The Strength, Fracture and Workability of Coal

A Monograph on Basic Work on Coal Winning carried out by the Mining Research Establishment, National Coal Board

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To the Memory of

N. B. TERRY

Friend and Colleague

Foreword

THE writing of this monograph was undertaken at the instigation of Mr. A.H.A.Wynn, Scientific Member of the National Coal Board. During the greater part of the time occupied by the writing and the work in general, the Mining Research Establishment was in the Scientific Department of the National Coal Board, and we were greatly encouraged in our efforts by the then Director-General of Research, Dr. W.Idris Jones.

Although largely written by the two titular authors, the monograph is based upon the work of a number of people, and we express our thanks to present and former colleagues of the Mining Research Establishment, and other Departments of the National Coal Board, for their co-operation in allowing us to make reference to their findings. In particular, we are indebted to Mr. W.T.A.Morgans for Chapter 2, which is virtually a verbatim reproduction of an original paper written by himself and the late Dr. N.B. Terry.

Many of the chapters were originally published as articles or original papers in scientific and technical periodicals, and we are indebted to the following journals or publishers for permission to reproduce the whole or part of papers that were first presented in this way: *Fuel*, *Colliery Engineering*, Institution of Chemical Engineers, *Colliery Guardian*, *Steel and Coal*, Pergamon Press.

The monograph deals essentially with the strength of coal and the way in which various aspects of strength enter into the processes of mechanical winning, the ultimate aim being the rationalization of the design of coal-winning machines for use in British mines and an increase of efficiency in existing machines. Most of the work described has been carried out at the Mining Research Establishment. The volume does not pretend to be a survey of all the work carried out in this field in the many coal-producing countries. Its terms of reference are more limited and personal and it proceeds in a rough-and-ready manner to the chosen goal as quickly as possible. If we have not referred to work on the properties of coal which may be regarded as standard in allied fields, it is not because we do not appreciate its worth, but simply that we have not been able to use it immediately to meet our ends. Although we may lose in omniscience by our standpoint, we hope we may have gained in communicating a sense of the excitement and urgency that has been felt at one establishment in the pursuit of a limited topic of great practical importance. We hope, too, that the work will be of interest not only to coal-mining scientists but to those whose interests lie in the more general field of the properties of materials, brittle materials especially.

The work would not have been possible if we had not had throughout the stimulus, encouragement and support of the Director of the Mining Research Establishment, Dr. L. C. Tyte, to whom we express our deep gratitude. Finally, our best intentions would have come to nothing if it had not been for the hard work of our respective secretaries, Mrs. Margaret Wallace and Miss Mary Reed.

I.E.
C.D.P.

CHAPTER 1

The Nature of Coal and the Elements of Long-wall Mining

COAL is a combustible sedimentary rock, of specific gravity in the region of 1·3–1·5, formed from plant remains which have undergone a remarkable transformation while still preserving some of their original features. The main agent of the process has been the pressure of superimposed strata, which has compacted the material and produced or initiated physical and chemical changes in it. The characteristics of different coals are due to differences in the original nature of the plant remains, to the extent of the changes which have taken place under different conditions during the course of geological history, and to the nature and quantity of inorganic impurities present.

The original mode of accumulation of the plant debris has given rise to a good deal of controversy, with two main schools of thought preponderating, one favouring the growth of the plants *in situ*, the other favouring the concept of accumulation of vegetation which has drifted from the zone of growth. A critical account of the controversy is given by Moore(1), who comes to the conclusion that the majority of coal seams owe their origin to *in situ* growth, although some less extensive deposits have developed from drifted material.

Periods of accumulation and deposition of organic matter from which coal has been subsequently produced have occurred at many times in the geological succession of the Palaeozoic, Mesozoic, Tertiary and Quaternary Eras. At the present time, the existence of peat bogs and tropical swamps such as the Everglades of Florida may be taken to indicate that this process is still at work.

In Great Britain the great coal-fields which are worked today were all laid down in the Carboniferous Period of the Palaeozoic Era, roughly about 200 million years ago. The concept of their origin now widely held is that of a cyclic process whereby extensive regions underwent subsidence, with a period of quiescence during which vegetation could flourish, followed again by subsidence during which flooding and the accumulation of inorganic detritus could take place. The result, at the present time, of sequences of this nature is that coal seams in Great Britain, only a few feet thick in themselves, may be dispersed throughout thousands of feet of inorganic beds, the whole sequence comprising the Coal Measures.

The succession of beds of plant remains and inorganic sediments of the Coal Measures was finally buried under great thicknesses of younger inorganic sediments. The heavy pressures produced by this overburden promoted the chemical changes which have produced coal. These changes have been most complex, and are even now little understood in detail. The general effect of "coalification" has been to increase the proportion of carbon in the organic matter at the expense of the hydrogen and oxygen, this increase being designated as an increase in the "rank" of the coal. "Rank" in the qualitative sense indicates the position of a coal in the more or less continuous series ranging from peat, through brown coal and bituminous coal, to anthracite; anthracite is designated as "high rank". A more quantitative assessment of rank is made in terms of the carbon content, or alternatively the volatile matter of the coal. The figures are quoted as carbon (or volatile matter) on a dry mineral-matter-free (d.m.m.f.) basis or sometimes, less correctly, on a dry ash-free basis. British coals range from low rank bituminous coals with about 80 per cent carbon and 45 per cent volatile matter to high rank anthracites with 94 per cent carbon and under 10 per cent volatile matter.

Differences in rank between coals do not necessarily signify differences in age, but frequently reflect differences in the tectonic and other physical processes to which the coal was subjected during the course of coalification. The formation of anthracite is an interesting case in point. In South Wales the self-same seams which yield bituminous coal in the east of the coalfield yield anthracite in the west. It is now generally accepted that special conditions of pressure and temperature occurred in the anthracite areas, for anthracite is also found elsewhere in Great Britain in small quantities in the vicinity of igneous intrusions in coal seams. Significant lateral variations in rank are found in most of the British coalfields. The lowest rank coals, non-caking and free-burning, are to be found in Warwickshire, South Staffordshire, Leicestershire, South Derbyshire and around Nottingham. Caking properties occur with increase in carbon content in the coals of North Derbyshire, North Nottinghamshire and South Yorkshire, where the usual range of carbon content is from 81 to 86 per cent and metallurgical coke of moderate quality is made. Further north, the carbon content of the famous coking coals in Durham is still higher at 88–89 per cent.

A general law or rule due originally to Hilt applies to the differences in rank found in a normal sequence of beds. The rule is that the volatile matter in a normal sequence of beds decreases (i.e. the rank increases) with the depth of the seam. In the Ruhr, for example, coals range from bituminous in the uppermost seams at shallow depths to anthracite in the lowest seams in the succession.

Structure and Petrology of Coal

The use of coal as a fuel and as a source of coke and chemicals has promoted much research into its structure and petrology. This has not the same importance from the point of view of the problems of coal-winning, but some of the concepts should be mentioned as they are a source of terminology which has carried over into wider fields.

Most specimens of coal have a banded structure, these bands being parallel to the planes of deposition of the original material. Two main types of substance persist throughout the whole range of coals and they can generally be distinguished by optical or chemical means.[(2)] In the bituminous coals the distinctions can be made with the unaided eye; bright bands which are derived mainly from wood or cortex, and dull bands formed from miscellaneous debris. In the low volatile coals and anthracites, and again in the brown coals, the two main components are not always well defined to the unaided eye, but they can be distinguished under the microscope.

Early recognition of the relationship between differences in appearance and chemical properties of the banded components present in British bituminous coals was due to Stopes[(3)], who recognized four different and separable bands or layers of bituminous coal in all seams which she named provisionally vitrain, clarain, durain and fusain.

Vitrain. This occurs in bands or lenticles of a few millimetres or upwards in thickness. Brilliant, glossy and vitreous in texture. It is macroscopically structureless and homogeneous and breaks with a well-marked rectangular fracture perpendicular to the bedding, and a conchoidal fracture in other directions.

Clarain. Bright, streaky or striated coal with a silky lustre. It is not so uniformly brilliant and homogeneous as vitrain and lacks its conchoidal fracture.

Durain. Hard, compact, dull coal. Macroscopically it is practically structureless and is greyish to dull black in colour. It occurs in both thick and thin bands. Fracture surfaces are finely granular and irregular.

Fusain. Fibrous, friable, dull charcoal-like material. It breaks down readily to fine dust which soils the hands, and is usually greyish-black in colour.

In the work to be described in this volume, no attempt has been made to measure the mechanical properties of these various constituents. Coal has been accepted as a physical material, its lack of homogeneity being one of its essential properties.

Structure of Coal in the Seam

The fracture of coal, and the forces required to break it from the seam, depend greatly upon the innate weaknesses. The bedding planes are one obvious source, where bonding between successive layers of deposition has not fully taken place. Successive hard bands may be separated, for example, by a very thin layer of fusain or of dirt which provides little cohesive properties.

Other planes of division occur at right angles to the stratification. The main divisions are known as "cleats" and the secondary divisions as "cross-cleats". These systems intersect at an angle of generally 80–90 deg. The cleats are always best developed in the bright parts of bituminous coal, and are less distinct, more widely spaced, or even non-apparent in durain. When the dull and bright parts are finely laminated, however, the cleat planes pass through the mass without interruption. A specimen of cleated coal is shown in Fig. 1.1.

A study of cleat(4) has made clear the general uniformity in its direction over a wide area, and the general agreement in the polarity of the cleat in seams that may be separated by great thicknesses of rock. Another remarkable attribute of cleat is its ubiquity; no bright band of coal, however thin, and no lenticle, however small, has been too insignificant to escape.

Kendall(5) suggested that the origin of the cleat of coal was to be sought in tidal action, for it is known that the combined gravitational pull of the moon and sun produces an elastic response of sensible magnitude in the earth's crust. The following qualitative description of the action of this mechanism in producing cleat has been given by Kendall and Briggs:

> A bed or series of beds is laid down on strata that are already jointed. At first soft enough or incoherent enough easily to adjust itself to the tidal wave without breaking, it is sooner or later covered up and gradually becomes consolidated by pressure, loss of moisture, inflow of cementing material, and, with coal, by chemical changes. Losing its initial plasticity and gaining brittleness, it then finds itself called upon to flex as a continuous elastic sheet under the alternating tidal torque. Eventually it fails through fatigue along the planes of maximum shear, and the joints thus come into existence.

Some doubt has been cast on this hypothesis by Moore(6), who obtained data for coal mines across North America and from Mexico to northern Canada, and failed to find any such uniformity as that postulated by Kendall. He found that the major cleats varied their direction with the structural features of different coal-fields, and that they were related in direction to these structures, indicating the influence of folding and other disturbances of terrestrial origin in the earth's crust.

The coal seams of South Wales and Kent exhibit a characteristic natural fracture, known as slip, which is quite distinct from cleating, and is seldom of importance in other British coal-fields(7). Slip fractures often traverse the full thickness of the seam and are usually inclined at about 25 to 45 deg to the vertical. There is little displacement along the fracture planes though they exhibit some evidence of movement under pressure. The interval between slip planes varies from several inches to a few feet. The seams in South Wales often exhibit a dominant and a secondary slip striking in different directions. The dominant slip usually strikes roughly parallel to the normal faulting (i.e. NNW./SSE.) but there are many exceptions to this generalization. Face slips and back slips are those which strike parallel to a particular working face, and dip from the roof towards the direction of advance or towards the waste respectively; end slips are at right angles to these; oblique slips are intermediate in direction. In general a well-developed slip is advantageous to the ease of working the coal, whether by hand or by mechanical means. The disposition and frequency of slip may sometimes be a decisive factor in planning the direction of a face line.

A photograph of slip in a South Wales seam is given in Fig. 1.2.

Coal Classification System

The volatile content of coal is a good indication of its properties from the point of view of combustion and other technological processes, but does not define the properties uniquely. Especially in the high volatile range, coals may have different properties at the same volatile content, and they are then differentiated according to their propensity for swelling or "caking" when heated. This swelling is related to the coking properties of the coal.

The classification system devised by the National Coal Board places the coals in arbitrary categories on the basis of two measurements: volatile content and type of coke produced in the Gray-King test. A simplified version of the classification is given in Table 1.1.

Table 1.1. The Coal Classification System used by the National Coal Board

Coal rank code number	Volatile matter (d.m.m.f.) (per cent)	General description
100	Under 9·1	Anthracites
200	9·1–19·5	Low-volatile steam coals
300	19·6–32·0	Medium-volatile coals
400–900	Over 32	High-volatile coals
400		Very strongly caking coals
500		Strongly caking coals
600		Medium caking coals
700		Weakly caking coals
800		Very weakly caking coals
900		Non-caking coals

In practice the Groups may be subdivided into Classes and Sub-classes by the addition of integers and letters to the Group code numbers, e.g. Group 200 has Classes 201, 202, 203, 204, 206, and Class 201 is subdivided into Sub-classes 201a and 201b. Sometimes the properties of a coal may span two classes and it is designated by a dual characteristic, e.g. 301/501 (or 3/501 for short). A detailed account is given in a pamphlet published by the National Coal Board[8].

Even in the absence of detailed investigation, it is possible to gain the impression that the strength of coal is related in some way to its volatile content; a specimen of anthracite (one not containing a major fissure of the kind that sometimes weakens this coal significantly) is strong, so also are specimens of many high-volatile bituminous coals. Between the two extremes, in the region of 20–30 per cent volatiles, coals are encountered that appear to be friable. Tentatively, a U-shaped distribution can be ascribed to strength as a function of volatile content.

These considerations have prompted the selection for Mining Research Establishment (M.R.E.) work of one coal, as nearly as possible, from each of the various Groups for experimental work in the laboratory, so that cover of most of the coal types to be found in Great Britain could be achieved. Consignments of large lumps from the particular seam have been sent regularly from each pit to M.R.E., where they have been stored under dry conditions until use, a period which has usually been of the order of weeks, and rarely more than a few months. As a face has progressed in the seam, the samples obtained from it have sometimes varied in quality, but this has been accepted as part of the general pattern of variability that has to be

TABLE 1.2. REPRESENTATIVE COALS USED BY THE MINING RESEARCH ESTABLISHMENT

Coal rank code number	Colliery	Seam	Percentage volatiles (d.m.m.f.)	Description
100a	Pentremawr	Pumpquart	5	Anthracite
201	Deep Duffryn	Five Feet†	12	Dry steam coal
301	Oakdale	Meadow	22	Metallurgical Coking coal
301/401	Cwmtillery	Garw	30	Coking coal
402/502	Markham	Black Shale	39	Bituminous coal
702	Rossington	Barnsley (Brights)	37	
702/802	Pleasley Teversal	Dunsil	37	
801	Rossington	Barnsley (Hards)	36	
902	Linby	High Main	39	

† The Five Feet seam is separated by only a dirt band of variable thickness from an upper seam, the Gellideg, and the two together are sometimes regarded as a simple seam and denoted by the joint name, i.e. the Five Feet/Gellideg.

delineated in dealing with coal. Occasionally a face or seam has run into difficulties in working, or into obviously non-representative conditions, e.g. by the incursion of dirt bands, and then equivalent samples from other seams in the same Classification Group have had to be used.

Occasionally other coals have been used for special purposes, e.g. 203/204 Coegnant Colliery, Gellideg seam, 17 per cent volatiles, which has the

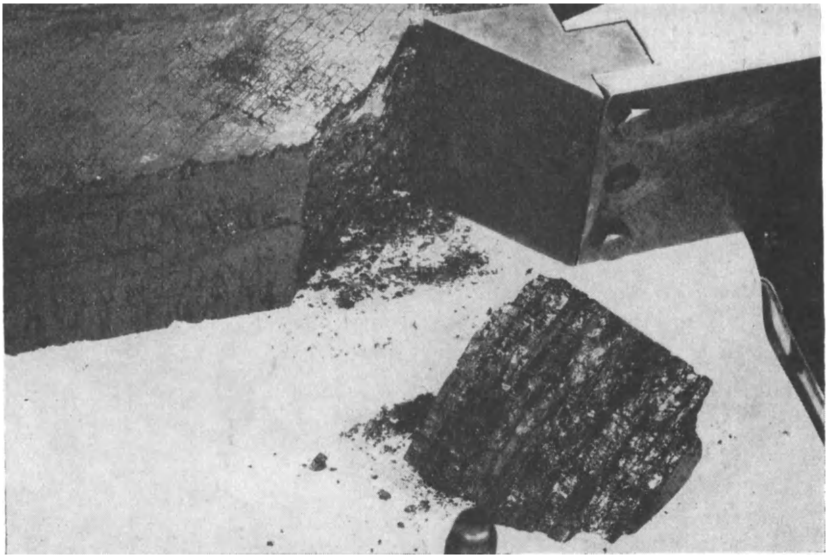


FIG. 1.1. Cleated coal (cleats observable on upper surface of unbroken slab).

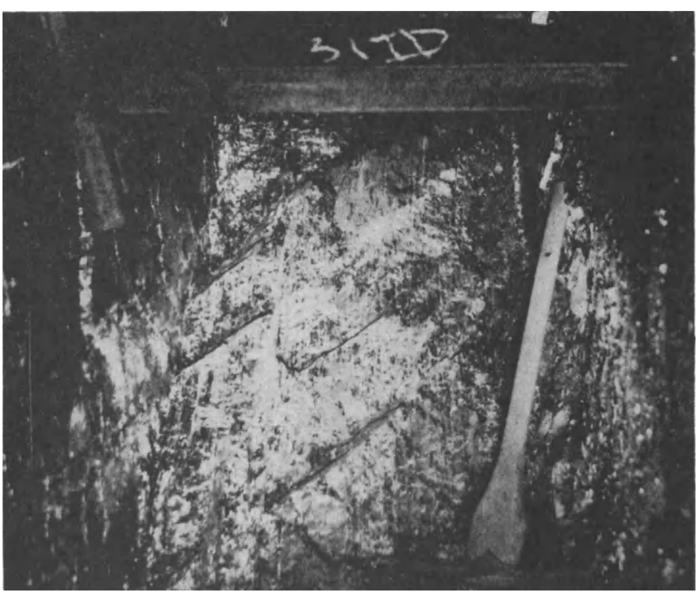


FIG. 1.2. Slip in a South Wales coal (Garw seam, Cwmtillery Colliery).

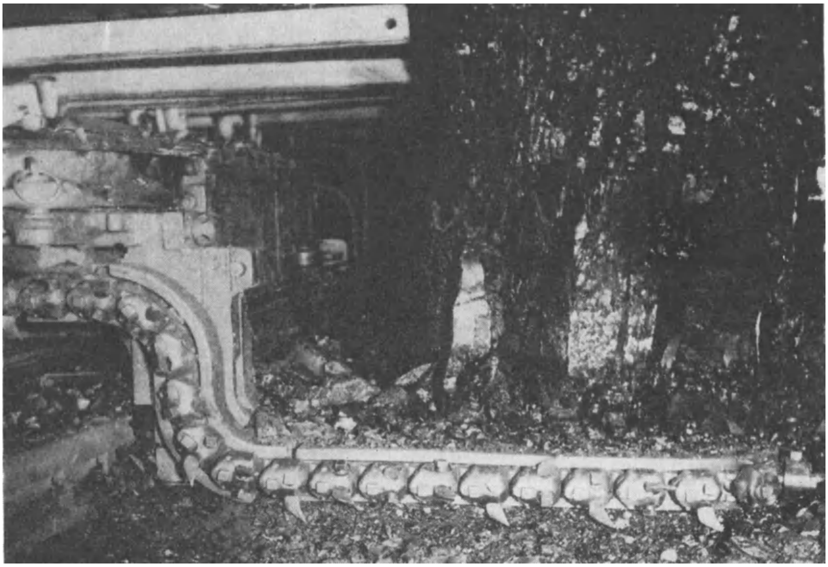


FIG. 1.3. Jib cutter.

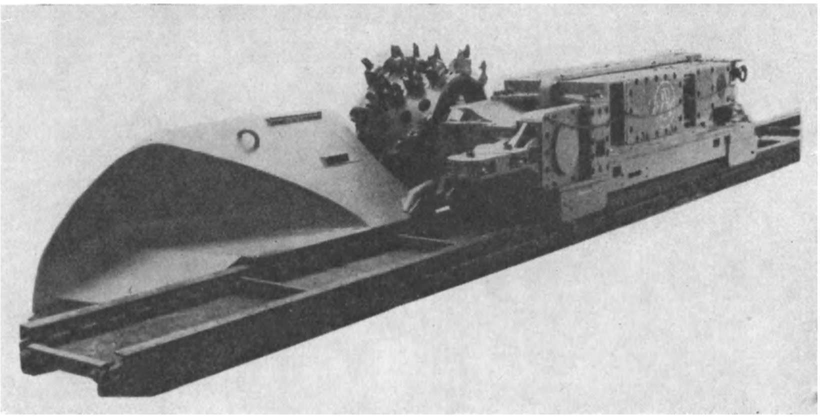


FIG. 1.4. Anderton shearer.

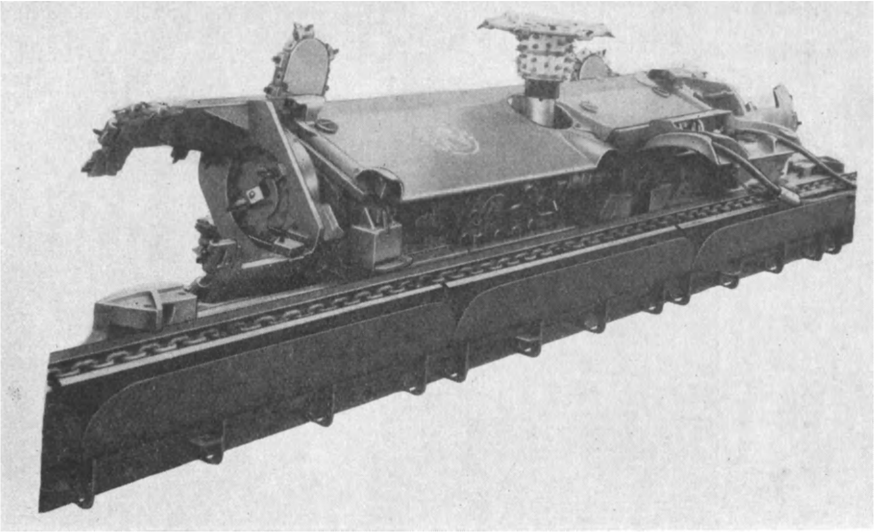


FIG. 1.5. Long-wall trepanner from waste side. (By courtesy of Anderson, Boyes & Co. Ltd.).

FIG. 1.7. Rapid plough at the coal face.

reputation of being particularly friable, and the following bituminous coals, which were highly cleated:

402 Trimdon Grange, Tilley seam, 37 % volatiles,
602/702 Sutton Tupton, Low Main seam, 38 % volatiles,
602 South Kirkby, Beamshaw seam, 35 % volatiles.

The values of volatile content quoted above and in Table 1.2 are all approximate, and may vary slightly from one sample of the coal concerned to another.

Of the representative coals, three in particular have been much used, anthracite from Pentremawr (Pumpquart seam), coking coal from Cwmtillery (Garw) and bituminous coal from Rossington (Barnsley Hards). In the tentative U-shape of strength against volatile matter, Pentremawr and Rossington fall at the extremities of the limbs of the U und Cwmtillery near the middle. The concentration of effort on these three merits a more detailed consideration of their characteristics.

Pentremawr anthracite, Pumpquart seam

A highly fusainic anthracite. Bedding is rudimentary and discontinuous, the general plane being represented by approximately parallel fusain lenses (the larger lenses are of the order of 0·3–0·7 in. in length and 0·025–0·1 in. in thickness). Hair line fractures (not cleat) make an angle of about 45 deg with the bedding plane. Because of the absence of cleat and continuous bedding, the coal is very compact and resistant to breaking.

Cwmtillery coal, Garw seam

A coal containing 60–80 per cent of bright constituents, with bands 0·08–0·2 in. thick. It has some fusainic lenses and a little pyrites in the vitrain bands. Cleat consists of fine cracks, usually less than 1·5 in. long. It is friable and parts readily along the bedding.

Rossington, Barnsley Hards seam

This coal is from the dull middle section of the famous Barnsley seam. It contains about 20–40 per cent of bright constituents. Grey durain bands are prominent and some fusain lenses are visible. It is a compact coal, only lightly cleated.

METHODS OF COAL-WINNING

The principal means of mechanized extraction used in Great Britain is the long-wall system, in which a face is opened between two parallel roadways, situated at a distance of 100–200 yd apart. The working space on the face has a width of about 10 ft and is supported by rows of props engaging with roof bars, the latter being disposed at right angles to the face. As the face advances the props are moved forward (in most recent practice they are self-advancing), and the region behind the back row is allowed to crumble into a waste or goaf by the process of "caving". Sometimes complete caving results in bad roof conditions and rapid convergence on the roadways, which are advanced with the face. Under these circumstances walls of stone ("packs") several yards in width are constructed at the waste edge between floor and roof at intervals along the face, and extended in the direction of advance as the face moves forward. Occasionally, where surface subsidence has to be mitigated, the waste is completely "stowed" by crushed rock which is placed by mechanical means or blown into position by compressed air.

The original, and still widely-practised method of machine mining is to use a cutting machine, which works along the space between the coal face and the first row of props. This machine is armed with picks mounted on a chain which runs around a jib (Fig. 1.3). As the machine is pulled along the face by means of a built-in winch which hauls in a steel rope anchored at the other end, the jib under-cuts the seam to a depth of about 5 ft and a height of 6 in. This operation usually takes one shift. On succeeding shifts the following operation takes place:

Shift 2. The coal above the undercut has holes drilled into it, in which explosives are placed. The coal is then broken down by blasting, and manually loaded on the conveyor, which runs along between the first and second rows of props.

Shift 3. A row of props is set at a distance of one machine width from the new face. The conveyor is dismantled and moved forward so as to sit again behind the first row. Packs are extended, and the last row of props, now being redundant, is dismantled or "drawn off".

The cyclic system has several disadvantages, of which the most pronounced are that coal is extracted on one shift only, and that a great deal of effort goes into the non-productive job of dismantling and reassembling the conveyor. An essential prerequisite of continuous mining was the development of a conveyor that was sufficiently strong and resistant to hard knocks to be used in the immediate vicinity of the face, i.e. between the face and the first row of props, and yet flexible enough to be pushed over segmentally, without being broken down, after a strip of coal had been cut and loaded by the machine. Satisfactory armoured flexible or "panzer" conveyors, the sobriquet paying tribute to their German origin, are now in widespread use in British mines. The present generation of coal-winning machines are mounted on and move along the armoured conveyor, cutting the coal and loading it automatically. All these machines work to a buttock, i.e. their cutting elements have in the first instance to be advanced beyond the line of the face in order to break down a "web" of coal as the machine travels along the face. The face ends are therefore enlarged into "stable-holes" where this and other operations concerned with power loading can take place. Irrespective of the machine on the face, the stable-holes are usually cut by jib-cutters.

The machines most commonly used for power-loading on long-wall faces in British pits at the present time are the Anderton shearer, the trepanner and the rapid plough.

The Anderton shearer

The machine was invented by Mr. J. Anderton, now Chairman of the North Western Division, National Coal Board, and has been developed by the British firms of Anderson, Boyes & Co. Ltd. and British Jeffrey–Diamond Ltd. The principle of the machine is the use of a standard coal-cutter mechanism to drive a horizontal shaft on which a drum is mounted (Fig. 1.4). As the machine is hauled along the face the drum rotates and the picks with which it is equipped take off a web of coal, which may vary in thickness from about 15 in. to 30 in. or more. Drum diameter varies according to the thickness of the seam, and the common range of cutting diameters (i.e. to pick points) is between 30 and 50 in. Drum speed is upwards of about 60 r.p.m. Haulage speed for cutting is commonly about 6 ft/min but higher speeds up to 30 ft/min are possible. The original machine cut in only one direction along the face; on its return path it ploughed on to the conveyor any broken coal that had fallen in the machine track. Shearers which cut and load in both directions have recently been developed.

The Anderton shearer has proved itself a reliable and versatile machine. Its main disadvantage lies in the fact that it tends to break the coal into small sizes; thus its main use is for winning coal where large size is not important, e.g. for power stations or for coke ovens. In the pit, the production of small sizes aggravates the dust problem.

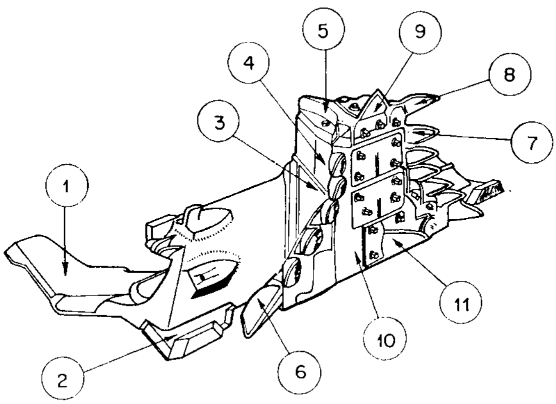
The Anderson–Boyes long-wall trepanner

In this bi-directional machine the cutting heads rotate about an axis parallel to the coal face. Each head has projecting arms on which picks are mounted so as to cut an annulus (Fig. 1.5). A core of coal is thus isolated during the cutting action, and broken up by picks mounted on the axis. Because of the cylindrical nature of the section cut out of the seam by the trepanning head, auxiliary jibs and roof cutters are required to trim the face and the roof to a more orthodox shape. The machine is conveyor mounted and hauls itself along on a chain with a rate of advance of about 3 ft/min for cutting. It is particularly suitable for seam sections from 37 in. to 48 in. in diameter, and yields a larger size of product than the Anderton shearer.

The rapid plough

This machine, developed initially by the German firm of Westfalia, is a device for peeling off a web of coal, of the order of 2–3 in. thick, from the coal face by an action that superficially resembles planing in wood or metal working. The plough has no rotating parts on the face itself, being hauled by an endless chain which is motivated in the stable-holes, the return path being through a tube bolted on the side of the conveyor.

The plough body consists of a base which is kept in firm contact with the conveyor by means of prong-like extension pieces which pass under it, and



1. PLOUGH BASE
2. FLOOR CUTTER OR FLOOR PRECUTTER
3. PLOUGH TOP PORTION
4. EXTENSION PIECE
5. COVER
6. BOTTOM BIT
7. CUTTING BIT
8. TOP BIT
9. TOP KNIFE
10. PRECUTTER PLATE
11. PRECUTTER

FIG. 1.6. Rapid plough body.

a cutting head which is bolted on to the base. The conveyor, and hence the plough, is held in to the face by means of forces exerted by pneumatic or hydraulic rams. A diagram of the plough body is given in Fig. 1.6, and of a plough working in the face in Fig. 1.7. The speed of the plough along the face is about 75 ft/min.

The plough is a simple device and gives a good size of product. Originally designed for soft German coals, it has experienced difficulties in winning harder British coals.

Research Work in Relation to Practical Problems

The machines just described are different in many details of their mode of action, but the basic principle is the same, viz. the attack upon coal by an array of wedge-shaped devices, usually called picks or blades, the latter name being particularly applied in relation to ploughs. The distinction between picks and blades is not marked, and is rapidly losing all significance in modern practice, as a large pick on a shearer looks very much like a blade on a rapid plough. The speeds of action may be different. The cutting point on a shearer moves at a speed which is in the region of 600 ft/min, whereas a plough blade cuts at 75 ft/min. It will be seen later that this difference is not important in the action of cutting. The fundamental problem is therefore the mode of breakage of coal by a wedge at various depths of cut, and this enquiry informs all the research work to be described in the following chapters. Ineffective cutting devices waste power, and under the conditions of mining this waste is especially deplorable in that it may go partly into producing dust and partly into producing heat, both dissipations tending to worsen the bad environmental conditions that the miner faces. Effective cutting would therefore have many benefits, larger coal, avoidance of over-powered and over-large machines, and less dust and heat. It would probably also make the machine easier to control in the seam.

The chapters to follow represent necessary elaborations on the main theme of breakage by wedges. The properties of coal as a material are investigated, the empirical tests which can characterize it, the aspects of strength which enter into breakage, the mode of fracture and the size of the product. Then follow investigations related specifically to wedge penetration and the mechanics of wedge breakage, including the effect of friction between tool and coal. Finally, there are chapters devoted to consideration of how these various aspects can be knitted into a simple theoretical framework, which will be of interest to scientists concerned with materials, and of particular use to engineers concerned with the design and use of coal-winning machines. The final aim is the economic production of coal for the benefit of the community, the amelioration of the task of the miners who produce it, and incidentally the acquirement of knowledge and insight into the physical properties of this strange and fascinating material.

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8. National Coal Board, Scientific Department, *The Coal Classification System used by the N.C.B.*, 1956.

CHAPTER 2

The Static and Dynamic Elastic Moduli of Coal

AS DISCUSSED in Chapter 1, coal possesses a stratified structure comprising a number of components. It is likely, therefore, that the elastic behaviour of bulk coal will in general be different from that of the basic coal substance. With possibly one exception[1], measurements by previous workers, as well as those described here, refer to the elastic properties of bulk coal. The relationship between the two sets of properties cannot easily be assessed, since it depends upon the distribution of cracks or capillaries, and the effect of these on the technique used for measurement.

For dynamic techniques, where small specimens are subjected to mechanical vibrations involving low strains (10^{-5}), the scatter is comparatively small; static methods of measurement, on the other hand, give rise to a larger scatter of results, but can be extended to the high strains (10^{-2}) occurring in mining processes. Because of the inhomogeneous nature of coal, unique values for the elastic moduli cannot be obtained and many measurements are required for representative evaluation. Very few data have in fact been published on the elastic constants of coal and, apart from the work of Inouye[2,3] with Tani on Japanese and American coals, a detailed examination of the same coals by both static and dynamic techniques does not seem to have been previously attempted. Moreover, in the literature available[2–9], little or no information is given of the number of specimens tested and of the scatter of results obtained.

TABLE 2.1. DESCRIPTION OF COALS

Colliery	Seam	Description	NCB coal rank code no.	Approx. volatile matter (%)	Density (in helium) (g/cm³)
Llandebie or Pentremawr	Pumpquart	Anthracite	100a	7	1·499
Rossington	Barnsley (Top Hards)	Strong, dull, bituminous coal	801	34	1·364

(A summary of measurements on other coals is also given later.)

In order that a reliable model for the mechanical behaviour of bulk coal may be eventually formulated it is considered that an examination of the elastic behaviour of the same coal types by more than one technique is essential. This chapter describes detailed measurements on a bituminous and an anthracite coal, using two distinct static techniques and one dynamic technique. Details of the two coals are given in Table 2.1.

STATIC COMPRESSION TESTS

Preliminary Considerations

The measurement of Young's modulus of coal by conventional static compression tests has recently been criticized[3,9] on the ground that flow taking place in the specimen under load would influence the values obtained. In order to investigate this, the variation of strain with time was measured for cubes of coal subjected to constant loads; a typical curve for Barnsley Hards is shown in Fig. 2.1. To a first approximation, the behaviour is that

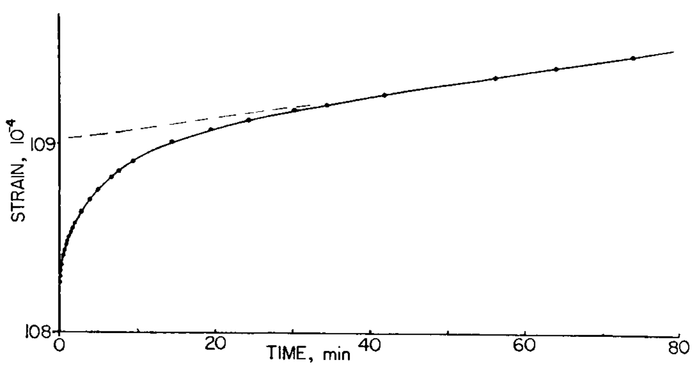

FIG. 2.1. Strain–time curve for Barnsley Hards $1\frac{1}{4}$in. cube. Load 1·98 tons. Compression perpendicular to the bedding plane.

which would be obtained from the Burgers spring and dashpot model (see Fig. 2.2) which is frequently used to represent viscoelastic materials. The application of a compressive stress, σ, to this model would result in a strain S_B which is given by

$$S_B = \sigma\left\{\frac{t}{\eta_1} + \frac{1}{E_1} + \frac{[1 - \exp(-tE_2/\eta_2)]}{E_2}\right\} \tag{2.1}$$

where t = time.

The component $\sigma t/\eta_1$ represents the irrecoverable strain due to viscous flow in the dashpot η_1. The slope of the linear portion of the curve in Fig. 2.1

gives a measure of the viscosity η_1, which for Barnsley Hards is found to be of the order of 10^{16} poises, and for anthracite is at least an order of magnitude higher.

The recoverable component σ/E_1 is the instantaneous strain occurring in the spring E_1 and the recoverable component $(\sigma/E_2)\,[1 - \exp(-tE_2/\eta_2)]$ is due to the closing of the spring E_2 against the action of the dashpot η_2. The true static modulus E_s is given by

$$1/E_s = 1/E_1 + 1/E_2 \qquad (2.2)$$

and may be obtained by measuring the total recoverable strain, which is given by the intercept of the linear portion of the curve on the strain axis.

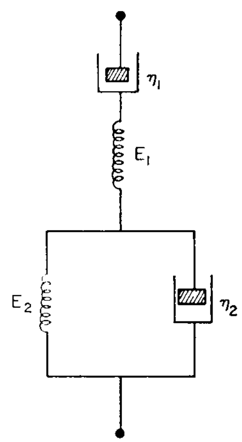

FIG. 2.2. Burgers model illustrating the behaviour of viscoelastic materials.

In practice, the strain is measured within a few seconds of the application of load, when the value is apparently steady. It can be seen from Fig. 2.1 that the measured strain will, in fact, be approximately 0·5 per cent lower than the total recoverable strain. It is clear, therefore, that neither of the viscous components will affect the measured modulus to any appreciable extent.

Previous attempts to derive a value of Young's modulus from stress/strain curves appear, however, to have met with but limited success. Lawall and Holland[5], Heywood[6], and Phillips[8] have reported that the graphs obtained have been continuously curved. There are several factors which may contribute to this non-linearity:

(*a*) The process of bedding-down between the compression platens and the specimen would give rise to apparently high increments of strain, for given stress increments, at very low stresses. This effect can, however, be eliminated by measuring the strain between two points on the surface of the specimen instead of between the platens;

(*b*) The application of load would tend to close up suitably oriented cracks in the specimen, also giving rise to high apparent rates of strain at low stresses;

(*c*) The stress–strain curve would be influenced by partial or complete breakdown of the specimen, giving high rates of strain at high stresses. Both Barnsley Hards and anthracite, however, in common with all other coals tested in this laboratory, fail explosively under the conditions normally present in a compression test, so that the stress–strain curve obtained would not be affected.

Compression Tests on $\frac{1}{2}$ in. Cubes

The apparatus used for the compression of $\frac{1}{2}$ in. cubes consists of a steel anvil and ram between which the specimen cube is compressed by means of an engineer's bench vice fitted with a reduction gear. The load applied to the coal is measured by means of resistance strain-gauges fitted to the ram. The longitudinal strain in the specimen is determined from the change in capacity of an air condenser, one element of which is fixed to the anvil, the other being carried on the end of the ram. Prior to the experiments described below, this apparatus was fitted with an optical extensometer designed to measure the strain in a direction perpendicular to the applied load.

A single layer of 24 cubes was cut from one lump of each type of coal, the layer lying parallel to the bedding plane. Each cube was tested in one of the following ways:

(*a*) load applied perpendicular ($\perp$) to bedding plane, lateral strain measured parallel ($\|$) to bedding plane;
(*b*) load applied $\|$ to bedding plane, lateral strain measured $\|$ to bedding plane;
(*c*) load applied $\|$ to bedding plane, lateral strain measured $\perp$ to bedding plane.

The graphs of stress against longitudinal strain, and lateral strain against longitudinal strain exhibited, in general, a curved initial portion, varying considerably from one cube to another, followed in almost every case by a substantially linear relationship which persisted up to the point at which failure of the specimen was imminent. Young's modulus and Poisson's ratio were determined from the linear portions of the curves, the mean values being given in Tables 2.2 and 2.3 respectively. The results are given in both c.g.s. and f.p.s. units, the convenient sub-units being 10^{10} dyne/cm^2 and 10^5 lb/in^2 respectively.

The scatter in the measured values of Poisson's ratio in compression is too large to show a significant dependence on the orientation of the specimen (see Table 2.3). The values obtained from individual cubes were frequently greater than 0·5, and for three anthracite cubes, greater than unity. If the coal is anisotropic, values greater than 0·5 can, of course, be obtained, but

TABLE 2.2. YOUNG'S MODULUS—½ IN. CUBES

Type of coal	Direction of stress	Number of specimens	Young's modulus 10^{10} dyne/cm^2 (10^5 lb/in^2)	Standard error 10^{10} dyne/cm^2 (10^5 lb/in^2)
Barnsley Hards	Perpendicular to bedding plane	8	3·77 (5·43)	0·25 (0·36)
Barnsley Hards	Parallel to bedding plane	16	4·13 (5·98)	0·24 (0·35)
Anthracite	Perpendicular to bedding plane	8	4·61 (6·68)	0·37 (0·54)
Anthracite	Parallel to bedding plane	16	4·43 (6·41)	0·20 (0·29)

the latter three determinations are obviously incorrect. It is well known that in the normal compression test a specimen does not deform under conditions of plane strain. Owing to friction at the platens, the lateral expansion of the specimen occurs as a barrelling effect, and the usual method of measuring the deformation (i.e. midway between the platens) gives rise to high values of Poisson's ratio. Filon(10) has calculated that for an ideal material with completely restrained ends the error is about 5 per cent, but some rough experiments in this laboratory have shown that for some materials the errors are liable to be much greater than this—e.g. errors of over 30 per cent have been observed with cubes of india-rubber.

TABLE 2.3. POISSON'S RATIO—½ IN. CUBES

Type of coal	Direction of stress	Direction in which lateral strain is measured	Number of specimens	Poisson's ratio	Standard error
Barnsley Hards	Perpendicular to bedding plane	Parallel to bedding plane	8	0·41	0·02
Barnsley Hards	Parallel to bedding plane	Parallel to bedding plane	8	0·37	0·03
Barnsley Hards	Parallel to bedding plane	Perpendicular to bedding plane	8	0·44	0·07
Anthracite	Perpendicular to bedding plane	Parallel to bedding plane	8	0·48	0·11
Anthracite	Parallel to bedding plane	Parallel to bedding plane	8	0·44	0·04
Anthracite	Parallel to bedding plane	Perpendicular to bedding plane	8	0·50	0·14

Compression Tests on $1\frac{1}{2}$ in. Cubes

Compression tests on $1\frac{1}{2}$ in. cubes were carried out in a 10-ton hydraulic testing machine, the longitudinal strain being measured between two points on the side of the specimen by means of an optical extensometer, similar in principle to the Lamb roller extensometer[(11)]. The lateral strain was not measured.

Ten cubes were cut from a single 5 in. lump of Barnsley Hards coal, and eleven cubes from a somewhat larger lump of anthracite. Each cube was loaded in each of the three possible directions in turn, the order in which

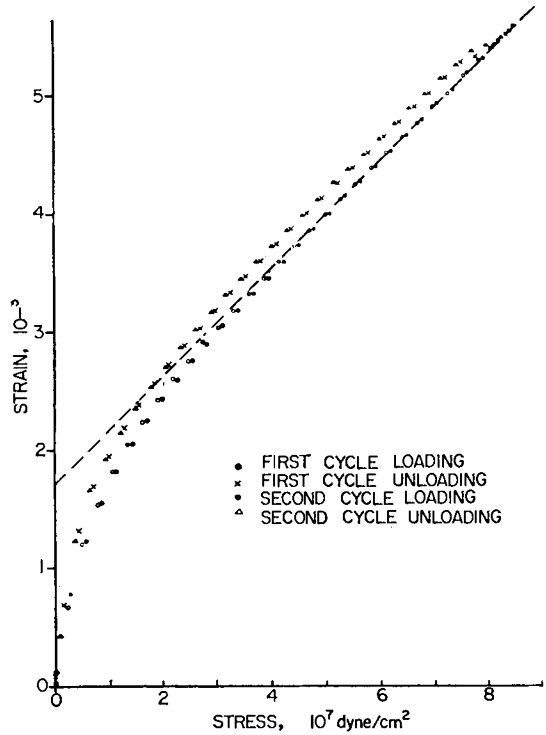

FIG. 2.3. Typical stress–strain curves for a cube of Barnsley Hards coal in compression.

the directions were chosen for any particular specimen being determined cyclically. The maximum load applied was well below that at which fracture would have been expected. A typical example of the stress/strain curve obtained from two cycles of loading and unloading is given in Fig. 2.3. The mean values obtained for Young's modulus are given in Table 2.4.

TABLE 2.4. YOUNG'S MODULUS—$1\frac{1}{2}$ IN. CUBES

Type of coal	Direction of stress	Number of specimens	Young's modulus 10^{10} dyne/cm^2 (10^5 lb/in^2)	Standard error 10^{10} dyne/cm^2 (10^5 lb/in^2)
Barnsley Hards	Perpendicular to bedding plane	10	2·70 (3·91)	0·08 (0·12)
Barnsley Hards	Parallel to bedding plane	20	3·35 (4·85)	0·09 (0·13)
Anthracite	Perpendicular to bedding plane	11	3·77 (5·45)	0·16 (0·23)
Anthracite	Parallel to bedding plane	22	4·04 (5·86)	0·19 (0·29)

It would appear that the only significant influence on the stress/strain curve is that due to the closing up of the effective cracks in the specimen as the load is increased, and the initial curvature of the graphs is therefore attributed to this effect. Moreover, the intercept on the strain axis of the extrapolated linear portion of the graph may be considered as a measure of the open cracks and/or layers of very soft material in the specimen. The mean values of these intercepts are shown in Table 2.5.

TABLE 2.5. STRAIN INTERCEPTS—$1\frac{1}{2}$ IN. CUBES

Type of coal	Direction of stress	Number of specimens	Strain intercept (units of 10^{-3})	Standard error (units of 10^{-3})
Barnsley Hards	Perpendicular to bedding plane	10	1·57	0·15
Barnsley Hards	Parallel to bedding plane	20	0·30	0·09
Anthracite	Perpendicular to bedding plane	11	0·70	0·15
Anthracite	Parallel to bedding plane	22	0·32	0·10

For both coals the value of the strain intercept for compression perpendicular to the bedding plane is significantly greater than that for compression parallel to the bedding plane. It should, however, be emphasized that neither of these coals exhibits extensive cleating. In a similar series of tests on a heavily cleated coal (Deep Duffryn, Gellideg seam) the strain intercept for parallel loading was significantly greater than that for perpendicular loading.

It might be expected that in a very extensively cracked coal the compressive strength may be so low that the coal would fail before all the cracks were closed, thus giving a continuously curved stress–strain relationship up to the fracture point. This effect has, in fact, been observed during recent tests on one of the most friable of British coals (Oakdale, Meadow seam) but it is likely to be exceptional, at any rate in this country.

Static Bending Tests

Preliminary Considerations

Previously published values of Young's modulus in bending are confined to those reported by Inouye[2], who investigated the behaviour of rectangular strips of Japanese and American coals supported on wedges near the ends and loaded at the centre. The strain in the coal was calculated from the slope of the ends of the deformed specimens. The stress–strain relationship obtained was continuously curved and, when the increment loads were removed after noting the total strain, the specimen did not return to its original position. It was suggested that appreciable plastic flow was occurring (at high loads almost 50 per cent of the total strain observed), and that the "true elastic strain" was, in fact, the difference between the total measured strain and the residual or "plastic" strain. The relationship between stress and the "elastic strain" was very closely linear.

The occurrence of such a large "plastic" deformation at ordinary room temperatures is not, however, substantiated by recent work carried out in these laboratories on British coals[12,13], which has shown that the strain/time relationship for a strip of Barnsley Hards subjected to a constant bending moment is similar in shape to the curve shown in Fig. 2.1. The creep viscosity in bending is again found to be about 10^{16} poises so that the irrecoverable strain produced during the time of a typical bending test would be negligible; it must, therefore, be assumed that either the coals used by Inouye have very much lower creep viscosities than Barnsley Hards, or that the "plastic strains" observed were, in fact, due to the propagation of gross cracks under tension.

For Barnsley Hards the delayed elastic strain in bending, however, takes several days to approach its maximum value, and the strain observed a few seconds after loading is about 5 per cent less than the total recoverable strain. The modulus determined from the slope of the stress–strain curve in bending, therefore, is about 5 per cent higher than the true static modulus.

Experimental Work

Owing to the difficulty of preparing, from single blocks of coal, large numbers of strips lying perpendicular to the bedding plane, tests have so far been restricted to specimens lying parallel to this plane.

Several series of tests have been carried out on strips of both Barnsley Hards and anthracite using an apparatus which had been designed primarily for measuring the tensile strength of coal, and is described in detail elsewhere[14]. It provides for the application of a uniform bending moment over the centre portion of the strip, the strain in the specimen being measured by means of an optical extensometer which indicates the deflection of the centre of the specimen from its initial, unstrained position. Simple beam theory was used to determine the corresponding values of stress and strain. The mean values of Young's modulus (parallel to the bedding plane) obtained from a typical series of tests on each coal are given in Table 2.6.

TABLE 2.6. YOUNG'S MODULUS PARALLEL TO THE BEDDING PLANE—BENDING TESTS ON STRIPS 0·1 IN. THICK

Type of coal	Width of specimen, (in.)	Number of specimens	Young's modulus 10^{10} dyne/cm^2 (10^5 lb/in^2)	Standard error 10^{10} dyne/cm^2 (10^5 lb/in^2)
Barnsley Hards	0·09	8	3·38 (4·90)	0·14 (0·20)
	0·15	8	3·21 (4·65)	0·13 (0·19)
	0·25	8	3·42 (4·95)	0·18 (0·26)
	0·40	8	3·38 (4·90)	0·15 (0·22)
	0·60	6	3·39 (4·91)	0·21 (0·30)
	0·90	6	3·26 (4·73)	0·15 (0·22)
	1·25	8	3·48 (5·05)	0·17 (0·25)
Anthracite	0·50	12	4·69 (6·80)	0·20 (0·29)

Estimates of Poisson's ratio in tension were made using resistance strain-gauges to measure the longitudinal and lateral surface strains in strips of coal during test. Only three specimens of each coal were tested in this way, the values obtained being 0·37, 0·32 and 0·38 for Barnsley Hards, and 0·24, 0·32 and 0·27 for anthracite; these values are markedly lower than those obtained from compression tests.

It would be expected that any cracks lying at an angle to an applied tensile stress would tend to open, thus giving rise to a high apparent strain and correspondingly low values for Young's modulus and Poisson's ratio. For the two coals tested, however, the values of Young's modulus in bending agree closely with those in compression. The error in measuring the tensile strain cannot, therefore, be very large, and the true values of Poisson's ratio must be close to those given above.

Dynamic Measurements

The Composite Oscillator Method

It is preferable to carry out measurements on small specimens since these may be selected so as to be free of macroscopic flaws, which might considerably influence the results. The composite oscillator method, using magnetostrictive excitation, is considered suitable for measurements on small specimens, and since it has been adequately described elsewhere[15–17], only a brief description will be included here.

The coal specimen, in the form of a small prism or cylinder, is cemented end-to-end to a nickel cylinder which is selected so as to have approximately the same natural mechanical frequency as the coal specimen. The nickel rod is magnetostrictively excited to mechanical vibration by placing it axially in a small coil in which an alternating current is flowing. The mechanical vibrations are transmitted to the coal specimen and the mechanical resonance vibrations of the composite rod are detected by a second small coil placed about the nickel rod. The motion at resonance is described by the following equations:

$$M_1 f_1 \tan \pi f/f_1 + M_2 f_2 \tan \pi f/f_2 = 0 \tag{2.3}$$

and

$$f_i = (1/2l_i)(K_i/\varrho_i)^{\frac{1}{2}} \tag{2.4}$$

where M_i is the mass of a rod ($i = 1$ and 2 for nickel transducer and coal specimen respectively), ϱ_i is the density of a rod, f is a mechanical resonance frequency of the composite rod, l_i is the length of a rod, and K_i is the appropriate elastic modulus.

Depending on the polarization of the nickel transducer, either longitudinal or torsional vibrations may be produced,[17] so that the Young's moduli and the rigidity moduli of the coal specimens may be determined. For longitudinal vibrations, eqs. (2.3) and (2.4) are applicable to thin prisms of any shape of cross section, and for convenience of preparation, prisms of approximately square cross section of side about $\frac{1}{8}$ in. were used. These were prepared from blocks of coal by means of a high-speed carborundum slitting wheel. For torsional vibrations, eqs. (2.3) and (2.4) apply only to cylinders of circular cross section. Suitable cylinders of coal, $\frac{3}{8}$ in. in diameter, were prepared in a lathe using a small grinding wheel. All the specimens used were 2–3 cm in length and were resonated at frequencies between 4 kc/s and 20 kc/s. The experimental accuracy of the method is limited by the precision with which the specimens can be prepared, but is somewhat better than 1 per cent.

As the values of the elastic moduli are affected by the moisture content of the coal, all the dynamic measurements were made on specimens which had been dried for several weeks over phosphorus pentoxide. For convenience the actual measurements were effected rapidly at room temperatures and humidities.

As far as could be ascertained all the Barnsley Hards specimens were free of macroscopic cracks. The anthracite contained a number of fine, but visible, cracks in spite of the fact that special care had been taken in its selection from the coal face. The small rods used for longitudinal excitation could, however, be selected so as to be fairly free of these cracks; for the $\frac{3}{8}$ in. diameter cylinders used in the measurement of the rigidity moduli this was not possible.

Investigation of Anisotropy

In order to obtain information about the elastic symmetry of the coal, specimens were cut with the major axis oriented in different directions relative to the bedding plane and, in the case of Barnsley Hards where well-defined cleating exists, relative to the cleat planes as well. All the specimens were cut from a single lump of each coal. A summary of the results obtained by the dynamic method is given in Table 2.7.

It can be seen that the scatter of the results is fairly small. Moreover, measurements on specimens cut from a later consignment of Barnsley Hards gave values for the elastic moduli which were within 5 per cent of those shown.

Table 2.7 shows that for specimens cut in the bedding plane the elastic modulus is independent of the direction of the axis relative to the cleat planes. Specimens cut perpendicularly to the bedding plane, however, show significantly lower moduli than those cut in the bedding plane. Thus the results for Barnsley Hards suggest that the elastic properties are symmetrical about an axis perpendicular to the bedding plane and differ in a direction parallel to this axis from those perpendicular to it. A homogeneous material possessing this form of symmetry is said to be "transversely isotropic"(18) and its elastic behaviour may be described in terms of five independent "moduli of compliance", s_{11}, s_{12}, s_{13}, s_{33} and s_{44}. If components σ_x, σ_y and σ_z represent tensile stresses acting along the x, y and z coordinate axes, τ_{yz}, τ_{zx}, and τ_{xy} represent shear stresses acting respectively in the yz, zx and xy planes, and $e_x \ldots \gamma_{yz} \ldots$ are the corresponding strains, then Hooke's law may be written:

$$
\begin{aligned}
e_x &= s_{11}\sigma_x + s_{12}\sigma_y + s_{13}\sigma_z \\
e_y &= s_{12}\sigma_x + s_{11}\sigma_y + s_{13}\sigma_z \\
e_z &= s_{13}\sigma_x + s_{13}\sigma_y + s_{33}\sigma_z \\
\gamma_{yz} &= s_{44}\tau_{yz} \\
\gamma_{zx} &= s_{44}\tau_{zx} \\
\gamma_{xy} &= 2(s_{11} - s_{12})\,\tau_{xy}
\end{aligned}
$$

where the z axis is the axis of symmetry.

TABLE 2.7. SUMMARIZED RESULTS—DYNAMIC MODULI

Coal	Orientation of major axis of specimen	Type of vibration	Young's modulus 10^{10} dyne/cm^2 (10^5 lb/in^2)	Rigidity modulus 10^{10} dyne/cm^2 (10^5 lb/in^2)	Number of specimens	Standard error 10^{10} dyne/cm^2 (10^5 lb/in^2)
Barnsley Hards	(a) Parallel to bedding plane, and to major cleat	Longitudinal	3·99 (5·79)	—	5	0·05 (0·07)
	(b) Parallel to bedding plane, perpendicular to major cleat	Longitudinal	4·08 (5·91)	—	4	0·11 (0·16)
	(c) Perpendicular to bedding plane	Longitudinal	3·27 (4·74)	—	6	0·08 (0·12)
	(d) 45° to bedding plane	Longitudinal	3·36 (4·86)	—	4	0·02 (0·03)
	(e) Parallel to bedding plane	Torsional	—	1·36 (1·97)	7	0·03 (0·04)
	(f) Perpendicular to bedding plane	Torsional	—	1·18 (1·71)	6	0·02 (0·03)
Anthracite	(g) Parallel to bedding plane	Longitudinal	5·42 (7·88)	—	10	0·07 (0·10)
	(h) Perpendicular to bedding plane	Longitudinal	4·59 (6·63)	—	11	0·08 (0·12)
	(i) 45° to bedding plane	Longitudinal	5·25 (7·61)	—	9	0·04 (0·06)
	(j) Parallel to bedding plane	Torsional	—	1·65 (2·39)	9	0·03 (0·04)
	(k) Perpendicular to bedding plane	Torsional	—	1·47 (2·13)	6	0·05 (0·07)

The relationships between the experimental moduli (i.e. Young's modulus E and rigidity modulus G) and the moduli of compliance[18] are given in Table 2.8.

Poisson's ratio is defined by

$$\nu_{ik} = -\frac{\text{Strain in the } k \text{ direction}}{\text{Strain in the } i \text{ direction}}$$

where the stress is applied in the i direction. For a transversely isotropic material there are three different Poisson's ratios, i.e.

$$\nu_{yx} = \nu_{xy} = -s_{12}/s_{11} \qquad \nu_{yz} = \nu_{xz} = -s_{13}/s_{11}$$

and

$$\nu_{zx} = \nu_{zy} = -s_{13}/s_{33} \tag{2.5}$$

The bulk modulus k is given by

$$k = 1/\{2(s_{11} + s_{12} + 2s_{13}) + s_{33}\} \tag{2.6}$$

TABLE 2.8. RELATIONSHIPS BETWEEN MODULI OF COMPLIANCE AND EXPERIMENTAL MODULI

Direction of measurement relative to bedding plane	Experimental modulus	Relationship between experimental modulus and moduli of compliance
Parallel	Young's	$E_{\parallel} = 1/s_{11}$
Parallel	Rigidity	$G_{\parallel} = 2/\{s_{44} + 2(s_{11} - s_{12})\}$
Perpendicular	Young's	$E_{\perp} = 1/s_{33}$
Perpendicular	Rigidity	$G_{\perp} = 1/s_{44}$
45°	Young's	$E = 4/(s_{11} + 2s_{13} + s_{33} + s_{44})$

Using the experimental results shown in Table 2.7, the relationships given in Table 2.8 and eqns. (2.5) and (2.6) it is possible to derive values of the moduli of compliance, Poisson's ratio and bulk modulus for Barnsley Hards. The values obtained are shown in Table 2.9.

TABLE 2.9. DERIVED VALUES OF POISSON'S RATIO AND BULK MODULUS—BARNSLEY HARDS

Modulus of compliance 10^{-11} cm²/dyne (10^{-6} in²/lb)	Poisson's ratio	Bulk modulus 10^{10} dyne/cm² (10^5 lb/in²)	Standard error (appropriate units)
$s_{11} =$ 2·48 (1·71)	—	—	0·03 (0·02)
$s_{33} =$ 3·06 (2·11)	—	—	0·08 (0·06)
$s_{44} =$ 8·47 (5·81)	—	—	0·15 (0·10)
$s_{12} =$ −0·64 (−0·44)	—	—	0·19 (0·13)
$s_{13} =$ −1.06 (−0.73)	—	—	0·10 (0·07)
—	$\nu_{xy} = 0{\cdot}26$	—	0·08
—	$\nu_{yz} = 0{\cdot}43$	—	0·04
—	$\nu_{zx} = 0{\cdot}35$	—	0·03
—	—	4·0 (5·81)	0·9 (1·3)

Although the scatter for the experimental moduli is small it may be seen from Table 2.9 that, for the derived values of Poisson's ratio and bulk modulus, the standard errors, which have been calculated by means of the usual equations[19], are comparatively large. The static measurements of Poisson's ratio (Table 2.3) show as reasonable agreement with the derived values given in Table 2.9 as can be expected; there are no published values for the bulk modulus of Barnsley Hards.

In view of the experimental difficulties of obtaining precise independent static measurements of Poisson's ratio and bulk modulus, and considering the large degree of uncertainty with which these may be derived from the dynamic measurements, the validity of the assumption of transversely isotropic symmetry cannot be considered as verified. In fact, in view of its inhomogeneous nature, it is unlikely that bulk coal would behave elastically in a perfectly symmetrical manner, and it is probably more correct to conclude that it merely tends towards transverse isotropy. The experimental results for anthracite do not conform with the theoretical relationships for a transversely isotropic material since application of these leads to a negative value for the bulk modulus. This anomaly may be due to the effect of the observed cracks in the cylinders of anthracite used for measurement of the rigidity moduli; these would reduce the value of the elastic wave velocity and therefore the calculated value of the modulus. It can be shown from Table 2.8 and eqn. (2.6) that this would, in fact, give a low and possibly negative derived value for the bulk modulus.

Considerations of the Elastic Anisotropy

The dynamic measurements clearly show that both Young's modulus and the rigidity modulus for a coal cylinder depend on the orientation of the cylinder axis with respect to the bedding plane. Except for the results obtained from 1½ in. cubes of Barnsley Hards (see Table 2.4), the scatter of the static measurements was too large to show a significant directional variation of Young's modulus. The dynamic measurements on Barnsley Hards also indicate that there are no significant differences between the elastic moduli of specimens cut in the bedding plane but in different directions relative to the cleat plane. This would suggest that the cleats do not arise as a consequence of a directional variation of the bond strength between molecular layers in the coal substance.

The difference between the elastic behaviour in the bedding plane and that in a perpendicular direction could be accounted for by several different mechanisms. In the first place, it could arise from a preferred molecular orientation (with its attendant variation of bond strength). This could be due either to the anisotropic nature of the original vegetable cells, or to the physical and chemical processes taking place during the formation of the coal; for example, the application of high stresses perpendicular to the bedding plane. It is worth noting that, from dynamic measurements on comparatively homogeneous pure vitrains, Schuyer, Dijkstra and van Krevelen[9] have found evidence of transverse isotropy in those vitrains having a carbon content greater than 93 per cent.

More important, possibly, is the formation of coal as a layered material. On a microscopic scale this could have resulted in the presence of a large number of flat cavities situated between the layered components of the coal; these would certainly account for a reduction in the elastic wave velocity in a direction perpendicular to the plane of the layers. Some evidence of the presence of such cracks or lenticular discontinuities is given by the large initial strains obtained by applying static stresses to cubes of coal in a direction perpendicular to the bedding plane (see Table 2.5). Flat cavities could, in fact, be produced by separation of the layers when the coal is removed from the influence of the high stresses found in the seam.

On the macroscopic scale some of the components such as vitrain, durain, clarain and fusain would be expected to differ from each other in elastic behaviour, although there has not as yet been a systematic attempt to determine their respective elastic moduli. These components could be individually either isotropic or anisotropic in their elastic properties. It can be shown[22], that even if they were isotropic, any differences between the layers themselves would give rise to transverse isotropy in the composite material, the magnitude of the effect depending on the scatter of the values of the modulus corresponding to the different layers.

Comparison of Static and Dynamic Moduli

A summary of the values obtained for Young's modulus is given in Table 2.10.

The differences between the static values and the corresponding dynamic values are comparatively small, and it is interesting that where these differences are statistically significant the static modulus is always lower than the dynamic modulus.

In order to determine the difference more precisely, a set of strips of Barnsley Hards (cross-section 0·25 in. × 0·12 in.) was prepared and tested by both the dynamic and the static bending techniques. The results obtained are given in Table 2.11.

A brass strip of similar dimensions gave values of $9{\cdot}76 \times 10^{11}$ and $9{\cdot}83 \times 10^{11}$ dyne/cm^2 for the dynamic modulus and the static modulus respectively.

Table 2.12 gives results obtained for a range of British coals.

The only other static and dynamic measurements carried out on the same types of coal are those reported by Inouye[2], who has determined Young's modulus for a number of American and Japanese coals. For the dynamic measurements prisms of rectangular cross section were excited into lateral vibration at frequencies in the kilocycle range and values were obtained of a similar order to those described here. The static moduli were determined from bending tests on the same specimens, and the values obtained were, in general, roughly an order of magnitude lower than the dynamic values.

TABLE 2.10. YOUNG'S MODULUS—COMPARISON OF STATIC AND DYNAMIC RESULTS

Type of coal	Direction of stress	Young's modulus 10^{10} dyne/cm² (10^5 lb/in²) Dynamic technique	Compression of ½ in. cubes	Compression of 1½ in. cubes	Bending tests	Number of specimens	Standard error 10^{10} dyne/cm² 10^5 lb/in²
Barnsley Hards	Parallel to bedding plane	4·03 (5·83)				9	0·05 (0·07)
			4·13 (6·00)			16	0·24 (0·35)
				3·35 (4·86)		20	0·06 (0·09)
					3·36 (4·88)	52	0·09 (0·13)
	Perpendicular to bedding plane	3·27 (4·73)				6	0·08 (0·12)
			3·77 (5·46)			8	0·25 (0·36)
				2·70 (3·91)		10	0·07 (0·10)
Anthracite	Parallel to bedding plane	5·42 (7·88)				10	0·07 (0·10)
			4·43 (6·41)			16	0·20 (0·29)
				4·04 (5·85)		22	0·13 (0·19)
					4·69 (6·80)	12	0·20 (0·29)
	Perpendicular to bedding plane	4·59 (6·14)				11	0·08 (0·12)
			4·61 (6·70)			8	0·37 (0·54)
				3·77 (5·45)		11	0·16 (0·23)

Inouye[20] attributes this difference to the "plastic" behaviour of the coal. It is clear, however, that for cracked coals, such as were used by Inouye, the effective thickness of the specimen would be somewhat less than the measured thickness and since the expression for calculating Young's modulus involves the cube of this dimension, a small degree of cracking would give rise to comparatively large errors in the measured modulus.

It is interesting, however, that Honda and Sanada[21], also using Japanese coals, have attempted to derive values for the static modulus from the results of indentation hardness tests and have obtained values which are of the same order of magnitude as those reported by Inouye.

The present results suggest that although the differences between the static and dynamic moduli of British coals are very much smaller than those

TABLE 2.11. YOUNG'S MODULUS PARALLEL TO THE BEDDING PLANE—COMPARISON OF DYNAMIC MODULI AND MODULI IN BEDDING

Specimen No.	Dynamic modulus 10^{10} dyne/cm^2 (10^5 lb/in^2)	Static modulus 10^{10} dyne/cm^2 (10^5 lb/in^2)	Ratio: $\frac{\text{Dynamic modulus}}{\text{Static modulus}}$
1	3·99 (5·79)	3·47 (5·02)	1·15
2	4·03 (5·83)	3·14 (4·55)	1·28
3	3·85 (5.60)	3·18 (4·61)	1·21
4	3·99 (5·79)	3·36 (4.86)	1·19
5	4·08 (5·91)	3·31 (4·80)	1·23
6	4·08 (5·91)	3·34 (4·84)	1·22
7	3·90 (5·65)	2·96 (4·30)	1·32
8	4·08 (5·91)	3·00 (4·35)	1·36
9	3·94 (5·70)	3·07 (4·44)	1·28
10	4·13 (5·98)	3·14 (4·55)	1·31
Mean	4·01 (5·81)	3·20 (4·64)	1·25
Standard error	0·03 (0·04)	0·05 (0·07)	—

obtained by Inouye for Japanese and American coals, they are nevertheless appreciable and it is worth considering them in more detail.

Dynamic techniques employ high rates of loading so that the amount of delayed elastic strain involved is very small. At kilocycle frequencies the value obtained for the modulus is, in fact, very close to E_1 (see Fig. 2.2), and by equation 2.2 will always be higher than the static modulus E_s.

The observed discrepancy between the static and dynamic moduli of Barnsley Hards is of the order of 20 per cent, and while much of this may be attributed to the viscoelastic nature of coal it is likely that other contributing factors may be found. Possibly the most important of these is the presence of large numbers of discontinuities in the coal substance.

It is very probable that the elastic properties of the basic coal material are considerably different from those of the bulk coal or its various layered components. Measurement of the elastic moduli of the basic coal material is experimentally very difficult. Bangham and Maggs[(1)] have, in fact, attempted this, making use of the fact that the adsorption of a liquid (methanol) by coal results in a swelling of the latter which is proportional to the area of contact between liquid and solid, and which is resisted by elastic forces within the coal. The values they obtained for various coals ranged from $4{\cdot}1 \times 10^9$ dyne/cm^2 to $5{\cdot}8 \times 10^{10}$ dyne/cm^2, but should be treated with some reserve since the method requires an estimation of the distribution of surface area within the coal.

Inouye[(3)] has suggested that the values of the elastic moduli of the bulk material are dependent upon the amount of mineral "ash" dispersed in the coal; that is to say, the dispersed mineral particles behave as a "filler" in the

TABLE 2.12. YOUNG'S MODULUS OF REPRESENTATIVE BRITISH COALS

Name of coal (see Table 1.2)	Volatile content % by weight (d.m.m.f.)	Density (g/cc)	Young's modulus, units of 10^{10} dynes/cm^2 (10^5 lb/in^2)					
			Perp. B.P.		Par. B.P.	Par. M.C.	Par. B.P.	Perp. M.C.
			dynamic	static	dynamic	static	dynamic	static
Pentremawr	5	1·499	4·59 (6·68)	3·77 (5·48)	5·41 (7·90)	4·05 (5·89)	—	4·04 (5·89)
Deep Duffryn	12	1·395	3·46 (5·02)	3·44 (5·00)	3·63 (5·28)	3·05 (4·43)	3·80 (5·51)	3·04 (4·41)
Oakdale	22	1·358	3·83 (5·58)	0·93†(1·35)	3·22 (4·68)	0·83†(1·21)	2·37 (3·44)	0·77†(1·12)
Cwmtillery	30	1·345	2·91 (4·23)	2·35 (3·41)	1·43 (1·94)	1·69 (1·01)	1·57 (2·28)	2·59 (3·76)
Barnsley Hards (Rossington)	36	1·364	3·28 (4·78)	2·69 (3·90)	3·99 (5·80)	3·34 (4·85)	4·08 (5·93)	3·34 (4·85)
Markham	39	1·333	2·38 (3·46)	4·04 (5·88)	4·35 (6·31)	5·32 (7·76)	4·27 (6·21)	3·45 (5·01)
Pleasley	37	1·306	1·55 (2·26)	2·69 (3·90)	4·00 (5·81)	3·66 (5·32)	3·77 (5·48)	3·18 (4·62)
Barnsley Brights (Rossington)	37	1·314	2·54 (3·70)	3·12 (4·55)	4·16 (6·04)	2·80 (4·06)	4·35 (6·31)	2·71 (3·95)
Linby	39	1·333	2·47 (3·58)	2·60 (3·78)	3·00 (4·36)	2·65 (3·85)	2·65 (3·85)	2·44 (3·54)

† Stress-strain relationship markedly curved up to failure
Perp. ≡ Perpendicular   Par. ≡ Parallel
B.P. ≡ Bedding Plane   M.C. ≡ Main Cleat

matrix of the basic coal substance. Since the mineral fillers have higher elastic moduli than the coal, the moduli of the bulk coal would be expected to increase as the dispersed mineral content increased – in fact, Inouye claims to have established a linear relationship between Young's modulus and the percentage by volume of mineral "ash".

It is likely, however, that the microscopic cracks and capillaries also play an important role in determining the elastic behaviour of the bulk coal. From this viewpoint the coal can be considered as having a structure bearing a resemblance to that of a sponge consisting of the basic coal material interspersed with fine cracks and pores. The resulting reduction in the apparent moduli would vary considerably from one coal to another and might well mask the increase due to the mineral content. In view of the lack of knowledge of the distribution of cracks and mineral deposits, the relationship between the moduli of the bulk coal and those of the basic material cannot at present be estimated.

It is, however, clear that in general the effect of the discontinuities will be different for dynamic methods of measurement than for static methods. The dynamic method consists essentially of measuring the frequency of a vibrating specimen, which is in effect a timing of the transit of elastic waves through the material. Since the velocity of the elastic wave in the coal is much greater than that in air, the transit time is largely determined by the path of the elastic waves travelling in the coal substance itself. The path will be increased by the presence of a crack system, and the measured velocity (and consequently the derived value of the elastic modulus) will be lowered.

The error introduced into the static measurements, on the other hand, is due to different mechanisms. Attention has already been drawn to the fact that the initial curvature observed in the stress–strain curves for cubes in compression suggests the closing of cracks. There is a likelihood, however, that even over the linear portion of the stress–strain curve, the cracks are, in some cases at least, only partially closed, and behave subsequently in a springlike manner, maintaining a linear relationship between stress and strain. The deformations would, however, be larger than would be the case if the cracks had closed completely, so that a low value of Young's modulus would be estimated. Moreover, because of the porous nature of the material, the stresses actually supported by the framework of the coal substance would be higher than those estimated by dividing the load by the apparent cross section. The under-estimation of the true static stresses involved would lead to an under-estimation of the Young's modulus.

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CHAPTER 3

The Compressive Strength of Coal

INTRODUCTION

Coal is an engineering material in the sense that it has to be won from the seam by the application of mechanical forces. At a later stage of its journey from pit to consumer it may have to be broken to sizes that are appropriate to ease of transport or to particular methods of combustion, and again the application of external force is necessary. The highest efficiency in the processes of winning and breaking can only be secured if the resistance of the material concerned to breakage, i.e., its strength, is known. With coal-winning becoming more and more highly mechanized, the importance of having reliable data on the strength of coal, and of having some insight into the mode of breakage of coal under stress, is increasing.

It is a curious state of affairs that adequate knowledge on the strength of its product should be lacking in an industry as old as the coal industry. There are probably two contributory factors; on the one hand the traditional pick-and-shovel methods were for a long while quite good enough to meet the needs of the industry; on the other hand coal has been a difficult and apparently unrewarding material to work with in the laboratory. The result is that at the moment the empirical development of coal-winning machines has outstripped the basic knowledge of their mode of action. This disparity must be made good if further advances are to be forthcoming.

DEFINITION OF STRENGTH

The strength of a material is defined as the stress at which permanent changes in its structure take place. Most solid materials, when subjected to stress, show a proportionate change in shape at first. This change is recoverable when the stress is removed and is termed "elastic". Beyond a certain limiting value of stress, complete recovery does not take place. Metals, for example, pass from the elastic state into one where considerable changes of shape may take place for small increments of stress, the plastic state; and these plastic changes are permanent, the metal acquiring a "permanent set". The transition from elasticity to plasticity is a gradual one and may not be accompanied by any macroscopic manifestations of change. In the case of metals there may not be even any obvious change in strength; indeed, modern methods of design of steel-framed structures deliberately bring parts of the load-bearing members into the plastic state; so long as these parts are

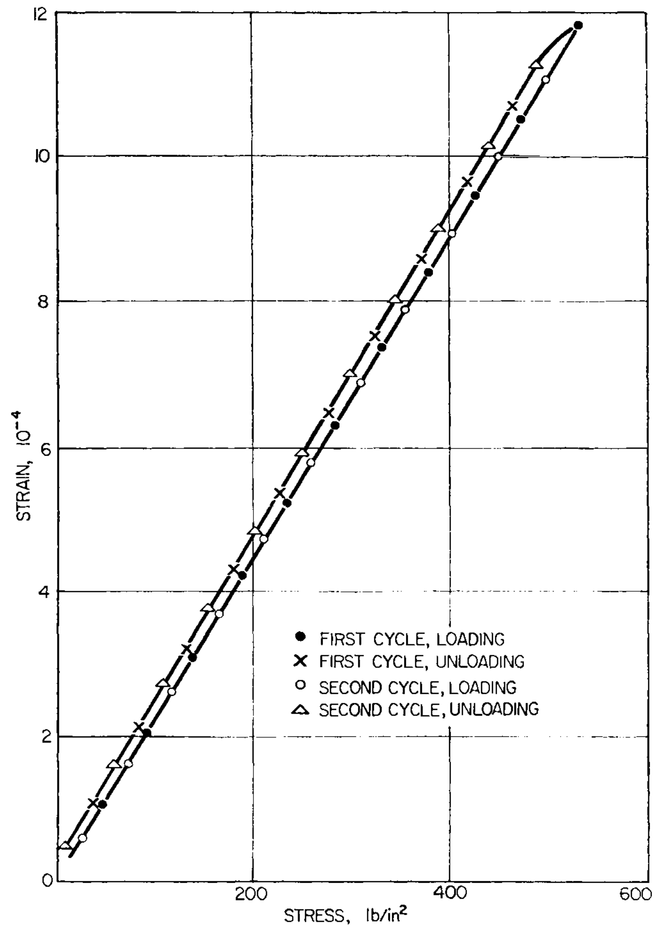

FIG. 3.1. Typical stress–strain curves for a thin strip of coal subjected to a bending test.

supported by elastic metal and prevented from inter-communicating, then the structure is stable. The strength of a metal is therefore largely a matter of definition, but it is nevertheless based upon the stress required to bring the metal to the limit of the elastic state.

Other materials behave in a quite different way, and coal is a notable exemplar of "brittle" failure. Here the limit of elastic behaviour is attended by a sudden and devastating change in structure; the material breaks and flies apart. The limit of strength is quite clear and quite specific. Figure 3.1 shows a stress–strain graph for a specimen of coal in tension. It can be seen that it is quite closely linear and that there is no departure from linearity in the approach to the elastic limit. Coal does show "creep" under protracted loading, but this has no significance when measured in the time-scale appropriate to breakage problems. From the latter point of view coal shows elastic behaviour and clear-cut breakage, but these are practically the only concessions to simplicity encountered in a study of its strength.

Systems of Stress

If breakage is defined as the limit of elastic equilibrium, then the concepts of the theory of elasticity can be used to define stress systems to which testing procedures should approximate as closely as possible.

If an elastic body is in equilibrium under the action of external forces, the force acting at a point can be described in terms of the stresses acting on an

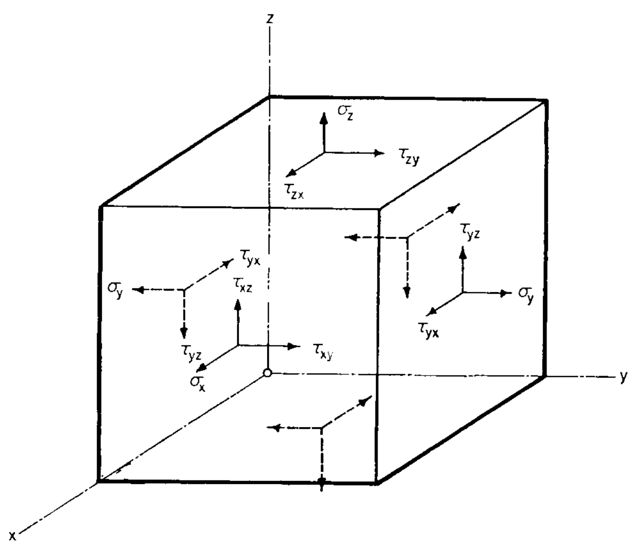

Fig. 3.2. Stress on an infinitesimal element.

elementary cube that contains the point (Fig. 3.2). There are three normal forces, acting parallel to the respective axes, σ_x, σ_y, σ_z, and three shear forces $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, $\tau_{zx} = \tau_{xz}$. The relative complexity of this system can be reduced very greatly by a suitable rotation of the axes of reference of the element. A position can be found that reduces the shear stresses to zero.

The normal stresses appropriate to this orientation σ_1, σ_2, σ_3 are known as principal stresses.

Thus, no matter what the complexity of the stress applied externally to an elastic body, the stress at a point can be regarded as the resultant of three mutually perpendicular stresses. The strength of a body is therefore usually defined in terms of various combinations of principal stress. Bearing in mind the common convention that tensile stresses are reckoned positive, the following test conditions can immediately be postulated:

(i) *Uniaxial tension*

$$\sigma_1 \text{ positive}, \sigma_2 = \sigma_3 = 0$$

Since this condition would apply equally if σ_2 were positive, and σ_1 and σ_3 zero, we may write in general

$$\sigma_i \text{ positive}, \sigma_j = \sigma_k = 0$$

where i, j, k are cyclic combinations of the numbers 1, 2 and 3.

(ii) *Uniaxial compression*

$$\sigma_i \text{ negative}, \sigma_j = \sigma_k = 0$$

(iii) *Pure shear*

$$\left.\begin{array}{l}\sigma_i \text{ positive} \\ \sigma_j \text{ negative}\end{array}\right\}, \quad \begin{array}{l}\sigma_i = -\sigma_j \\ \sigma_k = 0\end{array}$$

(iv) *Triaxial compression*

σ_i, σ_j and σ_k all negative, $|\sigma_i| > |\sigma_j| \geqq |\sigma_k|$.

Of the above systems, (iii) is difficult to achieve experimentally, and so also is (iv) when $|\sigma_j|$ and $|\sigma_k|$ are not equal, but approximations to the other cases can be made.

Perhaps the easiest test to do in a rough way is (ii), uniaxial compression, and this will be considered here.

Early Work on Compressive Strength

Compressive strength would, on the face of it, seem a simple enough measurement to make. All that is required is that the coal should be machined to a suitable shape, say a cube or a cylinder, and that the specimen should be put in a press and then crushed. The history of compressive strength measurements on coal goes back about 100 years, and many isolated measurements of this nature have been made. These results have had limited value because there is no means of gauging whether they are representative of the coal as a whole. The variability of coal is well known, both in chemical composition and in physical structure. Even where it is relatively "homogeneous" in the chemical or petrological sense, its physical structure is complex. The bedding planes are a characteristic feature, and are recognized to be planes of weakness. Other planes of weakness are the cleat planes, usually two families approximately at right angles, and approximately normal to the bedding planes. There may be other "slip" planes distinct from these systems. The strength of coal is sensitive to the direction of application of stress relative to these weaknesses. A satisfactory experiment on the measurement of coal strength ought therefore to contain a large number of individual readings, so as to establish the inherent variability of the material, and the tests should have been carried out so that the structure of the material is orientated relative to the applied stress in a consistent way. These conditions have not always been met. In many early measurements of strength (see Appendix) the number of individual tests has not been mentioned, and one can only conclude that in general the number was very small, perhaps in some cases no more than a single one. Equally, the direction of crushing relative to the structure is not mentioned. Evidently this was not thought important, and if a multiplicity of tests had, in fact, been carried out, the chances are that they were at random orientations to the structure. Thus many of the observations of the Appendix have only limited value, perhaps no more than to indicate the order of magnitude of strength.

However, it is interesting to note that where measurements on a particular coal were carried out at a variety of specimen sizes, there is a tendency for the larger specimens to be weaker in terms of load per unit area at breakage. Thus Rice and Enzian[1] found that while the compressive strength of cubes of bituminous coal in the size range $2\frac{1}{2}$–4 in. was about 2500 lb/in^2, a 54 in. cube of the same coal failed at 300 lb/in^2. The preparation and conveyance of a 54 in. cube of coal from face to laboratory must have been a major undertaking, and the specimen was in fact severely damaged before test. Nevertheless, the loss in specific strength was not due entirely to this damage. Greenwald, Howarth and Hartmann[2] confirmed a "size effect" by preparing and crushing pillars of coal at an experimental face. The pillars were

TABLE 3.1. CRUSHING OF DEEP DUFFRYN CUBES

Nominal side of cube (in.)	Mean side of cube (in.)	No. of cubes	Mean strength (lb/in^2)	Standard deviation of single observation (lb/in^2)
$\frac{1}{4}$	0·23	262	3700 ± 95†	1530
$\frac{1}{2}$	0·50	164	2860 ± 70	840
1	0·93	62	2380 ± 90	710
2	1·92	23	1820 ± 160	750

† Standard error of mean.

produced by over-cutting and side-cutting, and then crushed at right angles to the bedding planes by a hydraulic assembly inserted between the top of the pillar and the roof of the seam. A range of pillar sizes was tested. There was a decrease in strength with increasing height relative to the cross-sectional area, the breaking stress σ_c for small pillars approximately satisfying the relationship

$$\sigma_c \propto L^{\frac{1}{2}}$$

where L is the ratio of lateral dimension to height. A power law of this kind was also found by Burton and Phillips[3], who have studied the relationship between size and compressive strength of cubes in a more modest size range than did the Americans—from $\frac{1}{4}$ in. to 4 in. The cubes were compressed at right angles to the bedding plane between

(i) flat plates,
(ii) a rod and a flat plate,
(iii) a ball and a flat plate.

The relationship between the rupture load Q_c and the linear dimension a of the cubes is stated to be

$$Q_c \propto a^{1 \cdot 55 \pm 0 \cdot 05}$$

for each method of test, giving $\sigma_c \propto a^{-0 \cdot 45 \pm 0 \cdot 05}$.

On the whole it may be said that the work summarized in the Appendix and briefly reviewed here, has indicated that the compressive strength of coal is of the order of a few thousand pounds per square inch and that a relation for cubical specimens between stress and side of cube exists of the approximate form

$$\sigma_c \propto a^{-\frac{1}{2}}.$$

Effect of Testing Procedure

Apart from the variability of the coal, the result of a crushing test is affected by the mechanical conditions of the test, so that where a multiplicity of tests is contemplated the conditions should be standardized. When a specimen is crushed, for example, between smooth steel platens, the degree of roughness or smoothness of the specimens has a bearing on the measured strength. So also has the presence or absence of lubrication between the specimen and platens, and where it is present the degree of lubrication may cause apparent variation of strength. In short, measured strength is dependent upon the frictional force acting on the specimen at the boundary between the specimen and the test machine. This matter will be discussed later on in this chapter.

Further, many apparently brittle materials have viscous properties, which means that the apparent strength may be dependent upon the rate of application of load. A study of these properties in coal leads to the conclusion[4] that they would not cause an appreciable variation of strength over a wide range of rate of compression of the sample. They are therefore not given special attention in the work to be described.

The point to be borne in mind is that where a large number of specimens is to be tested and the results compared, the testing procedure should be standardized in an arbitrary but simply-achievable manner.

Work at the Mining Research Establishment

Work at M.R.E. on the crushing strength of cubes has been described by Evans and Pomeroy[5]. It is characterized, in the main, by the use of many more specimens than has been evident in the work summarized in the Appendix. Because of the numbers used it was impossible to prepare them all from a single lump of coal. The cubes were cut from lumps collected at the same time from run-of-mine supplies, and in many instances more than one size of cube was cut from each lump.

Preparation of Cubes

The mode of preparation of the cubes was as follows: the parent lump was cemented to a jig which could be mounted on a universal grinding machine. The jig was traversable past a rotating slitting wheel, made of resin-bonded carborundum, so that a series of parallel cuts were made in the coal. The jig was then turned successively to two other alternative positions which enabled further families of cuts to be made, orthogonal to their predecessors. In this way rectangular blocks of coal, or, in particular, cubes, could be made. The "cubes" were in general of sufficiently good shape to be used for test without any further preparation. They were made in several sizes ranging from 2 in. to $\frac{1}{8}$ in. side.

Test Procedure

In these tests the cubes were made so that the bedding planes lay parallel to one face, and they were crushed so that the applied force acted normal to this face. The testing machines were a Hounsfield Tensometer and an Avery Universal Testing Machine (10-ton load range). Both had smooth steel platens, and no lubricant was used between specimen and platens.

Coals Employed

Deep Duffryn, five feet	A friable steam coal permeated with cracks and weaknesses that are plainly visible on the surface. Volatile content 12 per cent.
Rossington, Barnsley Hards	A hard, smooth-looking and apparently "uniform" bituminous coal. Volatile content 36 per cent.

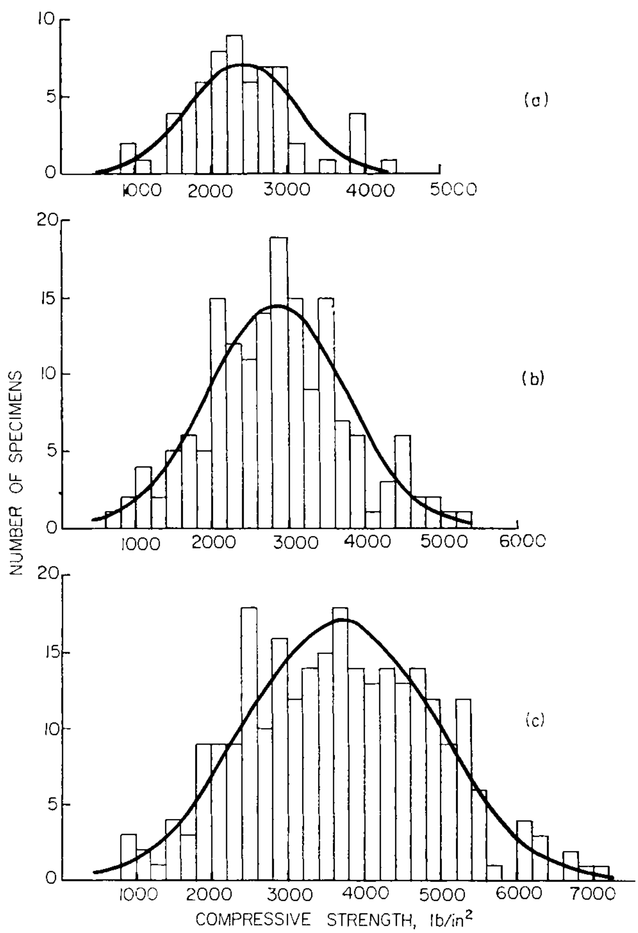

FIG. 3.3. Histograms of compressive strength of cubes of Deep Duffryn coal. (a) 1 in. cubes, total number 62, (b) $\frac{1}{2}$ in. cubes, total number 164, (c) $\frac{1}{4}$ in. cubes, total number 262.

Results of Crushing Tests on Deep Duffryn Coal

The results, shown in Table 3.1, are plotted in the form of histograms in Fig. 3.3, with smooth curves showing the general trends. The most noticeable feature of the results is the extremely wide spread of the observations. A large number of observations is therefore necessary to establish an average value of strength with any exactitude; a few results taken at random from the possible range of values might easily give a very different answer.

Despite the scatter of observations a certain regularity is discernible in the behaviour of the mean value with respect to specimen size. The variation is shown in Fig. 3.4. On a log-log scale the observations fall on a straight line, so that the relation is a power law of the form

$$\sigma_c \propto a^{-0\cdot32 \pm 0\cdot02}$$

where

$$\sigma_c = \text{crushing stress}$$

$$a = \text{side of cube}$$

(The uncertainty of the exponent is the standard error of the mean.)

Results of Crushing Tests on Barnsley Hards Coal

With Barnsley Hards, cubes of side down to $\frac{1}{8}$ in. could be cut. A greater number of different sizes was studied than for Deep Duffryn (Table 3.2).

TABLE 3.2. CRUSHING OF BARNSLEY HARDS CUBES

Side of cube (in.)	No. of cubes	Mean strength (lb/in^2)	Standard deviation of single observation (lb/in^2)
0·125	159	11,730 ± 180	2240
0·25	445	10,820 ± 80	1760
0·76	30	9030 ± 200	1090
1·00	78	8930 ± 160	1380
1·30	12	7870 ± 250	880
1·44	27	7090 ± 490	2560
1·60	76	7530 ± 210	1860

The results confirm those for Deep Duffryn in the following respects:

(i) For a particular size there is a wide spread of strength about the mean (Fig. 3.5).

(ii) Mean crushing strength decreases with increasing size of specimen, the law relating stress to size being of the form

$$\sigma_c \propto a^{-0\cdot17 \pm 0\cdot02} \quad \text{(Fig. 3.6)}$$

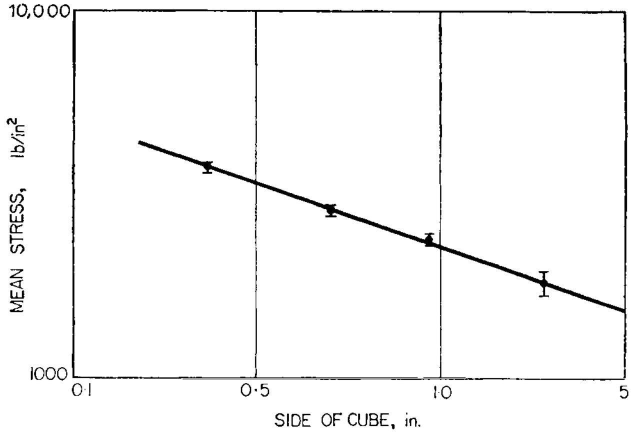

FIG. 3.4. Relation between mean compressive strength and side of cube for Deep Duffryn coal.

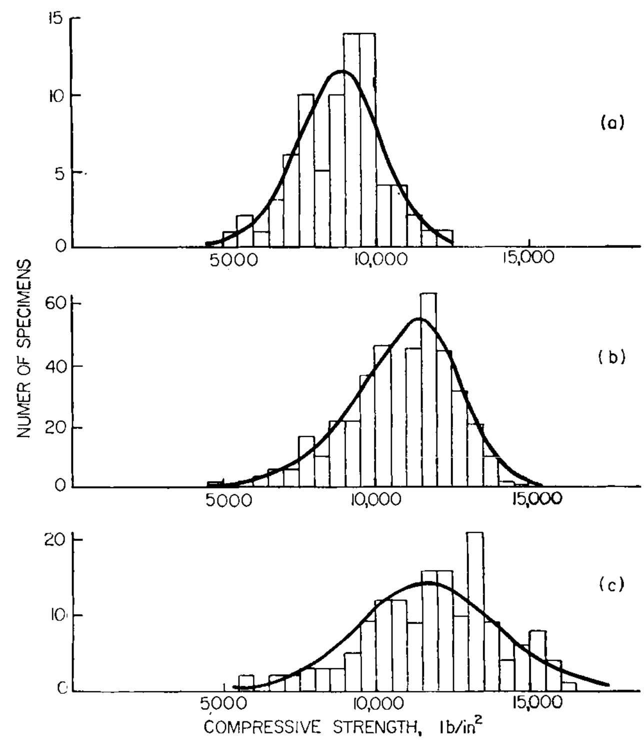

FIG. 3.5. Histograms of strength of cubes of Barnsley Hards coal. (a) 1 in. cubes, total number 78. (b) $\frac{1}{4}$in. cubes, total number 445, (c) $\frac{1}{8}$in. cubes, total number 159.

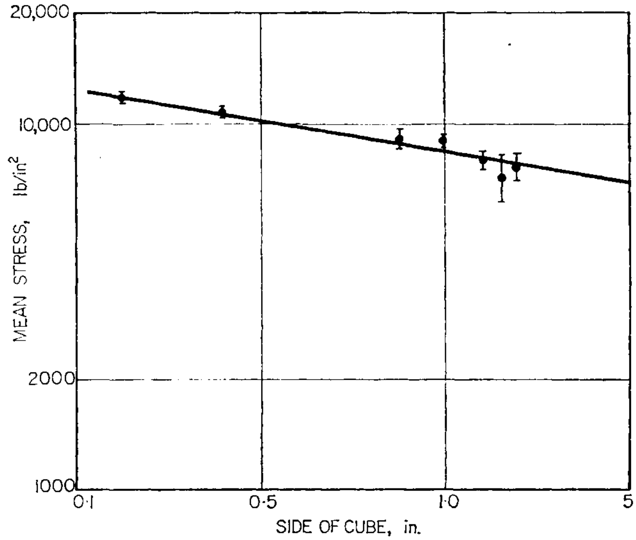

FIG. 3.6. Relation between compressive strength and side of cubes for Barnsley Hards coal.

CONSIDERATION OF RESULTS

Two important facts are evident in these results:

(i) The crushing strength of coal cubes of a particular size shows a wide variation.

(ii) Mean crushing strength decreases as side of cube increases.

These facts lead to the conclusion that the strength of coal can only effectively be dealt with by statistical methods such as have been applied to the strength of other natural materials, e.g. cotton threads(6). A simple quantitative approach is as follows: Consider a mechanical system comprised of a series of units joined end to end, and let the probabilities of these units surviving the application of a particular stress be P_1, P_2, ... P_n. Then if P is the probability of the system surviving the stress,

$$P = P_1 P_2 \cdots P_n. \tag{3.1}$$

(The probability of an event is the product of the probabilities of the independent components of which it is composed.)

Notice that if P_r ($r = 1$ to n) is zero for any particular stress, then P also is zero. Equation (3.1) is the quantitative statement of the proverb that the strength of a chain is the strength of the weakest link, and forms the basis of the so-called weakest-link theory.

If

$$P_1 = P_2 = P_3 = P_n = P_o,$$

then eqn. (3.1) becomes

$$P = P_o^n.$$

Let P_a be the probability of coal cubes of side a surviving a particular stress. It is desired to relate this to the probability of a cube of side b surviving the stress. It may be supposed that the cube "a" contains r_c elements of the kind to which the weakest-link theory applies and the cube "b" s_c elements. If P_o is the probability of an element surviving the stipulated stress, then

$$P_a = P_o^{r_c}$$

and

$$P_b = P_o^{s_c}$$

so that

$$P_b = P_a^{s_c/r_c}$$

s_c/r_c is clearly related to the relative size of the cubes, but it is not necessarily the ratio of the volumes; it may be a ratio of surface areas or of characteristic lengths. In any event it is a ratio of some attribute which is a function of length, and so may be put in the form $(b/a)^{\alpha_c}$ where α_c is likely to have the value 1, 2 or 3 according to whether the attribute is length, area or volume. The expected equation is therefore

$$P_b = P_a^{(b/a)^{\alpha_c}} \tag{3.2a}$$

If P_b and P_a are found from experiment, then α_c is given by

$$\alpha_c \log b/a = \log \frac{(\log P_b)}{(\log P_a)} \tag{3.2b}$$

Validity of Weakest-link Theory

The validity of eqn. (3.2a) has been investigated. From the histograms of Fig. 3.3, probability-summation curves (P-curves) have been constructed for Deep Duffryn cubes (Fig. 3.7). This graph also shows results, not given in Fig. 3.3, for cubes of side approximately 2 in. For Barnsley cubes similar curves are shown in Fig. 3.8, and here also additional results are given to those presented in Fig. 3.5. The steps which are formed by successive addition of the pillars of the histogram have been smoothed by eye in constructing the curves.

The extremities of the P-curves are defined by a few results corresponding to abnormally low or abnormally high values of strength, and so cannot be considered to be very accurate. Values of P have therefore been compared,

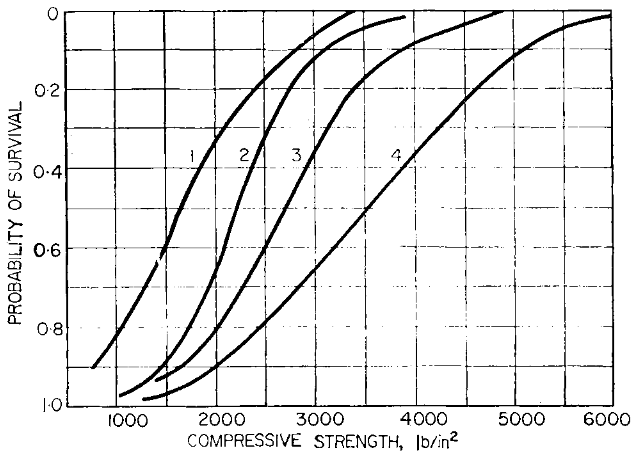

FIG. 3.7. Probability–summation curves for compressive breakage of Deep Duffryn coal. Mean side of cube (1) 1·92 in., (2) 0·93 in., (3) 0·50 in., (4) 0·23 in.

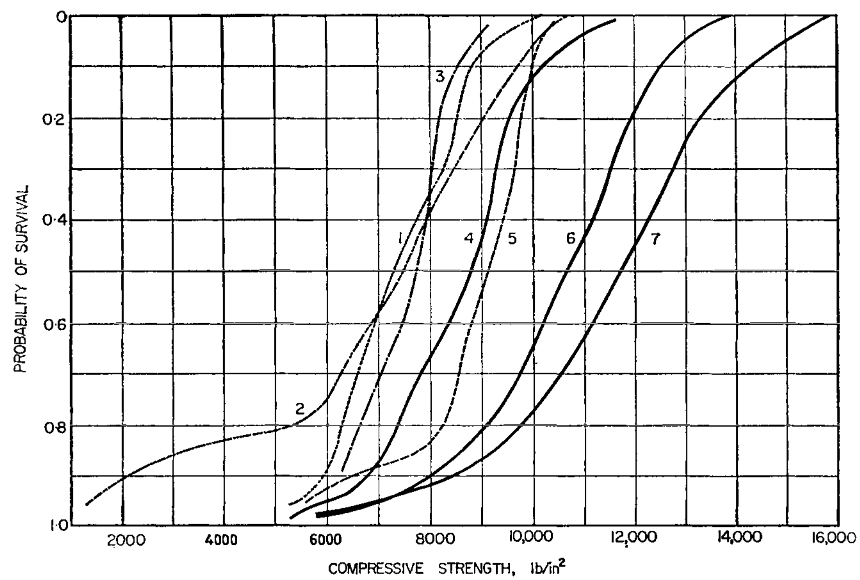

FIG. 3.8. Probability–summation curves for compressive breakage of cubes of Barnsley Hards coal. Mean side of cube (1) 1·60 in., (2) 1·44 in., (3) 1·30 in., (4) 1·00 in., (5) 0·76 in., (6) 0·25 in., (7) 0·125 in.

for Deep Duffryn coal, at stresses for which all four curves lie somewhere between the values $P = 0{\cdot}1$ and $P = 0{\cdot}9$. The value of P_a at a particular stress has been taken to be the one which lies nearest to 0·5; the corresponding values of P_b have been read from the other curves. A worked example will make this clear.

TABLE 3.3. COMPARISON OF P-VALUES FOR DEEP DUFFRYN CUBES AT STRESS OF 2600 lb/in²

Mean side of cube (in.)	P	b/a	$\frac{\log P_b}{\log P_a}$
1·92	0·15	3·84	3·6
0·93	0·28	1·86	2·4
0·50	0·59	1·00	1·0
0·23	0·73	0·46	0·60

It can be seen from Table 3·3 that P is nearest 0·5 for the $\frac{1}{2}$ in cubes, and hence $a = \frac{1}{2}$ in., $P_a = 0{\cdot}59$. The ratios b/a and $\log P_b/\log P_a$ follow immediately by combination with the other results.

In this way corresponding values of $\log P_b/\log P_a$ and b/a have been obtained for stresses 2000, 2200, 2400, 2600 and 2800 lb./in². for Deep Duffryn, and the values have been plotted on a log-log scale in Fig. 3·9. There is a linear regression between them, and the slope of the line is $1{\cdot}00 \pm 0{\cdot}15$. For Barnsley Hards coal (Fig. 3·10) the slope of a similar line is $1{\cdot}27 \pm 0{\cdot}12$. On the basis of equations (3·2) these results lead to the deduction that the attribute of cube size which enters into breakage theory is the length of the cube. This is an extremely important result in the study of the breakage mechanism of coal.

STRENGTH OF RECTANGULAR BLOCKS

The results given so far refer to cubes of coal, and so there is no precise indication of the significance of the linear dimension, i.e. whether it is the height or one of the base dimensions of the sample. With the aim of investigating this point further work was carried out consisting of measurements of the crushing strength of specimens of coal having the shape of right parallelepipeds, i.e. "rectangular blocks."

This work was carried out with Barnsley Hards. All specimens were cut with bedding plane parallel to one pair of faces of the rectangular prisms, and the load was applied perpendicular to the bedding plane. All experiments involved a range of specimen sizes, and where preparation involved several lumps, some specimens of all sizes were taken from each lump.

Preliminary experiments were concerned with the strength of rectangular prisms $\frac{1}{2}$ in. high and of various lateral dimensions up to 2 in. × 2 in., the speci-

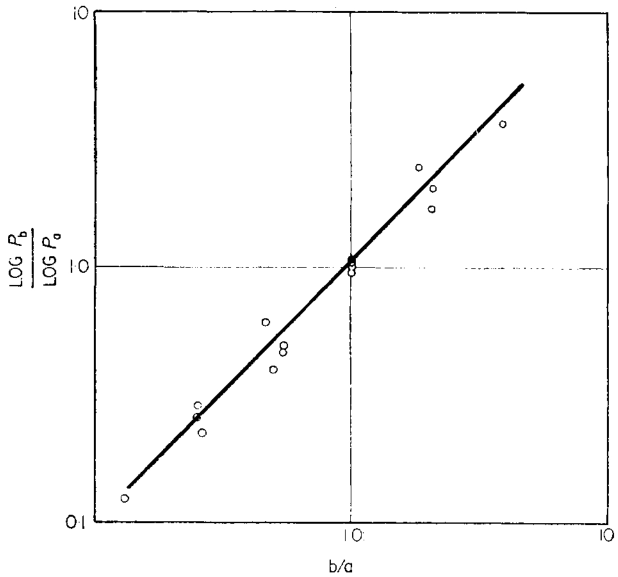

FIG. 3.9. Relation between probability of survival and size of cube (Deep Duffryn).

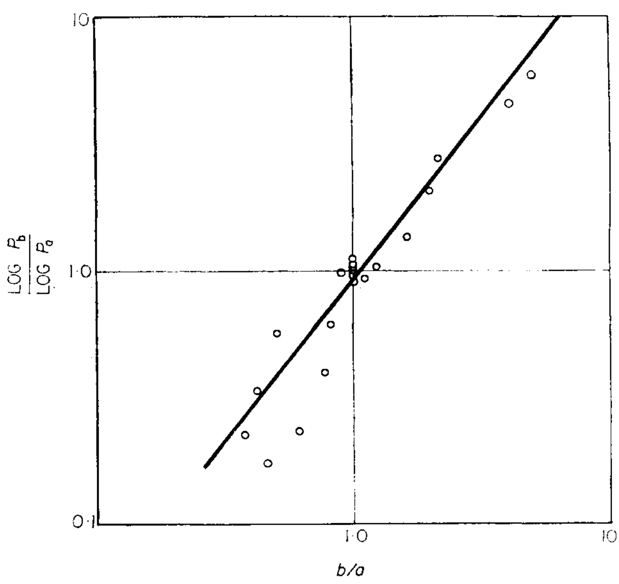

FIG. 3.10. Relation between probability of survival and size of cube (Barnsley Hards).

mens being crushed between ground steel platens, unlubricated. The results showed that there were no consistent trends in the strengths measured. Thus there were early indications that the lateral dimensions were not important, and height could be brought into the picture without more ado. A second, more complicated experiment was therefore planned, in which the strengths of specimens of three different heights, widths and lengths were compared, as shown in Table 3.4.

TABLE 3.4. COMPRESSIVE STRENGTH OF RECTANGULAR BLOCKS OF BARNSLEY HARDS

Lateral dimension (in.)		Specimen height (in.)					
Width	Length	0·5		1·0		2·0	
		(a) (b)	(c)	(a) (b)	(c)	(a) (b)	(c)
0·5	0·5	9580 ± 310	29	8290 ± 390	25	7060 ± 350	29
0·5	1·0	9410 ± 330	29	7680 ± 360	28	6960 ± 320	29
0·5	2·0	9540 ± 300	28	8260 ± 220	26	7170 ± 300	27
1·0	1·0	10,120 ± 250	29	8040 ± 310	28	7850 ± 260	29
1·0	2·0	9910 ± 240	27	8100 ± 300	29	7460 ± 260	29
2·0	2·0	10,890 ± 320	29	7590 ± 350	29	6630 ± 330	29

The figure given are (a) the mean compressive strength (lb/in^2), (b) the standard error of the mean (lb/in^2), and (c) the number of observations.

A full combination of these factors defines 3^3 different sizes of specimen, but these reduce to 18 if the assumption is made that length and breadth are indistinguishable in their effects e.g. that 1 in. long × $\frac{1}{2}$ in. wide is indistinguishable from $\frac{1}{2}$ in. long × 1 in. wide. A plan of sample preparation was devised so that one of each size of specimen was cut from each of 29 parent cubes of about 5 in. side. Figure 3.11 is a photograph of a typical set cut from a single lump of coal.

A measure of the success of the sample preparation is the small number of specimens rejected before test. Eleven sizes were cut with 100 per cent success and even in the worst cases a success rate of over 86 per cent was established. Some of the rejections were due to operators' errors in carrying out the complex machining plan, so that failures due to accidental breakage were relatively few.

Results of Experiments on Rectangular Blocks

These results show that height affects strength to a much greater extent than change in lateral dimensions, and the main effects present may be summarized as follows:

(i) Mean compressive strength decreases with increasing height for each cross-section.
(ii) The strength of specimens of height greater than 0·5in. shows no consistent trend with increase in lateral dimensions.
(iii) Mean strength increases with minimum lateral dimension for specimens of 0·5 in. height.

The results are illustrated in Figs. 3.12 and 3.13 in a manner that indicates the effects of height and lateral dimension.

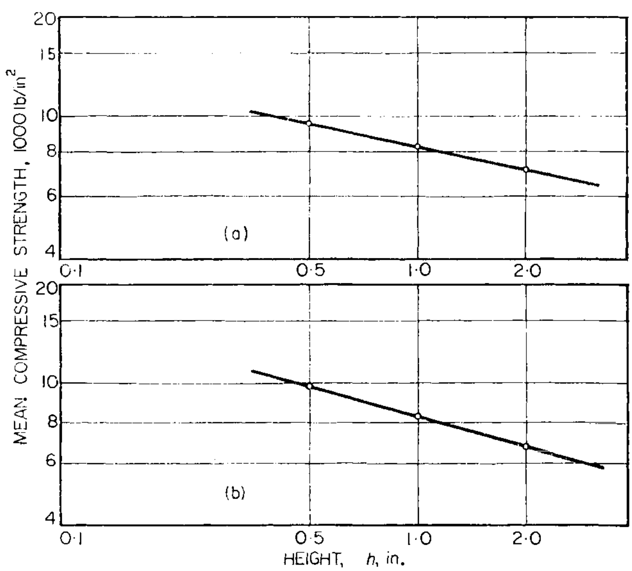

FIG. 3.12. Strength of rectangular blocks. Curve (a) shows the relationship between mean compressive strength and height for all specimens of width 0·5 in. Curve (b) shows the relationship between mean compressive strength and height for cubic specimens.

The three sets of points are well represented by straight lines on log-log scales. From Fig. 3.12 the relationships between strength σ_c and height h are:

(Fig. 3.12*a*) specimens 0·5 in. wide $\sigma_c \propto h^{-0 \cdot 22}$

(Fig. 3.12*b*) cubes $\sigma_c \propto h^{-0 \cdot 25}$

The exponent of h differs from the one determined earlier for cubes ($-0{\cdot}17$); this difference may be due to the variability of coal from one consignment to another.

The base dimensions of the specimen only appear to be important when the height is less than the minimum lateral dimension; where this situation prevails then crushing stress increases slightly with specimen size. This is an effect that has also been detected in the study of the strength of concrete[7], and appears to be associated with friction between specimen and crushing

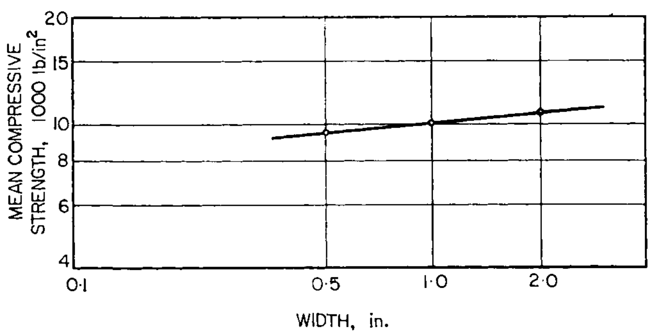

FIG. 3.13. The relationship between mean compressive strength and width for blocks of height 0·5 in.

platens. Where friction is largely eliminated by means of lubrication the effect disappears, as was shown for coal in further experiments in which a series of flat slabs of Barnsley Hards 0·25 in. high, and of 0·25, 0·5, 1·0 and 2·0 in. square cross-section were crushed. While the crushing stress of unlubricated specimens increased with size, that of graphite-lubricated specimens remained independent of size.

Applicability of Weakest-link Theory to Rectangular Blocks

The strength of rectangular blocks of height greater than the minimum lateral dimension depends therefore on height and not on length or breadth. It seems reasonable to identify the cube sides a and b of eqns. (3.2) with the heights of reactangular blocks. The verification of this postulate has been carried out in the manner described for cubes; first, the probability of survival curves for specimens of width 0·5 in. have been used to derive corresponding values of P_a and P_b at particular stresses, and then log (log P_b/log P_a) has been plotted against log (b/a). Although the results showed a fairly wide scatter, the slope of the best straigth line was $1{\cdot}14 \pm 0{\cdot}05$, and this is near enough to unity for the postulate to be acceptable.

A similar exercise was performed using the strength measurements on cubes. The slope of the best straight line for these cubes was 1·08 ± 0·17, which is not significantly different from that observed for the rectangular columns. It may be concluded, therefore, that the relationship $P_b = P_a^{b/a}$ observed for cubes is in fact a special case of the probability relationship for columns, the linear factor involved being the height of the column.

PHYSICAL IMPLICATIONS OF WEAKEST-LINK THEORY

The formal applicability of the weakest-link theory to the breakage of coal focuses attention upon tensile strength, since the theory is most directly related to tensile breakage and was indeed first propounded to account for the fact that the tensile strength of cotton threads decreases as the length of the thread increases.

The question arises—how are tensile stresses generated in a specimen when the conditions have been especially arranged, so far as is practicable, to promote uniform compressive stress? Of course complete uniformity is impossible to achieve, but lack of it is probably not the sole cause of the results just described.

A clue to a possible solution comes from a consideration of the nature of coal as a material containing cracks and weaknesses ranging from those that can be seen with the naked eye down to those which are submicroscopic in size. It is evident that coal is not a continuous medium, and even when the greatest care is taken to apply uniform stresses at the boundary of a regular specimen, these stresses may become non-uniform in the interior.

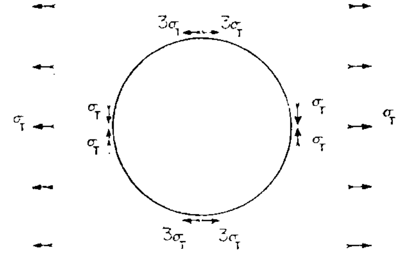

FIG. 3.14. Stresses in the vicinity of a circular hole in a plate subjected to tension.

An analytic solution due to Kirsch[8] has relevance here. Consider a plate containing a circular hole (Fig. 3.14), the boundaries of the plate being subjected to a uniform tension. Kirsch shows that this tension produces a tensile stress equal to $3\sigma_T$ at the ends of the diameter perpendicular to the tension and a tangential compressive stress of value σ_T at the other cardinal points of the hole.

If the applied tension is reversed so as to become a compression the solution still applies provided the direction of the stresses in the plate are also reversed. The stresses at the perpendicular diameter now become compressive stresses $3\sigma_T$, and the stresses at the other cardinal points tensile stresses, σ_T. Thus a uniform compressive stress applied to the boundary of a medium can generate tensile stresses if the medium is not uniform but contains circular holes.

A circular hole is possibly not an acceptable idealization of the larger cracks in coal. A more realistic one would be an opening of elliptical shape in which the ratio of major to minor axis tends towards a large value. Inglis[9] has shown that when a tensile stress is applied at right angles to the major axis of such a crack, higher tensile stresses are generated at the extremities of the crack. The crack acts as a "stress-raiser".

Griffith[10] has applied Inglis' solution to a variety of other stress situations, and in particular has obtained the remarkable result that tensile stresses can be obtained in the vicinity of the ends of a crack even when the applied stresses are compressive. For instance, a crack orientated parallel to a uniaxial compressive stress is subjected to tensile stresses at its extremities which act at right angles to the applied stress. These stresses are not, however, the greatest that can be generated; Griffith shows that these apply for a particular crack when it lies at an angle of 30 deg to the direction of the compressive stress.

Thus theory shows that the presence of cracks in a medium would serve to generate tensile stresses even when uniform compressive stress is exerted at the boundaries of the sample. The stress would be most potent on cracks orientated obliquely to the direction of the stress, but it would still occur even if the cracks were aligned parallel to the stress. If oblique cracks did not occur, or were few in number, then breakage in general would have to occur as the result of tensile stresses acting upon the "parallel" cracks.

Consider now the specimens of coal used in these investigations. The most prominent system of cracks is that associated with the direction of the bedding plane, which lay normal to the applied stress during the tests. These cracks could not therefore be subjected to high tensile stresses. The other system of cracks are those associated with the cleat, which is fairly prominent in Deep Duffryn coal, and present, although less prominently, in Barnsley Hards; and as is well known, the two cleat planes are approximately perpendicular to the bedding plane. Thus the cracks which suffer tensile stresses are in the main those lying parallel to the direction of the applied stress, and breakage must be associated with the propagation of these cracks. Some oblique cracks will certainly exist in the coal owing to the quirks of its structure, and they may propagate under the action of induced tensile stresses, but the general pattern of breakage will proceed parallel to the applied force, diverted here and there by excursions into obliquity.

The applicability of the weakest-link theory now becomes understandable.

The links in the chain are cracks situated in a potential line extending from the bottom of the sample to the top. These cracks will not all be the same, and their tendency to propagate under stress depends upon size and shape. One of these cracks, the "weakest-link", will propagate first, and this propagation will proceed from one crack to another, accelerating as the composite crack becomes increasingly long, until a chain of breakage extends from one end of the sample to another. The importance of the height of the sample in breakage theory now becomes evident.

Other workers have used Griffith's theory as an aid to explaining the results of uniaxial crushing tests. Millard, Newman and Phillips(11) have given an explanation of the fact that the power-law relating crushing stress and side of cube contains an exponent of size that is close to $-\frac{1}{2}$. Griffith showed that under conditions of plane strain a crack will propagate when the tensile stress normal to its length is proportional to $(\text{length})^{-1/2}$. Since the length of the longest crack is likely to be commensurate with the linear dimension of the specimen it is conceivable that the applied stress should be inversely proportional to the square root of the linear dimension, as was observed.

The defect of this simple explanation is that the exponent is not exactly $-\frac{1}{2}$, and the M.R.E. investigations give values which are significantly different from $-\frac{1}{2}$. A mathematical investigation of the consequences of the weakest-link theory(5) shows that a "$\frac{1}{2}$-power" law could only be expected to apply to a specimen so small as to contain only a single crack, i.e. to the "structural elements" of the material. It is also shown that the association of larger numbers of these elements in the form of a macroscopic sample will produce a relation between size and strength which is different from the simple "half-power" law. The matter can be looked at in this way; a flawless material would have a strength independent of size, i.e. $\sigma_c \propto a^0$. An otherwise flawless material containing a single Griffith crack, commensurate in size with the specimen would give $\sigma_c \propto a^{-1/2}$. A real material, containing many small flaws will give an exponent of a somewhere between 0 and $-\frac{1}{2}$.

Further, it has been shown that a relation between strength and size approximating to a power-law follows as a natural consequence of the weakest-link theory. The observed relation can be explained simply in qualitative terms along these lines: the larger the specimen the greater is the probability of its containing a gross crack that will precipitate breakage. Hence, breakage stress can be expected to decrease with increasing size.

Pattern of Breakage

The discussion just given will help in an understanding of the observed mode of breakage of compression specimens, and other related matters. Coal specimens, in general, break suddenly, almost explosively and a high-speed camera is necessary to catch details of the breakage. Figure 3.15 shows successive frames from a shot at 500 frames/sec of the disintegration of a cube of Barnsley Hards compressed normal to the bedding plane. The exploitation of the vertical cracks in the coal, giving the impression of a series of columns flying apart, can be clearly seen. The residue of breakage is quite often a pair of irregular conical fragments adjoining the top and bottom platens (Fig. 3.16). As noted in the last section, if oblique cracks exist in the coal then they will be subjected, for a particular crack size, to the greatest tensile stresses. At the platens, too, the break-up of the coal may be hindered by friction between platen and specimen, thus giving rise to the end-fragments, and also incidentally to an increase in measured strength of the sample.

The effect of several variants in the testing procedure has been investigated and the following conclusions reached:

(i) Lubrication of the interface between coal and platens reduces the strength by about 30 per cent. However, the weakest-link theory still applies to lubricated specimens.

TABLE 3.5. STRENGTH OF REPRESENTATIVE BRITISH COALS

Coal	Coal rank code no.	Volatile content (%)	Size of specimen	Compressive strength (lb/in^2)	Direction of crushing relative to bedding planes	No. of specimens
Pentremawr	100a	5	1 in.	5550 ± 400†	⊥	20
(Anthracite)				5280 ± 630	∥	20
Deep Duffryn	201	12	1 in.	2650 ± 190	⊥	24
(Gellideg seam)				2340 ± 200	∥	24
Oakdale	301	22	1 in.	1150 ± 90	⊥	24
(Meadow seam)				890 ± 70	∥	24
Cwmtillery	3/501	30	1 in.	2630 ± 95	⊥	24
(Garw seam)				1750 ± 120	∥	24
Markham	4/502	39	1 in.	4760 ± 370	⊥	24
(Black Shale seam)				4570 ± 270	∥	24
Rossington	702	37	1 in.	5290 ± 260	⊥	20
(Barnsley Brights)				3810 ± 280	∥	20
Pleasley	7/802	37	1 in.	8320 ± 520	⊥	24
(Dunsil seam)				5680 ± 20	∥	24
Rossington	801	36	1 in.	8080 ± 290	⊥	24
(Barnsley Hards)				4940 ± 190	∥	24
Linby	902	39	1 in.	7960 ± 420	⊥	24
(High Main seam)				4150 ± 270	∥	24

† Standard error of the mean. ⊥ Load applied across bedding planes. ∥ Load applied along bedding planes.

(ii) Compressive strength is dependent on the direction of the load relative to the bedding plane, the strength for loading normal to the bedding plane being generally greater than the strength for loading parallel to the bedding plane. The difference is small for high rank coals but tends to become larger with decreasing rank.

M.R.E. results for the compressive strength of representative British coals, determined by the technique described previously, are given in Table 3.5.

It will be noticed that compressive strength is roughly related to the rank of the coal as defined by the volatile content. Both low- and high-volatile coals are strong, while the coals of intermediate rank are weaker. The minimum strength of the representative coals is shown by Oakdale (Meadow seam) at a volatile content of 22.

The results in Table 3.5 for Deep Duffryn and Barnsley Hards coals differ slightly from those given in Tables 3.1 and 3.2. This difference illustrates the variation of strength that may be encountered from one batch of a particular coal to another.

Compressive Strength of Cubes Loaded Parallel to the Bedding Planes

The simple form of the weakest-link theory given in eqns. (3.2) applies to specimens in which the compressive stress was exerted normal to the bedding planes. Hobbs[12] has investigated the consequences of applying stress parallel to the bedding planes. He used rectangular blocks of Barnsley Hards coal of four heights, four widths perpendicular to the bedding planes and four lengths parallel to the bedding planes, the dimensional range in all cases being 0·25–2 in. He found that whereas mean strength decreased with height of specimen h and length parallel to the bedding planes l, it increased with width perpendicular to the bedding planes w.

A more general expression for the weakest link theory applies to this case, in the form

$$\log P_{w_1h_1l_1} = \left(\frac{w_1}{w_2}\right)^{-\frac{1}{2}} \left(\frac{h_1}{h_2}\right)^{2} \left(\frac{l_1}{l_2}\right)^{\frac{1}{2}} \log P_{w_2h_2l_2}$$

where $P_{w_1h_1l_1}$ and $P_{w_2h_2l_2}$ are the probabilities of a rectangular specimen of sides $(w_1h_1l_1)$ and $(w_2h_2l_2)$ surviving the same stress.

It is noticeable that breakage parallel to the bedding plane is more docile than breakage perpendicular to it. A crack lying between bedding planes is likely to propagate between the planes, instead of initiating a catastrophic rupture, and it is only when a succession of roughly parallel and adjacent breaks have occurred that total failure can be assumed to have taken place.

Hobbs considers the physical implications of the results and shows that they are best suited by a model in which the pattern of cracks in the bedding

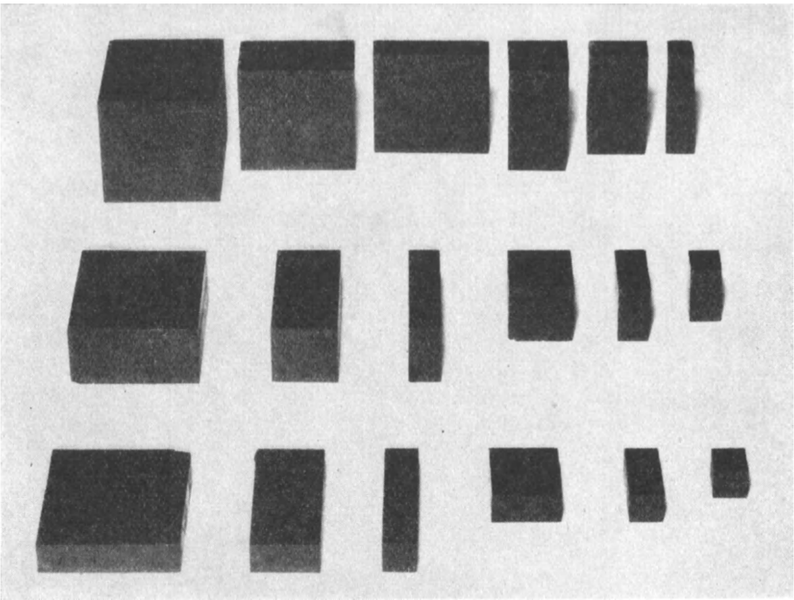

FIG. 3.11. Rectangular blocks cut from a single lump of Barnsley Hards.

FIG. 3.15. Breakage of a cube of Barnsley Hards.

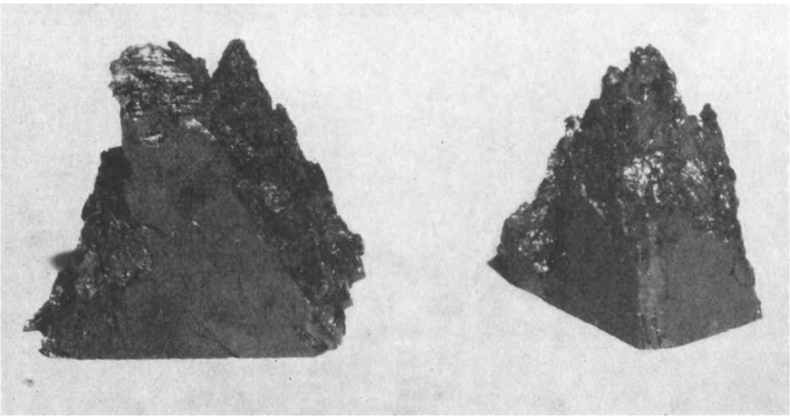

FIG. 3.16. Residue of breakage of cubes.

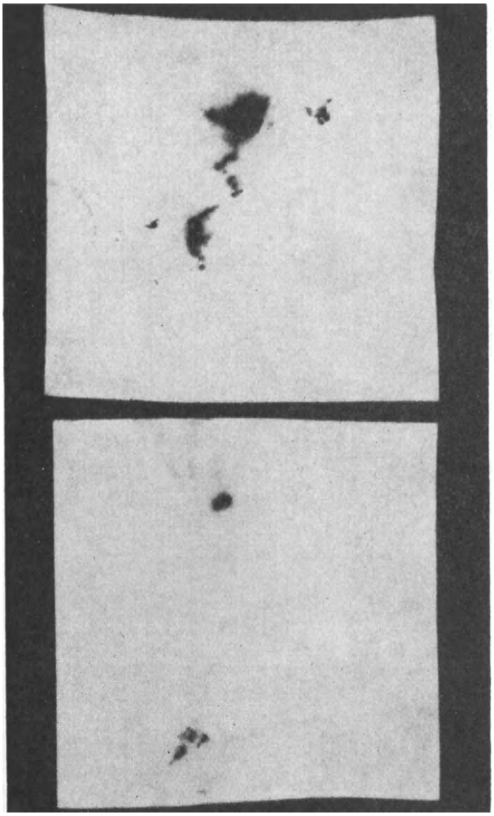

FIG. 3.17. Imprints of contact area between irregular lump and platens.

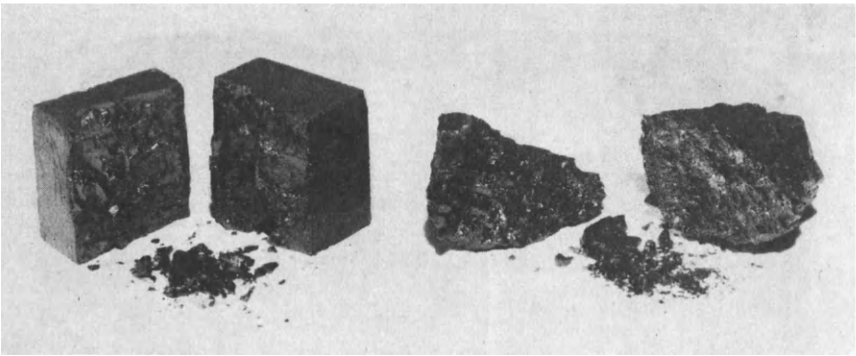

FIG. 3.21. Comparison of debris of breakage — cubes and irregular lumps.

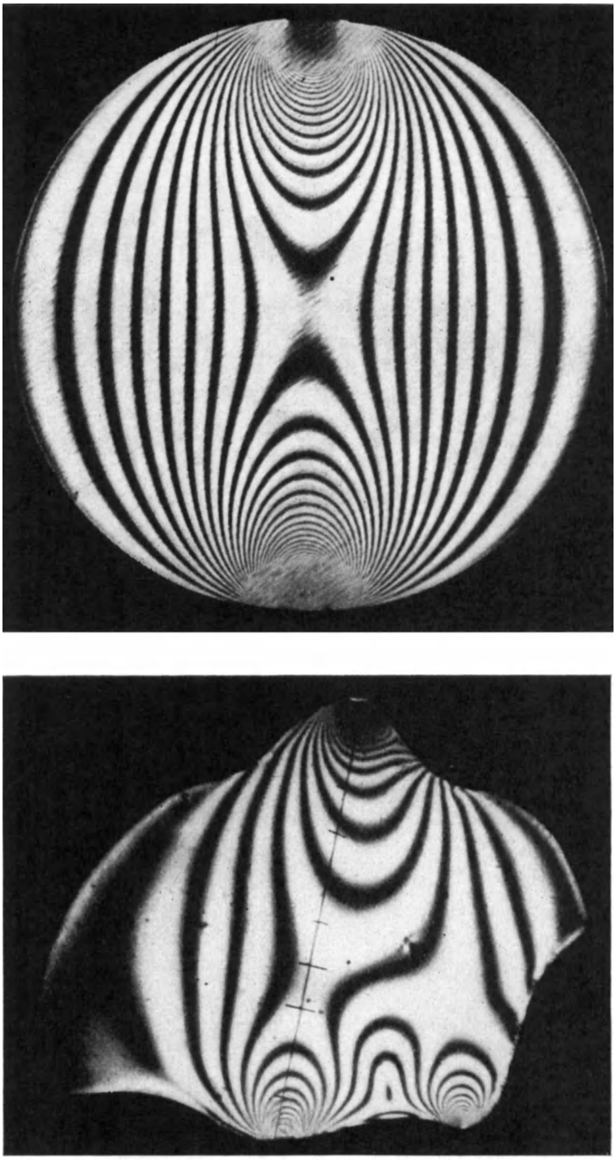

FIG. 3.23. Comparison of shear strength patterns—disc and flat irregular fragment.

planes forms not so much a potential chain as a potential mesh, with the links distributed in two dimensions. A specimen of coal crushed parallel to the bedding planes is a system of parallel meshes. For Barnsley Hards, Hobbs estimates that the length of the bedding plane crack that constitutes a link is of the order of 0·01 in.

Strength of Irregular Lumps

The study of the breakage of cubes is interesting as an academic exercise, but it is nececessary to ask to what extent it has relevance to processes of breakage that are met in the mining industry. One important type of breakage is that which occurs in the transport of coal from the face, a process that includes free fall at transfer points in conveyor belt systems. This breakage is associated with the jostling of rough lumps of coal, and with forces of impact between coal and more-or-less firm surfaces. Idealizing the matter to a point where it becomes amenable to laboratory investigation, one might say that degradation of coal is associated with the compressive breakage of irregular lumps.

The authors have investigated this matter, using irregularly-shaped lumps of Deep Duffryn coal selected from the debris from the crushing of heavier specimens. In some cases, random smashing of larger pieces with a hammer was resorted to in order to obtain greater numbers of specimens. It was found that there was no significant difference between the compressive strengths of specimens obtained in these two ways.

The Deep Duffryn coal specimens were very roughly cuboid, and when crushed in an Avery press or a Hounsfield tensometer made approximately equal contact with each of the parallel platens. No particular orientation of the bedding planes of the specimens with respect to the applied load was adhered to, since earlier work had indicated no significant strength difference for Deep Duffryn coal when this orientation was varied. In general, there was some crushing of minor projections, but the point of major breakage of the specimen was nearly always quite determinate.

Since it would seem important to measure the applied stress as well as the load, a simple method of measuring the areas of contact between a specimen and the crushing platens was adopted. Pieces of squared paper were placed between the specimen and the platens; during a test each sustains a well-marked blackened imprint, the area of which can be readily determined. Figure 3.17 shows a typical pair of imprints obtained. Careful observation of a number of specimens while undergoing crushing, together with examination of the imprints from regularly-shaped specimens, indicated that the imprinted area did indeed give a reliable measurement of the area of contact between the platens and the specimen.

The compressive strength of an irregular specimen may be defined as the crushing load divided by the mean area of contact of the two surfaces of the specimen in contact with the platens. With this definition, the mean compressive strength for 46 irregular specimens of Deep Duffryn coal, each of mass about 5 g, was found to be 4300 lb/in^2 with a standard deviation of 1100 lb/in^2, i.e. about 25 per cent. For the same specimens, the mean crushing load was 57 lb wt. and the standard deviation 24 lb wt.—that is, over 40 per cent. A standard deviation of 25 per cent was found for the strength of cubes of approximately the same volume, suggesting that the strength of lumps as defined in this way is a more fundamental quantity than the load.

In general, the procedure outlined above was adopted; that is, papers were placed on both sides of the specimen and the mean of the areas thus found taken as the area of contact of the load with the specimen. It was recognised that the stresses calculated from these areas are probably only indirectly related to the actual complex system of stress found in an irregular specimen, compressed at various localized projections, but it was considered that some significance could nevertheless be attached to this "compressive strength" as a measure of coal strength. Accordingly, this method was used in the work subsequently described on the compressive strength-size relationships for irregular lumps of coal.

Compressive Strength of Irregular Lumps of Deep Duffryn Coal

Measurements were made of crushing load and area of contact for five different mass groups of irregular Deep Duffryn specimens in the range 5–500 g. The results of these measurements are given in Table 3.6.

Figures 3.18 and 3.19 show mean compressive strength plotted against mean mass and mean area plotted against mean mass respectively.

From these results the following relationships were obtained:

$$\sigma_c' \propto V^{-0\cdot09\pm0\cdot03} \tag{3.3}$$

and

$$A \propto V^{0\cdot55\pm0\cdot05} \tag{3.4}$$

where σ_c' is the compressive strength as defined above, A is the area of contact as defined above, and V is the mean volume (or mass) of the specimens.

If a' is a representative linear dimension of an irregular lump then one might write $a'^3 = V$, so that the relationship (3.3) becomes

$$\sigma_c' \propto a'^{-0\cdot27\pm0\cdot09}$$

This relationship is not significantly different from that found for cubes of Deep Duffryn coal.

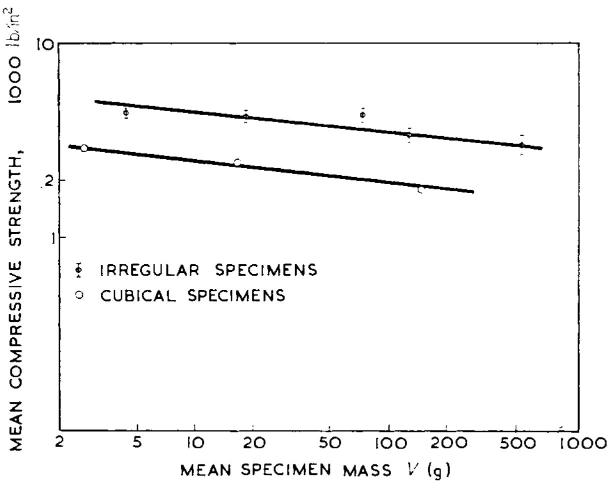

FIG. 3.18. Variation in mean compressive strength with mass for Deep Duffryn coal.

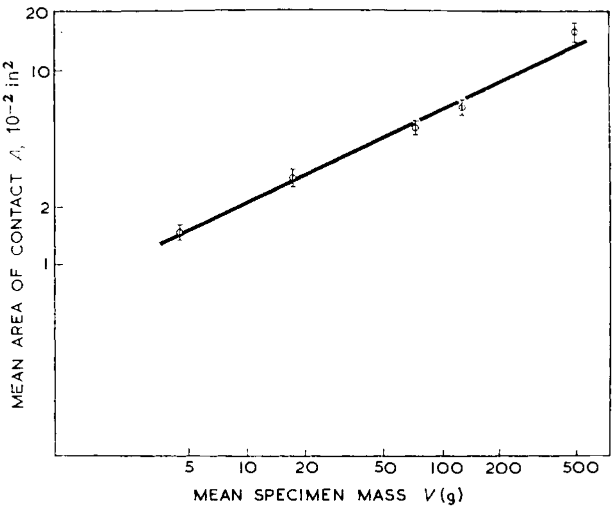

FIG. 3.19. Variation in mean area of contact with specimen mass for Deep Duffryn coal.

TABLE 3.6. STRENGTHS OF IRREGULAR LUMPS OF DEEP DUFFRYN COAL AND OTHER RELATED DATA

Mean mass (*g*)	Mass range (*g*)	No. of specimens	Mean area† of contact ($in^2 \times 10^{-3}$)	Mean† crushing load (lb)	Mean compressive† strength (lb/in^2)
4·5	4–5	46	14·7 ± 1·2	57 ± 3·5	4310 ± 160
19	18–20	27	27·9 ± 2·3	103 ± 9	4190 ± 250
75	70–80	32	49 ± 5	168 ± 14	4190 ± 330
130	120–140	32	64 ± 6	192 ± 12	3350 ± 260
500	450–550	19	158 ± 20	386 ± 40	2970 ± 320

† The uncertainties quoted represent standard error of the mean.

COMPRESSIVE STRENGTH OF IRREGULAR LUMPS OF BARNSLEY HARDS

Measurements of crushing load and area of contact were made on 10 groups of irregular specimens between 1 and 800 g in mass.

The results of these measurements are tabulated in Table 3.7. All measurements were made with the load applied perpendicular to the bedding plane.

From these results the following relationships connecting σ_c', V, A and W were found:

$$\sigma_c' \propto V^{-0.04 \pm 0.02}, \quad \text{(Fig. 3.20)} \tag{3.5}$$

$$A \propto V^{0.51 \pm 0.06}. \tag{3.6}$$

Equations (3.5.) gives rise to

$$\sigma_c' \propto a'^{-0.12 \pm 0.06}$$

where a', as before, is a representative linear dimension of an irregular sample. The equation is in agreement with the earlier result for cubes.

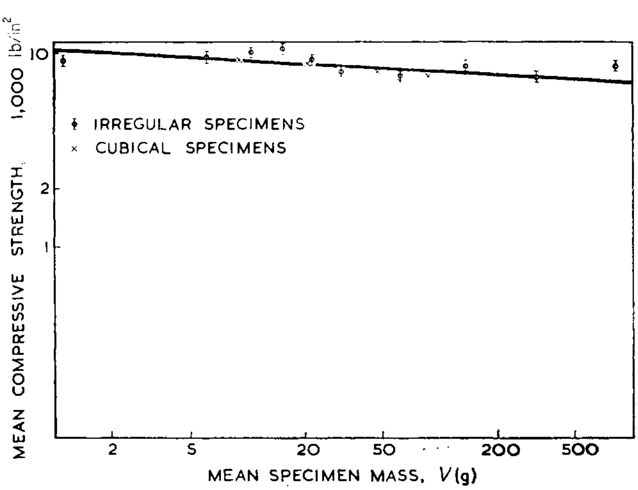

FIG. 3.20. Variation of mean compressive strength with mass of specimen for Barnsley Hards coal.

It will be noted that for both coals the area of contact A differs significantly from a proportionality to $V^{2/3}$ which one might expect on the grounds of simple geometric similitude between specimens of various sizes. This phenomenon can be linked with the elastic flattening of the coal that takes place at the areas of contact with the platens.

TABLE 3.7. STRENGTH OF IRREGULAR LUMPS OF BARNSLEY HARDS AND OTHER RELATED DATA

Mean mass (*g*)	Mass range (*g*)	No. of specimens	Mean crushing load† (lb)	Mean area† of contact ($in^2 \times 10^{-3}$)	Mean† compressive strength (lb/in^2)
1·2	1·05–1·35	62	89 ± 5	11·5 ± 1·0	9200 ± 450
6·5	6–7	47	222 ± 15	25·4 ± 1·9	9550 ± 520
11·0	10–12	108	246 ± 10	28·4 ± 1·2	10,000 ± 260
15·8	15–17	46	331 ± 21	37·4 ± 3·1	10,450 ± 470
22·0	20–25	29	382 ± 25	50 ± 6	9220 ± 650
32·5	30–35	35	484 ± 29	67 ± 6	8120 ± 430
65	60–70	42	523 ± 48	71 ± 6	7480 ± 390
140	130–150	22	924 ± 127	113 ± 15	8510 ± 510
320	300–350	17	1460 ± 180	200 ± 20	7430 ± 450
820	700–900	28	2230 ± 170	280 ± 26	8580 ± 520

† The uncertainties quoted represent the standard error of the mean.

Applicability of Weakest-link Theory to Irregular Lumps

When the areas of contact are known the mean crushing stress at the points of contact can be determined. The strength of lumps at a particular size shows the same sort of variation as was obtained with cubes, and by comparing the strength distribution at one size with that at another size, as was done for cubes, the applicability of the weakest-link theory can be investigated. The equation for cubes (see eqns. (3.2))

$$P_b = P_a^{b/a}$$

might be expected to become

$$P_b = P_a^{(V_b/V_a)^{1/3}}$$

where V_a and V_b are the volumes of irregular lumps. This equation has been checked against the results summarized in Tables 3.6 and 3.7 and it has been found that although showing a considerable experimental scatter, the results are not at variance with this expression. It can be taken therefore that the weakest-link theory is fundamental to the compressive breakage of coal, irrespective of the shape of the sample.

Compressive Strength and Indentation Strength

Figure 3.18 gives the variation of mean strength with size for lumps of Deep Duffryn coal with, in addition, the results already given for cubes. The strengths obtained for the crushing of the irregular lumps were about 1·7 to 2·0 times as large as those for the crushing of cubes of similar mass. For Barnsley Hards (Fig. 3.20) the compressive strengths of irregular and cubical specimens are about the same. Further work with Cwmtillery (Garw) coal, a coking coal of 30 per cent volatiles, showed the ratio to be about 2, as for Deep Duffryn.

A difference between the compressive strengths of cubical and irregular specimens of coal is not unexpected. It has been suggested that the irregular lumps of coal tested broke as a result of high stresses applied to a few projections, rather than as a result of a roughly uniform stress applied to the whole specimen, as for the cubes. Thus, a measurement of the strength of irregular lumps by the method described would be akin to a measurement of the indentation strength of the coal rather than the compressive strength, and there is no reason to assume that these are identical. In metals, for example, the indentation strength is about three times the compressive strength(13), while Evans and Murrell(14) have shown that the resistance of coal to wedge penetration, although of the same order as compressive strength, is not necessarily identical with it. Thus, it would appear likely that the agreement of the two stresses found in Barnsley Hards is coincidental.

Mode of Breakage of Irregular Lumps

A qualitative demonstration of the similarity of compressive breakage of irregular lumps and the indentation breakage of cubes was attempted by breaking cubes by the action of cylindrical projections of small area ("indenters") acting across opposite faces. Figure 3.21 shows a comparison between the breakage products of an irregular lump between platens and that of a cube between indenters, the initial size of the two specimens being about the same. The resemblance in size distribution of the breakage products is obvious; moreover, it has been shown at M.R.E. that the compressive strengths are also roughly the same when the surface areas to which the load is applied are the same.

This would appear to contradict the assumptions frequently made that the breakage of irregular lumps of coal is determined only by the weakness within the lump and not by the nature of applied force. Bennett[15], for

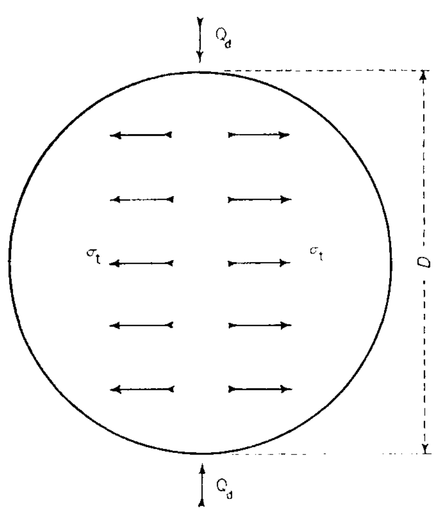

FIG. 3.22. Tensile stress in a disc subjected to compressive loading. $\sigma_t = 2Q_d/\pi D$, where Q_d = load per unit length at right angles to plane of diagram.

example, in discussing size distributions, assumed that fracture will take place at points of weakness irrespective of position, and in a later paper[16] derives the "ideal law of breakage" from the assumption that the lumps are broken by "forces sufficiently violent to make breakage equally likely at any points". It seems likely that the stresses actually encountered in coal mining, transportation and preparation will more closely approximate to the localized stresses applied in the work described here than to the nearly uniform stresses required by these assumptions, and that uniform stresses will only be found in work on prepared specimens which have plane faces in contact with plane loading surfaces.

As to the physical method of breakage, it has already been remarked that irregular lumps under compression do not merely crumble near the areas

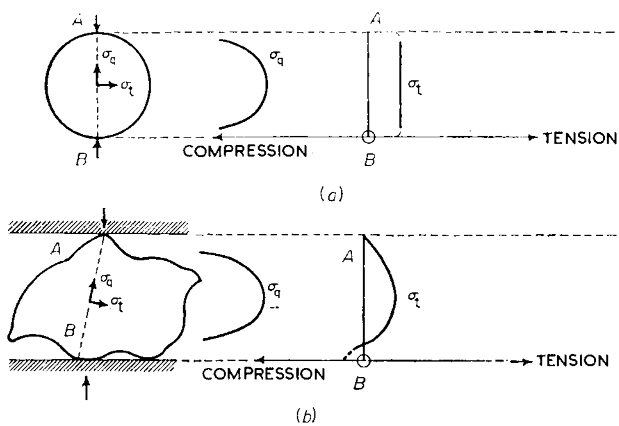

Fig. 3.24. Principal stresses on line joining points of application of load. (a) Disc in diametral compression, (b) irregularly shaped piece compressed between parallel plates.

of compression, but suffer a major and clear-cut breakage which splits the specimens into large pieces.

The reason for this behaviour can be indicated. If a disc of solid material is acted upon by opposing diametral loads (Fig. 3.22) it can be shown theoretically that the stress acting at right angles to the diameter is a uniform tension. Berenbaum and Brodie[17] have used this theory as the basis of measuring the tensile strength of coal. The line loads of the theory can be simulated with reasonable accuracy by loads acting over the areas of contact between the specimen and flat platens. Moreover, the specimen does not necessarily have to be circular in shape in order to generate tensile stresses; photo-elastic analysis with irregular specimens (Fig. 3.23) shows that the shear stress pattern has a strong qualitative resemblance to that of the circular disc, and it is a reasonable deduction that the tensile stress patterns are similar, too. This has been confirmed by calculation of the principal stresses along the line joining the contact areas (Fig. 3.24). It is a still further step from essentially two-dimensional theory and experiments to the problem of a solid lump of material under the action of indentation stresses, but here again it seems reasonable to suppose that the essential features of the stress pattern do not change, and that tensile stresses are set up in the core of the material. It can be at least postulated, with some degree of verisimilitude, that the stresses by which lumps of coal are shattered by impacts encountered in conveyance, free fall, and so on, are essentially tensile in nature. It will be noticed that the stresses are set up by virtue of the shape of the material and the disposition of the applied forces, and will apply whether or not any internal cracks exist. This is, in principle, a difference from the behaviour of cubes. However, the lumps finally break by the action of these tensile stresses upon the cracks, which as already pointed out, act as "stress-raisers", the tensile stresses at the end of a crack being greater than the mean tensile stress in the material. In general, therefore, the important cracks in irregular lumps will be those existing parallel to the direction of the applied compressive load, since they are at right angles to the major tensile stress, and the main breakage plane of the lump can be expected to be in this direction, so that the terminal mechanism of breakage is likely to be very similar to that of cubes of coal in uniform compression.

Summary of Results Relating to Uniaxial Compression

The strength of cubes of coal loaded perpendicular to the bedding plane between steel platens is very varied, even when the cubes are cut from the same lump. Strength is best dealt with on a statistical basis, and the principle which has been found applicable is a variant of the "weakest-link" theory, stating

$$P_b = P_a^{b/a}$$

where P_a = probability of survival of cubes of side a and P_b = probability of survival of cubes of side b.

For rectangular blocks the equation still holds, the linear dimensions concerned being the heights of the blocks. It also applies to irregular lumps of coal, the characteristic dimension now being proportional to the cube root of the volume.

The mean strength of specimens varies with characteristic dimension according to a power law. The exponent of the power law is the same for cubes and irregular lumps, provided that for the latter, strength (i.e. load per unit area) is measured in terms of actual area of contact, which is not a simple function of the volume of the specimen.

A consideration of the physical implications of the weakest-link theory leads to the conclusion that breakage is precipitated by tensile stresses generated in the vicinity of cracks by the applied compressive stresses. The cracks which are likely to propagate are those running parallel to the direction of the applied stress.

The breakage of irregular lumps is accomplished by compressive stresses acting over very small areas of contact. It is suggested that these compressive stresses produce tensile stresses in the body of the specimen independent of the existence of cracks. The action of these tensile stresses on the cracks produces the major breakage observed.

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Appendix

Summary of Reported Compressive Strength Measurements on Coal

Workers	Date	Coal	Size of specimen	Compressive strength lb/in^2	Direction of crushing relative to bedding planes	No. of specimens
See (g)	1875	German		2500		
H. Louis (a)	1904–5	—	3–4 in.	560–6360	$\perp$	
				690–3800	$\parallel$	
J. Daniels (b)	1907	Anthracite	2 in.	2370	—	
and			3 in.	3390	—	
L. D. Moore			4 in.	2000	—	
			6 in.	1550	—	
		Bituminous	6 in.	950	—	
A. N. Talbot (c)	1907	—	12 in.	1000–2090 mean 1480	—	
D. Bunting (d)	1911	Anthracite	—	—	—	
W. Griffith and E. T. Connor (e)	1912	Anthracite	2 in.	1740–7010	—	423
G. S. Rice	1929	Bituminous	$2\frac{1}{2}$–4 in.	2490	$\perp$	
and			7–8 in.	2170	$\perp$	
C. Enzian (f)			10–12 in.	2010	$\perp$	
			$12 \times 12 \times 18$ in	1150	$\perp$	1
			30 in.	820	$\perp$	1
			54 in.	310	$\perp$	1
O. Muller (g)	1930	Heinitz bed	—	3000	—	7
		Schumann vein	—	2350	—	6

$\perp$ Load applied across bedding planes. $\parallel$ Load applied along bedding planes.

APPENDIX—*Continued* SUMMARY OF REPORTED COMPRESSIVE STRENGTH MEASUREMENTS ON COAL

Workers	Date	Coal	Size of specimen	Compressive strength lb/in^2	Direction of crushing relative to bedding planes	No. of specimens
D. Penman (h)	1931	Indian	—	1000–4600	—	
H. Bode (j)	1933	>12% vitrain	—	320–1300	—	
		<12% vitrain		2300–4800	—	
H. Heywood (k)	1935	Anthracite (Gt. Mountain)	1–1½ in.	7280	∥	
				5460	⊥	
		Anthracite (Red vein)		3300	—	
				2350	—	
		Welsh Steam		2840	∥	
				1280	⊥	
		Barnsley Hards	1–1½ in.	5000	∥	
				9900	⊥	
		Barnsley Softs		5150	∥	
				6950	⊥	
		Illinois		2180	∥	
				3880	⊥	
		Cannel		5600	∥	
				9560	⊥	
		Carbonaceous shale		6670	∥	
				5510	⊥	
		Shale		5510	∥	
				6500	⊥	

⊥ Load applied across bedding planes. ∥ Load applied along bedding planes.

APPENDIX—*Continued* SUMMARY OF REPORTED COMPRESSIVE STRENGTH MEASUREMENTS ON COAL

Workers	Date	Coal	Size of specimen	Compressive strength lb/in^2	Direction of crushing relative to bedding planes	No. of specimens
C. E. Lawall	1937	Bituminous	3 in.	1160–6840	⊥	
and			3 in.	820–5010	∥	
C. T. Holland (l)			$1\frac{1}{2}$ in.	5340		
			3 in.	4730	⊥	
			3 in.	2420		
			10 in.	1390		
			$1\frac{1}{2}$ in.	4360		
			3 in.	3601	∥	
			3 in.	1660		
R. E. Gilmore and	1937			1480	∥	
J. H. H. Nichols (m)				5020	⊥	
Discussion L. A. Shipman						
J. Pulkrabek (n)	1939	Anthracite	1·6 in.	1260–5420	⊥	
				380–1200	∥	
		Lignites		2190–6560	⊥ and ∥	
		Dull coals		12400–13600	⊥ and ∥	
H. P. Greenwald,	1939	Pittsburgh	30×30×15 in.	500		1
H. C. Howarth			45×45×35 in.	600		1
and			64 in. cube	695		1
I. Hartmann (o)			32 in. cube	885		1
			31 in. cube	920		1
D. W. Phillips (p)	1948		$2\frac{1}{2}\times1\frac{1}{2}\times1\frac{1}{2}$ in.	3–6000		2
L. D. Burton	1952	Anthracite	4 in.	2330	⊥	1
and			2 in.	4050		3
J. W. Phillips (q)			1 in.	5210		6
			$\frac{1}{2}$ in.	8600		9
			$\frac{1}{4}$ in.	6130		10

⊥ Load applied across bedding planes. ∥ Load applied along bedding planes.

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CHAPTER 4

The Tensile Strength of Coal

DURING work on the compressive strength of coal,[1] summarized in the previous chapter, it was noted that even when compressive stresses are applied to coal specimens the breakage appears to be essentially tensile in nature. This concept cannot be checked directly but is inferred from a variety of evidence, the chief items of which are:

(i) the applicability of the weakest-link theory, a concept particular to tensile breakage, to the relationships between compressive strength and size for coal specimens, both regular and irregular in shape;
(ii) the demonstration by means of photo-elastic evidence that tensile stresses are produced in an irregular lamina by isolated compressive stresses acting in its plane at the boundary.

With the implication that tensile breakage will be an important factor in the winning of coal, it is essential that tensile strength should be studied in its own right.

METHODS OF TENSILE TESTING

The obvious test is the straight pull, as employed with metals. This cannot easily be used with coal owing to the difficulty of machining suitable specimens, which need to have a thinned-down central section. The test has other snags for brittle materials: breakage may start from the vicinity of the grip instead of from the intended site, and if a truly axial extension is not provided the slight amount of bending that the specimen has to undergo may induce large additional tensions in certain regions.

A more practicable test is the bending test. In this test a thin rectangular beam of the material is bent under the action of four parallel line loads until it breaks. The condition of loading can be contrived so that between the two inner lines the bending moment is constant, and by the application of thin beam theory the maximum tension acting on the strip can be calculated. This tension acts on the convex surface of the beam.

A simple apparatus based on this technique has been described by Pomeroy and Morgans[2]. It was designed to accommodate specimens with a minimum length of 4 in., a maximum thickness of 0·375 in., and to break specimens having a tensile strength of the order of 1000 lb/in^2 (Fig. 4.1). Bending of the strips is achieved by means of four equal line loads applied perpendicular to the length of the strip and symmetrically disposed with respect to its centre. The master load is applied by means of a weight which is pulled along a light steel beam *A* by a string attached to the spindle of a small electric motor. The beam is pivoted on a hardened steel knife edge *B*, and from its other end the load applied to the specimen is taken off by means of a triangular stirrup which holds the specimen on two supports, these supplying the inner line loads of the system acting on the beam.

The strip of coal under test is placed in the stirrup so that it rests on the two rollers *CC* and passes under the two rollers *DD*. A steadily increasing bending moment is applied to the coal until failure occurs, at which point the beam rotates through a small angle, contacts a micro-switch and stops the weight. From the position of the weight, the maximum bending moment, and hence the maximum tensile stress sustained by the coal, is calculated.

Results from Bending Tests

Measurements were made by Pomeroy and Morgans on the following coals:

(i) Llandebie anthracite (coal rank code no. 100*a*);
(ii) Deep Duffryn, steam coal (201);
(iii) Cwm "Mildred" coking coal (301);
(iv) Barnsley Hards (801).

The specimens were prepared by the same technique as for preparing cubes[1], but the coals differed greatly in their machining properties. Barnsley Hards could be cut to thinner than 0·1 in., but the steam and coking coals could not be systematically cut to less than $\frac{3}{8}$ in. The anthracite was variable; strong over much of its volume but interspersed with gross weaknesses.

Table 4.1. Tensile Strengths of Various Coals by Bending Beam Method

Coal	Approx. size of specimen (in.)	Tensile strength (lb/in^2)	
		Perpendicular to bedding planes	Parallel to bedding planes
Llandebie anthracite	$4\times\frac{1}{4}\times\frac{1}{4}$	880 (6)	360 (8)
Deep Duffryn steam coal	$4\times1\times\frac{3}{8}$	170 (6)	80 (7)
Cwm "Mildred" coking coal	$4\times\frac{3}{4}\times\frac{3}{8}$	—	30 (6)
Barnsley Hards	$4\times1\times\frac{1}{4}$	70 (15)	—
	$4\times1\times\frac{1}{10}$	—	820 (20)

(Bracketed figures give the number of measurements for each mean.)

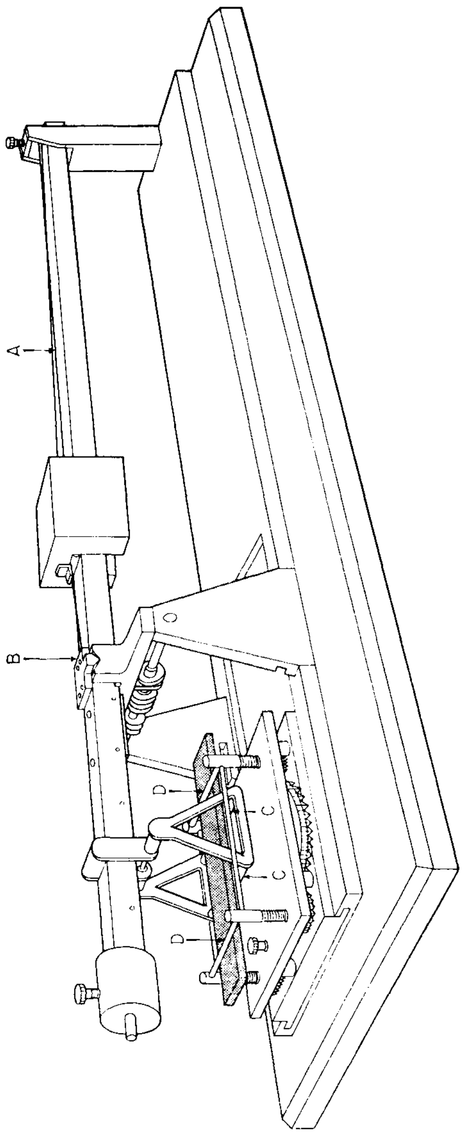

FIG. 4.1. Bending machine for tensile tests.

These measurements (Table 4.1) were not made in sufficient numbers for useful estimates of the standard deviation to be made, and since the size of the specimen was not the same in all tests they are not strictly comparable. Perhaps the only indisputable observations that can be permitted are that tensile strength changes with the direction of the applied load relative to the bedding planes and that the values obtained seem to be about an order of magnitude lower than compressive strength.

Consideration of Results from Bending Tests

The bending test suffers from the disadvantages that specimens are often not easy to prepare, but in addition another more substantial objection can be lodged against it. It has been pointed out(3) that where straight-pull and bending tests are both practicable with a particular brittle material the values of tensile strength observed in the latter case are frequently higher (often by a factor of two or more) than those obtained by the conventional test. This has been observed for concrete, porcelain and cast iron. The nature of the bending test suggests that quite large errors can arise as a result of modification of the surface of the material under test, since it is at the surface that the maximum tensile stresses are set up. This has, in fact, been shown experimentally by experiments on glazed and unglazed porcelain, in which a layer of glaze only one eightieth of the thickness of the specimen was sufficient to increase the bending strength by 60 per cent.

These drawbacks of the beam bending method have stimulated research into alternative methods of measuring tensile strength. This work was carried out at M.R.E. by Berenbaum and Brodie(3).

Method of Diametrical Compression

This method depends upon an analytic solution(4) that may be stated in the following terms: if a disc of isotropic elastic material is subjected to directly opposed uniform line loads at the extremities of a diameter, then a uniform principal tensile stress acts at right angles to the diameter. If Q_d is the applied load per unit length at right angles to the plane of the disc and D the diameter of the disc, the magnitude of the tensile stress is given by

$$\sigma_t = \frac{2Q_d}{\pi D}. \tag{4.1}$$

The other principal stress σ_q, acting along the diameter, is always compressive but varies in magnitude from point to point. This stress is given by

$$\sigma_q = -\frac{2Q_d}{\pi}\left(\frac{2}{D-2y} + \frac{2}{D+2y} - \frac{1}{D}\right) \tag{4.2}$$

where y is the distance along the diameter measured from the centre.

The theoretical line load cannot of course be achieved in practice, but compression of a disc between parallel planes appears to offer a suitable enough simulation for practical purposes. It can be shown that the stresses differ from the theoretical solution only at points separated from the applied load by distances of the same order as the width of the area of contact between the plane and the disc, provided this width is small compared with the diameter of the disc. The width of the zone of contact has been measured for coal discs of 1 in. diameter, and found to be less than 0·05 in. Hence it may be concluded that the value of the tensile stress which acts across the loaded diameter is to a first approximation of the constant magnitude given by eqns. (4.1), and that tensile strength can be calculated from the load applied when breakage takes place. It has been found that coal discs do indeed break along the diameter of loading when crushed in a press between platens.

A disadvantage of the disc test is that the tensile stress is associated with an orthogonal compressive stress, which increases from a minimum of three times the constant tensile principal stress at the centre to large values as the platens are approached. This combination of compressive and tensile principal stresses means that large shear stresses are set up in the material, and it is necessary to be certain that any breakage observed is a failure in tension and not in shear. Berenbaum and Brodie have been able to reassure themselves that with coal specimens this is so.

The experimental technique for the disc test is simple. First of all, cores of coal of 1 in. diameter are produced with a diamond coring bit, and then suitable discs for test can be sliced off with a carborundum slitting wheel. The slicing is done with the sample held in a slotted jig so that correct thickness is automatically obtained. Whilst various diameters and various thicknesses have been investigated, a convenient standard is a diameter of 1 in. and a width of 0·3 in.

With some friable coals a core may be successfully taken, but the samples break up during slicing. It has been found at M.R.E.[5] that this disruption may be prevented by covering the core with a thin coating of "Araldite" epoxy resin. Discs so obtained should have the coating scraped off before testing near the four cardinal points of the disc with respect to the applied load; the result for tensile strength is then not affected by the presence of the coating on the rim.

A special advantage of the disc method is the ease with which the variation of tensile strength with direction in the coal can be studied. Suppose, for example, disc specimens are prepared having the bedding planes perpendicular to the flat surface of the disc. Tensile strength parallel to the bedding planes can be obtained from a breakage test in which the bedding planes are horizontal while the applied load is vertical. Tensile strength perpendicular to the bedding planes can be obtained from a sample turned through a right-angle from this position. For the strength at any intermediate angle,

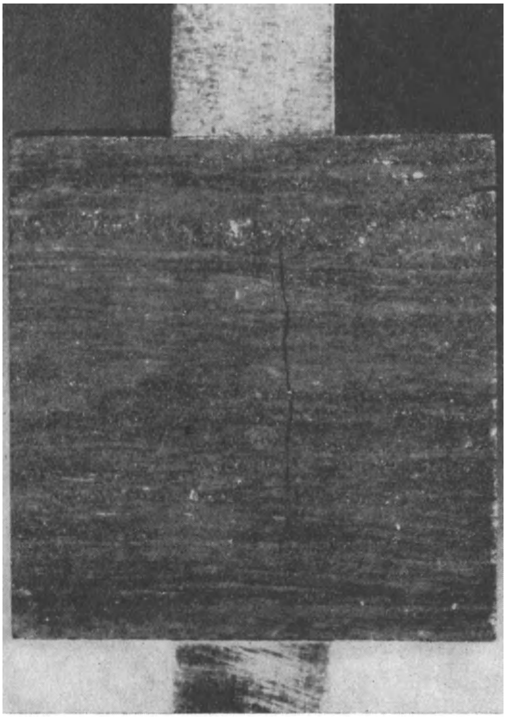

FIG. 4.2. Tensile breakage by indentation.

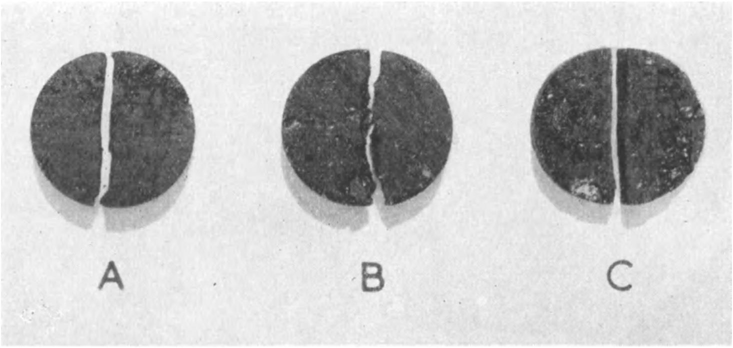

FIG. 4.4. Disc specimens broken by tensile stress.

all that is required is the appropriate orientation of the sample in the testing machine, and then the breakage in the normal way. This great facility contrasts markedly with the difficulty experienced in producing beams for the beam bending test. The stresses produced by a high speed grinding wheel render it virtually impossible to obtain samples having certain prescribed orientations of weakness planes, e.g. specimens of friable coal with bedding planes perpendicular to the axis.

Method of Indentation

An interesting variant of the disc test is that in which a flat square-shaped specimen of coal is subjected to loads from plane indenters applied at opposite edges. It has been shown[3] that the stress at right angles to the axis of loading is tensile near the centre of the specimen and compressive nearer the edge. If a specimen of coal is loaded in this way, then as the load is gradually increased a crack suddenly appears in the region in which the tensile stresses act (Fig. 4.2). The maximum tensile stress in the specimen when the crack appears, and hence the tensile strength of the material, can be deduced from the applied load, using the results of photoelastic analysis.

This method provides a vivid illustration of the thesis that tensile breakage consists in the propagation of cracks in the material, but in application it is less versatile than the disc method. Comparative results have shown that the two methods give substantially the same estimate of the tensile strength of coal.

Detailed Measurements of the Tensile Strength of Coal by Means of the Disc Method

Reference to the original paper of Berenbaum and Brodie[6] should be made for full details: only a summary will be attempted here. Their results are related to the following convention governing the direction of the applied tensile stress to the direction of the major weaknesses in the specimen: the angle made by a given direction with the bedding planes (i.e. the complement of the angle between the direction and the normal to the plane) is defined as α. In a similar way β is the angle with the main-cleat planes and γ the angle with the cross-cleat planes. α, β and γ are not of course independent but are related by the equation:

$$\cos^2\left(\frac{\pi}{2} - \alpha\right) + \cos^2\left(\frac{\pi}{2} - \beta\right) + \cos^2\left(\frac{\pi}{2} - \gamma\right) = 1$$

or

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2.$$

If a disc is cut which has, for example, the cross cleats parallel to its face, then in a disc test the tension acts parallel to these cleats, and so γ is taken to be zero. Either α or β can however be chosen to have any value; the other is then defined by the interrelation that $\cos^2 \alpha + \cos^2 \beta = 1$, i.e. $\alpha + \beta = 90°$. A diagram illustrating the types of specimen used for test is given in Fig. 4.3.

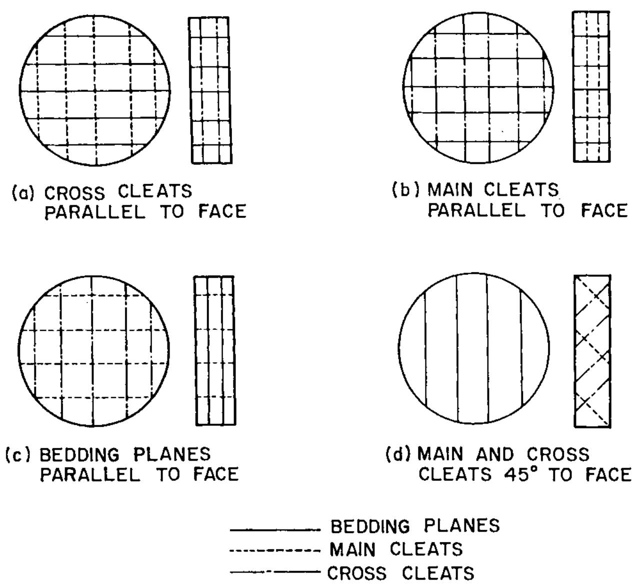

FIG. 4.3. Disc specimens for test on coal.

The discs used had a mean thickness of 0·31 ± 0·005 in. The mean diameter was 1·00 ± 0·005 in. The mean values of tensile strength given, accompanied by the standard error, are based on 20–25 specimens each.

Photographs of specimens broken by tensile stress are given in Fig. 4.4. The simplicity and delicacy of the disc method yields a reliable comparison, probably for the first time, between tensile strengths measured at various angles to the weakness planes. Graphs illustrating the results are given in Figs. 4.5 and 4.6. In each of the two coals the maximum variation in tensile strength with direction covers a range of about 3 to 1. Although absolute values of strength may alter from one lump of coal to another, the variation with direction seems to persist at strength ratios within the lump which are roughly unaltered.

Tensile strengths obtained by disc and indentation methods are not, in general, in agreement with those obtained by the bending of beams. For example, for Barnsley Hards, specimens obtained from the same block of coal gave:

Beam bending	620 ± 80 lb/in^2
Indentation method	345 ± 20 lb/in^2

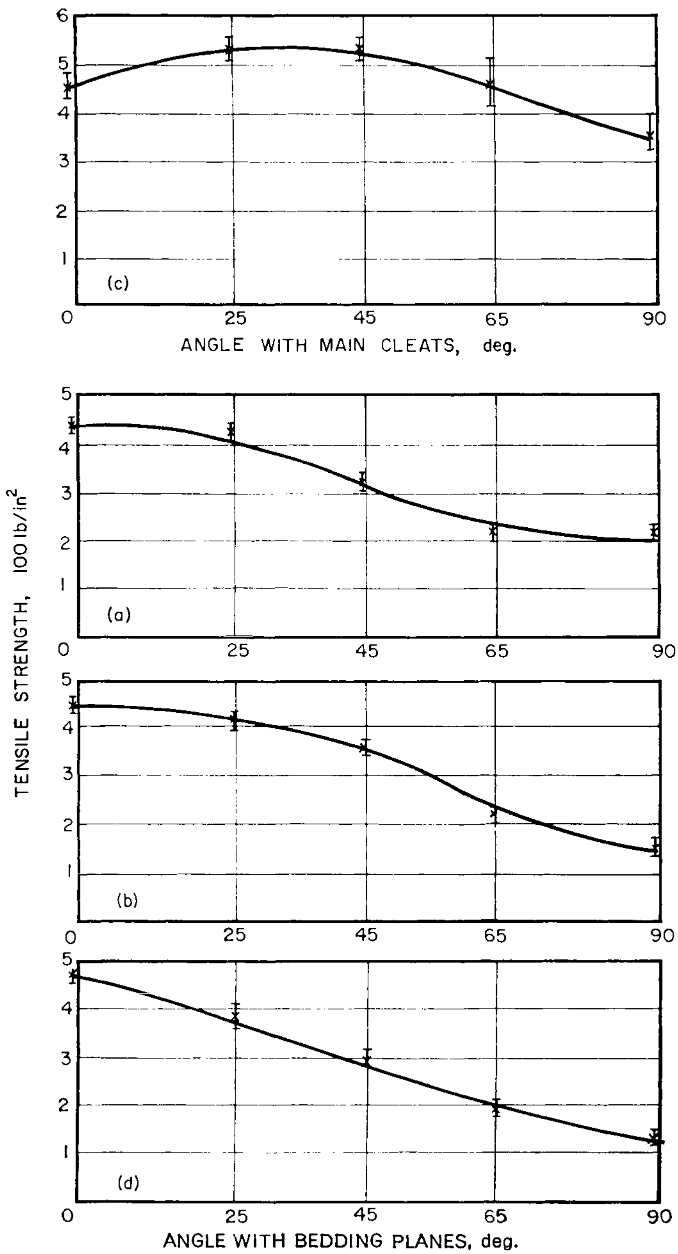

FIG. 4.5. Tensile strength results for Barnsley Hards.

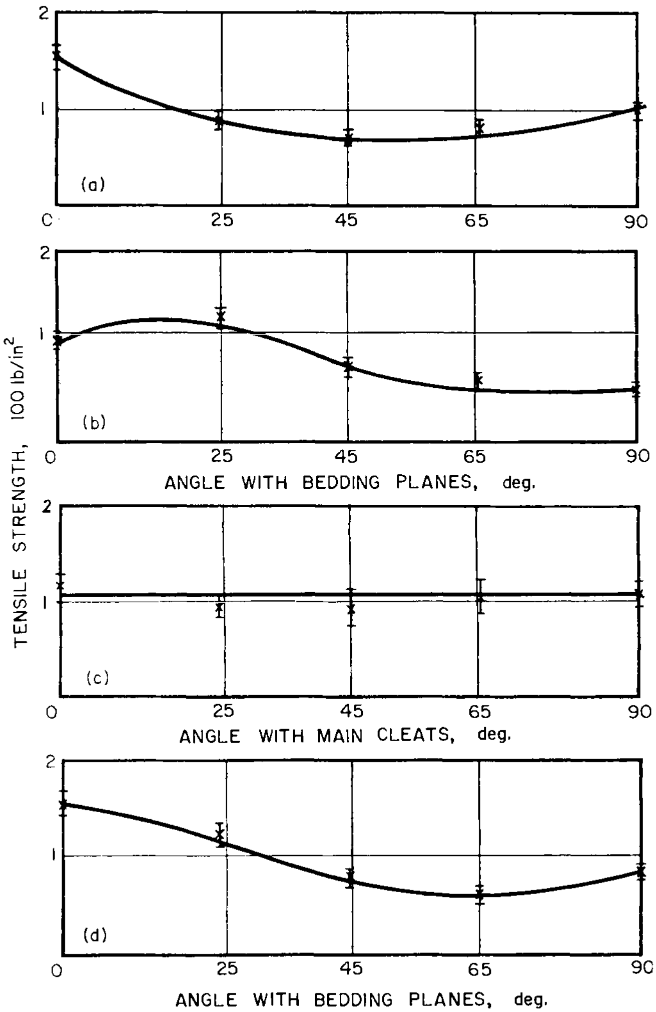

FIG. 4.6. Tensile strength results for Cwmtillery Garw coal.

(tensile strength measured in a direction parallel to bedding planes and perpendicular to main cleats in both cases).

Evidently the surface condition of the beam is affected in some way by the high speed grinding that is part of the preparation technique. It is quite possible that the temperature of the coal in contact with the grinding wheel is high enough to char or plasticize a very thin layer at the surface.

STRENGTH-SIZE EFFECT

It has been found that the compressive strength of cubes is dependent upon size, and this dependence has been traced to the intrinsic nature of coal as a material that contains many cracks and fissures. It could therefore be reasonably expected that tensile strength also would be dependent upon size, and this has been shown to be true in practice. The tests were carried out with discs of Barnsley Hards coal of orientation (a) of Tables 4.2 and 4.3 so that the tensile stress acted parallel to the bedding planes. The thickness of the

TABLE 4.2. TENSILE STRENGTH OF ROSSINGTON BARNSLEY HARDS (lb/in^2)

Inherent orientation	(a) (Cross cleats parallel to face)	(b) (Main cleats parallel to face)	(c) (Bedding planes parallel to face)	(d) (Both cleats at 45 deg to face)
Orientation of disc	(0, 90, 0)	(0, 0, 90)	(0, 0, 90)	(0, 45, 45)
Tensile strength	435 ± 12	443 ± 14	455 ± 27	474 ± 23
Orientation of disc	(25, 65, 0)	(25, 0, 65)	(0, 25, 65)	(25, 40, 40)
Tensile strength	414 ± 10	412 ± 21	533 ± 33	381 ± 23
Orientation of disc	(45, 45, 0)	(45, 0, 45)	(0, 45, 45)	(45, 30, 30)
Tensile strength	318 ± 16	355 ± 14	525 ± 25	293 ± 14
Orientation of disc	(65, 25, 0)	(65, 0, 25)	(0, 65, 25)	(65, 17½, 17½)
Tensile strength	209 ± 10	215 ± 16	457 ± 55	182 ± 16
Orientation of disc	(90, 0, 0)	(90, 0, 0)	(0, 90, 0)	(90, 0, 0)
Tensile strength	209 ± 12	144 ± 12	353 ± 35	125 ± 8

Table 4.3. TENSILE STRENGTH OF CWMTILLERY GARW COAL (lb/in^2)

Inherent orientation	(a) (Cross cleats parallel to face)	(b) (Main cleats parallel to face)	(c) (Bedding planes parallel to face)	(d) (Both cleats at 45 deg to face)
Orientation of disc	(0, 90, 0)	(0, 0, 90)	(0, 0, 90)	(0, 45, 45)
Tensile strength	152 ± 12	103 ± 8	115 ± 8	152 ± 14
Orientation of disc	(25, 65, 0)	(25, 0, 65)	(0, 25, 65)	(25, 40, 40)
Tensile strength	86 ± 8	127 ± 10	90 ± 9	121 ± 11
Orientation of disc	(45, 45, 0)	(45, 0, 45)	(0, 45, 45)	(45, 30, 30)
Tensile strength	68 ± 6	74 ± 9	90 ± 18	78 ± 8
Orientation of disc	(65, 25, 0)	(65, 0, 25)	(0, 65, 25)	(65, 17½, 17½)
Tensile strength	78 ± 7	59 ± 9	103 ± 16	59 ± 7
Orientation of disc	(90, 0, 0)	(90, 0, 0)	(0, 90, 0)	(90, 0, 0)
Tensile strength	96 ± 7	59 ± 8	105 ± 13	84 ± 6

discs varied between 0·31 and 1·92 in. It was found that tensile strength decreased with increasing specimen size according to the power law:

$$\text{tensile strength} \propto (\text{thickness})^{-0 \cdot 23 \pm 0 \cdot 03}$$

This exponent is not significantly different from that found for cubes of the same coal crushed perpendicular to the bedding plane. It has been suggested[1] that in a crushing test lateral tensile stresses are set up by the presence of cracks. These tensile stresses would, in the crushing tests just mentioned, act parallel to the bedding planes, and so the results may be compared with the present tensile results. The agreement of the exponents of size in the two cases may be taken to be another favourable indication of the validity of the suggestion.

An interesting facet of the dependence of strength upon size of specimen has been investigated by Pomeroy and Morgans[2]. A strip of coal which has been broken in the bending machine can be rejoined by a strong cement and resubmitted to test. Breakage takes place at the next most potent weakness, and by a succession of cementings the distribution of weaknesses in the specimen can be explored. Up to ten successive breakages were carried out on numbers of similar specimens, and it was shown that mean strength increased continually with successive breaks. It is physically impossible to carry out more than a few repetitions of this nature, but if it were feasible to carry out a large number there is no doubt that strength would continue to rise, for the smallest cracks in coal are of the order of a few Ångstrom units only in size, and the stress required to propagate them would be well above macroscopic tensile strength.

Tensile Strength of Representative Coals

In a laboratory investigation of the relation between ploughability and the mechanical properties of coal, Pomeroy and Foote[7] have measured the compressive and tensile strengths of samples of the M.R.E. representative coals. Their results are given in Table 4.4 and are again quoted in terms of mean and standard error.

The argument just used about the direction of the induced tensile stresses in a crushing test suggests that compressive strengths in direction (i) of the above Table should be compared with tensile strengths in directions (ii) and (iii). This is done in Fig. 4.7, the mean value of the two tensile strengths for each coal being employed. In the same way, compressive strength in direction (ii) is compared with tensile strength in direction (i). It can be seen that there is roughly a linear relation between the two sets of observations, and that compressive strength appears to be some 20–30 times as great as tensile strength.

TABLE 4.4

Colliery/seam	N.C.B. code no.	Volatiles (per cent d.a.f.)	Compressive strength lb/in^2		Tensile strength lb/in^2		
			(i)	(ii)	(i)	(ii)	(iii)
Pentremawr	100a	6	6630	6340	230	350	280
Pumpquart			±510	±300	±20	±20	±20
Deep Duffryn	201	12	2680	2110	100	140	100
Five Feet (Gellideg)			±80	±160	±10	±10	±10
Oakdale	301	22	1420	2110	70	110	90
Meadow			±90	±180	±10	±10	±10
Cwmtillery	301/	30	3110	1660	80	140	80
Garw	501		±90	±90	±10	±20	±10
Rossington	801	32	7320	4860	160	590	400
Barnsley Hards			±510	±200	±10	±30	±40
Teversal	7/802	35	5570	3950	160	450	330
Dunsil			±400	±210	±20	±40	±40
Markham	4/502	38	4950	4140	90	250	270
Blackshale			±350	±120	±10	±20	±20
Rossington	702	38	5450	3970	110	360	240
Barnsley Brights			±270	±130	±10	±30	±20
Linby	902	38	5570	4020	130	420	290
High Main			±320	±280	±10	±30	±30
			Stress applied: (i) Perpendicular. (ii) Parallel to bedding planes.		Stress applied: (i) Perpendicular to bedding planes. (90, 0, 0). (ii) Parallel to bedding planes and main cleat. (0, 0, 90). (iii) Parallel to bedding planes and perpendicular to main cleat. (0, 90, 0).		

CONSIDERATION OF RESULTS

The results of Berenbaum and Brodie show that tensile strength has anisotropic properties that are dependent upon the weakness system in the coal. The two authors, in their paper, mention that their results seem to be explicable in terms of the crack systems of Barnsley and Garw coals inferred by Terry and Morgans[(8)] from the results of examination by ultrasonic and other techniques, but they nevertheless still consider the problem of strength anisotropy to be complex, and to merit further study.

The next point of interest is the magnitude of tensile strength. Griffith's theory of the propagation of a crack was discussed in the earlier chapter on compressive strength: one of the derivations of that theory was that the

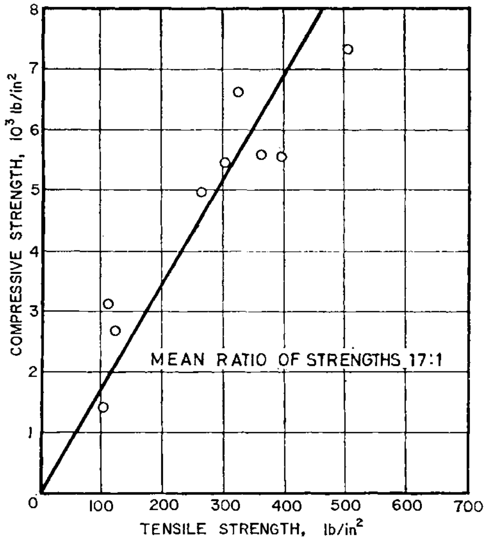

FIG. 4.7. Comparative results of compressive and tensile strengths of coal (after Pomeroy and Foote) (compressive strength ⊥ to bedding planes; tensile strength ∥ to bedding planes).

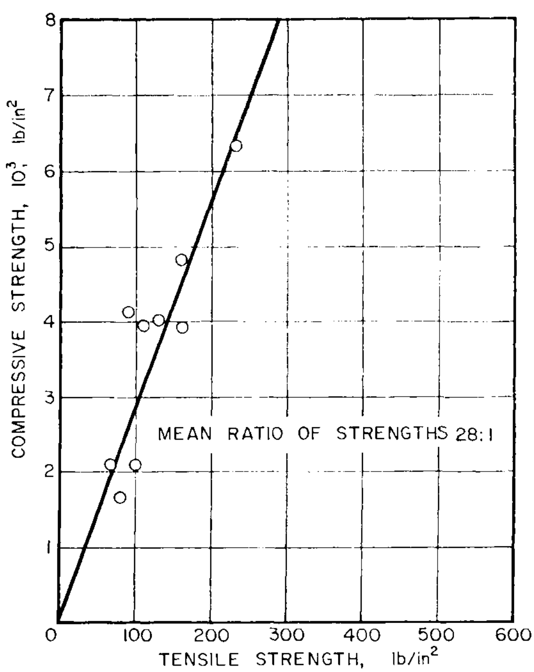

FIG. 4.8. Comparative results of compressive and tensile strengths of coal (after Pomeroy and Foote) (compressive strength ∥ to bedding planes; tensile strength ⊥ to bedding planes).

compressive force required to propagate a crack should be eight times the tensile force. As already noted, the ratio of compressive to tensile strengths for coal is greater than this, being in fact in the region of 20–30.

Several suggestions can be made to account for this discrepancy. In the first instance, Griffith's theory applies to a crack which is suitably orientated with respect to the applied compressive stress, the optimum orientation being oblique to the stress. If the nature of the material is such that these oblique cracks do not exist, then naturally they cannot be exploited, and those cracks which do propagate will require larger force. In coal crushed perpendicular to the bedding plane, for example, the cracks which are most liable to be exploited by the generated tensile stresses are the cleat cracks normal to the bedding, and so parallel to the applied stress. These cracks will require a higher force for propagation than would an oblique system.

Another possibility is this: if a serious weakness plane exists near and parallel to the edge of a specimen undergoing compressive test, the breakage which is generated by it may amount to no more than a surface spalling which would not be considered a complete disintegration. The applied load would be increased until a satisfactory breakage originating somewhere further into the interior of the specimen was generated. If, however, the specimen were used for a direct tensile test with the stress applied normal to the weakness plane, then the breakage there would be the definitive one from which tensile strength would be calculated. This effect would obviously be the more potent where the tensile stresses were applied perpendicular to the major weaknesses, thus accounting for the fact that the ratio of compressive to tensile strengths is greater in Fig. 4.8 than in Fig. 4.7.

Further, compressive strength measured between dry steel platens, as in the present case, is greater than that obtained when precautions are taken to reduce inter-facial friction to a small value, e.g. by suitable lubrication. The difference may amount to 30 per cent of "unlubricated" strength.

All in all, therefore, it is not surprising that the strength ratio is greater than the theoretical value of 8.

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CHAPTER 5

The Strength of Coal under Complex Stresses

IN THE work summarized in previous Chapters[1,2], the strength of coal in uniaxial compression and tension has been discussed in considerable detail. In practice, however, coal is only likely to be broken by such simple stress systems after it has been extracted from the seam. At the coal face itself and in the solid, unworked coal the stresses acting are much more complex.

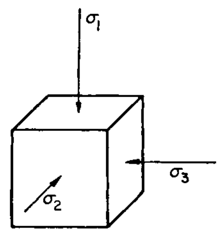


FIG. 5.1. Principal stresses acting on an elementary cube in a solid body.

Consider a small cube, situated well into the seam, subjected to stresses σ_1, σ_2 and σ_3 acting on the three pairs of faces as shown in Fig. 5.1. An orientation of cube exists in which all three stresses act normally to the faces of the cube, and in this case the stresses are known as principal stresses. If all of the principal stresses are equal, the material at the point under consideration is in a state of hydrostatic stress, but this state will not normally exist in the vicinity of mine workings. In fact, at the face itself the stress normal to the face is zero and the coal is said to be in a state of biaxial stress. At depths into the coal the three principal stresses are non-zero and usually of unequal magnitude and the coal is now in a state of triaxial stress. The stresses may all be tensile or compressive, or a combination of these. If two of the principal stresses are zero the coal is subjected either to pure tension or to pure compression. Measurement of the strength of coal under these ideal states has already been described[1,2].

It is virtually impossible to vary all three principal stresses independently and only the following combinations have been used. If the three principal stresses are σ_1, σ_2 and σ_3 and compressive stresses are negative, the systems studied are given by:

(i) biaxial compression, two of the stresses are negative and

$$|\sigma_1| \geqslant |\sigma_2|, \sigma_3 = 0$$

(ii) triaxial compression, all the stresses are negative and

$$|\sigma_1| > |\sigma_2| = |\sigma_3|$$

(If $|\sigma_1| = |\sigma_2| = |\sigma_3|$ the specimen is subjected to hydrostatic pressure.)

(iii) a complex stress system in which two of the stresses are negative (compressive) and the third is positive (tensile)

$$\sigma_1 > 0 > \sigma_2 \geqslant \sigma_3$$

Stress Systems Likely to be Experienced in Practice

Some measurements of the biaxial stresses acting in the coal face have been made by fixing measuring points on a coal face and recording the change in vertical and horizontal strain that takes place as the coal is destressed by a succession of cuts with a hand-held pneumatic chain saw. The cuts are made around the measuring points to a depth of about 12 in. The strain measurements cannot be related very accurately to the stresses acting on the coal, but they show that vertical and horizontal compressive stresses of the order of 2000 lb/in^2 can occur. This is the type of stress system represented by case (i) described in the third paragraph.

At points ahead of the face the exact stress distribution is unknown. It might be hydrostatic, but if circular symmetry is assumed, with the axis of symmetry vertical, it is likely that the coal is subjected to two equal compressive principal stresses parallel to the bedding planes and a third principal stress due to the loading from the overlying strata that is also compressive, but different in magnitude from the other two. This is the case (ii) described above.

A short distance in advance of the coal face the loads from the overlying strata frequently induce severe cracking in the coal, and cracks in brittle substances usually result from tensile stresses. The third system of stresses considered in the programme of tests (case (iii)) corresponds to this situation, at least approximately. In other words, the effect of orthogonal compressive stresses on tensile strength has been studied.

It can be seen, therefore, that a limited study of the strength of coal under complex stress systems should help in understanding what happens to coal at the face, behind the face and deep in the solid coal.

In addition to strength measurements some strain measurements were made for specimens subjected to the case (ii), triaxial compression, type of loading. This is the kind of fundamental information that can help strata control engineers to predict the effect of methods of working upon convergence and on the compaction of the seam as the face advances.

Although, at first sight, it might seem that biaxial strength tests are easier to do than triaxial ones, it will be shown that the converse is true and that the techniques used for the triaxial compression tests can be adapted to enable both the biaxial and the triaxial compressive/tensile (case (iii)) stress systems to be applied. The triaxial apparatus is therefore described first.

Triaxial Compressive Strength Apparatus

A cylindrical specimen of coal (2 in. long and 1 in. dia.) is contained in an oil-filled pressure vessel (Fig. 5.2)[3,4]. The specimen is placed between two steel end pieces and protected from the oil by an impermeable membrane. Hydraulic pressure is built up in the oil in the pressure chamber by loading

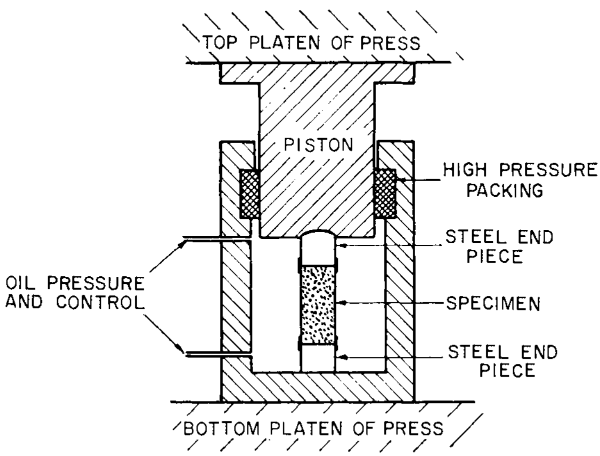


Fig. 5.2. Triaxial apparatus.

the piston, which is initially clear of the top end piece, and forcing it into the chamber. When the confining pressure reaches the predetermined working value it is kept constant by means of a needle valve. The piston is now allowed to come into contact with the top end piece and the axial load is increased until the specimen fractures. A spherical seating is used to ensure that the specimen is loaded axially. The onset of fracture is normally marked by the load remaining constant while the strain increases, although a sudden drop in load is sometimes observed.

The axial strain in the specimen is measured by recording the movement between the press platens with a dial gauge, a small correction being made for the strain induced in the apparatus as the load is applied. The specimens are usually loaded in a 10 ton or 100 ton Avery Universal testing machine at a rate of about 100 lb/sec. The frictional force required to push the piston past the high-pressure packing is measured at the start of each experiment and it is assumed that this remains constant throughout the experiment.

Preparation of Specimens for Triaxial Compression Tests

Experiments have been made on a range of nine British coals that include anthracite at one extreme, friable middle rank coals, and dull and relatively crack-free bituminous coals of low rank. One-inch diameter cores of the coals were cut with a water-flushed diamond coring bit and these cores were sliced into 2 in. lengths with a high-speed carborundum slitting wheel. For most of the coals studied samples were prepared with the three different orientations of bedding and cleat planes described in Table 5.1. About eight specimens of each orientation were tested at a number of different confining pressures.

TABLE 5.1. ORIENTATION OF WEAKNESS PLANES IN TRIAXIAL SPECIMENS

Plane of weakness	Orientation to specimen axis		
	1	2	3
Bedding	Parallel	Parallel	Perpendicular
Main cleat	Parallel	Perpendicular	Parallel
Cross cleat	Perpendicular	Parallel	Parallel

These three orientations can only be prepared if the bedding planes and the two dominant families of cleat planes are mutually orthogonal. This is not always true and, for some coals, particularly Linby High Main, different orientations had to be used.

For most coals, specimens could be prepared with a success rate of at least 70 per cent but some of the more heavily cracked coals could not be handled so successfully and the specimens probably represent only the strongest parts of the coal. Fortunately some uni-axial strength tests on specimens that were visually selected as strong and weak suggest that the sampling errors introduced are small.

Triaxial Compressive Strength Measurements

A typical set of stress–strain curves for specimens of Deep Duffryn, Five Feet, coal is given in Fig. 5.3. The strain measured is the differential axial strain, since before the tests started the specimens were subjected to a

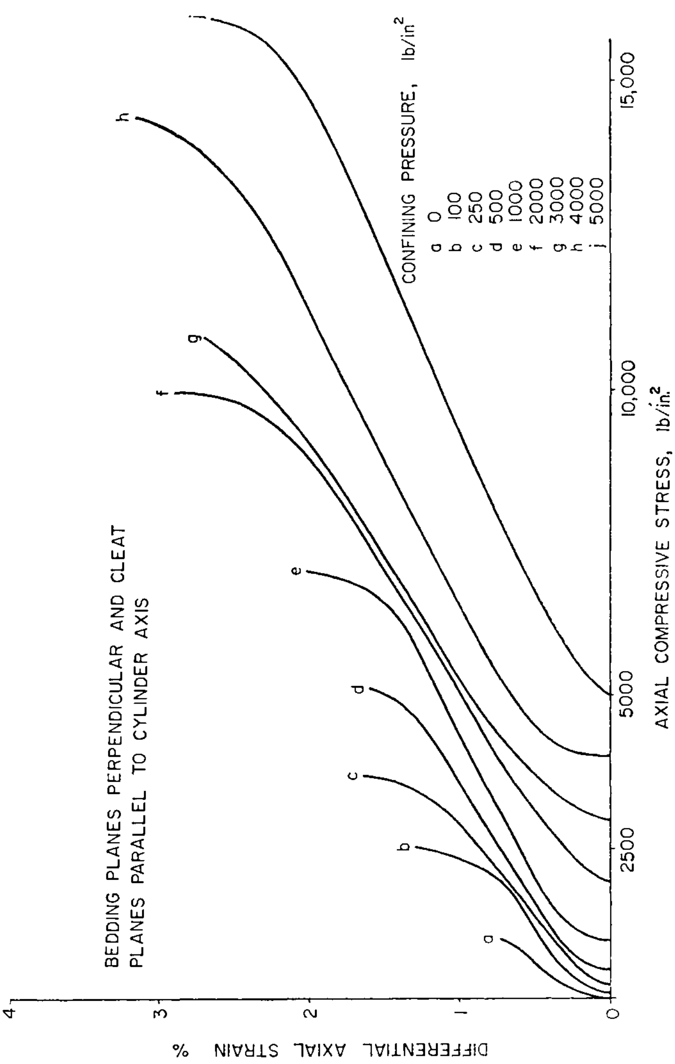


FIG. 5.3. Stress–strain curves for Deep Duffryn coal.

hydrostatic pressure and a strain measurement corresponding to the unstressed length of specimen could not be made.

The curves may be interpreted in three phases:

(i) An initial non-linear portion caused by elastic deformation of the basic coal material and the closing of the gross cracks in the coal.

(ii) A range of elastic linearity, from which Young's modulus can be determined.

(iii) A final non-linear portion which is attributed to pre-rupture cracking and plastic flow.

These three phases are not always evident and, in particular, at low confining pressures the linear component is missing for the friable middle rank coals (see curve (a) of Fig. 5.3). For these exceptions Young's modulus is determined at the point of inflexion on the stress–strain curve.

Some specimens, particularly of anthracite, are elastic up to failure and no final non-linear part of the curve is observed.

Yield and Fracture Stresses

The strengths of the cylinders were measured in two ways, as the axial fracture stress, which is the maximum stress sustained, and as the axial yield stress, which is the stress corresponding to the upper limit of proportionality on the stress–strain curves.

Both the yield and the fracture stresses increase with confining pressure for all coals, but the manner of increase varies from one coal to another. Some typical results are shown in Fig. 5.4, each point representing the average of at least seven tests. Results for other specimen orientations are similar. The coefficient of variation of the yield or fracture stress decreased with increasing confining pressure, even though the mean strength increased while the sensitivity of the method decreased.

The differential stresses at failure (the yield or fracture stress minus the confining pressure) also increase with confining pressure, although for the Linby coal the differential stress at the yield point increases only slightly at confining pressures above 1000 lb/in^2.

A graph, for one particular specimen orientation (3), of the failure stress at atmospheric pressure and at a confining pressure of 5000 lb/in^2 plotted against percentage volatiles shows (Fig. 5.5) that there is evidence of a U-shaped relationship for the fracture stress, whereas for low-rank coals the yield stress tends to be independent of percentage volatiles at the higher confining pressure. It is interesting to note that at atmospheric pressure there is a 14 : 1 ratio between the strongest and the weakest coals while at 5000 lb/in^2 confining pressure the ratio is only 2 : 1. This latter ratio is reduced to about 1·25 : 1 if the results for anthracite are excluded. In other words, the strengths of almost all coals are virtually indistinguishable at high confining pressures.

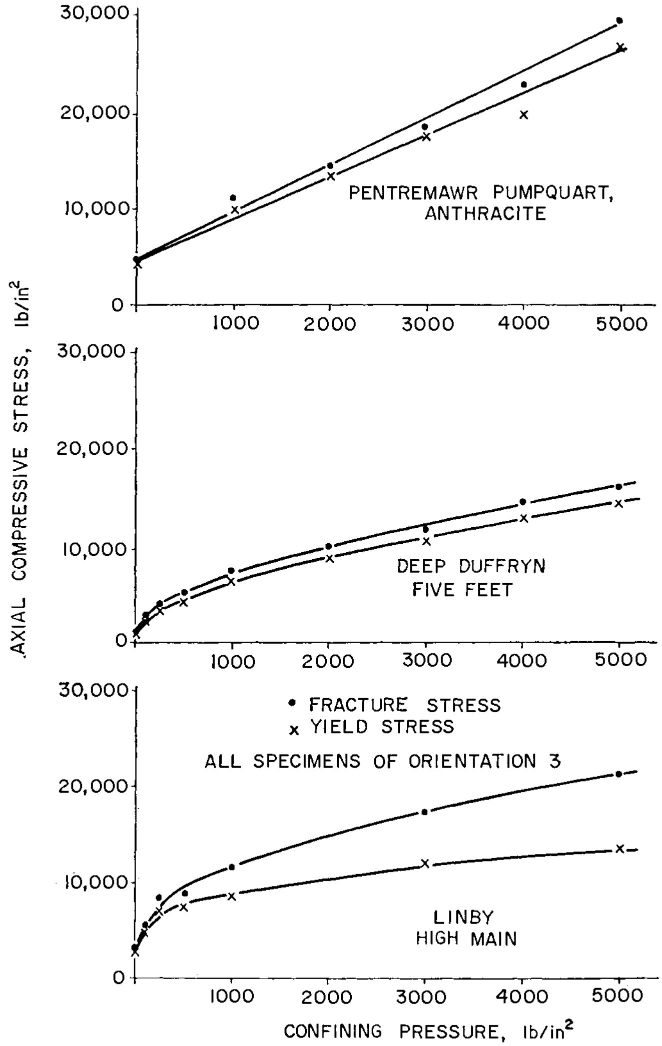


FIG. 5.4. Variation of axial compressive yield and fracture stress with confining pressure.

Strength anisotropy

In general the strength of the coals is independent of the direction of loading, except at the lowest confining pressures used. A good example of this is shown in Fig. 5.6, which shows that cylinders loaded perpendicular to the bedding planes are stronger than those of other orientations at confining pressures below 2000 lb/in^2, but that at higher pressures the strength is independent of orientation.

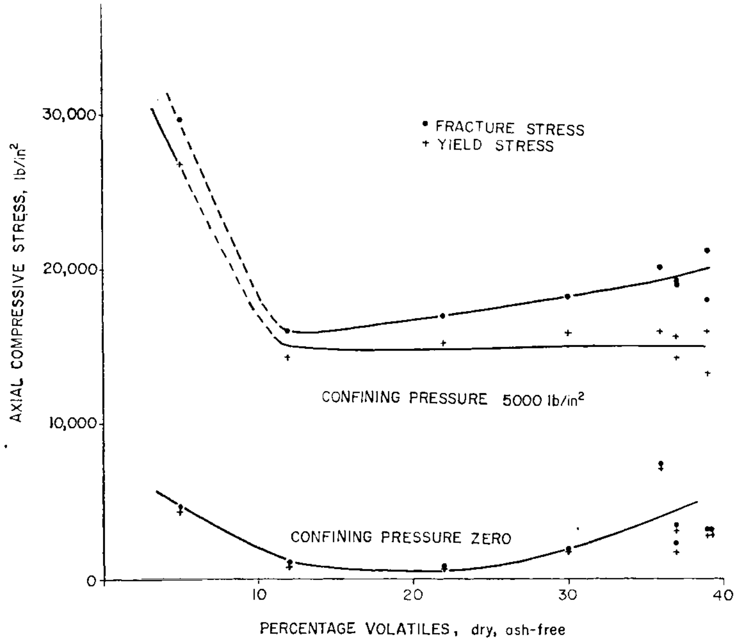


FIG. 5.5. Variation of axial compressive yield and fracture stress with coal rank (percentage volatiles).

There were exceptions; in particular the cylinders of Pentremawr anthracite loaded perpendicular to the bedding planes were stronger at all confining pressures than those loaded parallel to the bedding planes.

Mode of Fracture

The way in which the cylinders broke depended upon the type of the coal' the orientation of the weakness planes and the confining pressure. The friable middle-rank coals tended to fracture in a docile manner along one or more, roughly parallel, irregular planes, inclined to the axis of the specimen. A few specimens of these coals failed along irregular conical surfaces, and when on occasion more than one fracture plane occurred the planes intersected to form a wedge which split the cylinder by transverse tension. The fracture planes were obviously affected by the weakness planes since the intersecting normals to the fracture and weakness planes lay in planes parallel to the direction of axial loading.

At atmospheric pressure the less friable coals fractured along a number of small-angled intersecting wedges when the cylinders were loaded parallel

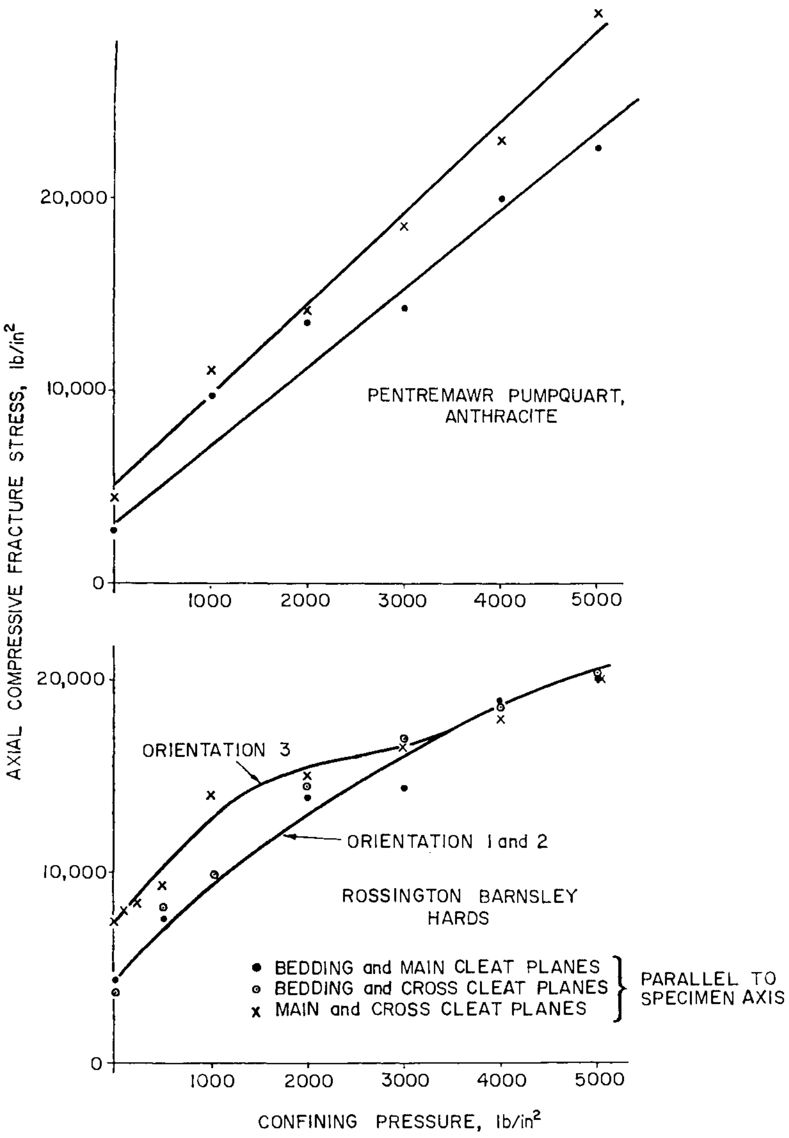


FIG. 5.6. Variation of axial compressive fracture stress with confining pressure for different specimen orientations.

to the bedding planes. For other specimen orientations and at all confining pressures the friable specimens either fractured along a single plane or cone, or shattered. Where a fracture surface was observed it was influenced by the weakness planes in the manner described above.

Angles of fracture

The angle between the normal to the fracture plane and a horizontal plane was measured whenever possible. This increased from about 10 to 15 deg at atmospheric pressure to 20–30 deg at 5000 lb/in^2 confining pressure, the angles of fracture generally being larger for specimens with the bedding planes perpendicular to the axis of the specimens.

Strain Measurements

As already explained, the stress-strain curve has three components, the first largely representing the closing of the gross cracks, the second linear portion representing the elastic component of strain, and the third the pre-rupture cracking and flow that occurs. The Young's modulus, derived from the middle linear portion (or from the point of inflexion), varies from one coal to another and increases with increasing confining pressure except for Barnsley Hards, for which the highest moduli recorded occur for the unconfined specimens. Some values of Young's modulus for the nine coals tested are given in Table 5.2, a more complete table being published elsewhere[5].

It can be seen that the variation in Young's modulus between coals is much greater when the specimens are unconfined than when subjected to confining pressures even as low as 500 lb/in^2. At the highest confining pressures used (5000 lb/in^2) there is very little difference between the different coals, nor is there any evidence of anisotropy, the small differences in modulus observed for the different directions of loading being statistically insignificant. This again emphasizes that all coals are virtually indistinguishable when confined in the solid and it is only when the coal is close to the face or has been extracted that large differences in behaviour are likely to be observed.

As can be seen from Table 5.3, the linear or elastic component of strain increases with confining pressure, the percentage elastic strain being about three times as large at 5000 lb/in^2 as when the cylinders are unconfined. There are no consistent directional effects for the various coals tested and Table 5.3 shows the mean variation of elastic strain with confining pressure for all three orientations tested. The friable South Wales coals (coals 2–4 in Table 5.3) have a slightly smaller elastic range than the other coals, at all confining pressures.

TABLE 5.2. PERCENTAGE VOLATILES AND YOUNG'S MODULUS E (10^5 lb/in^2)

Orientation of weakness planes to specimen axis	Confining pressure (lb/in^2)	Colliery and seam								
		Pentremawr Pumpquart	Deep Duffryn Five Feet	Oakdale Meadow	Cwmtillery Garw	Rossington Barnsley Hards	Rossington Barnsley Brights	Teversal Dunsil	Markham Blackshale	Linby High Main
	†	6	12	20	25	32	38	35	38	38
	‡	100a	201	301	3/501	801	702	7/802	4/502	902
Bedding planes parallel, main cleat planes parallel	0	5·3 ± 0·8	1·9 ± 0·3	0·6 ± 0·2	2·0 ± 0·3	5·5 ± 0·1	3·8 ± 0·2	2·4 ± 0·1	4·4 ± 0·9	2·0 ± 0·4
	500	—	5·0 ± 0·3	3·2 ± 0·4	4·7 ± 0·8	4·4 ± 0·3	4·7 ± 0·2	4·0 ± 0·2	5·4 ± 0·3	3·4 ± 0·1
	1000	6·2 ± 0·3	5·2 ± 0·5	3·2 ± 0·7	4·0 ± 0·3	4·4 ± 0·3	5·5 ± 0·4	4·3 ± 0·1	6·1 ± 0·2	3·7 ± 0·2
	5000	6·1 ± 0·5	6·5 ± 0·3	5·4 ± 0·3	6·6 ± 0·2	4·1 ± 0·2	5·1 ± 0·2	4·1 ± 0·2	6·3 ± 0·1	3·7 ± 0·1
Bedding planes parallel, main cleat planes perpendicular	0	—	3·8 ± 0·7	1·2 ± 0·2	3·2 ± 0·7	5·3 ± 0·2	3·4 ± 0·2	3·2 ± 0·2	—	3·5 ± 0·2
	500	—	4·9 ± 0·4	2·6 ± 0·4	4·6 ± 0·2	4·7 ± 0·2	4·4 ± 0·1	3·8 ± 0·2	—	4·4 ± 0·2
	1000	—	5·5 ± 0·6	3·8 ± 0·5	5·5 ± 0·8	4·6 ± 0·3	5·0 ± 0·1	3·9 ± 0·3	—	4·1 ± 0·1
	5000	—	6·4 ± 0·4	5·7 ± 0·2	5·8 ± 0·3	4·4 ± 0·2	4·8 ± 0·1	3·7 ± 0·3	—	3·7 ± 0·2
Bedding planes perpendicular, cleat planes parallel	0	3·9 ± 0·4	2·5 ± 0·1	1·7 ± 0·3	3·5 ± 0·7	4·2 ± 0·2	3·3 ± 0·3	1·7 ± 0·3	3·5 ± 0·3	4·7 ± 0·8
	500	—	3·8 ± 0·4	5·3 ± 0·5	4·4 ± 0·5	4·4 ± 0·4	4·2 ± 0·4	3·2 ± 0·4	5·0 ± 0·2	3·7 ± 0·1
	1000	5·9 ± 0·2	5·2 ± 0·3	4·2 ± 0·5	5·9 ± 0·5	4·9 ± 0·2	5·8 ± 0·4	3·8 ± 0·2	4·5 ± 0·5	3·8 ± 0·3
	5000	5·8 ± 0·2	5·7 ± 0·2	6·5 ± 0·2	6·0 ± 0·4	4·6 ± 0·1	5·0 ± 0·1	3·7 ± 0·2	5·2 ± 0·1	3·8 ± 0·2

† % Volatiles. ‡ NCB Code No.

TABLE 5.3. ELASTIC COMPONENT OF STRAIN (%)

Confining pressure (lb/in²)	Colliery and seam								
	Pentremawr Pumpquart	Deep Duffryn Five Feet	Oakdale Meadow	Cwmtillery Garw	Rossington Barnsley Hards	Rossington Barnsley Brights	Teversal Dunsil	Markham Blackshale	Linby High Main
	1	2	3	4	5	6	7	8	9
9	0·77	0·42	0·60	0·51	1·01	0·81	0·92	0·65	0·62
100	—	0·56	0·86	0·82	1·64	1·65	1·26	0·99	1·48
250	—	0·99	0·97	1·04	1·71	1·53	1·30	1·44	1·61
500	—	0·92	0·93	1·17	1·64	1·61	1·66	1·34	1·39
1000	1·54	0·93	1·04	1·10	1·89	1·56	1·88	1·42	1·70
2000	2·00	1·21	1·14	—	2·01	—	—	1·50	—
3000	2·08	1·49	1·41	1·60	2·25	2·22	2·53	1·81	1·88
4000	2·33	1·46	1·34	—	2·42	—	—	1·71	—
5000	3·08	1·46	1·57	1·76	2·54	2·20	2·60	1·92	2·01

Results for all three orientations combined.

Strain attributed to closure of cracks

The percentage strain attributed to the closure of cracks was almost independent of the confining pressure, and mean values only for each specimen orientation are given for the nine coals in Table 5.4.

Axial compression perpendicular to the bedding planes caused a larger component of strain attributable to cracks closing for most of the coals, particularly those of low rank (coals 5–9 in Table 5.4). This is in accordance with findings by Terry[6] that coal contains flat disc-shaped cracks orientated parallel to the bedding planes, and that these are closed by the initial increments of axial load.

It is also interesting to note that the strain attributed to crack closure is greatest for anthracite, Barnsley Brights, and Teversal Dunsil coal, whereas the most heavily cracked coals appear visually to be the friable South Wales coals such as Oakdale Meadow.

Strain attributed to yielding

The strain component attributed to yielding increases as the confining pressure is increased for all of the coals tested. Apart from Linby High Main, Barnsley Hards, and Teversal Dunsil coals, all of which showed marked flow before fracture, the other coals were indistinguishable and showed little anisotropy. With the exception of these three coals the percentage strain for yielding increased from about 0·2 to 0·3 at zero confining pressure to about 1·0 at 5000 lb/in^2. The Linby coal yielded as much as 5 per cent when the cylinders were subjected to a confining pressure of 5000 lb/in^2.

Discussion of strain measurements

The surprising feature of the strain measurements is the similarity between different coals, particularly in Young's modulus and in the strain attributed to crack closure. Some of the coals, however, showed marked flow before fracture at high confining pressures and these were all high-volatile dull coals.

The effect of strata loading on coals that yield or flow before fracture is likely to be very different from that on the more "elastic" coals. If the coal can flow and so absorb and dilute the strata stresses then it may become dead and "woody", and difficult to work. In other coals a build-up in strata load will leave the coal either heavily stressed or fractured. The aim from the mining standpoint must depend on local conditions, but if the face coal can be fractured without risk of roof damage the extraction will be much easier. From the strain measurements it seems that coals 1, 2, 3, 4, 6, and 8 of Tables 5.3 and 5.4 are those most likely to be broken by strata loading and to some extent this is borne out in practice.

TABLE 5.4. PERCENTAGE STRAIN ATTRIBUTED TO CLOSURE OF CRACKS

Orientation of weakness planes to specimen axis	Colliery and seam								
	Pentremawr Pumpquart	Deep Duffryn Five Feet	Oakdale Meadow	Cwmtillery Garw	Rossington Barnsley Hards	Rossington Barnsley Brights	Teversal Dunsil	Markham Blackshale	Linby High Main
	1	2	3	4	5	6	7	8	9
Bedding planes parallel, main cleat planes parallel	0·83	0·33	0·32	0·47	0·49	0·57	0·55	0·40	0·40
Bedding planes parallel, main cleat planes perpendicular	—	0·47	0·42	0·45	0·70	0·63	0·55	—	0·37
Bedding planes perpendicular, cleat planes parallel	0·88	0·37	0·43	0·42	0·66	0·76	0·74	0·62	0·46

Biaxial Compressive Strength Measurements

Two experimental techniques were used to measure "biaxial strengths". The first used the apparatus already described, but the specimen was a hollow cylinder instead of a solid one and the procedure adopted when making a test was modified. The second method employed different equipment which is described later.

At this stage, however, it is necessary to consider the stresses in a hollow cylinder subjected simultaneously to an external hydrostatic pressure and an axial loading so that predictions can be made of the expected modes of failure of the specimens. These were later verified by the experiments.

The Stresses in a Hollow Cylinder Subjected to External Pressure and Axial Loading

Consider a thick-walled hollow cylinder, closed at both ends, subjected to an external hydrostatic pressure σ_H and a zero internal pressure. The stress components, using conventional cylindrical coordinates (r, θ, z) are[7]

$$\sigma_r = \frac{\sigma_H r_2^2}{r_2^2 - r_1^2}\left(1 - \frac{r_1^2}{r^2}\right) \tag{5.1}$$

$$\sigma_\theta = \frac{\sigma_H r_2^2}{r_2^2 - r_1^2}\left(1 + \frac{r_1^2}{r^2}\right) \tag{5.2}$$

and, since the forces on the ends of the cylinder must balance,

$$\sigma_H \pi r_2^2 - \sigma_z \pi (r_2^2 - r_1^2) = 0$$

or

$$\sigma_z = \frac{\sigma_H r_2^2}{r_2^2 - r_1^2} \tag{5.3}$$

where r_2 is the external radius and r_1 the internal radius. It can be seen that the stress distribution is independent of the elastic constants.

The compressive stress σ_θ is a minimum on the external surface ($r = r_2$) and increases to $\sigma_{\theta_{\max}} = 2\sigma_H r_2^2/(r_2^2 - r_1^2)$ on the internal surface ($r = r_1$). The radial stress σ_r has its maximum (compressive) value on the external surface and decreases to zero on the internal surface. It follows from this that the maximum shear stress occurs on the internal surface, in a direction at 45 deg to the maximum and minimum principal stresses and is given by

$$\tau_{\max} = \frac{\sigma_H r_2^2}{r_2^2 - r_1^2} \tag{5.4}$$

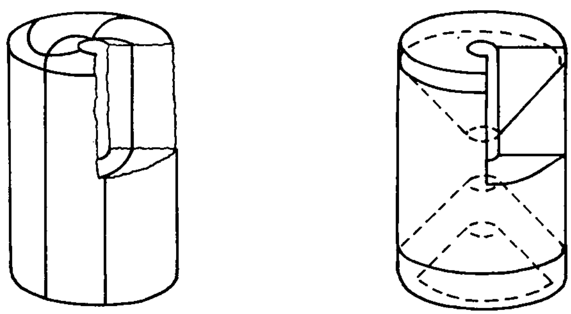
The tangential-radial maximum shear stress surfaces are shown in Fig. 5.7(a).

If an additional axial load Q_a is applied to the hollow cylinder

$$\sigma_z = \frac{\sigma_H r_2^2}{r_2^2 - r_1^2} + \frac{Q_a}{\pi(r_2^2 - r_1^2)} \tag{5.5}$$

and if $Q_a > \sigma_H \pi r_2^2$, then $\sigma_z > \sigma_\theta > \sigma_r$. The planes of maximum shear stress for this system are shown in Fig. 5.7(b).

It must be borne in mind that these formulae only apply to homogeneous, elastic, and isotropic bodies subjected to rigorous boundary conditions, but



(a) TANGENTIAL-RADIAL SHEAR SURFACES $\sigma_\theta > \sigma_z > \sigma_r$

(b) AXIAL-RADIAL SHEAR SURFACES $\sigma_z > \sigma_\theta > \sigma_r$

FIG. 5.7. Maximum shear stress surfaces.

Brodie and Berenbaum[8] have shown that coal fractures are frequently compatible with such simple elasticity theory.

A more complex analysis of the stresses in a hollow cylinder has been given by Jeffery[9], who has considered the effect of non-symmetry between the inner and outer surfaces of the cylinder. Quite large errors in symmetry have marginal effects on the maximum stresses in the cylinders. The material from which the end pieces are made can affect the tendency of the cylinders to barrel and so influence the radial and tangential stresses, but this can be overcome to a large degree by using end pieces with elastic properties similar to those of coal.

Experimental Procedure

The failure of hollow cylinders of coal subjected to an external hydrostatic pressure.

The hollow cylinder of coal was placed in the triaxial apparatus between two steel end pieces and protected from the oil by rubber sleeves which gripped on the end pieces. Hydraulic pressure was increased in the pressure chamber by advance of the steel piston until fracture occurred as evidenced by a loud crack and a sudden fall in pressure. The expressions for the stresses at failure are given by eqns. (5.1), (5.2), and (5.3), where $r = r_1$.

The failure of hollow cylinders subjected to an external hydrostatic pressure and an additional axial load.

The technique adopted for this system is identical with that used for the orthodox triaxial tests described under Triaxial Compressive Strength Apparatus. The expressions for the stresses at failure are now given by equations (5.1), (5.2), and (5.5) where $r = r_1$.

Application of uniformly distributed compressive stresses

The stresses applied to the hollow cylinders vary continuously throughout the walls, with maximum shear stresses occurring on the internal surface. The location of the onset of fracture is therefore localized so that the influence of the cracks and flaws that occur naturally in coal is not likely to be dominant.

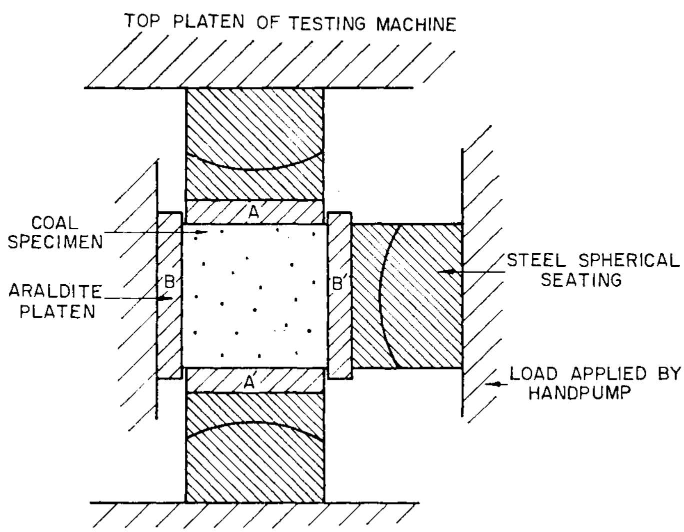


FIG. 5.8. Apparatus for fracturing cubes of coal under biaxial compression.

An alternative technique was therefore used in which cubes of coal were subjected simultaneously to two approximately uniform, but unequal, compressive loads between two pairs of faces.

A diagram of the apparatus is shown in Fig. 5.8. Three spherical seatings were incorporated in the apparatus to align the four Araldite D compression

TABLE 5.5. BIAXIAL STRENGTHS OF HOLLOW CYLINDERS

	Internal dia. (in.)	External dia. (in.)	Coal			
			Barnsley Hards		Cwmtillery Garw	
			Tangential stress, lb/in²	Axial stress, lb/in²	Tangential stress, lb/in²	Axial stress, lb/in²
			Intermediate	Fracture	Intermediate	Fracture
Circumferential stress maintained constant whilst axial load increased to fracture	0·66	1·64	0	2770 ± 190	0	790 ± 100
	0·66	1·64	590	7130 ± 290	240	2220 ± 70
	0·66	1·64	1190	8070 ± 370	600	3610 ± 70
	0·66	1·64	2390	9130 ± 690	1200	6010 ± 280
	0·66	1·64	4760	10,180 ± 590	2390	5840 ± 340
	0·66	1·64	7160	10,820 ± 880	4790	8240 ± 360
	0·66	1·64	9550	10,520 ± 410	7180	7810 ± 360
			Fracture	Intermediate	Fracture	Intermediate
Hydrostatic pressure increased until specimens collapsed	0·66	1·64	11690 ± 330	5820 ± 160	9420 ± 320	4690 ± 190
	0·76	1·65	10430 ± 570	5180 ± 290	—	—
	0·50	1·00	10050 ± 300	4930 ± 150	—	—
	0·65	1·00	5430 ± 480	2630 ± 280	—	—
	1·26	1·63	4940 ± 1350	2280 ± 600	—	—

platens and correct for slight lack of parallelism between the specimen faces. By definition the lower of the compressive stresses applied is known as the intermediate stress (since the third pair of faces are unloaded and so have a zero normal stress) and this was applied and maintained constant while the larger compressive stress was built up until the specimens fractured.

Experimental Results

Hollow cylinders

Measurements using both hollow cylinders and cubes have been made for Rossington, Barnsley Hards, and for Cwmtillery Garw coal. The hollow cylinders were cut with the bedding and main cleat planes parallel to the axis of the specimen.

The cylinders were fractured in one of two ways: either the circumferential stress was maintained constant and the axial load increased to failure, or the cylinder was subjected to an increasing hydrostatic pressure until it collapsed. In the first case the axial stress σ_z is the fracture stress and the tangential stress σ_θ is the intermediate principal stress and in the second the situation is reversed and the tangential stress is the fracture stress. The results are summarized in Table 5.5. Three specimens of Barnsley Hards and five of the Garw coal were tested at each value of intermediate stress.

The fracture surfaces of the specimens broken by hydrostatic pressure alone tended to follow the tangential radial maximum shear stress surfaces (Fig. 5.7(a)). Those for the cylinders broken by the combined circumferential stress and additional axial load followed the axial-radial maximum shear surfaces (Fig. 5.7(b)), although for low values of the intermediate stress the influence of the inherent weakness planes resulted in fracture along planes inclined to the specimen axis. The table shows clearly that the strength of the coals is affected by the intermediate stress, large increases in fracture stress occurring for quite small increases in the intermediate stress. This shows how coals can become extremely difficult to win if strata loading is incorrectly applied.

Cubes

Cubes of 1·5 in. were loaded biaxially in the manner described above, the compressive stresses applied being much more uniform than in the hollow cylinders. In all of the tests, using Barnsley Hards and Garw coals as before, the maximum compressive stress was applied parallel to the bedding planes and the intermediate stress was applied either parallel or perpendicular to the bedding planes. The results are given in Table 5.6.

TABLE 5.6. BIAXIAL STRENGTHS OF CUBES

Intermediate stress (lb/in²)	Coal			
	Barnsley Hards		Cwmtillery Garw	
	Fracture stress (lb/in²)		Fracture stress (lb/in²)	
	A	B	A	B
0	2720±550	2720±550	1730±160	1730±160
900	4190±380	8060±360	2590±200	3020±100
1900			3100±210	3490±230
2900	4830±240	7650±580	2640±230*	2490±160*
5000	3850±330*	6280±200		

Four specimens of the Barnsley Hards and five of the Garw coal were tested at each intermediate stress. The results given in column A refer to specimens for which the intermediate stress was applied parallel to the bedding planes, and those in column B when the intermediate stress was applied perpendicular to the bedding planes.

In the experiments it was not possible to increase both compressive loads simultaneously until the intermediate stress reached the predetermined constant value, and then increase the fracture stress until failure occurred. Instead the intermediate stress was built up more rapidly than the fracture stress, with the result that in some instances (indicated by asterisks in Table 5.6) the specimens failed before the fracture stress reached the "intermediate" stress. This means that the intermediate stress is in fact the fracture stress for these exceptional specimens.

The results show a smaller dependence of the fracture stress on the intermediate stress than do those for the hollow cylinders. They also suggest a decrease in the fracture stress at the higher values of intermediate stress used. There is also evidence that the specimens of Barnsley Hards, in particular, are stronger when the intermediate stress is applied perpendicular rather than parallel to the bedding planes, which is in accordance with uniaxial compressive strength measurements[10].

Comparison between results for hollow cylinders and cubes

The apparent contradiction between the influence of the intermediate stress on the fracture stress for hollow cylinders and cubes may be explained by a number of factors. The most important is that the stresses applied to the cubes were fairly uniform throughout the specimens, which means fracture can be initiated at any point of weakness, whereas for the hollow cylinders a stress concentration is imposed on the inner surface where the probability of the most damaging weakness and the peak stresses coinciding is obviously low. The hollow cylinders will therefore appear stronger than the cubes.

There are other factors that affect strength measurements, such as the modification of the surface of specimens during preparation, the difficulty in applying loads to specimens without frictional effects introducing unknown boundary stresses and the orientation of weakness planes relative to the principal stresses. It is therefore not surprising that the results for the two methods of biaxial stressing are not completely compatible.

Strength Anisotropy

Hollow cylinders of Barnsley Hards and Garw coal were cut with the bedding and cleat planes orientated in the directions given in Table 5.1. Some of these were fractured by hydrostatic pressure and the remainder by the combined circumferential and axial stress system. As can be seen from Table 5.7 the strengths of the cylinders collapsed by a hydrostatic pressure are dependent on weakness plane orientation, the cylinders with the bedding planes perpendicular to the axis being the strongest, in accordance with the findings from the triaxial strength tests at low confining pressures. The

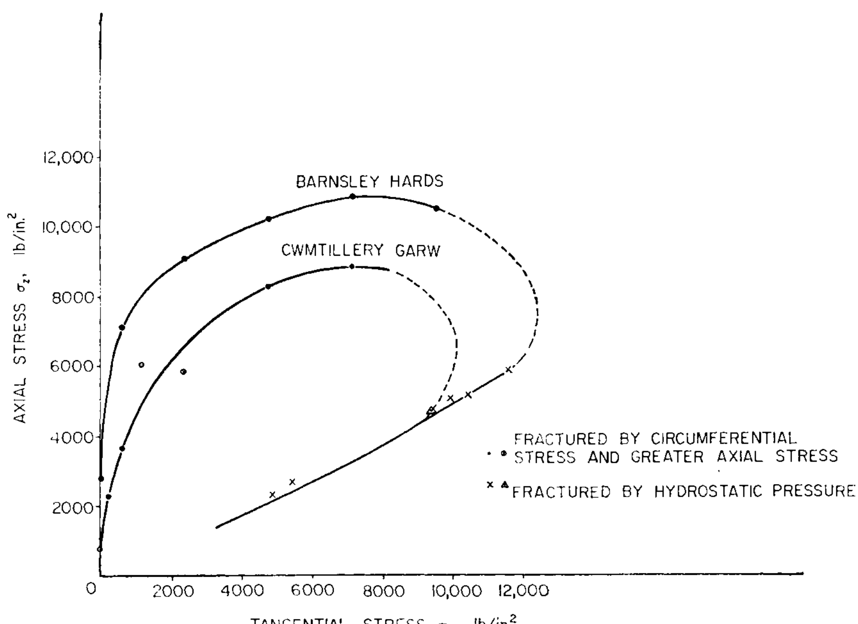


FIG. 5.9. Relationship between axial and tangential stresses at the fracture of hollow cylinders of Barnsley Hards and Garw coal.

specimens fractured by a combination of a circumferential and an axial stress show no dependence on weakness plane orientation.

Further evidence of the dependence of strength upon weakness plane alignment is shown in Fig. 5.9 in which the magnitudes of corresponding biaxial compressive stresses at failure are compared. The cylinders represented on this graph were all cut with both bedding and main cleat planes parallel to the axis of the specimen. It can be seen that if the intermediate stress is axial and is 2000 lb/in^2 the tangential stress at fracture is about 4000 lb/in^2, whereas if the intermediate stress is tangential and also 2000 lb/in^2 the axial stress at fracture is nearly 9000 lb/in^2. However, at higher values of intermediate stress the maximum compressive stress at fracture is virtually independent of the method of stressing the cylinders, which once again suggests that strength anisotropy is only dominant at low restraining pressures, as was observed in the triaxial tests.

TABLE 5.7. THE EFFECT OF WEAKNESS PLANE ORIENTATION ON BIAXIAL STRENGTH MEASUREMENTS

	Cylinder orientation†	Coal			
		Barnsley Hards		Cwmtillery Garw	
		Tangential stress (lb/in^2)	Axial stress (lb/in^2)	Tangential stress (lb/in^2)	Axial stress (lb/in^2)
		Intermediate	Fracture	Intermediate	Fracture
Circumferential stress maintained constant while axial load increased to fracture	1	4690 ± 60	10,410 ± 980	3600 ± 20	6610 ± 520
	2	4930 ± 30	9270 ± 670	3560 ± 40	5820 ± 420
	3	5000 ± 40	9820 ± 510	3640 ± 50	5760 ± 730
		Fracture	Intermediate	Fracture	Intermediate
Hydrostatic pressure increased until specimens collapsed	1	9600 ± 340	4730 ± 170	6830 ± 500	3340 ± 250
	2	9980 ± 250	4900 ± 120	7260 ± 400	3250 ± 210
	3	>14,630	>7,030	8780 ± 560	4280 ± 260

† See Table 5.1.

TENSILE STRENGTH MEASUREMENTS

It is difficult to measure the tensile strength of brittle materials since only a small strain can be sustained before fracture takes place. Hence it is virtually impossible to subject specimens to a uniform tensile stress if the normal pull or bending types of test are used[8] (see Chapter 4). However, if a disc is subjected to a compressive load between two ends of a diameter an almost constant tensile stress σ_t is set up perpendicular to this diameter and parallel to the surface of the disc, given by

$$\sigma_t = 2Q_d/\pi D$$

where Q_d is the applied load per unit thickness of the disc and D is the diameter.

The compressive strength along the loaded diameter, σ_q, is given by

$$\sigma_q = \frac{2Q_d}{\pi}\left[\frac{2}{D-2y} + \frac{2}{D+2y} - \frac{1}{D}\right],$$

where y is the distance along the diameter from the centre.

If the disc is subjected simultaneously to an axial load Q_d and a hydrostatic pressure σ_H, the state of stress along the loaded diameter is defined by the three principal stresses

(i) a tensile stress

$$\sigma_t = \frac{2Q_d}{\pi D} - \sigma_H$$

(ii) a compressive stress

$$\sigma_q = \frac{2Q_d}{\pi}\left[\frac{2}{D-2y} + \frac{2}{D+2y} - \frac{1}{D}\right] + \sigma_H$$

which has a minimum at the centre of the disc

$$\sigma_{q\min} = \frac{3Q_d}{\pi D} + \sigma_H$$

(iii) a compressive stress acting perpendicular to the face of the disc equal to the hydrostatic pressure σ_H.

Experimental Technique

This method of loading discs diametrically has been widely used for tensile strength tests on coal[2]. A few measurements have been made in which the combined axial loading and hydrostatic pressure have been used. The apparatus used is that described for the triaxial strength tests; the discs of coal, which had been sealed from ingress of oil by a rubber or Araldite coating, being placed between the loading platens inside the pressure chamber. The discs were carefully aligned so that the load was applied along a chosen diameter of the disc. The method of applying the hydrostatic pressure and then building up the diametral load to fracture the disc was the same as that used for the solid cylinders.

Measurements have been made on two coals, Barnsley Hards and Pentremawr anthracite, the discs being loaded so that the tensile stress was applied either parallel or perpendicular to the bedding planes.

Results

Results for the two coals are given in Table 5.8. There is little evidence that the tensile strength of the anthracite is affected by the other principal stresses, but there is a slight indication that the strength of the Barnsley Hards increases with the initial increase in the hydrostatic pressure. At high pressures the tensile stress at fracture falls, but this is because failure is no longer tensile along the diameter, but is probably initiated by shear close to the loading platens.

Tests on a larger size of Barnsley Hards disc loaded so that the tensile stress was parallel to the bedding planes confirmed the increase in tensile stress at fracture with increase in confining pressure.

TABLE 5.8. THE EFFECT OF COMPRESSIVE PRINCIPAL STRESSES ON TENSILE STRENGTH MEASUREMENTS

Direction of tensile stress	Hydrostatic pressure (lb/in^2)	Coal			
		Barnsley Hards		Pentremawr anthracite	
		Minimum compressive stress (lb/in^2)	Tensile stress (lb/in^2)	Minimum compressive stress (lb/in^2)	Tensile stress (lb/in^2)
Parallel to bedding planes	0	1050 ± 120	350 ± 40	810 ± 60	270 ± 20
	250	—	—	2140 ± 210	380 ± 70
	500	2870 ± 210	290 ± 70	2960 ± 330	320 ± 110
	750	4590 ± 450	530 ± 150	—	—
	1000	5170 ± 510	390 ± 170	4090 ± 390	30 ± 130
Perpendicular to bedding planes	0	690 ± 90	230 ± 30	900 ± 60	300 ± 20
	250	2530 ± 330	510 ± 110	2110 ± 210	370 ± 140
	1000	5350 ± 570	450 ± 190	4330 ± 480	110 ± 160

The discs were 1 in. in diameter and 0·33 in. thick.

Dumb-bell Method

An alternative technique was developed by Murrell[3] in which a dumbbell shaped coal specimen was subjected to hydrostatic pressure around the curved surfaces, the flat end faces being unsupported. The experimental arrangement was that shown in Fig. 5.10. A rubber membrane was used to protect the specimen from ingress of oil.

In this way the specimens were subjected to a lateral compressive stress and an induced tensile stress parallel to the axis which results from the stress applied to the shoulders of the dumb-bell. The ratio between them is given by:

$$\frac{\text{tensile stress}}{\text{compressive stress}} = \frac{d_2^2 - d_1^2}{d_2^2}$$

where d_2 is the diameter of the specimen ends and d_1 the diameter of the waist. This ratio can be varied by employing a number of different waist diameters. Measurements on Barnsley Hards recorded in Table 5.9, show a decrease in tensile strength as the lateral pressure is increased.

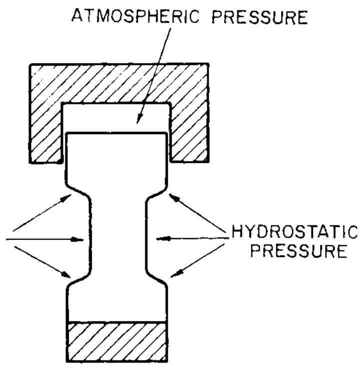


FIG. 5.10. Schematic diagram of dumb-bell specimen used to study tensile strength in a triaxial stress system.

TABLE 5.9. THE EFFECT OF COMPRESSIVE PRINCIPAL STRESSES ON TENSILE STRENGTH OF BARNSLEY HARDS, MEASURED USING THE DUMB-BELL METHOD

Lateral pressure (lb/in^2)	Axial tensile fracture stress (lb/in^2)	Diameter of waist (in.)
1110	210	0·915
580	270	0·825
450	350	0·749
320	490	0·623

These results conflict with those obtained on discs, but the stress systems are not identical, the two principal compressive stresses being equal for the dumb-bell tests, whereas there is a large difference in magnitude between the compressive principal stresses acting on the discs. Moreover, the decrease in strength with increase in lateral pressure found for the dumb-bell specimens may result from the simultaneous increase in waist diameter used to vary the ratio of the principal stresses since the strength of coal is known to decrease with increase in specimen size[1].

The studies of the effect of orthogonal compressive stresses on tensile strength measurements are far from conclusive, but the tests do show that tensile fracture is affected by the stress system employed. It is important to learn that the tensile stress needed to fracture coal may be doubled if compressive stresses act perpendicular to the applied tension.

Strength Theories

Strength is usually measured using a simple stress system, but, as has already been discussed, the stresses acting on coal in the seam are far from simple. It would be ideal if the simple strength measurements could be used to predict how coal would break in practice and attempts have been made to explain the results from the complex stress tests in terms of the many strength theories that have been developed[11]. The simplest of these, the maximum stress, maximum strain, and maximum shear stress theories do not satisfy the bi- and tri-axial measurements, since the fracture stresses and strains have been shown to increase with confining pressure.

It is usual to consider triaxial tests in terms of the Mohr theory[12], but the angles of fracture observed in the measurements on coal do not agree with those predicted from the Mohr envelope, nor does the theory imply that the intermediate stress influences fracture as was observed in the biaxial tests. In fact, the only criterion that applies at all satisfactorily to both bi- and tri-axial measurements is the strain energy of distortion theory, which can be written,

$$\tau_{oct} = \tfrac{1}{3}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}^{\frac{1}{2}} = \text{const.}$$

where the σ's are the principal stresses and $\sigma_1 > \sigma_2 > \sigma_3$. τ_{oct} is known as the octahedral stress.

According to this theory, yielding begins when the distortion energy reaches the distortion energy at the yield point in a simple tensile test which corresponds to $\sigma_1 = \sigma_{y.p.}$ and $\sigma_2 = \sigma_3 = 0$, so that the condition for yielding is

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{y.p.}^2.$$

In the two dimensional case $\sigma_3 = 0$ so that

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{y.p.}^2.$$

This is the equation of an ellipse which can be fitted to the results from the biaxial tests with fair agreement. The equation can only be expected to represent the results completely if coal is isotropic and the lack of symmetry shown by the experimental results (Fig. 5.9) reflects the influence of the bedding and cleat planes on strength.

Evidence of the validity of the strain energy of distortion criterion for the triaxial tests is shown in Table 5.10 (after Hobbs[5])

TABLE 5.10. STRAIN ENERGY OF DISTORTION AT FAILURE FOR SOLID CYLINDERS OF COAL SUBJECTED TO TRIAXIAL COMPRESSION† (ARBITRARY UNITS)

Cylinder orientation‡	Confining pressure lb/in²	Colliery and seam			
		Pentremawr Pumpquart	Oakdale Meadow	Rossington Barnsley Hards	Markham Blackshale
	0	56	7	135	44
	1000	511	80	500	420
1	2000	770	140	840	430
	3000	640	270	820	610
	4000	1080	340	1020	670
	5000	1020	360	1250	680
	0	245	15	600	110
	1000	610	200	1220	480
3	2000	1080	190	960	590
	3000	1560	340	900	770
	4000	1640	300	930	700
	5000	3430	520	880	790

† Strain energy of distortion.

$$W = \frac{1+\nu}{6E}((\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2 + (\sigma_\theta - \sigma_r)^2)$$

where E, Young's modulus, is the value calculated at the appropriate confining pressure and Poisson's ratio ν is assumed constant. σ_r, σ_θ and σ_z are the stress components acting on the cylinder, expressed in cylindrical coordinates.

‡ The orientation of the cylinders is given in Table 5.1.

DISCUSSION

It can be seen from Table 5.10 that at confining pressures above 1000 lb/in² the strain energy of distortion, with the exception of the anthracite specimens of orientation 3, either remains constant or increases slightly. The strain energy of distortion theory, which predicts failure when a constant value is reached, therefore satisfies the results fairly satisfactorily at the higher confining pressures. The influence of the cracks and weaknesses probably accounts for the low values of energy required to fracture specimens at low confining pressures.

A full study of the breakage of coal by complex stresses would take many years to complete. However, the measurements recorded in this chapter show that coals that are markedly different at atmospheric pressure become virtually indistinguishable when subjected to triaxial compressive loading, with the exception of the anthracite studied which was markedly stronger than the other coals at high confining pressures. It is conceivable, but unlikely, that a different conclusion would have been reached if the specimens had been cut with a family of weakness planes aligned parallel to the planes along which fracture was expected to occur. In practice, none of the fracture planes coincided with existing discontinuities, which means that it was primarily the strength of the coal between the major flaws that determined the fracture stress. However, the direction of the fracture planes is affected by the direction of the major cleat planes, at all confining pressures, so that the inherent weaknesses in the coals must play some part in determining fracture.

The studies of strength under biaxial stresses and of the influence of compressive principal stresses on tensile strength have shown that the mechanics of fracture is far from simple. Most of the common strength theories suggest that strength is not affected by the magnitude of the intermediate principal stress, but the experimental measurements on coal do not support this contention. In fact, the only criterion that satisfies the observations from both the triaxial and biaxial tests at all satisfactorily is the constant strain energy of distortion criterion and this does not apply to the strength tests on cylinders that were crushed uniaxially or at low confining pressures.

The fact that cracks affect the strength of unconfined or weakly confined cylinders, whereas they have little influence on the strength when high confining pressures are applied, may possibly account for this disagreement between observation and theory.

The strain measurements show the influence of cracks on the behaviour of loaded cylinders, the percentage strain attributed to closure generally being larger for cylinders cut with bedding planes perpendicular to the axis of the specimen. However, the Young's moduli of all the coals tested show remarkable agreement at high confining pressures, the differences between different specimen orientations being virtually nothing. This again shows that, within the elastic range, coals in the seam ahead of the coal face are all very similar in mechanical behaviour, which is most surprising in view of the marked differences that are easily seen between extracted coals. In fact, the only gross difference in the effect of an axial load on cylinders of the different coals subjected to confining pressures of about 5000 lb/in^2 is that a few of the low-rank coals show marked flow before they fracture, and in particular Linby, High Main coal can sustain strains in excess of 5 % before it breaks.

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CHAPTER 6

Routine Strength Tests on Coal

CHAPTERS 3, 4 and 5 deal with the strength of rectangular and cylindrical specimens subjected to uniaxial tension and compression and to simple bi- or tri-axial stress systems. The breakage of irregular lumps by a compressive load between parallel steel platens and by shatter in free fall have also been described[1,2,3,4]. These tests are not suitable for routine strength measurements on coal since they need expensive equipment and the majority of the tests also require careful preparation of regular coal samples. Moreover the bi- and tri-axial strength tests showed clearly that the strength of the coal on a coal face or embedded deeply into a coal seam bears little relation to uniaxial measurements, although if the strain energy contained in the coal is known at any particular spot the possibility of an additional load causing fracture can be predicted. Unfortunately, the strains present on a coal face are seldom known at all accurately so that the only way to get a reliable measure of the strength of coal in the seam is to measure it *in situ*, but as is shown later in this chapter, even measurements made on a coal face vary considerably with time and with location.

Routine strength measurements are wanted for two main purposes, to compare the properties of different coal faces and so provide a yardstick for assessing mechanization potential and possible difficulties, and to compare the strengths of extracted coals, so that the degradation that is likely to occur during handling can be predicted and mitigated when necessary. The first of these tests must be done at the face, and laboratory and underground development of two seam testers are described in this chapter. The degradation test is undertaken on extracted coal and the measurement and use of the Impact Strength Index for this purpose are also described.

SEAM TESTS

If a seam test is to have any practical value there are a number of basic requirements that must be met. The measurement obtained with the equipment must be reproducible and not dependent on the person who is making the test. The tests must be easily performed in the confined space on a coal face so that the equipment used must be both portable and pitworthy, the latter requirements tending to be contradictory. The measurements must reflect the state of the coal, as it exists on the face, so that little preparation of the coal, e.g. trimming off loose or overhanging coal, is necessary before a test is carried out. Furthermore, the measurements should be related to practical observations of machine performance or strata loading where possible.

Additionally, it would be useful if the tests could be used to study the variation in strength between different horizons on a coal face and to enable comparisons to be made with the strength of the roof and the floor close to the coal face, although these requirements are not essential to the development of an *in situ* test.

Both of the *in situ* tests that are described below meet the basic list of requirements, but only the first satisfies the additional ones. The first test is one in which a flat-ended rod, known as an indenter, is driven into the coal and a record made of the force required for increments of penetration. The other test, instead of pushing a rod into the coal, breaks a cone away from the face, and consists of embedding an expanded bolt (modified from a Rawlbolt) about 6in. into the coal and then breaking the cone of coal from the face by a direct pull on the bolt. Both of these tests were developed first in the laboratory and the design of equipment for underground use followed.

Historical Background

When the *in situ* tests were conceived existing forms of seam tests used in other countries were first considered and three main types seemed worthy of examination in detail. All three devices work in boreholes and the basic modes of action are as follow:

(*i*) *Russian Device.*[5] The back end of the borehole is reamed out to a larger diameter about 6–8in. from the open end of the borehole. A hydraulic mechanism is inserted which grips the coal in this enlarged region and pulls it away from the face. The force necessary is measured.

(*ii*) *Polish Device.*[6] A tube having a longitudinal slot is placed in the borehole. A miniature cutting pick is thrust down the hole, cutting the coal exposed by the slot, the force required being measured.

(*iii*) *German Device.*[7] A close-fitting hydraulic tube is placed in the borehole. From the side of the tube a small pin can be thrust out into the coal. The force is measured.

Copies of the Russian and German devices were made and the expanding-bolt test, described later, is a development from the Russian one. The original form of the test was unsatisfactory since it proved difficult to ream a shoulder inside the borehole that was strong enough to be gripped by the hydraulic device. In practice, the coal inside the borehole broke away locally and no large fragment was torn from the coal face.

The German test consists of the indentation of the coal inside the borehole by small rods. The depth of penetration was so shallow, however, that little confidence was felt in the test, surface irregularities causing wide variation between adjacent measurements.

No attempt was made to copy the Polish method, since this necessitates the measurement of grooving forces continuously as the pick is moved along the tube inserted in the borehole and special intrinsically safe equipment would be required. Moreover, the analysis of the force records would be a lengthy process and so make the time taken to complete a test unacceptable for a routine measurement.

M.R.E. Penetrometer

While the M.R.E. penetrometer(8) is an indentation type of test, it differs fundamentally from the German version. It is based on methods that have been used for many years to assess the strength or hardness of materials as different as metals and plastics, or soils and ceramics. These hardness tests consist of some device that measures the force required to push a penetrating rod, with a conical, ball or flat end, into the material. In some of the tests the indentation that remains when the rod is removed from the surface is measured, and various modifications to the test are made to suit different substances. The tests are usually made into a flat surface and depend to some extent on the rheological properties of the materials.

Coal, however, is a brittle substance, and shows little evidence of plasticity, particularly under the conditions that are found at the face during its extraction from the seam. The first work that had to be done, therefore, was to see if a penetration type of test for coal had any practical significance and to establish the exact form that the test would have to take. The initial development was undertaken in the laboratory.

(1) Laboratory Development

In the initial experiments the loads required to drive cylindrical rods with flat, hemispherical, or pointed ends perpendicularly into large, unconfined, lumps of a number of different coals were measured for increments of penetration. The insertions were made in specified directions relative to the bedding and dominant cleat plane orientations.

Subsequent investigations were concerned with the effect of changes in rod diameter on the penetration load and with the effect of a uniform compressive load applied perpendicular to the bedding planes to simulate the pressure due to the overbearing strata that act on coal in the seam. The latter factor was considered extremely important since practical observation has told mining engineers that coal strength at the face is highly dependent on the way the roof is supported, and seam tests must therefore be sensitive to changes in strength induced by strata loading.

(2) The Shape of the End of the Indenter

The relationships between the load on the indenter and the depth of penetration were dependent on the shape of the indenter end. The laboratory tests consisted of pushing cylindrical rods of various diameters ($\frac{1}{8}$, $\frac{3}{16}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$ in.) and having various shapes of end (conical, hemispherical, flat) into large blocks of coal that had been trimmed to have a flat surface normal to the rod. The measurements were made on the nine coals chosen to represent the coals commonly found in Britain. The insertions were made in two directions, either parallel or perpendicular to the bedding planes, but in general the form of the load/penetration graph depended on the type of coal rather than on the direction of entry, although the magnitude of the forces was directionally dependent for some coals. The typical forms of relationship for the differently-shaped indenters are:

(i) Cone-ended rods.

The load tended to build up continuously as penetration proceeded, although there were fluctuations as fragments of coal broke away. The cone tended to follow weaknesses, and so caused the indenters to buckle. The tests usually ended with the blocks splitting in transverse tension, even when the linear dimensions of the block of coal were more than thirty times the indenter diameter. Some typical penetration curves are given in Fig. 6.1.

(ii) Flat-ended rods.

The penetration characteristics were markedly different from those observed with the conical rods. The loads for friable coals, such as Cwmtillery, Garw, build up at first and then stay fairly constant as penetration continues (Fig. 6.2). The loads also build up initially when strong bituminous coals such as Barnsley Hards are used, and then fluctuate in a saw-tooth pattern as fragments of coal spall from the face of the block. (Fig. 6.3).

(iii) Hemispherical-ended rods

The penetration characteristics with this form of indenter were intermediate between those with the conical and flat ends. The rods still tended to follow weakness planes and buckle.

A test which results in a "plateau" or constant value has much in its favour, since a strength value can be obtained which is independent of the depth of insertion, provided the first build-up of force is ignored. Where the load fluctuates about a mean value a planimeter can be used to calculate the mean

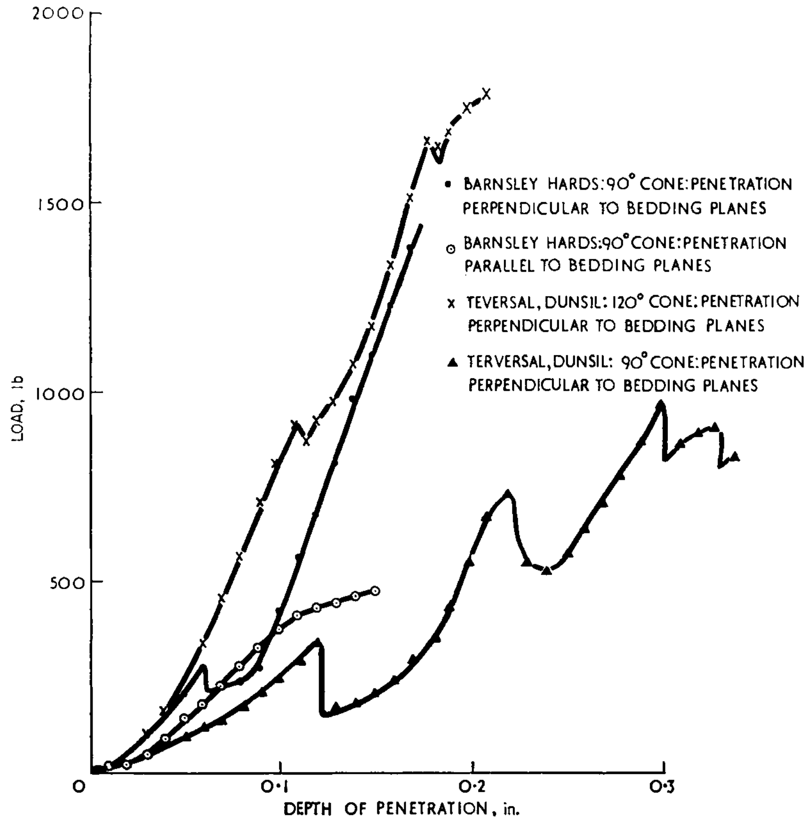

FIG. 6.1. Load–depth of penetration for conical indenters.

resistance to penetration (defined as the indenter load/cross-sectional area) corresponding to the plateau region. The penetration resistance calculated in this way is several times the compressive strength. For the Garw coal the ratio is about 5, which is remarkably near the value appropriate to plastic yield. For the hard bituminous coals the ratio is between 2 and 3, which is close to that found in plasticity for surface bearing capacity as distinct from deep penetration, and may be due to the fact that the indentation into the hard coals is accompanied by surface spalling as the test proceeds.

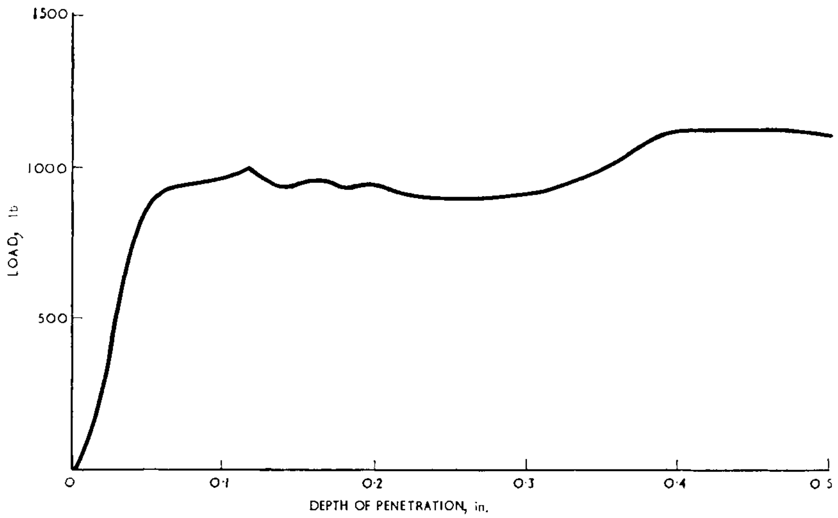

FIG. 6.2. Load–depth of penetration for the indentation of a rectangular block of Cwmtillery (Garw) coal, perpendicular to the bedding planes, by a 0·25 in. dia. flat-ended indenter.

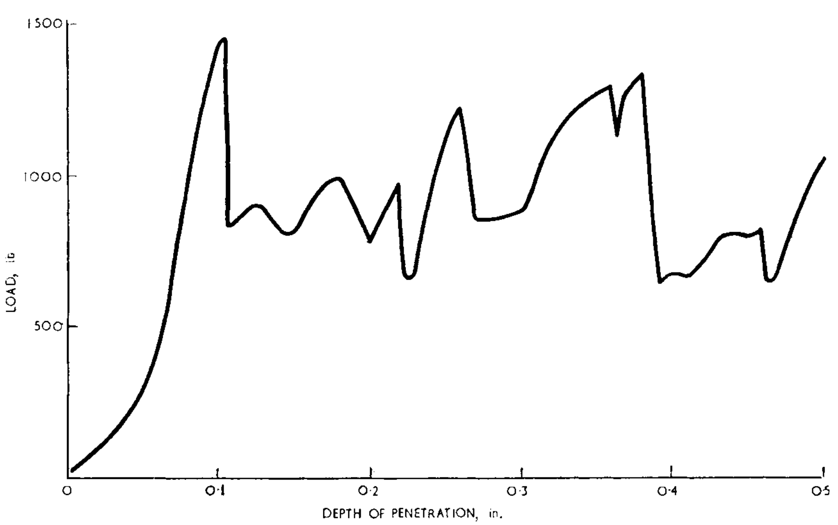

FIG. 6.3. Load–depth of penetration for the indentation of an unrestrained rectangular block of Barnsley Hards, perpendicular to the bedding planes, by a 0·25 in. dia. flat-ended indenter.

(3) The Effect of Overburden Pressure

Rectangular blocks of coal, cut with the bedding planes parallel to the largest faces of the blocks, were clamped between heavy steel plates in the manner shown in Fig. 6.4. The compressive stress applied perpendicular to the bedding planes could be varied between wide limits and held constant during the course of the test, which consisted of the insertion of a small flat-ended rod ($\frac{1}{4}$ in. dia.) to a depth of about 1 in. The plateau type of penetration

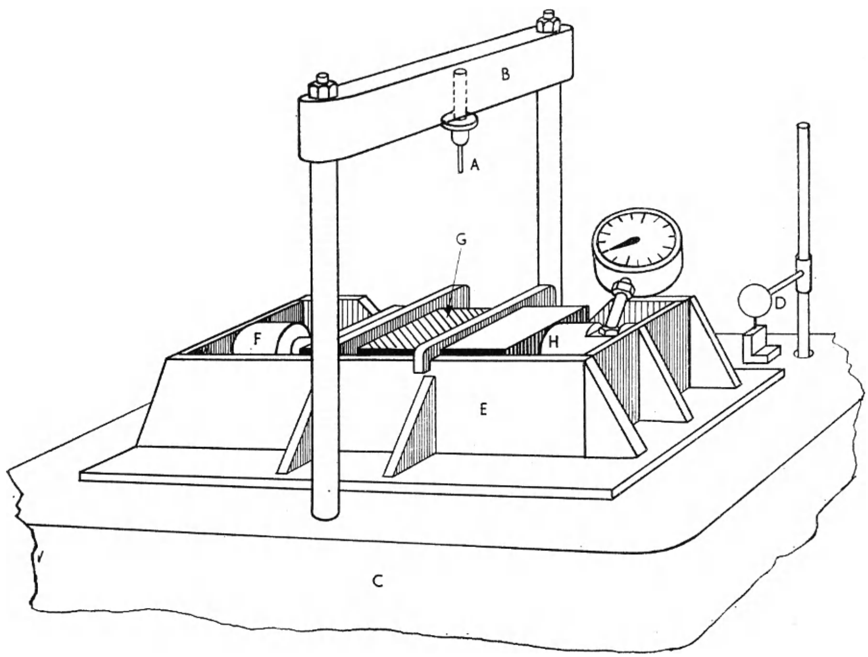

FIG. 6.4. Experimental arrangement.

curve was not always observed, but the load frequently built up continuously during the test, although the rate of increase was much less rapid than was observed when cone and ball-ended rods were forced into unconfined blocks of coal.

In general as the simulated overburden pressure was built up the resistance to penetration also increased, reaching a maximum and then subsequently falling away as the applied pressure approached the compressive strength of the coal (Fig. 6.5). This is similar to the relationships that have been observed between the forces required by a pick to cut grooves in rectangular blocks of coal that have a simulated overburden pressure acting on them[9].

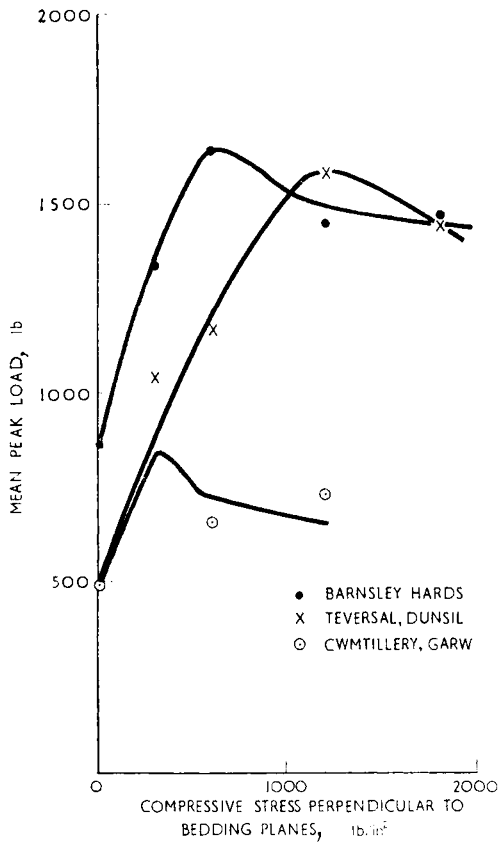

FIG. 6.5. Dependence of mean peak load on lateral pressure for $\frac{1}{4}$ in. dia. flat-ended rods inserted parallel to the bedding planes.

(4) Development of Underground Equipment

The laboratory experiments were thought sufficiently encouraging to warrant the design of equipment for underground trials. The indenter was chosen to be a $\frac{9}{16}$ in. dia. rod with a flat end, since both the cone- and ball-ended rods had proved to be more liable to buckle than the flat-ended ones, and also gave load–penetration curves that were more difficult to interpret. Calculation, based on Euler's theory for the critical load that causes buckling of a column, showed that an axial thrust of about 10 tons could be applied to a 9 in. rod of the chosen diameter without fear of buckling. However, the rods have been found to buckle in practice, since entry is made into the coal through a very uneven surface, so that a bending movement is superimposed on the axial load, thereby aggravating the problem. Methods to overcome this difficulty are described later.

The choice of indenter determines much of the remainder of the equipment. A means is required to drive the rod into the coal face and to do this some form of staking system is needed to take the thrust. Calculations, based on the laboratory work, showed that thrusts of up to 8 tons would be needed to indent the strongest coals and various designs of staking system were tried before a suitable pattern was evolved.

The present design of penetrometer equipment is shown in Fig. 6.6. The support to take the thrust to drive the indenters into the coal face is provided by two 8 ton hydraulic rams, each having a 6 in. stroke, which stand abreast on a floor plate. The rams support a roof plate, the requisite height being obtained by the addition or removal of extension tubes that are secured to the rams by screwed collars. The top and bottom end-pieces of both props are terminated in spherical caps which bear against spherical seatings in the roof and floor plates, so that lack of parallelism between roof and floor can be accommodated. The end plates are fitted with triangular studs which dig into the roof and floor to secure a good grip. An I-section girder, chosen to suit the seam, is loosely bolted between the end plates, as shown. An 8 ton hydraulic ram with a 12 in. stroke is used to force the circular indenter into the coal, the reaction to the thrust being taken against the I-beam. A locating stud at the rear end of the penetrometer ram is engaged in the girder in holes drilled at 3 in. intervals along its length. Extension rods may be added to the penetrometer ram to allow tests to be made over an armoured conveyor.

The three hydraulic rams are activated by a single hand-pump, through a three-way manifold fitted with control valves. The high-pressure hydraulic hoses that link the rams to the manifold are fitted with self-sealing couplings to facilitate the movement of the equipment and assembly at different points along a coal face.

The equipment is suitable for use in seams from about 1 ft 6 in. to 5 ft in section, but it has been used in thicker seams with a more robust staking system.

Test Procedure

A site for the test is selected on a coal face and the staking system is erected about 3 ft away from the face, the twin rams being pumped up to take their full load. The indenter ram is located against the I-girder and the rod is brought into light contact with the coal by a few strokes with the pump. This state is taken as the zero depth of penetration. The load to drive the indenter into the face is then recorded for $\frac{1}{4}$ in. increments of penetration, the extension of the ram from the hydraulic cylinder being taken to be equal to the penetration.

As stated earlier, many of the indenters were bent if the thrust needed to drive the rod into the coal was high, and two methods were successfully used to eliminate this trouble. In the first, a "steady" which fitted closely round

TABLE 6.1. MRE PENETROMETER MEASUREMENTS AT MARKHAM NO. 1 COLLIERY

Location of test	Distance above floor (in.)	Mean resistance to penetration (RP) (ton/in^2)	Maximum resistance to penetration (max. RP) (ton/in^2)
Station I, 20 yd from R.H. gate	Roof	8·0 (2 to 4)†	11·2 (4)‡
	52	16·0	19·2 (5)
	45	12·8	18·4 (5)
	40	8·8	12·8 (5)
	34	5·6	8·4 (5)
	28	4·8	6·8 (5)
	22	3·6	4·8 (5)
	17	4·8	5·6 (4·5)
	12	4·0	4·4 (5)
	Floor	No mean	21·2 (2)
Station II, 60 yd from R.H. gate	Roof	No mean	8·4 (3·5)
	60	13·2	17·6 (5)
	51	5·6	12·8 (5)
	43	15·2 (2 to 4)	17·6 (4)
	36	11·2	16·4 (5)
	26	11·8	14·0 (5)
	21	8·8 (2 to 4)	11·2 (4)
	13	4·0	5·2 (4)
	Coal floor	1·6 (2 to 4)	3·2 (3)
	Under floor	No mean	15·2 (2)
Station III, 120 yd from R.H. gate	Roof	No mean	12·8 (1·5)
	55	2·0	4·0 (1)
	41	6·8	10·0 (5)
	36	1·2 (2 to 4)	2·0 (4)
	27	0·8	1·2 (4)
	22	1·2	2·8 (4)
	14	5·6	7·6 (4)
	7	5·6	6·4 (4)
	Floor	No mean	14·0 (2)
Station IV, 10 yd from L.H. gate	Roof	No mean	21·3 (3)
	50	No mean	20·0 (3)
	46	14·0	18·8 (4·5)
	41	9·2	12·0 (4)
	26	5·2	8·4 (5)
	17	9·2	13·2 (4)
	15	4·4 (2 to 4)	4·8 (3)
	Floor	9·2 (2 to 4)	15·2 (4)

† Figures in brackets are depths of penetration (in.) for which mean is calculated, when different from 2 to 5 in. ‡ Depth (in.) at which maxima occurred given in brackets.

the indenter was screwed on to the outer case of the hydraulic ram, as shown in Fig. 6.7(a). In the other, only a short length of rod was allowed to protrude from the rod holder during the initial entry into the coal. When the exposed rod was fully embedded a pin fixing the rod in the holder was withdrawn, the indenter ram was retracted, leaving the rod in the coal, and relocated so that further penetration could be made. Fig. 6.7(b) shows more clearly how this is done.

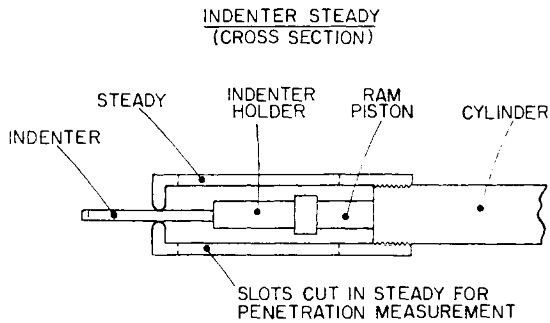

FIG. 6.7(a). Indenter "steady".

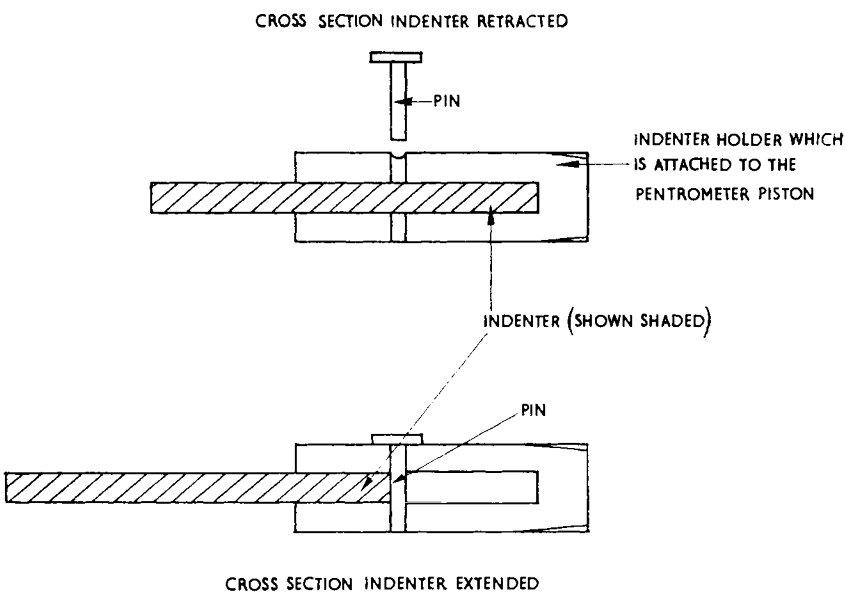

FIG. 6.7(b). Modified indenter holder.

Adaptation for Roof and Floor Tests

The penetrometer can be adapted very easily to measure the strength of roof or floor. The locating lug on the end of the penetrometer ram is replaced by a flat plate which bears against the roof as the indenter is forced into the floor or vice versa.

Measurements with the Penetrometer

There are two distinct purposes for which seam assessment may be required and each of these calls for a slightly different approach. In one case, assessment of a face might be undertaken to enable a given machine to be utilized on the face to best advantage. For this a detailed knowledge of the strength variation of coal, roof, and floor at different parts of the face is required and any local peculiarities that might affect machine performance are important. However, before such detailed analysis of a face is required a less precise assessment is needed that will enable the principal characteristics on one face to be compared with those on others. This restricted information can be used to estimate the performance of a machine on a new face from past experience, but the more detailed information may be required before the machine can be used with maximum efficiency.

The restricted appraisal is based upon a smaller number of observations than are made during a full study of face strength with the penetrometer, the tests being confined to a few positions on the face and to only one or two horizons. A typical set of measurements obtained from a full face assessment is recorded in Fig. 6.8. These measurements were made at Markham, No. 1 Colliery, in 6's face in the Blackshale Seam.

The seam varied in thickness and consisted of a section of 11–20 in. of dirt sandwiched between two sections of bright coal, each about 20 in. thick. The normal roof was a grey mudstone, but occasional grey siltstone roof rolls were present. There were usually a few inches of "flamper" (siltstone with interleaved coal bands) between the roll and the seam. Roof and floor parted readily from the coal. The main cleat, which was well developed, lay at an angle of 13 deg to the face line.

The penetrometer was used at four stations along the face and a visual comparison between the load/penetration curves in Fig. 6.8, shows that, except for Station III, the penetrometer loads for a given depth of penetration are highest near the top of the seam. At Station III the coal is equally weak at all horizons. The graphs also show that the floor is stronger than the bottom coal and that roof and top coal are of almost equal strength.

The graphs alone do not allow quantitative comparisons of coal strength to be made and Table 6.1 gives the mean resistance to penetration (RP) for

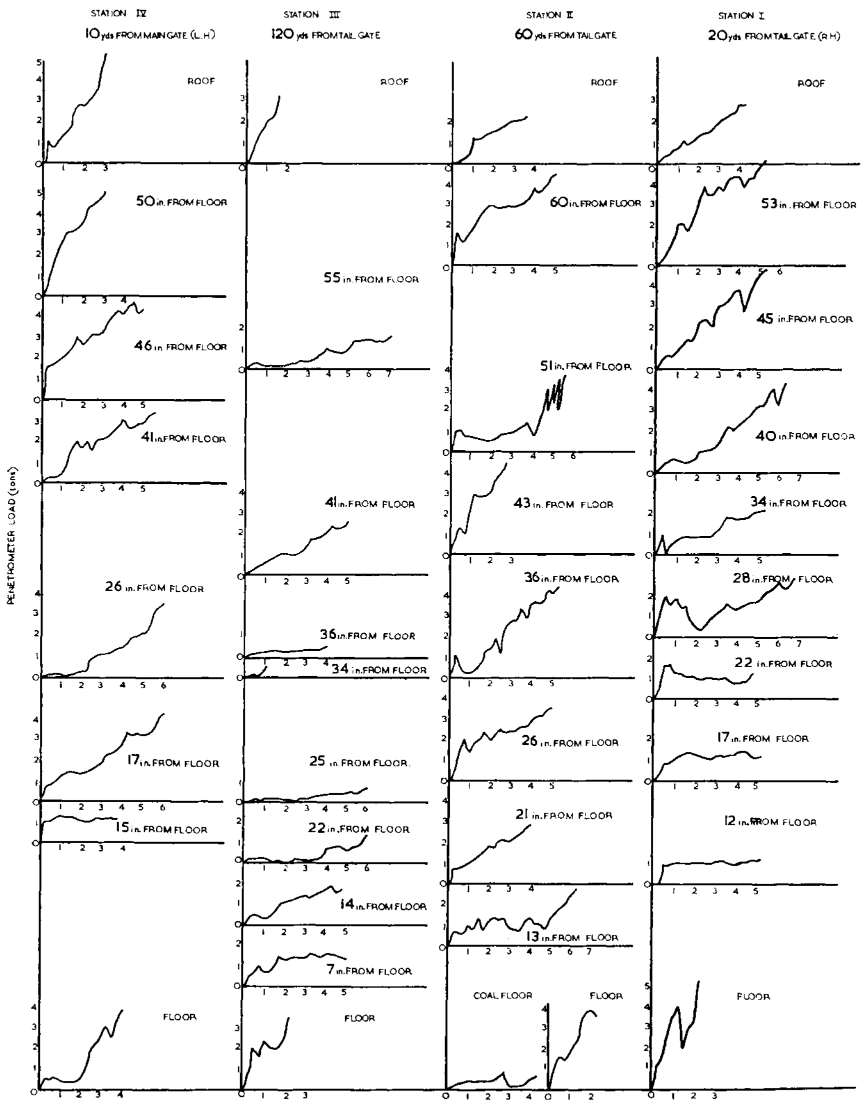

FIG. 6.8. Typical series of penetrometer measurements.

a depth of penetration from 2–5in. into the coal or rock and also of the maximum resistance to penetration (max. RP). If the mean and maximum values are almost identical the resistance to penetration is approximately constant and the load/penetration curve is of the plateau type; but if the values differ the graph shows an increase in resistance to penetration with depth of insertion. Even in the form used to record the data in Table 6.1 the interpretation of the measurements is difficult although local variations can be pin-pointed more accurately than is possible from a visual examination of the graphs.

The data are therefore grouped and averaged for different sections of the seam. The face was being worked by a rapid plough so that the bottom 20 in. of the seam (approx.) were cut by the plough blades, the top coal being left to fall freely. The groupings used, therefore, are the bottom 20 in. of the seam, and the middle and the top sections of approximately equal heights. The average and maximum values of RP for each station and for the whole face are given in Table 6.2. This table shows clearly that the bottom coal is weak at all stations and is much softer than the floor, so that little difficulty should be experienced in keeping the plough in its operational horizon. The gradual increase in strength from floor to roof will also help to keep the plough working at the correct level.

Some difficulty might be experienced if the top coal does not fall, but this is not likely to happen since visual examination of the face shows a clean parting between roof and floor.

TABLE 6.2. SUMMARY OF MRE PENETROMETER READINGS AT MARKHAM NO. 1 COLLIERY, 6S FACE

Location of test	Station I		Station II		Station III	
	Mean RP	Max. RP	Mean RP	Max. RP	Mean RP	Max. RP
Roof	8·0	11·2 (4)	—	8·4 (3·5)	—	12·8 (1·5)
Top coal	12·5	16·8 (5)	11·3	16·1 (5)	4·4	7·0 (5)
Middle coal	5·0	6·7 (5)	10·3	12·6 (5)	1·1	2·0 (4)
Bottom coal	4·4	5·0 (5)	4·0	5·2 (5)	5·6	7·0 (4)
Floor	—	21·2 (2)	1·6	3·2 (3)	—	14·0 (2)
Underfloor	—	—	—	15·2 (2)	—	—

Location of test	Station IV		Face average	
	Mean RP	Max. RP	Mean RP	Max. RP
Roof	—	21·2 (3)	—	13·4 (3)
Top coal	12·0	16·9 (4)	10·5	15·0 (5)
Middle coal	5·2	8·4 (4)	4·8	6·7 (4·5)
Bottom coal	6·8	9·0 (4)	4·7	5·7 (4·5)
Floor	9·2	15·2 (4)	—	16·4 (2·5)
Underfloor	—	—	—	—

Figures in brackets give depth (in.) at which maxima recorded.
RP = Resistance to penetration in ton/in^2.

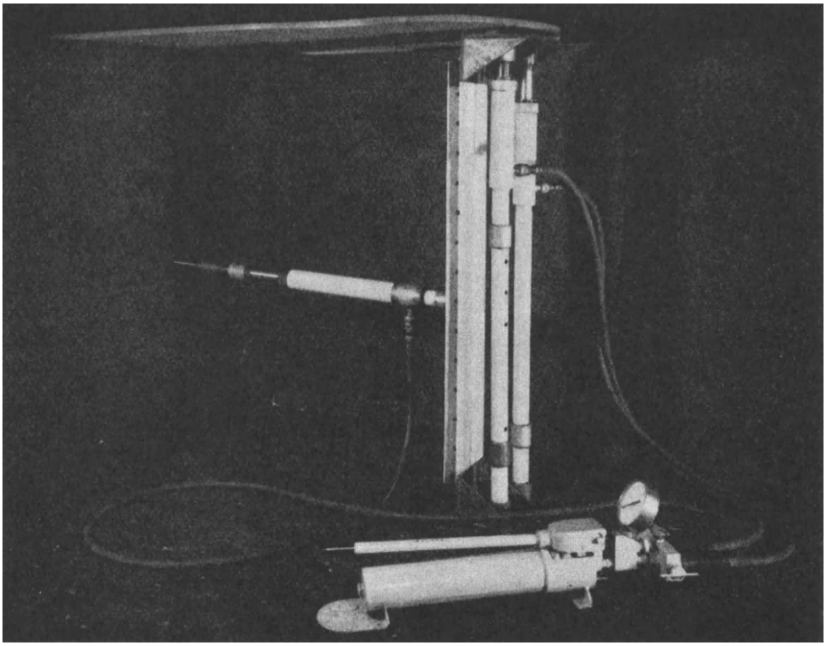

FIG. 6.6. M.R.E. Penetrometer and staking system.

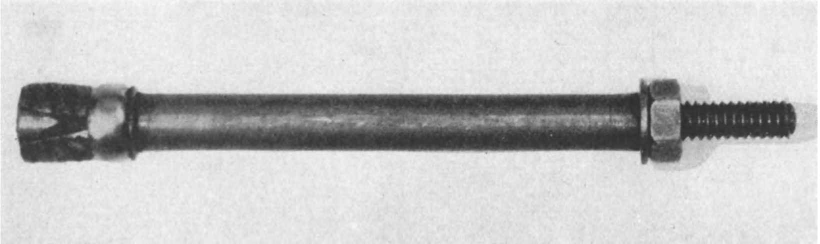

FIG. 6.10. Modified "Rawbolt" for laboratory tests.

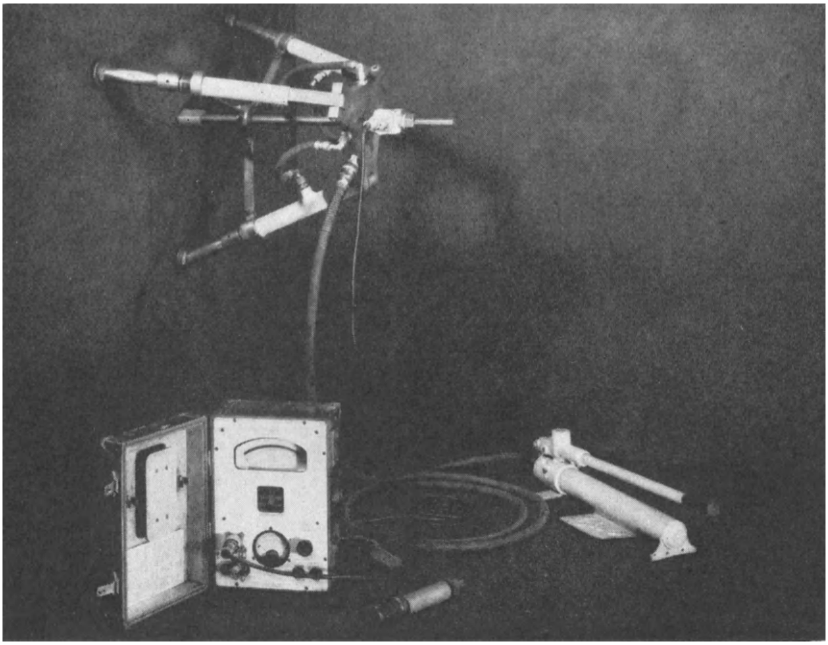

FIG. 6.12. The expanding-bolt seam tester.

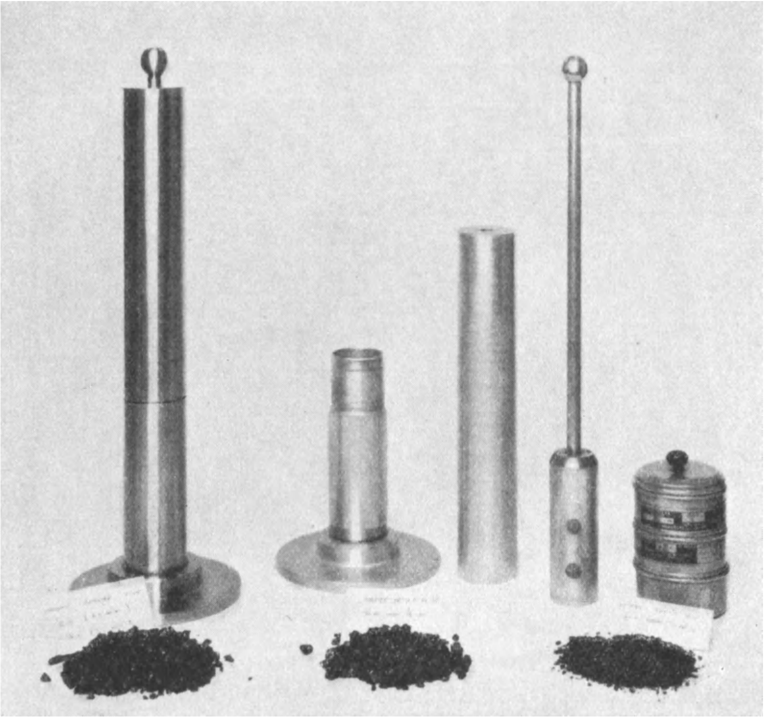

FIG. 6.13. Impact strength index apparatus.

More recent experience has suggested that it is not always possible to make a 5 in. penetration but that meaningful comparisons can be made between mean and maximum values of RP calculated over a penetration from 1 to 4in. into the coal or rock and this is now advocated as the standard procedure.

The wider applications of the penetrometer are discussed later, after the expanding-bolt and the ISI tests have been described.

Expanding-Bolt Seam Tester

Although the penetrometer provided a simple method for seam assessment, the mode of action is dissimilar from the action of cutting tools and plough blades, which tend to prise fragments of coal from the surface of the coal face or from surfaces of coal formed during the previous passage of picks. It has been suggested that the only way that pick performance can be studied is to use picks, but the test of the Polish variety[(6)] is deemed unsatisfactory for a number of reasons, the most important, apart from the complexities of the equipment, being the difficulty that there would be in

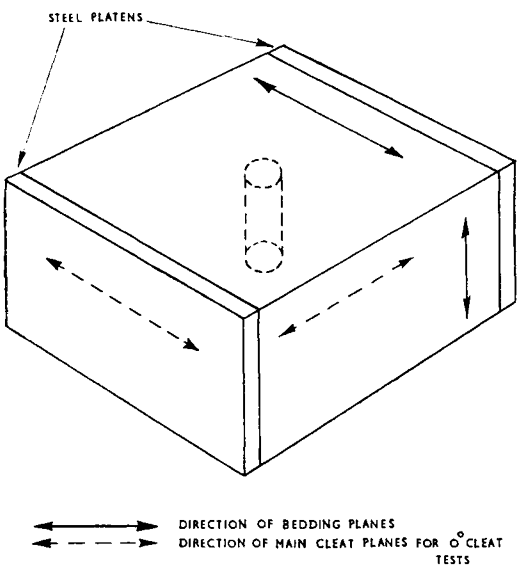

FIG. 6.9. Diagram of coal block for laboratory tests of the expanding bolt.

standardizing the test. Slight wear of any pick used to cut a groove can increase the cutting forces sufficiently to mask the differences between coals.[(10)]

The Russian test breaks a cone of coal from the face. Laboratory development of an *in situ* test based[(11)] on this method followed similar lines to the development of the penetrometer.

(1) *Laboratory development.* Experiments were undertaken on four markedly different types of coal which were machined to make 7 in. by 7 in. by 4 in. rectangular blocks. The bedding and main cleat planes were orientated as shown in Fig. 6.9, two orientations of cleat planes being used in different blocks. A hole, in which a 0·25 in. Rawlbolt could be fitted, was drilled in the middle of a 7 in. by 7 in. face of each block. The Rawlbolt was modified by shortening the segments and providing a tube to fit over the bolt shank, as shown in Fig. 6.10. The segments on the bolt were expanded to a constant extent by tightening the fixing nut with a torque wrench. The blocks of coal were held in a clamp attached to the bottom platen of a Universal testing machine. The clamping pressure was adjusted to simulate different overburden pressures. The bolt was pulled from the blocks and the maximum load during the extraction of the bolt was recorded. The size of the cone of coal broken was measured by making a plaster cast of the cavity.

The blocks of coal were clamped between flat steel plates to simulate the overburden pressure. Tests on unrestrained or lightly clamped blocks were unsatisfactory since, during the test, the block invariably split in two along the bedding planes. The range of clamping pressures that proved satisfactory was 200–1000 lb/in^2. Higher pressures crushed the weaker coals used.

Breakage Force

The results (Table 6.3) showed that the maximum force applied during extraction of the cone was sensitive to the different pressures applied to simulate strata loading, but the effect of cleat orientation on the measurements was not statistically significant.

TABLE 6.3. LABORATORY TESTS WITH AN EXPANDED BOLT

Colliery and seam	Breakage force (lb)		
	(a)	(b)	(c)
Cwmtillery, Garw	300 ± 40	380 ± 50	420 ± 40
Rossington, Barnsley Hards	750 ± 40	840 ± 30	800 ± 40
Teversal, Dunsil	430 ± 80	590 ± 60	610 ± 70
Markham, Blackshale	500 ± 50	560 ± 60	580 ± 30

Clamping pressures: (a) = 200 lb/in^2, (b) = 600 lb/in^2, (c) = 1000 lb/in^2.

Cone Size

The cone size was affected by the orientation of the main cleat planes during the test, the mean semi-angle of the cone being larger (*i.e.*, the extracted fragment was larger) when the cleat is orientated perpendicular to the bolt (0 deg. case) (Table 6.4).

TABLE 6.4. CONE SEMI-ANGLE

Colliery and seam	Cleat orientation	
	0°	90°
Cwmtillery, Garw	66°	62°
Rossington, Barnsley Hards	65°	62°
Teversal, Dunsil	66°	49°
Markham, Blackshale	64°	51°

Breakage Mechanism

Attempts have been made to explain the breakage process in physical terms, but so far no completely satisfactory theory has been formulated. From the practical standpoint, however, it is enough to know that a strength index has been derived(11) that is sensitive to overburden pressure and satisfactorily distinguishes between coals. As the cone size is cleat dependent, while the breakage force is not, it is tempting to derive an index σ_b based on the two measurements and the following expression has been found to be useful:

$$\sigma_b = \frac{3F_b}{\pi h_b(R_b + 2r_b)},$$

where F_b is the breakage force, R_b is the mean base diameter of the cone, r_b the radius of the hole in which the bolt is fitted and h_b the height of the cone extracted. σ_b correlates well with the shear strength of the coals, but as brittle materials usually break in tension this correlation may be fortuitous.

An alternative relationship based on tensile breakage theory has been derived(12) which states that

$$F_b = \text{constant.}\ tr_b h_b$$

where t is the tensile strength of the coal. At present, sufficient experimental evidence is not available to judge which, if either, of these formulae is acceptable.

Laboratory Grooving Tests

The breakage force (F_b) required to extract a cone from the four coals tested correlates well with the forces required to channel grooves in the plane surface of rectangular blocks of coal, as shown in Fig. 6.11. On the basis of this relationship, and in view of the fact that the breakage force reflects the influence of simulated overburden pressure on the strength of the coal it was thought worthwhile to develop an underground version of the equipment.

(2) *Development of underground equipment.* The equipment that has been evolved is shown in Fig. 6.12. It consists of an expanding bolt, of somewhat similar construction to a $\frac{3}{4}$ in. dia. Rawlbolt, but the expanding segments are much shorter (about 1 in. long) than those fitted to a Rawlbolt. The steel used in construction of the bolt is also stronger than that used in the commercial bolt, the segments being EN 25 steel heat-treated to 80 to 90 tons/in^2 tensile strength and the bolt core itself of annealed EN 25 steel.

The bolt can be inserted in a $1\frac{7}{16}$ in. dia. hole drilled to a depth of 6 in. into the coal face. The segments are expanded a standard distance and the bolt is

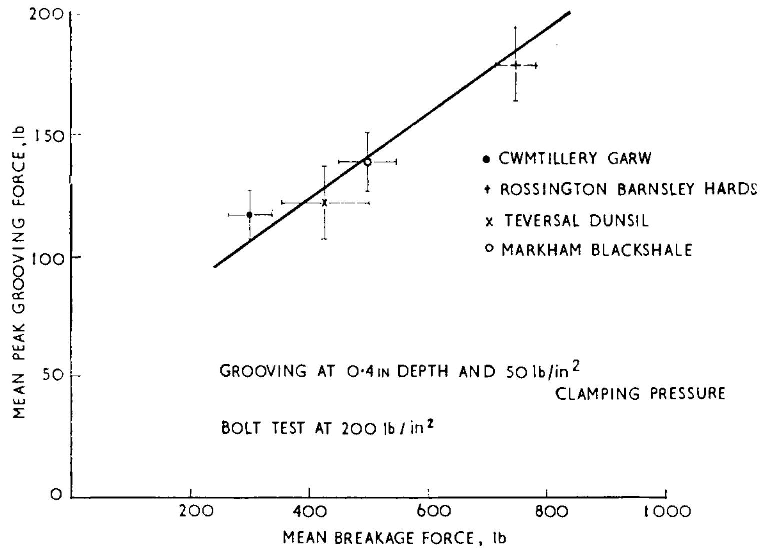

FIG. 6.11. Relationship between mean peak grooving force and mean breakage force.

pulled out of the coal by the tripod assembly. This tripod consists of three hydraulic rams which form the legs, the rams being operated by a hand pump via a common oil manifold. Self-sealing oil couplings link the components together to facilitate movement of the equipment along the coal face.

The expanding bolt is attached to the tripod by an extension rod which bears against a dynamometer that is used to measure the force applied to the bolt. Spherical seatings are used to ensure that the pull on the bolt is as axial as possible. The dynamometer is one that has been designed to measure the tension in roof bolts.[13] Wire resistance strain gauges provide the load-sensitive elements in the dynamometer, the loads being measured with an intrinsically safe transistorized galvanometer test set. The tripod is designed to apply loads of up to 10 tons to the bolt during withdrawal.

Practical Considerations

It is necessary to prepare a roughly flat face on which the tripod legs can rest when the bolt is pulled from the coal. As the legs open to about 15 in., the positions where the bolt can be inserted in the coal face are somewhat restricted. However, as the size of cone broken from the coal face is often at least 18 in. mean base diameter, the test tends to give an overall assessment of coal strength rather than the more localized indices obtained with the penetrometer test.

Tests are also restricted by the presence of overhanging top coal, but this is a problem that is likely to affect any form of *in situ* seam test.

Measurements are usually made immediately after a coaling shift, when any coal "hardening" that occurs during the daily advance is most effective and therefore the coal is tested in the worst condition. The test can easily be undertaken by two men, although a third man going ahead and preparing the shotholes to take the expanding bolt and roughly smoothing the coal face should enable tests to be made during a single shift at 20 yd intervals along a 200 yd longwall face. The tests can usually be made on non-coaling shifts without interfering with mining operations, such as belt-turning or ripping.

(3) *Measurements with the expanding-bolt seam tester.* The expanding-bolt tester has been used at a number of different collieries during development. In most cases penetrometer tests were undertaken at the same time, but this did not always prove practicable.

A summary of measurements for a number of different collieries is given in Table 6.5.

It is interesting to note from Table 6.5 that at Cardowan, Oxcroft, and Markham collieries the strength of coal in the rib side was much stronger than that in the stable or on the coal face. The tests in the rib side were made in a different direction to the cleat compared with those in the stable or on the face, but laboratory tests have shown that breakage force is not affected by cleat orientation, although cone size is. Underground, where the coal is much less regular than the samples used in the laboratory, it is not easy to measure the size of the conical fragment, but the force measurements alone suggest that the high strength of the coal in the rib side reflects the influence of strata loading on the coal. This feature is certainly encouraging, for the influence of the methods of roof support on coal strength is going to affect coal-face mechanization very considerably, and no reliable method of studying the relationship between support and strength exists.

(4) *Comparison between the penetrometer and expanding-bolt tests.* There is some evidence that the penetrometer also reflects changes in coal strength caused by strata loading, and at Cwmtillery Colliery, in the Garw Seam, the penetration resistance was found to fall from about 30 tons/in^2 to near zero when the coal was virtually destressed by undercutting the seam.

Moreover, at Cardowan and Donisthorpe collieries there is quite good correlation between the penetrometer and the expanding-bolt measurements (Table 6.5), particularly if only the results for the 17–19 in. horizon from Donisthorpe are considered. At the other horizon the coal was very heavily banded and the results were, in consequence, very variable.

At present it is not possible to say categorically that one test is better than the other. Laboratory experiments suggest that the expanding-bolt test is more sensitive to cleating than the penetrometer and is also more likely to reflect strength changes induced by strata loading, but more direct comparisons in the field are needed to decide this issue. It is likely that the techniques are complementary and that both will have their own uses.

The Impact Strength Index (I.S.I.)[14]

The penetrometer and the expanding-bolt seam test measure certain aspects of coal strength at the coal face and the magnitude of the measurements depend as much on the conditions that prevail at the face as on the coal type itself. However, once the coal is removed from the seam the natural strength of the coal controls the subsequent breakage. This may occur in loading the coal on to the face conveyor, during transfer from one conveyor to another or into storage bunkers, during loading into trams and skips, and during screening and washing. It is of interest to know whether difficulties found in maintaining satisfactory sizes after the coal leaves the face are attributable to the inherent strength of the coal.

A simple test, used widely in Russia, has been devised by Protodyakonov[15], but as it is described it has several faults that could give rise to error. The test consists basically of the crushing of a sample of coal fragments, larger than 10 mm, contained in a hollow cylinder closed at the bottom, by five successive drop-hammer blows. The percentage of coal that is broken down to pass a 5 mm sieve is used to calculate a "coefficient of strength". The faults in the method are concerned with the detailed specification of the test (e.g. sample size and selection) and not with the technique, on which the Impact Strength Index (I.S.I.) is itself based.

(1) *Development of I.S.I. test procedure.* Degradation of coal fragments by a succession of drop-hammer blows depends on a number of factors such as

(i) the size of the hammer,
(ii) the number of blows,
(iii) the height of fall,
(iv) the size of the original coal fragment,
(v) the quantity of coal in the sample,
(vi) the nature of the floor upon which the test is performed.

TABLE 6.5. TYPICAL MEASUREMENTS WITH THE EXPANDING-BOLT SEAM TESTER

Colliery, seam, and face	Method of working	Seam section	Horizon of test	Breakage force (ton)	Cone size Dia. (in.)	Cone size Depth (in.)	Cleat	Comments on test location	Max. resistance to penetration, ton/in²	ISI
Cardowan	Hand	27	9 in.	4·8	11 × 5	4½	4–24/ft	In rib side	18·8	63
Meikelhill	filled			3·6	—	5			13·6	63
Wee				2·5	—	5		Spaced along	9·2	64
No. 2 South				0·8	15 × 7	5	on bord	face	2·0	64
Oxcroft	Hand			4·4	7 × 7	4½		In rib side	—	
Clown	filled			0·9	12 × 11	4½		In stable hole	—	
21 s		46	24 in.	3·0	28 × 23	5	on bord	Spaced along	—	
				2·1	24 × 24	4½		face	—	
				2·4	16 × 17	5			—	
Parkhouse	Hand			1·1	25 × 20	4½		In stable hole	—	
Threequarters	filled			3·3	20 × 20	4	on bord	On face	—	57
50s		31	15 in.	2·7	20 × 14	4½			—	
Markham No. 1	Hand			0·5	—	5		In stable hole	—	
Threequarters	filled			0·2	—	5	on bord		—	63
T1 s		35	17 in.	4·3	—	4½		In rib side	—	
				3·4	—	5			—	
Donisthorpe	Anderton		17 in.	0·9		5			6	66
Woodfield	Shearer		32½ in.	5·6	18 × 13	5			16	68
Seam	Loader	46	19 in.	0·05	24 × 14	5			—	66
experimental			37 in.	0·3	45 × 30	5	24–48/ft	Spaced along	12	70
face			18½ in.	2·4	24 × 23	5	20° off	face	16	66
			33 in.	4·8	32 × 22	5	bord		24	68
			33 in.	3·0	26 × 14	5			10	68
			18 in.	1·8	32 × 28	5			12	66

Moreover, a test that is based on the degradation of sized fragments depends on the accurate measurement of fragment size, and the sieving methods used must be reproducible and cause the minimum of degradation.

As a result of extensive experiments involving comparisons between different coals, between different laboratories, and between different methods of sieving, in addition to the factors (i) to (vi) listed above, the following method was adopted for the routine test.

The apparatus consists of a vertical steel cylinder of 1·75 in. internal diameter, closed at the lower end by a screwed cap (see Fig. 6.13). A steel plunger, which weighs 4 lb and is $1\frac{11}{16}$ in. in diameter at the bottom, is fitted loosely inside the hollow cylinder. A steel cap, through which the plunger handle passes, is fitted to the cylinder and this cap has a dual purpose; it stops coal dust from escaping during the test and with the plunger raised to its full height it controls the height through which the plunger falls. This height is 12 in., measured from the bottom of the plunger to the inside surface of the base of the cylinder.

Basically the test consists of placing 100 g of coal in the $\frac{3}{8}$–$\frac{1}{8}$ in. size range in the cylinder and subjecting the assembly of fragments to 20 blows from the plunger, raised to and dropped from the maximum height each time. The amount of coal remaining in the initial size range after the test is defined as the I.S.I. The exact procedures used to prepare samples of coal and to perform the test are important and are given below.

(2) *Preparation of sample for test.* A sample of coal is taken from the face as a selected subsection, or from the surface as a run-of-mine sample. It is sieved through a 1 in. sieve, the oversize coal is spread on a concrete floor and individual lumps of coal are broken by an 8 lb flat-based steel earth rammer. The coal should be broken to give the maximum yield of fragments just under 1 in. size. Breakage and sieving are repeated consecutively until all of the coal passes the 1 in. mesh.

The $\frac{3}{8}$–$\frac{1}{8}$ in. fraction is then removed from the broken material by hand sieving, pieces trapped in the sieve meshes being rejected. Six fractions, each of about 125 g, are removed from the $\frac{3}{8}$–$\frac{1}{8}$ in. sample by incremental sampling, successive increments being added to different fractions. Each fraction is then sieved on a $\frac{1}{8}$ in. B.S. sieve (8 in. dia.) for 3 min using a Russell sieving machine (or a sieving machine that has been standardized against a Russell machine). 100 $\pm$ 0·05 g is then carefully weighed from the sieved fraction, care being taken to avoid further degradation.

(3) *Test procedure.* The 100 g sample is poured gently down inside the tilted hollow cylinder of the I.S.I. apparatus which is then placed vertically on a level solid floor (stone or concrete). The surface of the coal is levelled carefully with a spatula without shaking the coal. The base of the apparatus is steadied with the feet and the top cap is fitted with the plunger raised. Keeping the base steady with the feet the operator raises the plunger to the full extent and drops it freely 20 times. The impacting rate should not be

faster than one per 2 sec. Finally, the cap and the plunger are removed and the coal re-sieved through the $\frac{1}{8}$ in. B.S. sieve, using the same sieving machine. It is sometimes necessary to tap the side of the I.S.I. apparatus lightly to eject the material. The weight of coal in grammes remaining on the $\frac{1}{8}$ in. sieve, including the material trapped in the sieve, is the Impact Strength Index of the coal.

Six samples should be tested for each coal seam or subsection of each seam and the average value taken to be the I.S.I. I.S.I. values must differ

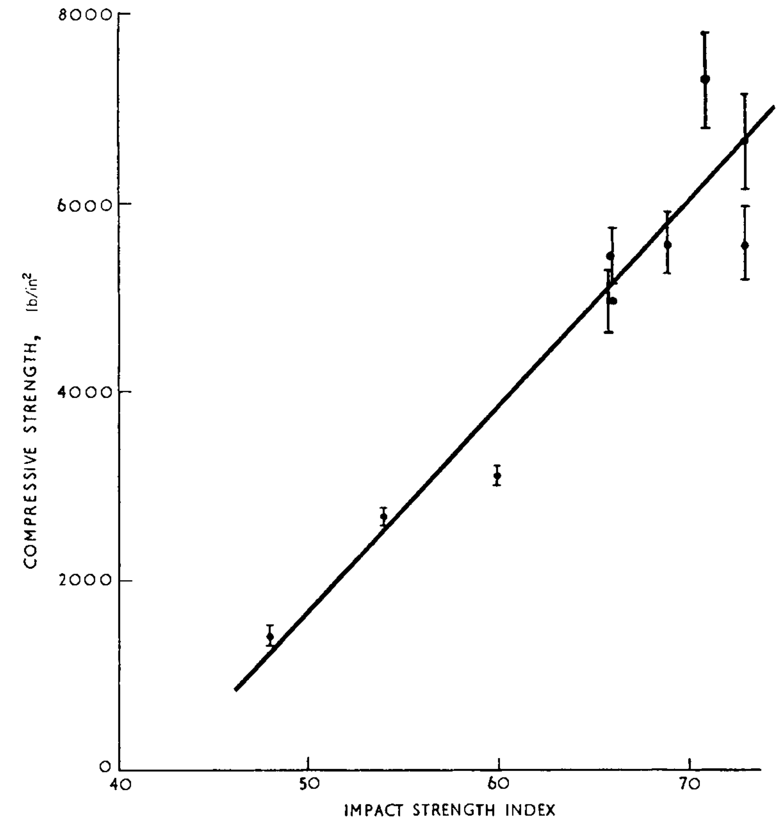

FIG. 6.14. Relationship between compressive strength of cubes of coal loaded perpendicular to bedding planes and I.S.I.

by at least three units before a strength difference is taken as real (or 4·5 units if the tests are done in different laboratories).

(4) *Relationship between I.S.I. and uniaxial strength measurements.* Measurements of I.S.I. have been related to compressive and tensile strength measurements made on samples prepared from a number of coals that have widely different mechanical properties. The best correlation has been found

between the I.S.I. and the compressive strength of 1 in. cubes loaded perpendicular to the bedding planes. This is shown in Fig. 6.14. A less linear relation is found with cubes compressed parallel to the bedding planes.

Tensile strength measurements are related to the I.S.I. values as shown in Fig. 6.15. It is interesting to note that the tensile strengths of coals of

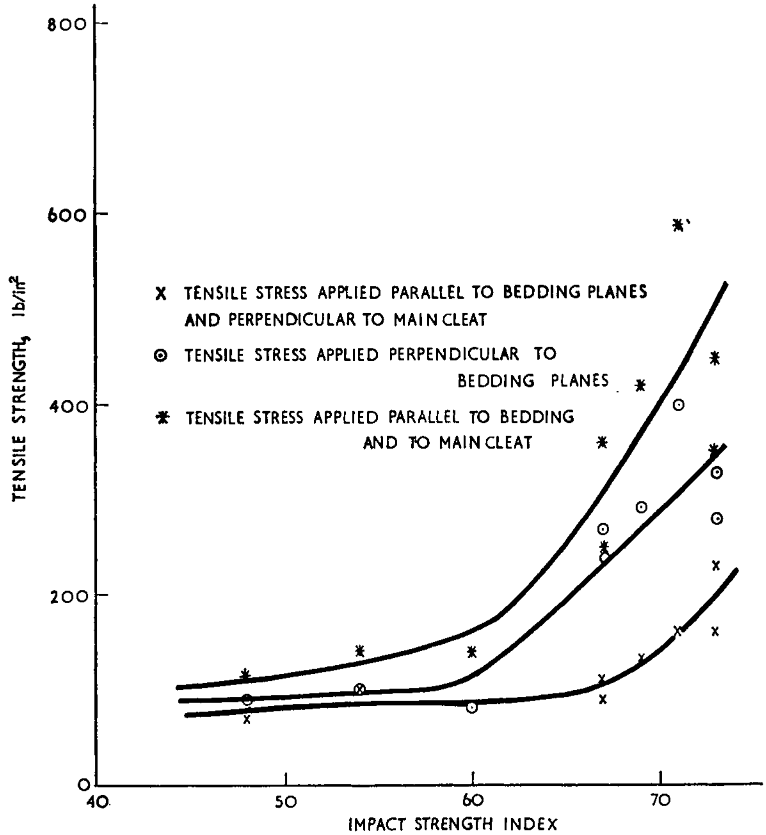

FIG. 6.15. Relationship between tensile strength and I.S.I.

I.S.I. less than about 60 are almost constant, but that the tensile strengths build up rapidly for I.S.I. values greater than 60. As brittle materials tend to break in tension we might expect, therefore, that coals with I.S.I.'s less than 60 are likely to be broken much more easily during transit than the coals of higher I.S.I.

Berenbaum[4] has recently derived a method for classifying coals broken by free fall in a shatter test. The strengths of the coals are measured in terms of the probability of survival (C) of unit sized lumps falling 8 ft onto a standard surface. The strengths of some of the coals increase if they are dropped several times in succession, the major cracks gradually being eliminated as the fragments break. Other coals retain an almost constant strength as they undergo a sequence of falls onto the shatter surface, each drop possibly introducing new weaknesses that cause the lumps to break in a subsequent fall. However, as shown in Fig. 6.16, C (plotted as $-\log C$) is closely related to I.S.I. particularly for samples of coal that have been dropped five times. A marked decrease in shatter strength occurs as I.S.I. decreases below about 60 to 65, as was suggested by the tensile strength observations.

It is apparent, therefore, that the I.S.I. tests provide a very good measure of the compressive strength of coal, when loaded perpendicular to the bedding planes, and also of the degradation that is likely to occur between the coal face and the consumer. Much greater care must be taken of coals of I.S.I. lower than 60 than of the stronger coals if there is not to be a marked decrease in coal size and hence in the market value of the product.

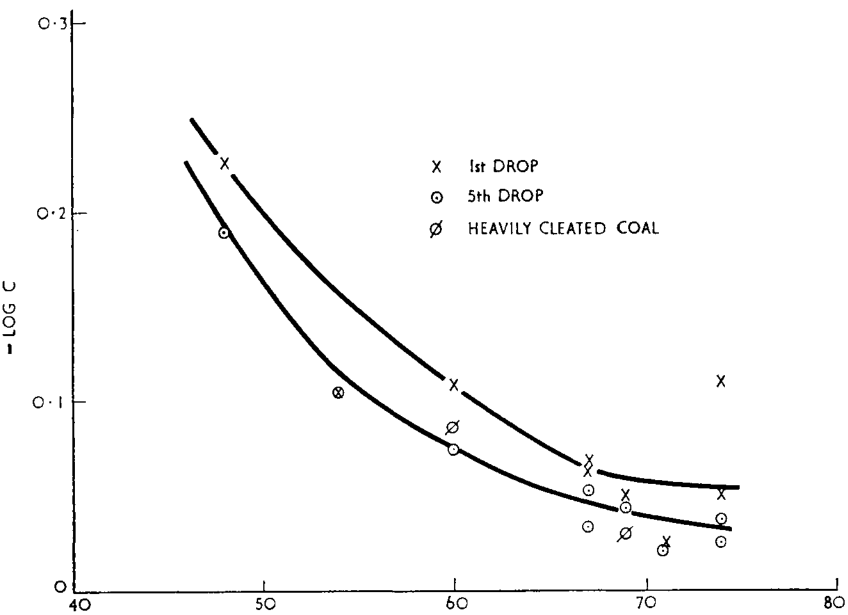

FIG. 6.16. Relationship between shatter strength and I.S.I. C is the probability of survival of a 1 in. lump of coal falling through a distance of 18 ft onto a standard surface.

Evaluation of Coal Strength Tests on Production Coal-plough Faces

If routine strength tests are to be of any value they must ultimately be related to production problems. This is difficult in practice since no two mining installations are identical and the nature of the coal, the seam thickness, the surrounding strata, the mining history in the district and many other uncontrollable factors will all affect the performance of the machines. However, by concentrating observations on one particular mining method, namely coal ploughing, it has proved possible to consider the usefulness of the I.S.I. and penetrometer measurements for assessing ploughability. The expanding-bolt test has only recently been developed and so a full comparison of this technique with the other two has not yet been attempted.

On nine coal-plough faces the forces in the leading and trailing chains were measured for different pressures in the pushing cylinders used to hold the armoured conveyor and plough against the coal face[16]. Variation in the pusher pressure controlled the depth of cut taken by the plough and in general there was a linear relationship between the haulage force and depth of cut on every installation studied (a typical example is given in Fig. 6.17). The slope of this line, S_l is taken to represent the potential performance of the installation. S_l has been related to the I.S.I. and penetrometer data

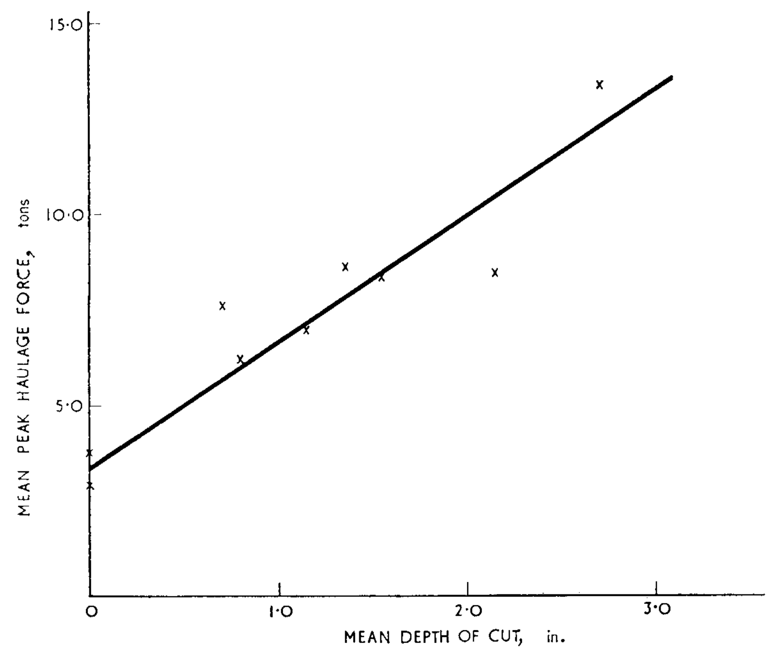

FIG. 6.17. Pentremawr Colliery, mean depth of cut versus mean peak haulage force.

obtained for the coal in the section actually extracted by the plough. *Sl* has also been related to the volatiles in the coal since these are known to be related to the mechanical properties of coal[14].

(1) *Relationship between S_l and resistance to penetration.* Figure 6.18 relates S_l, the haulage force per unit depth of cut, to the maximum resistance to

penetration (max. R.P.) and with the exception of the results for Granville Colliery there is an extremely close linear correlation.

The principal difference between the coal at Granville Colliery and the others is that it had only light cleating. It seems, therefore, that the penetrometer gives a very reliable comparison when the coal is fractured,

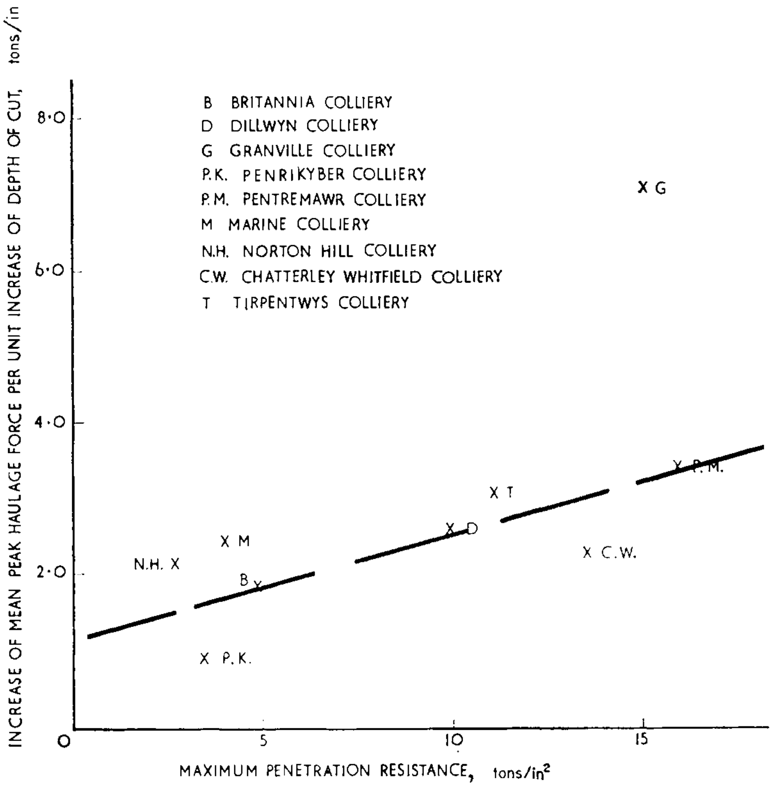

FIG. 6.18. Ploughing forces related to penetrometer measurements.

but it might be misleading if the coal remains unbroken, despite inherent weaknesses or stresses induced by the overlying strata.

The graph (Fig. 6.18) also shows that most of the coals studied can be ploughed by a force of less than 4 tons/in. of cut, and even though frictional and loading forces must be added to this figure to get the true haulage force in the plough chain there is no reason why cuts 3 in. deep should not be made consistently. In other words, provided the coal is cracked, by slips, cleats, or by strata loading, it will be ploughable if the maximum resistance to penetration is less than about 20 tons/in^2, within the depth of cut taken by the plough.

(2) *Relationship between S_t and Impact Strength Index.* Figure 6.19 relates S_t to I.S.I., and for the coals from South-Western Division there is a very good linear correlation. However, as these coals are heavily cracked both naturally and by strata loading it is possible that the strength of coal on the face and in the extracted state is comparable. The results for Granville and Chatterley Whitfield collieries, however, show that I.S.I. values can be misleading. Moreover, I.S.I.'s cannot reflect changes in strength caused by strata loading, so that the test cannot be advocated as a very reliable method of seam assessment.

(3) *Relationship between S_t and volatiles.* The percentage volatiles in coal can provide some guidance on strength, since both low- and high-volatile coals are strong, while those of middle range are weak[14]. In consequence, there

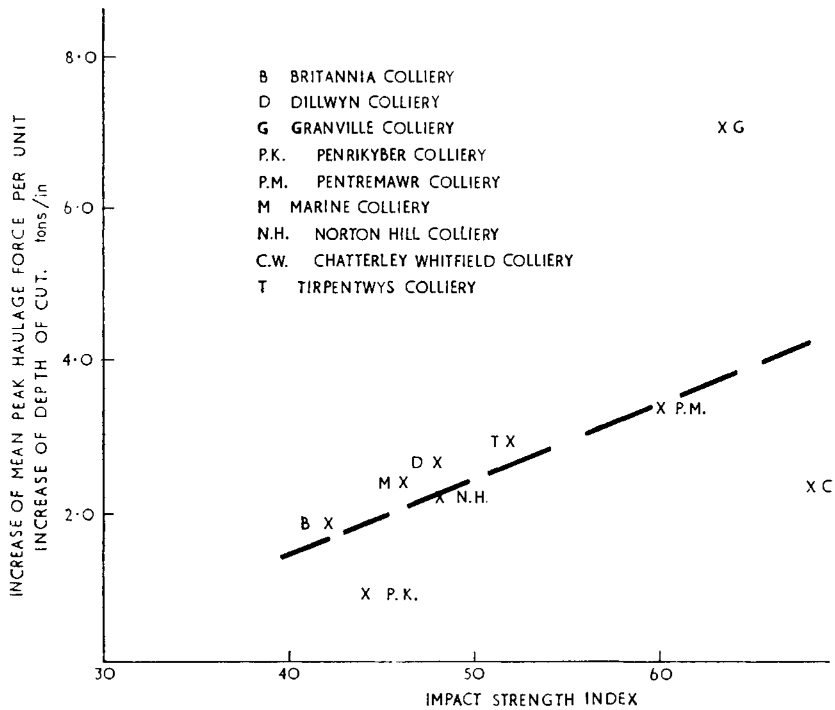

FIG. 6.19. Ploughing forces related to the I.S.I.

is a U-shaped relationship between S_t and volatiles, as shown in Fig. 6.20. Again the results for Granville Colliery are anomalous, and the scatter between results for the other high-volatile coals is large, but this tendency has been noted elsewhere[17]. However, the use of the volatiles in coal to provide a strength assessment provides no more than a rough guide, for the same reasons that the I.S.I. is unacceptable as a seam test.

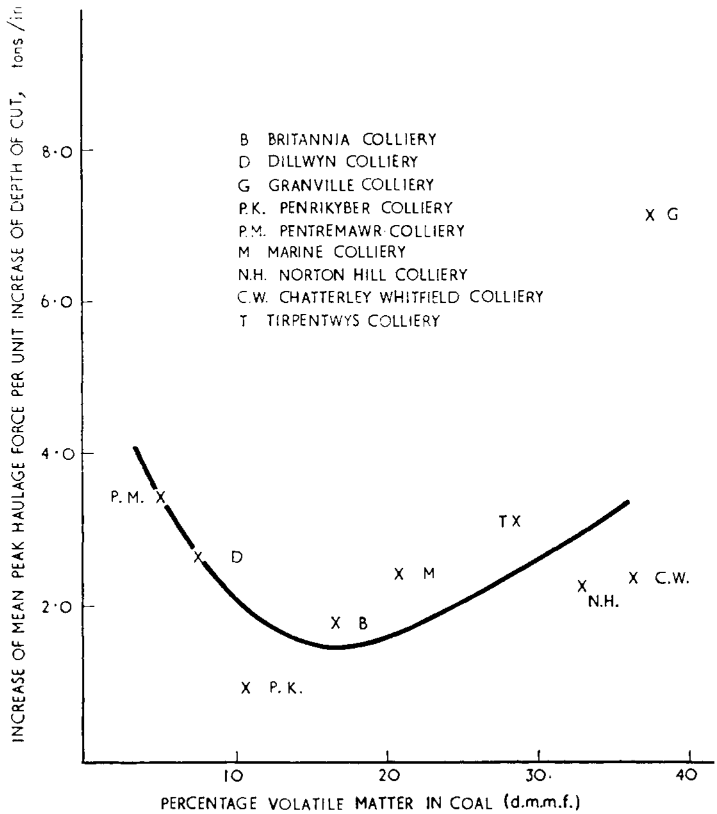

FIG. 6.20. Ploughing forces related to volatiles.

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CHAPTER 7

The Size Distribution of Run-of-mine Coal

THE distribution of sizes among broken coal shows certain regularities which have for a long while excited attention and speculation. If an analysis is made of size and weight produced during a breakage process, e.g. a shatter test or a particular mode of winning at the coal face, a graph resembling Fig. 7.1(*a*) is frequently found. In this graph the percentage by weight passing a particular sieve size is plotted against sieve size on a log-log scale. The distribution of finer sizes yields a smooth curve, while the upper portion of the curve may be irregular. As the breakage process becomes more severe the irregular

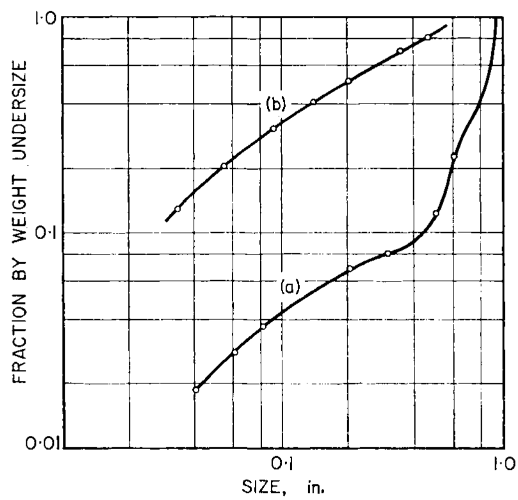

FIG. 7.1. Typical size distribution curves.

portion of the curve is virtually eliminated, and the distribution curve takes the smooth form of Fig. 7.1(b).

The irregularity of Fig. 7.1(a) can be explained qualitatively on the following grounds: breakage processes tend to impress localized applied forces on the initial material, whether this is coal in the seam, or a large lump that has already been won from the seam. In the former case the localized force is the stress concentration due to the cutting edge of a tool or wedge, and in the latter case it is the force generated by impact when the large lump falls over a transfer point on to a bearing surface and is arrested, the area of contact at impact being usually very small. These localized forces, compressive in nature at the points of contact, can generate tensile forces in the body of the material which lead to tearing and the breaking away of large fragments (Fig. 7.2). This major disruption is accompanied by a certain amount of detritus from the surfaces that are wrenched apart, and by localized crushing

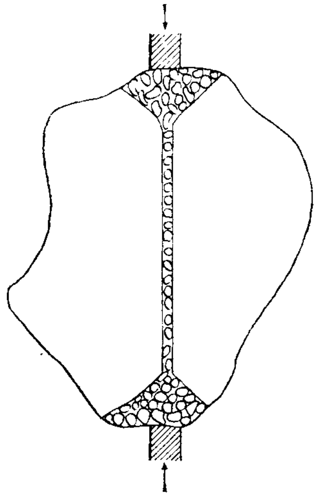

FIG. 7.2. Pattern of breakage for lump of coal broken by localized forces.

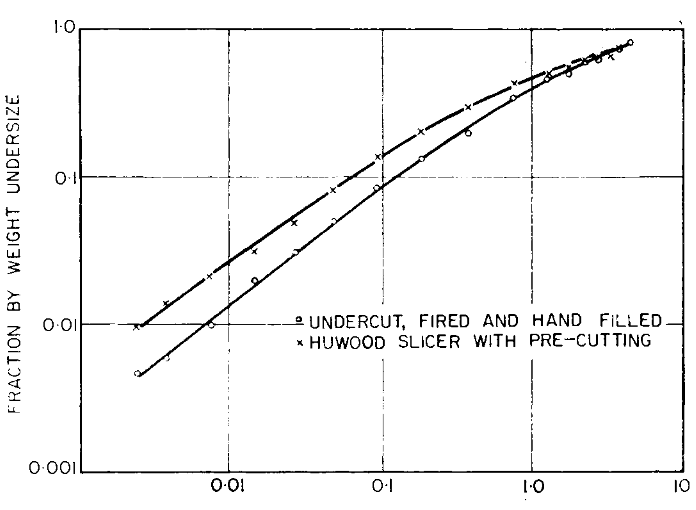

FIG. 7.3. Size distributions, Easington colliery, Low Main seam.

in the vicinity of the area of contact of the coal and the applied stresses. Thus two modes of breakage, the large and small sizes, are produced. As further breakage takes place the product is again broken in the same way, and the larger lumps are quickly reduced in size, with the result that the size distribution curve takes on a greater regularity.

The analysis of run-of-mine coal shows that the output is without exception so degraded that a smooth curve is obtained right up to the maximum size of coal that can be handled conveniently in this type of analysis, i.e. about 6 in. A typical example is shown in Fig. 7.3. The wonderfully regular curve defined by observations that extend from sizes of 4·5 in. down to sizes of 0·0025 in. is obvious. The general shape of the curve remains remarkably constant for various coals and various means of breakage, indicating that some regular mechanism is at work. The present chapter describes some ideas that are current in this field, with particular reference to those that have been developed at the Mining Research Establishment.

Distribution Laws for Size and Weight of Broken Coal

A number of laws for the size distribution of materials which have been fragmented in various ways have been suggested, most of them frankly empirical. Only a very few are worthy of notice in relation to broken coal.

Martin, Blyth and Tongue[1], in a study of the crushing of sand, showed that in every instance the frequency curve of size possessed the form

$$\frac{\delta N}{\delta x} = a_1 \exp(-b_1 x)$$

where x = diameter of particle, δN = number of particles in the size range x to $x + \delta x$, and a_1 and b_1 are constants depending on the sample of sand tested. Heywood[2] has shown that the equation applies approximately to certain powdered coals.

Gaudin[3] has suggested a cumulative weight distribution of the form

$$S_x = (x/x_{o1})^{n_1}$$

where S_x = fraction of material by weight finer than size x, and x_{o1} and n_1 are disposable constants.

This expression holds for small sizes, but is admittedly incorrect for coarse sizes.

A cumulative weight distribution curve which has gained wide acceptance is that due to Rosin and Rammler[4]. It may be written

$$R_x = \exp -(x/x_{o2})^{n_2},$$

where R_x = fraction by weight coarser than size x, and x_{o2} and n_2 are disposable constants.

The formulators of this equation state that it is "a universal law of size distribution valid for all powders, and irrespective of the nature of the material and the method of grinding". It does, in fact, appear to be a good fit to many weight distributions for broken coal. Obviously x_{o2} is the size above which a fraction $1/e$, or approximately 37 per cent by weight of material, is found. The significance of n_2 may be demonstrated in the following way: if log log $(1/R_x)$ is plotted against log x (a conventional way of plotting the Rosin–Rammler distribution), n_2 is the slope of the resulting straight line.

Clearly, Gaudin's equation is a special case of the Rosin–Rammler equation, for $(x/x_{o2})^{n_2}$ small.

Theoretical Attempts at Distribution Laws

Theoretical attempts to formulate distribution laws have been few, which is not surprising in view of the difficulties of the subject. It is appropriate to mention the attempt of Griffith[5] who achieves a distribution which, in a special instance, resembles Gaudin's equation. This is done by an approach via statistical mechanics in a manner resembling the application of this subject to the behaviour of gases. The application of the theory to macroscopic matter which is not itself necessarily in motion is ingenious, but perhaps a little dubious.

The breakage of coal is obviously a function of two main factors, the distribution of stress in the material and the distribution of weaknesses. If the distribution of stress is disregarded, or taken to be uniform, then the best hope of achieving a size distribution of the Rosin–Rammler type is to associate the distribution of weaknesses with algebraic terms which contain exponential functions. An attempt of this nature was made by Bennett[6]. He suggests that breakage occurs along planes of weakness in the coal, and that these planes can be assumed to belong to three sets, normal to mutually perpendicular axes. The distribution of weaknesses of a particular set along the axis intersecting them is assumed to be random, in that intersections are as liable to occur at any particular point as at any other. This leads to the Poisson law of distribution for distances between consecutive planes, and these intercepts, it is assumed, become the edges of fragments when breakage takes place.

Bennett deduces the equation

$$R_x = \exp -(x/x_{o3}),$$

and this law is propounded by Bennett and his colleagues as the "ideal law of breakage" of a single lump of material subjected to a single set of boundary forces suddenly applied. The extension of this result to the Rosin–Rammler equation is made by a semi-empirical correction.

However, Manning[7] has pointed out that serious objection can be taken to the mathematical development of Bennett's argument. He shows that even with the liberal aid of approximations the simplest form that can be made of the final equation is

$$R_x = \exp -(x/x_{o3})^2.$$

This equation, although it destroys the essential simplicity of Bennett's ideal law, could nevertheless be modified by the same semi-empirical correction to give the Rosin–Rammler equation. Long[8], who has made a further study of Bennett's theory, comes to the conclusion that if Manning's simplifying assumptions are rejected on physical grounds, the resulting distribution cannot be brought into agreement with the Rosin–Rammler equation.

Manning himself[9] has deduced a theoretical distribution of the form

$$R_x = \exp -(x/x_{o4})^3\{1 + (x/x_{o4})^3\}.$$

This is derived purely from statistical considerations. He does not relate his theory to any physical mechanism of breakage, nor has he investigated its applicability to broken coal in any great detail.

An Alternative Approach to the Problem

It may even be misguided effort to try to set up a theoretical model that will yield the Rosin–Rammler law, as the law is only an approximate fit (though the approximation is a good one) to available data, and there is no guarantee that it has any fundamental validity. In view of this, a different approach to the problem was made by Evans[10] which will be set out in what follows. The argument is very simple, but it is not mathematically rigorous, and in developing it some appeals to mathematical and physical intuition are made. It may well be that the problem is in fact intractable by rigorous analytical methods; certainly the interaction of non-uniform stress fields with materials of complicated shape that are ramified with weaknesses is a problem which seems to hold no promise of a simple theoretical outcome. Even so, there are some points of special interest in the attempt made by Evans to derive a law of size distribution, as it is based upon experimental observations on the relationship between size of specimen and strength.

The argument to be presented proceeds from a consideration of work on the strength of cubes of coal in uniaxial compression carried out by Evans and Pomeroy[11]. It has been shown that cubes cut from a lump of superficially uniform coal show a wide variation in compressive strength, so much so that strength can be adequately described only in statistical terms. If, nevertheless, attention is restricted temporarily to mean breaking stress, this quantity increases with decreasing size of specimen. It might be expected that a similar situation would exist in an assemblage of irregular fragments, that smaller fragments would on the whole be stronger than larger fragments, and that a pattern of size distribution might emerge from considerations of strength. The converse idea, that a pattern of strength might emerge from considerations of size, has in fact been studied by R.L. Brown[12], who showed that a relation existed between shatter strength and sieve size for lumps of coal.

The fundamental relation governing statistical considerations of strength has been shown to be as follows[11 13]: if for a particular coal, P_a is the probability of cubes of side a surviving a particular stress and P_b the probability of cubes of side b surviving the same stress, the relation between the two quantities is

$$P_b = P_a^{b/a}.$$

The equation represents a simple example of the "weakest-link" theory of breakage, in which the breakage of a system consisting of a number of elements linked end to end is precipitated by the rupture of a single element. The application of the theory to the breakage of coal seems to lie in terms of the propagation under tensile stress of cracks contained in the coal, the extension of the "weakest" crack (i.e. the one which propagates under the least stress) initiating a chain of breakage throughout the specimen.

It is not immediately obvious that irregular fragments should obey the weakest-link law, but the supposition becomes tenable when it is remembered that the law applies to the structural elements of the material, and hence to any fragment, whatever its shape. Moreover, Evans, Pomeroy and Berenbaum[13], who have investigated the compressive strength of irregular fragments, have shown that while direct corroborative results are difficult to obtain because of experimental scatter, observations are certainly not at variance with the assumption that the weakest-link theory applies. In this application, the representative dimension of a fragment has been taken to be the cube root of the volume.

These facts constitute the entire physical basis of Evans' theory. From there on the argument is intuitive and mathematical.

As a preliminary to the consideration of irregular fragments, consider an assemblage of cubes. Suppose the assemblage consists of equal numbers of cubes of side a, $2a$, $3a$ ... ma, and suppose that all cubes are subjected to uniaxial compression to a stress σ_c. Then the relative numbers surviving the stress will be proportional to P, P^2, P^3 ... P^m respectively, and the weight of coal surviving at the size will be proportional to Pa^3, $P^2(2a)^3$, $P^3(3a)^3$ ... $P^m(ma)^3$. These expressions have the form of products of powers of P and a.

Now suppose a start is made with a large number of cubes of side ma, all subjected to a stress σ_c. Some of the cubes will break; let it then be assumed that the breakage products are cubical, and are also subjected to breakage stress σ_c, and so on through a great range of size. A situation will be reached eventually when the cubes being crushed are many orders of magnitude different in size from ma, so that the distribution of sizes has attained a stable state over a wide range. In such a condition it can be postulated that the number of cubes of a particular size present is a function of two parameters. One is size itself, because it would seem to be a natural tendency for the numbers of a small cube to be larger than the numbers of a larger cube. This tendency is modified by the probability of the particular size surviving the applied stress, for the small cubes have a smaller probability of being broken by the stress. Thus the number of cubes of size a may be written $f(P, a)$, and the various numbers of cubes of related size are in the ratio $f(P^m, ma) \ldots f(P^2, 2a), f(P, a) \ldots$

One of the simplest forms of the function f which can be suggested empirically, and appears reasonable on the grounds already cited, is a product of powers of the two variables, i.e. of the form $(P^r)^{\alpha'}(ra)^{\beta'}$. The weight at a particular size is therefore proportional to $P^{r\alpha'}(ra)^{\beta'+3}$, and hence to $P^{r\alpha'}r^{\beta'+3}$, as a is regarded as an arbitrary constant in this analysis.

The argument can now be applied to an assemblage of fragments of various sizes and not of regular shape, such as may be found in the output from a coal-face or from a mine as a whole. It is assumed that the breakage process has reached a stage at which the large sizes giving an irregular distribution curve have been eliminated; on the views of Bennett, Brown and Crone(14), this would represent a state of affairs at which more or less uniform stresses have penetrated throughout the material. It seems reasonable, therefore, to attribute the characteristics of the assemblage to a particular stress level, this level being related to the severity of the breakage during the mining process and during conveyance. It can be suggested on the basis of the argument relating to an assemblage of cubes that for a continuum of size the weight of material in size range z to $z + dz$ (taking this to mean sieve sizes) may possibly be given by a function of the form $P^{qz}z^p dz$, where p and q are constants.

If the continuum of size extends theoretically from 0 to ∞, the fraction by weight under size z is given by

$$S_z = \frac{\int_0^z P^{qz} z^p \, dz}{\int_0^\infty P^{qz} z^p \, dz}. \tag{7.1}$$

These integrals may be transformed to a more convenient form. Write

$$P^{qz} = \exp(\log P^{qz}) = \exp(qz \log P) \quad \text{and let} \quad qz \log P = -x.$$

This is permissible, because P always lies between O and 1 and $\log P$ is therefore intrinsically negative. Then

$$S_z = \frac{\int_0^x e^{-x} x^p \, dx}{\int_0^\infty e^{-x} x^p \, dx}. \tag{7.2}$$

In this form the integrals are recognisable as gamma functions, the denominator as the complete gamma function, defined as $\Gamma(p + 1)$, and the numerator as the incomplete gamma function. The ratio given in eqn. (7.2) has been defined by Pearson[15] as the function $I(x, p)$. The lower limit of the argument p is -1.

Comparison with Experiment

For arithmetical convenience the function calculated by Pearson is not $I(x, p)$ but $I(u, p)$, where $u = x/(p + 1)^{1/2} = -qz \log P/(p + 1)^{1/2}$. Now a "fraction-undersize" graph is usually plotted on log-log paper, so that the substitution of u for x, and x for $-qz \log P$, makes no difference to the shape of the curve, but merely displaces it bodily. The function $I(u, p)$ is plotted, for various values of p, in Fig. 7.4. It is an extremely good fit to a variety of size distribution curves, for example those in Figs. 7.5, 7.6, and 7.7.

These are a few of many examples which could be cited.

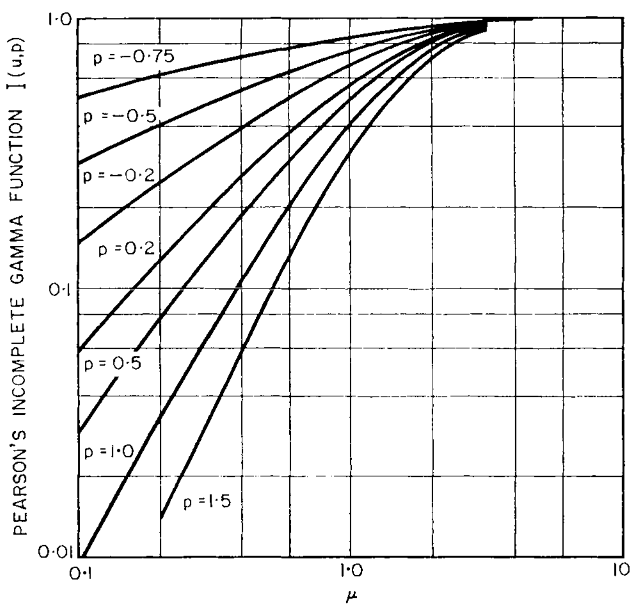

FIG. 7.4. Graph of the function $I(u, p)$.

Although the fit is a very good one, it will be observed that there is a tendency for the theoretical curve to lie slightly above the experimental one for small values of u. One cannot be sure whether or not this discrepancy is significant, but it is noteworthy that many of the acknowledged perturbations of size analysis, such as inefficient sieving, agglomeration of small particles,

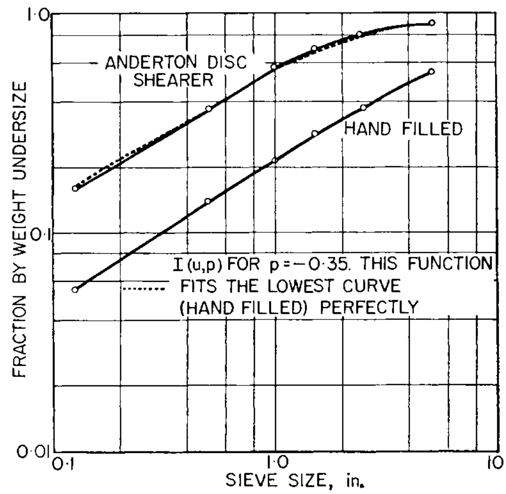

FIG. 7.5. Data from Gedling colliery (E. Midlands).

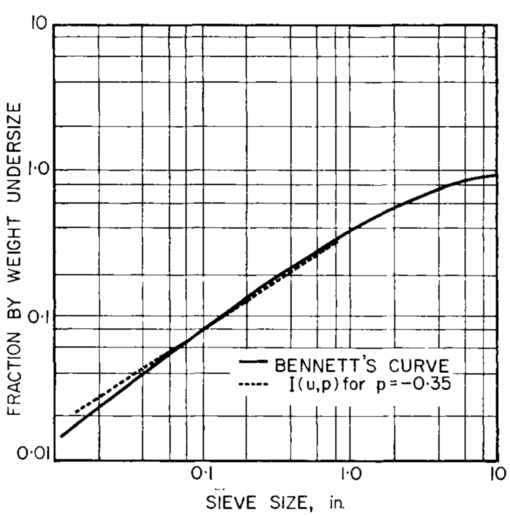

FIG. 7.6. Mean size distribution of coal from twenty different seams and collieries (after J. G. Bennett).

jamming of several small particles simultaneously in a mesh, would tend to reduce the apparent yield of small sizes. Against this trend would act the tendency of larger sizes to break as a result of sieving stresses, but the balance of advantage may well lie with the depopulation of smaller sizes.

If it is required to find the algebraic expression of a particular curve the procedure to be followed is given in the Appendix.

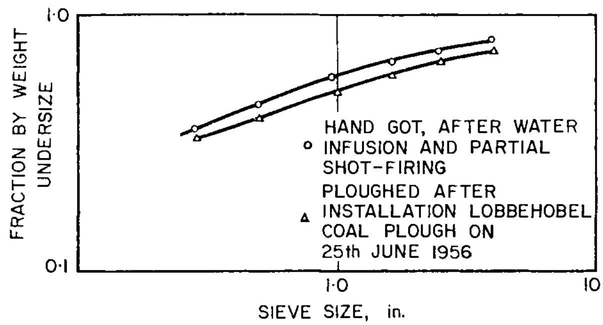

FIG. 7.7. Output from Cefn Coed colliery, Dulais seam.

RELATION OF INCOMPLETE GAMMA FUNCTION TO ROSIN–RAMMLER DISTRIBUTION

Since the Rosin–Rammler equation also appears to be a good fit to size distribution curves, $I(u, p)$ would be expected to approximate to it. It can be shown analytically that this should be so under the special conditions of u small and p small.

Consider $S_u = I(u, p) = C_u \int_o^u e^{-u} u^p \, du$, where C_u is a constant.

Now if u is small, we may write $e^{-u} \approx 1 - u$ so that

$$S_u \approx \frac{C_u u^{p+1}}{p+1}\left(1 - \frac{p+1}{p+2}u\right) \approx \frac{C_u u^{p+1}}{p+1}$$

Now the fraction by weight oversize R_u is equal to $1 - S_u$ so that

$$R_u = 1 - \frac{C_u u^{p+1}}{p+1}$$

and if p is small, $R_u \approx \exp -C_u u^{p+1}$, which is of the form of the Rosin–Rammler equation.

Size Distribution of Dust

The smoothness of the size distribution curve at larger sizes raises the question of whether the curve continues down to the smallest measurable sizes (of the order of microns). This question has practical implications of great importance, for airborne dust (i.e. smaller than about 60 μ) is liable to explode if present in sufficient quantity when a gas ignition takes place, and the respirable fraction (i.e. smaller than about 5 μ) provides one of the mining industry's great health hazards. If size distribution curves are continuous down to the smallest sizes, then the amount and size of dust is determined inevitably by breakage at larger sizes, and the fraction of large coal that is obtained from a particular method of winning will decide the amount of airborne dust that is present.

Theory enters in the following way: we have found from eqn. (7.2)

$$S_x = C_x \int_o^x e^{-x} x^p \, dx,$$

where C_x is a constant and

$$\frac{dS_x}{dx} = C_x e^{-x} x^p.$$

As x tends to very small sizes

$$\frac{dS_x}{dx} \to C_x x^p.$$

If δN is the number of fragments in the size range x to $x + \delta x$, then we may write

$$\delta S_x = \delta N \, . \, \lambda x^3,$$

where λ is an empirical shape factor.

Then

$$\frac{\delta N}{\delta x} \approx \frac{C_x}{\lambda x^{3-p}}. \tag{7.3}$$

Thus the continuity of the size distribution curve implies that size distribution in the dust range should exhibit a power law.

Hamilton and Knight[16] have carried out experiments that confirm this point. They broke coal by two different methods, slow compression and shatter, and analysed the distribution of sizes, using sieving for the larger sizes and microscopical counting for the smaller. Hamilton and Knight showed that the curve of fraction by weight undersize is continuous into the dust range. Owing to experimental difficulties their distribution curves are not very regular (see Fig. 7.8 for an example), and an $I(u, p)$ curve can be fitted only approximately, but the appropriate values of p appear to be as in Table 7.1.

TABLE 7.1. APPROPRIATE VALUES OF p

Coal	p	Exponent of $(1/x)$ from equation (7.3)
Ashington Plessey	0·7	2·3
Cwm Mildred	1·5	1·5
Llandebie anthracite	0·8	2·2

Thus, fitting a gamma function to the curves of Hamilton and Knight, a prediction can be made of the exponent that applies to the distribution of the very smallest sizes. This can be checked against the experimental value deduced from counting. Hamilton and Knight found in practice an exponent of 2·25 for all coals.

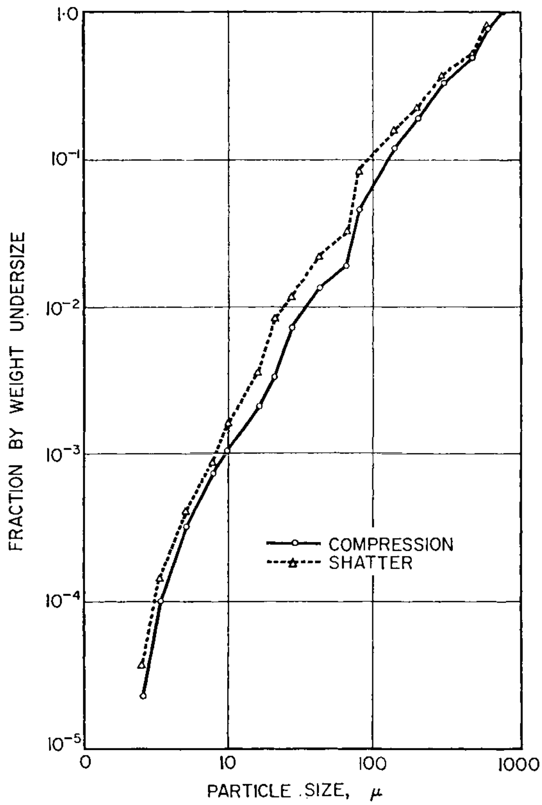

FIG. 7.8. Cumulative mass distribution curve for Ashington Plessey coal.

Hence there is reasonable evidence for the assertion that every breakage pattern in coal carries a tail that extends into the range of the very smallest examinable sizes. Hamilton and Knight show that the results of Wynn and Dawes[17] on the size distribution of underground airborne coal dust clouds can be interpreted on this assumption.

COMPARISON OF CURVES FOR RUN-OF-MINE COAL

It will be noticed that practically all the sizing curves for output from mines that have been given here have the same shape, corresponding in terms of the gamma function to an exponent p of value $-0{\cdot}35$ or a Rosin–Rammler exponent n of 0·75 (the curves of Fig. 7.7, to which a value of p of $-0{\cdot}65$ applies, relate to anthracite, a coal with unusual properties). Why this is so is a matter that has not yet been explained. If lumps of coal in the laboratory are subjected to a single incident of breakage the curves are steeper. Bennett, Brown and Crone have shown that when single large lumps of coal are broken by being dropped from a considerable height on to a solid floor the breakage products follow a simple exponential law corresponding to an n of unity or a p of zero. Hamilton's cumulative weight distribution curves, from the experiments already referred to, may be characterized by p values of between 0·7 and 1·5.

If the breakage products of such a test are subjected to successive incidents or "cycles" of breakage the cumulative curve of weight distribution will become less steep, i.e. the p value is reduced. If the winning of coal at the face is regarded as a "cycle" there are probably a large number of cycles involved before the coal emerges from the pit, each fall over a transfer point, each jolt in a mine car, contributing to the total. One must suppose that the value of p decreases with increasing cycles, but more and more slowly, and at about $p = -0{\cdot}35$ the form of the curve is evidently very insensitive to the exact number. There would appear to be considerable scope for a reduction of this effective number by attention to transfer and loading points so as to reduce the forces that cause breakage.

If eqn. (7.2) can be relied upon down to dust sizes, as the evidence mentioned in the previous section suggests, it contains astonishing implications about the amount of coal coming out of a pit that has been reduced to respirable sizes. When applied to the curve of Fig. 7.6, for example (see Appendix), it indicates that about $\frac{1}{700}$ part of the total output has been reduced to sizes below 5 μ. Whether this is true or not remains to be found out. It is possible to see one circumstance that would tend to reduce this amount, and that is air resistance during fall at transfer points, which would have a marked effect in buoying up very tiny particles, thereby allowing them to suffer less severe breakage forces than larger pieces.

Only a small fraction of this make of dust is actually dispersed in the air, hence becoming potentially respirable, the amount depending upon the violence of the breaking process; the greater this is, the greater the amount dispersed. Moreover, very small particles, even when formed, have the habit of agglomerating into larger ones under the action of surface forces. The amount of respirable dust in the air is therefore a complex matter, even when breakage processes that produce the dust are understood and a quantitative estimate of dust production can be made. However, the results of tentative calculations of this nature provide no comfort for those concerned with the possible dust problems that may be associated with intensive mining, where high outputs of coal that is severely degraded at the face itself are contemplated.

Mode of Breakage by Machines

Certain types of machine, e.g. ploughs and drum shearers, perform a breakage action that is essentially uniform in that the whole height of the seam is won by the same means (the contribution of top coal is neglected for the moment). For example, the plough takes off a slab of coal and the shearer a more complex layer determined by the path traced out in space by the picks under the action of linear and circular movements. Other machines or processes carry out a composite action, e.g. that of undercutting and blasting, where the size distribution of the gummings produced by the jib cutter may

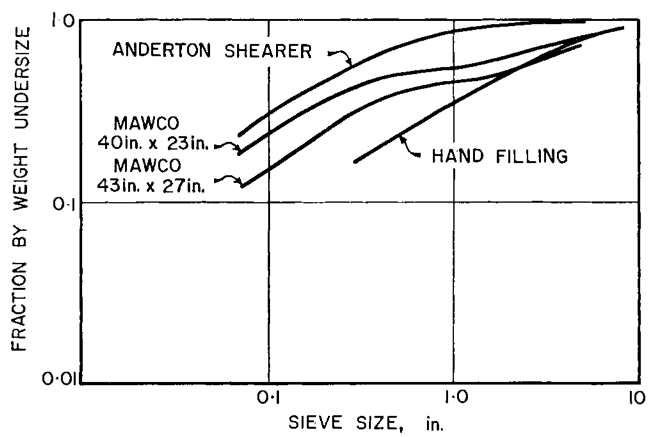

FIG. 7.9. Output by various means from Whitwell colliery.

be very different from that found for the coal broken by explosives. With the trepanner, the gummings from the annulus may be differently sized from the breakage products of the core. In general, though, the further breakage to which all components are subjected during conveyance from the face is sufficient to merge the two or more individual distribution curves associated with the process, so that no break in progression is seen in the final curve for the output. Occasionally such a break can be seen. The Mawco[18] is a machine that has a jib bent through two right angles so as to isolate virtually a rectangular beam of coal as it cuts along the seam. Size distribution curves for the Mawco (Fig. 7.9) show a distinct "wiggle" in the vicinity of 1 in., and it is plausible and probably correct to distinguish an upper portion of the curve that is attributable to breakage of the beam, the smaller fractions of the breakage being added to the gummings to form the lower portion of the curve. The junction of the two distributions has not been eradicated by breakage during conveyance.

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APPENDIX

Assume that the general form of the cumulative weight distribution curve is

$$S_y = \frac{\int_0^y e^{-\left(\frac{y}{y_0}\right)} \left(\frac{y}{y_0}\right)^p dx}{\int_0^\infty e^{-\left(\frac{y}{y_0}\right)} \left(\frac{y}{y_0}\right)^p dx}$$

where S_y is fraction undersize, y is size, and y_o and p are arbitrary constants. The equation may be written

$$S_y = A_y \int_o^y e^{-\left(\frac{y}{y_o}\right)} \left(\frac{y}{y_o}\right)^p \mathrm{d}y.$$

Let

$$x = \frac{y}{y_o}.$$

Then

$$\mathrm{d}x = \frac{1}{y_o}\,\mathrm{d}y$$

so that

$$S_x = A_y \int_o^x e^{-x} x^p y_o \,\mathrm{d}x.$$

Hence

$$A_y = \frac{1}{y_o \Gamma(p+1)}.$$

Take as a particular example the curve of Fig. 7.6 giving the mean distribution for twenty different seams and collieries. The appropriate value of p, found by matching incomplete gamma function curves (Fig. 7.4) to the graph is $-0{\cdot}35$. S_y has a value of 0·5 for $y = 1{\cdot}8$ in. Pearson's Tables show that for $I(u, p) = 0{\cdot}5$, $p = -0{\cdot}35$, $u = 0{\cdot}45$.

From Pearson's definition

$$u = \frac{x}{(p+1)^{\frac{1}{2}}}.$$

Hence

$$x = 0{\cdot}45 \times 0{\cdot}65^{\frac{1}{2}} = 0{\cdot}364$$

and

$$\frac{1{\cdot}8}{y_o} = 0{\cdot}364.$$

$$y_o = 4{\cdot}95$$

Also

$$\Gamma(p+1) = \Gamma(0{\cdot}65) = 1{\cdot}385,$$

$$A_y = \frac{1}{y_o \Gamma(p+1)} = \frac{1}{4{\cdot}95 \times 1{\cdot}385} = \frac{1}{6{\cdot}85}.$$

The final equation is therefore

$$S_y = \frac{1}{6{\cdot}85} \int_o^y e^{-\left(\frac{y}{4{\cdot}95}\right)} \left(\frac{y}{4{\cdot}95}\right)^{-0{\cdot}35} \mathrm{d}y.$$

CHAPTER 8

Numerical Methods of Studying Breakage: A Rational Index of Shatter

INTRODUCTION

Despite the interest of analytical size distribution laws, and the light they throw upon breakage processes, the fact must be faced that they cannot always be used as a help to the solution of practical problems. There are two main reasons:

(i) At the coal face, the existence of a small number of very large lumps makes for irregularities at the upper end of cumulative distribution curves that defy analytic description.

(ii) Analytical methods are of their nature somewhat cumbersome and not suited to routine use. A frequent practical demand is an "index" of breakage that describes the breakage properties of a particular coal, or a particular process. Analytic methods are unable to answer this need.

As an illustration of the practical significance of (ii) above we may mention the handling of coal, involving its movement from one conveyor to another at transfer points underground, and its ejection from hoppers and chutes into containers. During this movement size degradation takes place, which is a matter of concern, as breakage reduces the amount of the larger sizes which frequently command a higher price, and increases the amount of smaller sizes, thus accentuating the hazard to health that is provided by respirable dust.

The requirement for a simple index of breakage means that descriptions of size distribution must be in numerical terms. Something of this kind is attempted, in a crude way, in several types of "shatter index". A sample of material, prepared in a manner that standardizes its extreme sizes, is subjected to breakage in a simple apparatus, e.g. a drop-weight tester or a tumbling mill. The shatter index is obtained by comparing the amount of material above size "a" before a drop with the amount above size "a" afterwards. The ratio of the two gives a non-dimensional index greater than unity, and the greater the value the greater the friability of the material. Alternatively, the standardized samples (often referred to as the "feed") can be put in turn through a process that varies the breaking procedure. The shatter index can then be used to classify the procedures in the order of damage to the samples.

The disadvantages of this type of test are:

(i) Breakage is evaluated by a restricted number of levels of measurement, often just a single one corresponding to a single sieve size.
(ii) Shatter index is dependent upon the size distribution of the test material, even where the extreme sizes remain the same. Hence "feed" samples have to be very carefully prepared.

The disadvantage outlined in (i) above all calls for the use of many levels of measurement, provided they can be integrated into a satisfactory description of breakage. That in (ii) provides a barrier to the effective use and interpolation of empirical shatter indices in continuous processes such as conveying coal, where variation in the grading of the feed inevitably occurs. The breakage problems associated with these processes require an index of breakage that will not be limited to a particular size of feed, and which is less empirical and more satisfying in terms of the physical behaviour of the material than is the shatter index.

An index of this nature has been suggested by Berenbaum(1,2,3) on the basis of work carried out at the Mining Research Establishment. Fundamental to Berenbaum's argument are certain concepts of breakage that were worked out by Broadbent and Callcott(4,5) at the British Coal Utilisation Research Association, and to other work at M.R.E.(6)

Numerical Methods

The contribution of Broadbent and Callcott just referred to' amounts in practice to the treatment of breakage by means of a set of simultaneous equations that are most easily handled by the techniques of matrix algebra. This matrix method has been described fully by its authors, and only a simple reprise, sufficient to lead on to later developments made at the Mining Research Establishment, will be given. A fundamental concept used by Broadbent and Callcott is that of "selection for breakage", due to Epstein(7). When a particular feed is submitted to a breakage process the product will contain at all sizes a certain fraction of unbroken particles. In other words, a fraction of the feed has been selected for breakage and the product at a particular size will comprise the unbroken fraction plus the products that arrive at that size from the breakage of larger sizes. A particular particle will have a certain probability of being broken, but when large numbers of particles are involved this probability becomes the fraction of particles of that size that are broken.

Each of the broken particles will give a certain yield of smaller sizes, and a second assumption is that this yield, averaged over a large number of particles of the same size, can be given by a simple analytic expression, i.e. it resembles the smooth curve of Fig. 7.3 and can be described by a similar law. Broadbent and Callcott choose a law, given below, which seems to be tolerably accurate and which is very simple to handle.

The two assumptions can be summarized:

(i) the fraction selected for breakage is the same at all sizes,
(ii) the breakage law at each size is as follows: where a particle of unit size is broken, the fraction of the product smaller than x, $B(x)$, is given by:

$$B(x) = \frac{[1 - \exp(-x)]}{[1 - \exp(-1)]}. \tag{8.1}$$

The theory now involves the action of these two assumptions upon the feed as defined by the amounts retained between sieves of various sizes. Suppose that consecutive sieve sizes are related by a constant fraction a, and that the feed is defined as fractions f_1, f_2, f_3, etc., of the total that are found in the following size ranges or grades:

Grade	1 to a,	a to a^2,	a^2 to a^3	...	a^{n-1} to a^n.
Feed	f_1,	f_2,	f_3	...	f_n.

(It is not necessary that a set of sieves of this nature should be used in practice, for the data for a hypothetical set can be read off the size-distribution curve given by an actual set.)

The geometric mean size of the various grades, which can be taken to be representative of the particles in the grade, are given by $a^{1/2}$, $a^{3/2}$, $a^{5/2}$, ..., $a^{(2n-1)/2}$ respectively.

Suppose that the fraction of the topmost grade (1 to a) that is selected for breakage is given by the numerical value π. Then the amounts in the various grades that are actually broken are $\pi f_1, \pi f_2, \pi f_3$, etc., which for convenience can be written f_1', f_2', f_3'. The amount of the topmost grade (1 to a) that is selected for breakage will break down according to eqn. (8.1), yielding the fractions in each grade after breakage given in Table 8.1.

TABLE 8.1. FRACTION AFTER BREAKAGE OF SAMPLE IN RANGE (1 to a)

Grade	Fraction in grade after breakage	Symbol
1 to a	$[\exp(-a^{1/2}) - \exp(-1)]/[1 - \exp(-1)]$	b_1
a to a^2	$[\exp(-a^{3/2}) - \exp(-a^{1/2})]/[1 - \exp(-1)]$	b_2
a^2 to a^3	$[\exp(-a^{5/2}) - \exp(-a^{3/2}]/[1 - \exp(-1)]$	b_3
....	....	...
a^{n-1} to a^n	$[\exp(-a^{(2n-1)/2}) - \exp(-a^{(2n-3)/2})]/[1 - \exp(-1)]$	b^n

Since, so far as breakage is concerned, the effective feed in the top grade is f_1', then after breakage the product which, although broken, is still in this size range is $b_1 f_1'$, and the products in the second, third ... nth grades are $b_2 f_1', b_3 f_1' \dots b_n f_1'$, respectively. Now the "Fraction in grade after breakage" in Table 8.1 applies to an arbitrary unit size before breakage, and the example just given takes as this unit size the representative size of the first grade, $a^{1/2}$. The breakage law applies equally to the second grade, third grade, and so on, and summing up all the contributions the final products are:

Grade	
1 to a	$p_1' = b_1 f_1'$
a to a^2	$p_2' = b_1 f_2' + b_2 f_1'$
a^2 to a^3	$p_3' = b_1 f_3' + b_2 f_2' + b_3 f_1'$
....	....
a^{n-1} to a^n	$p_n' = b_1 f_n' + b_2 f_{n-1}' \dots b_n f_1'$.

Broadbent and Callcott point out that these equations can be summarized in a single equation in matrix algebra: if $\mathbf{f}'$ is the feed vector, defined by the elements f_1', f_2', f_3', etc., representing amounts in the consecutive size grades, and **B** the breakage matrix that is derived from eqn. (8.1), then the product vector $\mathbf{p}'$, defined as p_1', p_2', p_3', etc., is given by

$$\mathbf{p}' = \mathbf{B}\mathbf{f}'. \tag{8.2}$$

We now take into account that only a fraction π of the particles are broken, and that in each grade a fraction $(1 - \pi)$ must be added to the broken product. The final products, p_1, p_2, etc., are therefore given by:

Grade	
1 to a	$p_1 = \pi b_1 f_1 + (1 - \pi) f_1$
a to a^2	$p_2 = \pi(b_1 f_2 + b_2 f_1) + (1 - \pi) f_2$
a^2 to a^3	$p_3 = \pi(b_1 f_3 + b_2 f_2 + b_3 f_1) + (1 - \pi) f_3$
....	....
a^{n-1} to a^n	$p_n = \pi(b_1 f_n + b_2 f_{n-1} \dots b_n f_1) + (1 - \pi) f_n$.

The equations can be summarized in a single modification to eqn. (8.2):

$$\mathbf{p} = \pi \mathbf{B}\mathbf{f} + (1 - \pi)\,\mathbf{f}. \tag{8.3}$$

Equation (8.3) is discussed briefly in the Appendix.

In eqn. (8.3) the feed vector and the product vector are obtained from experimental observations on sizing, and the breakage matrix is defined by the quantities b_1, b_2, etc. of Table 8.1. The only unknown quantity is therefore π, which may be obtained as a weighted mean value from the set of simultaneous equations represented by eqn. (8.3). The breakage process is characterized by π. It is independent of the feed vector. It is therefore a breakage index which can take account of variations in feed, and as such is the type of index for which we are searching. Once it has been obtained for one particular feed it can be entered into the equations to calculate what breakage can be expected for any other feed, and in particular for a second cycle of breakage where the product is returned as a new feed. Broadbent and Callcott give several examples of the success of calculations of this nature, the agreement between experiment and theory being good.

It should be noted that apart from the breakage function, itself semi-empirical in nature, nothing related to the physics of breakage has entered into the argument. The assumed constancy of π means that for a given breaking procedure, all sizes of particle are equally liable to breakage. This is at variance with a known property of brittle materials, that particles become stronger as they become smaller. An amendment to the theory which is based upon this consideration has been proposed by Berenbaum and will now be discussed. Some development of this kind was foreshadowed by Broadbent and Callcott[4] in their remark: "Future developments in the physics of breakage may lead to more appropriate breakage and selection functions."

Variation of Selection for Breakage with Size

Berenbaum's contribution takes as its starting point the applicability, already referred to, of the weakest-link theory to the compressive strength of coal[6] (see Chapter 3). If P_a is the probability of cubes of side a surviving a particular stress, then it has been found that

$$(P_a)^{1/a} = \text{constant}. \tag{8.4}$$

The result was also shown to be true for the crushing of irregular lumps between platens, a being taken to be the cube root of the volume for the specimen concerned.

These experiments on both cubes and irregular lumps involved static crushing. It is a long step from these to breakage by shatter, where dynamic forces due to the deceleration of the fragment are brought into play. Nevertheless, shatter breakage is a form of crushing, and it might conceivably be the case that the same factors might apply, if not entirely at least in part, to breakage by fall as to breakage by static crushing. It becomes a reasonable postulate to link the probability of survival in static tests with selection for breakage in shatter tests. The selection factor could therefore be expected to vary with particle size, instead of being uniform. This postulate was made, and its mathematical consequences pursued, by Berenbaum. Its effect upon eqn. (8.3) is as follows: if π is not constant but varies with size, suppose it has values π_1, in the first grade, π_2 in the second grade, and so on.

Then:

$$p_1 = b_1\pi_1 f_1 + (1 - \pi_1)f_1 \tag{8.5a}$$

$$p_2 = b_1\pi_2 f_2 + b_2\pi_1 f_1 + (1 - \pi_2)f_2 \tag{8.5b}$$

$$p_3 = b_1\pi_3 f_3 + b_2\pi_2 f_2 + b_3\pi_1 f_1 + (1 - \pi_3)f_3. \tag{8.5c}$$

These equations can be summarized by the single relationship:

$$\mathbf{p} = \mathbf{B}\boldsymbol{\pi}\mathbf{f} + (\mathbf{I} - \boldsymbol{\pi})\mathbf{f} \tag{8.6}$$

where $\boldsymbol{\pi}$ is the selection matrix and $\mathbf{I}$ is a unit matrix. These again are defined in more detail in the Appendix.

The next step is to link π_1, π_2, etc., with the probability of survival in terms of the weakest link theory. Equation (8.4) can be rewritten

$$P_x = C^x,$$

where P_x is the probability of survival of a size x in a shatter breakage mechanism, and C is the probability of survival of unit size. If $\boldsymbol{\pi}_x$ is the fraction selected for breakage at this size then it is reasonable to write

$$\boldsymbol{\pi}_x = 1 - C^x, \tag{8.7}$$

so that equations (8.5) become:

$$\left.\begin{aligned} p_1 &= b_1(1 - C^{x_1})f_1 + C^{x_1}f_1 \\ p_2 &= b_1(1 - C^{x_2})f_2 + b_2(1 - C^{x_1})f_1 + C^{x_2}f_2 \\ p_3 &= b_1(1 - C^{x_3})f_3 + b_2(1 - C^{x_2})f_2 + b_3(1 - C^{x_1})f_1 + C^{x_3}f_3 \\ . &= \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ . &= \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{aligned}\right\} \tag{8.8}$$

For a particular breakage mechanism these equations allow the calculation of C, which in practical units is the probability of survival of pieces which are just retained by a sieve of 1 in. mesh. In practice it is more convenient to deal with $\log_{10} C$. A representative value of $\log C$ is obtained by weighting each individual value obtained from the eqns. (8.8) and determining the mean.

The validity of eqn. (8.7) has been established by Berenbaum. He took a series of batches of the same coal, each batch being closely graded about a particular mean size, the means ranging from about 4½ in. to about 0·1 in. The batches were put through an identical shatter test, and the π-value for each batch calculated. The values ranged from 0·621 to 0·019 respectively, and can in no way be regarded as random variations about a mean. They do in fact lie on a sensibly straight line when x is plotted against $\log(1 - \pi_x)$, thus showing agreement with eqn. (8.7). Berenbaum confirmed his results by putting various arbitrary assemblies of coal sizes through the same shatter test, and showing that the calculated values of C for the same coal were sensibly the same.

Index of Shatter

Since C is independent of the sizing of the feed it can be suggested as a convenient index for characterizing breakage processes. The derivation of the index is dealt with at length in Berenbaum's papers, and it will only be touched on here. It amounts, in fact, to no more than purely arithmetical manipulation upon the feed and product, defined by the quantities $f_1, f_2, f_3, \ldots, p_1, p_2, p_3, \ldots$, in the various grades. As an indication of how the procedure goes, we obtain from eqn. (8.5)

$$\pi_1 = \frac{p_1 - f_1}{f_1} \frac{1}{b_1 - 1}. \tag{8.9}$$

Of the quantities on the right-hand side of the equation, p_1 and f_1 are experimental results, and b_1 is a number which may be calculated from its definition in Table 1. Hence π_1 can be found.

Now from eqn. (8.7):

$$\pi_1 = 1 - C^{a^{\frac{1}{2}}}$$

$$\log C = \frac{\log(1 - \pi_1)}{a^{1/2}}. \tag{8.10}$$

Knowing π_1, we calculate π_2 from eqn. (8.5b) to give a second value of $\log C$, and so on for every grade of the experimental observations. The weighted mean value is defined as:

$$\overline{\log C} = \frac{\sum(\log C)_i f_i^2}{\sum f_i^2}, \tag{8.11}$$

where $(\log C)_i$ is the value calculated for the ith grade.

The calculations just given can be carried out in a more sophisticated manner by making use of the matrix relationship, eqn. (8.6). This can be handled according to the rules of matrix algebra, and $\boldsymbol{\pi}$ can be made the subject of the eqn. by re-writing it in the form

$$\boldsymbol{\pi}\mathbf{f} = (\mathbf{B} - \mathbf{I})^{-1}(\mathbf{p} - \mathbf{f}). \tag{8.12}$$

The inverse matrix $(\mathbf{B} - \mathbf{I})^{-1}$ is calculable, and hence $\boldsymbol{\pi}$ can be obtained.

Use of the Index

Preliminary

As a preliminary to discussing the use of the index, some results of Berenbaum can be quoted:

Value of C for Various Coals

The values of C given in Table 8.2 were determined as the mean from five successive experiments, each involving an eight-foot drop. The product of each drop formed the feed for the next experiment.

Table 8.2. Values of C for Drop of Eight Feet

Colliery and seam	Percentage volatiles (dry, ash free)	Mean value of $\log_{10} C$	C
Deep Duffryn, Five Feet, Gellideg	12	−0·104	0·79
Oakdale, Meadow	20	−0·189	0·65
Cwmtillery, Garw	25	−0·075	0·84
Rossington, Barnsley Hards	32	−0·022	0·95
Markham, Black Shale	38	−0·056	0·88
Rossington, Barnsley Brights	38	−0·043	0·91

Effect of Height of Drop

Five 5000 g samples of Barnsley Hards were prepared. One sample was dropped repeatedly (five times) at each of the following heights: $4\frac{3}{4}$ ft, 8 ft, 12 ft, 16 ft, and 20 ft. The results are shown in Fig. 8.3. On a logarithmic scale $\log C$ is linearly related to height of drop so that the following equation applies: $\log C \propto -h$ when h is the height of the drop. Thus $C = \exp(-\alpha h)$ where α is a constant.

From these results it will be noted that these coals are clearly differentiated in breakage propensities at a particular drop by the parameter C, and that there is a simple relationship between the parameter and the height of drop. A similar relationship has been found for Deep Duffryn coal and it will be surprising if it is found not to hold for the other coals when similar tests on them have been carried out. Now the kinetic energy of a fragment in free fall is directly proportional to h and the breakage force depends upon the rate of change of this energy at impact with respect to distance. This rate of change would be high for a hard elastic surface, e.g. a steel plate, lower for a softer material that had some ability to absorb energy itself, e.g. rubber which may have marked hysteresis losses in cyclic deformation. There would seem to be a promising field for research here in investigating the variation of C

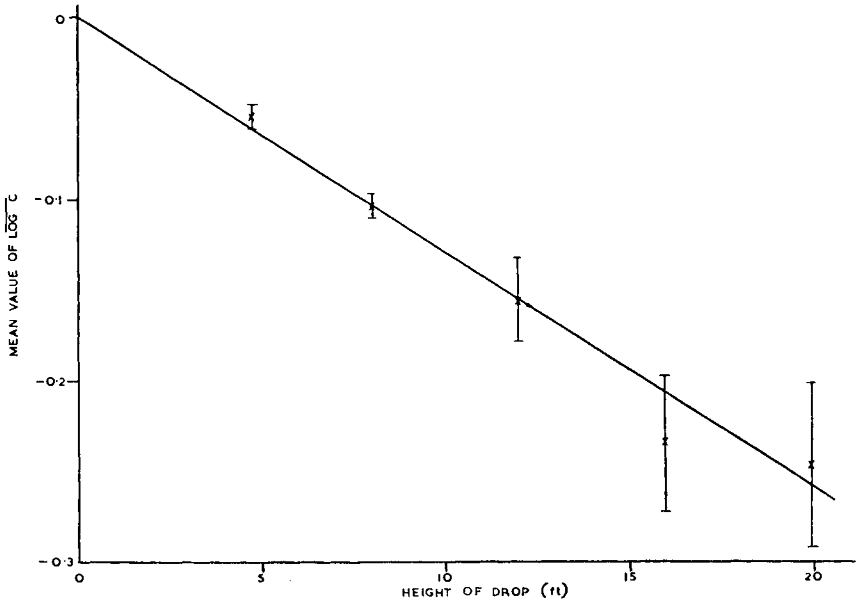

FIG. 8.1. Variation of log C with height of drop for Deep Duffryn coal.

with h and with type of receiving surface. Some important questions pose themselves immediately:

What are the characteristics of the receiving surface for least breakage?

To what extent are these characteristics modified by a permanent layer of broken material?

How does a cascade of small drops compare with a large drop?

Can breakage be alleviated by the use of chutes?

The concepts described in this chapter will be a useful adjunct to the answering of these questions.

APPENDIX

For information on the employment of elementary matrices, reference should be made to a text-book (e.g. *Elementary Matrices* by Frazer, Duncan, and Collar, Cambridge University Press, 1952).

In the equation

$$\mathbf{p} = \pi\mathbf{Bf} + (1 - \pi)\,\mathbf{f} \tag{A.1}$$

the vectors p and f are written as the column matrices

$$\{p_1, p_2, p_3 \cdots p_n\}, \quad \{f_1, f_2, f_3 \cdots f_n\},$$

B is the matrix:

$$\begin{bmatrix} b_1 & 0 & 0 & 0 & . & . & 0 \\ b_2 & b_1 & 0 & 0 & . & . & 0 \\ b_3 & b_2 & b_1 & 0 & . & . & . \\ . & . & . & . & . & . & . \\ b_n & b_{n-1} & . & . & . & . & b_1 \end{bmatrix}$$

When π is not constant, eqn. (A.1) becomes:

$$\mathbf{p} = \mathbf{B}\boldsymbol{\pi}\mathbf{f} + (\mathbf{I} - \boldsymbol{\pi})\mathbf{f}, \tag{A.2}$$

where $\boldsymbol{\pi}$ is the matrix:

$$\begin{bmatrix} \pi_1 & 0 & 0 & . & . & 0 \\ 0 & \pi_2 & 0 & . & . & 0 \\ 0 & 0 & \pi_3 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & \pi_n \end{bmatrix}$$

I is a unit matrix of similar form to $\boldsymbol{\pi}$, but having values of unity along the main diagonal.

As indicated in the text, eqn. (A.2) can be re-written:

$$\boldsymbol{\pi}\mathbf{f} = (\mathbf{B} - \mathbf{I})^{-1}(\mathbf{p} - \mathbf{f}) \tag{A.3}$$

$(\mathbf{B} - \mathbf{I})^{-1}$ is a matrix of the form

$$\begin{bmatrix} e_1 & 0 & 0 & . & . & . & 0 \\ e_2 & e_1 & 0 & . & . & . & 0 \\ e_3 & e_2 & e_1 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ e_n & e_{n-1} & e_{n-2} & . & . & . & e_1 \end{bmatrix}$$

For a sieve size ratio $a = \frac{1}{2}$, the first seven numbers of the first column are: $-1{\cdot}247$, $-0{\cdot}514$, $-0{\cdot}546$, $-0{\cdot}554$, $-0{\cdot}555$, $-0{\cdot}555$, $-0{\cdot}555$.

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CHAPTER 9

Friction between Coal and Metal Surfaces

IN COAL-MINING the space available at the coal face is generally very restricted, yet larger and larger mining machines have been developed when it is possible that more efficient use of existing machines would achieve the same purpose. This increase in the size of machines is particularly noticeable with drum shearers, the horse-power having been steadily increased from 60 to over 200. Admittedly some of the higher powered machines cut proportionately more coal, but this is not always true. One way to improve efficiency is to reduce the frictional losses, and this requires some knowledge of the likely magnitudes of frictional losses at different parts of the machines.

An underground study of the performance of rapid ploughs[1] has shown that only about one quarter of the pull by the chains on the plough base is usefully employed in breaking the coal down. A further quarter is used in loading the broken coal, and so is doing a useful job, but one half of the effort available is used in overcoming friction. If half of this could be saved the forces on the plough blades could be doubled, without altering the plough drives in any way.

Similar losses occur on other machines, particularly those using cutter chains. Further high friction losses can occur if blunt picks or plough blades are used, the efficiency of a cutting tool falling off rapidly as the sharp edges are worn away.

In order to understand the sources of frictional losses and to make positive suggestions to reduce these losses it is necessary to know something of the basic causes of friction between coal and metal surfaces. The experiments described in this chapter had this objective. A study has been made of the friction between a range of different coals sliding on steel and brass for different normal loads. Two different techniques were used, one in which the coal specimens were slid a succession of short distances over the same metal surface and the other in which specimens were held against a flat disc that was rotated continuously.

EXPERIMENTAL DETAILS

Intermittent Sliding

The apparatus used for intermittent sliding of coal on metal is shown in Fig. 9.1. The equipment consists essentially of two metal plates clamped between two coal specimens at normal loads of up to 1000 lb. The metal slider is moved between the two coal specimens by a hand-operated hydraulic ram which moves the slider one inch in eight equal increments. After each

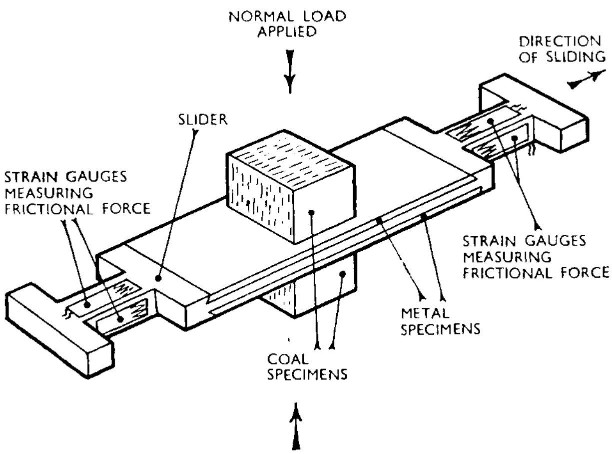

FIG. 9.1. Arrangement of coal and metal specimens for intermittent sliding.

inch of sliding the normal load is removed and the steel slider is returned to its original position. The normal load is then re-applied and the slider is again forced between the two coal specimens. The frictional force is measured with resistance strain gauges attached to the slider.

Continuous Sliding

The equipment used for continuous sliding is shown in Fig. 9.2. It consists of a small coal specimen clamped to one end of a cantilever and pressed by it against the flat horizontal face of a slowly rotating steel disc, the normal force being applied through a lever and pivot. The steel disc was rotated at a constant speed of 0·27 in./min.

The normal and frictional forces on a coal specimen were measured independently by two sets of strain gauges attached to the cantilever and the outputs from these were displayed on two moving coil galvanometers.

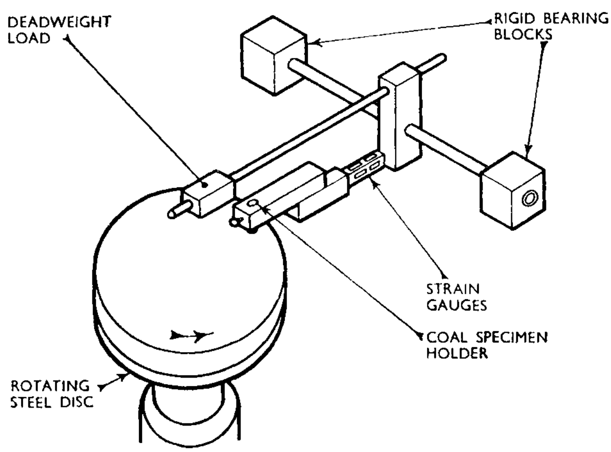

FIG. 9.2. Apparatus for continuous sliding.

Preparation of Coal and Metal Specimens

For both types of experiment the coal specimens were machined flat on precision grinders and the surfaces of the coal were cleaned with No. 4/0 S.I.A. precision paper immediately before use. The metal surfaces had to be cleaned thoroughly to reduce any oil contamination and in the case of the apparatus for successive sliding the metal specimens were first machined flat on precision grinders and were then de-greased in trichlorethylene vapour immediately before use. This was not possible with the large steel plate used for continuous sliding, but it was found that consistent results could be obtained if the metal surface was cleaned with S.I.A. No. 4/0 precision paper under distilled water immediately before the tests were started. The surface of the steel was dried by a jet of compressed air. After this treatment the metal surface "wetted" when water was applied showing that the steel was grease-free.

Details of Coals and Metals Used

Measurements were made with a number of different coals ranging from a high rank anthracite to a low rank bituminous coal. The same coals were used both for the intermittent sliding and the continuous sliding experiments. Details of the coals used are given in Table 9.1.

The metals used for the intermittent sliding were brass and steels of different hardness (150–600 V.D.P.H.). Only a mild steel was used for the continuous sliding experiments.

TABLE 9.1. DETAILS OF COALS USED

Colliery	Seam	N.C.B. Coal Rank Code No.	% Volatiles (dry, ash-free)	
			Batch 1	Batch 2
Pentremawr	Pumpquart	100a	7·2	6·2
Deep Duffryn	Five Feet	201	13·2	12·7
Oakdale	Meadow	301	21·9	21·8
Cwmtillery	Garw	3/501	28·0	25·8
Markham	Black Shale	4/502	38·9	38·7
Rossington	Barnsley Brights	702	44·1	42·4
Pleasley	Dunsil	7/802	42·5	—
Rossington	Barnsley Hards	801	33·9	36·6
Linby	High Main	902	43·1	—

RESULTS

Intermittent Sliding

The coefficient of friction was measured for seven coals subjected to a normal load of 1000 lb on a one inch square rubbing surface. Each pair of coal specimens was slid 20 times over two standard steel surfaces as described above. The coefficient of friction was measured during the 1st, 10th, 15th and 20th inch of sliding. The mean values of the coefficient of friction (μ) from a number of separate experiments on each coal are given in Table 9.2.

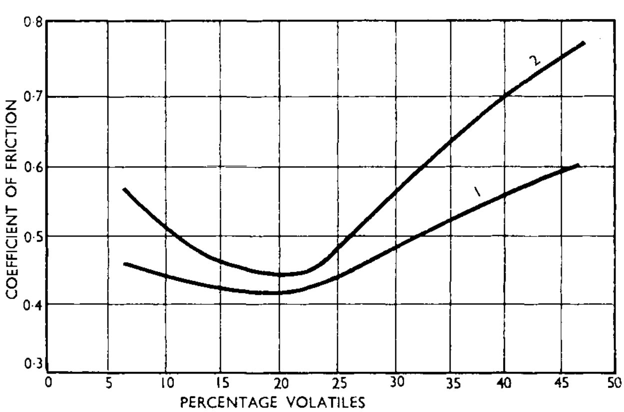

FIG. 9.3. The variation of the coefficient of friction with percentage volatiles (rank) for coal sliding on steel under 1000 lb/in^2 normal pressure. 1. First 0·25 in. sliding. 2. Twentieth inch sliding.

The relationships between percentage volatiles (normally used to characterize coals) and coefficient of friction for the first 0·25 in. and the 20th in. of sliding are shown in Fig. 9.3. The bedding planes were orientated in the same direction for all coals, but subsequent experiments have shown that this is of no importance.

TABLE 9.2. VARIATION OF COEFFICIENT OF FRICTION BETWEEN COAL AND STEEL WITH DISTANCE SLID. NORMAL PRESSURE 1000 lb/in^2

Coal		Distance slid, in.				
		0 to 0·25	0·75 to 1	9 to 10	14 to 15	19 to 20
Pentremawr	μ	0·46	0·48	0·54	0·55	0·55
	No. of tests	11	11	11	11	11
	Std. error	0·01	0·01	0·01	0·01	0·01
Deep Duffryn	μ	0·43	0·46	0·47	0·46	0·47
	No. of tests	10	10	10	10	10
	Std. error	0·01	0·01	0·02	0·01	0·01
Oakdale	μ	0·42	0·45	0·43	0·44	0·44
	No. of tests	6	6	7	7	7
	Std. error	0·01	0·01	0·01	0·01	0·01
Cwmtillery	μ	0·49	0·53	0·56	0·57	0·57
	No. of tests	12	12	12	12	12
	Std. error	0·01	0·01	0·01	0·01	0·01
Markham	μ	0·64	0·72	0·73	0·74	0·76
	No. of tests	9	9	9	9	7
	Std. error	0·02	0·02	0·02	0·01	0·02
Barnsley Brights	μ	0·53	0·65	0·77	0·75	0·74
	No. of tests	4	4	5	5	5
	Std. error	0·03	0·03	0·02	0·01	0·02
Barnsley Hards	μ	0·57	0·63	0·68	0·68	0·69
	No. of tests	12	12	12	12	12
	Std. error	0·01	0·01	0·02	0·02	0·01

After every experiment it was found that sliding had resulted in the formation of a dark brown film of coal on the steel. Moreover, for nearly every coal tested μ increased markedly with the distance that the coal had slid, reaching a constant value after about one foot of sliding.

This increase in coefficient of friction could arise from changes in the steel surface or in the coal surface or in both, and the following experiment was undertaken to show which of these possibilities was the true one.

One inch square specimens of Barnsley Hards were given a rough surface finish on a 60 grit Carborundum slitting disc and were slid on a standard steel surface under a load of 750 lb. After a pair of specimens had slid a distance of eight successive inches they were replaced by fresh ones, the steel remaining unchanged. Sliding continued for a further 8 in. after which both coal and steel were changed.

In the next experiment the steel was changed after the first 8 in. of sliding and the same coal specimens were used on new steel surfaces. These two sequences were repeated a number of times. It was found that if the coal specimens were changed, the coefficient of friction dropped to the original value and gradually increased again, whereas if the steel specimens were changed the results for the second 8 in. of sliding continued as if no change in specimen had been made. In other words, the increase in friction as sliding progressed could be attributed to a change in the surface of the coal rather than to a change in the surface of the metal.

The Effect of Changes in Normal Pressure

Measurements were made of the variation of coefficient of friction with normal pressure using $\frac{1}{2}$ in square specimens of different coals. Each specimen was first slid over the steel surface under a load of 200 lb so that the effect of changes in coefficient with distance slid was eliminated. During the next one

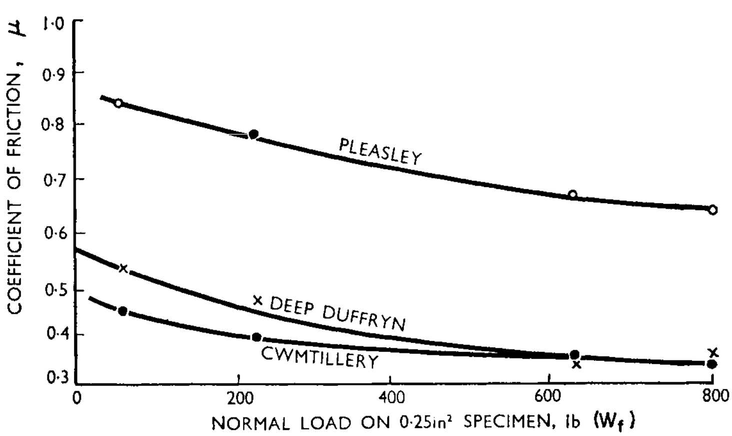

Fig. 9.4. Variation of the coefficient of friction with load for three typical coals sliding on steel.

inch movement between coal and slider the normal load was increased in steps from 25 to 800 lb, an increment of load being made between successive advances of the slider. The coefficient of friction falls with increase of normal load as shown in Fig. 9.4.

This decrease is satisfied by an equation of the form $\mu \propto (\text{load})^{\alpha_f}$ where α_f is about $-0{\cdot}1$ (see Table 9.3).

TABLE 9.3. SLOPES OF GRAPH OF $\log \mu/\log W_f$ AND CALCULATED VALUES OF n_f FOR COAL SLIDING ON STEEL

Coal	Slope $\log \mu/\log W_f = \alpha_f$	n_f
Pentremawr	−0·15	0·70
Deep Duffryn	−0·22	0·56
Oakdale	−0·11	0·78
Cwmtillery	−0·13	0·74
Markham	−0·08	0·84
Pleasley	−0·13	0·74
Barnsley Hards	−0·09	0·82
Linby	−0·11	0·78

(The significance of n_f is discussed on p. 180.)

THE EFFECT OF HARDNESS OF THE STEEL SURFACE

The coefficient of friction was found to be independent of the hardness of the steel (150–600 V.D.P.H.) for coals subjected to a mean pressure of 50 lb/in^2.

THE EFFECT OF LOADS LARGE ENOUGH TO CRUSH THE COAL

When high loads are applied to the coal specimens the coal at the surface is liable to be crushed even if the specimen does not completely disintegrate. The simultaneous effect of both the normal and the shear loads can cause this surface degradation. The coefficient of friction was measured between steel and crushed Barnsley Hards for a 1 in. square specimen subjected to a range of loads from 200 to 1800 lb. These loads cannot be converted to stress as the area over which the load was applied increased during the test as the coal was squeezed out over a larger area. In general, as the load increased, the coefficient of friction decreased as shown in Table 9.4. These results are consistent with a relationship between μ and the load of the form $\mu \propto (\text{load})^{\alpha_1}$ where α_1 lies between $-0{\cdot}05$ and $-0{\cdot}13$ and has the most probable value of $-0{\cdot}090$. It is interesting to note that the index α_1 has about the same value as that found when uncrushed coal was used.

TABLE 9.4. THE EFFECT OF NORMAL LOAD ON THE COEFFICIENT OF FRICTION BETWEEN CRUSHED BARNSLEY HARDS AND STEEL

Load (lb)	μ (Mean of 12 runs)	Standard error
200	0·36	0·03
450	0·33	0·02
900	0·31	0·01
1800	0·29	0·01

Friction between Coal and Brass

Some tests were made of the coefficient of friction between eight different coals and brass at a normal pressure of 500 lb/in². Results for this series are shown in Table 9.5. Figures for steel at the same normal pressure are included for comparison.

Table 9.5. Variation of Coefficient of Friction with Distance Slid for Coal Sliding on Brass and Steel
Normal pressure 500 lb/in²

Coal	Metal	Distance slid (in.)				
		0·0 to 0·2	0·8 to 1·0	9 to 10	14 to 15	19 to 20
Pentremawr	brass	0·48	0·49	0·46	0·48	0·46
	steel	0·53	0·64	0·64	0·63	0·63
Deep Duffryn	brass	0·54	0·56	0·61	0·62	0·62
	steel	0·48	0·52	0·56	0·57	0·59
Oakdale	brass	0·43	0·45	0·46	0·45	0·45
	steel	0·47	0·50	0·54	0·54	0·53
Cwmtillery	brass	0·45	0·44	0·46	0·46	0·46
	steel	0·44	0·48	0·53	0·54	0·54
Markham	brass	0·68	0·72	0·75	0·73	0·74
	steel	0·69	0·72	0·77	0·77	0·78
Barnsley Brights	brass	0·70	0·73	0·76	0·75	0·75
	steel	0·71	0·76	0·83	0·82	0·82
Barnsley Hards	brass	0·61	0·63	0·60	0·60	0·61
	steel	0·60	0·68	0·67	0·67	0·67
Linby	brass	0·66	0·71	0·65	0·63	0·61
	steel	0·67	0·75	0·76	0·75	0·73

A smear of coal was left on the metal surface after each run. In many of the tests on brass there was also some transfer of brass to the coal, but no trace of steel on the coal surface was observed for any of the specimens slid on steel.

Continuous Sliding

The experiments on continuous sliding were undertaken primarily to study the effect of water on the coefficient of friction. The experiments were carried out on all of the coals listed in Table 9.1. The specimens were subjected to a normal pressure of about 150 lb/in² and were held against the rotating steel surface for about 2 min in order to eliminate the change in coefficient of friction that occurs when coal is slid on steel. The increase in coefficient of friction after the 2 min was negligible. Immediately after the coefficient of friction was measured between the coal and the dry steel the surface was flooded with distilled water. For every coal the frictional force fell rapidly to an approximately constant value and ten seconds after the steel was wetted the coefficient of friction was measured again, when it appeared that the change in μ was complete.

The steel surface was then cleaned and the procedure repeated with a fresh coal specimen. Results for the nine coals tested are given in Table 9.6.

TABLE 9.6. THE EFFECT OF WATER ON THE COEFFICIENT OF FRICTION BETWEEN COAL AND STEEL

Colliery	Seam	μ dry	Standard error of mean	μ wet	Standard error of mean	$\frac{\mu \text{ wet}}{\mu \text{ dry}}$	Standard error of mean
Pentremawr	Pumpquart	0·48	0·01	0·32	†	0·67	0·01
Deep Duffryn	Five Feet	0·44	0·02	0·29	0·01	0·67	0·03
Oakdale	Meadow	0·42	0·03	0·28	0·01	0·70	0·03
Cwmtillery	Garw	0·53	0·01	0·35	0·01	0·66	0·01
Markham	Black Shale	0·67	0·01	0·48	†	0·72	0·01
Rossington	Barnsley Brights	0·69	0·01	0·50	0·01	0·72	0·02
Pleasley	Dunsil	0·61	0·01	0·44	†	0·73	0·02
Rossington	Barnsley Hards	0·58	0·01	0·40	0·01	0·69	0·02
Linby	High Main	0·59	0·01	0·42	0·01	0·72	0·01

(† indicates values $< 0{\cdot}005$)

It will be seen that the coefficient of friction when the surfaces are wet is about 0·7 times the value of μ dry for all of the coals tested. The variation in μ with percentage volatiles persists after the surfaces have been wetted although the magnitude of the coefficient is consistently lower.

DISCUSSION

When two surfaces are brought together contact is only made at the tips of the microscopic irregularities on each surface, so that the total area of contact is much smaller than the apparent area of contact. The actual area of contact will depend on the normal pressure applied and owing to the high local pressure that occurs at the asperities local bonds occur at some of the points of contact. When sliding takes place the areas of contact are sheared either at the actual interface or close to it in the weaker of the sliding materials. When coal slides on steel the fact that coal is transferred to the steel shows that the coals tested are all weaker than the steel, whereas in the experiments with brass the fact that in some cases brass is transferred to the coal implies that under the conditions of test the strength of the brass asperities can be lower than those of the coals. This is particularly true for the strongest of the coals used, namely, the Pentremawr anthracite and Barnsley Hards.

The observations for coal sliding on brass are somewhat inconclusive and whilst the coefficient of friction increased with distance slid for all coals when sliding on steel, this same observation is true only for a few of the coals sliding on brass. The friction process for coal sliding on brass is thus more complicated than for coal on steel and subsequent discussion is concerned only with coal–steel friction.

Dry sliding

If it is assumed that coal–steel friction between "dry" surfaces is caused by the shearing of the asperities of contact in the weaker material by a total force F_f, then

$$F_f = A_f s, \tag{9.1}$$

where A_f is the real area of contact and s the shear strength of the coal.

Bowden and Tabor[2] have shown that for metals the pressure at the points of contact will be sufficiently high to cause plastic deformation in the softer material until the area of contact is large enough to support the load without further plastic flow. Then assuming that elastic deformation is negligible the area of contact A_f and the load W_f are related by

$$W_f = A_f Y, \tag{9.2}$$

where Y is the pressure at which plastic flow starts (i.e. the yield pressure).

Usually the softer of the materials has the lower shear strength so that from eqns. (9.1) and (9.2)

$$\mu = \frac{F_f}{W_f} = \frac{s}{Y}. \tag{9.3}$$

This equation implies that the coefficient of friction is independent of the normal load between the mating surfaces, which is contrary to the observations for coal sliding on steel.

Consider now the elastic deformation of mating asperities. For analytic simplicity it is necessary to restrict the consideration to hemispherical points of contact. Hertz[3] has shown that the area of contact between a sphere and a plane surface is given by

$$A_f \propto W_f^{2/3}. \tag{9.4}$$

It may be suggested, therefore, that if two mating surfaces consist of matching hemispheres the true area of contact between the two surfaces will show a similar dependence on normal load.

Archard[4] has shown that this is only true if the number of points of contact is unchanged by an increase in the normal load. He has also shown that a more complex model can be postulated in which the true area of contact between the mating surfaces is proportional to the normal load without recourse to the assumption of plastic flow. In general, the more complex the assumed model the more nearly is this condition realized.

Suppose now that the real area of contact $A_f(W)$ between coal and steel is given by

$$A_f(W) \propto W_f^{n_f} \qquad 1 \geqslant n_f \geqslant \tfrac{2}{3}.$$

The limits of the exponent n_f correspond to the condition of plastic flow or the elastic deformation of a highly complex surface ($n_f = 1$) and to elastic deformation of a surface of matching hemispheres ($n_f = \frac{2}{3}$).

The mean normal stress σ over the asperities is given by

$$\sigma \propto W_f / W_f^{n_f} \propto W^{1-n_f}.$$

The shearing stress $s(W)$ required to break these junctions will be a function of the normal stress acting on them. Mohr's envelope derived in triaxial tests on coal shows that to a first approximation

$$s(W) \propto \sigma^{1/k_f},$$

where

$$k_f \approx 2.$$

Hence the force required to shear the asperities is

$$F_f(W) = A_f(W)\, s(W) \tag{9.5}$$

$$\propto W_f^{n_f}[W_f^{(1-n_f)}]^{1/k_f}$$

and so

$$\mu = \frac{F_f(W)}{W_f} \propto W_f^{(n_f-1)(k_f-1)/k_f}$$

and assuming

$$k_f = 2$$

$$\mu \propto W_f^{\frac{1}{2}(n_f-1)}.$$

Graphs of μ against W_f on logarithmic scales are linear (typical graphs are shown in Fig. 9.5). Values of n_f for eight distinct coals are given in Table 9.3.

It can be seen that apart from the result for Deep Duffryn coal the values of n_f all lie within the predicted range of 0·66 to unity. The anomalous result is possibly accounted for by the fact that this coal was broken by the higher loads applied and so the friction was measured between broken coal and steel.

Archard[5] has suggested that shearing does not occur at many of the points of contact, so that the force required to separate the junctions may not be load dependent. If this is so equation (9.5) becomes

$$F_f(W) \propto W_f^{n_f}$$

or

$$\mu \propto W_f^{n_f - 1}$$

and the theoretical limits to the slope of log μ/log W_f become $-0{\cdot}33$ and zero instead of $-0{\cdot}17$ and zero. The wider range of acceptable values includes the previously unacceptable value for the Deep Duffryn coal.

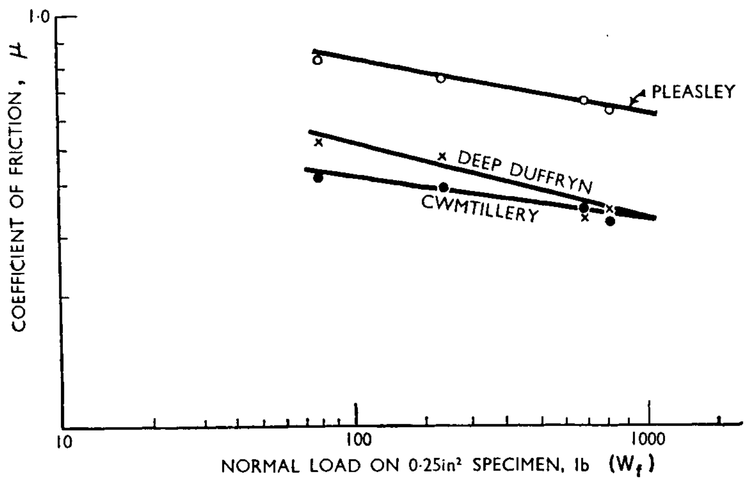

FIG. 9.5. Variation of the coefficient of friction with load for three typical coals sliding on steel (logarithmic scales).

Wet Sliding

The force required to shear at the point of contact will depend on surface films on the boundaries. Surface films of water only a few molecules thick, adsorbed or chemically bound to the sliding surfaces, will modify the friction process, for while the load is effectively borne by the underlying material, intimate contact is prevented unless the local pressure is sufficient to break the film.

In order to interpret the effect of water on coal–steel friction properly it would be necessary to make direct measurements of the size, number, and shear strengths of the points of contact, but this is not possible. A qualitative estimate of the way water can effect these factors may be made, however, and an idea of their relative importance inferred from comparison of the predictions with the measurements given in Table 9.6.

The reduction in coefficient of friction on wetting could be due to a change in the mechanical properties of the coal or to the formation or modification of the surface film of water molecules on the sliding surfaces, or possibly to a combination of both these factors. Measurements of the effect of soaking on coal strength have shown that the compressive strength of coal is generally reduced by about 5 per cent for Deep Duffryn coal and by about 20–25 per cent for high volatile coals. It is reasonable to assume that the shear strength of these coals will vary with wetting in a similar way so that the force required to shear the local points of contact will decrease as the coal is wetted. However, the fact that the effect of water on strength varies from one coal to another implies that the decrease in coefficient of friction on wetting should depend on the coal rank, but this has not been found to be so. It must be concluded, therefore, that either the shear strength of coal does not vary with wetting in the same way as compressive strength or that very few of the junctions shear in the coal itself. The latter possibility is preferred as it is supported by observations by Archard(6) who found for metals that the majority of junctions part cleanly at the points of contact, and that only a few per cent of the junctions are actually sheared.

Another physical parameter of the coal which must be considered is the modulus of elasticity, E, which has been found to decrease by about 7–10 per cent when various coals have been soaked in water, but if it is assumed that the surface asperities are hemispherical (see above), that all deform elastically and that their number is constant, then

$$A_f \propto (1/E)^{\frac{2}{3}}.$$

Thus the change of modulus when the coal is soaked would be expected to increase A_f and μ. This is contrary to experiment, and although the change in E must occur, the reduction of friction must be due to some quite separate process.

It seems likely then, that the reduction in coefficient of friction must be attributed to a modification in the film of water between the coal and steel. This is supported to some extent by work of other experimenters and Savage(7), for example, has shown that the slipperiness of graphite is entirely due to an adsorbed layer of water on the surface. Friction between coal and steel will almost certainly be reduced by the presence of such a layer but, as no special precautions were taken to protect the surfaces, these were undoubtedly covered with a film of adsorbed atmospheric moisture even during the "dry" measurements. When the bulk water was applied to the surfaces it seems likely that the thickness of the liquid film was increased and the local pressure between the high spots on the coal and steel was reduced by the presence of the liquid layer and this caused a reduction in the coefficient of friction.

CONCLUSIONS

The experiments show that the coefficient of friction between coal and steel can be as high as 0·8 and that the values increase steadily to a maximum as flat coal specimens are slid on steel. If the normal loads are high enough to crush the coal the friction coefficient falls to much lower values (about 0·3), but these values are higher than those calculated from penetration tests with wedges into coal. In general, there is a higher coefficient between coal and brass than between coal and steel, but this is not invariably so. The hardness of the steel used seems immaterial, and the change in coefficient of friction found when coal slides on steel is attributed to a change in the surface of the coal and not to that of the steel. If the points of contact are elastically deformed as suggested above, it is possible that the number and size of the coal asperities are modified during the first stage of sliding to a larger number of smaller points of contact which results in an increase in the true area of contact between the surfaces and hence in the frictional force required to break the junctions.

The coefficient of friction varies with coal rank and the variation persists after the specimens have been "run-in" and also when the measurements are made in the presence of water. As the strength of coal shows a similar variation with rank it seems probable that the shear breakage of the coal asperities must play some part in determining coal–metal friction, even if the majority of bonds between the mating surfaces part at the boundary and not in the weaker substance.

The effect of water is to reduce the friction for all of the coals tested. Although water reduces the strength of coal, the reductions that have been observed depend on coal type, so that it seems unlikely that it is the effect of water on coal strength that causes the uniform reduction in μ for different coals. It seems more likely that a water layer, formed between the mating surfaces, reduces the stress between contacting asperities and so lowers the friction.

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CHAPTER 10

Wedge Penetration into Coal

THE wedge is one of the oldest implements of mankind, and the relative abundance of flint axe heads that were used by our remote ancestors shows the importance of the tool at the dawn of civilization. The wedge still remains extremely important, especially in mining technology where it is the basis of most methods of mining. Cutter picks and plough blades of all types depend for their function upon a wedging action, and for any improvements in their design the mechanics of this action should be understood. This chapter describes some preliminary investigations on the mechanics of wedge breakage carried out at M.R.E.[1]

The wedge is usually used for prising off material in the vicinity of an external corner, e.g. a buttock in coal, but these two free surfaces meeting at an angle represent a somewhat complicated physical situation. The first experiments were therefore carried out with the intention of seeing what happens when a simple wedge is pushed into a piece of coal normal to a single surface. In practice comparatively small samples of coal must be used, but the penetrations have been very limited so that the results are probably independent of the size of sample. In physical parlance the experiments relate to the penetration of a wedge into a semi-infinite medium. The penetration rates were small (about 0·01 in./min) in order to avoid initially the complications of dynamic cutting. The following effects were investigated in the first instance:

(i) the load-penetration characteristic as a function of tool angle
(ii) the load-penetration characteristic as a function of the coal
(iii) the manner of break-up of the coal.

APPARATUS

Fig. 10.1 gives a schematic diagram of the apparatus used. The coal specimens were in the form of blocks of 1 in. square section and ½ in. thick, each specimen being mounted for test in a U-shaped holder with a thin cardboard gasket between coal and steel. One of the 1 in. × 1 in. faces of the coal bore against a piece of plate glass, the other against a steel supporting plate, so that the coal was supported on all surfaces except the top one. The tool was

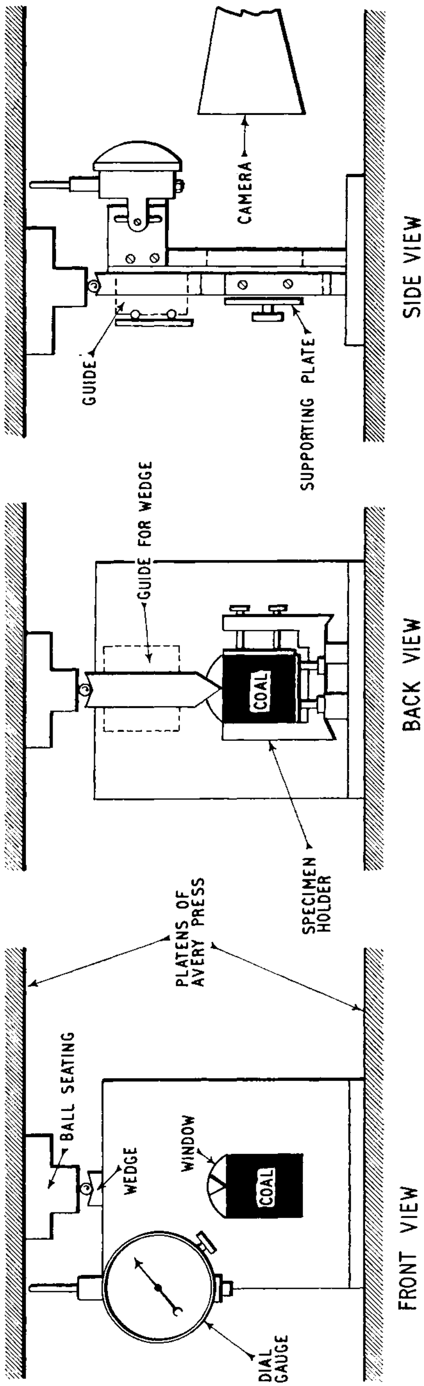

FIG. 10.1. Schematic diagram of apparatus.

forced in normal to this surface, so that conditions in the coal approximated to plane strain.

The tools were made from $\frac{1}{2}$ in. square section steel, and the ends were ground to the form of simple wedges. During the course of the work the following angles were employed: 10, 20, 30, 45, 75, 90 and 140 deg.

Coals Used

In the preliminary experiment the coal used was from Rossington colliery (Barnsley Hards). This is a hard low rank (801) coal. Further work was carried out with Cwmtillery, Garw seam, a 301/501 coal.

TYPICAL EXPERIMENT

The typical case chosen is that of a 20 deg tool pushed into a specimen of coal by means of a compression testing machine at the steady rate of about 0·01 in./min. The coal is Barnsley Hards, and the specimen was cut so that its bedding planes were parallel to the 1 in. × 1 in. surface, hence the edge of the tool lay in the direction normal to these planes. The specimen was photographed at intervals during penetration through the vertical plate glass window in order to obtain some qualitative ideas of the manner in which the coal breaks up. These photographs are shown as Fig. 10.2.

The breakage pattern is complicated and variable, but certain common features can be seen. The inception of breakage appears to be by a crack or series of cracks running radially from the tip of the wedge. The most prominent of these, in general, is the one running vertically into the specimen, along the line of action of the resultant load on the tool. After the cracks are propagated, the coal in the vicinity of the tip becomes fragmented. In some photographs there was at this phase very little contact between the tip of the wedge and the coal, and the main resistance to the tool must have acted on the sides of the wedge. The coal displaced by the wedge is forced outwards and upwards, causing the surface to heave.

During a penetration the load-penetration curve is approximately linear. Occasionally a fracture may develop to a greater or lesser extent, allowing a sudden increment of penetration at a particular load, or the load may even fall off a little, but on the whole the excursions of the load-penetration characteristic from a straight line through the origin are not pronounced.

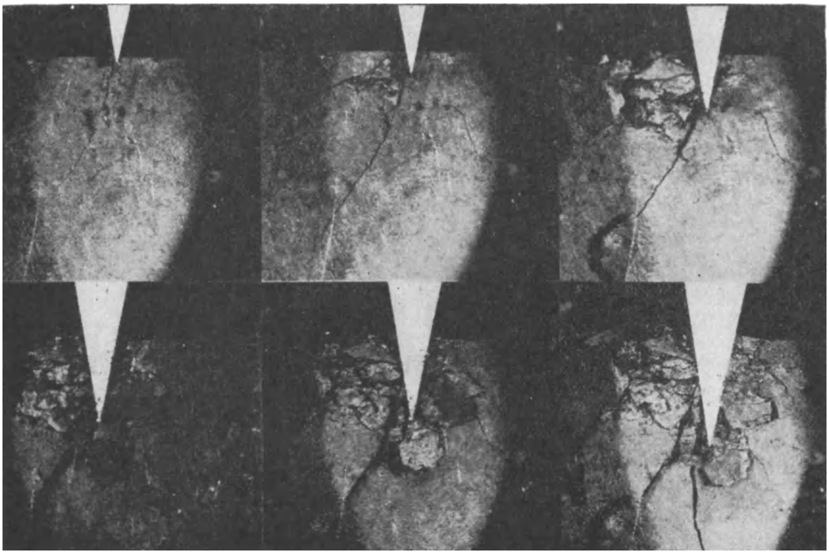

FIG. 10.2. Wedge penetration into Barnsley Hards.

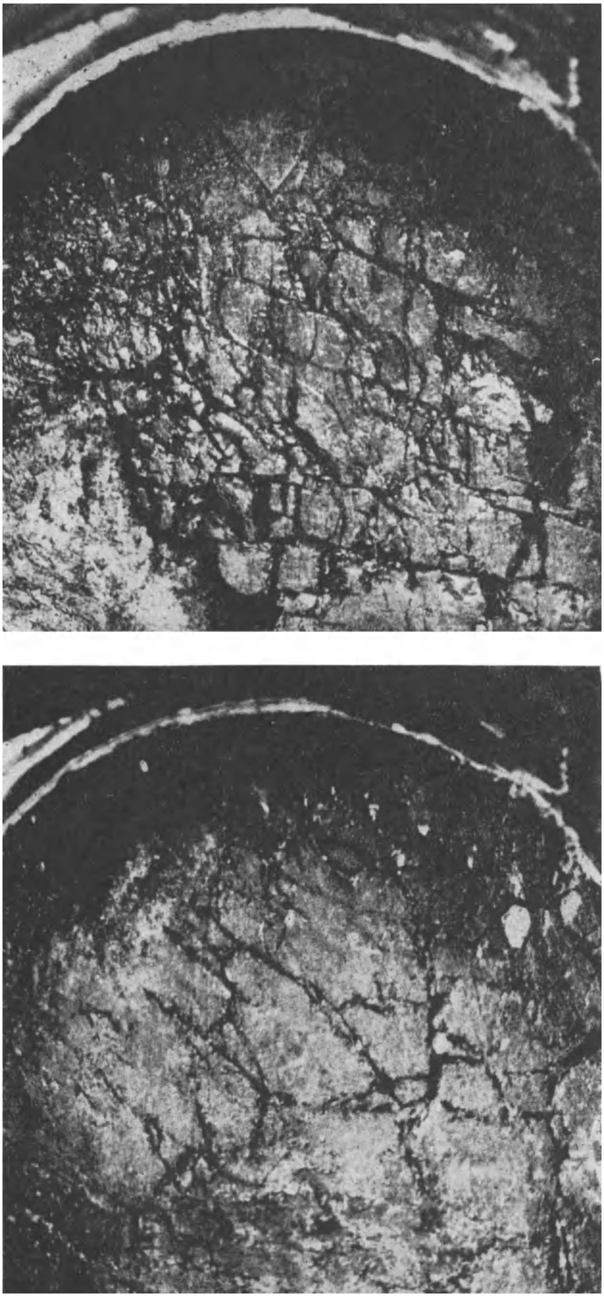

FIG. 10.3. Wedge penetration into Cwmtillery coal (top) 75 deg wedge, (bottom) 30 deg wedge.

Experimental Procedure

The main experiment consisted of a systematic investigation of penetration characteristics of the two coals for a wide range of wedge angles and for two orientations of the bedding planes,

(i) parallel to the square surface of the block so that the cutting edge was in the direction of the normal to the planes,

(ii) parallel to one pair of 1 in. $\times \frac{1}{2}$ in. faces of the block, and vertical in the apparatus, so that the cutting edge moved along a bedding plane.

For a particular coal, a particular orientation and a particular tool, five separate penetrations were carried out so as to obtain some measure of the scatter of the results. The penetration was read by a dial gauge at intervals of 0·005 in. of penetration. The maximum penetration achieved in each experiment was between 0·15 and 0·2 in.; this was regarded as sufficiently small a value in relation to the dimensions of the block to ensure that conditions in the vicinity of the wedge would not be influenced by the finite size of the block.

Specimens for a particular orientation of the bedding planes were in general prepared as one batch from a single lump of coal. As a measure of the compressive strength of the coal the strength in uniaxial compression of a number of specimens from the batch was observed, the load being applied normal to the 1 in. $\times$ 1 in. surfaces. At least five samples were taken from the batch for this purpose, the number in one instance being as many as 20.

Penetration of Cwmtillery Coal

Superficially, the process of penetration appears to differ from that for Barnsley coal, just as the coals themselves are superficially different. A machined specimen of Barnsley coal looks smooth and apparently uniform, whereas the surface of the Cwmtillery coal is marked with a network of cracks varying from fine hair-like structures to gross fissures. During the penetration of Cwmtillery coal the development of cracks radiating from the tip of the wedge, which is such a marked feature of the Barnsley coal experiments, is almost entirely absent (Fig. 10.3). Nevertheless, the load-penetration characteristics for the two coals were similar in general form.

Treatment of Experimental Observations

As already noted, the photographs of penetration indicate that the actual edge of the wedge is instrumental in initiating cracks in the coal and that thereafter, for considerable portions of the penetration curve, the edge may not even be in contact with the coal, the load being carried on the shoulders of the wedge. This suggests that the force on the wedge should be related to the effective bearing area of the wedge at the surface of the coal; i.e. if P_w is the load, h_w the penetration measured from the level of the undisturbed coal surface, and 2θ the angle of the wedge, it may be that

$$P_w \propto 2h_w \tan\theta$$

or

$$P_w/h_w \propto \tan\theta. \tag{10.1}$$

P_w/h_w may be taken to be the mean slope of the penetration curve of the wedge.

For a particular penetration it is required to find a representative value of P_w/h_w over a penetration arbitrarily standardized at 0·15 in. As observed in the preliminary experiment, the penetration characteristic is somewhat irregular, but it does not differ greatly from a straight line. For an irregular

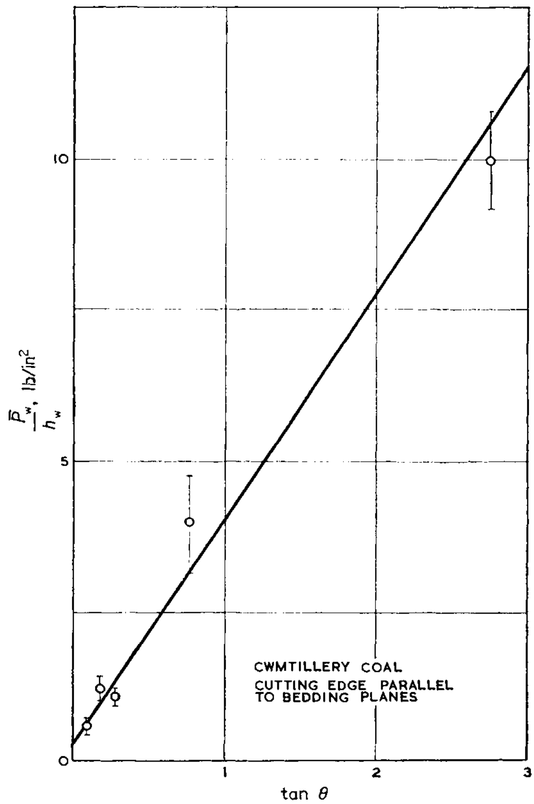

FIG. 10.4. Load per unit penetration as a function of tan θ.

characteristic the work done during a penetration h_w is $\int_0^{h_w} P_w \mathrm{d}h_w$. A representative value of P_w written $\bar{P}_w$ can then be defined as follows:

$$\int_0^{h_w} P_w \,\mathrm{d}h_w = \tfrac{1}{2}\bar{P}_w h_w.$$

A representative value of P_w/h_w can then be defined as

$$\frac{\bar{P}_w}{h_w} = \frac{2}{h_w^2}\int_0^{h_w} P_w \,\mathrm{d}h_w.$$

The integral can be evaluated numerically (since the curves do not depart greatly from linearity, this is most easily done by the simple process of counting squares) and the values of $\bar{P}_w/h_w$ for a particular experiment determined.

In Fig. 10.4 a typical plot of $\bar{P}_w/h_w$ against tan θ is given, and it can be seen that there is a linear relation between them.

Relation of Compressive Strength to Penetration Resistance

A further extension of the basic idea can be brought about by linking the compressive strength of the coal to the parameter P_w/h_w. It may be supposed, to a first approximation, that a "penetration resistance" of value q per unit area normal to the faces of the wedge is brought into play during wedge penetration. At a guess, this penetration resistance might be equal to, or close to, the compressive strength of the specimens, measured as described in a previous section.

Assuming, therefore, that penetration resistance q is equal to compressive strength q_0 per unit area, equation (10.1) can be modified as follows:

$$P_w = q_o 2h_w w \tan\theta$$

where w = length of cutting edge.

Hence

$$P_w/h_w q_o = 2w \tan\theta. \tag{10.2}$$

This relation should enable the results for the two coals to be brought to a common basis, despite the disparity between values of q_o illustrated in Table 10.1.

Values of $\bar{P}_w/h_w q_o$ have been calculated and are given in Fig. 10.5, and it can be seen that the experimental points for the two coals are in fact brought to a common basis and fall roughly together. As regards absolute agreement with eqn. (10.2), this is not achieved. If agreement were exact, the mean line would pass through the origin, but it does not, and differs from its theoretical position by a small horizontal displacement. The reason for this will now be discussed.

TABLE 10.1. MEAN VALUES OF COMPRESSIVE STRENGTH FOR BARNSLEY AND CWMTILLERY COALS

Direction of bedding planes of specimen	Mean compressive strength, lb/in^2	
	Barnsley Hards	Cwmtillery
Normal to cutting edge	9480 ± 380	3360 ± 560
Parallel to cutting edge	6660 ± 400	3160 ± 580

(The uncertainty represents the standard error of the mean.)

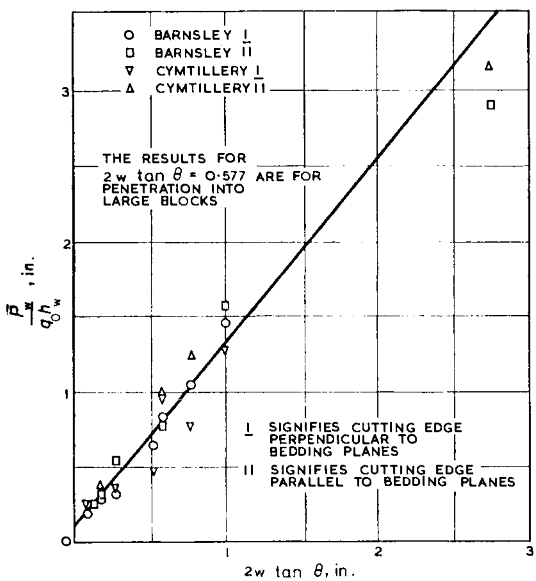

FIG. 10.5. Penetration characteristics as a function of compressive strength of coal.

EFFECT OF FRICTION BETWEEN COAL AND STEEL

One of the notable factors that has so far been omitted in the analysis is friction between steel wedge and coal. The total resistance to a wedge can be thought of as a combination of penetration resistance, as already defined, and frictional resistance. It is extremely difficult to make a numerical assessment of frictional resistance; the problem has not been fully solved even for wedge penetration into metals, where the properties are much more susceptible of analysis. Hankins[2] has produced a semi-empirical solution for the increase of penetration resistance of a cone penetrating into metal due to the presence of friction. For a wedge, Hankins's analysis may be paraphrased as follows:

Consider a wedge (Fig. 10.6) penetrating into a medium. There exists a coefficient of friction μ between wedge and medium. Let the normal reaction between each surface of the wedge and the material be $R/2$; then at limiting equilibrium the frictional resistance on each surface is $\mu R/2$.

Suppose that the load on the wedge for a penetration h_w is P_w; then, resolving vertically,

$$P_w = 2\left(\frac{R}{2}\sin\theta + \frac{\mu R}{2}\cos\theta\right)$$

$$= R\sin\theta\,(1 + \mu\cot\theta). \qquad (10.3)$$

This equation shows that friction may be taken into account by multiplying the penetrating force for zero friction by the factor $(1 + \mu \cot\theta)$. Hence in the experiments with coal

$$P_w = q\,.\,2wh_w\tan\theta(1 + \mu\cot\theta) = q\,.\,2wh_w(\tan\theta + \mu).$$

In algebraic terms, therefore, the introduction of μ means that $\overline{P}_w/q_o h_w$ should be linear with respect to $(\tan\theta + \mu)$ instead of to $\tan\theta$. It is interesting to note this is the kind of relationship shown in Fig. 10.5. The value of μ that would give the observed horizontal displacement of the mean line

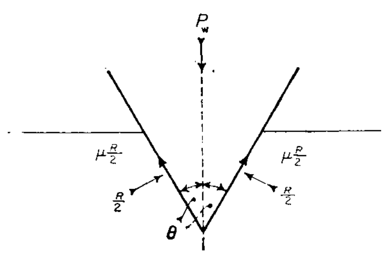

FIG. 10.6. Forces on a wedge.

is about 0·1. This, however, differs appreciably from the values of μ of about 0·5 determined in the laboratory for sliding between flat surfaces of coal and steel under loads of 1000 lb/in^2 (Chapter 9)[3].

The discrepancy may lie in the nature of the coal itself as a breakable material. In laboratory friction experiments the applied load was a good deal less than the compressive strength. During the course of penetration experiments the wedge is in contact not so much with solid coal as with the products of breakage; in fact a layer of powdered coal exists between the wedge and the solid coal. This layer must be continuously renewed as penetration proceeds, some of the powder working its way to the surface as more is ground from the solid. The frictional force in this case is therefore the force required to tear particles from the coal by means of a lateral movement, i.e. its shear strength, and the coefficient of friction μ is the ratio of shear strength to compressive strength. It is conceivable that this is smaller than the frictional force between steel and unbroken coal, and there are grounds for suggesting that the ratio is in the vicinity of the experimentally deduced value of 0·1[1].

Relation of Results to those for Extended Medium

These results have been obtained with a finite and in fact rather small piece of coal, and it is important to know how they are related to those applying to penetrations into lumps that may be considered to be semi-infinite in extent with respect to the indenter. The presence of a boundary will strongly affect the stresses in the vicinity of the boundary but may not greatly affect the stresses in the vicinity of the wedge, which are presumed to be the important ones. Penetration experiments were therefore carried out on large lumps of coal in order to see how they compared with those on the small specimens. It was prohibitive in time and labour to do more than a few tests, so measurements were confined to a single tool angle. The chosen tool angle was 60 deg and the load was applied (i) perpendicular, and (ii) parallel to the bedding planes for both coals. The results are as follows:

Coal	$\bar{P}_w/q_o h_w$	
	$\perp$ To bedding planes	$\parallel$ To bedding planes
Barnsley	0·84 ± 0·10	0·81 ± 0·13
Cwmtillery	0·97 ± 0·19	1·00 ± 0·20

These points lie close to the mean line of Fig. 10.5 (at $2w \tan \theta = 0{\cdot}577$) and are evidence, even if not conclusive, that the line is appropriate to the penetration of a wedge into a medium of semi-infinite extent.

Penetration of Ductile Materials

Practically all other recorded work on wedge penetration has been carried out using ductile materials, usually metals, as the penetrated medium. While the physical process involved is quite different from that which is observed with coal, some benefit may be obtained from a comparison of results.

The indentation of a metal by a tool takes place as a result of the displacement of metal in the vicinity of the working part of the tool by means of plastic flow. A theory of plane indentation of an ideally plastic metal by a hard frictionless wedge has been given by Hill, Lee and Tupper[4]. According to this theory the ratio between the pressure exerted by the wedge on the metal and the plastic yield strength (y) of the metal is a function only of the wedge angle. This function has a numerical value that increases from 1 to 2·6 as the wedge angle increases from near zero to 180 deg. The results obtained by Dugdale[5] using lubricated wedges with very low friction (μ = 0·02 to 0·03) were in good agreement with the theory.

Experiments on conical indenters were carried out by Hankins[2]. He assumed that in the absence of friction the indentation pressure would be the same for cones of all angles. This intrinsic yield pressure p_o would be increased by friction, according to a theory already mentioned in this chapter, giving an apparent indentation pressure p_i according to the relation

$$p_i = p_o(1 + \mu \cot \theta).$$

Hankins obtained good agreement with this expression. The intrinsic yield pressures obtained by Hankins by means of this formula were in good agreement with values he obtained using spherical indenters, and also with Brinell Hardness values. Tabor[6] shows that on theoretical grounds the latter are about 3y, where y, as before, is the tensile yield strength of the material.

Briefly, all experiments indicate that the indentation of work-hardened metal by a cone or a wedge is resisted by an indentation resistance p_o, which is roughly constant over a wide range of cone or wedge angles and has a value of the order of magnitude of the tensile yield stress. The total penetration resistance is given by $p_o(1 + \mu \cot \theta)$.

Penetration of a Brittle Material

The indentation of a brittle material, such as coal, by a wedge can only take place as a result of breakage and crushing of the coal in the vicinity of the wedge and the displacement of the broken coal to the free surface. The penetration of the wedge appears to be assisted by the low shear strength of coal, which allows the pulverized coal to be displaced to the surface between the wedge and the unbroken coal without exerting large tangential frictional forces on the faces of the wedge. In general, therefore, the cracks observed to emanate from the tip of the wedge in these experiments are not fundamental to the penetration except in so far as they facilitate the breakage of the coal in the vicinity of the wedge. The mean indentation pressure exerted between coal and wedge is constant over a wide range of wedge angles (10 to 140 deg) and equals the strength of the coal in compression. It should be borne in mind that the strength of coal is not unique, but varies with size of specimen. This matter has not been pressed in the present chapter, where change of size has not been important.

There is, therefore, an interesting similarity between the experimental results for wedge penetration in coal and the theoretical and experimental results for the penetration of wedges and cones into work-hardened metals, although the numerical factor connecting the indentation pressure with the strength differs somewhat in the two cases.

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CHAPTER 11

The Breakage of Coal by Wedge Action

1. FACTORS INFLUENCING BREAKAGE

MOST coal is broken from the seam by some form of wedge action. This is equally true of breakage by coal ploughs, by pick machines such as shearers or trepanners or by impact from a hand-held jigger pick. It follows that a knowledge of the mechanics of breakage of coal by wedge action should result in a greater mining efficiency. Such a knowledge can be obtained from "underground experience" which takes years to acquire, or from a combination of laboratory tests coupled with properly instrumented underground studies of coal winning machines. The latter is the better way to obtain this information, the many different aspects of wedge breakage being studied systematically.

The factors that affect wedge breakage fall roughly into two categories. There are those that affect breakage by any given shape or type of blade, and there are those that affect tool design. The first part of this paper is concerned with those factors that are not primarily concerned with tool shape. These include:

(i) the mechanical properties of the coal,
(ii) the cleat orientation,
(iii) the depth of cut,
(iv) the speed of cut,
(v) the size of tool,
(vi) the width of cut,
(vii) the pressure from the overbearing strata acting on the coal,
(viii) the effect of relief from other picks cutting in an array,
(ix) breakage by impact.

These factors have been studied in laboratory experiments in which cuts have been made in rectangular blocks of coal of various types. The same basic technique has been used throughout although a number of slightly different test rigs have been used. As the studies are based on many different experiments, definition of large and small coal sizes is not consistent throughout, but the conclusions are not affected in any way.

Experimental Technique

The basic experimental arrangement is shown in Fig. 11.1. A rectangular block of coal, which has been machined from a larger parent lump of run-of-mine coal is clamped to a heavy steel table that can be driven horizontally by an electric motor. The steel table passes under a gantry which supports a cutting tool. A groove can be cut in the top surface of the coal by suitably adjusting the height of the upper coal surface relative to the tool edge.

A dynamometer, fitted with electrical resistance strain gauges, is used to record the forces acting continuously on the cutting tool. In many of the experiments three orthogonal components of the grooving force were measured, in the cutting direction, normal to the surface of the coal and sideways on the pick. In some of the tests only the first or the first two components were measured.

The rectangular coal blocks were always machined and mounted on the table so that cuts were made in predetermined directions relative to the bedding

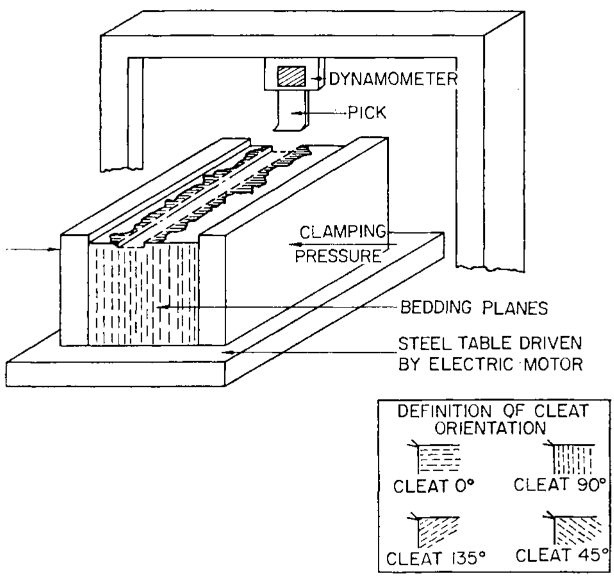

FIG. 11.1. Schematic diagram of single pick apparatus.

and cleat planes. The majority of the cuts were made parallel to the bedding planes and perpendicular to or parallel to the main cleat. The pressure between the steel plates used to hold the specimens on the table was of the order of 50 lb/in^2 unless otherwise stated.

Cutting Forces

The forces acting on a pick fluctuate continuously, and a typical record of cutting force is shown in Fig. 11.2. These records are interpreted in two ways. Either the average or mean force is calculated by integration of the trace, or the average of a small number (6–10) of the highest peaks is determined. The two values calculated are known as the mean cutting and mean peak cutting forces respectively.

FIG. 11.2. Typical force record.

Coal Produced

From each cut the broken coal was collected, sized and weighed.

The efficiency of cutting a groove is measured from the energy used in extracting a unit mass of coal (measured in ft-lb per lb). This is identically equal to the mean cutting force (in lb f) divided by the coal yield per ft advance of the tool (in lb f).

DETAILS OF THE EXPERIMENTS

The results given and discussed in this chapter were obtained from a large number of individual experiments, and for convenience the factors are being considered in the following sequence.

(i) Depth of cut.
(ii) Coal properties.
(iii) Tool size.
(iv) Other factors.
(v) Impact breakage.

Depth of Cut

As the depth of cut is increased the cutting forces increase and so does the coal yield. For most of the coals tested the peak cutting forces increase linearly with depth of cut whilst the mean forces increase approximately in proportion to $(\text{depth})^{1\cdot3-1\cdot4}$.

A typical set of results for a sharp cutter pick with a 30 deg rake angle and a back clearance angle greater than 5 deg, cutting in Barnsley Hards, a very strong bituminous coal, is plotted in Fig. 11.3. If the tool edge is not extremely

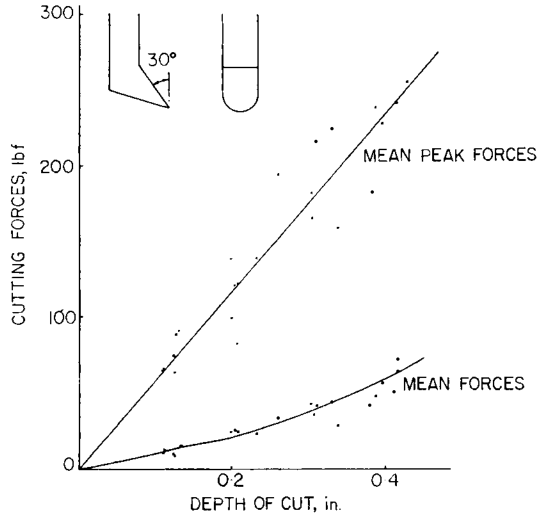

FIG. 11.3. Relationships between cutting forces and depth of cut for a sharp pick cutting in Barnsley Hards.

sharp, a substantial component of the cutting force is wasted in overcoming friction between the tool and the coal. In such a case, if the smoothed graph of mean force against depth of cut is extrapolated to zero depth, there is a positive force intercept on the ordinate, the blunter the tool the larger being the intercept.

The coal prised from the surface of a block of coal increases rapidly as the depth of cut is increased, the rate of increase in yield generally being more rapid than the corresponding increase in the mean cutting force. This implies that cutting becomes progressively more efficient as deeper cuts are made since the energy to extract a unit quantity of coal U is proportional to $\frac{\text{mean force}}{\text{coal yield}}$, and so to an exponent of depth of cut. This varies from one coal to another and also depends upon tool shape, normally being about $-0{\cdot}2$. The implication that deep cuts are best is almost invariably true.

Apart from the benefit of greater efficiency, larger sizes of coal and smaller quantities of dust also result from an increase in pick penetration (Fig. 11.4).

The quantity of fine coal produced during cutting is related to the energy expended. This is demonstrated by the graph on logarithmic scales of per-

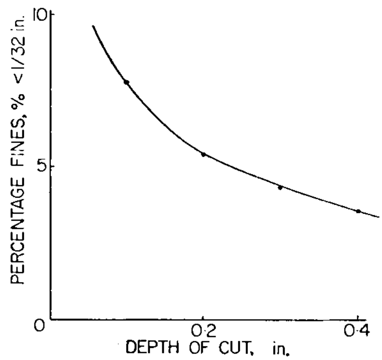

FIG. 11.4. Reduction in fine coal produced with increase in depth of cut. Pick cutting a groove in Barnsley Hards.

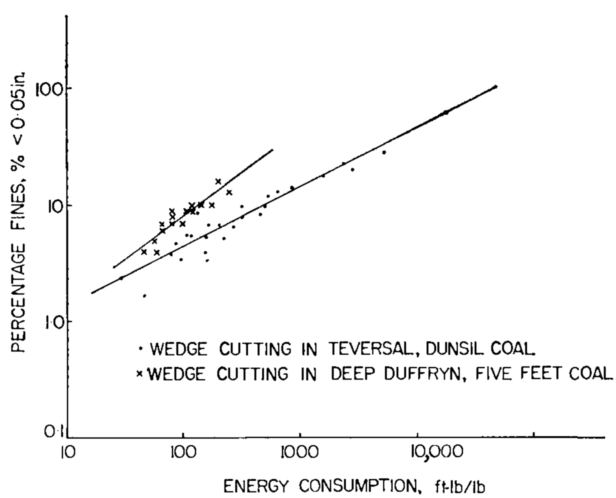

FIG. 11.5. Relationship between formation of fine coal and energy consumption.

centage fines against energy consumption per unit weight of coal extracted (Fig. 11.5). The relationship implies that any unnecessary power used in breaking coal from the seam is likely to result in unwanted fines and airborne dust.

Coal Properties

The types of coal that are found in Britain have a very wide range of mechanical properties. At one extreme there are the high rank, low volatile anthracites which are extremely strong and brittle and break with conchoidal fracture giving fragments with razor-sharp edges. In the seam these coals are frequently fractured by tectonic forces and extraction of the coal is largely a matter of exploitation of natural weaknesses and not the formation of new fracture surfaces. At the other extreme the low rank coals can be similar in strength to anthracite, particularly if the coals are free from cleat partings. A typical example of a strong low rank coal is Barnsley Hards, which is often almost cleat-free (but coals of similar rank which contain a dominant family of cleats may be much weaker since breakage of heavily cleated coals is controlled by exploitation of the natural fractures).

Intermediate coals are generally weaker and include the very friable Welsh steam and coking coals.

Simple grooving tests were undertaken on a wide range of coals chosen to represent the different types found in Britain. As cleat orientation affects breakage, cuts were made in different directions relative to the dominant cleat planes. The designation of the angles used is shown in Fig. 11.1. A list of the coals used is given in Table 11.1.

Table 11.1. Details of Coals Used

Colliery	Seam	Volatiles d.a.f. (per cent)	Cleat angles studied (°)
Pentremawr	Pumpquart	6	0, 90
Deep Duffryn	Five Feet	12	0, 90
Merthyr Vale	Five Feet	13	0, 45, 90, 135
Oakdale	Meadow	20	0, 90
Cwmtillery	Garw	25–30	0, 45, 90, 135
Rossington	Barnsley Hards	32	0, 90
Teversal	Dunsil	36	0, 45, 90, 135
Trimdon Grange	Tilley	37	0, 45, 90, 135
Rossington	Barnsley Brights	37	0, 45, 90, 135
Markham	Black Shale	38	0, 90
Sutton Tupton	Low Main	38	0, 45, 90, 135
Linby	High Main	38	0, 90
South Kirby	Beamshaw	35	0, 45, 90, 135

Apart from measuring the forces required to cut a prescribed groove with a given tool, and the coal yield, measurements were made of the basic mechanical properties of each coal. These included tensile and compressive strengths, the Impact Strength Index (I.S.I.) and the cleat frequency. The techniques adopted in making the measurements have been described earlier[1,2,3]. A frequent problem in the study of coal strength is the preparation of representative samples since the weakest coals are easily broken

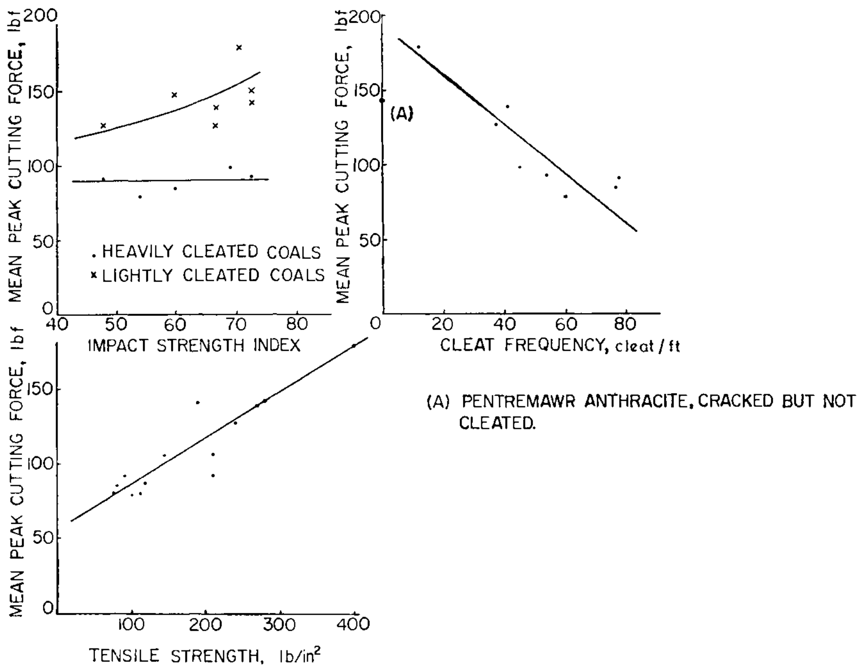

FIG. 11.6. Relationship between peak cutting forces and mechanical properties of coal. 0·4 in. deep cut, 0·5 in. wide made in coal blocks clamped at 50 lb/in^2.

during handling and the specimens that survive probably represent only the strongest parts.

However, despite the influence of sampling errors some good relationships have been established between the cutting forces and the coal properties[4] (Fig. 11.6). The effect of cleat is most pronounced for deep cuts, but the differences shown in Fig. 11.6, which are for 0·4 in. deep cuts, 0·5 in. wide, apply to shallower cuts as well.

It may be seen that the I.S.I. relates to the cutting forces in a manner that depends on the dominant cleat cracks in the coal. The data are plotted in two groups and for the measurements on heavily cleated coals the cutting forces are independent of coal strength. For less dominantly cleated coals, or for coals cut in such a way that cleat does not help breakage (i.e. cutting "on-end" or at 90 deg to the main cleat) the force required to make a specified cut increases with coal strength as measured by I.S.I.

Figure 11.6 also shows that the cutting forces are closely related to the frequency of cleats in the coal. The slightly anomalous value for Pentremawr anthracite is accounted for by the fact that whilst this coal is cleat-free it contains other natural cracks that have a large effect on breakage.

The final graph in Fig. 11.6 shows that tensile strength and cutting force are closely related. The tensile tests were made on specimens orientated so that the influence of the main cleats was exploited (i.e. the tensile stress was applied perpendicularly to the main cleat) and the grooves were also cut in a direction in which the cleats helped breakage. As cleat orientation and clamping pressure affect the forces needed to cut a groove, however, care must be taken in predicting cutting forces from tensile strength measurements.

Cleat Orientation

For a number of coals, carefully selected for the presence of a dominant main cleat system, grooves were cut in four different directions relative to the cleat, as shown in Fig. 11.1 and in Table 11.1.

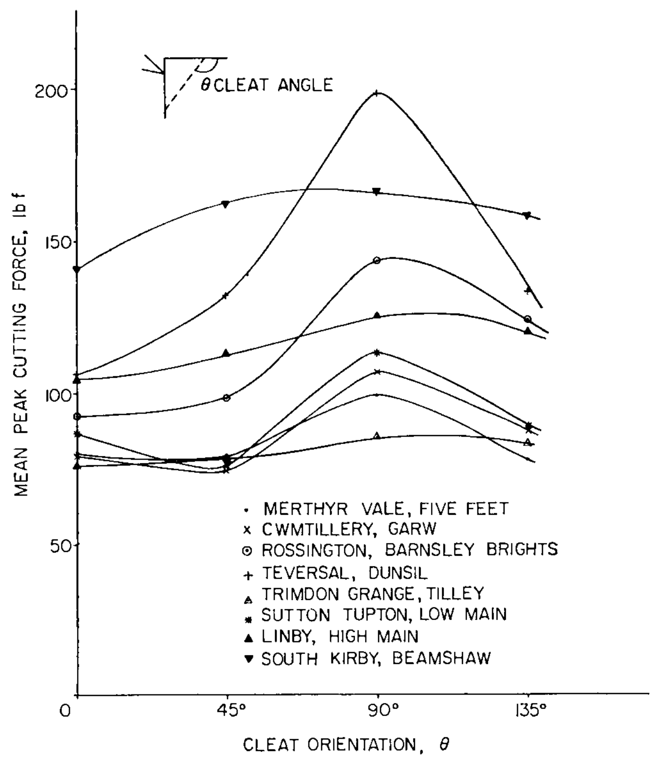

FIG. 11.7. The effect of cleat orientation on single pick cutting.

As may be seen from Fig. 11.7, it is generally most difficult to cut coal at 90 deg to the main cleat (i.e. "on-end") and that cutting "on-bord" (0 deg) or at 45 deg to the main cleat requires the lowest forces. The values plotted in Fig. 11.7 are the peak cutting forces and whilst the mean cutting forces are in broad agreement, there is some evidence that the 45 deg orientation is best. Averaging the data for all coals the cutting forces depend on cleat orientation as follows:

Cleat orientation (deg)	Mean peak cutting force (lb f)	Mean cutting force (lb f)
0	85	40
45	86	35
90	110	50
135	94	45

In other words the forces required to cut in the difficult direction (90 deg) are about 30 per cent higher than in the most favourable one. However, cutting force is not the sole criterion of cutting efficiency, but coal yield must also be considered. The greatest quantity of coal extracted also depends on cleat orientation and fortunately the greatest yield frequently coincides with the lowest forces. The overall effect (for all coals tested) of cleat orientation on cutting efficiency is:

Cleat orientation (deg)	Energy consumption (ft-lb per lb)
0	52
45	41
90	70
135	61

Cutting at 45 deg to the main cleat is thus shown to require only 60 per cent of the energy required to extract the same quantity of coal when cutting at 90 deg to the cleat. Formation of dust and production of large coal also benefit from improvement in cutting efficiency, so that when possible cuts should be made at 45 deg to the main cleat. Often, however, cuts are made both ways along a face so that a cut at 45 deg to the cleat one way is accompanied by one at 135 deg to the cleat the other. The total energy consumption for the two cuts is comparable with that for two cuts at 0 deg to the cleat, so that some of the benefits of the cut at 45 deg to the cleat are lost.

Explanation of Cleat Effects

In order to see how the cleats affect the propagation of cracks in coal, a rectangular block of coal was mounted against a plate glass window and a simple wedge was used to cut the coal adjacent to the glass. Different coal blocks were aligned with the cleats in the four cutting directions (0 deg, 45 deg, 90 deg and 135 deg) used earlier. A high-speed cine-camera that operated at about 2000 frames per sec was used to photograph the progression of the cut. Some typical stills from the film sequences are shown in Fig. 11.8, and Fig. 11.9 shows the most common types of failure that were observed. It can be seen why cutting at 45 deg to the main cleat results in a

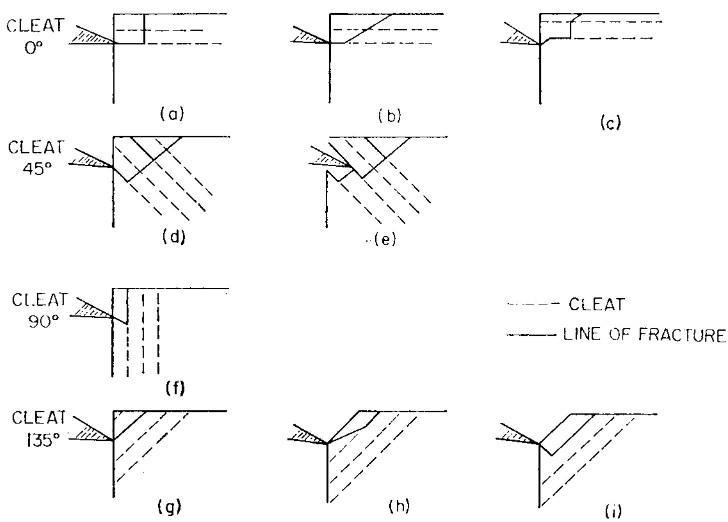

FIG. 11.9. Types of fracture in cleated coals.

larger product yield than for the other orientations. Moreover, as the initial crack from the wedge tip runs down into the coal along a cleat a low force is needed to start it. Breakage finally results in transverse bending of the "beams" of coal defined by the cleats.

In the 90 deg direction there are no existing cleats in the coal to assist crack formation (unless there is a marked cross cleat) and the cracks from the wedge run downwards into the coal until a gross main cleat is intersected, when a fragment breaks away. As the cracks are formed in solid coal the cutting forces are high, and unless the main cleats are widely spaced the product yield will be low and consist of relatively small fragments.

Breakage in the other two directions is almost self-explanatory. At 0 deg the cleats are oriented in a way that encourages cracks to run along the path of the

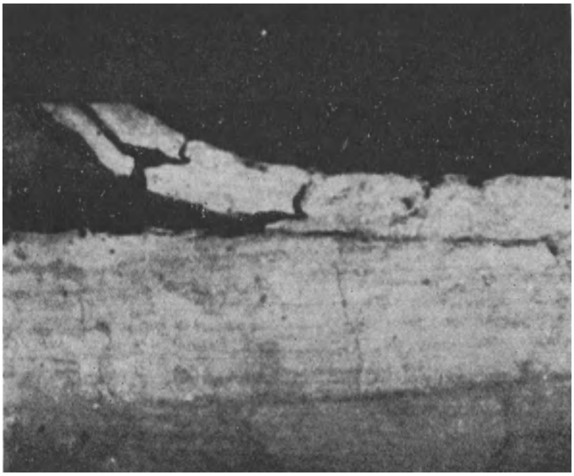

Fig. 11.8(a)

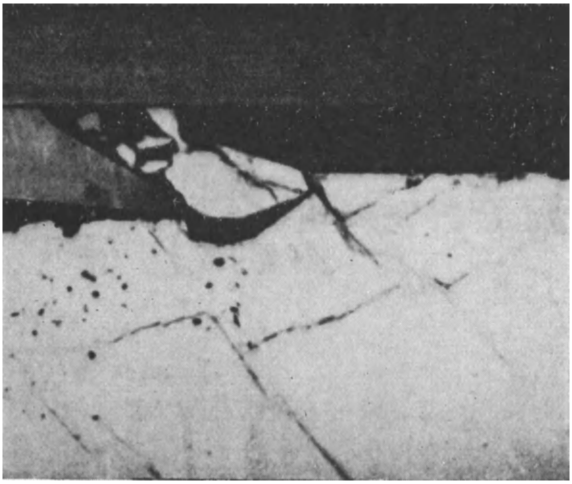

Fig. 11.8(b)

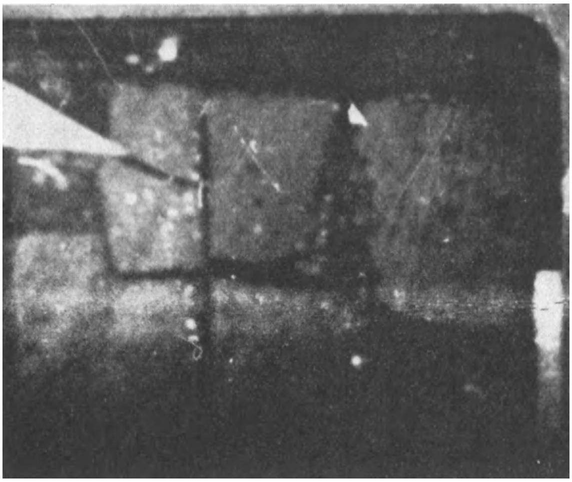

FIG. 11.8(c)

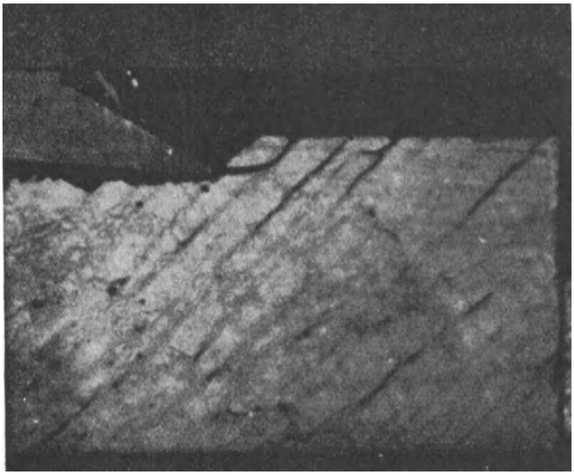

FIG. 11.8(d)

FIG. 11.8. Typical "stills" from high-speed photographs of wedges cutting coal. Cleat 0 deg. Cleat 45 deg. Cleat 90 deg. Cleat 135 deg.

wedge tip and so breakage forces and coal yield are both low, whilst at 135 deg a sequence of cracks runs from the wedge to the surface of the coal with very few excursions into the bulk of the coal. In this case the forces are again relatively low but the product is small in size and in quantity.

Tool Size

Laboratory experiments are usually carried out with a single size of cutting tool and it is essential to establish relationships between tool size and cutting performance if the results obtained with one size of pick are to be used to predict the performance of other sizes.

A number of experiments have been undertaken in which a range of sizes of simple wedge and of cutter pick have been used to groove different coals. The coals included strong and friable types. Cuts were made parallel and perpendicular to the main cleat directions (the 0 deg and 90 deg orientations shown in Fig. 11.1), but cleat direction has not been found to influence size effects significantly.

Basically different relationships were established for the two coal types. For a given shape of cutting tool (wedge, or pick) the mean cutting forces needed to groove the friable Blaenavon, Garw and Deep Duffryn, Five Feet coals increase in direct proportion to the cross-sectional area swept by the tool below the surface of the coal.

For the stronger coals, such as Rossington, Barnsley Hards and Teversal, Dunsil, cracks are formed at the tool tip which run rapidly ahead of and to the side of the tool and it is the length of the boundary crack that defines the mean cross-section of the groove that governs the cutting forces.

If we define the size or scale of the tools by g in direct linear proportion to some unit size of tool and designate the depth of cut by d we find that for the Garw coal the mean grooving force f_c is given by:

$$f_c \propto d^{1\cdot 2} g^{0\cdot 8} \tag{11.1}$$

and the mean peak grooving force F_c by

$$F_c \propto d g^{0\cdot 8}. \tag{11.2}$$

The equivalent equations for Barnsley Hards are

$$f_c \propto d^{1\cdot 35} g^{0\cdot 1} \tag{11.3}$$

$$F_c \propto d g^{0\cdot 2}. \tag{11.4}$$

Equation (11.1) is compatible with the mean force for friable coals being proportional to the area swept by the tool below the coal surface for the shape of tool used in the experiments. It can be seen from these equations that the size of the tool has a much greater effect on cutting a friable coal than it has on strong coal. Moreover, for both coals the peak cutting forces increase in direct proportion to the depth of cut for a given size and type of tool.

If coal yield is also taken into account it is found that for the Garw coal the energy required to remove a unit weight of coal is independent of both the depth of cut and the size of tool used. However, by taking deep cuts or by using a large tool there is a marked improvement in product size.

With Barnsley Hards there is a marked improvement in cutting efficiency as the depth of cut and scale of tool are increased, the equations obtained being:

$$U \propto d^{-0\cdot25} g^{-0\cdot25} \quad \text{for} \quad 5 \text{ deg rake angled picks}$$

$$U \propto d^{-0\cdot12} g^{-0\cdot15} \quad \text{for} \quad 30 \text{ deg rake angled picks}$$

where U is the work done in extracting unit weight of coal.

The strength of cubes of Barnsley Hards in compression (σ_c) has been shown[1] to decrease with increase in specimen size (a) according to:

$$\sigma_c \propto a^{-0\cdot17 \pm 0\cdot02}$$

and the effect of pick size on cutting efficiency is seen to be in broad agreement with this. It follows, therefore, that the improved cutting efficiency obtained by making deep cuts with large picks could result from a better exploitation of gross weaknesses in the coal.

There is a corresponding fall in the compressive strength of the Garw coal with increase in specimen size, and this should be reflected in an improved cutting efficiency when deep cuts with large picks are made in the friable coal. A wide scatter in the results may account for the lack of agreement, but the difference in the modes of breakage between the strong and the friable coals may be responsible. The local crushing around the pick tip that occurs in the friable coal may well conceal the advantage in cutting efficiency that would otherwise be gained from large picks biting deeply into the coal.

The experiments with simple wedges show an improvement in cutting efficiency with increase in wedge size and depth of cut for both types of coal so that, in general, the use of large picks taking deep cuts is recommended for efficient cutting, for the extraction of large sizes of coal and for the formation of the minimum amounts of fine coal.

Miscellaneous Factors

Pick Speed

The speed at which the cutting elements on power loaders cut into the coal varies widely from about 60 ft/min on a rapid coal plough to about 2000 ft/min on some of the drum and chain type machines. This full speed range has not yet been studied in the laboratory, but picks have been used to cut grooves in coal blocks at various speeds from 200 ft/min up to 600 ft/min. Two shapes of pick (a straight bar and a forged 30 deg rake angled pick) were used and cuts were made in the friable Garw and the strong Dunsil coals. Two cleat orientations (0 deg and 90 deg) were used for the Dunsil coal, but no consistent cleat orientations were used for the Garw coal.

The results given in Table 11.2 show that for both coals and for all four depths of cut studied there is no evidence that cutting speed has any effect on the forces needed to cut a groove to a prescribed depth. Additional experiments have extended the velocity range studied down to about 1 ft/min and even over this extended range pick speed does not affect the grooving forces. The amount and size of coal broken are also independent of cutting speed.

TABLE 11.2. THE EFFECT OF PICK SPEED ON CUTTING FORCES

Cutting speed (ft/min)	Mean cutting force (lb f)					
	Straight bar pick			Forged 30 deg rake angled pick		
	Dunsil (0 deg)	Dunsil (90 deg)	Garw	Dunsil (0 deg)	Dunsil (90 deg)	Garw
200	170	230	130	85	100	100
400	165	200	90	90	110	95
600	170	220	110	100	95	115

In applying these results to practical problems it must be remembered that speeding up a drum or a chain, without a comparable increase in the rate of advance of the machine, reduces the penetration made by each pick into solid coal. This reduces the efficiency of cutting, leads to a marked decrease in product size and the higher speed picks will throw more dust into the airstream with undesirable consequences.

However, for a given pick penetration the forces taken by individual picks and the total torque on the driving shaft or sprocket are independent of the pick speed.

Confirmation of these findings has come from laboratory experiments with a one-third scale jib cutter. A range of pick (or chain) speeds (100 ft/min to 250 ft/min) was used for different machine haulage rates and it was found that pick penetration was the only factor that affected the specific energy and the product size.

The high-speed photographs taken in the study of cleat effects showed that cracks propagate at speeds very much greater than the cutting speeds. The filming rate was about 2000 frames/sec and cracks usually propagated from the tool tip to the coal surface (a distance of about 1 in.) between two successive frames, that is the cracks spread at a speed of at least 200 ft/sec, compared with the highest cutting speed used of 600 ft/min. It seems that cutting speed will not affect the breakage process unless pick speeds approach the speed of crack propagation and this is not likely to happen.

Overburden Pressure

Coal at the face is not only subjected to stresses imposed by the cutting tools but also to those imposed by the overbearing strato. It has been shown[(5)] that biaxial compressive stresses of the order of 2000 lb/in^2 in vertical and horizontal directions can be experienced and stresses as large as this must affect the resistance of the coal to extraction in some way or another. If the stresses from the overburden are sufficiently large the face coal will spall off the solid and so no additional winning forces are needed, but if the stresses are not large enough to cause fracture it is difficult to predict what forces will be needed to cut the coal.

In the grooving experiments undertaken in the laboratory the rectangular coal blocks were held between rigid steel plates whilst the cuts were made. The bedding planes were aligned parallel to these plates (see Fig. 11.1) to which loads were applied to hold the coal blocks. These loads were varied to subject the coal blocks to different confining pressures, which may be considered to represent the overburden pressure acting on coal in the seam.

TABLE 11.3. THE EFFECT OF OVERBURDEN PRESSURE ON CUTTING FORCES FOR CUTS 0·4 in. DEEP

Mean peak cutting force (lb f)				
Colliery	Seam	Confining pressure		Percentage increase
		50 lb/in^2	500 lb/in^2	
Pentremawr	Pumpquart	142	152	7·0
Deep Duffryn	Five Feet	79	118	49·4
Oakdale	Meadow	(i) 91	118	29·7
		(ii) 127	143	12·6
Cwmtillery	Garw	(i) 85	127	49·4
		(ii) 148	127	−14·2
Rossington	Barnsley Hards	179	189	5·6
Teversal	Dunsil	(i) 93	111	19·4
		(ii) 150	167	11·3
Markham	Blackshale	139	171	23·0
Rossington	Barnsley Brights	127	131	3·1
Linby	High Main	99	131	32·3

(i) Cutting at 0 deg to main cleat.
(ii) Cutting at 90 deg to main cleat.

In one series of tests, cuts 0·4 in. deep were made into nine different coals and confining pressures of 50 lb/in^2 and 500 lb/in^2 were used. The upper figure was the highest value that could be used without breaking the weakest coal (Oakdale, Meadow) in compression. Cuts were made parallel to the bedding planes and either parallel (0 deg) or perpendicular (90 deg) to the main cleat. Where cleat significantly affected the results separate values are given in Table 11.3, which summarizes the results. Otherwise an average figure is given.

It can be seen that with one exception the coals are more difficult to groove at the higher confining pressure. The largest increases occur for the most heavily cleated coals and for the cuts made at 0 deg to the main cleat, when the cleat weaknesses are more easily exploited than in the 90 deg direction. The initial application of a confining pressure tends to close the existing cleat and bedding plane cracks and so reduces their effect on cutting.

A more detailed study has been made for three coals and the relationships between cutting force and confining pressure are shown in Fig. 11.10. For shallow cuts variation in pressure has little effect on the friable coals, possibly because the irregularities on the surfaces of the coal blocks masked differences due to the confining pressure.

For deeper cuts all of the coals become progressively more difficult to cut as the confining pressure is increased from zero. At a pressure of about 500 lb/in^2 for the Oakdale coal and about 750 lb/in^2 for the Deep Duffryn

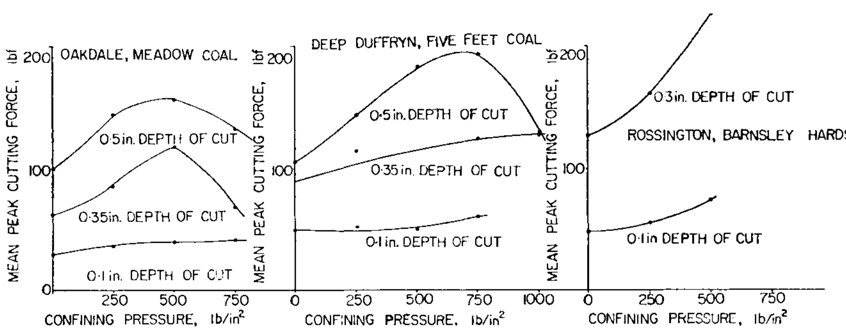

FIG. 11.10. The effect of variation in confining pressure (to simulate overburden pressure) on cutting force.

coal the cutting forces reach a maximum and any further increase in confining pressure weakens the coal. The confining pressures applied are approaching the compressive strengths of these coals and it is suggested that the stress from the overburden pressure supplements the force from the cutting tool, so that the magnitude of the cutting force falls. No maxima were observed for the Barnsley Hards, as the confining pressures applied to this coal were only a small percentage of the compressive strength.

Picks Cutting in Sequence

All the experiments described so far have been tests in which a single groove has been cut in a plane surface of a rectangular block of coal. In practice a number of picks cut at the same time and a systematic study is being made of the interplay between picks cutting in an array. Although these experiments are still continuing much useful information has already been obtained. The problem has been simplified into consideration of (i) what happens when a single groove is deepened and (ii) the effect of cutting one groove between parallel grooves previously cut by other picks. The sequence in which cuts were made is shown in Fig. 11.11. In these experiments the central cut was the same depth from the surface of the coal as the parallel grooves. The distance between adjacent grooves is L_s, the line spacing. The depth of the first cuts relative to the coal surface is d_1 and where a groove is deepened by a subsequent cut the pick advance is given by d_2 (see Fig. 11.11).

As shown in Fig. 11.12(a) it is much more difficult to deepen a groove than it is to make the original single cut. The deepened central cut gets some

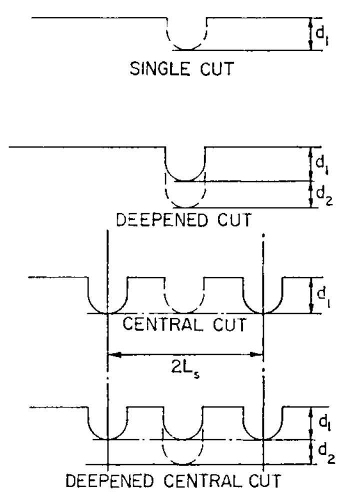

FIG. 11.11. Sequence of cuts. The dotted part refers to the named cut.

benefit from the adjacent parallel grooves and so, whilst more difficult to cut than the original single groove, requires much lower cutting forces than the deepened single groove.

The magnitude of the force required to cut a central groove depends on the amount of coal left to be broken by the preceding outer cuts. If the depth of cut d_1 is large enough the grooves cut by the outer picks will splay and give some relief to the central cut. The depth of cut that causes this relief depends on the spacing between adjacent lines of picks. It can be seen from Fig. 11.12(b) that the forces needed to cut the central groove are lower than those needed to make a single cut, and that these forces increase to a maximum and then decrease. The decrease occurs as the main benefit or relief from the outer grooves is gained. The maxima occur at gradually increasing depths of cut as the line spacing is increased and it may be seen that the

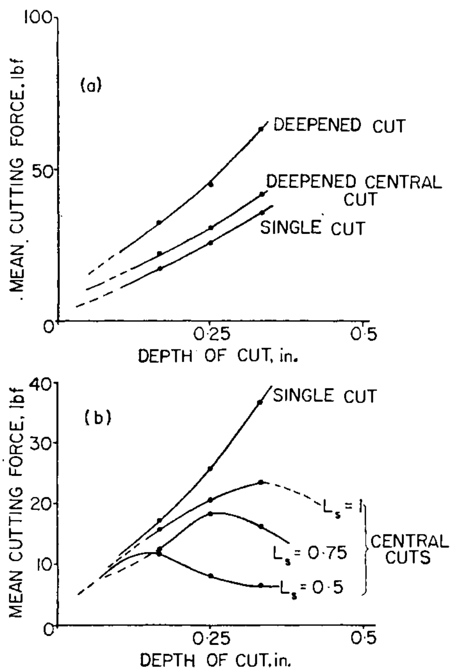

FIG. 11.12. The effect of preceding cuts on cutting force. L_s = line spacing.

lowest cutting forces decrease rapidly when the depth of cut becomes deeper than about one half the line spacing.

If the depth of cut is less than one-third the line spacing there is little breakage between pick lines to help the following patterns of picks. Successive patterns of picks will deepen the grooves cut already and the unacceptably high cutting forces needed to deepen a groove (Fig. 11.12(a)) will be experienced. Furthermore, a succession of shallow cuts tends to form ridges of coal between adjacent pick lines, and these ridges are very difficult to remove and often in practice foul the pick boxes.

As has already been emphasized cutting force values alone are not a measure of efficiency but coal yield must also be taken into account. The cutting forces obtained in the relief experiments, together with measurements of coal yield, have been used to calculate the work done per foot of a coal-cutter jib per in. advance in cutting a kerf 6 in. wide. The benefits from relief from the outer lines of picks are felt when the pick penetration is greater than one-third the line spacing. Cutting efficiency also increases as the line spacing is increased (see Fig. 11.13), so that deep cuts that are widely spaced should be used whenever possible on coal cutting machines. The total cutting force taken by individual picks will depend on the depths of cut taken so that it may

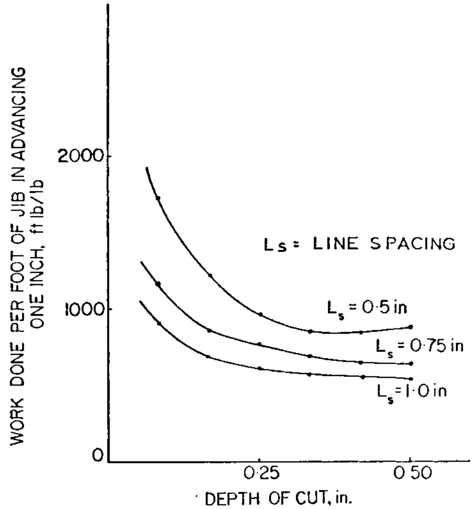

FIG. 11.13. Work done per foot of jib in advancing it by 1 in. when cutting a kerf 6 in. wide. (The depth of cut is the pick advance by successive picks in the same line.) L_s = line spacing.

be necessary to use stronger and larger picks than are normally employed if the full advantages of relief are to be gained.

The benefits of efficient cutting are not only reflected in the lower power required to extract a given quantity of coal, but the size of product is improved and the formation of dust reduced.

IMPACT BREAKAGE

The forces that act on a tool cutting in coal fluctuate widely (see Fig. 11.2) and the peak forces frequently exceed three times the mean cutting force. On multi-pick machines it is highly unlikely that all of the picks that are cutting at the same time will have peak forces acting on them simultaneously, so that the shaft or sprocket torque will not vary so much. On the other hand, on coal ploughs which have a small number of blades cutting at once the haulage force fluctuates continuously[6]. A possible way to reduce the load variation in the haulage chain would be to provide extra power to the blades when and only when peak loads occur. Experiments in the laboratory and underground have been undertaken to see if stored energy could be used to break the coal by impact when the ploughing forces become large. The experiments were designed to see what size of blow would be needed to break the coal and to see whether repeated blows of lower energy at one point would initiate and propagate cracks. Experiments were carried out on a friable South Wales steam coal and on the strong Barnsley Hards. Grooves were cut in the plane surface of coal blocks by a succession of impacts from a simple wedge attached to the end of a pendulum. The magnitude of the impulse was varied by altering the angle through which the pendulum swung and by varying the mass on the end of the pendulum. A diagram of the apparatus is shown in Fig. 11.14. A 0·5 in. wide wedge was used to cut a groove

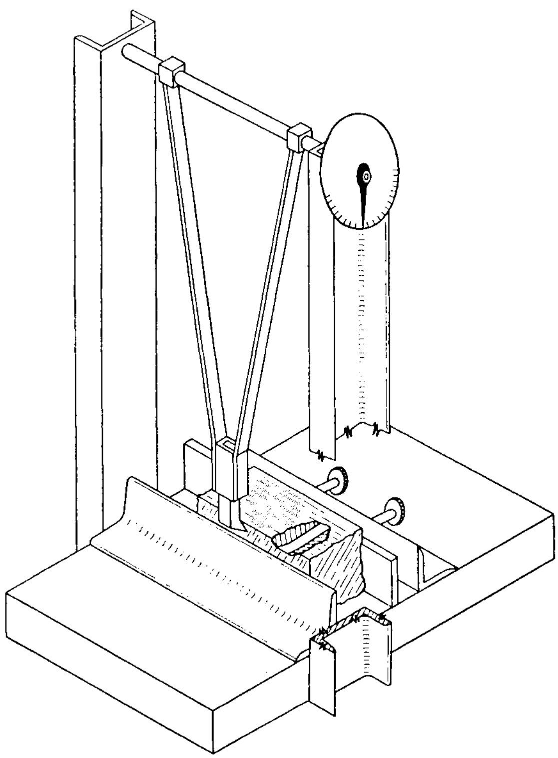

FIG. 11.14. Diagram of apparatus.

0·5 in. deep in the friable coal and a 0·375 in. wide wedge to cut grooves 0·125 in., 0·5 in. and 0·375 in. deep in Barnsley Hards.

The principal findings were:

(i) The energy required to break coal was at best comparable with the energy used in continuous cutting but for blows that were too light or too heavy impact breakage was highly inefficient (see Fig. 11.15(a)).

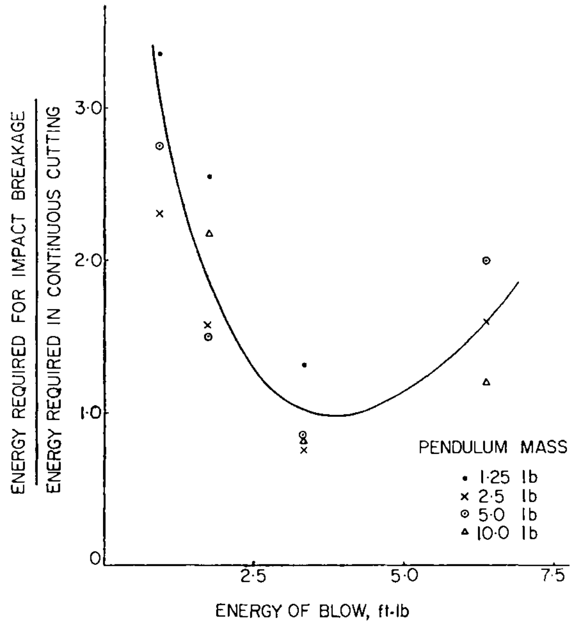

FIG. 11.15(a). The energy required to cut coal by a succession of impacts compared with the energy used on continuous cutting for different sizes of blow.

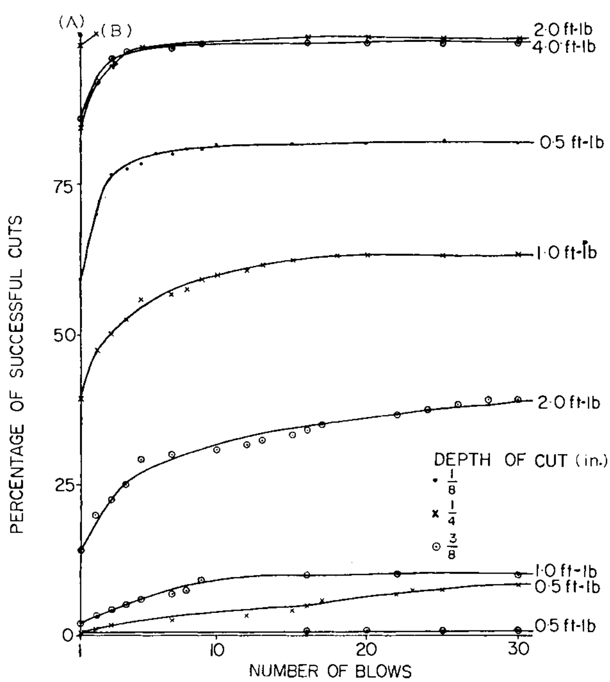

FIG. 11.15(b). The effect of repeated impact on breakage. (A) 1·0, 2·0 and 4·0 ft/lb blows, $\frac{1}{8}$ in. depth of cut. (B) 4·0 ft/lb blows, $\frac{1}{4}$ in. depth of cut.

(ii) If breakage is not effected by the first few blows, repeated impact will not cause breakage no matter how many blows are imparted (Fig. 11.15(b)).
(iii) A blow of at least 3000 ft-lb is needed to break a 6 in. by 6 in. section from Barnsley Hards in the seam. A lower value would be needed for weaker coals but these are relatively easily ploughed and so are not likely to need help from impact.

It is thought that light blows are ineffective because the first impacts form a cushion of fine coal around the tool tip and successive blows are absorbed by this cushion. Unnecessarily large impacts are inefficient since the excess power applied is partly absorbed in the machine and partly in unwanted degradation and dispersion of the coal.

These findings have been confirmed[7] in underground tests on a friable (Cwmtillery, Garw) and a strong (Blackwell B Winning, Three-quarter) coal. Impacts up to 1000 ft-lb were used to cut prepared buttocks of coal to depths of 4 in., 6 in. and 8 in., with blades 8·5 in. and 12 in. high. It was shown that even 1000 ft-lb blows did not often fracture the buttock of coal at the first attempt, even for the shallowest cuts. It was also shown that if repeated blows were needed to fracture the coal (and it was not always possible to do this) the energy required to break a given amount of coal rose rapidly.

As the use of unnecessary energy results in added degradation of the coal, a system that needs repeated impact to break the coal cannot be advocated.

Discussion

This chapter has outlined some of the principal factors that affect breakage of coal by pick action. The next one deals with other factors that affect tool design. These factors include the basic shape of the tools and the effects of wear on tool performance. However, before considering tool design it is worthwhile considering the implications of the findings reported so far. These may be summarized as follows:

(i) Shallow cuts (low pick penetrations) are highly inefficient and produce small coal and hence an abundance of dust.
(ii) Cutting efficiency (reciprocal of work done per unit coal yield) increases with increase in depth of cut.
(iii) The most efficient orientation of the cleats during cutting is 45 deg to the line of attack, mean cutting forces being low and coal yield high. Cleat orientation has little bearing on trepanners and shearers, where much of the cutting is done perpendicular to the bedding planes.
(iv) Provided the size and shape of the cutting tool are such that the channel of coal actually extracted is greater than and independent

of the area swept by the tool, tool size does not affect significantly the forces needed to groove a strong coal, whilst it has a large effect on cutting friable coals. In terms of cutting efficiency, however, the size of tool used in the friable coal is not important, whilst large tools and deep cuts lead to improved efficiency in cutting strong coals.

(v) Pick speed does not affect cutting forces or coal yield significantly, although high speed picks can throw additional dust into the air-stream.

(vi) Pressure from the overbearing strata can either weaken or strengthen coal at the face, so that throwing extra weight on the face will in some cases ease coal winning whilst in others it will make it more difficult.

(vii) Picks cutting in an array help one another, provided the penetration by individual picks is $\frac{1}{3}$–$\frac{1}{2}$ the line spacing. Moreover, wide line spacings, and hence deep cuts, are more efficient than the converse of shallow cuts from closely spaced picks.

(viii) Deepening an existing groove is highly inefficient, requiring much higher forces than cutting the same depth of cut in a plane surface, and results in severe coring or ridging of the coal.

(ix) Breakage by impact can be as efficient as continuous cutting but if the blows are either too light or too heavy efficiency suffers. If the blows are too light, repeated impacts will not cause breakage, no matter how many blows are applied.

(x) Impacts of the order of 3000 ft-lb are needed to ensure the breakage of a 6 in. by 6 in. section from a strong coal, such as Barnsley Hards.

(xi) The strength characteristic that appears to control the breakage forces closely is tensile strength, but as cleat orientation and over-burden pressure affect breakage forces, care must be taken in predicting breakage forces from laboratory strength measurements.

The general conclusion from these experiments, therefore, is that mining machines should be designed to take as deep a cut with each pick as is compatible with the strength of the picks and the strength and stability of the machine. The depth of cut taken by the picks defines the optimum spacing between adjacent lines of picks, which is two to three times the depth of cut.

The combination of a deep cut and a wide line spacing is ideal for the production of large sizes of coal and small quantities of dust. Many of the existing machines are unsuitable for use with large picks, since machine vibration from the high pick forces cannot be tolerated. There is, however, an optimum arrangement of picks on these machines, and the minimum number of picks should be used. An over-picked machine could even have greater loads on each pick, in view of the coring that takes place, than would be observed if some of the picks were removed.

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CHAPTER 12

The Breakage of Coal by Wedge Action

2. FACTORS AFFECTING TOOL DESIGN

IT HAS been shown[1] and restated in Chapter 11 that for efficient coal cutting, machines should be designed with widely spaced picks which bite deeply into the coal. In this way the size of the product is larger and the formation of fines is smaller than from a machine that contains a greater density of picks, and the cutting efficiency is high. The present Chapter describes work that has been undertaken at M.R.E. to find the most efficient designs of cutting tool. Cutting efficiency cannot be separated from the effects of tool wear and some aspects of wear are also considered.

The same basic types of experiment were undertaken as those described in Chapter 11. Single grooves were cut into a plane surface of a rectangular block of coal by wedges and cutting tools of different design. The forces acting on the tools were measured and related to the quantity and sizes of coal extracted. In all of the experiments cuts were made in specific directions relative to the bedding and cleat planes. Most of the cuts were made parallel to the bedding planes and perpendicular to the main cleat.

THE EFFECT OF FRONT RAKE AND BACK CLEARANCE ANGLE ON CUTTING

A survey of picks in current manufacture in Britain showed that a very wide range of tool shapes was in use and a pattern of pick was derived which permitted two of the fundamental features of these picks to be compared systematically; namely the front rake and the back clearance angles (Fig. 12.1)[2]. A series of twenty-four picks was made in which each pick had one of six rake angles (ε = 0, 5, 10, 20, 30 and 45 deg) and one of four clearance angles (δ = 0, 5, 15 and 30 deg). All twenty-four possible shapes of pick tip were used. The picks were also made so that the shape when viewed from the front was the same (see Fig. 12.1). All cutting edges were sharp.

Cuts of four different depths (0·1, 0·2, 0·3 and 0·4 in.) were made with each pick in a friable (Cwmtillery, Garw) and in a strong (Rossington, Barnsley Hards) coal. The forces acting on the cutting tools fluctuate continuously

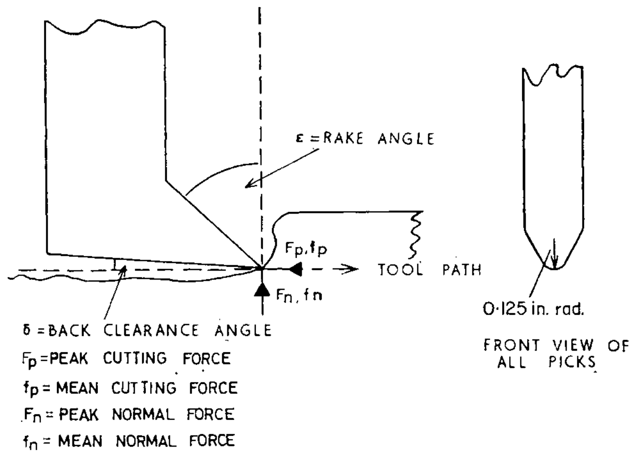

FIG. 12.1. Definition of tool angles and forces acting on tool during cutting.

during each cut and for analysis these are described by the mean cutting (or normal) force (obtained by integration of the force–time record) and the mean peak cutting force, which is the average of the highest forces recorded in six equal increments of the trace.

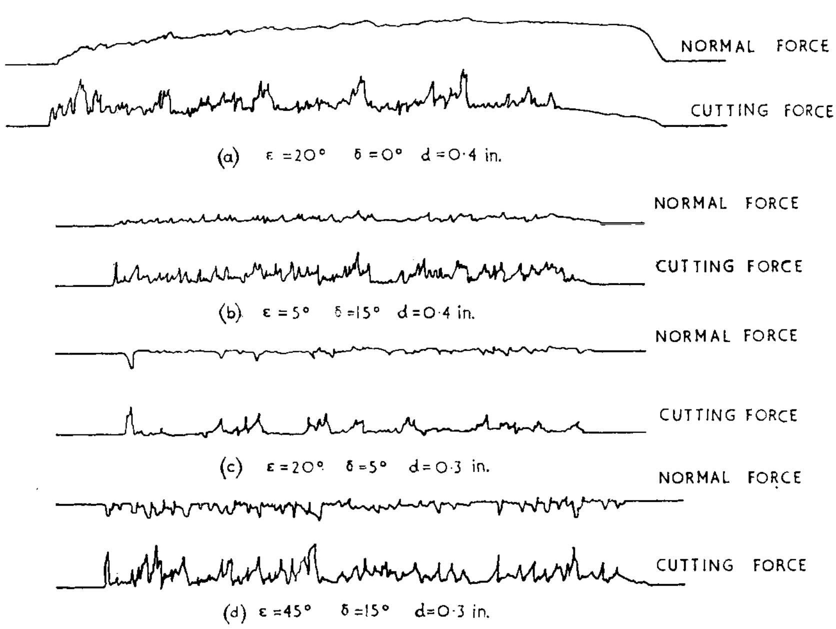

FIG. 12.2. Typical force records.

Visual Examination of the Force Records

Some typical force records are given in Fig. 12.2. The records show a number of markedly different characteristics for different shapes of cutting tool. If the back clearance angle is zero the cutting force fluctuates in a fairly orthodox manner (Fig. 12.2(a)), but the normal force rises to a high value, which may be comparable in magnitude with the cutting force, and remains fairly constant throughout the cut.

For all cuts using tools with at least 5 deg clearance the normal and cutting forces fluctuate roughly in phase (Fig. 12.2(b)), but when the cutting edge is extremely sharp the normal force corresponding to a high cutting force acts into the coal (Fig. 12.2(c)) particularly when the rake angle is large (Fig. 12.2(d)). If the edge of the cutting tool is blunted, or if the rake angle is low, the normal force remains in phase with the cutting force variations but usually acts outwards from the coal surface (Fig. 12.2(b)).

The Effect of Tip Angles on Cutting Performance

As the back clearance angle is increased from zero to 5 deg the cutting and the normal forces decrease substantially, but increases in the clearance angle beyond 5 deg cause little further reduction in the forces. The same effect is observed for picks of all rake angles (from 0 to 45 deg) and some typical results for a 30 deg rake pick, cutting in Barnsley Hards, are given in Fig. 12.3. Similar observations were obtained with the friable Garw coal.

The mean cutting force decreases continuously as the rake angle is increased (Fig. 12.4(a)), particularly with the picks which have a positive clearance angle. The normal forces acting on the pick also decrease with increasing rake angle (Fig. 12.4(b)), becoming almost zero for rake angle of 25 deg and above. In some instances, particularly when sharp picks are used, the normal force becomes negative (i.e. the forces acting on the pick tend to drag the pick into the coal).

Cutting forces alone do not measure cutting efficiency since a low cutting force might be associated with a low yield of coal. A useful measure of cutting efficiency is the specific energy, which is defined as the energy expended in breaking a unit weight of coal from the parent mass. In general, cutting efficiency increases (i.e. the specific energy decreases) as the depth of cut is increased but in the pick shape experiments the converse was observed for Barnsley Hards (Table 12.1). This reversal of the normal pattern is attributed to the fact that when shallow cuts are made in Barnsley Hards a thin plate-like section of coal is peeled from the coal surface, so that the coal yield is very much larger than the volume swept by the tool during these cuts. The specific energy is therefore low. When deeper cuts are made less surface spalling occurs and the specific energy rises.

With Cwmtillery Garw coal, however, there is no abnormal splitting parallel to the surface of the coal and the specific energy decreases as the cuts

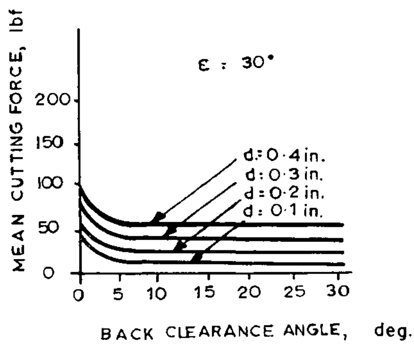

FIG. 12.3(a). Variation of mean cutting force with back clearance angle for a pick with rake angle 30 deg. at various depths of cut *d*.

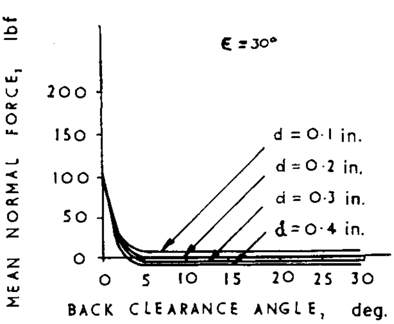

FIG. 12.3(b). Variation of mean normal force with back clearance angle for a pick with rake angle 30 deg. at various depths of cut *d*.

TABLE 12.1. THE EFFECT OF DEPTH OF CUT ON CUTTING EFFICIENCY

Depth of cut (in.)	Specific energy (ft-lb/lb)†	
	Barnsley Hards	Cwmtillery, Garw coal
0·1	670	610
0·2	900	330
0·3	910	300
0·4	1150	340

† Average values for all picks except those with zero clearance angles.

are made deeper. There is slight evidence that this procedure does not continue indefinitely, and it is possible that if cuts are made deeper than some specified depth the specific energy rises again (Table 12.1). This has been noticed in other laboratory experiments and is attributed to the fact that if very deep cuts are made in a friable coal local crushing occurs around the tool tip so that the effectiveness of the tool is reduced, an excessive quantity of fine coal is formed close to the tip and hence the specific energy rises.

A more consistent relationship was found for the two coals between specific energy and rake angle. The specific energy decreases steadily with

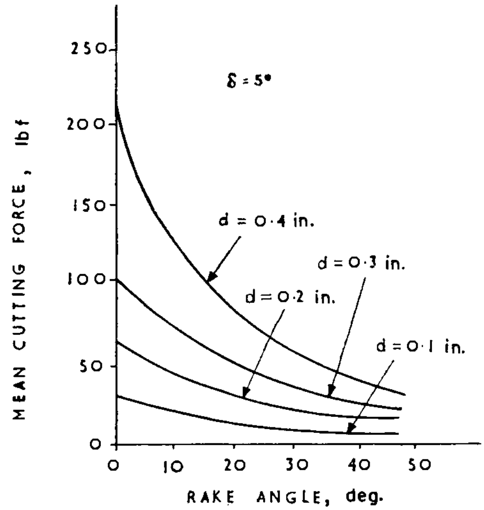

FIG. 12.4(a). Variation of mean cutting force with rake angle for a pick with back clearance angle 5 deg at various depths of cut *d*.

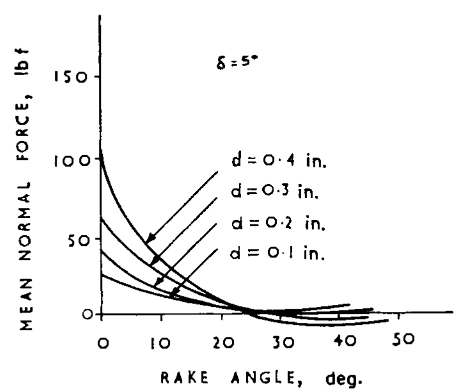

FIG. 12.4(b). Variation of mean normal force with rake angle for a pick with back clearance angle 5 deg, at various depths of cut *d*.

increase in rake angle as shown in Fig. 12.5, so that for efficient cutting, tools with as large a rake angle as possible should be used. Unfortunately, as the rake angle is increased the strength of the tool tip is weakened so that it is not possible to have the full benefits of cutting efficiency with a pick that is sufficiently robust to have an acceptable pit life. This aspect of tool design will be discussed in more detail later.

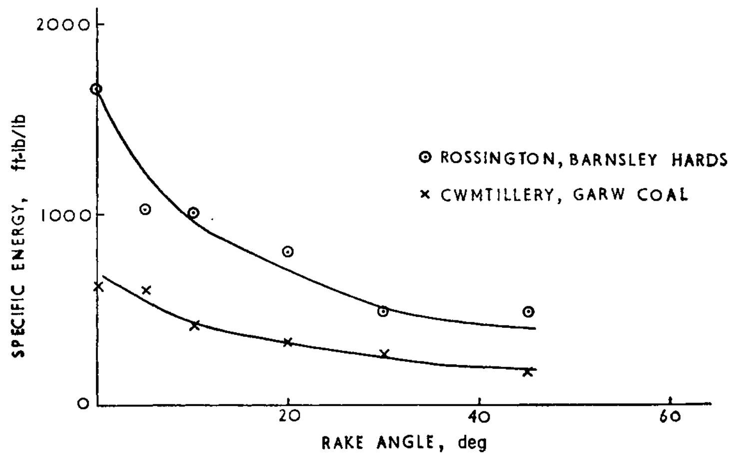

FIG. 12.5. The effect of rake angle on the energy used in cutting coal.

The Effect of Pick Profile on Cutting Performance

All of the picks used in the experiments described above had the same profile when viewed from ahead of the cut. The second phase of the systematic study of tool design was to see what effect variations in the profile of a flat-faced pick had on its performance. Two families of picks were generated from a pointed wedge with a 30 deg rake angle and a 6 deg back clearance angle. In one family the angle of the point was varied between 30 and 180 deg (a simple wedge or chisel tip). In the other, 60 deg pointed wedges had different radii ground on the pick tip. Details of the pick tips are shown in Fig. 12.6.

Using energy consumption again as the criterion of pick efficiency it can be seen (Fig. 12.7) that the more obtuse the pick tip the more efficient is the cutting process. The cutting forces acting on an individual pick are larger for the 180 deg or chisel pick than for the pointed ones, but the quantity of coal cut more than compensates for the increase in the cutting forces.

When a pointed pick is rounded at the tip the cutting efficiency increases as the point is rounded to a radius of about $\frac{1}{16}$ in. (Fig. 12.8). Subsequent broadening of the tip by increasing the tip radius tends to reduce the cutting efficiency, but, with the exception of the results for the $\frac{3}{16}$ in. radius tipped pick cutting in Dunsil coal, the decrease in efficiency is small. Some additional tests showed that the exceptionally high value of energy consumption is probably associated with wear on the tip of the $\frac{3}{16}$ in. radius pick. The dotted line (Fig. 12.8) shows the observed relationship when allowance is made for the high forces attributed to pick wear. It can be deduced therefore that acutely pointed picks are not efficient, and that a 60 deg pointed tool

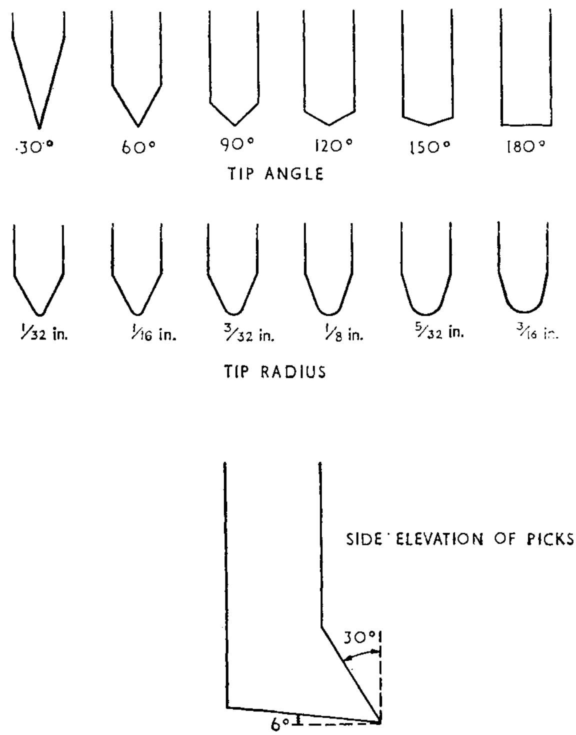

FIG. 12.6. Geometry of picks used.

should have a radius of at least $\frac{1}{16}$ in. ground on the tip. Further broadening of the tip by increasing the tip radius will not increase the cutting efficiency, but neither will it result in much of a decrease in efficiency.

As picks usually cut in an array and one pick helps others in the pattern, the picks which break most coal per cut have much in their favour. These are the broadest picks. It follows therefore that it is probably worth sacrificing a little of the efficiency of a single cut to gain the benefit from relief to other picks in the array, so that picks with a large tip radius, which approximate to a simple chisel-shaped wedge, are advocated.

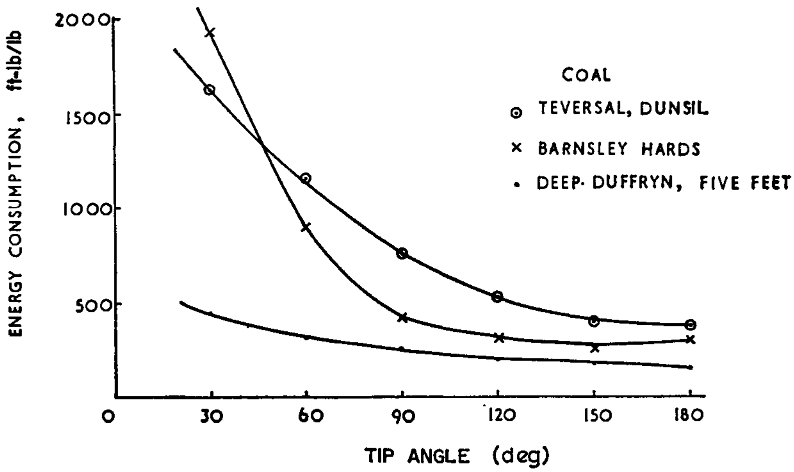

FIG. 12.7. The effect of pick tip angle on cutting efficiency.

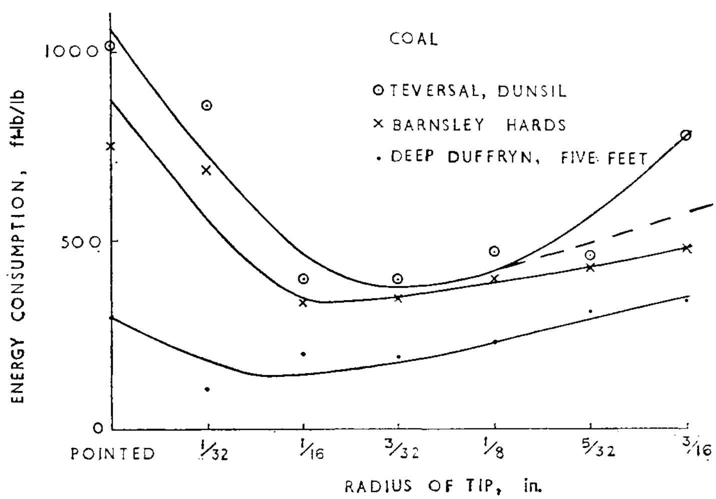

FIG. 12.8. The effect of tip radius on cutting efficiency.

The Effect on Cutting Performance of Ridging and Pointing Picks

Many of the picks that are currently in use do not have a flat rake face, such as was used in the experiments which have just been described. Instead they have a ridge along the front face or are of a pencil point shape. The next phase of the laboratory experiments included these features in the variables studied. Figures 12.9(a) and (b) show the tip designs that have been compared. The four pick types shown in Fig. 12.9(a) have been designated chisel, ridged-chisel, pointed and ridged-pointed, and a glance at the figure will justify the choice of name. The chisel and the pointed picks both have a flat rake face whilst the ridged blades have a sharp ridge along the centre of the rake face. The pencil point picks shown in Fig. 12.9(b) are also self descriptive, one pick having a negative clearance angle and the other two picks, with positive clearance, being of the straight bar and parrot beak types.

Grooves with the different picks, and with a 30 deg rake, 6 deg clearance angle pick of the type shown in Fig. 12.1 for comparison, were cut in a variety of coals and in Portland limestone. The latter substance was used in some of the tests since, whilst comparable in strength to many of the stronger coals, it was more uniform than coal and could be machined more easily into the required rectangular blocks for test purposes. In most of the experiments a number of parallel grooves were cut, so that allowance is made for the help afforded to one pick by cuts made by others, as was discussed in Chapter 11. This was deemed essential since the amount of coal broken for a given depth of cut by the different forms of pick tip might well affect the measurements significantly.

TABLE 12.2. THE ENERGY USED IN CUTTING A UNIT WEIGHT OF COAL (ft-lb/lb) FOR DIFFERENT TYPES OF BLADE. CWMTILLERY, GARW COAL

Depth of cut (in.)	Blade type				
	Chisel	Ridged chisel	Pointed	Ridged pointed	Mean
0·1	330	300	590	630	460
0·2	290	200	420	360	320
0·3	170	150	310	310	240
0·4	130	130	240	270	190
Mean	230	200	390	390	300

Type of cut		Blade type				
		Chisel	Ridged chisel	Pointed	Ridged pointed	Mean
Unrelieved		400	320	600	630	490
Cuts relieved by parallel cut (in. apart)	1	160	190	300	270	230
	$1\frac{1}{2}$	140	80	290	290	200
	2	220	190	380	380	290
Mean		230	200	390	390	300

It has been estimated theoretically[(3)] that ridging and pointing a wedge might result in an increase in the force needed to cut a groove of a prescribed depth. This has been confirmed for grooves cut in strong coals, such as Barnsley Hards or in Portland stone, the mean cutting force being nearly doubled in the coal. The differences in cutting force in Portland stone are less marked, but the quantity of material broken was more dependent on the tip shape than for the coal. The ridged pick did not cause so much breakout from around the path swept by the pick in the Portland stone, particularly in close proximity to the tip. Presumably the natural cracks and cleat planes in the coal directed the breakage paths away from the pick tip in favourable directions.

With the friable Garw coal, ridging the pick face did not affect cutting efficiency significantly (Table 12.2), except possibly when the chisel blade was ridged but not pointed and when the cuts benefited from full relief from preceding blades. Pointing, however, either with or without ridging, caused a marked decrease in cutting efficiency, the average for all depths of cut and for all line spacings showing that the pointed picks required twice the energy to cut a given quantity of coal. This is in full accord with the studies of pick profile cited above and again emphasizes that broad picks are the best.

The measurements in Barnsley Hards and Portland stone (Table 12.3) are in agreement with the findings for the friable coal, but ridging a pointed blade (Table 12.4) results in a significant decrease in cutting efficiency with Barnsley Hards.

TABLE 12.3. A COMPARISON BETWEEN THE ENERGY USED (ft-lb/lb) BY RIDGED-POINTED AND FLAT-FACED PICKS CUTTING IN COAL AND STONE

	Pick type			
	Flat-faced		Ridged-pointed (V-faced)	
	(a)	(b)	(a)	(b)
Barnsley Hards	780	850	2650	1650
Portland Stone	1900	1390	2520	1920

(a) = 0·2 in. depth of cut. (b) = 0·4 in. depth of cut.

Cuts with the pencil-pointed picks were even less efficient than those with the ridged-pointed blade. This was particularly true for the pencil point pick which had in effect a negative clearance angle (see Fig. 12.9(b)), the energy required to cut Portland stone (Table 12.5) being ten times that required by the ridged-pointed pick for shallow cuts and about three and a half times for deeper cuts, even though all these cuts were getting the maximum benefit

TABLE 12.4. THE EFFECT ON ENERGY CONSUMPTION (ft-lb/lb) OF RIDGING THE FRONT FACE OF A POINTED PICK, USED TO CUT BARNSLEY HARDS

Depth of cut (in.)	Pick type	
	Pointed	Ridged-pointed
0·1	1660	1920
0·2	1130	1370
0·3	820	840
0·4	640	750
Mean	1060	1220

Type of cut		Pick type	
		Pointed	Ridged-pointed
Unrelieved		1340	1730
Relieved by parallel cuts (in. apart)	½	720	730
	1	750	920
	1½	1020	1280
	2	1290	1460
Mean		1060	1220

from relief from other picks cutting ahead and on each side. When the pencil-point carbide tips were inserted to have a positive clearance angle, the cutting performance still remained worse than that of the ridged-pointed tool, the parrot beak pick being the poorer of the two positive clearance picks tested (see Fig. 12.9(b)), because of the differences in sharpness of the inserted tips.

TABLE 12.5. THE EFFECT OF POINTING PICKS ON THE ENERGY USED (ft-lb/lb) IN CUTTING PORTLAND LIMESTONE

		Pick type			
			Pencil point		
Depth of cut (in.)	Line spacing (in.)	Ridged-pointed (V-faced)	Negative clearance (straight bar)	Positive clearance (straight bar)	Positive clearance (parrot beak)
0·2	0·4	1080	10,600	1890	2570
0·4	0·8	1140	4,130	1370	2020

THE IDEAL PICK

At this point it is worth summarizing the findings from the laboratory experiments on pick design.

(1) The back clearance angle should be not less than 6 deg, but neither should it be much greater than this to avoid weakening the tip needlessly. There should also be side clearance of at least 3 deg.

(2) The front rake angle should be as large as possible compatible with a tolerable capacity for pick wear.

(3) Broad picks are better than pointed ones, but if for some reason pointed picks are used a radius of at least $\frac{1}{16}$ in. should be ground on the tip.

(4) A ridge on the front face of a pick brings no advantage and for strong coals (such as Barnsley Hards) can have a deleterious effect on cutting efficiency. This is equally true for chisel edged and for pointed tools.

(5) A pencil point pick is a highly inefficient cutting tool, particularly if there is a negative clearance angle between the tip point and the back of the pick.

The Effect of Tool Wear on Cutting Performance

Were it not for considerations of tool wear it would now be possible to design an "ideal pick" based on the recommendations listed above. However, in practice some of the most highly efficient picks have a very weak tip, wear rapidly and quickly lose their superiority over other pick shapes. Some studies of the influence of pick wear on cutting performance have recently been made[4] but the experiments are not sufficiently advanced to enable generally applicable conclusions to be reported. The experiments completed so far are:

(i) A study of the forces needed to initiate cracks in coal with symmetrical wedges, blunted by grinding known radii on the tips (see Fig. 12.10).

(ii) A study of the cutting forces on picks blunted by grinding known lengths of flat parallel to the tool path and perpendicular to the rake face (see Fig. 12.11).

The second series of experiments showed (Table 12.6) that blunting perpendicular to the rake face has a much larger effect on both the cutting and the normal forces than does blunting parallel to the tool path. It is worth noting that visual examination of picks worn underground shows that the wear land frequently approximates to the wear flat ground perpendicular to the rake face. This angle of wear forms a negative clearance angle at the pick

TABLE 12.6. THE EFFECT OF SYSTEMATIC BLUNTING ON THE FORCES REQUIRED TO CUT GROOVES IN CWMTILLERY, GARW COAL

Tool condition	Cutting force (lb f)	Normal force (lb f)
Sharp	38 ± 3	7 ± 1
1/16 in. flat perpendicular to rake face	88 ± 6	66 ± 6
1/8 in. flat parallel to tool path	35 ± 1	6 ± 1
1/16 in. flat perpendicular to rake face and 1/16 in. flat parallel to tool path	81 ± 5	58 ± 4

tip and as the extreme tip of the pick is all that penetrates the coal before a chip breaks away, the adverse effect found with the negative rake pencil point pick cited above (Table 12.5) is produced.

The experiments were extended to study the effect of variation in the widths of the wear lands either perpendicular to the rake face or parallel to the direction of cutting. Variation in the length parallel to the direction of

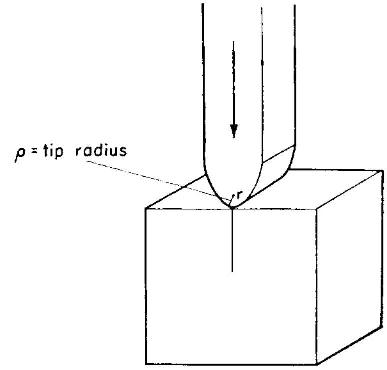

FIG. 12.10. Initiation of cracks with blunted wedges.

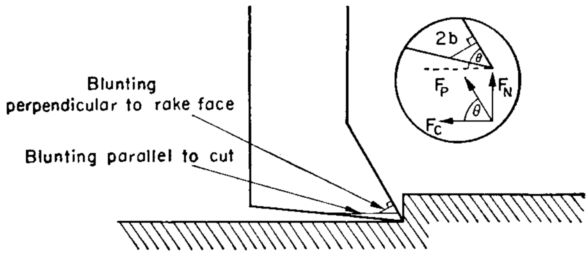

FIG. 12.11. Systematic blunting of wedges used to groove coal.

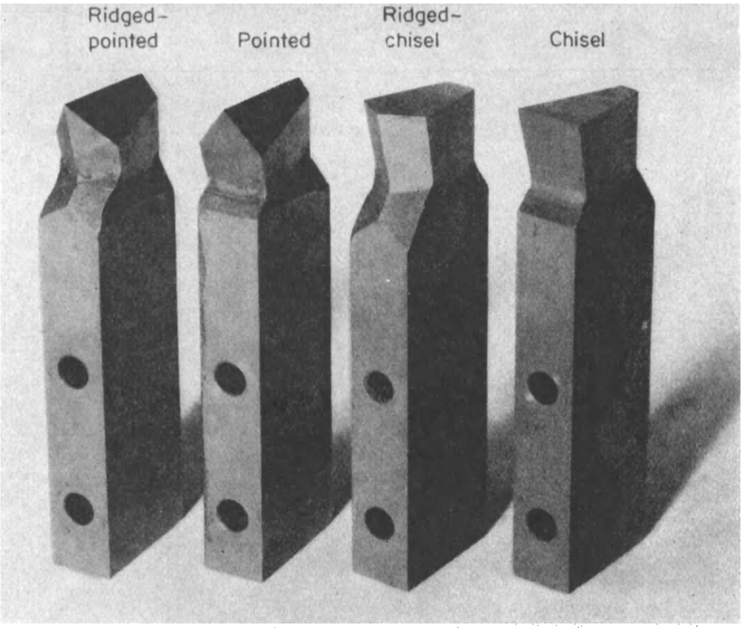

FIG. 12.9(a). Ridged picks.

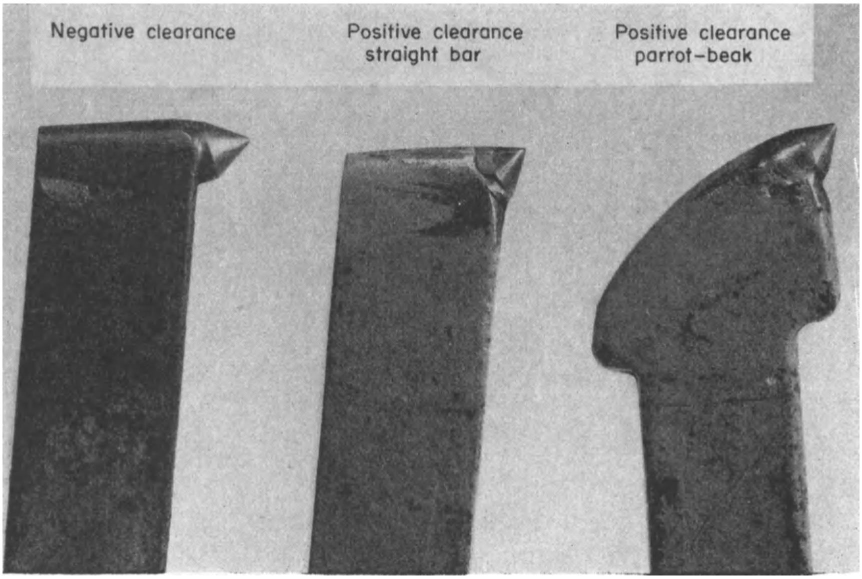

FIG. 12.9(b). Pencil-pointed picks.

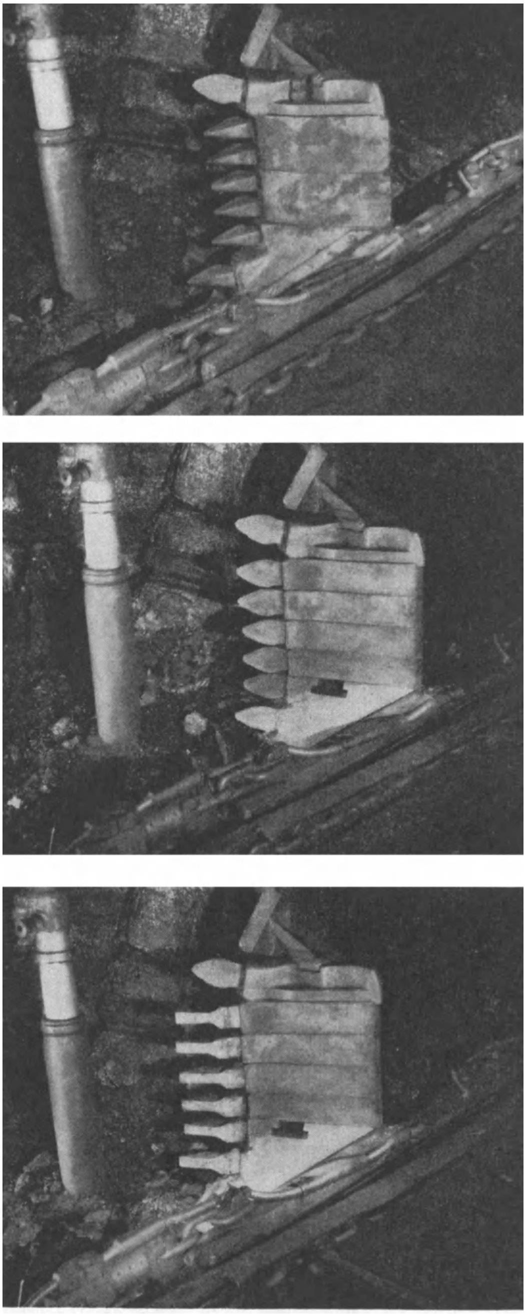

FIG. 12.13. Plough blades being compared underground. The haulage transducers used to measure the chain loads can be seen. TOP: K bits. CENTRE: Hard coal bits. BOTTOM: Kerfing or simple chisel-bits.

cutting did not produce any consistent effect. This result is attributed to the fact that a slight change in angle between the clearance face of the pick and the coal quickly transforms a negative clearance angle to a positive one, so that there is an equally rapid change from a rubbing process to one in which the tool clears the coal. As cuts are made the tools bend through a small angle and this can be large enough to affect the cutting force.

With blunting perpendicular to the rake face a definite relationship between the width of the wear land and the cutting forces has been established. Grooves 0·25 in. deep were cut in the plane surface of blocks of Cwmtillery, Garw coal and the cutting and normal forces were recorded continuously. The width of the wear land perpendicular to the rake face was varied from 2×10^{-3} to 122×10^{-3} in.

From the mean peak cutting (F_c) and normal (F_n) forces the peak forces acting perpendicular to the wear land (F_p) were calculated. The values of F_p are linearly related to the width of the wear land when the results are plotted on logarithmic scales (Fig. 12.12). The slope of the line is approximately 0·5, the equation relating F_p to b, the semi-width of the wear land (in in.), being

$$F_p = 1540b^{0\cdot 52 \pm 0\cdot 5}\,\text{lbf}.$$

The tests with the symmetrical wedges gave similar results, the force (F_p) required to initiate cracks in Barnsley Hards varying according to the equation

$$F_p = 1760\varrho^{0\cdot 48 \pm 0\cdot 04}\,\text{lbf},$$

where ϱ is the radius of the wedge tip in inches.

It seems logical, therefore, to consider that the effect of the wear land is equivalent to that of a radius ground at the pick tip, the fracture forces increasing approximately in proportion to $\varrho^{0\cdot 5}$. This implies that the first stages of blunting of a tool can have a marked effect on cutting efficiency, so that cutting tools must be changed regularly and kept sharp if the benefits gained from the use of an efficient cutting element are to be maintained.

It is interesting to speculate why breakage forces should be related to the tip radius by a half power law ($F_p \propto \varrho^{0\cdot 5}$). It immediately suggests some link with Griffith's theory of crack propagation[5] which showed that the stress (σ) needed to propagate a crack increased with the radius of the crack tip according to the relationship $\sigma \propto \varrho^{0\cdot 5}$. There are complications in applying Griffith's theory to wedge breakage and no attempt is being made here to do so, but the similarity between the equations merits comment (see Chapter 14).

Much work still remains to be done on the effect of tool wear on cutting efficiency and conversely the effect of different substances on tool wear. The metallurgical properties of tools are obviously important, but from the standpoint of cutting efficiency the nature or quality of the tool material does not enter. After an efficient tool has been designed it is then and only then that the metallurgist can say whether or not it can be made. It is possible that the use of a few very large picks on a machine, a logical conclusion from the work described in Chapter 12, will help the metallurgist since doubling the dimension of a tool increases its strength many fold.

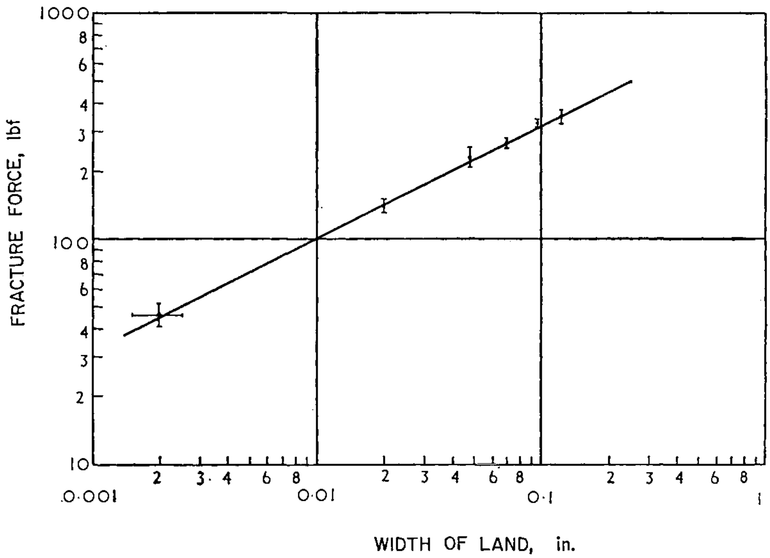

FIG. 12.12. Relationship between fracture force F_p and width of the wear land for grooves cut in Cwmtillery Garw coal.

Underground Trials of Coal Cutting Tools

The coal grooving experiments described have been undertaken in the laboratory, but a large proportion of the findings have been verified in underground trials, both on pick machines such as chain coal-cutters, trepanners and shearers and on coal ploughs. An additional link has been provided between the single pick tests and the application of the results underground by experiments with scaled-down models of a chain coal-cutter and a shearer in the laboratory.

In underground plough trials, in which the forces required to haul the plough along the face were recorded continuously, different types of plough blade have been compared. Three of the blade types are shown in Fig. 12.13. Some of the chisel shaped, or kerfing bits had a hard metal facing on the tip which was not acutely sharp; others had a tungsten carbide tip which compared much more closely with the sharp picks used in the laboratory experiments. For a given depth of cut into a strong anthracite Guppy and Johnson[6] have shown (Table 12.7) that the chisel bit, even when hard faced, is superior to the hard coal bit, in terms of the mean peak haulage force in the plough chain. The advantage of using the chisel bit is less marked for cuts in a friable Welsh coal, although the K type of blade (Fig. 12.13) is inferior to the other bits, particularly when blunt.

TABLE 12.7. COMPARISON OF PLOUGH BLADE SHAPES

Plough blade type	Mean peak haulage force per unit depth of cut (ton/in.)	
	Anthracite	Friable Welsh coal
Blunt K	–	5·4
Sharp K	–	4·2
Hard coal	6·9	3·9
Kerfing or chisel	4·2	3·8

In later experiments Guppy and Johnson have shown that in strong coals the use of a sharp tungsten carbide tip is beneficial. The gain in performance is not always reflected in a substantial reduction in the haulage force required to plough a unit depth of cut, but a given thrust from the hydraulic pushing rams results in a deeper bite into the coal when extremely sharp kerfing or chisel shaped blades are used. This is in accordance with the finding that the effect of blunting on the outward normal forces acting on a cutting tool is greater than the effect on the cutting forces.

Studies on conventional chain coal cutters and shearers have not as yet given such good confirmation of the laboratory conclusions, although the underground experiments have not resulted in findings that conflict with the laboratory predictions. It seems probable that gross energy losses in these machines mask the reductions in cutting force that would be expected from the use of more efficient picks. These losses include the use of power to re-circulate coal fragments or gummings that have already been cut, and the use of power to crush the coal during such recirculation. Friction losses on chain coal-cutters are also likely to be large, so that the overall gains from using a good design of pick might not show up clearly in terms of power reduction. However, this is only one criterion of cutting efficiency, and the benefits from using well designed cutting tools will show up in an improved size of coal produced and in a reduction in the volume of airborne dust formed.

Underground trials on chain coal-cutters have shown though (Table 12.8) that pencil point picks and negative rake picks are highly undesirable and that there should be no tongue of metal in front of a sharp carbide tip if the virtues of the tip shape are to be exploited(2).

The Table also shows that the recommended flat-faced pick with the large rake angle was the best in both trials, although at Wheldale colliery the V-faced pick had a comparable performance.

More recently, underground studies have started on shearers fitted with special drums that contain a small number of picks. The design of these drums is based on the recommendations given in the previous chapter and the special picks employed are flat faced, large rake angled picks in accordance with the recommendations given above.

TABLE 12.8. COMPARISON OF PICKS ON CHAIN COAL-CUTTERS

Pick type	Energy expended in cutting coal (kWh/ton)	
	Donisthorpe colliery	Wheldale colliery
Flat-faced, 30 deg rake and 6 deg clearance, carbide tipped	0·55	0·69
As above, with tongue of metal masking carbide	0·75	0·94
V-faced, positive rake and clearance	0·63	0·68
V-faced, negative rake, positive clearance	0·94	–
Straight bar, small rake and positive clearance, carbide tipped	0·76	–
As above with tongue of metal masking carbide	0·77	0·82
Pencil point pick, positive rake and clearance	1·40	–

SUMMARY

Chapters 11 and 12 have summarized some of the basic experiments that have been undertaken to find the best designs of picks and the best ways to use them.

It is worth remembering that, in general, efficient cutting implies not only the utilization of the minimum amount of energy, but also the formation of the minimum amount of fine coal for the particular pattern of picks and depths of cut that are being employed. It follows that good tool design should be married to the good design of the "cutting heads" in mining machines. Data are now available, both experimentally and theoretically[(3)], which permit the calculation to be made of the forces that are likely to be encountered on the cutting head. The first stage in the design of a new machine should therefore be the consideration of the cutting elements on the machine. Only when a satisfactory pattern has been evolved should the provision of power and the design of the machine structure be considered in detail. All too often the reverse procedure is adopted and the cutting elements are attached to a motor assembly and power is either wasted and the coal is needlessly degraded or the machine is underpowered and so is useless.

On the other hand, consideration of the conclusions from this work should enable existing machines to be operated with greater efficiency.

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CHAPTER 13

Theoretical Aspects of the Cutting and Ploughing of Coal

1. SHARP WEDGES AND BLADES

INTRODUCTION

Cutting picks and plough blades are essentially wedges. They break coal by traversing parallel to the free surface of an exposure of coal, so making grooves from which the broken product is ejected away from the free surface. It has been found that the speed of attack of such devices has negligible effect on the forces involved (Chapter 11), so that the action of both picks and blades can be idealized into that of a simple wedge which makes a groove in coal parallel to a free surface, the process being regarded as quasi-static. The component of the force required that acts parallel to the free surface is defined as the tractive or cutting force, and the force that acts outwards from the coal, at right angles to the tractive force, is defined as the transverse or normal force. Laboratory experiments indicate that both tractive and transverse forces vary in an irregular manner as cutting proceeds. We know that coal is far from being homogeneous; its mode of break-up depends upon the multiplicity of cracks that it contains, and the variability of the pattern of breakage can be seen clearly during a laboratory ploughing experiment. Sometimes a large piece is being gouged out, subsequently a number of remanent minor irregularities are scraped away, then the main ridge of the unbroken coal offers stouter resistance and is in turn prised off; this main sequence is repeated time and again with infinite minor variations. The variability is accompanied by variations in the tractive and transverse forces. Nevertheless, the cutting or ploughing process does have certain consistent features; for example, the mean force for a particular coal seems to be specific to the depth and to vary in a regular manner from one depth to another, being in fact roughly proportional to depth. A similar state of affairs applies to peak loads, as the mean of say the half a dozen highest peaks encountered during a cut is also related to depth of cut. Moreover, mean and peak forces seem to be greater the harder the coal, whether hardness be judged by tactile appreciation or by measurement of certain restricted aspects of strength in the laboratory. These facts provide encouragement for the seeking of a rational basis to the cutting process, and this chapter is aimed at investigating the mechanics of certain possible modes of interaction of coal and tool. The problem considered in the first instance is the simplest one—the force required by a simple two-dimensional wedge to break off a buttock of coal, a buttock being defined as a right-angled corner formed by the coal face and a surface excavated at right angles. The conditions stipulated resemble those applying when a plough first starts from a "stable-hole" after having been set at a depth from the face which is intended to apply throughout the cut. As such it is rather a special case, but one might expect the force to be related to the peak value subsequently encountered during a cut.

Consideration of Coal Strength

The "failure" of a homogeneous material under stress occurs when there is an appreciable departure from the linearity between stress and strain that is observed during elastic behaviour. Two main types of failure may be distinguished—plastic flow and brittle fracture. Plastic flow is a quasi-static phenomenon and is met with in the response of metals to extrusion and indentation, and of cohesive soils to overloaded foundations. It is now accepted that plastic flow in the vicinity of a point in a material is initiated when there is a certain definite relation between the principal stresses acting at the point. It is unnecessary to go into details here, but it may be said that a sufficiently good criterion is that of Tresca, postulating that failure takes place when the difference between the greatest and least principal stresses at a point reaches a certain critical value. This is equivalent to stating that failure occurs when the principal shear stress at the point reaches a particular level, defined as the shear strength of the material. Theoretical aspects of plasticity have developed very rapidly in recent years, and sufficient is known for solutions to have been reached for many of the simpler problems, e.g. the forces required for sheet rolling, wire extrusion, tube drawing, etc., these problems having simple boundary conditions which render the governing differential equations tractable. The solutions are associated with the existence of a plastic zone in the material throughout which the condition for plasticity is obeyed.

Brittle behaviour, on the other hand, is characterized by fracture. In compressive tests the stress–strain curve is linear, and there is a sudden termination at the rupture point, quite unlike the gradual development of a "yield value" that is characteristic of plastic materials. Brittle materials are considerably weaker in tension than in compression, and failure appears to be due to the propagation under tensile stresses of cracks contained in the material. Coal appears to be typically such a material; its breaking strain in compression is small (of the order of 0·5 per cent) and its stress–strain curves are closely linear. In uniaxial compression tests its break-up is very rapid, so much so as to be virtually explosive in nature, and in the cutting process broken material flies away from the cutting edge. All this is typical of brittle behaviour.

The removal of material by a planing process is important in metal working, and a theory of the phenomenon has been proposed by Merchant[1]. Although one is not too sanguine about the applicability of such theories to coal cutting, it is worthwhile reiterating the main points of the theory and examining its applicability.

The problem is to calculate the force on a wedge-shaped tool taking a cut of uniform depth d from the plane surface of a strip of metal. The tool moves in a direction at right angles to the cutting edge. Since the depth of cut is generally small compared with the width of the tool, a condition of plane strain can be assumed.

No analytic solution of this problem has been obtained, and the solution of Merchant, which will now be given, is semi-empirical in nature. He assumes

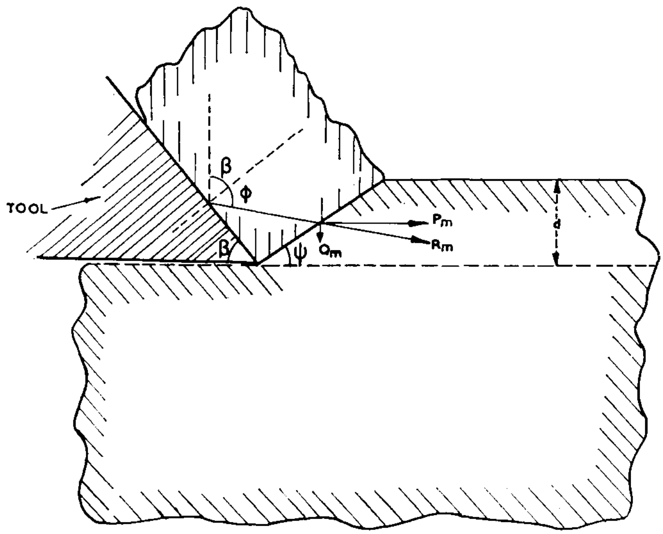

FIG. 13.1. Illustrating Merchant's theory of metal cutting.

(Fig. 13.1) that shear takes place over a line rising from the tip of the tool to the surface of the metal at an angle ψ with the horizontal. The angle that the front surface of the wedge makes with the horizontal is β; the actual angle of the wedge may be smaller than this by the amount of a small "clearance" angle. The strength of the metal is characterized by a shear strength c, and friction between tool and metal by a coefficient of friction μ associated with an angle of friction φ.

Merchant achieves his solution by considering the equilibrium of the "chip" of metal lying against the tool. Let R_m be the resultant force from the wedge acting on the chip, with P_m and Q_m its horizontal and vertical components respectively. Then

$$\frac{Q_m}{P_m} = -\cot(\beta + \varphi) \tag{13.1}$$

and resolving P_m and Q_m along the plane of shear,

$$P_m \cos \psi - Q_m \sin \psi = \frac{cd}{\sin \psi}$$

from which

$$P_m = \frac{cd \sin (\beta + \varphi)}{\sin \psi \sin (\beta + \varphi + \psi)}. \tag{13.2}$$

The angle ψ is obtained in terms of the other angles by means of an hypothesis of minimum work, i.e.

$$\frac{\partial P_m}{\partial \psi} = 0$$

giving

$$\psi = \frac{\pi}{2} - \tfrac{1}{2}(\beta + \varphi). \tag{13.3}$$

Merchant, in his experiments on metal cutting, determined ψ experimentally and used eqn. (13.3) to deduce a value for φ. If, however, we accept φ as being known,

$$P_{m\,\min} = 2cd \tan \tfrac{1}{2}(\beta + \varphi). \tag{13.4}$$

This function is plotted in Fig. 13.2.

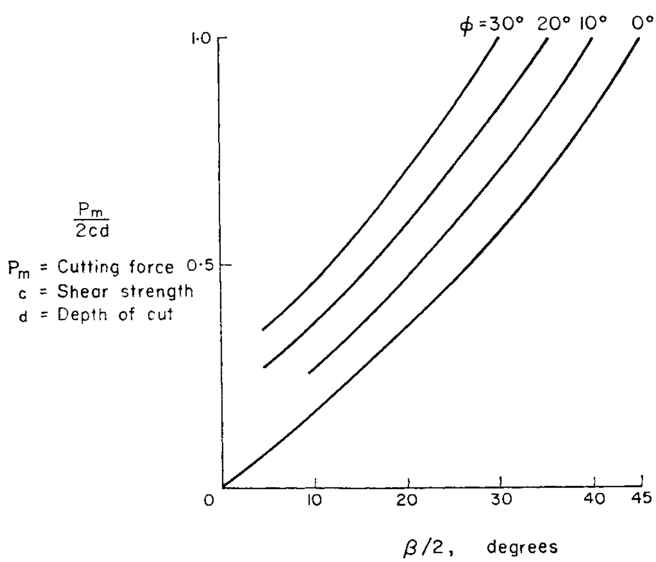

FIG. 13.2. Cutting force from Merchant's theory.

Cutting of a Brittle Material

Whatever the formal agreement between Merchant's theory and experimental results for coal (and this will be discussed later) there is no doubt that there is little physical resemblance between the behaviour of coal and metal in response to cutting tools. With coal there is no semblance of the formation of the chip that plays so important a part in Merchant's theory: if shear does take place it does so extremely locally, for there is no sign of plastic deformation in the vicinity of the surface of breakage.

Evans and Murrell[2] (Chapter 10) have shown that during the penetration of wedges normal to the surface in certain types of coal (e.g. Barnsley

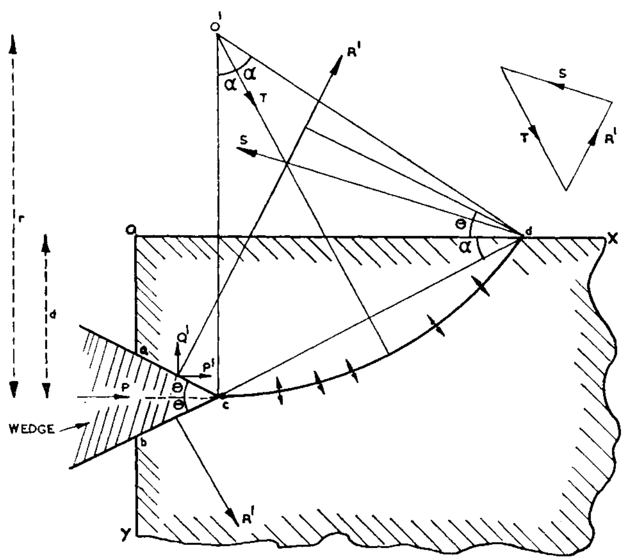

FIG. 13.4. Illustrating assumptions of tensile breakage theory.

Hards, a hard bituminous coal), cracks attributed to tensile breakage radiate from the tip of the wedge. Despite the existence of these cracks the mode of entry of the wedge appears to be primarily by the crushing of coal against the surface of the wedge, for the force required is related primarily to the compressive strength of the coal. Nevertheless these cracks do exist, and by spreading from the tip of the wedge to the free surface they can cause the fracture of the buttock. The process is illustrated in Fig. 13.3 by a laboratory experiment on a block of Barnsley Hards. This breakage will be essentially tensile in character.

On the basis of these observations a possible mode of breakage by wedges has been suggested. The full theory has been given elsewhere[3] and what

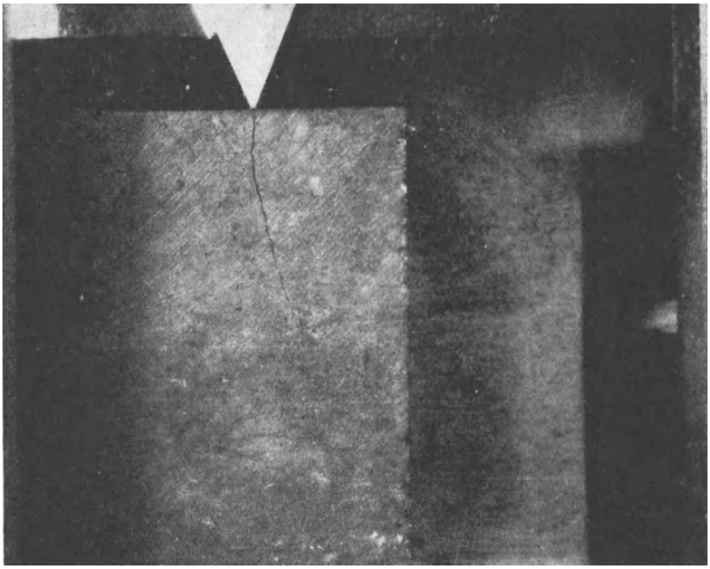

FIG. 13.3. Tensile breakage in a block of Barnsley Hards coal.

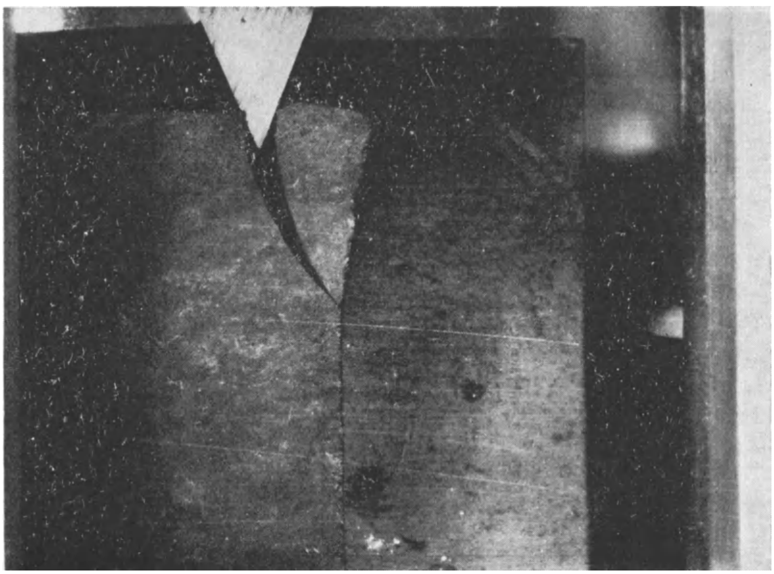

FIG. 13.5. Illustrating the action of the wedge in cutting a block of coal.

follows is a simplified version that retains most of the important features of the original. Consider a right-angled buttock of coal xOy (Fig. 13.4) with a wedge of angle 2θ entered at abc. It is assumed that breakage is caused by tearing of the coal along a curve cd. Since the work of Evans and Murrell shows that the most potent cracks are initiated from the wedge along the direction of the resultant load, it is assumed that the curve has a horizontal tangent at c. The curve is assumed to be a circular arc of radius r, so that its centre O' lies on the radius vector which is at right angles to Ox. The angle $cO'd$ is 2α.

It is assumed in the first instance that there is zero friction between the wedge and the coal. The forces acting on the buttock are

(i) The force R' acting normal to the surface of the wedge ac.
(ii) The resultant T of the tensile forces acting at right angles to cd.

A third force is required to maintain limiting equilibrium in the buttock. The action of the wedge tends to prise away the coal and rotate it in effect about the point d, and it is assumed that this force S acts through d. This assumption is illustrated in Fig. 13.5, again from a laboratory experiment. The buttock $Oacd$ is in equilibrium under the action of three forces which can therefore be represented in magnitude and direction by the three sides of a triangle. The relation between R' and T can be obtained by taking moments about d.

A state of plane strain is assumed, and we consider the equilibrium of a slice of coal of unit thickness normal to the plane of Fig. 13.4. If t is the tensile strength of the coal then

$$T = tr \int_{-\alpha}^{\alpha} \cos w dw = 2tr \sin \alpha$$

where rdw is an element of the arc cd making angle w with the radius of symmetry. Let d be the depth of cut. Assume that the penetration of the wedge is small, so that it may be neglected in comparison with d (this being the essential simplification of the original theory). Taking moments about d

$$R' \frac{d}{\sin \alpha} \cos(\alpha + \theta) = Tr \sin \alpha. \tag{13.5}$$

There is also the auxiliary geometric relation

$$r \sin \alpha = \frac{d}{2 \sin \alpha}. \tag{13.6}$$

Hence

$$R' = \frac{td}{2 \sin \alpha \cos(\theta + \alpha)}.$$

The horizontal component of R', P', is given by $R' \sin\theta$, and owing to the symmetry of the forces acting on the wedge the total cutting force F_c on the wedge is equal to $2P'$. Hence

$$F_c = 2R' \sin\theta = \frac{td \sin\theta}{\sin\alpha \cos(\theta + \alpha)}. \tag{13.7}$$

The assumption is now made that α will be such as to make F_c a minimum, i.e. that $dF_c/d\alpha = 0$, giving

$$\cos\alpha \cos(\theta + \alpha) - \sin\alpha \sin(\theta + \alpha) = 0$$

$$\cos(\theta + 2\alpha) = 0$$

or

$$\alpha = \frac{1}{2}\left(\frac{\pi}{2} - \theta\right).$$

Hence

$$F_c = \frac{2\,td \sin\theta}{1 - \sin\theta}. \tag{13.8}$$

The variation of F_c with friction can be found from the fact that each R' force acts at an angle φ with the normal to the corresponding face of the wedge, φ being the angle of friction between coal and wedge. This means that θ in eqn. (13.8) is replaced by $(\theta + \varphi)$, i.e.

$$F_c = \frac{2td \sin(\theta + \varphi)}{1 - \sin(\theta + \varphi)}. \tag{13.9}$$

This function is graphed in Fig. 13.6.

One further development of the theory should be noted. The tensile mode of breakage proceeds essentially by propagation of a crack, starting from the tip of the wedge, and the simple theoretical picture of a sudden uniform disruption is therefore probably not justified. It is much more likely to be a rapidly progressive failure. It is possible that only a fraction of the tensile strength is mobilized along most of the potential failure surface when the stress at the wedge tip has reached the level for disruption.

The variation of tensile stress in the vicinity of a wedge tip is no light problem, either experimental or theoretical, but some idea of the effect of such variation can be obtained in a relatively simple manner. Suppose (Fig. 13.7) that failure takes place along the line cd (designated length l) and that the tensile stress varies along this line according to the law $t(x'/l)^{n'}$ where x' is the distance from the point d and n' is an empirical exponent; thus tensile stress is equal to tensile strength at the wedge tip, and falls from this value at c to zero at d according to a power law.

Then taking moments about d:

$$R'l\cos(\alpha + \theta + \varphi) = t\int_0^l (x'/l)^{n'} x' dx' = \frac{tl^l}{n' + 2}$$

so that

$$F_c = 2R' \sin(\theta + \varphi) = \frac{2td}{n' + 2} \cdot \frac{\sin(\theta + \varphi)}{\sin\alpha \cos(\alpha + \theta + \varphi)}.$$

If F_c is minimized with respect to α in the manner already adopted, then we obtain finally

$$\frac{F_c}{2td} = \frac{2}{n' + 2} \cdot \frac{\sin(\theta + \varphi)}{1 - \sin(\theta + \varphi)} \tag{13.10}$$

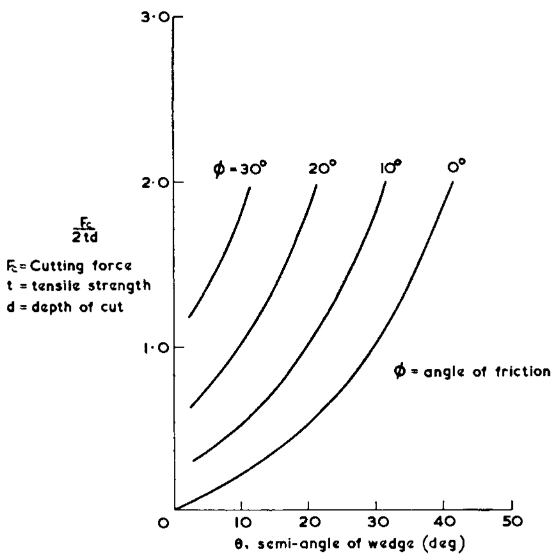

FIG. 13.6. Variation of cutting force with angle of wedge.

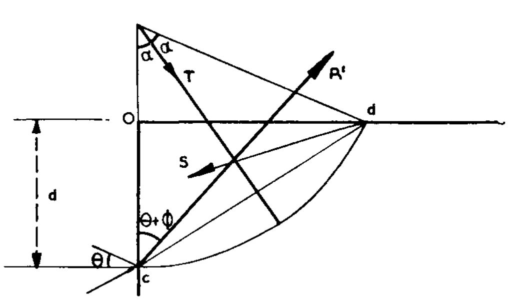

FIG. 13.7. Illustrating assumptions of simplified tensile breakage theory.

It will be noticed that this expression has exactly the same form as eqn. (13.9) except that the effective tensile strength of the material is reduced by the factor $2/(n' + 2)$.

Comparison with Experiment

Certain experiments have been carried out which can be compared with theory. In any comparison the following limitations of the theory should be noted:

(i) It is two-dimensional.
(ii) It deals with a material of uniform properties, hence neglecting the effect of cleats, bedding planes and other weaknesses.
(iii) Practical coal cutting tools are not symmetrical along the line of advance, but look more like the metal-cutting tools illustrated in Fig. 13.1.

The first is neglected on the grounds that the approximate concurrence of three-dimensional phenomena with planar theories has many examples in physical research. The second is dealt with by averaging available results over various angles of attack on the coal. The difference (iii) is considered in more detail in a later section, but for the moment it is neglected and it is assumed that θ can be equated with the half-angle of the wedge even when the wedge is advanced in an asymmetrical manner.

Comparison between the two theories is facilitated by the fact that the tensile theory gives a more rapid increase of tractive force with blade angle than Merchant's theory. The conclusion was drawn from the examination of early results(3) that hard coals (Barnsley Hards and Brights) favoured the tensile theory, while a soft coal (Cwmtillery Garw) favoured the shear theory. This is not entirely unexpected, as it has been shown(2) that Barnsley coal showed the tensile type of behaviour in wedge penetration normal to a surface, while Garw coal showed no such behaviour, as even when the wedge became embedded in the coal there was no sign of radiating cracks. This phenomenon has been related to the "granular" nature of the coal. An angle of friction of 10 deg seemed appropriate to both coals, and this again was linked with earlier observations related to wedge penetration.(2)

Further results in this field were given by Werblow(4) who carried out ploughing experiments in the laboratory using wedge-shaped blades of height 1 in. and a clearance angle of 4 deg. The included angles of the wedge were 30, 45, 55 and 65 deg. The coals used were Barnsley Hards and Deep Duffryn (Five Feet), a more friable steam coal. With the Barnsley Hards, ploughing was carried out with the main cleat on bord and on end, and the results quoted here are averaged for these two directions. In Deep Duffryn coal the cleats are not well defined, and no differentiation of direction for various experiments was possible. The experiments may therefore be regarded as carried out with cleat direction having random orientation relative to the direction of ploughing.

During these experiments the tractive force acting on the wedge was recorded as a function of wedge position. In every experiment the force has

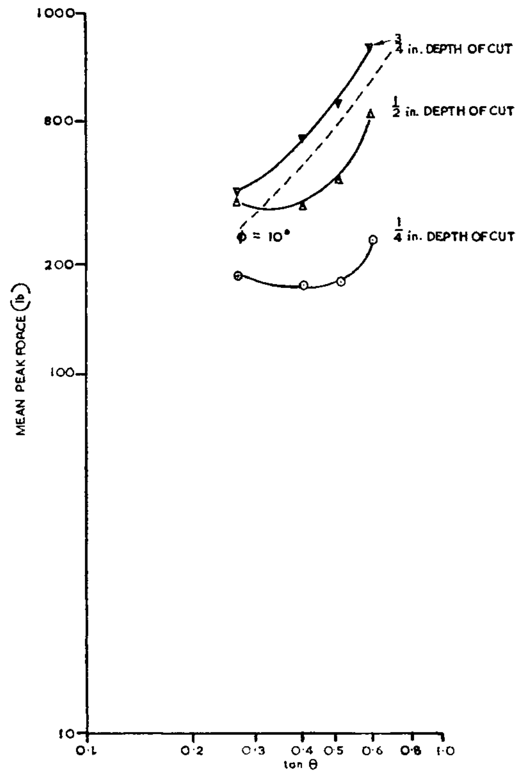

FIG. 13.8. Mean peak tractive force for continuous ploughing in Barnsley Hards. Dotted line gives shape of theoretical curve.

been found to vary in an irregular manner, but each experiment has been characterized by the "mean peak tractive or cutting force", that is, the mean of the ten largest values of force recorded during the cut, each cut occupying a length of between 6 and 7 in. This parameter is graphed in Figs. 13.8 and 13.9.

The results for both coals show similar trends. For shallow cuts ($\frac{1}{4}$ in. compared with a 1 in. high blade) there does not seem to be a very great variation of force with angle of wedge. The reason for this can only be guessed at, but an explanation might be that when the theoretical tensile failure arc in the coal is short, it is easily modified or diverted by the existing cracks. Hence breakage forces are reduced to a rough uniformity depending more upon the physical condition of the coal in the vicinity of the wedge than upon the wedge angle.

A more regular increase of force with wedge angle is shown by the deeper cuts. At $\frac{3}{4}$ in. depth of cut, a depth commensurate with the height of the wedge, the variation of force with angle is very close to the tensile theory for a φ of 10 deg.

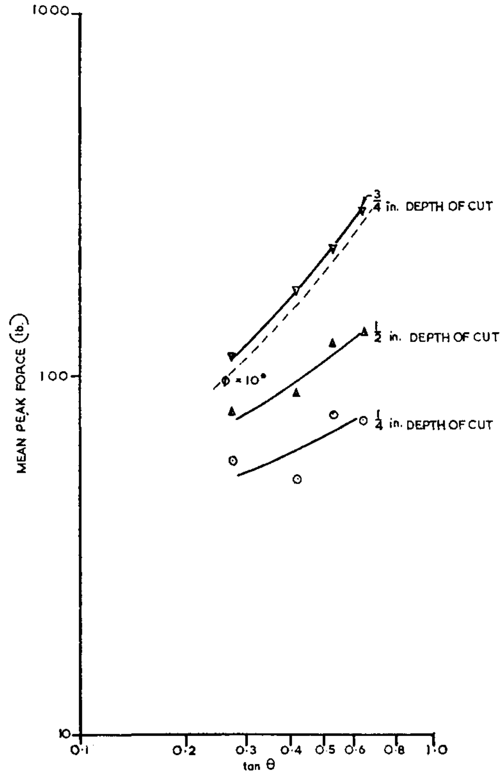

FIG. 13.9. Mean peak tractive force for continuous ploughing in Deep Duffryn coal. Dotted line gives shape of theoretical curve.

ABSOLUTE ACCURACY

It has been pointed out that under certain conditions the cutting or ploughing force for the coal studied seems to be roughly in agreement with a theoretical expression based on tensile breakage as regards variation with angle of wedge, and some study should be made of its absolute accuracy. A difficulty that arises is the uncertainty of the value of n' in eqn. (13.10). If the wedge is regarded as applying a uniform line load on the buttock, the principal elastic stress in the coal before breakage took place would fall off inversely as the radius vector—hence a value of $n' = 1$ would probably be the most likely. However, this supposition is so tentative that it would be presumptuous to use it without further evidence. It is best, in the first instance, to concentrate attention on eqn. (13.9), where $n' = 0$. This equation yields for Werblow's results the following values of tensile strength at $\frac{3}{4}$ in. depth of cut, for $\varphi = 10$ deg:

Barnsley Hards	290 lb/in^2,
Deep Duffryn	110 lb/in^2.

These values are of the same order as the experimental values (500 lb/in^2 and 130 lb/in^2 respectively). It is interesting to note that if we do take $n' = 1$ the theoretical values are raised to 435 lb/in^2 and 165 lb/in^2 respectively, which are in good agreement with experiment. At present, though, it is more circumspect to say that the theory of eqn. (13.9) gives absolute values of tractive forces, in terms of tensile strength, of the right order of magnitude, for cuts of approximately square section. This finding is made use of in certain practical calculations later.

For rough calculations it is useful to note from Fig. 13.6 that for wedge semi-angles of about 30 deg, the sort of thing which is practicable, $F_c/2td$ has the order of unity. Hence we may write that tractive force per unit depth of cut is of order $2t$.

Effect of Obliquity of Attack by a Wedge

The idealization studied so far is that of a wedge symmetrical about the line of advance, but practical cutting tools are not symmetrical. In general, the bisector of the angle of the wedge will make an angle, say η (Fig. 13.10), with the line of advance, $(\eta - \theta)$ being the clearance angle. It seems likely that for continuous cutting the clearance angle should be positive, for whereas the wedge used in the manner treated earlier could prise off the buttock, it could not cut continuously as it would tend to ride up the shoulder of unbroken coal. So far this skewness has been neglected, and the results of experiments for attack by an asymmetrical wedge have been compared direct with the theory for symmetrical attack, a presumption that seems to be justified. However, it is clear that an attempt ought to be made to try to modify the simple theory to take account of asymmetric attack. Further discussion is simplified by defining not only a clearance angle, as above, but a "rake" angle ε equal to $\{90° - (\eta + \theta)\}$ (see Fig. 13.10).

For the present purpose some clear physical picture of the function of the trailing, as against the leading, surface of the wedge must be obtained. When the wedge is used as a tool having a positive clearance angle the leading surface is actively engaged with the coal, but the trailing surface, judging from the simple geometric picture, need not necessarily have any contact with the coal. On this basis it might be assumed that the exact value of clearance angle is irrelevant to cutting action; provided it is positive then one value is as effective as another. This postulate would, however, seem to contradict the essentials of wedge action that have already been deduced, that the splitting action of a wedge is greatly dependent upon the wedge angle. This finding requires the postulate that, in the vicinity of the wedge until the moment that breakage of the buttock is initiated, the coal is in contact with both the wedge surfaces, being in a state of elastic strain.

Evans[3] has used this assumption to calculate the force required to break a buttock by a wedge entering along a skew path, making angle η with Ox. He then assumes that as a practical approximation this force is the same as that encountered when the same wedge enters parallel to Ox with a clearance angle of $(\eta - \theta)$. The problem is treated (Fig. 13.10) in the same way as for symmetrical attack: it is still assumed that the arc of breakage is circular, and that at the wedge tip it is tangential to the bisector of the wedge angle.

It is found in a particular case for which experimental results are available, that for attack by a wedge having a 4 deg clearance angle, the theory predicts a more rapid increase of cutting force with wedge angle than is found in practice. The reason for this is not immediately obvious. One suggestion that seems tenable is that in asymmetrical attack a wedge induces a strong shearing component along the surface of failure that increases the likelihood of rupture, this force being negligible in symmetrical attack. Whatever the cause, the effect is to nullify the increase of cutting force with increasing

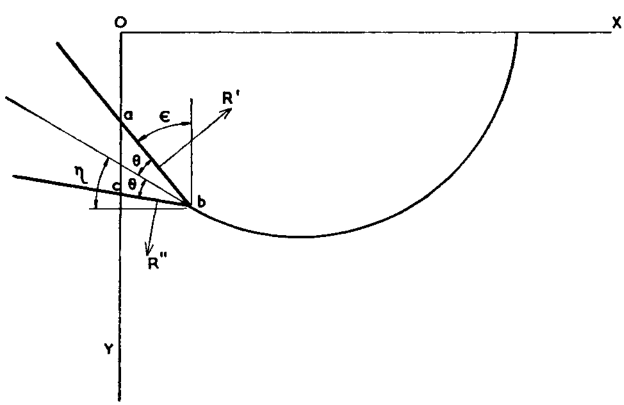

FIG. 13.10. Cutting by asymmetric wedge.

asymmetry that is predictable on the theory of pure tensile breakage, so that it leaves the theory of symmetrical attack still applicable, even for asymmetrical approach.

The effect of variation in clearance angle has been studied in some detail by Whittaker and his colleagues[5] who used a family of cutting picks of identical characteristics apart from clearance angle. The results are given in

Chapter 12. He found that for clearance angles greater than zero, the actual value of the angle made little difference to the tractive force. The reason for the lack of effect is likely to be this: the use of an asymmetric wedge for cutting or ploughing must rapidly produce a wear flat parallel to the direction of motion (Fig. 13.11). Thus a sharp wedge of rake angle ε and clearance angle δ has an effective wedge angle of not $90° - (\varepsilon + \delta)$, but $(90° - \varepsilon)$.

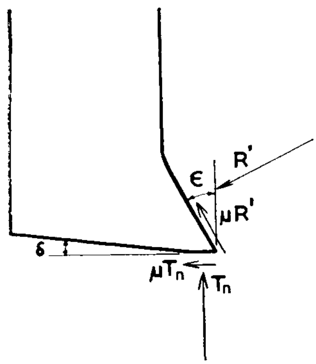

FIG. 13.11. Forces on asymmetric wedge due to development of elementary wear flat.

This is substantiated by the work of Dalziel and Davies[6] (see Chapter 12), who ground a flat land of $\frac{1}{8}$ in. width parallel to the direction of cutting on a sharp wedge. They reported that no significant increase could be detected in either the cutting or normal force. The geometrically sharp edge of a wedge cannot be sustained in any physical contact with a hard or abrasive material. From this point of view there is no justification for having a large clearance angle on picks or plough blades, and as a small one would enable the cutting edge to be backed by a more solid disposition of metal, there are good reasons for keeping the value small, say not more than 10 deg. So far as blade geometry is concerned, cutting force is best reduced to acceptable values by increasing the rake angle.

Whittaker's results for cutter picks, probably the most comprehensive in this field, give further evidence in favour of the tensile theory of breakage. His wedge angles varied from 15 to 90 deg and he found that the variation of cutting force with angle of wedge satisfied eqn. (13.9), where θ was taken to be $\frac{1}{2}(90° - \varepsilon)$, in accordance with the idea just stated.

TRANSVERSE FORCE ACTING ON CUTTING TOOLS

The transverse or normal force has been defined as that component of force acting on the face of the wedge which is at right angles to the tractive force. For symmetrical penetration it is obvious that the transverse force on one face is equal and opposite to that on the other, so that the net transverse force is zero.

This appears to be true, on the basis of the postulates that have been made in this calculation, for the asymmetrical case also. For, referring to Fig. 13.10,

Net transverse force

$$= R' \cos(\eta + \theta) - R'' \cos(\eta - \theta),$$

$$= q_0\{ab \cos(\eta + \theta) - bc \cos(\eta - \theta)\},$$

where q_0 = compressive strength of coal.

Now from the triangle abc

$$\frac{ab}{\sin\{\pi/2 + (\eta - \theta)\}} = \frac{bc}{\sin\{\pi/2 - (\eta + \theta)\}}.$$

Hence net transverse force

$$= q_0 ab\left\{\cos(\eta + \theta) - \frac{\cos(\eta + \theta)}{\cos(\eta - \theta)} \cdot \cos(\eta - \theta)\right\} = 0.$$

Simple theory therefore produces the clear-cut conclusion that the transverse force on a blade should be zero. In laboratory experiments a small trans-

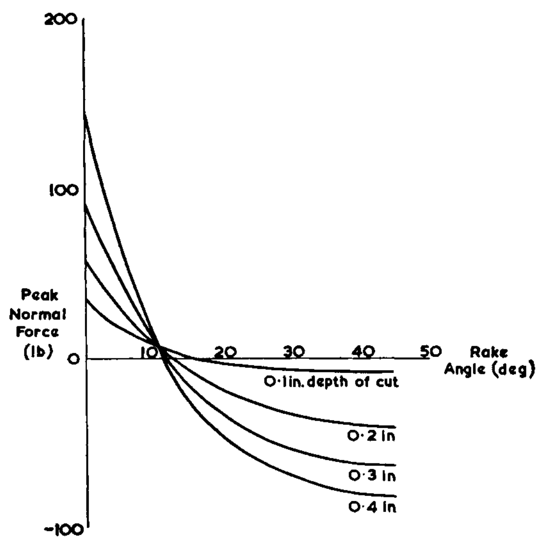

FIG. 13.12. Peak normal force in cutting with sharp picks as a function of rake angle.

verse force is found even with sharp blades. Figure 13.12 shows the results obtained by Whittaker for the transverse force on cutter picks as a function of the rake angle. The force is considerably less than the cutting force, and an interesting feature of it is that it changes sign; for small rake angles it is outward from the coal, but it becomes progressively smaller as rake angle increases, finally changing sign and acting into the coal.

One source of transverse force would be blunt blades. The effect of gross blunting will be discussed later, but it does not arise in overt form here, as Whittaker's picks were reasonably sharp. The existence of a transverse force must nevertheless be due to some modification of the forces acting on the trailing surface of the wedge, and hence probably to some kind of blunting. Tentatively it may be ascribed to the wear flat that was postulated in the previous section to account for the lack of dependence of tractive force upon clearance angle. Whittaker postulates that the system of forces shown in Fig. 13.11 acts on a blade used asymmetrically. T_n is the normal force acting on the wear flat and μ the coefficient of friction between coal and steel. F_c, the tractive or cutting force, and F_n the transverse or normal force on the wedge are the appropriate components of the system. Hence

$$F_c = R' \cos \varepsilon + \mu R' \sin \varepsilon + \mu T_n,$$

$$F_n = \mu R' \cos \varepsilon - R' \sin \varepsilon + T_n.$$

Now μT_n is probably a small component of F_c, whereas T_n is not a negligible component of F_n, hence the equations simplify to

$$F_c = \frac{R' \cos (\varphi - \varepsilon)}{\cos \varphi}$$

$$F_n = \frac{R' \sin (\varphi - \varepsilon)}{\cos \varphi} + T_n.$$

Hence

$$F_n = F_c \tan (\varphi - \varepsilon) + T_n.$$

We now make use of Whittaker's finding, already mentioned, that the value of F_c is in good agreement with the tensile breakage theory for asymmetrical attack, i.e.

$$F_c = \frac{2td \sin \{\frac{1}{2}(\pi/2 - \varepsilon) + \varphi\}}{1 - \sin \{\frac{1}{2}(\pi/2 - \varepsilon) + \varphi\}}$$

so that

$$F_n = \frac{2td \sin \{\frac{1}{2}(\pi/2 - \varepsilon) + \varphi\}}{1 - \sin \{\frac{1}{2}(\pi/2 - \varepsilon) + \varphi\}} \tan (\varphi - \varepsilon) + T_n.$$

This expression for F_n decreases monotonically as ε increases. For $\varepsilon = \varphi$ it has the small value T_n, and for slightly larger values of ε, F_n becomes negative. It therefore reproduces closely the form of the graphs of Fig. 13.12.

In practical ploughing large transverse forces acting out from the face are often met and have to be resisted by strong pneumatic or hydraulic rams.

These must be associated with blunting, either, as already discussed, with the wear flat of a nominally sharper blade, or the rounded forms of more severely blunted blades that are not infrequently found in practice.

Energy for Breakage by Impulsive Blow

The present theory enables a calculation to be made of the energy required to cause impulsive breakage such as may be used in a percussive plough. The assumption is made that the resistance of coal to breakage is not very susceptible to the rate at which cutting force is applied (see experimental evidence for this, Chapter 11). In the following calculation it is assumed that the energy for impulsive breakage by a plough blade is the same as that required for quasi-static ploughing.

Referring to Fig. 13.4, a force F_c acts on the plough blade when the apex is entered a distance h into the buttock. Since the force is linear with respect to the displacement, the energy U_c performed in the penetration is

$$U_c = \tfrac{1}{2}F_c h.$$

If q_0 is the compressive strength of the coal, then the load F_c is borne upon a bearing area $2h \tan \theta$, so that

$$q_0 = \frac{F_c}{2h}\cot \theta \quad \text{(cf. eqn. 10.2, Chapter 10).}$$

Hence

$$U_c = \frac{F_c^2 \cot \theta}{4q_0}.$$

In terms of the parameter of F_c that is graphed in Fig. 13.6, this may be written

$$U_c = t \cdot \left(\frac{F_c}{2td}\right)^2 \cdot \frac{d^2 \cot \theta}{k} \tag{13.11}$$

where

$$k = \frac{q_0}{t}$$

The numerical implication of this expression will be considered on the next page. In the meantime it is useful to enquire what is the form of variation of U_c with θ. This is not obvious from eqn. (13.11) as $F_c/2td$ is itself a function of θ.

However, we have an approximation for $F_c/2td$ from eqn. (13.9)

$$\frac{F_c}{2td} = \frac{\sin \theta}{1 - \sin \theta}, \quad \text{for} \quad \varphi = 0,$$

Hence

$$U_c \propto \frac{\sin\theta\cos\theta}{(1-\sin\theta)^2}$$

$$\propto \frac{\sin 2\theta}{(1-\sin\theta)^2}. \tag{13.12}$$

This function is graphed in Fig. 13.13. It can be seen that the energy to break the buttock increases rapidly with wedge angle, so much so that there might well be a case for having, in a percussive plough, a small-angle impacting wedge separate from the quasi-static wedge.

Experimental checks upon the validity of this expression are difficult to obtain, as very often in impulsive blows a great amount of energy is dissipated in other than effective work. Morgans[7] has carried out experiments in

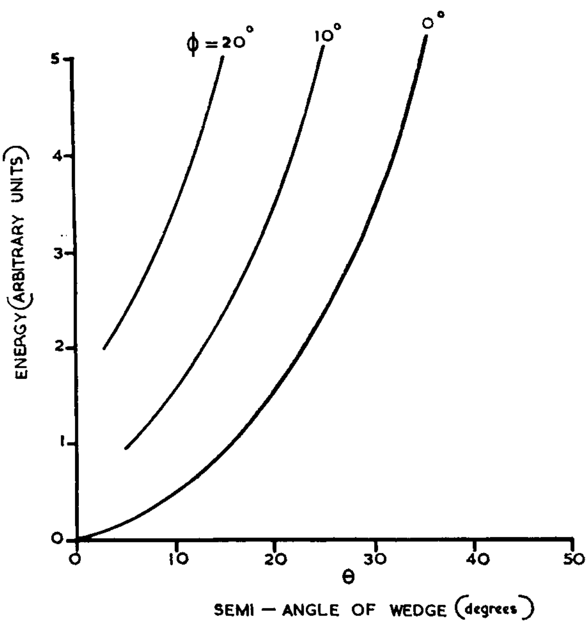

FIG. 13.13. Dependence of energy required for impulsive breakage upon wedge angle.

which an impulsive blow was struck at a block of coal by a tool mounted at the end of a pendulum. He found that, using Barnsley Hards, the energy required to effect breakage of 118 specimens out of the 120 tested was about 44 in-lb; on the other hand, the energy required in equivalent quasi-static breakage, based on the result of twelve experiments, was 9 ± 4·4 in-lb. There is a very great discrepancy between the two results, which must be attributed to the particular inefficiency of the pendulum as a means of delivering a blow.

Since the theory given above is based upon assumptions of quasi-static penetration, it must be compared with the results of quasi-static experiments. Morgans used a tool of 50 deg angle, with a blade height of $\frac{3}{8}$ in. Hence, in terms of theory, θ is 25 deg, and from Fig. 13.6 $(F_c/2td) \simeq 1{\cdot}2$. The depth of cut was $\frac{1}{4}$ in. The tensile strength of Barnsley Hards is taken to be 500 lb/in^2, and k, the ratio compressive strength/tensile strength, to be 10. Bearing in mind that the expression for U_c in eqn. (13.11) is appropriate to unit height of blade, then the calculated energy is

$$U_c = \frac{3}{8} \times 500 \times 1{\cdot}2^2 \times \frac{1}{16} \times \frac{2{\cdot}14}{10} = 3{\cdot}6 \text{ in-lb.}$$

This value must be compared with the mean value of 9 in-lb obtained experimentally. Although there appears to be a considerable discrepancy between the two, the standard error of the experimental mean is so large that the theoretical value falls well within the 95 per cent confidence limits.

The Effect of Pointed Blades

So far a plough blade in the form of a simple wedge has been considered. In practice, however, coal-winning machines that employ a plough-like action generally have a blade that is more or less pointed, or a series of such blades. The qualifying phrase "more or less" is employed here because a blade having a sharp point would not retain its pristine shape underground for long, and the virtues of a point, if they exist, must be sacrificed to some extent in favour of adequate life for the tool. Thus "pointed" plough blades are in fact fairly well rounded in the vicinity of the geometric apex.

In this section a simple type of pointed blade will be considered, and a description will be attempted of the basic mechanics of its action. This description will lead to a rough estimate of the forces acting on a pointed blade in comparison with those on a wedge-shaped blade, and hence to a discussion of the merits of the pointed blade.

The generation of a simple pointed blade from a wedge is illustrated in Fig. 13.14. The wedge has an angle λ, as shown by the two right sections *acd* and *bef*. The length of the edge is *ab*.

The right section *ogh* at *o*, the centre of the edge, is now drawn. The points *odfg* define a tetrahedron, which is taken to be a blade with a point at *o*. The blade is defined by the angles λ and v, $\angle\ dof = 2v$.

Consider the blade entered into a buttock of coal to a depth p (Fig. 13.15), the plane *dof* being vertical and the line *gh* horizontal. Although a geometric point cannot exist, its physical approximation enables an intense concentration of stress to be built up in the material in its vicinity. These stresses break up the material around the point.

Now if the action of the pointed blade resembles that of the wedge to any extent, any major breakage must be due to tensile failure across surfaces radiating from the edges of the blade. The edges which can initiate this action are *od* and *of*, and the blade can be regarded as the symmetrical juxtaposition of two elementary wedges *ofhg* and *odhg*. The effective angle o each wedge

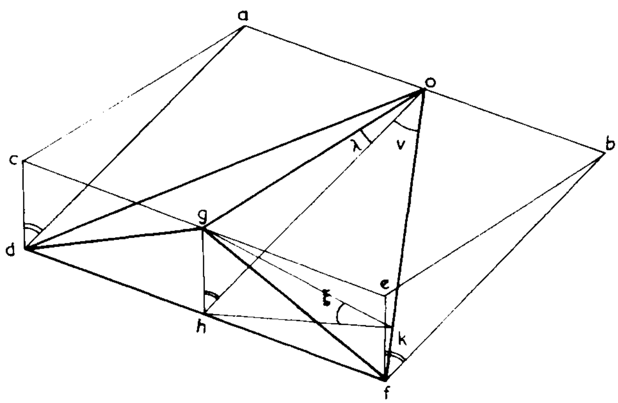

FIG. 13.14. A blade and its generating wedge.

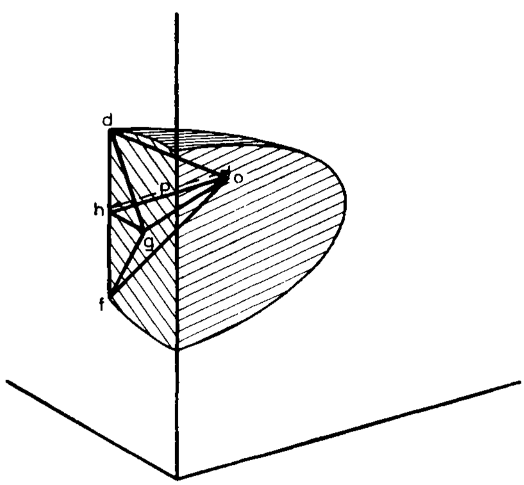

FIG. 13.15. Suggested pattern of breakage for blade entered into a buttock of coal.

can be calculated by dropping a perpendicular from h upon of, meeting of at k. Then if the penetration oh of the wedge into the buttock is p, and ξ is the angle of the elementary wedge,

$$\tan \xi = \frac{gh}{hk} = \frac{p \tan \lambda}{p \sin v} = \frac{\tan \lambda}{\sin v}.$$

This simple equation illustrates the fundamental principle of blade action as revealed by this train of thought: since ξ must always be greater than λ *the effective wedge angle of a blade is greater than the angle of the simple wedge from which it is derived.* For example, if $\lambda = 45°$ and $v = 30°$, then $\xi = \tan^{-1}(2) = 63{\cdot}4°$. Thus the benefits of concentration of stress around the point provided by a blade must be offset by the loss of effective wedging action. Not only is the effective wedge angle greater, but wedging action, being directed at right angles to the edges of the elementary wedges, is not in the line of advance of the tool. Thus, although the wedging action may crack the coal, it cannot follow up the action by the physical removal of the coal, such as is carried out by a simple wedge having an edge at right angles to its line of motion.

Suppose the wedging force on an elementary wedge is f per unit length of edge, this value of f being taken from Fig. 13.6 as being appropriate to a wedge angle ξ. The component of force in the direction of motion is f*of* sin v = fp tan v. This may be considered to act over a base-line *fh*, so that the force per unit length of the base is fp tan v/p tan v = f. If *df* represents the height of a web taken by the blade in a seam, the force required is that appropriate to a simple wedge of the same height, having an angle greater than the angle of the wedge which generates the blade.

It is possible that this value of f may even be an underestimate as the elementary wedges are not acting from a free face akin to the face *Oy* of Fig. 13.4. Such a face can only be obtained by tensile breakage along a horizontal plane passing through *go* (Fig. 13.15) and extra energy, and therefore force must be provided for actual breakoge.

To summarize: compare a blade with its generating wedge, both being entered into a buttock at the same depth of cut. The following factors would tend to make the blade force the less per unit height of coal removed.

(a) Easier initial penetration due to the point.
(b) Possibility of sideways splay of the breakage.

The following factors would tend to make the blade force the greater:

(c) The lower wedging efficiency of the blade.
(d) The more complicated breakage pattern, requiring extra force.

It might be concluded that, on balance, a blade would not confer any great diminution of force, compared with a wedge, and might even require a greater force.

It must be asked then: why do many plough-type devices have blades instead of wedges? One advantage that must be mentioned is that an undercutting blade may relieve stress to some extent for the blades above it, thus lessening the ploughing force. From other points of view no significant reduction of tractive force can be anticipated and recent practical experience has shown the superiority of wedge-shaped "chisel bits" over other shapes of blade (Chapter 12).

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CHAPTER 14

Theoretical Aspects of the Cutting and Ploughing of Coal

2. EFFECTS OF BLUNTING OF WEDGES

THE theoretical ideas that have been considered so far apply to sharp wedges. In practice these are inevitably blunted to a greater or lesser extent by use, even where hard materials such as tungsten carbide are employed at the tip, and coal in the pit is in effect won with blunted tools. It is highly important

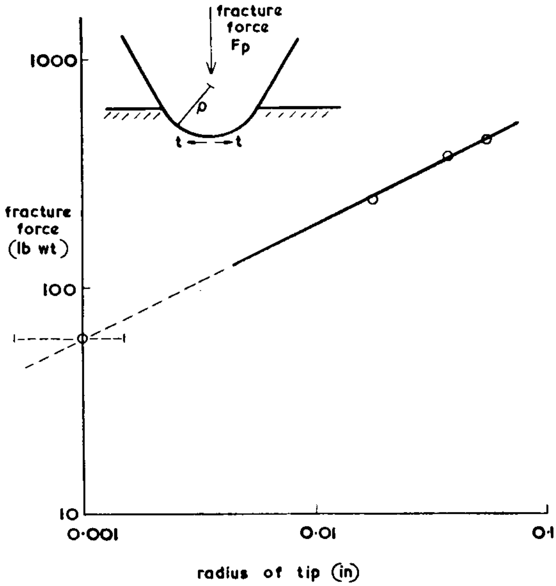

FIG. 14.1. Fracture force as function of radius of tip.

that these theoretical considerations should be extended to take account of bluntness in order that we may see to what extent the elimination of a sharp edge will result in an increase of the forces acting.

Some preliminary experiments of this nature have been carried out by Dalziel and Davies[1] (see Chapter 12). In the first instance a simple wedge having a radiused tip was pushed normally into a block of coal, the experiment being akin to that performed by Evans and Murrell[2] with sharp wedges. The load was increased until breakage took place, the outward sign of this being a sharp drop in the load-penetration curve. Typical results are given in Fig. 14.1 where the fracture force F_p is shown plotted against the radius of curvature ϱ. (The lowest point refers to the estimated radius of a nominally sharp blade). The straight line of a log–log plot indicates the existence of a power law of the form

$$F_p = K\varrho^{\xi}.$$

The exponent ξ was found to be

Barnsley Hards coal $0{\cdot}51 \pm 0{\cdot}02$

Cwmtillery Garw coal $0{\cdot}47 \pm 0{\cdot}03$

The values are not significantly different from 0·5.

Interpretation of Results in Terms of Griffith's Theory

This is an intriguing result and, as Dalziel and Davies show, the half power law invites interpretation in terms of Griffith's theory of the propagation of cracks. Griffith showed that if an elliptical crack is subjected to uniaxial tension at right angles to its major axis it will start to propagate when the tension reaches a value which is inversely proportional to the square root of the radius of curvature at the tip.

The following argument is a simplified version of that given by Dalziel and Davies.

Consider the blunted wedge being pushed into the coal. Before breakage takes place an indentation, elastic in nature, is formed in the coal, and this may be considered to be associated with a tensile stress exerted at the extremity of the indentation and at right angles to its axis of symmetry (Fig. 14.1). Employing Griffith's hypothesis it is possible to write this tensile stress t in terms of the radius of the tip

$$t \propto \frac{1}{\varrho^{\frac{1}{2}}}.$$

But this stress t is generated by the applied vertical force F_p and it is reasonable to suppose that the two are directly proportional, hence

$$t \propto \frac{F_p}{\varrho^{\frac{1}{2}}}.$$

If breakage takes place when t reaches a particular value, the tensile strength of the coal, then it can be seen from this equation that

$$F_p \propto \varrho^{\frac{1}{2}}.$$

These results have been confirmed by experiments by the same workers in which the cutting process was more nearly simulated. A wedge-shaped cutting tool of included angle 30 deg was deliberately blunted by having a flat land ground at its tip normal to its leading surface. Continuous cutting experiments were performed and the cutting and normal forces were recorded during a cut. The fracture force F_p is now defined as the components of the peak cutting force and the peak normal force that act perpendicular to the blunt land. A plot of F_p against the width of the land again showed that a power law applies and the exponent of this power law is $0{\cdot}52 \pm 0{\cdot}05$, i.e. again indistinguishable from 0·5.

These results do not admit to such an easy interpretation as the previous ones relating to simple wedge penetration, but the existence of the half power law suggests that the basic explanation is the same. It is possible, perhaps, to see the effect of the flat land as being equivalent to that of a rounded tip, where the radius corresponds to a circle that can be inscribed at the extremity of the tool. It can be visualized that the width of the land and the radius of the circle are directly proportional and hence, if the tensile stresses due to the land are proportional to those set up by the radiused tip, the half-power law relating force and width of land would result. It is of interest to note that for both experiments the results for tools having a nominally sharp tip are at one with those where deliberately fabricated radii or flattened portions exist. The results indicate the particularly deleterious effect on tools of the early blunting that takes place; the half-power law means that subsequent blunting does not have a proportionate effect.

Theory in Terms of Wedge Action

The explanation by Dalziel and Davies of the results with blunt wedges, while providing an interesting physical insight into the phenomena, does not lend itself very easily to practical calculation. However, their postulate provides the clue to the means whereby the theory of sharp wedges[3] (Chapter 13) may be extended to include blunt wedges[4]. The theory is again based upon the assumption that for the calculation of maximum tractive force, an asymmetric wedge can be replaced by a symmetrical one having the same total wedge angle.

Figure 14.2 shows a symmetrical wedge with a blunt land of width $2b$ ground normal to the direction of cutting. This will exert a compressive stress on the coal, and also generate a tensile stress, as in the work just described, at right angles to the compressive stress. By the latter means a tensile crack is initiated, which is propagated to the surface by the action of the force R' acting normal to the unworn surface of the wedge.

In the calculation that follows it is assumed that the compressive force on the land required to initiate the tensile crack is proportional to $(2b)^m$ where

m is a generalized parameter. It may differ to some extent from the value of $\frac{1}{2}$ found by Dalziel and Davies, as the experimental condition postulated differs slightly from theirs in the presence of a free face to the coal at right angles to the surface of entry of the wedge. If the compressive stress is q_0 then an expression for the force which has the right physical dimensions, and

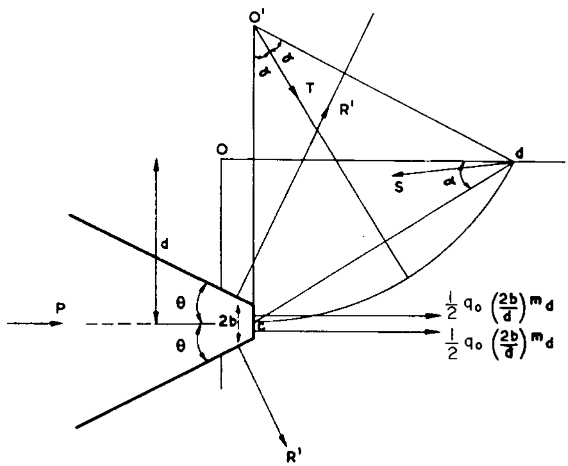

FIG. 14.2. Tensile breakage by blunt wedge.

includes all the parameters that are likely to be important is $q_0(2b)^m d^{1-m}$ or $q_0(2b/d)^m d$. The force on the half-land is therefore $q_0 2^{(m-1)}(b/d)^m d$ or $A_0 q_0 (b/d)^m d$ say, where $A_0 = 2^{(m-1)}$.

If we look at the problem in terms of the same physical mode of behaviour that was postulated earlier, viz. rotation of the buttock about the point d, it can be seen that the compressive force due to the blunt land produces a moment about d which is in the opposite sense to that produced by the force R'. We again neglect the depth of penetration of the wedge into the buttock in comparison with the depth of cut. At limiting equilibrium in the rotation of the coal about d we have

$$R' \frac{d}{\sin\alpha} \cos(\alpha + \theta) = \frac{1}{2} \frac{td^2}{\sin^2\alpha} + A_0 q_0 \left(\frac{b}{d}\right)^m d^2 \qquad (14.1)$$

or

$$R' = \frac{td}{2} \cdot \frac{1}{\sin\alpha\cos(\alpha+\theta)} + A_0 q_0 d \left(\frac{b}{d}\right)^m \frac{\sin\alpha}{\cos(\alpha+\theta)}.$$

As before, the principle of minimum work is invoked, giving

$$\frac{dR'}{d\alpha} = 0$$

or

$$\frac{td}{2}\left[-\frac{(\cos\alpha\cos\overline{\alpha+\theta}-\sin\alpha\sin\overline{\alpha+\theta})}{\sin^2\alpha\cos^2(\alpha+\theta)}\right]$$

$$+A_0q_0d\left(\frac{b}{d}\right)^m\left[\frac{\cos\alpha\cos(\alpha+\theta)+\sin\alpha\sin(\alpha+\theta)}{\cos^2(\alpha+\theta)}\right]=0$$

which simplifies to

$$\frac{\cos(2\alpha+\theta)}{1-\cos 2\alpha}=A_0\left(\frac{q_0}{t}\right)\left(\frac{b}{d}\right)^m\cos\theta. \tag{14.2}$$

From this equation the value of α corresponding to particular values of the parameter on the right hand side of the equation can be calculated.

[The confirmation that it yields a minimum value of R' would require that $(d^2R'/d\alpha^2)$ is positive. While this calculation could undoubtedly be carried through the algebra would be tedious, and as an alternative the value of R' has been graphed as a function of α for various particular cases. It has been confirmed that the value of α obtained from eqn. (14.2) does yield a minimum for R'.]

As before, the cutting force is taken to be the resolved component of R' and its companion force on the side of the wedge, with, in addition, the compressive force acting on the blunt land.

$$F_c=2R'\sin\theta+2A_0q_0\left(\frac{b}{d}\right)^m d$$

or

$$\frac{F_c}{2td}=\frac{\sin\theta}{2\sin\alpha\cos(\alpha+\theta)}+A_0\left(\frac{q_0}{t}\right)\left(\frac{b}{d}\right)^m\left[\frac{1+\sin\alpha\sin\theta}{\cos(\alpha+\theta)}\right] \tag{14.3}$$

If the angle of friction between coal and steel is φ then, as before, the effective semi-angle of the wedge becomes $(\theta+\varphi)$ so that the final equation is

$$\frac{F_c}{2td}=\frac{\sin(\theta+\varphi)}{2\sin\alpha\cos(\alpha+\theta+\varphi)}$$

$$+2^{(m-1)}\left(\frac{q_0}{t}\right)\left(\frac{b}{d}\right)^m\left[\frac{1+\sin\alpha\sin(\theta+\varphi)}{\cos(\alpha+\theta+\varphi)}\right], \tag{14.4}$$

where α is obtained from a similarly modified form of eqn. (2):

$$\frac{\cos(2\alpha+\theta+\varphi)}{1-\cos 2\alpha}=2^{(m-1)}\left(\frac{q_0}{t}\right)\left(\frac{b}{d}\right)^m\cos(\theta+\varphi). \tag{14.5}$$

Experimental

We consider in more detail the results obtained by Dalziel and Davies for continuous cutting with a wedge-shaped pick having a blunt land at right angles to its leading face. The pick had the following characteristics:

Rake angle 30 deg

Clearance angle 30 deg

Breadth normal to direction of cut $\frac{3}{8}$ in.

Cuts of depth $\frac{1}{4}$ in. were taken in Cwmtillery coal. The graph of cutting force against b/d is given in Fig. 14.3.

In order to obtain the theoretical value of cutting force from eqns. (14.4) and (14.5) it is necessary to enquire what values of the variables are likely to prevail. On simple theoretical grounds (q_0/t), the ratio of compressive to tensile strength of coal, ought to be in the vicinity of 10, and m ought to be in the vicinity of $\frac{1}{2}$. Apart from these rough directions, the best solution has to be found by trial and error. The mechanics of the solution are to solve eqn. (14.5) for α by graphical means; the value is then used to calculate $F_c/2td$ from eqn. (14.4).

It has been found that a very good approximation to experimental results can be obtained for $(q_0/t) = 6.25, m = \frac{2}{3}$ (Fig. 14.4). The calculated values are as follows:

Table 14.1. Calculated Cutting Force for $q_0/t = 6{\cdot}25$, $m = 2/3$, $\theta = 15$ deg, $\varphi = 10$ deg

b/d	α (deg)	$\dfrac{F_c}{2td}$
0	32·5	0·735
0·01	29·4	1·05
0·03	26·9	1·40
0·09	23·4	2·06
0·27	19·0	3·42

It may well be that other values of q_0/t and m which are in the region of the anticipated ones will also give good agreement with experiment. The most likely values would have to be decided on physical grounds. This problem is not tackled at this stage, and the values used here are assumed to be good enough for the time being.

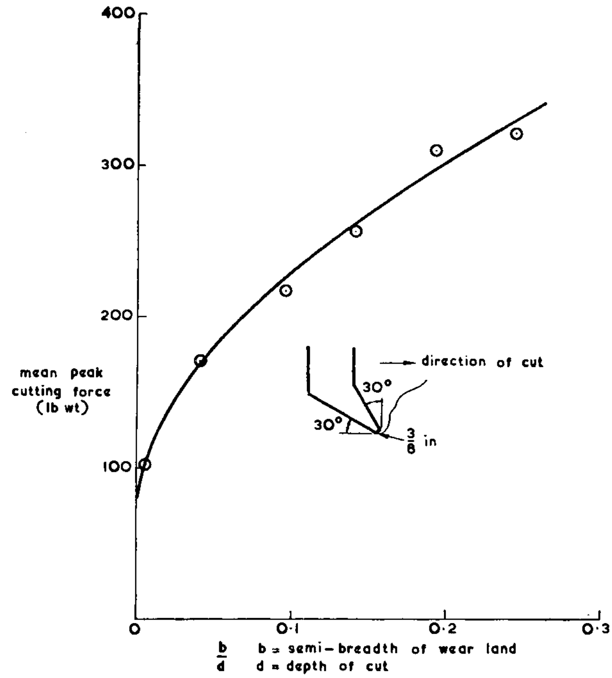

FIG. 14.3. Results of Dalziel and Davies for continuous cutting in Cwmtillery coal.

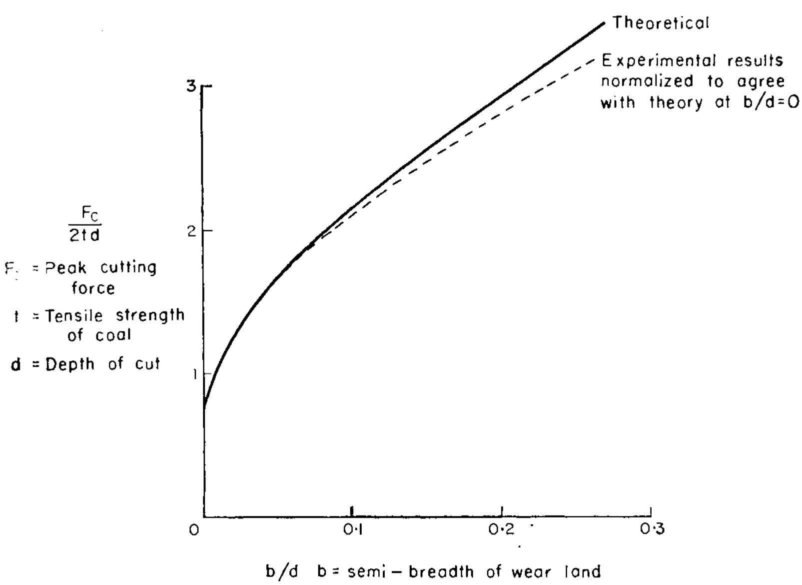

FIG. 14.4. Comparison between experimental and theoretical results, continuous cutting with blunted picks.

Illustrations of Use of Theory

If the validity of the theory is accepted provisionally, it can be used to generalize the limited observations of Dalziel and Davies.

(i) The effect of bluntness, for various wedge angles. Calculations in which θ, the semi-angle of the wedge, is varied, yield the following results:

Table 14.2. Calculated Cutting Force

$\frac{b}{d}$	$\frac{F_c}{2td}$ for $\frac{q_0}{t} = 6{\cdot}25$, $m = \frac{2}{3}$, $\varphi = 10°$		
	$\theta = 5°$	$\theta = 25°$	$\theta = 40$ deg
0	0·35	1·34	3·26
0·01	0·64	1·66	3·68
0·02	0·94	2·03	4·10
0·09	1·55	2·75	4.93
0·27	2·78	4·19	6·57

The results are graphed in Fig. 14.5.

(ii) From Fig. 14.5 the variations of cutting force with bluntness for a wedge of semi-angle $27\frac{1}{2}$ deg can be extracted (Fig. 14.6). This corresponds to a pick of rake angle 30 deg and clearance angle 5 deg. On

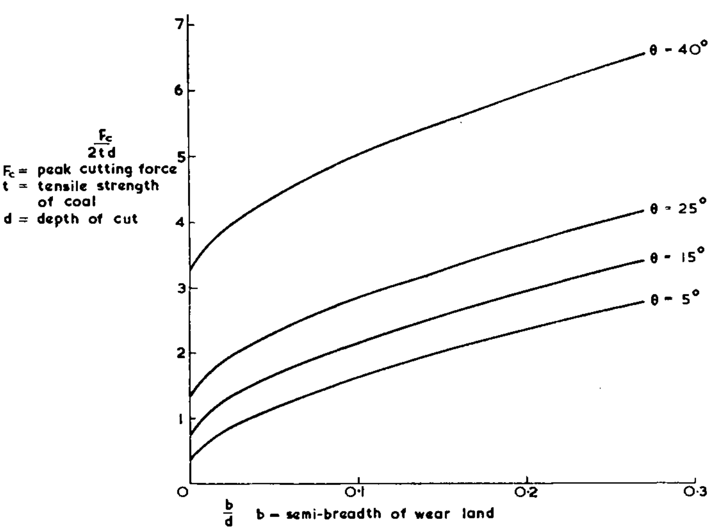

Fig. 14.5. Cutting force for blunt picks (theoretical for various wedge angles).

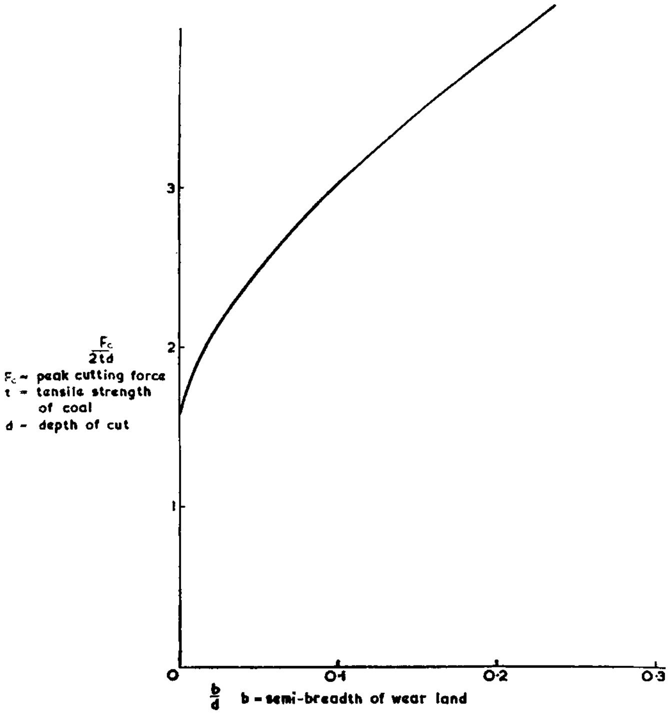

FIG. 14.6. Cutting force for blunt pick, $\theta = 27\frac{1}{2}$ deg. (theoretical).

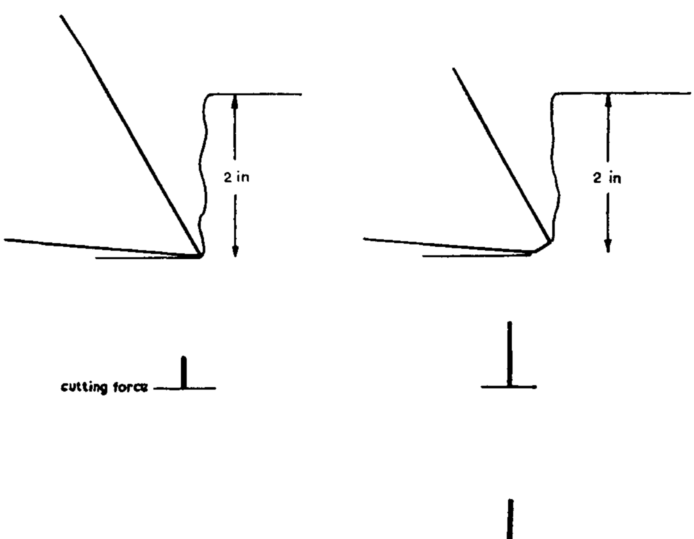

FIG. 14.7. Blunting that will double peak cutting force.

the basis of the work described in Chapter 11 and 12, a pick of this nature would be highly effective for cutting provided it were metallurgically feasible.

To give a visual impression of the amount of bluntness which produces a serious perturbation upon cutting force, Fig. 14.7 has been drawn. This shows the width of blunt land which would cause a pick of this recommended shape to double the cutting force for a 2 in. depth of cut. It can be seen that this amount of bluntness is easily detectable to casual inspection.

Normal Force

With a sharp pick having a clearance angle of more than 5 deg the force acting from the coal normally to the direction of cutting is generally much smaller in magnitude than the cutting force. Its variation with rake angle is such that it acts outwards from the coal for small rake angles; as the angle increases the force decreases in magnitude, passes through zero, and for high rake angles acts into the coal. This behaviour is explicable in terms of a postulated simple force system, as already discussed in Chapter 13.

With a blunt pick, the normal force attains a magnitude which is comparable with the cutting force. The results shown in Table 14.3 have been extracted from the work of Dalziel and Davies.

Table 14.3. Cutting and Normal Forces with a Blunt Pick ($\theta = 15$ deg, Depth of Cut $d = \frac{1}{2}$ in.). Cwmtillery Coal

Width of land (10^{-3} in.)	$\frac{b}{d}$	Mean peak cutting force (lb)	Mean peak normal force (lb)	$\frac{\text{Cutting force}}{\text{Normal force}}$
Sharp (2·0)	0·004	102·4	−9·75	−10·2
20	0·04	170·6	62·9	2·7
48	0·096	217·8	140·8	1·55
70	0·14	257·8	155·5	1·67
96	0·192	311·4	195·4	1·60
122	0·244	323·4	214·5	1·51

An interesting point is the ratio of the cutting and normal forces, which appears to attain a reasonable constant value of 1·6 as blunting increases. A practical consequence of the large normal forces found is the high ram forces required to keep a plough with blunt blades into the face.

The explanation of this phenomenon cannot be essayed on the basis of evidence available at the moment. It may well be concerned in some way with the crushing that goes on in the vicinity of a blunted tip. The tip is then surrounded by a granular material which is in a continuous state of motion as the pick proceeds, and which may be characterized by particular internal stress conditions, e.g. nearly hydrostatic, perhaps, or those pertaining to a granular material in a state of shear failure. The implications of these ideas have not yet been fully worked out, and this is one major task to which future experimental and theoretical work might be bent. The properties of coal under complex stresses, discussed in Chapter 5, become of consequence here. It is noted there that the axial breaking strengths of a variety of British coals, with the exception of anthracite, tend to roughly the same value as lateral pressures increase. The important possibility exists, therefore, that the approximate equality of cutting and normal forces for blunt picks may apply to most coals.

References

1. Dalziel, J. A. and Davies, E., *The Engineer*, Vol. 217, Jan. 1964, p. 217.
2. Evans, I. and Murrell, S. A. F., *Proc. of Conf. on Mechanical Properties of Non-metallic Brittle Materials*, London, Butterworths, 1958, p. 432.
3. *Ibid.*, p. 451.
4. Evans, I., *Int. J. Rock Mech. Mining Sci.*, Vol. 2, 1965, p. 1.

APPENDIX

Practical Applications of Theory

THE application of the theory given in Chapters 13 and 14 can best be illustrated by taking specific examples. Use is made of the following findings:

(i) The peak ploughing force for a simple wedge taking an approximately square-sectioned cut is given roughly by two-dimensional theory in the form

$$F_c = 2tdf(\theta, \varphi),$$

where F_c = cutting or tractive force per unit height of wedge normal to the two-dimensional plane,
t = tensile strength of coal,
d = depth of cut,
θ = semi-angle of wedge,
φ = angle of friction between steel and coal,
$f(\theta, \varphi)$ is a calcuable function.

(ii) When a linear array of wedges is used to take off a strip of coal of extended height, side-splay in breakage means that the wedges need occupy only a fraction (about one-third) of the total height of the strip (see Chapter 11).

(iii) An array of pointed blades requires the same force as an array of wedges, provided the blades and wedges can be regarded as being derived from the same basic extended wedge.

PROBLEMS

1. It is required to plough a hard bituminous coal, of tensile strength 500 lb/in^2. A linear array of wedges of angle 50 deg is to be used. The height of the array is 18 in. and a depth of cut of 3 in. is to be taken. What is the magnitude of the peak tractive force?

For ploughing at 3 in. depth of cut, wedges of height 3 in. should ideally be employed, and they need be placed at 9 in. centres. An array of three blades of this type would suffice.

For a wedge angle of 50 deg, $\theta = 25$ deg. For $\varphi = 10$ deg, $f(\theta, \varphi) = 1{\cdot}4$ (Theory (i), Fig. 13.6).

Hence

$F_c = 2 \times 500 \times 3 \times 1{\cdot}4$ per in. height of wedge $= 4200$ lb.

Force per individual wedge $= 3 \times 4200$ lb $= 12{,}600$ lb.

Force for array of wedges $= 3 \times 12{,}600$ lb $= 37{,}800$ lb ≈ 17 tons.

2. What is the transverse force for the preceding case? Theoretically, for a sharp wedge, the transverse force is effectively zero.

3. The wedges of Problem 1 have a wear land of breadth 0·2 in. at the tip. What are the values of peak cutting and peak transverse forces?

If the width of the wear land is written as $2b$,

$$\frac{b}{d} = 0{\cdot}033.$$

Taking this value of b/d in conjunction with $\theta = 25$ deg, $\varphi = 10$ deg, $f(\theta, \varphi) = 2{\cdot}0$. Hence cutting force is increased by a factor of $2{\cdot}0/1{\cdot}4$ compared with Problem 1, giving a value of 24 tons. For wedges worn to this extent, the transverse force could be expected to be of about the same magnitude as the cutting force, i.e. about 24 tons.

4. What impulse would be required to take a web of 2 ft height by 6 in. depth with an array of 50 deg wedges, for coal of tensile strength 150 lb/in^2?

We employ eqn. (13.11) of Theory (i)

$$U_c = t\left(\frac{F_c}{2td}\right)^2 \frac{d^2 \cot\theta}{k}.$$

k, the ratio $\dfrac{\text{compressive strength}}{\text{tensile strength}}$ of coal, is taken to be 10. An array of two blades, each of 6 in. height, at 18 in. centres, would give a breakage pattern of about the right size. The expression given above applies to unit height of blade, so that

$$\text{Energy per blade} = 6 \times 150 \times 1{\cdot}4^2 \times 6^2 \times \frac{2{\cdot}14}{10} \text{ in-lb} = 1130 \text{ ft-lb}.$$

Hence energy per array $= 2260$ ft-lb.

As already discussed, this is the energy that would be required for quasi-static breakage. An actual blow would involve a loss of energy that cannot be estimated. However, it seems likely, on the basis of laboratory experiments, that a blow would demand two or three times the amount of energy, and an estimate of 5000 ft-lb could reasonably be made.

The coal strength used as the basis of this calculation, 150 lb/in^2, is not very great. Impact ploughing would have its greatest application in hard coals, having tensile strengths of up to about three times this value. Hence it can be said that impact ploughing in hard coal would demand a blow of the order of magnitude of 10,000 ft-lb.

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