$F'_{bx} = F_{bx} C_D C_m C_t C_V + f_c \leq F_{bx} C_D C_m C_t C_L$$

$$F'_{bx} = (2000 \text{ psi})(1.0)(1.0)(1.0) C_V + f_c \leq (2000 \text{ psi})(1.0)(1.0)(1.0) C_L$$

$$F'_{bx} = (2000 \text{ psi}) C_V + f_c \leq (2000 \text{ psi}) C_L$$

$$F'_c = F^*_c C_P = F_c C_D C_M C_t C_P$$

$$F'_c = F^*_c C_P = (1400 \text{ psi})(1.0)(1.0)(1.0) C_P$$

$$F'_c = F^*_c C_P = (1400 \text{ psi}) C_P$$

$$E'_{x\ \min} = E_{x\ \min} C_M C_t$$

$$E'_{x\ \min} = (790{,}000 \text{ psi})(1.0)(1.0)$$

$$E'_{x\ \min} = 790{,}000 \text{ psi}$$

$$E'_{y\ \min} = E_{y\ \min} C_M C_t$$

$$E'_{y\ \min} = (790{,}000 \text{ psi})(1.0)(1.0)$$

$$E'_{y\ \min} = 790{,}000 \text{ psi}$$

Section properties:

$$A = bd = (5.0 \text{ in})(12.375 \text{ in}) = 61.9 \text{ in}^2$$

$$S_x = \frac{bd^2}{6} = \frac{(5.0 \text{ in})(12.375 \text{ in})^2}{6} = 128 \text{ in}^3$$

Compression stress:

$$f_c = \frac{P}{A}$$

$$f_c = \frac{20{,}000 \text{ lb}}{61.9 \text{ in}^2}$$

$$f_c = 323 \text{ psi}$$

Bending stress:

$$f_{bx} = \frac{M_x}{S_x} = \frac{P_1 \ell}{4 S_x}$$

$$f_{bx} = \frac{(2400 \text{ lb})(240 \text{ in})}{4(128 \text{ in}^3)}$$

$$f_{bx} = 1130 \text{ psi}$$

Unbraced length for lateral-torsional buckling:

$$\ell_u = \frac{\ell}{2} = \frac{20 \text{ ft}}{2} = 10 \text{ ft} = 120 \text{ in}$$

Effective length for lateral-torsional buckling (Table 3.4.3.1.1-1):

$$\ell_e = 1.11\ell_u = 1.11(120 \text{ in}) = 133.2 \text{ in}$$

Beam slenderness ratio (Equation 3.4.3.1.1-1):

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(133.2 \text{ in})(12.375 \text{ in})}{(5.0 \text{ in})^2}} = 8.12$$

Critical buckling design value for bending (Equation 3.4.3.1-2):

$$F_{bE} = \frac{1.20(790{,}000 \text{ psi})}{(8.12)^2} = 14{,}400 \text{ psi}$$

Beam stability factor (Equation 3.4.3.1-1):

$$C_L = \frac{1 + (F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1 + (F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{(F_{bE}/F_b^*)}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{14400 \text{ psi}}{2000 \text{ psi}}\right)}{1.9} - \sqrt{\left(\frac{1 + \left(\frac{14400 \text{ psi}}{2000 \text{ psi}}\right)}{1.9}\right)^2 - \frac{\left(\frac{14400 \text{ psi}}{2000 \text{ psi}}\right)}{0.95}}$$

$$C_L = 0.992$$

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125 \text{ in}}{b}\right)^{\frac{1}{20}} \left(\frac{12 \text{ in}}{d}\right)^{\frac{1}{20}} \left(\frac{21 \text{ ft}}{L}\right)^{\frac{1}{20}} \leq 1.0$$

$$C_V = \left(\frac{5.125 \text{ in}}{5.0 \text{ in}}\right)^{\frac{1}{20}} \left(\frac{12 \text{ in}}{12.375 \text{ in}}\right)^{\frac{1}{20}} \left(\frac{21 \text{ ft}}{20 \text{ ft}}\right)^{\frac{1}{20}} = 1.0 \leq 1.0$$

$$C_V = 1.0$$

Adjusted *x-x* bending design value (Equation 6.1.1-1):

$$F'_{bx} = F^*_{bx} C_V + f_c \leq F^*_{bx} C_L$$

$$F'_{bx} = (2000 \text{ psi}) C_V + f_c \leq (2000 \text{ psi}) C_L$$

$$F'_{bx} = (2000 \text{ psi})(1.0) + 323 \text{ psi} \leq (2000 \text{ psi})(0.992)$$

$$F'_{bx} = 2320 \text{ psi} \leq 1980 \text{ psi}$$

$$F'_{bx} = 1980 \text{ psi}$$

Effective column lengths:

$$\ell_{e1} = K_e \ell_{u1} = 1.0(20 \text{ ft}) = 20 \text{ ft} = 240 \text{ in}$$

$$\ell_{e2} = K_e \ell_{u2} = 1.0\,(10 \text{ ft}) = 10 \text{ ft} = 120 \text{ in}$$

Column slenderness ratios:

$$\frac{\ell_{e1}}{d_1} = \frac{240 \text{ in}}{12.375 \text{ in}} = 19.4 \leq 50 \qquad \therefore \text{ OK}$$

$$\frac{\ell_{e2}}{d_2} = \frac{120 \text{ in}}{5.0 \text{ in}} = 24.0 \leq 50 \qquad \therefore \text{ OK}$$

Critical buckling design values for column (Equation 3.4.3.9-2):

$$F_{cEx} = \frac{0.822 E'_{x \text{ min}}}{(\ell_e/d_1)^2} = \frac{(0.822)(790{,}000 \text{ psi})}{(19.4)^2} = 1720 \text{ psi}$$

$$F_{cEy} = \frac{0.822 E'_{y \text{ min}}}{(\ell_e/d_2)^2} = \frac{(0.822)(790{,}000 \text{ psi})}{(24.0)^2} = 1130 \text{ psi}$$

Buckling about the y-y axis will govern with $F_{cEy} = 1130$ psi.

Column stability factor (Equation 3.4.3.9-1):

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{(F_{cE}/F_c^*)}{c}}$$

$$C_p = \frac{1 + \left(\dfrac{1130 \text{ psi}}{1400 \text{ psi}}\right)}{2(0.9)} - \sqrt{\left[\frac{1 + \left(\dfrac{1130 \text{ psi}}{1400 \text{ psi}}\right)}{2(0.9)}\right]^2 - \frac{\left(\dfrac{1130 \text{ psi}}{1400 \text{ psi}}\right)}{0.9}}$$

$$C_p = 0.671$$

Adjusted compression design value:

$$F'_c = F_c^* C_P = (1400 \text{ psi})0.671 = 939 \text{ psi}$$

Stresses vs. critical buckling design values (Equations 6.2-5, 6.2-6, 6.2-7):

$$f_c = 323 \text{ psi} < F_{cE1} = F_{cEx} = 1720 \text{ psi} \qquad \therefore \text{ OK}$$

$$f_c = 323 \text{ psi} < F_{cE2} = F_{cEy} = 1130 \text{ psi} \qquad \therefore \text{ OK}$$

$$f_{b1} = 1130 \text{ psi} < F_{bE} = 14{,}400 \text{ psi} \qquad \therefore \text{ OK}$$

Combined stresses (Equation 6.4-1):

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}[1 - (f_c/F_{cE1})]} \leq 1.0$$

$$\left[\frac{323 \text{ psi}}{939 \text{ psi}}\right]^2 + \frac{1130 \text{ psi}}{(1980 \text{ psi})\left[1 - \left(\dfrac{323 \text{ psi}}{1720 \text{ psi}}\right)\right]} \overset{?}{\leq} 1.0$$

$$0.821 \leq 1.0 \qquad \therefore \text{ OK}$$

Answer: The 5 in. × $12\frac{3}{8}$ in. 20F-V3 SP member is adequate to carry the stated loads.

Discussion: the SP 20F-V3 layup is tabulated under the 20F-1.5E stress class in AITC 117 [1] Table A1-Expanded and Table 5A in the *NDS*® *Supplement* [2]. A number of the design values associated with that stress class are significantly lower than the ones used in the design checks above. As such, in this example, it would not be appropriate to specify Stress Class 20F-1.5E SP unless the member is reevaluated with the lower design values and still found suitable.

6.5 ECCENTRIC AXIAL COMPRESSION ONLY

In the case of eccentric axial compression without side-load bending, Equation 6.2-1 becomes Equation 6.5-1, and Equation 6.2-8 reduces to Equation 6.5-2.

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_c(6e_1/d_1)[1 + 0.234(f_c/F_{cE1})]}{F'_{b1}[1 - (f_c/F_{cE1})]} + \ldots$$

$$\ldots + \frac{f_c(6e_2/d_2)\left(1 + 0.234(f_c/F_{cE2}) + 0.234\left[\dfrac{f_c(6e_1/d_1)}{F_{bE}}\right]^2\right)}{F'_{b2}\left(1 - (f_c/F_{cE2}) - \left[\dfrac{f_c(6e_1/d_1)}{F_{bE}}\right]^2\right)} \leq 1.0 \quad (6.5\text{-}1)$$

$$\frac{f_c}{F_{cE2}} + \left(\frac{f_c(6e_1/d_1)}{F_{bE}}\right)^2 \leq 1.0 \quad (6.5\text{-}2)$$

6.6 AXIAL COMPRESSION ECCENTRICITY IN STRONG DIRECTION ONLY

If the axial compression load is eccentric only in the strong direction, and no side load bending occurs, Equation 6.5-1 reduces to Equation 6.6-1.

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_c(6e_1/d_1)[1 + 0.234(f_c/F_{cE1})]}{F'_{b1}[1 - (f_c/F_{cE1})]} \leq 1.0 \quad (6.6\text{-}1)$$

EXAMPLE 6.6-1 ECCENTRIC AXIAL COMPRESSION

Given: $5\frac{1}{8}$ in. × $5\frac{1}{2}$ in. × 13 ft, pressure-treated, Combination 50 SP glulam column has been proposed to support an eccentric ($e_1 = 1.75$ in.) axial load of $P = 10{,}000$ lb as illustrated in Figure 6.6-1. The column is subject to normal load duration and wet service conditions. Bracing is provided to prevent lateral movement in the x and y directions at both the top and bottom.

Wanted: Determine the adequacy of the column to resist the eccentric load.

Approach: Design values from AITC 117 [1] will be used with Equation 6.6-1. Wet service factors will be obtained from Table 3.4.2-1.

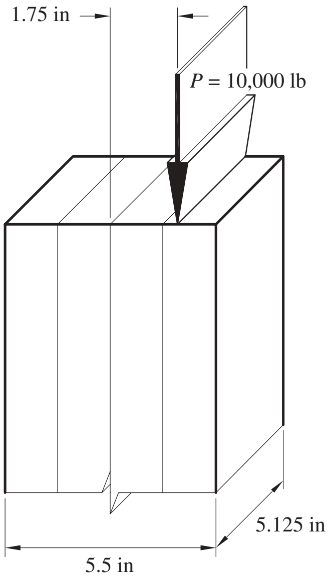


Figure 6.6-1 Eccentric column load—Example 6.6-1.

Solution:

Design values:

$$F'_c = F^*_c C_P = F_c C_D C_M C_t C_P$$
$$F'_c = 2300 \text{ psi}(1.0)(0.73)(1.0)C_P$$
$$F'_c = (1680 \text{ psi})C_P$$

$$F'_{bx} = F^*_{bx} C_V + f_c \leq F^*_{bx} C_L$$
$$F'_{bx} = F_{bx} C_D C_M C_t C_V + f_c \leq F_{bx} C_D C_M C_t C_L$$
$$F'_{bx} = (2100 \text{ psi})(1.0)(0.8)(1.0)C_V + f_c \leq (2100 \text{ psi})(1.0)(0.8)(1.0)C_L$$
$$F'_{bx} = (1680 \text{ psi})C_V + f_c \leq (1680 \text{ psi})C_L$$
$$E'_{\min} = E_{\min} C_M C_t = (1{,}000{,}000 \text{ psi})(0.833)(1.0) = 833{,}000 \text{ psi}$$

Section properties:

$$A = bd = (5.125\text{ in})(5.5\text{ in}) = 28.2\text{ in}^2$$

$$S_x = \frac{bd^2}{6} = \frac{(5.125\text{ in})(5.5\text{ in})^2}{6} = 25.8\text{ in}^3$$

Compression stress:

$$f_c = \frac{P}{A} = \frac{10{,}000\text{ lb}}{28.2\text{ in}^2} = 355\text{ psi}$$

Effective length of column ($K_e = 1.0$):

$$\ell_e = K_e \ell = 1.0(13\text{ ft}) = 13\text{ ft} = 156\text{ in}$$

Column slenderness ratios:

$$\frac{\ell_{e2}}{d_2} = \frac{156\text{ in}}{5.125\text{ in}} = 30.4 \leq 50 \qquad \therefore \text{OK}$$

$$\frac{\ell_{e1}}{d_1} = \frac{156\text{ in}}{5.5\text{ in}} = 28.4 \leq 50 \qquad \therefore \text{OK}$$

Critical buckling design values for compression (Equation 3.4.3.9-2):

$$F_{cE2} = \frac{0.822E'_{y\text{ min}}}{(\ell_{e2}/d_2)^2} = \frac{0.822(833{,}000\text{ psi})}{(30.4)^2} = 741\text{ psi}$$

$$F_{cE1} = \frac{0.822E'_{x\text{ min}}}{(\ell_{e1}/d_1)^2} = \frac{0.822(833{,}000\text{ psi})}{(28.4)^2} = 849\text{ psi}$$

The lower critical buckling design value, $F_{cE2} = 741$ psi, is used to determine the column stability factor.

Column stability factor (Equation 3.4.3.9-1):

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_c^*)}{2c}\right]^2 - \frac{(F_{cE}/F_c^*)}{c}}$$

$$C_P = \frac{1 + \left(\frac{741\text{ psi}}{1680\text{ psi}}\right)}{1.8} - \sqrt{\left(\frac{1 + \left(\frac{741\text{ psi}}{1680\text{ psi}}\right)}{1.8}\right)^2 - \frac{\left(\frac{741\text{ psi}}{1680\text{ psi}}\right)}{0.9}}$$

$$C_P = 0.412$$

Adjusted compression design value:

$$F'_c = F_c^* C_P = (1680\text{ psi})(0.412) = 692\text{ psi}$$

Unbraced length for lateral-torsional buckling:

$$\ell_u = \ell = 13 \text{ ft} = 156 \text{ in}$$

Effective length (Table 3.4.3.1.1-1, equal end moments):

$$\ell_e = 1.84\ell_u = 1.84(156 \text{ in}) = 287 \text{ in}$$

Beam slenderness ratio (Equation 3.4.3.1.1-1):

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{(287 \text{ in})(5.5 \text{ in})}{(5.125 \text{ in})^2}} = 7.75 \leq 50 \qquad \therefore \text{OK}$$

Critical buckling design value for beam (Equation 3.4.3.1-2):

$$F_{bE} = \frac{1.20(833{,}000 \text{ psi})}{(7.75)^2} = 16{,}640 \text{ psi}$$

Beam stability factor (Equation 3.4.3.1-1):

$$C_L = \frac{1 + (F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1 + (F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{16{,}640 \text{ psi}}{1680 \text{ psi}}\right)}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{16{,}640 \text{ psi}}{1680 \text{ psi}}\right)}{1.9}\right]^2 - \frac{\left(\frac{16{,}640 \text{ psi}}{1680 \text{ psi}}\right)}{0.95}}$$

$$C_L = 0.994$$

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125 \text{ in}}{b}\right)^{\frac{1}{10}} \left(\frac{12 \text{ in}}{d}\right)^{\frac{1}{10}} \left(\frac{21 \text{ ft}}{L}\right)^{\frac{1}{10}}$$

$$C_V = \left(\frac{5.125 \text{ in}}{5.125 \text{ in}}\right)^{\frac{1}{10}} \left(\frac{12 \text{ in}}{5.5 \text{ in}}\right)^{\frac{1}{10}} \left(\frac{21 \text{ ft}}{13 \text{ ft}}\right)^{\frac{1}{10}} = 1.13 \leq 1.0$$

$$C_V = 1.0$$

Adjusted bending design value (Equation 6.1.1-1):

$$F'_{bx} = F^*_{bx} C_V + f_c \leq F^*_{bx} C_L$$

$$F'_{bx} = (1680 \text{ psi}) C_V + f_c \leq (1680 \text{ psi}) C_L$$

$$F'_{bx} = (1680 \text{ psi})(1.0) + 355 \text{ psi} \leq (1680 \text{ psi})(0.994)$$

$$F'_{bx} = (1680 \text{ psi})(0.994)$$

$$F'_{bx} = 1670 \text{ psi}$$

Stresses vs. critical buckling design values (Equations 6.2-5, 6.2-6):

$$f_c = 355 \text{ psi} < F_{cE1} = 849 \text{ psi} \qquad \therefore \text{OK}$$

$$f_c = 355 \text{ psi} < F_{cE2} = 741 \text{ psi} \qquad \therefore \text{OK}$$

Combined stresses (Equation 6.6-1):

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_c(6e_1/d_1)[1 + 0.234(f_c/F_{cE1})]}{F'_{b1}[1 - (f_c/F_{cE1})]} \leq 1.0$$

$$\left[\frac{355 \text{ psi}}{692 \text{ psi}}\right]^2 + \frac{(355 \text{ psi})\left(\dfrac{6(1.75 \text{ in})}{5.5 \text{ in}}\right)\left[1 + 0.234\left(\dfrac{355 \text{ psi}}{849 \text{ psi}}\right)\right]}{(1670 \text{ psi})\left[1 - \left(\dfrac{355 \text{ psi}}{849 \text{ psi}}\right)\right]} \overset{?}{\leq} 1.0$$

$$1.00 \leq 1.0 \qquad \therefore \text{OK}$$

Answer: The $5\frac{1}{8}$ in. $\times$ $5\frac{1}{2}$ in. section is adequate to support the eccentric load.

6.7 COLUMNS WITH SIDE BRACKETS

Exact solutions of columns loaded by side brackets are complicated and laborious. For these reasons, special simplified design procedures are recommended when eccentric load is applied through a bracket within the upper quarter of the column length as shown in Figure 6.7-1. In the simplified procedure the bracket load P applied at distance a from the center of the column is replaced by the same load P centrally applied at the top of the columns plus a side load P_s applied at mid-height, $\ell/2$ (Figure 6.7-1).

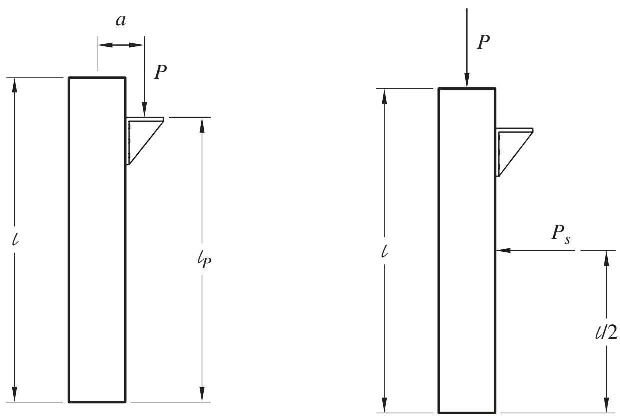


Figure 6.7-1 Column with load applied at through side bracket..

The load P_s is determined by the empirical Equation 6.7-1:

$$P_s = \frac{3Pa\ell_P}{\ell^2} \tag{6.7-1}$$

where:

P_s = assumed horizontal side load, placed mid-height of the column
P = actual load on bracket
a = horizontal distance from the bracket load to center of column
ℓ_P = distance from point of application of load on the bracket to the farthest end of the column
ℓ = length of column

The assumed axially applied load P should be added to other centric column loads, and the calculated horizontal side load should be used to determine the flexural stress. The solution is obtained by satisfying the appropriate stress interaction equation. This method assumes that the column is of such a size that the combination of loads is critical. The empirical method is to be used only in checking the combined axial and bending stress, and should not be used to determine shear or horizontal reaction.

6.8 COMBINED AXIAL TENSION AND BENDING

Equation 6.8-1 is used to evaluate axial tension combined with flexural tension and Equation 6.8-2 is used to evaluate axial tension combined with flexural compression.

$$\frac{f_t}{F'_t} + \frac{f_{b1}}{F^*_{b1}} + \frac{f_{b2}}{F'_{b2}} \leq 1.0 \tag{6.8-1}$$

$$\frac{f_{b1} - f_t}{F^{**}_{b1}} + \frac{f_{b2}}{F'_{b2}} \leq 1.0 \tag{6.8-2}$$

where:

f_t = axial tension stress
f_{b1} = strong axis bending stress
f_{b2} = weak axis bending stress
F'_t = adjusted tension design value
F^*_{b1} = strong axis bending design valued multiplied by all applicable adjustment factors except C_L
F'_{b2} = weak axis adjusted bending design value
F^{**}_{b1} = strong axis bending design value multiplied by all applicable adjustment factors except C_V

EXAMPLE 6.8-1 COMBINED AXIAL TENSION AND BENDING

Given: A $5\frac{1}{8}$ in. × $10\frac{1}{2}$ in. × 10 ft, Combination 3 DF glued laminated timber tension beam must resist a 30,000 lb axial tension load and equal end moments of 75,000 in-lb about the *x-x* axis. No lateral support is provided between the ends of the member. The member is subject to normal duration of load and dry conditions of use. The ends of the member are laterally supported to prevent rotation.

Wanted: Determine the suitability of the $5\frac{1}{8}$ in. × $10\frac{1}{2}$ in. Douglas fir member.

Solution:

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125\text{ in}}{b}\right)^{\frac{1}{x}} \left(\frac{12\text{ in}}{d}\right)^{\frac{1}{x}} \left(\frac{21\text{ ft}}{L}\right)^{\frac{1}{x}} \leq 1.0$$

$$C_V = \left(\frac{5.125\text{ in}}{5.125\text{ in}}\right)^{\frac{1}{10}} \left(\frac{12\text{ in}}{10.5\text{ in}}\right)^{\frac{1}{10}} \left(\frac{21\text{ ft}}{10\text{ ft}}\right)^{\frac{1}{10}} = 1.09 \leq 1.0$$

$$C_V = 1.0$$

Design values:

$$F_t' = F_t C_D C_M C_t = (1450\text{ psi})(1.0)(1.0)(1.0) = 1450\text{ psi}$$

$$F_{bx}^* = F_{bx} C_D C_M C_t C_V = (2000\text{ psi})(1.0)(1.0)(1.0)(1.0) = 2000\text{ psi}$$

$$E'_{\min} = E_{\min} = 1{,}000{,}000\text{ psi}$$

Section properties:

$$A = bd = (5.125\text{ in})(10.5\text{ in}) = 53.8\text{ in}^2$$

$$S = \frac{bd^2}{6} = \frac{(5.125\text{ in})(10.5\text{ in})^2}{6} = 94.2\text{ in}^3$$

Design stresses:

$$f_t = \frac{T}{A} = \frac{30{,}000\text{ lb}}{53.8\text{ in}^2} = 557\text{ psi}$$

$$f_{bx} = \frac{M_x}{S_x} = \frac{75{,}000\text{ lb-in}}{94.2\text{ in}^3} = 796\text{ psi}$$

Combined stresses (Equation 6.8-1):

$$\frac{f_t}{F_t'} + \frac{f_{bx}}{F_{bx}^*} = \frac{557\text{ psi}}{1450\text{ psi}} + \frac{796\text{ psi}}{2000\text{ psi}} = 0.783 \leq 1.0 \qquad \therefore \text{OK}$$

The axial tension stress does not fully offset the flexural compression stress, so Equation 6.8-2 must be used to investigate axial tension combined with flexural compression.

Beam slenderness ratio (Equation 3.4.3.1.2-1):

From Table 3.4.3.1.2-1 (equal end moments), $C_e = 1.0$ and $C_b = 2.30$.

$$R_B = \sqrt{\frac{1.84\ell_u d}{C_b C_e b^2}} = \sqrt{\frac{1.84(120\text{ in})(10.5\text{ in})}{(2.30)(1.0)(5.125\text{ in})^2}} = 6.20 \le 50 \qquad \therefore \text{OK}$$

Critical buckling design value for bending (Equation 3.4.3.1-2):

$$F_{bE} = \frac{1.20E'_{\min}}{R_B^2} = \frac{1.20(1{,}000{,}000\text{ psi})}{(6.20)^2} = 31{,}220\text{ psi}$$

Beam stability factor (Equation 3.4.3.1-1):

$$C_L = \frac{1 + (F_{bEx}/F^*_{bx})}{1.9} - \sqrt{\left[\frac{1 + (F_{bEx}/F^*_{bx})}{1.9}\right]^2 - \frac{(F_{bEx}/F^*_{bx})}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{31{,}220\text{ psi}}{2000\text{ psi}}\right)}{1.9} - \left[\frac{1 + \left(\frac{31{,}220\text{ psi}}{2000\text{ psi}}\right)}{1.9}\right]^2 - \frac{\left(\frac{31{,}220\text{ psi}}{2000\text{ psi}}\right)}{0.95}$$

$$C_L = 0.997$$

Adjusted bending design value:

$$F^{**}_{bx} = F_{bx}(C_D)(C_L) = 2000\text{ psi}(1.0)(0.997) = 1994\text{ psi}$$

Combined stresses (Equation 6.8-2):

$$\frac{f_{bx} - f_t}{F^{**}_{bx}} = \frac{(796\text{ psi} - 557\text{ psi})}{1994\text{ psi}} = \frac{239\text{ psi}}{1994\text{ psi}} = 0.120 \le 1.0 \qquad \therefore \text{OK}$$

<u>**Answer:**</u> The $5\frac{1}{8}$ in. × 10 in. DF Combination 3 member is satisfactory.

<u>**Discussion:**</u> Complete analysis of the member would require knowledge of the connection details at the ends. The effects of a reduced net section to accommodate bolts or other fasteners would have to be considered for both tension and flexure. Typically, the design of tension members is governed by the net section at the connections, rather than the full section away from the connection.

6.9 CONCLUSION

Members subject to combined stresses from axial loads and bending moments are common in timber construction. Several configurations are possible. Members subject to axial compression combined with flexure are generally referred to as *beam-columns*. Members subject to axial tension combined with flexure are referred to as *tension beams*. This chapter has presented formulas and examples for the analysis of both types of members.

CHAPTER 7

TAPERED BEAMS

7.1 INTRODUCTION

Glued laminated beams can be tapered to meet architectural requirements, to provide pitched roofs, to facilitate drainage, and to lower parapet wall height requirements at the end supports. Figure 7.1-1 illustrates the most common forms of these beams.

7.1.1 Stress Interaction, Volume, and Beam Stability Factors

Tapered members result in interactions along the tapered face between bending stress, perpendicular-to-grain stress, and shear stress. The stress interaction factor, C_I, described in Section 3.4.3.8, accounts for this stress interaction by reducing the allowable bending stress. This factor applies in tapered segments of all timber members. For members tapered on the compression face, the stress interaction factor is applied concurrently with the beam stability factor, but not with the volume factor. For members tapered on the tension face (such as for load reversals), the stress interaction factor is applied concurrently with the volume factor, but not with the beam stability factor.

As discussed in Section 3.4.3.3, the volume factor is applied to reduce the bending design values for large glulam timbers. For tapered beams, the volume factor is customarily calculated using the depth and width at the cross section of interest and the length taken as the span of the beam. This results in different volume factors for each section along the length. To simplify design, however, a single volume factor using the dimensions of the largest cross section can be conservatively used. The volume factor is not applied concurrently with the beam stability factor.

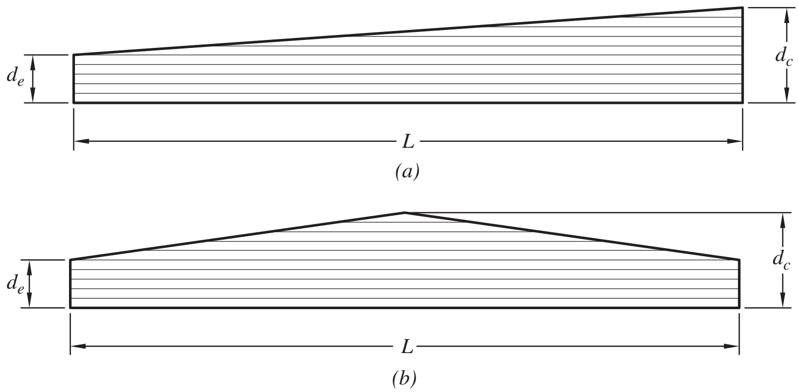


Figure 7.1-1 Tapered glulam beams.

Where possible, beams should be braced along the compression edge to prevent lateral-torsional buckling. Where full lateral bracing is not present, a beam stability factor must be calculated using Equation 3.4.3.1-1. A conservative approach is to calculate the beam stability factor using the deepest section between points of bracing. The beam stability factor is not applied concurrently with the volume factor.

7.1.2 Field Taper versus Manufactured Taper

Tapered beams are typically manufactured with the laminations parallel to the tension face and the taper sawn on the compression face. For members tapered in the laminating plant by the beam manufacturer, special techniques are used to ensure that the beam layup requirements are met all along the beam length. This allows the use of the published design values for the particular grade of beam without reduction for the loss of high-grade material removed by tapering.

Tapered beams can also be made by field-cutting straight beams. When this method is used, the taper should be cut on the compression face (TOP) of the beam layup. The removal of the compression laminations reduces the allowable bending stress, modulus of elasticity, and top face bearing stress for tapered segments. The shear reduction factor is also applied to shear design values in tapered segments. Reduced reference design values for beams with taper cuts on the compression face are listed in Table 7.1.2-1. AITC 117 [1] provides an expanded table of design values for individual combinations within the stress classes shown in Table 7.1.2-1.

The design values of Table 7.1.2-1 are intended for beams with relatively short taper cuts at the ends. Where taper cuts extend for significant portions of the beam length, the design values at any section of interest may be obtained by straight-line interpolation between the values at the end (from Table 7.1.2-1) and the full design values associated with no taper cut. Where the design values are

TABLE 7.1.2-1 Reference Design Values for Field-Tapered Glulam Beams[a]

Stress class	F_{bx}^{+} (psi)	E_x (10^6 psi)	$E_{x\min}$ (10^6 psi)	$F_{c\perp x,top}$ (psi)	$F_{vx}C_{vr}$ [(c)], psi
16F-1.3E	1,050	1.2	0.63	315	140
20F-1.5E	1,250	1.4	0.74	375	150
24F-1.7E	1,250	1.4	0.74	375	150
24F-1.8E	2,000	1.7	0.90	560	190
26F-1.9E	2,000	1.7	0.90	560	190
28F-2.1E	2,400	1.9	1.00	650	215
30F-2.1E	2,400	1.9	1.00	650	215

[a] Values are applicable to members that have up to $\frac{2}{3}$ the depth on the compression side removed by taper cutting.

so determined, the stress interaction factor at intermediate sections should also be calculated based on the interpolated design values.

7.1.3 Camber in Tapered Beams

Camber may be provided in beams that are taper cut on the compression face by bending the laminations to the required curvature during the bonding process and by sawing the compression face to maintain the required depth along the length of the beam. The designer should specify the required camber for the tension face and the required depth at several locations along the length of the beam (typically at 1- to 2 -foot intervals).

7.2 TAPERED BEAM DESIGN

Tapered beam design generally consists of choosing a trial size and evaluating the resulting member for adequacy. The roof slope and span are generally given by the architectural requirements, so a trial size can be established from these geometric requirements and from the required end depth based on shear. The size can then be increased if greater depth is needed to satisfy bending or deflection criteria.

7.2.1 End Depth Based on Shear

Equation 7.2.1-1 gives the minimum depth of the small end of a tapered beam:

$$d_e \geq \frac{3V}{2bF_v'} \qquad (7.2.1\text{-}1)$$

where:

d_e = depth at small end of tapered beam
V = shear force at end of beam

b = beam width
F'_v = adjusted shear design value

7.2.2 Location of Maximum Bending Stress

Tapered beams differ from prismatic beams in that the point of maximum moment may not correspond to the point of maximum stress. For example, for a tapered beam subject to uniform loading, the point of maximum stress is not at mid-span. Consequently, the section with the highest bending stress, f_b, must be located the analysis of a tapered beam. The location of the section of maximum bending stress can be obtained by simple calculation or by inspection in some members. In other members, it may be necessary to calculate the bending stress at several cross sections to determine the critical section.

7.2.2.1 Uniformly Loaded Tapered Beams The depth d_x of the section where maximum bending stress occurs for uniformly loaded tapered beams can be determined by Equation 7.2.2.1-1. This equation is valid for both single-tapered beams and symmetrical double-tapered beams.

$$d_x = 2d_e \frac{d_e + \ell \tan\theta}{2d_e + \ell \tan\theta} \tag{7.2.2.1-1}$$

where:

ℓ = span
θ = taper angle

The distance x from the small end to the point of maximum stress is given by Equation 7.2.2.1-2:

$$x = \frac{\ell d_e}{2d_e + \ell \tan\theta} \tag{7.2.2.1-2}$$

The bending stress $f_{b,x}$ at the point of maximum stress can be determined by Equation 7.2.2.1-3:

$$f_{b,x} = \frac{3\omega\ell^2}{4bd_e(d_e + \ell \tan\theta)} \tag{7.2.2.1-3}$$

where: ω = total total uniform load

7.2.2.1.1 Uniformly Loaded, Single-Tapered Beams For uniformly loaded, single tapered beams, Equations 7.2.2.1-1, 7.2.2.1-2, and 7.2.2.1-3 reduce to Equations 7.2.2.1.1-1, 7.2.2.1.1-2, and 7.2.2.1.1-3 for easier computation.

$$d_x = 2d_e \frac{d_c}{d_e + d_c} \tag{7.2.2.1.1-1}$$

$$x = \frac{\ell d_e}{(d_e + d_c)} \tag{7.2.2.1.1-2}$$

$$f_{b,x} = \frac{3\omega\ell^2}{4bd_e d_c} \tag{7.2.2.1.1-3}$$

where: d_c is the depth at the deeper end.

7.2.2.1.2 Uniformly Loaded, Double-Tapered Beams For symmetrical double-tapered beams subject to uniform loading, Equations 7.2.2.1-1, 7.2.2.1-2, and 7.2.2.1-3 reduce to Equations 7.2.2.1.2-1, 7.2.2.1.2-2, and 7.2.2.1.2-3 for easier computation:

$$d_x = \frac{d_e}{d_c}(2d_c - d_e) \tag{7.2.2.1.2-1}$$

$$x = \frac{\ell d_e}{2d_c} \tag{7.2.2.1.2-2}$$

$$f_{b,x} = \frac{3\omega\ell^2}{4bd_e(2d_c - d_e)} \tag{7.2.2.1.2-3}$$

where: d_c is the depth at mid-span.

7.2.2.2 Tapered Beams with a Concentrated Load For cases in which a concentrated load is applied at a section with depth greater than twice the end depth ($d_x > 2d_e$), the maximum bending stress will occur at a section where the depth is twice the end depth ($x = d_e/\tan\theta$). For cases in which a concentrated load is applied at a section where the depth is less than or equal to twice the end depth, the location of maximum bending stress occurs at the same section as the concentrated load.

7.2.2.3 Tapered Beams with Other Loads Where a tapered member supports loads other than a uniform load or a single concentrated load or if a nonsymmetrical double-tapered member is used, the simplified equations for locating the section of maximum stress do not apply. In these cases, it is necessary to evaluate the bending stress at various sections to ensure that the beam is structurally adequate.

7.2.3 Deflection of Tapered Beams

The shear deflection in tapered members is larger than in prismatic members, and the resulting total deflection including both bending deflection and shear deflection is slightly larger than that obtained by the customary methods of calculating deflection in prismatic members. Acceptable accuracy in determining the deflection of tapered members can be obtained by determining the deflection of an equivalent prismatic member. Equation 7.2.3-1 is used to determine the depth of an equivalent member of constant cross section of the same width that will have the same deflection as a tapered beam.

$$d = C_{dt} d_e \tag{7.2.3-1}$$

where: C_{dt} = empirical constant derived from relationship of equations for deflection of tapered beams and straight prismatic beams.

7.2.3.1 Uniformly Loaded, Double-Tapered Beams For symmetrical double-tapered beams subject to uniform loads, Equations 7.2.3.1-1 and 7.2.3.1-2 are used to determine the empirical constant for use in Equation 7.2.3-1.

$$C_{dt} = 1 + 0.66C_y \quad \text{where } 0 < C_y \leq 1 \tag{7.2.3.1-1}$$

$$C_{dt} = 1 + 0.62C_y \quad \text{where } 1 < C_y \leq 3 \tag{7.2.3.1-2}$$

where:

$$C_y = \frac{d_c - d_e}{d_e} \tag{7.2.3.1-3}$$

7.2.3.2 Uniformly Loaded, Single-Tapered Beams For uniformly loaded, single-tapered beams, Equations 7.2.3.2-1 and 7.2.3.2-2 are used to determine the empirical constant for use in Equation 7.2.3-1.

$$C_{dt} = 1 + 0.46C_y \quad \text{where } 0 < C_y \leq 1.1 \tag{7.2.3.2-1}$$

$$C_{dt} = 1 + 0.43C_y \quad \text{where } 1.1 < C_y \leq 2 \tag{7.2.3.2-2}$$

where: C_y is defined by Equation 7.2.3.1-3.

EXAMPLE 7.2.3.2-1 SYMMETRICAL DOUBLE-TAPERED BEAM

Given: Double-tapered straight, glued laminated timber roof beams, as illustrated in Figure 7.2-1, have been specified with the following requirements:

Length = 60 ft

Spacing (tributary width) = 16 ft

Roof slope = 1:12

Snow load = 30 psf

Dead load = 15 psf

Tops of beams braced by roof decking

Deflection under total load to not exceed $\frac{\ell}{180}$

Camber to be 1.5 times the dead load deflection

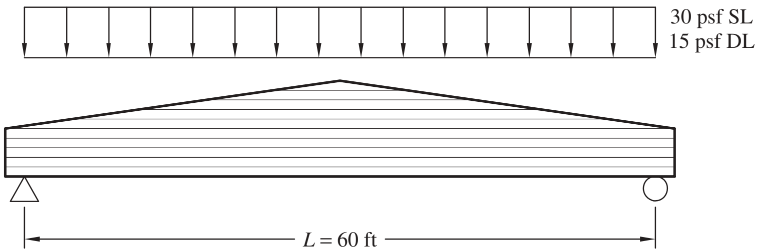


Figure 7.2-1 Tapered beam—Example 7.2-1.

Wanted: Determine suitable size member using the 20F-V3 DF combination.

Approach: A $5\frac{1}{8}$ in. wide section will be used. The beam depth will be determined based on shear requirements, then the trial size will be analyzed. The depth will be increased as necessary until bending and deflection criteria are satisfied. It will be assumed that the taper is manufactured into the beam with the layup requirements maintained along the full length (not field-tapered).

Solution:

Design values:

$$F'_v = F_v C_D C_M C_t C_{vr} = (265\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\,(0.72) = 219\,\text{psi}$$

$$E'_x = E_x C_M C_t = 1.6\left(10^6\,\text{psi}\right)(1.0)\,(1.0) = 1.6\left(10^6\,\text{psi}\right)$$

$$F'_{bx} = F_{bx} C_D C_M C_t \left(C_V \text{ or } C_I\right)$$

$$F'_{bx} = (2000\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\left(C_V \text{ or } C_I\right)$$

$$F'_{bx} = (2300\,\text{psi})\left(C_V \text{ or } C_I\right)$$

$$F_{cx\perp,top} = 560\,\text{psi}$$

End depth based on shear (Equation 7.2-1):

$$w = s\,(D + S) = (16\,\text{ft})\,(30\,\text{psf} + 15\,\text{psf}) = 720\,\text{plf}$$

$$V = \frac{w\ell}{2} = \frac{(720\,\text{plf})\,(60\,\text{ft})}{2} = 21{,}600\,\text{lb}$$

$$d_e \geq \frac{3V}{2bF'_v} = \frac{3\,(21{,}600\,\text{lb})}{2\,(5.125\,\text{in})\,(219\,\text{psi})} = 28.9\,\text{in} \quad \therefore \text{Try } d_e = 30\,\text{in}$$

Mid-span depth (based on geometry):

$$d_c = d_e + (\ell/2)\tan\theta = 30\,\text{in} + \frac{(60\,\text{ft})\,(12\,\text{in/ft})}{2}\left(\frac{1}{12}\right) = 60\,\text{in}$$

Deflection from total load:

$$C_y = \frac{d_c - d_e}{d_e} = \frac{60\,\text{in} - 30\,\text{in}}{30\,\text{in}} = 1.0$$

$$C_{dt} = 1 + 0.66C_y = 1 + 0.66\,(1.0) = 1.66$$

$$d_{\text{equivalent}} = d_e C_{dt} = (30\,\text{in})\,(1.66) = 49.8\,\text{in}$$

$$I_{\text{equivalent}} = \frac{(5.125\,\text{in})\,(49.8\,\text{in})^3}{12} = 52{,}800\,\text{in}^4$$

$$\Delta_{TL} = \frac{5\omega\ell^4}{384EI_{\text{equivalent}}}$$

$$\Delta_{TL} = \frac{5\,(720\,\text{plf})\,(60\,\text{ft})^4\,(12\,\text{in/ft})^3}{384\,(1{,}600{,}000\ \text{psi})\left(52{,}800\,\text{in}^4\right)}$$

$$\Delta_{TL} = 2.5\,\text{in}$$

Deflection limit:

$$\delta = \frac{l}{180} = \frac{(60\,\text{ft})\,(12\ \text{in/ft})}{180}$$

$$\delta = 4.0\,\text{in} \geq \Delta_{TL} = 2.5\,\text{in} \quad \therefore \text{OK}$$

Maximum bending stress (Equations 7.2.2.1.2-1, 7.2.2.1.2-2, 7.2.2.1.2-3):

$$d_x = \frac{d_\text{e}}{d_\text{c}}\left(2d_\text{c} - d_\text{e}\right) = \frac{(30\,\text{in})}{(60\,\text{in})}\,(2\,(60\,\text{in}) - 30\,\text{in}) = 45\,\text{in}$$

$$x = \frac{ld_e}{2d_c} = \frac{(720\,\text{in})\,(30\,\text{in})}{2\,(60\,\text{in})} = 180\,\text{in}$$

$$f_{b,x} = \frac{3\omega l^2}{4bd_e\left(2d_c - d_e\right)} = \frac{3\,(720\,\text{lb/ft})\,(60\,\text{ft})^2\,(12\ \text{in/ft})}{4\,(5.125\,\text{in})\,(30\,\text{in})\,(2\,(60\,\text{in}) - (30\,\text{in}))} = 1690\,\text{psi}$$

Volume factor (Equation 3.4.3.3-1) (using mid-span depth):

$$C_V = \left(\frac{5.125\,\text{in}}{b}\right)^{\frac{1}{x}}\left(\frac{12\,\text{in}}{d}\right)^{\frac{1}{x}}\left(\frac{21\,\text{ft}}{L}\right)^{\frac{1}{x}} \leq 1.0$$

$$C_V = \left(\frac{5.125\text{ in}}{5.125\text{ in}}\right)^{\frac{1}{10}} \left(\frac{12\text{ in}}{60\text{ in}}\right)^{\frac{1}{10}} \left(\frac{21\text{ ft}}{60\text{ ft}}\right)^{\frac{1}{10}} \leq 1.0$$

$$C_V = 0.767$$

Stress interaction factor (Equation 3.4.3.8-1):

$$C_I = \frac{1}{\sqrt{1 + \left(\frac{F_b \tan\theta}{F_v C_{vr}}\right)^2 + \left(\frac{F_b \tan^2\theta}{F_{c\perp}}\right)^2}}$$

$$C_I = \frac{1}{\sqrt{1 + \left(\frac{(2000\text{ psi})\left(\frac{1}{12}\right)}{(265\text{ psi})(0.72)}\right)^2 + \left(\frac{(2000\text{ psi})\left(\frac{1}{12}\right)^2}{560\text{ psi}}\right)^2}}$$

$$C_I = 0.753 \quad \therefore \text{Controls over } C_V$$

Adjusted bending stress:

$$F'_{bx} = (2300\text{ psi})\left(C_V \text{ or } C_I\right)$$

$$F'_{bx} = (2300\text{ psi})\, C_I$$

$$F'_{bx} = (2300\text{ psi})\,(0.753)$$

$$F'_{bx} = 1730\text{ psi} \geq f_{b,x} = 1690\text{ psi} \quad \therefore \text{OK}$$

Camber:

$$\Delta_{DL} = \Delta_{TL}\frac{w_{DL}}{w_{TL}} = (2.5\text{ in})\left(\frac{15\text{ psf}}{45\text{ psf}}\right) = 0.83\text{ in}$$

$$c = 1.5\Delta_{DL} = 1.5\,(0.83\text{ in}) = 1.25\text{ in}$$

Answer: The $5\frac{1}{8}$ in. wide, 20F-V3 DF tapered beam with $d_e = 30$ in. and $d_c = 60$ in. beam is satisfactory. Camber of 1.25 in. will be specified.

Discussion: This example assumed adequate attachment of the roof decking to provide lateral support of the compression face ($C_L = 1.0$). Such attachment should be verified by the designer and specified in the plans. The volume factor calculated for this example was based on maximum beam depth. If the volume factor were less than the stress interaction factor, it would be worthwhile to calculate this factor for the actual section of interest. However, the controlling section might not be at the section of maximum stress due to a smaller volume factor at the adjacent larger sections. Multiple sections should be checked where the volume factor controls and the design is near-optimum for bending stress.

7.3 BEAMS WITH TAPERED END CUTS

Taper cuts on the top at the end of beams are sometimes used to improve drainage, facilitate discharge of roof water, and reduce the height of the parapet wall (Figure 7.3-1). The lengths of taper cuts vary, depending on the length of span, the roof-framing systems, or other requirements. Sloping cuts from 4 to 8 ft long are commonly used. The depth of the end cut usually depends on the drainage requirements.

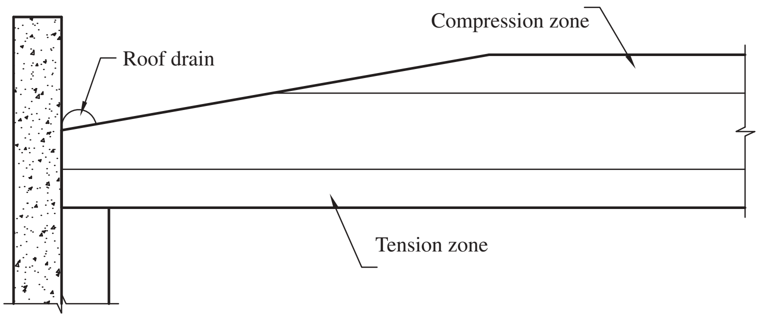


Figure 7.3-1 Beam with tapered end.

Beams with end taper are typically not manufactured with the layup requirements maintained through the tapered segment. They are typically field-tapered after the manufacture of the laminated timbers. The sloping cut is commonly made through the compression zone laminations exposing the lower strength core laminations and resulting in a member with reduced design values as discussed in Section 7.1.3. A maximum of two-thirds of the depth of the member is permitted to be removed by the taper cut, subject also to satisfying the structural requirements as illustrated in Example 7.3-1.

EXAMPLE 7.3-1 TAPER CUT ON COMPRESSION FACE AT END OF BEAM

Given: A simply supported southern pine (SP) beam spans 40 ft carrying 240 lb/ft roof live load (construction load duration) and 300 lb/ft dead load. The beam is laterally supported continuously along its length to resist buckling. The beam will be modified with a taper cut $7\frac{1}{2}$ in. × 6 ft, as shown in Figure 7.3-2. The roof is adequately sloped for drainage, so ponding need not be investigated. Deflection should be limited to $\frac{\ell}{240}$ for live load and $\frac{\ell}{180}$ for total load.

Wanted: Determine a suitable SP 24F-1.7E section, taking into consideration the taper cuts at the ends. Also, specify the appropriate camber for the beam.

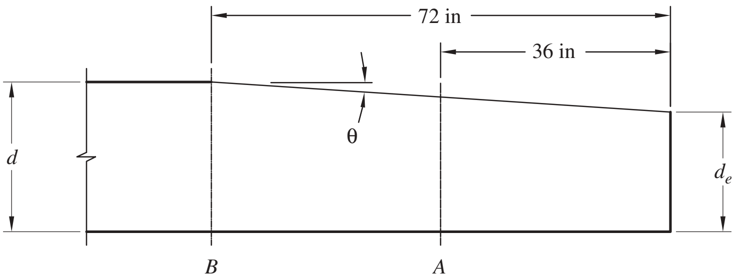


Figure 7.3-2 Beam end detail—Example 7.3-1.

Approach: The beam section will be determined based on deflection ignoring the taper at the ends. The bending stress will be checked at mid-span, and the effects of taper at the end will be checked using design values from Table 7.1.2-1 at sections A and B. Once an appropriate section has been verified, the appropriate camber will be calculated and specified.

Solution:

Deflection limits:

$$\delta_L = \frac{\ell}{240} = \frac{(40\,\text{ft})\,(12\,\text{in/ft})}{240} = 2\,\text{in}$$

$$\delta_{0.5D+L} = \frac{\ell}{180} = \frac{(40\,\text{ft})\,(12\,\text{in/ft})}{180} = 2.67\,\text{in}$$

Required moment of inertia, based on deflection limits:

$$I_{x,L} = \frac{5}{384}\frac{\omega\ell^4}{E\Delta_L} = \frac{5}{384}\frac{\left(240\,\text{lb/ft}\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)\right)\left(40\,\text{ft}\left(\frac{12\,\text{in}}{\text{ft}}\right)\right)^4}{(1{,}700{,}000\,\text{psi})\,(2\,\text{in})} = 4066\,\text{in}^4$$

$$I_{x,0.5D+L} = \frac{5}{384}\frac{\omega\ell^4}{E\Delta_{0.5D+L}}$$

$$I_{x,0.5D+L} = \frac{5}{384}\frac{\left([(0.5)\,300\,\text{lb/ft} + 240\,\text{lb/ft}]\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)\right)\left(40\,\text{ft}\left(\frac{12\,\text{in}}{\text{ft}}\right)\right)^4}{(1{,}700{,}000\,\text{psi})\,(2.67\,\text{in})}$$

$$I_{x,0.5D+L} = 4949\,\text{in}^4$$

Required depth (assuming $5\frac{1}{8}$ in. width):

$$d \geq \sqrt[3]{\frac{12I}{b}} = \sqrt[3]{\frac{12\left(4949\,\text{in}^4\right)}{5.125\,\text{in}}} = 22.6\,\text{in}$$

Try a 5-$\frac{1}{8}$ in. × 23-$\frac{3}{8}$ in. beam.

Volume factor:

$$C_V = \left(\frac{5.125\text{ in}}{5.125\text{ in}}\right)^{\frac{1}{20}}\left(\frac{12\text{ in}}{23.375\text{ in}}\right)^{\frac{1}{20}}\left(\frac{21\text{ ft}}{40\text{ft}}\right)^{\frac{1}{20}} = 0.937$$

Allowable bending stress at mid-span:

$$F_b' = F_b C_D C_V = 2400\text{ psi}\,(1.25)\,(0.94) = 2810\text{ psi}$$

Bending moment at mid-span:

$$M = \frac{\omega\ell^2}{8} = \frac{(300\text{ lb/ft} + 240\text{ lb/ft})\,(40\text{ ft})^2}{8} = 108{,}000\text{ lb-ft} = 1{,}296{,}000\text{ lb-in}$$

Bending stress at mid-span:

$$f_b = \frac{6M}{bd^2} = \frac{(6)\,(1{,}296{,}000\text{ lb-in})}{(5.125\text{ in})\,(23.375\text{ in})^2} = 2780\text{ psi} \le F_b' = 2810 \quad \therefore \text{OK}$$

Shear at tapered end:

$$V = \frac{\omega\ell}{2} = \frac{(540\text{ lb/ft})\,(40\text{ ft})}{2} = 10{,}800\text{ lb}$$

$$d_e = 23.375\text{ in} - 7.5\text{ in} = 15.875\text{ in}$$

$$f_v = \frac{3V}{2bd_e} = \frac{3}{2}\left[\frac{10{,}800\text{ lb}}{(5.5\text{ in})(15.875\text{ in})}\right] = 186\text{ psi}$$

$$F_v' = F_v C_{vr} C_D = \left[150\text{ psi}\right](1.25) = 188\text{ psi} > 186\text{ psi} \quad \therefore \text{OK}$$

Stress interaction factor (Equation 3.4.3.8-1):

$$\theta = \tan^{-1}\left(\frac{7.5\text{ in}}{6\text{ ft}}\left(\frac{1\text{ ft}}{12\text{ in}}\right)\right) = 5.95^\circ$$

$$C_I = \frac{1}{\sqrt{1 + (F_b \tan\theta F_v)^2 + (F_b \tan^2\theta F_{c\perp})^2}}$$

$$C_I = \frac{1}{\sqrt{1 + \left(\dfrac{(1250\text{ psi})\tan(5.95^\circ)}{150\text{ psi}}\right)^2 + \left(\dfrac{(1250\text{ psi})\tan^2(5.95^\circ)}{375\text{psi}}\right)^2}}$$

$$C_I = 0.755$$

Bending stress at section *A*:

$$M_A = \frac{\omega x}{2}(l - x)$$

$$M_A = \frac{(540\ \text{lb/ft})\ 3\ \text{ft}}{2}(40\ \text{ft} - 3\ \text{ft})$$

$$M_A = 29{,}970\ \text{lb-ft} = 359{,}600\ \text{lb-in}$$

$$d_A = 23.375\ \text{in} - \frac{7.5\ \text{in}}{2} = 19.625\ \text{in}$$

$$S_A = \frac{bd_A^2}{6} = \frac{(5.125\ \text{in})\ (19.625\ \text{in})^2}{6} = 329\ \text{in}^3$$

$$f_{b,A} = \frac{M_A}{S_A} = \frac{359{,}600\ \text{lb-in}}{329\ \text{in}^3} = 1090\ \text{psi}$$

$$C_{V,A} = \left(\frac{5.125\ \text{in}}{b}\right)^{\frac{1}{20}}\left(\frac{12\ \text{in}}{d}\right)^{\frac{1}{20}}\left(\frac{21\ \text{ft}}{L}\right)^{\frac{1}{20}} \leq 1.0$$

$$C_{V,A} = \left(\frac{5.125\ \text{in}}{5.125\ \text{in}}\right)^{\frac{1}{20}}\left(\frac{12\ \text{in}}{19.625\ \text{in}}\right)^{\frac{1}{20}}\left(\frac{21\ \text{ft}}{40\ \text{ft}}\right)^{\frac{1}{20}} \leq 1.0$$

$$C_{V,A} = 0.945$$

$$F'_{bx,A} = F_{bx} C_D C_I$$

$$F'_{bx,A} = 1250\ \text{psi}\ (1.25)\ (0.755)$$

$$F'_{bx,A} = 1180\ \text{psi} > 1090\ \text{psi} \qquad \therefore \text{OK}$$

Bending stress at section *B*:

$$M_B = \frac{\omega x}{2}(l - x)$$

$$M_B = \frac{(540\ \text{lb/ft})\ 6\ \text{ft}}{2}(40\ \text{ft} - 6\ \text{ft})$$

$$M_B = 55{,}080\ \text{lb-ft} = 660{,}960\ \text{lb-in}$$

$$S_B = 466.7\ \text{in}^3\ (\text{full section})$$

$$f_{b,B} = \frac{M_B}{S_B} = \frac{660{,}960\ \text{lb-in}}{466.7\ \text{in}^3} = 1420\ \text{psi}$$

$$C_{V,B} = 0.937\ (\text{full section})$$

$$F'_{bx,B} = F_{bx} C_D C_I$$
$$F'_{bx,B} = 1250\,\text{psi}\,(1.25)\,(0.755)$$
$$F'_{bx,B} = 1180\,\text{psi} < 1420\,\text{psi} \quad \therefore \text{ Not acceptable}$$

Required depth at *B* (estimated):

$$d_B = \sqrt{\frac{6M_B}{F'_{b,B}\,b}}$$
$$d_B = \sqrt{\frac{6\,(660{,}960\,\text{lb-in})}{(1180\,\text{psi})\,(5.125\,\text{in})}}$$
$$d_B = 25.6\,\text{in} \quad \therefore \text{Try } d_B = 26.125\,\text{in}$$

Bending stress for $d_B = 26\frac{1}{8}$ in:

$$S_B = \frac{bd_B^2}{6} = \frac{(5.125\,\text{in})\,(26.125\,\text{in})^2}{6} = 583\,\text{in}^3$$
$$f_{bB} = \frac{M_B}{S_B} = \frac{660{,}960\,\text{lb-in}}{583\,\text{in}^3} = 1134\,\text{psi}$$
$$f_b = 1134\,\text{psi} \le F'_b = 1180\,\text{psi} \quad \therefore \text{OK}$$

Moment of inertia of new section ($5\frac{1}{8}$ in $26\frac{1}{8}$ in):

$$I = \frac{bd^3}{12} = \frac{(5.125\,\text{in})\,(26.125\,\text{in})^3}{12} = 7615\,\text{in}^4$$

Camber (Section 3.6):

$$c = 1.5\Delta_{DL} = (1.5)\,\frac{5\omega\ell^4}{384EI_x}$$
$$c = (1.5)\,\frac{5\left((300\ \text{lb/ft})\left(\dfrac{1\,\text{ft}}{12\,\text{in}}\right)\right)\left(40\,\text{ft}\left(\dfrac{12\,\text{in}}{\text{ft}}\right)\right)^4}{384\,(1{,}700{,}000\,\text{psi})\,\left(7615\,\text{in}^4\right)} = 2.0\,\text{in}$$
$$R = \frac{L^2}{8c} = \frac{(40\,\text{ft})^2}{8\,(2.0\,\text{in})\left(\dfrac{1\,\text{ft}}{12\,\text{in}}\right)} = 1200\,\text{ft}$$

Answer: A $5\frac{1}{8}$ in $26\frac{1}{8}$ in SP 24F-1.7E beam is suitable for the stated conditions, including considerations for the taper cut at the end. The beam is to be manufactured with a camber of 2.0 in.

Discussion: Since the beam is simply supported, an unbalanced layup should be specified. Unbalanced beams are often available as stock items with premanufactured cambers. Availability of the beam at the required camber should be determined, or the suitability of another available camber evaluated. For example, a stock beam with a 3500 ft radius may be immediately available, while a beam with a 1200 ft radius may require a special order with increased costs. The designer would have to decide if the stock beam (0.7 in. camber) is suitable or if a beam will need to be custom manufactured with 2 inches of camber.

7.4 CONCLUSION

Timber beams, particularly glulam beams, are occasionally tapered for drainage, appearance, to reduce parapet wall height, or to save weight. The taper should be sawn on the compression face of the beam and consideration must be given to the effect of the taper on both design stresses and beam design values. Taper can be manufactured into the beam with the layup requirements maintained at every cross section along the length of the beam, or can be field-cut into a prismatic beam with reduced design values assigned in the tapered segments to account for the removal of high-grade laminations from the top of the beam. This chapter has presented methods for analysis and design of tapered beams of both types.

CHAPTER 8

CURVED GLULAM BEAMS

8.1 INTRODUCTION

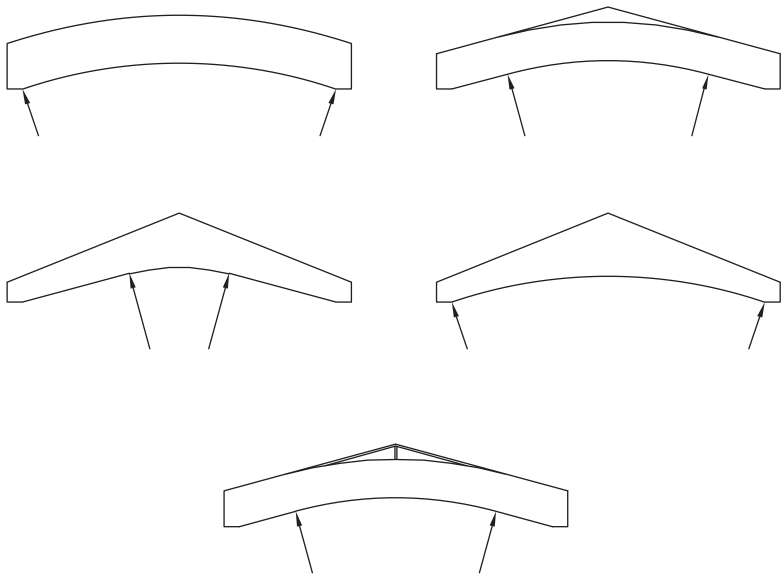
Structural glued laminated timbers are unique in that they can be manufactured in curved, pitched, and tapered forms to accommodate architectural and aesthetic purposes. To form curved members, laminations are bent to curved form and bonded to produce a glulam beam with permanent curvature. Common shapes of curved beams are shown in Figure 8.1-1.

8.1.1 Shear Reduction Factor, C_{vr}

For curved beams, the tabulated shear design values must be reduced by application of the shear reduction factor, C_{vr}, as described in Section 3.4.3.7. Otherwise, the shear design of curved beams is identical to the shear design of straight beams. The shear reduction factor is typically applied to the design of both curved and straight segments in a curved beam.

8.1.2 Curvature Factor, C_c

The process of manufacturing a curved beam causes residual stresses in the bent laminations. These residual stresses result in reduced bending capacity for the beam. The residual stresses are accounted for in design through application of the curvature factor, C_c, as described in Section 3.4.3.6. For members with both curved and straight segments, the curvature factor is only applied to the design values in the curved segments.



Arrows represent points of curvature.

Figure 8.1-1 Curved glulam beams.

8.1.3 Minimum Radii of Curvature

To minimize the stresses induced into the laminations during the process of manufacturing curved members, limits on the minimum radii of curvature are recommended for curved structural glued laminated timbers. Excessive bending can cause laminations to break or be irreparably damaged. For curved beams manufactured with nominal 2 in. thickness laminations, the minimum radius of curvature (at the inside face) is typically 18 feet for southern pine and 27 ft 6 in. for other softwood species. For tightly curved members manufactured with nominal 1 in. thickness laminations, typical minimum radii of curvature (at the inside face) are 7.0 ft for southern pine and 9 ft 4 in. for other softwood species. The manufacture of curved members with radii shorter than these typically requires standard-thickness laminations to be planed to a thinner dimension, resulting in more waste and less efficient use of materials. Therefore, it is recommended that the designer contact the laminator prior to specifying radii shorter than those just described.

8.1.4 Radial Stresses

Where curved members are subject to a bending moment, stresses develop parallel to the radius of curvature (perpendicular-to-grain). If the moment increases the radius of curvature, causing the member to become straighter, the stress is tension; if it decreases the radius, causing the member to become more sharply curved, the stress is compression.

The reference design value for radial compression stress, F_{rc}, is equal to the adjusted design value in compression perpendicular to grain, $F_{c\perp y}$, for the stress class or combination. The use of the y value accommodates the compression strength of the typically weaker inner laminations.

For southern pine and hardwoods, the design value for radial tension, F_{rt}, is equal to one-third of the shear design value for nonprismatic members ($F_{rt} = F_{vx}C_{vr}/3$). Radial reinforcement is not required.

For all other softwood species, the radial tension design value for unreinforced beams, subject to wind or earthquake load combinations, is one-third of the adjusted shear design value; $F'_{rt} = F'_{vx}/3$ (including adjustment by the shear reduction factor, C_{vr}). For other loads, the reference design value for radial tension, F_{rt}, in unreinforced curved beams is limited to 15 psi. Where the calculated radial tension stress, f_{rt}, exceeds the adjusted radial tension stress, $F'_{rt} = (15\text{ psi})\, C_D C_M C_t$, radial reinforcement is required. The radial tension design value for a radially reinforced member is limited to one third of the adjusted shear design value, $F'_{rt} = F'_{vx}/3$ (including adjustment by shear reduction factor, C_{vr}).

8.1.5 Radial Reinforcement

For softwood species other than southern pine, radial reinforcement is commonly used to resist radial tension. Where reinforcement is used, it must be designed to carry the full radial tension stress (no sharing of radial tension between reinforcement and wood). The reinforcement used must be designed on the basis of sound engineering principles.

Any type of reinforcement that effectively transfers the radial tension stresses between the wood and the reinforcement throughout the entire depth of embedment may be used. The method of bonding or attaching the reinforcement to the wood must be of a durable quality and capable of developing the required tensile capacity of the reinforcement. Common types of reinforcement include (1) fully threaded lag screws, mechanically attached to wood and (2) deformed steel bars, bonded by adhesive.

8.1.5.1 Installation of Radial Reinforcement Where radial reinforcement is used, it resists shrinkage that might occur between the time of installation of the reinforcement and the time the member reaches equilibrium. If the shrinkage is excessive, this restraint will cause cracking in the member. To minimize this condition, the moisture content of lumber used to manufacture radially reinforced members for use in dry conditions is not permitted to exceed 12%.

For members in which radial reinforcement is required, the reinforcement must be used throughout the entire curved segment(s). Radial reinforcement spacing may be uniform throughout the curved segment or may vary to accommodate different radial tension stresses along the length of the member. Spacing of reinforcement should generally not exceed half the depth of the member in the section of maximum moment.

Radial reinforcement should be shop-installed in prebored holes from the top of the member, centered across the width. Reinforcement should be installed normal to the axis of the laminations (perpendicular to the bond lines), parallel to the sides of the member, and should extend to within 2 to 3 inches of the bottom face.

For lag screws, the prebored lead hole should have a diameter not greater than 85% of the nominal diameter of the lag screw for species with a specific gravity of 0.50 or greater, and 80% for species with a specific gravity of less than 0.50. The lead holes for radial reinforcement are necessarily larger than for lag screws in ordinary application due to the longer screws necessary for radial reinforcement.

Long, thin lag screws may be difficult to install, and the holes required to accommodate large lag screws may reduce the effective section of the beam excessively for small beam widths. Recommended lag screw sizes are provided in Tables 8.1.5.1-1 and 8.1.5.1-2.

TABLE 8.1.5.1-1 Recommended Maximum Lengths for Lag Screws [1]

Lag Screw Diameter (in.)	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	1	$1\frac{1}{4}$
Lag Screw Length (in.)	20	30	40	66	66

TABLE 8.1.5.1-2 Maximum Size Lag Screw Based on Beam Width [1]

Beam Width (in.)	$3\frac{1}{8}$	$5\frac{1}{8}$	$6\frac{3}{4}$	$8\frac{3}{4}$	$10\frac{3}{4}$
Lag Screw Diameter (in.)	$\frac{1}{2}$	$\frac{3}{4}$	1	1	$1\frac{1}{4}$

8.1.5.2 Radial Tension Force in Each Screw or Bar The radial force to be carried by each lag screw or deformed bar is equal to the radial tension stress multiplied by the beam width and the spacing between reinforcing elements (Equation 8.1.5.2-1).

$$T_{radial} = f_{rt} bs \tag{8.1.5.2-1}$$

where:

T_{radial} = radial force in lag screw
f_{rt} = radial tension stress due to design loads
b = beam width
s = spacing between radial reinforcement

The magnitude of the radial tension force to be carried by each lag screw or deformed bar must not exceed either the allowable tension load for the reinforcement or the allowable load based on withdrawal of the reinforcement from the wood. The allowable tension load for the reinforcement must be calculated based on the root diameter of the reinforcement. The withdrawal resistance of the wood

TABLE 8.1.5.2-1 Lag Screw Reinforcement Data for Douglas Fir-Larch

Lag Screw Nominal Diameter (in.)	Recommended Lead Hole Diameter for Long Lag Screws (in.)	Steel		Wood		
		Net Area (in.^{2})	Allowable Tension Load (lb)	Allowable Withdrawal Load (lb/in)		
				Normal $C_D = 1.0$	Snow $C_D = 1.15$	7-Day $C_D = 1.25$
$\frac{1}{4}$	$\frac{3}{16}$	0.0235	470	191	220	239
$\frac{5}{16}$	$\frac{1}{4}$	0.0405	810	226	260	283
$\frac{3}{8}$	$\frac{5}{16}$	0.0552	1105	259	298	324
$\frac{7}{16}$	$\frac{3}{8}$	0.0845	1690	291	334	363
$\frac{1}{2}$	$\frac{3}{8}$	0.108	2160	321	369	402
$\frac{9}{16}$	$\frac{7}{16}$	0.149	2980	349	401	436
$\frac{5}{8}$	$\frac{1}{2}$	0.174	3485	380	437	475
$\frac{3}{4}$	$\frac{5}{8}$	0.263	5265	436	501	545
$\frac{7}{8}$	$\frac{11}{16}$	0.366	7330	490	563	612
1	$\frac{13}{16}$	0.478	9555	541	622	676
$1\frac{1}{8}$	$\frac{15}{16}$	0.618	12,360	591	679	736
$1\frac{1}{4}$	$1\frac{1}{16}$	0.804	16,090	639	735	799

is calculated using the embedment depth between the end of the reinforcement to the neutral axis of the beam (excluding the tapered tip for lag screws).

Design values and recommended lead hole sizes for lag screws used as radial reinforcement are provided for Douglas fir-larch in Table 8.1.5.2-1. The allowable tension load for each size screw is based on an allowable stress in the steel of 20,000 psi (60 ksi ultimate tensile strength steel). The withdrawal values (in pounds per inch of wood embedment) are 85% of the design values for lag screws in withdrawal from the *National Design Specification*® [2]. The reduction is due to the larger diameter lead holes required to facilitate installation of long lag screws. For other species, lag screw withdrawal values must similarly be multiplied by 85% for use as radial reinforcement.

AITC 404-2005 *Standard for Radially Reinforcing Curved Glued Laminated Timber Members to Resist Radial Tension* [1] gives procedures for determining the strength of adhesive bonds for epoxy-bonded bars and for establishing equivalency to lag screw reinforcement.

8.1.5.3 Number of Reinforcing Elements Required To determine the number of reinforcing elements needed, the length of the curve must be calculated based on the mid-depth radius at mid-span, R_m. This length can be calculated by Equation 8.1.5.3-1.

$$S_c = R_m\theta \tag{8.1.5.3-1}$$

where:

S_c = length along the curve
R_m = radius of curvature at mid-depth
θ = included angle of the curve

The number of reinforcing elements required is determined by Equation 8.1.5.3-2.

$$n = \frac{S_c}{s} \tag{8.1.5.3-2}$$

where:

n = number of reinforcing elements required

8.1.5.4 Reduced Section Modulus for Reinforcement The section modulus is reduced by the hole bored for the radial reinforcement. The net section on the tension side of the neutral axis is reduced by the area of the hole. On the compression side, the reinforcement fills the hole and transmits compression stresses, so no reduction is required. Equations 8.1.5.4-1, 8.1.5.4-2, and 8.1.5.4-3 are used to locate the neutral axis of the modified section and determine its section properties (Figure 8.1.5.4-1).

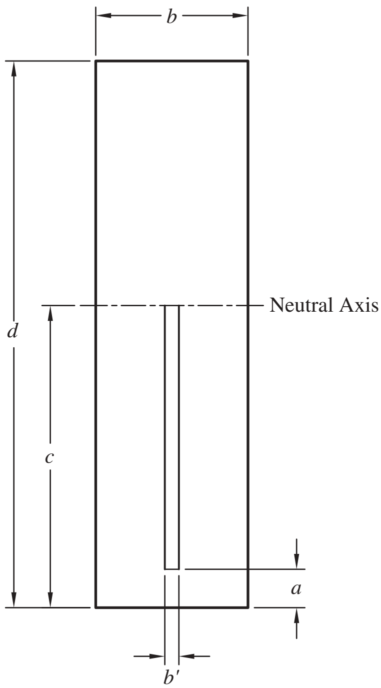


Figure 8.1.5.4-1 Section loss due to radial reinforcement.

The distance from the soffit face to the neutral axis of the modified section is determined by Equation 8.1.5.4-1:

$$c = \frac{bd^2 - b'c^2 + b'a^2}{2(bd - b'c + b'a)} \tag{8.1.5.4-1}$$

where:

c = distance from the soffit to the neutral axis of the modified section
b = member width
d = member depth
b' = effective width of lag screw hole (lag screw diameter)
a = distance from bottom of hole to bottom face of beam

The moment of inertia of the reduced section is calculated with Equation 8.1.5.4-2.

$$I_{x\ reduced} = \frac{b\,(d-c)^3}{3} + \frac{bc^3}{3} - \frac{b'\,(c-a)^3}{3} \tag{8.1.5.4-2}$$

The section modulus of the reduced section is calculated with Equation 8.1.5.4-3.

$$S_{x\ reduced} = \frac{I_{x\ reduced}}{c} \tag{8.1.5.4-3}$$

8.2 CURVED BEAMS WITH CONSTANT DEPTH

The design of curved beams with constant depth is similar to the design of straight beams, but with a couple of additional considerations. In addition to the design checks discussed in Chapter 4 for straight beams (shear, flexure, and vertical deflection), curved beams must also be evaluated for radial stresses and horizontal displacement of the beam ends.

8.2.1 Radial Stress

The equation for computing the radial stress, f_r, in members of constant depth through the curved segment is given by Equation 8.2.1-1.

$$f_r = \frac{3M}{2R_m bd} \tag{8.2.1-1}$$

where:

M = bending moment
b = width of rectangular member
d = depth of rectangular member
R_m = radius of curvature of centerline of member

Where the calculated radial stress, f_r, exceeds the adjusted radial stress, the design should be revised. The radial stress can be reduced by increasing the section size, or by increasing the beam radius.

8.2.2 Flexure

The bending stress in curved beams of constant depth is calculated from the flexure formula for extreme fiber stress in bending, Equations 4.2.1-1 and 4.2.1-2. This is identical to the procedures for straight beams in Chapter 4.

8.2.3 Deflection

Downward deflection of curved beams of constant depth can be estimated using the standard formulas for deflection for prismatic beams, however, the curved shape also results in horizontal displacement of the beam ends as the center of the beam deflects vertically. For a beam with center height above the wall height, downward deflection results in outward movement of the beam ends. The horizontal displacement (push-out) at each support can be estimated using Equation 8.2.3-1.

$$\Delta_H = \frac{2h\Delta_c}{\ell} \tag{8.2.3-1}$$

where:

Δ_H = calculated horizontal movement at each support
h = rise in mid-depth of beam from end to mid-span
Δ_c = deflection at mid-span
ℓ = beam span

The designer must detail the connections to the support structure to either accommodate the above displacement at each end, or the combined effect of both at one end only. AITC 104 [3] provides typical connection details that may be of assistance to the designer in this regard. If the calculated horizontal displacement is excessive, a stiffer section may need to be designed. The designer must also ensure that the beam ends have adequate bearing and are also properly anchored against lateral forces parallel and perpendicular to the beam axis as well any applicable uplift forces.

EXAMPLE 8.2-1 CONSTANT-DEPTH CURVED BEAM

Given: A $6\frac{3}{4}$ in. × $26\frac{1}{8}$ in., 24F-V3 SP curved beam illustrated in Figure 8.2-1 spans 40 ft and supports a total load of 780 lb/ft including the weight of the beam, dead load, and snow load. The beam is manufactured with $1\frac{3}{8}$ in. thick laminations. The bottom face of the beam has a radius of 75 ft (900 in.), and the

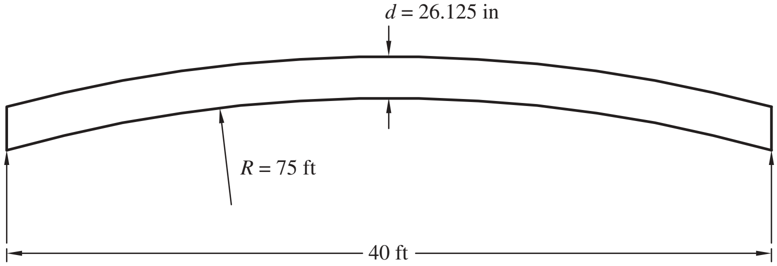


Figure 8.2-1 Curved beam—Example 8.2-1.

entire length of the beam is curved. The vertical deflection limit for the beam is $\delta_{D+S} = 2.7$ in. for total load.

Wanted: Evaluate the beam for shear, flexure, deflection, radial tension, and horizontal displacement of the beam ends.

Solution:

Volume factor (Equation 3.4.3.3-1):

$$C_V = \left(\frac{5.125\,\text{in}}{b}\right)^{\frac{1}{20}} \left(\frac{12\,\text{in}}{d}\right)^{\frac{1}{20}} \left(\frac{21\,\text{ft}}{L}\right)^{\frac{1}{20}} \leq 1.0$$

$$C_V = \left(\frac{5.125\,\text{in}}{6.75\,\text{in}}\right)^{\frac{1}{20}} \left(\frac{12\,\text{in}}{26.125\,\text{in}}\right)^{\frac{1}{20}} \left(\frac{21\,\text{ft}}{40\,\text{ft}}\right)^{\frac{1}{20}} \leq 1.0$$

$$C_V = 0.919$$

Curvature factor Equation (3.4.3.6-1):

$$C_c = 1 - 2000\,(t/R)^2$$

$$C_c = 1 - 2000\left(\frac{1.375\,\text{in}}{900\,\text{in}}\right)^2$$

$$C_c = 0.995$$

Design stresses:

$$F'_{bx} = F_{bx}\,C_D\,C_M\,C_t\,C_V\,C_c$$

$$F'_{bx} = (2400\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\,(0.919)\,(0.995)$$

$$F'_{bx} = 2520\,\text{psi}$$

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr}$$
$$F'_{vx} = (300\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\,(0.72)$$
$$F'_{vx} = 248\,\text{psi}$$

$$E'_x = F_{vx} C_M C_t$$
$$E'_x = 1.8\left(10^6\,\text{psi}\right)(1.0)\,(1.0)$$
$$E'_x = 1.8\left(10^6\,\text{psi}\right)$$

$$F'_{rt} = F_{rt} C_D C_M C_t$$
$$F'_{rt} = \frac{F_{vx} C_{vr}}{3} C_D C_M C_t$$
$$F'_{rt} = \frac{(300\,\text{psi})\,(0.72)}{3}\,(1.15)\,(1.0)\,(1.0)$$
$$F'_{rt} = 82.8\,\text{psi}$$

Design moment:

$$M = \frac{\omega \ell^2}{8}$$
$$M = \frac{(780\ \text{lb/ft})\,(40\,\text{ft})^2}{8}$$
$$M = 156{,}000\,\text{ft-lb} = 1.872\left(10^6\right)\ \text{in-lb}$$

Mid-depth radius:

$$R_m = R + \frac{d}{2}$$
$$R_m = (75\,\text{ft})\left(\frac{12\,\text{in}}{\text{ft}}\right) + \frac{26.125\,\text{in}}{2}$$
$$R_m = 913.1\,\text{in}$$

Radial tension stress:

$$f_{rt} = \frac{3M}{2R_m bd}$$
$$f_{rt} = \frac{3\left[1.872\left(10^6\right)\ \text{in-lb}\right]}{2\,(913.1\,\text{in})\,(6.75\,\text{in})\,(26.125\,\text{in})}$$
$$f_{rt} = 17.4\,\text{psi} \leq F'_{rt} = 82.8\,\text{psi} \qquad \therefore\ \text{OK}$$

Flexural stress:

$$f_{bx} = \frac{6M}{bd^2}$$

$$f_{bx} = \frac{6\left(1.872\left(10^6\right)\ \text{in-lb}\right)}{(6.75\,\text{in})\,(26.125\,\text{in})^2}$$

$$f_{bx} = 2440\,\text{psi} \le F'_{bx} = 2520\,\text{psi} \qquad \therefore\ \text{OK}$$

Shear stress:

$$V = \frac{\omega\ell}{2} = \frac{(780\,\text{lb/ft})\,(40\,\text{ft})}{2} = 15{,}600\,\text{lb}$$

$$f_{vx} = \frac{3V}{2bd}$$

$$f_{vx} = \frac{3\,(15{,}600\,\text{lb})}{2\,(6.75\,\text{in})\,(26.125\,\text{in})}$$

$$f_{vx} = 133\,\text{psi} \le F'_{vx} = 248\,\text{psi} \qquad \therefore\ \text{OK}$$

Vertical deflection:

$$\Delta = \frac{5\omega\ell^4}{384E'_xI_x} = \frac{5\omega\ell^4}{32E'_xbd^3}$$

$$\Delta = \frac{5\,(780\ \text{lb/ft})\,(40\,\text{ft})^4\left(\dfrac{1728\,\text{in}^3}{\text{ft}^3}\right)}{32\left(1.8\left(10^6\,\text{psi}\right)\right)(6.75\,\text{in})\,(26.125\,\text{in})^3}$$

$$\Delta = 2.5\,\text{in} \le \delta_{D+S} = 2.7\,\text{in} \qquad \therefore\ \text{OK}$$

Soffit height at mid-span (Equation 3.6-1):

$$h_s = c = R - \frac{\sqrt{4R^2 - \ell^2}}{2}$$

$$h_s = 900\,\text{in} - \frac{\sqrt{4\,(900\,\text{in})^2 - (480\,\text{in})^2}}{2}$$

$$h_s = 32.6\,\text{in}$$

Horizontal displacement at supports (Equation 8.2.3-1):

$$h \approx h_s$$

$$h \approx 32.6\,\text{in}$$

$$\Delta_H = \frac{2h\,\Delta_c}{\ell}$$

$$\Delta_H = \frac{2\,(32.6\,\text{in})\,2.5\,\text{in}}{480\,\text{in}}$$

$$\Delta_H = 0.34\,\text{in}$$

Answer: The curved beam is satisfactory for the stated design loads. The connections must be detailed to accommodate 0.35 in. of outward deflection at each support, or 0.7 in. at one support. In this case, the shear and radial tension stresses were low, and the beam was nearly optimal for both flexure and deflection.

8.3 PITCHED AND TAPERED CURVED BEAMS

Pitched and tapered curved glued laminated timber beams are among the most popular types of structural roof members where a sloping roof and maximum interior clearance are desired. The top edge of these beams slopes from the apex at the centerline toward the supports at angle ϕ_T as shown in Figure 8.3-1, and the lower edge is curved in the center and slopes at angle ϕ_B from the tangent points to the supports. The end portion of the beam is usually tapered ($\phi_T > \phi_B$), but may be of constant cross section ($\phi_T = \phi_B$), between the end and the tangent point. The geometric features for pitched and tapered curved beams are illustrated in Figure 8.3-1. These features are as defined in the sections that follow.

The bending and radial stresses in pitched and tapered curved beams are affected by the variable shape of the section, and their exact determination is complex. The procedures presented in this *Manual* are based on a simplification

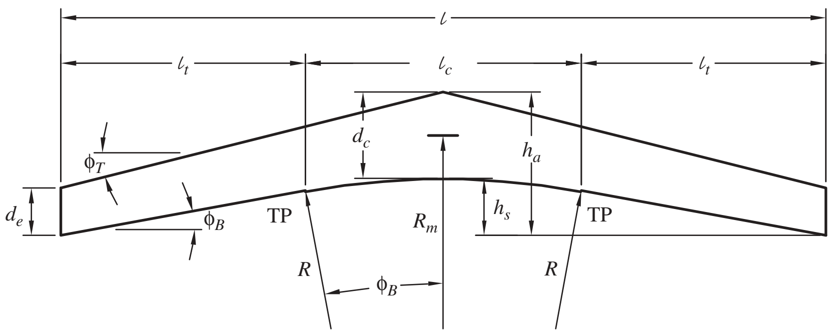


Figure 8.3-1 Pitched and tapered curved beam.

of those contained in *Behavior and Design of Double-Tapered Pitched and Curved Glulam Beams* [4]. Because the factors for determining bending and radial tension stresses are a function of the geometric configuration of the member, they cannot be accurately determined until the final size and shape are known. Therefore, the procedure presented in this *Manual* is a trial-and-error method whereby a trial size is determined and adjusted until a suitable size is obtained. Other methods may be used, provided the design criteria are satisfied.

8.3.1 Radial Stress

When designing a curved bending member of variable cross section such as a pitched and tapered curved beam, the radial stress f_r is computed by Equation 8.3.1-1:

$$f_r = K_{rs} C_{rs} \frac{6M}{bd_c^2} \quad (8.3.1\text{-}1)$$

where:

K_{rs} = radial stress factor obtained from Equation 8.3.1-2
C_{rs} = empirical load-shape radial stress reduction factor, obtained from Equation 8.3.1-3
M = bending moment at mid-span
b = width of cross section
d_c = depth of cross section at mid-span

$$K_{rs} = 0.29 \left(\frac{d_c}{R_m}\right)^2 + 0.32 \tan^{1.2} \phi_T \quad (8.3.1\text{-}2)$$

The K_{rs} formula was developed based on the assumption that the member was subjected to pure bending. It can be shown from equilibrium equations that for a given bending moment, the case of pure bending will result in higher values of radial stresses as compared to other, more common loading conditions such as a uniformly distributed load. Since structural roof members such as pitched and tapered curved beams are seldom designed for the case of pure bending, the C_{rs} factor was developed to allow the designer to reduce the K_{rs} value to account for uniform loading conditions.

The reduction factor, C_{rs}, is obtained from Equation 8.3.1-3 for uniformly loaded members with $d_c/R_m \leq 0.3$, and is taken as 1.0 for members subject to constant moment. C_{rs} can be conservatively taken as 1.0 for all other loading conditions.

$$C_{rs} = 0.27 \ln\left(\tan \phi_T\right) + 0.28 \ln\left(\frac{\ell}{\ell_c}\right) - 0.8\left(\frac{d_c}{R_m}\right) + 1 \leq 1.0 \quad (8.3.1\text{-}3)$$

For equal concentrated loads positioned at third points of the span and for single concentrated loads at mid-span, the value of C_{rs} as determined for the uniformly distributed load case is permitted to be adjusted by the values in Table 8.3.1-1, but need not exceed 1.0 in any case.

The adjustment factors for loading conditions other than uniformly distributed loads are approximate values based on a ratio of the area of the moment diagrams in the curved portion of the pitched and tapered curved member for equal maximum moment conditions. For other, more complex conditions, the designer may wish to determine these adjustment factors by comparing the area of the moment diagram in the curved portion of the pitched and tapered curved beam with the area for an equivalent uniformly distributed loading condition.

TABLE 8.3.1-1 Adjustment to C_{rs} for Curved Beams with Concentrated Loads

ℓ/ℓ_c	Multiply C_{rs} by
Equal Concentrated Loads at Third Points	
All ℓ/ℓ_c	1.05
Concentrated Load at Mid Span	
$\ell/\ell_c = 1.0$	0.75
$\ell/\ell_c = 2.0$	0.80
$\ell/\ell_c = 3.0$	0.85
$\ell/\ell_c = 4.0$	0.90

8.3.2 Bending Stress

The bending stress, f_b, for curved beams of variable cross section, such as pitched and tapered curved beams, is greater than for straight prismatic members and is increased by the bending stress factor K_θ (Equation 8.3.2-1) due to the shape of the member.

$$f_b = K_\theta \frac{6M}{bd_c^2} \quad (8.3.2\text{-}1)$$

where:

M = moment at mid-span
b = width
d_c = depth at mid-span
K_θ = empirical bending stress shape factor, as defined in Equation 8.3.2-2:

$$K_\theta = 1 + 2.7 \tan \phi_T \quad (8.3.2\text{-}2)$$

8.3.3 Deflection

The member's shape also affects deflection. Equation 8.3.3-1 provides an approximation of mid-span deflection based on test data. Other methods of determining deflection, such as finite elements or virtual work, may also be used.

$$\Delta_c = \frac{5\omega \ell^4}{32E'b\left(d_{equiv}\right)^3} \tag{8.3.3-1}$$

where:

Δ_c = deflection at mid-span
ω = uniform distributed load
ℓ = span
E' = modulus of elasticity multiplied by appropriate adjustment factors
b = beam width
d_{equiv} = depth of "equivalent" prismatic member as defined in Equation 8.3.3-2

$$d_{equiv} = \left(d_e + d_c\right)\left(0.5 + 0.735 \tan\phi_T\right) - 1.41\left(d_c\right)\tan\phi_B \tag{8.3.3-2}$$

where:

ϕ_B = slope of bottom (soffit), at ends
ϕ_T = slope of top
d_c = mid-span depth
d_e = depth at end of the member

8.3.4 Design Procedure for Pitched and Tapered Curved Beams

The procedure for the design of symmetrical pitched and tapered curved (PTC) glued laminated timber beams under uniform load follows. Other procedures may be used as long as the appropriate design stress and serviceability checks are met and the appropriate adjustments are made for various loading conditions. The procedure that follows is based on selection of a trial geometry and subsequent design checks. Trial geometry is determined based on a minimum curved portion radius. The trial or final geometry must also be checked for aesthetic appeal.

The design of PTC beams includes seven considerations:

1. Trial geometry
2. Shear stress analysis
3. Radial tension stress analysis
4. Bending stress analysis
5. Vertical deflection analysis
6. Horizontal displacement at supports
7. Radial reinforcement (if applicable)

8.3.4.1 Trial Geometry This section presents one method of establishing a trial geometry. The method presented is intended to facilitate incorporation of the method into a design spreadsheet. Other methods, including graphical methods (mechanical or computer-aided), may be used. Once the trial geometry is established, the subsequent design steps are identical.

The geometries of pitched and tapered beams are governed by architectural considerations such as top (roof) slope, bottom (soffit) slope, and overall appearance (length and radius of curved portion), as well as the section requirements (width and depth) to satisfy stress checks and serviceability criteria. The steps for determining the trial geometry are summarized as follows:

1. The span and angle of the top (roof) slope are determined.
2. A trial radius, soffit slope, end depth, and beam width are chosen by the designer.
3. Sections of interest (including mid-span, tangent points, and multiple sections in the tapered leg) are located and their depths are determined.

The minimum required end depth is typically determined based on shear. Beam width is often governed by support conditions and available standard widths. Experience has indicated that successful trial geometries are those for which the end depth based on shear is somewhat conservative, and for which the difference in the angles of the top and bottom slopes of the tapered ends is not excessive. Trial geometries for which end shear stress is at or near the allowable stress level may be unsatisfactory with regard to the other design checks. Geometries with large differences between the top and bottom slopes may be less efficient due to the relatively large reduction in allowable stresses due to the stress interaction factor.

The minimum end depth, d_e, based on shear is calculated using Equation 8.3.4.1-1.

$$d_e \geq \frac{3}{2}\frac{V}{bF'_{vx}} \tag{8.3.4.1-1}$$

The apex height (relative to the supports), h_a, is calculated with Equation 8.3.4.1-2.

$$h_a = \frac{\ell}{2}\tan\phi_T + d_e \tag{8.3.4.1-2}$$

The soffit height (relative to the supports), h_s, is calculated by Equation 8.3.4.1-3.

$$h_s = \frac{\ell}{2}\tan\phi_B - R\left(\sec\phi_B - 1\right) \tag{8.3.4.1-3}$$

The depth at mid-span, d_c, is calculated with Equation 8.3.4.1-4.

$$d_c = h_a - h_s \tag{8.3.4.1-4}$$

The length of the curved segment of the beam, ℓ_c, is calculated with Equation 8.3.4.1-5.

$$\ell_c = 2R \sin \phi_B \text{ (plan length)} \tag{8.3.4.1-5}$$

The length of each tapered end, ℓ_t, is given by Equation 8.3.4.1-6.

$$\ell_t = \frac{\ell - \ell_c}{2} \text{ (plan length)} \tag{8.3.4.1-6}$$

The radius of the curved portion of the beam at mid-depth and mid-span is calculated using Equation 8.3.4.1-7.

$$R_m = R + \frac{d_c}{2} \tag{8.3.4.1-7}$$

The depth of the beam along the straight tapered portion, d'_x, may be determined using Equation 8.3.4.1-8.

$$d'_x = \left[d_e + x\left(\tan\phi_T - \tan\phi_B\right)\right]\left[\cos\phi_B - \sin\phi_B \tan\left(\phi_T - \phi_B\right)\right] \tag{8.3.4.1-8}$$

where:

d'_x = the depth of the beam at x, measured perpendicular to the soffit
x = the distance from the end

8.3.4.2 *Shear Stress Analysis* If the trial end depth was based on shear requirements, as shown, it is not necessary to reanalyze the shear stresses. Otherwise, shear stresses are analyzed using methods previously presented (Equation 8.3.4.2-1).

$$f_v = \frac{3}{2}\frac{V}{bd_e} \leq F'_{vx} = F_{vx} C_D C_M C_t C_{vr} \tag{8.3.4.2-1}$$

8.3.4.3 *Radial Tension Stress Analysis* The radial tension stress is evaluated using Equation 8.3.4.3-1.

$$f_{rt} = K_{rs} C_{rs} \frac{6M}{bd_c^2} \leq F'_{rt} = F_{rt} C_D C_M C_t \tag{8.3.4.3-1}$$

Where the radial tension stress design check is not satisfied, the geometry must be adjusted to reduce the radial tension stress, or for softwood species except southern pine, radial reinforcement may be used to carry the radial tension stress. Increasing the radius of curvature and increasing the beam depth are both effective ways to reduce the radial tension stress.

8.3.4.4 Bending Stress Analysis Bending stress in the beam at mid-span is evaluated using Equation 8.3.4.4-1.

$$f_{b,midspan} = K_\theta \frac{6M}{bd_c^2} \leq F'_{bx,midspan} = F_{bx} C_D \left(C_V \text{ or } C_L\right) C_c C_M C_t \qquad (8.3.4.4\text{-}1)$$

The bending stress must also be checked along the tapered portions of the beam, including the tangent point (TP). In the tapered section, the stress interaction factor must be calculated and applied. At the tangent point, both the stress interaction factor and the curvature factor are applicable. As such, the tangent point may be the critical design point of a pitched and tapered curved beam. For many pitched and tapered beam applications, the roof is attached to the top of the beam in such a way as to provide lateral bracing, in which case the beam stability factor, C_L, may be taken to be 1.0. Equations 8.3.4.4-2 and 8.3.4.4-3 can be used to determine allowable stress.

$$f_{b,x} = \frac{6M}{bd_x'^2} \leq F'_{bx} = F_{bx} C_D \left(C_V \text{ or } C_L\right) C_I C_M C_t \qquad (8.3.4.4\text{-}2)$$

$$f_{b,TP} = \frac{6M}{b\left(d'_{TP}\right)^2} \leq F'_{bx,TP} = F_{bx} C_D \left(C_V \text{ or } C_L\right) C_I C_c C_M C_t \qquad (8.3.4.4\text{-}3)$$

Customarily, the bending stress is checked at mid-span, the tangent point, and at several points between the tangent point and the ends. If the bending stress check, $f_b \leq F'_b$, is not satisfied at all locations, the beam geometry must be adjusted to either reduce the bending stress or increase the allowable stress. Increasing the beam depth at the location of overstress will reduce the bending stress. Decreasing the angle of taper in the tapered leg will increase the allowable bending stress (through the stress interaction factor).

8.3.4.5 Vertical Deflection Analysis The deflection of a pitched and tapered curved beam under uniform load may be calculated using Equation 8.3.4.5-1.

$$\Delta_c = \frac{5\omega\ell^4}{32E'bd_{equiv}^3} \qquad (8.3.4.5\text{-}1)$$

where:

Δ_c = the mid-span deflection (typically due to either live load or total load)
ω = uniform load
ℓ = beam span

E' = adjusted modulus of elasticity
b = beam width
d_{equiv} = depth of a prismatic beam having a deflection equivalent to the pitched and tapered curved beam, given by Equation 8.3.4.5-2:

$$d_{equiv} = \left(d_e + d_c\right)\left(0.5 + 0.735 \tan \phi_T\right) - 1.41 \left(d_c\right) \tan \phi_B \qquad (8.3.4.5\text{-}2)$$

The deflection criteria must be satisfactory to the owner and building official and, as appropriate, include the time-dependent deflection (creep). Where deflection criteria are not satisfied, the beam geometry must be adjusted (typically by increased depth).

8.3.4.6 Horizontal Displacement at Supports The horizontal movement (push-out) at each support can be estimated using Equation 8.3.4.6-1.

$$\Delta_H = \frac{2h\Delta_c}{\ell} \qquad (8.3.4.6\text{-}1)$$

where:

Δ_H = calculated horizontal movement at each support
h = the rise in mid-depth of beam from end to mid-span, which may be calculated using Equation 8.3.4.6-2
Δ_c = vertical deflection at mid-span

$$h = h_a - \frac{d_c}{2} - \frac{d_e}{2} \qquad (8.3.4.6\text{-}2)$$

The designer must detail the connections to the support structure to accommodate either the above displacement at each end or the combined effect of both at one end only. AITC 104 [3] provides typical connection details that may be of assistance to the designer in this regard. If the calculated horizontal displacement is excessive, a stiffer section might need to be designed. The designer must also ensure that the beam ends have adequate bearing and are properly anchored against lateral forces parallel and perpendicular to the beam axis, as well as any applicable uplift forces.

8.3.4.7 Radial Reinforcement (If Applicable) For softwood species other than southern pine, it is common to use radial reinforcement to resist radial tension forces in curved beams. Requirements for the design and manufacture of beams with radial reinforcement are discussed in Section 8.1.5.

EXAMPLE 8.3-1 SOUTHERN PINE PITCHED AND TAPERED CURVED BEAM (WITHOUT RADIAL REINFORCEMENT)

Given: A roof system consisting of southern pine pitched and tapered curved glued laminated timber beams has been requested. The beams will span 45 ft (540 in.) and will be spaced 14 ft on center. A roof slope of 4:12 (18.43°) has been specified. The beams will support a snow load of 30 psf and a dead load of 15 psf in addition to their own weight. Timber decking will provide continuous lateral support to the top edge of the beam. A deflection limit of $\delta_{D+S} = \ell/180 = 3.0$ in. is required for total load. $1\frac{3}{8}$ in. thick laminations will be used with a minimum radius of 18 ft (216 in.) at the inside face.

Wanted: Using a width of $b = 5$ in., determine suitable section and geometric properties for the beam.

Approach: The design procedure from Section 8.3.4 for PTC beams will be used. An assumed weight of 50 plf will be used for the beam itself, in addition to the stated loads. A trial geometry will be established, with the end depth based on shear, a bottom slope of 14°, and a radius of 18 ft. The geometry will be revised as necessary to satisfy the design criteria for radial tension, flexure, and deflection.

Solution:

Design values:

$$F'_{bx} = F_b C_D C_M C_t \left(C_L \text{ or } C_V\right) C_I C_c$$
$$F'_{bx} = (2400\text{ psi})\,(1.15)\,(1.0)\,(1.0)\left(C_L \text{ or } C_V\right) C_I C_c$$
$$F'_{bx} = (2760\text{ psi})\left(C_L \text{ or } C_V\right) C_I C_c$$

$$F'_{vx} = F_v C_D C_M C_t C_{vr}$$
$$F'_{vx} = (300\text{ psi})\,(1.15)\,(1.0)\,(1.0)\,(0.72)$$
$$F'_{vx} = 248\text{ psi}$$

$$E'_x = E_x C_M C_t$$
$$E'_x = 1.8\left(10^6\text{ psi}\right)(1.0)\,(1.0) = 1{,}800{,}000\text{ psi}$$
$$E'_x = 1.8\left(10^6\text{ psi}\right)$$

$$F'_{rt} = F_{rt} C_D C_M C_t$$
$$F'_{rt} = \frac{F_{vx} C_{vr}}{3} C_D C_M C_t$$
$$F'_{rt} = \frac{(300\text{ psi})\,(0.72)}{3}(1.15)\,(1.0)\,(1.0)$$
$$F'_{rt} = 82.8\text{ psi}$$

$$F'_{c\perp} = F_{c\perp} C_M C_t$$

$$F'_{c\perp} = (650\,\text{psi})\,(1.0)\,(1.0)$$

$$F'_{c\perp} = 650\,\text{psi}$$

Total uniform load (self-weight estimated as 50 plf):

$$\omega = \omega_D + \omega_S + \omega_{sw}$$

$$\omega = (15\,\text{psf} + 30\,\text{psf})\,(14\,\text{ft}) + 50\,\text{plf}$$

$$\omega = 680\,\text{plf} = 56.7\,\text{lb/in}$$

Shear force:

$$V = \frac{\omega\ell}{2} = \frac{(680\,\text{plf})\,(45\,\text{ft})}{2} = 15{,}300\,\text{lb}$$

Minimum end depth (Equation 8.3.4.1-1):

$$d_e \geq \frac{3V}{2bF'_v} = \frac{(3)\,(15{,}300\,\text{lb})}{(2)\,(5\ \text{in})\,(248\,\text{psi})} = 18.5\,\text{in} \qquad \therefore\ \text{Use } 19\,\text{in}$$

Length of curved segment (Equation 8.3.4.1-5):

$$\ell_c = 2R\sin\phi_B = (2)\,(216\,\text{in})\left(\sin\ 14^\circ\right) = 104.5\,\text{in}$$

$$\frac{\ell}{\ell_c} = \frac{540\,\text{in}}{104.5\,\text{in}} = 5.17$$

Apex height (Equation 8.3.4.1-2):

$$h_a = \frac{\ell}{2}\tan\phi_T + d_e = \frac{540\,\text{in}}{2}\left(\tan 18.43^\circ\right) + 19\,\text{in} = 109.0\,\text{in}$$

Soffit height at mid-span (Equation 8.3.4.1-3):

$$h_s = \frac{\ell}{2}\tan\phi_B - R\left(\sec\phi_B - 1\right)$$

$$h_s = \frac{540\,\text{in}}{2}\left(\tan 14^\circ\right) - (216\,\text{in})\left(\sec\ 14^\circ - 1\right)$$

$$h_s = 60.7\,\text{in}$$

Depth at mid-span (Equation 8.3.4.1-4):

$$d_c = h_a - h_s = 109.0\,\text{in} - 60.7\ \text{in} = 48.3\,\text{in}$$

Radius at mid-depth at mid-span (Equation 8.3.4.1-7):

$$R_m = 216\,\text{in} + \frac{48.3\,\text{in}}{2} = 240.1\,\text{in}$$

$$\frac{d_c}{R_m} = \frac{48.3\,\text{in}}{240.1\,\text{in}} = 0.20$$

Radial tension stress analysis (Section 8.3.1):

$$M = \frac{\omega \ell^2}{8} = \frac{56.67\,\text{lb/in}\,(540\,\text{in})^2}{8} = 2{,}065{,}500\,\text{lb-in}$$

$$C_{rs} = 0.27 \ln\left(\tan \phi_T\right) + 0.28 \ln\left(\frac{\ell}{\ell_c}\right) - 0.8\left(\frac{d_c}{R_m}\right) + 1 \leq 1.0$$

$$C_{rs} = 0.27 \ln\left(\tan 18.4^\circ\right) + 0.28 \ln(5.17) - 0.8\,(0.2) + 1 = 1.00$$

$$C_{rs} = 1.0$$

$$K_{rs} = 0.29\left(\frac{d_c}{R_m}\right)^2 + 0.32 \tan^{1.2} \phi_T$$

$$K_{rs} = 0.29\,(0.20)^2 + 0.32 \tan^{1.2}\left(18.43^\circ\right) = 0.097$$

$$f_{rt} = K_{rs} C_{rs} \frac{6M}{bd_c^2}$$

$$f_{rt} = (0.097)\,(1.0)\,\frac{6\,(2{,}065{,}500\,\text{lb-in})}{(5\,\text{in})\,(48.3\,\text{in})^2}$$

$$f_{rt} = 103\,\text{psi} > F'_{rt} = 82.8\,\text{psi} \qquad \therefore \text{ Not acceptable}$$

The radial stress may be decreased by increasing the depth, width, or radius of curvature. Further design checks with the above geometry would also reveal that the beam is not acceptable with regard to bending in parts of the pitched and tapered section. Decreasing the bottom slope will increase the mid-span depth and reduce radial tension stress; however, decreased bottom slope will also decrease the allowable bending stress through the stress interaction factor. The increased end depth could have been established initially by being additionally conservative in the analysis of the end shear condition. Since determining a satisfactory geometry may involve a number of iterations and trial geometries, developing computational spreadsheets is recommended.

In this example, a new trial geometry will be used based on increased end depth and larger radius of curvature. The increased end depth will increase the overall depth and thus reduce the bending stress throughout. The increased radius of curvature will increase the length and depth of the curved portion, reduce radial stress, and increase the allowable bending stress through increasing the curvature factor.

Revised trial geometry:

$$d_e = 21 \text{ in}$$
$$R = 300 \text{ in}$$
$$\ell_c = 145 \text{ in}; \ \frac{\ell}{\ell_c} = 3.72$$
$$h_a = 111.0 \text{ in}$$
$$h_s = 58.1 \text{ in}$$
$$d_c = 52.8 \text{ in}$$
$$R_m = 326.4 \text{ in}; \ \frac{d_c}{R_m} = 0.162$$

Radial tension stress analysis (Section 8.3.1):

$$K_{rs} = 0.29\left(\frac{d_c}{R_m}\right)^2 + 0.32 \tan^{1.2} \phi_T$$
$$K_{rs} = 0.29\,(0.162)^2 + 0.32 \tan^{1.2}\left(18.43^\circ\right) = 0.093$$

$$C_{rs} = 0.27 \ln\left(\tan \phi_T\right) + 0.28 \ln\left(\frac{\ell}{\ell_c}\right) - 0.8\left(\frac{d_c}{R_m}\right) + 1 \leq 1.0$$
$$C_{rs} = 0.27 \ln\left(\tan 18.4^\circ\right) + 0.28 \ln (3.72) - 0.8\,(0.16) + 1$$
$$C_{rs} = 0.943$$

$$f_{rt} = K_{rs} C_{rs} \frac{6M}{bd_c^2}$$
$$f_{rt} = 0.093\,(0.943)\,\frac{6\,(2{,}065{,}500 \text{ lb - in})}{(5 \text{ in})\,(52.8 \text{ in})^2}$$
$$f_{rt} = 78.0 \text{ psi} \leq F'_{rt} = 82.8 \text{ psi} \qquad \therefore \ \text{OK}$$

Bending stress analysis (Section 8.3.2):

$$K_\theta = 1 + 2.7 \tan \phi_T = 1 + 2.7\left(\tan 18.4^\circ\right) = 1.90$$
$$f_{b,midspan} = K_\theta \frac{6M}{bd_c^2} = (1.90)\,\frac{6\,(2{,}065{,}500 \text{ lb-in})}{(5 \text{ in})\,(52.8 \text{ in})^2} = 1689 \text{ psi}$$
$$C_L = 1.0 \text{ (full lateral support at top of beam)}$$
$$C_V = \left(\frac{5.125 \text{ in}}{5 \text{ in}}\right)^{\frac{1}{20}} \left(\frac{12 \text{ in}}{52.8 \text{ in}}\right)^{\frac{1}{20}} \left(\frac{21 \text{ ft}}{45 \text{ ft}}\right)^{\frac{1}{20}} = 0.895$$

$$C_c = 1 - 2000\left(\frac{1.375}{300}\right)^2 = 0.958$$

$$F'_{bx,midspan} = 2400\,\text{psi}\,(1.15)\,(0.895)\,(0.958)$$

$$F'_{bx,midspan} = 2366\,\text{psi} \geq f_{b,midspan} = 1689\,\text{psi} \qquad \therefore \text{ OK}$$

The bending stress is also evaluated at four sections (Figure 8.3-2) along the straight tapered segments of the beam in Table 8.3-1 using Equations 8.3.4.4-2 and 8.3.4.4-3, as illustrated in Figure 8.3-2.

TABLE 8.3-1 Bending Stresses in Straight Tapered Segment—Example 8.3-1.

Section	x in.	d'_x in.	M_x lb-in.	S in^3	f_b psi	C_V	C_I	C_c	F'_{bx} psi	f_b/F'_{bx}
A	49.4	23.9	686,000	477	1439	0.931	0.758		2091	0.688
B	98.7	27.9	1,234,000	647	1908	0.924	0.758		2091	0.912
C	148	31.8	1,644,000	843	1951	0.918	0.758		2091	0.933
TP	197	35.8	1,916,000	1065	1800	0.913	0.758	0.958	2004	0.898

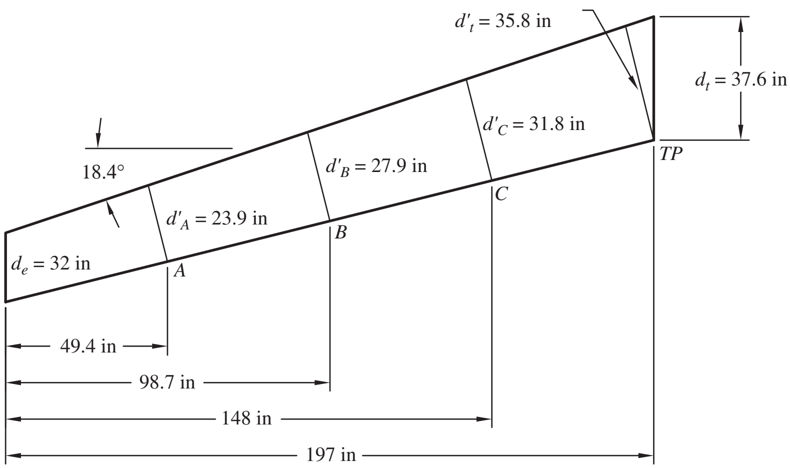


Figure 8.3-2 Straight tapered end segment of beam—Example 8.3-1.

Table 8.3-1 shows that the bending stresses are within acceptable limits at the selected locations along the straight tapered segments of the beam.

Vertical deflection (Equations 8.3.4.5-1 and 8.3.4.5-2):

$$d_{equiv} = \left(d_e + d_c\right)\left(0.5 + 0.735\tan\phi_T\right) - 1.41\left(d_c\right)\tan\phi_B$$

$$d_{equiv} = (21\,\text{in} + 52.8\,\text{in})\left(0.5 + 0.735\left(\tfrac{4}{12}\right)\right) - 1.41\,(52.8\,\text{in})\tan\left(14^\circ\right)$$

$$d_{equiv} = 36.4\,\text{in}$$

$$\Delta_c = \frac{5\omega\ell^4}{32E'b\left(d_{equiv}\right)^3}$$

$$\Delta_{c,SL} = \frac{5\,(35\text{ lb/in})\,(540\text{ in})^4}{32\,(1{,}800{,}000\text{ psi})\,(5\text{ in})\,(36.4\text{ in})^3} = 1.07\text{ in}$$

$$\Delta_{c,DL} = \frac{5\,(21.67\text{ lb/in})\,(540\text{ in})^4}{32\,(1{,}800{,}000\text{ psi})\,(5\text{ in})\,(36.4\text{ in})^3} = 0.66\text{ in}$$

$$\Delta_{c,TL} = \Delta_{c,DL} + \Delta_{c,SL}$$

$$\Delta_{c,TL} = 0.66\text{ in} + 1.07\text{ in}$$

$$\Delta_{c,TL} = 1.73\text{ in} \le \delta_{D+S} = 3.00\text{ in} \qquad \therefore\ \text{OK}$$

Horizontal displacement (Equations 8.3.4.6-1 and 8.3.4.6-2):

$$h = h_a - \frac{d_c}{2} - \frac{d_e}{2} = 111\text{ in} - \frac{52.8\text{ in}}{2} - \frac{21\text{ in}}{2} = 74.1\text{ in}$$

$$\Delta_{H,TL} = \frac{2h\,\Delta_c}{\ell} = \frac{2\,(74.1\text{ in})\,(1.73\text{ in})}{540\text{ in}} = 0.47\text{ in}$$

Answer: The southern pine pitched and tapered curved beam is satisfactory as follows (Figure 8.3-3):

End depth: $d_e = 21$ in.

Mid-span depth: $d_c = 52.8$ in.

Top slope: $\phi_T = 18.43^\circ$ $\left(\frac{4}{12}\text{ pitch}\right)$

Bottom slope: $\phi_B = 14.0^\circ$

Distance from end to point of tangency: $\ell_t = 197$ in.

Inside (soffit) radius: $R = 300\text{ in} = 25\text{ ft}$

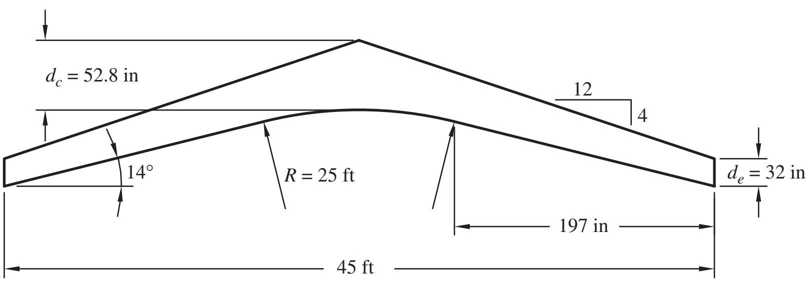


Figure 8.3-3 Southern pine PTC beam—Example 8.3-1.

EXAMPLE 8.3-2 DOUGLAS FIR PITCHED AND TAPERED CURVED BEAM (WITH RADIAL REINFORCEMENT)

Given: 24F-1.8E DF PTC roof beams will be spaced at 16 ft on center and span 60 ft (720 in.). The roof slope is 3:12 ($\phi_T = 14.0^\circ$). The beams will support a dead load of 15 psf and a snow load of 30 psf in addition to their own weight. The vertical deflection of the beams must be limited to $\delta_{D+S} = \ell/240 = 3.0$ in. The beams are supported laterally along the top edge by roof decking. The beams will be manufactured using $1\frac{1}{2}$ in.–thick laminations with a minimum radius of 27.5 ft.

Wanted: Determine a suitable Douglas fir pitched and tapered curved beam with a width of 6.75 in. Radial reinforcement may be used if necessary.

Approach: The design procedure from Section 8.3.4 for PTC beams will be used. An assumed weight of 100 plf will be used for the beam itself, in addition to the stated loads. A trial geometry will be established, with the end depth based on shear, a bottom slope of 9°, and a radius of 27.5 ft (330 in.). The geometry will be revised as necessary to satisfy the design criteria for radial tension, flexure, and deflection.

Solution:

Design values:

$$F'_{bx} = F_b C_D C_M C_t \left(C_L \text{ or } C_V\right) C_I C_c$$
$$F'_{bx} = (2400\text{ psi})\,(1.15)\,(1.0)\,(1.0)\left(C_L \text{ or } C_V\right) C_I C_c$$
$$F'_{bx} = (2760\text{ psi})\left(C_L \text{ or } C_V\right) C_I C_c$$

$$F'_{vx} = F_v C_D C_M C_t C_{vr}$$
$$F'_{vx} = (265\text{ psi})\,(1.15)\,(1.0)\,(1.0)\,(0.72)$$
$$F'_{vx} = 219\text{ psi}$$

$$E'_x = E_x C_M C_t$$
$$E'_x = 1.8\left(10^6\text{ psi}\right)(1.0)\,(1.0) = 1{,}800{,}000\text{ psi}$$
$$E'_x = 1.8\left(10^6\text{ psi}\right)$$

$$F'_{rt} = F_{rt} C_D C_M C_t$$
$$F'_{rt} = (15\text{ psi})\, C_D C_M C_t$$
$$F'_{rt} = (15\text{ psi})\,(1.15)\,(1.0)\,(1.0)$$
$$F'_{rt} = 17.2\text{ psi}$$

$$F'_{c\perp} = F_{c\perp} C_M C_t$$
$$F'_{c\perp} = (650\,\text{psi})\,(1.0)\,(1.0)$$
$$F'_{c\perp} = 650\,\text{psi}$$

Design load:

$$\omega = \omega_D + \omega_S + \omega_{sw}$$
$$\omega = (15\,\text{psf} + 30\,\text{psf})\,(16\,\text{ft}) + 100\,\text{plf} = 820\,\text{plf}$$

Minimum end depth:

$$V = \frac{\omega \ell}{2} = \frac{820\,\text{plf}\,(60\ \text{ft})}{2} = 24{,}600\,\text{lb}$$
$$d_e \geq \frac{3V}{2bF'_v} = \frac{3}{2}\frac{24{,}600\,\text{lb}}{(6.75\,\text{in})\,(219\ \text{psi})} = 25.0\,\text{in} \qquad \therefore\ \text{Try } 26\,\text{in}$$

Apex height (Equation 8.3.4.1-2):

$$h_a = \frac{\ell}{2}\tan\phi_T + d_e = \frac{720\,\text{in}}{2}\left(\frac{3}{12}\right) + 26\,\text{in} = 116\,\text{in}$$

Soffit height at mid-span (Equation 8.3.4.1-3):

$$h_s = \frac{\ell}{2}\tan\phi_B - R\left(\sec\phi_B - 1\right)$$
$$h_s = \frac{720\,\text{in}}{2}\tan 9^\circ - 330\,\text{in}\left(\sec 9^\circ - 1\right) = 52.9\,\text{in}$$

Depth at mid-span (Equation 8.3.4.1-4):

$$d_c = h_a - h_s = 116\ \text{in} - 52.9\ \text{in} = 63.1\,\text{in}$$

Length of curved segment (Equation 8.3.4.1-5):

$$\ell_c = 2R\sin\phi_B = 2\,(330\,\text{in})\sin\left(9^\circ\right) = 103.2\,\text{in}$$

Length of straight tapered segment (Equation 8.3.4.1-6):

$$\ell_t = \frac{\ell - \ell_c}{2} = \frac{720\,\text{in} - 103.2\,\text{in}}{2} = 308.4\,\text{in}$$

Radius at mid-depth at mid-span (Equation 8.3.4.1-7):

$$R_m = R + \frac{d_c}{2} = 330\,\text{in} + \frac{63.1\,\text{in}}{2} = 361.6\,\text{in}$$

Shear stress analysis:

$$f_v = \frac{3V}{2bd_e}$$

$$f_v = \frac{3\,(24{,}600\,\text{lb})}{2\,(6.75\,\text{in})\,(26.0\,\text{in})}$$

$$f_v = 210\,\text{psi} \leq F'_v = 219\,\text{psi} \qquad \therefore\ \text{OK}$$

Radial tension stress analysis (Section 8.3.1):

$$\frac{d_c}{R_m} = \frac{63.1\,\text{in}}{362\,\text{in}} = 0.174$$

$$K_{rs} = 0.29\left(\frac{d_c}{R_m}\right)^2 + 0.32\tan^{1.2}\phi_T$$

$$K_{rs} = 0.29\,(0.174)^2 + 0.32\tan^{1.2}\left(14^\circ\right) = 0.069$$

$$C_{rs} = 0.27\ln\left(\tan\phi_T\right) + 0.28\ln\left(\frac{\ell}{\ell_c}\right) - 0.8\left(\frac{d_c}{R_m}\right) + 1 \leq 1.0$$

$$C_{rs} = 0.27\ln\left(\tan\left(14^\circ\right)\right) + 0.28\ln\left(\frac{720\,\text{in}}{103.2\,\text{in}}\right) - 0.8\,(0.174) + 1 \leq 1.0$$

$$C_{rs} = 1.0$$

$$M_{midspan} = \frac{\omega\ell^2}{8}$$

$$M_{midspan} = \frac{820\,\text{lb/ft}\,(60\ \text{ft})^2}{8}$$

$$M_{midspan} = 369{,}000\,\text{lb-ft} = 4{,}428{,}000\,\text{lb-in}$$

$$f_{rt} = K_{rs}C_{rs}\frac{6M}{bd_c^2}$$

$$f_{rt} = (0.069)\,(1.0)\,\frac{6\,(4{,}428{,}000\,\text{lb-in})}{6.75\,\text{in}\,(63.1\,\text{in})^2}$$

$$f_{rt} = 68.2\ \text{psi}$$

$$\frac{1}{3}F'_v = \frac{1}{3}\,(219\ \text{psi}) = 73.0\ \text{psi}$$

$$F'_{rt} = 17.2\ \text{psi} < f_{rt} = 68.2\ \text{psi} \leq \frac{1}{3}F'_v = 73.0\ \text{psi}$$

$\therefore$ Use radial reinforcement.

Bending stress analysis (Section 8.3.2):

$$K_\theta = 1 + 2.7\tan\phi_T = 1 + 2.7\tan\left(14.0^\circ\right) = 1.67$$

$$f_{b,midspan} = K_\theta \frac{6M_{midspan}}{bd_c^2}$$

$$f_{b,midspan} = 1.67\left(\frac{6\,(4{,}428{,}000\,\text{lb-in})}{(6.75\,\text{in})\,(67.1\,\text{in})^2}\right)$$

$$f_{b,midspan} = 1460\,\text{psi}$$

$$C_L = 1.0 \text{ (full lateral support at top of beam)}$$

$$C_V = \left(\frac{5.125\,\text{in}}{6.75\,\text{in}}\right)^{\frac{1}{10}}\left(\frac{12\,\text{in}}{63.1\,\text{in}}\right)^{\frac{1}{10}}\left(\frac{21\,\text{ft}}{60\,\text{ft}}\right)^{\frac{1}{10}} = 0.742$$

$$C_c = 1 - 2000\left(\frac{1.5\,\text{in}}{330\,\text{in}}\right)^2 = 0.959$$

$$F'_{bx,midspan} = (2760\,\text{psi})\left(C_L \text{ or } C_V\right) C_I C_c$$

$$F'_{bx,midspan} = (2760\,\text{psi})\,(0.742)\,(1.0)\,(0.959)$$

$$F'_{bx,midspan} = 1964\,\text{psi} \geq f_{b,midspan} = 1460\,\text{psi} \qquad \therefore\ \text{OK}$$

The bending stress is also evaluated at four sections along the straight tapered segments of the beam in Table 8.3-2 using Equations 8.3.4.4-2 and 8.3.4.4-3, as illustrated in Figure 8.3-4.

TABLE 8.3-2 Bending Stresses in Straight Tapered Segment—Example 8.3-2

Section	x in.	d'_x in.	M_x lb-in	S in^3	f_b psi	C_V	C_I	C_c	F'_{bx} psi	f_b/F'_{bx}
A	77.1	32.2	1,693,000	1167	1451	0.794	0.668	1.0	1843	0.787
B	154.2	39.1	2,981,000	1719	1734	0.778	0.668	1.0	1843	0.941
C	231.3	46.0	3,862,000	2378	1624	0.766	0.668	1.0	1843	0.881
TP	308.4	52.9	4,337,000	3143	1380	0.755	0.668	0.959	1767	0.781

Table 8.3-2 shows that the bending stresses are acceptable at the selected locations in the straight tapered portions of the beam.

Vertical deflection (Equations 8.3.4.5-1and 8.3.4.5-2):

$$d_{equiv} = \left(d_e + d_c\right)\left(0.5 + 0.735\tan\phi_T\right) - 1.41\left(d_c\right)\tan\phi_B$$

$$d_{equiv} = (26\,\text{in} + 63.1\,\text{in})\left(0.5 + 0.735\left(\tfrac{3}{12}\right)\right) - 1.41\,(63.1\,\text{in})\tan\left(9^\circ\right)$$

$$d_{equiv} = 46.8\,\text{in}$$

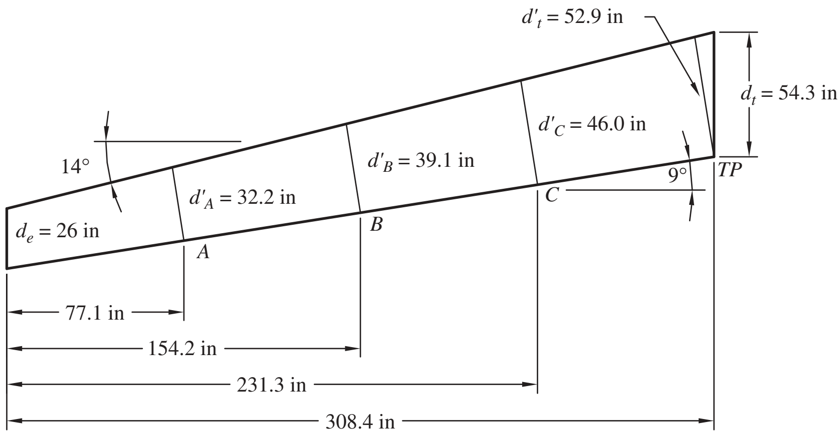


Figure 8.3-4 Straight tapered segment at beam end—Example 8.3-2.

$$\Delta_{c,SL} = \frac{5\omega\ell^4}{32E'b\left(d_{equiv}\right)^3} = \frac{5\,(40\,\text{lb/in})\,(720\,\text{in})^4}{32\,(1{,}800{,}000\ \text{psi})\,(6.75\,\text{in})\,(46.8\ \text{in})^3} = 1.3\,\text{in}$$

$$\Delta_{c,DL} = \frac{5\omega\ell^4}{32E'b\left(d_{equiv}\right)^3} = \frac{5\,(28.33\,\text{lb/in})\,(720\,\text{in})^4}{32\,(1{,}800{,}000\ \text{psi})\,(6.75\,\text{in})\,(46.8\ \text{in})^3} = 1.0\,\text{in}$$

$$\Delta_{c,TL} = \Delta_{c,LL} + \Delta_{c,DL} = 1.3 + 1.0 = 2.3 \leq \frac{\ell}{240} = \frac{720\,\text{in}}{240} = 3.0\,\text{in} \quad \therefore\ \text{OK}$$

Horizontal displacement (Equations 8.3.4.6-1 and 8.3.4.6-2):

$$h = h_a - \frac{d_c}{2} - \frac{d_e}{2} = 116\,\text{in} - \frac{63.1\,\text{in}}{2} - \frac{26\,\text{in}}{2} = 71.5\,\text{in}$$

$$\Delta_{H,TL} = \frac{2h\Delta_c}{\ell} = \frac{2\,(71.5\,\text{in})\,(2.3\,\text{in})}{720\ \text{in}} = 0.46\,\text{in}$$

Radial reinforcement ($\frac{3}{4}$ in. lag screws, Table 8.1.5.2-1, Equation 8.1.5.2-1):

$$T_{lag} = (501\,\text{lb/in})\,\ell_p \leq 5265\,\text{lb}$$

$$T_{lag} = (501\,\text{lb/in})\left(\frac{d_c}{2} - 3\,\text{in}\right) \leq 5265\,\text{lb}$$

$$T_{lag} = (501\,\text{lb/in})\left(\frac{63.1\,\text{in}}{2} - 3\,\text{in}\right) \leq 5265\,\text{lb}$$

$$T_{lag} = 14{,}300\,\text{lb} \leq 5265\,\text{lb}$$

$$T_{lag} = 5265\,\text{lb}$$

$$s = \frac{T_{lag}}{f_{rt} b} = \frac{5265\,\text{lb}}{(68.2\,\text{psi})\,(6.75\,\text{in})} = 11.4\,\text{in.} \qquad \therefore \text{ Use 10 in. spacing.}$$

Length along the curve (Equation 8.1.5.3-1):

$$S_c = R_m \theta$$

$$S_c = R_m \left(2\phi_B \frac{\pi}{180^\circ}\right)$$

$$S_c = (361.6\,\text{in}) \left(2\,(9.0^\circ)\,\frac{\pi}{180^\circ}\right)$$

$$S_c = 113.6\,\text{in}$$

Number of lag screws (Equation 8.1.5.3-2):

$$n = \frac{S_c}{s} = \frac{113.6\,\text{in}}{10\,\text{in}} = 11.4 \qquad \therefore \text{ Use 12 lag screws.}$$

Installation of lag screws:
Lag screws should be installed symmetrically about mid-span straddling the apex, so a screw does not coincide with the point of maximum moment. Place six lags on each side of mid-span, starting at 5 in. each side of mid-span. Each lag should be installed from the top to within 2 in. to 3 in. from the soffit face. The lag screws are to be installed perpendicular to the glue lines of the laminated timber.

Reduced section modulus at mid-span due to lag screws (Section 8.1.5.4):

$$c_1 = \frac{bd^2 - b'c_{trial}^2 + b'a^2}{2\left[bd - b'c_{trial} + b'a\right]}$$

$$c_1 = \frac{(6.75\,\text{in})\,(63.1\,\text{in})^2 - (0.75\,\text{in})\,(31.55\,\text{in})^2 + (0.75\,\text{in})\,(2\,\text{in})^2}{2\,[(6.75\,\text{in})\,(63.1\,\text{in}) - (0.75\,\text{in})\,(31.55\,\text{in}) + (0.75\,\text{in})\,(2\,\text{in})]}$$

$$c_1 = 32.36\,\text{in}$$

$$c_2 = \frac{bd^2 - b'c_1^2 + b'a^2}{2\left[bd - b'c_1 + b'a\right]}$$

$$c_2 = \frac{(6.75\,\text{in})\,(63.1\,\text{in})^2 - (0.75\,\text{in})\,(32.36\,\text{in})^2 + (0.75\,\text{in})\,(2\,\text{in})^2}{2\,[(6.75\,\text{in})\,(63.1\,\text{in}) - (0.75\,\text{in})\,(32.36\,\text{in}) + (0.75\,\text{in})\,(2\,\text{in})]}$$

$$c = c_2 = 32.36\,\text{in}$$

$$I_{x\ reduced} = \frac{b\,(d-c)^3}{3} + \frac{bc^3}{3} - \frac{b'\,(c-a)^3}{3}$$

$$I_{x\ reduced} = \frac{(6.75\,\text{in})\,(63.1\,\text{in} - 32.4\,\text{in})^3}{3} + \frac{(6.75\,\text{in})\,(32.4\,\text{in})^3}{3} - \frac{(0.75\,\text{in})\,(32.4\,\text{in} - 2\,\text{in})^3}{3}$$

$$I_{x\ reduced} = 134{,}600\,\text{in}^4$$

$$S_{x\ reduced} = \frac{I_{x\ reduced}}{c} = \frac{134{,}600\,\text{in}^4}{32.4\,\text{in}} = 4154\,\text{in}^3$$

Unmodified section modulus at mid-span:

$$S_x = \frac{bd^2}{6} = \frac{6.75\,\text{in}\,(63.1\ \text{in})^2}{6} = 4479\,\text{in}^3$$

Revised bending stress at mid-span:

$$f_b = (1655\,\text{psi})\left(\frac{4479\,\text{in}^3}{4154\,\text{in}^3}\right)$$

$$f_b = 1784\,\text{psi} \le\ F_b' = 1964\,\text{psi} \qquad \therefore\ \text{OK}$$

Revised bending stress at tangent point:
Similarly, at the tangent point, the modified section modulus becomes 2926 in^3, which is 93% of 3143 in^3 (unmodified section modulus). Since the ratio of the applied stress to allowable stress at the tangent point for the unmodified section was found to be 0.78, the section is still adequate, considering section loss due to reinforcement ($0.78/0.93 = 0.84 \le 1.0$).

Self-weight check:
Finally, the assumed self-weight should be checked. A prismatic beam with a section of 6.75 in. × 63.1 in. would have a self-weight of:

$$\omega_{s.w.} = \left(\frac{6.75\,\text{in}}{12\,\text{in/ft}}\right)\left(\frac{63.1\,\text{in}}{12\,\text{in/ft}}\right)(33\ \text{pcf}) = 97\,\text{plf}$$

Since most of the beam will actually have significantly less section, the use of 100 plf for self-weight should be suitable, indeed conservative.

Answer: The 24F-1.8E Douglas fir pitched and tapered curved beam is satisfactory as follows (Figure 8.3-5):

End depth: $d_e = 26.0\,\text{in}$

Mid-span depth: $d_c = 63.1\,\text{in}$

Top slope: $\phi_T = 14.0^\circ$ (3:12)

Bottom slope: $\phi_B = 9.0^\circ$

TABLE 8.3-3 Example Spreadsheet for PTC Beam Design

INPUT PARAMETERS

Reference Stresses			Adjustment Factors		Geometric Parameters			Loads		
F_{bx} =	2400	psi	C_D	1.15	ℓ =	48	ft	ω_L =	500	plf
F_{vx} =	216	psi	C_M (f_b)	1	b =	6.75	in	ω_D =	180	plf
$F_{cperp\ x}$ =	740	psi	C_M (f_v)	1	ϕ_T =	18.4	degrees	ω_{SW} =	100	plf
E_x =	1.80	10^6 psi	C_M ($f_{c\ perp}$)	1	ϕ_B =	11.4	degrees	ω_{Total} =	780	plf
F_{rt} =	72	psi	C_M (E)	1	d_e =	17	in			
Reinforcement			C_t (f_b, f_v, $f_{c\ perp}$)	1	R =	24	ft			
$D_{nominal}$ =	1	in	C_t (E)	1	t =	1.375	in			
D_{root} =	0.78	in	C_V exponent	0.05						
$f_{allowable}$ =	20,000	psi								
W =	622	lb/in								

PTC BEAM ANALYSIS

Beam Geometry			Shear Stress			Radial Tension Stress			Midspan Bending Stress		
h_a	112.8048486	in	V =	18720	lb	$M_{midspan}$ =	2,695,680	in-lb	K_θ =	1.898	
h_s	52.27	in	f_v =	245	psi	K_r =	0.0959		f_b =	1241	psi
d_c	60.53	in	F'_{vx} =	248	psi	C_r =	1.000		C_c =	0.954	
ℓ_c	113.85	in	f_v/F'_{vx} =	0.985		f_{rt} =	63	psi	C_V =	0.873	
ℓ_t	231.07	in				F'_{rt} =	83	psi	F'_{bx} =	2299	psi
R_M	318.27	in				$F'_{vx}/3$ =	83	psi	f_b/F'_{bx} =	0.540	
d_{equiv}	40.51	in				f_{rt}/F'_{rt} =	0.758				
						$3f_{rt}/F'_v$ =	0.758				

(continues)

TABLE 8.3-3 (*Continued*)

Bending Stresses in Legs

Section	x (in)	d'_x (in)	M (in-lb)	S (in^3)	f_b (psi)	C_V	C_I	C_c	F'_{bx} (psi)	f_b/F'_{bx}
A	57.77	23.49	972970	621	1568	0.915	0.591	1.000	1631	0.961
B	115.54	30.72	1729021	1062	1628	0.903	0.591	1.000	1631	0.998
C	173.31	37.96	2268152	1621	1399	0.893	0.591	1.000	1631	0.858
TP	231.07	45.20	2590364	2298	1127	0.886	0.591	0.954	1557	0.724

Vertical Deflection

E'_x = 1.80E+06 psi
$\Delta_{c,LL}$ = 0.89 in
$\Delta_{c,DL}$ = 0.50 in

Horizontal Deflection

h = 74.04 in
$\Delta_{H,LL}$ = 0.23 in
$\Delta_{H,DL}$ = 0.13 in

REINFORCEMENT DESIGN

Radial Reinforcement

s_{max} = 22.57 in
$s_{recommended}$ = 22 in
$n_{required}$ = 6

Revised Bending Stress at Midspan

c_{trial} = 30.2651 in
c_1 = 31.3154 in
c_2 = 31.3168 in
c_3 = 31.3168 in
c = 31.3168 in
I = 116802 in^4
S = 3729.68 in^3
f_b = 1372 psi
F'_{bx} = 2299 psi
f_b/F'_{bx} = 0.597

Revised Bending Stress at Tangents

c_{trial} = 22.5977 in
c_1 = 23.3434 in
c_2 = 23.3444 in
c_3 = 23.3444 in
c = 23.3444 in
I = 48857 in^4
S = 2093 in^3
f_b = 1238 psi
F'_{bx} = 1557 psi
f_b/F'_{bx} = 0.795

Distance from end to tangent point: $l_t = 308.4\,\text{in}$

Inside (soffit) radius: $R = 27.5\,\text{ft}$

Reinforcement: (12) $\frac{3}{4}$-in. lag screws @ 10 in. o.c. (6 each side) symmetric about mid-span.

Discussion: A complete design would consider all other relevant load conditions, including unbalanced snow loads and wind loads. This example showed hand calculations. An example of a simple spreadsheet with the same calculations is shown in Table 8.3-3.

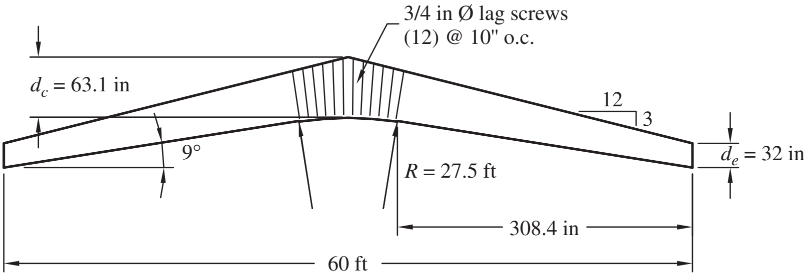


Figure 8.3-5 Douglas fir PTC beam—Example 8.3-2.

8.4 PITCHED AND TAPERED CURVED BEAMS WITH MECHANICALLY ATTACHED HAUNCH

Pitched and tapered curved beams may be designed with a mechanically attached haunch. The mechanically attached haunch is assumed to not contribute to beam stiffness, and consequently does not resist bending stresses. Such beams are less efficient than beams with integral haunches, but may be easier to manufacture and transport.

The haunch may be of nonstructural glued laminated timber attached mechanically, or may be framed. The design of such beams is generally accomplished by trial and error to produce a member that is aesthetically pleasing and structurally adequate.

The curved section of such beams is designed and constructed so that it has a constant depth, often with a large enough radius to eliminate the need for radial reinforcement. The depth through the curved section should be adjusted to ensure that it is equal to an integer multiple of the chosen lamination thickness. The straight segments may be prismatic or tapered to accommodate the geometric and structural requirements. Because the haunch is not integral with the beam, the factors K_θ, K_{rs} and C_{rs} are not applied to the centerline stresses.

EXAMPLE 8.4-1 CURVED BEAM WITH MECHANICALLY ATTACHED HAUNCH

Given: Pitched and tapered curved beams with roof slope of 3:12 will be spaced at 16 ft and will span 60 ft. The roof snow load is 25 psf and the roof dead load, not including the weight of the beams, is 15 psf. The deflection under total load must be limited to L/180.

Lamination thickness, $t = 1.5\,\text{in}$

Assume beam width of $b = 8.75\,\text{in}$

Wanted: Using a lamination thickness of 1.5 in and a beam width of 8.75 in., design a pitched and tapered curved beam with a mechanically attached haunch using design values of 24F-1.8E DF from AITC 117 [5] or *NDS*® [2]. Geometry should be such that no radial reinforcement is required.

Approach: It will be assumed that in the straight tapered section, the roof will be attached to top of the beam to provide lateral support. It will also be assumed that the haunch will *not* provide lateral support for the curved section. As such, for the tapered section, C_V, and C_I will be calculated, and the smaller value used. For the curved portion, C_c and the lesser of C_V and C_L will be applied. At the tangent point, C_c and the smaller of C_V, and C_I will be applied. The minimum end depth will be determined based on shear stress, and the minimum depth through the curved portion will be determined based on flexure.

Solution:

Design values:

$$F'_{bx} = F_b C_D C_M C_t \left(C_L \text{ or } C_V\right) C_I C_c$$
$$F'_{bx} = (2400\,\text{psi})\,(1.15)\,(1.0)\,(1.0) \left(C_L \text{ or } C_V\right) C_I C_c$$
$$F'_{bx} = (2760\,\text{psi}) \left(C_L \text{ or } C_V\right) C_I C_c$$

$$F'_{vx} = F_v C_D C_M C_t C_{vr}$$
$$F'_{vx} = (265\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\,(0.72)$$
$$F'_{vx} = 219\,\text{psi}$$

$$E'_x = E_x C_M C_t$$
$$E'_x = 1.8\left(10^6\,\text{psi}\right)(1.0)\,(1.0) = 1{,}800{,}000\,\text{psi}$$
$$E'_x = 1.8\left(10^6\,\text{psi}\right)$$

$$F'_{rt} = F_{rt} C_D C_M C_t$$
$$F'_{rt} = (15\,\text{psi})\,(1.15)\,(1.0)\,(1.0)$$

$$F'_{rt} = 17.2\,\text{psi}$$

$$F'_{c\perp} = F_{c\perp} C_M C_t$$
$$F'_{c\perp} = (650\,\text{psi})\,(1.0)\,(1.0)$$
$$F'_{c\perp} = 650\,\text{psi}$$

Total uniform load (self-weight estimated as 80 plf):

$$\omega = \omega_D + \omega_S + \omega_{sw}$$
$$\omega = (15\,\text{psf} + 25\,\text{psf})\,(16\,\text{ft}) + 80\,\text{plf} = 720\,\text{plf}$$

Shear load:

$$V = \frac{\omega L}{2} = \frac{(720\,\text{plf})\,(60\,\text{ft})}{2} = 21{,}600\,\text{lb}$$

Minimum end depth:

$$d_e \geq \frac{3V}{2bF'_v} = \frac{3\,(21{,}600\,\text{lb})}{2\,(8.75\,\text{in})\,(219\,\text{psi})} = 16.9\,\text{in} \qquad \therefore \text{ Use } d_e = 18\text{ in}$$

Apex height (Equation 8.3.4.1-2):

$$h_a = \frac{\ell}{2}\tan\phi_T + d_e = \frac{720\,\text{in}}{2}\left(\frac{3}{12}\right) + 18.0\,\text{in} = 108\,\text{in}$$

Bending moment at mid-span due to applied loads:

$$M = \frac{\omega L^2}{8} = \frac{(720\,\text{lb/ft})\,(60\,\text{ft})^2}{8} = 324{,}000\,\text{ft-lb} = 3{,}888{,}000\,\text{in-lb}$$

Minimum depth at mid-span (estimating (C_V or C_L) ≈ 0.8 and C_c ≈ 1.0):

$$d_c \approx \sqrt{\frac{6M}{bF_b C_D C_M C_t C_c \left(C_V \text{ or } C_L\right)}}$$

$$d_c \approx \sqrt{\frac{6\,(3{,}888{,}000\,\text{in-lb})}{(8.75\,\text{in})\,(2400\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\,(1.0)\,(0.8)}}$$

$$d_c \approx 34.8\,\text{in}$$

A radius of 100 ft and a soffit slope angle of 8.4° will result in a depth of 36 in. (24 laminations) between tangent points as shown with the following calculations.

Soffit height at mid-span (Equation 8.3.4.1-3):

$$h_s = \frac{\ell}{2} \tan \phi_B - R \left(\sec \phi_B - 1\right)$$
$$= \frac{720 \text{ in}}{2} \tan 8.4^\circ - 1200 \text{ in} \left(\sec 8.4^\circ - 1\right) = 40.1 \text{in}$$

Length of the curved segment (Equation 8.3.4.1-5):

$$\ell_c = 2R \sin \phi_B = 2\,(1200 \text{ in}) \sin \left(8.4^\circ\right) = 350.6 \text{ in}$$

Length of each tapered end, ℓ_t (Equation 8.3.4.1-6):

$$\ell_t = \frac{\ell - \ell_c}{2} = \frac{720 \text{ in} - 350.6 \text{ in}}{2} = 184.7 \text{ in}$$

Depth of beam between the tangent points (Equation 8.3.4.1-8):

$$d_t' = \left[d_e + \ell_t \left(\tan \phi_T - \tan \phi_B\right)\right] \left[\cos \phi_B - \sin \phi_B \tan \left(\phi_T - \phi_B\right)\right]$$
$$d_t' = \left[18.0 \text{ in} + 184.7 \text{ in} \left(\tan 14.04^\circ - \tan 8.4^\circ\right)\right]$$
$$\times \left[\cos 8.4^\circ - \sin 8.4^\circ \tan \left(5.64^\circ\right)\right]$$
$$d_t' = 36.0 \text{ in} = d_c$$

Radius of curved portion of beam at mid-depth and mid-span (Equation 8.3.4.1-7):

$$R_m = R + \frac{d_c}{2} = 1200 \text{ in} + \frac{36.0 \text{ in}}{2} = 1218 \text{ in}$$

Beam depth at $\ell_A = \ell_t/4$, $\ell_B = \ell_t/2$, and $\ell_C = 3\ell_t/4$ from the support (Equation 8.3.4.1-8):

$$d_A' = \left[d_e + \ell_A \left(\tan \phi_T - \tan \phi_B\right)\right] \left[\cos \phi_B - \sin \phi_B \tan \left(\phi_T - \phi_B\right)\right]$$
$$d_A' = \left[18 \text{ in} + 46.2 \text{ in} \left(\tan 14.0^\circ - \tan 8.4^\circ\right)\right] \left[\cos 8.4^\circ - \sin 8.4^\circ \tan \left(5.64^\circ\right)\right]$$
$$d_A' = 22.2 \text{ in}$$
$$d_B' = \left[d_e + \ell_B \left(\tan \phi_T - \tan \phi_B\right)\right] \left[\cos \phi_B - \sin \phi_B \tan \left(\phi_T - \phi_B\right)\right]$$
$$d_B' = \left[18 \text{ in} + 92.4 \text{ in} \left(\tan 14.0^\circ - \tan 8.4^\circ\right)\right] \left[\cos 8.4^\circ - \sin 8.4^\circ \tan \left(5.64^\circ\right)\right]$$
$$d_B' = 26.8 \text{ in}$$
$$d_C' = \left[d_e + \ell_A \left(\tan \phi_T - \tan \phi_B\right)\right] \left[\cos \phi_B - \sin \phi_B \tan \left(\phi_T - \phi_B\right)\right]$$
$$d_C' = \left[18 \text{ in} + 138.5 \text{ in} \left(\tan 14.0^\circ - \tan 8.4^\circ\right)\right] \left[\cos 8.4^\circ - \sin 8.4^\circ \tan \left(5.64^\circ\right)\right]$$
$$d_C' = 31.4 \text{ in}$$

Radial tension stress at mid-span (Equation 8.2.1-1):

$$M = \frac{\omega \ell^2}{8} = \frac{60\,\text{lb/in}\,(720\,\text{in})^2}{8} = 3{,}888{,}000\,\text{lb-in}$$

$$f_{rt} = \frac{3M}{2bdR_m} = \frac{3\,(3{,}888{,}000\,\text{lb-in})}{2\,(8.75\,\text{in})\,(36.0\,\text{in})\,(1218\,\text{in})} = 15.2\,\text{psi}$$

$$f_{rt} = 15.2\,\text{psi} \le F'_{rt} = 15\,\text{psi}\;(1.15) = 17.2\,\text{psi} \qquad \therefore\ \text{OK}$$

Volume factor and curvature factor:

$$C_V = \left(\frac{5.125\,\text{in}}{8.75\,\text{in}}\right)^{\frac{1}{10}} \left(\frac{12\,\text{in}}{36.0\,\text{in}}\right)^{\frac{1}{10}} \left(\frac{21\,\text{ft}}{60\,\text{ft}}\right)^{\frac{1}{10}} = 0.76$$

$$C_c = 1 - 2000\left(\frac{t}{R}\right)^2 = 1 - 2000\left(\frac{1.5\,\text{in}}{1200\,\text{in}}\right)^2 = 0.997$$

Beam stability factor (equivalent moment method):

$$C_b = 1.0$$

$$\eta = \frac{1.3kd}{\ell_u} = \frac{1.3\,(1.72)\,36.0\,\text{in}}{351\,\text{in}} = 0.229$$

$$C_e = \sqrt{\eta + 1} - \eta = \sqrt{0.229 + 1} - 0.229 = 0.88$$

$$R_B = \sqrt{\frac{1.84 l_u d}{C_b C_e b^2}} = \sqrt{\frac{1.84\,(351\,\text{in})\,(36.0\,\text{in})}{(1.0)\,(0.88)\,(8.75\,\text{in})^2}} = 18.6$$

$$F_{bE} = \frac{1.2E'_{\min}}{R_B^2} = \frac{1.2\left(0.95\left(10^6\,\text{psi}\right)\right)}{(18.6)^2} = 3295\,\text{psi}$$

$$F_b^* = F_b C_D C_C = 2400\,\text{psi}\,(1.15)\,(0.997) = 2752\,\text{psi}$$

$$C_L = \frac{1 + (F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1 + (F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{(F_{bE}/F_b^*)}{0.95}}$$

$$C_L = \frac{1 + \left(\frac{3295\,\text{psi}}{2752\,\text{psi}}\right)}{1.9} - \sqrt{\left[\frac{1 + \left(\frac{3295\,\text{psi}}{2752\,\text{psi}}\right)}{1.9}\right]^2 - \frac{\left(\frac{3295\,\text{psi}}{2752\,\text{psi}}\right)}{0.95}}$$

$$C_L = 0.88$$

Allowable bending stress (curved segment):

$$F'_{bx} = F_{bx}\left(C_D\right)\left(C_L \text{ or } C_V\right)\left(C_c\right) = F_b\left(C_D\right)\left(C_V\right)\left(C_c\right)$$
$$F'_{bx} = 2400\,\text{psi}\,(1.15)\,(0.76)\,(0.997) = 2090\,\text{psi}$$

Bending stress at mid-span:

$$f_b = \frac{M}{S} = \frac{6M}{bd_c^2}$$
$$f_b = \frac{6\,(3{,}888{,}000\,\text{lb-in})}{(8.75\ \text{in})\,(36.0\,\text{in})^2}$$
$$f_b = 2060\,\text{psi} \le F'_{bx} = 2090\ \text{psi} \qquad \therefore\ \text{OK}$$

Bending stresses in straight tapered segments:
The bending stress is also evaluated at four sections along the straight tapered segments of the beam in Table 8.4-1 using Equations 8.3.4.4-2 and 8.3.4.4-3.

TABLE 8.4-1 Bending Stresses in Straight Tapered Segments

Section	x in.	d'_x in.	M lb-in	S in^3	f_b psi	C_V	C_I	C_c	F'_{bx} psi	f_b/F'_{bx}
A	46.2	22.2	933,000	716	1304	0.803	0.625	1.0	1726	0.755
B	92.4	26.8	1,739,000	1045	1664	0.788	0.625	1.0	1726	0.964
C	138.5	31.4	2,416,000	1436	1683	0.775	0.625	1.0	1726	0.975
TP	184.7	36.0	2,966,000	1888	1571	0.765	0.625	0.997	1720	0.913

Vertical deflection:
Estimating that the deflection will be 10% higher than for a beam of constant depth, d_c:

$$\Delta \approx \frac{(1.1)\,5\omega_D \ell^4}{384E'_x I} = \frac{(1.1)\,5\omega_D \ell^4}{32E'_x bd_c^3}$$
$$\Delta_D \approx \frac{(1.1)\,5\omega_D \ell^4}{32E' bd_c^3} = \frac{(1.1)\,5\,(26.7\,\text{lb/in})\,(720\,\text{in})^4}{32\left(1.8\left(10^6\right)\ \text{psi}\right)(8.75\,\text{in})\,(36.0\,\text{in})^3} = 1.7\,\text{in}$$
$$\Delta_L \approx \frac{(1.1)\,5\omega_L \ell^4}{32E' bd_c^3} = \frac{(1.1)\,5\,(33.3\,\text{lb/in})\,(720\,\text{in})^4}{32\left(1.8\left(10^6\right)\ \text{psi}\right)(8.75\,\text{in})\,(36.0\,\text{in})^3} = 2.1\,\text{in}$$
$$\Delta_{D+L} = \Delta_D + \Delta_L = 1.7\,\text{in} + 2.1\,\text{in} = 3.8\,\text{in}$$
$$\Delta_{D+L} = 3.8\,\text{in} \le \frac{\ell}{180} = \frac{720\,\text{in}}{180} = 4.0\,\text{in} \qquad \therefore\ \text{OK}$$

Horizontal displacement (Equation 8.3.4.6-1):

$$h \approx h_s + \frac{d_c}{2} - \frac{d_e}{2} = 40.1\,\text{in} + \frac{36.0\,\text{in}}{2} - \frac{18\,\text{in}}{2} = 49.1\,\text{in}$$

$$\Delta_H = \frac{2h\Delta_c}{\ell} = \frac{2\,(49.1\,\text{in})\,(3.8\,\text{in})}{720\,\text{in}} = 0.52\,\text{in}$$

__Answer:__ The pitched and tapered curved beam with detached haunch is shown in Figure 8.4-1. The important geometric features follow:

Width: $b = 8.75\,\text{in}$

End depth: $d_e = 18.0\,\text{in}$

Apex height: $h_a = 108.0\,\text{in}$

Mid-span depth: $d_c = 36.0\,\text{in}$(depth of curved section)

Soffit height: $h_s = 40.1\,\text{in}$

Soffit radius: $R = 1200\,\text{in} = 100\,\text{ft}$

Length of tapered segment: $\ell_t = 185\,\text{in}$

Top slope: $\phi_T = 14.0^\circ(\frac{3}{12}$ pitch)

Bottom (soffit) slope: $\phi_B = 8.4^\circ$

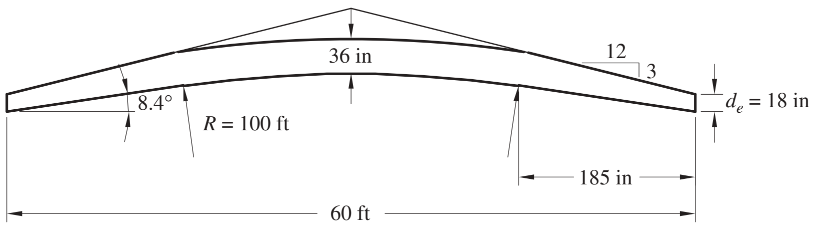


Figure 8.4-1 PTC beam with detached haunch—Example 8.4-1.

__Discussion:__ The beam design assumes that the tapered section will be manufactured into the beam such that the design values for 24F-1.8E in AITC 117 [5] are applicable. Connections at the supports must be designed to accommodate $\frac{1}{2}$ in. of displacement at each support or 1 in. at one support.

Once an acceptable geometry is determined, its aesthetic suitability must be approved by the owner and the constructability of the beam must be verified by the manufacturer. This example used hand calculations. An example of a simple spreadsheet with the same calculations is shown in Table 8.4-2.

TABLE 8.4-2 Example Spreadsheet for Design of PTC Beam with Detached Haunch

INPUT PARAMETERS

Reference Stresses			Adjustment Factors		Geometric Parameters			Loads		
F_{bx} =	2400	psi	C_D	1.15	ℓ =	60	ft	ω_L =	400	plf
F_{vx} =	190	psi	C_M (f_b)	1	b =	8.75	in	ω_D =	240	plf
$F_{cperp\ x}$ =	650	psi	C_M (f_v)	1	ϕ_T =	14.04	degrees	ω_{SW} =	80	plf
E_x =	1.80	10^6 psi	C_M ($f_{c\ perp}$)	1	ϕ_B =	8.4	degrees	ω_{Total} =	720	plf
F_{rt} =	15	psi	C_M (E)	1	d_e =	18	in			
Reinforcement			C_t (f_b,f_v, $f_{c\ perp}$)	1	R =	100	ft			
$D_{nominal}$ =	0	in	C_t (E)	1	t =	1.5	in			
D_{root} =	0	in	C_V exponent	0.1						
$f_{allowable}$ =	0	psi	C_b	1						
W =	0	lb/in								

PTC BEAM ANALYSIS

Beam Geometry			Shear Stress			Radial Tension Stress			Midspan Bending Stress		
h_a =	108.03	in	V =	21600	lb	$M_{midspan}$ =	3,888,000	in-lb	f_b =	2059	psi
h_s =	40.15	in	f_v =	206	psi	f_{rt} =	15	psi	C_c =	0.997	
$d_{TP} = d_c$ =	35.99	in	F'_{vx} =	219	psi	F'_{rt} =	17	psi	C_V =	0.765	
h_{roof} =	76.13	in	f_v/F'_{vx} =	0.941		$F'_{vx}/3$ =	73	psi	C_e =	0.796	
ℓ_c =	350.60	in				f_{rt}/F'_{rt} =	0.882		R_B =	19.5	
ℓ_t =	184.70	in				$3f_{rt}/F'_v$ =	0.209		F_{bE} =	2996	psi
R_M =	1217.99	in							F_b^* =	2751	psi
									C_L =	0.849	
									F'_{bx} =	2104	psi
									f_b/F'_{bx} =	0.979	

Bending Stresses in Legs

Section	x (in)	d'_x (in)	M (in-lb)	S (in^3)	f_b (psi)	C_V	C_I	C_c	F'_{bx} (psi)	f_b/F'_{bx}
A	46.18	22.16	933418	716	1304	0.803	0.625	1.000	1726	0.755
B	92.35	26.77	1738907	1045	1664	0.788	0.625	1.000	1726	0.964
C	138.53	31.38	2416468	1436	1683	0.775	0.625	1.000	1726	0.975
TP	184.70	35.99	2966101	1888	1571	0.765	0.625	0.997	1720	0.913

Vertical Deflection

$E'_x = 1.80E+06$ psi
$\Delta_{c,LL} = 2.10$ in
$\Delta_{c,DL} = 1.68$ in

Horizontal Deflection

$h = 49.14$ in
$\Delta_{H,LL} = 0.29$ in
$\Delta_{H,DL} = 0.23$ in

REINFORCEMENT DESIGN

Radial Reinforcement

$s_{max} = 0.00$ in
$s_{recommended} = 0$ in
$n_{required} =$ #DIV/0!

Revised Bending Stress at Midspan

$c_{trial} = 17.9926$ in
$c_1 = 17.9926$ in
$c_2 = 17.9926$ in
$c_3 = 17.9926$ in
$c = 17.9926$ in
$I = 33978.2$ in^4
$S = 1888.45$ in^3
$f_b = 2059$ psi
$F'_{bx} = 2104$ psi
$f_b/F'_{bx} = 0.979$

Revised Bending Stress at Tangents

$c_{trial} = 17.9926$ in
$c_1 = 17.9926$ in
$c_2 = 17.9926$ in
$c_3 = 17.9926$ in
$c = 17.9926$ in
$I = 33978$ in^4
$S = 1888$ in^3
$f_b = 1571$ psi
$F'_{bx} = 1720$ psi
$f_b/F'_{bx} = 0.913$

8.5 CONCLUSION

Structural glued laminated timber can be manufactured in curved shapes, expanding their architectural versatility. Similar to prismatic beams, curved beams must satisfy shear, flexure, and deflection criteria. In addition, curved beams experience radial stresses, and vertical deflections cause the ends of curved beams to move horizontally relative to their supports. These effects must be considered in design.

For softwood species other than southern pine, the reference radial tension design value is prescriptively limited to 15 psi. To increase the radial tension capacity of these species, radial reinforcement is commonly used. The reinforcement typically consists of long lag screws or epoxy-embedded steel bars placed perpendicular to the bond lines in the curved portions of the beam.

A common form of curved beam is the pitched and tapered curved (PTC) beam. This shape provides a sloped roof line with a sloping and curved bottom face. The discontinuity at the apex of PTC beams results in modified flexure and radial stress distributions that are accounted for in design through empirical factors.

This chapter presented methods and examples for the design of both constant-depth curved beams and PTC beams. The design of radial reinforcement was also covered.

CHAPTER 9

GLULAM ARCHES

9.1 INTRODUCTION

Glulam arches have been successfully used in the United States since the 1930s and continue to be popular for use in large open structures such as churches and gymnasiums, because of their excellent structural performance, inherent fire resistance, and aesthetic appeal. Glulam arches have also been successfully used in vehicle and pedestrian bridges. Common arch configurations are illustrated in Chapter 1. The most common arch configuration in use today is the three-hinged Tudor arch. It provides a vertical wall frame and sloping roof that are commonly used in modern structures. Its appearance is also pleasing to most people. Because of its popularity, the three-hinged Tudor arch is the focus of this chapter; however, similar principles can be used to design other types of glulam arches.

The design of arches is an iterative process. Experience will lead to less design iterations. Design checks should be made at several sections along the arch. Because the number of calculations required is large, the use of a spreadsheet or structural analysis software is recommended for arch design and analysis. Detailed procedures for Tudor arch design are available in the AITC publication *Design of Tudor Arches with Structural Glued Laminated Timber* [1]. The following preliminary design procedure was excerpted from that publication.

9.2 PRELIMINARY DESIGN PROCEDURE

For preliminary design, the arch geometry is established based on the $D + S$ loading. (For areas without snow load, roof live load can be substituted). The required depths in the arch leg are determined based on the balanced snow loading or the unbalanced loading, whichever results in larger base reaction forces. The depths in the arch arm are controlled by the unbalanced load case. The following steps are used to establish the geometry.

9.2.1 Choose Angles of Taper, Radius of Curvature, Wall Height, Peak Height, Span, and Arch Width

The outside arch geometry (wall height, peak height, and span) will generally be dictated by the building dimensions (Figure 9.2.1-1). The angles of taper should generally be less than 3° for the arch arm and less than 5° for the arch leg (steeper tapers can be used if they are properly accounted for in design). The radius of curvature of the inside face is commonly chosen as 7 ft 0 in. for southern pine and 9 ft 4 in. for other softwood species. The arch width is often chosen so that it exceeds the minimum requirements for *heavy timber construction* in addition to structural requirements.

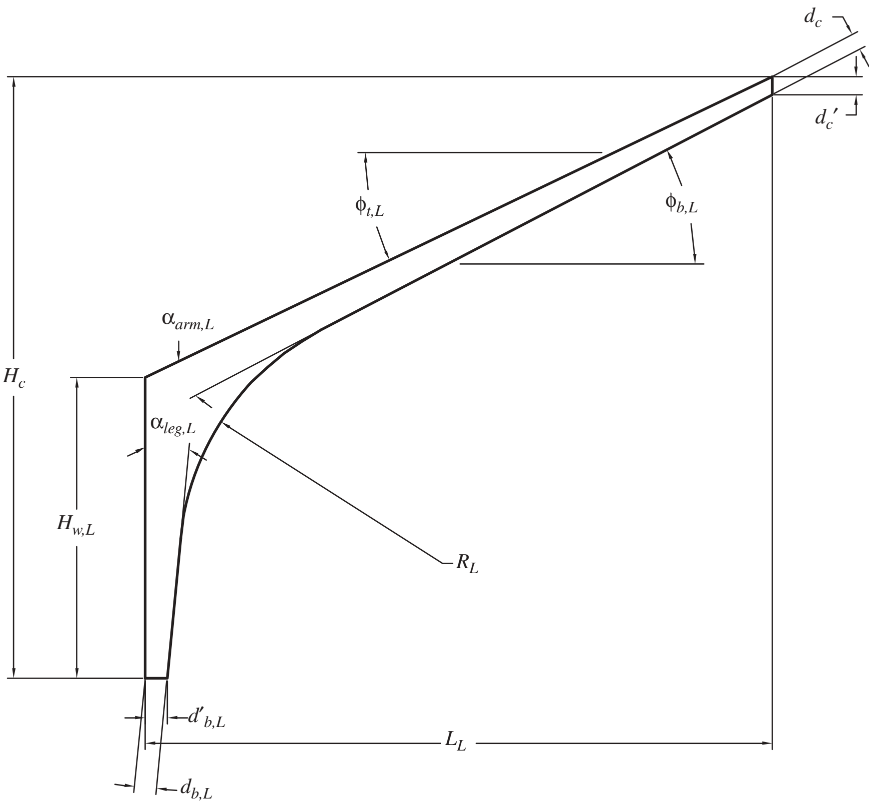


Figure 9.2.1-1 Arch geometry (only left half shown for clarity).

9.2.2 Choose Material Properties (Species, Design Stresses: F_{bx}, F_{vx})

For Douglas fir arches, common design values are: $F_{bx} = 2400$ psi and $F_{vx} C_{vr} = 190$ psi. For southern pine arches, common properties are: $F_{bx} = 2400$ psi and $F_{vx} C_{vr} = 215$ psi.

9.2.3 Locate Approximate Tangent Points

Figure 9.2.3-1 illustrates the approximate tangent point locations.

$$A = R \tan\left(\frac{90 - \phi}{2}\right) \tag{9.2.3-1}$$

$$x_2 = A \cos(\phi) \tag{9.2.3-2}$$

$$y_2 = H_w + A \sin(\phi) \tag{9.2.3-3}$$

$$y_1 = H_w - A \tag{9.2.3-4}$$

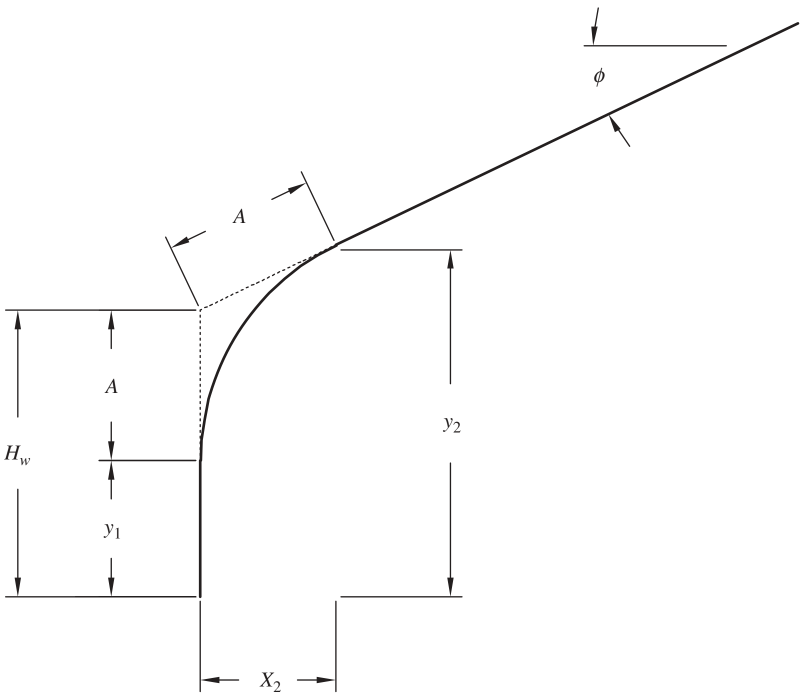


Figure 9.2.3-1 Location of approximate tangent points.

9.2.4 Calculate the Reactions and Pin Forces for Balanced and Unbalanced Snow Loads

For preliminary design, the reaction and pin forces are calculated based on the outside geometry of the arch, because the actual centerline of the members is not known. This produces a reasonably accurate estimate of the forces.

9.2.4.1 Balanced Snow Load Base reactions for the balanced snow load case (Figure 9.2.4.1-1) are calculated with Equations 9.2.4.1-1 and 9.2.4.1-2

$$R_{y,R} = R_{y,L} = (\omega_D + \omega_S)L_L \tag{9.2.4.1-1}$$

$$R_{x,R} = R_{x,L} = \frac{1}{H_c}\left[R_{y,L}L_L - \frac{(\omega_D + \omega_S)L_L^2}{2}\right] \tag{9.2.4.1-2}$$

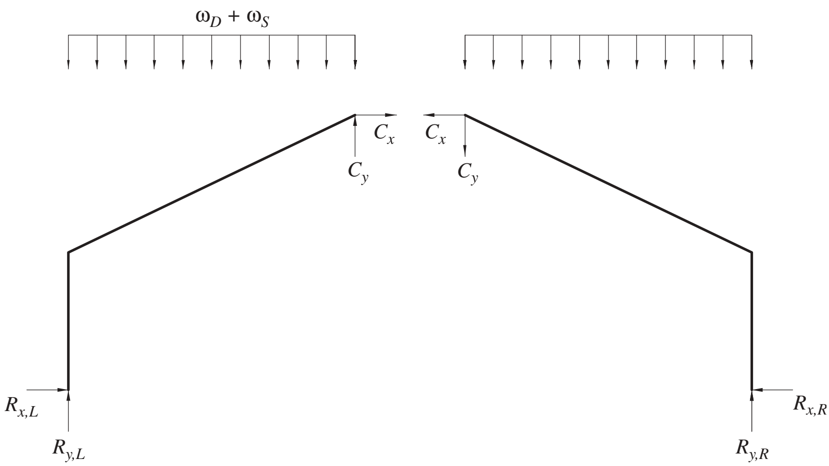


Figure 9.2.4.1-1 Dead load plus balanced snow load.

9.2.4.2 Unbalanced Snow Load Base reactions and the shear force at the peak are calculated using Equations 9.2.4.2-1 and 9.2.4.2-2 for the unbalanced load case.

$$R_{y,L} = \frac{(k_1\omega_S + \omega_D)(3L_L) + (k_2\omega_S + \omega_D)(L_L)}{4} \tag{9.2.4.2-1}$$

$$C_y = (k_1\omega_S + \omega_D)L_L - R_{y,L} \tag{9.2.4.2-2}$$

$$R_{x,L} = \frac{R_{y,L}L_L + (k_1\omega_S + \omega_D)\frac{L_L^2}{2}}{H_c} \tag{9.2.4.2-3}$$

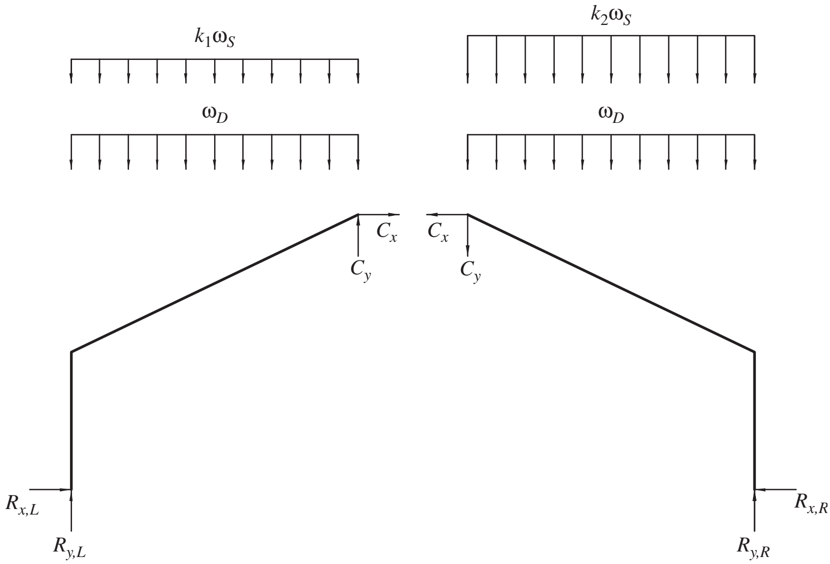


Figure 9.2.4.2-1 Dead load plus unbalanced snow load (simplified).

9.2.5 Estimate the Required Depth at the Lower Tangent Point, d_{LT}

The minimum depth at the lower tangent point is estimated based on the bending moment created by the horizontal reaction (from the balanced snow load or unbalanced snow load, whichever gives larger reaction) applied at a distance, y_1, from the approximate tangent point. The depth should be increased approximately 5% over that calculated based on moment alone to account for compression-flexure interaction (Equation 9.2.5-1).

$$d_{LT} > 1.05\sqrt{\frac{6y_1 R_{x,L}}{bF_{bx} C_D C_I C_V C_M C_t}} \tag{9.2.5-1}$$

C_D is 1.15 for snow loading (1.25 for roof live load). C_I is calculated based on the chosen material properties and leg taper angle. A rough estimate of the volume factor, C_V, at the tangent point can be obtained from one of the following expressions: $C_V \approx 0.995 - 0.0015L_L$ for southern pine or $C_V \approx 0.90 - 0.0025L_L$ for other species, where L_L is expressed in units of feet.

The depth at the lower tangent point should not be greater than six times the width for arches braced against lateral buckling by decking, sheathing, or closely spaced girts or purlins and should not be greater than five times the width for arches without such lateral bracing. If the required depth calculated by Equation 9.2.5-1 is not within these limitations, the width must be increased.

9.2.6 Estimate the Required Depth at the Base

The minimum depth at the base is estimated based on the horizontal reaction force and is also determined based on the estimated tangent point depth and the amount of taper chosen for the leg. The greater depth from these two calculations should be chosen. Generally, the base depth is also chosen to be a minimum of 1.5 times the arch width. The base depth may also be chosen based on the minimum size required for heavy timber construction.

9.2.6.1 Required Base Depth for Shear

$$d_b' > \frac{3R_x}{2bF_{vx}C_{vr}C_D C_M C_t} \tag{9.2.6.1-1}$$

9.2.6.2 Required Base Depth to Get the Required Lower Tangent Depth with Chosen Leg Taper Angle

$$d_b' > d_{LT} - y_1 \tan\alpha_{leg} \tag{9.2.6.2-1}$$

Occasionally, the required depth at the base for shear may be greater than the depth required at the tangent point. In such cases, the base depth required for shear should be chosen and the leg taper angle should be set to zero to avoid excessive depth at the tangent point.

The base depth should generally not be chosen as less than 1.5 times the arch width. If this criterion governs the selection of the depth, the leg taper angle can be reduced to avoid excessive depth at the lower tangent point.

9.2.7 Estimate the Required Depth at the Crown

The minimum depth at the crown is estimated based on the shear plate connector requirements to resist the vertical shear force at the crown due to *unbalanced* snow loading and based on flexure in the loaded arm due to unbalanced snow load.

9.2.7.1 Required Crown Depth Based on Shear Plate Capacity

- Calculate reference capacity for a single shear plate in sloped-end grain using Hankinson's formula for the chosen roof slope and arm taper:

$$N = \frac{PQ_{90}}{P\sin^2(90^\circ - \phi - \alpha_{arm}) + Q_{90}\cos^2(90^\circ - \phi - \alpha_{arm})} \tag{9.2.7.1-1}$$

- *Adjust the capacity for load duration and apply the geometry factor for reduced unloaded edge distance*. Shear plate design values are tabulated for unloaded edge distances of 1.75 in. and 2.75 in. for $2\frac{5}{8}$ in. and 4 in. shear plates, respectively. For narrower edge distances typical of standard glulam arches, a reduction in capacity is required. Geometry factors for reduced unloaded edge distance are shown in Table 9.2.7.1-1 for standard arch widths. The effect of group action will generally be small and can be ignored at this point.

$$N^* = NC_D C_M C_t C_{\Delta,u} \tag{9.2.7.1-2}$$

TABLE 9.2.7.1-1 Geometry Factors, $C_{\Delta,u}$, for Shear Plate Capacity Based on Arch Width

	Geometry Factors, $C_{\Delta,u}$	
Arch width b (in.)	$2\frac{5}{8}$ in. Connectors	4 in. Connectors
3	0.88	–
$3\frac{1}{8}$	0.91	–
5	1.0	0.93
$5\frac{1}{8}$	1.0	0.95
$5\frac{1}{2}$ or wider	1.0	1.0

- Determine the minimum number of connectors, $n_{\min}$, to transfer the load by dividing the vertical crown reaction force by the partially adjusted capacity of an individual connector (round up to nearest whole number).

$$n_{\min} = \frac{|C_y|}{N^*} \tag{9.2.7.1-3}$$

- *Determine the number of rows of connectors, n_x.* Table 9.2.7.1-2 gives the maximum number of rows of connectors based on arch width. (Rows are oriented vertically.)
- Calculate the required number of connectors per row, n_y.

$$n_y = \frac{n_{\min}}{n_x} \tag{9.2.7.1-4}$$

Except for very shallow members, a minimum of two shear plates is generally required per row to prevent significant reductions in member shear capacity due to notch effects (NDS 3.4.3.3).

- Determine the geometry factor for optimal spacing and edge distance:

$$0.5 \le C_\Delta = \frac{|C_y|}{nN^*} \le C_{\Delta,u} \tag{9.2.7.1-5}$$

- Determine the required spacing for the cases of $C_\Delta = 0.5$ and $C_\Delta = 1.0$

TABLE 9.2.7.1-2 Maximum Number of Rows of Connectors Based on Arch Width

	Maximum Rows of Connectors, n_x	
Arch Width b (in.)	$2\frac{5}{8}$ in. Connectors	4 in. Connectors
$3, 3\frac{1}{8}, 3\frac{1}{2}$	1	–
$5, 5\frac{1}{8}, 5\frac{1}{2}$	1	1
$6\frac{3}{4}$	1	1
$8\frac{1}{2}, 8\frac{3}{4}$	2	1
$10\frac{1}{2}, 10\frac{3}{4}$	3	2

The required spacing for $C_\Delta = 1.0$ is determined based on the angle of the connector with respect to the grain of the member, using Equation 9.2.7.1-6.

$$S_\alpha = \frac{S_C S_D}{\sqrt{S_C^2 \sin^2(90 - \phi - \alpha_{arm}) + S_D^2 \cos^2(90 - \phi - \alpha_{arm})}} \qquad (9.2.7.1\text{-}6)$$

where:

S_α = Required spacing of connectors
S_C = 6.75 in. for $2\frac{5}{8}$ in. shear plates
= 9 in. for 4 in. shear plates
S_D = 4.25 in. for $2\frac{5}{8}$ in. shear plates
= 6 for 4 in. shear plates

The required minimum spacing for $C_\Delta = 0.5$ is 3.5 in. for $2\frac{5}{8}$ in. shear plates and is 5.0 in. for 4 in. shear plates.

- Calculate the required loaded edge distance for $C_\Delta = 0.83$ and $C_\Delta = 1.0$

$$E_\alpha = \frac{E_C E_D}{\sqrt{E_C^2 \sin^2(90 - \phi - \alpha_{arm}) + E_D^2 \cos^2(90 - \phi - \alpha_{arm})}} \qquad (9.2.7.1\text{-}7)$$

E_α = required loaded edge distance
E_C = 5.5 in. for $2\frac{5}{8}$ in. shear plates and $C_\Delta = 1.0$
C_Δ = 4.25 in. for $2\frac{5}{8}$ in. shear plates and $C_\Delta = 0.83$
= 7 in.for 4 in. shear plates and $C_\Delta = 1.0$
= 5.4 in. for 4 in. shear plates and $C_\Delta = 0.83$
E_D = 2.75 in. for $2\frac{5}{8}$ in. shear plates and $C_\Delta = 1.0$
= 1.5 in.for $2\frac{5}{8}$ in. shear plates and $C_\Delta = 0.83$
= 3.75 in. for 4 in. shear plates and $C_\Delta = 1.0$
= 2.5 in. for 4 in. shear plates and $C_\Delta = 0.83$

- Determine the required spacing and loaded edge distance corresponding to the optimal geometry factor using linear interpolation. If the optimal geometry factor is less than 0.83, use the minimum edge distance determined for $C_\Delta = 0.83$

$$S_{\alpha,C_\Delta} = \left(\frac{C_\Delta - 0.5}{1 - 0.5}\right)\left(S_{\alpha,1.0} - S_{\alpha,0.5}\right) + S_{\alpha,0.5} \quad (9.2.7.1\text{-}8)$$

$$E_{\alpha,C_\Delta} = \left(\frac{C_\Delta - 0.83}{1 - 0.83}\right)\left(E_{\alpha,1.0} - E_{\alpha,0.83}\right) + E_{\alpha,0.83} \quad (9.2.7.1\text{-}9)$$

Choose spacing, S, and end distance, E, equal to or greater than those calculated. The chosen spacing and end distance should be rounded up to the nearest $\frac{1}{16}$ in. or other practical unit of measure to facilitate fabrication.

- Calculate the required depth based on connector spacing and edge distance:

$$d'_c \geq d'_{c\,\text{shear plates}} = (n_y - 1)S_\alpha + 2E_\alpha \quad (9.2.7.1\text{-}10)$$

9.2.7.2 Required Crown Depth Based on Flexure in the Arm and Chosen Arm Taper Angle Estimating that the point of inflection occurs near the upper tangent point of the loaded arch for the unbalanced snow loading and that the crown depth is approximated by the depth required for the shear plates, the location of the critical section in the upper arm of the loaded arch (measured from the crown) can be determined from the following relationship, which was derived for this case from Equation 7.2.2.1-2 (x is measured horizontally from the crown).

$$x \approx \frac{\cos(\phi)(L_R - x_2)\left(\dfrac{d'_c}{\cos(\alpha_{arm})}\right)}{2\left(d'_c\dfrac{\cos(\phi)}{\cos(\alpha_{arm})}\right) + \dfrac{(L_R - x_2)}{\cos(\phi)}\tan\alpha_{arm}} \quad (9.2.7.2\text{-}1)$$

The required depth at that section can be approximated as:

$$d_x > \sqrt{\frac{3\left(\omega_S + \omega_D\right)\left(L_R - x_2\right)x\left(1 - \dfrac{x}{L_R - x_2}\right)}{bF_{bx}C_D C_I C_M C_t}} \quad (9.2.7.2\text{-}2)$$

For tapered arms, the reference flexural stress, F_{bx}, used in Equation 9.2.7.2-2 should be reduced to account for the loss of high-strength material at the surface due to tapering, unless the arch is laid up with a uniform-grade layup or the high-grade material is maintained throughout the length of the arch [1]. Either of these options may increase the cost of the arch. For standard arches with tapered arms, a 10% reduction is suitable for preliminary design purposes.

The corresponding depth at the crown can be estimated as follows:

$$d_c > d_x - \left(\frac{x}{\cos\phi}\right)\tan\alpha_{arm} \tag{9.2.7.2-3}$$

$$d_c' = \frac{d_c\cos(\alpha_{arm})}{\cos\phi_t} \tag{9.2.7.2-4}$$

The greater depth from connector requirements or from arm flexure is chosen as the minimum depth of the arch at the crown.

9.2.8 Finalize Trial Geometry and Draw Arch to Scale

With the preceding steps completed, a trial arch geometry is defined. The trial geometry should be further adjusted to ensure that the upper and lower tangent point depths are within approximately 10% of each other. The trial arch should be drawn to scale and the geometry should be adjusted if necessary for aesthetic considerations.

At this point, the preliminary design of the arch is completed. The final design must include a thorough analysis of the arch subject to all applicable loads and load combinations by the procedures discussed in *Design of Tudor Arches with Structural Glued Laminated Timber* [1].

EXAMPLE 9.2-1 PRELIMINARY DESIGN OF TUDOR ARCH

Given: A building with outside dimension of 50 ft × 60 ft is to be constructed using southern pine Tudor arches spaced at 15 ft. The arches will be subject to normal temperatures and dry-use conditions. The arches will be symmetric with a maximum roof height of 24 ft and a wall height of 12 ft, as shown in Figure 9.2-1. The curved portion of the arches will have a radius of 10.5 ft at the inside face and a lamination thickness of $\frac{3}{4}$ in. The arch has continuous lateral bracing along the wall and roof. The building has no shear walls at the ends, so the arches will be assumed to support all vertical and lateral loads acting parallel to the planes of the arches.

Material properties: $F_{bx}^{+} = 2000\,\text{psi}$, $F_{vx}C_{vr} = 215\,\text{psi}$

Loads: the loads have been determined as shown in Figures 9.2-2 through 9.2-4. The wall dead loads are assumed to act at the outer face of the arch.

Wanted: Perform *preliminary* design of a Tudor arch with a width of 6.75 in. based on the dead plus snow load combinations, considering both balanced and unbalanced loading.

Solution: The required depth at the lower tangent is commonly governed by the balanced snow loading. The depths in the arch arm are commonly controlled by the unbalanced snow loading.

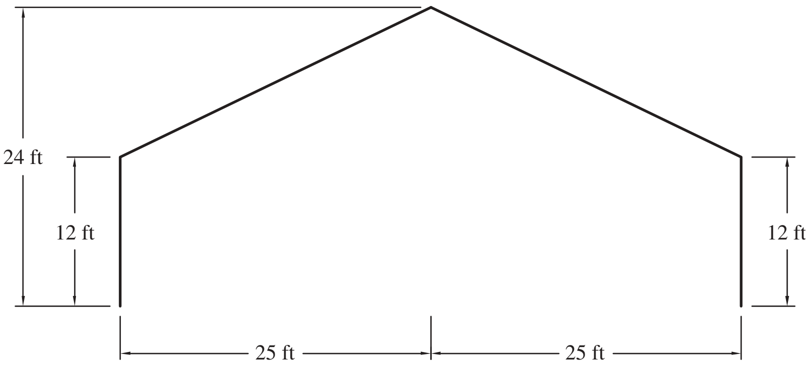


Figure 9.2-1 Outside geometry of arch—Example 9.2-1.

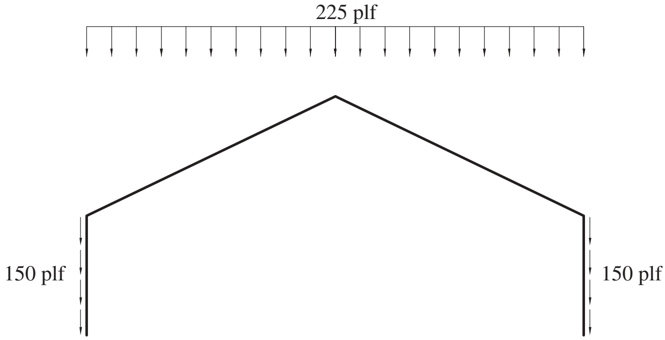


Figure 9.2-2 Dead loads—Example 9.2-1.

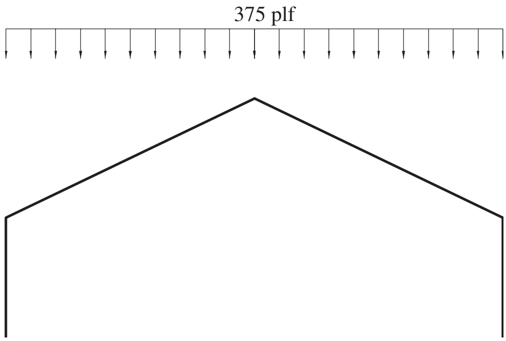


Figure 9.2-3 Balanced snow load—Example 9.2-1.

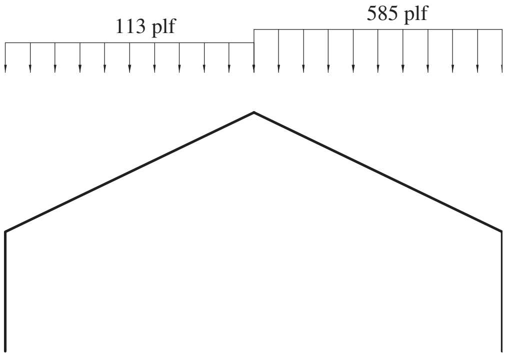


Figure 9.2-4 Unbalanced snow load—Example 9.2-1.

Table 9.2-1 shows the approximate reaction forces (using the outside arch geometry).

TABLE 9.2-1 Approximate Reactions for $D + S$ Load Combinations

Load	$R_{y,L}$	$R_{x,L}$	C_y	C_x	$R_{y,R}$	$R_{x,R}$
$D + S$ (Balanced)	16800	7810	0	–7810	16800	7810
$D + S$ (Unbalanced)	13200	7470	–2950	–7470	19100	7470

Required depth at the crown based on connection requirements:

The minimum depth at the crown is estimated based on the vertical reaction force at the peak connection due to unbalanced snow loading and the number of shear plates required to transfer the load.

For 4 in. shear plates in southern pine:

$$P = 4360\,\text{lb}$$

$$Q = 3040\,\text{lb}$$

$$Q_{90} = 0.6Q = 0.6(3040\,\text{lb}) = 1824\,\text{lb}$$

The shear plates will be loaded at an angle to the grain of $90^\circ - \phi - \alpha_{arm}$.

$$\phi = \arctan\left(\frac{12\,\text{ft}}{25\,\text{ft}}\right) = 25.64^\circ$$

$$\alpha_{arm} = 2.5^\circ$$

$$90^\circ - \phi - \alpha_{arm} = 90^\circ - 25.64^\circ - 2.5^\circ = 61.86^\circ$$

Hankinson's formula can be used to determine the reference capacity for a single shear plate loaded at an angle to the grain of $90^\circ - \phi - \alpha_{arm}$.

$$N = \frac{PQ_{90}}{P\sin^2\left(90^\circ - \phi - \alpha_{arm}\right) + Q_{90}\cos^2\left(90^\circ - \phi - \alpha_{arm}\right)}$$

$$N = \frac{(4360\,\text{lb})\,(1824\,\text{lb})}{(4360\,\text{lb})\sin^2(61.86^\circ) + (1824\,\text{lb})\cos^2(61.86^\circ)}$$

$$N = 2095\,\text{lb}$$

The partially adjusted capacity (dry-use, normal temperatures, snow load) for a single shear plate is:

$$N^* = NC_D C_{\Delta,u} = (2095\,\text{lb})(1.15)(1.0) = 2410\,\text{lb}$$

Based on the vertical reaction at the crown of $C_y = -2950\,\text{lb}$, a minimum of two shear plates will be required:

$$n_{\min} = \frac{|C_y|}{N^*} = \frac{2950\,\text{lb}}{2410\,\text{lb}} = 1.22 \qquad \therefore \text{Use two shear plates}$$

The geometry factor for optimal spacing and edge distance is:

$$0.83 \le C_\Delta = \frac{|C_y|}{nN^*} \le C_{\Delta,u}$$

$$C_\Delta = \frac{|C_y|}{nN} = \frac{2950\ \text{lb}}{2\,(2410\,\text{lb})} = 0.612 \ge 0.5 \qquad \therefore C_\Delta = 0.61$$

The required spacing between shear plates for $C_\Delta = 1.0$ is calculated as:

$$S_{\alpha,1.0} = \frac{S_C S_D}{\sqrt{S_C^2\sin^2(90^\circ - \phi - \alpha_{arm}) + S_D^2\cos^2(90^\circ - \phi - \alpha_{arm})}}$$

$$S_{\alpha,1.0} = \frac{(9\ \text{in})(6\ \text{in})}{\sqrt{(9\ \text{in})^2\sin^2(61.86^\circ) + (6\ \text{in})^2\cos^2(61.86^\circ)}}$$

$$S_{\alpha,1.0} = 6.41\ \text{in}$$

The required spacing between shear plates for $C_\Delta = 0.5$ for 4 in. shear plates is:

$$S_{\alpha,0.5} = 5.0\ \text{in}$$

The required spacing between shear plates for $C_\Delta = 0.61$ is determined by linear interpolation:

$$S_{\alpha,0.61} = \left(\frac{C_\Delta - 0.5}{1 - 0.5}\right)\left(S_{\alpha,1.0} - S_{\alpha,0.5}\right) + S_{\alpha,0.5}$$

$$S_{\alpha,0.61} = \left(\frac{0.61 - 0.5}{1 - 0.5}\right)(6.41\,\text{in} - 5.0\,\text{in}) + 5.0\,\text{in}$$

$$S_{\alpha,0.61} = 5.3\,\text{in} \quad \therefore \text{Choose } S_\alpha = 5.5\ \text{in}$$

The actual geometry factor based on the spacing of 5.5 in. is:

$$C_\Delta = \left(\frac{S_\alpha - S_{\alpha,0.5}}{S_{\alpha,1.0} - S_{\alpha,0.5}}\right)(1.0 - 0.5) + 0.5$$

$$= \left(\frac{5.5 - 5.0}{6.41 - 5.0}\right)(1.0 - 0.5) + 0.5 = 0.68$$

The minimum loaded edge distance for $C_\Delta = 0.83$ is calculated as:

$$E_\alpha = \frac{E_C E_D}{\sqrt{E_C^2 \sin^2\left(90^\circ - \phi_t - \alpha_{arm}\right) + E_D^2 \cos^2\left(90^\circ - \phi_t - \alpha_{arm}\right)}}$$

$$E_{\alpha,0.83} = \frac{(5.4\,\text{in})\,(2.5\,\text{in})}{\sqrt{(5.4\,\text{in})^2 \sin^2(61.86^\circ) + (2.5\,\text{in})^2 \cos^2(61.86^\circ)}}$$

$$E_{\alpha,0.83} = 2.75\,\text{in} \quad \therefore \text{Choose } E_\alpha = 2.75\ \text{in}$$

The minimum vertical depth at the crown to accommodate the two 4 in. shear plates is:

$$d_c' = S_\alpha + 2E_\alpha = 5.5\,\text{in} + 2\,(2.75\,\text{in}) = 11.0\,\text{in}$$

The depth measured perpendicular to the laminations at the crown is:

$$d_c = \frac{d_c' \cos\phi}{\cos\alpha_{arm}} = \frac{(11.0\ \text{in})\cos\left(25.6^\circ\right)}{\cos(2.5^\circ)} = 9.9\ \text{in}$$

A depth, d_c', of 11.0 inches is sufficient to accommodate the use of two 4 in. shear plates. The geometry factor will be the lesser of that determined for spacing and edge distance, $C_\Delta = 0.68$. The adjusted capacity (dry-use, normal temperatures, snow load) of the connection is:

$$N' = 2\left(P_{90^\circ-\phi-\alpha} C_D C_\Delta C_g\right) = 2\,(2095\ \text{lb})\,(1.15)\,(0.68)\,(1.0)$$

$$N' = 3280\,\text{lb} \geq C_y = 2950\,\text{lb} \quad \therefore \text{OK}$$

Required depth at the crown based on flexure in the arm:

The location of the upper tangent point of the loaded arch half can be approximated by:

$$x_2 = A\cos(\phi) = (79.3\text{ in})\cos\left(25.64^\circ\right) = 71.5\text{ in}$$

Estimating that the point of inflection will occur near the upper tangent point of the loaded arch for the unbalanced snow loading and that the crown depth is approximately equal to the depth required to accommodate the shear plates at the crown, the location of the critical section in the upper arm of the loaded arch (measured from the crown) can be determined from the following relationship:

$$x \approx \frac{\cos(\phi)\left(L_R - x_2\right)\left(\dfrac{d_c'}{\cos\left(\alpha_{arm}\right)}\right)}{2\left(d_c'\dfrac{\cos(\phi)}{\cos\left(\alpha_{arm}\right)}\right) + \dfrac{\left(L_R - x_2\right)}{\cos(\phi)}\tan\alpha_{arm}}$$

$$x \approx \frac{\cos\left(25.64^\circ\right)(300\text{ in} - 71.5\text{ in})\left(\dfrac{12.5\text{ in}}{\cos(2.5^\circ)}\right)}{2\left(12.5\text{ in}\dfrac{\cos\left(25.64^\circ\right)}{\cos(2.5^\circ)}\right) + \dfrac{(300\text{ in} - 71.5\text{ in})}{\cos(25.64^\circ)}\tan\left(2.5^\circ\right)}$$

$$x \approx 76.7\text{ in}$$

The required depth at that section can be approximated as:

$$d_x > \sqrt{\frac{3\omega_{y,R}\left(L_R - x_2\right)x\left(1 - \dfrac{x}{L_R - x_2}\right)}{bF_{bx}C_D C_I}}$$

$$d_x > \sqrt{\frac{3\,(67.5\text{ lb/in})\,(300\text{ in} - 71.5\text{ in})\,(76.7\text{ in})\left(1 - \dfrac{76.7\text{ in}}{300\text{ in} - 71.5\text{ in}}\right)}{(6.75\text{ in})\,(2000\text{ psi})\,(0.9)\,(1.15)\,(0.926)}}$$
$$= 13.5\text{ in}$$

The corresponding depth at the crown can be estimated as:

$$d_c \approx d_x - \left(\frac{x}{\cos\phi}\right)\tan\alpha_{arm} = 13.5\text{ in} - \left(\frac{76.7\text{ in}}{\cos(25.6^\circ)}\right)\tan\left(2.5^\circ\right) = 9.8\text{ in}$$

$$d_c' = \frac{d_c\cos\left(\alpha_{arm}\right)}{\cos\phi} = \frac{9.8\cos\left(2.5^\circ\right)}{\cos(25.6^\circ)} = 10.9\text{ in}$$

A trial crown depth, d_c', of 11.0 inches will be chosen (controlled by connector design). The depth of the crown (measured perpendicular to the laminations) is $d_c = 9.9\,\text{in.}$, as previously calculated.

Depth at the upper tangent based on chosen crown depth and taper angle:

$$d_{UT} \approx d_c + \left(\frac{L_R - x_2}{\cos\phi}\right)\tan\alpha_{arm} = 9.9\,\text{in}$$
$$+\left(\frac{300\,\text{in} - 71.5\,\text{in}}{\cos(25.6^\circ)}\right)\tan\left(2.5^\circ\right) = 21.0\,\text{in}$$

Required depth of the lower tangent point:

The minimum depth at the lower tangent point will be estimated based on the moment created by the horizontal reaction applied at a distance, $y_{1,L}$, from the approximate tangent point. The leg taper is chosen as 4° for the determination of C_I. The volume factor will be estimated as 0.95.

$$A = R\tan\left(\frac{90^\circ - \phi}{2}\right) = (126\,\text{in})\left(\tan\left(\frac{90^\circ - 25.64^\circ}{2}\right)\right) = 79.28\,\text{in}$$

$$y_1 = H_w - A = 144\,\text{in} - 79.28\,\text{in} = 64.72\,\text{in}$$

$$d_{LT} > \sqrt{\frac{6y_1R_{x,L}}{bF_{bx}C_DC_IC_V}} = \sqrt{\frac{6\,(64.72\,\text{in})\,(7812.5\,\text{lb})}{(6.75\,\text{in})\,(2000\,\text{psi})\,(1.15)\,(0.83)\,(0.95)}}$$

$$d_{LT} > 15.74\,\text{in}$$

The trial depth should be a minimum of 5% larger than calculated based on flexure alone, to accommodate the interaction of compression and bending stresses. Additionally, it is recommended that the upper and lower tangent point depths be within approximately 10% of each other. To satisfy both objectives, a trial depth of 19 in. will be chosen.

Required depth at the base:

The minimum depth at the base will be estimated based on the horizontal shear reaction and also determined based on the estimated tangent point depth and the amount of taper chosen.

$$d_b' > \frac{3R_{x,L}}{2bF_{vx}C_D} = \frac{3\,(7812.5\,\text{lb})}{2\,(6.75\,\text{in})\,(215\,\text{psi})\,(1.15)} = 7.022\,\text{in}$$

$$d_b' > d_{LT} - y_1\tan\alpha_{leg} = 19\,\text{in} - (64.7\,\text{in})\tan\left(4^\circ\right) = 14.5\,\text{in}$$

A trial depth of 14.5 inches will be chosen for the base.

Results: A drawing of the trial arch is shown in Figure 9.2-5. The appearance is determined to be acceptable. The preliminary design is complete.

$$b = 6.75\,\text{in}$$
$$H_{w,L} = H_{w,R} = 144\,\text{in} = 12\,\text{ft}$$
$$H_c = 288\,\text{in} = 24\,\text{ft}$$
$$d'_{b,L} = d'_{b,R} = 14.5\,\text{in}$$
$$d'_c = 11.0\,\text{in}$$
$$R_L = R_R = 126\,\text{in} = 10.5\,\text{ft}$$
$$\alpha_{leg,L} = \alpha_{leg,R} = 4^\circ$$
$$\alpha_{arm,L} = \alpha_{arm,R} = 2.5^\circ$$

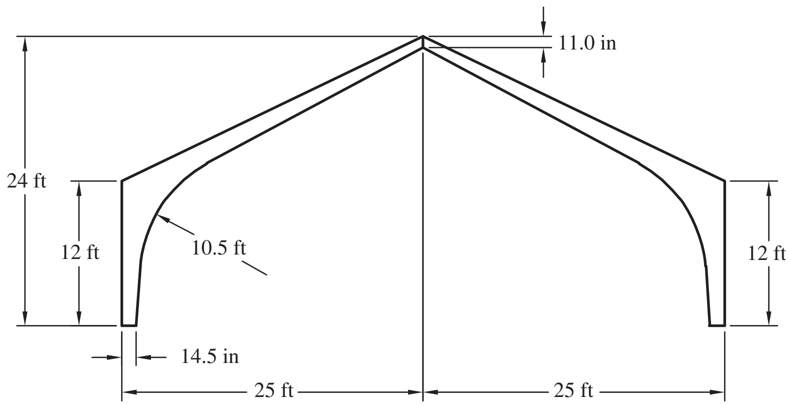


Figure 9.2-5 Trial arch with $\alpha_{arm} = 2.5^\circ$ and $\alpha_{leg} = 4^\circ$—Example 9.2-1.

Discussion: The design presented in this example is preliminary. For final design, the precise geometry should be modeled and analyzed for all relevant load combinations.

9.3 CONCLUSION

Glulam arches have been successfully used in the United States since the 1930s. They continue to be popular because of their excellent structural performance, inherent fire resistance, and pleasing aesthetics.

Because of complex geometries and loadings, arch design can be challenging. This chapter has presented a preliminary design procedure for Tudor arches to enable the architect or engineer of record to visualize the arch form, get a preliminary estimate of costs, and provide a starting point for arch analysis. Final design is commonly subcontracted to custom glulam manufacturers or engineers specializing in timber design. Additional information regarding Tudor arch design is available in *Design of Tudor Arches with Structural Glued Laminated Timber* [1].

CHAPTER 10

HEAVY TIMBER DECKING

10.1 INTRODUCTION

Heavy timber decking is a specialty lumber product, constituting an important part of modern timber construction. It complements exposed timber systems and is especially well-adapted for use with glulam arches and girders. Heavy timber decking typically spans 4 to 18 ft between supporting members, often eliminating the need for purlins or other secondary framing members.

Most heavy timber decking is sawn lumber with edges shaped to a tongue-and groove profile. Laminated decking is also available. Chapter 1 provides general information about these products and illustrates the common profiles. This chapter covers the structural design of heavy timber decking.

10.2 INSTALLATION REQUIREMENTS

Decking pieces should be installed with the tongues upward on sloping roofs. The pattern face is installed downward and is typically exposed to view from below.

10.2.1 Nailing Schedule

Each course of two-inch decking should be nailed with two 16d common nails per support: one through the face and the other toenailed through the tongue. Each course of three-inch or four-inch decking should be toenailed through the tongue with a 40d common nail and face-nailed with a 60d common nail at each support. In addition, three-inch and four-inch decking must be fastened to adjacent courses with 8-inch spikes driven through predrilled edge holes. These spikes must be placed within 10 inches of the end of the piece and spaced at a maximum of 30 inches. Nailing patterns for laminated decking are provided by the manufacturer. Typically, each course of laminated decking is face nailed at the supports and adjacent courses of decking are slant-nailed to each other at intervals of 30 inches or less.

10.2.2 Moisture Content

The maximum moisture content of any piece of two-inch sawn decking must not exceed 15% at the time of installation. The maximum moisture content of any piece of three-inch or four-inch sawn decking must not exceed 19% at the time of installation.

10.2.3 Installation Patterns

Heavy timber decking may be installed in any of five standard patterns (Figure 10.2.3-1): (1) simple span, (2) two-span continuous, (3) combination simple and two-span continuous, (4) cantilevered pieces intermixed, and (5) controlled random. Other patterns are permitted if substantiated by engineering analysis.

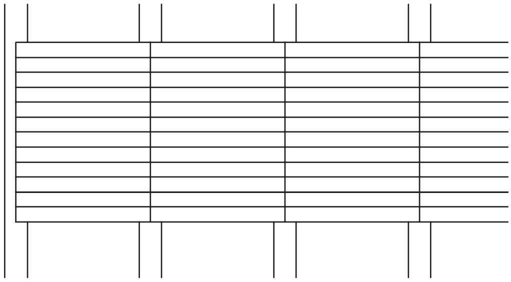
10.2.3.1 Simple Span Pattern For the simple span pattern, each piece must be supported by two supports.

10.2.3.2 Two-span Continuous Pattern For the two-span continuous pattern, all pieces must be supported by three supports, and all end joints must occur in line on alternating supports. Supporting members must be designed to accommodate the load redistribution caused by this pattern.

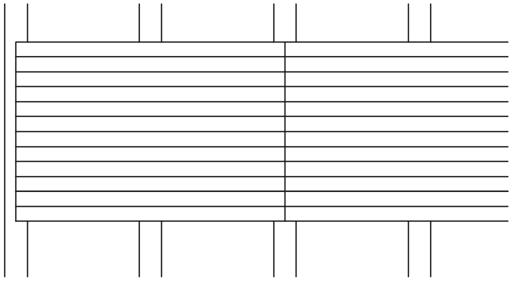
10.2.3.3 Combination Simple and Two-span Continuous Pattern For the combination simple and two-span continuous pattern, courses in the end spans must be alternating simple span pattern and two-span continuous pattern. End joints are staggered in adjacent courses and bear on supports.

10.2.3.4 Cantilevered Pieces Intermixed Pattern For the cantilevered pieces intermixed pattern, the decking must extend across a minimum of four supports (three spans). Each starter course and every third course are arranged in a simple span pattern. Pieces in other courses are cantilevered over the supports with end joints at alternating quarter or third points of the spans. Each piece must bear on at least one support.

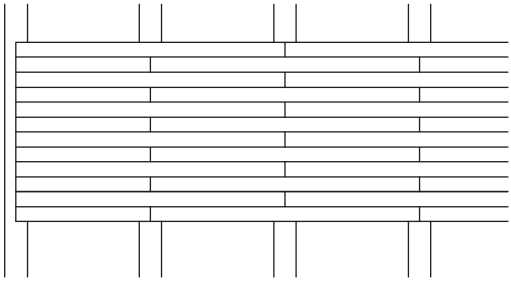
10.2.3.5 Controlled Random Pattern For the controlled random pattern, the decking must extend across a minimum of four supports (three spans). End joints of pieces within 6 inches of the end joints of the adjacent pieces in either direction must be separated by at least two intervening courses. In the end bays, each piece must bear on at least one support. Where an end joint occurs in an end bay, the next piece in the same course must continue over the first inner support for at least 24 inches. Other details of the controlled random pattern vary depending on the decking material.



Simple Span Pattern (Type1)

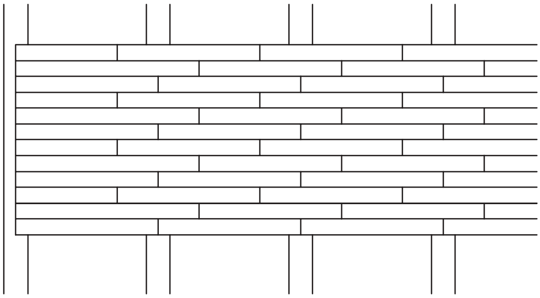


Two-span Continuous Pattern (Type 2)

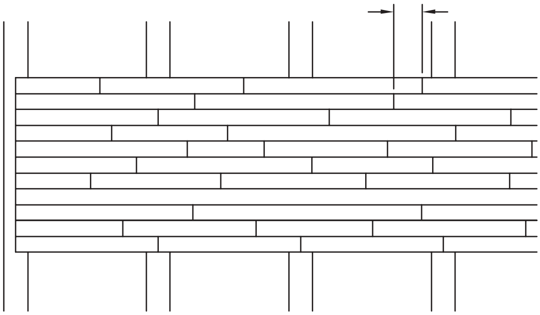


Combination Simple and Two-span
Continuous Pattern (Type 3)

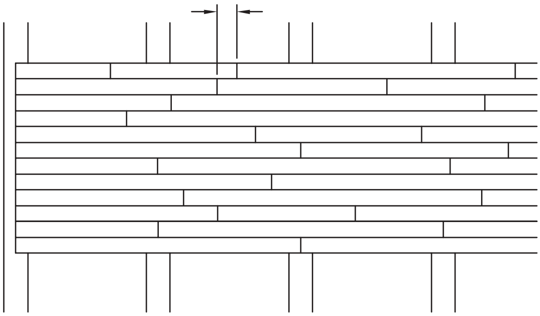
Figure 10.2.3-1 Standard installation patterns for timber decking.



Cantilevered Pieces Intermixed Pattern
(Type 4)



Controlled Random Pattern (Type 5)
(3 and 4 Inch Decking)



Controlled Random Pattern (Type 5)
(2 Inch Decking)

Figure 10.2.3-1 (*Continued*)

Decking that cantilevers beyond a support for a distance greater than 18 inches, 24 inches, or 36 inches for two-inch, three-inch, and four-inch nominal thickness decking, respectively, must meet the following requirements:

1. The maximum cantilevered length is 30% of the length of the first adjacent interior span.
2. A structural fascia must be fastened to each decking piece to maintain a continuous straight line.
3. No end joints are permitted in the decking between the cantilevered end of the decking and the centerline of the first adjacent interior span.

10.2.3.5.1 Two-inch Decking Additional Requirements For two-inch nominal thickness sawn decking, a minimum distance of 24 inches is required between end joints in adjacent courses. The pieces in the first and second courses must bear on at least two supports with end joints in these two courses occurring on alternate supports. A maximum of seven intervening courses is permitted before this pattern is repeated.

A minimum of 40% of the pieces must be 14 feet or longer. A maximum of 10% of the pieces are permitted to be less than 10 feet. A maximum of 1% of the pieces are permitted to be 4 to 5 feet long. However, the minimum length permitted is 75% of the span.

10.2.3.5.2 Three-inch and Four-inch Decking Additional Requirements For three-inch and four-inch nominal thickness sawn decking, a minimum distance of 48 inches is required between end joints in adjacent courses. Pieces not bearing on a support are permitted to be located in interior bays, provided the adjacent pieces in the same course continue over the support for at least 24 inches. This condition must not occur more than once in every six courses in each interior bay.

For three-inch decking, a minimum of 40% of the pieces must be 14 feet or longer with at least 20% of the pieces equal to or greater in length than the maximum span. No more than 10% of the pieces are permitted to be less than 10 feet, and a maximum of 1% of the pieces can be 4 to 5 feet long.

For four-inch decking, a minimum of 25% of the pieces must be 16 feet or longer with at least 20% of the pieces equal to or greater in length than the maximum span. A minimum of 50% of the pieces must be 15 feet or longer. No more than 10% of the pieces are permitted to be 5 to 10 feet long. A maximum of 1% of the pieces can be 4- to 5 feet long.

10.3 DESIGN FORMULAS

Heavy timber decking essentially forms a beam and must be analyzed as such. The basic criteria for bending, deflection, and shear must be satisfied. However, due to the typical flat-wise orientation of the decking pieces, shear rarely controls the design. The equations in Table 10.3-1 may be used to compute the allowable uniform load for the standard patterns. Additional checks might be necessary for point loads and short spans with heavy loads where shear becomes significant. Except for the simple span and two-span continuous patterns, the equations of Table 10.3-1 may not be used for the loads on individual pieces, because the equations are dependent on load transfer between adjacent courses.

TABLE 10.3-1 Design Formulas for Heavy Timber Decking[a]

	Allowable Area Load[b,c]	
Pattern Type	Based on Bending	Based on Deflection
Simple span	$\sigma_b = \frac{8F_b'}{\ell^2}\frac{d^2}{6}$	$\sigma_\Delta = \frac{384\Delta E'}{5\ell^4}\frac{d^3}{12}$
Two-span continuous	$\sigma_b = \frac{8F_b'}{\ell^2}\frac{d^2}{6}$	$\sigma_\Delta = \frac{185\Delta E'}{\ell^4}\frac{d^3}{12}$
Combination simple- and two-span continuous	$\sigma_b = \frac{8F_b'}{\ell^2}\frac{d^2}{6}$	$\sigma_\Delta = \frac{131\Delta E'}{\ell^4}\frac{d^3}{12}$
Cantilevered pieces intermixed	$\sigma_b = \frac{20F_b'}{3\ell^2}\frac{d^2}{6}$	$\sigma_\Delta = \frac{105\Delta E'}{\ell^4}\frac{d^3}{12}$
Controlled random layup		
Nominal 2 in. decking	$\sigma_b = \frac{20F_b'}{3\ell^2}\frac{d^2}{6}$	$\sigma_\Delta = \frac{100\Delta E'}{\ell^4}\frac{d^3}{12}$
Nom. 3 in. and 4 in. decking	$\sigma_b = \frac{20F_b'}{3\ell^2}\frac{d^2}{6}$	$\sigma_\Delta = \frac{116\Delta E'}{\ell^4}\frac{d^3}{12}$
Glued laminated decking [d]	$\sigma_b = \frac{20F_b'}{3\ell^2}\frac{d^2}{6}$	$\sigma_\Delta = \frac{100\Delta E'}{\ell^4}\frac{d^3}{12}$

[a] σ_b = allowable total uniform area load limited by bending; σ_Δ = allowable total uniform area load limited by deflection.
[b] d = actual decking thickness.
[c] Design formulas assume solid section and do not account for section loss due to beveled edges.
[d] Higher stiffness values may be provided by manufacturers with additional construction requirements.

Manufacturers of laminated decking may provide a deflection equation for the controlled random pattern that is stiffer than the equation in Table 10.3-1. Such additional stiffness provided by the manufacturer is generally the result of additional installation requirements for the pattern required by the manufacturer, such as reducing or eliminating the end joints in the end spans or reducing the length of the end spans.

10.4 SECTION PROPERTIES

Section modulus and moment of inertia of decking may be determined for individual deck pieces, or may be provided by the manufacturer. Where manufacturers provide section modulus and moment of inertia per unit of width of deck, these values should be used in place of $d^2/6$ and $d^3/12$ in Table 10.3-1. Section moduli and moments of inertia provided by manufacturers typically account for section loss from beveled edges.

10.5 DECKING DESIGN VALUES

Design values for sawn timber decking are published by the lumber grading agencies including the Southern Pine Inspection Bureau (SPIB) [1], the West Coast Lumber Inspection Bureau (WCLIB)[2], the Western Wood Products Association (WWPA) [3], and the National Lumber Grades Authority (NLGA) [4]. The design values are also reprinted in the National Design Specification [5] for convenience. The published design values commonly incorporate adjustments for flat use and repetitive use, so these adjustments should not be applied again by the designer. Adjustments for load duration, temperature, size, and wet service are typically applied by the designer.

Design values for laminated timber decking are published by the individual manufacturer. These design values typically do not require further adjustment by the designer, except for load duration and wet service.

EXAMPLE 10-1 TIMBER DECKING

Given: The following design properties are provided by the manufacturer:

$$F_b = 2250\,\text{psi}$$
$$E = 1{,}800{,}000\,\text{psi}$$
$$I/b = 10.29\,\text{in}^4/\text{ft}$$
$$S/b = 9.39\,\text{in}^3/\text{ft}$$

Decking weight: 6 psf

Approach: It will be assumed that the design values provided for bending include consideration of flat use and size; thus, the bending design value will be adjusted for load duration only.

Solution:

Design values:

$$F_b' = F_b(C_D) = 2250 \text{ psi}(1.15) = 2585 \text{ psi}$$
$$E' = E = 1.8(10^6 \text{ psi})$$

Deflection limits:

$$\delta_S = \frac{\ell}{240} = \frac{10 \text{ ft}(12 \text{ in/ft})}{240} = 0.50 \text{ in}$$
$$\delta_{0.5D+S} = \frac{\ell}{180} = \frac{10 \text{ ft}(12 \text{ in/ft})}{180} = 0.67 \text{ in}$$

Flexure design (combination simple and two-span pattern):

$$\sigma_b = \frac{8F_b'}{\ell^2}\frac{d^2}{6} = \frac{8(2585\ \text{lb/in}^2)}{(10\ \text{ft})^2}(9.39\ \text{in}^3/\text{ft})\left(\frac{\text{ft}}{12\ \text{in}}\right)$$

$$\sigma_b = 162\ \text{psf} \geq 45\ \text{psf} + 12\ \text{psf} = 57\ \text{psf} \qquad \therefore \text{OK}$$

Deflection design (S only) (combination simple and two-span pattern):

$$\sigma_\Delta = \frac{131\Delta E'}{\ell^4}\frac{d^3}{12} = \frac{131(0.50\ \text{in})(1{,}800{,}000\ \text{lb/in}^2)}{(120\ \text{in})^4}\left(\frac{10.29\ \text{in}^4}{\text{ft}}\right)(12\ \text{in/ft})$$

$$\sigma_\Delta = 70.2\ \text{psf} \geq 45\ \text{psf} \qquad \therefore \text{OK}$$

Deflection design ($\mathbf{0.5D + S}$) (combination simple and two-span pattern):

$$\sigma_\Delta = \frac{131\Delta E'}{\ell^4}\frac{d^3}{12} = \frac{131(0.67\ \text{in})(1{,}800{,}000\ \text{lb/in}^2)}{(120\ \text{in})^4}\left(\frac{10.29\ \text{in}^4}{\text{ft}}\right)(12\ \text{in/ft})$$

$$\sigma_\Delta = 94.1\ \text{psf} \geq 45\ \text{psf} + 6\ \text{psf} = 51\ \text{psf} \qquad \therefore \text{OK}$$

Flexure design (controlled random pattern):

$$\sigma_b = \frac{20F_b'}{3\ell^2}\frac{d^2}{6} = \frac{20(2585\ \text{lb/in}^2)}{3(10\ \text{ft})^2}(9.39\ \text{in}^3/\text{ft})\left(\frac{\text{ft}}{12\ \text{in}}\right)$$

$$\sigma_b = 135\ \text{psf} \geq 45\ \text{psf} + 12\ \text{psf} = 57\ \text{psf} \qquad \therefore \text{OK}$$

Deflection design (S only) (combination simple and two-span pattern):

$$\sigma_\Delta = \frac{100\Delta E'}{\ell^4}\frac{I}{b} = \frac{100(0.50\ \text{in})(1{,}800{,}000\ \text{psi})}{(120\ \text{in})^4}\left(\frac{10.29\ \text{in}^4}{\text{ft}}\right)(12\ \text{in/ft})$$

$$\sigma_\Delta = 53.6\ \text{psf} \geq 45\ \text{psf} \qquad \therefore \text{OK}$$

Deflection design $(0.5D + S)$ **(combination simple and two-span pattern):**

$$\sigma_\Delta = \frac{100\Delta E'}{l^4}\frac{d^3}{12} = \frac{100\,(0.67\,\text{in})\left(1{,}800{,}000\,\text{lb/in}^2\right)}{(120\,\text{in})^4}\left(\frac{10.29\,\text{in}^4}{\text{ft}}\right)(12\ \text{in/ft})$$

$$\sigma_\Delta = 71.8\,\text{psf} \geq 45\ \text{psf} + 6\,\text{psf} = 51\,\text{psf} \therefore \text{OK}$$

Discussion: In this example, a deflection criterion of $\ell/180$ with respect to applied load plus 50% of dead load was used. The deflection value calculated in association with this criterion is *not* the actual deflection that would be expected from the applied live and dead loads. With regard to deflection, the designer must be sure to satisfy the deflection criteria of the governing building code, and ensure that actual deflections, including creep, are satisfactory to the owner. This example illustrates that decking applications of moderate and long span typically are governed by deflection criteria.

10.6 CONCLUSION

Heavy timber decking is an important element in timber construction. Both sawn and laminated options are available. Timber decking is typically installed as an exposed wood product forming the finished ceiling of the space below. As such, it complements exposed timber members and generally produces a warm aesthetic. This chapter has presented installation and design requirements for timber decking, including a design example.

CHAPTER 11

CONNECTIONS IN TIMBER STRUCTURES

11.1 INTRODUCTION

Connections between structural elements are an integral part of any structure. Connections involving timber members generally incorporate one or more fasteners, may also incorporate bearing, and may or may not utilize metal side plates. With the exception of specially designed moment splices, connections in timber are generally not expected to transfer moments between members.

This chapter provides an overview of fastener design, including connection detailing principles, fastener types, reference design values, and adjustment factors. Chapter 12 discusses potential member failure modes at connections and presents methods to account for these in design. Chapters 13 and 14 present design procedures for common fastener types. Chapter 15 covers the design of moment splices between timber members.

11.2 CONNECTION DETAILING PRINCIPLES

A variety of methods are used to connect timber members and transfer loads within a structure. The simplest type of connection uses direct bearing to transfer loads. Bearing type connections are very effective and should be used where possible. However, bearing connections are not feasible for all situations, so other methods must be used. The most common is through the use of various mechanical fasteners, typically steel. Nails, lag screws, bolts, and timber connectors are commonly used with timber members. Timber rivets provide another option for glulam connections. In general, the fasteners in a connection should not be mixed, but rather should be of the same size and type.

Regardless of the type of connection used, every connection must be designed and detailed to resist all anticipated loads throughout the life of the structure. In addition to transferring the structural loads, connections must accommodate changes in wood dimensions due to variations in moisture content. Care must also be taken in moist or wet environments to avoid trapped moisture at connections. Connections that cause tension perpendicular-to-grain stresses in timber members should be avoided, because of wood's inherent weakness in tension perpendicular-to-grain. Careful detailing will reduce problems and ensure the continued good performance of the connection. AITC 104 *Typical Construction Details* [1] illustrates recommended details and also shows details which tend to be problematic.

11.2.1 Accommodating Member Shrinkage

Understanding the wood-moisture relationships discussed in Chapter 2 of this manual is critical to proper detailing. Most timber members are used in dry conditions and will shrink in service. A reasonable estimate of indoor equilibrium moisture content for most design purposes is 7% to 8%. The average moisture content of structural glued laminated timber members can be estimated as 12 to 13% at the time of manufacturing. The average moisture content of kiln-dried lumber is normally estimated as 15%. For green lumber, the moisture content (MC) may be greater than 30%.

Based on the rule of thumb presented in Chapter 2 (5% drop in MC = 1% shrinkage), a structural glued laminated timber member is normally expected to shrink by 1% as it dries in service. Kiln-dried sawn timbers may shrink two to three times more than glulam depending on the orientation of the growth rings. Green timbers can be expected to shrink five to six times more than glulam. For very dry environments, shrinkage may be more.

Shrinkage generally does not cause problems by itself; however, relative shrinkage between connected parts can result in splitting in the timber members at the connection. Care must be taken in detailing connections so that the connection does not restrain the cross-grain shrinkage of the timber member, creating tension perpendicular-to-grain stresses. Tension strength perpendicular to the grain in wood is very small and can practically be considered as zero for most design purposes. Particular care must be exercised when using metal plates or when connecting nonparallel timber members directly to each other.

In general, The *NDS*® [2] limits the *overall* spacing of rows of fasteners parallel to a sawn timber member to 5 in. where the fasteners attach to a single metal side or splice plate (Figure 11.2.1-1), unless special detailing is provided to accommodate shrinkage. For structural glued laminated timber, the *NDS*® [2] limits overall spacing between outer rows of bolts on a single plate to 10 in. unless analysis demonstrates that more shrinkage can be tolerated or special detailing is provided (Figure 11.2.1-1). The *NDS*® [2] limit assumes an in-service moisture content of 6%, representative of very dry climates such as the southwestern United States. Example 11.2.1-1 demonstrates a simple analysis that can be used

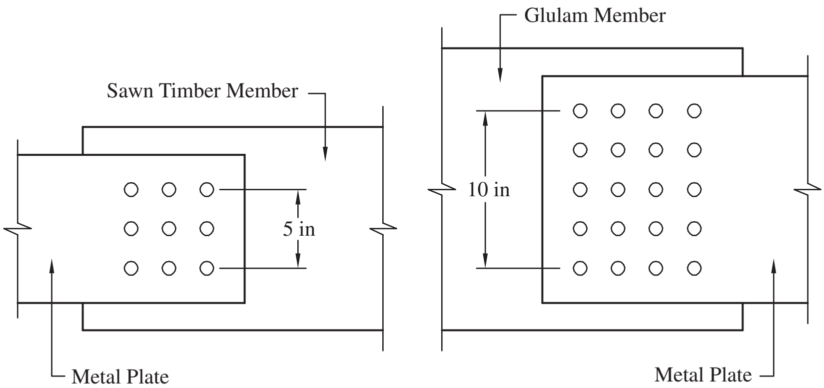


Figure 11.2.1-1 *NDS*® limits for overall spacing between rows of fasteners without special detailing or further analysis.

to determine spacing requirements for an in-service equilibrium moisture content of 7% to 8%.

EXAMPLE 11.2.1-1 TENSION CONNECTION USING BOLTS WITH STEEL SIDE PLATES

Given: A tension connection will be necessary in the bottom chord of a structural glued laminated timber truss of an undetermined size (Figure 11.2.1-2). Steel side plates and bolts will be used for the splice. Standard size holes ($\frac{1}{16}''$ oversize) will be used in the steel plates and timber member.

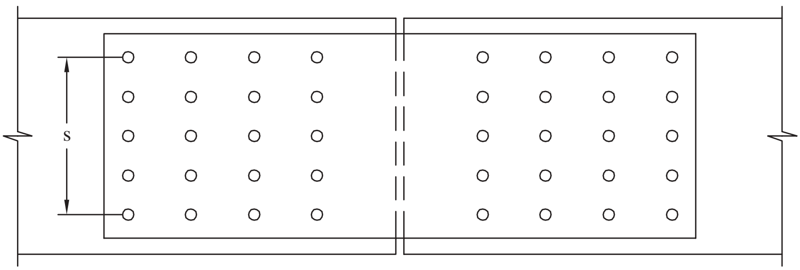


Figure 11.2.1-2 Maximum spacing between outer rows of bolts in glulam—Example 11.2.1-1.

Wanted: Determine the maximum spacing, s, that can be used between the outer rows of bolts without inducing tension perpendicular-to-grain stresses due to member shrinkage.

Solution: For tension perpendicular-to-grain to develop, the bolts must be in contact with the outer edges of the holes in the timber member and in contact with the inner edges of the holes in the steel plates. Assuming that the holes are aligned at the centers at the time of fabrication, standard size holes will allow for a total of $\frac{1}{8}$ in. shrinkage before tension perpendicular-to-grain develops. Assuming the glulam beam will shrink 1% across the grain after fabrication gives:

$$\frac{\Delta s}{s} = 0.01$$

$$s = \frac{\Delta s}{0.01} = \frac{0.12 \text{ in}}{0.01} = 12 \text{ in}$$

where:

s = maximum spacing between outer rows of bolts across the grain of the timber

Δs = the change in dimension that is allowed between the outer rows of bolts

Result: The bolt spacing between outer rows of bolts should be limited to 12 in. or less to accommodate cross-grain shrinkage.

11.2.2 Avoiding Tension Perpendicular-to-Grain Loading

Tension perpendicular-to-grain can be induced in timber members in a number of ways. Using bolts or other fasteners to support a beam, rather than using a bearing connection, creates tension perpendicular-to-grain stresses. Eccentric truss connections create tension perpendicular-to-grain. Rigid truss connections that restrict joint rotation cause tension perpendicular-to-grain. Hanging loads from the bottom of a beam also creates tension perpendicular-to-grain. Connections of these types should be avoided to prevent splitting of the member and capacity loss.

Where possible, loads should be transferred using direct bearing, rather than through fasteners. Truss connections should be concentric and allow rotation of the individual members. As illustrated in the previous example, the overall dimension between fasteners in connections with rigid side plates or other rigid elements should not be excessive.

11.2.3 Minimizing Moisture Problems

Moisture can cause a number of problems in timber members. Dimensional changes associated with changes in moisture content can induce stresses in the member, leading to checking and possible splitting of the member. Extended or repeated wetting of the wood may also lead to decay of the timber member, drastically reducing the capacity of affected sections. Proper detailing will minimize these problems.

For timbers that will be exposed to the weather, pressure-preservative treatment or naturally durable wood should be specified to prevent decay of the member. Where possible, all fabrication of the member, including connections, should be performed before the treatment process. For field-fabrication, holes and cuts should be field-treated with an approved preservative to minimize decay problems at the connections. AITC Technical Note 12 [3] and Standard 109 [4] give more information regarding preservative treatments.

Most timbers that are protected from the weather will not have problems with moisture, however, even in protected structures, improper detailing can cause moisture problems. In particular, timber members should not be placed in direct contact with concrete or masonry. Contact with these materials allows moisture to migrate from the concrete or masonry into the wood, leading to decay. The bases of timber columns should not be recessed below the concrete floor level or be configured in any other way that will trap moisture. Natatoriums, wet-process facilities, water storage structures, and other high-moisture indoor environments require special consideration and may require preservative-treated or naturally durable wood.

11.2.4 Construction Documents

Construction documents must contain enough detail to ensure proper materials and installation. Construction documents should clearly show fastener locations including edge distances, end distances, spacing of fasteners in rows and spacing of rows, and also any clearance requirements for shrinkage/swelling or for fire protection requirements. Edge, end, and spacing distances must be clearly shown or specified. Where possible, consistent fastener size and spacing values should be used within individual connections and from connection to connection. Size and grade of metal parts (bolts, split rings, shear plates, rivets, and metal plate) and welding details must be clearly communicated. Minimum bearing lengths and anchorage requirements must be shown. Where nails or similar fasteners are used, diameter and length must be clearly identified. Wood to concrete connections must include minimum concrete compressive strength and the appropriate edge distances, end distances, and penetration (embedment) into concrete to achieve the values used in design. Where required by the designer or by local building code, connections requiring special inspection must also be identified.

11.3 TYPES OF FASTENERS

Mechanical fasteners are commonly used to transfer loads to and from timber members. Fasteners are typically steel with occasional use of other metals or even wood. Design procedures in this manual focus on steel fasteners. This manual covers dowel-type fasteners, shear plates and split rings. A brief overview is given of other fastener types. In general, the fasteners in a connection should not be mixed, but rather should be of the same size and type.

11.3.1 Dowel-Type Fasteners

Dowel-type fasteners include nails, spikes, wood screws, lag screws, bolts, and drift pins. These fasteners resist loads perpendicular to the fastener axis through bearing and bending mechanisms. Certain dowel-type fasteners also resist withdrawal loading through friction or interlocking of threads with wood.

11.3.1.1 Bolts Steel bolts with nuts and washers are commonly used to connect timber members. Bolts typically transfer load perpendicular to the bolt axis, but they can be used to transfer loads parallel to the bolt axis through bearing of the washer on the wood surface. For timber construction, bolts typically range from $\frac{1}{2}$ in. to 1 in. in diameter. For bolts of this size, holes should be drilled $\frac{1}{16}$ in. larger in diameter than the bolt.

11.3.1.1.1 Bolt Size Limitations The maximum bolt size permitted by the *NDS*® [2] is 1 inch in diameter [5]. Research supporting this limitation indicated that negative effects of drying and fabrication imperfections were more pronounced with large, stiff fasteners than for smaller diameter fasteners [6]. Other researchers found close correlation between test results of 1 in. diameter bolts and the yield theory, but that $1\frac{1}{2}$ in. diameter bolts did not perform as well as predicted [7].

The results of a test program [8] consisting of single $1\frac{1}{4}$ in. diameter bolts in $4\frac{1}{2}$ in. wide glued laminated timber with $\frac{1}{2}$ in. steel or $2\frac{1}{4}$ in. wood side members compared favorably with fastener capacities determined using the NDS procedures, however, tests were not performed on multiple-fastener joints or in other configurations. Therefore, AITC recommends against the use of bolts larger than 1 in. diameter without further research.

11.3.1.1.2 Bolt Strength ASTM A307—Grade A [9] or SAE J429—Grade 1 [10] ($F_y = 36,000$ ksi, $F_u = 60,000$ ksi) bolts are typically used in timber construction, and fastener capacities published in the *NDS*® [2] are based on bolts of this strength. A bending yield strength of 45,000 psi is used in design calculations for these fasteners.

High-strength bolts may be used to connect timber members. The bending yield strength (F_{yb}) is determined from tests in accordance with ASTM F1575 [11] or may be estimated as the average of the ultimate tensile strength and the tensile yield stress (NDS® Appendix I [2]).

11.3.1.1.3 Nut Tightening For all grades of bolts used in timber construction, nuts should be tightened to the *snug-tight* condition. This is generally sufficient to bring the faces of the joined members into firm contact. However, for U-shaped hangers or saddles, where the side plates are welded to the bearing plate, nuts should be tightened only enough to bring the nut and bolt head into firm contact with the side plates. For bolts in slotted holes (such as to accommodate cross-grain shrinkage of the timber member or displacement at the end of a pitched beam), nuts should be installed *finger-tight* to allow the members to slide past each other with minimal resistance.

There is no minimum torque or bolt tension requirement, even when high-strength bolts are used. Bolt tension will generally be lost due to subsequent shrinkage of the timber members due to drying. *Slip-critical* connections are not used in timber construction, because it is impossible to maintain the bolt tension over time. Over-tightening of nuts should be avoided to prevent localized crushing of the wood fibers or damage to the bolts.

11.3.1.1.4 Washers A washer at least as large as a standard cut washer is required between the wood and the bolt head and between the wood and the nut. It is not necessary to use a washer if a steel plate or strap is placed between the bolt head or nut and the wood member. Hardened washers are not required.

11.3.1.1.5 Bolt Holes Standard holes in metal and glulam members in bolted connections are typically $\frac{1}{16}$ in. larger than the bolt diameter. Occasionally, over-sized or slotted holes are specified in the metal member to allow for cross-grain shrinkage of the wood member. These holes should not be confused with over-sized or slotted holes sometimes used in steel construction for slip-critical connections. As such, washers are not required between the bolt head or nut and the steel plate when slotted holes are used in timber connections, unless specified for aesthetic purposes.

11.3.1.2 Lag Screws Lag screws are sometimes referred to as "lag bolts" because their size more closely approximates that of bolts than that of wood screws. Sizes range from $\frac{1}{4}$ in. diameter to $1\frac{1}{4}$ in. diameter and with lengths up to 12 in. For special purposes such as reinforcing pitched and tapered curved beams for radial tension, specially made lag screws may be 4 ft or even longer. Lag screws for timber construction should conform to ANSI/ASME Standard B18.2.1 [12].

Lag screws, except as noted below, must be inserted in prebored lead holes. The lead holes for the shank must be the same diameter as the shank and extend the same depth as the depth of penetration of the unthreaded shank. Dimensions of standard lag screws including shank length, threaded portion, and tapered tip are included in the appendix. The lead hole for the threaded portion should have a diameter equal to 65% to 85% of the shank diameter in wood with $G > 0.60$, 60% to 75% in wood with $0.50 < G \leq 0.60$, and 40% to 70% for wood with $G \leq 0.50$. The larger percentages in each range should apply to the larger diameter lag screws. Lead holes are not required for $\frac{3}{8}$ in. diameter and smaller lag screws used in wood with a $G \leq 0.50$ provided end distance, edge distance, and spacing are such that unusual splitting does not occur.

Lag screws must be installed in the lead hole by turning, not by driving, for example, with a hammer. Soap or other lubricant should be used to facilitate installation, particularly of long lag screws. Washers of proper size or a metal plate or strap should be installed between the wood and the bolt head. Washer or plate or strap area must be of sufficient size to accommodate the axial load of the fastener without damage to wood or metal parts.

11.3.1.3 Wood Screws Wood screws discussed herein are those conforming to ANSI/ASME Standard B18.6.1 [13]. The capacity of wood screws is calculated based on the root diameter D_r of the screw. Wood screws are not permitted to be used to resist withdrawal from end grain. Wood screw sizes and dimensions are given in the appendix. Wood screws are typically available in various lengths, of which approximately two-thirds of the screw length is threaded. Dowel bending yield strength values for wood screws are also provided in the appendix.

For wood screws loaded laterally in wood members with $G > 0.60$, the part of the lead hole receiving the shank should have approximately the same diameter as the shank, and the part receiving the threaded portion should have approximately the same diameter as the diameter at the root of the thread. For members with $G \leq 0.60$, the part of the lead hole receiving the shank should be approximately seven-eighths the shank diameter, and the portion receiving the threads should be about seven-eighths the diameter of the screw at the root of the thread. Root diameters for wood screws are provided in the appendix.

For wood screws loaded in withdrawal, wood members with $G > 0.60$ should have a lead hole of approximately 90% of the wood screw root diameter and for wood members with $0.50 < G \leq 0.60$ the lead hole should be approximately 70% of the wood screw root diameter. For wood members with $G \leq 0.5$, no lead hole is necessary. Edge distance, end distance, and spacing of wood screws must be sufficient to prevent splitting of the wood member.

11.3.1.4 Nails and Spikes Sizes of nails and spikes commonly used in timber construction are shown in the appendix. Because of the variety of nails and spikes available, it is necessary in all engineered construction that the diameter and length of all nail fasteners be clearly specified in the construction documents. Dowel bending yield strengths for nails and spikes may be found in the appendix.

Prebored holes may be used to avoid splitting of the wood. Where prebored holes are used, the holes may not be larger than 90% of the nail or spike diameter for species with a specific gravity greater than 0.6 nor larger than 75% of the nail or spike diameter for species with a specific gravity less than or equal to 0.6.

Nails or spikes should not be loaded in withdrawal in general and are *not permitted* to be loaded in withdrawal from end grain. Where withdrawal from side grain must be considered, the reference withdrawal load in pounds per inch of penetration may be calculated by equations presented in Chapter 13 or may be obtained in tabular form from the *National Design Specification*® [2]. Clinched wire nails provide greater resistance to withdrawal than unclinched nails. However, since proper clinching of nails in the field may be difficult to control and inspect, using increased design values considering clinching is not recommended.

For single shear applications, nails and spikes must penetrate the main member at least six fastener diameters. For double shear applications, the fastener must penetrate the opposite side member at least six fastener diameters except in the case of symmetric double shear connections with side members at least $\frac{3}{8}$ in., where 12d or smaller nails extend at least three diameters from the opposite side member and are clinched.

11.3.1.4.1 Toenailed Fasteners Toenailed fasteners should be driven at an angle of approximately 30° with the face of the side member as shown in Figure 11.3.1.4.1-1 and must start approximately one-third the length of the fastener from the edge. Care must be taken to avoid splitting the side member for closely spaced toe-nailed fasteners. Some building codes may not allow toenailed fasteners to be loaded in withdrawal for some load or load combinations.

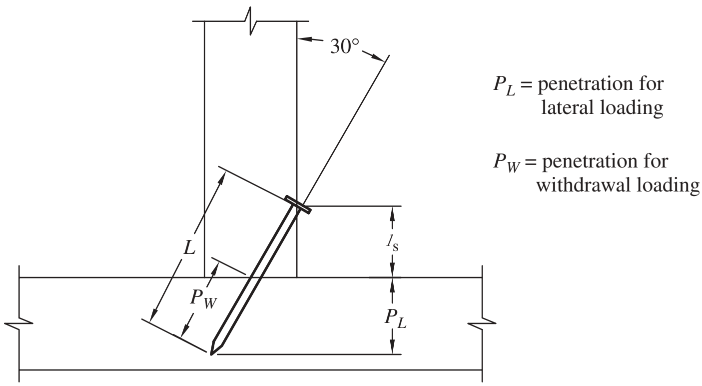


Figure 11.3.1.4.1-1 Common toenail connection.

For lateral loading of toenailed fasteners, the side member thickness and main member dowel or penetration lengths are taken to be the projection of the fastener lengths perpendicular to the face of the main member. For withdrawal loading, the actual penetration length in the main member is used.

11.3.1.5 Drift Pins Drift pins are long, unthreaded bolts or pins that are occasionally used to fasten large timbers. Drift pins are driven into lead holes that are smaller than the pins themselves and held in place through friction. For lateral loading, lead holes for drift pins should be drilled $\frac{1}{32}$ in. smaller than the actual pin diameter for lateral loading. However, if withdrawal loading is required, the lead holes must be drilled $\frac{1}{8}$ in. smaller than the actual pin diameter. Drift pins should not be used in unseasoned wood or in applications where significant moisture cycling may be expected.

Design values for drift pins have not been studied as extensively as those for other fasteners and are not usually included in codes. Design values for drift pins in lateral load should not be taken to be greater than 75% of the design values for bolts of the same diameter and same main member bearing length. The same considerations and requirements for bolts may be applied to drift pins, with the main differences being that drift pins generally do not fully penetrate the receiving member and may also be loaded in withdrawal.

11.3.2 Shear Plates and Split Rings

Split ring and shear plate connectors used in timber construction are illustrated in Figure 11.3.2-1. Connectors discussed in this manual have dimensions and specifications shown in the appendix and are made from SAE 1010 hot-rolled carbon steel (split rings and $2\frac{5}{8}$ in. shear plates) or Grade 32510 ASTM A47 [14] malleable iron (4 in. shear plates).

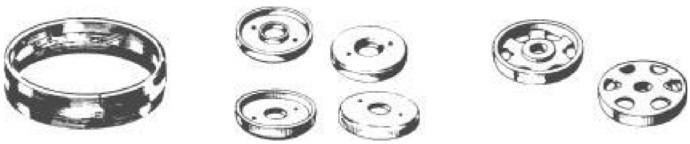


Figure 11.3.2-1 Shear plates and split rings. Reprinted with permission from *National Design Specification® for Wood Construction* [2]. Copyright © 2012. Courtesy American Wood Council, Leesburg, Virginia.

Connected members using split ring and shear plate connectors must be brought into snug contact with one another, and washers must be used between wood members and bolt heads. Bolts or lag screws are used to hold the connected members together.

Where wood members are not seasoned to the in-service moisture content conditions, the connector assembly must be periodically tightened until equilibrium is reached. Split ring and shear plate connectors are not intended to resist loads out of plane with the fastener or connected wood surfaces.

Design values for split ring and shear plate connectors are for a single split ring or set of shear plates. Where more than one connector is used in a connection, the group action factor must be applied and row and group tear-out also considered. Where the end of a connected member is not square, the end distance, measured parallel to the member axis, may be taken from a point $D/4$ from the fastener center as shown in Figure 11.3.2-2. The minimum edge distance must also be maintained at the non-square end. Spacing of multiple connectors is generally defined as the center to center spacing of connectors without respect to orientation of grain or member axis (Figure 11.3.2-2).

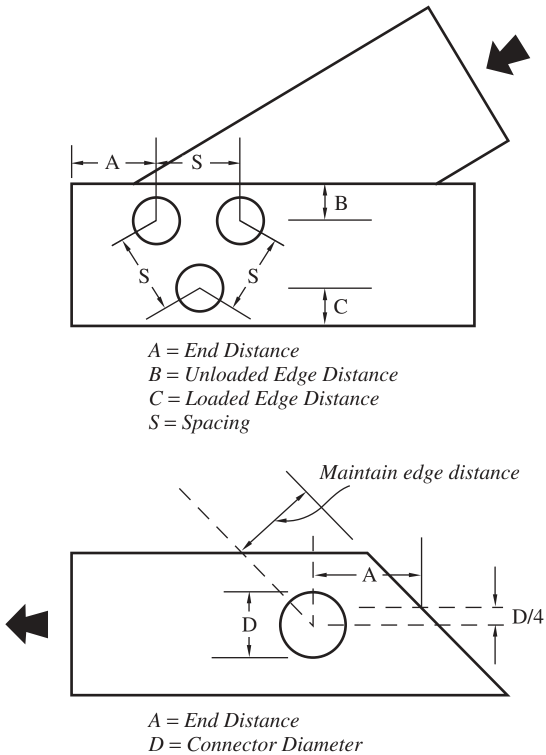


Figure 11.3.2-2 Edge, end, and spacing distances for split-ring and shear plate connectors. Reprinted with permission from *National Design Specification® for Wood Construction* [2]. Copyright © 2012. Courtesy American Wood Council, Leesburg, Virginia.

11.3.3 Other Fasteners

Other fasteners that are used in timber construction include timber rivets, staples, light metal framing devices, and metal truss plate connectors. A brief overview of these follows.

11.3.3.1 Timber Rivets Timber rivets, also called glulam rivets, are special hardened-steel nails with a shape that is nearly rectangular in cross section. They are used in conjunction with steel side plates to transfer loads. They are installed with the long cross-sectional axis along the grain of the wood, regardless of the plate orientation or load orientation.

Typical timber rivets and side plates for timber rivet connections are shown in the appendix. Timber rivets are made from AISI 1035 [15] steel with Rockwell C32-39 [16] hardness and minimum ultimate tensile stress of 145,000 psi according to ASTM A370 [17]. Side plates are ASTM Standard A36 [18] steel with thickness ranging from $\frac{1}{8}$ in. to $\frac{1}{4}$ in.

Design values for timber rivets are determined as the lesser of the nominal rivet capacity per rivet multiplied by the total number of rivets or the nominal wood capacity based on the number of rows of rivets and the number of rivets in each row. The nominal wood capacity values take into consideration group action and row and group tear-out; therefore, the group action factor and row and group tear-out need not be independently considered for timber rivet connections. Rivet and wood capacities are dependent on load orientation with respect to grain. The *National Design Specification®* [2] should be consulted for timber rivet design.

11.3.3.2 Staples Because staples and nails are similar in nature, the loads for staples may be determined in a manner similar to that for nails. The design value for one staple of a given diameter equals twice the value for a nail of equal diameter, provided that the staple leg spacing (or crown width) is adequate, and that the penetration of both legs of the staple into the member receiving the points is approximately two-thirds of their length. In general, nail penetration requirements and other provisions regarding seasoning of members, service conditions, and so on, apply equally to staples.

11.3.3.3 Light Metal Framing Devices A great many framing anchors and other fastening devices are commercially available for the wood framing industry. These fasteners are generally made of metal with predrilled holes for special fasteners. Allowable loads for such fasteners are provided by the manufacturer and are generally already adjusted for the common load duration conditions of the expected end use. Such fasteners have become increasingly popular in resisting wind and seismic loads where typical construction nailing is insufficient.

Fasteners should be selected that meet appropriate code requirements. Installation of such fasteners must be in accordance with the manufacturer requirements. Such fasteners should not be modified or used in ways other than as specified by the manufacturer, as modification or such alternate uses will generally void warranty and liability of the manufacturer for such products.

11.3.3.4 Metal Truss Plate Connections The fabrication and design of light gauge metal plates commonly used to connect dimensional lumber in trusses for residential and light commercial structures are typically proprietary and are included in preengineered and premanufactured truss systems. Although connections for such trusses may be designed in accordance with the procedures of this manual, preengineered/premanufactured systems are typically more economical.

11.3.4 Wood Joinery

A renewed interest in traditional timber framing in recent years is requiring designers in the timber engineering field to provide rational approaches to designing or determining the capacity of all wood joinery. Design provisions for wood construction are in many cases restrictive or not applicable to such joinery (mortise and tenon joints, dovetail joints, spline connections, wood-peg dowels, and the like). Such joinery often involves large losses of section due to mortises, notching, and housings. Connection design may be further complicated by the relatively large dimension changes involved where unseasoned wood is used.

Whereas bearing-only type joints may be designed with the methods and provisions of this manual, other joints involving mortises, tenons, wood pegs, or large slots for splines, in general, may not. For such applications, designers often employ various metal fasteners (hidden, if necessary) to resist the loads and for providing the anchorage requirements of modern building codes. Lateral loads, for example, in many of the newer timber frame structures are resisted by means other than the timber frames themselves—for example, by structural panels or structural insulated panels.

11.4 REFERENCE DESIGN VALUES FOR FASTENERS

Reference design values for a single fastener can be calculated, or they can be obtained from published tables in the *National Design Specification® for Wood Construction* [2]. The reference design values are then adjusted for the specific conditions of service and the connection configuration to give the adjusted design value for individual fasteners in the connection.

For fasteners such as bolts, lag screws, split ring and shear plate connectors, and timber rivets, the angle of load to grain must be considered; in others, such as with nails and wood screws, fastener design values are independent of direction of load with respect to grain. Reference design values for loads parallel-to-grain are generally denoted P or $Z_{\perp}$. Reference design values for loads perpendicular-to-grain are generally denoted by Q or $Z_{\perp}$. Design values associated with loading the fastener in withdrawal are denoted W. Adjusted design values are denoted P', Q', Z', and W'.

Reference design values are typically dependent on the specific gravity of the connected members and on the angle of load with respect to the grain. These common issues are discussed in this section. Provisions specific to each fastener type will be considered in subsequent chapters.

11.4.1 Specific Gravity

Timber fastener design values are dependent on the specific gravity of the wood members being connected. Table 2.2-1 includes species groups of wood used in timber construction and their accepted specific gravity values. Where the wood members in a connection are of different species and specific gravity, the lesser of the specific gravity values is generally used in design.

11.4.2 Angle of Load to Grain

For dowel-type fasteners smaller than $\frac{1}{4}$ in. diameter (nails and wood screws), the design capacities are considered to be independent of load direction with respect to grain. However, the effect of large numbers of such fasteners causing stresses perpendicular-to-grain near a loaded edge should be avoided.

For other fasteners, including dowel-type fasteners with diameters equal to and greater than $\frac{1}{4}$ in. (bolts, lag screws, etc.), shear plate and split ring connectors, and timber rivets, the capacities of the fasteners loaded laterally are directly related to angle of load with respect to the grain. The *National Design Specification®* [2] publishes tabulated nominal design capacities for these connectors loaded both parallel-to-grain and perpendicular-to-grain. These fasteners in general have greater capacity loaded parallel-to-grain than perpendicular-to-grain.

The angle of load with respect to grain, θ, may be different for the various members being connected by the same fastener. For example, in Figure 11.4.2-1, the angle of load with respect to grain for the side members is 0° (parallel-to-grain), whereas the angle of load to grain with respect to the main member is 90° (perpendicular-to-grain), or θ.

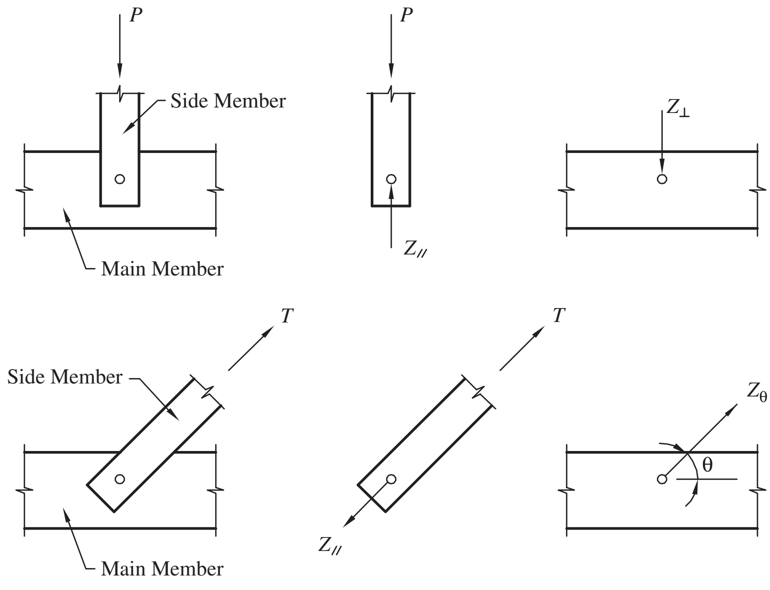


Figure 11.4.2-1 Fastener loaded at an angle to the grain.

11.4.2.1 Bolts and Lag Screws Reference fastener design values for bolts and lag screws are tabulated in the *National Design Specification®* [2] for the cases of both side and main members loaded parallel-to-grain, both loaded perpendicular-to-grain, and one member loaded perpendicular-to-grain and the other parallel. Reference values can also be calculated. In cases where the angle of load to grain in either member is at an angle other than 0° or 90°, the design value must be calculated taking into consideration the dowel bearing strengths in the direction of load of each member by utilizing the bearing strengths parallel- and perpendicular-to-grain and the Hankinson formula (Equation 11.4.2.1-1) .

$$F_{e\theta} = \frac{F_{e\parallel} F_{e\perp}}{F_{e\parallel} \sin^2 \theta + F_{e\perp} \cos^2 \theta} \qquad (11.4.2.1\text{-}1)$$

where:

$F_{e\theta}$ = dowel bearing strength at angle θ to grain
$F_{e\parallel}$ = dowel bearing strength parallel-to-grain
$F_{e\perp}$ = dowel bearing strength perpendicular-to-grain

11.4.2.2 Shear Plates and Split Rings The design values for split rings and shear plates provided in the *National Design Specification*® [2] are for loads parallel- and perpendicular-to- grain with fasteners installed into side grain. For load at angles to grain other than parallel or perpendicular, the reference design values for parallel and perpendicular loading (P and Q, respectively) are adjusted by all applicable factors to obtain adjusted design values P' and Q', and then used in Equation 11.4.2.2-1 (the Hankinson formula) to obtain the allowable load at angle to grain, N'.

$$N' = \frac{P'Q'}{P' \sin^2 \theta + Q' \cos^2 \theta} \qquad (11.4.2.2\text{-}1)$$

where:

N' = adjusted design value for load at angle θ to grain
P' = adjusted design value for load parallel-to-grain
Q' = adjusted design value for load perpendicular-to-grain
θ = angle of load to grain

11.5 ADJUSTMENT FACTORS

Reference design values are generally for normal load duration for a single fastener in dry wood, fabricated with the necessary edge distance, end distance, and spacing requirements to provide full design value. Where other conditions exist, reference design values must be multiplied by appropriate adjustment factors. Where more than one fastener is used, group action must be considered and is treated as an adjustment factor for the design value for the single fastener. The capacity of the fasteners is then taken to be the single fastener capacity multiplied by the number of fasteners. The capacity of the fasteners, however, cannot be taken to exceed the capacities of the other components or parts of the connection.

The applicability of adjustment factors with various fastener types is shown in Table 11.5-1. A general description of the adjustment factors for fasteners follows. The adjustment factors for the fasteners are applied generally in the same manner as the adjustment factors for strength properties of wood in that the allowable value based on the fastener is equal to the reference design value multiplied by the applicable adjustment factors.

TABLE 11.5-1 Applicability of Adjustment Factors for Fasteners

Fastener Type	Design Values and Adjustment Factors
Bolts	$Z' = ZC_D C_M C_t C_g C_\Delta$
Lag screws	$Z' = ZC_D C_M C_t C_g C_\Delta C_{eg}$ $W' = WC_D C_M C_t C_{eg}$
Drift pins	$Z' = ZC_D C_M C_t C_g C_\Delta C_{eg}$ $W' = WC_D C_M C_t$
Nails, spikes, and wood screws	$Z' = ZC_D C_M C_t C_g C_\Delta C_{eg} C_{di} C_{tn}$ $W' = WC_D C_M C_t C_{eg} C_{tn}$
Shear plates and split rings	$P' = PC_D C_M C_t C_g C_\Delta C_d C_{st}$ $Q' = QC_D C_M C_t C_g C_\Delta C_d$ $Q'_{90} = Q_{90} C_D C_M C_t C_g C_\Delta C_d$
Timber rivets	$P' = PC_D C_M C_t C_{st}$ $Q' = QC_D C_M C_t C_\Delta C_{st}$

11.5.1 Load Duration Factor, C_D

The load duration factors applicable to the strength properties of wood (see Table 3.4.1-1) are generally applicable to fasteners, with three exceptions:

1. The load duration factor for fasteners cannot exceed 1.6.
2. The load duration factor is not applicable to bearing stresses perpendicular-to-grain.
3. The load duration factor may not be applied where it will produce greater fastener capacity than is determined for the nonwood parts of the connection.

11.5.2 Wet-Service Factor, C_M

The nominal design values for connections in wood are based on wood that is seasoned to a moisture content of 19% or less and used under continuously dry conditions. When the wood is unseasoned or partially seasoned at the time of fastener fabrication, or when the connection is exposed to wet service conditions, the nominal design value must be multiplied by the wet-service factors in Table 11.5.2-1. These factors apply to both sawn lumber and glued laminated timber. Sawn lumber may be manufactured in the dry, partially seasoned, or wet condition and may be in service in dry, wet, partially seasoned, or exposed to

TABLE 11.5.2-1 Wet Service Factors, C_M, for Connections (from *NDS*® [2])

Fastener Type	Moisture Content: At Time of Fabrication	Moisture Content: In-service	C_M
Lateral Loads			
Dowel-type fasteners	≤19%	≤19%	1.0
	>19%	≤19%	0.4[a]
	any	>19%	0.7
Shear plates and split rings	≤19%	≤19%	1.0
	>19%	≤19%	0.8
	any	>19%	0.7
Timber rivets	≤19%	≤19%	1.0
	≤19%	>19%	0.8
Withdrawal Loads			
Lag screws and wood screws	any	≤19%	1.0
	any	>19%	0.7
Nails and spikes	≤19%	≤19%	1.0
	>19%	≤19%	0.25
	≤19%	>19%	0.25
	>19%	>19%	1.0
Threaded hardened nails	any	any	1.0

[a] $C_M = 0.7$ for dowel-type fasteners with diameter, D, less than $\frac{1}{4}$ in. $C_M = 1.0$ for dowel-type fastener connections with (1) one fastener only, (2) two or more fasteners placed in a single row parallel-to-grain, or (3) fasteners placed in two or more rows parallel-to-grain with separate splice plates for each row.

weather conditions. Glued laminated timber is manufactured dry and may be used under dry, wet, or exposed to weather conditions.

11.5.3 Temperature Factor, C_t

In cases where the wood controls the connection capacity and the fasteners are subject to sustained elevated temperatures, the temperature factor of Table 11.5.3-1 must be applied.

TABLE 11.5.3-1 Temperature Factors, C_t, for Connections (from *NDS*® [2])

In-Service Moisture Conditions	C_t: $T \le 100°$F	C_t: $100\,°F < T \le 125\,°F$	C_t: $125\,°F < T \le 150\,°F$
Dry	1.0	0.8	0.7
Wet	1.0	0.7	0.5

11.5.4 Group Action Factor, C_g

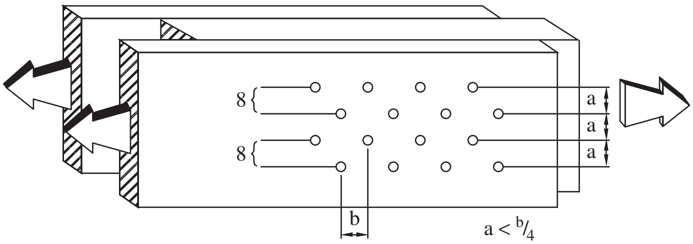
Research has shown that the load carried by a row of fasteners such as bolts, lag screws, drift pins, split rings, and shear plates is not equally divided among the fasteners. End fasteners in a row tend to carry a larger portion of the load than the intermediate fasteners. The distribution of load among fasteners is dependent on the stiffness of the fasteners, the relative stiffness of the main member and the side member(s), and the fastener spacing. The unequal load sharing is accounted for by the group action factor, C_g.

With regard to determining the group action factor, C_g, five conditions apply:

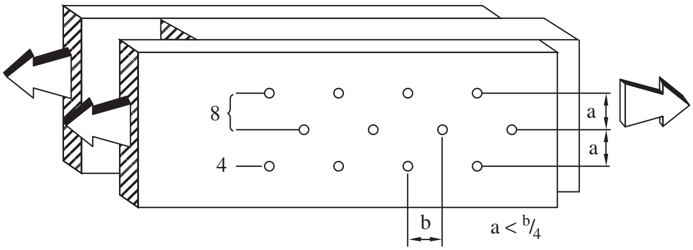
1. A group of fasteners consists of one or more rows of fasteners.
2. A row of fasteners consists of either two or more bolts loaded in single or multiple shear or two or more split rings, shear plates, or lag screws loaded in single shear. The row is aligned with the direction of the load.
3. Where fasteners in adjacent rows are staggered, and the distance between the rows is less than one-fourth of the spacing between the closest fasteners in an adjacent row (as shown in Figure 11.5.4-1), the fasteners in adjacent rows should be considered as one row.
4. The load for each row of fasteners is determined by summing the individual loads for each fastener in the row and then multiplying this value by the adjustment factor C_g. For convenience, C_g may be applied to individual fastener values prior to summation of values. The design value for the group of fasteners is the sum of the design values of the rows in the group.
5. Where a member is loaded perpendicular-to-grain, its equivalent cross-sectional area is the product of the thickness of the member and the overall width of the fastener group for calculating cross-sectional area ratios. When only one row of fasteners is used, the width is equal to the minimum spacing for full load for the type of fastening used. In general, long rows of fasteners perpendicular-to-grain should be avoided.

The group action factor is taken to be unity ($C_g = 1.0$) for dowel-type fasteners with diameters less than $\frac{1}{4}$ in. (nails and wood screws), as these fasteners tend to deform and more uniformly accept load than the larger, stiffer fasteners. Design values for timber rivets already account for group action. The group action factor for dowel-type fasteners with $\frac{1}{4}$ in. or larger diameter and for split ring and shear plate connectors may be calculated by Equation 11.5.4-1. Tabulated values for specific fastener size, type, and spacing are provided in the *National Design Specification*® [2]. The tabulated values will be conservative for most cases; however, the equation is more versatile and is recommended.

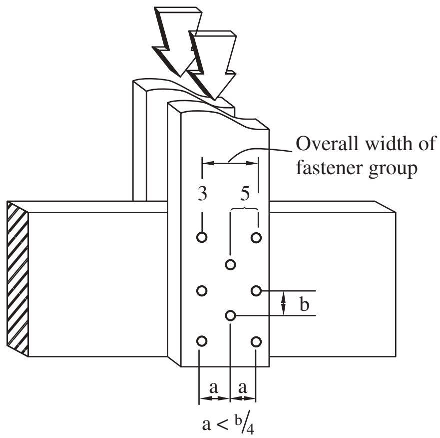
$$C_g = \left[\frac{m(1-m^{2n})}{n[(1+R_{\text{EA}}m^n)(1+m)-1+m^{2n}]}\right]\left[\frac{1+R_{\text{EA}}}{1-m}\right] \qquad (11.5.4\text{-}1)$$



Consider as 2 rows of 8 fasteners



Consider as 1 row of 8 fasteners and 1 row of 4 fasteners



Consider as 1 row of 5 fasteners and 1 row of 3 fasteners

Figure 11.5.4-1 Group action for staggered fasteners. Reprinted with permission from *National Design Specification® for Wood Construction* [2]. Copyright © 2012. Courtesy American Wood Council, Leesburg, Virginia.

where:

n = number of fasteners in a row

R_{EA} = the lesser of $\dfrac{E_s A_s}{E_m A_m}$ or $\dfrac{E_m A_m}{E_s A_s}$

E_m = modulus of elasticity of main member (psi)

E_s = modulus of elasticity of side members (psi)
A_m = gross cross-sectional area of main member (in^2)
A_s = sum of gross cross-sectional areas of side members (in^2)

$$m = u - \sqrt{u^2 - 1} \tag{11.5.4-2}$$

$$u = 1 + \gamma \frac{s}{2}\left[\frac{1}{E_m A_m} + \frac{1}{E_s A_s}\right] \tag{11.5.4-3}$$

s = center-to-center spacing between adjacent fasteners in a row (in.)
γ = load/slip modulus for a connection (lb/in.)
γ = 500,000 lb/in. for 4 in. split ring or shear plate connectors
γ = 400,000 lb/in. for $2\frac{1}{2}$ in. split ring or $2\frac{5}{8}$ in. shear plate connectors
γ = (180,000)($D^{1.5}$) for bolts or lag screws in wood-to-wood connections
γ = (270,000)($D^{1.5}$) for bolts or lag screws in wood-to-metal connections
D = diameter of bolt or lag screw (in.)

The NDS® group action factor will result in larger factors for large members with multiple-rows of fasteners than for a smaller member with the same number of fasteners in a row. The equation for u only considers one row of fasteners. For the case of multiple rows of fasteners, the load/slip modulus will increase proportionately to the number of rows of fasteners. It is hereby recommended that γ in the above equation be replaced with $n_j\gamma$, where n_j is the number of rows of fasteners. Furthermore, the term γ represents the load/slip modulus of connections tested in double shear. For single shear connections, it is appropriate to divide this value by 2.

11.5.4.1 Group Action Factor for Fasteners Loaded Perpendicular-to-Grain For members loaded in the perpendicular-to-grain direction, the modulus of elasticity and the load/slip modulus perpendicular-to-grain are not provided. The NDS Commentary [5] states that "it is standard practice to use the same group action factor as for fasteners aligned parallel-to-grain. This practice is based on the assumption that the member and connection stiffnesses perpendicular-to-grain ($E_\perp$ and $\gamma_\perp$) in NDS Equation 10.3-1 would result in similar group action factors."

In other words, it is assumed that each of the two moduli would be proportionately reduced from their corresponding parallel-to-grain values, so the net effect would be the same as using the parallel-to-grain values in the calculations. However, the paper *Design Equation for Multiple-Fastener Wood Connections* [19], on which the current NDS provisions are based, recommends that $E_\perp = E/20$ and that $\gamma_\perp = \gamma/2$. Because these values are not proportionately reduced from their corresponding parallel-to-grain values, the NDS Commentary recommendation results in a less conservative group action factor than Zahn's recommendation. Additionally, the NDS Commentary recommendation assumes that the two members are oriented in the same direction (i.e., both either parallel or perpendicular to the load). This assumption may result in significant error when connected members are loaded at different angles to the grain.

For load at an angle to grain other than 0° or 90°, the Hankinson formula can be used to calculate values of the modulus of elasticity, E_θ, and load-slip modulus, γ_θ. These values can then be used to calculate a group action factor for the connection.

11.5.4.2 Group Action Factor for Fasteners in End Grain For fasteners in end-grain surfaces such as shear plates at the peak connection of a three-hinged arch, the NDS does not give guidance on determination of the appropriate dimensions to use to calculate a member "cross-sectional area." Without further guidance from NDS, it is recommended that the effective cross section for the connectors in end grain be taken as equal to the width of the member multiplied by its depth, as illustrated in Figure 11.5.4.2-1.

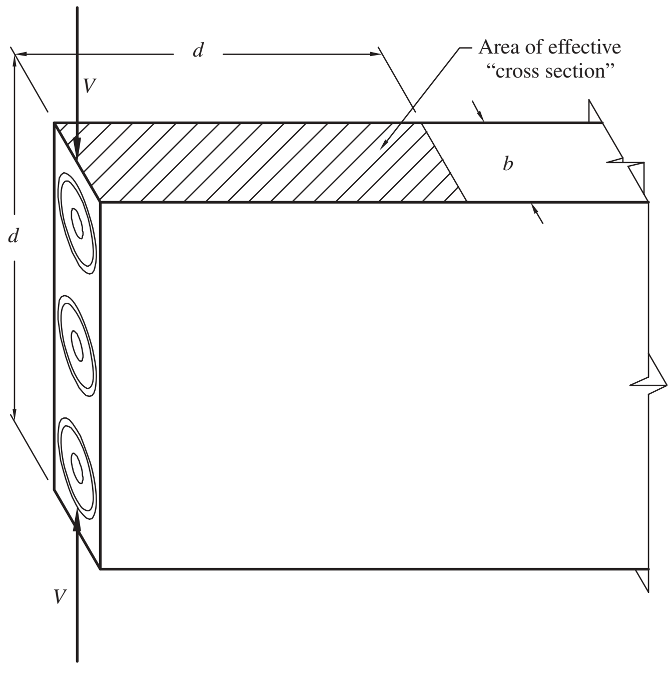


Figure 11.5.4.2-1 Effective area for group action factor calculation for connectors in end grain.

11.5.5 Geometry Factor, C_Δ

The reference design values for bolts, split rings, shear plates, lag screws, and drift pins are based on specific end distance, edge distance, and spacing requirements. In addition, minimum values are specified, below which no capacity is permitted. Edge, end, and spacing requirements for dowel-type fasteners are generally specified in terms of fastener diameter, *D,* and are measured from the fastener centerline. For shear plates and split rings, actual dimensions are stated, and are measured from the center of the fastener. Where the provided spacing, edge distance, or end distance is less than the required values for full design values, but not less than the minimum values, the design values for the fasteners are reduced by the geometry factor C_Δ.

The geometry factor is applicable to dowel-type fasteners with diameters equal to or greater than $\frac{1}{4}$ in., to split ring and shear plate connectors, and to timber rivet connections where the rivet design value is controlled by the capacity of the wood perpendicular-to-grain. It is not applicable to nails and wood screws. However, installation of nails and wood screws must be such that splitting of the wood members does not occur. Certain specific edge, end, and spacing requirements for nails and screws may be applicable, however, in the construction of structural diaphragms and as required by the manufacturer of proprietary engineered wood products.

Determination of a geometry factor generally requires interpolation between the value for $C_\Delta = 1.0$ and $C_{\Delta,\min}$, based on spacing, end distance, or edge distance. Details of the geometry factor will be discussed in the chapters for specific fasteners.

11.5.6 Penetration Depth Factor, C_d

The effect of penetration depth of dowel fasteners is taken into direct account with the yield mode equations of Chapter 13. However, tabular reference design values in the *National Design Specification*® [2] must be adjusted for penetration depth where fastener penetration is less than the values used to generate the tables, but greater than the minimum fastener penetration.

For split ring and shear plate connectors, a penetration depth factor, C_d, is applied where lag screws are used instead of bolts and the lag screw penetration is less than the penetration required for full design value but greater than or equal to the minimum penetration permitted. These factors are shown in Table 11.5.6-1.

TABLE 11.5.6-1 Penetration Depth Factors for Split Rings and Shear Plates Installed with Lag Screws (from *NDS*® [2]).

Connector	Side Member	Penetration Depth Factor, C_d	Penetration of Lag Screw into Main Member			
			Species Group A	Species Group B	Species Group C	Species Group D
4″ Split Ring	Wood	1.0	7D	8D	10D	11D
$2\frac{1}{2}$″ Split Ring		0.75	3D	3.5D	4D	4.5D
4″ Shear Plate	Wood or Metal	1.0	7D	8D	10D	11D
		0.75	3D	3.5D	4D	4.5D
$2\frac{5}{8}$″ Shear Plate	Wood	1.0	4D	5D	7D	8D
		0.75	3D	3.5D	4D	4.5D
	Metal	1.0	3D	3.5D	4D	4.5D

11.5.7 End Grain Factor, C_{eg}

The nominal design values for fasteners are based on the fasteners being installed in side grain. Dowel-type fasteners installed in end grain (fastener axis parallel-to-grain) require special consideration. The dowel-bearing strength for lateral loading of dowel type fasteners is taken to be the dowel-bearing strength perpendicular-to-grain. In addition, the end grain factor, $C_{eg} = 0.67$, is applied to the lateral design values.

With the exception of lag screws, dowel-type fasteners must not be loaded in withdrawal when installed in end grain. Wood screws, nails, and spikes must not be relied on for anchorage or resisting transient, incidental, or construction loads or movements where such would tend to load the fasteners in withdrawal from end grain. Such loads or movements must be resisted by other means.

Where lag screws are installed in end grain and loaded in withdrawal, the withdrawal design values are multiplied by $C_{eg} = 0.75$. Where lag screws are installed in end grain and are loaded both laterally and in withdrawal, *both* end grain factors apply.

Split rings, shear plates, and timber rivets may also be installed in end grain. Although the end grain factor, C_{eg}, is not explicitly applied to these fastener types, reference design values for these fasteners are reduced for end-grain installations. Withdrawal loading from end grain for these types of fasteners is not permitted. Considerations for split rings and shear plates installed in end grain are discussed in detail in Chapter 14.

11.5.8 Metal Side Plate Factor, C_{st}

The design values of 4 in. shear plates loaded parallel-to-grain and timber rivets, where capacity is controlled by the rivet, are adjusted by a metal side plate factor, C_{st}. These factors are discussed in greater detail in Chapter 14 for shear plates and in the *NDS*® [2] for timber rivets.

11.5.9 Diaphragm Factor, C_{di}

Where nails are used in a structural diaphragm, the lateral design values of the fasteners may be multiplied by the diaphragm factor, $C_{di} = 1.10$. This adjustment factor applies only to the loads related to the structural diaphragm and not for resistance of other forces the fasteners may also be dependent upon to resist.

11.5.10 Toenail Factor, C_{tn}

Where toenailed connections are used, the reference design values must be multiplied by the toenail factor, C_{tn}, which is equal to 0.83 for lateral loading and 0.67 for withdrawal loading. For toenailed nails and spikes loaded in withdrawal, the wet-service factor C_M need not be applied simultaneously with the toenail factor.

11.5.11 Effect of Treatment

No reduction in fastener design values is required for wood preservatively treated in accordance with AWPA standards [21]. However, fire-retardant treatments or other proprietary treatments may reduce the capacity of members. Where such treatments are applied, appropriate reduction values should be obtained from the company providing the treatment.

11.6 CONCLUSION

When designing and detailing connections in structural glued laminated timber members, a basic understanding of wood properties is essential. In particular, it is critical to understand wood's propensity for cross-grain shrinkage and its weakness to resist stresses in tension perpendicular-to-grain. Long-term or repeated exposure to moisture can cause decay that will greatly reduce the capacity of the member and connection.

Proper connection detailing will ensure that the connection will transfer the required loads satisfactorily over the life of the structure. AITC 104 [1] provides examples of both good and poor detailing. The following guidelines will help the designer minimize problems:

- Transfer loads in bearing wherever possible.
- Avoid tension perpendicular-to-grain.
- Allow for timber shrinkage in detailing.
- Keep wood dry.
- Don't install wood in direct contact with masonry or concrete.
- Don't trap incidental moisture.
- Use preservative treatments where wood can't be kept dry.

A number of different fastener types are used for timber construction. This manual includes design procedures and examples for dowel-type fasteners, shear plates, and split rings. The format of connection design in timber structures is similar to the design of the members themselves in that reference design values are adjusted through the application of various factors to account for effects of factors such as moisture, load duration, and connection geometry.

CHAPTER 12

MEMBER CAPACITY AT CONNECTIONS

12.1 INTRODUCTION

Several different failure types are possible at connections, and each must be considered. Failures can occur in the fasteners themselves, at the interface between the fasteners and wood, or in the timber member at the location of the connection due to local stress concentrations. The capacity of the connection to resist each failure mode is determined independently, and the lowest capacity is assigned to the connection. This chapter focuses on determining the capacity of members to resist failure in various modes at connections, including bearing connections where fasteners aren't used to transfer the loads.

12.2 MEMBER CAPACITY AT CONNECTIONS LOADED PERPENDICULAR-TO-GRAIN

For connections where forces are transferred perpendicular to the grain of a member, the shear capacity of the member must be sufficient to support the shear load at the connection. The shear capacity at a connection is dependent on the type of connection and must be determined by the appropriate equation. In addition, members supported by bearing must have sufficient bearing area to prevent excessive deformation and wood crushing at the contacting surfaces.

12.2.1 Bearing Connection with Unaltered Cross-Section

For the case of a simple bearing connection with an unaltered cross section (no notches or tapers), the requirements for shear and compression perpendicular-to-grain are given by Equations 12.2.1-1 and 12.2.1-2.

$$V \leq \frac{2}{3} b d F'_v \qquad (12.2.1\text{-}1)$$

$$V_r \leq b \ell_b F'_{c\perp} \qquad (12.2.1\text{-}2)$$

where:

V = design shear force in member
V_r = reaction shear force at connection
b = width of member
d = depth of member
ℓ_b = bearing length
F'_v = adjusted shear design value
$F'_{c\perp}$ = adjusted compression perpendicular-to-grain design value

EXAMPLE 12.2.1-1 BEARING LENGTH

Given: A 6 × 10 hem fir No. 1 simply supported timber beam has a reaction force of 2000 lb. The full width of the beam will be supported. The beam will be in dry use with normal temperatures.

Wanted: Determine the required bearing distance.

Approach: The bearing distance will be determined using the published compression perpendicular-to-grain value for hem-fir No. 1 timbers.

Solution:

Design value:

$$F'_{c\perp} = F_{c\perp} C_M C_t C_b = (405\,\text{psi})\,(1.0)\,(1.0)\,(1.0) = 405\,\text{psi}$$

Required bearing length (from Equation 12.2.1-2):

$$V_r \leq b \ell_b F'_{c\perp} \quad \Rightarrow \quad \ell_b = \frac{V_r}{bF'_{c\perp}} \quad \Rightarrow \quad \ell_b = \frac{V_r}{bf_{c\perp}}$$

$$\ell_b = \frac{V_r}{bf_{c\perp}} = \frac{2000\,\text{lb}}{(5.5\,\text{in})\,(405\,\text{psi})} = 0.90\,\text{in}$$

Answer: The minimum required bearing length for the timber beam is 0.90 in.

Discussion: Though the minimum bearing length is calculated to be 0.90 in., building codes may prescribe a greater minimum distance (1 in. or more). The designer may also specify a greater bearing distance for ease and safety during construction, to accommodate minor framing length discrepancies in the field, or to accommodate shrinkage of supporting members.

12.2.2 Bearing Connection with Tension-Face Notch

The notching of a bending member on the tension face results in a decrease in strength caused by stress concentrations around the notch, as well as a reduction of the area resisting the shear forces. The notch induces tension perpendicular-to-grain stresses, which interact with the shear parallel-to-grain creating a splitting tendency. For these reasons, the notching of large glued laminated timbers is not recommended and should be limited in smaller wood members.

For sawn timber members, tension-face notches up to 25% of the cross section are permitted at end supports. For glulam members, tension-face notches are limited to 10% of the member depth with an absolute maximum of 3 inches permitted at end supports. Generally these notches can be avoided by using a slightly shorter column or lower support. Where such a notch is used, the reaction shear force is limited by Equation 12.2.2-1.

$$V_r \leq \left[\frac{2}{3}F_v' b d_e\right]\left[\frac{d_e}{d}\right]^2 \tag{12.2.2-1}$$

where:

d_e = the depth of the member, minus the depth of the notch

The designer should also consider reducing the stress concentration that occurs when a member is notched by using a gradual tapered notch configuration in lieu of a square-cornered notch (Figure 12.2.2-1). The designer may also need to consider the use of reinforcement such as full threaded lag screws to resist the tendency of the member to split at the notch. Notching on the tension side of beams away from the ends is not permitted. Although the length of the notch is not explicitly included in the analysis, the notch should be confined to the immediate vicinity of the end of the beam.

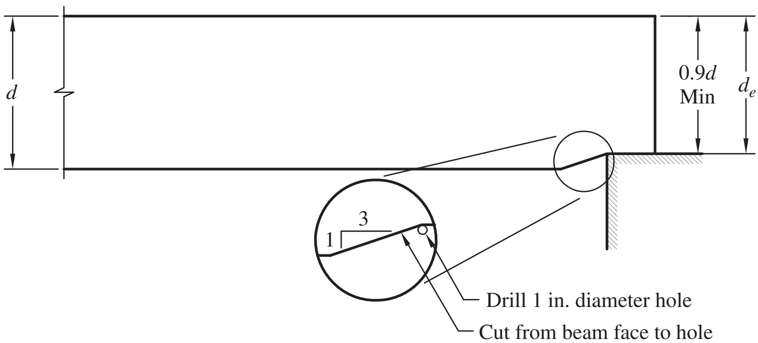


Figure 12.2.2-1 Beam with tension-face notch.

EXAMPLE 12.2.2-1 END NOTCHED MEMBER

Given: A $3\frac{1}{8}$ in. × 12 in. DF beam supports a shear load of 4500 lb at the end support. The load is caused by dead load plus snow load.

Wanted: Determine the suitability of a 1 in. notch on the tension face at one end of the beam

Approach: The depth of the notch will be checked against prescriptive limitations, and Equation 12.2.2-1 will be used to check the beam's shear capacity at the notch. For the analysis of shear stresses at the notch, the reference shear stress will be reduced by application of the shear reduction factor, $C_{vr} = 0.72$.

Solution:

Prescriptive notch limits:

$$d_n = 1.0\,\text{in} \leq 0.1d \leq 3\text{in}$$

$$d_n = 1.0\,\text{in} \leq 0.1\ (12\,\text{in}) \leq 3\,\text{in}$$

$$d_n = 1.0\,\text{in} \leq 1.2\,\text{in} \leq 3\,\text{in}$$

$$d_n = 1.0\,\text{in} \leq 1.2\,\text{in} \quad \therefore \text{OK}$$

Shear design values:

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr}$$

$$F'_{vx} = (265\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\,(0.72)$$

$$F'_{vx} = 219\,\text{psi}$$

Shear capacity at notch:

$$V' = \left[\frac{2}{3}F'_v b d_e\right]\left[\frac{d_e}{d}\right]^2$$

$$V' = \left[\frac{2}{3}\,(219\,\text{psi})\,(3.125\,\text{in})\,(11\,\text{in})\right]\left[\frac{11\,\text{in}}{12\,\text{in}}\right]^2$$

$$V' = 4220\,\text{lb} < V_r = 4500\,\text{lb} \therefore \quad \text{Not acceptable}$$

Answer: The $3\frac{1}{8}$ in. × 12 in. beam with a 1 in. notch on the tension face does not have adequate capacity to support the design load.

12.2.3 Bearing Connection with Compression-Face Notch

When a beam is notched or beveled on its upper (compression) side at the ends, a less severe condition from the standpoint of stress concentrations is realized. Consequently, larger notches may be permitted on the compression face at the end supports.

For sawn timber members, compression-face notches up to 25% of the cross section are permitted at end supports. For glulam members, compression-face notches up to 40% of the member depth or compression-face taper cuts up to $\frac{2}{3}$ of the member depth are allowed at the end bearings of bending members (Figure 12.2.3-1). When such a notch is used, the shear capacity is limited by Equations 12.2.3-1 and 12.2.3-2.

$$V_r \leq \frac{2}{3}F'_v b\left[d - \left(\frac{d-d_e}{d_e}\right)e\right] \quad \text{where } e \leq d_e \qquad (12.2.3\text{-}1)$$

$$V_r \leq \frac{2}{3}F'_v b d_e \quad \text{where } e > d_e \qquad (12.2.3\text{-}2)$$

where:

e = distance from the face of the support to the farthest edge of the notch
d_e = depth of the member at the face of the support

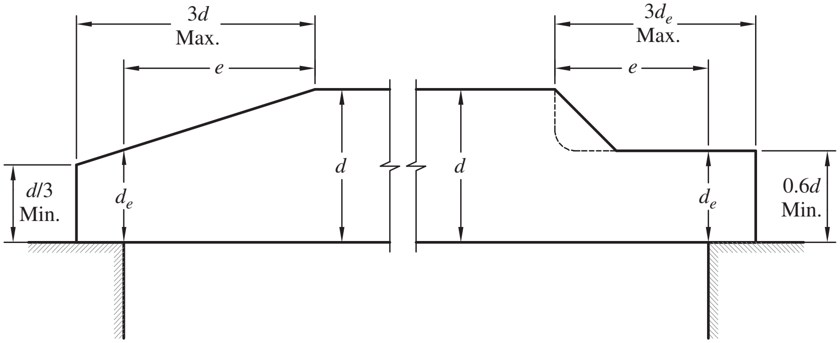


Figure 12.2.3-1 Compression-face notch limitations for glulam beams.

Compression-face notches subject to these provisions are limited to a maximum distance of $3d_e$ from the end of the beam. Taper cuts are permitted to extend to a maximum distance of $3d$ from the end of the beam without requiring special analysis of beam flexure. For taper cuts that extend farther from the support, a more extensive analysis is required following the provisions for tapered beams in Chapter 7. The reference shear design value should be reduced by the shear reduction factor, $C_{vr} = 0.72$, for the analysis of these members.

Generally, continuous span or cantilever span beams should not be notched over the interior support(s) or away from the supports. Under no circumstances should a member be notched on both top and bottom at a support.

12.2.4 Connections Using Mechanical Fasteners

The analysis of a beam supported at its end by fasteners is analogous to beams with a notch on the tension face, so the same equation is used. For loads more than five times the depth from the support, the member shear capacity at the connection is determined using the standard equation for shear capacity, but with a reduced effective depth. Both approaches reduce the design shear capacity of the section located at a connection. These reduced shear capacities must be considered for connections using any type of fasteners loaded perpendicular to the grain. Effective depths for various connection conditions are illustrated in Figure 12.2.4-1.

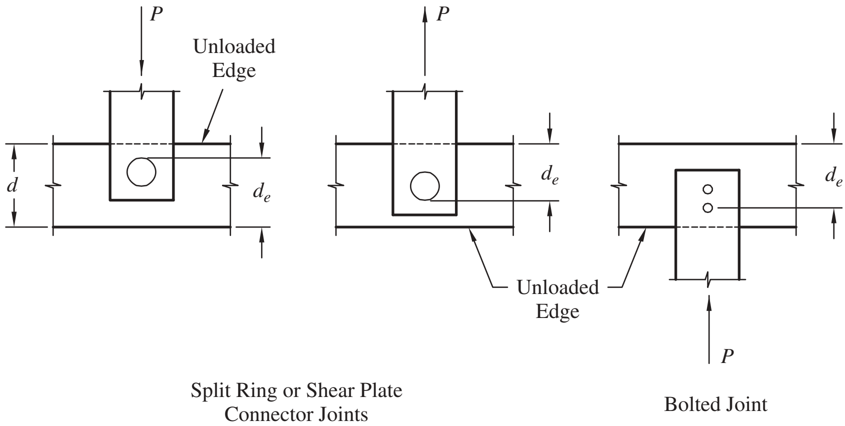


Figure 12.2.4-1 Effective depth for shear at connections.

In cases where the fasteners are located less than five times the depth of the member from the end of the member, the design shear force at the connection is limited by Equation 12.2.4-1. Where the connection is at least five times the depth from the end, the design shear force at the connection is limited by Equation 12.2.4-2.

$$V \le \left[\frac{2}{3}F'_v b d_e\right]\left[\frac{d_e}{d}\right]^2 \qquad (12.2.4\text{-}1)$$

$$V \le \frac{2}{3}F'_v b d_e \qquad (12.2.4\text{-}2)$$

EXAMPLE 12.2.4-1 EFFECTIVE DEPTH FOR SHEAR

<u>**Given:**</u> A 6 × 10 hem fir No. 1 timber beam ($F_v = 140$ psi) with applied load P at distance x from the left end as illustrated in Figure 12.2.4-2. The span of the beam is 15 ft, 6 in. taken center-to-center of the $5\frac{1}{2}$ in. supporting columns. Two values of x are considered: $x = 3$ ft and $x = 4$ ft. The beam will be in dry use and have loads of normal duration.

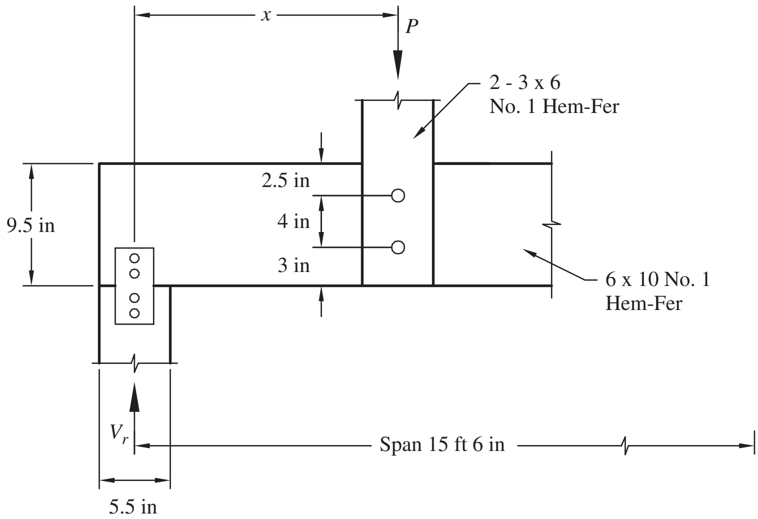


Figure 12.2.4-2 Joint detail—Example 12.2.4-1.

Wanted: Determine the maximum value of P based on shear at the bolted connection.

Approach: The effective depth will be used in conjunction with Equation 12.2.4-1 or 12.2.4-2, depending on the distance of the connection from the end.

Solution:

Shear design value:

$$F'_v = F_v C_D C_M C_t = 140\,\text{psi}\,(1.0) = 140\,\text{psi}$$

Effective depth:

$$d_e = d - 2.5\,\text{in}$$

$$d_e = 9.5\,\text{in} - 2.5\,\text{in}$$

$$d_e = 7.0\,\text{in}$$

Effective span:

The effective span may be taken to be not less than the distance between the inside faces of the supports plus half the required bearing distance at each support. For convenience, the span will be taken as the center-to-center column spacing, which assumes that half the required bearing distance is less than the distance from column centerline to inside face, which is often the case. Thus, $\ell = 15.5$ ft.

End reaction force and maximum load on connection:

$$V_r = \frac{P\,(\ell - x)}{\ell} \quad \Rightarrow \quad P = \frac{V_r \ell}{(\ell - x)}$$

Shear capacity for $x = 4$ ft:

$$\frac{x}{d} = \frac{(4\,\text{ft})\,(12\,\text{in/ft})}{9.5\,\text{ft}} = 5.05 > 5 \qquad \therefore \text{ use Equation 12.2.4-2}$$

$$V_r \leq V' = \frac{2}{3}F_v' b d_e = \frac{2}{3}\,(140\,\text{psi})\,(5.5\,\text{in})\,(7.0\,\text{in}) = 3590\,\text{lb}$$

Maximum load for $x = 4$ ft:

$$P = \frac{V_r \ell}{(\ell - x)} = \frac{(3590\,\text{lb})\,(15.5\,\text{ft})}{(15.5\,\text{ft} - 4\,\text{ft})} = 4840\,\text{lb}$$

Shear capacity for $x = 3$ ft:

$$\frac{x}{d} = \frac{(3\,\text{ft})\,(12\,\text{in/ft})}{9.5\,\text{ft}} = 3.79 \leq 5 \qquad \therefore \text{ use Equation 12.2.4-1}$$

$$V_r \leq V' = \left[\frac{2}{3}F_v' b d_e\right]\left[\frac{d_e}{d}\right]^2$$

$$V_r \leq V' = \frac{2}{3}(140\,\text{psi})(5.5\text{in})(7.0\,\text{in})\left[\frac{7.0\,\text{in}}{9.5\,\text{in}}\right]^2$$

$$V_r \leq V' = 1950\,\text{lb}$$

Maximum load for $x = 3$ ft:

$$P = \frac{V_r \ell}{(\ell - x)} = \frac{(1950\,\text{lb})\,(15.5\,\text{ft})}{(15.5\,\text{ft} - 3\,\text{ft})} = 2420\,\text{lb}$$

Answer: The allowable loads P for the connection shown, based on the effective depth of 7.0 in. for shear in the main member for the cases $x = 4$ ft and $x = 3$ ft, are 4840 lb and 2420 lb, respectively.

Discussion: The capacity of the connection based on the bolts must also be checked, taking into consideration edge distance in the main member (timber beam) and end and edge distances in the vertical wood plates.

12.3 MEMBER CAPACITY AT CONNECTIONS LOADED PARALLEL-TO-GRAIN

For connections transferring forces parallel to the grain of a timber member, three basic wood failure modes have been defined: *net section fracture*, *row tear-out*, and *group tear-out*. Net section fracture refers to failure through a cross section with reduced area due to the removal of material, such as for placement of fasteners.

Tear-out failure modes are characterized by shear failure or combined shear and tension failure in the connection. Groups of closely spaced bolts, lag screws, split rings, or shear plates loaded parallel-to-grain may produce stresses sufficient to tear the fastener row or group out of the member. Two conditions must be considered: first, the tearing out of single row of fasteners; and, second, the tearing out of a group of multiple rows of fasteners. Tear-out of timber rivets does not need to be considered independently, because the design values for timber rivets already consider group tear-out behavior.

12.3.1 Net Section Fracture

The installation of fasteners and other connection hardware reduces the carrying capacity of the member. A net section tension failure for a shear plate connection is illustrated in Figure 12.3.1-1. For determining the capacity, the net section must be considered, where the loss in wood section due to all boring, grooving, dapping, and other means is taken into consideration. Figure 12.3.1-2 illustrates common net section conditions. Where bolts, drift pins, or lag screws are staggered in adjacent rows, but where the parallel-to-grain spacing of fasteners in adjacent rows is less than 4 fastener diameters, the adjacent fasteners are considered to act at the same cross section for determining net section.

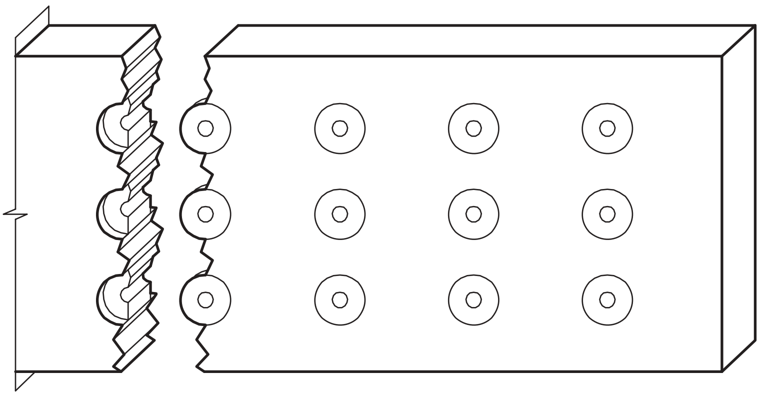
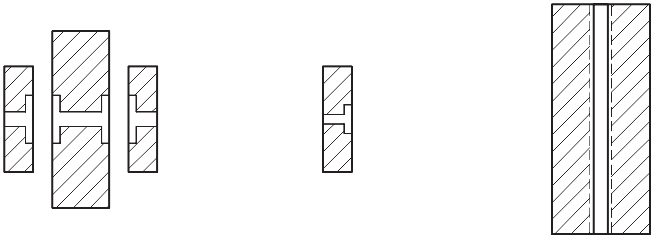


Figure 12.3.1-1 Net section fracture for a tension connection using shear plates or split rings.

12.3.1.1 Net Section Area The net section area is determined by subtracting the projected area of all material removed for the placement of bolts, connectors, or other fabrication from the gross cross-sectional area (Figure 12.3.1-2).

For bolted connections, the net area, A_n, of a cross section is calculated with Equation 12.3.1.1-1.

$$A_n = b\left(d - n_j D_{BH}\right) \qquad (12.3.1.1\text{-}1)$$



Split Ring or Shear Plate Connectors

Countersunk Bolt

Vertical Hole in Beam (effective hole diameter taken as 1.5 times actual hole diameter)

Figure 12.3.1-2 Net section area for common connections.

where:

A_n = net area of the cross section
b = width of the member
d = depth of the member
D_{BH} = diameter of the bolt hole
n_j = number of bolts in the cross section

For shear plates or split rings, the net area, A_n, of a cross section is calculated using Equation 12.3.1.1-2 for a member in double shear with connectors on both faces or Equation 12.3.1.1-3 for a member in single shear.

$$A_n = bd - n_j\left(2pD_O + D_{BH}\left(b - 2p\right)\right) \quad \text{double shear} \quad (12.3.1.1\text{-}2)$$

$$A_n = bd - n_j\left(pD_O + D_{BH}\left(b - p\right)\right) \quad \text{single shear} \quad (12.3.1.1\text{-}3)$$

where:

A_n = net area of the cross section
b = width of the member
d = depth of the member
p = depth of the groove cut to receive the connector
D_O = outside diameter of the connector
D_{BH} = diameter of the bolt hole
n_j = number of connectors in the cross section

12.3.1.2 Net Section Design Capacity Equations 12.3.1.2-1 and 12.3.1.2-2 govern the design of the member for net section at the connection.

$$\frac{P}{A_n} \le F_c' = F_c C_D C_M C_t \quad (12.3.1.2\text{-}1)$$

$$\frac{T}{A_n} \le F_t' = F_t C_D C_M C_t \quad (12.3.1.2\text{-}2)$$

12.3.2 Row Tear-Out

Row tear-out describes the condition where a row of fasteners tears out a "plug" of wood. The failure is due to inadequate shear area along the planes of failure. This condition is illustrated in Figures 12.3.2-1 and 12.3.2-2 for bolts and for timber connectors, respectively.

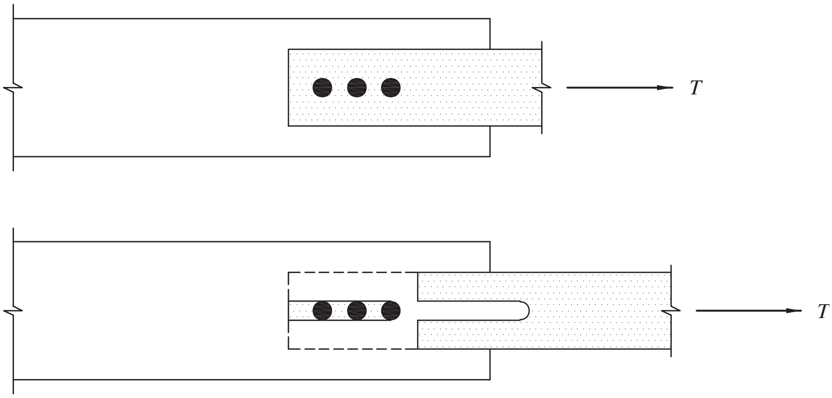


Figure 12.3.2-1 Tear-out of a row of bolts.

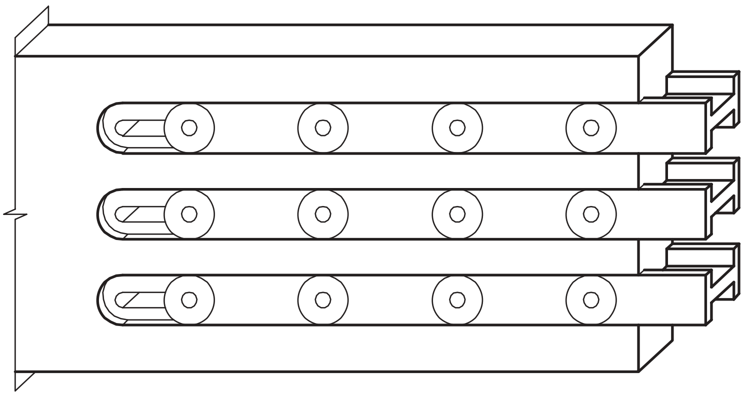


Figure 12.3.2-2 Row tear-out for bolts with shear plate or split ring connectors.

12.3.2.1 Critical Shear Area for Row Tear-out Each fastener in a connection is assigned the same critical shear area for row tear-out calculations. Therefore, the critical shear area for each fastener is determined as the lesser of the critical area based on end distance and the critical area based on spacing. For bolts, the critical shear area is calculated using Equations 12.3.2.1-1 and 12.3.2.1-2.

$$A_{crit\,shear,\ rt} \leq A_{crit,end,rt} = 2te \tag{12.3.2.1-1}$$

$$A_{crit\,shear,\ rt} \leq A_{crit,spacing,rt} = 2ts_1 \tag{12.3.2.1-2}$$

where:

$A_{crit,end,rt}$ = the critical shear area based on end distance
$A_{crit,spacing,rt}$ = the critical shear area based on spacing between fasteners in a row
t = member thickness
e = distance from end of piece to nearest fastener in the row
s_1 = spacing between fasteners in the row

Neglecting the contribution of the bolts to resist row tear-out, the critical shear area for split rings or shear plates is calculated using Equations 12.3.2.1-3 and 12.3.2.1-4.

$$A_{crit\ shear,rt} \leq A_{crit,end,rt} = \left(D_O + 2p\right) e + \frac{\pi}{4}\left(D_I^2 - \frac{D_O^2}{2} - D_{BH}^2\right) \tag{12.3.2.1-3}$$

$$A_{crit\ shear,rt} \leq A_{crit,spacing,rt} = \left(D_O + 2p\right) s_1 - \frac{\pi}{4}\left(D_O^2 + D_{BH}^2 - D_I^2\right) \tag{12.3.2.1-4}$$

where:

$A_{crit,\ end,\ rt}$ = critical shear area based on end distance
$A_{crit,\ spacing,\ rt}$ = critical shear area based on spacing
D_O = outside diameter of the connector
D_I = inside diameter of the connector
D_{BH} = diameter of the bolt hole
p = penetration of the connector into the member (groove depth)
e = end distance
s_1 = spacing between fasteners in the row

Equations 12.3.2.1-3 and 12.3.2.1-4 are based on current NDS recommendations. These equations conservatively neglect the contribution of the wood behind the bolt in resisting fastener tear-out. Consequently, these equations lead to greater spacing requirements for shear plates and split rings than those recommended by the primary research [1] used to develop the design loads for these timber connectors.

Scholten's descriptions of failures from split ring tests in southern pine indicate that the minimum spacings and end distances recommended for full design values resulted in fastener yielding and wood crushing. Shorter spacings and end distances resulted in shear failures prior to developing the full capacity of the fastener [1]. These results suggest that the minimum recommended spacings and end distances for full design values are sufficient to prevent row tear-out in medium grain southern pine members. However, other species within the same group with lower shear values may require greater spacing and end distance to prevent row tear-out below the full design load. The effect of shear strength on required spacing and connection capacity can be determined through the row tear-out analysis, including the contribution of the shear planes behind the bolt.

It is nonconservative to assume that the resisting shear planes behind the bolts extend across the full thickness of the member in the row tear-out analysis. Due to nonuniform stress distribution across the thickness of the member, only a small portion of the shear planes behind the bolt can effectively provide resistance. The contribution of the shear planes behind the bolt have not been well studied, however, the row tear-out analysis yields comparable spacing and end distance requirements for southern pine glulam when the width of the effective shear area behind the bolt is estimated to be equal to the penetration depth of the shear plate or split ring for each connector unit on the bolt.

Therefore, including the contribution of the bolts to resist row tear-out, the critical shear area for split rings or shear plates is estimated using Equations 12.3.2.1-5 and 12.3.2.1-6.

$$A_{crit\,shear,rt} \leq A_{crit,end,rt} = \left(D_O + 2p\right) e + \frac{\pi}{4}\left(D_I^2 - \frac{D_O^2}{2} - D_{BH}^2\right) + 2pe \tag{12.3.2.1-5}$$

$$A_{crit\,shear,rt} \leq A_{crit,spacing,rt} = \left(D_O + 2p\right) s_1 - \frac{\pi}{4}\left(D_O^2 + D_{BH}^2 - D_I^2\right) + 2ps_1 \tag{12.3.2.1-6}$$

The relationships in Equations 12.3.2.1-5 and 12.3.2.1-6 will be used for subsequent analyses in this manual.

12.3.2.2 Row Tear-out Design Capacity The row tear-out capacity for a connection is determined Equation 12.3.2.2-1.

$$T'_{rt} = \sum n_i \frac{F'_v}{2} A_{crit\,shear,rt} \tag{12.3.2.2-1}$$

where:

T'_{rt} = row tear-out capacity for a connection
$A_{crit\,shear,rt}$ = critical shear area for a single fastener
F'_v = adjusted shear stress for member design at connections
n_i = number of fasteners in each row

12.3.2.3 Row Tear-out for Fasteners Loaded at Angle to Fastener Axis Where the load on a fastener is at an angle other than perpendicular to the fastener axis (Figure 12.3.2.3-1), the fastener must be capable of transmitting the load component parallel to the axis, either by bearing of bolt head or nut and washer on wood or other surface, or by withdrawal capacity developed by threads or friction. Adequate end distance must be provided such that the equivalent shear area illustrated in Figure 12.3.2.3-1 for parallel members of the same main and side member thickness values satisfies the row tear-out provisions and the end distance requirements.

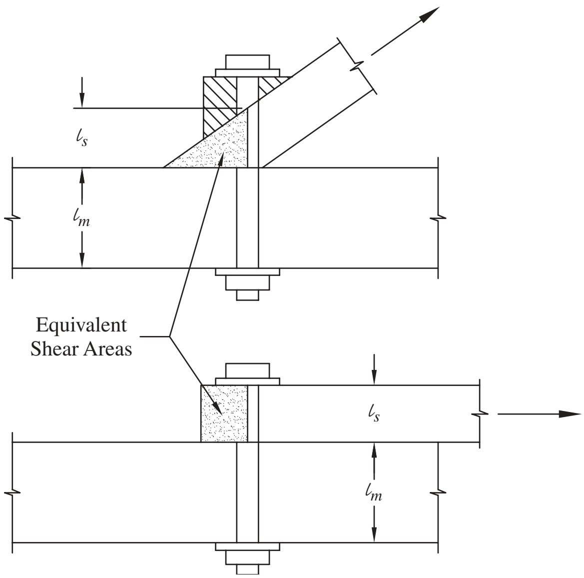


Figure 12.3.2.3-1 Shear area for load at angle to fastener axis.

12.3.3 Group Tear-Out

The group tear-out failure mode occurs when two or more rows of fasteners are closely spaced. Instead of individual plugs tearing out for each row, a single block of material tears out with the entire fastener group.

The group tear-out capacity of multiple rows of bolts is developed by two shear planes and a net area that develops tension stress (Figure 12.3.3-1). For split rings, shear plates, and other fasteners that do not penetrate the member fully, the failure acts over three shear planes and a net section across which tension stresses are developed (Figure 12.3.3-2).

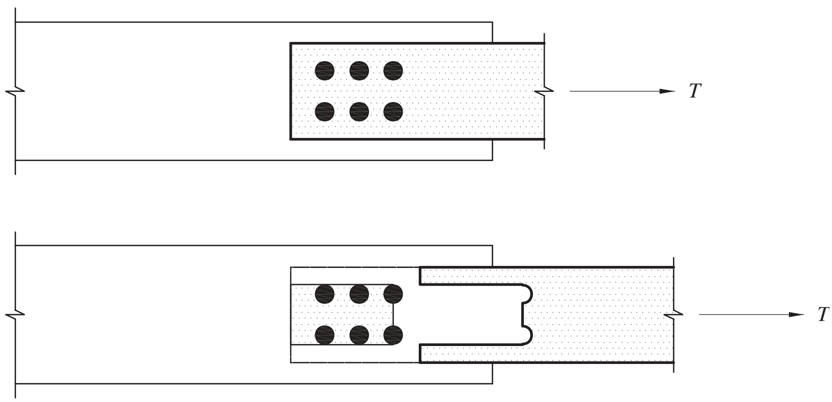


Figure 12.3.3-1 Group tear-out for bolted connection.

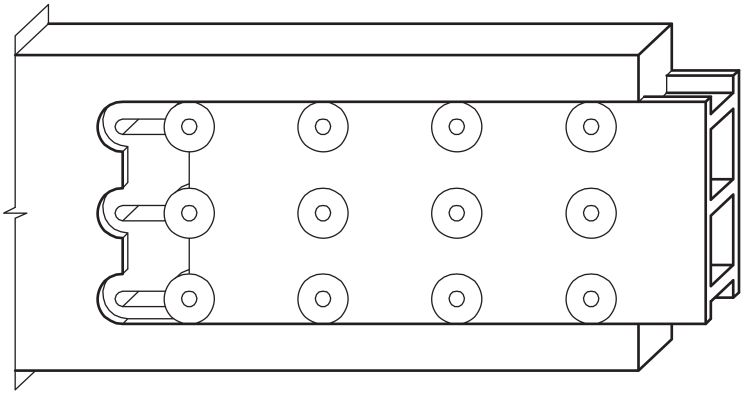


Figure 12.3.3-2 Group tear-out for bolts with shear plate or split ring connectors.

12.3.3.1 Critical Shear Area for Group Tear-out The critical shear area is determined between the outer rows for the group, using the smaller of the critical areas based on end distance or spacing (Equations 12.3.3.1-1 and 12.3.3.1-2).

$$A_{crit\ shear,\,gt} \leq A_{crit\ gt,\,end} \tag{12.3.3.1-1}$$

$$A_{crit\ shear,\,gt} \leq A_{crit\ gt,\,spacing} \tag{12.3.3.1-2}$$

where:

$A_{crit\ shear,\,gt}$ = the critical shear area for group tear-out
$A_{crit\ gt,\,end}$ = the critical shear area for group tear-out based on end distance
$A_{crit\ gt,\,spacing}$ = the critical shear area for group tear-out based on spacing

For bolted connections, the critical shear areas for group tear-out based on end distance and spacing are calculated using Equations 12.3.3.1-3 and 12.3.3.1-4.

$$A_{crit\ gt,\,end} = 2n_i te \tag{12.3.3.1-3}$$

$$A_{crit\ gt,\,spacing} = 2n_i ts_1 \tag{12.3.3.1-4}$$

Neglecting the contribution of shear resistance behind the bolt, the critical shear areas for group tear-out for shear plates or split rings, based on end distance and spacing are calculated using Equations 12.3.3.1-5 and 12.3.3.1-6.

$$A_{crit\ gt,\,end} = n_i\left(\left[2p + \left(n_j - 1\right)s_2 + D_O\right]e + \frac{\pi n_j}{4}\left[D_I^2 - \frac{D_O^2}{2} - D_{BH}^2\right]\right) \tag{12.3.3.1-5}$$

$$A_{crit\ gt,\,spacing} = n_i\left(\left[2p + \left(n_j - 1\right)s_2 + D_O\right]s_1 - \frac{\pi n_j}{4}\left[D_o^2 - D_I^2 - D_{BH}^2\right]\right) \tag{12.3.3.1-6}$$

Where the contribution of the shear planes behind the bolt is included, the critical shear areas for group tear-out for shear plates or split rings are calculated using Equations 12.3.3.1-7 and 12.3.3.1-8.

$$A_{crit\ gt,\ end} = n_i \left(\left[2p\left(1+n_j\right) + \left(n_j - 1\right)s_2 + D_O\right] e + \frac{\pi n_j}{4}\left[D_I^2 - \frac{D_O^2}{2} - D_{BH}^2\right]\right) \tag{12.3.3.1-7}$$

$$A_{crit\ gt,\ spacing} = n_i \left(\left[2p\left(1+n_j\right) + \left(n_j - 1\right)s_2 + D_O\right] s_1 - \frac{\pi n_j}{4}\left[D_o^2 - D_I^2 - D_{BH}^2\right]\right) \tag{12.3.3.1-8}$$

where:

$A_{crit\ gt,\ end}$ = critical shear area for group tear-out based on end distance
$A_{crit\ gt,\ spacing}$ = critical shear area for group tear-out based on spacing
D_O = outside diameter of the connector
D_I = inside diameter of the connector
D_{BH} = diameter of the bolt hole
e = end distance
n_i = number of fasteners in each row
n_j = number of rows in the connection
p = penetration of the connector into the member (groove depth)
s_1 = spacing between fasteners in the row
s_2 = spacing between rows in the connection

12.3.3.2 Effective Tension Area for Group Tear-out The effective tension area for bolts in group tear-out is calculated using Equation 12.3.3.2-1.

$$A_{eff\ tension} = t\left(s_2 - D_{BH}\right)\left(n_j - 1\right) \tag{12.3.3.2-1}$$

where:

$A_{eff\ tension}$ = net cross-sectional area of wood between outermost fasteners in the group
D_{BH} = diameter of the bolt hole
n_j = number of rows in the connection
s_2 = spacing between rows in the connection
t = member thickness

The effective tension area for shear plates or split rings in group tear-out is calculated using Equation 12.3.3.2-2.

$$A_{eff\ tension} = p\left(s_2 - D_O\right)\left(n_j - 1\right) \tag{12.3.3.2-2}$$

where:

$A_{eff\ tension}$ = net cross-sectional area of wood between outermost fasteners in the group
D_O = outside diameter of the connector
p = penetration of the connector into the member (groove depth)
n_j = number of rows in the connection
s_2 = spacing between rows in the connection

12.3.3.3 Group Tear-out Design Capacity The group tear-out capacity is calculated with Equation 12.3.3.3-1.

$$T'_{gt} = \frac{F'_v}{2} A_{crit\ shear,\,gt} + F'_t A_{eff\ tension} \quad (12.3.3.3\text{-}1)$$

where:

T'_{gt} = group tear-out capacity
F'_v = adjusted shear stress for member design at connections
$A_{crit\ shear,\,gt}$ = critical shear area for group tearout
F'_t = adjusted tension stress
$A_{eff\ tension}$ = the net cross-sectional area of wood between outermost fasteners in the group

EXAMPLE 12.3-1 MEMBER FAILURE MODES IN TENSION CONNECTION

Given: A $5\frac{1}{8}$ in. × $7\frac{1}{2}$ in. DF, Combination 5, glulam truss tension web is connected to $\frac{1}{4}$ in. steel gusset plates with 1 in. diameter bolts configured as shown in Figure 12.3-1. The yield mode capacity (normal load duration) of the fasteners based on the *NDS*® [2] is 44,000 lb.

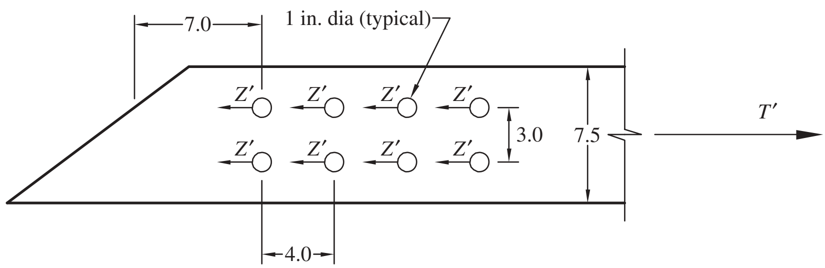


Figure 12.3-1 Bolted connection—Example 12.3-1.

Wanted: Calculate the capacity of the member to resist net section fracture, row tear-out, and group tear-out failure modes assuming normal load duration.

Solution:

Net section fracture (Figure 12.3-2, Section 12.3.1):

$$A_n = b\left(d - n_j D_{BH}\right)$$

$$A_n = (5.125\,\text{in})\,(7.5\,\text{in} - 2\,(1.0625\,\text{in})) = 27.5\,\text{in}^2$$

$$T'_n = F'_t A_n = (1600\,\text{psi})\left(27.5\,\text{in}^2\right) = 44,000\,\text{lb}$$

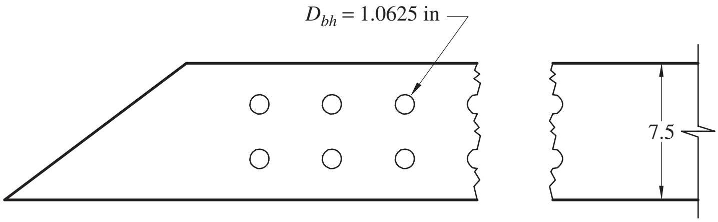


Figure 12.3-2 Net section fracture—Example 12.3-1.

Row tear-out (Figure 12.3-3, Section 12.3.2):

$$T'_{rt} = n_i F'_v t s_{critical} n_j$$

$$T'_{rt} = 4\,\frac{\text{bolts}}{\text{row}}\,(190\,\text{psi})\,(5.125\,\text{in})\,(4\,\text{in})\,(2\,\text{rows})$$

$$T'_{rt} = 31,160\,\text{lb}$$

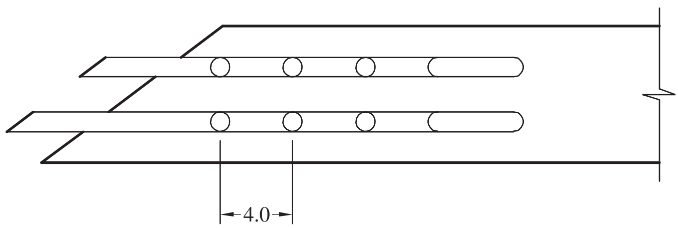


Figure 12.3-3 Row tear-out—Example 12.3-1.

Group tear-out (Figure 12.3-4, Section 12.3.3):

$$T'_{gt} = \frac{T'_{rt}}{2} + F'_t A_{\text{group net}}$$

$$T'_{gt} = \frac{31,160\,\text{lb}}{2} + (1600\,\text{psi})\,(5.125\,\text{in})\,(3\,\text{in} - 1.0625\,\text{in})$$

$$T'_{gt} = 31,470\,\text{lb}$$

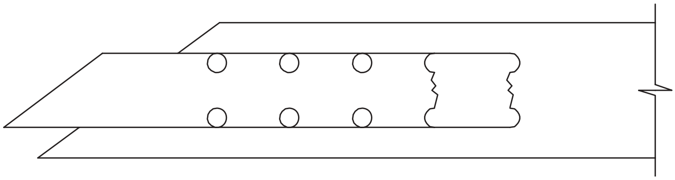


Figure 12.3-4 Group tear-out—Example 12.3-1.

Answer: The net-section capacity is 44,000 lb. The row tear-out capacity is 31,200 lb. The group tear-out capacity is 31,500 lb.

Discussion: The connection capacity is controlled by the row tear-out failure mode (31.2 kip). The capacity to resist group tear-out failure is not much higher (31.5 kip). Consideration of only the fastener yield and net section capacities would have been non-conservative by 30%. This detail was used in actual practice and failures were observed in both the row tear-out and group tear-out failure modes (Figures 12.3-5 and 12.3-6).

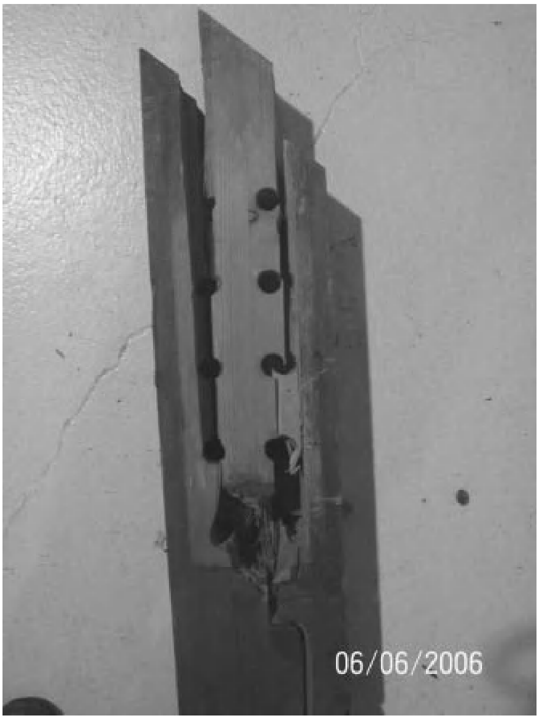


Figure 12.3-5 Group tear-out failure.

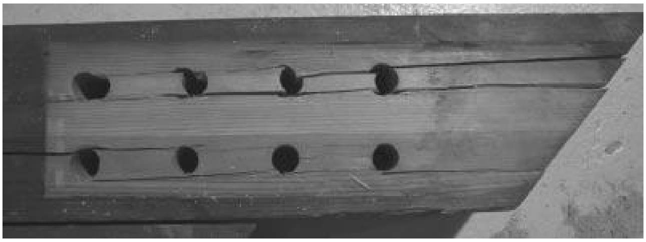


Figure 12.3-6 Row tear-out failure.

12.3.4 Design to Prevent Row and Group Tear-Out

Example 12.3-1 clearly demonstrates the importance of a member's capacity to resist row and group tear-out failure modes. However, rather than performing these calculations after the design is completed, possibly requiring multiple iterations for design, a method of determining the minimum required fastener spacings to prevent these failures is desirable.

12.3.4.1 Bolted Connections The critical spacing between bolts within a row, s_1, required to prevent row tear-out is determined from the row tear-out equation, resulting in Equation 12.3.4.1-1.

$$s_1 \geq \frac{Z'}{F'_v t} \tag{12.3.4.1-1}$$

where:

Z' = single bolt design yield capacity
F'_v = adjusted design value for connection shear
t = thickness of member (measured parallel to bolt axis)

The minimum spacing, s_2, between rows of bolts can be determined from the group tear-out equation, assuming that spacing between fasteners in a row has been adequately established to develop the capacity of the fasteners and to prevent row tear-out. This results in Equation 12.3.4.1-2.

$$s_2 \geq \frac{Z' n_i}{F'_t t} + D_{BH} \tag{12.3.4.1-2}$$

where:

n_i = number of bolts in a row
D_{BH} = diameter of hole for bolt
F'_t = reference design value for tension

EXAMPLE 12.3.4.1-1 REDESIGN OF EXAMPLE 12.3-1 CONNECTION TO PREVENT ROW AND GROUP TEAR-OUT

Given: The connection from Example 12.3-1.

Wanted: Redesign the connection to develop the design capacity of the fasteners (44,000 lb) and prevent failure in row or group tear-out.

Solution:

$$s_1 \geq \frac{Z'}{F_v' t} = \frac{5500\,\text{lb}}{(190\,\text{psi})\,(5.125\,\text{in})}$$

$$s_1 \geq 5.65\,\text{in} \quad \therefore \text{Use } 5.75\,\text{inches}$$

$$s_2 \geq \frac{Z' n_i}{F_t' t} + D_{BH} = \frac{(5500\,\text{psi})\,(4)}{(1600\,\text{psi})\,(5.125\,\text{in})} + 1.0625\,\text{in}$$

$$s_2 \geq 3.75\,\text{in} \quad \therefore \text{Use } 3.75\,\text{inches}$$

Answer: To prevent row and group tear-out, the bolts within each row should be spaced 5.75 inches apart, and the rows should be spaced 3.75 inches apart.

Discussion: With fasteners spaced at $s_1 = 5.75$ in. and $s_2 = 3.75$ in., the row tear-out capacity of the revised connection is 44.8 kips, and the group tear-out capacity is 44.4 kips. This is sufficient to develop the full design capacity of the bolts (44.0 kips). The revised edge distance of 1.875 in. exceeds the 1.5 in. minimum specified by NDS for 1 in. bolts. For the same size member and the same number of bolts, the capacity of the revised connection is more than 40% higher than the capacity of the original connection.

12.3.4.2 Shear Plate or Split Ring Connections The critical area, $A_{crit\,shear}$, for a single shear plate or split ring connector to develop the full fastener capacity and prevent row tear-out is determined from the row tear-out equation, resulting in Equation 12.3.4.2-1.

$$A_{crit\,shear,\,rt} = \frac{2P'}{F_v'} \tag{12.3.4.2-1}$$

Including the contribution of the shear planes behind the bolt, the corresponding minimum end distance, e, and spacing, s_1, to achieve full capacity while preventing row tear-out are calculated with Equations 12.3.4.2-2 and 12.3.4.2-3.

$$e \geq \frac{A_{crit\ shear,\ rt} - \dfrac{\pi}{4}\left(D_I^2 - \dfrac{D_O^2}{2} - D_{BH}^2\right)}{D_O + 4p} \tag{12.3.4.2-2}$$

$$s_1 \geq \frac{A_{crit\ shear,\ rt} + \frac{\pi}{4}\left(D_O^2 + D_{BH}^2 - D_I^2\right)}{D_O + 4p} \qquad (12.3.4.2\text{-}3)$$

As an alternative to Equation 12.3.3.3-1, the group tear-out capacity for multiple rows of shear plates or split rings can be determined starting with the row tear-out capacity, then adding the tension resistance and modifying the shear resistance to account for the different shear planes involved, resulting in Equation 12.3.4.2-4.

$$T'_{gt} = T'_{rt} + F'_t A_{eff\ tension} + \frac{F'_v}{2} n_i \left(n_j - 1\right)\left(s_2 - D_O - 2p\right)\left(s_1 \text{ or } e\right) \qquad (12.3.4.2\text{-}4)$$

By requiring the group tear-out capacity to equal or exceed the row tear-out capacity, the minimum spacing between rows is easily determined. With this criterion, using Equation 12.3.4.2-4, it is apparent that the additional resistance to tear-out must be greater than or equal to zero (Equation 12.3.4.2-5).

$$F'_t A_{eff\ tension} + \frac{F'_v}{2} n_i \left(n_j - 1\right)\left(s_2 - D_O - 2p\right)\left(s_1 \text{ or } e\right) \geq 0 \qquad (12.3.4.2\text{-}5)$$

Substituting Equation 12.3.3.2-2 into Equation 12.3.4.2-5 results in Equation 12.3.4.2-6.

$$F'_t p \left(s_2 - D_O\right)\left(n_j - 1\right) + \frac{F'_v}{2} n_i \left(n_j - 1\right)\left(s_2 - D_O - 2p\right)\left(s_1 \text{ or } e\right) \geq 0 \qquad (12.3.4.2\text{-}6)$$

The spacing between rows, s_2, to prevent group tear-out is then determined algebraically, resulting in Equation 12.3.4.2-7.

$$s_2 \geq \frac{p n_i \left(s_1 \text{ or } e\right) F'_v}{F'_t p + n_i \left(s_1 \text{ or } e\right) \frac{F'_v}{2}} + D_O \qquad (12.3.4.2\text{-}7)$$

where:

(s_1 or e) represents the spacing or the end distance. This parameter is determined depending on whether the critical area based on spacing, s_1, or end distance, e, governed the row tear-out calculation.

EXAMPLE 12.3.4.2-1 ROW TEAR-OUT OF SHEAR PLATE CONNECTORS

Given: A SP glulam main member is connected to two $\frac{1}{4}$ in. steel gusset plates with $2\frac{5}{8}$ in. shear plates with $\frac{3}{4}$ in. diameter bolts. The member is subject to normal load duration and dry use.

Wanted: Calculate the critical spacing and end distance to develop the full connector capacity while preventing row tear-out.

Solution:

Critical shear area for row tear-out (Equation 12.3.4.2-1):

$$A_{crit\ shear,\ rt} = \frac{2P'}{F'_v} = \frac{2PC_D C_M C_t C_\Delta}{F_v C_D C_M C_t C_{vr}}$$

$$A_{crit\ shear,\ rt} = \frac{2\,(2860\,\text{lb})\,(1.0)\,(1.0)\,(1.0)\,(1.0)}{(300\,\text{psi})\,(1.0)\,(1.0)\,(1.0)\,(0.72)} = 26.5\,\text{in}^2$$

Minimum end distance to prevent row tear-out (Equation 12.3.4.2-2):

$$e \geq \frac{A_{crit\ shear,\ rt} - \dfrac{\pi}{4}\left(D_I^2 - \dfrac{D_O^2}{2} - D_{BH}^2\right)}{D_O + 2p}$$

$$e \geq \frac{26.5\,\text{in}^2 - \dfrac{\pi}{4}\left((2.29\,\text{in})^2 - \dfrac{(2.63\,\text{in})^2}{2} - (0.81\,\text{in})^2\right)}{2.63\,\text{in} + 4\,(0.42\,\text{in})}$$

$$e \geq 5.94\,\text{in}$$

Note: The required end distance to prevent row tear-out for $2\frac{5}{8}$ in. shear plates exceeds the minimum end distance of 5.5 in. prescribed by the NDS for full design capacity.

Minimum spacing to prevent row tear-out (Equation 12.3.4.2-3):

$$s_1 \geq \frac{A_{crit\ shear,\ rt} + \dfrac{\pi}{4}\left(D_O^2 + D_{BH}^2 - D_I^2\right)}{D_O + 2p}$$

$$s_1 \geq \frac{26.5\,\text{in}^2 + \dfrac{\pi}{4}\left((2.63\,\text{in})^2 + (0.81\,\text{in})^2 - (2.29\,\text{in})^2\right)}{2.63\,\text{in} + 4\,(0.42\,\text{in})}$$

$$s_1 \geq 6.57\,\text{in}$$

Answer: An end distance of 5.9 inches and spacing of 6.6 inches will prevent row tear-out prior to development of the full design capacity of the fasteners.

Performing the calculation illustrated in Example 12.3.4.2-1 for $2\frac{1}{2}$ in. split rings, for 4 in. split rings, and for 4 in. shear plates with metal side plates gives the following minimum spacings and end distances to develop the full design capacity for timber connectors in southern pine glulam (Table 12.3.4.2-1).

Performing the same calculations for Douglas fir glulam gives the values in Table 12.3.4.2-2.

TABLE 12.3.4.2-1 Spacing and End Distance to Prevent Row Tear-Out in Southern Pine Glulam

Connector	Minimum Spacing (in.)	Minimum End Distance (in.)
$2\frac{5}{8}$ in. Shear Plate	6.6	5.9
4 in. Shear Plate	7.4	6.4
$2\frac{1}{2}$ in. Split Ring	6.1	5.4
4 in. Split Ring	8.0	6.8

TABLE 12.3.4.2-2 Spacing and End Distance to Prevent Row Tear-Out in Douglas Fir Glulam

Connector	Minimum Spacing (in.)	Minimum End Distance (in.)
$2\frac{5}{8}$ in. Shear Plate	7.4	6.7
4 in. Shear Plate	8.3	7.3
$2\frac{1}{2}$ in. Split Ring	6.9	6.1
4 in. Split Ring	9.0	7.8

The prescriptive spacing and end distance limitations from the NDS must also be followed.

12.4 MEMBER CAPACITY AT CONNECTIONS LOADED AT AN ANGLE TO GRAIN

Where loads are transferred at an angle to grain between 0° and 90°, forces must be separated into parallel-to-grain and perpendicular-to-grain components and compared to their respective components of member resistance. The parallel-to-grain force component is evaluated against the member resistance for net section, row tear-out, and group tear-out. The perpendicular-to-grain force component is compared to the member resistance to shear loads perpendicular-to-grain.

In addition, where bearing stresses occur at an angle to grain other than 0° or 90°, such as in Figure 12.4-1, the allowable bearing stress is calculated by the Hankinson formula, Equation 12.4-1.

$$F'_\theta = \frac{F_c^* F'_{c\perp}}{F_c^* \sin^2\theta + F'_{c\perp}\cos^2\theta} \qquad (12.4\text{-}1)$$

where:

F'_θ = allowable bearing stress at angle to grain

θ = angle between direction of load (perpendicular to bearing surface) and direction of grain

F_c^* = design value for compression parallel-to-grain multiplied by all applicable adjustment factors except C_P

$F'_{c\perp}$ = design value for compression perpendicular-to-grain multiplied by all applicable adjustment factors

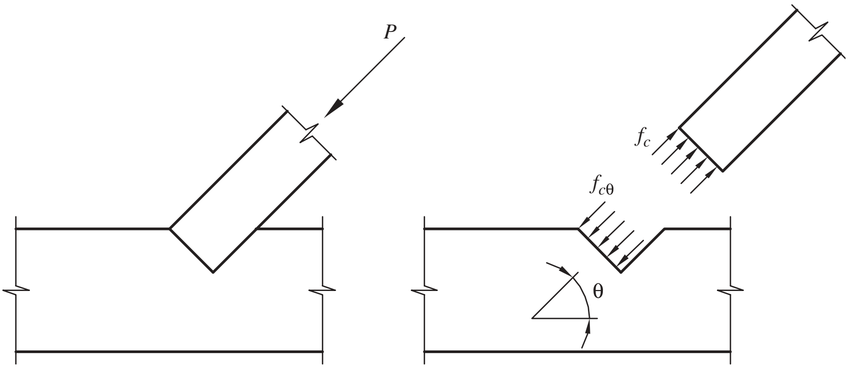


Figure 12.4-1 Bearing at an angle to the grain.

12.5 CONCLUSION

Multiple member failure modes must be evaluated in the connection design process. It is generally preferable for the fastener capacity to govern design, rather than the member capacity, where possible. This chapter presented procedures for the evaluation of member capacity at connections, considering several potential failure modes.

For loads perpendicular-to-grain, the member shear capacity must be evaluated taking into account the configuration of the connection. Where members are supported by bearing, the compression perpendicular-to-grain design capacity must also be evaluated. Notched members and members supported by fasteners have reduced capacity relative to an unmodified section supported by bearing.

For loads parallel-to-grain, failure modes of net section, row tear-out and group tear-out must be considered. Several formulas have been presented to facilitate evaluation of these potential failure modes. Use of these formulas will help achieve the full design capacity of the fasteners and avoid premature member failure.

CHAPTER 13

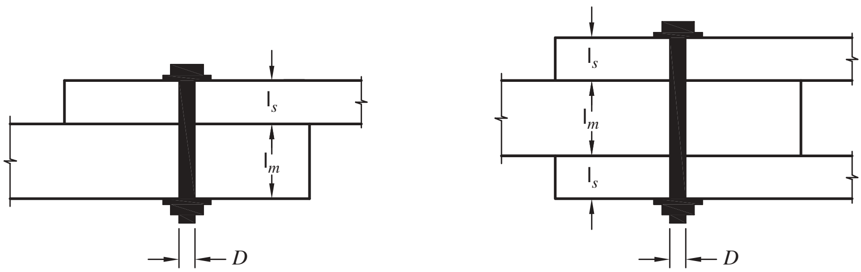
DOWEL-TYPE FASTENERS

13.1 INTRODUCTION

Dowel-type fasteners include bolts, lag screws, drift pins, nails, spikes and wood screws. This type of fastener may transfer load perpendicular to the fastener axis (lateral load) or parallel to the fastener axis. Loads parallel to the fastener axis are transferred through friction or threads (withdrawal loading), or by bearing of the bolt head or nut and washers. Loads perpendicular to the fastener axis (lateral loading) are transferred by bearing against the wood. This chapter presents equations for determining the capacity of dowel-type fasteners loaded laterally, in withdrawal, or both.

13.2 DOWEL-TYPE FASTENERS LOADED LATERALLY

Reference design values for lateral capacity, Z, for dowel type fasteners may be calculated directly or may be obtained from tables in the *National Design Specification®* [1] for single shear and double shear connections (Figure 13.2-1). Tabulated design values are based on the lowest or critical load considering the various yield modes shown in Figure 13.2-2. The reference design values, whether calculated or tabulated, assume that (1) the faces of the connected members are in contact; (2) the load acts perpendicular to the fastener axis; (3) the edge distance, end distance, and spacing values are sufficient to develop the full design value; and (4) the penetration of the fastener in the receiving member (main member for single shear or side member for double shear) is greater than or equal to the minimum penetration specified for the type of fastener.



Single-Shear Bolted Connection

Double-Shear Bolted Connection

Figure 13.2-1 Single- and double-shear connections.

The modes of fastener and member behavior illustrated in Figure 13.2-2 are described as follows:

- Mode I_m represents the bearing dominated yield of wood bearing on the main member.
- Mode I_s represents bearing dominated yield of the side member(s).
- Mode II represents fastener pivoting, producing localized crushing near the adjacent member faces.
- Mode III represents fastener yielding with one plastic hinge point per shear plane.
- Mode IV represents fastener yielding with two plastic hinge points per shear plane.

Design values for dowel-type fasteners are generally denoted by Z. For fasteners with diameter greater than or equal to $\frac{1}{4}$ in., design values are tabulated [1] for the cases of both members loaded parallel-to-grain ($Z_{\parallel}$), main member loaded parallel-to-grain and side member(s) loaded perpendicular-to-grain ($Z_{s\perp}$), main member loaded perpendicular-to-grain and side member(s) loaded parallel-to-grain ($Z_{m\perp}$), and both side and main members loaded perpendicular-to-grain ($Z_{\perp}$). Design values multiplied by all applicable adjustment factors are denoted $Z', Z'_{s\perp}, Z'_{m\perp}$, and $Z'_{\perp}$. Where the load in either member is at an angle other than parallel- or perpendicular-to-grain, the design value must be calculated based on the dowel bearing strength for the direction of load with respect to grain for each of the members.

Design values for dowel-type fasteners with diameters less than $\frac{1}{4}$ in. are assumed to be independent of load direction with respect to grain and are denoted simply Z, and the adjusted values Z'.

13.2.1 Special Considerations for Lag Screws and Wood Screws

For lag screws and wood screws, the fastener root diameter D_r is used for the calculation of lateral design capacities, and the main member bearing length

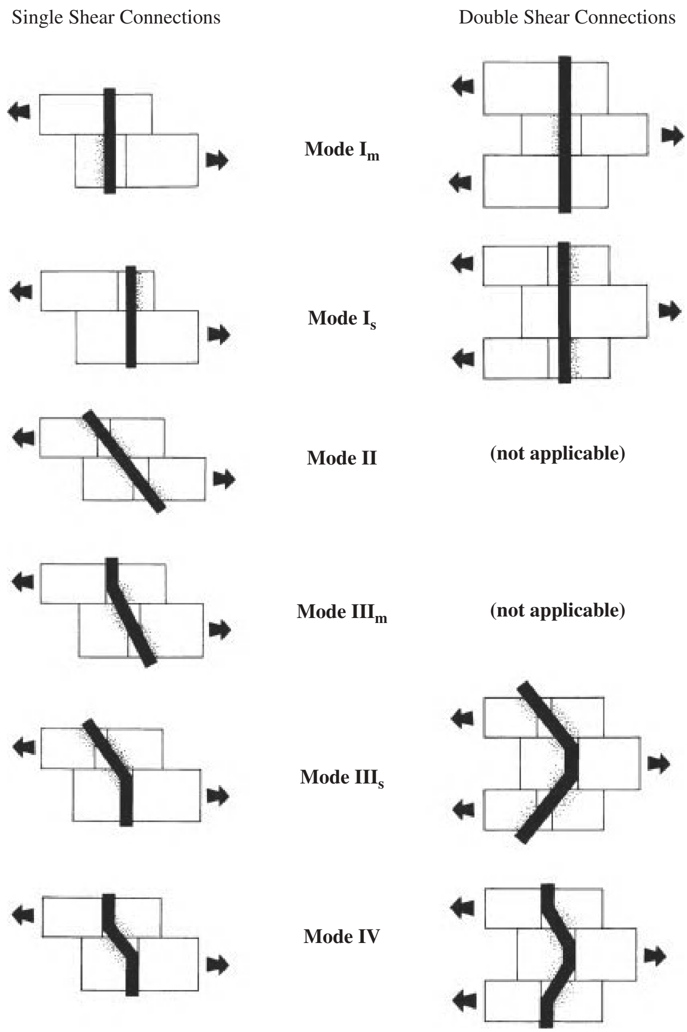


Figure 13.2-2 Connection yield modes. Reprinted with permission from *National Design Specification® for Wood Construction*. Copyright © 2012. Courtesy American Wood Council, Leesburg, Virginia.

in single shear connections and side member bearing length in double shear connections is taken to be the penetration depth. For lag screws, the penetration depth, *excluding* the tapered tip length, must be not less than four diameters (4D), based on full diameter. Tabular values for lag screws loaded laterally in the *National Design Specification®* [1] are based on 8D penetration but may be reduced for penetration values not less than 4D. Wood screws must penetrate into the main member at least six diameters (6D) for reduced design values and ten diameters (10D) for the full design values tabulated in the *NDS®* [1]. The required penetration depth *includes* the length of the tapered tip for wood screws. Dowel bending yield strength values for lag screws and wood screws are given in the appendix.

13.2.2 Special Considerations for Nails and Spikes

For nails and spikes, the main member bearing length in single shear connections and side member bearing length in double shear connections is taken to be the penetration depth in the member. For single-shear connections, nails and spikes must penetrate into the main member at least six diameters ($6D$) for reduced design values and ten diameters ($10D$) for the full design values tabulated in the *NDS*® [1]. For double-shear connections, nails and spikes must penetrate at least six diameters into the outside member receiving the point. The required penetration depth *includes* the length of the tapered tip for nails and spikes. Dowel bending yield strengths for nails and spikes are included in the appendix.

13.2.3 Geometry Factor for Bolts, Lag Screws, and Drift Pins

The geometry factor, C_Δ, can be subdivided into factors for spacing, edge distance, and end distance:

- $C_{\Delta s}$ = spacing factor
- $C_{\Delta e}$ = edge distance factor
- $C_{\Delta n}$ = end distance factor

These factors are not cumulative, and the smallest controls in determining C_Δ. Where fasteners are used in a group and any one of the fasteners is adjusted by any of the geometry factors $C_{\Delta e}$, $C_{\Delta n}$, $C_{\Delta s}$, the whole group must be adjusted by the lowest factor obtained.

13.2.3.1 Geometry Factor for Edge Distance, $C_{\Delta e}$ Edge distance requirements for dowel-type fasteners in wood members are shown in Table 13.2.3.1-1. Edge distance requirements are dependent on direction of load with respect to the nearest edge, direction of load with respect to grain, and relative length of fastener in member with respect to fastener diameter. Edge distances (as well as end distances and spacing values) are generally given in terms of fastener diameter, D. Figure 13.2.3.1-1 illustrates various edge, end, and spacing conditions. As such, for dowel-type fasteners, all of the edge distances of Table 13.2.3.1-1 are to be met in all circumstances, and the geometry factor for edge distance is not applicable ($C_{\Delta e} = 1.0$).

13.2.3.2 Geometry Factor for End Distance, $C_{\Delta n}$ End distance requirements are shown in Table 13.2.3.2-1. End distance requirements are dependent on species (softwood or hardwood), direction of load with respect to grain, and whether the load is directed away from or toward the end of the member.

TABLE 13.2.3.1-1 Edge Distance Requirements (from *NDS*® [1])

Direction of Loading		Minimum Edge Distance[a,b]
Parallel to grain:	when $\ell/D \leq 6$	$1.5D$
	when $\ell/D > 6$	$1.5D$ or $\frac{1}{2}$ the spacing between rows, whichever is greater
Perpendicular to grain:	loaded edge	$4D$
	unloaded edge	$1.5D$

[a] The ℓ/D ratio used to determine the minimum edge distance is the lesser of:

(a) length of fastener in wood main member divided by fastener diameter $= \ell_m/D$

(b) total length of fastener in wood side member(s) divided by fastener diameter $= \ell_s/D$

[b] Heavy or medium concentrated loads shall not be suspended below the neutral axis of a sawn lumber or glued laminated timber beam except where mechanical or equivalent reinforcement is provided to resist tension stresses perpendicular to grain.

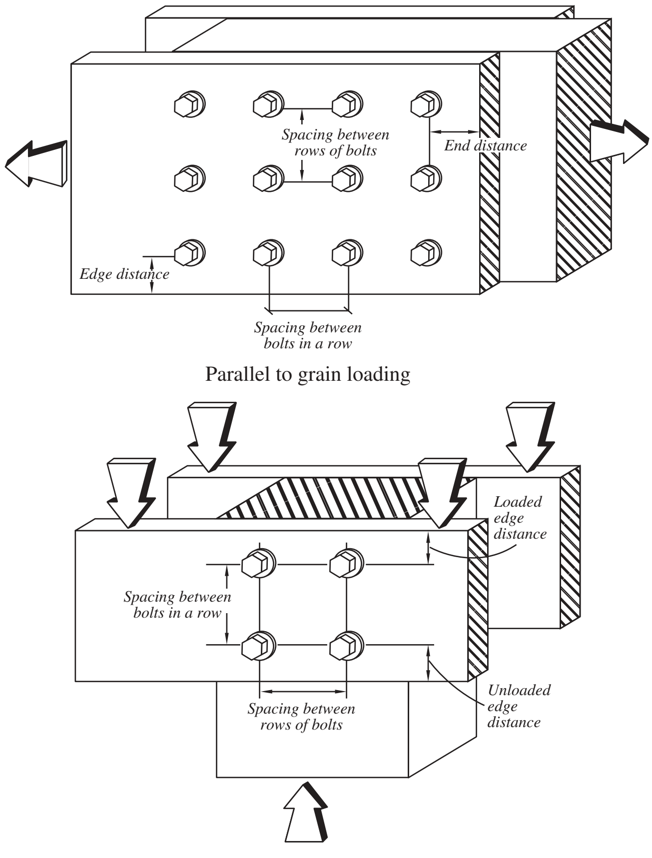


Figure 13.2.3.1-1 Dowel-type fastener connection geometry. Reprinted with permission from *National Design Specification® for Wood Construction*. Copyright © 2012. Courtesy American Wood Council, Leesburg, Virginia.

TABLE 13.2.3.2-1 End Distance Requirements (from *NDS*® [1])

Direction of Loading		Minimum End Distance for $C_\Delta = 0.5$	Minimum End Distance for $C_\Delta = 1.0$
Perpendicular-to-grain		$2D$	$4D$
Parallel-to-grain, compression: (fastener bearing away from member end)		$2D$	$4D$
Parallel-to-grain: (fastener bearing toward member end)	for softwoods	$3.5D$	$7D$
	for hardwoods	$2.5D$	$5D$

Table 13.2.3.2-1 gives the end distances required for use of the full design value of the fastener, as well as minimum end distance values for reduced fastener design values. In cases where the end distances for the full design values are met, no end distance geometry factor need be applied ($C_{\Delta n} = 1.0$). Where the end distance is less than required for $C_{\Delta n} = 1.0$, but not less than the minimum distance for $C_{\Delta n} = 0.5$, the geometry factor is applied as calculated by Equation 13.2.3.2-1.

$$C_{\Delta n} = \frac{\text{Actual end distance}}{\text{Minimum end distance for full design value}} \qquad (13.2.3.2\text{-}1)$$

In cases of nonparallel main and side members, as illustrated in Figure 12.3.2.3-1, the minimum end distance for the full design value for the fastener for any end is the end distance that produces the same shear area as would be determined by the end distance of Table 13.2.3.2-1 for parallel members with thicknesses equal to the length of fastener bearing in each member. In cases where the shear area is less than that required by the end distances for full design value in Table 13.2.3.2-1, the geometry factor for end distance is applied as calculated by Equation 13.2.3.2-2, except that in no case may a fastener be installed such that the shear area is less than half of the shear area associated with full end distance and full design value.

$$C_{\Delta n} = \frac{\text{Actual shear area}}{\text{Minimum shear area for full design value}} \qquad (13.2.3.2\text{-}2)$$

13.2.3.3 Geometry Factor for Spacing, $C_{\Delta s}$ Spacing requirements for fasteners in a row are shown in Table 13.2.3.3-1. Where the spacing requirements for full design value are met, no adjustment is necessary ($C_{\Delta s} = 1.0$). Where spacing is less than the value required for full design value, but not less than the minimum for reduced design value, the geometry factor is calculated by Equation 13.2.3.3-1.

$$C_{\Delta s} = \frac{\text{Actual spacing}}{\text{Minimum spacing for full design value}} \qquad (13.2.3.3\text{-}1)$$

TABLE 13.2.3.3-1 Spacing Requirements within a Row (from *NDS*® [1])

Direction of Loading	Minimum Spacing	Minimum Spacing for $C_\Delta = 1.0$
Parallel-to-grain	$3D$	$4D$
Perpendicular-to-grain	$3D$	Required spacing for attached members

TABLE 13.2.3.3-2 Spacing Requirements between Rows (from *NDS*® [1])

Direction of Loading	Minimum Spacing between Rows
Parallel-to-grain	$1.5D$
Perpendicular-to-grain[(1)]:	
when $\ell/D \leq 2$	$2.5D$
when $2 < \ell/D < 6$	$(5\ell + 10D)/8$
when $\ell/D \geq 6$	$5D$

[a] The ℓ/D ratio used to determine the minimum spacing between rows is the lesser of:

(a) ℓ_m/D
(b) ℓ_s/D

Minimum spacing requirements between rows are shown in Table 13.2.3.3-2. There are no provisions for a geometry factor based on reduced spacing between rows, so the geometry factor based on spacing between rows is always 1.0. However, the designer must be aware that group tear-out failure calculations (Chapter 12) often govern the design for closely spaced rows of fasteners.

13.2.4 Single Shear Connections

Design values for dowel-type fasteners in single shear are calculated using Equations 13.2.4-1 through 13.2.4-6. Values of Z are calculated for each mode, with the lowest value of Z used as the fastener design value.

$$Z_{\mathrm{Im}} = \frac{D\ell_m F_{em}}{R_d} \quad \text{Mode I}_\mathrm{m} \tag{13.2.4-1}$$

$$Z_{\mathrm{Is}} = \frac{D\ell_s F_{es}}{R_d} \quad \text{Mode I}_\mathrm{s} \tag{13.2.4-2}$$

$$Z_{\mathrm{II}} = \frac{k_1 D\ell_s F_{es}}{R_d} \quad \text{Mode II} \tag{13.2.4-3}$$

$$Z_{\text{IIIm}} = \frac{k_2 D \ell_m F_{em}}{(1 + 2R_e)R_d} \quad \text{Mode III}_\text{m} \tag{13.2.4-4}$$

$$Z_{\text{IIIs}} = \frac{k_3 D \ell_s F_{em}}{(2 + R_e)R_d} \quad \text{Mode III}_\text{s} \tag{13.2.4-5}$$

$$Z_{\text{IV}} = \frac{D^2}{R_d}\sqrt{\frac{2F_{em}F_{yb}}{3(1 + R_e)}} \quad \text{Mode IV} \tag{13.2.4-6}$$

where:

$$k_1 = \frac{\sqrt{R_e + 2R_e^2\left(1 + R_t + R_t^2\right) + R_t^2 R_e^3} - R_e\left(1 + R_t\right)}{1 + R_e} \tag{13.2.4-7}$$

$$k_2 = -1 + \sqrt{2\left(1 + R_e\right) + \frac{2F_{yb}\left(1 + 2R_e\right)D^2}{3F_{em}\ell_m^2}} \tag{13.2.4-8}$$

$$k_3 = -1 + \sqrt{\frac{2\left(1 + R_e\right)}{R_e} + \frac{2F_{yb}\left(2 + R_e\right)D^2}{3F_{em}\ell_s^2}} \tag{13.2.4-9}$$

D = fastener diameter (in.)
F_{em} = dowel bearing strength of main member (psi)
F_{es} = dowel bearing strength of side member (psi)
F_{yb} = dowel bending yield strength of fastener (psi)
ℓ_m = main member dowel bearing length (in.)
ℓ_s = side member dowel bearing length (in.)
R_d = reduction term from Table 13.2.4-1
$R_e = F_{em}/F_{es}$
$R_t = \ell_m/\ell_s$

The fastener diameter to be used in Equations 13.2.4-1 through 13.2.4-9 is the body diameter for full-body fasteners and the reduced or root diameter for threaded fasteners, except that the body diameter may be used if the threaded portion does not bear over more than one-fourth of the bearing length of the fastener. Dimensions for common fasteners including bolts, lag screws, wood screws, and nails are included in the appendix. Tabulated reference design values for fasteners in the *National Design Specification*® [1] assume full-body diameter for bolts, wood screws, and nails and assume root or reduced diameter for lag screws.

The reduction term, R_d, is a function of fastener diameter, yield mode, and direction of load with respect to grain, and is shown in Table 13.2.4-1.

The dowel bearing strength used in the yield limit equations may be obtained from Equations 13.2.4-10 through 13.2.4-12 for parallel- and perpendicular-to-grain loading. For loading at angle to grain, the dowel bearing strength is

TABLE 13.2.4-1 Reduction Factor, R_d, (from *NDS*® [1])

Fastener Size	Yield Mode	Reduction Term, R_d
0.25 in. $<$ D $\leq$1 in.	I_m, I_s	$4K_\theta$
	II	$3.6K_\theta$
	III_m, III_s, IV	$3.2K_\theta$
D $<$ 0.25 in.	I_m, I_s, II, III_m, III_s, IV	K_D [a]

Notes:

$K_\theta = 1 + 0.25\left(\theta/90^\circ\right)$

θ = maximum angle of load to grain $\left(0^\circ \leq \theta \leq 90^\circ\right)$ for any member in a connection

D = diameter (in.)

$K_D = 2.2$ for $D \leq 0.17$ in.

$K_D = 10D + 0.5$ for 0.17 in. $< D \leq 0.25$ in.

[a] For threaded fasteners where nominal diameter is greater than or equal to 0.25 in. and root diameter is less than 0.25 in., $R_d = K_D K_\theta$.

determined by use of the Hankinson formula, Equation 11.4.2.1-1. Dowel bending yield strength values, F_{yb}, for various fasteners are given in the appendix.

$$F_{e\|} = 11{,}200G \tag{13.2.4-10}$$

$$F_{e\perp} = \frac{6100G^{1.45}}{\sqrt{D}} \tag{13.2.4-11}$$

$$F_e = 16{,}600G^{1.84} \text{ for } D < \tfrac{1}{4}\text{ in} \tag{13.2.4-12}$$

EXAMPLE 13.2.4-1 SINGLE SHEAR CONNECTION

Given: A horizontal $5\frac{1}{8}$ in. × 12 in. 24F-1.8E DF glued laminated timber member $\left(E_{axial} = 1.7\left(10^6\text{ psi}\right)\right)$ is fastened to an $8\frac{3}{4}$ in. × 9 in. Combination 2 DF glued laminated timber column $\left(E_{axial} = 1.7\left(10^6\text{ psi}\right)\right)$ with two $\frac{3}{4}$ in. through bolts as shown in Figure 13.2.4-1.

Wanted: Determine the maximum downward load that can be transferred by the connection from the beam to the column assuming a load duration factor of 1.25.

Approach: Edge distance, end distance, and spacing will be evaluated, and the geometry factor will be applied if needed. The group action factor will be calculated. Reference design values will be calculated using Equations 13.2.4-1 through 13.2.4-6.

Solution: Assuming the bolts are centered on the column, the edge distances in the vertical member are $4\frac{1}{2}$ in. each. The vertical member is assumed to

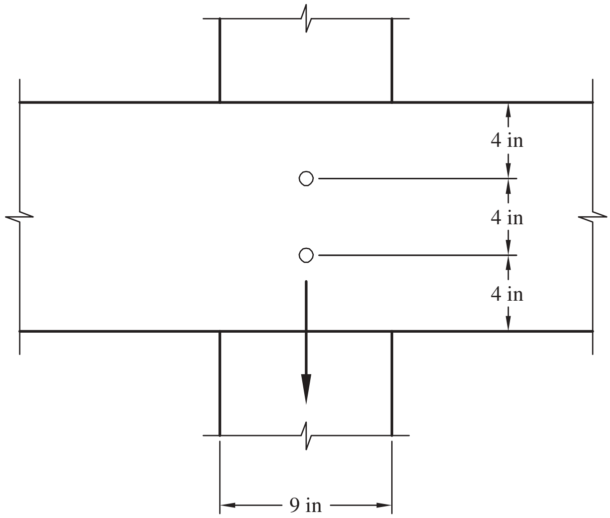


Figure 13.2.4-1 Single-shear connection, beam to column—Example 13.2.4-1.

extend upward past the connection; therefore, end distance requirements are not applicable in this case. The load direction is downward; therefore, for both members, there is a single row of two bolts.

Required edge distance for vertical member (Table 13.2.3.1-1):

$$\frac{\ell_m}{D} = \frac{8.75\,\text{in}}{0.75\,\text{in}} = 11.67$$

$$\frac{\ell_s}{D} = \frac{5.125\,\text{in}}{0.75\,\text{in}} = 6.83$$

$$edge = 4.5\ \text{in} \geq 1.5D = 1.5\,(0.75\,\text{in}) = 1.125\,\text{in} \qquad \therefore\ C_\Delta = 1.0$$

Required edge distance for horizontal member (13.2.3.1-1):

$$edge_{loaded} = 4\,\text{in} \geq 1.5D = 1.5\,(0.75\,\text{in}) = 1.125\,\text{in} \qquad \therefore\ C_\Delta = 1.0$$

$$edge_{unloaded} = 4.0\,\text{in} \geq 4D = 4\,(0.75\,\text{in}) = 3.00\,\text{in} \qquad \therefore\ C_\Delta = 1.0$$

Side member effective area:
For the side member, the equivalent cross-sectional area for group action factor calculations is the product of the member thickness and the minimum required spacing between rows loaded perpendicular-to-grain. For the case of $\ell/D = 6.83$, Table 13.2.3.3-2 indicates that the value to use is $5D$.

$$A_s = b\,(5D)$$

$$A_s = (5.125\,\text{in})\,(5)\,(0.75\,\text{in})$$

$$A_s = 19.22\,\text{in}^2$$

Main member area:

$$A_m = bd = (8.75\,\text{in})\,(9\,\text{in}) = 78.75\,\text{in}$$

Side member modulus of elasticity, perpendicular-to-grain (Section 11.5.4.1):

$$E'_{\perp s} = \frac{E'_s}{20} = \frac{1.7\left(10^6\,\text{psi}\right)}{20} = 85{,}000\,\text{psi}$$

Main member modulus of elasticity:

$$E'_m = 1.7\left(10^6\,\text{psi}\right)$$

Member stiffness ratio:

$$R_{EA} = \frac{E'_{\perp s}A_s}{E'_m A_m} = \frac{(85{,}000\,\text{psi})\,(19.22\,\text{in})}{1.7\left(10^6\,\text{psi}\right)(78.75\,\text{in})} = 0.012$$

Load-slip modulus (Section 11.5.4):
The fastener load-slip modulus perpendicular-to-grain is estimated as half of the parallel-to-grain value and divided by 2, because the connection is in single-shear:

$$\gamma_\perp = \frac{\gamma}{2\,(2)} = \frac{180{,}000D^{1.5}}{4} = \frac{(180{,}000)\left(0.75^{1.5}\right)}{4} = 29{,}200\,\text{lb/in}$$

Group action factor (Equation 11.5.4-1):

$$u = 1 + n_j\gamma\frac{s}{2}\left[\frac{1}{E_mA_m} + \frac{1}{E_sA_s}\right]$$

$$u = 1 + 1\left(29{,}200\frac{\text{lb}}{\text{in}}\right)\left(\frac{4\,\text{in}}{2}\right)$$

$$\times\left[\frac{1}{(1{,}700{,}000\,\text{psi})\left(78.75\,\text{in}^2\right)} + \frac{1}{(85{,}000\,\text{psi})\left(19.22\,\text{in}^2\right)}\right]$$

$$u = 1.037$$

$$m = u - \sqrt{u^2 1} = 1.037 - \sqrt{1.037^2 - 1} = 0.762;$$

$$C_g = \left[\frac{m(1-m^{2n})}{n[(1+R_{EA}m^n)(1+m) - 1 + m^{2n}]}\right]\left[\frac{1+R_{EA}}{1-m}\right]$$

$$C_g = \left[\frac{0.762(1-0.762^{2(2)})}{2[(1+(0.012)0.762^2)(1+0.762) - 1 + 0.762^{2(2)}]}\right]\left[\frac{1+0.012}{1-0.762}\right]$$

$$C_g = 0.966$$

Fastener yield calculation inputs:

$$F_{yb} = 45{,}000 \text{ psi from the appendix}$$

$$G = 0.50 \text{ from Table 2.2-1}$$

$$F_{em} = F_{e||} = 11{,}200G = 11{,}200\,(0.5) = 5600\,\text{psi}$$

$$F_{es} = F_{e\perp} = \frac{6100G^{1.45}}{\sqrt{D}} = \frac{6100\,(0.5)^{1.45}}{\sqrt{0.75}} = 2580\,\text{psi}$$

$$R_e = \frac{F_{em}}{F_{es}} = \frac{5600\,\text{psi}}{2580\,\text{psi}} = 2.17$$

$$\ell_m = 8.75 \text{ in}$$

$$\ell_s = 5.125 \text{ in}$$

$$K_\theta = 1 + 0.25(\theta/90) = 1 + 0.25(90/90) = 1.25$$

$R_d = 4K_\theta = 4(1.25) = 5.00$ for yield modes I_m and I_s;

$R_d = 3.6K_\theta = 3.6(1.25) = 4.50$ for yield mode II;

$R_d = 3.2K_\theta = 3.2(1.25) = 4.00$ for yield modes III_m, and III_s, and IV.

$$R_t = \ell_m/\ell_s = 8.75/5.125 = 1.707.$$

$$k_1 = \frac{\sqrt{R_e + 2R_e^2\left(1 + R_t + R_t^2\right) + R_t^2R_e^3} - R_e\left(1 + R_t\right)}{1 + R_e}$$

$$k_1 = \frac{\sqrt{\begin{array}{c}2.171 + 2\,(2.171)^2\left(1 + 1.707 + 1.707^2\right)\\ + (1.707)^2\,(2.171)^3\end{array}} - 2.171\,(1 + 1.707)}{1 + 2.171}$$

$$k_1 = 1.054$$

$$k_2 = -1 + \sqrt{2\left(1 + R_e\right) + \frac{2F_{yb}\left(1 + 2R_e\right)D^2}{3F_{em}\ell_m^2}}$$

$$k_2 = -1 + \sqrt{2\,(1 + 2.171) + \frac{2\,(45{,}000\,\text{psi})\,(1 + 2\,(2.171))\,(0.75\,\text{in})^2}{3\,(5600\,\text{psi})\,(8.75\,\text{in})^2}}$$

$$k_2 = 1.560$$

$$k_3 = -1 + \sqrt{\frac{2\left(1 + R_e\right)}{R_e} + \frac{2F_{yb}\left(2 + R_e\right)D^2}{3F_{em}\ell_s^2}}$$

$$k_3 = -1 + \sqrt{\frac{2\,(1 + 2.171)}{2.171} + \frac{2\,(45{,}000\,\text{psi})\,(2 + 2.171)\,(0.75\,\text{in})^2}{3\,(5600\,\text{psi})\,(5.125\,\text{in})^2}}$$

$$k_3 = 0.844$$

Fastener yield calculations (Equations 13.2.4-1 through 13.2.4-6):

$$Z_{\text{Im}} = \frac{D\ell_{\text{m}}F_{\text{em}}}{R_d} = \frac{(0.75\text{ in})(8.75\text{ in})(5600\text{ psi})}{5.00} = 7350\text{ lb}$$

$$Z_{\text{Is}} = \frac{D\ell_{\text{s}}F_{\text{es}}}{R_d} = \frac{(0.75\text{ in})(5.125\text{ in})(2580\text{ psi})}{5.00} = 1983\text{ lb}$$

$$Z_{\text{II}} = \frac{k_1 D\ell_{\text{s}}F_{\text{es}}}{R_d} = \frac{(1.054)(0.75\text{ in})(5.125\text{ in})(2580\text{ psi})}{4.50} = 2323\text{ lb}$$

$$Z_{\text{IIIm}} = \frac{k_2 D\ell_{\text{s}}F_{\text{em}}}{(1+2R_{\text{e}})R_d} = \frac{(1.560)(0.75\text{ in})(8.75\text{ in})(5600\text{ psi})}{(1+2(2.171))(4.00)} = 2683\text{ lb}$$

$$Z_{\text{IIIs}} = \frac{k_3 D\ell_{\text{s}}F_{\text{em}}}{(2+R_{\text{e}})R_d} = \frac{(0.844)(0.75\text{ in})(5.125\text{ in})(5600\text{ psi})}{(2+2.171)(4.00)} = 1089\text{ lb}$$

$$Z_{\text{IV}} = \frac{D^2}{R_d}\sqrt{\frac{2F_{\text{em}}F_{\text{yb}}}{3(1+R_{\text{e}})}} = \frac{(0.75\text{ in})^2}{4.00}\sqrt{\frac{2(5600\text{ psi})(45{,}000\text{ psi})}{3(1+2.171)}} = 1024\text{ lb}$$

Adjusted design value for $\frac{3}{4}$ in. bolt ($Z = Z_{\text{IV}}$):

$$Z' = ZC_D C_g = 1024\frac{\text{lb}}{\text{bolt}}\,(1.25)\,(0.966) = 1236\frac{\text{lb}}{\text{bolt}}$$

Total fastener capacity (2 bolts):

$$P' = (2\text{ bolts})\left(1236\frac{\text{lb}}{\text{bolt}}\right) = 2472\text{ lb}$$

<u>**Answer:**</u> The maximum vertical (downward) load that can be transferred from the horizontal member to the vertical column is 2,470 lb (load duration factor 1.25).

<u>**Discussion:**</u> Complete design must consider member failure modes discussed in Chapter 12. In addition, since the horizontal member is attached to the side of the main member, prying action should also be checked. The downward action of the side member on the bolts will cause the side member to bear on the main member at the bottom corner of the side member and will also tend to pull out the top bolt. Bearing stresses at the bottom should be checked to ensure that crushing does not occur, and sufficient washer size must be provided for the top bolt to resist the bolt head being pulled through or wood crushing under the washer. If the top bolt in such an arrangement is loaded in withdrawal, such as in the case of a lag screw, the combined effect of lateral and withdrawal load should be considered for the top bolt. Finally, the effect of eccentricity of the load on the vertical member should be considered in the analysis of the vertical member capacity.

13.2.5 Double Shear Connections

Reference design values for dowel-type fasteners in double shear may be calculated using Equations 13.2.5-1 through 13.2.5-4 or may be obtained for common cases from tables in the *National Design Specification*® [1]. As with single shear, values of Z are calculated for each mode, with the lowest value of Z used as the design value for the fastener. Calculated and tabulated values assume symmetric connections with side members of equal thickness and load orientation with respect to grain. Yield modes II and III_m are not applicable to double shear connections.

$$Z_{\text{Im}} = \frac{D \ell_{\text{m}} F_{\text{em}}}{R_d} \quad \text{Mode I}_\text{m} \tag{13.2.5-1}$$

$$Z_{\text{Is}} = \frac{2D \ell_{\text{s}} F_{\text{es}}}{R_d} \quad \text{Mode I}_\text{s} \tag{13.2.5-2}$$

$$Z_{\text{IIIs}} = \frac{2k_3 D \ell_{\text{s}} F_{\text{em}}}{(2 + R_{\text{e}})R_d} \quad \text{Mode III}_\text{s} \tag{13.2.5-3}$$

$$Z_{\text{IV}} = \frac{2D^2}{R_d}\sqrt{\frac{2F_{\text{em}}F_{\text{yb}}}{3(1 + R_{\text{e}})}} \quad \text{Mode IV} \tag{13.2.5-4}$$

It should be noted that ℓ_s is the effective dowel bearing length of one side member (assumed to be equal to the other). If side members of different thicknesses are used, ℓ_s should be chosen equal to the thinner of the side members.

13.2.6 Metal to Wood Connections

Lateral design values for wood connected to metal plates may be calculated using the yield mode equations of the previous sections or may be obtained in tabular form for common combinations of metal plate thickness and wood member size in the *National Design Specification*® [1]. The dowel bearing strength, F_e, of the steel plate(s) in the *NDS*® [1] is taken to be 87,000 psi for ASTM A36 steel. Metal side plates must also have adequate end and edge distances and be of sufficient thickness to develop the capacities of the fasteners.

EXAMPLE 13.2.6-1 TENSION SPLICE CONNECTION

Given: Two $\frac{3}{8}$ in. × 4 in. steel side plates are to be used to splice the ends of two $6\frac{3}{4}$ in. × $8\frac{1}{4}$ in. Combination 48 SP glued laminated timbers as illustrated in Figure 13.2.6-1. The timbers will carry a total tensile load (snow plus dead) of 38,000 lb. The governing load duration factor is $C_D = 1.15$.

Wanted: Design a suitable double-shear connection using 1 in. diameter through-bolts.

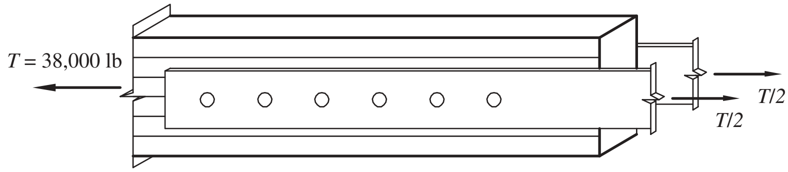


Figure 13.2.6-1 Tension splice connection—Example 13.2.6-1.

Approach: A preliminary design will be obtained using tabular design values for through bolts in double shear from the *National Design Specification®* [1] in glulam members with $\frac{1}{4}$ in. side plates. Reference design values for the through bolts will be calculated from Equations 13.2.5-1 through 13.2.5-4 and adjusted as required. Edge distance, end distance, and spacing requirements will be evaluated.

Solution: For a preliminary design, the design value of 1 in. diameter bolts in double shear for glued laminated timber for main member width of $6\frac{3}{4}$ in. and $\frac{1}{4}$ in. steel side plates is obtained from Table 11*I* of the *National Design Specification®* [1]. From the table, a value of 5960 lb is obtained, which assumes normal duration of load and dry service conditions. The value also assumes edge, end, and spacing requirements have been met, and is not adjusted for group action. The design value is based on metal side plate thickness of $\frac{1}{4}$ in., and may be considered conservative for thicker plate values.

Estimated bolt design value ($C_g \approx 0.95$):

$$Z'_{est} = ZC_D C_g = 5960\,\text{lb}\,(1.15)\,(0.95) = 6511\,\text{lb}$$

Estimated number of bolts required:

$$n \approx \frac{T}{Z'_{est}} = \frac{38{,}000\,\text{lb}}{6{,}511\ \text{lb/bolt}} = 5.8\ \text{bolts} \qquad \therefore\ \text{Use 6 bolts}$$

Glulam design values (*AITC 117* [2]):

$$E' = EC_M C_t = 1{,}700{,}000\,\text{psi}\,(1.0)\,(1.0) = 1{,}700{,}000\,\text{psi}$$

$$F'_t = F_t C_D C_M C_t = 1400\,\text{psi}\,(1.15)\,(1.0)(1.0) = 1610\,\text{psi}$$

$$F'_v = F_v C_D C_M C_t C_{vr} = 300\,\text{psi}\,(1.15)\,(1.0)(1.0)(0.72) = 248\,\text{psi}$$

Bolt spacing to prevent row tear-out (Section 12.3.4.1):

$$s_1 \geq \frac{Z'}{F'_v t} = \frac{6511\,\text{lb}}{(248\,\text{psi})\,(6.75\,\text{in})} = 3.89\,\text{in} \qquad \therefore\ \text{Use 4 in}$$

Member stiffness ratio (Section 11.5.4):

$$E_sA_s = 29.0\left(10^6\text{ psi}\right)2\,(0.375\text{ in})\,(4\text{ in}) = 29.0\left(10^6\text{ psi}\right)\left(3.0\text{ in}^2\right)$$
$$= 87.0\left(10^6\text{ lb}\right)$$
$$E_mA_m = 1.7\left(10^6\text{ psi}\right)(6.75\text{ in})\,(8.25\text{ in}) = 1.7\left(10^6\text{ psi}\right)\left(55.7\text{ in}^2\right)$$
$$= 94.7\left(10^6\text{ lb}\right)$$
$$R_{EA} = \frac{E_sA_s}{E_mA_m} = \frac{87.0\left(10^6\text{ lb}\right)}{94.7\left(10^6\text{ lb}\right)} = 0.919$$

Load-slip modulus (Section 11.5.4):

$$\gamma = 270{,}000D^{1.5} = (270{,}000)\left(1.0^{1.5}\right) = 270{,}000\text{ lb/in}$$

Group action factor (Section 11.5.4):

$$u = 1 + n_j\gamma\frac{s}{2}\left[\frac{1}{E_mA_m} + \frac{1}{E_sA_s}\right]$$
$$u = 1 + 1\left(270{,}000\frac{\text{lb}}{\text{in}}\right)\left(\frac{4\text{ in}}{2}\right)\left[\frac{1}{94.7\left(10^6\text{ lb}\right)} + \frac{1}{87.0\left(10^6\text{ lb}\right)}\right] = 1.012$$
$$m = u - \sqrt{u^2 - 1} = 1.012 - \sqrt{1.012^2 - 1} = 0.857;$$
$$C_g = \left[\frac{m(1 - m^{2n})}{n[(1 + R_{EA}m^n)(1 + m) - 1 + m^{2n}]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$
$$C_g = \left[\frac{0.857(1 - 0.857^{2(6)})}{6[(1 + (0.919)0.857^6)(1 + 0.857) - 1 + 0.857^{2(6)}]}\right]\left[\frac{1 + 0.919}{1 - 0.857}\right]$$
$$C_g = 0.956$$

Fastener yield calculation inputs:

$$D = 1\text{ in}$$
$$\ell_m = 6.75\text{ in}$$
$$\ell_s = 0.375\text{ in}$$
$$F_{em} = (11{,}200\text{ psi})\,G = (11{,}200\text{ psi})\,0.55 = 6150\text{ psi}$$
$$F_{es} = 87{,}000\text{ psi}$$

$$F_{yb} = 45{,}000\,\text{psi}$$

$$\theta_{\max} = 0^\circ$$

Fastener yield calculation results:

$$Z_{\text{Im}} = 10{,}378\,\text{lb} \;\; (\text{Mode I}_{\text{m}})$$

$$Z_{\text{Is}} = 16{,}313\,\text{lb} \;\; (\text{Mode I}_{\text{s}})$$

$$Z_{\text{IIIs}} = 6338\,\text{lb} \;\; (\text{Mode III}_{\text{s}})$$

$$Z_{\text{IV}} = 8204\,\text{lb} \;\; (\text{Mode IV})$$

Adjusted design value ($Z = Z_{\text{IIIs}}$):

$$Z' = Z(C_D)(C_g) = 6338\,\text{lb} \;(1.15)(0.956) = 6968\,\text{lb}$$

Adjusted design capacity based on fastener yielding:

$$T' = nZ' = 6\,(6968\,\text{lb}) = 41{,}800\,\text{lb} \geq T = 38{,}000\,\text{lb}$$

Edge distance:

$$edge = \frac{8.25\,\text{in}}{2} = 4.125\,\text{in} \geq 1.5D = 1.5\,(1.0\,\text{in}) = 1.5\,\text{in} \qquad \therefore \;\; C_\Delta = 1.0$$

Minimum end distance:

$$e \geq 7D = 7\,(1.0\,\text{in}) = 7.0\,\text{in}$$

Minimum spacing:

$$s_1 \geq 4D = 4\,(1.0\,\text{in}) = 4.0\,\text{in}$$

<u>**Answer:**</u> The double shear splice connection requires: six 1 in. diameter bolts on each side of the splice (12 bolts total), spaced at 4 in. and centered in two $\frac{3}{8}$ in. × 4 in. steel plates. A 7 in. end distance should be maintained in timbers.

<u>**Discussion:**</u> Since the allowable load for the bolts was greater than the design load, the bolt spacing or end distances could possibly be reduced. However, unless necessary to accommodate other constraints, end and spacing distances should not be detailed less than the minimum values for full design load. A complete design would also require evaluation of member failure modes according to Chapter 12. The side plates must also be examined in accordance with standard practices such as specified in the *Steel Construction Manual* [3].

EXAMPLE 13.2.6-2 FASTENER WITH LOAD AT ANGLE TO GRAIN

Given: A tension connection consists of two 3 in. × $\frac{1}{4}$ in. steel straps with a $\frac{3}{4}$ in. diameter through-bolt attached to a 5 in. × $6\frac{7}{8}$ in., Combination 48 SP, glued laminated timber as shown in Figure 13.2.6-2. The straps are at an angle of 30° with the main member.

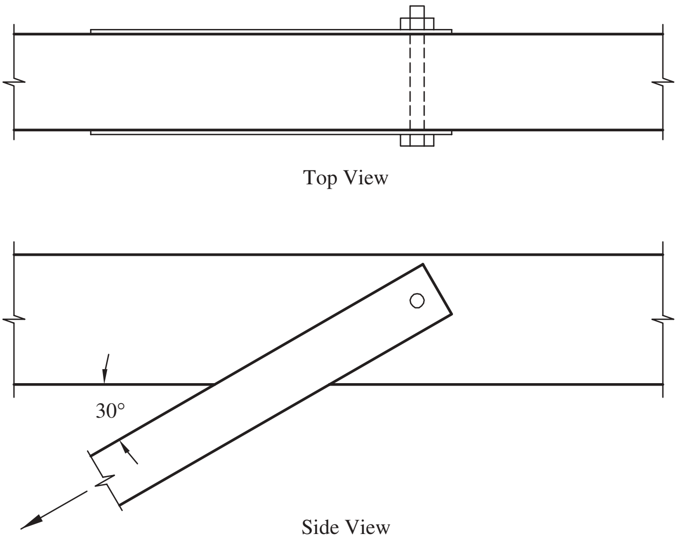


Figure 13.2.6-2 Tension splice connection—Example 13.2.6-2.

Wanted: Determine the capacity of the connection for normal load duration.

Approach: Since the load is at angle to grain in the main member, the yield mode equations will be used for double shear (Equations 13.2.5-1 through 13.2.5-4). For the main member, the dowel bearing stress will be calculated using Equation 11.4.2.1-1 for an angle of 30°.

Solution:

Dowel bearing design value for SP glulam (Equation 11.4.2.1-1):

$$F_{e\|} = (11{,}200\,\text{psi})\,G = (11{,}200\,\text{psi})\,0.55 = 6160\,\text{psi}$$

$$F_{e\|} = \frac{(6{,}100\,\text{psi})\,G^{1.45}}{\sqrt{D}} = \frac{(6{,}100\,\text{psi})\,(0.55)^{1.45}}{\sqrt{0.75}} = 2960\,\text{psi}$$

$$F_{em} = F_{e\theta} = F_{e30^\circ} = \frac{F_{e\|}F_{e\perp}}{F_{e\|}\sin^2\theta + F_{e\perp}\cos^2\theta}$$

$$F_{em} = \frac{(6160\text{ psi})(2190\text{ psi})}{(6160\text{ psi})\sin^2 30^\circ + (2190\text{ psi})\cos^2 30^\circ} = 4850\text{ psi}$$

Fastener yield calculation inputs:

$$\ell_m = 5.0\,\text{in}$$

$$\ell_s = 0.25\,\text{in}$$

$$F_{es} = 87{,}000\,\text{psi}$$

$$F_{yb} = 45{,}000\,\text{psi}$$

$$R_e = \frac{F_{\text{em}}}{F_{\text{es}}} = \frac{F_{\text{e}\theta}}{F_{\text{es}}} = \frac{4{,}850\ \text{psi}}{87{,}000\ \text{psi}} = 0.056$$

$$K_\theta = 1 + 0.25(\theta/90) = 1 + 0.25(30/90) = 1.083.$$

$R_d = 4K_\theta = 4(1.083) = 4.33$ for yield modes I_m and I_s;

$R_d = 3.2K_\theta = 3.2(1.083) = 3.47$ for yield modes III_m and III_s and IV.

$$R_t = \frac{\ell_m}{\ell_s} = \frac{5.00\,\text{in}}{0.25\,\text{in}} = 20.0$$

$$k_3 = -1 + \sqrt{\frac{2\left(1+R_e\right)}{R_e} + \frac{2F_{yb}\left(2+R_e\right)D^2}{3F_{em}\ell_s^2}}$$

$$k_3 = -1 + \sqrt{\frac{2\,(1+0.056)}{0.056} + \frac{2\,(45{,}000\,\text{psi})\,(2+0.056)\,(0.75\,\text{in})^2}{3\,(4850\,\text{psi})\,(0.25\,\text{in})^2}}$$

$$k_3 = 11.34$$

Fastener yield mode calculations (Equations 13.2.5-1 through 13.2.5-4):

$$Z_{\text{Im}} = \frac{D\ell_\text{m}F_{\text{em}}}{R_d} = \frac{0.75\,(5.0\ \text{in})(4850\ \text{psi})}{4.33} = 4{,}200\,\text{lb}$$

$$Z_{\text{Is}} = \frac{2D\ell_\text{s}F_{\text{es}}}{R_d} = \frac{2(0.75\ \text{in})\,(0.25\ \text{in})(87{,}000\ \text{psi})}{4.33} = 7{,}540\,\text{lb}$$

$$Z_{\text{IIIs}} = \frac{2K\ell_3D\ell_\text{s}F_{\text{em}}}{(2+R_e)R_d} = \frac{2(11.34)(0.75\ \text{in})\,(0.25\ \text{in})(4850\ \text{psi})}{(2+0.056)3.47} = 2{,}890\,\text{lb}$$

$$Z_{\text{IV}} = \frac{2D^2}{R_d}\sqrt{\frac{2F_{\text{em}}F_{\text{yb}}}{3(1+R_\text{e})}} = \frac{2(0.75\,\text{in})^2}{3.47}\sqrt{\frac{2(4850\,\text{psi})(45{,}000\,\text{psi})}{3(1+0.056)}} = 3{,}810\,\text{lb}$$

Loaded edge distance:

$$edge_{loaded} = \frac{6.88\,\text{in}}{2} = 3.44\,\text{in} \geq 4D = 4\,(0.75\ \text{in}) = 3.0\,\text{in} \qquad \therefore\ C_\Delta = 1.0$$

Unloaded edge distance:

$$edge_{unloaded} = \frac{6.88\,\text{in}}{2} = 3.44\,\text{in} \geq 1.5D$$
$$= 1.5\,(0.75\ \text{in}) = 1.125\,\text{in} \qquad \therefore\ C_{\Delta} = 1.0$$

Total design capacity ($Z = Z_{IIIs}$):

$$Z' = Z\left(C_D\right) = (2890\,\text{lb})\,(1.0) = 2890\,\text{lb}$$

Answer: The capacity of the given connection for normal load duration is 2890 lb. (Mode III_s behavior governs.)

Discussion: The capacity of the side plates themselves must be evaluated. In addition, effective section for shear as well as the net section for tension in the timber must be evaluated with consideration made for all of the loads on the member.

13.2.7 Wood to Concrete Connections

The lateral design values for embedded bolts connecting sawn lumber and structural composite lumber wood members to cast-in-place concrete may be found in tabular form in the *National Design Specification*® [1]. These values may also be calculated using the single shear yield mode equations of Section 13.2.4. In addition to the edge distance, end distance, and spacing requirements for the wood side member, bolts must have adequate embedment and adequate edge and end distances in concrete to prevent failure of concrete in accordance with the *Building Code Requirements for Structural Concrete* [4] by the American Concrete Institute. In general, the wood must also be preservative treated when used in contact with concrete. Proprietary fasteners and connectors for wood to concrete applications are also commercially available.

13.3 DOWEL-TYPE FASTENERS LOADED IN WITHDRAWAL

Lag screws, wood screws, nails, and spikes are commonly used to resist withdrawal loading conditions in wood construction. Drift pins are also occasionally loaded in withdrawal. Where drift pins are loaded in withdrawal, they must be installed in seasoned wood in prebored holes having a diameter $\frac{1}{8}$ in. smaller than the actual diameter of the pin.

Withdrawal capacity is a function of the tensile capacity of the fastener and the amount of force that can be transferred between fastener and wood either by threads or friction. The amount of load that can be safely transferred between wood and fastener is generally given in terms of W in pounds per inch of the threaded portion in the main member for lag screws and wood screws or pounds per inch of penetration depth in main member for nails, spikes, and drift pins. The design value is then multiplied by all applicable adjustment factors (Table 11.5-1) and then by the effective withdrawal length to give the total safe withdrawal force for the fastener. This force cannot, however, exceed the safe tensile capacity of the fastener itself. Design values (W) for lag screws, wood screws, nails, and spikes are tabulated in the *National Design Specification® for Wood Construction* [1] for various wood specific gravities and fastener size or diameter. Alternately, the safe withdrawal loads may be calculated using Equation 13.3-1.

$$P_W = W'p \leq F_t A_{net} \tag{13.3-1}$$

where:

P_W = allowable withdrawal load
W' = adjusted withdrawal design value
p = effective fastener penetration
$F_t A_{net}$ = allowable tensile capacity of fastener.

The design value W may be obtained in tabular form as mentioned above or calculated using Equations 13.3-2 through 13.3-6.

$$W = 1800 G^{\frac{3}{2}} D^{\frac{3}{4}} \text{ (for lag screws)} \tag{13.3-2}$$

$$W = 2850 G^2 D \text{ (for wood screws)} \tag{13.3-3}$$

$$W = 1380 G^{\frac{5}{2}} D \text{ (for nails and spikes)} \tag{13.3-4}$$

$$W = 1800 G^2 D \text{ (for post-frame ring-shank nails)} \tag{13.3-5}$$

$$W = 1200 G^2 D \text{ (for drift pins)} \tag{13.3-6}$$

where:

G = the specific gravity based on oven dry weight and oven dry volume (Table 2.2-1)
D = fastener diameter (in.)

13.3.1 Placement of Fasteners for Withdrawal

To ensure that the withdrawal capacity can be developed, fasteners must not be placed too close to other fasteners or to the end and edge of the piece. Nails, spikes, and wood screws must be installed such that splitting does not occur. Lag screws and drift pins must be placed a minimum of 1.5D from the nearest edge, 4D from the nearest end, and 4D from the nearest fastener when loaded in withdrawal without lateral loads. Increased distances may be required for fasteners subject to combined lateral and withdrawal loading.

13.3.2 Withdrawal from End Grain

Equations 13.3-2 through 13.3-6 give reference withdrawal design values for fasteners installed in the side grain of the member. Where lag screws are installed in the end grain of a member and are loaded in withdrawal (load applied parallel-to-grain), the end grain factor, $C_{eg} = 0.75$, must be applied. Wood screws, nails, spikes, and drift pins are not permitted to be used to resist withdrawal loads where installed in the end grain of members.

Typical dimensions of nails, spikes, lag screws, and wood screws, including the lengths of threaded portions and tapered tips, are included in the appendix.

13.4 DOWEL-TYPE FASTENERS LOADED LATERALLY AND IN WITHDRAWAL

Where dowel type fasteners are subject to combined lateral and withdrawal loading, Equations 13.4-1 and 13.4-2 are used to determine the design capacity.

$$Z'_\alpha = \frac{(W'p)\,Z'}{(W'p)\cos^2\alpha + Z'\sin^2\alpha} \quad \text{(lag screws and wood screws)} \qquad (13.4\text{-}1)$$

$$Z'_\alpha = \frac{(W'p)\,Z'}{(W'p)\cos\alpha + Z'\sin\alpha} \quad \text{(nails and spikes)} \qquad (13.4\text{-}2)$$

where:

$Z_\alpha{}'$ = combined allowable load at angle α
α = angle of load with respect to wood surface
$W'p$ = adjusted design value in withdrawal
Z' = adjusted design value for lateral load

EXAMPLE 13.4-1 LAG SCREW LOADED LATERALLY AND IN WITHDRAWAL

Given: A single $\frac{1}{2}$ in. × 5 in. lag screw is loaded by a steel plate at angle $\alpha = 30^\circ$ as shown in Figure 13.4-1. The wood member is $5\frac{1}{8}$ in. × $6\frac{7}{8}$ in., Combination 47 SP, glued laminated timber, and the fastener will be centered in the top of the member (through the wide faces of the laminations). The metal side plate thickness is $\frac{1}{4}$ in. The timber is preservative treated will be exposed to wet conditions. The design load will have load duration associated with snow loads. Assume that the fastener is not close to either end of the timber. The lateral component of the load acts parallel to grain in the main timber member.

Wanted: Determine the allowable load P_{30} for the fastener.

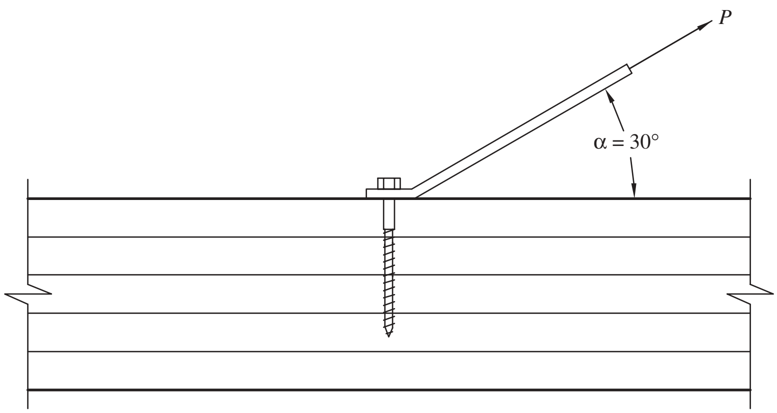


Figure 13.4-1 Lag screw loaded laterally and in withdrawal—Example 13.4-1.

Approach: The allowable load P_{30} will be calculated using Equations 13.2.4-1 through 13.2.4-6 for dowel-type fasteners (single shear), Equations 13.3-1 and 13.3-2 for lag screw withdrawal design values, and Equation 13.4-1 for combined lateral and withdrawal loading. The specific gravity for southern pine of 0.55 (Table 2.2-1) will be used for both lateral and withdrawal design value calculations.

Solution:

Main member dowel bearing length (lag screw dimensions from appendix): Assuming $\frac{1}{16}$ in. thick washer and $\frac{1}{4}$ in. thick steel member, the bearing length is calculated as:

$$\ell_m = L - \tfrac{1}{16}\,\text{in} - \tfrac{1}{4}\,\text{in} - E$$

$$\ell_m = 5.00\,\text{in} - \tfrac{1}{16}\,\text{in} - \tfrac{1}{4}\,\text{in} - \tfrac{5}{16}\,\text{in}$$

$$\ell_m = 4.38\,\text{in} \geq 4D = 4\,(0.5\,\text{in}) = 2.0\,\text{in} \qquad \therefore\ \text{OK}$$

Fastener yield equation inputs:

$$D = D_r = 0.371\,\text{in (from appendix)}$$

$$\theta = \text{ angle of lateral load to grain} = 0^\circ$$

$$F_{em} = F_{e\|} = (11{,}200\,\text{psi})\,G = (11{,}200\,\text{psi})\,0.55 = 6160\,\text{psi}$$

$$R_d = 4.0 = \text{ for modes } \text{I}_\text{m} \text{ and } \text{I}_\text{s}$$

$$R_d = 3.6 \text{ for mode II}$$

$$R_d = 3.2 \text{ for modes } \text{III}_\text{m}, \text{III}_\text{s}, \text{and IV}$$

$$\ell_s = 0.25\,\text{in}$$

$$F_{es} = 87{,}000 \text{ psi}$$

$$F_{yb} = 45{,}000 \text{ psi}$$

$$R_e = \frac{F_{em}}{F_{es}} = \frac{6{,}160 \text{ psi}}{87{,}000 \text{ psi}} = 0.071$$

$$R_t = \frac{\ell_m}{\ell_s} = \frac{4.38 \text{ in}}{0.25 \text{ in}} = 17.5$$

$$k_1 = \frac{\sqrt{R_e + 2R_e^2(1 + R_t + R_t^2) + R_t^2 R_e^3} - R_e(1 + R_t)}{1 + R_e}$$

$$k_1 = \frac{\sqrt{\begin{array}{c}0.071 + 2(0.071)^2(1 + 17.5 + 17.5^2)\\ +(17.5)^2(0.071)^3\end{array}} - 0.071(1 + 17.5)}{1 + 0.071}$$

$$k_1 = 0.509$$

$$k_2 = -1 + \sqrt{2(1 + R_e) + \frac{2F_{yb}(1 + 2R_e)D^2}{3F_{em}\ell_m^2}}$$

$$k_2 = -1 + \sqrt{2(1 + 0.071) + \frac{2(45{,}000 \text{ psi})(1 + 2(0.071))(0.5 \text{ in})^2}{3(6160 \text{ psi})(4.38 \text{ in})^2}}$$

$$k_2 = 0.488$$

$$k_3 = -1 + \sqrt{\frac{2(1 + R_e)}{R_e} + \frac{2F_{yb}(2 + R_e))D^2}{3F_{em}\ell_s^2}}$$

$$k_3 = -1 + \sqrt{\frac{2(1 + 0.071)}{0.071} + \frac{2(45{,}000 \text{ psi})(2 + 0.071)(0.5 \text{ in})^2}{3(6160 \text{ psi})(0.25 \text{ in})^2}}$$

$$k_3 = 7.40$$

Fastener yield calculations:

$$Z_{\text{Im}} = \frac{D\ell_m F_{em}}{R_d} = \frac{0.5 \text{ in}(4.38 \text{ in})(6160 \text{ psi})}{4.0} = 3373 \text{ lb}$$

$$Z_{\text{Is}} = \frac{D\ell_s F_{es}}{R_d} = \frac{0.5 \text{ in}(0.25 \text{ in})(87{,}000 \text{ psi})}{4.0} = 2719 \text{ lb}$$

$$Z_{\text{II}} = \frac{K_1 D\ell_s F_{es}}{R_d} = \frac{0.509(0.5 \text{ in})(0.25 \text{ in})(87{,}000 \text{ psi})}{3.6} = 1538 \text{ lb}$$

$$Z_{\text{IIIm}} = \frac{K_2 D\ell_m F_{em}}{(1 + 2R_e)R_d} = \frac{0.488(0.5 \text{ in})(4.38 \text{ in})(6160 \text{ psi})}{(1 + 2(0.071))(3.2)} = 1801 \text{ lb}$$

$$Z_{\text{IIIs}} = \frac{K_3 D \ell_s F_{em}}{(2+R_e)R_d} = \frac{7.4(0.5\text{ in})(0.25\text{ in})(6160\text{ psi})}{(2+0.071)(3.2)} = 860\text{ lb}$$

$$Z_{\text{IV}} = \frac{D^2}{R_d}\sqrt{\frac{2F_{em}F_{yb}}{3(1+R_e)}} = \frac{(0.5\text{ in})^2}{3.2}\sqrt{\frac{2(6160\text{ psi})(45{,}000\text{ psi})}{3(1+0.071)}} = 1026\text{ lb}$$

Wet-service factor:

From Table 11.5.2-1, $C_M = 0.7$.

Edge distance:

$$edge = \frac{5.125\text{ in}}{2} = 2.56\text{ in} \geq 1.5D = 1.5\,(0.5\text{ in}) = 0.75\text{ in} \quad \therefore\; C_\Delta = 1.0$$

Lateral capacity ($Z = Z_{\text{IIIs}}$):

$$Z' = ZC_DC_M = (860\text{ lb})\,(1.15)\,(0.7) = 692\text{ lb}$$

Withdrawal design value (Equation 13.3-2):

$$W = 1800G^{\frac{3}{2}}D^{\frac{3}{4}} = 1800\,(0.55)^{\frac{3}{2}}\,(0.50)^{\frac{3}{4}} = 437\text{ lb/in}$$

$$W' = WC_DC_M = 437\text{ lb/in}\,(1.15)\,(0.7) = 351\text{ lb/in}$$

Withdrawal capacity (Equation 13.3-1):

The penetration, p, of the threaded portion is given by the threaded portion length in the main member minus the tapered tip. From the appendix, the threaded portion length minus the tapered tip (T-E) is given to be $2\frac{11}{16}$ in. = 2.69 in. Since the fastener penetrates the main member 4.69 in., the threads are fully developed in withdrawal:

$$P'_W = W'p \leq F_tA_{net} = F_t\frac{\pi D_r^2}{4}$$

$$P'_W = 351\text{ lb/in}\,(2.69\text{ in}) \leq (20{,}000\text{ psi})\left(\frac{\pi\,(0.371\text{ in})^2}{4}\right)$$

$$P'_W = 944\text{ lb} \leq 2161\text{ lb}$$

$$P'_W = 944\text{ lb}$$

Combined lateral and withdrawal capacity (Equation 13.4-1):

$$Z'_\alpha = \frac{(W'p)(Z')}{W'p\cos^2\alpha + Z'\sin^2\alpha}$$

$$Z'_{30} = \frac{(944\text{ lb})(692\text{ lb})}{(944\text{ lb})\cos^2 30^\circ + (692\text{ lb})\sin^2 30^\circ} = 742\text{ lb}$$

Answer: The design capacity at an angle of 30° is 742 lb.

13.5 CONCLUSION

Dowel-type fasteners including bolts, lag screws, drift pins, nails, spikes, and wood screws are commonly used in timber connections. These fasteners can transfer loads perpendicular to the fastener axis (lateral load), parallel to the fastener axis (withdrawal loading), or a combination of both. This chapter presented equations for determining connection capacity based on fastener failure modes. Connection capacities based on fastener failure modes must be compared with capacities based on member failure modes and the least value taken as the design value for the connection.

CHAPTER 14

SHEAR PLATES AND SPLIT RINGS

14.1 INTRODUCTION

Shear plates and split rings are used in conjunction with bolts or lag screws to increase the capacity of timber connections. They function by increasing the bearing area of the fasteners at the surfaces of the connected members, where bearing stresses are highest.

Most of the provisions for shear plates and split rings can be traced back to a 1944 report, *Timber-Connector Joints: Their Strength and Design,* by John Scholten [1]. In that study, the author determined relationships between the capacity of various connectors to specific gravity of the wood and performed several experiments to determine the effects of connection geometric parameters, such as end distance, edge distance, and spacing.

For loads at an angle to grain with connector axes at an angle to grain, elliptical functions for spacing between connectors were developed based on research conducted by the Timber Engineering Company (TECO) at the Massachusetts Institute of Technology (MIT) some time prior to 1944 [2]. The connector axis is formed by a line joining the centers of any two adjacent connectors located in the same face of a member in a joint. The angle of axis of the connectors is the angle formed by the axis line of the connectors and the longitudinal axis of the member, as illustrated by angle β in Figure 14.1-1.

Gaps in the data have been filled in using engineering judgment. The 2011 NDS includes several revisions for the spacing, end distance, and edge distance requirements for fasteners installed in sloped end-grain cuts, such as are

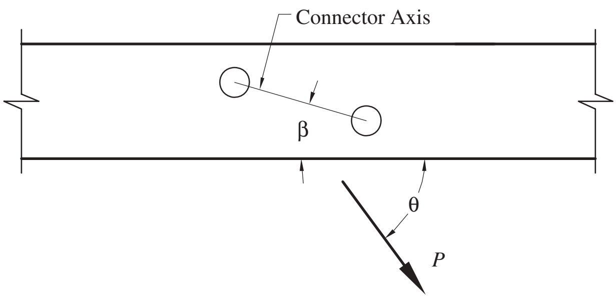


Figure 14.1-1 Angle of connector axis to the grain.

commonly used at arch peak connections. Minimum edge distance provisions and geometry factors for connectors installed in side grain have also been revised.

14.2 CONNECTORS IN SIDE GRAIN

Shear plates and split rings are commonly installed in side grain for use in tension splices and moment splices. For loads parallel- or perpendicular-to-grain, the design is simple and straightforward using tabulated reference design values. For loads at an angle to the grain, Hankinson's formula (Equation 11.4.2.2-1) is used to determine a reference design value. Reference design values for split ring and shear plate connectors require further adjustment by each of the applicable adjustment factors of Tables 11.5-1, except that the adjusted values for shear plates may not exceed maximum values based on failure of the metal parts.

14.2.1 Reference Design Values

The *National Design Specification*® (*NDS*®) *for Wood Construction* [3] includes reference design values (P, Q) for timber connectors loaded in either the parallel-to-grain (P) or perpendicular-to-grain (Q) direction (Tables 14.2.1-1 and 14.2.1-2). For loads acting at other angles to grain, the Hankinson formula is used to establish design values. Reference design values for connectors installed in end grain (Q_{90}) are 60% of the values for connectors installed in side grain with perpendicular-to-grain loading.

Reference design values are tabulated for different thicknesses of the wood member under consideration. Thin members have reduced capacities relative to thick members. Different values are tabulated for single-shear connections and double-shear connections as well.

Connector capacity is dependent on the specific gravity of the surrounding wood, and four species groups for connector design have been established (Table 14.2.1-3). Reference design values are tabulated for each group. Careful review of the Scholten report indicates that the specific gravity divisions presented in the NDS and Table 14.2.1-3 for these species groups is appropriate. Douglas fir and

TABLE 14.2.1-1 Split Ring Connector Unit Reference Design Values

Split Ring Diameter in.	Bolt Diameter in.	Number of Faces of Member with Connectors on Same Bolt	Net Thickness of Member in.	Loaded Parallel to Grain (0°) Design Value, P. per Connector Unit and Bolt, lbs.				Loaded Perpendicular to Grain (90°) Design Value, Q. per Connector Unit and Bolt. lbs.			
				Group A Species	Group B Species	Group C Species	Group D Species	Group A Species	Group B Species	Group C Species	Group D Species
$2\frac{1}{2}$	$\frac{1}{2}$	1	1″ minimum	2630	2270	1900	1640	1900	1620	1350	1160
			$1\frac{1}{2}$″ or thicker	3160	2730	2290	1960	2280	1940	1620	1390
		2	$1\frac{1}{2}$″ minimum	2430	2100	1760	1510	1750	1500	1250	1070
			2″ or thicker	3160	2730	2290	1960	2280	1940	1620	1390
4	$\frac{3}{4}$	1	1″ minimum	4090	3510	2920	2520	2840	2440	2040	1760
			$1\frac{1}{2}$″	6020	5160	4280	3710	4180	3590	2990	2580
			$1\frac{5}{8}$″ or thicker	6140	5260	4380	3790	4270	3660	3050	2630
		2	$1\frac{1}{2}$″ minimum	4110	3520	2940	2540	2980	2450	2040	1760
			2″	4950	4250	3540	3050	3440	2960	2460	2120
			$2\frac{1}{2}$″	5830	5000	4160	3600	4050	3480	2890	2500
			3″ or thicker	6140	5260	4380	3790	4270	3660	3050	2630

Tabulated Design Values[a] Apply to ONE Split Ring and Bolt in Single Shear.

[a]Tabulated lateral design values (P,Q) for split ring connector units shall be multiplied to all applicable adjustment factors (see Table 11.5-1).

TABLE 14.2.1-2 Shear Plate Connector Unit Reference Design Values

Tabulated Design Values[a,b,c] Apply to ONE Shear Plate and Bolt in Single Shear.

Split Plate Diameter	Bolt Diameter	Number of Faces of Member with Connectors on Same Bolt	Net Thickness of Member	Loaded Parallel to Grain (0°) Design Value, P. per Connector Unit and Bolt, lbs.				Loaded Perpendicular to Grain (90°) Design Value, Q. per Connector Unit and Bolt. lbs.			
in.	in.		in.	Group A Species	Group B Species	Group C Species	Group D Species	Group A Species	Group B Species	Group C Species	Group D Species
$2\frac{5}{8}$	$\frac{3}{4}$	1	$1\frac{1}{2}''$ minimum	3110*	2670	2220	2010	2170	1860	1550	1330
		2	$1\frac{1}{2}''$ minimum	2420	2080	1730	1500	1690	1450	1210	1040
			$2''$	3190*	2730	2270	1960	2220	1910	1580	1370
			$2\frac{1}{2}''$ or thicker	3330*	2860	2380	2060	2320	1990	1650	1440
4	$\frac{3}{4}$ or $\frac{7}{8}$	1	$1\frac{1}{2}''$ minimum	4370	3750	3130	2700	3040	2620	2170	1860
			$1\frac{3}{4}''$ or thicker	5090*	4360	3640	3140	3540	3040	2530	2200
		2	$1\frac{3}{4}''$ minimum	3390	2910	2420	2090	2360	2020	1680	1410
			$2''$	3790	3240	2700	2330	2640	2260	1880	1630
			$2\frac{1}{2}''$	4310	3690	3080	2660	3000	2550	2140	1850
			$3''$	4830*	4140	3450	2980	3360	2880	2400	2060
			$3\frac{1}{2}''$ or thicker	5030*	4320	3600	3110	3500	3000	2510	2160

[a]Tabulated lateral design values (P, Q) for shear plate connector units shall be multiplied to all applicable adjustment factors (see Table 11.5-1).

[b]Allowable design values for shear plate connector units shall net exceed the following:

(1) $2\frac{5}{8}''$ shear plate . . . . . 2990 pounds

(2) $4''$ shear plate with $\frac{3}{4}''$ bolt . . . . . 4400 pounds

(3) $4''$ shear plate with $\frac{7}{8}''$ bolt . . . . . 6000 pounds

The design values in footnote b shall be permitted to be increased in accordance with the American Institute of Steel Construction (AISC) *Manual of Steel Construction*, 9th edition, Section A.5.2 "Wind and Seismic Stresses," except when design loads have already been reduced by load combination factors (see NDS 10.2.3).

[c]Loads followed by an asterisk (*) exceed these permitted by footnote b, but are needed for determination of design values for other angles of load to grain. Footnote b limitations apply in all cases.

TABLE 14.2.1-3 Wood Specific Gravity Groups for Shear Plates and Split Rings

Specific Gravity Grouping	Specific Gravity, G
A	$G \geq 0.60$
B	$0.49 \leq G < 0.60$
C	$0.42 \leq G < 0.49$
D	$G < 0.42$

southern pine glulam are in species group B, while Alaska cedar, hem-fir, and spruce-pine-fir are in species group C.

14.2.2 Metal Side Plate Factor for Shear Plates

For 4 in. shear plates loaded parallel-to-grain, the reference design value is multiplied by the metal side plate factor (Table 14.2.2-1), C_{st}, to account for improved performance of these connectors when used in conjunction with metal side plates.

TABLE 14.2.2-1 Metal Side Plate Factor for 4 in. Shear Plates Loaded Parallel-to-Grain

Connector Species Group	C_{st}
A	1.18
B	1.11
C	1.05
D	1.00

14.2.3 Geometry Factor, C_Δ, for Split Rings and Shear Plates

In addition to the direction of load, specific gravity, and member thickness, the capacity of a timber connector depends on its placement relative to other connectors and to the ends and edges of the member. These effects are accounted for through the application of the geometry factor, C_Δ. In general, separate factors are calculated based on end distance, edge distance, and spacing with the lowest factor then applied to each connector in the group. The geometry factor is cumulative with the other applicable adjustment factors listed in Table 11.5-1.

The NDS® provides minimum spacing, end distance, and edge distance requirements for a connector unit to develop its full capacity ($C_\Delta = 1.0$) and also provides absolute minimum distances and spacings with corresponding reduced geometry factors. For intermediate spacings or distances, linear interpolation is used to determine a geometry factor. The required spacings, end distances, and edge distances depend on the direction of load and whether the connectors are installed in end grain, side grain, or sloping end grain surfaces. Wherever possible, it is recommended that connector joints be designed with edge distance, end distance, and spacing equal to or greater than the values required for full capacity ($C_\Delta = 1.0$).

TABLE 14.2.3.1-1 Geometry Factors, C_Δ, for Split Ring and Shear Plate Connectors

		$2\frac{1}{2}''$ Split Ring Connectors & $2\frac{5}{8}''$ Shear Plate Connectors				4″ Split Ring Connectors & 4″ Shear Plate Connectors			
		Parallel to grain loading		Perpendicular to grain loading		Parallel to grain loading		Perpendicular to grain loading	
		Minimum Value	Minimum for $C_\Delta = 1.0$	Minimum Value	Minimum for $C_\Delta = 1.0$	Minimum Value	Minimum for $C_\Delta = 1.0$	Minimum Value	Minimum for $C_\Delta = 1.0$
Edge Distance	Unloaded Edge	$1\frac{1}{2}''$	$1\frac{3}{4}''$	$1\frac{1}{2}''$	$1\frac{3}{4}''$	$2\frac{1}{2}''$	$2\frac{3}{4}''$	$2\frac{1}{2}''$	$2\frac{3}{4}''$
	C_Δ	0.88	1.0	0.88	1.0	0.93	1.0	0.93	1.0
	Loaded Edge	–	–	$1\frac{1}{2}''$	$2\frac{3}{4}''$	–	–	$2\frac{1}{2}''$	$3\frac{3}{4}''$
	C_Δ for Split Rings	–	–	0.70	1.0	–	–	0.70	1.0
	C_Δ for Shear Plates	–	–	0.83	1.0	–	–	0.83	1.0
End Distance	Tension Member	$2\frac{3}{4}''$	$5\frac{1}{2}''$	$2\frac{3}{4}''$	$5\frac{1}{2}''$	$3\frac{1}{2}''$	7″	$3\frac{1}{2}''$	7″
	C_Δ	0.63	1.0	0.63	1.0	0.63	1.0	0.63	1.0
	Compression Member	$2\frac{1}{2}''$	4″	$2\frac{3}{4}''$	$5\frac{1}{2}''$	$3\frac{1}{4}''$	$5\frac{1}{2}''$	$3\frac{1}{2}''$	7″
	C_Δ	0.63	1.0	0.63	1.0	0.63	1.0	0.63	1.0
Spacing	Spacing parallel to grain	$3\frac{1}{2}''$	$6\frac{3}{4}''$	$3\frac{1}{2}''$	$3\frac{1}{2}''$	5″	9″	5″	5″
	C_Δ	0.5	1.0	1.0	1.0	0.5	1.0	1.0	1.0
	Spacing perpendicular to grain	$3\frac{1}{2}''$	$3\frac{1}{2}''$	$3\frac{1}{2}''$	$4\frac{1}{4}''$	5″	5″	5″	6″
	C_Δ	1.0	1.0	0.5	1.0	1.0	1.0	0.5	1.0

14.2.3.1 Load Applied at Angle of 0° or 90° to Grain For connectors installed in side grain with loads applied in the parallel-to-grain direction or perpendicular-to-grain direction, determination of the geometry factor and corresponding requirements for spacing distance, edge distance, and end distance is simple and straightforward. Minimum permissible distances are tabulated with corresponding geometry factors, and requirements for full capacity are also tabulated (Table 14.2.3.1-1). Interpolation is used to determine geometry factors for intermediate distances.

EXAMPLE 14.2.3.1-1 TENSION CONNECTION WITH SPLIT RING CONNECTORS

Given: The split ring connection illustrated in Figure 14.2.3.1-1 must transfer 3,000 lb of dead load and 9,000 lb of snow load ($C_D = 1.15$). The members are sawn lumber, No. 1 and Btr grade, Douglas fir-larch with design values of $F_t = 800$ psi, $E = 1,800,000$ psi, and $F_v = 180$ psi. The main member is a 4×6 and the side members are 2×6. Connectors are $2\frac{1}{2}$ in. split rings. The connection is subject to dry conditions of use and normal temperatures.

Wanted: Design a suitable split ring connection.

Approach: First the net section will be checked in both main and side members, assuming the connectors will be placed in a single row. The edge distance, end distance, and spacing values for full design values will be determined. The required number of connectors will be estimated assuming a geometry and group action factors equal to 1.0. A trial geometry will be determined based on row tear-out provisions, and the group action and geometry factors will be determined for the trial connection. The capacity of the connection will finally be evaluated using the adjusted design value for the split rings.

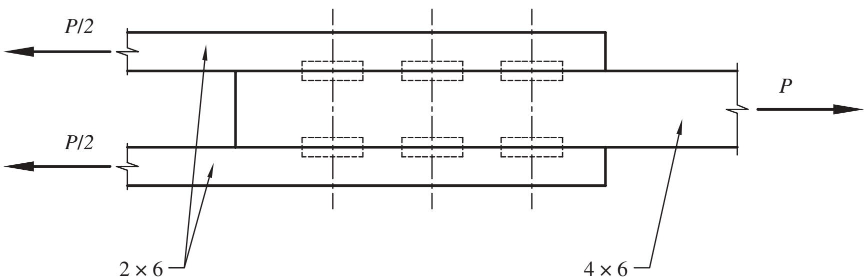


Figure 14.2.3.1-1 Tension connection with split ring connectors—Example 14.2.3.1-1.

Solution:

Net section area (Equations 12.3.1.1-2 and 12.3.1.1-3):

$$A_{n,4x6} = bd - n_j\left(2pD_O + D_{BH}(b - 2p)\right) \quad \text{for member in double-shear}$$

$$A_{n,4x6} = (3.5\,\text{in})(5.5\,\text{in}) - 1(2(0.375\,\text{in})(2.92\,\text{in}) + (0.56)(3.5\,\text{in} - 2(0.375\,\text{in})))$$

$$A_{n,4x6} = 19.25\,\text{in}^2 - 3.73\,\text{in}^2$$

$$A_{n,4x6} = 15.5\,\text{in}^2$$

$$A_{n,2x6} = bd - n_j\left(pD_O + D_{BH}(b - p)\right) \quad \text{for member in single-shear}$$

$$A_{n,2x6} = (1.5\,\text{in})(5.5\,\text{in}) - 1((0.375\,\text{in})(2.92\,\text{in}) + (0.56)(3.5\,\text{in} - 0.375\,\text{in}))$$

$$A_{n,2x6} = 8.25\,\text{in}^2 - 2.85\,\text{in}^2$$

$$A_{n,2x6} = 5.40\,\text{in}^2$$

Tension load:

$$T = T_D + T_S$$

$$T = 3,000\,\text{lb} + 9,000\,\text{lb}$$

$$T = 12,000\,\text{lb}$$

Allowable tension stress:

$$F_t' = F_t C_D C_F = (800\,\text{psi})(1.15)(1.3) = 1200\,\text{psi}$$

Net section tension stress:

$$f_{t,2x6} = \frac{T}{2A_{n,2x6}} = \frac{12,000\,\text{lb}}{2\left(5.40\,\text{in}^2\right)} = 1110\,\text{psi} \leq 1200\,\text{psi} \quad \therefore \text{OK}$$

$$f_{t,4x6} = \frac{T}{A_{n,4x6}} = \frac{12,000\,\text{lb}}{15.5\,\text{in}^2} = 770\,\text{psi} \leq 1200\,\text{psi} \quad \therefore \text{OK}$$

Spacing, end, and edge distances for $C_\Delta = 1.0$ (Table 14.2.3.1-1):

- Edge distance (unloaded edge, parallel-to-grain loading) = $1\frac{3}{4}$ in.
- End distance (tension member, parallel-to-grain loading) = $5\frac{1}{2}$ in.
- Spacing (parallel-to-grain loading) = $6\frac{3}{4}$ in.

Provided spacing, end, and edge distances:

$$edge_{unloaded} = \frac{5.5\,\text{in}}{2} = 2.75\,\text{in} \geq 1.75 \quad \therefore C_\Delta = 1.0$$

It will be assumed that the end distance and spacing requirements for full design value can be met, therefore, $C_\Delta = 1.0$.

Split ring design value (Tables 11.5-1 and 14.2.1-1):

$$P' = PC_D C_g C_\Delta = (2730\,\text{lb})\,(1.15)\,C_g\,(1.0) = (3140\,\text{psi})\,C_g$$

Number of connector units required (estimating $C_g \approx 1.0$):

$$n = \frac{T}{P'} = \frac{12{,}000\,\text{lb}}{3140\,\text{lb}} = 3.8 \quad \therefore \text{Use 4 connector units}$$

Critical shear area for one fastener to prevent row tear-out (Equation 12.3.4.2-1:

$$A_{crit\ shear,\ rt} = \frac{2P'}{F_v'} \approx \frac{2T}{nF_v'}$$

$$A_{crit\ shear,\ rt} = \frac{2\,(12,000\,\text{lb})}{4\,(180\,\text{psi})} = 33.3\,\text{in}^2$$

Required end distance to prevent row tear-out (Equation 12.3.4.2-2):

$$e \geq \frac{A_{crit\ shear,\ rt} - \dfrac{\pi}{4}\left(D_I^2 - \dfrac{D_O^2}{2} - D_{BH}^2\right)}{D_O + 4p}$$

$$e \geq \frac{33.3\,\text{in}^2 - \dfrac{\pi}{4}\left((2.5\,\text{in})^2 - \dfrac{(2.92\,\text{in})^2}{2} - (0.563\,\text{in})^2\right)}{2.92\,\text{in} + 4\,(0.375\,\text{in})}$$

$$e \geq 7.24\,\text{in}$$

Required spacing to prevent row tear-out (Equation 12.3.4.2-3):

$$s_1 \geq \frac{A_{crit\ shear,\ rt} + \dfrac{\pi}{4}\left(D_O^2 + D_{BH}^2 - D_I^2\right)}{D_O + 4p}$$

$$s_1 \geq \frac{33.3\,\text{in}^2 + \dfrac{\pi}{4}\left((2.92\,\text{in})^2 + (0.563\,\text{in})^2 - (2.5\,\text{in})^2\right)}{2.92\,\text{in} + 4\,(0.375\,\text{in})}$$

$$s_1 \geq 8.0\,\text{in}$$

An end distance of 7.25 inches and a spacing of 8.0 inches will be chosen. These values exceed the values for $C_\Delta = 1.0$.

Member stiffness ratio (Section 11.5.4):

$$E_sA_s = 1.8\left(10^6\,\text{psi}\right)2\,(1.5\,\text{in})\,(5.5\,\text{in})$$

$$E_sA_s = 1.8\left(10^6\,\text{psi}\right)\left(16.5\,\text{in}^2\right)$$

$$E_sA_s = 29.7\left(10^6\,\text{lb}\right)$$

$$E_mA_m = 1.8\left(10^6\,\text{psi}\right)(3.5\,\text{in})\,(5.5\,\text{in})$$

$$E_mA_m = 1.7\left(10^6\,\text{psi}\right)\left(19.25\,\text{in}^2\right)$$

$$E_mA_m = 34.7\left(10^6\,\text{lb}\right)$$

$$R_{EA} = \frac{E_sA_s}{E_mA_m} = \frac{29.7\left(10^6\,\text{lb}\right)}{34.7\left(10^6\,\text{lb}\right)} = 0.856$$

Fastener load slip modulus (Section 11.5.4):

$$\gamma = 400,000\,\text{lb/in}$$

Group action factor (Section 11.5.4):

$$u = 1 + n_j\gamma\frac{s}{2}\left[\frac{1}{E_mA_m} + \frac{1}{E_sA_s}\right]$$

$$u = 1 + 1\left(400,000\,\frac{\text{lb}}{\text{in}}\right)\left(\frac{8.0\,\text{in}}{2}\right)\left[\frac{1}{34.7\left(10^6\,\text{lb}\right)} + \frac{1}{29.7\left(10^6\,\text{lb}\right)}\right] = 1.100$$

$$m = u - \sqrt{u^2 - 1} = 1.100 - \sqrt{1.100^2 - 1} = 0.642;$$

$$C_g = \left[\frac{m\left(1 - m^{2n}\right)}{n\left[\left(1 + R_{EA}m^n\right)(1 + m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$

$$C_g = \left[\frac{0.642\left(1 - 0.642^{2(2)}\right)}{2\left[\left(1 + (0.856)\,0.642^{(2)}\right)(1 + 0.642) - 1 + 0.642^{2(2)}\right]}\right]\left[\frac{1 + 0.856}{1 - 0.642}\right]$$

$$C_g = 0.99$$

Adjusted design value (per connector):

$$P' = (3140\,\text{psi})\,C_g = (3140\,\text{psi})\,(0.99) = 3110\,\text{lb}$$

Design capacity:

$$T' = nP' = 2 \text{ sides} \left(\frac{2 \text{ connectors}}{\text{side}}\right)\left(\frac{3110 \text{ lb}}{\text{connector}}\right)$$

$$T' = 12{,}440 \text{ lb} \geq T = 12{,}000 \text{ lb} \quad \therefore \text{OK}$$

Answer: Four $2\frac{1}{2}$ in. split ring connectors (2 on each side) are required, with connectors placed in a single row, centered in the wide faces of the pieces, with 7.25 in. minimum end distance and 8.0 inch spacing.

14.2.3.2 Load Applied at Angle Other Than 0° or 90° to Grain For connectors installed in side grain with loads applied at an angle to grain, geometry factors for end and edge distance are calculated separately for the parallel-to-grain and perpendicular-to-grain components of the resistance, prior to using the Hankinson formula to establish a design capacity. Spacing requirements for the full design value ($C_\Delta = 1.0$) are determined in accordance with Equation 14.2.3.2-1 using factors from Table 14.2.3.2-1.

$$S_\beta = \frac{S_A S_B}{\sqrt{S_A^2 \sin^2 \beta + S_B^2 \cos^2 \beta}} \tag{14.2.3.2-1}$$

where:

S_β = required spacing along connector axis
S_A = factor from Table 14.2.3.2-1
S_B = factor from Table 14.2.3.2-1
β = angle of connector axis to the grain

TABLE 14.2.3.2-1 Factors for determining required spacing along connector axis for C_Δ = 1.0

Connector	Angle of Load to Grain[a] (degrees)	S_A (in.)	S_B (in.)
$2\frac{1}{2}$ in. split ring or $2\frac{5}{8}$ in. shear plate	0	6.75	3.50
	15	6.00	3.75
	30	5.13	3.88
	45	4.25	4.13
	60–90	3.5	4.25
4 in. split ring or 4 in. shear plate	0	9.00	5.00
	15	8.00	5.25
	30	7.00	5.50
	45	6.00	5.75
	60–90	5.00	6.00

[a] Interpolation is permitted for intermediate angles of load to grain.

The minimum spacing is 3.50 in. for $2\frac{1}{2}$ in. split rings and $2\frac{5}{8}$ in. shear plates and is 5.0 in. for 4 in. split ring or shear plate connectors. For this minimum spacing, $C_\Delta = 0.5$.

Where the actual spacing between split ring or shear plate connectors is greater than the minimum spacing, but less than the minimum spacing for $C_\Delta = 1.0$, the geometry factor, C_Δ, is determined by linear interpolation. The geometry factor calculated for spacing is applied to reference design values for both parallel- and perpendicular-to-grain components of the resistance. Where three or more connectors are used in one face of a member, the spacing between any two connectors should be checked.

EXAMPLE 14.2.3.2-1 SPACING AT ANGLE TO GRAIN

Given: $2\frac{5}{8}$ in. diameter shear plates are installed in a wood main member with steel side plates. They are installed with a 40° connector axis angle and load is applied at an angle of 30° relative to the member axis as shown in Figure 14.2.3.2-1.

Wanted: Determine the required spacing along the connector axis for full design value and the minimum reduced design value.

Approach: the spacing for full design value, S_β, will be calculated using Equation 14.2.3.2-1 and the values for S_A and S_B from Table 14.2.3.2-1.

Solution:

Values from Table 14.2.3.2-1 ($\theta = 30°$):

$$S_A = 5.125 \text{ in}$$
$$S_B = 3.875 \text{ in}$$

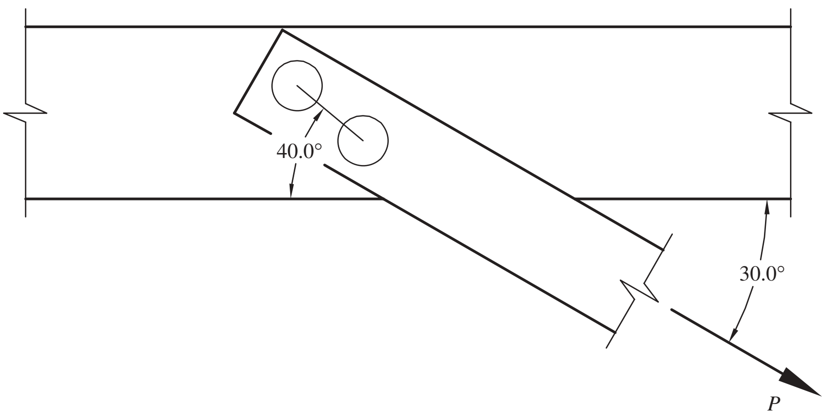


Figure 14.2.3.2-1 Connection detail—Example 14.2.3.2-1.

Minimum spacing along connector axis for $C_\Delta = 1.0$ (Equation 14.2.3.2-1):

$$S_\beta = \frac{S_A S_B}{\sqrt{S_A^2 \sin^2 \beta + S_B^2 \cos^2 \beta}}$$

$$S_\beta = \frac{(5.125\,\text{in})\,(3.875\,\text{in})}{\sqrt{(5.125\,\text{in})^2 \sin^2 40^\circ + (3.875\,\text{in})^2 \cos^2 40^\circ}}$$

$$S_\beta = 4.48\,\text{in}$$

Minimum spacing along connector axis for $C_\Delta = 0.5$ (Section 14.2.3.2):

$$S_\beta = 3.5\,\text{in}$$

Answer: For use of the full design value for the case described, the spacing along the connector axis must be 4.5 in. or more. The minimum permissible spacing is 3.5 in. for $C_\Delta = 0.50$.

14.3 TIMBER CONNECTORS IN END GRAIN

Where timber connectors are installed in end grain, such as in the square cut or sloping surface at the end of a member, the design values for the connectors must be reduced. Such conditions are illustrated in Figure 14.3-1. In addition, special provisions are required for the calculation of the geometry factor, and the members must be designed for shear in accordance with Equation 12.2.4-1.

14.3.1 Design Values for Connectors in End Grain

Where the end of a member is square cut as shown in Figure 14.3-1*a*, the design value, Q'_{90}, is 60% of the perpendicular-to-grain design value regardless of the direction of the load in the plane of the cut (Equation 14.3.1-1).

$$Q'_{90} = 0.60\,Q' \qquad (14.3.1\text{-}1)$$

where:

Q'_{90} = adjusted design value for a connector in end grain, loaded perpendicular-to-grain
Q' = adjusted design value for a connector in a side grain, loaded perpendicular-to-grain

For a sloped cut, the axis of the cut is defined as the intersection of a plane that is perpendicular to both the general direction of grain and the sloped surface. The angle of slope, α, of the sloped end is the angle between the sloped surface and the general longitudinal grain direction of the member.

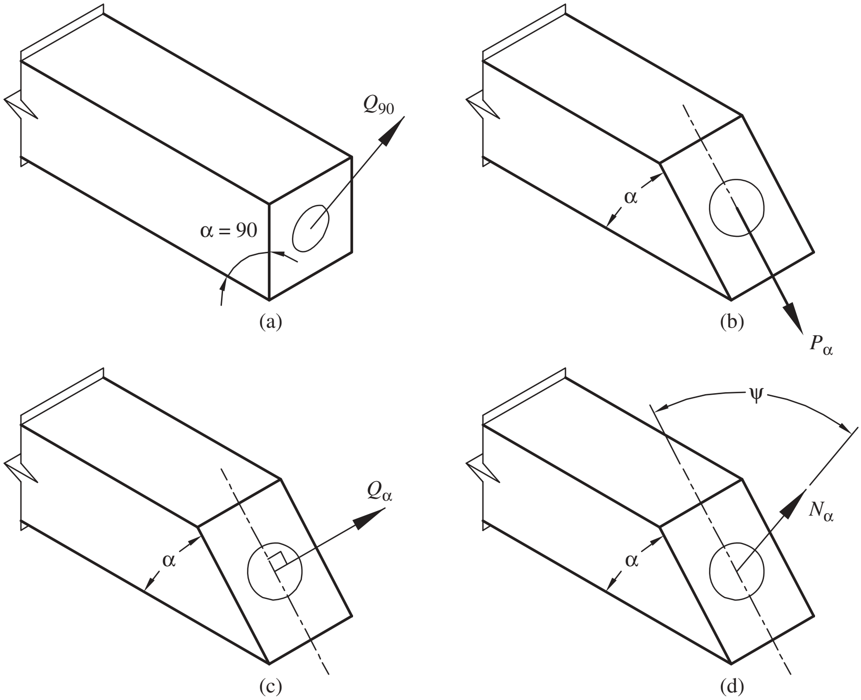


Figure 14.3-1 Timber connectors in end grain.

Where the load on the connector is parallel with the axis of the cut, as shown in Figure 14.3-1*b*, the design value P'_α is determined by Equation 14.3.1-2.

$$P'_\alpha = \frac{P'Q'_{90}}{P'\sin^2\alpha + Q'_{90}\cos^2\alpha} \tag{14.3.1-2}$$

where:

P'_α = adjusted design value for a connector in a sloping surface cut at angle α and loaded parallel to the axis of cut

P' = adjusted design value for a connector in a side grain surface, loaded parallel-to-grain

Q'_{90} = adjusted design value for a connector in end grain, loaded perpendicular-to-grain

α = slope of cut surface, as shown in Figure 14.3-1

Where the load on a connector in end grain is perpendicular to the axis of the cut (but parallel to cut surface), as shown in Figure 14.3-1c, the allowable load on the connector is determined by Equation 14.3.1-3.

$$Q'_\alpha = \frac{Q'Q'_{90}}{Q'\sin^2\alpha + Q'_{90}\cos^2\alpha} \tag{14.3.1-3}$$

where:

Q'_α = adjusted design value for a connector in a sloping surface, loaded perpendicular to the axis of cut
Q' = adjusted design value for a connector in side grain, loaded perpendicular-to-grain
Q'_{90} = adjusted design value for a connector in end grain, loaded perpendicular-to-grain
α = slope of cut surface

Where the load on a connector in end grain is at some angle ψ with respect to the slope axis (Figure 14.3-1d), the allowable load on the connector is determined by Equation 14.3.1-4.

$$N'_\alpha = \frac{P'_\alpha Q'_\alpha}{P'_\alpha \sin^2\psi + Q'_\alpha \cos^2\psi} \tag{14.3.1-4}$$

where:

N'_α = adjusted design value for connector in a sloping surface, loaded at an angle ψ to the axis of the cut

14.3.2 Geometry Factor for Connectors in End Grain

For shear plates and split rings installed in end grain, special provisions are applied to determine a geometry factor with its corresponding spacing and edge distances. A single geometry factor is determined and applied to reference design values for both parallel- and perpendicular-to-grain components of the resistance.

14.3.2.1 Connectors Installed in Square Cut End Grain Surface, Load in Any Direction For connectors installed in end grain in timbers with square-cut ends and loaded in any direction (Figure 14.3-1*a*), the provisions for perpendicular-to-grain loading for connectors installed in side grain apply, except for end distance provisions. This case is typical of shear plates in moment splice connections.

14.3.2.2 Connectors Installed in Sloped End Grain Surface, Load Parallel to Axis of Cut For connectors installed in sloped end grain surfaces and loaded parallel to the axis of the cut (Figure 14.3-1*b*), the spacing and edge distance requirements will be somewhere between the requirements for parallel-to-grain loading of connectors in side grain and perpendicular-to-grain loading of connectors in end grain. An elliptical transition function is used to determine the required spacing and edge distance based on the angle of the sloped cut. This case is typical for the peak connection of tudor arches, A-frames, and trusses.

14.3.2.2.1 Spacing Parallel to Axis of Cut The minimum spacing parallel to the axis of cut for $C_\Delta = 1.0$ is determined in accordance with Equation 14.3.2.2.1-1. The minimum spacing parallel to the axis of cut is 3.5 in. for $2\frac{1}{2}$ in.

split rings and $2\frac{5}{8}$ in. shear plates and is 5.0 in. for 4 in. split ring or shear plate connectors. For this minimum spacing, $C_\Delta = 0.5$. Where the actual spacing parallel to the axis of cut between split ring or shear plate connectors is greater than the minimum spacing for $C_\Delta = 0.5$ but less than the minimum spacing for $C_\Delta = 1.0$, the geometry factor, C_Δ is determined by linear interpolation.

$$S_\alpha = \frac{S_\parallel S_\perp}{\sqrt{S_\parallel^2 \sin^2\alpha + S_\perp^2 \cos^2\alpha}} \qquad (14.3.2.2.1\text{-}1)$$

where:

S_α = minimum spacing parallel to axis of cut
$S_\parallel$ = factor from Table 14.3.2.2.1-1
$S_\perp$ = factor from Table 14.3.2.2.1-1
α = angle of sloped cut (see Figure 14.3-1*b*)

TABLE 14.3.2.2.1-1 Factors for Determining Minimum Spacing along Axis of Cut of Sloping Surfaces

Connector	Geometry Factor	$S_\parallel$ (in.)	$S_\perp$ (in.)
$2\frac{1}{2}$ in. split ring or $2\frac{5}{8}$ in. shear plate	$C_\Delta = 1.0$	6.75	4.25
4 in. split ring or 4 in. shear plate	$C_\Delta = 1.0$	9.0	6.0

14.3.2.2.2 Loaded Edge Distance Parallel to Axis of Cut The minimum loaded edge distance parallel to the axis of cut for $C_\Delta = 1.0$ is determined in accordance with Equation 14.3.2.2.2-1. For split rings, the minimum loaded edge distance parallel to the axis of cut for $C_\Delta = 0.70$ is determined in accordance with Equation 14.3.2.2.2-1. For shear plates, the minimum loaded edge distance parallel to the axis of cut for $C_\Delta = 0.83$ is determined in accordance with Equation 14.3.2.2.2-1. Where the actual loaded edge distance parallel to the axis of cut is greater than the minimum loaded edge distance parallel to the axis of cut for $C_\Delta = 0.70$ for split rings or for $C_\Delta = 0.83$ for shear plates, but less than the minimum loaded edge distance parallel to the axis of cut for $C_\Delta = 1.0$, the geometry factor, C_Δ, is determined by linear interpolation.

$$E_\alpha = \frac{E_\parallel E_\perp}{\sqrt{E_\parallel^2 \sin^2\alpha + E_\perp^2 \cos^2\alpha}} \qquad (14.3.2.2.2\text{-}1)$$

where:

E_α = minimum loaded edge distance parallel to axis of cut
$E_\parallel$ = factor from Table 14.3.2.2.2-1
$E_\perp$ = factor from Table 14.3.2.2.2-1
α = angle of sloped cut (see Figure 14.3-1*b*)

TABLE 14.3.2.2.2-1 Factors for Determining Minimum Loaded Edge Distance for Connectors in End Grain

Connector	Geometry Factor	$E_{\parallel}$ (in.)	$E_{\perp}$ (in.)
$2\frac{1}{2}$ in. split ring	$C_{\Delta} = 1.0$	5.5	2.75
	$C_{\Delta} = 0.70$	3.3	1.5
$2\frac{5}{8}$ in. shear plate	$C_{\Delta} = 1.0$	5.5	2.75
	$C_{\Delta} = 0.83$	4.25	1.5
4 in. split ring	$C_{\Delta} = 1.0$	7.0	3.75
	$C_{\Delta} = 0.70$	4.2	2.5
4 in. shear plate	$C_{\Delta} = 1.0$	7.0	3.75
	$C_{\Delta} = 0.83$	5.4	2.5

14.3.2.2.3 Unloaded Edge Distance Parallel to Axis of Cut The minimum unloaded edge distance parallel to the axis of cut for $C_{\Delta} = 1.0$, is determined in accordance with Equation 14.3.2.2.3-1. The minimum unloaded edge distance parallel to the axis of cut for $C_{\Delta} = 0.63$ is determined in accordance with Equation 14.3.2.2.3-1. Where the actual unloaded edge distance parallel to the axis of cut is greater than the minimum unloaded edge distance for $C_{\Delta} = 0.63$ but less than the minimum unloaded edge distance for $C_{\Delta} = 1.0$, the geometry factor, C_{Δ}, is determined by linear interpolation.

$$U_{\alpha} = \frac{U_{\parallel} U_{\perp}}{\sqrt{U_{\parallel}^2 \sin^2 \alpha + U_{\perp}^2 \cos^2 \alpha}} \qquad (14.3.2.2.3\text{-}1)$$

where:

U_{α} = minimum unloaded edge distance parallel to axis of cut
$U_{\parallel}$ = factor from Table 14.3.2.2.3-1
$U_{\perp}$ = factor from Table 14.3.2.2.3-1
α = angle of sloped cut (see Figure 14.3-1*b*)

TABLE 14.3.2.2.3-1 Factors for Determining Minimum Unloaded Edge Distance

Connector	Geometry Factor	$U_{\parallel}$ (in.)	$U_{\perp}$ (in.)
$2\frac{1}{2}$ in. split ring or $2\frac{5}{8}$ in. shear plate	$C_{\Delta} = 1.0$	4.0	1.75
	$C_{\Delta} = 0.63$	2.5	1.5
4 in. split ring or 4 in. shear plate	$C_{\Delta} = 1.0$	5.5	2.75
	$C_{\Delta} = 0.63$	3.25	2.5

14.3.2.2.4 Unloaded Edge Distance and Spacing Perpendicular to Axis of Cut Geometry factors for unloaded edge distance perpendicular to the axis of cut and for spacing perpendicular to the axis of cut are determined following the provisions for unloaded edge distance and perpendicular-to-grain spacing for connectors installed in side grain and loaded parallel-to-grain.

14.3.2.3 Connectors Installed in Sloped End Grain Surface, Loaded Perpendicular to Axis of Cut For sloping end-grain surfaces loaded perpendicular to the axis of cut (Figure 14.3-1*c*), provisions for perpendicular-to-grain loading for connectors installed in end grain shall apply, except (1) the minimum end distance parallel to the axis of cut for $C_\Delta = 1.0$ is determined in accordance with Equation 14.3.2.3-1, (2) the minimum end distance parallel to the axis of cut for $C_\Delta = 0.63$ is determined in accordance with Equation 14.3.2.3-1, and (3) where the actual end distance parallel to the axis of cut is greater than the minimum end distance for $C_\Delta = 0.63$, but less than the minimum unloaded edge distance for $C_\Delta = 1.0$, the geometry factor, C_Δ, is determined by linear interpolation.

$$e_\alpha = \frac{E_\parallel U_\perp}{\sqrt{E_\parallel^2 \sin^2 \alpha + U_\perp^2 \cos^2 \alpha}} \tag{14.3.2.3-1}$$

where:

e_α = minimum end distance parallel to axis of cut
$E_\parallel$ = factor from Table 14.3.2.3-1
$U_\perp$ = factor from Table 14.3.2.3-1
α = angle of sloped cut (Figure 14.3-1*c*)

14.3.2.4 Connectors Installed in Sloped End Grain Surface, Load at an Angle ψ to Axis of Cut For connectors installed in sloped end grain surfaces with load at an angle ψ to the axis of the cut (Figure 14.3-1*d*), separate geometry factors, C_Δ, are determined separately for the components of resistance parallel and perpendicular to the axis of cut prior to applying the Hankinson formula (Equation 11.4.2.2-1).

TABLE 14.3.2.3-1 Factors for Determining Minimum End Distance

Connector	Geometry Factor	$E_\parallel$ (in.)	$U_\perp$ (in.)
$2\frac{1}{2}$ in. split ring or $2\frac{5}{8}$ in. shear plate	$C_\Delta = 1.0$	5.5	1.75
	$C_\Delta = 0.63$	2.75	1.5
4 in. split ring or 4 in. shear plate	$C_\Delta = 1.0$	7.0	2.75
	$C_\Delta = 0.63$	3.5	2.5

14.4 CONCLUSION

Shear plates and split rings are commonly used in bolted timber connections to increase the capacity relative to bolts alone. Design procedures for these connectors are based primarily on research reported in 1944 [1] with subsequent additions and modifications. Shear plates and split rings can be installed in either side grain or end grain, leading to several applications in design, including tension splices, moment splices, and peak connections of arches and trusses. This chapter presented design formulas and examples for the use of these timber connectors.

CHAPTER 15

MOMENT SPLICES

15.1 INTRODUCTION

The use of glued laminated timber has reduced the need for moment splices in timber beams and girders, because they can be manufactured to long lengths, limited only by transportation constraints. However, moment splices in timber members are sometimes used in glulam arches where the transportation of full-size structural frames may be impractical or uneconomical. In general, moment splices should be located at or near inflection points where bending moments are relatively small. Typical moment splice connections for glued laminated timber are illustrated in Figure 15.1-1.

Typical moment splices must be designed to transfer shear forces, bending moments, and axial forces. For splices where the axial compressive stress is large relative to the flexural stresses under all loading conditions, such that no net tension exists at the section, the moment splice may be designed simply to provide continuity at the section and to resist shear. Such is the case for which $P > 6M/d$ where P is the applied axial load (compression), M is the applied bending moment, and d is the member depth. For cases in which the bending moment is not relatively small, $M > Pd/6$, net tension exists across the section, and a moment splice must be designed.

Resistance to the applicable moments and forces may be achieved by various means. Tension forces across the splice are typically resisted by steel plates connected with bolts or shear plates. Compression forces are commonly transferred through direct end grain bearing of the members or through a metal bearing plate. Compression forces may instead be transferred through steel straps fastened

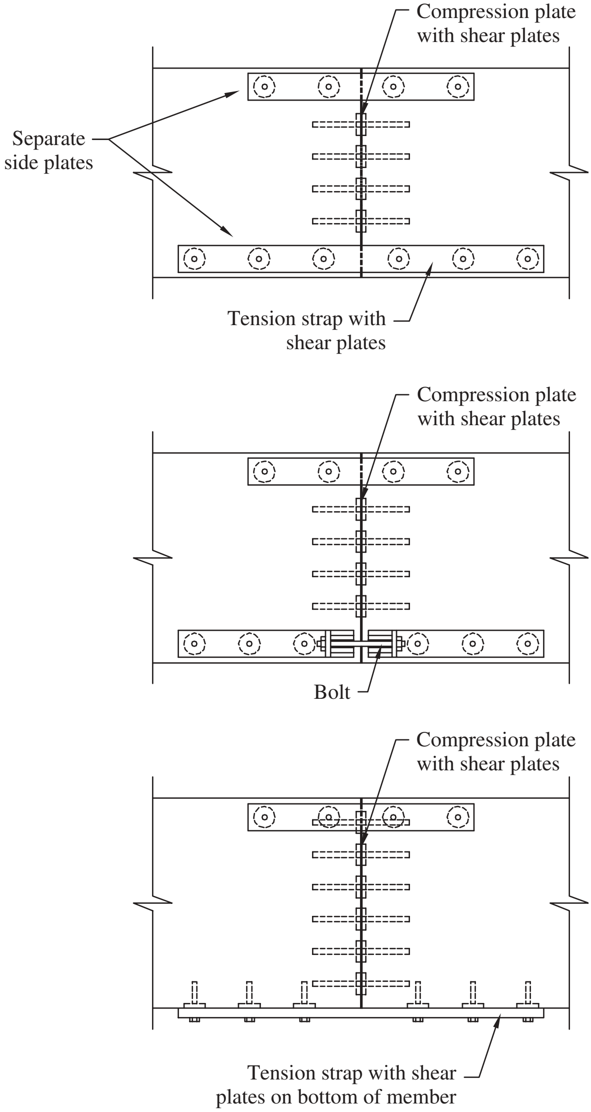


Figure 15.1-1 Moment splice connections.

across the splice. Shear between members is typically resisted with shear plates at the abutting faces.

Close tolerances are required in the fabrication of moment splices. Consideration should be given to inelastic as well as elastic deformation in the joint. Separate side plates should be considered for each row of connectors in order to minimize stresses due to shrinkage effects in the member. Initial slip in the tension splice plates can be offset by using a tension-type splice with bolts and nuts that can be tightened to eliminate the slip, as shown in Figure 15.1-1.

15.2 SHEAR TRANSFER

Shear transfer in moment splices is typically accomplished through shear plates installed in end grain on steel dowels. The number and placement of connector units must be such that the connection has the capacity to resist all loads applied to it. For large members with several connector units, the group action factor may be significant. Where possible, spacings and edge distances for full design values should be used. Small reductions in spacing or edge distance can significantly reduce the design capacity. In addition, the members themselves must be evaluated for notch effect using Equation 12.2.4-1.

15.3 MOMENT TRANSFER

Typical moment splices transfer compression forces by end bearing and tension forces through steel plates fastened across the splice. The magnitudes of the compression and tension forces are dependent on the stiffness of the connections. Therefore, a trial connection configuration must be selected and then analyzed. Iteration may be necessary to obtain an optimal solution.

An initial estimate of the tension force can be obtained from elastic analysis of the uncut section. This estimate can be used to establish a trial fastener configuration. The placement and stiffness of the fasteners determine the location of the neutral axis and the magnitude of the compression stress and tension forces. In general, the compression stress will exceed that obtained through analysis of the uncut section, so the compression stress must be evaluated in addition to the tension connections.

15.3.1 Elastic Analysis of the Uncut Section

Elastic analysis of the uncut section is a useful tool to estimate the tension force in the cross section. This force estimate can be used to approximate the number of fasteners required and develop a trial connection configuration. The actual forces in the tension connection(s) will be dependent on the placement and stiffness of the fasteners.

For the uncut section, the flexural stresses are added to the axial stresses to determine the combined stress distribution and the location of the neutral axis (Figure 15.3.1-1). The magnitude of the tension force is the resultant of the net tension stresses in the uncut section.

15.3.2 Fastener Stiffness and Placement

Fastener stiffness and location significantly affect the distribution of stresses and forces in the spliced section. Because the fasteners in the spliced section will be less stiff than the uncut section, the neutral axis of the spliced section will be closer to the compression face than the neutral axis in the uncut section. This

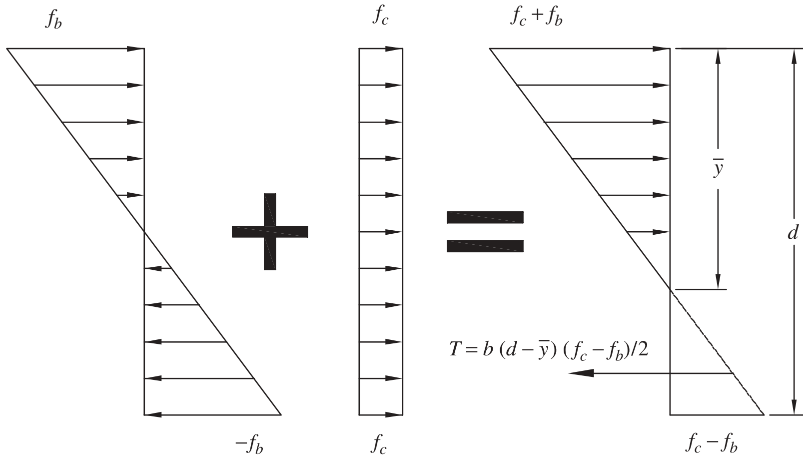


Figure 15.3.1-1 Stresses and neutral axis in uncut section.

results in higher stresses on the compression face than those calculated for the uncut section.

Stiffer fasteners and fasteners placed closer to the tension face cause the neutral axis to move toward the tension side, reducing the stresses on the compression face. The actual forces on each tension connection are dependent on the stiffness and location of the connection. Load/slip moduli presented in the group action factor section of this chapter and in the NDS can be used to estimate the stiffness of a row of fasteners. The values of γ represent load/slip moduli of double-shear connections, so they should be divided by 2 for single-shear connections.

The stiffness of a row of bolts (or bolts with shear plates or split rings) in double shear can be estimated using Equation 15.3.2-1.

$$(EA)_j \approx n_{bolts} \gamma s_{\min} \qquad (15.3.2\text{-}1)$$

where:

$(EA)_j$ = stiffness of j^{th} row of fasteners,
n_{bolts} = number of bolts in row
γ = load/slip modulus for fasteners in double shear
$s_{\min}$ = minimum spacing for $C_\Delta = 1.0$

Tension plates may be installed on the sides of the members or on the tension faces of the members (see Figure 15.1-1). Placement on the face results in more distance to the compression resultant; however, the fasteners will also be in single shear, so the stiffness of the fasteners will be reduced. Multiple rows of fasteners can be used where necessary to transfer the tension forces, however, the forces will not be distributed evenly to each row of fasteners. Only a thorough analysis can determine the effects of different fastener configurations.

15.3.3 Neutral Axis Location and Force Distribution

The location of the neutral axis must be determined to accurately evaluate the tension forces and compression stresses in the spliced section. It is dependent on the magnitudes of the bending moment and axial force on the section and on the placement and stiffness of each row of fasteners. The location of the neutral axis is derived from the free body diagram in Figure 15.3.3-1. Symbols used in the derivation are illustrated in Figure 15.3.3-1, except k, which is a constant that is determined through the derivation.

Assuming that plane sections remain plane, the force on each row of connections, T_j, is calculated with Equation 15.3.3-1.

$$T_j = (EA)_j k(y_j - \overline{y}) \tag{15.3.3-1}$$

The compression resultant, C, is calculated using Equation 15.3.3-2.

$$C = \frac{E_w b \overline{y}^2 k}{2} \tag{15.3.3-2}$$

Summation of forces in the x-direction gives Equation (15.3.3-3).

$$P_1 = C - \sum_{j=1}^{n} T_j \tag{15.3.3-3}$$

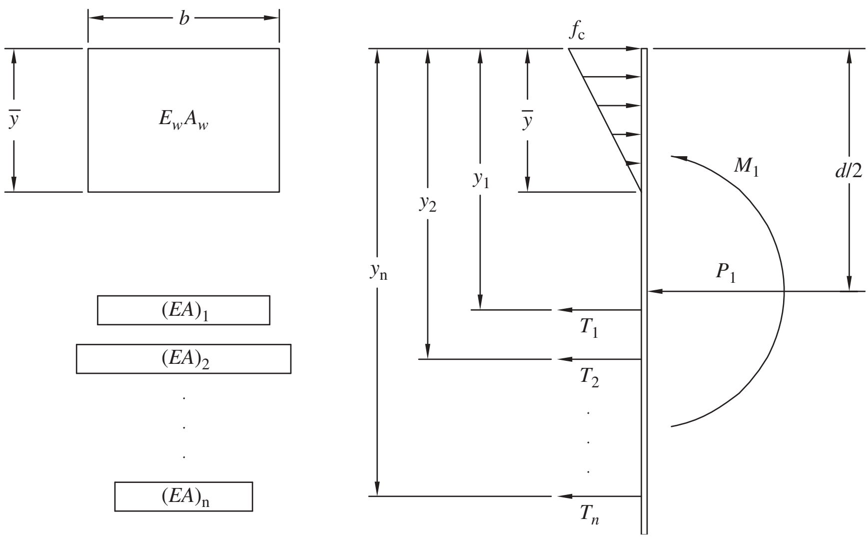


Figure 15.3.3-1 Neutral axis location in moment splice.

Combining Equations 15.3.3-1 and 15.3.3-2 with 15.3.3-3 gives Equation 15.3.3-4.

$$P_1 = k\left[\frac{E_w b\overline{y}^2}{2} - \sum_{j=1}^{n}(EA)_j\left(y_j - \overline{y}\right)\right] \qquad (15.3.3\text{-}4)$$

Summation of moments about mid-depth gives Equation15.3.3-5.

$$M_1 = k\left[\frac{E_w b\overline{y}^2}{2}\left(\frac{d}{2} - \frac{\overline{y}}{3}\right) + \sum_{j=1}^{n}(EA)_j\left(y_j - \overline{y}\right)\left(y_j - \frac{d}{2}\right)\right] \qquad (15.3.3\text{-}5)$$

Combining Equations 15.3.3-4 and 15.3.3-5 gives Equation 15.3.3-6.

$$\frac{P_1}{M_1}\frac{\left[\frac{E_w b\overline{y}^2}{2}\left(\frac{d}{2} - \frac{\overline{y}}{3}\right) + \sum_{j=1}^{n}(EA)_j\left(y_j - \overline{y}\right)\left(y_j - \frac{d}{2}\right)\right]}{\left[\frac{E_w b\overline{y}^2}{2} - \sum_{j=1}^{n}(EA)_j\left(y_j - \overline{y}\right)\right]} - 1 = 0 \qquad (15.3.3\text{-}6)$$

Equation 15.3.3-6 can be solved by iteration using a spreadsheet. The equation has a vertical asymptote at the location of the neutral axis for bending only. The added axial compression force moves the neutral axis farther away from the compression face. It is recommended that the left side of Equation 15.3.3-6 be plotted for $0 \leq \overline{y} \leq d$ to ensure that the correct point is obtained.

With the neutral axis located, k is determined using Equation 15.3.3-7.

$$k = \frac{P_1}{\left[\frac{E_w b\overline{y}^2}{2} - \sum_{j=1}^{n}(EA)_j\left(y_j - \overline{y}\right)\right]} \qquad (15.3.3\text{-}7)$$

k is then used with Equation 15.3.3-1 to determine the tension forces on each row of fasteners and the compression resultant. The compression stress is determined from Equation 15.3.3-8.

$$f_c = E_w k\overline{y} \qquad (15.3.3\text{-}8)$$

Equation 15.3.3-6 is not valid for the case of $P_1 = 0$. For the case of bending moment alone, the neutral axis is located by solving the following quadratic equation (Equation 15.3.3-9).

$$\frac{E_w b\overline{y}_m^2}{2} + \overline{y}_m\sum(EA)_j - \sum(EAy)_j = 0 \qquad (15.3.3\text{-}9)$$

For the case of bending moment alone, k is calculated using Equation 15.3.3-10.

$$k = \frac{M_1}{\dfrac{E_w b \overline{y}_m^3}{3} + \sum_{j=1}^{n} (EA)_j \left(y_j - \overline{y}_m\right)^2} \tag{15.3.3-10}$$

15.3.4 Evaluation of Connection

Once the compression stress on the section and the tension force for each row of fasteners are determined, the adequacy of the trial connection can be evaluated. The combined axial and flexural compression stress must not exceed the allowable bending stress for the section, and the tension force on each row of fasteners must not exceed the design capacity of the row. In cases where the combined axial and flexural compressive stress exceeds 75% of the adjusted design value in flexure, F'_b, a snug fitting metal bearing plate not thinner than 20 gauge should be installed between the abutting ends. To prevent restraint of shrinkage, multiple bearing plates should be used where the spacing between the top and bottom shear plates exceeds 10 to 12 inches, as discussed in Chapter 11.

EXAMPLE 15.3.4-1 MOMENT SPLICE CONNECTION

Given: A two-hinged arch consisting of 5 in. × $17\frac{7}{8}$ in. southern pine glued laminated timbers, Combination 24F-V3, requires a moment splice for shipping. At the location of the splice, the design positive moment is 255,000 in.-lb, and the axial compression force is 27,000 lb caused by $D + S$. The maximum shear is 5,000 lb caused by $D + S$ (unbalanced snow). The maximum negative moment is 15,000 in.-lb caused by wind (assuming no axial force). The reference design value for a single $\frac{3}{4}$ in. bolt is $Z = 3480$ lb. The reference design value for a $2\frac{5}{8}$ in. shear plate loaded perpendicular-to-grain is $Q = 1990$ lb.

Wanted: Design a suitable moment splice connection using $\frac{3}{4}$ in. bolts in double shear with steel side plates to transfer tension forces and $2\frac{5}{8}$ in. shear plates in end grain to transfer shear forces.

Solution:

Design values:

$$F'_b = F_b C_D C_M C_t$$

$$F'_b = (2400\text{ psi})(1.15)(1.0)(1.0)$$

$$F'_b = 2760\text{ psi} \qquad \text{for snow load}$$

$$F'_b = F_b C_D C_M C_t$$

$$F'_b = (2400\text{ psi})(1.6)(1.0)(1.0)$$

$$F'_b = 3840\text{ psi} \qquad \text{for wind load}$$

$$F_v' = F_v C_D C_M C_t C_{vr}$$
$$F_v' = (300\,\text{psi})\,(1.15)\,(1.0)\,(1.0)\,(0.72)$$
$$F_v' = 248\,\text{psi} \qquad \text{for snow load}$$

$$F_v' = F_v C_D C_M C_t C_{vr}$$
$$F_v' = (300\,\text{psi})\,(1.6)\,(1.0)\,(1.0)\,(0.72)$$
$$F_v' = 346\,\text{psi} \qquad \text{for wind load}$$

$$E' = EC_M C_t$$
$$E' = 1.8(10^6\,\text{psi})(1.0)(1.0)$$
$$E' = 1.8(10^6\,\text{psi})$$

$$Z' = ZC_D C_M C_t C_\Delta C_g$$
$$Z' = (3480\,\text{lb})(1.15)(1.0)(1.0)C_g$$
$$Z' = (4,002\,\text{lb})C_g \qquad \text{for snow load}$$

$$Z' = ZC_D C_M C_t C_\Delta C_g$$
$$Z' = (3480\,\text{lb})(1.6)(1.0)(1.0)C_g$$
$$Z' = (5,568\,\text{lb})C_g \qquad \text{for wind load}$$

$$Q_{90}' = 0.6QC_D C_M C_t C_\Delta C_g$$

$$Q_{90}' = 0.6(1990\,\text{lb})(1.15)(1.0)C_\Delta C_g$$
$$Q_{90}' = (1,373\,\text{lb})C_\Delta C_g \qquad \text{for snow load}$$

Required number of shear plates:

$$n = \frac{V}{Q_{90}'} = \frac{5,000\,\text{lb}}{(1,373\,\text{lb})C_\Delta C_g} = \frac{3.64}{C_\Delta C_g} \qquad \therefore \text{ Try 4 shear plates.}$$

Geometry factor (Section 14.3.2):
The unloaded edge distance from shear plate centers to the wide faces of the timbers is 2.5 in., which is greater than the minimum of 1.75 in. for full design load from Table 14.2.3.1-1; therefore $C_\Delta = 1.0$.

The required spacing for full design load for $2\frac{5}{8}$ in. shear plates is 4.25 in., also from Table 14.2.3.1-1.

Using a spacing of 4.25 in., the loaded edge distance, E, is calculated as:

$$E = \frac{d - (n-1)\,s}{2}$$

$$E = \frac{17.875\,\text{in} - (4-1)(4.25\,\text{in})}{2}$$
$$E = 2.56\,\text{in}$$

Since this value is less than the stated minimum for full design value (2.75 in.), but greater than the minimum for reduced design value (1.5 in.), a geometry factor is determined by interpolation as:

$$C_{\Delta E} = 0.83 + \frac{(1.0 - 0.83)(2.56\,\text{in} - 1.5\,\text{in})}{(2.75\,\text{in} - 1.5\,\text{in})} = 0.974$$

Group action factor for $n = 4$, $s = 4.25$ in (Section 11.5.4):

$$E'_{m\perp} = E'_{s\perp} = \frac{E'}{20} = \frac{1.8(10^6\,\text{psi})}{20} = 90,000\,\text{psi}$$
$$A_m = A_s = bd = (5\,\text{in})(17.875\,\text{in}) = 89.4\,\text{in}^2$$
$$E'_{m\perp}A_m = E'_{s\perp}A_s = (90,000\,\text{psi})(89.4\,\text{in}^2) = 8.05(10^6)\,\text{lb}$$
$$R_{EA} = \frac{E'_{s\perp}A_s}{E_mA_m} = 1.0$$

The load/slip modulus is given in the *NDS*® for fasteners loaded parallel-to-grain in double shear. For load perpendicular-to-grain, the modulus is divided by 2. For the single-shear connection in this example, the modulus is divided by 2, again.

$$\gamma_\perp = \frac{\gamma}{2\,(2)} = \frac{400,000\,\text{lb/in}}{4} = 100,000\,\text{lb/in}$$
$$u = 1 + n_j\gamma\frac{s}{2}\left[\frac{1}{E_mA_m} + \frac{1}{E_sA_s}\right]$$
$$u = 1 + 1\left(100,000\,\frac{\text{lb}}{\text{in}}\right)\left(\frac{4.25\,\text{in}}{2}\right)\left[\frac{2}{8.05\left(10^6\right)\,\text{lb}}\right]$$
$$u = 1.053$$
$$m = u - \sqrt{u^2 - 1} = 1.053 - \sqrt{1.053^2 - 1} = 0.723$$
$$C_g = \left[\frac{m\left(1 - m^{2n}\right)}{n\left[\left(1 + R_{EA}m^n\right)(1+m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$
$$C_g = \left[\frac{0.723\left(1 - 0.723^{2(4)}\right)}{4\left[\left(1 + (1.0)\,0.723^4\right)(1 + 0.723) - 1 + 0.723^{2(4)}\right]}\right]\left[\frac{1 + 1.0}{1 - 0.723}\right]$$
$$C_g = 0.952$$

Shear capacity of fasteners:

$$Q'_{90} = 0.6QC_D C_M C_t C_\Delta C_g$$
$$Q'_{90} = 0.6(1990\,\text{lb})(1.15)(1.0)(0.974)(0.952)$$
$$Q'_{90} = 1,273\,\text{lb}$$
$$V'_Q = nQ'_{90} = 4(1,273\,\text{lb}) = 5,092\,\text{lb} \geq V = 5,000\,\text{lb} \quad \therefore \text{ OK}$$

Dowel Requirements:

Since the shear plates are loaded in end grain, steel dowels will be used for the plate bolts. The dowel length in each member must be taken to be at least the required penetration for full design value of the shear plate with a lag screw fastener, 5 in., from the *National Design Specification*® (2). Thus, dowels of total length 2 (5 in. + 1 in.) = 12 in. will be used.

Member shear capacity (Section 12.2.4):

Shear at the splice must also be checked with respect to effective depth due to the fastener group. The effective depth is 17.875 in. − 2.563 in. + $\frac{1}{2}$ × 2.62 in. = 16.622 in.

$$d_e = d - E + D_O$$
$$d_e = 17.875\,\text{in} - 2.56\,\text{in} + \frac{2.62\,\text{in}}{2}$$
$$d_e = 16.63\,\text{in}$$
$$V'_r = \left[\frac{2}{3}F'_v b d_e\right]\left[\frac{d_e}{d}\right]^2$$
$$V'_r = \left[\frac{2}{3}(248\,\text{psi})(5\,\text{in})(16.63\,\text{in})\right]\left[\frac{16.63\,\text{in}}{17.875\,\text{in}}\right]^2$$
$$V'_r = 11,900\,\text{lb} \geq V = 5,000\,\text{lb} \quad \therefore \text{ OK}$$

Section properties of uncut section:

$$S = \frac{bd^2}{6} = \frac{(5\,\text{in})(17.875\,\text{in})^2}{6} = 266\,\text{in}^3$$
$$A = bd = (5\,\text{in})(17.875\,\text{in}) = 89.4\,\text{in}^2$$

Stresses on uncut section (Figure 15.3.4-1):

$$f_b = \frac{M}{S} = \frac{255,000\,\text{lb-in}}{266\,\text{in}^3} = 959\,\text{psi}$$
$$f_c = \frac{P}{A} = \frac{27,000\,\text{lb}}{89.4\,\text{in}^2} = 302\,\text{psi}$$

$$f_b + f_c = 959\,\text{psi} + 302\,\text{psi} = 1261\,\text{psi (compression)}$$

$$f_b - f_c = 959\,\text{psi} - 302\,\text{psi} = 657\,\text{psi (net tension)}$$

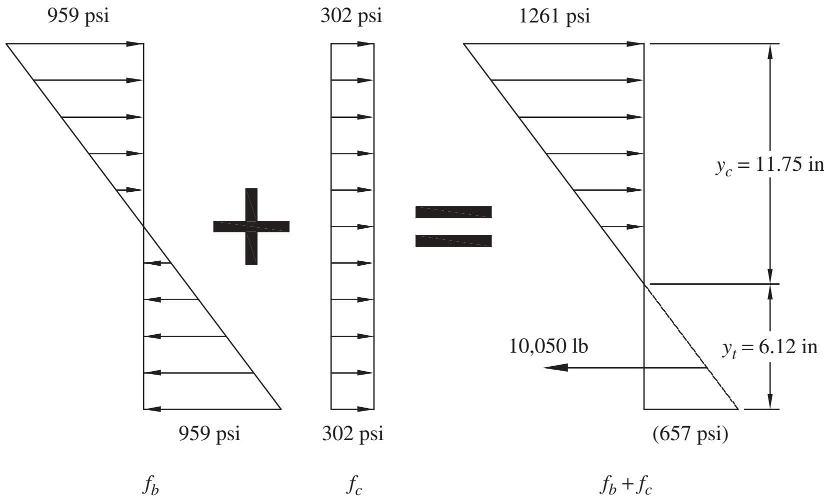


Figure 15.3.4-1 Stresses on uncut section—Example 15.3.4-1.

Neutral axis of uncut section (Figure 15.3.4-1):

$$y_t = \frac{657\,\text{psi}}{1261\,\text{psi}} y_c = 0.521 y_c$$

$$y_c + y_t = 17.875\,\text{in}$$

$$y_c + 0.521 y_c = 17.875\,\text{in}$$

$$y_c = 11.75\,\text{in} = \overline{y}$$

Tension force resultant on uncut section (Figure 15.3.1-1):

$$T = \frac{b\,(d - \overline{y})\,(f_b - f_c)}{2} = \frac{b y_t (f_b - f_c)}{2}$$

$$T = \frac{(5\,\text{in})(6.12\,\text{in})(657\,\text{psi})}{2} = 10,050\,\text{lb}$$

Estimated number of bolts required:

$$n \approx \frac{T}{Z'} = \frac{10,050\,\text{lb}}{(4,002\,\text{lb})C_g} = \frac{2.5}{C_g}$$

Try three bolts in a single row placed 1.5 in. from the bottom of the section (Figure 15.3.4-2).

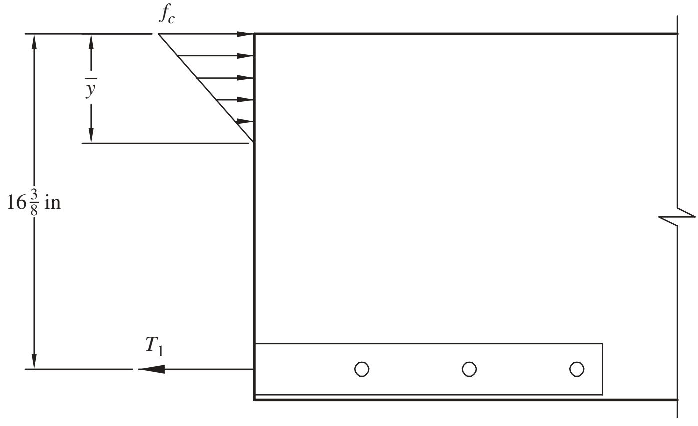


Figure 15.3.4-2 Location of bolted connection—Example 15.3.4-1.

Stiffness of row of fasteners (Equation 15.3.2-1):

$$(EA)_1 = n_{bolts}\gamma s_{\min}$$

$$(EA)_1 = n_{bolts}\left(270,000D^{1.5}\right)(4D)$$

$$(EA)_1 = 3\left(270,000\,(0.75)^{1.5}\ \text{lb/in}\right)(3.0\,\text{in}) = 1,580,000\,\text{lb}$$

Location of neutral axis in spliced section (Section 15.3.3):

$$E_w = E' = 1.8(10^6\ \text{psi})$$

$$b = 5\,\text{in}$$

$$d = 17.875\,\text{in}$$

$$y_1 = 16.375\,\text{in}$$

$$P_1 = 27,000\,\text{lb}$$

$$M_1 = 255,000\,\text{in-lb}$$

$$\frac{P_1}{M_1}\frac{\left[\dfrac{E_w b\overline{y}^2}{2}\left(\dfrac{d}{2}-\dfrac{\overline{y}}{3}\right)+(EA)_1\left(y_1-\overline{y}\right)\left(y_1-\dfrac{d}{2}\right)\right]}{\left[\dfrac{E_w b\overline{y}^2}{2}-(EA)_1\left(y_1-\overline{y}\right)\right]}-1=0$$

Using a spreadsheet to solve by iteration gives:

$$\bar{y} = 5.344\,\text{in}$$

Calculation of k (Equation 15.3.3-7):

$$k = \frac{P_1}{\left[\dfrac{E_w b\bar{y}^2}{2} - (EA)_1\left(y_1 - \bar{y}\right)\right]}$$

$$k = \frac{27,000\,\text{lb}}{\left[\dfrac{1.8(10^6\,\text{psi})(5\,\text{in})(5.344\,\text{in})^2}{2} - (1,580,000\,\text{lb})(16.375\,\text{in} - 5.344\,\text{in})\right]}$$

$$k = \frac{243\left(10^{-6}\right)}{\text{in}}$$

Tension force in steel plates (Equation 15.3.3-1):

$$T_1 = k\,(EA)_1\left(y_1 - \bar{y}\right)$$

$$T_1 = \frac{243\left(10^{-6}\right)}{\text{in}}\,(1,580,000\,\text{lb})\,(16.375\,\text{in} - 5.344\,\text{in})$$

$$T_1 = 4,240\,\text{lb}$$

Compression stress in splice (Equation 15.3.3-8):

$$f_c = E_w k\bar{y} = 1.8\left(10^6\,\text{psi}\right)\left[\frac{243\left(10^{-6}\right)}{\text{in}}\right](5.344\,\text{in}) = 2340\,\text{psi}$$

End grain bearing plate requirement (NDS 3.10.1.3):

$$\frac{f_c}{F_b'} = \frac{2340\,\text{psi}}{2760\,\text{psi}} = 0.85 > 0.75 \quad \therefore\ \text{Bearing plate is required}$$

Design of A36 steel side plates for gross section (Use plate width of 2.5 in.):

$$T_1 \le \frac{F_y A_g}{1.67}$$

$$4,240\,\text{lb} \le \frac{(36,000\,\text{psi})\,A_g}{1.67}$$

$$0.1967\,\text{in}^2 \le A_g = 2ht = 2\,(2.5\,\text{in})\,t \quad \Rightarrow \quad t = 0.039\,\text{in}$$

Design of A36 steel side plates for net section (Use plate width of 2.5 in.):

$$T_1 \le \frac{F_u A_n}{2}$$

$$4,240\,\text{lb} \le \frac{(58,000\,\text{psi})\,A_n}{2}$$

$$0.1462\,\text{in}^2 \le A_n = 2\,(h - (D + 0.0625\,\text{in}))\,t$$

$$0.1462\,\text{in}^2 \le A_n = 2\,(2.5\,\text{in} - (0.75\,\text{in} + 0.0625\,\text{in}))\,t \quad \Rightarrow \quad t = 0.043\,\text{in}$$

$\frac{1}{4}$ in. × 2.5 in. side plates are more than adequate.

Group action factor for $n = 3$, s = 4 in. (Section 11.5.4):

$$E'_m A_m = 1.8(10^6\,\text{psi})(5\,\text{in})(3\,\text{in}) = 27.0(10^6\,\text{lb})$$

$$E_s A_s = 29(10^6\,\text{psi})(2)(0.25\,\text{in})(2.5\,\text{in}) = 36.3(10^6\,\text{lb})$$

$$R_{EA} = \frac{E'_m A_m}{E_s A_s} = \frac{27.0(10^6\,\text{lb})}{36.3(10^6\,\text{lb})} = 0.744$$

$$u = 1 + n_j \gamma \frac{s}{2}\left[\frac{1}{E_m A_m} + \frac{1}{E_s A_s}\right]$$

$$u = 1 + 1\left(175,400\frac{\text{lb}}{\text{in}}\right)\left(\frac{4\,\text{in}}{2}\right)\left[\frac{1}{27.0(10^6\,\text{lb})} + \frac{1}{36.3(10^6\,\text{lb})}\right]$$

$$u = 1.023$$

$$m = u - \sqrt{u^2 - 1} = 1.023 - \sqrt{1.023^2 - 1} = 0.871\,;$$

$$C_g = \left[\frac{m\left(1 - m^{2n}\right)}{n\left[\left(1 + R_{EA} m^n\right)(1 + m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$

$$C_g = \left[\frac{0.871\left(1 - 0.871^{2(3)}\right)}{3\left[\left(1 + (0.744)\,0.871^3\right)(1 + 0.871) - 1 + 0.871^{2(3)}\right]}\right]\left[\frac{1 + 0.744}{1 - 0.871}\right]$$

$$C_g = 0.99$$

Geometry factor (Section 13.2.3):
The minimum end distance for full design load is $7D = 5.25$ in. The minimum edge distance for full design load is $1.5D = 1.125$ in., which is satisfied with the 2.0 in. distance from the bottom face. Spacing will be chosen as 4 in., which exceeds the spacing of $4D = 3.0$ in. required for full design value. Therefore, the geometry factor is $C_\Delta = 1.0$.

Fastener capacity:

$$T' = n_i Z'$$
$$T' = n_i Z C_D C_M C_t C_\Delta C_g$$
$$T' = 3(3480\,\text{lb})(1.15)(1.0)(1.0)(0.99)$$
$$T' = 11,890\,\text{lb} \geq T_1 = 4,240\,\text{lb} \quad \therefore \text{ OK}$$

Row tear-out capacity:

Assuming an end distance of 5.25 in. and spacing of 4.0 in., row tear-out is investigated using Equation 12.3.2.2-1 as follows:

$$T'_{rt} = \sum n_i \frac{F'_v}{2} A_{crit\ shear,\ rt}$$
$$T'_{rt} = n_j n_i \frac{F'_v}{2}\,(2ts)$$
$$T'_{rt} = (1)\,(3)\,(248\,\text{psi})\,(5\,\text{in})\,(4\,\text{in})$$
$$T'_{rt} = 14,880\,\text{lb} \geq T_1 = 4,240\,\text{lb} \quad \therefore \text{ OK}$$

Steel block shear analysis:

Each strap is checked with regard to block shear per the *Steel Construction Manual* [1], as follows, assuming the strap extends 2 in. past the end bolt.

$$R_{BS} = 0.3A_V F_u + 0.5A_t F_u$$
$$R_{BS} = 0.3A_V F_u + 0$$
$$R_{BS} = (0.3)(2)\,[(2)(2\,\text{in} + 5.25\,\text{in} + 5.25\,\text{in})(0.25\,\text{in})]\,(58,000\,\text{psi})$$
$$R_{BS} = 217,500\,\text{lb} \leq \text{T}_1 = 4,240\,\text{lb} \quad \therefore \text{ OK}$$

Load reversal:

The strap required to accommodate the positive design bending moment will be duplicated on the upper part of the splice to accommodate the negative design moment. The connection will be evaluated with respect to flexural stresses only.

Stiffness of row of fasteners for load reversal (Equation 15.3.2-1):

$$(EA)_1 = n_{bolts}\gamma s_{\min} = n_{bolts}\left(270,000D^{1.5}\right)(4D)$$
$$(EA)_1 = 3\left(270,000\,(0.75)^{1.5}\ \text{lb/in}\right)(3.0\,\text{in}) = 1,580,000\,\text{lb}$$

Location of neutral axis in spliced section (Section 15.3.3):

$$E_w = E' = 1.8\left(10^6\,\text{psi}\right)$$
$$b = 5\,\text{in}$$

$$y_1 = 16.375\,\text{in}$$
$$M_1 = 15,000\,\text{in-lb}$$

$$\frac{E_w b \overline{y}_m^2}{2} + \overline{y}_m \sum (EA)_j - \sum (EAy)_j = 0$$

$$\frac{1.8\left(10^6\,\text{psi}\right)(5\,\text{in})\,\overline{y}_m^2}{2} + \overline{y}_m\,(1,580,000\,\text{lb}) - (1,580,000\,\text{lb})\,(16.375\,\text{in}) = 0$$

$$4.50\left(10^6\,\text{lb/in}\right)\overline{y}_m^2 + \overline{y}_m\,(1,580,000\,\text{lb})\,\overline{y}_m - 25.87\left(10^6\,\text{lb-in}\right) = 0$$

$$\overline{y}_m = \frac{-1,580,000\,\text{lb} \pm \sqrt{(-1,580,000\,\text{lb})^2 - 4\left[4.50\left(10^6\,\text{lb/in}\right)\right]\left[-25.87\left(10^6\,\text{lb-in}\right)\right]}}{2\left(4.50\left(10^6\,\text{lb/in}\right)\right)}$$

$$\overline{y}_m = \frac{-1.58\left(10^6\right)\,\text{lb} \pm 21.64\left(10^6\right)\,\text{lb}}{9.00\left(10^6\,\text{lb/in}\right)} = 2.23\,\text{in}, -2.58\,\text{in}$$

$$\overline{y}_m = 2.23\,\text{in}$$

Calculation of k (Equation 15.3.3-10):

$$k = \frac{M_1}{\dfrac{E_w b \overline{y}_m^3}{3} + \sum_{j=1}^{n} (EA)_j \left(y_j - \overline{y}_m\right)^2}$$

$$k = \frac{15,000\,\text{in-lb}}{\dfrac{1.8\left(10^6\,\text{psi}\right)(5\,\text{in})\,(2.23\,\text{in})^3}{2} + (1,580,000\,\text{lb})\,(16.375\,\text{in} - 2.23\,\text{in})^2}$$

$$k = \frac{44.3\left(10^{-6}\right)}{\text{in}}$$

Tension force in straps (Equation 15.3.3-1):

$$T_1 = k\,(EA)_1\left(y_1 - \overline{y}\right)$$

$$T_1 = \frac{44.3\left(10^{-6}\right)}{\text{in}}\,(1,580,000\,\text{lb})\,(16.375\,\text{in} - 2.23\,\text{in})$$

$$T_1 = 990\,\text{lb}$$

Tension design capacity:

$$T' = n_i Z'$$
$$T' = n_i Z C_D C_M C_t C_\Delta C_g$$
$$T' = 3(3480\,\text{lb})(1.6)(1.0)(1.0)(0.99)$$
$$T' = 16{,}540\,\text{lb} \geq T_1 = 990\,\text{lb} \quad \therefore \text{ OK}$$

Compression stress in splice (Equation 15.3.3-8):

$$f_c = E_w k \bar{y} = 1.8\left(10^6\,\text{psi}\right)\left[\frac{44.3\left(10^{-6}\right)}{\text{in}}\right](2.23\,\text{in})$$
$$f_c = 180\,\text{psi} < F_b' = 3840\,\text{psi} \quad \therefore \text{ OK}$$

Answer: The salient features of the moment splice connection for the stated conditions are shown in Figure 15.3.4-3. The end distance for the bolts in the tension straps with respect to the wood members is increased to 7.0 in. to avoid interference with the dowels used for the shear plates, and the end distance with respect to the metal is taken to be 2.0 in., resulting in total strap length of 34 in. A 20-gauge steel bearing plate must also be inserted between the two members. Bolt holes in the bearing plate should be slotted or oversized to permit unrestrained shrinkage of the members, or two separate plates should be used.

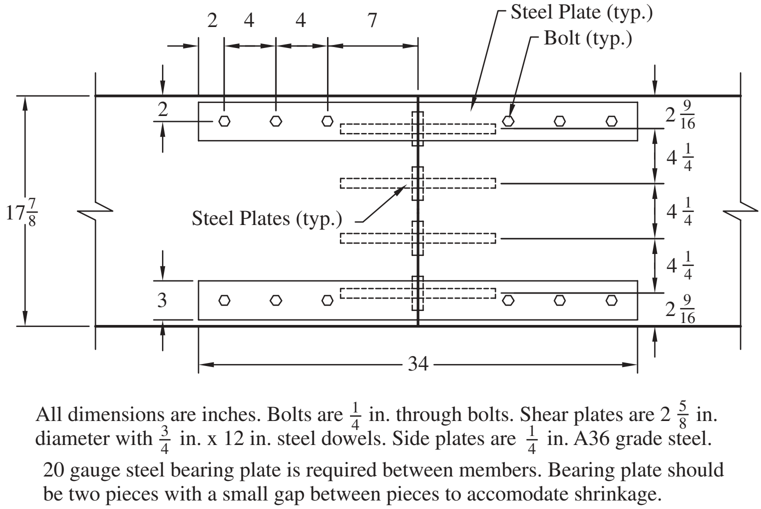


Figure 15.3.4-3 Moment splice—Example 15.3.4-1.

Discussion: In this example, the tension capacity of the connection significantly exceeded the tension load, so it is possible that two bolts could have been used on each side of the splice. However, reducing the number of bolts would reduce the stiffness of the connection, requiring reevaluation of the moment splice. Reduced connection stiffness would result in reduced tension force and increased compression stress. For convenience, the connection configuration determined for the bottom part of the connection was also used for the upper part, which in this case was more than adequate. A smaller strap or strap with fewer fasteners could have also been used, producing an asymmetric splice connection. It may also be possible to use narrower straps at both top and bottom, provided the minimum edge distances and the tensile stress requirements for the steel are still satisfied.

15.4 CONCLUSION

Due to the long lengths possible with structural glued laminated timber, moment splices are generally not required for straight members such as beams. However, arches commonly require moment splices to facilitate shipping. The design of moment splices is somewhat complex, because the forces are statically indeterminate.

Typical moment splices utilize shear plates in end grain to transfer the shear forces, bolts with steel straps to transfer tension forces, and end grain bearing to transfer compression forces. Shear plates may also be used to increase the capacity and stiffness of the tension connections. This chapter presented procedures, equations, and an example for the design of moment splices.

CHAPTER 16

LOAD AND RESISTANCE FACTOR DESIGN

16.1 INTRODUCTION

This chapter discusses load and resistance factor design (LRFD) as used in timber engineering. LRFD is similar to allowable stress design (ASD) already covered in this manual in that proper design results in structural members and connections that are capable of resisting the loads or forces that can be expected in the life of the structure with some factor of safety and without loss of serviceability.

In ASD, trial members are selected and stresses are calculated due to design loads. The stresses are compared to allowable stresses for the members and trial grades and sizes are considered acceptable when all design stresses due to applied loads do not exceed the allowable stresses. The allowable stresses are determined from design values adjusted to end use condition. These design values incorporate internal factors of safety, and are thus lower than the actual strengths of the materials. The (internal) factors of safety account for overload (legitimate uncertainty in the predicted load), possible under-strength (of the member), and the relative importance of the member in the structure (consequences and nature of possible failure). Design checks are performed at the service load (stress) level.

Using the LRFD methodology, structural members are selected such that their strength or resistance capacities exceed the structural demand from all factored design loading conditions.The member capacities are explicitly reduced by resistance factors (strength reduction factors) to account for possible under-strength, and for modes and consequences of possible failure. The design loading conditions are explicitly factored using prescribed LRFD load combinations to include uncertainty in future loading. Design checks for strength are thus performed at the limit levels or states instead of service load conditions. These checks can generally be performed in terms of stress versus strength or in terms of load versus capacity; examples of each are included in this chapter. In general, member capacity values are obtained by multiplying material strength values by the appropriate section properties. Thus, the design procedure in LRFD is to design or determine member size, grade, orientation, and bracing such that the reduced capacity exceeds or equals all anticipated factored loads.

16.2 DESIGN VALUES AND ADJUSTMENT FACTORS

Design values (reference design values) for LRFD are the same as those for ASD and are found in the *National Design Specification*® [1] and other sources. These values must be adjusted for end use (moisture content, bracing conditions, etc.) using the same adjustment factors as for ASD, with the exception of the *load duration factor* and using the *additional format conversion factor*, *resistance factor*, and *time effect factor*. Format conversion factors convert the reference design values to strength or limit state. Resistance factors account for possible under-strength and mode of failure. Time effect factors account for the effect of load duration.

16.2.1 Time Effect Factor

Wood is unique as a building material, because its strength properties depend on the duration of the applied load. Thus, for any structural member, and, in fact, any particular design check (shear, flexure, etc.), the *strength* of the member depends on the time effect (duration) of the load (or combination of loads) considered.

A key difference between ASD and LRFD methodologies is in the treatment of this important effect and its relationship to load combinations. ASD accounts for the time-dependence of strength using the load duration factor, C_D, which is applied based on the *shortest duration* load combination being considered. In LRFD, the load duration factor is functionally replaced with the time effect factor, λ, which is based on the *primary* load in the load combination under consideration.

It is important to note that while the load duration factor and the time effect factor both account for the effect of load duration on strength, *they are not identical.* For example, in ASD the load duration factor for occupancy live load (10 year duration) is 1.0 and for snow load (2 month duration) is 1.15. In LRFD, the time effect factors for occupancy live *and* snow are (both) 0.8.

These differences between the time effect factor and the load duration factor may lead to different designs required to satisfy the requirements of LRFD and ASD for certain cases. For example, the design of floors in ASD will generally be conservative relative to LRFD, due primarily to the difference in load duration/time effects.

Load combinations and the corresponding time effect factors for wood design using LRFD are shown in Table 16.2.1-1.

TABLE 16.2.1-1 Load Combinations and Time Effect Factors [1]

Load Combination[a]	Time Effect Factor, λ
$14D$	0.6
$12D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	0.7 when L is from storage 0.8 when L is from occupancy 1.25 when L is from impact[b]
$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + L \text{ or } 0.8W)$	0.8
$1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$	1.0
$1.2D + 1.0E + L + 0.2S$	1.0
$0.9D + 1.0W$	1.0
$0.9D + 1.0E$	1.0

[a] Load combinations and load factors consistent with ASCE 7-10 are listed for ease of reference.
[b] Time effect factors, λ, greater than 1.0 do not apply to connections or to structural members pressure treated with water-borne preservatives or fire retardant chemicals.

The loads in Table 16.2.1-1 are defined as follows.

D = Dead load
E = Earthquake load
L = Live load caused by storage, occupancy, or impact
L_r = Roof live load
R = Rain load
S = Snow load
W = Wind load

16.2.2 Format Conversion Factors for Use with NDS

Because ASD is more commonly chosen for timber design, the reference design values in the *NDS*® [1] are published on an ASD basis to minimize disruption for designers using that methodology with the dual-format *NDS*®. Consequently, LRFD design includes a format conversion factor to adjust the stresses to the appropriate level for LRFD. The format conversion factor converts the reference design value to the strength or limit state. Format conversion factors for the design of wood buildings using LRFD are provided in Table 16.2.2-1.

TABLE 16.2.2-1 Format Conversion Factors for Design of Wood Buildings Using LRFD [1]

Property or Design Value	Format Conversion Factor, K_F
F_b	2.54
F_t	2.70
F_c	2.40
F_v, F_{rt}, F_s	2.88
$F_{c\perp}$	1.67
$E_{\min}$	1.76
Connections	3.32

AASHTO [2] has adopted different format conversion factors for the design of wood bridges. The AASHTO format conversion factors are provided in Chapter 17.

16.2.3 Resistance Factors

Resistance factors for the design of wood buildings are listed in Table 16.2.3-1.

TABLE 16.2.3-1 Resistance Factors for Wood Design [1]

ϕ_c	Compression (parallel- and perpendicular-to-grain)	0.90
ϕ_b	Bending (flexure)	0.85
ϕ_s	Stability	0.85
ϕ_t	Tension	0.80
ϕ_v	Shear parallel to grain, rolling shear, torsion, and radial tension	0.75
ϕ_s	Modulus of elasticity for beam and column stability ($E_{\min}$)	0.85
ϕ_Z	Connections	0.65

16.2.4 Applicability of Adjustment Factors

Table 16.2.4-1 summarizes the applicability of the factors discussed with regard to LRFD with glued laminated timber.

Descriptions of the adjustment factors other than those described in Table 16.2.4-1 are found in Chapter 3. It can be noted from the Table 16.2.4-1 that the time effect factor is not applicable to the modulus of elasticity for deflection calculations (E) and stability calculations ($E_{\min}$). The format conversion factor and resistance factor are applicable to $E_{\min}$ used in stability calculations; as such, the beam stability factor (C_L) and column stability factors (C_P) may not be identical for any particular design check in LRFD as compared to ASD.

16.3 DESIGN CHECKS

Design checks may be performed on material stresses or member capacities. For example, if the design check is considering flexure, a member must be selected so that the reduced adjusted reference design value in bending is equal to or exceeds the bending stresses resulting from all applicable loading combinations (Equation 16.3-1).

$$F'_b \geq f_{bu} \tag{16.3-1}$$

where:

F'_b = LRFD adjusted design value in bending
f_{bu} = design stress based on factored loads

TABLE 16.2.4-1 Applicability of Adjustment Factors

Member Type and Loading Condition	Design Values and Adjustment Factors
Visually Graded Dimension Lumber	
Bending	$F'_b = F_b C_M C_F C_{fu} C_L C_r C_t C_i K_F \phi_b \lambda$
Tension parallel to grain	$F'_t = F_t C_M C_F C_t C_i K_F \phi_t \lambda$
Compression parallel to grain	$F'_c = F_c C_M C_F C_P C_t C_i K_F \phi_c \lambda$
Compression perpendicular to grain	$F'_{c\perp} = F_{c\perp} C_M C_t C_b K_F \phi_c$
Shear parallel to grain	$F'_v = F_v C_M C_t C_i K_F \phi_v \lambda$
Modulus of elasticity for deflection	$E' = E C_M C_t$
Modulus of elasticity for buckling	$E'_{\min} = E_{\min} C_M C_t K_F \phi_s$
Mechanically Graded Dimension Lumber	
Bending	$F'_b = F_b C_M C_L C_r C_t C_i K_F \phi_b \lambda$
Tension parallel to grain	$F'_t = F_t C_M C_t C_i K_F \phi_t \lambda$
Compression parallel to grain	$F'_c = F_c C_M C_P C_t C_i K_F \phi_c \lambda$
Compression perpendicular to grain	$F'_{c\perp} = F_{c\perp} C_M C_t C_b K_F \phi_c$
Shear parallel to grain	$F'_v = F_v C_M C_t C_i K_F \phi_v \lambda$
Modulus of elasticity for deflection	$E' = E C_M C_t$
Modulus of elasticity for buckling	$E'_{\min} = E_{\min} C_M C_t K_F \phi_s$
Sawn Timbers 5 in. × 5 in. and Larger	
Bending	$F'_b = F_b C_M C_F C_L C_t K_F \phi_b \lambda$
Tension parallel to grain	$F'_t = F_t C_M C_t C_i K_F \phi_t \lambda$
Compression parallel to grain	$F'_c = F_c C_M C_P C_t K_F \phi_c \lambda$
Compression perpendicular to grain	$F'_{c\perp} = F_{c\perp} C_M C_t C_b K_F \phi_c$
Shear parallel to grain	$F'_v = F_v C_M C_t K_F \phi_v \lambda$
Modulus of elasticity for deflection	$E' = E C_M C_t$
Modulus of elasticity for buckling	$E'_{\min} = E_{\min} C_M C_t K_F \phi_s$
Structural Glued Laminated Timber (Glulam)	
Bending	$F'_{bx} = F_{bx} C_M \left(C_V \text{ or } C_L\right) C_C C_I C_t K_F \phi_b \lambda$
Bending	$F'_{by} = F_{by} C_M C_L C_{fu} C_t K_F \phi_b \lambda$
Tension parallel to grain	$F'_t = F_t C_M C_t K_F \phi_t \lambda$
Compression parallel to grain	$F'_c = F_c C_M C_P C_t K_F \phi_c \lambda$
Compression perpendicular to grain	$F'_{c\perp} = F_{c\perp} C_M C_t C_b K_F \phi_c$
Shear parallel to grain	$F'_v = F_v C_M C_t C_{vr} K_F \phi_v \lambda$
Modulus of elasticity for deflection	$E' = E C_M C_t$
Modulus of elasticity for buckling	$E'_{\min} = E_{\min} C_M C_t K_F \phi_s$
Radial tension	$F'_{rt} = F_{rt} C_D C_M C_t K_F \phi_v \lambda$
Decking	
Bending	$F'_b = F_b C_M C_t K_F \phi_b \lambda$
Compression perpendicular to grain	$F'_{c\perp} = F_{c\perp} C_M C_t C_b K_F \phi_c$
Shear parallel to grain	$F'_v = F_v C_M C_t C_i K_F \phi_v \lambda$
Modulus of elasticity	$E' = E C_M C_t$

Alternately, in terms of member capacity, the design check may be as shown in Equation 16.3-2.

$$M' \geq M_u \tag{16.3-2}$$

where:

M' = adjusted design capacity of member
M_u = moment from factored loads

Designers familiar with nominal strength or capacity calculations using LRFD or strength design with other materials might wish to cast the previous equation in the following form (Equation 16.3-3):

$$M' = \phi_b M_n \geq M_u \tag{16.3-3}$$

where:

M_n = nominal moment capacity of the member

Deflection calculations in LRFD are performed in the same manner as with ASD, in that unfactored loads (or portions of loads) are used to determine deflections using average modulus of elasticity.

EXAMPLE 16.3-1 GLUED LAMINATED TIMBER BEAM

Given: A $5\frac{1}{8}$ in. × 12 in., 24F-1.8E DF glulam beam has been proposed to support a residential floor live load of 800 lb/ft and a dead load of 300 lb/ft in addition to its own weight of 14 lb/ft. The simply supported beam will span 12 ft from the centerline of one support to the centerline of the other. The top of the beam is laterally supported to prevent buckling. The beam will be subject to dry conditions of use and normal temperatures. (See also Example 4.3.1-1.)

Wanted: Evaluate the beam using LRFD.

Solution:

Design values:

$$F'_{bx} = F_{bx} C_M \left(C_V \text{ or } C_L\right) C_c C_I C_t K_F \phi_b \lambda$$

$$F'_{bx} = (2400\,\text{psi})\,(1.0)\,(1.0)\,(1.0)\,(1.0)\,(1.0)\,(2.54)\,(0.85)\,(0.8)$$

$$F'_{bx} = 4145\,\text{psi}$$

$$F'_{vx} = F_{vx} C_M C_t C_{vr} K_F \phi_v \lambda$$

$$F'_{vx} = (265\,\text{psi})\,(1.0)\,(1.0)\,(1.0)\,(2.88)\,(0.75)\,(0.8)$$

$$F'_{vx} = 458 \text{ psi}$$

$$E'_x = E_x C_M C_t = E_x = 1.8\left(10^6\right) \text{ psi}$$

Section properties:

$$S = \frac{bd^2}{6} = \frac{5.125 \text{ in } (12 \text{ in})^2}{6} = 123 \text{ in}^3$$

$$I = \frac{bd^3}{12} = \frac{5.125 \text{ in } (12 \text{ in})^3}{12} = 738 \text{ in}^4$$

Factored bending moment:

$$M_u = 1.2M_D + 1.6M_L$$

$$M_u = \frac{1.2w_D \ell^2}{8} + \frac{1.6w_L \ell^2}{8}$$

$$M_u = \frac{1.2 \text{ (314 lb/ft) (12 ft)}^2}{8} + \frac{1.6 \text{ (800 lb/ft) (12 ft)}^2}{8}$$

$$M_u = 28{,}820 \text{ ft-lb} = 357{,}900 \text{ in-lb}$$

Factored bending resistance:

$$M' = F'_{bx} S$$

$$M' = (4145 \text{ psi})\left(123 \text{ in}^3\right)$$

$$M' = 509{,}800 \text{ in-lb} \geq M_u = 357{,}900 \text{ in-lb} \qquad \therefore \text{ OK}$$

Factored shear load:

$$V_u = 1.2V_D + 1.6V_L$$

$$V_u = \frac{1.2w_D \ell}{2} + \frac{1.6w_D \ell}{2}$$

$$V_u = \frac{1.2 \text{ (314 lb/ft) (12 ft)}}{2} + \frac{1.2 \text{ (800 lb/ft) (12 ft)}}{2}$$

$$V_u = 8301 \text{ lb}$$

$$V_u = 8020 \text{ lb}$$

Factored shear resistance:

$$V' = F'_{vx}\left(\frac{2bd}{3}\right)$$

$$V' = (458\,\text{psi})\left(\frac{2\,(5.125\,\text{in})\,(12\,\text{in})}{3}\right)$$

$$V' = 18{,}780\,\text{lb} \geq V_u = 8020\,\text{lb} \qquad \therefore\ \text{OK,}$$

Live load deflection:

$$\Delta_L = \frac{5w_L\ell^4}{384EI}$$

$$\Delta_L = \frac{5\,(800\,\text{lb/ft})\,(12\,\text{ft})^4\left(\frac{1728\,\text{in}^3}{\text{ft}^4}\right)}{384\left(1.8\left(10^6\ \text{psi}\right)\right)\left(738\,\text{in}^3\right)}$$

$$\Delta_L = 0.28\,\text{in}$$

Live load deflection limit (Table 3.5-1):

$$\delta_L = \frac{\ell}{360} = \frac{144\text{in}}{360} = 0.40\,\text{in} \geq \Delta_L = 0.28\,\text{in} \qquad \therefore\ \text{OK}$$

***L* + 0.5*D* deflection:**

$$\Delta_L = \frac{5w_{L+0.5D}\,\ell^4}{384EI}$$

$$\Delta_L = \frac{5\,(800\,\text{lb/ft} + 0.5\,(314\,\text{lb/ft}))\,(12\,\text{ft})^4\left(\frac{1728\,\text{in}^3}{\text{ft}^3}\right)}{384\left(1.8\left(10^6\ \text{psi}\right)\right)\left(738\,\text{in}^4\right)}$$

$$\Delta_L = 0.34\,\text{in}$$

***L* + 0.5*D* deflection limit (Table 3.5-1):**

$$\delta_{L+0.5D} = \frac{\ell}{240} = \frac{144\,\text{in}}{240} = 0.60\,\text{in} \geq \Delta_{L+0.5D} = 0.34\,\text{in} \qquad \therefore\ \text{OK}$$

Result: Using the LRFD methodology, a $5\frac{1}{8}$ in. × 12 in. 24F-1.8 DF beam is satisfactory for the stated load and conditions.

EXAMPLE 16.3-2 STRUCTURAL GLUED LAMINATED TIMBER COLUMN

Given: An 18 ft long, $6\frac{3}{4}$ in. × $8\frac{1}{4}$ in., 16F-V2 southern pine glulam timber is to be used as a column. The top and bottom of the column are held to prevent translation, and lateral support is provided at mid-height to resist buckling about the *y-y* axis (weak direction). The column is subject to centric axial loads of 12,000 lb dead and 26,000 lb live and is used in a dry location. (See also Example 5.3.2-1.)

Wanted: Evaluate the given section subject to the stated dead and live loads using LRFD. (The live load is due to occupancy.)

Approach: From AITC 117 (1), the 16F-V2 SP layup is normally used as a beam. However, axial design values are provided in AITC 117 Table A1-Expanded [1], and they will be used to evaluate the column. Because the moduli of elasticity are different for the *x-x* and *y-y* directions, critical buckling design values will need to be calculated for both directions.

Solution:

Design values (AITC 117 [3]):

$$F_c' = F_c^* C_P = F_c C_M C_t C_P K_F \phi_c \lambda$$
$$F_c' = F_c^* C_P = (1300\,\text{psi})\,(1.0)\,(1.0)\,C_P\,(2.40)\,(0.9)\,(0.8)$$
$$F_c' = F_c^* C_P = (2246\,\text{psi})\,C_P$$

$$E'_{x\ \min} = E_{x\ \min} C_M C_t K_F \phi_s$$
$$E'_{x\ \min} = 790{,}000\,\text{psi}\,(1.0)\,(1.0)\,(1.76)\,(0.85)$$
$$E'_{x\ \min} = 1.18\left(10^6\,\text{psi}\right)$$

$$E'_{y\ \min} = E_{y\ \min} C_M C_t K_F \phi_s$$
$$E'_{y\ \min} = 740{,}000\,\text{psi}\,(1.0)\,(1.0)\,(1.76)\,(0.85)$$
$$E'_{y\ \min} = 1.11\left(10^6\,\text{psi}\right)$$

Effective lengths (Section 3.4.3.9.2):

$$\ell_{e1} = K_e \ell_1 = (1.0)\,(18\,\text{ft}) = 18\,\text{ft} = 216\,\text{in}$$
$$\ell_{e2} = K_e \ell_2 = (1.0)\,(9\,\text{ft}) = 9\,\text{ft} = 108\,\text{in}$$

Slenderness ratios (Section 3.4.3.9.1):

$$\frac{\ell_{e1}}{d_1} = \frac{216\,\text{in}}{8.25\,\text{in}} = 26.2 \le 50 \qquad \therefore\ \text{OK}$$
$$\frac{\ell_{e2}}{d_2} = \frac{108\,\text{in}}{6.75\,\text{in}} = 16.0 \le 50 \qquad \therefore\ \text{OK}$$

Critical buckling design values (Equation 3.4.3.9-2):

$$F_{cE1} = \frac{0.822\,(1{,}180{,}000\,\text{psi})}{(26.2)^2} = 1413\,\text{psi}$$

$$F_{cE2} = \frac{0.822\,(1{,}110{,}000\,\text{psi})}{(16.0)^2} = 3564\,\text{psi}$$

Column stability factor (Equation 3.4.3.9-1):

$$C_P = \frac{1 + \left(F_{cE}/F_c^*\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_c^*\right)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$C_P = \frac{1 + \left(\frac{1413\,\text{psi}}{2246\,\text{psi}}\right)}{2(0.9)} - \sqrt{\left[\frac{1 + \left(\frac{1413\,\text{psi}}{2246\,\text{psi}}\right)}{2(0.9)}\right]^2 - \frac{\left(\frac{1413\,\text{psi}}{2246\,\text{psi}}\right)}{0.9}}$$

$$C_P = 0.559$$

Adjusted compression design value:

$$F_c' = F_c^* C_P = (2246\,\text{psi})\,C_P$$

$$F_c' = (2246\,\text{psi})\,(0.559)$$

$$F_c' = 1256\,\text{psi}$$

Applied compression force:

$$P_u = 1.2P_D + 1.6P_L$$

$$P_u = 1.2\,(12{,}000\,\text{lb}) + 1.6\,(26{,}000\,\text{lb})$$

$$P_u = 56{,}000\,\text{lb}$$

Column capacity:

$$P' = F_c' bd$$

$$P' = (1256\,\text{psi})\,(6.75\,\text{in})\,(8.25\,\text{in})$$

$$P' = 69{,}940\,\text{lb} \geq P_u = 56{,}000\,\text{lb}$$

Answer: The 6.75 in. × 8.25 in., 16F-V2 SP timber is acceptable for the stated load condition. In this application a straight member must be specified, because unbalanced members are often manufactured with camber for beam applications.

Discussion: Comparison of the unity check of this example with that of Example 5.3.2-1 will show that the LRFD and ASD methods do not produce identical results. In the ASD example, the column is stressed to 84% of its capacity. In the LRFD example, the column is loaded to only 80% of its capacity. The example also illustrates that in LRFD the E_{min} values used in stability calculations (beam and column) are multiplied by the resistance and format conversion factors in addition to the applicable adjustment factors used in ASD.

16.4 CONCLUSION

In many cases, the load and resistance factor design (LRFD) methodology will produce identical or near identical results (member size selections) as with using the allowable stress design approach (ASD), particularly since structural members are manufactured in discrete sizes and commonly accepted grades. In cases where dead loads account for large proportions of total loads, the load and resistance factor design approach might result in smaller members since the LRFD approach distinguishes dead loads with lower load factors. The differences between application of the time effect factor (LRFD) and the load duration factor (ASD) might also lead to some differences in member selection.

Although some examples in this manual have been solved using both methods, in the design of any particular structure only one method should be used. Provision of the two methodologies is not intended to imply a design procedure that goes back and forth between the two.

CHAPTER 17

TIMBER BRIDGES

17.1 INTRODUCTION

From the earliest colonial days to the middle of the nineteenth century, wood was the predominant bridge building material used in America. The early covered bridges are a part of our nation's heritage. Great timber truss bridges with clear spans up to 340 feet in length testify to the skill of our early bridge designers and builders [1].

Commercial pressure-treating processes for the application of preservative chemicals were introduced in the middle of the nineteenth century and helped to assure the longevity of timber bridges [2]. Glued laminated timber was introduced into the United States in the 1930s. With the development of waterproof adhesives in the 1940s, it became practical to use glued laminated timber in bridges and other outdoor structures. Glued laminated timber has since become an important engineered material for vehicular, railway, and pedestrian bridges.

Timber bridge design incorporates the methods described in this manual in combination with the loading and other requirements put forth by organizations such as the American Association of State Highway and Transportation Officials (AASHTO) for highway bridges, and the American Railway Engineering and Maintenance-of-Way Association (AREMA) for railway bridges. Design of non-timber components and support structures (such as steel and concrete) must be done in accordance with the accepted design standards and practices associated with the specific materials. Though not covered in this manual, bridge design must also incorporate proper location, site preparation, roadway design, geotechnical investigation, foundation design, and hydrology/hydraulic investigations.

17.2 TYPES OF TIMBER BRIDGES

Timber bridges consist of seven basic types: (1) trestle, (2) longitudinal deck, (3) longitudinal stress-laminated, (4) girder, (5) truss, (6) arch, and (7) portable and temporary timber bridges, which are used only occasionally.

17.2.1 Trestles

The trestle is a simple type of timber bridge. Timber trestles consist of stringers (girders) supported by pile or frame bents. The bridge deck is applied to the stringers. Pile and frame bents are typically capped by nominal 12 in. × 12 in. or larger timbers, fastened to the tops of the piles or posts. Frame bents must rest on some type of foundation structure, such as concrete footings or piles. Sway bracing and longitudinal tower bracing, appropriate to the height of the bent, must be provided.

Spacing of bents is determined, in part, by the commercially available lengths of sawn timber stringers, which are typically available in even-foot increments. The ends of interior stringers are commonly lapped and fastened to the bearing on the caps, whereas exterior stringers are butted to the ends and spliced over the bent caps.

Stringers are designed as simple-span or continuous beams under the loadings recommended by AASHTO or AREMA. Sizes and spacing are determined by the span and loading conditions. Solid blocking should be provided at the ends of stringers to hold them in line and also to serve as a fire stop. Bridging should also be placed between stringers at mid-span and, on long spans, at additional intermediate locations.

17.2.2 Longitudinal Deck Bridges

The longitudinal deck bridge (Figure 17.2.2-1) offers a low-profile structure that is ideal for short spans and where clearance below the structure is limited. Longitudinal glulam panels offer an excellent alternative for deck replacement on existing bridges. Longitudinal glulam panels are commonly made in widths of 4 ft, but can be made in other widths as required for the design.

Crossties or stiffeners placed under the deck are required to tie the deck panels together and to distribute wheel loads between panels. For vehicular bridges, wheel loads are distributed laterally using transverse stiffeners. AASHTO requires the transverse stiffeners to have a minimum stiffness, *EI,* of 80,000 kip-in.^{2} and provides factors to determine the wheel load fraction supported by each panel [3, 4]. A stiffener must be placed at mid-span with additional stiffeners as required based on maximum spacing between stiffeners of 10 ft. Stiffeners can also be used as part of the guardrail support assembly. If a rail system is used that does not rely on the stiffeners as part of its support, the stiffeners can be reduced in size considerably [5, 6].

Figure 17.2.2-1 Longitudinal glulam deck vehicular bridge, 40-ft span. (Photo courtesy of Western Wood Structures, Inc., Tualatin, Oregon)

17.2.3 Longitudinal Stress-Laminated Girder Deck Bridges

The longitudinal stress-laminated girder bridge offers a low-profile structure for short to intermediate spans where clearance below the structure is limited. Longitudinal stress-laminated girder bridges consist of horizontally glued laminated girders placed side by side. The girders are supported on abutments and other supports as required by the design. High-strength rods are installed transverse to the girders and stressed to squeeze the girders together. The stress laminated longitudinal girders act as a large, continuous "plate" without joints. The differential movement between girders normally experienced in other bridge systems is eliminated and adverse effects on asphalt wearing surfaces are eliminated. Load transfer between the girders is by friction rather than by adhesive or mechanical connections. The stress-laminated girders also form the deck of the structure.

The stressing system consists of high-strength, hot-dipped galvanized threaded steel rods and nuts, and anchors specifically designed for the stressing system. The rods used are from $\frac{5}{8}$ in. to $1\frac{3}{8}$ in. in diameter. The stressing rods are spaced uniformly along the length of the girders. Prebored holes located near the mid-depth of the girders allow the rods to be installed in the field. Each rod is stressed in tension using hydraulic jacks. The tension force in each rod is generally in the range of 80,000 to 100,000 pounds. The anchorages must be large enough to prevent crushing of wood at the bearing points and often steel plates are used to distribute the force over a larger area. The initial level of stress will be reduced by creep and shrinkage in the wood making it necessary to restress the rods after one week and again at 4 to 6 weeks after the initial stressing. Periodic monitoring and re-stressing of the rods are typically required to ensure proper performance.

Girders used to form the bridge are of standard industry widths, or may include a combination of widths, but all girders must be of the same depth. Girders may be of any depth, but are generally based on multiples of the standard lamination thicknesses used in the industry ($1\frac{3}{8}$ in. for southern pine or $1\frac{1}{2}$ in. for other species).

17.2.4 Girder Bridges

The girder bridge is the most commonly used timber bridge system (Figure 17.2.4-1). Short span structures (less than 24 ft) are typically constructed from sawn lumber, while longer spans normally consist of glued laminated timber girders supporting a transverse bridge deck. Girders are also referred to as *stringers* and the terms may be used interchangeably. Substructures for girder bridges can be similar to those used for timber trestles.

The selection of decks for timber bridges is determined by density of traffic and economics. Plank decks may be used for light traffic or for temporary bridges. Glulam decks can be used for heavier traffic conditions. Asphalt wearing surfaces may be applied on glulam decks, but they are not usually applied over plank decks.

AASHTO limits transverse glulam deck panels to widths of 3 to 6 ft (Ref. [3], Section C9.9.4.1). However, to facilitate fabrication, panel widths are typically limited to half of the maximum spacing of the guardrail posts. This allows the guardrail to be attached to the center of alternating panels and avoids requiring every panel to be fabricated differently. An odd number of panels will allow the guardrail to be attached at the center of alternating panels with the posts symmetrical about the mid-span of the bridge.

Wet-use design values are used for the design of deck panels as the moisture content may exceed 16% in service. Dry-use design values are commonly used for the design of the glued laminated timber stringers supporting glulam deck

Figure 17.2.4-1 116 ft glulam girder vehicular bridge. (Photo courtesy of Western Wood Structures, Inc., Tualatin, Oregon)

panels, with the exception of compression perpendicular to grain stress, $F_{c\perp}$. The glued laminated timber deck panels, when properly treated and installed, typically provide a roof for the stringers, keeping them dry. Wind-driven moisture can cause a surface wetting on the exposed face of the edge stringers, but this superficial wetting should not affect the design. However, localized moisture accumulations can develop, and preservative treatment is required to protect the stringers against possible decay hazards such as at areas of steel connections and at bearing locations.

17.2.5 Truss Bridges

Truss bridges may be any of three types: deck-truss, through-truss, or pony-truss. Deck-truss bridges are characterized by trusses placed below the bridge deck and roadway (Figure 17.2.5-1). For through-truss bridges, the bridge deck and roadway are placed near the bottom of the trusses and pass between two parallel trusses. Overhead bracing is usually required to provide stability to the trusses. With pony-truss bridges, the deck and roadway are placed somewhere between the top and bottom of the truss, with no overhead bracing incorporated in the design. The deck-truss bridge is generally more economical, though the use of deck trusses may be precluded by under-clearance requirements. Through-trusses have the disadvantage of potential damage from vehicles.

Substructures for truss bridges may be similar to those for timber trestles or girder bridges; however, because the spans are greater, the bents must be capable of carrying higher loads. For heavily loaded or long-span trusses, timber, stone, or concrete piers may be required. Lateral forces also tend to be greater on truss bridges, and a carefully designed substructure sway bracing system is necessary.

Figure 17.2.5-1 Glulam deck-truss vehicular bridge. 148 ft span. (Photo courtesy of Western Wood Structures, Inc., Tualatin, Oregon)

Figure 17.2.5-2 88-ft covered vehicular bridge. (Photo courtesy of Western Wood Structures, Inc., Tualatin, Oregon)

The design of trusses for bridges is similar to that for roof trusses, the length of truss panel being determined by economical spacing of support beams, minimum number of joints, and commercially available lengths of timber. As in roof truss design, the joint design is an important consideration. Bridge truss joints should be designed to eliminate or minimize moisture entrapment.

Covered bridges (Figure 17.2.5-2) are typically through-truss bridges with roofs and walls added to keep the structure dry. More information on covered bridges can be obtained from the *Covered Bridge Manual* published by the U.S. Federal Highway Administration [7].

17.2.6 Arch Bridges

When site conditions are such that considerable height is required between the foundation and the roadway, or a relatively long clear span is required, an arch bridge may be most economical because of the lesser need for substructure framing. Arch bridges may be of the two-hinged or three-hinged type, two-hinged designs being more frequently used on short spans and three-hinged designs on long spans. Glued laminated timber arches may be fabricated to the desired shape and the ends built up to the level of the roadway by means of post bents. Post bents may be connected to the arch by means of steel gusset plates, which should be designed for erection loads, possible stress reversals, and lateral forces as well as for the anticipated bridge loads.

Arch bridges can be designed as (1) deck arches with the deck above the arch (Figure 17.2.6-1), (2) through arches where the arch springs from below the deck and rises above it at mid-span, or (3) through arches where the arch is above the roadway for the length of the arch (Figure 17.2.6-2). In any case, special

Figure 17.2.6-1 Horizontally curved glulam deck-arch bridge. (Photo courtesy of Western Wood Structures, Inc., Tualatin, Oregon)

Figure 17.2.6-2 Glulam through-arch bridge.

attention must be given to the lateral bracing requirements of the arches, because arches are predominantly compression members.

17.2.7 Portable and Temporary Bridges

Glued laminated timber bridges are ideal for use as portable or temporary bridges. Portable bridges have been used in military, forestry, utility, and other construction applications where a permanent bridge is being replaced and a temporary bypass is needed during the construction period. Portable bridges can serve as temporary structures during disaster situations, depending on the availability of materials. An example is when a flood washes out a highway bridge. There are many situations where temporary access is needed across streams in remote areas for the construction or maintenance of utility structures.

Compared to fords and culverts, properly installed portable timber bridges reduce environmental impacts at road stream crossings. Portable bridges decrease erosion and minimize the amount of sediment that is introduced into streams during activities such as logging or road construction. Bridges that completely span the waterway do not inhibit the movement of aquatic life in the stream. One standard design, for simple spans up to 40 ft, consists of longitudinal glulam decks that are placed across the stream. Depending on site conditions, these bridges may be erected on temporary mud sills or spread footings that may be placed directly on the stream banks. These designs can be quickly and easily installed at the stream-crossing site using typical construction or forestry equipment such as backhoes, hydraulic knuckleboom loaders, or log skidders. These timber bridges may be installed without operating the equipment in the stream to minimize site disturbance and associated erosion and sediment load on the stream.

Portable bridges using longitudinal glulam deck panels have been used successfully in temporary stream crossing applications for forestry and road construction activities. Design procedures for portable bridges are essentially the same as those described for permanent bridges. However, the designer may choose to make special provisions for this type of bridge. For example, if traffic rates are low or the bridge has a short design life, a wearing surface may not be needed. The designer may also allow for greater deflections than typically recommended for highway bridges. A curb may be substituted for a full guardrail if traffic rates and speeds are relatively low and conditions permit. Methods for interconnecting longitudinal glulam deck panels are equally suited to use in portable bridge applications. However, in cases where only two deck panels are used, deck panels may not need to be interconnected, thereby reducing costs of the bridge and making installation and removal easier. If the bridge is to be used only for a short time period, the designer may consider using untreated wood members. However, decay may develop within a few months when climatic conditions are favorable to decay. Standard plans for portable timber bridge systems are available through the USDA Forest Service [8].

17.3 ADVANTAGES OF GLUED LAMINATED TIMBER

Structural glued laminated timber (glulam) offers many advantages in the construction of bridges as follows:

- Components can be prefabricated at the laminating plant or fabrication shop, permitting rapid on-site assembly that reduces labor costs and construction time.
- A variety of configurations, including curved girders and arches, can be manufactured, thus allowing the bridge designer a wide degree of latitude in designing structural components.

- Wood is virtually unaffected by the chemicals or corrosive materials commonly applied to roadway surfaces, therefore de-icing salts do not corrode or deteriorate the decking as may occur with other bridge materials.
- Glued laminated timber is a relatively lightweight construction material that permits the transportation of large prefabricated structural units such as stringers, girder and deck panels. Prefabricated units may be transported and placed by helicopter in remote areas. Glulam's high strength-to-weight ratio in comparison to other materials permits the use of smaller mobile erection equipment and may reduce foundation costs.
- Assembly of modular units can be accomplished at the construction site by semiskilled labor.
- Aesthetically, timber fits most environments, particularly in rural and suburban areas where a natural appearance is desired.
- Wood exhibits excellent short-term duration of load characteristics to resist dynamic loads, such as vehicular loads, earthquake loads, and wind loads.
- Wood is a renewable natural resource.
- The use of waterproof adhesives, pressure preservative treatments, proper design and detailing, extends the service life to be comparable to that of other materials, such as steel and concrete.
- Structural glued laminated timber can be manufactured to large sizes, limited only by shipping constraints.

17.4 PRESERVATIVE TREATMENTS

Pressure preservative treatments have extended the service life of timber bridges to be comparable to that of other materials, such as steel and concrete. It is recommended that all timber bridge components, whether hardwoods or softwoods, be pressure preservative-treated in accordance with appropriate AWPA Standards [9]. Additional information on preservative treatment types, applications, retentions and penetration for glued laminated timber is included in AITC 109 [10]. It is also recommended that treated wood used in timber bridges conform to the Best Management Practices specified in *Best Management Practices for the Use of Treated Wood in Aquatic and Other Sensitive Environments* [11].

Deck panels should be pressure-treated with a preservative such as creosote, pentachlorophenol (penta), or copper naphthenate in a heavy oil carrier, because one of the functions of the glued laminated timber deck is to provide partial moisture protection for the stringers. These types of preservative treatments can be effective in reducing moisture movement through the panels, thus helping to minimize dimensional changes caused by swelling and shrinking of the wood with changes in moisture content. Waterborne or solvent-borne pressure treatments are also permitted, but may not be as effective at retarding moisture movement through the deck panels.

The stringers should be pressure-treated, because it is virtually impossible to eliminate moisture accumulation, particularly at areas of connections and at bearing locations. However, since the stringers are partially protected from the weather by the deck panels, treatment generally does not need to be limited to preservatives in heavy oil.

Complete fabrication of all components prior to pressure preservative treatment is recommended wherever possible. When it is not possible to complete fabrication and drilling of glued laminated timbers before treating or when field modifications are required, preservative treatment must be applied to any cut surfaces or holes in the field in accordance with AWPA Standard M4 [9].

17.5 WEARING SURFACES

Wearing surfaces are applied to bridge decks to prevent damage to the decks from vehicles, extending the life of the bridge. Two types of wearing surfaces are commonly used for timber bridges: (1) asphalt and (2) timber planks. Wearing surfaces are designed to be removed and replaced as needed.

17.5.1 Waterproof Membrane and Asphalt Wearing Surface

A waterproof barrier and an asphalt wearing surface are typically added to the wood deck roadway after the bridge structure has been completed. Waterproof barriers, such as Petrotac,® are reported to protect wood decks from moisture that can penetrate the asphalt wearing surface. These membranes are easy to install and provide an excellent bond with the asphalt overlay. The use of a waterproof membrane, in combination with proper maintenance of the asphalt overlay, allows the designer to use dry service conditions for the design of the wood girders or stringers. Wet-use design values are used for the design of deck panels, because tests in the field indicate the moisture content is likely to exceed 16% in service.

Where heavier retentions of preservatives are used for the wood deck panels, a period of time must be allowed for deck panels to exude preservatives and stabilize before application of the waterproof membrane. Exuding of preservative treatment (heavier retentions) may also require the use of “blotter” material, which should be placed on the surfaces to absorb excessive chemicals and then removed. This process is repeated until the wood surface is relatively free of exudation. A tack coat or primer (asphalt cement) should be applied prior to application of the waterproof membrane. When using creosote treatment, the designer should also consider having creosote treated deck panels steam cleaned to lessen the amount of exudation. After the waterproof membrane has been installed, a tack coat is applied.

The asphalt wearing surface, typically 3 in. thick, is installed using normal paving operations. The thickness of the asphalt may vary in thickness to provide crown and facilitate proper drainage. Design of the asphalt is beyond the scope of this manual. Specific recommendations for deck design, membranes, asphalt mix design and installation guidelines are published in *Guidelines for Design, Installation, and Maintenance of a Waterproof Wearing Surface for Timber Bridge Decks* [12]; *Timber Bridges: Design, Construction, Inspection, and Maintenance* [13]; and *Asphalt Paving of Treated Timber Bridge Decks* [14].

17.5.2 Timber Running Plank Wearing Surfaces

On low-traffic-volume roads, timber bridge decks are often protected with a timber running plank wear surface. With this system, longitudinal timber planks are laid side by side to form the wearing surface. These planks should extend across the entire roadway width to eliminate areas where debris can accumulate. The planks are attached to the structural deck with lag screws countersunk into the plank. The running plank should be pressure treated when truck traffic is expected to be minimal and wear surface life is expected to be long. For bridges with higher volume of truck traffic such as logging roads, the planks are often untreated and are replaced at regular intervals as they wear out.

Since a timber running plank wear surface is not watertight, the joints between deck panels must be flashed to provide waterproofing if the girders are designed using dry-service conditions. If the flashing is omitted, the girders must be designed assuming wet-service conditions.

17.6 GUARDRAILS

For vehicular bridges, performance and geometry requirements for traffic rails are specified by AASHTO (Ref. [3], Section 13; Ref. [4], Section 2.7). Static load design criteria have been used in the past, but new emphasis has been placed on full-scale crash testing.

17.6.1 Crash-Tested Guardrail Systems

Since August 28, 1986, the U.S. Federal Highway Administration (FHWA) has required that bridge railings used on federal-aid projects meet full-scale crash test criteria and has provided listings of those railings meeting these requirements [15] [16]. These rail systems prescribe specific member sizes, layout, spacing, connection details, etc. Further information on railing systems for timber bridges is provided by the National Center for Wood Transportation Structures [17].

17.6.2 Static Design of Guardrails

Typical design considerations for rail, post, and curb systems include outward, upward, and downward impact loads on rails, plus outward, upward, downward, and longitudinal impact loads on posts, and outward impact loads on curbs. Rails, curbs, and posts are typically designed using wet conditions of use with impact load duration factors.

A typical rail, curb, and post system for a transverse deck is shown in Figure 17.6.2-1. In the system shown in Figure 17.6.2-1, the post must be

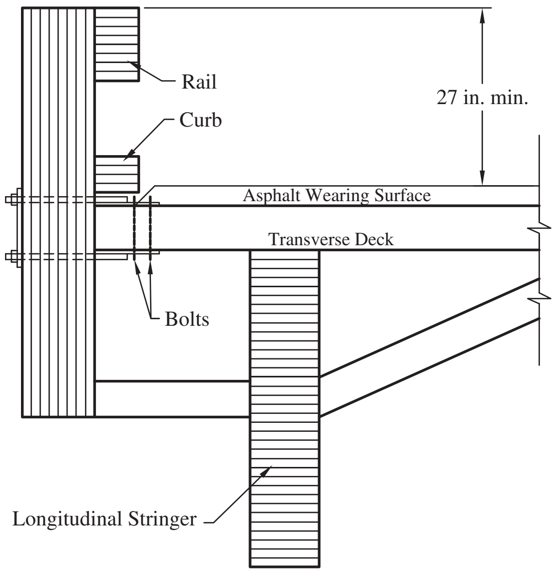


Figure 17.6.2-1 Typical rail and curb system for a transverse deck timber highway bridge.

adequately braced in reaction of the outward load, typically by attachment to the outside girder; and the girder likewise must be adequately braced.

17.7 DESIGN METHODS

As discussed previously in this manual, two different design methodologies are available to engineers: allowable stress design (ASD) and load and resistance factor design (LRFD). Both methods are considered acceptable for many bridge projects; however, the Federal Highway Administration (FHWA) mandates the use of LRFD for bridges to be built using federal funding.

In both methods, loads for design vehicles and traffic lane loads are assumed to occupy a width of 10 ft. The loads are required to be placed in 12-ft-wide traffic lanes spaced across the entire bridge roadway. The number of design lanes is generally determined by taking the integer part of the ratio $w/12$, where w is the clear roadway width in feet. The traffic lanes and the loads within the lanes are to be placed to create the maximum stress in the member under consideration. Bridges 20 to 24 ft in width are designed for two traffic lanes with each occupying half of the bridge width. Bridges of widths less than 20 ft are designed for one lane of traffic.

ASD requirements for highway bridges can be found in AASHTO's *Standard Specifications for Highway Bridges* [4]. LRFD requirements are published in the *AASHTO LRFD Bridge Design Specifications* [3]. These two methods have significant differences and can result in different designs; key differences are described next. Additionally, the format conversion factors for LRFD bridge design are different than the factors used for building design.

17.7.1 Differences Between AASHTO ASD and LRFD

The AASHTO specifications for allowable stress design (ASD) and for load and resistance factor design (LRFD) have significant differences in four areas: (1) design loads, (2) load distribution factors for interior girders, (3) deck requirements, and (4) railing requirements. For girders, trusses, and other main members, LRFD designs are generally conservative relative to ASD designs; however, design of glulam deck panels in ASD is typically conservative relative to LRFD.

17.7.1.1 Design Loads The first difference between the ASD and LRFD methods is the much higher design live load used by LRFD. The design live load for ASD (Ref. [4], Section 3.7) consists of the HS20-44 vehicle, or the HS20-44 lane loading, or the alternate military vehicle that is used for Interstate and other highways carrying heavy truck traffic. The truck loading and the lane loading are not required to be applied simultaneously.

LRFD (Ref. [3], Section 3.6.1.2) uses the HL-93 loading, which consists of the design truck (similar to the HS20-44 vehicle) or the design tandem (similar to the alternate military vehicle), whichever produces greater effect, *plus* a superimposed design lane load of 640 pounds per linear foot. The lane load is applied simultaneously with the design truck. The LRFD superimposed loads are always greater than the ASD HS20 loads, but they are intended to simulate current highway bridge loadings more closely.

The ASD alternate military vehicle and the LRFD tandem vehicle have the same configuration with two axles spaced 4 feet apart. The axle load is 24 kips for ASD and 25 kips for LRFD. The tandem vehicle typically controls designs of bridges from 10 to 38 ft long in LRFD and the alternate military vehicle controls for bridges from 11 to 32 ft in ASD.

17.7.1.2 Load Distribution Factors The methods used to distribute loads to interior longitudinal beams differ between ASD (Ref. [4], Section 3.23.2.2) and LRFD (Ref. [3], Section 4.6.2.2.2) specifications. ASD uses *wheel line loads* and distributes the loads to each longitudinal beam by multiplying by a distribution factor. LRFD uses *lane loads* and distributes the loads to beams by multiplying by a different distribution factor. The net effect is similar for both methodologies, but the magnitude of the LRFD factor is half of the magnitude of the ASD distribution factor, because there are two wheel lines per lane. In addition, LRFD requires live loads to be multiplied by the multiple presence factor to account for the probability of simultaneous lane occupation. LRFD approximate load distribution factor equations already include the multiple presence factor.

17.7.1.3 Deck Requirements The most significant difference in deck design between ASD and LRFD is the effective panel width used in design. The LRFD specifications allow significantly larger panel widths than those used for ASD. This difference generally results in shorter spans or thicker panels for ASD decks.

For transverse decks using glulam panels, the effective width for ASD design is calculated using Equation 17.7.1.3-1 or Equation 17.7.1.3-2.

$$b_{eff} = (15\,\text{in} + t) \le b_{panel} \quad \text{for non-interconnected decks} \quad (17.7.1.3\text{-}1)$$

$$b_{eff} = (15\,\text{in} + 2t) \le b_{panel} \quad \text{for interconnected decks} \quad (17.7.1.3\text{-}2)$$

For transverse decks designed using LRFD, the effective panel width is determined using Equation 17.7.1.3-3 or Equation 17.7.1.3-4.

$$b_{eff} = 2.0t + 40\,\text{in} \le b_{panel} \quad \text{for non-interconnected panels} \quad (17.7.1.3\text{-}3)$$

$$b_{eff} = 4.0t + 30\,\text{in} \quad \text{for interconnected panels} \quad (17.7.1.3\text{-}4)$$

Another difference in deck design requirements is in the loading. For design of transverse timber decks, the ASD provisions allow H-20 and HS-20 wheel loads to be reduced from 16,000 lb to 12,000 lb (Ref [4], Figures 3.7.6A and 3.7.7A). This effectively makes the design of these decks the same for H-15, HS-15, H-20, and HS-20 loads. There are no similar provisions in LRFD for deck design; the full axle load is used. However, the reduced loads for H-20 and HS-20 deck design do not typically offset the effect of smaller effective panel widths used in ASD, so ASD designs remain conservative relative to LRFD.

17.7.1.4 Railing Requirements The AASHTO ASD specifications [4] allow the use of statically designed guardrails with a prescriptive outward load of 10 kips at any point on the guardrail, regardless of the design vehicle load for the bridge. AASHTO LRFD specifications require the use of crash-tested rail systems and include six different test levels, depending on the traffic and site conditions (Ref [3], Section 13). The test levels are based on recommendations of the National Cooperative Highway Research Program [18].

17.8 CONCLUSION

Timber bridges have long been a part of America's infrastructure. From short-span trestle bridges to long-span arch or truss bridges, preservative-treated timber remains a viable option as a modern bridge material. Modern waterproof adhesives have expanded the capability of timber bridges through the use of structural glued laminated timber (glulam).

Seven types of timber bridges have been used successfully and were discussed in this chapter: (1) trestle, (2) longitudinal deck, (3) longitudinal stress-laminated, (4) girder, (5) truss, (6) arch, and (7) portable and temporary timber bridges. In addition, wear surfaces, guardrails, and design methodologies were discussed. Subsequent chapters include design procedures and examples for the more common timber bridge types. Chapter 18 covers LRFD design and Chapter 19 covers ASD design of timber bridges.

CHAPTER 18

LRFD BRIDGE DESIGN

18.1 INTRODUCTION

Starting in October 2007, the Federal Highway Administration (FHWA) has mandated the use of LRFD for bridges to be built using federal funding. LRFD requirements are published in the *AASHTO LRFD Bridge Design Specifications* [1]. This chapter presents design procedures and examples specific to the LRFD design methodology.

18.1.1 Loads and Load Combinations

In general, the *AASHTO LRFD Bridge Design Specifications* [1] require bridges to be designed to resist the following loads:

- Dead load
- Live load
- Dynamic effect of the live load
- Wind load
- Collision loads
- Other loads when they exist, including longitudinal forces, centrifugal force, thermal forces, earth pressure, buoyancy, shrinkage stresses, rib shortening, erection stresses, ice and current pressure, snow and ice loads, and earthquake stresses

For timber structures, the live load dynamic effect is not required to be considered. Load combinations are prescribed in the AASHTO LRFD Specifications.

For most timber bridge superstructures, the relevant LRFD load combinations are summarized in Table 18.1.1-1.

TABLE 18.1.1-1 LRFD Load Combinations for Timber Bridge Superstructures

AASHTO Designation	Load Combination	Time Effect Factor
Strength I	$1.25DC + 1.5DW + 1.75LL$	0.8
Strength II (Permit Vehicle)	$1.25DC + 1.5DW + 1.35LL$	1.0
Strength III (Wind)	$(0.9 \text{ or } 1.25)DC + (0.65 \text{ or } 1.5)DW + 1.4WS$	1.0
Strength V (Wind)	$(0.9 \text{ or } 1.25)DC + (0.65 \text{ or } 1.5)DW + 1.35LL + 0.4WS + 1.0WL$	1.0
Extreme Event I (Earthquake)	$(0.9 \text{ or } 1.25)DC + (0.65 \text{ or } 1.5)DW + 0.5LL + 1.0EQ$	1.0

DC = Dead load from structure
DW = Dead load from wear surface
LL = Vehicular live load
WS = Wind load on structure
WL = Windload on vehicles
EQ = Earthquake

18.1.2 Loading System

Standard truck and lane loads are used to represent vehicular loads on bridges. The AASHTO LRFD Specifications [1] designate the required loading scheme as HL-93, which consists of a combination of a design truck or design tandem applied simultaneously with the design lane load. The design truck is identical to the HS-20 design truck, and the design tandem is similar to the alternate military vehicle used in ASD, but the axle loads are 25 kips, rather than 24 kips. The design lane load is 640 lb/ft of length, distributed over a 10 ft width. Details of the required loads are provided in the AASHTO LRFD specifications [1].

18.1.3 Multiple Presence Factor

The multiple presence factor, m, is used to account for the probability that one vehicle heavier than the design vehicle may use the bridge. AASTHO LRFD [1], Section 3.6.1.1.2 explains this provision. For AASHTO LRFD, the designer is required to check the bridge for two load cases; the first is with one lane on the bridge loaded using the multiple presence factor of 1.2 and the second is for two or more lanes loaded with a lesser value for m.

The multiple presence factor has been incorporated into the approximate load distribution factors specified in Table 18.1.4-1; therefore, it is not applied separately when those factors are used. The multiple presence factor is used in conjunction with the lever rule for exterior stringer design.

18.1.4 Lane Load Distribution

For the calculation of bending moments and shear loads in longitudinal members, axle loads are not distributed along the length of the bridge, but are distributed laterally between stringers or longitudinal beams. For the calculation of load effects in transverse beams, wheel loads are not distributed across the width of the bridge, but are distributed between transverse beams. Lane load distribution factors, DF, depend on the type of deck and the size of the bridge and are typically prescribed in terms of a fraction of the beam spacing, S, in feet (Tables 18.1.4-1 and 18.1.4-2).

TABLE 18.1.4-1 Lane Load Distribution Factors for Interior Longitudinal Beams

Type of Deck	Bridge Designed for One Traffic Lane	Bridge Designed for Two or More Traffic Lanes	Range of Applicability
Timber Plank	$S/6.7$	$S/7.5$	$S \leq 5.0$ ft
Nail-Laminated	$S/8.3$	$S/8.5$	$S \leq 6.0$ ft
Glulam	$S/10.0$	$S/10.0$	$S \leq 6.0$ ft

TABLE 18.1.4-2 Lane Load Distribution Factors for Transverse Beams

Kind of Deck	Distribution Factor	Range of Applicability
Timber Plank	$S/4$	N/A
6″ Nominal Thickness or Thicker Glulam or Nail-lam	$S/5$	$S \leq 5.0$ ft

18.1.5 Camber

Glulam bridge girders are generally cambered for appearance and drainage. AASHTO LRFD [1] specifies that glulam girders be cambered a minimum of two times the dead load deflection at the service limit state. Stress-laminated timber deck bridges must be cambered for three times the dead load deflection at the service limit state.

18.1.6 Format Conversion Factors

Reference design values for timber construction are published at the allowable stress level. For use with the LRFD design methodology, it is necessary to multiply them by the format conversion factor, C_{KF}, provided in Table 18.1.6-1. It should be noted that the format conversion factor used for AASHTO LRFD bridge design is not identical to the format conversion factor used for LRFD building design.

TABLE 18.1.6-1 Format Conversion Factors for Design of Bridges Using AASHTO LRFD

Property or Design Value	Format Conversion Factor, C_{KF}
F_b	2.94
F_t	3.12
F_c	2.78
F_v, F_{rt}, F_S	3.33
$F_{c\perp}$	1.87
Connections	3.85

18.1.7 Resistance Factors

Resistance factors for use with LRFD bridge design are presented in Table 18.1.7-1. These factors are identical to those used in LRFD building design.

TABLE 18.1.7-1 Resistance Factors for Timber Bridge Design

ϕ_c	Compression (parallel- and perpendicular-to-grain)	0.90
ϕ_b	Bending (flexure)	0.85
ϕ_t	Tension	0.80
ϕ_v	Shear, torsion, and radial tension	0.75
ϕ_z	Connections	0.65

18.2 LONGITUDINAL STRINGERS

The design of longitudinal stringers is identical for interconnected (doweled) decks and non-interconnected decks. Given the overall bridge width, the designer must determine the desired width of the overhang and determine the number of stringers that will be used to support the deck. The stringer spacing is used to determine wheel load distribution factors and girder loads. Then the girders are designed by choosing a trial size based on flexure. The trial size is then analyzed for shear and deflection. Interior and exterior stringers must be considered separately.

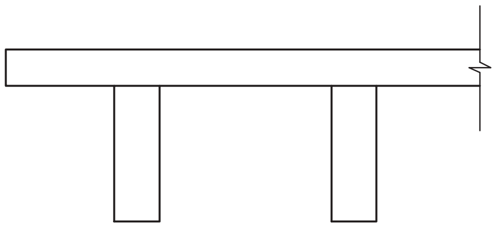
18.2.1 Stringer Spacing, *S*, and Deck Overhang, *O*

Given the overall width of the roadway, the designer must determine the number and spacing of stringers that will be used to support the deck. The deck panels are typically cantilevered somewhat past the exterior stringers. The overhang distance, O, and the stringer spacing, S, are commonly selected to minimize differences in moment between the interior and exterior stringers or to optimize deck design.

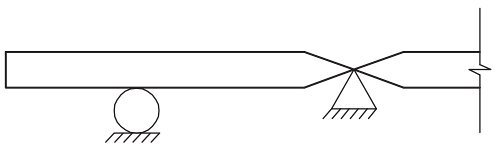
18.2.2 Distribution Factors

For interior stringers, the distribution factor (DF) is determined using the stringer spacing and Table 18.1.4-1. The multiple presence factor is not used with these distribution factors for interior stringers, because it is already incorporated into the factors. For the exterior stringers, the distribution factor is determined by comparing the distribution factor found using the lever rule and by assuming the cross section deflects and rotates as a rigid cross section (Ref. [1], Section 4.6.2.2.2d). The distribution factor calculated for each case (i.e., one lane loaded, two lanes loaded, etc.) is multiplied by the appropriate multiple presence factor. The largest product (mDF) of the multiple presence factor and distribution factor is used to design the exterior stringer.

18.2.2.1 Lever Rule (Ref. [1], Section 4.6.2.2.1) The lever rule is a simplified analysis procedure used to determine the load on an exterior stringer supporting a cantilevered deck or transverse beams. The deck (or transverse beams) is notionally modeled as having a pin connection at the first interior stringer and a roller connection at the exterior stringer (Figure 18.2.2.1-1). The reaction force calculated using this notional model is the load for which the exterior stringer is designed. The reaction force is easily determined by summing moments about the pin connection. To determine a distribution factor, the calculated load is divided by the design vehicle load. Because this calculation results in the load in both the numerator and denominator of the fraction, the load cancels out and the distribution factor can be expressed without including the load. For calculating the distribution factor for the exterior stringer, the outside wheel line of the truck is placed 2 ft from the face of the curb or rail (Ref. [1], Section 3.6.1.3.1). The lever rule can also be used to conservatively estimate the dead loads on the exterior stringer.



Girder Bridge



Notional Model for Applying Lever Rule

Figure 18.2.2.1-1 Lever rule.

For the case where both wheel lines are outside of the first interior stringer (Figure 18.2.2.1-2), the live load distribution factor for the exterior stringer is calculated using Equation 18.2.2.1-1. This occurs when the inside face of the guardrail is 8 ft-10 in. or more from the centerline of the first interior stringer.

$$DF_{\text{ext, lever rule}} = \frac{x_1 + x_2}{2S} \quad (18.2.2.1\text{-}1)$$

where:

x_1 = distance from the centerline of first interior stringer to centerline of outside wheel
x_2 = distance from centerline of first interior stringer to centerline of inside wheel

For the case where only one wheel line is outside of the first interior stringer (Figure 18.2.2.1-2), the live load distribution factor for the exterior stringer is calculated using Equation 18.2.2.1-2. This occurs when the inside face of the guardrail is 7 ft-2 in. or less from the centerline of the first interior stringer.

$$DF_{\text{ext,lever rule}} = \frac{x_1}{2S} \quad (18.2.2.1\text{-}2)$$

where:

x_1 = distance from centerline of first interior stringer to centerline of outside wheel

For the case where the inside wheel line is partially outside of the first interior stringer (Figure 18.2.2.1-2), the live load distribution factor for the exterior stringer is calculated using Equation 18.2.2.1-3. This occurs when the inside face of the guardrail is between 7 ft-2 in. and 8 ft-10 in. from the centerline of the first interior stringer.

$$DF_{\text{ext,lever rule}} = \frac{x_1 + \dfrac{2x_2^2}{1.67\text{ ft}}}{2S} \quad (18.2.2.1\text{-}3)$$

where:

x_1 = distance from centerline of first interior stringer to centerline of outside wheel
x_2 = half of the tire contact width outside of centerline of first interior stringer

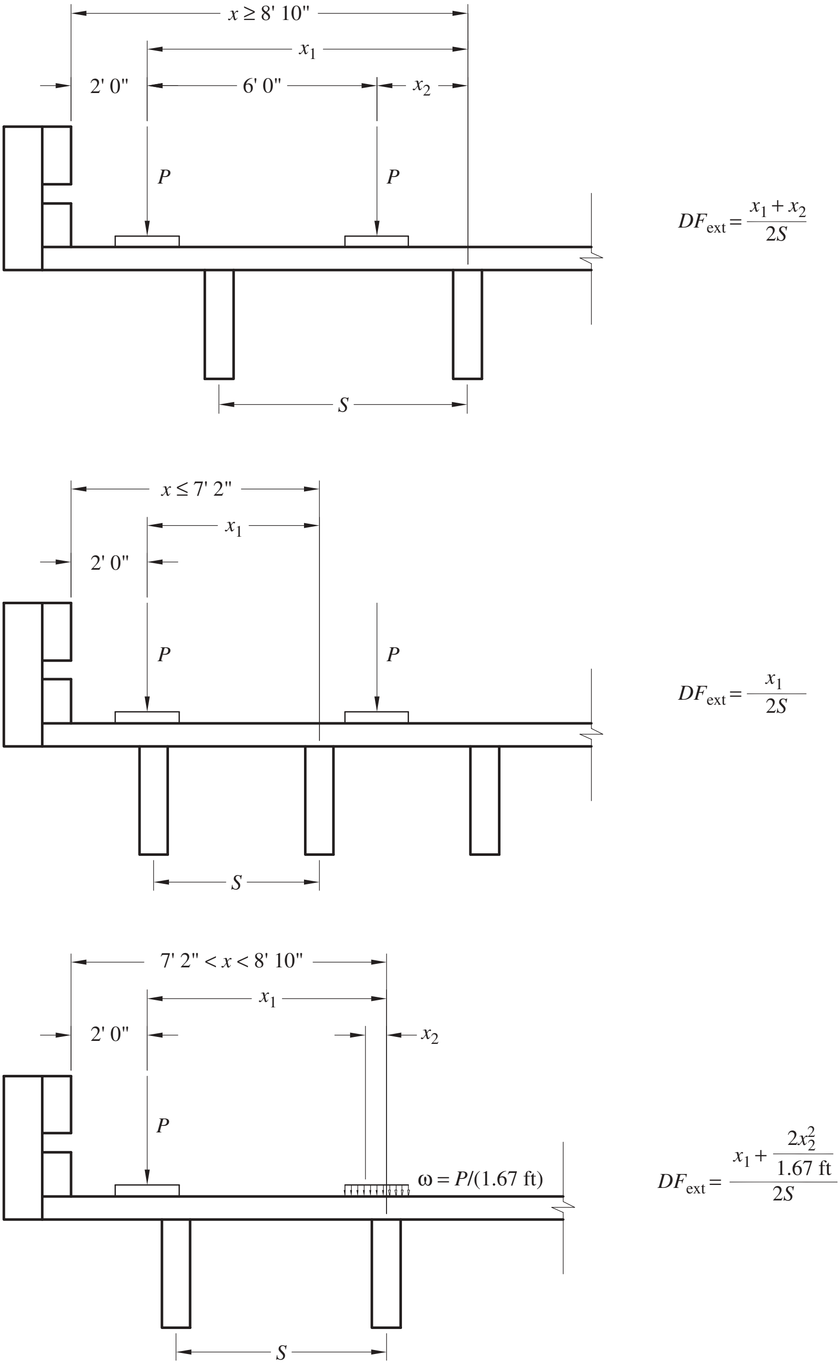


Figure 18.2.2.1-2 LRFD distribution factors for exterior stringer.

18.2.2.2 Rigid Body Rotation (Ref. [1], Section 4.6.2.2.2d) For girder bridges with diaphragms or cross-frames providing bracing between the girders, distribution factors for the exterior girder must be calculated assuming the cross section of the bridge deflects and rotates as a rigid section in addition to the factor calculated by the lever rule. Distribution factors must be calculated for each loading configuration considered (i.e., one lane loaded, two lanes loaded, etc.). The outside wheel line of the truck is placed 2 ft from the face of the curb or rail for this analysis. For cases with multiple lanes loaded, design vehicles are placed starting at 2 ft from the curb and at 12-ft intervals across the bridge (Figure 18.2.2.2-1). The distribution factor, assuming that the cross section of the bridge deflects and rotates as a rigid section, can be found using Equation 18.2.2.2-1.

$$DF_{\text{exterior, rotation rule}} = \frac{N_L}{N_b} + \frac{X_{EXT}\sum^{N_L} e}{\sum^{N_b} x^2} \qquad (18.2.2.2\text{-}1)$$

where:

N_L = number of lanes
N_b = number of girders
X_{EXT} = horizontal distance from center of gravity of pattern of girders to exterior girder
x = horizontal distance from center of gravity of pattern of girders to each girder
e = eccentricity of a design truck or design lane load from center of gravity of pattern of girders

The terms in Equation 18.2.2.2-1 are illustrated in Figure 18.2.2.2-2 for a bridge with $N_L = 3$ and $N_b = 6$.

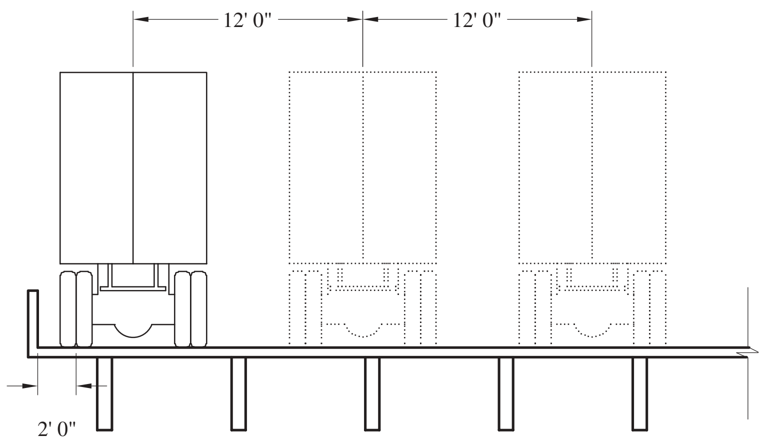


Figure 18.2.2.2-1 Placement of trucks for rotational rule.

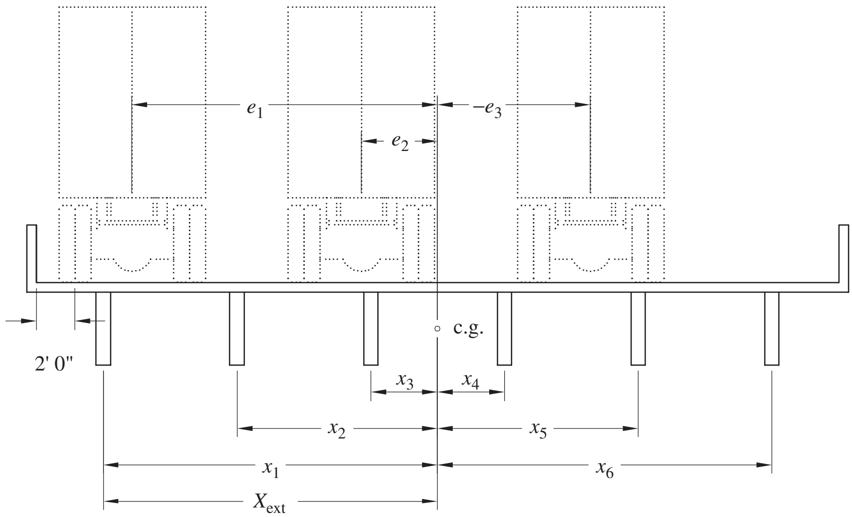


Figure 18.2.2.2-2 Dimensions for rotational rule calculation—three-lane bridge.

18.2.3 Design Bending Moments

The maximum bending moment due to the HL-93 design truck or tandem for a simple-span bridge can be determined using Equations 18.2.3-1 through 18.2.3-4 for the cases indicated.

One axle of design truck on bridge ($\ell \leq 10.0$ ft):

$$M_{vehicle} = (8.0\ \text{kip})\ell \tag{18.2.3-1}$$

Two axles of design tandem on bridge (10.0 ft $< \ell \leq$ 38.4 ft):

$$M_{vehicle} = (12.5\ \text{kip})\ell + \frac{50\ \text{kip-ft}^2}{\ell} - 50\ \text{kip-ft} \tag{18.2.3-2}$$

Two axles of design truck on bridge (38.4 ft $< \ell \leq$ 42.0 ft):

$$M_{vehicle} = (16.0\ \text{kip})\ell + \frac{1568\ \text{kip-ft}^2}{\ell} - 224\ \text{kip-ft} \tag{18.2.3-3}$$

Three axles of design tandem on bridge ($\ell >$ 42.0 ft):

$$M_{vehicle} = (18\ \text{kip})\ell + \frac{392\ \text{kip-ft}^2}{\ell} - 280\ \text{kip-ft} \tag{18.2.3-4}$$

The maximum bending moment due to the HL-93 design lane load is calculated using Equation 18.2.3-5.

$$M_{lane} = \left(0.08\ \frac{\text{kip}}{\text{ft}}\right)\ell^2 \tag{18.2.3-5}$$

The design bending moments caused by dead loads from the deck, wearing surface, and bridge components, such as the guardrail and girders themselves are calculated using conventional methods (Equation 18.2.3-6).

$$M_D = \frac{\omega_D \ell^2}{8} \tag{18.2.3-6}$$

The total factored bending moment for the Strength I load combination is calculated as shown in Equation 18.2.3.7.

$$\begin{aligned} M_u &= 1.25M_{DC} + 1.5M_{DW} + 1.75M_{LL} \\ M_u &= 1.25(M_{self} + M_{deck} + M_{guardrail}) + 1.5(M_{DW}) \\ &\quad + 1.75(M_{vehicle} + M_{lane})(mDF) \end{aligned} \tag{18.2.3-7}$$

18.2.4 Girder Design

The girders are designed to meet flexural requirements then evaluated for flexure, shear, and deflection. After selecting a trial width, the required depth can be calculated based on flexure using Equation 18.2.4-1. For preliminary sizing, the beam self-weight and the volume factor are estimated by the designer.

$$d = \sqrt{\frac{6M_u}{b\phi F_b C_\lambda C_{KF} C_V C_M C_t}} \tag{18.2.4-1}$$

Depending on the design, the exterior stringers may be the same size or larger than the interior stringers. Typically, the depth of all stringers is the same and a larger width is used for the exterior stringer if necessary. Therefore, the required depth is calculated for both interior and exterior stringers for a trial width(s) and the larger of the two depths is chosen for all of the girders.

18.2.5 Flexure Analysis

Once trial sizes are established, the girders must be evaluated for flexure. The actual self-weight(s) and volume factor(s) must be calculated for the girders and the factored design moment compared with the adjusted moment capacity (Equation 18.2.5-1).

$$M_u \le \phi_b M_n = \phi_b F_{bx} C_\lambda C_{KF}(C_V \text{ or } C_L) C_M C_t S_x \tag{18.2.5-1}$$

18.2.6 Shear Analysis

The girders must be evaluated to ensure that their adjusted shear capacity exceeds the factored design shear. The shear is evaluated at a distance from the support equal to the depth of the member. The shear force from distributed dead loads is calculated using Equation 18.2.6-1.

$$V_D = \omega_D \left(\frac{\ell}{2} - d\right) \tag{18.2.6-1}$$

The design vehicle is placed such that the maximum shear is produced with an axle at a distance from the support equal to the lesser of either three times the depth, $3d$, of the girder or one-quarter of the span , $\ell/4$. Once the placement of the design vehicle is determined, the distributed live load shear from the design vehicle or tandem is calculated using Equation 18.2.6-2.

$$V_{LL} = 0.5(0.6V_{LU} + V_{LD}) \tag{18.2.6-2}$$

where:

V_{LL} = distributed live load vertical shear
V_{LU} = maximum vertical shear at $3d$ or $\ell/4$ due to undistributed wheel loads
V_{LD} = maximum vertical shear at $3d$ or $\ell/4$ due to wheel loads distributed laterally as for moment

The distributed vertical shear due to the design lane loading is determined using Equation 18.2.6-3.

$$V_{lane} = \omega_{lane}\left(\frac{\ell}{2} - d\right) mDF \tag{18.2.6-3}$$

The total factored shear for the Strength I load combination is calculated as follows (Equation 18.2.6-4).

$$V_u = 1.25V_{DC} + 1.5V_{DW} + 1.75V_{LL}$$

$$V_u = 1.25(V_{self} + V_{deck} + V_{guardrail}) + 1.5(V_{DW}) + 1.75(V_{LL} + V_{lane}) \tag{18.2.6-4}$$

The factored shear must be less than the adjusted shear capacity of the beam (Equation 18.2.6-5).

$$V_u \leq \phi_v V_n = \phi_v F_{vx}(0.72)C_\lambda C_{KF} C_M C_t \left(\frac{2bd}{3}\right) \tag{18.2.6-5}$$

18.2.7 Deflection Analysis

AASTHO LRFD [1] Section 2.5.2.6.2 states, “When investigating the maximum absolute deflection for straight girder systems, all design lanes should be loaded, and all supporting components should be assumed to deflect equally.” Furthermore, Section 3.6.1.1.2 states, “If the Owner invokes the optional live load deflection criteria specified in Section 2.5.2.6.2, the deflection should be taken as the larger of:

- That resulting from the design truck alone, or
- That resulting from 25 percent of the design truck taken together with the design lane load.”

The live load deflection can be estimated by converting the live load moment to an equivalent uniformly distributed load (Equation 18.2.7-1).

$$\omega_{eq} = \frac{8M}{\ell^2} \tag{18.2.7-1}$$

The corresponding live load deflection due to the design vehicle is calculated using Equation 18.2.7-2.

$$\Delta_{LL} = \frac{N_L 5\omega_{eq}\ell^4}{384E' \sum \iota} \tag{18.2.7-2}$$

A live load deflection limit of $\ell/425$ is recommended for timber bridges carrying vehicular and pedestrian loading.

18.2.8 Camber

AASHTO LRFD [1] Section 8.12.1 states, "Glued laminated timber girders shall be cambered a minimum of two times the dead load deflection at the service limit state." The dead load due to components and the dead load due to the wearing surface should be combined for this calculation and the entire bridge should be considered to deflect uniformly. The deflection due to dead load is calculated using Equation 18.2.8-1.

$$\Delta_{DL} = \frac{5(\omega_{DC} + \omega_{DW})\ell^4}{384E' \sum \iota} \tag{18.2.8-1}$$

EXAMPLE 18.2-1 LONGITUDINAL STRINGER BRIDGE

Given: Glued laminated timber bridge with 50 ft span, 34 ft roadway width (inside guardrails), two lanes, with 2-in. asphalt wearing surface. Transverse decking is assumed to be structural glued laminated timber, 6.75 in. thick with 2 ft overhangs from the center of the exterior stringer to the inside face of the guardrail. A crash-tested guardrail system weighing 60 lb/ft will be installed. Six longitudinal stringers of Douglas fir glulam, Combination 24F-V4 DF will be spaced at 6 ft on center. (Figure 18.2-1) Steel diaphragms will be used as bracing between the stringers. Assume dry conditions for the stringers.

Wanted: Determine suitable girder sizes with AASHTO HL-93 loading. Consider the Strength I load combination.

Approach: Stringers will be designed in accordance with AASHTO LRFD specifications and design values for the glued laminated timber from AITC 117 [2].

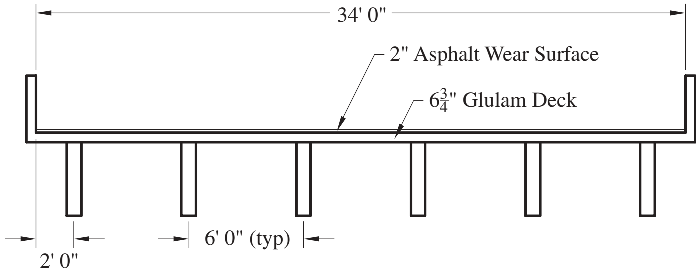


Figure 18.2-1 Typical girder bridge section—Example 18.2-1.

Solution:

Live load distribution factor, interior girder:

$$DF_{\text{int}} = \frac{S}{10\text{ ft}} = \frac{6\text{ ft}}{10\text{ ft}} = 0.60$$

Live load distribution factor, exterior girder, lever rule (Figure 18.2.2):

$$DF_{\text{exterior,lever rule}} = \frac{x_1 + \dfrac{2x_2^2}{1.67\text{ ft}}}{2S} = \frac{6.0\text{ ft} + \dfrac{2(0.42\text{ ft})^2}{1.67\text{ ft}}}{2(6\text{ ft})} = 0.52$$

$$m_1 DF_{\text{exterior,lever rule}} = 1.2(0.52) = 0.62$$

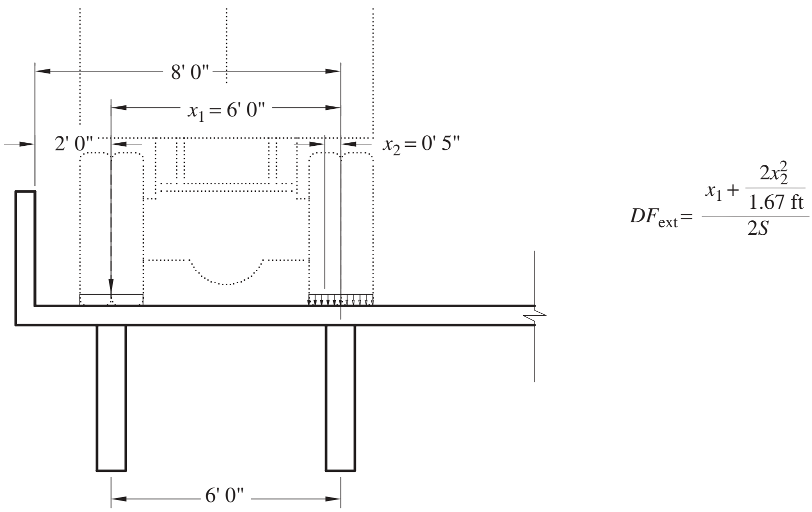


Figure 18.2-2 Position of design vehicle for determining the exterior stringer load distribution factor using the lever rule—Example 18.2-1.

Live load distribution factor, exterior girder, rotational rule, one lane loaded (Figure 18.2-3):

$$DF_{\text{exterior,rotation rule}} = \frac{N_L}{N_b} + \frac{X_{EXT}\sum^{N_L} e}{\sum^{N_b} x^2}$$

$$DF_{\text{exterior,rotation rule,1 lane loaded}} = \frac{1}{6} + \frac{(15\ \text{ft})(12\ \text{ft})}{2[(3\ \text{ft})^2 + (9\ \text{ft})^2 + (15\ \text{ft})^2]}$$

$$DF_{\text{exterior,rotation rule,1 lane loaded}} = 0.45$$

$$m_1 DF_{\text{exterior,rotation rule,1 lane loaded}} = 1.2(0.45) = 0.54$$

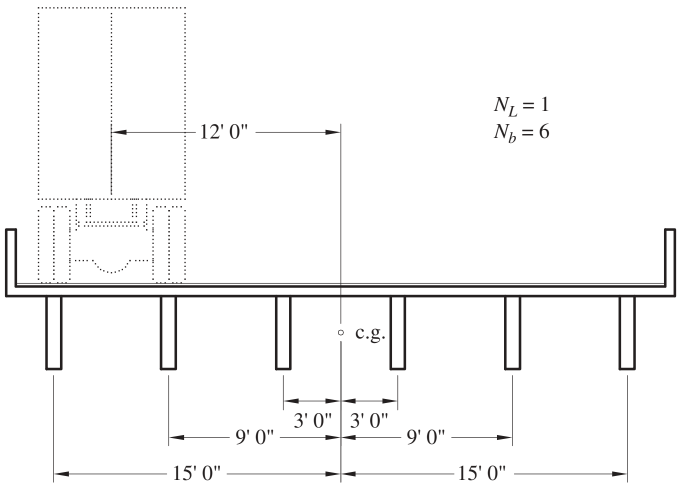


Figure 18.2-3 Rotational rule with one lane loaded—Example 18.2-1.

Live load distribution factor, exterior girder, rotational rule, two lanes loaded (Figure 18.2-4):

$$DF_{\text{exterior,rotation rule}} = \frac{N_L}{N_b} + \frac{X_{EXT}\sum^{N_L} e}{\sum^{N_b} x^2}$$

$$DF_{\text{exterior,rotation rule,2 lanes loaded}} = \frac{2}{6} + \frac{(15\ \text{ft})[12\ \text{ft} + 0\ \text{ft}]}{2[(3\ \text{ft})^2 + (9\ \text{ft})^2 + (15\ \text{ft})^2]}$$

$$DF_{\text{exterior,rotation rule,2 lanes loaded}} = 0.62$$

$$m_2 DF_{\text{exterior,rotation rule,2 lanes loaded}} = 1.0\,(0.62) = 0.62$$

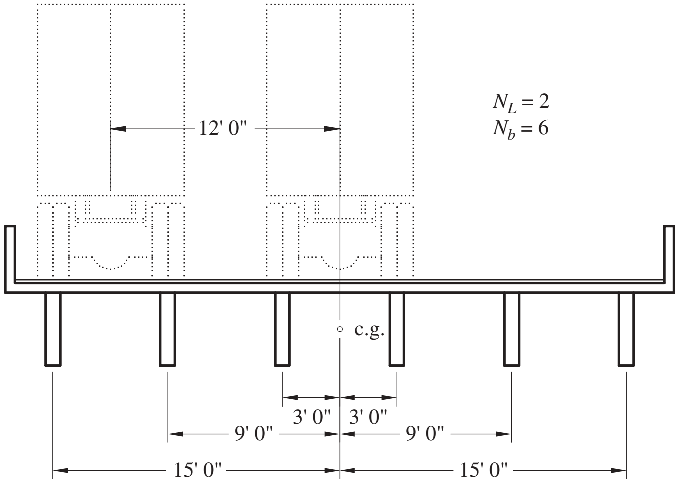


Figure 18.2-4 Rotational rule with two lanes loaded—Example 18.2-1.

Vehicular live load:

$$M_{vehicle} = (18\text{ kip})\ell + \frac{392\text{ kip-ft}^2}{\ell} - 280\text{ kip-ft}$$

$$M_{vehicle} = (18\text{ kip})(50\text{ ft}) + \frac{392\text{ kip-ft}^2}{(50\text{ ft})} - 280\text{ kip-ft}$$

$$M_{vehicle} = 628\text{ kip-ft}$$

Lane live load:

$$M_{lane} = \left(0.08\ \frac{\text{kip}}{\text{ft}}\right)\ell^2 = 200\text{ kip-ft}$$

Dead load of wear surface (Figure 18.2-5):

$$\omega_{DW,\text{int}} = \frac{150\text{ pcf }(2\text{ in})(72\text{ in})}{144\text{ in}^2/\text{ ft}^2} = 150\text{ plf (based on tributary width of 72 in)}$$

$$M_{DW,\text{int}} = \frac{\omega_{DW,\text{int}}\ell^2}{8} = \frac{150\text{ plf }(50\text{ ft})^2}{8}\frac{1\text{ kip}}{1000\text{ lb}} = 46.9\text{ kip-ft}$$

$$\omega_{DW,\text{ext}} = \frac{150\text{ pcf }(2\text{ in})(96\text{ in})^2}{2(72\text{ in})(144\text{ in}^2/\text{ft}^2)} = 133\text{ plf (based on lever rule)}$$

$$M_{DW,\text{ext}} = \frac{\omega_{DW,\text{ext}}\ell^2}{8} = \frac{133\text{ plf }(50\text{ ft})^2}{8}\frac{1\text{ kip}}{1000\text{ lb}} = 41.6\text{ kip-ft}$$

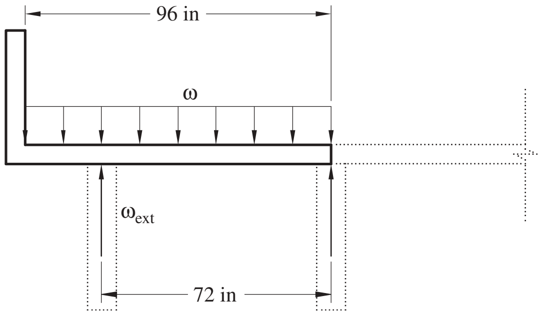


Figure 18.2-5 Lever rule for deck and asphalt loads —Example 18.2-1.

Dead load of deck based on tributary width of 72 in. (Figure 18.2-5):

$$\omega_{deck,\mathrm{int}} = \frac{50\ \mathrm{pcf}\ (6.75\ \mathrm{in})(72\ \mathrm{in})}{144\ \mathrm{in}^2/\mathrm{ft}^2} = 169\ \mathrm{plf}$$

$$M_{deck,\mathrm{int}} = \frac{\omega_{deck,\mathrm{interior}}\ell^2}{8} = \frac{169\ \mathrm{plf}\ (50\ \mathrm{ft})^2}{8}\frac{1\ \mathrm{kip}}{1000\ \mathrm{lb}} = 52.8\ \mathrm{kip\text{-}ft}$$

$$\omega_{deck,\mathrm{ext}} = \frac{50\ \mathrm{pcf}\ (6.75\ \mathrm{in})(96\ \mathrm{in})^2}{2(72\ \mathrm{in})(144\ \mathrm{in}^2/\mathrm{ft}^2)} = 150\ \mathrm{plf\ (based\ on\ lever\ rule)}$$

$$M_{deck,\mathrm{ext}} = \frac{\omega_{deck,\mathrm{ext}}L^2}{8} = \frac{150\ \mathrm{plf}\ (50\ \mathrm{ft})^2}{8}\frac{1\ \mathrm{kip}}{1000\ \mathrm{lb}} = 46.9\ \mathrm{kip\text{-}ft}$$

Dead load of guardrail:

$$\omega_{guardrail} = 60\ \mathrm{plf}$$

Dead load of guardrail on exterior stringer (lever rule) (Figure 18.2-6):

$$\omega_{guardrail,ext} = \frac{(108\ \mathrm{in})(\omega_{guardrail})}{72\ \mathrm{in}} = 1.5(60\ \mathrm{plf}) = 90\ \mathrm{plf}$$

$$M_{guardrail,\mathrm{ext}} = \frac{\omega_{guardrail,\mathrm{ext}}\ell^2}{8} = \frac{90\ \mathrm{plf}\ (50\ \mathrm{ft})^2}{8}\frac{1\ \mathrm{kip}}{1000\ \mathrm{lb}} = 28.1\ \mathrm{kip\text{-}ft}$$

Self-weight of beam (estimated):

$$\omega_{self} \approx 200\ \mathrm{plf}$$

$$M_{self} = \frac{\omega_{self}\ell^2}{8} = \frac{200\ \mathrm{plf}\ (50\ \mathrm{ft})^2}{8}\frac{1\ \mathrm{kip}}{1000\ \mathrm{lb}} = 62.5\ \mathrm{kip\text{-}ft}$$

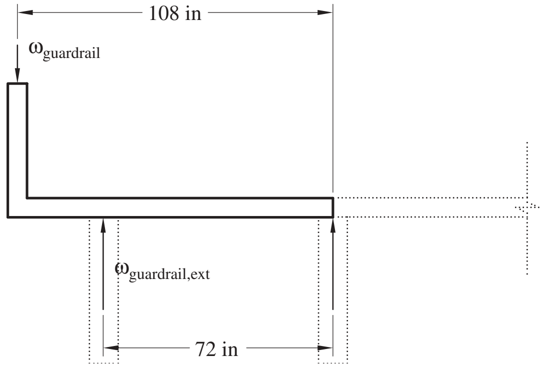


Figure 18.2-6 Lever rule for guardrail load.

Design bending moment for interior girder (with estimated self-weight):

$$M_{u,\text{int}} = 1.25M_{DC} + 1.5M_{DW} + 1.75M_{LL}$$

$$M_{u,\text{int}} = 1.25(M_{self} + M_{deck,\text{int}}) + 1.5(M_{DW,\text{int}}) + 1.75(M_{vehicle} + M_{lane})(DF_{\text{int}})$$

$$M_{u,\text{int}} = 1.25(62.5 + 52.8)\text{kip-ft} + 1.5(46.9)\text{kip-ft} + 1.75(628 + 200)\text{kip-ft}(0.60)$$

$$M_{u,\text{int}} = 1080 \text{ kip-ft}$$

Design bending moment for exterior girder (with estimated self-weight):

$$M_{u,\text{ext}} = 1.25M_{DC} + 1.5M_{DW} + 1.75M_{LL}$$

$$M_{u,\text{ext}} = 1.25(M_{self} + M_{deck,\text{ext}} + M_{guardrail,\text{ext}}) + 1.5(M_{DW,\text{ext}}) + 1.75(M_{vehicle} + M_{lane})(mDF_{\text{ext}})$$

$$M_{u,\text{ext}} = 1.25(62.5 + 46.9 + 28.1)\text{kip-ft} + 1.5(41.6)\text{kip-ft} + 1.75(628 + 200)\text{kip-ft}(0.62)$$

$$M_{u,\text{ext}} = 1130 \text{ kip-ft}$$

Girder design:

Because the moments are similar for both the interior and exterior girders, a single size will be determined for both. Using a trial girder width of $8\frac{3}{4}$ in. and estimating the volume factor as 0.75, the required depth is estimated as:

$$d \geq \sqrt{\frac{6M_u}{b\phi F_b C_\lambda C_{KF} C_V C_M C_t}}$$

$$d \geq \sqrt{\frac{6(1130 \text{ kip-ft})(12 \text{ in/ft})(1000 \text{ lb/kip})}{(8.75 \text{ in})(0.85)(2400 \text{ psi})(0.8)(2.9)(0.75)(1.0)(1.0)}}$$

$$d \geq 51.2 \text{ in} \qquad \therefore \text{ Try } 51.0 \text{ in depth}$$

Girder self-weight:

$$?_{self} = (8.75 \text{ in})(51.0 \text{ in})\left(\frac{\text{ft}^2}{144 \text{ in}^2}\right)(50 \text{ pcf}) = 155 \text{ plf}$$

$$M_{self} = \frac{\omega_{self}\ell^2}{8} = \frac{155 \text{ plf } (50 \text{ ft})^2}{8} \frac{1 \text{ kip}}{1000 \text{ lb}} = 48.4 \text{ kip-ft}$$

Revised design moment:

$$M_{u,\text{ext}} = 1.25M_{DC} + 1.5M_{DW} + 1.75M_{LL}$$

$$M_{u,\text{ext}} = 1.25(M_{self} + M_{deck,\text{ext}} + M_{guardrail,\text{ext}}) + 1.5(M_{DW,\text{ext}}) + 1.75(M_{vehicle} + M_{lane})(mDF_{\text{ext}})$$

$$M_{u,\text{ext}} = 1.25(48.4 + 46.9 + 28.1)\text{kip-ft} + 1.5(41.6) \text{ kip-ft} + 1.75(628 + 200) \text{ kip-ft } (0.62)$$

$$M_{u,\text{ext}} = 1120 \text{ kip-ft}$$

Volume factor:

$$C_V = \left[\left(\frac{5.125 \text{ in}}{8.75 \text{ in}}\right)^{\frac{1}{10}}\left(\frac{12 \text{ in}}{51.0 \text{ in}}\right)^{\frac{1}{10}}\left(\frac{21 \text{ ft}}{50 \text{ ft}}\right)^{\frac{1}{10}}\right] = 0.75$$

Flexural design check:

$$\phi M_n = \phi F_b C_\lambda C_{KF} C_V C_M S_x$$

$$\phi M_n = 0.85(2400 \text{ psi})(0.8)(2.9)(0.75)(1.0)\left(\frac{(8.75 \text{ in})(51.0 \text{ in})^2}{6}\right)$$

$$\phi M_n = 13{,}460{,}000 \text{ in-lb} = 1120 \text{ kip-ft} \geq M_{u,\text{ext}} = 1120 \text{ kip- ft} \qquad \therefore \text{OK}$$

Shear load from wearing surface (interior stringer):

$$V_{DW,\text{int}} = \omega_{DW,\text{int}}\left(\frac{\ell}{2} - d\right) = 150 \text{ plf}\left(\frac{50 \text{ ft}}{2} - \frac{52.5 \text{ in}}{12 \text{ in/ft}}\right) = 3090 \text{ lb}$$

Shear load from wearing surface (exterior stringer):

$$V_{DW,\text{ext}} = \omega_{DW,\text{ext}}\left(\frac{\ell}{2} - d\right) = 133\text{ plf}\left(\frac{50\text{ ft}}{2} - \frac{52.5\text{ in}}{12\text{ in/ft}}\right) = 2740\text{ lb}$$

Shear load from deck weight (interior stringer):

$$V_{deck,\text{int}} = \omega_{deck,\text{int}}\left(\frac{\ell}{2} - d\right) = 169\text{ plf}\left(\frac{50\text{ ft}}{2} - \frac{52.5\text{ in}}{12\text{ in./ft}}\right) = 3490\text{ lb}$$

Shear load from deck weight (exterior stringer):

$$V_{deck,\text{ext}} = \omega_{deck,\text{ext}}\left(\frac{\ell}{2} - d\right) = 150\text{ plf}\left(\frac{50\text{ ft}}{2} - \frac{52.5\text{ in}}{12\text{ in/ft}}\right) = 3090\text{ lb}$$

Shear load from guardrail (exterior stringer):

$$V_{guardrail,\text{ext}} = \omega_{guardrail,\text{ext}}\left(\frac{\ell}{2} - d\right) = 90\text{ plf}\left(\frac{50\text{ ft}}{2} - \frac{52.5\text{ in}}{12\text{ in/ft}}\right) = 1860\text{ lb}$$

Shear load from self weight:

$$V_{self} = \omega_{self}\left(\frac{\ell}{2} - d\right) = 155\text{ plf}\left(\frac{50\text{ ft}}{2} - \frac{52.5\text{ in}}{12\text{ in/ft}}\right) = 3200\text{ lb}$$

Placement of design vehicle for shear:

The design vehicle is placed such that the maximum shear is produced with an axle at a distance from the support equal to the lesser of either three times the depth, $3d$, of the girder or one-quarter of the span, $\ell/4$.

$$3d = 3(52.5\text{ in})/(12\text{ in/ft}) = 13.1\text{ ft}$$

$$\frac{\ell}{4} = \frac{50\text{ ft}}{4} = 12.5\text{ ft} \qquad \therefore \text{Controls}$$

The design vehicle or tandem will be placed on the bridge as shown in Figure 18.2-7 with an axle at 12.5 ft from the support.

Undistributed shear due to vehicle load (Figure 18.2-7):

$$V_{LU} = 40{,}600\text{ lb}$$

Distributed shear due to vehicle load (exterior stringer):

$$V_{LL,\text{ext}} = 0.5(0.6V_{LU} + V_{LD})$$

$$V_{LL,\text{ext}} = 0.5[0.6V_{LU} + mDF_{\text{ext}}(V_{LU})]$$

$$V_{LL,\text{ext}} = 0.5[0.6(40{,}600\text{ lb}) + 0.62(40{,}600\text{ lb})]$$

$$V_{LL,\text{ext}} = 24{,}800\text{ lb}$$

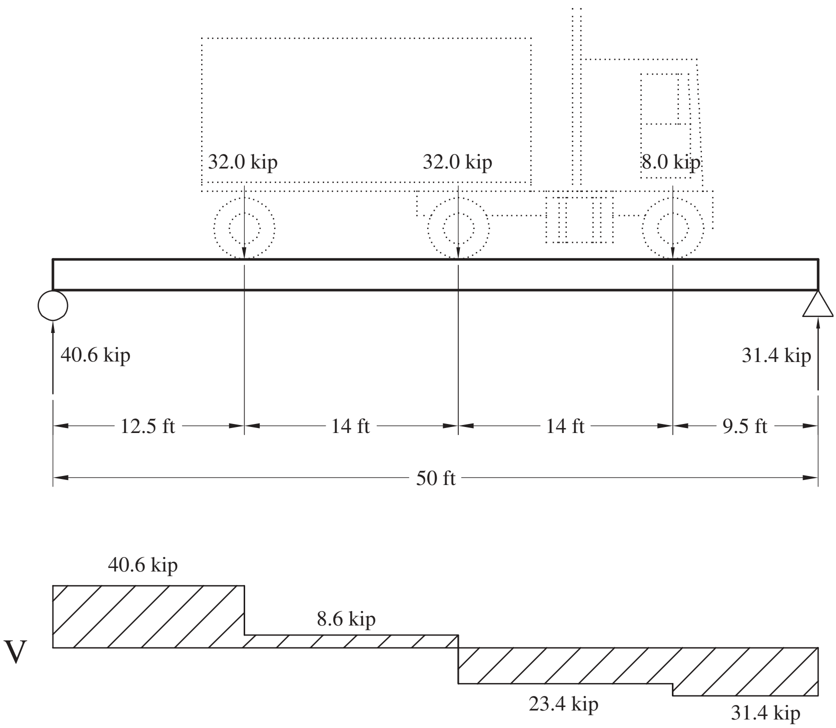


Figure 18.2-7 Design loading for shear.

Shear due to lane loading (exterior stringer):

$$V_{lane,\text{ext}} = \omega_{lane}\left(\frac{\ell}{2} - d\right) mDF_{ext}$$

$$V_{lane,\text{ext}} = 640 \text{ lb/ft}\left(\frac{50 \text{ ft}}{2} - \frac{52.5 \text{ in}}{12 \text{ in/ft}}\right) 0.62$$

$$V_{lane,\text{ext}} = 8180 \text{ lb}$$

Factored shear (exterior stringer):

$$V_u = 1.25V_{DC} + 1.5V_{DW} + 1.75V_{LL}$$

$$V_{u,ext} = 1.25(V_{self,\text{ext}} + V_{deck,\text{ext}} + V_{guardrail,\text{ext}}) + 1.5(V_{DW,\text{ext}}) + 1.75(V_{LL,\text{ext}} + V_{lane,\text{ext}})$$

$$V_{u,ext} = 1.25(3200 + 3090 + 1860) \text{ lb} + 1.5(2740 \text{ lb}) + 1.75(24{,}800 + 8180) \text{ lb}$$

$$V_{u,ext} = 72{,}000 \text{ lb}$$

Shear design check (exterior stringer):

$$\phi_v V_n = \phi_v F_{vx}(0.72) C_\lambda C_{KF} C_M C_t \left(\frac{2bd}{3}\right)$$

$$\phi_v V_n = 0.75(265 \text{ psi})(0.72)(0.8)(3.3)(1.0)(1.0)\left(\frac{2(8.75 \text{ in})(51.0 \text{ in})}{3}\right)$$

$$\phi_v V_n = 112{,}000 \text{ lb} \geq V_{u,ext} = 72{,}000 \text{ lb} \qquad \therefore \text{OK}$$

Distributed shear due to vehicle load (interior stringer):

$$V_{LL,\text{int}} = 0.5(0.6V_{LU} + V_{LD})$$

$$V_{LL,\text{int}} = 0.5[0.6V_{LU} + mDF_{\text{int}}(V_{LU})]$$

$$V_{LL,\text{int}} = 0.5[0.6(40{,}600 \text{ lb}) + 0.6(40{,}600 \text{ lb})]$$

$$V_{LL,\text{int}} = 24{,}360 \text{ lb}$$

Shear due to lane loading (interior stringer):

$$V_{lane,\text{int}} = \omega_{lane}\left(\frac{\ell}{2} - d\right) mDF_{\text{int}}$$

$$V_{lane,\text{int}} = 640 \text{ lb/ft}\left(\frac{50 \text{ ft}}{2} - \frac{52.5 \text{ in}}{12 \text{ in/ft}}\right) 0.60$$

$$V_{lane,\text{int}} = 7920 \text{ lb}$$

Factored shear (interior stringer):

$$V_u = 1.25V_{DC} + 1.5V_{DW} + 1.75V_{LL}$$

$$V_{u,\text{int}} = 1.25(V_{self,\text{int}} + V_{deck,\text{int}}) + 1.5(V_{DW,\text{int}}) + 1.75(V_{LL,\text{int}} + V_{lane,\text{int}})$$

$$V_{u,\text{int}} = 1.25(3200 + 3490)\text{lb} + 1.5(3090 \text{ lb}) + 1.75(24{,}360 + 7920)\text{lb}$$

$$V_{u,\text{int}} = 69{,}500 \text{ lb}$$

Shear design check (interior stringer):

$$\phi_v V_n = \phi_v F_{vx}(0.72) C_\lambda C_{KF} C_M C_t \left(\frac{2bd}{3}\right)$$

$$\phi_v V_n = 0.75(265 \text{ psi})(0.72)(0.8)(3.3)(1.0)(1.0)\left(\frac{2(8.75 \text{ in})(51.0 \text{ in})}{3}\right)$$

$$\phi_v V_n = 112{,}000 \text{ lb} \geq V_{u,\text{int}} = 69{,}500 \text{ lb} \qquad \therefore \text{OK}$$

Equivalent uniform load for deflection calculation:

$$\omega_{eq} = \frac{8M_{vehicle}}{\ell^2} = \frac{8(628 \text{ kip-ft})}{(50 \text{ ft})^2} = 2.00 \text{ kip/ft}$$

Deflection due to design vehicle (two lanes loaded):

$$\Delta_{LL} = \frac{(2)5\omega_{eq}\ell^4}{384E'\sum \iota}$$

$$\Delta_{LL} = \frac{(2)5(2{,}000 \text{ lb/ft})(50 \text{ ft})^4(1728 \text{ in}^3/\text{ft}^3)}{384(1.8(10^6 \text{ psi}))\left[(6)\dfrac{(8.75 \text{ in})(51.0 \text{ in})^3}{12}\right]}$$

$$\Delta_{LL} = 0.5 \text{ in} \le \frac{\ell}{425} = \frac{(50 \text{ ft})(12 \text{ in/ft})}{425} = 1.4 \text{ in} \qquad \therefore \text{ OK}$$

Dead load deflection:

$$\omega_{DW} = (150 \text{ pcf})\left(\frac{2 \text{ in}}{12 \text{ in./ft}}\right)(34 \text{ ft}) = 850 \text{ plf}$$

$$\omega_{deck} = (50 \text{ pcf})\left(\frac{6.75 \text{ in}}{12 \text{ in/ft}}\right)(36 \text{ ft}) = 1010 \text{ plf}$$

$$\omega_{DL} = \omega_{DC} + \omega_{DW}$$

$$\omega_{DL} = 6\omega_{self} + 2\omega_{guardrail} + \omega_{deck} + \omega_{DW}$$

$$\omega_{DL} = 6(155 \text{ plf}) + 2(60 \text{ plf}) + 1010 \text{ plf} + 850 \text{ plf}$$

$$\omega_{DL} = 2910 \text{ plf}$$

$$\Delta_{DL} = \frac{5\omega_{DL}\ell^4}{384E'\sum \iota}$$

$$\Delta_{DL} = \frac{5(2910 \text{ lb/ft})(50 \text{ ft})^4(1728 \text{ in}^3/\text{ft}^3)}{384(1.8(10^6 \text{ psi}))\left[(6)\dfrac{(8.75 \text{ in})(51.0 \text{ in})^3}{12}\right]}$$

$$\Delta_{DL} = 0.4 \text{ in}$$

Required camber:

$$c \ge 2\Delta_{DL} = 2(0.4 \text{ in}) = 0.8 \text{ in}$$

$$R \le \frac{L^2}{8c}$$

$$R \le \frac{(50 \text{ ft})^2}{8(0.8 \text{ in})\left(\dfrac{1 \text{ ft}}{12 \text{ in}}\right)} = 4688 \text{ ft} \qquad \therefore \text{ Use } R = 4500 \text{ ft}$$

Result: Six longitudinal stringers of Douglas fir glulam, Combination 24F-V4 DF will be spaced at 6 ft-0 in. on center with a total roadway width of 34 ft-0 in. The required size for the interior girders is $8\frac{3}{4}$ in. × 51 in. The girders will be cambered with a radius of 4500 ft.

18.3 TRANSVERSE GLULAM DECK PANELS

The AASTHO LRFD specifications [1] refer to deck panels connected with stiffener beams as interconnected decks. Stiffener beams are required for all glulam decks spanning 8 ft or more. Non-interconnected glulam decks are limited to use on secondary rural roads (Ref. [1], Section 9.9.4.4). In general, the only difference between design of interconnected decks (i.e., interconnected with stiffener beams) and non-interconnected decks is in the effective width of the panel considered. Interconnected decks are permitted to be designed for a greater effective panel width.

18.3.1 Strip Widths

AASHTO prescribes the maximum width of a strip of deck that may be considered effective for various deck systems. The strip width cannot be taken as larger than the actual width of the deck panel for non-interconnected decks. For glulam transverse deck bridges, the prescribed strip widths are determined by Equations 18.3.1-1 and 18.3.1-2.

$$b_{eff} = 2.0t + 40 \text{ in} \leq b_{panel} \qquad \text{for non-interconnected panels} \qquad (18.3.1\text{-}1)$$

$$b_{eff} = 4.0t + 30 \text{ in} \qquad \text{for interconnected panels} \qquad (18.3.1\text{-}2)$$

18.3.2 Shear Deformation

Typical spans for non-interconnected deck panels supporting asphalt are approximately 10 times the thickness of the panel or less. With span-to-depth ratios this low, shear deformation becomes significant and should be accounted for in design. For a span-to-depth ratio of 9.5:1, the apparent modulus of elasticity of the beams will be only 85% of the published modulus of elasticity, resulting in actual deflections approximately 17% greater than the deflections calculated using the published values for E_y. The deflection, including shear deformation, can be estimated for the design of transverse decks by multiplying the tabulated E_y value by 0.85.

18.3.3 Deck Design

Deck design is accomplished by placing the design vehicle such that the maximum stresses and deflections are produced. Because the required placement to produce the maximum stresses may not be obvious, an axle from the design vehicle is typically moved across the width of the bridge to determine vehicle placement(s) producing the maximum shear and bending stresses. The axle is placed with the centerline of the outside wheel at one foot from the face of the guardrail system (Ref. [1], Section 3.6.1.3.1), then moved incrementally across the bridge to determine maximum shear, moment, and deflection. Computer software is typically used to facilitate the calculations. For transverse decks, only the axles of the design vehicle are applied to the deck to determine the effects of live load; the lane load is omitted (Ref. [1], Section 3.6.1.3.3).

EXAMPLE 18.3-1 NON-INTERCONNECTED TRANSVERSE DECK

Given: A timber bridge will be constructed with six stringers spaced 6 ft on center with 2 ft overhangs on each side (Figure 18.2-1). A $6\frac{3}{4}$ in.-thick glulam deck using Douglas fir Combination 2 glulam panels has been proposed. The deck will consist of twelve 46.5 in.-wide panels and one 42 in. panel positioned at mid-span of the bridge.

Wanted: Evaluate the suitability of the proposed deck.

Solution:

Reference design values (AITC 117 [2]):

$$F_b = 1800 \text{ psi}$$
$$F_v = (230 \text{ psi})(0.72) = 166 \text{ psi}$$
$$E = 1.6(10^6) \text{ psi}$$

Deflection criterion: 0.10 in. differential displacement between panels.

Effective deck width:

$$b_{eff} = 2.0t + 40 \text{ in} \le b_{panel} \qquad \text{for non-interconnected panels}$$
$$b_{eff} = 2.0\,(6.75 \text{ in}) + 40 \text{ in} \le 42 \text{ in}$$
$$b_{eff} = 53.5 \text{ in} \le 42 \text{ in}$$
$$b_{eff} = 42 \text{ in}$$

Dead load moment and shear (Figures 18.3-1, 18.3-2, 18.3-3):
The moment and shear due to dead loads were determined for a 42 in.-wide strip of decking using structural analysis software. Shear and moment diagrams are shown in Figures 18.3-1, 18.3-2, and 18.3-3 for the loading due to the asphalt wear surface, deck, and guardrail, respectively.

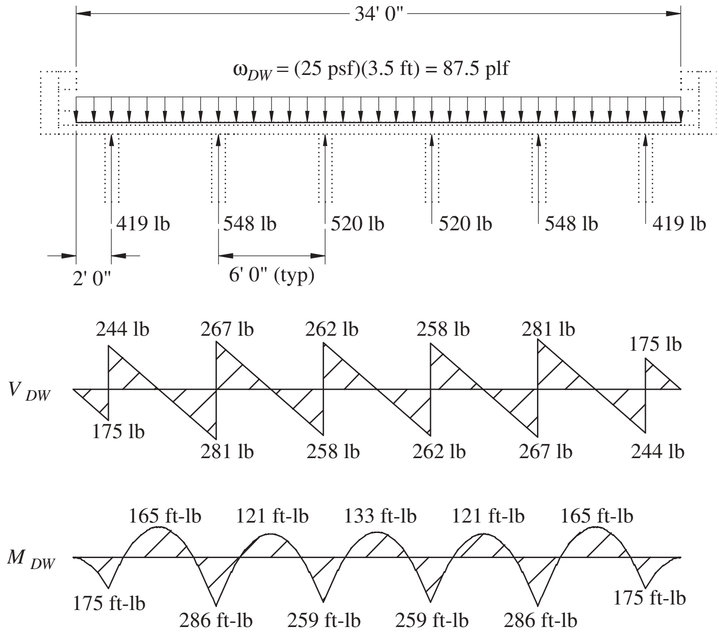


Figure 18.3-1 Dead load from asphalt wear surface.

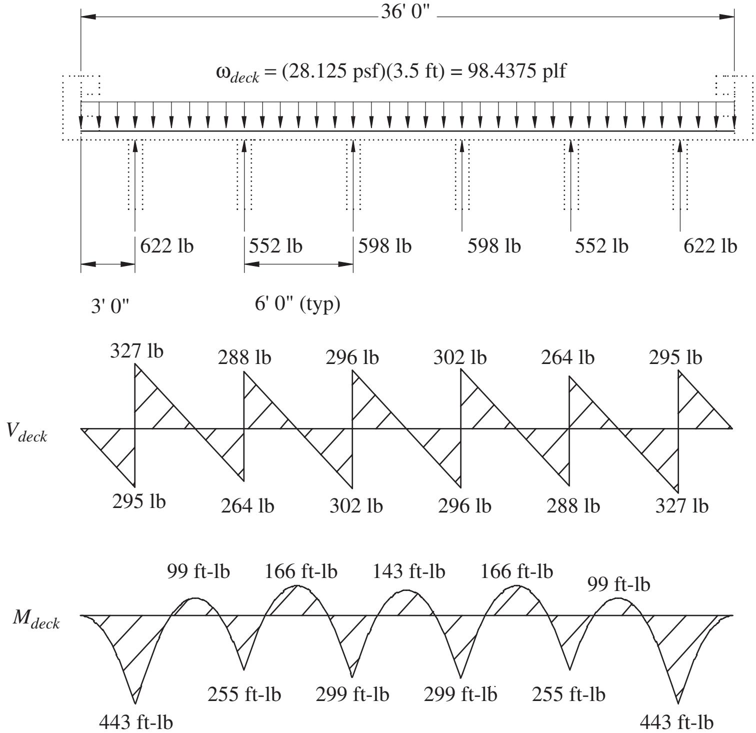


Figure 18.3-2 Dead load from deck.

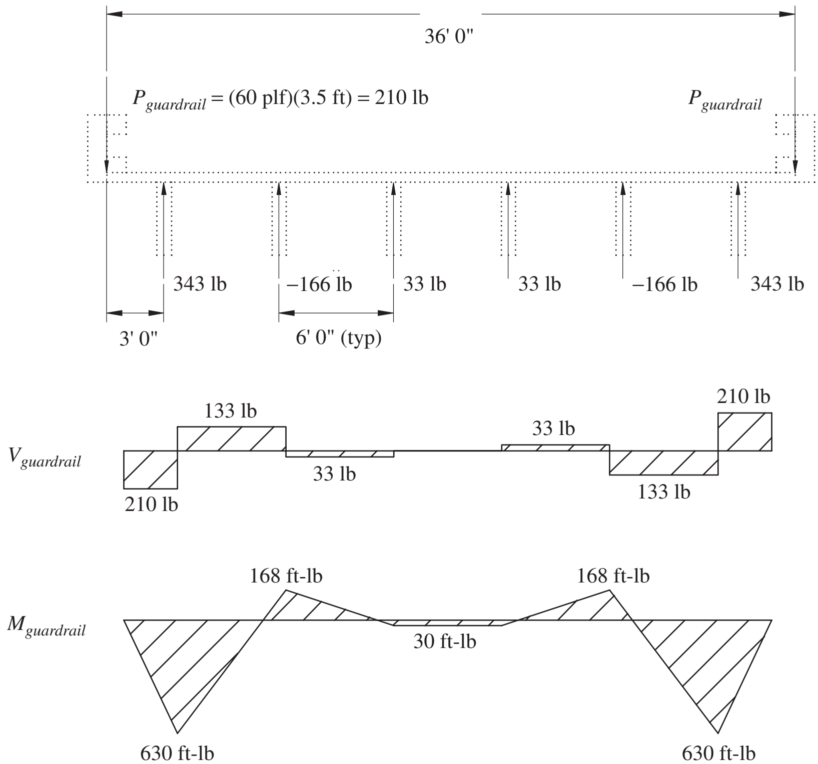


Figure 18.3-3 Dead load from guardrail.

Vehicular load, moment, shear, and deflection (Figures 18.3-4, 18.3-5, 18.3-6):

To determine the maximum moment, shear, and deflection in the overhang, the axle from the design truck was placed on the bridge with the centerline of the outside tire placed one foot from the face of the guardrail. The axle was then moved incrementally across the bridge to determine the maximum shear and moment in the interior spans. The maximum moment, shear, and deflection occurred with the truck placed one foot from the guardrail (Figure 18.3-4). The maximum moment in the interior spans occurred with the truck placed 5 ft-0 in. from the guardrail (Figure 18.3-5). The maximum shear in the interior spans occurred with the truck placed 7 ft-0 in. from the guardrail (Figure 18.3-6).

Factored design moment (Strength I):

$$M_u = 1.25M_{DC} + 1.5M_{DW} + 1.75M_{LL}$$

$$M_u = 1.25(M_{deck} + M_{guardrail}) + 1.5(M_{DW}) + 1.75(M_{vehicle})$$

$$M_u = 1.25(443 + 630) \text{ ft-lb} + 1.5(175 \text{ ft-lb}) + 1.75(16{,}000 \text{ ft-lb})$$

$$M_u = 1340 \text{ ft-lb} + 260 \text{ ft - lb} + 28{,}000 \text{ ft-lb}$$

$$M_u = 29{,}600 \text{ ft-lb}$$

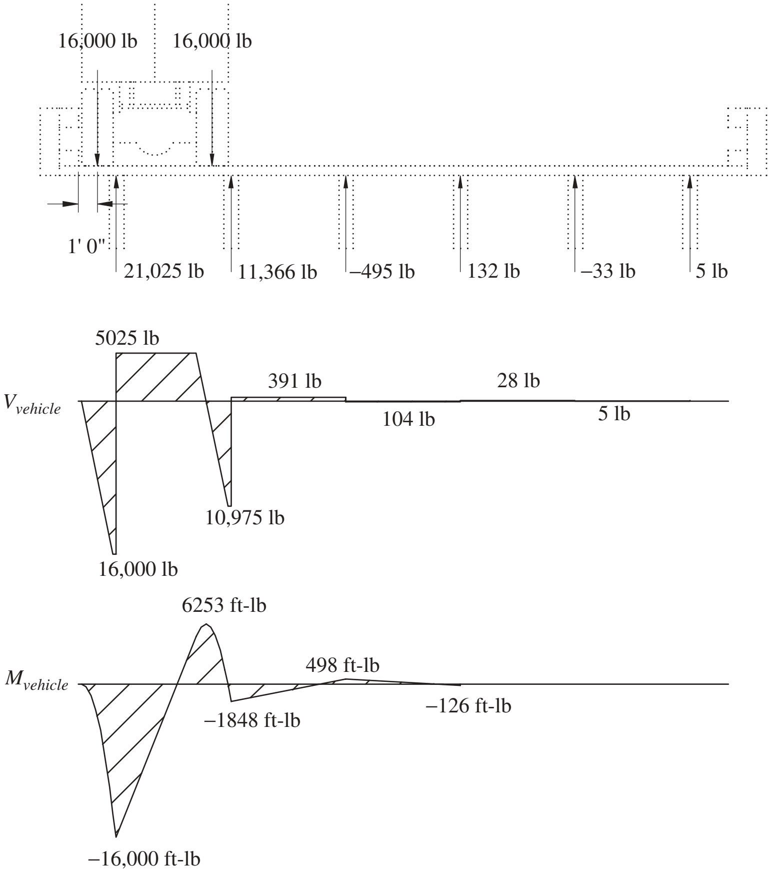


Figure 18.3-4 Deck shear and moment from truck placed 1 ft from guardrail.

Factored resisting moment:

$$\phi_b M_n = \phi_b F_{bx} C_\lambda C_{KF} C_{fu} C_M C_t S_y$$

$$\phi_b M_n = 0.85(1800 \text{ psi})(0.8)(2.9)\left(\frac{12 \text{ in}}{6.75 \text{ in}}\right)^{\frac{1}{9}}(0.8)(1.0)\left[\frac{(42 \text{ in})(6.75 \text{ in})^2}{6}\right]$$

$$\phi_b M_n = 965{,}500 \text{ in-lb} = 80{,}500 \text{ ft-lb} \geq M_u = 29{,}600 \text{ ft-lb} \qquad \therefore \text{OK}$$

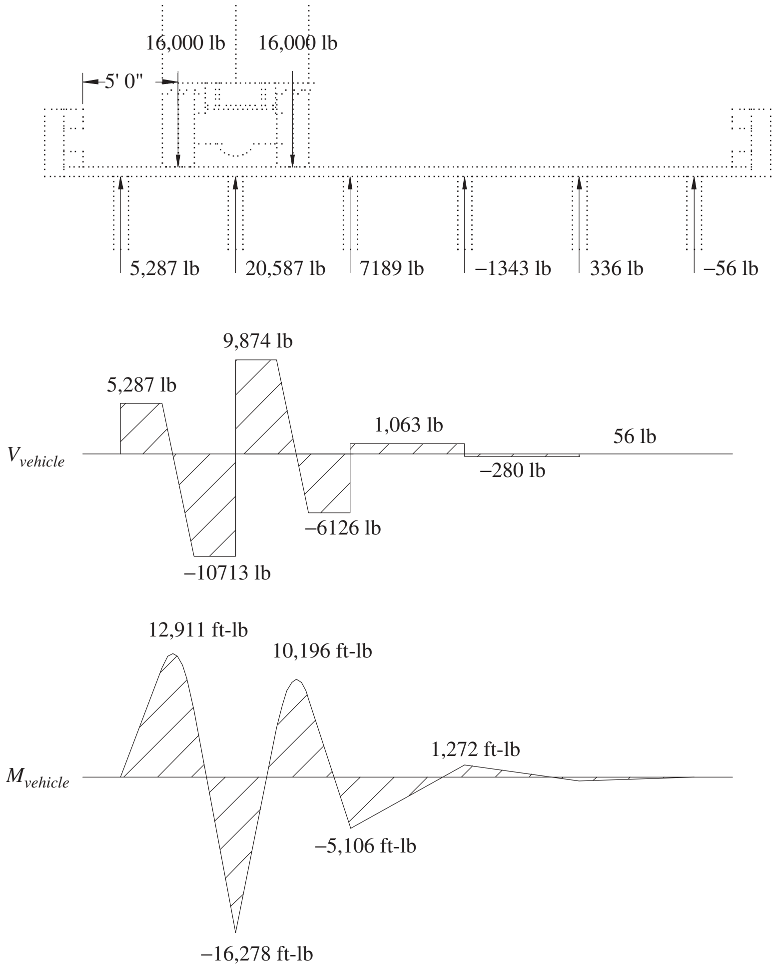


Figure 18.3-5 Deck shear and moment with truck placed 5 ft-0 in. from guardrail.

Factored design shear:

$$V_u = 1.25V_{DC} + 1.5V_{DW} + 1.75V_{LL}$$

$$V_u = 1.25(V_{deck} + V_{guardrail}) + 1.5(V_{DW}) + 1.75(V_{vehicle})$$

$$V_u = 1.25(327 + 210)\text{ lb} + 1.5(244\text{ lb}) + 1.75(16{,}000\text{ lb})$$

$$V_u = 29{,}000\text{ lb}$$

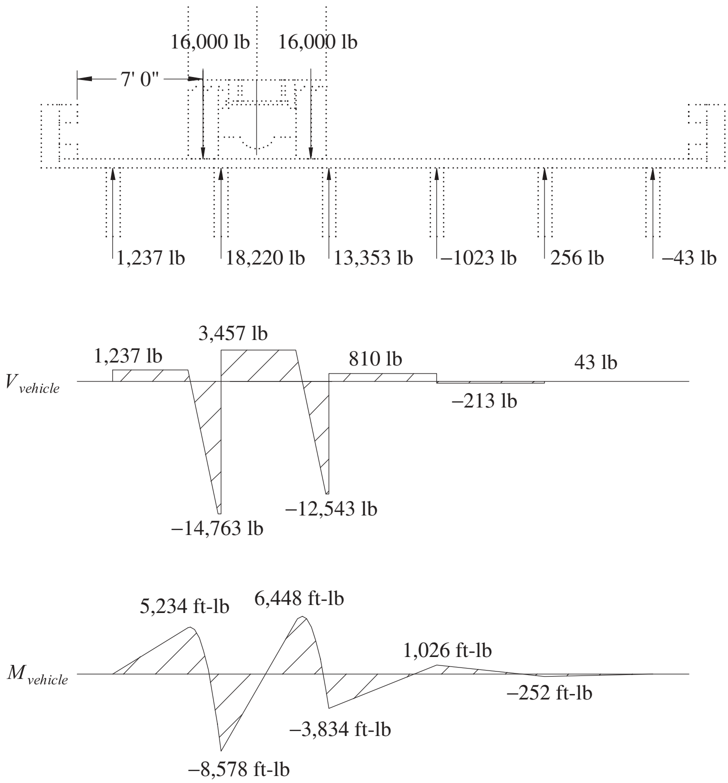


Figure 18.3-6 Deck shear and moment with truck placed 7 ft from guardrail.

Factored resisting shear:

$$\phi_v V_n = \phi_v F_{vx}(0.72) C_\lambda C_{KF} C_M C_t \left(\frac{2bd}{3}\right)$$

$$\phi_v V_n = 0.75(230 \text{ psi})(0.72)(0.8)(3.3)(0.875)(1.0)\left(\frac{2(42 \text{ in})(6.75 \text{ in})}{3}\right)$$

$$\phi_v V_n = 54{,}500 \text{ lb} \geq V_u = 29{,}000 \text{ lb} \qquad \therefore \text{ OK}$$

Live load deflection:

From the structural analysis, the maximum live load deflection occurred in the cantilever span with the design truck placed 1 ft from the curb. Its magnitude was 0.07 in. including shear deformation. The maximum deflection in the interior spans occurred with the design truck placed 7 feet from the curb and had a magnitude of 0.06 in. Because all deflections were less than 0.10 in., the deck is satisfactory.

Result: The $6\frac{3}{4}$ in. glulam deck is adequate for flexure, shear, and deflection for the bridge layout considered.

18.4 LONGITUDINAL DECK (WITH STIFFENERS)

For bridges with relatively short spans and/or light loading, longitudinal glulam deck panels may be used as both deck and superstructure for the bridge. For truss and arch bridges, longitudinal panels span across transverse beams to form the deck.

18.4.1 Design Bending Moments

The maximum bending moment due to the HL-93 design truck or tandem for a simple-span bridge can be determined using Equations 18.4.1-1 through 18.4.1-4 for the cases indicated.

One axle of design truck on bridge ($\ell \leq 10.0$ ft):

$$M_{vehicle} = (8.0 \text{ kip})\ell \tag{18.4.1-1}$$

Two axles of design tandem on bridge (10.0 ft $< \ell \leq 38.4$ ft):

$$M_{vehicle} = (12.5 \text{ kip})\ell + \frac{50 \text{ kip-ft}^2}{\ell} - 50 \text{ kip-ft} \tag{18.4.1-2}$$

Two axles of design truck on bridge (38.4 ft $< \ell \leq 42.0$ ft):

$$M_{vehicle} = (16.0 \text{ kip})\ell + \frac{1568 \text{ kip-ft}^2}{\ell} - 224 \text{ kip-ft} \tag{18.4.1-3}$$

Three axles of design tandem on bridge ($\ell > 42.0$ ft):

$$M_{vehicle} = (18 \text{ kip})\ell + \frac{392 \text{ kip-ft}^2}{\ell} - 280 \text{ kip-ft} \tag{18.4.1-4}$$

The HL-93 design lane load results in a maximum bending moment of:

$$M_{lane} = \left(0.80 \ \frac{\text{kip}}{\text{ft}}\right)\ell^2 \tag{18.4.1-5}$$

The design bending moments caused by dead loads from the deck, wearing surface, and bridge components such as the guardrail and girders themselves are calculated using conventional methods (Equation 18.4.1-6).

$$M_D = \frac{\omega_D \ell^2}{8} \tag{18.4.1-6}$$

The total factored bending moment for the Strength I load combination is:

$$\begin{aligned} M_u &= 1.25M_{DC} + 1.5M_{DW} + 1.75M_{LL} \\ M_u &= 1.25(M_{deck} + M_{guardrail}) + 1.5(M_{DW}) + 1.75(M_{vehicle} + M_{lane}) \end{aligned} \tag{18.4.1-7}$$

18.4.2 Strip Width

Longitudinal deck bridges with transverse stiffeners are considered to be "slab-type" bridges by AASHTO. Strip widths for slab-type bridges are prescribed in Section 4.6.2.3 [1]. The prescribed strip widths are calculated using Equations 18.4.2-1 and 18.4.2-2.

$$b_{strip} = 10.0\text{ in} + (5.0\text{ in/ft})\sqrt{\ell_1 W_1} \text{ per lane for one lane loaded} \quad (18.4.2\text{-}1)$$

$$b_{strip} = 84.0\text{ in} + (1.44\text{ in/ft})\sqrt{\ell_1 W_1} \leq \frac{(12\text{ in/ft})W}{N_L} \text{ per lane for multiple lanes loaded} \quad (18.4.2\text{-}2)$$

where:

b_{strip} is the width of the strip for the analysis (in.)
ℓ_1 is the lesser of the actual span length or 60 (ft)
W_1 is the lesser of the actual bridge width or 60 for multi-lane loading or is the lesser of the actual bridge width or 30 for single-lane loading (ft)
W is the actual bridge width (ft)
N_L is the number of design lanes

The smaller strip width should be used with the full truck load and lane load for the design of the bridge. The multiple presence factor has already been considered in Equations 18.4.2-1 and 18.4.2-2; therefore, it is not applied separately.

18.4.3 Deflection Design

The deck is designed to meet deflection requirements then evaluated for flexure and shear. Using the maximum vehicle moment calculated with equations from LRFD [1] Section 8.7.3.9.1, an equivalent point load at mid-span can be determined to facilitate deflection calculations (Equation 18.4.3-1).

$$P_{equiv} = \frac{4M_{vehicle}}{\ell} \quad (18.4.3\text{-}1)$$

The corresponding deflection with all design lanes loaded and all components resisting is calculated using Equation 18.4.3-2.

$$\Delta_{vehicle} = \frac{N_L P_{equiv} \ell^3}{4EWh^3} \quad (18.4.3\text{-}2)$$

A live load deflection limit of Span/425 is recommended for timber bridges carrying vehicular and pedestrian loading. The required depth can be calculated based on deflection using Equation 18.4.3-3.

$$h \geq \sqrt[3]{\frac{N_L P_{equiv} \ell^3}{4E'_y W \left(\frac{\ell}{425}\right)}} = \sqrt[3]{\frac{106 N_L P_{equiv} \ell^2}{E'_y W}} \tag{18.4.3-3}$$

18.4.4 Flexure Analysis

Once a trial depth is established, the deck must be evaluated for flexure. The actual self-weight and flat-use factor must be calculated and the factored design moment compared with the adjusted moment capacity (Equation 18.4.4-1).

$$M_u \leq \phi_b M_n = \phi_b F_{bx} C_\lambda C_{KF} C_{fu} C_M C_t S_y \tag{18.4.4-1}$$

18.4.5 Shear Analysis

The deck must be evaluated to ensure that its adjusted shear capacity exceeds the factored design shear. The shear is evaluated at a distance from the support equal to the depth of the member. The shear force from distributed dead loads is calculated using Equation 18.4.5-1.

$$V_D = \omega_D \left(\frac{\ell}{2} - h\right) \tag{18.4.5-1}$$

The design vehicle is placed such that the maximum shear is produced with the rear axle at a distance from the support equal to the lesser of either three times the depth, d, of the girder or one-quarter of the span ℓ. Once the placement of the design vehicle is determined, the live load shear from the design vehicle or tandem can be determined from the shear diagram.

The vertical shear due to the design lane loading is determined using Equation 18.4.5-2.

$$V_{lane} = \omega_{lane} \left(\frac{\ell}{2} - h\right) \tag{18.4.5-2}$$

The total factored shear for the Strength I load combination is calculated using Equation 18.4.5-3.

$$\begin{aligned} V_u &= 1.25 V_{DC} + 1.5 V_{DW} + 1.75 V_{LL} \\ V_u &= 1.25(V_{deck} + V_{guardrail} + V_{stiffeners}) + 1.5(V_{DW}) + 1.75(V_{LL} + V_{lane}) \end{aligned} \tag{18.4.5-3}$$

The factored shear must be less than the adjusted shear capacity of the deck (Equation 18.4.5-4).

$$V_u \leq \phi_v V_n = \phi_v F_{vy}\,(0.72)\,C_\lambda C_{KF} C_M C_t \left(\frac{2bh}{3}\right) \tag{18.4.5-4}$$

18.4.6 Transverse Stiffeners

Transverse stiffeners are required to transfer load and minimize deflections between adjacent panels. These stiffeners are prescriptively required to have a minimum stiffness of 80,000 kip-in.^{2} Stiffeners are required to be placed to with a maximum spacing of 8 ft (Ref. [1], Section 9.9.4.3). The stiffeners are typically spaced evenly within the span.

EXAMPLE 18.4-1 LONGITUDINAL DECK WITH TRANSVERSE STIFFENERS

Given: 20 ft simple span, 32 ft width, 2 lanes

Loading criteria: HL-93

Wearing surface: 3 in. asphalt

Wanted: Design a longitudinal deck and transverse stiffeners using Douglas fir Combination 2 panels with a 4 ft panel width.

Solution:

Adjustment factors:

$$C_\lambda = 0.80$$
$$C_M = 0.80 \text{ (for bending)}$$
$$C_M = 0.875 \text{ (for shear)}$$
$$C_M = 0.833 \text{ (for modulus of elasticity)}$$

Design values:

$$E'_y = E_y C_M = 1.6(10^6 \text{ psi})(0.833) = 1.33(10^6 \text{ psi})$$
$$F_{by} = 1800 \text{ psi}$$
$$F_{vy} = (230 \text{ psi})(0.72) = 166 \text{ psi}$$

Effective strip width (one lane loaded):

$$b_{strip,1} = 10.0 \text{ in} + (5.0 \text{ in/ft})\sqrt{\ell_1 W_1}$$
$$b_{strip,1} = 10.0 \text{ in} + (5.0 \text{ in/ft})\sqrt{(20 \text{ ft})(30 \text{ ft})}$$
$$b_{strip,1} = 132 \text{ in}$$

Effective strip width (multiple lanes loaded):

$$b_{strip,2} = 84.0 \text{ in} + (1.44 \text{ in/ft})\sqrt{\ell_1 W_1} \leq \frac{(12 \text{ in/ft})W}{N_L}$$

$$b_{strip,2} = 84.0 \text{ in} + (1.44 \text{ in/ft})\sqrt{(20 \text{ ft})(32 \text{ ft})} \leq \frac{(12 \text{ in/ft})(32 \text{ ft})}{2}$$

$$b_{strip,2} = 120 \text{ in} \leq 192 \text{ in}$$

$$b_{strip,2} = 120 \text{ in}$$

Choose smaller strip width:

$$b_{strip} = b_{strip,1} \leq b_{strip,2}$$

$$b_{strip} = 132 \text{ in} \leq 120 \text{ in}$$

$$b_{strip} = 120 \text{ in}$$

Transverse stiffener design (assuming dry-use):

One stiffener will be placed at mid-span, and stiffeners will be placed at quarter points. A shallow profile cross section will be investigated. Considering DF Combination 2, $6\frac{3}{4}$ in. × 4.5 in:

$$I_x = \frac{1}{12}(6.75 \text{ in})(4.5 \text{ in})^3 = 51.3 \text{ in}^4$$

$$E_x' = E_x = 1{,}600{,}000 \text{ psi}$$

$$E_x' I_x = (1{,}600{,}000 \text{ psi})(51.3 \text{ in}^4)$$

$$E_x' I_x = 82{,}000{,}000 \text{ lb-in}^2 \geq 80{,}000{,}000 \text{ lb-in}^2 \qquad \therefore \text{ OK}$$

Maximum vehicle moment:

$$M_{vehicle} = (12.5 \text{ kip})\ell + \frac{50 \text{ kip-ft}^2}{\ell} - 50 \text{ kip-ft}$$

$$M_{vehicle} = (12.5 \text{ kip})(20 \text{ ft}) + \frac{50 \text{ kip-ft}^2}{20 \text{ ft}} - 50 \text{ kip-ft}$$

$$M_{vehicle} = 203 \text{ kip-ft}$$

Maximum moment due to lane load:

$$M_{lane} = \left(0.08 \ \frac{\text{kip}}{\text{ft}}\right)\ell^2 = \left(0.08 \ \frac{\text{kip}}{\text{ft}}\right)(20 \text{ ft})^2 = 32.0 \text{ kip-ft}$$

Deflection design:

$$P_{equiv} = \frac{4M_{vehicle}}{\ell} = \frac{4(203 \text{ kip-ft})}{20 \text{ ft}} = 40.6 \text{ kip}$$

$$h \geq \sqrt[3]{\frac{106 N_L P_{equiv} \ell^2}{E'_y W}}$$

$$h \geq \sqrt[3]{\frac{106(2)(40{,}600\ \text{lb})(20\ \text{ft})^2(144\ \text{in}^2/\text{ft}^2)}{(1.33(10^6\ \text{psi}))(32\ \text{ft})(12\ \text{in/ft})}}$$

$$h \geq 9.9\ \text{in} \qquad \therefore \text{Try } 10\tfrac{3}{4} \text{ in thick deck}$$

Dead loads:

$$\omega_{DW} = (\gamma t)_{DW}\, b_{strip} = \left[(150\ \text{pcf})(3\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)\right](120\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 375\ \text{lb/ft}$$

$$\omega_{deck} = (\gamma h)_{deck}\, b_{strip} = \left[(50\ \text{pcf})(10.75\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)\right](120\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 448\ \text{lb/ft}$$

The weight of the stiffeners adds point loads at mid-span and quarter points of:

$$P_{stiffener} = (\gamma bd)_{stiffener}\, b_{strip}$$

$$P_{stiffener} = \left[(50\ \text{pcf})(6.75\ \text{in})(4.5\ \text{in})\left(\frac{\text{ft}^2}{144\ \text{in}^2}\right)\right](120\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) = 105\ \text{lb}$$

Dead load moments:

$$M_{DW} = \frac{\omega_{DW}\ell^2}{8} = \frac{375\ \text{lb/ft}(20\ \text{ft})^2}{8} = 18{,}750\ \text{lb-ft}$$

$$M_{deck} = \frac{\omega_{deck}\ell^2}{8} = \frac{448\ \text{plf}\,(20\ \text{ft})^2}{8} = 22{,}400\ \text{lb-ft}$$

$$M_{stiffeners} = \frac{2P_{stiffener}\ell}{4} = \frac{2(105\ \text{lb})(20\ \text{ft})}{4} = 1{,}050\ \text{lb-ft}$$

Factored design moment (Strength I):

$$M_u = 1.25M_{DC} + 1.5M_{DW} + 1.75M_{LL}$$

$$M_u = 1.25(M_{deck} + M_{stiffeners}) + 1.5(M_{DW}) + 1.75(M_{vehicle} + M_{lane})$$

$$M_u = 1.25(22{,}400 + 1{,}050)\text{ft-lb} + 1.5(18{,}750\ \text{ft-lb}) + 1.75(203{,}000\ \text{ft-lb} + 32{,}000\ \text{ft-lb})$$

$$M_u = 469{,}000\ \text{ft-lb}$$

Factored resisting moment:

$$\phi_b M_n = \phi_b F_{bx} C_\lambda C_{KF} C_{fu} C_M C_t S_y$$

$$\phi_b M_n = 0.85(1800\text{ psi})(0.8)(2.9) = \left(\frac{12\text{ in}}{10.75\text{ in}}\right)^{\frac{1}{9}}(1.0)(0.8)\left[\frac{(120\text{ in})(10.75\text{ in})^2}{6}\right]$$

$$\phi_b M_n = 6{,}640{,}000\text{ in-lb} = 553{,}000\text{ ft-lb} \geq M_u = 469{,}000\text{ ft-lb} \qquad \therefore\ \text{OK}$$

Dead load shear forces:

$$V_{DW} = \omega_{DW}\left(\frac{\ell}{2} - h\right) = 375\text{ plf}\left(\frac{20\text{ ft}}{2} - \frac{10.75\text{ in}}{12\text{ in/ft}}\right) = 3410\text{ lb.}$$

$$V_{deck} = \omega_{deck}\left(\frac{\ell}{2} - h\right) = 448\text{ plf}\left(\frac{20\text{ ft}}{2} - \frac{10.75\text{ in}}{12\text{ in/ft}}\right) = 4080\text{ lb}$$

$$V_{stiffeners} = \frac{3P_{stiffener}}{2} = \frac{(3)(105\text{ lb})}{2} = 158\text{ lb}$$

Live load shear forces:

The design live load shear is obtained by placing the maximum wheel load at the lesser of $3h$ or $\ell/4$ from the support; in this example, $3h = 3(10.75\text{ in}(1\text{ ft}/12\text{ in})) = 2.69$ ft, and $\ell/4 = 5$ ft, the lesser of which is 2.69 ft. The load placement and shear diagram are illustrated in Figure 18.4-1. The maximum shear force due to the design vehicle is $V = 33{,}100$ lb.

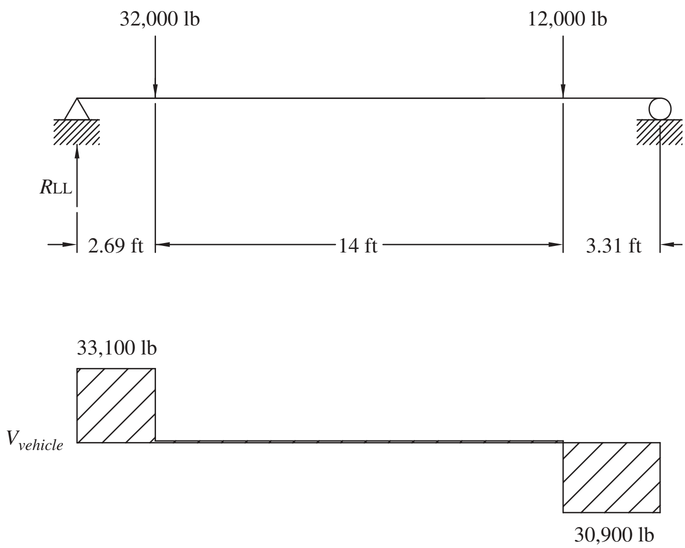


Figure 18.4-1 Shear force from design vehicle.

Shear from design lane load:

$$V_{lane} = \omega_{lane}\left(\frac{\ell}{2} - h\right) = 640 \text{ plf}\left(\frac{20 \text{ ft}}{2} - \frac{10.75 \text{ in}}{12 \text{ in/ft}}\right) = 5{,}830 \text{ lb}$$

Factored design shear:

$$V_u = 1.25V_{DC} + 1.5V_{DW} + 1.75V_{LL}$$

$$V_u = 1.25(V_{deck} + V_{stiffeners}) + 1.5(V_{DW}) + 1.75(V_{vehicle} + V_{lane})$$

$$V_u = 1.25(4080 + 158) \text{ lb} + 1.5(3410 \text{ lb}) + 1.75(33{,}100 + 5830) \text{ lb}$$

$$V_u = 78{,}500 \text{ lb}$$

Factored resisting shear:

$$\phi_v V_n = \phi_v F_{vy}(0.72)C_\lambda C_{KF} C_M C_t\left(\frac{2bd}{3}\right)$$

$$\phi_v V_n = 0.75(230 \text{ psi})(0.72)(0.8)(3.3)(0.875)(1.0)\left(\frac{2(120 \text{ in})(10.75 \text{ in})}{3}\right)$$

$$\phi_v V_n = 247{,}000 \text{ lb} \geq V_u = 78{,}500 \text{ lb} \qquad \therefore \text{OK}$$

Answer: Eight 48 in. × $10\frac{3}{4}$ in. Douglas fir Combination 2 panels will be used. $6\frac{3}{4}$ in. × 4.5 in. DF Combination 2 stiffeners will be used at mid-span and at the quarter points of the span.

18.5 CONCLUSION

Load and resistance factor design (LRFD) has become the standard method for bridge design. It is mandated for federally-funded highway projects and AASHTO is planning on promulgating LRFD as the sole method in future standards. This chapter presented design procedures and examples for the design of common bridge systems using LRFD. Chapter 19 presents procedures and examples with the allowable stress design (ASD) methodology. Chapter 17 discusses various types of timber bridges and highlights some of the major differences between ASD and LRFD design of bridges based on AASHTO specifications.

CHAPTER 19

ASD BRIDGE DESIGN

19.1 INTRODUCTION

This chapter discusses allowable stress design (ASD) for highway bridges. ASD requirements are found in the seventeenth edition AASHTO *Standard Specifications for Highway Bridges* [1]. However, the designer should be aware that AASHTO has adopted LRFD as its recommended design method and does not intend to publish any more editions of the ASD specifications.

19.1.1 Loads and Load Combinations

In general, the *Standard Specifications for Highway Bridges* [1] require bridges to be designed to resist the following loads:

- Dead load
- Vehicular live load
- Impact or dynamic effect of the live load
- Wind load
- Other loads when they exist, including longitudinal forces, centrifugal force, thermal forces, earth pressure, buoyancy, shrinkage stresses, rib shortening, erection stresses, ice and current pressure, and earthquake forces

Loads due to snow and ice buildup are not typically considered unless they may exceed the design vehicular loads. Snow and ice loads and are not expected to occur simultaneously with the design vehicular loads. For timber bridge structures, AASHTO does not require application of the impact factor in design, because the increased resistance of the wood due to the load duration factor exceeds the magnitude of the increased load due to the impact factor.

Load combinations (and accompanying load duration factors) are prescribed in the AASHTO Specifications [1]. For most timber bridge superstructures, the relevant load combinations are summarized in Table 19.1.1-1.

TABLE 19.1.1-1 Allowable Stress Load Combinations for Timber Bridge Superstructures

AASHTO Designation	Load Combination	Load Duration Factor
Group I	$D + L$	1.15
Group IA (Overload)	$(D + 2L)/1.5$	1.15
Group II	$(D + W)/1.25$	1.6
Group III	$(D + L + 0.3W + WL + LF)/1.25$	1.6
Group VII	$(D + EQ)/1.33$	1.6

D = Dead load
L = Live load (vehicular load)
W = Wind load on structure
WL = Wind load on vehicles
LF = Longitudinal force
EQ = Earthquake

19.1.2 Loading Systems

Standard truck and lane loads are used to represent vehicular loads on bridges. Two systems of loading are included in the AASHTO ASD specifications: H loadings and HS loadings. H loadings represent a two-axle truck or a corresponding lane load. HS loadings represent a tractor truck and trailer or a corresponding lane load. Four standard load classes include H 15-44, H 20-44, HS 15-44, and HS 20-44. The first numerical designation associated with the loading represents the gross weight of the truck in tons. The "44" in the designation corresponds to the 1944 edition of the AASHTO specifications that originally used these loadings. In addition, highway bridges designed for HS 20-44 loading must also be designed for an alternate military loading with a design vehicle represented by two axle loads of 24,000 lb placed 4 ft apart. Details of the required loads are provided in the AASHTO specifications [1].

For the design of timber decks for HS 20 and H 20 loadings, AASHTO permits the application of two axle loads of 16,000 lb placed 4 ft apart or a single axle load of 24,000 lb, whichever produces the greater stress. For the design of a transverse glulam deck panel, this results in a maximum HS 20 wheel load of 12,000 lb, rather than half of the maximum HS-20 axle load, which would be 16,000 lb (Ref [1], Figure 3.7.6A).

To simplify design, AASHTO tabulates maximum values for shear and moment for the design trucks on simple span bridges in Appendix A [1]. The higher loading designated "HS-25" is obtained by increasing all HS-20 loads by 25%.

The maximum bending moment due to the HS-20 design truck or alternate military vehicle for a simple-span bridge can be determined using Equations 19.1.2-1 through 19.1.2-4 for the cases indicated.

One axle of design truck on bridge ($\ell < 10.9$ ft):

$$M_{vehicle} = (8.0\,\text{kip})\,\ell \tag{19.1.2-1}$$

Two axles of design tandem on bridge ($10.0\,\text{ft} \le \ell \le 32.2\,\text{ft}$):

$$M_{vehicle} = (12.5\,\text{kip})\,\ell + \frac{50\,\text{kip - ft}^2}{\ell} - 50\,\text{kip - ft} \tag{19.1.2-2}$$

Two axles of design truck on bridge ($32.2\,\text{ft} < \ell \le 42.0\,\text{ft}$):

$$M_{vehicle} = (16.0\,\text{kip})\,\ell + \frac{1568\,\text{kip - ft}^2}{\ell} - 224\,\text{kip - ft} \tag{19.1.2-3}$$

Three axles of design tandem on bridge ($\ell > 42.0$ ft):

$$M_{vehicle} = (18\,\text{kip})\,\ell + \frac{392\,\text{kip - ft}^2}{\ell} - 280\,\text{kip - ft} \tag{19.1.2-4}$$

19.1.3 Wheel Load Distribution

For the calculation of bending moments and shear loads in longitudinal members, wheel loads are not distributed along the length of the bridge, but are distributed laterally between stringers or longitudinal beams. For the calculation of load effects in transverse beams, wheel loads are not distributed across the width of the bridge, but are distributed between transverse beams. Wheel load distribution factors depend on the type of deck and the size of the bridge and are typically prescribed in terms of a fraction of the beam spacing, S, in feet (Tables 19.1.3-1 and 19.1.3-2).

TABLE 19.1.3-1 Wheel Load Distribution Factors for Interior Longitudinal Beams

Type of Deck	Bridge Designed for One Traffic Lane	Bridge Designed for Two or More Traffic Lanes
Timber plank	$S/4.0$	$S/3.75$
4″ nominal thickness nail-laminated	$S/4.5$	$S/4.0$
6″ nominal thickness or thicker nail-laminated	$S/5.0$	$S/4.25$
4″ nominal [a] thickness glulam panels on glulam stringers	$S/4.5$	$S/4.0$
6″ nominal [a] thickness or thicker glulam panels on glulam stringers	$S/6.0$ If S exceeds 6’ use footnote [b].	$S/5.0$ If S exceeds 7.5’ use footnote [b].

S = Interior stringer (beam) spacing (ft)

[a] 4-in nominal thickness glulam refers to 3-in actual dimension and thicker; 6-in nominal refers to 5-in actual dimension and thicker.

[b] In this case, the load on each stringer shall be the reaction of the wheel loads, assuming the flooring between the stringers to act as a simple beam.

TABLE 19.1.3-2 Wheel Load Distribution Factors for Transverse Beams

Kind of Deck	Distribution Factor
Timber plank	$S/4$
4″ nominal thickness glulam or nail-lam	$S/4.5$
6″ nominal thickness or thicker glulam or nail-lam	$S/5$

S = Interior stringer (beam) spacing (ft)

19.1.4 Camber

Glulam bridge girders are generally cambered for appearance and drainage. AASHTO [1] specifies that a camber of three times the calculated dead load deflection is preferred and that glulam girders must be cambered a minimum of $\frac{1}{2}$ in.

19.2 LONGITUDINAL STRINGERS (GIRDERS)

As stated previously, the girder bridge is the most commonly used timber bridge system. This section presents a design method for glulam girders based on AASHTO's allowable stress design specifications [1].

19.2.1 Stringer Spacing, *S*, and Deck Overhang, *O*

Given the overall width of the roadway, the designer must determine the number and spacing of stringers that will be used to support the deck. The deck panels are typically cantilevered somewhat past the exterior stringers. The overhang distance, O, and the stringer spacing, S, are commonly selected to minimize differences in moment between the interior and exterior stringers or to optimize deck design.

The stringer spacing is used to determine wheel load distribution factors and girder loads, then the girders are designed by choosing a trial size based on flexure. The trial size is then analyzed for shear and deflection. Stresses must also be checked for the load combination representing overload. Interior and exterior stringers must be considered separately.

19.2.2 Flexure Design of Interior Stringers

Dead load on each interior stringer is typically calculated based on a tributary width equal to the spacing between stringers. Moment due to dead load is calculated based on conventional methods. Self-weight of the beam is estimated. The moment due to dead load on the interior stringer is calculated using Equation 19.2.2-1.

$$M_{D,\text{int}} = \frac{\left[\omega_{deck} + \omega_{DW} + \omega_{self}\right] \ell^2}{8} \qquad (19.2.2\text{-}1)$$

where:

$$\omega_{deck} = \gamma_{deck} t_{deck} s_{stringer}$$
$$\omega_{DW} = \gamma_{DW} t_{DW} s_{stringer}$$
$$\omega_{self} = \gamma_{stringer} bd$$

The maximum moment due to the design truck on one lane is obtained from AASHTO Appendix A [1]. The design live load moment on an interior stringer is calculated using Equation 19.2.2-2.

$$M_{L,\text{int}} = \frac{\text{(Moment/Lane)(Distribution factor)}}{\text{Wheel lines/Lane}} \tag{19.2.2-2}$$

The total design moment on an interior stringer is calculated using Equation 19.2.2-3.

$$M_{D+L,\text{int}} = M_{D,\text{int}} + M_{L,\text{int}} \tag{19.2.2-3}$$

After choosing a beam width and estimating the volume factor, the required depth can be calculated with Equation 19.2.2-4.

$$d = \sqrt{\frac{6M_{D+L,\text{int}}}{bF_b C_D C_V C_M}} \tag{19.2.2-4}$$

A trial depth should be selected as a standard depth. Values for girder self-weight and the volume factor are then calculated and the design is revised as necessary.

19.2.3 Shear Analysis, Interior Stringers

For the uniformly distributed dead load, the shear is calculated at a distance d from the support (Equation 19.2.3-1).

$$V_{DL} = \frac{\omega \ell}{2} - \omega_D (d) \tag{19.2.3-1}$$

For the live load, the wheel loads are taken to be half the axle loads and are placed to produce the maximum shear (V_{LU}) at the lesser of $3d$ or $\frac{\ell}{4}$ from the support. The live load design shear, from the AASHTO Specification [1], is then calculated with Equation 19.2.3-2.

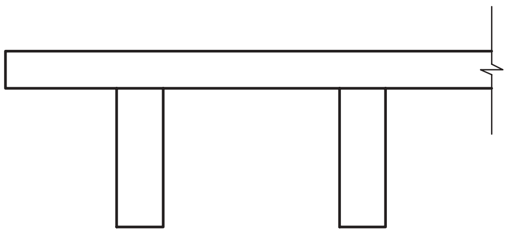
$$V_{LL} = 0.5\left[0.6\left(V_{LU}\right) + V_{LD}\right] = 0.5\,[0.6 + \text{DF}]\,V_{LU} \tag{19.2.3-2}$$

where:

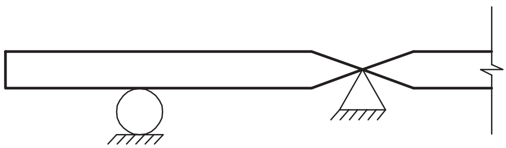
V_{LL} = live load design shear
V_{LU} = maximum vertical shear at $3d$ or $\frac{\ell}{4}$ due to undistributed wheel loads
V_{LD} = V_{LU} multiplied by the distribution factor for moment
DF = distribution factor for moment
d = stringer depth
ℓ = span

19.2.4 Exterior Stringers

AASHTO [1] requires that the exterior stringers have capacity equal to or greater than the interior stringers. Loads on the exterior stringer are determined based on modeling the bridge deck as a beam with a hinge support on the first interior stringer and a roller support on the exterior stringer (Figure 19.2.4-1). This model is also known as the *lever rule*. For this analysis, the wheel loads are placed two feet from the face of the curb. Live load distribution factors have been derived for this case, as shown in Figure 19.2.4-2. Three different cases are possible, depending on the stringer spacing and the overhang distance. The exterior girders must also be designed to support the guardrail and a portion of the bridge deck and wearing surface. The lever rule can be used to conservatively estimate the dead loads on the exterior stringer.



Girder Bridge



Notional Model for Applying Lever Rule

Figure 19.2.4-1 Lever rule.

19.2.5 Overload

In consideration of overload, the AASHTO Specifications [1] require the live load to be doubled for one lane, and other lanes may be considered to not be loaded concurrently. The calculated loads or the resulting stresses are reduced by dividing by 1.5. The distribution factor for overload is calculated assuming that a single lane is loaded.

The overload bending moment is calculated using Equation 19.2.5-1.

$$M_{overload} = \frac{M_D + 2M_{L,overload}}{1.5} \tag{19.2.5-1}$$

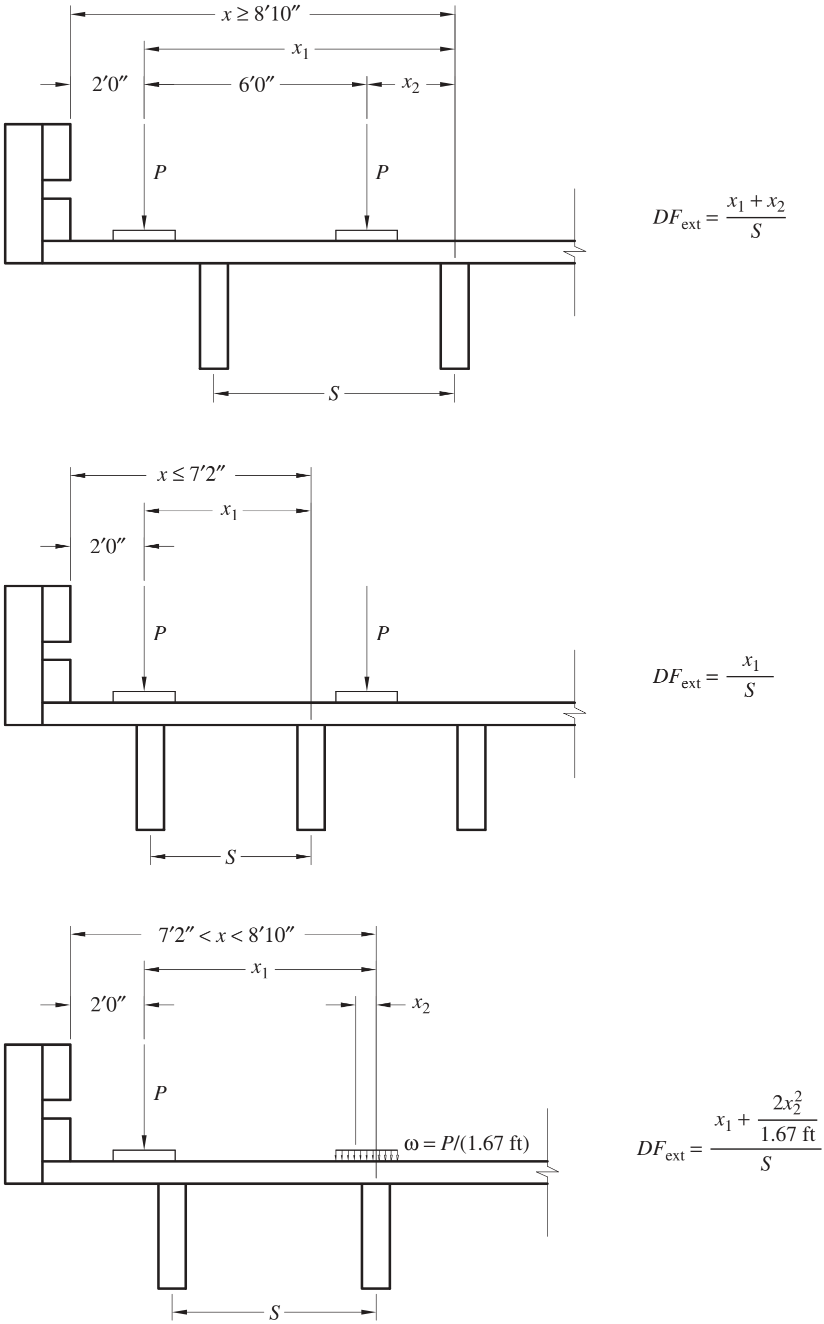


Figure 19.2.4-2 ASD distribution factors for exterior stringers based on lever rule.

The overload shear force is calculated using Equation 19.2.5-2.

$$V_{overload} = \frac{V_{DL} + 2V_{LL,overload}}{1.5} \tag{19.2.5-2}$$

where:

$V_{LL,overload}$ is calculated using the single lane distribution factor

19.2.6 Deflection

For the evaluation of deflection of the interior stringers, the distributed live load moment may be converted into an equivalent concentrated load placed at mid-span (Equation 19.2.6-1) and the corresponding deflection used to check the deflection criterion. Such analysis is conservative.

$$P_{equiv} = \frac{4M_{L,\text{int}}}{\ell} \tag{19.2.6-1}$$

The corresponding live load deflection is calculated using Equation 19.2.6-2.

$$\Delta_{LL} = \frac{P_{equiv}\,\ell^3}{48EI} = \frac{P_{equiv}\,\ell^3}{4Ebd^3} \tag{19.2.6-2}$$

The AASHTO specification for ASD [1] allows the deflection to be calculated using all stringers acting concurrently if there are cross braces with sufficient depth and strength to distribute the loads. Assuming adequate bracing, the deflection is calculated as follows.

The total live load bending moment is determined by multiplying the single-lane bending moment obtained from AASHTO Appendix A by the number of traffic lanes (Equation 19.2.6-3).

$$M_{L,total} = \frac{\text{Moment}}{\text{Lane}}\,(\text{number of lanes}) \tag{19.2.6-3}$$

The mid-span point load required to produce an equivalent moment is calculated with Equation 19.2.6-4.

$$P_{equiv,total} = \frac{4M_{L,total}}{\ell} \tag{19.2.6-4}$$

The corresponding live load deflection is determined using Equation 19.2.6-5.

$$\Delta_{LL} = \frac{P_{equiv,total}\,\ell^3}{4Enbd^3} \tag{19.2.6-5}$$

EXAMPLE 19.2-1 LONGITUDINAL STRINGER

Given: Glued laminated timber bridge with 40 ft-0 in span, 34 ft-0 in width, two lanes, with 3-in asphalt wearing surface (Figure 19.2-1). Transverse decking is assumed to be structural glued laminated timber, 5 in thick (6 in nominal thickness) and will overhang the exterior stringer by 2 ft.

A crash-tested guardrail system weighing 60 lb/ft will be installed. Six longitudinal stringers of southern pine glulam, Combination 24F-V3 SP/SP will be used. For vehicular live load, a load duration factor of 1.15 is used. Assume dry conditions for the stringers. Use $\ell/425$ for live load deflection.

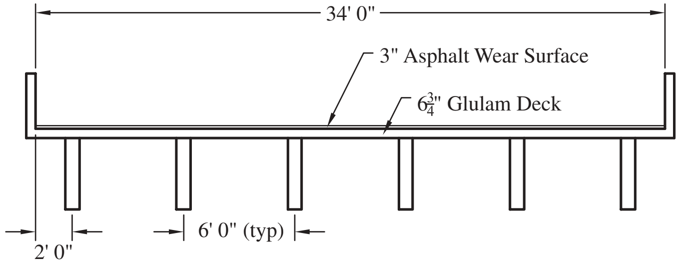


Figure 19.2-1 Bridge schematic—Example 19.2-1.

Wanted: Determine suitable girder sizes with AASHTO HS 15−44 vehicle load. Consider Group I and Group IA (overstress) load combinations.

Solution:

Tributary width (spacing) for interior stringer:

$$S = \frac{W - 2O}{n - 1} = \frac{34\,\text{ft} - 2\,(2\,\text{ft})}{6 - 1} = 6\,\text{ft}$$

Deck dead load on interior stringer (50 pcf):

$$\omega_{deck,\text{int}} = (50\,\text{pcf})\left(\frac{5\,\text{in}}{12\,\text{in/ft}}\right)(6\,\text{ft}) = 125\,\text{plf}$$

Asphalt wearing surface dead load on interior stringer (150 pcf):

$$\omega_{asphalt,\text{int}} = (150\,\text{pcf})\left(\frac{3.0\,\text{in}}{12\,\text{in/ft}}\right)(6\,\text{ ft}) = 225\,\text{plf}$$

Estimated stringer self-weight:

$$\omega_{self,\text{int}} = 100\,\text{plf}$$

Total estimated dead load on interior stringer:

$$\omega_{D,\text{int}} = \omega_{deck,\text{int}} + \omega_{asphalt,\text{int}} + \omega_{self,\text{int}}$$

$$\omega_{D,\text{int}} = 125\,\text{plf} + 225\,\text{plf} + 100\,\text{plf}$$

$$\omega_{D,\text{int}} = 450\,\text{plf}$$

Moment on interior stringer due to dead load:

$$M_{D,\text{int}} = \frac{\omega_D\,\ell^2}{8} = \frac{(450\,\text{plf})(40\,\text{ft})^2}{8} = 90{,}000\,\text{lb-ft} = 1{,}080{,}000\,\text{lb-in}$$

Number of traffic lanes for design (12 ft width per lane):

$$n \leq \frac{W}{12\,\text{ft}} = \frac{34\,\text{ft}}{12\,\text{ft}} = 2.8 \quad \therefore \text{ Use 2 lanes}$$

Interior stringer distribution factor for live load (two lanes, Table 19.1.3-1):

$$mDF = \frac{S}{5} = \frac{6}{5} = 1.2$$

Live load design moment on interior stringer:
From AASHTO Appendix A [1], the single lane moment for HS 15-44 is 337,400 lb-ft. The design moment on an interior stringer is calculated as:

$$M_{L,\text{int}} = \frac{(\text{Moment/Lane})(\text{Distribution factor})}{\text{Wheel lines/Lane}} = \frac{(337{,}400\,\text{lb-ft})\,(1.2)}{2}$$

$$M_{L,\text{int}} = 202{,}400\,\text{lb-ft} = 2{,}429{,}000\,\text{lb-in}$$

Total load moment on interior stringer (Group I load combination):

$$M_{D+L,\text{int}} = M_{D,\text{int}} + M_{L,\text{int}} = 1{,}080{,}000\,\text{lb-in} + 2{,}429{,}000\,\text{lb-in}$$

$$M_{D+L,\text{int}} = 3{,}509{,}000\,\text{lb-in}$$

Interior stringer design:
Using a girder width of $6\frac{3}{4}$ in. and estimating the volume factor as 0.9 the required depth can be estimated, based on the required flexure strength as:

$$d = \sqrt{\frac{6M_{D+L,\text{int}}}{bF_b C_D C_V C_M}} = \sqrt{\frac{6\,(3{,}509{,}000\,\text{lb-in})}{(6.75\,\text{in})\,(2400\,\text{psi})\,(1.15)\,(0.9)\,(1.0)}}$$

$$d = 35.4\,\text{in} \quad \therefore \text{ Try } 35.75\,\text{in}$$

Interior stringer self-weight:

$$\omega_{self,\text{int}} = (6.75\,\text{in})\,(35.75\,\text{in})\left(\frac{\text{ft}^2}{144\,\text{in}^2}\right)(50\,\text{pcf}) = 84\,\text{plf}$$

The actual self-weight is slightly less than the estimated self-weight of 100 plf. Because the difference is small, the dead load will not be recalculated.

Volume factor:

$$C_V = \left[\left(\frac{5.125\,\text{in}}{6.75\,\text{in}}\right)^{\frac{1}{20}}\left(\frac{12\,\text{in}}{35.75\,\text{in}}\right)^{\frac{1}{20}}\left(\frac{21\,\text{ft}}{40\,\text{ft}}\right)^{\frac{1}{20}}\right] = 0.904$$

Beam stability factor:
The deck is fastened to the stringers to provide lateral support; therefore, $C_L = 1.0$.

Adjusted bending design value:

$$F_b' = F_b C_D C_M \left(C_L \text{ or } C_V\right)$$
$$F_b' = (2400\,\text{psi})(1.15)(0.904)(1.0) = 2480\,\text{psi}$$

Design bending stress:

$$f_b = \frac{6M_{D+L}}{bd^2} = \frac{6\,(3{,}509{,}000\,\text{lb-in})}{(6.75\,\text{in})\,(35.75\,\text{in})^2}$$
$$f_b = 2440\,\text{psi} < F_b' = 2480\,\text{psi} \quad \therefore\ \text{OK}$$

Dead load design shear on interior stringer:

$$V_{DL} = \frac{\omega \ell}{32} - \omega(d) = \frac{450\,\text{plf}\,(40\,\text{ft})}{2} - 450\,\text{plf}\left(\frac{35.75\,\text{in}}{12\,\text{in/ft}}\right)$$
$$V_{DL} = 7660\,\text{lb}$$

Live load design shear on interior stringer:
For the live load, the wheel loads are taken to be half the axle loads and are placed to produce the maximum shear at the lesser of $3d$ or $\frac{\ell}{4}$.

$$3d = 3\,(35.75\text{in})\,\frac{1\,\text{ft}}{12\,\text{in}} = 8.94\,\text{ft}$$
$$\frac{\ell}{4} = \frac{40\,\text{ft}}{4} = 10\,\text{ft}$$

The placement of loads to produce maximum shear is shown in Figure 19.2-2 and the corresponding design shear is 14,700 lb.

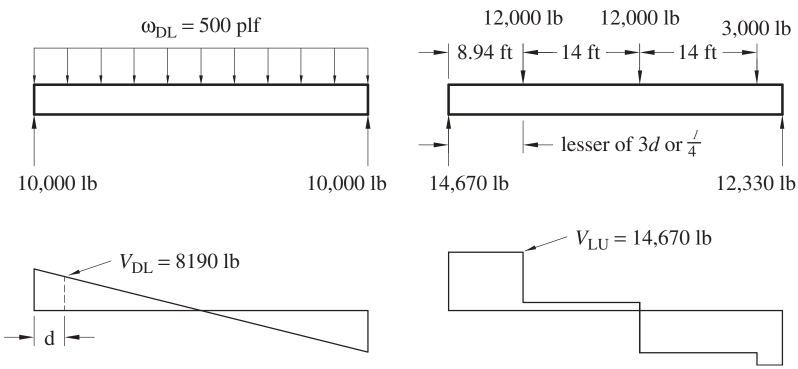


Figure 19.2-2 Shear loads on interior stringer—Example 19.2-1.

Live load design shear on interior stringer:

$$V_{LL} = 0.5\left[0.6\left(V_{LU}\right) + V_{LD}\right] = 0.5\,[0.6 + \text{DF}]\,V_{LU}$$
$$V_{LL} = 0.5\,[0.6 + 1.2]\,(14{,}700\,\text{lb}) = 13{,}230\,\text{lb}$$

Total design shear on interior stringer:

$$V_{D+L} = V_{DL} + V_{LL}$$
$$V_{D+L} = 7660\,\text{lb} + 13{,}230\,\text{lb} = 20{,}900\,\text{lb}$$

Design shear stress on interior stringer:

$$f_v = \frac{3V_{D+L}}{2bd} = \frac{(3)(20{,}900\text{lb})}{(2)(6.75\text{ in})(35.75\text{ in}} = 130\,\text{psi}$$

Adjusted shear design value (dry service assumed):

$$F'_v = F_v\ (0.72)\ C_D C_M$$
$$F'_v = 300\,\text{psi}\ (0.72)\ (1.15)\ (1.0)$$
$$F'_v = 248\,\text{psi} > f_b = 130\,\text{psi} \quad \therefore\ \text{OK}$$

Overload distribution factor for interior stringer (single lane loaded):

$$mDF = \frac{S}{6} = \frac{6}{6} = 1.0$$

Overload bending moment on interior stringer:

$$M_{L,OL} = \frac{(\text{Load/Lane})(\text{Distribution factor})}{W\text{heel lines/Lane}} = \frac{(337{,}400\,\text{lb-ft})\ (1.0)}{2}$$
$$M_{L,OL} = 168{,}700\,\text{lb-ft} = 2{,}024{,}000\,\text{lb-in}$$
$$M_{OL} = \frac{M_D + 2M_{L,OL}}{1.5} = \frac{1{,}080{,}000\,\text{lb-in} + 2\ (2{,}024{,}000\,\text{lb-in})}{1.5}$$
$$M_{OL} = 3{,}418{,}000\,\text{lb-in}$$

Overload bending stress on interior stringer:

$$f_b = \frac{6M_{OL}}{bd^2}$$
$$f_b = \frac{6\ (3{,}418{,}000\,\text{lb-in})}{(6.75\text{ in})\ (35.75\text{ in})^2}$$
$$f_b = 2378\,\text{psi} < F'_b = 2480\,\text{psi} \quad \therefore\ \text{OK}$$

Overload shear force on interior stringer:

$$V_{LL,OL} = 0.5\,[0.6\ (14{,}700\,\text{lb}) + 1.0\ (14{,}700\,\text{lb})] = 11760\,\text{lb}$$
$$V_{OL} = \frac{V_{DL} + 2V_{LL,OL}}{1.5}$$
$$V_{OL} = \frac{7660\,\text{lb} + 2\ (11{,}760\,\text{lb})}{1.5} = 20{,}800\,\text{lb}$$

Overload shear stress on interior stringer:

$$f_{\mathrm{OL}} = \frac{3V_{OL}}{2bd}$$

$$f_{\mathrm{OL}} = \frac{(3)(20{,}800\,\mathrm{lb})}{(2)(6.75\,\mathrm{in})(35.75\,\mathrm{in})}$$

$$f_{\mathrm{OL}} = 129\,\mathrm{psi} < F'_V = 247\,\mathrm{psi} \quad \therefore\ \mathrm{OK}$$

Guardrail dead load on exterior stringer:

$$\omega_{guardrail,ext} = \frac{(60\,\mathrm{plf})}{(6\,\mathrm{ft})}(6\,\mathrm{ft} + 2\,\mathrm{ft}) = 80\,\mathrm{plf}$$

Deck dead load on exterior stringer:

$$\omega_{deck,ext} = \frac{(50\,\mathrm{pcf})\left(\dfrac{5\,\mathrm{in}}{12\,\mathrm{in/ft}}\right)}{2\,(6\,\mathrm{ft})}(6\,\mathrm{ft} + 2\,\mathrm{ft})^2 = 110\,\mathrm{plf}$$

Dead load of asphalt wearing surface on interior stringer:

$$\omega_{asphalt,ext} = \frac{(150\,\mathrm{pcf})\left(\dfrac{3\,\mathrm{in}}{12\,\mathrm{in/ft}}\right)}{2\,(6\,\mathrm{ft})}(6\,\mathrm{ft} + 2\,\mathrm{ft})^2 = 200\,\mathrm{plf}$$

Estimated stringer self-weight:

$$\omega_{self,ext} = 100\,\mathrm{plf}$$

Total estimated dead load on exterior stringer:

$$\omega_{D,ext} = \omega_{guardrail,ext} + \omega_{deck,ext} + \omega_{asphalt,ext} + \omega_{self,ext}$$

$$\omega_{D,ext} = 80\,\mathrm{plf} + 110\,\mathrm{plf} + 200\,\mathrm{plf} + 100\,\mathrm{plf}$$

$$\omega_{D,ext} = 490\,\mathrm{plf}$$

Moment on exterior stringer due to dead load:

$$M_{D,ext} = \frac{\omega_{D,ext}L^2}{8} = \frac{(490\,\mathrm{plf})(40\,\mathrm{ft})^2}{8} = 98{,}000\,\mathrm{lb\text{-}ft} = 1{,}176{,}000\,\mathrm{lb\text{-}in}$$

Moment on exterior stringer due to live load:
When the outside wheel load is placed two feet from the curb, it is directly over the exterior girder. The other wheel is directly over the next interior girder. The result is a distribution factor of 1.0:

$$M_{L,ext} = \frac{\left(\dfrac{\mathrm{Load}}{\mathrm{Lane}}\right)(\mathrm{Distribution\ factor})}{\left(\dfrac{\mathrm{Wheel\ lines}}{\mathrm{Lane}}\right)} = \frac{(337{,}400\,\mathrm{lb\text{-}ft})\,(1.0)}{2}$$

$$M_{L,ext} = 168{,}700\,\mathrm{lb\text{-}ft} = 2{,}024{,}000\,\mathrm{lb\text{-}in}$$

Moment on exterior stringer due to total load (Group I Load Combination):

$$M_{D+L,ext} = M_{D,ext} + M_{L,ext}$$
$$M_{D+L,ext} = 1{,}176{,}000\,\text{lb-in} + 2{,}024{,}000\,\text{lb-in}$$
$$M_{D+L,ext} = 3{,}200{,}000\,\text{lb-in}$$

In this case, the design moment on the exterior girder is less than the design moment on the interior girders. The design shear force will also be less for the exterior stringers. AASHTO requires the exterior stringer to have equal or greater capacity than the interior stringers, so the same size beam will be used for the exterior stringers.

Live load deflection limit:

$$\delta_{LL} = \frac{\ell}{425} = \frac{480\,\text{in}}{425} = 1.1\,\text{in}$$

Deflection of interior stringer:

$$P_{equiv,ext} = \frac{4M_{L,ext}}{\ell} = \frac{4\,(202{,}400\,\text{lb-ft})}{(40\,\text{ft})} = 20{,}240\,\text{lb}$$
$$\Delta_{LL} = \frac{P_{equiv,ext}\,\ell^3}{4Ebd^3}$$
$$\Delta_{LL} = \frac{20{,}240\,\text{lb}\,(480\,\text{in})^3}{4\,(1{,}800{,}000\,\text{psi})\,(6.75\,\text{in})\,(35.75\,\text{in})^3}$$
$$\Delta_{LL} = 1.0\,\text{in} \le \delta_{LL} = 1.1\,\text{in} \quad \therefore\ \text{OK}$$

Total live load bending moment (two lanes loaded):

$$M_L = \left(337{,}400\ \frac{\text{lb-ft}}{\text{lane}}\right)(2\,\text{lanes}) = 674{,}800\,\text{lb-ft}$$

Mid-span point load required to produce equivalent moment:

$$P_{equiv} = \frac{4M_\ell}{\ell} = \frac{4\,(674{,}800\,\text{lb-ft})}{(40\,\text{ft})} = 67{,}480\,\text{lb}$$

Total live load deflection for bridge as a whole:

$$\Delta_{LL} = \frac{P\,\ell^3}{4Enbd^3}$$
$$\Delta_{LL} = \frac{67{,}480\,\text{lb}\,(480\,\text{in})^3}{4\,(1{,}800{,}000\,\text{psi})\,(6)\,(6.75\,\text{in})\,(35.75\,\text{in})^3}$$
$$\Delta_{LL} = 0.6\,\text{in} < \delta_{LL} = 1.1\,\text{in} \quad \therefore\ \text{OK}$$

__Results:__ The longitudinal stringers will be specified as $6\frac{3}{4}$ in $\times 35\frac{3}{4}$ in 24F-V3 southern pine. The stringers will be spaced at 6 ft o.c.

19.3 INTERCONNECTED TRANSVERSE DECK PANELS

Structural glued laminated timber transverse deck panels can be interconnected with steel dowels to minimize relative deflections between adjacent panels and associated cracking of the wearing surface. AASHTO [1] and Ritter [2] provide a method for designing interconnected transverse decks.

However, experience has demonstrated that fabrication of the precisely aligned holes required for this deck system is difficult and costly. Assembly is also difficult and often requires field modification, which reduces the effectiveness of the load transfer mechanism. Prior to specifying this system, the glulam fabricator should be consulted to determine feasibility and discuss other options.

19.4 NON-INTERCONNECTED TRANSVERSE DECK PANELS

For non-doweled decks supporting asphalt pavement, the maximum span of deck panels between stringers is generally limited by deflection considerations. Excessive differential deflection between adjacent panels causes pavement cracking. To minimize this problem, a maximum panel deflection of 0.1 in is recommended. For bridges with timber plank wearing surfaces, greater deflections can be tolerated.

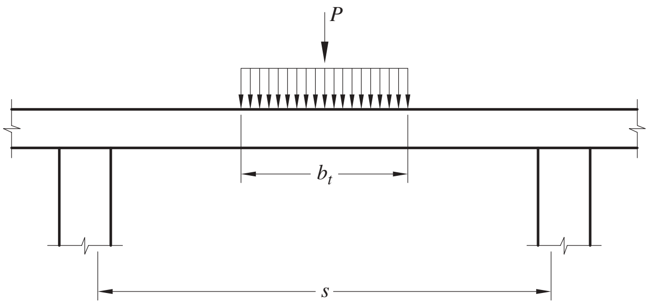
19.4.1 Load Placement

For ASD design, AASHTO provides simplified, approximate methods to facilitate hand calculations. Deck design is accomplished by placing the design wheel load on the deck, such that the maximum stresses and deflections are produced. The live load bending moment and deck deflection are obtained by placing the wheel load at mid-span, and the shear load is obtained by placing the wheel load at a distance equal to the deck thickness away from the support, as shown in Figure 19.4.1-1. For decks that are continuous across more than two spans, the moment and deflection in the interior spans are taken as 80% of the calculated values for a simply-supported deck to account for the continuity (Ref. [1], Section 3.25.4).

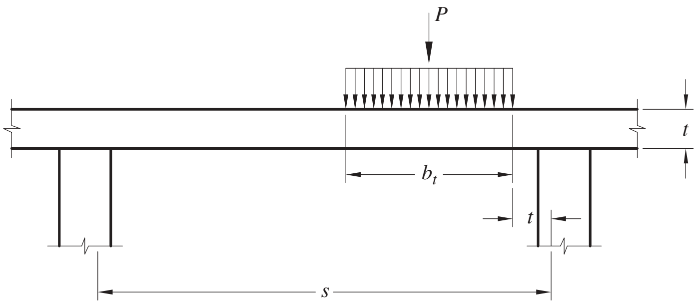
This approximation is not adequate for design of the cantilever spans at the edges of the bridge, where the bending moment will typically exceed 80% of the simple-span moment, except for very short cantilevers. AASHTO doesn't give any guidance for placement of the wheel load on the cantilevered span or modeling of the bridge deck to estimate cantilever stresses and deflections. Based on engineering judgment, the wheel load should be placed at a distance of 1 ft from the curb (Figure 19.4.1-1) for the evaluation of the deck overhang.

19.4.2 Wheel Load Distribution

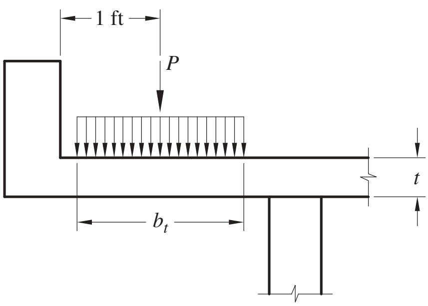
The design wheel load is distributed for 20 inches across the width of the bridge (Equation 19.4.2-1) and a distance of 15 inches plus the deck thickness along



Wheel load distribution for moment



Wheel load distribution for shear



Wheel load placement on overhang

Figure 19.4.1-1 Wheel load placement for deck design.

the length of the bridge (Equation 19.4.2-2), but not to exceed the width of the decking panel (Ref. [1], Section 3.25.1.1).

$$b_t = 20\,\text{in} \tag{19.4.2-1}$$

$$\ell_t = (15\,\text{in} + t) \le b_{panel} \tag{19.4.2-2}$$

19.4.3 Effective Span

The effective span, s, for the decking is the lesser of (1) the clear span, s_c, plus half the stringer width or (2) the clear span plus the thickness of the deck.

$$s_c = s_{stringer} - b_{stringer} \tag{19.4.3-1}$$

$$s = s_c + \frac{b_{stringer}}{2} \leq s_c + t \tag{19.4.3-2}$$

19.4.4 Live Load Deflection

For design purposes, the thickness of the deck is typically determined by live load deflection criteria and then evaluated for flexure and shear strength. Typical spans for non-interconnected deck panels supporting asphalt are approximately 10 times the thickness of the panel or less. With span-to-depth ratios this low, shear deformation becomes significant and should be accounted for in design. For a span-to-depth ratio of 9.5:1, the apparent modulus of elasticity of the beams will be only 85% of the published modulus of elasticity, resulting in actual deflections approximately 17% greater than the deflections calculated using the published values for E_y. The deflection, including shear deformation, can be estimated for the design of transverse decks by multiplying the tabulated E_y value by 0.85 prior to applying Equations 19.4.4-1, 19.4.4-2, and 19.4.4-4.

For a simply-supported deck panel, the design live load deflection is calculated using Equation 19.4.4-1.

$$\Delta_{LL,simple} = \frac{P\left(8s^3 - 4b_t s^2 + b_t^3\right)}{384EI} = \frac{P\left(8s^3 - 4b_t s^2 + b_t^3\right)}{32E\ell_t t^3} \tag{19.4.4-1}$$

For decking that is continuous across more than two spans, the design deflection is estimated as 80% of the simply supported deflection (Ref [1], Section 3.25.4) (Equation 19.4.4-2).

$$\Delta_{LL,continuous} = 0.8\Delta_{LL,simple} \tag{19.4.4-2}$$

AASHTO [1] does not provide guidance for calculating the deflection of the overhang. Unfortunately, there is no simple formula to calculate deflections for the overhang. One approach is to limit the live load bending moment on the overhang to be less than the live load bending moment in the interior spans. Limiting the overhang distance to 20% of the effective interior span plus 10 in (Equation 19.4.4-3) will satisfy this criterion.

$$O \leq 0.2s + 10\,\text{in} \tag{19.4.4-3}$$

When the interior spans are not optimized for deflection, the overhang distance can exceed this distance. In this case, the live load cantilever moment should be limited to the bending moment that would cause 0.1 in of deflection in the interior spans (Equation 19.4.4-4).

$$M_{LL,overhang} \leq \frac{384\,(0.1\,\text{in})\,EI}{0.8\left(32s^2 + \dfrac{8b_t^3}{2s - b_t}\right)} \tag{19.4.4-4}$$

A better approach is to use structural analysis software to calculate the deflection of the overhang. For the analysis, the axle of the design truck is placed on the bridge deck with the centerline of the outside wheel placed at one foot from the face of the curb.

19.4.5 Flexure Analysis

The live load bending moment in the interior spans of the decking is determined by placing the wheel of the design truck as indicated in Figure 19.4.1-1. For a simple-span deck, the moment is then calculated using Equation 19.4.5-1.

$$M_{LL,simple} = P\left(\frac{\ell}{4} - \frac{b_t}{8}\right) \tag{19.4.5-1}$$

For a simple-span deck, the dead load bending moment is calculated using Equations 19.4.5-2 and 19.4.5-3.

$$\omega_{DL} = \omega_{asphalt} + \omega_{deck} \tag{19.4.5-2}$$

$$M_{DL,simple} = \frac{\omega_{DL} s^2}{8} \tag{19.4.5-3}$$

The total bending moment for simple-span decks is calculated with Equation 19.4.5-4.

$$M_{simple} = M_{LL,simple} + M_{DL,simple} \tag{19.4.5-4}$$

For continuous decks, the total bending moment on the interior spans is calculated using Equation 19.4.5-5 (Ref. [1], Section 3.25.4).

$$M_{continuous} = 0.80\,M_{simple} \tag{19.4.5-5}$$

The negative moment in the cantilever span should be evaluated with the design vehicle placed with the centerline of the outside wheel located one foot from the face of the guardrail (Figure 19.4.1-1). The live load bending moment on the overhang is calculated with Equation 19.4.5-6. The dead load bending moment on the overhang is calculated with Equation 19.4.5-7. The total load bending moment is calculated with Equation 19.4.5-8.

$$M_{LL,overhang} = \frac{P\,(O - 2\,\text{in})^2}{40\,\text{in}} \quad \text{for } O \geq 2\,\text{in} \tag{19.4.5-6}$$

$$M_{DL,overhang} = P_{guardrail} x_{guardrail} + \frac{\omega_{DW} x_{DW}^2}{2} + \frac{\omega_{deck} x_{deck}^2}{2} \tag{19.4.5-7}$$

$$M_{overhang} = M_{LL,overhang} + M_{DL,overhang} \tag{19.4.5-8}$$

The bending stress on the deck is calculated with the effective width of the panel taken as ℓ_t (Equation 19.4.5-9). The stress on the interior spans is calculated with Equation 19.4.5-10. For the overhang, the stress is calculated using Equation 19.4.5-11.

$$S_y = \frac{\ell_t t^2}{6} \tag{19.4.5-9}$$

$$f_{b,\mathrm{int}} = \frac{M_{continuous}}{S_y} \qquad (19.4.5\text{-}10)$$

$$f_{b,overhang} = \frac{M_{overhang}}{S_y} \qquad (19.4.5\text{-}11)$$

19.4.6 Shear Analysis

The live load shear in interior spans is determined by placing the wheel of the design truck as indicated in Figure 19.4.1-1. The live load shear is then determined using Equation 19.4.6-1.

$$V_{LL,\mathrm{int}} = \frac{P\left(2s - 2t - b_t\right)}{2s} \qquad (19.4.6\text{-}1)$$

The dead load shear force on the interior spans is calculated at a distance, t, from the support using Equation 19.4.6-2.

$$V_{DL,\mathrm{int}} = \omega_{DL}\left(\frac{s}{2} - t\right) \qquad (19.4.6\text{-}2)$$

The total shear force in the interior spans is calculated with Equation 19.4.6-3.

$$V_{\mathrm{int}} = V_{LL,\mathrm{int}} + V_{DL,\mathrm{int}} \qquad (19.4.6\text{-}3)$$

The shear force in the cantilever span should be evaluated with the design vehicle placed with the centerline of the outside wheel located one foot from the face of the guardrail (Figure 19.4.1-1). The live load shear force on the overhang is calculated with Equation 19.4.6-4. The dead load shear force on the overhang is calculated with Equation 19.4.6-5. The total load shear force is calculated with Equation 19.4.6-6.

$$V_{LL,overhang} = \frac{P\,(O - 2\text{ in.})}{20\text{ in.}} \qquad (19.4.6\text{-}4)$$

$$V_{DL,overhang} = P_{guardrail} + \omega_{DW}\left(x_{DW} - t\right) + \omega_{deck}\left(x_{deck} - t\right) \qquad (19.4.6\text{-}5)$$

$$V_{overhang} = V_{LL,overhang} + V_{DL,overhang} \qquad (19.4.6\text{-}6)$$

The shear stress in the interior spans is calculated using Equation 19.4.6.7, and the shear stress in the overhang is calculated using Equation 19.4.6-8.

$$f_{v,\mathrm{int}} = \frac{3V_{\mathrm{int}}}{2l_t t} \qquad (19.4.6\text{-}7)$$

$$f_{v,overhang} = \frac{3V_{overhang}}{2l_t t} \qquad (19.4.6\text{-}8)$$

EXAMPLE 19.4-1 NON-INTERCONNECTED DECK

Given: A $6\frac{3}{4}$ in. thick glulam deck is proposed for a two-lane timber bridge with a span of 50 ft-0 in., roadway width of 34 ft-0 in., and 3 in-thick asphalt wearing surface. The bridge will be subject to AASHTO HS 20-44 loading. The deck will be supported by a total of six stringers, $10\frac{3}{4}$ in. wide each, spaced at 72 in. on center with overhangs of 24 in. on each end. Overload is not required to be analyzed for H 20 loading (Ref. [1], Section 3.5.1).

Wanted: Evaluate the proposed deck using Combination 1 Douglas fir glulam panels.

Solution:

Design values:

$$F_{by} = 1450\,\text{psi}$$
$$F_{vy} = 230\,\text{psi}\,(0.72) = 166\,\text{psi}$$
$$E = 1{,}500{,}000\,\text{psi}$$

Adjustment factors:

$$C_D = 1.15$$
$$C_M = 0.80 \text{ (for bending)}$$
$$C_M = 0.875 \text{ (for shear)}$$
$$C_M = 0.833 \text{ (for modulus of elasticity)}$$
$$C_{fu} = \left(\frac{12\,\text{in}}{6.75\,\text{in}}\right)^{\frac{1}{9}} = 1.07$$

Adjusted design values:

$$F'_{by} = F_{by} C_D C_{fu} C_M = (1450\,\text{psi})\,(1.15)\,(1.07)\,(0.8) = 1430\,\text{psi}$$
$$F'_{vy} = F_{vy} C_D C_M = (166\,\text{psi})\,(1.15)\,(0.875) = 167\,\text{psi}$$
$$E'_y = E' = EC_M = (1{,}500{,}000\,\text{psi})\,(0.833) = 1{,}250{,}000\,\text{psi}$$

Design wheel load (HS 20):

$$P = 12{,}000\,\text{lb}$$

Wheel load distribution:

$$b_t = 20\,\text{in}$$
$$l_t = (15\,\text{in} + t) \le b_{panel}$$
$$l_t = (15\,\text{in} + 6.75\,\text{in}) = 21.75\,\text{in}$$

Deck clear span:

$$s_c = s_{stringer} - b_{stringer} = 72\,\text{in} - 10.75\,\text{in} = 61.25\,\text{in}$$

Effective deck span:

$$s = s_c + \frac{b_{stringer}}{2} \le s_c + t$$
$$s = 61.25\,\text{in} + \frac{10.75\,\text{in}}{2} \le 61.25\,\text{in} + 6.75\,\text{in}$$

$$s = 66.63\text{ in} \le 68\text{ in}$$

$$s = 66.63\text{ in}$$

Live load deflection, simple-span, including shear deformation ($E = 0.85E'_y$):

$$\Delta_{LL,simple} = \frac{P\left(8s^3 - 4b_t s^2 + b_t^3\right)}{32E\, l_t t^3}$$

$$\Delta_{LL,simple} = \frac{(12{,}000\text{ lb})\left(8\,(66.63\text{ in})^3 - 4\,(20\text{ in})\,(66.63\text{ in})^2 + (20\text{ in})^3\right)}{32\left((0.85)\,1.25\left(10^6\right)\text{ psi}\right)(21.75\text{ in})\,(6.75\text{ in})^3}$$

$$\Delta_{LL,simple} = 0.11\text{ in}$$

Live load deflection, continuous deck, including shear deformation:

$$\Delta_{LL,continuous} = 0.8\Delta_{simple}$$

$$\Delta_{LL,continuous} = 0.8\,(0.11\text{ in}) = 0.09\text{ in} \le 0.1\text{ in} \quad \therefore\ \text{OK}$$

Live load deflection (cantilever span, including shear deformation):

$$O \overset{?}{\le} 0.2\,s + 10\text{ in}$$

$$24\text{ in} > 0.2\,(66.63\text{ in}) + 10\text{ in} = 23.33\text{ in}$$

$$\therefore\ \text{cantilever moment will be higher than interior span}$$

$$M_{LL,overhang} \le \frac{384\,(0.1\text{ in})\,EI}{\left(32s^2 + \dfrac{8b_t^3}{2s - b_t}\right)}$$

$$M_{LL,overhang} \le \frac{384\,(0.1\text{ in})\left((0.85)\,1.25\left(10^6\right)\text{ psi}\right)\left(\dfrac{(21.75\text{ in})\,(6.75\text{ in})^3}{12}\right)}{\left(32\,(66.63\text{ in})^2 + \dfrac{8\,(20\text{ in})^3}{2\,(66.63\text{ in}) - (20\text{ in})}\right)}$$

$$M_{LL,overhang} \le 159{,}400\text{ lb-in}$$

Live load bending moment, interior spans (simple-span):

$$M_{LL,simple} = P\left(\frac{s}{4} - \frac{b_t}{8}\right)$$

$$M_{LL,simple} = 12{,}000\text{ lb}\left(\frac{66.63\text{ in}}{4} - \frac{20\text{ in}}{8}\right) = 169{,}900\text{ lb-in}$$

Dead load bending moment, interior spans (simple-span):

$$\omega_{DL} = \omega_{asphalt} + \omega_{deck}$$

$$\omega_{DL} = \left[(3\,\text{in})\left(\frac{\text{ft}}{12\,\text{in}}\right)(150\,\text{pcf}) + (6.75\,\text{in})\left(\frac{\text{ft}}{12\,\text{in}}\right)(50\,\text{pcf})\right] \times (21.75\,\text{in})\left(\frac{\text{ft}}{12\,\text{in}}\right)$$

$$\omega_{DL} = 119\,\text{plf}$$

$$M_{DL,simple} = \frac{\omega_{DL} s^2}{8}$$

$$M_{DL,simple} = \frac{\left(119\frac{\text{lb}}{\text{ft}}\right)\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)(66.63\,\text{in})^2}{8}$$

$$M_{DL,simple} = 5500\,\text{lb-in}$$

Total bending moment, interior spans (simple-span):

$$M_{simple} = M_{LL,simple} + M_{DL,simple}$$

$$M_{simple} = 169{,}900\,\text{lb-in} + 5500\,\text{lb-in}$$

$$M_{simple} = 175{,}400\,\text{lb-in}$$

Total bending moment, interior spans (continuous deck):

$$M_{continuous} = 0.8 M_{simple}$$

$$M_{continuous} = 0.80(175,400\,\text{lb-in}$$

$$M_{continuous} = 140{,}300\,\text{lb-in}$$

Guardrail dead load (effective strip width of 21.75 in):

$$P_{guardrail} = \omega_{guardrail} l_t$$

$$P_{guardrail} = \left(\frac{60\,\text{lb}}{\text{ft}}\right)\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)(21.75\,\text{in})$$

$$P_{guardrail} = 109\,\text{lb}$$

Pavement dead load (effective strip width of 21.75 in):

$$\omega_{DW} = \gamma_{asphalt} t_{asphalt} l_t$$

$$\omega_{DW} = 150\,\text{pcf}\frac{(3\,\text{in})\,(21.75\,\text{in})}{144\,\text{in}^2/\text{ft}^2}$$

$$\omega_{DW} = 68.0\,\text{lb/ft} = 5.67\,\text{lb/in}$$

Deck dead load (effective strip width of 21.75 in):

$$\omega_{deck} = \gamma_{deck} t_{deck} l_t$$

$$\omega_{deck} = 50\,\text{pcf}\frac{(6.75\,\text{in})\,(21.75\,\text{in})}{144\,\text{in}^2/\text{ft}^2}$$

$$\omega_{deck} = 51.0\,\text{lb/ft} = 4.25\,\text{lb/in}$$

Dead load bending moment, cantilever span:

$$M_{DL,overhang} = P_{guardrail} x_{guardrail} + \frac{\omega_{DW} x_{DW}^2}{2} + \frac{\omega_{deck} x_{deck}^2}{2}$$

$$M_{DL,overhang} = (109\,\text{lb})\,(36\,\text{in}) + \frac{\left(5.67\,\frac{\text{lb}}{\text{in}}\right)(24\,\text{in})^2}{2} + \frac{\left(4.25\,\frac{\text{lb}}{\text{in}}\right)(36\,\text{in})^2}{2}$$

$$M_{DL,overhang} = 8310\,\text{lb-in}$$

Live load bending moment, cantilever span:

$$M_{LL,overhang} = \frac{P\,(O - 2\,\text{in})^2}{40\,\text{in}}$$

$$M_{LL,overhang} = \frac{12{,}000\,\text{lb}\,(24\,\text{in} - 2\,\text{in})^2}{40\,\text{in}}$$

$$M_{LL,overhang} = 145{,}200\,\text{lb-in} \le 159{,}400\,\text{lb-in} \quad \therefore\ \text{OK}$$

Total bending moment, cantilever span:

$$M_{overhang} = M_{LL,overhang} + M_{DL,overhang}$$

$$M_{overhang} = 145{,}200\,\text{lb-in} + 8310\,\text{lb-in}$$

$$M_{overhang} = 153{,}500\,\text{lb-in}$$

Section modulus (effective strip width of 21.75 in):

$$S_y = \frac{l_t t^2}{6} = \frac{(21.75\,\text{in})(6.75\,\text{in})^2}{6} = 165\,\text{in}^3$$

Design bending stresses:

$$f_{b,\text{int}} = \frac{M_{continuous}}{S_y}$$

$$f_{b,\text{int}} = \frac{140{,}300\,\text{in-lb}}{165\,\text{in}^3}$$

$$f_{b,\text{int}} = 850\,\text{psi} < 1440\,\text{psi} = F'_{by} \quad \therefore\ \text{OK}$$

$$f_{b,overhang} = \frac{M_{overhang}}{S_y}$$

$$f_{b,overhang} = \frac{153{,}500\,\text{in-lb}}{165\,\text{in}^3}$$

$$f_{b,overhang} = 930\,\text{psi} < 1440\,\text{psi} = F'_{by} \quad \therefore\ \text{OK}$$

Dead load shear force, interior spans:

$$V_{DL,\text{int}} = \omega_{DL}\left(\frac{s}{2} - t\right)$$

$$V_{DL,\text{int}} = 119\,\text{plf}\left(\frac{66.63\,\text{in}\left(\dfrac{1\,\text{ft}}{12\,\text{in}}\right)}{2} - 6.75\,\text{in}\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)\right)$$

$$V_{DL,\text{int}} = 263\,\text{lb}$$

Live load shear force, interior spans:

$$V_{LL,\text{int}} = \frac{P\left(2s - 2t - b_t\right)}{2s}$$

$$V_{LL,\text{int}} = \frac{(12{,}000\,\text{lb})\,[2\,(66.63\,\text{in}) - 2\,(6.75\,\text{in}) - 20\,\text{in}]}{[2\,(66.63\,\text{in})]}$$

$$V_{LL,\text{int}} = 8983\,\text{lb}$$

Total shear force, interior spans:

$$V_{\text{int}} = V_{LL,\text{int}} + V_{DL,\text{int}}$$

$$V_{\text{int}} = 8983\,\text{lb} + 263\,\text{lb}$$

$$V_{\text{int}} = 9246\,\text{lb}$$

Live load shear force, cantilever span:

$$V_{LL,overhang} = \frac{P\,(O - 2\,\text{in})}{20\,\text{in}} = \frac{(12{,}000\,\text{lb})\,(24\,\text{in} - 2\,\text{in})}{20\,\text{in}} = 13{,}200\,\text{lb}$$

Dead load shear force, cantilever span:

$$V_{DL,overhang} = P_{guardrail} + \omega_{DW}\left(x_{DW} - t\right) + \omega_{deck}\left(x_{deck} - t\right)$$

$$V_{DL,overhang} = 109\,\text{lb} + \left(5.67\frac{\text{lb}}{\text{in}}\right)(24\,\text{in} - 6.75\,\text{in}) + \left(4.25\frac{\text{lb}}{\text{in}}\right)(36\,\text{in} - 6.75\,\text{in})$$

$$V_{DL,overhang} = 330\,\text{lb}$$

Total shear force, cantilever span:

$$V_{overhang} = V_{LL,overhang} + V_{DL,overhang} = 13{,}200\,\text{lb} + 331\,\text{lb} = 13{,}530\,\text{lb}$$

Design shear stress (effective width = 21.75 in):

$$f_{v,\text{int}} = \frac{3V_{\text{int}}}{2l_t t}$$

$$f_{v,\text{int}} = \frac{3}{2}\left(\frac{9246\,\text{lb}}{21.75\,\text{in}\,(6.75\,\text{in})}\right)$$

$$f_{v,\text{int}} = 94\,\text{psi} < 167\,\text{psi} = F'_{vy} \quad \therefore\ \text{OK}$$

$$f_{v,overhang} = \frac{3V_{overhang}}{2l_t t}$$

$$f_{v,overhang} = \frac{3}{2}\left(\frac{13,530\,\text{lb}}{21.75\,\text{in}\,(6.75\,\text{in})}\right)$$

$$f_{v,overhang} = 138\,\text{psi} < 167\,\text{psi} = F'_{vy} \qquad \therefore\ \text{OK}$$

Deck Deflection with Structural Analysis Software:

The maximum live load deflection occurred in the cantilever span with the design truck placed so the centerline of the outside wheel was 1 ft from the curb. Its magnitude was 0.12 in. including shear deformation. The maximum deflection in the interior spans occurred with the design truck placed 7 ft from the curb and had a magnitude of 0.10 in. The design deflection of this deck exceeds 0.10 in in the cantilever span, so the proposed deck panel is not satisfactory.

Result: The proposed $6\frac{3}{4}$ in. thick, Combination 1 DF deck is adequate for flexure and shear in the interior spans and in the cantilever. The anticipated deflection of the deck slightly exceeds 0.10 in. in the cantilever span.

Discussion: To reduce the deflection, a stiffer deck should be considered. A Combination 3 DF glulam deck has a 27% higher modulus of elasticity than the Combination 1 DF panel evaluated. Consequently, specifying Combination 3 DF deck panels will reduce the cantilever live load deflection to 0.09 in, which is within acceptable limits.

19.5 LONGITUDINAL DECK (WITH STIFFENERS)

For bridges with relatively short spans and/or light loading, longitudinal glulam deck panels may be used as both deck and superstructure for the bridge. For truss and arch bridges, longitudinal panels may be used across transverse beams to form the deck. In either case, the design procedures are the same for simply-supported decks.

19.5.1 Load Placement

For simple-span decks with spans of less than 24 ft, the maximum moment for H or HS trucks occurs when the heaviest axle is at the center of the bridge, and the other axles are not on the bridge. For spans longer than 24 ft, the maximum moment occurs when both rear axles are placed on the bridge. Once the load placements are determined, the design shear, moment, and deflection are determined by standard engineering methods. To simplify design for simple-span bridges, AASHTO [1] has tabulated maximum moments and shears for simple-span bridges with standard H and HS loadings.

For continuous decks with multiple spans along the length of the bridge, placement of the loads can be determined using influence lines, trial and error, or computer software to identify the worst-case loadings. Deflection is evaluated with the wheel loads placed and distributed as for maximum moment. The design live load shear is obtained by placing the maximum wheel load at the lesser of $3d$ or $\frac{\ell}{4}$ from the support.

19.5.2 Transverse Stiffeners

Transverse stiffeners are required to transfer load between adjacent panel pieces and minimize differential deflections between panels. These stiffeners are prescriptively required to have a minimum stiffness of 80,000 kip-in^2 [1]. A stiffener must be placed at mid-span and additional stiffeners are required to ensure that the maximum spacing between stiffeners does not exceed 10 ft. The stiffeners are typically spaced evenly within the span. The stiffeners distribute wheel loads to adjacent panels, resulting in load sharing between panels.

19.5.3 Wheel Load Fractions

AASHTO accounts for load sharing between panels through the use of *wheel load fraction* factors, which are a function of the panel width and the number of traffic lanes on the bridge. Factors are determined separately for bending and shear. Factors for bending are determined using Equations 19.5.3-1 and 19.5.3-2 [1].

$$LF_{bending} = \frac{W_p}{3.75 + \frac{\ell}{28}} \geq \frac{W_p}{5.00} \quad \text{for two or more lanes} \qquad (19.5.3\text{-}1)$$

$$LF_{bending} = \frac{W_p}{4.25 + \frac{\ell}{28}} \geq \frac{W_p}{5.50} \quad \text{for one traffic lane} \qquad (19.5.3\text{-}2)$$

where:

W_p = width of panel, ft

ℓ = length of span for simple span bridges or length of shortest span for continuous bridges, ft

Wheel load fractions for shear in longitudinal deck panels are determined using Equation 19.5.3-3.

$$LF_{shear} = \frac{W_p}{4} \leq 1.0 \qquad (19.5.3\text{-}3)$$

EXAMPLE 19.5-1 LONGITUDINAL DECK WITH TRANSVERSE STIFFENERS

<u>**Given:**</u> A two lane bridge will span 20 ft, with a total roadway width of 32 ft. The bridge must be designed for HS 15-44 loading. The wearing surface will be 3 in. thick asphalt.

Wanted: Design a longitudinal deck bridge using 4 ft wide combination 2 DF glulam panels.

Approach: The longitudinal deck bridge will be designed for live load deflection, and then evaluated for flexure and shear.

Solution:

Adjustment factors:

$$C_D = 1.15$$

$$C_M = 0.80 \text{ (for bending)}$$

$$C_M = 0.875 \text{ (for shear)}$$

$$C_M = 0.833 \text{ (for modulus of elasticity)}$$

Design values (AITC 117 [3]):

$$E_y = E = 1.6\left(10^6 \text{ psi}\right)$$

$$F_{by} = 1800 \text{ psi (four or more laminations)}$$

$$F_{vy} = (230 \text{ psi})(0.72) = 166 \text{ psi} \quad \text{(cyclic and impact loading)}$$

Adjusted design values:

$$E'_y = E_y C_M = 1.6\left(10^6 \text{psi}\right)(0.833) = 1.33\left(10^6 \text{psi}\right)$$

$$F'_{by} = F_{by} C_D C_M C_{fu} = (1800 \text{ psi})(1.15)(0.8) C_{fu} = (1656 \text{ psi}) C_{fu}$$

$$F'_{vy} = F_{vy} C_D C_M = 166 \text{ psi} (1.15)(0.875) = 167 \text{ psi}$$

Transverse stiffener design (dry service):

Combination 2 DF will be used for the stiffener. A trial size of $6\frac{3}{4}$ in. × $4\frac{1}{2}$ in. will be selected.

$$I_x = \frac{(6.75 \text{ in})(4.5 \text{ in})^3}{12} = 51.3 \text{ in}^4$$

$$E'_x = E_x = 1,600,000 \text{ psi}$$

$$E'_x I_x = (1,600,000 \text{ psi})\left(51.3 \text{ in}^4\right)$$

$$E'_x I_x = 82,000,000 \text{ lb-in}^2 \geq 80,000,000 \text{ lb-in}^2 \quad \therefore \text{OK}$$

Design wheel load:

$$P = 12,000 \text{ lb}$$

Wheel load fraction for bending (two lanes):

$$LF_{bending} = \frac{W_p}{3.75 + \frac{\ell}{28}} \geq \frac{W_p}{5.00} \quad \text{for two or more lanes}$$

$$LF_{bending} = \frac{4}{3.75 + \frac{20}{28}} \geq \frac{4}{5.00}$$

$$LF_{bending} = 0.90 \geq 0.80$$
$$LF_{bending} = 0.90$$

Wheel load fraction for bending (Overload, one lane):

$$LF_{bending,OL} = \frac{W_p}{4.25 + \frac{L}{28}} \geq \frac{W_p}{5.50} \quad \text{for one traffic lane}$$
$$LF_{bending,OL} = \frac{4}{4.25 + \frac{20}{28}} \geq \frac{4}{5.50}$$
$$LF_{bending,OL} = 0.81 \geq 0.73$$
$$LF_{bending,OL} = 0.81$$

Deflection equation:
Deflection is evaluated with the wheel loads placed and distributed as for maximum moment. The deflection limit criterion is taken to be $\frac{\ell}{425}$. In this example, maximum moment occurs when the heaviest axle is placed at mid-span.

$$\Delta_{LL} = \frac{\left(LF_{bending}\right) P\,\ell^3}{48\,E_y' I} = \frac{\left(LF_{bending}\right) P\,\ell^3}{4\,E_y' b t^3}$$

Required deck thickness based on deflection:

$$t = \sqrt[3]{\frac{\left(LF_{bending}\right) P\,\ell^3}{4\,E_y' b\,\Delta_{LL}}}$$

$$t = \sqrt[3]{\frac{(0.9)\,(12,000\,\text{lb})\,(240\ \text{in})^3}{4\left(1.33\left(10^6\,\text{psi}\right)\right)(48\,\text{in})\left(\frac{240\,\text{in}}{425}\right)}} = 10.1\,\text{in} \qquad \therefore \text{Choose } t = 10.75\,\text{in}$$

Dead loads from asphalt and deck:

$$\omega_{asphalt} = (\gamma t)_{asphalt}\,W_p = \left[(150\ \text{pcf})\,(3\,\text{in})\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)\right](4\,\text{ft}) = 150\,\text{lb/ft}$$
$$\omega_{deck} = (\gamma t)_{deck}\,W_p = \left[(50\,\text{pcf})\,(10.75\,\text{in})\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)\right](4\,\text{ft}) = 179\,\text{lb/ft}$$
$$\omega_{DL} = \omega_{deck} + \omega_{asphalt} = 179\,\text{lb/ft} + 150\,\text{lb/ft} = 329\,\text{lb/ft}$$

Dead load from stiffener (point load at mid-span):

$$P_{stiffener} = (\gamma bd)_{stiffener}\,W_p$$
$$P_{stiffener} = \left[(50\,\text{pcf})\,(6.75\,\text{in})\,(4.5\,\text{in})\left(\frac{\text{ft}^2}{144\,\text{in}^2}\right)\right](4\,\text{ft}) = 42.2\,\text{lb}$$

Dead load bending moment:

$$M_{DL} = \frac{\omega_{DL} \ell^2}{8} + \frac{P_{stiffener} \ell}{4}$$

$$M_{DL} = \frac{329\,\text{plf}\,(20\text{ft})^2}{8} + \frac{(42.2\,\text{lb})\,(20\,\text{ft})}{4}$$

$$M_{DL} = 16,770\,\text{lb-ft}$$

Live load bending moment:

$$M_{LL} = \frac{\left(\text{LF}_{bending}\right) P \ell}{4}$$

$$M_{LL} = \frac{(0.90)\,(12,000\,\text{lb})\,(20\,\text{ft})}{4}$$

$$M_{LL} = 54,000\,\text{lb-ft}$$

Total load bending moment:

$$M_{TL} = M_{DL} + M_{LL}$$

$$M_{TL} = 16,770\,\text{lb-ft} + 54,000\,\text{lb-ft}$$

$$M_{TL} = 70,770\,\text{lb-ft} = 849,000\,\text{lb-in}$$

Adjusted bending design value (based on 10.75 in depth):

$$C_{fu} = \left(\frac{12\,\text{in}}{\text{d}}\right)^{\frac{1}{9}} = \left(\frac{12\,\text{in}}{10.75\,\text{in}}\right)^{\frac{1}{9}} = 1.01$$

$$F'_{by} = (1656\,\text{psi})\,C_{fu}$$

$$F'_{by} = (1656\,\text{psi})\,(1.01)$$

$$F'_{by} = 1673\,\text{psi}$$

Design bending stress:

$$f_b = \frac{M_{TL}}{S} = \frac{6M_{TL}}{bt^2}$$

$$f_b = \frac{6\,(849,000\,\text{lb-in})}{(48\,\text{in})\,(10.75\,\text{in})^2}$$

$$f_b = 918\,\text{psi} \le F'_{by} = 1673\,\text{psi} \quad \therefore \text{OK}$$

Overload bending moment (using load fractions from above):

$$M_{OL} = \frac{M_{DL} + 2M_{LL}}{1.5}$$

$$M_{OL} = \frac{16,770\,\text{lb-ft} + 2\,(54,000\,\text{lb-ft})\left(\dfrac{0.81}{0.90}\right)}{1.5}$$

$$M_{OL} = 76,000\,\text{lb-ft} = 912,000\,\text{lb-in}$$

Overload bending stress:

$$f_{b,OL} = \frac{M_{OL}}{S} = \frac{6M_{OL}}{bt^2}$$

$$f_{b,OL} = \frac{6\,(912{,}000\,\text{lb-in})}{(48\,\text{in})\,(10.75\,\text{in})^2}$$

$$f_{b,OL} = 986\,\text{psi} \le F'_{by} = 1673\,\text{psi} \quad \therefore \text{OK}$$

Wheel load fraction for shear:

$$LF_{shear} = \frac{W_p}{4\,\text{ft}} \le 1.0$$

$$LF_{shear} = \frac{4\,\text{ft}}{4\,\text{ft}} \le 1.0$$

$$LF_{shear} = 1.0.$$

Dead load shear force (at distance *d* from end):

$$V_{DL} = \omega_{DL}\left(\frac{\ell}{2} - d\right) + \frac{P_{stiffener}}{2} = 329\,\text{plf}\left(\frac{20\,\text{ft}}{2} - \frac{10.75\,\text{in}}{12\,\text{in/ft}}\right) + \frac{42.2\,\text{lb}}{2} = 3020\,\text{lb}$$

Live load shear force:

The design live load shear is obtained by placing the maximum wheel load at the lesser of $3d$ or $\frac{\ell}{4}$ from the support. In this example, $3d = 3(10.75\,\text{in}/12\,\text{in/ft}) = 2.69$ ft., and $\frac{\ell}{4} = 5$ ft, the lesser of which is 2.69 ft. In the case of the HS loading, two equal loads of 12,000 lb may act as close as 14 ft apart (Ref [1], Figure 3.7.7A) as shown in Figure 19.5-1. At a distance of 2.69 ft from one end, with one 12,000 lb force acting at that location, and another 12,000 lb force acting 14 ft from the first, the design live load shear force is $V = 12{,}400$ lb..

$$V_{LL} = \left(LF_{shear}\right) V_{vehicle} = (1.0)\,12{,}400\,\text{lb} = 12{,}400\,\text{lb}$$

Total shear force:

$$V_{TL} = V_{DL} + V_{LL} = 3{,}020\,\text{lb} + 12{,}400\,\text{lb} = 15{,}420\,\text{lb}$$

Design shear stress:

$$f_v = \frac{3}{2}\left(\frac{V}{bt}\right) = \frac{3}{2}\left(\frac{15{,}420\,\text{lb}}{48\,\text{in}\,(10.75\,\text{in})}\right) = 45\,\text{psi} \le F'_{vy} = 166\,\text{psi} \quad \therefore \text{OK}$$

Overload shear force:

$$V_{OL} = \frac{V_{DL} + 2V_{LL}}{1.5} = \frac{3{,}020\,\text{lb} + 2\,(12{,}400\,\text{lb})}{1.5} = 18{,}550\,\text{lb}$$

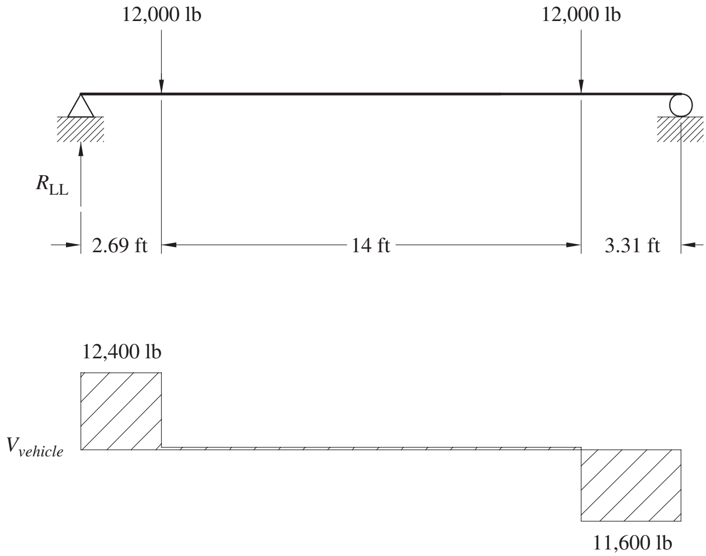


Figure 19.5-1 Design wheel loads for shear—Example 19.5-1.

Overload shear stress:

$$f_{v,OL} = \frac{3}{2}\left(\frac{V}{bt}\right) = \frac{3}{2}\left(\frac{18,550\,\text{lb}}{48\,\text{in}\,(10.75\,\text{in})}\right) = 54\,\text{psi} \leq F'_{vy} = 166\,\text{psi} \quad \therefore \text{OK}$$

Answer: Eight 48 in × $10\frac{3}{4}$ in Douglas fir Combination 2 panels will be used. One $6\frac{3}{4}$ in × 4.5 in DF Combination 2 stiffener will be used at mid-span.

19.6 STATIC DESIGN OF GUARDRAIL SYSTEM

A typical guardrail system for timber bridges consists of a curb and a horizontal rail supported by vertical posts, which are attached to the deck (Figure 17.6.2-1). AASHTO specifies construction requirements, including a smooth, continuous face and a minimum guardrail height of 27 in above the road surface.

Where permitted, timber guardrail systems may be designed using static force analysis. For this analysis, AASHTO [1] specifies a load duration factor of $C_D = 1.65$ for timber components. Guardrails less than 33 in high must be designed to resist an outward load of $P = 10$ kip applied anywhere along its length, simultaneously with a longitudinal load of $\frac{P}{2} = 5$ kip distributed to a maximum of four posts.

19.6.1 Rail Design

If multiple horizontal rails are used, the load is permitted to be distributed between rails. The horizontal rails must be designed for shear, flexure, and bearing. For flexure, the design load is assumed to be placed mid-span between posts. For continuous rails, AASHTO recommends a design moment of $M = \frac{P}{6}$, where L is the center-to-center post spacing. For shear and bearing, the design load is placed at the post, resulting in reaction force of $P = 10\text{k}$. The design load is assumed to act through the centroid of the rail.

19.6.2 Post Design

The posts must be designed to resist a load of $P = 10{,}000\,\text{lb}$ applied outward from the rail acting simultaneously with a longitudinal load of $0.5P = 5{,}000\,\text{lb}$ distributed over four posts ($\frac{P}{8}$ per post). This produces biaxial bending stresses in the posts. The post must also be designed for shear and bearing.

19.6.3 Rail-to-Post Connection Design

The rails are typically attached to the posts using bolts (Figure 19.6.3-1). The connection must be designed to resist a design load of $\frac{P}{4} = 1250\,\text{lb}$ independently applied inward, upward, and downward.

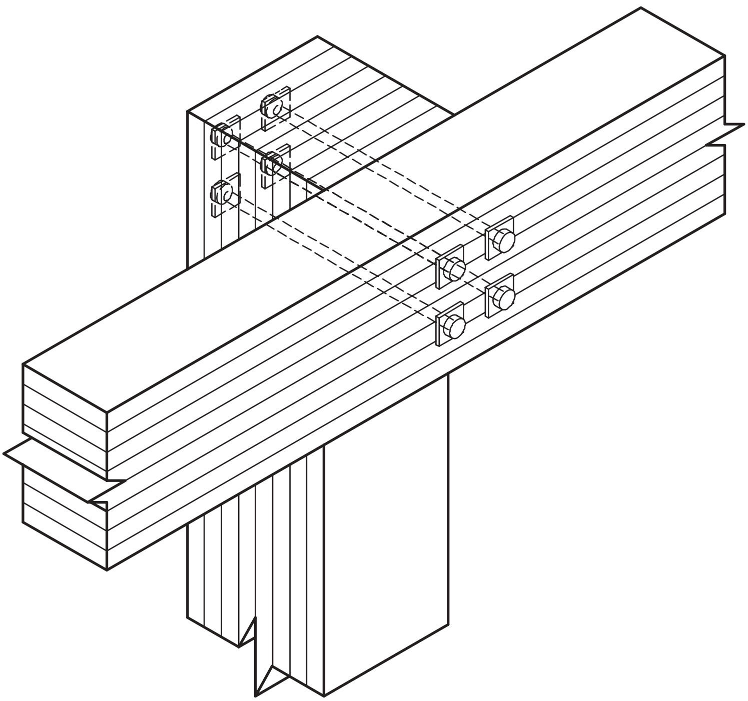


Figure 19.6.3-1 Typical rail-to-post connection.

19.6.4 Post-to-Deck Connection Design

One means of fastening the post to the deck is illustrated in Figure 19.6.4-1. Threaded rods are welded to steel plates, which are bolted to the deck (top and bottom). The rods extend alongside the post and connect to a back plate at the post and are welded to top and bottom plates on the deck. Through bolts are used to attach the top and bottom plates to the deck. When the outward load is applied, the post bears on the back plate, stressing the rods in tension. The through bolts transfer the loads from the top and bottom plates to the deck.

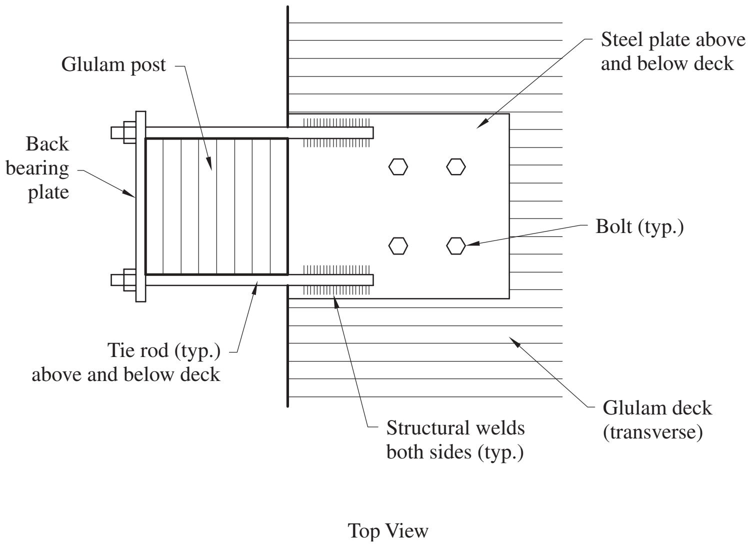


Figure 19.6.4-1 Typical post-to-deck connection.

19.6.5 Curb Design

Typically, the curb is designed only if the rail system is omitted. When both a curb and a rail system are used, the rail system will restrain the vehicles. If a curb is used without a guardrail, it must be designed to resist a lateral load of 500 lb/ft.

EXAMPLE 19.6-1 GUARDRAIL STATIC DESIGN EXAMPLE

Given: The guardrail system illustrated in Figure 19.6-1 has been proposed for a longitudinal stringer bridge with a transverse deck. The material used for the glulam rail, posts, and deck will be Combination 47 Southern Pine. Four $\frac{3}{4}$-in. A307 through bolts with 2 in. × 2 in. timber washers will be used to connect the rail to each post as illustrated in Figure 19.6-2. The posts will be attached to the deck as shown in Figure 19.6-3.

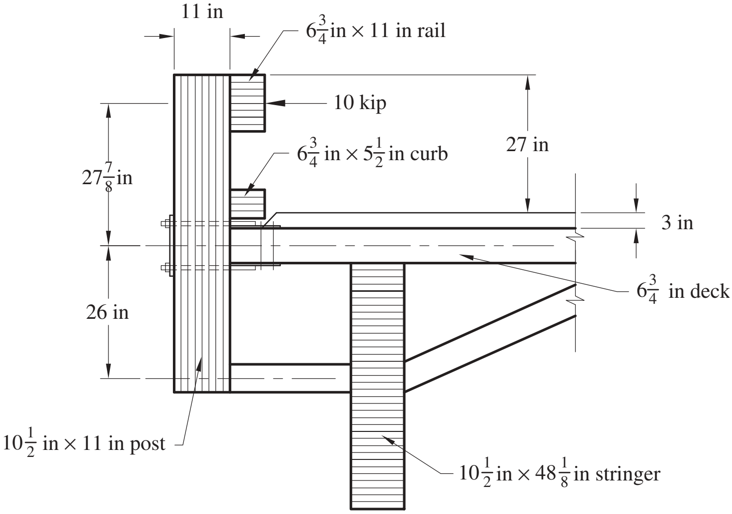


Figure 19.6-1 Guardrail system—Example 19.6-1.

Member dimensions:

$$\text{Rail: } 6\tfrac{3}{4}\text{ in} \times 11\text{ in}$$

$$\text{Post: } 10\tfrac{1}{2}\text{ in} \times 11\text{ in } @8\text{ ft o.c.}$$

$$\text{Curb: } 6\tfrac{3}{4}\text{ in} \times 5.5\text{ in}$$

Reference design values (AITC 117 [3]):

$$E = 1{,}400{,}000\text{ psi}$$

$$F_{bx} = 1400\text{ psi}$$

$$F_{by} = 1750\text{ psi (4 or more lams)}$$

$$F_{vx} = (300\text{ psi})(0.72) = 216\text{ psi (for impact and cyclic loads)}$$

$$F_{vy} = (260\text{ psi})(0.72) = 187\text{ psi (for impact and cyclic loads)}$$

$$F_c = 1900\text{ psi(4 or more lams)}$$

$$F_{c\perp} = 650\text{ psi}$$

$$F_t = 1900\text{ psi}$$

Wet-service factors:

$$C_M = 0.833 \quad \text{(modulus of elasticity)}$$

$$C_M = 0.8 \quad \text{(bending and tension)}$$

$$C_M = 0.875 \quad \text{(shear)}$$

$$C_M = 0.73 \quad \text{(compression parallel-to-grain)}$$
$$C_M = 0.53 \quad \text{(compression perpendicular-to-grain)}$$
$$C_M = 0.70 \quad \text{(dowel-type fasteners)}$$

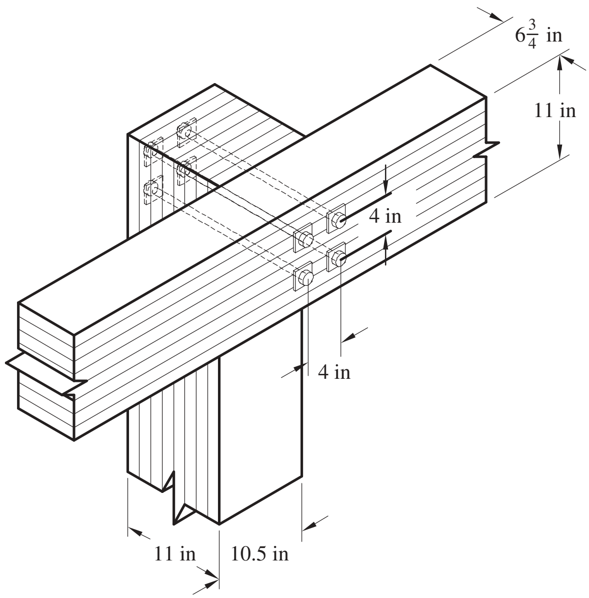


Figure 19.6-2 Rail-to-post connection—Example 19.6-1.

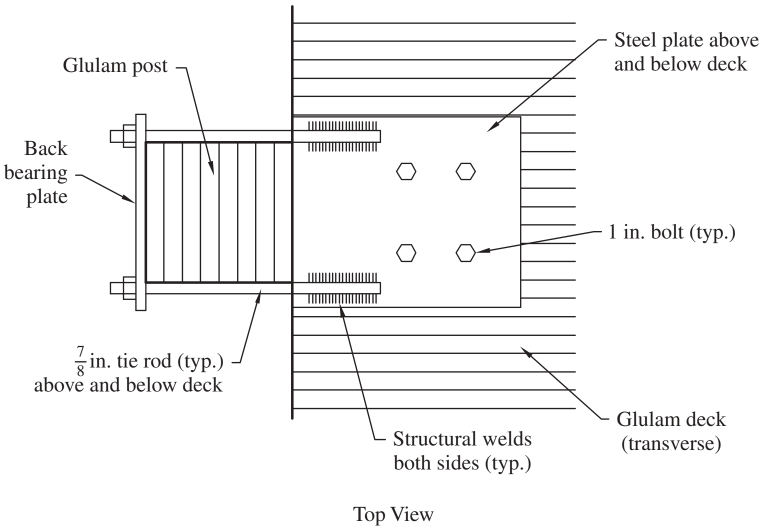


Figure 19.6-3 Post-to-deck connection—Example 19.6-1.

Wanted: Evaluate guardrail system.

Solution:

Rail bending:

$$M = \frac{P\ell}{6} = \frac{10{,}000\,\text{lb}\,(8\,\text{ft})}{6} = 13{,}333\,\text{lb-ft} = 160{,}000\,\text{lb-in}$$

$$S_y = \frac{bd^2}{6} = \frac{11\,\text{in}\,(6.75\,\text{in})^2}{6} = 83.5\,\text{in}^3$$

$$C_{fu} = \left(\frac{12\,\text{in}}{d}\right)^{\frac{1}{9}} = \left(\frac{12\,\text{in}}{6.75\,\text{in}}\right)^{\frac{1}{9}} = 1.066$$

$$F'_b = F_{by} C_D C_M C_{fu}$$

$$F'_b = 1750\,\text{psi}\,(1.65)\,(0.80)\,(1.066) = 2460\ \text{psi}$$

$$f_b = \frac{M}{S} = \frac{160{,}000\,\text{lb-in}}{83.5\,\text{in}^3} = 1916\,\text{psi} \le F'_b = 2460\,\text{psi} \quad \therefore \text{OK}$$

Rail shear:

$$F'_v = F_{vy} C_D C_M = 190\,\text{psi}\,(1.65)\,(0.875) = 274\,\text{psi}$$

$$f_v = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{10{,}000\,\text{lb}}{(11\,\text{in})\,(6.75\,\text{in})} = 202\,\text{psi} \le F'_v = 274\,\text{psi} \quad \therefore \text{OK}$$

Rail bearing:

$$F'_{c\perp} = F_{c\perp} C_M C_b$$

$$F'_{c\perp} = 650\,\text{psi}\,(0.53)\,(1.0) = 345\,\text{psi}$$

$$f_{c\perp} = \frac{P}{A} = \frac{10{,}000\,\text{lb}}{10.5\,\text{in}\,(11\,\text{in})} = 87\,\text{psi} \le F'_{c\perp} = 345\,\text{psi} \quad \therefore \text{OK}$$

Downward load on rail-to-post connection:
For the downward vertical load, the self-weight of the rail will be added to the vehicle load of 2500 lb.

$$P_{down} = \frac{P}{4} + \left(\gamma_{sw} b d \ell\right)_{rail}$$

$$P_{down} = 2500\,\text{lb} + \left(\frac{6.75\,\text{in}}{12\,\text{in/ft}}\right)\left(\frac{11\,\text{in}}{12\,\text{in/ft}}\right)(50\,\text{cf})\,(8\,\text{ft})$$

$$P_{down} = 2500\,\text{lb} + 206\,\text{lb} = 2706\,\text{lb}$$

Adjusted design value per bolt in rail-to-post connection (downward load):
The allowable load on one $\frac{3}{4}$-in. bolt, assuming all requirements for spacing (4D parallel to load, 5D perpendicular to load), end distance (4D in post, 4D in rail), and edge distance (4D in rail, 2.5D in post) are met for full design values

is determined by looking up the value for Z in the *NDS*® [4] and applying appropriate adjustment factors.

$$Z' = ZC_g C_D C_M$$
$$Z'_{down} = 1088\,\text{lb}\,(1.0)\,(1.65)\,(0.70) = 1257\,\text{lb}$$

Bolt design capacity of rail-to-post connection for downward load:

$$P'_{down} = nZ'_{down}$$
$$P'_{down} = 4\,(1257\,\text{lb})$$
$$P'_{down} = 5028\,\text{lb} \geq P_{down} = 2706\,\text{lb} \quad \therefore \text{OK}$$

Upward vertical load on rail-to-post connection:
For the upward vertical load, the self-weight of the rail will be subtracted from the vehicle load of 2500 lb.

$$P_{up} = \frac{P}{4} - \left(\gamma_{sw} bdL\right)_{rail}$$
$$P_{up} = 2500\,\text{lb} - \left(\frac{6.75\,\text{in}}{12\,\text{in/ft}}\right)\left(\frac{11\,\text{in}}{12\,\text{in/ft}}\right)(50\,\text{pcf})\,(8\,\text{ft})$$
$$P_{up} = 2500\,\text{lb} - 206\,\text{lb} = 2294\,\text{lb}$$

Geometry factor for upward vertical load:
For the upward vertical load, the end distance in the vertical post is 3.5 in The minimum end distance for reduced design value is $3.5D = 3.5\ (0.75\,\text{in}) = 2.75\,\text{in}$ and the minimum end distance for full design load is $7.0D = 5.25\,\text{in}$ Thus, the geometry factor is calculated using Equation 13.2.3.2-1.

$$C_{\Delta n} = \frac{\text{Actual end distance}}{\text{Min. end distance for full design value}} = \frac{3.5\,\text{in}}{5.25\,\text{in}} = 0.667$$

Adjusted design value per bolt in rail-to-post connection (upward load):

$$Z' = ZC_g C_D C_M C_\Delta$$
$$Z'_{up} = 1088\,\text{lb}\,(1.0)\,(1.65)\,(0.70)\,(0.67) = 842\,\text{lb}$$

Bolt design capacity of rail-to-post connection for upward load:

$$P'_{up} = nZ'_{up}$$
$$P'_{up} = 4\,(842\,\text{lb})$$
$$P'_{up} = 3368\,\text{lb} \geq P_{up} = 2294\,\text{lb} \quad \therefore \text{OK}$$

Row tear-out capacity of post for upward load:

$$T_{rt} = n_i F'_v t s_{critical} n_r$$

$$T_{rt} = \left(2\frac{\text{Bolts}}{\text{Row}}\right)(274\,\text{psi})\,(11\,\text{in})\,(3.5\,\text{in})\,(2\,\text{rows})$$

$$T_{rt} = 42{,}200\,\text{lb} \geq 2294\,\text{lb} \quad \therefore \text{OK}$$

Group tear-out capacity of post for upward load:

$$F_t' = F_t C_D C_M = 1200\,\text{psi}\,(1.65)\,(0.8) = 1584\,\text{psi}$$

$$T_{gt} = \frac{T_{rt}}{2} + F_t' A_{\text{group net}}$$

$$T_{gt} = \frac{42{,}200\,\text{lb}}{2} + (1584\,\text{psi})\,[(11\,\text{in})\,(4\,\text{in} - 0.8125\,\text{in})]$$

$$T_{gt} = 76{,}600\,\text{lb} \geq 2500\,\text{lb} \quad \therefore \text{OK}$$

Bolt tensile capacity for inward load:
With regard to the inward design load of 2500 lb, each of the (4) $\frac{3}{4}$-in bolts has a tensile capacity of 6200 lb, which may be increased by 33% for transient loading [5]. Thus, the tensile capacity of the bolts is more than sufficient; however, adequate bearing area must be provided by the washers.

Bearing area (diameter of hole in washer is $\frac{1}{16}$ in larger than bolt diameter):

$$A = (4)\left[(2\,\text{in})\,(2\,\text{in}) - \frac{\pi\,(0.8125\,\text{in})^2}{4}\right] = 13.9\,\text{in}^2$$

Bearing area factor:

$$C_b = \frac{(\ell_b + 0.375\,\text{in})}{\ell_b} = \frac{(2\,\text{in} + 0.375\,\text{in})}{2\,\text{in}} = 1.19$$

Adjusted bearing design value:

$$F_{c\perp}' = F_{c\perp} C_M C_b = 650\,\text{psi}\,(0.53)\,(1.19) = 410\,\text{psi}$$

Design compression stress:

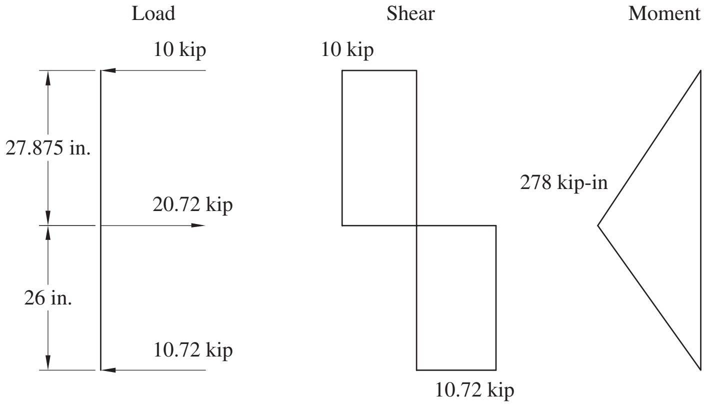
$$f_{c\perp} = \frac{P}{A} = \frac{2500\,\text{lb}}{13.9\,\text{in}^2} = 180\,\text{psi} \leq F_{c\perp}' = 410\,\text{psi} \quad \therefore \text{OK}$$

Post loading:
Figure 19.6-4a shows the outward acting load and reaction and resulting shear and moment diagrams for the post due to the outward load at the mid-depth of the rail. Figure 19.6-4b shows the longitudinal load on a single post and the resulting reactions, shear, and moment.

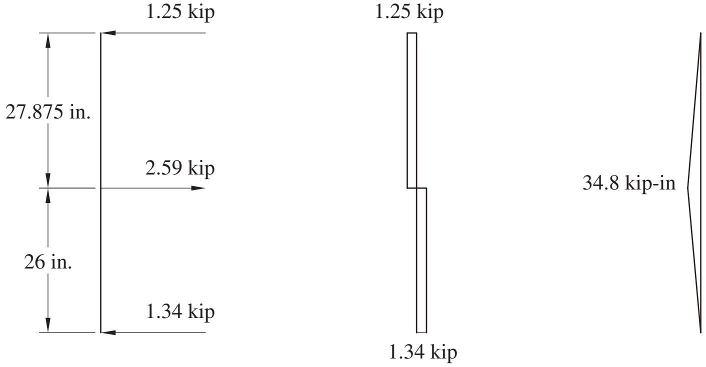
Shear and bending moment due to the outward force (Figure 19.6-4a):

$$V_x = 10{,}720\,\text{lb}$$

$$M_x = 278{,}800\,\text{lb-in}$$



a. Load, reactions, shear, and moment due to outward vehicle load on post at mid-depth of rail



b. Load, reactions, shear, and moment due to longitudinal load on post at mid-depth of rail

Figure 19.6-4 Shear and moment diagrams for post—Example 19.6-1.

Shear and bending moment due to the longitudinal force (Figure 19.6-4b):

$$V_y = 1250\,\text{lb}$$
$$M_y = 34{,}840\,\text{lb-in}$$

Section properties of post:

$$I_x = \frac{bd^3}{12} = \frac{(10.5\text{in})\,(11\text{in})^3}{12} = 1271\,\text{in}^4$$

$$S_x = \frac{I_x}{c_x} = \frac{1271\,\text{in}^4}{11\,\text{in}/\,2} = 231\,\text{in}^3$$

$$A = bd = (10.5\,\text{in})\,(11\,\text{in}) = 115.5\,\text{in}^2$$

$$I_y = \frac{db^3}{12} = \frac{(11\,\text{in})\,(10.5\,\text{in})^3}{12} = 1061\,\text{in}^4$$

$$S_y = \frac{I_y}{c_y} = \frac{1061\,\text{in}^4}{10.5\,\frac{\text{in}}{2}} = 202\,\text{in}^3$$

Design bending stresses in post:

$$f_{b1} = f_{bx} = \frac{M_x}{S_x} = \frac{278{,}800\,\text{lb-in}}{231\,\text{in}^3} = 1210\,\text{psi}$$

$$f_{b2} = f_{by} = \frac{M_y}{S_y} = \frac{34{,}840\,\text{lb-in}}{202\,\text{in}^3} = 172\,\text{psi}$$

Volume factor:

$$C_V = \left(\frac{5.125\,\text{in}}{10.5\,\text{in}}\right)^{\frac{1}{20}}\left(\frac{12\,\text{in}}{11\,\text{in}}\right)^{\frac{1}{20}}\left(\frac{21\,\text{ft}}{\frac{27}{12}\,\text{ft}}\right)^{\frac{1}{20}} \leq 1.0 = 1.0$$

Beam stability factor:

$$\ell_e = 1.87\ell_u = 1.87\,(27\,\text{in}) = 50.5\,\text{in}$$

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} = \sqrt{\frac{50.5\,\text{in}\,(11\,\text{in})}{(10.5\,\text{in})^2}} = 2.24$$

$$E'_{\text{min}} = E_{\text{min}}\left(C_M\right) = 740{,}000\,\text{psi}\,(0.833) = 620{,}000\,\text{psi}$$

$$F_{bE} = \frac{1.2E'_{\text{min}}}{R_B^2} = \frac{1.2\,(620{,}000\,\text{psi})}{(2.24)^2} = 150{,}000\,\text{psi}$$

$$F^*_{bx} = F_{bx}C_DC_M = 1400\,\text{psi}\,(1.65)\,(0.8) = 2888\,\text{psi}$$

$$C_L = \frac{1+\left(\frac{150{,}000\,\text{psi}}{2888\,\text{psi}}\right)}{1.9} - \sqrt{\left[\frac{1+\left(\frac{150{,}000\,\text{psi}}{2888\,\text{psi}}\right)}{1.9}\right]^2 - \frac{\frac{150{,}000\,\text{psi}}{2888\,\text{psi}}}{0.95}}$$

$$C_L = 0.999$$

Flat use factor:

$$C_{fu} = \left(\frac{12\,\text{in}}{10.5\,\text{in}}\right)^{\frac{1}{9}} = 1.01$$

Adjusted bending stresses:

$$F'_{b1} = F'_{bx} = F_{bx}C_DC_M\left(C_V \text{ or } C_L\right) = 1400\,\text{psi}\,(1.65)\,(0.80)\,(0.999) = 1846\,\text{psi}$$

$$F'_{b2} = F'_{by} = F_{by}C_DC_MC_{fu} = 1750\,\text{psi}\,(1.65)\,(0.80)\,(1.01) = 2333\,\text{psi}$$

Biaxial bending stress check:

$$\frac{f_{b1}}{F'_{b1}} + \frac{f_{b2}}{F'_{b2}[1-(f_{b1}/F_{bE})^2]} = \ldots$$

$$\ldots = \frac{1210\,\text{psi}}{1846\,\text{psi}} + \frac{172\,\text{psi}}{2333\,\text{psi}\left[1-\left(\frac{1210\,\text{psi}}{142{,}000\,\text{psi}}\right)^2\right]} = \ldots$$

$$\ldots = 0.66 + 0.07 = 0.73 \leq 1.0 \quad \therefore \text{OK}$$

Shear in post (outward rail force):

$$F'_v = F_{vx}C_D C_M = 215\,\text{psi}\,(1.65)\,(0.875) = 310\,\text{psi}$$

$$f_v = \frac{3V}{2A} = \frac{3\,(10{,}720\,\text{lb})}{2\left(115.5\,\text{in}^2\right)} = 139\,\text{psi} \leq F'_v = 310\,\text{psi} \quad \therefore \text{OK}$$

Loads on post bracket-to-deck connection:
The reaction at the deck due to the outward load is 20,720 lb from Figure 19.6-4a. The longitudinal reaction is 2590 lb (Figure 19.6-4b).
Combined outward and longitudinal reaction force:

$$R_{7.1^\circ} = \sqrt{(20{,}720\text{lb})^2 + (2590\,\text{lb})^2} = 20{,}880\,\text{lb}$$

Angle of load to grain:
The (4) 1-in through bolts are loaded at an angle to grain with the deck due to the combined outward and longitudinal loads.

$$\theta = \tan^{-1}\left(\frac{2590\,\text{lb}}{20{,}720\,\text{lb}}\right) = 7.1^\circ$$

Bolt design values (1 in diameter bolts):
Assuming bracket plate thickness of at least $\frac{1}{4}$ in, deck thickness of $6\frac{3}{4}$ in, specific gravity of 0.55 (for southern pine) and a group action factor, $C_g = 1.0$, the design values for the bolts may be determined from the equations of Chapter 13 or may be approximated using tabulated values for parallel- and perpendicular-to-grain loading from the *National Design Specification*® [4] and the Hankinson formula, as follows.

$$Z'_{\parallel} = Z_{\parallel} C_g C_D C_M = 5960\,\text{lb}\,(1.0)\,(1.65)\,(0.7) = 6880\,\text{lb}$$

$$Z'_{\perp} = Z_{\perp} C_g C_D C_M = 3180\,\text{lb}\,(1.0)\,(1.65)\,(0.7) = 3670\,\text{lb}$$

Hankinson formula:

$$Z'_\theta = \frac{Z'_{\parallel}\left(Z'_{\perp}\right)}{Z'_{\parallel}\sin^2\theta + Z'_{\perp}\cos^2\theta}$$

$$Z'_{7.1^\circ} = \frac{(6880\,\text{lb})\,(3670\,\text{lb})}{(6880\,\text{lb})\sin^2 7.1^\circ + (3670\,\text{lb})\cos^2 7.1^\circ} = 6790\,\text{lb}$$

Fastener capacity:

$$R' = nZ'_{7.1^\circ} = 4\,(6790\,\text{lb}) = 27{,}200\,\text{lb} \geq 20{,}880\,\text{lb} \quad \therefore \text{OK}$$

Row tear-out capacity (assume bolt spacing of 4*D* = 4 inches):

$$T_{rt} = n_i F'_v t s_{critical} n_r$$

$$T_{rt} = \left(2\frac{\text{Bolts}}{\text{Row}}\right)(274\,\text{psi})\,(6.75\,\text{in})\,(4\,\text{in})\,(2\,\text{rows})$$

$$T_{rt} = 29{,}600\,\text{lb} \geq 20{,}700\,\text{lb} \quad \therefore \text{OK}$$

Group tear-out capacity (assume row spacing of 6 inches):

$$T_{gt} = \frac{T_{rt}}{2} + F'_t A_{\text{group net}}$$

$$T_{gt} = \frac{29{,}600\,\text{lb}}{2} + (1584\,\text{psi})\,[(11\,\text{in})\,(6\,\text{in} - 1.0625\,\text{in})]$$

$$T_{gt} = 100{,}800\,\text{lb} \geq 20{,}700\text{lb} \quad \therefore \text{OK}$$

Answer: The proposed guardrail system and connections are satisfactory.

Discussion: The metal fasteners, plate thickness, weld size and lengths must be checked against the requirements of the American Institute of Steel Construction [5]. The bottom of the post must be braced against the adjacent stringer as illustrated in Figure 19.6-1 and the stringer further braced to the next stringer or deck. Furthermore, the post must be braced longitudinally to prevent rotation under longitudinal load. In the case of transverse deck, the combined outward and longitudinal load is nearly parallel-to-grain for the through bolts connecting the bracket to the deck. For a longitudinal deck, the same fasteners would have considerably less capacity, because the primarily outward load would be oriented nearly perpendicular-to-grain

19.7 CONCLUSION

This chapter has presented design procedures and examples for timber bridges using allowable stress design (ASD). This method has been largely replaced by LRFD and is no longer permitted for highway projects, however, these procedures may provide a viable option for bridges with lesser design loads than those used for highways.

CHAPTER 20

FIRE SAFETY

20.1 INTRODUCTION

Fire safety is one of the primary considerations for the design and use of buildings. The primary focus of fire-safe design is to ensure that a building's occupants can exit safely in the event of a fire. A structure should also permit rescue activities without disproportionately endangering the lives of the firefighters. As such, building codes typically require increased fire resistance for large structures, for structures intended for large groups of people, and for structures with high fire hazard. The building code requirements are based on historical experience, research, and judgment.

Perhaps the most important feature of fire-safe design is the presence of adequate and accessible exits. Consequently, building codes typically include extensive requirements relating to means of egress, particularly from public buildings. It is critical that the structure does not collapse before the occupants have sufficient time to escape. It is also important to ensure that exit pathways are protected from heat and smoke. To achieve these objectives, building codes typically prescribe requirements for (1) the number and location of exits, (2) acceptable parameters for exit pathways, and (3) fire-resistive construction of structural elements and systems (walls, roofs, floors, beams and columns, etc.), and (4) fire protection features, such as fire detection systems and fire suppression systems.

Buildings are also designed to minimize or slow the spread of fire. Two concepts are used to meet this objective: separation and compartmentalization. For example, the required fire-resistivity of exterior walls may be dictated by a structure's proximity to other structures. The intent of the code provisions relating to separation distances is to prevent a fire from transferring to and from buildings, causing a conflagration. Adequate separation also improves access to the

Timber Construction Manual American Institute of Timber Construction

structure for firefighting activities. Similarly, buildings can be compartmentalized with fire-resistance-rated walls and ceilings to prevent rapid spread of fire from one space to another. If the fire is contained within a limited area, occupants have a better chance to escape.

These multiple strategies work together to minimize the loss of life in fire events. This chapter will focus on the design and construction of timber structural elements and systems, particularly those utilizing structural glued laminated timber components.

20.2 TYPES OF CONSTRUCTION

Building codes have traditionally classified buildings into types based on the expected fire performance of the structural systems used. The allowable height and area of a building are typically limited based on the type of construction and the intended occupancy of the building. The *International Building Code (2009)* [1] classifies buildings into Types I through V. Types I and II are primarily constructed of noncombustible materials. Type III requires noncombustible exterior walls, but allows any material for interior elements. Type IV construction is also known as *heavy timber construction*, in which the exterior walls are non-combustible and minimum sizes are prescribed for timber structural elements in the interior of the building. *Heavy timber construction* also requires the elimination of concealed spaces where fire could start and spread unobserved. Type V construction permits the use of any material allowed otherwise by the code. Each type of construction (except *heavy timber construction*) is further subdivided into categories A and B. With the exception of Type I construction, category A is for fire-resistance-rated construction and category B is for nonrated construction. In Type I construction, both categories A and B are fire-resistance rated, but A has more stringent requirements.

Combustibility of the building's structural materials is only one factor when considering fire safety. No structural material is immune to the effects of fire: wood burns, steel softens and buckles, concrete spalls, and so on. The duration of time that the structure will continue to function, allowing egress and rescue activities is much more important than the mode of failure. The height and area limitation in the building codes are prescribed with this in mind.

20.3 LESSONS FROM ACTUAL FIRES

It is important to note that the code provisions do not result in "fire-proof" buildings, which do not exist. A building's combustible contents pose the greatest potential fire hazard and will burn with sufficient intensity to destroy a building constructed with noncombustible framing. An example of such an event occurred at the McCormick Place exhibition hall in Chicago in 1967.

However, the presence of combustible framing members does not necessarily indicate that a structure will perform poorly in a fire. The 1959 Zion Baptist

Temple fire in Chicago provides an example of the good performance of large timber members in a fire.

20.3.1 McCormick Place Fire (Noncombustible Construction)

With a nine-acre (395,000 ft^2) footprint, the three-story McCormick Place was the largest exhibition hall in the United States. All of its structural components were of noncombustible construction. The exterior walls were built from reinforced concrete. The interior columns were made from structural steel, with fire protection applied within 20 ft from the floor. The roof trusses were 30 to 40 ft above the floor and were constructed of unprotected steel. [2]

On January 16, 1967, a large exhibit with 1200 exhibitors was scheduled in the McCormick Place exhibition hall in Chicago. At approximately 2:05 A.M., workers saw a small fire. After unsuccessfully trying to extinguish the fire, workers preparing for the day's exhibition triggered the alarm at 2:11 A.M. Firefighters arrived on site at 2:14 A.M., and sounded a second alarm at 2:16 A.M. A total of five alarms were sounded by 2:30 A.M. Witness accounts indicate that the roof collapsed within 30 to 45 minutes of the first alarm (Figure 20.3.1-1). Although an estimated 475 firefighters responded to the fire with approximately 100 pieces of equipment, the fire was not extinguished until 9:46 A.M. [2].

Figure 20.3.1-1 Noncombustible structure collapsed after 30 to 45 minutes of fire exposure at McCormick Place.

Fortunately, only about 125 people were in the building when the fire started. All but one escaped. Had the fire occurred later in the day, 12,000 to 15,000 visitors would have been at risk. The building and its contents represented a property loss of approximately $150 million (approximately $980 million in 2010 based on consumer price index). Economic losses due to the destruction of the exhibit hall were estimated to be as high as $450 million (approximately $2.9 billion in 2010 based on consumer price index) [3].

20.3.2 Zion Baptist Temple

Early in the morning on February 25, 1959, a heating boiler exploded, sending flaming debris throughout the Zion Baptist Temple in Chicago, Illinois. An alarm was called in at 5:08 A.M. The first firefighters arrived on the scene within five minutes and sounded another alarm at 5:15 A.M. Two more alarms were sounded by 5:27 A.M. A total of 23 fire companies responded to the fire. The fire was considered under control by 7:30 A.M. and completely extinguished by 10:55 A.M. [4].

Built in 1950, the structure consisted of six parabolic glulam arches supporting glulam purlins and heavy timber decking. The intensity and duration of the fire resulted in a complete loss of the structure and heavy charring on the timber members; however, the laminated timber arches and purlins did not collapse even after more than two hours of fire exposure (Figure 20.3.2-1).

Figure 20.3.2-1 Glulam framework remains standing after devastating fire at Zion Baptist Temple.

20.4 PERFORMANCE OF WOOD IN FIRE

Although wood burns, large timbers perform well in fires and are recognized for such in the building codes. *Heavy timber construction* (with exposed timbers) is permitted for buildings of larger heights and areas than for Type IIB (unrated, noncombustible construction), and is permitted for similar size buildings as those of Type IIA (one-hour fire-rated, noncombustible construction). Exposed timbers can also be designed for one-hour fire-resistance ratings where required in Types IIIA and VA fire-resistance-rated construction.

Wood will ignite and burn at temperatures above approximately 500°F. The residue created by the combustion of wood is referred to as *char*. As wood burns, char develops at a rate of approximately 1.5 inches per hour [5]. The growing char layer acts as an insulator, greatly reducing the temperature of the underlying wood surface. A typical building fire will reach temperatures of 1290°F to 1650°F, but the interface between the char and the wood will be reduced to approximately 550°F. Due to the insulating nature of the wood itself, the temperature drops to 360°F [6] at a distance of $\frac{1}{4}$ in. ahead of the char front and drops to 210°F at $\frac{1}{2}$ in. ahead of the char front. The core of the timber remains relatively cool and maintains its ability to carry loads. The capacity of the member is reduced only as the outer layer of material is lost due to the charring. Consequently, large timbers perform well in fires.

20.5 WOOD VERSUS STEEL

To illustrate how well large timbers perform alongside other materials, the National Lumber Manufacturers Association (now American Wood Council) conducted comparative fire tests between a glulam beam and a steel beam with similar capacities [7]. A $7'' \times 21''$ glulam beam and a W16×40 steel beam spanning 43 ft, 3 in. were subjected side-by-side to a standardized fire exposure under identical design loads. The glulam beam was initially stressed to 1550 psi (~65% of allowable design stress), and the steel beam was initially stressed to 12,350 psi (~51% of allowable design stress). The calculated deflection of the glulam beam under the design load was 2.3 inches. For the steel beam, the calculated initial deflection was 1.5 inches.

Upon commencing the test, the effects of the high temperatures were immediately evident on the steel beam, which deflected an additional 5.5 inches in the first 10 minutes. In contrast, the glulam beam deflection increased by only $\frac{3}{4}$ in. during the same time. After 29 minutes of fire exposure, the steel beam had deflected 36 inches beyond its initial deflection, while the timber beam had deflected only 2 inches beyond its initial deflection. The test was concluded after 30 minutes of fire exposure when the steel beam collapsed into the furnace. Flaming of the glulam beam promptly ceased when the burners were turned off. The glulam beam continued to support its load until it was removed by workmen. Figure 20.5-1 shows the condition of the beams after the conclusion of the test.

The test clearly demonstrates that, unprotected steel is not fireproof, even though it is noncombustible. It also illustrates the slow progression of char in the glulam beam (Figures 20.5-1 and 20.5-2) with a correspondingly slow loss of capacity. The results indicate that large timbers can perform successfully under fire exposure.

In addition to laboratory tests, fires in real structures illustrate the comparative differences in performance between timber and steel. Figure 20.5-3 shows a

Figure 20.5-1 After 30 minutes, glulam remained straight and supported its load while steel beam collapsed.

Figure 20.5-2 Char measured approximately $\frac{3}{4}$ in. on each of three exposed sides after 30-minute fire.

nail-laminated 12″ × 16″ timber beam after a fire in a manufacturing plant for casein adhesives in Frankfort, New York. Steel girders collapsed from the heat, but the timber beam remained intact for hours until the fire was extinguished. Figure 20.5-4 shows fire damage to the Winter Garden Pavilion in Cranston Rhode Island. The original building used unprotected steel trusses, but had been expanded using glulam trusses. After a fire destroyed the building, the glulam trusses continued to support the roof while the steel trusses collapsed, pulling the roof and walls down. In rebuilding the Pavilion, timber trusses were used exclusively [8].

Figure 20.5-3 Nail-laminated 12″ × 16″ timber beam maintained integrity while steel girders collapsed.

Figure 20.5-4 Glulam trusses (left) supported roof throughout fire, while steel trusses (right) collapsed.

These examples are provided to illustrate that "noncombustible" does not mean "fire-proof" or "safe" and that properly designed timber construction (combustible) will perform well and support its loads during fire events. However, it should not be inferred that all steel structures perform poorly in fires or that all timber structures will perform well. As discussed previously, several design tactics are used to ensure adequate performance in the event of fire. The choice of structural materials is only one aspect of fire safety.

20.6 HEAVY TIMBER CONSTRUCTION

Heavy timber construction, as defined in the building codes, provides fire resistance through the use of large timber sections and the avoidance of concealed spaces where a fire could potentially ignite and spread unnoticed. In addition to the requirements of structural design, members are required to meet the minimum sizes in Table 20.6-1. Structural requirements may indeed dictate larger member sizes.

Structural glued laminated timber is permitted to be used in *heavy timber construction*. Equivalent net finished widths and depths for glulam members corresponding to the minimum nominal widths and depths of solid sawn lumber are shown in Table 20.6-2. These minimum dimensions were selected based on comparable dimensions and equivalent cross-sectional areas. The values in

TABLE 20.6-1 Minimum Size Requirements for Heavy Timber Construction

Structural Element	Minimum Requirement
Exterior Walls	Noncombustible or FRT[a] 2-hour rated
Columns	
Supporting floors	8×8 (nominal)
Supporting roofs	6×8 (nominal)
Floor Framing	
Beams and girders	6×10 (nominal)
Arches and trusses	8×8 (nominal)
Roof Framing	
Arches springing from floor	6×8 lower half, 6×6 upper half
Framing from walls or abutments	4×6 (nominal)
Trusses	4×6 (nominal)
Other roof framing	4×6 (nominal)
Structural floor	3 in. T&G or splined decking 4 in. nail-laminated decking
Floor overlay	1 in. T&G flooring or 0.5 in. particleboard or $\frac{15}{32}$ structural panels[b]
Roofs	2 in. T&G or splined decking or $1\frac{1}{8}$ in. structural panel or 3 in. nail-laminated decking
Partitions	2 layers of 1 in. matched boards or 4 in. laminated construction (nail-laminated) or 1-hour fire-rated assembly

[a] FRT is Fire-retardant treated
[b] $\frac{15}{32}$ structural panels not permitted as overlay for 3 in. decking

TABLE 20.6-2 Equivalent Glulam Sizes to Meet Heavy Timber Construction Requirements

Minimum Nominal Solid Sawn Size		Minimum Glued-Laminated Net Size	
Width, inch	Depth, inch	Width, inch	Depth, inch
8	8	$6\frac{3}{4}$	$8\frac{1}{4}$
6	10	5	$10\frac{1}{2}$
6	8	5	$8\frac{1}{4}$
6	6	5	6
4	6	3	$6\frac{7}{8}$

the table reflect the fact that glulam dimensions do not generally match the dimensions of solid-sawn timbers. Glulam sizes must likewise be checked for structural adequacy.

The inclusion of *heavy timber construction* in the building codes is based on excellent historical performance in building fires. Although individual members do not necessarily meet the requirements for one-hour fire-rated construction, the system as a whole (including the exclusion of concealed spaces and minimum timber sizes) performs very well. Based on its excellent record, *heavy timber* roofs are permitted to be used in all construction types and occupancies that otherwise require a one-hour or less fire-resistance-rated roof (IBC Table 601, Footnote c and 603.1.18 [1]).

In all types of construction, where a horizontal separation of 20 feet or more is provided, wood columns and arches conforming to *heavy timber* sizes are permitted to be used externally (2009 IBC 602.4.7 [1]). Balconies and similar projections are also permitted to be *heavy timber construction* in lieu of fire-resistance-rated construction (2009 IBC 1406.3 [1]).

Heavy timber construction is exempted from flame-spread requirements for interior finishes (2009 IBC 803.3 [1]). This allows for the exposed timber to be used without application of additional finish materials. However, finish materials are permitted to be applied directly to the heavy timber members or to furring strips attached to the members, provided that the space between furring strips is adequately fireblocked (2009 IBC 803.11.3 [1]).

20.7 FIRE-RESISTANCE-RATED CONSTRUCTION

Fire-resistance-rated construction achieves its fire resistance by requiring structural components and assemblies to meet defined fire performance standards. Rather than prescribing minimum sizes and eliminating concealed spaces, structural elements are assigned a specific fire-resistance rating based on approved tests or analysis procedures based on tests. IBC 2009 [1] defines *fire-resistance rating* as “the period of time a building element, component or assembly maintains the ability to confine a fire, continues to perform a given structural function, or both, as determined by the tests, or the methods based on tests, prescribed in Section 703.”

For a structural beam or column, a fire-resistance rating of one hour means that the member is expected to support its loads for at least an hour during a fire event. For a non–load-bearing partition wall, a fire-resistance rating of one hour means that a fire should not breach the wall for at least one hour. One-hour-rated

TABLE 20.7.2-1 Assigned Resistance Times for Components

Component Description	Assigned Time (Minutes)
Wallboard Membrane[a,b,c,d]	
$\frac{3}{8}$ Wood Structural Panel bonded with exterior glue	5
$\frac{15}{32}$-inch wood structural panel bonded with exterior glue	10
$\frac{19}{32}$-inch wood structural panel bonded with exterior glue	15
$\frac{3}{8}$-inch gypsum wallboard	10
$\frac{1}{2}$-inch gypsum wallboard	15
$\frac{5}{8}$-inch gypsum wallboard	30
$\frac{1}{2}$-inch Type X gypsum wallboard	25
$\frac{5}{8}$-inch Type X gypsum wallboard	40
Double $\frac{3}{8}$-inch gypsum wallboard	25
$\frac{1}{2}$-inch + $\frac{3}{8}$-inch gypsum wallboard	35
Double $\frac{1}{2}$-inch gypsum wallboard	40
Wood Framing[e]	
Wood studs @ 16 inches o.c. or less	20
Wood floor and roof joists @ 16 inches o.c. or less	10
Insulation	
Glass fiber mineral wool weighing not less than 2 pcf	15
Rockwool or slag material weighing not less than 3.3 pcf	15
Cellulose having a nominal density not less than 2.6 pcf	15

[a] Values apply only when membranes are installed on framing members which are spaced 16 inches o.c.

[b] Gypsum wallboard installed over framing or furring shall be installed so that all edges are supported, except $\frac{5}{8}$-inch Type X gypsum wallboard shall be permitted to be installed horizontally with the horizontal joints staggered 24 inches each side and unsupported but finished.

[c] On wood frame floor/ceiling or roof/ceiling assemblies, gypsum board shall be installed with the long dimensions perpendicular to framing members and shall have all joints finished.

[d] The membrane on the unexposed side shall not be included in determining the fire resistance of the assembly. When dissimilar membranes are used on a wall assembly, the calculation shall be made from the least fire-resistant side.

[e] All studs shall be nominal 2 × 4 or larger, and all joists shall have a nominal thickness of at least 2 inches.

load-bearing walls and floors should support their loads and prevent passage of fire for at least one hour.

20.7.1 Tabulated Assemblies

Wood materials can be used as components of fire-resistance rated assemblies, and in the case of large timbers and glulam, can achieve a fire-resistance rating as exposed members. The International Building Code (IBC) includes a table listing numerous approved fire-resistance-rated assemblies utilizing wood members. Tables are included for walls and partitions as well as floor and roof systems [1].

20.7.2 Component Additive Method

Additionally, systems can be designed and evaluated using the *component additive method* in which the membranes, framing, and insulation are assigned specific time values (Table 20.7.2-1). The fire-resistance rating of the assembly is then determined by summing the individual values [1]. AF&PA DCA 4 *Component Additive Method (CAM) for Calculating and Demonstrating Assembly Fire Endurance* [9] provides more detail on this method.

EXAMPLE 20.7.2-1 ONE-HOUR-FIRE-RATED PARTITION WALL

Given: A non–load-bearing partition wall is required to act as a one-hour-rated fire barrier in Type VA construction.

Wanted: Use the component additive method to design the wall with gypsum wallboard.

Solution: This method allows the fire-resistance rating to be determined as the sum of the resistances of the membrane(s) on the side of the wall exposed to fire, the framing members, and other protective measures such as insulation.

Wood studs spaced at 16 in. are assigned 20 minutes. (Table 20.7.2-1); $\frac{5}{8}$-inch Type X gypsum wallboard is assigned 40 minutes; and $\frac{1}{2}$-inch Type X gypsum wallboard is assigned 25 minutes (Table 20.7.2-1). Filling the cavities in the wall with rockwool insulation adds 15 minutes (Table 20.7.2-1).

Two options are considered. Both will have symmetrical construction, anticipating that the fire could occur on either side.

Option 1:

$$\tfrac{5}{8}\text{ in. Type X drywall} + 2 \times 4\text{ studs @ 16 in o.c.} + \tfrac{5}{8}\text{ in. Type X drywall}$$

$$40\,\text{min} + 20\,\text{min} = 60\,\text{min} \quad \text{(backside membrane is not added)}$$

Option 2:

$$\tfrac{1}{2}\text{ in. Type X drywall} + 2 \times 4\text{ studs @ 16 in o.c.} + \text{rockwool}$$
$$+\tfrac{1}{2}\text{ in. Type X drywall}$$
$$25\,\text{min} + 20\,\text{min} + 15\,\text{min} = 60\,\text{min} \quad \text{(backside membrane is not added)}$$

Result: Two options are given for obtaining the required one-hour fire-resistance rating.

20.7.3 Members Supporting More Than Two Levels

Primary structural framing members requiring fire-resistance ratings must meet the required fire-resistance rating independently or with encasement protection applied directly to the member and connections. Primary members include all members supporting more than two stories and all columns. These members cannot be protected only by the membrane enclosing a ceiling. The member or the combination of the member plus applied protection must achieve the required fire-resistance rating separately from the floor/ceiling assembly or roof/ceiling assembly just as if it were placed below the floor joists and ceiling (2009 IBC, 704.2, 704.3 [1]).

Secondary members that are required to have a fire-resistance rating are permitted to achieve that rating independently, by individual encasement protection, by the assembly membrane, or by any combination thereof. Secondary members include roof joists, floor joists, and beams supporting two stories or less. King studs and boundary elements in light frame construction built integrally with load-bearing walls are permitted to achieve their fire-resistance rating by the membrane protection provided for the load-bearing wall (2009 IBC, 704.4 [1]).

20.7.4 Fire Rating of Unprotected Timber Members

Unprotected timber members can be used in fire-resistance-rated construction. Procedures for designing one-hour-rated beams and columns can be found in AITC Technical Note 7 *Calculation of Fire Resistance of Glued Laminated Timbers* [10]. A newer procedure has been developed and included in the *National Design Specification® (NDS®) for Wood Construction* [11]. The latter procedure includes provisions for all exposed timber members (not just beams and columns) and has provisions for $1\frac{1}{2}$-hour and 2-hour fire-rated members. Development of the NDS procedure is detailed in AF&PA Technical Report 10 *Calculating the Fire Resistance of Exposed Wood Members* [5]. Both procedures are valid and were derived based on similar concepts; however, because the NDS procedure is more generally applicable and somewhat simpler to understand, it will be demonstrated in this manual.

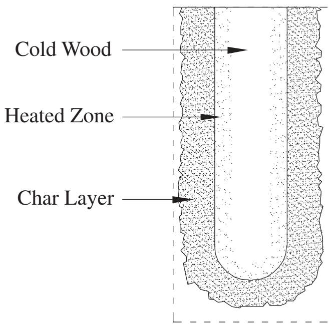


Figure 20.7.4.1-1 Charring of a wood member exposed to fire on three sides.

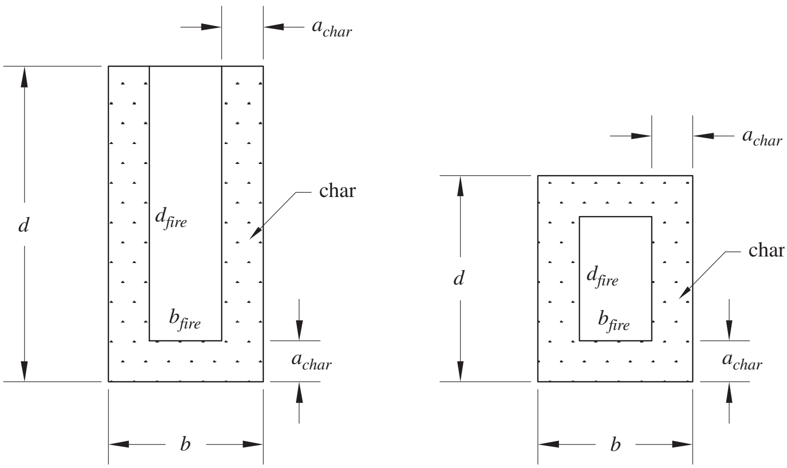


Figure 20.7.4.1-2 Effective dimensions of members exposed to fire: prior to fire (b,d), after fire (b_{fire}, d_{fire}).

20.7.4.1 Section Loss When a timber member is exposed to fire, material is lost from the surfaces due to charring (Figure 20.7.4.1-1). The strength of the heated wood just ahead of the char front is also reduced. The net effect of the charring and heat is a reduction in the effective section properties (area, section modulus, moment of inertia) of the member. The rate of material loss can be adequately predicted, so the capacity of the remaining section can be calculated based on the member's required fire-resistance rating. The thickness lost to charring and heating of the wood is referred to as the *effective char layer thickness* and is illustrated in Figure 20.7.4.1-2 for timbers exposed on three or four sides to fire.

The effective char layer thickness for fires of 1-hour, $1\frac{1}{2}$-hours, and 2 hours from the NDS are given in Table 20.7.4.1-1. These values were calculated based

TABLE 20.7.4.1-1 Effective Char Layer Thicknesses

Required Fire-Resistance Rating	Effective Char Layer Thickness, a_{char} (in.)
1-hour	1.8
$1\frac{1}{2}$-hour	2.5
2-hour	3.2

on a nominal char rate of $1\frac{1}{2}$ in. per hour and account for some damage from heat ahead of the actual char front.

For three-sided fire exposure, the dimensions at the end of the fire are estimated using Equations 20.7.4.1-1 and 20.7.4.1-2.

$$b_{fire} = b - 2a_{char} \tag{20.7.4.1-1}$$

$$d_{fire} = d - a_{char} \tag{20.7.4.1-2}$$

For four-sided fire exposure, the dimensions at the end of the fire are estimated using Equations 20.7.4.1-3 and 20.7.4.1-4.

$$b_{fire} = b - 2a_{char} \tag{20.7.4.1-3}$$

$$d_{fire} = d - 2a_{char} \tag{20.7.4.1-4}$$

The section properties at the end of the fire are calculated using Equations 20.7.4.1-5 through 20.7.4.1-7.

$$A_{fire} = b_{fire} d_{fire} \tag{20.7.4.1-5}$$

$$S_{fire} = \frac{b_{fire} d_{fire}^2}{6} \tag{20.7.4.1-6}$$

$$I_{fire} = \frac{b_{fire} d_{fire}^3}{12} \tag{20.7.4.1-7}$$

20.7.4.2 Design Values Published design values for timber design (cold) include factors of safety and adjustment to a 10-year (normal) load duration. As such, the average ultimate strength of glulam beams in testing is significantly higher than the published design value. The average ultimate strength can be estimated as 2.95 times the published (ASD) reference design value for bending [12]. Due to higher variability in sawn timbers, the expected average ultimate strength is more than three times the published ASD reference design value.

For fire endurance calculations, design stresses are increased to a level close to the expected average ultimate strength. Equations 20.7.4.2-1 through 20.7.4.2-6 show the design strengths used to calculate the fire endurance of timber members.

Note that load duration effects are already included in the factors; therefore, the load duration factor is not applied separately for fire design.

$$F'_{bx\ fire} = 2.85F_{bx}\left(C_v \text{ or } C_L\right) \tag{20.7.4.2-1}$$

$$F'_{by\ fire} = 2.85F_{by}\left(C_{fu} \text{ or } C_L\right) \tag{20.7.4.2-2}$$

$$F'_{t\ fire} = 2.85F_t \tag{20.7.4.2-3}$$

$$F'_{c\ fire} = 2.58F_c C_P \tag{20.7.4.2-4}$$

$$F'_{bE\ fire} = 2.03F_{bE\ fire} \tag{20.7.4.2-5}$$

$$F'_{cE\ fire} = 2.03F_{cE\ fire} \tag{20.7.4.2-6}$$

where:

C_V and C_{fu} are calculated based on pre-fire dimensions
C_L and C_P are calculated based on post-fire dimensions
$F_{bE\ fire}$ and $F_{cE\ fire}$ are calculated based on post-fire dimensions

20.7.4.3 Member Capacity The expected member capacities in bending, tension, and compression at the end of the fire event are calculated using Equation 20.7.4.3-1 through 20.7.4.3-4.

$$M'_{x\ fire} = F'_{bx\ fire} S_{x\ fire} = 2.85F_{bx}\left(C_v \text{ or } \mathrm{C}_L\right)\frac{b_{fire} d_{fire}^2}{6} \tag{20.7.4.3-1}$$

$$M'_{y\ fire} = F'_{by\ fire} S_{y\ fire} = 2.85F_{by}\left(C_{fu} \text{ or } \mathrm{C}_L\right)\frac{d_{fire} b_{fire}^2}{6} \tag{20.7.4.3-2}$$

$$T'_{fire} = F'_{t\ fire} A_{fire} = 2.85F_t b_{fire} d_{fire} \tag{20.7.4.3-3}$$

$$P'_{fire} = F'_{c\ fire} A_{fire} = 2.58F_c C_P b_{fire} d_{fire} \tag{20.7.4.3-4}$$

20.7.4.4 Glulam Layup Modifications Structural glued laminated timber is commonly manufactured with the highest quality laminations placed at the outer surfaces, with decreasing quality permitted as the laminations get closer to the core. In the event of a fire, outer laminations are destroyed first. To prevent a loss of design strength, such members require layup modifications. For a one-hour fire-resistance-rated member, one nominal 2-in.-thick lamination is removed from the core of the beam (lowest grade zone) and another nominal 2-in. thickness surface-quality lamination is placed at the surface that will be exposed to fire (Figure 20.7.4.4-1) [13]. For timbers that will be exposed to fire on all four sides, two core laminations are removed and additional high-grade laminations are placed at both surfaces (i.e., top and bottom). For $1\frac{1}{2}$- or 2-hour rated beams, two additional high-quality laminations are added to each top or bottom face that will be exposed to fire, with the same number of lower-grade laminations removed from the core (Figure 20.7.4.4-1). Timbers that have been appropriately

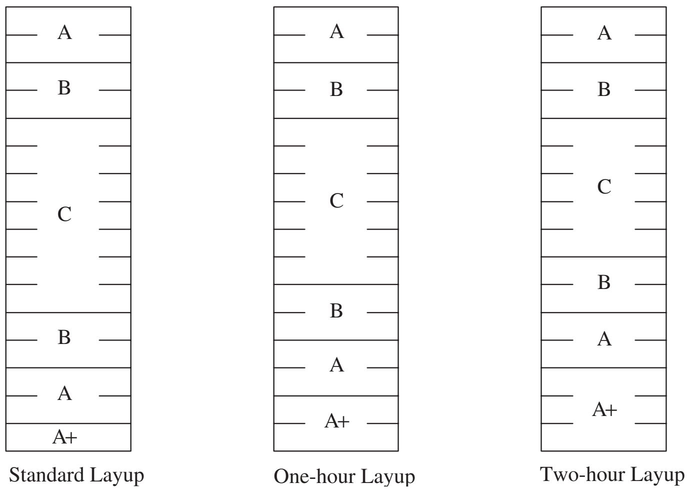


Figure 20.7.4.4-1 Layup modifications for fire-resistance-rated beams (fire on three sides). Letters represent relative quality of laminations, with C as the lowest; they are not actual grade designations.

modified are marked by the manufacturer with the appropriate fire-resistance rating. Layup modifications are not required for glulam timbers used to meet the prescriptive requirements of *heavy timber construction*.

20.7.4.5 Loads for Fire Design Traditionally, engineers have applied the full allowable stress design gravity load when designing for fire-resistance ratings. Members and assemblies are tested under full allowable design load, while exposed to standardized fire conditions. Members or systems that can support their full design load throughout the required time period have been "deemed to comply" for fire-resistance ratings of the same or lesser time.

Recently, however, load combinations for extraordinary events, such as fire, have been developed and published in ASCE 7 [14]. These load combinations were developed based on a probabilistic analysis similar to that used to develop other LRFD load combinations. Additionally, there is a trend toward better defining the fire behavior of structures. It is expected that new design procedures will be developed for fire design of timber members. However, such procedures are not currently available. Therefore, in this manual, the full design gravity load will be assumed for design examples.

20.7.4.6 Protection of Metal Fasteners For fire-resistance-rated construction, both the members and the fasteners must be designed and constructed to achieve the required fire-resistance rating. IBC 2009, Section 721.6.3.3 Fastener Protection [1] states, "Where minimum 1-hour fire resistance is required, connectors and fasteners shall be protected from fire exposure by $1\frac{1}{2}$ inches of wood, or other approved covering or coating for a 1-hour rating. Typical details for commonly used fasteners and connectors are shown in AITC Technical Note 7 [10].”

Where the designer does not want to encase the steel connections with wood or otherwise conceal them, intumescent paints may be permitted for use as protection for structural steel members subject to approval by the building official. These paints foam when subjected to the high heat of fires, providing insulation to the metal components. Information regarding the appropriate coating thickness should be obtained from the coating manufacturer.

EXAMPLE 20.7.4-1 ANALYSIS OF TRUSS TENSION CHORD FOR ONE-HOUR-FIRE RATING

Given: A $6\frac{3}{4}$ in $\times$ 12 in glulam timber will be used as a tension chord on a large exposed truss. The truss is required to have a one-hour fire-resistance rating. The chord is made using DF 5 (all L1 laminations) with $F_t = 1600$ psi. The member and connections have been designed for a snow load of $S =$ 80,000 lb and a dead load of $D = 20{,}000$ lb. The tension chord will be exposed to fire on four sides. The connections will have one-hour rated fire protection.

Wanted: Evaluate the member with regard to a one-hour design fire.

Solution:

Post-fire dimensions (four-sided exposure):

$$b_{fire} = b - 2a_{char}$$

$$b_{fire} = 6.75\,\text{in} - 2\,(1.8\,\text{in}) = 3.15\,\text{in}$$

$$d_{fire} = d - 2a_{char}$$

$$d_{fire} = 12\,\text{in} - 2\,(1.8\,\text{in}) = 8.4\,\text{in}$$

Post-fire capacity (Equation 20.7.4.3-3):

$$T'_{fire} = F'_{t\ fire} A_{fire} = 2.85 F_t b_{fire} d_{fire}$$

$$T'_{fire} = 2.85\,(1600\,\text{psi})\,(3.15\,\text{in})\,(8.4\,\text{in})$$

$$T'_{fire} = 120{,}700\,\text{lb}$$

Required capacity for one-hour fire (assuming full design load):

$$T_{fire,required} = D + S = 20{,}000\,\text{lb} + 80{,}000\,\text{lb} = 100{,}000\,\text{lb}$$

$$T_{fire,required} = 100{,}000\,\text{lb} \le T'_{fire} = 120{,}700\,\text{lb} \quad \therefore \text{OK}$$

Result: The glulam tension chord is adequate for a one-hour design fire using the full allowable design load.

EXAMPLE 20.7.4-2 ANALYSIS OF GLULAM BEAM FOR ONE-HOUR-FIRE RATING

Given: A $6\frac{3}{4}$ in × $13\frac{3}{4}$ in 24F-V3 SP simply-supported glulam beam spans 20 ft and supports a one-hour-fire-resistance-rated floor assembly. The attachment to the floor provides lateral support of the top of the beam. The layup of the beam has been modified, and the beam has been marked for a one-hour fire-resistance rating. The beam will be exposed on three sides.

Wanted: Determine the maximum load that the beam can be expected to support for the duration of the one-hour design fire.

Solution:

Post-fire dimensions (three-sided exposure):

$$b_{fire} = b - 2a_{char}$$

$$b_{fire} = 6.75\,\text{in} - 2\,(1.8\,\text{in}) = 3.15\,\text{in}$$

$$d_{fire} = d - a_{char}$$

$$d_{fire} = 13.75\,\text{in} - (1.8\,\text{in}) = 11.95\,\text{in}$$

Beam stability factor:
The beam will be assumed to be laterally braced along the compression edge through the fire while supporting the floor, so, $C_L = 1.0$.

Volume factor (pre-fire dimensions):

$$C_V = \left(\frac{5.125\,\text{in}}{6.75\,\text{in}}\right)^{\frac{1}{20}} \left(\frac{12\,\text{in}}{13.75\,\text{in}}\right)^{\frac{1}{20}} \left(\frac{21\,\text{ft}}{20\,\text{ft}}\right)^{\frac{1}{20}} = 0.982$$

Post-fire bending capacity (Equation 20.7.4.3-1):

$$M'_{x\ fire} = F'_{bx\ fire} S_{x\ fire} = 2.85 F_{bx} \left(C_v \text{ or } \mathrm{C}_L\right) \frac{b_{fire} d_{fire}^2}{6}$$

$$M'_{x\ fire} = 2.85\,(2400\,\text{psi})\,(0.982)\,\frac{(3.15\,\text{in})\,(11.95\,\text{in})^2}{6}$$

$$M'_{x\ fire} = 503{,}600\,\text{in-lb}$$

Maximum load at end of one-hour fire:

$$\omega = \frac{8M_{x\ fire}}{L^2} = \frac{8\,(503{,}600\,\text{in} - \text{lb})}{(240\,\text{in})^2} = 69.9\,\frac{\text{lb}}{\text{in}} = 839\,\frac{\text{lb}}{\text{ft}}$$

Result: The glulam beam is expected to support a load of 839 lb/ft throughout a one-hour design fire, provided that the beam is manufactured with a one-hour-fire layup.

In cases where lateral support for the beam is not provided during a fire event, the diminished section properties would be used to determine the beam stability factor, C_L. Reduced section and the resulting change in slenderness for a column subject to fire is illustrated in the following example.

EXAMPLE 20.7.4-3 ANALYSIS OF GLULAM COLUMN FOR ONE-HOUR-FIRE RATING

Given: A $6\frac{3}{4}$ in × $8\frac{1}{4}$ in, Number 47 SP column with an effective length of 10 ft supports a one-hour-fire-resistance-rated floor assembly. The column will be exposed to fire on all four sides.

Wanted: Determine the maximum load that the column can be expected to support for the duration of the one-hour design fire.

Solution:

Post-fire dimensions (four-sided exposure):

$$b_{fire} = b - 2a_{char}$$

$$b_{fire} = 6.75\text{ in} - 2\,(1.8\text{ in}) = 3.15\text{ in}$$

$$d_{fire} = d - 2a_{char}$$

$$d_{fire} = 8.25\text{ in} - 2\,(1.8\text{ in}) = 4.65\text{ in}$$

Critical buckling design value (post-fire dimensions):

$$F'_{cE\ \text{fire}} = 2.03\left(\frac{0.822E_{\min}}{\left(L_e/b_{fire}\right)^2}\right) = \frac{(2.03)\,0.822\left(0.74\left(10^6\right)\ \text{psi}\right)}{((120\text{ in})/(3.15\text{ in}))^2}$$

$$= (2.03)419\text{ psi} = 851\text{ psi}$$

$$F^*_{c\ fire} = 2.58F_c = 2.58\,(1900\text{ psi}) = 4902\text{ psi}$$

Column stability factor:

$$C_P = \frac{1 + \left(F'_{cE}/F_{c\ fire}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F'_{cE}/F_{c\ fire}\right)}{2c}\right]^2 - \frac{F'_{cE}/F_{c\ fire}}{c}}$$

$$C_P = \frac{1 + \left(\dfrac{851\text{ psi}}{4900\text{ psi}}\right)}{2\,(0.9)} - \sqrt{\left[\frac{1 + \left(\dfrac{851\text{ psi}}{4900\text{ psi}}\right)}{2\,(0.9)}\right]^2 - \frac{\dfrac{851\text{ psi}}{4900\text{ psi}}}{(0.9)}}$$

$$C_P = 0.170$$

Post-fire compression capacity:

$$P'_{fire} = F'_{c\ fire} A_{fire}$$
$$P'_{fire} = 2.58 F_c C_P b_{fire} d_{fire}$$
$$P'_{fire} = 2.58\,(1900\,\text{psi})\,(0.170)\,(3.15\,\text{in})\,(4.65\,\text{in})$$
$$P'_{fire} = 12{,}200\,\text{lb}$$

Result: The glulam column can support a load of 12,200 lb for a one-hour fire.

EXAMPLE 20.7.4-4 ANALYSIS OF HEAVY TIMBER DECKING FOR ONE-HOUR-FIRE RATING

Given: 4 × 6, Commercial grade, DF-L roof decking will span between glulam arches spaced at 16 ft. The decking will be installed in a controlled random layup pattern. The reference bending design value for the decking is 1650 psi, and the reference modulus of elasticity is 1.7 million psi.

Wanted: Determine the maximum load that the decking can be expected to support for the duration of a one-hour design fire.

Solution:

Post-fire dimensions:
The decking will be exposed to the fire on the bottom face only; therefore, only the thickness will be reduced by the fire. The thickness of the member after a one-hour fire can be estimated as follows:

$$d_{fire} = d - a_{char}$$
$$d_{fire} = 3.5\,\text{in} - 1.8\,\text{in} = 1.7\,\text{in}$$

Maximum post-fire load:

$$\sigma_{fire} = \frac{20 F'_{b\ fire}}{3\ell^2}\,\frac{d_{fire}^2}{6} = \frac{20\,(2.85\,(1650\,\text{psi}))}{3\,(192\,\text{in})^2}\,\frac{(1.7\,\text{in})^2}{6}$$
$$\sigma_{fire} = 0.41\,\text{psi} = 59\,\text{psf}$$

Result: The 4 in. heavy timber decking can support a load of 59 psf throughout a one-hour fire.

20.8 USE OF STOCK GLULAM BEAMS IN FIRE RATED CONSTRUCTION

Example 20.7.4-2 illustrated the capacity of a beam with its layup specifically modified for fire-rating and for which the published design value of F_b was used with Equation 20.7.4.3-1. Members manufactured without the modified layup are subject to lesser design values due to the unreplaced loss of higher-grade material. The reduced design stress for a standard or stock glulam beam with one lamination notionally removed (i.e., resulting from a one-hour fire) from the bottom of the beam can be estimated as 70% of the original design stress of the member. This reduced design stress should be used when evaluating the fire-resistance rating of off-the-shelf or stock glulam beams. This procedure is limited to fire-resistance ratings of one hour or less. Due to the reduction in design strength, stock layups are necessarily less efficient than beams that have been specifically modified for one-hour fire resistance.

20.9 FIRE RETARDANT TREATMENT

Pressure-impregnated fire-retardant-treated (FRT) wood is permitted for use in specific construction applications where combustible materials are generally prohibited. Typically, FRT wood is dimension lumber or plywood. Large timbers are typically not treated with fire-retardant chemicals. The laminated timber industry generally does not recommend the use of fire-retardant-treated glulam.

Fire-retardant treatments are applied to wood to reduce its flame-spread characteristics (i.e., to slow the advance of flames along its surface). However, typical pressure-impregnated fire-retardant treatments significantly reduce the strength of wood, increase corrosivity on fasteners, and increase the rate of char formation. FRT wood is also generally limited to interior use.

Due to the deleterious effect of fire-retardant treatments on structural wood and connections, they should only be used where specific information can be provided by the fire-retardant manufacturer on the effects of the treatment and where these effects are properly taken into account.

20.10 CONCLUSION

Fire safety is one of the most important aspects of building design and construction. The primary concern in fire-safe design of structural elements and systems is to provide for egress of the building occupants through the prevention of structural collapse and especially progressive collapse. A secondary objective is to facilitate firefighting and rescue operations. The structural system must provide adequate time for egress and rescue without collapsing on the occupants or emergency personnel. Consequently, much of the building code is dedicated to the regulation of structural elements with regard to fire performance.

Experience with actual building fires has demonstrated that no material (combustible or noncombustible) is immune to the effects of fires. The specification of noncombustible structural materials may not provide adequate fire safety. Conversely, fire-safe buildings can be built with combustible materials. In particular, large timber members perform very well in fires. Large timber members burn slowly and can continue to support loads even after sustaining significant damage from surface charring.

The excellent performance of large timber members has led to larger allowable heights and areas for *heavy timber construction* than many other building types. This method of construction includes prescriptive rules for minimum sizes of timber elements and requires the elimination of concealed spaces.

In addition to *heavy timber construction*, large timber members can be explicitly designed to meet the requirements of prescribed fire-resistance ratings. The design procedures presented herein allow fire-resistance ratings of up to two hours.

The design of structural glued laminated timber members for a fire-resistance rating has two steps: (1) the designer must size the member appropriately, and (2) the manufacturer must modify the layup to ensure adequate performance as the outer laminations are lost due to charring. The designer and manufacturer must clearly communicate to ensure that both steps are completed. Alternatively, the designer may design a stock beam (without layup modification) for a prescribed fire-resistance rating by taking into account the reduction in design strength due to the loss of high-grade material.

Fire-retardant-treated wood has some applications in building construction, particularly as studs and plywood in walls that otherwise require noncombustible materials. However, large timbers are typically not pressure-impregnated with fire-retardant chemicals, because of negative side effects that typically offset the benefits obtained from the treatment. The glulam industry does not generally recommend the use of fire-retardant treatments with structural glued laminated timber.

APPENDIX A

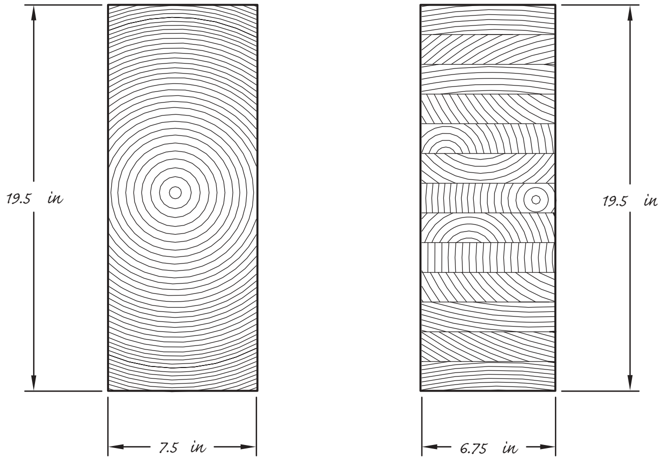
DESIGN EXAMPLES

INTRODUCTION

This chapter contains example problems in a format similar to what a designer might use when performing hand calculations. Each problem is intended to serve as a quick reference for the procedures on a particular topic. Problems are not intended as a primary learning tool, but, rather, to augment the content of previous chapters for key topics. As such, explanatory text is not included in the examples. However, the heading to each problem includes a list of sections from the text where relevant information can be found.

COMPARATIVE SHRINKAGE OF SAWN TIMBER AND GLULAM BEAMS
(See also Section 2.3.1)

Given: An 8 × 20 (green) DF-L sawn timber beam and a $6\frac{3}{4}$ in. × $19\frac{1}{2}$ in. DF glulam beam are being considered for use in a structure with an expected equilibrium moisture content of 8%.



Wanted: Determine the expected shrinkage of each member through the depth.

Assumed: Each member has a shrinkage rate of 6% from green to oven-dry (combination of radial and tangential shrinkage). The initial moisture content is 30% for the green sawn lumber and 13% for the glulam.

Solution:

$$s_{glulam} = 6.0\% \left(\frac{13\% - 8\%}{30\%} \right) = 1.0\%$$

$$s_{sawn} = 6.0\% \left(\frac{30\% - 8\%}{30\%} \right) = 4.4\%$$

$$\Delta d_{glulam} = s_{glulam} d = (0.01)(19.5 \text{ in}) = 0.20 \text{ in}$$

$$\Delta d_{sawn} = s_{sawn} d = (0.044)(19.5 \text{ in}) = 0.86 \text{ in}$$

Result:

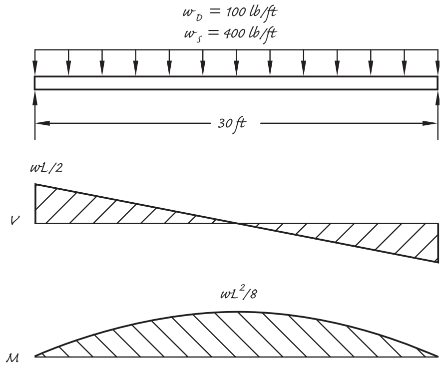
The sawn lumber beam will shrink nearly 0.9 in. in service. The glulam beam will shrink approximately 0.2 in. The sawn beam shrinkage is almost $4\frac{1}{2}$ times that of the glulam beam. Furthermore, significant checking and possible warp is expected to occur as the sawn beam dries. These shrinkage-related issues must be considered in design.

SIMPLE BEAM DESIGN (ASD Method)
(See also Sections 4.1, 4.2, 4.3)

Given: Simply-supported beam spanning 30 ft with an estimated uniform dead load of 100 lb/ft (including self weight) and a uniform snow load of 400 lb/ft. Deflection limits are $\delta_{SL} = L/240 = 1.5$ in. for snow load and $\delta_{TL} = L/180 = 2$ in. for total load including creep. Beam is fully braced at the ends and along the top edge.

Wanted: Design a $5\frac{1}{8}$ in. wide, 24F-1.8E DF glulam beam to support D + S load combination.

Solution:



From Appendix B.1.1 (case 1)

$$V_{TL} = \frac{w_{TL}L}{2} = \frac{(500\ lb/ft)(30\ ft)}{2}$$

$$V_{TL} = 7500\ lb$$

$$M_{TL} = \frac{w_{TL}L^2}{8} = \frac{(500\ lb/ft)(30\ ft)^2}{8}$$

$$M_{TL} = 56{,}250\ lb\text{-}ft = 675{,}000\ lb\text{-}in$$

$$\Delta = \frac{5wL^4}{384EI} = \frac{5wL^4}{384E}\left(\frac{12}{bd^3}\right)$$

Required depth based on total load deflection (including creep):

$$w_{TL+creeo} = 1.5w_D + w_S = 1.5\,(100\ lb/ft) + 400\ lb/ft = 550\ lb/ft$$

$$d_{TL} \geq \sqrt[3]{\frac{15w_{TL+creep}L^4}{96Eb\delta_{TL}}} = \sqrt[3]{\frac{15\,(550\ lb/ft)(30\ ft)^4\left(\frac{1728\ in^3}{ft^3}\right)}{96\,(1.8\,(10^6)\ psi)(5.125\ in)(2\ in)}} = 18.7\ in$$

Required depth based on snow load deflection:

$$d_S \geq \sqrt[3]{\frac{15w_SL^4}{96Eb\delta_{SL}}} = \sqrt[3]{\frac{15\,(400\ lb/ft)(30\ ft)^4\left(\frac{1728\ in^3}{ft^3}\right)}{96\,(1.8\,(10^6)\ psi)(5.125\ in)(1.5\ in)}} = 18.5\ in$$

Check flexure for trial depth of $19\frac{1}{2}$ in. (13 lams, 1.5 in/lam):

$$C_V = \left(\frac{5.125\ in}{5.125\ in}\right)^{\frac{1}{10}} \left(\frac{12\ in}{19.5\ in}\right)^{\frac{1}{10}} \left(\frac{21\ ft}{30\ ft}\right) = 0.92$$

$$F'_{bx} = F_{bx} C_D C_M C_t \left(C_V \text{ or } C_L\right) = 2400\ psi(1.15)(1.0)(1.0)(0.92) = 2540\ psi$$

$$f_{bx} = \frac{M_{TL}}{S_x} = \frac{6\left(675{,}000\ lb\text{-}in\right)}{(5.125\ in)(19.5\ in)^2} = 2080\ psi$$

$$\frac{f_{bx}}{F'_{bx}} = \frac{2080\ psi}{2540\ psi} = 0.819 \leq 1.0 \qquad \therefore \text{ OK for flexure.}$$

Check shear for trial depth of 19.5 in. (neglecting allowed shear reduction):

$$F'_{vx} = F_{vx} C_D C_M C_t = 265\ psi(1.15)(1.0)(1.0) = 305\ psi$$

$$f_{vx} = \frac{3V_{TL}}{2bd} = \frac{3\left(7500\ lb\right)}{2\left(5.125\ in\right)(19.5\ in)} = 113\ psi$$

$$\frac{f_{vx}}{F'_{vx}} = \frac{113\ psi}{305\ psi} = 0.370 \leq 1.0 \qquad \therefore \text{ OK for shear.}$$

Calculate required bearing length at support:

$$l_b = \frac{R}{F'_{c\perp} b} = \frac{7500\ lb}{\left(650\ psi\right)\left(5.125\ in\right)} = 2.25\ in$$

Calculate deflection due to dead load plus creep:

$$\Delta_{DL+creep} = 1.5\left(\frac{5 w_D L^4}{384E}\left(\frac{12}{bd^3}\right)\right)$$

$$\Delta_{DL+creep} = 1.5\left(\frac{5\left(100\ lb/ft\right)\left(30\ ft\right)^4\left(\frac{1728\ in^3}{ft^3}\right)}{384\left(18\left(10^6\right)\ psi\right)}\frac{12}{(5.125\ in)(19.5\ in)^3}\right)$$

$$\Delta_{DL+creep} = 0.48\ in$$

Calculate radius of curvature for 0.5 in. of camber:

$$R = \frac{L^2}{80} = \frac{(360\ in)^2}{8\left(0.5\ in\right)} = 32{,}400\ in = 2700\ ft$$

Result:

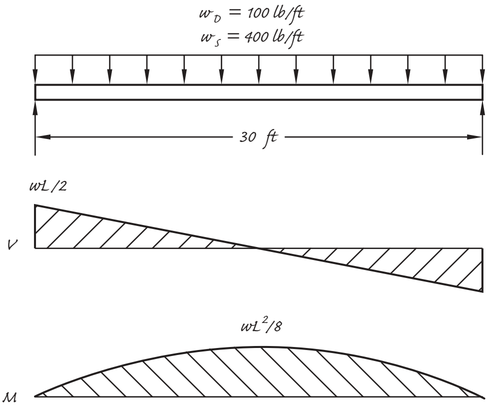
$5\frac{1}{8}$ in. × $19\frac{1}{2}$ in. 24F-1.8E DF glulam beam is adequate for stated conditions. A bearing length of 2.25 in. or more is required at each end. A 2700 ft radius would give 0.5 in. of camber to offset the expected long term deflection due to dead loads and creep.

UPSIDE-DOWN BEAM ANALYSIS (ASD Method)
(See also Section 4.3.2)

Given: *A $5\frac{1}{8}$ in. × 19.5 in. 24F-V4 DF beam was specified to span 30 ft to support a uniform dead load of 100 lb/ft (including self weight) and a uniform snow load of 400 lb/ft. However, the beam was inadvertently installed upside-down. The beam is fully braced at the ends and along the top edge.*

Wanted: *Analyze the beam for flexure as installed.*

Solution:



$$M_{TL} = \frac{w_{TL}L^2}{8} = \frac{(500\ lb/ft)(30\ ft)^2}{8}$$

$$M_{TL} = 56{,}250\ lb\text{-}ft = 675{,}000\ lb\text{-}in$$

Volume factor:

$$C_V = \left(\frac{5.125\ in}{b}\right)^{\frac{1}{10}} \left(\frac{12\ in}{d}\right)^{\frac{1}{10}} \times \left(\frac{21\ ft}{L}\right)^{\frac{1}{10}}$$

$$C_V = \left(\frac{5.125\ in}{5.125\ in}\right)^{\frac{1}{10}} \left(\frac{12\ in}{19.5\ in}\right)^{\frac{1}{10}} \times \left(\frac{21\ ft}{30\ ft}\right)^{\frac{1}{10}}$$

$$C_V = 0.92$$

Allowable stress:

$$F'_{bx} = F_{bx} C_D C_M C_t\ (C_V \text{ or } C_L)$$

$$F'_{bx} = 1850\ psi(1.15)(1.0)(1.0)(0.92)$$

$$F'_{bx} = 1960\ psi$$

Stress due to applied loads:

$$f_{bx} = \frac{M_{TL}}{S_x} = \frac{6\ (675{,}000\ lb\text{-}in)}{(5.125\ in)(19.5\ in)^2} = 2080\ psi$$

Stress ratio:

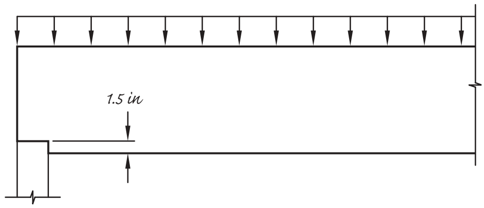
$$\frac{f_{bx}}{F'_{bx}} = \frac{2080\ psi}{1960\ psi} = 1.06 > 1.0 \qquad \therefore \text{Not acceptable.}$$

Result:

The beam is overstressed in flexure when installed in the upside-down orientation. In addition, any camber manufactured into the beam would now be downward, giving a sagging appearance and potentially leading to ponding issues. The beam must be removed and reinstalled or replaced.

TENSION-FACE NOTCH (ASD Method)
(See also Section 12.2.2)

Given: *A 5 in. × $23\frac{3}{8}$ in. 24F-1.8E SP glulam floor beam was specified to support a design live load of $w_L = 1200$ plf and a design dead load of $w_{D+SW} = 530$ plf (including self-weight) across a 20 ft span. The beam was designed with full lateral bracing ($C_L = 1.0$). The contractor has requested authorization to cut a notch on the bottom face of the beam at one end support, as illustrated.*



Wanted: *Evaluate the shear capacity of the beam considering the proposed notch.*

Solution:

Allowable design value (AITC 117-2010):

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 300\ psi(1.0)(1.0)(1.0)(0.72) = 216\ psi$$

Prescriptive notch limits:

$$d_{notch} = 1.5\ in \leq 0.1d = 0.1(23.375\ in) = 2.34\ in \qquad \therefore\ OK$$

$$d_{notch} = 1.5\ in \leq 3\ in \qquad \therefore\ OK$$

Reaction force:

$$R_v = \frac{(w_{D+SW} + w_L)\, l}{2} = \frac{(530\ plf + 1200\ plf)(20\ ft)}{2} = 17{,}300\ lb$$

Design shear stress:

$$f_v = \frac{3R_v}{2bd_e}\left[\frac{d}{d_e}\right]^2$$

$$f_v = \frac{3(17{,}300\ lb)}{2(5\ in)(23.375\ in - 1.5\ in)}\left[\frac{23.375\ in}{23.375\ in - 1.5\ in}\right]^2 = 271\ psi$$

Stress ratio:

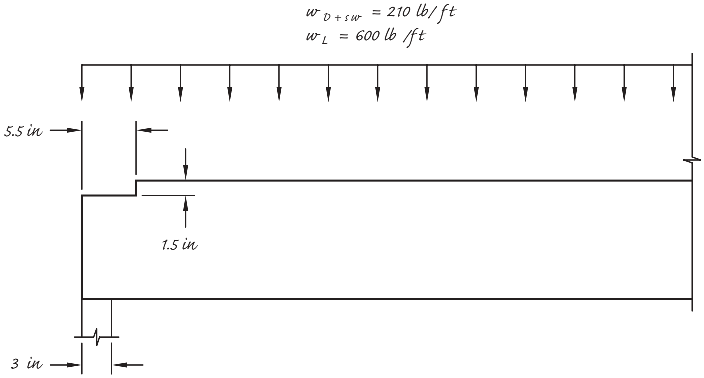
$$\frac{f_v}{F'_{vx}} = \frac{271\ psi}{216\ psi} = 1.26 > 1.0 \qquad \therefore\ \text{Not Acceptable}$$

Result:

The proposed notch is within the prescriptive limits of the code, but the notched beam is not adequate to resist the design loads.

COMPRESSION-FACE NOTCH (ASD Method)
(See also Section 12.2.3)

Given: A $3\frac{1}{8}$ in. × 12 in. 24 F-1.8E DF glulam floor beam was specified to support a design live load of $w_L = 600$ plf and a design dead load of $w_{D+SW} = 210$ plf (including self-weight) across a 12 ft span. The beam was designed with full lateral bracing ($C_L = 1.0$). The contractor has requested authorization to cut a notch on the top face of the beam at one end support, as illustrated.



Wanted: Evaluate the shear capacity of the beam considering the proposed notch.

Solution:

Notch size limits:

$d_{notch} = 1.5\ in \leq 0.4d = 0.4\,(12\ in) = 4.8\ in$ ∴ OK

Notch does not extend into middle third of the span ∴ OK

Reaction force:

$$R_v = \frac{\left(w_{D+SW} + w_L\right) l}{2} = \frac{\left(210\ plf + 600\ plf\right)\left(12\ ft\right)}{2} = 4{,}860\ lb$$

Design shear stress:

$d_e = 12\ in - 1.5\ in = 10.5\ in$

$e = 5.5\ in - 3.0\ in = 2.5\ in$

$$f_v = \frac{3R_v}{2b\left(d - \left(\frac{d - d_e}{d_e}\right) e\right)}$$

$$f_v = \frac{3\,(4{,}860\ lb)}{2\,(3.125\ in)\left(12\ in - \left(\frac{12\ in - 10.5\ in}{10.5\ in}\right)(2.5\ in)\right)} = 200\ psi$$

Allowable design value:

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 265 \text{ psi}(1.0)(1.0)(1.0)(0.72)$$

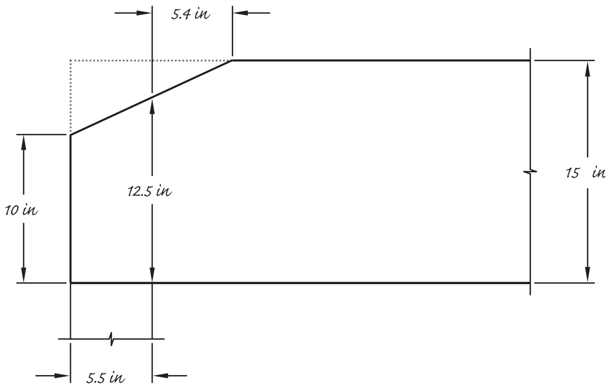
$$F'_{vx} = 191 \text{ psi} < f_v = 200 \text{ psi} \qquad \therefore \text{ Not Acceptable}$$

Result:

The proposed notch is within the notch size limits of the code; however, the increased shear stress resulting from the notch is excessive. The proposed notch cannot be permitted.

SLOPED END CUT (ASD Method)
(See also Section 12.2.3)

Given: A $5\frac{1}{8}$ in. × 15 in. 24F-1.8E DF glulam roof beam spans 16 ft and supports a design snow load of $w_S = 925$ plf and a design dead load of $w_{D+SW} = 323$ plf (including self-weight). Framing considerations require a sloped out on the top face of the beam at the end, as illustrated. The beam is adequate for flexure and deflection.



Wanted: Evaluate the shear capacity of the beam considering the sloped cut.

Solution:

Allowable design value (AITC 117-2010):

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 265 \text{ psi} (1.15)(1.0)(1.0)(0.72) = 219 \text{ psi}$$

Prescriptive notch limit:

$$d_{notch} = 5 \text{ in} \leq \frac{2d}{3} = \frac{2}{3}(15 \text{ in}) = 10 \text{ in} \qquad \therefore \text{OK}$$

Reaction force:

$$R_v = \frac{\left(w_{D+SW} + w_L\right) l}{2} = \frac{\left(323 \text{ plf} + 925 \text{ plf}\right)\left(16 \text{ ft}\right)}{2} = 9984 \text{ lb}$$

Design shear stress ($e < d_e$):

$$f_v = \frac{3R_v}{2b\left[d - \left(\dfrac{d - d_e}{d_e}\right) e\right]}$$

$$f_v = \frac{3\,(9984 \text{ lb})}{2\,(5.125 \text{ in})\left[15 \text{ in} - \left(\dfrac{15 \text{ in} - 12.5 \text{ in}}{15 \text{ in}}\right)(5.4 \text{ in})\right]} = 207 \text{ psi}$$

Stress ratio:

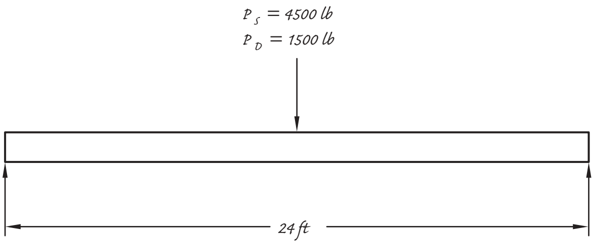
$$\frac{f_v}{F'_{vx}} = \frac{207\ psi}{219\ psi} = 0.95 < 1.0 \quad \therefore OK$$

Result:

The beam is adequate to resist the design loads.

BEAM STABILITY (Effective Length Method, ASD)
(See also Sections 3.4.3.1, 4.3.4)

Given: A 24F-1.8E DF glulam beam will span $l = 24$ ft with a point load at mid-span from 4500 lb of snow and 1500 lb of dead weight (not including the self-weight of the beam). The beam must be designed for a deflection limit of $l/240$ under snow load alone. The beam will be laterally braced to prevent displacement and rotation at the supports only.



Wanted: Design a $5\frac{1}{8}$ in. wide beam to support the design loads plus self-weight.

Solution:

Reference design values (AITC 117-2010):

$$F_{bx} = 2400 \text{ psi}$$
$$E_x = 1.8\left(10^6\right) \text{ psi}$$
$$E_{y\ min} = 0.85\left(10^6\right) \text{ psi}$$

Adjusted design values:

$$F'_{bx} = F_{bx}C_DC_MC_t\,(C_v \text{ or } C_L) = 2400 \text{ psi }(1.15)(1.0)(1.0)(C_v \text{ or } C_L)$$
$$F'_{bx} = 2760 \text{ psi }(C_v \text{ or } C_L)$$
$$E'_x = E_xC_MC_t = \left[1.8\left(10^6\right) \text{ psi}\right](1.0)(1.0) = 1.8\left(10^6\right) \text{ psi}$$
$$E'_{y\ min} = E_{y\ min}C_MC_t = \left[0.85\left(10^6\right) \text{ psi}\right](1.0)(1.0) = 0.85\left(10^6\right) \text{ psi}$$

Required depth based on deflection limit:

$$\Delta = \frac{l}{240} = \frac{P_Sl^3}{48E'_xI_x}$$

$$I_x = \frac{240}{l}\frac{P_Sl^3}{48E'_x} = \frac{5P_Sl^2}{E'_x} = \frac{5\,(4500 \text{ lb})\left(24 \text{ ft}\right)^2\left(\dfrac{144 \text{ in}^2}{\text{ft}^2}\right)}{1.8\left(10^6\right) \text{ psi}} = 1037 \text{ in}^4$$

$$I_x = \frac{bd^3}{12} = \frac{(5.125 \text{ in})\,d^3}{12} = 1037 \text{ in}^4$$

$$d = \sqrt[3]{\frac{(1037 \text{ in})(12)}{5.125 \text{ in}}} = 13.4 \text{ in}$$

Estimate depth based on flexural stress (estimate lower of C_V or C_L to be 0.8 and ignore self-weight):

$$F_b' = (2760\ psi)(C_L \text{ or } C_V) \approx (2760\ psi)(0.8) = 2210\ psi$$

$$M_{D+S} = \frac{Pl}{4} = \frac{(6{,}000\ lb)(24\ ft)}{4} = 36{,}000\ ft\text{-}lb = 432\left(10^3\right)\ in\text{-}lb$$

$$S_x = \frac{bd^2}{6} \approx \frac{M_{D+S}}{F'_{bx,est}} \quad \Rightarrow \quad d \approx \sqrt{\frac{6M_{D+S}}{bF'_{bx,est}}} = \sqrt{\frac{6\left(432\left(10^3\right)\ in\text{-}lb\right)}{(5.125\ in)(2210\ psi)}}$$

$$S_x = 15.1\ in \quad (\text{Try } d = 15\ in)$$

Self-weight:

$$w_{SW} = (5.125\ in)(15\ in)\left(33\frac{lb}{ft^3}\right)\left(\frac{1\ ft^2}{144\ in^2}\right) = 17.6\frac{lb}{ft} \quad (\text{use } 18\ plf)$$

Maximum bending moment and bending stress due to design loads including self-weight:

$$M = \frac{w_{SW}l^2}{8} + \frac{Pl}{4} = \frac{(18\ plf)(24\ ft)^2}{8} + \frac{(6{,}000\ lb)(24\ ft)}{4}$$

$$M = 37{,}300\ ft\text{-}lb = 448\left(10^3\right)\ in\text{-}lb$$

$$f_b = \frac{6M}{bd^2} = \frac{6\left(448\left(10^3\right)\ in\text{-}lb\right)}{(5.125\ in)(15.0\ in)^2} = 2330\ psi$$

Beam stability factor:

$$\frac{l_u}{d} = \frac{288\ in}{15\ in} = 19.2 > 7$$

$$l_e = 1.37l_u + 3d = 1.37\,(288\ in) + 3\,(15\ in) = 440\ in$$

$$R_B = \sqrt{\frac{l_e d}{b^2}} = \sqrt{\frac{(440\ in)\ 15.0\ in}{(5.125\ in)^2}} = \sqrt{251.3} = 15.9$$

$$F_{bE} = \frac{1.20\left(E'_{min}\right)}{R_B^2} = \frac{1.20\left(0.85\left(10^6\right)\ psi\right)}{251.3} = 4059\ psi$$

$$\frac{F_{bE}}{F_b^*} = \frac{4059\ psi}{2760\ psi} = 1.47$$

$$C_L = \frac{1 + \frac{F_{bE}}{F_b^*}}{1.9} - \sqrt{\left(\frac{1 + \frac{F_{bE}}{F_b^*}}{1.9}\right)^2 - \frac{\frac{F_{bE}}{F_b^*}}{0.95}}$$

$$C_L = \frac{1 + 1.47}{1.9} - \sqrt{\left(\frac{1 + 1.47}{1.9}\right)^2 - \frac{1.47}{0.95}} = 0.922$$

Volume factor:

$$C_V = \left[\left(\frac{5.125\ in}{b}\right)\left(\frac{12\ in}{d}\right)\left(\frac{21\ ft}{l}\right)\right]^{\frac{1}{10}}$$

$$C_V = \left[\left(\frac{5.125\ in}{5.125\ in}\right)\left(\frac{12\ in}{15\ in}\right)\left(\frac{21\ ft}{24\ ft}\right)\right]^{\frac{1}{10}} = 0.965 \qquad (C_L\ controls)$$

Allowable stress:

$$F'_{bx} = 2760\ psi(C_V\ or\ C_L) = 2760\ psi(C_L) = 2760\ psi\ (0.922)$$

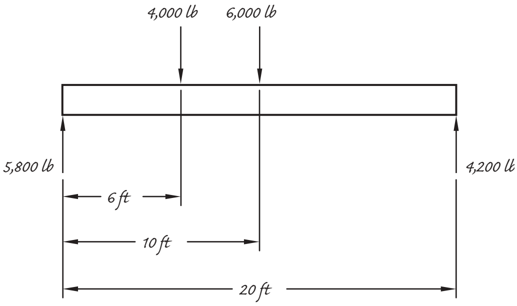
$$F'_{bx} = 2545\ psi > f_b = 2330\ psi \qquad \therefore OK$$

Result:

A $5\frac{1}{8}$ in. × 15 in. glulam beam is adequate to support the design loads.

BEAM STABILITY (Equivalent Moment Method, ASD)
(See also Sections 3.4.3.1, 4.3.4)

Given: A simply-supported 5 in. × 16.5 in. 24F-V3 SP glulam beam will span $l = 20$ ft with two point loads as illustrated. The beam will be braced to prevent lateral displacement and rotation at the supports only.



The bending moment, including self weight, is defined as follows:

$$M = \begin{cases} (6{,}010 \text{ lb})\,x - (10.5 \text{ plf})\,x^2 & \text{for } x \le 6 \text{ ft} \\ (2{,}010 \text{ lb})\,x - (10.5 \text{ plf})\,x^2 + 24{,}000 \text{ ft-lb} & \text{for } 6 \text{ ft} < x \le 10 \text{ ft} \\ 84{,}000 \text{ ft-lb} - (3{,}990 \text{ lb})\,x - (10.5 \text{ plf})\,x^2 & \text{for } x > 10 \text{ ft} \end{cases}$$

Wanted: Evaluate the adequacy of the beam for flexure.

Solution:

Adjusted design values:

$$F'_{bx} = F_{bx} C_D C_M C_t \,(C_v \text{ or } C_L) = 2400 \text{ psi}(1.15)(1.0)(1.0)(C_v \text{ or } C_L)$$

$$F'_{bx} = 2760 \text{ psi}(C_v \text{ or } C_L)$$

$$E'_{y\,min} = E_{y\,min} C_M C_t = \left[0.85\left(10^6\right) \text{ psi}\right](1.0)(1.0) = 0.85\left(10^6\right) \text{ psi}$$

Maximum Bending Moment (at $x = 10$ ft) and Corresponding Stress:

$$M_{max} = M_{10} = (2{,}010 \text{ lb})(10 \text{ ft}) - (10.5 \text{ plf})(10 \text{ ft})^2 + 24{,}000 \text{ ft-lb}$$

$$M_{max} = M_{10} = 43{,}050 \text{ ft-lb} = 516{,}600 \text{ in-lb}$$

$$f_b = \frac{6M}{bd^2} = \frac{6\left(516.6\left(10^3\right) \text{ in-lb}\right)}{(50.0 \text{ in})(16.5 \text{ in})^2} = 2280 \text{ psi}$$

Bending Moments, M_A, M_B, M_C:

$$M_A = M_5 = (6{,}010 \text{ lb})(5 \text{ ft}) - (10.5 \text{ plf})(5 \text{ ft})^2 = 29{,}790 \text{ ft-lb}$$

$$M_B = M_{10} = 43{,}050 \text{ ft-lb}$$

$$M_C = M_{15} = 84{,}000 \text{ ft-lb} - (3{,}990 \text{ lb})(15 \text{ ft}) - (10.5 \text{ plf})(15 \text{ ft})^2$$

$$M_C = 21{,}790 \text{ ft-lb}$$

Beam stability factor:

$$C_b = \frac{12.5M_{max}}{3M_A + 4M_B + 3M_C + 2.5M_{max}}$$

$$C_b = \frac{12.5\left(43,050\ \text{ft-lb}\right)}{3\left(29,790\ \text{ft-lb}\right) + 4\left(43,050\ \text{ft-lb}\right) + 3\left(21,790\ \text{ft-lb}\right) + 2.5\left(43,050\ \text{ft-lb}\right)}$$

$$C_b = 1.24$$

$$k = 1.72$$

$$\eta = \frac{1.3kd}{l_u} = \frac{1.3(1.72)(16.5\ \text{in})}{(20\ \text{ft})(12\ \text{in/ft})} = 0.154$$

$$C_e = \sqrt{\eta^2 + 1} - \eta = \sqrt{0.154^2 + 1} - 0.154 = 0.858$$

$$R_B = \sqrt{\frac{1.84 l_u d}{C_b C_e b^2}} = \sqrt{\frac{1.84\,(240\ \text{in})\ 16.5\ \text{in}}{(1.24)(0.858)(5\ \text{in})^2}} = \sqrt{274} = 16.6$$

$$F_{bE} = \frac{1.20\left(E'_{min}\right)}{R_B^2} = \frac{1.20\left(0.85\,(10^6)\ \text{psi}\right)}{274} = 3720\ \text{psi}$$

$$\frac{F_{bE}}{F_b^*} = \frac{3720\ \text{psi}}{2760\ \text{psi}} = 1.35$$

$$C_L = \frac{1 + \frac{F_{bE}}{F_b^*}}{1.9} - \sqrt{\left(\frac{1 + \frac{F_{bE}}{F_b^*}}{1.9}\right)^2 - \frac{\frac{F_{bE}}{F_b^*}}{0.95}}$$

$$C_L = \frac{1 + 1.35}{1.9} - \sqrt{\left(\frac{1 + 1.35}{1.9}\right)^2 - \frac{1.35}{0.95}} = 0.907$$

Volume factor:

$$C_V = \left[\left(\frac{5.125\ \text{in}}{b}\right)\left(\frac{12\ \text{in}}{d}\right)\left(\frac{21\ \text{ft}}{l}\right)\right]^{\frac{1}{20}}$$

$$C_V = \left[\left(\frac{5.125\ \text{in}}{5\ \text{in}}\right)\left(\frac{12\ \text{in}}{16.5\ \text{in}}\right)\left(\frac{21\ \text{ft}}{20\ \text{ft}}\right)\right]^{\frac{1}{20}} = 0.988 \qquad (C_L\ \text{controls})$$

Allowable stress:

$$F'_{bx} = 2760\ \text{psi}(C_V\ \text{or}\ C_L) = 2760\ \text{psi}(C_L) = 2760\ \text{psi}(0.907)$$

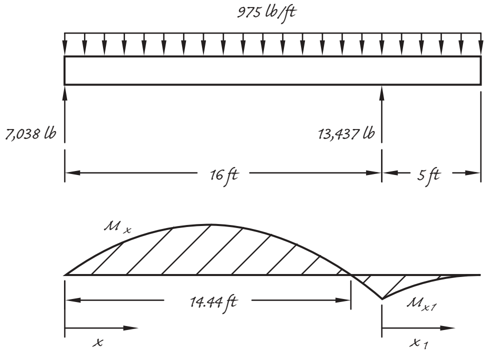
$$F'_{bx} = 2500\ \text{psi} > f_b = 2280\ \text{psi} \qquad \therefore\ \text{OK}$$

Result:

The 5 in. × 16.5 in. SP glulam beam is adequate to support the design loads.

CANTILEVER BEAM STABILITY (Equivalent Moment Method, ASD)
(See also Section 3.4.3.1, 4.3, 4.4)

Given: A $3\frac{1}{8}$ in. × 18 in., 24F-V4 DF, glulam floor beam has a main span of 16 ft with a cantilever span of 5 ft as illustrated. The beam will be braced to prevent lateral displacement along the top edge and rotation at the supports. The beam has been previously evaluated and is adequate for positive moment including pattern loading with the cantilever span unloaded.



The bending moment, including self weight, is defined as follows:

$$M_x = (7038\ lb)\ x - (487.5\ lb/ft)\ x^2$$
$$M_{x_1} = -\left[12{,}190\ ft\text{-}lb - (4875\ lb)\ x_1 + (487.5\ lb/ft)\ x_1^2\right]$$

Wanted: Determine beam stability factors for negative flexure in the main span and in the cantilever span, and evaluate the adequacy of the beam for negative flexure.

Solution:

Adjusted design values:

$$\bar{F}_{bx}^{-\prime} = \bar{F}_{bx}^{-} C_D C_M C_t\ (C_V\ or\ C_L) = 1850\ psi(1.0)(1.0)(1.0)(C_V\ or\ C_L)$$
$$\bar{F}_{bx}^{-\prime} = 1850\ psi(C_V\ or\ C_L)$$
$$E'_{y\ min} = E_{y\ min} C_M C_t = \left[0.85\left(10^6\right)\ psi\right](1.0)(1.0) = 0.85\left(10^6\right)\ psi$$

Maximum Negative Bending Moment and Corresponding Stress:

$$M^{-}_{max} = (7038\ lb)(16\ ft) - (487.5\ lb/ft)(16\ ft)^2$$
$$M^{-}_{max} = -12{,}190\ ft\text{-}lb = -146{,}300\ in\text{-}lb$$
$$f_b = \frac{6M}{bd^2} = \frac{6\ (146.3\ (10^3)\ \ in\text{-}lb)}{(3.125\ in)(18\ in)^2} = 867\ psi$$

Main Span Bending Moments, M_0, M_1, and M_{CL}:

$$M_0 = M^{-}_{max} = -12{,}190 \text{ ft-lb}$$

$$M_1 = 0 \text{ ft-lb}$$

$$M_{CL} = (7038 \text{ lb})(8 \text{ ft}) - (487.5 \text{ lb/ft})(8 \text{ ft})^2 = 25{,}100 \text{ ft-lb}$$

Beam Stability Factor (cantilever span):

$$k = 0.9$$

$$C_b = 2.05$$

$$\eta = \frac{1.3kd}{l_u} = \frac{1.3\,(0.9)(18 \text{ in})}{(5 \text{ ft})(12 \text{ in/ft})} = 0.351$$

$$C_e = \sqrt{\eta^2 + 1} - \eta = \sqrt{0.351^2 + 1} - 0.351 = 0.709$$

$$R_B = \sqrt{\frac{1.84 l_u d}{C_b C_e b^2}} = \sqrt{\frac{1.84\,(60 \text{ in})\ 18 \text{ in}}{(2.05)(0.709)(3.125 \text{ in})^2}}$$

$$R_B = \sqrt{140} = 11.83 \leq 50 \qquad \therefore \text{ OK}$$

$$F_{bE} = \frac{1.20\left(E'_{min}\right)}{R_B^2} = \frac{1.20\left(0.85\,(10^6)\ \text{psi}\right)}{140} = 7{,}286 \text{ psi}$$

$$\frac{F_{bE}}{F_b^*} = \frac{7{,}286 \text{ psi}}{1850 \text{ psi}} = 3.94$$

$$C_L = \frac{1 + F_{bE}/F_b^*}{1.9} - \sqrt{\left(\frac{1 + F_{bE}/F_b^*}{1.9}\right)^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$C_L = \frac{1 + 3.94}{1.9} - \sqrt{\left(\frac{1 + 3.94}{1.9}\right)^2 - \frac{3.94}{0.95}} = 0.984$$

Beam Stability Factor (negative moment in main span):

$$k = 1.72$$

$$C_b = 3.0 - \frac{2}{3}\left(\frac{M_1}{M_0}\right) - \frac{8}{3}\frac{M_{CL}}{(M_0 + M_1)^*}$$

*Take $M_1 = 0$ in this term if M_1 is negative.

$$C_b = 3.0 - \frac{2}{3}\left(\frac{0}{-12{,}190 \text{ ft-lb}}\right) - \frac{8}{3}\frac{25{,}100 \text{ ft-lb}}{(-12{,}190 \text{ ft-lb} + 0 \text{ ft-lb})} = 8.49$$

$$\eta = \frac{1.3kd}{l_u} = \frac{1.3\,(1.72)(18 \text{ in})}{(16 \text{ ft})(12 \text{ in/ft})} = 0.210$$

$$C_e = \sqrt{\eta^2 + 1} - \eta = \sqrt{0.210^2 + 1} - 0.210 = 0.812$$

$$R_B = \sqrt{\frac{1.84 l_u d}{C_b C_e b^2}} = \sqrt{\frac{1.84\,(192\text{ in})\;18\text{ in}}{(8.49)(0.812)(3.125\text{ in})^2}}$$

$$R_B = \sqrt{94.46} = 9.72 \leq 50 \quad \therefore \text{ OK}$$

$$F_{bE} = \frac{1.20\left(E'_{min}\right)}{R_B^2} = \frac{1.20\left(0.85\,(10^6)\text{ psi}\right)}{94.46} = 10{,}800\text{ psi}$$

$$\frac{F_{bE}}{F_b^*} = \frac{10{,}800\text{ psi}}{1850\text{ psi}} = 5.84$$

$$C_L = \frac{1 + F_{bE}/F_b^*}{1.9} - \sqrt{\left(\frac{1 + F_{bE}/F_b^*}{1.9}\right)^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$C_L = \frac{1 + 5.84}{1.9} - \sqrt{\left(\frac{1 + 5.84}{1.9}\right)^2 - \frac{5.84}{0.95}} = 0.990$$

Volume Factor (negative bending):

$$\left[\left(\frac{5.125\text{ in}}{b}\right)\left(\frac{12\text{ in}}{d}\right)\left(\frac{21\text{ ft}}{l}\right)\right]^{\frac{1}{10}}$$

$$= \left[\left(\frac{5.125\text{ in}}{3.125\text{ in}}\right)\left(\frac{12\text{ in}}{18\text{ in}}\right)\left(\frac{21\text{ ft}}{21\text{ ft} - 14.44\text{ ft}}\right)\right]^{\frac{1}{10}} = 1.13 \quad \therefore C_V = 1.0$$

Allowable Stress:

$$\bar{F}'_{bx} = 1850\text{ psi}(1.0)(1.0)(1.0)\;C_L = 1850\text{ psi}(0.984)$$

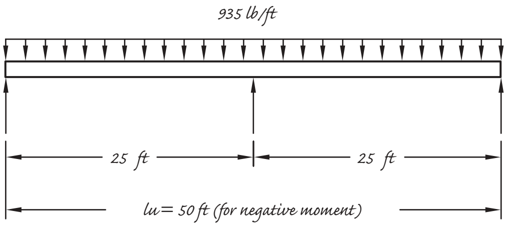
$$\bar{F}'_{bx} = 1820\text{ psi} > f_b = 867\text{ psi} \qquad \therefore \text{ OK}$$

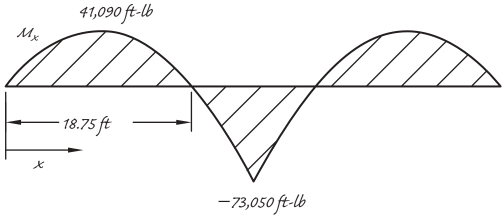
Result:

The 3.125 in. × 18 in. DF glulam beam is adequate for negative bending.

TWO-SPAN CONTINUOUS BEAM STABILITY (Equivalent Moment Method, ASD)
(See also Sections 3.4.3.1, 4.4)

Given: A $6\frac{3}{4}$ in. × 18 in. 24F-1.8E DF balanced glulam beam is continuous over two spans of 25 ft as illustrated. The beam is braced to prevent lateral displacement along the top edge and rotation at the end supports only. No lateral bracing is provided by the center support. The beam supports 935 lb/ft of combined snow and dead load, including self-weight.





The bending moment, including self weight, is defined as follows:

$$M_x = (8766\ lb)\,x - \left(467.5\ lb/ft\right)x^2$$

Wanted: Determine beam stability factor and evaluate the adequacy of the beam for negative flexure.

Solution:

Adjusted design values:

$$\bar{F}_{bx}' = \bar{F}_{bx} C_D C_M C_t\,(C_V \text{ or } C_L) = 2400\ psi(1.15)(1.0)(1.0)(C_V \text{ or } C_L)$$

$$\bar{F}_{bx}' = 2760\ psi(C_V \text{ or } C_L)$$

$$E_{y\ min}' = E_{y\ min} C_M C_t = \left[0.85\left(10^6\right)\ psi\right](1.0)(1.0) = 0.85\left(10^6\right)\ psi$$

Maximum Negative Bending Moment and Corresponding Stress:

$$\bar{M}_{max} = -73{,}050\ ft\text{-}lb$$

$$f_b = \frac{6M}{bd^2} = \frac{6\left(73{,}050\ ft\text{-}lb\right)\left(12\ in/ft\right)}{(6.75\ in)(18\ in)^2} = 2410\ psi$$

Beam Stability Factor (setting positive moments equal to zero):

$$C_b = \frac{12.5M_{max}}{3M_A + 4M_B + 3M_C + 2.5M_{max}}$$

$$C_b = \frac{12.5\left(73{,}050\ ft\text{-}lb\right)}{3\left(0\ ft\text{-}lb\right) + 4\left(73{,}050\ ft\text{-}lb\right) + 3\left(0\ ft\text{-}lb\right) + 2.5\left(73{,}050\ ft\text{-}lb\right)}$$

$$C_b = 1.92$$

$$C_e = 1.0 \qquad \left(\text{all loads are at brace points or on tension face}\right)$$

$$R_B = \sqrt{\frac{1.84 l_u d}{C_b C_e b^2}} = \sqrt{\frac{1.84\,(600\ in)\ 18\ in}{(1.92)(1.0)(6.75\ in)^2}}$$

$$R_B = \sqrt{227.2} = 15.1 \leq 50 \qquad \therefore OK$$

$$F_{bE} = \frac{1.20\left(E'_{min}\right)}{R_B^2} = \frac{1.20\left(0.85\ (10^6)\ psi\right)}{227.2} = 4{,}489\ psi$$

$$\frac{F_{bE}}{F_b^*} = \frac{4{,}489\ psi}{2760\ psi} = 1.63$$

$$C_L = \frac{1 + \frac{F_{bE}}{F_b^*}}{1.9} - \sqrt{\left(\frac{1 + \frac{F_{bE}}{F_b^*}}{1.9}\right)^2 - \frac{\frac{F_{bE}}{F_b^*}}{0.95}}$$

$$C_L = \frac{1 + 1.63}{1.9} - \sqrt{\left(\frac{1 + 1.63}{1.9}\right)^2 - \frac{1.63}{0.95}}$$

$$C_L = 0.94$$

Volume Factor (negative bending):

$$C_V = \left[\left(\frac{5.125\ in}{b}\right)\left(\frac{12\ in}{d}\right)\left(\frac{21\ ft}{l}\right)\right]^{\frac{1}{10}} \leq 1.0$$

$$C_V = \left[\left(\frac{5.125\ in}{6.75\ in}\right)\left(\frac{12\ in}{18\ in}\right)\left(\frac{21\ ft}{50\ ft - 2\left(18.75\ ft\right)}\right)\right]^{\frac{1}{10}} = 0.98 \leq 1.0$$

$$C_V = 0.98 \qquad (C_L \text{ controls})$$

Allowable Stress:

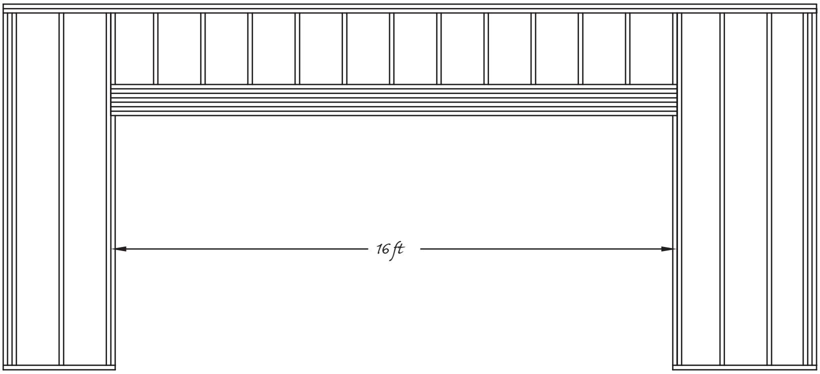
$$\overline{F}_{bx}' = \left(2760\ psi\right) C_L = 2760\ psi\,(0.934) = 2590\ psi > f_b = 2410\ psi \qquad \therefore OK$$

Result:

The $6\frac{3}{4}$ in. × 18 in. DF glulam beam is adequate for negative bending.

BIAXIAL BENDING (ASD Method)
(See also Section 4.5)

Given: A 5.125 in. × 10.5 in. 24F-V4 DF glulam garage door header was selected based on the D+S load combination. The beam spans 16 ft and is framed as a dropped header (see illustration) with no lateral support except at the beam ends. The beam is in dry conditions and normal temperatures.



Loads on the beam:

$w_W = 125\ plf \quad (y\text{-}y\ direction)$

$w_S = 300\ plf \quad (x\text{-}x\ direction)$

$w_D = 150\ plf \quad (x\text{-}x\ direction)$

Wanted: Evaluate the beam subject to the $D + 0.75S + 0.75W$ load combination.

Solution:

Adjusted design values:

$$F'_{bx} = F_{bx}C_D C_M C_t\ (C_V\ or\ C_L) = 2400\ psi(1.6)(1.0)(1.0)\ C_L = 3840\ psi(C_L)$$

$$F'_{by} = F_{by}C_D C_M C_t C_{fu} = 1450\ psi(1.6)(1.0)(1.0)\left(\frac{12\ in}{5.125\ in}\right)^{\frac{1}{9}} = 2550\ psi$$

$$E'_x = E_x C_M C_t = \left(1.8\left(10^6\right)\ psi\right)(1.0)(1.0) = 1.8\left(10^6\right)\ psi$$

$$E'_y = E_y C_M C_t = \left(1.6\left(10^6\right)\ psi\right)(1.0)(1.0) = 1.6\left(10^6\right)\ psi$$

$$E'_{y\ min} = E_{y\ min} C_M C_t = \left(0.85\left(10^6\right)\ psi\right)(1.0)(1.0) = 0.85\left(10^6\right)\ psi$$

Section Properties:

$$S_x = \frac{bd^2}{6} = \frac{(5.125\ in)(10.5\ in)^2}{6} = 94.2\ in^3$$

$$I_x = \frac{bd^3}{12} = \frac{(5.125\ in)(10.5\ in)^3}{12} = 494\ in^3$$

$$S_y = \frac{db^2}{6} = \frac{(10.5\ in)(5.125\ in)^2}{6} = 46.0\ in^3$$

$$I_y = \frac{db^3}{12} = \frac{(10.5\ in)(5.125\ in)^3}{12} = 118\ in^3$$

Beam self-weight:

$$w_{SW} = bd\gamma = (5.125\ in)(10.5\ in)\left(33\frac{lb}{ft^3}\right)\left(\frac{1\ ft^3}{144\ in^2}\right)$$

$$w_{SW} = 12.3\ plf \quad (use\ 13\ plf)$$

Beam stability factor:

$$\frac{l_u}{d} = \frac{192\ in}{10.5\ in} = 18.3 > 7$$

$$l_e = 1.63 l_u + 3d = 1.63\,(192\ in) + 3\,(10.5\ in) = 345\ in$$

$$R_B = \sqrt{\frac{l_e d}{b^2}} = \sqrt{\frac{(345\ in)\ 10.5\ in}{(5.125\ in)^2}} = \sqrt{137.9} = 11.7$$

$$F_{bE} = \frac{1.20\left(E'_{min}\right)}{R_B^2} = \frac{1.20\left(0.85\,(10^6)\ psi\right)}{137.9} = 7397\ psi$$

$$\frac{F_{bE}}{F_b^*} = \frac{7397\ psi}{3840\ psi} = 1.93$$

$$C_L = \frac{1 + F_{bE}/F_b^*}{1.9} - \sqrt{\left(\frac{1 + F_{bE}/F_b^*}{1.9}\right)^2 - \frac{F_{bE}/F_b^*}{0.95}}$$

$$C_L = \frac{1 + 1.93}{1.9} - \sqrt{\left(\frac{1 + 1.93}{1.9}\right)^2 - \frac{1.93}{0.95}} = 0.953$$

$$F'_{bx} = 3840\ psi\ (C_L) = 3840\ psi\,(0.953) = 3660\ psi$$

x-x axis bending moments and stresses:

$$M_{x,D+0.75S} = \frac{(w_{SW} + w_D + 0.75 w_S)\ l^2}{8}$$

$$M_{x,D+0.75S} = \frac{\left(13\ plf + 150\ plf + 0.75\left(300\ plf\right)\right)\left(16\ ft\right)^2}{8}$$

$$M_{x,D+0.75S} = 14{,}820\ ft\text{-}lb = 177{,}800\ in\text{-}lb$$

$$f_{bx,D+0.75S} = \frac{M_{x,D+0.75S}}{S_x} = \frac{177{,}800\ in\text{-}lb}{94.2\ in^3} = 2000\ psi$$

y-y axis bending moments and stresses:

$$M_{y,0.75W} = \frac{(0.75w_W)\,l^2}{8} = \frac{\left(0.75\left(125\ plf\right)\right)\left(16\ ft\right)^2}{8}$$

$$M_{y,0.75W} = 3000\ ft\text{-}lb = 36{,}000\ in\text{-}lb$$

$$f_{by,0.75W} = \frac{M_{y,0.75w}}{S_y} = \frac{36{,}000\ in\text{-}lb}{46.0\ in^3} = 783\ psi$$

Biaxial bending $(D + 0.75S + 0.75w)$:

$$\frac{f_{bx,D+0.75S}}{F'_{bx}} + \frac{f_{by,0.75W}}{F'_{by}\left[1-\left(\dfrac{f_{bx,D+0.75S}}{F_{bE}}\right)^2\right]}$$

$$= \frac{2000\ psi}{3660\ psi} + \frac{783\ psi}{\left(2550\ psi\right)\left[1-\left(\dfrac{2000\ psi}{7397\ psi}\right)^2\right]} = 0.88 < 1.0 \quad \therefore\ OK$$

Result:

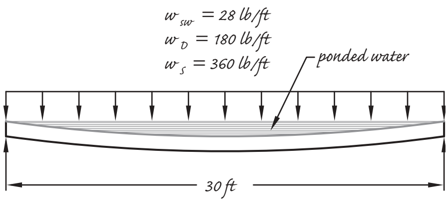
The $5\frac{1}{8}$ in. $\times$ 10.5 in. beam is adequate to support the combined bending loads.

BEAM WITH PONDING LOAD (ASD Method)
(See also Section 3.7)

Given: $6\frac{3}{4}$ in. × 18 in. 24F-1.8E DF glulam beams are proposed to support a flat roof with a dead load of 15 psf and a snow load of 30 psf. The beams are spaced at 12 ft o.c. and span 30 ft. Full lateral support is provided to prevent beam buckling. The beams are subject to dry use and normal temperatures. The beams are adequate to support the design loads without consideration of ponding.

Wanted: Considering the effect of rain ponding on the roof in conjunction with the maximum snow load, evaluate the existing beam size. If the beams are inadequate, determine the required size.

Solution:



Design values are obtained from AITC 117 or NDS:

$$E'_x = E_x C_M C_t = 1.8\left(10^6\right) \text{psi}\,(1.0)(1.0) = 1.8\left(10^6\right) \text{psi}$$

$$E'_{x,05} = E'_x\,(1 - 1.645\,COV_E) = 1.8\left(10^6 \text{psi}\right)(1 - 1.645\,(0.10)) = 1.5\left(10^6 \text{ psi}\right)$$

$$F'_{vx} = F_{vx} C_D C_M C_t = 265 \text{ psi}\,(1.15)(1.0)(1.0) = 305 \text{ psi}$$

$$C_V = \left[\left(\frac{5.125 \text{ in}}{b}\right)\left(\frac{12 \text{ in}}{d}\right)\left(\frac{21 \text{ ft}}{L}\right)\right]^{\frac{1}{10}}$$

$$C_V = \left[\left(\frac{5.125 \text{ in}}{6.75 \text{ in}}\right)\left(\frac{12 \text{ in}}{18 \text{ in}}\right)\left(\frac{21 \text{ ft}}{30 \text{ ft}}\right)\right]^{\frac{1}{10}} = 0.901$$

$$F'_{bx} = F_{bx} C_D C_M C_t\,(C_V \text{ or } C_L) = 2400 \text{ psi}\,(1.15)(1.0)(1.0)\,C_V$$

$$F'_{bx} = 2760 \text{ psi}\,(0.901) = 2490 \text{ psi}$$

Section properties:

$$I_x = \frac{bd^3}{12} = \frac{(6.75 \text{ in})(18 \text{ in})^3}{12} = 3280 \text{ in}^4$$

$$S_x = \frac{bd^2}{6} = \frac{(6.75 \text{ in})(18 \text{ in})^2}{6} = 365 \text{ in}^3$$

$$w_{SW} = bd\left(33 \text{ pcf}\right) = (6.75 \text{ in})(18 \text{ in})\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)\left(33\frac{\text{lb}}{\text{ft}^3}\right) = 28\frac{\text{lb}}{\text{ft}}$$

Specific weight of ponding fluid (water):

$$\gamma = 62.4\frac{lb}{ft^3} = 0.036\frac{lb}{in^3}$$

Ponding magnification factor:

$$MF = \frac{1}{\left(1 - \frac{\lambda\gamma S L^4}{\pi^4 E'_{x,05} I_x}\right)} = \frac{1}{\left(1 - \frac{1.5\,(0.036\ lb/in^3)(144\ in)(360\ in)^4}{\pi^4\,(1.5\,(10^6\ lb/in^2))(3280\ in^4)}\right)}$$

$$MF = 1.38$$

Shear stress:

$$V = (MF)\left(\frac{wL}{2}\right) = (MF)\,\frac{(w_{SW} + w_D + w_S)\,L}{2}$$

$$V = (1.38)\,\frac{\left(28\ lb/ft + 180\ lb/ft + 360\ lb/ft\right)\left(30\ ft\right)}{2}$$

$$V = 11,800\ lb$$

$$f_{vx} = \frac{3V}{2bd} = \frac{3\,(11,800\ lb)}{2\,(6.75\ in)(18\ in)} = 146\ psi \le F'_{vx} = 305\ psi \quad \therefore\ OK$$

Bending stress:

$$M = (MF)\,\frac{wL^2}{8} = (MF)\,\frac{(w_{SW} + w_D + w_S)\,L^2}{8}$$

$$M = (1.38)\,\frac{\left(28\ lb/ft + 180\ lb/ft + 360\ lb/ft\right)\left(30\ ft\right)^2}{8}$$

$$M = 88,180\ ft\text{-}lb = 1.058\left(10^6\right)\ in\text{-}lb$$

$$f_{bx} = \frac{M}{S} = \frac{1.058\,(10^6)\ in\text{-}lb}{365\ in^3} = 2900\ psi > F'_{bx} = 2490\ psi$$

Try $6\frac{3}{4}$ in. × $19\frac{1}{2}$ in. section

$$w_{SW} = bd\left(33\ pcf\right) = (6.75\ in)(19.5\ in)\left(\frac{1\ ft^2}{144\ in^2}\right)\left(33\ lb/ft^3\right) = 30\ lb/ft$$

Section properties:

$$I_x = \frac{bd^3}{12} = \frac{(6.75\ in)(19.5\ in)^3}{12} = 4170\ in^4$$

$$S_x = \frac{bd^2}{6} = \frac{(6.75\ in)(19.5\ in)^2}{6} = 428\ in^3$$

Allowable bending stress:

$$C_V = \left[\left(\frac{5.125\ in}{b}\right)\left(\frac{12\ in}{d}\right)\left(\frac{21\ ft}{L}\right)\right]^{\frac{1}{10}}$$

$$C_V = \left[\left(\frac{5.125\ in}{6.75\ in}\right)\left(\frac{12\ in}{19.5\ in}\right)\left(\frac{21\ ft}{30\ ft}\right)\right]^{\frac{1}{10}} = 0.894$$

$$F'_{bx} = F_{bx} C_D C_M C_t\ (C_V \text{ or } C_L) = 2400\ psi (1.15)(1.0)(1.0)\ C_V$$

$$F'_{bx} = 2760\ psi (0.894) = 2470\ psi$$

Ponding magnification factor:

$$MF = \frac{1}{\left(1 - \dfrac{\lambda \gamma S L^4}{\pi^4 E'_{x,05} I_x}\right)} = \frac{1}{\left(1 - \dfrac{1.5\ (0.0361\ lb/in^3)(144\ in)(360\ in)^4}{\pi^4\ (1.5\ (10^6\ lb/in^2))(4170\ in^4)}\right)}$$

$$MF = 1.27$$

Bending stress:

$$M = (MF)\ \frac{wL^2}{8} = (MF)\ \frac{(w_{SW} + w_D + w_S)\ L^2}{8}$$

$$M = (1.27)\ \frac{\left(30\ lb/ft + 180\ lb/ft + 360\ lb/ft\right)\left(30\ ft\right)^2}{8}$$

$$M = 81{,}450\ ft\text{-}lb = 977\left(10^3\right)\ in\text{-}lb$$

$$f_{bx} = \frac{M}{S} = \frac{977\ (10^3)\ in\text{-}lb}{428\ in^3} = 2280\ psi < F'_{bx} = 2470\ psi \quad \therefore\ OK$$

Result:

The $6\frac{3}{4}$ in. × 18 in. section was adequate to support the design loads without consideration of ponding, but it was not adequate to support the ponded water in addition to the design loads. A $6\frac{3}{4}$ in. × $19\frac{1}{2}$ in. section is required to support the design loads plus the ponded water.

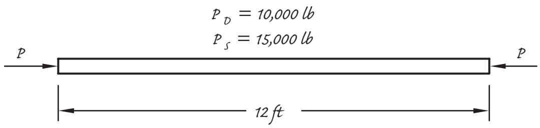
COMPRESSION WEB DESIGN (ASD Method)
(See also Sections 3.4.3.9, 5.2, 5.3)

Given: A truss web with total compression load of 25 kip ($D = 10$ kip and $S = 15$ kip) will be used in dry conditions with normal temperatures. The web is 12 ft long and will not be braced except at the ends. The truss will be manufactured using 5 in. wide structural glued laminated timber members. Load will be transferred into the chord by two rows of 1 in. diameter bolts loaded in double shear.

Wanted: Design the web to support the required load using SP glulam combination 47 1:8.

Approach: Truss web will be modeled as pinned at both ends ($K_e = 1.0$). Trial size will be selected such that f_c is somewhat less than F_{CE}. The trial size will be checked with the column stability factor and the gross section properties and will also be checked at the net section without the column stability factor.

Solution:



Determine minimum depth for two rows of 1 in. bolts:

$$d_{min} = 1.5D + 3D + 1.5D = 1.5\,(1\ in) + 3\,(1\ in) + 1.5\,(1\ in) = 6\ in$$

Assuming 4 laminations or more, design values are obtained from AITC 117 or NDS:

$$F'_c = F^*_c C_P = F_c C_D C_M C_t C_P = 1500\ psi\ (1.15)(1.0)(1.0)\ C_P = 1725\ psi\ (C_P)$$

$$E'_{x\ min} = E'_{y\ min} = E_{y\ min} C_M C_t = 0.74\left(10^6\right) psi\ (1.0)(1.0) = 0.74\left(10^6\right) psi$$

Because $l_{ey} = l_{ex}$ and $E_{y\ min} = E_{x\ min}$, buckling about the weak geometric axis (y-y axis in this case) will control.

$$\frac{l_{ey}}{b} = \frac{144\ in}{5\ in} = 28.8$$

$$F_{cEy} = \frac{0.822 E'_{y\ min}}{\left(\frac{l_{ey}}{b}\right)^2}$$

$$F_{cEy} = \frac{0.822\left[0.74\left(10^6\right) psi\right]}{(28.8)^2}$$

$$F_{cEy} = 733\ psi$$

Try 6.875 in. deep member (5 lams @ 1.375 in. per lam)

$$f_c = \frac{P}{bd} = \frac{25,000 \text{ lb}}{(5 \text{ in})(6.875 \text{ in})}$$

$$f_c = 727 \text{ psi} \approx F_{cEy} = 733 \text{ psi} \quad \therefore \text{ Larger section probably needed}$$

Try 8.25 in. deep member (6 lams @ 1.375 in. per lam)

$$f_c = \frac{P}{bd} = \frac{25,000 \text{ lb}}{(5 \text{ in})(8.25 \text{ in})} = 606 \text{ psi} < F_{cEy} = 733 \text{ psi} \quad \therefore \text{ Try this section.}$$

Calculate column stability factor:

$$C_P = \frac{1 + F_{cE}/F_c^*}{2c} - \sqrt{\left(\frac{1 + F_{cE}/F_c^*}{2c}\right)^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$C_P = \frac{1 + 733 \text{ psi}/1725 \text{ psi}}{2(0.9)} - \sqrt{\left(\frac{1 + 733 \text{ psi}/1725 \text{ psi}}{2(0.9)}\right)^2 - \frac{733 \text{ psi}/1725 \text{ psi}}{0.9}}$$

$$C_P = 0.399$$

Calculate the allowable compression stress and stress ratio:

$$F_c' = F_c^* C_P = 1725 \text{ psi}(0.399) = 688 \text{ psi}$$

$$\frac{f_c}{F_c'} = \frac{606 \text{ psi}}{688 \text{ psi}} = 0.88 \le 1.0 \quad \therefore \text{ OK for compression.}$$

Check the compression stresses on the net section at the connection:

$$A_{net} = b(d - 2D_h) = (5 \text{ in})(8.25 \text{ in} - 2(1.0625 \text{ in})) = 30.6 \text{ in}$$

$$f_{cnet} = \frac{P}{A_{net}} = \frac{25,000 \text{ lb}}{30.6 \text{ in}^2} = 817 \text{ psi}$$

$$\frac{f_{cnet}}{F_c^*} = \frac{817 \text{ psi}}{1725 \text{ psi}} = 0.474 \le 1.0 \quad \therefore \text{ OK for compression.}$$

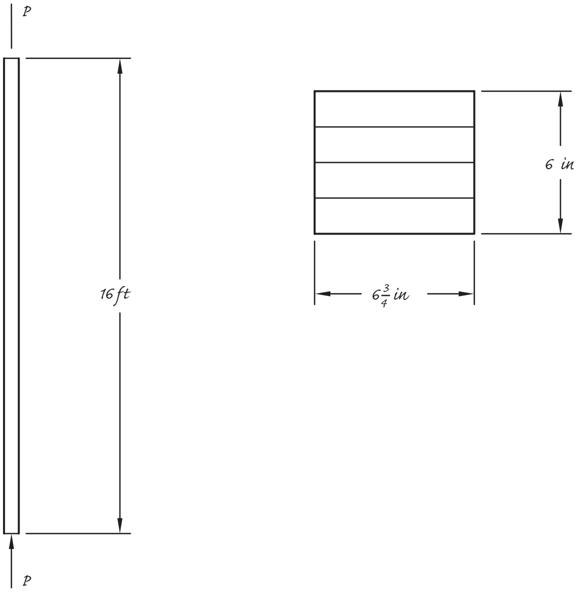
Result:

5 in. × $8\frac{1}{4}$ in. Combination 47 1:8 SP glulam timber is adequate for the stated conditions. The stress ratios are:

$$\frac{f_c}{F_c'} = 0.881 \qquad \frac{f_{cnet}}{F_c^*} = 0.474$$

COLUMN WITH CENTRIC LOAD, BEAM LAY-UP (ASD Method)
(See also Sections 3.4.3.9, 5.2, 5.3)

Given: A $6\frac{3}{4}$ in. × 6 in. 24F-V8 DF glulam column supports an upper floor. The column height is 16 ft. The ends are held in position, but no other bracing occurs along the length. The column is subject to wet-use and normal temperatures.



Wanted: Determine the allowable capacity of the column.

Solution:

Design values are obtained from AITC 117 or NDS:

$$F_c' = F_c^* C_P = F_c C_D C_M C_t C_P = 1650\ psi\ (1.0)(0.73)(1.0)\ C_P = 1205\ psi\ (C_P)$$

$$E_{x\ min}' = E_{x\ min} C_M C_t = 0.95\left(10^6\right) psi\ (0.833)(1.0) = 0.79\left(10^6\right) psi$$

$$E_{y\ min}' = E_{y\ min} C_M C_t = 0.85\left(10^6\right) psi\ (0.833)(1.0) = 0.71\left(10^6\right) psi$$

Effective length ratios:

$$\frac{l_{ex}}{d} = \frac{192\ in}{6\ in} = 32 \leq 50 \quad \therefore\ OK$$

$$\frac{l_{ey}}{b} = \frac{192\ in}{6.75\ in} = 28.4 \leq 50 \quad \therefore\ OK$$

Critical buckling design values:

$$F_{cEx} = \frac{0.822E'_{x\ min}}{\left(\frac{l_{ex}}{d}\right)^2} = \frac{0.822\left[0.79\left(10^6\right)\ psi\right]}{(32)^2}$$

$$\boxed{F_{cEx} = 634\ psi}$$

$$F_{cEy} = \frac{0.822E'_{y\ min}}{\left(\frac{l_{ey}}{b}\right)^2} = \frac{0.822\left[0.71\left(10^6\right)psi\right]}{(28.4)^2}$$

$$F_{cEy} = 724\ psi$$

Column stability factor (use lowest value of F_{cE}):

$$C_P = \frac{1 + F_{cE}/F'_c}{2c} - \sqrt{\left(\frac{1 + F_{cE}/F^*_c}{2c}\right)^2 - \frac{F_{cE}/F^*_c}{c}}$$

$$C_P = \frac{1 + \frac{634\ psi}{1205\ psi}}{2\left(0.9\right)} - \sqrt{\left(\frac{1 + \frac{634\ psi}{1205\ psi}}{2\left(0.9\right)}\right)^2 - \frac{\frac{634\ psi}{1205\ psi}}{0.9}}$$

$$C_P = 0.481$$

Allowable compression stress:

$$F'_c = F^*_c C_P = 1205\ psi\left(0.481\right) = 580\ psi$$

Allowable capacity:

$$P' = F'_c bd = \left(580\ psi\right)\left(6.75\ in\right)\left(6\ in\right) = 23,500\ lb$$

Result:

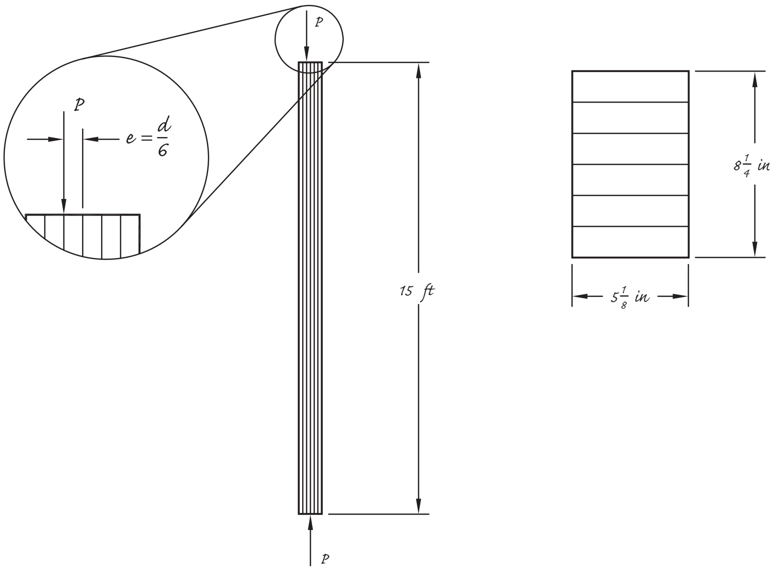
$6\frac{3}{4}$ in. × 6 in. 24F-V8 DF column subject to wet-use can support a total floor load of 23,500 lb. The column should be pressure-treated with an appropriate preservative to protect against decay in the wet environment.

COLUMN WITH ECCENTRIC LOAD, BEAM LAY-UP (ASD Method)
(See also Sections 3.4.3.9, 6.1.2, 6.5)

Given: A $5\frac{1}{8}$ in. × $8\frac{1}{4}$ in. × 15 ft 24F-V8 SP column is continuously braced to prevent buckling about its weak axis (y-y axis). The column supports the end of a beam, which will apply an axial load consisting of D = 8000 lb and S = 18000 lb. The ends are held in position, but no other bracing occurs to prevent buckling about the strong axis (x-x axis). The column will be subject to dry-use and normal temperatures.

Wanted: Determine the suitability of the column assuming that the load will be applied with an eccentricity of d/6 = 1.375 in.

Solution:



Design values are obtained from AITC 117 or NDS:

$$F_c' = F_c^* C_P = F_c C_D C_M C_t C_P = 1650\ psi\ (1.15)(1.0)(1.0)\ C_P = 1900\ psi\ (C_P)$$

$$E_{x\ min}' = E_{x\ min} C_M C_t = 0.95\left(10^6\right) psi\ (1.0)(1.0) = 0.95\left(10^6\right) psi$$

Effective length ratio:

$$\frac{l_{ex}}{d} = \frac{K_e l_u}{d} = \frac{(1.0)(180\ in)}{8.25\ in} = 21.8 \leq 50 \quad \therefore\ OK$$

Critical buckling design value:

$$F_{cEx} = \frac{0.822 E_{x\ min}'}{\left(\frac{l_{ex}}{d}\right)^2} = \frac{0.822\left[0.95\left(10^6\right) psi\right]}{(21.8)^2} = 1640\ psi$$

Column stability factor:

$$C_P = \frac{1 + F_{cE}/F_c^*}{2c} - \sqrt{\left(\frac{1 + F_{cE}/F_c^*}{2c}\right)^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$C_P = \frac{1 + \frac{1640\ psi}{1900\ psi}}{2(0.9)} - \sqrt{\left(\frac{1 + \frac{1640\ psi}{1900\ psi}}{2(0.9)}\right)^2 - \frac{\frac{1640\ psi}{1900\ psi}}{0.9}} = 0.700$$

Allowable compression stress:

$$F_c' = F_c^* C_P = 1900\ psi\,(0.700) = 1330\ psi$$

Compression stress due to applied load:

$$f_c = \frac{P}{bd} = \frac{26000\ lb}{(5.125\ in)(8.25\ in)} = 615\ psi$$

Allowable bending stress:

$$C_V = \left(\frac{5.125\ in}{b}\right)^{\frac{1}{20}} \left(\frac{12\ in}{d}\right)^{\frac{1}{20}} \left(\frac{21\ ft}{L}\right)^{\frac{1}{20}} \le 1.0$$

$$C_V = \left(\frac{5.125\ in}{5.125\ in}\right)^{\frac{1}{20}} \left(\frac{12\ in}{8.25\ in}\right)^{\frac{1}{20}} \left(\frac{21\ ft}{15\ ft}\right)^{\frac{1}{20}} = 1.04 \quad \therefore \text{ use } C_V = 1.0$$

$$F_{bx}' = F_{bx} C_D C_M C_t C_V = 2400\ psi\,(1.15)(1.0)(1.0)(1.0) = 2760\ psi$$

Combined stresses:

$$\left(\frac{f_c}{F_c'}\right)^2 + \frac{f_c\left(\frac{6e_1}{d}\right)\left[1 + 0.234\left(\frac{f_c}{F_{cE1}}\right)\right]}{F_{b1}'\left[1 - \frac{f_c}{F_{cE1}}\right]} \overset{???}{\le} 1.0$$

$$\left(\frac{615\ psi}{1330\ psi}\right)^2 + \frac{(615\ psi)\left(\frac{6\,(1.375\ in)}{8.25\ in}\right)\left[1 + 0.234\left(\frac{615\ psi}{1640\ psi}\right)\right]}{(2760\ psi)\left[1 - \frac{615\ psi}{1640\ psi}\right]} \overset{???}{\le} 0$$

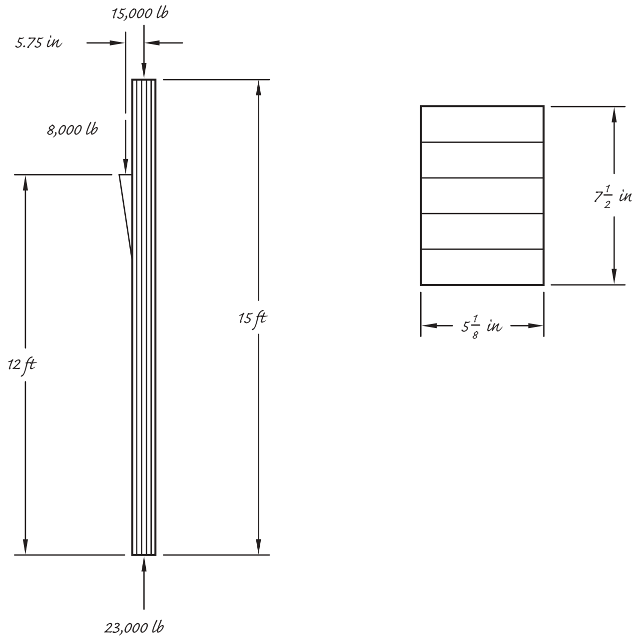
$$0.214 + 0.388 = 0.602 < 1.0 \quad \therefore \text{ OK}$$

Result:

$5\frac{1}{8}$ in. × $8\frac{1}{4}$ in. 24F-V8 SP column is adequate.

COLUMN WITH SIDE BRACKET, UNIFORM GRADE LAYUP (ASD Method)
(See also Section 6.7)

Given: *A $5\frac{1}{8}$ in. × $7\frac{1}{2}$ in. × 15 ft Combination 2 DF column is continuously braced to prevent buckling about its weak axis (y-y axis). The column supports a centric axial load of 8,000 lb and a 4,000 lb load on a side bracket as illustrated. The loads are primarily due to snow. The ends are held in position, but no other bracing prevents buckling about the strong axis (x-x axis). The column will be subject to dry-use and normal temperatures.*



Wanted: *Determine the suitability of the column to resist the applied loads.*

Solution:

Design values are obtained from AITC 117 or NDS:

$$F_c' = F_c^* C_P = F_c C_D C_M C_t C_P = 1950 \text{ psi } (1.15)(1.0)(1.0)\ C_P = 2240 \text{ psi } (C_P)$$

$$E'_{x\ min} = E_{x\ min} C_M C_t = 0.85\left(10^6\right) \text{ psi } (1.0)(1.0) = 0.85\left(10^6\right) \text{ psi}$$

Effective Length Ratio:

$$\frac{l_{ex}}{d} = \frac{K_e l_u}{d} = \frac{(1.0)(180 \text{ in})}{7.5 \text{ in}} = 24 \le 50 \quad \therefore \text{OK}$$

COLUMN WITH SIDE BRACKET, UNIFORM GRADE LAYUP (ASD Method) continued ...

Critical Buckling Design Value:

$$F_{cEx} = \frac{0.822E'_{x\ min}}{\left(\frac{l_{ex}}{d}\right)^2} = \frac{0.822\left[0.85\left(10^6\right) psi\right]}{(24)^2} = 1210\ psi$$

Column stability factor:

$$C_P = \frac{1 + F_{cE}/F^*_c}{2c} - \sqrt{\left(\frac{1 + F_{cE}/F^*_c}{2c}\right)^2 - \frac{F_{cE}/F^*_c}{c}}$$

$$C_P = \frac{1 + 1210\ psi/2240\ psi}{2(0.9)} - \sqrt{\left(\frac{1 + 1210\ psi/2240\ psi}{2(0.9)}\right)^2 - \frac{1210\ psi/2240\ psi}{0.9}} = 0.492$$

Allowable compression stress:

$$F'_c = F^*_c C_P = 2240\ psi\ (0.492) = 1100\ psi$$

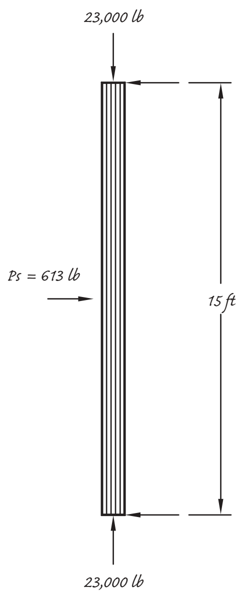
Allowable bending stress:

$$C_V = \left(\frac{5.125\ in}{b}\right)^{\frac{1}{10}} \left(\frac{12\ in}{d}\right)^{\frac{1}{10}} \left(\frac{21\ ft}{L}\right)^{\frac{1}{10}} \le 1.0$$

$$C_V = \left(\frac{5.125\ in}{5.125\ in}\right)^{\frac{1}{10}} \left(\frac{12\ in}{7.5\ in}\right)^{\frac{1}{10}} \left(\frac{21\ ft}{15\ ft}\right)^{\frac{1}{10}} = 1.08 \quad \therefore\ use\ C_V = 1.0$$

$$F'_{bx} = F_{bx} C_D C_M C_t C_V = 1700\ psi\ (1.15)(1.0)(1.0)(1.0) = 1960\ psi$$

Model as Beam-Column with Side Load (see drawing):



$$P_s = \frac{3Pal_P}{l^2} = \frac{3\ (8,000\ lb)(5.75\ in)(144\ in)}{(180\ in)^2} = 613\ lb$$

Compression stress due to applied loads:

$$f_c = \frac{P}{bd} = \frac{23,000\ lb}{(5.125\ in)(7.5\ in)} = 598\ psi$$

Bending stress due to applied loads:

$$f_b = \frac{6P_s l}{4bd^2} = \frac{6\ (613\ lb)\ 180\ in}{4\ (5.125\ in)(7.5\ in)^2} = 574\ psi$$

Check combined stresses:

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{bx}}{F'_{bx}\left[1-\left(\frac{f_c}{F_{cEx}}\right)\right]} \overset{???}{\leq} 1.0$$

$$\left(\frac{598\ psi}{1100\ psi}\right)^2 + \frac{574\ psi}{(1960\ psi)\left[1-\left(\frac{598\ psi}{1210\ psi}\right)\right]} \overset{???}{\leq} 1.0$$

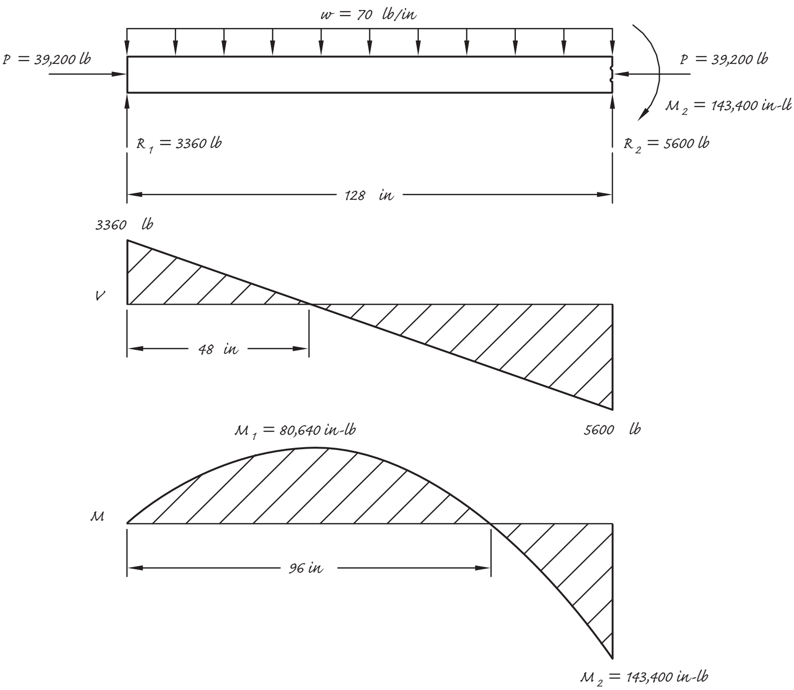
$$0.296 + 0.579 = 0.875 < 1.0 \quad \therefore \text{ GOOD}$$

Result:

$5\frac{1}{8}$ in. × $7\frac{1}{2}$ in. Combination 2 DF Column is adequate.

CONTINUOUS TRUSS CHORD, BEAM LAY-UP (ASD Method)
(See also Sections 6.4, 12.3.1)

Given: A 5 in. × $9\frac{5}{8}$ in. 24F-V5 SP glulam beam is used as a top chord of a truss. The truss chord is continuous across panel points and is continuously braced to prevent buckling about its weak axis (y-y axis). Structural analysis gives the loads shown below for the bottom segment of the truss chord (see illustration). The section at the panel point is reduced to accommodate two $\frac{3}{4}$ in. diameter bolts placed 3.25 inches from the top and bottom edges. The truss is subject to snow loads (plus dead) dry-use and normal temperatures.



Wanted: Evaluate the truss chord for the combined compression and bending stresses.

Solution: The combined stresses must be checked at two locations: 1) point of maximum positive moment considering column stability (buckling in the x-x direction), and 2) point of maximum negative moment considering reduced net section at the panel point.

Design values (AITC 117 or NDS):

$$F_c' = F_c^* C_P = F_c C_D C_M C_t C_P = 1650\ psi\ (1.15)(1.0)(1.0)\ C_P = 1900\ psi\ (C_P)$$

$$F_{bx}' = F_{bx} C_D C_M C_t\ (C_L \text{ or } C_V) = 2400\ psi\ (1.15)(1.0)(1.0)(1.0) = 2760\ psi$$

$$E_{x\ min}' = E_{x\ min} C_M C_t = 0.95\left(10^6\right) psi\ (1.0)(1.0) = 0.95\left(10^6\right) psi$$

POINT OF MAXIMUM POSITIVE MOMENT:

Effective Length Ratio:

$$\frac{l_{ex}}{d} = \frac{K_e l_u}{d} = \frac{(0.8)(125\ in)}{9.625\ in} = 10.5 < 50 \quad \therefore\ OK$$

Critical Buckling Design Value:

$$F_{cEx} = \frac{0.822 E'_{x\ min}}{\left(\frac{l_{ex}}{d}\right)^2} = \frac{0.822\left[0.95\left(10^6\right)\ psi\right]}{(10.6)^2} = 6950\ psi$$

Column stability factor:

$$C_P = \frac{1 + F_{cE}/F_c^*}{2c} - \sqrt{\left(\frac{1 + F_{cE}/F_c^*}{2c}\right)^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$C_P = \frac{1 + \frac{6950 psi}{1900\ psi}}{2(0.9)} - \sqrt{\left(\frac{1 + \frac{6950 psi}{1900\ psi}}{2(0.9)}\right)^2 - \frac{\frac{6950 psi}{1900\ psi}}{0.9}}$$

$$C_P = 0.965$$

Allowable compression stress:

$$F'_c = F_c^* C_P = 1900\ psi\ (0.965) = 1830\ psi$$

Compression stress due to applied load:

$$f_c = \frac{P}{bd} = \frac{39,200\ lb}{(5\ in)(9.625\ in)} = 815\ psi$$

Bending stress due to applied load:

$$f_b = \frac{6M}{bd^2} = \frac{6\ (80,640\ in\text{-}lb)}{(5\ in)(9.625\ in)^2} = 1050\ psi$$

Combined stresses:

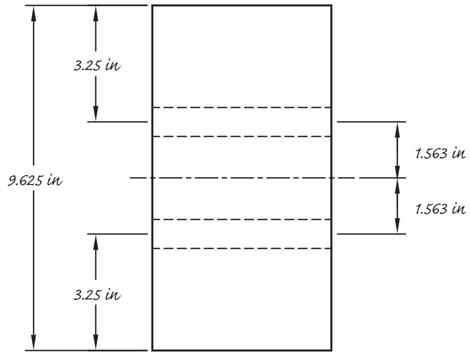
$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_b}{F'_{b1}\left[1 - \left(\frac{f_c}{F_{cE1}}\right)\right]} \overset{???}{\leq} 1.0$$

$$\left(\frac{815\ psi}{1830\ psi}\right)^2 + \frac{1050\ psi}{(2760\ psi)\left[1 - \left(\frac{815\ psi}{6950\ psi}\right)\right]} \overset{???}{\leq} 1.0$$

$$0.198 + 0.431 = 0.629 < 1.0 \quad \therefore\ OK$$

POINT OF MAXIMUM NEGATIVE MOMENT:

Net Section Properties:



$$A_{net} = b\left(d - 2D_{hole}\right)$$

$$A_{net} = (5\ in)(9.625\ in - (2)\ 0.813\ in)$$

$$A_{net} = 40.0\ in^2$$

$$I_{net} = \frac{bd^3}{12} - 2\left(\frac{bD_{hole}^3}{12} + bD_{hole}y^2\right)$$

$$I_{net} = \frac{(5\ in)(9.625\ in)^3}{12} - 2\left(\frac{(5\ in)(0.813\ in)^3}{12} + (5\ in)(0.813\ in)(1.563\ in)^2\right)$$

$$I_{net} = 351\ in^4$$

$$S_{net} = \frac{I_{net}}{c} = \frac{351\ in^4}{4.813\ in} = 72.9\ in^3$$

Bending and compression stresses on net section due to applied loads:

$$f_b = \frac{M}{S_{net}} = \frac{143,400\ in\text{-}lb}{72.9\ in^3} = 1970\ psi$$

$$f_c = \frac{P}{A_{net}} = \frac{39,200\ lb}{40.0\ in^2} = 980\ psi$$

Combined stresses (restrained against buckling, $F_{cE1} = \infty$):

$$\left(\frac{f_c}{F_c^*}\right)^2 + \frac{f_b}{F'_{b1}\left[1 - \left(\frac{f_c}{F_{cE1}}\right)\right]}$$

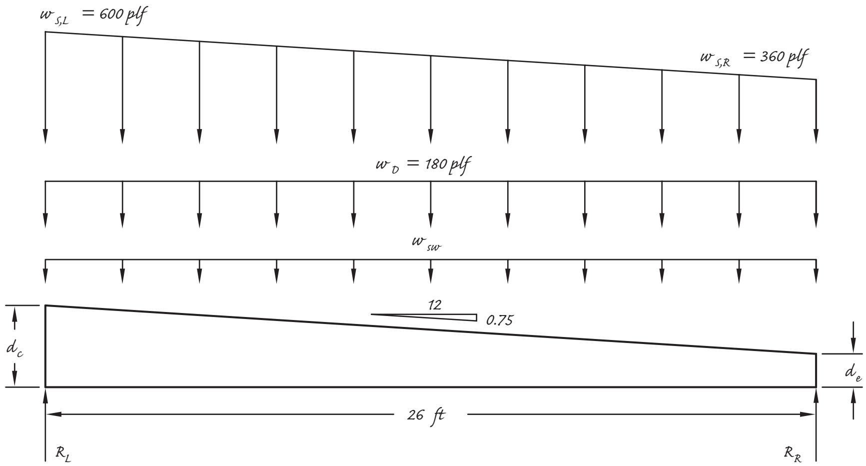
$$= \left(\frac{980\ \text{psi}}{1900\ \text{psi}}\right)^2 + \frac{1970\ \text{psi}}{\left(2760\ \text{psi}\right)[1 - 0]} = 0.980 \leq 1.0 \quad \therefore \text{OK}$$

Result:

The 5 in. × $9\frac{5}{8}$ in. 24F-V5 SP member is adequate to support the combined bending and compression loads. The combined stresses on the net section at the panel point controlled the design.

SINGLE-TAPERED STRAIGHT BEAM (ASD Method)
(See also Sections 3.4.3.8, 7.2)

Given: *A 24F-1.8E DF tapered glulam beam will span 26 ft and support a trapezoidal snow load of 600 plf at the heavy end and 360 plf at the light end and a uniform dead load of 180 plf in addition to its own weight. The top of the beam will be tapered with a slope of 0.75 in. per ft with the deeper end of the beam more heavily loaded as illustrated. The beam is braced to prevent rotation at the supports and braced laterally along the compression edge. The beam width was chosen as* $5\frac{1}{8}$ *in. The beam will be manufactured with the desired taper in the laminating plant, so layup requirements will be maintained for all sections along the length. The deflection limit is l/240 for snow load.*



Wanted: *Determine the required beam depths.*

Solution:

Allowable design values (AITC 117-2010):

$$F'_{c\perp x,top} = F_{c\perp x,top} C_M C_t = 650 \text{ psi } (1.0)(1.0) = 650 \text{ psi}$$

$$E'_x = E_x C_M C_t = 1.8\left(10^6 \text{ psi}\right)(1.0)(1.0) = 1.8\left(10^6 \text{ psi}\right)$$

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 265 \text{ psi } (1.15)(1.0)(1.0)(0.72) = 219 \text{ psi}$$

$$F'_{bx} = F_{bx} C_D C_M C_t \, (C_V \text{ or } C_I) = 2400 \text{ psi } (1.15)(1.0)(1.0)(C_V \text{ or } C_I)$$

$$F'_{bx} = 2760 \text{ psi } (C_V \text{ or } C_I)$$

$$C_I = \sqrt{\frac{1}{1 + \left(\dfrac{F_{bx}\tan\theta}{F_{vx}}\right)^2 + \left(\dfrac{F_{bx}\tan^2\theta}{F_{c\perp}}\right)^2}}$$

$$C_I = \sqrt{\frac{1}{1 + \left(\frac{(2400\ psi)\left(\frac{0.75}{12}\right)}{191\ psi}\right)^2 + \left(\frac{(2400\ psi)\left(\frac{0.75}{12}\right)^2}{650\ psi}\right)^2}}$$

$$C_I = 0.79$$

The load is defined by the following equation:

$$w_x = w_o - \frac{\Delta w}{l}x$$

Estimate self-weight to be a uniform 50 plf and solve for w_o, $\frac{\Delta w}{l}$, R_L, and R_R:

$$w_o = w_{S,L} + w_D + w_{SW} = 600\ plf + 180\ plf + 50\ plf = 830\ plf$$

$$\frac{\Delta w}{l} = \frac{w_{S,L} - w_{S,R}}{l} = \frac{600\ plf - 360\ plf}{26\ ft} = \frac{240\ plf}{26\ ft}$$

$$R_L = \frac{(w_D + w_{S,R})\ l}{2} + \left(\frac{2}{3}\right)\frac{\Delta w\, l}{2} + \frac{w_{sw}\, l}{2}$$

$$R_L = \frac{(180\ plf + 360\ plf)(26\ ft)}{2} + \left(\frac{2}{3}\right)\frac{(240\ plf)(26\ ft)}{2} + \frac{(50\ plf)(26\ ft)}{2} = 9750\ lb$$

$$R_R = \frac{(w_D + w_{S,R})\ l}{2} + \left(\frac{1}{3}\right)\frac{\Delta w\, l}{2} + \frac{w_{sw}\, l}{2}$$

$$R_R = \frac{(180\ plf + 360\ plf)(26\ ft)}{2} + \left(\frac{1}{3}\right)\frac{(240\ plf)(26\ ft)}{2} + \frac{w_{sw}\ (26\ ft)}{2} = 8710\ lb$$

Write equations for load, shear, and moment:

$$w_x = w_o - \frac{\Delta w}{l}x = 830\ plf - \left(\frac{240\ plf}{26\ ft}\right)x$$

$$V_x = R_L - w_o x + \frac{\Delta w}{l}\left(\frac{x^2}{2}\right)$$

$$V_x = 9750\ plf - (830\ plf)\ x + \left(\frac{240\ plf}{26\ ft}\right)\left(\frac{x^2}{2}\right)$$

$$M_x = R_L x - w_o\left(\frac{x^2}{2}\right) + \frac{\Delta w}{l}\left(\frac{x^2}{6}\right)$$

$$M_x = (9750\ plf)\ x - (830\ plf)\left(\frac{x^2}{2}\right) + \left(\frac{240\ plf}{26\ ft}\right)\left(\frac{x^2}{6}\right)$$

Calculate minimum end depths based on shear and corresponding depth at opposite end:

$$d_{c,min} = \frac{3R_L}{2bF'_{vx}} = \frac{3\,(9750\ lb)}{2\,(5.125\ in)(219\ psi)} = 26.1\ in$$

$$\text{The corresponding } d_e = d_{c,min} - \left(0.75\frac{in}{ft}\right)(26\ ft) = 26.1\ in - 19.5\ in = 6.6\ in$$

$$d_{e,min} = \frac{3R_R}{2bF'_{vx}} = \frac{3\,(8710\ lb)}{2\,(5.125\ in)(219\ psi)} = 11.6\ in$$

$$\text{The corresponding } d_c = d_{e,min} + \left(0.75\frac{in}{ft}\right)(26\ ft) = 11.6\ in + 19.5\ in = 31.1\ in$$

Choose a trial depth of $d_c = 31.5$ in, and calculate smallest volume factor:

$$C_V = \left[\left(\frac{5.125\ in}{b}\right)\left(\frac{12\ in}{d}\right)\left(\frac{21\ ft}{l}\right)\right]^{\frac{1}{10}}$$

$$C_V = \left[\left(\frac{5.125\ in}{5.125\ in}\right)\left(\frac{12\ in}{31.5\ in}\right)\left(\frac{21\ ft}{26\ ft}\right)\right]^{\frac{1}{10}} = 0.89$$

Verify that $d_e \geq d_c/3$:

$$d_e = 12\ in \leq \frac{d_c}{3} = \frac{31.5\ in}{3} = 10.5\ in \quad \therefore\ OK$$

Allowable bending design value:

$$F'_{bx} = 2760\ psi\ (C_V \text{ or } C_I) = 2760\ psi\ (C_I) = 2760\ psi\ (0.79) = 2180\ psi$$

Write equation for depth along the length of the beam:

$$d_x = d_L - \left(0.75\frac{in}{ft}\right)x = 31.5\ in - \left(0.75\frac{in}{ft}\right)x$$

Using a spreadsheet, calculate the flexural stress on sections at selected intervals:

x (ft)	M_x (ft-lb)	M_x (in-lb)	d_x (in.)	S_x (in^3)	f_b (psi)	f_b/F'_{bx}
0	0	0	31.5	848	0	0.000
2	17,852	214,228	30	769	279	0.128
4	32,458	389,502	28.5	694	561	0.258
6	43,892	526,708	27	623	846	0.388
8	52,228	626,732	25.5	555	1128	0.518

x (ft)	M_x (ft-lb)	M_x (in-lb)	d_x (in.)	S_x (in^3)	f_b (psi)	f_b/F'_{bx}
10	57,538	690,462	24	492	1403	0.644
12	59,898	718,782	22.5	432	1662	0.762
14	59,382	712,578	21	377	1892	0.868
16	56,062	672,738	19.5	325	2071	0.950
17	53,373	640,482	18.75	300	2133	0.978
18	50,012	600,148	18	277	2169	0.995
18.6	47,676	572,117	17.55	263	2175	0.998
19	45,987	551,848	17.25	254	2171	0.996
20	41,308	495,692	16.5	233	2132	0.978
22	30,022	360,258	15	192	1875	0.860
24	16,228	194,732	13.5	156	1251	0.574
26	0	0	12	123	0	0.000

The beam is adequate for flexure at all sections. The critical section is located 18.6 ft from the left end.

Calculate deflection estimating snow load as a uniform 480 plf:

$$C_y = \frac{d_c - d_e}{d_e} = \frac{31.5\ in - 12.0\ in}{12.0\ in} = 1.63$$

$$C_{dt} = \begin{cases} 1 + 0.46C_y & \text{where } 0 < C_y \le 1.1 \\ 1 + 0.43C_y & \text{where } 1.1 < C_y \le 2 \end{cases}$$

$$C_{dt} = 1 + 0.43C_y = 1 + 0.43(1.63) = 1.70$$

$$d_{equiv} = C_{dt}d_e = 1.70(12.0\ in) = 20.4\ in$$

$$I_{equiv} = \frac{bd_{equiv}^3}{12} = \frac{(5.125\ in)(20.4\ in)^3}{12} = 3626\ in^4$$

$$\Delta_s = \frac{5w_s l^4}{384EI} = \frac{5(480\ plf)(26\ ft)^4\left(\frac{1728\ in^3}{ft^3}\right)}{384\left(1.8\left(10^6\ psi\right)\right)(3626\ in^4)}$$

$$\Delta_s = 0.76\ in < \frac{l}{240} = \frac{312\ in}{240} = 1.3\ in \quad \therefore\ OK$$

Check estimate of self-weight:

$$w_{sw} = bd\gamma = (5.125\ in)(31.5\ in)\left(33\frac{lb}{ft^3}\right)\left(\frac{1\ ft^2}{144\ in^2}\right)$$

$$w_{sw} = 37\ plf \le 50\ plf\ \text{assumed} \quad \therefore\ OK$$

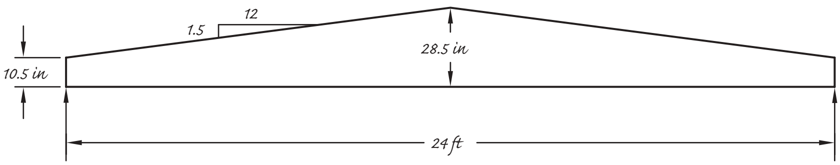
Result:

The required end depths are: $d_e = 12$ in. and $d_c = 31.5$ in.

DOUBLE-TAPERED STRAIGHT BEAM (ASD Method)
(See also Sections 3.4.3.8, 7.2)

Given: The design calls for 5 in. wide, 24F-V3 SP, double-tapered, straight, roof beams spanning 24 ft. The original specification requires the beams to be custom manufactured with lamination grade requirements maintained for the full length of the beam. The beam is braced to prevent rotation at the supports and braced laterally along the compression edge. In place of the custom beams specified, the contractor has proposed to use $5\frac{1}{8}$ in. × $28\frac{7}{8}$ in. stock beams and to saw the taper himself.

Each beam supports a snow load of 570 plf and a dead load of 153 plf (including self-weight). The deflection limit is l/240 based on snow load. The beam geometry is as illustrated.



Wanted: Evaluate the field-tapered beams to determine if the proposed change is acceptable.

Solution:

Allowable design values for field-tapered beam (AITC 117-2010):

$$F'_{c\perp x,top} = F_{c\perp x,top} C_M C_t$$

$$F'_{c\perp x,top} = 650\text{ psi}\,(1.0)(1.0) = 650\text{ psi} \quad \left(\text{reference value reduced because of taper cut}\right)$$

$$E'_x = E_x C_M C_t$$

$$E'_x = 1.7\left(10^6\text{ psi}\right)(1.0)(1.0) = 1.7\left(10^6\text{ psi}\right) \quad \left(\text{ref. value reduced because of taper cut}\right)$$

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 300\text{ psi}\,(1.15)(1.0)(1.0)(0.72) = 248\text{ psi}$$

$$C_V = \left[\left(\frac{5.125\text{ in}}{b}\right)\left(\frac{12\text{ in}}{d}\right)\left(\frac{21\text{ ft}}{l}\right)\right]^{\frac{1}{20}}$$

$$C_V = \left[\left(\frac{5.125\text{ in}}{5.125\text{ in}}\right)\left(\frac{12\text{ in}}{28.5\text{ in}}\right)\left(\frac{21\text{ ft}}{24\text{ ft}}\right)\right]^{\frac{1}{20}} = 0.95$$

$$C_I = \sqrt{\frac{1}{1 + \left(\frac{F_{bx}\tan\theta}{F_{vx}}\right)^2 + \left(\frac{F_{bx}\tan^2\theta}{F_{c\perp}}\right)^2}}$$

$$C_I = \sqrt{\frac{1}{1 + \left(\frac{(2100\ psi)\left(\frac{1.5}{12}\right)}{216\ psi}\right)^2 + \left(\frac{(2100\ psi)\left(\frac{1.5}{12}\right)^2}{650\ psi}\right)^2}} = 0.64$$

$$F'_{bx} = F_{bx} C_D C_M C_t \,(C_V \text{ or } C_I)$$

$$F'_{bx} = 2100psi\ (1.15)(1.0)(1.0)(0.64) \quad (\text{reference value reduced because of taper cut})$$

$$F'_{bx} = 1546\ psi$$

Shear:

$$V = \frac{wl}{2} = \frac{(570\ plf + 153\ plf)(24\ ft)}{2} = 8680\ lb$$

$$f_v = \frac{3V}{2bd} = \frac{3\,(8680\ lb)}{2\,(5.125\ in)(10.5\ in)} = 242\ psi < F'_{vx} = 248\ psi \quad \therefore\ OK$$

Deflection:

$$C_y = \frac{d_c - d_e}{d_e} = \frac{28.5\ in - 10.5\ in}{10.5\ in} = 1.71$$

$$C_{dt} = \begin{cases} 1 + 0.66C_y & \text{where } 0 < C_y \le 1 \\ 1 + 0.62C_y & \text{where } 1 < C_y < 3 \end{cases}$$

$$C_{dt} = 1 + 0.62C_y = 1 + 0.62\,(1.71) = 2.06$$

$$d_{equiv} = C_{dt} d_e = 2.06\,(10.5\ in) = 21.6\ in$$

$$I_{equiv} = \frac{bd_{equiv}^3}{12} = \frac{(5.125\ in)(21.6\ in)^3}{12} = 4304\ in^4$$

$$\Delta_s = \frac{5w_s l^4}{384EI} = \frac{5\,(570\ plf)(24\ ft)^4\left(\frac{1728\ in^3}{ft^3}\right)}{384\,(1.7\,(10^6\ psi))(4304\ in^4)}$$

$$\Delta_s = 0.58\ in < \frac{l}{240} = \frac{288\ in}{240} = 1.2\ in \quad \therefore\ OK$$

Flexure:

Depth, location, and section modulus of section with maximum stress

$$d = \frac{d_e}{d_c}(2d_c - d_e) = \frac{10.5\ in}{28.5\ in}(2\,(28.5\ in) - 10.5\ in) = 17.1\ in$$

$$x = \frac{ld_e}{2d_c} = \frac{(24\ ft)\left(\frac{12\ in}{ft}\right)(10.5\ in)}{2\,(28.5\ in)} = 53.1\ in = 4.42\ ft \quad (\text{from end})$$

$$S = \frac{bd^2}{6} = \frac{(5.125\ in)(17.1\ in)^2}{6} = 250\ in^3$$

Moment at section with maximum stress

$$M = \frac{wx}{2}(l - x) = \frac{(570\ plf + 153\ plf)(4.42\ ft)}{2}(24\ ft - 4.42\ ft)$$

$$M = 31{,}290\ ft\text{-}lb = 375{,}400\ in\text{-}lb$$

Stress on section

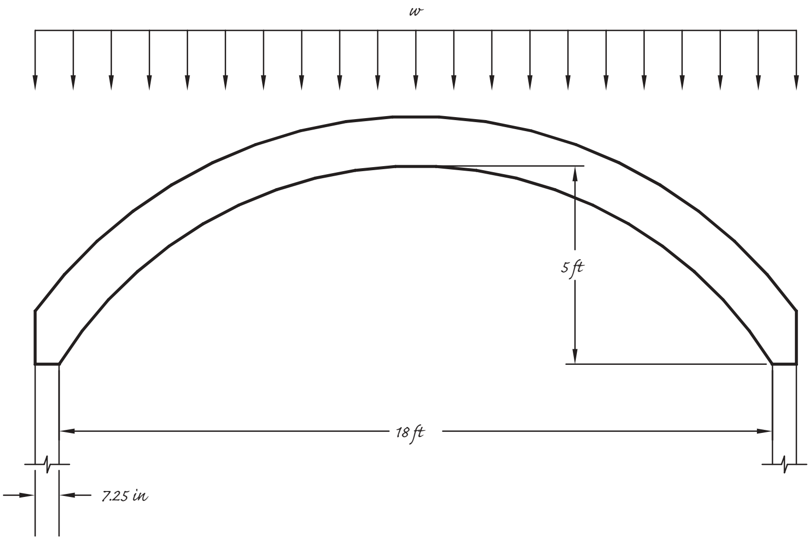
$$f_{bx} = \frac{M}{S} = \frac{375{,}400\ in\text{-}lb}{250\ in^3} = 1502\ psi < F'_{bx} = 1546\ psi \quad \therefore\ OK$$

Result:

The field-tapered beam is adequate to resist the design loads.

CONSTANT-DEPTH CURVED BEAM (ASD Method)
(See also Sections 8.1, 8.2)

Given: *The architectural design calls for a barrel-arch roof with $6\frac{3}{4}$ in. wide curved DF glulam beams spaced at 5 ft on center. The beams span 18 ft between the inside faces of 7.25 in. deep columns, which are framed inside of walls. The design calls for a circular curve with a height of 5 ft from the top of the wall to the bottom of the beam at mid-span. The roof supports a snow load of 40 lb/ft^2 and a dead load of 12 lb/ft^2, excluding the beam weight. The beam is braced to prevent rotation at the supports and braced laterally along the top edge. The vertical deflection limit is $\frac{l}{240}$ for snow load. Horizontal displacement at the beam ends is limited to 0.25 in. at each end for snow load.*



Wanted: *Determine the required depth for DF Combination 1 (L3 DF) with no radial reinforcement.*

Solution:

Allowable design values (AITC 117-2010):

$$E'_x = E_x C_M C_t = 1.5\left(10^6\ psi\right)(1.0)(1.0) = 1.5\left(10^6\ psi\right)$$

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 265\ psi(1.15)(1.0)(1.0)(0.72) = 219\ psi$$

$$F'_{rt} = F_{rt} C_D C_M C_t = 15 \text{ psi} (1.15)(1.0)(1.0) \quad \left(\text{no radial reinforcement}\right)$$

$$F'_{rt} = 17.3 \text{ psi}$$

$$F'_{bx} = F_{bx} C_D C_M C_t \left(C_V \text{ or } C_L\right) C_c$$

$$F'_{bx} = \left(1250 \text{ psi}\right)(1.15)(1.0)(1.0)\, C_V C_c = \left(1440 \text{ psi}\right) C_V C_c$$

Loads on beam (estimate self weight to be 25 lb/ft):

$$w = w_S + w_D + w_{sw}$$

$$w = \left(5 \text{ ft}\right)\left(40 \text{ lb/ft}^2\right) + \left(5 \text{ ft}\right)\left(12 \text{lb/ft}^2\right) + w_{sw}$$

$$w \approx 200 \text{ lb/ft} + 60 \text{ lb/ft} + 25 \text{ lb/ft}$$

$$w \approx 285 \text{ lb/ft}$$

Radius of curvature of the inside face:

$$R = \frac{L^2}{8c} + \frac{c}{2} = \frac{\left(18 \text{ ft}\right)^2}{8\left(5 \text{ ft}\right)} + \frac{\left(5 \text{ ft}\right)}{2} = 10.60 \text{ ft} = 127.2 \text{ in}$$

Maximum moment and shear:

$$M = \frac{wl^2}{8} = \frac{\left(\dfrac{285 \text{ lb/ft}}{12 \text{ in/ft}}\right)\left(18 \text{ ft}\left(12 \text{ in/ft}\right) + 7.25 \text{ in}\right)^2}{8} = 148{,}000 \text{ in-lb}$$

$$V = \frac{wl}{2} = \frac{\left(285 \text{ lb/ft}\right)\left(18 \text{ ft} + \frac{7.25}{12} \text{ ft}\right)}{2} = 2{,}650 \text{ lb}$$

Required depth for radial tension:

$$d \geq \frac{3M}{2R_m b F'_{rt}} = \frac{3M}{2\left(R + \frac{d}{2}\right) b F'_{rt}}$$

$$d\left(R + \frac{d}{2}\right) \geq \frac{3M}{2bF'_{rt}} \;\Rightarrow\; \frac{d^2}{2} + Rd - \frac{3M}{2bF'_{rt}} \geq 0$$

$$\frac{d^2}{2} + \left(127.2 \text{ in}\right) d - \frac{3\left(148{,}000 \text{ in-lb}\right)}{2\left(6.75 \text{ in}\right)\left(17.3 \text{ psi}\right)} \geq 0 \Rightarrow d \geq 14.2 \text{ in} \qquad \therefore \text{ Use 15 in}$$

Self-weight of beam at mid-span:

$$w_{sw} = bd\gamma = (6.75\ in)(15\ in)\left(33\frac{lb}{ft^3}\right)\left(\frac{1\ ft^2}{144\ in^2}\right)$$

$$w_{sw} = 23.2\ plf < 25\ plf\ \text{assumed} \quad \therefore\ OK$$

Self-weight of beam at ends (vertical depth determined graphically to be 16.1 in.):

$$w_{sw} = bd\gamma = (6.75\ in)(16.1\ in)\left(33\frac{lb}{ft^3}\right)\left(\frac{1\ ft^2}{144\ in^2}\right)$$

$$w_{sw} = 24.9\ plf < 25\ plf\ \text{assumed} \quad \therefore\ OK$$

Calculate volume factor:

$$C_V = \left[\left(\frac{5.125\ in}{b}\right)\left(\frac{12\ in}{d}\right)\left(\frac{21\ ft}{l}\right)\right]^{\frac{1}{10}}$$

$$C_V = \left[\left(\frac{5.125\ in}{6.75\ in}\right)\left(\frac{12\ in}{15\ in}\right)\left(\frac{21\ ft}{18\ ft}\right)\right]^{\frac{1}{10}} = 0.966$$

Calculate curvature factor:

$$C_c = 1 - 2000\left(\frac{t}{R}\right)^2 = 1 - 2000\left(\frac{0.75\ in}{127.2\ in}\right)^2 = 0.930$$

Allowable bending design value:

$$F'_{bx} = (1440\ psi)\ C_V C_c = 1440\ psi(0.966)(0.930) = 1290\ psi$$

Bending stress:

$$f_{bx} = \frac{6M}{bd^2} = \frac{6\ (148{,}000\ in\text{-}lb)}{(6.75\ in)(15\ in)^2} = 585\ psi \le F'_b = 1290\ psi \quad \therefore\ OK$$

Shear stress:

$$f_{vx} = \frac{3V}{2bd} = \frac{3\ (2{,}650\ lb)}{2\ (6.75\ in)(15\ in)} = 39.3\ psi \le F'_{vx} = 219\ psi \quad \therefore\ OK$$

Vertical and horizontal deflections for snow load:

$$\Delta_s = \frac{5w_s l^4}{32E'bd^3} = \frac{5\ (200\ plf)(18.6\ ft)^4\ (1728\ in^3/ft^3)}{32\ (1.5\ (10^6\ psi))(6.75\ in)(15\ in)^3}$$

$$\Delta_S = 0.19\ in < \frac{l}{240} = \frac{216\ in}{240} = 0.9\ in \quad \therefore\ OK$$

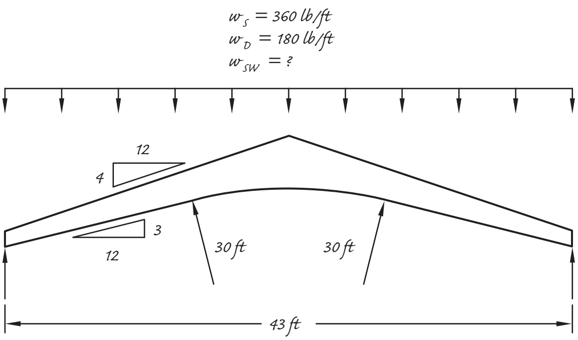
$$\Delta_H = \frac{2h\Delta_s}{l} = \frac{2(60\ in)(0.19\ in)}{223.25\ in} = 0.10\ in < 0.25\ in \quad \therefore\ OK$$

Result:

A $6\frac{3}{4}$ in. × 15 in. beam is adequate. (Depth governed by radial stress)

PITCHED AND TAPERED CURVED DF BEAM (ASD Method)
(See also Section 8.3)

Given: The architectural design calls for Douglas Fir pitched and tapered curved glulam beams spanning 43 ft with a 4:12 roof pitch and a 3:12 soffit pitch. Each beam will support a snow load of 360 lb/ft and a dead load of 180 lb/ft, excluding the beam weight. The beam is braced to prevent rotation at the supports and braced laterally along the top edge. The vertical deflection limit is $\frac{l}{240}$ for snow load. Horizontal displacement at the beam ends is limited to 0.5 in. at each end for snow load.



Wanted: Design PTC beam with a 30 ft radius using Combination 24F-V4 DF.

Solution:

Allowable design values (AITC 117-2010):

$$E'_x = E_x C_M C_t = 1.8\left(10^6 \text{ psi}\right)(1.0)(1.0) = 1.8\left(10^6 \text{ psi}\right)$$

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 265 \text{ psi}(1.15)(1.0)(1.0)(0.72) = 219 \text{ psi}$$

$$F'_{rt} = F_{rt} C_D C_M C_t = \frac{F_v C_{vr}}{3} C_D C_M C_t = \frac{\left(265 \text{ psi}\right)(0.72)}{3}(1.15)(1.0)(1.0)$$

$$F'_{rt} = 73 \text{ psi} \quad \left(\text{with reinforcement}\right)$$

$$F'_{bx} = F_{bx} C_D C_M C_t \left(C_V \text{ or } C_I\right) C_c = \left(2400 \text{ psi}\right)(1.15)(1.0)(1.0)\left(C_V \text{ or } C_I\right) C_c$$

$$F'_{bx} = \left(2760 \text{ psi}\right)\left(C_V \text{ or } C_I\right) C_c$$

Load on beam (estimate self weight to be 75 lb/ft):

$$w = w_S + w_D + w_{sw}$$

$$w \approx 360 \text{ lb/ft} + 180 \text{ lb/ft} + 75 \text{ lb/ft}$$

$$w \approx 615 \text{ lb/ft}$$

Maximum moment and shear:

$$M = \frac{w l^2}{8} = \frac{\left(615 \text{ lb/ft}\right)\left(43 \text{ ft}\right)^2\left(12 \text{ in/ft}\right)}{8} = 1.706\left(10^6\right) \text{ in-lb}$$

$$V = \frac{w \cdot l}{2} = \frac{(615\ lb/ft)(43\ ft)}{2} = 13,220\ lb$$

End depth based on shear:

$$d_e = \frac{3V}{2bF_b'} = \frac{3\,(13{,}220\ lb)}{2\,(6.75\ in)(219\ psi)} = 13.4\ in \quad \therefore\ use\ 15\ in$$

Beam geometry:

$$\phi_T = tan^{-1}\left(\frac{4}{12}\right) = 18.4^\circ$$

$$\phi_B = tan^{-1}\left(\frac{3}{12}\right) = 14.0^\circ$$

$$h_a = \frac{l}{2} tan(\phi_T) + d_e = \left(\frac{516\ in}{2}\right) tan(18.4^\circ) + 15\ in = 100.8\ in$$

$$h_s = \frac{l}{2} tan\,\phi_B - R\,(sec\,\phi_B - 1) = \left(\frac{516\ in}{2}\right) tan(14.0^\circ) - 360\ in\,(sec(14.0^\circ) - 1)$$

$$h_s = 53.3\ in$$

$$d_c = h_a - h_s = 100.8\ in - 53.3\ in = 47.5\ in$$

$$l_c = 2R\ sin\,\phi_B = 2\,(360\ in)\ sin(14.0^\circ) = 174.2\ in$$

$$l_t = \frac{l - l_c}{2} = \frac{516\ in - 174.2\ in}{2} = 170.9\ in$$

$$R_m = R + \frac{d_c}{2} = 360\ in + \frac{47.5\ in}{2} = 383.8\ in$$

Self-weight of beam at midspan:

$$w_{sw} = bd_c\gamma = (6.75\ in)(47.5\ in)\left(33\frac{lb}{ft^3}\right)\left(\frac{1\ ft^2}{144\ in^2}\right)$$

$$w_{sw} = 73.5\ plf < 75\ plf\ assumed \quad \therefore\ OK$$

Radial stress:

$$K_{rs} = 0.29\left(\frac{d_c}{R_m}\right)^2 + 0.32\ tan^{1.2}\ \phi_T$$

$$K_{rs} = 0.29\left(\frac{47.5\ in}{383.8\ in}\right)^2 + 0.32\ tan^{1.2}\ (18.4^\circ)$$

$$K_{rs} = 0.090$$

$$C_{rs} = 0.27\ ln(tan\,\phi_T) + 0.28\ ln\left(\frac{l}{l_c}\right) - 0.8\left(\frac{d_c}{R_m}\right) + 1 \le 1.0$$

$$C_{rs} = 0.27\ ln\left(\frac{4}{12}\right) + 0.28\ ln\left(\frac{516\ in}{174.2\ in}\right) - 0.8\left(\frac{47.5\ in}{383.8\ in}\right) + 1 \le 1.0$$

$$C_{rs} = 0.908$$

$$C_{rs} = 0.908$$

$$f_{rt} = K_{rs}C_{rs}\frac{6M}{bd_c^2} = (0.090)(0.908)\left(\frac{6\left(1.706\left(10^6\right)\ in\text{-}lb\right)}{(6.75\ in)(47.5\ in)^2}\right)$$

$$f_{rt} = 54.9\ psi < F'_{rt} = 73\ psi \quad \therefore\ OK$$

Bending stress at midspan:

$$C_V = \left[\left(\frac{5.125\ in}{b}\right)\left(\frac{12\ in}{d}\right)\left(\frac{21\ ft}{l}\right)\right]^{\frac{1}{10}}$$

$$C_V = \left[\left(\frac{5.125\ in}{6.75\ in}\right)\left(\frac{12\ in}{47.5\ in}\right)\left(\frac{21\ ft}{43\ ft}\right)\right]^{\frac{1}{10}} = 0.789$$

$$C_c = 1 - 2000\left(\frac{t}{R}\right)^2 = 1 - 2000\left(\frac{1.5\ in}{360\ in}\right)^2 = 0.965$$

$$F'_{bx} = \left(2760\ psi\right)C_VC_c = \left(2760\ psi\right)(0.789)(0.965) = 2100\ psi$$

$$K_\theta = 1.0 + 2.7tan\phi_T = 1.0 + 2.7\ tan\left(18.4^\circ\right) = 1.898$$

$$f_{bx} = K_\theta\frac{6M}{bd^2} = (1.898)\ \frac{6\left(1.706\left(10^6\right)\ in\text{-}lb\right)}{(6.75\ in)(47.5\ in)^2}$$

$$f_{bx} = 1280\ psi \le F'_{bx} = 2100\ psi \quad \therefore\ OK$$

Stress Interaction Factor in straight tapered segment:

$$C_I = \sqrt{\frac{1}{1 + \left(\frac{F_{bx}tan\theta}{F_{vx}C_{vr}}\right)^2 + \left(\frac{F_{bx}tan^2\theta}{F_{c\perp}}\right)^2}}$$

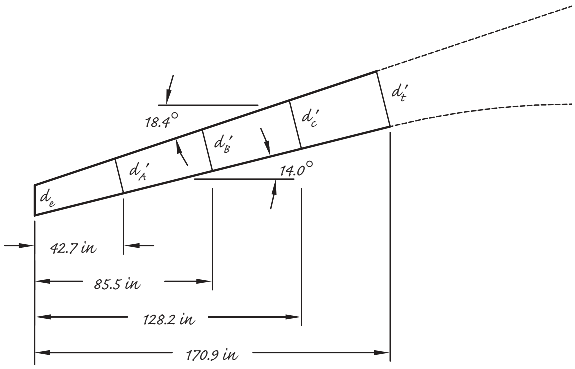
$$C_I = \sqrt{\frac{1}{1 + \left(\frac{\left(2400\ psi\right)\ tan\left(18.4^\circ - 14.0^\circ\right)}{191\ psi}\right)^2 + \left(\frac{\left(2400\ psi\right)\ tan^2\left(18.4^\circ - 14.0^\circ\right)}{650\ psi}\right)^2}}$$

$$C_I = 0.719$$

Depth and bending moment equations for straight tapered segment:

$$d'_x = \left[d_e + x\left(tan\phi_T - tan\phi_B\right)\right]\left[cos\phi_B - sin\phi_B tan\left(\phi_T - \phi_B\right)\right]$$

$$M_x = \frac{wx}{2}\left(l - x\right)$$



Bending stress at tangent point:

$$d_T' = \left[d_e + l_t\left(\tan\phi_T - \tan\phi_B\right)\right]\left[\cos\phi_B - \sin\phi_B\tan\left(\phi_T - \phi_B\right)\right]$$
$$d_T' = \left[15\ in + (170.9\ in)\left(\tan 18.4^\circ - \tan 14.0^\circ\right)\right]\left[\cos 14.0^\circ - \sin 14.0^\circ \tan 4.4^\circ\right]$$
$$d_T' = 27.8\ in$$

$$M_T = \frac{w\,l_t}{2}(l - l_t) = \frac{\left(\frac{615\ lb/ft}{12\ in/ft}\right)(170.9\ in)}{2}(516\ in - 170.9\ in)$$
$$M_T = 1.511\left(10^6\right)\ in\text{-}lb$$
$$F_{bx}' = (2760\ psi)(C_V \text{ or } C_I)\ C_c = (2760\ psi)(0.719)\ 0.965$$
$$F_{bx}' = 1910\ psi$$
$$f_{bx} = \frac{6M_T}{bd_T^2} = \frac{6\left(1.511\left(10^6\right)\ in\text{-}lb\right)}{(6.75\ in)(27.8\ in)^2} = 1740\ psi \le F_{bx}' = 1910\ psi \quad \therefore\ OK$$

Bending stress at $x_A = \frac{l_t}{4} = 42.7\ in$:

$$d_A' = \left[d_e + \frac{l_t}{4}\left(\tan\phi_T - \tan\phi_B\right)\right]\left[\cos\phi_B - \sin\phi_B\tan\left(\phi_T - \phi_B\right)\right]$$
$$d_A' = \left[15\ in + \left(\frac{170.9\ in}{4}\right)\left(\tan 18.4^\circ - \tan 14.0^\circ\right)\right]\left[\cos 14.0^\circ - \sin 14.0^\circ \tan 4.4^\circ\right]$$
$$d_A' = 17.7\ in$$
$$M_A = \frac{w x_A}{2}(l - x_A) = \frac{\left(\frac{615\ lb/ft}{12\ in/ft}\right)\left(\frac{170.9\ in}{4}\right)}{2}\left(516\ in - \frac{170.9\ in}{4}\right)$$

$$M_A = 518.2\left(10^3\right)\ in\text{-}lb$$
$$F'_{bx} = \left(2760\ psi\right)(C_V or C_I)\ C_c = \left(2760\ psi\right)(0.719)(1.0)$$
$$F'_{bx} = 1980\ psi$$
$$f_{bx} = \frac{6M_A}{bd_A^2} = \frac{6\ (518.2\ (10^3)\ \ in\text{-}lb)}{(6.75\ in)(17.7\ in)^2} = 1470\ psi \le F'_{bx} = 1980\ psi \quad \therefore\ OK$$

Bending stress at $x_B = \frac{l_t}{2} = 85.5\ in$:

$$d'_B = \left[d_e + \frac{l_t}{2}\ (\tan\phi_T - \tan\phi_B)\right]\left[\cos\phi_B - \sin\phi_B \tan\ (\phi_T - \phi_B)\right]$$
$$d'_B = \left[15\ in + \left(\frac{170.9\ in}{2}\right)(\tan\ 18.4^\circ - \tan\ 14.0^\circ)\right]$$
$$\times\left[\cos\ 14.0^\circ - \sin\ 14.0^\circ \tan\ 4.4^\circ\right]$$
$$d'_B = 21.0\ in$$
$$M_B = \frac{wx_B}{2}\ (l - x_B) = \frac{\left(\frac{615\ lb/ft}{12\ in/ft}\right)\left(\frac{170.9\ in}{2}\right)}{2}\left(516\ in - \frac{170.9\ in}{2}\right)$$
$$M_B = 942.8\left(10^3\right)\ in\text{-}lb$$
$$F'_{bx} = \left(2760\ psi\right)(C_V\ or\ C_I)\ C_c = \left(2760\ psi\right)(0.719)(1.0)$$
$$F'_{bx} = 1980\ psi$$
$$f_{bx} = \frac{6M_B}{bd_B^2} = \frac{6\ (942.8\ (10^3)\ \ in\text{-}lb)}{(6.75\ in)(21.0\ in)^2} = 1900\ psi \le F'_{bx} = 1980\ psi \quad \therefore\ OK$$

Bending stress at $x_C = \frac{3l_t}{4} = 128.2\ in$:

$$d'_C = \left[d_e + \frac{3l_t}{4}\ (\tan\phi_T - \tan\phi_B)\right]\left[\cos\phi_B - \sin\phi_B \tan\ (\phi_T - \phi_B)\right]$$
$$d'_C = \left[15\ in + \left(\frac{3}{4}\ (170.9\ in)\right)(\tan\ 18.4^\circ - \tan\ 14.0^\circ)\right]$$
$$\times\left[\cos\ 14.0^\circ - \sin\ 14.0^\circ \tan\ 4.4^\circ\right]$$
$$d'_C = 24.4\ in$$
$$M_C = \frac{wx_C}{2}\ (l - x_C)$$
$$M_C = \frac{\left(\frac{615\ lb/ft}{12\ in/ft}\right)\left(\left(\frac{3}{4}\right)\ 170.9\ in\right)}{2}\left(516\ in - \left(\frac{3}{4}\right)\ 170.9\ in\right)$$

$$M_C = 1.274\left(10^6\right) \text{ in-lb}$$

$$F'_{bx} = \left(2760 \text{ psi}\right)(C_V \text{ or } C_I)\ C_c = \left(2760 \text{ psi}\right)(0.719)(1.0)$$

$$F'_{bx} = 1980 \text{ psi}$$

$$f_{bx} = \frac{6M_C}{bd_C^2} = \frac{6\left(1.274\,(10)^6 \text{ in-lb}\right)}{(6.75 \text{ in})(24.4 \text{ in})^2} = 1900 \text{ psi} \leq F'_{bx} = 1980 \text{ psi} \quad \therefore \text{OK}$$

Vertical deflection for snow load:

$$d_{equiv} = (d_e + d_c)(0.5 + 0.735\tan\phi_T) - 1.41 d_c \tan\phi_B$$

$$d_{equiv} = (15 \text{ in} + 47.5 \text{ in})\left(0.5 + 0.735\left(\frac{4}{12}\right)\right) - 1.41\,(47.5)\left(\frac{3}{12}\right)$$

$$d_{equiv} = 29.8 \text{ in}$$

$$\Delta_S = \frac{5w_S l^4}{32E' bd_{equiv}^3} = \frac{5\left(360 \text{ plf}\right)\left(43 \text{ ft}\right)^4\left(\frac{1728 \text{ in}^3}{\text{ft}^3}\right)}{32\left(1.8\left(10^6 \text{ psi}\right)\right)(6.75 \text{ in})(29.8 \text{ in})^3}$$

$$\Delta_S = 1.0 \text{ in} < \frac{l}{240} = \frac{516 \text{ in}}{240} = 2.2 \text{ in} \quad \therefore \text{OK}$$

Horizontal deflection for snow load (at each support):

$$h = h_a - \frac{d_c}{2} - \frac{d_e}{2} = 100.8 \text{ in} - \frac{47.5 \text{ in}}{2} - \frac{15 \text{ in}}{2}$$

$$h = 69.6 \text{ in}$$

$$\Delta_H = \frac{2h\Delta_s}{l} = \frac{2\,(69.6 \text{ in})(1.0 \text{ in})}{516 \text{ in}} = 0.27 \text{ in} < 0.5 \text{ in} \quad \therefore \text{OK}$$

Radial reinforcement (use 1 in. diameter lag screws):

$$l_p = \frac{d'_t}{2} - 3 \text{ in} = \frac{27.8 \text{ in}}{2} - 3 \text{ in} = 10.8 \text{ in}$$

$$T_{screw} = w' l_p \leq F_{steel}\frac{\pi D_r^2}{4}$$

$$T_{screw} = 0.85\left(1800 G^{\frac{3}{2}} D^{\frac{3}{4}}\right) C_D C_M C_t l_p \leq F_{steel}\frac{\pi D_r^2}{4}$$

$$T_{screw} = \left[0.85\left(1800\,(0.5)^{\frac{3}{2}}\,(1.0)^{\frac{3}{4}}\right) \text{ lb/in}\right]\left[10.8 \text{ in}\right]$$

$$\leq \left(20{,}000 \text{ psi}\right)\left[\frac{\pi\,(0.780 \text{ in})^2}{4}\right]$$

$$T_{screw} = 5842 \text{ lb} \leq 9557 \text{ lb}$$

$$T_{screw} = 5842 \text{ lb}$$

$$s_{max} = \frac{T_{screw}}{f_{rt} b} = \frac{5842\ lb}{(54.9\ psi)(6.75\ in)} = 15.75\ in \quad \therefore\ use\ s = 15\ in.$$

$$S_c = 2R_m \phi_B \frac{\pi}{180^\circ} = 2(383.8\ in)(14.0^\circ)\left(\frac{\pi}{180^\circ}\right) = 187.6\ in$$

$$n_{screws} \geq \frac{S_c}{s} + 1 = \frac{187.6\ in}{15\ in} + 1$$

$$n_{screws} \geq 13.5 \quad \therefore\ use\ 14\ screws\ (7\ on\ each\ side\ of\ mid\text{-}span)$$

Bending stress at midspan with section modulus reduced for lag screw:

$$c_1 = \frac{bd_c^2 - b' c_{trial}^2 + b' a^2}{2\left[bd_c - b' c_{trial} + b' a\right]}$$

$$c_1 = \frac{(6.75\ in)(47.5\ in)^2 - (1.0\ in)(23.75\ in)^2 + (1.0\ in)(2\ in)^2}{2\left[(6.75\ in)(47.5\ in) - (1.0\ in)(23.75\ in) + (1.0\ in)(2\ in)\right]}$$

$$c_1 = 24.54\ in$$

$$c_2 = \frac{bd_c^2 - b' c_1^2 + b' a^2}{2\left[bd_c - b' c_1 + b' a\right]}$$

$$c_2 = \frac{(6.75\ in)(47.5\ in)^2 - (1.0\ in)(24.54\ in)^2 + (1.0\ in)(2\ in)^2}{2\left[(6.75\ in)(47.5\ in) - (1.0\ in)(24.54\ in) + (1.0\ in)(2\ in)\right]}$$

$$c_2 = 24.54\ in = c$$

$$I_{x\ reduced} = \frac{b(d-c)^3}{3} + \frac{bc^3}{3} - \frac{b'(c-a)^3}{3}$$

$$I_{x\ reduced} = \frac{(6.75\ in)(47.5\ in - 24.54\ in)^3}{3} + \frac{(6.75\ in)(24.54\ in)^3}{3} - \frac{(1.0\ in)(24.54\ in - 2\ in)^3}{3}$$

$$I_{x\ reduced} = 56.67\left(10^3\right)\ in^4$$

$$S_{x\ reduced} = \frac{I_{x\ reduced}}{c} = \frac{56.67\left(10^3\right)\ in^4}{24.54\ in} = 2309\ in^3$$

$$f_b = K_\theta \frac{M}{S_{x\ reduced}} = (1.898)\frac{1.706\left(10^6\right)\ in\text{-}lb}{2309\ in^3}$$

$$f_b = 1402\ psi \leq F'_{bx} = 2100\ psi \quad \therefore\ OK$$

Bending stress at tangent point with section modulus reduced for lag screw:

$$c_1 = \frac{bd_t'^2 - b' c_{trial}^2 + b' a^2}{2\left[bd_t' - b' c_{trial} + b' a\right]}$$

$$c_1 = \frac{(6.75\ in)(27.8\ in)^2 - (1.0\ in)(13.9\ in)^2 + (1.0\ in)(2\ in)^2}{2\left[(6.75\ in)(27.8\ in) - (1.0\ in)(13.9\ in) + (1.0\ in)(2\ in)\right]}$$

$$c_1 = 14.30\ in$$

$$c_2 = \frac{bd_t'^2 - b' c_1^2 + b' a^2}{2\left[bd_t' - b' c_1 + b' a\right]}$$

$$c_2 = \frac{(6.75\ in)(27.8\ in)^2 - (1.0\ in)(14.3\ in)^2 + (1.0\ in)(2\ in)^2}{2\left[(6.75\ in)(27.8\ in) - (1.0\ in)(14.3\ in) + (1.0\ in)(2\ in)\right]}$$

$$c_2 = 14.30\ in = c$$

$$I_{x\ reduced} = \frac{b\,(d-c)^3}{3} + \frac{bc^3}{3} - \frac{b'\,(c-a)^3}{3}$$

$$I_{x\ reduced} = \frac{(6.75\ in)(27.8\ in - 14.3\ in)^3}{3} + \frac{(6.75\ in)(14.3\ in)^3}{3} - \frac{(1.0\ in)(14.3\ in - 2\ in)^3}{3}$$

$$I_{x\ reduced} = 11.50\left(10^3\right)\ in^4$$

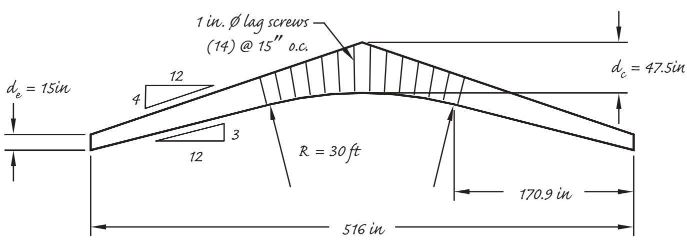
$$S_{x\ reduced} = \frac{I_{x\ reduced}}{c} = \frac{11.50\,(10^3)\ in^4}{14.30\ in} = 804\ in^3$$

$$f_b = \frac{M_T}{S_{x\ reduced}} = \frac{1.511\,(10^6)\ in\text{-}lb}{804\ in^3}$$

$$f_b = 1879\ psi \leq F_{bx}' = 1910\ psi \quad \therefore\ OK$$

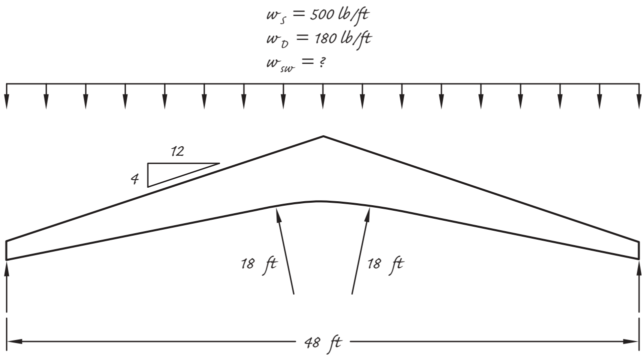
Result:

The $6\frac{3}{4}$ in. wide beam is illustrated below:



PITCHED AND TAPERED CURVED SP BEAM (ASD Method)
(See also Section 8.3)

Given: The architectural design calls for Southern Pine pitched and tapered curved glulam beams spanning 48 ft with a 4:12 roof pitch. Each beam will support a snow load of 500 lb/ft and a dead load of 180 lb/ft, excluding the beam weight. The beam is braced to prevent rotation at the supports and braced laterally along the top edge. The vertical deflection limit is l/240 for snow load. Horizontal displacement at the beam ends is limited to 0.5 in. at each end for snow load.



Wanted: Design PTC beam with an 18 ft radius using Combination 24F-V3 SP.

Solution:

Allowable design values (AITC 117-2010):

$$E'_x = E_x C_M C_t = 1.8\left(10^6\ psi\right)(1.0)(1.0) = 1.8\left(10^6\ psi\right)$$

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 300\ psi(1.15)(1.0)(1.0)(0.72) = 248\ psi$$

$$F'_{rt} = F_{rt} C_D C_M C_t = \frac{F_v C_{vr}}{3} C_D C_M C_t = \frac{(300\ psi)(0.72)}{3}(1.15)(1.0)(1.0) = 83\ psi$$

$$F'_{bx} = F_{bx} C_D C_M C_t\ (C_V \text{ or } C_I)\ C_c = (2400\ psi)(1.15)(1.0)(1.0)(C_V \text{ or } C_I)\ C_c$$

$$F'_{bx} = (2760\ psi)(C_V \text{ or } C_I)\ C_c$$

Load on beam (estimate self weight to be 100 lb/ft):

$$w = w_S + w_D + w_{sw}$$

$$w \approx 500\ lb/ft + 180\ lb/ft + 100\ lb/ft$$

$$w \approx 780\ lb/ft$$

Maximum moment and shear:

$$M = \frac{wl^2}{8} = \frac{(780\ lb/ft)(48\ ft)^2\ (12\ in/ft)}{8} = 2.696\left(10^6\right)\ in\text{-}lb$$

$$V = \frac{wl}{2} = \frac{(780\ lb/ft)(48\ ft)}{2} = 18{,}720\ lb$$

End depth based on shear:

$$d_e = \frac{3V}{2bF_b'} = \frac{3\,(18{,}720\ lb)}{2\,(6.75\ in)\,(248\ psi)} = 16.7\ in \quad \therefore\ use\ 17\ in$$

Beam geometry (use taper angle of 7°):

$$\phi_T = tan^{-1}\left(\frac{4}{12}\right) = 18.4^\circ$$

$$\phi_B = 18.4^\circ - 7^\circ = 11.4^\circ$$

$$h_a = \frac{l}{2}tan\,(\phi_T) + d_e = \left(\frac{576\ in}{2}\right) tan\,(18.4^\circ) + 17\ in = 112.8\ in$$

$$h_s = \frac{l}{2}tan\phi_B - R\,(sec\phi_B - 1) = \left(\frac{576\ in}{2}\right) tan\,(11.4^\circ)$$

$$- 216\ in\,(sec\,(11.4^\circ) - 1) = 53.7\ in$$

$$d_c = h_a - h_s = 112.8\ in - 53.7\ in = 59.1\ in$$

$$l_c = 2Rsin\phi_B = 2\,(216 in)\ sec\,(11.4^\circ) = 85.4\ in$$

$$l_t = \frac{l - l_c}{2} = \frac{576\ in - 85.4\ in}{2} = 245.3\ in$$

$$R_m = R + \frac{d_c}{2} = 216\ in + \frac{59.1\ in}{2} = 245.6\ in$$

Self-weight of beam at midspan:

$$w_{sw} = bd_c\gamma = (6.75\ in)(59.1\ in)\left(36\frac{lb}{ft^3}\right)\left(\frac{1\ ft^2}{144\ in^2}\right)$$

$$w_{sw} = 99.7\ plf = 100\ plf\ assumed \quad \therefore\ OK$$

Radial stress:

$$K_{rs} = 0.29\left(\frac{d_c}{R_m}\right)^2 + 0.32tan^{1.2}\phi_T$$

$$K_{rs} = 0.29\left(\frac{59.1\ in}{245.6\ in}\right)^2 + 0.32tan^{1.2}\,(18.4^\circ)$$

$$K_{rs} = 0.102$$

$$C_{rs} = 0.27ln\,(tan\phi_T) + 0.28ln\left(\frac{l}{l_c}\right) - 0.8\left(\frac{d_c}{R_m}\right) + 1 \leq 1.0$$

$$C_{rs} = 0.27ln\left(\frac{4}{12}\right) + 0.28ln\left(\frac{576\ in}{85.4\ in}\right) - 0.8\left(\frac{59.1\ in}{245.6\ in}\right) + 1$$

$$C_{rs} = 1.045 \leq 1.0$$

$$C_{rs} = 1.0$$

$$f_{rt} = K_{rs}C_{rs}\frac{6M}{bd_c^2} = (0.102)(1.0)\left(\frac{6\,(2.696\,(10^6)\ in\text{-}lb)}{(6.75\ in)(59.1\ in)^2}\right)$$

$$f_{rt} = 70.0\ psi < F'_{rt} = 83\ psi \quad \therefore\ OK$$

Bending stress at midspan:

$$C_V = \left[\left(\frac{5.125\ in}{b}\right)\left(\frac{12\ in}{d}\right)\left(\frac{21\ ft}{l}\right)\right]^{\frac{1}{20}}$$

$$C_V = \left[\left(\frac{5.125\ in}{6.75\ in}\right)\left(\frac{12\ in}{59.1\ in}\right)\left(\frac{21\ ft}{48\ ft}\right)\right]^{\frac{1}{20}}$$

$$C_V = 0.874$$

$$C_c = 1 - 2000\left(\frac{t}{R}\right)^2 = 1 - 2000\left(\frac{1.375\ in}{216\ in}\right)^2$$

$$C_c = 0.919$$

$$F'_{bx} = (2760\ psi)\ C_V C_c = (2760\ psi)(0.874)\ 0.919 = 2220\ psi$$

$$K_\theta = 1.0 + 2.7\tan\phi_T = 1.0 + 2.7\tan(18.4^\circ) = 1.898$$

$$f_{bx} = K_\theta\frac{6M}{bd^2} = (1.898)\ \frac{6\,(2.696\,(10^6)\ in\text{-}lb)}{(6.75\ in)(59.1\ in)^2}$$

$$f_{bx} = 1300\ psi \le F'_{bx} = 2220\ psi \quad \therefore\ OK$$

Stress Interaction Factor in straight tapered segment:

$$C_I = \sqrt{\frac{1}{1 + \left(\frac{F_{bx}\tan\theta}{F_{vx}C_{vr}}\right)^2 + \left(\frac{F_{bx}\tan^2\theta}{F_{c\perp}}\right)^2}}$$

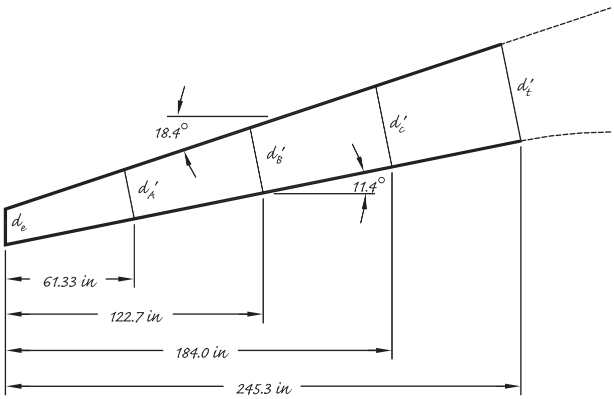
$$C_I = \sqrt{\frac{1}{1 + \left(\frac{(2400\ psi)\ \tan(7^\circ)}{216\ psi}\right)^2 + \left(\frac{(2400\ psi)\ \tan^2(7^\circ)}{740\ psi}\right)^2}}$$

$$C_I = 0.591$$

Depth and bending moment equations for straight tapered segment:

$$d'_x = \left[d_e + x(\tan\phi_T - \tan\phi_B)\right]\left[\cos\phi_B - \sin\phi_B\tan(\phi_T - \phi_B)\right]$$

$$M_x = \frac{wx}{2}(l - x)$$



Bending stress at tangent point:

$$d'_T = \left[d_e + l_t\left(\tan\phi_T - \tan\phi_B\right)\right]\left[\cos\phi_B - \sin\phi_B \tan\left(\phi_T - \phi_B\right)\right]$$

$$d'_T = \left[17\ in + (245.3\ in)(\tan 18.4^\circ - \tan 11.4^\circ)\right]\left[\cos 11.4^\circ - \sin 11.4^\circ \tan 7^\circ\right]$$

$$d'_T = 47.0\ in$$

$$M_T = \frac{w l_t}{2}\left(l - l_t\right) = \frac{\left(\frac{780\ lb/ft}{12\ in/ft}\right)(245.3\ in)}{2}(576\ in - 245.3\ in)$$

$$F'_{bx} = (2760\ psi)(C_V \text{ or } C_I)\ C_c = (2760\ psi)(0.591)\ 0.919$$

$$F'_{bx} = 1500\ psi$$

$$f_{bx} = \frac{6M_T}{bd_T^2} = \frac{6\left(2.636\left(10^6\right)\ in\text{-}lb\right)}{(6.75\ in)(47.0\ in)^2} = 1060\ psi \le F'_{bx} = 1500\ psi \quad \therefore OK$$

Bending Stress at $x_A = \frac{l_t}{4} = 61.33 in$:

$$d'_A = \left[d_e + \frac{l_t}{4}\left(\tan\phi_T - \tan\phi_B\right)\right]\left[\cos\phi_B - \sin\phi_B \tan\left(\phi_T - \phi_B\right)\right]$$

$$d'_A = \left[17\ in + \left(\frac{245.3\ in}{4}\right)(\tan 18.4^\circ - \tan 11.4^\circ)\right]\left[\cos 11.4^\circ - \sin 11.4^\circ \tan 7^\circ\right]$$

$$d'_A = 23.9\ in$$

$$M_A = \frac{w x_A}{2}\left(l - x_A\right) = \frac{\left(\frac{780\ lb/ft}{12\ in/ft}\right)\left(\frac{245.3\ in}{4}\right)}{2}\left(576\ in - \frac{245.3\ in}{4}\right)$$

$$M_A = 1.026\left(10^6\right)\ in\text{-}lb$$

$$F'_{bx} = (2760\ psi)(C_V\ or\ C_I)\ C_c = (2760\ psi)(0.591)(1.0)$$
$$F'_{bx} = 1630\ psi$$
$$f_{bx} = \frac{6M_A}{bd_A^2} = \frac{6\,(1.026\,(10^6)\ in\text{-}lb)}{(6.75\ in)(23.9\ in)^2} = 1600\ psi \leq F'_{bx} = 1630\ psi \quad \therefore OK$$

Bending stress at $x_B = \frac{l_t}{2} = 122.7\ in$:

$$d'_B = \left[d_e + \frac{l_t}{2}(\tan\phi_T - \tan\phi_B)\right]\left[\cos\phi_B - \sin\phi_B \tan(\phi_T - \phi_B)\right]$$
$$d'_B = \left[17\ in + \left(\frac{245.3\ in}{2}\right)(\tan 18.4^\circ - \tan 11.4^\circ)\right]\left[\cos 11.4^\circ - \sin 11.4^\circ \tan 7^\circ\right]$$
$$d'_B = 31.6\ in$$
$$M_B = \frac{wx_B}{2}(l - x_B) = \frac{\left(\frac{780\ lb/ft}{12\ in/ft}\right)\left(\frac{245.3\ in}{2}\right)}{2}\left(576\ in - \frac{245.3\ in}{2}\right)$$
$$M_B = 1.807\,(10^6)\ in\text{-}lb$$
$$F'_{bx} = (2760\ psi)(C_V\ or\ C_I)\ C_c = (2760\ psi)(0.591)(1.0)$$
$$F'_{bx} = 1630\ psi$$
$$f_{bx} = \frac{6M_B}{bd_B^2} = \frac{6\,(1.807\,(10^6)\ in\text{-}lb)}{(6.75\ in)(31.6\ in)^2} = 1610\ psi \leq F'_{bx} = 1630\ psi \quad \therefore OK$$

Bending stress at $x_C = \frac{3l_t}{4} = 184.0\ in$:

$$d'_C = \left[d_e + \frac{3l_t}{4}(\tan\phi_T - \tan\phi_B)\right]\left[\cos\phi_B - \sin\phi_B \tan(\phi_T - \phi_B)\right]$$
$$d'_C = \left[17\ in + \left(\frac{3}{4}(245.3\ in)\right)(\tan 18.4^\circ - \tan 11.4^\circ)\right]\left[\cos 11.4^\circ - \sin 11.4^\circ \tan 7^\circ\right]$$
$$d'_C = 39.3\ in$$
$$M_C = \frac{wx_C}{2}(l - x_C)$$
$$M_C = \frac{\left(\frac{780\ lb/ft}{12\ in/ft}\right)\left(\left(\frac{3}{4}\right)245.3\ in\right)}{2}\left(576\ in - \left(\frac{3}{4}\right)245.3\ in\right)$$
$$M_C = 2.344\,(10^6)\ in\text{-}lb$$

$$F'_{bx} = (2760\ psi)(C_V \text{ or } C_I)\ C_c = (2760\ psi)(0.591)(1.0)$$
$$F'_{bx} = 1630\ psi$$
$$f_{bx} = \frac{6M_C}{bd_C^2} = \frac{6\,(2.344\,(10^6)\ in\text{-}lb)}{(6.75\ in)(39.3\ in)^2} = 1350\ psi \le F'_{bx} = 1630\ psi \quad \therefore OK$$

Vertical deflection for snow load:

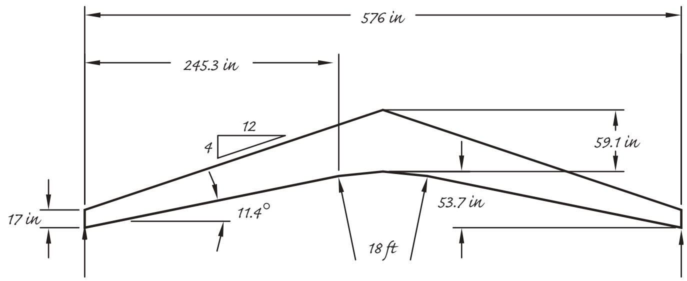
$$d_{equiv} = (d_e + d_c)(0.5 + 0.735\tan\phi_T) - 1.41 d_c \tan\phi_B$$
$$d_{equiv} = (17\ in + 59.1\ in)\left(0.5 + 0.735\left(\frac{4}{12}\right)\right) - 1.41\,(59.1)\tan(11.4^\circ)$$
$$d_{equiv} = 39.9\ in$$
$$\Delta_S = \frac{5w_S l^4}{32E' bd_{equiv}^3} = \frac{5\,(500\ plf)(48\ ft)^4\left(\frac{1728\ in^3}{ft^3}\right)}{32\left(1.8\left(10^6\ psi\right)\right)(6.75\ in)(39.9\ in)^3}$$
$$\Delta_S = 0.93\ in < \frac{l}{240} = \frac{576\ in}{240} = 2.4\ in \quad \therefore OK$$

Horizontal deflection for snow load (at each support):

$$h = h_a - \frac{d_c}{2} - \frac{d_e}{2} = 112.8\ in - \frac{59.1\ in}{2} - \frac{17\ in}{2}$$
$$h = 74.8\ in$$
$$\Delta_H = \frac{2h\Delta_S}{l} = \frac{2\,(74.8\ in)(0.93\ in)}{576\ in} = 0.24\ in < 0.5\ in \quad \therefore OK$$

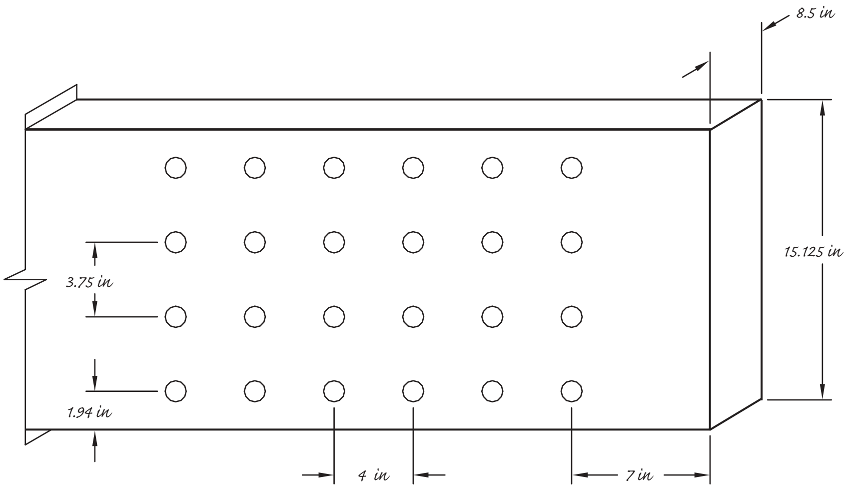
Result:

The $6\frac{3}{4}$ in. wide beam is illustrated below:



BOLTED TENSION CONNECTION WITH STEEL SIDE PLATES (ASD Method)
(See also Sections 12.3, 13.2)

Given: An $8\frac{1}{2}$ in. × 15.125 in. Combination 50 SP member has a tension splice consisting of $\frac{5}{16}$ in. × 14 in. steel side plates and four rows of six 1 in. bolts loaded in double shear and positioned as illustrated. The member is subject to construction loading (plus dead), normal temperatures and dry conditions of use.



Wanted: Determine the capacity of the connection considering fastener capacity and member capacity to resist failure modes of net section fracture, row tear-out, and group tear-out.

Solution:

Glulam Design values (AITC 117-2010):

$$F_t' = F_c C_D C_M C_t = 1550\ psi(1.25)(1.0)(1.0) = 1940\ psi$$

$$F_{vx}' = F_{vx} C_D C_M C_t C_{vr} = 300\ psi(1.25)(1.0)(1.0)(0.72) = 270\ psi$$

$$E' = EC_M C_t = 1.9\left(10^6\ psi\right)(1.0)(1.0) = 1.9\left(10^6\ psi\right)$$

NET SECTION FRACTURE:

$$A_n = b\left(d - n_j D_{BH}\right) = (8.5\ in)(15.125\ in - (4)\ 1.0625\ in) = 92.4\ in^2$$

$$T_n' = F_t' A_n = \left(1940\ psi\right)\left(92.4\ in^2\right) = \boxed{179{,}300\ lb = T_n'}$$

ROW TEAR-OUT:

$$A_{crit\ shear,rt} \leq A_{crit,\ end,\ rt} = 2te = 2\,(8.5\ in)(7\ in) = 119\ in^2$$

$$A_{crit\ shear,\ rt} \leq A_{crit,\ spacing,\ rt} = 2ts_1 = 2\,(8.5\ in)(4.0\ in)$$

$$A_{crit\ shear,\ rt} \leq 68.0\ in^2 \qquad \therefore\ controls$$

$$T'_{rt} = n_j n_i \frac{F'_v}{2} A_{crit\ shear,\ it}$$

$$T'_{rt} = (4)(6)\left(\frac{270\ psi}{2}\right)(68.0\ in^2)$$

$$\boxed{T'_{rt} = 220{,}300\ lb}$$

GROUP TEAR-OUT:

$$A_{crit\ shear,\ gt} = A_{crit\ gt,\ spacing} = 2n_i t s_1 = 2(6)(8.5\ in)(4.0\ in) = 408\ in^2$$

$$A_{eff\ tension} = t\,(s_2 - D_{BH})\left(n_j - 1\right)$$

$$A_{eff\ tension} = (8.5\ in)(3.75\ in - 1.0625\ in)(4 - 1) = 68.5\ in^2$$

$$T'_{gt} = \frac{F'_v}{2} A_{crit\ shear,\ gt} + F'_t A_{eff\ tension}$$

$$T'_{gt} = \left(\frac{270\ psi}{2}\right)(408\ in^2) + (1940\ psi)(68.5\ in^2) = \boxed{188{,}000\ lb = T'_{gt}}$$

FASTENER CAPACITY:

Reference Design Value:

$$F_{em} = 11{,}200G\ psi = 11{,}200\,(0.55)\ psi = 6160\ psi$$

$$R_e = \frac{F_{em}}{F_{es}} = \frac{6160\ psi}{87{,}000\ psi} = 0.0708$$

$$k_3 = -1 + \sqrt{\frac{2\,(1 + R_e)}{R_e} + \frac{2F_{yb}\,(2 + R_e)\,D^2}{3F_{em}l_s^2}}$$

$$k_3 = -1 + \sqrt{\frac{2\,(1 + 0.0708)}{0.0708} + \frac{2\,(45{,}000\ psi)(2 + 0.0708)(1.0\ in)^2}{3\,(6160\ psi)(0.3125\ in)^2}}$$

$$k_3 = 10.56$$

$$Z_{Im} = \frac{Dl_m F_{em}}{R_d} = \frac{(1.0\ in)(8.5\ in)(6160\ psi)}{4.0} = 13{,}090\ lb \qquad (Mode\ I_m)$$

$$Z_{Is} = \frac{2Dl_s F_{es}}{R_d} = \frac{2\,(1.0\ in)(0.3125\ in)(87{,}000\ psi)}{4.0} = 13{,}590\ lb \qquad (Mode\ I_s)$$

$$Z_{III} = \frac{2k_3 Dl_s F_{em}}{(2 + R_e)\,R_d} = \frac{2\,(10.56)(1.0\ in)(0.3125\ in)(6160\ psi)}{(2 + 0.0708)(3.2)}$$

$$Z_{III} = 6130\ lb \qquad (Mode\ III)$$

$$Z_{IV} = \frac{2D^2}{R_d}\sqrt{\frac{2F_{em}F_{yb}}{3(1+R_e)}} = \frac{2(1.0\ in)^2}{3.2}\sqrt{\frac{2(6160\ psi)(45{,}000\ psi)}{3(1+0.0708)}}$$

$$Z_{IV} = 8{,}210\ lb \qquad (Mode\ IV)$$

Geometry factor:

$$e = 7\ in = 7D \qquad \therefore C_{\Delta e} = 1.0$$

$$s_1 = 4.0\ in = 4D \qquad \therefore C_{\Delta s1} = 1.0$$

$$s_2 = 3.75\ in = 3.75D \geq 1.5D = 1.5\ in \qquad \therefore C_{\Delta s2} = 1.0$$

$$u = 1.94\ in \geq \frac{s_2}{2} = \left(\frac{3.75\ in}{2}\right) = 1.875\ in \qquad \therefore C_{\Delta u} = 1.0$$

$$C_\Delta = 1.0$$

Group action factor:

$$E'_m A_m = 1.9\left(10^6\ psi\right)(8.5\ in)(15.125\ in) = 244.3\left(10^6\ lb\right)$$

$$E_s A_s = 29\left(10^6\ psi\right)(14\ in)(0.3125\ in)\ 2 = 253.8\left(10^6\ lb\right)$$

$$R_{EA} = \frac{E'_m A_m}{E_s A_s} = \frac{244.3\ (10^6\ lb)}{253.8\ (10^6\ lb)} = 0.962$$

$$\gamma = 270{,}000D^{1.5}\ lb/in = 270{,}000\ (1.0)^{1.5}\ lb/in = 270{,}000\ lb/in$$

$$u = 1 + n_j\gamma\frac{s_1}{2}\left[\frac{1}{E'_m A_m} + \frac{1}{E_s A_s}\right]$$

$$u = 1 + 4\,(270{,}000\ lb/in)\ \frac{4\ in}{2}\left[\frac{1}{244.3\ (10^6\ lb)} + \frac{1}{253.8\ (10^6\ lb)}\right]$$

$$u = 1.017$$

$$m = u - \sqrt{u^2 - 1} = 1.017 - \sqrt{(1.017)^2 - 1} = 0.832$$

$$C_g = \left[\frac{m\,(1 - m^{2n})}{n\left[(1 + R_{EA}m^n)(1 + m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$

$$C_g = \left[\frac{0.832\left(1 - 0.832^{2(6)}\right)}{6\left[(1 + (0.962)\ 0.832^6)(1 + 0.832) - 1 + 0.832^{2(6)}\right]}\right]\left[\frac{1 + 0.962}{1 - 0.832}\right]$$

$$C_g = 0.944$$

Capacity of a single bolt:

$$Z' = ZC_D C_M C_t C_\Delta C_g$$

$$Z' = (6130\ lb)(1.25)(1.0)(1.0)(1.0)(0.94)$$

$$Z' = 7200\ lb$$

BOLTED TENSION CONNECTION WITH STEEL SIDE PLATES (ASD Method) continued ...

Total capacity of all bolts:

$$T' = n_j n_i Z'$$

$$T' = (4)(6)(7200\ lb)$$

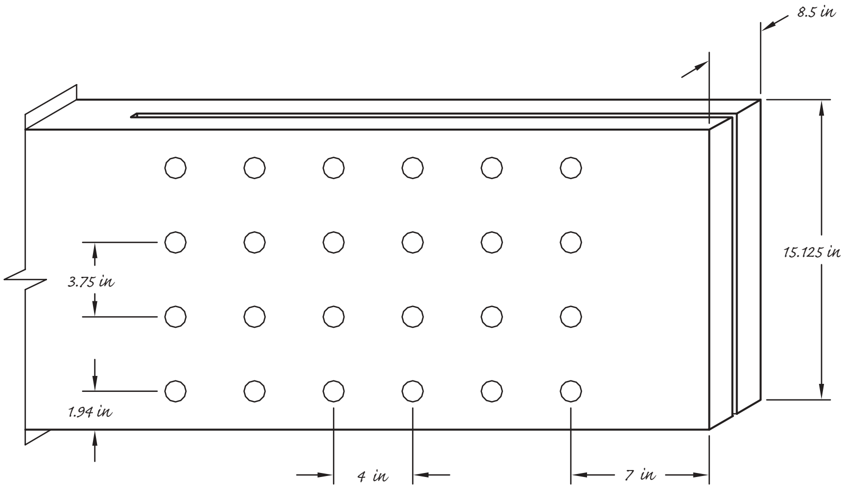
$$T' = 172{,}800\ lb$$

Result:

The connection can resist a maximum of 172,800 lb as controlled by mode III fastener yielding. The capacity of the steel side plates must also be verified using appropriate steel design procedures.

BOLTED TENSION CONNECTION WITH STEEL KERF PLATE (ASD Method)
(See also Sections 12.3, 13.2)

Given: An $8\frac{1}{2}$ in. × 15.125 in. Combination 50 SP member has a tension splice consisting of a $\frac{5}{8}$ in. × 14 in. steel plate in an $\frac{11}{16}$ in. wide vertical kerf cut into the end of the member and four rows of six 1 in. bolts loaded in double shear and positioned as illustrated. The member is subject to construction loading, normal temperatures and dry conditions of use.



Wanted: Determine the capacity of the connection considering fastener capacity and member capacity to resist failure modes of net section fracture, row tear-out, and group tear-out.

Solution:

Glulam Design values (AITC 117-2010):

$$F_t' = F_c C_D C_M C_t = 1550\ psi\ (1.25)(1.0)(1.0) = 1940\ psi$$

$$F_{vx}' = F_{vx} C_D C_M C_t C_{vr} = 300\ psi\ (1.25)(1.0)(1.0)(0.72) = 270\ psi$$

$$E' = EC_M C_t = 1.9\left(10^6\ psi\right)(1.0)(1.0) = 1.9\left(10^6\ psi\right)$$

NET SECTION FRACTURE (bolt hole diameter is $\frac{1}{16}$ in. larger than bolt):

$$A_n = \left(b - t_{kerf}\right)\left(d - n_j D_{BH}\right)$$

$$A_n = (8.5\ in - 0.688\ in)(15.125\ in - (4)\ 1.0625\ in) = 85.0\ in^2$$

$$T_n' = F_t' A_n = \left(1940\ psi\right)\left(85.6\ in^2\right) = \boxed{166{,}100\ lb = T_n'}$$

ROW TEAR-OUT:

$$A_{crit\ shear,rt} \leq A_{crit,\ end,\ rt}$$

$$A_{crit\ shear,rt} \leq 2\left(t - t_{kerf}\right) e = 2\,(8.5\ in - 0.688\ in)(7\ in) = 109\ in^2$$

$$A_{crit\ shear,\ rt} \leq A_{crit,\ spacing,\ rt} = 2\left(t - t_{kerf}\right) s_1$$

$$A_{crit\ shear,\ rt} \leq 2\,(8.5\ in - 0.688\ in)(4.0\ in) = 62.5\ in^2 \quad \therefore\ controls$$

$$T'_{rt} = n_j n_i \frac{F'_v}{2} A_{crit\ shear,\ it}$$

$$T'_{rt} = (4)(6)\left(\frac{270\ psi}{2}\right)\left(62.5\ in^2\right)$$

$$\boxed{T'_{rt} = 202,500\ lb}$$

GROUP TEAR-OUT:

$$A_{critshear,\ gt} = A_{crit\ gt,\ spacing}$$

$$A_{critshear,\ gt} = 2n_i\left(t - t_{kerf}\right) s_1 = 2\,(6)(8.5\ in - 0.688\ in)(4.0\ in) = 375\ in^2$$

$$A_{eff\ tension} = \left(t - t_{kerf}\right)(s_2 - D_{BH})\left(n_j - 1\right)$$

$$A_{eff\ tension} = (8.5\ in - 0.688\ in)(3.75\ in - 1.0625\ in)(4 - 1) = 63.0\ in^2$$

$$T'_{gt} = \frac{F'_v}{2} A_{crit\ shear,\ gt} + F'_t A_{eff\ tension}$$

$$T'_{gt} = \left(\frac{270\ psi}{2}\right)\left(375\ in^2\right) + \left(1940\ psi\right)\left(63.0\ in^2\right) = \boxed{172{,}800\ lb = T'_{gt}}$$

FASTENER CAPACITY:

Reference Design Value:

$$F_{es} = 11,200\ G\ psi = 11,200\ (0.55)\ psi = 6160\ psi$$

$$R_e = \frac{F_{em}}{F_{es}} = \frac{87,000\ psi}{6160\ psi} = 14.12$$

$$k_3 = -1 + \sqrt{\frac{2\,(1 + R_e)}{R_e} + \frac{2F_{yb}\,(2 + R_e)\,D^2}{3F_{em}\,l_s^2}}$$

$$k_3 = -1 + \sqrt{\frac{2\,(1 + 14.12)}{14.12} + \frac{2\left(45{,}000\ psi\right)(2 + 14.12)(1.0\ in)^2}{3\left(87{,}000\ psi\right)(3.91\ in)^2}}$$

$$k_3 = 0.583$$

$$Z_{Im} = \frac{D l_m F_{em}}{R_d} = \frac{(1.0\ in)(0.625\ in)\left(87{,}000\ psi\right)}{4.0}$$

$$Z_{Im} = 13{,}590\ lb \qquad (Mode\ I_m)$$

$$Z_{Is} = \frac{2 D l_s F_{es}}{R_d} = \frac{2\,(1.0\ in)(3.91\ in)\left(6160\ psi\right)}{4.0} = 12{,}040\ lb \qquad (Mode\ I_s)$$

$$Z_{III} = \frac{2k_3 D l_s F_{em}}{(2 + R_e)\, R_d} = \frac{2\,(0.583)(1.0\ in)(3.91\ in)\left(87{,}000\ psi\right)}{(2 + 14.12)(3.2)}$$

$$Z_{III} = 7{,}690\ lb \qquad (Mode\ III)$$

$$Z_{IV} = \frac{2D^2}{R_d}\sqrt{\frac{2F_{em}F_{yb}}{3\,(1 + R_e)}} = \frac{2\,(1.0\ in)^2}{3.2}\sqrt{\frac{2\left(87{,}000\ psi\right)\left(45{,}000\ psi\right)}{3\,(1 + 14.12)}}$$

$$Z_{IV} = 8{,}210\ lb \qquad (Mode\ IV)$$

Geometry factor:

$$e = 7\ in = 7D \qquad \therefore\ C_{\Delta e} = 1.0$$

$$s_1 = 4.0\ in = 4D \qquad \therefore\ C_{\Delta s1} = 1.0$$

$$s_2 = 3.75\ in = 3.75D \geq 1.5D = 1.5\ in \qquad \therefore\ C_{\Delta s2} = 1.0$$

$$u = 1.94\ in \geq \frac{s_2}{2} = \left(\frac{3.75\ in}{2}\right) = 1.875\ in \qquad \therefore\ C_{\Delta u} = 1.0$$

$$C_\Delta = 1.0$$

Group action factor:

$$E_s'A_s = 1.9\left(10^6\ psi\right)(8.5\ in - 0.688\ in)(15.125\ in) = 224.5\left(10^6\ lb\right)$$

$$E_mA_m = 29\left(10^6\ psi\right)(14\ in)(0.625\ in) = 253.8\left(10^6\ lb\right)$$

$$R_{EA} = \frac{E_m'A_m}{E_sA_s} = \frac{224.5\ (10^6\ lb)}{253.8\ (10^6\ lb)} = 0.885$$

$$\gamma = 270{,}000D^{1.5}\ lb/in = 270{,}000\,(1.0)^{1.5}\ lb/in = 270{,}000\ lb/in$$

$$u = 1 + n_j\gamma\frac{s_1}{2}\left[\frac{1}{E_mA_m} + \frac{1}{E_s'A_s}\right]$$

$$u = 1 + 4\,(270{,}000\ lb/in)\,\frac{4\ in}{2}\left[\frac{1}{253.8\ (10^6\ lb)} + \frac{1}{224.5\ (10^6\ lb)}\right]$$

$$u = 1.018$$

$$m = u - \sqrt{u^2 - 1} = 1.018 - \sqrt{(1.018)^2 - 1} = 0.827$$

$$C_g = \left[\frac{m\,(1 - m^{2n})}{n\left[(1 + R_{EA}m^n)(1 + m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$

$$C_g = \left[\frac{0.827\left(1 - 0.827^{2(6)}\right)}{6\left[(1 + (0.885)\ 0.827^6)(1 + 0.827) - 1 + 0.827^{2(6)}\right]}\right]\left[\frac{1 + 0.885}{1 - 0.827}\right]$$

$$C_g = 0.93$$

<u>Capacity of a single bolt:</u>

$$Z' = ZC_D C_M C_t C_\Delta C_g$$

$$Z' = (7,690\ lb)(1.25)(1.0)(1.0)(1.0)(0.93)$$

$$Z' = 9,250\ lb$$

<u>Total capacity of all bolts:</u>

$$T' = n_j n_i Z'$$

$$T' = (4)(6)(9250\ lb)$$

$$T' = 222,000\ lb$$

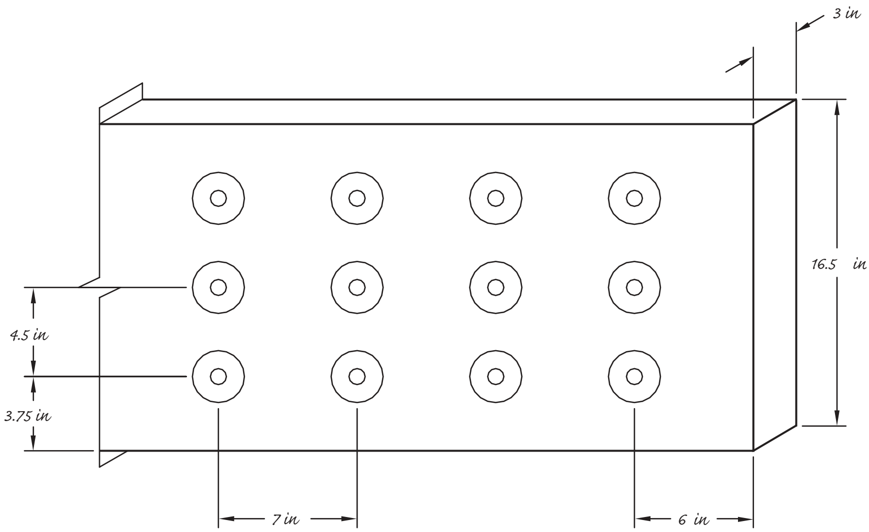
Result:

The connection can resist a maximum of 166,100 lb as controlled net section fracture. The capacity of the steel plate must also be evaluated using appropriate steel design procedures.

SHEAR PLATE TENSION CONNECTION (ASD Method)

(See also Sections 12.3, 14.2.3.1)

Given: A 3 in. × $16\frac{1}{2}$ in. Combination 48 SP member has a tension splice consisting of $\frac{1}{4}$ in. × 13 in. steel side plates and three rows of four $\frac{3}{4}$ in. bolts, as illustrated. The bolts are loaded in double shear with $2\frac{5}{8}$ in. shear plates installed in each shear plane. The member is subject to snow loading, normal temperatures and dry conditions of use.



Wanted: Determine the capacity of the connection considering fastener capacity and member capacity to resist failure modes of net section fracture, row tear-out, and group tear-out.

Solution:

Glulam Design values (AITC 117-2010):

$$F_t' = F_t C_D C_M C_t = 1400 \text{ psi } (1.15)(1.0)(1.0) = 1610 \text{ psi}$$

$$F_{vx}' = F_{vx} C_D C_M C_t C_{vr} = 300 \text{ psi } (1.15)(1.0)(1.0)(0.72) = 248 \text{ psi}$$

$$E' = EC_M C_t = 1.7\left(10^6 \text{ psi}\right)(1.0)(1.0) = 1.7\left(10^6 \text{ psi}\right)$$

NET SECTION FRACTURE:

$$A_n = bd - n_j\left(2pD_o + D_{BH}\left(b - 2p\right)\right) \quad \text{for member in double shear}$$

$$A_n = (3 \text{ in})(16.5 \text{ in}) - (3)\left[2\,(0.42 \text{ in})(2.62 \text{ in}) + (0.81 \text{ in})(3 \text{ in} - 2\,(0.42 \text{ in}))\right]$$

$$A_n = 37.7 \text{ in}^2$$

$$T_n' = F_t' A_n = \left(1610 \text{ psi}\right)\left(37.7 \text{ in}^2\right) = 60,700 \text{ lb}$$

ROW TEAR-OUT:

$$A_{crit,\ spacing,\ rt} = \left(D_o + 2p\right) s_1 + \frac{\pi}{4}\left(D_I^2 - D_o^2 - D_{BH}^2\right) + 2ps_1$$

$$A_{crit,\ spacing,\ rt} = \left[2.62 + 2\left(0.42\right)\right]\left(7\right)\ in^2 + \frac{\pi}{4}\left[\left(2.28\right)^2 - \left(2.62\right)^2 - \left(0.81\right)^2\right]\ in^2 + 2\left(0.42\right)\left(7\right)\ in^2$$

$$A_{crit,\ spacing,\ rt} = 28.3\ in^2$$

$$A_{crit,\ end,\ rt} = \left(D_o + 2p\right) e + \frac{\pi}{4}\left(D_I^2 - \frac{D_o^2}{2} - D_{BH}^2\right) + 2pe$$

$$A_{crit,\ end,\ rt} = \left[2.62 + 2\left(0.42\right)\right]\left(6\right)\ in^2 + \frac{\pi}{4}\left[\left(2.28\right)^2 - \frac{\left(2.62\right)^2}{2} - \left(0.81\right)^2\right]\ in^2 + 2\left(0.42\right)\left(6\right)\ in^2$$

$$A_{crit,\ end,\ rt} = 26.7\ in^2 \qquad \therefore \text{ end distance controls}$$

$$A_{crit\ shear,\ rt} = A_{crit,\ end,\ rt} = 26.7\ in^2$$

$$T'_{rt} = n_{shear\ planes} n_j n_i \frac{F'_v}{2} A_{crit\ shear,\ rt}$$

$$T'_{rt} = \left(2\right)\left(3\right)\left(4\right)\left(\frac{248\ psi}{2}\right)\left(26.7\ in^2\right)$$

$$T'_{rt} = 79{,}460\ lb$$

GROUP TEAR-OUT

$$A_{crit\ gt,\ end} = n_i\left(\left[2p\left(1 + n_j\right) + \left(n_j - 1\right) s_2 + D_o\right] e + \frac{\pi n_j}{4}\left[D_I^2 - \frac{D_o^2}{2} - D_{BH}^2\right]\right)$$

$$A_{crit\ gt,\ end} = 4\left(\left[2\left(0.42\right)\left(1 + 3\right) + \left(3 - 1\right) 7 + 2.62\right]\left(6\right) + \frac{\pi\left(3\right)}{4}\left[\left(2.28\right)^2 - \frac{\left(2.62\right)^2}{2} - \left(0.81\right)^2\right]\right)\ in^2$$

$$A_{crit\ gt,\ end} = 490\ in^2 = A_{crit\ shear,\ gt}$$

$$A_{efftension} = p\left(s_2 - D_{BH}\right)\left(n_j - 1\right)$$

$$A_{efftension} = 2\left(0.42\ in\right)\left(4.5\ in - 2.62\ in\right)\left(3 - 1\right)$$

$$A_{efftension} = 1.58\ in^2$$

$$T'_{gt} = n_{shear\ planes}\left[\frac{F'_v}{2}A_{crit\ shear,\ gt} + F'_t A_{eff\ tension}\right]$$
$$T'_{gt} = 2\left[\left(\frac{248\ psi}{2}\right)\left(490\ in^2\right) + \left(1610\ psi\right)\left(1.58\ in^2\right)\right]$$
$$T'_{gt} = 127,000\ lb$$

FASTENER CAPACITY:

Geometry factor:

$$e = 6\ in \geq e_{C_\Delta=1.0} = 5.5\ in \qquad \therefore\ C_{\Delta e} = 1.0$$
$$s_1 = 7\ in \geq s_{1,C_\Delta=1.0} = 6.75\ in \qquad \therefore\ C_{\Delta s1} = 1.0$$
$$s_2 = 4.5\ in \geq s_{2,C_\Delta=1.0} = 3.5\ in \qquad \therefore\ C_{\Delta s2} = 1.0$$
$$u = 3.75\ in \geq u_{C_\Delta=1.0} = 1.75\ in \qquad \therefore\ C_{\Delta u} = 1.0$$
$$C_\Delta = 1.0$$

Group action factor for $n = 4,\ s_1 = 7\ in$

$$E'_m A_m = 1.7\left(10^6\ psi\right)(3\ in)(16.5\ in) = 84.2\left(10^6\ lb\right)$$
$$E_s A_s = 29\left(10^6\ psi\right)(13\ in)(0.25\ in)\ 2 = 189\left(10^6\ lb\right)$$
$$R_{EA} = \frac{E'_m A_m}{E_s A_s} = \frac{84.2\left(10^6\ lb\right)}{189\left(10^6\ lb\right)} = 0.446$$
$$\gamma = 400,000\ lb/in$$
$$u = 1 + n_j\gamma\frac{s_1}{2}\left[\frac{1}{E'_m A_m} + \frac{1}{E_s A_s}\right]$$
$$u = 1 + 3\left(400,000\ lb/in\right)\frac{7\ in}{2}\left[\frac{1}{84.2\left(10^6\ lb\right)} + \frac{1}{189\left(10^6\ lb\right)}\right]$$
$$u = 1.072$$
$$m = u - \sqrt{u^2 - 1} = 1.072 - \sqrt{(1.072)^2 - 1} = 0.686$$
$$C_g = \left[\frac{m\left(1 - m^{2n}\right)}{n\left[\left(1 + R_{EA}m^n\right)(1 + m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$
$$C_g = \left[\frac{0.686\left(1 - 0.686^{2(4)}\right)}{4\left[\left(1 + (0.446)\ 0.686^4\right)(1 + 0.686) - 1 + 0.686^{2(4)}\right]}\right]\left[\frac{1 + 0.446}{1 - 0.686}\right]$$
$$C_g = 0.83$$

Capacity of a single connector unit:

$$P' = PC_D C_M C_t C_\Delta C_g C_{st}$$

$$P' = (2860\ lb)(1.15)(1.0)(1.0)(1.0)(0.83)(1.0)$$

$$P' = 2730\ lb$$

Capacity of all connectors:

$$T' = n_{shear\ planes} n_j n_i P'$$

$$T' = (2)(3)(4)(2730\ lb)$$

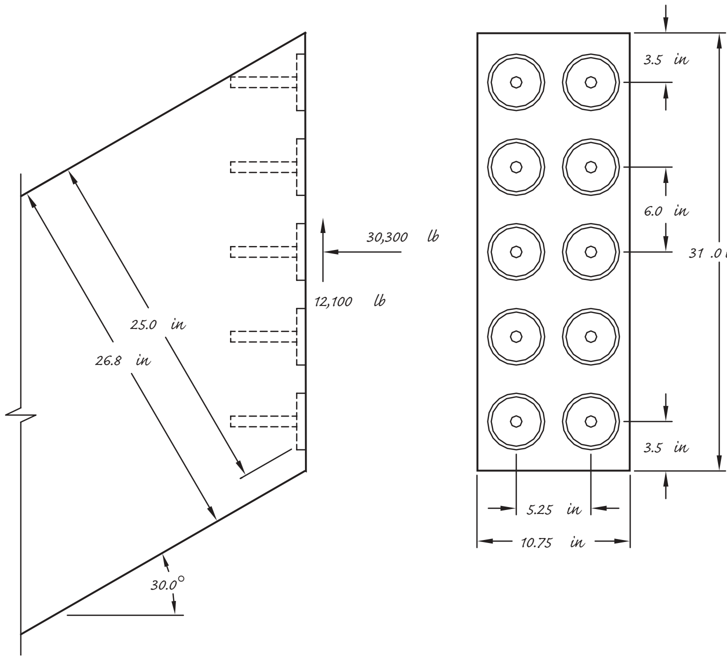
$$T' = 65,500\ lb$$

Result:

The connection can resist a maximum of 60,700 lb as controlled by net section fracture.

TUDOR ARCH PEAK SHEAR PLATE CONNECTION (ASD Method)
(See also Sections 11.5.4, 14.3)

Given: A proposed connection for the peak of a large Tudor arch is illustrated. The arch members are 10.75 in. wide with a depth of 31 inches along the vertical cut. The 4 inch shear plates are spaced 6 inches apart within each row, and the rows are spaced 5.25 inches apart. The soffit slope of the arch is 30°. The connection must transfer 30,300 lb compression and 12,100 lb shear.



Wanted: Evaluate the proposed connection. Consider the member shear capacity and fastener capacity.

Solution:

MEMBER SHEAR CAPACITY:

$$F'_{vx} = F_{vx} C_D C_M C_t C_{vr} = 265\ psi\ (1.15)(1.0)(1.0)(0.72) = 219\ psi$$

Notched capacity at the connection:

$$V' = \left[\frac{2}{3} F'_{vx} b d_e\right]\left[\frac{d_e}{d}\right]^2$$

$$V' = \frac{2}{3}(219\ psi)(10.75\ in)(25.0\ in)\left[\frac{25.0\ in}{26.8\ in}\right]^2$$

$$V' = 34,100\ lb$$

Shear force perpendicular to the member:

$$V = (30,300\ lb)\cos(30^\circ) + (12,100\ lb)\sin(30^\circ)$$

$$V = 32,300\ lb < V' = 34,100\ lb \quad \therefore\ OK$$

FASTENER CAPACITY:

Geometry factor based on spacing, $s_\perp$, and edge distance, $u_\perp$, perpendicular to axis of cut:

$$u_\perp = 2.75\ in \geq u_{\perp,1.0} = 2.75\ in \quad \therefore\ C_{\Delta,u\perp} = 1.0$$

$$s_\perp = 5.25\ in \geq s_{\perp,1.0} = 5.0\ in \quad \therefore\ C_{\Delta,s\perp} = 1.0$$

Geometry factor based on loaded edge distance, E_α:

$$E_\alpha = \frac{E_\parallel E_\perp}{\sqrt{E_\parallel^2 \sin^2\alpha + E_\perp^2 \cos^2\alpha}}$$

$$E_{60^\circ,1.0} = \frac{(7\ in)(3.75\ in)}{\sqrt{(7\ in)^2 \sin^2(60^\circ) + (3.75\ in)^2 \cos^2(60^\circ)}}$$

$$E_{60^\circ,1.0} = 4.1\ in \quad for\ C_\Delta = 1.0$$

$$E_{60^\circ,0.83} = \frac{(5.4\ in)(2.5\ in)}{\sqrt{(5.4\ in)^2 \sin^2(60^\circ) + (2.5\ in)^2 \cos^2(60^\circ)}}$$

$$E_{60^\circ,0.83} = 2.8\ in \quad for\ C_\Delta = 0.83$$

$$C_{\Delta,\alpha,E} = C_{\Delta,\alpha,E,min} + \left(\frac{E_\alpha - E_{\alpha,min}}{E_{\alpha,1.0} - E_{\alpha,min}}\right)(1.0 - C_{\Delta,\alpha,E,min})$$

$$C_{\Delta,\alpha,E} = 0.83 + \left(\frac{3.5\ in - 2.8\ in}{4.1\ in - 2.8\ in}\right)(1.0 - 0.83)$$

$$C_{\Delta,\alpha,E} = 0.92$$

Geometry factor based on spacing parallel to axis of cut, s_α:

$$s_\alpha = \frac{s_\parallel s_\perp}{\sqrt{s_\parallel^2 \sin^2\alpha + s_\perp^2 \cos^2\alpha}}$$

$$s_{30^\circ,1.0} = \frac{(9\ in)(6\ in)}{\sqrt{(9\ in)^2 \sin^2(60^\circ) + (6\ in)^2 \cos^2(60^\circ)}}$$

$$s_{30^\circ,1.0} = 6.5\ in \quad for\ C_\Delta = 1.0$$

$$s_{min} = 5.0 \quad for\ C_\Delta = 0.5$$

$$C_{\Delta,\alpha,s} = C_{\Delta,\alpha,s,min} + \left(\frac{s_\alpha - s_{\alpha,min}}{s_{\alpha,1.0} - s_{\alpha,min}}\right)(1.0 - C_{\Delta,\alpha,E,min})$$

$$C_{\Delta,\alpha,s} = 0.5 + \left(\frac{6\ in - 5\ in}{6.5\ in - 5\ in}\right)(1.0 - 0.5)$$

$$C_{\Delta,\alpha,s} = 0.83 \qquad \boxed{C_\Delta = 0.83}$$

Group action factor:

$$E'_x = E_x C_M C_t = 2.0\left(10^6\ psi\right)(1.0)(1.0) = 2.0\left(10^6\ psi\right)$$

$$\gamma = \frac{500,000\ lb/in}{2} = 250,000\ lb/in \qquad \text{(divide by 2 for single shear)}$$

$$E'_\perp = \frac{E'_x}{20} = \frac{2.0\left(10^6\ psi\right)}{20} = 100,000\ psi$$

$$\gamma_\perp = \frac{\gamma}{2} = \frac{250,000\ lb/in}{2} = 125,000\ lb/in$$

$$\alpha = 90^\circ - 30^\circ = 60^\circ$$

$$E_\alpha = \frac{EE_\perp}{E\sin^2\alpha + E_\perp\cos^2\alpha}$$

$$E_\alpha = \frac{\left[2.0\left(10^6\ psi\right)\right]\left[100\left(10^3\ psi\right)\right]}{\left[2.0\left(10^6\ psi\right)\right]\sin^2(60^\circ) + \left[100\left(10^3\ psi\right)\right]\cos^2(60^\circ)}$$

$$E_\alpha = 131,000\ psi$$

$$\gamma_\alpha = \frac{\gamma\gamma_\perp}{\lambda\sin^2\alpha + \gamma_\perp\cos^2\alpha}$$

$$\gamma_\alpha = \frac{\left[250,000\ lb\right]\left[125,000\ lb\right]}{\left[250,000\ lb\right]\sin^2(60^\circ) + \left[125,000\ lb\right]\cos^2(60^\circ)}$$

$$\gamma_\alpha = 143,000\ lb$$

$$A_s = A_m = bd' = (10.75\ in)(31\ in) = 333\ in^2$$

$$R_{EA} = \frac{E_s A_s}{E_m A_m} = \frac{\left(131,000\ psi\right)\left(333\ in^2\right)}{\left(131,000\ psi\right)\left(333\ in^2\right)} = \frac{43.6\left(10^6\ lb\right)}{43.6\left(10^6\ lb\right)} = 1.0$$

$$u = 1 + n_j\gamma_\alpha\frac{s}{2}\left[\frac{1}{E_m A_m} + \frac{1}{E_s A_s}\right]$$

$$u = 1 + (2)(143,000\ lb)\frac{(6\ in)}{2}\left[\frac{2}{43.6\left(10^6\ lb\right)}\right] = 1.039$$

$$m = u - \sqrt{u^2 - 1} = 1.039 - \sqrt{1.039^2 - 1} = 0.757$$

$$C_g = \left[\frac{m\left(1 - m^{2n}\right)}{n\left[\left(1 + R_{EA}m^n\right)(1 + m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$

$$C_g = \left[\frac{0.757\left(1-(0.757)^{2(5)}\right)}{5\left[\left(1+(1)(0.757)^5\right)(1+0.757)-1+(0.757)^{2(5)}\right]}\right]\left[\frac{1+1}{1-0.757}\right]$$

$$C_g = 0.93$$

Connector Capacity:

$$Q_{90} = 0.6Q = 0.6\,(3040\ lb) = 1824\ lb$$

$$N = \frac{PQ_{90}}{P\sin^2\alpha + Q_{90}\sin^2\alpha}$$

$$N = \frac{(4360\ lb)(1824\ lb)}{(4360\ lb)\sin^2(60°) + (1824\ lb)\cos^2(60°)} = 2130\ lb$$

$$N' = NC_DC_MC_tC_gC_\Delta = 2130\ lb\,(1.15)(1.0)(1.0)(0.93)(0.83) = 1890\ lb$$

$$V' = nN' = 10\,(1890\ lb)$$

$$V' = 18{,}900\ lb > 12{,}100\ lb \quad \therefore\ \text{OK, but significant excess capacity}$$

Using the same edge distance and increasing the spacing will cause both the geometry factor and the group action factor to increase and will permit the use of fewer fasteners. Neglecting the change in the group action factor, and increasing the spacing so that the geometry factor is controlled by edge distance ($C_\Delta = 0.92$), the required number of fasteners can be estimated as:

$$N' = NC_DC_MC_tC_gC_\Delta = 2130\ lb\,(1.15)(1.0)(1.0)(0.93)(0.92) = 2096\ lb$$

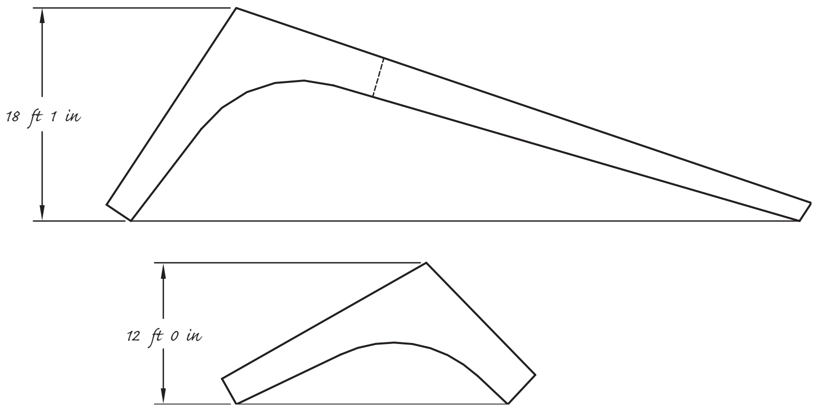
$$n \geq \frac{V}{N'} = \frac{12{,}100\ lb}{2096\ lb} = 5.8\ \text{connector units} \qquad (\text{use 6 connector units})$$

Result:

The proposed connection is adequate, but it is not efficient. Only six connector units are needed to transfer the design force. A spacing of 12 inches within each row should be specified with 5.25-inch spacing between rows. The edge distance of 3.5 inches is critical. A larger edge distance may reduce the member shear capacity below the required resistance.

MOMENT SPLICE (ASD Method)
(See also Chapters 9, 14, 15)

Given: A moment splice is required to facilitate shipping of an $8\frac{1}{2}$ in. wide, 24F-V8 SP glulam arch. Without the splice, the shipping width of the arch would be 18 ft 1 in. as illustrated. By using a moment splice, the shipping width will be reduced to 12 ft 0 in. The arch arm has a 2.5° taper. The arch will be subject to normal temperatures and dry conditions of use.



At the location of the splice, the arch is 41 in. deep. The splice must resist a maximum shear force of $V = 17{,}000$ lb due to the unbalanced snow load combination and must resist the following combinations of compression load and moment (positive moment = compression on outside face of arch):

Primary Load	C_D	M (in-lb)	P (lb)
Snow	1.15	−2,600,000	25,400 (C)
Wind	1.6	−2,400,000	22,400 (C)
Seismic	1.6	356,000	2,265 (C)

Wanted: Design a moment splice using two rows of $2\frac{5}{8}$ in. shear plates installed in end grain to transfer the shear force and $2\frac{5}{8}$ in. shear plates with steel side plates to transfer tension forces.

Solution:

The small taper angle will be neglected in connection design calculations and tension straps will be placed parallel to the tension surface (2.5° angle with grain).

Glulam Design values (AITC 117-2010):

$$F_c' = F_c C_D C_M C_t = 1650\ psi(1.15)(1.0)(1.0) = 1900\ psi \quad \text{for snow load}$$
$$F_c' = F_c C_D C_M C_t = 1650\ psi(1.6)(1.0)(1.0)$$
$$F_c' = 2640\ psi \quad \text{for wind and seismic loads}$$
$$F_{bx}' = F_{bx} C_D C_M C_t = 2400\ psi(1.15)(1.0)(1.0) = 2760\ psi \quad \text{for snow load}$$
$$F_{bx}' = F_{bx} C_D C_M C_t = 2400\ psi(1.6)(1.0)(1.0)$$
$$F_{bx}' = 3840\ psi \quad \text{for wind and seismic loads}$$
$$E_x' = E_x C_M C_t = 1.8\left(10^6\ psi\right)(1.0)(1.0) = 1.8\left(10^6\ psi\right)$$
$$F_{vx}' = F_{vx} C_D C_M C_t C_{vr} = 300\ psi(1.15)(1.0)(1.0)(0.72) = 248\ psi \quad \text{for snow load}$$

SHEAR TRANSFER:

Shear plate design values (NDS 2011, species group B):

$$Q_{90}' = 0.6 Q C_D C_M C_t C_\Delta C_g = 0.6\ (1990\ lb)(1.15)(1.0)(1.0)\ C_\Delta C_g = (1373\ lb)\ C_\Delta C_g$$

Geometry factors for shear plates in end grain based on loaded edge distance, E, and spacing, S:

E (in.)	C_Δ
$1\frac{1}{2}$	0.83
$2\frac{3}{4}$	1.0

S (in.)	C_Δ
$3\frac{1}{2}$	0.5
$4\frac{1}{4}$	1.0

Number of shear plates required in end grain if $C_\Delta = 1.0$, and $C_g \approx 0.75$:

$$n = \frac{V}{Q_{90}'} = \frac{17{,}000\ lb}{(1373\ lb)\ C_\Delta C_g} = \frac{17{,}000\ lb}{(1373\ lb)(1.0)(0.75)} = 16.5 \quad \therefore \text{Try 2 rows of 9}$$

Minimum depth for $C_\Delta = 1.0$:

$$d_{1.0} = 2E_{1.0} + (n-1)\ S_{1.0} = 2\ (2.75\ in) + (9-1)(4.25\ in)$$
$$d_{1.0} = 39.5\ in \leq 41\ in \qquad \therefore OK$$

Loaded edge distance if $S = 4\frac{1}{4}$ in:

$$E = \frac{d-(n-1)\ S}{2} = \frac{41\ in - (9-1)(4.25\ in)}{2} = 3.5\ in$$

Member shear capacity if $E = 3\frac{1}{2}$ in.:

$$d_e = d - E + \frac{D_I}{2} = 41\ in - 3.5\ in + \frac{2.29\ in}{2} = 38.7\ in$$

$$V' = \frac{2}{3} F'_{vx} b d_e \left[\frac{d_e}{d}\right]^2 = \frac{2}{3}(248\ psi)(8.5\ in)(38.7\ in)\left[\frac{38.7\ in}{41\ in}\right]^2$$

$$V' = 48{,}460\ lb > 17000\ lb \quad \therefore OK$$

Group action factor for $n = 9$ and $s = 4.25$ in.:

$$A_m = A_s = bd = (8.5\ in)(41\ in) = 174\ in^2$$

$$E'_{m\perp} = E'_{s\perp} = \frac{E'_m}{20} = \frac{E'_s}{20} = \frac{1.8\left(10^6\ psi\right)}{20} = 90{,}000\ psi$$

$$\gamma_\perp = \frac{\gamma}{2(2)} = \frac{400{,}000\ lb/in}{4} = 100{,}000\ lb/in \quad (\gamma \text{ divided by 2 for single shear})$$

$$R_{EA} = \frac{E_s A_s}{E_m A_m} = 1.0$$

$$u = 1 + n_j \gamma \frac{s}{2}\left[\frac{1}{E_m A_m} + \frac{1}{E_s A_s}\right]$$

$$u = 1 + 2(100{,}000\ lb/in)\frac{4.25\ in}{2}\left[\frac{1}{(90{,}000\ psi)(348\ in^2)} + \frac{1}{(90{,}000\ psi)(348\ in^2)}\right]$$

$$u = 1.027$$

$$m = u - \sqrt{u^2 - 1} = 1.027 - \sqrt{(1.027)^2 - 1} = 0.793$$

$$C_g = \left[\frac{m\left(1 - m^{2n}\right)}{n\left[\left(1 + R_{EA} m^n\right)(1 + m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$

$$C_g = \left[\frac{0.793\left(1 - 0.793^{2(9)}\right)}{9\left[\left(1 + (1.0)\ 0.793^9\right)(1 + 0.793) - 1 + 0.793^{2(9)}\right]}\right]\left[\frac{1 + 1.0}{1 - 0.793}\right]$$

$$C_g = 0.813$$

Design capacity of shear plate connectors in end grain:

$$Q'_{90} = (1373\ lb)\ C_\Delta C_g = (1373\ lb)(1.0)(0.813) = 1{,}116\ lb$$

$$n_i n_j Q'_{90} = 9(2)(1{,}116\ lb) = 20{,}090\ lb \geq V = 17{,}000\ lb \quad \therefore OK$$

MOMENT TRANSFER FOR SNOW LOAD COMBINATION:

Shear plate design values (NDS 2011, species group B):

$$P' = PC_D C_M C_t C_\Delta C_g = (2860\ lb)(1.15)(1.0)(1.0)\ C_\Delta C_g = (3289\ lb)\ C_\Delta C_g$$

Area and section modulus for **uncut** section:

$$A = bd = (8.5\ in)(41\ in) = 349\ in^2$$

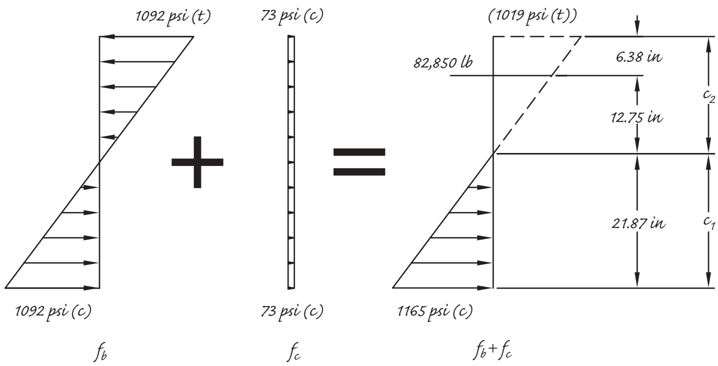
$$S_x = \frac{bd^2}{6} = \frac{(8.5\ in)(41\ in)^2}{6} = 2381\ in^3$$

Axial and flexure stresses for uncut section:

Primary Load	$f_b = \frac{M}{S}$ (psi)	$f_c = \frac{P}{A}$ (psi)	Top face stress $f_c + f_b$	Bottom face stress $f_c - f_b$
Snow	− 1,092	73	− 1,019 (T)	1165 (C)
Wind	− 1,008	90	− 918 (T)	1098 (C)
Seismic	150	7	157 (C)	− 143 (T)

The tension stresses are highest for the snow load case, and the load duration factor is lowest, so the snow load case will control the design of the top strap and fasteners. The requirements for the bottom strap and fasteners will be determined by the seismic load.

Stress distribution, neutral axis location, and resultant tension force on uncut section (snow load):



Location of neutral axis in **uncut** section:

$$\frac{c_1}{1165} = \frac{c_2}{1019} \qquad c_1 + c_2 = 41\ in$$

$$c_1 = \frac{1165c_2}{1019} \qquad \frac{1165c_2}{1019} + c_2 = 41\ in \rightarrow c_2 = 19.13\ in \qquad c_1 = \frac{1165c_2}{1019} = \frac{1165\,(19.13\ in)}{1019} \rightarrow c_1 = 21.87\ in$$

Magnitude of tension force resultant in uncut section:

$$T_{uncut} = \frac{bc_2\left(f_b - f_c\right)}{2} = \frac{(8.5\ in)(19.13\ in)\left(1019\ psi\right)}{2} = 82{,}850\ lb$$

Location of resultant tension force in uncut section (measured from top of section):

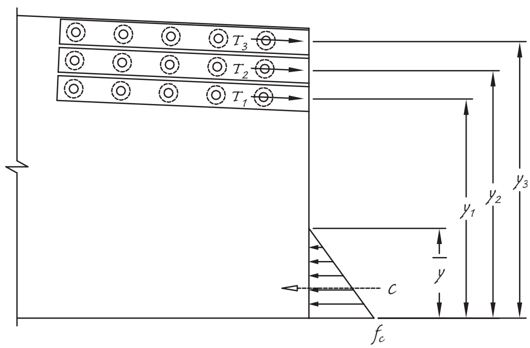
$$\frac{c_2}{3} = \frac{19.13\ in}{3} = 6.38\ in$$

Estimated number of connectors required (Estimate $C_g \approx 0.85$):

$$n \approx \frac{T_{uncut}}{P'} = \frac{82{,}850\ lb}{(3289\ lb)\ C_\Delta C_g} \approx \frac{82{,}850\ lb}{(3289\ lb)(1.0)(0.85)} = 30$$

Trial Design:

Three rows of 5 bolts per row with 2 connectors per bolt will be evaluated. One row of connectors will be placed at $y_1 = 31$ in. from the bottom, a second row will be placed at $y_2 = 35$ in. from the bottom, and a third row will be placed at $y_3 = 39$ in. from the bottom.



Stiffness of Each Row of Connectors:

$$(EA)_1 = (EA)_2 = (EA)_3 = \gamma s_{min} n_{bolts} = \left(400{,}000 \frac{lb}{in}\right)(6.75\ in)(5)$$

$$(EA)_1 = (EA)_2 = (EA)_3 = 13.5\left(10^6\ lb\right)$$

Location of neutral axis in **spliced** section:

$$\frac{P_1\left[\frac{E_w b\bar{y}^2}{2}\left(\frac{d}{2} - \frac{\bar{y}}{3}\right)\sum (EA)_i\left(y_i - \bar{y}\right)\left(y_i - \frac{d}{2}\right)\right]}{M_1\left[\frac{E_w b\bar{y}^2}{2} - \sum (EA)_i\left(y_i - \bar{y}\right)\right]} - 1 = 0$$

Using a spreadsheet, the equation is solved for $\bar{y}$ by iteration as:

$$\bar{y} = 12.69\ in$$

Calculation of "k":

$$k = \frac{P_1}{\frac{E_w b\bar{y}^2}{2} - \sum (\gamma s_{min} n)_i\left(y_i - \bar{y}\right)}$$

Using a spreadsheet to solve the equation, k is calculated as:

$$k = \frac{77.45\left(10^{-6}\right)}{in}$$

Magnitude of tension forces, T_y across splice:

$$T_i = k\,(EA)_i\left(y_i - \bar{y}\right)$$

$$T_1 = \frac{77.45\left(10^{-6}\right)}{in}\left[13.5\left(10^6\ lb\right)\right](31.0\ in - 12.69\ in) = 19,100\ lb$$

$$T_2 = \frac{77.45\left(10^{-6}\right)}{in}\left[13.5\left(10^6\ lb\right)\right](35.0\ in - 12.69\ in) = 23,300\ lb$$

$$T_3 = \frac{77.45\left(10^{-6}\right)}{in}\left[13.5\left(10^6\ lb\right)\right](39.0\ in - 12.69\ in) = 27,500\ lb$$

Compression force resultant, C, and compression stress, f_c on spliced section:

$$C = \frac{E_w b\bar{y}^2 k}{2} = \frac{1.8\left(10^6\ psi\right)(8.5\ in)(12.69\ in)^2\left(\frac{77.45\left(10^{-6}\right)}{in}\right)}{2} = 95{,}400\ lb$$

$$f_c = \frac{2C}{\bar{y}b} = \frac{2\,(95{,}400\ lb)}{(12.69\ in)(8.5\ in)} = 1{,}769\ psi \le F_b' = 2760\ psi \quad \therefore\ OK$$

End grain bearing plate requirement (NDS 3.10.1.3):

$$\frac{f_c}{F_{bx}'} = \frac{1769\ psi}{2760\ psi} = 0.64 \le 0.75 \qquad \therefore\ \text{No bearing plate is required}$$

Design of A36 grade steel side plates based on gross section (use plate width of 3.5 in.):

$$T_3 \le \frac{F_y A_g}{1.67}$$

$$27{,}500\ lb \le \frac{\left(36{,}000\ psi\right) A_g}{1.67}$$

$$1.276\ in^2 \le A_g = 2ht = 2\,(3.5\ in)\,t \Rightarrow t \ge 0.182\ in$$

Design of A36 grade steel side plates based on net section (use plate width of 3.5 in.):

$$T_3 \le \frac{F_u A_n}{2}$$

$$27{,}500\ lb \le \frac{\left(58{,}000\ psi\right) A_n}{2}$$

$$0.948\ in^2 \le A_n = 2\,(h - (D + 0.0625\ in))\,t$$

$$0.948\ in^2 \le A_n = 2\,(3.5\ in - (0.75\ in + 0.0625\ in))\,t \Rightarrow t \ge 0.176\ in$$

$\frac{1}{4}$ in $\times$ 3.5 in A36 steel plates are adequate for each row of connectors

Group action factor for $n = 5, S = 6.75\ in$

$$E_m' A_m = 1.8\left(10^6\ psi\right)(8.5\ in)(4\ in) = 61.2\left(10^6\ lb\right)$$

$$E_s A_s = 29\left(10^6\ psi\right)(3.5\ in)(0.25\ in)\,2 = 50.8\left(10^6\ lb\right)$$

$$R_{EA} = \frac{E_s A_s}{E_m A_m} = \frac{50.8\left(10^6\ lb\right)}{61.2\left(10^6\ lb\right)} = 0.830$$

$$\gamma = 400{,}000\ lb/in$$

$$u = 1 + n_j \gamma \frac{s}{2}\left[\frac{1}{E_m A_m} + \frac{1}{E_s A_s}\right]$$

$$u = 1 + 1\left(400{,}000\ lb/in\right)\frac{6.75\ in}{2}\left[\frac{1}{61.2\left(10^6\ lb\right)} + \frac{1}{50.8\left(10^6\ lb\right)}\right]$$

$$u = 1.049$$

$$m = u - \sqrt{u^2 - 1} = 1.049 - \sqrt{(1.049)^2 - 1} = 0.732$$

$$C_g = \left[\frac{m\left(1 - m^{2n}\right)}{n\left[\left(1 + R_{EA}m^n\right)(1 + m) - 1 + m^{2n}\right]}\right]\left[\frac{1 + R_{EA}}{1 - m}\right]$$

$$C_g = \left[\frac{0.732\left(1 - 0.732^{2(5)}\right)}{5\left[\left(1 + (0.830)\ 0.732^5\right)(1 + 0.732) - 1 + 0.732^{2(5)}\right]}\right]\left[\frac{1 + 0.830}{1 - 0.732}\right]$$

$$C_g = 0.89$$

Design capacity of each row of shear plate connectors in tension splice:

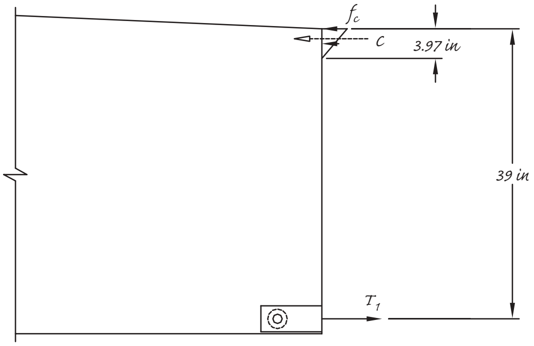
$$P' = PC_D C_M C_t C_\Delta C_g = (2860\ lb)(1.15)(1.0)(1.0)(1.0)(0.89) = 2930\ lb$$

$$T' = nP' = 10\left(2930\ lb\right) = 29{,}300\ lb \geq T_3 = 27{,}500\ lb \therefore\ OK$$

The design capacity of the connectors also exceeds the loads T_1 and T_2.

MOMENT TRANSFER FOR SEISMIC LOAD COMBINATION (LOAD REVERSAL):

Because the stresses are very low for the seismic load combination, a single bolt with two shear plates will be considered. The bolt will be placed at $y_1 = 39$ in. from the top of the section.



Stiffness of Connectors:

$$(EA)_1 = \gamma s_{min} n_{bolts} = \left(400{,}000\frac{lb}{in}\right)(6.75\ in)(1) = 2.7\left(10^6\ lb\right)$$

Location of neutral axis in **spliced** section:

$$\frac{P_1\left[\frac{E_w b\bar{y}^2}{2}\left(\frac{d}{2}-\frac{\bar{y}}{3}\right)\sum (EA)_i\left(y_i-\bar{y}\right)\left(y_i-\frac{d}{2}\right)\right]}{M_1\left[\frac{E_w b\bar{y}^2}{2}-\sum (EA)_i\left(y_i-\bar{y}\right)\right]}-1=0$$

Using a spreadsheet, the equation is solved for $\bar{y}$ by iteration as: $\bar{y} = 3.97\ in$

Calculation of "k":

$$k=\frac{P_1}{\frac{E_w b\bar{y}^2}{2}-\sum\left(\gamma s_{min} n\right)_i\left(y_i-\bar{y}\right)}$$

Using a spreadsheet to solve the equation, k is calculated as:

$$k=\frac{87.71\left(10^{-6}\right)}{in}$$

Magnitude of tension force, T_1, across splice:

$$T_i = k\,(EA)_i\left(y_i-\bar{y}\right)$$

$$T_1=\frac{87.71\left(10^{-6}\right)}{in}\left[2.7\left(10^6\ lb\right)\right](39.0\ in - 3.97\ in) = 8{,}300\ lb$$

Compression force resultant, C, and compression stress, f_c on spliced section:

$$C=\frac{E_w b\bar{y}^2 k}{2}=\frac{1.8\left(10^6\ psi\right)(8.5\ in)(3.97\ in)^2\left(\frac{87.71\left(10^{-6}\right)}{in}\right)}{2}=10{,}600\ lb$$

$$f_c=\frac{2C}{\bar{y}b}=\frac{2\,(10{,}600\ lb)}{(3.97\ in)(8.5\ in)}=630\ psi \le F_b' = 3840\ psi \quad \therefore OK$$

Design capacity of connectors:

$$P' = PC_D C_M C_t C_\Delta C_g = (2860\ lb)(1.6)(1.0)(1.0)(1.0)(1.0) = 4{,}576\ lb$$

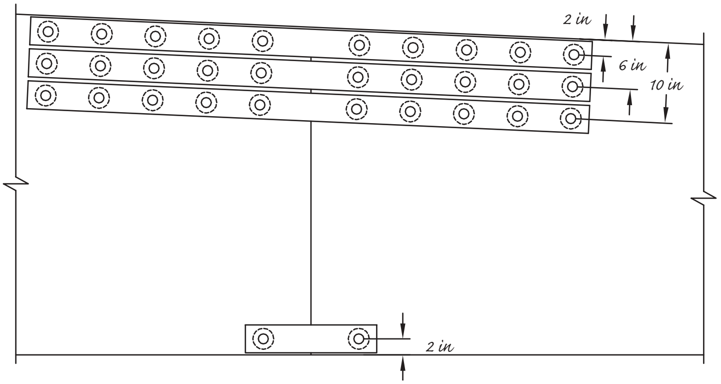
$$T' = nP' = 2\,(4{,}576\ lb) = 9{,}150\ lb \ge T_1 = 8{,}300\ lb \therefore OK$$

Result:

The connection can be detailed as illustrated below; or a symmetrical connection can be specified for aesthetic reasons. If a symmetrical appearance is desired, the connection for the load reversal case could possibly be redesigned using bolts without shear plates.

Spacing between shear plates within a row should be 6.75 in. and the end distance should be a minimum of 6 in. These dimensions will satisfy row tear-out provisions with a full design value.

18 shear plate connector units must be installed in the end grain in two rows of nine, with 4 inches between rows and shear plates within a row spaced at 4.25 inches and centered in the depth.



ONE-HOUR FIRE-RATED BEAM ANALYSIS

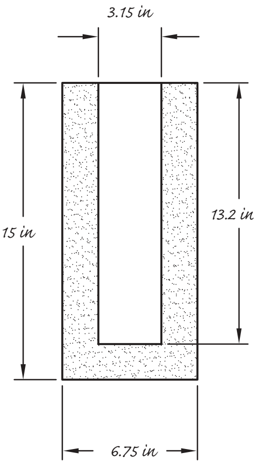
(See also Section 20.7.4)

Given: A $6\frac{3}{4}$ in. × 15 in. 24F-1.8E DF beam was designed to carry a floor live load of 400 plf and a dead load of 180 plf in addition to its own weight over a span of 21 ft. The beam is fully braced at the ends and along the top edge to prevent buckling. The floor will protect the top of the beam from char.

Wanted: Analyze the beam for a one-hour fire resistance rating using the full design load. Evaluate the case of a layup modified for fire resistance and the case of an unmodified layup.

Solution:

Determine post-fire dimensions and weight of beam:



$$b_{fire} = b - 2a = 6.75\ in - 2\,(1.8\ in) = 3.15\ in$$
$$d_{fire} = d - a = 15\ in - 1.8\ in = 13.2\ in$$
$$w_{sw} = b_{fire} d_{fire} \gamma$$
$$w_{sw} = (3.15\ in)(13.2\ in)\left(33\frac{lb}{ft^3}\right)\left(\frac{1\ ft^2}{144\ in^2}\right) = 10\ lb/ft$$

Calculate design moment and post-fire bending stress:

$$M = \frac{wl^2}{8} = \frac{(w_L + w_D + w_{sw})\ l^2}{8}$$
$$M = \frac{\left(400\ lb/ft + 180\ lb/ft + 10\ lb/ft\right)\left(21\ ft\right)^2}{8}$$
$$M = 32,520\ ft\text{-}lb = 390,300\ in\text{-}lb$$
$$f_{bx} = \frac{M}{S_{fire}} = \frac{6M}{b_{fire} d_{fire}^2} = \frac{6\,(390,300\ in\text{-}lb)}{(3.15\ in)(13.2\ in)^2} = 4270\ psi$$

Volume Factor (based on original dimensions):

$$C_V = \left(\frac{5.125 in}{6.75 in}\right)^{\frac{1}{10}} \left(\frac{12 in}{15 in}\right)^{\frac{1}{10}} \left(\frac{21 ft}{21 ft}\right)^{\frac{1}{10}} = 0.95$$

Check flexure for modified Section:

$$F'_{bx,modified\ section} = 2.85 F_{bx} C_D C_M C_t\ (C_V \text{ or } C_L)$$
$$F'_{bx,modified\ section} = 2.85\left(2400\ psi\right)(1.0)(1.0)(1.0)(0.95)$$
$$F'_{bx,modified\ section} = 6500\ psi \geq f_{bx} = 4270\ psi \qquad \therefore\ OK$$

Check flexure for unmodified Section:

$$F'_{bx,unmodified\ section} = 2.85 F_{bx} C_D C_M C_t\ (C_V \text{ or } C_L)$$
$$F'_{bx,unmodified\ section} = 2.85\left(2400\ psi\ (0.70)\right)(1.0)(1.0)(1.0)(0.95)$$
$$F'_{bx,unmodified\ section} = 4549\ psi \geq f_{bx} = 4270\ psi \qquad \therefore\ OK$$

Result:

Both the modified and unmodified layups are adequate to resists the design loads for the one-hour fire duration, so it is not necessary to specify that the layup be modified for one-hour fire resistance.

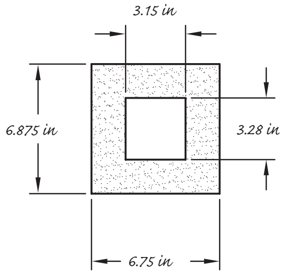
ONE-HOUR FIRE-RATED COLUMN ANALYSIS
(See also Section 20.7.4)

Given: *A $6\frac{3}{4}$ in. × $6\frac{7}{8}$ in. Combination 50 SP column with an effective length of 12 ft has been proposed to support a floor live load of 30,000 lb and a dead load of 6,000 lb throughout a one-hour fire. All four sides of the column will be exposed to fire. The loads are centric on the column.*

Wanted: *Analyze the column for a one-hour fire resistance rating.*

Solution:

Determine post-fire dimensions (see drawing):



$$b_{fire} = b - 2a = 6.75\ in - 2\,(1.8\ in) = 3.15\ in$$

$$d_{fire} = d - 2a = 6.875\ in - 2\,(1.8\ in) = 3.28\ in$$

Calculate post-fire compression stress:

$$f_c = \frac{P}{b_{fire}d_{fire}} = \frac{36,000\ lb}{(3.15\ in)(3.28\ in)} = 3480\ psi$$

Calculate adjusted post-fire design value:

$$F_c' = 2.58F_cC_P = 2.58\left(2300\ psi\right)C_P = \left(5930\ psi\right)C_P = F_c^*C_P$$

$$F_{cE} = 2.03\left[\frac{0.822\left(E_{min}'\right)}{\left(l_e/b_{fire}\right)^2}\right]$$

$$F_{cE} = 2.03\left[\frac{0.822\left(1.00\left(10^6\right)\ psi\right)}{\left(144\ in/3.15\ in\right)^2}\right]$$

$$F_{cE} = 798\ psi$$

$$\frac{F_{cE}}{F_c^*} = \frac{798\ psi}{1160\ psi} = 0.688$$

$$C_P = \frac{1+F_{cE}/F_c^*}{2c} - \sqrt{\left(\frac{1+F_{cE}/F_c^*}{2c}\right)^2 - \frac{F_{cE}/F_c^*}{c}}$$

$$C_P = \frac{1+0.688}{2\,(0.9)} - \sqrt{\left(\frac{1+0.688}{2\,(0.9)}\right)^2 - \frac{0.688}{0.9}}$$

$$C_P = 0.599$$

$$F_c' = F_c^*C_P = \left(5930\ psi\right)(0.599) = 3550\ psi\ f_c = 3480\ psi \qquad \therefore\ OK$$

Result:

The $6\frac{3}{4}$ in. × $6\frac{7}{8}$ in. column is adequate to support the loads for the one-hour fire duration.

HEAVY TIMBER ROOF DECKING (ASD Method)
(See also Chapter 10)

Given: Douglas Fir-Larch, Commercial grade, tongue and groove, heavy timber decking installed perpendicular to the supports

Available thicknesses: 1.5 in. (2×6), 2.5 in. (3×6), 3.5 in. (4×6)

Reference Design values (From NDS):

$$F_b = 1650\ psi$$
$$E = 1.7\left(10^6\right)\ psi$$

Live load deflection limit is $l/180$. Design loads are: $S = 30$ psf and $D = 15$ psf (including decking)

Wanted: Determine maximum spans for roof decking assuming controlled random layup pattern.

Solution:

Adjusted Design Values

$$F_b' = F_b C_D C_m C_t C_F$$
$$F_{b,2\times6}' = 1650\ psi(1.15)(1.0)(1.0)(1.10) = 2087\ psi$$
$$F_{b,3\times6}' = 1650\ psi(1.15)(1.0)(1.0)(1.04) = 1973\ psi$$
$$F_{b,4\times6}' = 1650\ psi(1.15)(1.0)(1.0)(1.0) = 1898\ psi$$

$$E' = EC_m C_t$$
$$E' = \left(1.7\left(10^6\right)\ psi\right)(1.0)(1.0) = 1.7\left(10^6\right)\ psi$$

Maximum span based on flexure:

$$\sigma_b = \frac{20F_b'}{3l^2}\frac{d^2}{6}$$
$$l \leq \sqrt{\frac{20F_b'}{3\sigma_b}\frac{d^2}{6}}$$
$$l_{2\times6} \leq \sqrt{\frac{20\left(2087\ psi\right)}{3\left(30\ psf + 15\ psf\right)\left(\frac{1\ ft^2}{144\ in^2}\right)}\frac{(1.5\ in)^2}{6}}$$
$$l_{2\times6} \leq 129\ in\ \text{for}\ 2\times6\ \text{decking}$$

$$l_{3\times 6} \leq \sqrt{\frac{20\left(1973\ psi\right)}{3\left(30\ psf + 15\ psf\right)\left(\frac{1\ ft^2}{144 in^2}\right)}\frac{\left(2.5\ in\right)^2}{6}}$$

$$l_{3\times 6} \leq 209\ in\ for\ 3 \times 6\ decking$$

$$l_{4\times 6} \leq \sqrt{\frac{20\left(1898\ psi\right)}{3\left(30\ psf + 15\ psf\right)\left(\frac{1\ ft^2}{144\ in^2}\right)}\frac{\left(3.5\ in\right)^2}{6}}$$

$$l_{4\times 6} \leq 287\ in\ for\ 4 \times 6\ decking$$

<u>Maximum span based on deflection for 2×6 decking:</u>

$$\sigma_\Delta \leq \frac{100\Delta E'}{l^4}\frac{d^3}{12} = \frac{100\left(\frac{l}{180}\right)E'}{l^4}\frac{d^3}{12} = \frac{100E'}{180l^3}\frac{d^3}{12}$$

$$l_{2\times 6} \leq \sqrt[3]{\frac{100E'}{180\sigma_\Delta}\frac{d^3}{12}} = \sqrt[3]{\frac{100\left(1.7\left(10^6\right)\ psi\right)}{180\left(30\ psf\right)\left(\frac{1\ ft^2}{144\ in^2}\right)}\frac{\left(1.5\ in\right)^3}{12}}$$

$$l_{2\times 6} \leq 108\ in\ for\ 2 \times 6\ decking$$

<u>Maximum span based on deflection for 3×6 and 4×6 decking:</u>

$$\sigma_\Delta \leq \frac{116\Delta E'}{l^4}\frac{d^3}{12} = \frac{116\left(\frac{l}{180}\right)E'}{l^4}\frac{d^3}{12} = \frac{116E'}{180l^3}\frac{d^3}{12}$$

$$l \leq \sqrt[3]{\frac{116E'}{180\sigma_\Delta}\frac{d^3}{12}} = \sqrt[3]{\frac{116\left(1.7\left(10^6\right)\ psi\right)}{180\left(30\ psf\right)\left(\frac{1\ ft^2}{144\ in^2}\right)}\frac{d^3}{12}} = 76.0d$$

$$l_{3\times 6} \leq 76.0\left(2.5\ in\right) = 190\ in\ for\ 3 \times 6\ decking$$

$$l_{4\times 6} \leq 76.0\left(3.5\ in\right) = 266\ in\ for\ 4 \times 6\ decking$$

<u>Result:</u>

The maximum spans for the roof decking are all controlled by deflection and are:

$$l_{2\times 6} = 108\ in\ for\ 2 \times 6\ decking$$

$$l_{3\times 6} = 190\ in\ for\ 3 \times 6\ decking$$

$$l_{4\times 6} = 266\ in\ for\ 4 \times 6\ decking$$

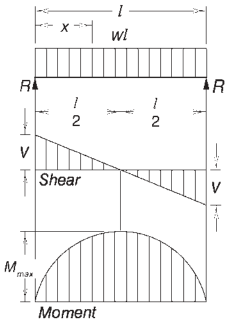
APPENDIX B

REFERENCE INFORMATION

B.1 BEAM DIAGRAMS AND FORMULAS

B.1.1 Single-Beam Diagrams and Formulas

1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \dfrac{wl}{2}$

V_x .... $= w\left(\dfrac{l}{2} - x\right)$

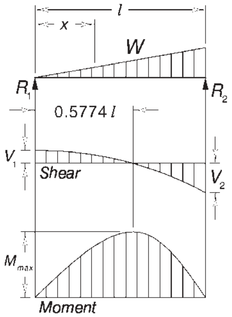
M_{max} (at center) .... $= \dfrac{wl^2}{8}$

M_x .... $= \dfrac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \dfrac{5wl^4}{384EI}$

Δ_x .... $= \dfrac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

2. SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO ONE END



Total Equiv. Uniform Load .... $= \dfrac{16W}{9\sqrt{3}} = 1.03W$

$R_1 = V_1$ .... $= \dfrac{W}{3}$

$R_2 = V_2 = V_{max}$ .... $= \dfrac{2W}{3}$

V_x .... $= \dfrac{W}{3} - \dfrac{Wx^2}{l^2}$

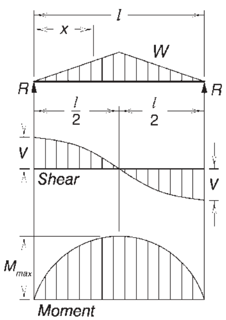
M_{max} $\left(\text{at } x = \dfrac{l}{\sqrt{3}} = 0.557\, l\right)$ .... $= \dfrac{2Wl}{9\sqrt{3}} = 0.128\, Wl$

M_x .... $= \dfrac{Wx}{3l^2}(l^2 - x^2)$

Δ_{max} $\left(\text{at } x = l\sqrt{1 - \sqrt{\dfrac{8}{15}}} = 0.519\, l\right)$ .... $= 0.0130\dfrac{Wl^3}{EI}$

Δ_x .... $= \dfrac{Wx}{180EIl^2}\left(3x^4 - 10l^2x^2 + 7l^4\right)$

3. SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO CENTER



Total Equiv. Uniform Load .... $= \dfrac{4W}{3}$

$R = V$ .... $= \dfrac{W}{2}$

V_x $\left(\text{when } x < \dfrac{l}{2}\right)$ .... $= \dfrac{W}{2l^2}\left(l^2 - 4x^2\right)$

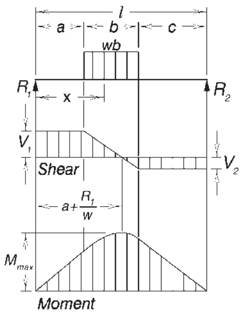
M_{max} (at center) .... $= \dfrac{Wl}{6}$

M_x $\left(\text{when } x < \dfrac{l}{2}\right)$ .... $= Wx\left(\dfrac{1}{2} - \dfrac{2x^2}{3l^2}\right)$

Δ_{max} (at center) .... $= \dfrac{Wl^3}{60EI}$

Δ_x $\left(\text{when } x < \dfrac{l}{2}\right)$ .... $= \dfrac{Wx}{480EIl^2}\left(5l^2 - 4x^2\right)^2$

4. SIMPLE BEAM — UNIFORM LOAD PARTIALLY DISTRIBUTED



$R_1 = V_1$ (max. when $a < c$) .... $= \frac{wb}{2l}(2c+b)$

$R_2 = V_2$ (max. when $a > c$) .... $= \frac{wb}{2l}(2a+b)$

V_x (when $x > a$ and $< (a+b)$) .... $= R_1 - w(x-a)$

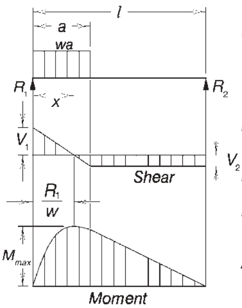
M_{max} $\left(\text{at } x = a + \frac{R_1}{w}\right)$ .... $= R_1\left(a + \frac{R_1}{2w}\right)$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $x > a$ and $< (a+b)$) .... $= R_1 x - \frac{w}{2}(x-a)^2$

M_x (when $x > (a+b)$) .... $= R_2 (l-x)$

5. SIMPLE BEAM — UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_1 = V_{max}$ .... $= \frac{wa}{2l}(2l-a)$

$R_2 = V_2$ .... $= \frac{wa^2}{2l}$

V_x (when $x < a$) .... $= R_1 - wx$

M_{max} $\left(\text{at } x = \frac{R_1}{w}\right)$ .... $= \frac{R_1^2}{2w}$

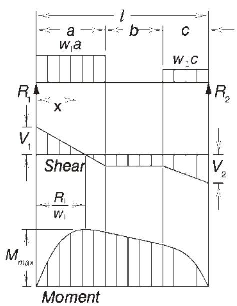
M_x (when $x < a$) .... $= R_1 x - \frac{wx^2}{2}$

M_x (when $x > a$) .... $= R_2 (l-x)$

Δ_x (when $x < a$) .... $= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$

Δ_x (when $x > a$) .... $= \frac{wa^2(l-x)}{24EIl}\left(4xl - 2x^2 - a^2\right)$

6. SIMPLE BEAM — UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



$R_1 = V_1$ .... $= \frac{w_1 a(2l-a) + w_2 c^2}{2l}$

$R_2 = V_2$ .... $= \frac{w_2 c(2l-c) + w_1 a^2}{2l}$

V_x (when $x < a$) .... $= R_1 - w_1 x$

V_x (when $a < x < (a+b)$) .... $= R_1 - w_1 a$

V_x (when $x > (a+b)$) .... $= R_2 - w_2 (l-x)$

M_{max} $\left(\text{at } x = \frac{R_1}{w_1}, \text{when } R_1 < w_1 a\right)$ .... $= \frac{R_1^2}{2w_1}$

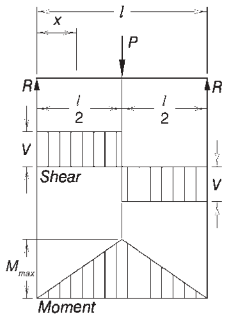
M_{max} $\left(\text{at } x = l - \frac{R_2}{w_2}, \text{when } R_2 < w_2 c\right)$ $= \frac{R_2^2}{2w_2}$

M_x (when $x < a$) .... $= R_1 x - \frac{w_1 x^2}{2}$

M_x (when $a < x < (a+b)$) .... $= R_1 x - \frac{w_1 a}{2}(2x-a)$

M_x (when $x > (a+b)$) .... $= R_2 (l-x) - \frac{w_2 (l-x)^2}{2}$

7. SIMPLE BEAM — CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load .... $= 2P$

$R = V$ .... $= \frac{P}{2}$

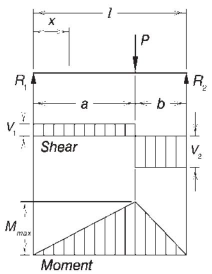
M_{max} (at point of load) .... $= \frac{Pl}{4}$

M_x (when $x < \frac{l}{2}$) .... $= \frac{Px}{2}$

Δ_{max} (at point of load) .... $= \frac{Pl^3}{48EI}$

Δ_x (when $x < \frac{l}{2}$) .... $= \frac{Px}{48EI}(3l^2 - 4x^2)$

8. SIMPLE BEAM — CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load .... $= \frac{8Pab}{l^2}$

$R_1 = V_1$ ($= V_{max}$ when $a < b$) .... $= \frac{Pb}{l}$

$R_2 = V_2$ ($= V_{max}$ when $a > b$) .... $= \frac{Pa}{l}$

M_{max} (at point of load) .... $= \frac{Pab}{l}$

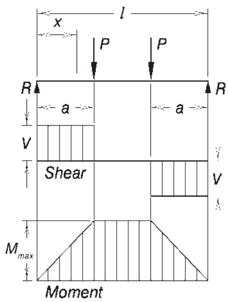
M_x (when $x < a$) .... $= \frac{Pbx}{l}$

Δ_{max} $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}}, \text{when } a > b\right)$ .... $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$

Δ_a (at point of load) .... $= \frac{Pa^2b^2}{3EIl}$

Δ_x (when $x < a$) .... $= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

9. SIMPLE BEAM — TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Total Equiv. Uniform Load .... $= \frac{8Pa}{l}$

$R = V$ .... $= P$

M_{max} (between loads) .... $= Pa$

M_x (when $x < a$) .... $= Px$

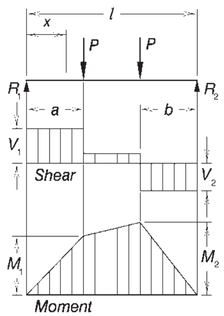
Δ_{max} (at center) .... $= \frac{Pa}{24EI}(3l^2 - 4a^2)$

Δ_{max} (when $a = \frac{l}{3}$) .... $= \frac{Pl^3}{28EI}$

Δ_x (when $x < a$) .... $= \frac{Px}{6EI}(3la - 3a^2 - x^2)$

Δ_x (when $a < x < (l - a)$) .... $= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

10. SIMPLE BEAM — TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1\ (= V_{max}$ when $a < b)$ .... $= \frac{P}{l}(l - a + b)$

$R_2 = V_2\ (= V_{max}$ when $a > b)$ .... $= \frac{P}{l}(l - b + a)$

V_x (when $a < x < (l - b)$) .... $= \frac{P}{l}(b - a)$

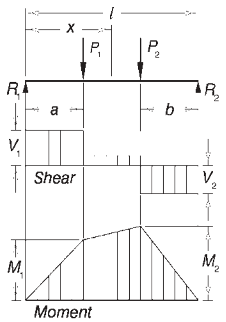
M_1 $(= M_{max}$ when $a > b)$ .... $= R_1 a$

M_2 $(= M_{max}$ when $a < b)$ .... $= R_2 b$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $a < x < (l - b)$) .... $= R_1 x - P(x - a)$

11. SIMPLE BEAM — TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ .... $= \frac{P_1(l - a) + P_2 b}{l}$

$R_2 = V_2$ .... $= \frac{P_1 a + P_2(l - b)}{l}$

V_x (when $a < x < (l - b)$) .... $= R_1 - P_1$

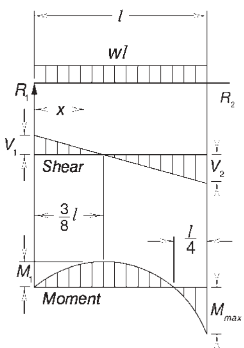
M_1 $(= M_{max}$ when $R_1 < P_1)$ .... $= R_1 a$

M_2 $(= M_{max}$ when $R_2 < P_2)$ .... $= R_2 b$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $a < x < (l - b)$) .... $= R_1 x - P_1(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER — UNIFORMLY DISTRUBTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R_1 = V_1$ .... $= \frac{3wl}{8}$

$R_2 = V_2 = V_{max}$ .... $= \frac{5wl}{8}$

V_x .... $= R_1 - wx$

M_{max} .... $= \frac{wl^2}{8}$

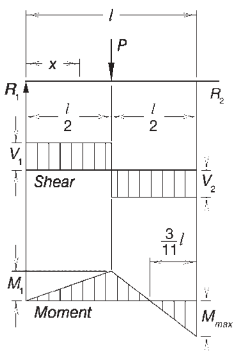
M_1 (at $x = \frac{3}{8}l$) .... $= \frac{9}{128}wl^2$

M_x .... $= R_1 x - \frac{wx^2}{2}$

Δ_{max} (at $x = \frac{l}{16}\left(1 + \sqrt{33}\right) = 0.422\, l$) .... $= \frac{wl^4}{185\, EI}$

Δ_x .... $= \frac{wx}{48\, EI}\left(l^3 - 3lx^2 + 2x^3\right)$

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER — CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load .... $= \frac{3P}{2}$

$R_1 = V_1$ .... $= \frac{5P}{16}$

$R_2 = V_2 = V_{max}$ .... $= \frac{11P}{16}$

M_{max} (at fixed end) .... $= \frac{3Pl}{16}$

M_1 (at point of load) .... $= \frac{5Pl}{32}$

M_x (at $x < \frac{l}{2}$) .... $= \frac{5Px}{16}$

M_x (when $x > \frac{l}{2}$) .... $= P\left(\frac{l}{2} - \frac{11x}{16}\right)$

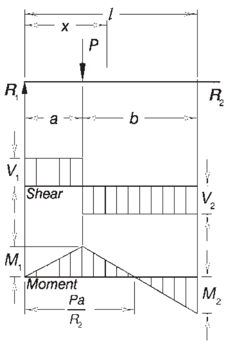
Δ_{max} (at $x = \frac{l}{\sqrt{5}} = 0.447\,l$) .... $= \frac{Pl^3}{48EI\sqrt{5}} = 0.00932\frac{Pl^3}{EI}$

Δ_x (at point of load) .... $= \frac{7Pl^3}{768EI}$

Δ_x (at $x < \frac{l}{2}$) .... $= \frac{Px}{96EI}\left(3l^2 \quad 5x^2\right)$

Δ_x (at $x > \frac{l}{2}$) .... $= \frac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT THE OTHER — CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ .... $= \frac{Pb^2}{2l^3}(a + 2l)$

$R_2 = V_2$ .... $= \frac{Pa}{2l^3}\left(3l^2 - a^2\right)$

M_1 (at point of load) .... $= R_1 a$

M_2 (at fixed end) .... $= \frac{Pab}{2l^2}(a + l)$

M_x (at $x < a$) .... $= R_1 x$

M_x (when $x > a$) .... $= R_1 x - P(x - a)$

Δ_{max} $\left(\text{when } a < 0.414l \text{ at } x = l\frac{(l^2 + a^2)}{(3l^2 - a^2)}\right)$ .. $= \frac{Pa}{3EI}\frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$

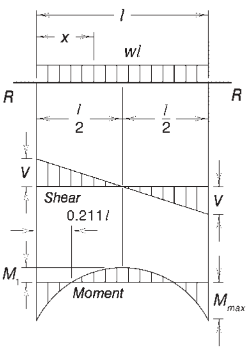
Δ_{max} $\left(\text{when } a > 0.414l \text{ at } x = l\sqrt{\frac{a}{2l + a}}\right)$ ..... $= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l + a}}$

Δ_a (at point of load) .... $= \frac{Pa^2b^3}{12EIl^3}(3l + a)$

Δ_x (when $x < a$) .... $= \frac{Pb^2x}{12EIl^3}\left(3al^2 - 2lx^2 - ax^2\right)$

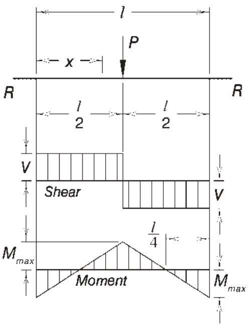
Δ_x (when $x > a$) .... $= \frac{Pa}{12EIl^3}(l - x)^2\left(3l^2x - a^2x - 2a^2l\right)$

15. BEAM FIXED AT BOTH ENDS — UNIFORMLY DISTRIBUTED LOADS



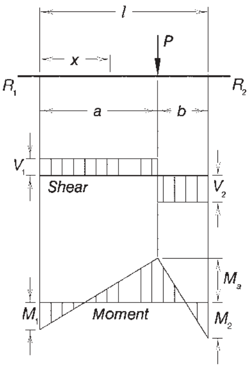
Total Equiv. Uniform Load	$= \frac{2wl}{3}$
$R = V$	$= \frac{wl}{2}$
V_x	$= w\left(\frac{l}{2} - x\right)$
M_{max} (at ends)	$= \frac{wl^2}{12}$
M_1 (at center)	$= \frac{wl^2}{24}$
M_x	$= \frac{w}{12}(6lx \quad l^2 \quad 6x^2)$
Δ_{max} (at center)	$= \frac{wl^4}{384EI}$
Δ_x	$= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS — CONCENTRATED LOAD AT CENTER



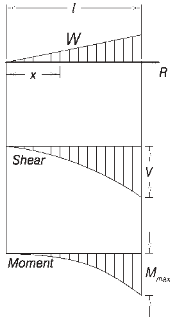
Total Equiv. Uniform Load	$= P$
$R = V$	$= \frac{P}{2}$
M_{max} (at center and ends)	$= \frac{Pl}{8}$
M_x (when $x < \frac{l}{2}$)	$= \frac{P}{8}(4x - l)$
Δ_{max} (at center)	$= \frac{Pl^3}{192EI}$
Δ_x (when $x < \frac{l}{2}$)	$= \frac{Px^2}{48EI}(3l - 4x)$

17. BEAM FIXED AT BOTH ENDS — CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1 (= V_{max}$ when $a < b)$	$= \frac{Pb^2}{l^3}(3a + b)$
$R_2 = V_2 (= V_{max}$ when $a > b)$	$= \frac{Pa^2}{l^3}(a + 3b)$
M_1 $(= M_{max}$ when $a < b)$	$= \frac{Pab^2}{l^2}$
M_2 $(= M_{max}$ when $a > b)$	$= \frac{Pa^2b}{l^2}$
M_a (at point of load)	$= \frac{2Pa^2b^2}{l^3}$
M_x (when $x < a$)	$= R_1x - \frac{Pab^2}{l^2}$
Δ_{max} (when $a > b$ at $x = \frac{2al}{3a + b}$)	$= \frac{2Pa^3b^2}{3EI(3a + b)^2}$
Δ_a (at point of load)	$= \frac{Pa^3b^3}{3EIl^3}$
Δ_x (when $x < a$)	$= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

18. CANTILEVERED BEAM — LOAD INCREASING UNIFORMLY TO FIXED END



Total Equiv. Uniform Load .... $= \frac{8}{3}W$

$R = V$ .... $= W$

V_x .... $= W\frac{x^2}{l^2}$

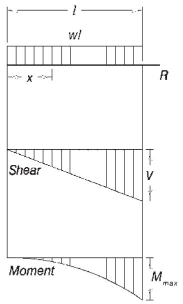
M_{max} (at fixed end) .... $= \frac{Wl}{3}$

M_x .... $= \frac{Wx^3}{3l^2}$

Δ_{max} (at free end) .... $= \frac{Wl^3}{15EI}$

Δ_x .... $= \frac{W}{60EIl^2}\left(x^5 - 5l^4x + 4l^5\right)$

19. CANTILEVERED BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= 4wl$

$R = V$ .... $= wl$

V_x .... $= wx$

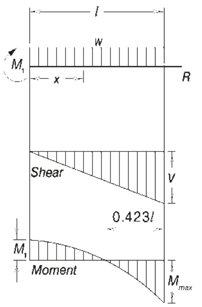
M_{max} (at fixed end) .... $= \frac{wl^2}{2}$

M_x .... $= \frac{wx^2}{2}$

Δ_{max} (at free end) .... $= \frac{wl^4}{8EI}$

Δ_x .... $= \frac{w}{24EI}\left(x^4 - 4l^3x + 3l^4\right)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= \frac{8}{3}wl$

$R = V$ .... $= wl$

V_x .... $= wx$

M_1 (at deflected end) .... $= \frac{wl^2}{6}$

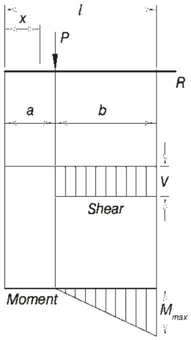
M_{max} (at fixed end) .... $= \frac{wl^2}{3}$

M_x .... $= \frac{w}{6}\left(l^2 \quad 3x^2\right)$

Δ_{max} (at deflected end) .... $= \frac{wl^4}{24EI}$

Δ_x .... $= \frac{w\left(l^2 - x^2\right)^2}{24EI}$

21. CANTILEVERED BEAM — CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load .... $= \frac{8Pb}{l}$

$R = V$ .... $= P$

M_{max} (at fixed end) .... $= Pb$

M_x (when $x > a$) .... $= P(x - a)$

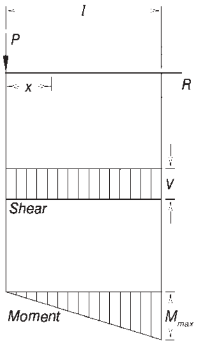
Δ_{max} (at free end) .... $= \frac{Pb^2}{6EI}(3l - b)$

Δ_a (at point of load) .... $= \frac{Pb^3}{3EI}$

Δ_x (when $x < a$) .... $= \frac{Pb^2}{6EI}(3l - 3x - b)$

Δ_x (when $x > a$) .... $= \frac{P(l - x)^2}{6EI}(3b - l + x)$

22. CANTILEVERED BEAM — CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load .... $= 8P$

$R = V$ .... $= P$

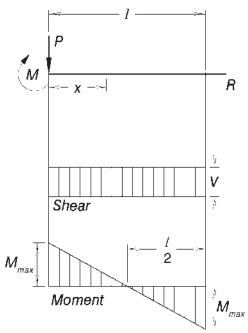
M_{max} (at fixed end) .... $= Pl$

M_x .... $= Px$

Δ_{max} (at free end) .... $= \frac{Pl^3}{3EI}$

Δ_x .... $= \frac{P}{6EI}\left(2l^3 - 3l^2 x + x^3\right)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER — CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load .... $= 4P$

$R=V$ .... $= P$

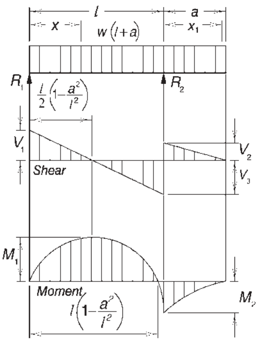
M_{max} (at both ends) .... $= \frac{Pl}{2}$

M_x .... $= P\left(\frac{l}{2} - x\right)$

Δ_{max} (at deflected end) .... $= \frac{Pl^3}{12EI}$

Δ_x .... $= \frac{P(l - x)^2}{12EI}(l + 2x)$

24. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD



$R_1 = V_1$ .... $= \frac{w}{2l}\left(l^2 - a^2\right)$

$R_2 = V_2 + V_3$ .... $= \frac{w}{2l}(l + a)^2$

V_2 .... $= wa$

V_3 .... $= \frac{w}{2l}\left(l^2 + a^2\right)$

V_x (between supports) .... $= R_1 - wx$

V_{x_1} (for overhang) .... $= w(a - x_1)$

M_1 $\left(\text{at } x = \frac{l}{2}\left[1 - \frac{a^2}{l^2}\right]\right)$ .... $= \frac{w}{8l^2}(l + a)^2 (l - a)^2$

M_2 (at R_2) .... $= \frac{wa^2}{2}$

M_x (between supports) .... $= \frac{wx}{2l}\left(l^2 - a^2 - xl\right)$

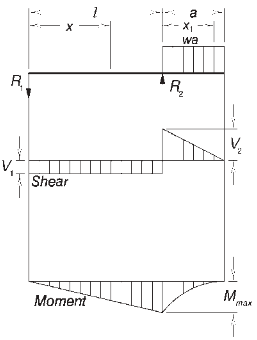
M_{x_1} (for overhang) .... $= \frac{w}{2}(a - x_1)^2$

Δ_x (between supports) .... $= \frac{wx}{24EIl}\left(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2\right)$

Δ_{x_1} (for overhang) .... $= \frac{wx_1}{24EI}\left(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3\right)$

NOTE: For a negative value of Δ_x, deflection is upward.

25. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD ON OVERHANG



$R_1 = V_1$ .... $= \frac{wa^2}{2l}$

$R_2 = V_1 + V_2$ .... $= \frac{wa}{2l}(2l + a)$

V_2 .... $= wa$

V_{x_1} (for overhang) .... $= w(a - x_1)$

M_{max} (at R_2) .... $= \frac{wa^2}{2}$

M_x (between supports) .... $= \frac{wa^2x}{2l}$

M_{x_1} (for overhang) .... $= \frac{w}{2}(a - x_1)^2$

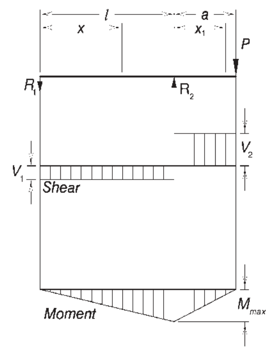
Δ_{max} $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{wa^2l^2}{18\sqrt{3}EI} = 0.0321\frac{wa^2l^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) .... $= \frac{wa^3}{24EI}(4l + 3a)$

Δ_x (between supports) .... $= \frac{wa^2x}{12EIl}\left(l^2 - x^2\right)$

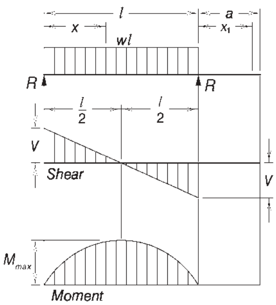
Δ_{x_1} (for overhang) .... $= \frac{wx_1}{24EI}\left(4a^2l + 6a^2x_1 \quad 4ax_1^2 + x_1^3\right)$

26. BEAM OVERHANGING ONE SUPPORT — CONCENTRATED LOAD AT END OF OVERHANG



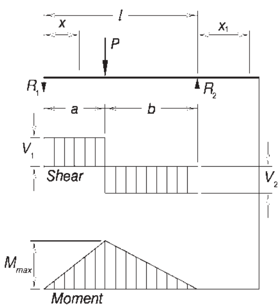
- $R_1 = V_1$ .... $= \frac{Pa}{l}$
- $R_2 = V_1 + V_2$ .... $= \frac{P}{l}(l+a)$
- V_2 .... $= P$
- M_{max} (at R_2) .... $= Pa$
- M_x (between supports) .... $= \frac{Pax}{l}$
- M_{x_1} (for overhang) .... $= P(a - x_1)$
- Δ_{max} $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right)$ .... $= \frac{Pal^2}{9\sqrt{3}EI} = 0.0642\frac{Pal^2}{EI}$
- Δ_{max} (for overhang at $x_1 = a$) .... $= \frac{Pa^2}{3EI}(l+a)$
- Δ_x (between supports) .... $= \frac{Pax}{6EIl}(l^2 - x^2)$
- Δ_{x_1} (for overhang) .... $= \frac{Px_1}{6EI}(2al + 3ax_1 \; x_1^2)$

27. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



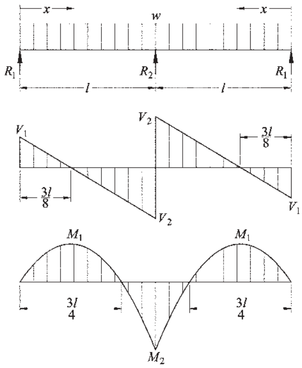
- Total Equiv. Uniform Load .... $= wl$
- $R = V$ .... $= \frac{wl}{2}$
- V_x .... $= w\left(\frac{l}{2} - x\right)$
- M_{max} (at center) .... $= \frac{wl^2}{8}$
- M_x .... $= \frac{wx}{2}(l - x)$
- Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$
- Δ_x .... $= \frac{wx}{24EI}(l^3 \quad 2lx^2 + x^3)$
- Δ_{x_1} .... $= \frac{wl^3x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT — CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



- Total Equiv. Uniform Load .... $= \frac{8Pab}{l^2}$
- $R_1 = V_1$ (= V_{max} when $a < b$) .... $= \frac{Pb}{l}$
- $R_2 = V_2$ (= V_{max} when $a > b$) .... $= \frac{Pa}{l}$
- M_{max} (at point of load) .... $= \frac{Pab}{l}$
- M_x (when $x < a$) .... $= \frac{Pbx}{l}$
- Δ_{max} $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right)$ .... $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$
- Δ_a (at point of load) .... $= \frac{Pa^2b^2}{3EIl}$
- Δ_x (when $x < a$) .... $= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$
- Δ_x (when $x > a$) .... $= \frac{Pa(l-x)}{6EIl}(2lx \quad x^2 \quad a^2)$
- Δ_{x_1} .... $= \frac{Pabx_1}{6EIl}(l+a)$

29a. CONTINUOUS BEAM – TWO EQUAL SPANS – UNIFORM LOAD ON BOTH SPANS



$R_1 = V_1$ .... $= \frac{3wl}{8}$

R_2 .... $= \frac{10wl}{8}$

V_2 .... $= \frac{5wl}{8}$

M_1 .... $= \frac{9wl^2}{128}$

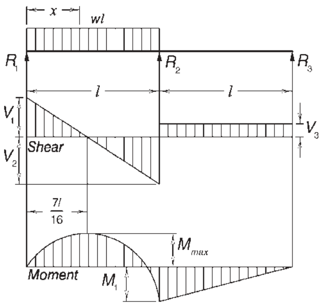
M_2 .... $= -\frac{wl^2}{8}$

M_x .... $= R_1 x - \frac{wx^2}{2}$

Δ_{max} (at $x = 0.4215l$) .... $= \frac{wl^4}{185EI}$

Δ_x .... $= \frac{wx}{48EI}\left(l^3 - 3lx^2 + 2x^3\right)$

29b. CONTINUOUS BEAM — TWO EQUAL SPANS — UNIFORM LOAD ON ONE SPAN



Total Equiv. Uniform Load .... $= \frac{49}{64} wl$

$R_1 = V_1$ .... $= \frac{7}{16} wl$

$R_2 = V_2 + V_3$ .... $= \frac{5}{8} wl$

$R_3 = V_3$ .... $= -\frac{1}{16} wl$

V_2 .... $= \frac{9}{16} wl$

M_{max} (at $x = \frac{7}{16} l$) .... $= \frac{49}{512} wl^2$

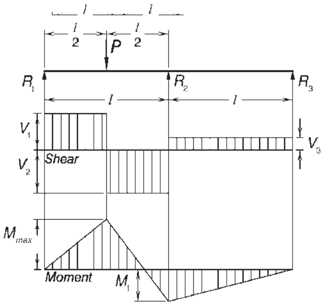
M_1 (at support R_2) .... $= \frac{1}{16} wl^2$

M_x (when $x < l$) .... $= \frac{wx}{16}(7l - 8x)$

Δ_{max} (at 0.472 l from R_1) .... $= \frac{0.0092\, wl^4}{EI}$

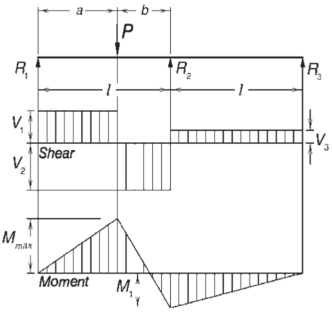
29a. from AITC.

30. CONTINUOUS BEAM — TWO EQUAL SPANS — CONCENTRATED LOAD AT CENTER OF ONE SPAN



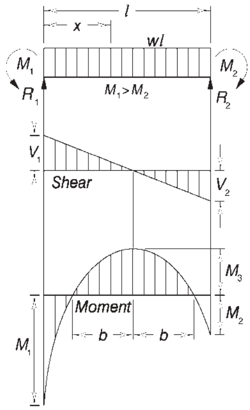
Total Equiv. Uniform Load	$= \frac{13}{8}P$
$R_1 = V_1$	$= \frac{13}{32}P$
$R_2 = V_2 + V_3$	$= \frac{11}{16}P$
$R_3 = V_3$	$= -\frac{3}{32}P$
V_2	$= \frac{19}{32}P$
M_{max} (at point of load)	$= \frac{13}{64}Pl$
M_1 (at support R_2)	$= \frac{3}{32}Pl$
Δ_{max} (at 0.480 l from R_1)	$= \frac{0.015\,Pl^3}{EI}$

31. CONTINUOUS BEAM – TWO EQUAL SPANS — CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$	$= \frac{Pb}{4l^3}\left(4l^2 - a(l+a)\right)$
$R_2 = V_2 + V_3$	$= \frac{Pa}{2l^3}\left(2l^2 + b(l+a)\right)$
$R_3 = V_3$	$= -\frac{Pab}{4l^3}(l+a)$
V_2	$= \frac{Pa}{4l^3}\left(4l^2 + b(l+a)\right)$
M_{max} (at point of load)	$= \frac{Pab}{4l^3}\left(4l^2 - a(l+a)\right)$
M_1 (at support R_2)	$= \frac{Pab}{4l^2}(l+a)$

32. BEAM — UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS



$R_1 = V_1$ .... $= \dfrac{wl}{2} + \dfrac{M_1 - M_2}{l}$

$R_2 = V_2$ .... $= \dfrac{wl}{2} - \dfrac{M_1 - M_2}{l}$

V_x .... $= w\left(\dfrac{l}{2} - x\right) + \dfrac{M_1 - M_2}{l}$

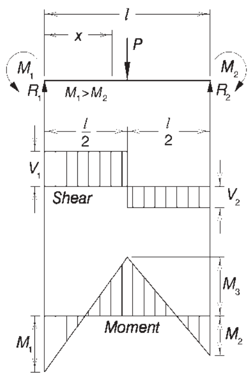
M_3 (at $x = \dfrac{l}{2} + \dfrac{M_1 - M_2}{wl}$) .... $= \dfrac{wl^2}{8} - \dfrac{M_1 + M_2}{2} + \dfrac{(M_1 - M_2)^2}{2wl^2}$

M_x .... $= \dfrac{wx}{2}(l - x) + \left(\dfrac{M_1 - M_2}{l}\right)x - M_1$

b (to locate inflection points) .... $= \sqrt{\dfrac{l^2}{4} - \left(\dfrac{M_1 + M_2}{w}\right) + \left(\dfrac{M_1 - M_2}{wl}\right)^2}$

$$\Delta_x = \frac{wx}{24EI}\left[x^3 - \left(2l + \frac{4M_1}{wl} - \frac{4M_2}{wl}\right)x^2 + \frac{12M_1}{w}x + l^3 - \frac{8M_1 l}{w} - \frac{4M_2 l}{w}\right]$$

33. BEAM — CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS



$R_1 = V_1$ .... $= \dfrac{P}{2} + \dfrac{M_1 - M_2}{l}$

$R_2 = V_2$ .... $= \dfrac{P}{2} - \dfrac{M_1 - M_2}{l}$

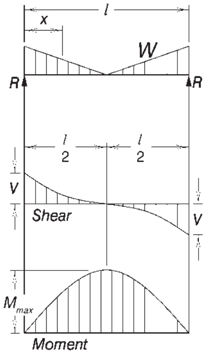
M_3 (at center) .... $= \dfrac{Pl}{4} - \dfrac{M_1 + M_2}{2}$

M_x (when $x < \dfrac{l}{2}$) .... $= \left(\dfrac{P}{2} + \dfrac{M_1 - M_2}{l}\right)x - M_1$

M_x (when $x > \dfrac{l}{2}$) .... $= \dfrac{P}{2}(l - x) + \dfrac{(M_1 - M_2)x}{l} - M_1$

Δ_x (when $x < \dfrac{l}{2}$) $= \dfrac{Px}{48EI}\left(3l^2 - 4x^2 - \dfrac{8(l - x)}{Pl}\left[M_1(2l - x) + M_2(l + x)\right]\right)$

34. SIMPLE BEAM — LOAD INCREASING UNIFORMLY FROM CENTER



Total Equiv. Uniform Load .... $= \frac{2W}{3}$

$R = V$ .... $= \frac{W}{2}$

V_x (when $x < \frac{l}{2}$) .... $= \frac{W}{2}\left(\frac{l-2x}{l}\right)^2$

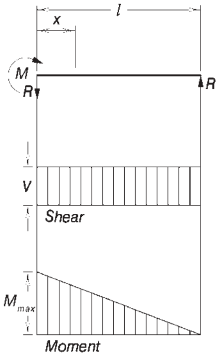
M_{max} (at center) .... $= \frac{Wl}{12}$

M_x (when $x < \frac{l}{2}$) .... $= \frac{W}{2}\left(x - \frac{2x^2}{l} + \frac{4x^3}{3l^2}\right)$

Δ_{max} (at center) .... $= \frac{3Wl^3}{320EI}$

Δ_x (when $x < \frac{l}{2}$) .... $= \frac{W}{12EI}\left(x^3 - \frac{x^4}{l} + \frac{2x^5}{5l^2} - \frac{3l^2x}{8}\right)$

35. SIMPLE BEAM — CONCENTRATED MOMENT AT END



Total Equiv. Uniform Load .... $= \frac{8M}{l}$

$R = V$ .... $= \frac{M}{l}$

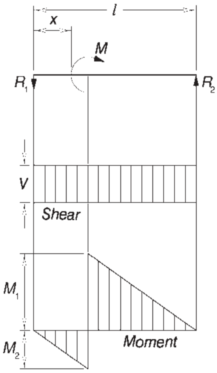
M_{max} .... $= M$

M_x .... $= M\left(1 - \frac{x}{l}\right)$

Δ_{max} (at $x = 0.423\,l$) .... $= 0.0642\frac{Ml^2}{EI}$

Δ_x .... $= \frac{M}{6EI}\left(3x^2 - \frac{x^3}{l} - 2lx\right)$

36. SIMPLE BEAM — CONCENTRATED MOMENT AT ANY POINT



Total Equiv. Uniform Load .... $= \frac{8M}{l}$

$R = V$ .... $= \frac{M}{l}$

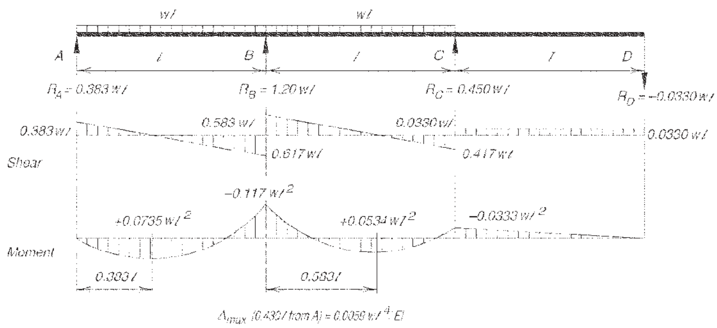
M_x (when $x < a$) .... $= Rx$

M_x (when $x > a$) .... $= M + Rx$

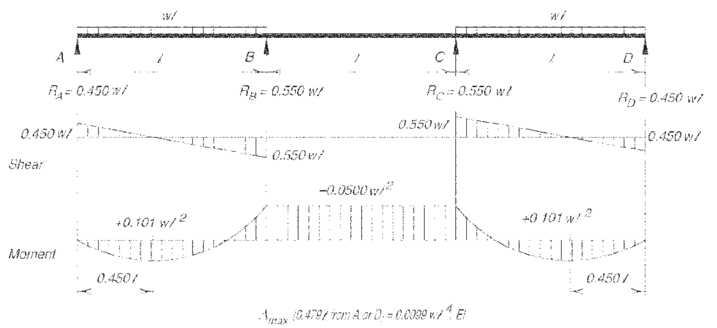
Δ_x (when $x < a$) .... $= \frac{M}{6EI}\left[\left(6a - \frac{3a^2}{l} - 2l\right)x - \frac{x^3}{l}\right]$

Δ_x .... $= \frac{M}{6EI}\left[3\left(a^2 + x^2\right) - \frac{x^3}{l} - \left(2l + \frac{3a^2}{l}\right)x\right]$

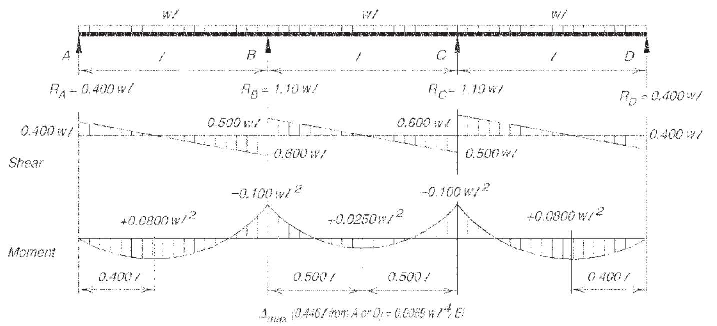
37. CONTINUOUS BEAM—THREE EQUAL SPANS—ONE END SPAN UNLOADED



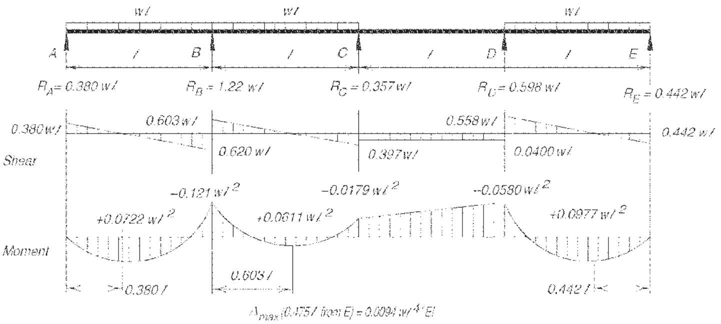
38. CONTINUOUS BEAM—THREE EQUAL SPANS—END SPANS LOADED



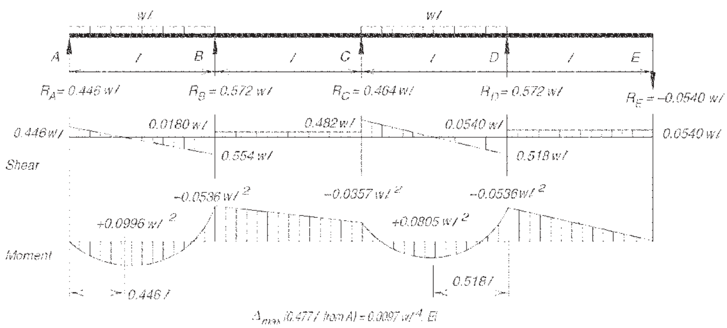
39. CONTINUOUS BEAM—THREE EQUAL SPANS—ALL SPANS LOADED



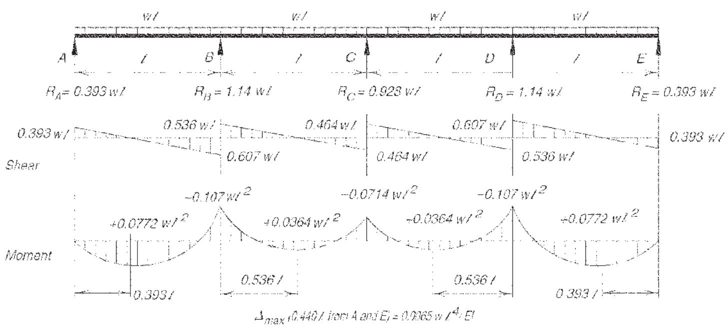
40. CONTINUOUS BEAM—FOUR EQUAL SPANS—THIRD SPAN UNLOADED



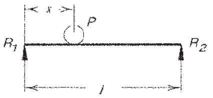
41. CONTINUOUS BEAM—FOUR EQUAL SPANS—LOAD FIRST AND THIRD SPANS



42. CONTINUOUS BEAM—FOUR EQUAL SPANS—ALL SPANS LOADED



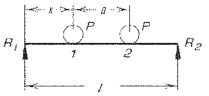
43. SIMPLE BEAM—ONE CONCENTRATED MOVING LOAD



$R_{1\,max} = V_{1\,max}(\text{at } x = 0) \ldots\ldots = P$

$M_{max}\left(\text{at point of load, when } x = \frac{l}{2}\right) \ldots\ldots = \frac{Pl}{4}$

44. SIMPLE BEAM—TWO EQUAL CONCENTRATED MOVING LOADS

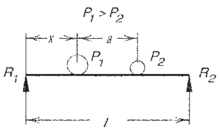


$R_{1\,max} = V_{1\,max}(\text{at } x = 0) \ldots\ldots = P\left(2 - \frac{a}{l}\right)$

M_{max}:

- $\left[\text{when } a < (2 - \sqrt{2})l = 0.586l \text{; under load 1 at } x = \frac{1}{2}\left(l - \frac{a}{2}\right)\right] \ldots\ldots = \frac{P}{2l}\left(l - \frac{a}{2}\right)^2$
- $\left[\text{when } a > (2 - \sqrt{2})l = 0.586l \text{; with one load at center of span (Case 43)}\right] \ldots\ldots = \frac{Pl}{4}$

45. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED MOVING LOADS

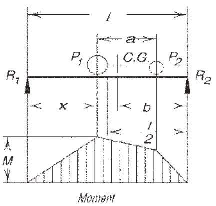


$R_{1\,max} = V_{1\,max}(\text{at } x = 0) \ldots\ldots = P_1 + P_2\frac{l - a}{l}$

M_{max}:

- $\left[\text{under } P_1\text{, at } x = \frac{1}{2}\left(l - \frac{P_2 a}{P_1 + P_2}\right)\right] \ldots\ldots = (P_1 + P_2)\frac{x^2}{l}$
- $\left[M_{max} \text{ may occur with larger load at center of span and other load off span (Case 43)}\right] \ldots\ldots = \frac{P_1 l}{4}$

GENERAL RULES FOR SIMPLE BEAMS CARRYING MOVING CONCENTRATED LOADS



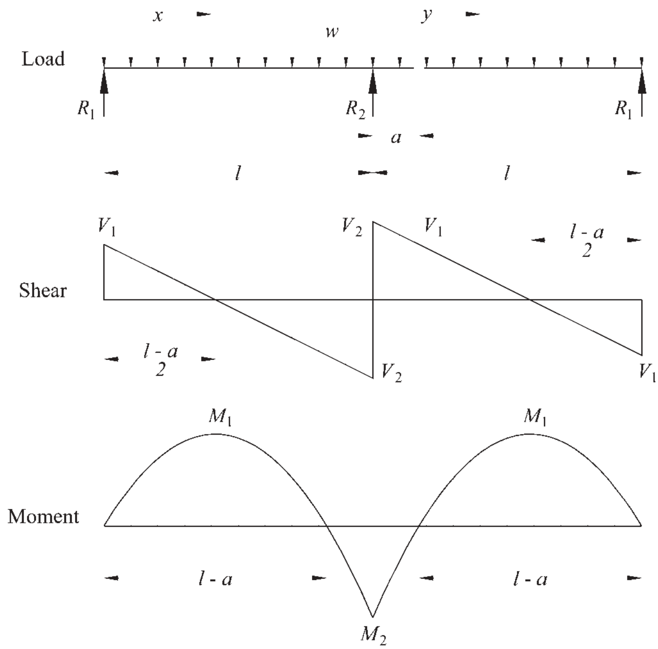
The maximum shear due to moving concentrated loads occurs at one support when one of the loads is at that support. With several moving loads, the location that will produce maximum shear must be determined by trial.

The maximum bending moment produced by moving concentrated loads occurs under one of the loads when that load is as far from one support as the center of gravity of all the moving loads on the beam is from the other support.

In the accompanying diagram, the maximum bending moment occurs under load P_1 when $x = b$. It should also be noted that this condition occurs when the center-line of the span is midway between the center of gravity of loads and the nearest concentrated load.

B.1.2 Cantilever Beam System Diagrams and Formulas

B.1.2.1 Two Equal Spans – Single Cantilever



General Formulas:

$$R_1 = \frac{w}{2}(l-a) \qquad R_2 = w(l+a)$$

$$V_1 = \pm\frac{w}{2}(l-a) \qquad V_2 = \pm\frac{w}{2}(l+a)$$

$$V_x = \frac{w}{2}(l-a-2x) \qquad V_y = \frac{w}{2}(l+a-2y)$$

$$M_1 = \frac{w}{8}(l-a)^2 \qquad M_2 = -\frac{wla}{2}$$

$$M_x = \frac{wx}{2}(l-a-x) \qquad M_y = -\frac{w}{2}(y-a)(l-y)$$

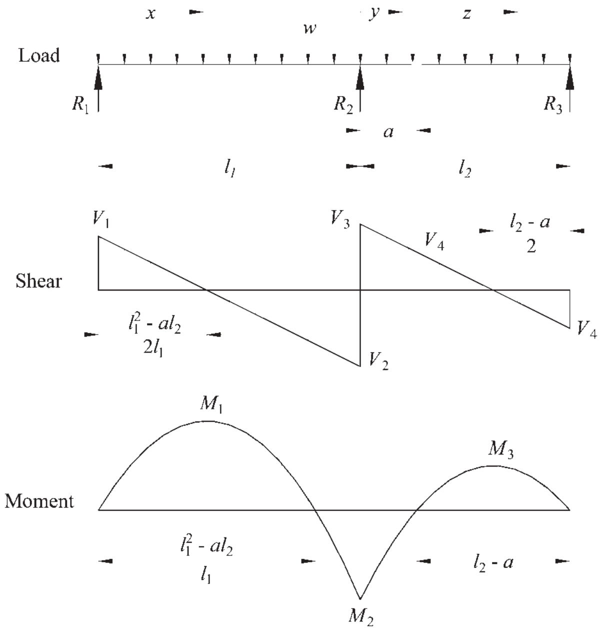
Special case where $a = 0.1716l$:

$M_1 = -M_2 = 0.08579wl^2$ $\quad R_1 = 0.4142wl$ $\quad R_2 = 1.1716wl$

$V_1 = \pm 0.4142wl$ $\quad V_2 = \pm 0.5858wl$

$\Delta_{\text{left span}} = -\dfrac{wl^4}{129.8EI}$ $\quad \Delta_{\text{hinge}} = +\dfrac{wl^4}{750EI}$ $\quad \Delta_{\text{right beam}} = -\dfrac{wl^4}{156.9EI}$

B.1.2.2 Two Unequal Spans – Single Cantilever



General Formulas:

$$R_1 = \frac{w}{2l_1}\left(l_1^2 - al_2\right) \qquad R_2 = \frac{w}{2l_1}\left(l_1 + a\right)\left(l_1 + l_2\right) \qquad R_3 = \frac{w}{2}\left(l_2 - a\right)$$

$$V_1 = \frac{w}{2l_1}\left(l_1^2 - al_2\right) \qquad V_2 = -\frac{w}{2l_1}\left(l_1^2 + al_2\right) \qquad V_3 = \frac{w}{2}\left(l_2 + a\right)$$

$$V_4 = \pm\frac{w}{2}\left(l_2 - a\right) \qquad V_x = \frac{w}{2l_1}\left(l_1^2 - al_2\right) - wx \qquad V_y = \frac{w}{2}\left(l_2 + a\right) - wy$$

$$V_z = \frac{w}{2}\left(l_2 - a\right) - wz \qquad M_1 = \frac{w}{8l_1^2}\left(l_1^2 - al_2\right)^2 \qquad M_2 = -\frac{wl_2a}{2}$$

$$M_3 = \frac{w}{8}\left(l_2 - a\right)^2 \qquad M_x = \frac{wx}{2l_1}\left(l_1^2 - xl_1 - al_2\right) \qquad M_y = \frac{w}{2}\left(l_2 - y\right)\left(y - a\right)$$

$$M_z = \frac{wz}{2}\left(l_2 - a - z\right)$$

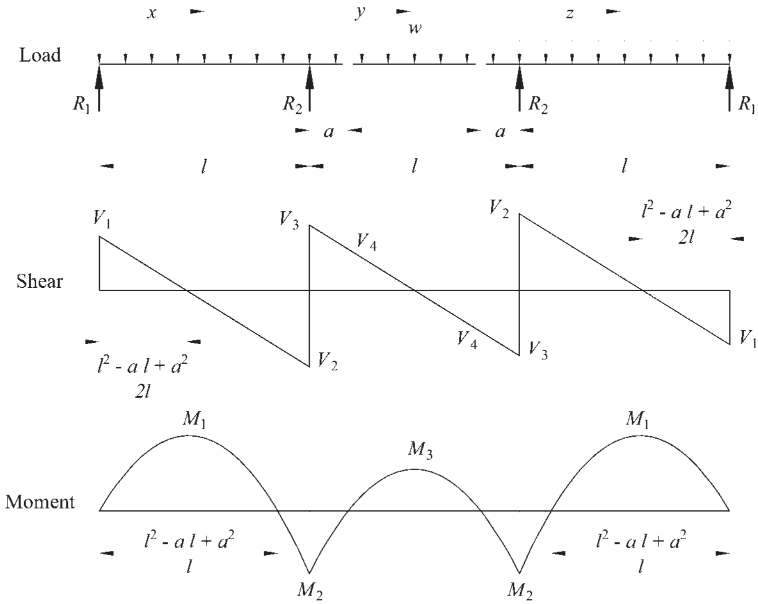
Special case where $a = 0.1716 l_1^2/l_2$ (consider when $l_1 > l_2$):

$$M_1 = -M_2 = 0.08579wl_1^2 \qquad R_1 = V_1 = 0.4142wl_1 \qquad V_3 = 0.5858wl_1$$

Special case where $a = 0.1716 l_2$ (consider when $l_1 < l_2$):

$$M_3 = -M_2 = 0.08579wl_2^2 \qquad R_3 = V_4 = 0.4142wl_2 \qquad V_2 = -0.5858wl_2$$

B.1.2.3 Three Equal Spans – Single Cantilever on End Beams



General Formulas:

$R_1 = \frac{w}{2l}\left(l^2 - al + a^2\right)$ $\quad R_2 = \frac{w}{2l}\left(2l^2 + al - a^2\right)$ $\quad V_1 = \pm\frac{w}{2l}\left(l^2 - al + a^2\right)$

$V_2 = \frac{w}{2l}\left(l^2 + al - a^2\right)$ $\quad V_3 = \pm\frac{wl}{2}$ $\quad V_4 = \pm\frac{w}{2}\left(l - 2a\right)$

$V_x = \frac{w}{2l}\left(l^2 - al + a^2\right) - wx$ $\quad V_y = \frac{w}{2}\left(l - 2y\right)$ $\quad V_z = \frac{w}{2l}\left(l^2 + al - a^2\right) - wz$

$M_1 = \frac{w}{8l^2}\left(l^2 - al + a^2\right)^2$ $\quad M_2 = -\frac{w}{2}\left(al - a^2\right)$ $\quad M_3 = \frac{w}{8}\left(l - 2a\right)^2$

$M_x = \frac{wx}{2l}\left(l^2 - al + a^2\right) - \frac{wx^2}{2}$ $\quad M_y = \frac{w}{2}\left(y - a\right)\left(l - y - a\right)$ $\quad M_z = \frac{w}{2}\left(l - z\right)\left(\frac{a^2}{l} + z - a\right)$

Special case where $a = 0.2200l$:

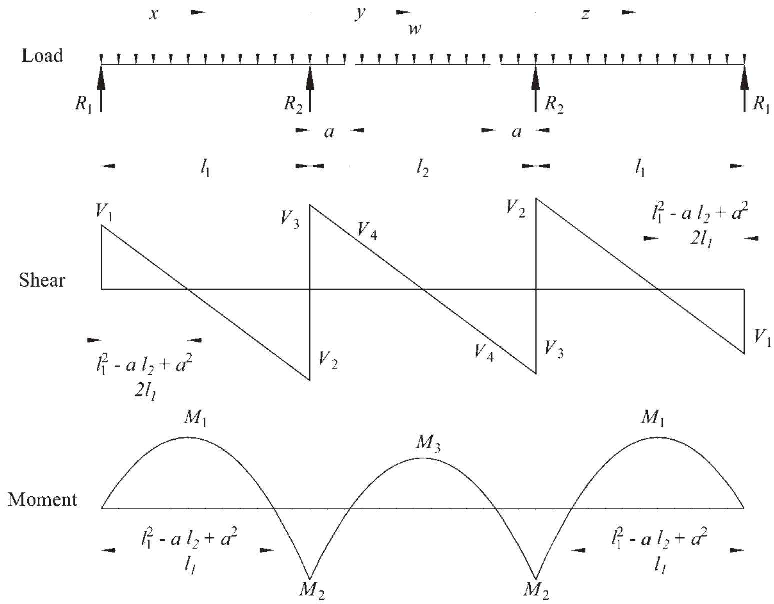
$M_1 = -M_2 = 0.08579wl^2$ $\quad M_3 = 0.03921wl^2$

$R_1 = 0.4142wl$ $\quad R_2 = 1.0858wl$ $\quad V_1 = 0.4142wl$

$V_2 = \pm 0.5858wl$ $\quad V_3 = \pm 0.5wl$ $\quad V_4 = \pm 0.2800wl$

$\Delta_{\text{end span}} = -\frac{wl^4}{129.8EI}$ $\quad \Delta_{\text{hinge}} = +\frac{wl^4}{640EI}$

B.1.2.4 Three Spans – End Spans Equal – Single Cantilever on End Beams



General Formulas:

$R_1 = \frac{w}{2l_1}\left(l_1^2 - al_2 + a^2\right)$ $\quad R_2 = \frac{w}{2l_1}(l_1 + a)(l_1 + l_2 - a)$ $\quad V_1 = \pm\frac{w}{2l_1}\left(l_1^2 - al_2 + a^2\right)$

$V_2 = \pm\frac{w}{2l_1}\left(l_1^2 + al_2 - a^2\right)$ $\quad V_3 = \pm\frac{wl_2}{2}$ $\quad V_4 = \pm\frac{w}{2}(l_2 - 2a)$

$V_x = \frac{w}{2l_1}\left(l_1^2 - al_2 + a^2\right) - wx$ $\quad V_y = \frac{w}{2}(l_2 - 2y)$ $\quad V_z = \frac{w}{2l_1}\left(l_1^2 + al_2 - a^2\right) - wz$

$M_1 = \frac{w}{8l_1^2}\left(l_1^2 - al_2 + a^2\right)^2$ $\quad M_2 = -\frac{w}{2}\left(al_2 - a^2\right)$ $\quad M_3 = \frac{w}{8}(l_2 - 2a)^2$

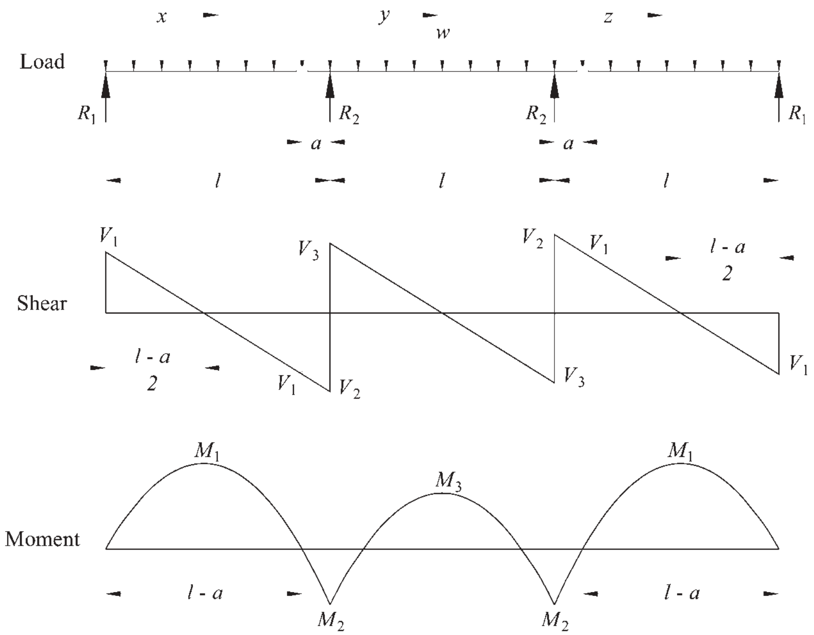
$M_x = \frac{wx}{2l_1}\left(l_1^2 - al_2 + a^2\right) - \frac{wx^2}{2}$ $\quad M_y = \frac{w}{2}(y - a)(l_2 - y - a)$ $\quad M_z = \frac{w}{2l_1}(l_1 - z)\left(l_1 z - al_2 + a^2\right)$

Special case where $a = \frac{l_2 - \sqrt{l_2^2 - 0.6863l_1^2}}{2}$:

$M_1 = -M_2 = 0.08579wl_1^2$ $\quad R_1 = 0.4142wl_1$

$V_1 = \pm 0.4142wl_1$ $\quad V_2 = \pm 0.5858wl_1$ $\quad V_3 = \pm 0.5wl_2$

B.1.2.5 Three Equal Spans – Double Cantilever on Center Beam



General Formulas:

$R_1 = \frac{w}{2}(l-a)$ $\quad R_2 = \frac{w}{2}(2l+a)$

$V_1 = \pm\frac{w}{2}(l-a)$ $\quad V_2 = \pm\frac{w}{2}(l+a)$ $\quad V_3 = \pm\frac{wl}{2}$

$V_x = \frac{w}{2}(l-a-2x)$ $\quad V_y = \frac{w}{2}(l-2y)$ $\quad V_z = \frac{w}{2}(l+a-2z)$

$M_1 = \frac{w}{8}(l-a)^2$ $\quad M_2 = -\frac{wla}{2}$ $\quad M_3 = \frac{w}{8}(l^2-4al)$

$M_x = \frac{wx}{2}(l-a-x)$ $\quad M_y = \frac{w}{2}(ly-y^2-al)$ $\quad M_z = \frac{w}{2}(l-z)(z-a)$

Special case where $a = \frac{l}{8}$:

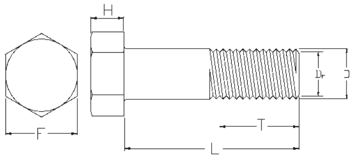
$M_3 = -M_2 = 0.0625wl^2$ $\quad M_1 = 0.09570wl^2$

$V_1 = \pm 0.4375wl$ $\quad V_2 = \pm 0.5625wl$ $\quad V_3 = \pm 0.5wl$

$R_1 = 0.4375wl$ $\quad R_2 = 1.0625wl$ $\quad \Delta_{\text{center span}} = -\frac{wl^4}{192EI}$

B.2 TYPICAL FASTENER DIMENSIONS AND YIELD STRENGTHS

B.2.1 Standard Hex Bolts[1]



D = diameter
D_r = root diameter
T = thread length
L = bolt length
F = width of head across flats
H = height of head

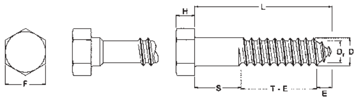
		Diameter, D							
		1/4"	**5/16"**	**3/8"**	**1/2"**	**5/8"**	**3/4"**	**7/8"**	**1"**
D_r		0.189"	0.245"	0.298"	0.406"	0.514"	0.627"	0.739"	0.847"
F		7/16"	1/2"	9/16"	3/4"	15/16"	1-1/8"	1-5/16"	1-1/2"
H		11/64"	7/32"	1/4"	11/32"	27/64"	1/2"	37/64"	43/64"
T	L ≤ 6 in.	3/4"	7/8"	1"	1-1/4"	1-1/2"	1-3/4"	2"	2-1/4"
	L > 6 in.	1"	1-1/8"	1-1/4"	1-1/2"	1-3/4"	2"	2-1/4"	2-1/2"

1. Tolerances are specified in ANSI/ASME B18.2.1. Full-body diameter bolt is shown. Root diameter based on UNC thread series (see ANSI/ASME B1.1).

Source: Reprinted with permission from National Design Specification® for Wood Construction.

B.2.2 Standard Hex Lag Screws[1]

D = diameter
D_r = root diameter
S = unthreaded body length
T = minimum thread length[2]



Reduced Body Diameter   Full-Body Diameter

E = length of tapered tip
L = lag screw length
N = number of threads/inch
F = width of head across flats
H = height of head

Length, L		Diameter, D										
		1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1-1/8"	1-1/4"
	D_r	0.173"	0.227"	0.265"	0.328"	0.371"	0.471"	0.579"	0.683"	0.780"	0.887"	1.012"
	E	5/32"	3/16"	7/32"	9/32"	5/16"	13/32"	1/2"	19/32"	11/16"	25/32"	7/8"
	H	11/64"	7/32"	1/4"	19/64"	11/32"	27/64"	1/2"	37/64"	43/64"	3/4"	27/32"
	F	7/16"	1/2"	9/16"	5/8"	3/4"	15/16"	1-1/8"	1-5/16"	1-1/2"	1-11/16"	1-7/8"
	N	10	9	7	7	6	5	4-1/2	4	3-1/2	3-1/4	3-1/4
	S	1/4"	1/4"	1/4"	1/4"	1/4"						
1"	T	3/4"	3/4"	3/4"	3/4"	3/4"						
	T-E	19/32"	9/16"	17/32"	15/32"	7/16"						
	S	1/4"	1/4"	1/4"	1/4"	1/4"						
1-1/2"	T	1-1/4"	1-1/4"	1-1/4"	1-1/4"	1-1/4"						
	T-E	1-3/32"	1-1/16"	1-1/32"	31/32"	15/16"						
	S	1/2"	1/2"	1/2"	1/2"	1/2"	1/2"					
2"	T	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"					
	T-E	1-11/32"	1-5/16"	1-9/32"	1-7/32"	1-3/16"	1-3/32"					
	S	3/4"	3/4"	3/4"	3/4"	3/4"	3/4"					
2-1/2"	T	1-3/4"	1-3/4"	1-3/4"	1-3/4"	1-3/4"	1-3/4"					
	T-E	1-19/32"	1-9/16"	1-17/32"	1-15/32"	1-7/16"	1-11/32"					
	S	1"	1"	1"	1"	1"	1"	1"	1"	1"		
3	T	2"	2"	2"	2"	2"	2"	2"	2"	2"		
	T-E	1-27/32"	1-13/16"	1-25/32"	1-23/32"	1-11/16"	1-19/32"	1-1/2"	1-13/32"	1-5/16"		
	S	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"	1-1/2"
4"	T	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"
	T-E	2-11/32"	2-5/16"	2-9/32"	2-7/32"	2-3/16"	2-3/32"	2"	1-29/32"	1-13/16"	1-23/32"	1-5/8"
	S	2"	2"	2"	2"	2"	2"	2"	2"	2"	2"	2"
5"	T	3"	3"	3"	3"	3"	3"	3"	3"	3"	3"	3"
	T-E	2-27/32"	2-13/16"	2-25/32"	2-23/32"	2-11/16"	2-19/32"	2-1/2"	2-13/32"	2-5/16"	2-7/32"	2-1/8"
	S	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"	2-1/2"
6"	T	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"
	T-E	3-11/32"	3-5/16"	3-9/32"	3-7/32"	3-3/16"	3-3/32"	3"	2-29/32"	2-13/16"	2-23/32"	2-5/8"
	S	3"	3"	3"	3"	3"	3"	3"	3"	3"	3"	3"
7"	T	4"	4"	4"	4"	4"	4"	4"	4"	4"	4"	4"
	T-E	3-27/32"	3-13/16"	3-25/32"	3-23/32"	3-11/16"	3-19/32"	3-1/2"	3-13/32"	3-5/16"	3-7/32"	3-1/8"
	S	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"	3-1/2"
8"	T	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"
	T-E	4-11/32"	4-5/16"	4-9/32"	4-7/32"	4-3/16"	4-3/32"	4"	3-29/32"	3-13/16"	3-23/32"	3-5/8"
	S	4"	4"	4"	4"	4"	4"	4"	4"	4"	4"	4"
9"	T	5"	5"	5"	5"	5"	5"	5"	5"	5"	5"	5"
	T-E	4-27/32"	4-13/16"	4-25/32"	4-23/32"	4-11/16"	4-19/32"	4-1/2"	4-13/32"	4-5/16"	4-7/32"	4-1/8"
	S	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"	4-1/2"
10"	T	5-1/2"	5-1/2"	5-1/2"	5-1/2"	5-1/2"	5-1/2"	5-1/2"	5-1/2"	5-1/2"	5-1/2"	5-1/2"
	T-E	5-11/32"	5-5/16"	5-9/32"	5-7/32"	5-3/16"	5-3/32"	5"	4-29/32"	4-13/16"	4-23/32"	4-5/8"
	S	5"	5"	5"	5"	5"	5"	5"	5"	5"	5"	5"
11"	T	6"	6"	6"	6"	6"	6"	6"	6"	6"	6"	6"
	T-E	5-27/32"	5-13/16"	5-25/32"	5-23/32"	5-11/16"	5-19/32"	5-1/2"	5-13/32"	5-5/16"	5-7/32"	5-1/8"
	S	6"	6"	6"	6"	6"	6"	6"	6"	6"	6"	6"
12"	T	6"	6"	6"	6"	6"	6"	6"	6"	6"	6"	6"
	T-E	5-27/32"	5-13/16"	5-25/32"	5-23/32"	5-11/16"	5-19/32"	5-1/2"	5-13/32"	5-5/16"	5-7/32"	5-1/8"

1. Tolerances are specified in ANSI/ASME B18.2.1. Full-body diameter and reduced body diameter lag screws are shown. For reduced body diameter lag screws, the unthreaded body diameter may be reduced to approximately the root diameter, D_r.
2. Minimum thread length (T) for lag screw lengths (L) is 6" or 1/2 the lag screw length plus 0.5", whichever is less. Thread lengths may exceed these minimums up to the full lag screw length (L).

Source: Reprinted with permission from National Design Specification® for Wood Construction.

B.2.3 Standard Wood Screws[1,5]

D = diameter
D_r = root diameter
L = wood length
T = thread length

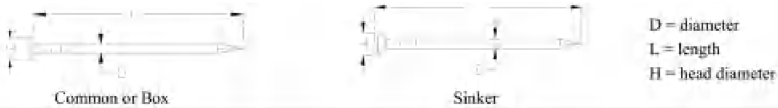
Cut Thread[2] Rolled Thread[3]

	Wood Screw Number										
	6	7	8	9	10	12	14	16	18	20	24
D	0.138"	0.151"	0.164"	0.177"	0.19"	0.216"	0.242"	0.268"	0.294"	0.32"	0.372"
D_r[4]	0.113"	0.122"	0.131"	0.142"	0.152"	0.171"	0.196"	0.209"	0.232"	0.255"	0.298"

1. Tolerances are specified in ANSI/ASME B18.6.1
2. Thread length on cut thread wood screws is approximately 2/3 of the wood screw length, L.
3. Single lead thread shown. Thread length is at least four times the screw diameter or 2/3 of the wood screw length, L, whichever is greater. Wood screws which are too short to accommodate the minimum thread length, have threads extending as close to the underside of the head as practicable.
4. Taken as the average of the specified maximum and minimum limits for body diameter of rolled thread wood screws.
5. It is permitted to assume the length of the tapered tip is 2D.

Source: Reprinted with permission from National Design Specification® for Wood Construction.

B.2.4 Standard Common, Box, and Sinker Steel Wire Nails[1,2]



Type		Pennyweight										
		6d	7d	8d	10d	12d	16d	20d	30d	40d	50d	60d
Common	L	2"	2-1/4"	2-1/2"	3"	3-1/4"	3-1/2"	4"	4-1/2"	5"	5-1/2"	6"
	D	0.113"	0.113"	0.131"	0.148"	0.148"	0.162"	0.192"	0.207"	0.225"	0.244"	0.263"
	H	0.266"	0.266"	0.281"	0.312"	0.312"	0.344"	0.406"	0.438"	0.469"	0.5"	0.531"
Box	L	2"	2-1/4"	2-1/2"	3"	3-1/4"	3-1/2"	4"	4-1/2"	5"		
	D	0.099"	0.099"	0.113"	0.128"	0.128"	0.135"	0.148"	0.148"	0.162"		
	H	0.266"	0.266"	0.297"	0.312"	0.312"	0.344"	0.375"	0.375"	0.406"		
Sinker	L	1-7/8"	2-1/8"	2-3/8"	2-7/8"	3-1/8"	3-1/4"	3-3/4"	4-1/4"	4-3/4"		5-3/4"
	D	0.092"	0.099"	0.113"	0.12"	0.135"	0.148"	0.177"	0.192"	0.207"		0.244"
	H	0.234"	0.250"	0.266"	0.281"	0.312"	0.344"	0.375"	0.406"	0.438"		0.5"

1. Tolerances are specified in ASTM F1667. Typical shape of common, box, and sinker steel wire nails shown. See ASTM F 1667 for other nail types.
2. It is permitted to assume the length of the tapered tip is 2D.

Source: Reprinted with permission from National Design Specification® for Wood Construction.

B.2.5 (Nonmandatory) Typical Dimensions for Split Ring and Shear Plate Connectors

	Split Rings[a]	
	$2\frac{1}{2}$ in.	4 in.
Split ring		
Inside diameter at center when closed (in.)	2.5	4
Thickness of ring at center (in.)	0.163	0.193
Thickness of ring at edge (in.)	0.123	0.133
Depth (in.)	0.75	1
Groove[b]		
Inside diameter (in.)	2.56	4.08
Outside diameter (in.)	2.92	4.5
Width (in.)	0.18	0.21
Depth (in.)	0.375	0.5
Bolt		
Diameter of bolt (in.)	0.5	0.75
Diameter of bolt hole (in.)	0.56	0.81
Washers		
Round, cast or malleable iron		
Diameter (in.)	2.125	3
Square, plate		
Length of side (in.)	2	3
Thickness (in.)	0.125	0.188

	Shear Plates[c]			
	$2\frac{5}{8}$ in.	$2\frac{5}{8}$ in.	4 in.	4 in.
Shear plate				
Material	Pressed steel	Malleable iron	Malleable iron	Malleable iron
Diameter of plate (in.)	2.62	2.62	4.02	4.02
Diameter of bolt hole in plate (in.)	0.81	0.81	0.81	0.93
Depth (in.)	0.42	0.42	0.62	0.62
Groove[b]				
Inside diameter (in.)	2.29	2.31	3.54	3.54
Outside diameter (in.)	2.63	2.63	4.02	4.02
Width (in.)	0.17	0.16	0.24	0.24
Depth (in.)	0.42	0.42	0.62	0.62
Bolt				
Diameter of bolt (in.)	0.75	0.75	0.75	0.88
Diameter of bolt hole in timber or metal member (in.)	0.81	0.81	0.81	0.94
Washers				
Round, cast or malleable iron				
Diameter (in.)	3	3	3	3.5
Square, plate				
Length of side (in.)	3	3	3	3
Thickness (in.)	0.25	0.25	0.25	0.25

[a] Split-ring dimensions are from *Design Manual for TECO Timber Connectors*. Cleveland Steel Specialty Company, Bedford Heights, OH, 1997.

[b] Groove dimensions have been estimated and are provided to facilitate calculation of connection row tear-out and group tear-out failure modes.

[c] Shear plate dimensions are from ASTM D5933-96, *Standard Specification for 2-5/8 in. and 4 in. Diameter Metal Shear Plates for Use in Wood Constructions*, American Society for Testing and Materials, West Conshohocken, PA, 2001.

B.2.6 Fastener Bending Yield Strengths

Fastener Type	F_{yb} (psi)
Bolt, lag screw (with D ≥ 3/8"), drift pin (SAE J429 Grade 1 - F_y = 36,000 psi and F_u = 60,000 psi)	45,000
Common, box, or sinker nail, spike, lag screw, wood screw (low to medium carbon steel)	
0.099" ≤ D ≤ 0.142"	100,000
0.142" < D ≤ 0.177"	90,000
0.177" < D ≤ 0.236"	80,000
0.236" < D ≤ 0.273"	70,000
0.273" < D ≤ 0.344"	60,000
0.344" < D ≤ 0.375"	45,000
Hardened steel nail (medium carbon steel) including post-frame ring shank nails	
0.120" ≤ D ≤ 0.142"	130,000
0.142" < D ≤ 0.192"	115,000
0.192" < D ≤ 0.207"	100,000

Source: Reprinted with permission from National Design Specification® for Wood Construction. Copyright © 2012. Courtesy American wood Council, Leesburg, Virginia.

B.3 STRUCTURAL GLUED LAMINATED TIMBER REFERENCE DESIGN VALUES

B.3.1 AITC 117-Table A1 — Reference Design Values for Structural Glued Laminated Softwood Timber

Table A1 - Reference Design Values for Structural Glued Laminated Softwood Timber

(Members stressed primarily in bending) (Tabulated design values are for normal load duration and dry service conditions.)

	Bending About X-X Axis Loaded Perpendicular to Wide Faces of Laminations						Bending About Y-Y Axis Loaded Parallel to Wide Faces of Laminations					Axially Loaded		Fasteners
	Extreme Fiber in Bending		Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Extreme Fiber in Bending	Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Tension Parallel to Grain	Compression Parallel to Grain	Specific Gravity for Fastener Design
	Bottom of Beam Stressed in Tension (Positive Bending)	Top of Beam Stressed in Tension (Negative Bending)			For Deflection Calculations	For Stability Calculations				For Deflection Calculations	For Stability Calculations			
Stress Class	F_{bx}^{+} (psi)	F_{bx}^{-} (1) (psi)	$F_{c\perp x}$ (psi)	F_{vx} (4) (psi)	E_x (10^6 psi)	$E_{x\ min}$ (10^6 psi)	F_{by} (psi)	$F_{c\perp y}$ (psi)	F_{vy} (4)(5) (psi)	E_y (10^6 psi)	$E_{y\ min}$ (10^6 psi)	F_t (psi)	F_c (psi)	G
16F-1.3E	1600	925	315	195	1.3	0.69	800	315	170	1.1	0.58	675	925	0.41
20F-1.5E	2000	1100	425	195 **(6)**	1.5	0.79	800	315	170	1.2	0.63	725	925	0.41
24F-1.7E	2400	1450	500	210 **(6)**	1.7	0.90	1050	315	185	1.3	0.69	775	1000	0.42
24F-1.8E	2400	1450**(2)**	650	265 **(3)**	1.8	0.95	1450	560	230 **(3)**	1.6	0.85	1100	1600	0.50 **(10)**

(*continues*)

	Bending About X-X Axis Loaded Perpendicular to Wide Faces of Laminations						Bending About Y-Y Axis Loaded Parallel to Wide Faces of Laminations					Axially Loaded		Fasteners
	Extreme Fiber in Bending		Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Extreme Fiber in Bending	Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Tension Parallel to Grain	Compression Parallel to Grain	Specific Gravity for Fastener Design
	Bottom of Beam Stressed in Tension (Positive Bending)	Top of Beam Stressed in Tension (Negative Bending)			For Deflection Calculations	For Stability Calculations				For Deflection Calculations	For Stability Calculations			
Stress Class	F_{bx}^{+} (psi)	F_{bx}^{-} (1) (psi)	$F_{c\perp x}$ (psi)	F_{vx} (4) (psi)	E_x (10^6 psi)	$E_{x\ min}$ (10^6 psi)	F_{by} (psi)	$F_{c\perp y}$ (psi)	F_{vy} (4)(5) (psi)	E_y (10^6 psi)	$E_{y\ min}$ (10^6 psi)	F_t (psi)	F_c (psi)	G
26F-1.9E (7)	2600	1950	650	265 **(3)**	1.9	1.00	1600	560	230 **(3)**	1.6	0.85	1150	1600	0.50 **(10)**
28F-2.1E SP (7)	2800	2300	805	300	2.1 **(9)**	1.11	1600	650	260	1.7	0.90	1250	1750	0.55
30F-2.1E SP (7)(8)	3000	2400	805	300	2.1 **(9)**	1.11	1750	650	260	1.7	0.90	1250	1750	0.55

1. For balanced layups, $\mathbf{F_{bx}^{-}}$ shall be equal to $\mathbf{F_{bx}^{+}}$ for the stress class. Designer shall specify when balanced layup is required.
2. Negative bending stress, $\mathbf{F_{bx}^{-}}$, is permitted to be increased to 1850 psi for Douglas Fir and to 1950 psi for Southern Pine for specific combinations. Designer shall specify when these increased stresses are required.
3. For structural glued laminated timber of **Southern Pine**, the basic shear design values, $\mathbf{F_{vx}}$ and $\mathbf{F_{vy}}$, are permitted to be increased to **300 psi**, and **260 psi**, respectively.
4. The design values for shear, $\mathbf{F_{vx}}$ and $\mathbf{F_{vy}}$, shall be decreased by multiplying by a factor of 0.72 for non-prismatic members, notched members, and for all members subject to impact or cyclic loading. The reduced design value shall be used for design of members at connections that transfer shear by mechanical fasteners. The reduced design value shall also be used for determination of design values for radial tension and torsion.
5. Design values are for timbers with laminations made from a single piece of lumber across the width or multiple pieces that have been edge bonded. For timbers manufactured from multiple piece laminations (across width) that are not edge bonded, value shall be multiplied by 0.4 for members with 5, 7, or 9 laminations or by 0.5 for all other members. This reduction shall be cumulative with the adjustment in footnote (4).
6. Certain Southern Pine combinations may contain lumber with wane. If lumber with wane is used, the design value for shear parallel to grain, $\mathbf{F_{vx}}$, shall be multiplied by 0.67 if wane is allowed on both sides. If wane is limited to one side, $\mathbf{F_{vx}}$ shall be multiplied by 0.83. This reduction shall be cumulative with the adjustment in footnote (4).
7. 26F, 28F, and 30F beams are not produced by all manufacturers, therefore, availability may be limited. Contact supplier or manufacturer for details.
8. 30F combinations are restricted to a maximum 6 in. nominal width unless the manufacturer has qualified for wider widths based on full-scale tests subject to approval by an accredited inspection agency.
9. For 28F and 30F members with more than 15 laminations, $\mathbf{E_x} = 2.0$ million psi.
10. For structural glued laminated timber of Southern Pine, specific gravity for fastener design is permitted to be increased to 0.55.

Stress classes represent groups of similar glued laminated timber combinations. Values for individual combinations are included in Table A1 - Expanded. Design values are for members with 4 or more laminations. For 2 and 3 lamination members, see Table A2. Some stress classes are not available in all species. Contact manufacturer for availability.

B.3.2 AITC 117-Table A1-Expanded — Reference Design Values for Structural Glued Laminated Softwood Timber Combinations[1]

Table A1 - Expanded—Reference Design Values for Structural Glued Laminated Softwood Timber Combinations (Members stressed primarily in bending) (Tabulated design values are for normal load duration and dry service conditions.

		Bending About X-X Axis (Loaded Perpendicular to Wide Faces of Laminations)							Bending About Y-Y Axis (Loaded Parallel to Wide Faces of Laminations)					Axially Loaded		Fasteners	
		Extreme Fiber in Bending		Compression Perpendicular to Grain		Shear Parallel to Grain	Modulus of Elasticity		Extreme Fiber in Bending	Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Tension Parallel to Grain	Compression Parallel to Grain	Specific Gravity for Fastener Design	
		Bottom of beam Stressed in tension (Positive Bending)	Top of Beam Stressed in Tension (Negative Bending)	Tension Face	Compression Face											Top or Bottom Face	Side Face
Combination Symbol	Species Outer/Core	F_{bx}^{+} (psi)	F_{bx}^{-} (psi)	$F_{c\perp x}$ (psi)		F_{vx} (2) (psi)	E_x (10^6 psi)	$E_{x\,min}$ (10^6 psi)	F_{by} (psi)	$F_{c\perp y}$ (psi)	F_{vy} (2)(3) (psi)	E_y (10^6 psi)	$E_{y\,min}$ (10^6 psi)	F_t (psi)	F_c (psi)	G	
16F-1.3E		**1600**	**925**	**315**		**195**	**1.3**	**0.69**	**800**	**315**	**170**	**1.1**	**0.58**	**675**	**925**	**0.41**	
16F-V3	DF/DF	1600	1250	560	560	265	1.5	0.79	1450	560	230	1.5	0.79	975	1500	0.5	0.5
16F-V6	DF/DF	1600	1600	560	560	265	1.6	0.85	1450	560	230	1.5	0.79	1000	1600	0.5	0.5
16F-E2	HF/HF	1600	1050	375	375	215	1.4	0.74	1200	375	190	1.3	0.69	825	1150	0.43	0.43
16F-E3	DF/DF	1600	1200	560	560	265	1.6	0.85	1400	560	230	1.5	0.79	975	1600	0.5	0.5
16F-E6	DF/DF	1600	1600	560	560	265	1.6	0.85	1550	560	230	1.5	0.79	1000	1600	0.5	0.5
16F-E7	HF/HF	1600	1600	375	375	215	1.4	0.74	1350	375	190	1.3	0.74	875	1250	0.43	0.43
16F-V2	SP/SP	1600	1400	740	650	300	1.5	0.79	1450	650	260	1.4	0.74	1000	1300	0.55	0.55
16F-V3	SP/SP	1600	1450	740	740	300	1.4	0.74	1450	650	260	1.4	0.74	975	1400	0.55	0.55
16F-V5	SP/SP	1600	1600	650	650	300	1.6	0.85	1600	650	260	1.5	0.79	1000	1550	0.55	0.55
16F-E1	SP/SP	1600	1250	650	650	300	1.6	0.85	1400	650	260	1.6	0.85	1050	1550	0.55	0.55
16F-E3	SP/SP	1600	1600	650	650	300	1.7	0.90	1650	650	260	1.6	0.85	1100	1550	0.55	0.55
20F-1.5E		**2000**	**1100**	**425**		**195**	**1.5**	**0.79**	**800**	**315**	**170**	**1.2**	**0.63**	**725**	**925**	**0.41**	
20F-V3	DF/DF	2000	1450	650	560	265	1.6	0.85	1450	560	230	1.5	0.79	1000	1550	0.5	0.5
20F-V7	DF/DF	2000	2000	650	650	265	1.6	0.85	1450	560	230	1.6	0.85	1050	1600	0.5	0.5
20F-V12	AC/AC	2000	1400	560	560	265	1.5	0.79	1250	470	230	1.4	0.74	925	1500	0.46	0.46
20F-V13	AC/AC	2000	2000	560	560	265	1.5	0.79	1250	470	230	1.4	0.74	950	1550	0.46	0.46
20F-V14	POC/POC	2000	1450	560	560	265	1.5	0.79	1300	470	230	1.4	0.74	900	1600	0.46	0.46

(continues)

		Bending About X-X Axis (Loaded Perpendicular to Wide Faces of Laminations)							Bending About Y-Y Axis (Loaded Parallel to Wide Faces of Laminations)					Axially Loaded		Fasteners	
		Extreme Fiber in Bending		Compression Perpendicular to Grain		Shear Parallel to Grain	Modulus of Elasticity		Extreme Fiber in Bending	Compression Perpendicular to Grain	Shear Parallel to Grain	Modulus of Elasticity		Tension Parallel to Grain	Compression Parallel to Grain	Specific Gravity for Fastener Design	
		Bottom of beam Stressed in tension (Positive Bending)	Top of Beam Stressed in Tension (Negative Bending)	Tension Face	Compression Face											Top or Bottom Face	Side Face
Combination Symbol	Species Outer/Core	F_{bx}^{+} (psi)	F_{bx}^{-} (psi)	$F_{c\perp x}$ (psi)		F_{vx} (2) (psi)	E_x (10^6 psi)	$E_{x\,min}$ (10^6 psi)	F_{by} (psi)	$F_{c\perp y}$ (psi)	F_{vy} (2)(3) (psi)	E_y (10^6 psi)	$E_{y\,min}$ (10^6 psi)	F_t (psi)	F_c (psi)	G	
20F-V15	POC/POC	2000	2000	560	560	265	1.5	0.79	1300	470	230	1.4	0.74	900	1600	0.46	0.46
20F-E2	HF/HF	2000	1400	500	500	215	1.6	0.85	1200	375	190	1.4	0.74	925	1350	0.43	0.43
20F-E3	DF/DF	2000	1200	560	560	265	1.7	0.90	1400	560	230	1.6	0.85	1050	1600	0.5	0.5
20F-E6	DF/DF	2000	2000	560	560	265	1.7	0.90	1550	560	230	1.6	0.85	1150	1650	0.5	0.5
20F-E7	HF/HF	2000	2000	500	500	215	1.6	0.85	1450	375	190	1.4	0.74	1050	1450	0.43	0.43
20F-E8	ES/ES	2000	1300	450	450	200	1.5	0.79	1000	315	175	1.4	0.74	825	1100	0.41	0.41
24F-E/SPF1	SPF/SPF	2400	2400	560	560	215	1.6	0.85	1150	470	190	1.6	0.85	1150	2000	0.42	0.42
24F-E/SPF3	SPF/SPF	2400	1550	560	650	215	1.6	0.85	1200	470	195	1.5	0.79	900	1750	0.42	0.42
20F-V2	SP/SP	2000	1550	740	650	300	1.5	0.79	1450	650	260	1.4	0.74	1000	1400	0.55	0.55
20F-V3	SP/SP	2000	1450	650	650	300	1.5	0.79	1600	650	260	1.5	0.79	1000	1400	0.55	0.55
20F-V5	SP/SP	2000	2000	740	740	300	1.6	0.85	1450	650	260	1.4	0.74	1050	1500	0.55	0.55
20F-E1	SP/SP	2000	1300	650	650	300	1.7	0.90	1400	650	260	1.6	0.85	1050	1550	0.55	0.55
20F-E3	SP/SP	2000	2000	650	650	300	1.7	0.90	1700	650	260	1.6	0.85	1150	1600	0.55	0.55
24F-1.7E		**2400**	**1450**	**500**		**210**	**1.7**	**0.90**	**1050**	**315**	**185**	**1.3**	**0.69**	**775**	**1000**	**0.42**	
24F-V5	DF/HF	2400	1600	650	650	215	1.7	0.90	1350	375	200	1.5	0.79	1100	1450	0.5	0.43
24F-V10	DF/HF	2400	2400	650	650	215	1.8	0.95	1450	375	200	1.5	0.79	1150	1550	0.5	0.43
24F-E11	HF/HF	2400	2400	500	500	215	1.8	0.95	1550	375	190	1.5	0.79	1150	1550	0.43	0.43
24F-E15	HF/HF	2400	1600	500	500	215	1.7	0.95	1200	375	190	1.5	0.79	975	1500	0.43	0.43
24F-V1	SP/SP	2400	1750	740	650	300	1.7	0.90	1450	650	260	1.5	0.79	1100	1500	0.55	0.55
24F-V4 (4)	SP/SP	2400	1650	740	650	210	1.7	0.90	1350	470	230	1.5	0.79	975	1350	0.55	0.43
24F-V5	SP/SP	2400	2400	740	740	300	1.7	0.90	1700	650	260	1.6	0.85	1150	1600	0.55	0.55
24F-1.8E		**2400**	**1450**	**650**		**265**	**1.8**	**0.95**	**1450**	**560**	**230**	**1.6**	**0.85**	**1100**	**1600**	**0.5**	
24F-V4	DF/DF	2400	1850	650	650	265	1.8	0.95	1450	560	230	1.6	0.85	1100	1650	0.5	0.5
24F-V8	DF/DF	2400	2400	650	650	265	1.8	0.95	1550	560	230	1.6	0.85	1100	1650	0.5	0.5

24F-E4	DF/DF	2400	1450	650	650	265	1.8	0.95	1400	560	230	1.7	0.90	1100	1700	0.5	0.5
24F-E13	DF/DF	2400	2400	650	650	265	1.8	0.95	1750	560	230	1.7	0.90	1250	1700	0.5	0.5
24F-E18	DF/DF	2400	2400	650	650	265	1.8	0.95	1550	560	230	1.7	0.90	975	1700	0.5	0.5
24F-V3	SP/SP	2400	2000	740	740	300	1.8	0.95	1700	650	260	1.6	0.85	1150	1650	0.55	0.55
24F-V8	SP/SP	2400	2400	740	740	300	1.8	0.95	1700	650	260	1.6	0.85	1150	1650	0.55	0.55
24F-E1	SP/SP	2400	1450	805	650	300	1.8	0.95	1550	650	260	1.7	0.90	1150	1600	0.55	0.55
24F-E4	SP/SP	2400	2400	805	805	300	1.9	1.00	1850	650	260	1.8	0.95	1450	1750	0.55	0.55
26F-1.9E (5)		**2600**	**1950**	**650**		**265**	**1.9**	**1.00**	**1600**	**560**	**230**	**1.6**	**0.85**	**1150**	**1600**	**0.5**	
26F-V1	DF/DF	2600	1950	650	650	265	2.0	1.06	1850	560	230	1.8	0.95	1350	1850	0.5	0.5
26F-V2	DF/DF	2600	2600	650	650	265	2.0	1.06	1850	560	230	1.8	0.95	1350	1850	0.5	0.5
26F-V1	SP/SP	2600	2000	740	740	300	1.8	0.95	1700	650	260	1.6	0.85	1150	1600	0.55	0.55
26F-V2	SP/SP	2600	2100	740	740	300	1.9	1.00	1950	740	260	1.8	0.95	1300	1850	0.55	0.55
26F-V3	SP/SP	2600	2100	740	740	300	1.9	1.00	1950	650	260	1.8	0.95	1250	1800	0.55	0.55
26F-V4	SP/SP	2600	2600	740	740	300	1.9	1.00	1700	650	260	1.8	0.95	1200	1600	0.55	0.55
26F-V5	SP/SP	2600	2600	740	740	300	1.9	1.00	1950	650	260	1.8	0.95	1300	1850	0.55	0.55
28F-2.1E SP(5)		**2800**	**2300**	**805**		**300**	**2.1** (7)	**1.11**	**1600**	**650**	**260**	**1.7**	**0.90**	**1250**	**1750**	**0.55**	
28F-E1	SP/SP	2800	2300	805	805	300	2.1 (7)	1.11	1600	650	260	1.7	0.90	1300	1850	0.55	0.55
28F-E2	SP/SP	2800	2800	805	805	300	2.1 (7)	1.11	2000	650	260	1.7	0.90	1300	1850	0.55	0.55
30F-2.1E SP(5)(6)		**3000**	**2400**	**805**		**300**	**2.1** (7)	**1.11**	**1750**	**650**	**260**	**1.7**	**0.90**	**1250**	**1750**	**0.55**	
30F-E1	SP/SP	3000	2400	805	805	300	2.1 (7)	1.11	1750	650	260	1.7	0.90	1250	1750	0.55	0.55
30F-E2	SP/SP	3000	3000	805	805	300	2.1 (7)	1.11	1750	650	260	1.7	0.90	1350	1750	0.55	0.55

Footnotes to Table A1:

1. The combinations in this table are applicable to members consisting of 4 or more laminations and are intended primarily for members stressed in bending due to loads applied perpendicular to the wide faces of the laminations. the laminations. However, design values are tabulated for loading both perpendicular and parallel to the wide faces of the laminations. For combinations and design values applicable to members loaded primarily axially or parallel to the wide faces of the laminations, see Table A2. For members of 2 or 3 laminations, see Table A2.
2. The design values for shear, $\mathbf{F}_{vx}$ and $\mathbf{F}_{vy}$, shall be decreased by multiplying by a factor of 0.72 for non-prismatic members, notched members, and for all members subject to impact or cyclic loading. The reduced design value shall be used for design of members at connections that transfer shear by mechanical fasteners. The reduced design value shall also be used for determination of design values for radial tension and torsion.
3. Design values are for timbers with laminations made from a single piece of lumber across the width or multiple pieces that have been edge bonded. For timber manufactured from multiple piece laminations (across width) that are not edge-bonded, value shall be multiplied by 0.4 for members with 5, 7, or 9 laminations or by 0.5 for all other members. This reduction shall be cumulative with the adjustment in footnote 3.
4. This combination may contain lumber with wane. If lumber with wane is used, the design value for shear parallel to grain, $F_{VX,}$ shall be multiplied by 0.67 if wane is allowed on both sides. If wane is limited to one side, $\mathbf{F}_{vx}$ shall be multiplied by 0.83. This reduction shall be cumulative with the adjustment in footnote 3.
5. 26F, 28F, and 30F beams are not produced by all manufacturers, therefore, availability may be limited. Contact supplier or manufacturer for details.
6. 30F combinations are restricted to a maximum 6 in. nominal width unless the manufacturer has qualified for wider widths based on full-scale tests subject to approval by an accredited inspection agency.
7. For 28F and 30F members with more than 15 laminations, $\mathbf{E}_x = 2.0$ million psi.

B.3.3 AITC 117-Table A2 — Reference Design Values for Structural Glued Laminated Softwood Timber

Reference Design Values for Structural Glued Laminated Softwood Timber (Members stressed primarily in axial tension or compression) (Tabulated design values are for normal load duration and dry service conditions.)

Combination Symbol	Species	Grade	All Loading: Modulus of Elasticity		All Loading: Compression Perpendicular to Grain	Axially Loaded: Tension Parallel to Grain	Axially Loaded: Compression Parallel to Grain		Bending about Y-Y Axis Loaded Parallel to Wide Faces of Laminations: Bending			Bending about Y-Y Axis: Shear Parallel to Grain(1)(2)(3)	Bending About X-X Axis Loaded Perpendicular to Wide Faces of Laminations: Bending	Bending About X-X Axis: Shear Parallel to Grain(3)
						2 or More Laminations	4 or More Laminations	2 or 3 Laminations	4 or More Laminations	3 Laminations	2 Laminations		2 Laminations to 15 in. Deep(4)	
			E (10^6 psi)	E_{min} (10^6 psi)	$F_{c\perp}$ (psi)	F_t (psi)	F_c (psi)	F_c (psi)	F_{by} (psi)	F_{by} (psi)	F_{by} (psi)	F_{vy} (psi)	F_{bx} (psi)	F_{vx} (psi)
Visually Graded Western Species														
1	DF	L3	1.5	0.79	560	950	1550	1250	1450	1250	1000	230	1250	265
2	DF	L2	1.6	0.85	560	1250	1950	1600	1800	1600	1300	230	1700	265
3	DF	L2D	1.9	1.00	650	1450	2300	1900	2100	1850	1550	230	2000	265
4	DF	L1CL	1.9	1.00	590	1400	2100	1950	2200	2000	1650	230	2100	265
5	DF	L1	2.0	1.06	650	1650	2400	2100	2400	2100	1800	230	2200	265
14	HF	L3	1.3	0.69	375	800	1100	1050	1200	1050	850	190	1100	215
15	HF	L2	1.4	0.74	375	1050	1350	1350	1500	1350	1100	190	1450	215
16	HF	L1	1.6	0.85	375	1200	1500	1500	1750	1550	1300	190	1600	215
17	HF	L1D	1.7	0.90	500	1400	1750	1750	2000	1850	1550	190	1900	215
22(5)	SW	L3	1.0	0.53	315	525	850	725	800	700	575	170	725	195
69	AC	L3	1.2	0.63	470	725	1150	1100	1100	975	775	230	1000	265
70	AC	L2	1.3	0.69	470	975	1450	1450	1400	1250	1000	230	1350	265
71	AC	L1D	1.6	0.85	560	1250	1900	1900	1850	1650	1400	230	1750	265
72	AC	L1S	1.6	0.85	560	1250	1900	1900	1850	1650	1400	230	1900	265
73	POC	L3	1.3	0.69	470	775	1500	1200	1200	1050	825	230	1050	265
74	POC	L2	1.4	0.74	470	1050	1900	1550	1450	1300	1100	230	1400	265
75	POC	L1D	1.7	0.90	560	1350	2300	2050	1950	1750	1500	230	1850	265

Visually Graded Southern Pine														
47	SP	N2M12	1.4	0.74	650	1200	1900	1150	1750	1550	1300	260	1400	300
47 1:10	SP	N2M10	1.4	0.74	650	1150	1700	1150	1750	1550	1300	260	1400	300
47 1:8	SP	N2M	1.4	0.74	650	1000	1500	1150	1600	1550	1300	260	1400	300
48	SP	N2D12	1.7	0.90	740	1400	2200	1350	2000	1800	1500	260	1600	300
48 1:10	SP	N2D10	1.7	0.90	740	1350	2000	1350	2000	1800	1500	260	1600	300
48 1:8	SP	N2D	1.7	0.90	740	1150	1750	1350	1850	1800	1500	260	1600	300
49	SP	N1M16	1.7	0.90	650	1350	2100	1450	1950	1750	1500	260	1800	300
49 1:14	SP	N1M14	1.7	0.90	650	1350	2000	1450	1950	1750	1500	260	1800	300
49 1:12	SP	N1M12	1.7	0.90	650	1300	1900	1450	1950	1750	1500	260	1800	300
49 1:10	SP	N1M	1.7	0.90	650	1150	1700	1450	1850	1750	1500	260	1800	300
50	SP	N1D14	1.9	1.00	740	1550	2300	1700	2300	2100	1750	260	2100	300
50 1:12	SP	N1D12	1.9	1.00	740	1500	2200	1700	2300	2100	1750	260	2100	300
50 1:10	SP	N1D	1.9	1.00	740	1350	2000	1700	2100	2100	1750	260	2100	300

Footnotes to Table A2:

1. For members with 2 or 3 laminations, the shear design value for transverse loads parallel to the wide faces of the laminations, $\mathbf{F_{vy}}$, shall be reduced by multiplying by a factor of 0.84 or 0.95, respectively.
2. The shear design value for transverse loads applied parallel to the wide faces of the laminations, $\mathbf{F_{vy}}$, shall be multiplied by 0.4 for members with 5, 7, or 9 laminations manufactured from multiple piece laminations (across width) that are not edge bonded. The shear design value, $\mathbf{F_{vy}}$, shall be multiplied by 0.5 for all other members manufactured from multiple piece laminations with unbonded edge joints. This reduction shall be cumulative with the adjustment in footnote (1).
3. The design values for shear, $\mathbf{F_{vx}}$ and $\mathbf{F_{vy}}$, shall be decreased by multiplying by a factor of 0.72 for non-prismatic members, notched members, and for all members subject to impact or cyclic loading. The reduced design value shall be used for design of members at connections that transfer shear by mechanical fasteners. The reduced design value shall also be used for determination of design values for radial tension and torsion.
4. For members greater than 15 in. deep, the bending design value, $\mathbf{F_{bx}}$, shall be reduced by multiplying by a factor of 0.88.
5. When Western Cedars, Western Cedars (North), Western Woods, and Redwood (open grain) are used in combinations for Softwood Species (SW), the design value for modulus of elasticity shall be reduced by 100,000 psi. When Coast Sitka Spruce, Coast Species, Western White Pine, and Eastern White Pine are used in combinations for Softwood Species (SW) tabulated design values for shear parallel to grain, $\mathbf{F_{vx}}$ and $\mathbf{F_{vy}}$, shall be reduced by 10 psi, before applying any other adjustments.

INDEX

REFERENCES

Chapter 1

1. American Lumber Standards Committee, Inc. (ALSC), P.O. Box 210, Germantown, MD 20875.
2. Southern Pine Inspection Bureau (SPIB), *Standard Grading Rules for Southern Pine Lumber*, Pensacola, FL, 2002.
3. West Coast Lumber Inspection Bureau (WCLIB), Standard no. 17 *Grading Rules for West Coast Lumber*. Portland, OR, 2000.
4. Western Wood Products Association (WWPA), *Western Lumber Grading Rules*, Portland, OR, 2005.
5. National Lumber Grades Authority (NLGA), *Standard Grading Rules for Canadian Lumber*. New Westminster, BC (Canada), 2003.
6. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.
7. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.
8. American Institute of Timber Construction, *American National Standard for Wood Products—Structural Glued Laminated Timber*, ANSI/AITC A190.1, Centennial, CO, 2007.
9. American Institute of Timber Construction, *Manufacturing Quality Control Systems Manual*, AITC 200, Centennial, CO, 2009.
10. American Institute of Timber Construction, *Standard Appearance Grades for Structural Glued Laminated Timber*, AITC 110, Englewood, CO, 2001.
11. International Code Council, *International Building Code*, Country Club Hills, IL, 2009.
12. American Institute of Timber Construction, *AITC Quality Control Program*, Technical Note 10, Centennial, CO, 2005.

13. American Wood Council, *Wood Frame Construction Manual for One- and Two-Family Dwellings*, Leesburg, VA, 2012.
14. United States Department of Commerce, *Structural Plywood*. PS 1–09, Washington, DC, 2010.
15. United States Department of Commerce, *Performance Standard for Wood-Based Structural-Use Panels*, PS 2–10, Washington, DC, 2011.
16. American Society for Testing and Materials, *Standard Specification and Test Method for Establishing Recommended Design Stresses for Round Timber Construction*, ASTM D3200, West Conshohoken, PA, 2005.
17. American Society for Testing and Materials, *Standard Specification for Round Timber Piles*, ASTM D25, West Conshohoken, PA, 2005.
18. American Society for Testing and Materials, *Standard Practice for Establishing Allowable Stresses for Round Timber Piles*, ASTM D2899, West Conshohoken, PA, 2003.
19. American National Standards Institute, Inc. (ANSI), *Wood Poles—Specifications and Dimensions*, ANSI O5.1, New York, NY, 2008.
20. American Wood Protection Association, *Book of Standards*, Birmingham, AL, 2011.
21. American Society for Testing and Materials, *Standard Practices for Establishing Stress Grades for Structural Members Used in Log Buildings*, ASTM D3957, West Conshohocken, PA, 2009.
22. International Code Council, *International Residential Code*, Country Club Hills, IL, 2009.
23. American Wood Protection Association, *Standard for the Care of Preservative-Treated Wood Products*, AWPA M4, Birmingham, AL, 2008.
24. National Frame Builders Association, *Post-Frame Building Design Manual*, Lawrence, KS, 2000.
25. R. J. Hoyle and F. E. Woeste, *Wood Technology in the Design of Structures*, Iowa State University Press, Ames, IA, 1989.
26. T. D. Skaggs, F. E. Woeste, and D. A. Bender, A Simple Analysis for Calculating Post Forces, Pages *Applied Engineering in Agriculture*, Vol. 9 (2), American Society of Agricultural Engineers, St. Joseph, MI, 1993, 253–259.
27. American Wood Preservers Institute, *Pile Foundations Know-How*, Vienna, VA, 1969.
28. American Institute of Timber Construction, *Design of Tudor Arches with Structural Glued Laminated Timber*, Centennial, CO, 2011.
29. American Wood Council, *Special Design Provisions for Wind and Seismic*, Leesburg, VA. 2008.
30. American Institute of Timber Construction, *Typical Construction Details*, AITC 104, Centennial, CO, 2003.
31. American Institute of Timber Construction, *Designing Structural Glued Laminated Timber for Permanence*, Technical Note 12, Centennial, CO, 2002.
32. American Institute of Timber Construction, *Standard for Preservative Treatment of Structural Glued Laminated Timber*, AITC 109, Centennial, CO, 2007.
33. American Society of Heating, Refrigerating and Air-conditioning Engineers, Inc., *ASHRAE Fundamentals Handbook*, Atlanta, GA, 1989.
34. United States Department of Agriculture, Forest Service, Forest Products Laboratory, *Wood Handbook: Wood as an Engineering Material*, Madison, WI, 2010.

35. American Institute of Timber Construction, *Checking in Glued Laminated Timbers*, Technical Note 11, Centennial, CO, 1987.
36. American Institute of Timber Construction, *Evaluation of Checks in Structural Glued Laminated Timbers*, Technical Note 18, Centennial, CO, 2011.
37. American Society for Testing and Materials, *Standard Practice for Establishing Structural Grades and Related Allowable Properties for Visually Graded Lumber*, ASTM D245, Conshohoken, PA, 2000.
38. American Institute of Steel Construction, *Steel Construction Manual*, 13th ed., Chicago, IL, 2005.
39. American Welding Society, *Structural Welding Code*, D1.1, Miami, FL, 2010.
40. American Institute of Timber Construction, *Recommended Practice for Protection of Structural Glued Laminated Timber During Transit, Storage and Erection*, AITC 111, Centennial, CO, 2005.

Chapter 2

1. United States Department of Agriculture, Forest Service, Forest Products Laboratory, *Wood Handbook: Wood as an Engineering Material*, Madison, WI, 2010.
2. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.
3. American Wood Protection Association, *Book of Standards*, Birmingham, AL, 2011.
4. American Society of Heating, Refrigerating and Air-conditioning Engineers, Inc., *ASHRAE Fundamentals Handbook*, Atlanta, GA, 1989.

Chapter 3

1. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, *AITC 117*, Centennial, CO, 2010.
2. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.
3. International Code Council, *International Building Code*, Country Club Hills, IL, 2009.
4. International Code Council, *International Residential Code*, Country Club Hills, IL, 2009.
5. American Society of Civil Engineers, *SEI/ASCE 7–10—Minimum Design Loads for Buildings and Other Structures*, Reston, VA, 2010.
6. American Association of State Highway and Transportation Officials, *Standard Specifications for Highway Bridges*, 17^{th} ed., Washington, DC, 2002.
7. American Association of State Highway and Transportation Officials, *AASHTO LRFD Bridge Design Specifications*, 5th ed., Washington, DC, 2010.
8. American Railway Engineering and Maintenance-of-Way Association, *Manual for Railway Engineering*, Landover, MD, 2003.
9. American Society of Civil Engineers, *ASCE Paper No. 2470, Design Considerations for Fatigue in Timber Structures*, Reston, VA, 1960.

10. United States Department of Agriculture, Forest Service, Forest Products Laboratory, Report No. 2236, *Fatigue Resistance of Quarter-Scale Bridge Stringers in Flexure and Shear*, Madison, WI, 1962.
11. American Society of Civil Engineers, Manual No. 42, *Design of Structures to Resist Nuclear Weapons Effects*, Reston, VA, 1985.
12. American Wood Council, *Designing for Lateral-Torsional Stability in Wood Members*, TR14, Leesburg, VA, 2003.
13. J.A. Yura, *Bracing for Stability—State of the Art, Proceedings of the Structures Congress XIII Restructuring: American and Beyond*. American Society of Civil Engineers, New York, NY, 1995, pp. 88–103.
14. United States Department of Agriculture, Forest Service, *Forest Products Laboratory, Research Paper FPL 34, Deflection and Stresses of Tapered Wood Beams*, Madison, WI, 1965.
15. American Wood Protection Association, *Book of Standards*, Birmingham, AL, 2011.
16. American Wood Council, *Special Design Provisions for Wind and Seismic*, Leesburg, VA, 2008.
17. E. Kuenzi and B. Bohannan, "Increases in Deflection and Stresses Caused by Ponding of Water on Roofs." *Forest Products Journal*, Vol. 14, No. 9, 1964, 421–424.
18. United States Department of Agriculture, Forest Service, *Forest Products Laboratory, Wood Handbook: Wood as an Engineering Material*. Madison, WI, 2010.

Chapter 4

1. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.
2. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*. AITC 117, Centennial, CO, 2010.
3. G. W. Thayer and H. W. March, *The Torsion of Members Having Sections Common in Aircraft Construction*. U.S. Department of Agriculture, Forest Products Laboratory, Madison, WI, 1929.
4. American Institute of Timber Construction Technical Advisory Committee recommendations based on the proportional limit for torsion from The Wood Handbook, U.S. Department of Agriculture, 1999, and the relationship between torsional shear strength and shear parallel to grain reported by Gupta et al., "Experimental Evaluation of the Torsion Test for determining Shear Strength of Structural Lumber," *Journal of Testing and Evaluation*, Vol. 30, No. 4, July 2002, pp. 283–290, and D. S. Ryanto and R. Gupta, "A Comparison of Test Methods for evaluating Shear Strength of Structural Lumber," *Forest Products Journal*, Vol. 48, No. 2, 1998, pp. 83–90, and the provisions of AF&PA/ASCE *Standard for Load and Resistance Factor Design for Engineered Wood Construction, American Society of Civil Engineers*, 1996, Section 5.5, pp. 16–95.

Chapter 5

1. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, *AITC 117*, Centennial, CO, 2010.

2. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.

Chapter 6

1. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.
2. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.

Chapter 7

1. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.

Chapter 8

1. American Institute of Timber Construction, *Standard for Radially Reinforcing Curved Glued Laminated Timber Members to Resist Radial Tension*, AITC 404, Centennial, CO, 2005.
2. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.
3. American Institute of Timber Construction, *Typical Construction Details*, AITC 104, Centennial, CO, 2003.
4. Colorado State University, Civil Engineering Department, Structural Research Report No. 16, *Behavior and Design of Double-Tapered Pitched and Curved Glulam Beams*, Ft. Collins, CO, 1976.
5. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.

Chapter 9

1. American Institute of Timber Construction, *Design of Tudor Arches with Structural Glued Laminated Timber*, Centennial, CO, 2011.

Chapter 10

1. Southern Pine Inspection Bureau (SPIB), *Standard Grading Rules for Southern Pine Lumber*, Pensacola, FL, 2002.
2. West Coast Lumber Inspection Bureau (WCLIB), *Standard no. 17 Grading Rules for West Coast Lumber*. Portland, OR, 2000.
3. Western Wood Products Association (WWPA), *Western Lumber Grading Rules*, Portland, OR, 2005.
4. National Lumber Grades Authority (NLGA), *Standard Grading Rules for Canadian Lumber*. New Westminster, BC (Canada) 2003.

5. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.

Chapter 11

1. American Institute of Timber Construction, *Typical Construction Details*, AITC 104, Centennial, CO, 2003.
2. American Wood Council, National Design Specification® for Wood Construction, Leesburg, VA, 2012.
3. American Institute of Timber Construction, Designing Structural Glued Laminated Timber for Permanence, Technical Note 12, Centennial, CO, 2002.
4. American Institute of Timber Construction, *Standard for Preservative Treatment of Structural Glued Laminated Timber*, AITC 109, Centennial, CO, 2007.
5. American Wood Council, *NDS Commentary*, Leesburg, VA, 2012.
6. S. K. Suddarth. "Test Performance of $1\frac{1}{2}$ Inch Bolts in Glulam—Row Effect and Effect of Subsequent Drying," *Wood Design Focus*, Issue 1, Volume 1, 1990.
7. R. D. Call and R. Bjorhawde. "Wood Connections with Heavy Bolts and Steel Plates," *ASCE Journal of Structural Engineering*, Vol. 116, No. 11, 1990.
8. W. H. Khushefati. Performance of Bolted Joints Comprised of Glue-Laminated Wood Members Connected with Large Diameter ($1\frac{1}{4}$ inch) Bolts. Master's thesis, Cornell University, 1985.
9. American Society for Testing and Materials, *Standard Specification for Carbon Steel Bolts and Studs—60,000 psi Tensile Strength*, ASTM A307, West Conshohoken, PA, 2010.
10. Society of Automotive Engineers. *Mechanical and Material Requirements for Externally Threaded Fasteners*, Warrendale, PA, 1999.
11. American Society for Testing and Materials, *Standard Test Method for Determining Bending Yield Moment of Nails*, ASTM F1575, West Conshohoken, PA, 2003.
12. American Society of Mechanical Engineers. Standard B18.2.1–1996, *Square and Hex Bolts and Screws (Inch Series)*, New York, NY, 1997.
13. American Society of Mechanical Engineers. Standard B18.6.1–1981, *Wood Screws (Inch Series)*, New York, NY, 1982.
14. American Society for Testing and Materials, *Specification for Ferritic Malleable Iron Castings*, ASTM A47, West Conshohoken, PA, 1999.
15. American Iron and Steel Institute, *Standard Steels*, AISI 1035, Washington, DC, 1985.
16. American Society for Testing and Materials, *Standard Test Methods for Rockwell Hardness of Metallic Materials*, ASTM E18, West Conshohoken, PA, 2008.
17. American Society for Testing and Materials, *Standard Test Methods and Definitions for Mechanical Testing of Steel Products*, ASTM A370, West Conshohoken, PA, 2003.
18. American Society for Testing and Materials, *Specification for Standard Structural Steel*, ASTM A36, West Conshohoken, PA, 2004.
19. J. Zahn, "Design Equation for Multiple-Fastener Wood Connections," *Journal of Structural Engineering*, Vol. 117, No. 11, 1991.
20. American Wood Protection Association, *Book of Standards*, Birmingham, AL, 2011.

Chapter 12

1. J. Scholten, *Timber-Connector Joints: Their Strength and Design*, Technical Bulletin No. 865, USDA Forest Products Laboratory, Madison, WI, 1944.
2. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.

Chapter 13

1. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.
2. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.
3. American Institute of Steel Construction, *Steel Construction Manual*, Chicago, IL, 2005.
4. American Concrete Institute, *Building Code Requirements for Structural Concrete and Commentary*, ACI 318, Farmington Hills, MI, 2011.

Chapter 14

1. J. Scholten, *Timber-Connector Joints: Their Strength and Design*, Technical Bulletin No. 865, USDA Forest Products Laboratory, Madison, WI, 1944.
2. Timber Engineering Company, *Timely Timber Topics No. 4*, Washington, DC, 1944.
3. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.

Chapter 15

1. American Institute of Steel Construction, *Steel Construction Manual*, Chicago, IL, 2005.

Chapter 16

1. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.
2. American Association of State Highway and Transportation Officials, *AASHTO LRFD Bridge Design Specifications*, 5th ed., Washington, DC, 2010.
3. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.

Chapter 17

1. American Society of Civil Engineers, *American Wooden Bridges, Committee on History and Heritage of American Civil Engineering*, ASCE Historical Publication No. 4, New York, 1976.

2. J. Wilkinson, *Industrial Timber Preservation*, Associated Business Press, London, 1979.
3. American Association of State Highway and Transportation Officials, *AASHTO LRFD Bridge Design Specifications*, 5th ed., Washington, DC, 2010.
4. American Association of State Highway and Transportation Officials, *Standard Specifications for Highway Bridges*, 17th ed., Washington, DC, 2002.
5. M. Ritter, R. Faller, S. Bunnell, P. Hilbrich Lee, and B. Rosson, *Plans for Crash-Tested Bridge Railings for Longitudinal Wood Decks on Low Volume Roads*, General Technical Report FPL-GTR-107. USDA Forest Product Laboratory, Madison, WI, 1998.
6. J. Wacker and M. Smith, *Standard Plans for Timber Bridge Superstructures*, General Technical Report FPL-GTR-125. USDA Forest Product Laboratory, Madison, Wisconsin, 2001.
7. Federal Highway Administration, *Covered Bridge Manual*, Publication Number FHWA-HRT-04–098. Washington, DC, 2005.
8. E. Cesa, J. Bejune, and M. Strothers, *Portable Timber Bridges as a Best Management Practice in Forest Management*, USDA Forest Service, Northeastern Area State & Private Forestry, National Wood in Transportation Information Center, Morgantown, WV, NA-TP-04–04. 2004.
9. American Wood Protection Association, *2010 AWPA Book of Standards*, Birmingham, AL, 2010.
10. American Institute of Timber Construction, *Standard for Preservative Treatment of Structural Glued Laminated Timber*, AITC 109, Centennial, CO. 2007.
11. Western Wood Preservers Institute, Wood Preservation Canada, Southern Pressure Treaters' Association, and Timber Piling Council, *Best Management Practices for the Use of Treated Wood in Aquatic and Other Sensitive Environments*, 2006.
12. R. Weyers, J. Loferski, D. Dolan, J. Haramis, J. Howard, and L. Hislop, *Guidelines for Design, Installation, and Maintenance of a Waterproof Wearing Surface for Timber Bridge Decks*, General Technical Report FPL-GTR-123. USDA Forest Product Laboratory, Madison, WI, 2001.
13. M. Ritter, *Timber Bridges: Design, Construction, Inspection, and Maintenance*, USDA Forest Products Laboratory. Madison, WI, 1990.
14. M. Ericksson, H. Wheeler, and S. Kosmalski. *Asphalt Paving of Treated Timber Bridge Decks*. USDA Forest Service Technology and Development Program, Missoula, MT, 2003.
15. FHWA Caltrans Bridge Rail Guide 2005. http://www.fhwa.dot.gov/bridge/bridgerail/index.cfm.
16. http://safety.fhwa.dot.gov/roadway_dept/policy_guide/road_hardware/barriers/bridgerailings/
17. National Center for Wood Transportation Structures, http://www.woodcenter.org/library/allpubs.cfm?topicID=17
18. H. Ross, D. Sicking, R. Zimmer, and J. Michie. NCHRP Report 350, *Recommended Procedures for the Safety Performance Evaluation of Highway Features*, National Cooperative Highway Research Program. Transportation Research Board. National Research Council. National Academy Press. 1993.

Chapter 18

1. American Association of State Highway and Transportation Officials, *AASHTO LRFD Bridge Design Specifications*, 5th ed., Washington, DC, 2010.
2. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.

Chapter 19

1. American Association of State Highway and Transportation Officials, *Standard Specifications for Highway Bridges*, 17th ed., Washington, DC, 2002.
2. M. Ritter, *Timber Bridges: Design, Construction, Inspection, and Maintenance* USDA Forest Products Laboratory, Madison, WI, 1990.
3. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.
4. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.
5. American Institute of Steel Construction, *Steel Construction Manual*, Chicago, IL, 2005.

Chapter 20

1. International Code Council. *International Building Code*, Country Club Hills, IL, 2009.
2. Report of Investigation of the McCormick Place Fire of January 16, 1967. Mayor's Committee to Investigate McCormick Place Fire. Chicago, Illinois.
3. National Automatic Sprinkler and Fire Control Association, Inc., News Bulletin Number 229, March–April 1967.
4. R. Dorman. Zion Baptist Temple Fire. Personal Correspondence with William Ganser. AITC files. August 10, 1959.
5. American Wood Council, *Calculating the Fire Resistance of Exposed Wood Members*, Technical Report 10, Leesburg, VA, 2012.
6. H. Fleisher, *The Performance of Wood in Fire, Report No. 2202*. USDA Forest Products Laboratory, Madison, WI, 1960.
7. National Lumber Manufacturers Association, *Comparative Fire Test of Timber and Steel Beams*, Washington, DC, 8 pages.
8. E. Ellis, "Fire Resistivity: Wood vs. Steel," *Winter Garden Pavillion. General Bulletin* 1140. Timber Structures, Inc. 1956.
9. American Wood Council, *Component Additive Method (CAM) for Calculating and Demonstrating Assembly Fire Endurance*, DCA 4, Leesburg, VA, 2001.
10. American Institute of Timber Construction, *Calculation of Fire Resistance of Glued Laminated Timbers*, Technical Note 7, Centennial, CO, 1996.
11. American Wood Council, *National Design Specification® for Wood Construction*, Leesburg, VA, 2012.

12. R. Hernandez, R. Moody, and R. Falk, *Fiber Stress Values for Design of Glulam Timber Utility Structures*, Research Paper FPL-RP-532. USDA Forest Products Laboratory, Madison, WI. 1995.
13. American Institute of Timber Construction, *Standard Specification for Structural Glued Laminated Timber of Softwood Species*, AITC 117, Centennial, CO, 2010.
14. American Society of Civil Engineers, *Minimum Design Loads for Buildings and Other Structures*. ASCE 7–10, American Society of Civil Engineers. 2010.