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Math Problems With Solution

maths solutions

$\sum f(x)$

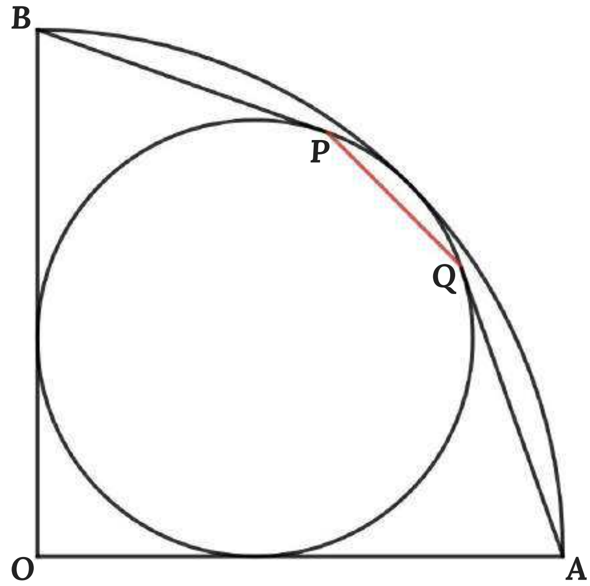
Geometry

2

Contents

Questions

QUESTION 01



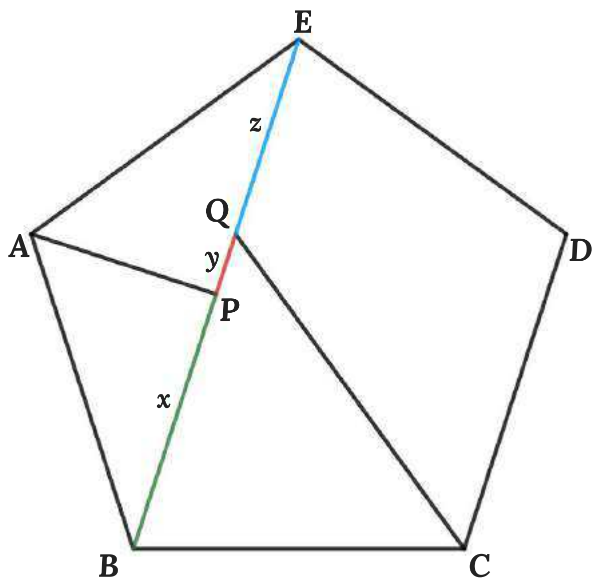
In figure

- A circle is inscribed inside the quarter circle
- The radius of the quarter circle is 3 cm
- QA & PB are tangent of the circle

Find **PQ**

{Solution: Page 022}

QUESTION 02



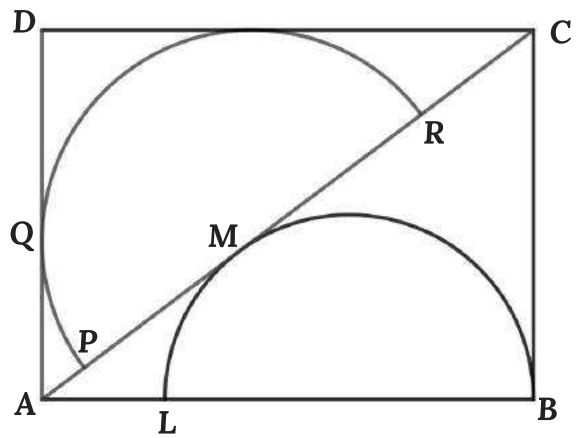
In figure

- ABCDE is a regular pentagon
- AP is the angle bisector of ∠BAE
- CQ is the angle bisector of ∠BCD

Find *x* : *y* : *z*

{Solution: Page 024}

QUESTION 03



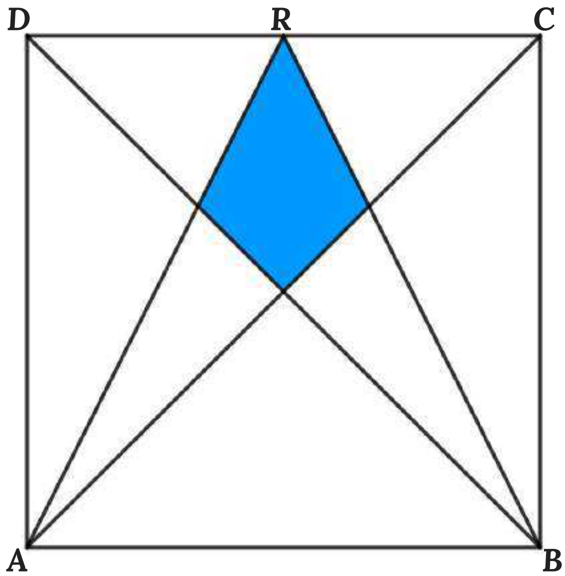
In figure

- Radius of the semicircle PQR = 24 cm
- Radius of the semicircle BML = 21 cm

Find the ***area of rectangle ABCD***

{Solution: Page 026}

QUESTION 04



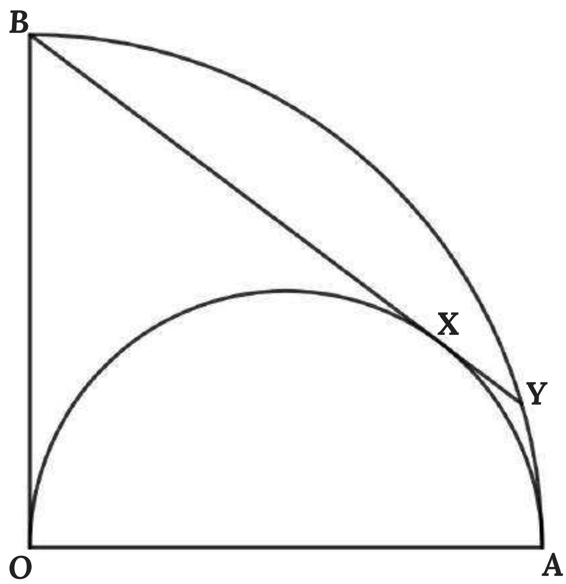
In figure

- RC = RD
- Blue area = 10 cm²

Find the ***area of the square***

{*Solution: Page 028*}

QUESTION 05



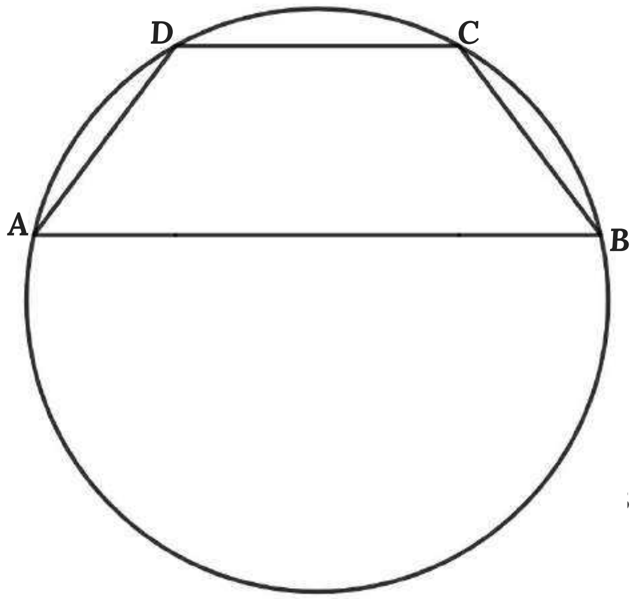
In figure

- AOB is a quarter circle
- OA = 10 cm
- OXA is a semicircle
- BY is the tangent of the semicircle

Find the ***value of XY***

{*Solution: Page 030*}

QUESTION 06



In figure

- AB // CD
- AD = BC = 5 cm
- CD = 6 cm & AB = 12 cm

Find the ***radius of the circle***

{*Solution: Page 031*}

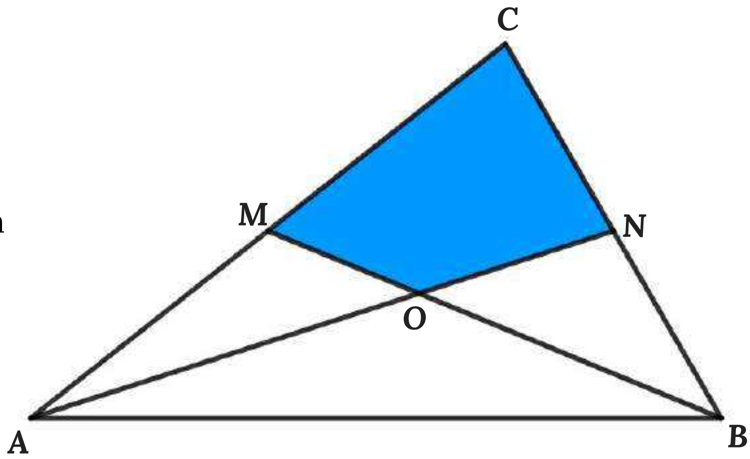
QUESTION 07

In figure

- AB = 16 cm, AC = 14 cm & BC = 10 cm
- AM = CM & CN = BN

Find the ***area of quadrilateral***

{*Solution: Page* 032}



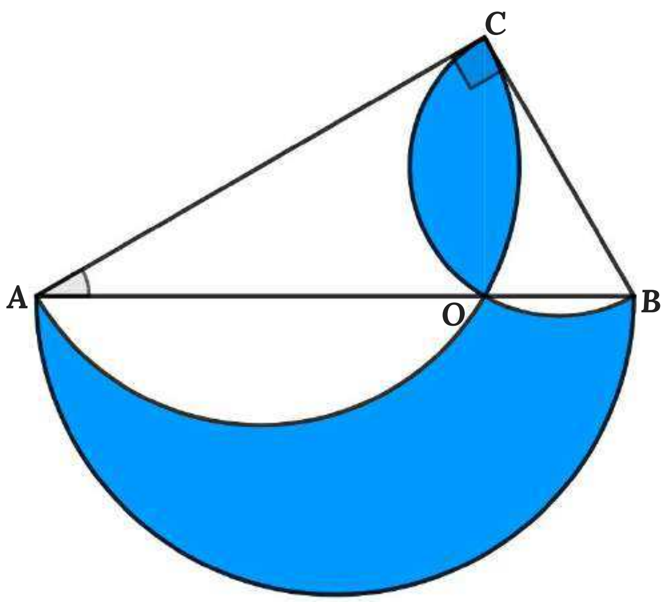
QUESTION 08

In figure

- ABC is a right angle triangle
- AB = 8 cm & ∠BAC = 30°
- Three semicircles

Find the ***area of the blue region***

{*Solution: Page* 034}



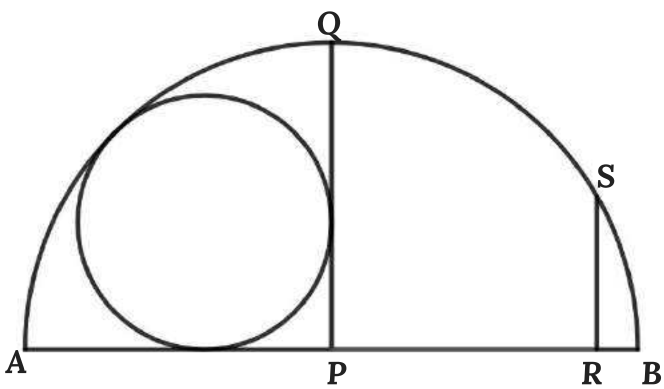
QUESTION 09

In figure

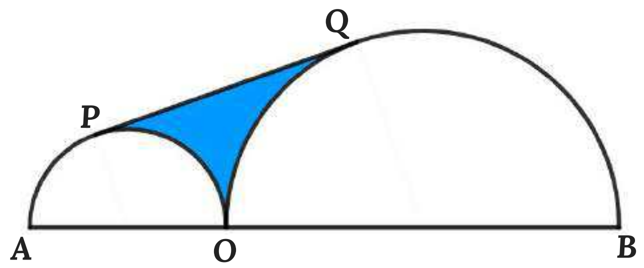
- AQB is a semicircle
- PQ // RS
- PA = PB = PQ = 2 × RS
- PR = 10 cm

Find the ***radius of the circle***

{*Solution: Page* 036}



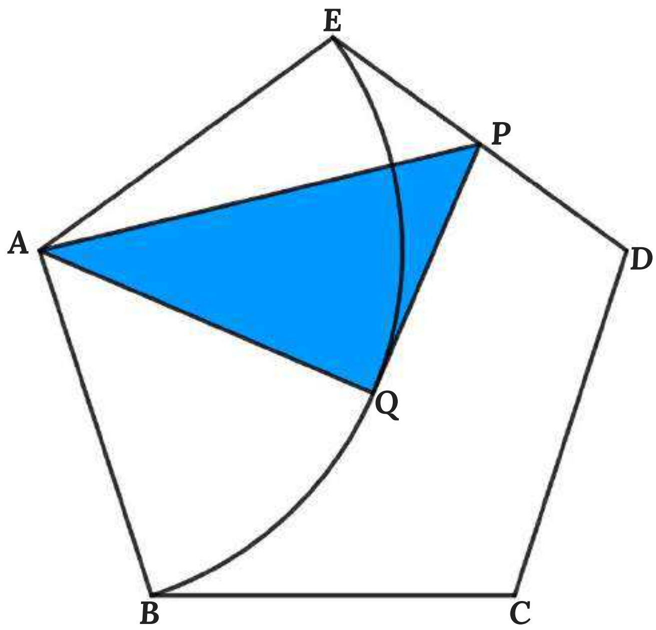
QUESTION 10



In figure
- Two semicircles
- AO = 4 cm & BO = 8 cm
- PQ is tangent of both semicircles

Find the ***area of the blue region***
{*Solution*: *Page* 037}

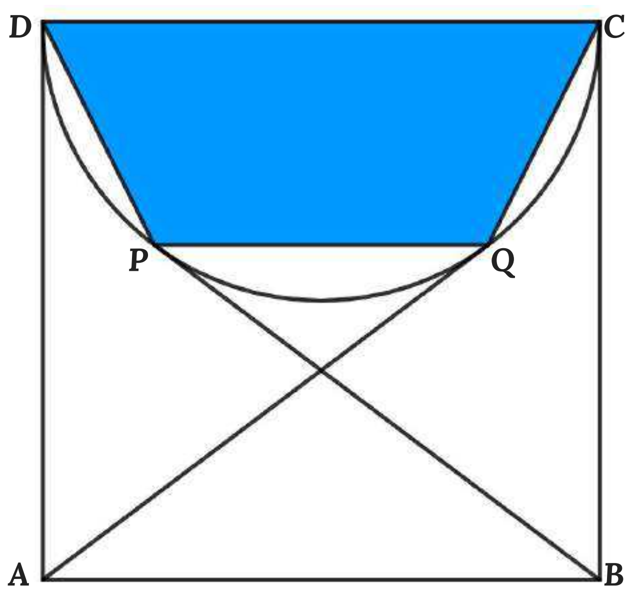
QUESTION 11



In figure
- ABCDE is a regular pentagon
- AB = 6 cm
- PD = PE
- PQ is the tangent of arc(BQE)

Find the ***area of*** **ΔPAQ**
{*Solution*: *Page* 039}

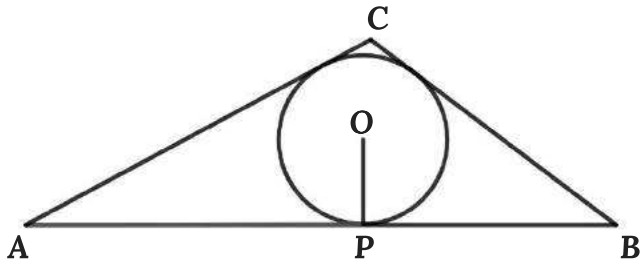
QUESTION 12



In figure
- ABCD is a square
- AB = 20 cm
- AQ & BP are tangent of the semicircle

Blue area =?
{*Solution*: *Page* 040}

QUESTION 13



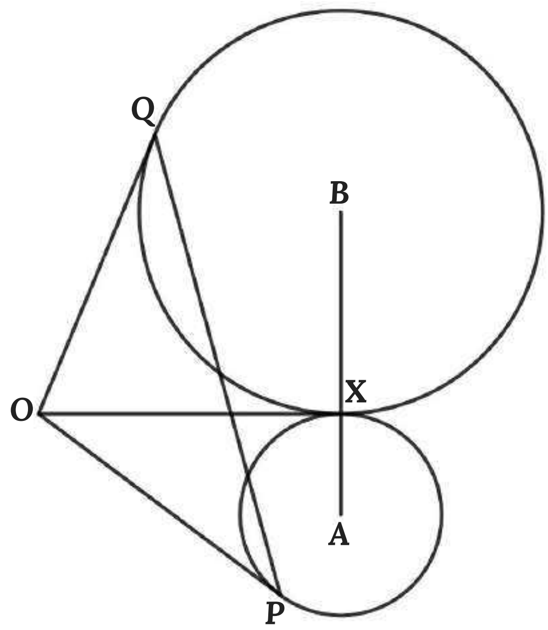
In figure

- Radius of incircle = 1 cm
- PA = 4 cm & PB = 3 cm

Find the ***area of*** **ΔABC**

{*Solution: Page 042*}

QUESTION 14



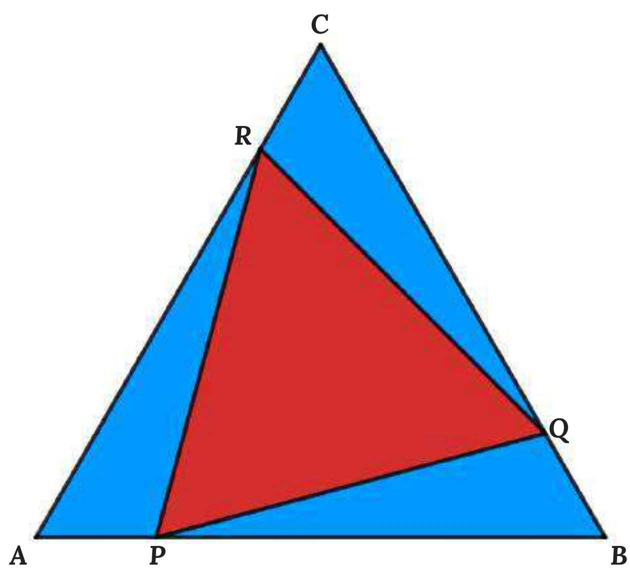
In figure

- AX = 3 cm & BX = 6 cm
- OX = 9 cm
- OP, OX & OQ are tangents

Find PQ

{*Solution: Page 044*}

QUESTION 15



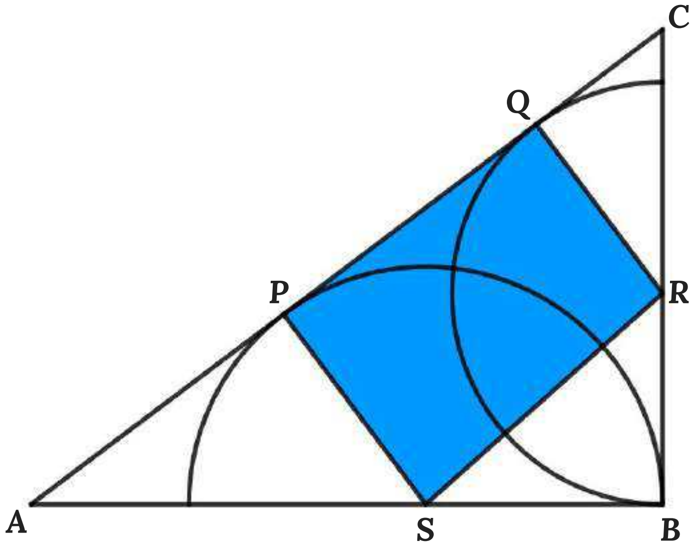
In figure

- ΔABC & ΔPQR are equilateral triangles
- Blue Area = Red Area

Find CQ : BQ

{*Solution: Page 046*}

QUESTION 16



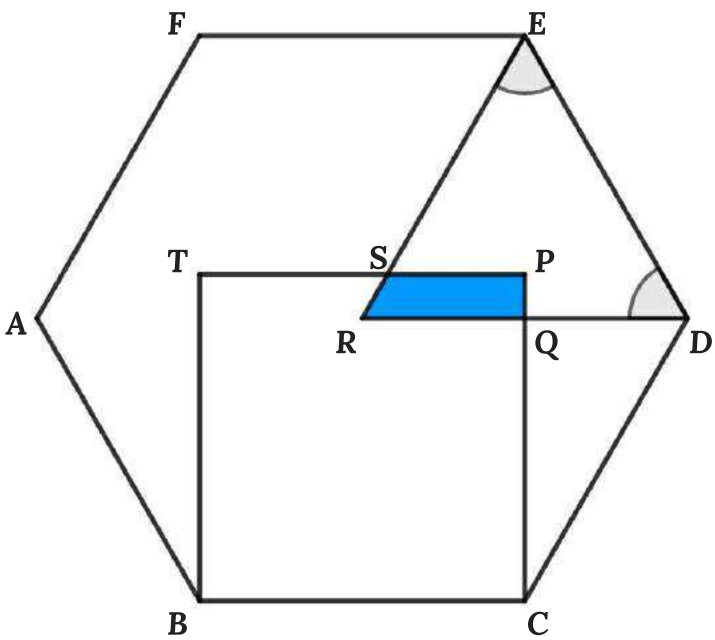
In figure

- AB = 24 cm, BC = 18 cm & AC = 30 cm
- AC is tangent of semicircles
- S & R are centres of the semicircles

Find the ***area of the blue rectangle***

{*Solution: Page 048*}

QUESTION 17



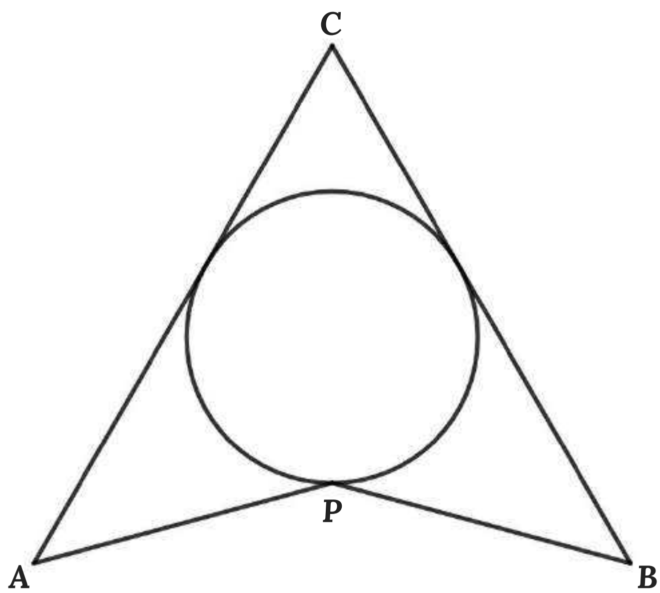
In figure

- ABCDEF is a regular hexagon
- BCPT is a square
- RDE is an equilateral triangle
- BC = 12 cm

Find the ***area of the blue region***

{*Solution: Page 050*}

QUESTION 18



In figure

- $\angle PAC = \angle PBC = 45°$
- AC = BC = 6 cm
- $\angle C = 60°$
- AC & BC are tangent of the circle

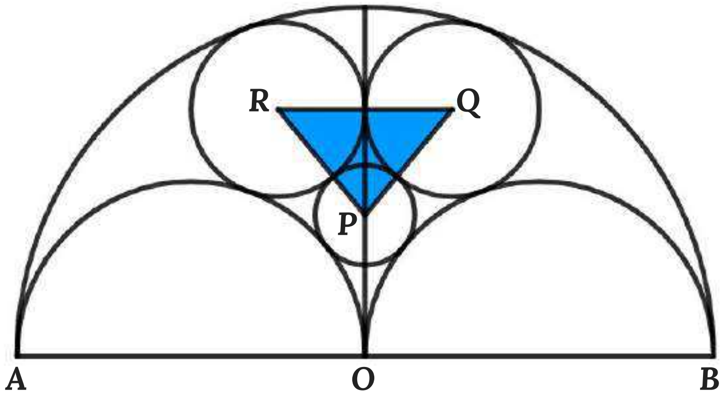
Find the ***radius of the circle***

{*Solution: Page 051*}

QUESTION 19

In figure

- O is the centre of a semicircle
- P, Q & R are centres of corresponding circles
- AB = 56 cm



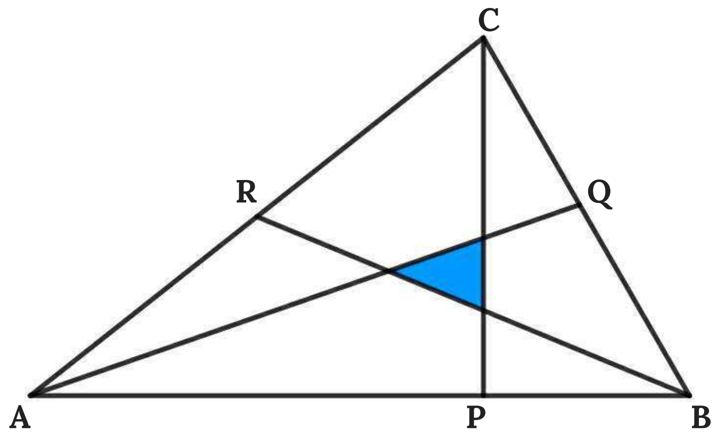
Find the ***area of the blue triangle***

{Solution: Page 053}

QUESTION 20

In figure

- AB = 16 cm, BC = 10 cm & AC = 14 cm
- RA = RC
- AB ⊥ PC
- AQ is angle bisector of ∠BAC



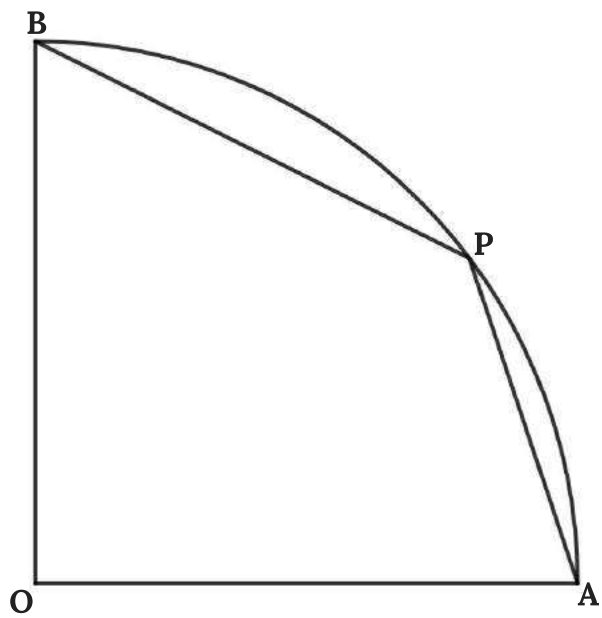
Find the ***area of the blue triangle***

{Solution: Page 056}

QUESTION 21

In figure

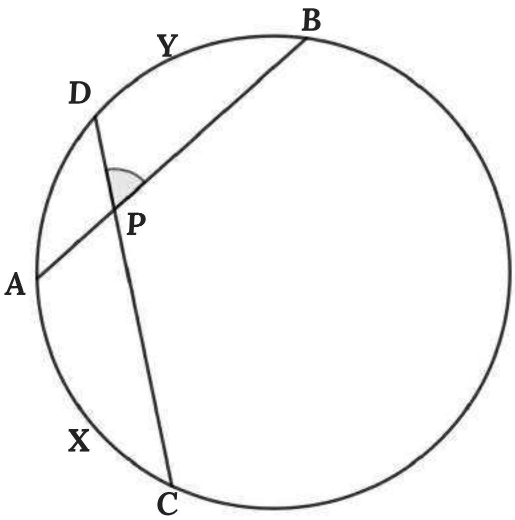
- A quarter circle
- PA = 1 cm & PB = $\sqrt{2}$ cm



Find the ***area of the quarter circle***

{Solution: Page 059}

QUESTION 22



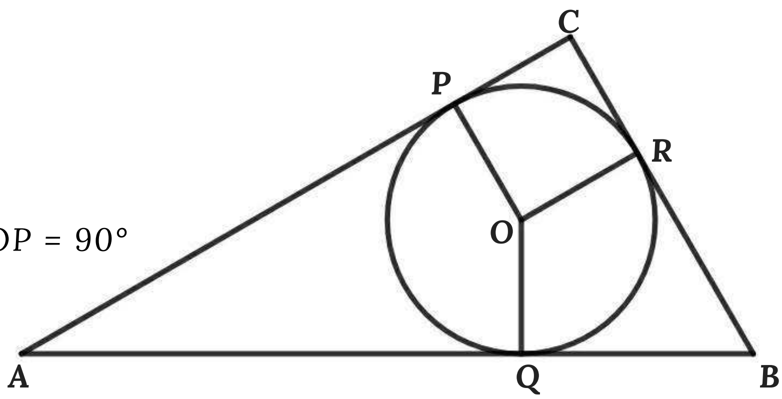
In figure

- $\angle BPD = 60°$
- Radius = 1 cm

Find the ***length of* (*arc*(AXC) + *arc*(BYD))**

{*Solution: Page 060*}

QUESTION 23



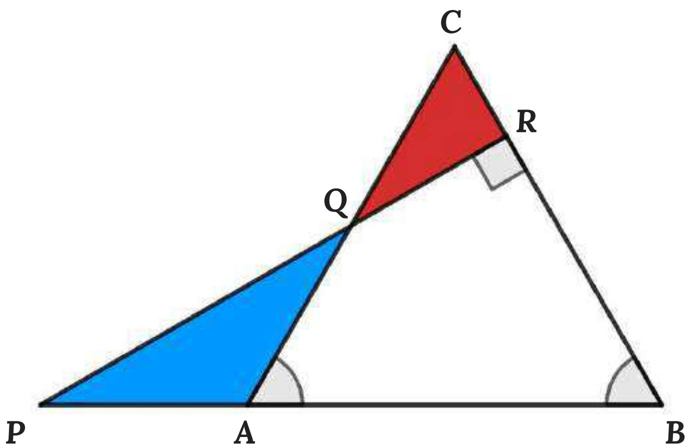
In figure

- Radius of incircle = 2 cm
- O is the centre of the circle
- $\angle POQ = 150°$, $\angle QOR = 120°$ & $\angle ROP = 90°$

Find the ***area of* ΔABC**

{*Solution: Page 061*}

QUESTION 24



In figure

- AP = CQ
- $\angle BAC = \angle ABC = 60°$
- $\angle PRB = 90°$

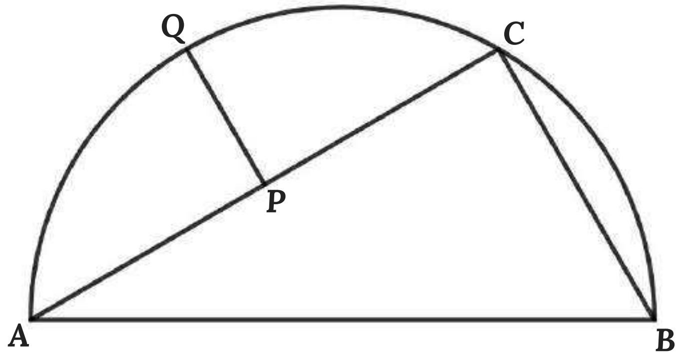
Find ***Blue area* : *Red area***

{*Solution: Page 063*}

QUESTION 25

In figure

- ACB is a semicircle
- PQ = 1 cm
- ∠ABC = 60°
- PA = PC



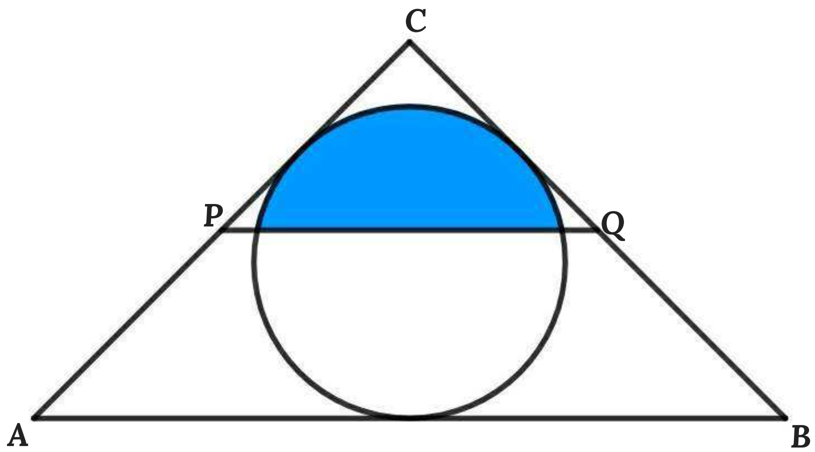
Find the ***radius of the semicircle***

{Solution: Page 065}

QUESTION 26

In figure

- AC ⊥ BC & AC = BC
- AB // PQ
- AB = 8 cm & PQ = 4 cm



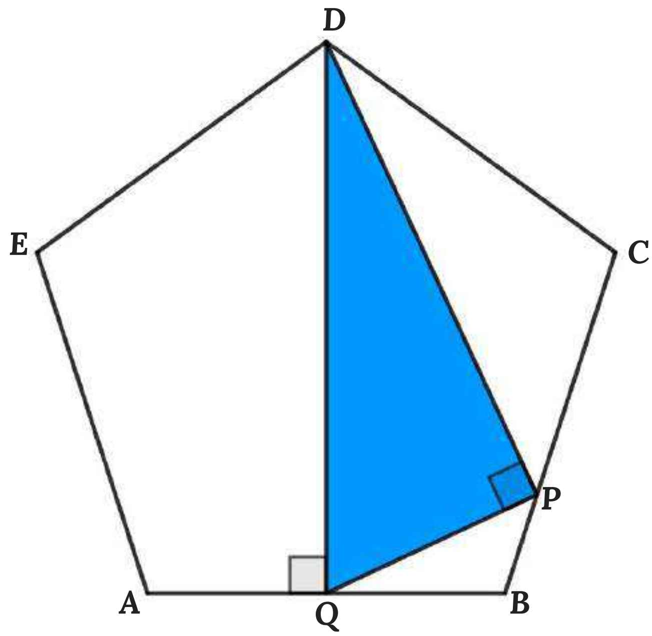
Find the ***area of the blue region***

{Solution: Page 067}

QUESTION 27

In figure

- ABCDE is a regular pentagon
- AB = 10 cm
- AB ⊥ QC



Find the ***area of the triangle***

{Solution: Page 070}

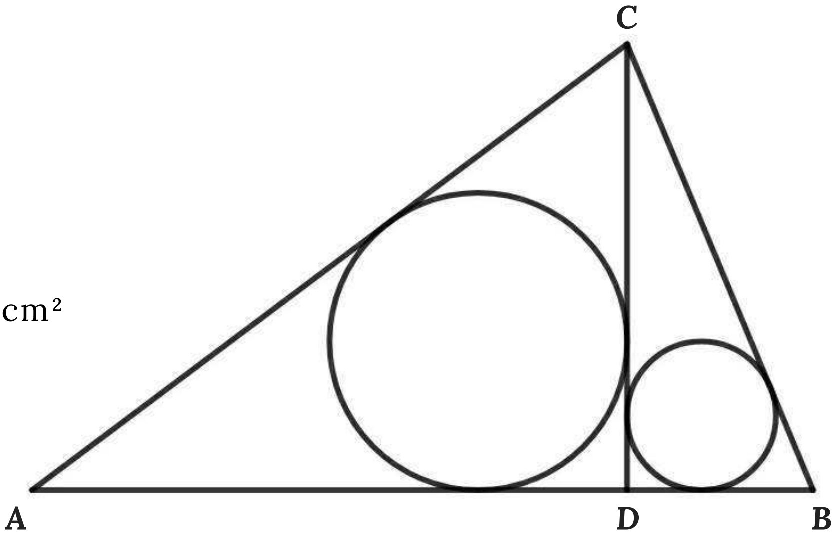
QUESTION 28

In figure

- All lines are tangents
- AB ⊥ CD
- Area of circles are 4π cm² & π cm²
- CD = 6 cm

Find the ***area of* ΔABC**

{*Solution: Page* 073}



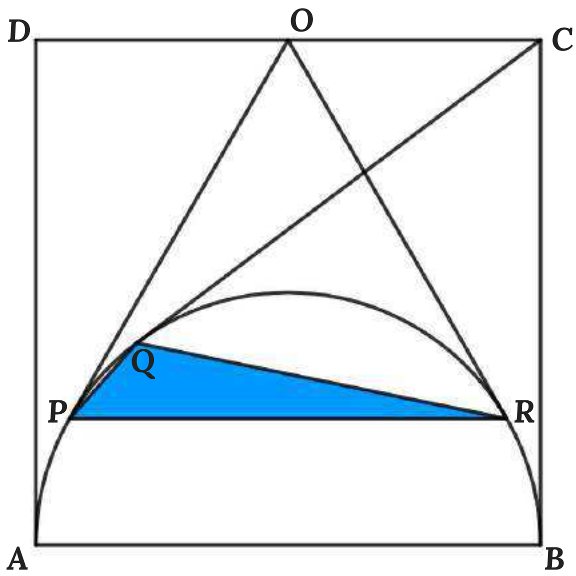
QUESTION 29

In figure

- ABCD is a square
- AB = 60 cm
- OC = OD
- OP, OR & CQ are tangent of a semicircle

Find the ***area of* ΔPQR**

{*Solution: Page* 075}



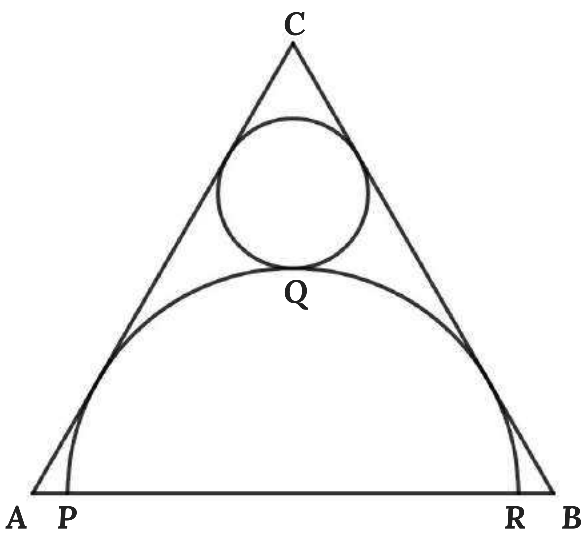
QUESTION 30

In figure

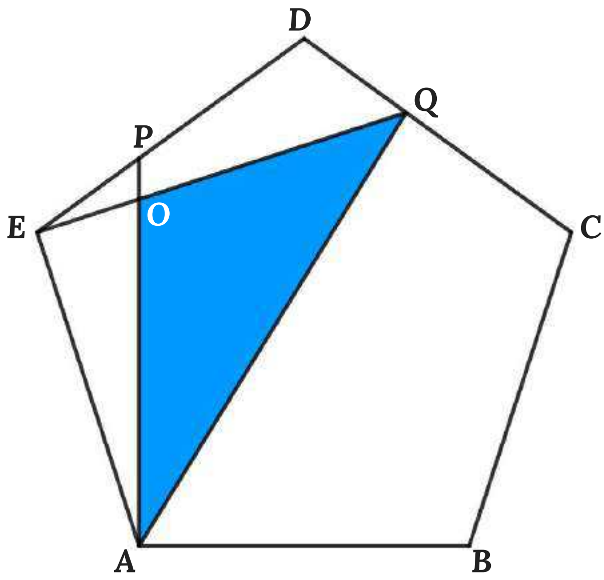
- ∠A = ∠B = ∠C = 60°
- AB = 12 cm
- AB & BC are tangents of circle and semicircle

Find the ***area of the circle***

{*Solution: Page* 077}



QUESTION 31



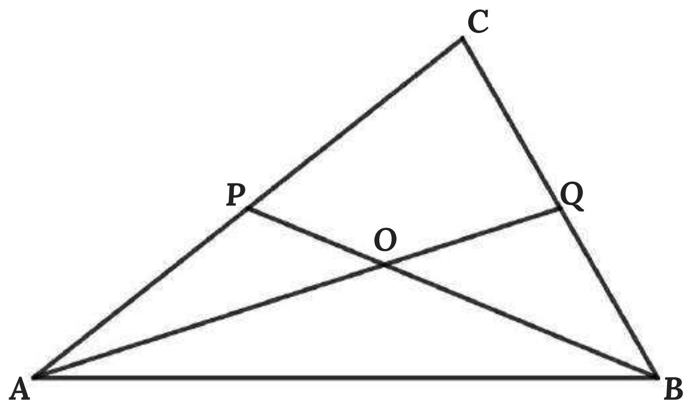
In figure

- ABCDE is a regular pentagon
- AB ⊥ AP & EA ⊥ EQ
- AB = 2 cm

Find the ***area of* ΔAOQ**

{*Solution: Page* 079}

QUESTION 32



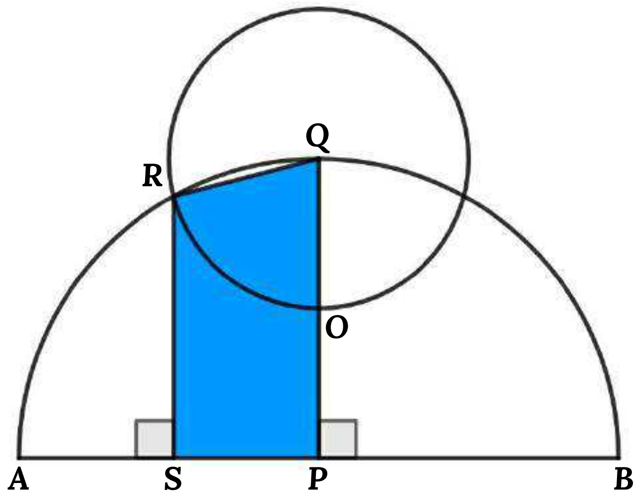
In figure

- AB = 16 cm, AC = 14 cm & BC = 10 cm
- AP = PC
- ∠QAC = ∠QAB

Find cos ∠AOB

{*Solution: Page* 081}

QUESTION 33



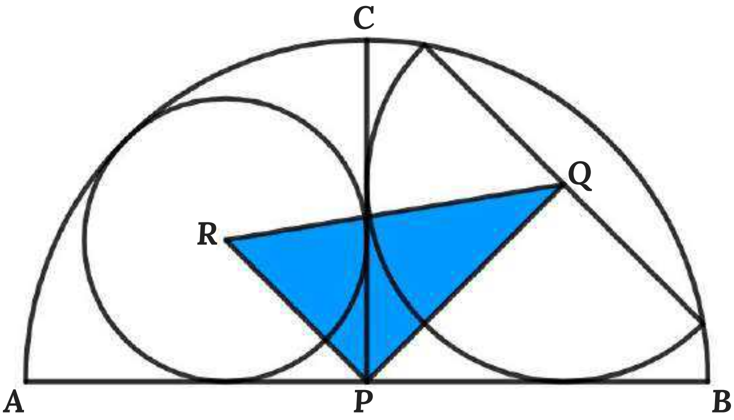
In figure

- APB is a semicircle with centre at P
- OP = OQ
- The radius of the semicircle = 16 cm

Find the ***area of* PQRS**

{*Solution: Page* 083}

QUESTION 34



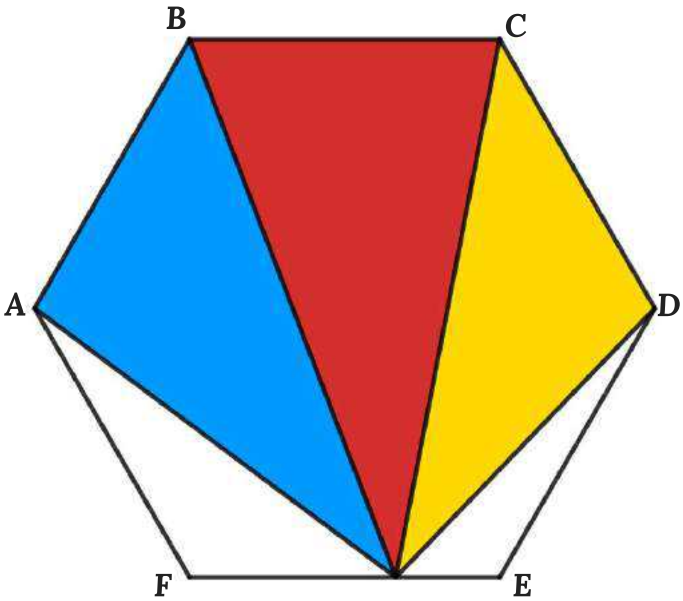
In figure

- P & Q are centres of the semicircles
- R is the centre of the circle
- AB ⊥ PC
- AB = 6 cm

Find the ***area of ΔPQR***

{*Solution: Page* 085}

QUESTION 35



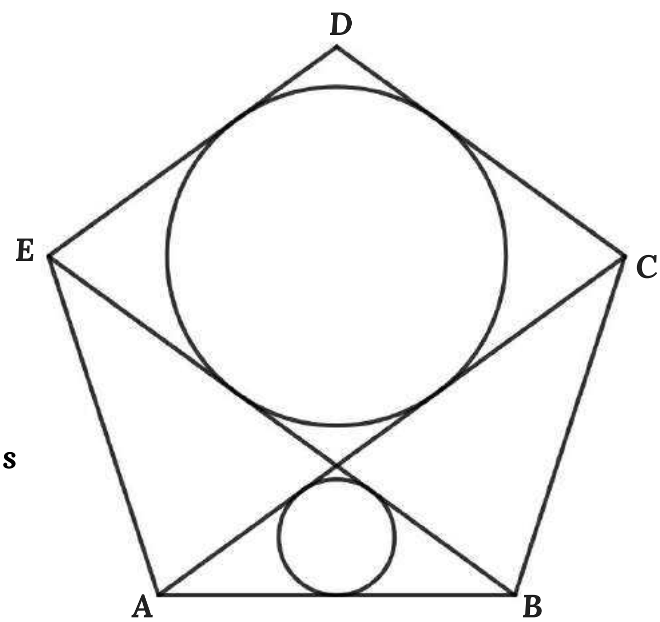
In figure

- ABCDEF is a regular hexagon
- Red Area = 64 cm^2
- Yellow Area = 42 cm^2

Find the ***blue area***

{*Solution: Page* 087}

QUESTION 36



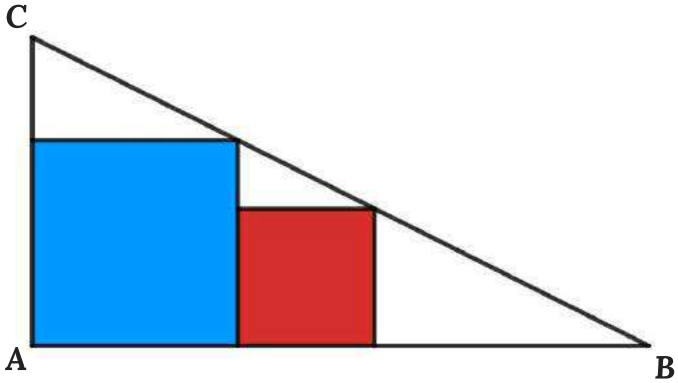
In figure

- ABCDE is a regular pentagon
- AC & BE are diagonals

Find the ***relation between the radius of circles***

{*Solution: Page* 089}

QUESTION 37



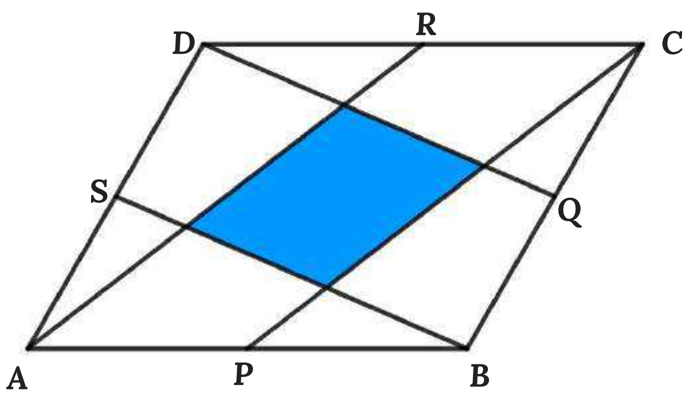
In figure

- Area of red square = 16 cm^2
- Area of blue square = 36 cm^2
- $\angle BAC = 90°$

Find the ***area of*** **ΔABC**

{*Solution: Page* 092}

QUESTION 38



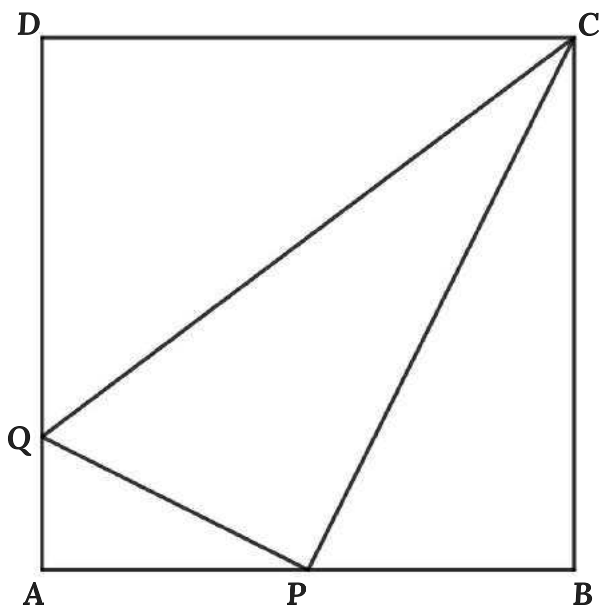
In figure

- ABCD is a parallelogram
- AB = BC = CD = DA = 10 cm
- $\angle BAD = 60°$
- P, Q, R, S are midpoints

Find the area of the ***blue parallelogram***

{*Solution: Page* 094}

QUESTION 39



In figure

- ABCD is a square
- $\angle CPQ = 90°$
- PC = 2 cm & PQ = 1 cm

Find the ***area of the square***

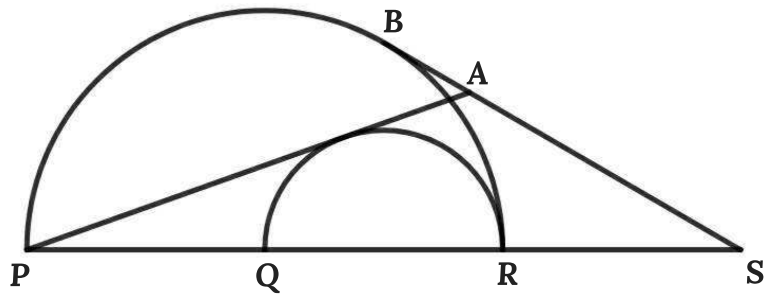
{*Solution: Page* 096}

QUESTION 40

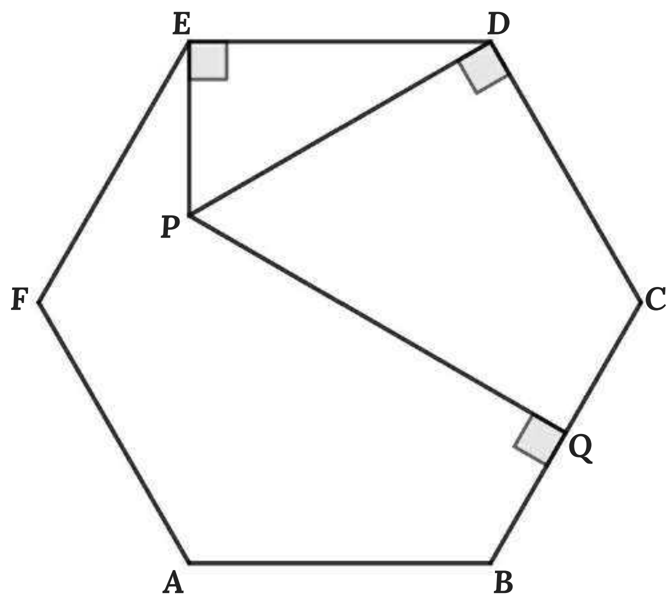
In figure

- Two semicircles
- PQ = QR = RS = 10 cm
- AP & BS are tangents

Find the ***length of AB***
{Solution: Page 098}



QUESTION 41

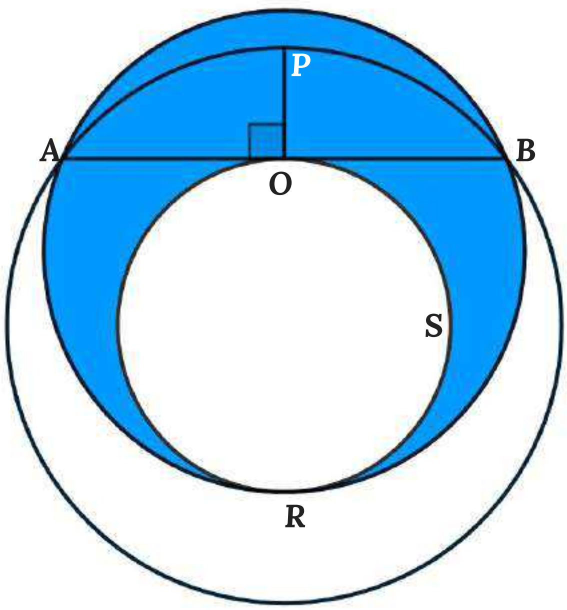


In figure

- ABCDE is a regular hexagon
- PQ ⊥ BC, PD ⊥ CD & PE ⊥ DE

Find **PQ : PD : PE**
{Solution: Page 100}

QUESTION 42

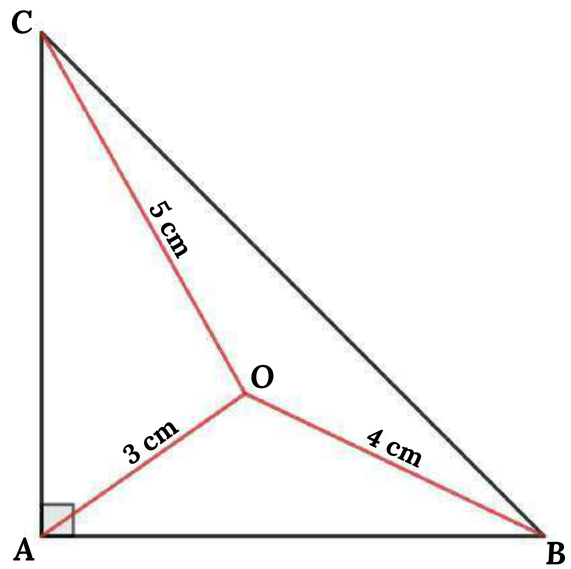


In figure

- AB ⊥ OP
- circle ORS & APB are concentric circles
- AB = 24 cm
- OP = 6 cm

Find the ***area of the blue region***
{Solution: Page 102}

QUESTION 43



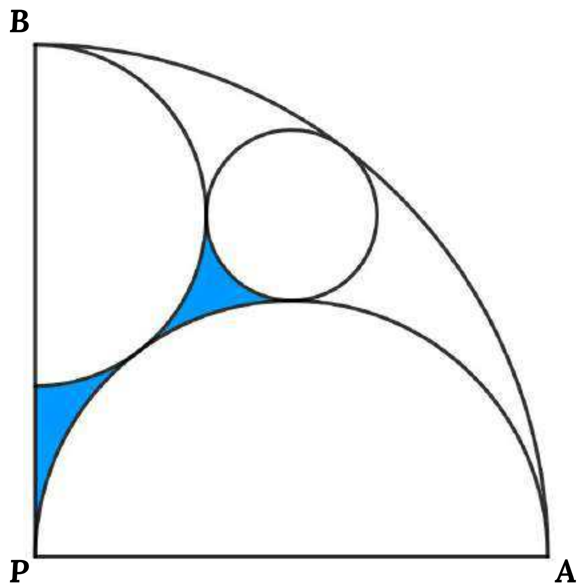
In figure

- AB ⊥ AC
- AB = AC
- OA = 3 cm, OB = 4 cm & OC = 5 cm

Find the ***area of ΔABC***

{Solution: Page 104}

QUESTION 44



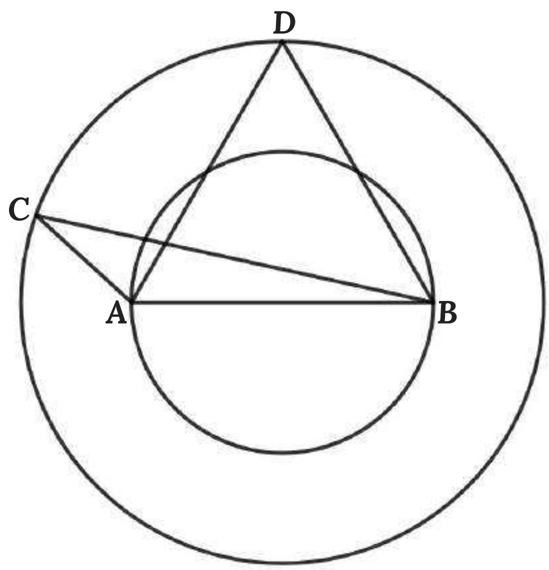
In figure

- A quarter-circle, two semicircles & a circle
- AO = 12 cm

Find the ***area of the blue region***

{Solution: Page 106}

QUESTION 45



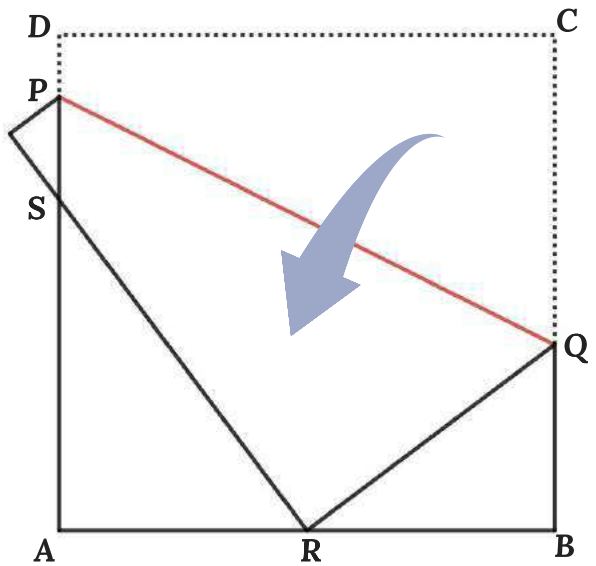
In figure

- Both circles are concentric
- ΔABC is an equilateral triangle
- AB = 6 cm
- ∠ACB = 30°
- AB is the diameter of the smaller circle

Find the ***area of ΔABC***

{Solution: Page 110}

QUESTION 46



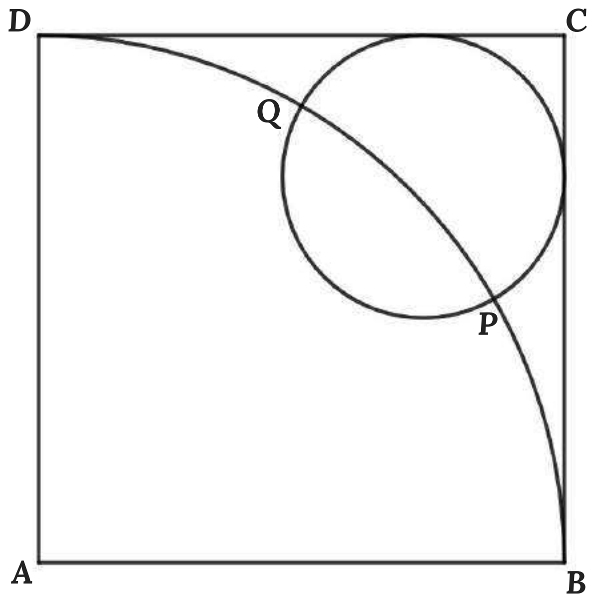
In figure

- ABCD is a square folded through PQ
- AB = 24 cm
- R is the midpoint of AB

Find the ***area of PQRS & length of PQ***

{*Solution: Page 113*}

QUESTION 47



In figure

- ABCD is a square
- BP = PQ = QD
- AB = 4 cm

Find the ***radius of the circle***

{*Solution: Page 116*}

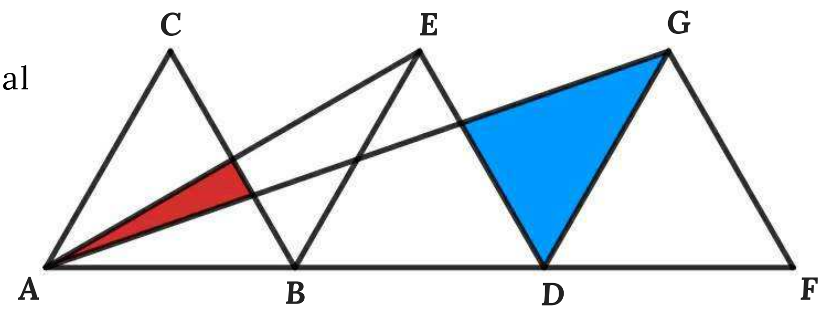
QUESTION 48

In figure

- ΔABC, ΔBDE & DFG are equilatral triangles
- AB = BD = DF

Find ***Red area : Blue area***

{*Solution: Page 118*}



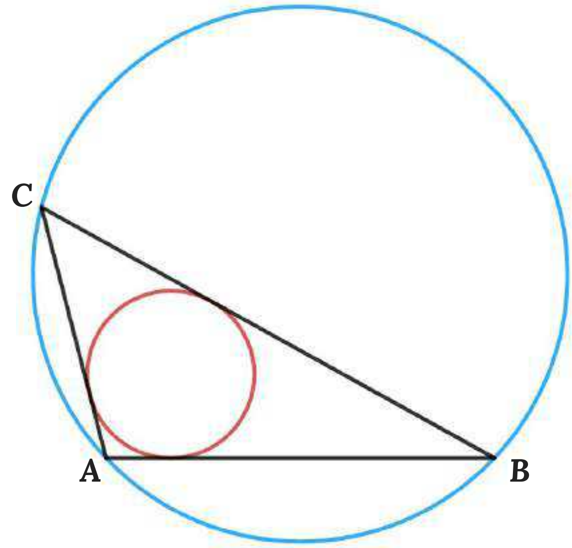
QUESTION 49

In figure

- Blue circle is the circumcircle of ΔABC
- Red circle is the incircle of ΔABC
- AB/BC = ¾ & AC/BC = ½

Find the ***relation between the radius of circles***

{*Solution: Page 120*}



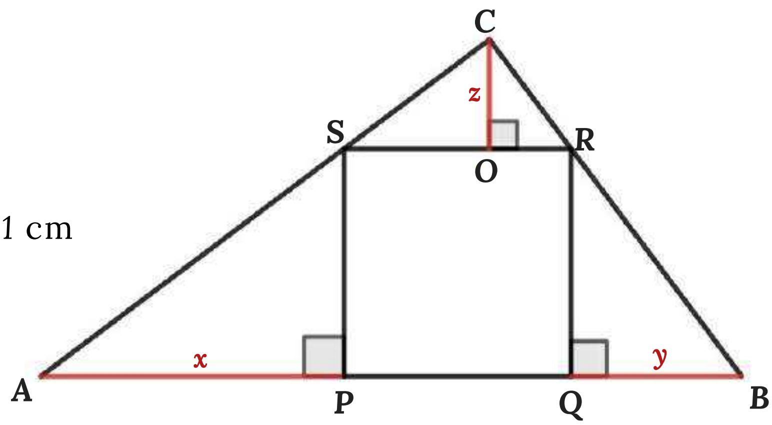
QUESTION 50

In figure

- PQRS is a square
- AB = 185 cm, AC = 148 cm & BC = 111 cm

Find the value of ***x+y+z***

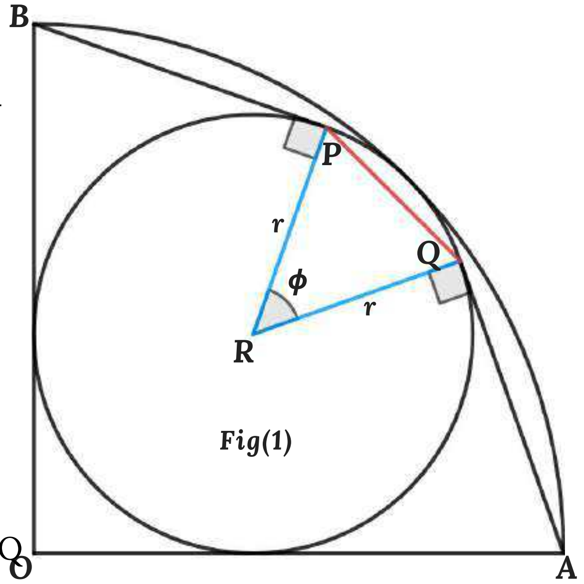
{*Solution: Page 122*}



Solutions

SOLUTION 01

Let $\angle PRQ = \phi$ & r is the radius of the circle



Fig(1)

From ΔPQR

$PQ^2 = PR^2 + QR^2 - 2 \times PR \times QR \times \cos\phi$ {*Cosine rule*}

$\Rightarrow PQ^2 = r^2 + r^2 - 2 \times r \times r \times \cos\phi$

$= 2r^2 - 2r^2 \cos\phi$

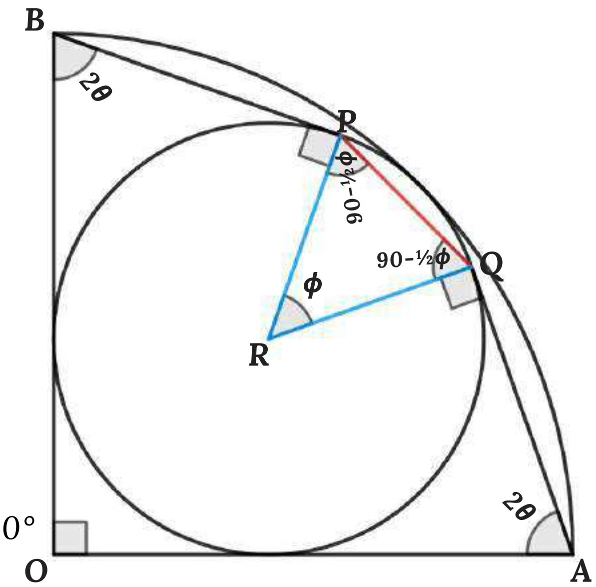
$= 2r^2(1 - \cos\phi)$

$= 2r^2 \times 2\sin^2(\tfrac{1}{2}\phi)$

$= 4r^2 \sin^2(\tfrac{1}{2}\phi)$

$\Rightarrow$ $\mathbf{PQ = 2r\sin(\tfrac{1}{2}\phi)}$

We need to find $\sin(\tfrac{1}{2}\phi)$ & r to find the length PQ



Fig(2)

From *Fig(2)*

Let $\angle PBO = 2\theta$ then $\angle QAO = 2\theta$ {*symmetry*}

$\angle BPR = 90°$ {*PB is a tangent*}

$\angle AQR = 90°$ {*AQ is a tangent*}

$\angle RQP = \angle RPQ = \tfrac{1}{2}(180-\phi)$

$\Rightarrow$ $\mathbf{\angle RQP = \angle RPQ = 90-\tfrac{1}{2}\phi}$

OAQPB is a pentagon, so the sum of angles is 540°

That is

$\angle AOB + \angle QAO + \angle AQP + \angle QPB + \angle PBO = 540$

$\Rightarrow 90 + 2\theta + (90+\tfrac{1}{2}(180-\phi)) + (90+\tfrac{1}{2}(180-\phi)) + 2\theta = 540$

$\Rightarrow 4\theta - \phi = 90$

$\Rightarrow$ $\boldsymbol{\phi = 4\theta - 90}$

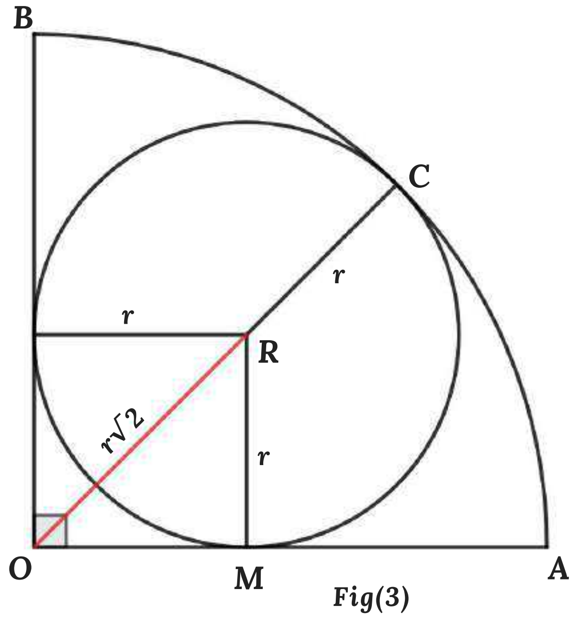
From Fig(3)

OA = OB = OC = 3 cm

$OR^2 = OM^2 + RM^2$

⇒ $OR^2 = r^2 + r^2$

⇒ $OR = r\sqrt{2}$



Fig(3)

OC = OR + RC

⇒ $3 = r\sqrt{2} + r$

⇒ $r = 3/(\sqrt{2} + 1)$

⇒ $\mathbf{r = 3\sqrt{2} - 3}$ **cm**

From Fig(4)

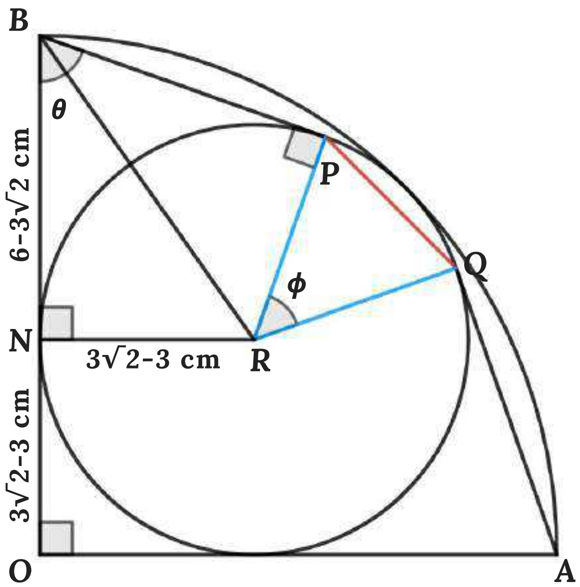
BN = OB - ON

⇒ $BN = 3 - (3\sqrt{2} - 3)$

⇒ $\mathbf{BN = 6-3\sqrt{2}}$ **cm**

We Know $\angle PBO = 2\theta$

⇒ $\angle NBR = \angle PBR = \theta$ {symmetry}



Fig(4)

$\tan\theta = NR/BN$

⇒ $\tan\theta = (3\sqrt{2}-3)/(6-3\sqrt{2})$

⇒ $\mathbf{\tan\theta = \tfrac{1}{2}\sqrt{2}}$

$\tan 2\theta = (2\tan\theta)/(1-\tan^2\theta)$

⇒ $\tan 2\theta = (2\times\tfrac{1}{2}\sqrt{2})/(1-(\tfrac{1}{2}\sqrt{2})^2)$

$= 2\sqrt{2}$

⇒ $\mathbf{\sin 2\theta = \tfrac{2}{3}\sqrt{2}}$ & $\mathbf{\cos 2\theta = \tfrac{1}{3}}$

$\phi = 4\theta - 90$

⇒ $\sin(\tfrac{1}{2}\phi) = \sin(2\theta - 45)$

$= \sin 2\theta \cos 45 - \cos 2\theta \sin 45$

$= \tfrac{2}{3}\sqrt{2} \times \tfrac{1}{2}\sqrt{2} - \tfrac{1}{3} \times \tfrac{1}{2}\sqrt{2}$

⇒ $\mathbf{\sin(\tfrac{1}{2}\phi) = \tfrac{1}{6}(4-\sqrt{2})}$

$PQ = 2r\sin(\tfrac{1}{2}\phi)$

⇒ $PQ = 2(3\sqrt{2}-3)\times\tfrac{1}{6}(4-\sqrt{2})$

⇒ $\mathbf{PQ = 5\sqrt{2}-6}$ **cm**

SOLUTION 02

From Figure

∠AOX = ∠AOX = 108° {ABCDE *is a regular pentagon*}

∠PAB = 108/2 {AP *is the angle bisector of* ∠BAE}

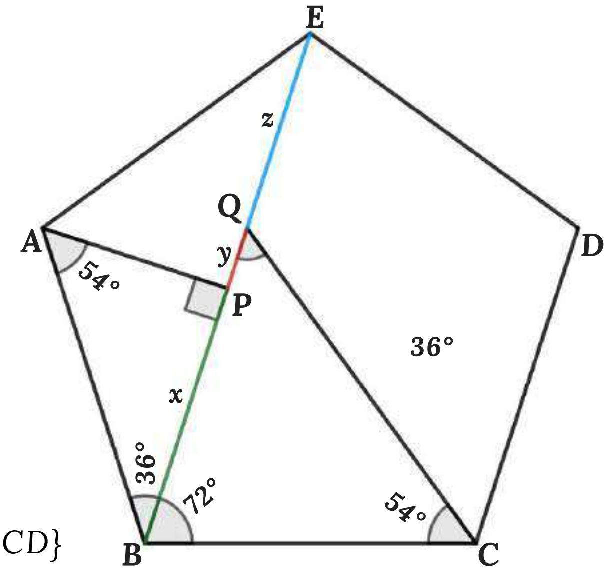
⇒ ∠PAB = 54°

∠APB = 90° {AB = AE & ∠PAB = ∠PAE}

∠PBA = 180 - (∠APB + ∠PAB)

⇒ ∠PBA = 180 - (90 + 54)

⇒ ∠PBA = 36°



Let assume u is the sides of the pentagon, then

AB = u

PB = AB sin 54

⇒ $x = u \sin 54$

⇒ $\mathbf{x = \frac{1}{4}u(1+\sqrt{5})}$

From ΔBCQ

∠BCQ = 108/2 {CQ *is the angle bisector of* ∠BCD}

⇒ ∠BCQ = 54°

∠CBQ = ∠ABC - ∠ABQ

⇒ ∠BCQ = 108 - 36

⇒ **∠BCQ = 72°**

∠BQC = 180 - (∠CBQ + ∠BCQ)

⇒ ∠BQC = 180 - (72 + 54)

⇒ ∠PBA = 54°

⇒ ∠PBA = ∠BCQ

⇒ **BQ = BC = u**

From Figure

BQ = PB + PQ

⇒ $u = x + y$

⇒ $y = u - x$

$= u - \frac{1}{4}u(1+\sqrt{5})$

⇒ $\mathbf{y = \frac{1}{4}u(3-\sqrt{5})}$

From Figure

PB = PE {AB = AE & ∠PAB = ∠PAE}

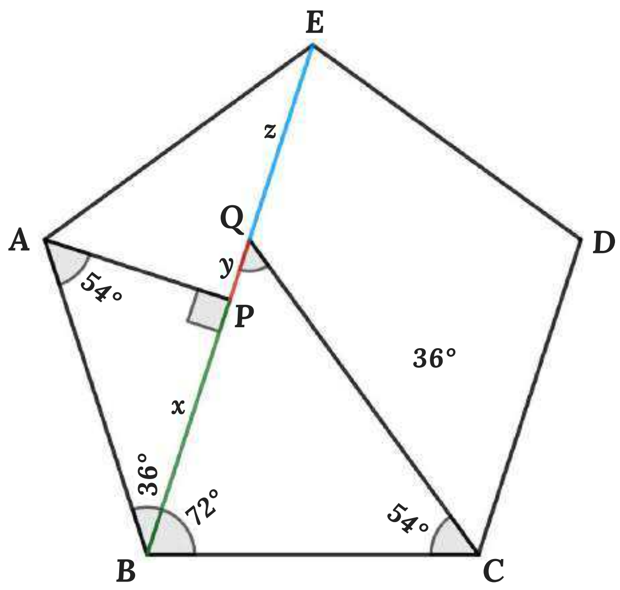
⇒ $x = y+z$

⇒ $z = x - y$

$= \frac{1}{4}u(1+\sqrt{5}) - \frac{1}{4}u(3-\sqrt{5})$

$= \frac{1}{4}u[1+\sqrt{5}-3+\sqrt{5}]$

⇒ $\mathbf{z = \frac{1}{4}u(2\sqrt{5} - 2)}$



$x : y : z = \frac{1}{4}u(1+\sqrt{5}) : \frac{1}{4}u(3-\sqrt{5}) : \frac{1}{4}u(2\sqrt{5} - 2)$

⇒ $x : y : z = 1+\sqrt{5} : 3-\sqrt{5} : 2\sqrt{5}-2$

⇒ $\mathbf{x : y : z = 1 : \sqrt{5}-2 : 3-\sqrt{5}}$

SOLUTION 03

Let AD = BC = x & AB = CD = y, then

Area of rectangle = xy

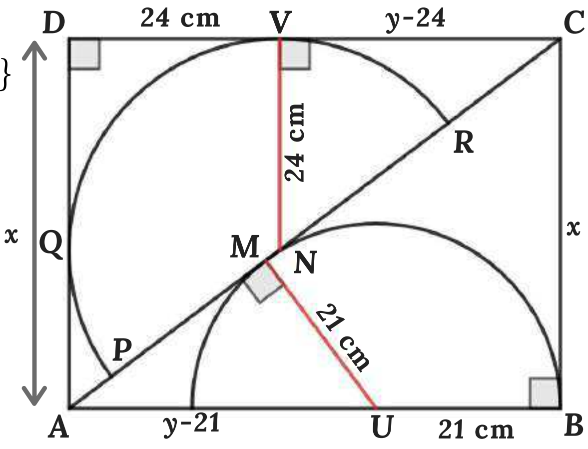
From ΔACD & NCV

∠ADC = ∠NVC = 90°

∠ACD = ∠NCV {*common angles of* ΔACD & NCV}

⇒ ∠CAD = ∠CNV

⇒ ΔACD & ΔMCV are similar triangles



that is, **AD/VN = AC/NC = CD/VC**

⇒ $x/24$ = AC/NC = $y/(y-24)$

⇒ $x/24 = y/(y-24)$

⇒ $x(y-24) = 24y$

⇒ $xy - 24x = 24y$

⇒ $24x+24y = xy$

⇒ $\mathbf{xy = 24x+24y}$......................................*eq*(1)

From ΔABC & AMU

∠ABC = ∠AMU = 90°

∠CAB = ∠MAU {*common angles of* ΔABC & AMU}

⇒ ∠ACB = ∠AUM

⇒ ΔABC & AMU are similar triangles

that is, **BC/MU = AB/AM = AC/AU**

⇒ $x/21$ = AB/AM = AC/$(y-21)$

⇒ $x/21$ = AC/$(y-21)$

⇒ $x(y-21)$ = 21AC

⇒ $xy-21x$ = 21AC

⇒ $\mathbf{21 = xy/(x+AC)}$...............................*eq*(2)

From ΔABC

$AC^2 = AB^2 + BC^2$

⇒ $AC^2 = y^2+x^2$

⇒ $\mathbf{AC = \sqrt{(x^2+y^2)}}$................................*eq*(3)

From $eq(2)$ & $eq(2)$

$21 = xy/(21x+21AC)$

⇒ $21 = xy/(x+\sqrt{(x^2+y^2)})$

⇒ $\mathbf{xy = 21x + 21\sqrt{(x^2+y^2)}}$....................$eq(4)$

From $eq(1)$ & $eq(4)$

$xy = 24x+24y = 21x + 21\sqrt{(x^2+y^2)}$

⇒ $3x+24y = 21\sqrt{(x^2+y^2)}$

⇒ $x+8y = 7\sqrt{(x^2+y^2)}$

$(x+8y)^2 = (7\sqrt{(x^2+y^2)})^2$

⇒ $x^2 + 16xy + 64y^2 = 49(x^2+y^2)$

$= 49x^2 + 49y^2$

⇒ $16xy + 15y^2 = 48x^2$

⇒ $48x^2 - 16xy - 15y^2 = 0$

⇒ $x = (1/2\times48)(16y\pm\sqrt{((-16y)^2 - 4\times48(-15y^2))})$

$= (^1/_{96})(16y\pm\sqrt{(3136y^2)})$

$= (^1/_{96})(16y\pm56y)$

$= (^1/_{96})(16y+56y)$ {*sides of a rectangle are always positive*}

$= (^1/_{96})72y$

⇒ $\mathbf{x = ¾y}$

From $eq(1)$

$xy = 24x+24y$

⇒ $¾y\times y = 24\times¾y + 24y$

⇒ $¾y^2 = 18y + 24y$

$= 42y$

⇒ $¾y = 42$

⇒ $\mathbf{y = 56\ cm}$

$x = ¾y$

⇒ $\mathbf{x = 42\ cm}$

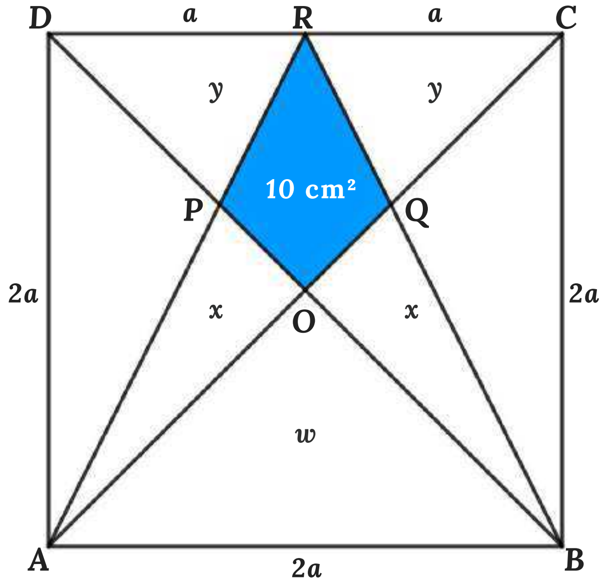
Area of rectangle $= xy$

⇒ $xy = 42\times56$

⇒ $xy = 2352\ cm^2$

⇒ ***Area of rectangle = 2352 cm²***

SOLUTION 04



From Figure

Blue Area = 10 cm^2

Area of square = $4a^2$

Area of ΔAOB = w

Area of ΔAOP = x

Area of ΔBOQ = x

Area of ΔPDR = y

Area of ΔQCR = y

ΔAOB isosceles triangle with ∠AOB = 90° *{AC & BD are diagonals of the square}*

$w = \frac{1}{2}AO^2$

⇒ $w = \frac{1}{2}(a\sqrt{2})^2$

⇒ $w = a^2$

⇒ **Area of square = $4w$**

Area of ΔAOB = Area of ΔCOD *{symmetry}*

⇒ $w = 2y + 10$

⇒ **$y = \frac{1}{2}(w-10)$**.......................................*eq(3)*

From ΔABR

Area of ΔABR = $\frac{1}{2}\times 2a \times 2a$

⇒ $w+2x+10 = 2a^2$

⇒ $w+2x+10 = 2w$

⇒ $2x+10 = w$

⇒ **$x = \frac{1}{2}(w-10)$**.......................................*eq(3)*

From ΔAQB & ΔCQR

∠AQB = ∠CQR *{Opposite angles}*

∠BAQ = ∠RCQ *{AB ⫽ RC}*

∠ABQ = ∠CRQ *{AB ⫽ RC}*

⇒ ΔAQB & ΔCQR are similar triangles

so, Area of $\Delta AQB = (AB/RC)^2 \times$ *Area of* ΔAQB

⇒ $w+x = (2a/a)^2\, y$

⇒ $\mathbf{4y = w+x}$...$eq(3)$

From $eq(1)$, $eq(2)$ & $eq(3)$

⇒ $4y = w+x$

⇒ $4(½(w-10)) = w + ½(w-10)$

⇒ $2w - 20 = w + ½w - 5$

Multiply both sides with 2

⇒ $4w - 40 = 2w + w - 10$

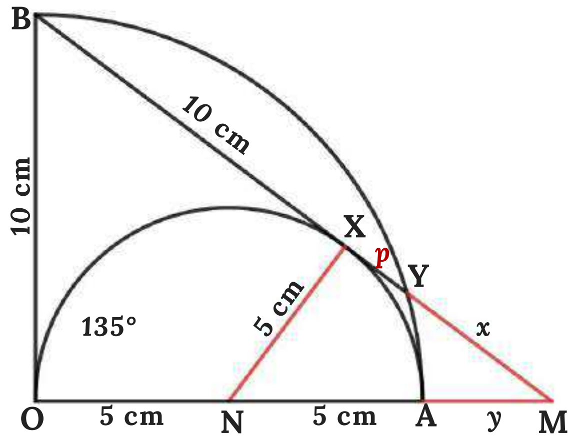
⇒ $\mathbf{w = 30}$

Area of the square = $4w$

⇒ Area of the square = 4×30

⇒ ***Area of the square = 120 cm²***

SOLUTION 05



From ΔOBM & ΔNXM

∠O = ∠NXM = 90°

∠OMB = ∠NMX

Then ∠OBM = ∠MNX

So ΔOBM & ΔNXM are similar triangles

OB = BX = 10 cm {BO & BX *are tangents*}

Let MY = x, AM = y & XY = p

From ΔOBM & ΔNXM

OB/NX = BM/NM = OM/XM

10/5 = (10+p+x)/(5+y) = (10+y)/(p+x)

(10+y)/(p+x) = 10/5 = 2

⇒ **2(p+x) = 10+y**...*eq*(1)

(10+p+x)/(5+y) = 10/5 = 2

⇒ **p+x = 2y**...*eq*(2)

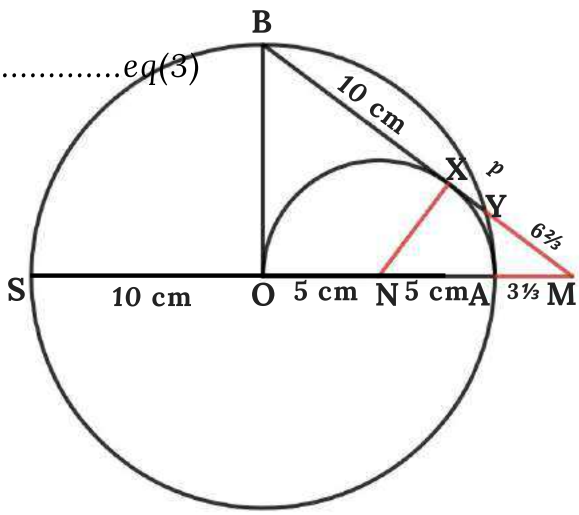
From *eq*(1) & *eq*(2)

4y = 10+y

⇒ **y = 3⅓ cm**

From *eq*(1)

p+x = 2(3⅓) = 6⅔ cm..*eq*(3)



From Figure

MA×MS = MY×MB

y(20+y) = x(10+p+x)

⇒ 3⅓ (20+3⅓) = x(10+6⅔)

⇒ **x = 4⅔ cm**

From *eq*(3)

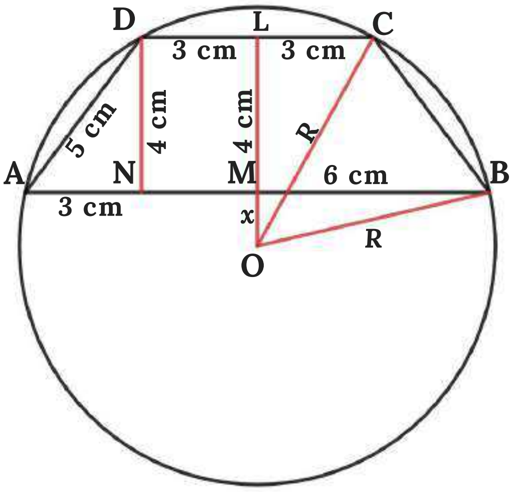
p+x = 6⅔

⇒ p+4⅔ = 6⅔

⇒ p = 2 cm

⇒ **XY = 2 cm**

SOLUTION 06



Let the radius of the circle = R

From ΔADN

$DN^2 = AD^2 - AN^2$

⇒ $DN^2 = 5^2 - 3^2$

$= 25 - 9$

$= 16$

⇒ DN = 4 cm

⇒ **LM = DN = 4 cm**

From ΔOBM

$OB^2 = OM^2 + BM^2$

⇒ $OB^2 = x^2 + 6^2$

⇒ $\mathbf{R^2 = x^2 + 36}$..*eq*(1)

From ΔOLC

$OC^2 = OL^2 + CL^2$

⇒ $OC^2 = (4+x)^2 + 3^2$

⇒ $\mathbf{R^2 = x^2 + 8x + 25}$..*eq*(2)

From *eq*(1) & *eq*(2)

$x^2 + 36 = x^2 + 8x + 25$

⇒ $8x = 11$

⇒ $\mathbf{x = 11/8}$ cm

From *eq*(1)

$R^2 = x^2 + 36$

⇒ $R^2 = (11/8)^2 + 36$

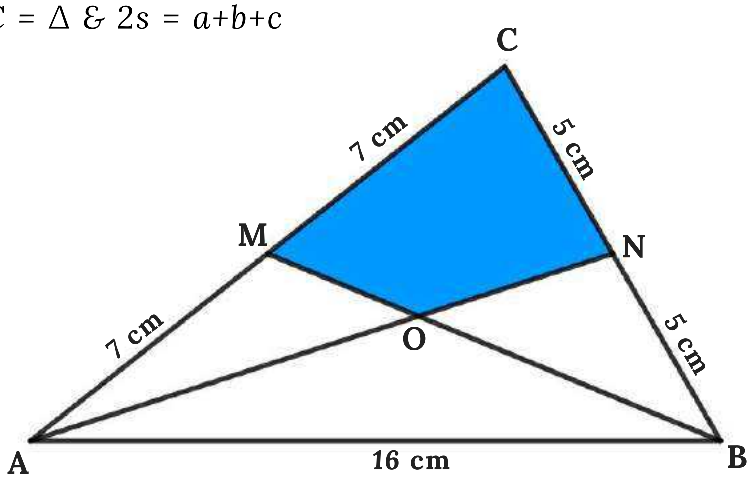
$= 2425/64$

⇒ $R = \frac{5}{8}\sqrt{97}$ cm

⇒ **Radius = $\frac{5}{8}\sqrt{97}$ cm**

SOLUTION 07

Let BC = a, AC = b, AB = c, *Area of* $\Delta ABC = \Delta$ & $2s = a+b+c$



$2s = 10+14+16 = 40$

⇒ $s = 20$

$\Delta^2 = s(s-a)(s-b)(s-c)$ {*heron's formula*}

⇒ $\Delta^2 = 20(20-10)(20-14)(20-16)$

$= 20\times10\times6\times4$

$= 4800$

⇒ $\mathbf{\Delta = 40\sqrt{3}\ cm^2}$

From ΔABN & ΔACN

BN = CN & height of both triangles are the same

then the area of triangles are the same

⇒ Area of ΔABN = Area of ΔACN

$= \frac{1}{2}\times40\sqrt{3}$

⇒ **Area of ΔABN = $20\sqrt{3}$ cm²**

From ΔAOB & ΔBON

AO : ON = 2 : 1 {BM & AN *are median of the triangle*}

Height of triangles is OB, then the area of triangles are in 2:1 ratio

From the figure, Area of ΔABN = Area of ΔAOB + Area of ΔBON

⇒ Area of ΔAOB + Area of ΔBON = $20\sqrt{3}$ cm²

Area of ΔBON = $\frac{1}{3}\times 20\sqrt{3}$

⇒ **Area of ΔBON = $20/\sqrt{3}$ cm²**

Area of ΔAOB = $\frac{2}{3}\times 20\sqrt{3}$

⇒ **Area of ΔAOB = $40/\sqrt{3}$ cm²**

From ΔAOB & ΔAOM

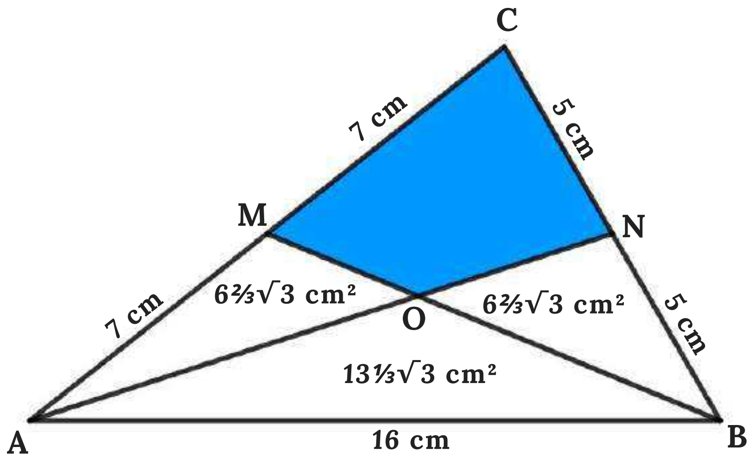
BO : MO = 2 : 1 {BM & AN *are median of the triangle*}

Height of triangles is OA, then the area of triangles are in 2:1 ratio

⇒ Area of ΔAOM = ½(Area of ΔAOB)

$= \frac{1}{2}\times 40/\sqrt{3}$

⇒ **Area of ΔAOM = $20/\sqrt{3}$ cm²**



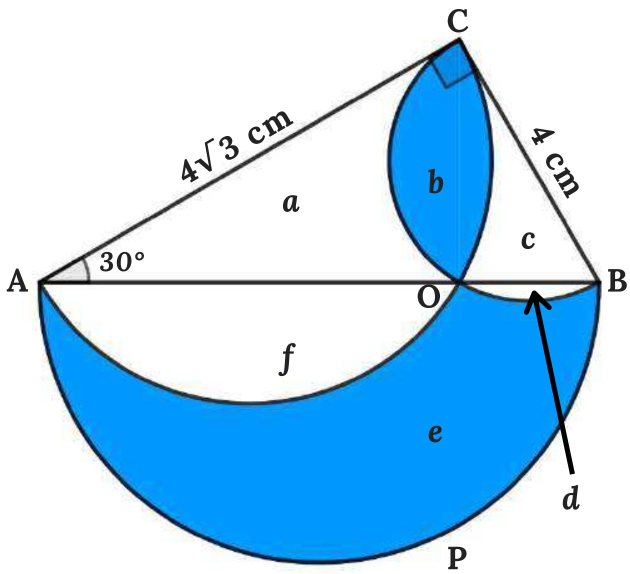
Area of Quadrilateral = Δ - Area of ΔAOM - Area of ΔAOB - Area of ΔBON

$= 40\sqrt{3} - 20/\sqrt{3} - 40/\sqrt{3} - 20/\sqrt{3}$

⇒ **Area of Quadrilateral = $40/\sqrt{3}$ cm²**

SOLUTION 08

We know the figure is divided into 6 parts,
Let the area of each part are ***a*, *b*, *c*, *d*, *e* & *f***



Fig(1)

From *Fig*(1)

Blue Area = *b*+*e*

We know AB = 8 cm, so

sin 30 = BC/AB

⇒ **BC** = AB sin 30 = 8×½ = **4 cm**

cos 30 = AC/AB

⇒ **AC** = AB cos 30 = 8×½√3 = **4√3 cm**

Area of ΔABC = ½×BC×AC = ½×4×4√3 = $8\sqrt{3}$ cm²

⇒ ***a*+*b*+*c* = 8√3 cm²**...*eq*(1)

Area of Semicircle AOC = ½π(½AC)² = ½π(2√3)² = 6π cm²

⇒ ***a*+*b*+*f* = 6π cm²**...*eq*(2)

Area of Semicircle BOC = ½π(½BC)² = ½π(2)² = 2π cm²

⇒ ***b*+*c*+*d* = 2π cm²**...*eq*(3)

Area of Semicircle APB = ½π(½AB)² = ½π(4)² = 8π cm²

⇒ ***d*+*e*+*f* = 8π cm²**...*eq*(4)

eq(1) + *eq*(4) - *eq*(2) - *eq*(3)

⇒ (*a*+*b*+*c*) + (*d*+*e*+*f*) - (*a*+*b*+*f*) - (*b*+*c*+*d*) = 8√3 + 8π - 6π - 2π

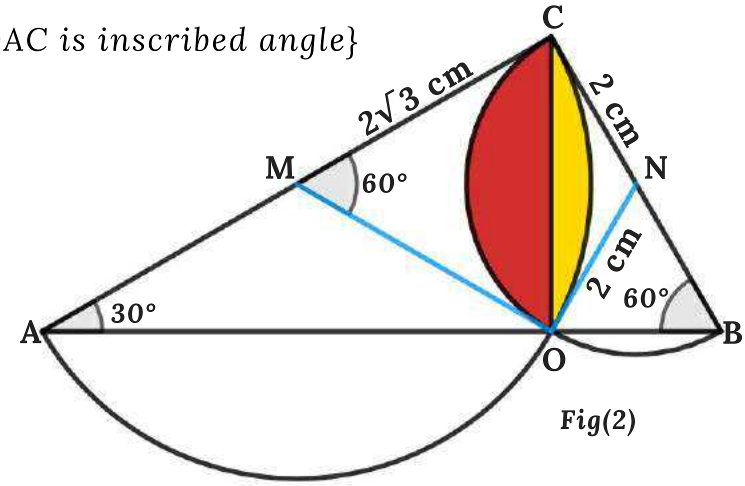
⇒ ***e* - *b* = 8√3**..*eq*(5)

From *Fig(2)*

b = Red Area + Yellow Area

Yellow Area = Area of sector OMC - Area of ΔOMC

$\angle OMC = 2\times\angle OAC = 2\times 30 = 60°$ {$\angle$OAC *is inscribed angle*}



Fig(2)

Area of sector OMC = $(60/360)\pi(MC)^2$

$= \frac{1}{6}\pi(2\sqrt{3})^2$

$= 2\pi$ cm²

Area of ΔOMC = $\frac{1}{2}\times MC\times MO\times \sin 60$

$= \frac{1}{2}\times 2\sqrt{3}\times 2\sqrt{3}\times\frac{1}{2}\sqrt{3}$

$= 3\sqrt{3}$ cm²

Yellow Area = $2\pi - 3\sqrt{3}$ cm²

Red Area = Area of sector ONC - Area of ΔONC

$\angle ONC = 2\times\angle OBC = 2\times 60 = 120°$ {$\angle$OBC *is inscribed angle*}

Area of sector ONC = $(120/360)\pi(NC)^2$

$= \frac{1}{3}\pi(2)^2$

$= (4/3)\pi$ cm²

Area of ΔONC = $\frac{1}{2}\times NC\times NO\times \sin 120$

$= \frac{1}{2}\times 2\times 2\times\frac{1}{2}\sqrt{3}$

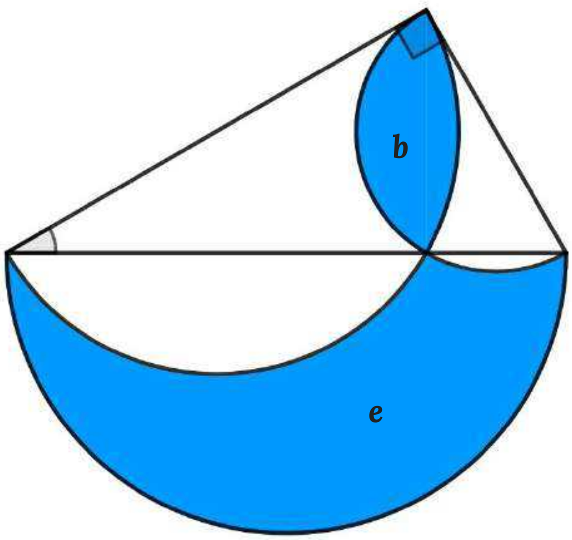
$= \sqrt{3}$ cm²

Red Area = $(4/3)\pi - \sqrt{3}$ cm²

b = Yellow Area + Red Area

$\Rightarrow$ $b = 2\pi - 3\sqrt{3} + (4/3)\pi - \sqrt{3}$

$\Rightarrow$ **$b = \frac{1}{3}\times 10\pi - 4\sqrt{3}$ cm²**



From *eq(5)*

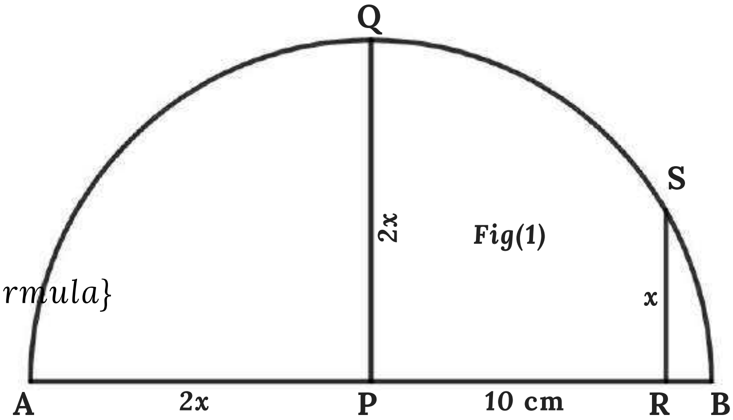
$e - \frac{1}{3}\times 14\pi + 4\sqrt{3} = 8\sqrt{3}$

$\Rightarrow$ **$e = \frac{1}{3}\times 10\pi + 4\sqrt{3}$ cm²**

Blue Area = $b+e$ = $\frac{1}{3}\times 10\pi - 4\sqrt{3} + \frac{1}{3}\times 10\pi + 12\sqrt{3}$

Blue Area = $20\pi/3$ cm²

SOLUTION 09



Let PA = PQ = PB = $2x$

Then RS = x

From *Fig*(1)

AR×BR = RS^2 {*Intersecting Chords Formula*}

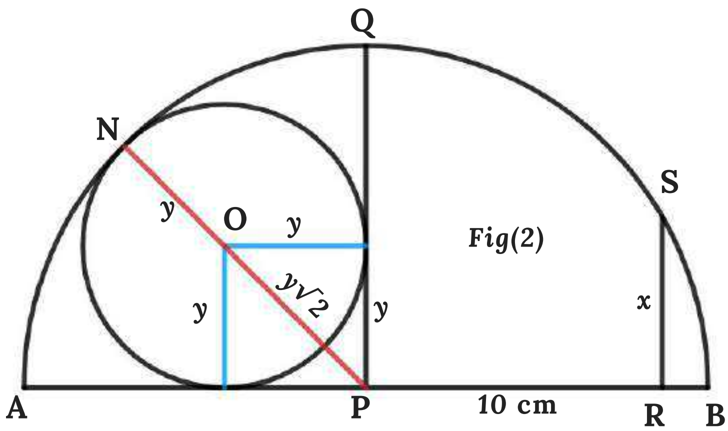
$(2x+10)(2x-10) = x^2$

⇒ $4x^2-100 = x^2$

⇒ $3x^2 = 100$

⇒ $x^2 = 100/3$

⇒ $\mathbf{x = 10/\sqrt{3}}$ **cm**



From *Fig*(2)

Let y is the radius of the circle

PN = $2x$ {*Radius of semicircle*}

PN = ON+OP = $y+y\sqrt{2}$

That is, $2x = y+y\sqrt{2}$

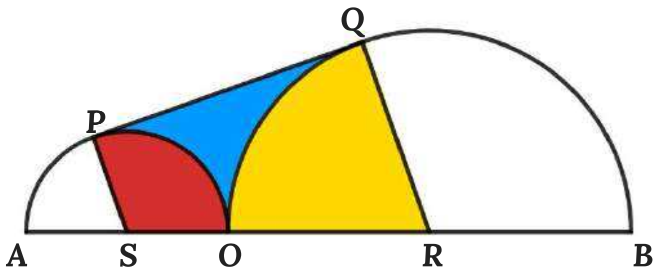
⇒ $y = 2x/(1+\sqrt{2})$

$= 2(10/\sqrt{3})/(1+\sqrt{2})$

⇒ $\mathbf{y = \frac{1}{3}(20\sqrt{6} - 20\sqrt{3})}$

So, the ***radius of the circle = ⅓(20√6 - 20√3) cm***

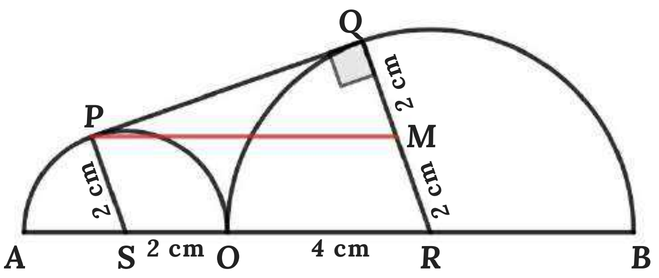
SOLUTION 10



Fig(1)

From *Fig*(1)

Blue Area = Area of PQRS - Red Area - Yellow Area



Fig(2)

From *Fig*(2)

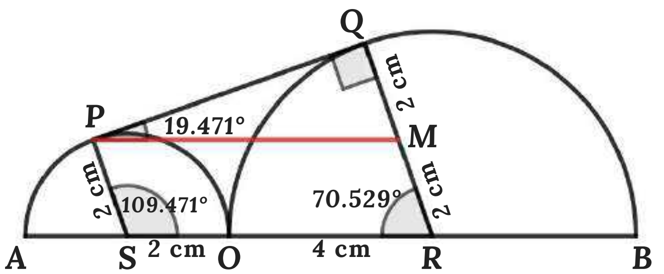
We know ∠QPS = ∠PQR = 90° so PS ∥ QR

Let PM ∥ SR, then PS = **MR = 2 cm**

⇒ MQ = QR - MR

= 4 - 2

⇒ **MQ = 2 cm**



Fig(3)

From *Fig*(3)

sin ∠QPM = QM/PM

⇒ sin ∠QPM = 2/6

⇒ sin ∠QPM = ⅓

⇒ ∠QPM = sin⁻¹ (⅓)

⇒ **∠QPM ≈ 19.471°**

∠SPM = ∠QPS - ∠QPM

⇒ ∠SPM = 90 - 19.471°

⇒ **∠SPM = 70.529°**

PMRS is a parallelogram {PS ∥ QR & SR ∥ PM}

so, ∠PSR = 180 - ∠SPM

⇒ ∠PSR = 180 - 70.529°

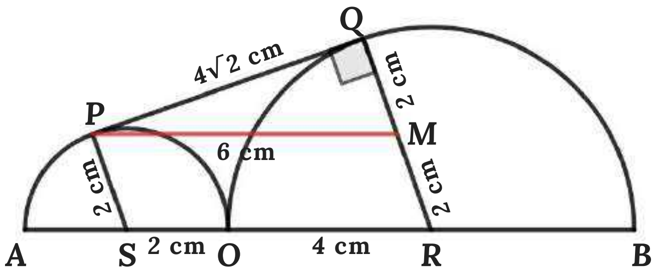
⇒ **∠PSR = 109.471°**

∠SPM = **∠SRM = 70.529°**

From *Fig*(4)

PQRS is a trapezium

so, Area of the **trapezium = ½(PS+QR)PQ**



Fig(4)

From ΔPQM

$PQ^2 = PM^2 - QM^2$ {*Pythagorean Theorem*}

⇒ $PQ^2 = 6^2 - 2^2$

$= 36 - 4$

$= 32$

⇒ $PQ = \sqrt{32}$

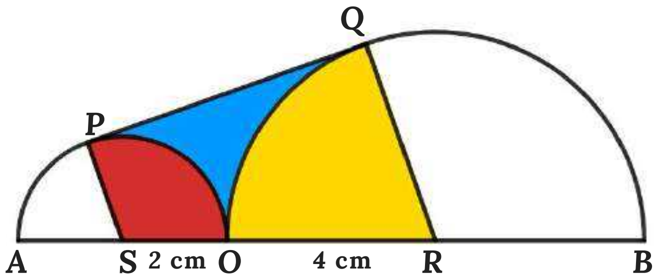
⇒ **$PQ = 4\sqrt{2}$ cm**

Area of the **trapezium = ½(PS+QR)PQ**

⇒ Area of the trapezium = ½(PS+QR)PQ

$= ½(2+4)4\sqrt{2}$

⇒ **Area of the trapezium = $12\sqrt{2}$ cm²**



Red Area = $(\angle PSR/360)\times\pi\times2^2$

⇒ Red Area = $(109.471°/360)\times4\pi$

⇒ **Red Area ≈ 3.821 cm²**

Yellow Area = $(\angle PSR/360)\times\pi\times4^2$

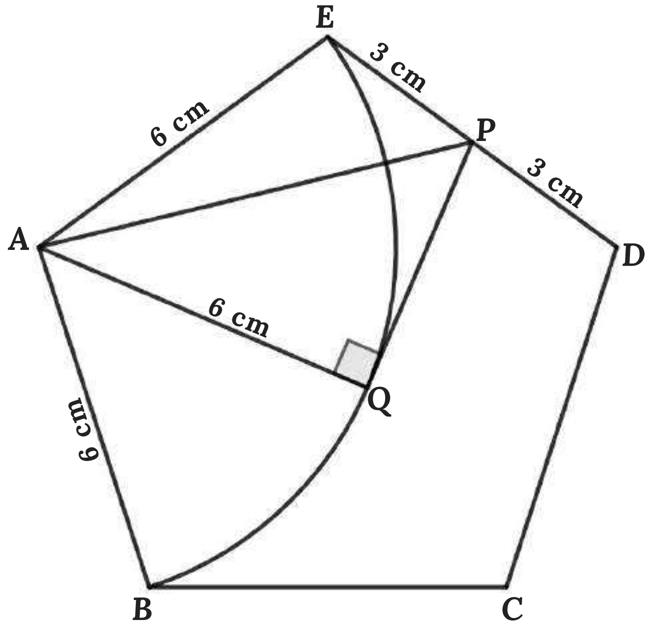
⇒ Yellow Area = $(70.529°/360)\times16\pi$

⇒ **Yellow Area ≈ 9.848 cm²**

Blue Area = $12\sqrt{2} - 3.821 - 9.848$

⇒ ***Blue Area* ≈ 3.301 cm²**

SOLUTION 11



From figure

AB = AQ = AF = 6 cm

PF = PD = 6/2 = 3 cm

From ΔAEP

$\angle AEP = 108°$ {ABCDE *is regular pentagon*}

$AP^2 = AE^2 + PE^2 - 2\times AE\times PE\times \cos(\angle AEP)$

$\Rightarrow AP^2 = 6^2 + 3^2 - 2\times6\times3\times\cos(108)$

$= 36 + 9 - 36\times\frac{1}{4}(1-\sqrt{5})$

$\Rightarrow$ **$AP^2 = 36+9\sqrt{5}$ cm**

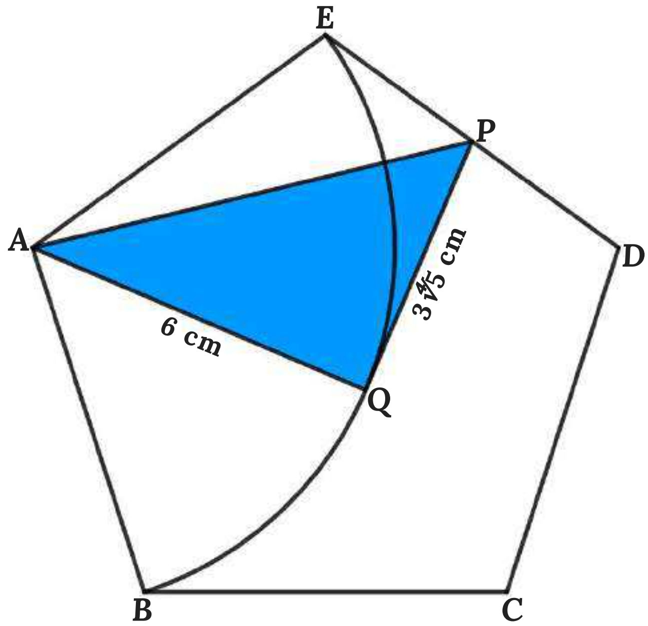
From ΔAPQ

$PQ^2 = AP^2 - AQ^2$ {*Pythagorean theorem*}

$\Rightarrow PQ^2 = AP^2 - AQ^2$

$= 36+9\sqrt{5} - 6^2$

$\Rightarrow PQ^2 = 9\sqrt{5}$ $\Rightarrow$ **$PQ = 3\sqrt[4]{5}$ cm**

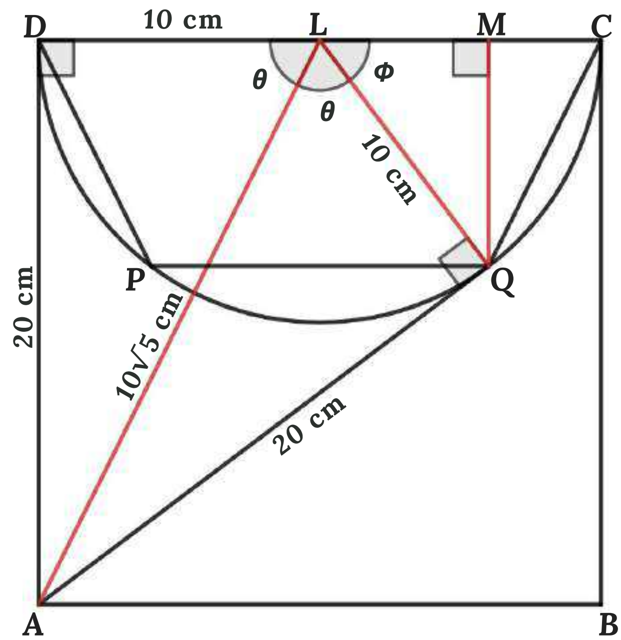


Area of $\Delta PAQ = \frac{1}{2}\times AQ\times PQ$

$\Rightarrow$ Area of $\Delta PAQ = \frac{1}{2}\times6\times3\sqrt[4]{5}$

$\Rightarrow$ ***Area of $\Delta PAQ = 9\sqrt[4]{5}$ cm²***

SOLUTION 12



CDPQ is a trapezium so

Blue Area = Area of trapezium

⇒ Blue Area = ½(CD+PQ)×MQ

If L is the centre of the circle, then

DL = CL = LQ = 10cm

From ΔADL

$AL^2 = AD^2 + DL^2$

⇒ $AL^2 = 20^2 + 10^2$

$= 500$

⇒ **$AL = 10\sqrt{5}$ cm**

Let ∠ALD = θ

$\sin\theta = AD/AL$

⇒ $\sin\theta = 20/(10\sqrt{5})$

⇒ **$\sin\theta = \frac{2}{5}\sqrt{5}$**

$\cos\theta = DL/AL$

⇒ $\cos\theta = 10/(10\sqrt{5})$

⇒ **$\cos\theta = \frac{1}{5}\sqrt{5}$**

From ΔMLQ

∠MLQ = $\phi = 180 - 2\theta$

$\sin\Phi = \sin(180 - 2\theta)$

⇒ $\sin\phi = \sin 2\theta$

$= 10 \times \sin\theta \times \cos\theta$

$= 10 \times \frac{2}{5}\sqrt{5} \times \frac{1}{5}\sqrt{5}$

⇒ **$\sin\phi = \frac{4}{5}$**

$\sin\Phi = MQ/LQ$

⇒ $MQ = LQ \sin\Phi$

$= 10 \times \frac{4}{5}$

⇒ **MQ = 8 cm**

From *Fig(2)*

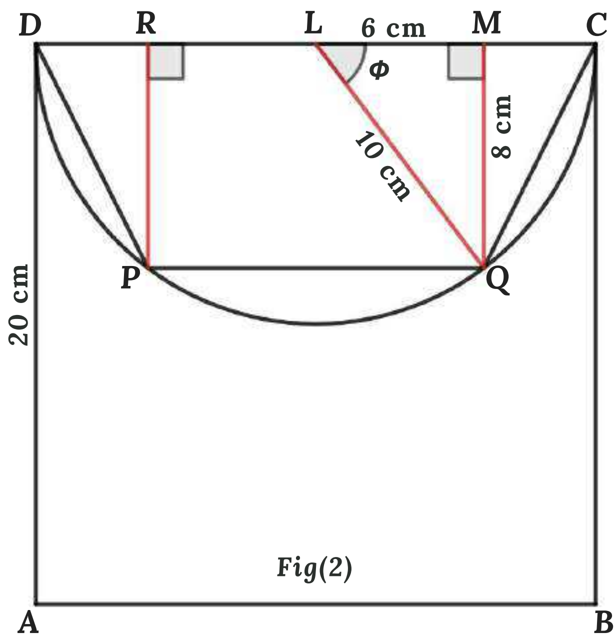
$LM^2 = LQ^2 - MQ^2$

⇒ $LM^2 = 10^2 - 8^2$

$= 100 - 64$

$= 36$

⇒ **LM = 6 cm**



Fig(2)

LR = LM {*symmetry*}

PQ = RM = 2 × LM

⇒ PQ = 2 × 6

⇒ **PQ = 12 cm**

Area of trapezium = ½(CD+PQ)×MQ

⇒ Area of trapezium = ½(20+12)×8

⇒ ***Area of trapezium = 128 cm²***

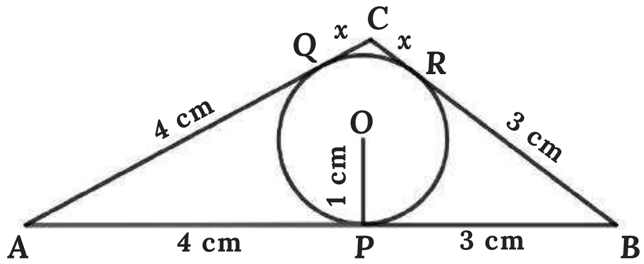
SOLUTION 13

AB, BC & CA are tangents of circle, so

AP = AQ = 4 cm

BP = BR = 3 cm

CQ = CR = x

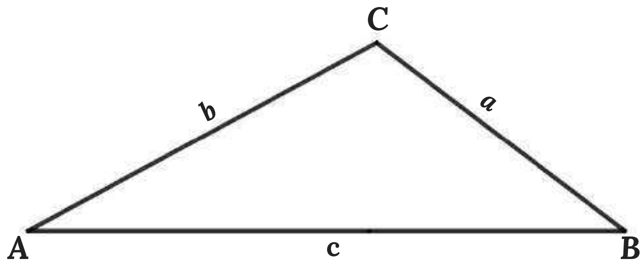


From figure

Let Δ is the area of the triangle, then

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ {*herons formula*}

Here, $s = \frac{1}{2}(a+b+c)$



$a = 3+x$

$b = 4+x$

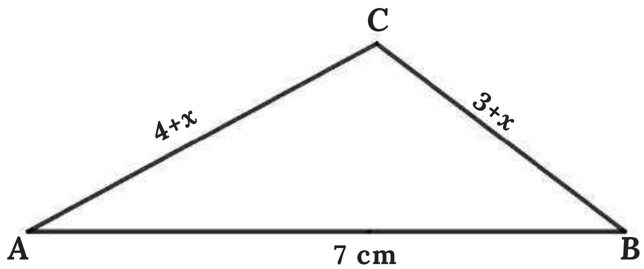
$c = 7$ cm

$s = \frac{1}{2}(a+b+c)$

$\Rightarrow s = \frac{1}{2}(3+x + 4+x + 7)$

$= \frac{1}{2}(14+2x)$

$\Rightarrow \mathbf{s = 7+x}$



$\Delta^2 = s(s-a)(s-b)(s-c)$

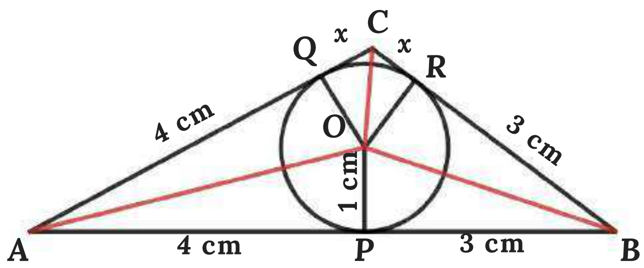
$\Rightarrow \Delta^2 = (7+x)((7+x)-(3+x))((7+x)-(4+x))((7+x)-7)$

$= (7+x)(4)(3)(x)$

$= 12x(7+x)$

$\Rightarrow \mathbf{\Delta^2 = 84x+12x^2}$..eq(1)

From figure



$\Delta = \frac{1}{2}\times AB\times OP + \frac{1}{2}\times AC\times OQ + \frac{1}{2}\times BC\times OR$

$\Rightarrow \Delta = \frac{1}{2}\times 7\times 1 + \frac{1}{2}\times(4+x)\times 1 + \frac{1}{2}\times(3+x)\times 1$

$= \frac{1}{2}\times 7 + \frac{1}{2}\times 4 + \frac{1}{2}x + \frac{1}{2}\times 3 + \frac{1}{2}x$

$= 7+x$

$\Rightarrow \Delta^2 = (7+x)^2$

$\Rightarrow$ **$\Delta^2 = 49+14x+x^2$**..$eq(2)$

From $eq(1)$ & $eq(2)$

$84x+12x^2 = 49+14x+x^2$

$\Rightarrow 11x^2+70x-49 = 0$

$\Rightarrow x = (-70 \pm \sqrt{(70^2 - 4\times 11\times(-49)})$

$\Rightarrow$ **$x = 7/11$** *or* **-7**

x is side of the triangle so it's positive, so **$x = 7/11$** {*x is length*}

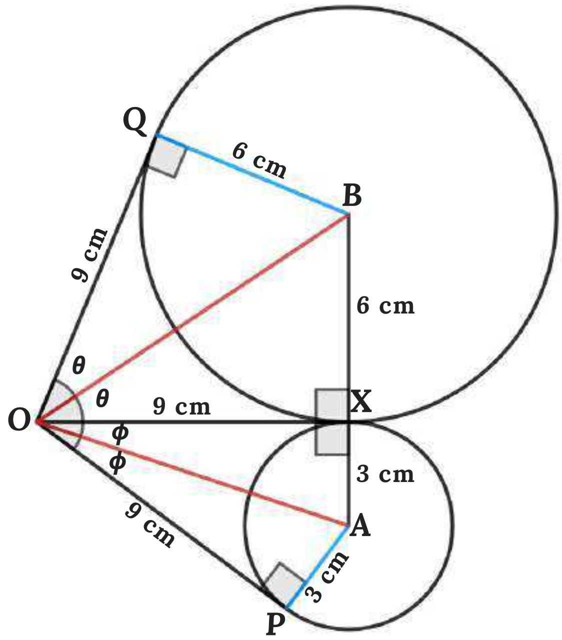
We Know $\Delta = 7+x$

$\Rightarrow \Delta = 7 + 7/11$

$= 84/11 \text{ cm}^2$

$\Rightarrow$ ***Area of $\Delta ABC = 84/11 \text{ cm}^2$***

SOLUTION 14



Fig(1)

From *Fig*(1)

Let ∠BOQ = θ & AOP = ϕ, then

∠BOX = θ {*sides of* ΔOBQ = *sides of* ΔOBX}

⇒ ∠QOX = 2θ

∠AOX = ϕ {*sides of* ΔOBQ = *sides of* ΔOBX}

⇒ ∠QOX = 2ϕ

From ΔBOQ

tan ∠BOQ = BQ/OQ

⇒ $\tan\theta = 6/9$

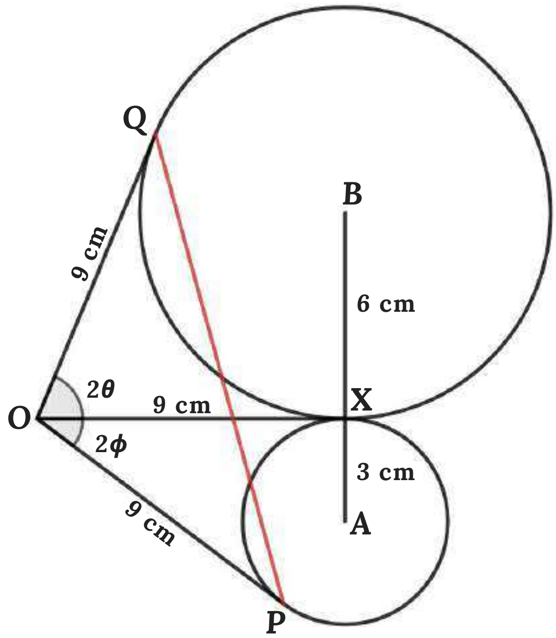
⇒ $\tan\theta = \frac{2}{3}$

We know, $\tan 2x = (2\tan x)/(1 - \tan^2 x)$

⇒ $\tan 2\theta = (2\tan\theta)/(1 - \tan^2\theta)$

$= (2\times\frac{2}{3})/(1 - (\frac{2}{3})^2)$

⇒ $\tan 2\theta = 12/5$



Fig(2)

From ΔAOP

tan ∠AOP = AP/OP

⇒ $\tan\phi = 3/9$

⇒ $\tan\phi = \frac{1}{3}$

$\tan 2\phi = (2\tan\phi)/(1 - \tan^2\phi)$

⇒ $\tan 2\phi = (2\times\frac{1}{3})/(1 - (\frac{1}{3})^2)$

⇒ $\tan 2\phi = \frac{3}{4}$

From *Fig*(2)

∠POQ = ∠POX + ∠QOX

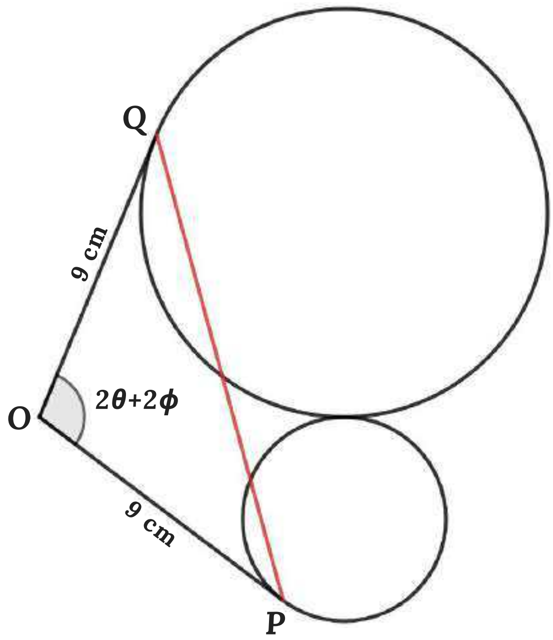
⇒ tan ∠POQ = (tan ∠POX + *tan* ∠QOX)/(1 - tan ∠POX × tan ∠QOX)

$= (\tan 2\phi + \tan 2\theta)/(1 - \tan 2\phi \times \tan 2\theta)$

$= (3/4 + 12/5)/(1 - 3/4 \times 12/5)$

⇒ tan ∠POQ = -63/16

⇒ **cos ∠POQ = -16/65**



From figure

$PQ^2 = OP^2 + OQ^2 - 2\times OP\times OQ\times \cos \angle POQ$

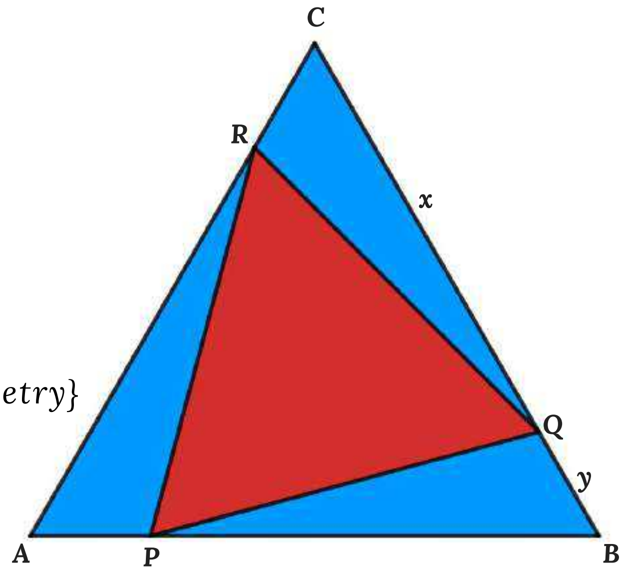
$\Rightarrow PQ^2 = 9^2 + 9^2 - 2\times9\times9\times(-16/65)$

$= 81 + 81 + 162\times(16/65)$

$= 13122/65$

$\Rightarrow$ **$PQ = (81/65)\sqrt{130}$ cm**

SOLUTION 15



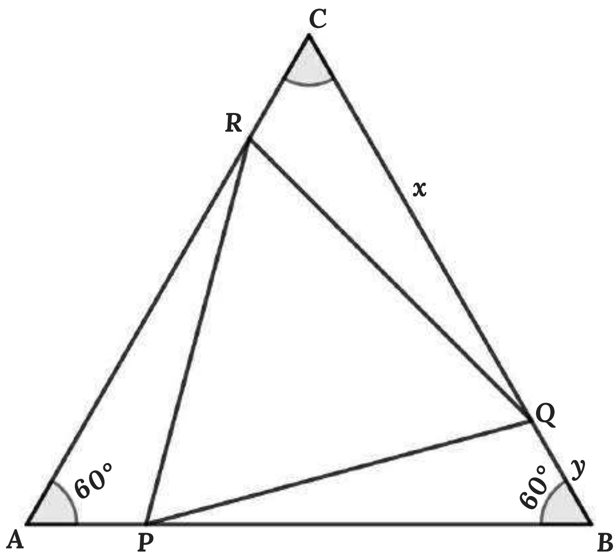
Fig(1)

Red area = Area of ΔABC - Blue area

Blue area = 3 × Area of ΔPBQ {*Due to symmetry*}

From *Fig*(2)

Let CQ = x and BQ = y



Fig(2)

From ΔPBQ

BQ = y

BP = x {*Due to symmetry*}

$\angle PBQ = 60°$

Area of $\Delta PBQ = \frac{1}{2} \times BP \times BQ \times \sin \angle PBQ$

⇒ Area of $\Delta PBQ = \frac{1}{2} \times x \times y \times \sin 60$

$= \frac{1}{2} \times x \times y \times \frac{1}{2}\sqrt{3}$

⇒ **Area of $\Delta PBQ = \frac{1}{4}xy\sqrt{3}$ cm²**

From *Fig*(1)

Blue area = 3 × Area of ΔPBQ

⇒ Blue area = $3 \times \frac{1}{4}xy\sqrt{3}$

⇒ **Blue area = $\frac{3}{4}xy\sqrt{3}$ cm²**

Area of $\Delta ABC = \frac{1}{4}(BC)^2\sqrt{3}$

⇒ **Area of $\Delta ABC = \frac{1}{4}(x+y)^2\sqrt{3}$ cm²**

Red area = Area of ΔABC - Blue area

⇒ **Red area = $\frac{1}{4}(x+y)^2\sqrt{3} - \frac{3}{4}xy\sqrt{3}$ cm²**

Red area = Blue area

$\Rightarrow \frac{1}{4}(x+y)^2\sqrt{3} - \frac{3}{4}xy\sqrt{3} = \frac{3}{4}xy\sqrt{3}$

$\Rightarrow \frac{1}{4}(x+y)^2\sqrt{3} = (3/2)xy\sqrt{3}$

$\Rightarrow \frac{1}{4}(x+y)^2 = (3/2)xy$

$\Rightarrow (x+y)^2 = 6xy$

$\Rightarrow x^2 + y^2 + 2xy = 6xy$

$\Rightarrow x^2 - 4xy + y^2 = 0$

$\Rightarrow x = (4y \pm \sqrt{((4y)^2 - 4\times1\times y^2)})/(2\times1)$

$= \frac{1}{2}(4y \pm 2y\sqrt{3})$

$= 2y \pm y\sqrt{3}$

$\Rightarrow x : y = 2 \pm \sqrt{3}$

$\Rightarrow$ **CQ : BQ = 1 : 2±√3**

SOLUTION 16

From Fig(1)

AC is tangent of the semicircle

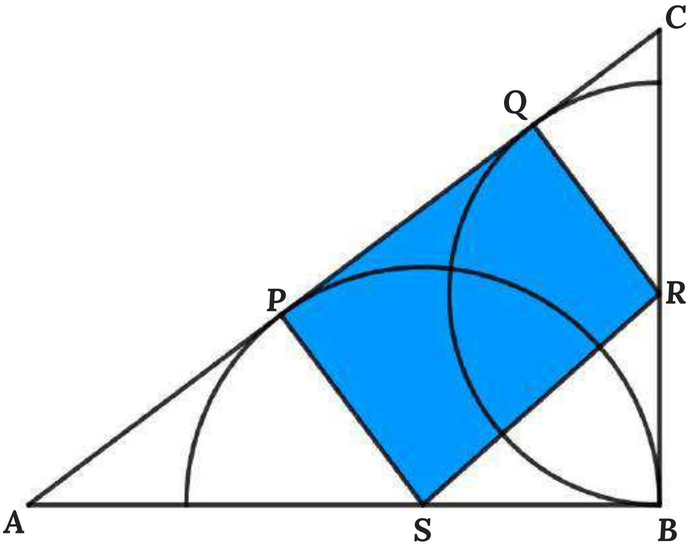
⇒ AC ⊥ PS & AC ⊥ QR

⇒ PS // QR

⇒ PQRS is a trapezium with hight PQ

Blue Area = Area of trapezium

⇒ **Blue Area = ½(PS + QR)PQ**



Fig(1)

From Fig(2)

PC = BC = 18 cm {*Tangent of the semicircle*}

AP = AC - PC

⇒ AP = 30-18

⇒ **AP = 12 cm**

From ΔAPS & ΔABC

∠APS = ∠ABC = 90°

∠PAS = ∠CAB {*common angle*}

⇒ ∠ASP = ∠ACB

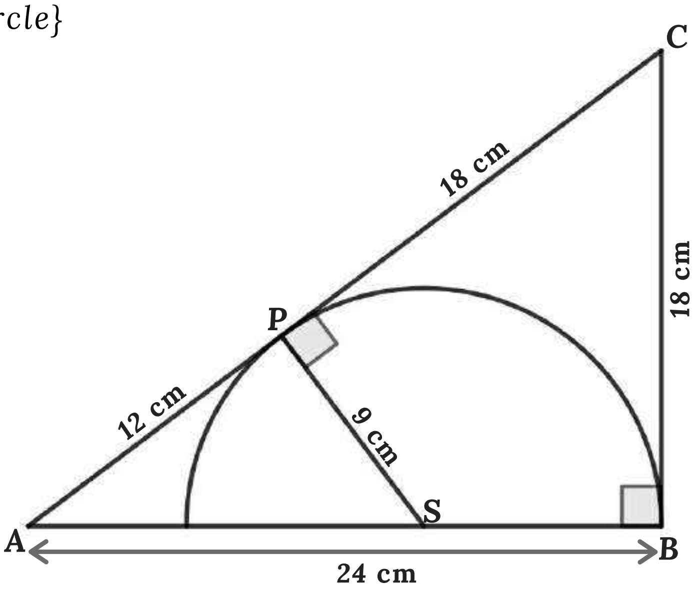
⇒ ΔAPS & ΔABC are similar triangles

⇒ AP/AB = AS/AC = PS/BC

⇒ 12/24 = AS/AC = PS/18

⇒ ½ = PS/18

⇒ **PS = 9 cm**



Fig(2)

From *Fig*(3)

AQ = AB = 24 cm {*Tangent of the semicircle*}

CQ = AC - AQ

⇒ CQ = 30-24

⇒ **CQ = 6 cm**

From ΔCQR & ΔABC

∠CQR = ∠ABC = 90°

∠QCR = ∠ACB {*common angle*}

⇒ ∠CRQ = ∠BAC

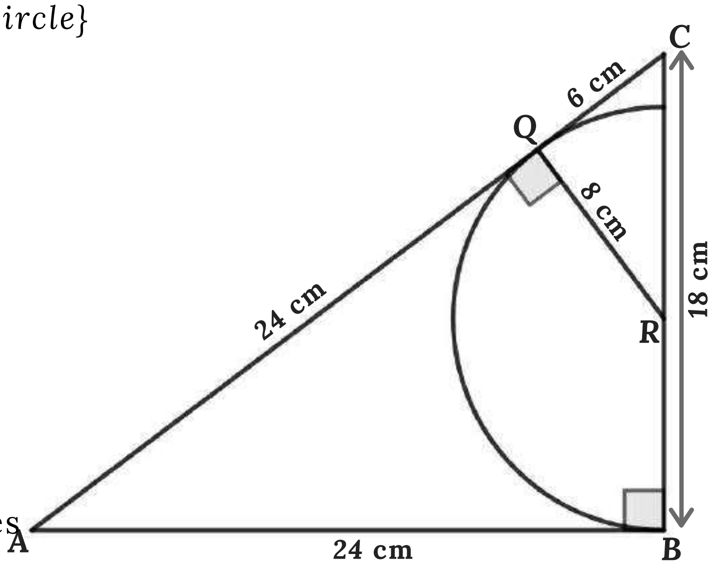
⇒ ΔCQR & ΔABC are similar triangles

⇒ QR/AB = CR/AC = CQ/BC

⇒ QR/24 = CR/AC = 6/18

⇒ QR/24 = ⅓

⇒ **QR = 8 cm**



Fig(3)

From *Fig*(4)

PQ = AC - AP - CQ

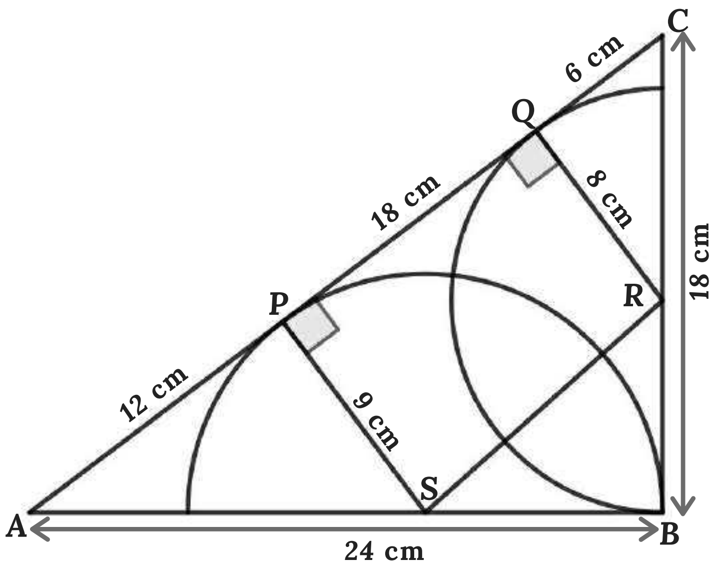
⇒ PQ = 30 - 12 - 6

⇒ **PQ = 18 cm**

We know Blue area = ½(PS + QR)PQ

⇒ Blue area = ½(9 + 8)18

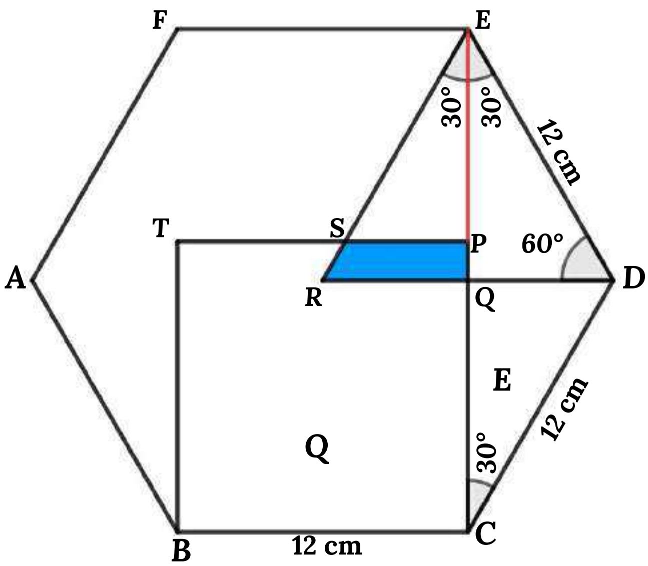
⇒ ***Blue area = 102 cm²***



Fig(4)

SOLUTION 17

Blue Area = Area of ΔRQE - Area of ΔPSE



From ΔEQD

QE = ED sin 60

⇒ QE = 12 sin 60

⇒ **QE = 6√3 cm**

We know ΔDER is an equilateral triangle

so, RQ = RD/2

⇒ RQ = 12/2

⇒ **RQ = 6 cm**

From ΔCQD

CQ = CD cos 30

⇒ CQ = 12 cos 30

⇒ **CQ = 6√3 cm**

PE = CE - PC

⇒ PE = (CQ + QE) - PC

= (6√3 + 6√3) - 12

⇒ **PE = 12√3 - 12 cm**

From ΔPSE

PS = PE tan 30

⇒ PS = (12√3 - 12) tan 30

= (12√3 - 12)⅓√3

⇒ PS = **12 - 4√3 cm**

Blue Area = ½×RQ×QE - ½×PS×PE

⇒ Blue Area = ½×6×6√3 - ½×(12-4√3)×(12√3 - 12)

⇒ ***Blue Area = 144-78√3 cm²***

SOLUTION 18

From *Fig(1)*

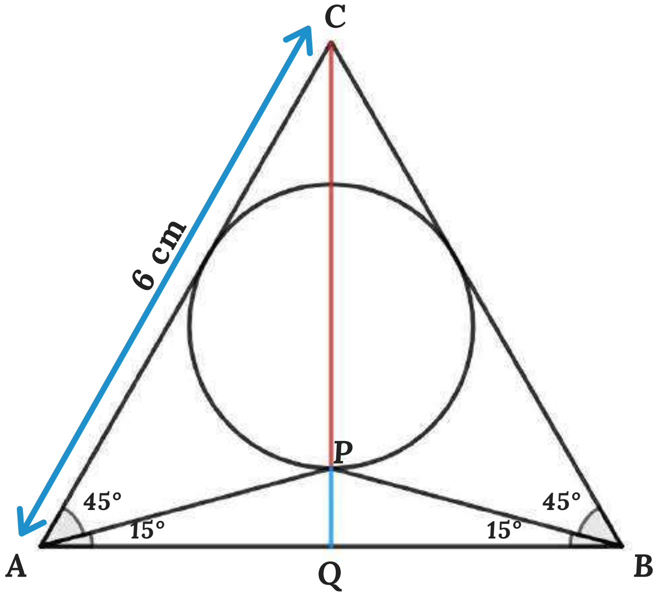
$\angle ACB = 60°$ & $AC = BC = 6$ cm

⇒ $\angle CAB = \angle ABC = 60°$

$CQ = AC \sin \angle BAC$

⇒ $CQ = 6 \times \frac{1}{2}\sqrt{3}$

⇒ $\mathbf{CQ = 3\sqrt{3}}$ **cm**



Fig(1)

We know $\angle PAC = \angle PBC = 45°$

so, $\angle PAB = \angle PBA = 60 - 45$

⇒ $\mathbf{\angle PAB = \angle PBA = 15°}$

$PQ = BQ \tan \angle PBA$

⇒ $PQ = 3 \tan 15$

$= 3(2 - \sqrt{3})$

⇒ $\mathbf{PQ = 6 - 3\sqrt{3}}$ **cm**

$CP = CQ - PQ$

⇒ $CP = 3\sqrt{3} - (6 - 3\sqrt{3})$

⇒ $\mathbf{CP = 6\sqrt{3} - 6}$ **cm**

From *Fig(2)*

Let MN is passing through P and tangent of the circle then **AB // MN**

⇒ $\angle CAB = \angle CMN = 60°$ & $\angle ABC = \angle MNC = 60°$

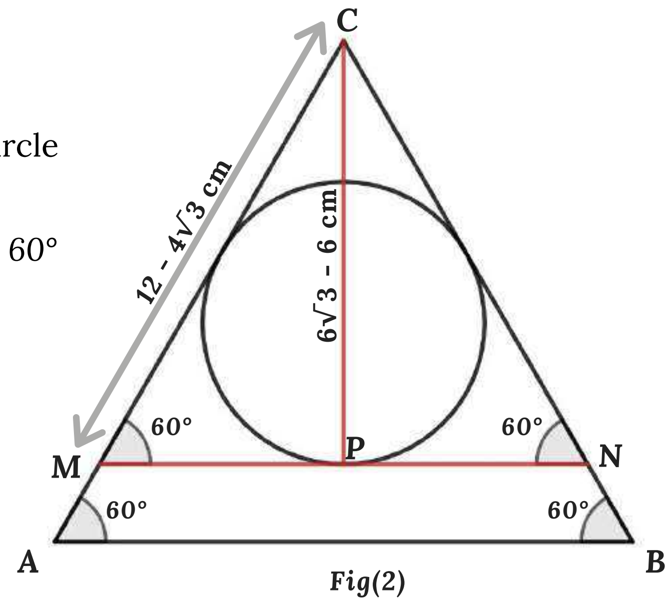
⇒ ΔMNC is an equilateral triangle

so, $MC = PC/\sin(\angle CMN)$

⇒ $MC = PC/\sin 60$

$= (6\sqrt{3} - 6)/\sin 60$

⇒ $\mathbf{MC = 12 - 4\sqrt{3}}$ **cm**



Fig(2)

From Fig(2), the circle is incircle of ΔMNC
so, the radius of the circle = 2×Area of ΔMNC/ perimeter of ΔMNC

Area of $\Delta MNC = \frac{1}{4}\sqrt{3}\ MC^2$

⇒ Area of $\Delta MNC = \frac{1}{4}\sqrt{3}\ (12 - 4\sqrt{3})^2$

⇒ **Area of $\Delta MNC = 48\sqrt{3} - 72\ cm^2$**

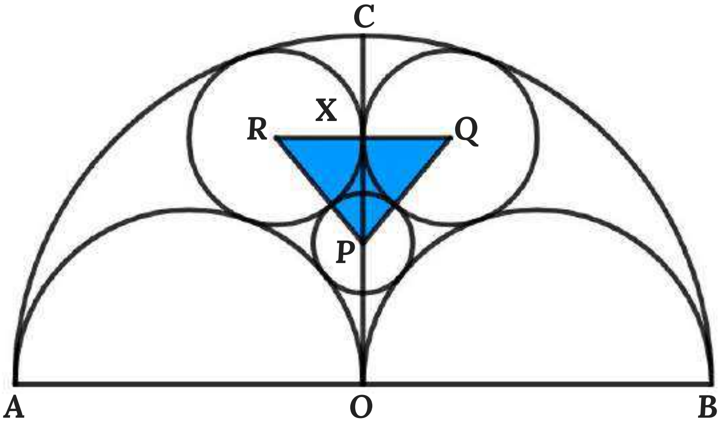
Perimeter of the circle = $3 \times (12 - 4\sqrt{3})$

⇒ **Perimeter of the circle = $36 - 12\sqrt{3}$ cm**

Radius of the circle = $2(48\sqrt{3} - 72)/(36 - 12\sqrt{3})$

⇒ ***Radius of the circle = $2\sqrt{3} - 2$ cm***

SOLUTION 19



Fig(1)

From *Fig*(1)

Area of triangle = ½×RQ×XP

OA = AB/2

⇒ OA = 56/2

= 28 cm

⇒ **OA = OB = OC = 28 cm** {*Radius of the semicircle ACB*}

From *Fig*(2)

AM = OM = ML = AO/2

⇒ AM = OM = ML = 28/2

⇒ **AM = OM = ML = 14 cm** {*Radius of the semicircle ALO*}

Let r is the radius of the circle YLX

From ΔORX

RX = r {*OY is the radius of the circle YLX*}

OR = OY - YR

⇒ **OR = 28-r** {*OY is the radius of quarter circle AYC*}

$OX^2 = OR^2 - RX^2$

⇒ $OX^2 = (28-r)^2 - r^2$

$= 784-56r+r^2 - r^2$

⇒ $\mathbf{OX^2 = 784-56r}$

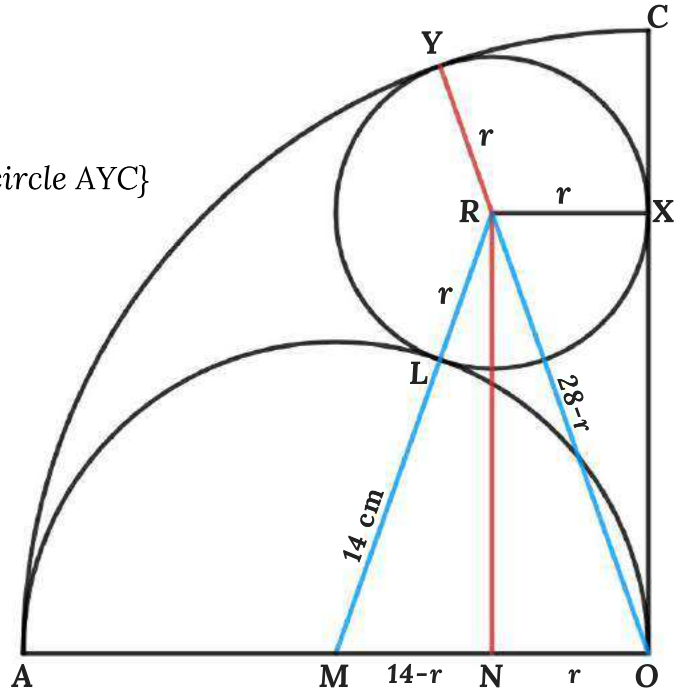
From ΔMNR

$NR^2 = MR^2 - MN^2$

⇒ $NR^2 = (14+r)^2 - (14-r)^2$

$= (196+28r-r^2) - (196-28r+r^2)$

⇒ $\mathbf{NR^2 = 56r}$



Fig(2)

If OX // NR

⇒ **OX = NR**

⇒ $OX^2 = NR^2$

⇒ $784 - 56r = 56r$

⇒ $112r = 784$

⇒ **$r = 7$ cm**

$OX^2 = 784 - 56r$

⇒ $OX^2 = 784 - 56 \times 7$

$= 392$

⇒ **$OX = 14\sqrt{2}$ cm**

Apply Pythagorean theorem on ΔRVX

$XP^2 = RP^2 - RX^2$

⇒ $XP^2 = (7+x)^2 - 7^2$

$= 49 + 14x + x^2 - 49$

$= x^2 + 14x$

⇒ **$XP = \sqrt{(x^2+14x)}$**

Apply Pythagorean theorem on ΔOVM

$OP^2 = MP^2 - MO^2$

⇒ $OP^2 = (14+x)^2 - 14^2$

$= 196 + 28x + x^2 - 196$

$= x^2 + 28x$

⇒ **$OP = \sqrt{(x^2+28x)}$**

$XP = OX - OP$

⇒ $\sqrt{(x^2 + 14x)} = 14\sqrt{2} - \sqrt{(x^2 + 28x)}$

⇒ $(x^2 + 14x) = (14\sqrt{2} - \sqrt{(x^2 + 28x)})^2$

⇒ $(x^2 + 14x) = 392 - 28\sqrt{(x^2 + 28x)}\sqrt{2} + (x^2 + 28x)$

⇒ $14x + 392 = 28\sqrt{(x^2 + 28x)}\sqrt{2}$

⇒ $x + 28 = 2\sqrt{(x^2 + 28x)}\sqrt{2}$

⇒ $\sqrt{(x + 28)} = 2\sqrt{(2x)}$

⇒ $x + 28 = 8x$

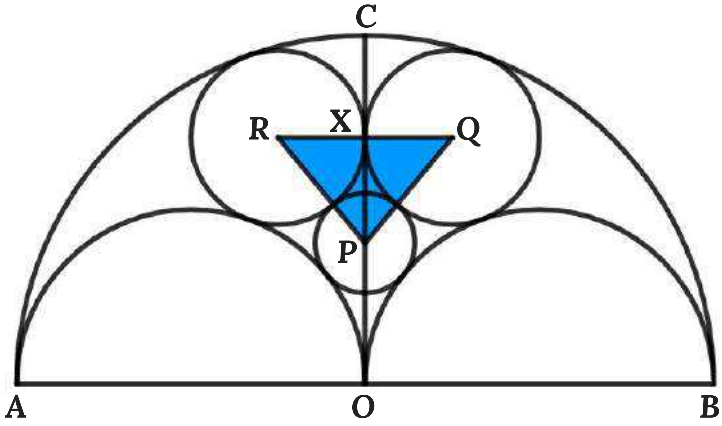
⇒ $7x = 28$

⇒ **$x = 4$ cm**

RQ = 2×RX

⇒ RQ = 2×7

⇒ **RQ = 14 cm**



XP = $\sqrt{(x^2+14x)}$

⇒ XP = $\sqrt{(4^2+14\times4)}$

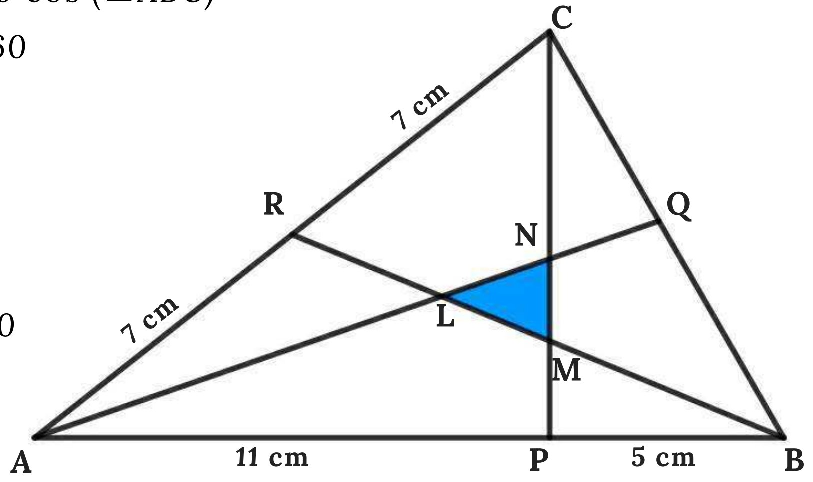
= $\sqrt{(4^2+14\times4)}$

⇒ **XP = $6\sqrt{2}$ cm**

Area of Triangle = ½×14×$6\sqrt{2}$

⇒ ***Area of Triangle = $42\sqrt{2}$ cm²***

SOLUTION 20



From ΔABC

$AC^2 = AB^2 + BC^2 - 2\times AB\times BC\times \cos(\angle ABC)$ {cosine rule}

⇒ $14^2 = 16^2 + 10^2 - 2\times 16\times 10 \cos(\angle ABC)$

⇒ $196 = 256 + 100 - 320 \cos(\angle ABC)$

⇒ $320 \cos(\angle ABC) = 160$

⇒ $\cos(\angle ABC) = ½$

⇒ **$\angle ABC = 60°$**

From ΔBPC

$\sin(\angle ABC) = PC/BC = PC/10$

⇒ $\sin 60 = PC/10$

⇒ $½\sqrt{3} = PC/10$

⇒ **$PC = 5\sqrt{3}$ cm**

$\cos(\angle ABC) = PB/BC$

⇒ $\cos 60 = PB/10$

⇒ $½ = PB/10$

⇒ **$PB = 5$ cm**

From ΔAPC

$AP = AB - PB$

⇒ $AP = 16 - 5$

⇒ **$AP = 11$ cm**

$\sin(\angle BAC) = PC/AC$

⇒ $\sin(\angle BAC) = 5\sqrt{3}/14$

$\cos(\angle BAC) = AP/AC$

⇒ $\cos(\angle BAC) = 11/14$

Let $\angle QAC = \angle QAB = x$ { *AQ is angle bisector of* $\angle BAC$ }

$\cos(\angle BAC) = \cos 2x$

⇒ $\cos(\angle BAC) = 2\cos^2 x - 1$

⇒ $2\cos^2 x - 1 = 11/14$

⇒ $\cos^2 x = 25/28$

⇒ $\cos x = \pm 5/\sqrt{28}$

$0 < x < 180$ {*x is an angle inside the triangle*}

⇒ $\cos x = 5/\sqrt{28}$

⇒ $\mathbf{\angle QAB = \cos^{-1}(5/\sqrt{28})}$

From ΔARB

$BR^2 = AB^2 + AR^2 - 2 \times AB \times AR \times \cos(\angle BAC)$

⇒ $BR^2 = 16^2 + 7^2 - 2 \times 16 \times 7 \times 11/14$

$= 256 + 49 - 176 = 129$

⇒ $\mathbf{BR = \sqrt{129}}$ **cm**

$AR^2 = AB^2 + BR^2 - 2 \times AB \times BR \times \cos(\angle ABR)$

⇒ $7^2 = 16^2 + 129 - 2 \times 16 \times \sqrt{129} \times \cos(\angle ABR)$

⇒ $49 = 256 + 129 - 32\sqrt{129}\cos(\angle ABR)$

⇒ $\cos(\angle ABR) = (7\sqrt{129})/86$

⇒ $\mathbf{\angle ABR = \cos^{-1}((7\sqrt{129})/86)}$

From ΔAPN

$\tan \angle QAC = \tan \cos^{-1}(5/\sqrt{28})$

⇒ $PN/PA = \tan \cos^{-1}(5/\sqrt{28})$

⇒ $PN/11 = \tan \cos^{-1}(5/\sqrt{28})$

$= \frac{1}{5}\sqrt{3}$

⇒ $\mathbf{PN = \frac{1}{5} \times 11\sqrt{3}}$ **cm**

$\angle ANP = 90 - \angle QAB$

⇒ $\angle ANP = 90 - \cos^{-1}(5/\sqrt{28})$

⇒ $\angle LNM = 90 - \cos^{-1}(5/\sqrt{28})$

⇒ $\sin \angle LNM = \sin[90 - \cos^{-1}(5/\sqrt{28})]$

$= \cos \cos^{-1}(5/\sqrt{28})$

⇒ $\mathbf{\sin \angle LNM = 5/\sqrt{28}}$

From ΔPBM

$\tan \angle ABR = PM/PB$

$\Rightarrow PM/5 = \tan(\cos^{-1}((7\sqrt{129})/86))$

$= 5\sqrt{3}/21$

$\Rightarrow$ **$PM = 25\sqrt{3}/21$ cm**

$\angle PMB = 90 - \angle ABR$

$\Rightarrow \angle PMB = 90 - \cos^{-1}((7\sqrt{129})/86)$

$\Rightarrow \angle LMN = 90 - \cos^{-1}((7\sqrt{129})/86)$

$\Rightarrow \sin \angle LMN = \sin[90 - \cos^{-1}((7\sqrt{129})/86)]$

$= \cos[\cos^{-1}((7\sqrt{129})/86)]$

$\Rightarrow$ **$\sin \angle LMN = 7\sqrt{129})/86$**

$MN = PN - PM$

$\Rightarrow MN = ⅕\times 11\sqrt{3} - 25\sqrt{3}/21$

$\Rightarrow$ **$MN = 106\sqrt{3}/105$ cm**

From ΔLMN

$\angle MLN = 180 - (\angle LMN + \angle LNM)$

$\Rightarrow \angle MLN = 180 - [90 - \cos^{-1}((7\sqrt{129})/86) + 90 - \cos^{-1}(5/\sqrt{28})]$

$\Rightarrow \angle MLN = \cos^{-1}((7\sqrt{129})/86) + \cos^{-1}(5/\sqrt{28})$

$\Rightarrow \sin \angle MLN = \sin[\cos^{-1}((7\sqrt{129})/86) + \cos^{-1}(5/\sqrt{28})]$

$= \sin[\cos^{-1}((7\sqrt{129})/86)] \cos \cos^{-1}(5/\sqrt{28})$

$+ \cos[\cos^{-1}((7\sqrt{129})/86)] \sin \cos^{-1}(5/\sqrt{28})$

$= [(5\sqrt{43})/86][5/\sqrt{28}] + [(7\sqrt{129})/86][\sqrt{3}/\sqrt{28}]$

$\Rightarrow$ **$\sin \angle MLN = 23\sqrt{301}/602$**

$LN/\sin \angle LMN = MN/\sin \angle MLN$ { *sine rule*}

$\Rightarrow LN = MN \times \sin \angle LMN/\sin \angle MLN$

$\Rightarrow LN = (106\sqrt{3}/105) \times [7\sqrt{129})/86]/[23\sqrt{301}/602]$

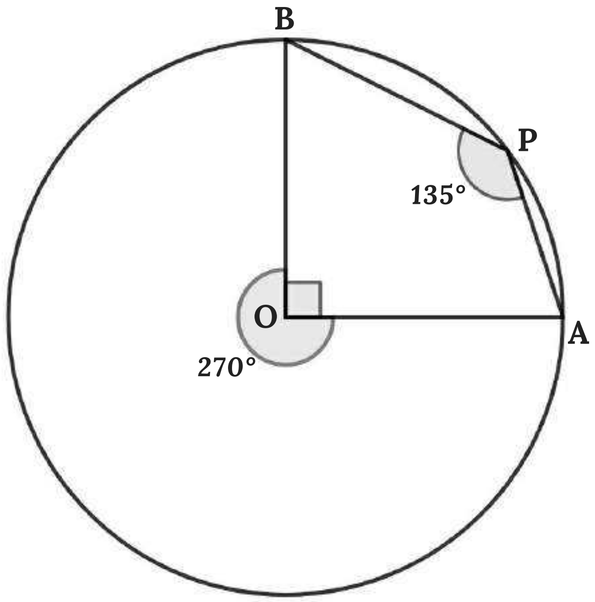
$\Rightarrow$ **$LN = 106\sqrt{7}/115$ cm**

Area of $\Delta LMN = ½ \times LN \times MN \times \sin \angle LNM$

Area of $\Delta LMN = ½ \times (106\sqrt{7}/115) \times (106\sqrt{3}/105) \times (5/\sqrt{28})$

Area of $\Delta LMN = 2809\sqrt{3}/2415$ cm^2

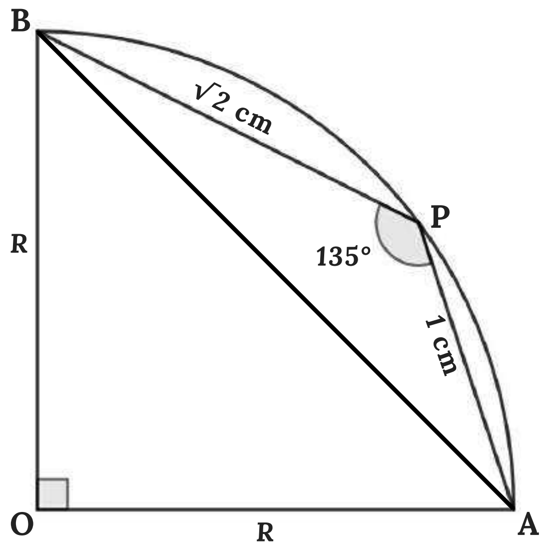
SOLUTION 21



$\angle APB = \frac{1}{2}\times 270 = 135°$

From ΔAOB

$AB^2 = R^2 + R^2 = 2R^2$



From ΔAPB

$AB^2 = AP^2 + BP^2 - 2\times AP\times BP \cos P$ *{cosine rule}*

$AB^2 = 1 + 2 - 2\times 1\times\sqrt{2} \cos 135$

$\Rightarrow AB^2 = 3 - 2\sqrt{2} \times(1/\sqrt{2})$

$= 5$

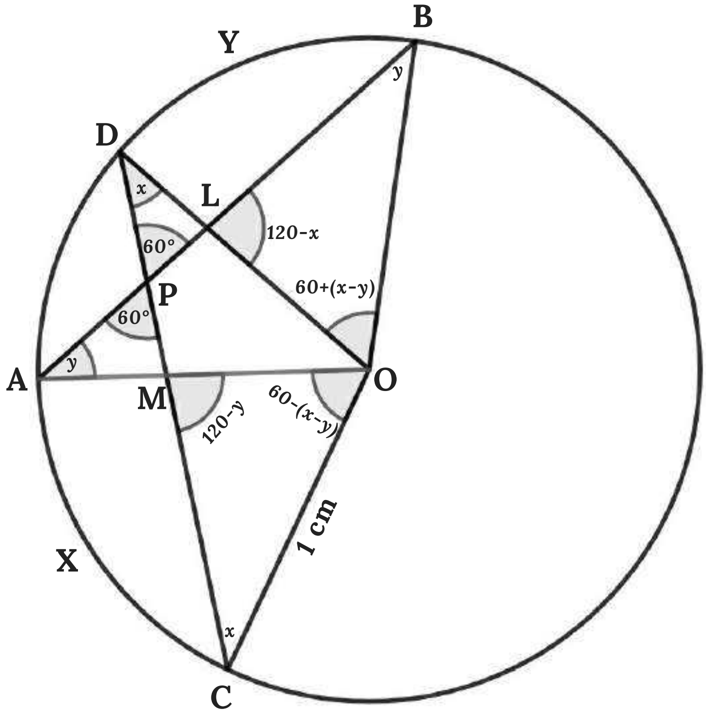
$2R^2 = 5$

$\Rightarrow$ $\mathbf{R^2 = 5/2}$

Area of quarter circle $= \frac{1}{4}\times\pi R^2 = \frac{1}{4}\pi\times 5/2$

Area of quarter circle = $\frac{5}{8}\pi$ cm²

SOLUTION 22



From figure

Let $\angle D = x$ & $\angle A = y$

then $\angle C = x$ & $\angle B = y$

From ΔPLD

$\angle PLD = 180 - (60+x)$

$\Rightarrow$ **$\angle PLD = 120-x$**

$\angle OLB = \angle PLD$

$\Rightarrow$ **$\angle OLB = 120-x$**

From ΔPMA

$\angle PMA = 180 - (60+y)$

$\Rightarrow$ **$\angle PMA = 120-y$**

$\angle OMC = \angle PMA$

$\Rightarrow$ **$\angle OMC = 120-y$**

From ΔOCM

$\angle COM = 180 - (\angle C + \angle PMA)$

$\Rightarrow$ $\angle COM = 180 - (x + 120-y)$

$= 60-(x-y)$

$\angle COA = \angle COM = 60-(x+y)$

From ΔOLB

$\angle BOL = 180 - (\angle B + \angle OLB)$

$\Rightarrow$ $\angle BOL = 180 - (y + 120-x)$

$= 60+(x-y)$

$\Rightarrow$ **$\angle BOD = \angle BOL = 60+(x-y)$**

Length of (*arc*(AXC) + *arc*(BYD)) = $(\angle COA/360)\times 2\pi R + (\angle BOD/360)\times 2\pi R$

$= ((60-(x-y))/360)\times 2\pi R + ((60+(x-y))/360)\times 2\pi R$

$= (120/360)\times 2\pi\times 1$ {R = 1 cm}

$= \frac{1}{3}\times 2\pi$

length of [arc(AXC) + arc(BYD)] = ⅔π cm

SOLUTION 23

From Fig(1)

∠BAC = 180 - ∠ROP

⇒ ∠BAC = 180 - 150

⇒ **∠BAC = 30°**

∠ABC = 180 - ∠ROQ

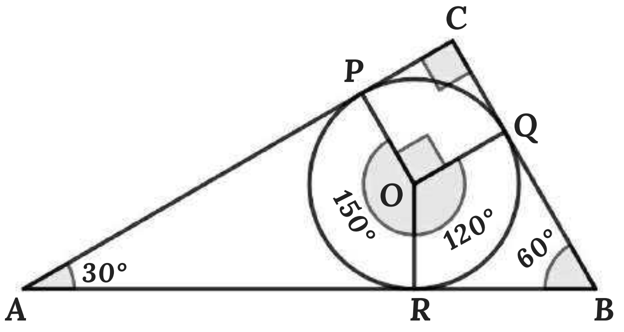
⇒ ∠ABC = 180 - 120

⇒ **∠ABC = 60°**

∠BCA = 180 - ∠POQ

⇒ ∠BCA = 180 - 90

⇒ **∠BCA = 90°**



Fig(1)

From Fig(2)

∠OAB = ∠BAC/2 {AR = AP & OR = OP}

⇒ ∠OAB = 30/2

⇒ **∠OAB = 15°**

∠OBA = ∠ABC/2 {BR = BQ & OR = OQ}

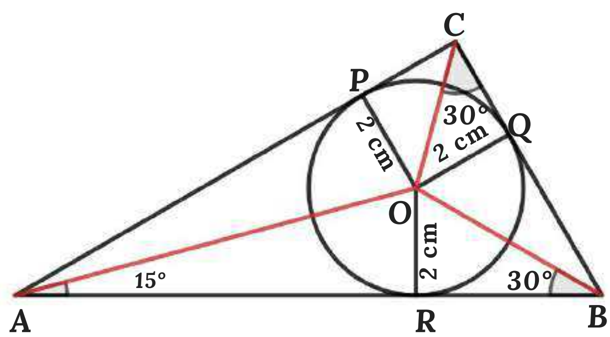
⇒ ∠OBA = 60/2

⇒ **∠OBA = 30°**

∠OCB = ∠BCA/2 {CQ = CP & OP = OQ}

⇒ ∠OCB = 90/2

⇒ **∠OCB = 45°**



Fig(2)

From ΔAOR

tan 15 = OR/AR

⇒ AR = 2/tan 15

= 4+2√3 cm

⇒ **AP = AR = 4+2√3 cm**

From ΔBOR

$\tan 30 = OR/BR$

⇒ $BR = 2/\tan 30$

$= 2\sqrt{3}$ cm

⇒ **$BQ = BR = 2\sqrt{3}$ cm**

From ΔCOQ

$\tan 45 = OQ/CQ$

⇒ $CQ = 2/\tan 45$

$= 2$ cm

⇒ **$CP = CQ = 2$ cm**

Area of ΔABC = Area of ΔAOB + Area of ΔBOC + Area of ΔAOC

Area of $\Delta AOB = \frac{1}{2} \times AB \times OR$

⇒ Area of $\Delta AOB = \frac{1}{2} \times (AR+BR) \times OR$

$= \frac{1}{2} \times (4+2\sqrt{3} + 2\sqrt{3}) \times 2$

⇒ **Area of $\Delta AOB = 4+4\sqrt{3}$ cm^2**

Area of $\Delta BOC = \frac{1}{2} \times BC \times OQ$

⇒ Area of $\Delta BOC = \frac{1}{2} \times (BQ+CQ) \times OQ$

$= \frac{1}{2} \times (2\sqrt{3}+2) \times 2$

⇒ **Area of $\Delta BOC = 2\sqrt{3}+2$ cm^2**

Area of $\Delta AOC = \frac{1}{2} \times AC \times OP$

⇒ Area of $\Delta AOC = \frac{1}{2} \times (AP+CP) \times OP$

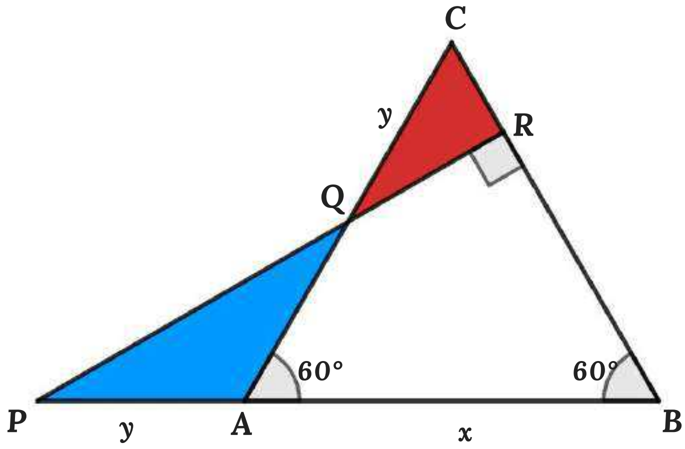
$= \frac{1}{2} \times (4+2\sqrt{3} + 2) \times 2$

⇒ **Area of $\Delta AOC = 6+2\sqrt{3}$ cm^2**

Area of $\Delta ABC = 4+4\sqrt{3}$ cm^2 + $2\sqrt{3}+2$ cm^2 + $6+2\sqrt{3}$ cm^2

⇒ ***Area of $\Delta ABC = 12+8\sqrt{3}$ cm^2***

SOLUTION 24



Red area = ½×RC×QR

Blue area = ½×PA×PQ×sin ∠APQ

Let AP = QC = y & AB = BC = AC = x

From figure

∠BAC = 60°

∠ABC = 60°

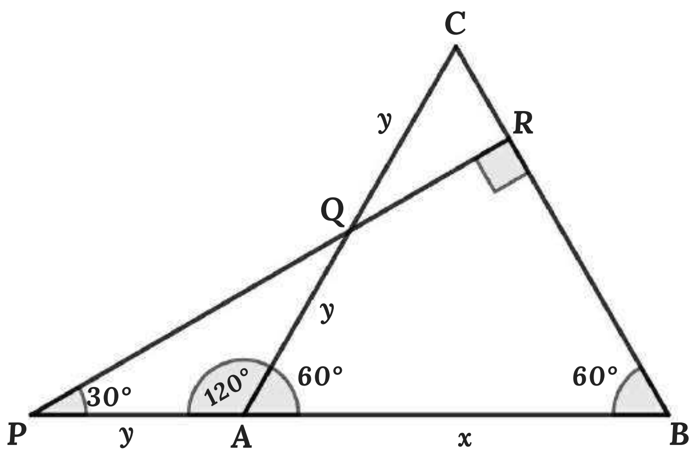
∠BRP = 90°

∠D = 180 - (∠BRP + ∠B)

⇒ ∠D = 180 - (90 + 60)

= 180 - 150

⇒ **∠D = 30°**



From ΔPAQ

∠PAQ = 180 - ∠BAC

⇒ ∠PAQ = 180 - 60

⇒ ∠PAQ = 120°

∠AQP = 180 - (∠QAP + ∠APQ)

⇒ ∠PAQ = 180 - (30 + 120)

⇒ ∠PAQ = 30°

⇒ ΔPAQ *is a isosceles triangle*

⇒ **AP = AQ = y**

AC = AQ + QC

⇒ $x = y + y$

$= 2y$

⇒ $\mathbf{y = ½x}$

From ΔPRB

sin 30 = BR/PB

⇒ BR = PB sin 30

= $(x+y)$ × ½

= ½(x+½x)

⇒ **BR = ¾x**

RC = BC - BR

⇒ RC = $x - \frac{3}{4}x$

⇒ **RC = $\frac{1}{4}x$**

Red area = $\frac{1}{2}$×RC×QC×sin ∠ACB

⇒ Red area = $\frac{1}{2} \times \frac{1}{2}x \times \frac{1}{4}x \times \sin 60$

⇒ **Red area = $(\sqrt{3}/32)x^2$**

Blue area = $\frac{1}{2}$×PA×PQ×sin ∠APQ

⇒ Blue area = $\frac{1}{2} \times \frac{1}{2}x \times \frac{1}{2}x \times \sin 120$

⇒ **Blue area = $(\sqrt{3}/16)x^2$**

Blue Area : Red area = $(\sqrt{3}/16)x^2 : (\sqrt{3}/32)x^2$

⇒ Blue area : Red Area = 1/16 : 1/32

⇒ ***Blue area : Red area = 2 : 1***

SOLUTION 25

From Fig(1)

$\angle ABC = 60°$

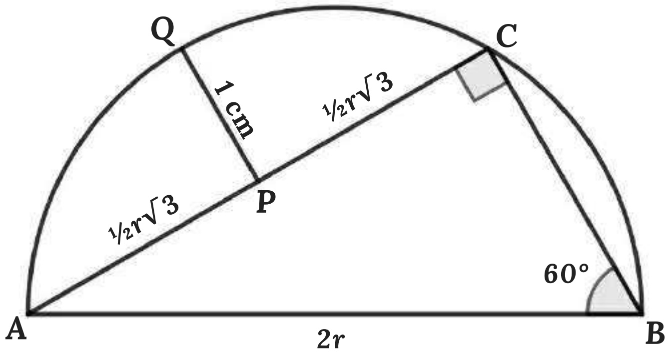
$\angle ACB = 90°$ {ACB is a semicircle}

$\sin \angle ABC = AC/AB$

⇒ $\sin 60 = AC/AB$

⇒ $\frac{1}{2}\sqrt{3} = AC/AB$

⇒ $AC = AB \times \frac{1}{2}\sqrt{3}$



Fig(1)

Let diameter of the semicircle = $2r$, then

$AB = 2r$

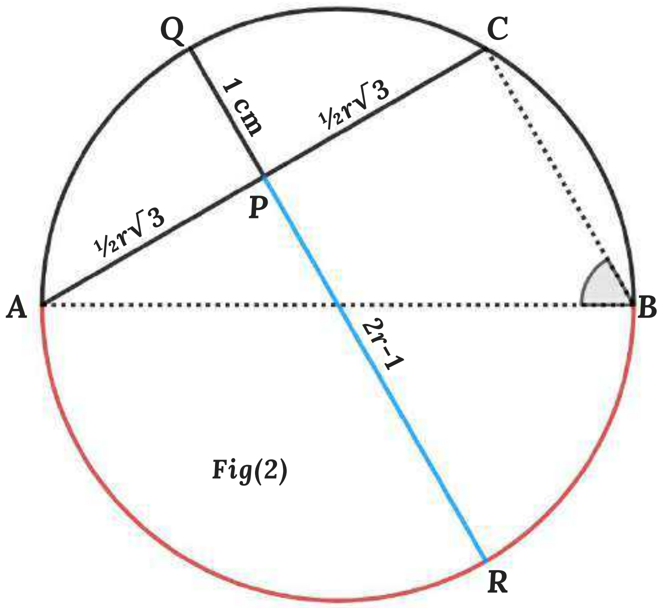
⇒ $AC = 2r \times \frac{1}{2}\sqrt{3}$

⇒ $AC = r\sqrt{3}$

We Know PA = PC, then

PA = PC = AC/2

⇒ $\mathbf{PA = PC = \frac{1}{2}r\sqrt{3}}$



Fig(2)

From Fig(2)

We can convert this semicircle to a circle, so

PQ×PR = PA×PC {intersecting chords theorem or chord theorem}

⇒ $1 \times PR = \frac{1}{2}r\sqrt{3} \times \frac{1}{2}r\sqrt{3}$

⇒ $\mathbf{PR = \frac{3}{4}r^2}$.................................eq(1)

$QR = 2r$ {Diameter of the circle}

PR = QR - PQ

⇒ $\mathbf{PR = 2r-1}$.................................eq(2)

From $eq(1)$ & $eq(2)$

$PR = 2r - 1 = \frac{3}{4}r^2$

$\Rightarrow 8r - 4 = 3r^2$

$\Rightarrow 3r^2 - 8r + 4 = 0$

$\Rightarrow r = \frac{1}{6}(8 \pm \sqrt{(64 - 4 \times 3 \times 4)}$

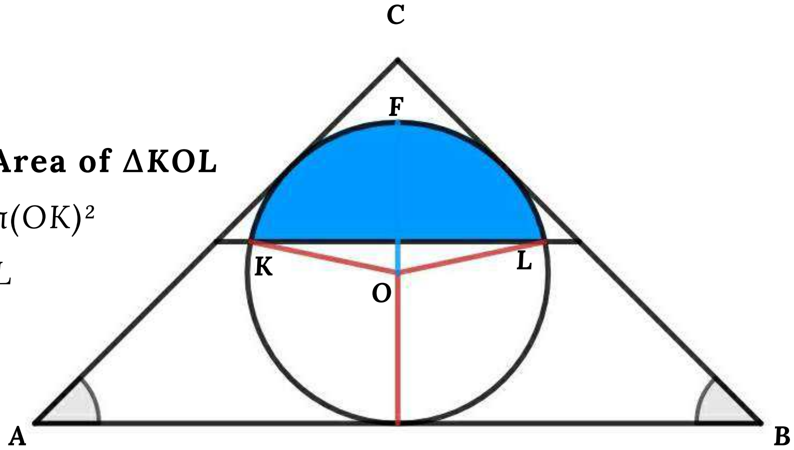
$= \frac{1}{6}(8 \pm \sqrt{16})$

$= \frac{1}{6}(8 \pm 4)$

$\Rightarrow r = 2$ cm $\{r > 1$ cm$\}$

$\Rightarrow$ ***Radius of the circle = 2 cm***

SOLUTION 26



Blue Area = Area of Sector (KFL) - Area of ΔKOL

Area of Sector (KFL) = (∠KOL/360)π$(OK)^2$

Area of ΔKOL = ½×OK×OL×sin ∠KOL

From *Fig*(2)

∠ACB = 90°

AC = BC

⇒ ΔABC is a isosceles triangle

⇒ ∠CAB = ∠ABC

= ½(180 - ∠ACB)

= ∠ABC = ½(180 - 90)

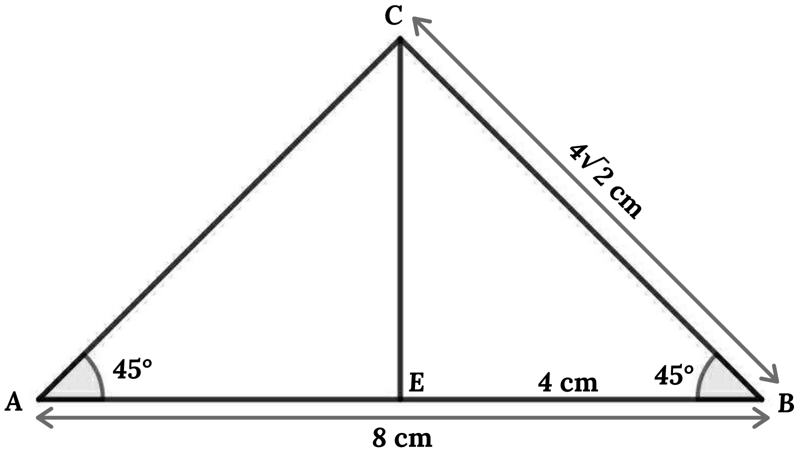
⇒ **∠CAB = ∠ABC = 45°**

If CE ⊥ AB

⇒ AE = BE = AB/2

= 8/2

⇒ **AE = BE = 4 cm**



Fig(2)

From ΔAEC

tan 45 = CE/AE

⇒ 1 = CE/4

⇒ **CE = 4 cm**

$BE^2 + CE^2 = BC^2$

⇒ $4^2 + 4^2 = BC^2$

⇒ **BC = 4√2 cm**

From $Fig(3)$

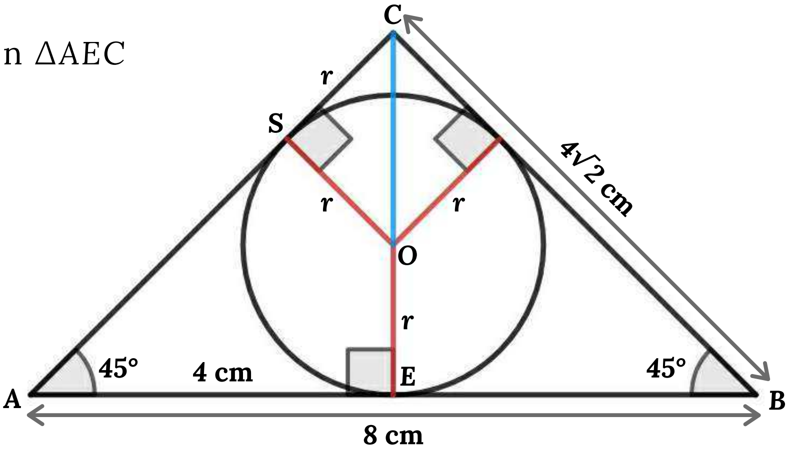
We can apply pythagorean theorem on ΔAEC

$OS^2 + CS^2 = OC^2$

⇒ $r^2 + r^2 = OC^2$

⇒ $OC = r\sqrt{2}$

⇒ $\mathbf{CE = r+r\sqrt{2}}$



Fig(3)

$CE = r+r\sqrt{2} = 4$

⇒ $r = 4/(1+\sqrt{2})$

⇒ $\mathbf{r = 4\sqrt{2} - 4 \text{ cm}}$

From $Fig(4)$

Let $XK = XL = x$

$XE \times XF = XK \times XL$ {*Intersecting chords theorem*}

⇒ $XE \times XF = x \times x$

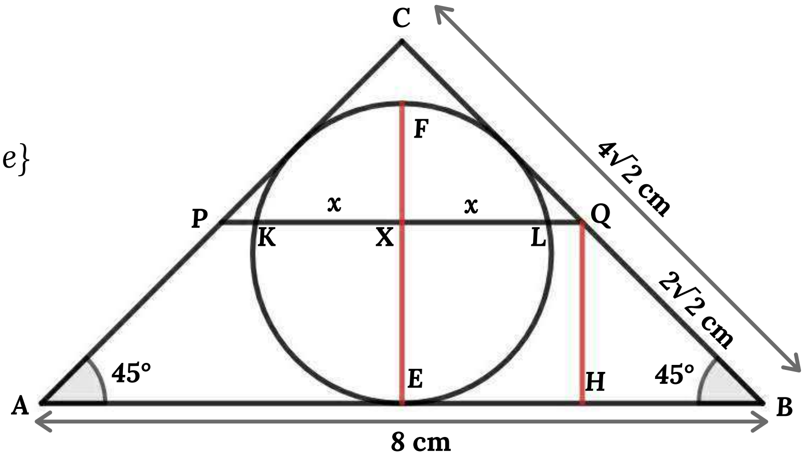
⇒ $\mathbf{x^2 = XE \times XF}$

$XE = HQ$ {*XEHQ is a rectangle*}

$\sin 45 = HQ/QB$

⇒ $\sin 45 = HQ/QB$

⇒ $\mathbf{HQ = QB \sin 45}$



Fig(4)

$QB = BC - QC$

⇒ $QB = 4\sqrt{2} - QC$

AB // PQ then, ΔABC & ΔPQC similar so

$BC/QC = AB/PQ$

⇒ $8/4 = 4\sqrt{2}/QC$

⇒ $QC = ½ \times 4\sqrt{2}$

⇒ $\mathbf{QC = 2\sqrt{2} \text{ cm}}$

$QB = 4\sqrt{2} - QC$

⇒ $QB = 4\sqrt{2} - 2\sqrt{2}$

⇒ $\mathbf{QB = 2\sqrt{2} \text{ cm}}$

HQ = QB sin 45

⇒ HQ = $2\sqrt{2} \times \frac{1}{2}\sqrt{2}$

= 2 cm

⇒ **XE = HQ = 2 cm**

EF = $2r$

⇒ EF = $2(4\sqrt{2} - 4))$

⇒ **EF = $8\sqrt{2} - 8$ cm**

XF = EF - XE

⇒ XF = $8\sqrt{2} - 8 - 2$

⇒ **XF = $8\sqrt{2}-10$ cm**

x^2 = XE × XF

⇒ $x^2 = 2(8\sqrt{2}-10)$

$= 16\sqrt{2} - 20$

⇒ **$x \approx 1.621$**

From *Fig*(5)

sin ∠XOQ = x/r

⇒ sin ∠XOQ = $1.621/(4\sqrt{2} - 4)$

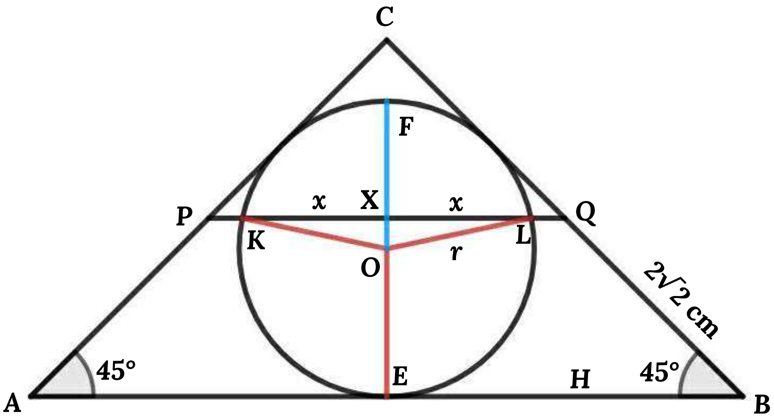
≈ 0.978

⇒ ∠XOQ = $\sin^{-1}$ (0.978)

= 78.047°

⇒ ∠KOQ = 2∠XOQ

⇒ **∠KOQ= 156.094°**



Fig(5)

Blue Area = Area of Sector (KFL) - Area of ΔKOL

⇒ Blue Area = $(156.094/360)\pi r^2 - \frac{1}{2} \times r \times r \sin 156.094$

= 3.739 - 0.556

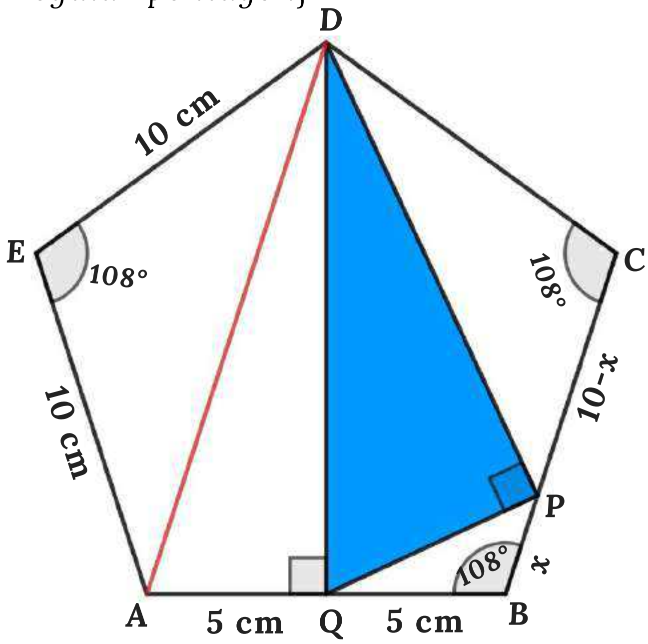
⇒ **Blue Area = 3.183 cm²**

SOLUTION 27

From figure

$\angle E = \angle B = \angle C = 108°$ *{Internal angles of the regular pentagon}*

Area of the blue triangle = ½×PQ×PD



Let $PB = x$, then $PC = 10-x$

From ΔADQ

$DQ^2 = AD^2 - AQ^2$ *{pythagorean theorem}*

Also from ΔPQD

$DQ^2 = PQ^2 + PD^2$ *{pythagorean theorem}*

That is $\mathbf{PQ^2 + PD^2 = AD^2 - AQ^2}$

From ΔAED

$AD^2 = EA^2 + ED^2 - 2 \times EA \times ED \times \cos \angle AED$ *{Cosine rule}*

$\Rightarrow AD^2 = 10^2 + 10^2 - 2\times10\times10\times\cos 108$

$\Rightarrow \mathbf{AD^2 = 200 - 200\cos 108}$

From ΔADQ

$DQ^2 = AD^2 - AQ^2$ *{pythagorean theorem}*

$\Rightarrow DQ^2 = 200 - 200\cos 108 - 5^2$

$\Rightarrow \mathbf{DQ^2 = 175 - 200\cos 108}$..*eq(1)*

From ΔPBQ

$PQ^2 = PB^2 + QB^2 - 2\times PB\times QB\times\cos\angle PBQ$

$\Rightarrow PQ^2 = x^2 + 5^2 - 2\times x\times5\cos 108$

$\Rightarrow \mathbf{PQ^2 = x^2 + 25 - 10x\cos 108}$

From ΔPCD

$PD^2 = CD^2 + PC^2 - 2\times CD\times PC\times\cos\angle PCD$

$\Rightarrow PD^2 = 10^2 + (10-x)^2 - 2\times10(10-x)\cos 108$

$= 10^2 + 100 - 20x + x^2 - 200\cos 108 + 20x\cos 108$

$\Rightarrow \mathbf{PD^2 = 200 - 20x + x^2 - 200\cos 108 + 20x\cos 108}$

From ΔPQD

$DQ^2 = PQ^2 + PD^2$ *{pythagorean theorem}*

$\Rightarrow DQ^2 = x^2 + 25 - 10x\cos 108 + 200 - 20x + x^2 - 200\cos 108 + 20x\cos 108$

$\Rightarrow$ **$DQ^2 = 2x^2 + 225 - 20x + 10x\cos 108 - 200\cos 108$**.........*eq(2)*

From *eq(1)* & *eq(2)*

$DQ^2 = 175 - 200\cos 108 = 2x^2 + 225 - 20x + 10x\cos 108 - 200\cos 108$

$\Rightarrow 0 = 2x^2 + 50 - 20x + 10x\cos 108$

$= 2x^2 + 50 - 20x + 10x(\frac{1}{4}(1-\sqrt{5}))$

$= 2x^2 + 50 - 20x + \frac{1}{2}(5x) - \frac{1}{2}(5x\sqrt{5})$

$\Rightarrow 4x^2 + 100 - 40x + 5x - 5x\sqrt{5} = 0$

$\Rightarrow 4x^2 - 35x - 5x\sqrt{5} + 100 = 0$

$\Rightarrow x = \frac{1}{8}(35+5\sqrt{5} \pm \sqrt{((35+5\sqrt{5})^2 - 4\times4\times100)})$

$= \frac{1}{8}(35+5\sqrt{5} \pm \sqrt{(1225 + 350\sqrt{5} + 125 - 1600)})$

$= \frac{1}{8}(35+5\sqrt{5} \pm \sqrt{(350\sqrt{5} - 250)})$

$\Rightarrow$ **$x_1 \approx 8.6574$ cm & $x_2 \approx 2.8877$ cm**

If $x = 8.6574$

$PQ^2 = x^2 + 25 - 10x\cos 108$

$\Rightarrow PQ^2 = 8.6574^2 + 25 - 10\times8.6574\times\cos 108$

≈ 126.703

$\Rightarrow$ **$PQ \approx 11.2562$ cm**

$PD^2 = 200 - 20x + x^2 - 200\cos 108 + 20x\cos 108$

$\Rightarrow PD^2 = 200 - 20\times8.6574 + 8.6574^2 - 200\cos 108 + 20\times8.6574\times\cos 108$

$= 110.0933$

$\Rightarrow$ **$PD = 10.4925$ cm**

Area of blue triangle = $\frac{1}{2}\times11.2562\times10.4925$

$\Rightarrow$ **Area of blue triangle = $59.0528\ cm^2$**

If $x = 2.8877$

$PQ^2 = x^2 + 25 - 10x\cos 108$

$\Rightarrow PQ^2 = 2.8877^2 + 25 - 10\times2.8877\times\cos 108$

≈ 42.2582

$\Rightarrow$ **PQ ≈ 6.5006 cm**

$PD^2 = 200 - 20x + x^2 - 200\cos 108 + 20x\cos 108$

$\Rightarrow PD^2 = 200 - 20\times2.8877 + 2.8877^2 - 200\cos 108 + 20\times2.8877\times\cos 108$

$= 194.5412$

$\Rightarrow$ **PD = 13.9478 cm**

Area of blue triangle = $\frac{1}{2}\times6.5006\times13.9478$

$\Rightarrow$ **Area of blue triangle = 45.3345 cm²**

Area of blue triangle = 59.0528 cm² or 45.3345 cm²

SOLUTION 28

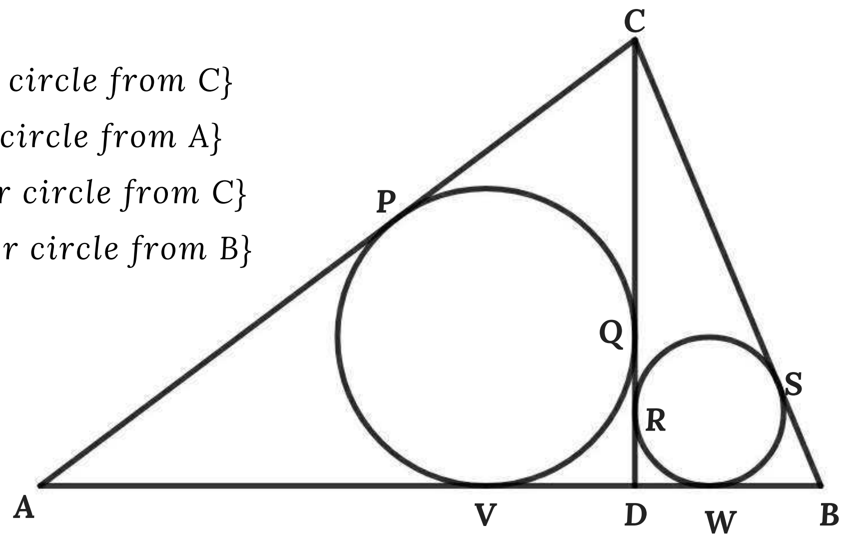
From figure

CP = CQ {*tangents of the larger circle from C*}

AP = AV {*tangents of the larger circle from A*}

CR = CS {*tangents of the smaller circle from C*}

BW = BS {*tangents of the smaller circle from B*}



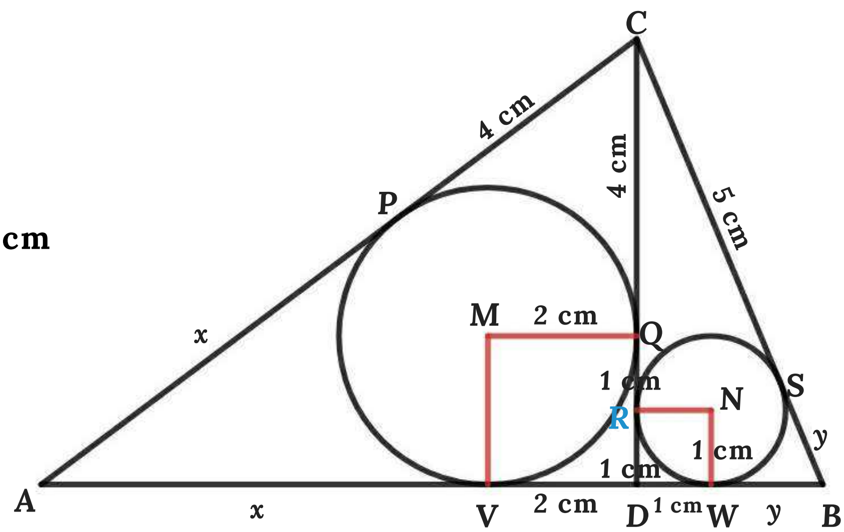
Area of circle = 4π cm²

⇒ $\pi\, MQ^2 = 4\pi$

⇒ $MQ^2 = 4$

⇒ MQ = 2 cm

⇒ **MQ = MV = VD = QD = 2 cm**



Area of circle = π cm²

⇒ $\pi\, RN^2 = \pi$

⇒ $RN^2 = 1$

⇒ RN = 1 cm

⇒ **RN = NW = WD = DR = 1 cm**

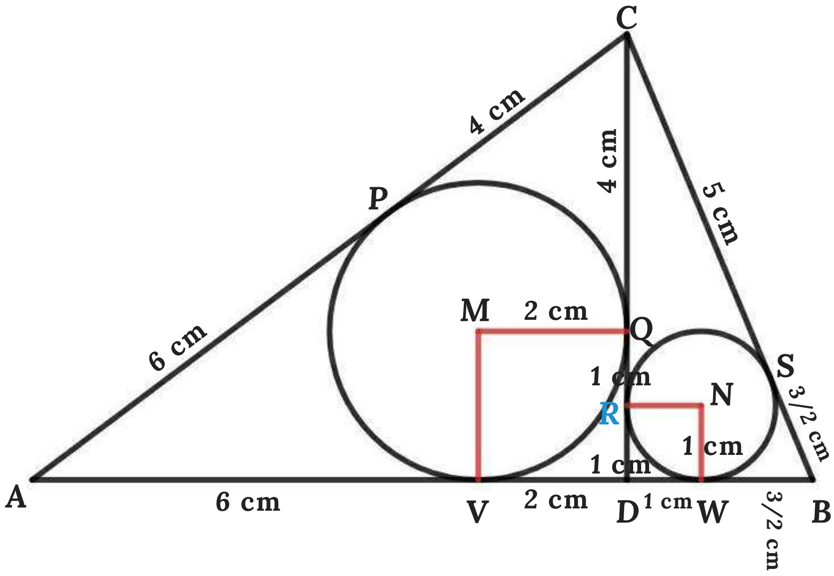
Let AP = x & BS = y

AP = AV = x

CR = CQ + QR

⇒ CR = 4+1

⇒ **CR = CS = 5 cm**



BS = BW = y

$AC^2 = AD^2 + CD^2$

$\Rightarrow (x+4)^2 = (x+2)^2 + 6^2$

$\Rightarrow x^2+8x+16 = x^2+4x+4 + 36$

$\Rightarrow 4x = 24$

$\Rightarrow$ **$x = 6$ cm**

$BC^2 = CD^2 + BD^2$

$\Rightarrow (y+5)^2 = 6^2 + (y+1)^2$

$\Rightarrow y^2+10y+25 = 36 + y^2+2y+1$

$\Rightarrow 8y = 12$

$\Rightarrow$ **$y = 3/2$**

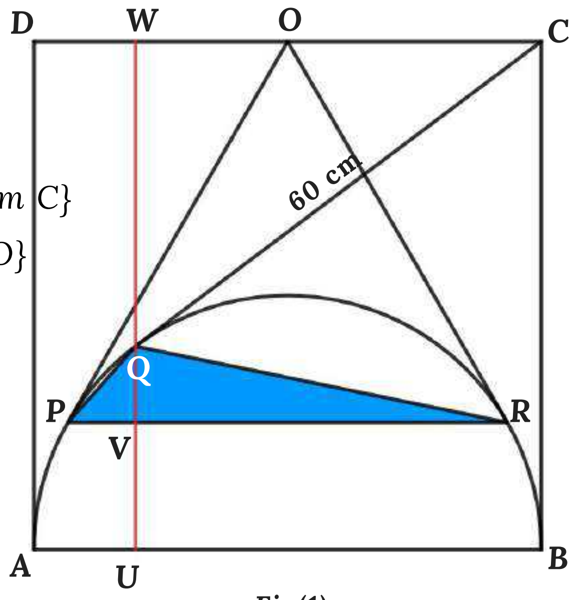
Area of the triangle = $\frac{1}{2}\times AB\times CD$

$\Rightarrow$ Area of the triangle = $\frac{1}{2}\times(x+3+y)\times 6$

$= \frac{1}{2}\times(6+3+3/2)\times 6$

$\Rightarrow$ ***Area of the triangle = 31.5 cm²***

SOLUTION 29



Fig(1)

Area of ΔPQR = ½×PQ×VQ

From *Fig*(1)

BC = CQ = 60 cm {*tangents of the semicircle from* C}

OP = OR {*tangents of the semicircle from* O}

Let ∠POT = ∠ROT = θ {*symmetry*}

OT = BC = 60 cm

From *Fig*(2)

sin θ = RT/OT

⇒ sin θ = 30/60

= ½

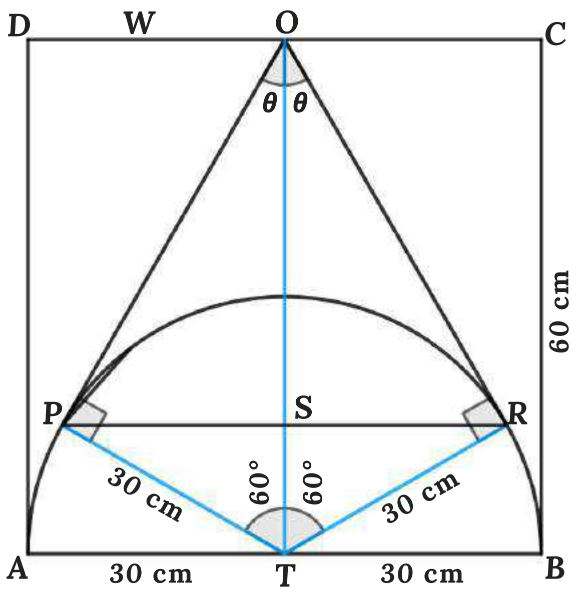
⇒ θ = $\sin^{-1}$ ½

= 30°

∠OTR = 180-(∠ROT + ∠ORT)

⇒ ∠OTR = 180-(30 + 90)

⇒ **∠OTR = 60°**



Fig(2)

From ΔPTR

PR = 2×RS {*symmetry*}

sin 60 = RS/RT

⇒ sin 60 = RS/30

⇒ RS = 30sin 60

= 30×½√3

= 30×½√3

⇒ RS = 15√3 cm

⇒ **PR = 30√3 cm**

cos 60 = ST/RT

⇒ cos 60 = ST/30

⇒ ST = 30cos 60

= 30×½

⇒ **ST = 15 cm**

From *Fig*(3)

$CT^2 = BC^2 + BT^2$

$\Rightarrow$ $CT^2 = 60^2 + 30^2$

$= 3600 + 900$

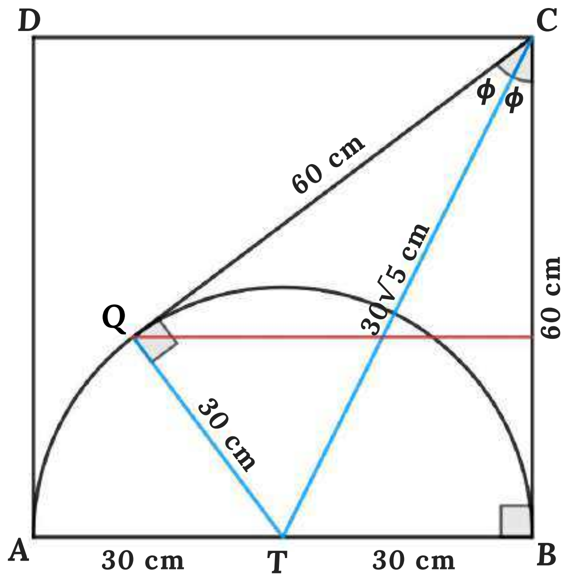
$= 4500$

$\Rightarrow$ $\mathbf{CT = 30\sqrt{5}\ cm}$

$\sin \phi = BT/CT$

$\Rightarrow$ $\sin \phi = 30/30\sqrt{5}$

$\Rightarrow$ $\mathbf{\sin \phi = \frac{1}{5}\sqrt{5}}$



Fig(3)

From ΔQCH

$\cos 2\Phi = CH/QC$

$\Rightarrow$ $CH = PC \times \cos 2\Phi$

$= 60 \times (1 - 2\sin^2 \Phi)$

$= 60 \times (1 - 2 \times \frac{1}{5})$

$= 60 \times \frac{3}{5}$

$\Rightarrow$ $\mathbf{CH = 36\ cm}$

From *Fig*(4)

$VQ = UW - (UV + QW)$

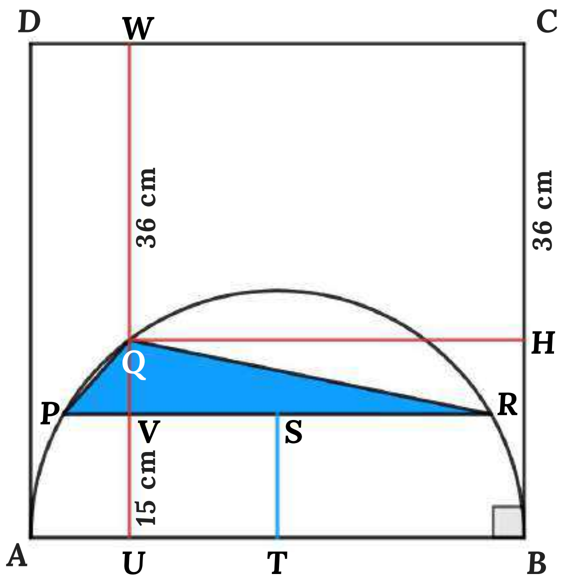
$\Rightarrow$ $VQ = 60 - (15 + 36)$

$\Rightarrow$ $\mathbf{VQ = 9\ cm}$

Area of $\Delta PQR = \frac{1}{2} \times PQ \times VQ$

$\Rightarrow$ Area of $\Delta PQR = \frac{1}{2} \times 30\sqrt{3} \times 9$

$\Rightarrow$ ***Area of $\Delta PQR = 135\sqrt{3}\ cm^2$***



Fig(4)

SOLUTION 30

From *Fig(1)*

$\angle C = 60°$

If $OC \perp AB$ then $OA = OB$

⇒ $OA = OB = 12/2$

⇒ **$OA = OB = 6$ cm**

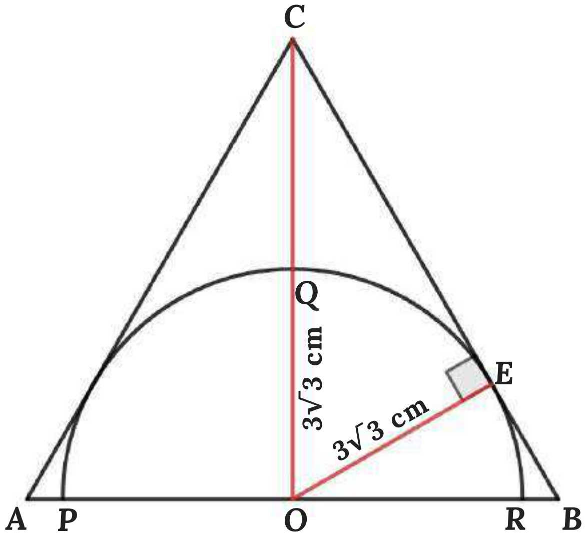
From ΔOBC

$OC^2 = BC^2 - OB^2$

⇒ $OC^2 = 12^2 - 6^2$

$= 108$

⇒ **$OC = 6\sqrt{3}$ cm**



Fig(1)

From ΔOEC

$\angle OCE = 30°$

$\sin \angle OCE = OE/OC$

⇒ $\sin 30 = OE/OC$

⇒ $OE = OC \sin 30$

$= 6\sqrt{3} \times ½$

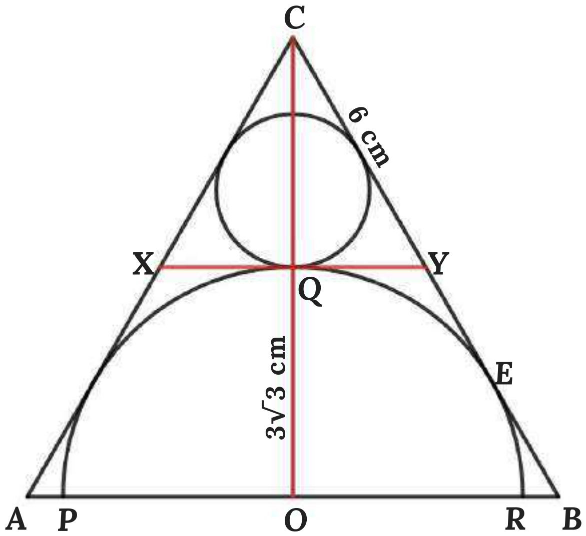
⇒ **$OE = 3\sqrt{3}$ cm**

From *Fig(2)*

$CQ = OC - OQ$

⇒ $CQ = 6\sqrt{3} - 3\sqrt{3}$

⇒ **$CQ = 3\sqrt{3}$ cm**



Fig(2)

If XY is the tangent of semicircle & circle

Then the circle is incircle of ΔCXY

ΔCXY is an equilateral triangle {$CX=CY$ & $\angle ACB = 60°$}

so $\angle CYX = 60°$

⇒ $\sin \angle CYX = CQ/CY$

⇒ $CY = CQ/\sin \angle CYX$

$= 3\sqrt{3}/\sin 60$

$= 3\sqrt{3}/(½\sqrt{3})$

⇒ **$CY = 6$ cm**

From ΔCXY

incircle radius = Area of $\Delta CXY/(\frac{1}{2}\times$Perimeter of $\Delta CXY\}$

Area of $\Delta CXY = \sqrt{3} \times 6^2/4$

⇒ **Area of $\Delta CXY = 9\sqrt{3}$ cm²**

Perimeter of $\Delta CXY = 3 \times 6$

⇒ **Perimeter of ΔCXY = 18 cm**

incircle radius = $9\sqrt{3}/(\frac{1}{2}\times 18)$

⇒ **incircle radius = $\sqrt{3}$ cm**

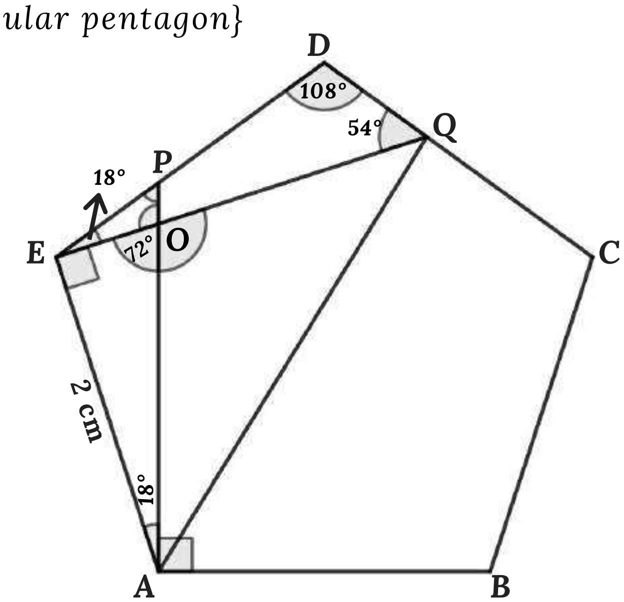
Area of circle = $\pi(\sqrt{3})^2$

⇒ ***Area of circle = 3π cm²***

SOLUTION 31

From figure

$\angle BAE = \angle AED = \angle EDC = 108°$ {ABCDE *is a regular pentagon*}



Area of ΔAOE = ½×OA×OQ sin ∠AOQ

OQ = EQ - EO

$\angle PAE = \angle BAE - \angle PAB$

⇒ $\angle PAE = 108 - 90$

⇒ $\mathbf{\angle PAE = 18°}$

$\angle AOE = 180 - (\angle AEQ + \angle PAE)$

⇒ $\angle PAE = 180 - (90+18)$

⇒ $\mathbf{\angle PAE = 72°}$

From ΔAOE

$AE/\sin \angle AOE = EO/\sin \angle OAE = OA/\sin \angle AEO$

⇒ $2/\sin 72 = EO/\sin 18 = OA/\sin 90$

⇒ $OA/\sin 90 = 2/\sin 72$

⇒ **OA = 2/sin 72 cm**

$2/\sin 72 = EO/\sin 18 = OA/\sin 90$

⇒ $2/\sin 72 = EO/\sin 18$

⇒ $EO = 2 \sin 18/\sin 72$

$= 2 \sin 18/\cos 18$ {$\sin (90-x) = \cos x$}

⇒ **EO = 2 tan 18 cm**

From ΔEDQ

$ED/\sin \angle EQD = EQ/\sin \angle EDQ = DQ/\sin \angle DEQ$

⇒ $2/\sin 54 = EQ/\sin 108 = DQ/\sin 18$

⇒ $2 \sin 108/\sin 54 = EQ$

⇒ $EQ = 2 \times 2 \times \sin 54 \times \cos 54/\sin 54$

⇒ **EQ = 4 cos 54 cm**

$OQ = EQ - EO$

⇒ **OQ = 4 cos 54 - 2 tan 18 cm**

Area of ΔAOE = ½×OA×OQ sin ∠AOQ

⇒ Area of ΔAOE = ½×(2/sin 72)×(4 cos 54 - 2 tan 18) × sin 108

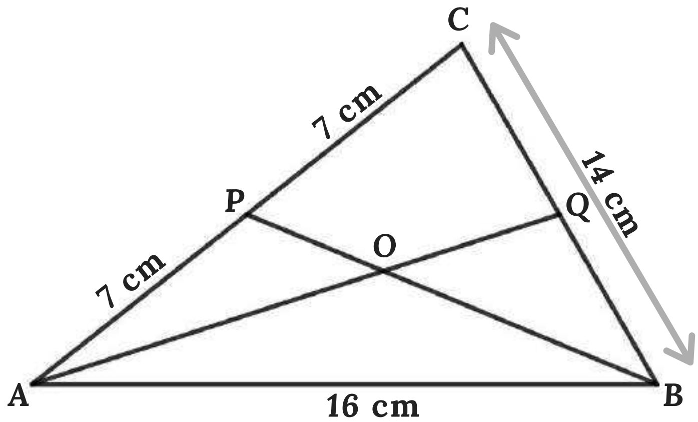
We know, sin (180-x) = sin x so sin 108 = sin 72

⇒ Area of ΔAOE = ½×(2/sin 72)×(4 cos 54 - 2 tan 18) × sin 72

⇒ ***Area of ΔAOE = 4 cos 54 - 2 tan 18***

⇒ ***Area of ΔAOE ≈ 1.7013 cm²***

SOLUTION 32



From figure

PA = PC = AC/2

⇒ PA = PC = AC/2

= 14/2

⇒ **PA = PC = 7 cm**

From ΔABC

$BC^2 = AB^2 + AC^2 - 2\times AB\times AC \cos \angle BAC$ *{cosine rule}*

⇒ $10^2 = 16^2 + 14^2 - 2\times16\times14 \cos \angle BAC$

⇒ $100 = 16^2 + 14^2 - 2\times16\times14 \cos \angle BAC$

$= 256 + 196 - 448 \cos \angle BAC$

$= 452 - 448 \cos \angle BAC$

⇒ $\cos \angle BAC = (452 - 100)/448$

$= 11/14$

⇒ $\angle BAC = \cos^{-1}(11/14)$

⇒ $\boldsymbol{\angle BAQ = \angle CAQ = \frac{1}{2}\cos^{-1}(11/14)}$

From ΔPAB

$PB^2 = AB^2 + AP^2 - 2\times AB\times AP \cos \angle BAC$ *{cosine rule}*

⇒ $PB^2 = 16^2 + 7^2 - 2\times16\times7 \cos \angle BAC$

⇒ $PB^2 = 256 + 49 - 224\times11/14$

⇒ $\boldsymbol{PB^2 = 129}$

$PA^2 = AB^2 + PB^2 - 2\times AB\times PB \cos \angle PAB$ *{cosine rule}*

⇒ $7^2 = 16^2 + 129 - 2\times16\times\sqrt{129} \cos \angle PAB$

⇒ $49 = 385 - 32\sqrt{129} \cos \angle PAB$

⇒ $\cos \angle PAB = (385-49)/(32\sqrt{129})$

⇒ $\boldsymbol{\cos \angle PAB = 21/(2\sqrt{129})}$

From ΔAOB

$\angle AOB = 180 - (\angle BAO + \angle ABO)$

$\Rightarrow \angle AOB = 180 - (\frac{1}{2}\cos^{-1}(11/14) + \cos^{-1}(21/(2\sqrt{129})))$

$\Rightarrow \cos \angle AOB = \cos[180 - (\frac{1}{2}\cos^{-1}(11/14) + \cos^{-1}(21/(2\sqrt{129})))]$

$= -\cos[\frac{1}{2}\cos^{-1}(11/14) + \cos^{-1}(21/(2\sqrt{129}))]$

$= -\cos(\frac{1}{2}\cos^{-1}(11/14))\cos\cos^{-1}(21/(2\sqrt{129}))$

$+ \sin(\frac{1}{2}\cos^{-1}(11/14))\sin\cos^{-1}(21/(2\sqrt{129}))$

We know $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ & $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ so,

$\cos^2[\frac{1}{2}\cos^{-1}(11/14)] = \frac{1}{2}(1+\cos 2[\frac{1}{2}\cos^{-1}(11/14)])$

$\Rightarrow \cos^2[\frac{1}{2}\cos^{-1}(11/14)] = \frac{1}{2}(1+\cos\cos^{-1}(11/14))$

$= \frac{1}{2}(1+ 11/14)$

$= 25/28$

$\Rightarrow$ **$\cos[\frac{1}{2}\cos^{-1}(11/14)] = 5/(2\sqrt{7})$**

$\sin^2[\frac{1}{2}\cos^{-1}(11/14)] = \frac{1}{2}(1-\cos 2[\frac{1}{2}\cos^{-1}(11/14)])$

$\Rightarrow \sin^2[\frac{1}{2}\cos^{-1}(11/14)] = \frac{1}{2}(1-\cos\cos^{-1}(11/14))$

$= \frac{1}{2}(1-(11/14))$

$= 3/28$

$\Rightarrow$ **$\sin[\frac{1}{2}\cos^{-1}(11/14)] = \sqrt{3}/2\sqrt{7}$**

also, $\sin\cos^{-1}(21/(2\sqrt{129})) = \cos^{-1}\sin(21/(2\sqrt{129}))$

$= \cos^{-1}\cos(5\sqrt{3}/(2\sqrt{129}))$

$\Rightarrow$ **$\sin\cos^{-1}(21/(2\sqrt{129})) = 5\sqrt{3}/(2\sqrt{129})$**

$\cos \angle AOB = -\cos(\frac{1}{2}\cos^{-1}(11/14))\cos\cos^{-1}(21/(2\sqrt{129}))$

$+ \sin(\frac{1}{2}\cos^{-1}(11/14))\sin\cos^{-1}(21/(2\sqrt{129}))$

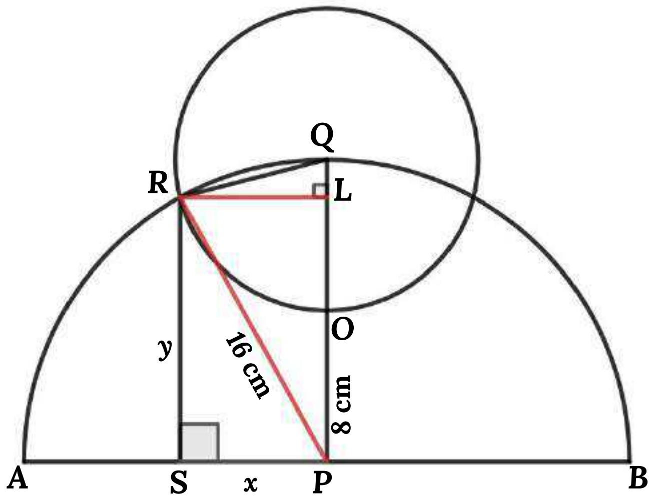
$\Rightarrow \cos \angle AOB = -[5/(2\sqrt{7})][21/(2\sqrt{129})] + [\sqrt{3}/2\sqrt{7}][5\sqrt{3}/(2\sqrt{129})]$

$= [15 - 105]/[4\sqrt{903}]$

$= -90/[4\sqrt{903}]$

$\Rightarrow$ **$\cos \angle AOB = -15\sqrt{903}/602$**

SOLUTION 33



From figure

PQ = PR = 16 cm {*Radius of the semicircle*}

OP = OQ = PQ/2

⇒ OP = OQ = 16/2

⇒ **OP = OQ = 8 cm**

⇒ **QR = 8 cm** {*Radius of the circle*}

Area of PQRS = Area of PLRS + Area of ΔRLQ

Let PS = x & RS = y, then

RS = LP = y

⇒ QL = PQ - LP

⇒ **QL = 16 - y**

RL = PS = x

Area of PLRS = PS×RS

⇒ Area of PLRS = xy

Area of ΔRLQ = ½×RL×QL

⇒ Area of ΔRLQ = ½$x(16-y)$

Area of PQRS = Area of PLRS + Area of ΔRLQ

⇒ **Area of PQRS = xy + ½$x(16-y)$**

From ΔRSP

$PS^2 + RS^2 = PR^2$

⇒ $x^2 + y^2 = 16^2$

⇒ $\mathbf{x^2+y^2 = 256}$.......................................*eq*(1)

From ΔQLR

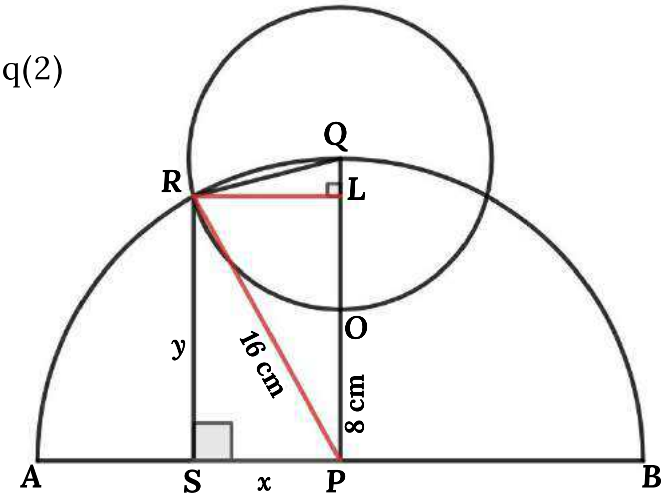
$RL^2 + QL^2 = RQ^2$

$\Rightarrow x^2 + (16-y)^2 = 8^2$

$\Rightarrow x^2 + 16^2 - 32y + y^2 = 8^2$

$\Rightarrow x^2 + 192 - 32y + y^2 = 0$

$\Rightarrow$ **$x^2+y^2 = 32y-192$**.................................eq(2)



From *eq*(1) & *eq*(2)

$x^2+y^2 = 256 = 32y-192$

$\Rightarrow 32y = 448$

$\Rightarrow$ **$y = 14$ cm**

From *eq*(1)

$x^2+y^2 = 256$

$\Rightarrow x^2+14^2 = 256$

$\Rightarrow x^2 = 256 - 196$

$= 60$

$\Rightarrow$ **$x = 2\sqrt{15}$ cm**

Area of PQRS = $xy + \frac{1}{2}x(16-y)$

$\Rightarrow$ Area of PQRS = $2\sqrt{15} \times 14 + \frac{1}{2} \times 2\sqrt{15}\ (16-14)$

$= 2\sqrt{15} \times 14 + \frac{1}{2} \times 2\sqrt{15}\ (16-14)$

$\Rightarrow$ ***Area of PQRS = $30\sqrt{15}$ cm***

SOLUTION 34

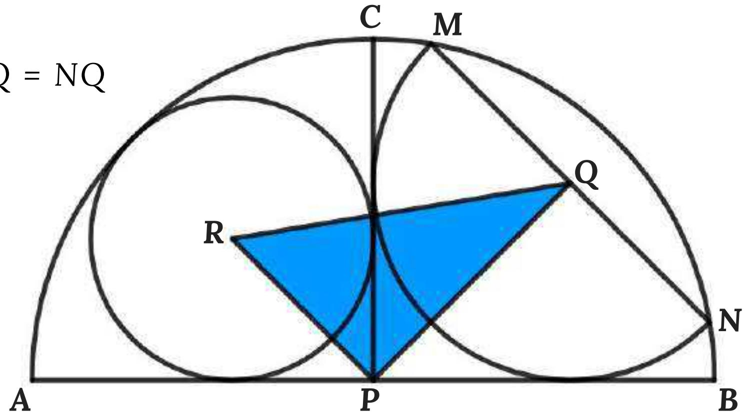
From Fig(1)

RP is pass-through incircle centre, that is RP is the angle bisector of $\angle APC$

$\Rightarrow \angle CPR = 45°$

MN is a chord of the semicircle and MQ = NQ

$\Rightarrow \angle CPQ = 45°$



Fig(1)

$\angle RPQ = \angle CPR + \angle CPQ$

$\Rightarrow \angle RPQ = 45 + 45 = 90°$

$= 90°$

$\Rightarrow$ **Area of ΔPQR = ½×PR×PQ**

From Fig(2)

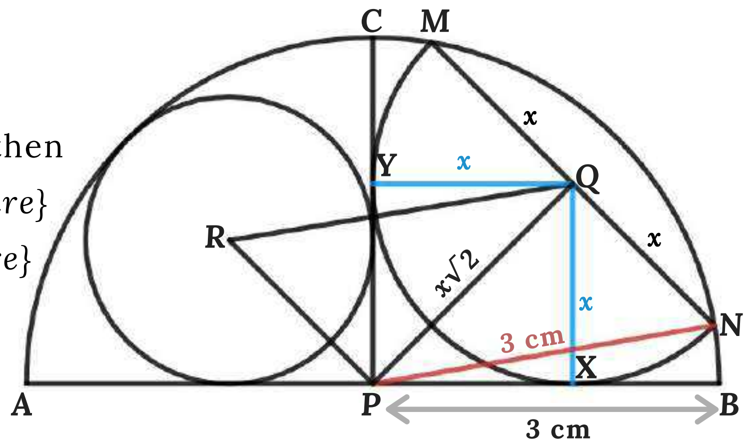
PB = 6/2 = 3 cm {*radius of semicircle*}

$\Rightarrow$ PN = 3 cm

Let the radius of small semicircle = x, then

PY = QY = QX = PX = x {*PXQY is a square*}

$\Rightarrow PQ = x\sqrt{2}$ {*PXQY is a square*}



Fig(2)

From ΔPQN

$PN^2 = PQ^2 + QN^2$

$\Rightarrow 3^2 = (x\sqrt{2})^2 + x^2$

$\Rightarrow 9 = 2x^2 + x^2$

$\Rightarrow x^2 = 3$

$\Rightarrow$ **$x = \sqrt{3}$ cm**

From Fig(3)

Let the radius of the circle = y, then

RH = RI = RJ = y {*radius of the circle*}

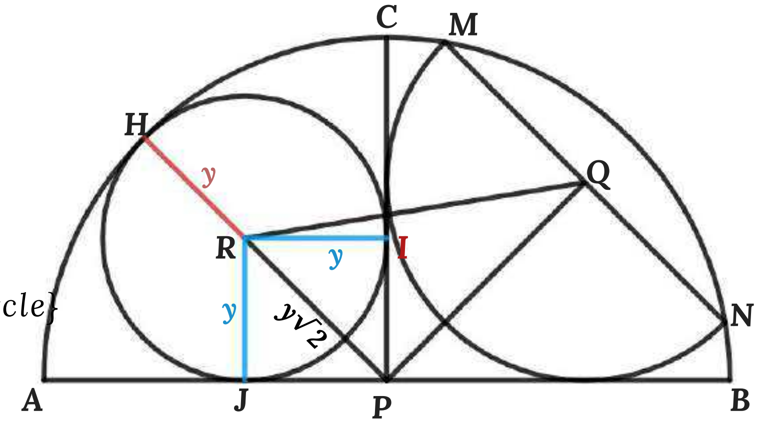
$\Rightarrow PR = y\sqrt{2}$

PH = $y + y\sqrt{2} = 3$ {*Radius of the semicircle*}

$\Rightarrow y + y\sqrt{2} = 3$

$\Rightarrow y = 3/(\sqrt{2} + 1)$

$\Rightarrow$ **$y = 3\sqrt{2} - 3$ cm**



Fig(3)

From *Fig*(4)

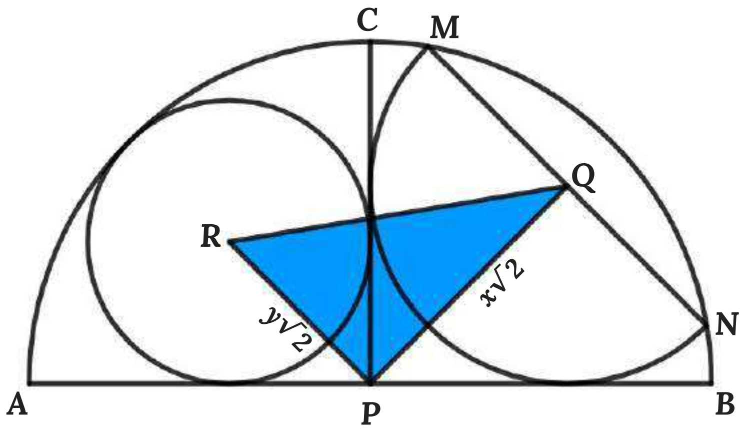
Area of $\Delta PQR = \frac{1}{2} \times PR \times PQ$

$\Rightarrow$ Area of $\Delta PQR = \frac{1}{2} \times y\sqrt{2} \times x\sqrt{2}$

$= xy$

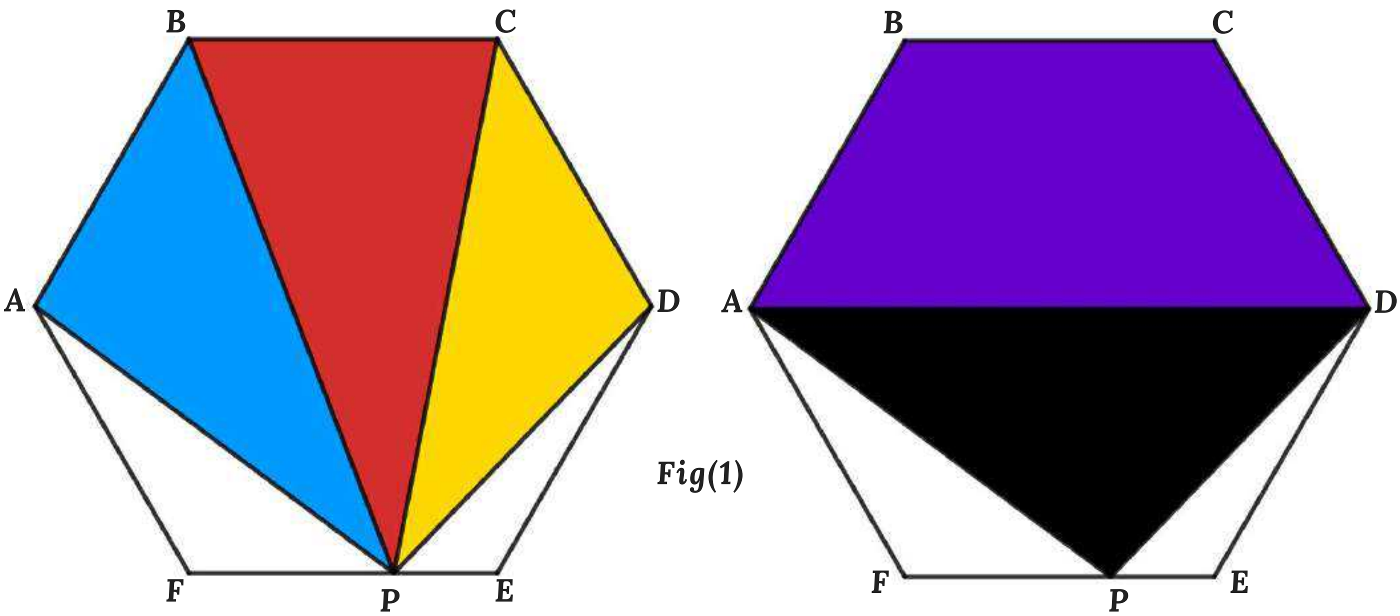
$= \sqrt{3} \times (3\sqrt{2} - 3)$

$\Rightarrow$ **Area of $\Delta PQR = 3\sqrt{6} - 3\sqrt{3}$ cm²**



Fig(4)

SOLUTION 35



***Fig*(1)**

From *Fig*(1)

Blue Area + Red Area + Yellow Area = Purple Area + Black Area

From *Fig*(2)

Red Area = Area of ΔPBC

⇒ **64 = ½×BC×PX**

PX = CE {*Parallel lines*}

Let"s sides of the hexagon = x, then

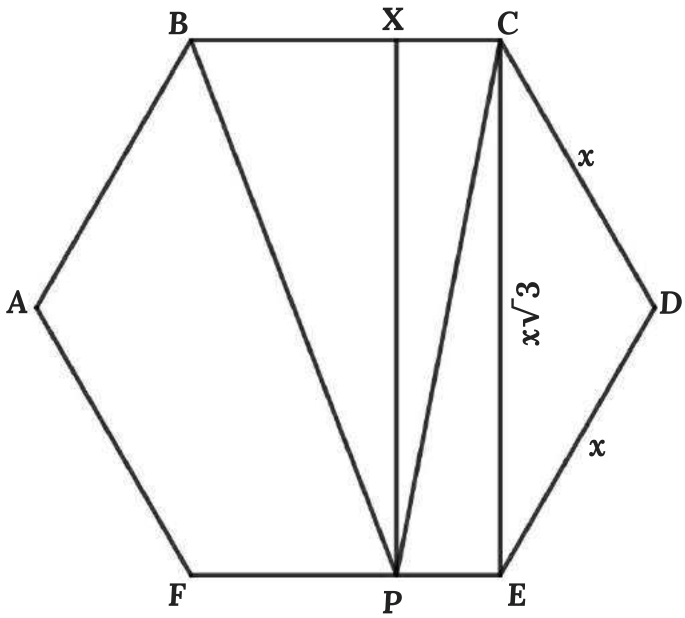
$CE^2 = CD^2 + ED^2 - 2 \times CD \times ED \cos D$

⇒ $CE^2 = x^2 + x^2 - 2 \times x \times x \cos 120$

$= 2x^2 - 2x^2 \times ½$

$= 3x^2$

⇒ **CE = PX =** $\mathbf{x\sqrt{3}}$



***Fig*(2)**

Red Area = Area of ΔPBC = ½ × BC × PX

⇒ $64 = ½ \times x \times x\sqrt{3} = ½x^2\sqrt{3}$

⇒ $\mathbf{x^2 = 128/\sqrt{3}}$

From *Fig*(3)

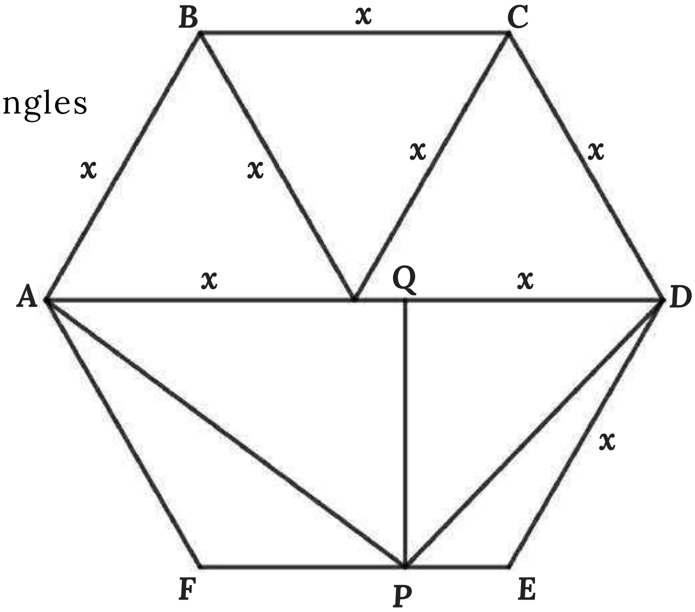
Purple Area = Area of ABCD

Area of ABCD = 3 × Area of equilateral triangles

⇒ Purple Area = $3 \times \frac{1}{4}x^2\sqrt{3}$

= $3 \times \frac{1}{4}(128/\sqrt{3})\sqrt{3}$

⇒ **Purple Area = 96 cm²**



***Fig*(3)**

Black Area = Area of ΔPAD

⇒ Black Area = ½×AD×PQ

PQ = ½PX

⇒ **PQ = ½x√3**

Black Area = ½×AD×PQ

⇒ Black Area = $\frac{1}{2}\times 2x \times \frac{1}{2}x\sqrt{3}$

= $\frac{1}{2}x^2\sqrt{3}$

= $\frac{1}{2}(128/\sqrt{3})\sqrt{3}$

⇒ **Black Area = 64 cm²**

Blue Area + Red Area + Yellow Area = Purple Area + Black Area

⇒ Blue Area + 64 + 42 = 96 + 64

⇒ Blue Area + 106 = 160

⇒ ***Blue Area* = 54 cm²**

SOLUTION 36

From *Fig*(1)

$\angle BAE = \angle AED = \angle EDC = \angle BCD = \angle ABC = 108°$

$AB = CB$

$\Rightarrow \angle BAC = \angle BCA = (180 - 108)/2$

$\Rightarrow$ **$\angle BAC = \angle BCA = 36°$**

Similarly, **$\angle PBA = 36°$**

$\angle ACD = 108 - \angle BCA = 108 - 36$

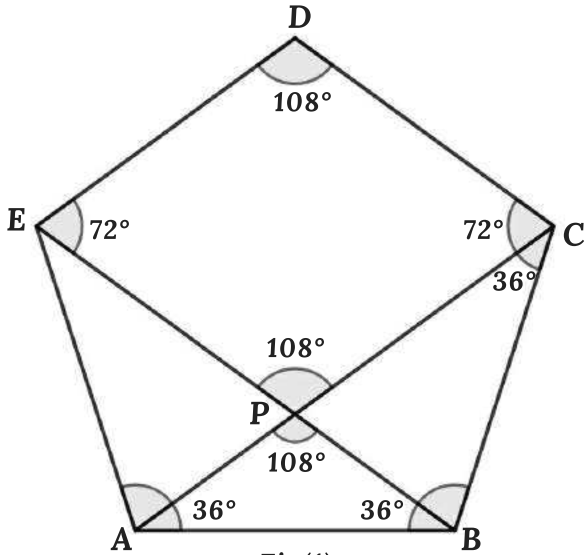
$\Rightarrow \angle ACD = 72°$

$\Rightarrow$ **$\angle ACD = \angle BED = 72°$**

$\angle CPE = 360 - (\angle ACD + \angle BED + \angle EDC)$

$\Rightarrow \angle CPE = 360 - (72 + 72 + 108) = 108°$

$\Rightarrow$**$\angle APB = \angle CPE = 108°$** {*opposite angles*}



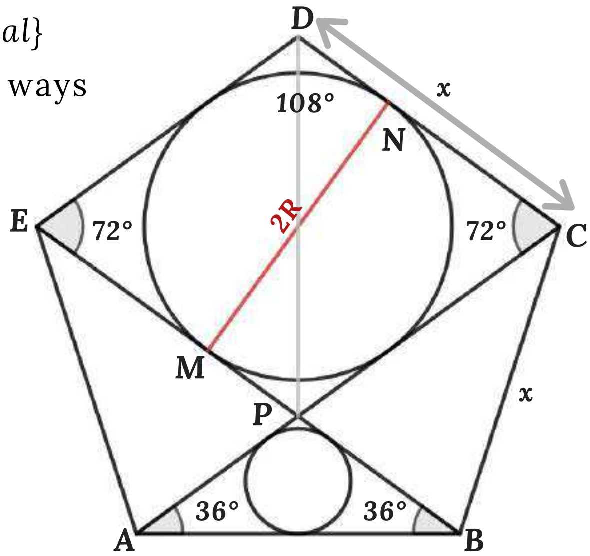
Fig(1)

From Figure, Let the radius of Large circle = R, Radius of small circle = *r*,

Sides of the pentagon = *x* & PA = PB = *y*

PCDE is a parallelogram {*opposite angles are equal*}

We can find the area of this parallelogram in two ways



Fig(2)

Method 1

Area of PCDE = Area of ΔPED + Area of ΔPCD

PE = CD = *x* {PCDE *is a parallelogram*}

PC = ED = *x* {PCDE *is a parallelogram*}

$\Rightarrow PE = CD = PC = ED = x$

$\Rightarrow$ ΔPED & ΔPCD are equal triangles

so, Area of PCDE = 2×Area of ΔPCD

$\Rightarrow$ Area of PCDE = $2 \times \frac{1}{2} \times PC \times CD \sin\angle PCD$

$= x \times x \sin 72$

$\Rightarrow$ **Area of PCDE = $x^2 \sin 72$**

Method 2

Area of PCDE = CD×MN {*MN is the diameter of the circle so MN ⊥ CD*}

⇒ Area of PCDE = x×2R

From methods 1 & 2

Area of PCDE = x×2R = x^2 sin 72

⇒ **R = ½x sin 72**

From ΔPAB

***r* = A/s** {*Here r = Inradius of ΔPAB, A = Area of ΔPAB & 2s = Perimeter of ΔPAB*}

A = ½×AB×PB sin∠ABE

⇒ **A = ½xy sin 36**

s = ½(AB + PB + PA)

⇒ s = ½(x + y + y)

⇒ **s = ½(x+2y)**

PA/sin ∠PBA = PB/sin ∠PAB = AB/sin ∠APB {*sine rule*}

⇒ y/sin 36 = y/sin 36 = x/sin 108

⇒ **y = x sin 36/sin 108**

A = ½xy sin 36

⇒ A = ½×x×(x sin 36/sin 108) × sin 36

⇒ **A = ½x² sin² 36/sin 108**

s = ½(x+2y)

⇒ s = ½[x+2(x sin 36/sin 108)]

⇒ **s = ½(x+2x sin 36/sin 108)**

r = A/s

⇒ r = [½x^2 $\sin^2$ 36/sin 108]/[½(x+2x sin 36/sin 108)]

= (x^2 $\sin^2$ 36/sin 108)/(x+2x sin 36/sin 108)

⇒ **r = (x $\sin^2$ 36)/(sin 108 + 2sin 36)**

$R/r = \frac{1}{2}x \sin 72 \,/\, [(x \sin^2 36)/(\sin 108 + 2 \sin 36)]$

$\Rightarrow R/r = (\frac{1}{2} \sin 72)(\sin 108 + 2 \sin 36)/\sin^2 36$

We know $\sin 2\theta = 2\sin\theta \cos\theta$ & $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

so $\sin 72 = 2\sin 36 \cos 36$ & $\sin 108 = 3\sin 36 - 4\sin^3 36$

$\Rightarrow R/r = (\frac{1}{2}\times 2\sin 36 \cos 36)(3\sin 36 - 4\sin^3 36 + 2\sin 36)/\sin^2 36$

$= \cos 36 (3 - 4\sin^2 36 + 2)$

$= \cos 36 (5 - 4\sin^2 36)$

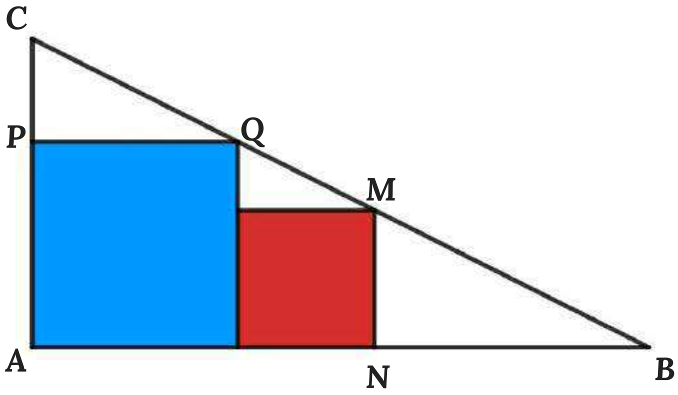
$= \cos 36 (1 + 4\cos^2 36)$

$= \frac{1}{4}(1+\sqrt{5})(1 + [\frac{1}{4}(1+\sqrt{5})]^2)$

$= \frac{1}{8}(1+\sqrt{5})(5+\sqrt{5})$

$\Rightarrow$ **$R/r = \frac{1}{4}(5+3\sqrt{5})$**

SOLUTION 37



Fig(1)

From *Fig*(1)

Blue area = 36 cm²

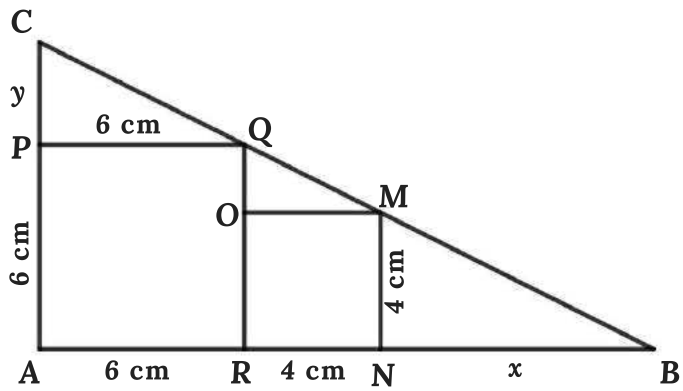
⇒ $PQ^2 = 36$ cm²

⇒ **PQ = AP = 6 cm**

Blue area = 16 cm²

⇒ $MN^2 = 16$ cm²

⇒ **MN = 4 cm**



Fig(2)

From *Fig*(2)

Let BN = x & PC = y

From ΔABC & ΔNBM

∠ABC = ∠NBM {*common angle*}

∠BAC = ∠BNM = 90° {*OMNR is a square*}

⇒ ∠ACB = ∠NMB

so ΔABC & ΔNBM are similar triangles

⇒ AC/MN = AB/BN = BC/BM

⇒ $(6+y)/4 = (10+x)/x$ = BC/BM

⇒ $(6+y)x = 4(10+x)$

⇒ $(6+y)x = 4(10+x)$

⇒ $6x+xy = 40+4x$

⇒ $\mathbf{xy = 40-2x}$...*eq*(1)

From ΔABC & ΔPQC

∠ACB = ∠PCQ {*common angle*}

∠BAC = ∠CPQ = 90° {*APQR is a square*}

⇒ ∠ABC = ∠PQC

so ΔABC & ΔPQC are similar triangles

⇒ PQ/AB = PC/AC = CQ/BC

⇒ $6/(10+x) = y/(6+y)$ = CQ/BC

⇒ $6(6+y) = y(10+x)$

⇒ $36+6y = 10y+xy$

⇒ $\mathbf{xy = 36-4y}$...*eq*(2)

From $eq(1)$ & $eq(2)$

$xy = 40-2x = 36-4y$

⇒ $4-2x = -4y$

⇒ $2-x = -2y$

⇒ $\mathbf{x = 2+2y}$..$eq(3)$

From $eq(1)$ & $eq(3)$

$xy = 40-2x$

⇒ $(2+2y)y = 40-2(2+2y)$

⇒ $2y+2y^2 = 40-4-4y$

⇒ $y^2+3y-18 = 0$

⇒ $y = \frac{1}{2}[-3\pm\sqrt{(9-4\times1\times(-18))}]$

⇒ $\mathbf{y = 3\ cm}$

From $eq(3)$

$x = 2+2y$

⇒ $x = 2+2\times3$

⇒ $\mathbf{x = 8\ cm}$

Area of $\Delta ABC = \frac{1}{2}\times AB\times AC$

⇒ Area of $\Delta ABC = \frac{1}{2}(10+x)(6+y)$

$= \frac{1}{2}(10+8)(6+3)$

$= \frac{1}{2}\times18\times9$

⇒ **Area of $\Delta ABC = 81\ cm^2$**

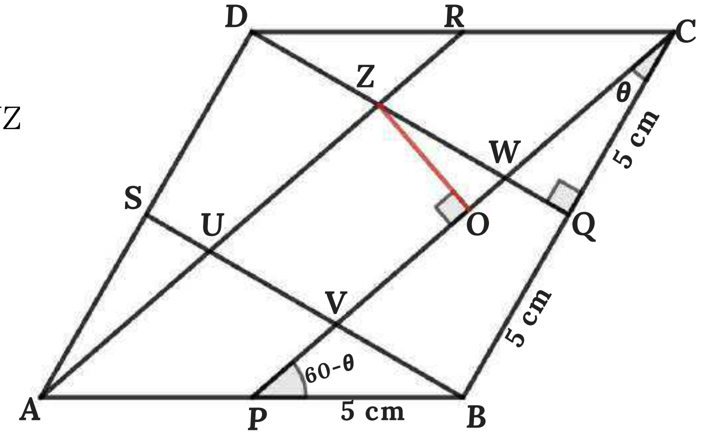
SOLUTION 38

From figure

$\angle BAD = \angle BCD = 60°$

⇒ $\angle ADC = \angle ABC = 120°$ {*UVWZ is a parallelogram*}

BC = 10 cm & PB = QB = QC = 5 cm



Area of blue parallelogram = Area of UVWZ

⇒ **Area of UVWZ = UZ×OZ**

From ΔPBC

$PC^2 = PB^2 + BC^2 - 2 \times PB \times BC \cos \angle ABC$

⇒ $PC^2 = 5^2 + 10^2 - 2 \times 5 \times 10 \cos 120$

$= 25 + 100 - 100(-½)$

$PC^2 = 175$

⇒ **$PC = 5\sqrt{7}$**

$PB^2 = BC^2 + PC^2 - 2 \times BC \times PC \cos \angle PCB$ {*Cosine Rule*}

⇒ $25 = 100 + 175 - 2 \times 10 \times 5\sqrt{7} \cos \theta$

⇒ $250 = 100\sqrt{7} \cos \theta$

⇒ **$\cos \theta = 5/(2\sqrt{7})$**

From *Fig*(2)

BD is a diagonal of the parallelogram

here sides of the parallelogram are equal so

BD is the angle bisector of ∠ABC

⇒ **$\angle ABD = \angle CBD = 120/2 = 60°$**

so ΔBCD is an equilateral triangle, then

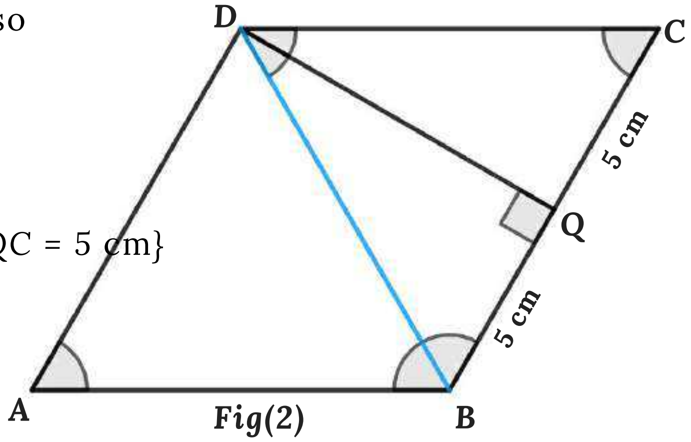
DQ is the angle bisector of ∠BDC {QB = QC = 5 cm}

⇒ **$\angle CDQ = \angle BDQ = 60/2 = 30°$**

⇒ $\angle CQD = 180 - (\angle CDQ + \angle BCD)$

$= 180 - (30 + 60)$

⇒ **$\angle CQD = 90°$**

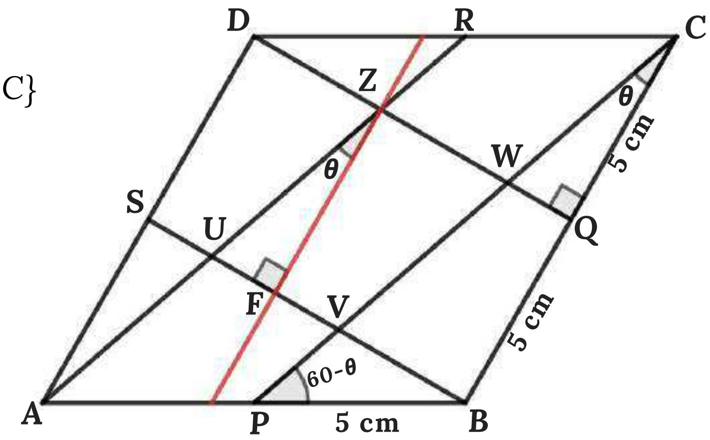


Fig(2)

From *Fig*(3)

$\angle PCB = \angle AZY = \theta$ {XY // BC & AR // PC}

FZ = BQ = 5 cm {BS // DQ}



Fig(3)

From ΔFUZ

$\cos\theta = FZ/UZ$

⇒ $5/(2\sqrt{7}) = 5/UZ$

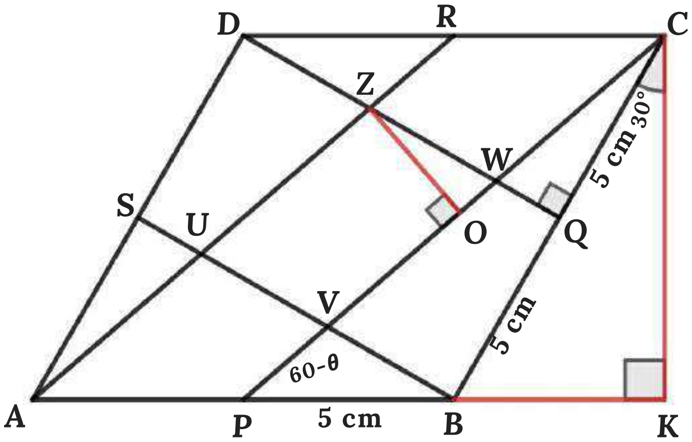
⇒ **$UZ = 2\sqrt{7}$ cm**

From *Fig*(4)

$\angle BCK = \angle DCK - \angle BCD$

⇒ $\angle BCK = 90 - 60$

⇒ **$\angle BCK = 30°$**



Fig(4)

From ΔBCK

$\cos 30 = CK/BC$

⇒ $\frac{1}{2}\sqrt{3} = CK/10$

⇒ **$CK = 5\sqrt{3}$ cm**

Area of Parallelogram APCR = AP×CK

$= 5\times5\sqrt{3}$

⇒ **Area of APCR = $25\sqrt{3}$ cm²**

We can also find the area of APCR in another method, that is

Area of APCR = PC×OZ

⇒ $25\sqrt{3} = 5\sqrt{7}\times OZ$

⇒ **$OZ = 5\sqrt{3}/\sqrt{7}$ cm**

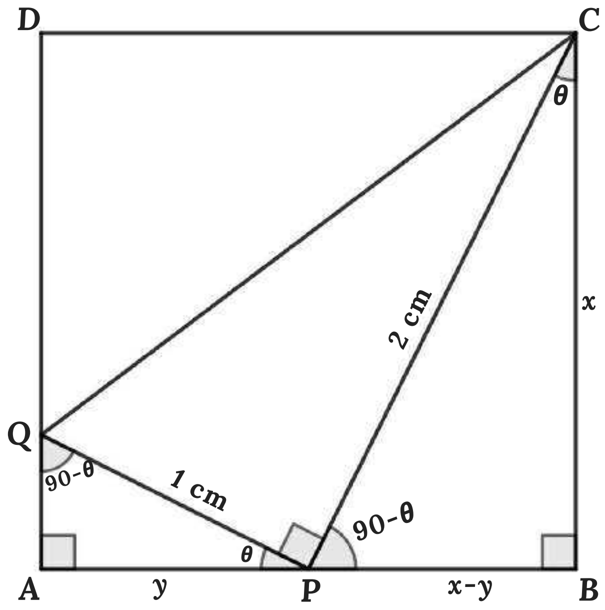
Area of UVWZ = UZ×OZ

$= 2\sqrt{7} \times 5\sqrt{3}/\sqrt{7}$

Area of UVWZ = $10\sqrt{3}$ cm²

Area of blue parallelogram = $10\sqrt{3}$ cm²

SOLUTION 39



Let ∠APQ = θ, BC = x & AP = y, then

AB = BC = CD = AD = x *{side of the square}*

Area of square = BC^2

⇒ **Area of square = x^2**

PB = AB - AP

⇒ **PB = $x-y$**

∠AQP = 180 - (∠APQ + ∠PAQ)

= 180 - (θ + 90)

⇒ **∠AQP = 90-θ**

∠BPC = 180 - (∠APQ + ∠QPC)

= 180 - (θ + 90)

⇒ **∠BPC = 90-θ**

∠PCB = 180 - (∠PBC + ∠BPC)

= 180 - (90 + 90-θ)

= 180 - (180-θ)

⇒ **∠PCB = θ**

From ΔPAQ & ΔPBC

∠APQ = ∠PCB = θ

∠AQP = ∠BPC = 90-θ

∠PAQ = ∠BPC = 90°

⇒ ΔPAQ & ΔPBC are similar triangles

so, **PQ/PC = PA/BC = AQ/PB**

⇒ ½ = y/x = AQ/($x-y$)

⇒ **z = ½x**

From ΔPBC

$PC^2 = PB^2 + BC^2$

$\Rightarrow 2^2 = (x-y)^2 + x^2$

$= (x-\frac{1}{2}x)^2 + x^2$

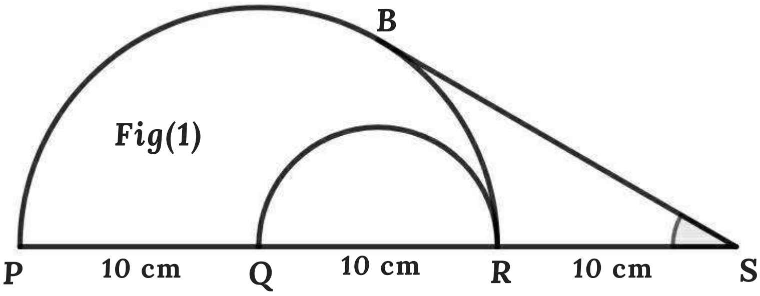
$= \frac{1}{4}x^2 + x^2$

$= 5x^2/4$

$\Rightarrow x^2 = 16/5 \text{ cm}^2$

Area of square = 16/5 cm²

SOLUTION 40



From *Fig(1)*

$BS^2 = SR \times SP$

$\Rightarrow BS^2 = 10 \times 30$

$= 300$

$\Rightarrow BS = 10\sqrt{3}$ cm

From *Fig(2)*

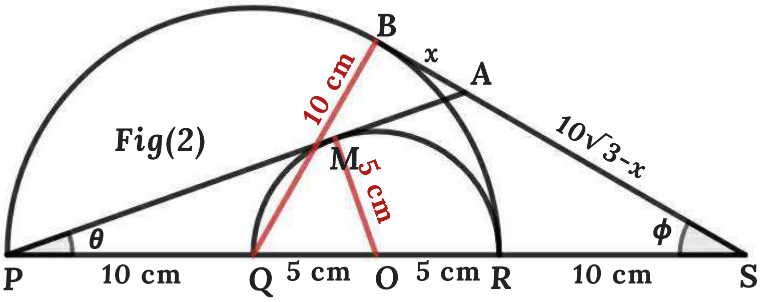
Let AB = x, $\angle APS = \theta$ & $\angle ASP = \phi$, then

AS = BS - AB

$\Rightarrow AS = 10\sqrt{3} - x$

OM = OQ = OR = 10/2 = 5 cm *{Radius of the semicircle}*

BQ = PQ = 10 cm *{Radius of the semicircle}*



From ΔPOM

$\sin \angle APS = OM/OP$

$\Rightarrow \sin \theta = 5/15$

$= \frac{1}{3}$

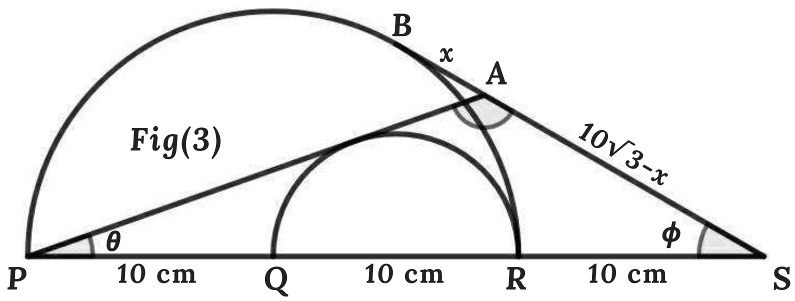
$\Rightarrow \boldsymbol{\theta = \sin^{-1} \frac{1}{3}}$

From ΔBQS

$\sin \angle BSP = 10/20$

$\Rightarrow \sin \phi = \frac{1}{2}$

$\Rightarrow \boldsymbol{\phi = \sin^{-1} \frac{1}{2}}$



From *Fig*(3)

$\angle PAS = 180 - \theta - \phi$

$= 180 - \sin^{-1} \frac{1}{3} - \sin^{-1} \frac{1}{2}$

$\sin \angle PAS = \sin (180 - \sin^{-1} \frac{1}{3} - \sin^{-1} \frac{1}{2})$

$= \sin (\sin^{-1} \frac{1}{3} + \sin^{-1} \frac{1}{2})$

$= \sin (\sin^{-1} \frac{1}{3}) \cos (\sin^{-1} \frac{1}{2}) + \cos (\sin^{-1} \frac{1}{3}) \sin (\sin^{-1} \frac{1}{2})$

$= \frac{1}{3} \times \frac{1}{2}\sqrt{3} + \frac{2}{3}\sqrt{2} \times \frac{1}{2}$

$\Rightarrow$ **$\sin \angle PAS = \frac{1}{6}(\sqrt{3} + 2\sqrt{2})$**

$\sin \theta / AS = \sin \angle PAS / PS$ {*Sine Rule*}

$\Rightarrow 10\sqrt{3}-x = PS \sin \theta / \sin \angle PAS$

$= 30 \times \frac{1}{3} / (\frac{1}{6}(\sqrt{3} + 2\sqrt{2}))$

$= 60/(\sqrt{3} + 2\sqrt{2})$

$= 24\sqrt{2} - 12\sqrt{3}$ cm

$\Rightarrow x = 22\sqrt{3} - 24\sqrt{2}$ cm

$\Rightarrow$ **$AB = 22\sqrt{3} - 24\sqrt{2}$ cm**

SOLUTION 41

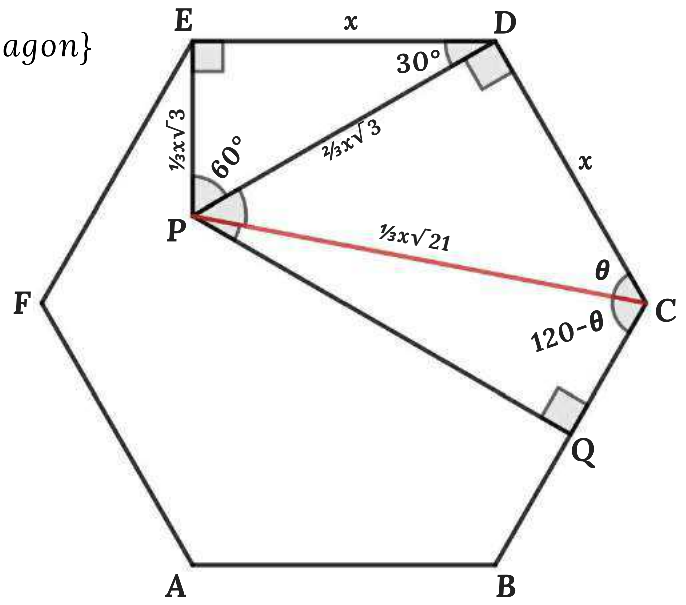
Let ∠PCD = θ & DE = x

From figure

∠PED = 90° {PE ⊥ ED}

∠PDC = 90° {PD ⊥ CD}

∠PQC = 90° {PQ ⊥ BC}



∠BCD = ∠CDE = 120° *{corner of the hexagon}*

∠PDE = ∠CDE - ∠PDC

= 120 - 90

⇒ **∠PDE = 30°**

From ΔPDE

∠DPE = 180-(∠PED + ∠PDE)

= 180-(30 + 90)

= 180 - 120

⇒ **∠DPE = 60°**

PE/sin ∠PDE = DE/sin ∠DPE = PD/sin ∠PED *{sine rule}*

⇒ PE/sin 30 = x/sin 60 = PD/sin 90

⇒ PE/sin 30 = x/sin 60

⇒ PE = x sin 30/sin 60

= $x \times \frac{1}{2}/(\frac{1}{2}\sqrt{3})$

⇒ **PE = $\frac{1}{3}x\sqrt{3}$**

x/sin 60 = PD/sin 90

⇒ $x/(\frac{1}{2}\sqrt{3})$ = PD/1

⇒ **PD = $\frac{2}{3}x\sqrt{3}$**

From ΔPCD

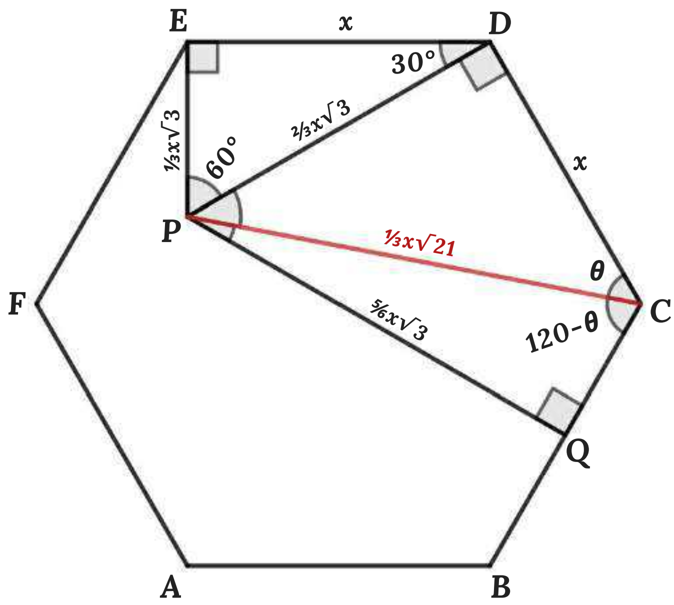
$PC^2 = PD^2 + CD^2$

⇒ $PC^2 = (\frac{2}{3}x\sqrt{3})^2 + x^2$

$= 4x^2/3 + x^2$

$= 7x^2/3$

⇒ **PC = $\frac{1}{3}x\sqrt{21}$**



sin θ = PD/PC

⇒ sin θ = ⅔x√3/(⅓x√21)

⇒ **sin θ = 2/√7**

cos θ = CD/PC

⇒ cos θ = x/(⅓x√21)

⇒ **cos θ = √3/√7**

From ΔPCQ

PC/sin ∠PQC = PQ/sin ∠PCQ

⇒ (⅓x√21)/sin 90 = PQ/sin (120-θ)

⇒ PQ = (⅓x√21) sin (120-θ)

= (⅓x√21)[sin 120 cos θ - cos 120 sin θ]

= (⅓x√21)(½√3 × (√3/√7) - [-½ × (2/√7)])

= (⅓x√21)[5/(2√7)]

⇒ **PQ = ⅚x√3**

PQ : PD : PE = ⅚x√3 : ⅔x√3 : ⅓x√3

⇒ PQ : PD : PE = ⅚ : ⅔ : ⅓

⇒ **PQ : PD : PE = 5 : 4 : 2**

SOLUTION 42

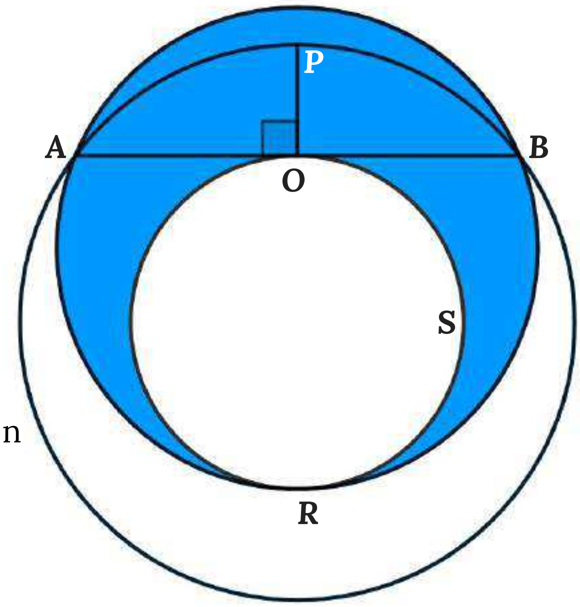
Blue Area = Area of the circle ARB

- Area of the circle ORS

Let the radius of the circle ORS = r

& radius of the circle ORS =R, then

Blue Area = $\pi R^2 - \pi r^2$



Fig(1)

From *Fig(2)*

OP = RQ = 6 cm {ORS & ARB *are concentric circles*}

OA = OB = 24/2 = 12 cm

OP × OQ = OA × OB {*Chord theorem*}

⇒ 6 × OQ = 12 × 12

⇒ OQ = 144/6

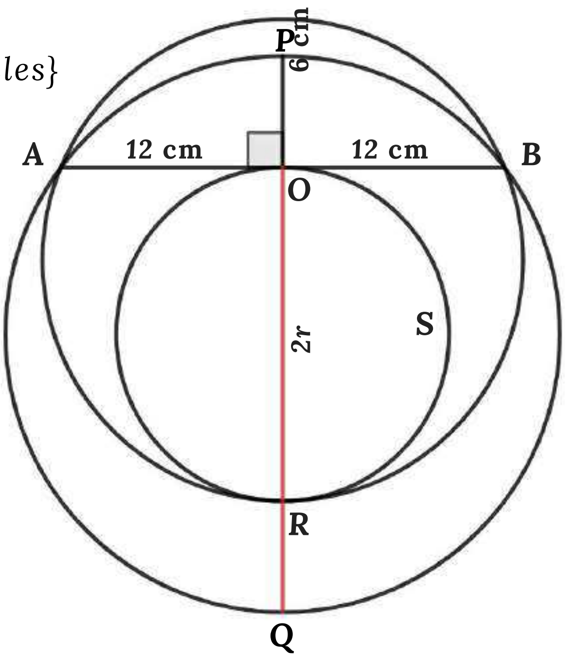
⇒ **OQ = 24 cm**

OR = OQ - RQ

⇒ $2r$ = 24 - 6

⇒ $2r$ = 18

⇒ **r = 9 cm**



Fig(2)

From *Fig*(3)

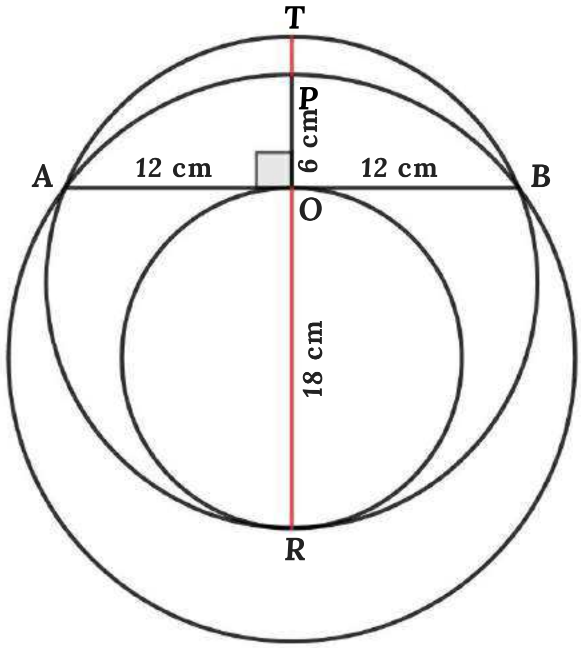
TR = 2R {*Diameter of the circle*}

OR × OT = OA × OB {*Chord theorem*}

⇒ 18 × OT = 12 × 12

⇒ OT = 144/18

⇒ **OT = 8 cm**



***Fig*(3)**

TR = OR + OT

⇒ 2R = 18 + 8

⇒ **R = 13 cm**

Blue Area = $\pi R^2 - \pi r^2$

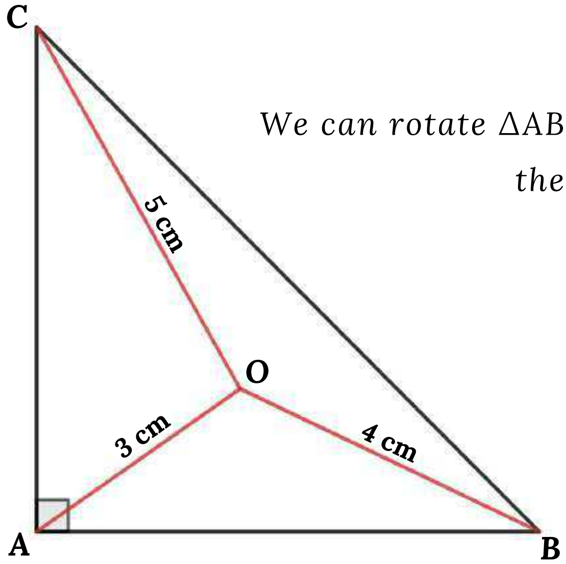
⇒ Blue Area = $\pi\times13^2 - \pi\times9^2$

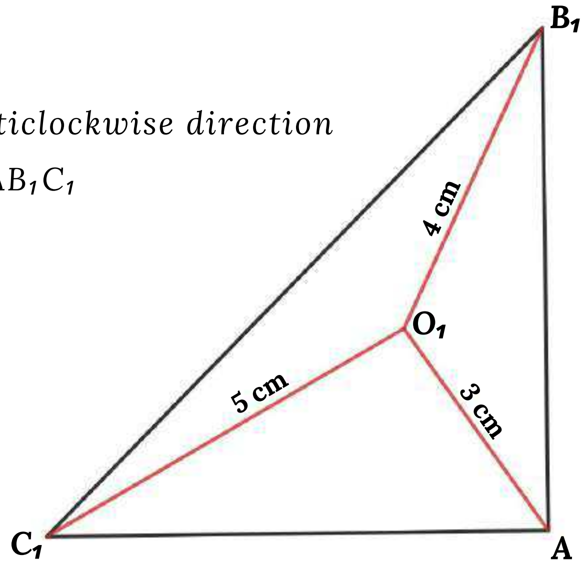
= $169\pi - 81\pi$

⇒ ***Blue Area = 88π cm²***

SOLUTION 43

We can rotate ΔABC in the anticlockwise direction then we get ΔAB_1C_1





Fig(3)

Merge these two triangles then we get *Fig*(2)

From *Fig*(2)

$\angle ABC = 90°$

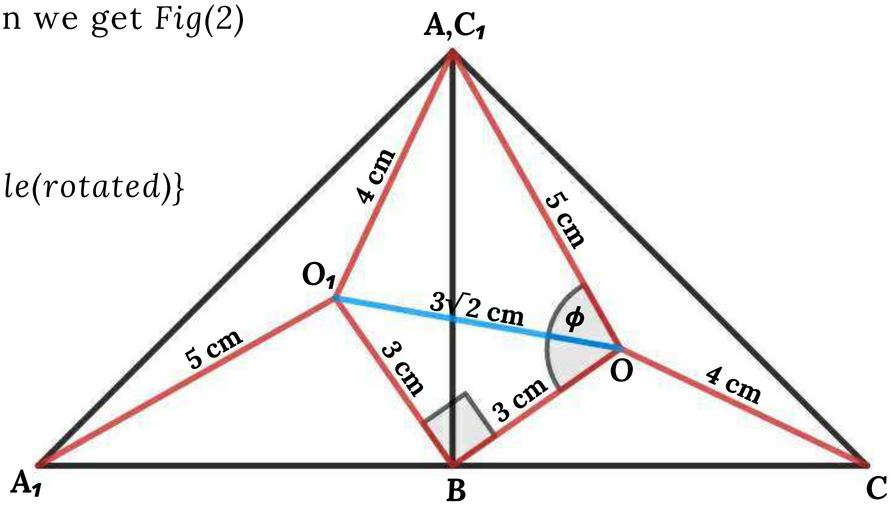
$\angle CBO = \angle ABO_1$ *{same angle(rotated)}*

$\angle OBO_1 = \angle ABO + \angle ABO_1$

$= \angle ABO + \angle CBO$

$= \angle ABC$

$\Rightarrow$ $\boldsymbol{\angle OBO_1 = 90°}$



Fig(2)

From ΔBOO_1

$O_1O^2 = OB^2 + O_1B^2$

$\Rightarrow$ $O_1O^2 = 3^2 + 3^2$

$= 18$

$\Rightarrow$ $\boldsymbol{O_1O = 3\sqrt{2}\ \text{cm}}$

Let $\angle O_1OA = \phi$, then from ΔABO_1

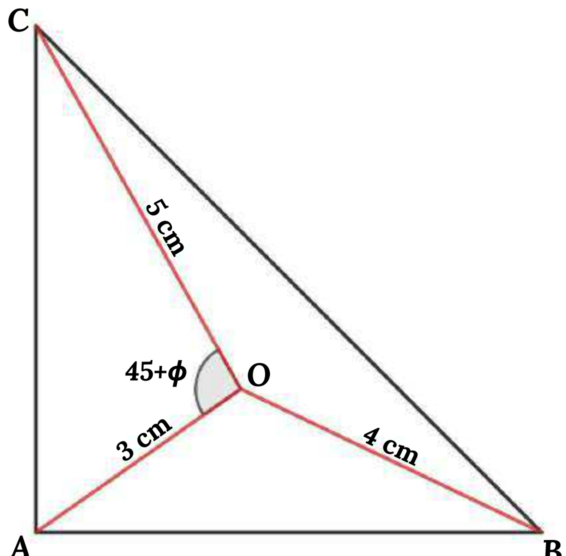
$O_1A^2 = O_1O^2 + OA^2 - 2\times O_1O\times OA\times\cos\phi$ *{cosine rule}*

$\Rightarrow$ $4^2 = 18 + 5^2 - 2\times3\sqrt{2}\times5\times\cos\phi$

$\Rightarrow$ $16 = 18 + 25 - 30\sqrt{2}\times\cos\phi$

$\Rightarrow$ $\boldsymbol{\cos\phi = (^9/_{20})\sqrt{2}}$

$\Rightarrow$ $\boldsymbol{\sin\phi = (^1/_{20})\sqrt{119}}$



Fig(3)

From ΔAOC

∠AOC = 45+ϕ

$AC^2 = OA^2 + OC^2 - 2\times OA\times OC\times\cos(45+\phi)$

⇒ $AC^2 = 3^2 + 5^2 - 2\times3\times5\times(\cos 45 \cos\phi - \sin 45 \sin\phi)$

⇒ $AC^2 = 34 - 30(\frac{1}{2}\sqrt{2} \times (\frac{9}{20})\sqrt{2} - \frac{1}{2}\sqrt{2} \times (\frac{1}{20})\sqrt{238})$

⇒ $AC^2 = 34 - \frac{3}{4}(18-2\sqrt{119})$

⇒ $\mathbf{AC^2 = \frac{1}{2}(41+3\sqrt{119})}$

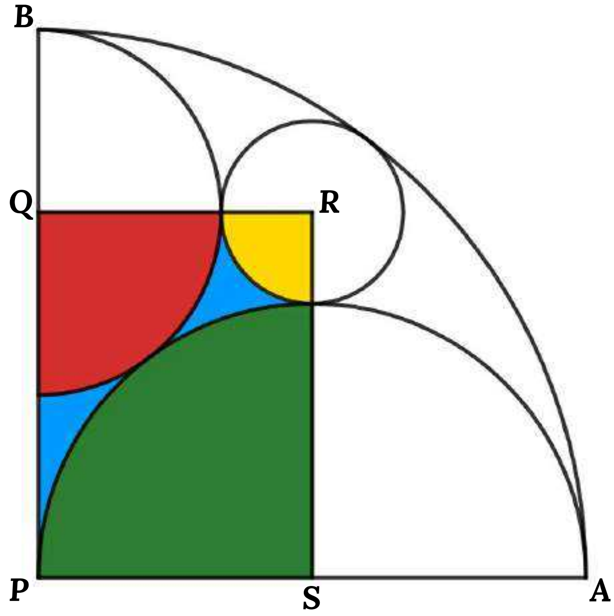
Area of ΔAOC = $\frac{1}{2}\times AC^2$

⇒ ***Area of ΔAOC = $\frac{1}{4}(41+3\sqrt{119})$ cm²***

SOLUTION 44

From *Fig*(1)

Blue area = Area of PQRS - Red area - Yellow area - Green Area



Fig(1)

From *Fig*(2)

SA = SP = SD = PA/2 = 12/2 = 6 cm

Let QD = R, then

QD = QB = R

PQ = PB - QB

$\Rightarrow$ PQ = 12-R

From ΔPQS

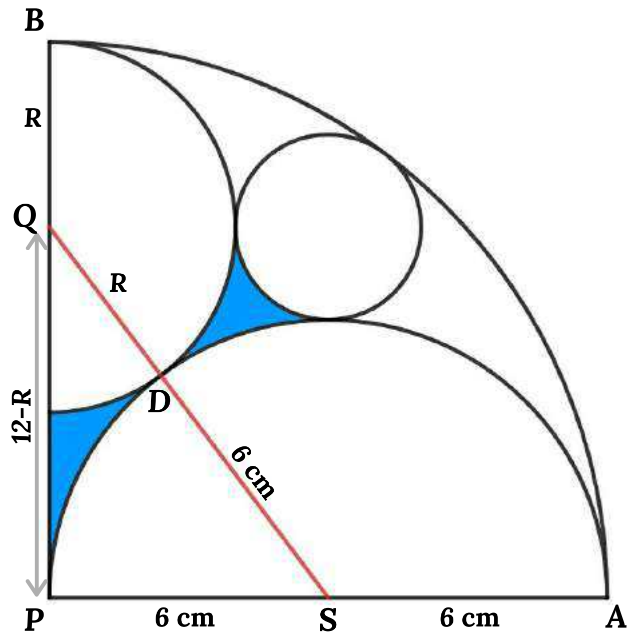
$PS^2 + PQ^2 = QS^2$

$\Rightarrow 6^2 + (12-R)^2 = (6+R)^2$

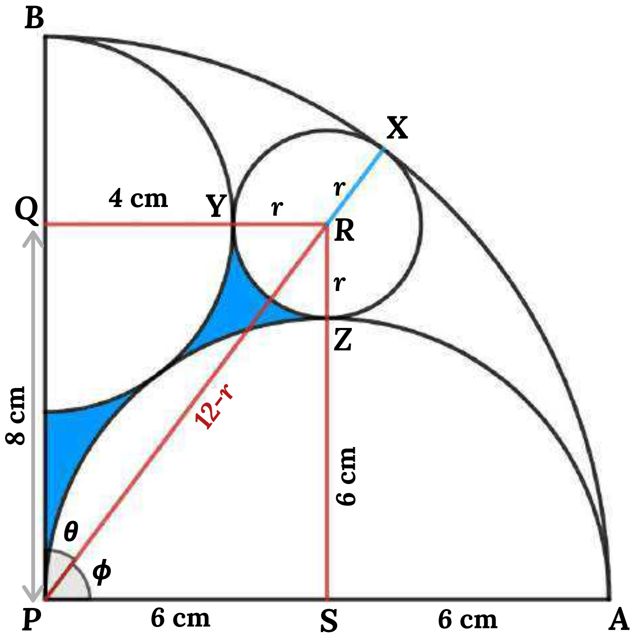
$\Rightarrow 36 + 144 - 24R + R^2 = 36 + 12R + R^2$

$\Rightarrow 144 = 36R$

$\Rightarrow$ **R = 4 cm**



Fig(2)



Fig(3)

From Fig(3)

Let RX = r, $\angle RPQ = \theta$ & $\angle RPS = \phi$

RX = RY = RZ = r {Radius of the circle}

$\Rightarrow$ QR = $4+r$

SR = $6+r$

PR = $12-r$

We can apply cosine rule in ΔPSR, then

$SR^2 = PS^2 + PR^2 - 2\times PS\times PR\times \cos \angle RPS$

$\Rightarrow (6+r)^2 = 6^2 + (12-r)^2 - 2\times 6\times(12-r)\times\cos\phi$

$\Rightarrow 36+12r+r^2 = 36 + 144-24r+r^2 - 12(12-r)\times\cos\phi$

$\Rightarrow 12(12-r)\times\cos\phi = 144-36r$

$\Rightarrow$ **$\cos\phi = (12-3r)/(12-r)$**

From ΔPQR

$QR^2 = PQ^2 + PR^2 - 2\times PQ\times PR\times\cos\angle RPQ$ {cosine rule}

$\Rightarrow (4+r)^2 = 8^2 + (12-r)^2 - 2\times 8\times(12-r)\times\cos\theta$

$\Rightarrow 16+8r+r^2 = 64 + 144-24r+r^2 - 16(12-r)\times\cos\theta$

$\Rightarrow 16(12-r)\times\cos\theta = 192-32r$

$\Rightarrow$ **$\cos\theta = (12-2r)/(12-r)$**

$\theta+\phi = 90°$

$\Rightarrow \sin\phi = \cos\theta$

$\Rightarrow$ **$\sin\phi = (12-2r)/(12-r)$**

We Know, $\sin^2\phi + \cos^2\phi = 1$

$[(12-2r)/(12-r)]^2 + [(12-3r)/(12-r)]^2 = 1$

$\Rightarrow (12-2r)^2 + (12-3r)^2 = (12-r)^2$

$\Rightarrow 144-48r+4r^2 + 144-72r+9r^2 = 144-24r+r^2$

$\Rightarrow 12r^2-96r+144 = 0$

$\Rightarrow r^2-8r+12 = 0$

$\Rightarrow r = ½[8\pm\sqrt{(8^2 - 4\times 12)}]$

$\Rightarrow$ **$r = 4\pm 2$**

If r = 6 cm, then

PR = 12-r = 12-6 = 6 cm

PS = 6 cm

RS = 6+r = 6+6 = 12 cm

"In a triangle sum of the small sides are always greater than the larger side otherwise it's a straight line"

So **PRS is in a straight line**

If r = 2 cm, then

PR = 12-r = 12-2 = 10 cm

PS = 6 cm

RS = 6+r = 6+2 = 8 cm

Here 6 cm, 8 cm & 10 cm are pythagorean triples so ∠PSR = 90°

We saw PQ = RS = 8 cm, QR = PS = 6 cm & ∠QPS = 90° so PQRS is a rectangle

⇒ **∠QPS = ∠PSR = ∠SRQ = ∠PQR = 90°**

From *Fig(4)*

Area of PQRS = PQ×PS

= 8×6

⇒ **Area of PQRS = 48 cm²**

Red area = ¼π×QY²

= ¼π×4²

= ¼π×16

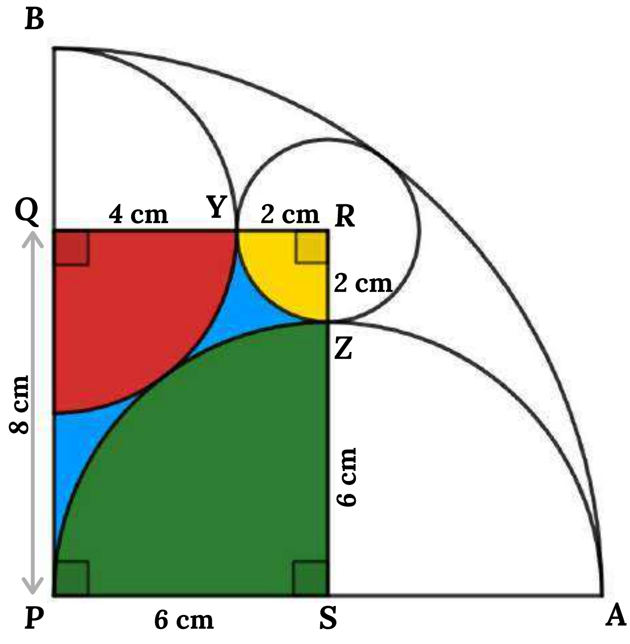
⇒ **Red area = 4π cm²**

Yellow area = ¼π×RY²

= ¼π×2²

= ¼π×4

⇒ **Yellow area = π cm²**



Fig(4)

Green area = $\frac{1}{4}\pi \times SP^2$

= $\frac{1}{4}\pi \times 6^2$

= $\frac{1}{4}\pi \times 36$

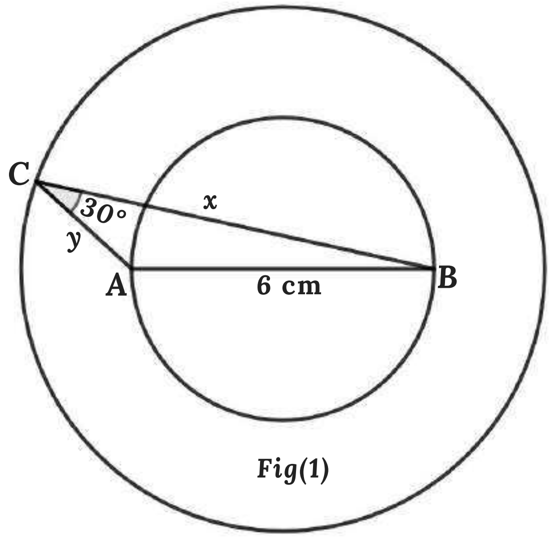
⇒ **Green area = 9π cm²**

Blue area = Area of PQRS - Red area - Yellow area - Green Area

= $48 - 4\pi - \pi - 9\pi$

⇒ ***Blue area = $48-14\pi$ cm²***

SOLUTION 45



Fig(1)

From *Fig*(1)

Area of ΔABC = ½×AC×BC×sin ∠ACB

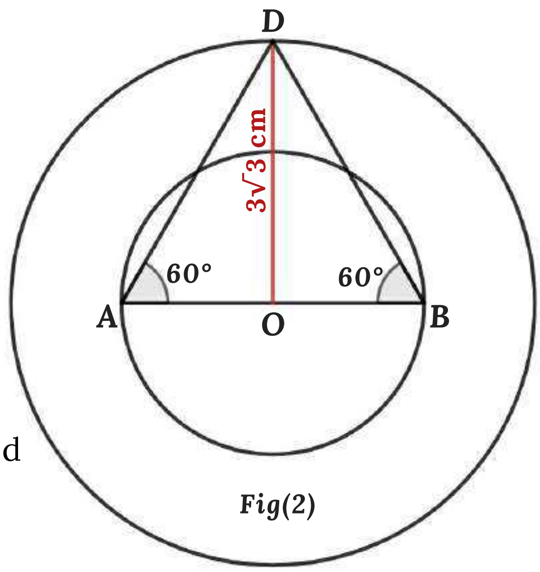
= ½×AC×BC×sin 30

⇒ Area of ΔABC = ¼×AC×BC

Let AC = x & BC = y, then

Area of ΔABC = ¼×AC×BC

⇒ **Area of ΔABC = ¼xy**



Fig(2)

From *Fig*(2)

We know ABD is an equilateral triangle, so

AB = AD = BD = 6 cm

∠BAD = ∠ADB = ∠ABD = 60°

Let r is the radius of the inner-circle and R is the radius of the outer circle. If O is the centre of the circles, then OD is the radius of the outer circle and OA is the radius of the inner-circle

OD = BD sin ∠ABD

⇒ R = 6 sin 60

= 6×½√3

⇒ **R = 3√3 cm**

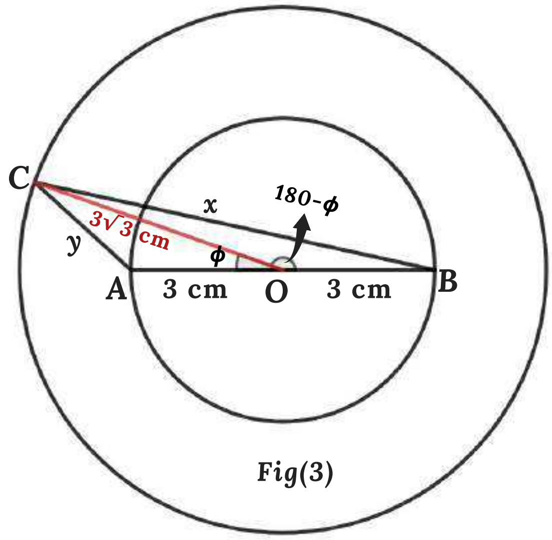
OA = OB = 6/2 = 3 cm

⇒ **r = 3 cm**

From *Fig*(3)

Let $\angle AOC = \phi$, then

$\angle BOC = 180-\phi$



Fig(3)

We can apply cosine rule in ΔAOC, then

$AC^2 = OA^2 + OC^2 - 2\times OA\times OC\times \cos \angle AOC$

$\Rightarrow y^2 = 3^2 + (3\sqrt{3})^2 - 2\times 3\times 3\sqrt{3}\times \cos \phi$

$= 9 + 27 - 18\sqrt{3}\times \cos \phi$

$\Rightarrow$ **$\cos \phi = (36-y^2)/(18\sqrt{3})$**........................*eq*(1)

From ΔBOC

$BC^2 = OB^2 + OC^2 - 2\times OB\times OC\times \cos \angle BOC$

$\Rightarrow x^2 = 3^2 + (3\sqrt{3})^2 - 2\times 3\times 3\sqrt{3}\times \cos (180-\phi)$

$= 9 + 27 - 18\sqrt{3}\times \cos (180-\phi)$

$\Rightarrow \cos (180-\phi) = (36-x^2)/(18\sqrt{3})$

We know $\cos (180-\phi) = -\cos \phi$, so

$\cos (180-\phi) = (36-x^2)/(18\sqrt{3})$

$\Rightarrow -\cos \phi = (36-x^2)/(18\sqrt{3})$

$\Rightarrow$ **$\cos \phi = (x^2-36)/(18\sqrt{3})$**.........................*eq*(2)

From *eq*(1) & *eq*(2)

$\cos \phi = (36-y^2)/(18\sqrt{3}) = (x^2-36)/(18\sqrt{3})$

$\Rightarrow 36-y^2 = x^2-36$

$\Rightarrow x^2+y^2 = 72$

From *Fig*(4)

Apply cosine rule in ΔABC, then

$AB^2 = AC^2 + BC^2 - 2\times AC\times BC\times \cos \angle ACB$

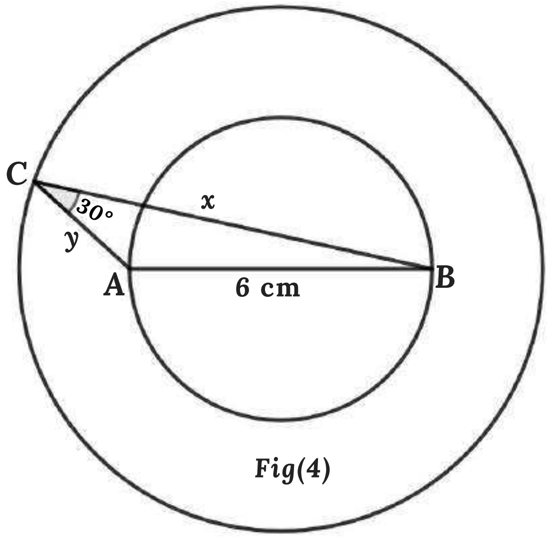
⇒ $6^2 = y^2 + x^2 - 2\times yx \cos 30$

⇒ $36 = x^2+y^2 - 2\times xy\times ½\sqrt{3}$

$= 72 - xy\sqrt{3}$

⇒ $xy = 36/\sqrt{3}$

⇒ $\mathbf{xy = 12\sqrt{3}}$



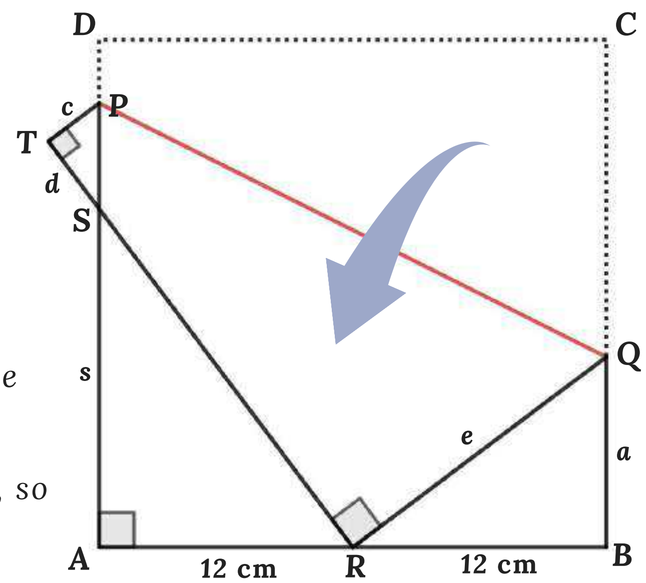
Fig(4)

Area of ΔABC = $¼xy$

$= ¼\times 12\sqrt{3}$

⇒ ***Area of ΔABC = $3\sqrt{3}$ cm²***

SOLUTION 46



Fig(1)

From Fig(1)

Area of PQRS = Area of PQRT - Area of ΔPST

Area of ΔPST = ½×PT×ST

PQRT is a trapezoid with hight RT so

Area of PQRT = ½×RT×(RQ+PT)

If BQ = a, AS = s, PA = b,PT = c, ST = d & RQ = e

then, Area of ΔPST = ½cd

We know RT is the side of the square (*folded*), so

Area of PQRT = ½×24×(e+c) = 12(e+c)

⇒ **Area of PQRS = 12(e+c) - ½cd**

From Fig(2)

RQ = CQ = e {*folded*}

BQ + RQ = 24 {*side of the square*}

⇒ a+e = 24 cm

⇒ **e = 24-a**

PD = PT = c {*folded*}

AP + PD = 24 {*side of the square*}

⇒ b+c = 24 cm

⇒ **c = 24-b**

From ΔBQR

$BR^2 + BQ^2 = RQ^2$

⇒ $12^2 + a^2 = e^2$

⇒ $144 + a^2 = (24-a)^2$

⇒ $144 + a^2 = 576-48a+a^2$

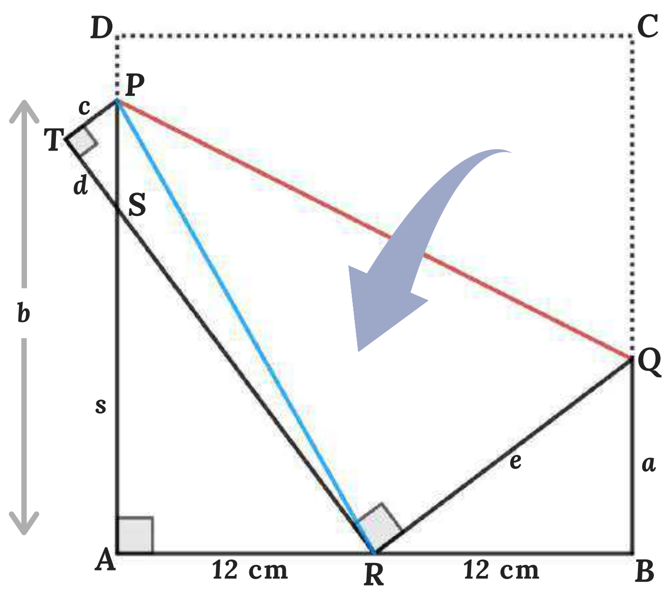
⇒ $48a = 432$

⇒ **a = 9 cm**

$e = 24-a$

⇒ $e = 24-9$

⇒ **e = 15 cm**



Fig(2)

From ΔPTR

$PR^2 = PT^2 + TR^2$

$\Rightarrow PR^2 = c^2 + 24^2$

$= (24-b)^2 + 24^2$

$\Rightarrow PR^2 = 1152-48b+b^2$

From ΔPAR

$PR^2 = PA^2 + AR^2$

$\Rightarrow PR^2 = b^2 + 12^2$

$= b^2+144$

$= 1152-48b+b^2$

$\Rightarrow 48b = 1008$

$\Rightarrow$ **$b = 21$ cm**

$c = 24-b$

$\Rightarrow c = 24-21$

$\Rightarrow$ **$c = 3$ cm**

From Fig(3)

Consider ΔPTS & ΔASR

$\angle PTS = \angle RAS = 90°$

$\angle PST = \angle ASR$

$\Rightarrow \angle SPT = \angle ARS$

$\Rightarrow \Delta PTS$ & ΔASR are similar triangles

$\Rightarrow$ **$PS/RS = PT/AR = TS/AS$**

$\Rightarrow (b-s)/(24-d) = c/12 = d/s$

$\Rightarrow (21-s)/(24-d) = 3/12 = d/s$

$\Rightarrow (21-s)/(24-d) = ¼ = d/s$

$¼ = d/s$

$\Rightarrow$ **$s = 4d$**

$(21-s)/(24-d) = ¼$

$\Rightarrow 4(21-s) = 24-d$

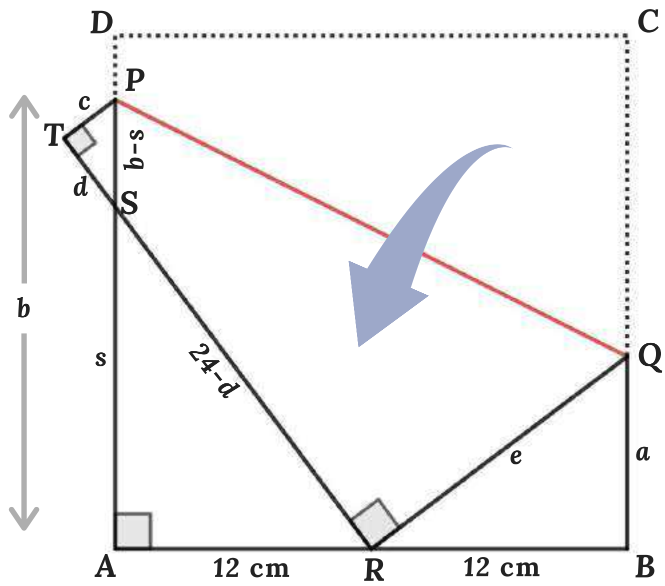
$\Rightarrow 84-4s = 24-d$

$\Rightarrow 4s-d = 60$

$\Rightarrow 4\times4d-d = 60$

$\Rightarrow 15d = 60$

$\Rightarrow$ **$d = 4$ cm**



Fig(3)

Area of PQRS = $12(e+c) - \frac{1}{2}cd$

⇒ Area of PQRS = $12(15+3) - \frac{1}{2}\times3\times4$

⇒ ***Area of PQRS = 210 cm²***

From *Fig*(4)

VQ = RQ - RV

⇒ VQ = $e-c$ = 15-3

⇒ VQ = 12 cm

PV = RT = 24 cm

Apply Pythagorean theorem in ΔPQV, then

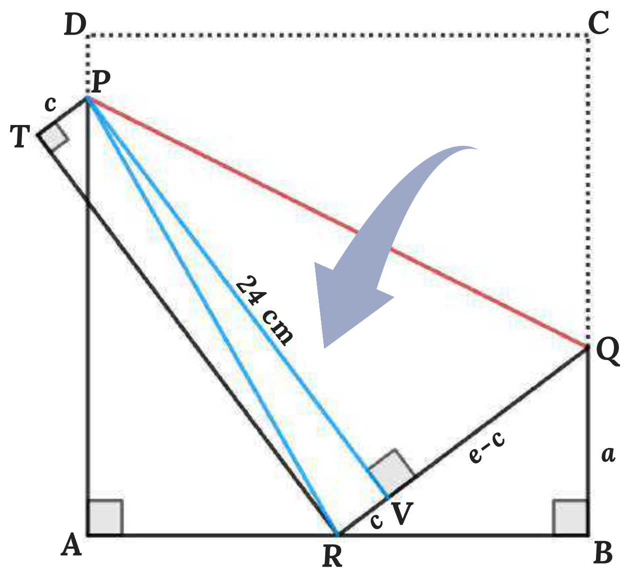
$PQ^2 = PV^2 + VQ^2$

⇒ $PQ^2 = 24^2 + 12^2$

$= 576 + 144$

⇒ $PQ^2 = 720$

⇒ **PQ = $12\sqrt{5}$ cm**

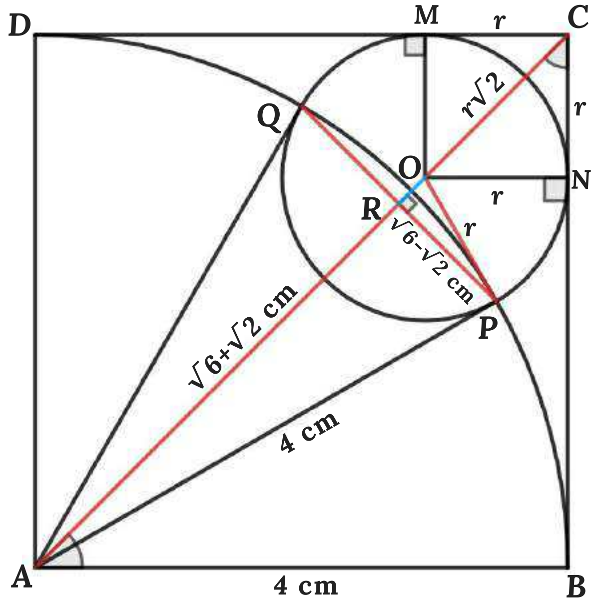


Fig(4)

SOLUTION 47

Let r is the radius of the circle

From Figure



AP = AB = 4 cm {*radius of the quarter circle*}

OP = ON = OM = r {*radius of the circle*

ON ⊥ BC {*tangent of the circle*}

OM ⊥ CD {*tangent of the circle*}

⇒ ONCM is a square

⇒ ∠OCN = 45°

⇒ OC = $r\sqrt{2}$

∠BAC = 180 - (∠ABC + ∠ACB)

= 180 - (90 + 45)

⇒ **∠BAC = 45°**

∠PAQ = 90/3 = 30° {BP = PQ = QD}

∠PAR = ∠BAC - ∠PAQ

= 45 - 30

⇒ **∠PAR = 15°**

⇒ **∠QAR = 15°**

From ΔPAQ

PA = PR {*radius of the circle*}

∠PAR = ∠QAR = 15°

⇒ ΔPAQ *is an isosceles triangle & AR is angle bisector of* ∠PAQ so **AR ⊥ PQ**

⇒ ∠ARP = 90°

From ΔPAQ

sin 15 = PR/AP

⇒ sin 15 = PR/4

⇒ PR = 4 sin 15

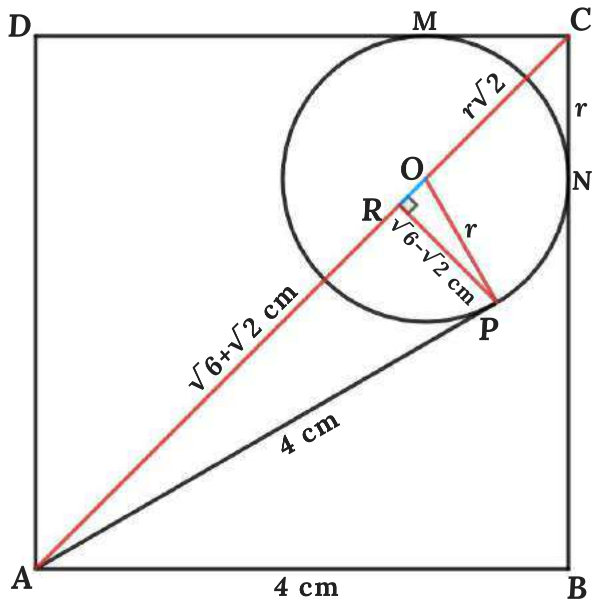
⇒ **PR = $\sqrt{6}-\sqrt{2}$ cm**

cos 15 = AR/AP

⇒ cos 15 = AR/4

⇒ AR = 4 cos 15

⇒ **AR = $\sqrt{6} + \sqrt{2}$ cm**



From ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 4^2 + 4^2$

$\Rightarrow AC^2 = 4\sqrt{2}$ cm

From ΔPOR

$OR = AC - AR - OC$

$\Rightarrow OR = 4\sqrt{2} - (\sqrt{6} + \sqrt{2}) - r\sqrt{2}$

$\Rightarrow \mathbf{OR = 3\sqrt{2} - \sqrt{6} - r\sqrt{2}}$

$OP^2 = OR^2 + PR^2$

$\Rightarrow r^2 = (3\sqrt{2} - \sqrt{6} - r\sqrt{2})^2 + (\sqrt{6} - \sqrt{2})^2$

$= (18 + 6 + 2r^2 - 12\sqrt{3} - 12r + 4r\sqrt{3}) + (6 - 4\sqrt{3} + 2)$

$= 32 + 2r^2 - 16\sqrt{3} - 12r + 4r\sqrt{3}$

$\Rightarrow r^2 - (12-4\sqrt{3}) + (32-16\sqrt{3}) = 0$

$\Rightarrow r = \frac{1}{2}[12-4\sqrt{3} \pm \sqrt{((12-4\sqrt{3})^2 - 4\times1\times(32-16\sqrt{3}))}]$

$= \frac{1}{2}[12-4\sqrt{3} \pm \sqrt{(144-96\sqrt{3}+48 - 128+64\sqrt{3})}]$

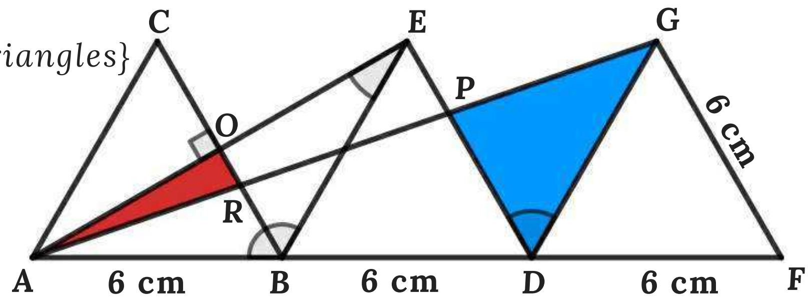
$= \frac{1}{2}[12-4\sqrt{3} \pm \sqrt{(64 - 32\sqrt{3})}]$

$\Rightarrow$ **r = 4 cm** or **$r = 8-4\sqrt{3}$ cm**

r is less than 4 cm

so, ***Radius of the circle = $8-4\sqrt{3}$ cm***

SOLUTION 48



From Figure

∠ABC = ∠DBE = ∠EDB = ∠GDF = ∠BED = 60°

∠ABE = 180 - ∠DBE

= 180 - 60

⇒ **∠ABE = 120°**

AB = BE {ΔABC & ΔBDE *are equal triangles*}

⇒ ∠AEB = ∠BAE = ½(180-120)

⇒ **∠AEB = 30°**

∠AED = ∠AEB + ∠BED

= 30+60

⇒ ∠AED = 90°

⇒ **∠AOB = 90°**

∠EDG = 180 - ∠EDB - ∠GDF

= 180 - 60 - 60

⇒ **∠EDG = 60°**

Blue Area = ½×PD×DG×sin ∠EDG

Red area = Area of ΔAOB - Area of ΔARB

From ΔAGF & ΔAPD

∠AGF = ∠APD {ED // GF}

∠AFG = ∠ADP {ED // GF}

∠GAF = ∠PAD *{common angles}*

⇒ ΔAGF & ΔAPD are similar triangles

⇒ GF/PD = AF/AD = AG/AP

⇒ x/PD = 3x/(2x) = AG/AP

⇒ x/PD = 3/2

⇒ **PD = ⅔x cm**

From ΔAOB

$\sin 30 = OB/AB$

⇒ $OB = AB \sin 30$

⇒ **$OB = \frac{1}{2}x$**

$\cos 30 = AO/AB$

⇒ $AO = AB \cos 30$

⇒ **$AO = \frac{1}{2}x\sqrt{3}$**

From ΔARB & ΔAGF

$\angle ABR = \angle AFG$ {CB ∥ FG}

$\angle ARB = AGF$ {CB ∥ FG}

$\angle RAB = \angle GAF$ {*common angles*}

⇒ ΔAGF & ΔARB are similar triangles

⇒ $RB/GF = AR/AG = AB/AF$

⇒ $RB/x = AR/AG = x/(3x)$

⇒ $RB/x = \frac{1}{3}$

⇒ **$RB = \frac{1}{3}x$**

Blue Area = $\frac{1}{2} \times PD \times DG \times \sin \angle EDG$

$= \frac{1}{2} \times \frac{2}{3}x \times x \times \sin 60$

⇒ **Blue Area = $\frac{1}{6}x^2\sqrt{3}$**

Area of $\Delta AOB = \frac{1}{2} \times AO \times OB$

⇒ Area of $\Delta AOB = \frac{1}{2} \times \frac{1}{2}x\sqrt{3} \times \frac{1}{2}x$

⇒ **Area of $\Delta AOB = \frac{1}{8}x^2\sqrt{3}$**

Area of $\Delta ARB = \frac{1}{2} \times AB \times RB \times \sin \angle ABR$

⇒ Area of $\Delta ARB = \frac{1}{2} \times x \times \frac{1}{3}x \times \sin 60$

⇒ **Area of $\Delta ARB = \frac{1}{12}x^2\sqrt{3}$**

Red area = Area of ΔAOB - Area of ΔARB

$= \frac{1}{8}x^2\sqrt{3} - \frac{1}{12}x^2\sqrt{3}$

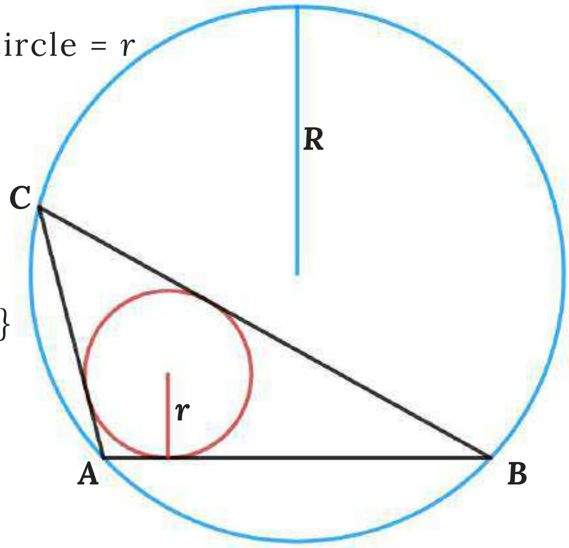
⇒ **Red area = $\frac{1}{24}x^2\sqrt{3}$**

Red Area : Blue Area = $\frac{1}{24}x^2\sqrt{3} : \frac{1}{6}x^2\sqrt{3}$

⇒ ***Red Area : Blue Area = 1 : 4***

SOLUTION 49

Let the radius of circumcircle = R & radius of incircle = r
then



r = Area of ΔABC/(½ Perimeter of ΔABC)

AC/sin B = AB/sin C = BC/sin A = 2R *{sine rule}*

⇒ **R = BC/(2 sin A)**

AB/BC = ¾

⇒ **AB = ¾ BC**

& AC/BC = ½

⇒ **AC = ½ BC**

Let **BC = 4*x***, then

AB = 3*x*

AC = 2*x*

Area of ΔABC = ½×AB×AC sin A

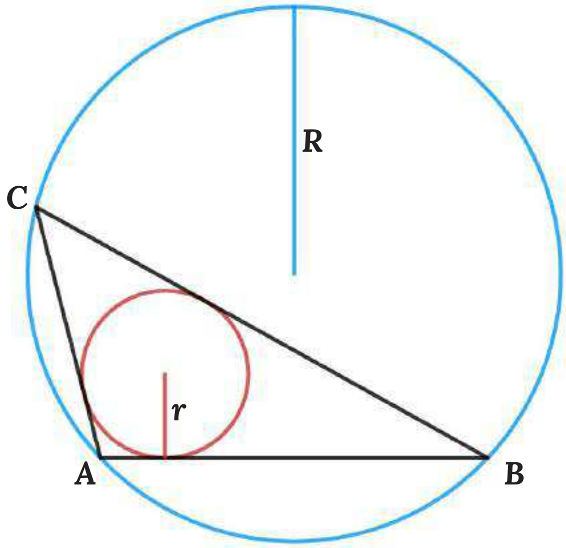
= ½×3*x*×2*x* sin A

⇒ **Area of ΔABC = 3*x*² sin A**

Perimeter of ΔABC = AB + BC + AC

= 3*x* + 4*x* + 2*x*

⇒ **Perimeter of ΔABC = 9*x***



r = Area of ΔABC/(½ Perimeter of ΔABC)

= 3*x*² sin A /(½×9*x*)

⇒ **r = ⅔ *x* sin A**

R = BC/(2 sin A)

= 4*x*/(2 sin A)

⇒ **R = 2*x* /sin A**

$r/R = [\frac{2}{3}\,x \sin A]/[2x / \sin A]$

$= \frac{1}{3} \sin^2 A$

Apply cosine rule in ΔABC

$BC^2 = AB^2 + AC^2 - 2\times AB\times AC\times \cos A$

$\Rightarrow (4x)^2 = (3x)^2 + (2x)^2 - 2\times 3x\times 2x\times \cos A$

$\Rightarrow 16x^2 = 9x^2 + 4x^2 - 12x^2\times \cos A$

$\Rightarrow 16 = 9 + 4 - 12\times \cos A$

$\Rightarrow \cos A = -\frac{1}{4}$

$\Rightarrow \cos^2 A = \frac{1}{16}$

$\Rightarrow \sin^2 A = 1-\frac{1}{16}$

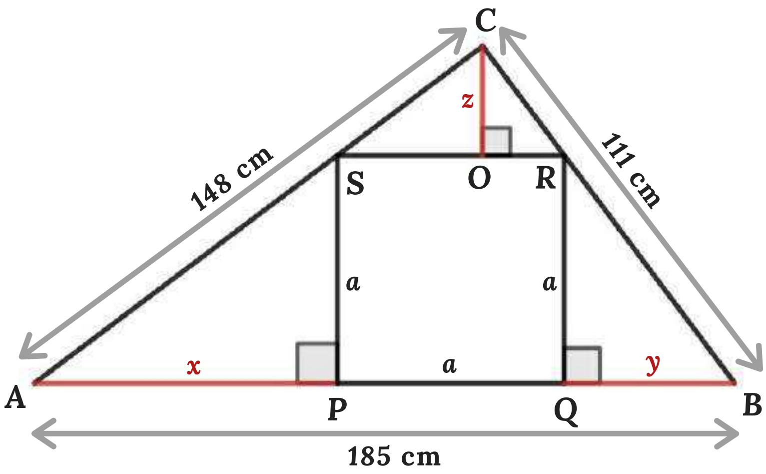
$\Rightarrow \sin^2 A = 15/16$

$r/R = \frac{1}{3}\times 15/16$

$\Rightarrow r/R = \frac{5}{16}$

$\Rightarrow$ **$r : R = 5 : 16$**

SOLUTION 50



Let the side of the square = a, then

$148^2 + 111^2 = 34225 = 185^2$

⇒ $AB^2 = AC^2 + BC^2$

⇒ ΔABC is a right angle triangle

From ΔABC & ΔAPS

∠BAC = ∠PAS {*common angles of ΔABC & ΔAPS*}

∠ACB = ∠APS = 90°

⇒ ∠ABC = ∠ASP

⇒ ΔABC & ΔAPS are similar triangles

⇒ AS/AB = AP/AC = PS/BC

⇒ AS/AB = $x/148 = a/111$

⇒ **$x = 148a/111$**

From ΔABC & ΔBQR

∠ABC = ∠QBR {*common angles of ΔABC & ΔBQR*}

∠ACB = ∠BQR = 90°

⇒ ∠BAC = ∠BRQ

⇒ ΔABC & ΔBQR are similar triangles

⇒ AC/QR = AB/BR = BC/BQ

⇒ $148/a$ = AB/BR = $111/y$

⇒ **$y = 111a/148$**

From figure

AP + PQ + QB = 185 cm

⇒ $148a/111 + a + 111a/148 = 185$

⇒ $21904a + 16428a + 12321a = 3039180$

⇒ $50653a = 3039180$

⇒ **$a = 60$ cm**

Area of ΔABC = Area of ΔAPS + Area of ΔBQR + Area of ΔCSR + Area of square

⇒ $\frac{1}{2} \times AC \times BC = \frac{1}{2} \times AP \times SP + \frac{1}{2} \times BQ \times RQ + \frac{1}{2} \times RS \times OC + PQ^2$

⇒ $\frac{1}{2} \times 148 \times 111 = \frac{1}{2} \times x \times 60 + \frac{1}{2} \times y \times 60 + \frac{1}{2} \times 60 \times z + 60^2$

⇒ $8214 = 30x + 30y + 30z + 3600$

⇒ $30x + 30y + 30z = 4614$

⇒ **$x+y+z = 153.8$ cm**