CONCRETE BRIDGE PRACTICE

Analysis, Design and Economics

SECOND EDITION

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RCARRRZJDQDYX

. . . for the development of the so-called
THIRD WORLD COUNTRIES
many of which, in the past were
physically conquered,
politically subjugated,
culturally oppressed,
and
economically exploited.

I dedicate this work to

THE INTERNATIONAL BANK FOR RECONSTRUCTION AND DEVELOPMENT

and

THE UNITED NATIONS ORGANISATION

for their pious and commendable efforts

. . . whether 'they' be the 'wealthy' poor, the 'well-to-do' poor, or the 'rock-bottom' poor, their continued dependence on alien ownership of technology must be checked so that these peoples are reasonably protected against commercial exploitation. There is no alternative for these countries but to search from within to strengthen their own infrastructures! We, as today's well-wishers, have an urgent job to do. We must take a leap-of-faith and remember that the only thing dark about these countries is our own ignorance about them! Technologies must be bent to suit the indigenously available manpower, materials and equipment as far as viable, and simultaneously the infrastructure must be strengthened by massive manpower training programmes . . .

Foreword

The twentieth century heralded a new era in bridge building concepts with large improvements in materials and methods. Structural steel and reinforced concrete began to be used extensively. Rapid developments in the theory of structures along with the advent of the computer made it possible to pioneer innovative designs. Sophisticated mathematical and model analysis was increasingly used to predict the behaviour of structures. With the massive knowledge explosion and the eagerness of creative men to develop large and more daring spans, there have been many pioneering achievements in the USA and Europe, which are indeed marvels of engineering.

The design of bridge structures has become intricate with the changeover from the conventional girder slab bridges to the complex interchanges requiring curved units or cable stays or suspended units. The analysis of such structures, having different forms and shapes, requires ingenuity of a high order as research may lag behind practical possibilities. How then can we, builders of bridges, calculate and design those daring structures to safely support the loads of railway trains or heavy vehicles and to withstand the often unpredictable forces of wind and water.

The emphasis on theory and too little consideration for structural detailing and on-site realities have resulted in bridge collapses in the not-too-distant past. To desist from using new methods and materials seems to be a wise way of reducing the risk of error and consequent failures. Thanks to the dynamism of the professionals, bridge builders continue to build spans larger or attempt forms of construction more elegant and more economical than ever before. In the words of Paul Bonatz and Fritz Leonard, "Every new difficulty challenges the human spirit to think of new solutions which in turn push forward the threshold of what can be done."

Undoubtedly, the public and even some engineers believe that the ultimate in bridge design and construction has been reached for the present. Of course, this remains to be seen; history would indicate otherwise.

The engineering work on large bridge projects of today is so complex that many engineers are involved in the design and construction activities. The credit cannot go primarily to one person as it did in the past. Large firms of specialists in many areas are involved. Nevertheless, one man has to make the final decision. These decisions, of which there may be several, call for a wealth of technical knowledge and sound judgement based on many years of experience.

This book by Dr V K Raina—*Concrete Bridge Practice: Analysis, Design and Economics*, the fourth book in his series of six books treating various facets of bridge engineering—provides a comprehensive coverage on the subject for both the designer and the constructor. The book is like a programmed text giving discrete steps to decision making. It is based on the author's experience both in India and abroad. His present assignment is as United Nations Expert in Saudi Arabia. He says, "It is no exaggeration to state that more bridges of many varieties, including some flyovers exceeding 10 km in length each, have been designed and built in Saudi Arabia in the late seventies and eighties than anywhere else in the world. If there ever existed in the world a feast of designing and building prestressed and reinforced concrete bridges, it was in Saudi Arabia during this period and it was not for nothing that so many internationally operating consultants and contractors converged in."

It is this setting that inspired Dr Raina to write this fantastic book providing an integrated coverage of the structural analysis and design of both conventional and modern bridges. It was his passion for the bridge art, which led him to take up this study. Dr Raina has written in the language of the engineer. The reader, on going through this book, will acquire a far-reaching insight into design procedures and methods and the interpretation and use of design specifications. ***It furnishes the practising engineer with much needed data to meet the challenges in his work life. It is an excellent source of reference.***

Dr T N Subba Rao
Managing Director
Gammon India Limited, Bombay

A Word to the Reader

Talking alone never pulled out a stump! Many try to throw about the weight of their purely academic degrees, non-productive publications, classroom or staid-office experiences, and even the thunder of their committee-memberships. In the end, only those that have finally actually been moulded on the professional anvil, are of real value—those that have had prolonged but successful exposures to furiously result-oriented and profit-bearing competitive practical commercial experiences where the next month's survival depends on the previous month's turnover.

My aim in writing this book is two-fold. One is to benefit those who may wish to receive exposure to actual professional practice from the 'scene of action' standpoint as distinct from a 'theoretical' classroom hyperbole that belongs to an almost imaginary 'air-conditioned' world that is far and remote from the sleeves up workman-like life-size actuality! Second is to try and 'talk' to the engineer in short straight steps, explaining the subtleties en route, in the vein of a story narrated informally, without 'mystifying' him with exotica. Descriptions have been written with clarity and brevity so that the engineer is neither overawed nor bored with jargon that is either too theoretical or oozing with impressive looking useless detail. This book takes the reader by the finger through the labyrinths of the subject in a workman-like manner, and thus caters for the contractor, the client and the practice-oriented engineer student alike!

Of the numerous works that have been written upon the subject of bridge analysis and design, many are excellent examples of mathematical gymnastics rather than of engineering application!

In this book the steps of the reader are guided in paths often trodden by and therefore familiar to the author, who, thereby, is able to recommend a straight course without the designer having to waste time in search for a route. If I have succeeded in some measure, it is not only by being encyclopaedic, but because the presentation is fresh in treatment, and, above all, easy to study and follow. It concisely provides what the designer/engineer wants, without making demands on his energy. However, the subject being what it is, and the work involved being awesome (as suggested by the title of the book), I have had to presume that the reader already has some experience in the analysis, design and detailing of concrete bridges, with a reasonable exposure to competitive professional practice.

Engineering is not just doing theoretical sums, nor is it a matter of blind adherence to graphs and formulae. One can run the danger of becoming too concerned with 'learning' and not be concerned enough with 'practical realities.' ***It is more meaningful to have an approximate solution to an exact problem than an exact solution to an approximated problem. A useful book does not have to be the graveyard of dead Ph Ds!*** As a prolific practitioner who has operated in so many countries and has worked with myriads of contractors and consultants, I am disturbed if a book purports to be 'practical' when it is packed with pages of iterative empirics and impressive looking graphs that are only of very restricted use and, worse still, is written by someone who has never stayed, survived and surfaced in the merciless world of competitive practice in construction and design. That is where all the fun is and where one grapples with the survival situations that can cause ulcers! ***A good musician is far superior to a music-critic!*** Practical engineers must be conceptual more than perceptual, creative more than analytical and more visual than merely mathematical. They have to have a wide breadth of experience rather than an isolated narrow specialisation alone. Originality comes out of understanding, and understanding comes out of relentless practice, not from mere information. ***Last but not the least, good judgement comes out of experience and experience often comes out of bad judgement!***

Dr V K Raina

Acknowledgements

As acknowledged in my other books, one of the prices a professional practitioner has to pay is that he, ***unlike those involved in research and laboratory work, classroom lectures, or staid-office work,*** hardly has time to write. A chronic practitioner would rather spend the time in designing (and still more designing) and constructing (and still more constructing) than just writing! But of course it would be very useful ***if such a real-life practising-professional, who has his fingers on the pulse of practice and in fact has a lot to*** write about, could squeeze time in order to 'also write' for the profession, however hard it might be for him to find that time! ***It would be even more meaningful if he, additionally, had a practical research background that would help him sift grain from husk.***

Fired by this feeling, I took up writing the present book in the humble hope that it may provide an amalgam of practice and theory, with the former subordinating the latter in order that the book be of gainful use to the practising engineer.

With the mind boggling and unparalleled spree of fast growth of world-class super-expressway and highway networks in Saudi Arabia, I, seconded by the United Nations (Department of Technical Cooperation for Development) as the in-house adviser to the Ministry of Communications of the Saudi Government, was involved first-hand with optimised design and construction of several bridges in many interchanges, crossings and flyovers, of various types, spans, skews and curves, with many consultants and contractors. (It is no exaggeration to state here that more bridges of many varieties, including some flyovers exceeding 10 km in length each, have been designed and built in Saudi Arabia in the late seventies and the eighties than anywhere else in the world! If there ever existed in the world a feast of designing and building prestressed and reinforced concrete bridges, it was in Saudi Arabia during this period and it was not for nothing that so many internationally operating consultants and contractors converged in.) I am indeed grateful both to the United Nations and to the Saudi Government for this challenging responsibility and the additional satisfying and revealing practical experience this opportunity afforded. In turn, I gratefully acknowledge the trust, confidence and appreciation shown by them. If I have acquitted myself creditably in their eyes, the credit is all theirs. It is this additional important experience that further prompted me to write this book.

Apart from drawing upon my own experience and interaction with others, in preparing this book, I have also drawn on some of the material published by the British Standards, the Indian Standards, the American Concrete Institute, the American Association of State Highway and Transportation Officials, the C&CA London, the FIP, CEB, the Indian National Group of the IABSE (Zurich), the Indian Roads Congress, New Delhi, the *British Steel Designer's Manual*, CS Reynolds, AH Allen, AA Witecki, B Richardson et al., GH Ryder, Podolny & Muller, Freyssinet International, Paris, PK Thomas, GN Smith and EN Pole, E Pennels, MJ Tomlinson, PW Ables et al., Fisher Cassie, W Teng, and various proprietary firms, to all of whom I am greatly indebted and owe grateful thanks.

I am thankful to various consulting engineering firms and contractors (Dar Al-Handasah, London, UK), Wilson Murrow (Salina, USA), Ital-Consult (Rome, Italy), Sauti-Renardet (Rome, Italy), Technic (Rome, Italy), Saudi-Consult Riyadh (KSA), R. Travers Morgan and Ptns, (London, UK), Arabian Engg: Bureau (Riyadh, KSA), Rhein-Ruhr Ingenieur (Dortmund, W. Germany), Dar Al-Riyadh (KSA), Doxiadis (Athens, Greece), Ove—Arup (London, UK), Gilcon PS Ltd. (New Delhi, India) Gammon India Ltd. (Bombay, India), US Dugal & Co. P. Ltd. (New Delhi, India), CCC (Lebanon), Al-Mashrik Contracting (KSA), Naser Haza & Bros. (KSA), Han Yang (S Korea), Edok-Eter (Athens, Greece), Tanmia (Riyadh, KSA) and J & P (Cyprus), to name only some), with whom I worked in different capacities in different countries. I also wish to acknowledge the opportunity I got of designing some of the first class curved and skew continuous prestressed concrete bridges in Canada while working with the Ontario Department of Transportation (previously, The Department of Highways), Toronto. These experiences I assiduously sifted and stored over the long years with a view to sharing them with others through this book.

Last but certainly not the least, I wish to express my heartfelt gratitude to Vinita, my dear wife, for her limit state endurance. While we both tried to serve the underserved through our respective professions, engineering and medicine (she has a Doctorate from London University in Bone Pathology), but perhaps we did this too devotedly...since this led to the neglect of priorities on our domestic front. Perhaps the cost to us in terms of common and worldly-mundane equations of understanding has been high, ***but it took us strength to stand the oneness of self-inflicted individual solitudes and solitude is always an exercise in agony!*** I can never pardon myself enough for causing to my wife (and to some extent to my parents) ***silent suffering and loneliness, with all its consequences and resulting despatch from (what to most people are) essential worldly norms though in reality only transitory, indeed illusory, in the ultimate mortal sense***! This was due to my professional commitments taking me away often-times, often to different lands, for long periods of time for years not just months, while her own professional commitments kept her tied back. The only solace was that in the resulting void I invested the time for five years in writing this book and my other four books (and various papers), utilising literally each available minute every single day (outside my crowded official work schedule), shunning the time-consuming and generally frivolous social get-togethers, and denying myself even the minor indiscretions of relaxation. (Only a bit of yoga kept me going.) ***Judgement is left to posterity***.

DR V K RAINA

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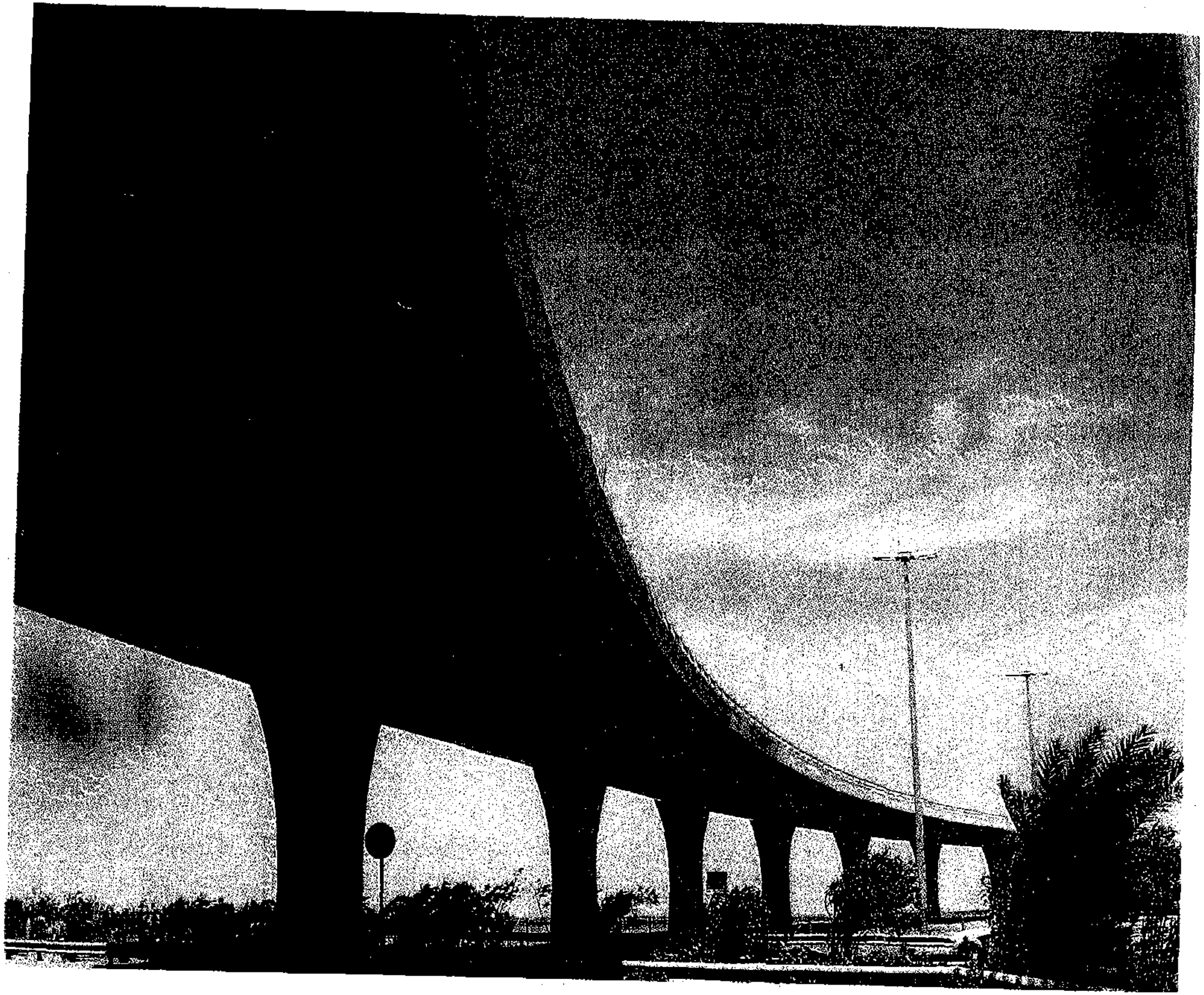

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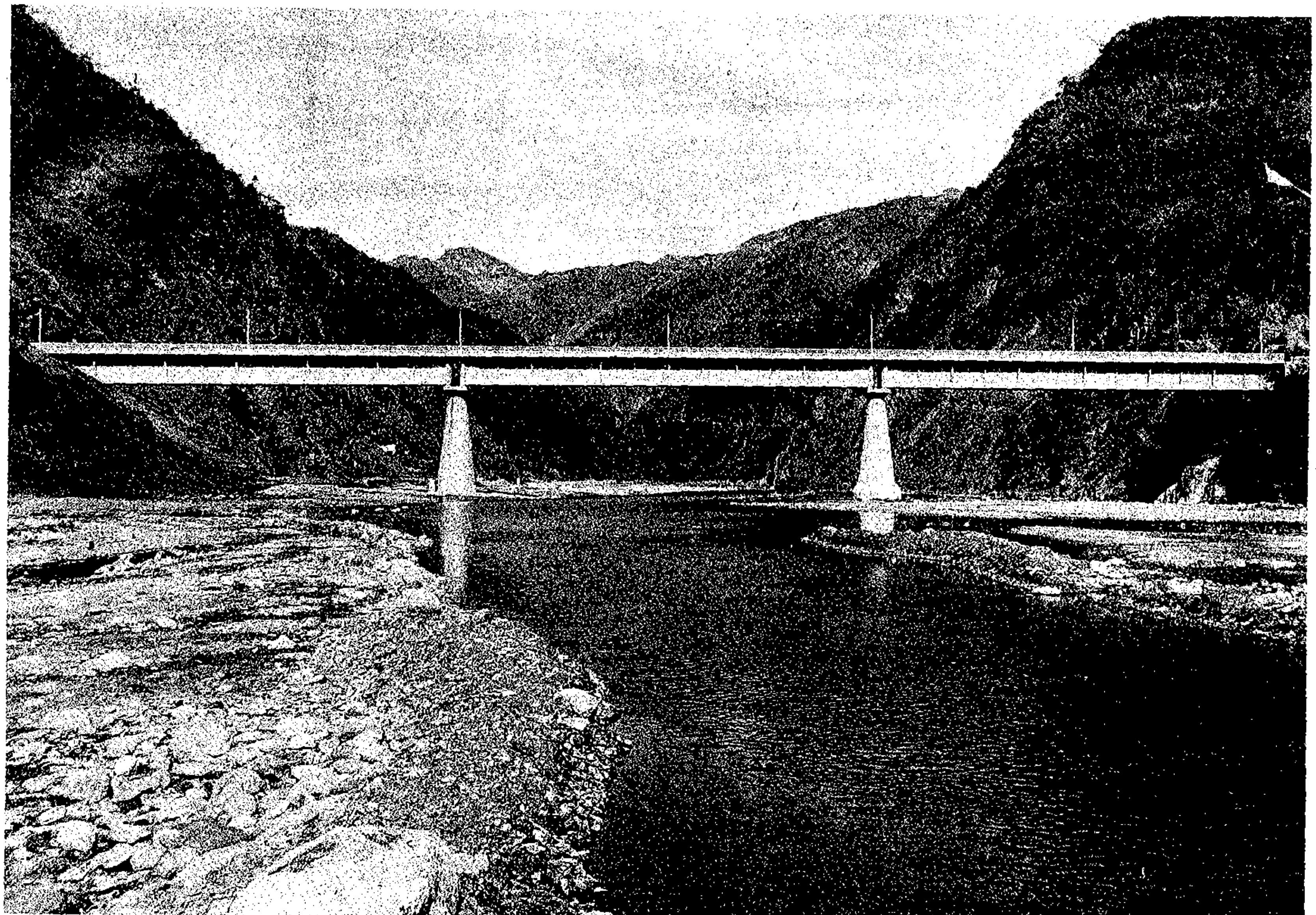

Giri Bridge (near Paonta, H.P., India)

CHAPTER 1

The Basic Principle of Practical Structural Analysis and Design

1.1 INTRODUCTION

After deciding the type of bridge, span arrangement and span lengths, assume suitable first-trial cross-sections of foundations and deck in concert with the method of construction. Hence, establish the loading sequence. For each load in the sequence see what it acts on, or what span or spans it acts on and under what end-conditions. From this find out what moments, etc. it causes at various sections, and which of these act on what section properties at those sections, and hence cause what stresses. The resultant stresses at every load stage at each section must not exceed their permissible values that are set out in the relevant Code of Practice (the design specification). This, in a nutshell, is the essence of structural analysis and design.

A good design can be produced only if developed along with scaled sketches and drawings with an eye for practical detail. Indeed design is guided by drawings made in parallel. For a regular workman-like practical design, the designer must not lose sight of the overall requirement which is, 'to produce a workable and practical structure in a limited amount of time at the minimum cost' keeping in mind the recent state of art and the contractually binding specifications. There is a large gap between a purely theoretical approach and a down-to-earth competitive practising professional's approach. It is more useful to carry out a practical design and produce a workman-like detailing in an execution drawing, rather than, for instance, merely be able to lecture on the 'ultimate strength of a nut' for hours! One has to appreciate the difference between husk and the grain, between the so-called 'coach' and the actual player, between the music critic sitting on the sidelines and the actual performing musician, between the class-room teacher and the practising professional working in a commercial, result- and profit-oriented scene of action. At the end of the day it is more important to have an approximate solution to an exact problem rather than try for an exact solution to an approximated problem. Very few theoreticians, lecturers, and those in staid office services have ever produced competitive practical structures themselves. Indeed, it should not be surprising if some of them, left to themselves, may even find the existing structures unsafe! In fact merely writing of papers and books, based on little practical experience and with no step-by-step, tool-kit applicability for a competitive practical end-product, often times is a manifestation of frustrated academics, and is no substitute for intelligent, long and hard drawn, cold, commercial practical experience resulting from swimming against the current covering the whole gamut of work from reconnaissance and feasibility through alternative working designs and actual construction and maintenance. Creativity cannot be taught. It is an experience one must live through to learn with one's own two hands in a cold competitive practice. (For more on this subject, see the 'Reflections' on 'Design Education' in Ch. 46 of this book.)

1.2 SUMMARY OF THE TOOL-KIT APPROACH FOR ANALYSIS AND DESIGN OF A USUAL TYPE OF BRIDGE

General Steps

Step 1 Knowing the required road-formation level, establish the permissible structural deck-depth (after allowing for (i) the minimum vertical clearance needed between the affluxed high flood level and deck, soffit, and (ii) the wearing-coat thickness below the road-formation level).

Step 2 Depending upon the depth of foundations, the height of deck above bed level (and above low water level), average depth of standing water during construction season, method of construction adequately suited to the site and the construction expertise available, decide: the type of bridge, span-lengths and arrangement, the type of foundations, the type and cross-section of the deck, method of construction and the loading sequence in the entire construction. (Considerations described in Chs. 7, 18 and 42 have direct bearing here.) Finally, the optimum type of bridge may well have to be decided by weighing between relevant alternatives.

Step 3 Decide the first-trial cross-sections and sizes of various elements of the substructure and superstructure, draw these to scale and establish the Preliminary General Arrangement Drawing (PGAD) of the bridge. (Some sizes and proportions, when seen to scale, will attract modifications and will be decided better through such scaled drawings. This is necessary so as to 'feel' the order in the transmission of forces and moments and the flow of stress trajectories that are to be surrounded by elegant enveloping proportions by practical detailing. Various trials lead to a structural form with optimum placement of its load-masses. Relative proportions and approximate sizes of certain members as well as their shapes will be best decided only through these scaled sketches, provided they are drawn by an experienced practical designer with an eye for detail.) Decide the type of bearings to be used and their locations (fixed, free, etc.).

Establish the preliminary member sections and sizes of various structural elements from a quick preliminary analysis and design. This is necessary for the subsequent detailed analysis and design work.

Substructure Design Steps

Step 4 Establish deck dead load reaction and the maximum and minimum live load reactions on the pier/abutment under consideration. Also estimate the co-existing moments due to these loads, about the transverse and longitudinal axes of the bridge due to the maximum possible eccentric transfer of these dead and live load reactions, as also the braking and temperature forces.

Step 5 Estimate total vertical load at base of foundation (at soffit of pile cap in case of piles) under maximum, minimum and no live load conditions, taking into account the upward buoyancy force equal to 100 or 50% weight of water in volume equal to that of the submerged mass (100% in case of saturated soil and fissured or weak rock, 50% in case of good rock).

Step 6 Estimate total moment about the bridge's longitudinal axis and horizontal force in the transverse direction, at various levels and at the base of the foundation, due to possible eccentricity of live and dead loads, flood water force and afflux if any (usually 10–15 cm).

Step 7 Estimate total moment about the bridge's transverse axis and horizontal force in longitudinal direction, at various levels and at the base of the foundation, due to possible eccentricity of live and dead loads, braking and temperature forces, and flood water force (or alternatively in case of an unerodible bed, the cross current effect of 25 cm static head difference across pier thickness, if this is greater than the flood water effect). (i) In case of simply supported spans on rocker/roller bearings, braking force from the live load on one complete span may be assumed to go to its rocker bearing alone so that the foundation under it will take either 'braking-temperature'* or 'half of braking + temperature'*, the latter usually applying to an abutment or a pier supporting unequal spans. (ii) In case of simple spans with identical neoprene bearings under each end of an individual span, the foundation will take the sum of half of braking from the live loads on each of the two spans supported by it and the horizontal temperature force equal to $(S_L m_L - S_R \cdot m_R)$, where S_L is the shear-rating of the neoprene bearings supporting the left side span, S_R that of those supporting the right side span, and m_L and m_R the deck movements above them, respectively. (iii) Refer to Chs. 8 and 9 for the method of distributing the externally applied as well as the self-induced horizontal forces among various bridge supports with different types of bearings (taking into account both the shear-rating of each support as well as the location of the zero-movement-point in the deck) the deck being continuous, or curved and/or skewed (simply supported or continuous), respectively.

Step 8 Estimate the wind force in the transverse direction that can be attracted by the exposed surface area of the bridge with or without the live load on deck. Generally, it is enough to consider wind on the deck surface area between its soffit and top of the solid parapet or up to mid-height of parapet in case of an open-type parapet, and that on the body of live load at the rate of 300 kg/m length of live load under maximum and minimum live load conditions. Wind pressure on deck surface area depends on the height of the centroid of its exposed surface area (indicated above) above the *mean retarding surface*, i.e. above the high flood level or the bed level, as the case may be. With live load on bridge, a wind force of 450 kg/m length of deck alone (ignoring that on live load) is also considered an alternative to 'wind on the deck exposed area and on the body of the live load'. Under 'no live load' conditions directly the effect of a wind pressure of 240 kg/m^2 on the exposed area of the deck is

* Temperature force here will be equal to $\mu(V - V')$, where μ = coeff. of friction at the roller (or sliding) bearing, and V and V' are the 'dead + live' load reactions at the two roller (sliding) bearings on the two sides of the rocker bearing on the foundation under consideration.

considered, assuming no live loads would ply under such a wind. (In coastal and certain specific areas, higher wind pressures (generally 100% extra or as pertinent) have to be considered.)

Since wind can also hit the bridge obliquely, therefore, as an alternative to the above-mentioned purely transverse wind condition, a combination of simultaneous wind forces in transverse and longitudinal directions, in magnitudes respectively equal to 67 and 33% of the said purely transverse wind force, should also be considered.

Step 9 (i) Estimate the static equivalent of the horizontal seismic force as can be attracted by the mass of the structure above the *embedment level* (maximum scour level in case of hydraulic bridges). Earthquake force is based on the full weight, even of the submerged portions of the structure (so long as they are above the embedment level).

(ii) Horizontal seismic force on a mass may be taken as a certain fraction of its weight, acting through its centroid. This fraction may vary between 0.10 for severe seismic zones to 0.05 for moderate seismic zones to zero for non-seismic zones, depending on the seismicity of the area. In addition, a vertical seismic force, upward or downward, equal to half the aforementioned horizontal value, can also co-exist and should be catered for—particularly when maximum or minimum base pressures are critical.

(iii) *If the earthquake is in bridge transverse direction*, then, as far as the contributions of the live load and the deck dead load to it are concerned, the aforementioned fraction may be applied on the magnitude of their reactions on the support under consideration, acting respectively at 1.2 m height above road surface and at the deck centroid level. (The lines of bearings in this direction effectively act as one fixed bearing.)

However, if the *earthquake is in bridge longitudinal direction*, then, the contribution of live load to it can be ignored since braking force is already considered and any further longitudinal horizontal force on live load will only cause skidding of its wheels. As for the longitudinal seismic force coming on the foundation from the weight of the deck and footpath live load, it will depend on whether the bearings are rocker and roller-rocker (sliding) type or shearing (elastomeric) type. In the former case the seismic fraction may be applied on the entire weight of a simple span deck on the rocker (fixed) bearing and the footpath live load on it, and this be assumed to go to the rocker bearing and to the foundation under it (roller-rocker bearing only takes the temperature force). However, in the latter case the fraction may be applied on the sum of the deck dead load and footpath live load reactions from the two simple spans sitting on the foundation under consideration. In each case, the point of action will be the bearing level.

NOTE that for distribution of externally applied longitudinal horizontal forces (e.g., seismic, wind and braking) in straight, simple or continuous decks and in curved and skewed (simple or continuous) decks, reference may be made to Chs. 8 and 9 as indicated earlier.

Step 10 Estimate the 'active' earth pressure force and moment (at various levels and at the base of foundation) on account of the retained fill above the soffit level of footing/pile cap. Passive relief from the front fill is generally to be ignored, but if it is well protected dependably then a fraction of it may be taken as dependable (but accounting for the negative surcharge angle effect if sloping downwards). This depends on the actual conditions *in situ*, case by case, the fraction may be such as to limit the magnitude of passive coefficient equal to the active coefficient.

If part of the foundation has been taken down well into permanent and unexcavated soil* (e.g., a caisson taken below the maximum scour level), estimate and take into account the net 'passive less active' earth pressure relief (force and moment) from such assisting soil grip. For this purpose, reference may be made to Ch. 13.

Step 11 (i) Summarise the net vertical load, the net horizontal forces in the two orthogonal directions and the net moments about these two directions, at the base of the foundation (at soffit of pile cap in case of piles), under each critical load combination. In other than piled foundations, establish the base pressures and the safety factors for stability against overturning and sliding and ensure that the requirements are satisfied, and if necessary redesign with revised dimensions. (Guidance on substrata bearing capacity... from Ch. 12.)

(ii) In case of piled foundations estimate the maximum and minimum axial loads in the piles by the traditional rivet-group approach (taking account of rakes if piles are raked). Ensure that no pile is

* For minimum depth of foundation refer to the book *Consultancy and Construction Agreements for Bridges including Field Investigations* by the author.

in tension unless tension-piles are provided. Ensure the structural design of the pile-section for the incumbent load and moment combinations on it (i.e. the moment resulting from the multilegged frame-action of the pile group under the action of the orthogonal horizontal forces, with the top and bottom fixity points in pile defined).

Check for the adequacy of soil resistance around an individual pile, the block failure and group action of the pile group. (Reference may be made to Ch. 11 for more details on this subject.)

(iii) Having then ensured the stability of the foundation, work out vertical load and moments at various intermediate critical sections (including at the maximum bending moment section in a caisson) in the foundation structure (this includes whole pier/abutment structure), and then structurally design these sections. Also design various other structural elements of the foundation structure (as they exist), taking due account of the critical load combinations incumbent for them.

Step 12 Estimate the vertical loads on each bearing under various load combinations, together with the co-existing orthogonal horizontal forces and rotations. Design the bearings for these effects. Use may be made of the standard manufacturers' catalogues for certain standard bearings. For details regarding bearings refer to Ch. 17 which also gives design procedure for elastomeric neoprene bearings and concrete hinge bearings.

Superstructure Design Steps

NOTE that for butterfly (i.e., double cantilever) decks, constructed essentially in free cantilever, as also for various other design considerations, refer to Chs. 18 and 37 and App. 6.

Step 13 Analyse and design the transverse-deck-slab and its cantilever wings, unless the superstructure is a purely longitudinally reinforced solid slab with no cantilevering wings. This is necessary as this decides the top flange thickness of the deck section which is essential to know in order to work out the deck section properties for the subsequent longitudinal design work.

Step 14 (i) Work out dead load and live load bending moments at each critical section (e.g. $^1/_2$, $^1/_4$ and $^1/_8$th span sections in simply supported spans up to about 30 m—one-tenth span sections for longer spans particularly in continuous and balanced cantilever decks, in the longitudinals of the deck).

(ii) In order to know the maximum and minimum live load effects that a particular longitudinal can receive, carry out the transverse load distribution for live load placed in various lanes (maximum, minimum and most eccentric placements). This* may be done by the simple *wheel-spacing method* given in the 1983 AASHTO specifications or by Dr Courbone's method (essentially if span to width ratio is between 2 and 4) or by Little and Morice's method, which basically is the Guyon-Massonet method. (A somewhat similar method suggested by Hendry and Jaeger somehow has not been commonly used.) Alternatively, use may be made of the *Plane-Grid method* which involves using one of the many standard computer programs (e.g., STRUDL program which is extremely powerful). The Plane Grid method is basically a *finite element method*. Though time consuming in writing the input data, it is nevertheless very powerful as it can easily take account of any skew effect and even the effect of curve in plan. (The curve is broken down into a series of jointed straight chord members.) In relatively narrow box section decks with full-depth end cross girders (and if their spacing exceeds about 45 m, then intermediate cross girders as well) the entire load effect may be assumed to be taken by almost full section evenly, owing to almost complete maturation of the section, ignoring only outer parts of the deck slab cantilevers (beyond a distance equal to six times the average deck slab thickness from the outer faces of the box section) due to shear-lag effect. For wide and multi-cell boxes the transverse live load distribution may be studied by the finite element method but it is time consuming. Alternatively, the total live load effect on them may be suitably increased (generally by 8–15%, depending on width and number of cells) and taken as acting on the modified box section properties mentioned above. The 1983 AASHTO specifications are much simpler to apply even for boxes.

(iii) Design against bending the aforementioned critical sections, in reinforced or in prestressed concrete as the case may be. In case of reinforced concrete—first a quick approximate design and detailing may be made and then the stresses and crack-widths checked accurately and then the ultimate moment capacities ensured, modifying the detailing appropriately. In case of prestressed concrete—basically, the relevant steps indicated in Ch. 18 and App. 6 may be followed. Also

* See more details in Ch. 19. Also refer Ch. 31 for transverse deck section design of certain special types of box sections.

include the effect of temperature stresses, as discussed in Ch. 30.

Step 15 (i) Work out dead load and live load shear forces at each critical section [e.g., at face of end-block (which in reinforced concrete deck may be taken at a distance equal to effective depth from centre of bearing), at $^1/_8$th and $^1/_4$th span sections and sections near intermediate supports] in the longitudinals of the deck. (Transverse live load distribution as in Step 14.)

(ii) Design the sections and reinforcements for shear on load factor basis adopting the cracked section approach discussed in Ch. 24.

Step 16 Design the reinforcement and its detailing in the anchorage zones (in case of prestressed concrete) and also above and below the bearings. Now design the remaining structural elements of the deck (e.g. cross girders, brackets, parapets, footpath-slabs, road-kerbs, etc.). Design the expansion joints for the appropriate deck movement (for some details see Ch. 32).

Step 17 Ensure the vibration characteristics of the deck (natural frequency and amplitude of vibration) so that it preferably falls below the strongly perceptible limit in Lenzen's criteria. Refer Ch. 39. However, it is to be noted that vibration is generally never a problem in concrete bridges, and even if the aforementioned vibration characteristic falls above the strongly perceptible limit, this alone is of no discomfiture to the driving public—unless either it is a pedestrian bridge or it has cycle tracks and footpaths with significant foot-movement traffic.

CHAPTER 2

Forces to be Considered in the Analysis for the Design of a Bridge

2.1 MAIN FORCES

The following is a list of the main forces whose effects should be analysed to estimate the load-effects (moments, shears, etc.) at all critical sections in the structure. Only then the structure should be designed for these load-effects to decide the section size, reinforcements, prestress, etc., so as to resist these forces at the specified stress levels, and serviceability criteria (crack-widths, deflections, etc.):

- Dead load of the structure (self-weight may come in stages)
- Live load (on roadway, cycle tracks and footpaths)
- Impact effect of moving live load
- Braking force (generated by the application of brakes on the live load)
- Wind load
- Earthquake force
- Lateral horizontal loads on parapets and kerbs
- Centrifugal force in horizontally curved decks
- Flood water current force in the bridge longitudinal and transverse directions (which is different from the static water pressure)
- Effect of afflux head (created locally on a pier as it obstructs the flood flow)
- Effect of cross-current force in bridge longitudinal direction (the effect of about 25 cm static head of water across the pier, looking at the bridge length), where the river bed is unerodible (rock) at scour level. (This force is an alternative to the flood water force in bridge longitudinal direction.)
- Buoyancy (floatation)
- Earth pressure
- Self-induced horizontal force caused at bearings by movement/rotation of deck due to temperature variation, creep and shrinkage of deck concrete, elastic-shortening of deck due to prestress, etc.)
- Thermal effect (comprising (i) the effect of non-linear distribution of temperature through the deck depth (leading to eigen stress resulting from the difference between final linear thermal strain gradient and the unrestrained non-linear thermal strain gradient), and (ii) the indeterminacy effect (comprising (a) effect of restraint to change in body mean temperature, and (b) effect of restraint to angular movements at the supports))
- Secondary effects (e.g., effect of eccentric connections, and shrinkage and creep of deck concrete)
- Effect of possible differential settlement of supports
- Loads resulting from temporary erection conditions and partial span-dislodgement conditions.

2.2 SOME RELEVANT CONSIDERATIONS

The following recommendations are intended to be used in conjunction with conditions prevailing at the sites and design loads and criteria (i.e. the particular codes of practice) specified by the organizations sponsoring the projects.

Dead Loads

(i) *Structure dead loads*: Structure dead loads are loads imposed on a member by its own weight and the weight of other structural elements that it supports including rails, sidewalks, slabs, and beams.

(ii) *Superimposed dead loads*: In addition to the structure dead loads, members should be designed to support the weight of superimposed dead loads including footpaths, earth-fill, wearing course, stay-in-place forms, ballast, water-proofing, signs, architectural ornamentation, pipes, conduits, cables and any other immovable appurtenances installed on the structure.

Construction, Handling, and Erection Loads

Consideration should be given to the effect of temporary loads imposed by sequence of construction stages, forming, falsework and construction equipment and the stresses created by lifting or placing precast members. Whereas a construction scheme should be considered for the feasibility of the project, it should be recognized that the contractor should be left to his own ingenuity for developing the construction procedure. The stability of precast members during and after construction should be investigated and provisions made for stability as needed. Effects of member-shortening and redistribution of loads during prestressing should be considered. Owing to long term creep effect the distribution of moments, shears, etc. in a continuous structure initially built in parts with different span arrangement for each casting becomes nearly same as if the whole structure was cast monolithically in one go. This calls for a workman-like understanding of true stress build up as against a mere numerical superimposition of stresses.

Shrinkage and Creep

Shrinkage of unreinforced concrete varies according to age, moisture conditions, temperature, water-cement ratio of mix, size and quality of aggregate and chemical composition of cement. Because a major portion of the shrinkage occurs soon after the concrete sets, shrinkage stresses of restrained concrete structures may be reduced by scheduling the placement of concrete. For example, alternate sections of continuous members may be placed and allowed to shrink prior to placing concrete between the sections to complete the continuity. Forces or displacements due to shrinkage of conventionally reinforced concrete may be evaluated assuming a strain of 0.0002.

Because shrinkage is not effectively restricted by the reinforcement in members such as arch ribs or prestressed beams that are subjected primarily to compression, the shrinkage strain can be significantly greater. For design of such members a shrinkage coefficient equivalent to a drop in temperature of about 30 to 80°F, and even that corresponding to strains of 0.0018 to 0.0048 has been suggested by some.

Creep may be assumed proportional to the sustained stress. For conventionally reinforced concrete members the ratio of creep deformation to the instantaneous deformation is also dependent upon the shape of the cross-section and the amount of compression reinforcement.

The average creep deformation due to sustained load at the end of 20 years is approximately two to three times the instantaneous elastic deformation. The approximate percentage of this strain occurring after application of sustained load are as follows:

25%	2 weeks
50%	2 months
75%	1 year
100%	20 years

More accurate prediction of creep and shrinkage values accounting for the shape of the cross-section, properties of concrete, its age at loading, the time under loading and the environmental conditions, can be made by the use of standard references. Also see Ch. 26 ahead.

Thermal Effects

Refer Ch. 30 of this book.

Earth Pressure Effects

Refer Chs. 11, 13, 14 of this book.

Bed Scour and Minimum Founding Level

Reference may be made to the author's book—*Consultancy and Construction Agreements for Bridges including Field Investigations.*

Differential Settlement of Supports

Its magnitude should be estimated from the considerations reported in the soil investigation report for the project site and then the effect of settlement of each individual support may be computed by the method explained in Ch. 22. Thereafter, the worst combinations of simultaneous settlement of different supports can be picked up for maximum effects at individual sections. (However, long term creep reduces this effect in the long run.)

Wind, Earthquake, Buoyancy and Vibration Effects

Reference may be made to the relevant mandatory codes of practice and Ch. 39 of this book.

Effects of Horizontal Forces—Externally Applied and Self Induced

For the effects of the externally applied horizontal forces (e.g. braking, seismic and wind) and self induced horizontal forces (caused by movement/rotation of the deck due to temperature variation, shrinkage and creep of deck concrete and due to elastic shortening of the deck due to prestress), reference may be made to Chs. 8 and 9. Here one finds the method for evaluating the horizontal forces caused at the various bridge supports as a result of the distribution of these forces as effected by the type of bearings, shear ratings of the supports, and location of the zero movement point in the deck.

Flood Water Force on Substructure

The flood current, flowing at a maximum velocity V m/s (or a maximum mean velocity v m/s) may be assumed to strike the substructure at $\theta = 20°$ to the normal flow direction, so that flood strikes the width of pier @ $V \cos\theta$ m/s and the length of pier @ $V \sin\theta$ m/s.

The resulting dynamic stream pressures on the substructure in the two orthogonal directions, respectively, may be assumed to vary linearly from zero at maximum scour level to the maxima of $52\ K(V\cos\theta)^2$ and $52\ K(V\sin\theta)^2$ kg/m^2 at the high flood level, where K = a coefficient, taken as 1.4 for square ended (i.e., square nosed) substructure, 0.5 if the noses include an angle of 30° or less and 0.67 for noses that are circular. V^2 is taken equal to $2v^2$,

V and v in m/s. (Note that this dynamic water pressure triangle is unlike the static pressure triangle which varies from zero at the surface to maximum at the bottom of water depth.)

Ice Pressures and Associated Forces

Reference may be made to the latest AASHTO design specifications.

Traffic Lanes

Lane-loading or standard-truck are generally considered to have a width of 1.8 m centres of wheels and 3.00 m overall. These loads should be placed in the design traffic lanes, spaced across the entire bridge roadway-width, in numbers and positions required to produce the maximum stress in the member under consideration. Roadway-width should be the distance between kerbs. Fractional parts of design-lanes should not be used. Roadway-width in metres, should be divided by 3.65 and the quotient approximated to the nearest whole number which then represents the Number of Design Traffic Lanes (NDTL), i.e. the number of lanes that can be assumed to be loaded for the purposes of bridge design. The said roadway-width should include shoulders also.

The lane-loadings or standard-trucks should be assumed to occupy any position within their individual design traffic lane-width, which will produce the maximum stress, maintaining generally 3 m as-centre to centre distance between adjacent trucks in cross section.

Reduction in Live Load Intensity

Where maximum stresses are produced in any member by loading more than one traffic lane simultaneously (as is generally the case), the following percentages of the resultant live load effects should be used in view of improbably coincident number of lanes loaded:

One or two lanes	100%
Three lanes	90%
Four lanes or more	75%

The reduction in intensity of floor beam loads should be determined as in the case of main trusses or girders, using the width of roadway which must be loaded to produce maximum stresses in the floor beam.

CHAPTER 3

Live Load on Road Bridges

3.1 GENERAL BACKGROUND

Loading specifications have a history which goes back to well before the general use of methods of structural analysis capable of simulating plate action, and some loading specifications were drawn up to enable the strength requirements for slab elements within steel girder bridges to be evaluated by means of simple hand calculations. However, methods of analysis which evaluate the design moments due to complex loading cases are now in widespread use, and it is therefore no longer considered necessary to enhance the distributed loading on short spans to give the appropriate design forces, because these can be evaluated directly from the concentrated wheel loads.

For major roads, and those giving access to certain types of industrial installations, provision has to be made for moving abnormal loads.

Abnormal loading has to be accommodated on all motorways and trunk roads.

The application of the normal *line load* is taken parallel to the supports. This is because the knife-edge load does not specifically represent an axle but is a load which, when combined with the distributed loads specified for the span effect, gives rise to design forces appropriate to the strength requirement for an element of a deck structure.

For the design of local structural elements within a bridge deck, such as the slabs spanning between longitudinal members, the requirement of a single wheel-axle load is common. Concentrated loads govern for short spans.

The distribution of the moments arising from concentrated wheel loads at the edges of a slab which is fixed along the line of support (as is commonly the case in a cellular or box deck) is subject to sharp peaks. Plotting the design bending moment along the length of a support will show the peak and illustrate how the area over which the load is applied becomes significant in evaluating design moments. It is therefore relevant to take the thickness of finishes and the wheel contact area into account in evaluating the load dispersion dimensions.

Simple types of bridge deck rarely produce stability problems but where narrow pier arrangements are used, as is frequently the case with lengths of elevated roadway, it becomes important to check that a structure remains stable with heavy vehicles on the outer extremities of the deck. Another form of instability is for uplift to develop at the bearings under some loading conditions where there are marked differences in the span on each side of a support.

The value of accuracy and refinement in design methods is very much diminished if it is not matched by similar qualities in the assessment of design live loads.

The early loading standards in some countries were not applied nationally; they were generally specified by local authorities who took into consideration the traffic which was likely to use the bridge concerned. These loadings often consisted of steam-rollers or some form of traction-engine.

The needs of military transport and its heavy equipment caused consideration to be given to the specification of a loading train for bridges representative of the actual and envisaged vehicles they would have to carry. This resulted in the introduction of the (then) Ministry of Transport's first 'standard loading train' in the UK in 1922, and the original loading standards of many other European countries at about the same time. This trend in the introduction of standard highway bridge loadings was almost a universal trend arising out of the technological advancements and industrial developments that were taking place at a fast rate all over the world.[1]

It was, however, the 1930s which witnessed the introduction of the most standard bridge loadings applied nationally and on a more scientific basis than hitherto. For example, in the UK the equivalent 'Ministry of Transport standard loading curve' was introduced in 1932, this was a new approach which proved to be very popular. Contemporary with this there was the British Standards (BS) train of loading. These two standards formed the basis of the present type HA loading of BS:153. In the USA, a loading standard consisting of truck-trains and equivalent loads was introduced by the American Association of State Highway Officials (AASHO)* in 1935. It is significant that even in some of the developing countries, like India, loading standards were introduced nationally during this period (in 1937).

* This name and abbreviation is now changed to the Association of American State Highway and Transportation Officials (AASHTO).

The highway bridge loading standards of most countries have developed gradually often with little regard to the standards prevalent in other countries. This has resulted in wide variations in loading standards, even among neighbouring countries, which is inhibiting the proper development of 'through' road transport all over the world. Therefore, there is a fundamental need to know how the loading standards of one country differ from those of others. Galambos[2], in a recent study made for the International Road Federation, has reported that many countries are currently engaged in revising their highway bridge loading standards.

Comparative studies reported[3-4] at ACI's second international symposium on *Concrete Bridge Design* (1969) were based on the loadings of different countries. However, in at least one of them, the dynamic effects were not considered. As the impact allowance varies considerably in different countries, it has to be added to the basic values for a realistic comparison. In one of them there are some obvious anomalies in the results. For example, the loading in the UK is shown to be more severe than that of West Germany and in fact, the heaviest, which is not the case.

3.2 LOADINGS OF DIFFERENT COUNTRIES

AASHTO Loadings

The Association of American State Highway and Transportation Officials (AASHTO), Washington DC, specified heaviest loading, designated as HS 20-44, comprises a tractor truck with a semi-trailer having a total load of 320.3 kN or the corresponding lane loading. The lane loading is made up of a ud load of 9.3 kN/m and a knife edge load of 80 kN for bending moment and 115.7 kN for shear. Impact is to be added in both the cases as per the formula given in the AASHTO specifications.

For the design of bridges, both the truck and lane loading are to be considered and the one which gives the worst effect is to be adopted. With the truck loading, only one truck is considered for each traffic lane for the whole of its length. There is no reduction in load intensity for up to two lanes of traffic loaded.

BS Loadings

The British Standards (BS), London specify two types of loading known as the type HA and type HB loadings. The HA type is also followed in Malaysia, Sri Lanka, Kenya, Zambia, Zimbabwe, etc. (with minor changes in some cases). The HA type is the normal design loading and consists of a uniformly distributed lane loading varying from 318.6 kN/m for 1 m loaded length (span) to 5.8 kN/m for 900 m loaded length (span), and a knife-edge load of 120 kN per lane. The values given are inclusive of impact. There is no reduction in the intensity of HA loading for up to two lanes of traffic loaded. An alternative axle load is also specified in this chapter on which impact must be considered.

The HB type is an abnormal unit loading. The number of units per axle (four axles in all) specified in the UK for bridges carrying the heaviest class of load is 45, amounting to a total load of 1800 kN. This is an idealised load on four axles which allows for the weight of the tractors accompanying trailers. With this loading, an overstress of 25% is allowed. No allowance is to be made for impact. Only one lane is to be loaded with type HB loading, all other lanes being considered as occupied by one-third full lane HA loading (latter only if its presence gives worst effect).

The type HA and HB loadings are currently under revision. In the UK, the Department of Environment has already effected certain changes in the type HA loading.

IRC Loadings

The Indian Roads Congress (IRC) specifies three classes of loads, designated as Class 70-R, Class AA and Class A for the design of permanent bridges, and all of them are followed in India. Pakistan has adopted Class AA* and Class A loadings for the design of bridges.

The Class 70-R and Class AA are of two types each. The first is a 700 kN tracked vehicle which is common to both the classes; the only difference is in the loaded length, which is slightly more for the Class 70-R. The second, which is of the wheeled type is a 1000 kN train of vehicles on seven axles for the Class 70-R, and a 400 kN vehicle on two closely spaced axles for the class AA. The Class A loading is a 554 kN train of wheeled vehicles on eight axles. Impact is to be allowed for in all the loadings as per the formulae given. The formulae are different for steel and concrete bridges.

All the three classes of loads are to be separately considered in the design and the worst effect is to be taken. For the design of two-lane bridges, only one lane of Class 70-R or Class AA load is considered, whereas both the lanes are assumed to be occupied by Class A loading if that gives worst effects.

Loadings of France

There are two normal systems of loads known as the System A and System B loads. The System A loading consists of a ud load which varies from 18.7 kN/m^2 for 10 m loaded length to 4 kN/m^2 for 199 m loaded length. This load, which is inclusive of impact, is given by a formula in terms of the loaded length. The System B comprises three types known as Systems B_c, B_r and B_t. While System B_c consists

* Tracked vehicle only.

of two trucks of 300 kN each per lane, B_r is a single wheel load of 100 kN, and B_t, a tandem axle of 320 kN. Impact is to be added to these according to a formula which takes into consideration the dead load of the structure as well.

In the design of bridges, systems A and B are to be considered successively and the worst effect is to be taken. There is no reduction in load intensity for up to two lanes of traffic loaded.

Loadings of West Germany

For federal autobahns (expressways), federal highways and rural highways of the first order, designated as Class 60, the loading per lane consists of a 600 kN vehicle on three axles and a ud load of 5 kN/m^2 in the remaining portion of the carriageway. Allowance for impact is to be made as per the formula given.

An equivalent ud load is also given as a substitute for the design vehicle.

Loadings of Japan

The live load specified for the design of main girders of the first class of bridges is known as the L-20 loading. There is a corresponding truck loading called T-20 loading for the design of floor systems. For a lane width of 5.5 m or less, the L-20 loading consists of a knife-edge load of 50 kN/m and a ud load of 3.5 kN/m^2 for spans up to 80 m reducing to 3 kN/m^2 for greater span lengths. For bridges having a width of more than 5.5 m, the knife edge load and the ud load are assumed to be reduced by one half on the portion of the roadway in excess of 5.5 m width. Impact is to be added as per the formula given.

Loadings of New Zealand

The HS 20-44 truck and the lane loading of the AASHTO and another truck loading designated H20-S16-T16 design vehicle are specified. The latter is the same as the HS 20-44 truck with a 142 kN trailer attached to it. Both the loadings are to be considered in the design and the one which gives the worst effect is to be taken. Impact is to be allowed as per the formula given; but for shear force, it is taken as 30% for all the spans. There is no reduction in load intensity for up to two lanes of traffic loaded.

Loadings of Sweden

Sweden specifies two types of loads. One is a lane loading, consisting of a 140 kN axle plus a ud load varying from 24 kN/m for 10 m span and less to 11 kN/m for a 90 m span and above. The other is a 1000 kN single truck on five axles. Impact is to be added only to the axle load of the lane loading. For the design of two lane bridges, either the lane loading in both the lanes or the single truck loading is considered.

NOTE: More details of the above loads, as also of certain other National (Highway Bridge) loads are given ahead.

3.3 SOME INTERESTING COMPARISONS[1] IN THE DIFFERENT TYPES OF LOADINGS

While many other countries specify the same ud load for bending and shear, Italy gives different values, those for shear being more than those for bending. This is understandable as there is no knife-edge load or axle load with it. It is, however, significant to note that France and West Germany do not distinguish between bending and shear in their equivalent ud load values, although they have no knife-edge load or axle load with the ud load.

Unlike other countries which specify ud loads for the full width of the traffic lane, Finland and Sweden specify it as a strip load in two strips of 0.6 m each running for the entire loaded length. As an alternative, Sweden allows the ud load to be applied uniformly over a width of 2.4 m.

The countries which specify a knife-edge load in combination with a ud load fall into two groups. While the HA type group gives the same value of knife-edge load for both bending and shear, the AASHTO type group, Iran and New Zealand, specify different values. In the latter case, the knife-edge load for shear is always more than that for bending.

With the exception of the HA type group, all countries which have an equivalent ud load system, have at least an alternative truck loading which is also to be considered in the design. Even in the HA type group, designs are generally to be checked for the type HB loading, although the number of units of the HB vehicle to be taken may vary from country to country. For example, the specifications of Kenya require the designs to be checked for 25 and 30 units respectively of the HB vehicle as against the UK practice of checking the designs of all the important bridges for 45 units for trunk roads and 37 1/2 units for principal roads. BS:153 permits an overstress of 25% with the type HB loading. There is, however, an ambiguity in the present provision in the code permitting this overstress, as it is not related to the number of units of the HB vehicle.

Italy, like some other countries, has separate civil and military loadings and all its important bridges are designed for the latter, which is heavier. Though not explicitly, military loadings are however covered in the standards of many other countries. In this category comes the IRC Class 70-R of India, the Caterpillar of Austria and the NK-80 loading of the USSR.

Lane Width

The design lane width of 5.5 m followed in Japan is an unusual one. Except for it, the lane width lies in the range 2.5–4.0 m, the most common value being 3 m. Norway specifies a range for the design lane widths and Sweden gives different widths for different types of loading. These practices do not conform to the ideal concept of a 'standard design lane width' and 'lane loading'. In countries like India, Pakistan and USSR, where there is no standard lane loading, only the minimum widths of carriageways for different number of lanes are specified. Therefore, they do not have any standard design lane width as such.

Impact Allowance

(i) BS:153 specifies an impact allowance of 25% to be added to the axle load (the pair of adjacent wheels) if it produces the greatest bending moment or shear, in the HA load case. The stipulations in the Norwegian standard are similar; the only difference is that instead of 25%, 38.5% impact is added to the heaviest axle load.

(ii) In the majority of countries, impact is related to the loaded length (span length) although the exact relationship varies considerably from country to country.

(iii) Some countries like Austria and India specify different impact factors for concrete and steel bridges, the factor for steel being more than that for concrete. This apparently is based on the principle that a lighter structure will be subjected to a more dynamic effect. Finland too has a similar approach, but it distinguishes only timber bridges from the others by specifying a lower impact factor for them presumably on account of the damping effect of timber.

(iv) At least West Germany and Italy ignore impact when the span length exceeds 50 and 100 m, respectively. But even for longer spans, countries like Australia and India specify certain minimum values of impact.

(v) Various standards give an upper limit for the impact allowance either with respect to the type of vehicle (tracked or wheeled) or in relation to the type of bridge (concrete, steel or timber) and the value of this varies from 25 to 64% in different standards.

(vi) Unlike other countries, Belgium and France relate the impact factor to the dead load of the bridge structure. The principle behind this is, however, implicit in the impact formulae being used by many other countries which relate impact to the type of bridge and length of span. The impact formula of Belgium is further complicated by including the speed of the vehicle in it.

(vii) Austria specifies different impact factors for the directly loaded and indirectly loaded main girders of concrete bridges, the factors for the former being more than that for the latter. Again in the case of steel bridges, it distinguishes between the first and second lanes of steel bridges, specifying higher impact factors for the first than the second and allowing no impact for lanes in excess of two.

Quantitative Comparison of Road Live Load from Various Countries

The following are the bases of comparison[1] of road live load:

(i) For comparing the loadings from the quantitative point of view, the maximum bending moment and shear force that would be caused by them in simply supported spans are taken as the basis. Simple spans were chosen since it was presumed that they are more common and are also indicative of what is likely to happen in other types of construction. A span range of 5–100 m was expected to cover the great majority of simple span bridges.

(ii) As the impact allowance varies considerably for different loadings, it is added to the calculated values of bending moment and shear force, and a comparison is made of the total values. Wherever the same standard gives different impact formulae for steel and concrete bridges, the one which gives the higher value is taken.

Computation of Bending Moment and Shear Force (Comparative Picture)

On the basis of the above assumptions, the values of the maximum bending moment and shear force, including impact, were calculated for each loading separately, for single and double lanes, for spans up to 100 m in 5 m increments. The results obtained are given in Tables 3.1 to 3.4[1].

Considering the predominant range of spans, the following general observations can, however, be made from the results of Tables 3.1 to 3.4:

(i) Although the IRC loadings appear to be the heaviest for a single lane, they are lighter than the French, West German, Japanese and Type HA loadings when two lanes are considered.

(ii) The West German loading, which is lighter than the IRC and Japanese loadings for a single lane is the heaviest when two lanes are considered.

(iii) For both single and double lanes, the AASHTO loading gives the minimum effect in bending and shear, being only about one half of that given by the West German loading.

(iv) The Type HA and French loadings are almost identical in effect for spans up to about 50 m, beyond that, the latter gives slightly higher values.

(v) In the higher span ranges, the global effect of Type

Table 3.1 *Simply Supported Span Versus Maximum Bending Moment for One Lane*

Span (m)	Maximum bending moment for one lane, including impact allowance (kN-m)								
	Loadings of New Zealand	L-20 Loading of Japan	Loadings of France	Class 60 Loading of W. Germany	IRC Loadings	HS 20-44 Loading of AASHTO	BS Loadings		Loadings of Sweden†
							Type HA	Type HB**	
5	231	551	390‡	612	687	231@	243	756	450
10	573	1237	816	1624	1548	573@	694	1863	1300
15	1137	2057	1539	2690	2725	1073@	1336	3331	2550
20	1797	3006	2371	3804	4198	1552@	2175	5654	3800
25	2485	4083	3290	4952	5680	2022@	3156	7862	5050
30	3157	5285	4280	6125	7058	2481@	4151	10085	6300
35	3817	6612	5338	7310	8412	2935@	5184	12315	7550
40	4465	8065	6454	8497	9739	3379@	6340	14550	8800
45	5107	9647	7637	9674	11059	3863	7501	16788	10050
50	5738	11344	8870	10830	12496	4597	8656	19029	11300
55	6367	13162	10151	12452	13933	5391	9780	21271	12550
60	6990	15115	11498	14168	15371	6243	11070	23515	13800
65	7607	17182	12884	15977	16808	7155	12354	25759	15050
70	8299	19376	14342	17880	18245	8125	13738	28005	16300
75	9155*	21682	15848	19877	19683	9155	15117	30251	17550
80	10244*	24119	17388	21968	22339	10244	16560	32497	18800
85	11392*	26387	18997	24152	25138	11392	17993	34744	20050
90	12631*	28714	20660	26430	27945	12631	19508	36992	21300
95	13872*	31080	22348	28802	30759	13872	21069	39239	22550
100	15184*	33494	24106	31268	33580	15184	22875	41487	23800

* Based on lane loading. Otherwise H20-S16-T16 truck loading governs
** 45 units. An overstress of 25% is permissible under this loading
† Truck loading governs throughout
‡ Based on System B_c loading. Otherwise System A loading governs
@ Based on standard HS truck loading. Otherwise standard lane loading governs

Table 3.2 *Simply Supported Span Versus Maximum Shear Force for One Lane*

Span (m)	Maximum shear force for one lane, including impact allowance (kN)								
	Loadings of New Zealand	L-20 Loading of Japan	Loadings of France	Class 60 Loading of W. Germany	IRC Loadings	HS 20-44 Loading of AASHTO	BS Loadings		Loadings of Sweden†
							Type HA	Type HB**	
5	212	441	359‡	571	549	212@	199	738	420
10	304	495	369‡	689	633	298@	278	927	590
15	375	549	454‡	743	820	337@	356	1218	700
20	420	601	523‡	779	927	358@	435	1364	775
25	447	653	569‡	807	978	369@	505	1451	820
30	465	705	592‡	828	997	375@	554	1509	850
35	478	756	610	845	1009	379@	592	1551	871
40	488	807	645	858	1015	387@	634	1582	888
45	495	858	679	867	1027	390@	667	1606	900
50	501	908	710	872	1090	410	693	1625	910
55	509	957	738	912	1181	433	711	1641	918
60	518	1008	767	950	1272	457	738	1655	925
65	545	1057	793	988	1351	481	760	1666	931
70	575*	1107	820	1026	1419	505	785	1675	936
75	606*	1156	845	1064	1477	529	806	1684	940
80	636*	1206	869	1102	1529	552	828	1691	944
85	666*	1242	894	1140	1574	576	847	1697	947
90	697*	1276	918	1178	1625	600	867	1703	950
95	727*	1309	941	1216	1695	623	887	1708	953
100	758*	1340	964	1254	1776	647	915	1713	955

* Based on lane loading. Otherwise H20-S16-T16 truck loading governs
** 45 units. An overstress of 25% is permissible under this loading
† Truck loading governs throughout
‡ Based on System B_c loading. Otherwise System A loading governs
@ Based on standard HS truck loading. Otherwise standard lane loading governs

Table 3.3 *Simply Supported Span Versus Maximum Bending Moment for Two Lanes*

Span (m)	Maximum bending moment for two lanes, including impact allowance (kN-m)								
	Loadings of New Zealand	L-20 Loading of Japan	Loadings of France	Class 60 Loading of W. Germany	IRC Loadings	HS 20-44 Loading of AASHTO	BS Loadings		Loadings of Sweden†
							Type HA	Type HB**	
5	462	827	780‡	1224	687	462@	488	838	640
10	1146	1856	1632	3248	1548	1146@	1388	2095	1580
15	2274	3086	3078	5380	2725	2146@	2672	3776	2775
20	3594	4509	4742	7608	4198	3104@	4350	6379	4198
25	4970	6125	6580	9904	5680	4044@	6312	8914	5819
30	6314	7928	8560	12250	7058	4962@	8302	11468	7609
35	7634	9918	10676	14620	8412	5870@	10368	14043	9537
40	8930	12098	12098	16994	9739	6758@	12680	16663	11572
45	10214	14471	15274	19348	11059	7726	15002	19288	13679
50	11476	17016	17740	21660	12496	9194	17312	21914	15838
55	12734	19743	20302	24904	14232	10782	19560	24531	18012
60	13980	22672	22996	28336	15598	12486	22140	27205	20172
65	15214	25773	25768	31954	17166	14310	24708	29877	22277
70	16598	29064	28684	35760	20046	16250	27476	32584	24316
75	18310*	32523	31696	39754	23486	18310	30234	35290	26250
80	20488*	36178	34776	43936	26650	20488	33120	38017	28048
85	22784*	39580	37994	48304	29908	22784	35986	40742	29662
90	25262*	43071	41320	52860	33620	25262	39016	43494	31095
95	27744*	46621	44696	57604	38050	27744	42138	46262	34129
100	30368*	50241	48212	62536	42880	30368	45750	49112	37300

* Based on lane loading. Otherwise H-20-S16-T16 truck loading governs
** One lane Type HB full and the other lane 1/3 Type HA as per the Code. An overstress of 25% is permissible under this loading
† Lane-loading governs throughout
‡ Based on System B_c loading. Otherwise System A loading governs
@ Based on standard HS truck loading. Otherwise standard lane loading governs

Table 3.4 *Simply Supported Span Versus Maximum Shear Force for Two Lanes*

Span (m)	Maximum shear force for two lanes, including impact allowance (kN)								
	Loadings of New Zealand	L-20 Loading of Japan	Loadings of France	Class 60 Loading of W. Germany	IRC Loadings	HS 20-44 Loading of AASHTO	BS Loadings		Loadings of Sweden†
							Type HA	Type HB**	
5	424	661	718‡	1142	594	424@	398	804	512
10	608	742	738‡	1378	692	596@	556	1020	632
15	750	823	908‡	1486	820	674@	712	1337	740
20	840	902	1046‡	1558	927	716@	870	1509	840
25	894	980	1138‡	1614	978	738@	1010	1619	931
30	930	1057	1184‡	1656	997	750@	1108	1694	1015
35	956	1134	1220	1690	1009	758@	1184	1748	1090
40	976	1210	1290	1716	1015	774@	1268	1793	1157
45	990	1286	1358	1734	1082	780@	1334	1828	1207
50	1002	1361	1420	1744	1174	820	1386	1856	1267
55	1018	1436	1476	1824	1274	866	1422	1878	1300
60	1036	1512	1534	1900	1368	914	1476	1901	1345
65	1090	1586	1586	1976	1446	962	1520	1919	1371
70	1150*	1661	1640	2052	1512	1010	1570	1937	1390
75	1212*	1735	1690	2128	1572	1058	1612	1952	1400
80	1272*	1809	1738	2204	1642	1104	1656	1967	1402
85	1332*	1863	1788	2280	1728	1152	1694	1980	1402
90	1394*	1914	1836	2356	1816	1200	1734	1992	1402
95	1454*	1963	1882	2432	1908	1246	1774	2004	1437
100	1516*	2010	1928	2508	1996	1294	1830	2018	1492

* Based on lane loading. Otherwise H20-S16-T16 truck loading governs
** One lane Type HB full and the other lane 1/3 Type HA as per BS 153. An overstress of 25% is permissible under this loading
† Lane loading governs throughout
‡ Based on System B_c loading. Otherwise System A loading governs
@ Based on standard HS truck loading. Otherwise standard lane loading governs

HB loading (consisting of 45 units of the HB vehicle in one lane and one-third of type HA in the second) is less than that of full type HA in both the lanes, when the former is adjusted for the permissible increase in stress of 25%.

(vi) The New Zealand loading is somewhat heavier than that of AASHTO for spans up to about 70 m. Beyond that, it gives the same values of bending moment as for the AASHTO loading; but in the case of shear force, it gives higher values throughout.

The loadings of different countries vary considerably both qualitatively and quantitatively. While the qualitative differences are understandable, it is difficult to see that such wide variations in intensity are warranted. This brings out the need for systematic surveys of vehicular loads on bridges. Apart from the intensity of traffic, the aspect of safety is closely linked with design loading and needs due consideration.

Of the different types of loadings, the equivalent ud load system appears to be the most popular one, possibly because it is simpler to apply. This explains the adoption of the Type HA and AASHTO loadings by many countries.

There are basic differences in the approach of different countries for assessing the dynamic effect of live loads on the bridge structure. The impact allowance formulae specified by some are unnecessarily complicated and not fully justified, particularly since the effect of live load on the bridge is comparatively less than that of dead load for span lengths of approximately 25 m and above. Thus there is need for more research in this field to ascertain the actual behaviour of bridge structures under dynamic loads and to evolve suitable design procedures (refer Ch. 39).

From a consideration of the simplicity of loading and the ease of its application in design, the Type HA loading appears to be the most favourable.

There is wide variation in the highway bridge loading standards of different countries. The extreme example is that of the USA where, for the range of spans covered, the effect of the design live load is only about one half of that in West Germany. (In fact, the loading of the Netherlands is even more severe than that of West Germany.) The loadings of the other countries generally fall in between the AASHTO and West German loadings.

3.4 DETAILS OF SOME NATIONAL (HIGHWAY) BRIDGE LOADINGS

<u>*AASHTO Loadings*</u> (Figs. 3.1 and 3.2)

(USA, Australia, Bangladesh, Canada, Ethiopia, Philippines, Turkey*)

Truck Loading

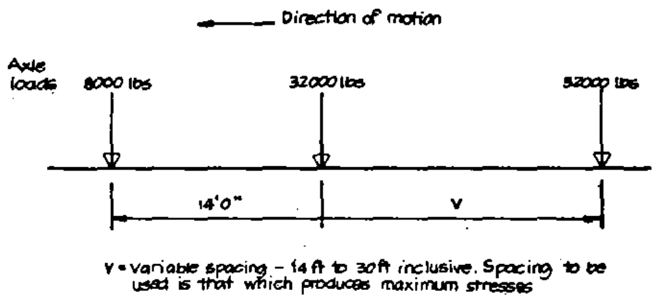

Fig. 3.1 *Standard HS 20-44 truck*

Impact Allowance

Impact allowance** is $50/(L+125)$ where L is the length in feet of the portion of the span to produce the maximum stress in the member. Maximum† impact allowed is 30%. For shear due to truck loads, L is taken as the loaded part of the span from the point being considered to the reaction, except for cantilever arms where the impact allowance is 30%.

Lane Loading

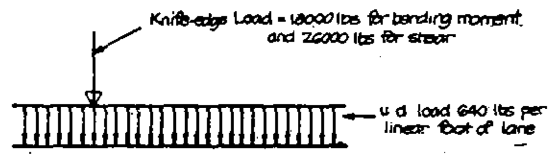

Fig. 3.2 *Standard HS 20-44 lane loading*

<u>*BS Loadings*</u> (Figs. 3.3 and 3.4)

(UK, Malaysia‡, Sri Lanka‡, Kenya‡§, Rhodesia‡)

Type HA Loading

The Type HA loading consists of:

(i) A ud lane loading as per the loading curve given plus a knife-edge load of 120 kN uniformly distributed across the width of the traffic lane, or

(ii) Two wheel loads§ each of 112 kN in line transversely to the direction of traffic flow, spaced at 0.9 m.

The ud load has a constant value of 31.5 kN per metre run of one lane for loaded lengths from 6.5 m to 23.0 m. For spans below 6.5 m, BS:153 gives separate curves for the ud load.

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* Axle loads followed in Turkey are 4 and 16 t in place of 8000 and 32,000 lbs respectively.

** Turkey specifies the impact allowance as $15/(L+37)$, where L is the span length in metres.

† Australia specifies also a minimum impact allowance of 10%.

‡ For Type HA loading only.

§ In Kenya, the wheel loads are specified as 40 kN each at the spacing of 1.0 m.

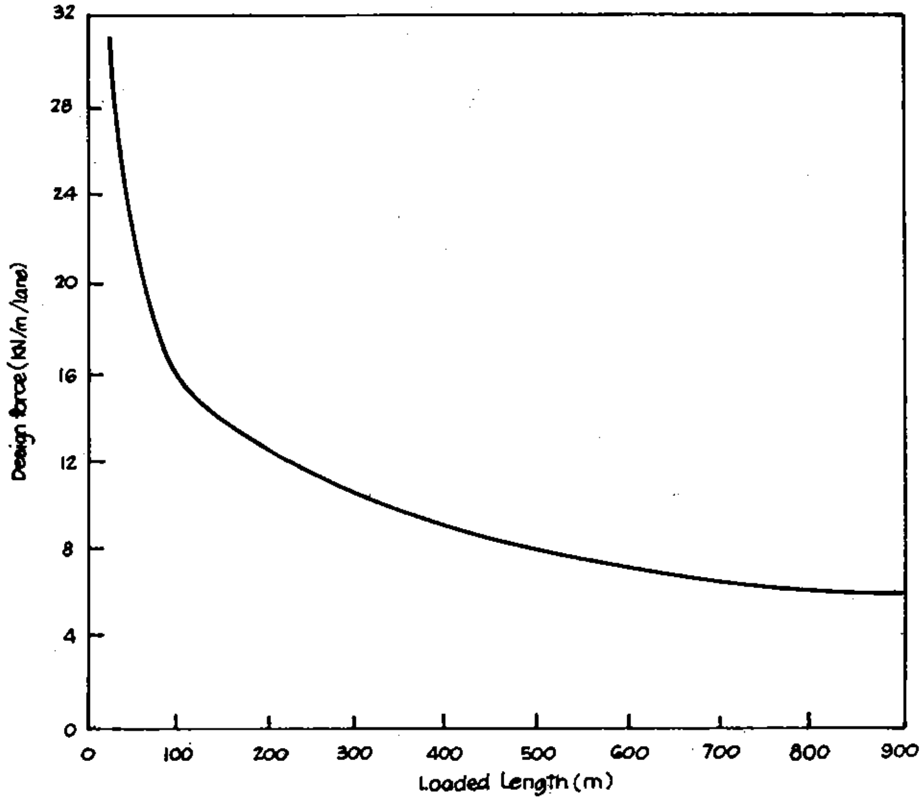


Fig. 3.3 *Loading curve for type HA loading*

Two lanes are always considered as occupied by full type HA loading while all other lanes in excess of two are considered as occupied by one-third the full lane loading.

The standard design lane width is 3.0 m. The number of traffic lanes to be considered for different widths of carriageway are specified.

In considering the effects of the 112 kN wheel loads, an overstress of 25% is permitted.

Impact Allowance

Type HB loading: no allowance is to be made for impact. Type HA loading incorporates an impact allowance of 25% on the heaviest axle in the train of vehicles from which the loading has been derived.

Type HB Loading

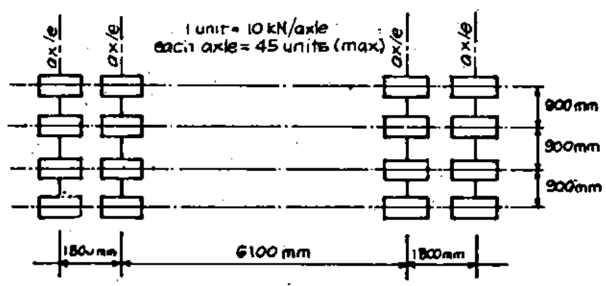


Fig. 3.4 *Plan view of type HB loading*

IRC Loadings (India and Pakistan*) (Figs. 3.5–3.10)

Class 70-R Loading

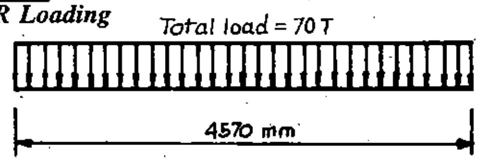


Fig. 3.5 *Class 70-R 'Tracked' loading*

* For the Class AA tracked and Class A loadings only.

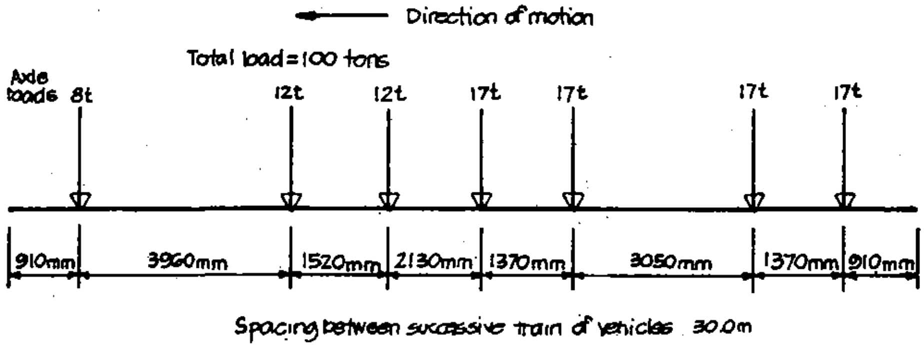


Fig. 3.6 *Class 70-R 'Wheeled' loading*

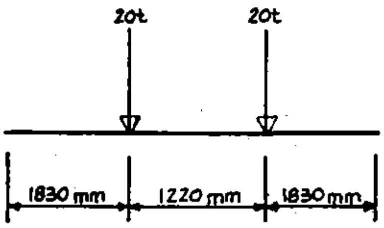


Fig. 3.7 *Class 70-R 'Bogie' loading*

Class AA Loading

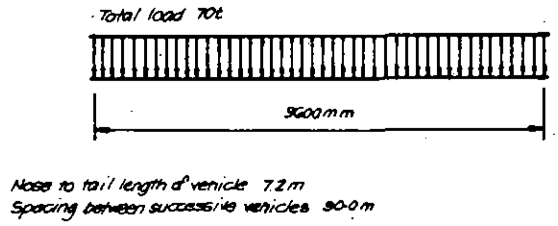


Fig. 3.8 *Class AA 'Tracked' loading*

Impact Allowance

When L is the length of span in metres, impact allowance for concrete bridges is equal to $4.5/(6 + L)$ subject to a maximum of 50% and minimum of 8.8%.

The impact allowance for steel bridges is $9/(13.5 + L)$ subject to a maximum of 54.5% and minimum of 15.4% with the following exceptions in the case of Class 70-R loading and Class AA loading:

(i) *For spans less than 9 m:*

• Tracked vehicles	25% for spans up to 5 m reducing to 10% for spans of 9 m.
• Wheeled vehicles	25%

(ii) *For spans of 9 m or more:*

• Tracked vehicles on concrete bridges	10% up to a span of 40 m
• Wheeled vehicles on concrete bridges	25% for spans up to 12 m

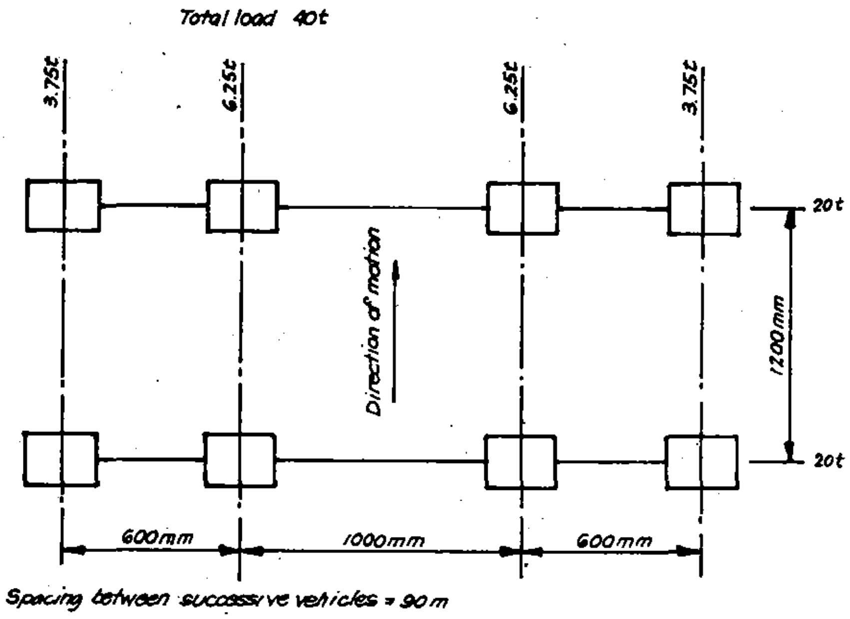


Fig. 3.9 *Plan view of the class AA 'Wheeled' vehicle*

Class A Loading

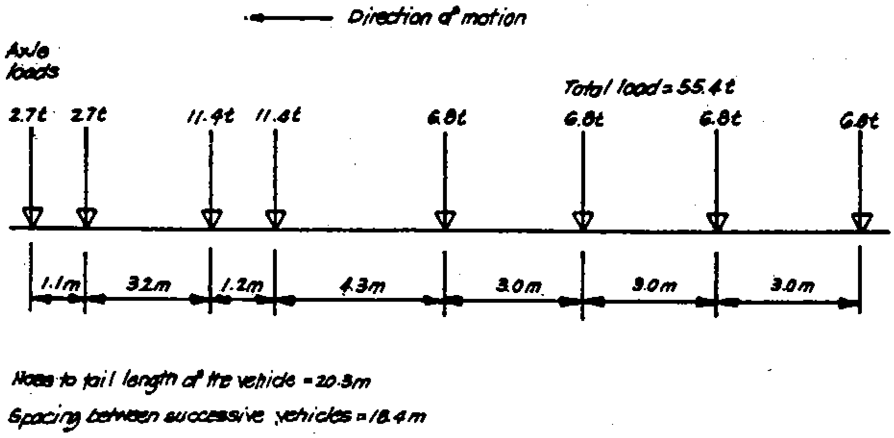


Fig. 3.10 *Class A train of vehicles*

• Tracked vehicles on steel bridges	10% for all spans
• Wheeled vehicles on steel bridges	25% for spans up to 23 m

Loadings of France (Figs. 3.11 and 3.12)

System A Loads

$$\text{ud load } A(l) = 230 + \frac{36{,}000}{l + 12} \text{ kg/m}^2$$

where l is the loaded length in metres.

The ud load $A(l)$ obtained from the above formula is to be multiplied by a coefficient a_1 whose value is 1.0 up to two lanes and then reduces with an increase in the number of lanes.

$a_1 A(l)$ is not to be less than $(400 - 0.2\, l)$ kg/m^2.

For class 1 roads, if the lane width is different from the standard lane width of 3.50 m, the value of $A(l)$ is to be multiplied by a coefficient a_2 also, so as to keep the total load per linear metre of lane unaltered for any loaded length.

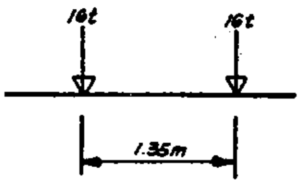


Fig. 3.12 *System B_t tandem axle loading*

System B Loads

System B_r is a single wheel load of 10 t.

Impact Allowance

System A loading is inclusive of impact. For System B loading, the impact factor, δ, is given by the formula

$$\delta = 1 + \frac{0.4}{1 + 0.2L} + \frac{0.6}{1 + 4\frac{G}{s}}$$

where L = length of the element in metres

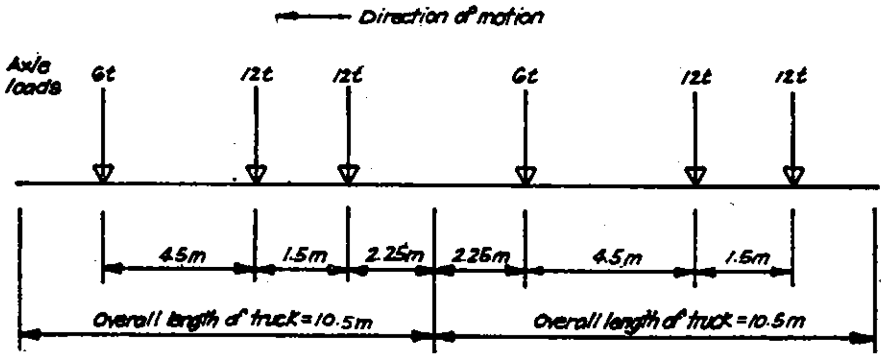


Fig. 3.11 *System B_c truck loading*

G = permanent weight of the bridge
s = maximum load of the truck

Loadings of West Germany (Fig. 3.13)

Class 60 Loading

The Class 60 loading consists of a 60 t heavy truck and a ud load of 0.5 t/m^2 in the portion of the lane not occupied by the truck. The substitute ud load for the 60 t heavy truck is 3.33 t/m^2.

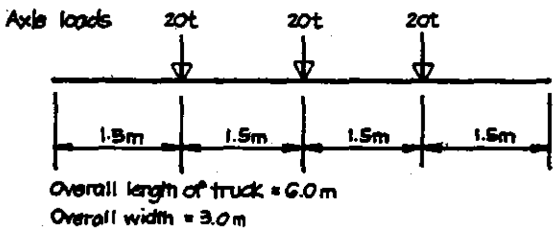


Fig. 3.13 *60 t heavy truck (SLW)*

The standard design lane width is 3.0 m. There is no reduction in intensity of load for up to two lanes of traffic. A ud load of 0.3 t/m^2 is specified in the area outside the main lanes.

Impact Allowance

The impact factor ϕ with which the live load values are to be multiplied is given by the formula

$$\phi = 1.4 - 0.008\, l_\phi \text{ but } \geq 1.0$$

where l_ϕ is the governing length in metres.

Loadings of Japan (Fig. 3.14)

L-20 Loading

For a lane width of 5.5 m or less, the L-20 loading consists of a knife-edge (line) load, P, of 5000 kg/m and a ud load, p, which has the following values,

For $l < 80$ m, $p = 350$ kg/m^2
For $l > 80$ m, $p = 430 - l$ but ≥ 300 kg/m^2

For bridges with a width of more than 5.5 m, the values of P and p are to be reduced by one-half on the portion of the roadway in excess of 5.5 m.

The full values of P and p are known as 'main loads' and the reduced values (50% of the main loads) are known as 'subloads'. The main loads are to be so placed on a 5.5 m wide part of the roadway and the subloads in the remaining part of the roadway as to produce maximum stresses.

In the expressions for the ud load p, l denotes the span length in metres.

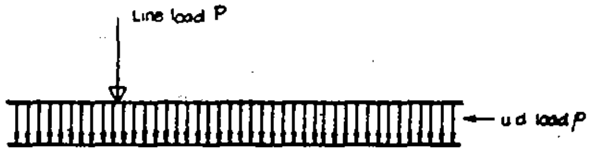


Fig. 3.14 *Composition of L-20 loadin~*

Impact Allowance

Impact allowance i is determined by the formula,

$$i = \frac{20}{50 + l}$$

where l is the length of the element in metres.

Loadings of New Zealand (Fig. 3.15)

Design Load

The design load per lane consists of the HS 20-44 truck and lane loading of the AASHTO or the H20-S16-T16 truck loading whichever gives the worst effects. The standard design lane width is 10 ft. (Fig. 3.15).

Impact Allowance

The impact allowance specified is the same as that given by the AASHTO, but there is, however, no upper limit to it.

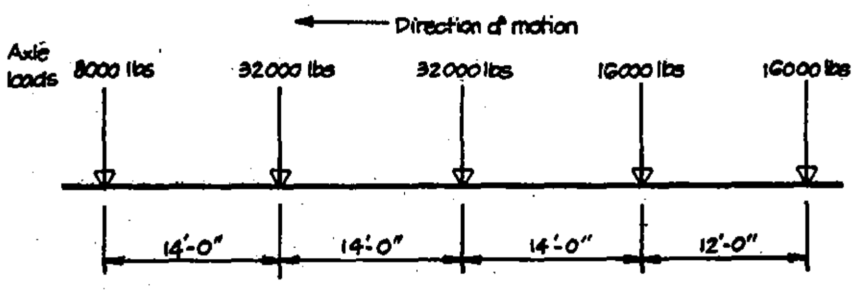


Fig. 3.15 *H20-S16-T16 truck*

Loadings of Sweden (Figs. 3.16 and 3.17)

Lane Loading

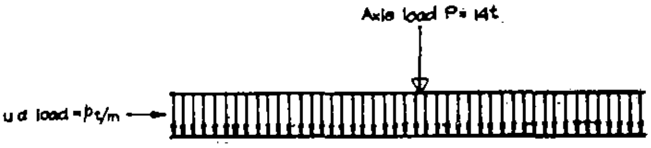


Fig. 3.16 *Composition of lane loading*

$$p = 2.4 \text{ t/m for } L < 10 \text{ m}$$

$$p = 2.4 - \frac{1.3(L - 10)}{80} \text{ t/m for } 10 < L < 90 \text{ m}$$

$$p = 1.1 \text{ t/m for } L > 90 \text{ m}$$

where L = loaded length in metres
Design lane width = 3.0 m

Truck Loading (see Fig. 3.17)

Two lane bridges are designed with lane loading in both the lanes or only with single truck loading whichever gives the worst results. For continuous structures, there is a separate loading consisting of two axle loads and a ud load.

Impact Allowance

The axle load, P, of the lane loading is to be increased by 40% for impact effects. No allowance for impact is to be made for the ud load and for the single truck loading.

Loadings of Austria (Figs. 3.18 and 3.19)

Tracked Loading

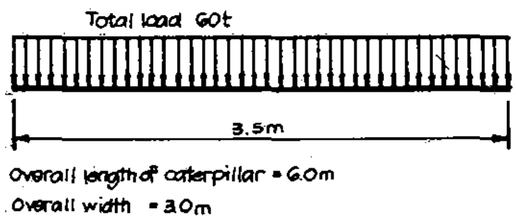


Fig. 3.18 *60 t caterpillar*

Truck Loading

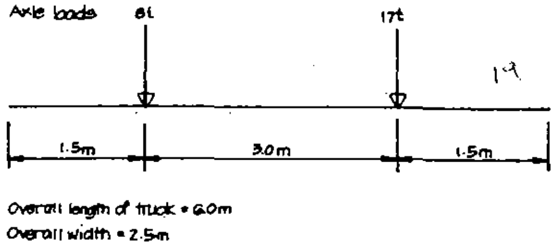


Fig. 3.19 *25 t truck*

A ud load of 0.5 t/m^2 is to be assumed on the portion of the lane not occupied by the truck.

On two adjoining lanes, the truck loading is assumed in both the lanes, the portion of the carriageway not occupied by the trucks being assumed to be carrying a ud load of 0.5 t/m^2. With the tracked loading, however, only one caterpillar is to be assumed for the whole carriageway and there will be no ud load with it. Both the cases are to be tried and the worst effect taken in the design.

The specifications also give the following equivalent weights of the caterpillar and truck loading which are to be used for the design of spans more than 30 m.

60 t Caterpillar	3.33 t/m^2
25 t Truck	1.67 t/m^2

Impact Allowance

The following are the impact factors given:
(i) Concrete bridges

Impact factor for different spans

Span of structural part (m)	0	10	30	50	70
Direct loaded main girder	1.40	1.30	1.20	1.10	1.00
Indirect loaded main girder	1.40	1.25	1.10	1.00	1.00

Impact factor for floor slab = 1.4

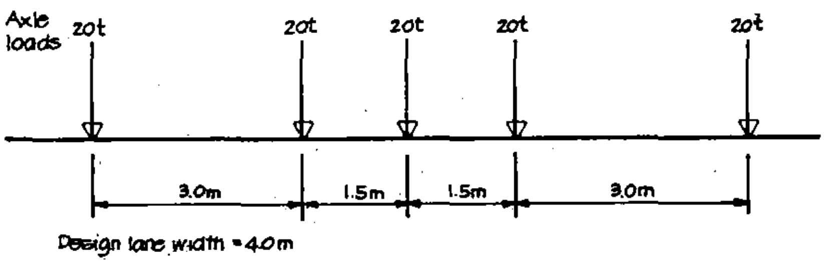


Fig. 3.17 *100 t single truck*

(ii) Steel bridges

Impact factor for different spans

Span of structural part (m)	2	6	10	20	40	60	80	100
Lane I	1.64	1.41	1.30	1.18	1.10	1.07	1.05	1.04
Lane II	1.32	1.20	1.15	1.09	1.05	1.03	1.02	1.02

For all remaining lanes impact factor is 1.00.

Loadings of Belgium (Figs. 3.20 and 3.21)

Normal Truck Loading

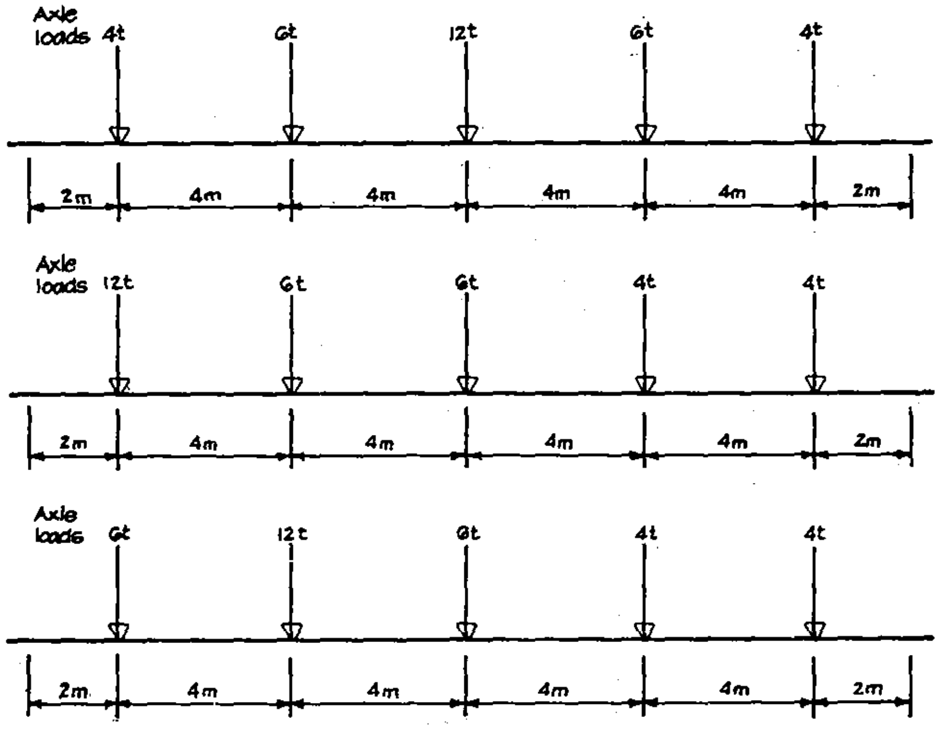


Fig. 3.20 *Different combinations of 32 t normal truck*

Heavy Truck Loading

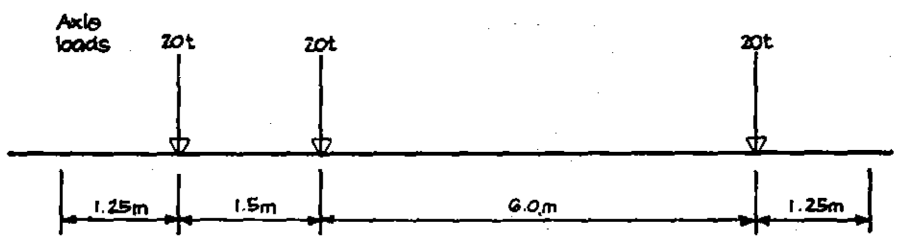


Fig. 3.21 *60 t heavy truck*

For two lane bridges on important roads, the loading consists of a 60 t truck in one lane in combination with a 32 t truck plus a ud load of 400 kg/m^2 in the other lane.

Impact Allowance

The impact factor, ϕ, is given by the formula:

$$\phi = 1 + \frac{0.377v}{\sqrt{l\alpha}} \cdot \sqrt{1 + \frac{2Q}{P}}$$

where v = the speed in km per hour (always greater than 60)

l = the distance between supports in metres

$\alpha = \frac{l}{fs}$

fs = static deflection in metres due to dead weight

Q = moving loads on the bridge deck in tonnes

P = dead weight of the bridge in tonnes

Loadings of Italy (Figs. 3.22, 3.23 and 3.24)

Design Loading

For the design of category 1 bridges, any one of the following three types of loads, flanked by one or more trains of 12 t trucks, producing the worst effect is to be taken:

The width of the three types of loadings is 3.5 m. An equivalent ud load having different values for bending moment and shear is also specified in tabular form.

Impact Allowance

For spans up to 100 m, the impact factor ϕ is given by the formula,

$$\phi = 1 + \frac{(100 - L)^2}{100(250 - L)}$$

where L is the span of the bridge in metres.

For spans exceeding 100 m, ϕ is assumed to be unity.

Loadings of Netherlands (Fig. 3.25)

Class 60 Loading

This is the highest class of loading and it consists of a 600 kN vehicle on three axles of 200 kN each plus a ud load as shown in Fig. 3.25. The standard design lane width for this loading is 3.0 m.

Impact Allowance

The magnitude of the impact coefficient, S, for bridges carrying normal traffic is given by the formula,

$$S = 1 + \frac{40}{100 + L}$$

where L is the span in metres.

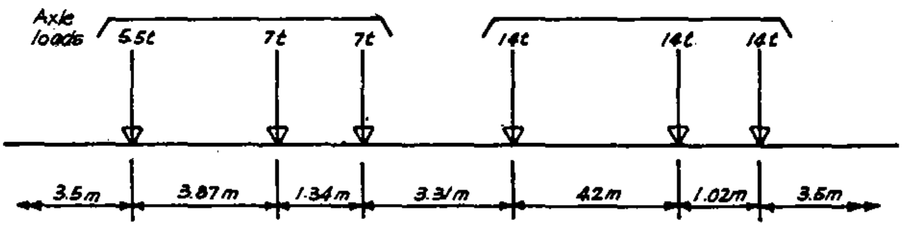


Fig. 3.22 *Continuous train of military load of 61.5 t*

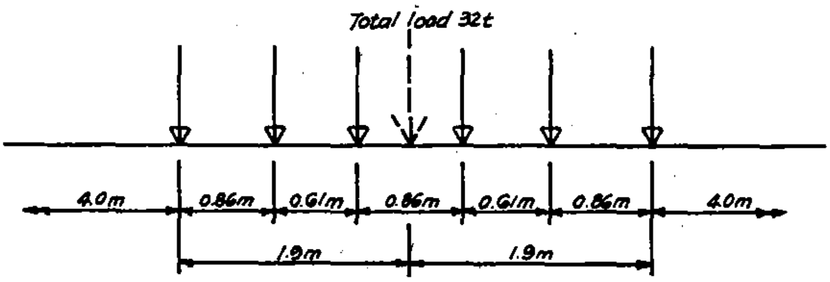


Fig. 3.23 *Continuous train of military load of 32 t*

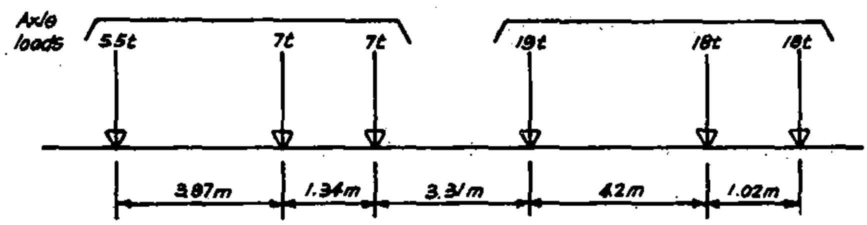


Fig. 3.24 *Single military load of 74.5 t*

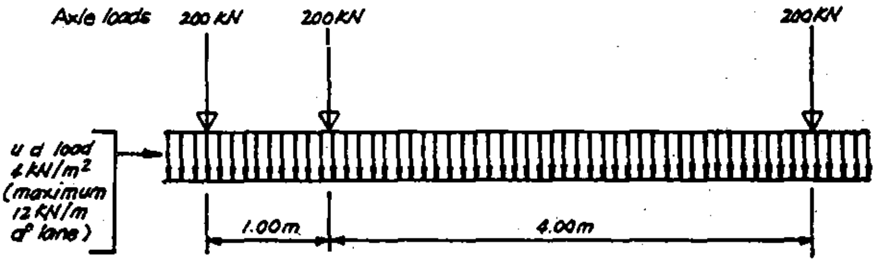


Fig. 3.25 *Class 60 loading*

Loadings of Norway (Fig. 3.26)

Lane Loading

The equivalent lane loading per lane for Class 1 bridges consists of a knife-edge load, A, and a ud load, p, as shown in Fig. 3.26.

$$A = 12 + \frac{8x}{L} \text{ t}$$

$$p = 0.5 + \frac{35}{L+5} \text{ t/m of lane}$$

L = Actual loaded length of lane in metres

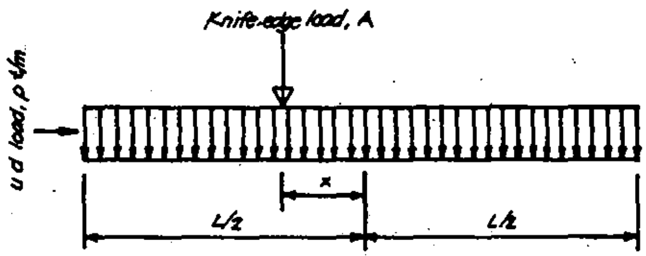


Fig. 3.26 *Equivalent lane loading per lane*

The above lane loadings are normally considered over lane widths from 3.0 to 3.75 m. For two lane bridges, the full equivalent loading is assumed in both the lanes. Besides the lane loading, the structure is designed for a local loading of two axles, each of 13 t.

Impact Allowance

It is assumed that 38.5% impact is to be added to the heaviest axle and it is unnecessary to add any impact to the remaining axles. The values of the knife-edge load, A and ud load, p, are inclusive of impact calculated on this basis. An impact of 38.5% is to be added to the axle loads.

Loadings of the USSR (Figs. 3.27 and 3.28)

Wheel Loadings

There are three types of wheeled loadings (Fig. 3.27).

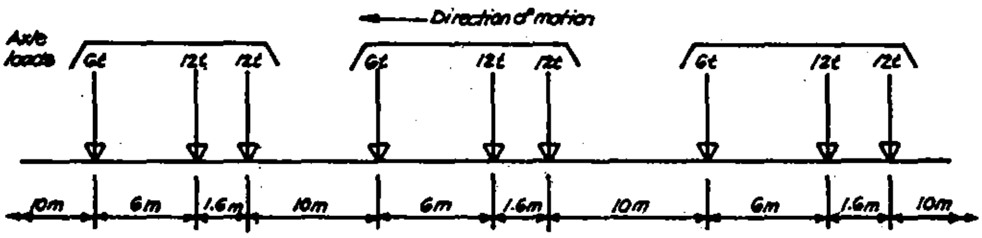


Fig. 3.27(a) *N-30 loading*

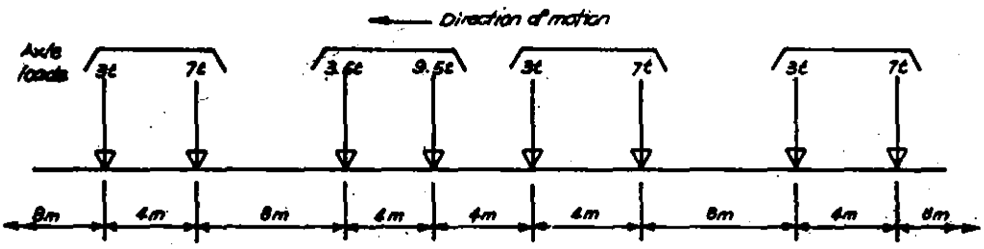


Fig. 3.27(b) *N-10 loading*

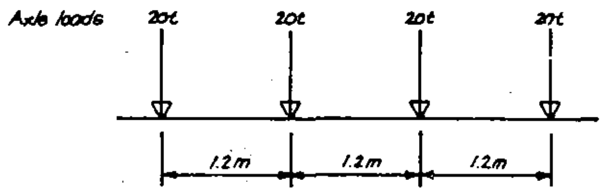


Fig. 3.27(c) *NK-80 loading*

Tracked Loading

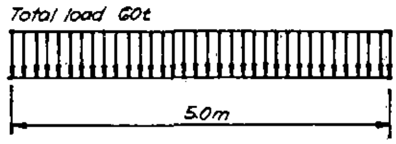


Fig. 3.28 *NG-60 caterpillar loading*

Saudi-Arabian Highway Bridge Loading (Fig. 3.29)

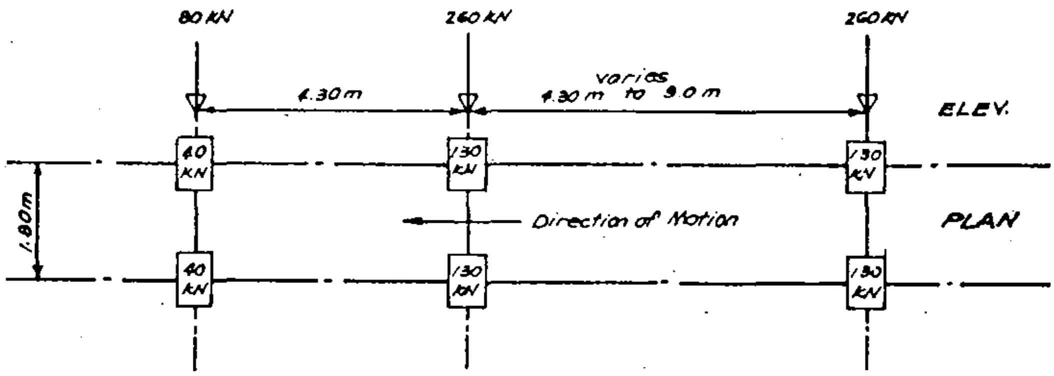


Fig. 3.29(a) *600 kN Truck (each lane can be loaded by a truck but only one truck/lane longitudinally)*

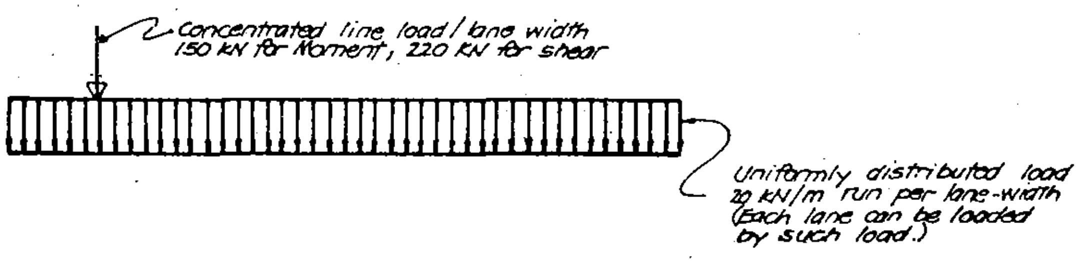


Fig. 3.29(b) *Uniformly distributed lane load plus one line load*

NOTE

(i) Design Load either (a) or (b), whichever produces the maximum effect.

(ii) Impact allowance as per AASHTO specifications.

(iii) For more than two lanes loaded, reduction factor on live load effect as per AASHTO specifications.

REFERENCES

1. Thomas, P.K., "A Comparative Study of Highway Bridge Loadings in Different Countries", *Supplementary Report 135UC*, Crowthorne, 1975. Transport and Road Research Laboratory.
2. Galambos, C.F., "International Road Federation In-depth Study on Fatigue, Fracture and Stress Corrosion Problems of Highway Bridges", *World Survey of Current Research and Development on Roads and Road Transport*. (International Road Federation, Washington, DC. 1972). pp. 332–365.
3. Seni, A., "Comparison of Live Loads Used in Highway Bridge Design in North America with Those Used in Western Europe", *Second International Symposium on Concrete Bridge Design* (Chicago, April 1969), (American Concrete Institute), Detroit, 1971, pp. 1–34.
4. Rajagopalan, K.S., Comparison of Loads Around the World for Design of Highway Bridges", *Second International Symposium on Concrete Bridge Design* (Chicago, April 1969), American Concrete Institute, Detroit, 1971, pp. 35–48.
5. Rowe, R.E., *Concrete Bridge Design* (CR Books, London, 1962), Wiley, New York, 1963.

CHAPTER 4

Structural Concrete

4.1 CONCRETE FOR CONSTRUCTION

This topic has been dealt with in detail in the author's book *Concrete for Construction—Facts and Practice*. Therefore, only a list of the chapter contents of this book is given here and reference may be made to the book itself. (The latter part of the present chapter contains some rough-and-ready information.)

The Concrete Mix

• Historical note • Specifications for concrete • Coarse aggregate size • Air entrainment in the concrete mix • Structural requirements of concrete • Portland cements (details, types, properties, manufacture) • Aggregates (coarse, fine, types, choice, tests, grading, sieve analysis, practical gradings) • Water • Admixtures, as an aid to making concrete • Selecting mix proportions • Batching, mixing, transporting, placing and curing of concrete including vibrating, and finishing the concrete • Control of surface evaporation and its importance • Construction, expansion and cold joints • Depositing concrete under water • Inspecting, sampling and testing for concrete • Curing and protection of concrete.

Strength and Properties of Concrete

• General considerations • Effect of mix proportions, water-cement ratio and curing • Effect of cement content and type • Effect of aggregate types and characteristics • Effect of admixtures • Bleeding and 'setting shrinkage' • Cement hydration • Effect of continuous moist storage • Effect of curing method • Effect of thermal changes • Effect of age at test • Relations between various types of concrete strength • Effect of materials on concrete properties • Effect of construction practices on concrete properties • Effect of sampling and testing procedure on observed concrete properties • Effect of type of compression specimen: cylinder or cube? • Effect of form and size of specimen • Effect of moisture content in test specimen at the time of testing • Effect of end bearing conditions of test specimens on observed concrete strength • Effect of lateral restraint on crushing strength • Effect of temperature at time of test • Effect of rate of loading while testing • Predicting 28-day strength of concrete • Causes of low rate of strength-gain • How to accelerate strength-gain of concrete • Plastic shrinkage • Drying shrinkage (details, effects) • Carbonation shrinkage • Creep (details, effects, etc.) • Modulus of elasticity of concrete (general, its determination, effects of variables, relationship with compressive strength) • Poisson's ratio of concrete • Cracking of concrete • Fatigue strength of concrete • Toughness (shock resistance) of concrete • Non-destructive tests on concrete ~

Schmidt hammer (indentation rebound) test; sonic tests (for vibration characteristics for dynamic modulus of elasticity); pulse-transmission tests at sonic and ultrasonic frequencies; radioactive tests (for density and presence of reinforcement, etc.); penetration probe tests (for compressive strength)

• Tests on composition of hardened concrete (determination of cement content; determination of original water-cement ratio) • Non-destructive methods of detecting defects in concrete structures • Rebound and penetration methods; stress-wave method; • Magnetic methods (for locating reinforcements); Electrical methods (for resistivity of concrete and corrosion of steel); Chemical methods (for depth of carbonation and amount of chloride ion); Infrared thermography (to detect delaminations/discontinuities to identify deteriorated areas); Radiographic technique with gamma rays • Thermal properties (and heat resistance) of concrete.

Deterioration and Durability of Concrete

• Effect of weathering on concrete • Effect of entrained air on freezing and thawing characteristic of concrete • Effect of de-icing agents on concrete • Sulphate attack and effect of sewage on concrete (and protective measures) • Effect of acids on concrete • Effect of leaching (by water and particularly snow-water) on concrete (by the dissolution of soluble calcium hydroxide) • Effect of sulphates on concrete • Effect of sea-water on concrete • Deterioration of concrete through rusting of steel • Deterioration of concrete from the actions of aluminium, copper, lead and zinc with cement • Corrosion of steel in reinforced concrete structures in marine environment and coastal regions • Effect of carbonation of concrete • Effect of free calcium oxide and magnesium oxide on concrete • Effect of reactive aggregates (alkali-aggregate reaction) • Efflorescence in concrete • Rocks and minerals which may be deleterious in concrete aggregates • Unduly large deflections (due to restrained shrinkage) in reinforced concrete • Surface cracking and crazing • Crumbling of concretes and plasters • Abrasion resistance (wear) of concrete • Effects of various substances on concrete and recommended protective treatments and precautions ~

Magnesium fluosilicate or zinc fluosilicate; Acids; Alkalies; Salts; Petroleum oils; Coal-tar distillates; Vegetable oils; Fats and fatty acids: Sodium silicate; Drying oils; Cumar; Varnishes and paints; Bituminous or Coal-tar paints, tar and pitches; Bituminous enamel; Bituminous mastic; Vitrified brick or tile; Glass, Lead; Synthetic resin, rubber and synthetic rubber.

Concrete in Desert Regions with Hot-Dry Humid Climates and Concrete in Sea-Water Surroundings

• Introduction • Causes of rapid deteriorative damage affecting concrete structures in hot climates • Influence of hot-climate on making concrete ~

Influence of temperature on destructive processes that affect hardened concrete; Influence of moisture on destructive processes that affect hardened concrete; Effect of surrounding water; Effect of biological attack; Effect of erosion; Effect of sea water and marine environment; Conclusion

- Influence of cement choice on durability ~
  Types of cement used; Durability problems in relation to choice of cement; Corrosion of reinforcement due to chloride penetration; Sulphate attack; Alkali-aggregate reactivity; Thermal deformation; Storage of cement; Conclusion
- Suitability of aggregates ~
  Types of rock for aggregates; Alkali-silica reaction; Alkali-carbonate reaction; Contamination by salts; Contamination by clay, mica and dust; Grading and particle shape of aggregates; Other durability aspects; Conclusions
- Admixtures, as an aid to making concrete ~
  Principal types, their characteristics; Experiences (in various middle eastern countries on different types of projects); Concluding remarks
- Effect of climate and working conditions (and various ways in which steel can pick up corrosion; concrete production vis-a-vis temperatures of atmosphere, materials; and precautions)
- Curing (special aspects)
- Recommendations for: Type of cement to be used (and the applicable requirements); (requirements of) coarse and fine aggregates; Water for the mix and for curing; Limitations on water-cement ratio, temperature of concrete mix, chloride-content, sulphate-content and alkali-content in concrete; Admixtures (to be used); Ensuring low permeability of concrete; Grading, measuring (batching), mixing, transporting, placing, compacting, finishing and curing of concrete; Concrete cover to reinforcement bars; Criterion for acceptability of test results

Design of a Concrete Mix and Statistical Control of Concrete Quality

• Introduction • Average and minimum strengths • Degree of workability of concrete • Grading of aggregates • Method of combining aggregates • Design of a concrete mix • Examples of high-strength concrete mix design • Statistical control of concrete quality ~

The normal curve (standard deviation, coefficient of variation, average strength, minimum strength); Standard deviation, for different degrees of quality control; Computation of the correct average or mean strength from 'works test results', and their standard deviation; Saving in cement through quality control; Minimum concrete strength; Factors effecting standard deviation and concrete strength

Pumped Concrete and Mortar, 'Pneumatically Sprayed' Concrete or Mortar, 'Non-Shrink' Concrete or Mortar

• Pumped concrete and pumped mortar versus shotcrete and gunite • Pumped concrete and mortar (equipment, pipes, mixes, pumping) • 'Pneumatically applied' concrete and mortar • Non-shrink or expansive concrete and mortar (prepacked concrete, expansive materials).

Control of General Cracking in Concrete by Controlling the Mix and the Making of Concrete

• Cure against cracking of concrete — not by calculations alone (various considerations from the standpoints of adequacy of materials — quality and grading; correctness of construction practices with regard to making, placing and curing the concrete, etc.).

Cracks in Concrete Due to Other than Structural Loading (Types, Causes, Prevention and Remedies)

• Introduction (intrinsic cracking, structural cracks) • Intrinsic cracks • Factors affecting cracking (water, cement, aggregates, admixture, bleeding, placing, curing, temperature, exposure, restraint, etc.) • Types of intrinsic cracks (and their times of appearance) • Frequency of occurrence of various intrinsic cracks • Causes and remedies of plastic cracks • Early thermal contraction cracks • Long-

term drying shrinkage cracks • Crazing type of intrinsic cracking • Other types of intrinsic cracks • Calcium chloride and corrosion of reinforcement; Sulphate-attack cracks; Alkali-aggregate reaction cracks
• Repair Techniques of cracks • Classification of cracks; Materials for repairs (rigid fillers for dormant cracks, and flexible-fillers for live cracks); Repairs to dormant cracks (fine dormant cracks, wide dormant cracks, dormant fractures, dormant multiple cracks); Repairs to live cracks (mastics, thermoplastics, elastomers, surface sealing, membrane sealing)
• References (from ACI and ASTM).

Crack and Wound Repair in Concrete by Epoxy-adhesive Injection and Mortars

• Introduction (and Caution) • Development • Structural applications • Product range (some epoxy preparations in INSTRUCTION SHEET format) • Supplementary data on long-term behaviour • Adhesion of new concrete to old (EMPA-test) • Heat resistance • Repair by shotcrete and gunite.

Repair and Strengthening of Concrete-structures by Externally Bonded Steel Plates

• Introduction • Information from some tests conducted at EMPA • Some interesting examples of repair strengthening works actually carried out (building floor slabs, reinforced concrete frames, bridges) • Some comments on execution of bonding work (steel, concrete, pretreatment, bonding procedures, adhesive, maturity of bond, testing of adhesive, work to be done after bonding) • Butt joints in tension-carrying bonding plates • Recommendations regarding: bonding adhesive, anti-corrosive primer, anti-weathering paint • Some considerations affecting strength of repair (effect of surface conditions on bond, effect of adhesive thickness) • Background information on the adhesives • Some cautions • References.

4.2 SOME ROUGH-AND-READY INFORMATION

Properties of Concrete

Compressive Strength

The compressive cube strength f_{cu} of concretes made with the same cement and cured and tested under the same conditions have been shown by Feret to comply approximately with the expression,

$$f_{cu} = \alpha_1 \left(\frac{B_c}{B_c + B_w + B_v} \right)^2$$

in which B_c, B_w and B_v are respectively the net volumes of cement, water and voids in a unit volume of mixed concrete, i.e., if B_a is the net volume of aggregate in a unit volume of mixed concrete $B_c + B_a + B_w + B_v = 1$. The coefficient α_1 is a numerical factor determined from tests and depends on the nature of the materials.

Tensile Strength

The ratio of the direct tensile strength f_{cut} to the compressive strength f_{cu} varies from 0.05 to more than 0.1, the relation being approximately of the form $f_{cut} = \alpha_2 (f_{cu})^{\beta}$, where β is between 0.5 and 1.0. A formula derived by Feret is $f_{cut} = \alpha_3 \sqrt{f_{cu}} - \alpha_4$ (N/mm²). Coefficients α_3 and α_4 are obtained by testing and depend

Table 4.1 *Some Mix Proportions*

Prescribed mixes: proportions per 50 kg bag of cement

Concrete grade*	Nom. Max. agg. size (mm)	40		20		14		10	
	Workability	medium	high	medium	high	medium	high	medium	high
20	Total aggregate (kg)	305	270	280	250	255	220	240	200
	Sand : Zone 1 (%)	35	40	40	45	45	50	50	55
	Zone 2 (%)	30	35	35	40	40	45	45	50
	Zone 3 (%)	30	30	30	35	35	40	40	45
	Vol. of finished concrete (m³)	0.165	0.155	0.156	0.143	0.146	0.130	0.137	0.121
25	Total aggregate (kg)	265	240	240	215	220	195	210	175
	Sand : Zone 1 (%)	35	40	40	45	45	50	50	55
	Zone 2 (%)	30	35	35	40	40	45	45	50
	Zone 3 (%)	30	30	30	35	35	40	40	45
	Vol. of finished concrete (m³)	0.147	0.137	0.137	0.127	0.130	0.118	0.124	0.110
30	Total aggregate (kg)	235	215	210	190	195	170	180	150
	Sand : Zone 1 (%)	35	40	40	45	45	50	50	55
	Zone 2 (%)	30	35	35	40	40	45	45	50
	Zone 3 (%)	30	30	30	35	35	40	40	45
	Vol. of finished concrete (m³)	0.134	0.127	0.124	0.115	0.115	0.106	0.109	0.097

Percentages are by weight of fine aggregate in weight of total dry aggregates. Volumes of finished concrete are approximate only.
* Grade represents standard cube crushing strength f_{cu} (N/mm²) at 28 days.

on the nature of the cement, the type, grading and maximum size of aggregate, the amount of water, the conditions of curing and the method of testing employed.

The British draft Unified Code gives values of indirect tensile strength at 28 days for concretes having various crushing strengths. These correspond approximately to the expression:

$$\text{Indirect tensile strength} = \frac{2}{3} + \frac{1}{15}f_{cu} - \frac{1}{2600}f_{cu}^2 \ (\text{N/mm}^2)$$

A very simple approximate relationship is,

$$\text{Indirect tensile strength} = \frac{1}{2}\sqrt{f_{cu}} \ (\text{N/mm}^2)$$

Note: f_{cu} in N/mm².

Flexural Strength (Modulus of Rupture)

For the values obtained for various concretes to be comparable, the test pieces must be of standard dimensions, say 100 × 100 × 400 mm or 4 × 4 × 16 in. If such a specimen is supported over a span of 300 mm or 12 in with a centrally-applied load, the modulus of rupture $f_{cur} = 0.00425\ F\text{N/mm}^2$ or $0.28125\ F\text{lb/in}^2$, where F is the load in kg or lb that causes the test piece to break. These expressions comply with CP 114 (BS). Factors that contribute to a high compressive strength f_{cu} also lead to an increase in the modulus of rupture. The relation of f_{cur} to f_{cu} is given by Feret as approximately.

$$f_{cur} = \alpha_5\sqrt{f_{cu}} - \alpha_6 \ (\text{N/mm}^2)$$

where α_5 and α_6 are parameters affected by the same conditions that affect α_3 and α_4 for direct tensile strength.

Data for estimating the flexural strength f_{cuf} at 28 days from a specified crushing strength given in the British draft Unified Code correspond closely to the relationship.

$$f_{cuf} = \frac{1}{2} + \frac{1}{10}f_{cu} - \frac{1}{2000}f_{cu}^2 (\text{N/mm}^2)$$

The flexural strength of a concrete is about 1½ times the cylinder splitting strength.

Modulus of Elasticity

The modulus of elasticity of concrete E_c increases with increases in cement content, age, repetition of stress, and various other factors, actual values ranging between 21 and 28 kN/mm²

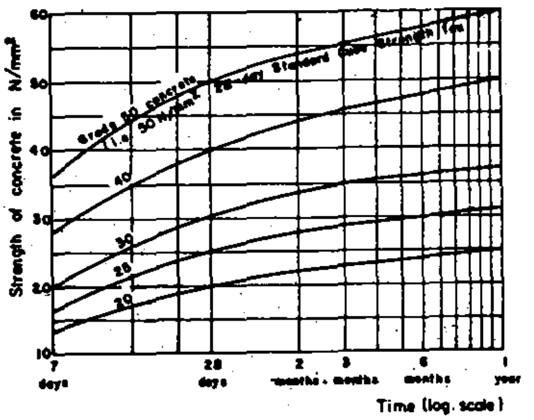


Fig. 4.1 *Concrete properties per CP 110. Increase of strength with time*

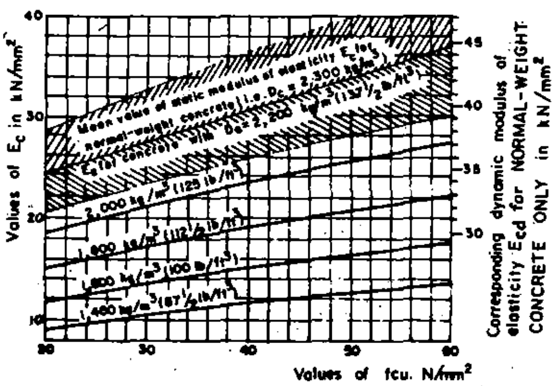


Fig. 4.2 *Variation of modulus of elasticity with strength and density of concrete*

Table 4.2 *Concrete: Properties and Stresses: Metric Values per CP 114 (BS)*

Mix	Type of cement	Nominal proportions or grade	Quantities of aggregates* per 50 kg of cement: Fine (dry)	Quantities of aggregates* per 50 kg of cement: Coarse (19 mm)	Strength N/mm²: Crushing: Age in days	Strength N/mm²: Crushing: Works cubes f_{cu}‡	Strength N/mm²: Modulus of rupture	Permissible stress (N/mm²): Compression: Bending f_{cr}	Permissible stress (N/mm²): Compression: Direct f_{cr}	Permissible stress (N/mm²): Shear v_d	Permissible stress (N/mm²): Bond: Anchorage: Plain bars	Permissible stress (N/mm²): Bond: Anchorage: High-bond bars	Permissible stress (N/mm²): Bond: Local: Plain bars	Permissible stress (N/mm²): Bond: Local: High-bond bars
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Nominal	Portland per BS 12 or BS 146	1 : 2 : 4	cu.m 0.07	cu.m 0.14	3 7 28	— 14 21	1.7 2.4 —	7	5.3	0.7	0.83	1.16	1.25	1.75
Nominal	Portland per BS 12 or BS 146	1 : 1½ : 3	0.05	0.10	3 7 28	— 17 25.5	1.9 2.75	8.5	6.5	0.8	0.93	1.3	1.4	1.96
	High** alumina	1 : 1 : 2	0.035	0.07	3 7 28	— 14 30	2.05 3.10 —	10	7.6	0.9	1.0	1.4	1.5	2.1
	High** alumina	1 : 2 : 4	0.07	0.14	1 3 7	40 — —	— 3.10 3.45	10	7.6	0.9	1.0	1.4	1.5	2.1
Standard†	Portland per BS 12 or BS 146	Grade / Work-ability / Slump mm	kg	kg										
Standard†	Portland per BS 12 or BS 146	A: Low 12–25 Med. 25–50 High 50–125	90 90 90	190 155 135	28	21	—	7.7	5.8	0.7	0.83	1.16	1.25	1.75
Standard†	Portland per BS 12 or BS 146	B: Low 12–25 Med. 25–50 High 50–125	80 80 80	165 135 110	28	25.5	—	9.3	7.0	0.8	0.93	1.3	1.4	1.96
Standard†	Portland per BS 12 or BS 146	C: Low 12–25 Med. 25–50 High 50–125	65 65 65	145 110 90	28	30	—	11.0	8.2	0.9	1.0	1.4	1.5	2.1
Designed	Portland per BS 12 or BS 146	A B C	Proportions determined by trial mixes (or otherwise). Cement content ≮ 240 ≯ 540 kg/m³ of compacted concrete		As for standard mixes									
Designed	Portland per BS 12 or BS 146	D E			28 28	40 50	— —	14.7 15.3	11.0 13.7	0.9 0.9	1.0 1.0	1.4 1.4	1.5 1.5	2.1 2.1
Designed	Portland per BS 12 or BS 146	Inter-mediate			28	f_{cu}	—	$\frac{f_{cu}}{2.73}$	$\frac{f_{cu}}{3.65}$	By linear proportions with respect to f_{cu}				
Designed	High** alumina	Ditto But cement content ≮ 270 ≯ 420 kg/m³			1	40 50	— —	14.7 18.3	11.0 13.7	0.9 0.9	1.0 1.0	1.4 1.4	1.5 1.5	2.1 2.1

Modifications to tabulated stresses							
Narrow beams $\frac{\text{span}}{\text{breadth}} : 30 < \frac{l}{b} \not> 60$ Reduced bending stress $= (1\frac{3}{4} - \frac{l}{40b}) f_{cr}$	Age of loading (Portland-cement concrete only)	Months: 1	2	3	6	12	Wind: If bending moments and forces include effects of wind increase stresses by 25% (Normal stresses apply for wind effects excluded.)
	Increase compressive@ stresses f_{cr} and f_{ce} by	0	10	16	20	24 per cent	

* Aggregates conforming to BS 882 or BS 1047.
** Approval for use of high-alumina cement in structural concrete has at present been withdrawn from CP 114.
† Maximum size of aggregates 19 mm and standard deviation 7 N/mm²
‡ f_{cu} at 28 days.
@ Not true under certain weather and exposure conditions.

Table 4.3 *Weights of Concrete*

Ordinary concrete (dense aggregates)				kN/m³	lb/ft³
	Non-reinforced plain or mass concrete	Nominal weight		22.6	144
		Aggregate: limestone		21.2-23.6	135-150
		gravel		22.0-23.6	140-150
		broken brick		19.6 (av.)	125 (av.)
		other crushed stone		22.8-24.4	145-155
	Reinforced concrete	Nominal weight		23.6	150
		Reinforcement: 1%		22.6-24.2	144-154
		2%		23.1-24.7	147-157
		4%		24.0-25.6	153-163
		Solid slabs	Thickness	kN/m²	lb/ft²
		(floors, walls	75 mm or 3 in	1.80	37.5
		etc.)	100 mm or 4 in	2.40	50
			150 mm or 6 in	3.6	75
			250 mm or 10 in	6.00	125
			300 mm or 12 in	7.20	150
		Ribbed slabs	125 mm or 5 in	2.00	42
			150 mm or 6 in	2.15	45
			225 mm or 9 in	2.75	57
			300 mm or 12 in	3.35	70
Lightweight concrete		Compressive strength			
	Aggregate or type	N/mm²	lb/in²	kN/m³	lb/ft³
	Clinker (1 : 8)	2.1-6.2	300-900	10.2-14.9	65-95
	Pumice (1 : 6 semi-dry)	1.4-3.8	200-550	7.1-11.0	45-70
	Formed blast-furnace slag	1.4-5.5	200-800	9.4-14.9	60-95
	ditto structural	13.8-34.5	2000-5000	16.5-20.4	105-130
	Expanded clay or shale	5.6-8.4	800-1200	9.4-11.8	60-75
	ditto structural	13.8-34.5	2000-5000	13.4-18.1	85-115
	Vermiculite (expanded mica)	0.5-3.5	70-500	3.9-11.0	25-70
	Pulverized fuel ash (sintered)	2.8-6.9	400-1000	11.0-12.6	70-80
	ditto structural	13.8-34.5	2000-5000	13.4-17.3	85-110
	No-fines (gravel)	—	—	15.7-18.9	100-120
	Cellular (aerated or gas concrete)	1.4	200	3.9 (min.)	25 (min.)
	ditto structural	10.3-15.5	1500-2250	14.1-15.7	90-100
Special concretes, etc.	Heavy concrete	Aggregates: barytes, magnatite,		31.5 (min.)	200 (min.)
		steel shot punchings		51.8	330
	Lean mixes	Dry-lean (gravel aggregate)		22.0	140
		Soil-cement (normal mix)		15.7	100
	Finishes, etc.	Rendering, screed, etc. } Granolithic, terrazzo }		N/m² per mm thick 18.9 to 23.6	lb/ft² per in thick 10 to 12.5
		Glass-block (hollow) concrete		17.0 (approx.)	9 (approx.)
	Prestressed concrete	Weights as for reinforced concrete (upper limits)			
	Air-entrained concrete	Weights as for plain or reinforced concrete			
Construction with concrete products	Concrete block and brick walls	Blockwork: 200 mm or 8 in thick		kN/m²	lb/ft²
		Stone aggregates: solid		4.31	90
		hollow		2.87	60
		Lightweight aggregates: solid		2.63	55
		hollow		2.15	45
	Weights of walls of other thickness	Cellular (aerated gas)		1.15-1.53	24-32
	pro rata	Brickwork: 120 mm or 4 1/2 in (nominal)		2.6	54
	Other products	Paving slabs (flags) 50 mm or 2 in thick		1.15	24
		Roofing tiles: plain		0.6-0.9	12.5-19
		interlocking		0.6	12.5

To convert values in kN to values in kg multiply by 102.

CHAPTER 5

Details of Structural Reinforcement Bars and Mesh Fabrics

(i) Practical details of reinforcement for design and construction purposes, as per ACI practice, are given at the end of App. 6.

(ii) Properties of and normal stresses allowed in reinforcement bars as per BS are presented in Tables 5.1 and 5.2.

(iii) Design aids (e.g. section areas, perimeters, weights, section areas for various bar arrangements, etc.) pertinent to reinforcement bars and mesh-fabrics are presented in Tables 5.3 to 5.9.

Table 5.1 *Reinforcement: Properties and Stresses*

Type of reinforcement				Minimum tensile properties per specified BS		Normal permissible stresses per CP 114 (BS)			
Type of bar			Size	Yield or characteristic strength f_y		Tensile		Compressive	
				N/mm^2	lb/in^2	N/mm^2	lb/in^2	N/mm^2	lb/in^2
Hot rolled bars	Plain round mild-steel bars	Metric BS 4449	mm						
			$\ngtr$ 40	250	—	140	—	125	—
			> 40	250	—	125	—	110	—
		Imperial BS 785	in						
			$\ngtr$ 1 1/2	—	36,000	—	20,000	—	18,000
			> 1 1/2	—	33,000	—	18,000	—	16,000
	Deformed high-yield bars	Metric BS 4449	mm						
			$\ngtr$ 16	460	—	230*	—	175*	—
			16 to 20	425	—	230*	—	—	—
			> 20	425	—	210*	—	175*	—
		Imperial BS 785	in						
			$\ngtr$ 7/8	—	60,000	—	33,000*	—	25,000*
			> 7/8	—	60,000	—	30,000*	—	25,000*
Cold worked bars (including twisted square bars)		Metric BS 4461	mm						
			$\ngtr$	460	—	230*	—	175*	—
			> 16 to 20	425	—	230*	—	175*	—
			> 20	425	—	210*	—	175*	—
		Imperial BS 1144	in						
			$\ngtr$ 5/8	—	66,000*	—	33,000*	—	25,000*
			5/8 to 7/8	—	60,000	—	33,000*	—	25,000*
			> 7/8	—	60,000	—	30,000*	—	25,000*
Hard drawn mild-steel wire		Metric BS 4482	$\ngtr$ 12 mm	485	—	230	—	—	—
		Imperial BS 785	all sizes	—	70,000	—	—	—	—

* $\ngtr 0.55 f_y$

Table 5.2 *Reinforcement Bond BS CP 110 Requirements (Ultimate Values)*

Anchorage bond: minimum lengths in millimetres for normal-weight concrete

Dia of bar mm	Minimum* 12ϕ mm	Minimum Lap† Tension mm	Minimum Lap† Comp. mm	Type of hook		f_{cu} = 20 N/mm²			f_{cu} = 25 N/mm²			f_{cu} = 30 N/mm²			f_{cu} = 40 N/mm²		
						f_y = 250	f_y = 425/460 Type 1	Type 2	f_y = 250	f_y = 425/460 Type 1	Type 2	f_y = 250	f_y = 425/460 Type 1	Type 2	f_y = 250	f_y = 425/460 Type 1	Type 2
6	75	300	270	Tension	0	275	355	275	235	320	245	220	275	210	175	235	180
					90°	225	285	200	185	245	175	170	205	140	125	160	110
					180°	180	210	130	140	175	100	125	130	70	80	90	35
				Compression		200	240	185	175	210	165	155	190	145	130	160	125
8	100	350	310	Tension	0	365	475	365	315	425	325	290	365	280	230	310	240
					90°	300	375	270	250	330	230	230	270	185	165	215	140
					180°	235	280	170	185	230	135	165	175	90	105	120	45
				Compression		265	320	245	235	280	215	210	250	190	175	210	165
10	120	400	350	Tension	0	455	590	455	390	530	405	365	455	350	290	385	300
					90°	375	470	335	310	410	285	285	335	230	210	265	180
					180°	295	350	215	230	290	165	205	215	110	130	145	60
				Compression		330	400	310	290	350	270	260	310	240	215	265	205
12	145	450	390	Tension	0	545	710	545	470	635	490	435	550	420	345	465	360
					90°	450	565	400	370	490	345	340	405	280	250	320	215
					180°	355	420	255	275	345	200	245	260	135	155	175	70
				Compression		395	480	370	350	420	325	310	375	285	260	315	245
16	195	550	470	Tension	0	725	945	725	625	845	650	580	730	560	460	620	475
					90°	600	750	535	495	655	460	455	540	370	330	425	285
					180°	470	560	340	370	460	265	325	345	180	205	235	90
				Compression		525	635	490	465	560	430	415	495	380	345	420	325
20	240	650	550	Tension	0	910	1.090	840	780	975	750	725	840	650	575	715	550
					90°	750	850	600	620	735	510	565	600	410	415	475	310
					180°	590	610	360	460	495	270	405	360	170	255	235	70
				Compression		655	745	575	580	650	500	520	580	445	430	490	375
25	300	775	650	Tension	0	1.135	1.360	1.050	975	1.220	940	910	1.050	810	720	890	685
					90°	935	1.060	750	775	920	640	710	750	510	520	590	385
					180°	735	760	450	575	620	340	510	450	210	320	290	85
				Compression		820	930	715	725	815	625	645	725	560	535	610	470
32	385	950	790	Tension	0	1.450	1.740	1.340	1.245	1.560	1.200	1.160	1.345	1.035	920	1.140	875
					90°	1.195	1.360	955	990	1.175	815	905	960	650	660	755	495
					180°	940	975	570	735	790	430	650	580	270	405	370	110
				Compression		1.050	1.190	915	925	1.040	800	830	925	715	685	780	600

* Minimum stopping-off length $= 12\phi$ or d whichever is greater.

† Minimum lap in tension: The greater of $25\phi + 150$ mm or anchorage length of smaller bar (mild steel) or 1½ times anchorage length of smaller bar (high-yield steel).
Minimum lap in compression: The greater of $20\phi + 150$ mm *or* anchorage length of smaller bar.

NOTE

1. $f_y = 250$ indicates mild steel. $f_y = 425/460$ indicates high yield bars.
2. All lengths rounded to 5 mm value above exact figure.
3. Values for hooks correspond to internal radius of 2 ϕ for mild steel bars and 3 ϕ for high-yield steel bars.
4. Bar must extend a minimum distance of 4 ϕ beyond bend.
5. Lengths given correspond to maximum design stresses in steel of $0.87f_y$ in tension and $2{,}000f_y/(2{,}300+f_y)$ in compression. For lower design stresses at point beyond which anchorage is to be provided, determine length required from 'no hook' value on pro rata basis. Then if hook is provided, subtract length equal to difference between appropriate values given in table.

Table 5.3 *Reinforcement: Weights at Specified Spacings and unit Weights*

Weights of metric (millimetre) bars in kilograms per square metre

Size (mm)	Weight per m (kg)	Length per tonne (m)	Spacing of bars in millimetres									
			75	100	125	150	175	200	225	250	275	300
6	0.222	4,505	2.960	2.220	1.776	1.480	1.269	1.110	0.987	0.888	0.807	0.740
8	0.395	2,532	5.267	3.950	3.160	2.633	2.257	1.975	1.756	1.580	1.436	1.317
10	0.616	1,623	8.213	6.160	4.928	4.107	3.520	3.080	2.738	2.464	2.240	2.053
12	0.888	1,126	11.84	8.880	7.104	5.920	5.074	4.440	3.947	3.552	3.229	2.960
16	1.579	633	21.05	15.79	12.63	10.53	9.023	7.895	7.018	6.316	5.742	5.263
20	2.466	406	32.88	24.66	19.73	16.44	14.09	12.33	10.96	9.864	8.967	8.220
25	3.854	259	51.39	38.54	30.83	25.69	22.02	19.27	17.13	15.42	14.01	12.85
32	6.313	158	—	63.13	50.50	42.09	36.07	31.57	28.06	25.25	22.96	21.04
40	9.864	101	—	—	78.91	65.76	56.37	49.32	43.84	39.46	35.87	32.88

Basic weight = 0.00785 $kg/mm^2/m$
Weight per metre = $0.006165\ \phi^2$ (kg)
Weight per mm^2 at spacing s (mm) = $6.165\phi^2/s$ (kg)
ϕ = diameter of bar in millimetres

Weights of imperial (inch) bars in pounds per square foot

Size (in)	Weight per foot (lb)	Length per ton (ft)	Spacing of bars in inches												
			3	3½	4	4½	5	5½	6	7	7½	8	9	10½	12
1/4	0.1669	13,421	0.688	0.572	0.501	0.445	0.401	0.364	0.334	0.286	0.267	0.250	0.223	0.191	0.167
5/16	0.2608	8,590	1.043	0.894	0.782	0.695	0.626	0.569	0.522	0.447	0.417	0.391	0.348	0.298	0.261
3/8	0.3755	5,965	1.502	1.287	1.127	1.001	0.901	0.819	0.751	0.644	0.601	0.563	0.501	0.429	0.376
7/16	0.5111	4,383	2.044	1.752	1.533	1.363	1.227	1.115	1.022	0.876	0.818	0.767	0.681	0.584	0.511
1/2	0.6676	3,355	2.670	2.289	2.003	1.780	1.602	1.457	1.335	1.144	1.068	1.001	0.890	0.763	0.668
5/8	1.0431	2,147	4.712	3.576	3.129	2.782	2.503	2.276	2.086	1.788	1.669	1.565	1.391	1.192	1.043
3/4	1.5021	1,491	6.008	5.150	4.506	4.006	3.605	3.277	3.004	2.575	2.403	2.253	2.003	1.717	1.502
7/8	2.0445	1,096	8.178	7.010	6.133	5.452	4.907	4.461	4.089	3.505	3.271	3.067	2.726	2.337	2.044
1	2.6704	839	10.68	9.155	8.011	7.121	6.409	5.826	5.341	4.578	4.273	4.006	3.560	3.052	2.670
1⅛	3.3797	663	—	11.59	10.14	9.012	8.111	7.374	6.759	5.794	5.407	5.069	4.506	3.862	3.380
1¼	4.1724	537	—	—	12.52	11.13	10.01	9.103	8.345	7.153	6.676	6.259	5.563	4.768	4.172
1½	6.0083	373	—	—	—	16.02	14.42	13.11	12.02	10.30	9.613	9.012	8.011	6.867	6.008

Basic weight = 3.4 $lb/in^2/ft$
Weight per foot = $2.6704\ \phi^2$ (lb)
Weight per ft^2 at spacing s (in) = $32.044\phi^2/s$ (lb)
ϕ = diameter of bar (in inches)

Plain round bars The weights tabulated are basically for plain round bars.

Deformed (high-bond bars) The weights tabulated apply to deformed (high-bond) bars on uniform cross-sectional area if the specified size (effective diameter) of the bar is the diameter of a circle of the same cross-sectional area.

Twisted square bars The weights tabulated apply to small non-chamfered and larger chamfered twisted square bars if the specified size is based on 'round area' but do not apply if based on 'square area'.

Table 5.4 *Reinforcement: Combinations of Metric Bars at Specific Spacings*

Cross sectional area	Bar arrangement
94	6 @ 300
102	6 @ 275
113	6 @ 250
125	6 @ 225
130	6/8 @ 300
141	6 @ 200
142	6/8 @ 275
157	6/8 @ 250
161	6 @ 175
167	8 @ 300
174	6/8 @ 225
178	6/10 @ 300
182	8 @ 275
188	6 @ 150
194	6/10 @ 275
196	6/8 @ 200
201	8 @ 250
213	6/10 @ 250
214	8/10 @ 300
223	8 @ 225
224	6/8 @ 175
226	6 @ 125
234	8/10 @ 275
237	6/10 @ 225
251	8 @ 200
257	8/10 @ 250
261	6/8 @ 150
	10 @ 300
267	6/10 @ 200
272	8/12 @ 300
282	6 @ 100
285	10 @ 275
286	8/10 @ 225
287	8 @ 175
297	8/12 @ 275
305	6/10 @ 175
314	6/8 @ 125
	10 @ 250
319	10/12 @ 300
322	8/10 @ 200
326	8/12 @ 250
335	8 @ 150
348	10/12 @ 275
349	10 @ 225
356	6/10 @ 150
363	8/12 @ 225
368	8/10 @ 175
376	6 @ 75
	12 @ 300
383	10/12 @ 250
392	6/8 @ 100
	10 @ 200
402	8 @ 125
408	8/12 @ 200
411	12 @ 275

Cross sectional area	Bar arrangement
425	10/12 @ 225
427	6/10 @ 125
429	8/10 @ 150
448	10 @ 175
452	12 @ 250
466	8/12 @ 175
	10/16 @ 300
479	10/12 @ 200
502	8 @ 100
	12 @ 225
508	10/16 @ 275
515	8/10 @ 125
523	6/8 @ 75
	12/16 @ 300
	10 @ 150
534	6/10 @ 100
544	8/12 @ 150
547	10/12 @ 175
559	10/16 @ 250
565	12 @ 200
571	12/16 @ 275
621	10/16 @ 225
628	12/16 @ 250
	10 @ 125
638	10/12 @ 150
644	8/10 @ 100
646	12 @ 175
653	8/12 @ 125
670	8 @ 75
	16 @ 300
698	12/16 @ 225
699	10/16 @ 200
712	6/10 @ 75
	12/20 @ 300
731	16 @ 275
753	12 @ 150
766	10/12 @ 125
776	12/20 @ 275
785	12/16 @ 200
	10 @ 100
798	10/16 @ 175
804	16 @ 250
816	8/12 @ 100
854	12/20 @ 250
858	8/10 @ 75
858	16/20 @ 300
893	16 @ 225
897	12/16 @ 175
904	12 @ 125
932	10/16 @ 150
936	16/20 @ 275
949	12/20 @ 225
958	10/12 @ 100
1,005	16 @ 200
1,030	16/20 @ 250

Cross sectional area	Bar arrangement
1,047	12/16 @ 150
	10 @ 75
	20 @ 300
1,068	12/20 @ 200
1,089	8/12 @ 75
1,118	10/16 @ 125
1,130	12 @ 100
1,142	20 @ 275
1,144	16/20 @ 225
1,148	16 @ 175
1,153	16/25 @ 300
1,220	12/20 @ 175
1,256	12/16 @ 125
	20 @ 250
1,258	16/25 @ 275
1,277	10/12 @ 75
1,288	16/20 @ 200
1,340	16 @ 150
1,341	20/25 @ 300
1,383	16/25 @ 250
1,396	20 @ 225
1,398	10/16 @ 100
1,424	12/20 @ 150
1,463	20/25 @ 275
1,472	16/20 @ 175
1,507	12 @ 75
1,537	16/25 @ 225
1,570	12/16 @ 100
	20 @ 200
1,608	16 @ 125
1,610	20/25 @ 250
1,636	25 @ 300
1,709	12/20 @ 125
1,717	16/20 @ 150
1,729	16/25 @ 200
1,784	25 @ 275
1,788	20/25 @ 225
1,795	20 @ 175
1,864	10/16 @ 75
	20/32 @ 300
1,963	20 @ 250
1,976	16/25 @ 175
2,010	16 @ 100
2,012	20/25 @ 200
2,033	20/32 @ 275
2,060	16/20 @ 125
2,094	12/16 @ 75
	20 @ 150
2,136	12/20 @ 100
2,158	25/32 @ 300
2,181	25 @ 225
2,236	20/32 @ 250
2,300	20/25 @ 175
2,306	16/25 @ 150
2,354	25/32 @ 275

Cross sectional area	Bar arrangement
2,454	25 @ 200
2,485	20/32 @ 225
2,513	20 @ 125
2,576	16/20 @ 100
2,590	25/32 @ 250
2,680	16 @ 75
	32 @ 300
2,683	20/25 @ 150
2,767	16/25 @ 125
2,796	20/32 @ 200
2,804	25 @ 175
2,848	12/20 @ 75
2,878	25/32 @ 225
2,924	32 @ 275
3,141	20 @ 100
3,195	20/32 @ 175
3,216	32 @ 250
3,220	20/25 @ 125
3,237	25/32 @ 200
3,272	25 @ 150
4,473	16/20 @ 75
3,459	16/25 @ 100
3,574	32 @ 225
3,700	25/32 @ 175
3,728	20/32 @ 150
3,926	25 @ 125
4,021	32 @ 200
4,025	20/25 @ 100
4,188	20 @ 75
4,317	25/32 @ 150
4,473	20/32 @ 125
4,595	32 @ 175
4,612	16/25 @ 75
4,908	25 @ 100
5,180	25/32 @ 125
5,361	32 @ 150
5,366	20/25 @ 75
5,592	20/32 @ 100
6,433	32 @ 125
6,475	25/32 @ 100
6,544	25 @ 75
7,456	20/32 @ 75
8,042	32 @ 100

NOTE Cross-sectional areas of metric bars in mm² per m width. 10 at the rate of 75, etc., denotes 10 mm bars at 75 mm centres, etc., 10/16 at the rate of 75, etc., denotes 10 mm and 16 mm bars alternately at 75 mm centres, etc. Only combinations of bars not differing by more than two sizes and spaced at multiples of 25 mm are tabulated. All areas are rounded to value in mm² below exact value.

Table 5.5 *Reinforcement: Areas of Combinations of Metric Bars*

Cross sectional area	Bar arrangement
113	1/12
201	1/16
226	2/12
314	1/20 1/12 + 1/16
339	3/12
402	2/16
427	1/12 + 1/20 2/12 + 1/16
452	4/12
490	1/25
515	1/12 + 2/16 1/16 + 1/20
540	2/12 + 1/20 3/12 + 1/16
565	5/12
603	3/16
628	2/12 + 2/16 2/20
653	3/12 + 1/20 4/12 + 1/16
678	6/12
691	1/16 + 1/25
716	1/12 + 3/16 2/16 + 1/20
741	1/12 + 2/20 3/12 + 2/16
766	4/12 + 1/20 5/12 + 1/16
791	7/12
804	4/16 1/32
805	1/20 + 1/25
829	1/16 + 2/20 2/12 + 3/16
854	2/12 + 2/20 4/12 + 2/16
879	5/12 + 1/20
892	2/16 + 1/25
904	8/12
917	1/12 + 4/16 3/16 + 1/20
942	3/12 + 3/16 3/20
967	5/12 + 2/16 3/12 + 2/20
981	2/25
1,005	5/16
1,017	9/12
1,030	2/12 + 4/16 2/16 + 2/20
1,055	1/12 + 3/20 4/12 + 3/16
1,080	4/12 + 2/20
1,094	3/16 + 1/25
1,118	1/12 + 5/16 4/16 + 1/20 1/20 + 1/32
1,119	2/20 + 1/25
1,130	10/12
1,143	3/12 + 4/16 1/16 + 3/20
1,168	5/12 + 3/16 2/12 + 3/20
1,182	1/16 + 2/25
1,193	5/12 + 2/20
1,206	6/16
1,231	2/12 + 5/16 3/16 + 2/20
1,256	4/12 + 4/16 4/20
1,281	3/12 + 3/20
1,295	4/16 + 1/25 1/20 + 2/25 1/25 + 1/32
1,319	5/16 + 1/20
1,344	3/12 + 5/16 2/16 + 3/20
1,369	1/12 + 4/20 5/12 + 4/16
1,383	2/16 + 2/25
1,394	4/12 + 3/20
1,407	7/16
1,432	4/16 + 2/20 2/20 + 1/32
1,433	3/20 + 1/25
1,457	1/16 + 4/20 4/12 + 5/16
1,472	3/25
1,482	2/12 + 4/20
1,496	5/16 + 1/25
1,507	5/12 + 3/20
1,545	3/16 + 3/20
1,570	5/20 5/12 + 5/16
1,584	3/16 + 2/25
1,595	3/12 + 4/20
1,608	8/16 2/32
1,610	2/20 + 2/25
1,633	5/16 + 2/20
1,658	2/16 + 4/20
1,673	1/16 + 3/25
1,683	1/12 + 5/20
1,709	4/12 + 4/20
1,746	3/20 + 1/32 4/16 + 3/20
1,747	4/20 + 1/25
1,771	1/16 + 5/20
1,785	4/16 + 2/25 2/25 + 1/32
1,795	1/20 + 3/25
1,796	2/12 + 5/20
1,809	9/16
1,822	5/12 + 4/20
1,859	3/16 + 4/20
1,874	2/16 + 3/25
1,884	6/20
1,910	3/12 + 5/20
1,922	1/20 + 2/32
1,924	3/20 + 2/25
1,947	5/16 + 3/20
1,963	4/25
1,972	2/16 + 5/20
1,987	5/16 + 2/25
2,010	10/16
2,023	4/12 + 5/20
2,060	4/16 + 4/20 4/20 + 1/32
2,061	5/20 + 1/25
2,075	3/16 + 3/25
2,099	1/25 + 2/32
2,100	2/20 + 3/25
2,136	5/12 + 5/20
2,164	1/16 + 4/25
2,173	3/16 + 5/20
2,199	7/20
2,236	2/20 + 2/32
2,238	4/20 + 2/25
2,261	5/16 + 4/20
2,276	3/25 + 1/32 4/16 + 3/25
2,277	1/20 + 4/25
2,365	2/16 + 4/25
2,375	4/16 + 5/20 5/20 + 1/32
2,412	3/32
2,415	3/20 + 3/25
2,454	5/25
2,477	5/16 + 3/25
2,513	8/20
2,550	3/20 + 2/32
2,552	5/20 + 2/25
2,566	3/16 + 4/25
2,576	5/16 + 5/20
2,590	2/25 + 2/32
2,591	2/20 + 4/25
2,655	1/16 + 5/25
2,726	1/20 + 3/32
2,729	4/20 + 3/25
2,767	4/25 + 1/32 4/16 + 4/25
2,768	1/20 + 5/25
2,827	9/20
2,856	2/16 + 5/25
2,865	4/20 + 2/32
2,903	1/25 + 3/32
2,905	3/20 + 4/25
2,945	6/25
2,968	5/16 + 4/25
3,041	2/20 + 3/32
3,043	5/20 + 3/25
3,057	3/16 + 5/25
3,081	3/25 + 2/32
3,082	2/20 + 5/25
3,141	10/20
3,179	5/20 + 2/32
3,216	4/32
3,220	4/20 + 4/25
3,258	5/25 + 1/32 4/16 + 5/25
3,355	3/20 + 3/32
3,394	2/25 + 3/32
3,436	7/25
3,459	5/16 + 5/25
3,531	1/20 + 4/32
3,534	5/20 + 4/25
3,571	4/25 + 2/32
3,669	4/20 + 3/32
3,707	1/25 + 4/32
3,711	4/20 + 5/25
3,845	2/20 + 4/32
3,885	3/25 + 3/32
3,926	8/25
3,983	5/20 + 3/32
4,021	5/32
4,025	5/20 + 5/25
4,062	5/25 + 2/32
4,159	3/20 + 4/32
4,198	2/25 + 4/32
4,335	1/20 + 5/32
4,376	4/25 + 3/32
4,417	9/25
4,473	4/20 + 4/32
4,512	1/25 + 5/32
4,649	2/20 + 5/32
4,689	3/25 + 4/32
4,787	5/20 + 4/32
4,825	6/32
4,867	5/25 + 3/32
4,908	10/25
4,963	3/20 + 5/32
5,002	2/25 + 5/32
5,180	4/25 + 4/32
5,277	4/20 + 5/32
5,493	3/25 + 5/32
5,592	5/20 + 5/32
5,629	7/32
5,671	5/25 + 4/32
5,984	4/25 + 5/32
6,433	8/32
6,475	5/25 + 5/32
7,238	9/32
8,042	10/32

NOTE Cross-sectional areas of metric bars in mm^2. 4/16 + 3/25, etc. denotes combination of four 16 mm bars plus three 25 mm bars, etc. Only combinations of up to five bars of two diameters differing by not more than two sizes (or ten bars of a single size) are considered. All areas are rounded to value in mm^2 below exact value.

Table 5.6 *Reinforcement: Metric Bar Data*

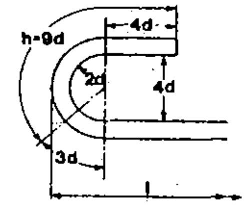
			Bar size in millimetres									
			6	8	10	12	16	20	25	32	40	50
Cross sectional area of bars/m at specific spacings	Bar spacing in millimetres (non-preferred spacings shown in italics)	*75*	*376*	670	1,047	1,507	2,680	4,188	6,544	—	—	—
		80	*353*	*628*	*981*	*1,413*	*2,513*	*3,926*	*6,135*	—	—	—
		90	*314*	*558*	*872*	*1,256*	*2,234*	*3,490*	*5,454*	—	—	—
		100	282	502	785	1,130	2,010	3,141	4,908	8,042	—	—
		110	*257*	*456*	*713*	*1,028*	*1,827*	*2,855*	*4,462*	*7,311*	—	—
		120	*235*	*418*	*654*	942	*1,675*	*2,617*	*4,090*	6,702	*10,471*	—
		125	226	402	628	904	1,608	2,513	3,926	6,433	10,053	—
		130	*217*	*386*	*604*	*869*	*1,546*	*2,416*	*3,775*	*6,186*	*9,666*	—
		140	*201*	*359*	*560*	*807*	*1,436*	*2,243*	*3,506*	*5,744*	*8,975*	—
		150	188	335	523	753	1,340	2,094	3,272	5,361	8,377	13,090
		160	*176*	*314*	*490*	*706*	*1,256*	*1,963*	*3,067*	*5,026*	*7,853*	*12,272*
		175	161	287	448	646	1,148	1,795	2,804	4,595	7,180	11,220
		180	*157*	*279*	*436*	*628*	*1,117*	*1,745*	*2,727*	*4,468*	*6,981*	*10,908*
		200	141	251	392	565	1,005	1,570	2,454	4,021	6,283	9,817
		220	*128*	*228*	*356*	*514*	*913*	*1,427*	*2,231*	*3,655*	*5,711*	*8,925*
		225	125	223	349	502	893	1,396	2,181	3,574	5,585	8,727
		240	*117*	*209*	*327*	*471*	*837*	*1,308*	*2,045*	*3,351*	*5,235*	*8,181*
		250	113	201	314	452	804	1,256	1,963	3,216	5,026	7,854
		275	102	182	285	411	731	1,142	1,784	2,924	4,569	7,140
		300	94	167	261	376	670	1,047	1,636	2,680	4,188	6,545
	Number of bars	1	28.3	50.3	78.5	113.1	201.1	314.2	490.9	804.2	1,257	1,963
		2	56.5	100.5	157.1	226.2	402.1	628.3	981.7	1,608	2,513	3,927
		3	84.8	150.8	235.6	339.3	603.2	942.5	1,473	2,413	3,770	5,890
		4	113.1	201.1	314.2	452.4	804.2	1,257	1,963	3,217	5,027	7,854
		5	141.4	251.3	392.7	565.5	1,005	1,571	2,454	4,021	6,283	9,817
		6	169.6	301.6	471.2	678.6	1,206	1,885	2,945	4,825	7,540	11,781
		7	197.9	351.9	549.8	791.7	1,407	2,199	3,436	5,630	8,796	13,744
		8	226.2	402.1	628.3	904.8	1,608	2,513	3,927	6,434	10,053	15,708
		9	254.5	452.4	706.9	1,018	1,810	2,827	4,418	7,238	11,310	17,671
		10	282.7	502.7	785.4	1,131	2,011	3,142	4,909	8,042	12,566	19,635
		11	311.0	552.9	863.9	1,244	2,212	3,456	5,400	8,847	13,823	21,598
		12	339.3	603.2	942.5	1,357	2,413	3,770	5,890	9,651	15,080	23,562
		13	367.6	653.5	1,021	1,470	2,614	4,084	6,381	10,455	16,336	25,525
		14	395.8	703.7	1,100	1,583	2,815	4,398	6,872	11,259	17,593	27,489
		15	424.1	754.0	1,178	1,696	3,016	4,712	7,363	12,064	18,850	29,452
		16	452.4	804.2	1,257	1,810	3,217	5,027	7,854	12,868	20,106	31,416
		17	480.7	854.5	1,335	1,923	3,418	5,341	8,345	13,672	21,363	33,379
		18	508.9	904.8	1,414	2,036	3,619	5,655	8,836	14,476	22,619	35,343
		19	537.2	955.0	1,492	2,149	3,820	5,969	9,327	15,281	23,876	37,306
		20	565.5	1,005	1,571	2,262	4,021	6,283	9,817	16,085	25,133	39,270
Perimeters of specific numbers of bars	Number of bars	1	18.8	25.1	31.4	37.6	50.2	62.8	78.5	100.5	125.6	157.1
		2	37.6	50.2	62.8	75.3	100.5	125.6	157.0	201.0	251.3	314.2
		3	56.5	75.3	94.2	113.0	150.7	188.4	235.6	301.5	376.9	471.2
		4	75.3	100.5	125.6	150.7	201.0	251.3	314.1	402.1	502.6	628.3
		5	94.2	125.6	157.0	188.4	251.3	314.1	392.6	502.6	628.3	785.4
		6	113.0	150.7	188.4	226.1	301.5	376.9	471.2	603.1	753.9	942.5
		7	131.9	175.9	219.9	263.8	351.8	439.8	549.7	703.7	879.6	1,100
		8	150.7	201.0	251.3	301.5	402.1	502.6	628.3	804.2	1,005	1,257
		9	169.6	226.1	282.7	339.2	452.3	565.4	706.8	904.7	1,130	1,414
		10	188.4	251.3	314.1	376.9	502.6	628.3	785.3	1,005	1,256	1,571

NOTE Areas are given in square millimetres, perimeters in millimetres

Table 5.7 *BS4466 Preferred shapes*

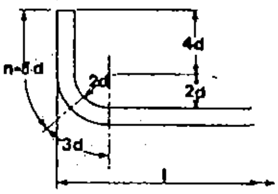
MINIMUM HOOK AND BEND ALLOWANCES FOR MILD STEEL BARS TO BE BS4449*

Semi-circular hooks for use with shape codes 32, 33 and 72 only



h = hook allowance = 9*d* (min.) taken to the nearest 10 mm over, or not less than 100 mm, to be added to dimension *L*. Hook length (min.) = *h* + 3*d*.

Bends forming end anchorages for use with shape codes 34, 35 and 42 only



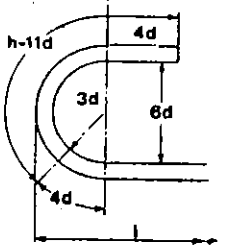
n = bend allowance = 5*d* (min.) taken to the nearest 10 mm over or not less than 100 mm, to be added to dimension *L*.

Bar size (mm)	*d*	6	8	10	12	16	20	25	32	40
Hook allowance (mm)	*h*	100	100	100	110	150	180	230	290	360
Bend allowance (mm)	*n*	100	100	100	100	100	100	130	160	200

NOTE. For intermediate sizes the dimensions and radii for the next larger size should be used.

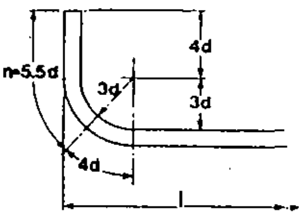
MINIMUM HOOK AND BEND ALLOWANCES FOR HOT ROLLED HIGH YIELD BARS COMPLYING WITH BS4449* AND COLD WORKED HIGH YIELD BARS COMPLYING WITH BS4461†

Semi-circular hooks for use with shape codes 32, 33 and 72 only



h = hook allowance = 11*d* (min.) taken to the nearest 10 mm over, or not less than 100 mm to be added to dimension *L*. Hook length (min.) = *h* + 4*d*.

Bends forming end anchorages for use with shape codes 34, 35 and 42 only

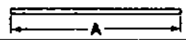
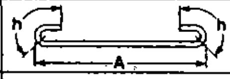
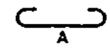
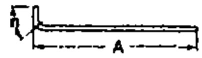
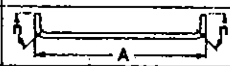
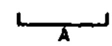
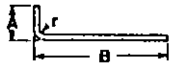
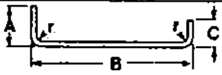
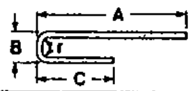
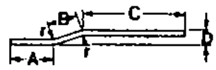
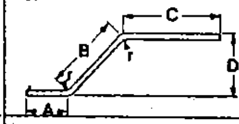
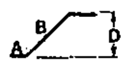
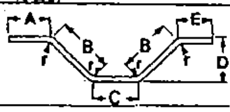
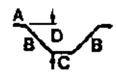
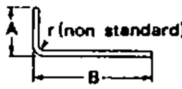
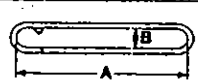
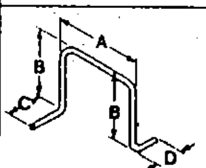


n = bend allowance = 5.5*d* (min.) taken to the nearest 10 mm over, or not less than 100 mm to be added to dimension *L*.

Bar size (mm)	*d*	6	8	10	12	16	20	25	32	40
Hook allowance (mm)	*h*	100	100	110	140	180	220	280	360	440
Bend allowance (mm)	*n*	100	100	100	100	100	110	140	180	220

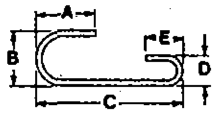
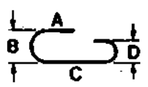
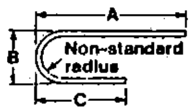
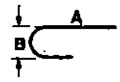
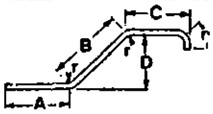
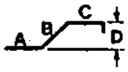
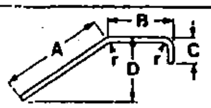
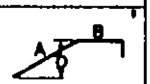
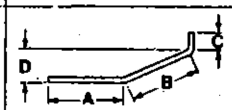
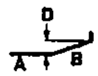
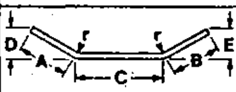
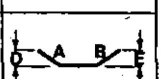
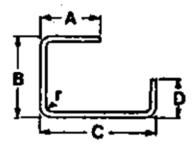
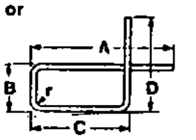
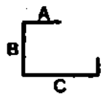
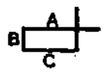
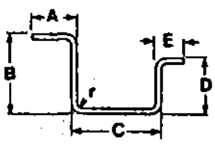
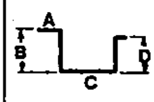
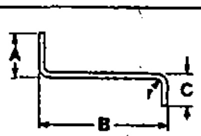
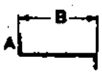
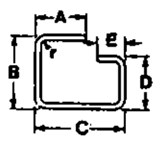
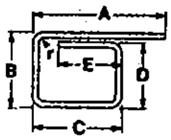
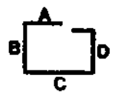
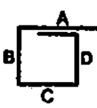
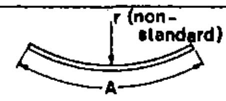
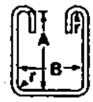
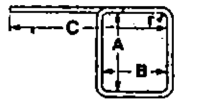
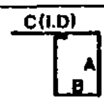
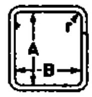
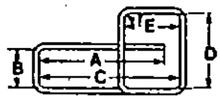
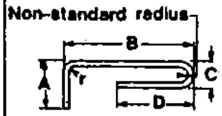
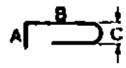
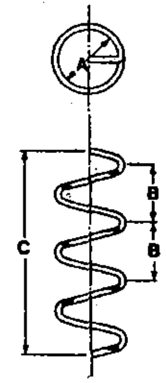
NOTE. For intermediate sizes the dimensions and radii for the next larger size should be used.

* BS4449, "Hot rolled steel bars for the reinforcement of concrete". (Metric units).
† BS4461, "Cold worked steel bars for the reinforcement of concrete". (Metric units)

Shape code	Method of measurement of bending dimensions	Total length of bar (*L*) measured along centre line	Dimensions to be given in schedule
20		A	Straight
32		A+h	
33		A+2h	
34		A+n	
35		A+2n	
37		If *r* is non-standard use shape code 51 A+B-½r-d	
38	 or 	A+B+C-r-2d A+B+C-r-2d	 or 
41	 D shall be at least 2d or 	A+B+C	 or 
43		If angle with horizontal is 45° or less A+2B+C+E	
51		If *r* is standard use shape code 37 A+B-½r-d	
60		2(A+B)+20d	
62		If angle with horizontal is 45° or less A+C	
81		2A+3B+22d	
83	 Isometric view	A+2B+C+D-2r-4d	 Isometric view

INSTRUCTION. Generally an inside or outside dimension shall be indicated by the position of the bending dimensions in the sketch. If the form of the bar is such that there may be doubt as to which is the inside of the bar, arrows should be shown on the bending schedule or the dimension stated with the suffix O.D. or I.D. (outside or inside dimension).

Table 5.8 *BS4466 Other shapes*

Shape code	Method of measurement of bending dimensions	Total length of bar (L) measured along centre line	Dimensions to be given in schedule
36		(A+C+E)+ 0.57(B+D)−3.14d where d = size of bar	
39		A+0.57B+C− 1.57d	
42		If angle with horizontal is 45° or less A+B+C+n	
45		A+B+C−½r−d	
48		A+B+C	
49		If angle with horizontal is 45° or less A+B+C	
52	 or 	A+B+C+D− 1½r−3d	 or 
53		A+B+C+D+ E−2r−4d	
54		A+B+C−r−2d	
58	 or 	A+B+C+D+E− 2r−4d	 or 
65		A	
72	 These dimensions shall be assumed to be internal unless noted otherwise by the use of arrows or the suffix "O.D.".	2A+B+20d	
73	 These dimensions shall be assumed to be internal unless noted otherwise by the use of arrows or the suffix "O.D.". Generally an inside or outside dimension shall be indicated by the position of the bending dimensions in the sketch. If the form of the bar is such that there may be doubt as to which is the inside of the bar, arrows should be shown on the bending schedule or the dimension stated with the suffix O.D. or I.D. (outside or inside dimension).	2A+B+C+10d	
74	 The dimensions shall be assumed to be internal unless noted otherwise by the use of arrows or the suffix "O.D.".	2A+3B+20d	
75	 These dimensions shall be assumed to be internal unless noted otherwise by the use of arrows or the suffix "O.D.". Generally an inside or outside dimension shall be indicated by the position of the bending dimension in the sketch. If the form of the bar is such that there may be doubt as to which is the inside of the bar, arrows should be shown on the bending schedule or the dimension stated with the suffix O.D. or I.D. (outside or inside dimension).	A+B+C+2D+ E+10d	
85	Non-standard radius 	A+B+0.57C+D− ½r−2.57d	
86		Where B is not greater than A/5 $\frac{C}{B}\pi(A+d)+8d$ Where d=size of bar	Helix A=Internal dia B=Pitch of helix C=Overall height of helix Dimensions (mm)
99	All other shapes		A dimensioned sketch of the shape shall be given on the schedule

r = standard radius of bend unless otherwise stated

Table 5.9 *Reinforcement: Fabrics and Wire*

	Type of fabric	Size of mesh (mm× mm)	BS ref. no.	Weight (kg/m²)	Main wires: Size (mm)	Main wires: Cross-sectional area (mm²/m)	Transverse wires: Size (mm)	Transverse wires: Cross-sectional area (mm²/m)	Notes
Fabrics	Square mesh	200 × 200	A98†	1.54	5	98	5	98	Fabrics made in either hard-drawn wire (BS 4482) or cold-worked bars (BS 4461) *Exceptions*: A98 and all long-mesh fabrics and main wires of B196: plain hard drawn wire only. *Preferred sizes* Sheets: 2.4 m wide 4.8 m long Rolls: 2.4 m wide 48 m long (indicated thus*) or 72 m long (indicated thus†)
			A142*	2.22	6	142	6	142	
			A193*	3.02	7	193	7	193	
			A252	3.95	8	252	8	252	
			A393	6.16	10	393	10	393	
	Structural	100 × 200	B196†	3.05	5	196	7	193	
			B283*	3.73	6	283			
			B385*	4.53	7	385			
			B503	5.93	8	503	8	252	
			B785	8.14	10	785			
			B1131	10.90	12	1131			
	Long mesh	100 × 400	C283	2.61	6	283	5	49	
			C385*	3.41	7	385			
			C503	4.34	8	503			
			C636	5.55	9	636	6	71	
			C785	6.72	10	785			
	Wrapping	100 × 100	D31	0.49	2	31	2	31	Stock sheets
			D49	0.77	2.5	49	2.5	49	2.4 m × 1.2 m

Wire		SWG. No	6g	5g	4g	3g	2g	1g	1/0g	2/0g	3/0g	4/0g	5/0g
	Size	in	0.192	0.212	0.232	0.252	0.276	0.300	0.324	0.348	0.372	0.400	0.432
		mm	4.9	5.4	5.9	6.4	7.0	7.6	8.2	8.8	9.5	10.2	11.0
	Cross sectional area	in²	0.029	0.035	0.042	0.050	0.060	0.071	0.082	0.095	0.109	0.126	0.146
		mm²	19	23	27	32	39	46	53	61	70	81	95

CHAPTER 6

Details of Prestressing-steel, Tendons and Anchorages*

6.1 TYPES OF STEEL

Steel for prestressed concrete must have high tensile strength and adequate ductility. These qualities are found in

(i) carbon or alloy steel, hot rolled, but otherwise untreated,
(ii) cold worked steel, which is drawn or deformed, and preferably tempered, and
(iii) hot rolled and tempered steel.

Carbon or alloy steel has a carbon content not greater than 1% and this is mainly responsible for its high tensile strength. Alloying elements such as manganese, nickel and chromium may also be added to improve the mechanical properties of the steel, and various heat treatments have a beneficial effect. These treatments make use of the fact that if steel is heated to a temperature higher than about 850°C (1550°F) (termed the *transformation temperature*), its final structure and the extent to which its properties are improved depend on the rate of cooling. If the steel is cooled slowly from its transformation temperature, the treatment is termed *annealing*: if it is allowed to cool from the transformation temperature at its normal rate, the treatment is termed *normalizing*. If, on the other hand, steel is suddenly cooled from above the transformation temperature to room temperature by immersion or *quenching* in oil, its hardness and brittleness are appreciably increased. Quenching is usually followed by a tempering process in which the steel is reheated to about 400°C (750°F) and allowed to cool in air. This reduces the brittleness of the steel. If the steel is rapidly cooled from above the transformation temperature to about 450°C (850°F) and then allowed to cool slowly to room temperature the process is termed *patenting*, and has an effect similar to that of quenching and tempering. The term *stress relieving* is used to describe heat treatment for a prolonged period at about 260°C (500°F) or a short period at about 500°C (950°F). The term *stabilizing* denotes heat treatment at about 400°C (750°F) combined with a tensile stress of about 65% of the ultimate strength of the steel.

Cold working of steel increases its strength, and is mainly carried out by drawing wire through a series of dies, with progressive reductions in the diameter of each die, and consequently of the wire. Rolling may also be used to produce the same result. Rolling, whether hot or cold, enables the steel to be deformed or indented, if required.

It is usual to apply heat treatment to all prestressing steels except those of natural hardness, and secret or proprietary processes are often used. Ordinary oil quenching is generally considered to be unsatisfactory. Other methods, termed *mar tempering* are used and cooling is often carried out in lead, salt or oil baths.

Although the effects of the foregoing processes are known qualitatively, the actual properties of any steel can be determined only by tests. It is essential that sufficient satisfactory data should be available before any type of steel is used for prestressing.

Strength of Prestressing Steel

The practice of specifying a minimum strength for prestressing steel has been superseded, in recent British Standards and Codes of Practice, by the concept of a *characteristic strength* defined as that value below which not more than 5% of test results fall. Typical values are 210 000–240 000 lbf/in^2 (14 800–16 900 kgf/cm^2; 1450–1660 N/mm^2) for wire; 240 000–260 000 lbf/in^2 (16 900–18 300 kgf/cm^2; 1660–1800 N/mm^2) for strand; and 150 000 lbf/in^2 (10 550 kgf/cm^2; 1040 N/mm^2) for alloy bar.

Stress-Strain Relationship

An ideal stress-strain diagram for prestressing steel is shown in Fig. 6.1 which meets the following requirements:

(i) It is imperative to have a high tensile stress which must be accompanied by only a small amount of creep. This is achieved if the permanent elongation at the working stress is small, and the type of steel for which the stress-strain diagram is linear for a large proportion of the ultimate load is used. This property is measured by the *proof stress* which

* The author wishes to gratefully acknowledge with thanks P.W. Ables, B. Roy et al. from whose monumental book on *Prestressed Concrete Design* the material in this chapter has been taken for compact presentation, material that is otherwise well known and exists already in standard references on the subject.

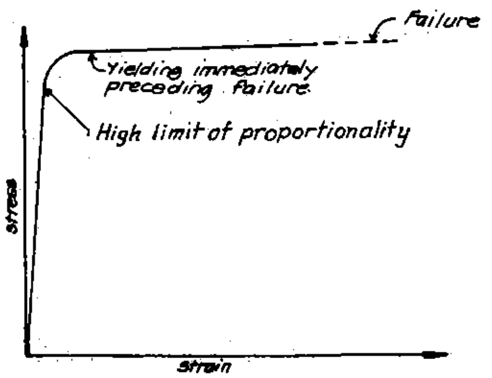
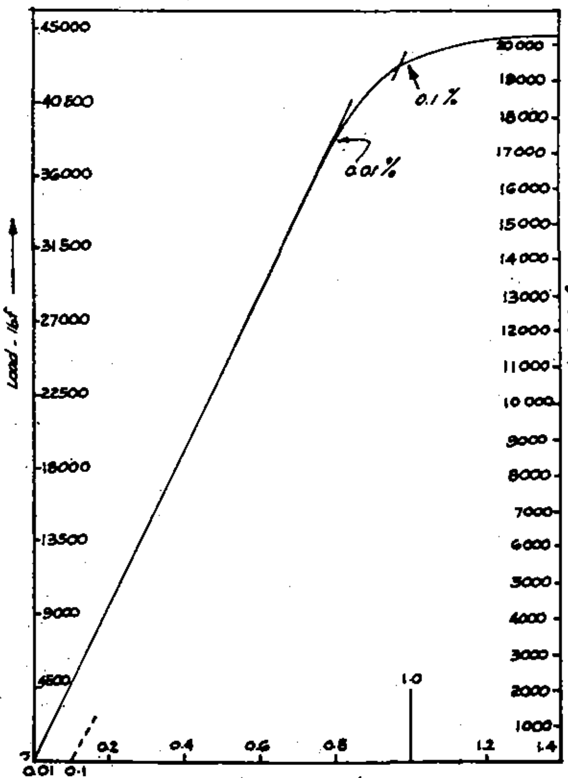


Fig. 6.1 *Ideal stress-strain diagram for prestressing steel*

is defined as the stress which produces a certain permanent deformation (usually 0.2% but sometimes 0.1%) on first loading, and a steel which is suitable for prestressing should have a high proof stress (Fig. 6.2).



0.5 in (12.70 mm) diameter Dyform strand; 0.01% off-set 38.250 lbf (17.350 kgf); 0.1% off-set 42,800 lbf (19,414 kgf); Load at 1% ext. 43.100 lbf (19,550 kgf); Breaking load 48,000 lbf (21,772 kgf); Modulus of elasticity; 27.88 × 10^6 lbf/in^2 (19,602 kgf/mm^2) Area = 0.174 in^2 (112.25 mm^2)

Fig. 6.2 *Load extension diagram (proof load)*

(ii) It is also most desirable that an ultimate elongation of appreciable magnitude should be obtained in order to reduce as much as possible the chance of sudden fracture; this may occur, for example, with piano wire, which has a very small elongation at failure. Prestressing wire and strand have a minimum elongation of between 3 and 5%, which is quite sufficient with satisfactory bond; the value for alloy bars is about 10%.

The stress-strain diagrams for various types of steel in Fig. 6.3 indicate that the ultimate elongation tends to decrease as the ultimate strength increases. It is clear, therefore, that piano wire is not entirely suitable for prestressing, despite its high proof stress, as its ultimate elongation is very limited. On the other hand, mild steel and deformed bars, which have a large ultimate elongation, are unsuitable because of their low yield point or proof stress. Figure 6.3 also shows that a distinct yield point occurs in low-alloy bars, and this influences the ultimate strength of structures with bonded steel in which the steel is the weaker part and failure is initiated by its excessive deformation in some cases by its fracture.

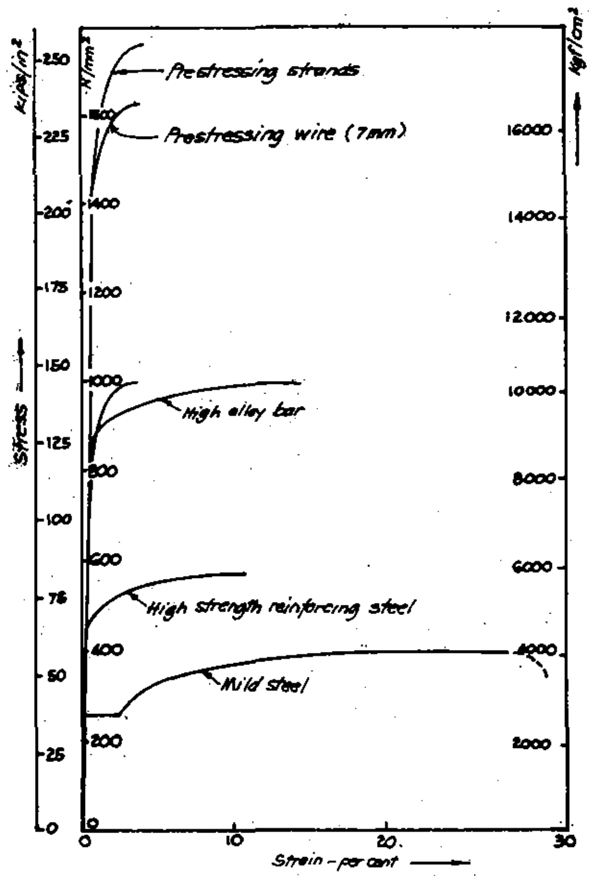


Fig. 6.3 *Stress-strain diagram for various steels*

The modulus of elasticity for prestressing steel depends on the type of steel employed, and values should be obtained from the supplier of the steel. Typical values are 25×10^6 lbf/in^2 (1.76×10^6 kgf/cm^2; 0.173×10^6 N/mm^2) for low-alloy bars, 28×10^6 lbf/in^2 (1.97×10^6 kgf/cm^2; 0.194×10^6 N/mm^2) for carbon steel wires, and between 23.5×10^6 and 29×10^6 lbf/in^2 (1.65×10^6 kgf/cm^2; $0.163 \times 10^6 - 0.2 \times 10^6$ N/mm^2) for strands. A typical load-extension curve for strands remains linear for only about a half of its length, and a typical 0.2 per cent proof stress is between 85% and 95% of the breaking load.

6.2 PRESTRESSING TENDONS

Prestressing tendons normally take the form of separate wires, wires spun together helically to form strands, or bars. For pre-tensioned steel, wires, strands, and occasionally bars are used singly, to permit the concrete to bond directly to them; when post-tensioning is used, it is common practice to group the separate tendons together, so as to reduce the number of anchorages and ducts required to accommodate them. When grouped in this way, the tendons in each duct are usually termed a *cable*.

British Standards for Tendons

The requirements for prestressing tendons are set out in the following British Standards:

BS 2691:1969: Steel wire for prestressed concrete
BS 3617:1971: Stress relieved seven-wire steel strand for prestressed concrete
BS 4757:1971: Nineteen-wire steel strand for prestressed concrete
BS 4486:1969: Cold worked high tensile alloy steel bars for prestressed concrete

Wires and Strands

The wire is required to be cold drawn from plain carbon steel (BS 2691) or patented plain carbon steel (BS 3617 and 4486); the chemical composition is shown in Table 6.1.

Table 6.1 *Chemical Composition of Alloying Elements of Prestressing Wires and Strands*

Element	Minimum %	Maximum %
Carbon	0.60	0.90
Silicon	0.10	0.35
Manganese	0.50	0.90
Sulphur	—	0.05
Phosphorus	—	0.05

The drawn wire is to be free from surface or other defects, and the finished wire or strand must be free from oil and grease unless otherwise specified by the purchaser. Superficial rusting is allowed, provided that there is no visible pitting of the surface. In the case of *wire (BS 2691)*, the coils supplied to the purchaser must not include welds; except that, by agreement between the buyer and the supplier, special lengths may be drawn from rods welded before the patenting process is applied. For *strands*, no length of strand may be joined to another by any method, though separate wires within the strand may be welded together prior to patenting. No welding is allowed after patenting or during or after wire drawing. If special lengths of seven-wire strand (BS 3617) are required, and provided the user is made fully aware of the reduced mechanical properties involved. not more than one wire in any 40 m (130 ft) may be welded after patenting or drawing; this relaxation applies to seven-wire strand only.

The tolerance on the nominal diameter of *prestressing wire (BS 2691)* is ±0.025 mm (±0.001 in.) for wires under 2.5 mm (0.104 in.) in diameter, and ±0.050 mm (±0.002 in.) for wires of 2.5 mm (0.104 in.) or more in diameter. If the wire is to pay out straight from the coil, the internal diameter of the coil shall not be less than 1.8 m (6 ft) for wires of 7 mm (0.276 in.) diameter or greater, or 1.2 m (4 ft) for wires less than 7 mm (0.276 in.) in diameter. In *seven-wire strands (BS 3617)* the nominal diameter of the centre wire is to be at least 2% greater than that of the surrounding wires; after heat treatment, it is to be wound onto coils of such a size [and not less than 600 mm (2 ft) in any case] that it pays off reasonably straight. The tolerances on the nominal diameter of the finished strand are + 0.4 mm (+ 0.016 in.) and −0.2 mm (− 0.008 in.) in all cases. For *nineteen-wire strand (BS 4757)*, different requirements are laid down for treated and 'as-spun' strands. A treated strand has a nominal diameter of 18 mm (0.725 in.) with tolerances on diameter of ±0.5 mm (±0.02 in.) and −0.25 mm (−0.01 in.) The treatment comprises low-temperature heating as a continuous linear process, after which it is to be wound onto coils with a minimum diameter of 900 mm (3 ft), from which it pays off 'substantially straight'. 'As-spun' strands, with nominal diameters of 25.4 mm (1 in.), 28.6 mm (1.125 in.) and 31.8 mm (1.25 in.) have tolerances on diameter of + 0.6 mm (+ 0.024 in.) and −0.25 mm (− 0.01 in.) in all cases; no heat treatment is required, and the minimum coil diameter is 1.5 m (5 ft).

Testing

In the case of *wire (BS 2691)* the manufacturer is required to provide one load/extension curve for each parcel of wire, a parcel being defined as any quantity of finished wire presented for testing at any one time. Tests are to be made on samples taken from the end of one coil in every five within the parcel, but the results of these are only required to be kept available for inspection by the purchaser.

Table 6.2 *Mechanical Properties of Wires (BS 2691)*

Nominal wire diameter		Specified characteristic strength			Reverse bend radius		Conditions in which wire supplied (see below)
mm	in.	N/mm^2	kgf/mm^2	lbf/in^2	mm	in.	
7	0.276	1470	150	214000	20	0.8	1,2
*7	0.276	1570	160	228000	20	0.8	1,2
*5	0.197	1570	160	228000	15	0.6	1,2,3
5	0.197	1720	175	248000	15	0.6	1,2,3
4.5	0.177	1620	165	235000	15	0.6	2,3,
*4	0.1575	1720	175	248000	12.5	0.5	1,2,3
*3.25	0.128	1720	175	248000	10	0.4	3
3.25	0.128	1870	190	270000	10	0.4	3
*3	0.118	1720	175	248000	10	0.4	3
2.65	0.104	1870	190	270000	7.5	0.3	3
2	0.079	2020	205	291000	5	0.2	3

Conditions in which wire is supplied:

Description	Number		
	1	2	3
	Cold drawn, pre-straightened, normal relaxation	Cold drawn, pre-straightened, low relaxation	Cold drawn
0.2% Proof stress, as percentage of specified characteristic strength	85	90	75
Maximum relaxation after 1000 hours from:			
70% Initial stress	5%	2%	—
80% Initial stress	8.5%	3%	—

* Preferred specified characteristic strengths.

Specimens are tested for characteristic strength, proof stress, and reverse bend tests; relaxation tests may also be called for. The specified values are shown in Table 6.2; the wire is deemed to comply with the requirements for specified characteristic strength provided that not more than two of any 40 consecutive results fall below the specified value, no results are less than 95% of the specified value, and none are more than 230 N/mm^2 (24 kgf/mm^2; 33 600 lbf/in^2) above it.

For Strands

The manufacturer is required to provide dated test certificates prepared from the relevant test results. Tests are to be made on samples cut from each coil; they comprise a tensile test, an elongation test, a proof-load test (for seven-wire strand only), and, if required, relaxation test results. For seven-wire strand, proof-load tests and load-extension curves are called for only for one test piece in every five; for nineteen-wire strands a proof-load test is required only for treated strands; for these, the test and the plotting of a load-extension curve, are specified for one test piece in every three. For 'as-spun' strand, load-extension curves are to be plotted for every test piece. The values specified are summarized in Table 6.3; the specified characteristic strength is defined in the same way as that for wire, except that no upper limiting value is imposed. The minimum elongation at failure is specified as 3.5%, except for 'as-spun' nineteen-wire strand; no value is specified for this.

All three specifications included provisions for re-testing, in the event of failure of a sample to meet the requirements.

Bars

No chemical composition is given for the steel, except that sulphur and phosphorus must not exceed 0.05%, but the manufacturer is required to provide the chemical analysis on request. Threads, if provided, are to be cold-rolled; no welds are permitted, and the bars are to be protected at all times from the effects of local heat. Tolerances are specified only on the mass; on the basis that the density of the steel is 7850 kg/m^3 (0.283 lb/in^3) the variations permitted are + 4% and −2% for a batch (defined as a number of lengths of one size from one cast) and + 6% and −2% for any one bar.

From the purchaser's viewpoint, the requirements for testing are less satisfactory than those included in the standards for wires and strands. The manufacturer is not required to provide any documentary evidence of the test results obtained, though the records of the tests must be 'available for inspection by the purchaser or his representative.' Further, unlike the standards for wires and strands, no option of independent testing before delivery

Table 6.3 *Mechanical Properties of Strands (BS 3617, 4757)*

Nominal diameter of strand		Nominal area of steel		Specified characteristic load			Conditions in which strand supplied	BS No.
(mm)	(in.)	(mm^2)	(in^2)	(kN)	(kgf)	(lbf)		
6.4	0.253	24.5	0.038	44.5	4540	10000	1,2	3617 (7-wire strand)
7.9	0.312	37.4	0.058	69.0	7040	15500	1,2	
9.3	0.366	52.3	0.083	93.5	9530	21000	1,2	
10.9	0.430	71.0	0.110	125.0	12750	28100	1,2	
12.5	0.492	94.2	0.146	165.0	16820	37100	1,2	
15.2	0.600	138.7	0.216	227.0	23150	51000	1,2	
18	0.7	210	0.325	370	37730	83180	1,2	4757 (19-wire strand)
25.4	1.0	423	0.656	659	67200	148150	3	
28.6	1 1/8	535	0.830	823	83920	185000	3	
31.8	1 1/4	660	1.020	979	99830	220100	3	

Condition in which strand is supplied:

Description	Number 1	Number 2	Number 3
	Normal relaxation heat treated	Low relaxation heat treated	As spun
0.2% proof stress as percentage of specific characteristic strength	85	90	—
Maximum relaxation after 1000 hours from:			
70% Initial stress	7%	2.5%	9%
80% Initial stress	12%	3.5%	14%

is available to the purchaser. The routine tests comprise a tensile test, a proof-load test and a minimum elongation (of 6%); one sample is to be taken from each 5 metric tonnes within a batch. The samples may be cut from the ends of processed tendons, or from off-cuts produced during processing. The specified characteristic load and the 0.2% proof load (defined as the load at 0.7% total strain) are given in Table 6.4; the breaking load for the thread is required to be at least 95% of that for the bar. The modulus of elasticity is to be determined from the test readings and recorded. The breaking load is required to be not less than 95% of the specified characteristic load, and not more than two out of the last 40 test results may be less than the specified values; no upper limiting value is specified.

Provision is made in the standard whereby a purchaser may, if he wishes, check that a batch attains the specified characteristic load. After delivery, ten bars are selected at random, and test pieces are cut from one end of each bar; if one should fail at less than 95% of the specified value, that bar is rejected. If two fall below this value, the whole batch is deemed not to comply with the requirements of the standard.

The purchaser may require the maker to provide evidence of the relaxation properties of the tendon. The standard also includes provisions for re-testing, if samples tested by the maker should fail to meet the requirements.

Wires Strands and Bars for Pre-Tensioning

It was thought at one time that a satisfactory bond between tensioned steel and concrete could be obtained only by the use of wires of small diameter, and piano wire of 2 mm (0.08 in.) diameter is still used occasionally. This type of wire has a smooth hard surface which for large

Table 6.4 *Properties of Cold-Worked High-Tensile Alloy Steel Bars (BS 4486)*

Nominal size		Specified characteristic load			Minimum 0.2% proof load		
(mm)	(in.)	(kN)	(kgf)	(ibf)	(kN)	(kgf)	(lbf)
*20	0.78	325	32850	73100	275	27750	61900
22	0.87	375	37900	84400	325	32850	73100
*25	0.985	500	50600	112500	425	42900	95800
28	1.11	625	63000	140600	525	53000	118100
*32	1.26	800	80900	180200	675	68050	152500
*40	1.57	1250	126200	281500	1050	106000	236500

* Preferred sizes.

wires prevents the development of good bond, and its unsatisfactory behaviour at ultimate load has been described earlier.

These disadvantages led to the use of indented wires. Single smooth wires of 0.2 in. and 0.276 in. (5 and 7 mm) diameters were introduced in Britain in 1939 and 1952 respectively and have proved satisfactory for pretensioning since their surface conditions are such as to ensure good bond. This is due to a very slight corrosion of the surface of the wire, such that no peeling of the surface is likely. Indented wires also provide a good bond, but the indentations must not be so large as to reduce appreciably the cross-sectional area of the wire or cause fatigue failure at the notches. Seven-wire strands are also widely used for pretensioning; in addition to the normal surface bond they provide a mechanical bond with the concrete, because of the configuration of the wires comprising the strand.

With pre-tensioned steel, a certain minimum embedded length, termed the *transmission length*, is necessary, along which the force is gradually developed in the concrete by bond. The transmission length required increases when the diameter of the wire increases and also to some extent when the strength of the concrete is reduced. With small wires the prestress in the concrete is developed over a very short length, but with larger sizes the required length may be 2 to 3 ft (0.67 to 0.9 m). It should be noted that the rate of transmission is not uniform. More than half of the prestressing force is transferred to the concrete in the first quarter of the transmission length and up to 85% may be transferred in the first half. In CP 110, it is noted that the transmission length for wire may vary between 50 and 160 diameters, and the following general recommendations are given:

Plain or lightly crimped wire: 100 diameters; 80% transfer in first 70 diameters
Heavily-crimped wire: 65 diameters; 80% transfer in first 54 diameters
Strand, 9.3 mm (0.366 in.) diameter: 200 mm ± 25 mm (8 in. ± 1 in.)
Strand, 12.5 mm (0.492 in.) diameter: 330 mm ± 25 mm (13 in. ± 1 in.)
Strand, 17.8 mm (0.7 in.) diameter: 500 mm ± 50 mm (20 in. ± 2 in.)

A special strand, known as Dyform, is made by British Ropes Ltd. The strand is first formed in the normal way and is then compacted to form the cross-section shown in Fig. 6.4. In this case, the objective is primarily to increase the force which a strand of given diameter can apply; there is no gain in transmission properties, but space taken is reduced.

High-alloy steel bars with special indentations have also been used for pretensioning. In this case the bar is tensioned in a manner similar to that described ahead for post-tensioned alloy bars, but after the concrete hardens, the end anchors are removed and the prestressing force is transmitted solely by bond, as for any pre-tensioning. When selecting a suitable size of wire it is desirable to ensure that the number of wires is sufficient to distribute the compressive stress uniformly over the concrete but not so great as to impede the placing of the concrete. In general it appears desirable to provide at least six wires in the tensile zone, but to avoid the adoption of a multitude of wires. If the number of wires is less than six, the failure of one would greatly reduce the factor of safety of the member. For this reason, the use of only two or three wires in a member should be avoided where possible.

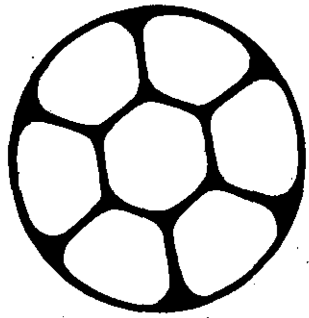


Fig. 6.4 *Cross-section of dyform strand*

Details of prestressing wires, strands and bars are given ahead.

Tendons for Post-Tensioning

Many cables with different arrangements of wires and strands and different methods of anchorage are available for post-tensioning. The main types are briefly described here; more data is given ahead.

Historically, the two basic types are represented first by the Freyssinet cable and later by the Magnel cable. In the Freyssinet cable, the wires, which usually number twelve, are closely spaced around a central spring, or core helix, thus forming a cable of annular cross-section. The cables may be very small and special care is then necessary to ensure satisfactory grouting, neat cement and water being used for the purpose. The spacing of cables should also be considered. In the Magnel cable, multiples of four or eight wires are provided in horizontal layers and the wires are well separated by spacers which allow easy grouting with cement mortar. With this type of cable a considerable prestressing force may be concentrated in a single cable. The Magnel system itself is no longer commercially very much in use now.

Many other types are available, including CCL Systems and PSC (Great Britain) and Leoba, Holzmann, and Beton-und-Monier Bau Gesellschaft (Germany), which more or less follow the Magnel principle with regard to the use of spacers; Systems Franki-Smet (Belgium), Morandi (Italy), Hochtief, and Grun and Bilfinger (Germany), which more or less follow the Freyssinet principle. Other early types like Gifford-Udall and Gifford-Burrow systems are no longer commercially available. BBRV and Losinger VSL (Switzerland), PI and Prescon (USA), represent a type intermediate between Magnel and Freyssinet. In some systems of this type, the wires may be distanced by spacers, but are not necessarily separated by them. In the latter case, if the wires are bent up they touch each other, forming a group into which the grout cannot be inserted.

In some types of cable, spacing may be obtained automatically if (Neptun or Sigma) wires with diagonal cross ribs are used instead of round wires. The cross ribs of adjacent wires touch each other at points only, since the ribs on opposite sides of a wire slope in opposite directions, and sufficient space is available between the wires to allow the easy admission of grout.

The cables may be inserted into holes formed in the concrete or placed in ducts (tubes or sheaths). The Baur-Leonhardt cable (Germany) may also be placed around the outside of the concrete, forming closed loops. This cable consists of closely spaced stranded wires, and is therefore of the Magnel type.

Cables comprising single or multiple strands, which can be inserted in holes or placed in tubes, have been introduced by Anderson and Roebling (USA), Rheinhausen (Germany), Freyssinet and SEEE (France), PSC, Stress Block and CCL Systems (Great Britain), and most suppliers of post-tensioning systems now provide components for anchoring strands. In order to obtain the greatest possible strength of a large stranded cable it is necessary that the outer wires should be stranded in the same direction as those forming the inner core; such strands are described as *parallel lay*. As a consequence, an untwisting torque may occur during tensioning in systems in which the jack is restrained from rotating (though most strand jacks now permit rotation during stressing). At transfer, this torque may be transmitted to the prestressed unit, and it may occasionally be large enough to warrant consideration in the design!

In addition to the medium and large cables already described, there are several types of cables with two, three, four, six and eight wires which are used to provide smaller prestressing forces. In all these cables, wires of 0.2 and 0.276 in. (5 and 7 mm) diameter are generally used. The two-wire and four-wire ducts of the PSC system are the smallest in size as no spacers are used, the arrangement of the wires being such that the space available for grout is not less than that obtained with spacers.

Bars for Post-Tensioning

Two types of steel bar have been developed for use in post-tensioning. They are used in the Macalloy system (Britain), Stressteel in the USA and the Dywidag system (Germany). In the Macalloy system, high-alloy steel bars from 18 mm (3/4 in.) up to 40 mm (1-5/8 in.) diameter are used. In the Dywidag system, the bars are of low-alloy steel of natural hardness, but with a definite yield point and are usually 25 mm (1 in.) in diameter although bars of lesser diameter are also available. In the Dywidag system high-alloy bars with greater strength have also been introduced.

The bars may be inserted into holes or placed within tubes or sheaths in the concrete, in the same way as cables; the Macalloy system also includes a four-bar tendon. It is possible to obtain a good bond if the grouting is carefully done with neat cement and water. The bars may be placed relatively close to one another, in the same way as Freyssinet cables.

Anchoring Prestressing Steel

There are four basic methods of anchoring the steel after it has been tensioned. Three of these are represented by the Freyssinet, Magnel and Macalloy methods respectively; the fourth is represented by CCL, PSC and BBRV systems.

In the Freyssinet system all the wires or strands of the cable are wedged (locked) between a cylinder which is embedded in the concrete and a cone which is inserted therein. In the Magnel system pairs of wires are anchored by flat wedges to plates, termed *sandwich plates*, which in turn transfer the prestressing force to the concrete through an anchor-plate; as previously mentioned, this system is no longer available. In the Macalloy system, the prestressing force is also transferred to the concrete through an anchor-plate, by means of a nut tightened on a thread, rolled on to the end of the bar. With SEEE system, soft steel cylinders, containing the strands are pushed through a die; threads are rolled onto the swaging cylinders, and nuts are tightened to anchor the tendons. In the Dywidag system, bars with threads throughout their length are available.

In the fourth method, single wires or strands are secured to cylindrical grips by means of one or more wedges, or alternatively by button heads formed on the wire (BBRV and Prescon). The wedge system has also been adopted for bars (Stressteel and Macalloy alternative anchorage) and for cables of single or multiple strands (CCL Systems, PSC, Stress Block, and Anderson.)

When the wires are secured by wedges, whether they be concrete or steel cones or steel wedges, some slipping is unavoidable when the pull on the prestressing steel is relaxed. This may affect the tensioning stress substantially if the prestressing tendon is short.* When the steel is secured by a nut the process is simple and no slip occurs during transfer. Moreover, no difficulty is experienced in regulating the prestressing force at any time. Because of the many advantages of this method of anchoring, it is employed in several of the cable systems previously mentioned, including some systems which use strand. In several systems, the separate wires of the cable are secured to a threaded anchor-head before tensioning. The wires are then tensioned simultaneously and anchored by means of a nut. The wires are connected to the anchor-head by upsetting and enlarging the ends of the wires (BBRV, Prescon), by concreting the wires into the head (Beton-und-Monier Bau, Holzmann A.G.), or by looping them around a cross-bar with a threaded hole (Leoba). In some of these systems it is also possible to employ a temporary anchor-plate and dispense with it as soon as cement mortar or concrete, which is inserted round the anchor-head, hardens and secures the head to the concrete. The anchor-plate is then removed. The anchor-head is usually conical. The advantage of a positive anchorage which will not slip is thereby retained without the cost of a permanent anchor-plate.

In the original Holzmann large-cable system (Germany), the cable comprises layers of four oval-shaped well-spaced wires, and is secured by means of a wedge to a large prefabricated member (corresponding to a female cone or large grip) which also forms the anchor-plate. In the improved Holzmann KA system (Klem Anker, i.e., clamp anchor system), now being used, up to forty oval-shaped or rectangular wires with diagonal ribs are clamped by means of transverse bolts and nuts which are tightened against the outer plates.

In the Losinger VSL system (Switzerland), up to 36 wires are secured to an anchor head by means of a single conical wedge with circumferential grooves in which the wires are housed.

A continuous cable is used in the Baur-Leonhardt system, in which the tensioning is done by jacking apart two parts of the structure around which the cables are looped. Alternatively, separate cables may be used, one end being anchored in the structure and the other in a movable anchor-block to which the jacks are applied.

Another post-tensioning system, which is in fact the oldest, is that developed by Coyne, and used mainly for retaining walls, dams, and barrages. It comprises a straight cable of 600 to 800 wires of 0.2 in. (5 mm) diameter, strapped together to form a bundle. One end is embedded in a bulkhead of concrete and held by bond, and the other end is fixed to a large steel drum filled with cement mortar, thereby forming an anchor head to which the jacking force is applied. This system was used to prestress the first prestressed concrete pressure vessels for nuclear reactors, at Marcoule (France).

Cables may have the same type of anchorage at both ends, or the wires may be embedded in the concrete at one end before they are tensioned; loops or other shapes which ensure a satisfactory anchorage may be formed at the dead end of the cable.

The systems described in the foregoing can also be used for prestressing circular containers or pipes by arranging the cables in overlapping arcs; special anchors are available with some systems to simplify the work. Circular structures can also be prestressed by means of wire under tension wound around them in the form of a continuous helix (Preload, BBRV and Dywidag systems). The British contractors Taylor Woodrow Ltd., have also developed a wire-winding system for large pressure vessels.

* In a short tendon the extension may not be very much more than the slip!

Relaxation of Stress in Steel

When a high tensile steel wire is stretched and maintained at a constant strain, the initial force in the wire does not remain constant but decreases with time. The decrease of stress in steel at constant strain is termed as *relaxation*. In a prestressed member, the high tensile steel between the anchorages is more or less in a state of constant strain. However, the actual relaxation will be rather less than that indicated by a test of a tendon at constant length, as there will be a shortening of the member due to other causes. With the high tensile steels at elevated stresses the relaxation of stress has been observed and it increases with the magnitude of initial stress. If the stress is maintained constantly, the material exhibits a plastic strain over and above the initial elastic strain, generally referred to as *creep*.

The cold drawn steels creep more than heat treated or tempered steels due to their lower magnitude of proof stress. The phenomenon of creep is influenced by the chemical composition. Micro-structure, grain size and variables in the manufacturing process, which results in changes in the internal crystal structure. Several hypotheses for explaining the mechanism of creep in steel are presented by several investigators.

The steel in a prestressed concrete member strictly does not remain under a constant condition of either stress or strain. The most severe condition generally occurs at the stage of initial stressing; subsequently, the strain in the steel reduces as the concrete deforms under the prestressing force.

The code provision for the relaxation of stress in steel is based on the results of the 1000 hours relaxation test on specimens. Experience has shown that the loss recorded over a period of about 1000 hours from an initial stress of 70% of the tensile strength is about the same as the loss experienced over a period of four years from an initial stress of 60% of the tensile strength. According to Stussi the relaxation curves obtained over 1000 hours can be extrapolated by a logarithmic plot. The Indian Standard specification I.S. 1785 prescribes the 1000-hour relaxation test with a relaxation of stress not exceeding 70 N/mm^2 for cold drawn stress relieved wires. In the absence of this, the 100-hour relaxation test is also provided for with a limiting value of relaxation stress of 46.7 N/mm^2.

Experiments have shown that a reduction in relaxation stress is possible by preliminary overstressing! A preliminary overstress of 5–10% maintained for two or three minutes results in a considerable reduction in the magnitude of relaxation. Some of the codes permit temporary overstressing with correspondingly lower magnitudes of relaxation stress.

Stress Corrosion

The phenomenon of stress corrosion in steel is particularly dangerous since it results in sudden brittle fractures. Stress corrosion cracking results from the combined action of corrosion and static tensile stress, which may be either residual or externally applied. This type of attack in alloys is due to the internal metallurgical structure which is influenced by composition, heat treatment and mechanical processing. The causes of the susceptibility of high tensile steels to stress corrosion are manifold. Schwier has reported that heat treated wires are specially prone to stress corrosion fractures when compared to drawn wires. If the ducts of post-tensioned members are not quickly grouted, there is the possibility of stress corrosion leading to a catastrophic failure of the structure.

There are other common types of corrosion frequently encountered in prestressed concrete constructions such as *pitting corrosion* and *chloride corrosion*. A critical review of the different types of corrosion of high tensile steel in structural concrete is reported elsewhere. Some of the important protective measures to prevent stress corrosion include protection from chemical contamination, protective coatings for high tensile steel and grouting of ducts immediately after prestressing operations.

Hydrogen Embrittlement

Atomic hydrogen is liberated due to the action of acids on high tensile steels. This penetrates into the steel surface making it brittle and resulting in fractures on being subjected to tensile stress. Even small amounts of hydrogen are sufficient to cause considerable deterioration in the tensile strength of high tensile steel wires.

Use of high alumina cement, blast furnace slag cement which is rich in sulphides, when used to make prestressed concrete is likely to give rise to hydrogen embrittlement. Use of dissimilar metals such as aluminium and zinc for sheaths to house high tensile steel wires also results in hydrogen embrittlement. Minute traces of sulphur which come in contact with high tensile steel wires in the presence of moisture results in reduction in the strength due to hydrogen embrittlement.

Table 6.5 *Strand Data*

BS3617 Normal-relaxation strand

Nominal diameter of strand	Nominal area of steel	Nominal mass per 1000 m run	Specified characteristic load	Minimum 0.2% proof load	Minimum elongation	Maximum relaxation after 1000 hours from initial load of 70% of the specified characteristic load	Maximum relaxation after 1000 hours from initial load of 80% of the specified characteristic load
mm	mm^2	kg	kN	kN	%	%	%
9.3	52.3	411	93.5	79.5	3.5	7	12
10.9	71.0	564	125.0	106.3			
12.5	94.2	744	165.0	140.3			
15.2	138.7	1101	227.0	193.0			

BS3617 Low-relaxation strand

Nominal diameter of strand	Nominal area of steel	Nominal mass per 1000 m run	Specified characteristic load	Minimum 0.2% proof load	Minimum elongation	Maximum relaxation after 1000 hours from initial load of 70% of the specified characteristic load	Maximum relaxation after 1000 hours from initial load of 80% of the specified characteristic load
mm	mm^2	kg	kN	kN	%	%	%
9.3	52.3	411	93.5	84.1	3.5	2.5	3.5
10.9	71.0	564	125.0	112.5			
12.5	94.2	744	165.0	148.5			
15.2	138.7	1101	227.0	204.3			

(Contd.)

Table 6.5 *(Contd.)*

Dyform L-R prestressing strand

Nominal diameter of strand mm	Nominal area of steel mm^2	Nominal mass per 1000 m run kg	Specified characteristic load kN	Specified characteristic load lbf	Minimum load at 1% relaxation: Normal-relaxation strand kN	Minimum load at 1% relaxation: Low-relaxation strand kN
12.7	112.0	890	209.0	4.6985	—	181
15.2	165.0	1300	300.0	6.7443	—	260
18.0	223.0	1750	380.0	8.5427	—	330

Bridon SUPA-7 prestressing strand

Nominal diameter of strand mm	Nominal area of steel mm^2	Nominal mass per 1000 m run kg	Specified characteristic load kN	Specified characteristic load lbf	Minimum load at 1% relaxation: Normal-relaxation strand kN	Minimum load at 1% relaxation: Low-relaxation strand kN
9.6	56.0	440	102.5	23.043	87.1	92.3
11.3	76.0	600	138.0	31.024	117.3	124.2
12.9	100.5	800	184.0	41.365	156.4	165.6
15.4	143.2	1130	250.0	56.202	212.5	225.0

In order to prevent hydrogen embrittlement, it is essential that steel is properly protected from the action of acids. Protective coverings like bituminous crepe paper covering during transport reduces the chances of contamination. The steel should be protected from rain and excessive humidity by storing it in dry conditions.

Strand Data

There are several grades of prestressing strand available. All prestressing strands are stress relieved, but further processes are often employed to reduce the losses arising from the relaxation of the steel. These processes involve a combination of applied heat and stress, carried out under such varying trade names as thermalising, normalising, etc.

Compact strand is pulled through a die after being spun as a stranded cable, which not only physically modifies the cross-sectional shape, but also enhances the strength characteristics of the stranded cable as a result of the further cold working.

Table 6.6 *Forces in Different Types, Numbers and Sizes of Strands*

Tendon: Strand size mm	Tendon: Type	Number of strands	Cross-sectional area (mm^2)	Force: Specified characteristic load in kN 100%	Force: Specified characteristic load in kN 70%
12.7	DYF	7	784	1463	1024
18.0	DYF	4	892	1520	1064
15.2	STD	7	970	1589	1112
15.4	SUPA	7	1002	1750	1225
12.5	STD	12	1130	1980	1386
15.2	DYF	7	1155	2100	1470
12.9	SUPA	12	1206	2208	1545
12.7	DYF	12	1344	2508	1755
18.0	DYF	7	1561	2660	1862
15.2	STD	12	1664	2724	1906
15.4	SUPA	12	1718	3000	2100
12.5	STD	19	1789	3135	2194
15.2	STD	15	2080	3405	2383
12.9	SUPA	19	1909	3496	2447
15.2	DYF	12	1980	3600	2520
15.2	DYF	13	2145	3900	2730
12.5	STD	25	2355	4125	2887
15.2	STD	19	2635	4313	3019
12.9	SUPA	25	2512	4600	3220
15.4	SUPA	19	2720	4750	3325
12.5	STD	31	2920	5115	3580
15.2	DYF	19	3135	5700	3990
12.9	SUPA	31	3115	5704	3992
18.0	DYF	19	4237	7220	5054

For, the purposes of assessing prestressing strand extensions, calculations should be based on values of E taken from tests on specimens of the actual strand used. For design purposes a figure of 200 kN/mm^2 may be used.

Couplers An economic range of couplers has been designed for simple assembly on site. The first-stage tendon is stressed and anchored in the normal way using standard equipment and the dead-end of the second tendon is assembled around it, using swaged grips on each strand to afford maximum security.

The coupler assembly is enclosed with a conical cover which has a grout access point for second stage grouting.

'Freyssinet' Multi-Wire 12/7 mm and 12/8 mm
Prestressing Anchorages (All Tendons Tensioned Together)

Internal female cones				
Size	Diameter		Length	
	mm	in.	mm	in
12/7	120	$4\frac{3}{4}$	125	5
12/8	150	6	125	5

External female cones				
Size	Diameter		Length	
	mm	in.	mm	in
12/7	140	$5\frac{1}{2}$	125	5
12/8	150	6	125	5
External cones should be specially ordered.				

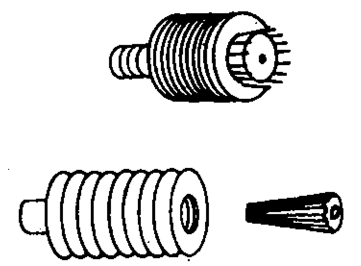
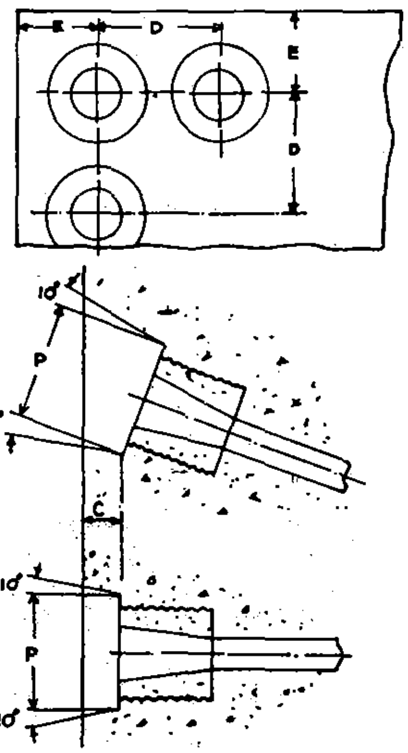
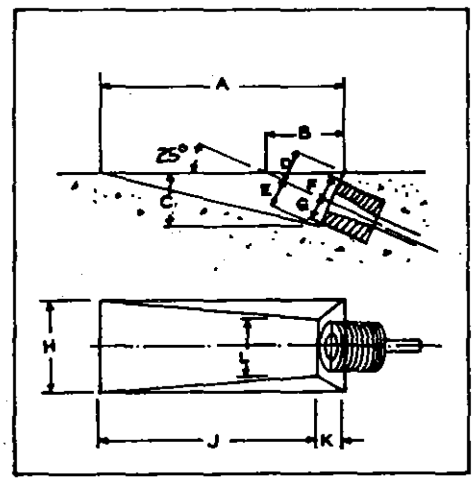


Fig. 6.5

Cone size		A	B	C	D	E	F	G	H	J	K	L
12/7	mm	600	200	140	85	70	60	60	220	550	65	140
	in.	24	$7\frac{7}{8}$	$5\frac{1}{2}$	$3\frac{1}{4}$	$2\frac{3}{4}$	$2\frac{3}{8}$	$2\frac{3}{8}$	$8\frac{3}{4}$	$21\frac{1}{2}$	$2\frac{1}{2}$	$5\frac{1}{2}$
12/8	mm	650	275	200	115	100	75	75	300	550	90	200
	in.	25	$10\frac{3}{4}$	$7\frac{3}{4}$	$4\frac{1}{2}$	4	3	3	12	$21\frac{1}{2}$	$3\frac{1}{2}$	8





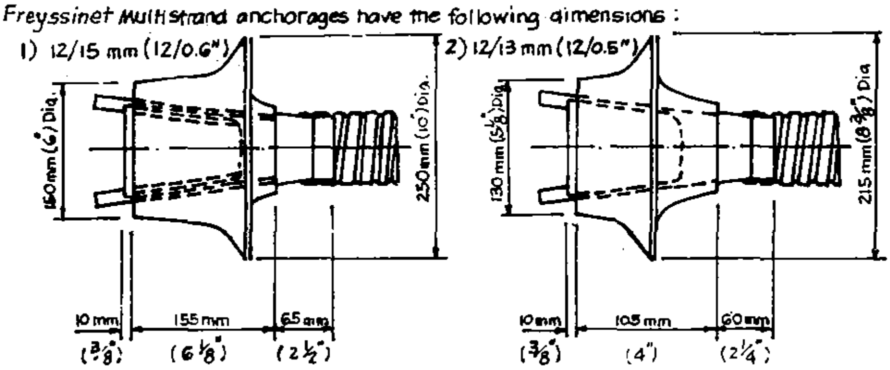
Typical details of the recesses for jacking upswept Freyssinet cables.

Fig. 6.6

'Freyssinet' Multistrand Anchorage

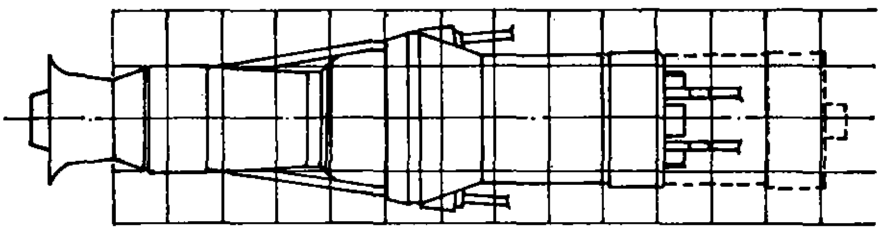
Cable Characteristics

Cable Type		Cable Diameter		Initial Design Force (80% ult.)		Initial Design Force (70% ult.)	
mm	in.	mm	in.	kN	kip force	kN	kip force
12/15	12/0.6	62	$2^1/_2$	2 180	490	1 907	428
12/13	12/0.5	52	$2^1/_8$	1 584	355	1 386	311



Anchorage Centres and Edge Distances

Anchorage size		Centre to Centre		Centre to Edge	
mm	in.	mm	in.	mm	in.
12/15	12/0.6	325	13	200	8
12/13	12/0.5	270	$10^1/_2$	150	6



JACK CLEARANCE DIAGRAM
SCALE : ONE SQUARE REPRESENTS 100 mm (4 in)

Fig. 6.7 *Anchorage detailing*

Blind-End Anchorages The normal anchorage can be used in accessible dead-end positions, but for situations where the anchorages are to be cast into the concrete, or are inaccessible, a range of blind-end anchorages is provided.

Looped Anchorages This is the preferred solution where a bond length is available, but due allowance has to be made in the overall design for the slow build-up of stress along the length of strand cast into the concrete. This anchorage is only suitable for small cables.

A saddle is fixed in position to space the strands and to assist in the distribution of the load to the concrete. A grout access point must be provided near to the sheath termination.

Swaged Anchorages This anchorage permits a rapid build-up of stress behind the guide. Swaged grips are used to ensure maximum security and the whole assembly is completed with a cast-iron cap containing a grout access point. This type of anchorage can be used for any size of cable.

Swaged Grips The swaged grip consists of a hardened steel inner coil over which a ductile steel body is compressed by drawing it through a die using a light, portable and robust jack designed for continuous site work. These swaged grips are used in couplers and blind end anchorages.

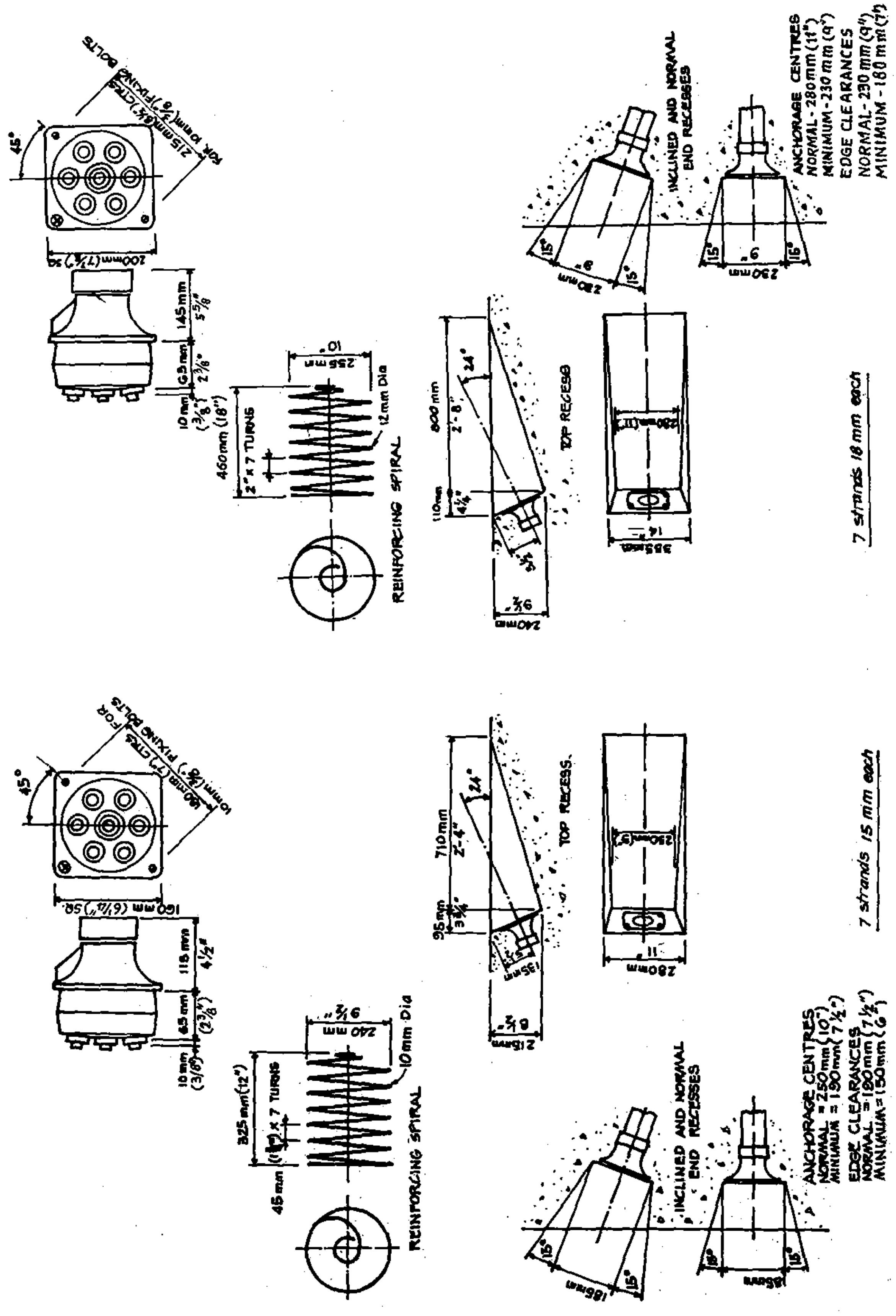
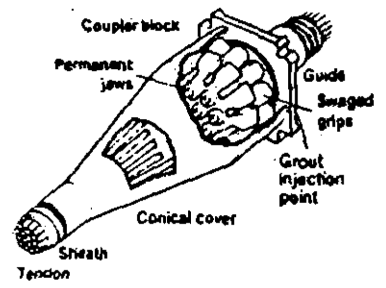
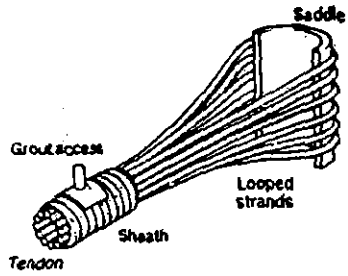


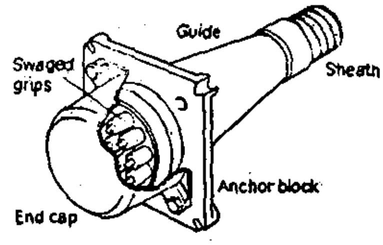
Fig. 6.8 'Freyssinet' monostrand anchorages (single tendon tensioning)



(a) The 'K' range coupler



(b) The looped anchorage



(C) The swaged and capped anchorage

Fig. 6.9(a), (b), (c) *Freyssinet system*

External Prestressing

Removable External Prestressing

Placing of prestressing cables on the outside of structural concrete is by no means a new idea. Many applications of this type over the past fortyfive years, in various countries of the world, are based on this idea.

The use of external prestressing has proved to be of particular interest for strengthening of structures, whether for the purpose of adapting them to new loading regulations, or in order to make them comply with new design regulations and to completely restore their capability of resisting applied loads.

It was however, in France, in the course of the past fifteen years and at the instigation of SETRA* that external prestressing was given pride of place among the modern techniques used in the construction of new structures.

Freyssinet has designed and developed a new range of anchorages which takes into account dismantling requirements, safety in placing and under load and reinforced corrosion protection which ensures the durability of the prestressing.

A New Range of Anchorages—Two Systems of Cables

This new range of external prestressing anchorages offered by FREYSSINET INTERNATIONAL allows a choice between two systems of cables.

System 1: Ordinary Strand Cables (normal or super grade)

The cable is formed of ordinary strands threaded in the traditional manner into a thick, high density polyethylene (HDPE) sheath.

* French Ministerial Service for Technical Design of Roads and Motorways.

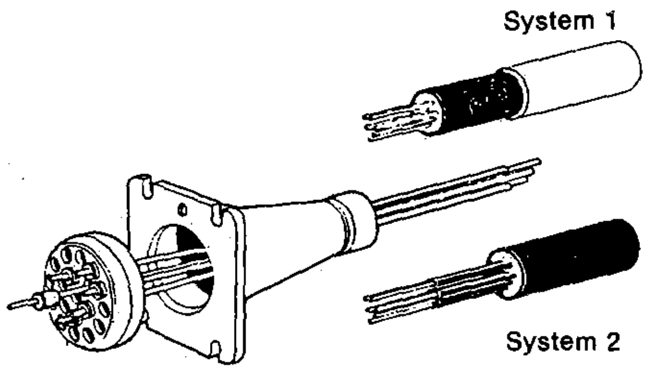


Fig. 6.10 *12 K 15 External prestressing anchorage (Freyssinet)*

The duct, which is continuous from one anchorage to the other, passes freely through the intermediate concrete cross-beams by means of sleeves, generally of metal, cast into the concrete of the structure during pouring.

These sleeves may also act as deviators.

Stressing and grouting, with cement grout, are then carried out in the traditional manner with standard equipment.

This method offers numerous advantages:

- As the duct is external to the structure, the quality of sheath placing and its watertightness may be checked at any moment.
- The cable/sheath friction coefficients are low and the high coefficient of transmission results in an appreciable improvement in the efficiency of the prestressing.
- The system is easily dismantable and allows replacement of the cable if necessary.

System 2: Plastic Coated, Greased Strand Cables

The cable is formed of greased strands, individually coated with a layer of high density, heat-extruded polyethylene and grouped together inside a thick HDPE sheath.

When the structure is cast *in situ*, the duct, which is continuous from one anchorage to another, passes through the deviation cross-beams of which it forms an integral part.

When the structure is precast special arrangements must be envisaged and defined case by case.

The originality of this method, which is proposed and patented by Freyssinet International, resides in the fact that the sheath is injected with cement grout prior to stressing, which prevents all interaction between strands during stressing and avoids damage to their individual protection system (grease + plastic coating).

Very low coefficients of friction and safety against corrosion are thereby guaranteed.

Stressing is then carried out strand by strand, in stages using a monostrand jack or in the traditional manner with a multistrand jack.

There are multiple advantages to this method:

- Strand/sheath friction coefficients are extremely low and the high value of the coefficient of transmission of the cable results in highly efficient prestressing.
- Stressing strand by strand allows the use of lighter, and therefore, more easily handled stressing equipment. The size of the jack does not constitute an obstacle to the instalment of large prestressing tendons.

  The use of a monostrand jack — less bulky than a multistrand jack — allows positioning of the axis of the cable closer to the wall of the structure, which is particularly advantageous when the cables are anchored in internal blisters (either cast-in-place or bolted on).
- Fourfold protection (HDPE sheath; cement grout; polyethylene coating and grease) guarantees a very high level of safety against corrosion.
- Ulterior adjustment of the prestress, during the life of the structure, is always possible — provided that the jacking lengths of strand, allowing gripping by the jack, are left uncut after the initial stressing operation.

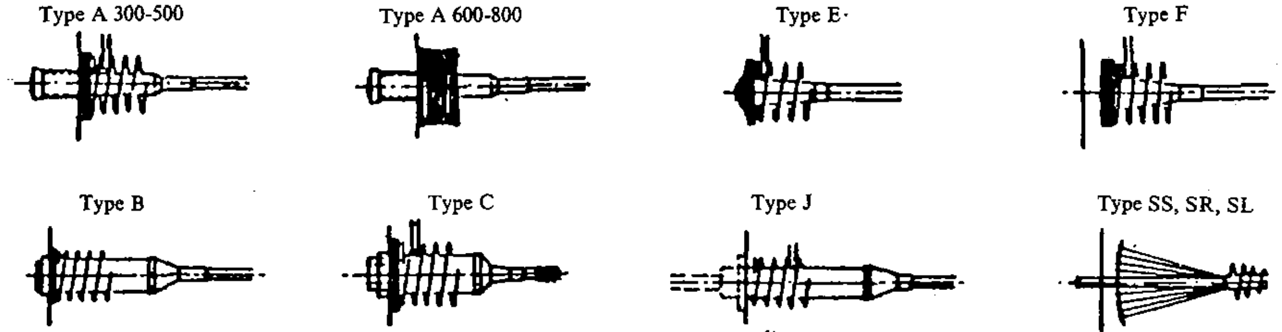
'B B R V' System

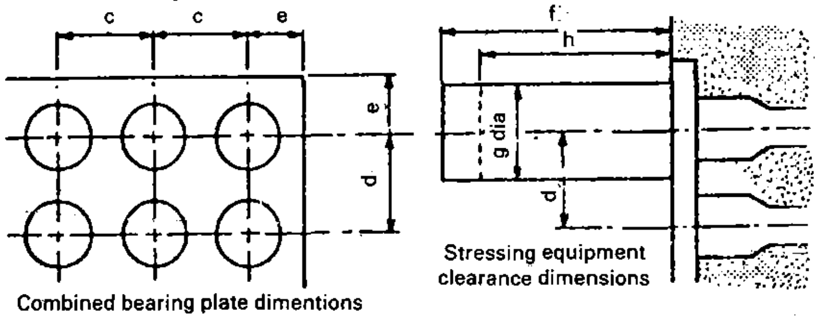
Small capacity tendons up to 2658 kN

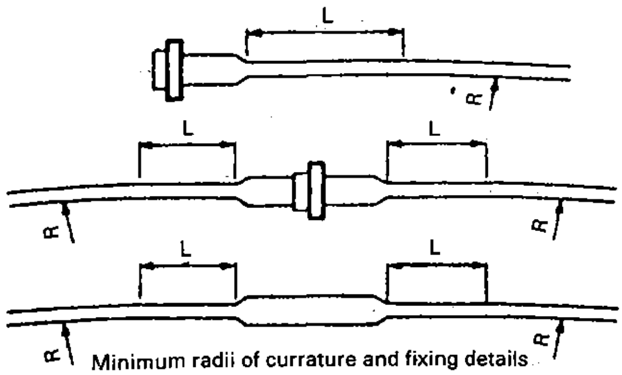
Maximum working load (80% f_u)	Number of 7 mm dia. wires	Anchorages: Stressing	Anchorages: Fixed	Anchorages: Nominal reference	Standard single bearing plate	Combined bearing plate: Minimum centres of anchorages	Combined bearing plate: Minimum concrete edge distance	Sim-Tube sheathing internal dia.	Stressing equipment clearance dimensions with dynamometer	without dynamometer
kN	No.	Type	Type	Ref. No.	mm	mm	mm	mm	mm	mm
					$a \times b$	$c \times d$	e		$f \times g$ dia	$h \times g$ dia
387	8	B J	F SR SL	32	150 × 150 150 × 150 120 × 120 120 × 150 60 × 300	138 × 138	76	30	1524 × 310	1344 × 310
773	16	B J	F SR SL	64	175 × 175 175 × 175 160 × 160 150 × 220 80 × 400	171 × 171	89	40	1524 × 310	1344 × 310
1160	24	B J	F SS SR SL	100	200 × 200 200 × 200 220 × 220 220 × 220 160 × 300 80 × 560	197 × 197	108	50	1524 × 310	1344 × 310
1498	31	C	E SS SR SL	130	250 × 250 235 dia. 260 × 260 180 × 360 120 × 560	235 × 235	127	55	1524 × 340	1324 × 340
1643	34	B J	F SS SR SL	138	250 × 250 250 × 250 260 × 260 260 × 260 180 × 360 120 × 560	235 × 235	127	55	1524 × 340	1324 × 340
2029	42	C	E SS SR SL	170	280 × 280 270 dia. 300 × 300 200 × 450 140 × 650	241 × 267	152	65	1880 × 440	1580 × 440
2658	55	C	E SS SR SL	220	300 × 300 300 dia. 340 × 340 220 × 500 160 × 700	267 × 305	152	75	1880 × 440	1580 × 440

*Large capacity tendons up to 9036 kN Type A**

Maximum working load (80% f_u)	Number of 7 mm dia. wires	Anchorage nominal reference	Standard single bearing plate	Sheating internal diameter	Stressing equipment clearance dimensions with dynamometer	Preferred sizes
kN	No.	Ref. No.	mm	mm	mm	
			$a \times b$		$f \times a$ dia.	
3237	67	300	435 × 435	90	2500 × 550	
3817	79	350	435 × 435	95	2500 × 550	←
4397	91	400	435 × 435	105	2500 × 550	
4977	103	450	435 × 435	105	2500 × 550	←
5557	115	500	640 × 640	120	2600 × 800	
6716	139	600	640 × 640	130	2600 × 800	
7876	163	700	640 × 640	140	2600 × 800	←
9036	187	800	640 × 640	150	2600 × 800	







Nom. Ref.	Length L min.	Radius R min.
No.	mm	mm
32	480	1980
64	700	3050
100	920	3200
130	1090	3960
138	1090	3960
170	1320	4500
220	1520	5030
300	1550	5500
350	1650	5900
400	1750	6100
450	1800	6500
500	1900	7000
600	2000	7100
700	2100	7500
800	2200	8000

Fig. 6.11

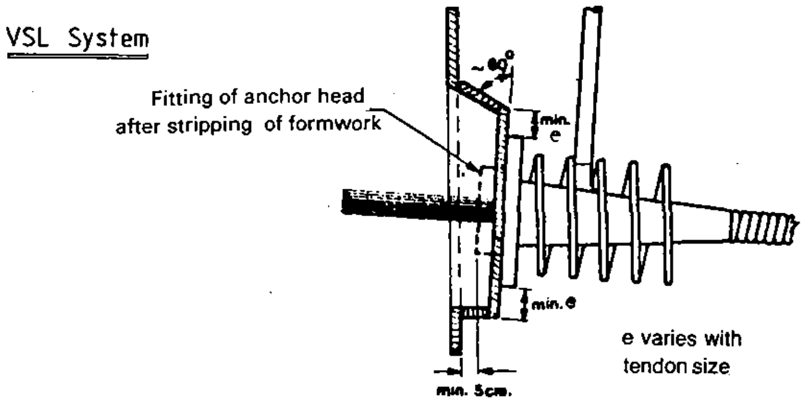


Fig. 6.12 *VSL system*

(Refer to Table 6.7)

Table 6.7

*13 mm (0.5 in.) STRAND										
Unit	No. of Strands	Duct Diameter mm		Standard fu = 165 kN		Super fu = 184 kN		Dyform Compact 209 kN		Jack Used
				A = 94.2	W = 0.744	A = 100.5	W = 0.80	A = 112.0	W = 0.89	
		Factory Assembled	Pull-through	0.7 fu kN	0.8 fu kN	0.7 fu kN	0.8 fu kN	0.7 fu kN	0.8 fu kN	
5-1	1	18/20	20/25	115	132	128	147	146	167	ZPE-30
5-3	2	30/35	30/35	231	264	257	294	292	334	ZPE-60
	3	35/40	35/40	346	396	386	441	438	501	
5-7	4	40/45	40/45	462	528	515	588	585	668	
	5	40/45	45/50	577	660	644	736	731	836	ZPE-100
	6	45/50	50/55	693	792	772	883	877	1003	
	7	50/55	55/60	808	924	901	1030	1024	1170	
5-12	8	50/55	60/67	924	1056	1030	1177	1170	1337	
	9	55/60	60/67	1039	1188	1159	1324	1316	1504	ZPE-200
	10	60/67	65/72	1155	1320	1288	1472	1463	1672	
	11	60/67	65/72	1270	1452	1416	1619	1609	1839	
	12	65/72	70/77	1386	1584	1545	1766	1755	2006	
5-19	13	65/72	70/77	1501	1716	1674	1913	1901	2173	
	14	70/77	75/82	1617	1848	1803	2060	2048	2340	
	15	70/77	75/82	1732	1980	1932	2208	2194	2508	
	16	75/82	80/87	1848	2112	2060	2355	2340	2675	ZPE-340
	17	75/82	80/87	1963	2244	2189	2502	2487	2842	
	18	80/87	85/92	2079	2376	2318	2649	2633	3009	
	19	80/87	85/92	2194	2508	2447	2796	2779	3176	
5-22	20	80/87	90/97	2310	2640	2576	2944	2926	3344	
	21	85/92	90/97	2425	2772	2704	3091	3072	3511	
	22	85/92	95/102	2541	2904	2833	3238	3218	3678	
5-31	23	85/92	95/102	2656	3036	2962	3385	3364	3845	ZPE-500
	24	90/97	100/107	2772	3168	3091	3532	3511	4012	
	25	90/97	100/107	2887	3300	3220	3680	3657	4180	
	26	95/102	100/107	3003	3432	3448	3827	3803	4347	
	27	95/102	110/117	3118	3564	3477	3974	3950	4514	
	28	95/102	110/117	3234	3696	3606	4121	4096	4681	
	29	100/107	110/117	3349	3828	3735	4268	4242	4848	
	30	100/107	110/117	3465	3960	3864	4416	4389	5016	
	31	100/107	110/117	3580	4092	3992	4563	4535	5183	
5-42	32	110/117	120/127	3696	4224	4121	4710	4681	5350	
	33	110/117	120/127	3811	4356	4250	4857	4827	5417	
	34	110/117	120/127	3927	4488	4379	5004	4974	5684	
	35	110/117	120/127	4042	4620	4508	5152	5120	5872	
	36	110/117	120/127	4158	4752	4636	5299	5266	6019	ZPE-1000
	37	120/127	120/127	4273	4884	4765	5446	5413	6186	
	38	120/127	130/137	4389	5016	4894	5593	5559	6353	
	39	120/127	130/137	4504	5148	5023	5740	5705	6520	
	40	120/127	130/137	4620	5280	5152	5888	5852	6688	
	41	120/127	130/137	4735	5412	5280	6035	5998	6855	
	42	120/127	130/137	4851	5544	5409	6182	6144	6922	
5-55	55	140/150	150/160	6352	7260	7084	8096	8046	9196	

A = nominal steel area per strand (mm^2) }
W = nominal weight per strand (kg/m) } These values are approximate only and may vary slightly depending on the source of supply.

*Similar tables exist for 15 and 18 mm strands. For particulars see firm's catalogue.

CHAPTER 7

The Substructure

7.1 INTRODUCTION

Usually *substructure* of a bridge refers to that part of it which supports the structure that carries the roadway (called *superstructure*). Thus the substructure covers pier and abutment bodies together with their foundations, and also the arrangements above the piers and abutments through which the superstructure sits, i.e., bears on the substructure. The latter are called the bearings. These have been dealt with separately.

The more usual types of foundation for substructure are briefly discussed below:

Shallow Type

These are foundations generally placed after open excavation, and are called *open foundations*. Examples of such foundations are isolated footing, combined footing, strip footing, raft and cellular monolith. The latter is constructed at or near the ground level by building up its body in portions (lifts) and sinking down the monolith by openly excavating from within its cells and finally sealing or plugging its bottom and capping its top effectively to support the pier or abutment body on it.

Deep Type

These are constructed by various special means. Deep foundations are *piles* and *caissons* (or *wells*). 'Piles' are essentially giant-sized nails (of concrete, steel or timber) that are either driven into the subsoil (in which case they *displace* the soil in their place) or are placed-in after boring holes in subsoil (in which case they *replace* the soil in their place). These giant-sized 'nails' can be square, rectangular, H-shaped or circular in section (20 to 200 cm or more in diameter), and can range in length from about 4 to 40 m or more. A group of piles is capped together at top (usually by a reinforced concrete cap) to support the pier or abutment body above.

'Caisson' is basically constructed at the open surface level in portions and sunk downwards by essentially mechanically excavating soil from within its dredge-hole all the way till its cutting edge reaches the desired founding level, after which the well is effectively sealed (i.e., plugged) at bottom, then filled by sand (at least partly), and then capped at or near the surface level. The pier or abutment body is then constructed on the cap.

Owing to interaction between the bridge deck and its supporting structure, it is essential that the two be considered together in formulating the overall proposal. Ground conditions with significant differential foundation settlement possibility may rule out the use of structural forms involving continuous spans.

Soil investigation should be concluded by means of adequate boreholes, penetration tests, and complementary tests on appropriate soil samples. Investigation carried out without proper supervision and understanding may be of little value, and can even be misleading and may give rise to problems during and even after construction.

7.2 IMPORTANT DEFINITIONS

Abutments

These are the first and last supports (i.e., the end supports) of a bridge. Mass concrete construction is generally economical for small heights, but is not competitive with other available alternatives in reinforced concrete for taller heights. Upstanding cantilevered reinforced concrete walls are probably the most widely used form of construction for typical highway bridge abutments. For tall heights it is more economical to shape the plan-section of the wall-stem into a series of T-junctions. This allows use of wall panels of the minimum practical thickness in combination with cantilevered upstanding T-beams to act as counterforts.

For large abutments where the ground is rising away from the bridge, there can be advantages in using a *hollow abutment*. This consists of four walls forming a box in plan and supporting a deck of simple cast *in situ* reinforced concrete beam-and-slab construction. The front and side walls simply act as supports to the deck, while the rear wall retains the earthfill of the approach embankment. The potential advantage of this arrangement is that the height of the retaining wall at the rear of the hollow abutment is much less than would be required if the retaining wall were the front wall of the abutment.

Piers

The bridge-supports inbetween the abutment-supports are referred to as *piers*. The choice of construction of the bridge deck will dictate the choice of type of pier. If support is required at intervals across the full width of the bridge deck, then some form of supporting wall or portal frame is made for the pier. However, where a deck has some capacity within itself to span transversely at intermediate-support positions by means of a diaphragm within the depth of the deck, there is a wider choice available for the type of pier.

Simplicity in the formation of a pier not only has the merit of providing easier and more economical construction, but is also likely to produce a more attractive result. But for special cases, complex shapes may be adopted. In this case the bearings are placed at the heads or feet of the piers. A monolithic connection between the head of a pier and bridge deck looks clean, but bearings at the foot of a pier require a chamber, and may cause drainage problems which could create additional expenses. There are also problems of providing stability to the pier during construction. That is why bearings are usually preferred at the heads of piers.

Bank Seat (Dwarf Abutment Seat)

At the end of a riding span (the short-end span generally about 5 m.) of the bridge which is supported at the head of a slope formed by a cutting or embankment, the foundation may be a strip footing, a buried skeletal abutment, or a piled bank seat, depending upon the level of suitable founding strata. It is possible to detail this foundation in such a way that it enables the deck profile to continue into the earthworks without the supporting foundation being visible.

Piling

It may become necessary to employ piled foundations for bridge works where ground nearer the surface is too soft to sustain spread (acceptable sized) footings and is hence susceptible to substantial settlement. In addition to providing a means of supporting the foundation loads, the use of piling can make it possible for other groundworks (such as the construction of pile caps in place of spread footings) to be carried out at levels higher than might otherwise be possible. This can be beneficial where the foundation is to be built adjacent to a waterway or in waterlogged ground.

The choice of type of pile to be used is influenced by the ground conditions. Where rock or some other hard-bearing stratum occurs at an accessible depth, preformed piles driven to provide an end-bearing is an attractive proposition. Steel H-piles are more easily driven, cut, and extended, than reinforced concrete precast piles. However, it is self-evident that reinforced concrete is a more suitable material when corrosive conditions exist. Preformed piles can be driven at a rake of up to 1 : 4, thereby better absorbing the horizontal forces.

Piles of a large diameter are normally installed vertically. But it is still possible to absorb horizontal loads in this position, though this gives rise to bending in the piles. Methods of assessing the horizontal-load capacity of large-diameter piles have been developed and these utilize subgrade resistance in combination with stiffness of the pile. The techniques of constructing large-diameter bored piles are best suited to cohesive soils. Granular layers near to the surface can be successfully dealt with, but at greater depths the risks of the shaft-sides collapsing, become greater.

Piling adds to the cost of a bridge, so that the practicability of providing traditional footings always merits careful investigation. Even where the soil will only permit low-bearing pressures, it is cheaper to provide extensive spread footings than to employ piles. But individual cases differ and settlement problems may not permit footings.

For more details on piles see Sec. 7.4 and Sec. 7.5 of this chapter. This is also discussed in Ch. 11 of this book.

Under-Reamed Small-Diameter (up to 50 cm diameter) Bored Piles

Under-reamed small-diameter piles are bored cast *in situ* concrete piles, having one or more bulbs formed near the bottom by enlarging the bore hole for the pile-stem by an under-reaming tool. These piles find applications in certain land situations in different types of soils, where foundations are required to be taken down to a certain depth to avoid the undesirable effect of seasonal moisture changes (as in expansive soils, e.g., black cotton soils) or to reach strata or obtain adequate capacity for downward, upward and lateral loads. In expansive soils the pile cap should be cast 2 to 5 cm above the finished soil level.

A pile with only one bulb is known as single under-reamed pile, while one with more than one bulb is known as multi under-reamed pile. Generally, the diameter of under-reamed bulbs is kept equal to 2.5 times the diameter of pile-stem. However, it may vary from 2 to 3 times the stem diameter, if required, depending upon the design requirements and feasibility of construction.

Under-Reamed Large-Diameter (more than 100 cm diameter) Bored Piles (or Drilled Caissons)

Urban area flyovers sometimes demand large-diameter bored piles, with or without enlarged or 'belled-out' bases. The various aspects of design, construction, testing, and economics of large-diameter bored piles are basically same as those for other piles.

The savings in cost incurred by the use of under-reamed large-diameter bored piles are mainly due to savings in material excavated from the pile borehole, and in concrete used to replace the excavated material. However, in difficult ground such as boulder clays containing lenses of silt or sand, or in cohesionless soil, it is difficult to form a large diameter under-ream. Also, because of the longer time taken to form an under-ream by mechanical means or hand excavation (compared to the time taken to drill straight-sided piles), the economic advantages of the under-reamed large-diameter bored pile are somewhat marginal even in favourable ground such as clay.

Excavation for the under-ream is achieved by a belling bucket rotated by drill rods. Two types of belling buckets are used. The one generally favoured has arms which are hinged at the top of the bucket (Fig. 7.1) and are actuated by drill rods. The arms are provided with cutting teeth and the soil excavated is removed with the bucket. This type cuts to a conical shape, which has the advantage of maintaining stability in fissured soils. Besides, the arms are forced back into the bucket when it is raised from the hole.

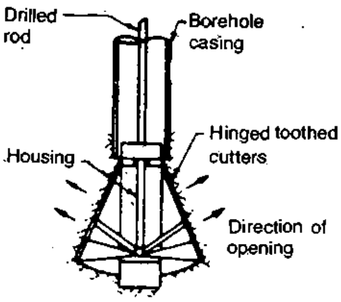


Fig. 7.1 *Top-hinged belling bucket*

The other type (Fig. 7.2) has arms hinged at the bottom of the bucket. This type has the advantage of being capable of cutting a larger bell than the top-hinged type; and because the action is always on the base of the hole, it produces a cleaner base with less loose and softened material. However, the hemispherical upper surface of the bell is less stable than the conical surface, and the bottom-hinged arms have a tendency to jam in the hole when raising the bucket.

Belling buckets normally cut to base diameters of up to 3700 mm, although diameters of as much as 7300 mm, are possible with special equipment. It is not usually practicable to form bells on piles having shafts of less than 760 mm diameter. Although the base of a mechanically under-reamed pile can be cleaned by specially designed mechanical tools, this is a somewhat tedious operation, and it is generally preferable to clean out the base by hand so that all soil crumbs and softened material are removed.

Enlarged bases can be formed in stable and relatively dry soils or rocks by hand excavation. This requires some form of support to the roof of the bell, to ensure safety of the workmen. One method of support which has been used is in the form of a "spider" consisting of a number of hinged steel ribs. The assembly is lowered down the borehole and the ribs are then expanded to force them into contact with the roof of the bell. If very large base areas are required, tunnels can be driven to connect the bells, and the whole base area can then be filled with concrete, which is suitably reinforced to achieve the required beam action.

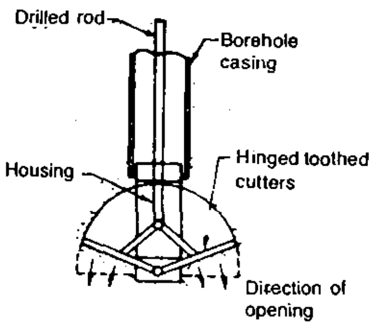


Fig. 7.2 *Bottom-hinged belling bucket*

It is often impossible to predict from ordinary site investigation boreholes, all the difficulties which may be encountered in attempting to form under-reamed bases on large-diameter piles. For this reason, it is a good practice to include an item in piling contracts for drilling a trial pile borehole in advance of the main piling contract. This item can be expensive as the selected piling contractor must bring his men and equipment onto the site and take them away again. But this procedure can often save a considerable amount of money at the main piling stage, since difficulties can be foreseen and any modifications to the pile design can be made, and if necessary, the idea of under-reaming the piles can be abandoned in favour of adopting deeper straight-sided piles. Test loading to check design assumptions can also be made at this preliminary trial stage.

It is desirable to make a close inspection of the base of all boreholes, with or without bells, to ensure that they are clean and free of softened material, and that the walls of the shafts are in a stable condition. Large-diameter bored piles are frequently used on a 'one-column-one-pile' basis, and it is unusual to provide more than three piles to a column. In these conditions, failure, even of a single pile due to faulty construction methods would have disastrous consequences. Therefore, during site supervision of piling contracts, the engineer should treat each pile in the same way as he would an ordinary pad foundation. That means the piling contractor must not be permitted to place concrete until the resident engineer or clerk of works has satisfied himself that the soil is not weaker than that taken as the basis for the pile design and that the hole is in a fit condition to receive concrete.

The following precautions are recommended in the construction and subsequent inspection of bored piles:

(i) The pile shaft should be supported by casing through soft or loose soils to prevent the walls of the shaft from collapsing. This casing may be extracted after concreting the shaft, depending upon whether or not the soil is too soft.

(ii) Casing should be provided to seal off soft water-bearing soil layers. Any soil, adhering to the inside of the casing, should be cleaned off, before inserting the casing in the borehole, and again before placing the concrete. The casing should be drilled into an impervious soil layer beneath the water-bearing layer to a sufficient depth to maintain the seal effect until the remainder of the borehole is completed, and until the concrete is brought up above ground-water level.

(iii) Soil or rock cuttings removed from the pile borehole should be compared and correlated with the descriptions stated on the site investigation borehole records, as otherwise more investigation could be warranted.

(iv) Shear-strength tests should be made when necessary on undisturbed soil samples taken from the bottom of selected piles, as a check on design data.

(v) Bores for shallow piles, if too small to enable a man to go down and inspect the base, should be inspected by shining a light down the shaft. Any loose crumbs or lumps of soil should be cleaned out before the concrete is placed.

(vi) All deep pile holes should be 'plumbed' to the bottom immediately before concreting by lowering a cage to the full depth. The plumbed depth should be compared to the depth accurately measured immediately after completion of drilling. This check will ensure that no soil has collapsed into the borehole.

(vii) All piles having a shaft large enough to admit a man should be inspected immediately before placing concrete. Any loose soil adhering to sides should be cleaned off.

Caisson (or Well) Foundations

Such foundations are advisable in rivers where a heavy scour at flood time would otherwise bare the piles and lead to buckling. In such conditions, it could also be possible for the floating debris to get entangled inbetween the naked pile shafts which can exert unpredictable forces in them. Caissons are relatively easy to construct, and other things being equal, could more than compete with piled solutions. Their stability essentially derives from the considerable passive resistance mobilised from the soil grip below the maximum scour level and the huge end-bearing resistance.

Diaphragm Walls

For vertical cuttings, such as those required for lengths of sunken road, the work of excavation can often be minimized by using such constructional techniques as *contiguous bored piling* or *diaphragm-wall construction*, in place of conventional retaining walls. Since these techniques are usually associated with particularly difficult ground conditions, such as those arising with over-consolidated clays, the design approach will involve consultation with authoritative experts.

The construction of a diaphragm wall requires excavation of a deep trench in short lengths, using a bentonite slurry to support the faces of the excavation where necessary. A prefabricated cage of reinforcement is lowered into the excavation, and concrete is placed by tremie. Each short length forms a panel, and the joints between panels introduce some measure of structural discontinuity into the wall.

Reinforced Earth Construction for Abutments and Retaining Walls

A rapidly constructed and lighter form of retaining wall construction has been developed in recent years. This is based on the use of facing panels that are stacked without any attempt to provide fixity or bond with adjacent units, but each panel is tied back to the earth-fill, by straps that are buried in the retained embankment during construction. The facing to a reinforced earth wall consists of precast concrete panels. In addition to giving a lighter wall than could be achieved in traditional reinforced concrete construction, this technique has the merit of allowing construction to proceed on ground which may not be suitable to form the foundation for a conventional wall. Joints between the facing panels are usually made to accept movements which may arise due to settlement. Such a flexibility of the finished construction makes it highly tolerant to differential settlement without affecting its structural integrity. The technique has been used for bridge abutments as well as free-standing walls. Some settlement is likely to occur, although this is nominal where the ground conditions are firm. In circumstances where the use of conventional abutments would involve extensive groundworks associated with foundations, it may be found that the use of reinforced earth could provide a solution which makes substantial savings by eliminating much of the groundwork. However, since there are only a few agencies who own patents for this type of construction, the cost may not always be as low as it should be relatively.

Analysis and design of Reinforced Earth Retaining Walls and Abutments is discussed in Ch. 16 of this book.

7.3 OPEN FOUNDATIONS

These foundations are generally of the following type:

(a) Isolated, combined, and strip footings.

(b) Raft foundations.

The design of the open foundations is based on complete subsoil investigations. But in the case of low safe-bearing capacity of soil, such foundations may have to be disallowed, owing to the possibility of problems of long-term settlement. However, some of the important features are given below for guidance:

(i) The selection of the appropriate type of open foundation will normally depend upon the magnitude and disposition of structural loads, requirements of structures (settlement characteristics, etc.), type of soil or rock encountered, allowable bearing pressures, etc. Where rocky stratum is encountered at shallow depths, it may be preferable to adopt open foundations because of its advantage in permitting proper seating over rock and speed of construction work.

(ii) In case the two adjoining foundations are at different levels, the horizontal distance between the foundations should be kept sufficiently large depending upon the type of soil on which the foundations are resting, to avoid overlapping of foundation pressure zones and consequent distress during construction.

(iii) For footings and raft foundations in erodible strata, protection against scour should be provided by means of suitably designed flooring, cut-off walls and launching aprons.

(iv) In the case of plain concrete, brick, or stone masonry footing, the load from the pier or column should be taken as dispersed through the footing at an angle of less than 45°C.

(v) For reinforced concrete footings, the design should satisfy the relevant provisions of standard design method.

(vi) The design of raft should be based on the assumption that it is resting on elastic soil medium, and guidance may be taken from standard literature on the subject.

(vii) The minimum thickness of the footing should not be less than 300 mm.

(viii) The protective works should be completed before the floods so that the foundation does not get undermined.

(ix) Excavation for open foundations should be done after taking the necessary precautions.

(x) Where blasting is required to be done for carrying out excavation in rock and there is likelihood of danger to the adjoining foundations or other structures, necessary precautions should be taken to prevent any such damage.

(xi) For laying the foundation concrete after de-watering, either of the following procedures should be adopted:

(a) A pit, moat, or trench, deeper than the foundation level as necessary, may be dug around the foundation pit so that the water is kept below the foundation level until after the concrete has suitably set.

(b) The water-table is depressed by *well-point* or other methods, until after the concrete has suitably set.

(xii) For laying the foundation concrete under water, if the percolation of water is heavy, it is advisable to lay the foundation concrete by skip boxes or tremie pipe. In the case of flowing water or artesian conditions, the flow should be stopped or reduced as far as possible during placing of concrete until after it has set. No pumping out of water should be permitted from the time of placing of concrete up to at least 24 hours after placement, for it may blow up due to the removal of balancing pressure against upward water pressure.

(xiii) All spaces excavated and not occupied by abutments, piers, or other permanent works, should be refilled with earth up to the surface of the surrounding ground with sufficient allowance for settlement. All backfill should be thoroughly compacted and in general, its top surface should be neatly graded.

(xiv) In case of excavation in rock, the annular space around the footing should be filled up to the rock surface with lean concrete of 1:3:6 mix.

7.4 PILE FOUNDATIONS

Pile foundations are suited for adoption in the following situations:

(i) Availability of good founding strata below large depth of soft soil.

(ii) Need to have very deep foundations beyond the limit of pneumatic operations (usually depth beyond 35 m or so).

(iii) Founding strata underlying deep 'standing' water, the strata being very hard not permitting easy sinking of wells.

(iv) Economic factors deciding the use of piles as compared to wells.

Classification of Piles

(i) Precast driven piles (soil displacement type)
(ii) Driven cast *in situ* piles (soil displacement type)
(iii) Bored cast *in situ* piles (soil replacement type)
(iv) Bored precast piles (soil replacement type)
(v) Driven steel piles (soil displacement type).

Precast Driven Piles

Advantages

(i) ensured quality of concrete
(ii) correct disposition of reinforcement
(iii) least deviations in structural dimensions
(iv) fast pace of construction.

Disadvantages

(i) not suitable for depths greater than about 24 m, as lengthening of piles is difficult and may involve proprietary systems for jointing
(ii) difficult to be driven through stiff layers, or layers containing, boulders
(iii) unsuitable if piles are to be anchored or keyed to rocky strata
(iv) limited load carrying capacity on account of relatively small structural size.

Driven Cast in situ Piles

Advantages

(i) suitable for larger depths of the order of 50 m or so
(ii) can penetrate a harder strata by virtue of the steel shoe at the end.

Disadvantages

(i) in the cast *in situ* concreting operations, there is a likelihood of segregation of concrete due to dropping it from height and interference by the reinforcement
(ii) in uncased piles 'necking' takes place each time the casing is lifted, while concreting is being done to fill up the hole
(iii) in softer soils with uncased piles, the caving in of the soil, as also waving, is possible
(iv) these piles are also generally of small diameter (under 100 cm) and therefore may not be very competitive particularly when negative friction is expected to develop and/or keying with rock is required.

Bored Cast in situ Piles

Advantages

(i) very large depths can be achieved
(ii) larger diameter piles are possible, achieving higher load carrying capacity
(iii) no disturbance of the surrounding mass of soil and hence no reduction in soil resistance
(iv) specially suited where harder and stiffer strata have to be penetrated (e.g., stiff clay)
(v) easy for keying into rock
(vi) in view of larger load carrying capacity the number of piles can be reduced under each foundation, which may reduce construction time.

Disadvantages and difficulties

(i) larger equipment cost
(ii) precise constructional control on concreting procedure, concrete mix, etc.
(iii) maintenance of correct specific gravity of bentonite slurry at about 1.1 to 1.2, otherwise sides may collapse and cave in
(iv) reduced skin-friction (since the pile is not driven into the soil).

Safe Load Carrying Capacity

It shall be lesser of the following two values:

(a) ultimate load carrying capacity based on the soil parameters surrounding the pile divided by a suitable factor of safety.
(b) structural strength of the pile.

These shall be assessed as given here.

Ultimate Load Carrying Capacity Based on Soil Parameters

Static Formula (based on soil data)

$$R_u = R_b + R_f - W - R'_f$$

where R_u = ultimate load carrying capacity
R_b = ultimate base resistance
R_f = ultimate positive skin resistance
W = self weight of pile
R'_f = ultimate negative skin friction
$R_b = A_b \times f_{bu}$
where A_b = plan area of base of pile
f_{bu} = ultimate bearing capacity of the soil at the pile base
and $R_f = A_s \times f_s$
$R'_f = A'_s \times f'_s$
A_s and A'_s = surface areas of pile in positive and negative skin friction zones, respectively (latter, for instance, exists from normal bed level down to max. scour level).
f_s and f'_s = average skin friction or 'adhesion factor times cohesion value' per unit area of the pile surface (depending on soil type) in positive and negative skin friction zones, respectively.

For working out safe load carrying capacity of the pile, a factor of safety of 2.5 may be adopted.

In addition, 'block failure' and 'group action' must be considered. For details see Ch. 11 ahead.

Structural Strength of Pile

The structural strength of a pile section shall be assessed based on its axial load and moment (caused by lateral loads) based on short or long column action, depending on the location of upper and lower points of fixity in it. (See the earlier mentioned reference for more details, including the effects of pile cap movement, defective construction, buckling effect, etc.)

In addition to the above points the following general features shall also be satisfied in design and construction of piles.

Spacing of Piles

(a) Friction piles—spacing, centre to centre, not less than perimeter of the pile.
(b) End bearing piles—spacing, centre to centre, not less than twice the least width of pile.
(c) Generally—2.5 times the bigger dimension of pile section in plan.

Size of Concrete Piles

Not less than 0.75 m diameter or equivalent section area for bridge foundations in major river, and not less than 0.4 m diameter or equivalent section for other locations, viz., wing walls, foundations for flyovers, etc.

Rake in Piles

The maximum rake normally should not be more flat than the following:

(i) 1 in 8 for pile diameter 0.75 m and above
(ii) 1 in 5 for smaller diameter bored piles
(iii) 1 in 4 for smaller diameter driven piles.

Tolerance in Pile Alignment

(i) for vertical piles: 75 mm at piling platform level and tilt not exceeding 1 in 50
(ii) for raking piles: tolerance of up to 5% in rake and up to 75 mm in position at platform level
(iii) where raking piles are installed from a level significantly different/higher than the ground level, then the 75 mm lateral tolerance mentioned above shall be suitably increased due to the effect of error in the rake.

Pile Cap

All pile caps shall be in reinforced concrete, and their sizes fixed, taking into consideration, the allowable tolerances mentioned above. A minimum off-set of 150 mm shall be provided beyond the outer faces of the outermost piles in the group. For pile caps resting on earth a levelling course of minimum 80 mm thickness on lean concrete shall be provided. The attachment of the pile-head to the cap shall be adequate for transmission of loads and forces. Concrete piles are stripped off at the top so that their reinforcement is exposed and anchored into the cap. About 50 to 100 mm of the pile itself should project into the pile cap, and the bottom reinforcement in the cap suitably adjusted. In marine conditions, or in areas exposed to the action of harmful chemicals, etc., apart from the use of densely compacted concrete of a higher grade, the pile cap and the piles up to about 600 mm above low water or tide level should be protected with a suitable anti-corrosive paint. High alumina cement, i.e., quick-setting cement shall not be used in marine construction, instead blast furnace slag cement is preferable. Pile caps should be designed against shear as well as bending due to pile forces. Critical section for shear is where a 45° plane from column face meets the mid depth plane of cap. Piles falling 150 mm or more inside such section may be ignored in contributing shear at this section.

Mix of Concrete

The concrete used in the piles shall not be leaner than 1 : $1^1/_2$: 3 or equivalent controlled concrete with cement content not less than 350 kg/cu.m of concrete from durability considerations.

Driven Precast Concrete Piles

Longitudinal reinforcement The section area of longitudinal reinforcement shall be based on the actual design but shall not be less than the following percentages of the cross-sectional area of the piles:

(i) for piles with a length less than 30 times the least width, 1.25%
(ii) for piles with a length of 30 to 40 times the least width, 1.5%
(iii) for piles with a length greater than 40 times the least width, 2%

The curtailment of the reinforcement along the length of pile shall avoid sudden discontinuity which may cause cracks during heavy driving.

Lateral reinforcement The lateral reinforcement is of particular value in restraining driving stresses and should be in the form of hoops, spirals or closed links. The minimum diameter of bars for this purpose shall not be less than 6 mm. The lateral reinforcement in the pile from each end for a distance of about 3 times the least width or diameter shall not be less than 0.6% of the gross volume, and in the body of the pile not less than 0.2% of the gross volume.

Driven Cast in situ Concrete Piles

(i) Reinforcement in the pile should preferably be provided in the entire length and shall be based on design requirements. However, longitudinal reinforcement within the pile shaft shall not be less than 0.4% of the cross-sectional area.

(ii) Where the casing pipe is withdrawn for the formation of cast *in situ* piles, the concreting should be done with necessary precaution to minimise the softening of the soil by excess water. Cast *in situ* piles shall not be allowed where mud flow conditions exist.

Bored Cast in situ Piles

(a) Reinforcement in the pile shall be provided in the entire length depending upon the manner of transmission of the load by the pile to the soil, and shall not be less than that specified above for driven cast *in situ* concrete piles.

(b) In soils which are stable it may often be possible to drill an unlined hole and place the concrete without having a casing. In such cases even if 1 : $1\frac{1}{2}$: 3 concrete is used, the permissible stresses shall be limited to 90% of the design value. In cases in which side soil can fall into the hole, it is necessary to stabilize the sides of the bore hole with drilling mud, e.g., bentonite, and where possible a suitable steel lining may be used. The liner may be withdrawn when the concrete is poured in or it may be left in position permanently especially in cases where the aggressive action of the ground water is to be avoided or in the case of piles built in marine/muddy conditions.

Precautions for Concreting under Water*

(i) The concreting of the pile must be completed in one continuous operation, using tremie method.

(ii) The concrete should be easily workable, rich in cement (not less than 370 kg/m^3) and of slump not less than 150 mm.

(iii) When concreting is being carried out under water, a temporary casing should be installed to the full depth of soil, except the portion in rock, in order the fragments of soil cannot drop from the sides of the hole into the concrete as it is placed.

(iv) The tremie pipe will have to be large enough with due regard to the size of aggregate. For 20 mm aggregate the tremie pipe should be of diameter not less than 150 mm, and for larger aggregate larger diameter tremie pipes are required.

(v) The first charge of concrete should be placed with a sliding plug pushed down the tube ahead of concrete, so as to prevent mixing of concrete with water in pipe.

(vi) The tremie pipe should always penetrate well into the placed concrete, with an adequate margin against accidental withdrawal if the pipe is surged to discharge the concrete.

(vii) The pile should be concreted wholly by tremie and the method of deposition should not be changed partway of the pile (to prevent the laitance from being entrapped within the pile).

(viii) All tremie tubes should be scrupulously cleaned after use (and in any case before any concrete in them sets).

(ix) The top of concrete in a pile shall be brought above the cut-off level to permit removal of all laitance and weak concrete before pile cap is laid. This will ensure good concrete at the cut-off level after stripping open its bars for embedment into cap.

Load Tests on Piles

Ultimate and working load tests shall be carried out as outlined in various standards as applicable to the particular contract in question.

7.5 SMALL DIAMETER SINGLE AND DOUBLE UNDER-REAMED AND RELATIVELY SHORT BORED PILES (... I.S.2911 — PART III, 1973)

Construction of Small Diameter Under-reamed Piles

The various stages involved in the construction of small diameter short under-reamed piles are as given here:

(a) Boring by augers
(b) Under-reaming by under-reamer
(c) Placing reinforcement cage in position
(d) Concreting of pile
(e) Concreting of pile caps.

One of the equipments used for boring and under-reaming has been developed by the Central Building Research Institute, Roorkee, India. It is covered by an Indian patent and is licensed to M/s MSJ (Engineers) and Co., Roorkee. Boring is done with the help of a spiral auger. The use of a boring guide is essential to keep the bore holes vertical and in position. Each guide is provided with a circular collar and four arms. The collar is fixed to the boring guide on the lower side and it does not allow the mouth of the bore hole to widen due to frequent insertion and removal of the auger and other boring tools. After setting the guide assembly in position, the spiral auger is introduced into the circular collar of the guide by opening out the two sets of flaps of the guide assembly. The auger is pressed down and rotated manually until the spirals are half full of earth. The auger is then taken out and earth removed. The auger is again introduced and the boring process repeated till the required depth is reached.

Under-reaming or locally bulbing the stem of bore hole at the required depth is achieved by means of the under-reamer consisting of an assembly of two collapsible blades

* Also refer to the author's book "Concrete for Construction — Facts and Practice" for more details regarding 'under water concreting by tremie'.

fixed around the central shaft with a detachable bucket for receiving the cut soil. The equipment is attached to extension rods and lowered down the hole (which has already been bored to the required depth) until the bucket rests at the bottom of the bore hole. The guide flaps are then closed. The tool is pressed down constantly, and at the right elevation, rotated slowly. The cutting blades of the tool open out and start cutting the sides of hole. The loose earth is collected in the bucket at the bottom. When the bucket is full, the assembly is pulled out and the bucket is emptied. The depth of the bore hole is checked each time before insertion of the under-reamer so that any loose earth spilled from the bucket is removed (otherwise the bucket position will get shifted upwards due to loose soil lying at the bottom, and this will shift the position of the bulb). The under-reamer is then lowered into the bore hole and the process repeated until the cutting blades have expanded fully and no further earth is cut by the blades. Generally, removal of about eight buckets full of earth is required for completion of one under-ream for an average sized bulb (pile stem diameter up to 50 cm).

In the case of double under-reamed pile, further boring is done, after the first bulb is formed. After boring to the required depth, under-reaming for the second bulb is carried out. The dimensions of the bulb can be checked by means of a graduated GI pipe assembly. After the bore hole and under-ream are checked, the reinforcement cage, already fabricated, is lowered into the hole. Concreting of the pile is carried out through a concreting funnel placed at the mouth of the bore hole and care is taken to see that the top of the pile shaft is 5 cm higher than the bottom of the pile cap to be cast on it. Care must be taken against possibility of segregation in poured concrete.

Details of Pile and Under-reamed Bulb

In deep layers of expansive soil a minimum pile length of 3.5 m is recommended. Where the ground movements become negligible, in shallow depths of expansive soils, and other poor soils, the length may be reduced depending upon the load requirements, and the piles taken-down to at least 50 cm in the stable zone (i.e., a zone where there are no ground movements due to seasonal moisture changes). The pile length may be increased for higher loads.

The diameter of manually bored piles ranges from 20 to 37.5 cm. The spacing of the piles should be considered in relation to the nature of the ground, the type of piles, and the manner in which the piles transfer loads to the ground. Generally, the centre to centre spacing for under-reamed piles should not be less than 2 D_u (where D_u is the under-reamed diameter). It may be reduced to 1.5 D_u when a reduction in load carrying capacity of 10% is allowed. For the spacing of 2 D_u the bearing capacity of pile group may be taken equal to the number of piles multiplied by the bearing capacity of the individual pile. If the adjacent piles are of different diameters, an average value for spacing should be taken. The maximum spacing of the under-reamed piles should not normally exceed 2 $^1/_2$ metres so as to avoid heavy caps.

In double under-reamed piles of stem diameter less than 30 cm, the center-to-center vertical spacing between the two under-reams may be kept equal to 1.5 D_u, while for piles of 30 cm and more, this distance may be reduced to 1.25 D_u. The upper bulb should not be placed too close to the ground. The minimum desirable depth of the centre of this bulb is 1.5 m or 2 D_u, whichever is greater.

Load Carrying Capacity of Small Diameter Under-reamed Piles Based on Soil Properties

The under-reamed pile is nominally reinforced with longitudinal bars of 10 to 12 mm diameters and 6 mm diameter rings. The details of the carrying capacity and *minimum reinforcement* are shown in Table 7.1, but structural design of the stem section should be carried out for actual stresses in order to decide the amount of steel and grade of concrete. A clear cover of 4 cm is generally provided to the reinforcement.

Load Test on Small Diameter Under-reamed Piles

Piles may be tested for determining their load carrying capacity in compression, tension and lateral loading. Two categories of tests are conducted,

(i) initial test
(ii) routine tests

Initial tests should be carried out on test piles, not on working piles. In case the initial tests show consistently higher or lower values than the estimated safe allowable loads on piles, designs should be re-examined and necessary modifications should be made. Routine tests are carried out as checks on working piles.

Procedure for Initial Test (Axial Compression)

Following are the recommendations of Indian Standard IS:2911 (Part III) - 1973.

1. The test shall be carried out by applying a series of loads to the pile unaided by any other support. Pile groups may be tested as free-standing piles or piled foundations, as specified.

   The load shall preferably be applied by means of a hydraulic jack reacting against a loaded platform, or rolled steel joists or suitable load frame held down by soil anchors and piles or other anchorage. The anchor piles may also be working piles, but they shall be sufficient in number and adequately reinforced to take the full tension with a proper factor of safety. The reaction available for loading should not be less

Table 7.1 *Safe loads for vertical under-reamed piles in sandy and clayey soils including black cotton soils as recommended in the IS:2911 Part III-1973*

		'Minm.' Reinforcement* (See Notes 3 and 13 below)			Safe Loads									
		Longitudinal		Spacing	Bearing Resistance (Compn.)				Uplift Resistance (Tension)				Lateral thrust	
Dia. of pile (D)	Under-ream Dia (D_u)	No. of bars	Dia.	of 6 mm dia. rings	Single under-reamed	Double under-reamed	Increase per 30 cm length	Decrease per 30 cm length	Single under-reamed	Double under-reamed	Increase per 30 cm length	Decrease per 30 cm length	Single under-reamed	Double under-reamed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
cm	cm		mm	cm	t	t	t	t	t	t	t	t	t	t
20	50	3	10	18	8	12	0.9	0.7	4	6	0.65	0.55	1.0	1.2
25	62.5	4	10	22	12	18	1.15	0.9	6	9	0.85	0.70	1.5	1.8
30	75	4	12	25	16	24	1.4	1.1	8	12	1.05	0.85	2.0	2.4
37.5	94	5	12	30	24	36	1.8	1.4	12	18	1.35	1.10	3.0	3.6
40	100	6	12	30	28	42	1.9	1.5	14	21	1.45	1.15	3.4	4.0
45	112.5	7	12	30	35	52.5	2.15	1.7	17.5	25.75	1.60	1.30	4.0	4.8
50	125	9	12	30	42	63	2.4	1.9	21	31.5	1.80	1.45	4.5	5.4

* Mild Steel

NOTE

1. The value of bearing resistance, uplift resistance and lateral thrust, given in the table are for a minimum pile length of 3.5 m, except in double under-reamed piles of more than 30 cm diameter. In such double-reamed piles, the minimum recommended lengths for 37.5, 40, 45 and 50 cm piles will normally be 3.75, 4.0, 4.5 and 5.0 m, respectively, so as to suitably accommodate the bulbs at specified distances.
2. Longitudinal bars may be curtailed or eliminated towards the toe depending upon the stresses in pile section.
3. For under-reamed piles subjected to a pull and/or lateral thrust, the requisite amount of steel should be provided as per proper design.
4. Values given in columns 14 and 15 for lateral thrust may not be reduced for changes in pile lengths and are fairly conservative. Higher values may be adopted after conducting lateral load tests on single or group of piles for surety.
5. In 25 and 30 cm dia., normal under-reamed piles when concreting is done by tremie, equivalent reinforcement in shape of single angle iron piece placed centrally may also be considered.
6. When a pile designed for a certain safe load is found to be just short of the load required to be carried by it, an over load of up to 10% may be allowed on it.
7. For working out the safe load for a group of the above piles, the safe load of individual piles is multiplied with the number of piles in the group. This would be applicable for piles taking lateral thrusts also.
8. Only 75% of the above safe loads should be taken for piles in which the bore holes are full of sub-soil water during concreting. When water is confined to the bucket portion only, no such reduction need be made.
9. In sandy soils when boring and under-reaming under water, minimum size of pile recommended is 25 cm.
10. In multi-under-reamed piles the depth of the center of upper bulb below ground level shall be kept a minimum of two times the diameter of the under-ream bulbs.
11. The values given above may be increased by 50% for broken wire condition in the design of transmission line tower footings.
12. Safe loads for multi-under-reamed piles may be worked out from the table by allowing additional 50% of the load as per col. (6) for each additional bulb. Increase in capacity due to increase in length will be as per col. (8).
13. Based on actual structural design, either the pile shaft should be suitably increased in diameter and/or additional reinforcement provided (treating it as a short column) to control the stresses in section design. The indicated reinforcement is only the minimum but actual amount must be structurally designed, to suit.
14. The load carrying capacity of an under-reamed pile may be determined from load test. In the absence of actual tests, the safe loads allowed on piles under-reamed to 2.5 D may be taken from above table (IS:2911, part III-1973). The safe loads given in the table apply to both medium compact sandy soil and clayey soils of medium consistency. For dense sandy ($N \geq 30$) and stiff clayey ($N \geq 8$) soils, the loads may be increased by 25%. However, the values of the lateral thrust should not be increased unless stability of the top soil (i.e., strata to a depth of about three times the stem diameter) is ascertained. On the other hand, a 25% reduction should be made in case of loose sandy ($N \leq 10$) and soft clayey ($N \leq 4$) soils. However, these are only rough guidelines to arrive at a first shot design which must be substantiated by actual pile load tests which alone can decide the acceptable design.

than three times the estimated safe load carrying capacity of the pile. The jack should be of adequate capacity, preferably with a remote control pump, and should have a pressure gauge or other suitable device for reading the applied loads.

2. Readings of settlement and rebound shall be recorded with the help of at least three dial gauges of 0.02 mm sensitivity, positioned at equal distances around the pile. The dial gauges shall be fixed to datum bars resting on non-movable supports at least $5D$ (subject to a maximum of 2.5 m) away from the piles where D is the pile stem diameter.
3. The test load shall be applied in increments of about 1/5 of the estimated safe load. At each stage of

loading/unloading, the load shall be maintained till the rate of movement of the pile top is not more than about 0.02 mm per hour.

4. Loading shall generally be continued up to 2 $^1/_2$ times the estimated safe load or to a settlement of 7.5% of the bulb diameter, whichever is earlier.
5. The safe load on the pile shall be the least of the following:
   (i) Two-thirds of the final load at which the total settlement attains a value of 12 mm, unless it is established that a total settlement different from 12 mm is permissible in a given case on the basis of nature and type of the structure; in the latter case the actual total settlement permissible shall be used for assessing the safe load instead of 12 mm.
   (ii) Fifty per cent of the final load at which the total settlement equals 7.5% of the bulb diameter.

Procedure for Routine Test (Axial Compression)

Loading shall be carried out up to 1 $^1/_2$ times the working safe load. The procedure followed for the test and determination of the allowable load shall be the same as per initial test excluding item (ii) above.

7.6 CAISSONS OR 'WELL' FOUNDATIONS

Suitability

Caisson construction is almost restricted to major foundation works, where other types of foundations cannot satisfy the requirements economically. Usually a caisson is advantageous as opposed to other types of deep foundations when some of the following conditions exist:

(i) A massive substructure is required to extend to well below the river bed in order to attract necessary net soil resistance against overturning, heavy scour, rolling boulders, and floating debris. Under such conditions piles would obviously be unsuitable.
(ii) The substrata contains large boulders which obstruct penetration of piles.
(iii) The foundation is subjected to large lateral forces.

Historical Note

The essential feature of caissons is that they are constructed above the ground or water level and then sunk as a single unit to the required depth, and also that this unit forms part of the permanent works. Because extensive temporary works, such as sheet piled cofferdams, are not required, they are specially suited to work in deep and fast-flowing waterways. Open well-type caissons were used by the inhabitants of India, Burma (now Myanmar) and Egypt for many centuries for the foundations of river bridges. The masonry of the wells was built on timber curbs, and the caissons sunk by hand excavation from within the wells. Skin divers doing the excavation could not work deeper than 6 m, which limited the usefulness of caisson foundations of this type to sites where a firm inerodible stratum could be reached within this depth. However, the engineers who were responsible for bridging over mighty rivers in India in the late nineteenth and early twentieth centuries, adopted the methods of using grabs and sand-pumps for underwater excavation in the wells. By these methods they were able to sink caissons to depths of more than 30 m. A notable example of this construction was the caissons for the Hardinge Bridge over the Lower Ganges, where the river piers were sunk to depths varying between 32 and 36 m below river bed level.

Compressed air was first used in bridge caissons by John Wright in 1851 for the piers of Rochester Bridge, and a few years later by Isambard Brunel at Saltash Bridge. Its first use for the foundations of very large bridges was by James B Eads for the St Louis Bridge over the river Mississippi, commenced in 1869. The two river piers were sunk under compressed air to depths of about 30 m, which was a notable achievement, since the physiological effects of working under high air pressures were more or less unknown at that time. The sinking methods devised by Eads have only been changed in matters of detail up to the present day. A new development in caisson construction known as the floatation caisson principle was introduced in 1936 by Daniel E Moran for the San Francisco-Oakland Bay bridge.

Usually the limiting depth of cofferdams is about 20 m. Caissons are therefore essential for constructing foundations through water or through unstable shifting ground to depths greater than this.

Caisson Construction and Sinking Methods (also refer to the author's other Book: "Concrete Bridge Practice... Constructions, Maintenance and Rehabilitation".)

Construction of Well Curb (Shoe)

The normal practice in caisson construction is to build the 'shoe' on land and slide or lower it into the water for floating, out to the site, or to construct it in a dry dock which is subsequently flooded to float out the shoe. Land caissons are of course constructed directly in their final position. Caisson shoes, constructed on the bank of a river or other waterway, are slid down launching ways into the water, or rolled out on a horizontal track and then lowered vertically by a system of jacks and suspended links. Gently sloping banks on a waterway with a high tidal range favour construction on sloping launching ways, whereas steep banks either in tidal or non-tidal conditions, usually require construction by rolling out on a horizontal track.

Care must be taken to avoid distortion of the shoe during construction. On poor ground the usual practice is to lay a thick blanket of crushed stone or brick rubble over the building site and to support the launching ways on timber or steel piles.

Economy in temporary works is given by constructing caissons in their final position. This can be done for land caissons, and for river work by constructing caissons on the dry river bed or on sand-islands (in up to 6 m of water). This is only advisable when the low water periods can be predicted reasonably accurately, and there is no risk of sudden 'flash' floods.

Towing a Floating Caisson to Sinking Site

The operation of towing a floated caisson from the construction site to its final location must be carefully planned. Soundings must be taken along the route to ensure an adequate depth of water at the particular state of tide or river stage at which the towing is planned to take place. An essential requisite of the launching, towing, and sinking programme is a 'stability diagram' for the caisson. This shows the draught at each stage of construction. In these diagrams the draught is plotted against the weight of the caisson for various conditions of free floating or floating with compressed air in the working chamber. The weight of each strake of skin plating and concrete within its walls to be added to give a desired draught can be read from the diagram. Also, the air pressures in the working chamber required to give any desired internal water level can be read off the appropriate lines.

Bed Preparation

The first operation is to take soundings over the sinking location to determine whether any dredging or filling is required to give a level bed for the caisson. A study should be made of the regime of the waterway to determine whether any bed movement is caused by vagaries of current. Such movement can cause difficulty in keeping a caisson plumb when landing it on the bottom, especially at the last stages when increased velocity below the cutting edge may cause non-uniform scour. Difficulties with bed movement can be overcome by sinking flexible mattresses of crushed stone on the sinking site, and sometimes by constructing control-dykes (or spurs) upstream and downstream that can alter the bed level considerably and also even it up.

Sometimes constructing a dyke (normal or inclined to the river bank) upstream of the site can silt up the area and very significantly reduce the water depth and velocity, which can be of immense help.

Supporting Structures

The various methods used to hold a caisson in position during its lowering include

(a) an enclosure formed by piling
(b) dolphins formed from groups of piles or circular sheet pile cells
(c) sinking through a sand island
(d) wire cables attached to submerged anchors.

The choice of method depends on the size of caisson, the depth of water, and particularly on the stability of the bed of the waterway. River bed conditions at the site of the Mackinac Bridge, USA, were favourable for the construction of a piled enclosure for the circular caissons on two pier sites. The 4.6 m wide space between the two steel shells was divided radially into eight watertight compartments. Four steel tubular towers were spaced at equal distances around the caissons. The towers were prefabricated and taken by barge to the site where they were lowered onto the river bed. Then steel H-beam piles were lowered down each of the pipe piles and driven to refusal. The space between the pipes and piles was grouted. Three towers were constructed in this way and connected by horizontal box-type trusses. After floating in the caisson the fourth tower was constructed and the enclosure completed by additional connecting trusses. A clearance of 30 cm was provided between the caisson and the towers.

The 67 m by 29 m caisson for the West anchorage of the Delaware Memorial Bridge, USA, was enclosed by a rectangular pen formed by two 9 m diameter sheet pile cells filled with sand on each long side of the caisson, and two steel pile dolphins formed from three vertical and three battered piles on the short (shoreward) side. Fendering spanned between the cells and dolphins to give a 1.2 m clearance around the caisson. After towing in the caisson another pair of dolphins was driven to complete the fourth side of the enclosure.

Sand islands were used for four of the caisson piers of the Baton Rouge bridge over the Mississippi river. The fast flowing river was known to cause deep scour, and bed protection was given to the sites of the two deepest piers in the form of 137 m by 76 m woven board mattresses. The islands were 34 and 37 m, respectively, in diameter, and were formed by steel plate sheets filled with sand. The shells were surrounded by a double row of piles. The sand islands narrowed the waterway, and this caused deep scour which the mattresses did little to prevent. The scour at Pier 3 was 12 m deep, and a similar depth of scour at Pier 4 caused the whole of the sand filling in the island to disappear in 2 to 3 minutes!

The external water pressure on the shell then pushed in the 9.5 mm plating which was torn apart. The caisson, which at that time had only penetrated 4.6 m into the river bed, tilted by 2.1 m in line with the bridge and 0.6 m in the other direction, and was plumbed with great difficulty. These experiences emphasize the hazards resulting from obstructions to flow caused by substantial temporary works in a river with an erodible bed.

The minimum of temporary construction and the lowest risk of bed erosion is given by the method of securing a floating caisson to submerged anchors; the caisson being moored between floating pontoons. This method is particularly suited to a multi-span structure when the high capital cost of an elaborate pontoon-mounted sinking plant can be spread over a number of caissons; whereas, if fixed stagings are provided for the piers of a multi-span structure, much time will be spent in driving and extracting piles for construction of the stagings at each pier site, with inevitable damage due to repeated re-use. A floating plant is highly mobile, and can be rapidly switched from one pier site to another to suit changing conditions of river level and accessibility at low water stages. It is advantageous in these conditions to design the floating plant to be adaptable to working in the dry.

Lowering Caissons

Four main methods are used for maintaining position and verticality of caissons during sinking. These are

(a) free sinking, using guides between caissons, and fixed stagings or floating plant
(b) lowering by block and tackle from piled stagings or floating plant
(c) lowering by suspension links and jacks from piled stagings or floating plant
(d) lowering without guides but controlling verticality by use of air domes.

Sinking Open Well Caissons

When a caisson reaches the stage where concrete has to be added to maintain downward movement, the rate of sinking should be governed by a fixed cycle of operations. The usual procedure is to maintain a 24-hour cycle comprising excavation from the wells, erecting steel plating or formwork in the walls (steining) and concreting a 1.2 to 1.5 m lift of the walls.

In floated caissons, the top of the skin plating should always be maintained at least about 1 m above water level to guard against an unexpected rise in level. However, the freeboard should not be so much that the centre of gravity is too high to give proper control of verticality.

Control of verticality can be achieved by one or a combination of the following methods:

(a) adding concrete on one side or the other
(b) differential dredging from beneath the cutting edge
(c) pulling by block and tackle to anchorages
(d) jetting under the cutting edge on the 'handing' side
(e) placing kentledge on one side or the other.

More details of this have been discussed in Ch. 7 of the author's other book, *Concrete Bridge Practice — Construction, Maintenance and Rehabilitation*, under 'construction considerations'.

Excavation Method

Grabbing is the most commonly used method of excavating from the open wells although ejectors, operated by compressed air or water pressure, have been used in sandy soils.

De-watering for Freeing a 'Hanging' Caisson, and the Phenomena of Sand Blow

If excavation becomes difficult, a caisson can be partially de-watered or pumped out. This increases its 'effective' weight, so increasing the sinking effort. The procedure may be dangerous where the cutting edge has only just penetrated a clay stratum overlain by waterbearing sand. The water in the sand may then force its way in through a limited thickness of the clay, causing a localized *blow* in and up the caisson, followed by tilting of the caisson which is not easy to rectify. In the process the caisson actually hangs on one side, which may even lead to cracking.

Explosives fired in the wells can be used to cause a jerk, creating a temporary breakdown in skin friction; but they are rarely effective in breaking down stiff material from beneath the cutting edge. Explosive charges, carefully placed by divers, can be used to break up boulders or other obstructions to sinking.

Water jetting is not usually effective in freeing hanging caissons, since 'sticky' sinking conditions usually occur in stiff clays or boulder clays which are not amenable to removal by jetting.

Jetting and Lubrication

Sometimes, to facilitate the sinking, a film of grease is applied to the exterior surface of the caisson, and/or water jetting is used. Jet pipes, $1\frac{1}{2}$ to 2 inches diameter, with nozzles, are cast in the concrete, usually one series of jet pipes is provided on the sloping surface immediately above the cutting edge, and one or two series on the periphery of the caisson at several meters above the bottom of the cutting edge. All jets are arranged symmetrically to induce straight sinking. Since fixed jet pipes can readily become plugged, movable jets have been found more efficient. Eight inch diameter pipes may be cast in the concrete for inserting movable jet pipes for inside jetting.

Rectifying TILT in Wells

Caissons can never be sunk perfectly in plumb and true to position. A certain amount of deviation from the planned location should be allowed for in design and permitted. For a deep caisson, the actual center may even be 30 cm or more from the required location. It is important to keep the caisson in the vertical position during the entire process of sinking. As soon as it tilts corrective measures should be taken. These can be:

(i) Excavating on the high side ahead of the low side, but not stopping excavation on the low side
(ii) Dredging on the outside of the high side
(iii) Jetting on the outside and inside of the high side
(iv) Pulling the caisson by wire ropes (using timber sleeper-packings)—attach cables to a deadman or dolphin, and apply tension as the sinking proceeds
(v) Blocking under the cutting edge on the low side; this can be done more readily in pneumatic sinking.

It should be noted that it is impossible to plumb a caisson without lowering it as a whole, and this may also lower the final founding level for no fault of the client who otherwise has to pay for increased sinking.

Skin Friction in Caissons

A conservative approach of ignoring skin friction should be used, when assessing the contribution of skin friction to the carrying capacity of a caisson in service, but when considering the dead weight to be provided to sink the caisson, the skin friction should not be under-estimated. Some caisson details incorporate water and air jets near and above cutting edge level to reduce skin friction during sinking. The main disadvantage of external jet pipes is that they readily become clogged, especially when sinking through tight ground.

Generally engineers view built-in jetting arrangements with suspicion, doubting their effectiveness, and many hold the view that independently operated external jet pipes worked down the outside of the caisson wall is the only sure method. Present day practice is to inject thixotropic clay slurries (e.g., bentonite) above the cutting edge or shoe level, thus providing a membrane of slurry around the walls. Successive injections are made as the caisson is sunk to its final level.

This reduces the skin friction very considerably and enables the dead weight of the caisson to be reduced, with the likely elimination of the need for kentledge to assist sinking. Pipes in caisson walls used for slurry injection or for air/water jetting should be interconnected by a header at shoe level because individual vertical pipes may become damaged or blocked as the walls are built up. Problems can arise if the circulation of bentonite up the outside of the caisson is interrupted, for example, by erosion of the river bed around the caisson. If the circulation cannot be restored, the slurry will lose its effectiveness as a means of reducing skin friction.

Design Features of Open Caissons and Monoliths

The principal design features of open caissons and monoliths are shown in Fig. 7.3. The cutting edge forms the lowermost extremity of the shoe (or well curb). The latter usually has vertical steel outer skin plates and sloping inner steel haunch plates (or cant plates). The skin plates (if tall enough) are braced internally with steel trusses, members, or heavy reinforcement, in vertical and horizontal planes. The trusses prevent distortion of the shoe during fabrication, towing to site, and the early stages of sinking. As soon as possible before or after the initial sinking, the space between the skin plates is filled with concrete (steining). When the structure has attained sufficient rigidity by reason of the concrete filling, the skin plating can be terminated and the steining carried up in reinforced concrete placed between formwork. At or below low water level in a bridge the caisson proper is completed and capped and then the pier/abutment carried up in concrete masonry, or brickwork. If the water level rises above the top of the caisson at its finally sunk level, a cofferdam or temporary false steining is constructed above the caisson for ease of operations. The dredging space within the walls may form a single dredging well or it may be divided by cross walls into a number of wells.

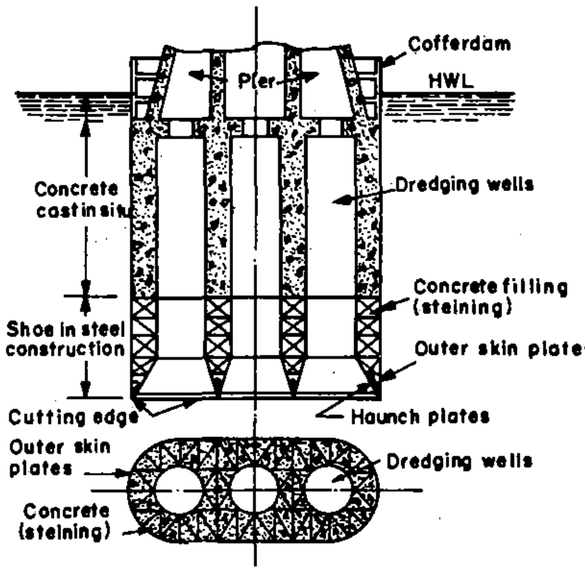


Fig. 7.3 *Design features of open well caisson*

In the following comments on the design of open caissons it must be realized that in most practical cases there is no ideal solution to the problem, and the final design is usually a compromise brought about by a number of conflicting requirements. For example, thick heavy walls may be desirable to provide maximum weight for easier sinking through stiff ground, but thick walls add to cost and may mean small dredge holes and then the grabs may not reach beneath the haunch plates to remove the stiff ground. Lightness of weight is desirable in the first stages if floating out the caisson, but this can only be obtained at the expense of rigidity and sinking effort which are so essential at the second stage of sinking through the upper layers of soil when the caisson is semi-buoyant and may not have uniform bearing, and when stresses due to sagging of the structure consequent on differential dredging levels may be critical. Maximum height of skin plates is desirable when sinking caissons in a waterway where there is a high tidal range, but the extra height of plates may mean excessive draught for towing to site, not to mention increased cost of plates. The shape of a caisson will, in most cases, be dictated by the requirements of the superstructure. The ideal shape for ease in sinking is circular in plan, since this gives the minimum surface area in skin friction for a given base area. However, the structural function of the caisson is, in most cases, the deciding factor.

The size and layout of the dredging wells is dependent mainly on the type of soil. For sinking through dense sands, or firm to stiff clays, the number and thickness of the cross walls, and the thickness of the outer walls, should be kept to a minimum consistent with the need for weight to aid in sinking and for rigidity against distortion. Grabs can excavate close to the cutting edge in caissons that have thin walls, assuming the outer dimension as unchanged. This is important in firm or stiff clays, since these soils do not easily slump towards the centre of a dredging well; whereas in sands and soft silts, grabbing below cutting edge level will cause the ground to readily slump away from the haunch plates towards the deepest part of the excavation sump, especially if assisted by water jetting. However, as already noted, thin walls mean reduced sinking effort and it is inconvenient to have to take kentledge on and off the top of the steining (for each lift of concrete that is placed) for assisting sinking.

Control of verticality in large caissons is facilitated by the provision of a number of dredge-holes. To give control in two directions at right angles to one another they should be disposed on both sides of the centre lines (Fig. 7.4), but for a narrow caisson there may be only room for one row, since sufficient width must be provided for a grab to work. Heavy monoliths sunk through soft material onto but barely into a firm or hard stratum need only have small dredge holes. Experiences in sinking open caissons for the piers of the Lower Zambezi Bridge showed that straight walls were preferable to circular walls when sinking through stiff clay, since with circular walls there was a tendency for the clay to arch and wedge itself around the cutting edge, rather than be forced towards the centre of the well.

If occasional obstructions in sinking are encountered

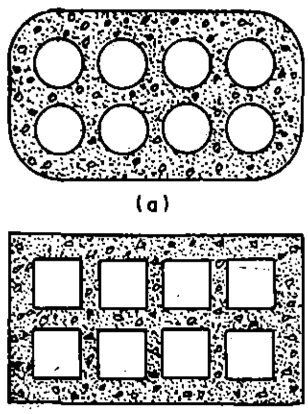


Fig. 7.4 *Layout of dredging wells in caisson (a) Circular wells (b) Square wells*

(e.g., old logs, sunken barges, boulders, etc.) then, down to about 20 m, workers wearing air-lock helmets connected to compressed air can dive down and break the obstructions underwater, using pneumatic tools.

Materials for Caissons and 'Sinking Cycle'

The desirable material for caisson construction is steel in the form of a double skin of plating which is subsequently filled with concrete. The concrete in the lower part of the shoe should be of high quality since it is required to develop high early strength to resist stresses developed in the 'tender' early stages of sinking. The cement content, however, should be sufficiently high to give it resistance to attack by sea or river water and the all important denseness.

Reinforced concrete has been used for caisson shoes but it has the disadvantage of being too heavy at the early stages of construction where lightness is needed for floatation and handling. The quantity of concrete-filling in a steel caisson may be somewhat greater than the volume of concrete in a reinforced concrete caisson, but due to the use of a slightly leaner mix, the ease in placing and the elimination of formwork, the unit cost of the concrete-filling in a steel caisson is appreciably lesser. Steel caisson is essential where it has to be floated out; however steel plating may be discontinued after grounding.

Reinforced concrete caissons utilize concrete to provide the structural strength as well as the weight for sinking. They are often more economical than the steel caissons. However, concrete caissons must be poured in sections (lifts), and the sinking operation must be interrupted while pouring each lift and while waiting for the concrete to mature. Every time the sinking is started from a stationary position, additional effort is required to overcome the static friction. Furthermore, the cyclic operation of stops and starts takes a long time to sink the caisson, and unless extreme scientific care is taken, the successive concrete lifts may inadvertently get cast vertically individually with the previous one off-plumb owing to well-tilt, thus giving a slightly zig-zag shaped well in elevation, which, with all its consequent 'necks', can prove difficult for sinking. Each lift should be cast in line with the previous lift even if the line is interimly tilted and the tilt should be controlled as a whole.

General Arrangement of Pneumatically Sunk Caissons

Pneumatic caissons are used in preference to open-well caissons in situations where dredging from open-wells would cause loss of ground around the caisson resulting in settlement of adjacent structures, or when sinking through variable ground or through ground containing obstructions (very hard lenses, conglomerates, etc.) where an open caisson would tend to tilt or refuse further sinking. Pneumatic caissons have the advantage that excavation can be carried out by hand in the 'dry' working chamber, and obstructions such as tree trunks or boulders can be broken out from beneath the cutting edge. Also, the soil at the foundation level can be inspected and, if necessary, bearing tests made directly upon it. The foundation concrete is placed under ideal conditions in the dry, whereas with open-well caissons the final excavation and sealing (plugging) with concrete is almost always carried out under water.

Pneumatic caissons have the disadvantage, compared with open-well caissons, of requiring more plant and labour for their sinking, and the rate of sinking is much lower. There is also the important limitation that men cannot work in air pressures much higher than 3.5 kg/cm^2, which limits the depth of sinking to about 35 m below the water-table, unless some form of ground water lowering is used outside the caisson. If such methods are used to reduce air pressures in the working chamber they must be entirely reliable, and the de-watering wells must be placed at a sufficient distance from the caisson to be unaffected by the ground movement caused by caisson sinking.

The development of large diameter cylindrical foundations installed by rotary drilling and the limitation in sinking depth due to considerations of limiting air pressure that the human lungs can accept, means that pneumatic caissons are only rarely used.

Design Features of Pneumatically Sunk Caissons

When caissons are designed to be sunk wholly under compressed air, it is usual to provide a single large working chamber (Fig. 7.5) instead of having a number of separate working chambers separated by cross walls. The single chamber is a convenient arrangement for minimising bearing resistance to sinking, since this resistance is then only given by the outer walls. Control of sinking by differential excavation from a number of cells is not necessary since control of position and verticality can readily be achieved by other means, for example by the use of shores and wedges beneath the cutting edge, or by differential excavation beneath the cutting edge.

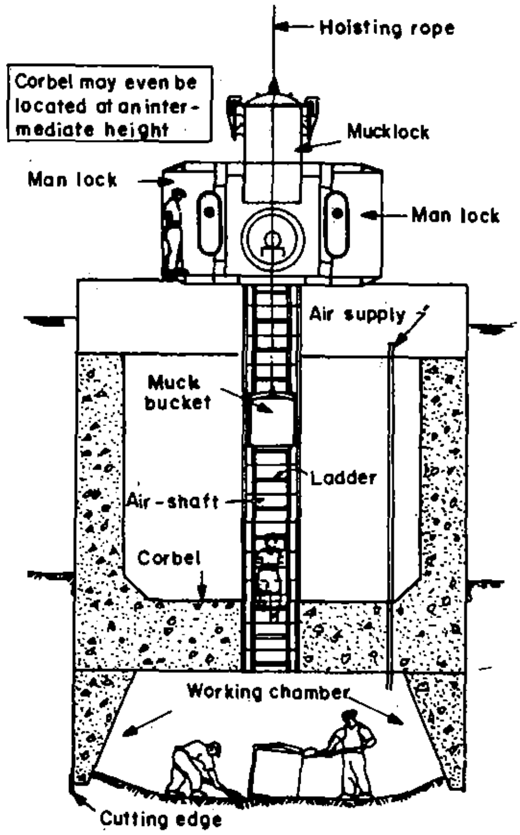


Fig. 7.5 *General arrangement of pneumatic caisson*

The working chamber is usually 2.5 to 10 m high, although where the caisson chamber is sunk to a limited penetration the height may be somewhat smaller. The roof of the working chamber (called *corbel*), must be strongly built as it may have to resist high air pressures over a wide span.

Access to the working chamber is through shafts. Since all excavated material must be lifted through the shafts, the shafts must have adequate capacity in size and numbers to pass the required quantity of spoil in buckets through the air-locks to meet the programmed rate of sinking. The air-shaft is usually oval in plan and is divided into two compartments by a vertical ladder. One compartment is used for hoisting and lowering spoil buckets and the other is for the workmen. The shaft is built up in 1.5 or 3 m lengths to permit its heightening as the caisson sinks down. The air-lock is mounted on top of the shaft, and it is essential for the safety of the workmen to ensure that the lock is always above the highest tidal or river flood levels, with sufficient safety margin to allow for unexpected rapidity in sinking of the caisson. Alternatively, the air-locks can be protected against flooding by building up the skin plating or providing a cofferdam around the top of the caisson to the required height.

Design Details for Compressed Air Sinking

Air-locks The number of air-locks required in a caisson depends on the number of men employed in any one working chamber in the caisson. The size of the air-locks and air-shafts is governed largely by the quantity of material to be excavated, i.e., by the size of the 'muck bucket'.

(1) For excavating in hard material, one man can be effectively employed in about 3 m^2 of working area of a caisson, but in loose material such as sand or gravel one man may be allowed in 6 to 7 m^2 area.
(2) Generally, the number of men in a caisson per shaft will range from 5 to 10 except in the smallest caissons. The optimum number is 10 men per shaft.
(3) Sufficient air-locks should be provided to allow the whole shaft to pass out of the caisson in reasonable time. This depends on the working pressure. For moderate pressures (under 2.5 kg/cm^2), an air-lock should be provided for every 90-100 m^2 of base area. For high pressures two locks would be considered for 90-100 m^2 of base area, because of the longer time required in 'locking' each shaft in and out of the caisson.
(4) The size of the lock is governed by the rate of excavation and the number of men to be accommodated. Thus the main chamber (or muck lock) has to accommodate a skip of sufficient size to pass through it the excavated material at the programmed rate. For example, with a base area of about 90 m^2 per lock, a large air-lock can deal with about 9 m^3 of spoil per hour; for continuous shift working at say 2.0 kg/cm^2, 180 m^3 of material would be passed through in 20 hours of 'effective' working. This gives about 100 m^3 of material 'in the solid' or 1.14 m of sinking per effective 3-shift day. With a smaller air-lock under similar conditions the output would be about 6 m^3/h or 120 m^3 of material in 20 effective hours, corresponding to a sinking rate of about 0.70 m per effective 3-shift day. The plant for the production and supply of compressed air to the working chamber shall deliver a supply sufficient to provide at the pressure in the chamber 0.30 m^3 of fresh air per minute per person for the time being in the chamber. BS CP 2004 recommends that whenever work involving compressed air at pressures greater than 1 kg/cm^2 above atmospheric pressure is undertaken, the Medical Research Council's Decompression Sickness Panel should be consulted for advice on decompression rates.

If the air supplied in accordance with the above rule is more than the amount lost under the cutting edge and through the air-locks, the surplus should be exhausted from the caisson through a control valve.

Compressors for air supply are usually stationary types. Ideally, they should be driven by variable speed motors to enable the supply to be progressively increased as the caisson sinks deeper. The type of plant in general use is a twin-cylinder single stage piston compressor of 8.5 m^3/min capacity motor-driven through a vee belt. Rotary compressors can be used for supplying low-pressure air. At least 50% spare compressor capacity should be provided for emergency purposes. Consideration should be given to alternative means of power supply, for example diesel generators for electrically-driven compressors normally supplied from the mains system, or a standby steam plant. The total available air supply may require to be twice the actual requirement if failure is liable to cause danger to life or property. However, a standby supply need not be provided if the loss of air pressure will not endanger the workmen: for example, if the caisson is being sunk on to a hard stratum which will remain stable, and if workmen have ample time to escape from the working chamber.

Air treatment Improved working conditions and greater immunity to caisson sickness is given by treatment of the air supply. The air-conditioning plant should aim to remove moisture and oil, and to warm the air for cold weather working, or to cool it for working in hot climates. The need to supply cool dry air is especially important for compressed air work in hot and humid climates. In cool climates it is advantageous to provide heating in man-locks since the cooling of the air which always takes place during decompression can cause discomfort to the occupants.

*Pneumatic Sinking of Caissons**

Control of position and verticality of pneumatic-caissons is more readily attainable than with open-well caissons. It is possible to maintain control by careful adjustments of the excavation beneath the cutting edge, and if this is insufficient, raking shores can be used in the working chamber, or the caisson can be moved bodily at early stages of sinking by placing sliding wedges or 'kickers' beneath the cutting edge.

Excavation in the working chamber is usually undertaken by men hand-shovelling into crane skips, compressed-air

* For estimation of bursting tension in well-steining see App. 5 of this book.

tools such as clay spades or breakers being used in stiff clays or boulder clays. When excavations are in sands or gravels, hand-held water jets can be used to sluice the material into a sump from where it is raised to the surface through a shaft. The latter can also be an open-ended pipe with its lower end dipping into water in the sump. By opening a valve on this 'snorer' pipe, the water and soil are forced by the air pressure in the working chamber out of the caisson. The snorer also performs a useful function in clearing water from the floor of the excavation if the soil is too impermeable for the air pressure to easily drive the water down into it.

The compressed air supply must be regulated to provide adequate ventilation for the workmen. In permeable ground this is readily attained by allowing it to escape through the soil and beneath the cutting edge. However, when sinking in impermeable clays and silts, ventilation must be maintained by opening a valve to allow air to escape through the caisson roof. Careful regulation of air pressure is necessary when sinking in ground affected by changes in tidal water levels.

Smoking or naked lights should not be permitted in the working chamber because of the risk of encountering explosive gases, e.g., methane (marsh gas), during sinking. A careful watch should be maintained in neighbouring excavations. Accidents have been known to happen by compressed air passing through beds of peat and becoming deprived of oxygen due to oxidation of the peat. The escape of this oxygen-deficient air into the confined spaces of excavations has caused asphyxiation of the workmen in them.

In very permeable ground the escape of air may be so great as to overtax the compressor plant. The quantity escaping can be greatly reduced by pre-grouting the well steining and even the ground, with cement or clay. However, each site has its own problems which must be looked into carefully in advance.

Blowing Down a Pneumatically Sunk Caisson

If a pneumatic caisson stops sinking due to build-up of skin friction, it can be induced to move by the process known as 'blowing down'. This involves reducing the air pressure to increase the effective weight of the caisson, so increasing the sinking effort. The process is ineffective if the ground is so permeable that air escapes from beneath the cutting edge at a faster rate than can be achieved by opening a valve and thus buoyancy-creating water is let in.

The procedure in blowing down a caisson is first to remove the men from the working chamber. The control valve is then opened and the caisson should soon begin to move. If it does not do so, the skin friction is too high, and either kentledge must be added or further excavation should be done below the cutting edge.

Careful control should be exercised when blowing down in ground containing boulders, or when blowing down a caisson to land it on an uneven rock bed. In some circumstances it may be necessary to excavate high spots in the rock and fill them with clay and then blow the caisson down into the clay.

An example of difficulties in sinking on to rock is the pneumatic caisson pier for a pipe bridge over the Mississippi River at Grand Tower, Illinois, which has been described by Newell.

If at all possible, a caisson should not be blown down in soft or loose ground, as this might result in soil surging into the working chamber, so increasing the quantity to be excavated. There is also the risk of penetration of ground into the working chamber, causing settlement of adjacent structures. It must be remembered that pneumatic caissons are, in many instances, used as a safeguard against such settlement. Blowing down, if properly controlled, is a safe procedure in a stiff clay.

If a caisson is sinking freely without the need for blowing down, measures must be taken to arrest the sinking on reaching founding level. This can be achieved by casting concrete blocks in pits excavated at each corner of the working chamber at such a level that the caisson comes to rest on the blocks at the desired founding level.

Safety Problems

For the safety and welfare of workmen, the following precautions should be exercised:

1. *Accurate control of air pressure* A gauge-tender should watch the pressure gauge constantly, and the gauge should be accurate, regularly calibrated, and in good working condition.
2. *Sufficient air circulation* To avoid the air in the working chamber becoming stale, fresh air must be circulated into the working chamber constantly. This may be done by opening a valve in the air lock. In granular soils, where certain amount of leakage takes place below the cutting edge and through the soil, the air is automatically circulated.
3. *Slow decompression* Men working under compressed air must be decompressed slowly. If coming out too fast, they are subjected to caisson disease. This disease is due to air bubbles formed in the blood and body tissues which are compressed while working under pressure. A period of about one-half hour is necessary for decompression from a pressure of 3.5 kg/cm^2.
4. *Duplicate and spare equipment* A spare or duplicate set of air compressors and other equipment for pneumatic operation should be provided in case of contingency.
5. *A medical doctor* should be available at site all

the time. All the workers must be medically fit to work inside a pneumatic caisson. No worker should work inside it for longer than about 4 hours per 24 hours, no more than 2 hours at a stretch. The figures reduce as the depth increases. Extreme care has to be taken against the caisson disease (called 'the bends') because of which the workers can loose control over their joints, vomit blood or suffer from nasal bleeding, paralysis, 'bubbles in skin', and even death. This calls for very controlled acclimatisation of the workers in the air-locks both for compression as well as for decompression.

Structural Design of Steining

In addition to the usual design forces for which the caisson must be designed for 'bridge in service' condition, the caisson, during its construction, is likely to be subjected to certain odd loading conditions, the effects of which may not be easily controllable. Some of these are outlined below:

(a) The caisson is hung up from near the top by skin friction necking. The lower portion of the caisson is then subjected to tension. Sufficient strength should be provided in the caisson to carry the weight of its lower portion (vague loading condition);

(b) The caisson is held on one side only or on two opposite points only over some length. This can lead to vertical cracks and, in the limit, may require refilling the cracked caisson and then sinking a new caisson inside it;

(c) The caisson is subjected to unbalanced earth pressures;

(d) The caisson has to be pulled into its correct position if it has tilted during construction. Large racking force and earth pressure would be introduced by pulling; and

(e) The caisson is 'dropped' suddenly during sinking, sometime owing to massive sand blow, and sometimes due to sudden break in skin friction while the sump is deep.

In addition, where the caisson is likely to be sunk pneumatically, additional hoop reinforcement may be required in the steining to withstand the bursting ring tension. The details of its calculation are dealt with in App. 5 of this book.

Some Considerations in the Dimensioning and Design of Wells

Some of the important features are given below for guidance:

(i) The minimum dimension of any dredge hole shall not be less than 2 metres.

(ii) For plain concrete wells, the mix of concrete in the steining shall not be leaner than 1 : 3 : 6. For wells located in marine areas or other similar adverse conditions of exposure, the concrete in steining shall not be leaner than 1 : 2 : 4 with cement content not less than 350 kg/m^3 of concrete and water-cement ratio not more than 0.45. In the case of plain and reinforced concrete single circular wells, the diameter of wells shall not normally exceed 12 metres.

(iii) The external diameter of brick masonry wells shall not exceed 6 m and such wells shall not be used for depths exceeding 20 m. For brick masonry wells, bricks of highest quality shall be used in cement mortar not leaner than 1 : 3 for steining.

(iv) Steining with two or more shells of different materials should not be permitted as experience has shown that these develop splitting cracks (differential shrinkage, etc.).

(v) The minimum thickness of well steining shall not be less than 500 mm and should satisfy the following relationship:

$$h = Kd\sqrt{l}$$

h = minimum thickness of steining in meters.

d = external diameter of circular well or dumb-bell shaped well or the smaller dimension in plan of twin 'D' well, in metres.

l = depth of well in metres below LWL or ground level whichever is higher.

K = a coefficient, with following values:

(1) Single circular or dumb-bell shaped well in cement concrete
K = 0.030 for predominantly sandy strata
K = 0.033 for predominantly clayey strata

(2) Twin-D well in cement concrete
K = 0.039 for predominantly sandy strata
K = 0.043 for predominantly clayey strata

(3) Single circular or dumb-bell shaped well in brick masonry
K = 0.047 for predominantly sandy strata
K = 0.052 for predominantly clayey strata

(4) Twin-D well in brick masonry
K = 0.062 for predominantly sandy strata
K = 0.068 for predominantly clayey strata

NOTE (a) For bouldery strata or for wells resting on rock where blasting may be involved, higher thickness of steining, better grade of concrete, heavier reinforcement, use of steel strake plates in the lower portions, etc., are advisable.

(b) For wells passing through very soft clayey strata, the steining thickness may be reduced based on local experience and in accordance with the decision of the engineer-in-charge to prevent the well from sinking by its own weight. In such cases, the steining may require relatively more reinforcement.

(vi) For plain concrete wells, the minimum reinforcement shall not be less than as indicated below:
Vertical reinforcements: 0.12% of gross sectional area of actual steining provided, distributed equally on both faces of the steining.
Hoop reinforcement: 0.04% of the volume per unit length of steining.

(vii) RC wells shall be designed as RC columns for combined axial load and bending. The reinforcements arrived at shall, however, not be less than as indicated below:
Vertical reinforcement: 0.2% (for either mild steel or deformed bars) of actual gross cross-sectional area of steining of which at least one-third shall be on inner face.
Hoop reinforcement: 0.04% of the volume per unit length of steining.

(viii) For brick masonry wells, the reinforcement shall not be less than as indicated below:
Vertical bond rods: 0.10% of cross-sectional area and shall be encased into cement concrete 1 : 2 : 4 bands of size 150 mm × 150 mm.
Hoop reinforcement: 0.04% of volume per unit length, provided in a concrete ring band and at spacing of 4 times the thickness of steining or 3 m, whichever is less (for details see Fig. 7.6).

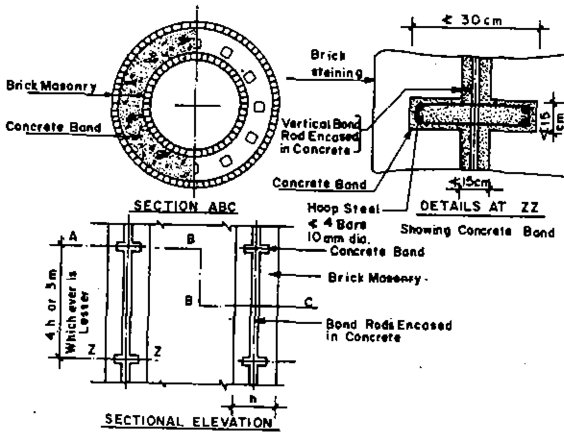


Fig. 7.6 *Details of brick masonry well steining.*

NOTE The horizontal RC ring bands shall not be less than 300 mm wide and 150 mm high reinforced with bars of diameter not less than 10 mm placed at the corners and tied with 6 mm dia stirrups at 300 mm centres.

(ix) Mild steel cutting edge of weight not less than 40 kg/m shall be provided properly anchored to the well curb. In case of wells with two or more compartments, the lower end of the cutting edge of middle stems shall be kept 300 mm above that of the outer walls to prevent rocking. Heavier cutting edge (85 kg/m or more) is required in hard and bouldery sub-strata.

(x) The well curb shall always be of reinforced concrete of mix not leaner than 1 : 1.5 : 3, with minimum reinforcement of 72 kg/m^3 excluding the bond rods of steining. A typical arrangement is shown in Fig. 7.7.

(xi) The angle contained by the vertical and the inclined surfaces of the well-curb should be around 30° and the curb height about 1.8 times the steining thickness.

(xii) Where blasting is anticipated, the outer faces of the well curb shall be protected with suitable plates of thickness not less than 6 mm up to half the height of the well curb and inner face with plate not less than 10 mm thick up to top of well curb and 6 mm to a height of 3 metres above top of the well curb. The steel plates shall be properly anchored to the curb and steining. The well curb shall be provided with additional 10 mm diameter hoop reinforcement at 150 mm centres up to a height of 3 m into the well steining in which portion the mix of concrete shall also be not leaner than 1 : 1.5 : 3 (by volume).

(xiii) Bottom plug shall be provided in all wells, and shall extend up to 300 mm above the well curb. The concrete mix shall be 1 : 2 : 4 with a minimum cement content of 330 kg/m^3 and a slump of about 150 mm. Where grouted concrete (colcrete) is used, the grout mix shall not be leaner than 1 : 2.

(xiv) Well dredge-hole shall be filled up to the top plug (placed at maximum scour level) with sand or excavated material free of organic matter.

(xv) A top plug 500 mm thick in c.c. 1 : 3 : 6 shall be provided over the dredge-hole filling.

(xvi) All wells shall be provided with a RC well cap with its bottom surface preferably at LWL.

(xvii) All structural design shall in advance account for a tilt of at least 1 in 80 and shift of at least 150 mm (in the resultant direction) in the body of well. Final founding level shall be decided after taking into account the moments due to actual values of tilts and shifts in the two orthogonal directions.

Bottom Plugging

The sealing (bottom-plugging) concrete in open-wells is placed by tremie pipe or bottom-opening skip to the required thickness and sometimes roughly levelled by a diver (after

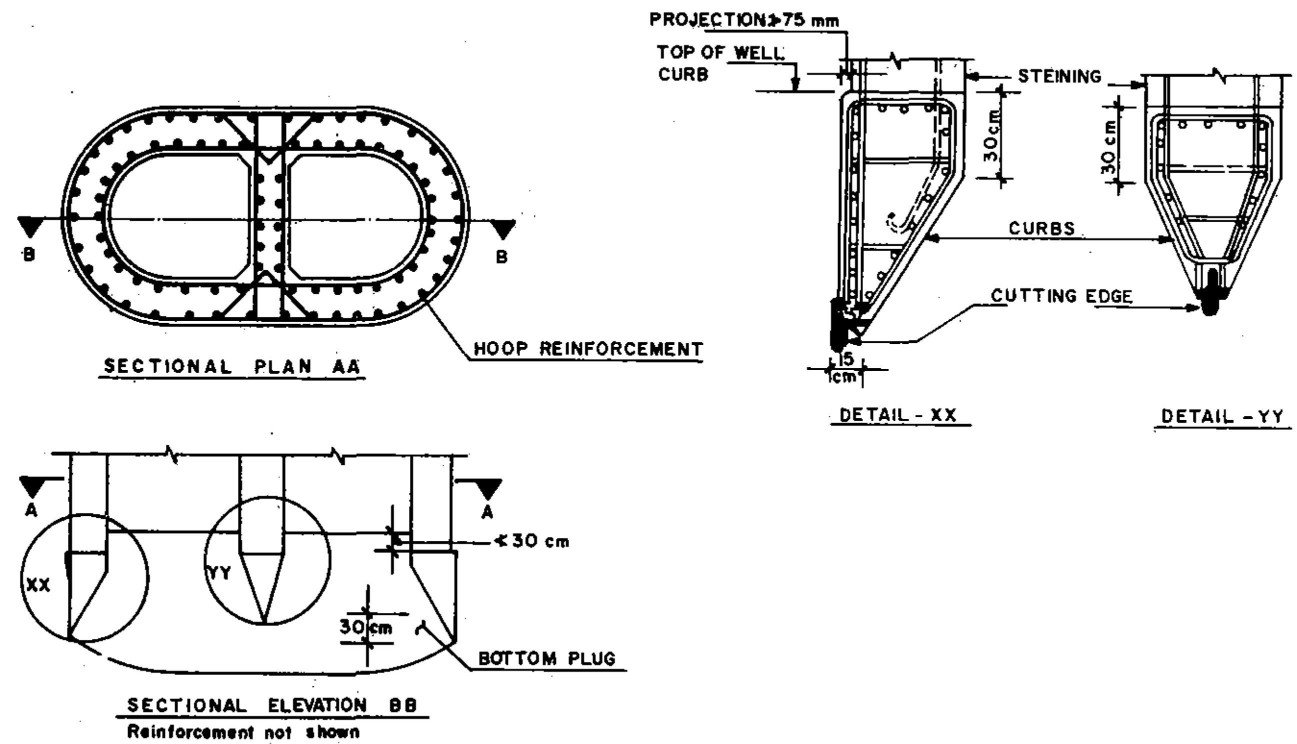


Fig. 7.7 *Details of well curb and cutting edge*

which sand is poured in to fill the remaining space in the wells up to the design level).

7.7 PIER AND ABUTMENT

Pier and abutment bodies can be constructed in plain concrete, reinforced concrete or in masonry (brick, stone or block), and in some special cases in timber, steel or prestressed concrete. Selection of a particular type depends upon the span and type of superstructure, height of substructure, the magnitude of loads and forces to be transmitted, availability of type of construction material and construction equipment at site, time for construction, and minimal cost.

In general, the shape of piers and abutments should be such as to cause minimum obstruction to the flow of water. For fast flowing rivers, highly surcharged with particles of abrasive nature, special precautions like masonry jacketing or steel lining to a suitable height above the normal scour level or bed level may have to be adopted.

Piers and abutments shall be designed to be safe under the worst combination of loads and forces during construction and service.

While finalising the designs of the substructure, however, the following general considerations shall also be satisfied.

(i) In case of plain concrete substructure, on exposed faces a surface reinforcement at the rate of 5 kg/m^2 shall be provided. This total reinforcement shall be made of small size bars spaced equally, both horizontally and vertically or orthogonally, in equal proportion. The spacing shall not exceed 200 mm.

(ii) The width of abutment-cap and pier-cap shall be sufficient to accommodate not only the bearings but also an off-set of 150 mm beyond the edges of the bearings and, in case of abutment, to carry dirt wall also. The thickness of such caps shall not be less than 225 mm up to a span of 25 m and 300 mm for longer spans. In addition, they should afford enough room and strength to accommodate lifting jacks, etc., for attending to the replacement of bearings and any future lifting of deck, etc.

(iii) Suitably designed cut and ease waters should be provided in piers up to affluxed HFL or higher (from consideration of waves) in order to reduce the current forces.

(iv) Abutment-piers may have to be provided at locations where there may be need of increasing the waterway subsequently. The design of such an abutment-pier shall be such that it should be possible to convert it to the similar shape as piers in the active channel as well as should be safe subsequently as a pier.

(v) Piers may be rigid or flexible. In case of framed type connections for pier, the base of such frames should be above the HFL. For bridges having multicolumn piers across rivers carrying floating trees or timber, they shall be braced by means of diaphragm walls of thickness not less than 150 mm, extending at least up to HFL.

(vi) The outer diameter of a single hollow circular pier should not be less than 2.75 m. It should be at least of reinforced concrete of strength not less than 200 kg/cm^2. Preferably the thickness of shell shall not be less than 600 mm and vertical reinforcement not less than 0.4% of the cross-sectional area of pier shaft.

The lateral reinforcement in the walls of hollow RC pier should not be less than 0.3% of the sectional area of the wall of the pier. This lateral reinforcement shall be distributed on both faces, 60% on the outer face and 40% on the inner face.

(vii) Where supports are made with 2 or more piles, or the columns are spaced closer than 2 metres across the direction of flow, the group should be treated as a solid pier of the same overall width, and value of shape factor (K) taken as 1.25 for working out the intensity of water pressure due to flood.

(viii) Spill-through type abutments should be provided with adequately designed pitched protection in front with apron/toe wall such that the entire system is safe from considerations of slipping, undermining, etc. Usually the pitching is provided in a minimum slope of $1\frac{1}{2}$H : 1V. The screen or cut-off wall should extend at least down to 500 mm depth into the fill so as to help prevent depression and squeeze-forward of the backfill—particularly during rains. Spill-through type abutments may be provided only in case solid abutments are not necessary. It is always preferable to adopt solid abutments, unless their cost is prohibitive and spill-through type is acceptable.

(ix) The top of wing/return walls shall preferably be carried 100 mm above the top of the slope of embankment to prevent any soil from being blown or washed away by rain.

(x) Cantilever-returns, where adopted, shall preferably not be of length more than 4 metres.

(xi) All abutments should be designed for a live load surcharge equivalent to 1.2 m height of earthfill unless a suitable RC approach slab at least 3.5 m long is provided with its one end sitting on the abutment and its remaining length resting evenly on compacted approach fill.

(xii) All wing walls and/or return walls provided for full height of approaches should be designed to withstand a live load surcharge equivalent to 0.6 m height of earthfill.

(xiii) In the case of spill-through type abutments, the active earth pressure calculated on the width of columns should be increased by 100% (to cater for the effect of arching action between columns).

(xiv) Structures designed to retain earthfill should be proportioned to withstand earth pressure calculated in accordance with any rational theory. Coulomb's theory shall be acceptable, subject to the modification that the centre of pressure exerted by the backfill is located at an elevation of 0.42 of the height of the wall above the base instead of 0.33 of that height. No structure shall however be designed to withstand a horizontal pressure less than that exerted by a fluid weighing 480 kg/m^3.

(xv) The fill behind abutments, wing walls, and return walls should conform to the standard specifications, and adequate weep-holes shall be provided in these structures for draining out static water pressure.

(xvi) In skew bridges where bearings are placed at right angles to the longitudinal axis of the bridge, the top width of the piers/abutments has to be more compared to right bridges in order to have a clear distance of 150 mm beyond the edge of the bearings.

(xvii) In the case of navigational streams, the effect of barge impact has to be considered in the design of piers and suitable fenders may have to be provided around the piers.

(xviii) For spill-through type of abutments, having columns and the cap beam supporting the deck, and dirt wall and the fly wings, it may be desirable to have the fly-wings directly resting on two extra columns in line with the wings, instead of connecting the fly wings to the cantilever projections of the abutment cap. This is to avoid unnecessary twist in the abutment cap. Further, for spill-through type of abutments, which are more prone to undergo deflections at top towards the river side due to earth pressure, the position of bearings has to be fixed adequately taking this probable deflection into account.

(xix) For skew bridges provided with square returns parallel to the bridge axis, the length of the wings at the obtuse angled corners will have to be more than at the acute corners to ensure that the revetment does not protrude into the waterway.

(xx) Piers should be designed for the possibility of one side span collapse (or unconstructed) case, if the structural system be such.

Achieving Economy in Design

The following measures often help in achieving economy in the design:

(i) For heights up to 5 to 6 m and spans up to 20 m, usually solid plain mass concrete or masonry piers may be suitable.

(ii) For heights above 6 m and spans beyond 20 m, RC piers are suitable.

(iii) For skew crossings, adoption of circular pier with a skew cap reduces the adverse effects of eddies, etc. In case the heights are large, hollow sections become economical.

(iv) At abutment locations, provision of sliding bearings or roller-cum-rocker bearings instead of fixed bearings helps in reducing the horizontal force (at bearings) on the abutment which is already under large horizontal earth pressure force.

(v) For abutments located on wells or pile foundations, the general practice is to keep them eccentric towards the backfill so that stabilizing moment increases.

(vi) Use of relieving shelves, etc., may be resorted to for reducing the effect of earth pressure in the design of the abutments.

(vii) In case of rivers hugging the banks, and in rivers with large depth of flow, it will be desirable to provide solid abutments with solid wings or box returns. However, where channels have shallow depth of flow near the banks and height of abutments is not much, cantilever returns offer cheaper solutions.

CHAPTER 8

Distribution of Externally-Applied and Self-Induced Horizontal Forces among Bridge-Supports in Straight-Decks

- Simple Spans on Unyielding Supports
- Simple Spans and Continuous Spans on Unyielding or Flexible Supports

8.1 INTRODUCTION

(i) The distribution of longitudinal horizontal forces among bridge supports is effected by the horizontal deformation of bearings, flexing of the supports and rotation of the foundations.

(ii) In simple non-skew straight decks, resting on similar stiff supports, the distribution of forces among supports may be assumed as indicated in 8.2 below.

(iii) In simple and continuous decks on flexible (or stiff) supports the distribution of longitudinal horizontal forces (e.g. the 'self-induced' horizontal forces due to change of temperature, shrinkage, creep and elastic shortening of deck, and the 'applied' horizontal forces such as braking, earthquake and wind) among bridge supports shall be adequately estimated after taking due account of deformation of bearings, flexing of piers and abutments and rotation of foundations, as well as the location of the Zero-Movement Point (Z.M.P.) of the deck as explained in 8.3 ahead. However, in the case of simple spans on stiff (i.e. unyielding) supports, the simplified method outlined in 8.2 below is accurate enough and that the refined method outlined in 8.3 is not called for in such cases. (In curved and skewed decks, in addition, the effect of in-plan meander may also have to be considered; see Chapter 9 for curved/skew decks.)

8.2 SIMPLY SUPPORTED NON-SKEW STRAIGHT DECKS ON UNYIELDING SUPPORTS

(i) For a simply supported span with fixed (Rocker) and free (Roller-Rocker or sliding plate) bearings (i.e. not elastomeric type) on stiff supports, horizontal forces at the deck-ends, in the bridge longitudinal direction, shall be as follows:

at the FIXED BEARINGS	*at the FREE BEARINGS*
$\{F_h - \mu(R_g + R_q)\}$	$\mu(R_g + R_q)$
or $\left\{\dfrac{F_h}{2} + \mu(R_g + R_q)\right\}$	

whichever greater.

Where:

F_h = Applied horizontal force on the Deck on the span under consideration (e.g. Braking, Earthquake, wind)

R_g = Reaction at the free end due to dead load

R_q = Reaction at free end due to live load

and μ = Coefficient of friction at the free bearing which shall be assumed to have the following values:

(a) For steel roller bearings	0.03*
(b) For concrete roller bearings	0.05
(c) For sliding bearings:	
• Steel on cast iron or steel on steel	0.50
• Grey cast iron on grey cast iron (Mehanite)	0.40
• Concrete over concrete with bitumen layer in between	0.60
• Teflon on stainless steel	0.05

(ii) For simply supported reinforced concrete and prestressed concrete superstructures, the span up to which plate bearings can be used shall be limited to 15 m.

(iii) In case of simply supported small spans up to 7.5 meters resting on unyielding supports and where no bearings** are provided, horizontal force in the longitudinal direction at each deck-end shall be taken as

$$\frac{F_h}{2} \text{ or } (\mu \cdot R_g), \text{ whichever greater.}$$

(iv) For a simply supported span on identical elastomeric bearings at each end, resting on unyielding supports, the longitudinal horizontal force at each deck-end shall be taken as:

$$\frac{F_h}{2} + V \cdot \Delta$$

* 0.05 if more than two rollers.

** It is usual to provide felt-pad or thick bitumen-impregnated paper layers for bearings in such spans.

where:

V = sum of the shear ratings of all the elastomeric bearings at one end,

and Δ = movement of deck above the bearings other than that due to 'applied loads' (moving length being only half of the span length assuming the Zero Movement Point is at midspan in the present case).

FORCE TRANSMITTED TO A SUPPORT

Indicated in items (i), (iii) and (iv) above are the forces caused at different deck-ends under simply supported decks. The forces transmitted to the supporting PIERS and ABUTMENTS should thereafter be calculated appropriately. *As an example*, if an intermediate pier supports simple spans from its right (r) and left (l) sides through 'Rocker' and 'Rocker-Roller' type of Bearings, respectively, then the longitudinal horizontal force transmitted to this pier shall be taken as the greater of

$$\{F_{h_r} - \mu(R_{g_r} + R_{q_r} - R_{g_l} - R_{q_l})\}$$

and

$$\left\{\frac{F_{h_r}}{2} + \mu(R_{g_r} + R_{q_r} - R_{g_l} - R_{q_l})\right\}$$

F_{h_r} being the Applied Horizontal Force on the deck of right hand span, suffixes 'r' and 'l' refer to the values at 'free' Bearings in the right and left spans. (Since the left hand span sits on this pier through a 'Rocker-Roller' Bearing — which can transmit only a μR type of force, F_{h_l} is assumed not to be transmitted to this pier.)

However, if the Bearings are of Elastomeric type, then the longitudinal force transmitted to this pier would be:

$$\left(\frac{F_{h_r}}{2}\right) + \left(\frac{F_{h_l}}{2}\right) + \{(V_r\Delta_r) - (V_l \cdot \Delta_l)\}$$

where:

V_r and V_l are the values of V of the right-hand and left-hand span Bearings on this pier, respectively,

and Δ_r and Δ_l are the values of deck movements above the right hand and left hand span Bearings on this pier, respectively.

NOTE If the elastomeric bearings at the two ends are not identical such that a very thin elastomeric pad (6–8 mm) is provided at one end (which acts like a rocker/fixed bearing as its shear rating is consequently enormous), then such a case is akin to case (i) above only that $\mu \cdot (R_g + R_q)$ should be replaced by $V \cdot \Delta$ where V = 'shear rating' of elastomeric bearings on the 'free' side and Δ = movement of full span length since the null-point will be assumed at the fixed end in this case.

8.3 DISTRIBUTION OF LONGITUDINAL HORIZONTAL FORCES AMONG BRIDGE-SUPPORTS IN STRAIGHT-DECKS IN:

- SIMPLE-SPANS AND CONTINUOUS-SPANS ON UNYIELDING OR FLEXIBLE-SUPPORTS

A method[1] is presented for estimating the distribution of longitudinal horizontal forces (acting on the bridge superstructure) among the bridge supports, based on the effect of the location of the point of zero-movement of the Deck, which does not necessarily coincide with the position of the fixed bearing. The position of this point of zero-movement has a direct influence on the magnitude of horizontal forces induced at the top of the supports. The method analyzed takes into account all possible external and internal forces, e.g. temperature, creep, shrinkage, elastic shortening, braking forces, wind forces, etc., acting on the superstructure and their transmission to the bridge supports.

The distribution of longitudinal horizontal forces among bridge supports is affected by the horizontal deformation of bearings, bending of support shafts and rotation of foundations. The problem of distribution becomes all the more acute when different supports have different stiffnesses, as may well be the case in view of the increasing number of multilevel interchanges where all piers may not be identical because of minimum sight distance and clearance restrictions.

Some supports may have to be single columns while others could be double columns or shafts. This difference in the type of supports may cause bearings of different types to be used, e.g., rocking-sliding bearings on heavily loaded single-column piers and elastomeric rubber bearings on double-column or shaft piers. Moreover, different supports may have different types of foundations (spread footings, piles, etc.) dictated by differing subsoil conditions. The analysis of the problem takes into account all these characteristics by considering the shear-rating of each support.

Notation

Type A support = support with a bearing which produces μR type force

Type B support = support with a elastomeric bearing

L_n = distance from extreme left-hand support, '0' to any support 'n'

Δ_n = movement of the deck over the support n relative to the point of zero-movement.

x = distance from support '0' to the point of zero-movement.

c = movement coefficient due to temperature, shrinkage, creep, and elastic shortening of deck

s = shear rating of a Type B support, i.e., horizontal force required to move the top of a Type B support through a unit distance, taking into account horizontal deformation of elastomeric bearings, bending of support shaft and rotation of foundation

μ = coefficient of friction in a bearing used on Type A supports

R = dead and live load reaction on a support

with subscripts l and r to denote, respectively, left and right.

Analysis

Consider any continuous-beam bridge deck as shown in Fig. 8.1 and let the point of zero-movement be at a distance x from support '0'. Let n be the total number of supports.

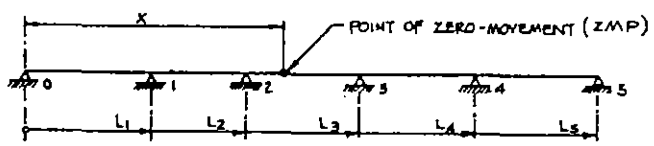


Fig. 8.1 *Continuous-Beam bridge deck showing point of zero longitudinal movement*

Therefore,

$$n = \Sigma \text{ Type } A + \Sigma \text{ type } B \tag{8.1}$$

Then deck movement at any support n to the left of the point of zero-movement is

$$\varDelta_{ln} = c(x - L_{ln}) \tag{8.2}$$

and deck movement at any support n to the right of the point of zero-movement is

$$\varDelta_{rn} = c(L_{rn} - x) \tag{8.3}$$

Noting also that at 'Type A supports' μR type forces are produced, while at 'Type B supports' $s\varDelta_n$ type forces are produced, and that the total force to the left of the zero-movement point must balance the total force to its right, the equilibrium equation can be written as follows:

$$[\Sigma\mu R + \Sigma s\varDelta_n]_l = [\Sigma\mu R + \Sigma s\varDelta_n]_r \tag{8.4}$$

where suffixes l and r refer to left and right of the zero-movement point.

Substituting for $\varDelta_n$,

$$[\Sigma\mu R+\Sigma sc(x-L_{ln})]_l = [\Sigma\mu R+\Sigma sc(L_{rn}-x)]_r \tag{8.5}$$

Transposing and noting that the sum of similar forces on left and right of the zero-movement point equals the overall total of such forces,

$$\Sigma sc(x - L_n) = \Sigma\mu R \tag{8.6}$$

which gives the zero-movement point location:

$$x = \frac{c\Sigma sL_n + \Sigma\mu R}{c\Sigma s} \tag{8.7}$$

where

ΣsL_n = arithmetic sum of products of shear rating s and distance from extreme left hand support, for all of the Type B supports

Σs = arithmetic sum of shear ratings of all of the Type B supports

$\Sigma\mu R$ = algebraic sum of μR forces at all of the Type A supports; + sign for those on right of point of zero-movement; − sign for those on left of point of zero-movement

The above formula gives the distance of the point of zero-movement from the extreme left-hand support. The actual movement of the deck at any support (c × distance of the support from the point of zero-movement) and also the actual force over each Type B support (s × actual movement) can therefore be calculated. As for any Type A support, it is assumed that the deck movement is at least equal to or more than the deflection of the support due to μR so that the limiting force of friction μR is mobilized.

There are, in fact, two types of longitudinal forces, viz., that caused by the deck movement due to temperature shrinkage, creep and elastic shortening, and that applied externally (e.g., braking force, longitudinal wind on deck, longitudinal wind on live load, etc.). Distribution of the former depends both on the actual location of the point of zero-movement as well as on the shear ratings of the type B supports, while distribution of the latter depends only on the shear ratings of type B supports. Consequently, the second type of longitudinal force is distributed among Type B supports only, because Type A supports are assumed to have reached their maximum capacity through deck movement as explained earlier.

8.4 APPLICATION

Step 1 Find s for each type B support system as follows:

(a) Assume a unit horizontal force at the top of the support system. If the support shaft and foundation are assumed restrained against any movement, then only the elastomeric bearing is deformed horizontally by a distance m_1. Releasing the support shaft and allowing it to deflect, its top will move further by a distance m_2 under the action of the applied unit force transferred through the elastomeric bearing.

(b) Now, allowing the foundation to rotate under the effect of existing unit force at the top of the support system, the top of the support shaft will move further horizontally by a distance m_3.

(c) Thus, application of unit horizontal force at the top of the support system will cause a total movement of $m_1 + m_2 + m_3$. It is this total movement that will affect the distribution of the horizontal forces. The shear rating is then given by:

$$s = \frac{1}{m_1 + m_2 + m_3} \tag{8.8}$$

Step 2 Assume (by judgement) the span in which the point of zero-movement lies.

Step 3 Find movement coefficient c.

Step 4 Calculate μR values for each Type A support.

Step 5 Work out x in the tabular manner shown in Table 8.1 and proceed to Step 6 if assumption made in Step 2 holds; otherwise, make a fresh assumption and work out a new x until the assumption proves correct. The point of zero-movement location is defined by Eq. (8.7).

Table 8.1 *Summation of factors for calculation of zero-movement point*

Support	Type A or B	μR	s	L_n	sL_n
0					
1					
2					
3					
...					
...					
	Sum	$\Sigma\mu R$	Σs		ΣsL_n

Step 6 Calculate the forces at various supports as follows:

(i) Horizontal force produced at the top of Type A support = μR

(ii) Horizontal force produced at the top of Type B support = $s \times c \times$ distance from zero-movement point.

Check $\Sigma\mu R + \Sigma sc$ (distance from point of zero-movement) = 0

Caution regarding sign of μR forces.

Step 7 Calculate any externally applied longitudinal force, H on the superstructure, e.g., braking force, longitudinal wind on superstructure, longitudinal wind on live load. Knowing Σs and s values from earlier steps (Table 8.1), distribute the force H among Type B supports in the tabular manner shown in Table 8.2. These longitudinal forces, H, are not shared by Type A supports since these supports are already assumed to have reached their limiting capacity μR as a result of deck movement (Steps 1 to 6).

Table 8.2 *Distribution of externally applied horizontal forces*

Support	Type A or B	Type B Supports Only		
		Shear Rating	Total Force	Distributed Force
		s	H	$H \times \frac{s}{\Sigma s}$
0				
1				
2				
3				
...				
...				
		Σs		

Step 8 Add the results of Step 6 and Step 7, at corresponding supports, to obtain total distributed longitudinal horizontal force at each support.

Step 9 Design the necessary reinforcement in the supporting structure after combining the above calculated forces with those resulting from any agencies other than the ones described here.

Numerical Examples

EXAMPLE 1 The structure is a three-span continuous bridge, post-tensioned prestressed concrete slab superstructure with circular voids (Fig. 8.2). Supports 1 and 2 are circular reinforced concrete single columns (Fig. 8.3). Supports 0 and 3 are very rigid abutments. Elastomeric rubber bearings on supports 0 and 3; Rotaflon rocking sliding bearing on support 2; fixed bearing on support 1. Supports 1 and 2 founded on spread footings on hard strata. Supports 0, 1 and 3 are Type B, and support 2 is Type A.

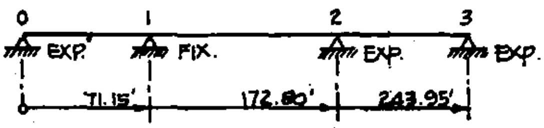


Fig. 8.2 *Structure of Example 1 (Three-span continuous bridge)*

Step 1 Determine shear rating s for each support. Assuming a unit horizontal force at the top of the support system (Fig. 8.3) compute:

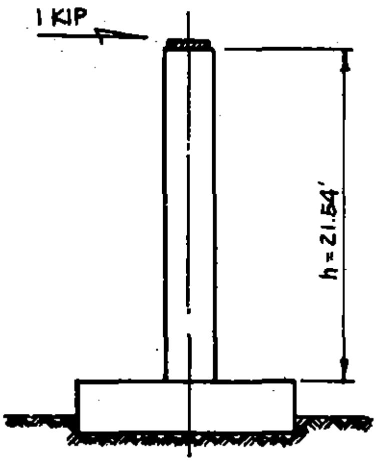


Fig. 8.3 *Intermediate support of bridge Example 1*

(a) *Horizontal deformation of bearings — supports 0 and 3*
If p = number of elastomeric bearings on a support,
q = shear stiffness of each bearing in kips/in.,
then

$$m_1 = \frac{1}{pq} \text{ (in.)}$$

(b) *Horizontal movement of top of column due to deflection of pier — support 1* For a column section having an $I = 13.41$ ft^4 and an $E = 619,000$ ksf, when acted upon by the applied unit kip force transferred through the fixed bearing, the top of the pier column moves horizontally by:

$$m_2 = \frac{h^3}{3EI} = \frac{21.54^3(12)}{3(619,000)(13.41)} = 0.0048 \text{ in.}$$

(c) *Horizontal movement of top of column due to rotation of pier foundation — support 1*
Since the foundation is on solid rock, there is no rotation of the footing and hence no related movement at the top of pier column. Thus,

$$m_3 = 0$$

The shear rating is then given by:

$$s = \frac{1}{m_1 + m_2 + m_3} = \frac{1}{\dfrac{1}{pq} + 0.0048} \text{ (kips/in.)}$$

Work out shear rating values s in the following tabular manner:

Support	p	q	s
0	2	25.6	51.2
1	—	—	208.5
2	—	—	—
3	2	15.5	31.0

Step 2 Assume that the point of zero-movement lies in the central span.

Step 3 Determine movement coefficient c.

$$c = at + \epsilon_s + \epsilon_c + \epsilon_{es}$$

where

a = coefficient of linear expansion or contraction $= 6 \times 10^{-6}$ per °F
t = change in temperature = 45°F
ϵ_s = shrinkage strain
ϵ_c = creep strain
ϵ_{es} = strain due to elastic shortening under prestress force

From the relevant bridge calculations:

$$\epsilon_s + \epsilon_c + \epsilon_{es} = 0.00064$$

Substituting these values:

$$c = 0.00091$$

Step 4 For support 2, which is the only one of Type A, μR = 22.3 kips from the relevant bridge calculations.

Step 5 Calculate position of zero-movement point.

Support	Type A or B	μR	s	L_n	sL_n
0	B	—	51.2	0	0
1	B	—	208.5	71.15	14834.78
2	A	22.3	—	172.80	—
3	B	—	31.0	243.95	7562.45
Sum		22.3	290.7	—	22397.23 × 12

$$x = \frac{0.00091(22397.23)(12) + 22.30}{0.00091(290.7)} = 84.08(12) \text{ in.}$$

i.e., x = 84.08 ft which means the point of zero-movement lies in the central span as assumed in Step 2.

Step 6 Calculate horizontal force produced by deck movement (for $c = 0.00091$).

Support	Type A or B	μR (1)	s (2)	Distance from point of zero-movement (3)	Distributed force, kips (1) or (2) × (3) × c
0	B	—	51.2	84.08 × 12	– 47.00
1	B	—	208.5	12.93 × 12	– 29.44
2	A	+ 22.3	—	88.72 × 12	+ 22.30
3	B	—	31.0	159.87 × 12	+ 54.12
				Total	0.00 OK

Step 7 Total externally applied longitudinal force due to braking action is 13.4 kips (from the actual calculation pertaining to this particular bridge).
Horizontal force produced by above braking force:

Support	s	H	Distributed force, kips
0	51.2	13.4	2.37
1	208.5		9.61
2	—		—
3	31.0		1.42
		Total	13.40 OK

Step 8 Add the results of Step 6 and Step 7 to obtain total distributed longitudinal horizontal force at each support.

Support	Horizontal distributed force, kips: Due to temp., shrinkage, creep	Due to braking	Total force, kips
0	47.00	2.37	49.37
1	29.44	9.61	39.05
2	22.30	—	22.30
3	54.12	1.42	55.54

Step 9 Design each support to resist the corresponding force calculated above.

EXAMPLE 2 The structure is a six-span semi-continuous bridge with prestressed pre-tensioned AASHO beams supporting a 7-in, reinforced concrete deck slab (Fig. 8.4). Supports 1 to 5 are triangulated piles (Fig. 8.5). Supports 0 and 6 are very rigid abutments. Elastomeric rubber bearings on all supports except on support 3, which carries a "fixed" bearing.

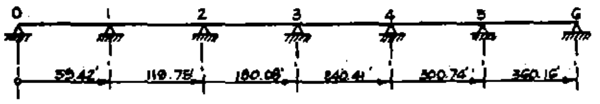


Fig. 8.4 *Structure of Example 2 (six-span semi-continuous bridge)*

Step 1 Determine shear rating s for each support. Assuming a unit horizontal force at the top of the support system (Fig. 8.5) compute:

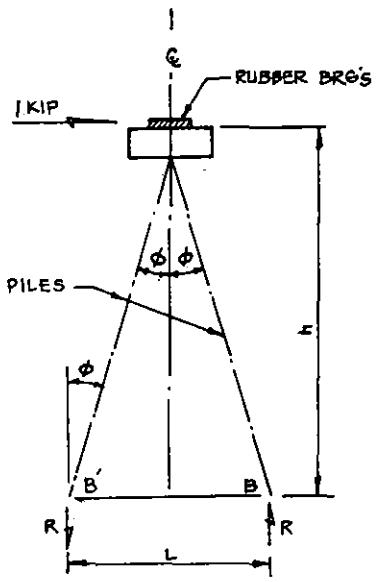


Fig. 8.5(a) *Intermediate support of bridge Example 2*

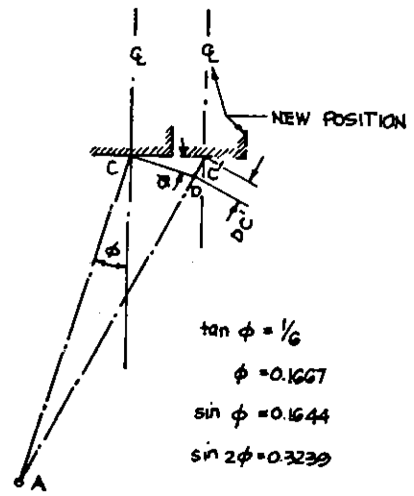


Fig. 8.5(b) *Horizontal movement due to pile deformation (bridge Example 2)*

(a) *Horizontal deformation of bearings*
If p = number of elastomeric bearings on a support,
q = shear stiffness of each bearing in kips/in.,
then

$$m_1 = \frac{1}{pq} \text{ (in.)}$$

(b) *Horizontal movement of top of pier due to deformation of piles*
Acted upon by the applied unit force transferred through the rubber bearings, the top of the pier

moves horizontally by:

$$m_2 = CC' = DC'/\sin\phi$$

The above relationship is easily established from Fig. 8.5(b). The small error due to the assumption of angle ϕ is negligible. If

$$DC' = \frac{L}{2\sin\phi} \times \frac{F}{AE} \text{ (i.e. original length} \times \text{strain)}$$

and

$$F = \frac{R}{\cos\phi} = \frac{h}{L\cos\phi} \text{ (axial force in pile)}$$

Then

$$DC' = \frac{L}{2\sin\phi} \times \frac{h}{AEL\cos\phi} = \frac{h}{AE\sin 2\phi}$$

Therefore

$$m_2 = \frac{h}{AE\sin\phi\sin 2\phi}$$

where
A = section area of pile = 493 in^2.
E = mod. of elasticity of pile material = 3300 ksi

$$m_2 = \frac{h}{493(3300)(0.1644)(0.3239)} = \frac{h}{86,600} \text{ (in.)}$$

The shear rating is then given by:

$$s = \frac{1}{m_1 + m_2} = \frac{1}{\frac{1}{pq} + \frac{1}{86,600}} \text{ (kips/in.)}$$

Work out shear rating values s in the following tabular manner:

Support	p	q	h	s
0, 6	5	4.3	very rigid	21.5
1, 5	10	6.0	480	45.0
2, 4	10	6.0	600	42.4
3	10	∞	600	144.4

Step 2 Because of complete symmetry about the 'fixed' support 3, it is evident that the point of zero-movement coincides with it. However, this will be confirmed numerically in Step 5.

Step 3 Determine movement coefficient c.

$$c = at + \epsilon_s + \epsilon_c$$

where

a = coefficient of linear expansion or contraction = 6×10^{-6} per °F.
t = change in temperature = 45°F
ϵ_s = shrinkage strain
ϵ_c = creep strain

From the actual calculation pertaining to this particular bridge:

$$\epsilon_s + \epsilon_c = 0.00025$$

Substituting these values:

$$c = 0.000,006(45) + 0.00025 = 0.00052$$

Step 4 μR values are zero because no support is of Type A.

Step 5 Calculate position of zero-movement point.

Support	Type A or B	μR	s	L_n	sL_n
0	B	—	21.5	0	0
1	B	—	45.0	59.42	2670
2	B	—	42.4	119.75	5070
3	B	—	144.4	180.08	26000
4	B	—	42.4	240.41	10200
5	B	—	45.0	300.74	13500
6	B	—	21.5	360.16	7760
	Sum	—	362.4	—	65200 × 12

$$x = \frac{0.00052(65,200)(12)}{0.00052(362.4)} = 180.08(12) \text{ in.}$$

Step 6 Calculate horizontal force produced by deck movement (for $c = 0.00052$).

Support	s	Distance from point of zero-movement	Distributed force, kips
	(1)	(2)	(1) × (2) × c
0, 6	21.5	180.08(12)	24.1
1, 5	45.0	120.66(12)	33.8
2, 4	42.4	60.33(12)	16.0
3	144.4	0.00	0.0

Step 7 Total externally applied longitudinal force due to braking action is 25.65 kips, from relevant bridge calculation.

Horizontal force produced by above braking force:

Support	s	H	Distributed force, kips
0,6	21.5		1.52 each
1,5	45.0		3.18 each
2,4	42.4	25.65	3.00 each
3	144.4		10.25
sum	362.4	25.65	25.65

Step 8 Add the results of Step 6 and Step 7 to obtain total distributed longitudinal horizontal force at each support.

Support	Horizontal distributed force, kips		Total force kips
	Due to temp., shrinkage, creep	Due to braking	
0,6	24.1	1.52	25.62
1,5	33.8	3.18	36.98
2,4	16.0	3.00	19.00
3	0.00	10.25	10.25

Step 9 Design each support to resist the corresponding force calculated above.

EXAMPLE 3 (General) This example shows how to compute m_3, the movement at the top of a support due to rotation of: (A) spread-footing foundation, and (B) piled foundation.

(A) Case of Spread-footing foundation

If the footing rotates through an angle ϕ under the bending moment created by the unit horizontal force applied as shown in Fig. 8.6, then $m_3 = h\phi$, approximately, and, in the absence of a more accurate formula, ϕ can be estimated

$$\tan\phi = \frac{12M}{ab^3c}$$

where

$M = 1.00h = h$

$c =$ coefficient of subgrade reaction of soil

$a =$ plan dimension of footing normal to bridge

$b =$ plan dimension of footing parallel to bridge

(B) Case of a piled foundation (Fig. 8.7)

If the axial force in the pile that is distant x_n from the pier centerline is

$$F = \frac{Mx_n}{I} = \frac{hx_n}{\Sigma x^2}$$

then the corresponding axial deformation of the pile is,

$$y = d\frac{F}{AE} = \frac{hx_n d}{AE\Sigma x^2}$$

so that: $m_3 = h\phi$

where:

$\phi = \dfrac{y}{x_n}$

$A =$ area of pile section

$E =$ modulus of elasticity of pile material

$d =$ length of pile

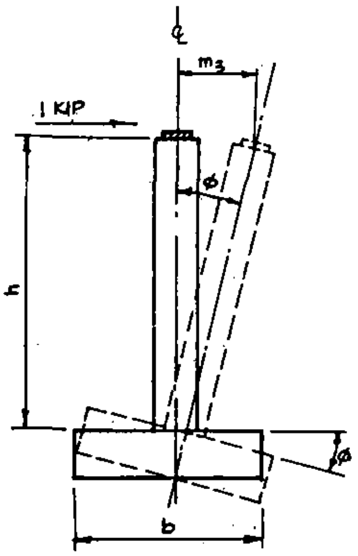


Fig. 8.6 *Rotation of footing caused by horizontal force at top of support (Example 3A)*

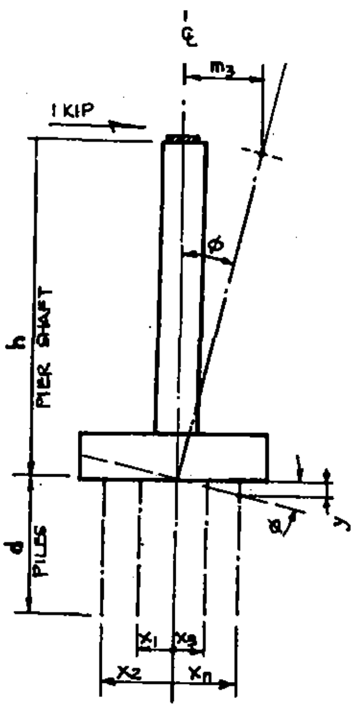


Fig. 8.7 *Axial deformation of pile caused by rotation of pier (Example 3B)*

8.5 CONCLUSION

In designing the bridge supports, it is very useful to distribute the longitudinal horizontal forces by taking into account the actual location of the zero-movement point and the shear ratings of all support systems. This is so because this realistically reduces the burden on the support with fixed bearing and thereby avoids its being overdesigned unnecessarily, while preventing the other supports from being underdesigned innocently.

REFERENCE

1. A.W. Witecki and V.K. Raina, "Distribution of Longitudinal Horizontal Forces among Bridge Supports (taking into account Deformation of Bearings, Flexing of Piers and Rotation of Foundations)", *1st Intnl. Symp. on Concrete Bridge Design*, A.C.I. April 1967 (Toronto), ACI Spl. Publication No. 23.

CHAPTER 9

Distribution of Externally Applied and Self-induced Horizontal Forces among Bridge-Supports in Curved and/or Skewed Decks (Simple or Continuous Spans)

Synopsis

The phenomenon of superstructure movement in a curved and/or skewed bridge deck, as affected by shrinkage, creep and elastic-shortening of concrete and any temperature variations, is studied and its effect on distribution of the horizontal forces it causes among the supports is analysed[1]. The analysis considers the actual stiffnesses of individual supports and their influence on the location of the zero-movement point, which does not necessarily coincide with the position of any of the supports. The concept of in-plan meandering of deck, as caused by differing shear ratings of any support in two orthogonal directions, is introduced in computing the location of zero-movement point. At the end a practical example is included in an endeavour to illustrate the application.

Notation

Type A support = support with a bearing which produces μR type force.

Type B support = support either with a bearing which produces Δs type force or a support which is monolithic with superstructure,

c = movement coefficient due to temperature change and shrinkage, creep and elastic-shortening of concrete.

Δ = deck movement,

s = shear rating of a Type B support, i.e., horizontal force required to move the top of a Type B support through a unit distance, taking into account horizontal deformation of its bearing, bending of pier shaft or column and rotation of its foundation,

μ = coefficient of friction of a bearing used on Type A support.

R = dead and live load reaction on a support

F = horizontal force caused at a Type B support

t = torsion rating of a support, i.e., twisting moment required to cause a unit radian twist in-plan in the support.

Support U = any arbitrarily chosen support, as a centre of (a, b) coordinate system, with respect to which the coordinates (a, b) of all other supports are known

x, y = coordinates of any support with respect to point of zero-movement; x and y being the coordinate axes through the point of zero-movement

$x_0 y_0$ = coordinates of support U with respect to point of zero-movement (along x and y axes)

ϕ = in-plan meander of superstructure, i.e., the angle between the two sets of coordinate axes (a, b) and (x, y).

T = in-plan torque at a support

M = in-plan moment

θ = angle used in derivation of Δ_x and Δ_y, see Fig. 9.1

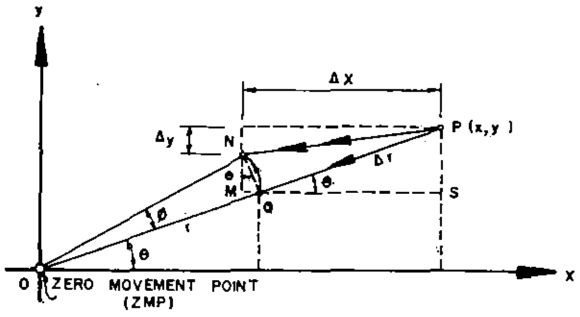
ZMP = zero-movement point,

r and Δ_r = a radial distance and a radial movement respectively as shown in Fig. 9.1

NOTE Suffixes x and y represent component effects in x and y directions, respectively.

9.1 INTRODUCTION

Shrinkage, creep and elastic shortening of concrete, as well as temperature variation, cause volume change in a bridge superstructure, resulting in its deformation in various directions and consequently affecting the distribution of horizontal forces among its supports. Similar volume change effects can occur among the supports of industrial concrete slab structures. The only significant deformations are in-plan movements. These are translatory (linear) in the case of straight bridges, but translatory and rotatory (curvilinear) in the case of curved bridges. The zero-movement point in



NQ ASSUMED NORMAL TO *OP* (ϕ SMALL)
$PQ = \Delta r$
$OQ = r$

Fig. 9.1 *Movement of support due to translation and deck meander*

the former case is consequently a full-width section normal to the bridge length, as against truly a point in the latter case. Thus, locating the point of zero-movement in a curved bridge is much more involved than in case of a straight bridge. It not only involves balancing the horizontal forces in two orthogonal directions but also determining the in-plan meandering twist in the deck resulting from the imbalance in the shear ratings of the supports in the two orthogonal directions.

The distribution of horizontal forces among bridge supports is affected by the horizontal deformation of bearings, bending of pier shafts or columns and rotation of their foundations. Externally applied horizontal forces like braking force, earthquake force, wind force, etc., may be distributed among the supports in proportion to their indirection shear ratings. However, the horizontal forces, induced by volume change due to shrinkage, creep, elastic shortening and temperature, are of a type, the distribution of which depends not only on shear ratings of individual supports but also on the location of zero-movement point. The resultant movement in a curved and/or skewed superstructure at any support takes place along the vector joining that support with the zero-movement point, in case of no in-plan deck meander. In case of straight bridge, the volume change effect on distribution of horizontal forces has already been investigated, analysed and reported by the Author and Witecki in the *First International Symposium on Concrete Bridge Design* published by American Concrete Institute (also refer Ch. 8). The present discussion investigates and analyses the above-mentioned effect in the case of a curved and/or skewed bridge, the most general case.

9.2 ANALYSIS

Consider a curved bridge superstructure subjected to temperature change and shrinking, creeping and elastic shortening of concrete, such that a support $P(x, y)$ moves to $N(x - \Delta_x, y - \Delta_y)$—due to translation Δ_r and in-plan deck meander of ϕ, as shown in Fig. 9.1. Generally angle ϕ is small enough to assume NQ to be normal to OP, so that,

$$\angle POX = \angle PQS = \theta \approx \angle QNM$$

and $$\text{chord } NQ = \text{arc } NQ = r\phi$$

Further the radial movement PQ is equal to the product of movement coefficient c and the distance OP, i.e.,

$$\Delta_r = c(r + \Delta_r)$$

and neglecting the second order term $c\Delta_r$,

$$\Delta_r = cr$$

From Fig. 9.1, the x-direction movement is,

$$\Delta_x = SQ + QM$$

Incorporating the above substitutions:

$$\Delta_x = cr\cos\theta + \phi r\sin\theta$$

and finally because,

$$r\cos\theta = x - QS = x - \Delta_r\cos\theta$$

and $$r\sin\theta = y - PS = y - \Delta_r\sin\theta$$

the above equation yields,

$$\Delta_x = cx - c\Delta_r\cos\theta + \phi y - \phi\Delta_r\sin\theta$$

However, knowing that the products, $c\Delta_r$ and $\phi\Delta_r$ are negligible as c, ϕ and Δ_r are individually very small magnitudes, the final movement of the point $P(x, y)$ in the x-direction is,

$$\Delta_x = cx + \phi y \tag{9.1}$$

Again from Fig. 9.1, the y-direction movement is,

$$\Delta_y = PS - NM$$

Incorporating the various substitutions described earlier and proceeding on similar lines as in the case of Δx, the above equation yields,

$$\Delta_y = cy - c\Delta_r\sin\theta - \phi x + \phi\Delta_r\cos\theta$$

and neglecting the second order terms as before, the final movement of the point $P(x, y)$ in the y-direction is,

$$\Delta_y = cy - \phi x \tag{9.2}$$

Due to deck movement of Δ_x and Δ_y at a (Type B) support $P(x, y)$, the horizontal forces generated are,

in x-direction

$$F_x = s_x \Delta_x = s_x(cx + \phi y) \tag{9.3}$$

in y-direction

$$F_y = s_y \Delta_y = s_y(cy - \phi x) \tag{9.4}$$

and resultant vectorial horizontal force equals

$$\sqrt{(F_x)^2 + (F_y)^2}$$

The x and y coordinates of all the supports with respect to ZMP are unknown, as well as different for different supports. In order to reduce the number of these unknowns to a minimum, it is advantageous to:

(i) establish, and therefore, know the coordinates of all the supports with respect to any arbitrarily chosen support U (0, 0) in the a, b coordinate system.
(ii) relate this support U to the ZMP by its unknown coordinates x_0, y_0 in the x, y coordinate system.
(iii) and finally relate each support to the ZMP by the principle of transformation of axes in terms of known a and b values and unknowns x_0 and y_0.

From the principle of transformation of coordinate axes, if (a, b) are the coordinates of a support $P(a, b)$ with respect to support U (0, 0), and (x, y) are coordinates with respect to ZMP, then, from Fig. 9.2,

$$x = x_0 + a\cos\phi + b\sin\phi \tag{9.5}$$

and

$$y = y_0 + b\cos\phi - a\sin\phi \tag{9.6}$$

It should be noted that Eqs. (9.5) and (9.6) are derived from Fig. 9.2 on the arbitrary assumption that the ZMP lies to the left of support U and below it.

The horizontal forces in the x and y directions, as given by Eqs. (9.3) and (9.4) can then be expressed as follows,

$$F_x = s_x[c(x_0 + a\cos\phi + b\sin\phi) + \phi(y_0 + b\cos\phi - a\sin\phi)] \tag{9.7}$$

$$F_y = s_y[c(y_0 + b\cos\phi - a\sin\phi) - \phi(x_0 + a\cos\phi + b\sin\phi)] \tag{9.8}$$

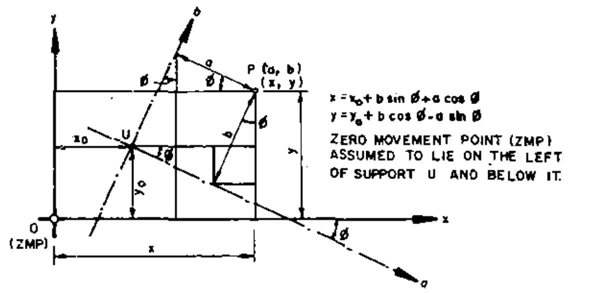


Fig. 9.2 *Relationship between the coordinates of a support P with reference to support U and ZMP*

Further neglecting the second order terms. Eqs. (9.7) and (9.8) may be reduced and re-written in the final form as follows,

$$F_x = s_x[cx_0 + \phi y_0 + (ca + \phi b)\cos\phi] \tag{9.9}$$

and

$$F_y = s_y[cy_0 - \phi x_0 + (cb - \phi a)\cos\phi] \tag{9.10}$$

The three unknowns x_0, y_0 and ϕ locate the ZMP. For locating it, it is necessary to consider the equilibrium of the in-plan horizontal forces in the two orthogonal directions x and y, together with the in-plan moment-equilibrium, viz.

$$\Sigma(F + \mu R)_x = 0 \tag{9.11}$$

$$\Sigma(F + \mu R)_y = 0 \tag{9.12}$$

and

$$\Sigma M = 0 \tag{9.13}$$

Equations (9.11) and (9.12) yield,

$$\Sigma\mu R_x + \Sigma s_x[cx_0 + \phi y_0 + (ca + \phi b)\cos\phi] = 0 \tag{9.14}$$

and

$$\Sigma\mu R_y + \Sigma s_y[cy_0 - \phi x_0 + (cb - \phi a)\cos\phi] = 0 \tag{9.15}$$

where

$$\mu R_x = \mu R\left[1 + \left(\frac{\Delta_y}{\Delta_x}\right)^2\right]^{-1/2} \tag{9.16}$$

$$\mu R_y = \mu R\left[1 + \left(\frac{\Delta_x}{\Delta_y}\right)^2\right]^{-1/2} \tag{9.17}$$

$$\Delta_x = cx_0 + \phi y_0 + (ca + \phi b)\cos\phi \tag{9.18}$$

(from Eqs. (9.1), (9.5) and (9.6))*

and

$$\Delta_y = cy_0 - \phi x_0 + (cb - \phi a)\cos\phi \tag{9.19}$$

(from Eqs. (9.2), (9.5) and (9.6))*

* Neglecting the second order terms.

Note that Eqs. (9.16) and (9.17) are derived from the relation between the force and displacement vector diagrams.

Taking moments about ZMP, Eq. (9.13) yields,

$$\Sigma F_x y + \Sigma F_y x + \Sigma t\phi + \Sigma \mu R_x y + \Sigma \mu R_y x = 0 \quad (9.20)$$

Substituting for the various terms in Eqs. (9.14), (9.15) and (9.20), and neglecting the second order terms, the following three simultaneous equations of equilibrium for evaluating x_0, y_0 and ϕ are arrived at,

$$\Sigma \mu R \left\{ 1 + \left[\frac{cy_0 - \phi x_0 + (cb - \phi a)\cos\phi}{cx_0 + \phi y_0 + (ca + \phi b)\cos\phi} \right]^2 \right\}^{-1/2} + \Sigma s_x [cx_0 + \phi y_0 + (ca + \phi b)\cos\phi] = 0 \quad (9.21)$$

$$\Sigma \mu R \left\{ 1 + \left[\frac{cx_0 + \phi y_0 + (ca + \phi b)\cos\phi}{cy_0 - \phi x_0 + (cb - \phi a)\cos\phi} \right]^2 \right\}^{-1/2} + \Sigma s_y [cy_0 - \phi x_0 + (cb - \phi a)\cos\phi] = 0 \quad (9.22)$$

and

$$\begin{aligned} &\Sigma s_x (y_0 + b\cos\phi)[cx_0 + \phi y_0 + (ca + \phi b)\cos\phi] + \\ &\Sigma s_y (x_0 + a\cos\phi)[cy_0 - \phi x_0 + (cb - \phi a)\cos\phi] + \\ &\Sigma \mu R \left\{ 1 + \left[\frac{cy_0 - \phi x_0 + (cb - \phi a)\cos\phi}{cx_0 + \phi y_0 + (ca + \phi b)\cos\phi} \right]^2 \right\}^{-1/2} \\ &\qquad \times (y_0 + b\cos\phi - a\sin\phi) + \\ &\Sigma \mu R \left\{ 1 + \left[\frac{cx_0 + \phi y_0 + (ca + \phi b)\cos\phi}{cy_0 - \phi x_0 + (cb - \phi a)\cos\phi} \right]^2 \right\}^{-1/2} \\ &\qquad \times (x_0 + a\cos\phi + b\sin\phi) + \Sigma t\phi = 0 \quad (9.23) \end{aligned}$$

These three equations are by no means simple to tackle in an average bridge design office. Fortunately they reduce to a simple set of two Eqs. (9.28) and (9.29) derived ahead in most practical bridge cases where ϕ equals zero. There are two specific cases, presented below, where ϕ equals zero and above-mentioned simplification in Eqs. (9.21), (9.22) and (9.23) takes place.

Case 1

The in-plan meander of deck is zero if its each support individually has its shear ratings in x and y directions equal, i.e.,

$\phi = 0$ if $s_x = s_y$ for every support individually.

Generally this is the case in majority of the bridges because their supports are usually of the following types:

(i) Deck monolithic with circular and/or square column supports.

(ii) Deck sitting on circular and/or square columns through bearings which absorb same effort in deforming equally in all directions.

(iii) Deck sitting on very stiff abutments through bearings which absorb same effort in deforming equally in all directions.

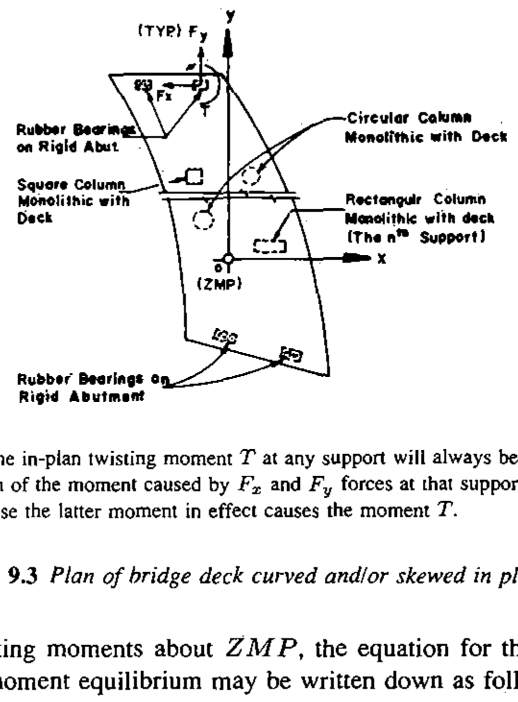
Case 2

The in-plan meander of deck is zero even if there are some supports whose shear ratings are different in different directions, but are so located that at least one of their coordinates with respect to ZMP is zero.

The statements in both these cases can be mathematically proved as follows:

Consider any curved and/or skewed bridge deck on n supports, in general, as shown in the plan in Fig. 9.3. Assume that the shear ratings in the x and y directions of all but the n-th support are equal. i.e.,

$s_x = s_y$ for each of the supports 1 to $n - 1$, equals s (say), and $s_x \neq s_y$ for the n-th support.



NOTE The in-plan twisting moment T at any support will always be in the direction of the moment caused by F_x and F_y forces at that support. This is because the latter moment in effect causes the moment T.

Fig. 9.3 *Plan of bridge deck curved and/or skewed in plan*

Taking moments about ZMP, the equation for the in-plan moment equilibrium may be written down as follows,

$$\sum_1^n [F_x y - F_y x + T] = 0$$

Noting that,

$$F_x = s_x(cx + \phi y)$$
$$F_y = s_y(cy - \phi x)$$

and $$T = t\phi$$

then the above equation may be re-written as,

$$\sum_{1}^{n}\Sigma[s_x(cx+\phi y)y - s_y(cy-\phi x)x + t\phi] = 0$$

Considering the supports 1 to $n-1$ as a separate group from the n-th support, then the equation may be split up and re-written as follows,

$$\sum_{1}^{n-1}[s_x(cx+\phi y)y - s_y(cy-\phi x)x + t\phi] + [s_x(cx+\phi y)y - s_y(cy-\phi x)x + t\phi]nth = 0$$

Noting that $s_x = s_y = s$ for each of the supports 1 to $n-1$, the above equation simplifies into,

$$\sum_{1}^{n-1}\Sigma[s(x^2+y^2)\phi + t\phi] + [x(s_x yc + s_y x\phi) + y(s_x y\phi - s_y xc) + t\phi]nth = 0 \qquad (9.24)$$

If x-coordinate of the n-th support equals zero, then Eq. (9.24) reduces to

$$\phi\left\{\sum_{1}^{n-1}[s(x^2+y^2)+t] + (s_x y^2 + t)nth\right\} = 0 \qquad (9.25)$$

If y-coordinate of the n-th support equals zero, then Eq. (9.24) reduces to,

$$\phi\left\{\sum_{1}^{n-1}[s(x^2+y^2)+t] + (s_y x^2 + t)nth\right\} = 0 \qquad (9.26)$$

And if both x and y coordinates of n-th support equal zero, then Eq. (9.24) reduces to,

$$\phi\left\{\sum_{1}^{n-1}[s(x^2+y^2)+t] + (t)nth\right\} = 0 \qquad (9.27)$$

Since all the terms within the parentheses in Eqs. (9.25), (9.26) and (9.27) are always positive and finite, therefore, in each of them, it is ϕ which must equal zero.

In other words ϕ equals zero either if each support has its s_x equal to its s_y, or even if only some supports are of this type but the rest are so located as to fall at least on one coordinate axis through the ZMP.

Thus, with ϕ equal to zero only the two Eqs. (9.21) and (9.22) need be considered in order to solve remaining two unknowns x_0 and y_0. After the proper substitution and simplification, the two final simultaneous equations, as a solution to the problem of locating ZMP, are as follows,

$$\Sigma\mu R\left\{1+\left[\frac{y_0+b}{x_0+a}\right]^2\right\}^{-1/2} + \Sigma s_x(cx_0+ca) = 0 \qquad (9.28)$$

and

$$\Sigma\mu R\left\{1+\left[\frac{x_0+a}{y_0+b}\right]^2\right\}^{-1/2} + \Sigma s_y(cy_0+cb) = 0 \qquad (9.29)$$

For the particular case of a straight bridge, where always not only ϕ equals zero but also only one coordinate axis, viz., x (same as a) axis, need be considered, Eq. (9.28) yields,

$$\Sigma\mu R + \Sigma s_x c(x_0+a) = 0$$

$$\therefore \quad x_0 = -\frac{\Sigma\mu R + c\Sigma s_x a}{c\Sigma s_x} \qquad (9.30)$$

The $-$ve sign in Eq. (9.30) indicates that the ZMP lies in a direction opposite to that assumed in Fig. 9.2, i.e., to the right of the support U. In the case of a straight bridge if the extreme left-hand support is taken as the support U, then all the other supports lie to its right and obviously the ZMP must also lie to its right. This is pointed out by the $-$ve sign in Eq. (9.30).

It is interesting to note the location of the ZMP as given by the formula in Eq. (9.30) above, is exactly what had already been arrived at for the case of straight bridge in Ch. 8 of this book.

9.3 APPLICATION

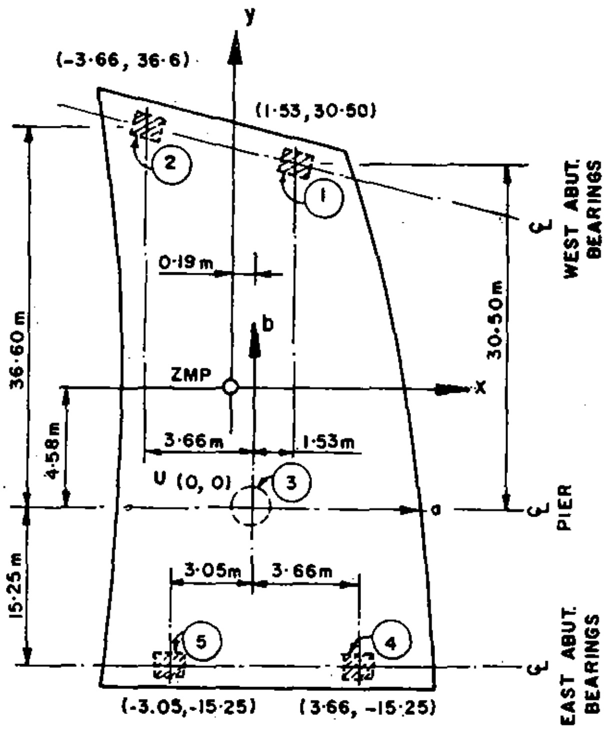
In order to analyse the distribution of horizontal forces that are induced in bridge-supports by the volume changes in a curved and/or skewed concrete bridge superstructure, proceed step by step as follows:

(i) Calculate the shear ratings in x and y directions of each Type B support (same procedure may be followed as explained earlier in Ch. 8).
(ii) Locate the ZMP using Eqs. (9.21), (9.22) and (9.23), unless ϕ equals zero, in which case use Eqs. (9.28) and (9.29) instead.
(iii) Calculate the horizontal forces F_x and F_y in the Type B supports using Eqs. (9.9) and (9.10).
(iv) Calculate the horizontal forces μR_x and μR_y in the Type A supports using Eqs. (9.16) and (9.17) with Δ_x and Δ_y from Eqs. (9.18) and (9.19).
(v) Check in-plan equilibrium by using Eqs. (9.11) and (9.12).

Numerical Example

This example is primarily intended to illustrate the procedure in a practical case.

The structure is a two-span, post-tensioned prestressed concrete bridge deck, curved and skewed in plan, as shown in Fig. 9.4. The superstructure is monolithic with the circular concrete column (support 3), and at the ends rests on rigid abutments, through rubber-steel elastomeric bearings (supports 1, 2, 4 and 5). Hence all supports are of Type B, and as it happens, every support has its s_x equal to its s_y. This means that the in-plan deck meander ϕ is zero.



1, 2, 4 & 5 are Elastomeric (Rubber-Steel) Bearings on Rigid Abutments.
3 is a Circular Column-Pier, Monolithic with Deck. Bridge is Multi-curved and Multiskewed Type.

Fig. 9.4 *Plan of a curved skewed bridge deck*

From the relevant bridge-design calculations,

- for supports 1, 2, 4 and 5, each, $s_x = s_y = 3.57$ T/cm.
- for support 3 $s_x = s_y = 14.28$ T/cm.
- movement coefficient, based on full shrinkage, creep and elastic-shortening of concrete, and a 7.2°C variation in surrounding temperature, $c = 0.001$ cm/cm.

Assume the support 3 to be the arbitrarily chosen support U, the centre of (a, b) coordinate system. From the known geometry of the bridge, the coordinates (a, b) to the supports 1, 2, 4 and 5, are then worked out. These are marked as shown in Fig. 9.4.

Proceeding step-by-step as explained in the beginning of this section, the horizontal forces at various supports as caused by volume change in the deck, may be evaluated as follows,

Step 1 Shear ratings of supports—as detailed above.

Step 2 Locating ZMP: use Eqs. (9.28) and (9.29) since ϕ equals zero. Noting that there are no μR type forces here because there are no Type A supports, and calling s_x and s_y simply as s since s_x equals s_y for every support, Eqs. (9.28) and (9.29) reduce to,

$$\sum_1^5 s\ c(x_0 + a) = 0$$

and

$$\sum_1^5 s\ c(y_0 + b) = 0$$

The summations on the left-hand sides of these equations may be worked out in a tabular manner as shown in Table 9.1.

Table 9.1

Support no:	s T/cm	a cm	b cm	$s.c.(x_0 + a)$ kg	$s.c.(y_0 + b)$ kg
1	3.57	153	3050	3.57 $(x_0 + 153)$	3.57 $(x_0 + 3050)$
2	3.57	−366	3660	3.57 $(x_0 - 366)$	3.57 $(y_0 + 3660)$
3	14.28	0	0	14.28 x_0	14.28 y_0
4	3.57	366	−1525	3.57 $(x_0 + 366)$	3.57 $(y_0 - 1525)$
5	3.57	− 305	−1525	3.57 $(x_0 + 305)$	3.57 $(y_0 - 1525)$
Note: c = 0.001 cm/cm			Total	$(28.56\ x_0 - 543)$	$(28.56\ y_0 + 13300)$

Then $\quad 28.56x_0 - 543 = 0$

and $\quad 28.56y_0 + 13330 = 0$

giving $x_0 = 543/28.56 = 19.1$ cm; i.e., 0.191 m

and $\quad y_0 = -13300/28.56 = -458$ cm, i.e., −4.58 m

Remembering that (x_0, y_0) are the coordinates of the support U with respect to ZMP on x, y coordinate system (Fig. 9.2), then, conversely, $(-x_0 - y_0)$ are the coordinates of the ZMP with respect to the support U, since, for ϕ equal to zero, (a, b) and (x, y) are parallel coordinate systems.

Step 3 Horizontal forces F_x and F_y may now be calculated at each Type B support using Eqs. (9.9) and (9.10). As ϕ equals zero, these equations reduce to,

$$F_x = s\ c(x_0 + a)$$

and

$$F_y = s\ c(y_0 + b)$$

These forces are worked out in a tabular manner in Table 9.2.

Table 9.2

$c = 0.001$ cm/cm, $x_0 = 0.191$m, $y_0 = -4.58$ m							
Support No	s T/cm	a cm	b cm	$(x_0 + a)$ cm	$(y_0 + b)$ cm	$F_x = sc(x_0 + a)$ kg	$F_y = sc(y_0 + b)$ kg
1	3.57	153	3050	172	2522	613	9250
2	3.57	−366	3660	−347	3202	−1239	11450
3	14.28	0	0	19	− 458	272	− 6550
4	3.57	366	−1525	385	− 1983	1372	− 7075
5	3.57	−305	−1525	−286	−1983	−1018	− 7075
					Total	0.00 O.K.	0.00 O.K.

Step 4 Horizontal forces μR_x and μR_y do not exist in the present case as there is no Type A support.

Step 5 The in-plan equilibrium is obviously existing since the summations in Table 9.2 show that,

$$\Sigma F_x = 0$$

and $$\Sigma F_y = 0$$

9.4 CONCLUSION

In view of the increasing number of highway bridges and particularly the multilevel interchanges, where not only all piers may not be identical because of minimum sight distance and clearance requirements but also the geometry often requires curved and skewed superstructures, the problem of distribution of horizontal forces among the supports due to vectorial deck movements attains all the more importance. In designing these supports it is therefore very useful to understand the true phenomenon of force distribution and accordingly design for the actual forces. This realistically reduces the burden on some supports and thereby avoids their being 'overdesigned' unnecessarily, while preventing the other supports from being 'underdesigned' innocently.

The concept outlined in this chapter is not restricted to bridges only. It can be used as a powerful tool in analysing the volume change stresses in any given set of supports under a concrete slab of any shape. Multilevel car parks and buildings and auditoria are only a few examples.

REFERENCE

1. V.K. Raina and A.W. Witecki, "Volume Change Effect on Distribution of Horizontal Forces among Supports in Curved and Skewed Concrete Bridges", *The Bridge and Structural Engineer*, The Indian National Group of the I.A.B.S.E. (Zurich), 1972.

CHAPTER 10

Estimation of 'Design Values' of Axial Load and Bending Moment in a Tall Slender Bridge Support—Guarding against Buckling Effect

When a gap has to be bridged at a high elevation with bridge supports rising vertically from the ground below, be it a crossing over a huge body of water with navigational requirements, an elevated highway over a valley or an elevated flyover over a part of a township—aesthetics, and perhaps economy (depending on the method of construction), may demand slenderlooking tall supports.

In the case of 'short' columns, where the ratio of effective length to least gyrational radius is under about 50, the sections may be designed for values of M (bending moment) and P (vertical load) as worked out from normal simple theory of the first order, i.e., by ignoring the effect of buckling deflections. However, in the case of so-called 'long' columns, where the above ratio is exceeded, the effect of buckling deflections may be significant. In such cases, the sections may be designed, using normal working stresses, either for increased M and P equal to those obtained as in above but divided by appropriate reduction factors described in the relevant design specifications, or for the actual value of P and an exact value of M obtained according to the theory of the second order.[1]

Since the application of the theory of the second order is relatively time consuming, for relatively lower ratios of effective length to least gyrational radius, resort may conveniently be taken to the practice of increasing the simple M and P values by dividing them by the appropriate reduction factors. For relatively higher ratios, however, it may be well worth to go into the intricacies of buckling analysis, because, beyond a certain range, the said factors may yield unrealistically high values of the increased M and P. In fact, in the case of the slender pier-columns of the Tasman Bridge at Hobart,[2] the ratio of effective length to least gyrational radius is so high that according to the appropriate reduction factors it would have been almost impossible to get away with such slender supports.

The Approach

The suggested approach for establishing the design values of bending moment M and axial load P in concrete columns subjected to bending and axial thrust is as follows:

Step 1 *Determine l_e/r Value (ratio of 'effective length' to least radius of gyration)*

- Establish effective length l_e (see following table)

Type of Column	l_e
Properly restrained at both ends in position and direction	0.75 L
Properly restrained at both ends in position and imperfectly restrained in direction at one or both ends	a value between 0.75 L and 1.00 L, depending on the efficiency of the directional restraint.
Properly restrained at one end in position and direction and imperfectly restrained in both position and direction at the other end	a value between 1.00 L and 2.00 L depending upon the efficiency of the imperfect restraint (l_e = 2.00 L for free cantilever)

NOTE L = actual length of column

- Compute least radius of gyration r of the section, $= \sqrt{\dfrac{I_{\min}}{A}}$, where $I_{\min}$ is the minimum second moment of area of full section area A
- Compute l_e/r value

Step 2 *Procedure to Establish the Design Values of M and P*

(i) If $l_e/r \leq 50$, then buckling effect is insignificant. Therefore find M and P from simple 1st order theory of bending.

(ii) If $l_e/r > 50$ but ≤ 100, then buckling effect can be significant. Therefore establish design values of M and P by any of the following two approaches (second one is more accurate and more realistic)

(a) *Method 1*: Divide the 1st order theory values of M and P by reduction factor ϕ, where

$$\phi = (1.5 - l_e/100\ r)$$

(b) *Method 2*: For the given values of P, the lateral load, the movement at column top, the amount of out of plumb construction, and the amount of built-in sinusoidal curvature, compute M from the 2nd order theory taking account of buckling (the increase of deflection owing to increasing eccentricity between load line and section cg line).

(iii) If $l_e/r > 100$, then either revise the section to control l_e/r to less than 100 and then proceed as in method (1) or otherwise proceed directly as per the 2nd order theory referred to in method (2).

10.1 2ND ORDER THEORY

This section analyses a tall bridge support according to the theory of second order, taking into account:

(i) Vertical load
(ii) Uniformly distributed lateral wind.
(iii) Horizontal movement at the top of the support owing to deck movement (assuming a rocker connection between the deck and the support)
(iv) Defective construction of the support resulting in its being out of plumb along a curve.
(v) Defective construction of the support resulting in a built-in initial curvature.

Notation

EI = modulus of rigidity
w = uniformly distributed wind load
x = distance measured along length direction, top downwards.
l = length of bridge support,
M = bending moment
P = vertical load
V = support reaction
A, B = constants of integration
CF = complimentary function in the solution of a differential equation
PI = particular integral in the solution of a differential equation
CS = complete solution of a differential equation
y = deflection
$\frac{dy}{dx}$ = slope
D^2y or $\frac{d^2y}{dx^2}$ = curvature,
$\alpha^2 = \frac{P}{EI}$
a = deck movement at top of support
b = out of plumb offset at top of support.
c = maximum offset in the built-in sinusoidal curvature

Analysis

In an initially curvatured axially compressed column-member, which in addition is subjected to a certain lateral loading, the total deflection may be obtained by superimposing on the deflections due to initial curvature and compression the deflection due to lateral loading calculated as for an initially straight compressed column.[3] Such a superimposition is valid because the effect of an initial curvature can be replaced by the effect of an equivalent lateral load. However, it is essential in calculating the deflections produced by each kind of lateral loading, to assume the presence of the compressive force P with each.

On the basis of this principle of superimposition, the combined effect of the actions (i) to (v), listed earlier, is therefore, treated as a superimposition of the following four cases, taken one at a time:

Case (A) Axial compression and lateral wind
Case (B) Axial compression and initial curvature due to deck movement
Case (C) Axial compression and initial curvature due to out of plumb defective construction
Case (D) Axial compression and initial curvature due to built-in curvature due to defective construction.

NOTE For each of these cases (A to D) we will assume that the bridge support is fixed at base and pin* supporting the deck above.

Case (A)

See Fig. 10.1. According to the differential equation of flexure,

$$EI\frac{d^2y}{dx^2} = -Py - Vx + \frac{wx^2}{2} \qquad (10.1)$$

or $$\frac{d^2y}{dx^2} + \frac{P}{EI}y = -\frac{Vx}{EI} + \frac{wx^2}{2EI}$$

or $$(D^2 + \alpha^2)y = -\frac{Vx}{EI} + \frac{wx^2}{2EI}$$

$$CF \text{ is } y = A\sin\alpha x + B\cos\alpha x$$

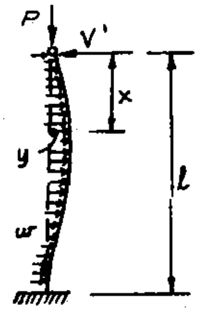


Fig. 10.1

* If the pier is monolithic with deck (i.e., fixed at top also), then an additional unknown moment M is introduced into the analysis. 2nd order theory analysis of such a column (subjected to axial load, horizontal force at top, a given deck movement at top, out of plumb construction, built-in curvature and transverse udl) can be carried out similarly as has been exemplified in the case of tall (slender) exposed piles in Chapter 11 in this book.

$$PI \text{ is } y = \frac{1}{(D^2+\alpha^2)}\left\{-\frac{Vx}{EI}+\frac{wx^2}{2EI}\right\}$$

$$= -\frac{V}{EI}\left\{\frac{1}{\alpha^2\left(1+\frac{D^2}{\alpha^2}\right)}x\right\}+\frac{w}{2EI}\left\{\frac{1}{\alpha^2\left(1+\frac{D^2}{\alpha^2}\right)}x^2\right\}$$

$$= -\frac{V}{\alpha^2 EI}\left\{\left(1+\frac{D^2}{\alpha^2}\right)^{-1}x\right\}$$

$$+\frac{w}{2\alpha^2 EI}\left\{\left(1+\frac{D^2}{\alpha^2}\right)^{-1}x^2\right\}$$

$$= -\frac{V}{P}\left\{\left(1-\frac{D^2}{\alpha^2}+\cdots\right)x\right\}$$

$$+\frac{w}{2P}\left\{\left(1-\frac{D^2}{\alpha^2}+\cdots\right)x^2\right\}$$

$$= -\frac{V}{P}\{x-0\}+\frac{w}{2P}\left\{x^2-\frac{2}{\alpha^2}\right\}$$

$$= -\frac{Vx}{P}+\frac{wx^2}{2P}-\frac{w}{\alpha^2 P}$$

$$CS \text{ is } y = A\sin\alpha x + B\cos\alpha x + \frac{wx^2}{2P}-\frac{Vx}{P}-\frac{w}{\alpha^2 P} \quad (10.2)$$

at $x = 0$, $y = 0$,

$$\therefore \text{ from Eq. (10.2) } B = \frac{w}{\alpha^2 P} \quad (10.3)$$

at $x = l$, $y = 0$, $\therefore$ from Eq. (10.2)

$$0 = A\sin\alpha l + B\cos\alpha l + \frac{w}{2P}l^2 - \frac{V}{P}l - \frac{w}{\alpha^2 P} \quad (10.4)$$

Also, from Eq. (10.2)

$$\frac{dy}{dx} = \alpha A\cos\alpha x - \alpha B\sin\alpha x + \frac{wx}{P}-\frac{V}{P}$$

at $x = l$, $\frac{dy}{dx} = 0$ so that,

$$0 = \alpha A\cos\alpha l - \alpha B\sin\alpha l + \frac{wl}{P}-\frac{V}{P} \quad (10.5)$$

Solving Eqs. (10.5) and (10.4) simultaneously, we obtain:

$$A = \frac{w}{\alpha^2 P}\left\{\frac{l^2\alpha^2+2(1-\cos\alpha l-\alpha l\sin\alpha l)}{2(\sin\alpha l-\alpha l\cos\alpha l)}\right\} \quad (10.6)$$

and $$V = \frac{w}{\alpha}\left\{\frac{l^2\alpha^2+2(1-\cos\alpha l-\alpha l\sin\alpha l)}{2(\tan\alpha l-\alpha l)}\right\}$$

$$-\frac{w}{\alpha}\sin\alpha l + wl \quad (10.7)$$

Thus knowing A, B and V, deflection can be evaluated from Eq. (10.2) and then bending moment from Eq. (10.1), at any section.

Greatest M occurs at base ($x = l$ and $y = 0$) and, after a lengthy simplification, it comes to

$$M_{\text{base}} = \frac{wl}{\alpha}\left\{\frac{\left(\frac{\alpha l}{2}-\tan\frac{\alpha l}{2}\right)\tan\alpha l}{(\tan\alpha l-\alpha l)}\right\} \quad (10.8)$$

Case (B)

See Fig. 10.2. After the deck has moved by an amount a the top of the support will be prevented by it from further movement due to P. So an unknown reaction V will be called into play.

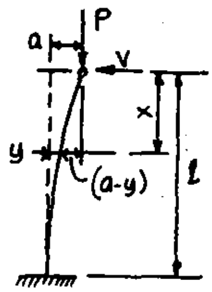


Fig. 10.2

According to the differential equation of flexure,

$$EI\frac{d^2y}{dx^2} = P(a-y) - Vx \quad (10.9)$$

or $$\frac{d^2y}{dx^2}+\frac{P}{EI}y = \frac{P}{EI}a - \frac{V}{EI}x$$

or $$(D^2+\alpha^2)y = -\frac{V}{EI}x + \alpha^2 a$$

$$CF \text{ is } y = A\sin\alpha x + B\cos\alpha x$$

$$PI \text{ is } y = \frac{1}{(D^2+\alpha^2)}\left\{-\frac{V}{EI}x+\alpha^2 a\right\}$$

Proceeding as in Case (A), this gives PI as $y = a - \frac{Vx}{P}$

$$\therefore CS \text{ is } y = A\sin\alpha x + B\cos\alpha x - \frac{Vx}{P} + a \quad (10.10)$$

at $x = 0$, $y = a$, $\therefore$ from Eq. (10.10)

$$B = 0 \quad (10.11)$$

at $x = l$, $y = 0$, $\therefore$ from Eq. (10.10)

$$0 = A \sin \alpha l - \frac{Vl}{P} + a \qquad (10.12)$$

Also, from Eq. (10.10)

$$\frac{dy}{dx} = \alpha A \cos \alpha x - \frac{V}{P} \qquad (10.13)$$

at $x = l$, $\frac{dy}{dx} = 0$, $\therefore$ from Eq. (10.13)

$$0 = \alpha A \cos \alpha l - \frac{V}{P} \qquad (10.14)$$

Solving Eqs. (10.14) and (10.12) simultaneously, we obtain,

$$A = \frac{a \sec \alpha l}{\alpha l - \tan \alpha l} \qquad (10.15)$$

and

$$V = \frac{a \alpha P}{\alpha l - \tan \alpha l} \qquad (10.16)$$

Thus knowing A, B and V, deflection can be evaluated from Eq. (10.10) and then bending moment from Eq. (10.9), at any section.

Greatest M occurs at base ($x = l$, $y = 0$) and comes to

$$M_{\text{base}} = \frac{P_a}{(1 - \alpha l \cot \alpha l)} \qquad (10.17)$$

Case (C)

This case may be analysed on the same lines as case (B) above, only replacing 'a' by 'b', the out of plumb offset at top of support.

Case (D)

See Fig. 10.3. Assume a built-in curvature of sinusoidal form $c \sin \frac{\pi x}{l}$, having a maximum offset of c, and a built-in slope of $-\frac{c\pi}{l}$

From the differential equation of flexure

$$EI \frac{d^2}{dx^2}\left(y - c \sin \frac{\pi x}{l}\right) = -Py - Vx \qquad (10.18)$$

or $\frac{d^2 y}{dx^2} + c\left(\frac{\pi}{l}\right)^2 \sin \frac{\pi x}{l} + \frac{P}{EI} y = -\frac{Vx}{EI}$

or $(D^2 + \alpha^2) y = -\frac{Vx}{EI} - c\left(\frac{\pi}{l}\right)^2 \sin \frac{\pi x}{l}$

CF is $y = A \sin \alpha x + B \cos \alpha x$

PI is $y = \frac{1}{(D^2 + \alpha^2)}\left\{-\frac{Vx}{EI} - c\left(\frac{\pi}{l}\right)^2 \sin \frac{\pi x}{l}\right\}$

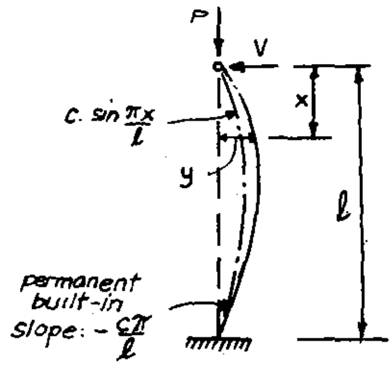


Fig. 10.3

$$= -\frac{V}{EI}\left\{\frac{1}{\alpha^2\left(1 + \frac{D^2}{\alpha^2}\right)} \cdot x\right\}$$

$$- c\left(\frac{\pi}{l}\right)\left\{\frac{1}{(D^2 + \alpha^2)} \cdot \sin \frac{\pi x}{l}\right\}$$

$$= -\frac{V}{P}\left\{\left(1 + \frac{D^2}{\alpha^2}\right)^{-1} \cdot x\right\}$$

$$- c\left(\frac{\pi}{l}\right)^2 \cdot \left\{\frac{\sin \frac{\pi x}{l}}{-\left(\frac{\pi}{l}\right)^2 + \alpha^2}\right\}.$$

$$= -\frac{V}{P}\left\{\left(1 - \frac{D^2}{\alpha^2} + \ldots\right) x\right\} + \frac{c \sin \frac{\pi x}{l}}{1 - \left(\frac{\alpha l}{\pi}\right)^2}$$

$$= -\frac{Vx}{P} + k \sin \frac{\pi x}{l}$$

where $k = \frac{c}{1 - \left(\frac{\alpha l}{\pi}\right)^2}$ (10.19)

$$\therefore CS \text{ is } y = A \sin \alpha x + B \cos \alpha x - \frac{Vx}{P} + k \sin \frac{\pi x}{l} \qquad (10.20)$$

at $x = 0$, $y = 0$, $\therefore$ from Eq. (10.20),

$$B = 0 \qquad (10.21)$$

at $x = l$, $y = 0$, $\therefore$ from Eq. (10.20),

$$0 = A \sin \alpha l - \frac{Vl}{P} \qquad (10.22)$$

Also, from Eq. (10.20)

$$\frac{dy}{dx} = \alpha A \cos \alpha x - \frac{V}{P} + k\left(\frac{\pi}{l}\right)\cos\frac{\pi}{l}x \qquad (10.23)$$

at $x = l$, $\dfrac{dy}{dx} = -\dfrac{c\pi}{l}$, $\therefore$ from Eq. (10.23)

$$-\frac{c\pi}{l} = \alpha A \cos \alpha l - \frac{V}{P} - k\left(\frac{\pi}{l}\right) \qquad (10.24)$$

Solving Eqs. (10.24) and (10.22) simultaneously, we obtain,

$$A = \frac{\pi(c-k)}{(\sin \alpha l - \alpha l \cos \alpha l)} \qquad (10.25)$$

and

$$V = \frac{\pi(c-k)P}{l(1 - \alpha l \cot \alpha l)} \qquad (10.26)$$

Thus knowing A, B and V, deflection can be evaluated from Eq. (10.20), and then bending moment from Eq. (10.18), at any section.

Bending moment at base ($x = l$, $y = 0$) comes to

$$M_{\text{base}} = \frac{\pi(c-k)P}{(1 - \alpha l \cot \alpha l)} \qquad (10.27)$$

where k is as defined in Eq. (10.19).

Numerical Example

Consider a 110 ft tall concrete bridge support of constant rectangular section, 7 ft wide × 5 ft deep, subjected to a total vertical load of 1610 kips, a lateral wind of constant intensity 30 lb/s ft, a deck movement of 5 in., an out of plumb offset of 1 in. at top, and a sinusoidal curvature having a maximum offset of 4 in.

$P = 1.61 \times 10^6$ lb., $w = 30 \times 7 \times \dfrac{1}{12} = 17.5$ lb/in. run,
$l = 110 \times 12 = 1320$ in.

Concrete is such that $EI = 6.20 \times 10^{12}$ lb in.^{2}

$$\alpha = \sqrt{\frac{P}{EI}} = 0.509 \times 10^{-3}\ \text{in.}^{-1},\ \alpha l = 0.6719\ \text{radian},$$

$$\left(\frac{\alpha l}{\pi}\right)^2 = .0456,$$

$$a = 5'',\ b = 1'',\ c = 4'',\ k = \frac{c}{1 - \left(\dfrac{\alpha l}{\pi}\right)^2} = 4.2\ \text{in. units.}$$

Case (A)

Evaluating A, B and V from Eqs. (10.6), (10.3) and (10.7), we get,

$$A = 10.80,\quad B = 41.9541 \text{ and } V = 8618.97$$

Then deflection and bending moment are evaluated from Eqs. (10.2) and (10.1) for various values of x. These values are recorded in Table 10.1.

Case (B)

Evaluating A and V from Eqs. (10.15) and (10.16) and noting that $B = 0$ in this case, we get:

$$A = -51.889,\quad V = -33285.6$$

Then deflection and bending moment are evaluated from Eqs. (10.10) and (10.9) for various values of x. These values are recorded in Table 10.1.

Case (C)

As $b/a = 1/5$, the deflections and bending moments in this case are simply 1/5th of those in Case (B) above.

Case (D)

Evaluating A and V from Eqs. (10.25) and (10.26) and

Table 10.1

	x (ft)	Deflection (in)					Bending Moment ($\times 10^6$ lb. in)				
		Case A	Case B	Case C	Case D	Total ...	Case A	Case B	Case C	Case D	Total ...
Top	0	0	5.00	1.00	0.00	6.000	0	0	0	0	0
↓	10	.015	4.32	.864	1.15	6.349	− .93	5.10	1.02	−1.26	3.93
	20	.030	3.64	.728	2.21	6.608	−1.62	10.18	2.03	−2.37	8.22
	30	.044	2.99	.598	3.09	6.722	−2.04	15.22	3.04	−3.20	13.02
	40	.049	2.37	.474	3.72	6.613	−2.20	20.21	4.04	−3.62	18.42
	50	.051	1.80	.360	4.03	6.241	−2.10	25.12	5.02	−3.52	24.52
	60	.048	1.34	.268	4.04	5.696	−1.75	29.86	5.97	−2.95	31.13
	70	.041	.85	.170	3.72	4.781	−1.13	34.65	6.93	−1.84	38.61
	80	.028	.48	.096	3.07	3.674	− .26	39.23	7.85	− .20	46.62
	90	.016	.22	.044	2.20	2.480	.87	43.65	8.73	1.80	55.05
	100	.012	.05	.010	1.15	1.222	2.24	47.92	9.58	4.08	63.82
Base	110	0	0	0	0	0	3.87	52.00	10.40	6.52	72.79

noting that $B = 0$ in this case, we get:

$$A = -6.51, \quad V = -4940$$

Then deflection and bending moment are evaluated from Eqs. (10.20) and (10.18) for various values of x. These values are recorded in Table 10.1.

10.2 CONCLUSION

The usefulness of this analysis can immediately be concluded by comparing the M and P values, say at the base section as obtained in the above examples, with their design values as obtained by the first order theory modified by the appropriate reduction factors.

Treating the pier as a propped cantilever subjected to a lateral wind of $(30 \times 7 =)$ 210 lb/ft run, a prop-sinking of $(5 + 1 =)6$ in. due to deck movement and out of plumb construction, and a longitudinal (vertical) load of 1610 kips causing bending moment at the base owing to prop sinking, then,

$$M_{\text{base}} = \left(\frac{3EIa}{l^2}\right) + P \cdot a + \left(\frac{wl^2}{8}\right)$$

$$= \left(\frac{3 \times 6.20 \times 10^{12} \times 6}{1320 \times 1320}\right) + (1610 \times 10^3 \times 6) + \left(\frac{210 \times 110 \times 1320}{8}\right) = 77.58 \times 10.6 \text{ lb in.}$$

For 7 ft $\times$ 5 ft section:

$$\frac{I_{\text{min}}}{A} = \frac{1}{12} \times 7 \times 5^3 \times \frac{1}{7 \times 5} = 2.08 \text{ ft}^2, \text{ so that}$$

$$r = \sqrt{2.08} = 1.44 \text{ ft}$$

$$\therefore \frac{l_{\text{eff}}}{r} = \frac{0.9 \times 110}{1.44} = 68.75$$

which is > 50, hence reduction factor to be applied.

Reduction Factor

$$\phi = 1.5 - l_{\text{eff}}/100\,r = 1.5 - \frac{0.9 \times 110}{100 \times 1.44}$$

$$= 0.8125$$

$\therefore$ design values of M and P, at base-section are,

$$M = \left(\frac{77.58 \times 10^6}{0.8125}\right) = 95.38 \times 10^6 \text{ lb in.}$$

and $$P = \left[\frac{1.610 \times 10^6}{0.8125}\right] = 1.98 \times 10^6 \text{ lb}$$

as compared to $(72.79 \times 10^6 - 6.52 \times 10^6 =)\ 66.27 \times 10^6$ lb in. and 1.61×10^6 lb respectively from the second order theory analysis.

Therefore, it can be seen that even in the present case of not too slender a pier ($\frac{l_{\text{eff}}}{r}$ only 68.75) the second order theory analysis can save about 44% on M and 23% on P, which is some saving—thanks to the 2nd order theory.

REFERENCES

1. International Code of Practice for reinforced concrete," CEB.
2. "The Superstructure of Tasman Bridge, Hobart," Discussion in the Structural Engineer, vol. 45, No. 10, London, Oct. 1967.
3. Timoshenko, "Theory of Elastic Stability"

CHAPTER 11

Analysis and Design of Slender Exposed Piles in a Group

Synopsis

Most of the methods available for analysing slender exposed (reinforced concrete) piles subjected to axial thrust, sway and lateral forces, are at best terribly theoretical and at worst inaccurate. This leaves a practising designer to the mercy of established and conservative methods whose scope is limited — leaving little room for bold engineering design. Since computerised calculations cannot be performed for every case by every designer, there is obviously need for a relatively easily applicable method, which simultaneously is also reasonably accurate. This chapter very simply derives the general expression for bending effects in long slender piles using second order theory for taking account of buckling deflections for various combinations of axial thrust and lateral forces. As well as this, it presents a general appreciation of various parameters enveloping the field of analysis and design of RC piles in scourable soil medium, a step by step application for analysis and design for RC piles and various recommendations. This is followed by a practical numerical example illustrating the entire procedure.

11.1 INTRODUCTION

Exposed piles are relatively long and slender structural members used for transmitting loads and lateral forces to deeper and more dependable subsoil strata. RC piles can either be precast and then driven into the ground, or driven and cast *in situ* or bored and cast *in situ*.

Piles in a group may either be all vertical or some vertical and some raked or all raked — either in opposite directions or in the same direction, depending on the functional requirements.

Owing to the slender columnar behaviour of exposed piles (and therefore being prone to buckling), the established and conservative method is to affix an effective length L to the column on the basis of its assumed end conditions, calculate the reduction factor, and then design the section using normal working stresses for increased bending moment and direct load equal to those obtained from first order analysis divided by the reduction factor. However, where the pile is exposed for a considerable length, the above referred reduction factor may be so low that it may almost be ridiculous if not impossible to design the pile section by this approach. Moreover, at any rate, affixing the effective length value to the column may in itself be ambiguous and unrealistic. All this may cumulatively lead to gross overdesign.

So long as the longitudinal reinforcement of piles is adequately embedded into the pile cap, the pile may be assumed to be fixed at top. The lower fixity point of the pile may be assumed at a distance below ground (or scour level) equal to either 10% of its exposed length or half the depth of soft strata whichever is greater. However, even in the case of soft marine clay, the lower fixity point does not lie more than 3–4 m below the ground (or scour) level.

The practice of designing pile sections for axial load alone so long as the net resultant lateral force on the pile group (after taking into account horizontal resistance of raked piles if any) does not exceed 5% of the total vertical load on the pile group, appears to be a rather crude way of terminating the process of design. This is not to say that the buried portions of the piles will not mobilise a dependable passive resistance equal to at least 5% of the total vertical load (in fact in most cases it is even more), but it is just that moments are set in the piles in the process of transmitting the horizontal forces through their exposed body, the effect of which should not be ignored in the design of the pile section before being optimistic about the subsequent horizontal soil-resistance.

Structural Strength of Pile

In the analysis of exposed pile groups, often the apparent mathematical exactitude displayed through the so-called sophisticated methods of analysis is diffused through the indefiniteness of the very assumptions on which they are based. This is particularly so when dealing with the abstruse. Apportioning of vertical loads among piles (resulting from direct load and orthogonal bending moments on the pile group) by the simple rivet group analysis, in the limit, is no more inaccurate than most other methods. It is the study of the combined effect of the thus calculated axial load on a pile and the lateral forces on it (latter estimated on the basis of all piles sharing the total lateral force equally owing to the relatively extremely stiff pile cap transom), that merits an in-depth mathematical investigation. The pile section should be adequately designed for the bending moment and direct load resulting from such analysis based on the second order theory (which takes into account the effect of buckling deflections), using normal working stresses. This saves having to sermonise in the field of ambiguity so inherent in the process of trying to affix effective lengths and reduction factors for really slender exposed piles.

Soil Strength

Ultimate soil resistance around an individual pile comprises the end bearing capacity and the shaft resistance due to friction or cohesion depending on the type of soil. However, the negative skin friction due to fill above ground (or scour level) and the functional self weight of pile should be deducted from it.

Ultimate soil resistance around a pile group against block failure can be estimated as in the case of an individual pile as described in the previous section but considering the whole group as one big pile of perimeter encircling all the piles in the group. From this block failure consideration, the ultimate soil resistance around an individual pile may be taken as an nth of the above, n being the total number of piles in the group.

In cohesive soils, since the ultimate soil resistance of the pile group of n piles is always lesser than n times the ultimate soil resistance around an individual pile, it is necessary in such soils to also ensure that the total vertical load on the pile group is lesser than n times the individual soil resistance around a pile multiplied by an efficiency factor[1] whose value ranges from 0.7 to 0.9, depending on the spacing of piles within the group. In cohesionless soils, the efficiency factor is more than unity owing to higher compaction.

The dependable carrying capacity of a pile may be taken as the least of that given in the above two paras divided by a factor of safety of 2.5, and this, in turn, should not be less than the maximum axial load actually coming on the pile.

However, the dependable soil resistance around a pile should not be taken to be more than the minimum working load carrying capacity of the pile as ascertained from load tests.

Where piles are founded on rock, their load carrying capacity should generally be based only on end bearing resistance even if they are surrounded by nonscourable soil. In fact, the surrounding soil mass, owing to its continued consolidation activity, loads the pile (negative skin friction) instead of supporting it. This is because the pile itself cannot move downwards relative to soil once it is seated on rock. In the absence of nonscourable soil surrounding these piles, the lower fixity point may be assumed at the foot of the pile provided the pile base is suitably anchored to rock, but this needs a very careful study for fear of adopting unrealistic moments.

11.2 ANALYSIS FOR COLUMN ACTION BY SECOND ORDER THEORY

In an initially curvatured axially compressed column member, which, in addition is subjected to certain lateral loading, the total deflection may be obtained by superimposing on the deflections due to initial curvature and compression the deflection due to lateral loading calculated as for an initially straight compressed column. Such a superimposition is valid because the effect on initial curvature can be replaced by the effect of equivalent lateral load. However, it is essential in calculating the deflection produced by each system of lateral loading to assume the presence of the axial compressive force.[2] On the basis of the principle of this superimposition, the combined effect of axial compression, lateral load and initial curvature, can therefore be treated as the superimposition of the following cases:

> ***Case A:*** Axial compression P and lateral force H at the end of the pile.
> ***Case B:*** Axial compression P and initial curvature due to possible inaccuracies in construction.
> ***Case C:*** Axial compression P and uniform transverse load w (if there is any).

Case A

See Fig. 11.1. According to the differential equation of flexure,

$$EI\frac{d^2y}{dx^2} = P(q-y) - Hx - M \tag{11.1}$$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{P}{EI}(q-y) - \frac{H}{EI}x - \frac{M}{EI}$$

If $\dfrac{P}{EI} = \alpha^2$, then $\dfrac{d^2y}{dx^2} = \alpha^2(q-y) - \alpha^2\dfrac{H}{P}x - \alpha^2\dfrac{M}{P}$

or $\quad D^2y = \alpha^2(q-y) - \alpha^2\dfrac{H}{P}x - \alpha^2\dfrac{M}{P}$

or $\quad (D^2+\alpha^2)y = \alpha^2\left\{q - \dfrac{H}{P}x - \dfrac{M}{P}\right\}$

Solving this differential equation, complimentary function (CF) is $y = A\sin\alpha x + B\cos\alpha x$ and particular integral (PI) is

$$y = \frac{1}{(D^2+\alpha^2)}\alpha^2\left[q - \frac{H}{P}x - \frac{M}{P}\right]$$

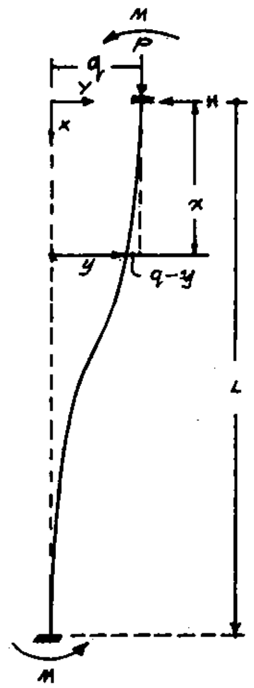


Fig. 11.1

$$= \frac{1}{\alpha^2\left(1+\frac{D^2}{\alpha^2}\right)}\alpha^2\left[q-\frac{H}{P}x-\frac{M}{P}\right]$$
$$= \left(1-\frac{D^2}{\alpha^2}+\ldots\right)\left[q-\frac{H}{P}x-\frac{M}{P}\right]$$
$$= \left[q-\frac{H}{P}x-\frac{M}{P}\right]$$

$\therefore$ Complete solution is $CF + PI$, i.e.,

$$y = A\sin\alpha x + B\cos\alpha x + q - \frac{H}{P}x - \frac{M}{P} \quad (11.2)$$

and
$$\frac{dy}{dx} = \alpha A\cos\alpha x - \alpha B\sin\alpha x - \frac{H}{P} \quad (11.3)$$

The four unknowns A, B, q and M are worked out from the above two equations for the following end conditions:

$$\text{at } x=0,\ \ y=q \text{ and } \frac{dy}{dx}=0$$
$$\text{at } x=L,\ \ y=0 \text{ and } \frac{dy}{dx}=0$$

giving $A = \dfrac{H}{\alpha P}$, $B = -\dfrac{H}{\alpha P}\tan\alpha L/2,$

$$q = -\frac{H}{\alpha P}(\alpha L - 2\tan\alpha L/2) \text{ and } M = -\frac{H}{\alpha}\tan\alpha L/2$$

Hence deflection y and BM: $EI\dfrac{d^2y}{dx^2}$ at any section can be evaluated from Eqs. (11.2) and (11.1).

Maximum BM occurs at ends and is given by;

$$\left[EI\frac{d^2y}{dx^2}\right]_{\substack{x=L\\y=0}} = Pq - HL - M$$
$$= -\frac{H}{\alpha}\tan\alpha L/2 \quad (11.4)$$

Case B

See Fig. 11.2. Assume a built-in curvature (due to possible inaccuracies in construction) of sinusoidal form $c\sin\dfrac{\pi x}{L}$ having a maximum offset c and a built-in slope $-\dfrac{c\pi}{L}$ at the lower end (at $x = L$) and $\dfrac{c\pi}{L}$ at the upper end (at $x = 0$).

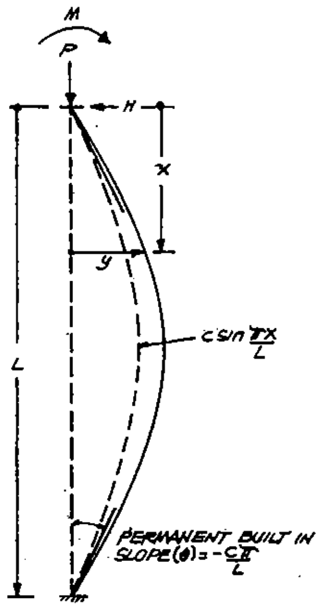


Fig. 11.2

From the differential equation of flexure:

$$EI\frac{d^2}{dx^2}\left(y - c\sin\frac{\pi x}{L}\right) = -Py - Hx + M \quad (11.5)$$

or
$$\frac{d^2}{dx^2}\left(y - c\sin\frac{\pi x}{L}\right) = -\frac{P}{EI}y - \frac{H}{EI}x + \frac{M}{EI}$$

If $\dfrac{P}{EI} = \alpha^2$, then $D^2\left(y - c\sin\dfrac{\pi x}{L}\right) = -\alpha^2 y - \alpha^2\dfrac{H}{P}x + \alpha^2\dfrac{M}{P}$

or
$$D^2y + c\left(\frac{\pi}{L}\right)^2\sin\frac{\pi x}{L} = -\alpha^2 y + \alpha^2\left[\frac{M}{P} - \frac{H}{P}x\right]$$

i.e., $(D^2+\alpha^2)y = \alpha^2\left[\frac{M}{P}-\frac{H}{P}x\right] - c\left(\frac{\pi}{L}\right)^2 \sin\frac{\pi x}{L}$

Solving this differential equation,
CF is $y = A\sin\alpha x + B\cos\alpha x$,

$$\text{and } PI \text{ is } y = \frac{1}{(D^2+\alpha^2)}\left[\alpha^2\left(\frac{M}{P}-\frac{H}{P}x\right) - c\left(\frac{\pi}{L}\right)^2\sin\frac{\pi x}{L}\right]$$

$$= \frac{1}{\alpha^2\left(1+\frac{D^2}{\alpha^2}\right)}\alpha^2\left(\frac{M}{P}-\frac{H}{P}x\right) - c\left(\frac{\pi}{L}\right)^2\frac{1}{(D^2+\alpha^2)}\sin\frac{\pi x}{L}$$

$$= \left(1-\frac{D^2}{\alpha^2}+\ldots\right)\left(\frac{M}{P}-\frac{H}{P}x\right) + c\left(\frac{\pi}{L}\right)^2\frac{\sin\frac{\pi x}{L}}{(\pi/L)^2-\alpha^2}$$

$$= \frac{M}{P}-\frac{H}{P}x+\frac{c\sin\pi x/L}{1-(\alpha L/\pi)^2}$$

$$= \frac{M}{P}-\frac{H}{P}x + k\sin\frac{\pi x}{L'}$$

$$\text{where } k = \frac{c}{1-(\alpha L/\pi)^2}$$

Complete solution is $CF + PI$, i.e.,

$$y = A\sin\alpha x + B\cos\alpha x + \frac{M}{P} - \frac{H}{P}x + k\sin\frac{\pi x}{L} \quad (11.6)$$

$$\frac{dy}{dx} = \alpha A\cos\alpha x - \alpha B\sin\alpha x - \frac{H}{P} + \frac{k\pi}{L}\cos\frac{\pi x}{L} \quad (11.7)$$

The four unknowns A, B, H and M are worked out from the above two equations for the following four end conditions:

$$\text{at } x = 0,\quad y = 0 \text{ and } \frac{dy}{dx} = \frac{c\pi}{L}$$

$$\text{at } x = L,\quad y = 0 \text{ and } \frac{dy}{dx} = -\frac{c\pi}{L}$$

$$\text{giving } A = \frac{\pi(c-k)}{\alpha L},\quad B = -\frac{\pi(k-c)}{\alpha L}\cot\frac{\alpha L}{2}$$

$$H = P\left[\alpha A + \frac{\pi}{L}(k-c)\right],\ = 0$$

$$\text{and}\quad M = \frac{P\pi(k-c)}{\alpha L}\cot\alpha L/2$$

Hence deflection y and BM: $EI\frac{d^2}{dx^2}\left(y - c\sin\frac{\pi x}{L}\right)$ at any section can be evaluated from Eqs. (11.6) and (11.5).

BM at each end,

$$M_{\substack{x=0\\x=L}} = \frac{P\pi(k-c)}{\alpha L}\cot\alpha L/2 \quad (11.8)$$

Case C

See Fig. 11.3. Proceeding as in previous cases, maximum BM (which occurs at ends) is given by,

$$M = \frac{w}{\alpha^2}\left[1-\frac{\alpha L/2}{\tan\alpha L/2}\right] \quad (11.9)$$

w being the uniformly distributed lateral loading and $\alpha^2 = \frac{P}{EI}$ as before.

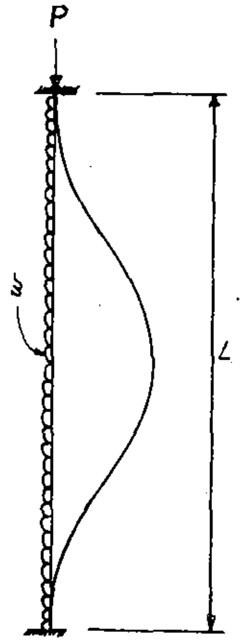


Fig. 11.3

11.3 APPLICATION

In order to analyse the pile group and design the pile, proceed step by step as follows,

Step 1 Estimate at the soffit of pile cap the total load W, total moments ΣM_T and ΣM_L about transverse and longitudinal directions of the structure and horizontal forces ΣH_L and ΣH_T along these directions.

Step 2 (a) Calculate the vertical load (and hence the axial load) in the extreme piles as per rivet group analysis. Axial load in a vertical pile is,

$$P = \frac{W}{n} \pm \frac{\Sigma M_T}{Z_T} \pm \frac{\Sigma M_L}{Z_L}$$

where Z_T and Z_L are section moduli $(\Sigma x^2/x_i)$ to the pile under consideration about T and L directions. (b) Calculate H_T $(= \Sigma H_T/n)$ and H_L $(= \Sigma H_L/n)$ per pile.

Step 3 Establish L i.e., the length of pile between its fixity points, assuming upper fixity point at the soffit of pile cap and lower fixity point as explained earlier.

Step 4 Calculate the maximum bending moments M_T and M_L in the pile section from Eqs. (11.4), (11.8) and (11.9).

Step 5 Design the reinforced concrete pile section for direct compression P and bending moments M_T and M_L calculated above.

Step 6 Ensure that the dependable load carrying capacity of the pile from the considerations of soil resistance around individual pile and block failure is not less than the maximum axial load actually coming on the pile (see under section 'Soil Strength' earlier).

Step 7 Ensure that the total vertical load coming on the pile group is not more than n times the individual soil resistance around a pile multiplied by the relevant efficiency factor as described under 'Soil Strength' earlier.

Step 8 Check for the long term settlement of the pile group by any of the standard methods.

11.4 RECOMMENDATIONS

Types of RC Piles

RC piles may usually be of the following types:

(a) Precast in full and then driven
(b) Precast in reasonable lengths and jointed[3] while driving
(c) Partly precast and partly cast *in situ* during driving
(d) Casing driven, pile cast *in situ* and casing withdrawn
(e) Bored and cast *in situ*.

Types (a), (b) and (c) are recommended in loose soil and in standing water. Type (d) is recommended for structures on land in loose soil. Type (e) piles are recommended in stiff clay or on rock and are convenient when piling has to be done in the proximity of existing and heavily loaded structures. Piling plant is less cumbersome and more easily manoeuvrable in the case of bored piles.

For piles cast *in situ*, in standing water or slushy conditions, a 3 mm thick mild steel liner should be used for encasing the concrete for protection.

In precast piles, the precast length is usually governed by the structural strength of pile section to resist handling and hoisting stresses. The precast pile unit can be handled by supporting it from single point or two point suspension, but while hoisting, it is usually suspended by a single point system only. Since a precast piece may require handling quite a few times (depending on the distance between precasting yard and location of pile foundation) it is recommended to allow only normal stresses in concrete and steel during handling. However, since hoisting is expected to be done only once and this operation lasts for a relatively much shorter duration, the stresses in concrete and steel may be allowed to be 25% higher than the normal permissible stresses during hoisting.

Also refer to Annexure (i) at the end of this chapter.

Detailing

Concrete for cast *in situ* piles (bored or casing driven type) should be cast by Tremie, and can be of 1 : 2 : 4 grade but the mix should be richened to preferably 1 : $1\frac{1}{2}$: 3 grade when such piling is done in water-logged and slushy conditions. Concrete in precast piles should preferably be not leaner than M250 grade. Concrete must be very dense (use of microsilica or pozolona or Portland Blast Furnace Slag Cement deserves a serious consideration). Minimum cement content should be 350 to 450 kg/cubic meter of concrete, depending on ground conditions and agressiveness of the surrounding environment.

The longitudinal reinforcement in a cast *in situ* pile should preferably be not less than 0.8% of the gross sectional area of concrete and this need not extend very much beyond the lower fixity point. This longitudinal steel should preferably be detailed as a six or eight bar arrangement. The transverse reinforcement (links or hoops) in a cast *in situ* pile should be provided on the same basis as for a regular RC column.

The longitudinal reinforcement in a precast pile should preferably be not less than $1\frac{1}{2}$% of the gross sectional area of concrete. Additional reinforcement may be required in critical zones from the considerations of handling and hoisting. This longitudinal steel in square section piles should preferably be detailed as a twelve bar arrangement with sturdy bars at the four corners. In order to utilise these bars to the full during handling and hoisting, the precast unit should be so handled as to cause bending about a face than about a diagonal. The volume of transverse reinforcement in the body of the precast pile should not be less than 0.2% of the gross volume of the pile. This should be increased to at least 0.6% at its ends for a length of about three times the lateral dimension of pile to account for driving stresses. The longitudinal corner bars should be held apart by diagonal fork shaped bars at about 2 m centres. (For more details see Ch. 7 earlier.)

All driven piles should be provided with a steel shoe at the bottom end to protect the pile-toe as well as to ease the driving.

While the longitudinal bars coming out from the pile head should be adequately anchored into the pile cap, the embedment of pile head into the pile cap should be between 50–75 mm. Deeper embedment can be dangerous as the same can function as a crack former.

Driving and Boring

A pile driving plant essentially involves a pile frame (raised from ground or mounted on a pontoon), winches and a driving hammer (monkey). The pile frame is of steel construction. The hammer can be activated by simple gravity, steam, compressed air or diesel. For a more efficient transfer of the driving shock wave, it is preferable to use a heavier hammer and a shorter drop and not the converse. The usual weight of hammer is between 3.5 to 5.0 tonnes and the usual drop is around 1.2 m. However, the weight of the hammer should preferably be not less than half the weight of the pile or casing being driven. To protect the pile head against the hammer hit, a cast steel helmet along with dolly and timber packing should be used during driving. Concrete in the top 60–90 cm, portion of the pile being driven, should be chipped off after driving it as this portion generally shows distress due to spalling effects.

For bored piles the hole in the ground is formed either by screwing an auger or by chiselling. In either case, generally, the sides of the hole are prevented from caving in either by a retractable mild steel casing or by the use of activated bentonite slurry. Bentonite slurry is basically a high density thixotropic fluidy mud. After the reinforcement cage is lowered into the hole, concrete should be placed by tremie beneath the bentonite ensuring that the delivery end of the tremie pipe always remains slightly beneath the top of freshly poured column of concrete.

Investigation for Soil Characteristics

The Dutch Cone Penetrometer test is highly recommended for estimating the soil strength around piles. The apparatus for this has been developed by the Government Soil Mechanics Laboratory, Delft. It essentially consists of a standard steel cone attached to rods which are protected by a sleeve. As the cone is driven into the soil under hydraulic pressure, the thrust on the rods and on the sleeve is measured separately. Readings are taken at regular intervals and this way a continuous serrated graph is drawn showing the end bearing and frictional resistance of the soil at various depths. The ultimate bearing capacity of soil at any depth may be taken equal to the cone resistance at that depth. However, Van der Veen recommends that the cone resistance at any depth should be taken as the average value over a depth equal to three times the lateral dimension of the pile above the pile founding level and one lateral dimension of the pile below the pile founding level.

The skin friction on the pile shaft in cohesive soil is not equal to the cohesion of the soil since the driving or boring into the cohesive medium relieves its adhesion characteristics. The reduction in adhesion factor is, however, less in the case of driven piles than in the case of bored piles. The actual skin friction mobilised around the pile shaft is expressed as the product of cohesion of soil and the adhesion factor and the latter varies almost inversely with the former. For cohesion values of 2.5 to 10 t/m^2, the adhesion factor varies from 0.9 to 0.5 for driven piles.[1] In the case of bored and cast *in situ* piles in clay, the adhesion factor may be as low as 0.45.

In piles through compressible fill, the consolidation of fill causes additional load on piles as the latter remain stationary relative to the consolidating fill. This additional load transmitted to the piles by reverse friction, termed negative skin friction, is a fraction of the cohesion, because of the sensitivity factor of the consolidating soil. This sensitivity factor generally varies between 2 and 4 for most clays.[4]

Since it is not practical to test load every working pile, it is a common practice to test load at least one to two per cent of the working piles (routine vertical test load). Such working piles to be tested may be loaded to 1.5 times their maximum working load they have to carry. However, the safe load carrying capacity of the pile should be taken as only two-thirds of the load at which the pile settles by 12 mm. It is also essential that a few piles are driven at representative locations but outside the working piles and are tested for ultimate load. Such ultimate load tests are essential in order to ensure the true safe load carrying capacity of the piles. That vertical load at which the pile settles by 10% of its lateral dimension may be taken as the ultimate load capacity of the pile. The safe working load capacity of the pile may be taken as 40% of the ultimate load capacity corresponding to a safety factor of 2.5.

In the case of driven piles (be they casing-driven and then cast *in situ* or precast and driven), in addition to the above-mentioned ultimate and routine load tests it is also possible to have an approximate idea about the load carrying capacity from the amount the empty casing or the precast pile (as the case may be) penetrates into ground under the last few hammer blows. This penetration or 'set' is governed by the dynamic impact of the elastic bodies. Based on equating the energy of the hammer blow to the work done in overcoming the ground resistance and penetration of the pile and allowing for losses of energy due to elastic contraction of the casing (or the precast pile as the case may be), its dolly arrangement and the subsoil, and taking due account of the inertia of the pile mass, a general formula of the following type can be derived:

$$U = \frac{k \times W \times h}{p + c}$$

where U = ultimate resistance of pile

c and k = empirical coefficients depending on the system of piling, pile driving plant, total number of blows imparted to the pile, inertia of pile mass, efficiency of blow, units of various parameters in the formula, etc.

W = weight of hammer

h = height of drop

p = average penetration per blow during the last few blows.

NOTE Also see Annexures (ii) and (iii) at the end of this chapter.

Duplex Driving for Consolidation Piling

Where the soil is loosely compacted, the shaft resistance as well as the end bearing resistance of casing driven piles can be improved by the *Duplex redriving* process. After driving the casing together with its shoe as deep as possible, the hole is filled by lean concrete while the casing is continuously withdrawn in jerks leaving the shoe behind. Another shoe is then placed on top of this freshly poured column of lean concrete and the casing is redriven together with the new shoe through this column of concrete, thereby forcing this concrete sideways and down until an acceptable set is obtained. Thereafter, the operations of placing the reinforcement, concreting and pulling out the casing afresh are executed as in a usual driven and cast *in situ* pile.

Settlement

The estimation of the total settlement of a pile group, comprising of immediate and long term settlements, is very complex as it depends on a multitude of parameters whose accurate prediction is almost impossible. Some of these parameters are pile shortening, relative movement between piles and surrounding soil, yielding of soil below the pile base, depth of fill undergoing settlement, void ratio and compression index of soil, true shape of load spread from piles into soil, unequal loading on piles, overlapping of pile pressure bulbs, etc. Therefore, it can be readily inferred that even if some values are assigned to these parameters, the magnitude of the long term settlement of a pile group so estimated should not be viewed with any great concern. Teng[5] recommends the method given by Peck wherein the load is assumed to spread out at one horizontal to two vertical from a depth of two-thirds the total depth of grip. The long-term settlement may then be estimated as per Terzaghi's theory of consolidation as further modified by Skempton and Bjerrum.

11.5 NUMERICAL EXAMPLE

Step 1 Consider the pile foundation as shown in Fig. 11.4 under a bridge pier in scourable cohesive soil. The layout of piles in plan and the values of direct load, moments and lateral forces at the soffit of pile cap are as follows:

Total vertical load	=	592.9 t
Total Moment about LL axis	=	109.2 mt
Total Moment about TT axis	=	53.0 mt
Total Horizontal force along LL axis	=	8.3 t
Total Horizontal force along TT axis	=	17.2 t

Step 2 max./min. vertical load on a pile

$$= \frac{592.9}{20} \pm \frac{53.0 \times 2.9}{94.0} \pm \frac{109.2 \times 3.8}{144.4}$$

$$= 29.65 \pm 1.64 \pm 2.88$$

$$= 34.17 \text{ t}/25.13 \text{ t}$$

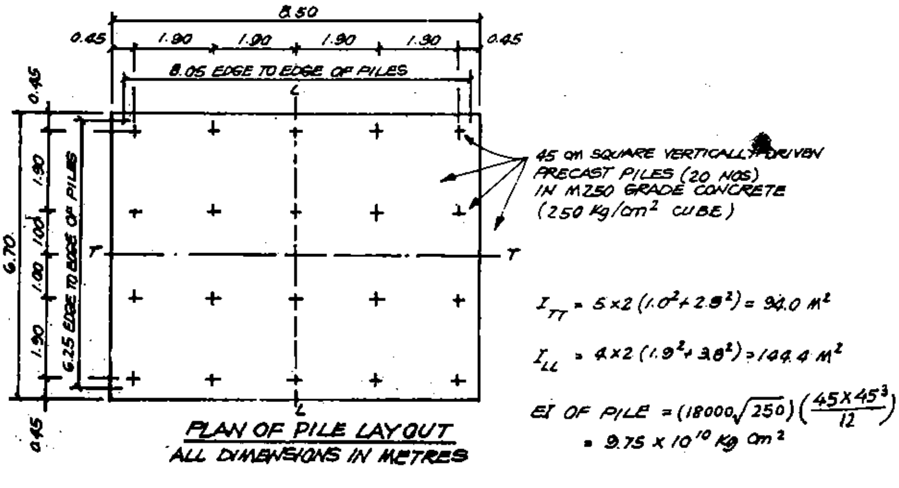


Fig. 11.4

H_T per pile $= \dfrac{17.2}{20} = 0.86$ t

H_L per pile $= \dfrac{8.3}{20} = 0.42$ t

Step 3 RL of soffit of pile cap = + 1.75 m.
Maximum scour level = − 10.80 m.
∴ Exposed length of pile = 12.55 m.

10% of exposed pile length = 1.255 m; half the depth of soft strata below scour level in the present case = 6 m. Hence assume lower fixity point to be at 4 m below scour level.

∴ RL of lower fixity point $= -10.80 - 4.00$
$= -14.80$ m

Length of pile between fixity points = L
$= 14.80 + 1.75 = 16.55$ m

Step 4 Considering the minimum loaded pile:

(i) $\alpha = \sqrt{\dfrac{P}{EI}} = \sqrt{\dfrac{25130}{9.75 \times 10^{10}}}$
$= 50.7 \times 10^{-5}\ \text{cm}^{-1}$, i.e., $50.7 \times 10^{-3}\ \text{m}^{-1}$

(ii) M_T per pile due to $H_L = (H_L/\alpha)\tan \alpha L/2$

$$= 0.42 \times \frac{\tan(50.7 \times 10^{-3} \times 16.55/2)}{50.7 \times 10^{-3}}$$
$= 3.66$ mt

(iii) M_L per pile due to $H_T = (H_T/\alpha)\tan \alpha L/2$

$$= 0.86 \times \frac{\tan(50.7 \times 10^{-3} \times 16.55/2)}{50.7 \times 10^{-3}}$$
$= 7.58$ mt

(iv) Effect of flood water force

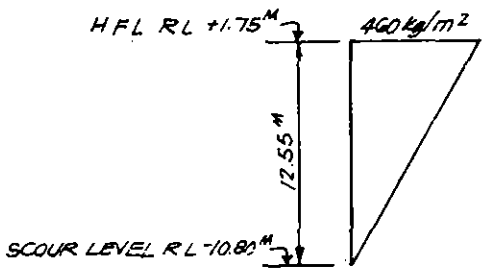


Fig. 11.5

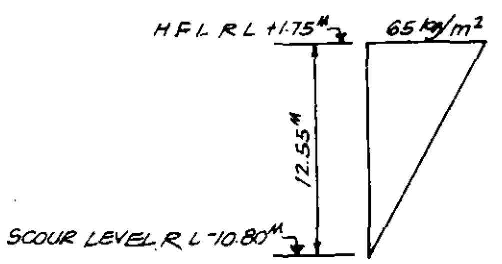


Fig. 11.6

In the present case, the max. mean velocity of flow at HFL (RL + 1.75) = 2.0 m/s

Component of max. velocity along TT direction
$= \sqrt{2} \times 2.0 \times \cos 20°$
$= 2.65$ m/s

Pressure at HFL $= 52 \times 1.25 \times 2.65^2$
$= 460\ \text{kg/m}^2$

(refer to clauses 213.2 and 213.7 of IRC Specifications—Section II)

∴ Total force on the pile group along TT axis
$= 1/2 \times 460 \times 12.55 \times 6.25 \times 10^{-3}$
$= 18.0$ t

∴ per pile $= \dfrac{18.0}{20} = 0.9$ t

Component of max. velocity along LL direction.
$= \sqrt{2} \times 2.0 \times \sin 20°$
$= 1.00$ m/s.

Pressure at HFL $= 52 \times 1.25 \times 1.0^2$
$= 65\ \text{kg/m}^2$

∴ Total force on the pile group along LL axis
$= 1/2 \times 65 \times 12.55 \times 8.05 \times 10^{-3}$
$= 3.20$ t

∴ per pile $= \dfrac{3.20}{20} = 0.16$ t

Assuming these forces as uniformly distributed along the height of pile (the effects as can be seen ahead are too small, hence this simplification),

$$M \text{ per pile} = (w/\alpha^2)\left[1 - \frac{\alpha L/2}{\tan \alpha L/2}\right]$$

$$\therefore M_L \text{ per pile} = \frac{0.90}{16.55 \times (50.7 \times 10^{-3})^2} \times \left[1 - \frac{50.7 \times 10^{-3} \times 16.55/2}{2\tan(50.7 \times 10^{-3} \times 16.55/2)}\right] = 1.36 \text{ mt}$$

and M_T per pile $= \dfrac{0.16}{0.90} \times 1.36 = 0.24$ mt (by proportion)

(v) Moment per pile due to built-in curvature: assuming $c = 5$ cm,

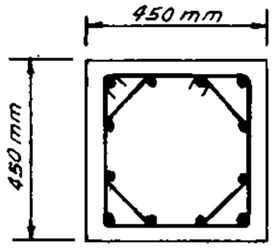
$$k = \frac{c}{1 - (\alpha L/\pi)^2}$$
$$= \frac{0.05}{1 - \left[\dfrac{50.7 \times 10^{-3} \times 16.55}{\pi}\right]^2}$$
$= 0.0539$ m

$$M = \frac{P\pi(k - c)}{\alpha L}\cot(\alpha L/2)$$
$$= \frac{25.13 \times \pi(0.0539 - 0.05)}{50.7 \times 10^{-3} \times 16.55}\cot(50.7 \times 10^{-3} \times 16.55/2)$$
$= 0.82$ mt

This moment may be assumed to act about LL axis for worst effects.

(vi) Total M_L per pile $= 7.58 + 1.36 + 0.82$
$= 9.76$ mt
Total M_T per pile $= 3.66 + 0.24$
$= 3.90$ mt

Step 5 Designing[6] the pile section for combined axial load of $P = 25.13$ t and biaxial bending of $M_T = 3.90$ mt and $M_L = 9.76$ mt, the stresses in reinforcement and concrete shown in Fig. 11.7 are found to be within permissible limits.



CONCRETE = M 250 GRADE
LONGITUDINAL REINFORCEMENT
OF 4 NOS. ϕ 40mm BARS AT CORNERS
+ 8 NOS. ϕ 25mm BARS AT FACES.

Fig. 11.7

Step 6 Approximate RL of bed $= -1.30$ m
Pile founding level $= -40.80$ m
Ultimate cohesion $= 2.1$ t/m^2 and adhesion factor $= 0.90$
Ultimate bearing capacity of soil at founding level $= 450$ t/m^2
Sensitivity factor for the cohesive soil $= 3$

Soil resistance around a single pile

Ultimate shaft resistance from maximum scour level to founding level

$= 2.1 \times 0.9 \times (4 \times 0.45) \times (40.8 - 10.8)$
$= 102.0$ t

Ultimate end bearing resistance $= 0.45 \times 0.45 \times 450$
$= 91.1$ t

Ultimate negative skin friction between bed level and scour level $= \frac{2.1}{3} \times 0.9 \times (4 \times 0.45) \times (10.8 - 1.3)$
$= 10.8$ t

Buoyant weight of the pile $= 0.45 \times 0.45 \times 42.55 \times (2.4 - 1.0)$
$= 12.1$ t

$\therefore$ Ultimate soil resistance around a pile $= 102.0 + 91.1 - 10.8 - 12.1$
$= 170.29$ t

$\therefore$ Net working load carrying capacity of the pile $= \frac{170.2}{2.5} = 68.1$ t,
> 34.17 t, ok

Check for block failure

Ultimate shaft resistance around pile group $= 2.1 \times 0.9 \times (8.05 + 6.25)2 \times (40.8 - 10.8)$
$= 1620$ t

Ultimate end bearing resistance $= 8.05 \times 6.25 \times 450$
$= 22650$ t

Buoyant weight of soil between founding level and bed level $= (8.05 \times 6.25 - 0.45 \times 0.45 \times 20) \times (40.8 - 1.3) \times (1.6 - 1.0)$
$= 1095.5$ t

Ultimate negative skin friction from bed level to scour level $= \frac{2.1}{3} \times 0.9 \times (8.05 + 6.25)2 \times (10.8 - 1.3)$
$= 171$ t

Buoyant weight of piles $= 12.1 \times 20 = 242$ t

$\therefore$ Ultimate soil strength around pile group $= 1620 + 22650 - 1095.5 - 171 - 242 = 22761.5$ t

$\therefore$ Working load carrying capacity of a pile in the group $= \frac{22761.5}{20 \times 2.5}$
$= 454.5$ t, > 34.17 t, ok

Step 7 Assuming an efficiency factor of 0.7

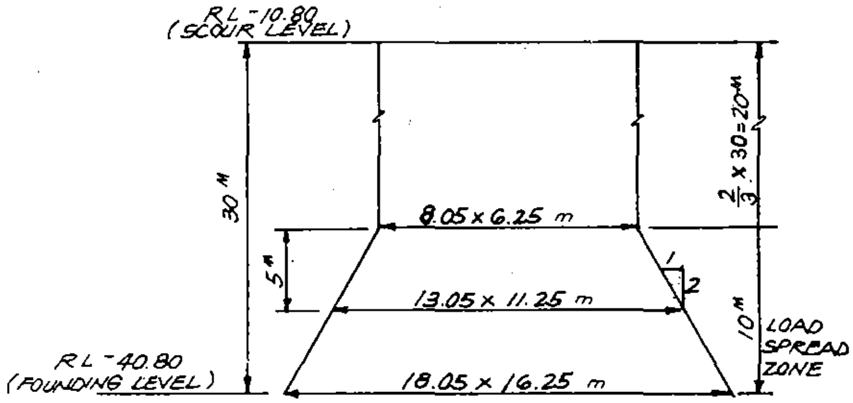


Fig. 11.8

Vertical load capacity of
pile group $= 68.1 \times 0.7 \times 20 = 957$ t
> 592.9 t, ok

Step 8 Assuming compression index of soil (C_s) = 1.12, void ratio (e_0) = 2.30, coeff. $k = 0.3$ (heavily over consolidated clay), and submerged soil density (γ_{sub}) = 0.6 t/m^3

Vertical pressure due to weight of soil at mid depth of load spread zone $p_0 = (30.0 - 10.0/2)0.6$
$= 15$ t/m^2

Overburden pressure due to permanent dead

load of bridge $\Delta p = \dfrac{592.9}{13.05 \times 11.25}$
$= 4.03$ t/m^2

∴ Long term settlement

$$S = \frac{C_s H}{1+e_0} \log_{10}\left(\frac{p_0 + \Delta p}{p_0}\right) k$$

$$= 1.12 \times \frac{10 \times 100}{1+2.3} \log_{10}\left(\frac{15+4.03}{15}\right) \times 0.3$$

$$= 10.6 \text{ cm}$$

NOTE Estimation of settlement in cohesionless soils may be made from the following formula:

$$S = \frac{H}{C} \log_e \left(\frac{p_0 + \Delta p}{p_0}\right)$$

where

S = final consolidation settlement of a layer of thickness H.

C = constant of compressibility

$= 1.5 \dfrac{C_{kd}}{p_0}$ (where C_{kd} = Static Cone Resistance) other symbols as defined earlier.

REFERENCES

1. Tomlinson, M.J., *Foundation Design and Construction*, 2nd edn, Isaac Pitman Publication, pp. 390, 407, 410.
2. Timoshenko and Gere, *Theory of Elastic Stability*. 2nd edn, McGraw-Hill Publication, pp. 7, 8, 35, 37.
3. Bruce and Herbert, "Splicing of precast piles", *PCI Journal*, Nos. 5 and 6, Vol. 19, *PCI Special Report*, 1974.
4. Terzaghi and Peck, *Soil Mechanics in Engineering Practice*, Asia Edition, 1962, p. 31.
5. Teng, W.C., *Foundation Design*, Prentice-Hall Publication, 1964, p. 219.
6. Raina, V.K., R C Section Subjected to Axial Thrust and Any-axis Bending—Simplified Practical Design, *The Bridge and Structural Engineer*, Vol. 5, No. 4, December, 1975. (reproduced in Ch. 25 of this book).

ANNEXURES

(i) Usual Pile-Types

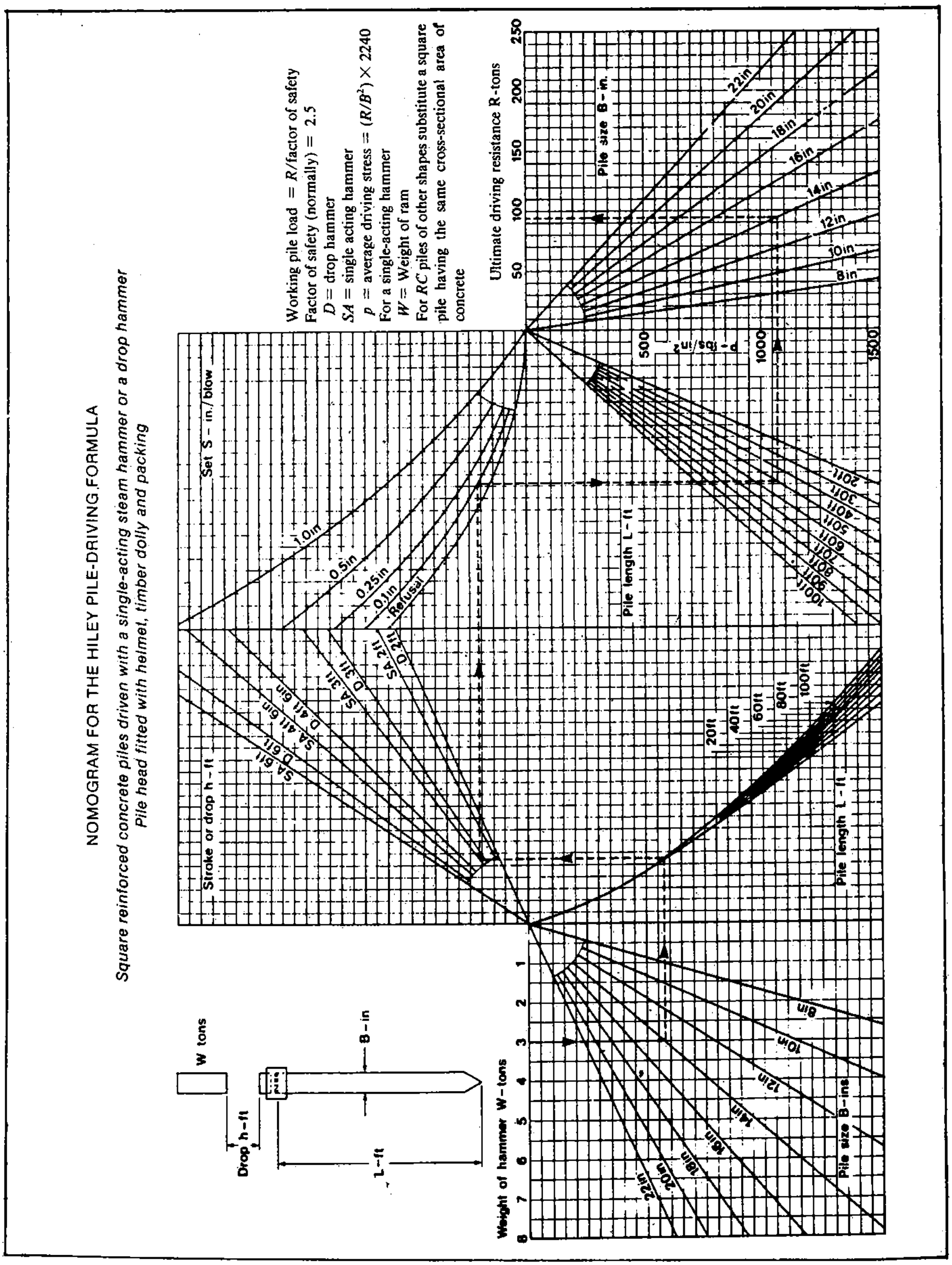
(a) Displacement piles (Driven piles)

Pile type			Normal range of sizes available		Normal range of load
			Cross-section	Length	
Preformed	Timber		Up to 400 mm× 400 mm	Up to 20 m	Up to 600 kN
	Concrete	Normal reinforced	Up to 450 × 400 mm	Up to 27 m	Up to 1000 kN
		Prestressed	Up to 400 mm square Up to 750 mm dia. hollow	Up to 27 m	Up to 1000 kN
	Steel	Box	Rendhex standard Frodingham octagonal Sheet pile fabrication	Up to 36 m	Up to 1500 kN
		Tubular	Heavy gauge up to 900 mm dia.	Up to 36 m	Up to 1500 kN
		H-section	200 mm × 200 mm to 300 mm × 300 mm	Up to 36 m	Up to 1700 kN
		Screw	600 to 2400 mm dia. helices	Up to 24 m	Up to 2500 kN
Partially Preformed	Precast and cast *in situ* concrete		450 to 600 mm dia.	Up to 50 m	Up to 2000 kN
	Steel and cast *in situ* concrete		250 to 500 mm dia.	Up to 18 m	Up to 800 kN
Driven *in situ*	Concrete		250 to 600 mm dia.	Up to 24 m	Up to 1500 kN

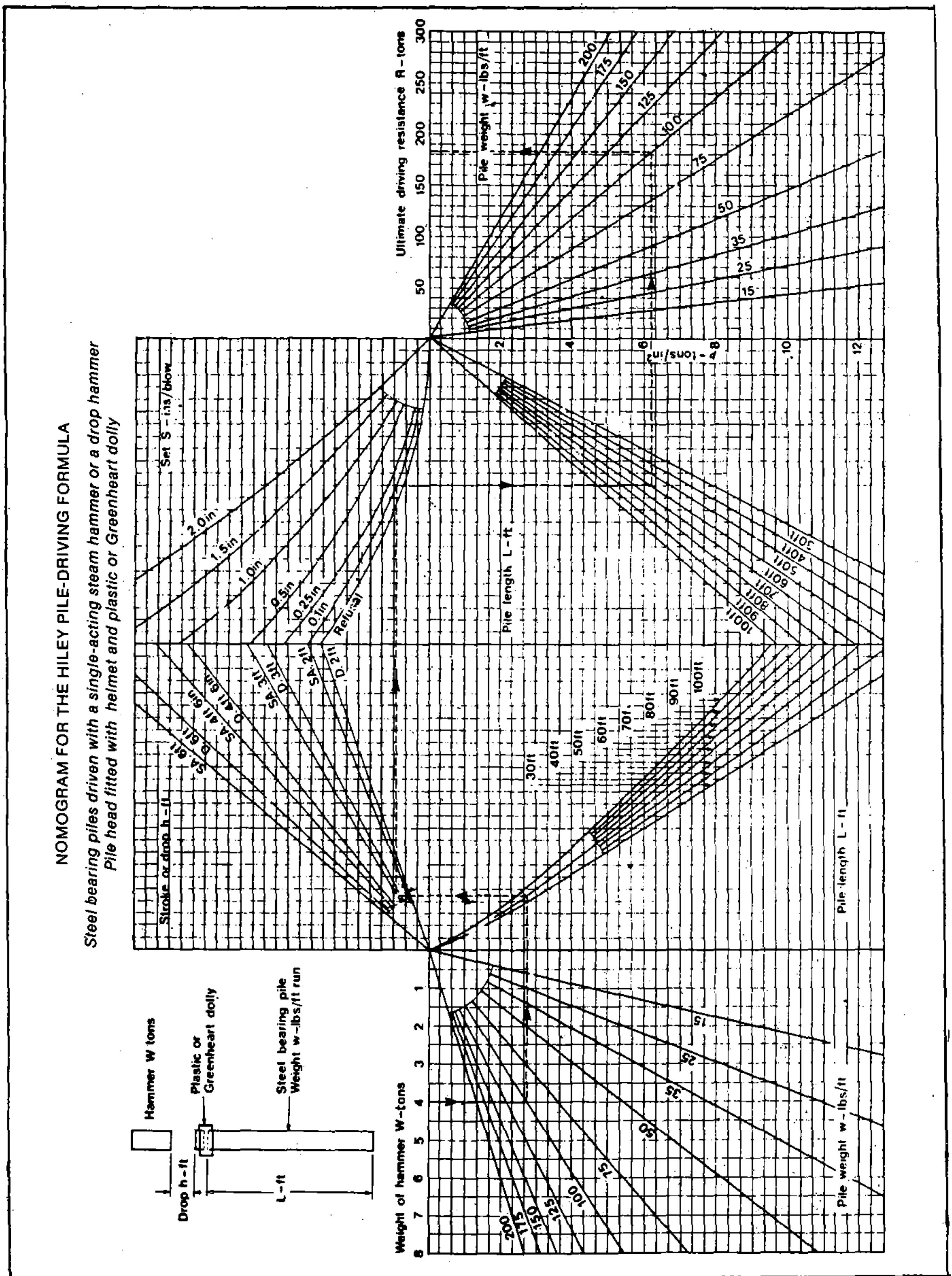
(b) Replacement piles (Bored piles)

Pile type			Normal range of sizes available		Normal range of load
			Cross-section	Length	
Percussion bored	Small diameter		450 to 600 mm dia.	Up to 24 m	Up to 1200 kN
Flush bored	Large diameter		600 mm dia. and over	Up to 45 m	Up to 10000 kN
Rotary bored	Large diameter	Straight shaft	600 to 1800 mm dia.	Up to 45 m	Up to 10000 kN
		Under-reamed base	As above with bell up to 3 times shaft diameter	Up to 45 m	Very high loads possible
	Small diameter		225 to 550 mm dia.	Up to 36 m	Up to 1000 kN

(ii) Precast Concrete Piles



(iii) Steel Bearing Piles



CHAPTER 12

A Practitioner's Guide to ~ Estimating Safe Bearing Capacity of Soils for Footings, Caissons and Piles

Synopsis

This chapter is deliberately presented in two parts. PART-I describes the actual step by step method for estimating the bearing capacity of soils (for footings, caissons, and piles) in a workman-like manner using a tool-kit-approach. ***(It is hoped that this clear-cut approach will help the engineer to 'stay on course for reaching the end of the problem' without feeling lost in the academic plethora that this subject is heaped in.)*** PART-II then separately tries to explain various subsidiary items (e.g. 'how to improve the bearing capacity of soils', 'various *in situ* penetration tests employed in the estimation of sub-strata bearing capacity', 'plate-load-bearing tests', 'bearing capacity of rocky SUBSTRATA', and a host of 'typical soil parameters') useful to a practising professional designer. ***The engineer thus has a clear choice of separating the presentation given in PART-I in order to identify the 'steps' involved in the chain of command for estimating the bearing capacity.***

12.1 INTRODUCTION

The bearing capacity of a soil depends upon the physical characteristics of the soil particles (i.e., size, shape, cohesive properties, frictional resistance and the power to retain moisture, etc.), moisture content and the changes brought in by the atmospheric influences such as heat, rain, etc. The finer the soil particles, the more variable are the cohesive and frictional properties of the soil under field conditions. In general, the heavier the unit weight of soil the greater the strength, and also the lesser the voids, the greater the strength.

With structures built on sands and gravels the settlement is likely to be practically completed at the end of construction, but when the site is underlain by clays or silts, settlement is likely to continue for a long time after construction and cracks may appear many years after completion.

All foundations settle under load and the general tendency is for some parts of a structure to settle more than others causing relative movement. The critical factor in the settlement of a structure is not the amount of settlement but the differential settlement between the different parts of a structure itself. Excessive pressure is a comparatively uncommon cause of settlement.

In clays, the ultimate bearing capacity under spread foundations is calculated using total stress parameters. This gives the end-of-construction case, which is the worst condition, and allows the design to be based on 'undrained' shear strength test, which are quick and inexpensive. For granular soils, where the dissipation of pore water pressures is rapid, 'effective' shear strength is used and, because of the difficulty in obtaining undisturbed samples, strength parameters are usually estimated from standard penetration test results.

PART I *(Workman-Like Approach)*

Contents ~

(A) Various PRELIMINARIES, and Obtaining 'Quickly' a Rough-estimate of the Safe Bearing Capacity of soil (S.B.C.) under a 'Footing' or a 'Caisson'

Step 1 Conduct Standard Penetration Tests (SPT) at short intervals (1 to 2 m) till about 10 m below a 'high-enough N-value zone (of say $N > 30$ to 50 if possible, for instance)' [where foundation is subject to scour, the founding level will have to be at least $x/3$ below the max. scour level (MSL) where x = depth from High Flood Level (HFL) to MSL; in such case conduct the SPT between MSL and a level about 10 to 15 m below min. founding-level].

Step 2 At the particular depth under consideration, note the SPT value and correct it for depth-effect as per Fig. 12.1 and further correct it in case of 'silts and fine-sands below water table' from the formula: Corrected $N = 15 + 1/2$ ("above corrected SPT value" -15) when the "above corrected SPT value" is greater than 15.

Step 3 Upon inspection of the soil samples retrieved by the Split-spoon-sampler from the successive SPTs decide whether soil is predominantly 'cohesive' type or 'non-cohesive' type.

Step 4 (a) If the soil is predominantly 'cohesive':
Find S.B.C. from Table 12.1 for the corrected N-value.

(b) If the soil is predominantly 'non-cohesive':
Find S.B.C. from Fig. 12.2 for the corrected N-value for an assumed approx. width of foundation.

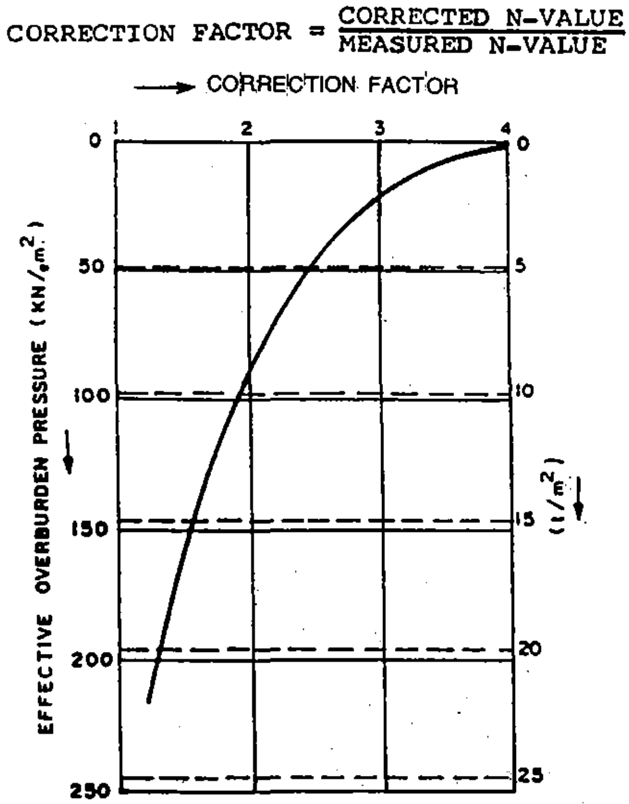


Fig. 12.1 *Depth correction factors for the measured SPT N-value (after Gibbs and Holtz)*

Figure 12.2 relates the settlement of foundations on sand to relative density, as determined by the standard penetration test. It shows the bearing pressure which will produce 25 mm of settlement for a given width of foundation. The relationship shown were established by Terzaghi and Peck from field observations and are intended only as a

Table 12.1 *Suggested allowable bearing values for clay*

N: Number of blows per 300 mm in standard penetration test.
c_u: Unconfined compressive strength.
q_d: Ultimate bearing capacity of continuous footing.
q_{ds}: Ultimate bearing capacity of square footing or circular footing.
q_a: Proposed 'allowable' bearing value (where $G_s = 3$).
G_s: Factor of safety with respect to base failure.

Description of clay	N (Corrected)	c_u	q_d	q_{ds}	q_a*,** Square $1.2\,c_u$ or circular	Continuous $0.9c_u$
Very soft	< 2	< 25	< 75	< 100	< 30	< 20
Soft	2 to 4	25 to 50	75 to 150	100 to 200	30 to 60	20 to 45
Medium	4 to 8	50 to 100	150 to 300	200 to 400	60 to 120	45 to 90
Stiff	8 to 15	100 to 200	300 to 600	400 to 800	120 to 240	90 to 180
Very stiff	15 to 30	200 to 400	600 to 1200	800 to 1600	240 to 480	180 to 360
Hard		> 400	> 1200	> 1600	> 480	> 360

All values in kN/m^2 (c_u, q_d, q_{ds}, q_a)

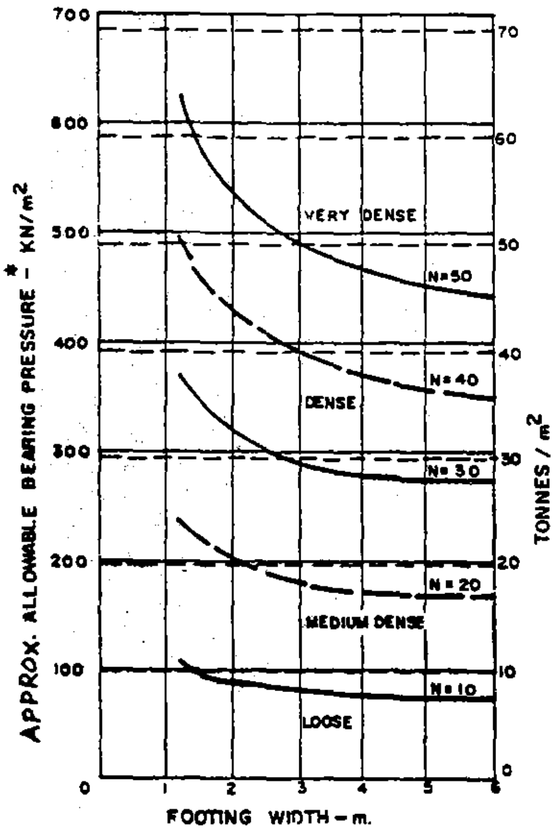
*q_a for square circular footings $= \dfrac{7.4c}{G_s} = \dfrac{3.7c_u}{G_s} = 1.2c_u \quad (G_s = 3)$

**q_a for continuous footings $= \dfrac{5.7c}{G_s} = \dfrac{2.85c_u}{G_s} = 0.9c_u \cdot (G_s = 3)$

Cohesion c being equal to $1/2\ c_u$.

Also, the 'ultimate' bearing capacity of a rectangular or an oblong footing of width B and length L, is approximately $= 2.85\ c_u \times (1 + 0.3\ B/L)$, giving an 'allowable' value of $0.9\ c_u\ (1 + 0.3\ B/L)$.

NOTE: Where the soil can get saturated (flooded), divide the above q_a values by 2 (i.e. a total safety factor of 6).



* Subject to corrections stated here.

Fig. 12.2 *Chart for estimating allowable bearing pressure based on standard penetration test results. Continuous lines are based on the original chart given by Terzaghi and Peck; broken lines are interpolations*
Ref.: Soil Mechanics in Engineering Practice by Terzaghi and Peck, and Foundation Design and Construction by M.J. Tomlinson

rough guide. Some workers consider the bearing pressures obtained by using this chart to be too low, particularly for wider foundations.

- *Corrections*

1. N-values: For foundations on clean, dry sands, the N-value obtained from the standard penetration test is used directly (see 2 below) to obtain the bearing pressure on a strip or pad foundation which will cause 25 mm settlement. The settlement for a different bearing pressure can be obtained on a prorata basis, provided the bearing pressure is well within the bearing capacity of the sand.

2. Corrections to N-values: Corrections may need to be carried out to the N-value to allow for 'depth' of test and the presence of silt or fine sand. These corrections are described in Step A-2. The corrected N-value is then used when reading the chart.

3. Corrections for water table: If the water table is at least one foundation width beneath the base of the foundation then no correction is required. However, if the water table is close to or at the foundation level, then, for a shallow foundation, the bearing pressure which will give 25 mm settlement is half the value read from the chart.

Alternatively, the bearing pressure read from the chart will produce 50 mm settlement.

4. Corrections for large rigid foundations: The rigidity of rigid raft or deep pier foundations results in smaller settlements and half the settlement estimated from the chart is to be expected. Thus, for 25 mm settlement, twice the bearing pressure read from the chart can be used where the water table is low and the actual values read off can be used where it is near or above the underside of the foundation.

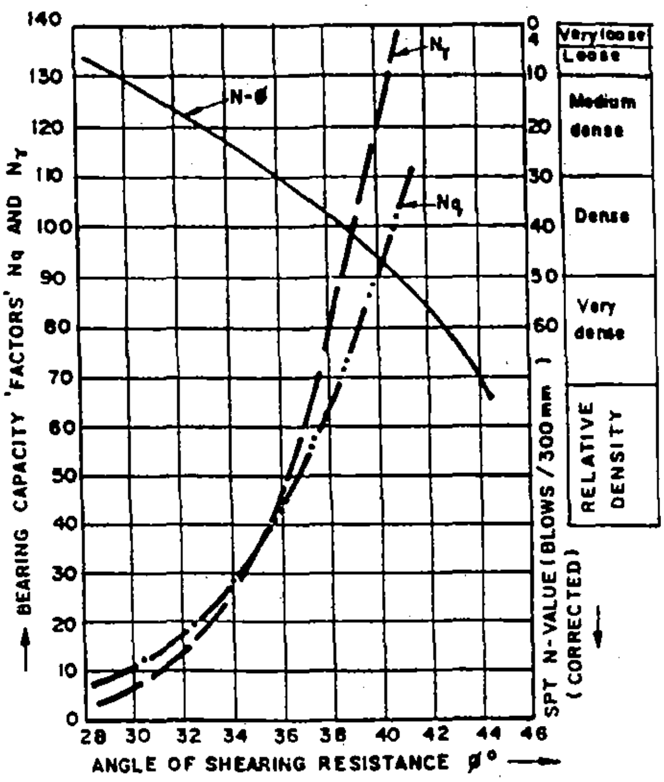


Fig. 12.3 *Correlation of values of ϕ, N_q and N_γ with SPT N-values (After Peck, Hanson and Thornburn)*

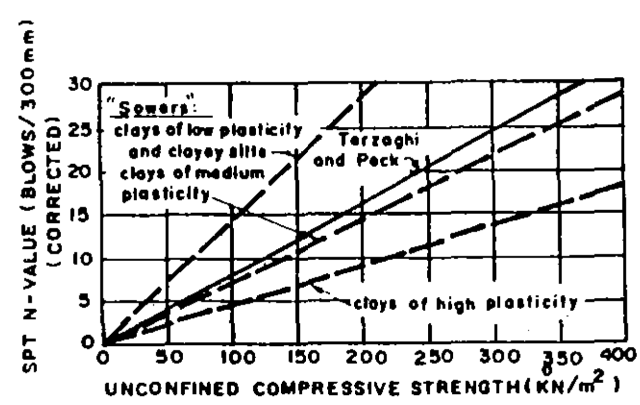


Fig. 12.4 *Approximate correlations between undrained shear strength c_u and SPT N-values, for clays. (Cohesion $c = \frac{1}{2}c_u$, c_u being the unconfined compressive strength of the soil)*

(B) More Accurate Estimation of S.B.C. of Soil under a 'Footing' or a 'Caisson'

Step 1. Same as Steps 1 and 3 of A above.

Step 2. *a) If the soil is found predominantly 'cohesive'*, then, using standard SHELBY's tubes, collect adequate number of 'undisturbed' soil-samples from the soil at each test-level, subject them to tri-axial compression test in the laboratory and thereby determine the values of cohesion c and angle of internal friction ϕ at the test-levels.

b) If the soil is found predominantly 'non-cohesive', then follow Step 2 of A above, and establish the corrected SPT N-value for each test-level. For this N-value (at any level) read off values of ϕ and c_u from Figs. 12.3 and 12.4 respectively. Hence ϕ and c are established (noting that cohesion $c = 1/2\ c_u$, where c_u is the unconfined compressive strength read off from Fig. 12.4).

Step 3. Estimate "ultimate" bearing capacity of soil at the concerned-level from *TERZAGHI'S* as well as *MEYERHOFF'S* approaches (explained ahead under I and II). Divide the smaller of these values by suitable safety-factor (generally 3 for 'dry' conditions and 6 if saturation possible) to obtain the S.B.C. i.e. the safe (permissible) bearing capacity value [but see '4' below].

Step 4. Also establish the S.B.C. from the "TOLERABLE SETTLEMENT approach" (explained ahead under III).

Step 5. Lesser of the two S.B.C. values, established in Steps '3' and '4' above, may then be taken as the acceptable S.B.C. value at the concerned test-level.

NOTE: In the case of a caisson it is usual to ignore the effects of skin-friction on its sides (because so much is unknown about the actual soil-characteristics that a conservative approach is preferred for *CAISSONS*).

... ***I.—Terzaghi's approach—***

The ultimate 'net' bearing capacity, p_{nu}, of a shallow foundation is given by:

(a) Strip Foundation:

$$p_{nu} = cN_c + p_o(N_q - 1) + \frac{1}{2}\gamma BN_\gamma$$

(b) Square or Circular Foundation:

$$p_{nu} = 1.2cN_c + p_o(N_q - 1) + 0.4\gamma BN_\gamma$$

where: γ is the bulk density of the soil 'below' the foundation level*

c is the shear strength of the soil (cohesion).

p_o is the 'effective' overburden pressure at foundation level ($= \gamma' \cdot D$).

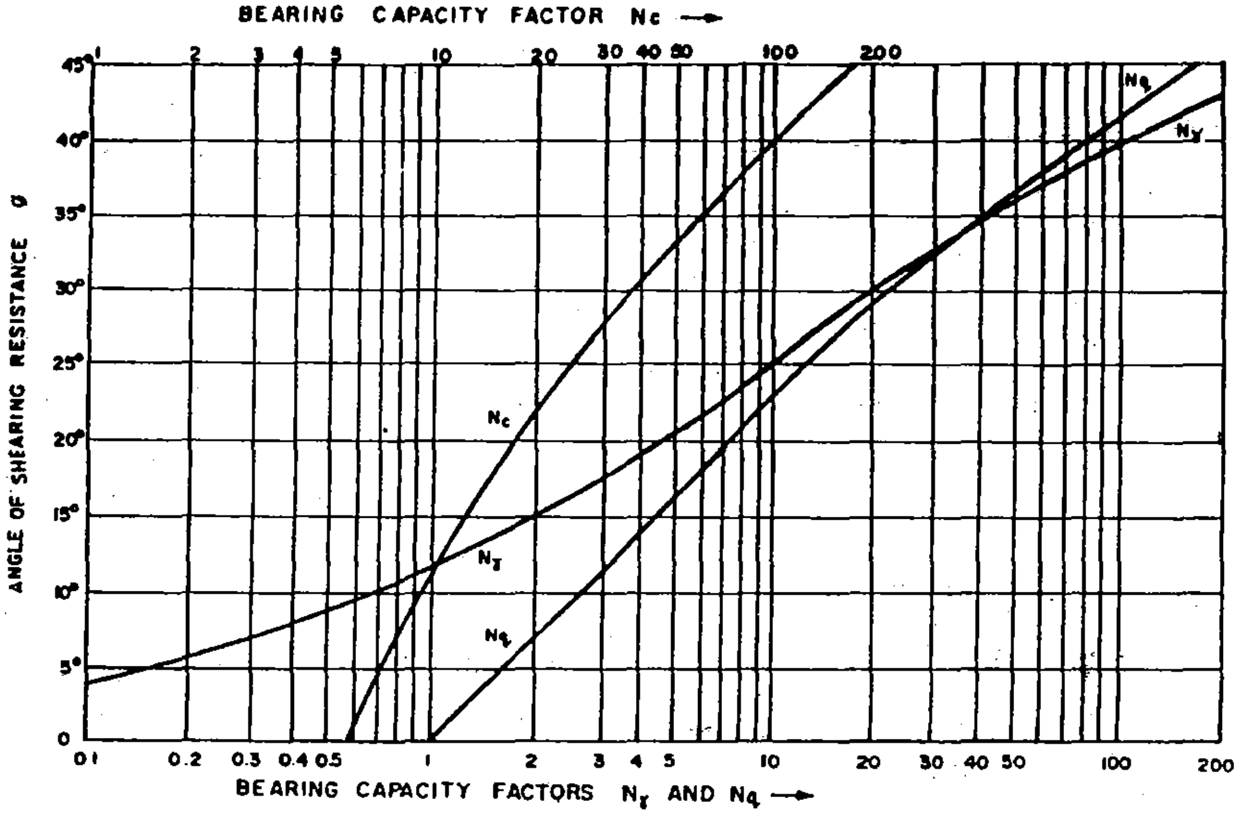


Fig. 12.5 *Terzaghi's bearing capacity factors*

Notes

(a) For clays, $\phi = 0$, giving $N_\gamma = 0$ $\quad N_q = 1 \quad N_c = 2 + \pi = 5.14$

(b) For sands, $c = 0$ so that the first term in the bearing capacity equations is zero and the value of N_c is not required. The value of ϕ may be obtained by direct testing but is usually estimated from SPT values.

* If the water table is at or above the founding level then the value of γ used in the bearing capacity calculations is the submerged density. Also an allowance must be made for the height of the water table above the founding level when calculating p_0.

B is the foundation width (or diameter).
γ' = appropriate density of soil 'within' depth D
N_c, N_q and N_γ *are Terzaghi's bearing capacity factors, obtained from Fig. 12.5.*
D is depth of foundation.

The ultimate bearing capacity is given by

$$p_u = p_{nu} + p$$

where p is the 'total' overburden pressure at the founding level ($= \gamma' \cdot D$ approximately).

... II—Meyerhoff's approach—

The ultimate 'net' bearing capacity under a *spread* foundation is given by:

$$p_{nu} = cN_c + p_o(N_q - 1) + \frac{1}{2}\gamma BN_\gamma$$

where N_c, N_q and N_γ are Meyerhoff's bearing capacity factors, obtained from Figs. 12.6, 12.7, 12.8 and 12.9.

It can be seen that this has the same form as Terzaghi's equation for a strip foundation. However, Meyerhoff's values of bearing capacity factors depend on the shape and depth of the foundation and on the roughness of the base.

• Meyerhoff's Bearing Capacity Factors—Figs. 12.6 to 12.9.

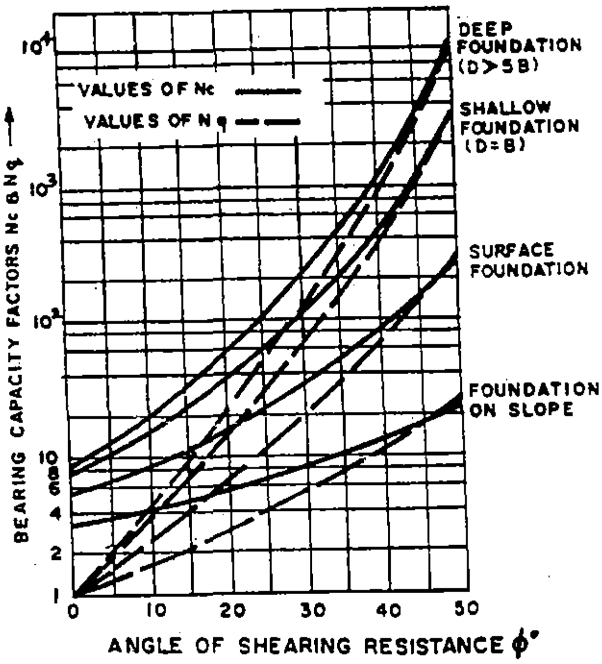


Fig. 12.6 *Values of N_c and N_q for a strip foundation —Meyerhoff's factors*

Values of N_c, N_q and N_γ for *strip* foundation are obtained using Figs. 12.6 and 12.7.

For *rectangular, square* and *circular* foundations, the values obtained from these graphs must be multiplied by a factor, λ, which depends on the shape and depth of the foundation, on the soil properties, and on the method of construction. Values of λ are obtained from Fig. 12.8.

If the *water table* is at or above the founding level, γ is replaced by γ_{sub} and the value of p_o is obtained as described for Terzaghi's equations.

Clay Soils (Meyerhoff) Using the usual procedure of total stress analysis, $\phi = 0$ so that $N_q = N_\gamma = 0$ and the bearing capacity equation reduces to:

$$p_{nu} = cN_c$$

For foundations on clay soils it is usual to use Meyerhoff's values for factor N_c and, for the special case of *purely cohesive* soils, values may be obtained from Fig. 12.9.

For *rectangular* foundations, values of N_c for a *strip* should usually be used.

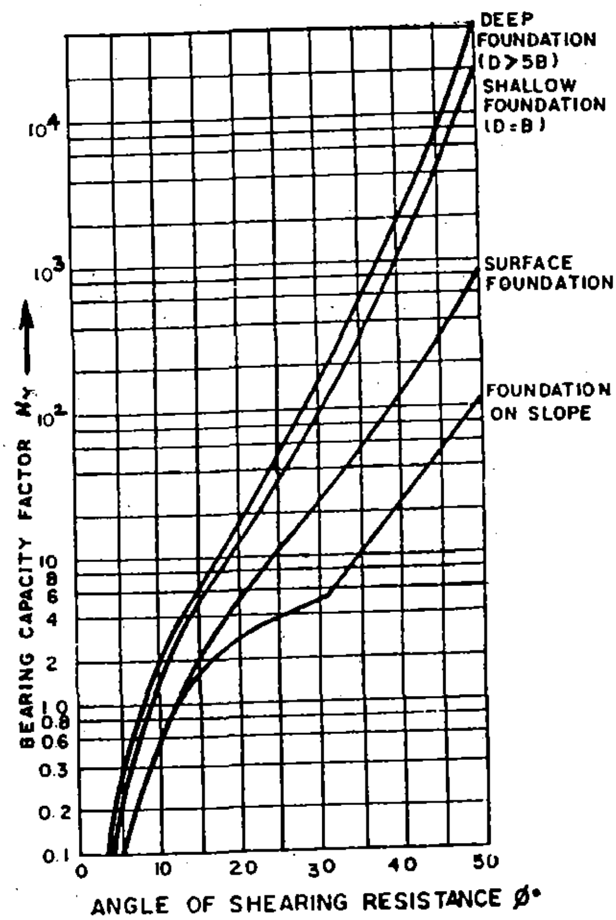


Fig. 12.7 *Values of N_γ for a strip foundation —Meyerhoff's factors*

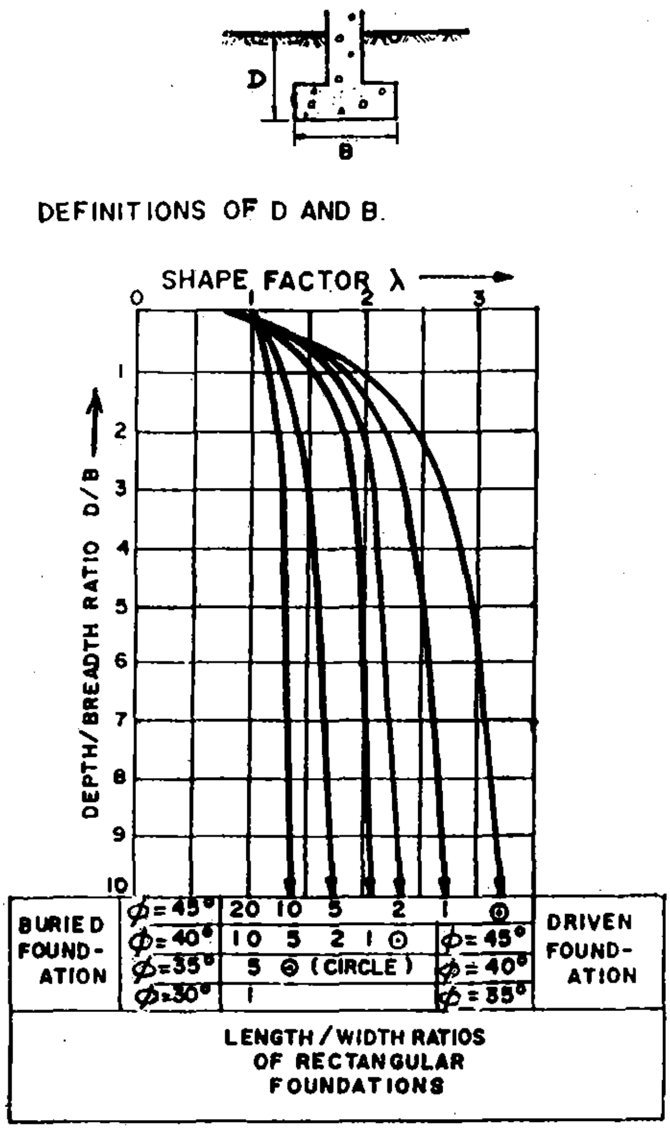


Fig. 12.8 *Values of shape factor* λ *for a rectangular foundations —Meyerhoff's factors*

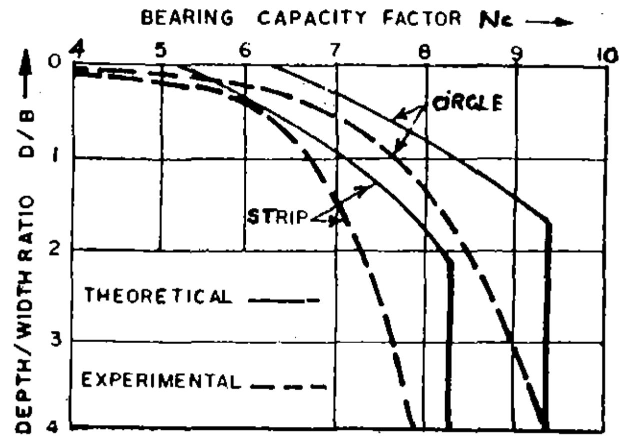


Fig. 12.9 *Values of N_c for foundations on a purely cohesive soil —Meyerhoff's factors*

α = 0.5, if this level is submersible.
= 1.0, if water level is always below this level by at least a depth in magnitude equal to dimension B defined earlier (values may be interpolated in between these limits).

$\beta = \left(1 + \frac{D}{5B}\right)$, but $\not> 1.20$

B and D... as defined earlier [D being the founding depth below the ground level (maximum scour level) at which SPT is done and the N-value established]

N = SPT value at the level considered (Standard Penetration Test).

W = Weight of the soil above the level at which N has been established in tonne/m^2. (For estimating this, use dry density for soil above water level and submerged density below water level, as in the case of p_o mentioned in I–earlier).

... *III—"TOLERABLE SETTLEMENT" approach—*

This 'allowable' bearing pressure has been established empirically (Terzaghi and Peck, 1948), and may be expressed by the equation.

$$p_s = 3.5(N - 3)\left(\frac{B + 0.3}{2B}\right)\alpha \cdot \beta + W$$

where p_s = safe bearing capacity of soil in tonne/m^2 at the 'level at which the SPT is done', for a maximum tolerable settlement of 2.5 cm.

GENERAL CAUTION

The bearing capacity under a footing is largely affected by the characteristics of the volume of soil within a depth equal to about 1 to $1\frac{1}{2}$ times the width of the footing. Unless the soil possesses some cohesion, the upper layer of one to two metres can be easily disturbed and loosened by construction operation. *Therefore, it is not advisable to use large bearing capacity for small or narrow footings such as those supporting continuous walls, even if the natural soil is very compact.*

(C) Soil Resistance to a PILE

1) Ultimate Value

(i) In 'cohesive' soil: The ultimate bearing capacity Q_u is made up of *adhesion* Q_s and *end bearing* Q_b, less *negative skin* friction Q_n (Adhesion, often called skin friction, is usually much greater than end bearing in clays.) Thus:

$$Q_u = Q_s + Q_b - Q_n$$

- The *adhesion* on a pile is given by

$$Q_s = \alpha \cdot \bar{c} \cdot A_s$$

Where A_s is the embedded surface area of the pile
$\bar{c}$ is the 'average' undrained shear strength of the clay along the sides of the pile.
α is an adhesion factor.

Researchers have found that the value of α can vary widely so that it is difficult to allocate a value to it. For 'driven' piles, values obtained by Nordlund, given in Table 12.2, are usually used. Values for 'bored' piles, discussed by Tomlinson, are also given in Table 12.2.

- The *end bearing* is obtained from Meyerhoff's equation for the bearing capacity of cohesive soils:

$$Q_b = c \cdot N_c \cdot A_b$$

where c is the undisturbed shear strength at the base of the pile.
A_b is the pile base area (thus for a circular pile, radius R, $A_b = \pi R^2$).
N_c is Meyerhoff's bearing capacity factor, usually taken as 9.

- The *negative skin friction*, Q_n resulting from the tendency of the fill material (or compressible soil)

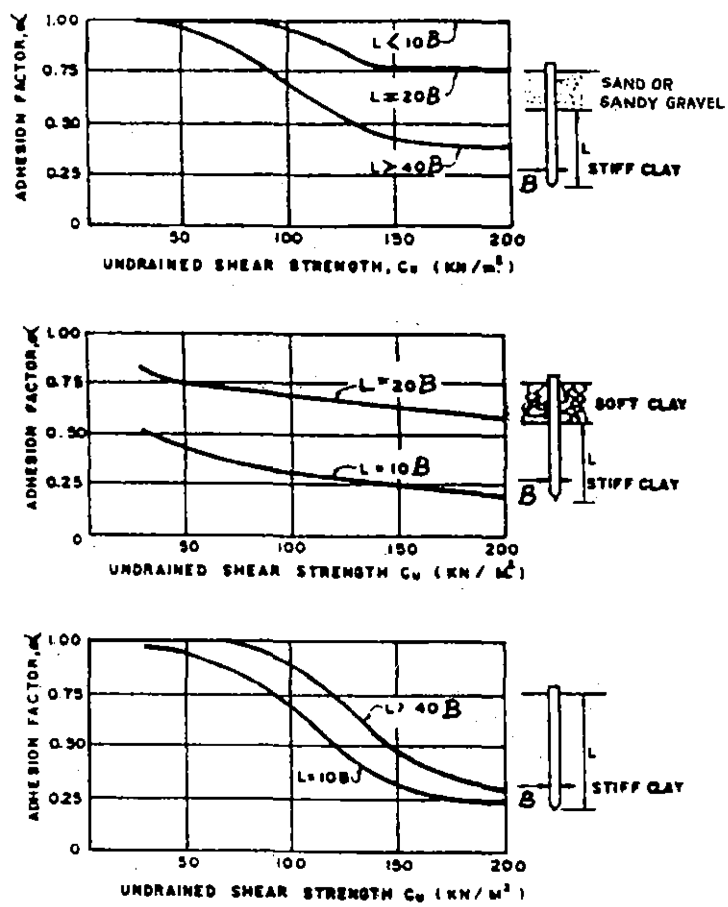
Table 12.2

Adhesion factors for 'bored' piles (After Tomlinson)	Adhesion factors for 'driven' piles (After Nordlund)

An adhesion factor α of 0.45 is used for bored piles in many clays, including London clay, although for short bored piles in London clay, where it may be heavily fissured, a value of 0.3 is more usual. Values of 0.49 to 0.52 have been reported for California clays but for hard lias clays α may be as low as 0.1.

Tomlinson recommends that, where there is no previous experience with a particular clay, a value of 0.45 should be adopted, up to a maximum adhesion value of 100 kN/m^2.

This may be conservative for soft clays but optimistic for very stiff fissured clays.



in the upper reaches of the pile to settle (or travel) down along the surface of the pile exposed to such material, is given by:

$$Q_n = 0.90 \cdot c' \cdot H \cdot S$$

where c' is the 'average' undrained shear strength of the clay (i.e. the compressible fill material) existing in the depth H, and, S the circumference of the pile. [For instance, where the pile is in a scourable medium, then H = depth between normal bed level and the max. scour level, and, for a circular sectioned pile of radius r, $S = 2\pi r$.] Note that the value 0.90 is an approximated magnitude of the product of the adhesion factor and the averaging factor of distribution of c'.

(ii) In 'non-cohesive' (or 'Granular') Soil: The ultimate bearing capacity Q_u is made up of *skin friction* Q_s and end bearing Q_b less *negative skin friction*, Q_n (end bearing is usually much greater than skin friction in granular soils). Thus:

$$Q_u = Q_s + Q_b - Q_n$$

- *Skin Friction* f, at depth z, is given by

$$f = K_s p_d \cdot \tan\delta$$

where K_s is an earth pressure coefficient; the ratio of the lateral to vertical earth pressure at the sides of the pile (higher than K_a but lower than K_p values).
p_d is the overburden pressure at depth z. Generally $P_d = \Sigma\gamma z$ where γ is the bulk density for strata above the water table and the submerged density below the water table as for spread foundation.
δ is the angle of wall friction (between the pile and the soil).

For a pile surrounded by granular soil between depths z_1 and z_2, the total skin friction is

$$Q_s = \frac{1}{2} K_s \gamma (z_1 + z_2) \tan\delta \cdot A_s$$

where A_s is the embedded area from z_1 to z_2 (thus, for a circular pile of radius R, $A_s = 2\pi R(z_2 - z_1)$. If the pile is partly submerged then contributions from above and below the water table must be calculated separately.

Values of K_s and δ, obtained by Broms, are given in Table 12.3. This is valid up to a skin friction value of 110 kN/m^2 which is the maximum skin friction value which should be used for any straight-sided pile.

Table 12.3 *Values of K_s and δ for driven piles*

Pile Material	δ	K_s Low Rel. Density ($\phi \le 35°$)	K_s High Rel. Density ($\phi > 35°$)
Steel	20°	0.5	1.0
Concrete	$3/4\phi$	1.0	2.0
Wood	$2/3\phi$	1.5	4.0

- In calculating *end resistance*, the third term (relating to base friction) in Meyerhoff's equation is relatively small for long slender piles and is usually ignored. Thus, base resistance is given by

$$Q_b = p_{ob} \cdot (N_q - 1) \cdot A_b$$

where p_{ob} is the effective overburden pressure at the pile 'base' level.
A_b is the area of the pile base (thus, for a circular pile of radius R, $A_b = \pi R^2$).

Meyerhoff's value of N_q (given in Fig. 12.6) tend to be unrealistically high for piled foundations, when compared with actual failures, and values obtained by Berezantsev, given in Fig. 12.10, are more suitable. The maximum value of end bearing which should be used in 1100 kN/m^2.

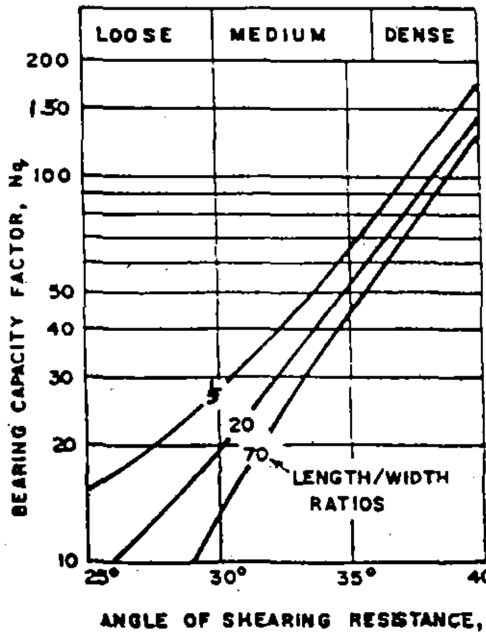


Fig. 12.10 *Berezantsev's bearing capacity factor N_q*

NOTE: When calculating both skin friction and end resistance of 'bored' piles in granular soil, a low relative density should always be assumed, whatever the initial state of the soil.

- The negative skin friction, Q_n, is as explained earlier, but, here, for non-cohesive fill material case, is given by:

$$Q_n = \left[\frac{1}{2} \cdot K_s \cdot \gamma \cdot z \cdot \tan\delta\right] \cdot H \cdot S$$

where: K_s is as explained earlier.

γ = bulk density of strata causing negative skin friction.

$H = z$ = depth of this strata.

(Note: $\gamma \cdot z$ may be treated as $\Sigma\gamma \cdot z$ where γ is the bulk density for strata above the water table and submerged density below the water table, as explained earlier.)

δ = angle of wall friction.

s = perimeter of the pile, as explained earlier.

2. *Safe Value*

The acceptable working-load value of the soil resistance to a pile, i.e. the SAFE value, is obtained by dividing the ULTIMATE value (calculated in (1) above) by a safety factor. While different authorities may recommend different values for this factor, a commonly adopted value is 2.5 or 3.0.

NOTE: Pile bearing capacity formulas should not be expected to give more than a rough indication of the ultimate load capacity of a pile and, except where piles are driven to refusal, it is usual to load test at least one pile at each site. Special test piles may be driven ahead of the main construction program and tested to failure. As a result of these tests, the engineer may decide to modify the pile lengths required.

— It is preferable to delay testing a pile for as long as possible after it has been driven to allow it to 'settle down'. This is not so important with piles in coarse granular soils, where time-dependant effects are negligible, but in silts and silty sands the ultimate capacity of a pile may be much higher immediately after driving than after it has been installed for a month or so. In clays, the reverse is usually (but not always) true; the carrying capacity increasing with time, particularly in soft or sensitive clays.

— Where piles are driven to refusal in rock or in very dense sands or gravels, the maximum allowable load is usually limited by the structural strength of the pile section rather than the support of the soil.

— Where piles pass through bands of different materials, the skin friction may be calculated for each band and the total skin friction taken as the sum of these values, unless very compressible layers are present. When calculating end bearing, care must be taken to check that weak material is not likely to occur near the tip, which would result in a decrease in end bearing capacity. If this is a possibility, it must be allowed for (by using a reduced value of N_c and N_q) when calculating end bearing.

— In $c-\phi$ soils, skin friction may be taken as the sum of friction and adhesion and end bearing may be taken as the sum of end bearing due to both cohesion and internal friction. However, results should be viewed with some scepticism because very little information is available about the behaviour of piles in $c - \phi$ soils.

(D) Soil Resistance to a 'Group' of Piles

Requirement # 1

Safe value of the soil-resistance to an individual pile (estimated as explained in (C) earlier) shall not be less than the max. axial compression load in the highest loaded pile in the group.

Requirement # 2

Block Failure The group of piles should also be assumed to act (hypothetically) as "one single large pile" of plan dimensions of the group, and behaving like a 'block'. Soil resistance (end bearing plus skin resistance less negative skin friction less self weight of piles less weight of soil within the group) should be estimated for this "block" or "large hypothetical pile" and this should not be less than greatest vertical load on a pile × no. of piles × factor of safety.

Requirement # 3

Group Action $\sim V \cdot nE_f \not< P$, where:

V = Safe value of the soil-resistance to an individual pile (esitmated as explained in (C) earlier).

n = No. of piles in the group.

E_f = Group Efficiency Factor (depending on type of soil, spacing between piles, etc.), as explained ahead*

P = Total vertical load on the group (working load value).

**Values of* E_f: *In Granular Soils*— 'Driven' piles compact the surrounding soil, increasing its bearing capacity, and model tests have shown that 'group efficiency' ratios of driven piles in sand can be as high as 2. With bored piles, the action of boring tends to reduce rather than increase compaction so that the group efficiency ratio of bored pile groups is unlikely to exceed 1.

For design purposes, a group efficiency ratio of 1 is typically used for all kinds of piles in granular soils, that is, the effects of the group are ignored when predicting bearing capacities. However, bored piles should not be spaced closer than 3 diameters (centre to centre).

In other words, *there is at least no reduction in the group load capacity* even if the pressure bulbs of individual piles may slightly overlap so long as piles are spaced at 3 diameters in 'bored' case and (even) 2.5 diameters in the 'driven' case, in non-cohesive soils.

In Cohesive Soils—In these soils the group efficiency factor is always less than 1.0. Figure 12.11 gives the results of model tests by Whitaker for 3 × 3 and 9 × 9 groups. It can be used as a method to account for the efficiency of group in cohesive soils.

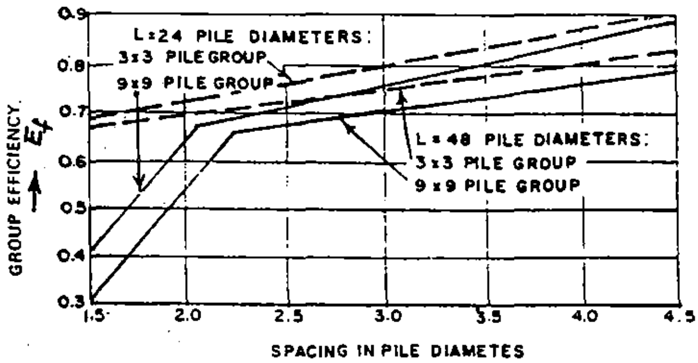


Fig. 12.11 *Group efficiencies for pile groups in 'cohesive' soil. (In grateful acknowledgement to the Controller of Her Majesty's Stationary Office, U.K.)*

NOTE: For a group of piles end-bearing on rock (either 'bored' into rock or 'driven-in' to refusal), the above-mentioned Requirement # 3 may be of no significance since the end-bearing value may be excessively high.

Requirement # 4

This requirement is only for pile-groups which are underlain by weak clay layers in which overstressing can occur. Hence a check should be made for this stress on such a weak layer as follows:

(a) Friction Piles in Clay: The load is assumed to spread out as shown in Fig. 12.12 from 2/3 of the way down the length of the pile embedded in the bearing stratum.

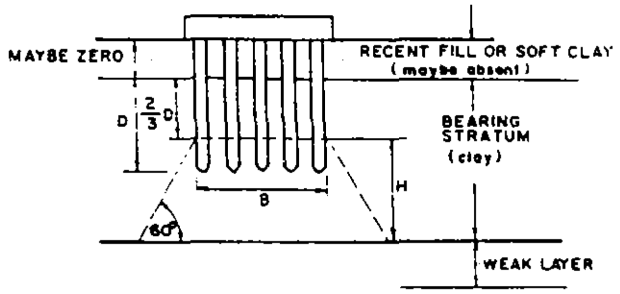


Fig. 12.12 *Spread of load for friction piles*

(b) End Bearing Piles in Sand or Gravel: The load is assumed to spread out as shown in Fig. 12.13 from the base of the piles.

... In each case above, for a pile group measuring L by B, in plan, supporting a load Q, the stressed area of weak material will be

$$(B + 2H \tan 30°) \cdot (L + 2H \tan 30°) = (B + 1.15H)(L + 1.5H)$$

Thus, the stress at the top of the weak layer will be

$$\frac{Q}{(B + 1.15H)(L + 1.15H)},$$ which should be ensured to be allowable there.

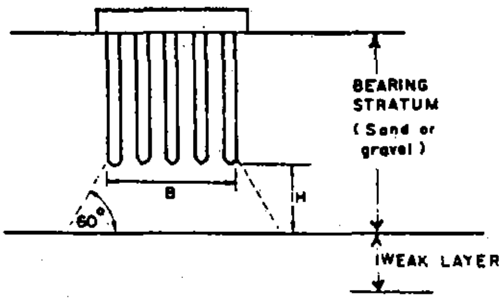


Fig. 12.13 *Spread of load for end-bearing piles*

Lengths of Closely-Spaced Piles

As far as possible, all piles on a pile cap should be of approximately equal length and capacity.

REFERENCES

1. Terzaghi, K., *Theoretical Soil Mechanics*, John Wiley (1943).,
2. Meyerhoff, G.G., "The Ultimate Bearing Capacity of Foundations", *Geotechnique*, Vol. 2, No. 4 (1952).
3. Whitaker, T., *The Design of Piled Foundations*, Pergamon Press, (2nd ed. 1976).
4. Nordlund, R.L., "Adhesion Factors from Bearing Capacity of Piles in Cohesionless Soils", *Proceedings of the American Society of Civil Engineers*, SM3 (May 1963).
5. Broms, B., "Methods of Calculating the Ultimate Bearing Capacity of Piles: A Summary", *Sols-Soils*, Vol. 5 (1966).
6. Berezantsev, V.G., "Load Bearing Capacity and Deformation of Piled Foundations", *Proceedings of the 5th International Conference on Soil Mechanics*, (Paris, 1961).
7. Tomlinson, M.J., *Foundation Design and Construction*, Pitman.
8. Whitaker, T., *Experiments with Model Pile Groups, Geotechnique*, Vol. 7 (1957).
9. Carter, M., *Geotechnical Engineering Handbook*, Pentech Press, 1983.
10. Raina, V.K., *Consultancy and Construction Agreements for Bridges, including FIELD-INVESTIGATIONS,*" Tata McGraw-Hills, New Delhi.

PART II *(Some Relevant Details)*...

Contents ~

(I) Improving the Bearing Capacity of Soil and Making Foundations on Weak Soils

If the foundations are left open for one rainy season it will enable the soil to settle down, and it will also be known whether the natural movements of the soil below, due to increment of moisture, are likely to cause any damage. Foundations in poor soils can be improved by:

(a) Increasing the depth of the foundation except when the material grows wetter as the depth increases.
(b) Compacting the soil by ramming.
(c) Ramming-in sand, gravels, moorum, broken stone or brick bats *in situ* between the foundation concrete and soil. This is useful for silt or black cotton soils and also clayey soils.
(d) Removing the poor soil and filling the gap with sand, rubble stone, gravel or other hard material. This will increase the bearing power to about twice its original value. In this method the foundation trenches are excavated for a depth of about 2 m and 1 m wider, and filled with the hard material to a thickness of about 0.5 m and heavily rammed with water so as to force the hard material in the soft soil. If the filled material is burried completely, then another layer of the hard material may be filled into a depth of about 0.2 m and well rammed. This method is especially useful for black cotton soils. (See ahead under "Foundations in Black-cotton Soils"). Cement grouting the rammed materials will make the foundations much harder.
(e) Draining out water from wet foundations (Well-point system).
(f) Driving piles, either of wood or concrete, or driving and withdrawing piles and filling the holes with sand or concrete. This will increase the density of the soil. (The method has been explained in detail ahead).
(g) Artificial Stabilization can be used to seal off permeable strata for deep excavations, or to give soft soils additional strength if they are likely to flow.

Cement grout: Water-bearing gravel and coarse sand can be made very much less permeable by pumping cement grout into them. The process is successful only on coarse sands and gravels where the grout can fill up the voids; finer sands necessitate some form of chemical or bituminous emulsion treatment. Grouting is of much use for deep excavations, such as tunnels.

• ***Making foundations on Weak Soils***

a) Grillage footings Consist of single or double tiers of steel beams or rails. The top tier is laid at right angles to the bottom tier. The beams are held in position by spacers placed between them 1 to 1.5 m apart. The stanchion is usually bolted to the top tier and the entire footing is filled solidly with concrete and encased in concrete with a minimum cover of 10 cm; a layer of concrete 20 cm in thickness is placed under the lower beams. The maximum spacing of beams should not be more than 0.5 m centre to centre. Overhang ends are designed as cantilevers subject to an upward uniform load equal to the pressure on the foundation. The working stresses of the uncased beams may be increased by $33\frac{1}{2}$ per cent. It is necessary with grillage beams to check the strength of the web for resistance to buckling, and also shear strength for short spans. This type of foundation is generally suitable for single column loads. Steel grillage footings have been largely replaced by reinforced concrete footings known as "Mat Foundations."

b) Column footings For light loads the column footings may be of plain concrete but most column footings are reinforced concrete footings with two-way reinforcing. Small-diameter closely spaced bars with hooked ends, should be used.

c) Raft or Mat Foundation These usually consist of either (1) thick reinforced concrete slabs covering the entire area occupied by the building and reinforced with layers of bars running at right angles to each other a few cm below the top surface of the mat, and another layer a few cm above the bottom, or (2) inverted T-beams of reinforced concrete, with the slab covering the entire foundation area. The beams run under both directions and intersect under columns and support wall loads, if any. Slab and beams are formed into a monolithic structure and act as a unit. Reinforcement is provided in the beams to support walls, if necessary. The basement floor is placed over the beams. Before the basement floor is placed, the space between these beams may be filled with cinders or some other material. This kind of foundations are used on soft natural ground or fill where the capacity of the soil is very low and where piles cannot be used advantageously. A raft should be so shaped and proportioned that the centre of area of the ground-bearing should, where practicable, be vertically under the centre of gravity of its imposed load.

Raft foundations are stable so long as the underground conditions are undisturbed. Rise and fall of the underground water-table is dangerous for such type of foundations. Where ground water pressure is likely to occur, relief holes should be left in the mat to relieve the water pressure.

Foundations of the above type are sometimes called *floating foundations* and the term is applied where the earth excavated to a depth that will make the weight of the earth removed about equal to the building load. The total vertical pressure on the soil under the building is about the same after the building is completed as it was before the site was excavated and the settlement is reduced to a minimum.

d) Piles: Piles have been described in detail in the author's other book, Analysis, Design and Economics and also ahead. As far as possible, a structure should be erected in such a manner that its whole weight is evenly distributed over the solid foundation below to avoid unequal settlement of the sub-soil. All settlement cannot be eliminated because there is a tendency for the central portion of the building to settle more than the outer portion. In order to reduce differential or uneven settlement to a minimum, foundations must be made very rigid. Heavily loaded parts of a building should be separated from the rest, and the higher and heavier parts treated as separate units with independent foundations fitting in such a manner that the whole structure will have equal settlement. The foundations have also to be separated if the soil underneath is of varying nature and different bearing capacities.

Where possible, the axis of the loads of a unit, i.e. the vertical line passing through the centre of gravity of the weight of the whole unit structure, should coincide with centroid of the area of the foundation of the unit. If there is an eccentricity, the intensity of pressure becomes uneven at the two ends producing more compression at one end and less at the other (or even tension and lifting up of the structure) and the structure, thus, assumes an inclined members such as cantilevers, thrust from an arch wind pressure, earthquake, etc.

- ***Sand Piling Under Foundations***

If the foundation soil is unsatisfactory it can be improved sand piling. Holes are made in the foundation soil with wooden pegs 15 cm in diameter and 1–1.5 m long driven 0.6 to 1.0 m into the ground. These holes are filled with saturated sand. The holes are spaced diagonally so that each hole is nearly 0.6 m apart from those adjacent to it. Work should proceed from the centre of the trench outwards. Sand-piling must never be resorted to in foundations subject to occasional floods, or in foundations where water is met within the course of excavation or bottom of driven pegs.

For big structures, holes are made about 0.3 m diameter and 3 m deep which are filled with sand. The spacing may be about 3 m according to the arrangements of the columns of the structure. The filled-in sand is thoroughly consolidated and a concrete slab laid on top of the piles. The concrete is also let into the pile holes for about 0.15 to 0.3 m so as to be monolithic with the slab.

On shrinkable clays, it may be more economical to use short bored piles and beam foundations to support the external walls.

- ***Shallow Foundations in Black Cotton Soils***

The following methods are generally adopted to meet the characteristics of this soil:

(i) Foundations loads are limited to 5 T/m^2 if water finds access to the foundations, otherwise it may be about 10 T/m^2.

(ii) Foundations are taken down to such depths to which the cracks do not extend.

(iii) Trenches are dug on the side of the foundations and filled with sand or other material to prevent intimate contact of the black cotton soil with the concrete and masonry of the foundations.
If the thickness of the black soil is only 1 to 1.5 m, it should be completely removed and foundation laid on the soil below.

(iv) For important buildings, raft foundations of reinforced concrete are provided.

For ordinary buildings, the foundation trench should be about 1.5 m wide and taken down to at least 0.2 m below the depth at which the cracks cease. The bottom of the trench should be well watered and thoroughly rammed with heavy rammers. On the rammed bed a 30 cm layer of good hard moorum or other such soil is spread in 15 cm layers, well watered and rammed. On top of the moorum about 0.5 m of sand is spread. Before spreading the sand and in order to keep it from running, when dry, into the cracks in the black cotton soil, a half-brick wall of mud or a thin skin of stone masonry is built along both sides of the trench.

On top of this sand the concrete foundation of the building is laid, the masonry to start 15 cm below ground level. Or alternatively, boulder filling may be done underneath the foundation concrete and sides filled with sand. Sand filled around the foundations is about 15 cm for compound walls and unimportant buildings and 0.5 m to 0.6 m for main walls.

Another method similar to the above is when trenches are excavated to a depth of about 2 m and width greater than the width of the bottom of footings by 0.5 m. Cement concrete is filled into a thickness of 25 cm on the sides of the trench bottom for a width of 25 cm on either side, thus leaving a space equal to the width of the bottom of the masonry and 25 cm high which is filled with sand. On the top of this (for full width of the trench) RC slab is built 15 cm thick. Masonry (foundation footings) is built on the RC slab and 25 cm space left on both sides of the foundation masonry is filled with sand. A vertical pipe of 75 mm diameter is passed through the plinth masonry to the sand under the RC slab (through the masonry and the slab) which is kept filled with sand. The sand in the tube will fill up the hollows created at the bottom. Such tubes can be built from 1.5 to 2 m apart and inspected at every change of season and filled up with sand if required.

Black cotton soil can be improved by blending it with granular material, or white clay and coarse sand in equal proportions, which is spread on top and rolled.

(II) Various *in-situ* Penetration Tests Employed in the Estimation of Sub-strata Bearing Capacity

Estimation of bearing capacity of sub-strata involves certain *in situ* operations of 'physically attempting to penetrate the strata'. These are different from the laboratory test* that may subsequently be done on the samples of soil collected from various depths from the said penetration tests.

These penetration tests may be classified as follows:

a) Standard Penetration Test (SPT)
b) Static Cone Penetration Test (SCPT)
c) Dynamic Cone Penetration Test (DCPT)
d) Plate Load Bearing Test (PLBT)

Out of the four tests mentioned above, the SPT is the most commonly employed. The SCPT (also referred to as the 'Dutch Cone Penetrometer Test'), developed by the Dutch Government Soil Mechanics Laboratory, Delft, is applied more to non-cohesive soils, and has been used mostly in Europe only. The DCPT is the least commonly employed penetration test. It has no direct application to shallow foundation design nor does it have a recognized correlation between dynamic probe resistance and bearing capacity or *in situ* density, etc. The PLBT is reasonably reliable only if the sub-strata is of uniform formation in extent and depth but even then it cannot take into effect the soil conditions beyond about 1.5 to 2.0 m below the level at which it is carried out.

a) Standard Penetration Test (SPT)

It is a very useful means of determining the approximate *in situ* density of non-cohesive soils, and to a considerable extent, of most of the regular engineering soils which are $C - \phi$ type.

This test, at a particular depth, is made inside the borehole that is first drilled to that depth. After making the necessary borehole to the depth at which SPT is to be conducted, the SPT apparatus is then lowered through the bore to its bottom. The apparatus essentially comprises a 51 mm diameter annular metallic bottom cutter piece. In gravels, where no soil sample need be 'spooned' out, it can be solid and 60° conical in shape. To this bottom piece is screwed on a 609 mm tall metallic tube made up of two semi-circular halves called the 'split' spoon (tube) sampler. To its upper end is attached a metallic top piece called the adaptor, to which is connected the 'connection rod' (built up of a series of extension rods) which provides the connection to the head piece at ground level.

Penetration of the bottom piece and the split tube sampler into the soil is achieved by repeated 'blows' of a 63.5 kg 'monkey' weight freely falling on the head piece through a 760 mm height. The number of blows required for a total penetration of 450 mm is recorded and out of these, the number of blows required for penetrating the final 300 mm (one foot) is noted and this represents N', the standard penetration resistance at that level. (N' = Number of blows for the last 30 cm penetration.)

This N' value is then corrected for the overburden (i.e. depth) effect to obtain the standard SPT N-value. (Corrected N is higher nearer surface, and the correction-multiplier reduces as depth increases, see Fig. 12.1). In case of dense silty sands, this N-value is further corrected if it exceeds 15, as explained ahead.

It is this corrected N-value which is used in all the (empirical) correlations with the angle of internal friction ϕ and cohesion (or unconfined compression c_u) in finally estimating the bearing capacity, etc. (Figs. 12.3 and 12.4). The same corrected N-value is used in Table 12.1.

It is standard practice to count the number of blows for every 75 mm of penetration in the full 450 mm of driving. By this means the depth of any disturbed soil in the bottom of the borehole can be assessed and the level at which any obstructions to driving such as cobbles, large gravel or cemented layers are met can be noted. Normally

* For example, Triaxial Compression Test (for determining the magnitudes of cohesion c and angle of internal friction ϕ), Consolidation Test (for estimating the magnitude and rate of consolidation settlement of soil beneath foundation), Permeability Test, Unconfined Compression Test, Particle-size distribution Test, etc.

not more than 50 blows (including the number of blows required to seat the sampler below the disturbed zone) are made in the test. If the full 300 mm penetration below the initial seating drive is not achieved, i.e. when 50 blows have been made before full 300 mm penetration is achieved, then both the depth at the start of the test and the depth at which it is concluded must be given in the borehole record, suitable symbols being used to denote whether the test was concluded within or below the initial seating drive. After withdrawal from the borehole the split tube is taken apart for examination of the soil contents, and although the sample is in a disturbed state it is often sufficiently intact to be able to see laminations or similar features.

The standard penetration test is used to determine the relative density of granular soils. Before starting the test, the bottom of the borehole must be carefully cleaned out to remove any disturbed material. In sands below the water-table, the sand may have a tendency to flow up into the borehole if the casing is not sufficiently far advanced, giving an unrealistically low value of density. If the casing is advanced too far, the sand below the borehole may get compacted. Thus, unless boring and testing are carried out carefully, the results of the test may be very misleading.

A standard split-spoon sampler, shown in Fig. 12.14, is driven 450 mm into the soil by repeated blows from a monkey of standard dimensions. The arrangement of monkey and sampler is shown in Fig. 12.15. The blows required to produce the first 150 mm penetration (termed the seating blows) are usually ignored and the number of blows required to drive the sampler a further 300 mm is recorded as the N-value.

Interpretation of the test is based on experience and on correlations carried out by various researchers. The various corrections and correlations commonly used when interpreting the test are given ahead next. When using the test in cemented soils, sands or gravels or weak rocks, these correlations are no longer valid and interpretation must be made with caution.

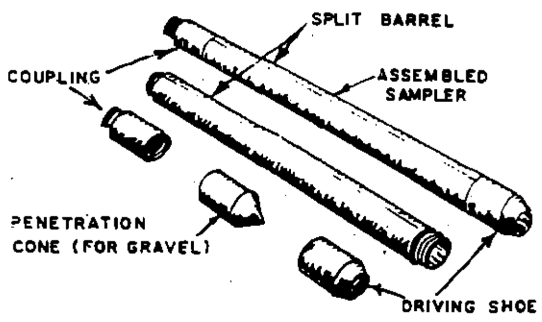


Fig. 12.14 *Split-spoon sampler*

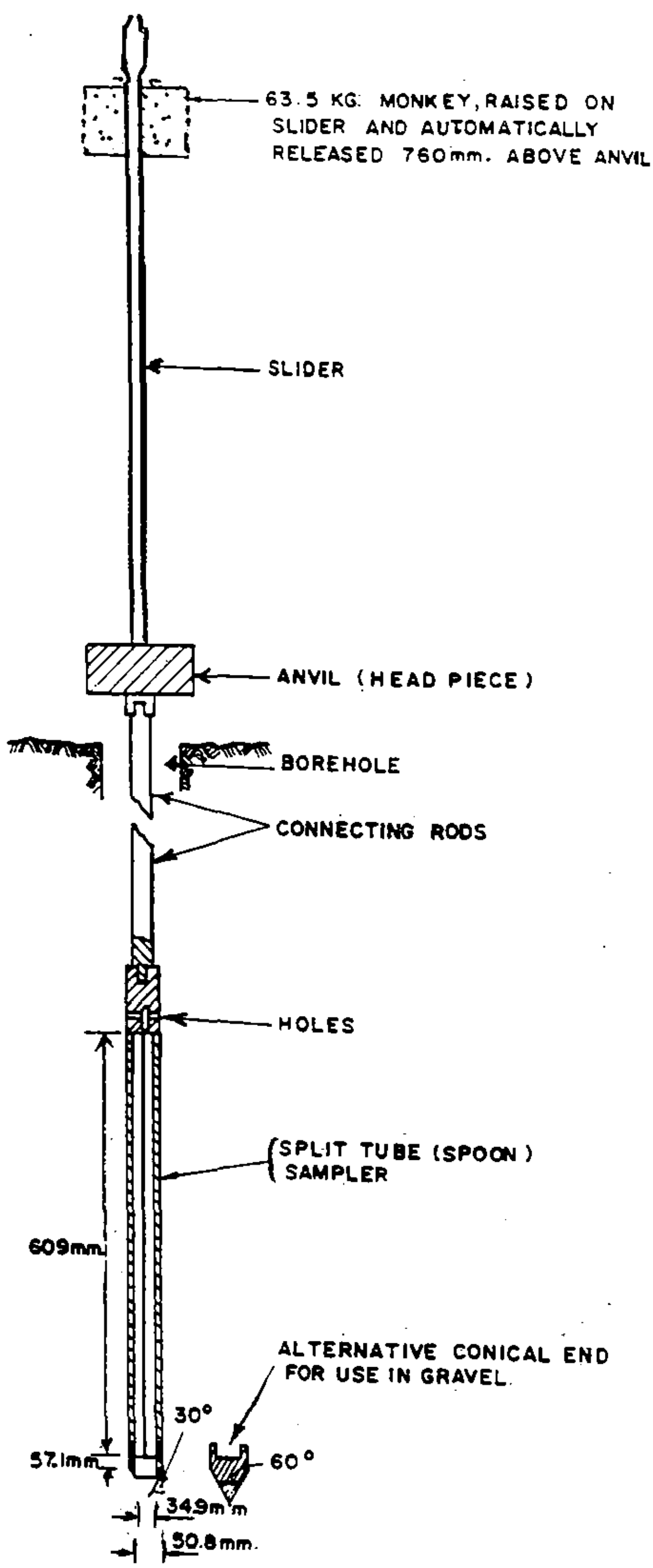


Fig. 12.15 *Schematic arrangement of standard penetration test equipment in borehole*

—Interpretation of Standard Penetration Test Results

SPT results are used primarily to assess the relative density of sands and gravels using a correlation by Terzaghi and Peck. They may also be used to estimate the angle of shearing resistance, θ, and Terzaghi's bearing capacity factors N_q and N_r, from correlations obtained by Peck, Hanson and Thornburn. All of these correlations are given in Fig. 12.3. In addition, SPT results may be used to give a rough indication of the settlement of spread foundations in sands, as indicated in Fig. 12.2.

Approximate relationships exist between the undrained shear strength of clays and N-values (Fig. 12.4) but it is usually better to obtain shear strengths by direct measurement (laboratory tests, e.g. triaxial compression test).

—Corrections in SPT Value

1. Depth effect: The standard penetration test tends to give underestimates of relative density near the surface and a correction factor is usually applied to measured N-value at shallow depths. Depth corrections obtained by Gibbs and Holtz are given in Fig. 12.1.

2. Silts and silty fine sands: In saturated dense silty sands the standard penetration test usually gives an over-estimate of relative density and it is usual to base relative density estimates on modified N-value, N_m, obtained from the expression—

$$N_m = 15 + 1/2(N - 15) \text{ if } N > 15$$

If N is less than 15, no correction is required.

b) Static Cone Penetration Test (SCPT)

As pointed out earlier, this test was developed by the Dutch, and they seem to use it the most. Long experience of its use and familiarity with their soil stratification have enabled the Dutch largely to dispense with conventional borings in their own country, and they rely almost entirely on this cone test for foundation design. The static cone test is also a valuable method of recording variations in the *in situ* density of loose sandy soils or laminated sands and clays in conditions where the *in situ* density is disturbed by boring operations, which tend to make the standard penetration test unreliable in evaluation. Only limited penetration can be achieved by the cone in coarse gravelly soils. The relationship between static cone tests and the standard penetration test shows that there is no unique relationship between them, but it appears to be related to particle size.

In the SCPT, a solid metallic cone is gradually 'pushed' down into the soil hydraulically (not by hammering, hence the word 'static' instead of 'dynamic'), without the need of any borehole to be made in advance (as required in the case of SPT). Hence, it provides a much more accurate and detailed record of the variation in the sub-strata.

Essentially the apparatus comprises a solid metallic cone (the bottom piece) which is connected through metallic extension 'sleeves' (tubes) to a head piece at ground level. The head piece is operated hydraulically, pressing the cone and the connecting tubes down, and additional sleeves are connected for making up the extension. The cone registers the end bearing resistance and the increasing length of sleeve tubes registers the skin friction! There are variations in the apparatus but the one most in use—the electrical cone type was developed by Fugro N.V. in Holland. In this, both the cone and the sleeve-tubes are jacked down together and continuously. The thrust on the cone-end and on a 120 mm length of the cylindrical sleeve are measured separately by electrical load-cells (strain gauges) installed at the lower end of the penetrometer. In modified versions, sensors in the cone read for bearing resistance and those in the tubes read for skin friction. The two values are then correlated and used to estimate the bearing capacity at various levels. However, the skin friction component may not be very reliable.

c) Dynamic Cone Penetration Test (DCPT)

This involves 'hammering' (and hence the word 'dynamic', not 'static') a penetrating metallic cone into sub-strata by the blows of a drop hammer on top of the connecting (extension) rods. The number of blows for a given distance of penetration is recorded. *The apparatus is like a mini pile-driver and is, in fact, used more by the piling contractors as a means of predicting empirically the driving resistance and hence the bearing capacity* (i.e. carrying capacity) *of piles.* It is, therefore, sometimes referred to as *'pre-piling test' apparatus.*

~ Static and Dynamic "Cones"

• *Basic principle of dynamic cone:* Like the standard penetration test, dynamic cones are driven into the ground using a standard falling hammer. However, friction between the drilling rods and soil limits the depth to which the standard sampler or cone can be driven and makes interpretation of results unreliable.

• *Example of a static cone:* The cone illustrated in Fig. 12.16 is pushed into the ground at a constant rate. Strain gauges 3a and 3b allow continuous separate measurements of sleeve friction and cone resistance, respectively.

• *Interpretation of results DCPT vs SPT:* Dynamic cone test results are usually converted to equivalent N-values of the standard penetration test. The relationship between the cone penetration test value N_c and the SPT N-value depends on the equipment used. For typical 60° push-fit cones, the relationship $N_c = 1.5\ N$ has often

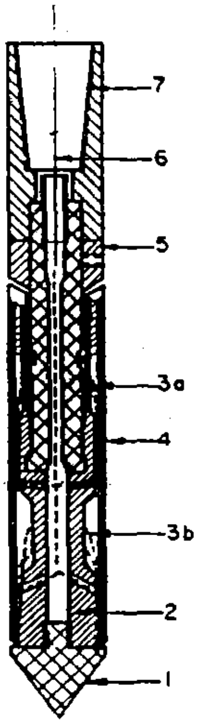


Fig. 12.16 *Example of a static cone: 1. Conical point 2. Load cell 3. Strain gauges 4. Friction sleeve 5. Adjustable ring 6. Cable 7. Connection with rods*

been quoted, for depths up to about 9 m, but it is not reliable.

d) Plate Load Bearing Test (PLBT)

Plate Load Bearing Tests give reliable results only when the soil condition is uniform from the bottom of the footing to a depth at least equal to the width of the largest footing, since settlement in cohesive and partially cohesive soils takes place in a long period of time, load bearing tests on such soils are not very practical. Fortunately, the bearing capacity and the settlement characteristics of such soils can be readily determined by laboratory tests on the relatively undisturbed sample.

The results of plate load bearing tests on soils are useful provided that the test is made with extreme care. The following are some of the factors that should be considered:

1. The test should be made on the loosest area contemplated to support any foundation.
2. The depth of ground water in the test case and in the actual cases should be comparable. Avoid making tests on a layer affected by capillary water.
3. In the 'maintained load test' each load increment is maintained until no further settlement of significant magnitude takes place, and only then the next loading is applied.
4. The ground is not frozen during the test.

There are many other factors which influence the test results. It is advisable to follow the standard test procedure of ASTM Designation D 1194.

The results of load bearing tests should be plotted in a graph similar to one shown in Fig. 12.17.

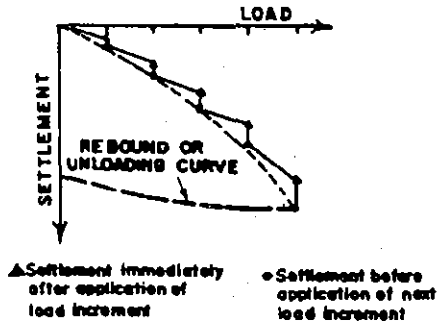


Fig. 12.17 *Graphical presentation of results of load bearing test*

More than one plate load bearing test should be made at the concerned foundation site. Because of variation in soil characteristics and other factors, two tests made under indentical conditions on a presumably uniform soil often show considerably different load-settlement curves. Therefore, results of load bearing tests require careful and expert interpretation.

Plate bearing tests are made by excavating a pit to the predetermined foundation level or other suitable depth below ground level, and then applying a static load to a plate set at the bottom of the pit. The load is applied in successive increments until failure of the ground in shear is attained or, more usually, until the bearing pressure on the plate reaches some multiple, say two or three, of the bearing pressure proposed for the full-scale foundations. The magnitude and rate of settlement under each increment of load is measured. After the maximum load is reached the pressure on the plate is reduced in successive decrements and the recovery of the plate is recorded at each stage of unloading.

This procedure is known as the maintained load test and is used to obtain the deformation characteristics of the ground. Alternatively, the load can be applied at a continuous and controlled rate to give a penetration of the plate of 2.5 mm/min. This is known as the constant rate of penetration test and is applicable to soils where the failure of the ground in undrained shear is required, as defined by gross settlement of the plate; or where there is no clear indication of failure with increasing load, the ultimate bearing capacity is defined by the load causing a settlement of 15 per cent of the plate diameter.

Although such tests appear to answer all the requirements of foundation design, the method is subject to serious limitations and in certain cases the information given by the tests can be wildly misleading! In the first place it is essential to have the bearing plate of a size which will take account of the effects of fissures or other discontinuities in the soil or rock, in plan area as well as depth.

A 300 mm plate is the minimum size which should be used which is suitable for obtaining the undrained shear strength of stiff fissured clays. If deformation characteristics are required from these soils, a *750 mm plate* should be provided in conjunction with the maintained load procedure. It is essential to make the plate tests in soil or rock of the same characteristics as will be stressed by the full-scale foundation. Misleading information will be given if, for examples, the tests are made in the stiff crust of weathered clay overlying a soft clay.

A 1000 mm plate is generally the economic limit, since a 1000 mm plate loaded say to 80 t/m^2 will require some 63 t of kentledge, which is expensive to hire including the costs of transport and handling. *The cost of a single plate-bearing test with a 300-600 mm plate with 50 t of kentledge is about three times the cost of a 12 m deep borehole (in soft ground) complete with in situ and laboratory testing.* A single plate bearing test on a site is, in any case, far from sufficient since the ground is generally variable in its characteristics both in depth and laterally. At least three tests, and preferably more, are required to obtain representative results.

Economies in plate bearing tests on rock can be made by jacking against cable or rod anchorages grouted into drill holes in the rock, instead of using kentledge. Even single anchors have been used successfully. The anchor cable, which is not bonded to the rock over its upper part, is passed through a hole drilled in the centre of the test plate. A test of this type can be made at the bottom of a borehole (pit).

The level of the water-table has an important effect on the bearing capacity and settlement of sands. Thus a plate bearing test made some distance above the water-table will indicate much more favourable results than will be felt by the large full-scale foundation which transmits stresses to the ground below the water-table. The plate bearing test gives no information whereby the magnitude and rate of long term consolidation settlement in clays may be calculated.

In spite of these drawbacks, the plate bearing test cannot be ruled out as ameans of site investigation.

Plate loading tests are best suited to investigating weak jointed rocks or soils containing large gravel or boulders in which *in situ* penetration tests cannot be made.

• *Plate Bearing Test*

A square or circular plate is seated on the stratum to be tested, usually at the bottom of a trial pit and loaded. The load is maintained until full consolidation settlement has taken place. The test is continued with further increments of load. A plot of settlement against load-intensity (to logarithmic scales) allows a zero correction to be made and sometimes allows the yield point of the soil to be determined. Another indication of failure of the soil is *steadily increasing settlement with time at a constant load, with no tendency to reach a limiting value!*

The settlement of a 'square' plate at a given load can be related to the settlement of a square footing by the following formula, proposed by Terzaghi (1948):

$$S_2 = S_1 \left(\frac{2B}{1+B} \right)^2$$

where S_1 is the settlement of the footing, of width B ft, and S_2 is the settlement of a 1 ft square plate.

Although this refers to a standard plate, 1 ft square, the expression can be used for any sized plate, measured in any units, if B is taken as the ratio of footing width to plate width. This relationship usually leads to an underestimate of settlement for large footings and Menard and Rousseau proposed the following relationship:

$$\frac{S_1}{S_2} = \left(\frac{B_2}{B_1} \right)^{\alpha}$$

where S_2 and S_1 are settlements of 'plate' and 'footing', B_2 and B_1 are their respective widths, and α is an exponent which depends on soil type. Values of α are:

sands and gravels	: 1/2 to 1/3
saturated silts	: 1/2
clays and dry silts	: 2/3 to 1/2
compacted fill	: 1

The ultimate bearing capacity of the footing can be assessed from that of the plate by using standard bearing capacity formulas. The ultimate bearing capacity of a foundation in clay is approximately equal to the ultimate bearing capacity of the plate, with no allowance being made for scale. In granular soils ultimate bearing capacity is proportional to width and, approximately.

$$\frac{Q_2}{Q_1} = \frac{B_2}{B_1}$$

where Q_2 and Q_1 are the ultimate bearing capacities under foundation and plate, respectively, and B_2 and B_1 are their respective widths.

Plate loading tests are particularly suitable for coarse granular materials which cannot be tested by normal laboratory means or by a penetration test. The main pitfall in predicting settlement from these tests is that the zone of stressed soil beneath the plate is much smaller than that beneath the larger foundation; so will be unaffected by deeper strata whose load bearing and settlement characteristics may critically affect the behaviour of the actual foundation. With clays, tests do not usually continue for long enough for consolidation to be completed so settlement cannot be predicted. In order to obtain reliable results, plates should be as large as possible and should never be less than 0.3 m wide.

• ***Relative Density and the 'Standard Penetration Test'***

The relative density of a granular soil is defined in terms of the loosest and densest states of compaction which can be achieved in the laboratory with that soil, although no specific laboratory tests are universally agreed. It is defined in terms of the voids ratio of the soil but can also be expressed in terms of dry densities. Thus:

$$D = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{\gamma_{max}}{\gamma} \cdot \frac{\gamma - \gamma_{min}}{\gamma_{max} - \gamma_{min}}$$

where D is the relative density,

γ, γ_{max} and γ_{min} are the dry densities in the field and at the densest and loosest states of compaction, respectively, and

e, e_{min} and e_{max} are the corresponding voids ratios, respectively.

Relative density in the field is usually assessed from standard penetration test results using N-values.

However, SPT results tend to overestimate relative densities in silts and fine sands below the water-table and the relative density is usually assessed using a modified N-value, N_m, obtained from the expression.

$$N_m = 15 + 1/2(N - 15) \text{ if } N > 15$$

If N is less than 15, no correction is required.

Descriptive terms for relative density and equivalent N-values are given in Table 12.4.

Table 12.4 *Descriptive terms for relative density and equivalent SPT N-value*

Relative Density (%)	Descriptive Term	N-values
0–15	Very loose	0–4
15–35	Loose	4–10
35–65	Medium dense	10–30
65–85	Dense	30–50
85–100	Very dense	50+

When relative densities are given in soil descriptions, it should always be made clear whether they are based on SPT results or assessments using a pick and shovel or driven stake.

(III) Safe Bearing Capacity of 'Rocky' Substrata

In case of rock, first the ultimate bearing capacity may be determined by testing a number of its 'representative' core specimens in a compression testing machine, under saturated soaked condition, and then the safe bearing capacity estimated by dividing the average ultimate value by a factor ranging in value between 5 and 20—depending on the *in situ* characteristics of the rock and the combined effects of long term saturation, permanent eccentric compression, possibility of leaching away of its softer constituents from within its vein-matrix, and the abrasion from the flowing sand. An engineering geologist's opinion should also be sought. A plate load test can also be used to determine the allowable bearing capacity of rocky strata. For some typical values see ahead. Many rocks can withstand a compressive stress higher than normally used concrete. Some of the exceptions are given below:

1. Limestones with cavities and fissures which may be filled with clay or silt.
2. Rocks with bedding planes, folds, faults or joints at an angle with the bottom of footing.
3. Soft rocks often reduce their strength after wetting. Weathered rocks can be very treacherous. Shales may become clay or silt in a matter of hours of soaking!

The common sandstones and limestones have modulus of elasticity varying from that of a poor concrete to that of high strength concrete. Very hard igneous and metamorphic rocks exhibit considerably greater value of modulus of elasticity.

Some Terms used in Rock Descriptions: (Based on B.S. 5930, 1981)

Texture and Structure: 'Texture' refers to the physical appearance of the rock crystals or grains. Terms include crystalline, cryptocrystalline, porphyritic, granular, amorphous and glassy.

'Structure' is usually applied to the rock mass and refers mainly to the arrangement of structural features and discontinuities. Structural features include bedding planes, laminations, foliations, flow-banding and cleavages. Discontinuities include joints, fissures, faults and shear planes. Where discontinuities occur in three dimensions, rock blocks are formed. Descriptive terms used for the frequency of discontinuities and structural features age given in Table 12.5.

Rock Strength: This refers to the strength of the rock material and is based on the unconfined compressive strength. Descriptive terms used and the corresponding strengths are given in Table 12.6. Strengths depend on the moisture content of the specimen at the time of test, anisotropic features in the specimen and the test procedure,

Table 12.5 *Terms used to describe the spacing of discontinuities in rock masses*

Spacing	Structural Features	Discontinuities in One Dimension	Discontinuities in Three Dimensions*
> 2 m	Very thick	Very widely spaced	Very large
600 mm–2 m	Thick	Widely spaced	Large
200–600 mm	Medium	Medium spaced	Medium
60–200 mm	Thin	Closely spaced	Small
20–60 mm	Very thin	Very closely spaced	Very small
6–20 mm	Thickly laminated (sedimentary) Narrow (metamorphic and igneous)		*Second term* Blocky-equidimensional Tabular—thickness much less than length or width
< 6 mm	Thinly laminated (sedimentary) Very narrow (metamorphic and igneous)	Extremely closely spaced	Columnar—height much greater than cross-section

*Relates to spacing of maximum dimension.

Table 12.6 *Descriptive terms of the compressive strength of rock material*

Compr. Strength (MN/m^2)	Description
< 1.25	Very weak
1.25–5.0	Weak
5.0–12.5	Moderately weak
12.5–50.0	Moderately strong
50–100	Strong
100–200	Very strong
> 200	Extremely strong

and are generally of little use in assessing the strength of the rock mass.

Grain Size and Rock Name: For engineering purposes, broad classifications are usually sufficient; detailed geological names are not necessary.

Recovery and Rock Quality Description (RQD): These terms refer to rock cores and are defined as follows:

$$\text{Recovery (\%)} = \frac{\text{length of core recovered}}{\text{length of core run (length drilled)}} \times 100$$

$$\text{RQD (\%)} = \frac{\text{length of rock recovered in sound lengths of 100 mm or more}}{\text{length of core run}} \times 100$$

They both give an indication of the strength of the rock mass but can be used only as rough guide because values obtained also depend on the diameter of the core, the method of drilling and the skill of the driller. Descriptive terms for RQD values are given in Table 12.7.

Table 12.7 *Descriptive terms for RQD† values*

RQD (%)	Description
0–25	Very poor
25–50	Poor
50–75	Fair
75–90	Good
90–100	Excellent

† Rock Quality Description

(IV) Soil Parameters: Some Typical Values

(i) Shearing Resistance of Cohensionless Soils: Estimates of the angle of shearing resistance can be obtained from Table 12.8 which gives typical values suggested by Terzaghi and Peck.

Table 12.8 *Typical values of the shearing resistance of cohesionless soils*

Material	ϕ-Degrees Loose	ϕ-Degrees Dense
Uniform sand, round grains	27	34
Well graded sand, angular grains	33	45
Sandy gravels	35	50
Silty sand	27–33	30–34
Inorganic silt	27–30	30–35

(ii) The Inter-Relationship between 'Cohesion', 'Internal Friction' and 'Stability': The unit shear resistance is composed of two parts, that furnished by the resistance of soil-grains to sliding over each other and that furnished by the cohesion existing between the soil particles. By experiments the cohesion c in lbs/sq ft and ϕ have been ascertained as given in Table 12.9.

Table 12.9 *Approx. values*

	Soil	c*	ϕ Deg.
• cohesive	Clay liquid	100	0
	Clay liquid very soft	200	2
	Clay liquid soft	400	4
	Clay liquid fairly stiff	1000	6
	Clay liquid stiff	1500	8
	Clay liquid very stiff	2000	12
	Silt	0	20
	Sand wet	0	10
	Sand wet dry or unmoved	0	34
	Sand wet predominating with some clay	400	30
	Cemented sand and gravel, wet	500	34
	Sand-gravel mixture cemented with clay, dry	1000	34

(Contd).

	Soil	Dry Density**	ϕ Deg.
• non-cohesive	Compact well-graded sands and gravel-sand mixtures	110–120	40–45
	Dense well-graded gravel	115–125	45–50
	Loose well-graded sands and gravel sand mixtures	100–110	35–40
	Dense sands	105–115	35–46
	Compact uniform sands	100–110	35–40
	Loose uniform sands	90–100	30–35
	Loose fine sands	90–100	28–34

*Cohesion lbs/sq.ft

**Dry density in lbs/c ft.

(iii) Friction and Adhesion at Interfaces

Table 12.10 *Typical values of friction and adhesion at interfaces*

Materials	Friction tan δ (δ = angle of wall-friction)	Adhesion C_s (kN/m^2)
• *Mass concrete or masonry against rocks and soils*		
Clean sound rock	0.7	
Clean gravel, gravel-sand mixtures, coarse sand	0.55–0.60	
Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel	0.45–0.55	
Clean fine sand, silty or clayey fine to medium sand	0.35–0.45	
Fine sandy silt, non-plastic silt	0.30–0.35	
Very stiff and hard residual or overconsolidated clay	0.40–0.50	
Stiff clay and silty clay	0.30–0.35	
• *Steel sheet piles against soils*		
Clean gravel, gravel-sand mixtures, well-graded rock fill	0.40	
Clean sand, silty sand-gravel mixtures, single size hard rock fill	0.30	
Silty sand, gravel or sand mixed with silt or clay	0.25	
Fine sandy silt, non-plastic silt	0.20	
Soft clay and clayey silt		5–30
Stiff and hard clay and clayey silt		30–60
• *Formed concrete or concrete sheet piling against soils*		
Clean gravel, gravel-sand mixtures, well-graded rock fill	0.40–0.50	
Clean sand, silty sand-gravel mixtures, single size hard rock fill	0.30–0.40	
Silty sand, gravel or sand mixed with silt or clay	0.30	
Fine sandy silt, non-plastic silt	0.25	
Soft clay and clayey silt		10–35
Stiff and hard clay and clayey silt		35–60
• *Various structural materials*		
Masonry or masonry, igneous and metamorphic rocks:		
dressed soft rock on dressed soft rock	0.70	
dressed hard rock on dressed soft rock	0.65	
dressed hard rock on dressed hard rock	0.55	
Masonry on wood (cross grain)	0.50	
Steel on steel at sheet pile interlocks	0.30	

(*Contd.*)

NOTE: The numbers are ultimate values and require sufficient movement for failure to occur. Where friction value alone is shown, the effect of adhesion is included in the friction factor. For data on adhesion on bearing piles, see Table 12.2.

• Typical Values of Allowable Bearing Pressure

Table 12.11 gives 'typical' values of allowable bearing pressures for shallow spread foundations subjected to vertical static loading. It is presumed that the founding level is at least 1 m deep in soils and that the ground surface is fairly level.

Table 12.11 *Typical values of allowable bearing pressure:*

	Type of Bearing Material	Allowable Bearing Pressure (kN/m^2)
Rocks	Massive hard crystalline igneous and metamorphic rocks	6000–10000
	Massive hard crystalline limestones; thoroughly-cemented sandstones and conglometers	4000–6000
	Unweathered schists and slates	3000–4000
	Hard shales and mudstones, moderate and weakly-cemented sandstones; hard unweathered marl or chalk	1500–2500
	Weathered and broken bedrock; clayey shales and soft mudstones	800–1200
Clays	Hard clays; cohesive strength > 300 kN/m^2	600
	Very stiff clays; cohesive strength 150–300 kN/m^2	300–600
	Stiff clays; cohesive strength 75–150 kN/m^2	150–300
	Firm clays; cohesive strength 35–75 kN/m^2	70–150
	Soft and very soft clays; cohesive strength < 35 kN/m^2	Negligible
Gravels	Very dense sands and gravels; SPT N-value > 50	400
	Dense sands and gravels; SPT N-value 30–50	300–400
	Medium dense sands and gravels; SPT N-value 10–30	100–300
	Loose sands and gravels; SPT N-value 5–10	50–100

• NOTES:

(1) *Approximate conversion:*
kN/m^2 to t/m^2, divide by 10
kN/m^2 to kg/cm^2 divide by 100
kN/m^2 to tons/ft^2 divide by 100

(2) *Clays:* A useful rule of thumb when dealing with shallow foundations for light structures is:

*under dry conditions: allowable bearing pressure = 2c = unconfined compressive strength, c_u.

*under saturated conditions: half of this value. The unconfined compressive strength can be estimated on site by taking numerous pocket penetrometer readings for borehole samples or in the sides of pits. Since most pocket penetrometers are graduated in terms of compressive strength, they give allowable bearing pressures directly. This convenient relationship arises because, from Meyerhoff's bearing capacity formula, net ultimate bearing capacity $p_{nu} = cN_c = 6c$ for shallow foundations. For dry conditions taking a factor of safety of 3, the allowable bearing capacity is $p_a = 1/3.\ p_{nu} = 2c$. For saturated conditions $p_a = 1/6.\ p_{nu} = c$.

(3) *Sands and gravels:*

(a) If SPT N-values are not available, the density of the sand and gravel may be judged by pick and shovel; if it can be excavated by shovel, it is loose; if a pick is required to excavate the material, it is compact. For compact sands and gravels, tested in this way, it is usual to make the conservative assumption that they are medium dense.

(b) The above bearing capacities are for foundations at least 1 m wide on dry sand; for saturated or submerged conditions, halve the values.

- **Broad Classification of Clays and Sands** (see Table 12.12)

Table 12.12

	Shear Strength/Relative Density			Structural Weathering	Colour	Particle Shape/ Composition/ Plasticity	Type of Particle			Inclusions
• Clays	*Clay*	*Cohesion $c(kN/m^2)$*	*Characteristics*				*Particle*		*Size (mm)*	
	Very soft	< 20	Exudes between fingers when squeezed	Intact	Grey	Angular	Clay		< 0.002	.. with shells.
				Fissured	Brown	Subangular	Silt	fine	0.002–0.006	.. with scattered cobbles and boulders.
						Subrounded				
	Soft	20–40	Moulded by light finger pressure.	Stratified	Blue-Gray	Rounded				
						Flat		medium	0.006–0.2	
	Firm	40–75	Moulded by strong finger pressure.	Laminated	Mottled yellow and brown	Elongated				.. with layers or lenses of fine sand.
						Irregular		coarse	0.02–0.06	
	Stiff	75–150	Can be indented by thumb.	Heterogeneous		Rough	Sand	fine	0.06–0.2	.. with some shell fragments.
						Smooth				
	Very stiff	150–300	Can be indented by thumb nail	Fibrous	Dark green	Polished				
								medium	0.2–0.6	
	Hard	> 300		Weathered	Yellowish	Sandstone				
						Limestone		coarse	0.6–2.0	
• Sands	*Sands*	*SPT**		etc.	etc.	Granite				etc.
	Very loose	< 4				Highly plastic	Gravel	fine	2–6	
	Loose	4–10	Can be dug by spade 50 mm peg easily driven.			Non-plastic				
								medium	6–20	
	Medium dense	10–30				etc.				
								coarse	20–60	
	Dense	30–50	Needs pick for excavation 50 mm peg hard to drive.				Cobbles		60–200	
	Very dense	> 50					Boulders		> 200	

*Standard penetration test *N-values*

Ref: B.S. 5930 (1981) for Site Investigations.

CHAPTER 13

Estimating the Net Dependable Passive Less Active' Earth-Pressure Relief from Undisturbed Soil Mass Gripping the Foundation-Bulkhead between the Maximum-Scour Level and the Founding-Level

NOTE: For details regarding the following:

(i) Minimum depth of foundations
(ii) Estimation of scour depth for design of piers and abutments
(iii) Estimation of linear waterway under a river bridge,

refer to the relevant ***data sheets*** in the author's book: *Consultancy and Construction Agreements for Bridges, including Field Investigations.*

The foundation (bulkhead) could be a very large diameter pile or a caisson. In order that the moments resulting from:

(i) its self weight acting in a slightly 'tilted' and possibly shifted condition (tilt and shift caused during actual construction of the bulkhead), and

(ii) those caused by possible local flexing of its body, do not enter the earth pressure computations, the bulkhead may be assumed weightless and rigid for the purpose of the calculations here. However, the tilt and shift moments and those due to any flexing must be computed separately and included along with the effect of other forces.

Two distinct cases are possible:

- *Case A:* Bulkhead resting on soil, with undisturbed soil mass gripping it below the maximum scour level.
- *Case B:* Bulkhead resting on rock, with undisturbed soil mass gripping it below the maximum scour level.

In case A, the soil under the foundation-base may be assumed to yield and give so that the foundation-base tends to move along an arc and kicks back into the soil. Thus the bulkhead tends to rotate about a point above its base.

In case B, if the rock under the foundation-base is assumed not to yield, then the bulkhead does not kick back into the soil. In such a case the base is the tending point of rotation, though not actually hinging physically at base.

The earth pressure distributions between these two cases differ vastly, and their pertinent details are briefly given in the following sections.

13.1 CASE A

Bulkhead rests on soil and tends to rotate about a point which lies above the base

Assumptions

(i) Bulkhead tends to rotate about a point which lies above its base.
(ii) Soil may be non-cohesive or cohesive, but the backfill (behind the abutment) above the MSL (maximum scour level) is always non-cohesive.
(iii) Consider the net passive-less-active earth pressure ordinate at each level above and below the point of rotation.
(iv) Horizontal forces (applied and passive-less-active pressure ones) are balanced, and the correspondingly generated net passive-less-active earth pressure moment relief is considered in the stability of the bulkhead.
(v) Ignore vertical components of passive and active earth pressure forces as also the skin friction and the force of friction generated at base.
(vi) Earth pressure on the portion of the structure above the MSL on account of the backfill earth mass above that level shall be considered separately (as another external force) but its surcharge effect shall be included in the present calculations. (See definition of H_R ahead, under 'Symbols'.)

Calculation of relief offered by the net passive-less-active earth pressure of soil, within the effective soil-grip around the bulkhead, is computed on the assumption that the bulkhead is a rigid body that derives its stability from the net earth pressure from the opposite sides owing to its tendency to rotate about a point slightly above its base. The point of rotation may be found from statical equilibrium between the external resultant horizontal force H_R at MSL and the net passive-less-active earth pressure force as worked out from the net pressure distribution in the relevant case. This is explained diagrammatically in Figs. 13.1 to 13.7 ahead (cases 1 to 6). After locating the point of rotation, the relieving net passive-less-active earth pressure moment at base (and at various intermediate levels) can then easily be worked out and the overall stability (as well as the material stresses at various sections) checked.

Symbols

K_a = horizontal component of active earth pressure coefficient of the soil below MSL

$$= \frac{\sin^2(\alpha+\phi)\cos\delta}{\sin^2\alpha\sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

(K_{a1} and K_{a2} pertain to the dry and submerged portions of the backfill)

K_p = horizontal component of passive earth pressure coefficient of the soil below MSL

$$= \frac{\sin^2(\alpha-\phi)\cos\delta}{\sin^2\alpha\sin(\alpha+\delta)\left[1-\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi+\beta)}{\sin(\alpha+\delta)\sin(\alpha+\beta)}}\right]^2}$$

(K_{p1} and K_{p2} pertain to the dry and submerged portions of the backfill)

P_a and P_p = active and passive earth pressure forces.
α = angle of inclination of wall with horizontal (= 90° for a vertical wall)
β = angle of inclination of top surface of soil with horizontal (i.e., surcharge angle, = 0 for horizontal bed)
δ = angle of wall friction
ϕ = angle of repose of soil (internal friction)
c = cohesion value of soil below MSL
γ = submerged density of soil below MSL
γ_2 = submerged density of backfill soil
γ_1 = dry density of backfill soil
MSL = maximum scour level
HFL = high flood level
FL = founding level
H_R = Σ of all the external horizontal forces at the MSL (including the total P_a from backfill above MSL in case of abutments) expressed per unit width of bulk head,. so that if η be the diameter of the bulk head then $H_R\eta$ represents the full total value of all the externally applied horizontal forces at MSL (including the total P_a from backfill above MSL in case of abutments) on the full width of the bulk head.

Method (Cases 1–6)

(i) Plot $(P_p - P_a)$ ordinate at various depths.
(ii) then,
- H_R per unit width of bulk head = area A − area B
- Hence find d (ref. Fig. 13.1)
- Then passive-less active moment relief, for example at base, = (moment of area A − moment of area B) about base.

(iii) Maximum bending moment occurs at a depth h below MSL where algebraic sum of the horizontal forces (H_R and part of A, or H_R and A and part of B, as the case may be—to be found by trial) is zero. For instance if $h < (H - d)$ in Fig. 13.1 then, $H_R \equiv (K_p - K_a)\gamma hh/2$
from where h can be found, and then the bending moment at h below MSL, i.e. the max. B.M., will be $M_0 + [H_R h - (K_p - K_a)\gamma h^3/6]\eta$

where M_0 = external BM at MSL (total)
η = diameter of bulk head

• ***Case 1***

Pier in non-cohesive soil (Fig. 13.1).

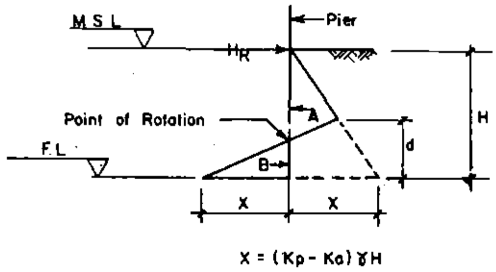


Fig. 13.1

• ***Case 2***

Pier in cohesive soil (Figs. 13.2 and 13.3).
(i) if depth of tension crack $d_c \geq H$ Figs. 13.2

i.e.,
$$\frac{2c}{\gamma\sqrt{K_a}} \geq H$$
$$X = 2c\sqrt{K_p}$$
$$Y = X + K_p\gamma H$$

(ii) If depth of tension crack $d_c < H$ Fig. 13.3

i.e.,
$$\frac{2c}{\gamma\sqrt{K_a}} < H$$
$$X = 2c\sqrt{K_p}$$
$$Y = X + K_p\gamma H - Z$$
$$Z = K_a\gamma H - 2c\sqrt{K_a}$$

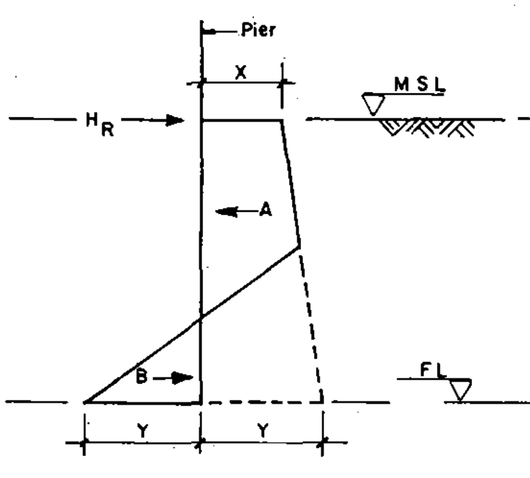


Fig. 13.2

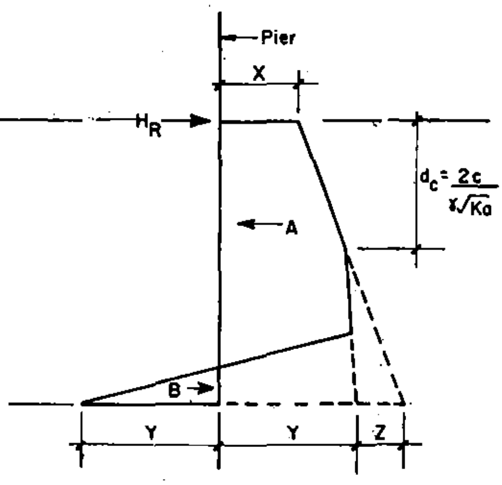


Fig. 13.3

- ***Case 3***

Abutment in non-cohesive soil (Fig. 13.4).

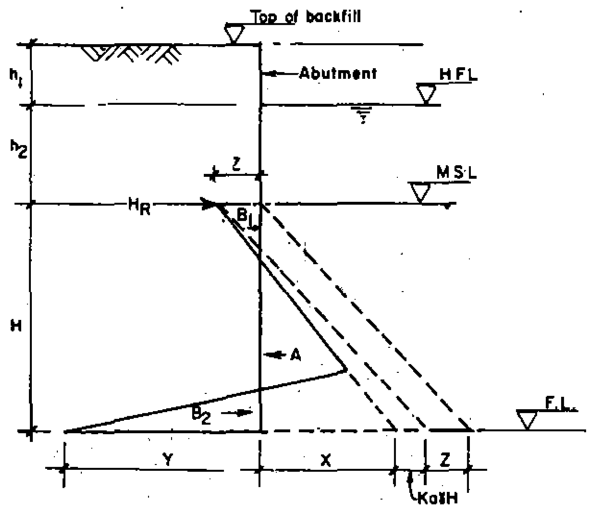


Fig. 13.4

$$Z = K_a(\gamma_1 h_1 + \gamma_2 h_2)$$
$$X = (K_p - K_a)\gamma H - Z$$
$$Y = (K_p - K_a)\gamma H + K_p(\gamma_1 h_1 + \gamma_2 h_2)$$

NOTE In case of abutments, in each case, the active earth pressure effects from the backfill above MSL shall be considered separately as if an additional external force. Its force-effect is already considered in the value of H_R, defined earlier. Also note that the backfill soil should always be non-cohesive.

- ***Case 4***

Abutment in cohesive soil, with depth of tension crack $d_c \geq H$ (Fig. 13.5).

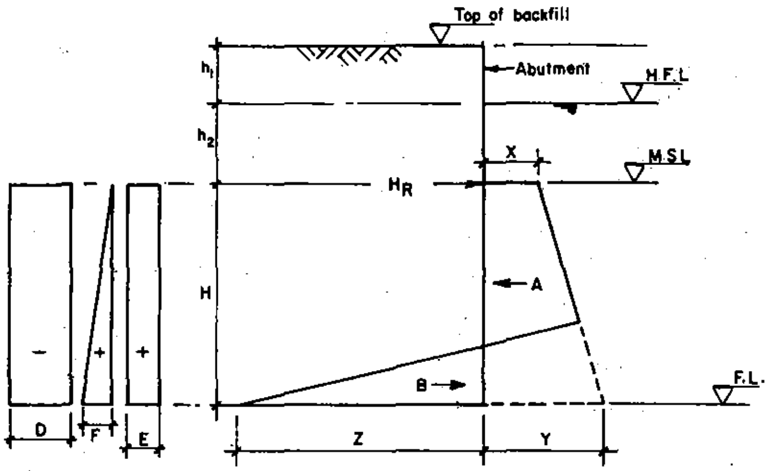


Fig. 13.5

$D = 2c\sqrt{K_a}$, $F = K_a\gamma H$, $E = Ka(\gamma_1 h_1 + \gamma_2 h_2)$

This means $D \geq (F + E)$, i.e.,

$2C\sqrt{K_a} \geq \{K_a\gamma H + K_a(\gamma_1 h_1 + \gamma_2 h_2)\}$

In such a case,

P_a ordinate below MSL is zero so that $(P_p - P_a)$ ordinate is only the P_p ordinate at any depth.

$$X = 2c\sqrt{K_p}$$
$$Y = X + K_p\gamma H$$

$$Z = Y + K_p(\gamma_1 h_1 + \gamma_2 h_2)$$

- ***Case 5***

Abutment in cohesive soil, with depth of tension crack $d_c < H$ (Fig. 13.6).

In such a case d_c the depth of tension crack is estimated from

$$2c\sqrt{K_a} \equiv (K_a \gamma d_c + K_a(\gamma_1 h_1 + \gamma_2 h_2))$$

giving $$d_c = \frac{2c}{\gamma\sqrt{K_a}} - \left\{\frac{(\gamma_1 h_1 + \gamma_2 h_2)}{\gamma}\right\}$$

and also that, $D > E$ but $< (E + F)$,

i.e., $2c\sqrt{K_a} > K_a(\gamma_1 h_1 + \gamma_2 h_2)$

but $< (K_a(\gamma_1 h_1 + \gamma_2 h_2) + K_a \gamma H)$

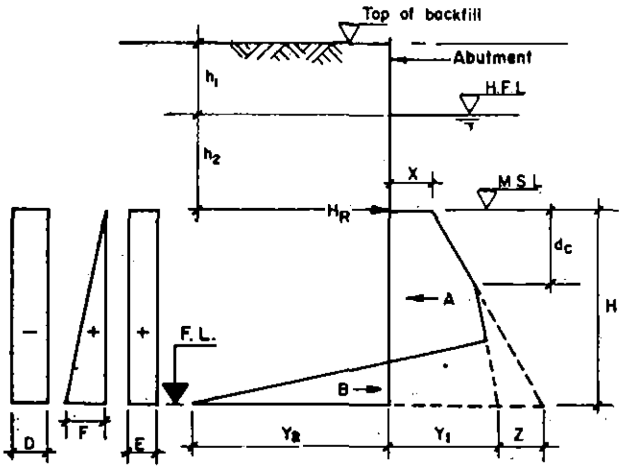


Fig. 13.6

$$D = 2c\sqrt{K_a}, \quad E = K_a(\gamma_1 h_1 + \gamma_2 h_2), \quad F = K_a \gamma H$$
$$X = 2c\sqrt{K_p}$$
$$Y_1 = X + K_p \gamma H - Z$$
$$Z = K_a(\gamma_1 h_1 + \gamma_2 h_2) + K_a \gamma H - 2c\sqrt{K_a}$$
$$Y_2 = K_p(\gamma_1 h_1 + \gamma_2 h_2) + 2c\sqrt{K_p} + K_p \gamma H - \underbrace{(K_a \gamma H - 2c\sqrt{K_a})}_{\text{ignore if } -\text{ve}}$$

- ***Case 6***

Abutment in cohesive soil, with no tension crack (Fig. 13.7).

This means $D < E$, i.e., $2c\sqrt{K_a} < K_a(\gamma_1 h_1 + \gamma_2 h_2)$, so that net P_a is a trapezium from the back.

$$D = 2c\sqrt{K_a}, \quad E = K_a(\gamma_1 h_1 + \gamma_2 h_2), \quad F = K_a \gamma H$$
$$X = 2c(\sqrt{K_p} + \sqrt{K_a}) - K_a(\gamma_1 h_1 + \gamma_2 h_2)$$
$$Y = X + K_p \gamma H - K_a \gamma H$$
$$Z = K_p(\gamma_1 h_1 + \gamma_2 h_2) + K_p \gamma H + 2c\sqrt{K_p} - \underbrace{(K_a \gamma H - 2c\sqrt{K_a})}_{\text{ignore if } -\text{ve}}$$

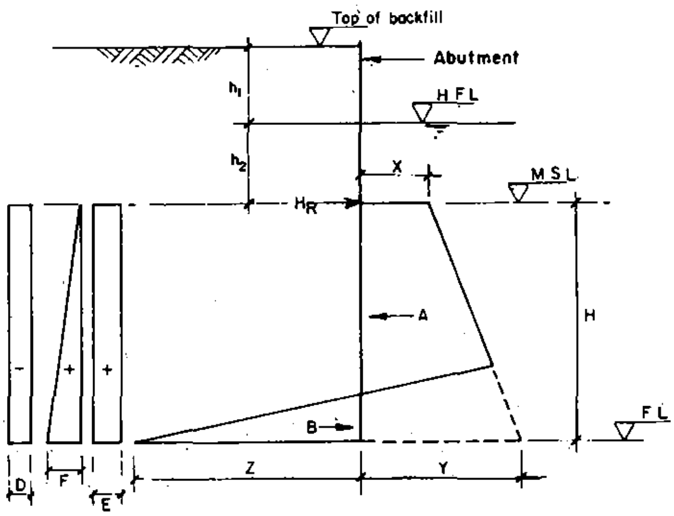


Fig. 13.7

13.2 CASE B

Bulkhead rests on rock* and therefore, it only tends to rotate about its base.

Assumptions

(i) Bulkhead tends to rotate about its base.

(ii) Soil may be non-cohesive or cohesive, but the backfill (behind the abutment) above the MSL is always non-cohesive.

(iii) Consider the passive and active earth pressure diagrams from opposite sides, with their maximum ordinates at base.

(iv) Since actual earth pressure distributions may well be slightly different, therefore, instead of attempting a statical equilibrium between the external resultant horizontal force H_R at MSL and the net passive-less-active earth pressure force, directly a dependable

* It should however be noted that this case B is applied sometimes even when the bulkhead is resting on soil (instead of rock), although not recommended.

value of net passive-less-active effects is assumed by dividing the net value by a factor of 3.0 in case of cohesive soils and 2.0 in case of non-cohesive soils under normal conditions and by 2.4 and 1.6, respectively, under seismic or wind or any other temporary condition.

(v) Ignore vertical components of passive and active earth pressure forces, as also the skin friction.

(vi) Earth pressure on the portion of the structure above the MSL on account of the backfill earth mass above that level shall be considered separately (as another external force) but its surcharge effect shall be included in the present calculations. (See definition of H_R earlier under 'Symbols'.)

Calculation of relief offered by the net passive-less-active earth pressure of soil, within the effective soil-grip around the bulkhead, is computed on the assumption that the bulkhead is a rigid body that derives its stability from the net earth pressure-distribution shown diagrammatically in Figs 13.8 to 13.11 ahead for the four individual cases (Cases 7 to 10), and its dependable value is estimated by dividing by the appropriate factor mentioned in (iv) above. Where this dependable moment is higher than the external moment at base level, revised value of the factor is computed by dividing the $P_p - P_a$ moment by the external moment, and this revised (higher) factor is used for subsequent calculations, and then the resultant moment transferred to base still is zero.

• ***Case 7***

Pier in non-cohesive soil (Fig. 13.8).

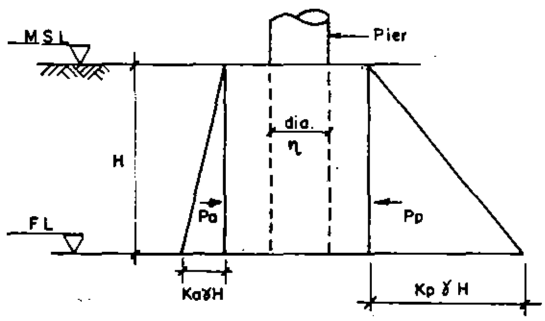


Fig. 13.8

•$M_{\text{dependable}}$ at base = dependable value of net passive-less-active earth pressure moment at base

$$= \frac{1}{f}\left\{\frac{\eta}{6}\gamma H^3(K_p - K_a)\right\}$$

• Factor f as per (iv) above.

and η = diameter (or width) of the bulkhead.

• maximum bending moment section and value of this moment can be found as explained in case 1.

• ***Case 8***

Pier in cohesive soil (Fig. 13.9).

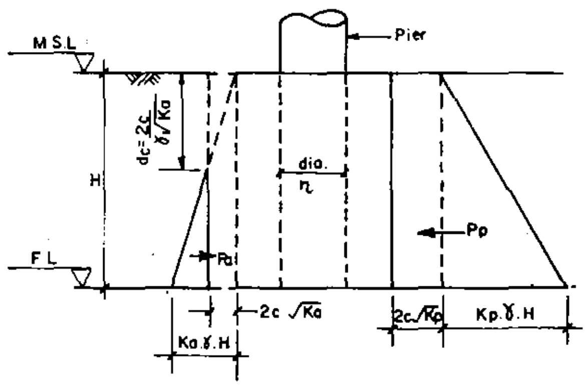


Fig. 13.9

• *Subcase (i)*

If depth of tension crack $\geq H$, i.e.

$$d_c \left(= \frac{2c}{\gamma\sqrt{K_a}}\right) \geq H,$$

then, $M_{\text{dependable}}$ at base

$$= \frac{1}{f}\left\{\frac{1}{6}K_p\gamma H^3 + c\sqrt{K_p}H^2\right\}\eta$$

• Factor 'f' as per (iv) above.

• *Subcase (ii)*

If $d_c \left(= \dfrac{2c}{\gamma\sqrt{K_a}}\right)$ is $< H$,

then, $M_{\text{dependable}}$ at base (approx.)

$$= \frac{1}{f}\left\{\frac{1}{6}\gamma H^3(K_p - K_a) + cH^2(\sqrt{K_p} + \sqrt{K_a})\right\}\eta$$

• Factor f as per (iv) above.

• ***Case 9***

Abutment in non-cohesive soil (Fig. 13.10).

• $M_{\text{dependable}}$ at base

$$= \frac{1}{f}\left\{\frac{1}{6}\gamma H^3(K_p - K_a) - p_o\frac{H^2}{2}\right\}\eta$$

• Factor f from (iv) above.

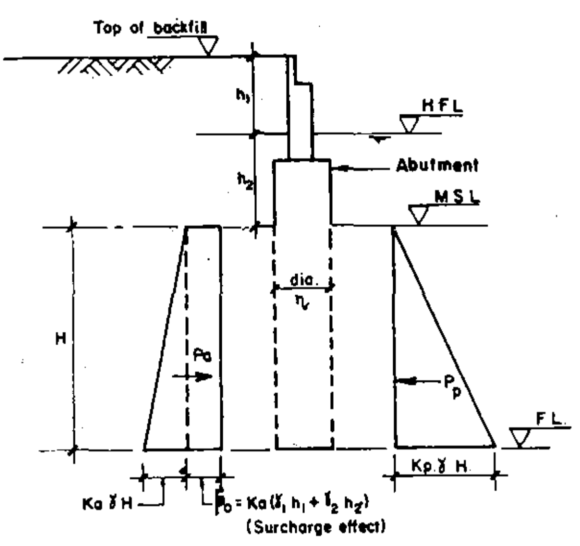


Fig. 13.10

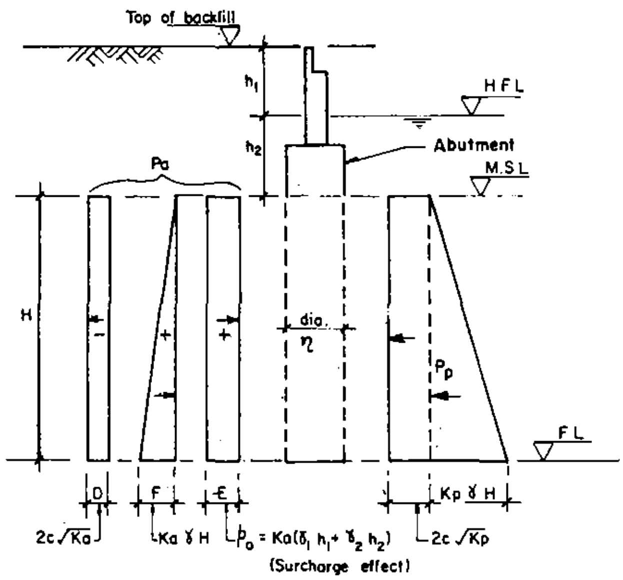


Fig. 13.11

- ***Case 10***

Abutment in cohesive soil (Fig. 13.11).
Tension crack extends down to a depth d_c where net $P_a \equiv 0$, i.e., where

$$E + F = D$$

i.e. $K_a(\gamma_1 h_1 + \gamma_2 h_2) + K_a \gamma d_c \equiv 2c\sqrt{K_a}$

giving $d_c = \dfrac{2c}{\gamma\sqrt{K_a}} - \left\{\dfrac{(\gamma_1 h_1 + \gamma_2 h_2)}{\gamma}\right\}$

- *Subcase (i)*

If $d_c \geq H$, i.e., if $D \geq E + F$,
then P_a is zero, so that

$$M_{\text{dependable}} \text{ at base } = \frac{1}{f}\left\{\frac{1}{6}K_p \gamma H^3 + c\sqrt{K_p}H^2\right\}\eta$$

- *Subcase (ii)*

If $d_c < H$, i.e., $D > E$ but $< E + F$, then P_a exists below d_c, so that,
$M_{\text{dependable}}$ at base (approx.)

$$= \frac{1}{f}\left\{\frac{1}{6}\gamma H^3(K_p - K_a) + cH^2(\sqrt{K_p} + \sqrt{K_a}) - p_o\frac{H^2}{2}\right\}\eta$$

- Factor 'f' from (iv) above.

- *Subcase (iii)*

If no tension crack, i.e., $D < E$, then net P_a is trapezoidal, so that $M_{\text{dependable}}$ at base

$$= \frac{1}{f}\left\{\frac{1}{6}\gamma H^3(K_p - K_a) + cH^2(\sqrt{K_p} + \sqrt{K_a}) - p_o\frac{H^2}{2}\right\}\eta$$

- Factor f from (iv) above.

13.3 ACTIVE EARTH-PRESSURE ON ABUTMENT OR ON RETAINING-WALL, FOUNDED ON FOOTING OR PILES

For active earth pressure on abutments on footings or on small diameter piles (diameter generally less than 1.2 m*) Coulomb's theory may be adopted subject to the modification that the centroid of the pressure exerted by the backfill, when considered dry, is located at an elevation of 0.42 of the height of the abutment above the soffit of footing/pile cap instead of at 0.333 of that height. In this dry condition this modification nearly approximates to the actual state that is between the earth pressure at 'rest' and the limiting 'active' pressure.

The structure shall, however, be designed to withstand a horizontal pressure not less than that exerted by a fluid weighing 480 kg/m^3.

* For diameters bigger than about 1.20 m the pile shaft may begin to act more like an earth-pressure-attracting-bulkhead, in which case that might cause flexing of the pile system, thereby altering the state of earth pressure acting on the structure above the piles.

CHAPTER 14

Evaluation of Base-Pressure and Contact-area under Foundations Subjected to Direct Load and Any-Axis Bending

A base, subjected to a direct load Q and moments M_L and M_B about two of its orthogonal axes L–L and B–B, suffers variable base-pressures. The pressure distribution is assumed linear in regular practical design. If there is no uplifting tensile base-pressure, then full base-area remains in contact with foundation-strata underlying the foundation; otherwise, if part base-pressure is of uplifting type and no balancing is done against this tendency, the base-pressures will redistribute on the 'net' contact area (i.e., on the portion of the full base area which still remains in contact with substrata) and laborious calculations may be involved in order to estimate first the net contact area and then the redistributed maximum base pressure on it.

Cases of rectangular (or square) and circular in plan bases have been worked out and results, as an aid to quicker analysis, are presented below. Areas shown shaded in the corresponding figures represent the *kern limits*, so that if the resultant eccentricity of the load Q lies within it, there is no uplift, and the full base-area remains in contact with the substrata.

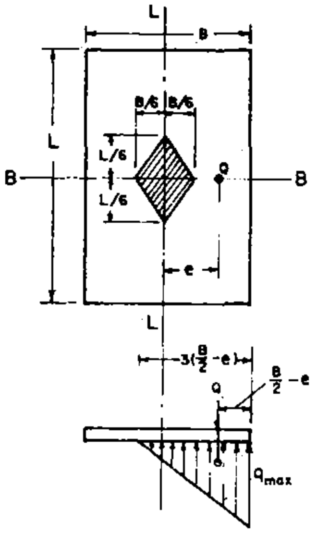
The figures and working details are self-explanatory as are the symbols. However, it may be noted that L–L axis is parallel to dimension L, B–B axis is parallel to dimension B, M_L represents moment about L–L axis and M_B represents moment about B–B axis. Accordingly,

$$e_l = \frac{M_B}{Q} \quad \text{and} \quad e_b = \frac{M_L}{Q}$$

where e_l and e_b represent the eccentricities along L–L and B–B axes if the load Q itself were placed eccentrically to cause the applied moments M_B and M_L, respectively.

Case of Rectangular (or Square) footing with moment applied about only one axis (see Fig. 14.1)

Load Q applied eccentric along B–B axis or footing subjected to M_L (moment about L–L axis). (Same procedure is used when Q applied eccentric along L–L axis or footing subjected to M_B. Only L replaces B in the formulae.)



A = B. L.

For $e \le \frac{B}{6}$, $q = \frac{Q}{A}\left[1 \pm 6\frac{e}{B}\right]$

For $e > \frac{B}{6}$, $q_{max} = \frac{Q}{A}\left[\frac{4B}{3B-6e}\right]$

$q_{min} = 0$ at a distance of $3\left(\frac{B}{2} - e\right)$ from compressed edge of footing

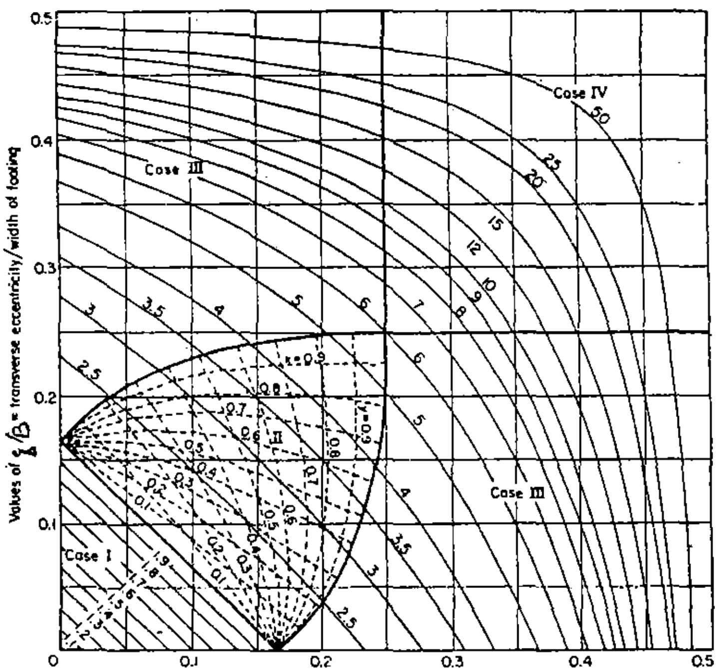
$\frac{B}{2} - e$ $\quad e = \frac{M_L}{Q}$, i.e., $e = e_b$

Fig. 14.1 *Rectangular footing, load on one of the center lines of footing*

Case of Rectangular (or Square) Footing with Moments Applied about both its Orthogonal Axes (refer to Figs. 14.2 and 14.3)

Case of a Circular Footing with Any-Axis Moment (Vectorial Resultant Moment) (Fig. 14.4)

In case $e > \frac{r}{4}$, so that there is uplifting base pressure (and consequently only part of the full circle in plan is in contact with the substrata) then the net base-area in contact with substrata, a sector, and its properties (e.g., area, position of its centroid, and its second moment of area about an axis parallel to the resultant bending axis and passing through its centroid) can be quickly evaluated from the data given below in Fig. 14.5.



Values of e_l/L = *Longitudinal eccentricity/length of footing* ⟶
Solid curves give values of K
Maximum pressure $P_{max} = K \times Q/BL$
Q = concentrated load on footing

Fig. 14.2

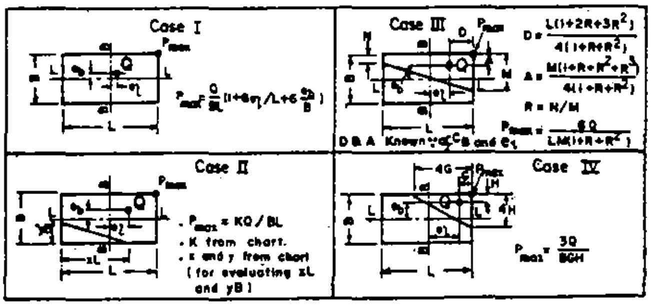
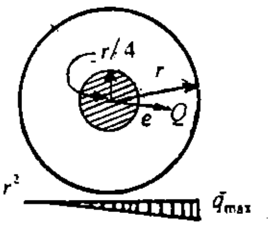


Fig. 14.3 *Rectangular footing, double eccentricity*

For $e \leq r/4, q = \frac{Q}{A}\,[1 \pm 4\,e/r\,]$
For $e > r/4, q_{max} = k\,\frac{Q}{A}$

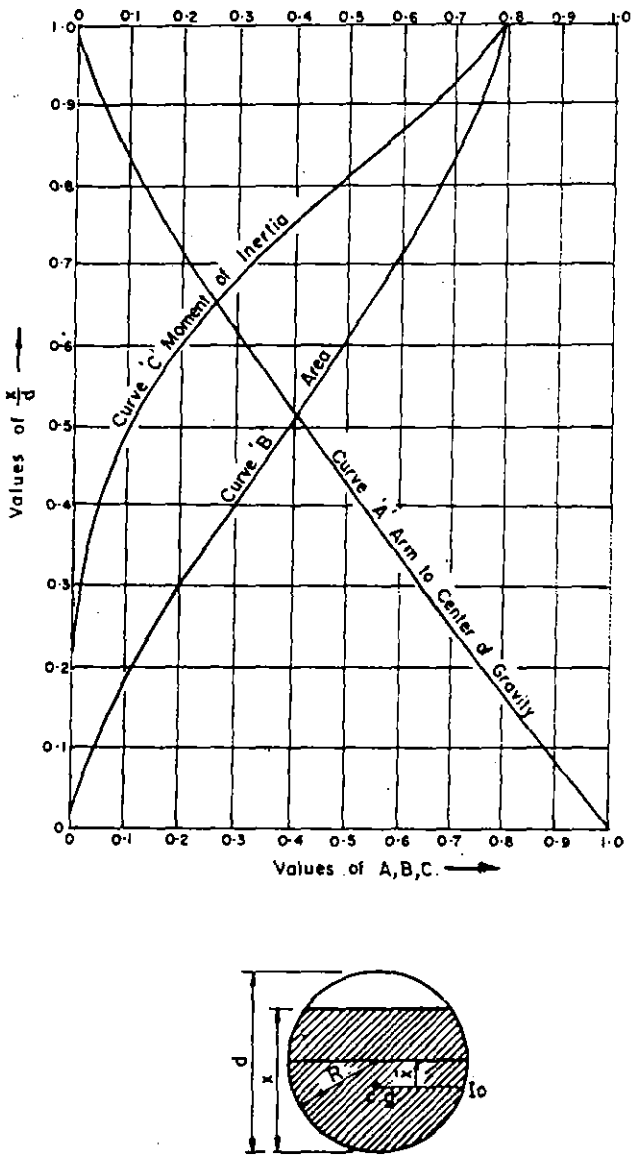


$A = \pi r^2$

k values are tabulated below.

e/r =	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.90
k =	2.00	2.20	2.43	2.70	3.10	3.55	4.22	4.92	5.90	7.20	9.20	13.0	80.0

Fig. 14.4 *Circular footing*



PROPERTIES OF SHADED SEGMENT

$\bar{x} = AR$

AREA = Bd^2 VALUES OF A, B, C TAKEN

$I_0 = CR^4$ FROM ABOVE CHART

Fig. 14.5

CHAPTER 15

Friction Slab for Stabilising Abutments and Retaining Walls

In principle a friction slab is a mere slab of plain concrete attached to the earth retaining structure on the backfill side by tension rods, and has earth fill on top of it so that any tendency of the structure to move (slide, rotate) would tend to drag this slab along which would immediately generate horizontal frictional force on its top as well as bottom surfaces since the slab is sitting on soil carrying the fill on it. This resisting frictional force would help stabilise the structure against sliding, and even against overturning if the friction slab is at a higher elevation than the foundation base.

15.1 PROCEDURE

Step 1 Decide the elevation at which the friction slab should be attached to the structure. Owing to the tendency of sliding of the earth wedge above the plane of rupture (inclined to the horizontal at $45° + \phi/2$, where ϕ is the angle of repose of the backfill material), the weight on the slab of only that part of soil mass which falls below and beyond this plane of rupture should be considered in evaluating the frictional force generated by the friction slab. Hence higher the elevation of the slab, longer will the slab have to be in order for it to catch the soil mass below and beyond the plane of rupture. Therefore, draw to scale the structure in elevation and the said plane (i.e., line) of rupture behind it, starting this line from the centre of the width of the foundation at the maximum scour level if the soil mass surrounds the structure below that level or from where the back-plane of the main back-wall meets the founding level if the entire soil mass causing the earth pressure is all a backfill and is kept in place by wing walls.

Step 2 Evaluate the weight on the slab of the earth-mass between the plane of rupture and the top of slab, and add to it the self weight of slab. This total load is R.

Step 3 Evaluate the total horizontal resisting frictional force, H, generated by the friction slab.

(i) If the fill above and below the slab is noncohesive then,

$$H = 2(\mu R), \text{ where } \mu = \tan\left(\frac{2}{3}\phi\right)$$

(ii) If the soil below the slab is cohesive but the backfill above the slab is non-cohesive (backfill should be a non-cohesive material), then,

$$H = \mu R' + 0.7\ Ac,$$

where A = plan area of the portion of friction slab below and beyond the plane of rupture

R' = R—weight of slab

c = cohesion value of the cohesive soil

$\mu = \tan\left(\frac{2}{3}\phi\right)$

Even so it is advisable to fill at least a meter depth below the slab with noncohesive soil, and saturate-compact it thoroughly, in 15 cm layers, prior to casting the slab on it.

However, only 70% of the above calculated horizontal resisting frictional force H, acting towards the backfill, may be used as the dependable value, and this be introduced as a stabilising horizontal force in the stability and stress-check calculations of the structure.

Step 4 Design the tension reinforcement (that passes the full length of the slab and is well anchored into the main structure) for the above calculated horizontal force H, allowing in it a tensile stress of only 70% of the normal working stress value. (This stress reduction is advisable owing to a slight chance of long term corrosion of the steel under the circumstances.)

Step 5 Detailing.

(i) Since the slab body may bend and crack in case the soil below it should differentially settle, it is advisable to make the friction slab with built-in partial-depth V-grooves (deliberate partial depth breaks) in the two orthogonal directions, sealed with bitumen filler. This gives the slab a checkered appearance, as these grooves break it into a rectangular grid. The main tension reinforcement goes under the grooves normal to it, and additional local parallel bars are placed at the grooves, extending about 60 bar-diameters on either side of a groove. These bars should be provided in level with the main tension bars, one for one, and of same diameter. The main tension reinforcement should be placed at middle of slab thickness.

(ii) As for the secondary reinforcement in the individual checkered panels, placed normal to the main tension reinforcement, under the particular circumstances of embedment, it is almost unnecessary. However, it may be provided at the rate of 25 kg/m^2 of plan area of the panel, in one layer, above or below the main tension reinforcement.

(iii) A slab thickness of 15 cm, cast with concrete of 250 kg/cm^2 standard 28-day cube crushing strength, is ample for the slab. However, the concrete should be thoroughly vibrated and should be dense. The aforementioned partial depth V-grooves may be 6 mm wide and 3 cm deep, from top downwards and from bottom upwards, sealed with bitumen filler/sheet.

CHAPTER 16

Reinforced Earth Structures

Reinforced earth 'wall' is a composite engineering 'mass' consisting of compacted soil, horizontal layers of reinforcement, and a form of facing to prevent erosion of the soil.

The increased use of this material is primarily due to its versatility, cost effectiveness and ease of construction.

Reinforced earth is a relatively new civil engineering material which has been used commercially for the past twenty years or so. Its main use has been in the construction of earth-retaining structures and bridge abutments. But now it is adopted into the field of foundation stabilisation, and its possible use in the future might even include the strengthening of cuttings.

The *external stability* of a reinforced soil mass (called wall) is easily investigated by assuming that it behaves as a rigid gravity mass-structure and conforms to the simple laws of statics. The analysis of *internal stability* is essentially one of designing the reinforcement against tension failure and ensuring that it has a sufficient anchorage length into the stable soil mass.

In 1966 Henri Vidal, a French architect, published a paper entitled *Terre-Armee*. This heralded an intensive programme of research, development and construction, which changed the image of reinforced earth from that of an engineering novelty to a construction technique of major significance. As the name implies, reinforced earth is a composite material which combines soil and strong (comparatively inextensible) tensile reinforcement to produce a mass having superior properties to soil alone. At its inception it was conceived as a construction material having a versatility on par with reinforced concrete, with Vidal illustrating the potentital use of reinforced earth over a wide spectrum of structures including beams, arches, tunnels and dams, as well as various forms of earth-retaining structures.

A simple form of reinforced earth wall is illustrated in Fig. 16.1. Brief descriptions of components listed in the figure are set in reference 1 given at the end of this chapter, from which material has been taken with grateful thanks.

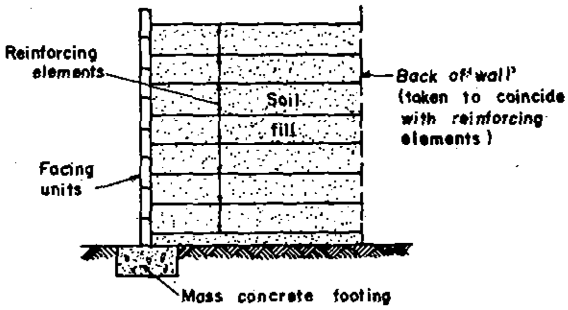


Fig. 16.1 *Reinforced earth retaining wall*

Soil Fill

Three types of soil fill are available.

Frictional fill Most practical reinforced earth structures that have been built have employed frictional or granular soil. The reasons are not far to seek as granular soil not only has good frictional resistance but is also free-draining and generally less corrosive than cohesive soil.

Cohesive frictional soil The UK department of transport memorandum (1978) permits the use of silty soils as reinforced earth-fill, subject to certain conditions listed in it.

Cohesive soil Research is being carried out into the possibility of using clay as a fill for reinforced earth. As clay is probably the most common soil, encouraging results from such research would be of interest, but any potential benefits arising from the local availability of such soil could be outweighed by the penalties that might arise, such as difficulty in handling, development of pore water pressures and a greater risk of corrosion.

Reinforcing Elements

Most of the reinforced earth structures that have been built so far have employed metallic reinforcing elements, the most common being galvanised steel. Each element is a thin strip of metal, typically 50–100 mm wide and up to 9 mm thick and several metres in length.

Other metals from which reinforcement strips have been prepared include stainless steel, aluminium, aluminium alloy and copper. Reinforcing elements made from reinforced concrete in the form of thin prestressed planks have been used occasionally.

Plastic materials show promise for the future, although more research is necessary. Two main types are presently available;

- *fibre reinforced plastic*, consisting of glass filaments *embedded in polyester resin.*
- *paraweb*, polyester filaments embedded in polyethylene.

Facing Units

At a free boundary of a reinforced earth structure it is necessary to provide some form of barrier so that the soil is contained. This 'skin' can be either flexible or stiff, but it must be strong enough to hold back the local soil and allow fastenings for the tension reinforcement to be attached.

The facing of a reinforced earth structure is usually prefabricated from units which are small and light enough to be manhandled for quick and easy construction. The units are generally made from steel, aluminium, reinforced concrete or plastic. Reinforced concrete is more common.

The facing units require a small foundation from which they can be built, generally consisting of a trench filled with mass concrete, giving a footing similar to those used in domestic housing.

16.1 PRINCIPLE OF REINFORCED EARTH

Consider a semi-infinite mass of cohesionless soil at rest. If the surface of the soil is horizontal then, at depth h below the surface,

$$\text{Vertical Stress} = \gamma h$$
$$\text{Lateral Stress} = K_0 \gamma h$$

where K_0 = coefficient of earth pressure at rest,
γ = unit weight of the soil.

According to Jaky (1944), for both normally consolidated clays and compacted soils, $K_0 \simeq (1 - \sin\phi)$ where ϕ = the angle of internal friction of the soil.

If the soil is allowed to expand laterally, the horizontal stress, $K_0\gamma h$, reduces to a limiting (or failure) value, $K_a \gamma h$, where K_a = coefficient of active earth pressure.

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2(45^\circ - \phi/2)$$
(ignoring the angle of wall friction)

If the soil is compressed laterally, the horizontal stress increases to the limiting value, $K_p \gamma h$,

where K_p = coefficient of passive earth pressure
$$= \frac{1 + \sin\phi}{1 - \sin\phi} = \tan^2(45^\circ + \phi/2)$$
(ignoring the angle of wall friction)

Now reconsider the soil mass with horizontal reinforcement strips embedded within it (Fig. 16.2).

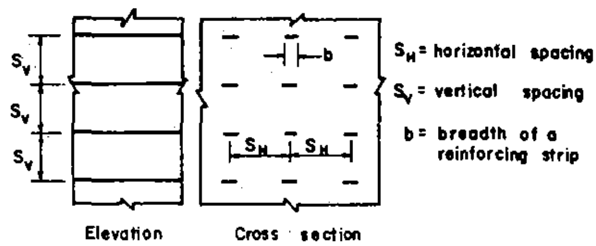


Fig. 16.2 *Arrangement of reinforcement strips*

Consider a soil layer between two reinforcement strips one above other. If enough friction is developed, the top and bottom of the layer will be attached to the reinforcements. If the strips are close enough then the whole soil layer will be more or less constrained and the maximum strain that it can experience in the direction of the reinforcements will be of the order of the strain in the reinforcements.

The types of reinforcement material available are discussed later but all have a Young's modulus much greater than that for the soil so that the resulting strains in the soil will be so small that the soil is essentially at rest and the lateral pressure within it can be assumed to be equal to $K_0\gamma h$. This fundamental idea of reinforced earth, was developed by Vidal in 1966. Reinforced earth, therefore, is a combination of soil which is weak in tension, and reinforcing elements which can carry the tensile forces transmitted from the soil. The composite material is strong in vertical compression and has tensile strength in the direction of the reinforcements. In this respect it is somewhat analogous to reinforced concrete.

It must be remembered that the tensile strength in reinforced earth is directional. If the soil is compressed laterally (instead of vertically), the horizontal reinforcement would have no effect.

The Rankine theory considers the equilibrium of an element in the soil mass and deduces that a network of potential failure planes at $(\phi/2 + 45^\circ)$ to the horizontal, exist behind the wall [Fig. 16.3(a)], for a wall supporting dry sand with a horizontal surface.

The Coulomb assumption considers the whole of the soil mass retained and assumes that failure will occur by a wedge of soil sliding down a failure-surface. For practical purposes the failure surface can be considered as a straight line and, for sand behind a wall, is inclined at $(\phi/2 + 45^\circ)$ to the horizontal [Fig. 16.3(b)].

The active pressure distribution behind a retaining wall approximates to either the Rankine or the Coulomb condition, depending upon the amount and type of yield

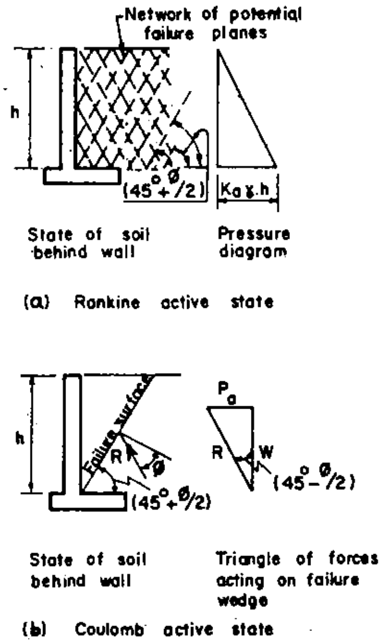


Fig. 16.3 *Rankine and Coulomb active states*

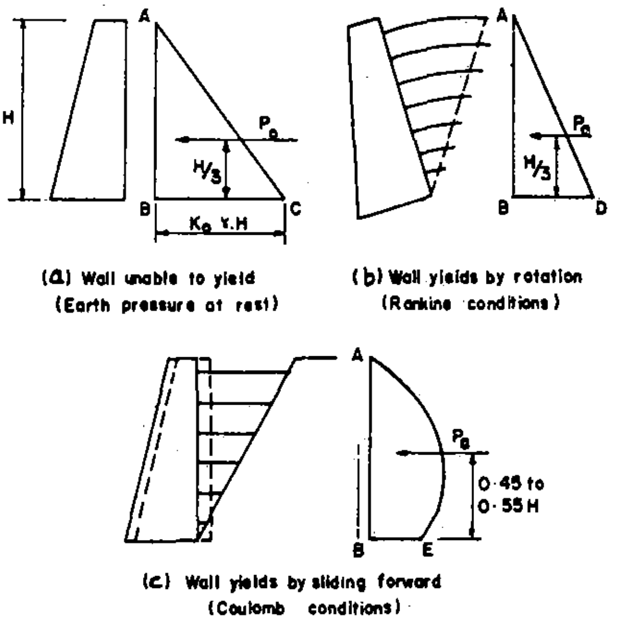


Fig. 16.4 *Influence of wall yield on pressure distributions behind a retaining wall*

that the wall has experienced.[1]

A wall can yield in one of the two ways—either by rotation about its lower edge or by sliding forward. Provided it yields sufficiently, a state of active pressure is reached and the total thrust on the back of the wall is P_a. The distribution that gives this P_a value can be very different and depends upon the way in which the wall has yielded.

Consider first a wall that is unable to yield [Fig. 16.4(a)]. The soil is at rest and pressure distribution is represented by line AC.

Consider now that the wall fails by rotation about its lower edge until the total active thrust is P_a [Fig. 16.4(b)]. This results in conditions approximating to the Rankine theory and is known as the totally active case (K_a applies instead of K_0).

Suppose, however, that the wall yields by sliding forward until active thrust conditions are achieved and the total thrust again equals P_a. A forward displacement hardly disturbs the upper layers of soil so that the top of the pressure diagram is similar to the earth pressure at rest diagram. As the total thrust on the wall is the same as for rotational yield, the pressure distribution must be similar to $AEBA$ in Fig. 16.4 (c). These conditions correspond to the Coulomb theory.

If we consider a frictional fill, angle of friction ϕ, unit weight γ, and assume a frictionless wall of height H, then both theories provide the same value for the total active thrust P_a, provided that the surface of the fill is horizontal and parallel with the back of the wall.

$$\text{Rankine } K_a = \tan^2\left(45° - \frac{\phi}{2}\right)$$

$$P_a = \text{area of pressure distribution diagram}$$

$$= \frac{1}{2}K_a\gamma H^2 = \frac{1}{2}\gamma H^2 \tan^2\left(45° - \frac{\phi}{2}\right)$$

Coulomb Area of the sliding triangular wedge

$$= \frac{1}{2}H^2 \tan\left(45° - \frac{\phi}{2}\right)$$

Weight of sliding wedge, W

$$= \frac{1}{2}\gamma H^2 \tan\left(45° - \frac{\phi}{2}\right)$$

$$\text{From triangle of forces, } P_a = W\tan\left(45° - \frac{\phi}{2}\right)$$

$$= \frac{1}{2}\gamma H^2 \tan^2\left(45° - \frac{\phi}{2}\right)$$

Tension Forces in Reinforcement Strips

Whether or not the theories of Rankine and Coulomb should be used in an unmodified form in the analysis of reinforced earth has been argued amongst researchers for several years. However, at this stage of development, there is little choice for designers but to use the Rankine and Coulomb assumptions. It should be remembered that thousands of reinforced earth structures designed with these theories have been constructed and have proved satisfactory. (It should be noted that there are other approaches, a summary of which has been prepared by Symons). In Fig. 16.5: $T = 1/2k_a\gamma H^2$ per m length of wall, where T = total of tensions in all the n tension strips holding the wall.

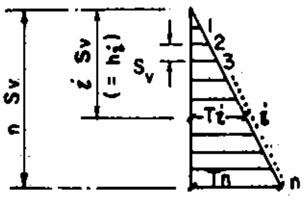


Fig. 16.5

This is the resultant of all the forces in the strips and must be distributed amongst them. This is achieved by assuming a triangular distribution, the forces increasing with depth, Fig. 16.5.

Consider strip number i counting down from top, we have from Fig. 16.5

$$\frac{T_i}{iSv} = \frac{T_n}{nSv}$$

$$\therefore \quad T_i = \frac{T_n}{n} i$$

Now, $T_1 + T_2 + T_3 + \ldots + T_{n-1} + T_n = T$

$$\therefore \frac{T_n}{n} + \frac{2T_n}{n} + \frac{3T_n}{n} + \cdots + \frac{(n-1)T_n}{n} + \frac{nT_n}{n} = T$$

$$\therefore \quad T_n(1 + 2 + 3 + \ldots + (n-1) + n) = nT$$

or $\quad T_n \sum n = nT$

Now, $\quad \sum n = \dfrac{n(n+1)}{2}$

$$\therefore \quad T_n = \frac{2nT}{n(n+1)} = \frac{2T}{n+1}$$

$$\therefore \quad T_i = \frac{2Ti}{n(n+1)} = \frac{i}{n(n+1)} K_a \gamma H^2$$

For large n, $T_i = iK_a\gamma(H/n)^2 = iK_a\gamma S_v^2$ (16.1)

It can also be written as,

$$T_i = K_a S_v(\gamma i \cdot S_v)$$
$$= K_a S_v \sigma_v \qquad (16.2)$$

where $\sigma_v = \gamma i S_v$ = vertical stress at level i where depth is iS_v.

Maximum Tension Line

It has been found, from both model tests and measurements on constructed works, that the tensile force in a reinforcement strip varies. It generally has a low (even a zero) value at the facing unit, reaches a maximum value at a short distance from the facing, and then tends towards zero at the unattached end, Fig. 16.6(a).

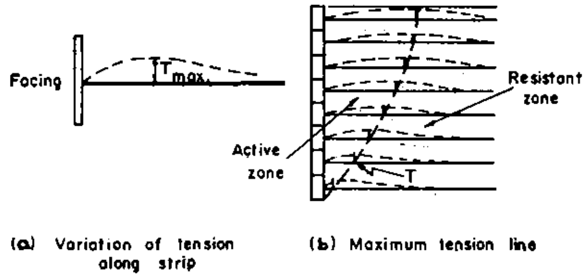


Fig. 16.6 *Maximum tension line*

If the points of maximum tensile force in various strips are joined, the imaginary line so formed is known as the maximum tension line, Fig. 16.6(b). It extends in a curve from the facing at the base of the wall and cuts the surface of the fill at some distance back from the facing. It is assumed that this line divides the reinforced fill into two zones, one on either side of it.

- *Active zone* in which the shearing stresses from the soil, on to the strips, act towards the facing, i.e., there is a tendency for the reinforcement to be pulled forward towards the facing.
- *Resistant zone* in which the shear stresses act away from the facing and tend to hold the reinforcement in the soil.

T_i as obtained from Eq. (16.1) is assumed to be the maximum tensile force in the strip, and therefore, occurs at some distance back from the facing.

Failure of a Reinforcing Strip

The tensile force in a reinforcing strip will tend to cause failure in one of the following two ways:

(i) Tensile failure—snapping of the strip.
(ii) Bond failure—slipping of the strip from within its surrounding soil.

Tensile Failure

The ultimate resistance of a reinforcing element to an axial tensile stress is equal to the ultimate axial tensile stress that the material can withstand ($p_{ult.}$) times the cross-section area of the strip,

$$R_t = p_{ult.} \times b \times t$$

where b = width of reinforcing element
t = thickness of reinforcing element.

Dividing p_{ult} by a suitable factor of safety gives the permissible axial tensile stress p_{at}.
Hence, when considering tensile failure effects, $T_i \ngtr p_{at}bt$.

Typical p_{at} values are,

	N/mm^2
Aluminium	120
Galvanised mild steel	120–190
Copper alloy	170
Stainless steel	120–220

Bond Failure

Just as a reinforcing bar in concrete requires a certain minimum bond length, so also a reinforcing element in reinforced earth must be long enough to prevent its slipping and pull out.

For a reinforcing element, the bond resistance, between it and the soil around it will be provided by one of the following three means, depending upon the type of soil fill,

(i) Frictional fill — by frictional resistance
(ii) Cohesive fill — by adhesion
(iii) Cohesive-frictional fill — both by frictional resistance and adhesion

(i) *Frictional resistance* Consider a strip at depth h_i and assume that the coefficient of friction between the soil and the reinforcing elements is μ.

Normal stress acting on strip = vertical stress = γh_i

∴ Normal force acting on strip = normal stress × area

$$= \gamma h_i bL$$

∴ Total frictional resistance available from strip

$$= 2\gamma h_i bL\mu$$

NOTE Just as the total peripheral area of a reinforcing bar in concrete is considered to provide bond resistance, so also the total area of the tension strip, i.e., its upper and lower surfaces are considered to provide bond resistance.

(ii) *Cohesion* Total resistance available from cohesion in a cohesive soil = $2cbL$.
where c = unit cohesion available along the length of the reinforcing element

NOTE μ and c values are obtained from shear box tests but μ is typically approximately equal to $\frac{1}{2}\tan\phi$ (some authorities take it as $\tan\left(\frac{2}{3}\phi\right)$, and c will lie somewhere between $\frac{1}{2}c_u$ and $1.0\, c_u$ (c_u = ultimate value of cohesion).

(iii) *Friction and adhesion* Generally for a $c-\phi$ fill: Bond resistance of a reinforcing element = $2(\gamma h_i \mu + c)bL$.

16.2 CURRENT DESIGN AND CONSTRUCTION SYSTEMS

Three proprietary systems are currently used in the UK. These are the reinforced earth system (due to Vidal), the DoE or York system (due to Jones), and the Websol system (due to Price). The former system is used universally, with over 3500 structures having been erected by Vidal's licence. In comparison, the latter systems have to date, been used largely mainly in the UK where approximately 200 structures have been built. Both the Vidal and the York systems have been bolstered by extensive research and development programmes culminating in the issue of formal design directives'. The basic requirements are the same in all three systems with the need for a facing unit to prevent surface erosion, a series of reinforcing strips or sheets, and a suitable backfill. Additionally, it is necessary to incorporate a mechanism which permits the fill and associated reinforcing strips to settle without inducing unacceptable stresses in the facing units and the reinforcing strips. With the advent of the UK department of transport technical memorandum, a number of ad hoc systems have appeared. However, these tend to lack the finesse of the proprietary systems.

Vidal developed two systems for constructing reinforced earth walls. The first comprises semi-elliptical cross-section facing units, typically 250 mm high, which have a locating slot formed along the bottom edge. Reinforcing strips are connected to the units by bolts passing through the strip and the interlocking edges of the facing units. The standard units are straight, measure up to 10 m long and weigh 115 kg. Shorter units and specials are supplied to form corners. Mild steel and galvanised mild steel are standard construction materials, these being typically 1.5 mm to 3.0 mm thick. These thicknesses are consistent with a vertical unit stiffness that allows flexure under vertical load. If the backfill and reinforcing strips suffer internal settlement, this vertical movement is reflected in the facing units which compress like a bellows, so obviating high stresses at the reinforcement connections that would otherwise be induced by differential settlement between the fill and the facing units.

The metal facing unit has now been largely superseded by a more substantial precast concrete unit which is cruciform shaped in front elevation. Standard units weigh approximately one tonne and are 1.5 m by 1.5 m with a total thickness of 180 mm. All edges of the unit are rebated to obviate any straight through joints, with the rebates doubling as guide rails to facilitate alignment of the units during construction. A further aid to alignment is in the form of a dowel bar extending from the upper and lower edge of one arm of the cruciform. These dowels are also used as pivot points for the construction of curved walls. Each unit is furnished with four steel lugs, cast *in situ* during manufacture. These lugs, which are usually at 1 m horizontal and 0.75 m vertical centres, are drilled to take the reinforcing strip connecting bolts. During construction a strip of compressible filler, such as cork board, is laid on the back edge of the horizontal joints before the next unit is placed. Frequent use is made of temporary wedges to form an open joint and aid vertical alignment. These construction techniques allow the facing unit to compress vertically in sympathy with any internal settlement of the fill.

The reinforcing strips are almost exclusively metal, usually galvanised steel. Unit 1975 plain strips, 60 mm or 80 mm wide and 3 mm thick were in common use. These were subsequently superseded by ribbed strips, 40 mm or 60 mm wide and 5 mm thick. In extremely corrosive fill environments, stainless steel may be used. The effect and rates of corrosion are still not totally predictable, however, recent research work suggests that the 5 mm thick galvanised steel strips offer a service life in excess of 100 years in all but the most aggressive environments, Darbin Etal (1978). Suitable fill material is generally of a granular nature with a limit of no more than 15% finer than 80 microns. The maximum particle size is restricted to 350 mm with no more than 25% of the fill being coarser than 150 mm, Long (1977).

The current design methods adopted by the Reinforced Earth Company have been set out in the papers presented at the 1978 Sydney conference by both Schlosser and Mckittrick.

The York System, which was largely developed by Jones, initially used facing units made of glass reinforced cement, providing a very light weight of 18 kg per unit. The units take the form of a hexagon-based pyramid 225 mm deep and 600 mm across the flats. One pair of diametrically opposite flanges on each unit is provided with a pair of large diameter holes which allow the units to be threaded onto vertical guide poles. These poles which serve as face reinforcement, are made up of short lengths of 35 mm diameter PVC tubing with spigot and socket connections. In the finished structure the poles are reinforced with mild steel bars, grouted *in situ* to make them rigid. The earth reinforcement, in the form of strips or plastic grids, is attached to the vertical pole reinforcement at any required vertical spacing. When any settlement occurs in the fill containing the reinforcement, the reinforcement simply slides down the vertical pole obviating any settlement-induced stresses at these connections.

Recent structures using the York sliding system have used full height facing units formed from double T bridge or flooring beams, standing on edge. The use of the vertical plastic poles can be retained; alternatively a short slot or pin is used to provide a critical movement connection for the reinforcement.

All aspects of the design and specifications for the component parts of reinforced earth walls are clearly set out in the UK department of transport document—Technical Memorandum (Bridges) BE3/78. Permitted reinforcing materials include aluminium alloy, galvanised carbon steel, copper and proprietary material awarded the UK Agreement Board Certificate. Two reinforcing strips falling into this latter category are *Fibretain* (a glass-fibre reinforced plastic) and *Paraweb* (a linear composite of Terylene fibre cores in an alkathene sheath). Allowance made for corrosion of the metallic reinforcement during the specified 120 year design life is dependent on the class of backfill used: Table 16.1.

Table 16.1 *Corrosion Allowances (Sacrificial Thickness)*

	Thickness to be allowed for on each surface exposed to corrosion (mm)	
Material	Frictional fill	Cohesive frictional fill
Aluminium alloy	0.15	0.30
Copper	0.15	0.30
Galvanised steel	0.75	1.25
Stainless steel	0.10	0.20

Both frictional and cohesive-frictional fill are limited to a maximum particle size of 125 mm. However, it is specified that frictional fill shall not contain more than 10% passing the 63 micron sieve. Conversely, the so-called cohesive-frictional fill may contain more than 10% finer than this size provided the liquid limit and plasticity index do not exceed 45% and 20% respectively. However, the clay fraction, i.e., 2 microns and finer, is limited to a maximum of 10%. The coefficient of friction between the soil and the reinforcement may either be measured directly using the shear-box or taken from the expression, $\mu = \alpha \tan \phi$, where α is in the range 0.45–0.50. The lengths of the various reinforcements are determined by calculation, however, in no circumstance is the reinforcement length to be less than the greater of 0.8 H or 5 m, H being the wall height.

The Websol *Paraweb* system differs from the Vidal and the York systems described previously in as much as it incorporates non-metallic reinforcement and thus eliminates the problem of corrosion as such. Each layer of reinforcement is a composite comprising a continuous layer of permeable non-woven fabric combined with strips of *Paraweb* which are placed doubled so as to form a loop at the wall end, with the loop secured by a short toggle bar passing through a pair of metal eyes cast into the back of the facing unit. The reinforcing strips are made from high tenacity synthetic fibres encased in a durable polyethylene sheath with the strength of the strip being developed in the fibres whilst the sheath gives the strip its required form, shape and protection. The fabric employed is one of a range of geotextiles having well-defined strengths and durability properties. The system has been approved by the UK Agreement Board and thus should conform to the UK department of transport design specifications which require a 120 years design life. (For author's comments see ahead). The facing panel is of precast concrete, 120 mm thick and is T-shaped in its front elevation. These units are slightly larger than those in the Vidal system with a face area of 3.2 m^2, as opposed to 2.2 m^2. Settlement-induced stresses between the facing panel and the reinforcement are again relieved by the provision of compressible cork packing along the horizontal panel joints.

Although the paraweb tension strips, which are polyester filament protected in polyethylene sheathing, have a certificate from the Agreement Board in UK and have built some reinforced earth structures, the performance of such strips over long periods of time under continuous moist conditions is still questionable, despite their claims to the contrary. Moisture is reported to seep through 1 mm thickness of polyethylene in about 6 months time (Ref: Study of Tergal Webbing at Beaulieu near Poitiers, France, *Terre Armee Report* 1, Feb. 1982). Thus the filament does get moist and tensile strength seems to deteriorate badly when moist. The above report says, "the strength is reduced by about 50% in just 10 years". For this purpose although *sealing* of the ends of the strip is done, apart from the fact that moisture seeps through polyethylene anyway, direct water ingress is possible through pores caused by pitting due to penetration of sharp soil particles during compaction (and the consequent biodegradation). Hence the use of plastics is not entirely unquestionable. Heavily galavanised (1000 gm/m^2) mild steel strips (with increased sacrificial thickness) are preferable, depending on individual merits (e.g., location, subsoil water level, proximity of sea water, atmospheric conditions, etc.).

Design Criteria

Basically the design of reinforced earth type walls and abutments can be done as per method outlined in:

(i) the French Ministry of Transport Recommendations and Rules of the *Art of Reinforced Earth Structures*, (Reference 3).
(ii) the UK Dept. of Transport Technical Memorandum, BE 3/78 (Reference 4).

Two major aspects must be considered when designing a reinforced earth retaining wall:

(i) *Internal Stability* within the reinforced earth monolith,
  (a) Tensile resistance
  (b) Bond resistance
  (c) Pressure on facing.
(ii) *External Stability*, i.e., stability of the reinforced earth monolith,
  (a) Rotational type failure
  (b) Failure by sliding as a rigid body
  (c) Bearing failure of supporting soil under the rigid body.

Internal Stability

There are two major approaches by which internal stability can be checked:

(i) Analysis involving the local stability of individual reinforcing elements,
(ii) Analysis involving the overall wedge-block stability of the reinforced earth monolith.

Method (i) has the advantage of being quick to use and the general practice appears to be to design a reinforced earth retaining wall by that method and then to check the result by method (ii), modifying the initial design if necessary.

• *Check for internal stability by method (i), i.e., by the local internal stability check method* For a wall such as illustrated in Fig. 16.1, with a granular fill, the maximum tensile force in the reinforcing elements at level i is obtained from Eq. (16.1) assuming S_v as the element height:
$T_i = iK_a\gamma S_v^2$/m run of wall (minus $2cS_v\sqrt{K_a}$ if cohesive frictional fill).
This T_i has to include the effect of at least the following (a), (b) and (c):

(a) *Treatment of 'uniform surcharge' on top of wall.* If the wall is uniformly loaded by a surcharge of q/m^2, there will be an increase in T_i due to the uniform pressure distribution K_aq induced within the soil.
Then $T_i = K_aS_v(\gamma iS_v + q)$/m length of wall (minus $2cS_v\sqrt{K_a}$ if cohesive frictional fill).

(b) *Treatment of 'line load' on top of wall.* Schlosser and Long carried out measurements in reinforced earth fill materials, both on models and on an experimental wall, to guage the effect of a line load acting on the surface of the fill and found that the load spreads through the reinforced earth at a slope (V/H) between 2/1 and 1/1. They proposed the following design assumptions:

If line load = S_L per unit length along wall, and its point of application is distance d back from the facing, then (referring to Fig. 16.7),

Vertical stress at level i (at depth h_i) due to $S_L = \dfrac{S_L}{d + 1/2h_i}$

Hence, increase in T_i due to $S_L = K_aS_v\dfrac{S_L}{(d + 1/2h_i)}$ (per unit length along the wall), taking $d \not> 0.5h_i$

The method becomes conservative when, as is usually the case, the line load can be considered as being applied through a continuous footing or the equivalent, e.g., railway track.

If the load is applied concentrically, then the spread width D_i at level i will be $(d + B + 1/2h_i)$ where

B = width of footing (Fig. 16.7b).

If S_L is applied eccentrically to the footing, it can be assumed that the bearing pressure distribution beneath it is trapezoidal. Then, for simplicity, one can assume that the maximum value of bearing pressure applies uniformly beneath the footing, i.e., that a uniform vertical pressure of,

$$p_{max} = \frac{S_L}{B}\left(1 + \frac{6e}{B}\right)$$

acts at the top of the wall over the distance B [Fig. 16.7(c)]. At depth h_i, the spread D_i, is again $(d + B + 1/2h_i)$ and the vertical pressure, σV_L can be taken as $[p_{max}.(B/D_i)]$

or $$\sigma V_L = \frac{S_L}{D_i}\left(1 + \frac{6e}{B}\right)$$

Hence, increase in T_i due to S_L (over element height S_v)

$$= K_a S_v \frac{S_L}{D_i}\left(1 + \frac{6e}{B}\right)$$

where d in the expression for D_i has the same limitation as noted earlier (i.e., $d \not> 0.5h_i$).

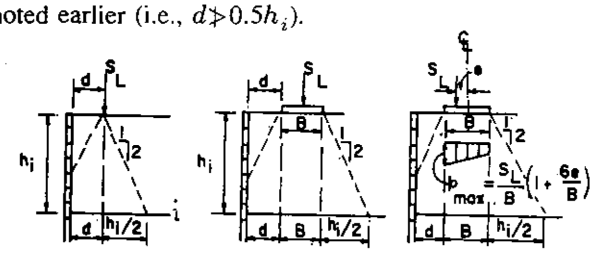


Fig. 16.7 *Treatment of line load*

(c) *Treatment of horizontal force at top of wall.* Quite often the traction forces of machinery running along the top of the wall can induce a horizontal shear force, F_L, which is applied through some form of foundation of width B, and may be regarded as continuous along the length of the wall (Fig. 16.8).

Treatment is to assume that the force F_L is carried by the tension elements which fall within the Coulomb wedge passing through the edge of the width B [Fig. 16.8(a)].

It is assumed that there is a triangular distribution of forces, decreasing with depth.

If h = height of Coulomb wedge, then

$$h = \frac{(d + B)}{\tan(45° - 1/2\phi)}$$

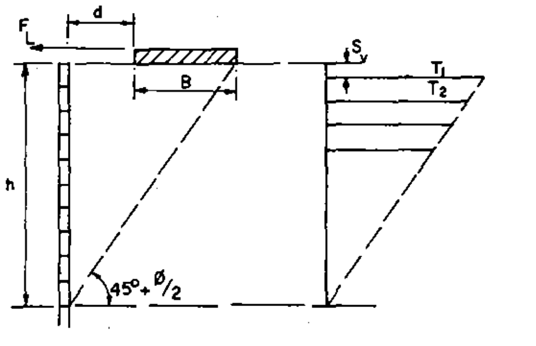


Fig. 16.8 *Treatment of horizontal force*

As tensile force distribution is assumed to be triangular [Fig. 16.8(b)].

$$\frac{T_i}{h - S_v} = \frac{T_i}{h - iS_v}$$

$$\therefore \quad T_i = \frac{T_1(h - iS_v)}{(h - S_v)} \tag{16.3}$$

Now $$T_1 + T_2 + T_3 + \ldots + T_n = F_L$$

$$\therefore \quad T_1[(h - S_v) + (h - 2S_v) + (h - 3S_v) + \cdots + (h - nS_v)] = F_L(h - S_v)$$

or $$T_1\left[nh - \frac{S_v}{2}n(n+1)\right] = F_L(h - S_v)$$

Since $n = \frac{h}{S_v}$, and if this substitution is made, the expression for T_1 becomes,

$$T_1 = \frac{2S_v F_L}{h}$$

and, substituting this in Eq. (16.3), we obtain,

$$T_i = \frac{2S_v F_L}{h}\left(\frac{h - iS_v}{h - S_v}\right) = \frac{2S_v F_L}{h}\left(\frac{h - h_i}{h - S_v}\right)$$

The S_v term in the denominator can be neglected with little loss in accuracy, hence,

$$T_i = \frac{2S_v F_L}{h}\left(1 - \frac{h_i}{h}\right), \text{ due to horizontal force } F_L.$$

Thus having evaluated total T_i (i.e., basic value, duly increased by the effects (a), (b) and (c) detailed above), a check is made to ensure against tensile failure and pull-out failure for each strip i. The required reinforcement perimeter and length per metre run of wall is then calculated.

• *Check for internal stability by method (ii), i.e., by the wedge block stability check method* As has been discussed earlier, the maximum tension line more or less marks the division of a reinforced earth fill into an active zone and a resistant zone. The block stability analysis assumes that the active zone consists of a failure wedge tending to pull away from the rest of the fill, whereas the resistant zone by gripping the ends of the reinforcing elements in the fill is anchoring the active zone in position.

Form of the failure wedge (or active zone): As a result of extensive observations by many research workers on both models and actual structures, it is now generally accepted that just before failure, a reinforced earth retaining wall develops some form of *failure wedge* and that the failure surface tends to follow the maximum tension line.

Obviously, if the exact shape of the failure wedge were known, it would lead to a relatively straightforward design method. However, as Schlosser points out, the boundary between the active and the resistant zones is variable, and depends upon the geometry, stress value, settlements within the subgrade and, possibly the most significant of all: the value of the factor of safety for the particular structure.

It must be remembered that the maximum tension line in a reinforced earth structure does not have a unique position; it varies, daily with the loading and the state of the weather.

For the almost theoretical case of a wall similar to that in Fig. 16.1, i.e., with an unloaded horizontal surface, the maximum tension line is nearly vertical for about one half the height of the wall, Fig. 16.9(b), an idealised form of which, suitable for design purposes, is shown in Fig. 16.9(c).

As can be seen, the volume of soil in the active zone of such a wall is considerably less than that contained in the Coulomb wedge [Fig. 16.9(a)]. Obviously, if the reinforcing elements are all of the same length, a much larger aggregate length is available for bond if the active zone is as in Fig. 16.9(c) than if the active zone approximates to the Coulomb wedge of Fig. 16.9(a). The question is whether it is safe to use the assumption of Fig. 16.9(c) for all the loadings that can be applied to a reinforced earth-retaining wall.

In view of Schlosser's findings such an assumption could be wrong. Murray observes that for the more complex situations involving concentrated loads and sloping backfill, a plane surface of failure is unlikely—it is more likely to be curved. Murray advocates, in view of the uncertainties involved, that the most direct approach to a solution would involve the analysis of a number of trial surfaces in order to determine the maximum thrust, as in normal retaining wall design.

Although the trial failure surfaces can be of any shape, the use of straight lines makes calculations easier.

Once the stability of each and every layer of reinforcement has been checked as shown earlier, the overall stability of several trial wedges is actually checked using a *graphical method*. The proposed technique is illustrated in Fig. 16.10 for a simple wall loaded by a uniform surcharge q and the self-weight of the backfill. As can be seen, a family of potential failure surfaces are assumed to originate from the face of the wall at a depth h_i. Several inclinations are assumed for the failure surface i.e., β_1, β_2, etc. For each value of β a triangle of forces is drawn to determine T the total tensile force to be resisted by the reinforcement cut by the failure plane under consideration. By evaluating T for several trial values of β it is possible to determine a critical value of β associated with a maximum value of T. This maximum value of T is compared with both allowable *tensile* and allowable *pull-out* resistance of the reinforcing strips within the said depth h_i. In this case, the length of each reinforcement considered is the effective bond length extending beyond the potential failure plane under consideration.

Satisfied that the stability is ensured at depth h_i, a further family of potential failure planes is investigated for another value of h_i. In fact, the BE 3/78 memorandum implies that up to five locations be checked down the face of the wall.

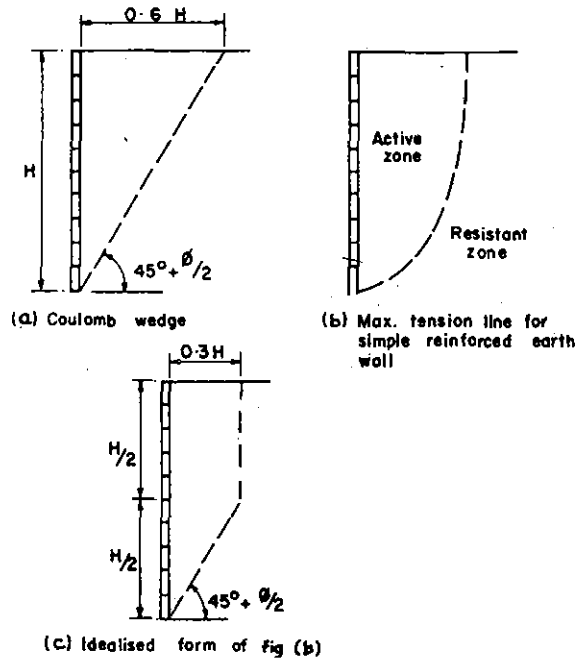


Fig. 16.9 *Form of failure wedge*

External Stability

The reinforced earth mass (behind the facing units) is assumed to act as one rigid monolith on which act its self weight, the applied loads, and the horizontal earth pressure

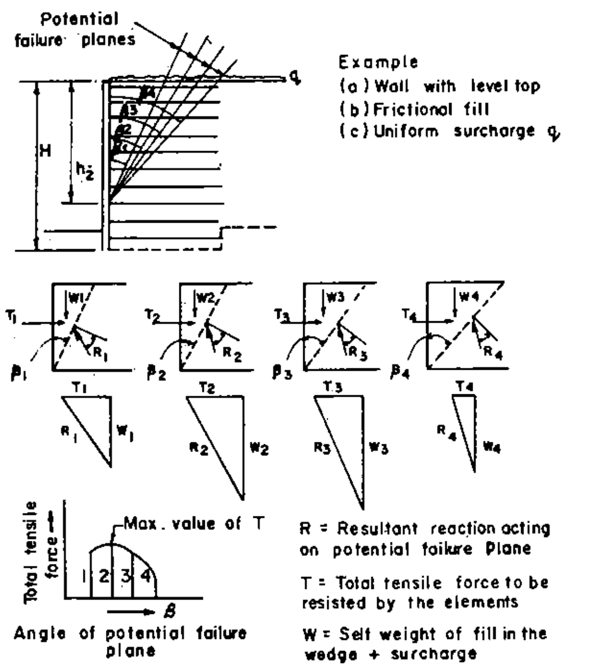


Fig. 16.10 *Wedge stability check calculation (Taken from UK Dot BE 3/78)*

force at its rear-end (from the soil there-beyond). Its stability against sliding, against base pressures and against over-turning has to be computed and ensured in the same manner as in the case of regular rigid reinforced concrete wall. The following may be noted:

- Factor of safety against sliding, $\mu W/H'$, should not be less than 2.0 (where W = total weight of the reinforced earth mass including any loads on it, μ = coeff. of friction against its sliding at its base, generally 0.5 and H' = total horizontal thrust on the back of the wall due to the retained material beyond and behind it.
- As per British practice the base pressures are calculated without allowing any uplift (i.e., tension), and the distribution is assumed linear (trapezoidal generally, but in the limit, triangular on full contact area). French practice recognises that since a reinforced earth wall is essentially flexible, the assumption of rigidity in bearing capacity calculations, described above, could be very unrealistic, particularly with weak subgrades where a more uniform bearing pressure distribution could be expected, justifying Meyerhoff's approach of partial uplift and redistributed rectangular pressure distribution. (Thus, there is a clear difference here between the British and the French practices.)
- Factor of safety against overturning about toe should also not be less than 2.0. Where topography so demands, check may be made against rotational type failure by the usual slip circle methodology.

Design Calculation

Design of Tension Strips and Stability of Reinforced Earth

Although fairly simple in principle, the actual arithmetics can be very lengthy, depending on the number of tension strips and the geometry, etc. This is, therefore, best done by using the commercially available computer programs, some of them are already run at some universities (e.g., Edinburgh, UK). However, for the sake of getting a feel of the mensuration, reference may be made to the numerical example given in Reference 1. Following may be noted:

- In these design calculations it may be noted that the bond resistance requirements in the upper reaches of the wall necessitate longer lengths of reinforcement strips than are required further down.
- It is apparent that the design can be made sophisticated by designing the wall in sections and using shorter reinforcement strips nearer the base, thus effecting a saving in reinforcement strips. But whether this effort would be worthwhile, bearing in mind the extra site supervision that would be required, is a matter for the designer's/contractor's understanding.
- Obviously the design procedure outlined above is very much on the safe side. However, this is not necessarily bad when one considers that the calculations cannot take account of the possible horizontal stress increases (possibly far above 'active' values) that can be induced by the compaction taking place during construction.
- For a frictional fill, with its immediate drainage, the long term and short term stability values are the same and the ϕ value used in calculations will be with respect to effective stresses.
- If a cohesive frictional fill is to be used, the ϕ and c value in the calculations must be with respect to effective stresses. This will give the long term stability of the wall but it may be necessary to check the possibility of pore water pressures affecting the short term properties of the soil fill.

Design of Facing Units

It is impossible to assess accurately the values of the true horizontal pressure acting on the back of the facing units.

As has been indicated earlier, the tensile force in a reinforcing element at the facing of a wall can have a low or a high magnitude (a state of affairs related to the relative length of the element).

A further indeterminate factor is the magnitude of the horizontal stresses that are induced during construction compaction.

With the present state of knowledge a designer has little option but to design the facings to withstand full active pressure values, i.e., to act as an *anchor plate* for the maximum tensile force in the reinforcement elements fastened to it.

It should be noted that a facing unit must also be substantial enough to withstand the weight of other units placed above it and that there is always the possibility of localised bending and shear effects if the fill settles down, relative to the facing.

This latter effect can be alleviated somewhat by the provision of some form of sliding joint, which illustrates a further advantage of the use of metallic strips. They are so much simpler to attach to the facing units than most other forms of tension strip reinforcement.

16.3 A NOTE ABOUT COSTING, CONSTRUCTION TIME, AND CARE NEEDED DURING CONSTRUCTION

Costing

Cost of a reinforced earth retaining wall, compared to that of a regular rigid RC wall, can be 20% to 30% cheaper provided the considerations of corrosion and availability of the appropriate fill material are not prohibitive. It is wrong to generalise that such structures are always cheaper. This is simply not true. Depending on the site and space clearances, such a structure, in the long run in certain cases may prove costlier in view of corrosion and the chances of dislodgement in the event of hits from trucks.

In fact the actual cost can well prove high unless the costs of both the alternative structures (reinforced earth as well as the rigid RC wall) are sought before the award of work and analysed. With increased facing area, the cost could actually come down, but the contractor may cleverly so apportion the cost of just even the moulds (for the facia units) that the final bill proves costlier. Hence a thorough cost analysis check is called for at the time of award of work.

Normally, the cost of reinforced earth structure involves the following:

(i) Detailed design and drawings (preparation of)
(ii) Anchor bars and dowels for shoulder connections
(iii) Closed cell polyethylene foam for vertical joints
(iv) Resin bonded cork for horizontal joints
(v) Moulds for precasting facing panels ($\approx$ 1 no. per 1000 m^2 facing area)
(vi) Lifting anchors (2 per panel)
(vii) Toggles and loops or tie strips, nuts and bolts
(viii) Facing units
(ix) Tension strips (reinforcements)
(x) The fill material (earth)
(xi) Compaction
(xii) Expert supervision, including various quality control checks.

In 1984 the cost, for instance in Riyadh area, for about 6.0 m tall reinforced earth abutments, designed to the French Practice, worked out to approximately SR 300/m^2 of facing area. It was also stipulated as follows,

SR	90/m^2	of facing elements, plus
SR	17/m	of 60 mm wide strip*, plus
SR	11/m	of 40 mm wide strip*

Construction Time

This is obviously lesser than that for the regular reinforced concrete rigid structure, but depends on,

- rate of fill delivery (depends on the lead)
- rate of placement
- rate of compaction

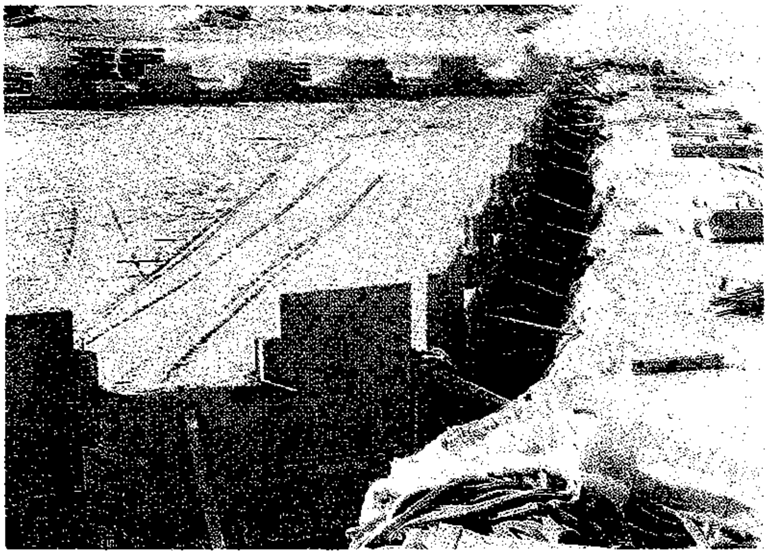
Care Needed During Construction

Considerable care has to be taken, during execution, about the following points:

- quality of fill material (clean/frictional, or as specified in design/preferably free draining type/should not contain more than permissible amounts of any salts, acids, alkalies, mineral oils, fungus, microbes, moisture and any other deleterious matters/and should be of specified pH value and specified resistivity)
- method of compaction
- thickness of soil layers
- construction procedure
- proximity of compactors to facing wall
- connection of strips to facing elements
- breakage and mishandling of elements, connections and strips
- deliberate and adequate backleaning of facing panels prior to compaction
- proper use of horizontal jointing (resin bonded cork) and vertical jointing (closed cell polyethelene foam) materials
- need for separation between the reinforced fill and the natural fill behind it in case the latter contains such of the impurities as may leach into the reinforced fill by capillary action and excite corrosion of tension strips.
- removal of slack from the tension strips.

* 5 mm thick galvanised mild steel strip.

Placing the
precast facing units



Placing the
precast facing units

Abutment
nearing completion

16.4 THE BRITISH VERSUS THE FRENCH CODES OF PRACTICE

The British code BE 3/78 of the UK DOE (DOT) is based on the use of working loads and permissible stresses, and requires the actual measured values of soil parameters (density, internal friction angle ϕ and cohesion c) to be used in the design calculations. Moreover, design to BE 3/78 is for the ultimate condition in that the coefficient of active soil pressure is used throughout in the calculations of horizontal thrust due to the fill material. It provides, as it admits, a conservative approach to the design of reinforced earth structures.

The French code does not require the actual measured values of soil parameters to be used in the design calculations. Also, design to the French code is for the working condition in so far as the horizontal thrust due to the fill material is calculated as it uses a coefficient which varies between the coefficient of earth pressure at rest and the active soil pressure coefficient, depending on the depth of soil being considered. The French code also requires that a design angle of internal friction, $\phi = 36°$, be used in the design calculations, subject to certain specified particle size distribution criteria being met and subject to the compaction of the reinforced earthfill being carried out in accordance with the stipulations in the French code requirements. If the criteria are not met the fill material is deemed to be unsuitable for use. Figure 16.11 describes the mechanical criteria for the backfill material.

A Brief Comparison Between the British and the French Codes The British practice seeks a greater 'sacrificial thickness' for the tension strip, requires 1000 g/m^2 of galvanisation of mild steel tension strips (cf. 500 g/m^2 by the French practice); assumes rigid body behaviour of the reinforced earth fill, allowing no tension in base pressure (cf. uplift allowed and distribution as per Meyerhoff by the French); assumes 'working loads' and 'allowable stress' (cf. limit state analysis by the French); assumes active pressure coefficient for the earth pressure thrust (cf. the French practice of adopting earth pressure between that 'at rest' and the 'active' value, depending on the depth at which calculation is being made); uses the soil properties of the soil actually used at site (cf. the French practice of directly assuming the friction angle as 36° but specifying the particle sizes and their distribution and the compaction requirements).

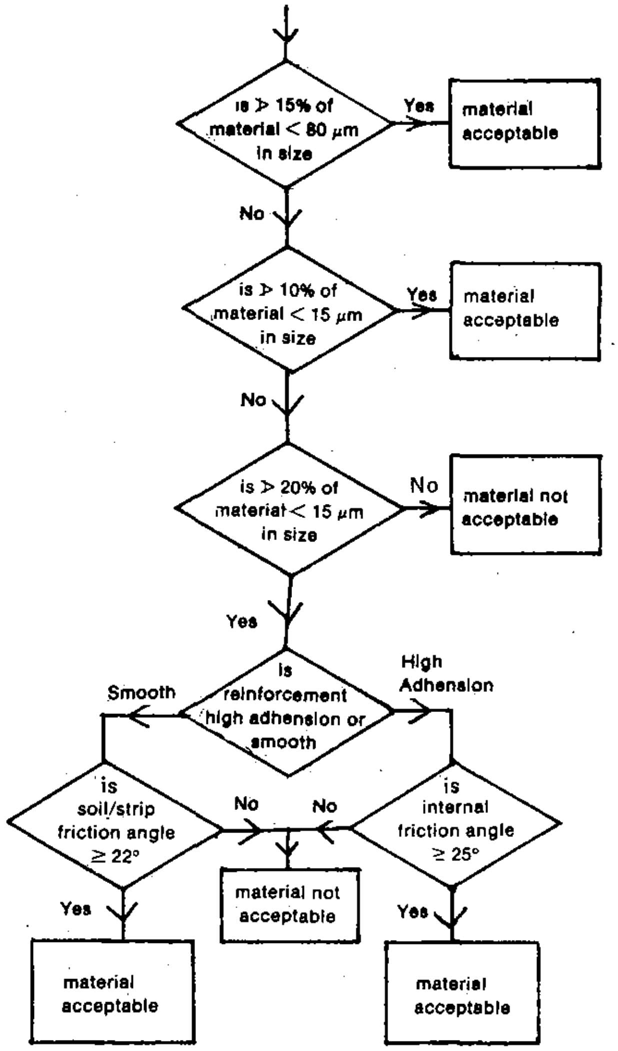


Fig. 16.11 *Mechanical criteria for acceptability of backfill material using French Code*

16.5 TESTING OF FILL MATERIAL

'Frictional' Soil (ϕ-soil, c = 0)

Determination of Angle of Friction, ϕ

Standard practice is to determine the angle of friction by means of the shear box test. The plan of the shear box should be 300 mm × 300 mm and have a depth of not less than 150 mm. Generally the soil sample should fill the box.

The maximum particle size of the soil should not exceed 1/8 of the height of the test sample which should be prepared to a dry density of 92% ± 2% of the maximum dry density as determined from the vibrating hammer test, described in BS 1377 (1975).

Determination of Coefficient of Friction, μ, between Reinforcement and the Soil

This is achieved by testing, also using the 300 mm shear box. A piece of the reinforcement material is carefully cut so that it fits exactly the inside of the shear box. By means of packing the material is arranged to be exactly level with the surface of the lower half of the box. The soil is placed in the upper half of the box and the maximum shearing force corresponding to a specific normal load is determined. The test is carried out three times at least and the average ratio of maximum shear stress to normal stress is taken as μ. The rate of shearing should not exceed 1.2 mm/minute. The normal load used in the test should correspond to the maximum vertical pressure that will operate within the fill, obtained from the design calculations.

> NOTE *The above is an abstract of the requirements of the UK DOT Memorandum (1978)* which also states that frictional fill should have the following properties if it is to be acceptable for reinforced earth. It should be well graded (uniformity coefficient $\nless 5$) with a maximum particle size of 125 mm and with not more than 10% fines (i.e., material passing the 0.063 mm sieve), with an angle of friction of not less than 25°

Cohesive-Frictional Soil
($c - \phi$ soil)

Determination of Effective Angle of Friction and Effective Cohesion

The test is carried out in the (300 mm) shear box with the sample compacted to a dry density of 92% + 2% of the maximum dry density value obtained from the laboratory compaction test (using the 4.5 kg rammer) BS 1377 (1975). The sample should fill the whole box and the maximum particle size should not be greater than 0.125 × the height of the sample.

A minimum of three samples are tested, first having allowed to soften under water for twenty-four hours, followed by twenty-four hours consolidation under the test normal stress. Normal stress values should roughly correspond to the vertical pressure in the fill of the completed structure, at the base, at quarter height and at half height.

The rate of shear should be slow enough to allow the dissipation of any pore water pressure so that the applied stresses are also effective stresses.

The results from the shear box tests are taken to give a measurement of the properties of the fill. For quality control, results from shear box tests using the 60 mm × 60 mm box are acceptable.

Determination of Friction and Adhesion between Reinforcement Strip and the Soil

The procedure is as for frictional fill, except for a possibly lower shearing rate. The adhesion is taken to be the intercept on the shear stress axis of the 'normal stress to shear stress' plot, and the angle of friction is taken as the angle of slope of the plot.

> NOTE Other requirements for a satisfactory cohesive-frictional soil fill for reinforced earth are: Liquid limit $\ngtr$ 45%; Plasticity index $\ngtr$ 20%; not more than 10% of the soil to have particle sizes smaller than 0.002 mm, and the angle of internal friction, with respect to effective stresses, not to be less than 20°.

Other Tests for Fill Material

Other necessary tests are chloride-ion content, sulphate content, pH value, and resistivity.

It should be noted that the maximum and minimum allowable values for these variables are affected by the type of reinforcement used in the structure.

> *Caution*: The use of chalk, unburnt colliery shale, pulverised fuel ash and other unsuitable material is not permitted for reinforced earth fills.

16.6 THE PROBLEM OF CORROSION

"...it appears that most engineers, either through their work or educational experience, have concluded that metal, buried in the earth, will corrode in a time period inversely proportional to their years of experience". Mckittrick (1979).

What is the definition of corrosion? What does it mean when it is said that a certain something has corroded?

There is little doubt that if corrosion creates a pin hole in an off-shore oil pipe the Government may be shaken, whereas if corrosion creates a pin hole in a reinforcing element of a reinforced earth structure the event is of much less consequence.

Perhaps if the tension strips were somehow eaten away, one would say that the event was of consequence. Pitting of the surface of a tension strip due to some years burial, might even improve the bond resistive qualities of the material, but of course the tension capacity will be effected.

However, Darbin *et al.* (1978), have illustrated that even in an aggressive soil environment, the type of galvanised steel reinforcement strips presently being used in construction will have a useful life of at least 120 years.

The UK DOT Memorandum (1978), gives guidance on the sacrificial thickness that should be added to the designed thickness of the metallic tension strips if full allowance is to be made for corrosive effects. This is indicated in the following table:

Reinforcement material	Total increase in strip thickness (mm)	
	Frictional fill	Cohesive frictional fill
Aluminium	0.3	0.6
Galvanised steel	1.5	2.5
Copper	0.3	0.6
Stainless steel	0.3	0.4

REFERENCES

1. Smith, G.N. and E.L. Pole, *Elements of Foundation Design*. Granada Publication, London.
2. Vidal, H., "La terre armee". *Armales del'Institure Technique du Batiment et des Travaux Publics* (19), August 1966.
3. French Ministry of Transport/LCPC, ('French Code'—English Translation), "Reinforced Earth Structures: Recommendations—Rules of the Art", 1979.
4. "Department of Transport, Reinforced Earth Retaining Walls and Bridge Abutments for Embankments—Technical Memorandum Bridges BE3/78," Department of Transport (DOE), London 1978.
5. Jaky. J., "The Coefficient of Earth Pressure at Rest", *J. Soc Hungarian Architects and Engineers*, 78(22), 1944.
6. Mckittrick, D.P., "Reinforced Earth: Application of Theory and Research to Practice", *Ground Engineering*, January 1979.
7. Murray, R.T., "Research at the Transport and Road Research Laboratory to Develop Design Criteria for Reinforced Earth", *Joint Symposium for Reinforced Earth*, TRRL and Heriot Watt University, 1977.
8. Boden, J.B., M.J. Irwin, and R.G. Pocock, "Construction of Experimental Reinforced Earth Wall at the TRRL", *Joint Symposium of Reinforced Earth*, TRRL and Heriot Watt University, 1977.
9. Darbin M., J.M. Jailoux and J. Montuelle, "Performance and Research on the Durability of Reinforced Earth Reinforcing Strips" *Symposium on Earth Reinforcement: ASCE Annual Convention*, Pittsburgh, USA, 1978.
10. Schlosser, F. and N.T. Long, "Recent Result in French Research on Reinforced Earth", *J. Am. Soc. Civ. Engrs.*, September 1974.
11. Schlosser, F. and H. Vidal, "Reinforced Earth", *Bulletin de Liaison des Laboratories des Ponts et Chaussées*, (41). November 1969.
12. Symons, N.F., "Reinforced Earth Retaining Walls", *Highways and Road Construction*, October 1973.
13. Kennard, R.M., P.T. Harmanm, M., Boyd, "Reinforced Earth in Hongkong Context", *SE Asia Geotechnical Society Meeting*, November 1982.
14. Jones, C.J.F.P., "Reintorced Earth : Practical Design Considerations", *Ground Engineering*, September, 1978, pp. 27-33.
15. Sims, F.A., and C.J.F.P. Jones, "The Use of Soil Reinforcement in Highway Schemes", *CR Coll Int. Rein. des Soils*, Paris, 1979.
16. Leece, R.B., "Reinforced Earth Highway Retaining Walls in New South Wales, Australia", *CR Coll. Int. Rein. des Soils*, Paris, 1979.
17. Jones, C.J.F.P. *Reinforced Earth and Soil Structures*, Butterworths Publication.
18. Carter, M, *Geotechnical Engineering Handbook*. Pentech Press Limited, Plymouth, 1983.
19. "Procs. of the Seventh South East Asian Geo. Conf., Hong Kong" Hong Kong Inst. of Engs./SE Asia, Geo. Soc., 1982.
20. Ingold, T.S. *Reinforced Earth*, Thomas Telford Limited, London, 1982.
21. "Int. Conf. on Soil Reinforcement, Paris", *Ecole Nationale Des Ponts Et Chaussees/LCPC*, 1979.
22. *Symposium on Earth Reinforcement*, Pittsburg, American Soc. of Civil Eng., 1978; "Reinforced Earth and other Composite Soil Techniques," TRRL *Supplementary Report* 457, 1979 (Procs. of Symposium Sponsored Jointly by the TRRL and Herriot-Watt University, Edinburgh, 1977.)
23. *Procs. of the 10th Int. Conf. on Soil Mechanics and Foundation Engineering*, Stockholm, 1981.
24. *Procs. of the 8th European Conf. on Soil Mechanics and Foundation Engineering*, Helsinki, 1983.
25. Jones, C.J.F.P., "The York Method of Reinforced Earth Construction", *Proc. ASCE Sympo. Earth Reinforcement*, Pittsburgh, 1978, pp 501-527.
26. Schlosser, F., "Experience on Reinforced Earth in France", *Proc. TRRL/Heriot-Watt Univ., Sympo. Reinforced Earth and other Composite Soil Techniques* (see TRRL Supplm. Rept. SR 457, 1979), 1977.

CHAPTER 17

Bearings for Bridges

17.1 BEARINGS

Basic Definitions

A *Fixed Bearing* is a point of connection between a structure and its support, designed to transmit vertical and horizontal loads and allow for rotations. A *Moveable Bearing*, in addition, allows for horizontal movements of the structure.

- *Fixed* bearings can be made of steel plates — sometimes flat and pinned, and sometimes curved to provide for rotation, sometimes of two curved plates with rocking lubricated surfaces, and sometimes of elastomeric material — with or without dowel pins through the bearing or preferably keys.
- *Sliding type* moveable bearings can be made of steel plates with lubricated surfaces, or a steel plate with special low friction material (e.g., PTFE) to enhance the sliding ability.
- *Rolling type* moveable bearings are usually made of hardened steel curved surfaces which roll on other hardened steel surfaces.

Limitations of Different Types of Bearings

- Fixed bearings allow negligible relative movement between the superstructure and substructure at the bearing location and hence may generate large forces at this point, resulting in costly details, anchor-piers or anchor-abutments.
- Sliding bearings, when they work satisfactorily, transmit a horizontal force which is dependent on the coefficient of sliding friction of the bearing and the vertical load on it at the time. This horizontal force may have little relation to the capacity of the pier to accept the force, and in some cases the horizontal force developed from the pier flexibility may be insufficient to cause sliding to take place. The designer only has limited control over the coefficient of friction which may vary widely in practice.
- Rolling bearings are limited to an even greater extent than sliding bearings in their ability to transmit horizontal forces as they are usually designed to have a very low rolling resistance.
- Elastomeric bearings also have limitations such as size, and in some cases problems due to co-existing small vertical loads, but they are the only type of bridge bearing where the designer can readily 'manipulate' the force which will be transmitted to the pier through the bearing. These bearings usually have a stiffness in shear which is about 10% of their stiffness in compression. This method allows the bridge designer to adjust the 'horizontal bearing stiffness' on a given pier to suit the capacity of the pier to take horizontal load or movement.

17.2 BRIDGE SUPERSTRUCTURE MOVEMENTS

The horizontal movements of a bridge superstructure are due to:

- shrinkage of concrete
- elastic shortening of concrete due to prestressing
- creep of concrete under prestress and other permanent loads
- temperature expansion and contraction
- movements due to induced external loads (e.g., earthquake, wind, vehicular braking etc.).

Shrinkage of Concrete

This is a shortening of the bridge superstructure which depends on a number of factors — concrete type and quality, size of the member, relative humidity and time after casting. Sometimes for simplicity of calculation, this movement is treated as a negative temperature effect.

Elastic Shortening

Elastic shortening of a prestressed concrete deck occurs during the process of prestressing. The amount of this shortening, which has to be accommodated by the bearing and pier, will depend on the stage at which the superstructure is placed on the bearings and also at what stage it is prestressed. At times, partial or complete prestressing may be carried out before the superstructure is placed on the bearings thus eliminating this shortening from affecting the bearings, at least partly.

Creep Shortening

Creep of concrete under prestressing and other permanent loads must be allowed for, and again, this is a shortening of the superstructure. It is a time-dependent effect which is readily calculable when all the variables which affect its value are determined. Again, for simplification, creep can also be treated as a negative temperature effect.

Temperature

Temperature variations cause expansion and shortening of the superstructure and need to be estimated as a plus and minus range about a mean structure temperature which occurs when the superstructure is placed on the bearings. Depending on the time of the year, this may or may not be the mean temperature for the locality. When construction occurs during the extreme temperature seasons, these two values may be many temperature degrees apart.

Temperature differentials will also occur in the superstructure namely from top to bottom and from one side to the other of the superstructure. Temperature differentials through the depth of the superstructure will usually have little effect on the bearings and piers but temperature differentials from one side of the superstructure to the other will cause the superstructure to bend in plan which will cause horizontal forces on the bearings and piers which may be of magnitude.

Load Movements

Movements due to applied horizontal loads on the superstructure can be either transverse or longitudinal or both, with respect to the bridge centre line.

(a) Loads transverse to bridge centre line: Wind or earthquake on both the bridge superstructure and on the traffic using the bridge, and also the centrifugal force from traffic using the bridge if curved.

(b) Loads parallel to the bridge centre line: Apart from earthquake or wind on the superstructure, there is the braking force (often as a percentage of the live load on the bridge).

17.3 DEVELOPMENT OF BEARINGS

Until the end of the 18th century all structures of any appreciable size were built of stone, brick or mixed masonry. These structures, generally massive, are little affected by environmental changes and any slight movements which may occur are compensated either by deformation of the constituent materials or by small displacement of the supports.

The 19th century, and the introduction of cast-iron and steel materials having the advantage of resisting tension, considerably enlarged the possibilities available to builders. An increase in structural spans is generally accompanied by a corresponding lightening of the structure which becomes more slender and flexible and loses some of the thermal inertia. They must then be fitted at their support points with simple devices, called *bearings*, which can withstand movement and, more particularly, the expansion or contraction due to temperature changes.

The first devices used at that time, consisted of either metal plates sliding one on the other or, of rollers or, of a combination of both. These appliances; gradually improved upon by the incorporation of swivel arrangements in order to provide rotational movement, were in general use on steel structures for more than a century.

The 20th century saw the introduction of reinforced concrete but, as was the case with masonry structures, the early structures built with this material were so massive that the support movements were of little significance. At the end of the Second World War the necessity of rebuilding rapidly the structures which had been destroyed, and also, no doubt, the shortage of steel for constructional purposes, favoured the rapid development of reinforced concrete and, even more so, of prestressed concrete.

With the advent of structures which were more and more slender, came the necessity to incorporate bearing devices to allow for movement and rotation. As a result, systems of entirely new conception, using the elastic properties of rubber and steel, appeared on the market. The use of this material in civil engineering was not new. As early as 1830, British railways, placed rubber shock absorbing pads between rails and sleepers. The idea was again taken up a century later by the French railways, first in 1932 when elastic pads were placed underneath the steel bearings of certain structures in order to absorb vibration, and again in 1936 when chief engineer Valette adopted the use of rubber pads as bearings underneath the steel deck of a railway bridge at La Plaine St Denis in the Paris suburbs, and finally in 1948 when the SNCF decided to lay the rails of certain tracks on rubber *sole-plates*.

It was Eugene Freyssinet who first had the excellent idea of making general use of rubber pads as bearing devices by combining steel and elastomer in a single product in order to improve their individual performances as a combined product.

As early as 1952 the first rubber bearings, consisting of a stack of elastomer layers and sheets of tinned metal grillage were manufactured by Freyssinet. A first patent was applied for in 1954 by Eugene Freyssinet, but it quickly became apparent that *grillage* bearings had a limited use because of their very low resistance in compression and too much permanent set. Therefore, in 1956, these bearings were abandoned and the metal grillage replaced by steel plates adhering to the rubber by vulcanization. Thus the first laminated elastomeric bearings were born.

At the request of Freyssinet International, Francois Conversy drew up a method of design and dimensioning, which was the subject of a report, in 1958 to the French Committee for Bridges and Structural Engineering and which was confirmed by a series of tests carried out under the sponsorship of the International Union of Railways between 1961 and 1965. This theory was to serve as a basis for most national regulations.

At the same time, industrialisation of the manufacturing process led to rapid improvement in the product which, since then, has never ceased to develop, thanks to discovering of new elastomer compounds (neoprene) allowing the use of the bearings in special climatic environments, the use of stainless steel reinforcing plates in order to resist corrosion in aggressive environments, and combining laminated elastomeric bearings with polytetrafluorethylene (PTFE) sliding sheets in order to allow large movements. The development of neoprene bearings and the realisation of PTFE as a very low friction element, which further led to more sophisticated bearing systems, is explained ahead in the chapter.

In Britain, bridgeworks carried out under the auspices of the Department of Transport (now called the Dept of Environment, DOE) have to comply with technical instructions issued by the Department Memorandum 557 issued in 1945(!). These stated that it was unnecessary to provide for expansion (or contraction) for bridges less than 30 ft span and recommended *roller* and *rocker-roller* bearings for spans above 50 ft (with *pin* or *rocker* bearings for the fixed end). Memorandum 802, published in 1962, gave provisional rules for the use of rubber bearings which were coming into general use at this time. Memorandum 557/1, published in 1966, gave rules for the design and use of Freyssinet concrete hinges in highway structures. These were updated by Technical Memorandum BE1/76 and 5/75, respectively. Interim Memorandum IM11 in 1970 dealt with PTFE in bridge bearings.

Up to the middle of this century, bridges relied on steel roller, rocker or metal sliding bearings to permit movement. With more advanced designs to make full use of the materials employed and increased use of skewed and curved bridges to carry modern high speed roads over obstructions, the need arose for bearings to take movement in more than one direction. New types of bearings have been developed taking advantage of the new materials arising from improved technology.

Because of the high friction values associated with metal to metal sliding surfaces and complete seizure if not kept lubricated or protected from corrosion, modern sliding bearings usually rely on PTFE or similar low friction non-corroding synthetic materials to allow assured very low friction in sliding.

In the business of bearings, rubber means either the natural product or a synthetic material with rubber-like characteristics. Some countries (e.g., Germany) do not permit the use of natural rubber because of its unfavourable ageing behaviour. A *laminated bearing* is a bearing consisting of some rubber slabs bonded to steel plates in between, so as to form a sandwich arrangement. A *rubber bearing* pad is a single unreinforced rubber slab. A laminated rubber or neoprene bearing has impregnated in it, during its vulcanization, the restraining steel plates which form the reinforcement.

Natural Rubber (NR) is one of the polymers reported to have almost all the properties needed to meet the requirements of bridge bearings except poorer ageing performance. Although synthetic rubbers like polychloroprene (CR, termed Neoprene) have found wide acceptance for making bearings for concrete structures, it is expected that in countries where natural rubber is abundantly available, the bearings made out of suitably compounded NR will be more economical, even though NR is relatively poor against ageing and may call for earlier replacement.

17.4 TYPES OF BEARINGS RECOMMENDED FOR VARIOUS SPAN-LENGTHS AND SUPPORT-FLEXIBILITY CONDITIONS

Bridge bearings must be designed to transmit all vertical loads and appropriate horizontal forces (depending on the functionality of the individual bearing).

(i) Generally where a simple span deck is supported over rigid supports (unlike the adjoining cantilever tips in segmental free-cantilever construction, which are flexible) and the span is less than about 7.5 m, no special bearing devices are necessary—only tar paper or a felt layer is adequate (after the mating concrete surfaces are smoothened by carborundum stone).

(ii) For spans between about 7.5 m to about 15 m, mild steel plate bearings (with top plate slightly curved), sliding-type over free supports and rocking type (with a rocking pin) over the fixed support, may be used. Alternatively, neoprene bearings (pads or strips or laminated type) may also be used.

(iii) For spans in excess of about 15 m, metallic rocker type bearing (top-plate appropriately curved and a rocker-pin placed between the top and bottom plates) is provided over the fixed support and a roller-cum-rocker type bearing (one roller between top and bottom plates, or, if the load is high so that more rollers are needed, then a curved-soffit top-plate (called saddle plate) placed on a flat intermediate-plate (with rocking pin in between) seated on rollers that rest on a flat bottom-plate) is provided over the free supports. Alternatively, suitably designed laminated neoprene bearings may be provided.

NOTE Even for short simple span decks, placed across flexible cantilever tips mentioned in (i) above, full-fledged rocker and roller-rocker metallic bearings [described in (iii) above] are advisable because of considerable deck-movement and deflection (and consequent rotation) at the tips of the long cantilevers. Alternative types of bearings may be used, but these normally would not include the laminated neoprene bearings if the vertical reaction is low and deformations high.

Types of Bearings

Basically two types of bearings are used. *Rocker* (i.e., hinge or fixed) type which permits only rotation—acting as a moment release. *Rocker-cum-roller* (i.e., free) type which permits rotation as well as translation-acting as a moment as well as a thrust release. From the materials point of view, these bearings can be made from metal, rubber, metal and elastomer and even concrete.

Metallic Bearings

These are usually made of mild steel and cast steel. Leadsheets, phosphorbronze and stainless steel are used sometimes for their respective special merits. Metallic free bearings can have the advantage of generating small to very small horizontal force because of low friction coefficient against sliding and very low against rolling. When the sliding or rolling surfaces are chemically coated with teflon (a rubber synthetic), the friction coefficient reduces even further. *Meehanite* bearings are made of cast steel of special tensile strength with special resistance to vibration and wear. Armoured steel bearings are basically of mild steel but specially treated so as to harden the surface to a depth of about 10 mm. With this costly treatment, the load carrying capacity of a roller is simultaneously improved. Load, in tons, carried by a forged steel roller placed between plain surfaces, as per Hertz's formula, approximately works out to 6.25 times the product of its length and diameter in inches. This carrying capacity reduces with increase in the number of rollers in the assembly. Hi-Load bearings, marketed initially in the UK, are made of a special high quality steel and can carry a load about 8 times that of the usual forged steel roller of the same diameter and length. (In mild steel bearings the roller is generally made of forged railway axles.)

Rubber Bearings

These are of natural rubber or synthetic rubber (neoprene), reinforced by impregnated mild steel plates, and have been widely used as bridge bearings. These elastomeric bearings are cheap, easy to maintain, and are good in shear under compression. However, on their own, they are not suitable where compressive load is low while rotation is high (e.g., under high torsion conditions) because they accept rotation only under significant load.

Working Principle of Neoprene Bearings Reinforced With Restraining Steel Plates (Laminated Neoprene Bearings)

When subjected to loads and/or displacement, an individual elastomer layer distorts (Fig. 17.1), tangential shear stresses occur in the plane of contact of the layer with its support, which depend not only on the magnitude of the loads applied but also on the shape of the layer—plan surface and lateral exposed surface, on the nature of the elastomer, the rate at which the loads are applied, and on the temperature. These tangential stresses are manifested by a tendency for inter-granular sliding, which is opposed by the adherence to the elastomer layer of the steel plates forming the reinforcing laminations. With equal thicknesses of elastomer, these plates have the further advantage of either reducing the set due to the normal load or increasing the admissible load.

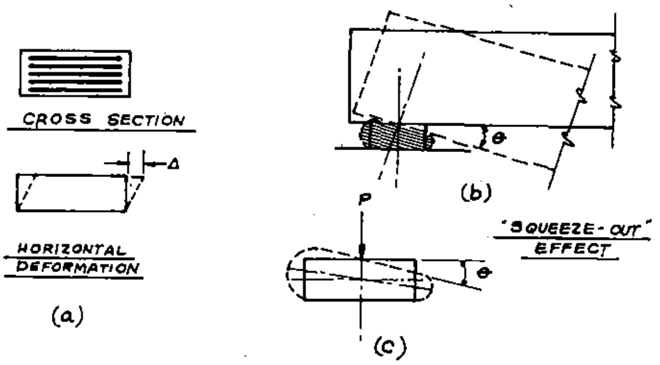


Fig. 17.1 *Elastomeric bearing*

In most countries regulations exist for calculating the stresses and distortion of the laminated elastomeric bearing pads and lay down the allowable limiting values which must not be exceeded.

Elastomer is the trade name of a synthetic rubber (CR or Neoprene), developed by Du-Pont Company. Mere solid rubber blocks cannot be used on account of 'squeeze-out' effect, shear failure and relatively early ageing problems. Interleaved layers of neoprene and steel-plates, bonded together, overcome these disadvantages. These bearings can be designed to be sufficiently soft horizontally so as to impose relatively small horizontal forces on the supports (= shear rating of the bearing × deck movement above it), but sufficiently stiff vertically to prevent appreciable changes in their height under variable loads. Rubber (and neoprene) are specified in terms of its 'shore' hardness. This is a measure of the penetration into the rubber by an indenting apparatus under a specified load and is, therefore, related to the elastic modulus of the rubber. For instance, a 60 shore hardness can correspond to an elastic modulus of 44.5 kg/cm^2, a shear modulus of 10.6 kg/cm^2, a bulk modulus of 20,000 kg/cm^2, and a maximum permissible shear strain of 0.7. The stiffness of rubber in compression depends on the *shape-factor* of its individual layer. This is the ratio of one loaded area to the total force-free (i.e., free-to-bulge) area of the individual layer. For stability, the least plan dimension should not be less than about four times the thickness of the bearing. Owing to the possibility of their deterioration with age and ozone attack, provision should be made in the bridge structure for replacement of the elastomeric bearings after about 25 years. Often it is advantageous to relieve these bearings after a few months of installation (or at least immediately after prestressing of a prestressed concrete deck) by momentarily jacking-up the deck so that the pads rebound back to their unstrained condition, then lowering the deck back on them. In this way they need be designed only for the balance movement (horizontal shear deformation) for permanent condition.

Elastomer and the Reinforcing Steel Plates in a Restrained (i.e., Laminated) Neoprene Bearing or a Rubber Bearing

- *Elastomer* The elastomer used for the manufacture of standard neoprene bearing pads is polychloroprene. This product was chosen, not only for its remarkable resistance to ageing, but also for its excellent behaviour in the presence of aggressive agents such as ozone, mineral oils or petrol, solvents and ultraviolet rays.

  Nevertheless, in cases where a particular specification so requires, natural rubber can be substituted for polychloroprene, and the inclusion, before vulcanisation, of certain additives, such as anti-oxidizing and anti-ozonizing agents, improves its resistance to ageing. It should be noted that several national regulations specify the use of one or the other or both these elastomers as basic components for the mixture used in the manufacture of these bearings.

  Design and production of special elastomer mixes can also be envisaged in order to meet the particular requirements of certain specifications—increased resistance to cold, and/or hydrocarbons, improved mechanical characteristics, etc.
- *Reinforcement Plates* The steel reinforcement sheets are of mild steel, conforming to current international regulations and norms. In the case of specific requirements, the thickness of the plates can be adapted to the required values.

  When used in an aggressive environment—maritime zones, corrosive chemical environments, etc., it is strongly recommended to use either fully-embedded bearings, or semi-embedded bearings with stainless steel reinforcement plates. The former are formed by vulcanizing the complete assembly of all the layers of neoprene and steel laminates in one single operation. Bonding together of individually vulcanized elements (in order later to form the total sandwich assembly) does not give as dependable a bearing as by the fully embedded method, and should not be permitted anymore.

Metal, rubber/elastomer and PTFE bearings combine all the advantages of each component. The so-called *pot* bearings (for example, the early German *Rota* and *Rotaflon* bearings, and Freyssinet's *Tetron disc* bearings D3 Series),

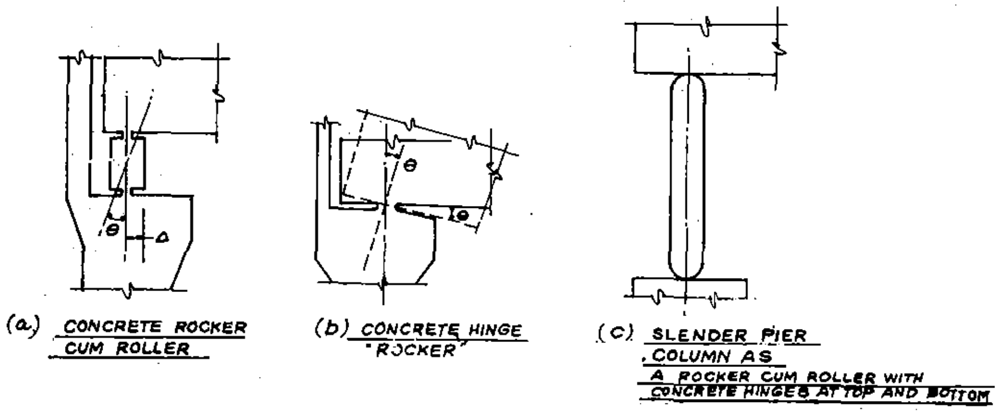


Fig. 17.2 *RC bearing*

essentially comprise of a circular steel pot, topped by a plunger steel plate, enclosing a round layer of high grade rubber (elastomer) in between. While rotation (i.e., rocking or moment-release) is attained by squeeze-deformation of the entrapped rubber medium that permits the plunger to rock on it in the pot, the movement in any direction is attained by low-friction-sliding between the plunger and the top plate on it. Three piece spherical PTFE bearings (Freyssinet's spherical *Tetron S3 series* bearings) allow movement in any direction and rotation about any axis through low friction contact surfaces. Three piece cylindrical PTFE bearing allows movement in any direction and rotation about the cylinder axis through low friction contact surfaces. Combined PTFE and laminated (i.e., plate-reinforced) neoprene bearings have been developed to allow high translation (through low friction sliding) and the required rotation (through deformation of plate-reinforced neoprene).

Concrete bearings and hinges are sometimes used for economy, but can be tricky in the execution of their construction. These (see Fig. 17.2) are usually in the form of shallow-necked hinges acting as rockers. Free bearings have two narrow throats (i.e., necks) and sometimes are of the full height of the column.

The advantages of concrete bearings are: low cost and little maintenance. The disadvantages are that they call for extreme care in construction, obstruct free movement of the deck, and create secondary stresses due to stiffness, require more room to construct, and suffer from an almost unknown internal stress distribution. The throat-width is designed so that under maximum working load the maximum compressive concrete stress can reach many times its 28-day crushing strength.

For fixed bearings, only one throat is provided. Guidelines for design of circular concrete hinges have been published by the UK DOE (as mentioned earlier). Reference may also be made to 'Design of Concrete Hinges' by Sims and Bridle in *Concrete and Constructional Engineering*, August, 1964. Details for design of linear concrete hinges are given ahead in the present chapter.

17.5 PRACTICAL CONSIDERATIONS IN THE SPECIFICATION, DESIGN, MANUFACTURE AND QUALITY CONTROL OF MECHANICAL BRIDGE BEARINGS

The requirement to build structures more economically led not only to improved techniques in the construction and design of bridges, but also to more sophisticated and advanced designs of bearings. Whilst some of the more simple bearings continue to be manufactured by general fabricators, most bearings are now supplied by specialist manufacturers.

Bearings constitute only a small fraction of the total cost of most structures, but the costs of malfunction and any subsequent rectification are high. It is therefore critical that serious attention is given to their selection and use. It is important for bridge designers and contractors to appreciate the extent to which bearing designs have advanced over the past 20 to 30 years. The principal reasons for the advances are listed below:

(i) Increasingly longer bridges and spans, often continuous, resulting in increased bearing loadings and movements.
(ii) The increasing occurrence of curved and skewed structures resulting in more complex movement and load distribution.
(iii) The availability of new materials such as polytetrafluoroethylene (PTFE).

Developments

The four developments which have had the greatest influence in the improvement of bearing designs since the Second World War have been:

(i) PTFE
(ii) Rubber pot and spherical bearings
(iii) High strength roller steels, and
(iv) Laminated elastomeric bearings.

PTFE

The copolymer polytetrafluoroethylene, PTFE, has excellent low frictional properties when properly designed into a sliding bearing assembly. In laboratory tests, coefficient of friction values as low as 0.002 have been achieved, which is 1/100th of the value which can be expected from lubricated bronze against steel. This compares well with roller bearings but with the added advantage that movement in any horizontal direction is possible.

Three basic principles should be observed when utilising PTFE:

- The mating surface must be smooth and flat or of the same curvature as the PTFE—and remain so in service. For this reason, polished stainless steel is the most commonly used mating surface. However, anodised aluminium, hard chromium plate (totally free of porosity) and acetal resin have all proven satisfactory. PTFE on PTFE is unsuitable.
- The PTFE should be mechanically retained on a rigid backing plate. Bonding PTFE to a backing plate is totally inadequate for most practical purposes. The PTFE sheet should be retained in a recess. When correctly designed, the recess will : (a) prevent excessive creep of the PTFE, and (b) secure the sheet and prevent its being inched out of the bearing due to structural movement.

- A self-aligning feature should be provided within the bearing assembly. Some misalignment will inevitably occur during service as a result of construction tolerances, settlement, creep, shrinkage, live loading, etc. Whilst PTFE has a small degree of elasticity, this is normally insufficient to prevent excessive edge loading due to misalignment. The alignment device should be such as to avoid high edge stresses on the internal (e.g., PTFE) and external (e.g., concrete) members.

Rubber Pot and Spherical Bearings

Typical rubber pot and spherical bearings are shown in Figs. 17.3 and 17.4.

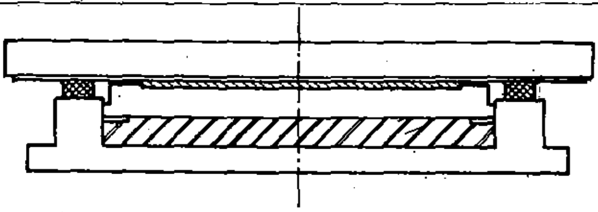


Fig. 17.3 *Rubber 'Pot' bearing (disc bearing)*

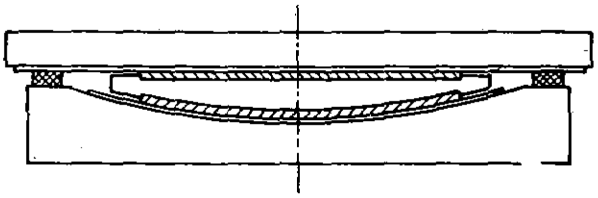


Fig. 17.4 *Spherical bearing*

In a rubber pot bearing the elastomer within the pot can be assumed to act as a fluid to accommodate rotation. The spherical bearing, utilising PTFE convex or concave surfaces, is self-aligning on the ball and socket principle. Both types of bearing have proven very successful throughout the world. The only differences in performance are with regard to torsional resistance under rotation and the degree of rotation available. The rubber pot bearing has a lower resistance to rotation for very small angles (say up to approximately 0.01 radians) but becomes increasingly more resistant with increasing rotation angles. The shperical bearing has a constant resistance to the friction in the sliding surfaces and will permit relatively large rotation angles.

High Strength Corrosion Resistant Roller Steels

Long term problems through fatigue and corrosion of rollers and plates have rendered many roller bearings ineffective. The commonly used materials have been forged railway axle steel, cast steel and mild steel, which would be designed in accordance with established codes. In practical terms, the load capacity of these bearings is limited to approximately 4 MN (900 kips). With the alloy used (for example) in the Glacier Cygnus roller bearings, the dimensions reduce considerably. The dimensions of a 4 MN (900 kips) roller would be:

	Mild Steel	Glacier Cygnus
Roller diameter (mm)	340	120
Roller length (mm)	1530	540

In this example, the volume of the mild steel roller is approximately 23 times that of the proprietary type. The cost savings result primarily from reduced machining times and the ease of handling of smaller components. Care must be taken in the selection of special steels and their heat treatment as embrittlement of very hard rollers has caused problems in service in the past.

Laminated Neoprene Bearings

See details ahead.

Specifying Bearing Requirements

To ensure a satisfactory proposal a bridge designer should specify clearly what his requirements are. Table 17.1 shows a typical bridge bearing performance schedule stating the important data required by the bearing manufacturer to select the most suitable bearing. This provides a basis for specification, but any special requirements should be indicated separately, e.g., abnormal high or low temperatures, abnormal construction loadings, etc. It is most important to give the bearing manufacturer drawings or sketches showing a plan view of the structure plus salient points such as space limitations, skew angles, etc.

Table 17.1 *Bridge Bearing Performance Schedule*

Bearing design data		
Bearing reference		
Quantity		
		Units
Load per bearing	$^*V_{max}$	kN
	$^*H_x$	kN
	$^*H_y$	kN
	V_{perm}	kN
	H_x	kN
	H_y	kN
	V_{min}	kN
	H_x	kN
	H_y	kN
Movements	$\pm ex$	mm
	$\pm ey$	mm
	$\pm \alpha x$	rads.
	$\pm \alpha y$	rads.

(Contd.)

Table 17.1 *(Contd.)*

Bearing interfaces

Maximum dimensions	Upper	mm	
	Lower	mm	
	Height	mm	
	Upper material		
	Lower material		
Allowable contact stress	Upper	Mean	N/mm^2
		Peak	N/mm^2
	Lower	Mean	N/mm^2
		Peak	N/mm^2
Fixing method	Upper**		
	Lower**		

* Serviceability and ultimate limit states should be quoted together with any special considerations, e.g. earthquake.

** e.g., bolts in sockets, friction only, etc.

Symbols used

V_{max} = maximum vertical load
$V_{perm.}$ = permanent vertical load
V_{min} = minimum vertical load
H_x = longitudinal horizontal load
H_y = transverse horizontal load
ex = longitudinal movement
ey = transverse movement .
αx = rotation about a longitudinal axis
αy = rotation about a transverse axis

NOTE The following details should also be included in this table:

(i) Drawings showing space available, skew angles, etc.
(ii) Bearing pre-set as appropriate
(iii) Any special considerations affecting bearing designs.

17.6 LESSONS FROM SOME ACTUAL DISTRESS EXPERIENCES[1]

(a) Bearing failure can result from a number of causes (e.g., damage or displacement following an accident, attack by chemicals, fire, and corrosion of contact surfaces) but probably the greatest cause of bearing malfunction, particularly of modern bearings, is due to inadequate or improper installation. It is not unknown for such simple looking bearings to be installed 90° out of phase or even upside down ! It cannot be stressed too strongly that care in the installation of bearings is of the utmost importance.

(b) Bridges are usually designed with an expected life in excess of 70 years. Modern bearings and bearing materials have not been proved in service for this length of time so it is advisable to make provision in the design of bridges for bearing replacement, should this be found to be necessary. Facilities for correcting the effects of differential settlements, etc., should be provided unless the structure has been designed to accommodate such effects.

(c) Regular inspection of the bearings should be made so that any potential trouble is detected before serious damage is done to the structure. There should be adequate space around bearings to allow for inspection and maintenance in service. In certain circumstances, such as when piers or abutments are high or over water, it may be advisable to incorporate some form of travelling staging in the bridge design to facilitate inspection.

(d) Elastomeric bearings will take a considerable amount of maltreatment before failure unless grossly inferior materials are used. However, localised overloading due, for example, to uneven seating can cause breakdown of the bond between elastomer and steel reinforcing plates. Unreinforced elastomer strips can squeeze or work their way out under certain circumstances. Small seating plinths can disintegrate under shear forces generated by elastomer bearings (the seatings should extend at least 50 mm beyond the edge of the bearing, preferably 100 mm).

(e) Disintegration of poorly prepared bearing seatings (pedestals) is one of the most common causes of bearing failure. This problem has recently been highlighted at the Gravelly Hill motorway inter-change outside Birmingham, England. Here, the bearing seatings have disintegrated and allowed the deck-support beams to drop, causing tension cracks in the locally unsupported deck-slab above.

(f) At another project, it is thought that incorrectly proportioned constituents (too much hardener plus a small quantity of water in the aggregate) led to the failure of 2″ high epoxy resin bearing plinths when the precast concrete beams were lowered onto the bearings.

(g) Incorrect installation procedures led to the failure of bearings supporting a viaduct over a river estuary. Here, large mechanical bearings were to be set on 12 mm thick pads of polyester resin mortar with a sheet of polythene placed on top of the mortar bed to break the bond between the bearing and the mortar. The mortar was domed, the intention being that surplus material would squeeze out when the fixing bolts were tightened down. In practice, the large quantity of resin mortar needed for each bearing required that it be made up in a number of mixes, and consequently the material could not be considered as entirely homogeneous. On removing the damaged bearings it was found that the polythene sheeting had unevenly curled. Both these results led to a non-uniform support to the bearing, causing failure.

(h) Leaking expansion joints can lead to corrosion of metal bearings. Unsuitable materials can give rise to problems. Many of the 18500 sliding rocker bearings installed in the Midland Links viaduct (UK) are not functioning as they should. The bearings are made of three rolled steel plates, the middle one heavily chamfered to allow the top plate to rotate. The steel deck beams rest directly on the top plate with no special sliding medium at the steel to steel interface apart from an initial coating of molybdenum disulphide. Some of the bearings have seized and those that still slide do so very reluctantly. Attempts to introduce lubricant between the sliding surfaces have proved ineffective.[2] This malfunction of rockers not rocking and rollers not rolling is a common feature in such steel bearings.

(i) In a similar manner, the steel deck beams of Vauxhall Bridge over the River Thames in London, built about 1906, rested directly on steel plates bedded on cill stones. Over the years these corroded and seized to the beams. Movement of the deck caused the front of the cill stones to break away. In 1976 the steel bearing plates were replaced by laminated rubber bearings set on new precast concrete bed stones.

(j) The abutment bearings of Wandsworth Bridge over the River Thames in London consisted of large knuckle leaf bearings supported on a bank of four flat sided forged steel (cut) rollers tied together with side bars bolted to each roller. The rollers ran on a bottom casting. As the bearings were subject to uplift, the lower casting of the leaf bearing was tied down to the bottom casting by four $1\,^1/_2$" diameter bolts which passed through slotted holes in the middle, or lower leaf bearing, casting. The bottom casting in turn was bolted down to the concrete abutment bearing shelf. The bridge was built in the late thirties and inspection of the bearings in 1973 indicated that although the main castings and forged steel knuckle pins were in good condition, the forged steel rollers were badly corroded with no sign of any lubrication having been applied or any protection against the entry of dirt or moisture. Several of the side bars had come adrift due to corrosion of the fixing bolts and a number of the tie-down bolts had broken or bent due to the heads binding on the intermediate casting. The bearings have subsequently been replaced by steel rocker bearings incorporating a PTFE/stainless steel sliding element. These have been set on new bearing plinths. No provision had been made for an expansion joint in the deck surfacing, which consequently cracked at the abutment, allowing water to penetrate down to the bearings.

(k) Other problems that have come to light include roller bearings which have overrun their design travel so that the gear pinions ran off the end of the guidance rack and were sheared off when trying to re-engage on their return; end flanges sheared off rollers due to insufficient allowance for side thrust on these bearings. Compatibility of steelwork fabrication with the drawings is necessary if the bearings are to function in accordance with the design.

(l) Replacing bearings can be a very difficult operation unless suitable provision has been made in the design of the bridge structure for proper access to the bearings and for jacking of the bridge-deck to be undertaken. Long[3] has dealt with the problems of replacing bridge bearings.

17.7 STRUCTURAL DESIGN OF VARIOUS TYPES OF BEARINGS

(based on BS 5400, Section 9.1, 1983)

Definitions

Refer Fig. 17.5.

Elastomer A compound containing natural or chloroprene rubber with properties similar to those of rubber.

Roller bearing A bearing consisting essentially of one or more steel rollers between parallel upper and lower steel plates [see Figs. 17.5(a), (b)].

Rocker bearing A bearing consisting essentially of a curved surface in contact with a flat or curved surface and constrained to prevent relative horizontal movement. The curved surfaces may be cylindrical or spherical. [Figs. 17.5(c), and (d)]. Rocker bearings permit rotation by rolling of one part on another.

Knuckle bearing A bearing consisting essentially of two or more members with mating curved surfaces. The curved surfaces may be cylindrical or spherical. [Figs. 17.5(e), (g) and (h)]. Knuckle bearings permit rotation by sliding of one part on another.

Leaf bearing A bearing consisting essentially of a pin passing through a number of interleaved plates fixed alternately to the upper and lower outer bearing plates. [Fig. 17.5(f)].

Sliding bearing A bearing consisting essentially of two surfaces sliding one on the other. [Fig. 17.5(i)].

Elastomeric bearing A bearing comprising of a block of elastomer that may be reinforced internally with steel plates (steel laminates)—which make it a laminated or restrained elastomeric bearing.

Laminated bearing An elastomeric bearing reinforced with steel plates [Fig. 17.5(j)].

Plain pad bearing An unreinforced elastomeric bearing.

Strip bearing A plain pad bearing for which the length is at least ten times the width.

Pot bearing A bearing consisting essentially of a metal piston supported by a disc of unreinforced elastomer that is confined within a metal cylinder or 'pot'. [Fig. 17.5(k)].

Symbols

The symbols used are as follows:

- A overall plan area of elastomeric bearing
- A_e effective plan area of elastomeric bearing
- A_l reduced effective plan area of elastomeric bearing
- b overall width of bearing (the shorter dimension of a rectangular bearing)
- b_c effective width of elastomeric bearing
- E modulus of elasticity
- E_b bulk modulus of elastomer
- G shear modulus of elastomer
- H horizontal force
- k a factor
- l overall length of bearing (the longer dimension of a rectangular bearing)

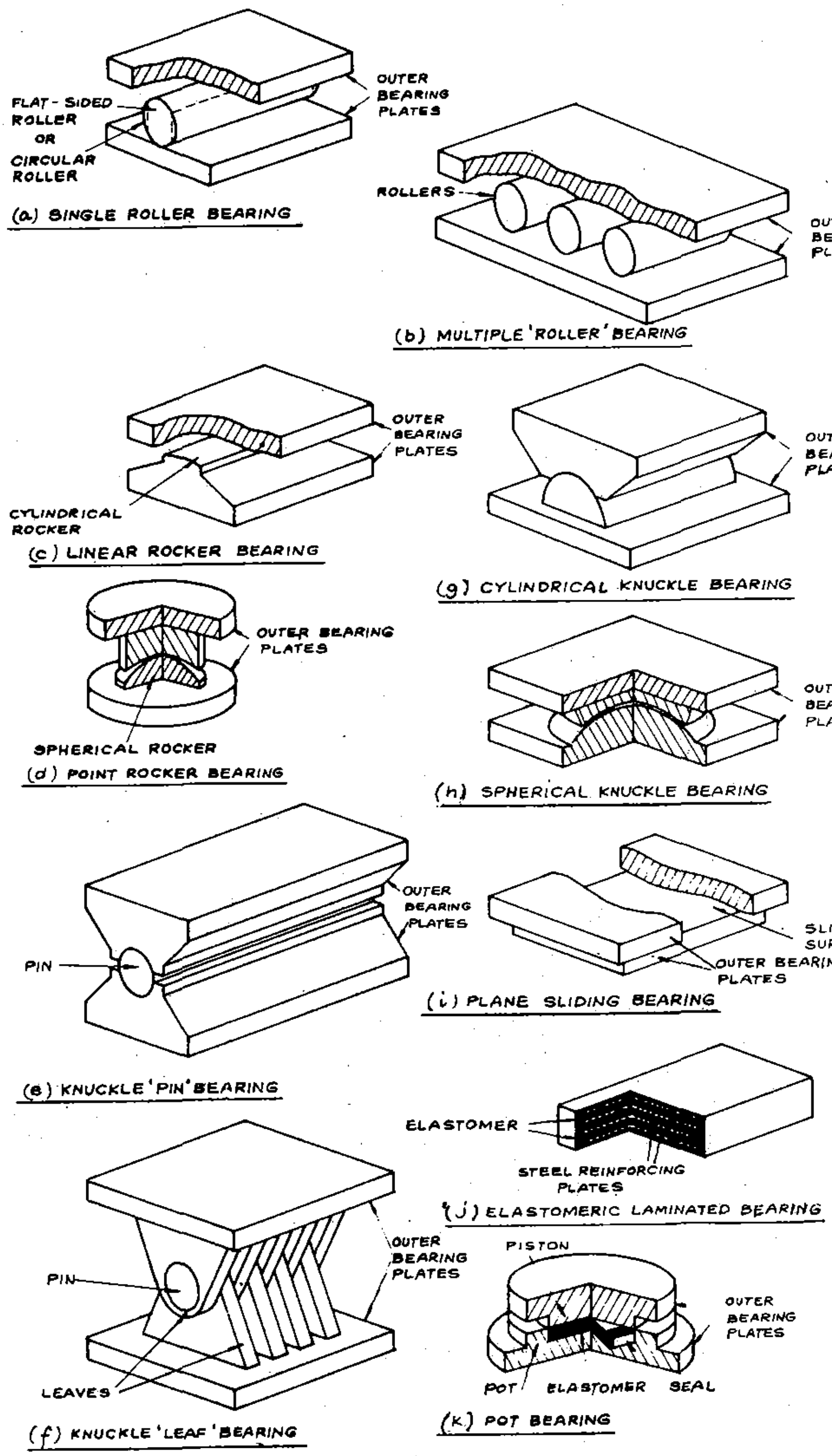


Fig. 17.5 *Types of bearings*

l_e effective length of elastomeric bearing
l_p force-free perimeter of elastomeric bearing
Q^* design loads
R radius of cylinder or sphere or convex surface
R_1 radius of concave surface
S shape factor
S' shape factor of thickest elastomer layer
S^* design load effects
T minimum shade air temperature
t thickness of a plain pad or strip bearing
t_1, t_2 thickness of adjacent elastomer layers
t_e effective thickness of elastomer in compression
t_i thickness of an individual elastomer layer in a laminated bearing
t_q total thickness of elastomer in shear
V vertical design load effect
α_b angular rotation across width b of bearing
α_l angular rotation across length l of bearing
Δ total vertical deflection
δ vertical deflection of individual elastomer layer
δ_b maximum horizontal relative displacement of parts of bearing in the direction of dimension b of the bearing
δ_l maximum horizontal relative displacement of parts of bearing in the direction of dimension 'l' of the bearing
δ_r maximum resultant horizontal relative displacement of parts of bearing obtained by vectorial addition of δ_b and δ_l.
ϵ_c nominal strain in elastomer slab due to compressive loads
ϵ_q shear strain in elastomer slab due to translational movement
ϵ_t total nominal strain in elastomer slab
ϵ_α nominal strain in elastomer due to angular rotation
σ_u nominal ultimate tensile strength of material
σ_s stress in steel

NOTE It is essential that the units used for these symbols in the formulae are compatible with each other.

Function of Bearings

The function of bearings, as explained earlier, is to provide a connection to control the interaction of loadings and movements between parts of a structure, usually between superstructure and substructure.

A guide to the suitability of various types of bearings for different functions is given in Table 17.2. To achieve the required degree of freedom it may be necessary to combine the characteristics of different types of bearings, the resultant bearing as a whole providing the required movements and load resistance, e.g., a plane sliding bearing to allow translation with a pot bearing to provide for rotation. The basic features of the various types of bearings have been illustrated in Fig. 17.5.

Table 17.2 *Bearing Function*

Types of bearing	Translation permitted		Rotation permitted			Loading resisted		
	Longitudinal	Transverse	Longitudinal*	Transverse†	Plan	Vertical	Longitudinal	Transverse
Roller								
single cylindrical	√	×	√	×	×	√	×	S
multiple cylindrical	√	×	×	×	×	√	×	S
non-cylindrical	√	×	√	×	×	√	×	S
Rocker								
linear	×	×	√	×	×	√	√	S
point	×	×	√	√	√	√	√	√
Knuckle								
pin	×	×	√	×	×	√	√	S
leaf	×	×	√	×	×	√	√	√
cylindrical	×	S	√	×	×	√	√	S
spherical	×	×	√	√	√	√	√	√
Plane *sliding*	√	√	×	×	√	√	S	S
Elastomeric								
unreinforced	√	√	√	√	√	√	√	√
laminated	√	√	√	√	√	√	√	√
Pot	×	×	√	√	S	√	√	√
Guide								
longitudinal	√	×	√	S	×	×	×	√
transverse	×	√	S	√	×	×	√	×

* Rotation about transverse axis.
† Rotation about longitudinal axis.
Key
√ suitable
× not suitable
S special consideration required

Provision for Handling

Where necessary, suitable handling attachments should be provided on bearings to facilitate their handling.

Movement Restraint

Where restraints are required to restrict the transitional movement of a structure, either totally, partially, or in a selected direction, they may be provided as part of or separate from the bearings and normally take the form of dowels, keys or side restraints.

In each case the restraints should still allow freedom of movement in the desired direction(s). The forces generated by the restraints should be considered in the design of the bearings and their connections and in the design of the structure. Where reliance is placed on friction to resist these forces, the lower bound value of friction coefficients obtained from available test data (appropriate to the surface condition in service) should be assumed.

Where bearing replacement may be required during the life of a structure, the provision of a restraint (e.g., dowels) through the bearing may cause difficulties, and alternative location of the restraints should be considered.

Uplift

If uplift can occur, bearings and their fixings should be designed to limit separation of the parts to a value agreed with the Engineer and to resist the consequent forces and actions.

Outer Bearing Plates or Spreader Plates

The outer plates of bearings should be so proportioned that concentrated loads are sufficiently distributed (or dispersed) to ensure that the permissible pressures on the adjacent bridge structure are not exceeded. The effective area for distributing a load may be taken as the contact area of the bearing member communicating the load to the plate plus the area within the uninterrupted dispersal lines drawn at a maximum of 60° to the line of application of the bearing reaction from the bearing contact area (Fig. 17.6). Where the adjacent structure is liable to deform significantly under load, the interaction of the structure and the bearing should be considered in the design of both.

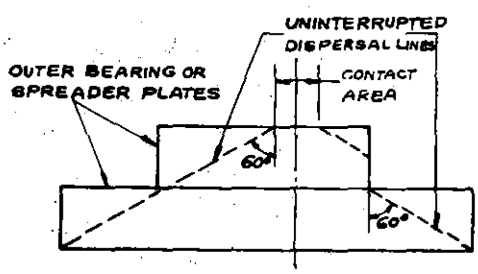


Fig. 17.6 *Load distribution through the metallic bearing plates*

Coefficient of Friction for Roller Bearings

For design purposes, the coefficient of friction for roller bearings should preferably be as given in Table 17.3(a).

Coefficient of Friction for Sliding Bearings

Recommended design coefficients of friction for bearings with stainless steel sliding on pure PTFE continuously lubricated are given in Table 17.3(b).

NOTE PTFE with lubricant contained in lubrication cavities (dimples) can be considered as continuously lubricated for the purposes of Table 17.3(b).

Table 17.3 (a) *Coefficient of friction of roller bearings*

	Type of roller bearing	Coefficient of friction
(a)	Roller bearing with one or two rollers in steel complying with BS 4360, or cast iron complying with BS 2789 with a hardness of 110 HB to 240 HB	0.03
(b)	Roller bearings as (a) but with more than two rollers	0.05
(c)	Single roller bearings with hardened steel contact surfaces with a hardness not less than 500 HB	0.02
(d)	Multiple roller bearings as (c)	0.03
(e)	Single roller bearings and bearing plates in special high tensile corrosion resistant steel hardened throughout with finely ground finish with a hardness not less than 350 HB	0.01
(f)	Multiple roller bearings as (e)	0.015

NOTE Values of hardness given in the above table are in accordance with BS 240:Part 1.

Table 17.3 (b) *Coefficient of friction for stainless steel sliding on pure PTFE continuously lubricated*

Bearing stress N/mm^2	Coefficient of friction
5	0.08
10	0.06
20	0.04
30 and over	0.03

NOTE

- Linear interpolation may be used for intermediate values. The load used for calculating the bearing stress should be that with which the coefficient of friction is being used.
- In the absence of test data, for design purposes, the coefficient of friction of pure unlubricated PTFE on stainless steel should be taken as twice the values given in Table 17.3. For PTFE sliding on any surface other than stainless steel, the coefficient of friction should be based on test data.

- The values given in Table 17.3(b) may be used for air temperatures down to $-24°C$.

Particular Recommendations for Roller and Rocker Bearings

(a) *Function* Roller bearings provide for transition in the direction of rolling only. Single rollers and rockers permit rotation about the line of contact, but multiple rollers require another element to provide for rotation, for example, a rocking saddle plate on top.

(b) *Curved surfaces* Any individual curved contact surface should have only one radius.

(c) *Surfaces in contract* Surfaces in contact should have the same nominal strength and hardness.

(d) *Length of rollers* The length of a roller should not be less than its diameter.

(e) *Guidance of rollers* Mechanical guidance should be provided to ensure that the axis of rolling is maintained in the desired orientation. Where gearing is used, the pitch circle diameter of the gear teeth should be the same as that of the rollers.

Allowable Loads on Steel and Cast Steel Roller and Rocker Bearings

(a) *Cylinder on Curved Surface* The design load effect per unit length on a cylinder of radius R running in a concave seating of redius R_1 should not exceed.

$$\frac{18\sigma_u^2}{E}\left(\frac{R_1 R}{R_1 - R}\right)$$

where, σ_u is the nominal ultimate tensile strength of the material

E is the modulus of elasticity of the material.

(b) *Cylinder on Flat Surface* The design load effect per unit length on a cylinder of radius R in contact with a flat surface should not exceed,

$$18R\sigma_u^2/E$$

where σ_u and E are as defined in (a) above.

(c) *Sphere in Spherical Seating* The vertical design load effect on a spherical surface of radius R in the concave seating of radius R_1 should not exceed,

$$\frac{170\sigma_u^3}{E^2}\left(\frac{R_1 R}{R_1 - R}\right)^2$$

where σ_u and E are as defined in (a) above.

(d) *Sphere on Flat Surface* The vertical design load effect on a spherical surface of radius R in contact with a flat surface should not exceed,

$$170R^2\sigma_u^3/E^2$$

where σ_u and E are as defined in (a) above.

Flat-sided (i.e., Cut) Rollers

If movement requirements permit, flat-sided rollers may be used. Such rollers should be symmetrical about the vertical plane passing through the centre. The minimum width should not be less than one-third of the diameter or such that the bearing contact does not fall outside the middle third of the rolling surface when the roller is at the extremes of movements. Flat-sided rollers can be mounted at closer centres than the circular rollers of the same load capacity, resulting in more compact bearings. However, they are not recommended in seismic areas.

Non-cylindrical Rollers

A single roller type of bearing with differing radii for the upper and lower curved surfaces of the roller can be designed using the appropriate expression given in (a) and (b) above. In all such designs, careful consideration should be given to the overall stability of the bearing. In particular, where the movement of the structure causes the line joining the upper and lower bearing contact points to depart from the vertical. a check should be made to ensure that the resulting horizontal force is resisted. Where the design of the bearing is such that horizontal movement is accompanied by a small vertical movement, the vertical movement should always be upward for horizontal movement on either side of the central position to ensure stability of the structure.

Multiple Rollers

For bearings having more than two rollers, the limiting values of design load effect should be taken as two-thirds of the value given by the expression in (b) above. This is so because more than two rollers may not be as fully effective as one or at best two in tandem might.

Particular Recommendations for Plane Sliding Bearings

(a) *Function* Plane sliding bearings normally provide for translation only; rotation can be permitted only with adequate provisions/incorporations.

(b) *Arrangement of Sliding Surfaces* Whenever possible, sliding bearings should have the larger of the sliding surfaces positioned above the smaller so that the sliding surfaces are kept clean.

(c) *Prevention of Rotation* Flat sliding surfaces should not be used to accommodate rotation other than about an axis perpendicular to the plane of sliding (e.g., in-plan rotation). Other provisions should be made for rotation about an axis in the plane of sliding.

Particular Recommendations for Sliding Elements with PTFE

(a) *Surfaces Mating with PTFE* Surfaces mating with PTFE should normally be stainless steel or hard anodized aluminium alloy; in all cases they should be harder than the PTFE and be corrosion resistant. The mating surface should normally form the upper component and overlap the PTFE at the extremes of movement.

(b) *Location of PTFE*

(i) *General* PTFE should be located either by confinement or by bonding. In either case it is essential that it is backed by a metal plate. The rigidity of this plate should be such that the plate retains its unloaded shape and resists shear forces under all loading conditions. The PTFE should be bonded or mechanically restrained in situations where the sliding surfaces can separate.

(ii) *Confined PTFE* Confined PTFE should be recessed into the metal backing plate. The shoulders of the recess should be sharp and square to restrict the flow of PTFE. The thickness of the PTFE and its protrusion from the recess should be related to its maximum plan dimension in accordance with Table 17.4.

Table 17.4 *Dimensions of confined PTFE*

Maximum dimension of PTFE (diameter or diagonal) (mm)	Minimum Th ckness (mm)	Maximum projection above races (mm)
≤ 600	4.5	2.0
$> 600, \leq 1200$	5.0	2.5
$> 1200, \leq 1500$	6.0	3.0

(iii) *Bonded PTFE* The thickness of the bonded PTFE sheet should be related to its maximum plan dimension in accordance with Table 17.5.

Table 17.5 *Thickness of bonded PTFE*

Maximum dimension of PTFE (diameter or diagonal) (mm)	Minimum thickness (mm)
≤ 600	1.0
$> 600, \leq 1200$ (max.)	1.5

(c) *Allowable Sliding Bearing Pressures for Pure PTFE*

(i) *Maximum sliding contact pressures* For pure PTFE in bearings, the average pressure and the extreme fibre pressure should not exceed the values given in Table 17.6 ahead.

(ii) *Contact area* For calculation of pressures, the contact surface may be taken as the gross area of the PTFE without deduction for the area occupied by lubrication cavities. In the case of curved surfaces, the gross area should be taken as the projected area of the contact surface.

Table 17.6 *Allowable sliding bearing pressures for pure PTFE*

Design load effects	Maximum average Contact pressure		Maximum extreme fibre pressure	
	Bonded PTFE (N/mm^2)	Confined PTFE (N/mm^2)	Bonded PTFE (N/mm^2)	Confined PTFE (N/mm^2)
Permanent design load effects	20	30	25	37.5
All design load effects	30	45	37.5	55

(d) *Thickness of Stainless Steel Sliding Surfaces* The thickness of the stainless steel sheet should be related to the difference between the PTFE and stainless steel dimension in the direction of movement in accordance with Table 17.7

Table 17.7 *Thickness of stainless steel sheet*

Dimensional difference between PTFE and stainless steel (mm)	Minimum thickness of stainless steel (mm)
≤ 300	1.5
$> 300, \leq 500$	2.0
$> 500, \leq 1500$	3.0

NOTE A dimensional difference in excess of 1500 mm requires special consideration.

(e) *Fixing of Stainless Steel Sheet*

(i) Stainless steel sheet should be attached to its backing plate by continuous welding along the edges or by fasteners supplemented by either peripheral sealing or full area bonding. It is essential that the method adopted ensures that the stainless steel sheet remains flat throughout its service life and interface corrosion cannot occur. The method of attachment should be capable of resisting the frictional forces set up in the bearing in the serviceability limit state.

(ii) *Attachment by welding* The backing plate should extend beyond the edges of the stainless steel sheet to accommodate the weld and the two should be attached by a continuous fillet weld along the edges. The weld should not be proud of the stainless steel sheet.

(iii) *Attachment by fasteners* Corrosion resistant fastenings, compatible with the stainless steel, should be used for securing the edges of the stainless steel sheet. They should be provided at all corners and along the outside edge, outside the area of contact with the PTFE sliding surface, with a maximum spacing of,

150 mm, for sheet 1.5 mm thick
300 mm, for sheet 2.0 mm thick
600 mm, for sheet 3.0 mm thick

Particular Recommendations for Elastomeric Bearings

(i) *Function* Elastomeric bearings can accommodate translational movements in any direction and rotational movements about any axis by elastic deformation. They should not be used in tension or when rotation is high and vertical load small. (Design stipulations, given ahead, will automatically draw the line for defining the latter while trying to satisfy the constraints.)

(ii) *Basis of design* The basis of the design is that the elastomer is an elastic material, the deflection of which under a compressive load is influenced by its shape (shape factor). Where reinforcing plates are included in the bearing, they should be bonded to the elastomer to prevent any relative movement at the steel/elastomer interface. Shape factor refers to an elastomer layer, not to the total assembly.

(iii) *Design recommendations* The design of elastomeric bearings should be such that:

- Their geometry satisfies the following conditions:
  - (i) The maximum strain of the elastomer due to translational movement does not exceed the limits given in the para ahead on Shear Strain.
  - (ii) The thickness of plain pad or strip bearings should not be less then 9 mm, to cater for irregularities in the seating surface.
  - (iii) The cover of elastomer to the steel interleaving plates in laminated bearings should be a minimum of 4.5 mm to all edges that would otherwise be exposed and a minimum of 2 mm to the contact surfaces, these values may need to be increased if there is a possibility of serious biological or chemical attack.
- They can resist the applied loads without exceeding:
  - (i) the mean pressure on plain pad or strip bearings (given ahead).
  - (ii) the maximum strain at any point in laminated bearings (given ahead).
  - (iii) the tensile stresses in the reinforcing plates (given ahead).
  - (iv) the stability criteria (given ahead).
- Their design movements satisfy the following conditions:
  - (i) the vertical deflection (calculated ahead) does not exceed the value specified by the engineer (i.e., the design authority).
  - (ii) the rotation of the bearing does not allow separation at the contact surfaces between the bearing and the structure; this may be deemed to be satisfied if the recommendations given ahead are met.
  - (iii) the force exerted on the structure by the bearing (resisting translational movement), calculated as follows, does not exceed the value specified by the engineer.

    Shear resistance of elastomeric bearings: For elastomeric bearings where horizontal movement is accommodated by shear in the elastomer, the nominal horizontal force H due to expansion or contraction is given by the expression,

$$H = AG\delta_r/t_q$$

where

A is the actual plan area of the individual elastomer slabs

G is the shear modulus of the elastomer

δ_r is the maximum resultant horizontal relative displacement of parts of the bearing.

t_q is the total thickness of elastomer in shear.

Typical values of G are given in Table 17.8. An allowance of $\pm 20\%$ should be made in the calculated value of H to give the most adverse effect.

Table 17.8 *Typical elastomer moduli*

Nominal hardness*	Shear modulus, G (N/mm^2)	Bulk modulus, E_b (N/mm^2)
IRHD		
50	0.6	2000
60	0.9	
70	1.2	

* Values of hardness in the above table are in accordance with BS 903: Part A26.

For movements due to live load effects on railway bridges, the value of G should be doubled (rapid deformation). Due allowance should be made in the value of G for temperature variation.

- Either they do not slip under the applied forces when checked in accordance with the data given ahead under 'Fixing of bearings' or they are mechanically fixed to the structure above and below.

(iv) *Design limit state* Elastomeric bearings should be designed to meet the provisions at the serviceability limit state only.

Shear Strain

The shear strain ϵ_q of the elastomer due to translational movement should not exceed 0.7, as given by the expression,

$$\epsilon_q = \delta_r/t_q$$

where δ_r is the maximum resultant horizontal relative displacement of parts of the bearing obtained by vectorial addition of δ_b and δ_l

δ_b is the maximum horizontal relative displacement of parts of the bearing in the direction of dimension b of the bearing due to all design load effects (Fig. 17.7)

δ_l is the maximum horizontal relative displacement of parts of the bearing in the direction of dimension l of the bearing due to all design load effects (Fig. 17.7).

t_q is the total thickness of the elastomer in shear.

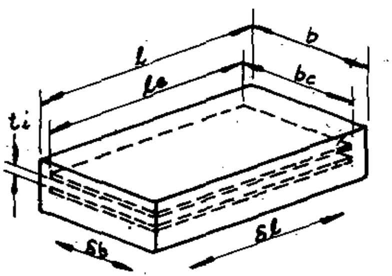


Fig. 17.7 *Elastomeric laminated bearing*

Shape Factor (S): Individual Layer of Elastomer

(i) The shape factor S is a means of taking account of the shape of the elastomer in strength and deflection calculations. It is the ratio of the effective plan area of an elastomeric layer to its force-free surface area (free to bulge) and is calculated as follows in (ii) to (iv).

Note that the factors associated with the effective thickness of the elastomer t_c in the expressions given in (ii) to (iv), allow for the fact that some slip will take place on faces restrained by friction only.

(ii) *Plain pad bearings* For plain pad bearings,

$$S = A/l_p t_c$$

where A is the overall plan area of the bearing

l_p is the force-free perimeter of the bearing, excluding that of any holes if these are not later effectively plugged

t_c is the effective thickness of elastomer in compression, which is taken as $1.8\ t$

t is the actual thickness of elastomer

Note that for a rectangular bearing without holes,

$$l_p = 2(l + b)$$

where l is the overall length of the bearing

b is the overall width of the bearing.

(iii) *Strip bearings* For strip bearings,

$$S = b/2t_e$$

where b and t_e are as defined in (ii) above.

(iv) *Laminated bearings* (Fig. 17.7) For laminated bearings, the shape factor S for each individual elastomer layer is given by the expression,

$$S = A_e/l_p t_e$$

where A_e is the effective plan area of the bearing, i.e., the plan area common to elastomer and steel plate, excluding the area of any holes if these are not later effectively plugged

l_p is as defined in (ii) above.

t_e is the effective thickness of an individual elastomer lamination in compression; it is taken as the actual thickness, t_i for inner layer, and $1.4t_i$ for outer layers.

t_i is the thickness of an individual elastomer layer

Note that for a rectangular bearing without holes,

$$A_e = l_e b_e$$
$$l_p = 2(l_e + b_e)$$

where l_e is the effective length of the bearing (= length of reinforcing plates)

b_e is the effective width of the bearing (= width of reinforcing plates)

Moduli of Elastomer

The shear modulus G should normally be obtained experimentally. Table 17.8 gives typical values of G and also an appropriate value for the bulk modulus E_b.

The variation of the shear modulus with low temperatures should be established by testing. For temperatures below 0°C, the values of G may, in the absence of test data, be taken as equal to the values in Table 17.8 multiplied by,

$$1 - \frac{T}{25}$$

where T is the minimum shade air temperature (in °C).

NOTE T is negative for temperatures below 0°C; the increased value of G applies only when variations in load and displacement take place at low temperature.

Design Pressure on Plain Pad and Strip Bearings

The mean design pressure (i.e., V/A) on a plain pad or strip bearing should not exceed GS or $5G$, whichever is the lesser,

where V is the vertical design load effect
A is the overall plan area of the bearing
G is the shear modulus of the elastomer
S is the shape factor of the elastomer slab (i.e., individual layer)

Maximum Design Strain in Laminated Bearings

At any point in the bearing the sum of the nominal strains due to all load effects, ϵ_t as given by the expression:

$$\epsilon_t = k(\epsilon_c + \epsilon_q + \epsilon_\alpha)$$

should not exceed 5.0 (see note at the end).

where k is a factor equal to 1.5 for live load effects 1.0 for all other effects (including wind and temperature),
ϵ_c is the nominal strain due to compressive loads where ϵ_c is given by the expression

$$\epsilon_c = 1.5V/GA_1S$$

ϵ_q is the shear strain due to translational movements as defined earlier.
ϵ_α is the nominal strain due to angular rotation, where ϵ_α is given by the expression,

$$\epsilon_\alpha = (b_e^2\alpha_b + l_e^2 \cdot \alpha_l)/2t_i\Sigma t_i$$

V and G are as defined earlier.
A_1 is the reduced effective plan area due to the loading effects, where A_1 is given by the expression

$$A_1 = A_e\left(1.0 - \frac{\delta_b}{b_e} - \frac{\delta_l}{l_e}\right)$$

A_e is as defined earlier
δ_b and δ_l are as defined earlier
b_e is the effective width of the bearing (Fig. 17.7)
l_e is the effective length of the bearing (Fig. 17.7)
S is the shape factor
α_b is the angle of rotation across the width b of the bearing (in radians)
α_l is the angle of rotation (if any) across the length l of the bearing (in radians)
t_i is the thickness of the individual layer of elastomer being checked
Σt_i is the total thickness of elastomer in the bearing

NOTE Value of 5.0 is an empirical value which has been found from fatigue tests on three types of elastomeric bearing to best fit the limiting criterion for a strain calculated by the method given here. It should not be taken to reflect the ultimate strain of the material.

Reinforcing Plate Thickness

To resist induced tensile stresses under load, the minimum thickness of the steel plates in a laminated bearing should be:

$$1.3V(t_1 + t_2)/A_1\sigma_s \text{ but not less than 2 mm}$$

where V and A_1 are as defined earlier.
t_1 and t_2 are the thickness of elastomer on either side of the plate
σ_s is the stress in the steel, which should be taken as not greater than the yield stress, nor greater than,
120 N/mm^2, for plates with holes
290 N/mm^2, for plates without holes

Stability

Elastomeric bearings will be stable if the recommendations of (i) and (ii) below are satisfied.

(i) *Plain pad and strip bearings.* For plain pad and strip bearings, the thickness should not exceed one-quarter of the least lateral dimension.

(ii) *Laminated bearings.* For laminated bearings the pressure V/A_1 should be less than $\{2b_eGS'/3\Sigma t_i\}$ This criterion will be satisfied automatically if

$$\Sigma t_i < b_e/4$$

where V, b_e, G, A_1 and Σt_i are as defined previously. S' is the shape factor for the thickest elastomer layer.

Vertical Deflection

(i) The vertical deflection of elastomeric bearings should be estimated from the expressions given ahead in (ii) to (iv). These expressions may be used to estimate the change in deflection between one-third of the total load and full load, with an accuracy of the order of ±25%.

NOTE (a) The actual deflection of bearing may include an initial bedding-down phase that can produce a deflection of approximately 2 mm. Thereafter, the stiffness of the bearing increases with increasing load. Where the vertical deflection under load is critical to the design of the structure, the stiffness of the bearing should be ascertained by tests. However, a variation of as much as ±20%

from the observed mean value may still occur. When a number of similar bearings are used at a support and the differential stiffness between the bearings is critical for the structure, a variation of compressive stiffness should be allowed for in the design, equal to either ±15% of the value estimated from (ii) to (iv) or ±15% of the mean value observed in tests.

(b) The calculations for the deflection of plain pad and strip bearings are likely to underestimate the deflection under permanent load and overestimate the deflection under transient loads.

(ii) *Plain pad bearings* The total vertical deflection of a plain pad bearing, Δ, is given by the expression,

$$\Delta = \frac{Vt}{5AGS^2} + \frac{Vt}{AE_b}$$

where V, G and S are as defined earlier
t and A are as defined earlier under 'Plain pad bearings'.
E_b is the bulk modulus of the elastomer

(iii) *Strip bearings* The total vertical deflection of a strip bearing, Δ, is given by the expression,

$$\Delta = Vt/5AGS^2$$

where V, G and S are as defined earlier
t and A are as defined earlier under 'Plain pad bearings'

(iv) *Laminated bearings* The total vertical deflection of a laminated bearing, Δ, is given by the expression,

$$\Delta = \Sigma\delta$$

where $\delta = \dfrac{Vt_i}{5A_eGS^2} + \dfrac{Vt_i}{A_eE_b}$

where
δ is the vertical deflection of an individual layer of elastomer
V, t_i, G and S are as defined earlier under 'Maximum design strain in laminated bearings'.
A_e is as defined earlier under 'Laminated bearings'.
E_b is the bulk modulus of the elastomer.

Rotational Limitation

The rotational limitation is satisfied if the recommendations (i) and (ii) given below are satisfied.

(i) *Plain pad and laminated bearings* For plain pad and laminated bearings, the total vertical deflection, Δ, should satisfy the expression:

$$\Delta > (b_e\alpha_b + l_e\alpha_l)/3$$

where b_e, α_b, l_e and α_l are as defined earlier.

(ii) *Strip bearings* For strip bearings, the total vertical deflection, Δ, should satisfy the expression:

$$\Delta > b_e\alpha_b/3$$

where b_e and α_b are as defined earlier.

Fixing of Bearings

If there is insufficient friction to prevent relative movement between the bearing and the structure under the most adverse loading conditions, positive means of location should be provided. Friction may be considered adequate if; under all loading conditions: numerically, $H < 0.1(V + 2A_1)$ and under permanent loads, $V/A_1 > (1 + b/l)$ for plain pad and strip bearings, and $V/A_1 > 2.0$ for laminated bearings.
Here all the terms and their units are as follows:
H is the design force exerted by the bearing to resist translational movement (in N)
V is the vertical design load effect (in N)
A_1 is the reduced effective plan area as defined earlier (in mm^2)
b is the overall width of the bearing (in mm) (Fig. 17.7)
l is the overall length of the bearing (in mm) (Fig. 17.7)

NOTE Positive means of location may limit the depth available for shear. This should be considered in the design of the bearing.

Particular Recommendations for Pot Bearings

(i) *Function* Pot bearings in themselves provide for rotational movements.

(ii) *Design* The stress in the elastomer in pot bearings due to the design load effects is limited by the effectiveness of the seal preventing it from extruding between the piston and the pot-wall, but it should not exceed 40 N/mm^2 at the serviceability limit state. The lateral pressure exerted on the confining cylinder walls resulting from vertical loading on the confined elastomer disc can be considered to be that produced by the disc acting as a fluid. Because details of pot bearings vary considerably and stress analysis is complex, their design should be verified by testing.

(iii) *Rotation* The rotation of pot bearings about a horizontal axis should be limited so that the vertical strain induced at the perimeter of the elastomeric pad, at the serviceability limit state, does not exceed 0.15.

NOTE The thickness and hardness of the elastomer have a direct relationship with the resistance of pot bearings to rotation, as does the friction between the piston and the pot. The latter is increased by increased force acting on the bearing. Sufficient test results should be available for a given elastomer stress, hardness and thickness to enable the resistance of the bearing to rotation to be calculated; otherwise, prototype tests should be made.

(iv) *Seal* A sealing device should be provided to prevent the elastomer extruding between the piston and the pot-wall. This seal should be effective under serviceability limit state loadings.

Particular Recommendations for Guides

1. *Function* Guides are used to constrain the movement of structures in a particular direction. They may be included in an independent guide bearing or may form a part of a bearing performing other functions.

2. *Sliding surfaces for guides*

(i) *PTFE Facing* Guides used for lateral restraint may be faced with unfilled or filled PTFE provided the frictional resistance to movement at the guides is either significantly smaller than that of the main bearing or the resulting frictional effects are taken into account. Commonly used materials for facing-guides are,

- unfilled PTFE
- PTFE filled with up to 25% by mass of glass fibres
- lead filled PTFE in a bronze matrix
- PTFE reinforced with a metal mesh

(a) *Lubrication* For this application (i.e., use in guides) lubrication of PTFE should not be considered to reduce friction.

(b) *Attachment* It is essential that all PTFE should be securely attached to the guides: reliance should not be placed on bonding alone for pure PTFE.

(ii) *Unfaced surfaces* For surfaces not intended to be in permanent contact, metal-to-metal contact may be permitted. The metal should be corrosion resistant.

3. *Allowable bearing pressure on guides*

(i) *PTFE* Under all serviceability design load effects, the average pressure on glass filled PTFE in guides should not exceed 45 N/mm^2, and on PTFE in a metal matrix 60 N/mm^2. Permissible values for other PTFE materials should be established by tests. In the absence of test data, the values for PTFE should be used.

For calculation of pressures, the contact surface may be taken as the gross area of the PTFE without deduction for the area occupied by any lubrication cavities (dimples).

(ii) *Bronze* At the serviceability limit state, the contact bearing stress for bronze should not exceed 30 N/mm^2.

17.8 DESIGN OF LINEAR CONCRETE HINGE (ROCKER) BEARING

Concrete hinges are simple and cheap to produce but require extremely careful construction and detail. They permit large rotations if they are constructed properly and accurately with correct dimensions. Such hinges do not require any corrosion protection and have the same life as that of the structure into which and along with which they are built. Such hinge bearings require no maintenance.

The design rules stipulated ahead are based on the experiments conducted at Stuttgart, West Germany, and the further work done at the EMPA (the Swiss Federal Laboratory for Testing Materials, etc.), Zurich.*

The functional form of a linear concrete hinge capable of rocking (rotating) about its length-axis, the directions of the three types of bursting forces (F_1, F_2 and F_3) set up in it, and the appropriate reinforcement details to resist these forces, are diagrammatically shown in the sketches in Fig. 17.8. The throat of the hinge (dimension a) should be narrow (generally 15 cm) so as to develop little resisting moment (M_{TT}) to rotation. For this purpose, the reinforcements against the three bursting forces should be adequate so as to permit a narrow throat of the hinge. A narrow throat and very low height of the hinge (generally about 2 cm), with circular curved faces all around, ensures an almost three dimensional confinement of hinge-concrete. This allows the hinge-concrete to accept even up to about 6 times its (standard 28-day cube) crushing compressive stress. Herein lies the secret of the working of a concrete hinge. Most of the cracking owing to rotation of permanent type and of the oscillating transient type heals itself owing to creep under the incumbent high compressive stress. Such hinges rarely require any reinforcements crossing the neck (along its longitudinal centreline) unless horizontal shear exceeds about one-eighth of the co-existing vertical load and/or longitudinal moment (M_{LL}) on the hinge causes tension after allowing for compression due to vertical load. In fact, ensuring accurate placement and positioning of such bars is not easy and it is advisable to create conditions whereby such reinforcement can be avoided. (Usually this is the case.)

It is reported that such hinges can rotate in opposite directions (oscillating rotation) without compromising safety. In such cases the entire throat cracks in effect, with part of its remaining closed at any one time. The joint surfaces which re-close again and again, remain fully effective. In the EMPA tests on concrete hinges of a large railway bridge (working load up to 450 tonnes on a throat plan-section of 15 cm × 70 cm) the hinge tolerated rotations of up to 0.012 radian millions of times. In all, 37 million rotational loadings of varying magnitudes were undertaken. After all that, the joint, under static load tests, did not show any sign of cracking even with a 900 tonne load (twice the working load) coupled with a rotation of 0.006 radian. The joint was brought to breaking point under an exceptional

* *References*: Betongelenke, F. Leonhardt: Vorlesungen-uber Massivbau Zweiter Teil. Sondefalle der Bemessung im Stahlbetonbau: F. Leonhardt and E. Monnig: Zweite Auflage, Springer-Verlag, Berlin-Heidelberg-New York, 1975; and the EMPA report on the subject.

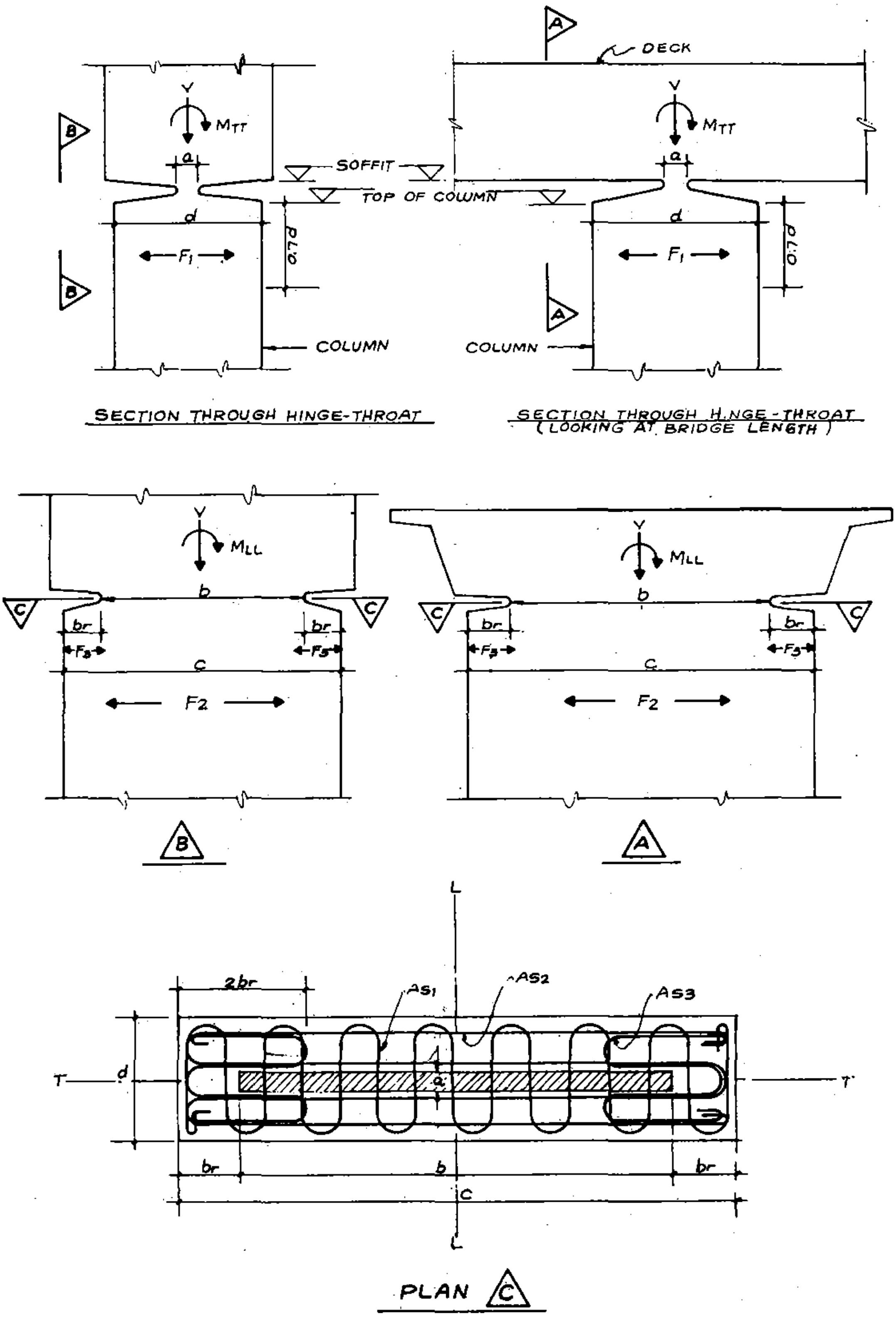

DECK
V
MTT
a
SOFFIT
TOP OF COLUMN
d
0.1d
F1
COLUMN
SECTION THROUGH HINGE-THROAT
SECTION THROUGH HINGE-THROAT (LOOKING AT BRIDGE LENGTH)
MLL
b
br
F3
c
F2
L
2br
AS1
AS2
AS3
T
PLAN

Fig. 17.8

rotation of 0.500 radian (coupled with a load of 250 tonnes). The experiments, therefore, indicate that such joints are capable of resisting large dynamic loadings coexisting with large rotations.

Geometry of the Hinge Throat

Following dimensional constraints are stipulated (see Figs. 17.8 and 17.9)

- $a \leq 0.3d$, generally 15 cm.
- $b_r \geq 0.7a$, but never less than 5 cm.
- $t \leq 0.2a$, but never more than 2 cm.
- $r = 0.5t$
- Tangent $\beta \leq 0.1$
- For details see Figs. 17.8 and 17.9.

Design Steps

Step 1 *Estimate*: V_p, V_t, $V_{\max}$, $V_{\min}$, M_{LL}, θ_p, θ_t, H_L and H_T at the hinge,

where V_p = permanent vertical load on hinge (dead load, secondary prestress, settlement effect, etc.)

V_t = transient vertical load on hinge (live load, temperature, etc.), various cases

$V_{\max}$, and $V_{\min}$ = maximum and minimum vertical loads on hinge, various cases

M_{LL} = moment on hinge, about bridge longitudinal axis, various cases

θ_p = permanent rotation at the hinge (due to dead load, prestress, creep, shrinkage, settlement, etc.)

θ_t = transient rotation at the hinge (due to live load, temperature, braking force, etc.), various cases

H_L, H_T = horizontal forces at the hinge, along the bridge longitudinal and transverse axes, under various cases

Step 2 Decide the 28-day cube crushing strength u of the concrete in the hinge. This should be same as for the deck with which it will be cast, and it is preferable to have same strength concrete at least in the top one meter height of the column below the hinge, u shall not be less than 250 kg/cm^2

Step 3 Select the various hinge-dimensions and its construction-joint detail. (Refer to Geometry of the Hinge-Throat discussed earlier, and decide the dimensions b, c and d, see Figs. 17.8 and 17.9.)

Step 4 Compute the plan section-area of the hinge-throat (ignoring any reinforcement passing through the hinge) $A = ab$, and ensure that this A is not more than $A_{\max}$ and not less than $A_{\min}$,

where $$A_{\max} = \frac{V_p}{1.25\theta\sqrt{u}}$$

$$A_{\min} = \frac{V_{\max}}{0.85u\left\{1 + \lambda\left(1 - \frac{1.47\theta\eta}{\sqrt{u}}\right)\right\}}$$

noting that

- $A_{\max}$ and $A_{\min}$ are in cm^2
- V_p and $V_{\max}$ are in kg ($V_p \leq 1.5V_{\min}$)
- u is in kg/cm^2 (cube strength at 28 days)

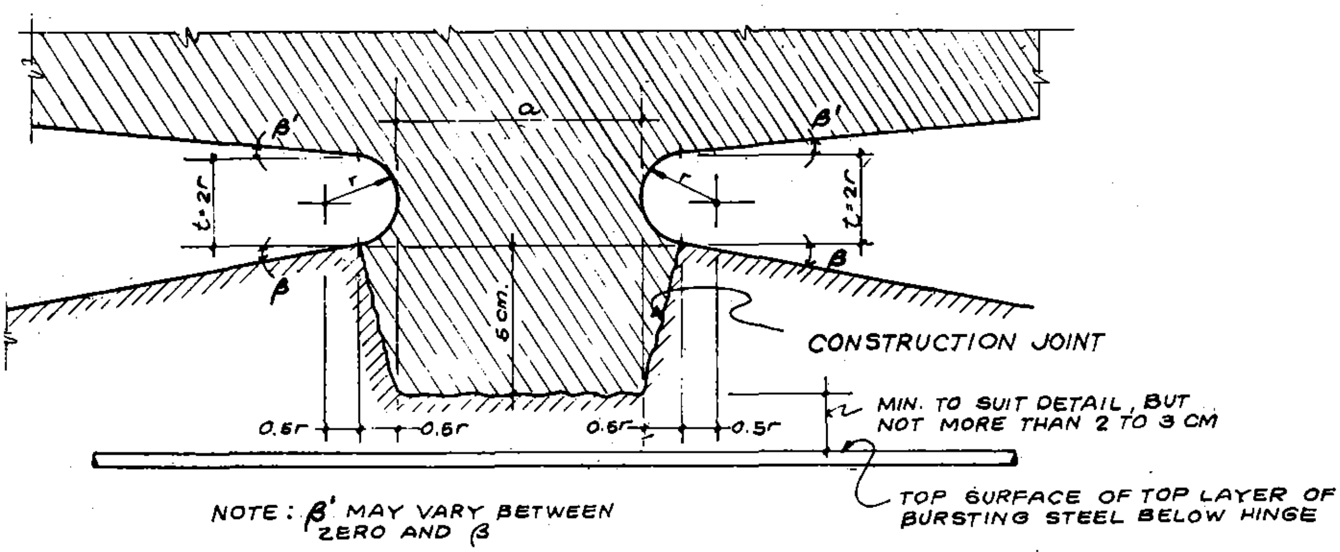


Fig. 17.9

- $\theta = (0.5\theta_p + \theta_t)$, in radians
- $\lambda = (1.2 - 4a/d)$, but never more than 0.8
- $\eta = \frac{V_{max}}{V_p}$, but never less than 1.0

Step 5 Estimate allowable rotation, θ_a and ensure that $(\theta_p + \theta_t)$ does not exceed it, noting that

$\theta_a = \pm\frac{0.8V}{A\sqrt{u}}$, but $\not> \pm$ 0.015 rad. ($\pm$ means 'allowable in either direction'),

where V = total vertical load on hinge under the loading condition considered for evaluating θ_p and θ_t (V may be taken as V_p, on the safer side, if V is greater than V_p).

Units: V (and V_p) in kg, A in cm^2, u in kg/cm^2, θ_a in radian

Step 6 Reinforcement in the column immediately below the hinge:

Estimate the steel reinforcements A_{s1}, A_{s2}, and A_{s3}, against the bursting forces F_1, F_2 and F_3 respectively, as follows:

(i) $F_1 = 0.3V_{max.}$, $\therefore A_{s1} = F_1/f_s$

(ii) $F_2 = 0.3\left(1 - \frac{b}{c}\right)V_{max.}$, $\therefore A_{s2} = F_2/f_s$

(iii) $F_3 = 0.3V_{max.}a/b$, $\therefore A_{s3} = F_3/f_s$

Note that,

- f_s = permissible normal working tensile stress in the steel reinforcement, which should not exceed 1800 kg/cm^2 in order that the cracks remain finer.
- while it is preferable to adopt smaller bar diameters to spread the splitting cracks (i.e., allow more but finer cracks), the detailing may be done using 16 mm bars for A_{s1} and 20 to 25 mm bars for A_{s2}, for ease in concreting. This is because F_1 being the biggest bursting force, A_{s1} controls the major bursting tendency and hence the above bar-diameter consideration is more strictly required for A_{s1} (i.e., smaller diameter for A_{s1}).
- for A_{s1} detailing, provide its each horizontal layer in the shape as indicated in Fig. 17.8, providing the necessary number of layers as required, but accommodating them all within a depth of 0.7 d below the top of the column.
- for A_{s2} detailing, the shape in each layer should be as indicated in Fig. 17.8, and the remaining details as for A_{s1} described above.
- detailing of A_{s3} presents no problems as its amount in relatively small, shape is as indicated in Fig. 17.8, and its amount can be easily accommodated within even less than 0.7 d below the top of the column.

Step 7 Estimate the rotational-resistance-moment, M_{TT}, of the hinge (since it is not actually a knife-edge rocker). This is obtained from the hinge-geometry, its concrete strength, and the rotation imposed on it. It is preferable to have $M_{TT}/V_{max.}$ not greater than $a/6$ to avoid tension due to this moment. Also, as this much moment will travel down into the column and the foundation, their respective designs should account for it.

$$M_{TT} = maV_{max.} \text{ kg cm}$$

where $m = \left(0.5 - \frac{1}{9\sqrt{\phi\theta}}\right)$, and preferably this $m \not> \frac{1}{6}$

$\theta = (0.5\theta_p + \theta_t)$ as defined earlier

$\phi = \frac{ab\sqrt{u}}{V_{max}}$

Units: a and b in cm, u in kg/cm^2, $V_{max.}$ in kg and θ in radian.

Note that this rotational-resistance-moment reduces with time owing to concrete creep.

Step 8 Check for horizontal shear in the hinge.

H_R is resultant horizontal force $(= \sqrt{H_L^2 + H_T^2})$ through the hinge.

(i) If $H_R \not> 0.125V_{max.}$ then no need to provide any vertical dowel bars through the hinge.

(ii) If $H_R > 0.125V_{max.}$ but $\leq 0.25V_{max.}$, then provide vertical dowel bars through the hinge along its linear axis, their section area in cm^2 being

$$A_s = \frac{H_R}{800} \qquad (H_R \text{ in kg})$$

(iii) If $H_R > 0.25V_{max.}$, it is preferable to revise the whole design and change over to other forms of rocker bearings. However, reference may also be made to the EMPA tests, referred to earlier.

Step 9 Check against tension in the hinge under M_{LL}. It is strongly recommended to ensure that the ratio of M_{LL} to the coexisting vertical load V on the hinge does not exceed $b/6$ so that there is no tension caused in the hinge. Generally this is the case. (If necessary, the hinge dimension b may be suitably altered, taking care then to review the Steps 1 to 8 for meeting their checks.) However, if tension cannot be avoided, then large diameter vertical dowel bars (armour bars) will have to be provided through the hinge near the ends of its dimension b at the middle of its throat-width, so that owing to the lever arm between them, they can take the whole of M_{LL}. Tensile stress in these armour bars may be restricted to 70% of their normal working stress value. These armour bars must be well anchored into the concrete above and below the hinge, but extreme care must be taken in ensuring their positions accurately. Therefore, their lengths should be short.

For this purpose, coarse threads may be cut at their ends and nuts screwed on them. However, the bonding zones of these armour bars should be assumed to commence only after a length equal to throat-width dimension a above and below the hinge-throat. The provision of the screwed-on nuts enhances bonding force, thereby permitting shorter armour bar lengths which facilitates their handling for accurate placement while casting the hinge concrete. The bonding zones should be secured with helical steel binders against local spalling. If large rotation coexists with M_{LL} then the above armour bars should additionally be sleeved with suitable plastic tubing up to the start of the helical binders to maintain them free from bending in order that they can rotate with the deck.

Step 10 Reinforcement immediately above the hinge:

(i) If the structure above the hinge is similar to that below it, then provide same reinforcement and detailing as that below the hinge, as in Step 6.

(ii) If the structure above the hinge is a deck, much wider than the hinge-length b, and relatively infinitely longer than the column dimension d then the area ab may be taken as if it is a standard bearing plate area (loading plate) on the deck and the necessary reinforcement against bursting be calculated as in a traditional prestressing end-block. Approximately, the bursting force is

$$F = 0.3V_{\max}.\left(1 - \frac{b}{2w}\right)$$

where w = deck-width at soffit

If six reinforcement grills are provided, the lowest at a distance x above the soffit and the remaining five respectively at x, $2x$, $3x$, $4x$, and $5x$ above the lowest grill (such that their centroid above the soffit falls at a height equal to $\frac{w}{4}$, giving $x = \frac{w}{14}$, but actual value to be adjusted to suit detail), then the effective area of bars in each of the two orthogonal directions in a grill may be taken as $\frac{F}{6f_s}$ where f_s is the normal working tensile stress in the steel (not exceeding 1800 kg/cm^2, as explained earlier).

Care in Constructing the Hinge

(i) Concrete in about one meter height at the top of the column (pier) below the hinge should preferably be of the same grade as that of the concrete in the hinge itself.

(ii) The hinge should be cast along with the deck, i.e., the structure above it, and the two should have the same grade of concrete (which should not be less than 250 kg/cm^2, standard cube strength nor less than that required by design of the deck).

(iii) Construction joint detail should preferably be as shown in Fig. 17.9

(iv) When concrete in the top portion of the column has set but not sufficiently hardened, the surface which will subsequently receive the *in situ* hinge and deck concrete should be brushed with a stiff wire brush (just sufficiently enough to remove the mortar skin and loose aggregates, exposing the larger aggregates without disturbing them) and the surface should be cleaned. Later, just prior to pouring concrete on it at the time of casting the hinge and the deck, this surface should again be thoroughly cleaned and wetcured with water for about two hours, thereafter blowing away the water collected in the recess with compressed air.

(v) Care should be taken not to disturb the hinge concrete and not to subject it to any kind of vibration, deformation or movement until after the deck is structurally ready for taking finishings and live load.

(vi) In its form-work and shuttering, the geometry of the concrete hinge should be followed strictly as per design details shown in the approved execution drawing. The form-work and shuttering for producing the hinge in all its details of dimensions, circular face, profiles, slopes, etc., should be almost 'sculpted' to give its exact geometry.

(vii) The temporary forms used to give the exact geometry of the concrete hinge should be made of a material such that:

   (a) the forms do not deform in any manner under the weight of wet concrete above, taking account of vibrations and moving about of workmen,

   (b) the forms can be removed reasonably easily without damage to the surrounded and the surrounding concrete.

(viii) The forms for the hinge that are sandwiched between the deck and the column should be carefully loosened at the time of prestressing of the deck only to the extent that the hinge is free to rotate during the prestressing operations.

Qualitative Comparison of Concrete Hinge with Dowelled Elastomeric Bearing and Fixed Pot Bearing

This is shown in the following Table 17.9, for a single bearing carrying approximately an average vertical load of 1600 tonnes.

17.9 DETAILS OF LAMINATED NEOPRENE BEARINGS

Some useful details about laminated (i.e., restrained neoprene bearings that are formed by vulcanizing all the restraining steel laminates and all the elastomer layers in one single operation altogether):

(i) *Description* The elastomeric bearings consist of rubber layers with steel plates vulcanized altogether in order to ensure a bond on their connection faces. The plates are completely embedded in the rubber to prevent any corrosion.

(ii) *Material specifications* See Tables 17.10(a) and 17.10(b).

(iii) *Handling and storage on site* Bearings should be stored under cover, away from sunlight, heat, oil and chemicals.

They should always be handled and stacked carefully. Damaged bearings, for example, with bent steel interleaving plates or partially debonded layers, should never be installed for use.

(iv) *Installation*

(a) *Seating* Where the support is concrete, its cast surface usually being irregular, the bearings should be placed on it accurately to line and level on a 6 mm thick bedding of stiff mortar. This can be ordinary sand/cement with a low water/cement ratio, or a mortar of fine dry sharp sand and chemical resin and hardener. In either case, the cube crushing strength of the mortar should be at least 20 N/mm^2. Where the support is steel, a rolled surface may be suitable for use directly, provided that it is

Table 17.9

Qualitative Comparisons	Concrete Hinge	Dowelled Elastomeric bearing	Fixed Pot Bearing
1. Vertical load carrying capacity	Good	Only very few elastomeric bearings have elastomers of strength to carry such heavy loads. Probably a special bearing is called for, unless numbers increase.	Good
2. Horizontal load carrying capacity	Good	Limited by crushing of concrete locally in pier and deck by local bearing of dowels against concrete.	Good
3. Appearance in elevation	Very small gap between top of pier column and deck gives the best detail in elevations.	Gap is deeper than in the case of hinge but the fine line between deck and column still maintained.	Fixed pot bearing itself requires a gap significantly deeper than elastomeric, but guided-bearings need 50% more gap (nearly)
4. Cost	Cheapest by far. Almost nil.	A little less expensive than the pot bearings but not by much.	Most expensive.
5. Delivery	No delivery problems for cast-in-place concrete.	Imported from special either one or at most two suppliers (high load). Hence supply and paperwork problems.	As in case of elastomeric bearings, but a slightly easier problem as pot bearings are manufactured by many suppliers.
6. Ease of construction	If the performed throat formwork is planned and made correctly and held in place during concreting, the hinge poured with deck becomes normal concreting.	Fixing dowels into top of column needs care and attention. Fixing bearings is a 2-stage operation. (a) grout-in dowels (b) bed-bearing slipping over top of dowels.	Mechanical fixings into top of column but, as deck is wide, then free sliding or guided sliding bearings are also required. Alignment of guides requires care.
7. Maintenance	Maintenance free.	If installed correctly, requires very little maintenance.	PTFE Sliding faces of sliding bearings have not yet proved themselves in terms of design life when fine airborne dust is considered.
8. Replacement	If abutment elastomeric bearings are to be replaced then each end of deck slightly jacked up with hinge holding down at the pier.	Producer as with pot bearings except jacking needs to be higher to clear dowels which sometimes stick—generally very inconvenient.	Whole deck is jacked up off central pier and abutments. Bearings removed and replaced.

reasonably smooth and true to level, but otherwise some surface preparation will be needed.

Trowelling often seems to produce a bedding that is slightly rounded on the top surface, and it is preferable to screed off or cast against a flat plate.

(b) *In situ superstructure* When the superstructure is to be in concrete which is cast *in situ*, the spaces around and between the bearings can be filled with expanded polystyrene or wellrammed damp sand covered with an impervious membrane such as polythene sheet. Extreme care must be taken not to disturb the bearings during casting, and a temporary bond to the substructure with an impact adhesive will help. After curing of the superstructure, the sand-infill can be washed away from around the bearings or the polystyrene can be broken up and blown out with compressed air. (It should not be dissolved, because the solvent may attack the elastomer in the bearings.)

(c) *Precast concrete superstructures* Where precast concrete beams are being used they should be lowered on to a mortar skim (2–3 mm), on the top of the bearings, to eliminate soffit irregularities and twist in the beams. The bearings should be so seated as to accommodate the rotation due to the precamber of the beams at this very low level of vertical loading, or the beams should be suitably propped near the bearings until the mortar skim has hardened into a wedge, so that the bearings are not rotated at this stage.

(v) Some suitable laminated (i.e., restrained) Neoprene bearings for different capacities (typical, for general guidance):

- The Tables 17.11 to 17.14 ahead are given for guidance only and allow the designer to make a preliminary selection within the range of Freyssinet International's standard bearings.
- For each table, the thickness of individual elastomer inner layer, and the associated thickness of the reinforcing steel plates are uniform as shown.
- The thickness of each outer elastomer layer is equal to half the thickness of the inner elastomer layer.
- The elastomer edge cover e is in every case equal to 5 mm.
- The values given in the tables are calculated in accordance with BS 5400:Section 9.1:1983, taking into account the following parameters and assumptions:
  - — Shear modulus of elastomer = G = 0.9 N/mm^2
  - — Bulk modulus of elastomer = E_b = 2000 N/mm^2
  - — Live load effect is similar to dead load effect (for load and rotation)
  - — Horizontal force on the bearing causes shear in the direction of the width of the bearing,
  - — Rotation is only across the width of the bearing (i.e., $\alpha_l = 0$)
  - The most important design criterion, and the one that normally governs, used for the calculations is:

  $k(\epsilon_c + \epsilon_q + \epsilon_\alpha) \leq 5.0$ (Symbols as explained earlier) where

  k is a factor equal to 1.5 for live load effects and 1.0 for all other effects.
- The maximum allowable load is given for each bearing for four cases, viz.,

  Case 1 No rotation, no shear
  $\epsilon_\alpha = 0$ and $\epsilon_q = 0$
  Case 2 No rotation, maximum shear
  $\epsilon_\alpha = 0$ and $\epsilon_q = 0.7$
  Case 3 Maximum rotation, no shear
  $\epsilon_\alpha = b_e^2\alpha_b/2t_i\Sigma t_i$ and $\epsilon_q = 0$
  Case 4 Maximum rotation, maximum shear
  ϵ_α = same as case 3 and $\epsilon_q = 0.7$
- The other two vertical-load design-limits are given by:

  Criterion of minimum thickness of the steel reinforcing plates, which is satisfied for the Freyssinet standard range of bearings

  Criterion of stability which is automatically satisfied for bearings for which $\Sigma t_i \leq b_e/4$ (symbols as explained earlier).
- The maximum value of shear movement is given by:

$$\epsilon_q = \frac{\delta_r}{t_q} = 0.7$$

- The shear-stiffness is calculated from the formula: $\dfrac{H}{\delta_r} = \dfrac{AG}{t_q}$, where G = 0.9 N/mm^2 (Also see the specification stated earlier for railway bridges.)
- The maximum allowable rotation, shown in Tables 17.11 to 17.14 has been calculated (following the expressions given in the specifications earlier) assuming V = 100 kN, so that actual max. allowable rotation will be equal to

$$\left\{\text{tabulated value} \times \left[\frac{\text{actual vertical load (kN)}}{100}\right]\right\}$$

in ($\times 10^{-3}$ radian) units.

Table 17.10(a) *Elastomer*

Property	Test standard	Specified value			
Raw polymer		natural rubber (NR) or chloroprene rubber (CR)			
Hardness (IRHD)	B.S. 903: Part A26 (Method N)	NR		(CR)	
		60 ± 5	70 ± 5	60 ± 5	70 ± 5
Shear modulus G(N/mm^2)	B.S. 903 : Part A14 (Shear strain = 0.25)	0.90 ± 0.14	1.20 ± 0.18	0.90 ± 0.14	1.20 ± 0.18
Tensile strength R(N/mm^2)	B.S. 903 : Part A2	min 15.5			
Elongation at break A (%)	B.S. 903 : Part A2	min. 300	min. 300	min. 350	min. 30
Compression set (%)	B.S. 903 : Part A6	(24 hrs. at 70°C) max. 30		(24 hrs. at 100°C) max. 35	
Ageing resistance ~	B.S. 903 : Part A19	(7 days at 70°C)		(3 days at 100°C)	
• change in hardness-(IRHD)		max. 10		max. 15	
• change in tensile-strength (%)		max. 15		max. 15	
• change in elongation at break (%)		max. 20		max. 40	
Ozone resistance	B.S. 903 : Part A43	No cracks (25 pphm/20% strain 96 hrs. at 30°C)			
Low temperature resistance					
• brittleness	B.S. 903 : Part A25	— brittleness temperature = max. − 25°C			
• stiffening	B.S. 5400 : 9.2	— change in hardness = max. 15			
• crystallization	B.S. 903: Part A39	— compression set = max. 65%			
Bonding of elastomer to metal	B.S. 903 : Part A21	bond peel strength = min. 7 N/mm^2			

Table 17.10(b) *Reinforcing Plates*

Property	Standard	Specified value
Raw material	B.S. 1449 : Part 1	Rolled mild steel shee
Tensile strength (N/mm^2)		min. 400
Yield strength (N/mm^2)		min. 300
Elongation at break (%)		min. 20

Table 17.11 • *Elastomer inner layer thickness t_i = 8 mm*
• *Reinforcing steel plate thickness t_s = 2 mm*

Plan size (mm)	Overall height (mm)	Maximum dead load + live load (kN): No rotation, No shear	No rotation, maximum shear	Maximum rotation, No shear	Maximum rotation, Maximum shear	Shear movement (mm)	Shear stiffness (kN/mm)	Max. allowable rotation about longer axis (10^{-1} rad/100 kN
1	2	3	4	5	6	7	8	9
150 × 100	10	104	80	75	60	5.6	1.69	20.714
150 × 100	20	104	75	60	47	11.2	0.84	61.886
150 × 100	30	97	70	57	43	16.8	0.56	103.059
150 × 100	40	73	55	55	40	22.4	0.42	144.231
150 × 100	50	58	40	54	38	28.0	0.34	185.403
200 × 100	10	157	121	119	94	5.6	2.25	12.434
200 × 100	20	157	113	97	75	11.2	1.13	36.997
200 × 100	30	147	105	91	69	16.8	0.75	61.561
200 × 100	40	110	83	89	65	22.4	0.56	86.124
200 × 100	50	88	61	87	61	28.0	0.45	110.688
200 × 150	10	322	255	220	178	5.6	3.38	3.087
200 × 150	20	322	244	177	142	11.2	1.69	9.051
200 × 150	30	322	234	166	131	16.8	1.13	15.016
200 × 150	40	322	223	162	125	22.4	0.84	20.980
200 × 150	50	281	212	159	121	28.0	0.68	26.945
200 × 150	60	235	178	157	117	33.6	0.56	32.909
250 × 150	10	446	353	317	256	5.6	4.22	2.074
250 × 150	20	446	338	258	205	11.2	2.11	6.041
250 × 150	30	446	324	242	190	16.8	1.41	10.008
250 × 150	40	446	309	236	182	22.4	1.05	13.976
250 × 150	50	390	294	232	175	28.0	0.84	17.943
250 × 150	60	325	274	229	170	33.6	0.70	21.910

(Contd.)

Table 17.11 (Contd).

1	2	3	4	5	6	7	8	9
300 × 150	10	575	455	419	338	5.6	5.06	1.531
300 × 150	20	575	436	343	273	11.2	2.53	4.437
300 × 150	30	575	410	324	253	16.8	1.69	7.342
300 × 150	40	575	399	315	242	22.4	1.27	10.247
300 × 150	50	503	380	310	233	28.0	1.01	13.152
300 × 150	60	419	319	306	226	33.6	0.84	16.058
250 × 200	20	725	563	380	306	11.2	2.81	2.365
250 × 200	30	725	546	357	285	16.8	1.88	3.904
250 × 200	40	725	528	347	274	22.4	1.41	5.444
250 × 200	50	725	510	341	266	28.0	1.13	6.984
250 × 200	60	718	493	337	260	33.6	0.94	8.523
250 × 200	70	615	475	334	254	39.2	0.80	10.063
250 × 200	80	538	411	332	249	44.8	0.70	11.603
300 × 200	20	949	737	516	416	11.2	3.38	1.704
300 × 200	30	949	714	486	388	16.8	2.25	2.808
300 × 200	40	949	690	472	372	22.4	1.69	3.912
300 × 200	50	949	667	465	362	28.0	1.35	5.017
300 × 200	60	939	644	460	353	33.6	1.13	6.121
300 × 200	70	805	621	456	346	39.2	0.96	7.225
300 × 200	80	704	538	453	339	44.8	0.84	8.329
350 × 200	20	1181	919	659	531	11.2	3.94	1.311
350 × 200	30	1181	888	622	496	16.8	2.63	2.158
350 × 200	40	1181	860	606	477	22.4	1.97	3.004
350 × 200	50	1181	831	596	463	28.0	1.58	3.850
350 × 200	60	1169	802	589	452	33.6	1.31	4.697
350 × 200	70	1002	773	585	443	39.2	1.13	5.543
350 × 200	80	877	670	582	434	44.8	0.98	6.390
400 × 200	20	1420	1103	808	651	11.2	4.50	1.056
400 × 200	30	1420	1068	764	608	16.8	3.00	1.735
400 × 200	40	1420	1033	744	585	22.4	2.25	2.414
400 × 200	50	1420	999	732	569	28.0	1.80	3.094
400 × 200	60	1405	964	725	555	33.6	1.50	3.773
400 × 200	70	1205	930	720	544	39.2	1.29	4.452
400 × 200	80	1054	805	716	532	44.8	1.13	5.132

Table 17.12 • *Elastomer inner layer thickness t_i = 10 mm*
• *Reinforcing steel plate thickness t_s = 3 mm*

Plan size (mm)	Overall height (mm)	Maximum dead load + live load (kN)				Shear movement (mm)	Shear stiffness (kN/mm)	Max. allowable rotation about longer axis (10^{-3} rad/100 kN)
		No rotation		Maximum rotation				
		No shear	maximum shear	No shear	Maximum shear			
1	2	3	4	5	6	7	8	9
300 × 250	26	1097	852	568	458	14.0	3.38	1.559
300 × 250	39	1097	826	534	426	21.0	2.25	2.574
300 × 250	52	1097	799	518	410	28.0	1.69	3.590
300 × 250	65	1097	773	509	398	35.0	1.35	4.605
300 × 250	78	1097	747	503	389	42.0	1.13	5.621
300 × 250	91	940	720	499	381	49.0	0.96	6.636
400 × 250	26	1669	1296	915	738	14.0	4.50	0.935
400 × 250	39	1669	1256	863	688	21.0	3.00	1.539
400 × 250	52	1669	1216	840	662	28.0	2.25	2.144
400 × 250	65	1669	1176	826	643	35.0	1.80	2.748
400 × 250	78	1669	1136	817	628	42.0	1.50	3.353
400 × 250	91	1430	1096	811	615	49.0	1.29	3.957
400 × 300	26	2257	1772	1154	935	14.0	5.40	0.529
400 × 300	39	2257	1728	1088	875	21.0	3.60	0.869
400 × 300	52	2257	1683	1058	843	28.0	2.70	1.209
400 × 300	65	2257	1638	1041	822	35.0	2.16	1.548
400 × 300	78	2257	1593	1029	806	42.0	1.80	1.888
400 × 300	91	2257	1548	1021	792	49.0	1.54	2.227
400 × 300	104	2046	1503	1016	779	56.0	1.35	2.567
400 × 300	117	1818	1423	1011	767	63.0	1.20	2.907

(Contd.)

Table 17.12

1	2	3	4	5	6	7	8	9
500 × 300	26	3107	2439	1647	1334	14.0	6.75	0.363
500 × 300	39	3107	2377	1558	1251	21.0	4.50	0.595
500 × 300	52	3107	2315	1516	1207	28.0	3.38	0.826
500 × 300	65	3107	2254	1493	1177	35.0	2.70	1.057
500 × 300	78	3107	2192	1477	1154	42.0	2.25	1.289
500 × 300	91	3107	2130	1467	1134	49.0	1.93	1.520
500 × 300	104	2815	2068	1459	1116	56.0	1.69	1.751
500 × 300	117	2503	1959	1452	1099	63.0	1.50	1.983
600 × 300	26	3992	3135	2167	1754	14.0	8.10	0.272
600 × 300	39	3992	3055	2054	1648	21.0	5.40	0.445
600 × 300	52	3992	2976	2001	1592	28.0	4.05	0.617
600 × 300	65	3992	2896	1971	1553	35.0	3.24	0.789
600 × 300	78	3992	2817	1952	1522	42.0	2.70	0.962
600 × 300	91	3992	2736	1938	1496	49.0	2.31	1.134
600 × 300	104	3618	2658	1928	1472	56.0	2.03	1.307
600 × 300	117	3216	2517	1920	1449	63.0	1.80	1.479

Table 17.13 • *Elastomer inner layer thickness $t_i = 12$ mm*
• *Reinforcing steel plate thickness $t_s = 3$ mm*

Plan size (mm)	Overall height (mm)	Maximum dead load + live load (kN): No rotation, No shear	Maximum dead load + live load (kN): No rotation, Maximum shear	Maximum dead load + live load (kN): Maximum rotation, No shear	Maximum dead load + live load (kN): Maximum rotation, Maximum shear	Shear movement (mm)	Shear stiffness (kN/mm)	Max. allowable rotation about longer axis (10^{-3} rad/100 kN)
450 × 350	30	2869	2250	1464	1186	16.8	5.91	0.438
450 × 350	45	2869	2192	1379	1108	25.2	3.94	0.719
450 × 350	60	2869	2133	1340	1067	33.6	2.95	1.001
450 × 350	75	2869	2075	1318	1040	42.0	2.36	1.283
450 × 350	90	2869	2016	1304	1019	50.4	1.97	1.564
450 × 350	105	2869	1958	1293	1001	58.8	1.69	1.846
450 × 350	120	2541	1899	1286	985	67.2	1.48	2.123
450 × 350	135	2258	1756	1280	970	75.6	1.31	2.409
500 × 400	45	4150	3203	1924	1552	25.2	5.00	0.399
500 × 400	60	4150	3129	1871	1499	33.6	3.75	0.554
500 × 400	75	4150	3055	1841	1464	42.0	3.00	0.709
500 × 400	90	4150	2981	1821	1437	50.4	2.50	0.865
500 × 400	105	4150	2908	1807	1414	58.8	2.14	1.020
500 × 400	120	4150	2834	1797	1394	67.2	1.88	1.175
500 × 400	135	3747	2760	1789	1376	75.6	1.67	1.330
600 × 400	45	5403	4169	2586	2086	25.2	6.00	0.292
600 × 400	60	5403	4073	2518	2016	33.6	4.50	0.405
600 × 400	75	5403	3977	2479	1969	42.0	3.60	0.518
600 × 400	90	5403	3881	2454	1933	50.4	3.00	0.632
600 × 400	105	5403	3785	2436	1903	58.8	2.57	0.745
600 × 400	120	5403	3689	2423	1876	67.2	2.25	0.858
600 × 400	135	4877	3593	2412	1851	75.6	2.00	0.971
600 × 450	45	6543	5039	2967	2401	25.2	6.75	0.205
600 × 450	60	6543	4986	2889	2324	33.6	5.06	0.285
600 × 450	75	6543	4883	2844	2273	42.0	4.05	0.864
600 × 450	90	6543	4780	2815	2235	50.4	3.38	0.443
600 × 450	105	6543	4677	2795	2204	58.8	2.89	0.522
600 × 450	120	6543	4574	2780	2177	67.2	2.53	0.602
600 × 450	135	6543	4471	2768	2151	75.6	2.25	0.681
600 × 450	150	5998	4367	2759	2128	84.0	2.03	0.760
600 × 450	165	5453	4264	2751	2105	92.4	1.84	0.839
600 × 500	45	7739	6056	3328	2702	25.2	7.50	0.151
600 × 500	60	7739	5947	3242	2618	33.6	5.63	0.209
600 × 500	75	7739	5837	3192	2564	42.0	4.50	0.267
600 × 500	90	7739	5728	3159	2524	50.4	3.75	0.325
600 × 500	105	7739	5618	3136	2491	58.8	3.21	0.384
600 × 500	120	7739	5509	3120	2463	67.2	2.81	0.442
600 × 500	135	7739	5400	3107	2438	75.6	2.50	0.500
600 × 500	150	7739	5290	3096	2415	84.0	2.25	0.558
600 × 500	165	7182	5181	3088	2392	92.4	2.05	0.616

17.10 SOME OF THE VERSATILE AND MODERN LARGER CAPACITY BEARINGS

The case may however arise where due to heavy loads, large displacements or rotation, laminated elastomeric bearing pads may not be suitable. In this case other devices must be adopted.

In order to satisfy such a requirement. Freyssinet (and indeed other specialist manufacturers too) manufacture and market other types of bearing devices:

- Sliding bearings: Neoflon
- Mechanical bearings (spherical type and pot or disc type): Tetron

Some Useful Details About Some Freyssinet Spherical Bearings (Tetron Type S3 Range)

Material Specifications

- Tetron 'S3' bases and rockers use maintenance-free aluminium alloy
- Sliding plates are made of mild steel, faced with high quality stainless steel
- Sliding surfaces are lined with pure PTFE, to BS. 3784
- Pins for side restraints are special spring-steel with minimum yield strength 1100 N/mm^2.
- All permanently exposed steel surfaces are corrosion protected with a metallic zinc rich epoxy coating, followed by chlorinated rubber paint.
- Full details of material specifications are available from the specialist manufacturers.
- *Contact stress* The average base contact stress of the bearings, illustrated ahead, approaches 17.5 N/mm^2. Direct contact between the aluminium parts of the bearings and dissimilar metals must be avoided.

Table 17.14 • *Elastomer inner layer thickness t_i = 12 mm*
• *Reinforcing steel plate thickness t_s = 3 mm*

Plan size (mm)	Overall height (mm)	Maximum dead load + live load (kN)				Shear movement (mm)	Shear stiffness (kN/mm)	Max. allowable rotation about longer axis (10^{-3} rad/100 kN)
		No rotation		Maximum rotation				
		No shear	Maximum shear	No shear	Maximum shear			
600 × 600	57	8215	6416	3458	2806	31.5	7.20	0.158
600 × 600	76	8215	6295	3362	2714	42.0	5.40	0.220
600 × 600	95	8215	6174	3306	2655	52.5	4.32	0.281
600 × 600	114	8215	6054	3271	2611	63.0	3.60	0.342
600 × 600	133	8215	5933	3245	2576	73.5	3.09	0.403
600 × 600	152	8215	5813	3227	2546	84.0	2.70	0.465
600 × 600	171	8215	5692	3213	2519	94.5	2.40	0.526
600 × 600	190	8078	5571	3201	2493	105.0	2.16	0.587
700 × 600	57	10358	8089	4490	3642	31.5	8.40	0.120
700 × 600	76	10358	7937	4371	3526	42.0	6.30	0.167
700 × 600	95	10358	7785	4302	3451	52.5	5.04	0.213
700 × 600	114	10358	7633	4257	3396	63.0	4.20	0.260
700 × 600	133	10358	7481	4226	3350	73.5	3.60	0.306
700 × 600	152	10358	7329	4203	3311	84.0	3.15	0.352
700 × 600	171	10358	7177	4185	3276	94.5	2.80	0.399
700 × 600	190	10186	7025	4171	3243	105.0	2.52	0.445
700 × 700	57	13140	10346	5248	4272	31.5	9.80	0.078
700 × 700	76	13140	10181	5109	4142	42.0	7.35	0.108
700 × 700	95	13140	10016	5029	4060	52.5	5.88	0.138
700 × 700	114	13140	9851	4978	4000	63.0	4.90	0.167
700 × 700	133	13140	9686	4941	3953	73.5	4.20	0.197
700 × 700	152	13140	9521	4914	3912	84.0	3.68	0.227
700 × 700	171	13140	9356	4894	3877	94.5	3.27	0.257
700 × 700	190	13140	9191	4877	3844	105.0	2.94	0.287
800 × 800	57	18557	15622	7437	6069	31.5	12.80	0.043
800 × 800	76	18557	15405	7251	5897	42.0	9.60	0.059
800 × 800	95	18557	15189	7144	5789	52.5	7.68	0.075
800 × 800	114	18557	14973	7074	5712	63.0	6.40	0.092
800 × 800	133	18557	14757	7025	5651	73.5	5.49	0.108
800 × 800	152	18557	14540	6989	5600	84.0	4.80	0.124
800 × 800	171	18557	14324	6961	5556	94.5	4.27	0.140
800 × 800	190	18557	14108	6939	5516	105.0	3.84	0.157

(a) Tetron S3T: Fixed (Table 17.15 & Fig. 17.10)

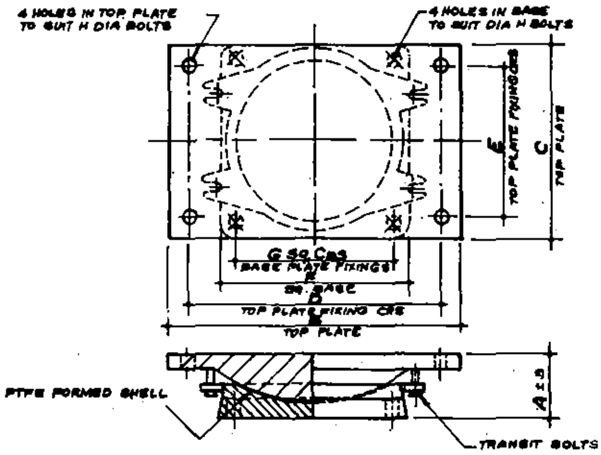


Fig. 17.10 *Tetron S3 T fixed*

Table 17.15

Bearing type	Principal dimensions (mm)								'Working stress' design (kN)		BS 5400: Section 9.1 Design Load effects (kN)			
									Max. load Vertical	Max. load Horizontal	Serviceability Limit State		Ultimate Limit State	
↓	A	B	C	D	E	F	G	H			Vertical	Horizontal	Vertical*	Horizontal
S3T70	70	400	215	340	145	230	177	M12	700	65	700	65	900	110
S3T150	100	475	305	410	235	305	254	M20	1500	150	1500	150	2000	210
S3T250	105	510	360	435	280	375	297	M20	2500	250	2500	250	3000	300
S3T300	115	570	400	490	310	420	336	M24	3000	280	3000	280	3800	400
S3T350	130	600	440	515	335	465	364	M24	3500	300	3500	330	4500	500
S3T400	140	660	485	565	355	500	396	M30	4000	360	4000	360	5000	600
S3T500	145	685	510	585	375	535	417	M30	5000	450	5000	450	6300	650
S3T600	155	760	565	640	430	600	463	M30	6000	500	6000	500	7500	710
S3T750	180	930	660	795	500	705	548	M30	7500	600	7500	600	9500	900
S3T1000	195	970	710	830	535	755	583	M30	10000	750	10000	750	12500	1100
S3T1200	215	1080	790	925	595	840	654	M30	12000	900	12000	900	15000	1300

* Assuming $\gamma_m = 1.2$ (partial material factor B S 5400, 9.1, 1983); Note: 1. 'Bearing type' column indicates maximum vertical design load in tonnes for 'Working Stress' design or B S 5400 serviceability limit state. 2. Larger bearings are also available.

(b) Tetron S3F sliding guided (Table 17.16 & Fig. 17.11)

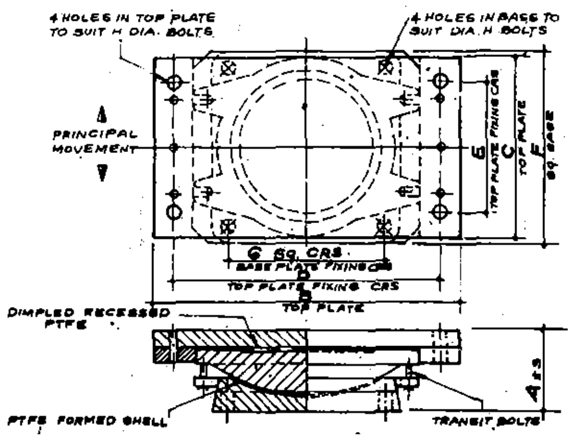


Fig. 17.11 *Tetron S 3 F sliding guided*

Table 17.16

Bearing type	Principal dimensions (mm)								'Working stress' design (kN)		BS 5400: Section 9.1 Design load effects (kN)			
									Max. load	Max. load	Serviceability Limit State		Ultimate Limit State	
↓	A	B	C	D	E	F	G	H	Vertical	Horizontal	Vertical	Horizontal	Vertical*	Horizontal
S3F70	100	351	270	286	190	230	177	M12	700	65	700	65	900	110
S3F150	130	497	360	432	280	305	254	M20	1500	150	1500	150	2000	210
S3F250	135	541	420	476	340	375	297	M20	2500	250	2500	250	3000	300
S3F300	145	636	460	546	360	420	336	M24	3000	280	3000	280	3800	400
S3F350	155	680	480	590	400	465	364	M24	3500	300	3500	300	4500	500
S3F400	170	718	540	628	460	500	396	M24	4000	360	4000	360	5000	600
S3F500	175	750	570	660	470	535	417	M30	5000	450	5000	450	6300	650
S3F600	190	814	625	724	525	600	463	M30	6000	500	6000	500	7500	710
S3F750	225	922	720	832	620	705	548	M30	7500	600	7500	600	9500	900
S3F1000	240	971	770	881	670	755	583	M30	10000	750	10000	750	12500	1100
S3F1200	265	1055	860	965	760	840	654	M30	12000	900	12000	900	15000	1300

* Assuming $\gamma_m = 1.2$ (partial material factor, BS 5400, 9.1, 1983)

NOTE

(i) Basic bearing as tabulated is for zero movement, and is specified typically as S3F250/00. For additional movements increase C and E dimensions by 100 mm per increment when bearing is described in a code, e.g. S3F250/10 (movement in cm)

(ii) 'Bearing type' column indicates maximum vertical design load in tonnes for 'working stress' design or BS 5400 serviceability limit state.

(iii) Larger bearings are also available.

(iv) ±3° (0.052 radian) rotation in any plane.

(c) Tetron S3E Free Sliding **(Table 17.17 & Fig. 17.12 a, b)**

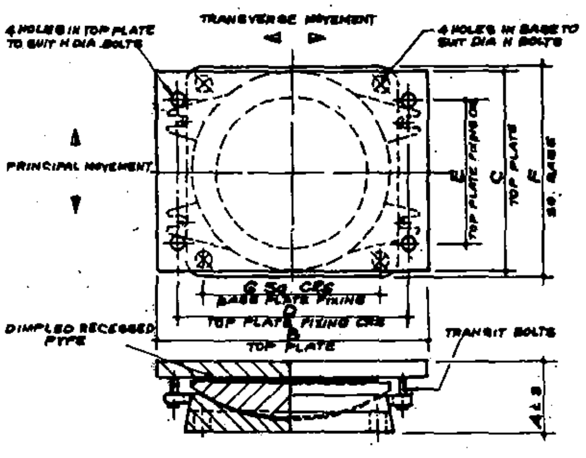


Fig. 17.12(a) *Tetron S3E free sliding*

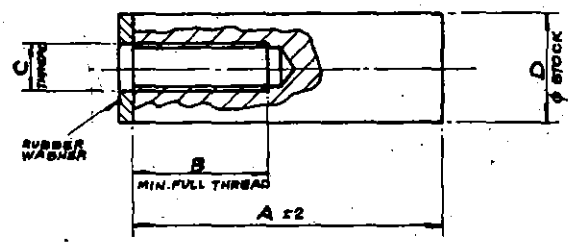


Fig. 17.12(b)

Standard socket detail

A	B	C	D
100	30	M12	40
100	50	M20	40
160	60	M24	40
220	75	M30	50

Fixing All bearings have provision for Grade 8.8 fixing bolts which are designed to cater for the horizontal force to which each specific type may be subjected, with some assistance from friction due to the minimum vertical load normally present in service. For ease of installation and to provide complete removability, bearing should be secured to cast-in sockets where possible.

Installation of Freyssinet Tetron Spherical (Type S3 Range) Bearings

In a bearing that is free sliding in all directions (i.e., the S3E range) positive fixing to the main structure may not be required if the bearing is always subjected to adequate vertical loading, because horizontal movement will occur on the plane of least resistance which is of course the bearing sliding surface. Nevertheless, in some cases it is prudent to provide fixings to guard against displacement during installation, impact, vibration and accidental unloading.

Most cases of bearing malfunction are attributable to faulty installation; and almost all bearing damage occurs during installation, or even earlier during handling and storage. Careless handling on site, and the ingress of dirt, can easily lead to abnormally high frictional resistance. Tetron 'S' bearings are normally delivered in a condition to discourage unnecessary dismantling, and bolts are used to connect together the upper and lower parts of the bearings. These temporary fixings, as well as excluding dirt during installation, prevent accidental relative displacement

Table 17.17

Bearing type	Principal dimensions (mm)								'Working stress' design (kN)	BS 5400: Section 9.1 Design Load (kN)	
									Max. load Vertical	Serviceability Limit State	Ultimate Limit State
	A	B	C	D	E	F	G	H		Vertical	Vertical*
S3E70	90	340	265	280	205	230	177	M12	700	700	900
S3E150	105	395	325	335	265	305	254	M20	1500	1500	2000
S3E250	120	485	385	410	315	375	297	M20	2500	2500	3000
S3E300	125	555	420	495	360	420	336	M20	3000	3000	3800
S3E350	140	620	460	560	400	465	364	M20	3500	3500	4500
S3E400	145	675	490	615	430	500	396	M20	4000	4000	5000
S3E500	150	710	520	650	460	535	417	M20	5000	5000	6300
S3E600	160	740	575	680	515	600	463	M20	6000	6000	7500
S3E750	190	815	660	755	600	705	548	M20	7500	7500	9500
S3E1000	200	910	720	840	650	755	583	M20	10000	10000	12500
S3E1200	220	1000	800	925	725	840	654	M20	12000	12000	15000

* Assuming $\gamma_m = 1.2$ (partial material factor, B S 5400, 9.1, 1983)

NOTE

(i) Basic bearing as tabulated is for zero movement in both the principal and transverse directions and is specified typically as S3E 150/0000.

(ii) For additional movement C and E increase by 100 mm per increment of principal movement and/or B and D increase by 100 mm per increment of transverse movement when bearing is described in a code. e.g.,

Transverse Movement (cm.)

S3E 150/ 10 10

Principal Movement (cm.)

(iii) 'Bearing type' column indicates maximum vertical design load in tonnes for 'Working stress' design or BS 5400 Serviceability Limit State.

(iv) Larger bearings are also available.

(v) ±3° (0.052 radian) rotation in any plane.

between the parts of the bearings, but they must be removed before the bearings are called upon to slide or rotate. However, they are not structural fixings and should be supplemented, for example, by wedges during installation.

Bearings should be clearly identified, by marking them with such details as type and location. Bearings should be transported and unloaded carefully and then stored under cover in clean, dry conditions until required.

As much preparatory work as possible should be carried out before bringing a bearing to its actual location. The seating should be level, and this usually necessitates the use of a mortar bedding, composed of sand and either cement, polyester resin or epoxy resin, with a cube crushing strength of at least 35 N/mm^2.

Base Fixings

An 'E' type bearing is free sliding in all directions, with a low coefficient of friction, so that the base may not require positive fixing, but see note above. All other types of bearing can resist horizontal loading and so their bases should be fixed.

There are several ways of achieving this. A recess can be left in the pier/abutment, into which the bearing base is placed bodily, bedded upon and surrounded with resin mortar (see Fig. 17.13). The recess must be correctly reinforced on all sides. Alternatively, small pockets can be left for dowels or bolts, which are then set accurately to

Fig. 17.13

position and level in grout using timber or light steel jig. The bearing base is then lowered over the dowels or bolts, on to a bed of wet mortar (Fig. 17.14).

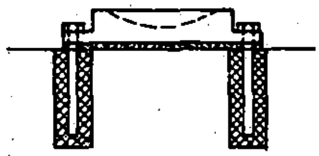


Fig. 17.14

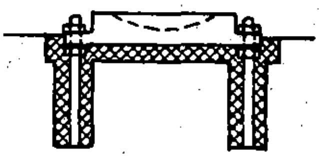


Fig. 17.15

NOTE

(i) Bearings should not be dismantled on site because the effects of dirt on the sliding and rotating surfaces are highly deleterious.

(ii) In all cases the transit bolts must be removed after the mortar has set and before the bearing is called upon to rotate or slide. These bolts are identified by metal labels.

It is possible to cast-in the fixing bolts and position the bearing in one operation, in which case the bearing is supported on rubber washers, over levelling nuts on the bolts, and a chemical resin grout is then poured around the bolts and under the bearing (Fig. 17.15). The fixings should not be over-tightened, when the grout has set, otherwise the bearing may be distorted due to compression in the washers. Nevertheless, rubber washers must be used, to prevent the fixing bolts carrying vertical loading.

Top Fixings

For similar reasons, top fixings must be provided for all bearings (optional for the *E* type).

A completed precast concrete structure may be lowered on to a skim of mortar on the top of the bearing, to eliminate soffit irregularities. The mortar mix needs careful control to ensure that it is not totally squeezed out by the weight of the superstructure, which must be supported until it has set. The fixing sockets for the top of the bearing should be ready cast into the soffit, but this requires accurate casting, using a jig drilled mould insert to match the bolting arrangement in the bearing. Alternatively, the sockets may be replaced by a single plate, cast in flush with the soffit, and tapped to receive the bearing fixing bolts.

Where a concrete superstructure is cast *in situ*, the bearing top plates, with dowels, can be built into the soffit shuttering. The area around the mould cut out must be carefully sealed to ensure that concrete mortar does not leak into the working parts of the bearing during placing, and all sliding plates must be propped to prevent them distorting under the weight of wet concrete.

Great care must be exercised to ensure that *F* range lateral restraint bearings are correctly oriented.

Some Useful Details About Freyssinet Pot Bearings (Tetron Disc Type, D3 Range)

Material Specification of Disc Bearings

Rubber disc Natural Rubber to BS1154 (+ anti-ozonants)

Ring High grade heat treated steel

Side Restraints (in D3F type only) Mild steel

Pins for side restraint Special spring-steel, minimum yield strength 1100 N/mm^2.

Seal and sliding surface High grade Stainless steel.

Top plate, rocker and base plate Mild Steel. PTFE BS.3784

Socket head cap screws BS.4168 Grade 12.9

Recommended HD bolt BS.3692 Grade 8.8 (zinc plated)

Cast-in sockets Mild Steel

All permanently exposed steel parts are corrosion protected with two-component metallic zinc rich epoxy coating followed by chlorinated rubber paint. Full technical data can be obtained from manufacturers.

Components

- Tetron Disc bases and rockers, and all sliding plates, are made of corrosion protected mild steel.
- Sliding plates are faced with a smooth surface of high quality stainless steel.
- Sliding surfaces are pure 'dimpled' PTFE, incorporating grease pockets which allows a permanent reservoir of lubricant.
- Elastomer used for rotational purposes in Tetron Disc bearings is high grade natural rubber to BS 1154.
- A mastic seal is provided around the rocker to prevent ingress of moisture into the base unit.
- These bearings can be removed from the structure with minimum of jacking when used in conjunction with special cast-in sockets or similar devices.

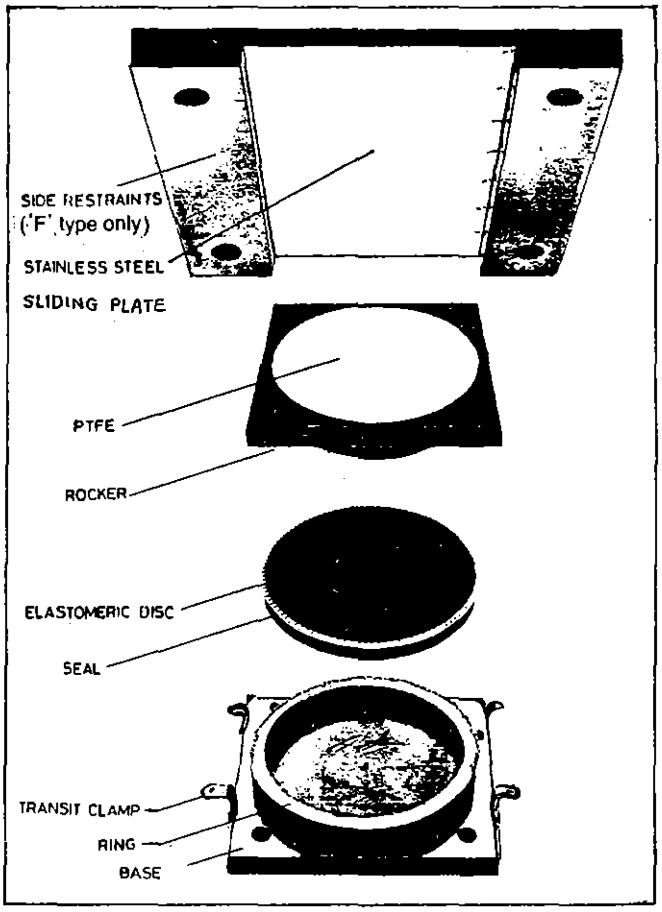


Fig. 17.16 *Tetron disc (pot) bearing*

(a) Tetron D3T Fixed (Table 17.18)

Fixed in all directions, free to rotate in all directions.

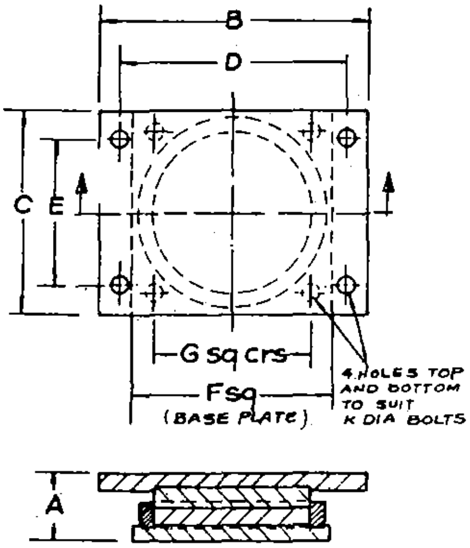


Fig. 17.17 *Tetron D3T fixed*

Table 17.18

Bearing type	Principal dimensions								Max. load vertical	Max. load horizontal
	A	B	C	D	E	F	G	K	(kN)	(kN)
D3T 50	58	235	170	195	120	170	130	M12	500	100
D3T 80	75	340	235	280	155	235	175	M20	800	150
D3T 100	80	355	250	295	170	250	190	M20	1000	150
D3T 125	83	375	270	315	190	270	210	M20	1250	190
D3T 160	92	395	290	335	210	290	230	M20	1600	220
D3T 200	97	460	335	385	240	335	260	M20	2000	250
D3T 250	97	485	360	410	270	360	285	M20	2500	280
D3T 325	116	575	410	475	280	410	310	M20	3250	300
D3T 400	127	615	450	515	330	450	350	M24	4000	360
D3T 500	132	680	515	580	390	515	410	M30	5000	500
D3T 650	141	770	570	645	420	570	440	M30	6500	600
D3T 800	156	835	635	710	490	635	510	M30	8000	650
D3T 1000	175	950	710	805	540	710	560	M30	10000	700
D3T 1250	179	1015	785	870	620	785	640	M30	12500	900
D3T 1600	203	1140	870	970	680	870	700	M30	16000	1000
D3T 2000	203	1260	985	1090	780	985	800	M36	20000	1300
D3T 2500	232	1425	1100	1220	875	1100	895	M42	25000	1600
D3T 3000	257	1550	1230	1350	1000	1230	1030	M42	30000	2000

Rotation as for D3E type (see ahead Table 17.19)

(See notes at the end of Table 17.20)

(b) Tetron D3E Free Sliding (Table 17.19)

Free sliding in all directions, free to rotate in all directions.

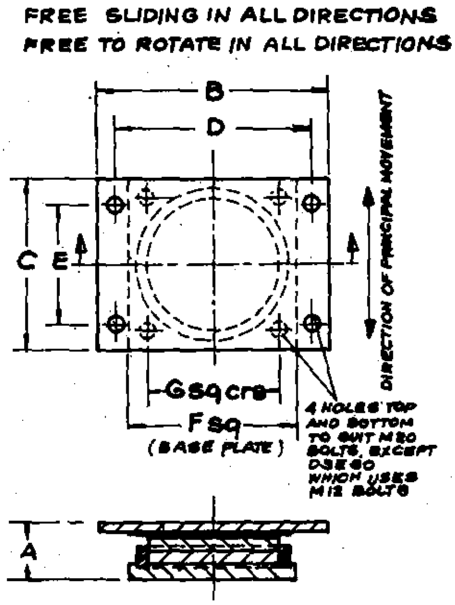


Fig. 17.18 *Tetron D3E free sliding*

Table 17.19

Bearing type	Principal dimensions							Max. load vertical (kN)	Normal rotation (Radian)
	A	B*	C*	D*	E*	F	G		
D3E 50	60	235	170	195	120	170	130	500	0.026
D3E 80	73	340	235	280	155	235	175	800	0.026
D3E 100	78	355	250	295	170	250	190	1000	0.026
D3E 125	82	375	270	315	190	270	210	1250	0.026
D3E 160	86	395	290	335	210	290	230	1600	0.026
D3E 200	92	460	335	385	240	335	260	2000	0.026
D3E 250	99	485	360	410	270	360	285	2500	0.024
D3E 325	112	515	375	455	280	410	310	3250	0.022
D3E 400	128	555	420	495	330	450	350	4000	0.022
D3E 500	128	620	465	560	390	515	410	5000	0.020
D3E 650	137	675	510	615	420	570	440	6500	0.018
D3E 800	147	740	575	680	490	635	510	8000	0.016
D3E 1000	162	815	635	755	540	710	560	10000	0.016
D3E 1250	168	910	700	840	620	785	640	12500	0.014
D3E 1600	183	1000	780	925	680	870	700	16000	0.012
D3E 2000	193	1155	875	1055	770	985	800	20000	0.012
D3E 2500	213	1270	970	1170	865	1100	895	25000	0.012
D3E 3000	228	1440	1080	1310	950	1230	1030	30000	0.012

* Dimensions B, C, D and E are for zero movement, and specified movement has to be added to above in increments of 100 mm

(See notes at the end of Table 17.20)

(c) *Tetron D3F Sliding guided* (Table 17.20)

Sliding guided in one direction, free to rotate in all directions.

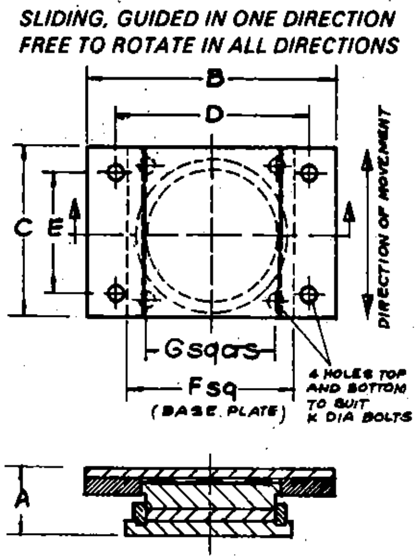


Fig. 17.19 *Tetron D3F sliding guided*

Table 17.20 *(for Tetron D3F sliding guided)*

Bearing type ↓	Principal dimensions A	B*	C*	D	E*	F	G	K	Max. load vertical (kN)	Max. load horizontal (kN)
D3F30	75	225	160	180	110	160	120	M12	300	80
D3F50	79	260	170	195	120	170	130	M12	500	100
D3F80	103	345	235	280	155	235	175	M20	800	150
D3F100	108	360	250	295	170	250	190	M20	1000	150
D3F125	117	405	270	315	190	270	210	M20	1250	190
D3F160	121	425	290	335	210	290	230	M20	1600	220
D3F200	129	475	335	385	240	335	260	M20	2000	250
D3F250	129	500	360	410	270	360	285	M20	2500	280
D3F325	138	565	410	475	280	410	310	M20	3250	300
D3F400	158	605	450	515	330	450	350	M24	4000	360
D3F500	158	680	515	580	390	515	410	M30	5000	500
D3F650	167	745	570	645	420	570	440	M30	6500	600
D3F800	177	810	635	710	490	635	510	M30	8000	650
D3F1000	192	905	710	805	540	710	560	M30	10000	700
D3F1250	198	970	785	870	620	785	640	M30	12500	900
D3F1600	213	1070	870	970	680	870	700	M30	16000	1000
D3F2000	227	1215	985	1090	780	985	800	M36	20000	1300
D3F2500	267	1360	1100	1220	875	1100	895	M42	25000	1600
D3F3000	282	1490	1230	1350	1000	1230	1030	M42	30000	2000

Rotation as for D3E type (see Table 17.19)

* Dimensions B, C ~~and E~~ are for zero movement, and specified movement has to be added to above in increments of 100 mm.

NOTES For Tables 17.18 to 17.20

(i) Base contact stress, of the bearings illustrated, approaches 20 N/mm^2.

(ii) Sliding plate dimensions shown are for zero movement. Add to these the amount of sliding required in increments of 100 mm. The bearings may then be described in a code, for example, thus:

BEARING   LATERALMOVEMENT (cm)   BEARING

D3E 650 / 10 10   OR   D3F 650 / 10

PRINCIPAL MOVEMENT (cm)   MOVEMENT (cm)

(iii) The height A is nominal; manufacturing tolerances give a variation of $\pm$ 3 mm on tabulated figure.

(iv) The size of fixing bolts listed in the tables assumes assistance from friction due to the minimum vertical load normally present in service.

(v) Larger capacity bearings are also available.

Small Bearings—Simplified Fixing (Freyssinet Tetron D3M Range)

- A simplified fixing method is often preferred for the smaller sizes of disc bearing where cast-in sockets are not specified.
- Both D3M and D3MT types are smaller overall than the corresponding D3E and D3T equivalent but are built to a similar specification.

(a) Tetron D3M Free Sliding (Fig. 17.20 and Table 17.21)

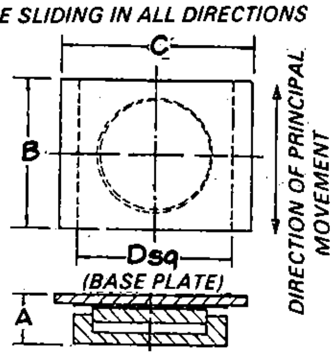


Fig. 17.20 *Tetron D3M free sliding*

Table 17.21

Bearing type ↓	Principal dimensions A	B*	C*	D	Max. load vertical (kN)	Max. rotation (Radian)
D3M 30	53	160	200	160	300	0.028
D3M 50	56	170	220	170	500	0.028
D3M 80	69	235	290	235	800	0.026
D3M 100	79	250	305	250	1000	0.026
D3M 125	83	270	325	270	1250	0.026
D3M 160	87	290	345	290	1600	0.026
D3M 200	96	335	390	335	2000	0.026
D3M 250	100	360	435	360	2500	0.024

• The D3M free sliding bearing is intended to locate by friction only, so adequate vertical loading must always be present to prevent slip.
* Dimensions B and C are for zero movement, and specified movement has to be added to above in increments of 50 mm.

(b) Tetron D3MT Fixed (Fig. 17.21 and Table 17.22)

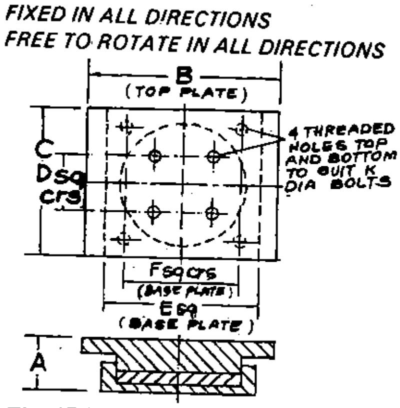


Fig. 17.21 *Tetron D3MT fixed*

Table 17.22

Bearing type ↓	Principal dimensions A	B	C	D	E	F	K	Max. load horizontal (kN)
D3MT 30	57	200	160	60	160	120	M12	80
D3MT 50	61	210	170	60	170	130	M12	80
D3MT 80	68	275	235	80	235	175	M16	110
D3MT 100	73	290	250	95	250	190	M16	120
D3MT 125	77	310	270	110	270	210	M16	140
D3MT 160	86	330	290	125	290	230	M16	160
D3MT 200	97	375	335	145	335	260	M20	210
D3MT 250	99	400	360	165	360	285	M20	250

• 'Rotation' and 'vertical-load' as for D3M type in Table 17.21.
• The D3MT fixed bearing is provided with threaded holes in top and base plates for simple bolt/dowel fixings. The size of fixing assumes assistance from friction due to vertical loading.

NOTES For Tables 17.21 to 17.22

1. Sliding plate dimensions shown are for zero movement. Add to these the amount of sliding required in increments of 50 mm. The bearing may then be described in a code, for example, thus

BEARING LATERAL MOVEMENT (cm)

D3M 80/ 05 05

PRINCIPAL MOVEMENT (cm)

2. Base contact stresses are approximately 20 N/mm² at maximum rated load.
3. The height *A* is nominal; manufacturing tolerances give a variation of ±3 mm on tabulated figures.

(c) Tetron D3MF Sliding guided

Where a sliding guided bearing is required use a D3F, Table 17.20.

Installation of Freyssinet Pot Bearings (Tetron Disc Type, D3 range)

In a bearing that is free sliding in all directions (e.g., the D3M and D3E ranges) positive fixing to the main structure may not be required at all because, if the bearing is always subjected to adequate vertical loading, horizontal movement will occur on the plane of least resistance which is, of course, the bearing sliding surface. Nevertheless, in some cases it is prudent to provide fixings to guard against displacement during installation, impact, vibration and accidental unloading. In a light structure, the bearing fixings must be vibration resistant, otherwise they may work loose.

Consideration should always be given to the practicability of removing and replacing the bearings, should this prove to be necessary.

Most cases of bearing malfunction (as indicated earlier) are attributable to faulty installation, and almost all bearing damage occurs during installation, or even earlier, during handling and storage. Careless handling on site, and the ingress of dirt, can easily lead to abnormally high frictional resistance. Tetron bearings are normally delivered in a condition to discourage unnecessary dismantling, and bolts or straps are used to connect together the upper and lower parts of the bearings. These temporary fixings, as well as excluding dirt during installation, prevent accidental relative displacement between the parts of the bearings, but they must be removed before the bearings are called upon to slide or rotate. However, as explained earlier, they are not structural fixings and should be supplemented, for example, by wedges during installation.

Bearings should be clearly identified by marking them with such details as type, and location. Marking is particularly important where the top plate is to be offset. Bearings should be transported and unloaded carefully and then stored under cover in clean, dry conditions until required. An inspection should be carried out shortly before installation, and bearings that have been damaged in store should not be accepted — almost certainly they will not work correctly, unless they are repaired.

As much preparatory work as possible should be carried out before bringing a bearing to its actual location. The seating should be level, and this usually necessitates the use of a mortar bedding composed of sand and either cement, polyester resin or epoxy resin, with a cube crushing strength of at least 35 N/mm^2, as stated earlier. If the bearing is located directly on steelwork, the seating area should be machined.

Base Fixings An *E* or *M* type bearing is free sliding in all directions, with a low coefficient of friction, so that theoretically the base does not require positive fixing, but see note above. All other types of bearing can resist horizontal loading and so their bases should be fixed.

There are several ways of achieving this. With some bearings a large recess can be left in the pier/abutment, into which the bearing base is placed bodily, bedded upon and surrounded with cement/sand or epoxy resin mortar (see Fig. 17.22). The recess must be correctly reinforced on all sides. Alternatively, small pockets can be left for dowels or bolts, which are then set accurately to position and level in grout using timber or light steel jig, made up from the bearing itself. The bearing base is then lowered over the dowels or bolts, on to a bed of wet mortar (Fig. 17.23).

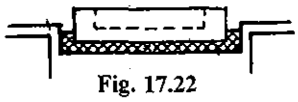


Fig. 17.22

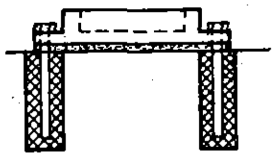


Fig. 17.23

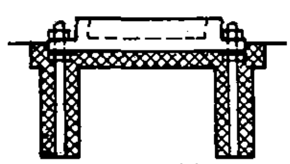


Fig. 17.24

It is possible to cast-in the fixing bolts and position the bearing in one operation, in which case the bearing is supported on rubber washers, over levelling nuts on the bolts, and a chemical resin grout is then poured around the bolts and under the bearing (Fig. 17.24). The fixing should not be over-tightened, when the grout has set, otherwise the bearing may be distorted, due to compression in the washers. Nevertheless, rubber washers must be used, to prevent the fixing bolts carrying vertical loading.

Sockets provide the easiest cast-in fixings with removal in view, for all types of bearing.

Top Fixings For similar reasons, top fixings must be provided for all bearings (optional for the *E* and *M* types). Fixed end bearings of the D3MT type are intended for use with steel or *in situ* concrete construction and have holes for dowels projecting into the rocker, whereas the D3T type has larger top plate with external holes for bolts, and is intended primarily for use with precast concrete, or steel structures, and to facilitate removal.

A completed precast concrete structure may be lowered on to a skim of mortar on the top of the bearing, to eliminate soffit irregulations. The mortar mix needs careful control to ensure that it is not totally squeezed out by the weight of the superstructure, which must be supported until it has set. The fixing sockets for the top of the bearing should be ready cast into the soffit, but this requires accurate casting, using a jig drilled mould insert to match the bolting arrangement in the bearing. Alternatively, the sockets may be replaced by a single plate, cast in flush with the soffit, and tapped to receive the bearing fixing bolts.

Where a concrete superstructure is cast *in situ*, the bearing top plates, with dowels, can be built into the soffit shuttering. The area around the mould cut out must be carefully sealed to ensure that concrete mortar does not leak into the working parts of the bearing during placing, and all sliding plates must be propped to prevent them distorting under the weight of wet concrete. A disc type bearing will compress by perhaps 2% of its total height during the casting operation, which must be provided for, to facilitate the removal of the shuttering.

Subsidiary spreader plates, with a special taper, may be required to accommodate a superstructure with an inclination or cross fall, if the bearings themselves are not capable of rotating to accept the requisite slope. When intermediate tapered plates are used, the plane of movement in sliding bearings may not coincide with the plane of the superstructure.

Great care must be exercised to ensure that *F* range lateral restraint bearings are correctly oriented.

NOTE

(i) It cannot be stressed too strongly that bearings should not be dismantled on site because the effects of dirt on the sliding and rotating surfaces are highly deleterious. Where appropriate, bearings should be requested with top plates and rockers preset to accommodate anticipated movements to prevent unnecessary dismantling on site.

(ii) In all cases the transit bolts connecting the bearing rocker to its base must be removed after the mortar has set and before the bearing is called upon to rotate or slide. These bolts are identified by metal labels.

17.11 ARTICULATION SYSTEMS

Choosing the type of bearing-connection (e.g., whether to provide fixed type, pinned type, sliding type, or elastomeric type, etc.) in order to attain an ideal Articulation system for different structural schemes of superstructures, keeping in view the effect of the location of the deck's zero-movement point is discussed here. Reference may be made to Chs. 8 and 9 for the mechanics of distribution of the applied and the self-induced horizontal forces among bridge supports.

~ Articulation by Elastic Restraint Throughout

This type of articulation uses elastomeric bearings throughout and makes use of the elastic properties of elastomeric bearings to restrain the superstructure adequately and yet share the horizontal loads and movements between the piers and abutments according to their individual abilities (shear ratings, i.e., shear stiffnesses) to take these loads and movements. There are no rigid connections between the bridge superstructure and substructure in such an elastically restrained bridge deck as there is no need for this type of connection here.

The system has been found to be particularly effective on long continuous bridges but can also be used on simply supported bridges to economic advantage.

• Articulation Systems for Continuous Superstructures

Case 1 Superstructure pinned at one support and sliding bearings used on the remaining supports

This type of articulation allows great distances between expansion joints in the superstructure. The sliding bearing limits the force that can be transmitted to the pier/abutment and hence any excess force has to be taken by the pinned pier/abutment. These often need special strengthening when the bridge is long. It is usually best to arrange for the pinned pier/abutment to be near the zero-movement point of the deck. When this is done, the reactions (from shortening and lengthening) from the piers with sliding bearings on either side of the pinned pier are in opposite directions and hence tend to cancel each other.

Case 2 Superstructure monolithic at one support and sliding bearings on the other supports

This type of articulation produces similar results to that in Case 1. The system can be made efficient by altering the coefficient of friction of the sliding bearings.

Case 3 Superstructure monolithic at one support and elastomeric bearings on the others

More than one pier can be made monolithic with the superstructure in special cases such as short bridges, or bridges with high (slender) piers. The articulation is quite efficient because all piers share the horizontal loads and all piers are subjected to temperature and other superstructure movements. With flexible piers, these movements can be accommodated for considereable distances between joints. However, as the piers get shorter (and stiffer), the forces induced in the piers and superstructure can become considerable and the effort to retain this type of articulation becomes uneconomical in such cases.

With this articulation, the monolithic pier tends to carry a considerable proportion of the total longitudinal force if it is much stiffer than the other piers.

Case 4 Superstructure pinned at one support and elastomeric bearings on the others

This type of articulation has advantages over the monolithic type because the pinned pier has a lower stiffness than the monolithic pier and hence attracts less horizontal force, thereby throwing more horizontal force into other supports. This results in a more even distribution of the horizontal forces. The system is very efficient particularly if the pinned pier is placed nearest to the deck's zero movement point. While the load sharing of this articulation system is particularly good, the only real disadvantage is the necessity to design a pin detail and its possible construction difficulties. This can be improved by installing a rocker type bearing in place of the pinning arrangement.

Case 5 Superstructure with elastomeric bearings on all supports

This is the articulation system particularly suited to long continuous structures; and has been described earlier.

• Articulation System for Simply-Supported Superstructures

Case 1 Span pinned at one end, sliding-bearing installed at the other end

This type of articulation usually transmits most of the applied longitudinal force to the pinned support. Some degree of control of the force transmitted to the sliding bearing pier can be obtained by varying the coefficient of friction of the sliding material. This is usually limited in its extent and not particularly accurate.

Some care must be taken in this type of design to analyse the system thoroughly to see if the sliding bearing will infact actually slide at all. If placed on a tall slender pier with low stiffness which cannot develop sufficient horizontal reaction to overcome the friction force, then the bearing may never slide.

Case 2 Span pinned at one end, and elastomeric bearings installed at the other end

This type of articulation is common and easy to construct. Characteristically the pinned pier usually takes most of the longitudinal force because it is usually stiffer than the combined stiffness of the pier and elastomeric bearing at the other end. However, if needed for pier design, by reducing the bearing thickness the shear rating of the elastomeric bearing can be increased to attract more horizontal force to its pier.

Shortening and lengthening movements produce pier forces which depend on the combined stiffness of the elastomeric bearing and that of the pier under it.

Case 3 Span with elastomeric bearings at both ends

The characteristics of this type of articulation are different from those already considered. Longitudinal forces on a span are shared by the piers at each end of the span in proportion to the combined stiffness of the pier and its bearing.

Shortening and lengthening movements apply forces to the supports in proportion to the combined support-bearing system stiffnesses. These forces tend to cancel if adjacent spans are similar, but no such cancellation can occur on the end-pier (abutment) which may have to be designed to transmit the greatest horizontal force or alternatively may have to be designed with a bearing of less stiffness than the other piers. As for an intermediate pier, if it is assumed rigid (as is many times the case), the deck-movements above it from the adjoining simple spans will cause in it a net force of only $(S_1\Delta_1 - S_2\Delta_2)$ where S_1 and S_2 are the shear ratings of the elastomeric bearings on the two sides of the pier and Δ_1 and Δ_2 the deck-movements above them. (S_1 represents the sum of shear ratings of all the bearings under the deck-end on one side of the pier, and S_2 that for the bearings under the other deck-end on the pier.)

• Conclusions Regarding Articulation Systems

The following conclusions can therefore be drawn from above:

(i) That, the procedure of using bearings stiffness as a variable in the design process (in order to match pier horizontal loads to pier capacities) can produce savings.

(ii) That, the elastomeric bearings are superior in achieving the most economic longitudinal load sharing amongst bridge supports in both continuous and simply supported bridges.

(iii) That, completely elastically restrained structures allow the greatest degree of freedom to the designer to achieve the best longitudinal load sharing between supports, hence the most economic bridge substructure of any of the bridge articulation systems.

(iv) That, improvements in longitudinal load sharing between supports can be achieved with sliding bearings if the coefficient of friction is treated as a controllable variable.

REFERENCES

1. *World Congress on Joints and Bearings*, American Concrete Institute, Publication SP-70, September 1981.
2. "Non-slip Bearings add to M6 Headaches", *New Civil Engineer*, 13th November, 1980.
3. Long, J.E. "Replacement of Bridge Bearings", Paper presented to Symposium on bearings for concrete bridges—Cement and Concrete Association, London, October 1980.
4. Shaw, J.D.N., "Bedding Materials for Bridge Bearings", *Symposium on Bearings for Concrete Bridges*, Cement and Concrete Association, London, October 1980.
5. "Spaghetti Junction to Stay Open", *New Civil Engineer*, 14th August, 1980.
6. Lee, D.J. *Theory and Practice of Bearings and Expansion Joints for Bridges*, Cement and Concrete Association, London 1971.
7. Long, J.E. *Bearings in Structural Engineering*, Newnes-Butterworth, 1974.
8. Dr. Ing V. Hakenjos Dipl. Ing. Richten MPA Stuttgart, "Experiments on Sliding Bearings" from *Lager in Bauwesen*, Eggert, Grote, Bauschke (figure 7.14 on page 306).
9. Institut fur Bautechnik, Richtlinien für die Zulassungs—prufüng von bewehrten Elastomer lagern, Berlin (1980) (in German); (Rules for approval tests for laminated elastomeric bearings).
10. DuPont, Elastomerblatter No. 102, Geneva, (in German) (Neoprene Notebook), 1975.

CHAPTER 18

The Superstructure

18.1 INTRODUCTION

The basic function of a bridge superstructure is to permit uninterrupted smooth passage of traffic over it and to transmit the loads and forces to the substructure safely through the bearings. Although it is difficult to stipulate the aesthetic requirements, it should, however, be ensured that the type of supersturcture adopted is simple, pleasing to the eye, and blends with the environment. No hard and fast rules can be laid regarding the economy in cost. The designer should, however, be able to evolve the most economical type of superstructure based on his judgement and experience given the particular conditions prevailing at the particular site at the particular time. (Also see Ch. 42.)

Materials Used for Constructing Superstructure

The following materials are generally adopted for constructing permanent bridge decks:

(a) Reinforced concrete
(b) Prestressed concrete
(c) Steel
(d) Masonry
(e) Composite construction using steel and reinforced concrete, or reinforced and prestressed concrete.

Reinforced Concrete Superstructure

The usual types of deck sections in this category are,

(a) Solid slab
(b) Slab and girder (T-beam)
(c) Hollow box girder.

The deck arrangement can be simple or continuous spans, or frame, arch, balanced cantilever or bow string type. Some components may also be precast.

Prestressed Concrete Superstructure

The usual types of deck sections in this category are:

(a) Voided slab
(b) Slab and girder (T-beam)
(c) Hollow box girder.

The deck arrangement can be simple or continuous spans, or frame, balanced cantilever, free cantilever: cast *in situ* precast or segmental.

Composite and Steel Superstructure

The usual types of deck in this category are:

(a) Longitudinal and transverse beams, with concrete slab
(b) Longitudinal plate girders, transverse beams and concrete slab
(c) Longitudinal box girder with concrete slab
(d) Steel truss.

The deck arrangement can be simple or continous spans, or arch or frame. The slab can be non-composite or can be made composite with the longitudinal beams by shear connectors.

Special Superstructures

(a) Orthotropic deck
(b) Cable stayed deck
(c) Suspension type deck.

Age old masonry superstructures are generally of arch type.

The composite type of decks generally involve combination of two dissimilar structural elements. Precast or prefabricated beams are connected to RC slab by special structural elements called shear connectors, acting together as a unit. The beams may be either:

(a) Prefabricated structural steel beams/girders, or
(b) Precast reinforced concrete, or precast prestressed concrete beams.

Basic Principle

The superstructure of any bridge must be designed such that it satisfies the geometric and load-carrying requirements set forth by its owner. (In distinct comparison, apart from any navigational requirements, the owner does not have to set forth any special requirements for the substructure.) The geometric requirements depend on the number and widths of traffic lanes and footpaths (and cycle tracks, if any) that have to be carried across. They also depend on the overall alignment and various horizontal and vertical clearances required above and below the roadway. Once the geometric considerations are decided, the superstructure has to be designed to meet various structural design requirements. These include considerations of strength, stiffness and stability. This first requires estimation of internal forces and moments and displacements which the externally applied forces will cause in the selected scheme and form of the structure (based on assumed first-trial dimensions of its elements), and then deciding the final section sizes, and reinforcements (and prestressing, if the structure is prestressed). The former process constitutes 'analysis' and the latter process constitutes 'design'. The analysis is generally done based on elastic behaviour of the structure. The design is then done either on elastic (i.e., working) strength basis or on load-factored (i.e., ultimate) strength basis, also ensuring the serviceability criteria (like limiting the flexural crack-widths, deflections and vibrations).

Some structural analysis work is done using empirical formulae. Sometimes, if the structure is very complicated, model analysis is resorted to. However, with the modern day analytical techniques, aided by fast computers, very complicated structural analyses can be accomplished. (Writing and perfecting the appropriate computer program for a particular analysis may consume time, and even preparing the input data for various load cases may take some time, but the actual analysis can be carried out relatively quickly. Indeed, nowadays ready-to-use programs are available in program libraries for a very large number of structural analysis (and even design) problems. This can simplify the work and make the drudgery more bearable.)

Geometrical Alignment

The horizontal and vertical alignment of a bridge should be governed by the geometrics of the highway, roadway or channel, it is crossing. A bridge may either be right or skewed, straight or curved, or any combination of these.

For girder type bridges, the girders may either be curved or straight, and may be aligned on chords between supports with the deck slab built on the curve. The following points require close examination when girders are aligned on a chord:

(a) Non-symmetrical deck cross-section
(b) Deck finish of the 'warped' surface
(c) Vertical alignment of the curbs and railings, to preclude visible discontinuities
(d) Proper development of super-elevation (cant).

Lighting

The lighting of the bridges should be in accordance with the provisions of the authority having jurisdiction on that area.

Drainage

The transverse drainage of the roadway should be accomplished by providing a suitable crown in the roadway surface, and the longitudinal drainage should be accomplished by camber or gradient. Water flowing downgrade in a gutter section, should be intercepted and not permitted to run onto the bridge. Short continuous span bridges, particularly over-passes, may be built without drain inlets and the water from the bridge surface carried off the bridge and downslope by open or closed chutes near the ends of the bridge structure. Special attention should be given to ensure that water coming off the end of the bridge is directed away from the structure without eroding the approach embankments. Such erosion has been a source of significant maintenance costs.

Longitudinal drainage on long bridges is accomplished by providing a longitudinal slope of the gutter (min. of 0.5% preferred), and draining to scuppers or inlets which should be of a size and number so as to drain the gutters adequately. At a minimum, scuppers should be located near each roadway joint. Downspouts, where required, should be of rigid corrosion resistant material not less than 100 mm (and preferably 150 mm) in the least dimension. The details of deck drains should be such as to prevent the discharge of drainage water against any portion of the structure and to prevent erosion at the outlet of the downspout. Overhanding portions of concrete deck should be provided with a drip bead or notch.

Traffic Lane Width, Road Width, Footpaths, and Clearance for Vehicles/Boats

Traffic lane width and design lane width have been discussed in Ch. 2 of this book. Road width, the distance between roadside faces of the kerbs, depends on the number (and width) of traffic lanes and the widths of the bounding hard shoulders.

Footpaths (i.e., walkways) are provided where pedestrian traffic is anticipated, but not on major arteries or in countrysides. Where provided, its width is 1.5 m generally, but may be as narrow as 0.60 m and as wide as 2.50 m depending on the requirements.

Minimum vertical clearance below a bridge deck, for vehicles to pass under it, is generally 5.50 m. Horizontal and vertical navigational clearances, under a bridge deck crossing a navigation channel, depend on the type of navigation, size of boats/ships expected to cross under, and high flood or high tide level, as the case may be. Horizontal clearance of 80 m with a vertical clearance of 20 m in the central half of the navigation spans is not unusual in some river crossings. However, actual values for a particular case have to be decided with the concerned waterway authorities.

Road Kerb, Crash-barrier, Parapet and Hand Rail

The road kerb is either 'surmountable' type (in which case the portion of the deck beyond it, e.g., footpath, is designed for an overriding point load of 4 to 5 tonnes, distributed over a 300 mm diameter contact area, allowing 25% overstress) or 'insurmountable' type. The latter is generally at least 225 mm high. In the absence of walkways, a road kerb is combined with the parapet. Parapets can be of many shapes and of variable sturdiness. They should be designed to prevent a fast moving vehicle of a given mass from shooting off the roadway (in the event of an accidental hit). Their height varies, but it should be at least 700 mm. They should be mounted by metal hand rail, about 350 mm high. Their road side face is best double-sloped, as shown in Fig. 18.1. This, sometimes referred to as the New Jersey barrier, has demonstrated superior safety aspects, and is presently being adopted for use by many highway authorities. Road kerbs, footpath slabs (and their supports), and the parapets should normally be provided with deliberate vertical cuts or other suitable details of discontinuity to prevent them from acting monolithically with the deck section, so that they do not share the deck moments and crack open over piers or spall in the midspan zones.

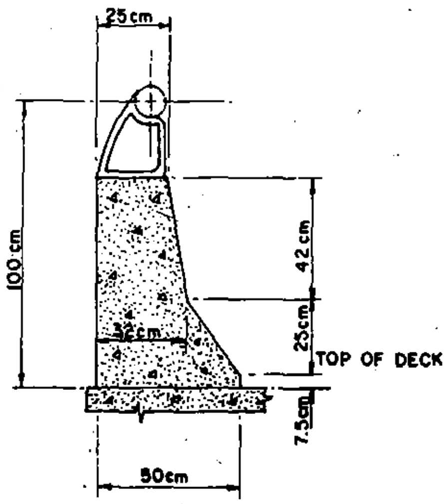


Fig. 18.1 *Parapet curb*

Sometimes walkways are protected from the erring vehicular traffic by crash barriers which act as insurmountable kerbs and deflect the hitting vehicles back into the traffic lane. A crash barrier essentially consists of corrugated or pressed metal sheet spanning horizontally between posts.

Expansion and Roadway Joints

To provide for expansion and contraction, joints should be provided at the movement end of spans, and at other points, where they may be desirable. Joints should preferably be sealed to prevent erosion and filling of debris. Presented below are some general details. More details are given in Ch. 32 of this book.

(a) *Field moulded joints (fillers)*

(i) *Mastics* The extension-compression range of these materials is approximately ±3%. These are used only where very small movements (±2 mm) are anticipated.

(ii) *Thermoplastics* The extension-compression range of these materials is generally ±5%. These are used only in horizontal joints where small movements (±4 mm) are anticipated and where first cost is a major factor.

(iii) *Thermosetting plastics* Sealants in this class are either one or two component systems, which cure by chemical reaction or by the release of solvent, changing from a liquid form into a solid state. They include polysulfide, silicone, urethane, chloro sulfonated polyethylene, butyl, neoprene and epoxy based materials. These materials have an extension-compression range of ±7% to ±25%, at a temperature range of −40° to 80°C. Their abrasion and indentation resistances are low. Joints are normally limited to 5 cm in width and must be protected if exposed to vehicular traffic.

(b) *Compression seals*

Compartmentalized neoprene, extruded to the desired configuration, is used for most compression seals. Neoprene compression seals used singly or as components in combined systems, can prove effective over wide ranges of movement. They must retain at least 15% compression at the widest opening. The allowable movement is approximately 40% of the uncompressed seal width. The maximum size of single neoprene compression seals is about 150 mm.

(c) *Compression-tension seals*

Neoprene expansive elements can be combined with encased steel-bearing plates and anchorage angles, to form a tension-compression joint. Such devices can be used for a range of movements from 30 to 700 mm. These joints, bolted directly to the bridge decks, transmit the wheel loads on the joint surface and seal the joint. Neoprene and metal combination joints can perform the expansion flexibility functions and simultaneously seal the joint effectively.

(d) *Steel plates and finger joints*

Steel joints of the sliding plate and finger conformation, commonly used for longer spans, are gradually being replaced by compression seals in combination with compression-tension seals.

To summarize, type (a) joints provide below average service and present excessive maintenance problems; type (b) joint should give good service; type (c) joints require careful consideration of thermal thrust transmitted by the joint; type (d) joints are inordinately costly and difficult to maintain. This is discussed in more detail in Ch. 32 of this book.

Medians

On expressways and freeways, the opposing traffic flows should be separated by median strips. Where possible, the lanes carrying opposing flows should be separated completely onto two distinct structures. However, where width limitations force the utilization of traffic separators (less than 1.2 m wide), the following median sections may be used:

(a) Parapet sections 300 to 700 mm in height, either integral or with a rail section. The bridge and approach parapets should have similar sections.
(b) Low rolled curb sections or double curb units with some form of paved surface in between.
(c) A series of transverse bars, rounded large blocks or buttons, placed on the deck along the centerline, as recommended by public agencies.

Super-elevation

The super-elevation of the surface of a bridge on a horizontal curve should be provided in accordance with the applicable standard for the highway. This should preferably not exceed 0.06 meter per meter, and never exceed 0.08 meter per meter.

Basic Structural Schemes

These are described in some detail in the remaining part of this chapter.

18.2 GENERAL COMPARISON OF REINFORCED CONCRETE AND PRESTRESSED CONCRETE SUPERSTRUCTURES

In recent years a great majority of bridges have been constructed in prestressed concrete. However, RC bridges have not been eliminated, and in certain circumstances still, are more economical.

Prestressed concrete bridges offer a high degree of freedom from cracks, low maintenance, long life; and they are particularly suitable for dynamic loads and vibrations, and offer great resistance to fatigue arising from repetitions of live loading. (Also see Ch. 37 of this book.)

In RC construction, live load normally approximately doubles the stress in steel, while in prestressed concrete this increase is very little. Position of neutral axis in RC (about which moments of areas from tension and compression side balance) remains fixed, but of that in prestressed concrete shifts with variation in applied load.

The greatest advantage of prestressed concrete for bridges is reduction in weight, slender appearance, possibility of precasting in factory conditions either in segments or even in whole spans, possibility of larger spans and freedom from tension and cracking. For long spans, prestressed concrete decks can be nearly comparable in weight with steel decks.

It is not possible to quote a minimum prestressed concrete span which can be most economical, it depends on local conditions.

The real meaning of economy of a bridge must include not only its initial cost of materials and labour, but also cost of maintenance, the life of the bridge, its depreciation, cost of attending to bearings and piers, sometimes adjustment of thrust, atmospheric deterioration, etc. The aesthetics of a bridge, however, cannot be measured in a similar way.

For short bridges the advantage of using smaller amount of concrete because of prestressing may be unimportant since there is always a certain basic cost which cannot be changed by the use of prestressing. In such cases the usual reinforced concrete construction may be used with equal advantage.

18.3 SLAB TYPE SUPERSTRUCTURE (SOLID OR VOIDED), STATICALLY DETERMINATE OR INDETERMINATE

Slab bridges are frequently used for small spans, they require more concrete and steel than girder bridges of the same spans, but construction cost is usually lower and their

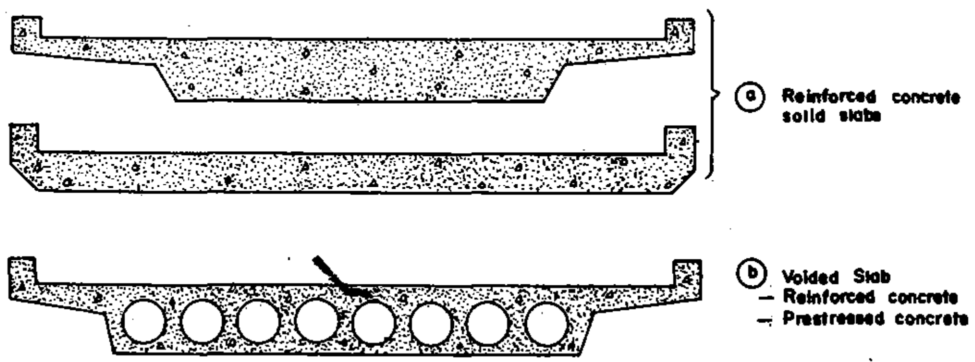


Fig. 18.2

formwork is simpler and less expensive.

The limit of span of slab bridges depends on the magnitude of load and the relative costs of formwork, materials and labour. The small headroom under slab bridges can also have some bearing on economy through cheaper formwork.

Slab bridges may be divided into three classes:

- slabs cast *in situ*,
- slabs built-up of precast elements, and
- composite slabs, in which precast elements are used in combination with *in situ* concrete filling.

Cast *in situ* slab bridges may be adopted from 6 to 30 m spans. For spans up to about 8 m, solid reinforced concrete slabs, with depths up to 60 cm, may be adopted. Voided RC slabs, with depths up to 80 cm, may be adopted for spans of about 8 to 15 m. However, for spans of 15 to 30 m, voided prestressed concrete slabs, of depths up to about 1.20 m, are cheaper.

Solid composite slab decks shown in Fig. 18.3 may be adopted for 8 to 15 m spans. These consist of precast units of various shapes. The units are either of inverted T or symmetrical I section, placed side by side, and stressed together transversely after the *in situ* filling or topping (Fig. 18.3). Shear connectors are used to achieve composite action between the precast elements and in filling concrete or top slab, as the case may be.

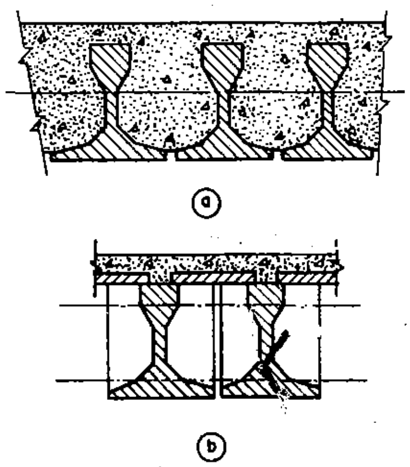


Fig. 18.3

Slab system is usually adopted when the erection of formwork presents no great difficulty. Cables or reinforcements are then placed in position, and after concrete is set, cables are stressed and anchored. Some cables are usually bent up, to reduce shear stresses and also for a convenient distribution of end anchorages. The advantage of this form is that the structure is monolithic and the stress distribution can be calculated. Disadvantages, however, are that the prestressing of a slab is generally uneconomical, the form of construction is often heavy, and the span lengths are limited.

The use of precast elements is of advantage when there is difficulty in supporting the formwork. A quick erection can be possible and very economical if large number of units is required.

Precast and prestressed units (forming a part of the total deck) can be temporarily used as formwork for *in situ* concrete. Additional cables are then often used and subsequently stressed on the composite section. Transverse prestressing is then often used to achieve transverse rigidity and unison of the whole section.

The main problems in the design of slab bridges are—the choice of maximum economical span, minimum depth, the choice whether to precast or cast *in situ*, the type of cross-section to be used, transverse load distribution, deflection and vibrations, and finally the choice of the method of erection.

18.4 BEAM-AND-SLAB AND BOX-TYPE SUPERSTRUCTURES

Cross-Section of Beam-and-Slab Decks

The type of cross-section of a bridge often governs the weight, maximum span, and cost of the bridge. No definite rules can be laid regarding the type of cross-section to be selected. Only a comparison of a few designs may supply the answer in a given case.

In recent years the beam-spacing has increased, and the number of beams in a cross-section has reduced. This is due to the availability of higher quality materials, better experience, and more exact methods of analysis. It is also due to more effective forms of beams developed.

However, this reduction of the number of beams and dead weight in relation to the live load has its limitations due to the following reasons:

- the deflections and vibrations of a light deck structure under live load may become unacceptable,
- demand on increase in permissible stresses may become too high, particularly compression and shear in concrete,
- the total amount of prestressing tendons for a given load, span and width of a bridge, seem to depend little on the number of beams used, and room has to be found to accommodate them,
- the minimum practical thickness of a web based on the practicability of placing concrete through it in the presence of congested reinforcement and tendons in it,
- transport and capacity of cranes sometimes can set limits to the size of the beams,
- the cost of shuttering and transverse stressing can have influence on type of cross-section,

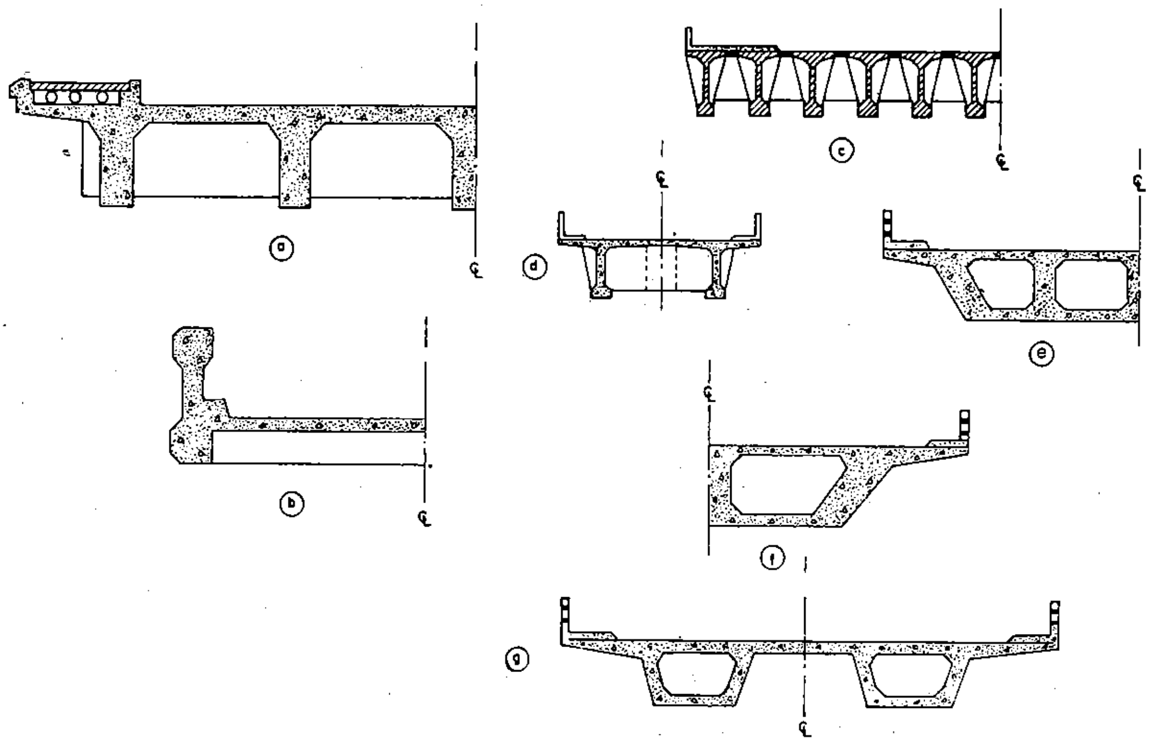


Fig. 18.4

(a) Multi-beam cross-section of the type shown in Fig. 18.4(a) is typical for many RC decks of moderate spans. Wide webs of these beams may not be suitable for pre-stressing as they will dilute the pre-stressing force.

(b) Upstanding beams shown in Fig. 18.4(b) are used when headroom is restricted and spans are small, for larger spans buckling of compression flange sets the limit on the span.

(c) Multi-beam pre-stressed concrete cross-sections of the types shown in Figs. 18.4(c) and 18.4(d) are economical for spans up to approximately 40 to 50 m in the very limit.

(d) Box type cross-section [Figs. 18.4(e), 18.4(f) and 18.4(g)] is uneconomical for simply-supported spans unless the span has necessarily to be large (45 m or more) and/or the construction depth is very limited. Its main advantage is that it facilitates placement of cables with maximum eccentricity, offers resolute section property for sagging as well as hogging moments, and is rigid for efficient transverse load distribution and torsion. For continuous decks however, box section is regarded as very good both from the elastic and safety points of view. If cables are not bonded (i.e., external cables), the ease of placing the cables, their bending up and stressing, are all in favour of box section.

Vertical prestressing of webs is sometimes resorted to on long spans in order to reduce shear stresses in webs.

18.5 ECONOMIC SPACING BETWEEN BEAMS

No definite rules can be given for the most economic spacing of beams and their number in the cross-section.

As a general guide however, and for preliminary design only, the following rules can be quoted (Fig. 18.5).

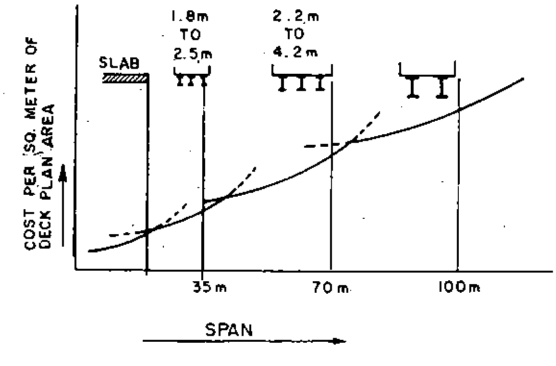


Fig. 18.5

- For small spans of about 9 to 15 m. particularly where headroom is restricted, a solid slab deck with precast U,T or I beams (infilled/topped) is probably the most economic solution.
- I-beams laid side by side are usually economical in the range of about 15 to 20 m. For larger spans up to about 30 m, the beams are usually spaced 1.8 to 2.5 m apart.
- For longer spans, three beams (even two beams) in the cross-section provide the cheapest solution, provided there is sufficient construction depth available.

18.6 BALANCED CANTILEVER TYPE SUPERSTRUCTURES

Balanced cantilever type decks offer some advantages when compared with simply supported decks, and compare well with fully continuous decks.

Hinges are usually positioned in such a way that the cantilevered span alternates with a simply-supported span as illustrated in Figs. 18.6(a) and 18.6(b). Occasionally (and very rarely only) one hinge per span is used [Fig. 18.6(c)], or a simply-supported span is supported by cantilevers from counterweighted abutments (either in mass concrete or in RC box filled with sand or gravel [Fig. 18.6(d)].

Hinges are positioned in the vicinity of low and zero bending moment (point of contraflexure) under dead load (usually at a distance of 0.18 to 0.20 units of the span).

By convenient location of hinges, the distribution of dead load bending moments can be made almost identical to that in continuous decks of the same shape and similar loading. However, the variation of live load bending moments in cantilever decks is not as favourable as in continuous decks in which even spans away from the live load contribute in carrying this load.

In the case of longer spans, however, it is not the variation, but maximum value of moments which matters. For long balanced cantilever spans the general arrangement is almost the same as for fully continuous decks, and hence so also the quantities of materials. (This is so because in long spans, dead load effect is more than live load effect.)

Other advantages of such cantilevered superstructures are: these type of bridges are unaffected by differential settlement of supports, by hogging or sagging due to difference in temperature between extreme fibres and by shrinkage or creep of deck concrete. They develop no parasitic prestress reactions, they conveniently allow segmental construction in certain zones which can require less formwork and scaffolding, and thus can be almost equally economical as continuous bridges, even though they may not have as good a riding quality because of more expansion joints.

The disadvantages could be summarized as follows: variation of bending moments is less favourable than in continuous spans; require more bearings, anchorages and expansion joints; shear stresses can be very high at hinges; hinges are very congested with steel and anchorages, and the joints are generally not pleasing to eye.

Cantilever decks are sometimes built determinate for dead load, and then are subsequently made fully continuous by means of stressing by cap cables or splice cables.

18.7 CONTINUOUS TYPE SUPERSTRUCTURES

Continuous girder decks have the following advantages over simply supported decks:

- longer spans are possible because of lesser span-moments,
- require much smaller ratio of the deck depth at the

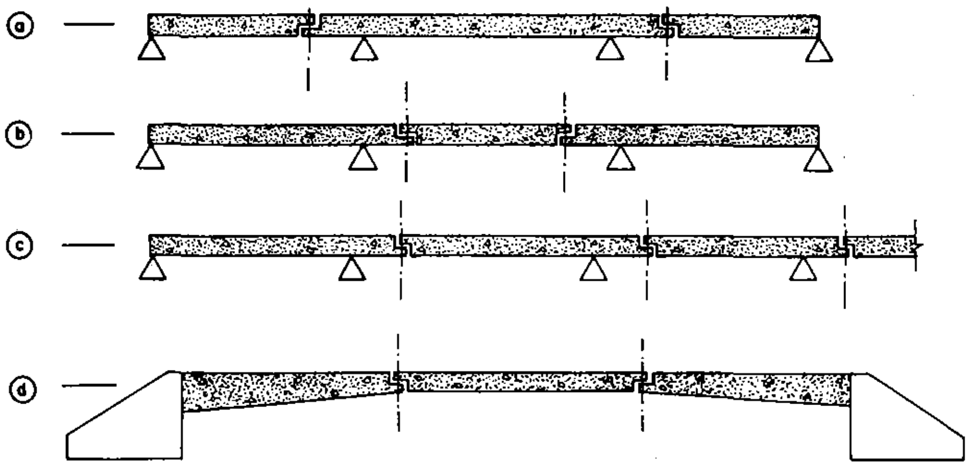


Fig. 18.6 *Position of hinges in balanced cantilever bridges*

centre of span to the span length,

- require fewer number of piers, bearings and expansion joints,
- have smaller deflections and vibrations,
- have a better riding quality, particularly because of lesser joints.

Disadvantages of continuous bridges are:

- sensitivity to the secondary stresses (settlement of supports, differential temperature, shrinkage and creep effects),
- prestressed concrete beams must either be designed with concordant cables or else the effect of lack of concordance must be carefully taken into account,
- continuous bridges require much greater skill in construction. (To facilitate construction, these type of decks are sometimes built as statically determinate and full continuity is restored subsequently.)

Continuity affords advantages for spans over about 35 m, particularly if a long and flexible span follows a short and stiff one. This ratio of short span to long span should ideally be about 0.3.

The number of spans between two consecutive expansion joints may be up to about 4 for a total length of about 150 m if all the cables have to be stressed (from both ends) in a single operation. Otherwise if couplers are used and maximum length of cable stressed at any one time is limited to about 35 to 45 m, the number of spans between two consecutive expansion joints will be limited only by the capacity and cost of the expansion joint.

18.8 SEGMENTAL DECK CONSTRUCTION

Bridge deck construction by free cantilever method, i.e., a butterfly deck built up segment by segment from both sides of the pier is discussed here (Fig. 18.7). Refer Chs. 36 and 37 of this book for more details.

General

Where conditions at the bridge site prohibit the erection of scaffolding and centering on riverbed and long spans are to be constructed to compensate for the high cost of tall piers and deep foundations, cantilever construction is elegantly convenient and competitive.

Cantilever construction is a method of progressive construction of a cantilever in segments and stitching them to the segments already completed, by prestressing. The cantilevering segments are constructed/erected from the pier outwards, one on either side, and stitched back simultaneously. The segments, normally 2.5 to 3.0 m long, can be either cast *in situ* on travelling gantries, or can be precast in yard and erected by launching truss or floating cranes. *In situ* construction is economical only in the case of a bridge having fewer spans. Usually it takes about 4 months to complete a 120 m 'butterfly' by cast *in situ* method. Hence for bridges where many long spans are involved, precasting can speed up the progress of work.

Though precasting involves additional investment on plant, machinery and organisation, for a longer bridge this investment ultimately proves economical over the cost of time saved. Usually in bridges where the foundations are deep, the superstructure is made monolithic with piers and at midspans either a short riding span or a hinge (connecting the two cantilevers) is provided. In bridges where the foundations rest on rock, the butterfly decks could instead be seated on bearings over piers and the cantilevers made continuous at midspan for superimposed deadloads and live loads. Many times two temporary supports are required, one on either side of a pier, to stabilise the deck during free cantilevering.

The *in situ* construction is done by a pair of travelling gantries, each weighing about 40 tonnes (for casting 2.5 to 3.0 m segments of a 2 lane deck). After constructing the pierhead unit (i.e., the portion of deck immediately above pier), a pair of gantry systems is erected on top, one on either side of the pier. The gantries project beyond the pierhead to support the hanging shuttering required for casting the next segment on either side. The external shuttering of the box section deck is supported directly from the gantry system. The internal shuttering is supported on a gantry-girder running inside the box along the length of bridge, which in turn is supported at its forward-end by the previously completed decking. Each travelling gantry system is counter weighted for supporting the shuttering system when it is moving from completed section to forward

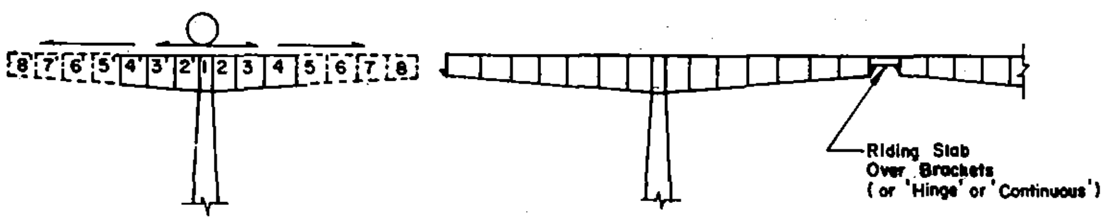


Fig. 18.7 *Segmental construction*

section. In addition to this, the reaction required to take up the weight and constructional loads of the unit to be cast by the cantilever shuttering is realised by means of suspenders passed through the decking and the bottom slab and anchored at the base of the previous unit. The gantry systems proceed in a systematic manner from section to section on either side of pier after the prestressing of the segments last cast. The gantry system also supports a suspended scaffolding for constructional convenience and labour safety.

Construction Cycle for in situ Working

Usually the construction cycle for *in situ* construction is a 10-day cycle, as summarised below:

(i) Shifting of travelling gantry system	1 day
(ii) Completion of entire shuttering and placing of reinforcement bars and prestressing cables for the segment	3 days
(iii) Casting of the segment	1 day
(iv) Curing	4 days
(v) Stressing and miscellaneous	1 day
Total	10 days

i.e., casting on Day-5, prestressing on Day-10, so that concrete in the segment is 5 days old when stressed by the cables anchored at its end. This periodicity is very important to know for the design calculations. However, the above construction cycle can be compressed into about 5 to 6 days as has been proved at various highly mechanized sites.

Choice Between Continuous, Hinge and Simple 'Riding Span'

Where differential settlement between foundations is not expected, the butterfly decks (constructed by free cantilever method) can either be made continuous (but calculations for continuous deck take longer), or a 'hinge' can be introduced between cantilever tips for shear transfer. Both these solutions result in a better riding quality. However, where significant differential settlement is possible, and/or contractor's construction experience is limited, a simply supported riding span may instead be provided between the adjacent cantilever tips. Despite the best construction care the adjacent cantilever tips generally do not 'mate', and, in order to safely 'hide' their level differences, a riding span is a simple alternative, as otherwise only additional stresses will be built in if the two cantilever tips are 'forced' to 'mate' while rendering continuity or building-in a hinge. However, where the bridge is long and precast construction is resorted to, at least a hinge solution may assist for rapid transportation of precast elements from one butterfly to the other (which otherwise is delayed if, instead, the casting of a riding span is involved).

Precast Segmental Construction for Long-span Bridges

The use of precast concrete segments offer the following advantages:

- control of high quality concrete
- manufacture of segments at a plant site instead of at site where the quality controls may not be as good
- lend themselves to new techniques and speed of erection
- great accuracy of section and profile can be obtained and problem of deflection during construction can be better overcome
- shrinkage can be practically eliminated and creep reduced by high quality of concrete.

The efficient construction of long-span prestressed concrete decks depends on:

(a) basic design scheme
(b) erection method
(c) suitable technique of jointing, stressing, etc.

Careful consideration must be given to ensure monolithic behaviour of the completed structure as well as flexibility of the structure during erection.

The weight of segments may vary from 5 to 100 t or more, and the length from 2.5 to 3.0 m, or more. Segments may be reinforced with mild steel along the bridge axis*, and are designed to be connected by post-tensioning after erection. Segments can also be prestressed in themselves, either by temporary cables to aid erection, or cables can be so arranged that the prestressing is efficient in the final condition as well. Transverse prestressing can also be applied to the segments at manufacture stage itself.

Short segments can be cast in end-on vertical position. Usually segments are match-cast. Steam curing is often used to attain high strength earlier and to reduce shrinkage. Strengths of 450 to 550 kg/cm^2 can be obtained.

Joints are of great importance in connecting the precast units, and to ensure monolithic behaviour. Very wide joints, 20 to 60 cm wide, have been used with reinforcement projecting to be lapped or welded. High strength concrete is then placed and well consolidated, and sometimes steam cured. This, however, is not done anymore.

Deep indentation shear keys are employed when high shear stresses must be transmitted. They are preferable even otherwise.

Normally, the ends of the segments are constructed as vertical planes with roughened surfaces. Shear keys, i.e., indentations, should preferably be formed in them. Tests show that reinforcing bars across the joints are of little

* With high yield deformed bars normal to traffic.

benefit to the ultimate strength, and shallow shear keys are also of no value.

Poured concrete joints: 7 to 10 cm wide joints have been found to be successful. Rich mixes using pea gravel for coarse aggregate, and high strength cement, are used.

Dry mortar packed joints: 3 cm wide, have been tried but it is extremely difficult to ensure their satisfactory performance.

Epoxy coatings have proved beneficial, and are extensively used as the jointing agents as an aid to bond the concrete joint.

Dry joints: where the segments but directly against each other, shear keys and epoxy painting can be resorted to. This requires special techniques and great care both in manufacture by match-casting of the consecutive segments and epoxy jointing with precisely time-controlled prestressing. This makes possible speedy and economical assembly on the site and assures a near perfect joint. The proven method, till now, is to cast each unit against the preceding segment. This generally requires a second handling of each segment in the casting yard and match-marking for erection.

All concreted joints must assure continuity of cable ducts across the joint (except when external cables are used). To form the duct in the joint, inflatable formers and plastic or metal sleeves have been used. While grouting the post-tensioned tendons, grout may escape at the joint and block the adjoining ducts. Grouting then is best postponed until all tendons are stressed but this can be dangerous since grouting should not be delayed.

Cables and Their Profiling

It is preferable to use bigger cables, e.g., a tendon of 12 strands (each strand 15 mm dia.) rather than the small 12/7 mm Freyssi wire type, as otherwise their number becomes inconveniently large which poses problems in profiling and handling. However, very big cables are not preferable as there should be at least one cable per web (preferably two) to stitch a segment.

Generally cables to be anchored at the end of a particular segment should be anchored at about three-quarters the depth of the segment below its top. Behind their anchorages these cables should rise 'straight inclined' through two or three segments to a plane which is convenient but a small depth s below the horizontal plane in which they travel from above the pier. This transition through the vertical drop of s should be accomplished preferably through one segment only. The value of s is adjustable—about 0.30 to 0.40 m for the above type of cables. This should permit about twice the minimum bending radius specified for the cables used (assuming this transition were a pure circle). The cable profile through the entire transition may in fact actually be specified as two 'inclined straights'.

Such a profile lends itself to easier construction as also to easier alterations, if required, while finalising the cable profiles in the design stage, and also permits easier calculation of angle-change. Beyond this transition, unless the cable has to remain in the vertical plane of its bending in web, it has to be suitably bent in plan while travelling in the top slab. This plan-bending (within the horizontal plane) may also be accomplished through a 'series of straights', each straight extended over at least one or more segments. The elevation and plan bending of cables in terms of straights makes not only the calculation and drawing easier but also the laying out at site.

Deck Section

A box section is the most ideal deck section for cantilever construction from the points of view of both construction and flow of stresses. Unlike a steel box, a well detailed concrete box is a 'thick walled' tube, has a tremendous torsional resistance and does not accept any significant distortion and warping. Depending, of course, on the section shape and the extent of cantilevering deck slab, almost a full cross-section of the concrete box of the more usual shape is effective for resisting the load effect. The more usual concrete box section may be assumed to permit complete maturation, provided it has full depth cross-girders at support points and at intermediate locations* such that the spacing between cross girders does not exceed about 45 to 55 m. In addition, the live load is placed away from the section at which its maximum bending effect is being considered, so that the load-effect trajectories get ample chance to flow into the full section. The torsional rigidity of the box greatly adds to this behaviour.

Soffit Surface

Soffit surface is generally a series of 'straights', segment to segment, while the soffit points at segment-ends may lie along a parabola if desired.

Instead of threading-in the longer cables right from the first segment onwards and keeping them dangling, it is enough to provide a cable-duct alone (sometimes filled with sand) for these cables all the way until their profile commences bending. It is only then that these cables need to be threaded-in (as subsequent cable-threading may be difficult through a bent-profile). However, the maximum dangling projection of a cable at any one time may be limited to about 10 to 15 m ahead of the preceding segment.

Air vents should be provided in the cable ducts over the pier and at points of change of curvature and at intervals

* Intermediate cross girders may be avoided if live and dead load effects are investigated in detail, e.g., by finite element method or by a model test.

of about 30 m. Neat cement grout (sometimes mixed with very fine sand and aluminium powder to give body and expansion to the grout) with about 0.5 w/c ratio should be injected simultaneously from both ends of a cable, against gravity, under about 7 kg/cm^2 pressure. The vents should be plugged when full grout issues through them. Care should be taken to see that grout from either side reaches the central vent by alternate pumping. (For more details for grouting operation, refer Ch. 7, 'Construction Considerations' of the author's book, *Concrete Bridge Practice—Construction, Maintenance and Rehabilitation*.)

Deflection and Pre-camber

Generally the order of deflection of a well-designed long span prestressed cantilever is about 1 in 600, which is very low.

To counter the long term creep 'dipping' effect of deflection under all dead loads and final prestress, it is necessary to calculate the deflection at various sections under this loading (based on the reduced modulus of elasticity of concrete due to creep) and give equal and opposite cambers at the soffit at these sections at the very time of construction. With this, it is hoped, that the structure will, within about 3 years (by when majority of losses due to creep would have occurred), finally take the intended profile, thus countering the dipping effect. These pre-cambers to be given during construction should be specified at the soffit level at each section. (Theoretically the variability of the atmospheric history during the entire period of construction will render it almost impossible to calculate these deflections correctly, but it is practically good enough to calculate these deflections taking E_c as half the instantaneous value.) Also refer to relevant details in Ch. 37.

The cambers may be laid out over the decking with theodolite located over the piers always at the same time immediately after the day-break, if only to somewhat offset the effect of temperature difference through the decking.

Expansion Joint

In order to admit relatively large longitudinal and vertical movements and rotation, usually a tongue and groove type steel joint or multi-grooved or layered elastomeric type expansion joint is adopted at expansion joint locations. Refer Ch. 32 of this book for relevant details.

Bearings

Bearings under the riding spans have to permit a large horizontal and vertical movement coupled with very significant rotation. For this purpose metallic rocker and full roller type bearings must be used. These bearings should be tied to the structure in order to prevent their dislocation since the relatively short riding spans may try to jump as the live load traverses from cantilever tip to it and vice versa. For this purpose, seismic attachment type tying arrangement may be adopted. Refer Ch. 17 of this book for further details.

Aesthetics

"The simplicity of form and proportion, with the load masses disposed precisely in locations where they are most effective, the long sweeps of the structure and the elegant arch of the soffit, all make the cantilever bridge deck a beautiful form of function, all well blended to give a pleasing architectural appearance."

Summary of Steps for Design of Butterfly Deck with Riding Spans

(i) Decide first-trial butterfly deck cross-section and sizes.

(ii) Design the cantilevering footpaths and do a quick design of top slab considering the box section action, taking the box section to be of an average depth (assuming approximate thicknesses for the webs and bottom slab).

(iii) Demarcate various segments and section numbers (starting from the cantilever-tip towards the root).

(iv) Decide arrangements of riding span and supporting brackets at the cantilever tip.

(v) Work out section properties at various sections.

(vi) Calculate weights of various segments and BMs and SFs at various sections due to self dead loads of the segments, the weight of finishings and due to dead load reaction at cantilever tip from riding span; calculate corresponding flexural stresses at various sections.

(vii) Calculate the governing live load BMs and SFs at various sections and the corresponding flexural stresses at these sections.

(viii) Decide the prestressing system; particulars of tendons and anchorages; minimum distance between cables, minimum distance from edge of concrete to centre of cable and to centre of anchorage; maximum jacking force allowable; grade of concrete and permissible flexural compressive and tensile stresses during construction and during service; estimation of losses due to shrinkage, creep and elastic shortening of concrete and relaxation of steel, coefficients of curvature and wobble friction; number and placement of cables above centre line of pier; profiling of cables, etc. (For convenience serial number of a cable may be taken as serial number of the section at which it is anchored plus one, e.g., cable anchored at Section 11 may be called cable number 12, and so on). For each cable calculate its eccentricity at various sections, cumulative angle turned by it in elevation and plan from jacking point up to each section. For each cable decide first trial anchorage force, calculate for friction, plot P_x values, estimate slips, extensions and jacking forces, read off P_i values at various sections, estimate values of '$P_i \cos\theta$', '$P_i \cos\theta e$' and '$P_i \sin\theta$'; calculate flexural stresses at various sections due to each cable; compose final flexural stress summaries at various sections and check against permissible flexural stresses, revise section and/or cables as necessary.

(ix) Check against ultimate moment of resistance at each section.

(x) Design against shear (revise section and/or cables if necessary, in which case recheck flexural stresses).

(xi) Transverse analysis of box at a few representative sections and design of reinforcement around the box.

(xii) Decide reinforcement detailing in box section vis-a-vis requirements from shear and transverse analysis and minimum reinforcements codally required.

(xiii) Anchorage zone reinforcement and detailing at various anchorages.

(xiv) Design of cross girders.

(xv) Design of brackets supporting riding span.

(xvi) Design of expansion joint.

(xvii) Design of riding span and its bearings.

(xviii) Design of parapet wall.

(xix) Calculation of pre-cambers to be given at various sections in the butterfly during construction; estimation of maximum tip deflection under service live load.

(xx) Ensure vibration characteristics under dynamic loading (details dealt in Ch. 39 in this book).

18.9 FRAME BRIDGES

In recent years there has been some revival of interest in this form of construction when applied to large span bridges, mainly because of the introduction of prestressing.

A portal frame is essentially an arch but of a shape very much different from the line of pressure, which is an ideal characteristic of an arch. Prestressing offers some advantages for these types of structures (see Fig. 18.8).

The main advantages of frame bridges in comparison with continuous beams can be summarised as follows:

(a) By prestressing the frames, the line of pressure can be kept within the middle third of the section.

(b) Frame bridges do not require expensive bearings at the supports.

(c) The stability of the supports is much greater than in the case of independent piers.

(d) Very large spans are possible (up to 100 m or more).

(e) The depth to span ratio can be as low as 1/50 at the crown.

(f) Frame bridges lend themselves to the use of jacks for correction of thrust and flat jacks for prestressing.

(g) Quantities of steel and concrete for long spans are relatively low in comparison with continuous beams.

(h) Can be even more economical than arches.

(i) Have pleasing appearance from aesthetic angle.

(j) Prestressed frames have greatly reduced horizontal thrust.

The disadvantages of frame bridges are:

(a) their sensitivity to secondary stresses, particularly horizontal and angular settlement of supports

(b) require greater skill in construction and high quality of materials

(c) stress distribution at corners is complex and may lead to serious redistribution of moments

It has been explained in a later chapter that the rate at which quantities increase with the span in the long spans is less for portal frames than for continuous bridges. In fact, concrete and steel quantities for portal frames theoretically need not necessarily increase with the span and this may be achieved by increase in horizontal thrust, which however is usually expensive to provide for.

Arch bridges are usually economical because their equilibrium can be maintained mainly by direct forces rather than by bending. But when comparing minimum construction depth, maximum headroom throughout the span, or general economy in the combined cost of substructure and superstructure, the frames may show advantages over arches.

For maximum efficiency the depth over the supports should be as much as even 3 times the depth at mid-span, although this is not always possible because of the

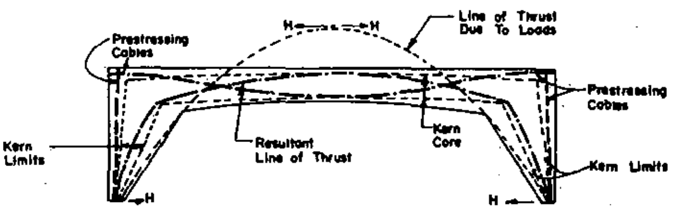


Fig. 18.8 *Prestressed portal*

appearance and constraints of headroom. In addition, at knees the total cable force would sometimes need to be increased to a value in excess of that required at mid-span. Alternatively, the increased prestressing force must be provided at these knee junction regions by cap cables.

The introduction of prestressing to frame bridges removed their main disadvantage of the line of pressure being otherwise far away from the centre line and reduced their sensitivity to settlement of supports. Also the greatly reduced horizontal thrust makes this form of construction economical.

The essential function of prestressing in frames is to superimpose a line of thrust upon the external load thrust line in such a way that the resultant thrust line is brought back into the section within certain limits.

At the corners, the leg cables and beam cables must cross, since the resultant thrust line must abruptly change its direction to pass from the narrow gap between the limit lines in the beam to that in the leg. This change of direction requires the action of a concentrated force, exerted by the resultant of anchorage forces at the corner. See Fig. 18.9.

For large spans, this crossing of two systems of cables causes problems of location of cables and anchorages. The outward thrust at this change of direction is balanced by the resultant of anchorage forces, and it is necessary to reinforce the corner by a diagonal rib to accommodate this balancing force. See Fig. 18.9.

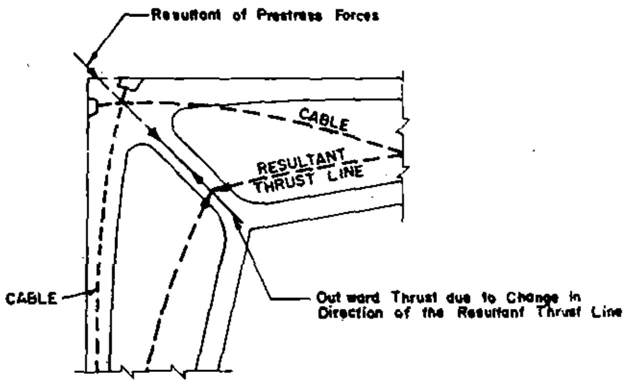


Fig. 18.9 *Forces in the portal knee*

However, the main disadvantages of frames lie in their sensitivity to differential settlement, particularly to a horizontal yielding of foundations under the action of horizontal thrust. Prestress greatly reduces the magnitude of horizontal thrust and therefore the horizontal settlement. To overcome the horizontal settlement, counterweights sometimes are used to minimize the horizontal thrust. However, heavy abutments should not be monolithically cast with the transome (Fig. 18.10). The ideal arrangement is to

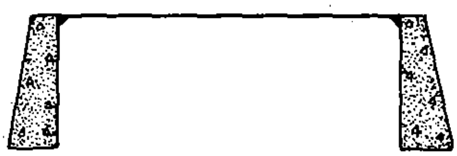


Fig. 18.10 *Wrong arrangement*

provide the tie if this is possible. Then only the vertical reactions are transferred to the ground, Fig. 18.11.

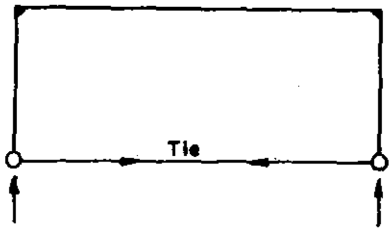


Fig. 18.11

If ties are not acceptable or possible, then the abutments should be disguised as separate units from the frame columns, thus counter-balancing horizontal thrust and making the systems statically clear and dependable, Fig. 18.12.

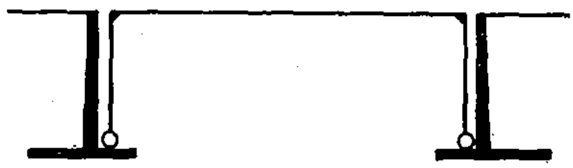


Fig. 18.12

As an alternative, counterweights have been used to reduce horizontal thrust, Fig. 18.13. The disadvantage however of this solution lies in great vibrations of weight as well as the main span. There are also difficulties in providing good joints between the ends of the counterweights and the roadway.

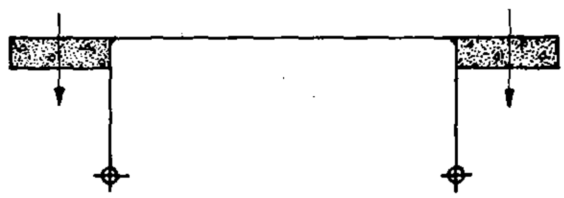


Fig. 18.13

Similar effect of reducing the horizontal thrust can be obtained by cantilevers at the level of the hinges. Dead weights of the fill above these cantilevers reduces the thrust so that essentially vertical component may be transferred to the foundations at normal working conditions, Figs. 18.14 and 18.15.

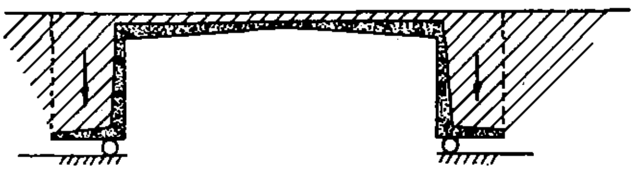


Fig. 18.14

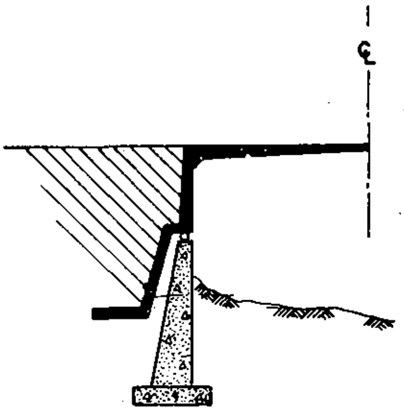


Fig. 18.15

Alternatively, hinges can be set inside, Fig. 18.16, with a similar effect of reducing the horizontal thrust.

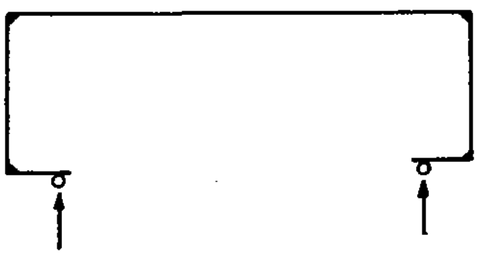


Fig. 18.16

Figures 18.17(a) to (e) show typical arrangements of abutments and columns in frame bridges.

Frames with Inclined Legs

Frames with inclined columns (Figs. 18.18 and 18.19) are used to improve visibility under the bridge, to reduce the main span, and to improve the appearance. These are usually in the form of a single-span bridge or three-span bridge. With certain arrangement of piers, horizontal thrust at ground level can also be greatly reduced.

One of the major factors in the cost of a portal frame is the wide gap between the funicular line of loads and the centroid lines of the frame. This gap is greatest at the corners of the frame and consequently requires great depths above the posts and very high prestressing forces. Reduction of that gap is most easily effected by sloping the legs of the portal outwards, providing that the specified headroom can still be maintained. Using a suitable slope of leg and adopting a convenient ratio of depth at mid-span to depth at the corner, well balanced designs may be obtained and materials (concrete and steel) can be utilised with the same efficiency in the two critical sections of the beam. There is still, in each case, an optimum value of the ratio of beam to leg stiffness, which is necessary to obtain the desired rise of the pressure line under external loads.

The length of the central span is usually about 0.7 of the distance between hinges. The ratio of depth at corner to the depth at mid-span is usually 2 to $2^1/_2$, Fig. 18.20.

Cantilevers are sometimes tied to the bottom of the leg or to the foundation. The problem of crossing of cables is much easier with inclined legs than with vertical legs, since the anchorages of the beam are at the ends of cantilevers, and the thrust line in the cantilevers intersects the thrust line of the leg some distance below the leg-cable anchorages (Fig. 18.21).

Frames with Triangulated Supports

Sharp corners inside the knees create a zone of extremely heavy and complicated compression stresses, which, in that locality in a redundant frame, can result in excessive strain and redistribution of bending moments, significantly different from the one calculated. The introduction of a sufficiently heavy triangulation strut can considerably alleviate this danger.

Another method frequently used is to split the column into two, forming a triangulated support. One column is then working as a strut and the other as a tie. High negative bending moments can be resisted by such system of supports.

Inclined columns essentially resist the horizontal and vertical forces by direct compression and tension only, and the bending moments in columns are either eliminated or are small, even in the case of complete continuity at the top.

Tension column is often pre-stressed.

Various arrangements of this type of bridges are shown diagrammatically in Fig. 18.22.

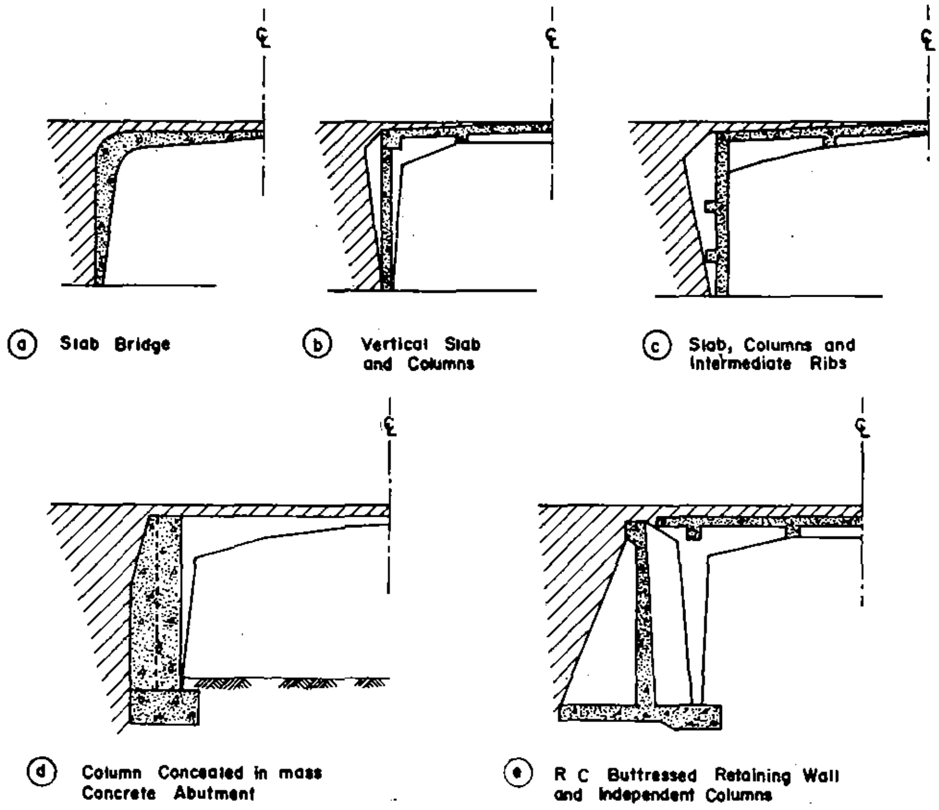


Fig. 18.17

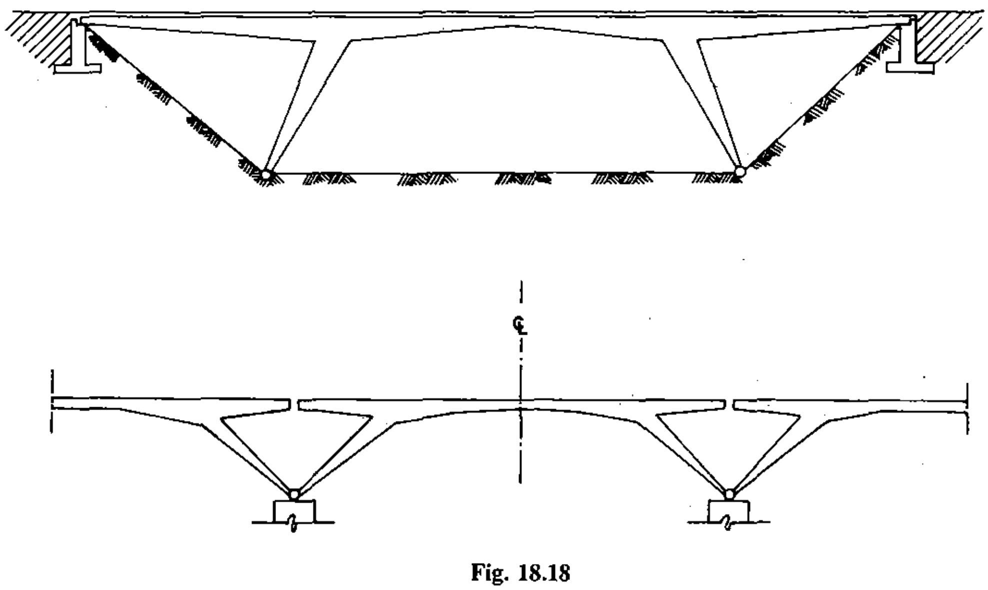


Fig. 18.18

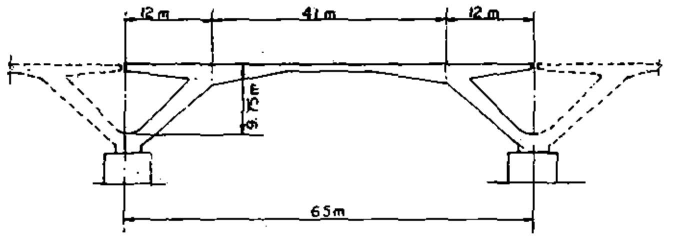


Fig. 18.19 *St. Michel Bridge at Toulouse (five spans of 65 m each)*

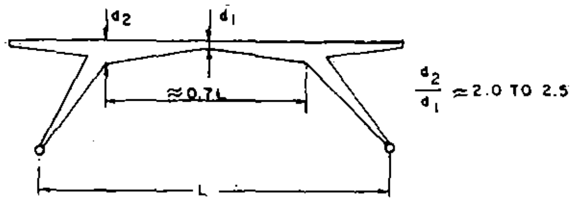


Fig. 18.20

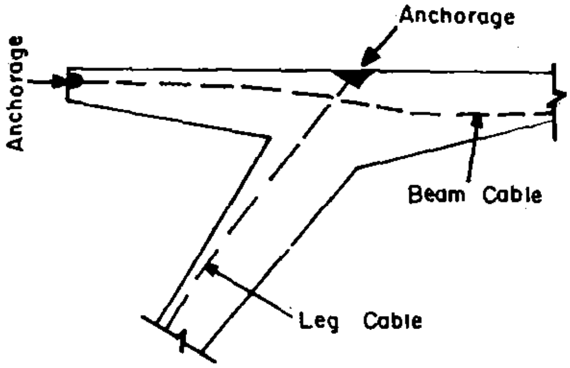


Fig. 18.21

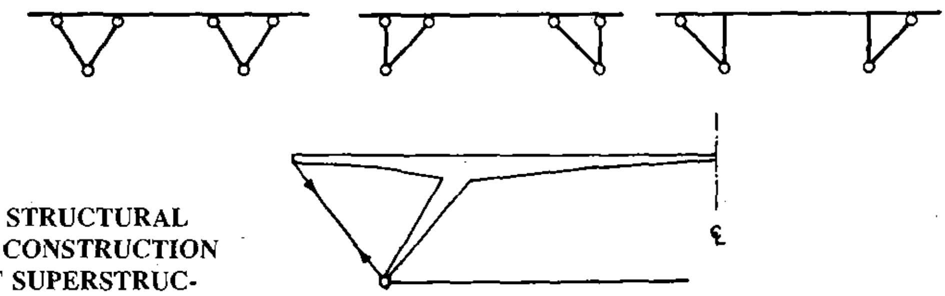


Fig. 18.22

18.10 BRIEF CHECK-LIST FOR STRUCTURAL ANALYSIS, DESIGN AND CONSTRUCTION FOR VARIOUS TYPES OF SUPERSTRUCTURES

Slab Bridges

In this simplest type of bridge superstructure, the deck slab also serves as the principal load carrying element. The concrete slab, which may be solid, voided, or ribbed, is supported directly on the substructures.

Analysis Method

Reinforced concrete bridge decks should be designed in accordance with recognised elastic analysis, assisted by certain empirical analysis. The analysis should take into consideration the configuration of the slab, the supporting conditions, and the effects of cracking on the slab stiffness.

Types of Construction

(i) *Cast in Place Slab Decks* Concrete slab decks may be cast monolithically together with the supporting elements. Alternatively, they may be cast separately but effectively bonded to achieve composite action under future loads.

(ii) *Precast Deck System* Precast concrete deck systems may be constructed in several different ways. These include:

(a) Precast panels extending the full width of the bridge attached to supporting members, without composite action.

(b) Precast panels extending the full width of the bridge and effectively bonded to the supporting members to develop composite action.

T-Beam and Box-Girder Decks

These types of superstructures consist of a deck slab supported by longitudinal beams or girders. The longitudinal beams or girders may in turn be supported by abutments, piers, bents or floor beams. Transverse intermediate diaphragms are used as may be required.

The longitudinal beams or girders may have any cross-section, with I, T or box sections being the most common, and they may have a prismatic or variable section along their span. Simple span or continuous structures may be used.

Either reinforced or prestressed concrete, or combinations thereof may be utilised. Also, the structures may be entirely cast in place, or constructed using precast beams or girders with a cast in place composite deck slab.

Analysis

The analysis of bridge superstructures should be based on elastic, empirical, or model analysis methods. All sections should be proportioned to resist the forces determined from the results of the analysis. The analysis should recognise stresses occurring from temperature, shrinkage, creep, transport and handling of units, prestress and other conditions which may affect the design. Torsional stresses should be considered in the design.

(i) *Elastic and Model Methods* The forces present in the component parts of the superstructure should be established in accordance with a recognised elastic analysis (analytical) or representative model analysis, except when an empirical method of analysis may be used.

(ii) *Empirical Methods* The empirical methods referred to here are those conforming with the recommendations presented in the "Standard Specifications for Highway Bridges," published by the American Association of State Highway and Transportation Officials.

Design Considerations

• *Simply Supported Spans with No Skew or Moderate Skew (less than* 20°*)* The loads and forces acting on simple spans include:

(a) Self weight (dead load),
(b) Superimposed dead load,
(c) Prestressing-Pretensioning and/or Post-tensioning,
(d) Live loads,
(e) Shrinkage, creep, and (for composite construction) differential shrinkage and differential creep.

In general, different combinations of the above loads act at various stages of construction and throughout the service life. If shoring for precast girders is used during construction, the effect of removal of shoring should be considered as another specific load combination. The cross-sections which resist these loads also vary. These loads and sections must be carefully identified to permit a meaningful analysis. Special attention is drawn to the forces caused by deflection of formwork, temperature effects, and incompatibility of materials. Deflection calculations should be carried out so that suitable camber can be provided in the reverse direction at the time of construction.

The analysis should take into consideration, where appropriate, the axial, flexural, and torsional stiffnesses for longitudinal girders and transverse medium at various stages of construction. The transverse medium consists of the deck slab and, if present, floor beams and diaphragms. The floor beams may be separate units or they may be monolithic parts of the slab.

• *Simply Supported Spans with Severe Skew* This condition requires, particularly in the case of multi-girder bridges, that recognition be given to the flexibility of supports since the magnitudes of reactions and bending moments may be affected.

The available analytical methods of analysis may be broadly classified into the following main groups:

(a) Analysis by anisotropic or orthotropic plate theory.
(b) Analysis as intersecting longitudinal and transverse members of particular flexural and torsional stiffnesses.
(c) Analysis by considering the longitudinal or primary members as interconnected by some form of cross-section which represents the behaviour of the transverse members.
(d) Analysis by the finite element technique, plane grid, etc.

The most suitable method of analysis should be selected for the solution of each particular problem. The accuracy of the results obtained depends on how closely the actual structure satisfies the assumptions inherent in the employed method of analysis. If analytical methods cannot be used or if available facilities offer a more convenient approach, model testing is recommended. Model testing may be used to advantage when skewed or irregularly shaped superstructures are encountered. A high degree of automation is possible. The modelling material may be plastic, micro-concrete or other materials which adequately approximate the behaviour of the prototype. The effect of scale must be considered. In addition, it should be ascertained that requirements pertaining to fatigue and vibration are satisfied.

• *Semi-continuous Spans* Semi-continuous spans identify structural configurations that consist of precast single-span beams laid consecutively over a number of spans, but where objectionable expansion joints are obviated by the introduction of continuity. This permits the development of support moments due to subsequent dead load and live load, shrinkage and creep. Partial continuity can be achieved as follows:

(a) Casting of transverse beams which fill the spaces between consecutive beam ends and are integral with the deck slab. Reinforcing should be provided within the longitudinal beams, for development of the positive moments generated by shrinkage and creep. Reinforcing, which extends across the joints and is capable of resisting the tension induced by the developed negative moments, should be placed in the slab.

(b) The same as (a) except that the concrete slab is prestressed over supports.

(c) Continuity is confined to the top slab only. Transverse beams are provided at the ends of girders, but the space between consecutive girder ends is not filled. Adjacent girder-ends are provided with separate bearings. Careful detailing is required to prevent slab crushing due to end rotation of girders.

In addition to the forces and effects listed for simply supported spans, effects introduced due to continuity should be considered (e.g., effect of settlement of supports, effect of temperature difference between extreme fibres, etc.).

• *Continuous Spans* Continuous spans identify structural configurations which are supported at intermediate points between the ends of main girders. Continuity may be achieved as follows:

(a) Unprestressed structure: The entire structure is cast in place on stationary form work. Full continuity is achieved for self weight, superimposed dead load, and live load.

(b) Prestressed structure:

- The superstructure is cast in place as in above, and full length post-tensioned tendons are used to prestress the deck. Jacking is done from both ends of the superstructure.
- The superstructure is cast in place, and is prestressed by the use of full length tendons and tendons with intermediate anchorages. This arrangement may be advantageous when the soffits of girders are curved. Jacking may take place from both ends and intermediate locations of the superstructure.
- The superstructure is cast in place, and is post-tensioned by the use of lapped tendons. Such an approach may be advantageous when a large number of spans are prestressed by the use of individual tendons of limited lengths. Lapping is used at supports, in the vicinity of which staggered intermediate anchorages are provided. Supplementary short tendons may also be provided at supports if needed.
- The superstructure is cast in place, and is post-tensioned by the use of coupled tendons. This scheme may be considered as supplementary to the one outlined above.
- The superstructure is built up from consecutively constructed segments, prestressed together. Segments of the superstructure are post-tensioned together by means of tendons threaded through preformed ducts. It is of paramount importance to consider all deformation and resulting stresses which are introduced during construction (particularly due to shrinkage and creep). Special effects due to construction and continuity should be considered in the analysis of continuous spans (settlement of supports, parasitic prestress effects, effect of unequal top and bottom fibre temperatures, etc.)

• *Cantilever Spans* If it is established that economy is achieved by designs which result in high negative moments over piers and relatively small positive moments, cantilever spans of prestressed concrete construction may be used to advantage. They may be cast *in situ* or assembled from precast segments. Cantilevers are fixed to stiff piers. Cantilever bridge structures may be further separated into the following types:

(a) Cantilevers extending from two adjacent piers and interconnected with a shear transfer device within the span length (a hinge).

(b) Cantilevers extending from two adjacent piers and interconnected at midspan in such a manner (by arrangement of tendons) that full continuity is established.

(c) Cantilevers in combination with simply supported suspended (riding) spans.

(d) Cantilevers in combination with suspended spans, which are made continuous with the cantilevers for live loads only.

NOTE (a) and (b) may finally rest on piers through discrete bearings, however.

Past experience indicates that types (a) and (b) are less economical than types (c) and (d), that type (b) is less economical than type (d), and that type (d) is less economical than type (c). Type (c) has more joints and consequently not as good a riding quality.

A considerable degree of engineering skill is required to make cantilever construction fully successful. All deformations and resulting stresses which are introduced during construction must be carefully considered. Deflections of cantilevered spans are influenced greatly by temperature, shrinkage and creep. In general, prestressing of webs is advantageous for increasing their shear capacity, but this is not easy. The analytical procedure should take into consideration axial, flexural and torsional stiffnesses, as described earlier. Also see 'Design of Substructure' given below.

• *Precast Segmental Box Girders* Except as otherwise noted here, the provisions of prestressed concrete apply as much to the analysis and design of precast segmental box girder bridges. Deck slabs without transverse post-tensioning are designed under the applicable provisions as for reinforced concrete deck slabs.

Elastic analysis and beam theory may be used in the design of precast segmental box girder structures. For box girders of unusual proportions, other methods of analysis which consider shear lag (full width of top slab may not be effective) shall be used to determine the portion of the cross-section to be used in resisting longitudinal bending, etc.

1. Design of Superstructure

(i) *Flexure* The transverse design of precast segments for flexure considers the segment as a rigid box frame. Top slabs are analyzed as variable depth sections considering the fillets between the top slab and webs. Wheel loads are positioned to provide maximum moments, and elastic analysis is used to determine the effective longitudinal distribution of wheel loads for each load location. Transverse post-tensioning of top slabs may be adopted. In the analysis of precast segmental box girder bridges no tension is generally permitted in the joints between segments during any stage of erection or service loading.

(ii) *Shear** Shear keys are provided in segment webs to transfer erection shears. Possible reverse shearing stresses in the shear keys shall be investigated, particularly in segments near a pier. At the time of erection, the shear stress (in p.s.i.) carried by the concrete section engaged by the shear keys should preferably not exceed $2\sqrt{f_c'}(f_c' = 28$ day cylinder crushing strength in p.s.i.), unless a more detailed analysis is made.

Design of web reinforcement for precast segmental box girder bridges is to be in accordance with the provisions of design for shear.

(iii) *Torsion** Consideration has to be given to the increase in web shear resulting from eccentric loading or geometry of structure.

(iv) *Deflections* Deflection calculations must consider dead load, prestressing, erection loads, concrete creep and shrinkage, and steel relaxation. Deflections are calculated prior to erection of segments (based on the anticipated segment production and erection schedule) as a guide against which construction deflection measurements are checked. The precamber calculation is necessary in order to give reverse deflection of estimated magnitude at each segment at the time of construction so that eventually (after creep over 2 to 3 years) the deck takes the correct desired profile as closely as possible. Also refer Ch. 37 of this book.

(v) *Details* Epoxy bonding agents for the match cast joints shall be thermosetting, 100% reactive, non-solvent compositions. They shall be formulated to permit erection of match cast segments at site air temperatures from 40°F (5°C) to 122°F (50°C). Epoxy bonding agents shall be relatively insensitive to damp conditions during application and, after curing, shall exhibit high bonding strength to cured concrete, good water resistivity, low creep characteristics, and tensile strength greater than that of concrete. In addition, the epoxy bonding agents shall function as a lubricant during the joining of the match cast segments, as a filler to perfectly match the surfaces of the segments being joined, and to provide a durable water tight bond at the joint.

2. Design of Substructure In addition to the usual substructure design considerations, unbalanced cantilever moments due to segment weights and erection loads have to be accommodated in pier and foundation design with auxiliary struts or using erection equipment that can eliminate these unbalanced moments. However, a suitable amount of unbalanced moment must be allowed for.

Rigid Frame Spans

Rigid frame is a unit consisting of a longitudinal continuous member rigidly connected with the vertical or inclined members upon which it rests. By *rigid connection* is meant a connection which is designed to resist bending moments, shears, and axial forces without relative displacement among the ends of various members meeting at the connection. Owing to rigid connections the stability of the supports in rigid frames is much greater than that of independent piers.

Each leg member of a frame should be connected with the foundation so as to be able to prevent horizontal movements at the base. Rigid frames should be considered free to sway longitudinally due to the application of vertical dead loads and vertically applied live loads, unless specifically prevented from movement by external restraints.

The effect of continuity of main girders should be included in the calculations of the reactions at the supports. For the analysis of the superstructure column bases should be assumed to be pinned unless they are known to be fully fixed.

The influence of braking forces, changes in temperature, shrinkage in rigid frame members, and yielding of foundations, can be of critical importance. It is also necessary to give proper attention to important changes in the moments of inertia of the frame elements. To determine the elastic properties of the frame, the moment of inertia of the entire superstructure section, excluding rail, curbs, etc., and that of the full cross-section of the bent or pier, should be used.

Provision should be made to prevent the frame from being subjected to a longitudinal horizontal force at the end of the deck due to an expanding road slab or other similar cause.

* For design against Shear and Torsion, refer to Ch. 24 of this book.

The members of a rigid frame have to be provided with shear reinforcement as required.

Special attention has to be given to stresses in the rigid frame corners.

Types of Rigid Frames

(i) *Barreled or Solid Slab Frame* The solid slab may be of uniform thickness throughout. Alternatively, the slab thickness may be increased at the supports by the introduction of haunches, or the bottom of the slab may be either segmental or parabolic.

For spans up to 7.5 m it is generally advisable to avoid ribs.

(ii) *Ribbed Rigid Frame* Ribbed frame reduces the dead load bending moments throughout the structure.

(iii) *Hinged Rigid Frame* If the foundation material is of yielding character or the footing is narrow, the frame should be designed on the assumption of hinged conditions of the footing.

(iv) *Fixed Rigid Frame* Full fixity at the footings may be assumed in the design of rigid frames if the foundations are sufficiently rigid and the dead load reaction produces a soil pressure as uniform as possible at the base of the foundations.

(v) *Skew Rigid Frame* In general, a skew frame must be analyzed in two dimensions. The finite element method is recommended for this analysis.

For preliminary design purposes, such a frame may be treated as a right angled frame with the calculated moments and trusts multiplied by a factor equal to the square of the secant of the skew angle.

Skews of more than 50 degrees are impractical for rigid frame type bridges.

The reinforcement at the obtuse corners should receive special consideration.

Arch Spans

Where site conditions are favourable to adoption and construction of an arch span, this type of bridge will usually result in an economically aesthetic solution. Arch spans should not be contemplated at sites where foundation conditions are not ideally suited to this type of bridge. Fixed arch spans should not be considered unless firm foundations prevail at the site for proper resistance to the arch reactions. Pile foundations can be used successfully but the economy of the selection is thus partially diminished. At sites where falsework may be difficult to construct, segmental construction using tie-backs can be considered.

Analysis

• *Short Spans (Elastic Method)* On short spans, the arch geometry should be such that the centroidal axis of the arch ring conforms, as nearly as practicable, to either the equilibrium polygon (line of pressure) for full dead load or the equilibrium polygon for full dead load plus one half live load over the full span, whichever produces the smallest bending stresses. On short spans, the deflection of the arch axis is small and the secondary moments (represented by the thrust times the deflection) may be neglected in the arch design.

• *Long Spans (Deflection Method)* On long spans (over 35 m), it is imperative that the arch centroidal axis coincides with the equilibrium polygon for full dead load.

Any live load system will disturb the above equilibrium polygon and cause an elastic deflection of the rib. This deflection creates additional moments which cause further deflection, thus creating still additional moments. If the rib is too flexible, the arch ring may not be able to regain equilibrium and fail radially by buckling. To prevent this type of failure the arch ring should be designed for the ultimate load and moments, including the elastic and deflection effects. This method of analysis is involved, but by the use of computers the deflection method taking into account the non-linear behaviour may be readily applied and a rapid and accurate analysis is possible. For the analysis, full consideration must be given to plastic flow, creep, temperature, variation in modulus of elasticity, possible geometric errors and other factors that cannot be precisely evaluated.

Span Lengths

The span length of the arch may be determined as follows:

(i) Two and three hinged arches: horizontal distance between the centres of the hinges at the supports.

(ii) Fixed arches: horizontal distance between the spring lines, l', adjusted for possible rotation at the support abutment. The adjusted span length, l, is dependent on the type of foundation material as follows:

$$\text{Solid rock } l = l' + 1.2h_r$$
$$\text{Medium rock } l = l' + 1.8h_r.$$

where h_r is the rib-thickness at the theoretical point of fixity. (On Gravel and sand: arch construction not recommended.)

Resistance to Transverse Buckling

Arch ribs are also investigated for resistance against transverse buckling. For buckling in the transverse direction, it may be assumed that the arch rib is a straight column with a length equal to the span and an axial load equal to the horizontal thrust. The resistance to buckling or the safety factor can then be based on column design.

Types of Arch Spans

• *Spandrel Arches* The spandrel arch may be either the open-spandrel column type or the filled-spandrel type. The arch should be designed for the combination of the dead load, live load, impact, the effect of temperature and rib shortening, and the effect of tractive, wind and centrifugal forces. In lieu of a complete analysis of the entire structure, an open spandrel arch may be designed for concentrated loads applied at the points of spandrel support. The amount of live load concentration at each point should be determined from the live load on the roadway deck, placed to produce maximum stress at the section under consideration. In filled spandrel construction, the equivalent uniform live load or distributed axle load may be used for the arch design.

(i) Arch ribs should have a minimum thickness as governed by architectural appearance as well as facility of construction and be reinforced longitudinally with two main layers of reinforcement. These two layers of reinforcement should have an area preferably not less than 0.2% of the arch section at the crown, and be tied together by a series of stirrups spaced not further apart than the thickness of the rib. Arch ribs and spandrel columns should conform to the column provisions. Transverse walls should be treated as columns with lateral ties.

Bearing seats for columns and transverse walls over arch rings or ribs should be horizontal with suitable dowels to anchor to the spandrel construction.

(ii) *Filled spandrel* The spandrel walls may be of gravity; cantilever or counterfort design or tied back with transverse walls. Counterfort or cantilever construction should be proportioned and reinforced to minimize transmission of torsional stresses to outer section of the arch barrel. The face of the spandrel walls has to be reinforced in two directions for temperature and shrinkage. The arch ring requirements listed in (i) above apply for this type also. The spandrel wall reinforcement in filled type arches shall preferably extend into the barrel with adequate anchorage to develop the strength of the bars in bond.

• *Three Hinged Arches* The three hinged arch is statically determinate and is recommended for short spans. Hinges may be formed by one of the following method:

(a) Structural Steel Pinned Shoe.
(b) Crossing the intradosal and extradosal reinforcement at the hinge centerline and reducing the depth of the ring or rib towards the hinge.

• *Two Hinged Arches* For arch spans founded on soft rock, piled foundations, or any foundation material other than sound rock, the two hinged design is recommended.

The hinge types (a) and (b) listed above are also applicable for two hinged construction.

• *Fixed Arches* Full fixity at the foundations may be assumed if these are sufficiently rigid and the dead load reaction produces a soil pressure as uniform as possible at the base of the foundations.

Truss Spans

To avoid congestion of reinforcement at joints. Vierendeel trusses should be used instead of triangular trusses. Vierendeel trusses may be simply supported, cantilevered, or continuous. They may have parallel chords, inclined upper chords (if through type), or inclined lower chords (if deck type). The deflection of Vierendeel trusses is usually smaller than girder bridges of similar span length. In a Vierendeel truss, inclined end posts with smaller bending stresses are better than upright end posts. Panels having length greater than the truss height are generally more economical than square or upright arrangement. In the analysis of Vierendeel trusses, the entire structure should be treated as a rigid frame to determine the axial, flexural, and shear loads in each member. (Design is rarely governed by shear.) Vierendeel trusses with upper (if through type) or lower (if deck type) lateral systems should be analyzed as rigid space frames.

Members of Vierendeel trusses may be precast in the plant and field-jointed. The availability of high strength concrete, high strength reinforcing bars and prestressing strands makes this type of bridge adaptable to moderately long spans. However, truss type bridges in concrete are seldom built.

Stayed and Suspension Spans

Stayed and suspension structures are best suited for spans exceeding 150 m. The stayed span is characterized by straight inclined cables which support the deck system at one or several locations (Fig. 18.23). The suspension span, on the other hand, is characterized by vertical suspenders and near parabolic suspension cables (Fig. 18.24). (For more details in cable stayed deck refer Ch. 38 of this book.)

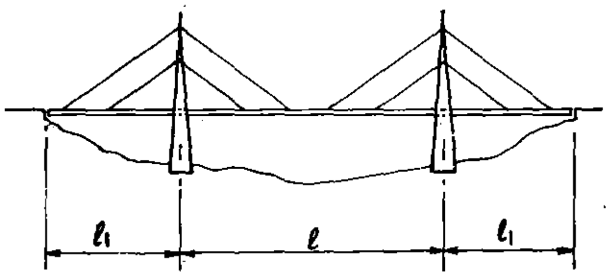


Fig. 18.23 *Stayed span*

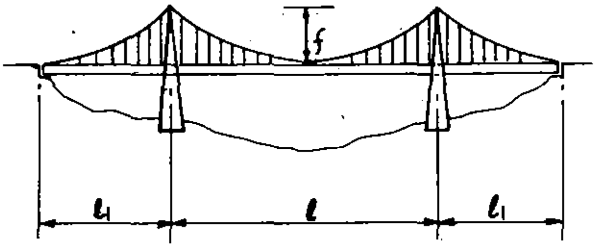


Fig. 18.24 *Suspension span*

Analysis

Short spans may be analyzed by the linear elastic theory. Longer spans, for which the deflections may be substantial, should be analyzed by the deflection theory (non-linear analysis).

Deck and Stiffening System

The deck and the stiffening system may perform either separate or combined functions. In the first case, the deck serves to transfer load to the stiffening and carrying systems. Examples are the conventional steel stiffening girder suspension bridges and some European concrete suspension bridges. In the second case, the deck is merged with the stiffening elements to serve dual purpose. Box girders and solid or voided prestressed concrete slabs are typical examples.

The stiffening system and the cable supporting system share (to various degrees, depending on relative stiffnesses) the live load superimposed on the bridge. In this regard, the stiffness factors must be carefully determined to avoid errors in the stress determination. The following guidelines may be used in the design of the stiffening system:

(i) Under dead load and median temperature, the system may be assumed to be supported by the bearings and cable supports.
(ii) Deflections from live load should not exceed $\frac{l}{800}$, where l = distance between non-yielding supports.
(iii) Live load and temperature effects absorbed by non-yielding supports at the bearings and yielding supports at the cable supports.
(iv) For suspension spans, the stiffening element, being a function of the dead load, becomes less critical and almost unnecessary in very long span structures where dead load exceeds ten times the live load.
(v) For self anchored stayed systems, the added longitudinal thrust in the stiffening element should be considered and can be utilized for the "prestressing".
(vi) The cable sag (f) to span (l) ratio should normally be between 0.09 and 0.14 for suspension spans.

Towers

The towers supporting the main cables may be one of the following types:

(i) Vertical legs with one or several cross girders to provide lateral stability.
(ii) Inclined legs with one or several cross girders for lateral stability. In this instance, the center line of a leg at the top coincides with the cable stay center line. The base width is dependent on the deck width and the lateral stability requirements.
(iii) Inclined legs meeting at a common apex. This type is used only for single cable systems.

In any of the above types, the towers may be hinged or fixed at the base. Fixed bases are more common as temporary support cables are not needed. On the other hand, fixed base results in substantial longitudinal bending moments in the tower legs.

The tower legs may be rectangular or cruciform in shape, their cross-section may be uniform or varied from top to bottom, and they may be prestressed or non-prestressed. Prestressed concrete tower legs are frequently constructed by segmental method. The legs should permit easy access to the top of the tower for maintenance, inspection of the cable saddle and connection, and navigation lights.

Cable Suspension System

The main suspension system on bridges may be one of the following types:

(i) *Parallel wire construction* In this type a series of parallel wires make up the circular sectioned cable.
(ii) *Parallel strand, closed construction* In this type a series of helically wound strands, either singly or more commonly in parallel, make up the circular section of the cable.
(iii) *Parallel strand, open construction* In this type a series of helically wound strands are separated into an open box like shape by means of spacers.

Types (i) and (ii) are nearly always wrapped tightly for protection against water and other corrosive agents. The wrapping usually consists of a skin of red lead paste and a tightly wound galvanized wire encircling the cable. A more recent innovation, which may become the standard method of protecting the cables, is a plastic covering. This utilises plastic filler pieces, zytel nylon film, glass-reinforced acrylic covering and a coating of lucite syrup.

In stayed spans, where the suspension system is virtually straight, concrete encasement of the cable is easier and advantageous. This encasement serves to protect the cable from corrosion, and also to reduce live load deflections, hence providing added safety against fatigue stresses at end connections. However, to fully realize these benefits, the cable must be pretensioned, prior to the concrete encasement, to such a level that tensile stress will not develop in the concrete under the full dead plus live load.

The design of the suspension system takes into account not only the axial dead load, live load and temperature stresses, but also the flexural stresses which could initiate fatigue failure. These flexural stresses normally originate at points of curvature-change and movement.

The ends of all strands are normally embedded in a poured zinc socket designed to develop the full strength of the strand in direct tension.

Suspenders, used in suspension bridges, are normally wire ropes or bridge strands socketed at both ends. The suspenders are fastened to the main cables by means of friction clamps. This design should be carefully checked to prevent slipping of the cable band and crushing of the cable wires by the clamping force.

Aerodynamic Stability

The suspended spans need to be designed so as to minimise the generation of vertical and torsional oscillations. Aerodynamic constants used for design should be carefully evaluated as the bridge site environment may greatly affect the direction and pressure of the wind. Model tests carried out in a wind tunnel for obtaining the most efficient cross-sectional shape of the bridge in its proposed environment are recommended.

NOTE For more details on 'Cantilever Construction' and 'Cable-stayed decks', refer to Chs. 36, 37 and 38 of this book.

CHAPTER 19

Transverse Distribution of Live Load among Deck Longitudinals

19.1 INTRODUCTION

When a concentrated load is applied over a single beam within the width of an open-spaced beam-and-slab deck, some load-sharing clearly takes place with adjacent beams, but the member directly under the load obviously deflects more than the others, and the slab which provides the transverse link between beams is therefore deformed. With a multi-celled box, similar deformations occur at the cross-section when the loads are applied over a single web, but the cross-section here is a closed-frame and the webs of a multi-celled box are not free to rotate in the way that is possible in a beam-and-slab deck, because they are tied laterally, at the top and bottom. The pattern of deformation and the resulting force system are akin to those of a Vierendeel girder.

The term *distortion* which is applied to this pattern of deformation can be misleading because it inevitably is assumed to be asscociated with torsion, which is not necessarily the case. In fact, by considering a twin-celled box subjected to distortion by a concentrated load over the central web, it is evident that the force system which develops can be in equilibrium without imparting any torsional rotation to the deck. Distortion is therefore essentially the effect of differential deflection between adjacent longitudinal members of the deck. This point should be kept in mind.

A bridge deck is basically a platform between piers (and abutments). It could be a slab, or a slab over a grid of longitudinal and transverse beams. Depending on certain considerations, the longitudinal beams could even be interconnected at their bottoms by a soffit slab, in which case the intermediate transverse beams (i.e., other than those at supports and at ends) could be avoided altogether. Whichever the structural scheme for this platform, the live load on it will be distributed among the platform elements in a certain mathematical proportion depending on their flexural and torsional rigidities, their material properties, and the platform geometry and its support conditions. For designing the platform elements it is obviously necessary to analyse their response to the applied load and estimate how much of the latter is apportioned to each one of them. The whole aim of the present chapter is to establish just this in the case of different types of deck-sections, and principally the apportionment of the applied (gravity) live load among the deck longitudinals.

Basically the various methods of analysis of grid and slab structures fall into three main categories. The first category covers those analyses which divide the structure into individual longitudinal and transverse members, each possessing the appropriate flexural and torsional stiffness; for each point of intersection of members, equations of deflection and slope compatibility can be set up and finally a set of governing simultaneous equations must be solved. Lazarides has used a method of this type. A variant of this method depends on the use of moment and torsion distribution or relaxation as a means of obtaining the solution to the simultaneous equations: Janssonius has developed the relaxation approach for bridge structures. This general approach, while of value in certain isolated cases, is extremely cumbersome, involving a great deal of arithmetical work and, of course, cannot be generalised. The advent of electronic digital computers has enabled the abundant numerical problems to be dealt with quickly and accurately, as Lightfoot and Sawko have shown, but it is difficult to produce a simple design procedure based upon this approach.

The second category covers those analyses which separate the longitudinal (or primary) members of the structure and consider some form of secondary cross connection which represents the behaviour of the transverse members. The theories in this category differ in the assumptions made; thus Hetenyi assumed that there was no rotation of individual members at an intersection and used a sine series to represent the load and deflection of the grillage in the direction of the longitudinal members; Pippard and de Waele assumed that the longitudinal members did not rotate and replaced the transverse members by a continuous medium; Leonhardt assumed that the transverse members could be replaced by a single member at mid-span with zero torsional stiffness. Most of these assumptions are invalid in practical bridge structures where the torsional stiffness of members, particularly in reinforced and prestressed concrete, may be considerable. Further, the methods again do not lend themselves to generalisation for an unspecified load position and are, as in the analyses in the first category, very cumbersome to use. Hendry and Jaeger thoroughly developed (during the early fifties) the basic approach outlined above, based upon only one simplifying assumption, which is, that the transverse members can be replaced by a uniform continuous transverse medium of equivalent stiffness. Their approach is to write down the differential equation for the loading on each longitudinal member including, where necessary, the effects of rotation and twisting. *Harmonic analysis* is then used to derive the amplitudes of the deflection and bending moment for each longitudinal member. The approach has been generalised to produce *distribution coefficients* applicable to many practical cases. These coefficients have been derived for bridges with various numbers of longitudinal members. This particular method of analysis is a considerable advance on the previous methods in this category and can be applied to various types of boundary conditions.

The third, and final category, covers those analyses which are based on anisotropic or orthotropic plate theory. These analyses replace the actual bridge deck structure by an equivalent orthotropic plate which is then treated according to the classical elastic theory. Guyon first developed this approach for grillages with members of negligible torsional stiffness and subsequently produced a similar analysis for isotropic slabs. This approach was then generalised by Massonnet to include the effects of torsion.

Extensions and developments of Guyon's and Massonnet's work have been produced, which generalise the use of this method and from which a design procedure has been formulated. This practicalising was done by Little and Morice at the C and CA (London) in the mid-fifties.

This particular approach has the merit that a single set of distribution coefficients for the two extreme cases of a no-torsion grillage and a full-torsion slab, enable the distribution behaviour of many types of deck structure to be found. Further, the implications of the analysis can readily be understood by the designer and hence the calculations become much more meaningful than being just a set of mathematical formulae with no practical applicability.

However, this approach (widely used where appropriate, and loosely referred to as the Little and Morice method) has its limitations — i.e., the deck should be either a slab or a pseudo-slab or a beam-and-slab and should preferably be simply supported with no skew. It cannot accommodate curved geometry. Nevertheless the method is applied for skew angle up to about 20° up to which the skew effect can be ignored without any significant sacrifice of engineering accuracy. Even continuous superstructures are handled by assuming the distance between the two consecutive contraflexure points in a span in the dead load bending moment diagram as the simple span length required in the analysis. (Details are given ahead.)

In 1950, M Courbon came up with an extremely simple approach for estimating the apportionment of live load among deck longitudinals in a beam-and-slab type of deck, but the deck had to be straight in plan, had to have an effective span-to-width ratio of between 2 and 4, had to have enough cross beams in it to make it behave rigidly in order to enable the live load transverse distribution on a rivet group analogy, etc. Courbon's method continues to be very widely used in certain countries, of course within its proclaimed limitations. (Details are given ahead.)

However, a practical method, powerful enough to tackle the load distribution among the curved and the skewed decks, even with box-deck section and variable moment of inertia, had still to appear on the bridge analysis scene.

The advent of electronic computers livened the interest, bringing in its wake the finite strip, finite beam-element and finite plate-element methods of global analyses. With time, these methods were streamlined, debugged and perfected for commercial use. Of these, the one that is good and powerful enough for use in bridge analysis is the *finite beam-element method*. This may also be referred to as the *grillage method* in a two-dimensional set-up. Here the deck is idealised into a grillage of linear beam members (i.e., the beam-elements), generally in two orthogonal directions, crossing each other at the nodes or junctions. Compatibility of deformations of the beam elements is ensured at these nodes. (In the more powerful *finite plate-element method* this compatibility would be ensured all along each edge of each plate-element, and hence the finite plate-element method is more time-consuming (and costly) both in formulating the input data as well as in meaningfully eking out the results from the printout.) More details are given ahead for the grillage method as it is today the most widely used method for the deck analysis where the appropriate computer programs and services are available.

Finite-Strip Method

This is a particular version of finite-element technique. The bridge deck is divided into strips which may all be in the horizontal plane for a plate deck, or in a three dimensional arrangement, as would be required for box construction, with the strips in the horizontal plane representing the slabs, and those in a vertical (or inclined) plane the webs.

This is a useful method, economical in computer time and giving good solutions for those structures which lie within its limitations. It can only represent bridge decks having constant cross-sections and with 'right' end supports. Irregular intermediate supports, can, however, be catered for by the technique of superimposing results to give zero deflections at the points of support on the beam originally considered as spanning between the end supports. (As already stated, it is not a popular method among the bridge analysts.)

Finite Plate-Elements Method

This analytical method is reputed to be very versatile, and is capable of representing complex structures acting in a complex manner. This flexibility inevitably leads to complex computer programs. It is expensive to use compared with the other methods, and its complexities mean that it is not the method for a design engineer who has not been steeped in computer lore and in the mathematics involved in the behaviour of plates—a highly complex mathematical drill.

The method was developed for the design of aircraft structures, where design and development costs dominate the total cost of a project. In that context such sophisticated methods make more sense. For the design of bridgeworks, this method is too sophisticated for regular use.

The ACI Committee 343 report (based mainly on AASHTO specifications) gives very simple rules for transverse distribution of live load among the deck longitudinals (subject to certain restricted dimensions in certain cases). These rules, despite being very simple to apply, obviously deserve to be highly respected, because a very large number of bridges have not only been and are being designed and built to these recommendations in the US, but they are also successfully standing in good service. Big computer programs and complicated formulae need not be necessary in situations where simple formulae simply applied can do for an otherwise complicated job. The details of these ACI recommendations are given ahead, and are strongly recommended for use.

A Word for the 'Practical' Designer

The subject of load distribution in bridge decks in fact can be very complicated. It is deeply rooted in the domains of higher mathematics and ideally involves orthotropic plate analysis. Fourier series is made use of for expressing the applied loads as continuous functions, etc. Rowe has reported on this very extensively, and so have many others. The reader is advised to study the literature indicated in the references (listed at the chapter end) for understanding what is involved.

However, it is important for the practical designer to maintain the realization that any mathematical analysis carried out is only an analogy that is not always truly representative of the way in which the real structure would behave. Too great a reliance on the quantitative answers arising from some form of mathematical analysis may even be indicative of a lack of engineering judgement.

Methods of analysis are evaluated by comparing the results with those given by laboratory tests carried out on models. Considerable progress has been made towards making models representative of prototype structures, but heavy reliance is still placed on measuring the deflected shape under load because of the difficulties in attempting to measure strains which may be more directly related to the stresses developed. The fact that a good comparison is achieved between the displacements obtained on a laboratory model and on an analytical mathematical model does not, in itself, mean that they are both predicting the same pattern of stresses. So the results of mathematical analyses still need to be approached with caution, intuition and (a resolute does of somewhat uncommon) common sense.

Grillage analysis is the most widely used mathematical tool, and the type of deck structure most difficult to represent in this way is a cellular deck, whether this is a voided slab or a box form of construction. The difficulties arise from the fact that the grillage is two-dimensional only, whereas a cellular deck behaves more clearly in a three-dimensional manner. A consideration of these differences can be a valuable aid to understanding both the cellular deck and the analytical limitations of a grillage. There is nothing wrong with making practical idealisations and, where necessary, it is better to go for an approximate solution to an exact problem rather than for an exact solution to an approximated problem. *Time-bound and result-oriented professional practice does not have much room for a frustrated engineer turned mathematician (he has his own place, but elsewhere).*

Consequently, restricting our jurisdiction to workman-like result-oriented professional practice, given in the remainder of this chapter are first certain basic features that influence the load distribution and must be understood, and then the details about the transverse distribution of live load among the deck longitudinals (various methods and deck sections).

19.2 BASIC FEATURES RELATED TO TRANSVERSE LOAD DISTRIBUTION

Shear Lag

Refer to Fig. 19.1. According to the basic assumptions of simple beam theory, where cross-sections are assumed to remain plane even after flexure, the distribution of stress across the top flange of a beam is uniform. In a broad-flanged T or I section, this assumption is not true except for sections which are far from a point of contraflexure. At a point of contraflexure the section is subjected to shearing force but no bending moment. Zero moment implies that there is no 'direct stress' in the flanges, while transverse shear on the section indicates that there are horizontal shearing stresses reducing in intensity toward the extremities of the section. In the case of a broad flanged I-section this means that the horizontal shear flow diminishes to zero at the outer-edges of the flange. Away from the point of contraflexure, direct stresses are present because of the moment on the section and the shearing stresses get modified. As with the case of simple bending theory for beams, the horizontal shear flow and direct stresses are inter-related, and what is happening may be visualized in terms of shear flow injecting direct stresses into the flange. The build up of these direct stresses resulting from the shear flow is not uniform across the width of the wide flange, but produces stresses which tail off toward the extremities, until a distance is attained that is far enough from the point of contraflexure for the pattern of stresses to have reached a balance which produces uniform direct stress. The effects associated with this change of distribution of direct stress are known as shear lag, and it consequently reduces the 'effectiveness' of the area of compression flange.

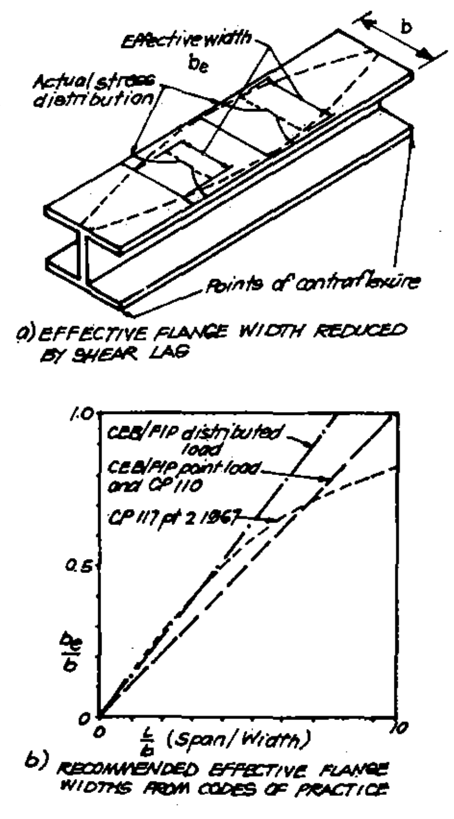


Fig. 19.1 *Shear lag*

In order to assess the peak stresses in such a section, the commonly adopted procedure is to calculate the sectional properties on the basis of a reduced (so-called effective) flange width. BS CP 110 and CP 117 (and indeed other codes too) recommend methods of assessing this reduced width. An exact assessment of the effects of shear lag involves recourse to plate theory, because the effect is a function of each particular loading case as well as of the plate dimensions.

With continuous bridge decks the effects of shear lag are most significant at and near the intermediate supports. The recommendation of CP 110 (i.e., that the effective flange widths are assessed on the assumption of points of contraflexure at distances of 0.7 of the span apart) can produce substantial errors at the support section. A reasonable compromise is to evaluate the effects of shear lag on the basis of the points of contraflexure which arise on a given structure under a uniformly distributed load (although, strictly speaking, these positions should be re-evaluated for each loading case, but that is impractical).

Since shear lag reduces the effective stiffness of a member in addition to modifying the distribution of stresses across it, improved accuracy can be obtained from a grillage analysis if the effective sectional properties, arising from shear lag, are used in the grillage input. This may redistribute some moment away from those sections modified by shear lag, thereby reducing the resultant stresses.

Where it is needed, a good picture of the distribution of stresses across the width of a plate can be obtained by plotting those stresses which arise from the net effective sectional properties, together with those arising from the gross sectional properties. The curved distribution of stress across the flange can then be sketched.

When applying shear lag to a box deck, the sectional properties of the reduced section can imply a different centroidal axis position to that which applies to the gross section. For a prestressed beam having varying sectional properties, it is well known that if the centroidal axis changes in level, this in itself, modifies the moment applied by the prestress. However, in the case of a change in the position of the centroidal axis due to shear lag, this is not the case.

The forces produced by the prestress can be considered as two separate elements — the horizontal force applied to the deck, and the bending moments produced by the eccentric cable profile. In the case of a beam with varying sectional properties a change in the position of the centroidal axis modifies the stresses produced by the horizontal force in the same way that a direct load on a column produces varying stresses if the sectional properties change — because of the changing eccentricity of the load at each section. This analogy cannot be adopted in shear lag calculations because differing rates of shear lag apply to the horizontal load due to the prestress, and to the bending moments. For the horizontal load, shear lag affects only the regions adjacent to the anchorages where the horizontal force is applied. The fact that the stresses induced by bending moments arising from the vertical force applied by changes in direction of the prestressing cable are subject to shear lag does not, in itself, modify the stresses produced by the horizontal force.

Not much adjustment is, therefore, needed to the stresses arising from axial prestress, even though the shear lag phenomenon suggests a change in the position of the centroidal axis when considering moments.

Torsion

Refer to Figs. 19.2 and 19.3. There are fundamental differences between a grillage and a cellular bridge deck in relation to the forces and stresses which arise due to torsion. Elementary considerations of equilibrium demand that a shearing stress in one plane can only co-exist with a shearing stress of equal intensity in the complementary plane. In the case of a simple beam this means that shearing stresses of equal intensity are present over a vertical cross-section and in the complementary horizontal plane. In the case of the top or bottom slab of a cellular deck, the implication is that shear flow arising from torsion is of equal intensity over both transverse and longitudinal sections through the deck, at any one point.

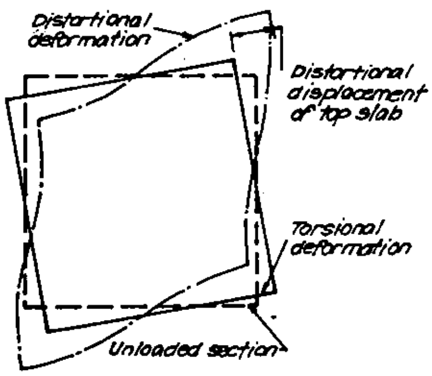


Fig. 19.2 *Torsional and distortional deformations of a box cell*

In contrast with this, there is no interdependence between the longitudinal and transverse torsional moments within a grillage. Balanced results can only be achieved by carefully evaluating the sectional properties. To obtain accurate values for torsion, using a grillage analysis, which might be directly applied to the real structure, theoretically it is necessary to re-assess the torsional stiffnesses of the component members for each individual loading case in the light of the pattern of deformation that is anticipated.

The torsional resistance of a cellular deck results from two primary components, shear flow in the webs and shear flow in the top and bottom slabs. A rectangular cell subjected to torsion develops shear flow of a pattern which means that the vertical (web) members make a contribution to the torsional resistance which is equal to that afforded by the top and bottom slabs.

A grillage subjected to torsion develops a system of reacting forces consisting of three components:

- a torsional shear in the longitudinal members in association with torsion in the transverse members
- a differential flexural shear in the longitudinal members, due to non-uniform load sharing
- torsional moments in the longitudinal members with associated flexure of the transverse members.

The torsional moments evaluated in the longitudinal members by using a grillage are inevitably underestimated and, assuming the stiffnesses of transverse members have been correctly evaluated, an appropriate amount of shear flow will develop in the webs. To this will be added the differential flexural shear reflecting the load-sharing pattern of the longitudinal members. A grillage will only use the torsional stiffness of the longitudinal members to make up the deficiency in equilibrium. If use is made of these longitudinal torsions to evaluate the flow of shear in the top and bottom plates, this will inevitably give a value which is less than that in the vertical webs, which must be equal in terms of force per unit perimeter.

Stiffnesses assigned to transverse members of the grillage have a twofold effect—they give rise to torsional moments which can be used to evaluate the flow of shear in the top and bottom slabs, and they modify the bending moments in the longitudinal grillage members, including shears which represent the flow of shear within the webs of the box. It is this which gives the grillage bending-moment diagrams their characteristic 'saw-toothed' shape.

In a grillage representing a multicelled box, the pattern of torsion in the transverse members within the various 'bays' of the grillage shows the build-up of torsion from the edges of the box inward. Intermediate longitudinal members are influenced only by the change in torsion along a transverse member, on each side of the node at which they intersect.

In considering a particular cross-section of deck, the transverse members reflect the torsion on the basis of the twist along that line. But the shear flow in the webs will be an outcome of the twist in the adjoining transverse members also, which may be some distance away and possibly subject to a significantly different twist, because of the deflected shape of the deck. To this extent, the shear flows evaluated are misrepresented.

It follows that the transverse members are a better source of values for the torsion within a grillage. However, if shear flexibility has been incorporated in the grillage sectional properties this will have the effect of increasing the differential deflection between the longitudinal members and, therefore, correspondingly increasing the twist in the transverse members connecting them. The torsion in the transverse members is therefore overstated.

Clearly, the prime contributors to torsional stiffness are

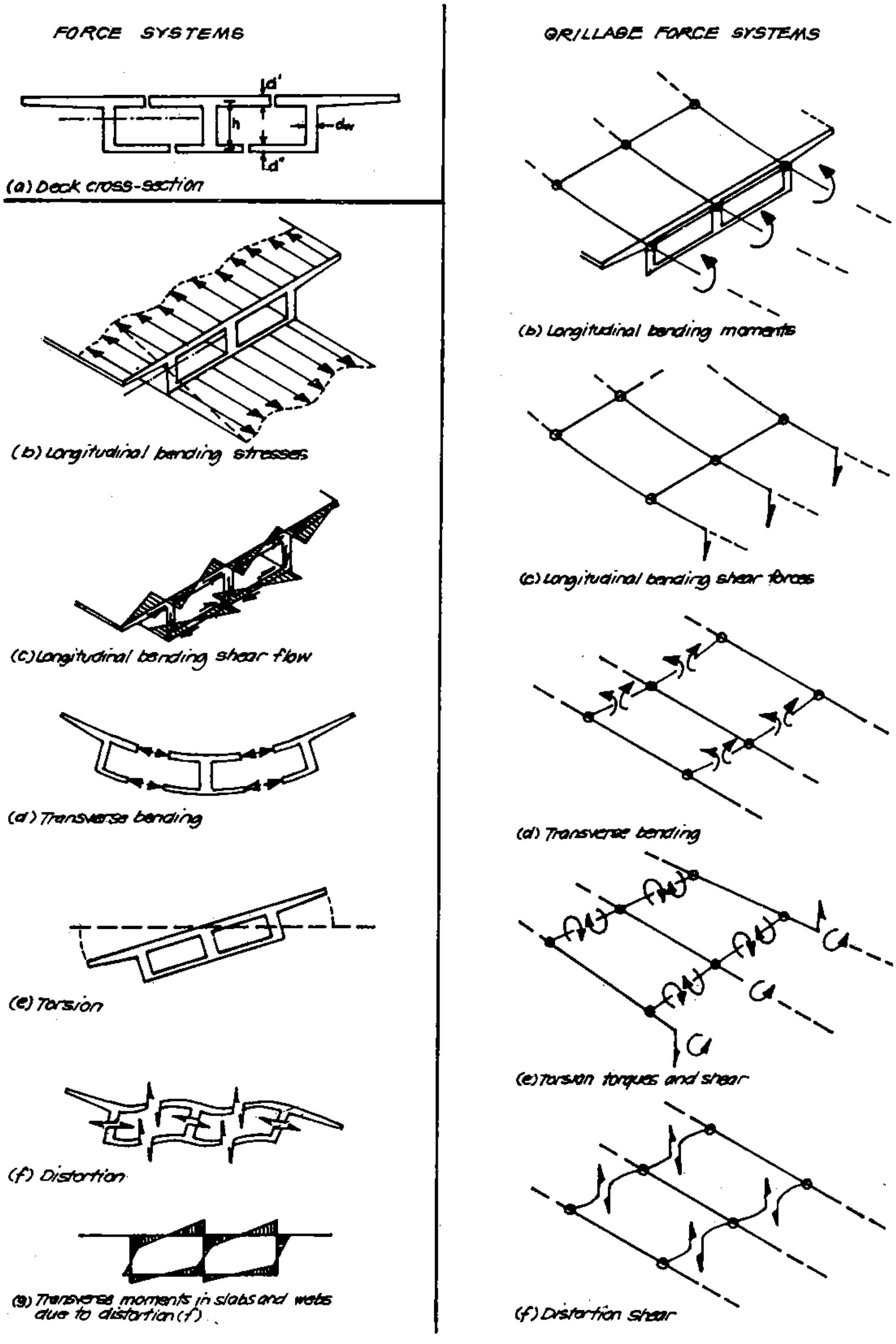


Fig. 19.3 *Grillage force system in a twin-cell box section*

the top and bottom slabs. Where diaphragms are present within the depth of a deck, a consideration of membrane analogy for torsion will readily show that the flow of torsional shear in a diaphragm member is only the difference in the flow of shear in the top and bottom slabs as they cross the diaphragm.

Distortion

Refer to Figs. 19.2 and 19.3. As explained earlier distortion of a cell section is essentially caused by the differential deflection between adjacent longitudinal members of the cell section, and is not necessarily associated with torsion.

Because of distortion, a cellular deck has a dual stiffness transversely. Circular bending across the deck results in direct tension and compression in the top and bottom slabs, thereby forming a couple to resist the bending moments. No distortion is involved. On the other hand, where there is a transfer of shear across the deck, it gives rise to distortion and the stiffness of the deck is now totally different from that which applies to circular bending. Most real loading cases produce a combination of these two effects. It is therefore essential that a grillage should be capable of handling this duality of stiffness.

In calculating the deflections of a beam, it is normal to consider only those deformations which arise due to bending. A full mathematical expression of the deflection in a beam also includes the deformations arising as a result of shear, in addition to those due to flexure. But shear deformations* are normally neglected because they are very small by comparison with those due to bending. To enable the transverse members of a grillage to take account of the dual stiffnesses applicable to bending and to shear, grillage programs have been written that include the full deflection equations, including shear. It is, therefore, possible to take account of the dual stiffness of a cellular section by calculating a shear area for the transverse members which would give the same rate of deformation as that which, in reality, arises from distortion.

The calculation of this 'effective shear area' is based on assumptions of how the cross-section deforms. This will, in fact, differ according to the loading case applied. In the case of a heavy load applied at one edge of a deck section, all the cells deform in a similar manner, including the flexure of the webs themselves. In contrast, a heavy load placed in the middle of the deck section, which is such that the force systems are in balance on each side of the point of application, means that the web under the load does not transversely flex (because there is no moment in it).

In the case of thin-webbed (i.e., steel) boxes, the presence or otherwise of flexure in the webs may significantly affect the stiffness of the cell. The shear area assigned to the transverse members can therefore only approximate the distortional stiffness, but in most practical bridge-deck sections the top and bottom slab elements are considerably more flexible than the webs, so this error is not serious.

Where there are variations in the shape of the cross-section, such as sloping webs, which complicate the assessment of the distortional flexibility, this can be assessed with the aid of a frame representing the cross-sectional shape. The deformation of the frame under the load provides a basis for assessing the 'effective shear area'.

Having obtained the grillage output, the force system arising from deformation is calculated by assuming that the points of contraflexure develop midway between the webs, and that the total shear from the grillage is shared between the top and bottom slabs in proportion to their stiffness.

In designing reinforcement or determining the prestress required to cater for the forces developing around the cross-section, the values are dominated by those forces which arise from local wheel loads. These are best evaluated by applying Pücher's (or a similar) design method for concentrated loads (details regarding the analysis and design of transverse section of deck are given in a separate chapter in this book). The distortional stresses are significant in thin walled (e.g. steel) boxes, and are small in comparison with the effects of local wheel loads.

19.3 TRANSVERSE DISTRIBUTION OF LOADS (AS PER ACI COMMITTEE 343 REPORT AND AASHTO SPECIFICATIONS)

Analysis based on the elastic theory is recommended to find the distribution in the transverse direction of the bending moment in the direction of the span. For the analysis, the structure may be idealized in one of the following ways:

(i) a system of interconnected beams forming a grid
(ii) an orthotropic plate
(iii) an assemblage of thin plate elements or thin plate elements and beams.

Several methods of analysis are available which can be applied with the use of a computer. In addition to the moments in the direction of the span, computer aided analyses can give moments in the transverse members also. A theoretical analysis is particularly recommended for bridges which have large skews or sharp curvatures.

• Beam-and-Slab (T-Beam or Precast I-Girder) and Box Girder Bridge Decks

In lieu of an analysis (based on elastic theory) for the distribution of live loads among longitudinal beams, the following empirical method authorized by AASHTO may be used for T-beam or precast I-girder bridges [see Figs. 19.4 (a) and (c)], and for box-girder bridges [Fig. 19.4 (b)]. The

* For details refer to Ch. 20 of this book.

distribution of shear should be determined by the method prescribed for moment.

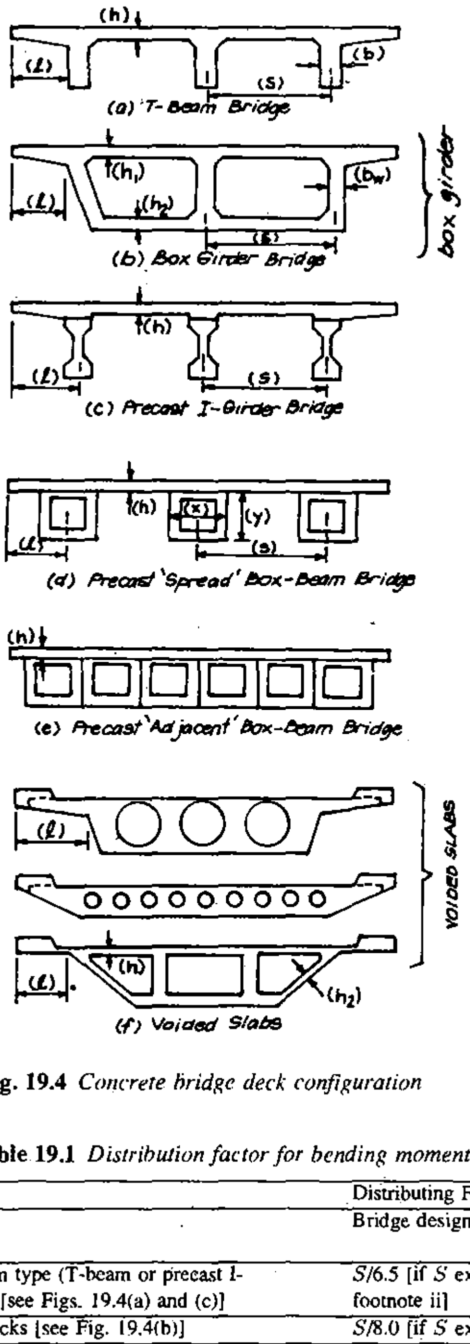


Fig. 19.4 *Concrete bridge deck configuration*

Interior Beams in Cases of Beam-and-Slab and Also Box Girder Decks

The live load bending moment for each interior longitudinal beam should be determined by applying to the beam the fraction (DF) of the wheel load (both front and rear) as defined in Table 19.1. In the present context wheel load (both front and rear) shall mean the effect of one wheel line i.e., half the effect of one truck.

Exterior Beams in Case of Beam-and-Slab Decks Only

The dead load considered as supported by the outside roadway beam is that portion of the floor slab as is carried by that stringer or beam. Curbs, railings and wearing surface, if placed after the slab has cured, may be considered equally distributed to all roadway stringers or beams.

The live load bending moment for outside roadway beams is to be determined by applying to the beam the reaction of the wheel loads obtained by assuming the flooring to act as simple spans transversely between the beams.

When the outside roadway beam supports the sidewalk live load as well as traffic live load and impact, the allowable stress in the beam may be increased 25% for the combination of dead load, traffic live load, and impact, provided the beam is of no less carrying capacity than would be required if there were no sidewalks.

In no case should an exterior beam have less carrying capacity than an interior beam.

Exterior Beams in Case of Box Girder Decks Only

The dead load considered as supported by the exterior girder here should be determined in the same manner as already given.

The wheel load distribution factor to the exterior girder should be taken as $w_e/7$. The width w_e to be used in determining the *wheel line distribution* to the exterior girder is the top slab width as measured from the midpoint between girders to the outside edge of the slab. The cantilever

Table 19.1 *Distribution factor for bending moments in interior main beams (for beam and slab and box-girder decks)*

	Distributing Factor (DF)	
Kind of floor	Bridge designed for one traffic lane	Bridge designed for two or more traffic lanes
Slab and beam type (T-beam or precast I-girder) decks [see Figs. 19.4(a) and (c)]	$S/6.5$ [if S exceeds 6 ft, use footnote ii]	$S/6.0$ [if S exceeds 10 ft, use footnote (ii)]
Box-girder decks [see Fig. 19.4(b)]	$S/8.0$ [if S exceeds 12 ft, use footnote (ii)]	$S/7.0$ [if S exceeds 16 ft, use footnote (ii)]
Multi-precast spread box-beam decks [see Fig. 19.4(d)]	(Given above in this chapter)	
Multi-precast adjacent box-beam decks [see Fig. 19.4(e)]	(Given ahead in this chapter)	

NOTE

(i) S = average beam spacing feet.

(ii) In this case the load on each beam will be the reaction of the wheel loads, assuming the flooring between the beams to act as simple beams.

dimension of any slab extending beyond the exterior girder should not exceed $S/2$, where S is the girder spacing in feet.

Total Capacity of Longitudinal Beams

The combined design load capacity of all the beams and stringers in a span should not be less than that required to support the total load in the span.

In Multi-Precast Spread Box-Beam Decks

For spread box beam superstructures [Fig. 19.4(d)], the lateral distribution of live load for bending moment can be determined as given here:

1. *In interior beams* The live load bending moment in each interior beam can be determined by applying to the beam the fraction (DF) of the wheel load (both front and rear) determined by the following equation:

$$DF = \frac{2n_l}{n_b} + k_l\frac{S}{l}$$

where n_l = number of design traffic lanes
n_b = number of beams ($4 \leq n_b \leq 10$)
S = beam spacing in feet ($6.75 \leq S \leq 11.00$)
l = span length in feet; for continuous structures use lengths to approximate points of zero moment under uniform load
$k_l = 0.07w_c - n(0.1n_l - 0.26) - 0.20n_b - 0.12$
w_c = roadway width between curbs in feet ($32 \leq w_c \leq 66$).

2. *In exterior beams* The live load bending moment in the exterior beams should be determined by applying to the beams the reaction of the wheel loads obtained by assuming the flooring to act as a simple span between beams, but the reaction should not be less than $2n_l/n_b$.

In Multi-Precast 'Adjacent' Box-Beam Decks

This type of deck [Fig. 19.4(e)], is constructed with precast reinforced or prestressed concrete beams which are placed in contact side by side on the supports. The interaction between the beams is developed by continuous longitudinal shear keys and lateral bolts which may, or may not, be prestressed.

In calculating bending moments in these beams, conventional or prestressed, no longitudinal distribution of wheel load should be assumed. The lateral distribution should be determined as given below.

The live load bending moment for each section should be determined by applying to the beam the fraction of a wheel load (both front and rear) determined by the following relations:

$$DF = \frac{(12n_l + 9)/n_b}{(5 + n_l/10) + (3 - 2n_l/7)(1 - C/3)^2}$$

or when C is 3 or greater, use

$$DF = \frac{(12n_l + 9)/n_b}{(5 + n_l/10)}$$

where n_l = number of design traffic lanes
n_b = number of beams
$C = K(w_c/l)$, a stiffness parameter
w_c = roadway width between curbs (in ft)
l = span length (in ft)

VALUES OF K TO BE USED IN $C = K(W_c/l)$

Beam Type and Deck Material	K
Nonvoided rectangular beams	0.7
Rectangular beams with circular voids	0.8
Box section beams	1.0
Channel beams	2.2

- **In Solid Slab Type Decks (with main Reinforcement Parallel to Traffic)**

In Simple Span Slab Deck

S = effective span length, in ft
E = width of slab in ft over which a wheel load is distributed

For wheel loads the distribution width E shall be ($4 + 0.06S$) but shall not exceed 7.0 ft. Lane loads are distributed over a width of $2E$.

For simple spans, the maximum live load moment per foot width of slab, without impact, is closely approximated by the following formulae:

(i) *HS 20 loading*
- for spans up to and including 50 ft LLM = $900S$ ft pounds/ft width
- for spans 50 to 100 ft LLM = 1000 ($1.3S - 20$) ft pounds/ft width
(LLM = live load moment)

(ii) *HS 15 loading* Use 3/4 of the values obtained from the formulae for HS 20 loading, mentioned above.

Moments in continuous spans shall be determined by suitable analysis using the truck or appropriate lane loading.

- **In Cantilevering Slab Deck (Cantilevering Parallel to Traffic Direction)** The distribution width for each wheel load on the cantilevering slab (cantilevering parallel to traffic) shall be as given below:

$E = 0.35X + 3.2$, but shall not exceed 7.0 ft and the live load moment per foot width of slab shall be $(P/E)\,X$ foot-pounds, where X is the distance in feet from load to point of support, and P is the load in pounds from one rear wheel of the truck (rear axle being the heaviest axle in the truck).

19.4 COURBON'S METHOD FOR ESTIMATING TRANSVERSE DISTRIBUTION OF LIVE LOAD AMONG DECK LONGITUDINALS IN A BEAM-AND-SLAB TYPE DECK

Reference may be made to J Courbon's *Application de la Resistance des Materiaux au Calculdes Ponts*, published by Dunod, Paris, in 1950.

In simple words this method for transverse load distribution among the deck longitudinals is applicable mainly to beam and slab type decks which are straight in plan (no skew, no curve). The longitudinal beams must be interconnected by full-depth (or almost full-depth) rigid cross beams that are at least five in number (one above each support, and, at least three intermediate cross beams, equally spaced) such that they are not more than about 9 m apart. The cross beams should preferably be cast monolithically with the longitudinals or should be cast at least before any other gravity loads (besides the self weight of the main beams) comes on. The longitudinal effective span ($2a$) should preferably be simply supported, and its ratio with the effective width ($2b$, equal to the product of the number of main beams and the spacing between main beams) should be between 2 and 4. However, the method can be applied even to longitudinally continuous decks in which case $2a$ must be taken as the distance between the two consecutive contraflexure points in a span in a uniformly loaded continuous condition. In addition, the factor ϕ should be less than 0.5, where

$$\phi = \frac{b}{2a}\sqrt[4]{\frac{a}{b}\frac{\Sigma(EI)\text{of main beams}}{\Sigma(EI)\text{of cross beams}}}$$

EI being the flexural rigidity.

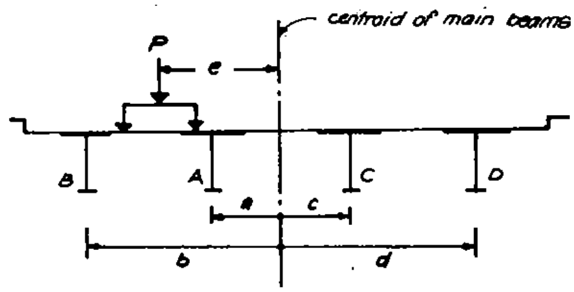
In principle, advantage is taken of the action of short (because of $2a/2b$ restriction) and deep diaphragms, action considered somewhat akin to that of a thick footing resting of piles and assumed to remain straight at all times. Thus, under a truck load P applied as shown in Fig. 19.5 on a deck section that satisfies the above specifications, the transverse distribution of the load P among the various main beams will be such that the reaction felt by a main beam will be equal to

$$\frac{P}{n} \pm \frac{Pe}{I}x$$

i.e.,

$$P\left(\frac{1}{n} \pm \frac{ex}{\Sigma x^2}\right)$$

where x is the distance of that beam from the centroid of the main beams, Σx^2 is the sum of the squares of the distances of various main beams from the said centroid, and n the number of main beams; +ve sign if the beam is on the load-



$I = \Sigma x^2$
$= (a^2 + b^2 + c^2 + d^2)$
in this case,
n = no. of main beams
= 4 in this case

Fig. 19.5

side relative to the centroid of the main beams, and −ve sign otherwise.

In Fig. 19.5. if the main beams are equally spaced, say at y centres, then

$$I = \Sigma x^2 = [(1.5y)^2 + (y/2)^2]2 = 5y^2$$

So that,

$$\text{Reaction on beam } B = P\left(\frac{1}{4} + \frac{e(1.5y)}{5y^2}\right)$$
$$= P(0.25 + 0.3e/y)$$
$$\text{Reaction on beam } A = P\left(\frac{1}{4} + \frac{e(0.5y)}{5y^2}\right)$$
$$= P(0.25 + 0.1e/y)$$
$$\text{Reaction on beam } C = P(0.25 - 0.1e/y)$$
$$\text{Reaction on beam } D = P(0.25 - 0.3e/y)$$

and, for example, if $e = 1.5y$, then,

Beam B takes: $P(0.25 + 0.3 \times 1.5) = 0.7\ P$
Beam A takes: $P(0.25 + 0.1 \times 1.5) = 0.4\ P$
Beam C takes: $P(0.25 - 0.1 \times 1.5) = 0.1\ P$
Beam D takes: $P(0.25 - 0.3 \times 1.5) = -0.2\ P$(i.e., upward effect)

Total = $1.0\ P$

Instead, if $e = 0$, i.e., load is centrally placed, then each beam takes 0.25 P, i.e., the beams are equally loaded.

However, the latter is not quite true since the central beams will tend to take more load under a centrally placed load.

Thus it is clear that Courbon's method slightly under-estimates the load on interior beams and over-estimates the load in the outer beams. However, deeper, the cross beams and more their number, lesser will be this inaccuracy. Nevertheless, within the restrictions specified earlier, Courbon's method is acceptable and many load tests carried out on completed decks have borne this out. Hundreds of decks have been analysed (and constructed) by this method and more continue to be analysed (and constructed). Test results vouch for the engineering accuracy of this method of transverse load distribution, limited to its stipulations stated earlier.

Caution

What has been described above is valid if the applied load is at least about 5 m away from the supports. For loads placed anywhere within the central $(2a - 10)$ m of the span ($2a$ being the said longitudinal span in meters), the reaction factors as already calculated can be used for apportioning moment, shear, reactions, etc., to the longitudinals. But for loads placed within about 5 m from the supports, the reaction factors for their transverse distribution (i.e., apportionment of moment, shear, reactions, etc., from them) to the various deck longitudinals should be done using the *reaction influence lines* for respective main-beam-support points, assuming the deck section is a continuous beam on rigid supports (supports being the longitudinal beams). This is so since the deck section tends to behave rather rigidly at and close to a support.

Analysis for Cross Beams

Instead of going into too much theoretical detail here, it is basically enough to remember the precaution as pointed out earlier. After working out the live load reaction attractable by the particular cross beam in question, the intermediate-one or the end-one, (for which an approximation may be made that the live load is simply supported longitudinally in between the successive cross beams), for analysing the moments and shears in the cross beam at its various sections. Courbon's live load reaction factors may be adopted for establishing the reactions from the supporting main beams in the case of intermediate cross beams and ordinary continuous beam reaction influence lines may be adopted in the case of (rigidly supported) end cross beams. Once the governing moments and shears at various sections in a cross beam have been established, then follows its section design. In the latter it is necessary to first establish the width of top slab that can be assumed to act along with the stem of the cross beam section in its sagging moment regions. (This will be helped by adequately detailing the flexural and shear reinforcements in order to achieve the combined action of the stem and the slab.) In its hogging moment regions the stem may be considered on its own as being the effective section of the cross beam.

19.5 LITTLE AND MORICE METHOD (FOR ESTIMATING TRANSVERSE DISTRIBUTION OF LIVE LOAD AMONG THE DECK LONGITUDINALS IN A SOLID-SLAB TYPE OR A PSEUDO-SLAB TYPE OR A BEAM-AND-SLAB TYPE DECK)

This is also called the method of distribution coefficients. The necessary background for this method has already been covered to the necessary extent earlier. The working details of the method have already been well documented by the C and CA (London) through their various publications (e.g., their publication Db. 11, first published in July, 1956, from which has gratefully been taken the information quoted ahead and in the associated tables and graphs). Its range of application is limited—it is only applicable to slab, pseudo-slab, and beam-and-slab types of construction having prismatic cross-sections, the spans being simply supported, with line supports and right spans only. In practice this analytical tool is of much wider use than what these limitations first imply. Its results can generally be accepted for skews of up to 20°, and a series of discrete supports can be accepted as representing line supports, provided there is no significant overhang beyond the outer bearings and the spanning effect between the bearings does not become dominant in terms of the behaviour of the deck.

Even where it is felt that this load-distribution, method would not provide a suitable final analysis for a deck, it can still be a useful tool for making a reasonable approximation of the load-distribution characteristics at an early stage in the calculations.

Until recently, this has been a method that was applied only by hand calculation, but the theory has now been reformulated in a manner which lends itself to computerized calculation, with the resulting benefit of improved accuracy and speed.

The basis of this method of distribution coefficients is the study of an equivalent elastic system obtained by transforming the stiffness of a number of beams, which may be considered as concentrated at nodal points [Fig. 19.6(a)], into a uniformly distributed system of the same overall stiffness. The effect of producing a distributed system is to introduce a structure width, $2b$, which is given by the number of original main beams multiplied by their spacing. This results in an 'equivalent width' greater than the original width in certain cases, (Fig. 19.6). The method may also be applied to slabs when the equivalent system is the same as the original system.

Since the equivalent systems are distributed, the section properties too must be expressed per unit width.

Thus, the longitudinal second moment of area per unit

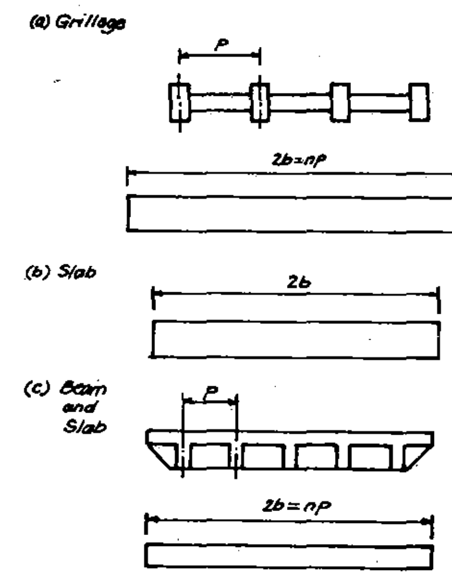


Fig. 19.6 *Actual and effective widths of various types of bridge deck structure*

width is,

$$i = \frac{I}{p}$$

where I = the second moment of area of a longitudinal beam section

and p = the longitudinal beam spacing.

Also, the transverse second moment of area per unit length is,

$$j = \frac{J}{q}$$

where J = the second moment of area of section of a transverse stiffener (i.e., diaphragm or cross beam)

q = the stiffener spacing.

In the case of a slab these per unit width section properties i and j are equal, and are given by

$$i = j = \frac{d^3}{12}$$

where d is the slab depth.

It is convenient to combine these geometrical properties into one parameter defining the relation between longitudinal and transverse properties, since this one parameter has a primary effect upon the distribution coefficients. The parameter, denoted by θ, is defined by,

$$\theta = \frac{b}{2a}\sqrt[4]{\frac{i}{j}} \qquad (19.1)$$

where $2a$ is the actual effective longitudinal span.

In the case of a slab the expression for θ is seen to reduce to $\frac{b}{2a}$.

In addition to the effects of θ, the distribution coefficients are also sensitive to the degree of torsional stiffness exhibited by the bridge deck.

The torsional properties, per unit width, are determined in the same way as those for bending (but see ahead), i_0 being the value for a transverse section and j_0 for a longitudinal section. These are combined in an overall torsional parameter α which is defined by,

$$\alpha = \frac{G(i_0 + j_0)}{2E\sqrt{ij}} \qquad (19.2)$$

where E = Young's modulus, } of the material of
G = torsional modulus } the bridge deck.

In the case of a slab α reduces to 1.0. (Refer to the section under *Calculation of torsional stiffness* ahead in this chapter.) So far the actual bridge deck structure has been replaced by a 'quasi' slab which has a width $2b$, not necessarily equal to the actual width of the structure, and a span $2a$, equal to the original span, and with its relative stiffness in bending and torsion specified by the non-dimensional parameters θ and α.

Now it is postulated, that within the accuracy of normal engineering design, the transverse deflection profile of the 'quasi' slab under any given loading is of constant form for all positions along the span of the slab.

This is the crux of the method, since it means that one set of relative arithmetical coefficients may be used for defining the deflected shape of all transverse sections. Thus all longitudinal deflection profiles are of the same shape, and it follows that the longitudinal bending moments are of the same form for any point across the width of the bridge. The set of relative arithmetical coefficients may therefore be used unaltered for deflections, longitudinal moments and longitudinal bending stresses. However, the maximum calculated longitudinal moments should be increased by 10% to correct them for the effect of having included only the first term in the harmonic analysis which forms the basis of the theory. When the arithmetical coefficients are expressed in units of the mean deflection (moment or stress), i.e., the deflection of the deck if the same load were uniformly distributed across the whole effective width, they are in the required form and are normally called *distribution coefficients, K.* The distribution coefficients are normally specified for single concentrated loads and will obviously depend upon the transverse eccentricity of the load. By the above postulation, they will be unaffected by the longitudinal or spanwise position of the load.

The merit of the method for *design office use* lies in the fact that these distribution coefficients (transverse deflection profiles) have been tabulated.

In order to make the system practical, the profile shape is given by distribution coefficients relative to the mean deflection of the whole section for stations at nine equally distributed standard positions across the width (Fig. 19.7), and the loading eccentricities are also given at these nine discrete positions $(-b, -3b/4, -b/2, -b/4, 0, b/4, b/2, 3b/4$ and $b)$.

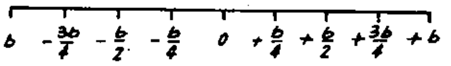


Fig. 19.7 *The nine standard positions on the effective width of a bridge*

The effect of θ upon these distribution coefficients K, is given by plotting them against θ.

The effect of α is introduced by providing two sets of curves, one for $\alpha = 0$. (Graphs 19.1 to 19.6, and the other for $\alpha = 1$, Graphs 19.7 to 19.11.) Any intermediate value of α is catered for by the interpolation expression for the distribution coefficients,

$$K_\alpha = K_0 + (K_1 - K_0)\sqrt{\alpha} \qquad (19.3)$$

where K_α = the distribution coefficient for general values of α

K_0 = the distribution coefficient for $\alpha = 0$

K_1 = the distribution coefficient for $\alpha = 1$

As a result of symmetry of the structure about the centre of a transverse section it is unnecessary to include four of the nine sets of curves, since a change of sign of both load position and reference station leaves the distribution coefficient unchanged. Thus,

K (load position $-3b/4$, reference station $+b/2$)
$= K$ (load position $+3b/4$, reference station $-b/2$)

Step by Step Procedure

The analysis starts with the determination of the elastic constants of the equivalent 'quasi' slab which corresponds to the actual structure (Fig. 19.8). This yields values for $2b$, θ and α; while the value of the span $(2a)$ and the total second moments of area are already known (see Table 19.2).

Next the loadings must be considered. In the first place the total bending moment must be determined (Fig. 19.10). Secondly, the actual loads vis-a-vis their positions must be reduced to statically equivalent sets (λP) acting at the standard positions (Fig. 19.11). These are recorded in Tables 19.3 and 19.4.

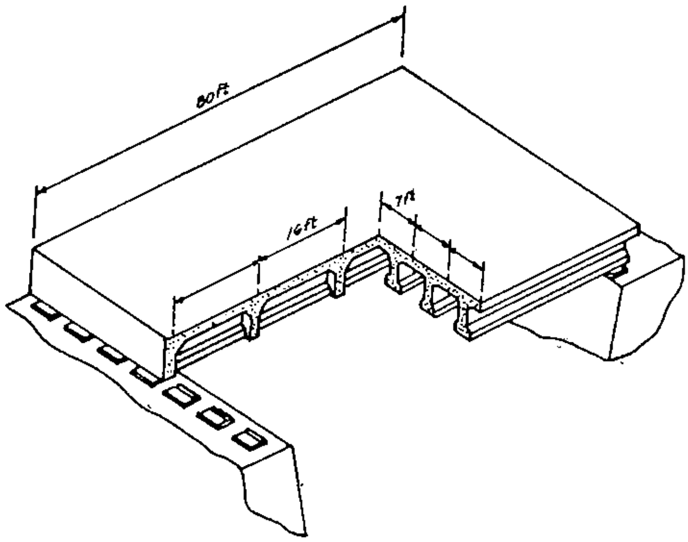


Fig. 19.8 *Actual form of beam and slab bridge structure*

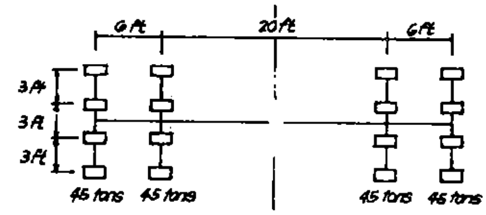


Fig. 19.9 *The wheel positions of the BS, HB loading in plan*

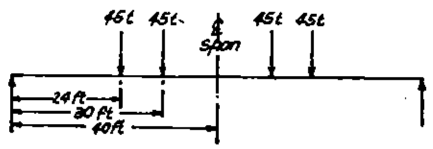


Fig. 19.10 *Longitudinal disposition of loading vehicle on beam and slab bridge*

The procedure may now be carried out in one of the two ways. Either the distribution coefficients may be read from the curves and immediately multiplied by the equivalent load factors, or a table of unit distribution coefficients may be prepared. The latter system is chosen here because it enables a symmetry check to be applied. Also, the calculation may be carried further before the loading is introduced. Thus the consideration of several loading conditions requires less labour.

For a deck of intermediate stiffness $(0 < \alpha < 1)$ the unit distribution coefficients will have to be prepared for

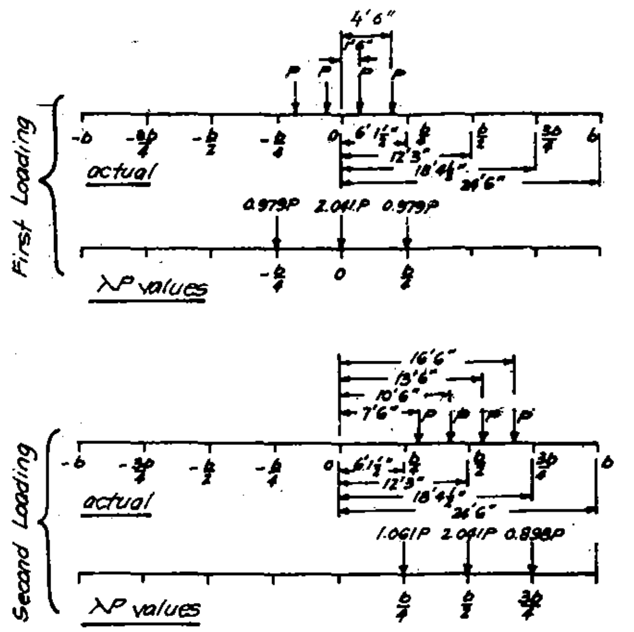


Fig. 19.11 *Transverse dispositions of loading vehicle on beam and slab bridge*

both $\alpha = 0$ and $\alpha = 1$. Two tables are obtained by reading off the values of K from the curves for the value of θ which has been calculated in Table 19.2. Graphs 19.1 to 19.6 give Table 19.5, Graph 19.1 giving the column for reference station 0. Graph 19.2 the column for reference station $+b/4$, and so on. It is now seen that the columns headed $-b$, $-3b/4$, $-b/2$ and $-b/4$ cannot be filled-in by direct reading from the curves. However, it has already been noted that the table possesses the property of symmetry about the diagonals, so that the values for the unfilled spaces may be obtained from those already filled-in. Thus K (reference station $-3b/4$ load position $+b/4$) is equal to K (reference station $+b/4$, load position $-3b/4$). Similarly Graphs 19.7 to 19.11 give Table 19.6, Graph 19.7 giving the fifth column, Graph 19.8 the sixth column, and so on. In practice it will often be found convenient to omit the rows of coefficients for the load positions $-b$ to $-b/4$. A check for these tables exists in the fact that if Simpson's rule is applied to the values in each row, the interval being taken as unity, the sum should equal 8.0. A tolerance in the sum from 7.8 to 8.2 can normally be permitted.

A combined table, Table 19.7, is then prepared from the previous two by use of the interpolation formula given in Eq. (19.3), for the appropriate value of α which has been calculated in Table 19.2. A table has not been prepared here for the computation, but, if required, one may be drawn up for the arithmetical steps K_0, K_1, (K_1-K_0), $(K_1-K_0)\sqrt{\alpha}$ and $K_0 + (K_1 - K_0)\sqrt{\alpha}$ with a space for each of the different values occurring in Table 19.7.

It is at this stage that the loadings are introduced by multiplying the unit coefficients of Table 19.7 by the appropriate loading values λP, which appeared in Table 19.3 for each of the loadings considered; in this case, two loading conditions are introduced. The final coefficients K' for each standard position may now be obtained by summing the value due to each load and dividing by the total load. This working for each loading condition is shown in Tables 19.8 and 19.9, respectively.

In fact we require the moment values in the beams, i.e., at the actual beam positions, and these are best obtained by plotting a curve (Fig. 19.12) of the transverse profile and inserting the actual beam positions found in Table 19.2, and reading the ordinates at these points. These results have been recorded in Table 19.10 for both loading conditions.

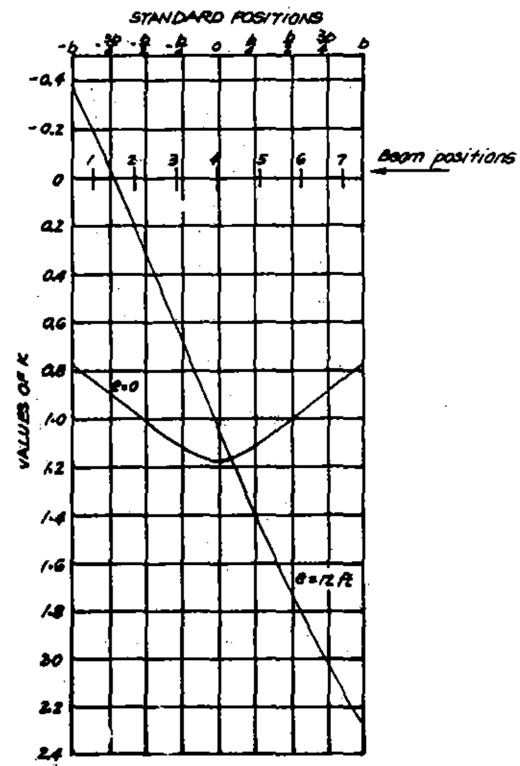


Fig. 19.12 *Transverse distribution profiles for beam and slab bridge*

Calculation of Torsional Stiffness

Equation (19.2), which defines the torsional parameter α, was based on the properties of a grillage for which it is sufficient to use the normal well-known formulae for determining torsional stiffness.

However, if these normal methods are applied to a slab, for example, using the thin rectangular section formula $i_0 = j_0 = d^3/3$ where d is the depth of the slab, it will be found that the resulting value of α is equal to 2 and not unity. This anomaly arises from the fact that the overall continuity of the slab in the longitudinal and transverse directions has been neglected. Actually the values of i_0 and j_0 to be used are equal to $d^3/6$ which results in a value of α equal to unity.

In a T-beam bridge deck, in each individual T-beam only the shear stresses parallel to the top surface can exist and if an individual T-beam is isolated, for convenience in determining the parameters, then the vertical shear stresses are not present. This means that only 50% of the torsional stiffness or inertia contributes to the torsional parameter α.

Thus the rule is that for that portion of the section forming part of the continuous slab portion of the structure, only half of the calculated torsional inertia may be considered in deducing α. This extends to structures intermediate in form between a grillage and a slab and a general rule is that in the determination of i_0 and j_0 the values for a continuous member, as calculated by the normal methods, should be halved, while the values for the non-continuous members should be retained. The total value of i_0 or j_0 is then obtained by summation. As an example in the beam-and-slab bridge deck analysed previously, the torsional stiffness of the web, being non-continuous, was calculated using normal formulae, but the torsional stiffness of the top flange, being continuous, was taken as half of that calculated by these formulae. The total value of i_0 and j_0 in each of the two principal directions was then found by summation of the component stiffness. The interpolation formula $K_\alpha = K_0 + (K_1 - K_0)\sqrt{\alpha}$ could then be applied directly.

For non-thin sections the relaxation method is recommended, the torsional stiffness of the slab again being halved.

A multi-webbed box-section deck is a unique case and i_0 should be calculated by the single cell formula:

$$i_0 = \frac{4A^2}{p\left\{\dfrac{p-2t_3}{t_1} + \dfrac{p-2t_3}{t_2} + \dfrac{2(d-t_1-t_2)}{t_3}\right\}}$$

where A = the area of the hole (Fig. 19.13).

Bending Moments in Cross Beams

A very detailed method has been described in the Db.11 Publication, quoted earlier, to which the reader may refer.

However, the method detailed earlier in *'Analysis for Cross Beams'* (under Courbon's Method) is good enough for the design of cross beams and may be adopted instead.

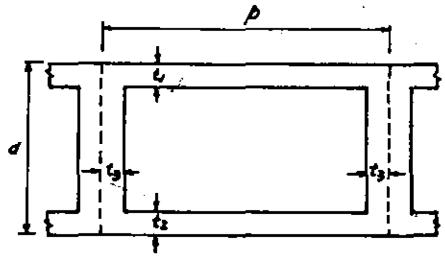


Fig. 19.13 *Nomenclature for torsional stiffness calculations of box section bridge beams*

19.6 GRILLAGE METHOD (FOR ESTIMATING THE APPORTIONMENT OF THE APPLIED LOAD* EFFECT IN THE LONGITUDINAL AND THE TRANSVERSE MEMBERS OF THE DECK)

The Grillage background has already been explained in Sec. 19.1 of this chapter, which may again be referred to before proceeding with this section.

Where a bridge deck is formed of a small number of cells, it is appropriate to place the longitudinal grillage members along the axes of the web members. Where there are sloping webs, the grillage members should be placed along the lines of intersection between the web and the top or bottom slab.

Excessive preoccupation with torsion might lead to the conclusion that longitudinal members would best be placed along the centre lines of the *cells*, but in fact it is better to place such members along the *web* axes to obtain the best representation of the transverse flexural characteristics of the deck. The torsional values included in the output are rarely directly applicable, in any case.

For a multi-celled deck which approaches the characteristics of an orthotropic plate in its behaviour, there is no need to align the grillage members with individual webs. Suggested guidelines for the selection of the number of longitudinal grillage members are that five or more members should be considered, and that the spacing of the longitudinal members should generally not exceed one-half of the width of a traffic lane, or one-and-a-half times the overall depth of the deck, whichever is lesser.

Transverse grillage members must be placed along the line of each diaphragm in a structure. Additional transverse members are also needed to reflect the load-sharing characteristics of the deck. The frequency of these members should be such that the differences between the analytical model and the real structure do not dominate the output, and thereby obscure interpretation. West has suggested that transverse members should be placed at

* Only those loads which appear after the 'grillage structural deck' commences to act as a 'grillage'.

Table 19.2 *Structural data*

Span	$2a$	80 ft.
Number of main beams	n	7
Main beam spacing	p	7 ft.
Effective width	$2b = np$	49 ft.
Number of cross-beams	m	6
Cross-beam spacing	q	16 ft.
Second moment of area of each main beam	I	720,251 in.^{4}
Distributed longitudinal stiffness	$i = I/p$	8,574 in.^{4}/in.
Second moment of area of each cross-beam	J	452,003 in.^{4}
Distributed transverse stiffness	$j = J/q$	2,354 in.^{4}/in.
Bending stiffness parameter	$\theta = \dfrac{b}{2a}\sqrt[4]{\dfrac{i}{j}}$	0.4230
Torsional stiffness of a main beam	I_0	13,329 in.^{4}
Distributed stiffness	$i_0 = \dfrac{I_0}{p}$	159 in.^{4}/in.
Torsional stiffness of a cross-beam	J_0	15,656 in.^{4}
Distributed stiffness	$j_0 = \dfrac{J_0}{q}$	82 in.^{4}/in.
Young's modulus	E	5×10^6 lb/sq. in.
Rigidity modulus	G	2×10^6 lb/sq. in.
Torsional parameter	$\alpha = \dfrac{G(i_0 + j_0)}{2E\sqrt{ij}}$	0.0107
		$(\sqrt{\alpha} = 0.1034)$

Actual beam positions in terms of effective width

Beam	1	2	3	4	5	6	7
	$\left(\dfrac{n-1}{n}\right)b$	$\left(\dfrac{n-3}{n}\right)b$	$\left(\dfrac{n-5}{n}\right)b$	$\left(\dfrac{n-7}{n}\right)b$	$\left(\dfrac{n-9}{n}\right)b$	$\left(\dfrac{n-11}{n}\right)b$	$\left(\dfrac{n-13}{n}\right)b$
Beam location	$-\dfrac{6}{7}b$	$-\dfrac{4}{7}b$	$-\dfrac{2}{7}b$	0	$+\dfrac{2}{7}b$	$+\dfrac{4}{7}b$	$+\dfrac{6}{7}b$

Table 19.3 *Applied loading*

Total vehicle load, w	180 tons
Number of wheels	16
Number of axles	4
Wheel load, P	11.25 tons
Total maximum longitudinal moment (on entire deck, all beams)	65.32×10^6 lb in.
Transverse wheel spacing	3 ft
• Eccentricity of vehicle centre of gravity with respect to effective width — 1st loading	0 ft
• Eccentricity of vehicle centre of gravity with respect to effective width — 2nd loading	12 ft

Equivalent loads λP at the nine standard positions — 1st loading

$-b$	$-\dfrac{3b}{4}$	$-\dfrac{b}{2}$	$-\dfrac{b}{4}$	0	$+\dfrac{b}{4}$	$+\dfrac{b}{2}$	$+\dfrac{3b}{4}$	$+b$
0	0	0	$0.978P$	$2.044P$	$0.978P$	0	0	0

Equivalent loads λP at the nine standard positions — 2nd loading

$-b$	$-\dfrac{3b}{4}$	$-\dfrac{b}{2}$	$-\dfrac{b}{4}$	0	$+\dfrac{b}{4}$	$+\dfrac{b}{2}$	$+\dfrac{3b}{4}$	$+b$
0	0	0	0	0	$1.061P$	$2.041P$	$0.898P$	0

Table 19.4 *Mean value of live load moment, M_{av}*

Average of the maximum longitudinal bending moment per beam : Mav.

$$= \frac{\text{total maximum longitudinal moment} : 65.32 \times 10^6}{\text{number of main beams} : 7} = 9{,}331{,}200 \text{ lb in.}$$

Table 19.5 *Beam-and-slab-bridge—unit load distribution coefficient K_0 ($\theta = 0.423,\ \alpha = 0$)*

Load position	Reference station									Row integral*
	$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$	
$-b$	4.35	3.39	2.37	1.55	0.69	0.10	−0.54	−1.06	−1.62	7.90
$-\frac{3b}{4}$	3.39	2.78	2.08	1.43	0.86	0.33	−0.17	−0.58	−1.06	7.90
$-\frac{b}{2}$	2.37	2.08	1.77	1.39	1.00	0.63	0.23	−0.17	−0.54	7.85
$-\frac{b}{4}$	1.55	1.43	1.39	1.30	1.14	0.92	0.63	0.33	0.10	7.96
0	0.69	0.86	1.00	1.14	1.23	1.14	1.00	0.86	0.69	7.95
$+\frac{b}{4}$	0.10	0.33	0.63	0.92	1.14	1.30	1.39	1.43	1.55	7.96
$+\frac{b}{2}$	−0.54	−0.17	0.23	0.63	1.00	1.39	1.77	2.08	2.37	7.85
$+\frac{3b}{4}$	−1.06	−0.58	−0.17	0.33	0.86	1.43	2.08	2.78	3.39	7.90
$+b$	−1.62	−1.06	−0.54	0.10	0.69	1.55	2.37	3.39	4.35	7.90

Table 19.6 *Beam-and-slab-bridge—unit load distribution coefficient K_1 ($\theta = 0.423, \alpha = 1$)*

Load position	Reference station									Row integral*
	$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$	
$-b$	1.91	1.59	1.31	1.08	0.90	0.75	0.64	0.55	0.47	7.99
$-\frac{3b}{4}$	1.59	1.44	1.28	1.10	0.95	0.82	0.71	0.63	0.55	7.99
$-\frac{b}{2}$	1.31	1.28	1.23	1.11	1.00	0.89	0.79	0.71	0.64	7.98
$-\frac{b}{4}$	1.08	1.10	1.11	1.11	1.06	0.97	0.89	0.82	0.75	7.98
0	0.90	0.95	1.00	1.06	1.08	1.06	1.00	0.95	0.90	8.01
$+\frac{b}{4}$	0.75	0.82	0.89	0.97	1.06	1.11	1.11	1.10	1.08	7.98
$+\frac{b}{2}$	0.64	0.71	0.79	0.89	1.00	1.11	1.23	1.28	1.31	7.98
$+\frac{3b}{4}$	0.55	0.63	0.71	0.82	0.95	1.10	1.28	1.44	1.59	7.99
$+b$	0.47	0.55	0.64	0.75	0.90	1.08	1.31	1.59	1.91	7.99

Table 19.7 *Beam-and-slab-bridge—unit load distribution coefficient K_α ($\theta = 0.423, \sqrt{\alpha} = 0.1034$), $K_\alpha = K_0 + (K_1 - K_0)\sqrt{\alpha}$*

Load position	Reference station									Row integral*
	$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$	
$-b$	4.10	3.20	2.26	1.50	0.71	0.17	−0.42	−0.89	−1.40	7.91
$-\frac{3b}{4}$	3.20	2.64	2.00	1.40	0.87	0.38	−0.08	−0.45	−0.89	7.92
$-\frac{b}{2}$	2.26	2.00	1.71	1.36	1.00	0.66	0.29	−0.08	−0.42	7.87

(Contd.)

* The row integral should theoretically be 8.00; since the values are taken from curves, however, some error is bound to occur and it is suggested that these values are typical of the tolerance which may be permitted.

$-\frac{b}{4}$	1.50	1.40	1.36	1.28	1.13	0.92	0.66	0.38	0.17	7.96
0	0.71	0.87	1.00	1.13	1.21	1.13	1.00	0.87	0.71	7.95
$+\frac{b}{4}$	0.17	0.38	0.66	0.92	1.13	1.28	1.36	1.40	1.50	7.96
$+\frac{b}{2}$	−0.42	−0.08	0.29	0.66	1.00	1.36	1.71	2.00	2.26	7.87
$+\frac{3b}{4}$	−0.89	−0.45	−0.08	0.38	0.87	1.40	2.00	2.64	3.20	7.92
$+b$	−1.40	−0.89	−0.42	0.17	0.71	1.50	2.26	3.20	4.10	7.91

Table 19.8 *Beam-and-slab-bridge—distribution coefficient K' for first loading*

Load position	Equivalent load multiplier λ from Table 19.3	Reference station								
		$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$
$-b$	0									
$-\frac{3b}{4}$	0									
$-\frac{b}{2}$	0									
$-\frac{b}{4}$	0.978	1.47	1.37	1.33	1.25	1.10	0.90	0.65	0.37	0.17
0	2.044	1.45	1.78	2.04	2.31	2.47	2.31	2.04	1.78	1.45
$+\frac{b}{4}$	0.978	0.17	0.37	0.65	0.90	1.10	1.25	1.33	1.37	1.47
$+\frac{b}{2}$	0									
$+\frac{3b}{4}$	0									
$+b$	0									
$\sum \lambda K\alpha$		3.09	3.52	4.02	4.46	4.67	4.46	4.02	3.52	3.09
	$K' = \frac{\sum \lambda K\alpha}{4}$	0.77	0.88	1.00	1.11	1.17	1.11	1.00	0.88	0.77

Note
(i) Throughout only two decimal places have been retained, as these give the desired degree of accuracy.
(ii) The denominator in the expression for K' is equal to the number of loads on the transverse section, i.e., 4 for the HB Loading under consideration.

Table 19.9 *Beam-and-slab-bridge—distribution coefficient K' for second loading*

Load position	Equivalent load multiplier λ from Table 19.3	Reference station								
		$-b$	$-\frac{3b}{4}$	$-\frac{b}{2}$	$-\frac{b}{4}$	0	$+\frac{b}{4}$	$+\frac{b}{2}$	$+\frac{3b}{4}$	$+b$
$-b$	0									
$-\frac{3b}{4}$	0									
$-\frac{b}{2}$	0									
$-\frac{b}{4}$	0									
0	0									

(Contd.)

$+\frac{b}{4}$	1.061	0.18	0.40	0.70	0.97	1.20	1.36	1.44	1.48	1.59
$+\frac{b}{2}$	2.041	−0.86	−0.16	0.59	1.35	2.04	2.77	3.49	4.08	4.62
$+\frac{3b}{4}$	0.898	−0.80	−0.40	−0.07	0.34	0.78	1.26	1.80	2.37	2.87
$+b$	0									
$\sum \lambda K \alpha$		−1.48	−0.16	1.22	2.66	4.02	5.39	6.73	7.93	9.08
$K' = \frac{\sum \lambda K \alpha}{4}$		−0.37	−0.04	0.30	0.66	1.00	1.40	1.68	1.98	2.27

Table 19.10 *Beam-and-slab-bridge — distribution coefficient at actual beam positions*

Beam	1	2	3	4	5	6	7
1st loading	0.83	0.97	1.10	1.17	1.10	0.97	0.83
2nd loading	−0.18	0.20	0.63	1.00	1.42	1.77	2.11

NOTE
Maximum bending moment is in beam 7, = 1.1 Mav. × 2.11 = 21,670,000 lb in.
(Mav. being the 'total' bending moment due to total live load on deck divided by the number of beams, Table 19.4).

intervals not exceeding twice the spacing of the longitudinal members. In any event, the adoption of less than five intermediate transverse members produces results in which the interpretation of the values at the intersection points of the grillage members could obscure the results.

Sectional Properties of Grillage Members

In assigning stiffnesses to the grillage members, it can appear for many decks that the arbitrary division of the deck into separate members by intersection lines at the mid-points between the members will give an adequate result. In the case of a deck having sloping webs, however, it is apparent that such an intersection line would result in the outer members having very low moments of inertia, with the position of the centroidal axis changing abruptly from one member to another, whereas it has been demonstrated that such changes in the level of the centroidal axis do not actually take place. Obviously the total stiffness should be correct, and this stiffness could be distributed in proportion to the respective areas of the members or purely as a matter of judgement.

Depending on the proportions of a structure, it may be necessary to take shear-lag (explained earlier) into account in assigning stiffnesses (effective section properties) to the grillage members, effective flange width in boxes, explained ahead.

The torsional stiffness is evaluated from the opposing shearing action of the top and bottom slabs, giving a torsional constant of one-half of that which arises from considering the closed section as a thin-walled box. This halving was found many years ago to best fit the experimental results, but no really satisfactory explanation has been given as to why this should be the case for a cellular deck when the full calculated figure is used for other forms of construction. The explanation* may be in the equal contributions of torsion in the members, and the adjusted shears in the intersecting members.

Incorporation of Skew

Because of the difficulties which arise in evaluating torsional parameters, the values obtainable for a grillage with members intersecting other than at right angles are subject to error. The skewed deck should, therefore, be represented by an orthogonal grillage, apart from the need to show the diaphragms in their real location.

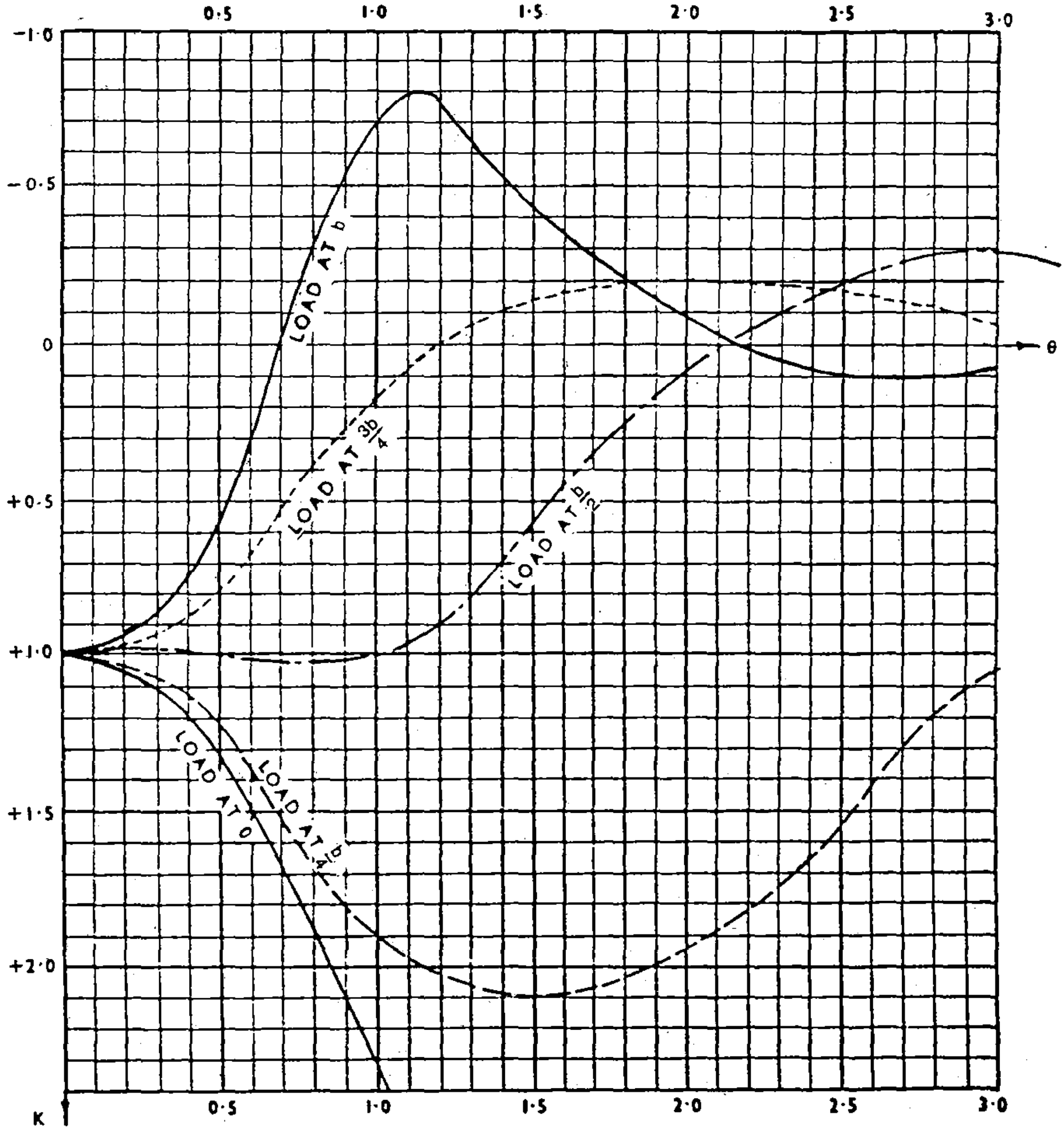
In many instances it is likely that the dominant difference in stiffness between the diaphragm and the other parts of the deck is the increased shearing stiffness. Although difficulties inevitably arise from overlapping members within a skewed deck, it is often sufficient to allow the orthogonal grillage members to represent the stiffness of the cellular section, and to superimpose on this a diaphragm — usually in a skewed direction — which is assigned a flexural stiffness equal to its own net dimensions, and a shear area equal to its actual cross-sectional area.

It is within a skewed deck that the accurate assessment of the torsional stiffness becomes more significant as otherwise the torsions introduced in the zones of skew support will not be evaluated realistically.

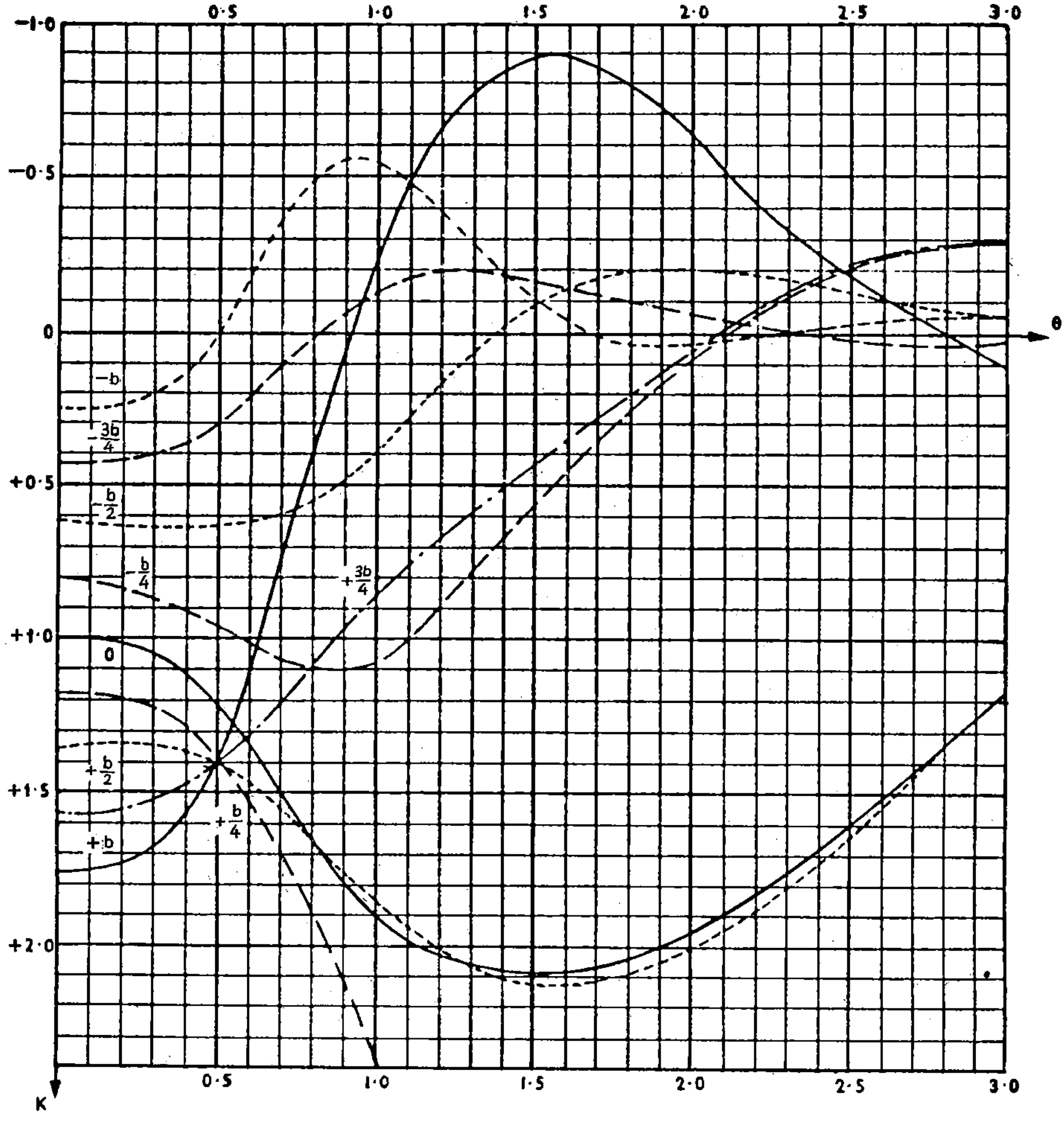
Variations in the plan geometry of a deck can introduce problems of tapering cells, which make the adoption of an orthogonal grillage impossible.

In a majority of cellular structures diaphragm members are contained within the depth of the deck. The force

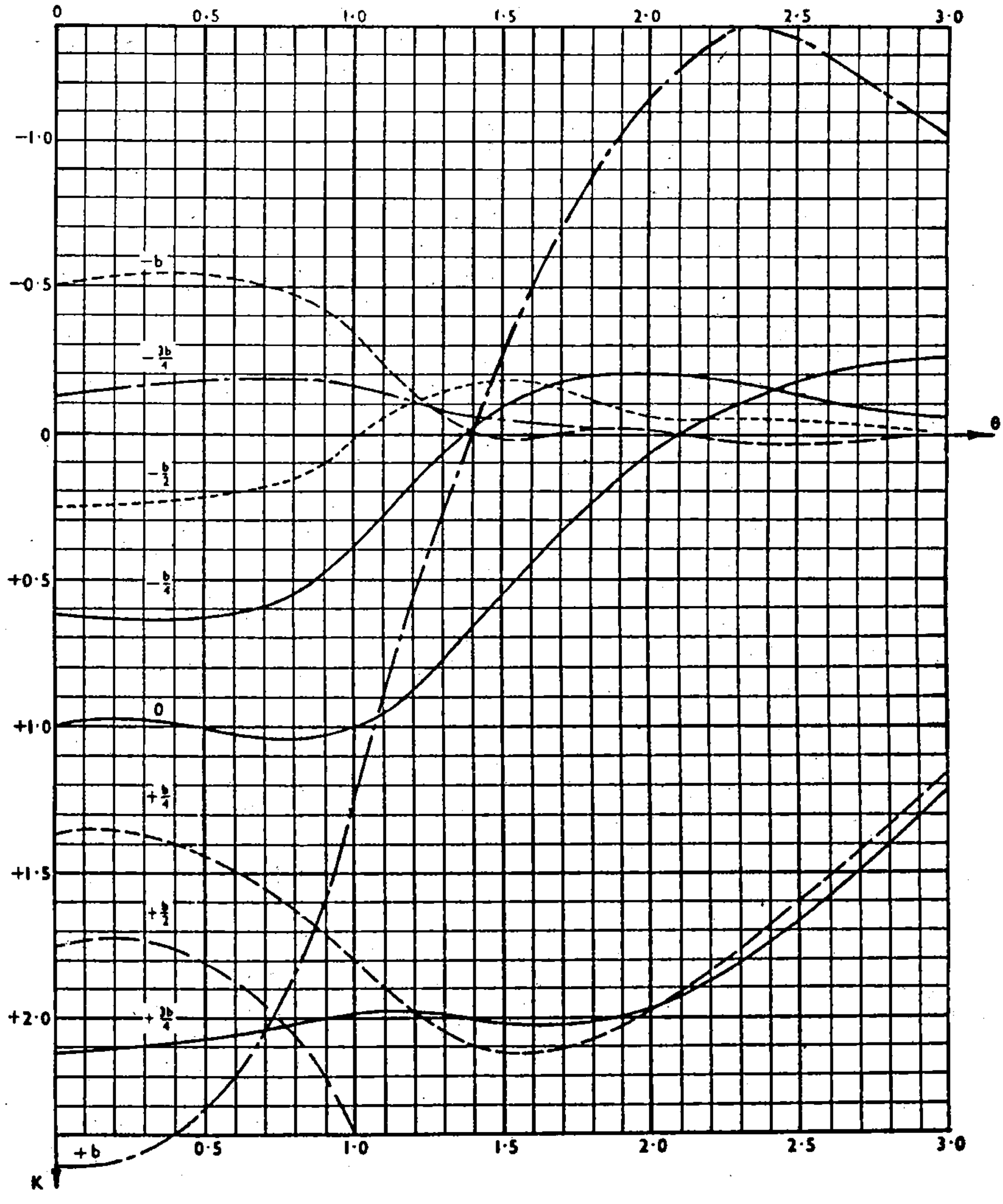
* Also refer to the explanation for 'Torsional Stiffness' in Section 19.5 earlier.



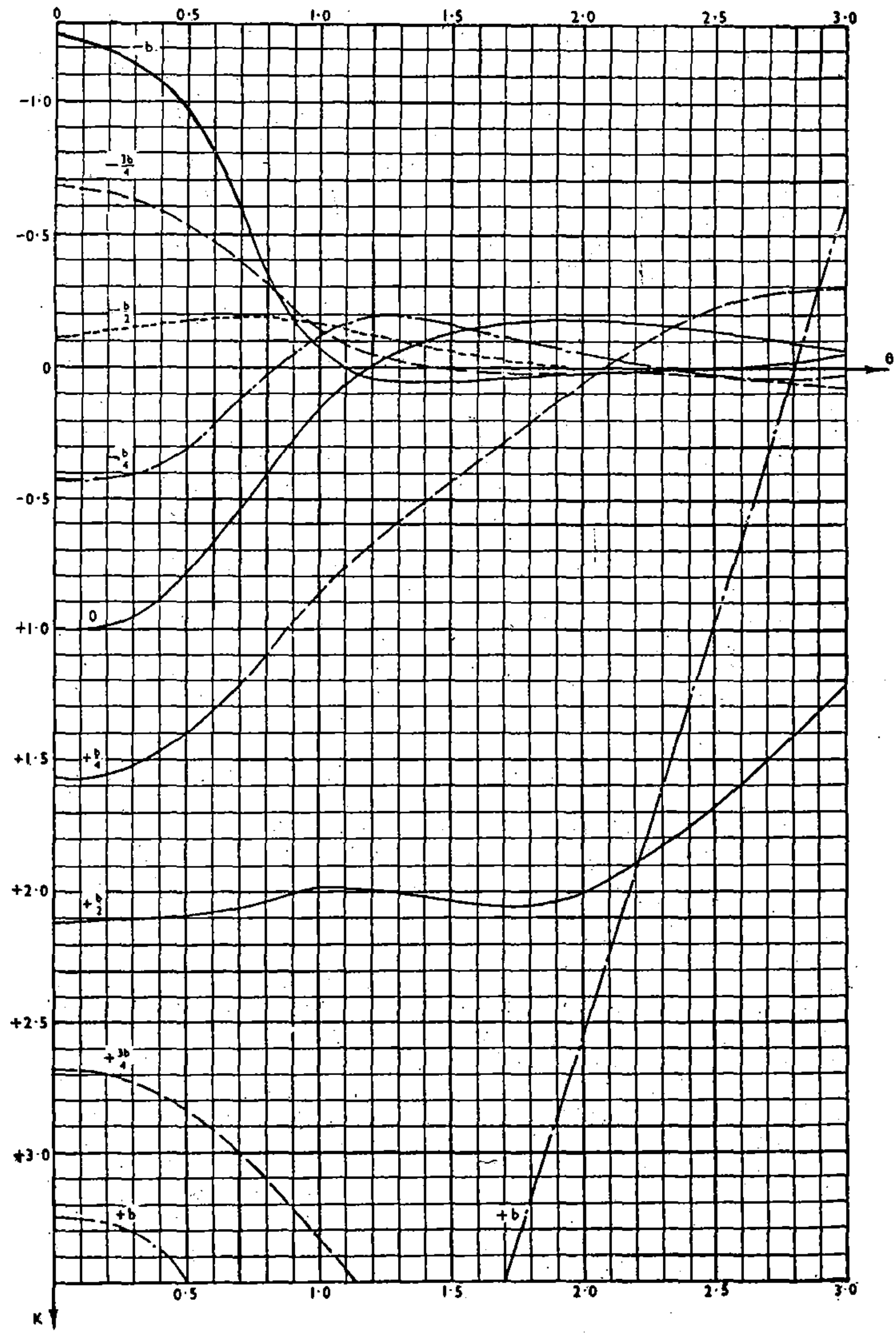
Graph 19.1 *Distribution coefficients K_0 at reference station 0 for various load eccentricities*



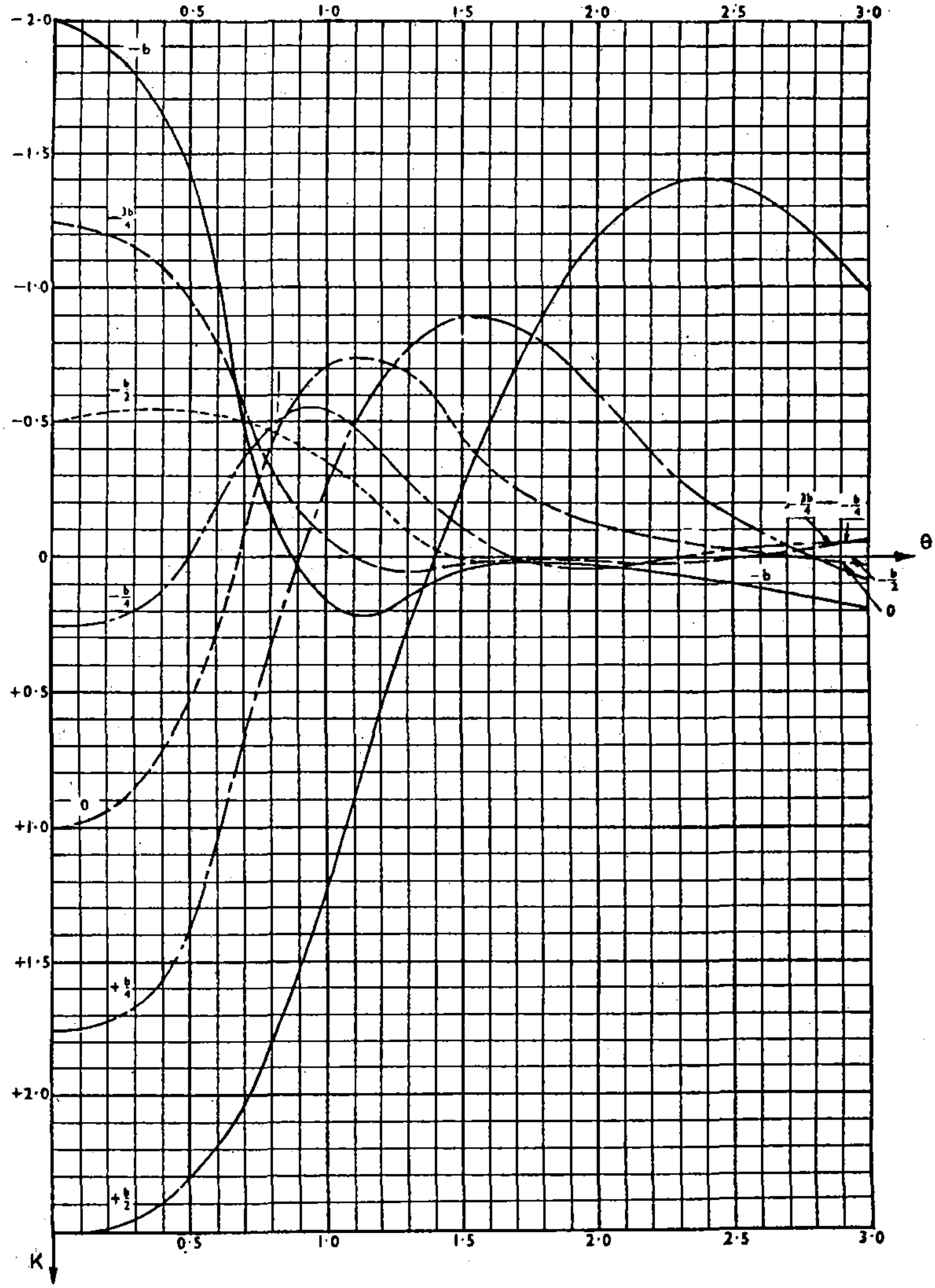
Graph 19.2 *Distribution coefficients* K_0 *at reference station* $\frac{b}{4}$ *for various load eccentricities*



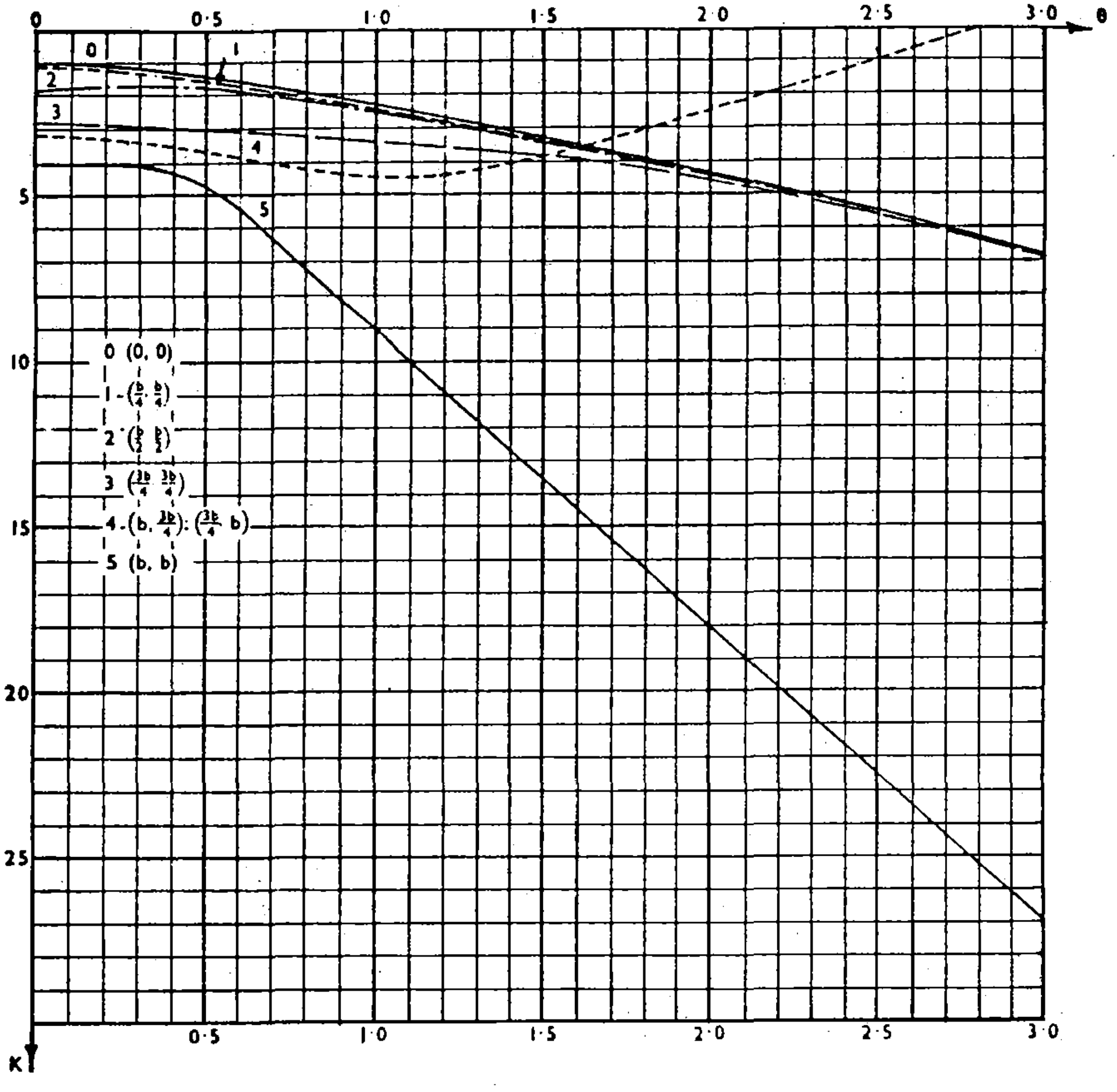
Graph 19.3 *Distribution coefficients K_0 at reference station $\frac{b}{2}$ for various load eccentricities*



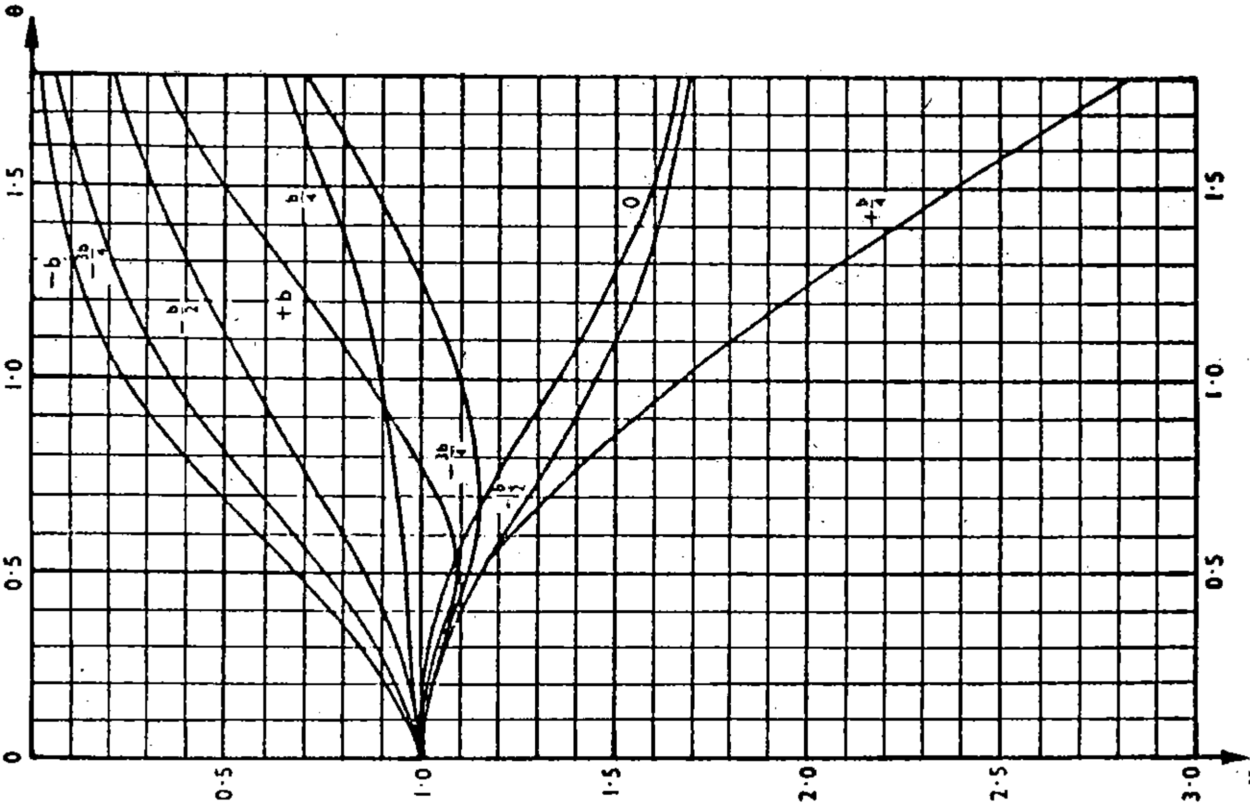
Graph 19.4 *Distribution coefficients K_0 at reference station $\frac{3b}{4}$ for various load eccentricities*



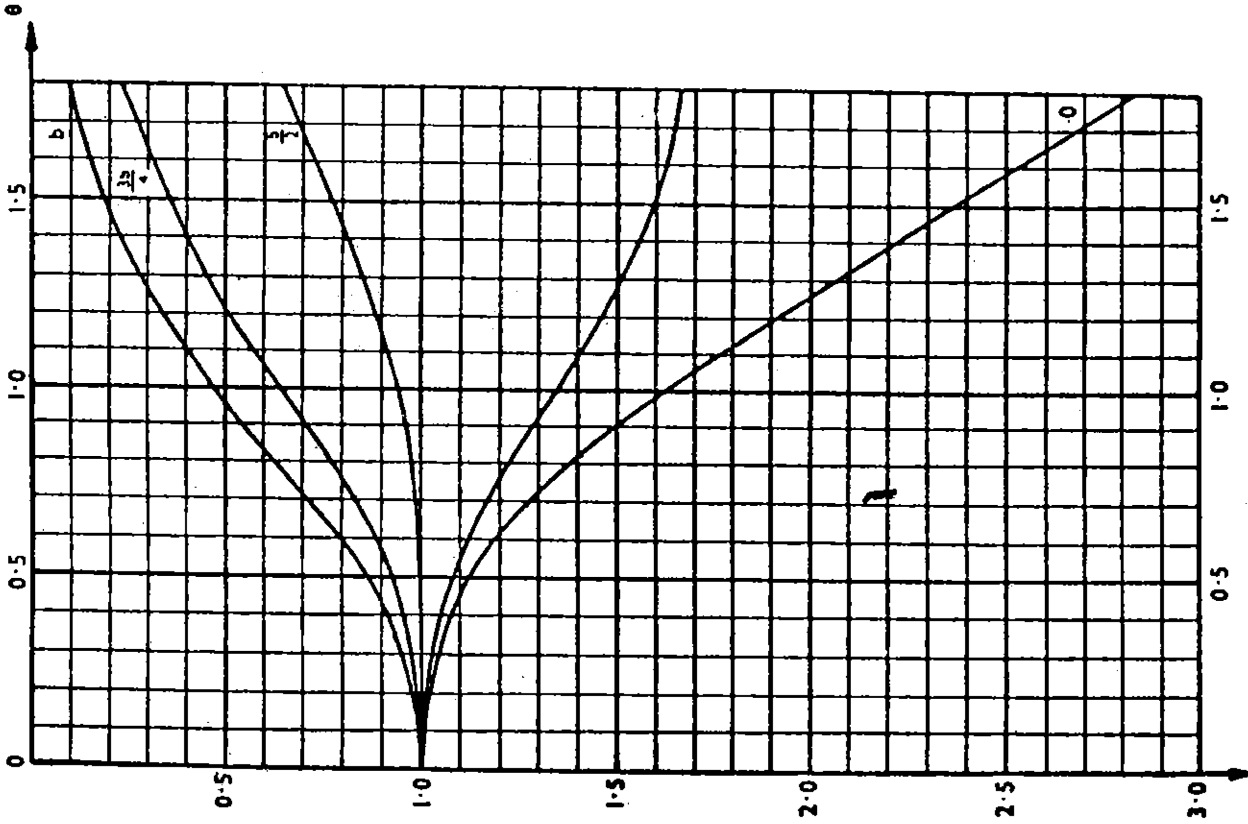
Graph 19.5 *Distribution coefficients K_0 at reference station b for various load eccentricities*



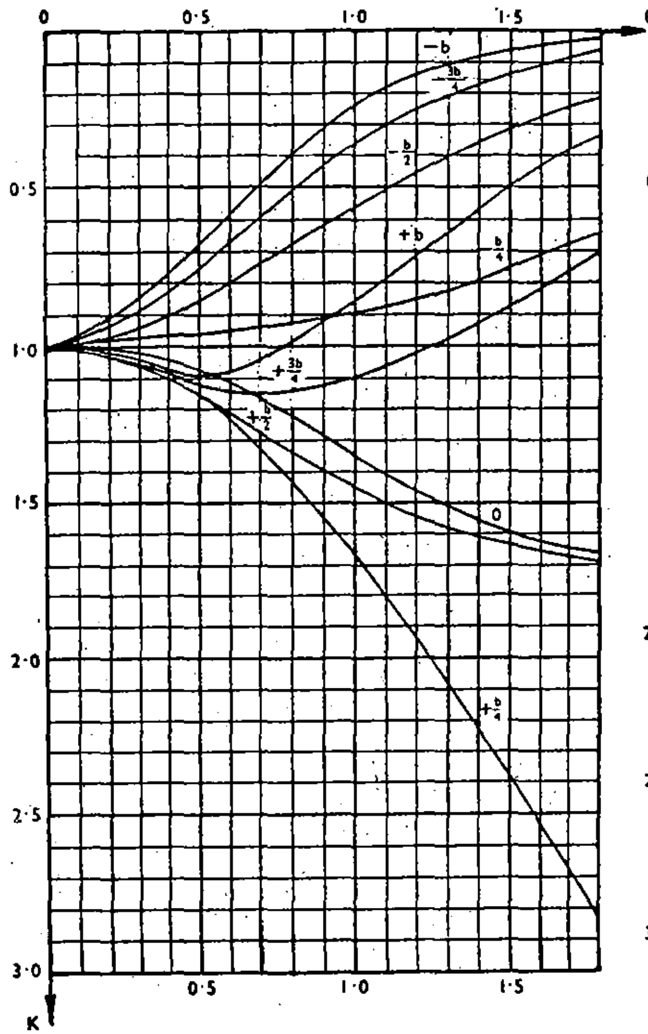
Graph 19.6 *Large range distribution coefficients K_0*



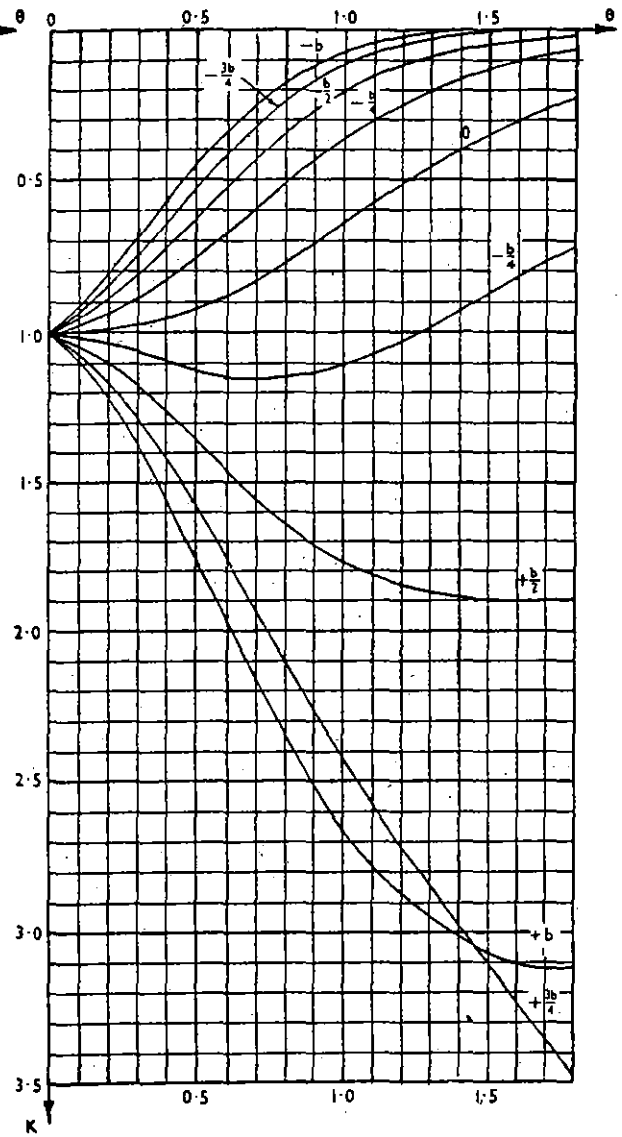
Graph 19.8 Distribution coefficients K_1 at reference station $\frac{b}{4}$ for various load eccentricities



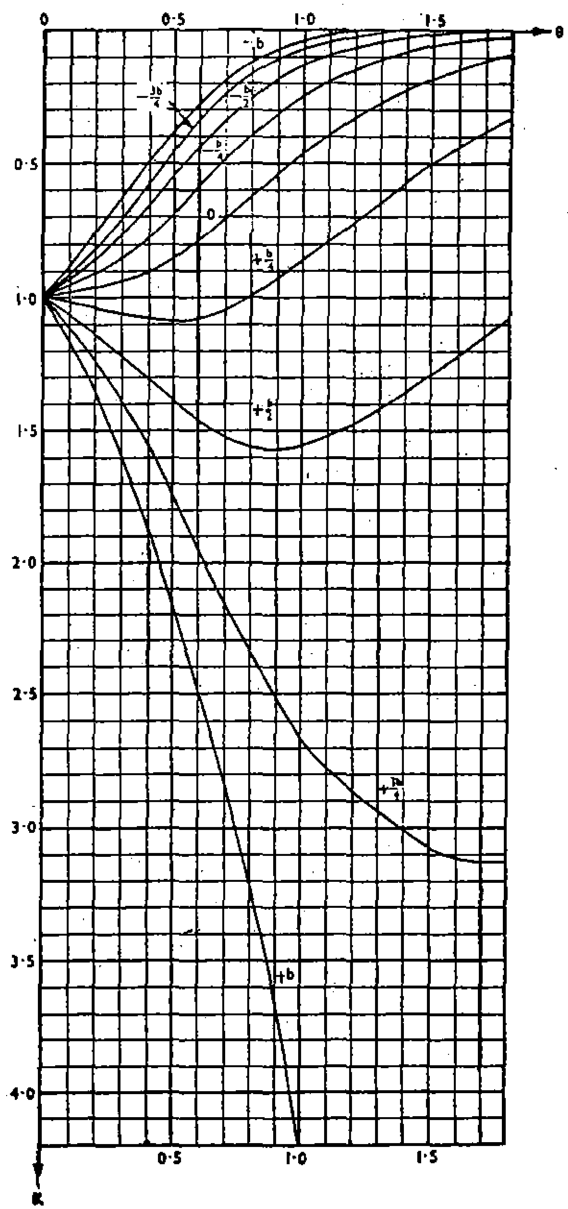
Graph 19.7 Distribution coefficients K_1 at reference station 0 for various load eccentricities



Graph 19.9 *Distribution coefficients K_1 at reference station $\frac{b}{2}$ for various load eccentricities.*



Graph 19.10 *Distribution coefficients K_1 at reference station $\frac{3b}{4}$ for various load eccentricities*



Graph 19.11 *Distribution coefficients K_1 at reference station b for various load eccentricities*

systems resulting from the grillage at the intersections between the diaphragm and the web members are not necessarily directly applicable to these members themselves. They represent the force field in the total deck and, in most instances, the web part of the diaphragm members will not be significantly affected by the torsions, which will be resisted by the top and bottom slabs. Any attempt to design an intersection between such members for the torsion which a grillage analysis implies is transmitted through the connection is, therefore, a misdirected effort.

Loading Cases, and Discretising the Grillage Elements

In any structure which is prestressed, the forces developed by the prestressing force are intended to oppose those arising from applied loading. Whenever the geometry of the deck is such that it will induce a complex system of forces, the intensity of these resultant forces in the real structure will still be reduced by the opposition between the prestress and the applied loading. It is, therefore, of fundamental importance to include the prestress as a loading case in any analysis.

In bridge decks having simple geometry it can be sufficient to use the grillage merely to get a picture of the distribution of moments across the width of the deck, and then to resort to continuous beam calculations to obtain the actual design figures.

In many instances the optimum value can be obtained from a grillage analysis by applying axle loads at intervals along the span as individual loading cases, which then form the basis of influence lines as it were. The benefit to be obtained from this is that any local effects can be more closely identified and evaluated.

In all cases it is recommended that the loads should (as far as possible) be applied to the longitudinal members forming the grillage, so that the transverse members reflect the forces arising from distortion, without adding the complication of the local effects.* This, however, is almost impossible in practice. In practice it becomes necessary to locally distribute to the longitudinal as well as to the transverse grillage elements those of the loads (and moments) that physically do not directly sit on them. Consequently, in such cases, it then becomes essential to combine the effects of the global (grillage) analysis with the local wheel-load-effects. This summation of the two effects is uncalled for where the grillage-grid is made small (additional longitudinal beam-elements are incorporated in between the physical longitudinal web members) so that the wheel loads nearly always fall *on* the grillage members rather than *between* them.

As a first step in a grillage analysis, the continuum of the deck must be idealized into a series of discrete elements. These elements are connected at joints (nodes) and it is at these joints that restraints to movement, i.e., supports or fixity, and loads, can be applied. Restraints may be applied at any joint. The members framing into a joint can be at any angle. It is thus possible to analyse a deck with any support conditions — simply supported, or skew. With all the available computer programs† it is possible to include some or all of the restraints as *elastic restraints*, thus simulating for instance the rubber-bearing deformation or the elastic shortening of support columns. It has been found from the analysis of experimental work that quite small movements of supports can have pronounced effects upon the moments and reactions in the slab. It is therefore desirable to have, in the grillage simulation, a true representation of the whole structure, including the supports.

An additional use of the option to apply elastic rotation restraints exists in a continuous structure. Rather than idealizing all the spans as grillage beams, the restraining effect of the deck which is more than one span away from the position at which moments are required can be simulated by an *elastic rotation restraint*.

Application of Loads to the Grillage

Computer programs vary regarding the types of load that it is possible to apply to the structure. All will permit the application of point loads and moments at the joints and some will allow point loads, distributed loads and moments on the members. Since any member-loading can be replaced by point loads and moments at the ends of the member, it is possible to apply any form of loading with any of the programs (but it is in estimating this distribution that the first approximation enters the computation, the other approximations come in while idealising the deck into the grillage and fixing the section properties of its members).

However, be that as it may, when a bridge deck, loaded with uniformly distributed loads or with a vehicle, is being analysed, it is sufficiently accurate to consider the wheel loads as *point loads* acting at the joints, e.g., for a point load acting within a quadrilateral formed by grillage members (Fig. 19.14), consider it statically proportioned to a pair of opposite members, then in the same way from these members to the joints as point loads.

* A load falling on slab in between the longitudinal and the transverse members of the grillage will first create a *local wheel load effect* by deforming the slab, and only then will it get 'conveyed' to the surrounding grillage members. There would be no such local effect if only there was a grillage member directly under the load.

† There are many computer programs in the market, commercially available (some are even suitable for personal computers, e.g., the Graphical Interactive Finite Element Total System — GIFTS/CASA. Tucson. AZ, USA. etc.). Three of the more powerful ones are the IBM-STRUDL (M.I.T.) program, the C and CA (London) program, and the LEAP program.

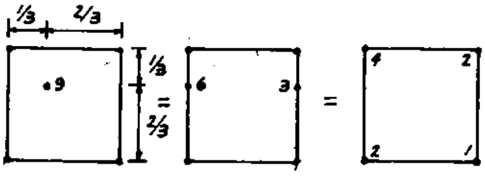


Fig. 19.14 *Distribution of load from a panel to the surrounding nodes*

Interpretation of Results From the Computer Output

The computer output should contain first a summary of the structure and also of the loadings applied. In programs where this is an option, it is advisable to request it in order to again check the input data. This is followed by lists of deflections and rotations at the joints and by shears and moments at the beam ends.

(i) *Bending moments* When examining the longitudinal and transverse bending moments, the user should bear in mind all the time the sign convention used by the program, which will be fully explained in the manual relating to the particular program. Where a grillage beam continues across a joint, the values of moment from end 2 of one member and end 1 of the continuation member will be different. If the two moments are of the same 'sense', the signs will be opposite. The method of dealing with these moments depends upon the actual structure. Where all the members meeting at the node are physical beams, there will be a genuine 'step' in the bending moments at this point and the actual values, output from the program, should be used. This also applies if the longitudinal grillage beams replace more than one physical beam. This method will always cause a slight over-estimate of the moment, because with every deck there will be some continuous slab present. If any of the grillage beams are hypothetical and represent sections of slab, the two moments may be averaged, since in the slab structure no step would occur in its moment diagram.

(ii) *Shear and reactions* Shear at any node should be evaluated from the output results in the same manner as the bending moments. If the reactions are not printed automatically by the program, they may be evaluated by summing the shear forces at the supported node.

(iii) *Twisting moments* The way in which twisting moments are catered for depends upon the type of deck under consideration. Full details are always given in the manual (*the user's manual*).

19.7 TRANSVERSE DISTRIBUTION OF LIVE LOAD IN BOX SECTIONS

A box section (i.e., a closed cell section) has better flexural and torsional properties than an open cell section, other things being equal. Therefore the applied load is distributed much more efficiently (and hence evenly) among the longitudinals of a box (single or multi-cell type, or even in multiple boxes that are interconnected by a common top-slab).

While diaphragms (cross girders or cross beams) in concrete boxes are necessary above their supports and at their ends, they are not required in between because concrete boxes always tend to be relatively more thick-walled (compared to the thin-walled steel box sections). However, intermediate diaphragms may be provided even in concrete boxes if the diaphragm spacing exceeds about 45 m., unless either a thorough (finite element) analysis or a model test is carried out or there is a successful precedence of a similar construction in the past.

In the usually adopted types of the single-cell concrete box decks, the applied load may be assumed to be shared equally by their longitudinal members. However, in the multi-cell concrete box decks the method outlined in Sec. 19.3 may be followed for estimating the apportionment of the applied load effect among the outer and inner longitudinal members. Alternatively, the total live load effect on the box (moment, shear and reactions) may be increased by the factor to which otherwise the heaviest apportioned longitudinal is to be designed, and then the total box section designed for this increased load effect.

Transverse Distribution (of Applied Load) among Various Boxes in the Decks Composed of Individual Boxes that are Interconnected by a Common Top-Slab and by the Diaphragms at the Ends and Over the Supports

Wide decks can conveniently be composed of two or three (and even more) separate boxes which are transversely interconnected by a common top-slab (and by the diaphragms at the ends and over the supports). Podolny and Muller, in their *Construction and Design of Prestressed Concrete Segmental Bridges* (John Wiley and Sons) report basically as follows on this subject:

A. A detailed analysis was made of such decks with regard to the distribution of live load between the various boxes. It was found that in normal structures of this type the combined effect of the flexural rigidity of the roadway slab acting transversely as a rigid frame with the webs and bottom slab of the various box girders, on one hand, and the torsional rigidity of such box girders on the other hand, would result in a very satisfactory transverse distribution of live loads between box girders. There is no need for diaphragms between girders as normally provided for I-girder bridges.

B. Comprehensive programs of load testing of several bridges, including accurate measurements of deflections for eccentric loading, fully confirmed the results of theoretical analysis. This analysis has been reported in various technical documents, and only selected results are presented here.

C. The first bridge analysed in this respect was the Choisy-le-Roi Bridge. A knife-edge load P is considered with a uniform longitudinal distribution along the span, Fig. 19.15. When this load travels crosswise from kerb to kerb, each position may be analysed with respect to the proportion of vertical load carried by each box girder, together with the corresponding torsional moment and transverse moment in the deck slab. These analyses have made it possible to draw transverse influence lines for each effect considered, such as longitudinal bending moments (over the support or at midspan), torsional moments, or transverse moments.

D. For the longitudinal moments it is convenient to use a dimensionless coefficient, Fig. 19.15(c), which represents the increase or decrease of the load carried by one box girder in comparison with the average load assuming an even distribution between both girders. Numerical results show that the transverse distribution of a knife-edge load placed on one side (next to the kerb) of a twin box deck produces bending moments in each box that are 1.4 and 0.6 times the average bending moment, i.e., 1.4 ($^1/_2$ Mp) and 0.6 ($^1/_2$ Mp), i.e., 0.7 and 0.3 Mp, where Mp = the bending moment due to full load P on the total deck of both boxes, and alternatively this would mean *Box Reaction-Factors* of 0.7 and 0.3 (operated on the effect of full P.) For the same configuration, a typical deck with 4 T-girders would have Beam Reaction-Factors of $0.7P$ and $0.4P$ for the two beams on loadside, meaning that these 2 beams together would take $1.1P$ effect which is very high compared to $0.7P$ that a two-webbed box is taking.

E. There are however, two side effects to such encouraging behaviour, which relate to torsion stresses and transverse bending of the deck slab.

F. *Torsional moments in the box girder* An unsymmetrical distribution of live loads in the transverse direction tends to warp the box girders and cause warping shear stresses but they are low in concrete boxes and it is the high torsional rigidity of boxes which produces a favourable distribution of loads in them. However, the maximum torsional moments usually occur when only one-half the structure (in cross-section) is loaded, and the resulting stresses do not cumulate with the shear stresses produced by the full live-load placement for governing shear force.

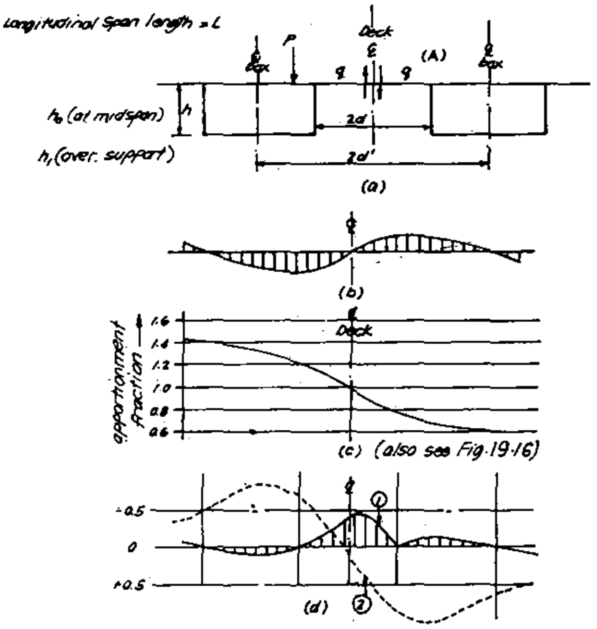


Fig. 19.15 *Principle of transverse distribution of loads between box-girders, (a) Dimensions, (b) Influence line of the shear in the connecting slab, (c) Transverse influence line of longitudinal moment, (d) Transverse bending influence line at section A*

G. *Transverse Moments in the Deck Slab* The deck slab cannot be considered as a 'continuous beam on fixed supports' because of the relative displacements of the two boxes due to unsymmetrical loading. Figure 19.15(d) shows the consequence. If the slab were resting on fixed supports, the influence line for the moment in a section such as, A in Fig. 19.15(a), would be the typical line 1, in Fig. 19.15(d). Because the box girders undergo certain deflections and rotations, the effect is to superimpose the coordinates of another line such as 2 on line 1.

Numerically, the difference is not as great as may be expected at the first sight, because line 1 pertains to the effect of local concentrated truck loads, while line 2, being the result of differential movements between box girders, pertains to the effect of uniformly distributed loads. In summary, deck slab moments may be increased by only 20 to 30% over their normal values if flexibility of the box girders is ignored.* As a matter of practical interest, the actual numerical values for several bridges in France with

* However, it should be noted that owing to the actual rotational behaviour of these boxes, sagging moments can develop in the top-slab at its supporting webs. For more details refer to Ch. 31.

two box girders, that have all shown excellent performance for more than 15 years so far, are presented in Fig. 19.16.

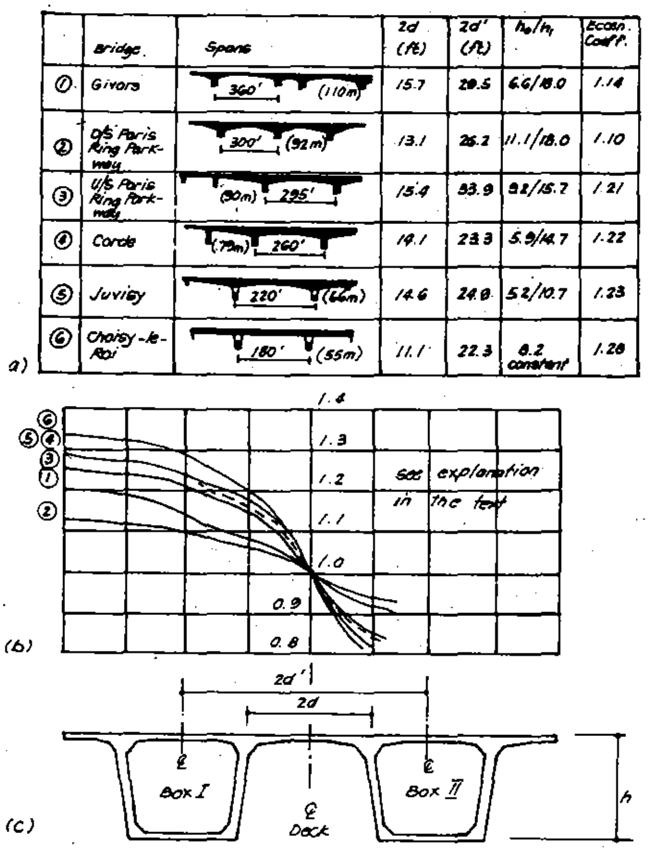


Fig. 19.16 *Transverse distribution of loads between box girders, numerical values for several two-box girders marked 1 to 6*

Explanation of the Influence Lines Shown in Fig. 19.16(b) and Fig. 19.15(c)

These Influence Lines show the *apportionment fraction* of applied load-effect in a box beam for various transverse placements of the load [in bridges 1 to 6 shown in Fig. 19.16(a)].

Example If load P is placed concentrically on deck (i.e., at zero eccentricity wrt ℄ of deck section), then the apportionment factor reads 1.0, meaning that each box takes as its share, $1.0 \times (P/2) = 0.5P$. However, if P is placed on ℄ of Box I (in case of Bridge 2 for example) then its Box I takes $1.1 \times (P/2)$, leaving the Box II to take the balance of $0.9 \times (P/2)$; and so on. The *transverse distribution* is somewhat like what simple statics would give—but not so severe, as can be noticed.

The reader may also refer to Ch. 31 (the analysis and design of certain types of deck cross-sections) and compare the above with the remarks given there regarding the transverse load distribution among the two boxes in a deck of the type shown in Fig. 19.16(c).

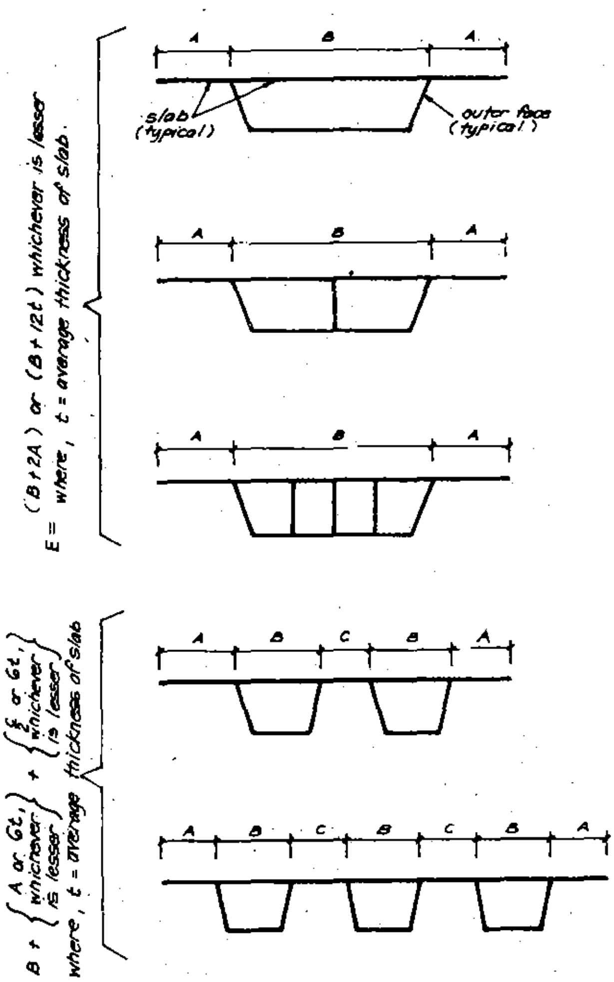


Fig. 19.17

Effective Flexural Section Properties of Box Sections

Owing to shear lag effect (explained earlier) the effective width of the top flange, particularly at and near supports, and the effective width of the compression flange, can turn out to be less than the physical width actually contributing to dead load of the structure. Various codes of practice give various formulae for estimating this effective width and the practising professional knows that some of these impressive looking formulae give values that are only as accurate (or inaccurate) as the very assumptions they are based on (refer Fig. 19.1). On the basis of various box girders actually designed (all of which are serving successfully), some of which were even either actually test-loaded or model studied. Fig. 19.17 gives simple but a good approach for working out this effective flange width E in various types of practically dimensioned box sections. In this, full-depth or almost full-depth diaphragms are assumed at ends and all supports.

REFERENCES

1. Massonnet, C., *Method of Calculation of Bridges with Several Longitudinal Beams Taking Into Account Their Torsional Resistance*. vol. 10, Publications, Internatl. Assn. for Bridges and Structural Engrg., 1950, pp. 147–182.
2. Rowe, R.E., *Concrete Bridge Design*, Applied Science Publishers, London 1962, p. 372.
3. Morice, P.B. and G., Little, *The Analysis of Right Bridge Decks Subjected to Abnormal Loading*, Cement and Concrete Association, Db 11. Publication No. 46.002., London, July 1956, p. 43.
4. West, R., *Recommendation on the use of Grillage Analaysis for Slab and Pseudo-slab Bridge Decks*, Wexham Springs, Cement and Concrete Association, Advisory Publication 46.017, 1973, p. 24.
5. Morice, P.B., Little, G. and Rowe, R.E. *Design Curves for the Effects of Concentrated Loads on Concrete*, Cement and Concrete Association. Publication No. 42.202, London, August 1955. p. 29.
6. Hambly, E.C., *Bridge Deck Behaviour*, Chapman Hall, London, 1976. p. 272.
7. Hambly E.C. and E. Pennells, "Grillage Analysis Applied to Cellular Bridge Decks", *The Structural Engineer*, vol. 53, No.7, July 1975, pp. 267-275.
8. Cusens, A.R., "Load Distribution in Concrete Bridge Decks." Construction Industry Research and Information Association, *Report 53*, London, December 1974, p. 38.
9. Cusens, A.R., and Pama, R.P., *Bridge Deck Analysis*, Wiley-Interscience, London, 1974, p. 278.
10. Best, B.C., *Methods of Analysis for Slab-type Structures*, Construction Industry Research and Information Association, *Technical Note 62*, London, December 1974, p. 96.
11. Maisel, B.I. and R.A. Swann. The Design of Concrete Box Spine-beam Bridges, Construction Industry Research and Information Association, *Report 52*, London, November 1974, p. 52.
12. Maisel, B.I. Rowe R.E. and R.A. Swann, "Concrete box girder bridges," *The Structural Engineer*, vol. 51, No. 10. October 1973, pp. 363–376, *Discussion:* vol. 52, no. 7, July 1974, p. 257–272.
13. Robinson, K.E., "The Behaviour of Simply Supported Skew Bridge Slabs Under Concentrated Loads", Cement and Concrete Association, *Research Report 8*, Publication No. 41.008., London, November 1959, p. 184.
14. Mehmel, A. and H. Weise Model Investigations on Skew Slabs on Elastically Yielding Point Supports, Cement and Concrete Association, *Library Translation 13*, Publication No. 61.123., London, December 1963, p. 88.
15. Rusch, H. and A. Hergenroder, *Influence Surfaces for Moments in Skew Slabs*, Cement and Concrete Association, Publication No. 624.073, London, 1964, p. 174.
16. Chapman, J.C. and J.S. Teraszkiewics, Research on Composite Construction at Imperial College, *Proceedings of the Conference on Steel Bridges*, British Constructional Steelwork Association, London, 1968, pp. 49–58.
17. Jones, L.L. and R.H. Wood, *Yield-line Analysis of Slabs*. Thames and Hudson/Chatto and Windus, London, 1967, p. 405.
18. Terrington, J.S., *Combined Bending and Torsion of Beams Girders*, British Constructional Steelwork Association, Publication No. 31., London, 1970, p. 132.
19. Sawko, F., Recent Developments in the Analysis of Steel Bridges using Electronic Computers, *Proceedings of the Conference on Steel Bridges*, British Constructional Steelwork Association, London, 1968, pp. 39–48.
20. Structural Engg. Research Centre, Roorkee, India: *Hand-Book for Prestressed Concrete Bridges*.
21. Scordelis, A.C., "Analysis of Simply Supported Box Girder Bridges". *Structures and Materials Research Report*, Division of Structural Engineering and Structural Mechanics, Department of Civil Engineering, University of California, Berkeley, SESM 66–17, Oct. 1966, pp. 1–120.
22. Duberg, J.E., N. Khachaturian, and R.E. Fradinger, "Method for Analysis of Multibeam Bridges", *Journal of the Structural Div.*, ASCE, vol. 86. no. ST7, Proc. Paper 2552, July 1960, pp. 109–138.
23. Spindel, J.E., "A Study of Bridge Slabs Having No Transverse Flexural Stiffness", *Thesis*, University of London, London, England 1961, in partial fulfilment of the requirements for the PhD degree.
24. Pama, R.P. and A.R. Cusens, "Edge Beam Stiffening of Multibeam Bridges," *Journal of the Structural Div.*, ASCE.
25. Roesli, A., "Lateral Load Distribution in Multibeam Bridges", *Prestressed Concrete Bridge Members Progress Report*, no. 10, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pa., July 1955.
26. Scordelis, A.C., Davis, R.E., K.S. Lo, "Load Distribution in Concrete Box Girder Bridges", *1st Intnl. Symp. on Design of Concrete Bridges*, ACT Special Publication No. SP-23, (1967).
27. Stevens, L.K., K.B. Gosbel, "Model Studies on a Beam-and-slab Concrete Bridge under Concentrated Loadings", *1st Intnl. Symp. on Design of Concrete Bridges*, ACI Special Publication No.: SP-23, 1967.

CHAPTER 20

Practical Structural Analysis

20.1 AIM

The final aim being to 'design' the structure (i.e., to decide on the basis of allowable material stresses the section sizes, concrete strengths and amounts and detailing of the 'untensioned' and 'tensioned' reinforcement), the first step obviously is to estimate the load-effects that can be caused at each critical section in the structure under design loads. These load effects are more commonly referred to as bending moment (BM), shear force (SF), thrust (N), torsion (T), reaction (R), deflection (Y) and rotation (θ). Estimation of these load effects, in simple words, is called *structural analysis*, and the methods employed for this analysis are the tools of the structural analyst. The aim, therefore, is first to know these tools, then use them to carry out the structural analysis in a practical and workman-like manner in order to do the *structural design*. (This is then followed by the preparation of *working drawings*, from which are prepared the *Bill of Quantities* of each item of work, which then is 'priced' to estimate the cost of the structure.)

This chapter deals briefly with structural analysis tools in simple language without trying to mystify the reader into numbness as it were, because classical theories of structural analysis can be profusely mathematical and mind-boggling. While the reader is free to go into these realms of mathematical gymnastics (for a deeper understanding of the subject), the aim of this chapter is only to assist him into practical application of the tools in day-to-day practice where results are required relatively quickly by the pressures of work and one has little desire to end up as a 'frustrated engineer turned mathematician.''

NOTE While certain other aspects of structural analysis and various aspects of structural design have been covered in their respective contexts in some of the other chapters of this book, the author presupposes that the reader is conversant with and up-to-date on the mundane matters of structural analysis and design. Certain facets may, therefore, look like already treaded paths in which case they are referenced merely as quick refreshers.

20.2 STRUCTURAL ANALYSIS—FUNDAMENTAL CONCEPTS

The bending moments and shearing forces on freely-supported beams and simple cantilevers are readily determined from simple statical rules but the solution of continuous beams and statically-indeterminate frames is more complex. Until fairly recently the techniques of structural analysis required to solve such problems were presented and employed as independent self-contained methods, the relationships between them being ignored or considered relatively unimportant. The choice of method used depended on its suitability to the type of problem concerned and also to some extent on its appeal to the particular designer involved.

Recently, the underlying inter-relationships between various analytical methods have become clearer. It is now realized that there are two basic types of method—*flexibility methods* (otherwise known as action methods, compatibility methods or force methods) where the behaviour of the structure is considered in terms of unknown forces, and *displacement methods* (otherwise known as stiffness methods or equilibrium methods) where the behaviour is considered in terms of unknown displacements. In each case, the complete solution consists of combining a particular solution, obtained by modifying the structure to make it statically determinate and then analysing it, with a complementary solution, in which the effects of each individual modification are determined. For example, for a continuous-beam system, with flexibility methods, the particular solution involves removing the redundant actions (*i.e.*, the continuity between the individual members) to leave a series of disconnected spans: with displacement methods the particular solution involves violating joint equilibrium by restricting the rotation and/or displacement that would otherwise occur at the joints.

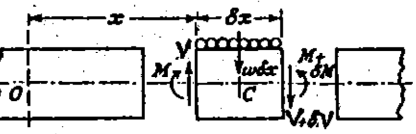
To clarify further the basic differences between the types of method, consider a propped cantilever: With the flexibility approach the procedure is to first remove the prop and calculate the deflection at the position of the prop due to the action of the load only: this gives the particular solution. Next, calculate the concentrated load that must be applied at the prop position to achieve an equal and opposite deflection: this is the complementary solution. The force obtained is the reaction in the prop and, when this is known, all the moments and forces in the propped cantilever can be calculated by combining the two solutions. If displacement methods are used, the span is considered fixed at both supports and the resulting moment acting at the end at which the prop occurs is found: this is the particular solution. The next step is to release this support and determine the moment that must then be applied at the pinned end of the cantilever to negate the fixing moment. Lastly, by summing both resulting moment diagrams the final moments are obtained and the reactions can be calculated.

In practical problems there are a number of unknowns and, irrespective of the method of solution adopted, the preparation and solution of a series of simultaneous equations is normally necessary. Whichever basic method of analysis is employed, the resulting relationship between forces and displacements embodies a series of coefficients which can be set out concisely in matrix form. If flexibility methods are used the resulting *flexibility matrix* is built up of *flexibility coefficients*, each of which represents a displacement produced by a unit action. Similarly, stiffness methods lead to the preparation of a *stiffness matrix* formed of *stiffness coefficients*, each of which represents an action produced by a unit displacement.

The solution of matrix equations, either by inverting the matrix or by a systematic elimination procedure, is ideally handled by machine aid and to this end, methods have been devised (so-called matrix stiffness and matrix flexibility methods) by means of which, not only are the necessary equations solved by computer but the machine is also used to set them up.

Relations between w, V and M

Figure 20.1 shows a short length δx imagined to be a slice cut out from a loaded beam at a distance x from a fixed origin O



w = uniform load, V = shear force
M = bending moment

Fig. 20.1

Let the shearing force at the section x be V, and at $x+\delta x$ be $V+\delta V$. Similarly, the bending moment is M at x, and $M+\delta M$ at $x+\delta x$. If w is the mean rate of loading on the length δx, the total load is $w\delta x$, acting approximately (exactly, if uniformly distributed) through the centre C. The element must be in equilibrium under the action of these forces and couples, and the following equations are obtained:

Taking moments about C.

$$M + V\delta x/2 + (V+\delta V)\delta x/2 = M + \delta M$$

Neglecting the product $\delta V \delta x$, and taking the limit, gives

$$V = dM/dx \qquad (20.1)$$

Resolving vertically.

$$w\delta x + V + \delta V = V$$

or
$$w = -dV/dx \qquad (20.2)$$
$$= -d^2M/dx^2 \text{ from Eq. (20.1)} \qquad (20.3)$$

From Eq. (20.1) it can be seen that, if M is varying continuously, zero shearing force corresponds to maximum or minimum bending moment, the latter usually indicating the greatest value of negative bending moment. It will be seen later, however, that 'peaks' in the bending moment diagram frequently occur at concentrated loads or reactions, and are not then given by $V = dM/dx = 0$, although they may represent the greatest bending moment on the beam. Consequently it is not always sufficient to investigate the points of zero shearing force when determining the maximum bending moment.

At a point on the beam where the type of bending is changing from sagging to hogging, the bending moment must be zero, and this is called a *point of inflection* or *contraflexure*.

By integrating Eq. (20.1) between two values of $x = a$ and b, then

$$M_b - M_a = \int_a^b V dx$$

showing that the increase in bending moment between two sections is given by the area under the shearing force diagram.

Similarly, integrating Eq. (20.2),

$$V_a - V_b = \int_a^b w dx$$

= the area under the load distribution diagram.
Integrating Eq. (20.3) gives

$$M_a - M_b = \int \int_a^b w dx dx$$

These relations prove very valuable when the rate of loading cannot be expressed in an algebraic form, and provide a means of graphical solution.

Deflection by Calculus

With usual symbols, the general equation of bending is,

$$M/EI = 1/R \tag{20.4}$$

and in terms of co-ordinates x and y.

$$\frac{1}{R} = \frac{\pm d^2y/dx^2}{[1+(dy/dx)^2]^{3/2}} \tag{20.5}$$

where the sign depends on the convention for axes. For beams met with in engineering practice the slope dy/dx is everywhere small, and may be neglected in comparison with 1 in the denominator.

Taking Y positive upwards, under the action of a positive bending moment the curvature of the beam is as shown in Fig. 20.2. It can be seen that dy/dx is increasing as x increases, i.e., d^2y/dx^2 is positive, and $1/R = d^2y/dx^2$ from Eq. (20.5).

Hence $\quad M/EI = d^2y/dx^2$ from Eq. (20.4)

or $$EId^2y/dx^2 = M \tag{20.6}$$

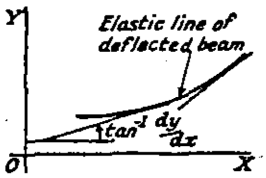


Fig. 20.2

Provided M can be expressed as a function of x, Eq. (20.6) can be integrated to give the slope dy/dx, and the deflection y, of the beam for any value of x. Two constants of integration will be involved, and these can be obtained by substituting known values of slope or deflection at particular points. A mathematical expression is thus obtained for the form of the deflected beam, or *elastic lines*.

- *Notes on application*
  - (i) Take the X axis through the level of the supports
  - (ii) Take the origin at one end, or at a point of zero slope
  - (iii) For a built-in or fixed end, or where the deflection is a maximum, the slope $dy/dx = 0$
  - (iv) For points on the X axis, usually supports, the deflection $y = 0$

Differentiating Eq. (20.6)

$$EId^3y/dx^3 = dM/dx = V$$

and $$EId^4y/dx^4 = dV/dx = -w$$

These forms are of use in some cases, though generally the bending moment relation is the most convenient.

EXAMPLE Obtain expressions for the maximum slope and deflection of a cantilever of length l carrying (a) a concentrated load W at its free end, (b) a uniformly distributed load w along its whole length.

(a) Taking the origin at the free end, the X axis through the fixed end, then at a distance x from the origin $M = -Wx$ (Fig. 20.3) and

$$EId^2y/dx^2 = M = -Wx \text{ from Eq. (20.6)}$$

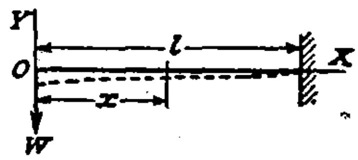


Fig. 20.3

Integrating $\quad EIdy/dx = -Wx^2/2 + A$

But $dy/dx = 0$ at $x = l$

$\therefore \quad A = Wl^2/2$

Integrate again

$$EIy = -Wx^3/6 + Wl^2x/2 + B$$

At $x = l, y = 0$

$$\therefore \quad B = Wl^3/6 - Wl^3/2 = -Wl^3/3$$

The slope and deflection at the free end (where they are a maximum) are given by the values of dy/dx and y when $x = 0$. i.e.,

$$\text{slope} = Wl^2/2EI$$

deflection $= -Wl^3/3EI$ (indicating downward). The deflected shape is shown dotted, Fig. 20.3.

(b) $EId^2y/dx^2 = M = -wx^2/2$ (Fig. 20.4)

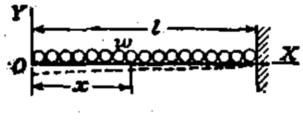


Fig. 20.4

Integrating $EIdy/dx = -wx^3/6 + A$

when $x = l, dy/dx = 0$

$\therefore \quad A = wl^3/6$

Integrating,

$EIy = -wx^4/24 + (wl^3/6)x + B$

when $x = l, y = 0$

$\therefore \qquad B = -wl^4/8$

Putting $x = 0$, maximum slope $= wl^3/6EI$
and maximum deflection $= -wl^4/8EI$

Macaulay's Method

In applying the above method normally a separate expression for bending moment must be obtained for each section of the beam between adjacent concentrated loads or reactions, each producing a different equation with its own constants of integration. It will be appreciated that in any but the simplest cases the work involved will be laborious, the separate equations being linked together by equating slopes and deflections given by the expressions on either side of each 'junction' point. However, a method devised by Macaulay enables one continuous expression for bending moment to be obtained, and provided certain rules are followed the constants of integration will be the same for all sections of the beam.

It is advisable to deal with the different types of loading separately.

(i) *Concentrated loads:* Measuring x from one end, write down an expression for the bending moment in the last section of the beam, enclosing all distances less than x in square brackets, i.e.,

$$EId^2y/dx^2 = M = -W_1x + R[x-a] - W_2[x-b] - W_3[x-c] \qquad \text{(Fig. 20.5)}$$

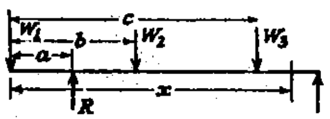


Fig. 20.5

Subject to the condition that all terms for which the quantity inside the square brackets is negative are omitted (i.e., given a value zero), this expression may be said to represent the bending moment for all values of x. If x is less than c the last term is omitted, if x is less than b then both the last two terms are omitted, and so on.

The brackets are to be integrated as a whole, i.e.,

$$EIdy/dx = -W_1x^2/2 + (R/2)[x-a]^2 - (W_2/2)[x-b]^2 - (W_3/2)[x-c]^2 + A$$

$$EIy = -W_1x^3/6 + (R/6)[x-a]^3 - (W_2/6)[x-b]^3 - (W_3/6)[x-c]^3 + Ax + B$$

By so doing it can be shown that the constants of integration are common to all sections of the beam, e.g., if $x = b - \Delta$

$$EIdy/dx = -(W_1/2)(b-\Delta)^2 + (R/2)(b-\Delta-a)^2 + A$$

and $EIy = -(W_1/6)(b-\Delta)^3 + (R/6)(b-\Delta-a)^3 + A(b-\Delta) + B$

and if $x = b + \Delta$

$$EIdy/dx = -(W_1/2)(b+\Delta)^2 + (R/2)(b+\Delta-a)^2 - (W_2/2)\Delta^2 + A'$$

and

$$EIy = -(W_1/6)(b-\Delta)^3 + (R/6)(b+\Delta-a)^3 - (W_2/6)\Delta^3 + A'(b+\Delta) + B'$$

Now as $\Delta \to 0$ these slope and deflection values must correspond (i.e., at $x = b$), from which it is seen that $A = A'$ and $B = B'$.

The values of A and B are found as before.

(ii) *Uniformly distributed loads:* Supposing a load w is stretched from a distance a to a distance b from one end (Fig. 20.6). Then in order to obtain an expression for the bending moment at a distance x from the end, which will apply for all values of x, it is necessary to continue the loading up to the section x, compensating with an equal negative load from b to x, i.e.,

$$M = Rx - (w/2)[x-a]^2 + (w/2)[x-b]^2$$

each length of loading acting at its centre of gravity, square brackets being interpreted as before.

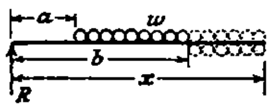


Fig. 20.6

For $x > a$ but $< b$, omit $[x-b]$, and $M = Rx - (w/2)(x-a)^2$, which is clearly correct.

The remaining steps of integration and constant enumeration are as before.

(iii) *Concentrated bending moment:* As shown in Fig. 20.7, write

$$EId^2y/dx^2 = M = -Rx + M_0[x-a]^0$$

then $\quad EIdy/dx = -Rx^2/2 + M_0[x-a] + A$, etc.

EXAMPLE A simply supported beam of length L carries a load W at a distance a from one end, b from the other $(a > b)$. Find the position and magnitude of the maximum deflection and show that the position is always within $L/13$, approximately, of the centre.

The maximum deflection (i.e., zero slope) will occur on the length a, since $a > b$.

Taking the axes as shown in Fig. 20.8,

$$EId^2y/dx^2 = M = (Wb/L)x - W[x-a]$$

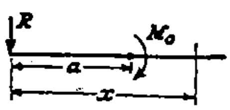


Fig. 20.7

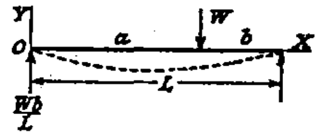


Fig. 20.8

$EI dy/dx = (Wb/L)(x^2/2) - (W/2)[x-a]^2 + A$ (i)
$EIy = (Wb/L)(x^3/6) - (W/6)[x-a]^3 + Ax + B$ (ii)
At $x = 0, y = 0,\ \therefore B = 0$
At $x = L, y = 0,\ \therefore AL = -(Wb/L)(L^3/6) + (W/6)b^3$
giving $\quad A = -(Wb/6L)(L^2 - b^2)$
$dy/dx = 0$ at a value of x given by
$(Wb/L)(x^2/2) - (Wb/6L)(L^2 - b^2) = 0$, from (i), omitting $[x-a]$ since $x < a$ for zero slope when $a > b$.

This gives $x = \sqrt{[(L^2 - b^2)/3]}$ at the point of maximum deflection.

Substituting in (ii) to find the value of the maximum deflection:

$$EIy = \frac{Wb}{L}\frac{(L^2-b^2)^{3/2}}{6\times 3\sqrt{3}} - \frac{Wb}{6L}\frac{(L^2-b^2)^{3/2}}{\sqrt{3}}$$

giving $y = -\dfrac{Wb(L^2-b^2)^{3/2}}{(9\sqrt{3})EIL}$

Distance of point of maximum deflection from centre $= \sqrt{[(L^2-b^2)/3]} - L/2$
which has a maximum value of $L/\sqrt{3} - L/2$, or approximately $L/13$.

Bending Moments and Shearing Forces: Basic Data

Basic Relationships

At any section:

(i) Shearing force

$$V = \sum\begin{bmatrix}\text{loads and reactions on}\\ \text{one side of section}\end{bmatrix} = \begin{matrix}\text{rate of}\\ \text{change of } M\end{matrix}$$

(ii) Bending moment

$$M = \sum\begin{bmatrix}\text{moments of loads and reactions}\\ \text{on one side of section}\end{bmatrix}$$
$$= \text{rate of change of } EI\theta$$

(iii) Slope

$$\theta = \int\frac{M}{EI} = \text{rate of change of } a \text{ or } y$$

(iv) Deflection (elastic)

$$y = a = \int\theta$$

where a or y = deflection
I = moment of inertia of member at section
E = elastic modulus of material

Material Constants

(i) Poisson's ratio $= 1/m$ = ratio of lateral strain to longitudinal strain, produced by a single stress.

(ii) Bulk Modulus $K = \dfrac{\text{fluid pressure } p}{\text{volumetric strain}}$, fluid pressure symbolising that three dimensional state of stress (equal in all directions) which causes the volumetric strain.

(iii) $E = 3K(1-2/m)$ = Young's Modulus of Elasticity

(iv) $U = p^2/2K$ per unit volume, U being the strain energy per unit volume

(v) $E = 9CK/(C+3K)$, C being the shear modulus of rigidity (shear stress/shear strain), also written as G.

(vi) $E = 2C(1 + 1/m)$

Deflection due to Shear

It is well known that shear stress is set up on transverse sections of a beam, and the accompanying shear strain will cause a distortion of the cross-section, and, since the shear stress varies from zero at the extreme fibres to a maximum at the neutral axis, cross-sections can no longer remain plane after bending.

In fact the warping will be of the form shown in Fig. 20.9, the left-hand view being for positive shear and the right-hand for negative shear. These strains are incompatible with the theory of pure bending, but nevertheless a good approximation to the deflection due to shear can be obtained by strain energy methods. It should also be noted that the shear distribution near to the application point of a concentrated load must differ considerably from that given by the theory since there can be no sudden change of shear strain from one type to the other, as would be implied for a simply supported beam with a central load, across the load position.

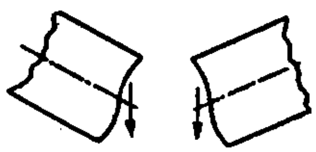


Fig. 20.9

Strain energy due to shear $= (s^2/2C)\times$ volume
where s = shear stress and C as defined earlier.

For the whole beam.

$$U_s = (1/2C)\int\int s^2 dA dx \qquad (20.7)$$

where dA is an element of cross-section and dx an element of length.

The integration can only be performed for particular cross-sections over which the variation of s is known, and rectangular and I-sections will be dealt with below.

Rectangular Section

From the usual formula: $s = \dfrac{VA\bar{Y}}{Ib}$ (shear stress) it follows that, $s = (6V/bd^3)(d^2/4 - y^2)$ where y is the distance from the neutral axis, $dA = bdy$, then

$$U_s = \frac{1}{2C}\int\left[\int_{-d/2}^{d/2}\frac{36V^2}{b^2d^6}\left(\frac{d^4}{16}-\frac{d^2y^2}{2}+y^4\right)bdy\right]dx$$

from Eq. (20.7)

$$= \frac{1}{2C}\int\frac{36V^2}{bd^6}\left[\frac{d^4y}{16}-\frac{d^2y^3}{6}+\frac{y^5}{5}\right]_{-d/2}^{d/2}dx$$

$$= \frac{18}{Cbd}\int V^2 2\left(\frac{1}{32}-\frac{1}{48}+\frac{1}{160}\right)dx$$

$$= \frac{3}{5Cbd}\int_0^l V^2 dx \qquad (20.8)$$

Cantilever with Load W at Free End

$$V = W$$

$$\therefore U_s = \frac{3W^2l}{5Cbd} \text{ from Eq. (20.8)}$$

But $U_s = 1/2W\delta_s$, where δ_s is the deflection due to shear
$\therefore \quad \delta_s = 6Wl/5Cbd$

Cantilever with Uniformly Distributed Load The load $w\delta x$, on a length δx at a distance x from the fixed end, treated as a concentrated load, will produce a deflection due to shear $= (6w\delta x x)/5Cbd$ at this point. For this load alone the distortion produced is indicated in Fig. 20.10, being uniform shear force over the length x and zero over $l - x$, hence the total deflection due to shear for all the distributed load.

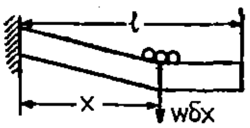


Fig. 20.10

$$= \int_0^l \frac{6wxdx}{5Cbd}$$
$$= 3wl^2/5Cbd$$

Simply Supported Beam with Central Load W

$$V = \pm W/2$$

$$U_s = \int_0^l \frac{3(W^2/4)}{5Cbd}dx \text{ from Eq. (20.8)}$$

$$= 3W^2l/20Cbd$$
$$= 1/2W\delta_s$$
$$\therefore \quad \delta_s = 3Wl/10Cbd$$

The 'simplified' deflection is as shown in the upper diagram of Fig. 20.11 and since the shearing force is constant over each half, this case is equivalent to a cantilever of length $l/2$ carrying an end load of $W/2$.

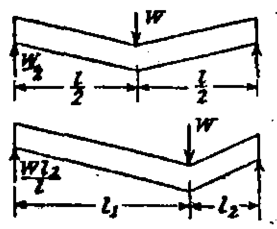


Fig. 20.11

If the load is not centrally applied, but divides the length into l_1 and l_2, then treating either section as a cantilever with an end load equal to the reaction on that side.

$$\delta_s = \frac{6(Wl_2/l)l_1}{5Cbd}$$

$= 6Wl_1l_2/5Cbdl$ under the load (Fig. 20.11)

Simply Supported Beam with Uniformly Distributed Load Due to a load $w\delta x$ only, at a distance x from one end $(x < l/2)$, the shear deflection at the load $= 6w\delta x(l - x)x/5Cbdl$ just proved.

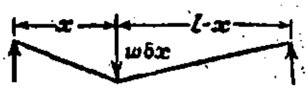


Fig. 20.12

By proportion, the deflection at the centre of the beam

$$= \frac{6w\delta x(l-x)}{5Cbdl}x\left(\frac{l/2}{l-x}\right) \text{ (Fig. 20.12)}$$

Then the total central deflection due to shear

$$= 2\int_0^{l/2} \frac{3wx\,dx}{5Cbd}$$
$$= 3wl^2/20Cbd$$

I-Section

Treating the shearing force as uniformly distributed over the web area bd,

$$s = V/bd \text{ and } \int dA = bd$$
$$\therefore \quad U_s = (1/2C)\int (V^2/b^2d^2)bd\,dx \text{ from Eq. (20.7)}$$
$$= \left(\int V^2dx\right)/2Cbd \qquad (20.9)$$

By methods similar to those employed for a rectangular section the deflections due to shear may be obtained as follows:

- Cantilever with end load, $\delta_s = Wl/Cbd$
- Cantilever with distributed load, $\delta_s = Wl/2Cbd$
- Simply supported beam with central load, $\delta_s = Wl/4Cbd$
- Simply supported beam with distributed load, $\delta_s = Wl/8Cbd$

The strain energy method known as *'Castigliano's Theorem'* (described ahead) may be used where a number of loads exist concurrently, or to find the deflection due to a distributed load by imposing a concentrated load at the deflection point and later giving it a value zero (i.e., $\delta_s = (\partial U_s/\partial P)_{P=0}$.

EXAMPLE For a given cantilever of rectangular cross-section, length l, and depth d, show that, if δ_s and δ_b are the deflections due to shear and bending due to a concentrated load at the free end, $\delta_s/\delta_b = k(d/l)^2$, and find the value of k for steel. $E = 30 \times 10^6$ lb/sq.in.; $C = 11.5 \times 10^6$ lb/sq.in.

Hence find the least value of l/d if the deflection due to shear is not to exceed 1% of the total.

It has been shown that,

$$\delta_s = 6Wl/5Cbd$$
and $$\delta_b = Wl^3/3EI = 4Wl^3/Ebd^3$$

for a rectangular section.

$$\delta_s/\delta_b = [6/(5 \times 4)](E/C)(d/l)^2 = k(d/l)^2$$
where $$k = (3/10)(E/C) = (3/10)(30/11.5)$$
$$= 0.783$$
If $\delta_s/(\delta_b + \delta_s) = 0.01$
$$\therefore \quad \delta_s/\delta_b = 0.01/0.99$$
and $\quad \equiv 0.783(d/l)^2$ from above,

i.e., Least value of $l/d = \sqrt{(0.783 \times 99)}$
$= 8.8$, which is an important conclusion for practice.

EXAMPLE A 10-in. by 6-in. RSJ with web 0.4 in., flanges 0.7 in. thick, acts as a horizontal cantilever 12ft. long and carries a load of 2 tons at 6 ft from the end. Assuming the shear force is carried by the web and is uniformly distributed, calculate the deflection at the end. $E = 12,800$ tons/sq.in.; $C = 5000$ tons/sq. in.

$$I = (6 \times 10^3 - 5.6 \times 8.6^3)/12 = 204 \text{ in.}^4$$

By the moment-area method (see later) the end deflection due to bending,

$$= \left(\frac{1}{2} \times 12 \times 6\right) \frac{10 \times 1728}{12,800 \times 204} = 0.238 \text{ in. (Fig. 20.13)}$$

Deflection due to shear at the load is given by

$$\frac{Wl}{Cbd} = \frac{2 \times 6 \times 12}{5000 \times 0.4 \times 8.6} = 0.0084 \text{ in.}$$

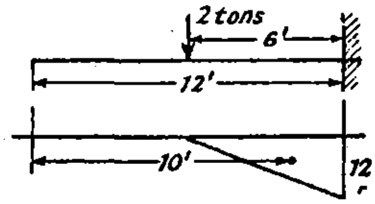


Fig. 20.13

But since the shearing force is zero beyond the load this is also the deflection due to shear at the free end (see also Fig. 20.10).

$$\text{Combined deflection at free end} = 0.238 + 0.0084$$
$$= 0.2464 \text{ in.}$$

Theories of Failure

The theory of elasticity and formulae derived are based on the assumption that the material obeys Hooke's law. Consequently no information can be derived from them if the material has passed beyond its elastic limit at any point in the member. In fact, when permanent (non-recoverable) deformations occur the material is said to have *failed*. Note that failure does not imply rupture or collapse.

It is natural to consider that in a simple tensile test, the elastic limit is associated with a certain value of the tensile stress; but at this stage other quantities, such as shear stress and strain energy, also attain definite values, and any one of these may be the deciding factor in the physical cause of failure.

In a complex stress system these quantities can be calculated from the known stresses and material constants, and the problem is to decide which quantity is the criterion of failure, i.e. the cause of the material passing beyond its elastic limit and taking up a permanent set. Having decided, the actual value of that particular factor which corresponds to the onset of failure is usually taken to be the value it reaches in the simple tension case at the elastic limit.

The principal theories of failure are outlined in detail below, in which f is the tensile stress at the elastic limit in simple tension, and f_1, f_2, f_3 the principal stresses in any complex system.

(i) *Maximum Principal Stress Theory (due to Rankine).* According to this theory failure will occur when the maximum principal stress in the complex system reaches the value of the maximum stress at the elastic limit in simple tension, i.e.,

$$f_1 = \frac{1}{2}(f_x + f_y) + \frac{1}{2}\sqrt{[(f_x - f_y)^2 + 4s^2]}$$

$$f_2 \text{ being } \frac{1}{2}(f_x + f_y) - \frac{1}{2}\sqrt{[(f_x - f_y)^2 + 4s^2]}$$

$= f$ in simple tension

where f_x, f_y and s are the stresses on given planes in the complex system.

(ii) *Maximum Shear Stress or Stress Difference Theory (due to Guest and Tresca)* This implies that failure will occur when the maximum shear stress q in the complex system reaches the value of the maximum shear stress in simple tension at the elastic limit. i.e.

$$q = \frac{1}{2}(f_2 - f_1) = \frac{1}{2}\sqrt{[(f_x - f_y)^2 + 4s^2]}$$

on the assumption that the maximum shear is greatest in the XY plane $= \frac{1}{2}f$ in simple tension

$$\text{or} \qquad f_2 - f_1 = f$$

(iii) *Strain Energy Theory (due to Haigh)*: This theory is based on the argument that as the strains are reversible up to the elastic limit, the energy absorbed by the material should be a single-valued function at failure, independent of the stress system causing it, i.e. strain energy per unit volume causing failure is equal to the strain energy at the elastic limit in simple tension

$$(1/2E)[f_1^2 + f_2^2 + f_3^2 - (2/m)(f_1f_2 + f_2f_3 + f_3f_1)] = f^2/2E$$

$$\text{or} \qquad [f_1^2 + f_2^2 + f_3^2 - (2/m)(f_1f_2 + f_2f_3 + f_3f_1)] = f^2$$

(iv) *Shear Strain Energy Theory (due to Mises and Hencky)*: At failure the shear strain energy in the complex system and in simple tension are equal, i.e.,

$$(1/12C)[(f_1 - f_2)^2 + (f_2 - f_3)^2 + (f_3 - f_1)^2] = f^2/6C$$

$$\text{or} \quad (f_1 - f_2)^2 + (f_2 - f_3)^2 + (f_3 - f_1)^2 = 2f^2$$

(The value in the simple tension case is found by putting the principal stresses equal to $f, 0, 0$.)

(v) *Maximum Principal Strain Theory (due to St. Venant)*: If e_1 is the maximum strain the complex stress system, then according to this theory,

$$e_1 = (1/E)(f_1 - F_2/m - F_3/m)$$

$$= f/E \text{ in simple tension}$$

$$\text{or} \qquad f_1 - F_2/m - F_3/m = f$$

Other theories have been put forward, but have not proved to be nearer the truth except perhaps for particular types of loading.

Conclusions

Considerable experimental work has been done on various stress systems, such as tubes under the action of internal pressure, end loads, and torsion; also on different materials. So far, however, no conclusive evidence has been produced in favour of any one theory.

It must be admitted that the cause of failure depends not only on the properties of the material but also on the stress system to which it is subjected, and it may not be possible to embody the results for all cases in one comprehensive formula. The following general conclusions may be used as a guide to design.

In the case of brittle materials such as cast iron the maximum principal stress theory should be used. For ductile materials the maximum shear stress or strain energy theories give a good approximation, but the shear strain energy theory is to be preferred, particularly when the mean principal stress is compressive. The maximum strain theory should not be used in general, as it only gives reliable results in particular cases.

It should be noted that, since the shear stress and shear strain energy theories depend only on stress differences, they are independent of the value of the mean stress and imply that a material will not fail under a hydrostatic stress system (i.e. $f_1 = f_2 = f_3$). In practice the effect of such a stress system is to produce a brittle type fracture in a normally ductile material, no plastic deformation having taken place.

EXAMPLE If the principal stresses at a point in an elastic material are $2f$ tensile, f tensile, and $\frac{1}{2}f$ compressive, calculate the value of f at failure according to five different theories.

The elastic limit in simple tension is 20 tons/sq. in and Poisson's ration = 0.3

(i) *Maximum Principal Stress Theory*

In the complex system, maximum stress = $2f$.

In simple tension, maximum stress = 20 tons/sq. in. Equating gives $f = 10$ tons/sq.in.

(ii) *Maximum Shear Stress Theory*

$$\begin{aligned}\text{Maximum shear stress} &= \text{Half difference between principal stresses}\\ &= \frac{1}{2}\left[2f - (-\tfrac{1}{2}f)\right]\\ &= \frac{5}{4}f\end{aligned}$$

In simple tension, principal stresses are 20, 0, 0 and maximum shear stress $= \frac{1}{2} \times 20$

$= 10$ tons/sq. in.

Equating gives $f = 8$ tons/sq. in.

(iii) *Strain Energy Theory*

In the complex system

$$\begin{aligned}U &= (1/2E)[(2f)^2 + f^2 + (-\tfrac{1}{2}f)^2 - 2 \times 0.3\\ &\qquad (2ff - ff/2 - f/2.2f)]\\ &= 4.95f^2/2E\end{aligned}$$

In simple tension $U = 20^2/2E$

Equating gives: $f = 20/\sqrt{4.95}$

$= 8.98$ tons/sq. in.

(iv) *Shear Strain Energy Theory*

In the complex system

$$\begin{aligned}U_s &= (1/12C)\left[(2f - f)^2 + \left(f + \frac{1}{2}f\right)^2 + \left(-\frac{1}{2}f - 2f\right)^2\right]\\ &= 9.5f^2/12C\end{aligned}$$

In simple tension (principal stresses 20, 0, 0)

$$U_s = 20^2/6C$$

Equating gives $f = 20/\sqrt{4.75}$

$= 9.17$ tons/sq. in.

(v) *Maximum Strain Theory*

Equating the maximum strain the complex and simple tension cases

$$(1/E)(2f - 0.3f + 0.3f/2) = 20/E$$

or $f = 20/1.85$

$= 10.8$ tons/sq. in.

EXAMPLE The load on a bolt consists of an axial pull of 1 ton together with a transverse shear force of $\frac{1}{2}$ ton. Estimate the diameter of bolt required according to (i) maximum principal stress theory, (ii) maximum shear stress theory, (iii) strain energy theory, (iv) shear strain energy theory. Elastic limit in tension is 18 tons/sq. in., and a factor of safety of 3 is to be applied. Poisson's ratio = 0.3.

The permissible simple tensile stress is 18/(Factor of safety = 6 tons/sq. in.

Let required diameter be d in., then the applied stresses are,

$$f = \frac{1}{\pi d^2/4} = \frac{4}{\pi d^2} \text{ tons/sq. in. tension}$$

and $s = \dfrac{1}{2\pi d^2/4} = \dfrac{2}{\pi d^2}$ tons/sq. in. shear (Fig. 20.14)

assuming uniform distribution over the cross-section.

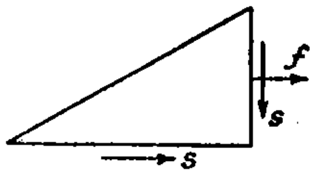


Fig. 20.14

(i) *Maximum Principal Stress in Bolt,*

$$\begin{aligned}&= \frac{1}{2}f + \frac{1}{2}\sqrt{(f^2 + 4s^2)} \qquad (f_x = f, f_y = 0)\\ &= \frac{1}{2}\cdot 4/\pi d^2 + \frac{1}{2}\sqrt{[(4/\pi d^2)^2 + 4(2/\pi d^2)^2]}\\ &= (2/\pi d^2)[1 + \sqrt{(1+1)}]\\ &= 4.829/\pi d^2\end{aligned}$$

Maximum stress in simple tension = 6. T/sq. in.

Equating to above gives

$$\begin{aligned}d &= \sqrt{(4.829/6\pi)}\\ &= 0.506 \text{ in. say } \frac{1}{2} \text{ in.}\end{aligned}$$

(ii) *Maximum Shear Stress* $= \frac{1}{2}\sqrt{(f^2 + 4s^2)}$

$= 2.829/\pi d^2$

and $\equiv 3$ in simple tension (T/sq. in.)

$\therefore\ d = \sqrt{(2.829/3\pi)}$

$= 0.548$ in. say $\frac{9}{16}$ in.

(iii) *Principal Stresses are* $\frac{1}{2}f \pm \frac{1}{2}\sqrt{(f^2 + 4s^2)}, 0$ i.e.,

$$4.829/\pi d^2, \quad -0.829/\pi d^2, 0$$

$$\begin{aligned}\text{Strain energy } &= (1/2E)(4.829^2 + 0.829^2 + 2 \times 0.3 \times \\ &\quad 4.829 \times 0.829)/\pi^2 d^4 \\ &= 26.4/(2E\pi^2 d^4)\end{aligned}$$

and $\quad \equiv 6^2/2E$ in simple tension (T/sq. in.)

$$\begin{aligned}\therefore d &= \sqrt[4]{(26.4/36\pi^2)} \\ &= 0.523 \text{ in. say } \frac{9}{16} \text{ in.}\end{aligned}$$

(iv) *Shear Strain Energy*

$$= (1/12C)[(4.829 + 0.829)^2 + 0.829^2 + 4.829^2]/(\pi^2 d^4)$$

and $\quad \equiv 6^2/6C$ in simple tension (T/sq. in.)

$$\begin{aligned}\therefore \quad d &= \sqrt[4]{[(56.0 \times 6)/(36\pi^2 \times 12)]} \\ &= 0.53 \text{ in., say } \frac{9}{16} \text{ in.}\end{aligned}$$

20.3 'AREA MOMENTS' METHOD OF ANALYSIS

The simplest method of analyzing the stresses in a bent beam, which is statically indeterminate to a low degree, is that of area moments. This method forms a good introduction to methods of wider application.

Principle of Area-Moments (Fig. 20.15)

(a) In any portion, AB, of a bent beam, the angle, ϕ, between the tangents to the beam at A and B is numerically equal to the area of the *M/EI* diagram between these points.

(b) In any portion, AB, of a bent beam, the displacement of A from the tangent to the beam at B is equal to the moment of the area of the *M/EI* diagram between A and B, taken about A.

Since the displacement, *d*, is always very small in relation to the length of the beam, it is immaterial whether it is assumed to be measured at right angles to the beam or at right angles to the tangent.

Two points of importance should be noted and remembered. When calculating displacement, take the moment of the *M/EI* area about the point where the displacement is required. The figures found by following the principle (b) above, do not necessarily indicate the deflection of the beam from its original position.

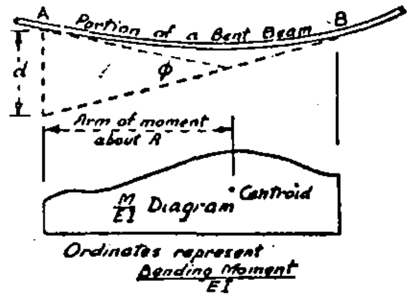


Fig. 20.15

Procedure

(i) Remove the statically indeterminate forces and moments so that the structure is left in a statically determinate condition.

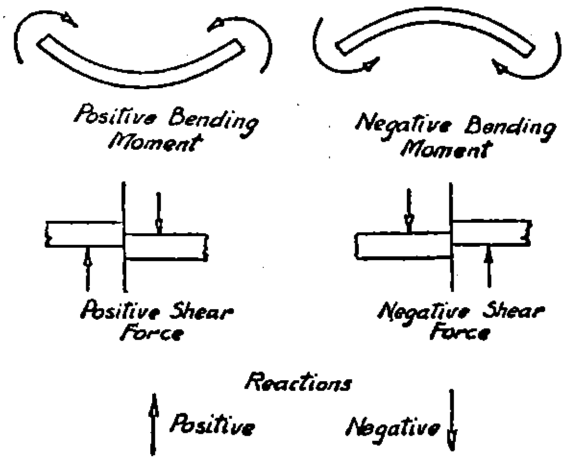
(ii) Apply the external loading and draw the bending moment diagram. This is often known as the free bending moment diagram.

(iii) Divide each ordinate of the bending moment diagram by the term EI relevant to each section and draw the *M/EI* diagram.

(iv) Remove the external loading from the beam and apply the statically indeterminate forces and moments. Draw the *M/EI* diagram for these forces and moments only.

(v) From the conditions of support the nett slope or deflection of the beam at one or more sections is usually known. Find the slope and/or deflection at these sections from (iii) in terms of the known loading, and from (iv) in terms of the unknown reactions or moments, and by comparison, determine the unknowns.

Convention of Signs



Moment-Area Method for Built-in Beams

A beam is said to be built-in or encastre when both its ends are rigidly fixed so that the slope remains horizontal. Usually also the ends are at the same level.

It follows from the moment-area method that, since the change of slope from end to end and the intercept are both zero:

$$\Sigma A = 0 \qquad (20.10)$$

and

$$\Sigma A\bar{x} = 0 \qquad (20.11)$$

It will be found convenient to show the bending moment diagram due to any loading such as [Fig 20.16(a)] as the algebraic sum of two parts, one due to the loads, treating the beam as simply supported [Fig. 20.16(b)], and the other due to the end moments introduced to bring the slopes back to zero [Fig. 20.16(c)].

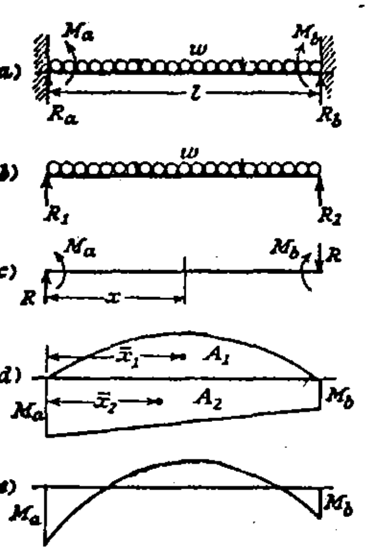


Fig. 20.16

The area and end reactions obtained it freely supported will be referred to as the *free moment diagram* and the *free reactions*, A_1, R_1 and R_2 respectively.

The fixing moments at the ends are M_a and M_b, and in order to maintain equilibrium when M_a and M_b are unequal, the reactions $R = (M_a - M_b)/l$ are introduced, being upwards at the left-hand end and downwards at the right-hand end. Due to M_a, M_b, and R, the bending moment at a distance x from the left-hand end

$$= -M_a + R \cdot x = M_a + [(M_a - M_b)/l]x$$

This gives a straight line going from a value $-M_a$ at $x = 0$ to $-M_b$ at $x = l$, and hence the *fixing moment diagram*, A_2 [Fig. 20.16(d)].

For downward loads, A_1 is a positive area (sagging BM), and A_2 a negative area (hogging BM) consequently the Eqs. (20.10) and (20.11) reduce to,

$$A_1 = A_2 \text{ from Eq. (20.10)}$$

and $A_1\bar{x}_1 = A_2\bar{x}_2$ (numerically) from Eq. (20.11)

i.e., Area of free moment diagram = Area of fixing moment diagram.

And moments of areas of free and fixing diagrams are equal.

It may be necessary to break down the areas still further to obtain convenient triangles and parabolas.

These two equations enable M_a and M_b to be found, and the total reactions at the ends, are,

$$R_a = R_1 + R$$
$$= R_1 + (M_a - M_b)/l$$

and

$$R_b = R_2 - R$$
$$= R_2 - (M_a - M_b)/l$$

Finally, the combined bending moment diagram is shown in Fig. 20.16 (e) as the algebraic sum of the two components.

EXAMPLE Obtain expressions for the maximum bending moment and deflection of a beam of length l and flexural rigidity EI, fixed horizontally at both ends, carrying a load W (a) Concentrated at midspan, (b) uniformly distributed over the whole beam.

Case (a): By symmetry $M_a = M_b = M$, say (Fig. 20.17)

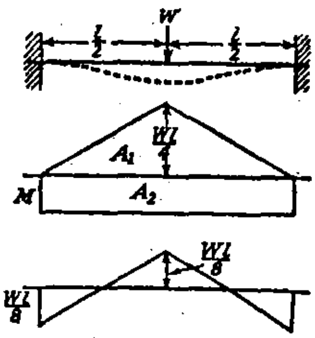


Fig. 20.17

The free moment diagram is a triangle with maximum ordinate $Wl/4$,

$$\therefore \quad \text{Area } A_1 = \frac{1}{2}(Wl/4)l$$
$$= Wl^2/8$$
$$\text{Area } A_2 = Ml$$

Equating $A_1 = A_2$ from Eq. (20.10) gives

$$M = Wl/8$$

The combined bending moment diagram is therefore as shown in the lower diagram, Fig. 20.17, and the maximum bending moment is $Wl/8$ occurring at the end (hogging), and the centre (sagging).

By taking moment-areas about one end for half the beam, the intercept gives the deflection, i.e.,

$$\delta = \frac{\left[\frac{1}{2}(Wl/4)(l/2)\right]\frac{2}{3}l/2 - M(l/2)l/4}{EI}$$

$$= Wl^3/192EI$$

Case (b): Free moment area

$$A_1 = \frac{2}{3}(wl^2/8)l = wl^3/12 \quad \text{(Fig. 20.18)}$$

Fixing moment area,

$$A_2 = Ml$$

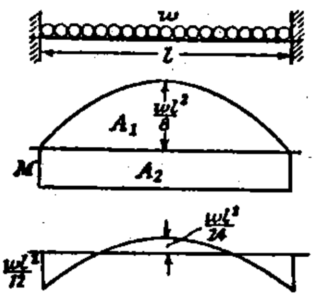


Fig. 20.18

Equating gives

$$M = wl^2/12$$

and this is the maximum bending moment.

Again, for half the beam, the intercept about one end gives the deflection, i.e.

$$\delta = \frac{\left[\frac{2}{3}(wl^2/8)(l/2)\right]\frac{5}{8}l/2 - M(l/2)l/4}{EI}$$

$$= wl^4/384EI$$

EXAMPLE A beam of span l ft has its ends fixed horizontally at the same level and carries a load W at a distance a ft from one end and b ft from the other. Deduce expressions for the fixing moments at the ends. Hence show that, for a distributed load on the same beam, the fixing moment at one end is given by $\int_0^l \frac{px(l-x)^2}{l^2}dx$

where p = load/ft. run at a distance of x ft from the end considered.

Apply the above result to find the fixing moments when l = 20 ft and p varies uniformly from zero at one end to 2 tons/ft at the other.

The free moment diagram is a triangle of height Wab/l, and the fixing moments are M_a and M_b (Fig. 20.19).

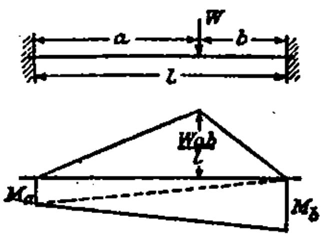


Fig. 20.19

Equating areas

$$\frac{1}{2}(M_a + M_b)l = \frac{1}{2}(Wab/l)l$$

i.e., $$M_a + M_b = Wab/l \qquad (20.12)$$

By moment-areas about the left-hand end, splitting each figure into two triangles

$$\left(\frac{1}{2}M_a l\right)l/3 + \left(\frac{1}{2}M_b l\right)2l/3 = \left[\left(\frac{1}{2}Wab/l\right)a\right]2a/3 + \left[\left(\frac{1}{2}Wab/l\right)b\right](a + b/3)$$

i.e., $(M_a + 2M_b)l^2/3 = 2Wa^3b/3l + (Wab^2/l)(a + b/3)$

or $$M_a + 2M_b = (Wab/l^3)(2a^2 + 3ab + b^2) \qquad (20.13)$$

Subtract Eq. (20.12), giving,

$$M_b = (Wab/l^3)(2a^2 + 3ab + b^2 - l^2)$$
$$= (Wab/l^3)(a^2 + ab), l = a + b$$
$$= (Wab/l^3)a(a + b)$$
$$= Wa^2b/l^2$$

From Eq. (20.12) $M_a = Wab/l - Wa^2b/l^2 = Wab^2/l^2$

For a distributed load the fixing moment δM_a due to the $p\delta x$ on a short length at a distance x from that end = $p\delta x x(l-x)^2/l^2$ from above.

Integrating for all the load

$$\therefore \quad M_a = \int_0^l \frac{px(l-x)^2dx}{l^2}$$

$$p = 2x/20 \text{ tons/ft}$$

$$\therefore \quad M_a = \int_0^{20} \frac{2x}{20}\frac{x(20-x)^2}{20^2}dx$$

$$= \frac{1}{4000}\int_0^{20}(400x^2 - 40x^3 + x^4)dx$$

$$= 26\frac{2}{3} \text{ tons-ft.}$$

Similarly

$$M_b = \int_0^{20} \frac{2x}{20}\frac{x^2(20-x)}{20^2}dx$$

$$= \frac{1}{4000}\int_0^{20}(20x^3 - x^4)dx$$

$$= 40 \text{ tons-ft}$$

It will be seen, therefore, that for standard cases the minimum bending moment occurs at one of the fixed ends. More complicated loadings may be built up by superposition and it may be accepted in general that, *for any combination of downward loads the maximum bending moment is given by the greater fixing moment.*

Moment Area Method for Continuous Beams

When a beam is carried on more than two supports it is said to be continuous. It is possible to employ an extension of the moment-area method to obtain a relation between the bending moments at three points (usually supports).

In Fig. 20.20 the areas A_1 and A_2 are "free" bending moment areas, treating the beam as simply supported over two separate spans l_1 and l_2. If the actual bending moments at these points are M_1, M_2 and M_3, a fixing moment diagram consisting of two trapezia will be introduced, the actual BM being the algebraic sum of the two diagrams.

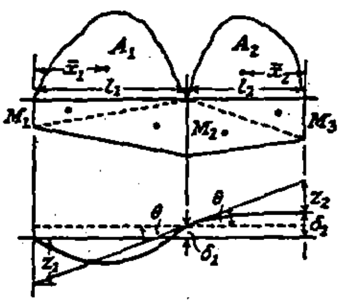


Fig. 20.20

In the lower figure the elastic line of the deflected beam is shown, the deflections δ_1 and δ_2 being relative to the left-hand support and positive upwards, θ is the slope of the beam over the centre support, and z_1 and z_2 the intercepts for l_1 and l_2.

Then $\quad \theta = (z_1 + \delta_1)/l_1 = (z_2 + \delta_2 - \delta_1)/l_2$

(slopes being everywhere small)

i.e., $$\frac{A_1\bar{x}_1 + (M_1l_1/2)(l_1/3) - (M_2l_1/2)(2l_1/3)}{EI_1l_1} + \frac{\delta_1}{l_1}$$

$$= -\frac{A_2\bar{x}_2 + (M_3l_2/2)(l_2/3) - (M_2l_2/2)(2l_2/3)}{EI_2l_2} + \frac{\delta_2 - \delta_1}{l_2}$$

(note that z_2 is a negative intercept)

or $$M_1l_1/I_1 + 2M_2(l_1/I_1 + l_2/I_2) + M_3l_2/I_2$$
$$= 6(A_1\bar{x}_1/I_1l_1 + A_2\bar{x}_2/I_2l_2) + 6E[\delta_1/l_1 + (\delta_1 - \delta_2)l_2] \quad (20.14)$$

If $I_1 = I_2$

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2$$
$$= 6(A_1\bar{x}_1/l_1 + A_2\bar{x}_2/l_2) + 6EI[\delta_1/l_1 + (\delta_1 - \delta_2)/l_2] \quad (20.15)$$

If the supports are at the same level

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2 = 6(A_1\bar{x}_1/l_1 + A_2\bar{x}_2/l_2) \quad (20.16)$$

and if the ends are simply supported ($M_1 = M_3 = 0$)

$$M_2(l_1 + l_2) = 3(A_1\bar{x}_1/l_1 + A_2\bar{x}_2/l_2) \quad (20.17)$$

Equation (20.14) is the most general form of the equation of three moments, also called *Clapeyron's equation*. The others are simplifications to meet particular cases. Eq. (20.16) being the form in which it is most frequently required.

EXAMPLE A beam AD, 60 ft long, rests on supports at A, B and C at the same level. AB = 24 ft; BC = 30 ft. The loading is 1 ton/ft throughout and in addition a concentrated load of 5 tons acts at the mid-point of AB and a load of 2 tons acts at D. Draw the SF and BM diagrams.

$M_a = 0$
$M_c = 2 \times 6 + 6 \times 3 = 30$ tons-ft

Applying Eq. (20.16) to the spans ABC (Fig. 20.21).

$$2M_b \times 54 + 30 \times 30$$

$$= 6\left(\frac{1}{2} \times \frac{5 \times 24}{4} \times 24\right) \times \frac{12}{24} + \left(\frac{2}{3} \times \frac{24^2}{8} \times 24\right) \times \frac{12}{24}$$

$$+ \left(\frac{2}{3} \times \frac{30^2}{8} \times 30\right) \times \frac{15}{30}$$

$$= 6 \times 1881$$

$$\therefore \quad M_b = 96.2 \text{ tons-ft.}$$

BM at mid-point of AB

$$= 5 \times 24/4 + 24^2/8 - M_b/2$$
$$= 53.9 \text{ tons-ft.}$$

BM at mid-point of BC

$$= 30^2/8 - \frac{1}{2}(M_b + 30)$$
$$= 49.4 \text{ tons-ft.}$$

To find the reactions at the supports, note that

$$M_b = -R_a \times 24 + 24 \times 12 + 5 \times 12 \text{ for } AB$$
$$= -R_c \times 30 + 36 \times 18 + 2 \times 36 \text{ for } BCD$$
$$\therefore R_a = (288{+}60{-}96.2)/24 = 10.49 \text{ tons, say } 10.5 \text{ tons.}$$

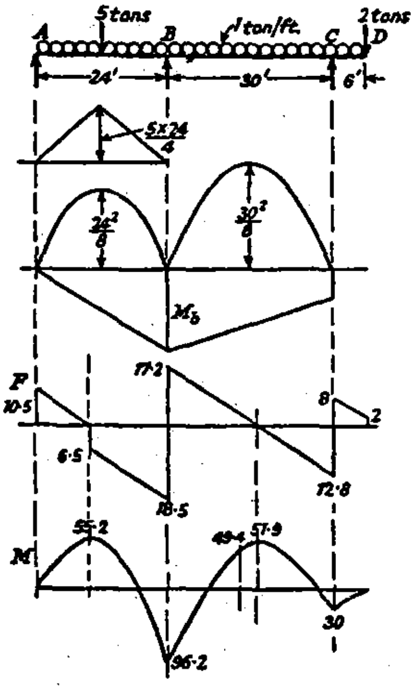


Fig. 20.21

and $R_c = (648 + 72 - 96.2)/30 = 20.8$ tons

By difference $R_b = 60 + 5 + 2 - 10.5 - 20.8$
$= 35.7$ tons

From the shear force diagram it can be worked out that the maximum bending moments occur.

(i) at a distance of 12.8 ft from C, where
$M = 21.5 \times 13.5 - 19.5^2/2 - 2 \times 19.5 = 51.9$ tons-ft

(ii) at a distance of 10.5 from A where
$M = 10.5 \times 10.5 - 10.5^2/2 = 55.2$ tons-ft

20.4 STRAIN ENERGY METHOD OF ANALYSIS

General Principles

The amount of strain energy of work stored in a loaded structure depends on the magnitude of the direct, shear and bending stresses imposed on the various parts of the structure. In pin-jointed frames, for example, where the members are in tension or compression, the work stored depends on direct forces only.

In rigid structures, direct stress, shear stress and bending stress may all occur at any section, and the total strain energy stored in the beam or frame depends on the magnitudes of the three types of stress. It is generally conceded, however, that the work done by the direct and shear forces is so small in comparison with that done by bending that only the latter need be considered when calculated statically indeterminate reactions or moments.

It must be remembered, however, that although the work done by shear and direct forces may be considered negligible, yet the values of the shear and direct stresses must be included in the final stress values when the strength of the structure is being checked.

In any member of a structure subjected to bending the total internal work or strain energy is

$$U = \int_0^l \frac{M^2 ds}{2EI}$$

Where M is the bending moment at any point on the member caused by the combined effect of the imposed loads and the supporting forces and moments. Whether statically determinate or not. The integration must be taken over the whole length of the member, of which ds is an element of length.

Castigliano showed that the partial differential coefficient of the strain energy in a structure, with respect to a load P acting on the structure, is equivalent to the displacement of P along its line of action.

$$\Delta_P = \frac{\partial U}{\partial P} = \int_0^l \frac{2M}{2EI}\frac{\partial M}{\partial P} ds = \int_0^l \frac{M}{EI}\frac{\partial M}{\partial P} ds$$

Similarly, the partial differential coefficient of the strain energy in a structure with respect to a moment acting on the structure is equivalent to the angle through which that portion of the structure rotates when the moment is applied.

$$\theta = \frac{\partial U}{\partial M_x} = \int_0^l \frac{M}{EI}\frac{\partial M}{\partial M_x} ds$$

It very often occurs that the forces P and the moments

M_x, are the supporting forces and moments of a statically indeterminate structure, and if the supports of the structure do not give way under the action of the loading, then there is no deformation of the structure at the points of support and the expressions just quoted can be equated to zero. If the differential coefficient of the strain energy is thus zero, then the strain energy itself is a minimum. The term *Method of Least Work* may therefore be found applied to Strain Energy determinations. Where there is rotation of the support the corresponding $\partial U/\partial M_x$ can be equated to that, and similarly if there is a movement in the direction of the reaction at the support the corresponding $\partial U/\partial P$ (or $\partial U/\partial H$) can be equated to that.

Beams and Frames Having One Redundant Reaction

EXAMPLE If the support R were removed, the structure (Fig. 20.22) would become a cantilever, and it is therefore statically indeterminate to the first degree. A single equation, in addition to those of statics, is sufficient to determine the stresses in the beam.

From the principle of Strain Energy the equation required is

$$\frac{1}{EI}\int_0^l M\frac{\partial M}{\partial R}dx = 0$$

EI is a constant whose value is not required and which can be cancelled from subsequent calculations.

The integration must be made in two sections, since the bending moment expression changes at half span. The work is most conveniently done in tabular form. Working from the right-hand end of the beam, the table is as follows:

	Bending Moment (M)	$\partial M/\partial R$	Limits
Right half	Rx	$+x$	0 to 5
Left half	$Rx - \frac{w}{2}(x-5)^2$	$+x$	5 to 10

It will be found easier to work each integration separately, than to try to evaluate all terms in one operation. By this means there is less chance of a mistake being made in the evaluation of the definite integrals.

$$\int_0^5 M\frac{\partial M}{\partial R}dx = \int_0^5 (Rx)x\,dx = 41.67R$$

$$\int_5^{10} M\frac{\partial M}{\partial R}dx = \int_5^{10} \{Rx - \frac{w}{2}(x-5)^2\}x\,dx =$$

$$291.7R - 364.6$$

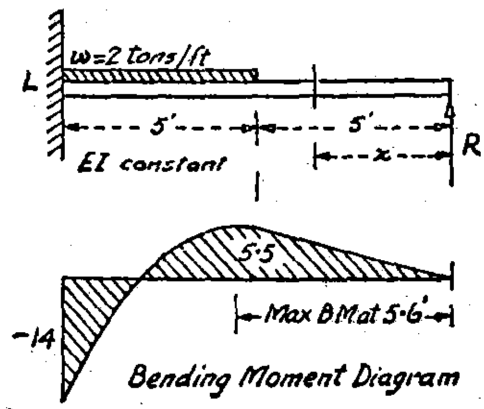


Fig. 20.22

summing and equating to zero,

$$\int_0^{10} M\frac{\partial M}{\partial R}dx = 333R - 364.6 = 0$$

$$R = +1.1 \text{ tons}$$

The bending moment diagram can now be drawn by substituting this value R in the bending moment column of the table.

EXAMPLE If any one of the three supports of the beam shown in Fig. 20.23 were to be removed, the beam would be statically determinate. Only one equation is therefore, required to find one of the reactions. Thereafter the methods of statics can be used to determinate the others.

By taking moments, two of the reactions are first expressed in terms of the third, so that only one unknown appears in the equation

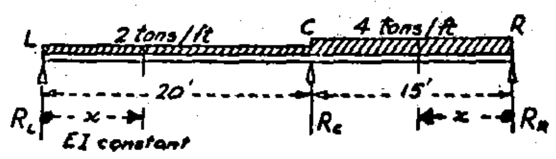


Fig. 20.23

Taking moments about R

$$35R_L + 15R_C = 1450 \quad \therefore R_L = 41.4 - 0.43R_C$$

Taking moments about L

$$35R_R + 20R_C = 2050 \quad R_R = 58.6 - 0.57R_C$$

It must be remembered that the integration must be taken over the whole length of the beam, and in this problem it is easier to work from both ends towards the centre than from one end only. EI is again constant over the whole length.

Member	Bending Moment	Moment rewritten	$\partial M/\partial R_C$	Limits
RC	$R_R x - 2x^2$	$58.6x - 0.57xR_C - 2x^2$	$-0.57x$	0 to 15
LC	$R_L x - x^2$	$41.4x - 0.43xR_C - x^2$	$-0.43x$	0 to 20

$$\int_0^{15} M\frac{\partial M}{\partial R_C}dx = \int_0^{15}(58.6x - 0.57xR_C - 2x^2)(-0.57x)dx$$
$$= 370R_C - 23{,}200$$

$$\int_0^{20} M\frac{\partial M}{\partial R_C}dx = \int_0^{20}(41.4x - 0.43xR_C - x^2)(-0.43x)dx$$
$$= 490R_C - 30{,}300$$

Summing and equating to zero

$$\int_0^{l} M\frac{\partial M}{\partial R_C}dx = 860R_C - 53{,}500 = 0 \quad R_C = +62.2 \text{ tons}$$

EXAMPLE The frame $ABCD$ in Fig. 20.24 has rigid joints at the corners B and C, and is pin-jointed at the supports A and D. This type of frame is used sometimes in bridge construction in steel and reinforced concrete, and is usually known as a Portal Frame.

When the frame is loaded as shown in Fig. 20.24, the points A and D have a tendency to move apart, and the horizontal force H is called into play. The vertical reactions V_A and V_D can be evaluated by the method of statics and H is the statically indeterminate force. The frame is indeterminate to the first degree.

The writing of the bending moment equations for this type of frame sometimes present difficulties to the beginner. A sheet of paper should be used to cover the frame except the portion to the right or left of the section being considered. The moments (about the edge of the paper) of all the forces which can be seen must then be written down, the values of any moments applied to the structure being also included.

$$V_A = V_D = 4.5 \text{ tons}, EI \text{ is constant}$$

Member	Bending Moment (M)	$\partial M/\partial H$	Limits
AB	$-Hy$	$-y$	0 to 15
CD	$-Hy$	$-y$	0 to 15
BC(left half)	$-15H + 4.5x$	-15	0 to 10
BC(right half)	$-15H + 4.5x$	-15	0 to 10

$$\frac{2}{EI}\int_0^{15} M\frac{\partial M}{\partial H}dy = \frac{2}{EI}\int_0^{15} Hy^2dy = \frac{2250}{EI}H$$

$$\frac{2}{EI}\int_0^{10} M\frac{\partial M}{\partial H}dx = \frac{2}{EI}\int_0^{10}(4.5x - 15H)(-15)dx =$$
$$\frac{4500}{EI}H - \frac{6750}{EI}$$

Summing and equating to zero,

$\int_0^{l} M\frac{\partial M}{\partial H}dx = 6750H - 6750 = 0, H = 1$ ton in the direction assumed.

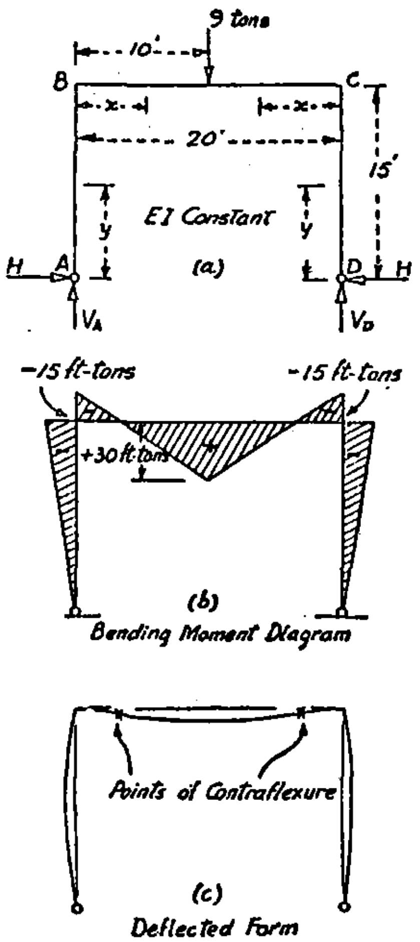


Fig. 20.24

Since the joints B and C are considered to be rigid, the bending moments in the beam and column at these points must be equal. The value of the bending moment of B and C is (from the table) $-15H = -15$ ft tons.

It is usual to plot the bending moment diagram on that 'side' of the beam or column which is in tension.

The reader should make a practice of drawing both the bending moment diagram and a diagram showing, to an exaggerated degree, the deflected form of the frame. Points of contraflexure occur where the bending moment diagram crosses the outline of the frame.

The work done by direct stress has not been included in the total strain energy, but the stress at any section is the sum of that due to the bending moment. [Fig. 20.24(b)] and that due to the direct stress, which must not be forgotten when stresses are being evaluated. At B, for example, the bending stress is obtained from the flexure formula, and the direct stress in the column AB is V_A divided by the cross-sectional area of AB.

Deflection from the Strain Energy (Castigliano's Theorem)

Theorem If U is the total strain energy of any structure due to the application of external loads $W_1, W_2, \ldots$ at $O_1, O_2 \ldots$ in the directions $O_1X_1, O_2X_2 \ldots$, and to couples $M_1, M_2 \ldots$ then the deflections at $O_1, O_2 \ldots$ in the directions $O_1X_1, O_2X_2 \ldots$, are $\partial U/\partial W_1, \partial U/\partial W_2 \ldots$, and the angular rotations of the couples are $\partial U/\partial M_1, \partial U/\partial M_2 \ldots$ at their applied points.

Proof for concentrated loads If the total displacements (in the direction of the loads) produced by gradually applied loads $W_1, W_2, W_3 \ldots$ and $X_1, X_2, X_3 \ldots$ then

$$U = \frac{1}{2}W_1x_1 + \frac{1}{2}W_2x_2 + \frac{1}{2}W_3x_3 + \ldots \qquad (20.18)$$

Let W_1 alone be increased by δW_1, then δU =increase in external work done

$$= (W_1 + \delta W_1/2)\delta x_1 + W_2\delta x_2 + W_3\delta x_3 + \ldots$$

(where $\delta x_1, \delta x_2, \delta x_3$ are increases in x_1, x_2 and x_3 due to application of δW_1)

$$= W_1\delta x_1 + W_2\delta x_2 + W_3\delta x_3 + \ldots \qquad (20.19)$$

neglecting the product $\frac{1}{2}\delta W_1\delta x_1$.

But if the loads $W_1 + \delta W_1, W_2, W_3 \ldots$ were applied gradually from zero, the total strain energy would have been:

$$U + \delta U = \frac{1}{2}(W_1 + \delta W_1)(x_1 + \delta x_1) + \frac{1}{2}W_2(x_2 + \delta x_2) + \frac{1}{2}W_3(x_3 + \delta x_3) + \ldots$$

Subtracting Eq. (20.18) and neglecting products of small quantities:

$$\delta U = \frac{1}{2}W_1\delta x_1 + \frac{1}{2}\delta W_1x_1 + \frac{1}{2}W_2\delta x_2 + \frac{1}{2}W_3\delta x_3 + \ldots \qquad (20.20)$$

or $\quad 2\delta U = W_1\delta x_1 + \delta W_1x_1 + W_2\delta x_2 + W_3\delta x_3 + \ldots$

Subtract Eq. (20.19), then $\delta U = \delta W_1x_1$

and in the limit $\quad \partial U/\partial W_1 = x_1$

Similarly for x_2 and x_3, and the proof can be extended to incorporate couples.

It is important to stress that U is the total strain energy, expressed in terms of the loads and not including statically determinate reactions, and that the partial derivative with respect to each load in turn (treating the others as constant) gives the deflection at the load point in the direction of the load.

The following principles should be observed in applying this theorem:

(i) In finding the deflection of curved beams and similar problems, only strain energy due to bending need normally be taken into account (i.e. $\int M^2ds/2EI$)

(ii) Treat all the loads as variables initially, carry out the partial differentiation and integration, putting in numerical values at the final stage.

(iii) If the deflection is to be found at a point where, or in a direction in which, there is no load, a load may be put in where required and given a value zero in the final reckoning (i.e. $x = (\partial U/\partial W)_{W=0}$).

Generally it will be found that the strain energy method requires less thought in application than the direct method, it being only necessary to obtain an expression for the bending moment; also there is no difficulty over the question of sign, as the strain energy is bound to be positive, and deflection is positive in the direction of the load. The only disadvantage occurs when a case such as note (iii) above has to be dealt with, when the direct method sometimes will probably be shorter.

EXAMPLE Obtain expression for the vertical displacement at A of the beam showing in Fig. 20.25.

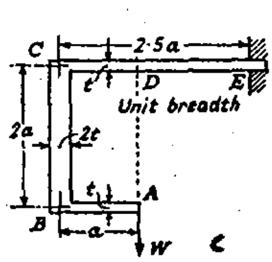


Fig. 20.25

If $a = 2$ in., $t = \frac{1}{4}$ in. find the displacement when $W =$ 5 lb $E = 30 \times 10^6$ lb/. sq.in. and section width is 1 in.

The bending moments in the various sections can be written as follows:

$AB, M = W \cdot x$, (at x from A)

$BC, M = W \cdot a$, constant

$CD, M = W \cdot x'$, at x' from D)

$DE, M = W \cdot x''$ at x'' from D)

$$U = M^2 ds/2EI$$

$$= \int_0^a \frac{W^2x^2dx}{2E \times t^3/12} + \int_0^{2a} \frac{W^2a^2ds}{2E \times (2t)^3/12} +$$

$$\int_0^a \frac{W^2x'^2dx'}{2E \times t^3/12} + \int_0^{1.5a} \frac{W^2x''^2 \cdot dx''}{2E \times t^3/12}$$

$$= (6W^2/Et^3)[a^3/3 + 2a^3/8 + a^3/3 + 1.5^3a^3/3]$$

$$= 24.5W^2a^3/2Et^3$$

Displacement of load at A $= \partial U/\partial W$ vertically

$$= 24.5Wa^3/Et^3$$

$$= (24.5 \times 5 \times 2^3)/\left[30 \times 10^6 \times \left(\frac{1}{4}\right)^3\right]$$

$$= 0.0021 \text{ in.}$$

An allowance could be made for the linear extension of the portion BC

$(W2a)/(2tE) = (5 \times 2)/\left(\frac{1}{4} \times 30 \times 10^6\right) = \frac{4}{3} \times 10^{-6}$ in. which is clearly negligible compared with the deflection due to bending.

20.5 'MOMENT DISTRIBUTION' METHOD OF ANALYSIS

Moment Distribution is a mechanical process of dealing with indeterminate structures by means of successive approximations in which the moments themselves are treated directly, the calculations involved being purely arithmetical. It was developed by Hardy-Cross.

The method is unique in that all joints are initially considered to be fixed against rotation. The fixed end moments are determined for each member as though it were an encastre beam and then the joints are allowed to rotate, either separately or all at once, the moments induced by the rotations being distributed among the members until the algebraic sum of the moments at each internal joint is zero.

The sign convention most commonly adopted for Moment Distribution is that all moments acting on individual members from supports or other members of a frame are positive if clockwise in application and negative if anti-clockwise. Before a BM diagram is drawn, this convention must be translated into the normal convention whereby in continuous beams, for example, sagging moments are positive and hogging moments are negative. The two conventions are compared in Fig. 20.26.

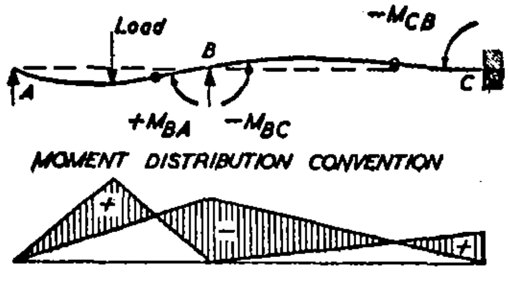


Fig. 20.26

It will be found that the operations of Moment Distribution are more readily understood and checked if the reader considers initially how the structure deflects under load. Consequently, deflection diagrams are incorporated in many of the examples.

Although the structural principles on which Moment Distribution is based are well known, it is advisable to consider them in a definite sequence.

Figure 20.27 shows a beam AB of constant cross-section, i.e. a prismatic beam, fixed in position and direction at A and fixed in position, but not in direction, at B. When the moment M_{BA} is applied at B a moment M_{AB} is induced at A. It can be shown that

$$M_{AB} = \frac{1}{2}M_{BA} \tag{20.21}$$

and

$$M_{BA} = 4E \tan\theta \frac{I}{L}$$

$$= 4E\theta\frac{I}{L} \text{(for small values of } \theta) \tag{20.22}$$

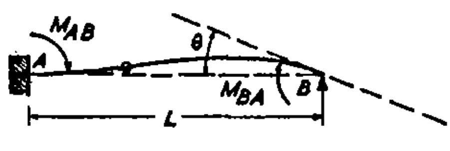


Fig. 20.27

Similarly, if A is fixed in position but not in direction, as in Fig. 20.28, then

$$M_{BA} = 3E\theta\frac{I}{L} \text{ (for small values of } \theta) \tag{20.23}$$

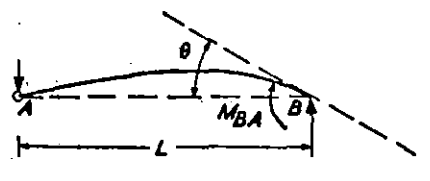


Fig. 20.28

These equations give the three fundamental principles of moment distribution applicable to continuous beams on unyielding supports.

- *Principle I,* Eq. (20.21) When a moment is applied at one end of a prismatic beam, that end remaining fixed in position but not in direction, the other end being fixed both in position and direction, a moment of half the amount and the same sign is induced at the second end.
- *Principle II,* Eq. (20.22) When one end of a beam remains fixed in position and direction, the moment required to produce a rotation of a given angle at the other end of the beam, which remains fixed in position, is proportional to the value I/L for the beam, provided that E is constant. The value I/L, known by the symbol K, is the stiffness factor for the particular beam in question.
- *Principle III,* (Eq. 20.23) When one end of a beam is rotated through a given angle, remaining fixed in position, and the other end remains fixed in position but not in direction, the moment required at the first end of $\frac{3}{4}$ of that required if the second end were fixed both in position and direction, i.e., the equivalent stiffness factor for the beam is $\frac{3}{4}I/L = \frac{3}{4}K$.

The three foregoing principles alone are applied when the supports do not yield. However, when the joints change their positions BMs have to be modified accordingly.

Beams with Support at Different Levels

The ends are assumed, as before, to be horizontal. The bent form of the unloaded beam as shown in Fig. 20.29 is similar to the bent form of two simple cantilevers which can be achieved by cutting the beam at the centre C, and placing downward and upward loads at the free ends of the cantilevers such that the deflection at the end of each cantilever is $d/2$.

Therefore, $\dfrac{d}{2} = \dfrac{P(L/2)^3}{3EI}$ (being the standard deflection formula)

or $$P = \frac{12EId}{L^3}$$

This load would cause a BM at A or B equal to

$$P \times \frac{L}{2} = \frac{12EId}{L^3} \times \frac{L}{2} = \frac{6EId}{L^2}$$

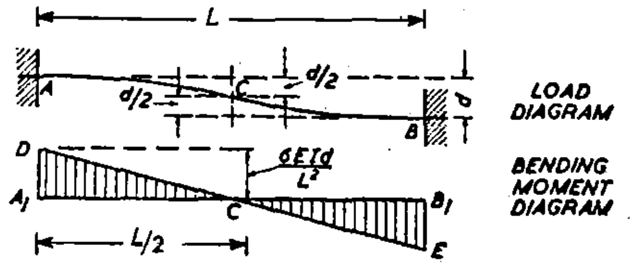


Fig. 20.29

The solution in any given case consists of adding to the ordinary diagram of BMs, the BM diagram A_1DCEB_1.

Shear Forces in Fixed Beams

It must be noted that in the case of fixed beams, it is necessary to evaluate the BMs before the SFs can be determined. This is the converse of the procedure in the case of simply supported beams.

The SF at the end of a beam is found in the following manner:

$$SF_A = \text{the simple support reaction at } A + \frac{M_A - M_B}{L}$$

$$SF_B = \text{the simple support reaction at } B + \frac{M_B - M_A}{L}$$

Consider Fig. 20.30 in which the end A of an encastre beam AB, of span L, has settled an amount d, the ends A and B remaining parallel in direction. As shown above.

$$M_{AB} = M_{BA} = \frac{6EId}{L^2} \qquad (20.24)$$

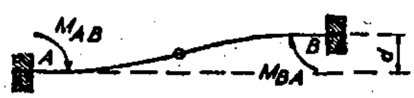


Fig. 20.30

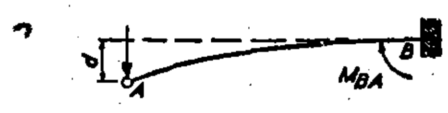


Fig. 20.31

Similarly, in Fig. 20.31 where the end A is hinged, i.e., not fixed in direction,

$$M_{BA} = \frac{3EId}{L^2} = \frac{6EId}{2L^2} \qquad (20.25)$$

From these equations, the following further principles may be derived.

- *Principle IV*, Eq. (20.24) When one end of a beam is deflected through a given distance, that end remaining parallel to its original position and the other remaining fixed in position and direction, equal moments of the same sign are induced at each end, proportional to the I/L^2 value of the beam.
- *Principle V*, Eq. (20.25) When a hinged end of a beam is deflected through a given distance, the other end remaining fixed in position and direction, a moment is induced at the second end, proportional to the $I/2L^2$ value of the beam.

Having stated the principles, the moment distribution processes may be explained by considering some simple examples.

Continuous Beams

EXAMPLE Figure 20.32 shows a continuous beam ABC, of constant cross-section, which is fixed in position and direction at A and C and simply supported at B and which carries uniformly distributed loads of 2 tons per ft on AB and 1 ton per ft on BC.

Under these loads the beam will rotate in an anti-clockwise direction at B and, as it is a fundamental assumption in the theory of continuous beams that the slope does not change over a support, the beam will rotate the same amount θ on either side of B. However, assume that the beam does not rotate at B, but through some locking device remains horizontal after the loads are applied. Then AB and BC are in effect two separate encastre beams and the moments at the end of each span are fixed-end moments (FEMs) depending only on the functions of the span and the loading.

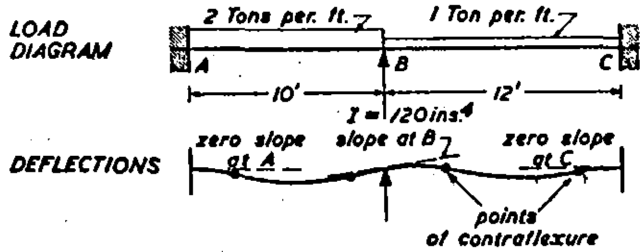


Fig. 20.32

The functions of a span are (i) L, its length, and (ii) I, moment of inertia of its sections.

When I is constant throughout a span, the process is straightforward. Hence, in this example I may be ignored, assuming it constant throughout (as also E).

DISTRIBUTION TABLE

	A	B	B	C
Distribution Factors		0.545	0.455	
Fixed End Moments	−16.67	+16.67	−12.00	+12.00
Distribution		−2.55	−2.12	
Carry Over	−1.27			−1.06
Final Moments	−17.94	+14.12	−14.12	+10.94

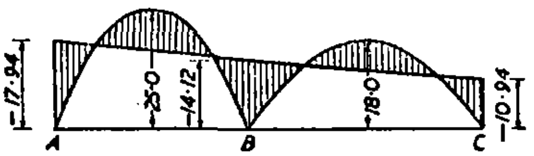


Fig. 20.33 *Final bending moments*

Consider the FEMs for the span AB. From the table given ahead in Fig. 20.37:

$$\begin{aligned} \text{FEM}_{AB} \text{ (being anti-clockwise)} &= -\frac{WL}{12} \\ &= \frac{-2 \times 10 \times 10}{12} \\ &= -16.67 \text{ tons ft} \\ \text{FEM}_{BA} \text{ (being clockwise)} &= +16.67 \text{ tons ft} \end{aligned}$$

Similarly, for the span BC,

$$\begin{aligned} \text{FEM}_{BC} &= -\frac{WL}{12} = \frac{1 \times 12 \times 12}{12} \\ &= -12 \text{ tons ft} \\ \text{FEM}_{CB} &= +12 \text{ tons ft} \end{aligned}$$

Having made these calculations the beam can be released at B and allowed to rotate in an anti-clockwise direction. Now the algebraic sum of the moments on either side of this support must be zero. However, when the beam was horizontal at B, the value of M_{AB} was +16.67 tons-ft and that of M_{BC} was −12 tons-ft. Therefore, to produce equilibrium at B, the total moment induced by the rotation of the beam there must be −4.67 tons-ft, since the moments are out of balance by +16.67 − 12 = +4.67 tons-ft.

But the span ends meeting at B rotate through the same angle θ. Consequently, the moments induced by rotation on either side of B are proportional to the stiffnesses of AB and BC (Principle II). In other words, the moment of −4.67 tons-ft is distributed between AB and BC in proportion to their stiffnesses, i.e., in the proportion

$K_{AB}/(K_{AB}+K_{BC})$ to the left and $K_{BC}/(K_{AB}+K_{BC})$ to the right.

These proportions are known as the distribution factors (DF) for the spans. Although it may sometimes be more accurate to employ fractions, these factors are usually expressed in decimals, but, in any case, the factors for a support or joint must always add up to unity.

Now $I = 120$ in.^{4}, $AB = 120$. in. and $BC = 144$ in.

$$\text{Hence} \quad K_{AB} = \frac{120}{120} = 1$$

$$\text{and} \quad K_{BC} = \frac{120}{144} = 0.833$$

$$\text{Therefore} \quad DF_{AB} = \frac{1}{1+0.833} = 0.545$$

$$DF_{BC} = \frac{0.833}{1+0.833} = 0.455$$

The operation of moment distribution is shown in the distribution table in Fig. 20.33, $0.545 \times -4.67 = -2.55$ tons-ft being added to the end BA and $0.455 \times -4.67 = -2.12$ tons-ft being added to the end BC.

From a consideration of Principle I moments are induced at the outer ends of the beam at A and C, equal to half the moments distributed between the spans at B and of the same signs.

Hence $0.5 \times -2.55 = -1.27$ tons-ft must be transferred to the end A and $0.5 \times -2.12 = -1.06$ tons-ft to the end C. This process, which is known as the *carry-over process* is shown in Fig. 20.33.

The final moments in the beam are found by adding each column algebraically. When constructing a BM diagram it is convenient to remember that the moment to the right of a support in a distribution table bears the same sign as the support moment in the BM diagram (in the normal sign convention). Therefore, the final moments at A, B and C are respectively -17.94, -14.12 and -10.94 tons-ft

The maximum static or 'free' BMs for AB and BC are obtained from the formula + $WL/8$ and equal 25 and 18 tons ft, respectively, and the net moment diagram is as shown shaded.

EXAMPLE The continuous beam *ABCDE*, which is shown in Fig. 20.34 is simply supported at A and over hangs the other outside support D. Consequently, the beam is free to rotate at A and D, although it is restrained to a certain extent at D by the load at E, and when deriving FEMs the beam is assumed to be fixed in a horizontal position at B and C only. Therefore, Principle III applies to spans AB and CD, and the stiffness factors for these spans equal $\frac{3}{4}$K. The stiffens factor for $BC = K$.

Now the moment of inertia I differs for each span, although it is constant throughout a span, as shown in Fig. 20.34.

$$\text{Hence,} \quad \frac{3}{4}K_{AB} = \frac{3 \times 205}{4 \times 168} = 0.915$$

$$K_{BC} = \frac{146}{144} = 1.014$$

$$\frac{3}{4}K_{CD} = \frac{3 \times 122}{4 \times 144} = 0.635$$

$$\text{DF}_{BA} = \frac{3}{4}K_{AB}\Big/\left(\frac{3}{4}K_{AB} + K_{BC}\right) = \frac{0.915}{0.915 + 1.014} = 0.474$$

$$\text{DF}_{BC} = K_{BC}\Big/\left(\frac{3}{4}K_{AB} + K_{BC}\right) = \frac{1.014}{0.915 + 1.014} = 0.526$$

$$\text{DF}_{CB} = K_{BC}/(K_{BC} + \frac{3}{4}K_{CD}) = \frac{1.014}{1.014 + 0.635} = 0.615$$

$$\text{DF}_{CD} = 1 - 0.615 = 0.385$$

Now AB and CD are treated as 'fixed at one end only', and accordingly from the tables of FEMs (given ahead in Figs. 20.37 to 20.40).

$$\text{FEM}_{BA} = +\frac{3PL}{16} = \frac{3 \times 12 \times 14}{16} = 31.50 \text{ tons ft}$$

$$\text{FEM}_{BC} = -\frac{2PL}{9} = -\frac{2 \times 7 \times 12}{9} = -18.67 \text{ tons ft}$$

$$\text{FEM}_{CB} = 18.67 \text{ tons ft}$$

$$\text{FEM}_{CD} = -\frac{WL}{8} = -\frac{22 \times 12}{8} = -33.00 \text{ tons ft}$$

Now under any circumstances in the remainder of the beam the moment of D can only be that due to the load on the cantilever.

The cantilever moment

$$M_{DE} = -M_{DC} = -5 \times 4 = -20.00 \text{ tons ft.}$$

Sufficient data have been accumulated now to analyse the beam by moment distribution as shown in Fig. 20.34.

The inexperienced M_{DE} and M_{DC} are inserted in the appropriate columns, and half M_{DC} is carried over to the other end of the span CD. Subsequently the support D is ignored until the final moments are summated.

The preliminary operations at support C demand some explanation. When the beam is released the unbalanced moment

$$= +10 + (18.67 - 33.00) = -4.33 \text{ tons ft}$$

To balance this a moment + 4.33 tons ft must be distributed between the ends CB and CD.

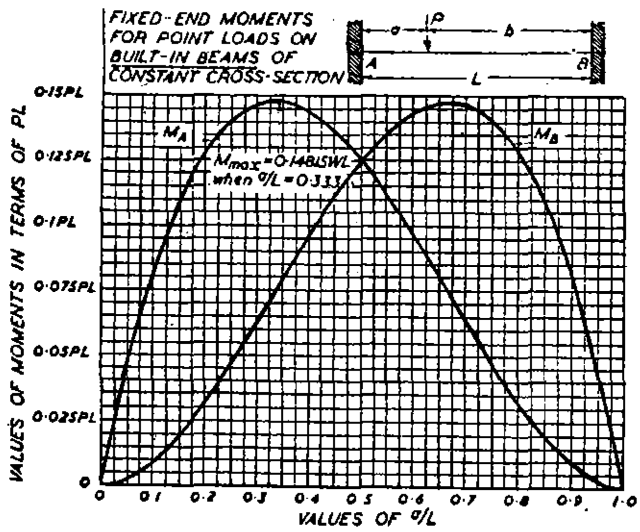
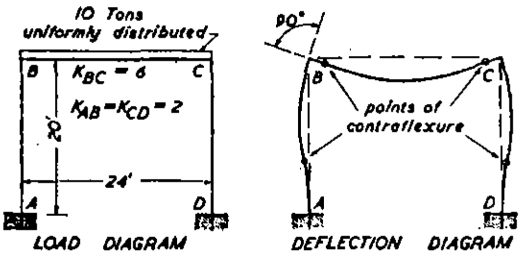


Fig. 20.38

It is appropriate at this stage to include a practical 'short cut' to reduce the amount of work in the distribution table. Just as it is convenient to consider that the equivalent stiffness of a simply supported end span is $3/4K$, so can one modify the stiffness factors of other members under certain conditions.



DISTRIBUTION TABLE

	A	B	B	C	C	D
D.F.		0·25	0·75	0·75	0·25	
F.E.M.			−20·00	+20·00		
Dist.		+5·00	+15·00	−15·00	−5·00	
C.O.	+2·50		−7·50	+7·50		−2·50
Dist.		+1·87	+5·63	Figures can be repeated here with opposite signs		
C.O.	+0·94		−2·81			
Dist.		+0·70	+2·11			
C.O.	+0·35		−1·05			
Dist.		+0·26	+0·79			
C.O.	+0·13		−0·39			
Dist.		+0·10	+0·29			
C.O.	+0·05					
Final Moments	+3·97 (Say 4)	+7·93 (8)	−7·93 (−8)	+7·93 (8)	−7·93 (−8)	−3·97 (−4)

Fig. 20.41

The portal frame of Fig. 20.41 and its load are symmetrical about the centre of the beam BC.

If, in this case, the stiffness factor for BC is taken as $K/2$, then there is no need to carry-over between B and C.

As $\quad K_{AB} : K_{BC}/2 : K_{CD} = 1 : 1.5 : 1,$

$$\mathrm{DF}_{BA} = \mathrm{DF}_{CD} = \frac{1}{1 + 1.5} = 0.4$$

and $\quad \mathrm{DF}_{BC} = \mathrm{DF}_{CB} = 1 - 0.4 = 0.6$

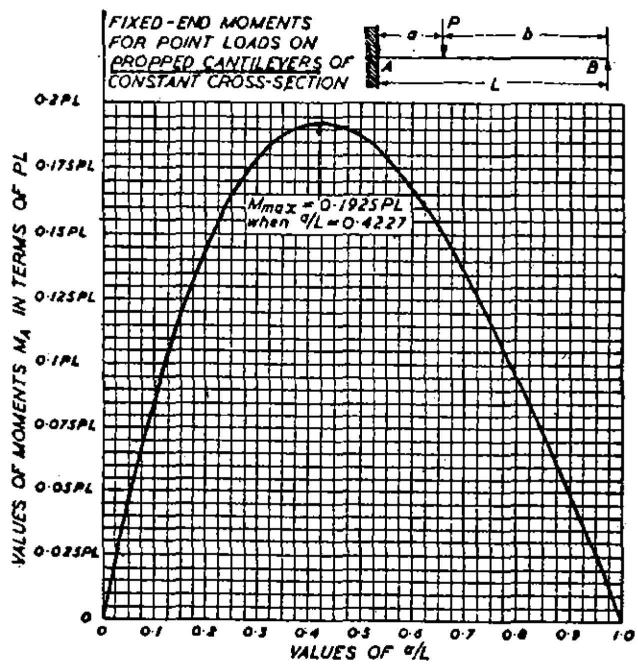


Fig. 20.40

Employing this method a very short distribution table results as shown in Fig. 20.42.

The final moments are shown in Fig. 20.42, as before.

The above procedure may be adopted for all members which are subject to *equal* end rotations in opposite directions.

DISTRIBUTION TABLE

	A	B	B	C	C	D
D.F.		0·4	0·6	0·6	0·4	
F.E.M.			−20·00	+20·00		
Dist.		+8·00	+12·00	−12·00	−8·00	
C.O	+4·00					−4·00
Final Moments	+4·00	+8·00	−8·00	+8·00	−8·00	−4·00

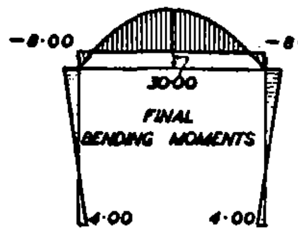


Fig. 20.42

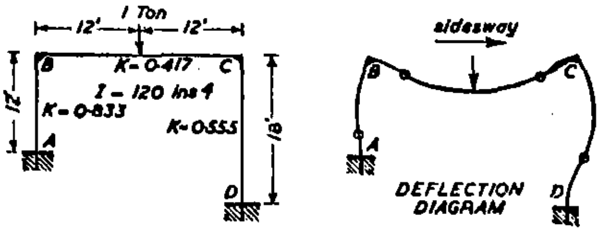


Fig. 20.43

FIXED-END MOMENTS

PROPPED CANTILEVERS OF CONSTANT CROSS-SECTION

Symmetrical loadings	Values of fixing moment M_A*
Total U.D.L. = W; A, B; L	$-\frac{WL}{8}$
W/2, W/2; A, B; a, L-2a, a	$-\frac{Wa}{8L}(3L-2a)$
W; A, B; a, b, a	$-\frac{W}{16L}(3L^2-b^2)$
W; A, B; L	$-\frac{5WL}{32}$
W; A, B; a, L-2a, a	$-\frac{W}{32L}(5L^2+4aL-4a^2)$
W/2, W/2; A, B	$-\frac{3WL}{32}$
W/2, W/2; A, B; a, L-2a, a	$-\frac{Wa}{8L}(2L-a)$
W/2, W/2; A, B; a, a	$-\frac{Wa}{8L}(4L-3a)$
W; A 2nd degree parabola B	$-\frac{3WL}{20}$
W/2, W/2; A complement of parabola B	$-\frac{3WL}{40}$
P; A, B	$-\frac{3PL}{16}$
P, P; A, B; a, L-2a, a	$-\frac{3Pa}{2L}(L-a)$
(n-1) forces; P, P, P, P, P; A, B; L/n	$-\frac{PL}{8n}(n^2-1)$
n forces; P, P, P, P, P, P; L/n, L/2n	$-\frac{PL}{16n}(2n^2+1)$
A Any symmetrical loading B	$-3A_s/2L$ Where A_s is the area of the 'free' bending moment diagram

* For cantilevers of opposite hand, the fixing moments M_B are of opposite sign.

FIXED-END MOMENTS

PROPPED CANTILEVERS OF CONSTANT CROSS-SECTION

Asymmetrical loadings	Values of fixing moment M_A**
W_1; A, B; L/2, L/2	$-\frac{9}{64}WL$
W_1; A, B; L/2, L/2	$-\frac{7}{64}WL$
W_1; a/L=m; A, B; a, L-a	$-\frac{Wa}{8}(2-m)^2$
b/L=m; W_1; A, B; a, b	$-\frac{Wb}{8}(2-m^2)$
W_1; A, B; a, b, c; d	$-\frac{W}{8L^2b}(d^2-c^2)(2L^2-c^2-d^2)$
W; A, B; L	$-\frac{2}{15}WL$
W; A, B	$-\frac{7}{60}WL$
W; A, B; a, b	$-\frac{Wa}{60L^2}(3a^2-15aL+20L^2)$
W; A, B; a, b	$-\frac{Wb}{15L^2}(5L^2-3b^2)$
W; a/L=m; A, B; a, b	$-Wa\left(\frac{m^2}{5}-\frac{3m}{4}+\frac{2}{3}\right)$
W; A, B; a, b	$-\frac{Wb}{60L^2}(10L^2-3b^2)$
a/L=n; +M; A, B; a, b	$+\frac{M}{2}(2-6n+3n^2)$
P; A, B; a, b	$-\frac{Pb}{2L^2}(L^2-b^2)$
W; A, B; $\bar{x}$; L	$-\frac{3A_s\bar{x}}{L^2}$ Where A_s is the area of the 'free' B.M. diagram, considering AB as a beam, and $\bar{x}$ is the distance from B to its centroid

** For cantilevers of opposite hand, the fixing moments M_B are of opposite sign.

Fig. 20.39

EXAMPLE when a portal frame is asymmetrical in shape or is asymmetrically loaded, it tends to sway to one side, and analysis by moment distribution has to be carried out in two stages. In the first stage the moments are derived assuming that the frame is propped against sway, while in the second moments induced by the sway are calculated.

Consider the frame shown in Fig. 20.43, which has a constant I of 120 in.^{4}

$$K_{AB} = \frac{120}{144} = 0.833$$
$$K_{BC} = \frac{120}{288} = 0.417$$
$$K_{CD} = \frac{120}{216} = 0.555$$

Hence $\quad \mathrm{DF}_{BA} = \dfrac{0.833}{0.833 + 0.417} = 0.67$

$$\mathrm{DF}_{BC} = 0.33$$
$$\mathrm{DF}_{CB} = \frac{0.417}{0.417 + 0.555} = 0.43$$
$$\mathrm{DF}_{CD} = 0.57$$

Now $\quad \mathrm{FEM}_{BC} = -FEM_{CB} = -\dfrac{WL}{8} = -\dfrac{1 \times 24}{8} = -3.00$ tons-ft

When the frame is prevented from swaying, the moments are those obtained in Fig. 20.44 (Stage I).

Distribution Table for Stage I Moments

	A	B		C		D
DF		0.67	0.33	0.43	0.57	
FEM			−3.00	+3.00		
Dist.		+2.00	+1.00	−1.29	−1.71	
CO	+1.00		−0.64	+0.50		−0.85
Dist.		+0.43	+0.21	−0.21	−0.29	
CO	+0.21		−0.11	+0.11		−0.15
Dist.		+0.07	+0.04	−0.05	−0.06	
CO	+0.04					−0.03
Stage I Moments	+1.25	+2.50	−2.50	+2.06	−2.06	−1.03

Fig. 20.44

Now, a frame sways because of unbalanced horizontal thrust.

$$\text{The thrust at A} = \frac{+1.25 + 2.50}{12} = 0.31 \text{ ton}$$

$$\text{while the thrust at D} = \frac{-2.06 - 1.03}{18} = -0.17 \text{ ton}$$

difference = 0.14T

Hence, the propping force equals 0.14 ton and is required in a horizontal direction from C towards B.

The second stage of the calculations is to find what moments result when a force of 0.14 ton acts in a horizontal direction from B towards C.

Unfortunately, there is no direct method of achieving this object. Nevertheless, within the elastic range of the material, the moments produced in a frame are proportional to the applied forces. Hence, if it can be calculated that a certain BM produces a known lateral force, then the bending moment resulting from another lateral force in the same place may be calculated by proportion.

Let the frame sway an amount d along the line BC, the joints B and C being prevented from rotation, as shown in Fig. 20.45.

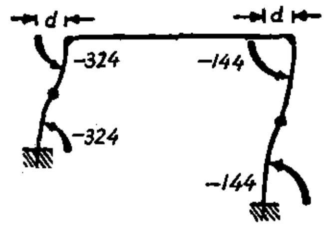


Fig. 20.45

Distribution Table for Sideway

	A	B		C		D
DF		0.67	0.33	0.43	0.57	
FEM	−324	−324	0	0	−144	−144
Dist.		+216	+108	+62	+82	
CO	+108		+31	+54		+41
Dist.		−20	−11	−23	−31	
CO	−10		−11	−5		−15
Dist.		+7	+4	+2	+3	
CO	+3					+1
Final Moments	−223	−121	+121	+90	−90	−117

Fig. 20.46

By Principle IV the moments induced in AB and CD are proportional to their I/L^2 values.

Hence,

$\text{FEM}_{AB} : \text{FEM}_{BA} : \text{FEM}_{CD} : \text{FEM}_{DC} = 1/12^2 : 1/12^2 : 1/18^2 : 1/18^2$
$= -324 : -324 : -144 : -144.$

Using these values as the arbitrary moments, release the joints B and C and calculate the resulting moments in the frame, as shown in Fig. 20.46.

The resulting shears sum up to

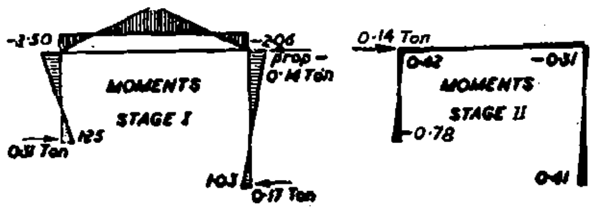
$$= \frac{-223 - 121}{12} + \frac{-90 - 117}{18}$$
$$= -40.2 \text{ tons}$$

This force is 40.2/0.14 = 287 times as greater as the propping force in Stage I. Hence, the Stage II moments are 1/287 of those calculated ahead.

The Stage I and Stage II moments are shown in Figs. 20.47 and 20.48. When added algebraically they provide the final BMs after including the simply-supported moments.

		A	B	C	D
Stage I	Moments	+1.25	−2.50	−2.06	+1.03
Stage II	Moments	−0.78	+0.42	−0.31	+0.41
Final	Moments	+0.47	−2.08	−2.37	+1.44

Fig. 20.47



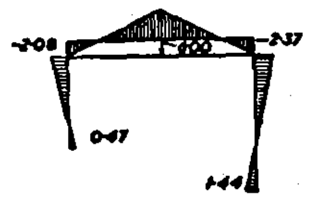


Fig. 20.48 *Final bending moments*

20.6 'SLOPE DEFLECTION' METHOD OF ANALYSIS

This method was made widely known by Maney and Wilson of Minnesota University in 1915, although it was based on the work of Mohr.

Joint rotations and deflections are treated as unknown quantities and, once these have been evaluated, the moments follow automatically by substituting the values in standard equations.

Suppose that the member AB in Fig. 20.49 is one unloaded span of a continuous beam and that the member is of constant moment of inertia. Then, for the conditions shown, the following fundamental equations hold,

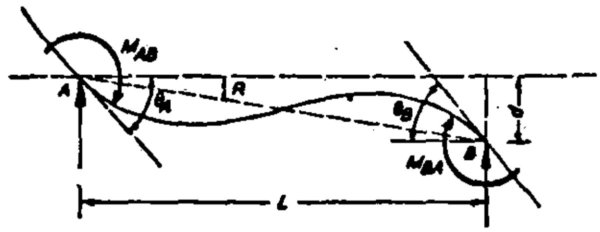


Fig. 20.49 *Slope deflection symbols*

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R)$$
and $$M_{BA} = 2EK(2\theta_B + \theta_A - 3R)$$

where E and K have the normal significance (K being I/L, the stiffness of the member).

θ_A and θ_B are the angles the joints make with the horizontal and R is the angle of rotation of B with respect to A when B sinks an amount d (i.e., $R = d/L$)

With regard to sign convention, following may be assumed here:

- θ is positive when the tangent to the beam rotates in a clockwise direction
- R is positive when the beam rotates in a clockwise direction
- M is positive when the moment acts in a clockwise direction on the beam.

Therefore, the various values of M, θ and R in Fig. 20.49 are all positive.

Suppose that the span AB carries a load acting downwards in the normal fashion. Then,

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) - \text{FEM}_{AB}$$
$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) + \text{FEM}_{BA}$$

FEM_{AB} and FEM_{BA} are the fixed-end moments which would exist if AB were a fixed-end beam. The values and signs used are precisely the same as in the moment distribution method and the tables given there (Figs 20.37 to 20.40) are equally of use for the Slope Deflection Method.

When the end A of a beam AB is hinged, the formula for the moment at the other end is modified as follows:

$$M_{BA} = EK(3\theta_B - 3R) \qquad \text{(unloaded condition)}$$
or $$M_{AB} = EK(3\theta_B - 3R) + \text{FEM}_{BA} \text{ (loaded condition)}$$

An analogy for this modification exists in moment distribution where the stiffness factor for a beam hinged at one end is reduced to $3/4K$. It should be noted that the value of the FEM is that applicable to beams hinged at one end and fixed in direction and position at the other.

The standard formulae will be applied to some of the examples which appeared in the section on moment distribution. The reader should examine especially the signs which are given to the rotation R.

As in most other methods of analysis the value of the modulus of elasticity E can be ignored in nearly every example.

When calculating the value of FEMs for loads acting downwards in the normal fashion the appropriate signs can be ignored because the fundamental formulae automatically provide the correct signs.

The final BM diagram is prepared by considering all hogging moments as negative and all sagging moments as positive.

When the method of slope deflection is used to find the moments in a continuous beam, the slope of the beam over each internal support is calculated. The value of the slopes may be useful in calculation the deflections in interior spans, but great care is needed with signs. In the slope-deflection calculations a positive value for the slope means that the beam has rotated in a clockwise direction. In the purely mathematical sense a positive slope is one 'going upwards to the right', i.e., dy/dx is positive.

Furthermore, it is essential that the units employed throughout the slope deflection calculations should be the same as those for E and I. Otherwise, the values of the slopes will not be related to the units employed in the deflection calculations.

Continuous Beams

EXAMPLE Consider the continuous beam in Fig. 20.50. Under the action of the loads, the joint B rotates in a anti-clockwise direction. When this rotation θ_B is calculated the whole beam can be analysed.

Let the suffixes 1 and 2 be applied to AB and BC respectively.

A and C are fixed in direction as well as position. Hence,

$$\theta_A = 0 = \theta_C$$

A, B and C are on the same level. Therefore,

$$R_1 = 0 = R_2$$

$$\text{FEM}_{AB} = \text{FEM}_{BA} = \frac{WL}{12} = 200 \text{ tons in.}$$

$$\text{FEM}_{BC} = \text{FEM}_{CB} = \frac{WL}{12} = 144 \text{ tons in.}$$

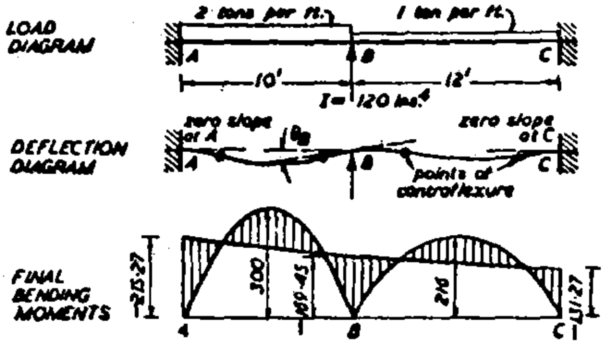


Fig. 20.50

Now $\qquad M_{BA} + M_{BC} = 0$

Hence, $2EK_1(2\theta_B + \theta_A - 3R_1) + 200 + 2EK_2(2\theta_B + \theta_C - 3R_2) - 144 = 0$

But $\qquad \theta_A = 0 = \theta_C$

$$R_1 = 0 = R_2$$

$$K_1 = \frac{I}{L} = \frac{120}{120}$$

and $\qquad K_2 = \dfrac{I}{L} = \dfrac{120}{144}$

Therefore substitution gives,

$$\left(2E \times \frac{120}{120} \times 2\theta_B\right) + \left(2E \times \frac{120}{144} \times 2\theta_B\right) = 144 - 200$$

so that $E\theta_B = -7.6364$

Using the basic formulae,

$$\begin{aligned}
M_{AB} &= 2EK_1(\theta_B) - 200 \\
&= (-2 \times 1 \times 7.6364) - 200 \\
&= -215.27 \quad \text{tons in.} \\
M_{BA} &= -M_{BC} \\
&= 2EK_1(2\theta_B) + 200 \\
&= (-2 \times 1 \times 2 \times 7.6364) + 200) \\
&= +169.45 \quad \text{tons in.} \\
M_{CB} &= 2EK_2(\theta_B) + 144 \\
&= (-2 \times 0.833 \times 7.6364) + 144 \\
&= 131.27 \quad \text{tons in.}
\end{aligned}$$

Symmetrical Portal Frames

EXAMPLE Rigid frames which are symmetrical in shape and symmetrically loaded are easy to analyse by the slope deflection method. The portal frame in Fig. 20.51 will be analysed as an example.

$$FEB_{BC} = FEM_{CB} = \frac{10 \times 24 \times 12}{12}$$
$$= 240 \text{ tons in.}$$

Let the suffixes 1, 2 and 3 be applied to the members AB, BC and CD respectively.
Then
$K_1 : K_2 : K_3 = 1 : 3 : 1$ (seeing the K values given in Fig. 20.51)

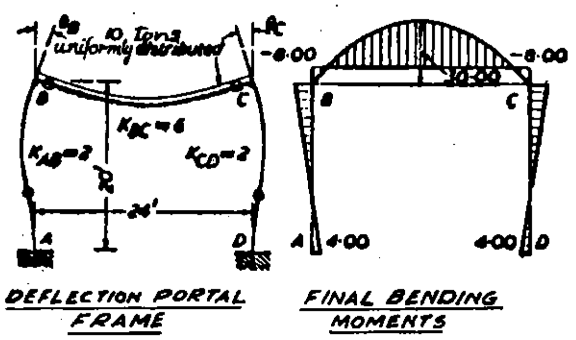


Fig. 20.51

Being symmetrical, the frame does not sway under load. In addition, it is assumed that the lengths of the members do not change.

Hence, $$R_1 = 0 = R_2 = R_3$$
$$\theta_B = -\theta_C$$

As A and D are fixed in direction as well as position,

$$\theta_A = 0 = \theta_D$$

Now $$M_{BA} + M_{BC} = 0,$$

i.e., $$2EK_1(2\theta_B) + 2EK_2(2\theta_B + \theta_C) - FEM_{BC} = 0$$
$$2EK_1(2\theta_B) + 2E(3K_1)(\theta_B) - 240 = 0$$
$$10EK_1\theta_B = 240$$
$$EK_1\theta_B = 24$$

Using the fundamental formula

$$M_{AB} = 2EK_1(\theta_B)$$
$$= 48 \text{ tons in.} = 4 \text{ tons ft.}$$

and $$M_{BA} = 2EK_1(2\theta_B)$$
$$= 96 \text{ tons in.} = 8 \text{ tons ft.}$$

Similarly, from the condition that $M_{CD} + M_{CB} = 0$, proceeding as in above we get,

$$M_{CD} = -8 \text{ tons ft}$$
$$M_{DC} = -4 \text{ tons ft}$$

The final bending moment diagram, after including the 'simply supported' moments, is drawn in Fig. 20.51.

Asymmetrical Portal Frames

EXAMPLE As the frame shown in Fig. 20.52 is not symmetrical it is less Easy to analyse than the previous example.

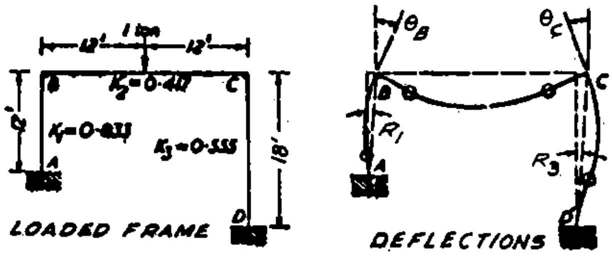


Fig. 20.52

Let the suffixes 1, 2 and 3 be applied to the members AB, BC and CD respectively.

From the data there is no slope at A and D, i.e.,

$$\theta_A = 0 = \theta_D$$

B and C are at the same height and under load it is assumed that they retain their positions relative to one another, i.e.,

$$R_2 = 0$$

$FEM_{BC} = FEM_{CB} = 36$ tons in.

The conditions of equilibrium require that,

$$\Sigma M_B = 0, \Sigma M_C = 0, \text{ and } \Sigma H = 0$$

Hence, $$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

and $$\frac{M_{AB} + M_{BA}}{L_1} + \frac{M_{CD} + M_{DC}}{L_3} = 0$$

The side-sway of B equals that of C. Hence,

$$R_1L_1 = R_3L_3$$

Using the basic formulae

$$2EK_1(2\theta_B - 3R_1) + 2EK_2(2\theta_B + \theta_C) - 36 = 0$$
$$2EK_2(2\theta_C - \theta_B) + 36 + 2EK_3(2\theta_C - 3R_3) = 0,$$

and

$$\frac{2EK_1(\theta_B - 3R_1) + 2EK_1(2\theta_B - 3R_1)}{L_1} + \frac{2EK_3(2\theta_C - 3R_3) + 2EK_3(\theta_C - 3R_3)}{L_3} = 0$$

Now $K_1 = 0.833, K_2 = 0.417, K_3' = 0.555$
and $R_1 = 1.5R_3$
Hence,

$$2\times0.833E(2\theta_B - 4.5R_3) + 2\times0.417E(2\theta_B + \theta_C) - 36 = 0$$

$$2\times0.417E(2\theta_C + \theta_B) + 2\times0.555E(2\theta_C - 3R_3) + 36 = 0,\ \text{and}$$

$$\frac{2\times0.833E}{12}(3\theta_B - 9R_3) + \frac{2\times0.555E}{18}(3\theta_C - 6R_3) = 0$$

That is

$$5E\theta_B + 0.833E\theta_C - 7.5ER_3 - 36 = 0 \quad \text{(i)}$$

$$0.833E\theta_B + 3.888E\theta_C - 3.33ER_3 + 36 = 0 \quad \text{(ii)}$$

$$\text{and } 0.417E\theta_B + 0.185E\theta_C - 1.62ER_3 = 0 \quad \text{(iii)}$$

Multiplying (ii) by 2.25,

$$1.875E\theta_B + 8.75E\theta_C - 7.5R_3 + 81 = 0 \quad \text{(iv)}$$

Subtracting (iv) from (i),

$$3.125E\theta_B - 7.917E\theta_C - 117 = 0 \quad \text{(v)}$$

Multiplying (iii) by 4.63.

$$1.93E\theta_B + 0.858E\theta_C - 7.5ER_3 = 0 \quad \text{(vi)}$$

Subtracting (vi) from (i)

$$3.07E\theta_B - 0.025E\theta_C - 36 = 0 \quad \text{(vii)}$$

Multiplying (vii) by 1.018,

$$3.125E\theta_B - 0.025E\theta_C - 36.65 = 0 \quad \text{(viii)}$$

Subtracting (viii) from (v),

$$-7.892E\theta_C - 80.35 = 0$$

Whence,

$$E\theta_C = -10.181$$
$$E\theta_B = 11.643$$
$$ER_3 = 1.831$$

and

$$ER_1 = 2.746$$

Using these values in the basic formulae

$$\begin{aligned}
M_{AB} &= 2EK_1(\theta_B - 3R_1)\\
&= 2\times0.833(11.643 - 8.238)\\
&= 5.675 \text{ tons in} = 0.47 \text{ tons ft}\\
M_{BA} &= 2EK_1(2\theta_B - 3R_1)\\
&= 2\times0.833(23.286 - 8.238)\\
&= 25.08 \text{ tons in.} = 2.09 \text{ tons ft}\\
M_{CD} &= 2EK_3(2\theta_C - 3R_3)\\
&= 2\times0.555(-20.362 - 5.493)\\
&= -28.715 \text{ tons in.} = -2.39 \text{ tons ft}\\
M_{DC} &= 2EK_3(\theta_C - 3R_3)\\
&= 2\times0.555(-10.181 - 5.493)\\
&= -17.414 \text{ tons in.} = -1.45 \text{ tons ft}
\end{aligned}$$

The final BM diagram is shown in Fig. 20.53 after including the 'simply-supported moments'

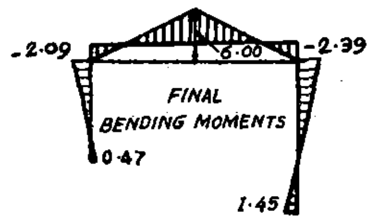


Fig. 20.53

20.7 'FLEXIBILITY' METHOD OF ANALYSIS (δ_{ik} OR V_{rs} METHOD)

This method of elastic analysis developed by Müller-Breslau and his contemporaries, known as the δ_{ik} method, provided continental engineers with a most powerful tool in structural analysis many years before it was accepted in Britain. With its basis in Influence-Coefficients, this method is becoming increasingly popular, particularly for computerised calculations. Jenkins' work on the use of matrices in the analysis of shells probably paved the way for considerable research in the use of this algebraic form in the analysis of beam and frame structures.

The result of all this work has been to produce a most versatile and comprehensive aid to structural analysis, a synthesis of δ_{ik} and matrix algebra. Briefly, the method may be explained physically as follows:

For an n times statically indeterminate structure behaving elastically under a given loading, the n compatibility equations of elastic stability are

$$V_{10} + V_{11}p_1 + V_{12}p_2 + \dots + V_{1n}p_n = 0$$
$$V_{20} + V_{21}p_1 + V_{22}p_2 + \dots + V_{2n}p_n = 0$$

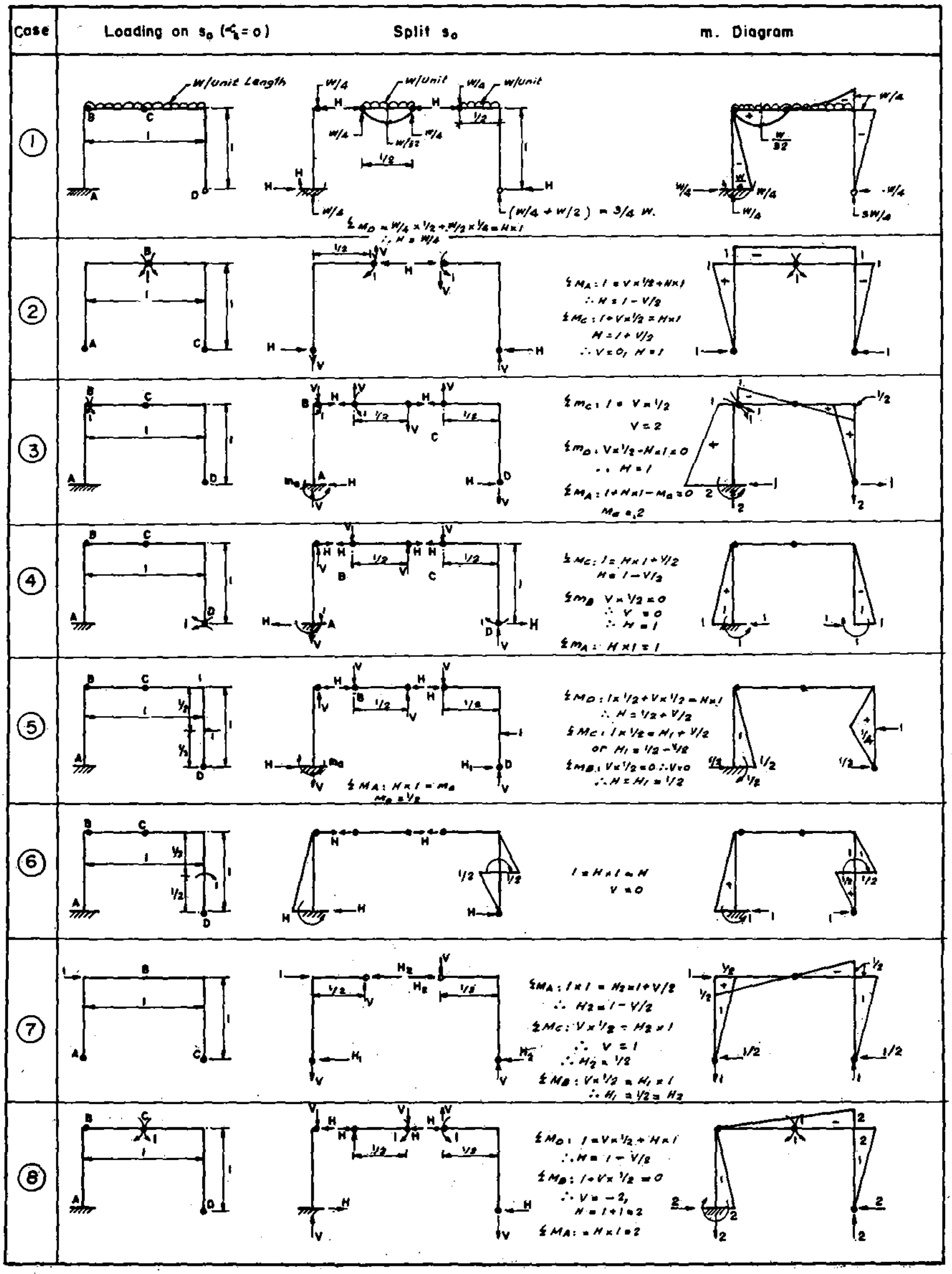


Fig. 20.55

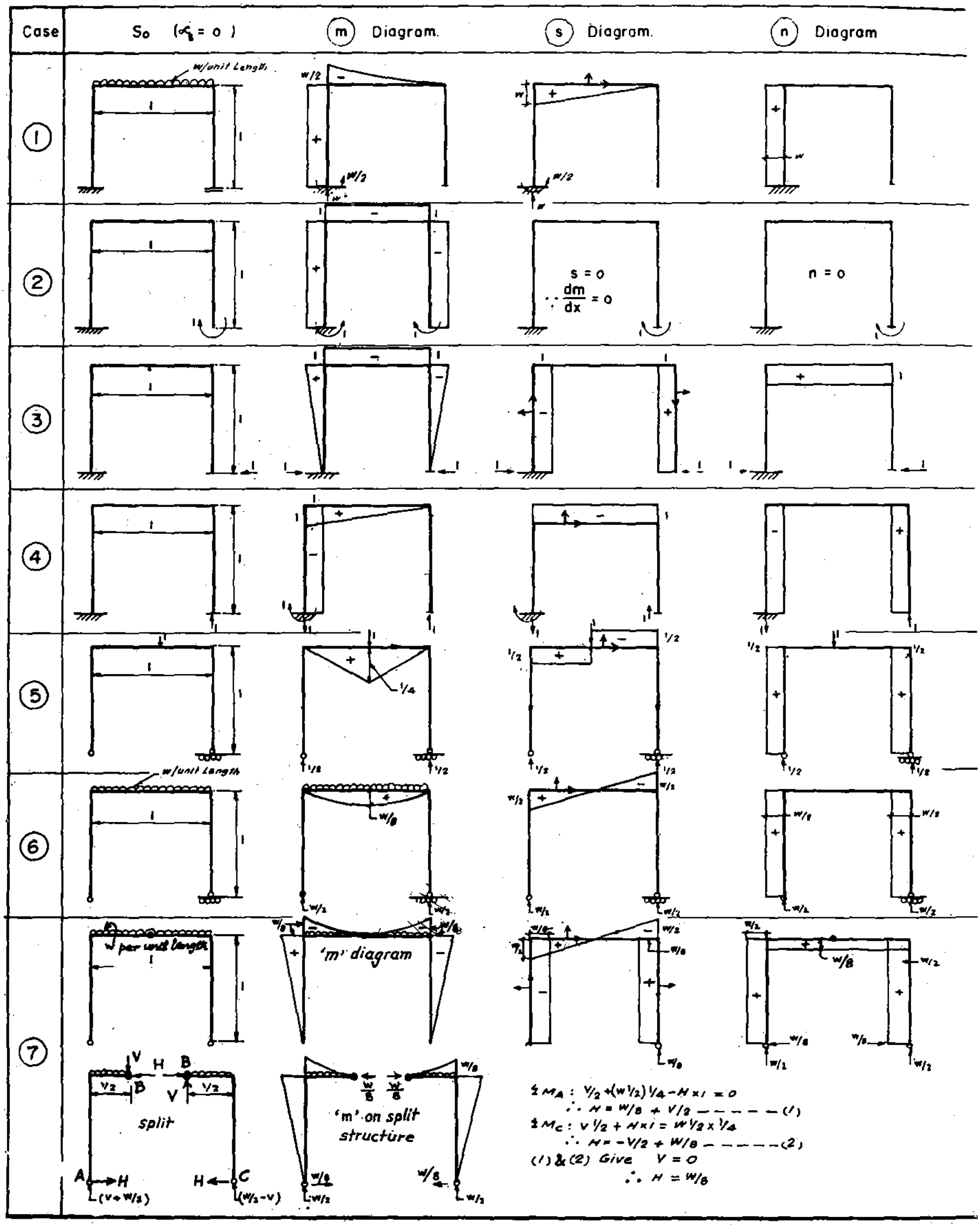

Case
So (α_s = 0)
(m) Diagram.
(s) Diagram.
(n) Diagram
w/unit Length
s = o
∴ dm/dx = o
n = o
w/unit Length
w per unit length
'm' diagram
split
'm' on split structure
ΣM_A : V/2 + (wl/2) l/4 − H × l = 0
∴ H = w/8 + V/2 ------ (1)
ΣM_C : V l/2 + H × l = w l/2 × l/4
∴ H = −V/2 + w/8 ------ (2)
(1) & (2) Give V = 0
∴ H = w/8

Fig. 20.56

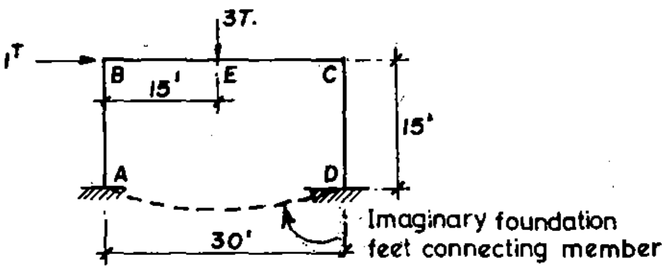


Fig. 20.57

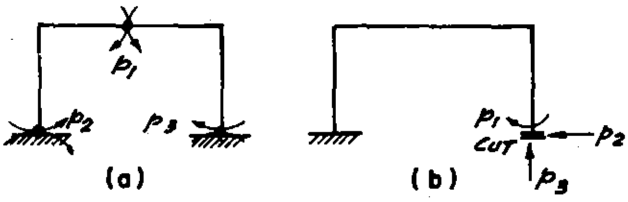


Fig. 20.58

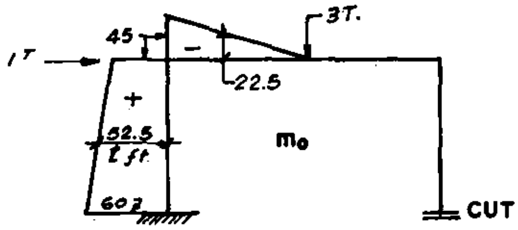


Fig. 20.59

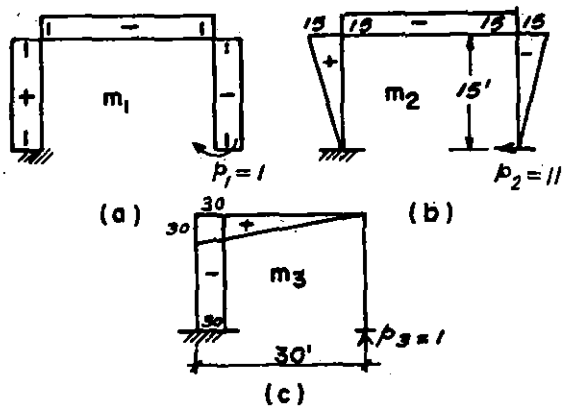


Fig. 20.60

$V_{10}, V_{20}, V_{30}, V_{11}, V_{12}, V_{13}, V_{21}, V_{22}, V_{23}, V_{31}, V_{32}$, and V_{33},

for:
$$V_{10} + V_{11}p_1 + V_{12}p_2 + V_{13}p_3 = 0$$
$$V_{20} + V_{21}p_1 + V_{22}p_2 + V_{23}p_3 = 0$$
$$V_{30} + V_{31}p_1 + V_{32}p_2 + V_{33}p_3 = 0$$

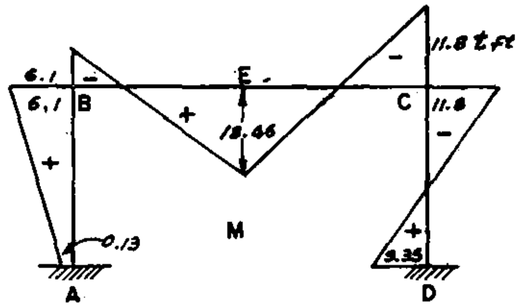


Fig. 20.61 *Final BMD*

Using Simpson's Rule (explained earlier), and retaining all units in Tons and Feet, we have,

$$V_{10} = \frac{1}{EI}\int m_1 m_0 ds = \frac{1}{EI}\Sigma m_1 m_0 ds$$
$$= \frac{1}{EI}\left[\frac{15/2}{3}\{60 \times 1 + 45 \times 1 + 52.5 \times 1\}\right.$$
$$+ \frac{15/2}{3}\{-1 \times -45 + 0 + 4\times$$
$$\left.(-22.5) \times (-1)\} + 0\right]$$
$$= 1125/EI$$

$$V_{20} = \frac{1}{EI}\int m_2 m_0 ds = \frac{1}{EI}\Sigma m_2 m_0 ds$$
$$= \frac{I}{EI}\left[\frac{15/2}{3}\{0 + 15 \times 45 + 4 \times 15/2 \times 52.5\}\right.$$
$$+ \frac{15/2}{3}\{-15 \times -45 + 0 + 4 \times (-15)\times$$
$$\left.(-22.5)\} + 0\right]$$
$$= 10,687.5/EI$$

$$V_{30} = \frac{1}{EI}\int m_3 m_0 ds = \frac{1}{EI}\Sigma m_3 m_0 ds$$
$$= \frac{1}{EI}\left[\frac{15/2}{3}\{-30 \times 60 + (-30) \times 45 + 4\times\right.$$
$$(-30) \times 52.5\} + \frac{15/2}{3}\{30 \times (-45) + 0 + 4\times$$
$$\left.22.5 \times (-22.5)\} + 0\right]$$
$$= -32062.5/EI$$

$$V_{11} = \frac{1}{EI}\int m_1 m_1 ds = \frac{1}{EI}\Sigma m_1 m_1 ds$$
$$= \frac{1}{EI}\left[\frac{15/2}{3}\left\{1 \times 1 + 1 \times 1 + 4 \times 1 \times 1\right\} + \right.$$

$$\frac{30/2}{3}\{-1\times-1+(-1)\times(-1)+4(-1)\times(-1)\}+\frac{15/2}{3}\{-1\times-1+(-1)\times(-1)+4(-1)\times(-1)\}]$$

$$=60/EI$$

$$V_{12}=\frac{1}{EI}\int m_1m_2ds=\frac{1}{EI}\Sigma m_1m_2ds$$

$$=\frac{1}{EI}\left[\frac{15/2}{3}\left\{0+1\times15+4\times1\times\frac{15}{2}\right\}+\frac{30/2}{3}\{-1\times-15+(-1)\times(-15)+4(-1)\times(-15)\}+\frac{15/2}{3}\left\{0+(-15)\times(-1)+4(-1)\times\left(-\frac{15}{2}\right)\right\}\right]$$

$$=675/EI$$

$$V_{13}=\frac{1}{EI}\int m_1m_3ds=\frac{1}{EI}\Sigma m_1m_3ds$$

$$=\frac{1}{EI}\left[\frac{15/2}{3}\{-30\times1+(-30)\times1+4(-30)\times1\}+\frac{30/2}{3}\{30\times(-1)+0+4\times\frac{30}{2}\times(-1)\}+0\right]$$

$$=-900/EI$$

$V_{21}\equiv V_{12}$, already found above.

$$V_{22}=\frac{1}{EI}\int m_2m_2ds=\frac{1}{EI}\Sigma m_2m_2ds$$

$$=\frac{1}{EI}\left[\frac{15/2}{3}\{0+15\times15+4\times\frac{15}{2}\times\frac{15}{2}\}\times2+\frac{30/2}{3}\{(-15)(-15)+(-15)\times(-15)+4(-15)(-15)\}\right]$$

$$=9000/EI$$

$$V_{23}=\frac{1}{EI}\int m_2m_3ds=\frac{1}{EI}\Sigma m_2m_3ds$$

$$=\frac{1}{EI}\left[\frac{15/2}{3}\{0+15(-30)+4(-30)\times\frac{15}{2}\}+\frac{30/2}{3}\{30(-15)+0+4(30/2)\times(-15)\}+0\right]$$

$$=10125/EI$$

$V_{31}\equiv V_{13}$ already found above.

$V_{32}\equiv V_{23}$ already found above.

$$V_{33}=\frac{1}{EI}\int m_3m_3ds=\frac{1}{EI}\Sigma m_3m_3ds$$

$$=\frac{1}{EI}\left[\frac{15/2}{3}\{(-30)\times(-30)+(-30)\times(-30)+4(-30)(-30)\}+\frac{30/2}{3}\{30\times30+0+4\times\frac{30}{2}\times\frac{30}{2}\}+0\right]$$

$$=22500/EI$$

Step 4 Substitute the values of influence coefficients in the three compatibility equations described in step 3 above:

$$1125+60p_1+675p_2+(-900)p_3=0$$
$$10,687.5+675p_1+9000p_2+(-10,125)p_3=0$$
$$-32,062.5+(-900)p_1+(-10,125)p_2+22500p_3=0$$

(Note that $1/EI$ is a common factor, hence removed.) Solving these three simultaneous equations, we get:

$$p_1=-9.35$$
$$p_2=1.41$$
$$p_3=1.6875$$

Step 5 Evaluate bending moment at any section from $m_1=m_o+(m_1p_1+m_2p_2+m_3p_3)$ at the section

$$m_{t_A}=60+1\times(-9.35)+0+(-30)\times1.6875=0.13\text{ t ft}$$

$$m_{t_{BB-A}}=45+1\times(-9.35)+15\times1.41+(-30)\times1.6875=6.1\text{ t ft}$$

$$m_{t_{CC-B}}=0+(-1)(-9.35)+(-15)\times1.41+0=-11.8\text{ t ft}$$

$$m_{t_{CC-D}}=m_{t_{CC-B}}=-11.8\text{ t ft}$$

$$m_{t_D}=0+(-1)(-9.35)+0+0=-9.35\text{ t ft}$$

$$m_{t_E}=0+(-1)(-9.35)+(-15)\times1.41+\frac{30}{2}\times1.6875=13.46\text{ t ft}$$

The final BMD is shown in Fig. 20.61.

Check

If computations are all right then $\int\frac{Mm_1}{EI}ds\equiv\int\frac{Mm_2}{EI}ds\equiv\int\frac{Mm_3}{EI}ds\equiv0$. Check any one of the three, let us say the first one,

$$\frac{1}{EI}\int Mm_1 ds = \frac{1}{EI}\begin{bmatrix} AB: & \frac{15/2}{3}\left\{0.13\times 1 + 6.1\times 1 + 4\times 1\times\left(\frac{6.1+0.13}{2}\right)\right\}+ \\ BE: & \frac{15/2}{3}\left\{-6.1\times(-1) + 13.46\times(-1) + 4\times(-1)\times\left(\frac{13.46-6.1}{2}\right)\right\}+ \\ EC: & \frac{15/2}{3}\left\{13.46\times(-1) + (-11.8)(-1) + 4\times(-1)\times\left(\frac{13.46-11.8}{2}\right)\right\}+ \\ CD: & \frac{15/2}{3}\left\{9.35\times(-1) + (11.8)(-1) + 4\times(-1)\times\left(\frac{-11.8+9.35}{2}\right)\right\} \end{bmatrix}$$

$=-2.5/EI \simeq 0$, hence computations practically OK.

(Very slight deviation from zero indicates slight inaccuracy in the mensuration work done above.)

Note that total shear and thrust at any section can be easily estimated from $s_t = s_0 + s_1p_1 + s_2p_2 + s_3p_3$ and $n_1 = n_0 + n_1p_1 + n_2p_2 + n_3p_3$ after drawing the $s_1, s_2, s_3, s_0, n_1, n_2, n_3$ and n_0 diagrams as explained earlier.

Deflection δ and Rotation θ at Any Point

As per Castigliano, partial derivative of strain energy with respect to a force gives displacement (or deflection) due to the force, at the point of its application and in the direction of force

i.e. $$\delta = \frac{\partial U}{\partial P}$$

And, partial derivative of strain energy with respect to a moment gives rotation due to the moment, at the point of its application, and in direction of moment

i.e. $$\theta = \frac{\partial U}{\partial M}$$

If in any structure S_α deflection δ at any point in any direction (usually vertical or horizontal) is to be found, then apply a load P at that point in the direction of δ. With m_δ being the BM diagram on S_0 due to unit P(S_0 being the statically made determinate structure), the total BM at that point = $M = (m_0 + m_1p_1 + m_2p_2 + \ldots + m_np_n) + m_\delta P$, so that $\frac{\partial M}{\partial P} = m_\delta$

$$U = \int \frac{M^2}{2EI}ds,$$

$$\delta_{\text{due to}P} = \frac{\partial U}{\partial P} = \int \frac{M}{EI}\frac{\partial M}{\partial P}ds = \int \frac{M}{EI}m_\delta ds$$

with $P = 0$ deflection in the structure at the point where it was applied, is $\left.\right\}\ \delta = \int \frac{Mm_\delta}{EI}ds$

where $M = m_0 + m_1p_1 + m_2p_2 + \ldots + m_np_n)$ only.

Similarly, if rotation θ at any point in any direction in $S\alpha$ is to be found, apply a moment M' at the point in that direction. With m_θ = BMD on S_0 due to unit M', the total BM at that point = M = $(m_0 + m_1p_1 + m_2p_2 + m_3p_3 + \ldots + m_np_n) + m_\theta M'$, so that $\frac{\partial M}{\partial M'} = m_\theta$.

$$U = \int \frac{M^2}{2EI}ds,$$

$$\theta_{\text{due to}M'} = \frac{\partial U}{\partial M'} = \int \frac{M}{EI}\frac{\partial M}{\partial M'}ds = \int \frac{M}{EI}m_\theta ds$$

with $M' = 0$ rotation in the structure at the point where M' was applied, is $\left.\right\}\ \theta = \int \frac{Mm_\theta}{EI}ds$

where $M = m_o + m_1p_1 + m_2p_2 + \ldots + m_np_n)$ only.

Hence

(i) Deflection at any point is $\delta = \int \frac{Mm_\delta}{EI}ds$
where m_δ = BMD on S_0 due to a unit load applied at the point of δ, in direction of δ.

(ii) Rotation at any point is $\theta = \int \frac{Mm_\theta}{EI}ds$
where m_θ = BMD on S_0 due to a unit moment applied at the point of θ, in direction of θ.

EXAMPLE A steel tube having outside diameter 2 in., bore 1 1/2 in., is bent into a quadrant of 6 ft radius. One end is rigidly attached to horizontal base plate to which a tangent to that end is perpendicular, and the free end supports a load of 100 lb. Determine the vertical and horizontal deflections of the free end under this load; $E = 30 \times 10^6$ lb./sq. in

$$I = (\pi/64)(2^4 - 1.5^4)$$
$$= (\pi/64)(4 - 2.25)(4 + 2.25)$$
$$= 0.537 \text{ in.}^4$$
$$x = 72\sin\theta \text{ (Fig. 20.62)}$$
$$y = 72(1 - \cos\theta)$$

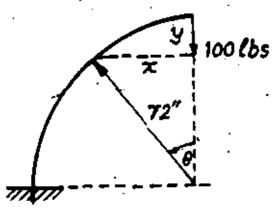


Fig. 20.62

$$M = 100x = 100 \times 72 \sin\theta$$
$$ds = 72 d\theta \text{ (from } R\theta)$$

$m_{\delta_v} = 1 \cdot x = x$, ∴ Vertical deflection $= \int \dfrac{M m_{\delta_v}}{EI} ds$

$$= \int M x ds/EI = \int_0^{\pi/2} = \frac{100 \times 72^3 \sin^2\theta d\theta}{30 \times 10^6 \times 0.537}$$
$$= 2.32 \int_0^{\pi/2} \frac{1 - \cos 2\theta}{2} d\theta$$
$$= 2.32 \times \pi/4$$
$$= 1.82 \text{ in.}$$

$m_{\delta_H} = 1 \cdot y = y$, ∴ Horizontal deflection $= \int \dfrac{M m_{\delta_H}}{EI} ds$

$$= \int M y ds/EI = \frac{100 \times 72^3}{30 \times 10^6 \times 0.537}$$

$$\int_0^{\pi/2} \sin\theta(1 - \cos\theta) d\theta$$
$$= 2.32[-\cos\theta + 1/4 \cos 2\theta]_0^{\pi/2}$$
$$= 2.32 \times 1/2$$
$$= 1.16 \text{ in.}$$

EXAMPLE Find moments set up in the frame shown in Fig. 20.63(a) (fixed at A and pinned at D), the horizontal deflection δ_C at C and the rotation θ_D at D. Assume EI is constant throughout.

In any case it will be necessary to first solve the structure and draw the bending moment diagram M whereafter mating this M-diagram with m_δ and $1/EI$ diagrams will give deflection and mating it with m_θ and $1/EI$ diagrams will give rotation

$$\delta_C = \int \frac{M m_\delta}{EI} ds$$
$$\theta_D = \int \frac{M m_\theta}{EI} ds$$

where $m\delta$ is the BMD on the released (i.e., statically made determinate) structure S_0 due to a unit load applied at C in the direction of the required deflection (horizontal in the present case).

m_θ is the BMD on S_0 due to a unit couple applied at D in the direction of the required rotation.

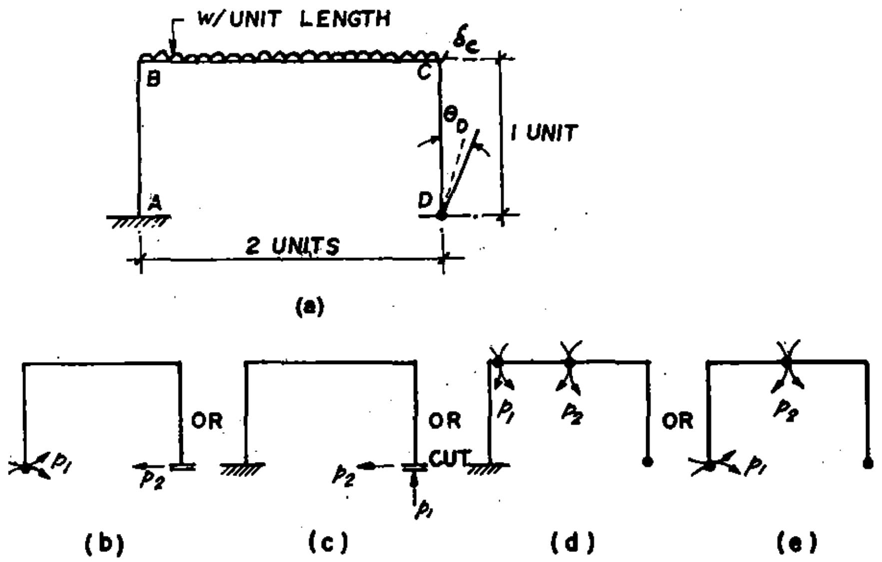


Fig. 20.63

So first we analyse the structure to obtain M, the final BMD, due to the applied loading.

$$\alpha_s = 3(M - N + 1) - R = 3(4 - 4 + 1) - 1 = 2$$

The 2 releases could be introduced in any of the 4 ways shown in Fig. 20.63(b) to 20.63(e) [easiest one is system (c)]. However, for the sake of understanding how to draw the more complicated statical diagrams m_0, m_1 and m_2, let us select the release system shown in Fig. 20.63(d). The corresponding m_0, m_1 and m_2 diagrams on S_0, are shown in Fig. 20.64.

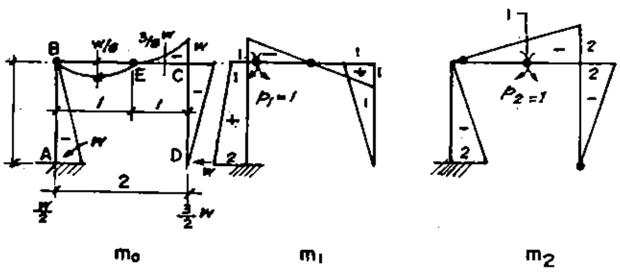


Fig. 20.64

Two compatibility equations are:

$$V_{10} + V_{11}p_1 + V_{12}p_2 = 0$$

and $$V_{20} + V_{21}p_1 + V_{22}p_2 = 0$$

Now first evaluate the involved influence coefficients

$$V_{10} = \frac{1}{EI}\int m_1 \cdot m_0\, ds = \frac{1}{EI}\Sigma m_1 m_0\, ds$$

$$= \frac{1}{EI}\left[\underbrace{\frac{1/2}{3}\left\{-2w + 0 - 4 \times \frac{w}{2} \times \frac{3}{2}\right\}}_{AB} + \overbrace{\frac{1/2}{3}\left\{0 + 0 - 4 \times \frac{w}{8} \times \frac{1}{2}\right\}}^{BE} + \overbrace{\frac{1/2}{3}\left\{0 - 4 \times \frac{3}{8}w \times \frac{1}{2} - w\right\}}^{EC} + \overbrace{\frac{1/2}{3}\left\{-w + 0 - 4 \times \frac{w}{2} \times \frac{1}{2}\right\}}^{CD}\right]$$

$$= -\frac{9w}{6EI}$$

$$V_{20} = \frac{1}{EI}\int m_2 m_0\, ds = \frac{1}{EI}\Sigma m_2 m_0\, ds$$

$$= \frac{1}{EI}\left[\overbrace{\frac{1/2}{3}\left\{2w + 0 + 4 \times \frac{w}{2} \times 1\right\}}^{AB} + \underbrace{\frac{1/2}{3}\left\{0 + 0 - 4 \times \frac{w}{8} \times \frac{1}{2}\right\}}_{BE} + \underbrace{\frac{1/2}{3}\left\{0 + 2w + 4 \times \frac{3}{8}w \times \frac{3}{2}\right\}}_{EC} + \underbrace{\frac{1/2}{3}\left\{2w + 0 + 4 \times \frac{w}{2} \times 1\right\}}_{CD}\right]$$

$$= \frac{12w}{6EI}$$

$$V_{11} = \frac{1}{EI}\int m_1 m_1\, ds = \frac{1}{EI}\Sigma m_1 m_1\, ds$$

$$= \frac{1}{EI}\left[\overbrace{\frac{1/2}{3}\left\{2 \times 2 + 1 \times 1 + 4 \times \frac{3}{2} \times \frac{3}{2}\right\}}^{AB} + \underbrace{\frac{2/2}{3}\{1 \times 1 + 0 + 1\}}_{BC} + \underbrace{\frac{1/2}{3}\left\{1 + 0 + 4 \times \frac{1}{2} \times \frac{1}{2}\right\}}_{CD}\right]$$

$$= \frac{20}{6EI}$$

$$V_{12} = \frac{1}{EI}\int m_1 m_2\, ds = \frac{1}{EI}\Sigma m_1 m_2\, ds$$

$$= \frac{1}{EI}\left[\underbrace{\frac{1/2}{3}\left\{-4 + 0 - 4 \times \frac{3}{2}\right\}}_{AB} + \overbrace{\frac{2/2}{3}\{0 + 0 - 2\}}^{BC} + \underbrace{\frac{1/2}{3}\left\{-2 + 0 - 4 \times 1 \times \frac{1}{2}\right\}}_{CD}\right]$$

$$= -\frac{18}{6EI} \equiv V_{21}$$

$$V_{22} = \frac{1}{EI}\int m_2 m_2\, ds = \frac{1}{EI}\Sigma m_2 m_2\, ds$$

$$= \frac{1}{EI}\left[\frac{1/2}{3}\{2\times 2+0+4\times 1\times 1\}\right.$$
$$+\frac{2/2}{3}\{0+2\times 2+4\times 1\times 1\}$$
$$\left.+\frac{1/2}{3}\{2\times 2+0+4\times 1\times 1\}\right]$$
$$=\frac{32}{6EI}$$

Substituting in the aforementioned two compatibility equations:

$$\left.\begin{aligned}-\frac{9w}{6}+\frac{20}{6}p_1-\frac{18}{6}p_2 &= 0\\ \frac{12w}{6}-\frac{18}{6}p_1+\frac{32}{6}p_2 &= 0\end{aligned}\right\}\ \text{whence}\ p_1=\frac{18w}{79},\quad p_2=\frac{-39w}{158}$$

Hence total moments:

$$m_{tA} = -w+2\times\frac{18w}{79}-2\times\frac{-39w}{158}$$
$$= -\frac{4w}{79}$$
$$m_{tB_{B-A}} = 0+1\times\frac{18w}{79}+0=\frac{18w}{79}$$
$$m_{t_C} = -w+1\times\frac{18w}{79}-2\times-\frac{39w}{158}=\frac{22w}{79}$$
$$m_{t_D} = 0+0+0=0$$
$$m_{t_E} = 0+0+(-1)\times\frac{-39w}{158}$$
$$=\frac{39w}{158}$$

The final BMD is therefore as shown in Fig. 20.65, and this represents the M diagram for the subsequent matings in the deflection and rotation computations as shown below.

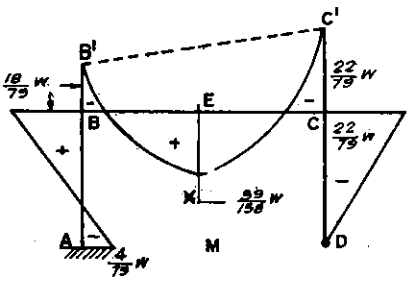


Fig. 20.65 *Final BMD*

Deflection at C in horizontal direction (δ_c) Figure 20.66 shows the m_δ diagram due to a unit horizontal force applied at C in horizontal direction, in the released structure S_0 (a simpler release system is considered here, the one shown in Fig. 20.63(c), hence:

$$\delta_c = \int\frac{Mm\delta}{EI}ds = \frac{1}{EI}\Sigma Mm\delta\, ds$$
$$= \frac{1}{EI}\left[\frac{1/2}{3}\left\{1\times\left(\frac{-4}{79}w\right)\right.\right.$$
$$\left.\left.+\ 0+4\times\frac{1}{2}\times\left(\frac{18-4}{79\times 2}\right)w\right\}\right]$$
$$=\frac{5w}{237EI}\ \text{in ft. units if } w \text{ in } t/ft \text{ and}$$

E and I in t and ft. units.

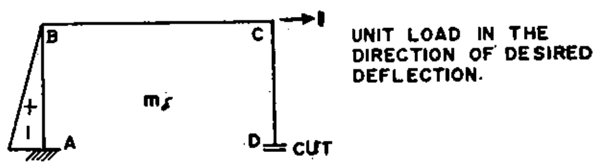


Fig. 20.66

Rotation at D in clock-wise direction: θ_D Figure 20.67 shows the m_θ diagram due to a unit couple applied at D in the clockwise direction, in the released structure (release system as in Figure 20.63(c), hence:

$$\theta_D = \int\frac{Mm_\theta}{EI}ds = \frac{1}{EI}\Sigma Mm_\theta\, ds$$
$$= \frac{1}{EI}\left[\frac{1/2}{3}\left\{1\times\left(\frac{-4}{79}w\right)+1\times\frac{18w}{79}\right.\right.$$
$$\left.+\ 4\times 1\times\left(\frac{18-4}{79\times 2}\right)w\right\}$$
$$+\frac{2/2}{3}\left\{1\times\frac{18w}{79}+1\times\frac{22w}{79}\right.$$
$$\left.+\ 4\times 1\times\left(\frac{18+22}{79\times 2}\right)w\right\}\ \text{with } BB'C'C$$
$$-\frac{2/2}{3}\{-1\times 0-1\times 0+4\times(-1)\times$$
$$\left.\left(\frac{39}{158}w+\frac{20}{79}w\right)\right\}\ \text{with parabola } B'\,X\,C'$$
$$\left.+\frac{1/2}{3}\left\{0+\frac{22}{79}w+4\times 1\times\frac{22}{79\times 2}w\right\}\right]$$
$$=\frac{16w}{237EI}\ \text{radians (if } w \text{ in t/ft, and } E \text{ and } I \text{ in}$$

t and ft. units, respectively)

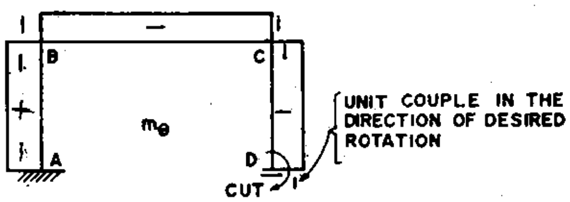


Fig. 20.67

COMBINED FLEXURE AND DIRECT STRESS IN STRUCTURES

• ***Trussed Beam***

A structure may have its ability to resist the effects of external loading conditions reinforced by the inclusion of some element or elements functioning in a different way from the original structure. As an example, a simply supported beam may be strengthened by adding a simple truss (Fig. 20.68). The added elements can take direct stress only, and the beam, while having less bending moment to resist, now has a compressive thrust as well. An extension of this early reinforcement is that which allows an initial prestress to be placed in the truss causing reverse bending to occur in the beam, see Fig. 20.69.

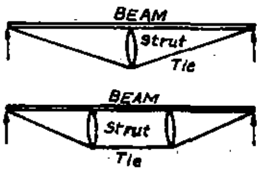


Fig. 20.68

Fig. 20.69

EXAMPLE Consider a timber beam, which has had its live load increased beyond that which it alone can safely carry. It is proposed that it be strengthened and converted to a trussed beam using steel components. Find the bending moments and forces in the resulting structures shown in Fig. 20.70.

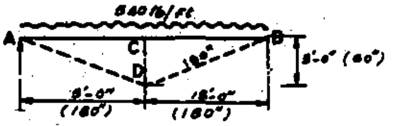


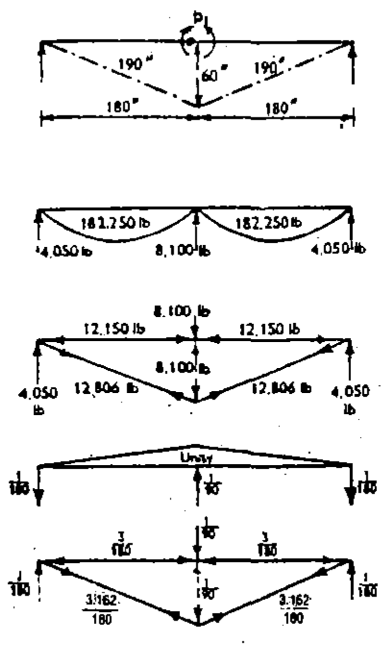
Fig. 20.70

$E_T = 1.5 \times 10^6$ p.s.i. (timber) $\quad E_S = 30 \times 10^6$ p.s.i. (Steel)

$I_T = 144 \text{ in}^4 \quad A_{ST} = 0.5 \text{ in}^2 \quad \dfrac{E_S}{E_T} = 20$

$A_T = 32 \text{ in}^2 \quad A_{SC} = 2.0 \text{ in}^2$

$$V_{10} = \int_A^B \frac{m_1 m_0}{EI}\,dx + \Sigma\frac{n_1 n_0 L}{EA} \text{ (see Fig. 20.71)}$$



Primary structure S_0, with one needed release introduced (biaction p_1)

diagram m_0 (BMD due to applied loading on released structure S_0)

diagram n_0 (due to m_0)

diagram m_1 (due to unit p_1 on S_0)

diagram n_1 (due to unit p_1 on S_0, i.e., due to m_1)

Fig. 20.71

A common factor may be removed from the term $1/E$, and the modular ratio used, but actual value for $1/I$ and $1/A$ must be used in the calculations.

$$\int_A^B \frac{m_1 m_0}{I}\,dx = -2 \times \frac{180}{3} \times \frac{182{,}250}{144} = -151{,}800$$

$$\text{and } \Sigma\frac{n_1 n_0 L}{20A} =$$

$$\left(\begin{matrix}\text{one}\\\text{strut}\end{matrix}\right) 8{,}100 \times \frac{1}{90} \times \frac{60}{2 \times 20} = +\quad 135$$

$$+\left(\begin{matrix}\text{two}\\\text{ties}\end{matrix}\right) \frac{2 \times 12{,}806 \times 3{,}162 \times 190}{0.5 \times 180 \times 20} = +\quad 8{,}610$$

$$+\left(\begin{matrix}\text{two}\\\text{beams}\end{matrix}\right) \frac{2 \times 12{,}150 \times 3 \times 180}{180 \times 32} = +\quad 2{,}278$$

$$V_{10} = -140{,}777$$

Table 20.1 *Area Integrals*

$$\int M_j \; M_k \, dx = (\text{tabulated value}) \times l$$

Nr.	TYP l	k k	k	k_1 k_2	k $-k$	k $-\frac{k}{2}$	k αl βl	$\int j^2 dx$
1	j j	jk	$\frac{1}{2} jk$	$\frac{1}{2} j(k_1+k_2)$	0	$\frac{1}{4} jk$	$\frac{1}{2} jk$	j^2
2	j	$\frac{1}{2} jk$	$\frac{1}{3} jk$	$\frac{1}{6} j(k_1+2k_2)$	$-\frac{1}{6} jk$	0	$\frac{1}{6} jk(1+\alpha)$	$\frac{1}{3} j^2$
3	j	$\frac{1}{2} jk$	$\frac{1}{6} jk$	$\frac{1}{6} j(2k_1+k_2)$	$\frac{1}{6} jk$	$\frac{1}{4} jk$	$\frac{1}{6} jk(1+\beta)$	$\frac{1}{3} j^2$
4	j_1 j_2	$\frac{1}{2} k(j_1+j_2)$	$\frac{1}{6} k(j_1+2j_2)$	$\frac{1}{6} [j_1(2k_1+k_2) + j_2(k_1+2k_2)]$	$\frac{1}{6} k(j_1-j_2)$	$\frac{1}{4} j_1 k$	$\frac{1}{6} k[j_1(1+\beta) + j_2(1+\alpha)]$	$\frac{1}{3}(j_1^2 + j_1 j_2 + j_2^2)$
5	j $-j$	0	$-\frac{1}{6} jk$	$\frac{1}{6} j(k_1-k_2)$	$\frac{1}{3} jk$	$\frac{1}{4} jk$	$\frac{1}{6} jk(1-2\alpha)$	$\frac{1}{3} j^2$
6	j $-\frac{j}{2}$	$\frac{1}{4} jk$	0	$\frac{1}{4} jk_1$	$\frac{1}{4} jk$	$\frac{1}{4} jk$	$\frac{1}{4} jk\beta$	$\frac{1}{4} j^2$
7	$-\frac{j}{2}$ j	$\frac{1}{4} jk$	$\frac{1}{4} jk$	$\frac{1}{4} jk_2$	$-\frac{1}{4} jk$	$-\frac{1}{8} jk$	$\frac{1}{4} jk\alpha$	$\frac{1}{4} j^2$
8	$\frac{l}{2}$ j	$\frac{1}{2} jk$	$\frac{1}{4} jk$	$\frac{1}{4} j(k_1+k_2)$	0	$\frac{1}{8} jk$	$\frac{jk}{12\beta}(3-4\alpha^2)$	$\frac{1}{3} j^2$
9	γl δl j	$\frac{1}{2} jk$	$\frac{1}{6} jk(1+\gamma)$	$\frac{1}{6} j[k_1(1+\delta) + k_2(1+\gamma)]$	$\frac{1}{6} jk(1-2\gamma)$	$\frac{1}{4} jk\delta$	$\frac{jk}{6\beta\gamma}(2\gamma-\gamma^2-\alpha^2)$ $\gamma \geq \alpha$	$\frac{1}{3} j^2$
10	j Quadr Parabola	$\frac{2}{3} jk$	$\frac{1}{3} jk$	$\frac{1}{3} j(k_1+k_2)$	0	$\frac{1}{6} jk$	$\frac{1}{3} jk(1+\alpha\beta)$	$\frac{8}{15} j^2$
11	j j Quadr Parabola	$\frac{1}{3} jk$	$\frac{1}{6} jk$	$\frac{1}{6} j(k_1+k_2)$	0	$\frac{1}{12} jk$	$\frac{1}{6} jk(1-2\alpha\beta)$	$\frac{1}{5} j^2$
12	j Quadr Parabola	$\frac{2}{3} jk$	$\frac{1}{4} jk$	$\frac{1}{12} j(5k_1+3k_2)$	$\frac{1}{6} jk$	$\frac{7}{24} jk$	$\frac{1}{12} jk(5-\alpha-\alpha^2)$	$\frac{8}{15} j^2$
13	j Quadr Parabola	$\frac{2}{3} jk$	$\frac{5}{12} jk$	$\frac{1}{12} j(3k_1+5k_2)$	$-\frac{1}{6} jk$	$\frac{1}{24} jk$	$\frac{1}{12} jk(5-\beta-\beta^2)$	$\frac{8}{15} j^2$

14	j Quadr Parabola	$\frac{1}{3} jK$	$\frac{1}{4} jK$	$\frac{1}{12} j(K_1+3K_2)$	$-\frac{1}{6} jK$	$-\frac{1}{24} jK$	$\frac{1}{12} jK(1+\kappa+\kappa^2)$	$\frac{1}{5} j^2$
15	j Quadr. Parabola	$\frac{1}{3} jK$	$\frac{1}{12} jK$	$\frac{1}{12} j(3K_1+K_2)$	$\frac{1}{6} jK$	$\frac{5}{24} jK$	$\frac{1}{12} jK(1+\beta+\beta^2)$	$\frac{1}{5} j^2$
16	$l/2$, $-j$, j Quadr Parabola	$\frac{1}{6} jK$	$\frac{1}{6} jK$	$\frac{1}{6} jK_2$	$-\frac{1}{6} jK$	$-\frac{1}{12} jK$	$\frac{1}{6} jK\kappa(1+2\beta)$	$\frac{1}{5} j^2$
17	$l/2$, j, $-j$ Quadr Parabola	$\frac{1}{6} jK$	0	$\frac{1}{6} jK_1$	$\frac{1}{6} jK$	$\frac{1}{6} jK$	$\frac{1}{6} jK\beta(1+2\alpha)$	$\frac{1}{5} j^2$
18	$l/2$, j_1, j_2, j_3 Quadr Parabola	$\frac{1}{6} K(j_1+4j_2+j_3)$	$\frac{1}{6} K(2j_2+j_3)$	$\frac{1}{6}[j_1K_1+2j_2(K_1+K_2)+j_3K_2]$	$\frac{1}{6} K(j_1-j_3)$	$\frac{1}{12} K(2j_1+2j_2-j_3)$	$\frac{1}{6} K[j_1\beta+2j_2+j_3\kappa-\kappa\beta(j_1-2j_2+j_3)]$	$\frac{1}{15}[2(j_1^2+4j_2^2+j_3^2)+2j_1j_2+2j_2j_3-j_1j_3]$
19	j Cubic Parabola	$\frac{1}{4} jK$	$\frac{1}{5} jK$	$\frac{1}{20} j(K_1+4K_2)$	$-\frac{3}{20} jK$	$-\frac{1}{20} jK$	$\frac{1}{20} jK(1+\kappa)(1+\kappa^2)$	$\frac{1}{7} j^2$
20	j Cubic Parabola	$\frac{1}{4} jK$	$\frac{1}{20} jK$	$\frac{1}{20} j(4K_1+K_2)$	$\frac{3}{20} jK$	$\frac{7}{40} jK$	$\frac{1}{20} jK(1+\beta)(1+\beta^2)$	$\frac{1}{7} j^2$
21	j Cubic Parabola	$\frac{1}{4} jK$	$\frac{2}{15} jK$	$\frac{1}{60} j(7K_1+8K_2)$	$-\frac{1}{60} jK$	$\frac{1}{20} jK$	$\frac{1}{20} jK(1+\alpha)\left(\frac{7}{3}-\alpha^2\right)$	$\frac{8}{105} j^2$
22	j Cubic Parabola	$\frac{1}{4} jK$	$\frac{7}{60} jK$	$\frac{1}{60} j(8K_1+7K_2)$	$\frac{1}{60} jK$	$\frac{3}{40} jK$	$\frac{1}{10} jK(1+\beta)\left(\frac{7}{3}-\beta^2\right)$	$\frac{8}{105} j^2$

		M_0, M_1, origin, l
23	m, l	$\frac{1}{3} m_1(2M_0-M_1)$
24	m_0, l	$\frac{1}{12} m_0(5M_0-M_1)$
25	m_1, l	$\frac{1}{4} m_1(M_0-M_1)$
26	m_0, m_1, l	$\frac{1}{12}[m_0(3M_0-M_1)+3m_1(M_0-M_1)]$

	a, l	d, $d+c$, l
l, o, b Quadr Parabola	$\frac{c}{3}(2a+b)$	$\left\{\frac{d}{3}(2a+b)+\frac{c}{4}(a+b)\right\}$
	27	28

$\int M_j \; M_k \; dx = (\text{tabulated value}) \times l$

$$V_{11} = \int_A^B \frac{m_1 m_1}{EI}\,dx + \Sigma \frac{n_1 n_1 L}{AE}$$

$$\int_A^B \frac{m_1 m_1}{I}\,dx = 2 \times \frac{180}{3} \times \frac{1^2}{144} = 0.8333$$

$$\text{and } \Sigma \frac{n_1 n_1 L}{20A} = \begin{pmatrix}\text{one}\\ \text{strut}\end{pmatrix} \frac{1}{90^2} \times \frac{60}{2 \times 20} = 0.0002$$

$$+\begin{pmatrix}\text{two}\\ \text{ties}\end{pmatrix} \frac{2 \times 3.162^2}{180^2} \times \frac{190}{0.5 \times 20} = 0.0117$$

$$+\begin{pmatrix}\text{two}\\ \text{beams}\end{pmatrix} \frac{2 \times 1}{60^2} \times \frac{180}{32} = 0.0313$$

$$V_{11} = 0.8765$$

$$p_1 = \frac{140{,}780}{0.8765} = 160{,}700 \text{ lb in.}$$

By making the appropriate substitutions the final bending moment diagram and final forces in the truss members can be found now.

Cable Stayed Beam

EXAMPLE A beam ABC is fixed at A and supported at B and C by steel ties anchored at a point D. Find the bending moments at A and B, and the forces in the ties (Fig. 20.72) E is constant, $I = 100$ in^4 and $A = 6.0$ in^2 for Beam. For Ties: A = 2.0 in^2.

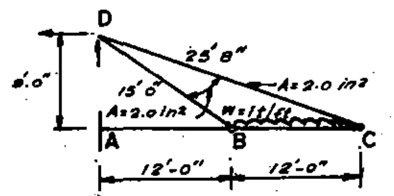


Fig. 20.72

$$V_{11} = \int_A^B \frac{m_1 m_1}{EI}\,dx + \Sigma \frac{n_1 n_1 L}{AE} \text{ (see Fig 20.73)}$$

$$\int_A^B \frac{m_1 m_1}{I}\,dx = \frac{144}{3} \times \frac{1}{100} = 0.48$$

$$\text{and } \Sigma \frac{n_1 n_1 L}{A} : (BD) = \frac{1.67^2}{144^2} \times \frac{180}{2} = 0.0121$$

$$(AB) = \frac{1.33^2}{144^2} \times \frac{144}{6} = 0.002$$

$$V_{11} = 0.4941$$

$$V_{12} = \int_A^B \frac{m_1 m_1}{EI}\,dx + \Sigma \frac{n_1 n_1 L}{AE}$$

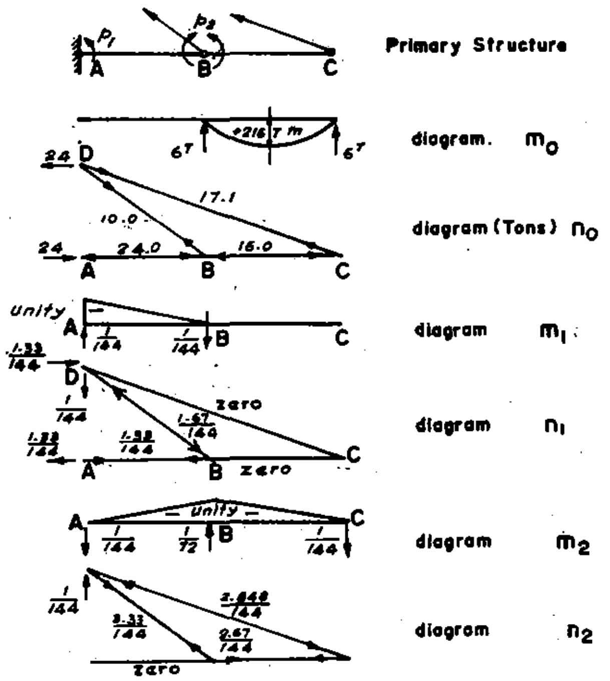


Fig. 20.73

$$\int_A^B \frac{m_1 m_2}{I}\,dx = \frac{144}{6} \times \frac{1}{100} = 0.24$$

$$\text{and } \Sigma \frac{n_1 n_2 L}{A} : (BD) = -\frac{1.67 \times 3.33}{144^2} \times \frac{180}{2} = -0.0241$$

$$V_{12} = 0.2159$$

$$V_{22} = \int_A^C \frac{m_2 m_2}{EI}\,dx + \Sigma \frac{n_2 n_2 L}{AE}$$

$$\int_A^C \frac{m_2 m_2}{I}\,dx = 2 \times \frac{144}{3} \times \frac{1}{100} = 0.96$$

$$\text{and } \Sigma \frac{n_2 n_2 L}{A} : (BD) = \frac{3.33^2}{144^2} \times \frac{180}{2} = 0.0482$$

$$(CD) = \frac{2.848^2}{144^2} \times \frac{308}{2} = 0.0595$$

$$(BC) = \frac{2.67^2}{144^2} \times \frac{144}{6} = 0.0082$$

$$V_{22} = 1.0759$$

$$V_{10} = \int_L \frac{m_1 m_0}{EI}\,dx + \Sigma \frac{n_1 n_0 L}{AE}$$

$$\int_L \frac{m_1 m_0}{I}\,dx = 0$$

and $\Sigma\frac{n_1 n_0 L}{A}$: $(BD) = -10 \times \frac{1.67}{144} \times \frac{180}{2} = -10.417$

$(AB) = -24 \times \frac{1.33}{144} \times \frac{144}{6} = -\ 5.333$

$V_{10} = -15.75$

$$V_{20} = \int_B^C \frac{m_2 m_0}{EI}\,dx + \Sigma\frac{n_2 n_0 L}{AE}$$

$$\int_B^C \frac{m_2 m_0}{I}\,dx = -\frac{144}{3} \times \frac{216}{100} = -103.68$$

and $\Sigma\frac{n_2 n_0 L}{A}$: $(BD) = \frac{3.33}{144} \times 10 \times \frac{180}{2} = +\ 20.833$

$(CD) = -\frac{2.848}{144} \times 17.1 \times \frac{308}{2} = -\ 52.1$

$(BC) = -\frac{2.67}{144} \times 16 \times \frac{144}{6} = -\ \ 7.111$

$V_{20} = -142.06$

Compatibility Equations

$$0.494p_1 + 0.216p_2 = 15.75$$
$$0.216p_1 + 1.076p_2 = 142.1$$

From which $\quad p_1 = -13.95$ ton in.
$\quad p_2 = 137.5$ ton in.

Again, by substitution the final forces and bending moments can be derived. It would seem as if in this case, the combination of flexural stress and direct stress in the beam would necessitate a review of that member's sizes.

20.8 BEAMS ON ELASTIC FOUNDATIONS

There are many problems in which a beam is supported on a compressible foundation which exerts a distributed reaction on the beam, of intensity proportional to the compressibility. In some cases the foundation can exert upward forces only, and the beam may, if sufficiently long, lose considerable contact with the foundation; in others pressure may be exerted either way. Again, the support may not be truly continuous (such as holding down a railway line) but can be replaced by an equivalent distributed support.

If y is the upward deflection of the foundation at any point and the rate of its upward reaction is $-ky$, then from $EId^2y/dx^2 = M$, $V = \frac{dM}{dx}$, and $w = \frac{dV}{dx}$, we have,

$$EId^4y/dx^4 = -ky$$

or

$$d^4y/dx^4 = -4\alpha^4 y \qquad (1)$$

where $\alpha^4 = k/4EI$, and k is the reaction force per unit length of beam per unit of its deflection from unloaded position.

A number of standard cases will now be considered.

(a) Long Beam Carrying Central Load W [Fig. 20.74(a)]

Assuming that the foundation can exert upward forces only, let $2l$ be the length of beam in contact with the foundation, and take the origin O at the left-hand end.

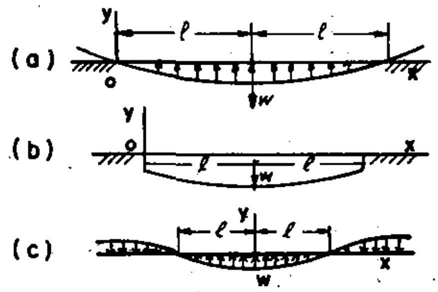


Fig. 20.74

The solution to (1) can be written as,

$$y = A\sin\alpha x\sinh\alpha x + B\cos\alpha x\sinh\alpha x + C\sin\alpha x\cosh\alpha x + D\cos\alpha x\cosh\alpha x$$

At $x = 0, y = 0$ $\qquad \therefore D = 0$

and $M = EId^2y/dx^2 = 0$ $\qquad \therefore A = 0$

also $V = EId^3y/dx^3 = 0$

giving

$$EI2\alpha^3[B(-\cos 0\cosh 0 - \sin 0\sinh 0) + C(-\sin 0\sinh 0 + \cos 0\cosh 0)] = 0$$

i.e. $\qquad C = B$

The equation is now reduced to

$$y = B(\cos\alpha x\sinh\alpha x + \sin\alpha x + \sin\alpha x\cosh\alpha x)$$

At $x = l$, $dy/dx = 0$

$$\therefore B\alpha\cos\alpha l\cosh\alpha l = 0$$

The least solution of this is $\alpha l = \pi/2$ which determines the length in contact with the ground. The value of the constant B is obtained from the condition that the shear force at the centre is $W/2$, since by symmetry it must be numerically the same on either side of the load and it must change by an amount W on passing through the load. Hence,

$$\begin{aligned} W/2 &= EId^3y/dx^3 \text{ at midspan} \\ &\quad (\text{where } x = l \text{ and } \alpha x = \pi/2) \\ &= -EI4\alpha^3 B \sin\alpha l \sinh\alpha l \\ \text{or } B &= -W\alpha/2k \sinh \pi/2 \end{aligned}$$

The maximum deflection and bending moment are at the centre, $\alpha x = \pi/2$, and are

$$\begin{aligned} \widehat{y} &= -(W\alpha/2k) \coth \pi/2 \\ M &= EI(W\alpha^3/k) \coth \pi/2 \\ &= (W/4\alpha) \coth \pi/2 \end{aligned}$$

(b) Short Beam Carrying Central Load W [Fig. 20.74(b)]

If $\alpha l < \pi/2$ in case (a), the beam will sink below the unstressed level of the foundation at all points. Again taking the origin at the left-hand end and the overall length of beam as $2l$, the following conditions are obtained for the constants of integration of the general solution of the previous paragraph.

At $\quad x = 0, d^2y/dx^2 = 0 \qquad \therefore A = 0$

and $\quad d^3y/dx^3 = 0 \qquad \therefore B = C$

and

$$y = B(\cos\alpha x \sinh\alpha x + \sin\alpha x \cosh\alpha x) + D\cos\alpha x \cosh\alpha x$$

At $x = l, dy/dx = 0$ giving

$$B \cdot 2\sin\alpha l \sinh\alpha l + D(\sin\alpha l \cosh\alpha l + \cos\alpha l \sinh\alpha l) = 0$$

and $\quad EId^3y/dx^3 = W/2$

giving

$$-B \cdot 2\sin\alpha l \sinh\alpha l - D(\sin\alpha l \cosh\alpha l + \cos\alpha l + \sinh\alpha l) = W/4EI\alpha^3 = W\alpha/k$$

Solving for B and D gives,

$$B = -\frac{W\alpha}{k} \cdot \frac{\sin\alpha l \cosh\alpha l + \cos\alpha l \cdot \sinh\alpha l}{\sin 2\alpha l + \sinh 2\alpha l}$$

$$\text{and} \quad D = -\frac{2W\alpha}{k} \cdot \frac{\cos\alpha l + \cosh\alpha l}{\sin 2\alpha l + \sinh 2\alpha l}$$

The complete solution for y is now known, the maximum deflection and bending moment being under the load.

(c) Infinite Beam Carrying Load W [Fig. 20.74(c)]

Assuming that the support can exert pressure either upwards or downwards, and taking the Y axis through the load and X axis at the undeformed level, a solution of equation (1) can be written in the form

$$y = e^{\alpha x}(A\sin\alpha x + B\cos\alpha x) + e^{-\alpha x}(C\sin\alpha x + D\cos\alpha x)$$

For the length to the right of W, since $y \to 0$ as $x \to \infty$, $A = B = 0$.

At $\quad x = 0, dy/dx = 0 \quad \therefore C = D$

and $\quad EId^3y/dx^3 = -W/2$

giving $\quad C = -W/8\alpha^3EI = -W\alpha/2k$

and $\quad y = -(W\alpha/2k)e^{-\alpha x}(\sin\alpha x + \cos\alpha x)$

The distance from the load at which $y = 0$ is given by

$$\sin\alpha l + \cos\alpha l = 0$$

the least solution being $\alpha l = 3\pi/4$ (giving $2l = 3\pi/2\alpha$)

The maximum deflection and bending moment are at $x = 0$,

$$\widehat{y} = -W\alpha/2k$$

and $\quad \widehat{M} = EIW\alpha^3/k = W/4\alpha$

EXAMPLE A steel railway track is supported on timber sleepers which exert an equivalent load of 400 lb./in.length of rail per inch deflection from its unloaded position. For each rail $I = 30$ in.^{4} $Z = 10$ in.^{4} and $E = 30 \times 10^6$ lb/sq.in. If a point load of 10 tons acts on each rail, find the length of rail over which the sleepers are depressed and the maximum bending stress in the rail:

$$\begin{aligned} \alpha^4 &= k/4EI \\ &= \frac{400}{4 \times 30 \times 10^6 \times 30} \end{aligned}$$

giving $\quad \alpha = 1/54.8$

Each rail can be treated as an infinitely long beam, for which the length over which downward deflection occurs is given in (c) above:

$$\begin{aligned} 2l &= 3\pi/2\alpha \\ &= 3\pi \times 54.8/2 \\ &= 258 \text{ in.} \end{aligned}$$

$$\text{and} \quad \begin{aligned} \widehat{M} &= W/4\alpha \\ &= 10 \times 54.8/4 \\ &= 137 \text{ tons-in.} \\ f &= \widehat{M}/Z \\ &= 13.7 \text{ tons/sq. in.} \end{aligned}$$

20.9 SIMPLIFIED AIDS FOR RAPID HAND-ANALYSIS

Given here (in Tables 20.2 to 20.44) are self-explanatory sets of certain formulae, and influence lines for enabling a rapid hand-analysis of various load-effects in different types of cantilevers and beams under various types of loadings. (In propped cantilevers, fixed-beams and continuous-beams, I the moment of inertia is assumed to be constant throughout; in frames it is constant within each individual member but can be different in different members, as clearly indicated there. Young's modulus of elasticity, E, is assumed constant throughout in each case.)

Summary of these tables is given below:

• Tables 20.2 and 20.3	Free and propped cantilevers.
• Tables 20.4 and 20.5	Simply supported and fixed beams.
• Tables 20.6 and 20.7	Simple beams ~ max. moments and deflections under various loadings.
• Table 20.8	Fixed end moment coefficients (general data).
• Table 20.9 and 20.10	Fixed end moment coefficients (various loadings).
• Table 20.11	Effects of moments applied at end-supports in equal-span continuous beams
• Tables 20.12 to 20.14	Continuous beams—moments and shears from equal loads on equal spans
• Table 20.15	Rectangular box culvert (as a rigid frame cell) put to different loads, sitting on compressible or rigid ground.
• Tables 20.16 to 20.20	Three types of single-story single-bay portal frames, put to different loadings.
• Tables 20.21 to 20.44	Influence lines for bending moment, shear force and reactions for 2, 3 and 4 span continuous beams of various span ratios.

Influence Lines (Tables 20.21 to 20.44)

By Maxwell's Theorem, if a unit load at YY causes a displacement of δ at XX, then a unit load at XX causes the same displacement, δ, at YY. Influence Lines for various load effects can be evaluated by solving the structure for the load effect for each placement of a unit load on its spans one by one. (For example: the unit load may be placed at each tenth point of each span, turn by turn, and the structure analysed for each placement.) Flexibility Method of analysis is recommended.

Given in Tables 20.21 to 20.44 are coefficients by means of which the moment (positive or negative) at any interior support or at any tenth point along all spans, produced by a unit load P placed at the same or any other tenth point in the spans, can be computed. In order to list all possible values of these coefficients a horizontal tabulation is given opposite all support and tenth points for all spans. However, due to the symmetry of the structures, all moment values can be tabulated in a lesser number of vertical columns.

Values given along any one horizontal line are ordinates to the bending moment diagram produced by a unit load placed at the load point, shown at the left of the table, opposite which they are tabulated, considering the length of the shorter span as equal to unity and that of the longer spans as equal to N. Taken vertically, the values in any one column are ordinates to the influence line for the point under which they are tabulated.

Values shown in the heavily outlined frames are the largest possible at the point on the continuous beam under which they are tabulated and are produced when the load P is placed at this point. Hence, these values are ordinates to an envelope of the maximum positive moments produced by a single moving concentrated load.

The lowest line in these tables designated "Total Area", gives ordinates to the moment diagram produced by a load, uniformly distributed along the entire structure and having a value of unity per unit of shorter span length. The two lines immediately above give, respectively, the largest positive and negative moments produced by partial distribution of the unit uniform load w.

Also included in these tables are influence coefficients for all reactions, and shears adjacent to these reactions.

The following rules for use of these tables are summarized:

(i) Reactions and Shears Due to Concentrated Load—Multiply the tabulated coefficient by the weight of the concentrated load.

(ii) Reactions and Shears Due to Uniform Load—Multiply the tabulated coefficient by the product of the weight per unit length of the uniform load and the length of the *Shorter* span.

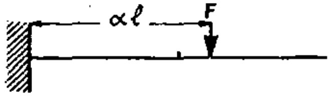
(iii) Moments Due to Concentrated Load—Multiply the tabulated moment coefficient by the product of the weight of the concentrated load and the length of the *Shorter* span.

(iv) Moments Due to Uniform Load—Multiply the tabulated moment area coefficient by the product of the weight per unit length of the uniform load and the square of the length of the *Shorter* span.

NOTE Moment of inertia is assumed constant throughout.

Table 20.2 *Moments: shears: deflections: General cases for cantilevers*

Concentrated load



Shearing forces: when $x < \alpha$, $V_x = F$;

when $x > \alpha$, $V_x = 0$

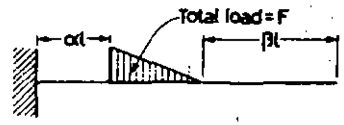
Bending moments: when $x \leqslant \alpha$, $M_x = -Fl(\alpha - x)$;

when $x \geqslant \alpha$, $M_x = 0$

Deflections: when $x \leqslant \alpha$, $a_x = -\dfrac{Fl^3}{6EI}x^2(3\alpha - x)$;

when $x \geqslant \alpha$, $a_x = -\dfrac{Fl^3}{6EI}\alpha^2(3x - \alpha)$

Partial triangular load *apex at l.h. end*



Shearing forces: when $x \leqslant \alpha$, $V_x = F$;

when $\alpha \leqslant x \leqslant (1-\beta)$, $V_x = F\left[1 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\left(2 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\right)\right]$; when $x \geqslant (1-\beta)$, $V_x = 0$

Bending moments: when $x \leqslant \alpha$, $M_x = -\tfrac{1}{3}Fl(1 + 2\alpha - \beta - 3x)$; when $x \geqslant (1-\beta)$, $M_x = 0$

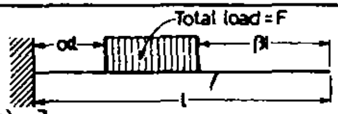
when $\alpha \leqslant x \leqslant (1-\beta)$, $M_x = -\tfrac{1}{3}Fl\left[(1 + 2\alpha - \beta - 3x) + \dfrac{(x-\alpha)^2}{(1-\alpha-\beta)}\left(3 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\right)\right]$;

Deflections: when $x \leqslant \alpha$, $a_x = (1 + 2\alpha - \beta - x)Fl^3x^2/6EI(1 + 2\alpha - \beta - x)$;

when $\alpha \leqslant x \leqslant (1-\beta)$, $a_x = -\dfrac{Fl^3}{60EI}\left\{10x^2(1 + 2\alpha - \beta - x) + \dfrac{(x-\alpha)^4}{(1-\alpha-\beta)}\left[5 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\right]\right\}$;

when $x \geqslant (1-\beta)$, $a_x = -\dfrac{Fl^3}{60EI}\{\alpha[(1-\beta)^2 + 10x\alpha - 3\alpha^2] - (1+\alpha-\beta)^2(1+\alpha-\beta-5x)\}$

Partial uniform load



Shearing forces: when $x \leqslant \alpha$, $V_x = F$;

When: $\alpha \leqslant x \leqslant (1-\beta)$, $V_x = F\left[1 - \dfrac{(x-\alpha)}{(1-\alpha-\beta)}\right]$; when $x \geqslant (1-\beta)$, $V_x = 0$

Bending movements when. $x \leqslant \alpha$, $M_x = -Fl[\tfrac{1}{2}(1 + \alpha - \beta) - x]$;

When $(\alpha \leqslant x \leqslant 1-\beta)$, $M_x = -\tfrac{1}{2}Fl\left[(1 + \alpha - \beta - 2x) + \dfrac{(x-\alpha)^2}{(1-\alpha-\beta)}\right]$; when $x \geqslant (1-\beta)$, $M_x = 0$

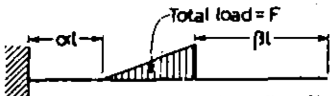
Deflections: when $x \leqslant \alpha$, $a_x = -\dfrac{Fl^3x^2}{12EI}[3(1 + \alpha - \beta) - 2x]$;

when $\alpha \leqslant x \leqslant (1-\beta)$, $a_x = -\dfrac{Fl^3}{24EI}\left[6x^2(1 + \alpha - \beta - \tfrac{2}{3}x) + \dfrac{(x-\alpha)^4}{(1-\alpha-\beta)}\right]$;

when $x \geqslant (1-\beta)$,

$$a_x = -\frac{Fl^3}{24EI}\left[2\alpha(1-\beta)(1 + \alpha - \beta - 2x) - (1 + \alpha - \beta)^2(1 + \alpha - \beta - 4x)\right]$$

Partial triangular load *apex at r.h. end*



Shearing forces: when $x \leqslant \alpha$, $V_x = F$;

when $\alpha \leqslant x \leqslant (1-\beta)$, $V_x = F\left[1 - \dfrac{(x-\alpha)^2}{(1-\alpha-\beta)^2}\right]$; when $x \geqslant (1-\beta)$, $V_x = 0$

Bending moments: when $x \leqslant \alpha$, $M_x = -\tfrac{1}{3}Fl(2 + \alpha - 2\beta - 3x)$;

when $\alpha \leqslant x \leqslant (1-\beta)$, $M_x = -\tfrac{1}{3}Fl\left[(2 + \alpha - 2\beta - 3x) + \dfrac{(x-\alpha)^3}{(1-\alpha-\beta)^2}\right]$

when $x \geqslant (1-\beta)$, $M_x = 0$

Deflections: when $x \leqslant \alpha$, $a_x = -\dfrac{Fl^3x^2}{6EI}(2 + \alpha - 2\beta - x)$;

when $\alpha \leqslant x \leqslant (1-\beta)$, $a_x = -\dfrac{Fl^3}{60EI}\left[10x^2(2 + \alpha - 2\beta - x) + \dfrac{(x-\alpha)^5}{(1-\alpha-\beta)^2}\right]$;

when $x \geqslant (1-\beta)$,

$$a_x = -(Fl^3/60EI)\{(1-\beta)[\alpha^2 + 10x(1-\beta) - 3(1-\beta)^2] - (1+\alpha-\beta)^2(1+\alpha-\beta-5x)\}$$

Key to sign convention for Tables 20.2 to 20.5

	Reaction	Shearing force	Bending moment	Slope	Deflection
Positive	↑	↑ ↓			
Negative	↓	↓ ↑			

For Tables 20.2 to 20.5 ℓ = span

F total load

x distance of point considered from left-hand support in terms of l

Members with fixed ends To determine deflection, moment, etc, for member with one or both ends fixed or continuous, first calculate deflection, moment, etc, for freely-supported span. Next, determine deflection, moment, etc, throughout span due to action of support moments only. Lastly, obtain final values of deflection, moment, etc, by summing foregoing results algebraically.

Slope To determine slope at any point, distance xl from left-hand support, differentiate expression for deflection with respect to x.

Table 20.3 *Moments: shears: deflections: Special cases for cantilevers*

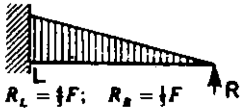
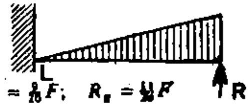
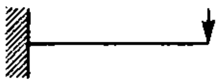
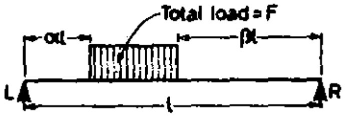
	Simple cantilever (fixed at left-hand end)		Propped cantilever (fixed at left-hand support)	
Uniform load	 Shearing forces: $V_x = F(1-x)$ Bending moments: $M_x = -\frac{1}{2}Fl(1-x)^2$ $M_{x\,max} = -\frac{1}{2}Fl$ at $x = 0$ Slopes: $\theta_L = 0$; $\theta_R = -\frac{Fl^2}{6EI}$ Deflections: $a_x = -\frac{Fl^3x^2}{24EI}(6-4x+x^2)$ $a_{x\,max} = -\frac{Fl^3}{8EI}$ at $x = 1$		Reactions: $R_L = \frac{5}{8}F$; $R_R = \frac{3}{8}F$ Shearing forces: $V_x = F(\frac{5}{8} - x)$ Bending moments: $M_L = -\frac{FL}{8}$, $M_R = 0$ $M_x = \frac{Fl}{8}[x(5-4x)-1]$  $M_{x\,max} = \frac{9Fl}{128}$ at $x = \frac{5}{8}$ Slopes: $\theta_L = 0$; $\theta_R = \frac{Fl^2}{48EI}$ Deflections: $a_x = -\frac{Fl^3}{48EI}x^2(1-x)(3-2x)$ $a_{x\,max} \simeq -\frac{Fl^3}{185EI}$ at $x = 0.5785$ from L	
Triangular load	*Apex at l.h. end*  Shearing forces: $V_x = F(1-x)^2$ Bending moments: $M_x = -\frac{1}{3}Fl(1-x)^3$ $M_{x\,max} = -\frac{1}{3}Fl$ at $x = 0$ Slopes: $\theta_L = 0$; $\theta_R = -Fl^2/12EI$ Deflections: $a_x = -\frac{Fl^3x^2}{60EI}(10-10x+5x^2-x^3)$ $a_{x\,max} = -\frac{Fl^3}{15EI}$ at $x = 1$	*Apex at r.h. end*  Shearing forces: $V_x = F(1-x^2)$ Bending moments: $M_x = -\frac{1}{3}Fl(1-x)^2(2+x)$ $M_{x\,max} = -\frac{2}{3}Fl$ at $x = 0$ Slopes: $\theta_L = 0$: $\theta_R = -Fl^2/4EI$ Deflections: $a_x = -\frac{Fl^3x^2}{60EI}(20-10x+x^3)$ $a_{x\,max} = -\frac{11Fl^3}{60EI}$ at $x = 1$	*Apex at l.h. end*  Reactions: $R_L = \frac{4}{5}F$; $R_R = \frac{1}{5}F$ Shearing forces: $V_x = F(\frac{4}{5} - 2x + x^2)$ Bending moments: $M_L = -2Fl/15$; $M_R = 0$ $M_x = -(5x^2 - 10x + 2)(1-x)Fl/15$ $M_{x\,max} \simeq Fl/16.77$ at $x = 0.5528$ from L Slopes: $\theta_L = 0$; $\theta_R = Fl^2/60EI$ Deflections: $a_x = -x^2(1-x)(2-x)^2Fl^3/60EI$ $a_{x\,max} \simeq -\frac{Fl^3}{209.6EI}$ at $x = 0.5528$ from L	*Apex at r.h. end*  Reactions: $R_L = \frac{9}{20}F$; $R_R = \frac{11}{20}F$ Shearing forces: $V_x = F(\frac{9}{20} - x^2)$ Bending moments: $M_L = -7Fl/60$; $M_R = 0.$ $M_x = -(20x^3 - 27x + 7)Fl/60$ $M_{x\,max} \simeq Fl/11.82$ at $x = \sqrt{0.45}$ from L Slopes: $\theta_L = 0$; $\theta_R = Fl^2/40EI$ Deflections: $a_x = -x^2(1-x)(7-2x-2x^2)Fl^3/120EI$ $a_{x\,max} \simeq -\frac{Fl^3}{164EI}$ at $x = 0.5975$ from L
Concentrated load	Load at unsupported end  Shearing forces: $V_x = F$ Bending moments: $M_L = -Fl$ $M_x = -F(l-x)$ at $x = 0$ Slope: $\theta_L = 0$; $\theta_R = -\frac{Fl^2}{2EI}$ Deflections: $a_x = -\frac{Fl^3x^2}{6EI}(3-x)$ $a_{x\,max} = -\frac{Fl^3}{3EI}$ at $x = 1$		Load at centre  Reactions: $R_L = \frac{11}{16}F$; $R_R = \frac{5}{16}F$ Shearing forces: when $x < \frac{1}{2}$, $V_x = \frac{11}{16}F$; when $x > \frac{1}{2}$, $V_x = -\frac{5}{16}F$ Bending moments: $M_L = -\frac{3}{16}Fl$. $M_R = 0.$ $M_{x\,max} = \frac{5}{32}Fl$ beneath load Slopes: $\theta_L = 0$; $\theta_R = Fl^2/32EI$ Deflections: when $x < \frac{1}{2}$, $a_x = -x^2(9-11x)Fl^3/96EI$ when $x > \frac{1}{2}$, $a_x = -\frac{Fl^3}{96EI}(x-1)(5x^2-10x+2)$ $a_{x\,max} = -\frac{Fl^3}{48\sqrt{5}EI}$ at $x = \left(1 - \frac{1}{\sqrt{5}}\right)$ from L	

Table 20.4 *Moments: shears: deflections: Special cases for beams*

Partial uniform load



Reactions: $R_L = \frac{1}{2}F(1 - \alpha + \beta)$; $R_R = \frac{1}{2}F(1 + \alpha - \beta)$

Shearing forces: when $x < \alpha$, $V_x = \frac{1}{2}F(1 - \alpha + \beta)$;

when $\alpha < x < (1 - \beta)$, $V_x = F\left[\frac{1}{2}(1 - \alpha + \beta) + \frac{(\alpha - x)}{(1 - \alpha - \beta)}\right]$;

when $x > (1 - \beta)$, $V_x = -\frac{1}{2}F(1 + \alpha - \beta)$

Bending moments: when $x < \alpha$, $M_x = \frac{1}{2}Flx(1 - \alpha + \beta)$;

when $\alpha < x < (1 - \beta)$, $M_x = \frac{1}{2}Fl\left[x(1 - \alpha + \beta) - \frac{(x - \alpha)^2}{(1 - \alpha - \beta)}\right]$;

when $x > (1 - \beta)$, $M_x = \frac{1}{2}Fl(1 - x)(1 + \alpha - \beta)$

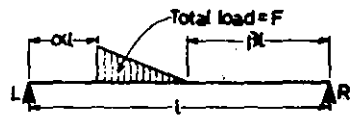
$M_{x\,max} = \frac{1}{8}(1 + \alpha + \beta)(1 + \alpha - \beta)(1 - \alpha + \beta)Fl$ at $\frac{1}{2}(1 + \alpha^2 - \beta^2)l$

Deflections: when $x < \alpha$, $a_x = -\frac{Fl^3(1 - \alpha + \beta)}{24EI}\left[(1 + 2\alpha - \alpha^2 - \beta^2)x - 2x^3\right] = a_1$;

when $\alpha < x < (1 - \beta)$, $a_x = a_1 - \frac{Fl^3(x - \alpha)^4}{24EI(1 - \alpha - \beta)}$;

when $x > (1 - \beta)$, use formula for a_1, transpose α and β and substitute $(1 - x)$ for x

Partial triangular load



Reactions: $R_L = \frac{1}{3}F(2 - 2\alpha + \beta)$; $R_R = \frac{1}{3}F(1 + 2\alpha - \beta)$

Shearing forces: when $x < \alpha$, $V_x = \frac{1}{3}F(2 - 2\alpha + \beta)$;

when $\alpha < x < (1 - \beta)$, $V_x = -F\left[\frac{1}{3}(1 + 2\alpha - \beta) - \frac{(1 - x - \beta)^2}{(1 - \alpha - \beta)^2}\right]$;

when $x > (1 - \beta)$, $V_x = -\frac{1}{3}F(1 + 2\alpha - \beta)$

Bending moments: when $x < \alpha$, $M_x = \frac{1}{3}Flx(2 - 2\alpha + \beta)$;

when $\alpha < x < (1 - \beta)$, $M_x = \frac{1}{3}Fl\left[(1 + 2\alpha - \beta)(1 - x) - \frac{(1 - x - \beta)^3}{(1 - \alpha - \beta)^2}\right]$;

when $x > (1 - \beta)$, $M_x = \frac{1}{3}Fl(1 - x)(1 + 2\alpha - \beta)$

Deflections: when $x < \alpha$,

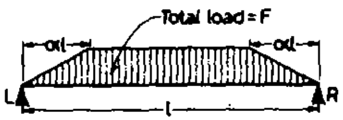
$$a_x = -\frac{Fl^3x(2 - 2\alpha + \beta)}{162EI}\left[9(1 + x)(1 - x) - (2 - 2\alpha + \beta)^2 - (1 - \alpha - \beta)^2\left(\frac{3}{2} - \frac{(1 - \alpha - \beta)}{5(2 - 2\alpha + \beta)}\right)\right] = a_1;$$

when $\alpha < x < (1 - \beta)$, $a_x = a_1 - \frac{Fl^3(x - \alpha)^4}{60EI(1 - \alpha - \beta)}\left(5 - \frac{(x - \alpha)}{(1 - \alpha - \beta)}\right)$;

when $x > (1 - \beta)$,

$$a_x = -\frac{Fl^3(1 - x)(1 + 2\alpha - \beta)}{162EI}\left[9x(2 - x) - (1 + 2\alpha - \beta)^2 - (1 - \alpha - \beta)^2\left(\frac{3}{2} + \frac{(1 - \alpha - \beta)}{5(1 + 2\alpha - \beta)}\right)\right]$$

Trapezoidal load



Reactions: $R_L = R_R = \frac{1}{2}F$

Shearing forces:

when $x < \alpha$, $V_x = \frac{1}{2}F\left(1 - \frac{x^2}{\alpha(1 - \alpha)}\right)$; when $\alpha < x \leqslant \frac{1}{2}$, $V_x = \frac{1}{2}F\frac{(1 - 2x)}{(1 - \alpha)}$

Bending moments: when $x < \alpha$, $M_x = \frac{1}{6}Flx\left(3 - \frac{x^2}{\alpha(1 - \alpha)}\right)$;

when $\alpha < x \leqslant \frac{1}{2}$, $M_x = \frac{1}{6}Fl\left[\frac{3x(1 - x) - \alpha^2}{(1 - \alpha)}\right]$ $M_{x\,max} = \frac{(3 - 4\alpha^2)}{24(1 - \alpha)}Fl$ at midspan

Deflections: when $x < \alpha$, $a_x = -\frac{Fl^3x}{24EI}\left[(1 + \alpha - \alpha^2) - 2x^2 + \frac{x^4}{5\alpha(1 - \alpha)}\right]$;

when $\alpha < x \leqslant \frac{1}{2}$, $a_x = -\frac{Fl^3}{24EI(1 - \alpha)}\left\{[1 - 2\alpha^2 + x(1 - x)]x(1 - x) + \frac{\alpha^4}{5}\right\}$

$a_{x\,max} = -\frac{(4\alpha^2 - 5)^2Fl^3}{1920(1 - \alpha)EI}$ at midspan

Concentrated load

Reactions: $R_L = F(1 - \alpha)$; $R_R = F\alpha$

Shearing forces: when $x < \alpha$, $V_x = F(1 - \alpha)$; when $x > \alpha$, $V_x = -F\alpha$

Bending moments: when $x < \alpha$, $M_x = Fl(1 - \alpha)x$;

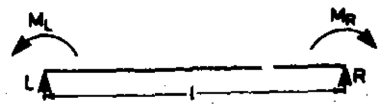
when $x > \alpha$, $M_x = Fl\alpha(1 - x)$; $M_{x\,max} = Fl\alpha(1 - \alpha)$ beneath load

Deflections:

when $x < \alpha$, $a_x = -\frac{Fl^3(1 - \alpha)x}{6EI}\left[\alpha(2 - \alpha) - x^2\right]$; when $x > \alpha$, $a_x = -\frac{Fl^3\alpha(1 - x)}{6EI}\left[x(2 - x) - \alpha^2\right]$

when $\alpha \leqslant \frac{1}{2}$, $a_{x\,max} = -\frac{Fl^3\alpha(1 - \alpha^2)^{3/2}}{9\sqrt{3}EI}$ at $\sqrt{\frac{1 - \alpha^2}{3}}\,l$ from R

Support moments



Reactions: $R_L = \frac{M_L - M_R}{l}$; $R_R = \frac{M_R - M_L}{l}$

Shearing force: $V_x = \frac{M_L - M_R}{l}$ **Bending moments:** $M_x = M_L(1 - x) + M_Rx$

Deflections: $a_x = -\frac{x(1 - x)l^2}{6EI}\,(2 - x)M_L + (1 + x)M_R$

Table 20.5 *Moments: shears: deflections: Special cases for beams.*

	Freely supported span	Fully fixed span
Uniform load	 Reactions: $R_L = R_R = \frac{1}{2}F$ Shearing forces: $V_x = F(\frac{1}{2} - x)$ Bending moments: $M_L = M_R = 0$ $M_x = \frac{1}{2}x(1-x)Fl$ $M_{x\,max} = \frac{1}{8}Fl$ at $\frac{1}{2}l$ Slopes: $\theta_L = \theta_R = \pm \frac{Fl^2}{24EI}$ Deflections: $a_x = -\frac{Fl^3 x}{24EI}(1-x)(1+x-x^2)$ $a_{x\,max} = -\frac{5Fl^3}{384EI}$ at midspan	 Reactions: $R_L = R_R = \frac{1}{2}F$ Shearing forces: $V_x = F\left(\frac{1}{2} - \frac{x}{l}\right)$ Bending moments: $M_L = M_R = -\frac{1}{12}Fl$ $M_x = \frac{1}{2}Fl[x(1-x) - \frac{1}{6}]$ $M_{x\,max} = \frac{1}{24}Fl$ at midspan Slopes: $\theta_L = \theta_R = 0.$ Deflections: $a_x = -\frac{Fl^3x^2(1-x)^2}{24EI}$ $\quad a_{x\,max} = -\frac{Fl^3}{384EI}$ at midspan
Triangular load	*Apex at l.h. end*  Reactions: $R_L = \frac{2}{3}F; \quad R_R = \frac{1}{3}F$ Shearing forces: $V_x = \frac{F}{3}(2 - 6x + 3x^2)$ Bending moments: $M_L = M_R = 0 \qquad M_x = \frac{1}{3}Flx(1-x)(2-x)$ $M_{x\,max} = \frac{2Fl}{9\sqrt{3}}$ at $\left(1 - \frac{1}{\sqrt{3}}\right)l$ from L Slopes: $\theta_L = -\frac{2Fl^2}{45EI}; \quad \theta_R = +\frac{7Fl^2}{180EI}$ Deflections: $a_x = -x(1-x)(2-x)(4+6x-3x^2)Fl^3/180EI$ $a_{x\,max} \simeq -(Fl^3/76.7EI)$ at $x \simeq 0.4807\,l$ from L	*Apex at l.h. end*  Reactions: $R_L = \frac{7}{10}F; \quad R_R = \frac{3}{10}F$ Shearing forces: $V_x = \frac{F}{10}(7 - 20x + 10x^2)$ Bending moments $M_L = -\frac{Fl}{10} \quad M_R = -\frac{Fl}{15} \qquad M_x = \frac{Fl}{30}(10x^3 - 30x^2 + 21x - 3)$ $M_{x\,max} \simeq \frac{Fl}{23.32}$ at $(1 - \sqrt{0.3})l$ from L Slopes: $\theta_L = \theta_R = 0$ Deflections: $a_x = -\frac{Fl^3}{60EI}x^2(1-x)^2(3-x) \quad a_{x\,max} \simeq -\frac{Fl^3}{382EI}$ at $x \simeq 0.4753\,l$ from L
Central concentrated load	 Reactions: $R_L = R_R = \frac{1}{2}F$ Shearing forces: when $x < \frac{1}{2}$, $V_x = \frac{1}{2}F$; when $x > \frac{1}{2}$, $V_x = -\frac{1}{2}F$ Bending moments: $M_L = M_R = 0. \qquad M_{x\,max} = \frac{1}{4}Fl$ at midspan Slopes: $\theta_L = -\frac{Fl^2}{16EI}; \quad \theta_R = +\frac{Fl^2}{16EI}$ Deflections: when $x \leqslant \frac{1}{2}$ $\quad a_x = -\frac{Fl^3}{48EI}x(3-4x^2)$ $a_{x\,max} = -\frac{Fl^3}{48EI}$ at midspan	 Reactions: $R_L = R_R = \frac{1}{2}F$ Shearing forces: when $x < \frac{1}{2}$, $V_x = \frac{1}{2}F$; when $x > \frac{1}{2}$, $V_x = -\frac{1}{2}F$ Bending moments: $M_L = M_R = -\frac{1}{8}Fl$ when $x \leqslant \frac{1}{2}$, $M_x = \frac{Fl}{8}(4x-1)$; $\qquad$ when $x \geqslant \frac{1}{2}$, $M_x = \frac{Fl}{8}(3-4x)$ $M_{x\,max} = \frac{1}{8}Fl$ at midspan Slopes: $\theta_L = \theta_R = 0$ Deflections: when $x \leqslant \frac{1}{2}$, $a_x = -\frac{Fl^3}{48EI}x^2(3-4x)$ $a_{x\,max} = -\frac{Fl^3}{192EI}$ at midspan

Table 20.6 *Freely-supported beams: Maximum moments*

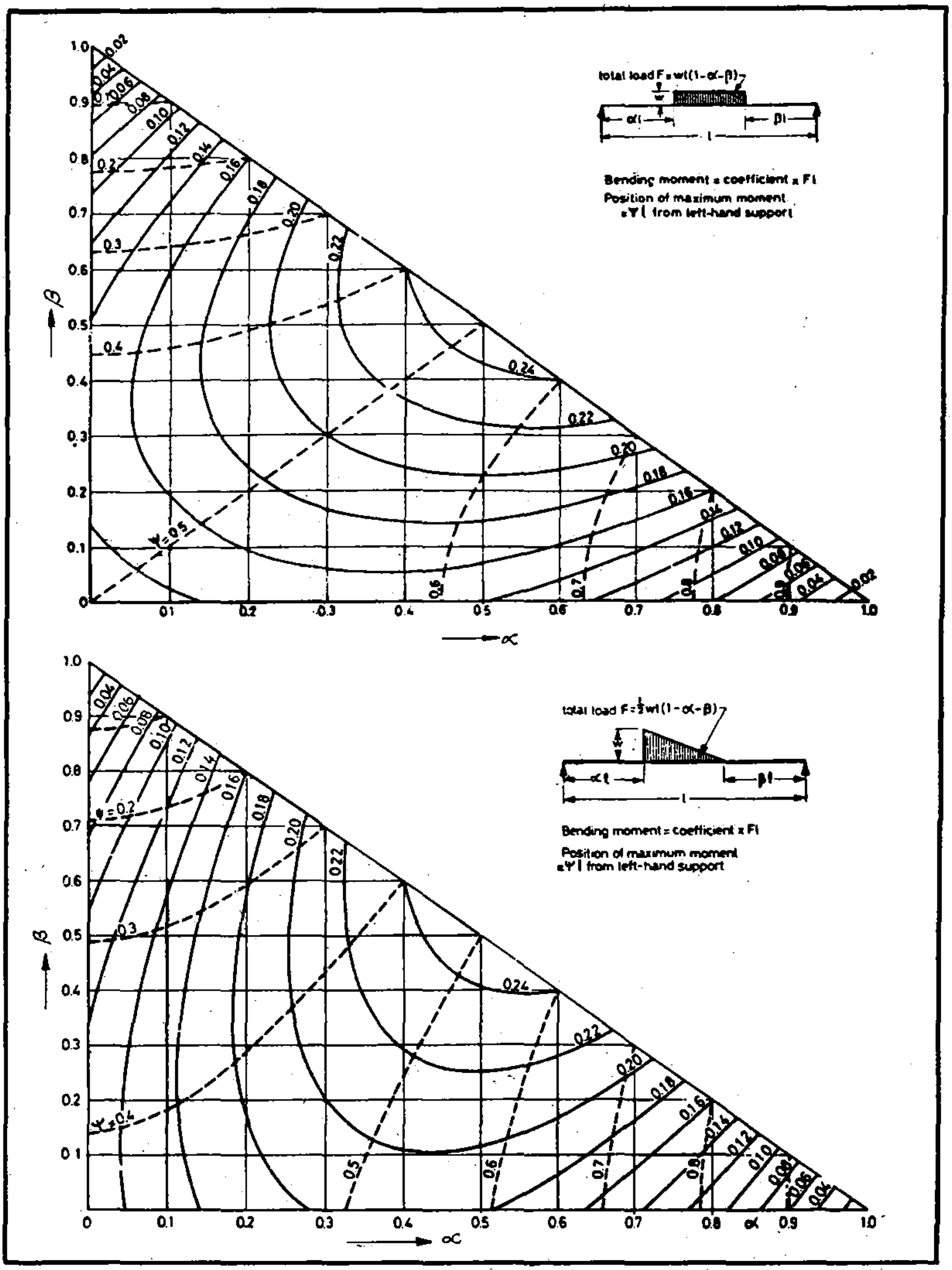


Table 20.7 *Freely-supported beams: Maximum deflections*

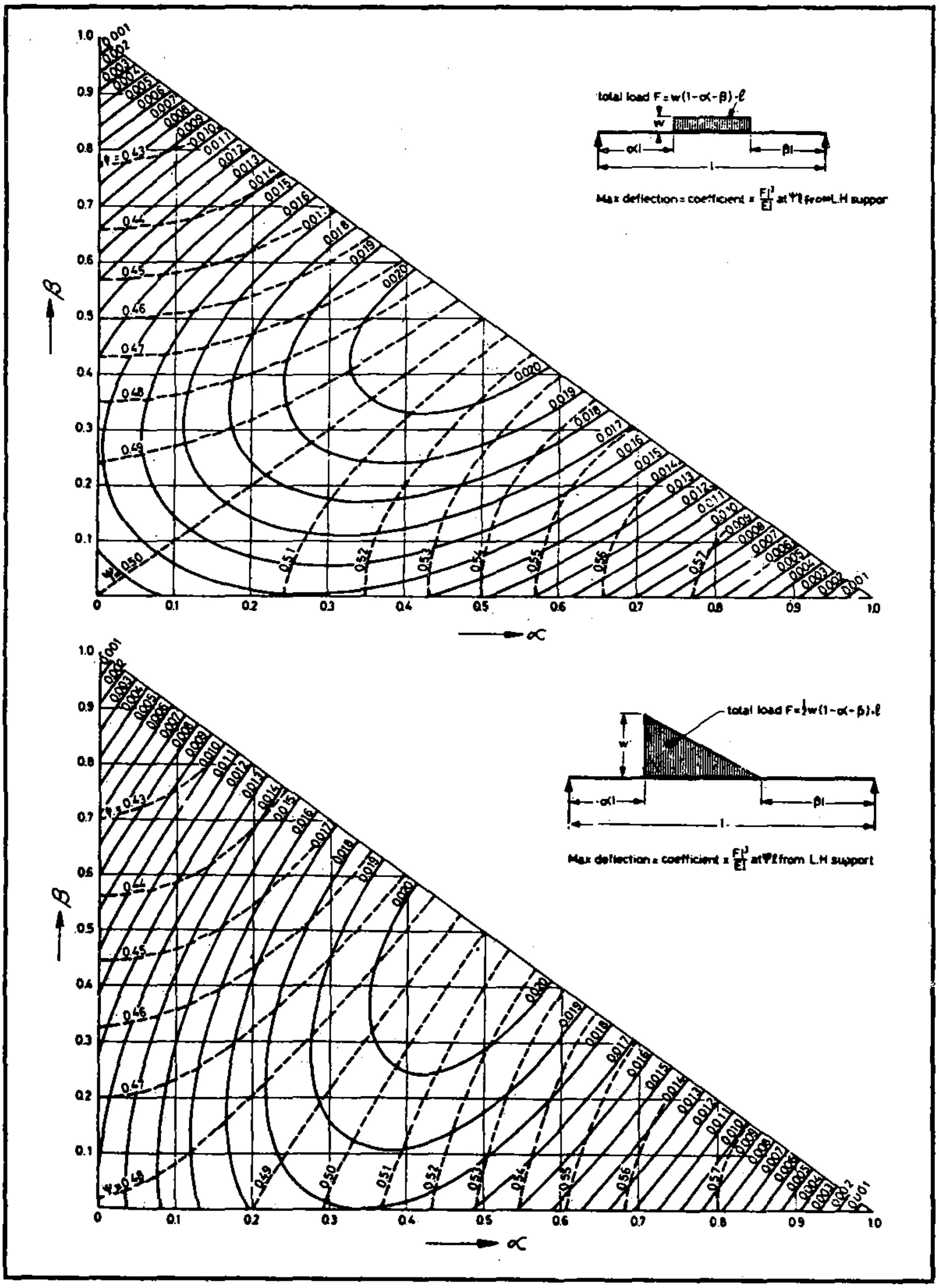
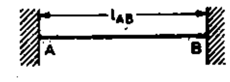
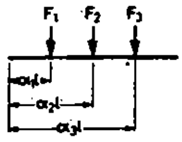
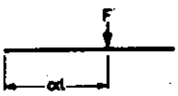
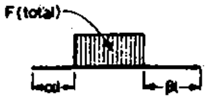
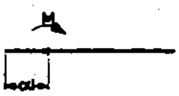


Table 20.8 *Fixed-end moment coefficients: General data*

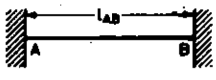
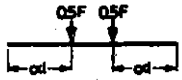
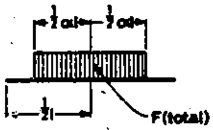
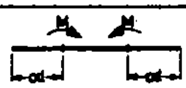
The fixed end moment coefficients C_{AB} and C_{BA} can be used as follows.
To obtain bending moments at supports of single-span beams fixed at both ends

$M_{AB} = -C_{AB}l_{AB}$ $M_{BA} = -C_{BA}l_{AB}$ With symmetrical load $M_{AB} = M_{BA}$

Unsymmetrical loading

	Fixed-end moment coefficients C_{AB}	Fixed-end moment coefficients C_{BA}
 Any number of loads	$\sum \alpha(1-\alpha)^2 F$	$\sum \alpha^2(1-\alpha)F$
	$\alpha(1-\alpha)^2 F$	$\alpha^2(1-\alpha)F$
	Read values from Table 20·10	
	Read values from Table 20·9	
	Read values from Table 20·10	
	$\frac{M}{l}(3\alpha-1)(\alpha-1)$	$\frac{M\alpha}{l}(2-3\alpha)$

Symmetrical loading

	Fixed-end moment coefficients $C_{AB} = C_{BA}$
 Any number of loads (j) equally spaced	$\frac{(j+2)}{12(j+1)}F$ j | factor: 1 | 0.125 F; 2 | 0.111 F; 3 | 0.104 F; 4 | 0.100 F
	$\frac{\alpha}{2}(1-\alpha)F$
	$\frac{1}{8}\left(1-\frac{\alpha^2}{3}\right)F$
	$\frac{1}{12}F$
	$\frac{5}{48}F$
 Parabolic	$\frac{1}{10}F$
	$\frac{(1+\alpha-\alpha^2)}{12}F$
	$\frac{M}{l}(1-2\alpha)$

Other loadings can generally be considered by combining tabulated cases, thus:

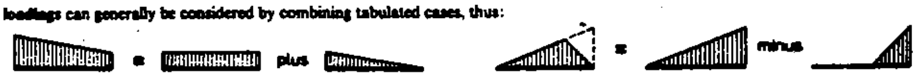


Table 20.9 *Fixed-end moments coefficients: Partial triangular loads*

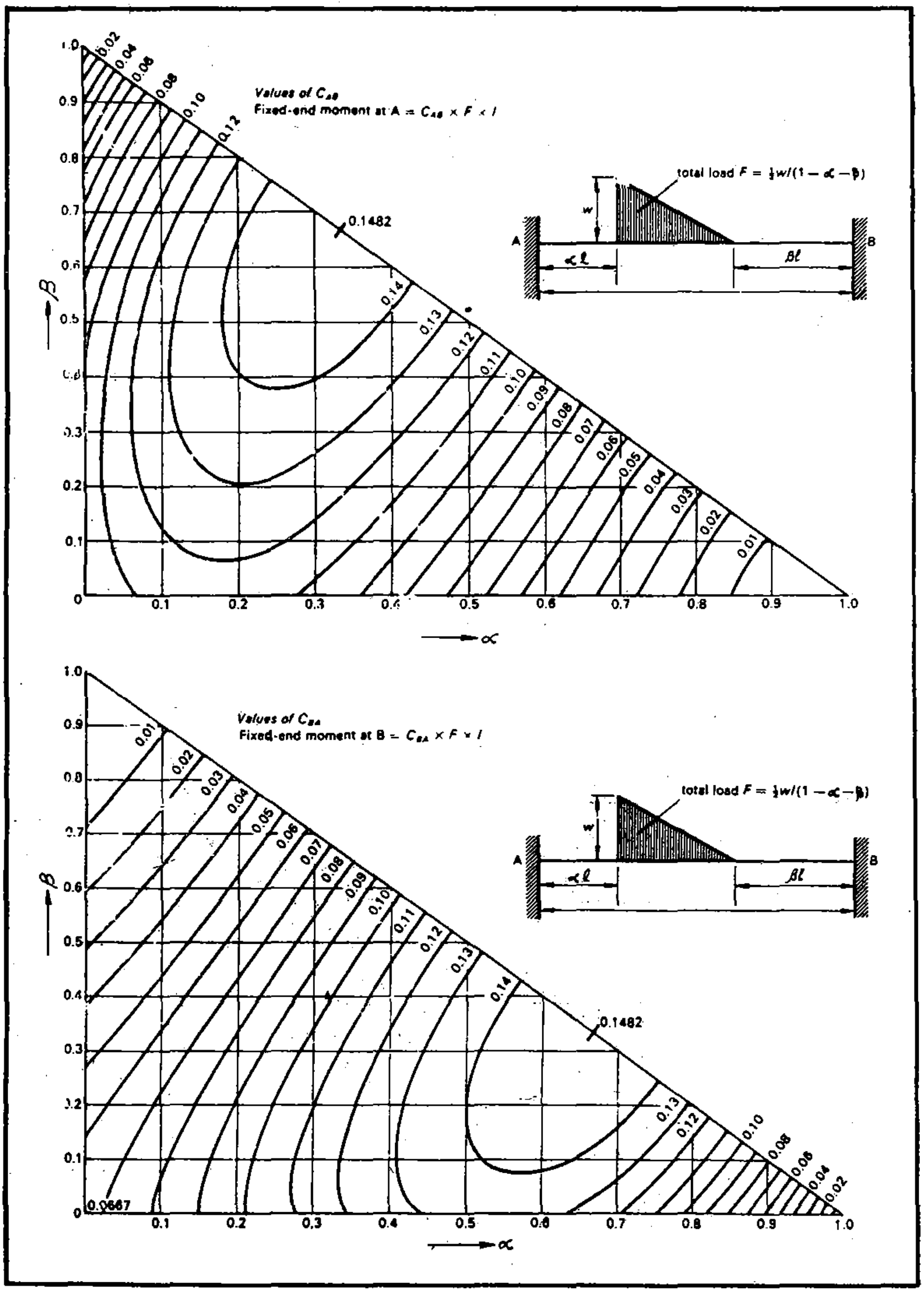


Table 20.10 *Fixed-end moment coefficients: Partial uniform and trapezoidal loads*

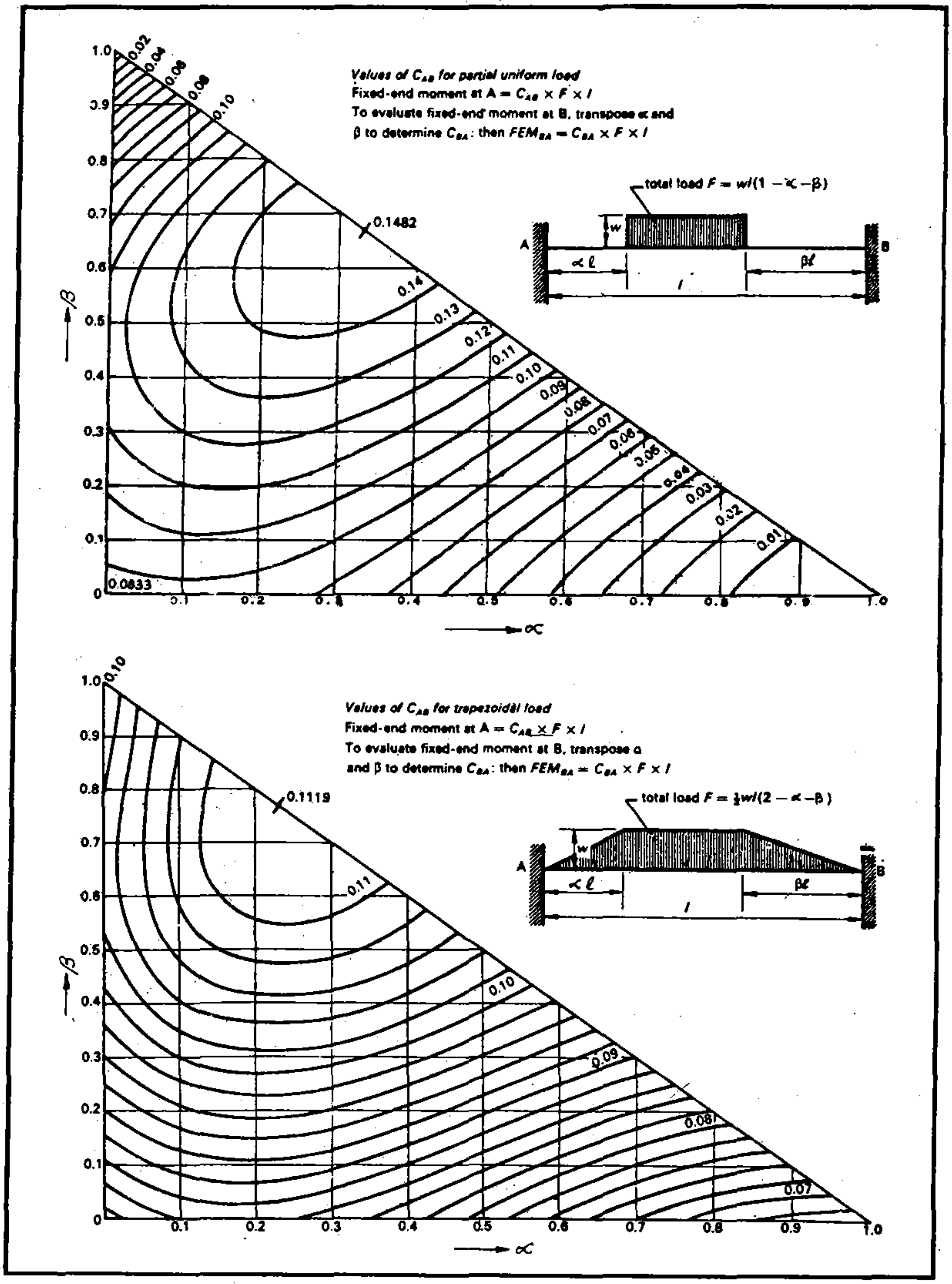
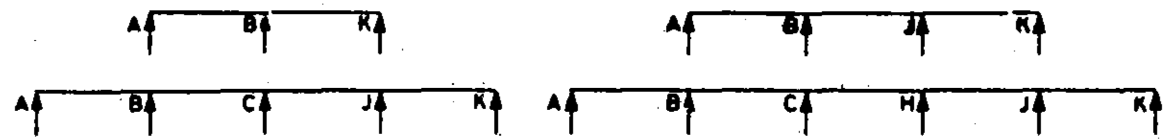


Table 20.11 *Effects of moments applied at end-supports in continuous beams of equal spans*

		Bending moment applied at A only				Equal bending moments applied at A and K			
Number of spans		2	3	4	5	2	3	4	5
Bending moment	M_A	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000
	M_B	+0.250	+0.267	+0.268	+0.268	+0.500	+0.200	+0.286	+0.263
	M_C	—	—	−0.071	−0.072	—	—	−0.143	−0.053
	M_H	—	—	—	+0.019	—	—	—	−0.053
	M_J	—	−0.067	+0.018	−0.005	—	+0.200	+0.286	+0.263
	M_K	0	0	0	0	−1.000	−1.000	−1.000	−1.000
Shearing force	V_{AR}	+1.250	+1.267	+1.268	+1.268	+1.500	+1.200	+1.286	+1.263
	V_{BL}	−1.250	−1.267	−1.268	−1.268	−1.500	−1.200	−1.286	−1.263
	V_{BR}	−0.250	−0.333	−0.339	−0.340	−1.500	0	−0.429	−0.316
	V_{CL}	—	—	+0.339	+0.340	—	—	+0.429	+0.316
	V_{CR}	—	—	+0.089	+0.091	—	—	—	0
	V_{HL}	—	—	—	−0.091	—	—	—	0
	V_{HR}	—	—	—	−0.024	—	—	+0.429	+0.316
	V_{JL}	—	+0.333	−0.089	+0.024	—	0	−0.429	−0.316
	V_{JR}	—	+0.067	−0.018	+0.005	—	−1.200	−1.286	−1.263
	V_{KL}	+0.250	−0.067	+0.018	−0.005	+1.500	+1.200	+1.286	+1.263

Key:



Notes: Adjustment to bending moment = *M*-coefficient × applied bending moment

$$\text{Adjustment to shearing force} = \frac{V\text{-coefficient} \times \text{applied bending moment}}{\text{span}}$$

Table 20.12 *Continuous beams: Moments from equal loads on equal spans—I*

Load	All spans loaded (e.g. dead load)	Imposed load (sequence of loaded spans to give max. bending moment)
0.4l / 0.4l	0.155 0.094 0.094	0.155 0.127 0.127
	0.124 0.124 0.107 0.040 0.107	0.145 0.145 0.134 0.102 0.134
	0.133 0.089 0.133 0.103 0.054 0.054 0.103	(0.144) (0.133) (0.144) 0.149 0.133 0.149 0.132 0.109 0.109 0.132
	0.131 0.098 0.096 0.131 0.104 0.050 0.066 0.050 0.104	(0.144) (0.132) (0.132) (0.144) 0.149 0.138 0.138 0.149 0.133 0.107 0.115 0.107 0.133
0.5l / 0.5l	0.156 0.095 0.095	0.156 0.129 0.129
	0.125 0.125 0.108 0.042 0.108	0.146 0.146 0.136 0.104 0.136
	0.134 0.089 0.134 0.104 0.056 0.056 0.104	(0.145) (0.134) (0.145) 0.151 0.134 0.151 0.134 0.111 0.111 0.134
	0.132 0.099 0.099 0.132 0.105 0.051 0.068 0.051 0.105	(0.145) (0.133) (0.133) (0.145) 0.150 0.139 0.139 0.150 0.135 0.109 0.117 0.109 0.135
Concentrated at midspan	0.188 0.156 0.156	0.188 0.203 0.203
	0.150 0.150 0.175 0.100 0.175	0.175 0.175 0.213 0.175 0.213
	0.161 0.107 0.161 0.170 0.116 0.116 0.170	(0.174) (0.161) (0.174) 0.181 0.161 0.181 0.210 0.183* 0.183 0.210
	0.158 0.118 0.118 0.158 0.171 0.112 0.132 0.112 0.171	(0.174) (0.160) (0.160) (0.174) 0.179 0.167 0.167 0.179 0.211 0.181 0.191 0.181 0.211
Concentrated at third points	0.167 0.111 0.111	0.167 0.139 0.139
	0.133 0.133 0.122 0.033 0.122	0.156 0.156 0.144 0.100 0.144
	0.143 0.095 0.143 0.119 0.056 0.056 0.119	(0.155) (0.143) (0.155) 0.160 0.144 0.160 0.143 0.111 0.111 0.143
	0.140 0.105 0.105 0.140 0.120 0.050 0.061 0.050 0.120	(0.155) (0.142) (0.142) (0.155) 0.159 0.148 0.148 0.159 0.144 0.108 0.115 0.108 0.144

Bending moment = (coefficient) × (total load on one span) × (span).
Bending moment coefficients:
above line apply to negative bending moment at supports.
below line apply to positive bending moment in span.
Coefficients apply when all spans are equal (or shortest ≯ 15% less than longest). Loads on each loaded span are equal. Moment of inertia same throughout all spans.
Bending moments coefficients in brackets (imposed load) apply if two spans only are loaded.

Table 20.13 *Continuous beams: Moments from equal loads on equal spans – 2*

(NOTE: See note in Table 20·12

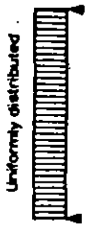
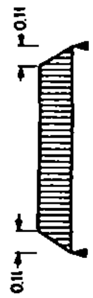
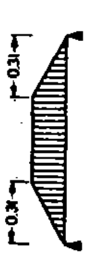
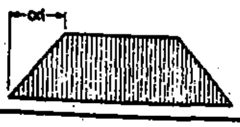
Load	All spans loaded (e.g. dead load)	Imposed load (sequence of loaded spans to give max. bending moment)
	0.125 0.070 0.070	0.125 0.096 0.096
	0.100 0.100 0.080 0.025 0.080	0.117 0.117 0.101 0.075 0.101
	0.107 0.071 0.107 0.077 0.036 0.036 0.077	(0.116) (0.107) (0.116) 0.121 0.107 0.121 0.099 0.081 0.081 0.099
	0.105 0.079 0.079 0.105 0.078 0.033 0.046 0.033 0.078	(0.116) (0.106) (0.106) (0.116) 0.120 0.111 0.111 0.120 0.100 0.079 0.086 0.079 0.100
	0.136 0.077 0.077	0.136 0.105 0.105
	0.109 0.109 0.088 0.028 0.088	0.127 0.127 0.111 0.083 0.111
	0.117 0.078 0.117 0.085 0.040 0.040 0.085	(0.127) (0.117) (0.127) 0.131 0.117 0.131 0.109 0.089 0.089 0.109
	0.115 0.086 0.086 0.115 0.086 0.037 0.051 0.037 0.086	(0.12[illegible]) (0.116) (0.116) (0.126) 0.131 0.121 0.121 0.131 0.110 0.087 0.094 0.087 0.110
	0.145 0.084 0.084	0.145 0.114 0.114
	0.116 0.116 0.095 0.032 0.095	0.135 0.135 0.120 0.090 0.120
	0.124 0.083 0.124 0.092 0.045 0.045 0.092	(0.135) (0.124) (0.135) 0.140 0.124 0.140 0.118 0.096 0.096 0.118
	0.122 0.092 0.092 0.122 0.093 0.041 0.056 0.041 0.093	(0.135) (0.123) (0.123) (0.135) 0.139 0.129 0.129 0.139 0.119 0.095 0.102 0.095 0.119
	0.151 0.090 0.090	0.151 0.121 0.121
	0.121 0.121 0.102 0.036 0.102	0.141 0.141 0.128 0.097 0.128
	0.130 0.086 0.130 0.098 0.050 0.050 0.098	(0.140) (0.130) (0.140) 0.146 0.130 0.146 0.126 0.103 0.103 0.126
	0.127 0.096 0.096 0.127 0.099 0.046 0.062 0.046 0.099	(0.140) (0.129) (0.129) (0.140) 0.145 0.135 0.135 0.145 0.127 0.102 0.109 0.102 0.127

Table 20.14 *Continuous beams: Shears from equal loads on equal spans*

Load	All spans loaded (e.g. dead load)	Imposed load (sequence of loaded spans to give max. shearing force)
Uniformly-distributed	0.375 0.625 0.625 0.375	0.438 0.625 0.625 0.438
	0.400 0.500 0.600 0.600 0.500 0.400	0.450 0.583 0.617 0.617 0.583 0.450
	0.393 0.536 0.464 0.607 0.607 0.464 0.536 0.393	0.446 0.603 0.571 0.621 0.621 0.571 0.603 0.446
	0.395 0.526 0.500 0.474 0.605 0.605 0.474 0.500 0.526 0.395	0.447 0.598 0.591 0.576 0.620 0.620 0.576 0.591 0.598 0.447
Triangularly-distributed	0.344 0.656 0.656 0.344	0.422 0.656 0.656 0.422
	0.375 0.500 0.625 0.625 0.500	0.437 0.605 0.646 0.646 0.605 0.437
	0.366 0.545 0.455 0.634 0.634 0.455 0.545 0.366	0.433 0.628 0.589 0.651 0.651 0.589 0.628 0.433
	0.369 0.532 0.500 0.468 0.631 0.631 0.468 0.500 0.532 0.369	0.434 0.622 0.614 0.595 0.649 0.649 0.595 0.514 0.622 0.434
Concentrated at midspan	0.313 0.688 0.688 0.313	0.406 0.688 0.688 0.406
	0.350 0.500 0.650 0.650 0.500 0.350	0.425 0.625 0.675 0.675 0.625 0.425
	0.339 0.554 0.446 0.661 0.661 0.446 0.554 0.339	0.420 0.654 0.607 0.681 0.681 0.607 [illegible] 0.420
	0.342 0.540 0.500 0.460 0.658 0.658 0.460 0.500 0.540 0.342	0.421 0.647 0.636 0.615 0.679 0.679 0.615 0.636 0.647 0.421
Concentrated at third points	0.333 0.667 0.667 0.333	0.417 0.667 0.667 0.417
	0.367 0.500 0.633 0.633 0.500 0.367	0.433 0.611 0.656 0.656 0.611 0.433
	0.357 0.548 0.452 0.643 0.643 0.452 0.548 0.357	0.429 0.637 0.595 0.661 0.661 0.595 0.637 0.429
	0.360 0.535 0.500 0.465 0.640 0.640 0.465 0.500 0.535 0.360	0.430 0.631 0.621 0.602 0.659 0.659 0.602 0.621 0.631 0.480

For any trapezoidal load.



S F Coefficient = $(k - \frac{1}{2})(1 + \alpha - \alpha^2) + \frac{1}{2}$ where k is S F coefficient for uniform load, read from above table.

e.g. if $\alpha = 0.5$, coefficient at central support of two-span beam is $(0.625 - 0.5)(1 + 0.5 - 0.25) + 0.5 = 0.656$.

Table 20.15 *Rectangular box culverts*

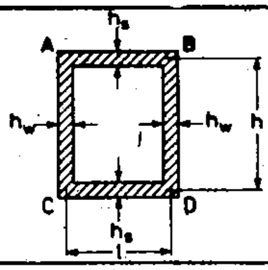
Bending moments (per unit length of culvert)

$M_A = M_B \qquad M_C = M_D$

Pressures and uniform loads are per unit area of walls or slab.

Loads F and G are total loads per unit length of culvert. h and l are measured between centres of walls or slabs.

q_1 = pressure transferred to soil.



$k = \frac{h}{l}\left(\frac{h_s}{h_w}\right)^3$ $\qquad K_4 = 4k + 9$

$K_5 = 2k + 3$

$K_1 = k + 1 \qquad K_6 = k + 6$

$K_2 = k + 2 \qquad K_7 = 2k + 7$

$K_3 = k + 3 \qquad K_8 = 3k + 8$

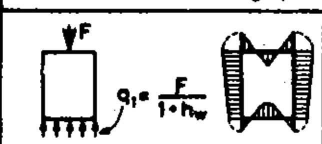
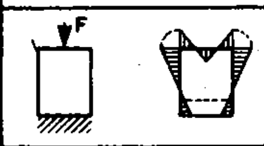
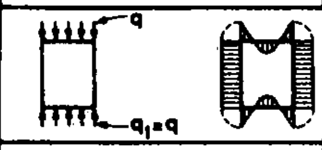
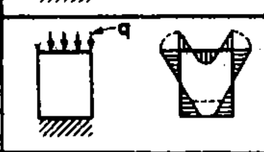
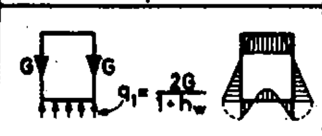
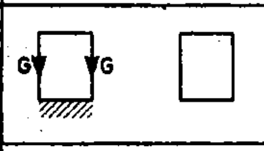
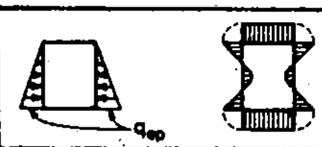
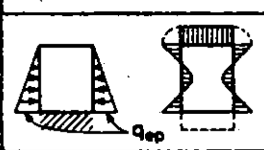
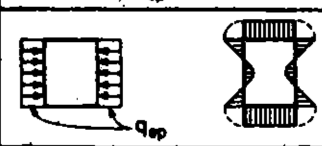
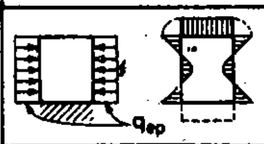
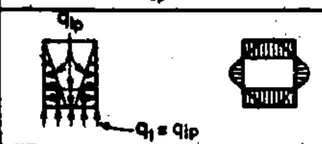
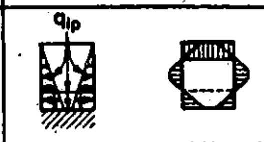
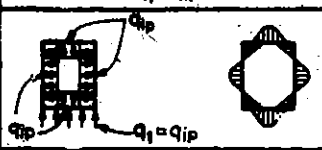
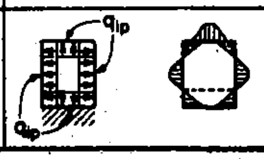
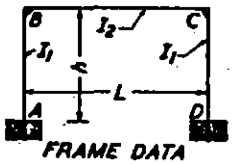
Loading	Condition of supporting ground (limiting cases)			
	Highly-compressible		Non-compressible	
Concentrated load on roof		$M_A = -\frac{FlK_4}{24K_1K_3}$ $M_C = \frac{K_5}{K_4} M_A$		$M_A = -\frac{Fl}{4K_2}$ $M_C = -\frac{M_A}{2}$
Uniform load on roof		$\left.\begin{matrix}M_A\\M_C\end{matrix}\right\} = -\frac{ql^2}{12K_1}$		$M_A = -\frac{ql^2}{6K_2}$ $M_C = -\frac{M_A}{2}$
Weight of walls		$M_A = +\frac{q_1 l^2 k}{12K_1K_3}$ $M_C = -\frac{K_6}{k} M_A$		$M_A = M_C = 0$
Earth pressure on walls		$M_A = -\frac{q_{ep}h^2kK_7}{60K_1K_3}$ $M_C = \frac{K_8}{K_7} M_A$		$M_A = -\frac{q_{ep}h^2k}{30K_2}$ $M_C = \frac{K_8}{2k} M_A$
Earth (surcharge) pressure on walls		$\left.\begin{matrix}M_A\\M_C\end{matrix}\right\} = -\frac{q_{ep}h^2k}{12K_1}$		$M_A = -\frac{q_{ep}h^2k}{12K_2}$ $M_C = \frac{K_3}{k} M_A$
Hydrostatic (internal) pressure		$M_A = +\frac{q_{ip}h^2kK_7}{60K_1K_3}$ $M_C = \frac{K_8}{K_7} M_A$		$M_c = +\frac{q_{ip}h^2k}{30K_2}$ $M_C = \frac{K_8}{2k} M_A$
Excess hydrostatic (internal) pressure		$M_A = +\frac{q_{ip}(h^2kK_3 + l^2K_8)}{12K_1K_3}$ $M_C = +\frac{q_{ip}k(h^2K_3 - l^2)}{12K_1K_3}$		$M_A = +\frac{q_{ip}(h^2k + 2l^2)}{12K_2}$ $M_C = +\frac{q_{ip}(h^2K_3 - l^2)}{12K_2}$

Table 20.16 *Formulae for rigid frames*

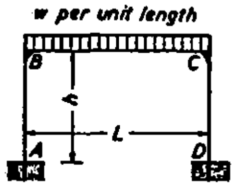
Frame – I

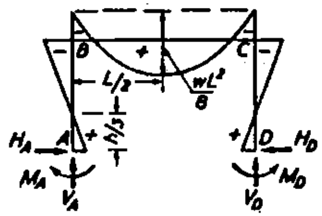


Coefficients:

$k = \frac{I_2}{I_1}\,\frac{h}{L}$

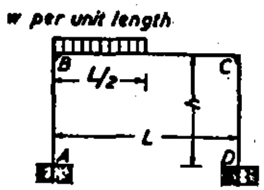
$N_1 = k + 2 \qquad N_2 = 6k + 1$

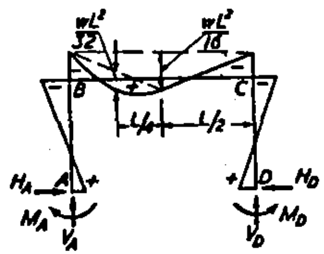




$$M_A = M_D = \frac{wL^2}{12N_1} \qquad M_B = M_C = -\frac{wL^2}{6N_1} = -2M_A$$

$$M_{max} = \frac{wL^2}{8} + M_B \qquad V_A = V_D = \frac{wL}{2} \qquad H_A = H_D = \frac{3M_A}{h}$$

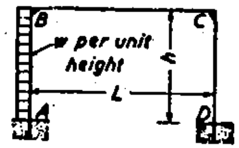


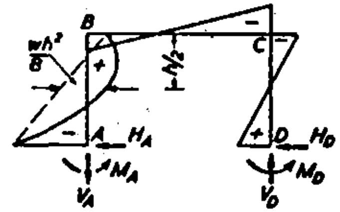


$$M_A = \frac{wL^2}{8}\left[\frac{1}{3N_1} - \frac{1}{8N_2}\right] \qquad M_B = -\frac{wL^2}{8}\left[\frac{2}{3N_1} + \frac{1}{8N_2}\right]$$

$$M_D = \frac{wL^2}{8}\left[\frac{1}{3N_1} + \frac{1}{8N_2}\right] \qquad M_C = -\frac{wL^2}{8}\left[\frac{2}{3N_1} - \frac{1}{8N_2}\right]$$

$$V_D = \frac{wL}{8}\left[1 - \frac{1}{4N_2}\right] \qquad V_A = \frac{wL}{2} - V_D \qquad H_A = H_D = \frac{wL^2}{8hN_1}$$

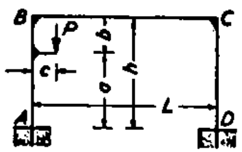


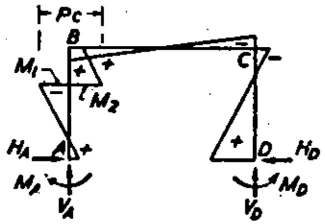


$$M_A = \frac{wh^2}{4}\left[-\frac{k+3}{6N_1} - \frac{4k+1}{N_2}\right] \qquad M_B = \frac{wh^2}{4}\left[-\frac{k}{6N_1} + \frac{2k}{N_2}\right]$$

$$M_D = \frac{wh^2}{4}\left[-\frac{k+3}{6N_1} + \frac{4k+1}{N_2}\right] \qquad M_C = \frac{wh^2}{4}\left[-\frac{k}{6N_1} - \frac{2k}{N_2}\right]$$

$$H_D = \frac{wh(2k+3)}{8N_1} \qquad H_A = -(wh - H_D) \qquad V_A = -V_D = -\frac{wh^2k}{LN_2}$$





Constants: $a_1 = \frac{a}{h} \qquad b_1 = \frac{b}{h}$

$$X_1 = \frac{Pc}{2N_1}\left[1 + 2b_1k - 3b_1^2(k+1)\right] \qquad X_2 = \frac{Pcka_1(3a_1 - 2)}{2N_1}$$

$$X_3 = \frac{3Pcka_1}{N_2}$$

$$M_A = +X_1 - \left(\frac{Pc}{2} - X_3\right) \qquad M_B = +X_2 + X_3$$

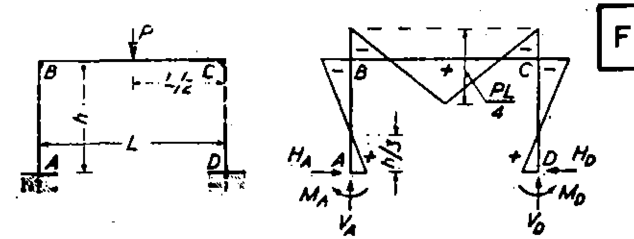
$$M_D = +X_1 + \left(\frac{Pc}{2} - X_3\right) \qquad M_C = +X_2 - X_3$$

$$H_A = H_D = \frac{Pc}{2h} + \frac{X_1 - X_2}{h} \qquad V_D = \frac{2X_3}{L} \qquad V_A = P - V_D$$

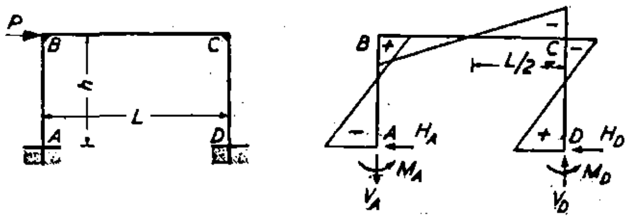
$$M_1 = M_A - H_Aa \qquad M_2 = M_B + H_Db$$

Table 20.17 *Formulae for rigid frames*

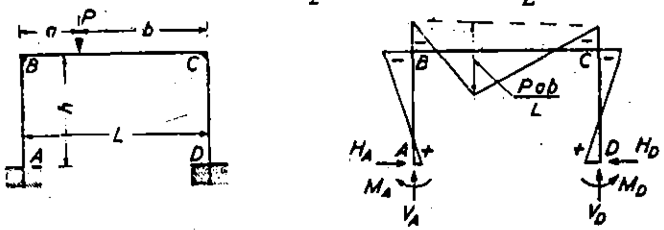
Frame-I contd.



$$M_A = M_D = +\frac{PL}{8N_1} \qquad M_B = M_C = -2M_A$$
$$V_A = V_D = \frac{P}{2} \qquad H_A = H_D = \frac{3M_A}{h}$$

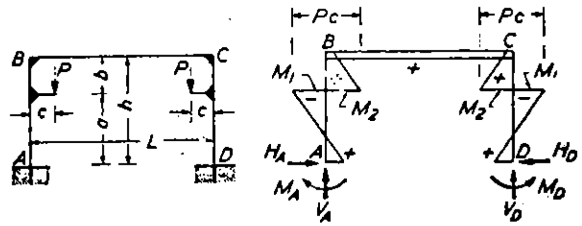


$$M_A = -\frac{Ph}{2}\,\frac{3k+1}{N_2} \qquad M_B = +\frac{Ph}{2}\,\frac{3k}{N_2}$$
$$M_D = +\frac{Ph}{2}\,\frac{3k+1}{N_2} \qquad M_C = -\frac{Ph}{2}\,\frac{3k}{N_2}$$
$$H_A = -H_D = -\frac{P}{2} \qquad V_A = -V_D = -\frac{2M_B}{L}$$



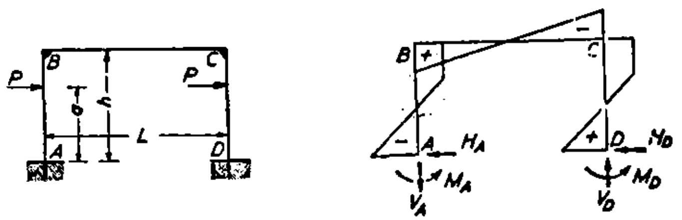
Constants: $a_1 = a/L \qquad b_1 = b/L$

$$M_A = +\frac{Pab}{L}\left[\frac{1}{2N_1} - \frac{b_1 - a_1}{2N_2}\right] \qquad M = -\frac{Pab}{L}\left[\frac{1}{N_1} + \frac{b_1 - a_1}{2N_2}\right]$$
$$M_D = +\frac{Pab}{L}\left[\frac{1}{2N_1} + \frac{b_1 - a_1}{2N_2}\right] \qquad M_C = -\frac{Pab}{L}\left[\frac{1}{N_1} - \frac{b_1 - a_1}{2N_2}\right]$$
$$V_A = Pb_1\left[1 + \frac{a_1(b_1 - a_1)}{N_2}\right] \qquad V_D = P - V_A \qquad H_A = H_D = \frac{3Pab}{2LhN_1}$$



Constants: $a_1 = \frac{a}{h} \qquad b_1 = \frac{b}{h}$

$$X_1 = \frac{Pc}{2N_1}[1 + 2b_1k - 3b_1^2(k+1)] \qquad X_2 = \frac{Pcka_1(3a_1 - 2)}{2N_1}$$
$$M_A = M_D = \frac{Pc}{N_1}[1 + 2b_1k - 3b_1^2(k+1)] = 2X_1$$
$$M_B = M_C = \frac{Pcka_1(3a_1 - 2)}{N_1} = 2X_2$$
$$V_A = V_D = P \qquad H_A = H_D = \frac{Pc + M_A - M_B}{h}$$
$$M_1 = M_A - H_A a \qquad M_2 = M_B + H_D b$$

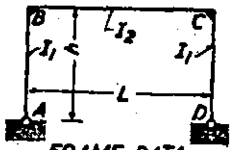


Constants: $a_1 = \frac{a}{h} \qquad X_1 = \frac{3Paa_1k}{N_2}$

$$M_A = -Pa + X_1 \qquad M_B = X_1$$
$$M_D = +Pa - X_1 \qquad M_C = -X_1$$
$$V_A = -V_D = -\frac{2X_1}{L} \qquad H_A = -H_D = -P$$

Table 20.18 *Formulae for rigid frames*

Frame - II



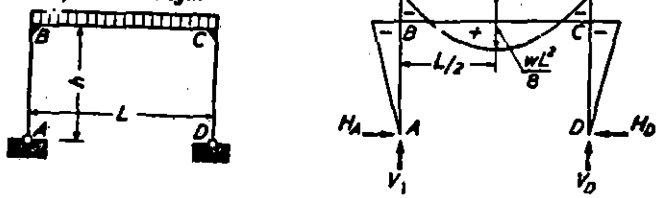
FRAME DATA

Coefficients:

$k = \frac{I_2}{I_1} \frac{h}{L}$

$N = 2k + 3$

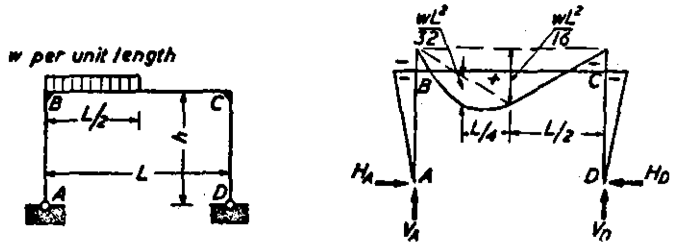
w per unit length



$M_B = M_C = -\frac{wL^2}{4N}$ $M_{max} = \frac{wL^2}{8} + M_B$

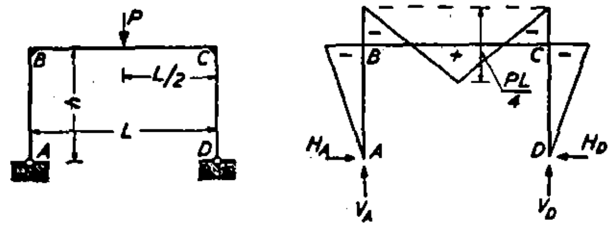
$V_A = V_D = \frac{wL}{2}$ $H_A = H_D = -\frac{M_B}{h}$

w per unit length

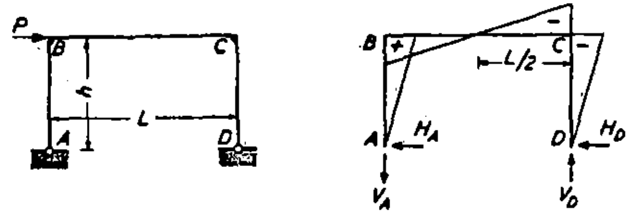


$M_B = M_C = -\frac{wL^2}{8N}$

$V_A = \frac{3wL}{8}$ $V_D = \frac{wL}{8}$ $H_A = H_D = -\frac{M_B}{h}$

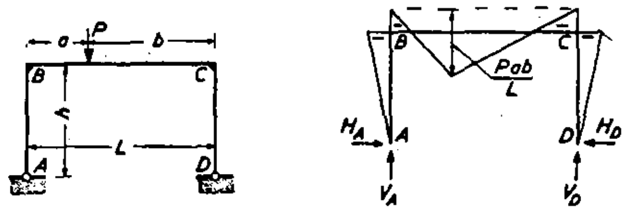


$M_B = M_C = -\frac{3PL}{8N}$ $V_A = V_D = \frac{P}{2}$ $H_A = H_D = -\frac{M_B}{h}$



$M_B = -M_C = +\frac{Ph}{2}$

$V_A = -V_D = -\frac{Ph}{L}$ $H_A = -H_D = -\frac{P}{2}$

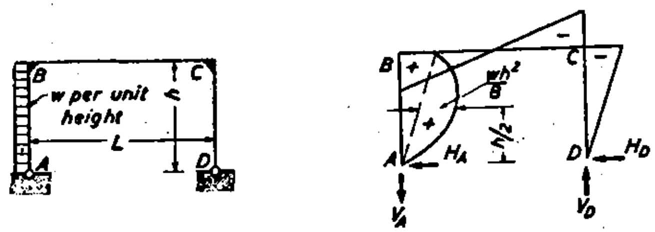


$M_B = M_C = -\frac{Pab}{L} \frac{3}{2N}$

$V_A = \frac{Pb}{L}$ $V_D = \frac{Pa}{L}$ $H_A = H_D = -\frac{M_B}{h}$

Table 20.19 *Formulae for rigid frames*

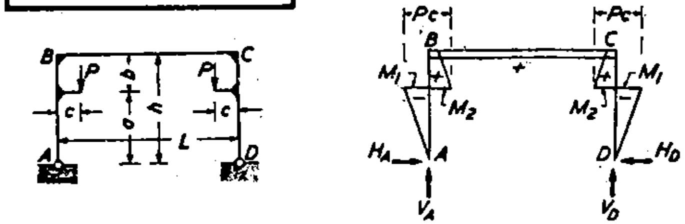
Frame – II contd.



$$M_B = \frac{wh^2}{4}\left[-\frac{k}{2N}+1\right] \qquad H_D = -\frac{M_C}{h}$$

$$M_C = \frac{wh^2}{4}\left[-\frac{k}{2N}-1\right] \qquad H_A = -(wh - H_D)$$

$$V_A = -V_D = -\frac{wh^2}{2L}$$

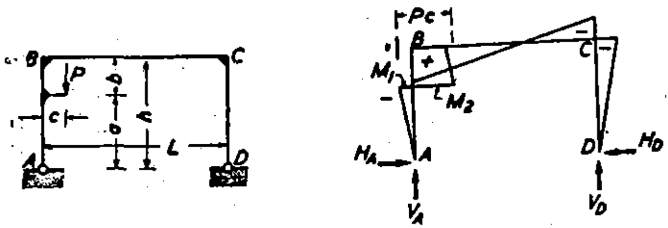


Constant: $a_1 = \frac{a}{h}$

$$M_B = M_C = \frac{Pc(3a_1^2-1)k}{N}$$

$$H_A = H_D = \frac{Pc - M_B}{h} \qquad V_A = V_D = P$$

$$M_1 = -H_A a \qquad M_2 = Pc - H_A a$$



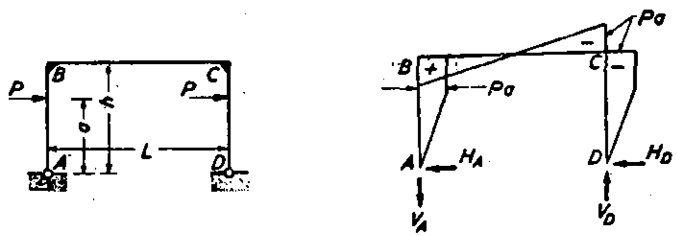
Constant: $a_1 = \frac{a}{h}$

$$M_B = \frac{Pc}{2}\left[\frac{(3a_1^2-1)k}{N}+1\right] \qquad H_A = H_D = -\frac{M_C}{h}$$

$$M_C = \frac{Pc}{2}\left[\frac{(3a_1^2-1)k}{N}-1\right]$$

$$V_D = \frac{Pc}{L} \qquad V_A = P - V_D$$

$$M_1 = -H_A a \qquad M_2 = Pc - H_A a$$



$$M_B = -M_C = Pa \qquad H_A = H_D = P$$

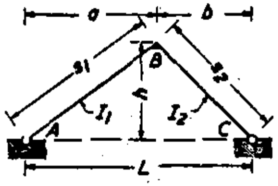
$$V_A = -V_D = -\frac{2Pa}{L}$$

Moment at loads $= \pm Pa$

Table 20.20 *Formulae for rigid frames*

(Extracts: Kleinlogel, Rahmenformeln 11. Auflage Berlin—Verlag von Wilhelm Ernest and Sohn)

FRAME - III



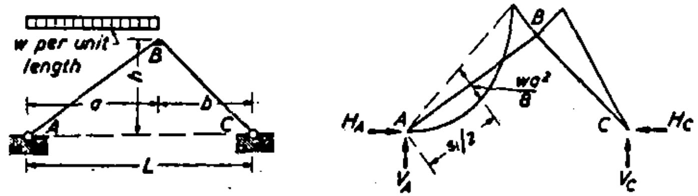
FRAME DATA

Coefficients:

$k = \frac{I_1}{I_2} \cdot \frac{s_2}{s_1}$ $\quad N = 1 + k$

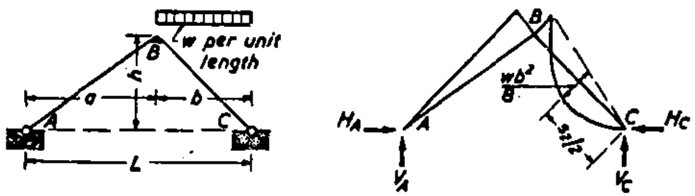
$a_1 = \frac{a}{L}$ $\quad b_1 = \frac{b}{L}$

$(a_1 + b_1 = 1)$



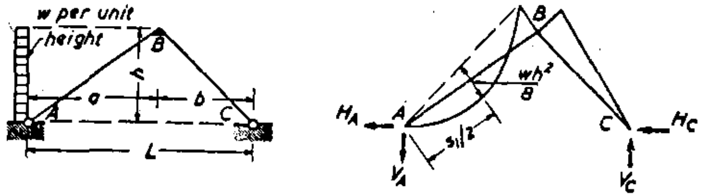
$M_B = -\frac{wa^2}{8N}$ $\quad V_C = \frac{wa^2}{2L}$ $\quad V_A = wa - V_C$

$H_A = H_C = \frac{wa^2 b_1}{2h} - \frac{M_B}{h}$



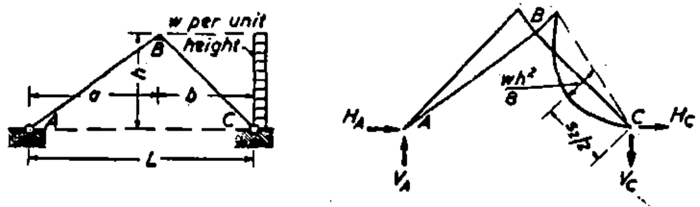
$M_B = -\frac{wb^2 k}{8N}$ $\quad V_A = \frac{wb^2}{2L}$ $\quad V_C = wb - V_A$

$H_A = H_C = \frac{wb^2 a_1}{2h} - \frac{M_B}{h}$



$M_B = -\frac{wh^2}{8N}$ $\quad V_A = -V_C = -\frac{wh^2}{2L}$

$H_C = \frac{whb_1}{2} - \frac{M_B}{h}$ $\quad H_A = -(wh - H_C)$



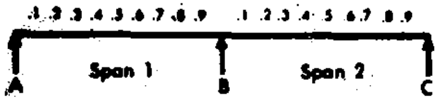
$M_B = -\frac{wh^2 k}{8N}$ $\quad V_A = -V_C = \frac{wh^2}{2L}$

$H_A = \frac{wha_1}{2} - \frac{M_B}{h}$ $\quad H_C = -(wh - H_A)$

Table 20.21

Unit load at		MOMENTS/PL																				
		SPAN 1											SPAN 2									
		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0875	.0751	.0626	.0501	.0376	.0252	.0127	.0002	-.0123	-.0248	-.0223	-.0198	-.0173	-.0149	-.0124	-.0099	-.0074	-.0050	-.0025	0
	.2	0	.0752	.1504	.1256	.1008	.0760	.0512	.0264	.0016	-.0232	-.0480	-.0432	-.0384	-.0336	-.0288	-.0240	-.0192	-.0144	-.0096	-.0048	0
	.3	0	.0632	.1264	.1895	.1527	.1159	.0791	.0422	.0054	-.0314	-.0683	-.0614	-.0546	-.0478	-.0410	-.0341	-.0273	-.0205	-.0137	-.0068	0
	.4	0	.0516	.1032	.1548	.2064	.1580	.1096	.0612	.0128	-.0356	-.0840	-.0756	-.0672	-.0588	-.0504	-.0420	-.0336	-.0252	-.0168	-.0084	0
	.5	0	.0406	.0813	.1219	.1625	.2031	.1438	.0844	.0250	-.0344	-.0938	-.0844	-.0750	-.0656	-.0563	-.0469	-.0375	-.0281	-.0188	-.0094	0
	.6	0	.0304	.0608	.0912	.1216	.1520	.1824	.1128	.0432	-.0264	-.0960	-.0864	-.0768	-.0672	-.0576	-.0480	-.0384	-.0288	-.0192	-.0096	0
	.7	0	.0211	.0422	.0632	.0843	.1054	.1265	.1475	.0686	-.0103	-.0893	-.0803	-.0714	-.0625	-.0536	-.0446	-.0357	-.0268	-.0179	-.0089	0
	.8	0	.0128	.0256	.0384	.0512	.0640	.0768	.0896	.1024	.0152	-.0720	-.0648	-.0576	-.0504	-.0432	-.0360	-.0288	-.0216	-.0144	-.0072	0
	.9	0	.0057	.0115	.0172	.0229	.0286	.0344	.0401	.0458	.0515	-.0428	-.0385	-.0342	-.0299	-.0257	-.0214	-.0171	-.0128	-.0086	-.0043	0
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0043	-.0086	-.0128	-.0171	-.0214	-.0257	-.0299	-.0342	-.0385	-.0428	.0515	.0458	.0401	.0344	.0286	.0229	.0172	.0115	.0057	0
	.2	0	-.0072	-.0144	-.0216	-.0288	-.0360	-.0432	-.0504	-.0576	-.0648	-.0720	.0152	.1024	.0896	.0768	.0640	.0512	.0384	.0256	.0128	0
	.3	0	-.0089	-.0179	-.0268	-.0357	-.0446	-.0536	-.0625	-.0714	-.0803	-.0893	-.0103	.0686	.1475	.1265	.1054	.0843	.0632	.0422	.0211	0
	.4	0	-.0096	-.0192	-.0288	-.0384	-.0480	-.0576	-.0672	-.0768	-.0864	-.0960	-.0264	.0432	.1128	.1824	.1520	.1216	.0912	.0608	.0304	0
	.5	0	-.0094	-.0188	-.0281	-.0375	-.0469	-.0563	-.0656	-.0750	-.0844	-.0938	-.0344	.0250	.0844	.1438	.2031	.1625	.1219	.0813	.0406	0
	.6	0	-.0084	-.0168	-.0252	-.0336	-.0420	-.0504	-.0588	-.0672	-.0756	-.0840	-.0356	.0128	.0612	.1096	.1580	.2064	.1548	.1032	.0516	0
	.7	0	-.0068	-.0137	-.0205	-.0273	-.0341	-.0410	-.0478	-.0546	-.0614	-.0683	-.0314	.0054	.0422	.0791	.1159	.1527	.1895	.1264	.0632	0
	.8	0	-.0048	-.0096	-.0144	-.0192	-.0240	-.0288	-.0336	-.0384	-.0432	-.0480	-.0232	.0016	.0264	.0512	.0760	.1008	.1256	.1504	.0752	0
	.9	0	-.0025	-.0050	-.0074	-.0099	-.0124	-.0149	-.0173	-.0198	-.0223	-.0248	-.0123	.0002	.0127	.0252	.0376	.0501	.0626	.0751	.0875	0
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0388	.0675	.0863	.0950	.0938	.0825	.0613	.0300	.0061	0	.0061	.0300	.0613	.0825	.0938	.0950	.0863	.0675	.0388	0
− Area		0	-.0063	-.0125	-.0188	-.0250	-.0313	-.0375	-.0438	-.0500	-.0736	-.1250	-.0736	-.0500	-.0438	-.0375	-.0313	-.0250	-.0188	-.0125	-.0063	0
Total Area		0	.0325	.0550	.0675	.0700	.0625	.0450	.0175	-.0200	-.0675	-.1250	-.0675	-.0200	.0175	.0450	.0625	.0700	.0675	.0550	.0325	0

Unit load at		REACTIONS/P			SHEARS/P			
		R_A	R_B	R_C	V_{AB}	V_{BA}	V_{BC}	V_{CB}
SPAN 1	A	1.0	0	0	1.0	0	0	0
	.1	.8753	.1495	-.0248	.8753	-.1247	.0248	.0248
	.2	.7520	.2960	-.0480	.7520	-.2480	.0480	.0480
	.3	.6318	.4365	-.0683	.6318	-.3682	.0683	.0683
	.4	.5160	.5680	-.0840	.5160	-.4840	.0840	.0840
	.5	.4063	.6875	-.0938	.4063	-.5937	.0938	.0938
	.6	.3040	.7920	-.0960	.3040	-.6960	.0960	.0960
	.7	.2108	.8785	-.0893	.2108	-.7892	.0893	.0893
	.8	.1280	.9440	-.0720	.1280	-.8720	.0720	.0720
	.9	.0573	.9855	-.0428	.0573	-.9427	.0428	.0428
	B	0	1.0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 2	.1	-.0428	.9855	.0573	-.0428	-.0428	.9427	-.0573
	.2	-.0720	.9440	.1280	-.0720	-.0720	.8720	-.1280
	.3	-.0893	.8785	.2108	-.0893	-.0893	.7892	-.2108
	.4	-.0960	.7920	.3040	-.0960	-.0960	.6960	-.3040
	.5	-.0938	.6875	.4063	-.0938	-.0938	.5937	-.4063
	.6	-.0840	.5680	.5160	-.0840	-.0840	.4840	-.5160
	.7	-.0683	.4365	.6318	-.0683	-.0683	.3682	-.6318
	.8	-.0480	.2960	.7520	-.0480	-.0480	.2480	-.7520
	.9	-.0248	.1495	.8753	-.0248	-.0248	.1247	-.8753
	C	0	0	1.0	0	0	0	-1.0
+ Area		.4375	1.2500	.4375	.4375	0	.6250	.0625
− Area		-.0625	0	-.0625	-.0625	-.6250	0	-.4375
Total Area		.3750	1.2500	.3750	.3750	-.6250	.6250	-.3750



Influence coefficients — Two continuous spans.

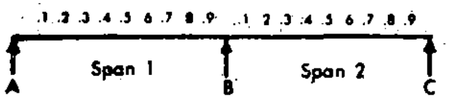
L = Length of SHORTER span; length of LONGER span = NL.

N = 1.0

Table 20.22

Unit load at		MOMENTS/PL																				
		SPAN 1										SPAN 2										
		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0876	.0753	.0629	.0506	.0382	.0259	.0135	.0011	-.0112	-.0236	-.0212	-.0189	-.0165	-.0141	-.0118	-.0094	-.0071	-.0047	-.0024	0
	.2	0	.0754	.1509	.1263	.1017	.0771	.0526	.0280	.0034	-.0211	-.0457	-.0411	-.0366	-.0320	-.0274	-.0229	-.0183	-.0137	-.0091	-.0046	0
	.3	0	.0635	.1270	.1905	.1540	.1175	.0810	.0445	.0080	-.0285	-.0650	-.0585	-.0520	-.0455	-.0390	-.0325	-.0260	-.0195	-.0130	-.0065	0
	.4	0	.0520	.1040	.1560	.2080	.1600	.1120	.0640	.0160	-.0320	-.0800	-.0720	-.0640	-.0560	-.0480	-.0400	-.0320	-.0240	-.0160	-.0080	0
	.5	0	.0411	.0821	.1232	.1643	.2054	.1464	.0875	.0286	-.0304	-.0893	-.0804	-.0714	-.0625	-.0536	-.0446	-.0357	-.0268	-.0179	-.0089	0
	.6	0	.0309	.0617	.0926	.1234	.1543	.1851	.1160	.0469	-.0223	-.0914	-.0823	-.0731	-.0640	-.0549	-.0457	-.0366	-.0274	-.0183	-.0091	0
	.7	0	.0215	.0430	.0645	.0860	.1075	.1290	.1505	.0720	-.0065	-.0850	-.0765	-.0680	-.0595	-.0510	-.0425	-.0340	-.0255	-.0170	-.0085	0
	.8	0	.0131	.0263	.0394	.0526	.0657	.0789	.0920	.1051	.0183	-.0686	-.0617	-.0549	-.0480	-.0411	-.0343	-.0274	-.0206	-.0137	-.0069	0
	.9	0	.0059	.0119	.0178	.0237	.0296	.0356	.0415	.0474	.0534	-.0407	-.0366	-.0326	-.0285	-.0244	-.0204	-.0163	-.0122	-.0081	-.0041	0
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0049	-.0099	-.0148	-.0197	-.0246	-.0296	-.0345	-.0394	-.0443	-.0493	.0547	.0486	.0425	.0364	.0304	.0243	.0182	.0121	.0061	0
	.2	0	-.0083	-.0166	-.0249	-.0332	-.0415	-.0498	-.0581	-.0664	-.0747	-.0830	.0133	.1096	.0959	.0822	.0685	.0548	.0411	.0274	.0137	0
	.3	0	-.0103	-.0206	-.0309	-.0411	-.0514	-.0617	-.0720	-.0823	-.0926	-.1028	-.0156	.0717	.1590	.1363	.1136	.0909	.0681	.0454	.0227	0
	.4	0	-.0111	-.0221	-.0332	-.0443	-.0553	-.0664	-.0774	-.0885	-.0996	-.1106	-.0336	.0435	.1206	.1976	.1647	.1317	.0988	.0659	.0329	0
	.5	0	-.0108	-.0216	-.0324	-.0432	-.0540	-.0648	-.0756	-.0864	-.0972	-.1080	-.0422	.0236	.0894	.1552	.2210	.1768	.1326	.0884	.0442	0
	.6	0	-.0097	-.0194	-.0290	-.0387	-.0484	-.0581	-.0678	-.0774	-.0871	-.0968	-.0431	.0106	.0642	.1179	.1716	.2253	.1690	.1126	.0563	0
	.7	0	-.0079	-.0157	-.0236	-.0315	-.0393	-.0472	-.0551	-.0629	-.0708	-.0786	-.0378	.0031	.0439	.0848	.1257	.1665	.2074	.1383	.0691	0
	.8	0	-.0055	-.0111	-.0166	-.0221	-.0277	-.0332	-.0387	-.0443	-.0498	-.0553	-.0278	-.0003	.0273	.0548	.0823	.1098	.1374	.1649	.0825	0
	.9	0	-.0029	-.0057	-.0086	-.0114	-.0143	-.0171	-.0200	-.0228	-.0257	-.0285	-.0147	-.0008	.0130	.0269	.0407	.0546	.0684	.0823	.0961	0
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+Area		0	.0390	.0681	.0871	.0962	.0952	.0843	.0633	.0324	.0067	0	.0068	.0336	.0716	.0977	.1116	.1135	.1033	.0810	.0465	0
-Area		0	-.0079	-.0158	-.0238	-.0317	-.0396	-.0475	-.0555	-.0634	-.0865	-.1387	-.0772	-.0478	-.0417	-.0357	-.0298	-.0238	-.0179	-.0119	-.0060	0
Total Area		0	.0311	.0523	.0633	.0645	.0556	.0368	-.0078	-.0310	-.0798	-.1387	-.0704	-.0142	.0299	.0620	.0819	.0897	.0854	.0691	.0405	0

Unit load at		REACTIONS/P			SHEARS/P			
		R_A	R_B	R_C	V_{AB}	V_{BA}	V_{BC}	V_{CB}
SPAN 1	A	1.0000	0	0	1.0000	0	0	0
	.1	.8764	.1450	-.0214	.8764	-.1236	.0214	.0214
	.2	.7543	.2873	-.0416	.7543	-.2457	.0416	.0416
	.3	.6350	.4241	-.0591	.6350	-.3650	.0591	.0591
	.4	.5200	.5527	-.0727	.5200	-.4800	.0727	.0727
	.5	.4107	.6705	-.0812	.4107	-.5893	.0812	.0812
	.6	.3086	.7745	-.0831	.3086	-.6914	.0831	.0831
	.7	.2150	.8623	-.0773	.2150	-.7850	.0773	.0773
	.8	.1314	.9309	-.0623	.1314	-.8686	.0623	.0623
	.9	.0593	.9777	-.0370	.0593	-.9407	.0370	.0370
	B	0	1.0000	0	0	-1.0 / 0	0 / 1.0	0
SPAN 2	.1	-.0493	.9941	.0552	-.0493	-.0493	.9448	-.0552
	.2	-.0830	.9584	.1246	-.0830	-.0830	.8754	-.1246
	.3	-.1028	.8963	.2065	-.1028	-.1028	.7935	-.2065
	.4	-.1106	.8112	.2994	-.1106	-.1106	.7006	-.2994
	.5	-.1080	.7062	.4018	-.1080	-.1080	.5982	-.4018
	.6	-.0968	.5848	.5120	-.0968	-.0968	.4880	-.5120
	.7	-.0786	.4501	.6285	-.0786	-.0786	.3715	-.6285
	.8	-.0553	.3056	.7497	-.0553	-.0553	.2503	-.7497
	.9	-.0285	.1545	.8741	-.0285	-.0285	.1259	-.8741
	C	0	0	1.0000	0	0	0	-1.0000
+Area		.4405	1.3149	.4780	.4405	0	.6761	.0541
-Area		-.0792	0	-.0541	-.0792	-.6388	0	-.4780
Total Area		.3613	1.3149	.4239	.3613	-.6388	.6761	-.4239



Influence coefficients — Two continuous spans.

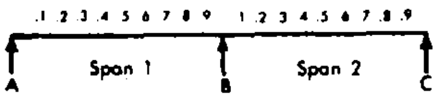
L = Length of SHORTER span length of LONGER span = NL.

N = 1.1

Table 20.23

Unit load at		MOMENTS/PL SPAN 1 A.	.1	.2	.3	.4	.5	.6	.7	.8	.9	B.	SPAN 2 .1	.2	.3	.4	.5	.6	.7	.8	.9	C.
SPAN 1	A.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0877	.0755	.0633	.0510	.0388	.0265	.0143	.0020	-.0102	-.0225	-.0203	-.0180	-.0157	-.0135	-.0113	-.0090	-.0067	-.0045	-.0023	0
	.2	0	.0756	.1513	.1269	.1025	.0782	.0538	.0295	.0051	-.0193	-.0436	-.0393	-.0349	-.0305	-.0262	-.0218	-.0175	-.0131	-.0087	-.0044	0
	.3	0	.0638	.1276	.1914	.1552	.1190	.0828	.0466	.0104	-.0258	-.0620	-.0558	-.0496	-.0434	-.0372	-.0310	-.0248	-.0186	-.0124	-.0062	0
	.4	0	.0524	.1047	.1571	.2095	.1618	.1142	.0665	.0189	-.0287	-.0764	-.0687	-.0611	-.0535	-.0458	-.0382	-.0305	-.0229	-.0153	-.0076	0
	.5	0	.0415	.0830	.1244	.1659	.2074	.1489	.0903	.0318	-.0267	-.0852	-.0767	-.0682	-.0597	-.0511	-.0426	-.0341	-.0256	-.0170	-.0085	0
	.6	0	.0313	.0625	.0938	.1251	.1564	.1876	.1189	.0502	-.0185	-.0873	-.0785	-.0698	-.0611	-.0524	-.0436	-.0349	-.0262	-.0175	-.0087	0
	.7	0	.0219	.0438	.0657	.0875	.1094	.1313	.1532	.0751	-.0030	-.0811	-.0730	-.0649	-.0568	-.0487	-.0406	-.0325	-.0243	-.0162	-.0081	0
	.8	0	.0135	.0269	.0404	.0538	.0673	.0807	.0942	.1076	.0211	-.0655	-.0589	-.0524	-.0458	-.0393	-.0327	-.0262	-.0196	-.0131	-.0065	0
	.9	0	.0061	.0122	.0183	.0245	.0306	.0367	.0428	.0489	.0550	-.0389	-.0350	-.0311	-.0272	-.0233	-.0194	-.0155	-.0117	-.0078	-.0039	0
	B.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0056	-.0112	-.0168	-.0224	-.0280	-.0336	-.0392	-.0448	-.0504	-.0560	.0576	.0512	.0448	.0384	.0320	.0256	.0192	.0128	.0064	0
	.2	0	-.0094	-.0189	-.0283	-.0377	-.0471	-.0566	-.0660	-.0754	-.0848	-.0943	.0112	.1166	.1020	.0874	.0729	.0583	.0437	.0291	.0146	0
	.3	0	-.0117	-.0234	-.0351	-.0467	-.0584	-.0701	-.0818	-.0935	-.1052	-.1168	-.0212	.0745	.1702	.1459	.1216	.0973	.0729	.0486	.0243	0
	.4	0	-.0126	-.0251	-.0377	-.0503	-.0628	-.0754	-.0880	-.1005	-.1131	-.1257	-.0411	.0435	.1280	.2126	.1772	.1417	.1063	.0709	.0354	0
	.5	0	-.0123	-.0245	-.0368	-.0491	-.0614	-.0736	-.0859	-.0982	-.1105	-.1227	-.0505	.0218	.0941	.1664	.2388	.1909	.1432	.0955	.0477	0
	.6	0	-.0110	-.0220	-.0330	.0440	-.0550	-.0660	-.0770	-.0880	-.0990	-.1100	-.0510	.0080	.0670	.1260	.1850	.2440	.1830	.1220	.0610	0
	.7	0	-.0089	-.0179	-.0268	-.0357	-.0447	-.0536	-.0625	-.0715	-.0804	-.0893	-.0444	.0005	.0455	.0904	.1353	.1803	.2252	.1501	.0751	0
	.8	0	-.0063	-.0126	-.0189	-.0251	-.0314	-.0377	-.0440	-.0503	-.0566	-.0628	-.0326	.0023	.0280	.0583	.0886	.1189	.1491	.1794	.0897	0
	.9	0	-.0032	-.0065	-.0097	-.0130	-.0162	-.0194	-.0227	-.0259	-.0292	-.0324	-.0172	.0019	.0133	.0286	.0438	.0590	.0743	.0895	.1048	0
	C.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+Area		0	.0393	.0686	.0880	.0973	.0966	.0859	.0652	.0345	.0072	0	.0075	.0372	.0825	.1139	.1309	.1335	.1217	.0956	.0550	0
-Area		0	-.0098	-.0196	-.0295	-.0393	-.0491	-.0589	-.0687	-.0785	-.1017	-.1550	-.0822	-.0460	-.0398	-.0341	-.0284	-.0227	-.0170	-.0114	-.0057	0
Total Area		0	.0295	.0490	.0585	.0580	.0475	.0270	-.0035	-.0440	-.0945	-.1550	-.0747	-.0088	.0427	.0798	.1025	.1108	.1047	.0842	.0493	0

Unit load at		REACTIONS/P R_A	R_B	R_C	SHEARS/P V_{AB}	V_{BA}	V_{BC}	V_{CB}
SPAN 1	A.	1.0000	0	0	1.0000	0	0	0
	.1	.8775	.1413	-.0187	.8775	-.1225	.0187	.0187
	.2	[illegible]	[illegible]	-.0364	.7564	-.2436	.0364	.0364
	.3	.6380	.4138	-.0517	.6380	-.3620	.0517	.0517
	.4	.5236	.5400	-.0636	.5236	-.4764	.0636	.0636
	.5	.4148	.6563	-.0710	.4148	-.5852	.0710	.0710
	.6	.3127	.7600	-.0727	.3127	-.6873	.0727	.0727
	.7	.2189	.8487	-.0676	.2189	-.7811	.0676	.0676
	.8	.1345	.9200	-.0545	.1345	-.8655	.0545	.0545
	.9	.0611	.9713	-.0324	.0611	-.9389	.0324	.0324
	B.	0	1.0000	0	0	-1.0 / 0	0 / 1.0	0
SPAN 2	.1	-.0560	1.0026	.0534	-.0560	-.0560	.9466	-.0534
	.2	-.0943	.9728	.1215	-.0943	-.0943	.8785	-.1215
	.3	-.1168	.9142	.2026	-.1168	-.1168	.7974	-.2026
	.4	-.1257	.8304	.2953	-.1257	-.1257	.7047	-.2953
	.5	-.1227	.7250	.3977	-.1227	-.1227	.6023	-.3977
	.6	-.1100	.6016	.5084	-.1100	-.1100	.4916	-.5084
	.7	-.0893	.4638	.6255	-.0893	-.0893	.3745	-.6255
	.8	-.0628	.3152	.7476	-.0628	-.0628	.2524	-.7476
	.9	-.0324	.1594	.8730	-.0324	-.0324	.1270	-.8730
	C.	0	0	1.0000	0	0	0	-1.0000
+Area		.4432	1.3842	.5182	.4432	0	.7292	.0473
-Area		-.0982	0	-.0473	-.0982	-.6550	0	-.5182
Total Area		.3450	1.3842	.4709	.3450	-.6550	.7292	-.4709



Influence coefficients — Two continuous spans.

L = Length of SHORTER span length of LONGER span = NL

N = 1.2

Table 20.24

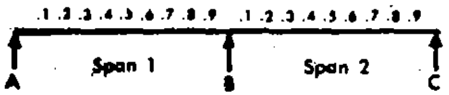
MOMENTS/PL — SPAN 1

Unit load at		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0878	.0757	.0635	.0514	.0392	.0271	.0149	.0028	-.0094	-.0215
	.2	0	.0758	.1517	.1275	.1033	.0791	.0550	.0308	.0066	-.0176	-.0417
	.3	0	.0641	.1281	.1922	.1563	.1203	.0844	.0485	.0125	-.0234	-.0593
	.4	0	.0527	.1054	.1581	.2108	.1635	.1162	.0689	.0216	-.0257	-.0730
	.5	0	.0418	.0837	.1255	.1674	.2092	.1511	.0929	.0348	-.0234	-.0815
	.6	0	.0317	.0633	.0950	.1266	.1583	.1899	.1216	.0532	-.0151	-.0835
	.7	0	.0222	.0445	.0667	.0890	.1112	.1334	.1557	.0779	.0002	-.0776
	.8	0	.0137	.0275	.0412	.0550	.0687	.0824	.0962	.1099	.0237	-.0626
	.9	0	.0063	.0126	.0188	.0251	.0314	.0377	.0440	.0503	.0565	-.0372
	B	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0063	-.0126	-.0188	-.0251	-.0314	-.0377	-.0440	-.0503	-.0565	-.0628
	.2	0	-.0106	-.0212	-.0317	-.0423	-.0529	-.0635	-.0741	-.0846	-.0952	-.1058
	.3	0	-.0131	-.0262	-.0393	-.0525	-.0656	-.0787	-.0918	-.1049	-.1180	-.1312
	.4	0	-.0141	-.0282	-.0423	-.0564	-.0705	-.0846	-.0988	-.1129	-.1270	-.1411
	.5	0	-.0138	-.0276	-.0413	-.0551	-.0689	-.0827	-.0964	-.1102	-.1240	-.1378
	.6	0	-.0123	-.0247	-.0370	-.0494	-.0617	-.0741	-.0[illegible]34	-.0988	-.1111	-.1234
	.7	0	-.0100	-.0201	-.0301	-.0401	-.0501	-.0602	-.0702	-.0802	-.0903	-.1003
	.8	0	-.0071	-.0141	-.0212	-.0282	-.0353	-.0423	-.0494	-.0564	-.0635	-.0705
	.9	0	-.0036	-.0073	-.0109	-.0145	-.0182	-.0218	-.0255	-.0291	-.0327	-.0364
	C	0	0	0	0	0	0	0	0	0	0	0
+Area		0	.0396	.0691	.0887	.0983	.0978	.0874	.0670	.0365	.0078	.0
-Area		0	-.0119	-.0239	-.0358	-.0478	-.0597	-.0716	-.0836	-.0955	-.1192	-.1737
Total Area		0	.0277	.0452	.0529	.0505	.0381	.0158	-.0166	-.0590	-.1114	-.1737

MOMENTS/PL — SPAN 2

Unit load at		.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0
	.1	-.0194	-.0172	-.0151	-.0129	-.0108	-.0086	-.0065	-.0043	-.0022	0
	.2	-.0376	-.0334	-.0292	-.0250	-.0209	-.0167	-.0125	-.0083	-.0042	0
	.3	-.0534	-.0475	-.0415	-.0356	-.0297	-.0237	-.0178	-.0119	-.0059	0
	.4	-.0657	-.0584	-.0511	-.0438	-.0365	-.0292	-.0219	-.0146	-.0073	0
	.5	-.0734	-.0652	-.0571	-.0489	-.0408	-.0326	-.0245	-.0163	-.0082	0
	.6	-.0751	-.0668	-.0584	-.0501	-.0417	-.0334	-.0250	-.0167	-.0083	0
	.7	-.0698	-.0621	-.0543	-.0466	-.0388	-.0310	-.0233	-.0155	-.0078	0
	.8	-.0563	-.0501	-.0438	-.0376	-.0313	-.0250	-.0188	-.0125	-.0063	0
	.9	-.0335	-.0297	-.0260	-.0223	-.0186	-.0149	-.0112	-.0074	-.0037	0
	B	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	.0605	.0537	.0470	.0403	.0336	.0269	.0202	.0134	.0067	0
	.2	.0088	.1234	.1079	.0925	.0771	.0617	.0463	.0308	.0154	0
	.3	-.0270	.0771	.1812	.1553	.1294	.1035	.0777	.0518	.0259	0
	.4	-.0490	.0431	.1352	.2274	.1895	.1516	.1137	.0758	.0379	0
	.5	-.0590	.0198	.0986	.1773	.2561	.2049	.1537	.1024	.0512	0
	.6	-.0591	.0052	.0696	.1339	.1983	.2626	.1970	.1313	.0657	0
	.7	-.0513	-.0022	.0468	.0958	.1449	.1939	.2429	.1619	.0810	0
	.8	-.0375	-.0044	.0286	.0617	.0947	.1278	.1608	.1939	.0969	0
	.9	-.0197	-.0031	.0135	.0302	.0468	.0635	.0801	.0967	.1134	0
	C	0	0	0	0	0	0	0	0	0	0
+Area		.0082	.0410	.0939	.1312	.1515	.1550	.1416	.1113	.0641	0
-Area		-.0885	-.0448	-.0380	-.0326	-.0272	-.0217	-.0163	-.0109	-.0054	0
Total Area		-.0803	-.0038	.0559	.0986	.1243	.1333	.1253	.1004	.0587	0

Unit load at		REACTIONS/P R_A	R_B	R_C	SHEARS/P V_{AB}	V_{BA}	V_{BC}	V_{CB}
SPAN 1	A	1.0000	0	0	1.0000	0	0	0
	.1	.8785	.1381	-.0166	.8785	-.1215	.0166	.0166
	.2	.7583	.2738	-.0321	.7583	-.2417	.0321	.0321
	.3	.6407	.4050	-.0457	.6407	-.3593	.0457	.0457
	.4	.5270	.5292	-.0562	.5270	-.4730	.0562	.0562
	.5	.4185	.6442	-.0627	.4185	-.5815	.0627	.0627
	.6	.3165	.7477	-.0642	.3165	-.6835	.0642	.0642
	.7	.2224	.8373	-.0597	.2224	-.7776	.0597	.0597
	.8	.1374	.9108	-.0482	.1374	-.8626	.0482	.0482
	.9	.0628	.9658	-.0286	.0628	-.9372	.0286	.0281
	B	0	1.0000	0	0	-1.0 / 0	0 / 1.0	0
SPAN 2	.1	-.0628	1.0112	.0517	-.0628	-.0628	.9483	-.0517
	.2	-.1058	.9872	.1186	-.1058	-.1058	.8814	-.1186
	.3	-.1312	.9320	.1991	-.1312	-.1312	.8009	-.1991
	.4	-.1411	.8496	.2915	-.1411	-.1411	.7085	-.2915
	.5	-.1378	.7438	.3940	-.1378	-.1378	.6060	-.3940
	.6	-.1234	.6184	.5050	-.1234	-.1234	.4950	-.5050
	.7	-.1003	.4774	.6228	-.1003	-.1003	.3772	-.6228
	.8	-.0705	.3248	.7457	-.0705	-.0705	.2543	-.7457
	.9	-.0364	.1644	.8720	-.0364	-.0364	.1280	-.8720
	C	0	0	1.0000	0	0	0	-1.0000
+Area		.4457	1.4574	.5582	.4457	0	.7837	.0418
-Area		-.1194	0	-.0418	.1194	-.6737	0	-.5582
Total Area		.3263	1.4574	.5164	.3263	-.6737	.7837	-.5164



Influence coefficients — Two continuous spans.

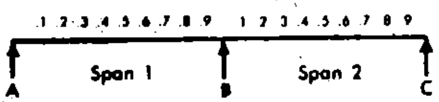
L = Length of SHORTER span ; length of LONGER span = NL

N = 1.3

Table 20.25

	Unit load at	MOMENTS/PL SPAN 1 A.	.1	.2	.3	.4	.5	.6	.7	.8	.9	B.	SPAN 2 .1	.2	.3	.4	.5	.6	.7	.8	.9	C.
SPAN 1	A.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0879	.0759	.0638	.0518	.0397	.0276	.0156	.0035	-.0086	-.0206	-.0186	-.0165	-.0144	-.0124	-.0103	-.0082	-.0062	-.0041	-.0021	0
	.2	0	.0760	.1520	.1280	.1040	.0800	.0560	.0320	.0080	-.0160	-.0400	-.0360	-.0320	-.0280	-.0240	-.0200	-.0160	-.0120	-.0080	-.0040	0
	.3	0	.0643	.1286	.1929	.1573	.1216	.0859	.0502	.0145	-.0212	-.0569	-.0512	-.0455	-.0398	-.0341	-.0284	-.0227	-.0171	-.0114	-.0057	0
	.4	0	.0530	.1060	.1590	.2120	.1650	.1180	.0710	.0240	-.0230	-.0700	-.0630	-.0560	-.0490	-.0420	-.0350	-.0280	-.0210	-.0140	-.0070	0
	.5	0	.0422	.0844	.1266	.1688	.2109	.1531	.0953	.0375	-.0203	-.0781	-.0703	-.0625	-.0547	-.0469	-.0391	-.0312	-.0234	-.0156	-.0078	0
	.6	0	.0320	.0640	.0960	.1280	.1600	.1920	.1240	.0560	-.0120	-.0800	-.0720	-.0640	-.0560	-.0480	-.0400	-.0320	-.0240	-.0160	-.0080	0
	.7	0	.0226	.0451	.0677	.0903	.1128	.1354	.1579	.0805	.0031	-.0744	-.0669	-.0595	-.0521	-.0446	-.0372	-.0297	-.0223	-.0149	-.0074	0
	.8	0	.0140	.0280	.0420	.0560	.0700	.0840	.0980	.1120	.0260	-.0600	-.0540	-.0480	-.0420	-.0360	-.0300	-.0240	-.0180	-.0120	-.0060	0
	.9	0	.0064	.0129	.0193	.0257	.0322	.0386	.0451	.0515	.0579	-.0356	-.0321	-.0285	-.0249	-.0214	-.0178	-.0143	-.0107	-.0071	-.0036	0
	B.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0070	-.0140	-.0209	-.0279	-.0349	-.0419	-.0489	-.0559	-.0628	-.0698	.0632	.0561	.0491	.0421	.0351	.0281	.0211	.0140	.0070	0
	.2	0	-.0118	-.0235	-.0353	-.0470	-.0588	-.0706	-.0823	-.0941	-.1058	-.1176	.0062	.1299	.1137	.0974	.0812	.0650	.0487	.0325	.0162	0
	.3	0	-.0146	-.0292	-.0437	-.0583	-.0729	-.0875	-.1020	-.1166	-.1312	-.1458	-.0332	.0794	.1920	.1645	.1371	.1097	.0823	.0548	.0274	0
	.4	0	-.0157	-.0314	-.0470	-.0627	-.0784	-.0941	-.1098	-.1254	-.1411	-.1568	-.0571	.0426	.1422	.2419	.2016	.1613	.1210	.0806	.0403	0
	.5	0	-.0153	-.0306	-.0459	-.0612	-.0766	-.0919	-.1072	-.1225	-.1378	-.1531	-.0678	.0175	.1028	.1881	.2734	.2188	.1641	.1094	.0547	0
	.6	0	-.0137	-.0274	-.0412	-.0549	-.0686	-.0823	-.0960	-.1098	-.1235	-.1372	-.0675	.0022	.0720	.1417	.2114	.2811	.2108	.1406	.0703	0
	.7	0	-.0111	-.0223	-.0334	-.0446	-.0557	-.0669	-.0780	-.0892	-.1003	-.1115	-.0583	-.0052	.0480	.1011	.1543	.2074	.2606	.1737	.0869	0
	.8	0	-.0078	-.0157	-.0235	-.0314	-.0392	-.0470	-.0549	-.0627	-.0706	-.0784	-.0426	-.0067	.0291	.0650	.1008	.1366	.1725	.2083	.1042	0
	.9	0	-.0040	-.0081	-.0121	-.0162	-.0202	-.0243	-.0283	-.0323	-.0364	-.0404	-.0224	-.0043	.0137	.0317	.0498	.0678	.0859	.1039	.1220	0
	C.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+Area		0	.0398	.0696	.0894	.0992	.0990	.0888	.0685	.0383	.0083	0	.0089	.0448	.1058	.1495	.1735	.1780	.1629	.1282	.0739	0
-Area		0	-.0143	-.0286	-.0429	-.0572	-.0715	-.0858	-.1000	-.1143	-.1388	-.1950	-.0962	-.0440	-.0365	-.0312	-.0260	-.0208	-.0156	-.0104	-.0052	0
Total Area		0	.0255	.0410	.0465	.0420	.0275	.0030	-.0315	-.0760	-.1305	-.1950	-.0873	.0008	.0693	.1183	.1475	.1572	.1473	.1178	.0687	0

	Unit load at	REACTIONS/P R_A	R_B	R_C	SHEARS/P V_{AB}	V_{BA}	V_{BC}	V_{CB}
SPAN 1	A.	1.0000	0	0	1.0000	0	0	0
	.1	.8794	-.1354	-.0147	.8794	-.1206	.0147	.0147
	.2	.7600	.2686	-.0286	.7600	-.2400	.0286	.0286
	.3	.6431	.3975	-.0406	.6431	-.3569	.0406	.0406
	.4	.5300	.5200	-.0500	.5300	-.4700	.0500	.0500
	.5	.4219	.6339	-.0558	.4219	-.5781	.0558	.0558
	.6	.3200	.7371	-.0571	.3200	-.6800	.0571	.0571
	.7	.2256	.8275	-.0531	.2256	-.7744	.0531	.0531
	.8	.1400	.9029	-.0429	.1400	-.8600	.0429	.0429
	.9	.0644	.9611	-.0254	.0644	-.9356	.0254	.0254
	B.	0	1.0000	0	0	-1.0 / 0	0 / 1.0	0
SPAN 2	.1	-.0698	1.0197	.0501	-.0698	-.0698	.9498	-.0501
	.2	-.1176	1.0016	.1160	-.1176	-.1176	.8840	-.1160
	.3	-.1458	.9499	.1959	-.1458	-.1458	.8041	-.1959
	.4	-.1568	.8688	.2880	-.1568	-.1568	.7120	-.2880
	.5	-.1531	.7625	.3906	-.1531	-.1531	.6094	-.3906
	.6	-.1372	.6352	.5020	-.1372	-.1372	.4980	-.5020
	.7	-.1115	.4911	.6204	-.1115	-.1115	.3796	-.6204
	.8	-.0784	.3344	.7440	-.0784	-.0784	.2560	-.7440
	.9	-.0404	.1693	.8711	-.0404	-.0404	.1289	-.8711
	C.	0	0	1.0000	0	0	0	-1.0000
+Area		.4479	1.5343	.5979	.4479	0	.8393	.0372
-Area		-.1429	0	-.0372	-.1429	-.6950	0	-.5979
Total Area		.3050	1.5343	.5607	.3050	-.6950	.8393	-.5607



Influence coefficients — Two continuous spans.

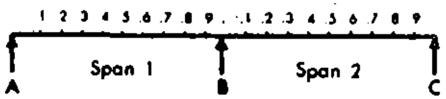
L = Length of SHORTER span ; length of LONGER span = NL

N = 1.4

Table 20.26

Unit load at		MOMENTS/PL																				
		SPAN 1											SPAN 2									
		A.	.1	.2	.3	.4	.5	.6	.7	.8	.9	B.	.1	.2	.3	.4	.5	.6	.7	.8	.9	C.
SPAN 1	A.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0880	.0760	.0641	.0521	.0401	.0281	.0161	.0042	-.0078	-.0198	-.0178	-.0158	-.0139	-.0119	-.0099	-.0079	-.0059	-.0040	-.0020	0
	.2	0	.0762	.1523	.1285	.1046	.0808	.0570	.0331	.0093	-.0146	-.0384	-.0346	-.0307	-.0269	-.0230	-.0192	-.0154	-.0115	-.0077	-.0038	0
	.3	0	.0645	.1291	.1936	.1582	.1227	.0872	.0518	.0163	-.0191	-.0546	-.0491	-.0437	-.0382	-.0328	-.0273	-.0218	-.0164	-.0109	-.0055	0
	.4	0	.0533	.1066	.1598	.2131	.1664	.1197	.0730	.0262	-.0205	-.0672	-.0605	-.0538	-.0470	-.0403	-.0336	-.0269	-.0202	-.0134	-.0067	0
	.5	0	.0425	.0850	.1275	.1700	.2125	.1550	.0975	.0400	-.0175	-.0750	-.0675	-.0600	-.0525	-.0450	-.0375	-.0300	-.0225	-.0150	-.0075	0
	.6	0	.0323	.0646	.0970	.1293	.1616	.1939	.1262	.0586	-.0091	-.0708	-.0691	-.0614	-.0538	-.0461	-.0384	-.0307	-.0230	-.0154	-.0077	0
	.7	0	.0229	.0457	.0686	.0914	.1143	.1372	.1600	.0829	.0057	-.0714	-.0643	-.0571	-.0500	-.0428	-.0357	-.0286	-.0214	-.0143	-.0071	0
	.8	0	.0142	.0285	.0427	.0570	.0712	.0854	.0997	.1139	.0282	-.0576	-.0518	-.0461	-.0403	-.0346	-.0288	-.0230	-.0173	-.0115	-.0058	0
	.9	0	.0066	.0132	.0197	.0263	.0329	.0395	.0461	.0526	.0592	-.0342	-.0308	-.0274	-.0239	-.0205	-.0171	-.0137	-.0103	-.0068	-.0034	0
	B.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0077	-.0154	-.0231	-.0308	-.0385	-.0462	-.0539	-.0616	-.0693	-.0770	.0657	.0584	.0511	.0438	.0365	.0292	.0219	.0146	.0073	0
	.2	0	-.0130	-.0259	-.0389	-.0518	-.0648	-.0778	-.0907	-.1037	-.1166	-.1296	.0034	.1363	.1193	.1022	.0852	.0682	.0511	.0341	.0170	0
	.3	0	-.0161	-.0321	-.0482	-.0643	-.0803	-.0964	-.1125	-.1285	-.1446	-.1607	-.0396	.0815	.2025	.1736	.1447	.1157	.0868	.0579	.0289	0
	.4	0	-.0173	-.0346	-.0518	-.0691	-.0864	-.1037	-.1210	-.1382	-.1555	-.1728	-.0655	.0418	.1490	.2563	.2136	.1709	.1282	.0854	.0427	0
	.5	0	-.0169	-.0337	-.0506	-.0675	-.0844	-.1013	-.1181	-.1350	-.1519	-.1688	-.0769	.0150	.1069	.1988	.2906	.2325	.1744	.1163	.0581	0
	.6	0	-.0151	-.0302	-.0454	-.0605	-.0756	-.0907	-.1058	-.1210	-.1361	-.1512	-.0761	-.0010	.0742	.1493	.2244	.2995	.2246	.1498	.0749	0
	.7	0	-.0123	-.0246	-.0369	-.0491	-.0614	-.0737	-.0860	-.0983	-.1106	-.1228	-.0656	-.0083	.0490	.1063	.1636	.2209	.2781	.1854	.0927	0
	.8	0	-.0086	-.0173	-.0259	-.0346	-.0432	-.0518	-.0605	-.0691	-.0778	-.0864	-.0478	-.0091	.0295	.0682	.1068	.1454	.1841	.2227	.1114	0
	.9	0	-.0045	-.0089	-.0134	-.0178	-.0223	-.0267	-.0312	-.0356	-.0401	-.0445	-.0251	-.0056	.0138	.0333	.0527	.0722	.0916	.1111	.1305	0
	C.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+Area		0	.0400	.0700	.0900	.1000	.1000	.0900	.0700	.0400	.0069	0	.0096	.0486	.1181	.1688	.1969	.2025	.1856	.1463	.0844	0
-Area		0	-.0169	-.0337	-.0506	-.0675	-.0844	-.1012	-.1181	-.1350	-.1608	-.2186	-.1052	-.0436	-.0350	-.0300	-.0250	-.0200	-.0150	-.0100	-.0050	0
Total Area		0	.0231	.0363	.0394	.0325	.0156	-.0112	-.0481	-.0950	-.1519	-.2186	-.0956	.0050	.0831	.1388	.1719	.1825	.1706	.1363	.0794	0

Unit load at		REACTIONS/P			SHEARS/P			
		R_A	R_B	R_C	V_{AB}	V_{BA}	V_{BC}	V_{CB}
SPAN 1	A.	1.0000	0	0	1.0000	0	0	0
	.1	.8802	.1330	-.0132	.8802	-.1198	.0132	.0132
	.2	.7616	.2640	-.0256	.7616	-.2384	.0256	.0256
	.3	.6454	.3910	-.0364	.6454	-.3546	.0364	.0364
	.4	.5328	.5120	-.0448	.5328	-.4672	.0448	.0448
	.5	.4250	.6250	-.0500	.4250	-.5750	.0500	.0500
	.6	.3232	.7280	-.0512	.3232	-.6768	.0512	.0512
	.7	.2286	.8190	-.0476	.2286	-.7714	.0476	.0476
	.8	.1424	.8960	-.0384	.1424	-.8576	.0384	.0384
	.9	.0658	.9570	-.0228	.0658	-.9342	.0228	.0228
	B.	0	1.0000	0	0	-1.0 / 0	0 / 1.0	0
SPAN 2	.1	-.0770	1.0282	.0487	-.0770	-.0770	.9513	-.0487
	.2	-.1296	1.0160	.1136	-.1296	-.1296	.8864	-.1136
	.3	-.1607	.9677	.1929	-.1607	-.1607	.8071	-.1929
	.4	-.1728	.8880	.2848	-.1728	-.1728	.7152	-.2848
	.5	-.1688	.7813	.3875	-.1688	-.1688	.6125	-.3875
	.6	-.1512	.6520	.4992	-.1512	-.1512	.5008	-.4992
	.7	-.1229	.5047	.6181	-.1229	-.1229	.3819	-.6181
	.8	-.0864	.3440	.7424	-.0864	-.0864	.2576	-.7424
	.9	-.0446	.1743	.8703	-.0446	-.0446	.1297	-.8703
	C.	0	0	1.0000	0	0	0	-1.0000
+Area		.4500	1.6146	.6375	.4500	.0	.8958	.0333
-Area		-.1688	0	-.0333	-.1688	-.7188	0	-.6375
Total Area		.2812	1.6146	.6042	.2812	-.7188	.8958	-.6042



Influence coefficients — Two continuous spans.

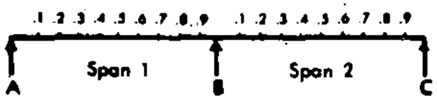
L = Length of SHORTER span ; length of LONGER span = NL

N = 1.5

Table 20.27

Unit load at		MOMENTS/PL																				
		SPAN 1											SPAN 2									
		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0881	.0762	.0643	.0524	.0405	.0286	.0167	.0048	-.0071	-.0190	-.0171	-.0152	-.0133	-.0114	-.0095	-.0076	-.0057	-.0038	-.0019	0
	.2	0	.0763	.1526	.1289	.1052	.0815	.0578	.0342	.0105	-.0132	-.0369	-.0332	-.0295	-.0258	-.0222	-.0185	-.0148	-.0111	-.0074	-.0037	0
	.3	0	.0648	.1295	.1943	.1590	.1238	.0885	.0533	.0180	-.0172	-.0525	-.0472	-.0420	-.0367	-.0315	-.0262	-.0210	-.0157	-.0105	-.0052	0
	.4	0	.0535	.1071	.1606	.2142	.1677	.1212	.0748	.0283	-.0182	-.0646	-.0582	-.0517	-.0452	-.0388	-.0323	-.0258	-.0194	-.0129	-.0065	0
	.5	0	.0428	.0856	.1284	.1712	.2139	.1567	.0995	.0423	-.0149	-.0721	-.0649	-.0577	-.0505	-.0433	-.0361	-.0288	-.0216	-.0144	-.0072	0
	.6	0	.0326	.0652	.0978	.1305	.1631	.1957	.1283	.0609	-.0065	-.0738	-.0665	-.0591	-.0517	-.0443	-.0369	-.0295	-.0222	-.0148	-.0074	0
	.7	0	.0231	.0463	.0694	.0925	.1157	.1388	.1619	.0851	.0082	-.0687	-.0618	-.0549	-.0481	-.0412	-.0343	-.0275	-.0206	-.0137	-.0069	0
	.8	0	.0145	.0289	.0434	.0578	.0723	.0868	.1012	.1157	.0302	-.0554	-.0498	-.0443	-.0388	-.0332	-.0277	-.0222	-.0166	-.0111	-.0055	0
	.9	0	.0067	.0134	.0201	.0268	.0336	.0403	.0470	.0537	.0604	-.0329	-.0296	-.0263	-.0230	-.0197	-.0164	-.0132	-.0099	-.0066	-.0033	0
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0084	-.0168	-.0253	-.0337	-.0421	-.0505	-.0589	-.0673	-.0758	-.0842	.0682	.0607	.0531	.0455	.0379	.0303	.0227	.0152	.0076	0
	.2	0	-.0142	-.0284	-.0425	-.0567	-.0709	-.0851	-.0992	-.1134	-.1276	-.1418	.0004	.1426	.1248	.1069	.0891	.0713	.0535	.0356	.0178	0
	.3	0	-.0176	-.0352	-.0527	-.0703	-.0879	-.1055	-.1230	-.1406	-.1582	-.1758	-.0462	.0834	.2130	.1825	.1521	.1217	.0913	.0608	.0304	0
	.4	0	-.0189	-.0378	-.0567	-.0756	-.0945	-.1134	-.1323	-.1512	-.1701	-.1890	-.0741	.0408	.1557	.2706	.2255	.1804	.1353	.0902	.0451	0
	.5	0	-.0185	-.0369	-.0554	-.0738	-.0923	-.1108	-.1292	-.1477	-.1662	-.1846	-.0862	.0123	.1108	.2092	.3077	.2462	.1846	.1231	.0615	0
	.6	0	-.0165	-.0331	-.0496	-.0662	-.0827	-.0992	-.1158	-.1323	-.1489	-.1654	-.0849	-.0043	.0762	.1568	.2373	.3178	.2384	.1589	.0795	0
	.7	0	-.0134	-.0269	-.0403	-.0538	-.0672	-.0806	-.0941	-.1075	-.1210	-.1344	-.0730	-.0115	.0499	.1114	.1728	.2342	.2957	.1971	.0986	0
	.8	0	-.0095	-.0189	-.0284	-.0378	-.0473	-.0567	-.0662	-.0756	-.0851	-.0945	-.0531	-.0116	.0298	.0713	.1127	.1542	.1956	.2371	.1185	0
	.9	0	-.0049	-.0097	-.0146	-.0195	-.0244	-.0292	-.0341	-.0390	-.0439	-.0487	-.0279	-.0070	.0139	.0348	.0556	.0765	.0974	.1183	.1391	0
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+Area		0	.0402	.0704	.0906	.1008	.1010	.0912	.0713	.0415	.0094	.0	.0103	.0528	.1310	.1890	.2215	.2284	.2097	.1654	.0955	0
-Area		0	-.0197	-.0394	-.0591	-.0788	-.0985	-.1182	-.1378	-.1575	-.1849	-.2450	-.1156	-.0440	-.0337	-.0288	-.0240	-.0192	-.0144	-.0096	-.0048	0
Total Area		0	.0205	.0310	.0315	.0220	.0025	-.0270	-.0665	-.1160	-.1755	-.2450	-.1053	.0088	.0973	.1602	.1975	.2092	.1953	.1558	.0907	0

Unit load at		REACTIONS/P			SHEARS/P			
		R_A	R_B	R_C	V_{AB}	V_{BA}	V_{BC}	V_{CB}
SPAN 1	A	1.0000	0	0	1.0000	0	0	0
	.1	.8810	.1309	-.0119	.8810	-.1190	.0119	.0119
	.2	.7631	.2600	-.0231	.7631	-.2369	.0231	.0231
	.3	.6475	.3853	-.0328	.6475	-.3525	.0328	.0328
	.4	.5354	.5050	-.0404	.5354	-.4646	.0404	.0404
	.5	.4279	.6172	-.0451	.4279	-.5721	.0451	.0451
	.6	.3262	.7200	-.0462	.3262	-.6738	.0462	.0462
	.7	.2313	.8116	-.0429	.2313	-.7687	.0429	.0429
	.8	.1446	.8900	-.0346	.1446	-.8554	.0346	.0346
	.9	.0671	.9534	-.0206	.0671	-.9329	.0206	.0206
	B	0	1.0000	0	0	-1.0 / 0	0 / 1.0	0
SPAN 2	.1	-.0842	1.0368	.0474	-.0842	-.0842	.9526	-.0474
	.2	-.1418	1.0304	.1114	-.1418	-.1418	.8886	-.1114
	.3	-.1758	.9856	.1902	-.1758	-.1758	.8098	-.1902
	.4	-.1890	.9072	.2818	-.1890	-.1890	.7182	-.2818
	.5	-.1846	.8000	.3846	-.1846	-.1846	.6154	-.3846
	.6	-.1654	.6688	.4966	-.1654	-.1654	.5034	-.4966
	.7	-.1344	.5184	.6160	-.1344	-.1344	.3840	-.6160
	.8	-.0945	.3536	.7409	-.0945	-.0945	.2591	-.7409
	.9	-.0487	.1792	.8695	-.0487	-.0487	.1305	-.8695
	C	0	0	1.0000	0	0	0	-1.0000
+Area		.4519	1.6981	.6769	.4519	.0	.9531	.0300
-Area		-.1969	0	-.0300	-.1969	-.7450	0	-.6769
Total Area		.2550	1.6981	.6469	.2550	-.7450	.9531	-.6469



Influence coefficients — Two continuous spans.

L = Length of SHORTER span ; length of LONGER span = NL

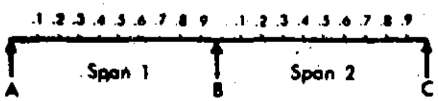
N = 1.6

Table 20.28

Unit load at		MOMENTS/PL SPAN 1 A.	.1	.2	.3	.4	.5	.6	.7	.8	.9	B.
SPAN 1	A.	0	0	0	0	0	0	0	0	0	0	0
SPAN 1	.1	0	.0882	.0763	.0645	.0527	.0408	.0290	.0172	.0053	-.0065	-.0183
SPAN 1	.2	0	.0764	.1529	.1293	.1058	.0822	.0587	.0351	.0116	-.0120	-.0356
SPAN 1	.3	0	.0649	.1299	.1948	.1598	.1247	.0897	.0546	.0196	-.0155	-.0506
SPAN 1	.4	0	.0538	.1076	.1613	.2151	.1689	.1227	.0764	.0302	-.0160	-.0622
SPAN 1	.5	0	.0431	.0861	.1292	.1722	.2153	.1583	.1014	.0444	-.0125	-.0694
SPAN 1	.6	0	.0329	.0658	.0987	.1316	.1644	.1973	.1302	.0631	-.0040	-.0711
SPAN 1	.7	0	.0234	.0468	.0702	.0936	.1169	.1403	.1637	.0871	.0105	-.0661
SPAN 1	.8	0	.0147	.0293	.0440	.0587	.0733	.0880	.1027	.1173	.0320	-.0533
SPAN 1	.9	0	.0068	.0137	.0205	.0273	.0342	.0410	.0478	.0547	.0615	-.0317
	B.	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0092	-.0183	-.0275	-.0366	-.0458	-.0549	-.0641	-.0732	-.0824	-.0915
SPAN 2	.2	0	-.0154	-.0308	-.0462	-.0617	-.0771	-.0925	-.1079	-.1233	-.1387	-.1541
SPAN 2	.3	0	-.0191	-.0382	-.0573	-.0764	-.0955	-.1146	-.1337	-.1528	-.1720	-.1911
SPAN 2	.4	0	-.0206	-.0411	-.0617	-.0822	-.1028	-.1233	-.1439	-.1644	-.1850	-.2055
SPAN 2	.5	0	-.0201	-.0401	-.0602	-.0803	-.1003	-.1204	-.1405	-.1606	-.1806	-.2007
SPAN 2	.6	0	-.0180	-.0360	-.0539	-.0719	-.0899	-.1079	-.1259	-.1439	-.1618	-.1798
SPAN 2	.7	0	-.0146	-.0292	-.0438	-.0584	-.0731	-.0877	-.1023	-.1169	-.1315	-.1461
SPAN 2	.8	0	-.0103	-.0206	-.0308	-.0411	-.0514	-.0617	-.0719	-.0822	-.0925	-.1028
SPAN 2	.9	0	-.0053	-.0106	-.0159	-.0212	-.0265	-.0318	-.0371	-.0424	-.0477	-.0530
	C.	0	0	0	0	0	0	0	0	0	0	0
+Area		0	.0404	.0707	.0911	.1015	.1019	.0922	.0726	.0430	.0100	0
-Area		0	-.0227	-.0455	-.0682	-.0910	-.1137	-.1365	-.1592	-.1820	-.2114	-.2737
Total Area		0	.0177	.0252	.0229	.0105	-.0118	-.0443	-.0866	-.1390	-.2014	-.2737

Unit load at		MOMENTS/PL SPAN 2 .1	.2	.3	.4	.5	.6	.7	.8	.9	C.
SPAN 1	A.	0	0	0	0	0	0	0	0	0	0
SPAN 1	.1	-.0165	-.0147	-.0128	-.0110	-.0092	-.0073	-.0055	-.0037	-.0018	0
SPAN 1	.2	-.0320	-.0284	-.0249	-.0213	-.0178	-.0142	-.0107	-.0071	-.0036	0
SPAN 1	.3	-.0455	-.0404	-.0354	-.0303	-.0253	-.0202	-.0152	-.0101	-.0051	0
SPAN 1	.4	-.0560	-.0498	-.0436	-.0373	-.0311	-.0249	-.0187	-.0124	-.0062	0
SPAN 1	.5	-.0625	-.0556	-.0486	-.0417	-.0347	-.0278	-.0208	-.0139	-.0069	0
SPAN 1	.6	-.0640	-.0569	-.0498	-.0427	-.0356	-.0284	-.0213	-.0142	-.0071	0
SPAN 1	.7	-.0595	-.0529	-.0463	-.0397	-.0331	-.0264	-.0198	-.0132	-.0066	0
SPAN 1	.8	-.0480	-.0427	-.0373	-.0320	-.0267	-.0213	-.0160	-.0107	-.0053	0
SPAN 1	.9	-.0285	-.0253	-.0222	-.0190	-.0158	-.0127	-.0095	-.0063	-.0032	0
	B.	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	.0706	.0628	.0549	.0471	.0392	.0314	.0235	.0157	.0078	0
SPAN 2	.2	-.0027	.1487	.1301	.1115	.0929	.0743	.0558	.0372	.0186	0
SPAN 2	.3	-.0530	.0852	.2233	.1914	.1595	.1276	.0957	.0638	.0319	0
SPAN 2	.4	-.0830	.0396	.1621	.2847	.2372	.1898	.1423	.0949	.0474	0
SPAN 2	.5	-.0956	.0094	.1145	.2196	.3247	.2597	.1948	.1299	.0649	0
SPAN 2	.6	-.0938	-.0079	.0781	.1641	.2801	.3361	.2521	.1680	.0840	0
SPAN 2	.7	-.0805	-.0149	.0507	.1163	.1819	.2476	.3132	.2088	.1044	0
SPAN 2	.8	-.0585	-.0142	.0301	.0743	.1186	.1629	.2072	.2514	.1257	0
SPAN 2	.9	-.0307	-.0084	.0139	.0362	.0585	.0808	.1031	.1254	.1477	0
	C.	0	0	0	0	0	0	0	0	0	0
+Area		.0111	.0570	.1442	.2103	.2475	.2558	.2352	.1857	.1073	0
-Area		-.1274	-.0448	-.0324	-.0278	-.0231	-.0185	-.0139	-.0093	-.0046	0
Total Area		-.1163	.0122	.1118	.1825	.2244	.2373	.2213	.1764	.1027	0

Unit load at		REACTIONS/P R_A	R_B	R_C	SHEARS/P V_{AB}	V_{BA}	V_{BC}	V_{CB}
SPAN 1	A.	1.0000	0	0	1.0000	0	0	0
SPAN 1	.1	.8817	.1291	-.0108	.8817	-.1183	.0108	.0108
SPAN 1	.2	.7644	.2565	-.0209	.7644	-.2356	.0209	.0209
SPAN 1	.3	.6494	.3803	-.0297	.6494	-.3506	.0297	.0297
SPAN 1	.4	.5378	.4988	-.0366	.5378	-.4622	.0366	.0366
SPAN 1	.5	.4306	.6103	-.0408	.4306	-.5694	.0408	.0408
SPAN 1	.6	.3289	.7129	-.0418	.3289	-.6711	.0418	.0418
SPAN 1	.7	.2339	.8050	-.0389	.2339	-.7661	.0389	.0389
SPAN 1	.8	.146[illegible]	.8847	-.0314	.1467	-.8533	.0314	.0314
SPAN 1	.9	.0683	.9503	-.0186	.0683	-.9317	.0186	.0186
	B.	0	1.0000	0	0	-1.0 / 0	0 / 1.0	0
SPAN 2	.1	-.0915	1.0454	.0462	-.0915	-.0915	.9538	-.0462
SPAN 2	.2	-.1541	1.0448	.1093	-.1541	-.1541	.8907	-.1093
SPAN 2	.3	-.1911	1.0035	.1876	-.1911	-.1911	.8124	-.1876
SPAN 2	.4	-.2055	.9264	.2791	-.2055	-.2055	.7209	-.2791
SPAN 2	.5	-.2007	.8187	.3819	-.2007	-.2007	.6181	-.3819
SPAN 2	.6	-.1798	.6856	.4942	-.1798	-.1798	.5058	-.4942
SPAN 2	.7	-.1461	.5320	.6141	-.1461	-.1461	.3859	-.6141
SPAN 2	.8	-.1028	.3632	.7396	-.1028	-.1028	.2604	-.7396
SPAN 2	.9	-.0530	.1841	.8688	-.0530	-.0530	.1312	-.8688
	C.	0	0	1.0000	0	0	0	-1.0000
+Area		.4537	1.7848	.7162	.4537	.0	1.0110	.0272
-Area		-.2275	0	-.0272	-.2275	-.7738	0	-.7162
Total Area		.2263	1.7848	.6890	.2263	-.7738	1.0110	-.6890



Influence coefficients — Two continuous spans.

L = Length of SHORTER span ; length of LONGER span = NL

N = 1.7

Table 20.29

MOMENTS/PL — SPAN 1

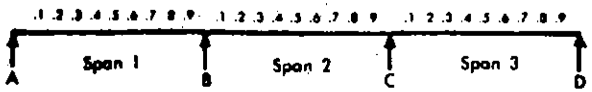
Unit load at		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0874	.0747	.0621	.0494	.0368	.0242	.0115	-.0011	-.0138	-.0264
	.2	0	.0749	.1498	.1246	.0995	.0744	.0493	.0242	-.0010	-.0261	-.0512
	.3	0	.0627	.1254	.1882	.1509	.1136	.0763	.0390	.0018	-.0355	-.0728
	.4	0	.0510	.1021	.1531	.2042	.1552	.1062	.0573	.0083	-.0406	-.0896
	.5	0	.0400	.0800	.1200	.1600	.2000	.1400	.0800	.0200	-.0400	-.1000
	.6	0	.0298	.0595	.0893	.1190	.1488	.1786	.1084	.0381	-.0322	-.1024
	.7	0	.0205	.0410	.0614	.0819	.1024	.1229	.1434	.0638	-.0157	-.0952
	.8	0	.0123	.0246	.0370	.0493	.0616	.0739	.0862	.0986	.0109	-.0768
	.9	0	.0054	.0109	.0163	.0218	.0272	.0326	.0381	.0435	.0490	-.0456
	B	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0039	-.0078	-.0117	-.0156	-.0195	-.0234	-.0273	-.0312	-.0351	-.0390
	.2	0	-.0064	-.0128	-.0192	-.0256	-.0320	-.0384	-.0448	-.0512	-.0576	-.0640
	.3	0	-.0077	-.0154	-.0231	-.0308	-.0385	-.0462	-.0539	-.0616	-.0639	-.0770
	.4	0	-.0080	-.0160	-.0240	-.0320	-.0400	-.0480	-.0560	-.0640	-.0720	-.0800
	.5	0	-.0075	-.0150	-.0225	-.0300	-.0375	-.0450	-.0525	-.0600	-.0675	-.0750
	.6	0	-.0064	-.0128	-.0192	-.0256	-.0320	-.0384	-.0448	-.0512	-.0576	-.0640
	.7	0	-.0049	-.0098	-.0147	-.0196	-.0245	-.0294	-.0343	-.0392	-.0441	-.0490
	.8	0	-.0032	-.0064	-.0096	-.0128	-.0160	-.0192	-.0224	-.0256	-.0288	-.0320
	.9	0	-.0015	-.0030	-.0045	-.0060	-.0075	-.0090	-.0105	-.0120	-.0135	-.0150
	C	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0011	.0023	.0034	.0046	.0057	.0068	.0080	.0091	.0103	.0114
	.2	0	.0019	.0038	.0058	.0077	.0096	.0115	.0134	.0154	.0173	.0192
	.3	0	.0024	.0048	.0071	.0095	.0119	.0143	.0167	.0190	.0214	.0238
	.4	0	.0026	.0051	.0077	.0102	.0128	.0154	.0179	.0205	.0230	.0256
	.5	0	.0025	.0050	.0075	.0100	.0125	.0150	.0175	.0200	.0225	.0250
	.6	0	.0022	.0045	.0067	.0090	.0112	.0134	.0157	.0179	.0202	.0224
	.7	0	.0018	.0036	.0055	.0073	.0091	.0109	.0127	.0146	.0164	.0182
	.8	0	.0013	.0026	.0038	.0051	.0064	.0077	.0090	.0102	.0115	.0128
	.9	0	.0007	.0013	.0020	.0026	.0033	.0040	.0046	.0053	.0059	.0066
	D	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0400	.0700	.0900	.1000	.1000	.0900	.0700	.0402	.0204	.0167
− Area		0	-.0050	-.0100	-.0150	-.0200	-.0250	-.0300	-.0350	-.0402	-.0654	-.1167
Total Area		0	.0350	.0600	.0750	.0800	.0750	.0600	.0350	.0000	-.0450	-.1000

MOMENTS/PL — SPAN 2

Unit load at		.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0
	.1	-.0231	-.0198	-.0165	-.0132	-.0099	-.0066	-.0033	.0000	.0033	.0066
	.2	-.0448	-.0384	-.0320	-.0256	-.0192	-.0128	-.0064	.0000	.0064	.0128
	.3	-.0637	-.0546	-.0455	-.0364	-.0273	-.0182	-.0091	.0000	.0091	.0182
	.4	-.0784	-.0672	-.0560	-.0448	-.0336	-.0224	-.0112	.0000	.0112	.0224
	.5	-.0875	-.0750	-.0625	-.0500	-.0375	-.0250	-.0125	.0000	.0125	.0250
	.6	-.0896	-.0768	-.0640	-.0512	-.0384	-.0256	-.0128	.0000	.0128	.0256
	.7	-.0833	-.0714	-.0595	-.0476	-.0357	-.0238	-.0119	.0000	.0119	.0238
	.8	-.0672	-.0576	-.0480	-.0384	-.0288	-.0192	-.0096	.0000	.0096	.0192
	.9	-.0399	-.0342	-.0285	-.0228	-.0171	-.0114	-.0057	.0000	.0057	.0114
	B	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	.0534	.0458	.0382	.0306	.0230	.0154	.0073	.0002	-.0074	-.0150
	.2	.0192	.1024	.0856	.0688	.0520	.0352	.0184	.0016	-.0152	-.0320
	.3	-.0042	.0684	.1414	.1142	.0870	.0598	.0326	.0054	-.0218	-.0490
	.4	-.0184	.0432	.1048	.1664	.1280	.0896	.0512	.0128	-.0256	-.0640
	.5	-.0250	.0250	.0750	.1250	.1750	.1250	.0750	.0250	-.0250	-.0750
	.6	-.0256	.0128	.0512	.0896	.1280	.1664	.1048	.0432	-.0184	-.0800
	.7	-.0218	.0054	.0326	.0598	.0870	.1142	.1414	.0686	-.0042	-.0770
	.8	-.0152	.0016	.0184	.0352	.0520	.0688	.0856	.1024	.0192	-.0640
	.9	-.0074	.0002	.0078	.0154	.0230	.0306	.0382	.0458	.0534	-.0390
	C	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	.0057	.0000	-.0057	-.0114	-.0171	-.0228	-.0285	-.0342	-.0399	-.0456
	.2	.0096	.0000	-.0096	-.0192	-.0288	-.0384	-.0480	-.0576	-.0672	-.0768
	.3	.0119	.0000	-.0119	-.0238	-.0357	-.0476	-.0595	-.0714	-.0833	-.0952
	.4	.0128	.0000	-.0128	-.0256	-.0384	-.0512	-.0640	-.0768	-.0896	-.1024
	.5	.0125	.0000	-.0125	-.0250	-.0375	-.0500	-.0625	-.0750	-.0875	-.1000
	.6	.0112	.0000	-.0112	-.0224	-.0336	-.0448	-.0560	-.0672	-.0784	-.0896
	.7	.0091	.0000	-.0091	-.0182	-.0273	-.0364	-.0455	-.0546	-.0637	-.0728
	.8	.0064	.0000	-.0064	-.0128	-.0192	-.0256	-.0320	-.0384	-.0448	-.0512
	.9	.0033	.0000	-.0033	-.0066	-.0099	-.0132	-.0165	-.0198	-.0231	-.0264
	D	0	0	0	0	0	0	0	0	0	0
+ Area		.0152	.0300	.0550	.0700	.0750	.0700	.0550	.0300	.0152	.0167
− Area		-.0702	-.0500	-.0500	-.0500	-.0500	-.0500	-.0500	-.0500	-.0702	-.1167
Total Area		-.0550	-.0200	.0050	.0200	.0250	.0200	.0050	-.0200	-.0550	-.1000

REACTIONS/P and SHEARS/P

Unit load at		R_A	R_B	R_C	R_D	V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}
SPAN 1	A	1.0	0	0	0	1.0	0	0	0	0	0
	.1	.8736	.1584	-.0396	.0066	.8736	-.1264	.0330	.0330	-.0066	-.0066
	.2	.7488	.3152	-.0768	.0128	.7488	-.2512	.0640	.0640	-.0128	-.0128
	.3	.6272	.4638	-.1092	.0182	.6272	-.3728	.0910	.0910	-.0182	-.0182
	.4	.5104	.6016	-.1344	.0224	.5104	-.4896	.1120	.1120	-.0224	-.0224
	.5	.4000	.7250	-.1500	.0250	.4000	-.6000	.1250	.1250	-.0250	-.0250
	.6	.2976	.8304	-.1536	.0256	.2976	-.7024	.1280	.1280	-.0256	-.0256
	.7	.2048	.9142	-.1428	.0238	.2048	-.7952	.1190	.1190	-.0238	-.0238
	.8	.1232	.9728	-.1152	.0192	.1232	-.8768	.0960	.0960	-.0192	-.0192
	.9	.0544	1.0026	-.0684	.0114	.0544	-.9456	.0570	.0570	-.0114	-.0114
	B	0	1.0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 2	.1	-.0390	.9630	.0810	-.0150	-.0390	-.0390	.9240	-.0760	.0150	.0150
	.2	-.0640	.8960	.2000	-.0320	-.0640	-.0640	.8320	-.1680	.0320	.0320
	.3	-.0770	.8050	.3210	-.0490	-.0770	-.0770	.7280	-.2720	.0490	.0490
	.4	-.0800	.6960	.4480	-.0640	-.0800	-.0800	.6160	-.3840	.0640	.0640
	.5	-.0750	.5750	.5750	-.0750	-.0750	-.0750	.5000	-.5000	.0750	.0750
	.6	-.0640	.4480	.6960	-.0800	-.0640	-.0640	.3840	-.6160	.0800	.0800
	.7	-.0490	.3210	.8050	-.0770	-.0490	-.0490	.2720	-.7280	.0770	.0770
	.8	-.0320	.2000	.8960	-.0640	-.0320	-.0320	.1680	-.8320	.0640	.0640
	.9	-.0150	.0810	.9630	-.0390	-.0150	-.0150	.0760	-.9240	.0390	.0390
	C	0	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 3	.1	.0114	-.0684	1.0026	.0544	.0114	.0114	-.0570	-.0570	.9456	-.0544
	.2	.0192	-.1152	.9728	.1232	.0192	.0192	-.0960	-.0960	.8768	-.1232
	.3	.0238	-.1428	.9142	.2048	.0238	.0238	-.1190	-.1190	.7952	-.2048
	.4	.0256	-.1536	.8304	.2976	.0256	.0256	-.1280	-.1280	.7024	-.2976
	.5	.0250	-.1500	.7250	.4000	.0250	.0250	-.1250	-.1250	.6000	-.4000
	.6	.0224	-.1344	.6016	.5104	.0224	.0224	-.1120	-.1120	.4896	-.5104
	.7	.0182	-.1092	.4638	.6272	.0182	.0182	-.0910	-.0910	.3728	-.6272
	.8	.0128	-.0768	.3152	.7488	.0128	.0128	-.0640	-.0640	.2512	-.7488
	.9	.0066	-.0396	.1594	.8736	.0066	.0066	-.0330	-.0330	.1264	-.8736
	D	0	0	0	1.0	0	0	0	0	0	-1.0
+ Area		.4500	1.2000	1.2000	.4500	.4500	.0167	.5833	.0832	.6167	.0500
− Area		-.0500	-.1000	-.1000	-.0500	-.0500	-.6167	-.0833	-.5833	-.0167	-.4500
Total Area		.4000	1.1000	1.1000	.4000	.4000	-.6000	.5000	-.5000	.6000	-.4000



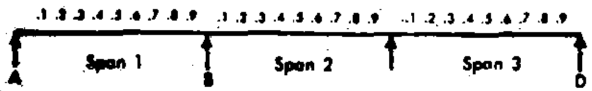
Influence coefficients — Three continuous spans.

L = Length of EXTERIOR spans; length of interior span = NL

N = 1.0

Table 20.30

Unit load at		MOMENTS/PL: SPAN 1: A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	SPAN 2: .1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0875	.0749	.0624	.0499	.0373	.0248	.0123	-.0002	-.0128	-.0253	-.0221	-.0189	-.0157	-.0125	-.0093	-.0061	-.0030	.0002	.0034	.0066
	.2	0	.0751	.1502	.1253	.1004	.0755	.0506	.0256	.0007	-.0242	-.0491	-.0429	-.0367	-.0305	-.0243	-.0181	-.0119	-.0057	.0005	.0067	.0129
	.3	0	.0630	.1260	.1891	.1521	.1151	.0781	.0411	.0042	-.0328	-.0698	-.0610	-.0522	-.0434	-.0346	-.0258	-.0169	-.0081	.0007	.0095	.0183
	.4	0	.0514	.1028	.1542	.2056	.1571	.1085	.0599	.0113	-.0373	-.0859	-.0751	-.0642	-.0534	-.0425	-.0317	-.0209	-.0100	.0008	.0117	.0225
	.5	0	.0404	.0808	.1212	.1617	.2021	.1425	.0829	.0233	-.0363	-.0959	-.0838	-.0717	-.0596	-.0475	-.0354	-.0233	-.0112	.0009	.0130	.0251
	.6	0	.0302	.0604	.0906	.1207	.1509	.1811	.1113	.0415	-.0283	-.0982	-.0858	-.0734	-.0610	-.0486	-.0362	-.0238	-.0115	.0009	.0133	.0257
	.7	0	.0209	.0417	.0626	.0835	.1044	.1252	.1461	.0670	-.0121	-.0913	-.0797	-.0682	-.0567	-.0452	-.0337	-.0222	-.0106	.0008	.0124	.0239
	.8	0	.0126	.0253	.0379	.0506	.0632	.0758	.0885	.1011	.0137	-.0736	-.0643	-.0550	-.0458	-.0365	-.0272	-.0179	-.0086	.0007	.0100	.0193
	.9	0	.0056	.0113	.0169	.0225	.0281	.0338	.0394	.0450	.0507	-.0437	-.0382	-.0327	-.0272	-.0216	-.0161	-.0106	-.0051	.0004	.0059	.0114
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0045	-.0090	-.0135	-.0179	-.0224	-.0269	-.0314	-.0359	-.0404	-.0449	.0569	.0487	.0406	.0324	.0242	.0160	.0078	-.0004	-.0086	-.0168
	.2	0	-.0074	-.0147	-.0221	-.0294	-.0368	-.0441	-.0515	-.0588	-.0662	-.0735	.0182	.1100	.0917	.0735	.0552	.0370	.0187	.0004	-.0178	-.0361
	.3	0	-.0088	-.0177	-.0265	-.0353	-.0442	-.0530	-.0618	-.0706	-.0795	-.0883	-.0080	.0722	.1525	.1228	.0931	.0634	.0336	.0039	-.0258	-.0555
	.4	0	-.0092	-.0183	-.0275	-.0366	-.0458	-.0549	-.0641	-.0732	-.0824	-.0916	-.0237	.0442	.1121	.1799	.1378	.0957	.0536	.0114	-.0307	-.0728
	.5	0	-.0086	-.0171	-.0257	-.0342	-.0428	-.0514	-.0599	-.0685	-.0771	-.0856	-.0306	.0244	.0794	.1344	.1894	.1344	.0794	.0244	-.0306	-.0856
	.6	0	-.0073	-.0146	-.0218	-.0291	-.0364	-.0437	-.0510	-.0583	-.0655	-.0728	-.0307	.0114	.0536	.0957	.1378	.1799	.1121	.0442	-.0237	-.0916
	.7	0	-.0055	-.0111	-.0167	-.0222	-.0278	-.0333	-.0389	-.0444	-.0500	-.0555	-.0258	.0039	.0336	.0634	.0931	.1228	.1525	.0722	-.0080	-.0883
	.8	0	-.0036	-.0072	-.0108	-.0144	-.0180	-.0216	-.0252	-.0288	-.0325	-.0361	-.0178	.0004	.0187	.0370	.0552	.0735	.0917	.1100	.0182	-.0735
	.9	0	-.0017	-.0034	-.0050	-.0067	-.0084	-.0101	-.0117	-.0134	-.0151	-.0168	-.0086	-.0004	.0078	.0160	.0242	.0324	.0406	.0487	.0569	-.0449
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0011	.0023	.0034	.0046	.0057	.0068	.0080	.0091	.0103	.0114	.0059	-.0004	-.0051	-.0106	-.0161	-.0216	-.0272	-.0327	-.0382	-.0437
	.2	0	.0019	.0039	.0058	.0077	.0097	.0116	.0135	.0154	.0174	.0193	.0100	.0007	-.0086	-.0179	-.0272	-.0365	-.0458	-.0550	-.0643	-.0736
	.3	0	.0024	.0048	.0072	.0096	.0120	.0143	.0167	.0191	.0215	.0239	.0124	.0009	-.0106	-.0222	-.0337	-.0452	-.0567	-.0682	-.0797	-.0913
	.4	0	.0026	.0051	.0077	.0103	.0128	.0154	.0180	.0206	.0231	.0257	.0133	.0008	-.0115	-.0238	-.0362	-.0486	-.0610	-.0734	-.0858	-.0982
	.5	0	.0025	.0050	.0075	.0100	.0126	.0151	.0176	.0201	.0226	.0251	.0130	.0009	-.0112	-.0233	-.0354	-.0475	-.0596	-.0717	-.0838	-.0959
	.6	0	.0023	.0045	.0068	.0090	.0113	.0135	.0158	.0180	.0203	.0225	.0117	.0008	-.0100	-.0209	-.0317	-.0425	-.0534	-.0642	-.0751	-.0859
	.7	0	.0018	.0037	.0055	.0073	.0092	.0110	.0128	.0146	.0165	.0183	.0095	.0007	-.0081	-.0169	-.0258	-.0346	-.0434	-.0522	-.0610	-.0698
	.8	0	.0013	.0026	.0039	.0052	.0065	.0077	.0090	.0103	.0116	.0129	.0067	.0005	-.0057	-.0119	-.0181	-.0243	-.0305	-.0367	-.0429	-.0491
	.9	0	.0007	.0013	.0020	.0026	.0033	.0040	.0046	.0053	.0059	.0066	.0034	.0002	-.0030	-.0061	-.0093	-.0125	-.0157	-.0188	-.0221	-.0253
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0403	.0705	.0908	.1011	.1015	.0917	.0720	.0423	.0209	.0167	.0163	.0346	.0643	.0824	.0865	.0824	.0643	.0346	.0163	.0167
− Area		0	-.0063	-.0125	-.0188	-.0251	-.0315	-.0377	-.0440	-.0503	-.0749	-.1267	-.0718	-.0478	-.0472	-.0472	-.0472	-.0472	-.0472	-.0478	-.0718	-.1267
Total Area		0	.0340	.0580	.0720	.0760	.0700	.0540	.0280	-.0080	-.0540	-.1100	-.0555	-.0132	.0171	.0352	.0413	.0352	.0171	-.0132	-.0555	-.1100



Unit load at		REACTIONS/P: R_A	R_B	R_C	R_D	SHEARS/P: V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}
SPAN 1	A	1.0	0	0	0	1.0	0	0	0	0	0
	.1	.8747	.1543	-.0356	.0066	.8747	-.1253	.0290	.0290	-.0066	-.0066
	.2	.7509	.3054	-.0692	.0129	.7509	-.2491	.0563	.0563	-.0129	-.0129
	.3	.6302	.4498	-.0983	.0183	.6302	-.3698	.0800	.0800	-.0183	-.0183
	.4	.5141	.5844	-.1210	.0225	.5141	-.4859	.0985	.0985	-.0225	-.0225
	.5	.4041	.7058	-.1350	.0251	.4041	-.5959	.1099	.1099	-.0251	-.0251
	.6	.3018	.8108	-.1383	.0257	.3018	-.6982	.1126	.1126	-.0257	-.0257
	.7	.2087	.8960	-.1286	.0239	.2087	-.7913	.1047	.1047	-.0239	-.0239
	.8	.1264	.9581	-.1038	.0193	.1264	-.8736	.0845	.0845	-.0193	-.0193
	.9	.0563	.9939	-.0616	.0114	.0563	-.9437	.0502	.0502	-.0114	-.0114
	B	0	1.0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 2	.1	-.0449	.9704	.0913	-.0168	-.0449	-.0449	.9255	-.0745	.0168	.0168
	.2	-.0735	.9076	.2020	-.0361	-.0735	-.0735	.8341	-.1659	.0361	.0361
	.3	-.0833	.8181	.3257	-.0555	-.0883	-.0883	.7298	-.2702	.0555	.0555
	.4	-.0916	.7086	.4558	-.0728	-.0916	-.0916	.6170	-.3830	.0728	.0728
	.5	-.0856	.5856	.5856	-.0856	-.0856	-.0856	.5000	-.5000	.0856	.0856
	.6	-.0728	.4558	.7086	-.0916	-.0728	-.0728	.3830	-.6170	.0916	.0916
	.7	-.0555	.3257	.8181	-.0883	-.0555	-.0555	.2702	-.7298	.0883	.0883
	.8	-.0361	.2020	.9076	-.0735	-.0361	-.0361	.1659	-.8341	.0735	.0735
	.9	-.0168	.0913	.9704	-.0449	-.0168	-.0168	.0745	-.9255	.0449	.0449
	C	0	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 3	.1	.0114	-.0616	.9939	.0563	.0114	.0114	-.0502	-.0502	.9437	-.0563
	.2	.0193	-.1038	.9581	.1264	.0193	.0193	-.0845	-.0845	.8736	-.1264
	.3	.0239	-.1286	.8960	.2087	.0239	.0239	-.1047	-.1047	.7913	-.2087
	.4	.0257	-.1383	.8108	.3018	.0257	.0257	-.1126	-.1126	.6982	-.3018
	.5	.0251	-.1350	.7058	.4041	.0251	.0251	-.1099	-.1099	.5959	-.4041
	.6	.0225	-.1210	.5844	.5141	.0225	.0225	-.0985	-.0985	.4859	-.5141
	.7	.0183	-.0983	.4498	.6302	.0183	.0183	-.0800	-.0800	.3698	-.6302
	.8	.0129	-.0692	.3054	.7509	.0129	.0129	-.0563	-.0563	.2491	-.7509
	.9	.0066	-.0356	.1543	.8747	.0066	.0066	-.0290	-.0290	.1253	-.8747
	D	0	0	0	1.0	0	0	0	0	0	0
+ Area		.4528	1.2500	1.2500	.4528	.4528	.0167	.6233	.0733	.6267	.0628
− Area		-.0628	-.0900	-.0900	-.0628	-.0628	-.6267	-.0733	-.6233	-.0167	-.4528
Total Area		.3900	1.1600	1.1600	.3900	.3900	-.6100	.5500	-.5500	.6100	-.3900

Influence coefficients — Three continuous spans.

L = Length of EXTERIOR spans; length of interior span = NL

N = 1.1

Table 20.31

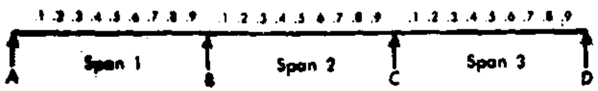
MOMENTS/PL — SPAN 1

Unit load at		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0876	.0781	.0657	.0503	.0378	.0254	.0130	.0006	-.0119	-.0243
	.2	0	.0753	.1506	.1259	.1011	.0764	.0517	.0270	.0023	-.0224	-.0471
	.3	0	.0633	.1266	.1899	.1532	.1165	.0798	.0431	.0064	-.0303	-.0670
	.4	0	.0518	.1035	.1553	.2070	.1588	.1105	.0623	.0140	-.0343	-.0825
	.5	0	.0408	.0816	.1224	.1632	.2040	.1448	.0855	.0263	-.0329	-.0921
	.6	0	.0306	.0611	.0917	.1223	.1529	.1834	.1140	.0446	-.0249	-.0943
	.7	0	.0212	.0425	.0637	.0849	.1062	.1274	.1486	.0699	-.0089	-.0877
	.8	0	.0129	.0259	.0388	.0517	.0646	.0776	.0905	.1034	.0164	-.0707
	.9	0	.0058	.0116	.0174	.0232	.0290	.0348	.0406	.0464	.0522	-.0430
	B	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0051	-.0102	-.0153	-.0204	-.0255	-.0305	-.0356	-.0407	-.0458	-.0509
	.2	0	-.0083	-.0167	-.0250	-.0333	-.0417	-.0500	-.0583	-.0667	-.0750	-.0833
	.3	0	-.0100	-.0200	-.0300	-.0400	-.0500	-.0599	-.0699	-.0799	-.0899	-.0999
	.4	0	-.0103	-.0207	-.0310	-.0413	-.0517	-.0620	-.0724	-.0827	-.0930	-.1034
	.5	0	-.0096	-.0193	-.0289	-.0386	-.0482	-.0579	-.0675	-.0771	-.0868	-.0964
	.6	0	-.0082	-.0164	-.0245	-.0327	-.0409	-.0491	-.0572	-.0654	-.0736	-.0818
	.7	0	-.0062	-.0124	-.0186	-.0248	-.0311	-.0373	-.0435	-.0497	-.0559	-.0621
	.8	0	-.0040	-.0080	-.0120	-.0160	-.0201	-.0241	-.0281	-.0321	-.0361	-.0401
	.9	0	-.0019	-.0037	-.0056	-.0074	-.0093	-.0111	-.0130	-.0148	-.0167	-.0185
	C	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0012	.0023	.0035	.0046	.0058	.0069	.0081	.0092	.0104	.0115
	.2	0	.0019	.0039	.0058	.0077	.0097	.0116	.0135	.0154	.0174	.0193
	.3	0	.0024	.0048	.0072	.0096	.0120	.0143	.0167	.0191	.0215	.0239
	.4	0	.0026	.0051	.0077	.0103	.0129	.0154	.0180	.0206	.0231	.0257
	.5	0	.0025	.0050	.0075	.0100	.0126	.0151	.0176	.0201	.0226	.0251
	.6	0	.0023	.0045	.0068	.0090	.0113	.0135	.0158	.0180	.0203	.0225
	.7	0	.0018	.0037	.0055	.0073	.0092	.0110	.0128	.0146	.0165	.0183
	.8	0	.0013	.0026	.0039	.0052	.0065	.0077	.0090	.0103	.0116	.0129
	.9	0	.0007	.0013	.0020	.0026	.0033	.0040	.0046	.0053	.0059	.0066
	D	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0406	.0710	.0916	.1022	.1027	.0932	.0737	.0443	.0214	.0167
− Area		0	-.0078	-.0154	-.0231	-.0309	-.0386	-.0463	-.0539	-.0617	-.0860	-.1385
Total Area		0	.0328	.0556	.0685	.0713	.0641	.0469	.0198	-.0174	-.0646	-.1218

MOMENTS/PL — SPAN 2

Unit load at		.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0
	.1	-.0212	-.0181	-.0150	-.0119	-.0088	-.0057	-.0027	.0004	.0035	.0066
	.2	-.0411	-.0351	-.0291	-.0231	-.0171	-.0111	-.0051	.0009	.0069	.0129
	.3	-.0585	-.0500	-.0414	-.0329	-.0244	-.0158	-.0073	.0012	.0098	.0183
	.4	-.0720	-.0615	-.0510	-.0405	-.0300	-.0195	-.0090	.0015	.0120	.0225
	.5	-.0804	-.0686	-.0569	-.0452	-.0335	-.0218	-.0100	.0017	.0134	.0251
	.6	-.0823	-.0703	-.0583	-.0463	-.0343	-.0223	-.0103	.0017	.0137	.0257
	.7	-.0765	-.0653	-.0542	-.0430	-.0319	-.0207	-.0096	.0016	.0128	.0239
	.8	-.0617	-.0527	-.0437	-.0347	-.0257	-.0167	-.0077	.0013	.0103	.0193
	.9	-.0366	-.0313	-.0260	-.0206	-.0153	-.0099	-.0046	.0008	.0061	.0115
	B	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	.0603	.0516	.0428	.0340	.0253	.0165	.0078	-.0010	-.0098	-.0185
	.2	.0170	.1173	.0976	.0780	.0583	.0386	.0189	-.0008	.0204	-.0401
	.3	-.0121	.0757	.1634	.1312	.0990	.0668	.0346	.0023	.0299	-.0621
	.4	-.0292	.0449	.1191	.1933	.1474	.1016	.0557	.0099	.0359	-.0818
	.5	-.0364	.0236	.0836	.1436	.2036	.1436	.0836	.0236	.0364	-.0964
	.6	-.0359	.0099	.0557	.1016	.1474	.1933	.1191	.0449	.0292	-.1034
	.7	-.0299	.0023	.0346	.0668	.0990	.1312	.1634	.0757	.0121	-.0999
	.8	-.0204	-.0008	.0189	.0386	.0583	.0780	.0976	.1173	.0170	-.0833
	.9	-.0098	-.0010	.0078	.0165	.0253	.0340	.0428	.0516	.0603	-.0509
	C	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	.0061	.0008	-.0046	-.0099	-.0153	-.0206	-.0260	-.0313	-.0366	-.0420
	.2	.0103	.0013	-.0077	-.0167	-.0257	-.0347	-.0437	-.0527	-.0617	-.0707
	.3	.0128	.0016	-.0096	-.0207	-.0319	-.0430	-.0542	-.0653	-.0765	-.0877
	.4	.0137	.0017	-.0103	-.0223	-.0343	-.0463	-.0583	-.0703	-.0823	-.0943
	.5	.0134	.0017	-.0100	-.0218	-.0335	-.0452	-.0569	-.0686	-.0804	-.0921
	.6	.0120	.0015	-.0090	-.0195	-.0300	-.0405	-.0510	-.0615	-.0720	-.0825
	.7	.0098	.0012	-.0073	-.0158	.0244	-.0329	-.0414	-.0500	-.0585	-.0670
	.8	.0069	.0009	-.0051	-.0111	-.0171	-.0231	-.0291	-.0351	-.0411	-.0471
	.9	.0033	.0004	-.0027	-.0057	-.0088	-.0119	-.0150	-.0181	-.0212	-.0243
	D	0	0	0	0	0	0	0	0	0	0
+ Area		.0174	.0394	.0740	.0956	.1028	.0956	.0740	.0394	.0174	.0167
− Area		-.0744	-.0480	-.0446	-.0446	-.0446	-.0446	-.0446	-.0480	-.0744	-.1385
Total Area		-.0570	-.0086	.0294	.0510	.0582	.0510	.0294	-.0086	-.0570	-.1218

Unit load at		REACTIONS/P R_A	R_B	R_C	R_D	SHEARS/P V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}
SPAN 1	A	1.0	0	0	0	1.0	0	0	0	0	0
	.1	.8757	.1501	-.0324	.0066	.8757	-.1243	.0258	.0258	-.0066	-.0066
	.2	.7529	.2871	-.0829	.0129	.7529	-.2471	.0600	.0600	-.0129	-.0129
	.3	.6330	.4381	-.0894	.0183	.6330	-.3670	.0711	.0711	-.0183	-.0183
	.4	.5175	.5700	-.1100	.0225	.5175	-.4825	.0875	.0875	-.0225	-.0225
	.5	.4079	.6897	-.1227	.0251	.4079	-.5921	.0976	.0976	-.0251	-.0251
	.6	.3057	.7943	-.1257	.0257	.3057	-.6943	.1000	.1000	-.0257	-.0257
	.7	.2123	.8806	-.1168	.0239	.2123	-.7877	.0929	.0929	-.0239	-.0239
	.8	.1293	.9457	-.0943	.0193	.1293	-.8707	.0750	.0750	-.0193	-.0193
	.9	.0580	.9865	-.0560	.0115	.0580	-.9420	.0445	.0445	-.0115	-.0115
	B	0	1.0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 2	.1	-.0509	.9779	.0915	-.0185	-.0509	-.0509	.9270	-.0730	.0185	.0185
	.2	-.0833	.9193	.2041	-.0401	-.0833	-.0833	.8360	-.1640	.0401	.0401
	.3	-.0999	.8314	.3306	-.0621	-.0999	-.0999	.7315	-.2685	.0621	.0621
	.4	-.1034	.7214	.4638	-.0818	-.1034	-.1034	.6180	-.3820	.0818	.0818
	.5	-.0964	.5964	.5964	-.0964	-.0964	-.0964	.5000	-.5000	.0964	.0964
	.6	-.0818	.4638	.7214	-.1034	-.0818	-.0818	.3820	-.6180	.1034	.1034
	.7	-.0621	.3306	.8314	-.0999	-.0621	-.0621	.2685	-.7315	.0999	.0999
	.8	-.0401	.2041	.9193	-.0833	-.0401	-.0401	.1640	-.8360	.0833	.0833
	.9	-.0185	.0915	.9779	-.0509	-.0185	-.0185	.0730	-.9270	.0509	.0509
	C	0	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 3	.1	.0115	-.0560	.9865	.0580	.0115	.0115	-.0445	-.0445	.9420	-.0580
	.2	.0193	-.0943	.9457	.1293	.0193	.0193	-.0750	-.0750	.8707	-.1293
	.3	.0239	-.1168	.8806	.2123	.0239	.0239	-.0929	-.0929	.7877	-.2123
	.4	.0257	-.1257	.7943	.3057	.0257	.0257	-.1000	-.1000	.6943	-.3057
	.5	.0251	-.1227	.6897	.4079	.0251	.0251	-.0976	-.0976	.5921	-.4079
	.6	.0225	-.1100	.5700	.5175	.0225	.0225	-.0875	-.0875	.4825	-.5175
	.7	.0183	-.0894	.4381	.6330	.0183	.0183	-.0711	-.0711	.3670	-.6330
	.8	.0129	-.0829	.2871	.7529	.0129	.0129	-.0500	-.0500	.2471	-.7529
	.9	.0066	-.0324	.1501	.8757	.0066	.0066	-.0258	-.0258	.1243	-.8757
	D	0	0	0	1.0	0	0	0	0	0	-1.0
+ Area		.4554	1.3036	1.3036	.4554	.4554	.0167	.6851	.0651	.6385	.0772
− Area		-.0772	-.0818	-.0818	-.0772	-.0772	-.6385	-.0651	-.6851	-.0167	-.4554
Total Area		.3782	1.2218	1.2218	.3782	.3782	-.6218	.6000	-.6000	.6218	-.3782



Influence coefficients — Three continuous spans.

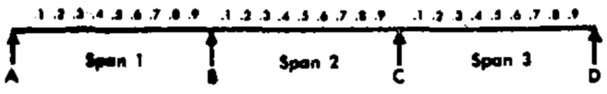
L = Length of EXTERIOR spans; length of interior span = NL

N = 1.2

Table 20.32

Unit load at		MOMENTS/PL SPAN 1 A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	SPAN 2 .1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0877	.0753	.0630	.0506	.0383	.0260	.0136	.0013	-.0111	-.0234	-.0204	-.0174	-.0144	-.0114	-.0084	-.0054	-.0024	.0006	.0036	.0066
	.2	0	.0755	.1509	.1264	.1019	.0773	.0528	.0282	.0037	.0208	-.0454	-.0395	-.0337	-.0279	-.0221	-.0163	-.0105	-.0046	.0012	.0070	.0128
	.3	0	.0636	.1271	.1907	.1542	.1178	.0813	.0449	.0084	-.0280	-.0645	-.0562	-.0480	-.0397	-.0314	-.0231	-.0149	-.0066	.0017	.0100	.0182
	.4	0	.0521	.1041	.1562	.2082	.1603	.1124	.0644	.0165	-.0314	-.0794	-.0692	-.0590	-.0488	-.0387	-.0285	-.0183	-.0081	.0021	.0123	.0224
	.5	0	.0411	.0823	.1234	.1646	.2057	.1466	.0880	.0291	-.0297	-.0886	-.0772	-.0659	-.0545	-.0431	-.0318	-.0204	-.0091	.0023	.0137	.0250
	.6	0	.0309	.0619	.0928	.1237	.1546	.1856	.1166	.0474	-.0217	-.0907	-.0791	-.0675	-.0558	-.0442	-.0325	-.0209	-.0093	.0024	.0140	.0256
	.7	0	.0216	.0431	.0647	.0863	.1078	.1294	.1510	.0725	-.0059	-.0843	-.0735	-.0627	-.0519	-.0411	-.0303	-.0194	-.0086	.0022	.0130	.0238
	.8	0	.0132	.0264	.0396	.0528	.0660	.0792	.0924	.1056	.0188	-.0680	-.0593	-.0506	-.0419	-.0331	-.0244	-.0157	-.0070	.0018	.0105	.0192
	.9	0	.0060	.0119	.0179	.0238	.0298	.0358	.0417	.0477	.0536	-.0404	-.0352	-.0300	-.0249	-.0197	-.0145	-.0093	-.0041	.0011	.0062	.0114
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0057	-.0114	-.0171	-.0228	-.0286	-.0343	-.0400	-.0457	-.0514	-.0571	.0636	.0543	.0450	.0356	.0263	.0170	.0077	-.0016	-.0109	-.0202
	.2	0	-.0093	-.0187	-.0280	-.0373	-.0467	-.0560	-.0653	-.0747	-.0840	-.0933	.0156	.1245	.1034	.0823	.0613	.0402	.0191	-.0020	-.0231	-.0442
	.3	0	-.0112	-.0223	-.0335	-.0447	-.0559	-.0670	-.0782	-.0894	-.1006	-.1117	-.0164	.0789	.1742	.1395	.1048	.0701	.0354	.0007	-.0340	-.0687
	.4	0	-.0115	-.0231	-.0346	-.0462	-.0577	-.0692	-.0808	-.0923	-.1039	-.1154	-.0350	.0455	.1260	.2064	.1369	.1073	.0578	.0083	.0413	-.0908
	.5	0	-.0107	-.0215	-.0322	-.0430	-.0537	-.0644	-.0752	-.0859	-.0967	-.1074	-.0424	.0226	.0876	.1526	.2176	.1526	.0876	.0226	-.0424	-.1074
	.6	0	-.0091	-.0182	-.0272	-.0363	-.0454	-.0545	-.0636	-.0727	-.0817	-.0908	-.0413	.0083	.0578	.1073	.1569	.2064	.1260	.0455	-.0350	-.1154
	.7	0	-.0069	-.0137	-.0206	-.0275	-.0344	-.0412	-.0481	-.0550	-.0618	-.0687	-.0340	.0007	.0354	.0701	.1048	.1395	.1742	.0789	-.0164	-.1117
	.8	0	-.0044	-.0088	-.0132	-.0177	-.0221	-.0265	-.0309	-.0353	-.0397	-.0442	-.0231	-.0020	.0191	.0409	.0613	.0823	.1034	.1245	.0156	-.0933
	.9	0	-.0020	-.0040	-.0061	-.0081	-.0101	-.0121	-.0142	-.0162	-.0182	-.0202	-.0109	-.0016	.0077	.0170	.0263	.0356	.0450	.0543	.0636	-.0571
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0011	.0023	.0034	.0046	.0057	.0068	.0080	.0091	.0103	.0114	.0082	.0011	-.0041	-.0093	-.0145	-.0197	-.0249	-.0300	-.0352	.0404
	.2	0	.0019	.0038	.0058	.0077	.0096	.0115	.0134	.0154	.0173	.0192	.0105	.0018	-.0070	-.0157	-.0244	-.0331	-.0419	-.0506	-.0593	-.0680
	.3	0	.0024	.0048	.0071	.0095	.0119	.0143	.0167	.0190	.0214	.0238	.0130	.0022	-.0086	-.0194	-.0303	-.0411	-.0519	-.0627	-.0735	-.0843
	.4	0	.0026	.0051	.0077	.0102	.0128	.0154	.0179	.0205	.0230	.0256	.0140	.0024	-.0093	-.0209	-.0325	-.0442	-.0558	-.0675	-.0791	-.0907
	.5	0	.0025	.0050	.0075	.0100	.0125	.0150	.0175	.0200	.0225	.0250	.0137	.0023	-.0091	-.0204	-.0318	-.0431	-.0545	-.0659	-.0772	-.0886
	.6	0	.0022	.0045	.0067	.0090	.0112	.0134	.0157	.0179	.0202	.0224	.0123	.0021	-.0081	-.0183	-.0285	-.0387	-.0488	-.0590	-.0692	-.0794
	.7	0	.0018	.0036	.0055	.0073	.0091	.0109	.0127	.0146	.0164	.0182	.0100	.0017	-.0066	-.0149	-.0231	-.0314	-.0397	-.0480	-.0562	-.0645
	.8	0	.0013	.0026	.0038	.0051	.0064	.0077	.0090	.0102	.0115	.0128	.0070	.0012	-.0046	-.0105	-.0163	-.0221	-.0279	-.0337	-.0395	-.0454
	.9	0	.0007	.0013	.0020	.0026	.0033	.0040	.0046	.0053	.0059	.0066	.0036	.0006	-.0024	-.0054	-.0084	-.0114	-.0144	-.0174	-.0204	-.0234
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0408	.0715	.0923	.1031	.1038	.0946	.0754	.0461	.0218	.0167	.0184	.0441	.0844	.1097	.1182	.1097	.0844	.0441	.0184	.0167
– Area		0	-.0093	-.0186	-.0279	-.0373	-.0465	-.0559	-.0652	-.0745	-.0987	-.1522	-.0778	-.0444	-.0424	-.0424	-.0424	-.0424	-.0424	-.0444	-.0778	-.1522
Total Area		0	.0315	.0529	.0644	.0658	.0573	.0387	.0102	-.0284	-.0769	-.1355	-.0594	-.0003	.0420	.0673	.0758	.0673	.0420	-.0003	-.0594	-.1355

Unit load at		REACTIONS/P R_A	R_B	R_C	R_D	SHEARS/P V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}
SPAN 1	A	1.0	0	0	0	1.0	0	0	0	0	0
	.1	.8766	.1465	-.0297	.0066	.8766	-.1234	.0231	.0231	-.0066	-.0066
	.2	.7546	.2901	-.0575	.0128	.7546	-.2454	.0447	.0447	-.0128	-.0128
	.3	.6355	.4281	-.0818	.0182	.6355	-.3645	.0636	.0636	-.0182	-.0182
	.4	.5206	.5577	-.1007	.0224	.5206	-.4794	.0783	.0783	-.0224	-.0224
	.5	.4114	.6760	-.1124	.0250	.4114	-.5886	.0874	.0874	-.0250	-.0250
	.6	.3093	.7802	-.1151	.0256	.3093	-.6907	.0895	.0895	-.0256	-.0256
	.7	.2157	.8676	-.1071	.0238	.2157	-.7843	.0833	.0833	-.0238	-.0238
	.8	.1320	.9352	-.0864	.0192	.1320	-.8680	.0672	.0672	-.0192	-.0192
	.9	.0596	.9803	-.0513	.0114	.0596	-.9404	.0399	.0399	-.0114	-.0114
	B	0	1.0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 2	.1	-.0571	.9855	.0918	-.0202	-.0571	-.0571	.9284	-.0716	.0202	.0202
	.2	-.0933	.9311	.2064	-.0442	-.0933	-.0933	.8378	-.1622	.0442	.0442
	.3	-.1117	.8448	.3356	-.0687	-.1117	-.1117	.7331	-.2669	.0687	.0687
	.4	-.1154	.7343	.4719	-.0908	-.1154	-.1154	.6189	-.3811	.0908	.0908
	.5	-.1074	.6074	.6074	-.1074	-.1074	-.1074	.5000	-.5000	.1074	.1074
	.6	-.0908	.4719	.7343	-.1154	-.0908	-.0908	.3811	-.6189	.1154	.1154
	.7	-.0687	.3356	.8448	-.1117	-.0687	-.0687	.2669	-.7331	.1117	.1117
	.8	-.0442	.2064	.9311	-.0933	-.0442	-.0442	.1622	-.8378	.0933	.0933
	.9	-.0202	.0918	.9855	-.0571	-.0202	-.0202	.0716	-.9284	.0571	.0571
	C	0	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 3	.1	.0114	-.0513	.9803	.0596	.0114	.0114	-.0399	-.0399	.9404	-.0596
	.2	.0192	-.0864	.9352	.1320	.0192	.0192	-.0672	-.0672	.8680	-.1320
	.3	.0238	-.1071	.8676	.2157	.0238	.0238	-.0833	-.0833	.7843	-.2157
	.4	.0256	-.1151	.7802	.3093	.0256	.0256	-.0895	-.0895	.6907	-.3093
	.5	.0250	-.1124	.6760	.4114	.0250	.0250	-.0874	-.0874	.5886	-.4114
	.6	.0224	-.1007	.5577	.5206	.0224	.0224	-.0783	-.0783	.4794	-.5206
	.7	.0182	-.0818	.4281	.6355	.0182	.0182	-.0636	-.0636	.3645	-.6355
	.8	.0128	-.0575	.2901	.7546	.0128	.0128	-.0447	-.0447	.2454	-.7546
	.9	.0066	-.0297	.1465	.8766	.0066	.0066	-.0231	-.0231	.1234	-.8766
	D	0	0	0	1.0	0	0	0	0	0	-1.0
+ Area		.4576	1.3604	1.3604	.4576	.4576	.0187	.7083	.0583	.6522	.0931
– Area		-.0931	-.0749	-.0749	-.0931	-.0931	-.6522	-.0583	-.7083	-.0187	-.4576
Total Area		.3645	1.2855	1.2855	.3645	.3645	-.6355	.6500	-.6500	.6355	-.3645



Influence coefficients — Three continuous spans.

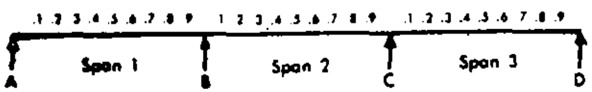
L = Length of EXTERIOR spans; length of interior span = NL

N = 1.3

Table 20.33

Unit load at		Moments/PL, Span 1: A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	Span 2: .1	.2	.3	.4	.5	.6	.7	.8	.9	C
Span 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0877	.0755	.0632	.0510	.0387	.0265	.0142	.0020	-.0103	-.0225	-.0196	-.0167	-.0138	-.0109	-.0080	-.0051	-.0022	.0008	.0037	.0066
	.2	0	.0756	.1513	.1269	.1025	.0781	.0538	.0294	.0050	-.0193	-.0437	-.0381	-.0324	-.0266	-.0211	-.0155	-.0098	-.0042	.0015	.0071	.0128
	.3	0	.0638	.1276	.1914	.1551	.1189	.0827	.0465	.0103	-.0259	-.0622	-.0541	-.0461	-.0381	-.0300	-.0220	-.0140	-.0060	.0021	.0101	.0181
	.4	0	.0523	.1047	.1570	.2094	.1617	.1141	.0664	.0188	-.0289	-.0765	-.0664	-.0567	-.0469	-.0370	-.0271	-.0172	-.0073	.0026	.0124	.0223
	.5	0	.0415	.0829	.1244	.1658	.2073	.1488	.0902	.0317	-.0269	-.0854	-.0744	-.0633	-.0523	-.0413	-.0302	-.0192	-.0082	.0028	.0139	.0249
	.6	0	.0313	.0625	.0938	.1250	.1563	.1875	.1188	.0500	-.0187	-.0874	-.0761	-.0648	-.0536	-.0423	-.0310	-.0197	-.0084	.0029	.0142	.0255
	.7	0	.0219	.0437	.0656	.0875	.1094	.1312	.1531	.0750	-.0032	-.0813	-.0708	-.0603	-.0498	-.0393	-.0288	-.0183	-.0078	.0027	.0132	.0237
	.8	0	.0134	.0269	.0403	.0538	.0672	.0807	.0941	.1075	.0210	-.0656	-.0571	-.0486	-.0402	-.0317	-.0232	-.0148	-.0063	.0022	.0107	.0191
	.9	0	.0061	.0122	.0183	.0244	.0305	.0366	.0427	.0489	.0550	-.0389	-.0339	-.0289	-.0238	-.0188	-.0138	-.0088	-.0037	.0013	.0063	.0114
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Span 2	.1	0	-.0063	-.0127	-.0190	-.0254	-.0317	-.0381	-.0444	-.0508	-.0571	-.0634	.0667	.0569	.0470	.0372	.0273	.0175	.0076	-.0022	-.0121	-.0219
	.2	0	-.0104	-.0207	-.0311	-.0414	-.0518	-.0621	-.0725	-.0828	-.0932	-.1035	.0140	.1315	.1091	.0866	.0641	.0417	.0192	-.0033	-.0257	-.0482
	.3	0	-.0124	-.0248	-.0371	-.0495	-.0619	-.0743	-.0867	-.0990	-.1114	-.1236	-.0210	.0819	.1847	.1476	.1104	.0733	.0361	-.0011	-.0382	-.0754
	.4	0	-.0128	-.0255	-.0383	-.0511	-.0638	-.0766	-.0893	-.1021	-.1149	-.1276	-.0409	.0459	.1327	.2194	.1662	.1130	.0597	.0065	-.0487	-.1000
	.5	0	-.0119	-.0237	-.0356	-.0474	-.0593	-.0711	-.0830	-.0948	-.1067	-.1185	-.0485	.0215	.0915	.1615	.2315	.1615	.0915	.0215	-.0485	-.1185
	.6	0	-.0100	-.0200	-.0300	-.0400	-.0500	-.0600	-.0700	-.0800	-.0900	-.1000	-.0467	.0065	.0597	.1130	.1662	.2194	.1327	.0459	-.0409	-.1276
	.7	0	-.0075	-.0151	-.0226	-.0301	-.0377	-.0452	-.0528	-.0603	-.0678	-.0754	-.0382	-.0011	.0361	.0733	.1104	.1476	.1847	.0819	-.0210	-.1236
	.8	0	-.0048	-.0096	-.0145	-.0193	-.0241	-.0289	-.0337	-.0386	-.0434	-.0482	-.0257	-.0033	.0192	.0417	.0641	.0866	.1091	.1315	.0140	-.1035
	.9	0	-.0022	-.0044	-.0066	-.0088	-.0110	-.0132	-.0153	-.0175	-.0197	-.0219	-.0121	-.0022	-.0076	.0175	.0273	.0372	.0470	.0569	.0667	-.0634
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Span 3	.1	0	.0011	.0023	.0034	.0046	.0057	.0068	.0080	.0091	.0103	.0114	.0063	.0013	-.0037	-.0088	-.0138	-.0188	-.0238	-.0289	-.0339	-.0389
	.2	0	.0019	.0038	.0057	.0076	.0096	.0115	.0134	.0153	.0172	.0191	.0107	.0022	-.0063	-.0148	-.0232	-.0317	-.0402	-.0486	-.0571	-.0656
	.3	0	.0024	.0047	.0071	.0095	.0119	.0142	.0166	.0190	.0213	.0237	.0132	.0027	-.0078	-.0183	-.0288	-.0393	-.0498	-.0603	-.0708	-.0813
	.4	0	.0026	.0051	.0077	.0102	.0128	.0153	.0179	.0204	.0230	.0255	.0142	.0029	-.0084	-.0197	-.0310	-.0423	-.0536	-.0649	-.0761	-.0874
	.5	0	.0025	.0050	.0075	.0100	.0125	.0149	.0174	.0199	.0224	.0249	.0139	.0028	-.0082	-.0192	-.0302	-.0413	-.0523	-.0633	-.0744	-.0854
	.6	0	.0022	.0045	.0067	.0089	.0112	.0134	.0156	.0178	.0201	.0223	.0124	.0026	-.0073	-.0172	-.0271	-.0370	-.0469	-.0567	-.0666	-.0765
	.7	0	.0018	.0036	.0054	.0072	.0091	.0109	.0127	.0145	.0163	.0181	.0101	.0021	-.0060	-.0140	-.0220	-.0300	-.0381	-.0461	-.0541	-.0622
	.8	0	.0013	.0026	.0038	.0051	.0064	.0077	.0090	.0102	.0115	.0128	.0071	.0015	-.0042	-.0098	-.0155	-.0211	-.0268	-.0324	-.0381	-.0437
	.9	0	.0007	.0013	.0020	.0026	.0033	.0040	.0046	.0053	.0059	.0066	.0037	.0008	-.0022	-.0051	-.0080	-.0109	-.0138	-.0167	-.0196	-.0225
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0410	.0719	.0929	.1038	.1048	.0958	.0768	.0477	.0221	.0166	.0194	.0490	.0952	.1246	.1344	.1246	.0952	.0490	.0194	.0166
− Area		0	-.0111	-.0221	-.0332	-.0442	-.0553	-.0664	-.0775	-.0885	-.1130	-.1676	-.0822	-.0432	-.0404	-.0404	-.0404	-.0404	-.0404	-.0432	-.0822	-.1676
Total Area		0	.0299	.0498	.0597	.0596	.0495	.0294	-.0007	-.0408	-.0909	-.1510	-.0628	.0058	.0548	.0842	.0940	.0842	.0548	.0058	.0628	.1510

Unit load at		Reactions/P: R_A	R_B	R_C	R_D	Shears/P: V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}
Span 1	A	1.0	0	0	0	1.0	0	0	0	0	0
	.1	.8774	.1433	-.0273	.0066	.8774	-.1226	.0207	.0207	-.0066	-.0066
	.2	.7563	.2841	-.0532	.0128	.7563	-.2437	.0404	.0404	-.0128	-.0128
	.3	.6376	.4195	-.0754	.0181	.6376	-.3622	.0573	.0573	-.0181	-.0181
	.4	.5235	.5471	-.0929	.0223	.5235	-.4765	.0706	.0706	-.0223	-.0223
	.5	.4146	.6642	-.1037	.0249	.4146	-.5854	.0788	.0788	-.0249	-.0249
	.6	.3126	.7681	-.1062	.0255	.3126	-.6874	.0807	.0807	-.0255	-.0255
	.7	.2187	.8563	-.0987	.0237	.2187	-.7813	.0750	.0750	-.0237	-.0237
	.8	.1344	.9261	-.0796	.0191	.1344	-.8656	.0605	.0605	-.0191	-.0191
	.9	.0611	.9749	-.0474	.0114	.0611	-.9389	.0360	.0360	-.0114	-.0114
	B	0	1.0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
Span 2	.1	-.0634	.9931	.0922	-.0219	-.0634	-.0634	.9297	-.0703	.0219	.0219
	.2	-.1035	.9431	.2086	-.0482	-.1035	-.1035	.8396	-.1604	.0482	.0482
	.3	-.1236	.8584	.3406	-.0754	-.1236	-.1236	.7346	-.2654	.0754	.0754
	.4	-.1276	.7474	.4802	-.1000	-.1276	-.1276	.6198	-.3802	.1000	.1000
	.5	-.1185	.6185	.6185	-.1185	-.1185	-.1185	.5000	-.5000	.1185	.1185
	.6	-.1000	.4802	.7474	-.1276	-.1000	-.1000	.3802	-.6198	.1276	.1276
	.7	-.0754	.3406	.8584	-.1236	-.0754	-.0754	.2654	-.7346	.1236	.1236
	.8	-.0482	.2086	.9431	-.1035	-.0482	-.0482	.1604	-.8396	.1035	.1035
	.9	-.0219	.0922	.9931	-.0634	-.0219	-.0219	.0703	-.9297	.0634	.0634
	C	0	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0
Span 3	.1	.0114	-.0474	.9749	.0611	.0114	.0114	-.0360	-.0360	.9389	-.0611
	.2	.0191	-.0796	.9261	.1344	.0191	.0191	-.0605	-.0605	.8656	-.1344
	.3	.0237	-.0987	.8563	.2187	.0237	.0237	-.0750	-.0750	.7813	-.2187
	.4	.0255	-.1062	.7681	.3126	.0255	.0255	-.0807	-.0807	.6874	-.3126
	.5	.0249	-.1037	.6642	.4146	.0249	.0249	-.0788	-.0788	.5854	-.4146
	.6	.0223	-.0929	.5471	.5235	.0223	.0223	-.0706	-.0706	.4765	-.5235
	.7	.0181	-.0754	.4195	.6376	.0181	.0181	-.0573	-.0573	.3622	-.6376
	.8	.0128	-.0532	.2841	.7563	.0128	.0128	-.0404	-.0404	.2437	-.7563
	.9	.0066	-.0273	.1433	.8774	.0066	.0066	-.0207	-.0207	.1226	-.8774
	D	0	0	0	1.0	0	0	0	0	0	-1.0
+ Area		.4597	1.4201	1.4201	.4597	.4597	.0166	.7525	.0525	.6676	.1107
− Area		-.1107	-.0691	-.0691	-.1107	-.1107	-.6676	-.0525	-.7525	-.0166	-.4597
Total Area		.3490	1.3510	1.3510	.3490	.3490	-.6510	.7000	-.7000	.6510	-.3490



Influence coefficients — Three continuous spans.

L = Length of EXTERIOR spans, length of interior span = NL

N = 1.4

Table 20.34

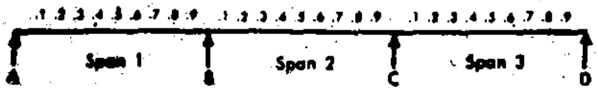
MOMENTS/PL — SPAN 1

Unit load at		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0878	.0756	.0635	.0513	.0391	.0269	.0148	.0026	-.0096	-.0218
	.2	0	.0758	.1516	.1273	.1031	.0789	.0547	.0305	.0062	-.0180	-.0422
	.3	0	.0640	.1280	.1920	.1560	.1200	.0840	.0480	.0120	-.0240	-.0600
	.4	0	.0526	.1052	.1578	.2105	.1631	.1157	.0683	.0209	-.0265	-.0738
	.5	0	.0418	.0835	.1253	.1670	.2088	.1505	.0923	.0341	-.0242	-.0824
	.6	0	.0316	.0631	.0947	.1262	.1578	.1894	.1209	.0525	-.0160	-.0844
	.7	0	.0222	.0443	.0665	.0886	.1108	.1329	.1551	.0772	-.0006	-.0785
	.8	0	.0137	.0273	.0410	.0547	.0684	.0820	.0957	.1094	.0230	-.0633
	.9	0	.0062	.0125	.0187	.0250	.0312	.0375	.0437	.0499	.0562	-.0376
	B	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0070	-.0140	-.0210	-.0279	-.0349	-.0419	-.0489	-.0559	-.0629	-.0699
	.2	0	-.0114	-.0228	-.0342	-.0456	-.0570	-.0684	-.0798	-.0911	-.1025	-.1139
	.3	0	-.0136	-.0272	-.0408	-.0544	-.0680	-.0816	-.0952	-.1088	-.1224	-.1360
	.4	0	-.0140	-.0280	-.0420	-.0560	-.0700	-.0840	-.0980	-.1120	-.1260	-.1400
	.5	0	-.0130	-.0260	-.0389	-.0519	-.0649	-.0779	-.0909	-.1038	-.1168	-.1298
	.6	0	-.0109	-.0218	-.0328	-.0437	-.0546	-.0655	-.0764	-.0873	-.0983	-.1092
	.7	0	-.0082	-.0164	-.0246	-.0328	-.0410	-.0492	-.0574	-.0656	-.0738	-.0820
	.8	0	-.0052	-.0104	-.0157	-.0209	-.0261	-.0313	-.0366	-.0418	-.0470	-.0522
	.9	0	-.0024	-.0047	-.0071	-.0094	-.0118	-.0142	-.0165	-.0189	-.0212	-.0236
	C	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0011	.0023	.0034	.0045	.0057	.0068	.0079	.0090	.0102	.0113
	.2	0	.0019	.0038	.0057	.0076	.0095	.0114	.0133	.0152	.0171	.0190
	.3	0	.0024	.0047	.0071	.0094	.0118	.0141	.0165	.0188	.0212	.0235
	.4	0	.0025	.0051	.0076	.0101	.0127	.0152	.0177	.0202	.0228	.0253
	.5	0	.0025	.0049	.0074	.0099	.0124	.0148	.0173	.0198	.0222	.0247
	.6	0	.0022	.0044	.0067	.0089	.0111	.0133	.0155	.0178	.0200	.0222
	.7	0	.0018	.0036	.0054	.0072	.0090	.0108	.0126	.0144	.0162	.0180
	.8	0	.0013	.0025	.0038	.0051	.0064	.0076	.0089	.0102	.0114	.0127
	.9	0	.0007	.0013	.0020	.0026	.0033	.0039	.0046	.0052	.0059	.0065
	D	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0412	.0723	.0934	.1046	.1057	.0969	.0781	.0492	.0225	.0168
− Area		0	-.0130	-.0260	-.0389	-.0519	-.0648	-.0779	-.0909	-.1038	-.1289	-.1848
Total Area		0	.0282	.0463	.0545	.0527	.0409	.0190	-.0128	-.0546	-.1064	-.1882

MOMENTS/PL — SPAN 2

Unit load at		.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0
	.1	-.0189	-.0161	-.0133	-.0104	-.0076	-.0048	-.0020	.0009	.0037	.0065
	.2	-.0367	-.0312	-.0257	-.0203	-.0148	-.0093	-.0038	.0017	.0072	.0127
	.3	-.0522	-.0444	-.0366	-.0288	-.0210	-.0132	-.0054	.0024	.0102	.0180
	.4	-.0643	-.0546	-.0450	-.0354	-.0258	-.0162	-.0066	.0030	.0126	.0222
	.5	-.0717	-.0610	-.0503	-.0396	-.0288	-.0181	-.0074	.0033	.0140	.0247
	.6	-.0734	-.0625	-.0515	-.0406	-.0295	-.0186	-.0076	.0034	.0143	.0253
	.7	-.0683	-.0581	-.0479	-.0377	-.0275	-.0173	-.0071	.0031	.0133	.0235
	.8	-.0551	-.0468	-.0386	-.0304	-.0222	-.0139	-.0057	.0025	.0108	.0190
	.9	-.0327	-.0278	-.0229	-.0180	-.0132	-.0083	-.0034	.0015	.0064	.0113
	B	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	.0698	.0594	.0490	.0386	.0283	.0179	.0075	-.0028	-.0132	-.0236
	.2	.0122	.1384	.1146	.0908	.0669	.0431	.0193	-.0046	-.0284	-.0522
	.3	-.0256	.0848	.1952	.1556	.1160	.0764	.0368	-.0028	-.0424	-.0820
	.4	-.0470	.0461	.1392	.2323	.1754	.1185	.0616	.0046	-.0523	-.1092
	.5	-.0548	.0202	.0952	.1702	.2432	.1702	.0952	.0202	-.0548	-.1298
	.6	-.0523	.0046	.0616	.1185	.1754	.2323	.1392	.0461	-.0470	-.1400
	.7	-.0424	-.0028	.0368	.0764	.1160	.1556	.1952	.0848	-.0256	-.1360
	.8	-.0284	-.0046	.0193	.0431	.0669	.0908	.1146	.1384	.0122	-.1139
	.9	-.0132	-.0028	.0075	.0179	.0283	.0386	.0490	.0594	.0698	-.0699
	C	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	.0064	.0015	-.0034	-.0083	-.0132	-.0180	-.0229	-.0278	-.0327	-.0376
	.2	.0108	.0025	-.0057	-.0139	-.0222	-.0304	-.0386	-.0468	-.0551	-.0633
	.3	.0133	.0031	-.0071	-.0173	-.0275	-.0377	-.0479	-.0581	-.0683	-.0785
	.4	.0143	.0034	-.0076	-.0186	-.0295	-.0406	-.0515	-.0625	-.0734	-.0844
	.5	.0140	.0033	-.0074	-.0181	-.0288	-.0396	-.0503	-.0610	-.0717	-.0824
	.6	.0126	.0030	-.0066	-.0162	-.0258	-.0354	-.0450	-.0546	-.0642	-.0738
	.7	.0102	.0024	-.0054	-.0132	-.0210	-.0288	-.0366	-.0444	-.0522	-.0600
	.8	.0072	.0017	-.0038	-.0093	-.0148	-.0203	-.0257	-.0312	-.0367	-.0422
	.9	.0037	.0009	-.0020	-.0048	-.0076	-.0104	-.0133	-.0161	-.0189	-.0218
	D	0	0	0	0	0	0	0	0	0	0
+ Area		.0204	.0539	.1065	.1402	.1515	.1402	.1065	.0539	.0204	.0188
− Area		-.0874	-.0422	-.0385	-.0385	-.0385	-.0385	-.0385	-.0422	-.0874	-.1848
Total Area		-.0670	.0117	.0680	.1017	.1130	.1017	.0680	.0117	-.0670	-.1882

Unit load at		REACTIONS/P R_A	R_B	R_C	R_D	SHEARS/P V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}
SPAN 1	A	1.0	0	0	0	1.0	0	0	0	0	0
	.1	.8782	.1406	-.0253	.0065	.8782	-.1218	.0188	.0188	-.0065	-.0065
	.2	.7578	.2788	-.0493	.0127	.7578	-.2422	.0366	.0366	-.0127	-.0127
	.3	.6400	.4120	-.0700	.0180	.6400	-.3600	.0520	.0520	-.0180	-.0180
	.4	.5262	.5378	-.0862	.0222	.5262	-.4738	.0640	.0640	-.0222	-.0222
	.5	.4176	.6538	-.0961	.0247	.4176	-.5824	.0714	.0714	-.0247	-.0247
	.6	.3156	.7575	-.0984	.0253	.3156	-.6844	.0731	.0731	-.0253	-.0253
	.7	.2215	.8465	-.0915	.0235	.2215	-.7785	.0680	.0680	-.0235	-.0235
	.8	.1367	.9182	-.0739	.0190	.1367	-.8633	.0549	.0549	-.0190	-.0190
	.9	.0624	.9702	-.0439	.0113	.0624	-.9376	.0326	.0326	-.0113	-.0113
	B	0	1.0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 2	.1	-.0699	1.0007	.0928	-.0236	-.0699	-.0699	.9308	-.0692	.0236	.0236
	.2	-.1139	.9551	.2110	-.0522	-.1139	-.1139	.8412	-.1588	.0522	.0522
	.3	-.1360	.8720	.3460	-.0820	-.1360	-.1360	.7360	-.2640	.0820	.0820
	.4	-.1400	.7606	.4886	-.1092	-.1400	-.1400	.6206	-.3794	.1092	.1092
	.5	-.1298	.6298	.6298	-.1298	-.1298	-.1298	.5000	-.5000	.1298	.1298
	.6	-.1092	.4886	.7606	-.1400	-.1092	-.1092	.3794	-.6206	.1400	.1400
	.7	-.0820	.3460	.8720	-.1360	-.0820	-.0820	.2640	-.7360	.1360	.1360
	.8	-.0522	.2110	.9551	-.1139	-.0522	-.0522	.1588	-.8412	.1139	.1139
	.9	-.0236	.0928	1.0007	-.0699	-.0236	-.0236	.0692	-.9308	.0699	.0699
	C	0	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 3	.1	.0113	-.0439	.9702	.0624	.0113	.0113	-.0326	-.0326	.9376	-.0624
	.2	.0190	-.0739	.9182	.1367	.0190	.0190	-.0549	-.0549	.8633	-.1367
	.3	.0235	-.0915	.8465	.2215	.0235	.0235	-.0680	-.0680	.7785	-.2215
	.4	.0253	-.0984	.7575	.3156	.0253	.0253	-.0731	-.0731	.6844	-.3156
	.5	.0247	-.0961	.6538	.4176	.0247	.0247	-.0714	-.0714	.5824	-.4176
	.6	.0222	-.0862	.5378	.5262	.0222	.0222	-.0640	-.0640	.4738	-.5262
	.7	.0180	-.0700	.4120	.6400	.0180	.0180	-.0520	-.0520	.3600	-.6400
	.8	.0127	-.0493	.2788	.7578	.0127	.0127	-.0366	-.0366	.2422	-.7578
	.9	.0065	-.0253	.1406	.8782	.0065	.0065	-.0188	-.0188	.1218	-.8782
	D	0	0	0	1.0	0	0	0	0	0	-1.0
+ Area		.4615	1.4824	1.4824	.4615	.4615	.0165	.7976	.0476	.6847	.1298
− Area		-.1298	-.0641	-.0641	-.1298	-.1298	-.6847	-.0476	-.7976	-.0165	-.4615
Total Area		.3317	1.4183	1.4183	.3317	.3317	-.6682	.7500	-.7500	.6682	-.3317



Influence coefficients — Three continuous spans.

L = Length of EXTERIOR spans; length of interior span = NL

N = 1.5

Table 20.35

MOMENTS/PL — SPAN 1

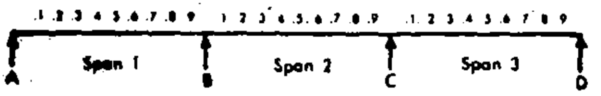
Span	Unit load at	A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B
SPAN 1	A	0	0	0	0	0	0	0	0	.0	0	0
	.1	0	.0879	.0758	.0637	.0516	.0395	.0274	.0153	.0032	-.0089	-.0210
	.2	0	.0759	.1518	.1278	.1037	.0796	.0555	.0315	.0074	-.0167	-.0408
	.3	0	.0642	.1284	.1926	.1568	.1210	.0852	.0494	.0136	-.0222	-.0580
	.4	0	.0529	.1057	.1586	.2115	.1643	.1172	.0700	.0229	-.0242	-.0714
	.5	0	.0420	.0841	.1261	.1681	.2102	.1522	.0942	.0363	-.0217	-.0797
	.6	0	.0318	.0637	.0955	.1274	.1592	.1911	.1229	.0547	-.0134	-.0816
	.7	0	.0224	.0448	.0673	.0897	.1121	.1345	.1569	.0793	.0018	-.0758
	.8	0	.0139	.0278	.0416	.0555	.0694	.0833	.0972	.1111	.0249	-.0612
	.9	0	.0064	.0127	.0191	.0255	.0318	.0382	.0446	.0509	.0573	-.0363
	B	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0076	-.0153	-.0229	-.0306	-.0382	-.0459	-.0535	-.0611	-.0688	-.0764
	.2	0	-.0124	-.0249	-.0373	-.0498	-.0622	-.0747	-.0871	-.0996	-.1120	-.1245
	.3	0	-.0148	-.0297	-.0445	-.0594	-.0742	-.0891	-.1039	-.1188	-.1336	-.1485
	.4	0	-.0153	-.0305	-.0458	-.0610	-.0763	-.0916	-.1068	-.1221	-.1373	-.1526
	.5	0	-.0141	-.0282	-.0424	-.0565	-.0706	-.0847	-.0988	-.1129	-.1271	-.1412
	.6	0	-.0118	-.0237	-.0355	-.0474	-.0592	-.0711	-.0829	-.0948	-.1066	-.1183
	.7	0	-.0089	-.0177	-.0266	-.0355	-.0444	-.0532	-.0621	-.0710	-.0798	-.0887
	.8	0	-.0056	-.0112	-.0169	-.0225	-.0281	-.0337	-.0394	-.0450	-.0506	-.0562
	.9	0	-.0025	-.0050	-.0076	-.0101	-.0126	-.0151	-.0177	-.0202	-.0227	-.0252
	C	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0011	.0022	.0034	.0045	.0056	.0067	.0078	.0090	.0101	.0112
	.2	0	.0019	.0038	.0056	.0075	.0094	.0113	.0132	.0150	.0169	.0188
	.3	0	.0023	.0047	.0070	.0093	.0117	.0140	.0163	.0186	.0210	.0233
	.4	0	.0025	.0050	.0075	.0100	.0126	.0151	.0176	.0201	.0226	.0251
	.5	0	.0025	.0049	.0074	.0098	.0123	.0147	.0172	.0196	.0221	.0245
	.6	0	.0022	.0044	.0066	.0088	.0110	.0132	.0154	.0176	.0198	.0220
	.7	0	.0018	.0036	.0053	.0071	.0089	.0107	.0125	.0142	.0160	.0178
	.8	0	.0013	.0025	.0038	.0050	.0063	.0075	.0088	.0100	.0112	.0125
	.9	0	.0007	.0013	.0020	.0026	.0033	.0039	.0046	.0052	.0059	.0065
	D	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0413	.0727	.0940	.1053	.1066	.0979	.0793	.0506	.0228	.0163
− Area		0	-.0150	-.0302	-.0452	-.0602	-.0753	-.0903	-.1054	-.1205	-.1464	-.2037
Total Area		0	.0263	.0425	.0488	.0451	.0313	.0076	-.0261	-.0699	-.1236	-.1874

MOMENTS/PL — SPAN 2

Span	Unit load at	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0
	.1	-.0183	-.0155	-.0128	-.0100	-.0073	-.0045	-.0018	.0010	.0037	.0065
	.2	-.0355	-.0301	-.0248	-.0195	-.0141	-.0088	-.0035	.0019	.0072	.0125
	.3	-.0504	-.0428	-.0352	-.0277	-.0201	-.0125	-.0049	.0027	.0103	.0178
	.4	-.0620	-.0527	-.0434	-.0340	-.0247	-.0154	-.0060	.0033	.0126	.0220
	.5	-.0692	-.0588	-.0484	-.0380	-.0276	-.0172	-.0067	.0037	.0141	.0245
	.6	-.0709	-.0602	-.0496	-.0389	-.0282	-.0176	-.0069	.0038	.0144	.0251
	.7	-.0659	-.0560	-.0461	-.0362	-.0262	-.0163	-.0064	.0035	.0134	.0233
	.8	-.0532	-.0452	-.0372	-.0292	-.0212	-.0132	-.0052	.0028	.0108	.0188
	.9	-.0316	-.0268	-.0221	-.0173	-.0125	-.0078	-.0031	.0017	.0064	.0112
	B	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	.0727	.0618	.0509	.0401	.0292	.0183	.0074	-.0035	-.0143	-.0252
	.2	.0103	.1452	.1200	.0948	.0696	.0445	.0193	-.0059	-.0310	-.0562
	.3	-.0305	.0875	.2055	.1634	.1214	.0794	.0374	-.0047	-.0467	-.0887
	.4	-.0532	.0462	.1456	.2451	.1845	.1239	.0633	.0027	-.0579	-.1185
	.5	-.0612	.0188	.0988	.1788	.2588	.1788	.0988	.0188	-.0612	-.1412
	.6	-.0579	.0027	.0633	.1239	.1845	.2451	.1456	.0462	-.0532	-.1526
	.7	-.0467	-.0047	.0374	.0794	.1214	.1634	.2055	.0875	-.0305	-.1485
	.8	-.0310	-.0059	.0193	.0445	.0696	.0948	.1200	.1452	.0103	-.1245
	.9	-.0143	-.0035	.0074	.0183	.0292	.0401	.0509	.0618	.0727	-.0764
	C	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	.0064	.0017	-.0031	-.0078	-.0125	-.0173	-.0221	-.0268	-.0316	-.0363
	.2	.0108	.0028	-.0052	-.0132	-.0212	-.0292	-.0372	-.0452	-.0532	-.0612
	.3	.0134	.0035	-.0064	-.0163	-.0262	-.0362	-.0461	-.0560	-.0659	-.0758
	.4	.0144	.0038	-.0069	-.0176	-.0282	-.0389	-.0496	-.0602	-.0709	-.0816
	.5	.0141	.0037	-.0067	-.0172	-.0276	-.0380	-.0484	-.0588	-.0692	-.0797
	.6	.0126	.0033	-.0060	-.0154	-.0247	-.0340	-.0434	-.0527	-.0620	-.0714
	.7	.0103	.0027	-.0049	-.0125	-.0201	-.0277	-.0352	-.0428	-.0504	-.0580
	.8	.0072	.0019	-.0035	-.0088	-.0141	-.0195	-.0248	-.0301	-.0355	-.0408
	.9	.0037	.0010	-.0018	-.0045	-.0073	-.0100	-.0128	-.0155	-.0183	-.0210
	D	0	0	0	0	0	0	0	0	0	0
+ Area		.0214	.0589	.1182	.1566	.1694	.1566	.1182	.0589	.0214	.0163
− Area		-.0935	-.0415	-.0368	-.0368	-.0368	-.0368	-.0368	-.0415	-.0935	-.2037
Total Area		-.0721	-.0174	.0814	.1198	.1326	.1198	.0814	.0174	-.0721	-.1874

REACTIONS/P and SHEARS/P

Span	Unit load at	R_A	R_B	R_C	R_D	V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}
SPAN 1	A	1.0	0	0	0	1.0	0	0	0	0	0
	.1	.8790	.1362	-.0237	.0065	.8790	-.1210	.0172	.0172	-.0065	-.0065
	.2	.7592	.2741	-.0458	.0125	.7592	-.2408	.0333	.0333	-.0125	-.0125
	.3	.6420	.4054	-.0652	.0178	.6420	-.3580	.0474	.0474	-.0178	-.0178
	.4	.5286	.5297	-.0803	.0220	.5286	-.4714	.0583	.0583	-.0220	-.0220
	.5	.4203	.6448	-.0896	.0245	.4203	-.5797	.0651	.0651	-.0245	-.0245
	.6	.3134	.7482	-.0917	.0251	.3184	-.6816	.0666	.0666	-.0251	-.0251
	.7	.2242	.8376	-.0853	.0233	.2242	-.7758	.0620	.0620	-.0233	-.0233
	.8	.1388	.9112	-.0688	.0188	.1388	-.8612	.0500	.0500	-.0188	-.0188
	.9	.0637	.9660	-.0409	.0112	.0637	-.9363	.0297	.0297	-.0112	-.0112
	B	0	1.0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 2	.1	-.0764	1.0084	.0932	-.0252	-.0764	-.0764	.9320	-.0680	.0252	.0252
	.2	-.1245	.9672	.2135	-.0562	-.1245	-.1245	.8427	-.1573	-.0562	.0562
	.3	-.1485	.8858	.3514	-.0887	-.1485	-.1485	.7373	-.2627	.0887	.0887
	.4	-.1526	.7739	.4972	-.1185	-.1526	-.1526	.6213	-.3787	.1185	.1185
	.5	-.1412	.6412	.6412	-.1412	-.1412	-.1412	.5000	.5000	.1412	.1412
	.6	-.1185	.4972	.7739	-.1526	-.1185	.1185	.3787	-.6213	.1526	.1526
	.7	-.0887	.3514	.8858	-.1485	-.0887	-.0887	.2627	-.7373	.1485	.1485
	.8	-.0562	.2135	.9672	-.1245	-.0562	-.0562	.1373	-.8427	.1245	.1245
	.9	-.0252	.0932	1.0084	-.0764	-.0252	-.0252	.0680	-.9320	.0764	.0764
	C	0	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 3	.1	.0112	-.0409	.9660	.0637	.0112	.0112	-.0297	-.0297	.9363	-.0637
	.2	.0188	-.0688	.9112	.1388	.0188	.0188	-.0500	-.0500	.8612	-.1388
	.3	.0233	-.0853	.8376	.2242	.0233	.0233	-.0620	-.0620	.7758	-.2242
	.4	.0251	-.0917	.7482	.3184	.0251	.0251	-.0666	-.0666	.6816	-.3184
	.5	.0245	-.0896	.6448	.4203	.0245	.0245	-.0651	-.0651	.5797	-.4203
	.6	.0220	-.0803	.5297	.5286	.0220	.0220	-.0583	-.0583	.4714	-.5286
	.7	.0178	-.0652	.4054	.6420	.0178	.0178	-.0474	-.0474	.3580	-.6420
	.8	.0125	-.0458	.2741	.7592	.0125	.0125	-.0333	-.0333	.2408	-.7592
	.9	.0065	-.0237	.1382	.8790	.0065	.0065	-.0172	-.0172	.1210	-.8790
	D	0	0	0	1.0	0	0	0	0	0	-1.0
+ Area		.4632	1.5471	1.5471	.4632	.4632	.0163	.8434	.0434	.7037	.1506
− Area		-.1506	-.0597	-.0597	-.1506	-.1506	-.7037	-.0434	-.8434	-.0163	-.4632
Total Area		.3126	1.4874	1.4874	.3126	.3126	-.6874	.8000	-.8000	.6874	-.3126



Influence coefficients — Three continuous spans.

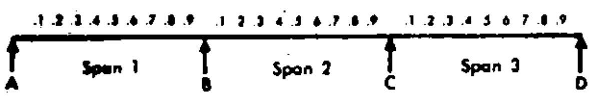
L = Length of EXTERIOR spans: length of interior span = NL

N = 1.6

Table 20.36

Span	Unit load at	A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
		MOMENTS/PL																				
		SPAN 1											SPAN 2									
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0880	.0759	.0639	.0519	.0398	.0278	.0158	.0037	-.0083	-.0204	-.0177	-.0150	-.0123	-.0096	-.0070	-.0043	-.0016	.0011	.0037	.0064
	.2	0	.0761	.1521	.1282	.1042	.0803	.0563	.0324	.0084	-.0155	-.0395	-.0343	-.0281	-.0239	-.0187	-.0135	-.0083	-.0031	.0020	.0072	.0124
	.3	0	.0644	.1288	.1932	.1576	.1219	.0863	.0507	.0151	-.0205	-.0561	-.0487	-.0414	-.0340	-.0266	-.0192	-.0118	-.0045	.0029	.0103	.0177
	.4	0	.0531	.1062	.1593	.2124	.1655	.1186	.0717	.0247	-.0222	-.0691	-.0600	-.0509	-.0418	-.0327	-.0237	-.0146	-.0055	.0036	.0127	.0217
	.5	0	.0423	.0846	.1269	.1692	.2115	.1537	.0960	.0383	-.0194	-.0771	-.0669	-.0563	-.0467	-.0365	-.0264	-.0163	-.0061	.0040	.0141	.0243
	.6	0	.0321	.0642	.0963	.1284	.1605	.1926	.1247	.0569	-.0110	-.0789	-.0686	-.0582	-.0478	-.0374	-.0270	-.0167	-.0063	.0041	.0145	.0248
	.7	0	.0227	.0453	.0680	.0906	.1133	.1360	.1586	.0813	.0040	-.0734	-.0637	-.0541	-.0444	-.0348	-.0251	-.0155	-.0058	.0038	.0135	.0231
	.8	0	.0141	.0282	.0422	.0563	.0704	.0845	.0986	.1126	.0267	-.0592	-.0514	-.0436	-.0358	-.0281	-.0203	-.0125	-.0047	.0031	.0109	.0186
	.9	0	.0065	.0130	.0195	.0259	.0324	.0389	.0454	.0519	.0584	-.0352	-.0305	-.0259	-.0213	-.0167	-.0120	-.0074	-.0028	.0018	.0064	.0111
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0083	-.0166	-.0249	-.0332	-.0415	-.0498	-.0581	-.0665	-.0748	-.0831	.0756	.0642	.0528	.0414	.0300	.0187	.0073	-.0041	-.0155	-.0268
	.2	0	-.0135	-.0270	-.0406	-.0541	-.0676	-.0811	-.0946	-.1081	-.1217	-.1352	.0083	.1518	.1253	.0988	.0723	.0458	.0193	-.0072	-.0337	-.0602
	.3	0	-.0161	-.0322	-.0483	-.0644	-.0805	-.0966	-.1127	-.1288	-.1449	-.1610	-.0355	.0901	.2157	.1712	.1268	.0823	.0379	-.0065	-.0510	-.0954
	.4	0	-.0165	-.0331	-.0496	-.0661	-.0826	-.0992	-.1157	-.1322	-.1488	-.1653	-.0595	.0462	.1520	.2577	.1935	.1292	.0650	.0007	-.0635	-.1278
	.5	0	-.0153	-.0305	-.0458	-.0611	-.0763	-.0916	-.1068	-.1221	-.1374	-.1526	-.0678	.0174	.1024	.1874	.2724	.1874	.1024	.0174	-.0676	-.1526
	.6	0	-.0128	-.0256	-.0383	-.0511	-.0639	-.0767	-.0895	-.1022	-.1150	-.1278	-.0635	.0007	.0650	.1292	.1935	.2577	.1520	.0462	-.0595	-.1653
	.7	0	-.0095	-.0191	-.0286	-.0382	-.0477	-.0572	-.0668	-.0763	-.0859	-.0954	-.0510	-.0065	.0379	.0823	.1268	.1712	.2157	.0901	-.0355	-.1610
	.8	0	-.0060	-.0120	-.0181	-.0241	-.0301	-.0361	-.0421	-.0482	-.0542	-.0602	-.0337	-.0072	.0193	.0458	.0723	.0988	.1253	.1518	.0083	-.1352
	.9	0	-.0027	-.0054	-.0080	-.0107	-.0134	-.0161	-.0188	-.0215	-.0241	-.0268	-.0155	-.0041	.0073	.0187	.0300	.0414	.0528	.0642	.0756	-.0831
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0011	.0022	.0033	.0044	.0056	.0067	.0078	.0089	.0100	.0111	.0064	.0018	-.0028	-.0074	-.0120	-.0167	-.0213	-.0259	-.0305	-.0352
	.2	0	.0019	.0037	.0056	.0074	.0093	.0112	.0130	.0149	.0167	.0186	.0109	.0031	-.0047	-.0125	-.0203	-.0281	-.0358	-.0436	-.0514	-.0592
	.3	0	.0023	.0046	.0069	.0092	.0116	.0139	.0162	.0185	.0208	.0231	.0135	.0038	-.0058	-.0155	-.0251	-.0348	-.0444	-.0541	-.0637	-.0734
	.4	0	.0025	.0050	.0074	.0099	.0124	.0149	.0174	.0198	.0223	.0249	.0145	.0041	-.0063	-.0167	-.0270	-.0374	-.0478	-.0582	-.0686	-.0789
	.5	0	.0024	.0049	.0073	.0097	.0122	.0146	.0170	.0194	.0219	.0243	.0141	.0040	-.0061	-.0163	-.0264	-.0365	-.0467	-.0568	-.0669	-.0771
	.6	0	.0022	.0043	.0065	.0087	.0109	.0130	.0152	.0174	.0195	.0217	.0127	.0036	-.0055	-.0146	-.0237	-.0327	-.0418	-.0509	-.0600	-.0691
	.7	0	.0018	.0035	.0053	.0071	.0089	.0106	.0124	.0142	.0159	.0177	.0103	.0029	-.0045	-.0118	-.0192	-.0266	-.0340	-.0414	-.0487	-.0561
	.8	0	.0012	.0025	.0037	.0050	.0062	.0074	.0087	.0099	.0112	.0124	.0072	.0020	-.0031	-.0083	-.0135	-.0187	-.0239	-.0291	-.0343	-.0395
	.9	0	.0006	.0013	.0019	.0026	.0032	.0038	.0045	.0051	.0058	.0064	.0037	.0011	-.0016	-.0043	-.0070	-.0096	-.0123	-.0150	-.0177	-.0204
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0415	.0729	.0945	.1060	.1074	.0989	.0804	.0518	.0231	.0162	.0224	.0641	.1304	.1738	.1882	.1738	.1304	.0641	.0224	.0162
− Area		0	-.0173	-.0345	-.0520	-.0693	-.0865	-.1038	-.1211	-.1384	-.1655	-.2244	-.1006	-.0411	-.0352	-.0352	-.0352	-.0352	-.0352	-.0411	-.1006	-.2244
Total Area		0	.0242	.0384	.0425	-.0367	.0209	-.0049	-.0407	-.0886	-.1424	-.2082	-.0782	.0230	.0952	.1386	.1530	.1386	.0952	.0230	-.0782	-.2082

Span	Unit load at	R_A	R_B	R_C	R_D	V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}
		REACTIONS/P				SHEARS/P					
SPAN 1	A	1.0	0	0	0	1.0	0	0	0	0	0
	.1	.8796	.1361	-.0221	.0064	.8796	-.1204	.0157	.0157	-.0064	-.0064
	.2	.7605	.2700	-.0429	.0124	.7605	-.2395	.0305	.0305	-.0124	-.0124
	.3	.6439	.3995	-.0611	.0177	.6439	-.3561	.0434	.0434	-.0177	-.0177
	.4	.5309	.5225	-.0751	.0217	.5309	-.4691	.0534	.0534	-.0217	-.0217
	.5	.4229	.6367	-.0839	.0243	.4229	-.5771	.0596	.0596	-.0243	-.0243
	.6	.3211	.7400	-.0859	.0248	.3211	-.6789	.0611	.0611	-.0248	-.0248
	.7	.2266	.8301	-.0798	.0231	.2266	-.7734	.0567	.0567	-.0231	-.0231
	.8	.1408	.9050	-.0644	.0186	.1408	-.8592	.0458	.0458	-.0186	-.0186
	.9	.0648	.9623	-.0382	.0111	.0648	-.9352	.0271	.0271	-.0111	-.0111
	B	0	1.0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 2	.1	-.0831	1.0161	.0938	-.0268	-.0831	-.0831	.9330	-.0670	.0268	.0268
	.2	-.1352	.9793	.2161	-.0602	-.1352	-.1352	.8441	-.1559	.0602	.0602
	.3	-.1610	.8996	.3568	-.0954	-.1610	-.1610	.7386	-.2614	.0954	.0954
	.4	-.1653	.7874	.5057	-.1278	-.1653	-.1653	.6221	-.3779	.1278	.1278
	.5	-.1526	.6526	.6526	-.1526	-.1526	-.1526	.5000	-.5000	.1526	.1526
	.6	-.1278	.5057	.7874	-.1653	-.1278	-.1278	.3779	-.6221	.1653	.1653
	.7	-.0954	.3568	.8996	-.1610	-.0954	-.0954	.2614	-.7386	.1610	.1610
	.8	-.0602	.2161	.9793	-.1352	-.0602	-.0602	.1559	-.8441	.1352	.1352
	.9	-.0268	.0938	1.0161	-.0831	-.0268	-.0268	.0670	-.9330	.0831	.0831
	C	0	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 3	.1	.0111	-.0382	.9623	.0648	.0111	.0111	-.0271	-.0271	.9352	-.0648
	.2	.0186	-.0644	.9050	.1408	.0186	.0186	-.0458	-.0458	.8592	-.1408
	.3	.0231	-.0798	.8301	.2266	.0231	.0231	-.0567	-.0567	.7734	-.2266
	.4	.0248	-.0859	.7400	.3211	.0248	.0248	-.0611	-.0611	.6789	-.3211
	.5	.0243	-.0839	.6367	.4229	.0243	.0243	-.0596	-.0596	.5771	-.4229
	.6	.0217	-.0751	.5225	.5309	.0217	.0217	-.0534	-.0534	.4691	-.5309
	.7	.0177	-.0611	.3995	.6439	.0177	.0177	-.0434	-.0434	.3561	-.6439
	.8	.0124	-.0429	.2700	.7605	.0124	.0124	-.0305	-.0305	.2395	-.7605
	.9	.0064	-.0221	.1361	.8796	.0064	.0064	-.0157	-.0157	.1204	-.8796
	D	0	0	0	1.0	0	0	0	0	0	-1.0
+ Area		.4648	1.6141	1.6141	.4648	.4648	.0162	.8897	.0397	.7244	.1730
− Area		-.1730	-.0559	-.0559	-.1730	-.1730	-.7244	-.0397	-.8897	-.0162	-.4648
Total Area		.2918	1.5582	1.5582	.2918	.2918	-.7082	.8500	-.8500	.7082	-.2918



Influence coefficients — Three continuous spans.

L = Length of EXTERIOR spans; length of interior span = NL

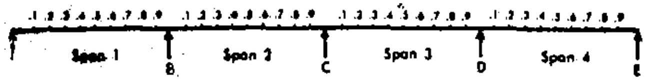
N = 1.7

Table 20.37

MOMENTS/PL

Span	Unit load at	A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
		SPAN 1											SPAN 2									
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0873	.0747	.0620	.0494	.0367	.0241	.0114	-.0012	-.0139	-.0265	-.0232	-.0198	-.0164	-.0131	-.0097	-.0064	-.0030	.0004	.0037	.0071
	.2	0	.0749	.1497	.1246	.0994	.0743	.0491	.0240	-.0011	-.0263	-.0514	-.0449	-.0384	-.0319	-.0254	-.0189	-.0123	-.0058	.0007	.0072	.0137
	.3	0	.0627	.1254	.1881	.1508	.1134	.0761	.0388	.0015	-.0358	-.0731	-.0639	-.0546	-.0453	-.0361	-.0268	-.0176	-.0083	.0010	.0102	.0195
	.4	0	.0510	.1020	.1530	.2040	.1550	.1060	.0570	.0080	-.0410	-.0900	-.0786	-.0672	-.0558	-.0444	-.0330	-.0216	-.0102	.0012	.0126	.0240
	.5	0	.0400	.0799	.1199	.1598	.1998	.1397	.0797	.0196	-.0404	-.1004	-.0877	-.0750	-.0623	-.0496	-.0368	-.0241	-.0114	.0013	.0141	.0268
	.6	0	.0297	.0594	.0891	.1189	.1486	.1783	.1080	.0377	-.0326	-.1029	-.0898	-.0768	-.0638	-.0507	-.0377	-.0247	-.0117	.0014	.0144	.0274
	.7	0	.0204	.0409	.0613	.0818	.1022	.1226	.1431	.0635	-.0161	-.0956	-.0835	-.0714	-.0593	-.0472	-.0351	-.0230	-.0108	.0013	.0134	.0255
	.8	0	.0123	.0246	.0369	.0491	.0614	.0737	.0860	.0983	.0106	-.0771	-.0674	-.0576	-.0478	-.0381	-.0283	-.0185	-.0087	.0010	.0108	.0206
	.9	0	.0054	.0108	.0163	.0217	.0271	.0325	.0379	.0434	.0488	-.0458	-.0400	-.0342	-.0284	-.0226	-.0168	-.0110	-.0052	.0006	.0064	.0122
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0039	-.0077	-.0116	-.0155	-.0194	-.0232	-.0271	-.0310	-.0349	-.0387	.0535	.0458	.0381	.0303	.0226	.0149	.0071	-.0006	-.0083	-.0161
	.2	0	-.0063	-.0127	-.0190	-.0254	-.0317	-.0381	-.0444	-.0507	-.0571	-.0634	.0195	.1024	.0853	.0682	.0511	.0341	.0170	-.0001	-.0172	-.0343
	.3	0	-.0076	-.0152	-.0228	-.0305	-.0381	-.0457	-.0533	-.0609	-.0685	-.0761	-.0038	.0686	.1410	.1133	.0857	.0581	.0304	.0028	-.0249	-.0525
	.4	0	-.0079	-.0158	-.0237	-.0315	-.0394	-.0473	-.0552	-.0631	-.0710	-.0789	-.0178	.0432	.1042	.1653	.1263	.0873	.0483	.0094	-.0296	-.0686
	.5	0	-.0074	-.0147	-.0221	-.0295	-.0368	-.0442	-.0516	-.0589	-.0663	-.0737	-.0243	.0250	.0743	.1237	.1730	.1223	.0717	.0210	-.0297	-.0804
	.6	0	-.0063	-.0125	-.0188	-.0250	-.0313	-.0375	-.0438	-.0501	-.0563	-.0626	-.0249	.0128	.0505	.0882	.1259	.1635	.1012	.0389	-.0234	-.0857
	.7	0	-.0048	-.0095	-.0143	-.0191	-.0238	-.0286	-.0333	-.0381	-.0429	-.0476	-.0211	.0054	.0319	.0584	.0848	.1115	.1380	.0645	-.0090	-.0825
	.8	0	-.0031	-.0062	-.0093	-.0123	-.0154	-.0185	-.0216	-.0247	-.0278	-.0309	-.0146	.0016	.0178	.0341	.0503	.0665	.0827	.0990	.0152	-.0686
	.9	0	-.0014	-.0029	-.0043	-.0057	-.0072	-.0086	-.0100	-.0114	-.0129	-.0143	-.0071	.0002	.0075	.0147	.0220	.0292	.0365	.0437	.0510	-.0418
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0010	.0021	.0031	.0042	.0052	.0063	.0073	.0084	.0094	.0104	.0052	.0000	-.0052	-.0104	-.0157	-.0209	-.0261	-.0313	-.0366	-.0418
	.2	0	.0017	.0034	.0051	.0069	.0086	.0103	.0120	.0137	.0154	.0171	.0086	.0000	-.0086	-.0171	-.0257	-.0343	-.0429	-.0514	-.0600	-.0686
	.3	0	.0021	.0041	.0062	.0083	.0103	.0124	.0144	.0165	.0186	.0206	.0103	.0000	-.0103	-.0206	-.0309	-.0413	-.0516	-.0619	-.0722	-.0825
	.4	0	.0021	.0043	.0064	.0086	.0107	.0129	.0150	.0171	.0193	.0214	.0107	.0000	-.0107	-.0214	-.0321	-.0429	-.0536	-.0643	-.0750	-.0857
	.5	0	.0020	.0040	.0060	.0080	.0100	.0121	.0141	.0161	.0181	.0201	.0100	.0000	-.0100	-.0201	-.0301	-.0402	-.0502	-.0603	-.0703	-.0804
	.6	0	.0017	.0034	.0051	.0069	.0086	.0103	.0120	.0137	.0154	.0171	.0086	.0000	-.0086	-.0171	-.0257	-.0343	-.0429	-.0514	-.0600	-.0686
	.7	0	.0013	.0026	.0039	.0053	.0066	.0079	.0092	.0105	.0118	.0131	.0066	.0000	-.0066	-.0131	-.0197	-.0263	-.0328	-.0394	-.0459	-.0525
	.8	0	.0009	.0017	.0026	.0034	.0043	.0051	.0060	.0069	.0077	.0086	.0043	.0000	-.0043	-.0086	-.0129	-.0171	-.0214	-.0257	-.0300	-.0343
	.9	0	.0004	.0008	.0012	.0016	.0020	.0024	.0028	.0032	.0036	.0040	.0020	.0000	-.0020	-.0040	-.0060	-.0080	-.0100	-.0121	-.0141	-.0161
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	0	-.0003	-.0006	-.0009	-.0012	-.0015	-.0018	-.0021	-.0024	-.0027	-.0031	-.0015	.0000	.0015	.0031	.0046	.0061	.0076	.0092	.0107	.0122
	.2	0	-.0005	-.0010	-.0015	-.0021	-.0026	-.0031	-.0036	-.0041	-.0046	-.0051	-.0026	.0000	.0026	.0051	.0077	.0103	.0129	.0154	.0180	.0206
	.3	0	-.0006	-.0013	-.0019	-.0026	-.0032	-.0038	-.0045	-.0051	-.0057	-.0064	-.0032	.0000	.0032	.0064	.0096	.0128	.0159	.0191	.0223	.0255
	.4	0	-.0007	-.0014	-.0021	-.0027	-.0034	-.0041	-.0048	-.0055	-.0062	-.0069	-.0034	.0000	.0034	.0069	.0103	.0137	.0171	.0206	.0240	.0274
	.5	0	-.0007	-.0013	-.0020	-.0027	-.0033	-.0040	-.0047	-.0054	-.0060	-.0067	-.0033	.0000	.0033	.0067	.0100	.0134	.0167	.0201	.0234	.0268
	.6	0	-.0006	-.0012	-.0018	-.0024	-.0030	-.0036	-.0042	-.0048	-.0054	-.0060	-.0030	.0000	.0030	.0060	.0090	.0120	.0150	.0180	.0210	.0240
	.7	0	-.0005	-.0010	-.0015	-.0020	-.0024	-.0029	-.0034	-.0039	-.0044	-.0049	-.0024	.0000	.0024	.0049	.0073	.0098	.0122	.0146	.0171	.0195
	.8	0	-.0003	-.0007	-.0010	-.0014	-.0017	-.0021	-.0024	-.0027	-.0031	-.0034	-.0017	.0000	.0017	.0034	.0051	.0069	.0086	.0103	.0120	.0137
	.9	0	-.0002	-.0004	-.0005	-.0007	-.0009	-.0011	-.0012	-.0014	-.0016	-.0018	-.0009	.0000	.0009	.0018	.0027	.0035	.0044	.0053	.0062	.0071
	E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0396	.0693	.0889	.0986	.0982	.0878	.0675	.0374	.0174	.0134	.0135	.0300	.0588	.0736	.0804	.0771	.0639	.0417	.0311	.0357
− Area		0	-.0053	-.0107	-.0161	-.0214	-.0268	-.0321	-.0375	-.0431	-.0488	-.1205	-.0721	-.0500	-.0482	-.0464	-.0447	-.0429	-.0411	-.0403	-.0611	-.1071
Total Area		0	.0343	.0586	.0728	.0772	.0714	.0557	.0300	-.0057	-.0314	-.1071	-.0586	-.0200	.0086	.0272	.0357	.0343	.0228	.0014	-.0300	-.0714

Span	Unit load at	R_A	R_B	R_C	R_D	R_E	V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}	V_{DE}	V_{ED}
		REACTIONS/P					SHEARS/P							
SPAN 1	A	1.0	0	0	0	0	1.0	0	0	0	0	0	0	0
	.1	.8735	.1601	-.0424	.0106	-.0018	.8735	-.1265	.0336	.0336	-.0088	-.0088	.0018	.0018
	.2	.7486	.3166	-.0823	.0206	-.0034	.7486	-.2514	.0651	.0651	-.0171	-.0171	.0034	.0034
	.3	.6269	.4658	-.1170	.0292	-.0049	.6269	-.3731	.0926	.0926	-.0244	-.0244	.0049	.0049
	.4	.5100	.6040	-.1440	.0360	-.0060	.5100	-.4900	.1140	.1140	-.0300	-.0300	.0060	.0060
	.5	.3996	.7277	-.1607	.0402	-.0067	.3996	-.6004	.1272	.1272	-.0335	-.0335	.0067	.0067
	.6	.2971	.8331	-.1646	.0411	-.0069	.2971	-.7029	.1303	.1303	-.0343	-.0343	.0069	.0069
	.7	.2044	.9168	-.1530	.0383	-.0064	.2044	-.7956	.1211	.1211	-.0319	-.0319	.0064	.0064
	.8	.1229	.9749	-.1234	.0309	-.0051	.1229	-.8771	.0977	.0977	-.0257	-.0257	.0051	.0051
	.9	.0542	1.0038	-.0733	.0183	-.0031	.0542	-.9458	.0580	.0580	-.0153	-.0153	.0031	.0031
	B	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0	0	0
SPAN 2	.1	-.0387	.9614	.0974	-.0241	.0040	-.0387	-.0387	.9227	-.0773	.0201	.0201	-.0040	-.0040
	.2	-.0634	.8926	.2137	-.0514	.0086	-.0634	-.0634	.8291	-.1709	.0429	.0429	-.0086	-.0086
	.3	-.0761	.7998	.3420	-.0768	.0131	-.0761	-.0761	.7236	-.2764	.0656	.0656	-.0131	-.0131
	.4	-.0789	.6891	.4754	-.1029	.0171	-.0789	-.0789	.6103	-.3897	.0857	.0857	-.0171	-.0171
	.5	-.0737	.5670	.6071	-.1205	.0201	-.0737	-.0737	.4933	-.5067	.1004	.1004	-.0201	-.0201
	.6	-.0626	.4394	.7303	-.1286	.0214	-.0626	-.0626	.3769	-.6231	.1071	.1071	-.0214	-.0214
	.7	-.0476	.3128	.8380	-.1238	.0206	-.0476	-.0476	.2651	-.7349	.1031	.1031	-.0206	-.0206
	.8	-.0309	.1931	.9234	-.1029	.0171	-.0309	-.0309	.1623	-.8377	.0857	.0857	-.0171	-.0171
	.9	-.0143	.0868	.9797	-.0627	.0104	-.0143	-.0143	.0725	-.9275	.0522	.0522	-.0104	-.0104
	C	0	0	1.0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 3	.1	.0104	-.0627	.9797	.0868	-.0143	.0104	.0104	-.0522	-.0522	.9275	-.0725	.0143	.0143
	.2	.0171	-.1029	.9234	.1931	-.0309	.0171	.0171	-.0857	-.0857	.8377	-.1623	.0309	.0309
	.3	.0206	-.1238	.8380	.3128	-.0476	.0206	.0206	-.1031	-.1031	.7349	-.2651	.0476	.0476
	.4	.0214	-.1286	.7303	.4394	-.0626	.0214	.0214	-.1071	-.1071	.6231	-.3769	.0626	.0626
	.5	.0201	-.1205	.6071	.5670	-.0737	.0201	.0201	-.1004	-.1004	.5067	-.4933	.0737	.0737
	.6	.0171	-.1029	.4754	.6891	-.0789	.0171	.0171	-.0857	-.0857	.3897	-.6103	.0789	.0789
	.7	.0131	-.0768	.3420	.7998	-.0761	.0131	.0131	-.0656	-.0656	.2764	-.7236	.0761	.0761
	.8	.0086	-.0514	.2137	.8926	-.0634	.0086	.0086	-.0429	-.0429	.1709	-.8291	.0634	.0634
	.9	.0040	-.0241	.0974	.9614	-.0387	.0040	.0040	-.0201	-.0201	.0773	-.9227	.0387	.0387
	D	0	0	0	1.0	0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 4	.1	-.0031	.0183	-.0733	1.0038	.0542	-.0031	-.0031	.0153	.0153	-.0580	-.0580	.9458	-.0542
	.2	-.0051	.0309	-.1234	.9749	.1229	-.0051	-.0051	.0257	.0257	-.0977	-.0977	.8771	-.1229
	.3	-.0064	.0383	-.1530	.9168	.2044	-.0064	-.0064	.0319	.0319	-.1211	-.1211	.7956	-.2044
	.4	-.0069	.0411	-.1646	.8331	.2971	-.0069	-.0069	.0343	.0343	-.1303	-.1303	.7029	-.2971
	.5	-.0067	.0402	-.1607	.7277	.3996	-.0067	-.0067	.0335	.0335	-.1272	-.1272	.6004	-.3996
	.6	-.0060	.0360	-.1440	.6040	.5100	-.0060	-.0060	.0300	.0300	-.1140	-.1140	.4900	-.5100
	.7	-.0049	.0292	-.1170	.4658	.6269	-.0049	-.0049	.0244	.0244	-.0926	-.0926	.3731	-.6269
	.8	-.0034	.0206	-.0823	.3166	.7486	-.0034	-.0034	.0171	.0171	-.0651	-.0651	.2514	-.7486
	.9	-.0018	.0106	-.0424	.1601	.8735	-.0018	-.0018	.0088	.0088	-.0336	-.0336	.1265	-.8735
	E	0	0	0	0	1.0	0	0	0	0	0	0	0	-1.0
+ Area		.4464	1.2232	1.1429	1.2232	.4464	.4464	.0134	.6027	.1071	.5714	.0670	.6205	.0536
− Area		-.0536	-.0804	-.2143	-.0804	-.0536	-.0536	-.6205	-.0670	-.5714	-.1071	-.6027	-.0134	-.4464
Total Area		.3928	1.1428	.9286	1.1428	.3928	.3928	-.6071	.5357	-.4643	.4643	-.5357	.6071	-.3928



Influence coefficients — Four continuous spans.

L = Length of EXTERIOR spans; length of interior spans = NL

N = 1.0

Table 20.38

MOMENTS/PL — SPAN 1

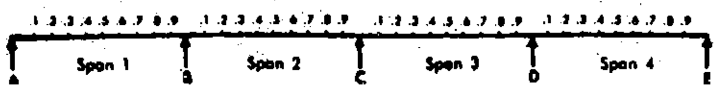
Span	Unit load at	A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0873	.0748	.0624	.0499	.0373	.0248	.0123	-.0003	-.0128	-.0253
	.2	0	.0751	.1502	.1253	.1003	.0754	.0505	.0256	.0007	-.0242	-.0492
	.3	0	.0630	.1260	.1890	.1520	.1151	.0781	.0411	.0041	-.0329	-.0699
	.4	0	.0514	.1028	.1542	.2056	.1570	.1084	.0598	.0112	-.0374	-.0860
	.5	0	.0404	.0808	.1212	.1616	.2020	.1424	.0828	.0232	-.0364	-.0960
	.6	0	.0302	.0603	.0905	.1207	.1508	.1810	.1112	.0413	-.0285	-.0983
	.7	0	.0209	.0417	.0626	.0834	.1043	.1252	.1460	.0669	-.0123	-.0914
	.8	0	.0126	.0253	.0379	.0506	.0631	.0758	.0884	.1010	.0136	-.0737
	.9	0	.0056	.0112	.0169	.0225	.0281	.0337	.0394	.0450	.0506	-.0438
	B	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0045	-.0090	-.0134	-.0179	-.0224	-.0269	-.0313	-.0358	-.0403	-.0448
	.2	0	-.0073	-.0147	-.0220	-.0293	-.0367	-.0440	-.0513	-.0586	-.0660	-.0733
	.3	0	-.0088	-.0176	-.0264	-.0352	-.0440	-.0528	-.0616	-.0704	-.0792	-.0880
	.4	0	-.0091	-.0182	-.0273	-.0364	-.0456	-.0547	-.0638	-.0729	-.0820	-.0911
	.5	0	-.0085	-.0170	-.0255	-.0340	-.0425	-.0511	-.0596	-.0681	-.0766	-.0851
	.6	0	-.0072	-.0145	-.0217	-.0289	-.0361	-.0434	-.0506	-.0578	-.0650	-.0723
	.7	0	-.0055	-.0110	-.0165	-.0220	-.0275	-.0330	-.0385	-.0440	-.0495	-.0550
	.8	0	-.0036	-.0071	-.0107	-.0142	-.0178	-.0214	-.0249	-.0285	-.0321	-.0356
	.9	0	-.0016	-.0033	-.0049	-.0066	-.0082	-.0099	-.0115	-.0132	-.0148	-.0165
	C	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0012	.0024	.0036	.0048	.0060	.0072	.0084	.0096	.0108	.0120
	.2	0	.0020	.0039	.0059	.0079	.0099	.0118	.0138	.0158	.0177	.0197
	.3	0	.0024	.0047	.0071	.0095	.0118	.0142	.0166	.0189	.0213	.0237
	.4	0	.0025	.0049	.0074	.0098	.0123	.0147	.0172	.0196	.0221	.0245
	.5	0	.0023	.0046	.0069	.0092	.0115	.0138	.0161	.0184	.0206	.0229
	.6	0	.0020	.0039	.0058	.0078	.0098	.0117	.0137	.0156	.0176	.0195
	.7	0	.0015	.0030	.0045	.0060	.0074	.0089	.0104	.0119	.0134	.0149
	.8	0	.0010	.0019	.0029	.0039	.0048	.0058	.0068	.0077	.0087	.0097
	.9	0	.0004	.0009	.0013	.0018	.0022	.0027	.0031	.0036	.0040	.0045
	D	0	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	0	-.0003	-.0006	-.0009	-.0012	-.0015	-.0018	-.0021	-.0025	-.0028	-.0031
	.2	0	-.0005	-.0010	-.0015	-.0021	-.0026	-.0031	-.0036	-.0041	-.0046	-.0052
	.3	0	-.0006	-.0013	-.0019	-.0026	-.0032	-.0038	-.0045	-.0051	-.0058	-.0064
	.4	0	-.0007	-.0014	-.0021	-.0028	-.0034	-.0041	-.0048	-.0055	-.0062	-.0069
	.5	0	-.0007	-.0013	-.0020	-.0027	-.0034	-.0040	-.0047	-.0054	-.0061	-.0067
	.6	0	-.0006	-.0012	-.0018	-.0024	-.0030	-.0036	-.0042	-.0048	-.0054	-.0060
	.7	0	-.0005	-.0010	-.0015	-.0020	-.0024	-.0029	-.0034	-.0039	-.0044	-.0049
	.8	0	-.0003	-.0007	-.0010	-.0014	-.0017	-.0021	-.0024	-.0028	-.0031	-.0034
	.9	0	-.0002	-.0004	-.0005	-.0007	-.0009	-.0011	-.0012	-.0014	-.0016	-.0018
	E	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0403	.0706	.0908	.1011	.1014	.0917	.0720	.0423	.0210	.0168
− Area		0	-.0067	-.0134	-.0200	-.0267	-.0334	-.0401	-.0468	-.0535	-.0787	-.1309
Total Area		0	.0336	.0572	.0708	.0744	.0680	.0516	.0252	-.0112	-.0577	-.1141

MOMENTS/PL — SPAN 2

Span	Unit load at	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0
	.1	-.0231	-.0189	-.0157	-.0125	-.0093	-.0061	-.0029	.0004	.0036	.0068
	.2	-.0429	-.0367	-.0305	-.0243	-.0180	-.0118	-.0055	.0007	.0069	.0132
	.3	-.0610	-.0522	-.0433	-.0345	-.0256	-.0167	-.0079	.0010	.0098	.0187
	.4	-.0751	-.0642	-.0533	-.0424	-.0315	-.0206	-.0097	.0012	.0121	.0230
	.5	-.0838	-.0717	-.0595	-.0473	-.0352	-.0230	-.0108	.0013	.0135	.0257
	.6	-.0859	-.0734	-.0609	-.0485	-.0360	-.0235	-.0111	.0014	.0138	.0263
	.7	-.0798	-.0682	-.0566	-.0451	-.0335	-.0219	-.0103	.0013	.0129	.0245
	.8	-.0644	-.0550	-.0457	-.0364	-.0270	-.0177	.0083	-.0010	.0104	.0197
	.9	-.0382	-.0327	-.0271	-.0216	-.0160	-.0105	-.0049	.0006	.0062	.0117
	B	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	.0370	.0488	.0405	.0323	.0240	.0158	.0076	-.0007	-.0089	-.0172
	.2	.0183	.1100	.0916	.0733	.0549	.0365	.0182	-.0002	-.0185	-.0369
	.3	-.0079	.0723	.1524	.1225	.0926	.0627	.0328	.0030	-.0269	-.0568
	.4	-.0235	.0442	.1119	.1795	.1372	.0949	.0525	.0102	-.0322	-.0745
	.5	-.0303	.0244	.0792	.1339	.1887	.1334	.0782	.0229	-.0323	-.0876
	.6	-.0304	.0113	.0533	.0952	.1370	.1789	.1108	.0428	-.0255	-.0937
	.7	-.0255	.0334	.0334	.0629	.0923	.1218	.1513	.0707	-.0098	-.0903
	.8	-.0176	.0005	.0185	.0365	.0546	.0726	.0907	.1087	.0167	-.0752
	.9	-.0084	-.0004	.0077	.0157	.0238	.0319	.0399	.0480	.0560	-.0459
	C	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	.0062	.0004	-.0054	-.0111	-.0169	-.0227	-.0285	-.0343	-.0401	-.0459
	.2	.0102	.0007	-.0088	-.0183	-.0278	-.0373	-.0467	-.0562	-.0657	-.0752
	.3	.0123	.0009	-.0105	-.0219	-.0333	-.0447	-.0561	-.0675	-.0789	-.0903
	.4	.0127	.0009	-.0109	-.0227	-.0346	-.0464	-.0582	-.0700	-.0818	-.0937
	.5	.0119	.0008	-.0102	-.0213	-.0323	-.0434	-.0544	-.0655	-.0765	-.0876
	.6	.0101	.0007	-.0087	-.0181	-.0275	-.0369	-.0463	-.0557	-.0651	-.0745
	.7	.0077	.0005	-.0066	-.0138	-.0210	-.0281	-.0353	-.0425	-.0496	-.0568
	.8	.0050	.0004	-.0043	-.0090	-.0136	-.0183	-.0229	-.0276	-.0322	-.0369
	.9	.0023	.0002	-.0020	-.0042	-.0063	-.0085	-.0107	-.0128	-.0150	-.0172
	D	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	-.0016	-.0001	.0014	.0028	.0043	.0058	.0073	.0088	.0102	.0117
	.2	-.0027	-.0002	.0023	.0048	.0073	.0098	.0123	.0147	.0172	.0197
	.3	-.0033	-.0002	.0029	.0059	.0090	.0121	.0152	.0183	.0214	.0245
	.4	-.0036	-.0002	.0031	.0064	.0097	.0130	.0163	.0197	.0230	.0263
	.5	-.0035	-.0002	.0030	.0062	.0095	.0127	.0160	.0192	.0224	.0257
	.6	-.0031	-.0002	.0027	.0056	.0085	.0114	.0143	.0172	.0201	.0230
	.7	-.0025	-.0002	.0022	.0045	.0069	.0093	.0116	.0140	.0163	.0187
	.8	-.0018	-.0001	.0015	.0032	.0049	.0065	.0082	.0098	.0115	.0132
	.9	-.0009	-.0001	.0008	.0016	.0025	.0034	.0042	.0051	.0059	.0068
	E	0	0	0	0	0	0	0	0	0	0
+ Area		.0164	.0347	.0661	.0862	.0942	.0902	.0740	.0467	.0313	.0343
− Area		-.0740	-.0480	-.0472	-.0471	-.0471	-.0471	-.0471	-.0481	-.0730	-.1285
Total Area		-.0576	-.0133	.0189	.0391	.0471	.0431	.0269	-.0014	-.0417	-.0942

REACTIONS/P and SHEARS/P

Span	Unit load at	R_A	R_B	R_C	R_D	R_E	V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}	V_{DE}	V_{ED}
SPAN 1	A	1.0	0	0	0	0	1.0	0	0	0	0	0	0	0
	.1	.8747	.1546	-.0370	.0094	-.0018	.8747	-.1253	.0292	.0292	-.0078	-.0078	.0018	.0018
	.2	.7508	.3059	-.0717	.0185	-.0034	.7508	-.2492	.0566	.0566	-.0151	-.0151	.0034	.0034
	.3	.6301	.4504	-.1020	.0264	-.0049	.6301	-.3699	.0805	.0805	-.0218	-.0218	.0049	.0049
	.4	.5140	.5852	-.1255	.0324	-.0060	.5140	-.4860	.0991	.0991	-.0264	-.0264	.0060	.0060
	.5	.4040	.7066	-.1401	.0362	-.0067	.4040	-.5960	.1106	.1106	-.0298	-.0298	.0067	.0067
	.6	.3017	.8116	-.1435	.0371	-.0069	.3017	-.6983	.1133	.1133	-.0302	-.0302	.0069	.0069
	.7	.2086	.8947	-.1334	.0345	-.0064	.2086	-.7914	.1053	.1053	-.0281	-.0281	.0064	.0064
	.8	.1263	.9587	-.1076	.0278	-.0052	.1263	-.8737	.0850	.0850	-.0228	-.0228	.0052	.0052
	.9	.0562	.9942	-.0639	.0165	-.0031	.0562	-.9438	.0505	.0505	-.0134	-.0134	.0031	.0031
	B	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0	0	0
SPAN 2	.1	-.0448	.9899	.0946	-.0242	.0045	-.0448	-.0448	.9251	-.0749	.0197	.0197	-.0045	-.0045
	.2	-.0733	.9064	.2092	-.0520	.0097	-.0733	-.0733	.8331	-.1669	.0423	.0423	-.0097	-.0097
	.3	-.0880	.8163	.3368	-.0800	.0149	-.0880	-.0880	.7283	-.2717	.0652	.0652	-.0149	-.0149
	.4	-.0911	.7062	.4704	-.1050	.0195	-.0911	-.0911	.6151	-.3849	.0855	.0855	-.0195	-.0195
	.5	-.0851	.5828	.6027	-.1234	.0229	-.0851	-.0851	.4977	-.5023	.1005	.1005	-.0229	-.0229
	.6	-.0723	.4528	.7268	-.1320	.0245	-.0723	-.0723	.3806	-.6195	.1075	.1075	-.0245	-.0245
	.7	-.0550	.3228	.8358	-.1273	.0237	-.0550	-.0550	.2679	-.7321	.1036	.1036	-.0237	-.0237
	.8	-.0356	.1996	.9223	-.1060	.0197	-.0356	-.0156	.1640	-.8360	.0863	.0863	-.0197	-.0197
	.9	-.0165	.0898	.9794	-.0647	.0120	-.0165	-.0165	.0733	-.9267	.0527	.0527	-.0120	-.0120
	C	0	0	1.0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 3	.1	.0120	-.0647	.9794	.0898	-.0165	.0120	.0120	-.0527	-.0527	.9267	-.0733	.0165	.0165
	.2	.0197	-.1060	.9223	.1996	-.0356	.0197	.0197	-.0863	-.0863	.8360	-.1640	.0356	.0356
	.3	.0237	-.1273	.8358	.3229	-.0550	.0237	.0237	-.1036	-.1036	.7321	-.2679	.0550	.0550
	.4	.0245	-.1320	.7269	.4528	-.0723	.0245	.0245	-.1075	-.1075	.6195	-.3806	.0723	.0723
	.5	.0229	-.1234	.6027	.5828	-.0851	.0229	.0229	-.1005	-.1005	.5023	-.4977	.0851	.0851
	.6	.0195	-.1050	.4704	.7062	-.0911	.0195	.0195	-.0855	-.0855	.3849	-.6151	.0911	.0911
	.7	.0149	-.0800	.3368	.8163	-.0880	.0149	.0149	-.0652	-.0652	.2717	-.7283	.0880	.0880
	.8	.0097	-.0520	.2092	.9064	-.0733	.0097	.0097	-.0423	-.0423	.1669	-.8331	.0733	.0733
	.9	.0045	-.0242	.0946	.9699	-.0448	.0045	.0045	-.0197	-.0197	.0749	-.9251	.0448	.0448
	D	0	0	0	1.0	0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 4	.1	-.0031	.0165	-.0639	.9942	.0562	-.0031	-.0031	.0134	.0134	-.0505	-.0505	.9438	-.0562
	.2	-.0052	.0278	-.1076	.9587	.1263	-.0052	-.0052	.0226	.0226	-.0850	-.0850	.8737	-.1263
	.3	-.0064	.0345	-.1334	.8967	.2086	-.0064	-.0064	.0281	.0281	-.1053	-.1053	.7914	-.2086
	.4	-.0069	.0371	-.1435	.8116	.3017	-.0069	-.0069	.0302	.0302	-.1133	-.1133	.6983	-.3017
	.5	-.0067	.0362	-.1401	.7066	.4040	-.0067	-.0067	.0295	.0295	-.1106	-.1106	.5960	-.4040
	.6	-.0060	.0324	-.1255	.5852	.5140	-.0060	-.0060	.0264	.0264	-.0991	-.0991	.4860	-.5140
	.7	-.0049	.0264	-.1020	.4504	.6301	-.0049	-.0049	.0215	.0215	-.0805	-.0805	.3699	-.6301
	.8	-.0034	.0185	-.0717	.3058	.7508	-.0034	-.0034	.0151	.0151	-.0566	-.0566	.2492	-.7508
	.9	-.0018	.0096	-.0370	.1546	.8747	-.0018	-.0018	.0078	.0078	-.0292	-.0292	.1253	-.8747
	E	0	0	0	0	1.0	0	0	0	0	0	0	0	-1.0
+ Area		.4528	1.2726	1.2507	1.2726	.4528	.4528	.0168	.6417	.0934	.6253	.0737	.6309	.0669
− Area		-.0669	-.0905	-.1868	-.0905	-.0669	-.0669	-.6309	-.0737	-.6253	-.0934	-.6417	-.0168	-.4528
Total Area		.3859	1.1821	1.0639	1.1821	.3859	.3859	-.6141	.5680	-.5319	-.5319	-.5680	.6141	-.3859



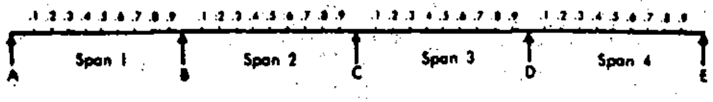
Influence coefficients — Four continuous spans.

L = Length of EXTERIOR spans; length of interior spans = NL

Table 20.39

Unit load at		MOMENTS/PL																				
		SPAN 1										SPAN 2										
		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0876	.0751	.0627	.0503	.0379	.0254	.0130	.0006	-.0119	-.0243	-.0212	-.0181	-.0150	-.0120	-.0089	-.0058	-.0027	.0004	.0034	.0065
	.2	0	.0753	.1506	.1259	.1012	.0765	.0518	.0270	.0023	-.0224	-.0471	-.0411	-.0351	-.0292	-.0232	-.0172	-.0113	-.0053	.0007	.0067	.0126
	.3	0	.0633	.1266	.1899	.1532	.1165	.0798	.0431	.0064	-.0303	-.0669	-.0585	-.0500	-.0415	-.0330	-.0245	-.0160	-.0075	.0010	.0095	.0180
	.4	0	.0518	.1035	.1553	.2070	.1588	.1106	.0623	.0141	-.0342	-.0824	-.0719	-.0615	-.0510	-.0406	-.0301	-.0197	-.0092	.0012	.0117	.0221
	.5	0	.0408	.0816	.1224	.1632	.2040	.1448	.0856	.0264	-.0328	-.0920	-.0803	-.0686	-.0570	-.0453	-.0336	-.0220	-.0103	.0013	.0130	.0247
	.6	0	.0306	.0612	.0918	.1223	.1529	.1835	.1141	.0447	-.0247	-.0942	-.0822	-.0703	-.0583	-.0464	-.0345	-.0225	-.0106	.0014	.0133	.0253
	.7	0	.0212	.0425	.0637	.0850	.1062	.1275	.1487	.0700	-.0088	-.0875	-.0764	-.0653	-.0542	-.0431	-.0320	-.0209	-.0098	.0013	.0124	.0235
	.8	0	.0129	.0259	.0388	.0518	.0647	.0776	.0906	.1035	.0164	-.0706	-.0617	-.0527	-.0438	-.0348	-.0258	-.0169	-.0079	.0010	.0100	.0189
	.9	0	.0058	.0116	.0174	.0232	.0290	.0348	.0406	.0465	.0523	-.0419	-.0366	-.0313	-.0260	-.0207	-.0153	-.0100	-.0047	.0006	.0059	.0112
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0051	-.0102	-.0153	-.0204	-.0255	-.0306	-.0357	-.0408	-.0459	-.0510	.0603	.0516	.0428	.0341	.0254	.0167	.0080	-.0008	-.0095	-.0182
	.2	0	-.0084	-.0167	-.0251	-.0334	-.0418	-.0501	-.0585	-.0668	-.0752	-.0835	.0169	.1173	.0977	.0781	.0585	.0390	.0194	-.0002	-.0198	-.0394
	.3	0	-.0100	-.0200	-.0301	-.0401	-.0501	-.0601	-.0701	-.0802	-.0902	-.1002	-.0123	.0756	.1636	.1315	.0994	.0673	.0352	.0032	-.0289	-.0610
	.4	0	-.0104	-.0208	-.0311	-.0415	-.0519	-.0623	-.0726	-.0830	-.0934	-.1038	-.0294	.0449	.1193	.1936	.1480	.1023	.0566	.0110	-.0347	-.0803
	.5	0	-.0097	-.0194	-.0291	-.0388	-.0484	-.0581	-.0678	-.0775	-.0872	-.0969	-.0367	.0235	.0838	.1440	.2042	.1444	.0846	.0248	-.0350	-.0947
	.6	0	-.0082	-.0165	-.0247	-.0329	-.0411	-.0494	-.0576	-.0658	-.0740	-.0823	-.0362	.0099	.0559	.1020	.1481	.1942	.1202	.0463	-.0276	-.1016
	.7	0	-.0063	-.0125	-.0188	-.0250	-.0313	-.0375	-.0438	-.0501	-.0563	-.0626	-.0301	.0023	.0348	.0672	.0996	.1321	.1645	.0770	-.0106	-.0981
	.8	0	-.0041	-.0081	-.0122	-.0162	-.0203	-.0243	-.0284	-.0324	-.0365	-.0405	-.0206	-.0008	.0191	.0390	.0588	.0787	.0985	.1184	.0183	-.0819
	.9	0	-.0019	-.0038	-.0056	-.0075	-.0094	-.0113	-.0131	-.0150	-.0169	-.0188	-.0099	-.0010	.0079	.0167	.0256	.0345	.0434	.0522	.0611	-.0500
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0014	.0027	.0041	.0055	.0068	.0082	.0095	.0109	.0123	.0136	.0073	.0009	-.0055	-.0118	-.0182	-.0246	-.0309	-.0373	-.0437	-.0500
	.2	0	.0022	.0045	.0067	.0089	.0112	.0134	.0156	.0178	.0201	.0223	.0119	.0015	-.0089	-.0193	-.0298	-.0402	-.0506	-.0610	-.0714	-.0819
	.3	0	.0027	.0054	.0080	.0107	.0134	.0161	.0187	.0214	.0241	.0268	.0143	.0018	-.0107	-.0232	-.0357	-.0482	-.0607	-.0732	-.0857	-.0981
	.4	0	.0028	.0055	.0083	.0111	.0138	.0166	.0194	.0222	.0249	.0277	.0148	.0018	-.0111	-.0240	-.0369	-.0499	-.0628	-.0757	-.0886	-.1016
	.5	0	.0026	.0052	.0078	.0103	.0129	.0155	.0181	.0207	.0233	.0258	.0138	.0017	-.0103	-.0224	-.0345	-.0465	-.0586	-.0706	-.0827	-.0947
	.6	0	.0022	.0044	.0066	.0088	.0110	.0131	.0153	.0175	.0197	.0219	.0117	.0015	-.0088	-.0190	-.0292	-.0394	-.0497	-.0599	-.0701	-.0803
	.7	0	.0017	.0033	.0050	.0067	.0083	.0100	.0116	.0133	.0150	.0166	.0089	.0011	-.0067	-.0144	-.0222	-.0300	-.0377	-.0455	-.0532	-.0610
	.8	0	.0011	.0021	.0032	.0043	.0054	.0064	.0075	.0086	.0097	.0107	.0057	.0007	-.0043	-.0093	-.0143	-.0193	-.0244	-.0294	-.0344	-.0394
	.9	0	.0005	.0010	.0015	.0020	.0025	.0030	.0035	.0040	.0045	.0050	.0026	.0003	-.0020	-.0043	-.0066	-.0089	-.0112	-.0136	-.0159	-.0182
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	0	-.0003	-.0006	-.0009	-.0012	-.0015	-.0018	-.0021	-.0025	-.0028	-.0031	-.0016	-.0002	.0012	.0027	.0041	.0055	.0070	.0084	.0098	.0112
	.2	0	-.0005	-.0010	-.0016	-.0021	-.0026	-.0031	-.0036	-.0041	-.0047	-.0052	-.0028	-.0003	.0021	.0045	.0069	.0093	.0117	.0141	.0165	.0189
	.3	0	-.0006	-.0013	-.0019	-.0026	-.0032	-.0038	-.0045	-.0051	-.0058	-.0064	-.0034	-.0004	.0026	.0056	.0085	.0115	.0145	.0175	.0205	.0235
	.4	0	-.0007	-.0014	-.0021	-.0028	-.0034	-.0041	-.0048	-.0055	-.0062	-.0069	-.0037	-.0005	.0028	.0060	.0092	.0124	.0156	.0188	.0220	.0253
	.5	0	-.0007	-.0013	-.0020	-.0027	-.0034	-.0040	-.0047	-.0054	-.0061	-.0067	-.0036	-.0004	.0027	.0058	.0090	.0121	.0153	.0184	.0215	.0247
	.6	0	-.0006	-.0012	-.0018	-.0024	-.0030	-.0036	-.0042	-.0048	-.0054	-.0060	-.0032	-.0004	.0024	.0052	.0080	.0109	.0137	.0165	.0193	.0221
	.7	0	-.0005	-.0010	-.0015	-.0020	-.0024	-.0029	-.0034	-.0039	-.0044	-.0049	-.0026	-.0003	.0020	.0042	.0065	.0088	.0111	.0134	.0157	.0180
	.8	0	-.0003	-.0007	-.0010	-.0014	-.0017	-.0021	-.0024	-.0028	-.0031	-.0034	-.0018	-.0002	.0014	.0030	.0046	.0062	.0078	.0094	.0110	.0126
	.9	0	-.0002	-.0004	-.0005	-.0007	-.0009	-.0011	-.0012	-.0014	-.0016	-.0018	-.0009	-.0001	.0007	.0015	.0024	.0032	.0040	.0049	.0057	.0065
	E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0409	.0719	.0928	.1037	.1047	.0956	.0766	.0475	.0249	.0207	.0195	.0396	.0760	.0999	.1093	.1044	.0850	.0523	.0317	.0329
– Area		0	-.0082	-.0164	-.0246	-.0328	-.0410	-.0492	-.0574	-.0656	-.0903	-.1433	-.0769	-.0463	-.0462	-.0481	-.0500	-.0519	-.0537	-.0566	-.0860	-.1516
Total Area		0	.0327	.0555	.0682	.0709	.0637	.0464	.0192	-.0181	-.0654	-.1226	-.0574	-.0067	.0298	.0518	.0593	.0525	.0313	-.0043	-.0543	-.1187

Unit load at		REACTIONS/P					SHEARS/P							
		R_A	R_B	R_C	R_D	R_E	V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}	V_{DE}	V_{ED}
SPAN 1	A	1.0	0	0	0	0	1.0	0	0	0	0	0	0	0
	.1	.8757	.1499	-.0326	.0087	-.0018	.8757	-.1243	.0257	.0257	-.0069	-.0069	.0018	.0018
	.2	.7529	.2968	-.0632	.0169	-.0034	.7529	-.2471	.0498	.0498	-.0134	-.0134	.0034	.0034
	.3	.6331	.4377	-.0898	.0239	-.0049	.6331	-.3669	.0708	.0708	-.0190	-.0190	.0049	.0049
	.4	.5176	.5695	-.1105	.0295	-.0060	.5176	-.4824	.0871	.0871	-.0234	-.0234	.0060	.0060
	.5	.4080	.6891	-.1234	.0329	-.0067	.4080	-.5920	.0972	.0972	-.0262	-.0262	.0067	.0067
	.6	.3058	.7937	-.1263	.0337	-.0069	.3058	-.6942	.0995	.0995	-.0268	-.0268	.0069	.0069
	.7	.2125	.8801	-.1174	.0313	-.0064	.2125	-.7875	.0925	.0925	-.0249	-.0249	.0064	.0064
	.8	.1294	.9452	-.0947	.0253	-.0052	.1294	-.8706	.0746	.0746	-.0201	-.0201	.0052	.0052
	.9	.0581	.9863	-.0543	.0150	-.0031	.0581	-.9419	.0443	.0443	-.0119	-.0119	.0031	.0031
	B	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0	0	0
SPAN 2	.1	-.0510	.9783	.0919	-.0242	.0050	-.0510	-.0510	.9273	-.0727	.0193	.0193	-.0050	-.0050
	.2	-.0835	.9202	.2051	-.0525	.0107	-.0835	-.0835	.8367	-.1633	.0418	.0418	-.0107	-.0107
	.3	-.1002	.8329	.3320	-.0813	.0166	-.1002	-.1002	.7327	-.2673	.0647	.0647	-.0166	-.0166
	.4	-.1038	.7233	.4657	-.1071	.0219	-.1038	-.1038	.6195	-.3805	.0852	.0852	-.0219	-.0219
	.5	-.0969	.5987	.5987	-.1263	.0258	-.0969	-.0969	.5018	-.4982	.1005	.1005	-.0258	-.0258
	.6	-.0823	.4662	.7238	-.1354	.0277	-.0823	-.0823	.3839	-.6161	.1077	.1077	-.0277	-.0277
	.7	-.0626	.3329	.8337	-.1309	.0268	-.0626	-.0626	.2704	-.7296	.1041	.1041	-.0268	-.0268
	.8	-.0405	.2061	.9213	-.1091	.0223	-.0405	-.0405	.1656	-.8344	.0868	.0868	-.0223	-.0223
	.9	-.0188	.0927	.9791	-.0667	.0136	-.0188	-.0188	.0739	-.9261	.0531	.0531	-.0136	-.0136
	C	0	0	1.0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 3	.1	.0136	-.0667	.9791	.0927	-.0188	.0136	.0136	-.0531	-.0531	.9261	-.0739	.0188	.0188
	.2	.0223	-.1091	.9213	.2061	-.0405	.0223	.0223	-.0868	-.0868	.8344	-.1656	.0405	.0405
	.3	.0268	-.1309	.8337	.3329	-.0626	.0268	.0268	-.1041	-.1041	.7296	-.2704	.0626	.0626
	.4	.0277	-.1354	.7238	.4662	-.0823	.0277	.0277	-.1077	-.1077	.6161	-.3839	.0823	.0823
	.5	.0258	-.1263	.5987	.5987	-.0969	.0258	.0258	-.1005	-.1005	.4982	-.5018	.0969	.0969
	.6	.0219	-.1071	.4657	.7233	-.1038	.0219	.0219	-.0852	-.0852	.3805	-.6195	.1038	.1038
	.7	.0166	-.0813	.3320	.8329	-.1002	.0166	.0166	-.0647	-.0647	.2673	-.7327	.1002	.1002
	.8	.0107	-.0525	.2051	.9202	-.0835	.0107	.0107	-.0418	-.0418	.1633	-.8367	.0835	.0835
	.9	.0050	-.0242	.0919	.9783	-.0510	.0050	.0050	-.0193	-.0193	.0727	-.9273	.0510	.0510
	D	0	0	0	1.0	0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 4	.1	-.0031	.0150	-.0563	.9863	.0581	-.0031	-.0031	.0119	.0119	-.0443	-.0443	.9419	-.0581
	.2	-.0052	.0253	-.0947	.9453	.1294	-.0052	-.0052	.0201	.0201	-.0746	-.0746	.8706	-.1294
	.3	-.0064	.0313	-.1174	.8801	.2125	-.0064	-.0064	.0249	.0249	-.0925	-.0925	.7875	-.2125
	.4	-.0069	.0337	-.1263	.7937	.3058	-.0069	-.0069	.0268	.0268	-.0995	-.0995	.6942	-.3058
	.5	-.0067	.0329	-.1234	.6891	.4080	-.0067	-.0067	.0262	.0262	-.0972	-.0972	.5920	-.4080
	.6	-.0060	.0295	-.1105	.5695	.5176	-.0060	-.0060	.0234	.0234	-.0871	-.0871	.4824	-.5176
	.7	-.0049	.0239	-.0898	.4377	.6331	-.0049	-.0049	.0190	.0190	-.0708	-.0708	.3669	-.6331
	.8	-.0034	.0169	-.0632	.2968	.7529	-.0034	-.0034	.0134	.0134	-.0498	-.0498	.2471	-.7529
	.9	-.0018	.0087	-.0326	.1499	.8757	-.0018	-.0018	.0069	.0069	-.0257	-.0257	.1243	-.8757
	E	0	0	0	0	1.0	0	0	0	0	0	0	0	-1.0
+ Area		.4594	1.3270	1.3579	1.3270	.4594	.4594	.0207	.6837	.0822	.6789	.0804	.6433	.0820
– Area		-.0820	-.1011	-.1643	-.1011	-.0820	-.0820	-.6433	-.0804	-.6789	-.0822	-.6837	-.0207	-.4594
Total Area		.3774	1.2259	1.1934	1.2259	.3774	.3774	-.6226	.6033	-.5967	.5967	-.6033	.6226	-.3774



Influence coefficients — Four continuous spans.

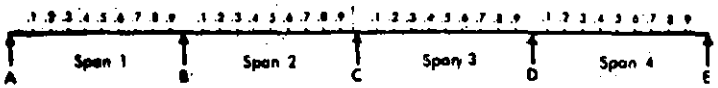
L = Length of EXTERIOR spans; length of interior spans = NL

N = 1.2

Table 20.40

Unit load at		MOMENTS/PL: SPAN 1: A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	SPAN 2: .1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0877	.0753	.0630	.0507	.0384	.0260	.0137	.0014	-.0110	-.0233	-.0203	-.0174	-.0144	-.0115	-.0085	-.0056	-.0026	.0004	.0033	.0063
	.2	0	.0755	.1510	.1264	.1019	.0774	.0529	.0284	.0039	-.0207	-.0452	-.0394	-.0337	-.0280	-.0222	-.0165	-.0108	-.0050	.0007	.0064	.0122
	.3	0	.0638	.1272	.1907	.1543	.1178	.0815	.0450	.0086	-.0278	-.0642	-.0561	-.0479	-.0398	-.0316	-.0235	-.0153	-.0072	.0010	.0091	.0173
	.4	0	.0521	.1042	.1563	.2084	.1605	.1126	.0647	.0168	-.0311	-.0791	-.0690	-.0590	-.0490	-.0389	-.0289	-.0189	-.0088	.0012	.0112	.0213
	.5	0	.0412	.0824	.1235	.1647	.2059	.1471	.0882	.0294	-.0294	-.0882	-.0770	-.0658	-.0546	-.0434	-.0322	-.0211	-.0099	.0013	.0125	.0237
	.6	0	.0310	.0619	.0929	.1239	.1548	.1858	.1168	.0477	-.0213	-.0903	-.0788	-.0674	-.0560	-.0445	-.0330	-.0216	-.0101	.0014	.0128	.0243
	.7	0	.0216	.0432	.0648	.0864	.1080	.1296	.1512	.0728	-.0056	-.0840	-.0733	-.0627	-.0520	-.0414	-.0307	-.0200	-.0094	.0013	.0119	.0226
	.8	0	.0132	.0264	.0397	.0529	.0661	.0793	.0926	.1058	.0190	-.0678	-.0592	-.0506	-.0420	-.0334	-.0248	-.0162	-.0076	.0010	.0096	.0182
	.9	0	.0060	.0120	.0179	.0239	.0299	.0359	.0418	.0478	.0538	-.0402	-.0351	-.0300	-.0248	-.0198	-.0147	-.0096	-.0045	.0006	.0057	.0108
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0057	-.0115	-.0172	-.0230	-.0287	-.0344	-.0402	-.0459	-.0517	-.0574	.0634	.0542	.0451	.0359	.0267	.0175	.0084	-.0008	-.0100	-.0192
	.2	0	-.0094	-.0188	-.0282	-.0376	-.0470	-.0564	-.0658	-.0752	-.0846	-.0940	.0152	.1244	.1037	.0829	.0621	.0413	.0205	-.0003	-.0211	-.0419
	.3	0	-.0113	-.0225	-.0338	-.0451	-.0564	-.0676	-.0789	-.0902	-.1015	-.1127	-.0170	.0788	.1745	.1403	.1061	.0718	.0376	.0033	-.0309	-.0651
	.4	0	-.0117	-.0233	-.0350	-.0467	-.0584	-.0700	-.0817	-.0934	-.1051	-.1167	-.0357	.0454	.1264	.2075	.1586	.1096	.0607	.0118	-.0372	-.0861
	.5	0	-.0109	-.0218	-.0327	-.0436	-.0545	-.0654	-.0763	-.0872	-.0981	-.1090	-.0433	.0224	.0882	.1539	.2196	.1553	.0910	.0267	-.0375	-.1018
	.6	0	-.0093	-.0185	-.0278	-.0370	-.0463	-.0555	-.0648	-.0740	-.0833	-.0925	-.0422	.0081	.0584	.1087	.1590	.2094	.1297	.0500	-.0297	-.1094
	.7	0	-.0070	-.0141	-.0211	-.0281	-.0352	-.0422	-.0493	-.0563	-.0633	-.0704	-.0349	.0005	.0360	.0714	.1069	.1423	.1777	.0832	-.0114	-.1059
	.8	0	-.0046	-.0091	-.0137	-.0182	-.0228	-.0273	-.0319	-.0364	-.0410	-.0455	-.0238	-.0021	.0196	.0413	.0630	.0847	.1064	.1281	.0198	-.0885
	.9	0	-.0021	-.0042	-.0063	-.0084	-.0105	-.0126	-.0148	-.0169	-.0190	-.0211	-.0114	-.0017	.0080	.0177	.0274	.0371	.0468	.0565	.0662	-.0541
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0015	.0031	.0046	.0061	.0076	.0092	.0107	.0122	.0138	.0153	.0084	.0014	-.0055	-.0125	-.0194	-.0264	-.0333	-.0402	-.0472	-.0541
	.2	0	.0025	.0050	.0075	.0100	.0125	.0150	.0175	.0200	.0225	.0250	.0137	.0023	-.0090	-.0204	-.0317	-.0431	-.0544	-.0658	-.0771	-.0885
	.3	0	.0030	.0060	.0090	.0120	.0150	.0180	.0210	.0239	.0269	.0299	.0163	.0228	-.0108	-.0244	-.0380	-.0516	-.0652	-.0787	-.0923	-.1059
	.4	0	.0031	.0062	.0093	.0124	.0155	.0186	.0216	.0247	.0278	.0309	.0169	.0028	-.0112	-.0252	-.0392	-.0533	-.0673	-.0813	-.0954	-.1094
	.5	0	.0029	.0058	.0086	.0115	.0144	.0173	.0201	.0230	.0259	.0288	.0157	.0027	-.0104	-.0235	-.0365	-.0496	-.0626	-.0757	-.0888	-.1018
	.6	0	.0024	.0049	.0073	.0097	.0122	.0146	.0170	.0195	.0219	.0243	.0133	.0022	-.0088	-.0198	-.0309	-.0419	-.0530	-.0640	-.0751	-.0861
	.7	0	.0018	.0037	.0055	.0074	.0092	.0110	.0129	.0147	.0166	.0184	.0101	.0017	-.0067	-.0150	-.0234	-.0317	-.0401	-.0484	-.0568	-.0651
	.8	0	.0012	.0024	.0035	.0047	.0059	.0071	.0083	.0095	.0106	.0118	.0065	.0011	-.0043	-.0096	-.0150	-.0204	-.0258	-.0311	-.0365	-.0419
	.9	0	.0005	.0011	.0016	.0022	.0027	.0033	.0038	.0043	.0049	.0054	.0030	.0005	-.0020	-.0044	-.0069	-.0093	-.0118	-.0143	-.0167	-.0192
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	0	-.0003	-.0006	-.0009	-.0012	-.0015	-.0018	-.0021	-.0024	-.0028	-.0031	-.0017	-.0003	.0011	.0025	.0039	.0053	.0067	.0080	.0094	.0108
	.2	0	-.0005	-.0010	-.0015	-.0021	-.0026	-.0031	-.0036	-.0041	-.0046	-.0052	-.0028	-.0005	.0019	.0042	.0065	.0089	.0112	.0136	.0159	.0182
	.3	0	-.0008	-.0013	-.0019	-.0026	-.0032	-.0038	-.0045	-.0051	-.0057	-.0064	-.0035	-.0006	.0023	.0052	.0081	.0110	.0139	.0168	.0197	.0226
	.4	0	-.0007	-.0014	-.0021	-.0027	-.0034	-.0041	-.0048	-.0055	-.0062	-.0069	-.0038	-.0006	.0025	.0056	.0087	.0118	.0150	.0181	.0212	.0243
	.5	0	-.0007	-.0013	-.0020	-.0027	-.0034	-.0040	-.0047	-.0054	-.0060	-.0067	-.0037	-.0006	.0024	.0055	.0085	.0116	.0146	.0176	.0207	.0237
	.6	0	-.0006	-.0012	-.0018	-.0024	-.0030	-.0036	-.0042	-.0048	-.0054	-.0060	-.0033	-.0006	.0022	.0049	.0076	.0104	.0131	.0158	.0185	.0213
	.7	0	-.0005	-.0010	-.0015	-.0020	-.0024	-.0029	-.0034	-.0039	-.0044	-.0049	-.0027	-.0005	.0018	.0040	.0062	.0084	.0106	.0128	.0151	.0173
	.8	0	-.0003	-.0007	-.0010	-.0014	-.0017	-.0021	-.0024	-.0027	-.0031	-.0034	-.0019	-.0003	.0012	.0028	.0044	.0059	.0075	.0090	.0106	.0122
	.9	0	-.0002	-.0004	-.0005	-.0007	-.0009	-.0011	-.0012	-.0014	-.0016	-.0018	-.0010	-.0002	.0006	.0014	.0022	.0031	.0039	.0047	.0055	.0063
	E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0416	.0732	.0948	.1065	.1081	.0997	.0813	.0529	.0293	.0249	.0329	.0448	.0865	.1145	.1256	.1198	.0971	.0585	.0324	.0316
− Area		0	-.0099	-.0198	-.0297	-.0396	-.0495	-.0594	-.0693	-.0791	-.1038	-.1577	-.0808	-.0448	-.0455	-.0493	-.0532	-.0570	-.0609	-.0657	-.1000	-.1765
Total Area		0	.0317	.0534	.0651	.0669	.0586	.0403	.0120	-.0262	-.0745	-.1328	-.0579	.0000	-.0410	.0652	.0724	.0628	.0362	-.0072	-.0676	-.1449

Unit load at		REACTIONS/P: R_A	R_B	R_C	R_D	R_E	SHEARS/P: V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}	V_{DE}	V_{ED}
SPAN 1	A	1.0	0	0	0	0	1.0	0	0	0	0	0	0	0
	.1	.8767	.1460	-.0289	.0079	-.0018	.8767	-.1233	.0227	.0227	-.0062	-.0062	.0018	.0018
	.2	.7548	.2893	-.0561	.0134	-.0034	.7548	-.2452	.0441	.0441	-.0120	-.0120	.0034	.0034
	.3	.6358	.4269	-.0797	.0219	-.0049	.6358	-.3642	.0627	.0627	-.0170	-.0170	.0049	.0049
	.4	.5209	.5562	-.0981	.0270	-.0060	.5209	-.4791	.0772	.0772	-.0210	-.0210	.0060	.0060
	.5	.4118	.6744	-.1095	.0301	-.0067	.4118	-.5882	.0861	.0861	-.0234	-.0234	.0067	.0067
	.6	.3097	.7785	-.1122	.0308	-.0069	.3097	-.6903	.0882	.0882	-.0240	-.0240	.0069	.0069
	.7	.2160	.8660	-.1043	.0287	-.0064	.2160	-.7840	.0820	.0820	-.0223	-.0223	.0064	.0064
	.8	.1322	.9339	-.0841	.0231	-.0052	.1322	-.8678	.0661	.0661	-.0180	-.0180	.0052	.0052
	.9	.0598	.9795	-.0500	.0137	-.0031	.0598	-.9402	.0393	.0393	-.0107	-.0107	.0031	.0031
	B	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0	0	0
SPAN 2	.1	-.0574	.9868	.0895	-.0243	.0054	-.0574	-.0574	.9294	-.0706	.0189	.0189	-.0054	-.0054
	.2	-.0940	.9341	.2012	-.0531	.0118	-.0940	-.0940	.8401	-.1599	.0413	.0413	-.0118	-.0118
	.3	-.1127	.8494	.3276	-.0827	.0184	-.1127	-.1127	.7366	-.2634	.0643	.0643	-.0184	-.0184
	.4	-.1167	.7403	.4614	-.1093	.0243	-.1167	-.1167	.6236	-.3764	.0849	.0849	-.0243	-.0243
	.5	-.1090	.6145	.5949	-.1292	.0288	-.1090	-.1090	.5055	-.4945	.1005	.1005	-.0288	-.0288
	.6	-.0925	.4795	.7209	-.1389	.0309	-.0925	-.0925	.3870	-.6130	.1079	.1079	-.0309	-.0309
	.7	-.0704	.3430	.8318	-.1344	.0299	-.0704	-.0704	.2727	-.7273	.1045	.1045	-.0299	-.0299
	.8	-.0455	.2125	.9203	-.1123	.0250	-.0455	-.0455	.1670	-.8330	.0873	.0873	-.0250	-.0250
	.9	-.0211	.0956	.9788	-.0687	.0153	-.0211	-.0211	.0746	-.9254	.0534	.0534	-.0153	-.0153
	C	0	0	1.0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 3	.1	.0153	-.0687	.9788	.0956	-.0211	.0153	.0153	-.0534	-.0534	.9254	-.0746	.0211	.0211
	.2	.0250	-.1123	.9203	.2125	-.0455	.0250	.0250	-.0873	-.0873	.8330	-.1670	.0455	.0455
	.3	.0299	-.1344	.8318	.3430	-.0704	.0299	.0299	-.1045	-.1045	.7273	-.2727	.0704	.0704
	.4	.0309	-.1389	.7209	.4795	-.0925	.0309	.0309	-.1079	-.1079	.6130	-.3870	.0925	.0925
	.5	.0288	-.1292	.5949	.6145	-.1090	.0288	.0288	-.1005	-.1005	.4945	-.5055	.1090	.1090
	.6	.0243	-.1093	.4614	.7403	-.1167	.0243	.0243	-.0849	-.0849	.3764	-.6236	.1167	.1167
	.7	.0184	-.0827	.3276	.8494	-.1127	.0184	.0184	-.0643	-.0643	.2634	-.7366	.1127	.1127
	.8	.0118	-.0531	.2012	.9341	-.0940	.0118	.0118	-.0413	-.0413	.1599	-.8401	.0940	.0940
	.9	.0054	-.0243	.0895	.9868	-.0574	.0054	.0054	-.0189	-.0189	.0706	-.9294	.0574	.0574
	D	0	0	0	1.0	0	00	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 4	.1	-.0031	.0137	-.0500	.9795	.0598	-.0031	-.0031	.0107	.0107	-.0393	-.0393	.9402	-.0598
	.2	-.0052	.0231	-.0841	.9339	.1322	-.0052	-.0052	.0150	.0180	-.0661	-.0661	.8678	-.1322
	.3	-.0064	.0287	-.1043	.8660	.2160	-.0064	-.0064	.0223	.0223	-.0820	-.0820	.7840	-.2160
	.4	-.0069	.0308	-.1122	.7785	.3097	-.0069	-.0069	.0240	.0240	-.0882	-.0882	.6903	-.3097
	.5	-.0067	.0301	-.1095	.6744	.4118	-.0067	-.0067	.0234	.0234	-.0861	-.0861	.5882	-.4118
	.6	-.0060	.0270	-.0981	.5562	.5209	-.0060	-.0060	.0210	.0210	-.0772	-.0772	.4791	-.5209
	.7	-.0049	.0219	-.0797	.4269	.6358	-.0049	-.0049	.0170	.0170	-.0627	-.0627	.3642	-.6358
	.8	-.0034	.0154	-.0561	.2893	.7548	-.0034	-.0034	.0120	.0120	-.0441	-.0441	.2452	-.7548
	.9	-.0018	.0079	-.0289	.1460	.8767	-.0018	-.0018	.0062	.0062	-.0227	-.0227	.1233	-.8767
	E	0	0	0	0	1.0	0	0	0	0	0	0	0	-1.0
+ Area		.4661	1.3856	1.4646	1.3856	.4661	.4661	.0249	.7276	.0730	.7323	.0871	.8577	.0989
− Area		-.0989	-.1120	-.1461	-.1120	-.0989	-.0989	-.8577	-.0871	-.7323	-.0730	-.7278	-.0249	-.4661
Total Area		.3672	1.2736	1.3185	1.2736	.3672	.3672	-.8328	.6407	-.6593	.6593	-.6407	.8328	-.3672



Influence coefficients — Four continuous spans.

L = Length of EXTERIOR spans; length of interior spans = NL

N = 1.3

Table 20.41

MOMENTS/PL — SPAN 1

	Unit load at	A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0878	.0755	.0633	.0510	.0388	.0266	.0143	.0021	-.0101	-.0224
	.2	0	.0757	.1513	.1270	.1026	.0783	.0540	.0296	.0053	-.0191	-.0434
	.3	0	.0638	.1277	.1915	.1553	.1191	.0830	.0468	.0106	-.0256	-.0617
	.4	0	.0524	.1048	.1572	.2096	.1620	.1144	.0668	.0192	-.0284	-.0760
	.5	0	.0415	.0830	.1246	.1661	.2076	.1491	.0906	.0322	-.0263	-.0848
	.6	0	.0313	.0626	.0940	.1253	.1566	.1879	.1192	.0505	-.0181	-.0868
	.7	0	.0219	.0438	.0658	.0877	.1096	.1316	.1535	.0754	-.0027	-.0807
	.8	0	.0135	.0270	.0405	.0540	.0674	.0809	.0944	.1079	.0214	-.0651
	.9	0	.0061	.0123	.0184	.0245	.0307	.0368	.0429	.0491	.0552	-.0367
	B	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0064	-.0128	-.0192	-.0256	-.0320	-.0384	-.0448	-.0512	-.0576	-.0640
	.2	0	-.0105	-.0209	-.0314	-.0419	-.0523	-.0628	-.0733	-.0838	-.0942	-.1047
	.3	0	-.0126	-.0251	-.0377	-.0502	-.0628	-.0754	-.0879	-.1005	-.1130	-.1256
	.4	0	-.0130	-.0260	-.0390	-.0520	-.0650	-.0780	-.0910	-.1040	-.1170	-.1300
	.5	0	-.0121	-.0243	-.0364	-.0486	-.0607	-.0728	-.0850	-.0971	-.1092	-.1214
	.6	0	-.0103	-.0206	-.0309	-.0412	-.0515	-.0618	-.0721	-.0824	-.0927	-.1030
	.7	0	-.0078	-.0157	-.0235	-.0313	-.0392	-.0470	-.0548	-.0627	-.0705	-.0783
	.8	0	-.0051	-.0101	-.0152	-.0203	-.0253	-.0304	-.0355	-.0405	-.0456	-.0507
	.9	0	-.0023	-.0047	-.0070	-.0094	-.0117	-.0141	-.0164	-.0188	-.0211	-.0234
	C	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0017	.0034	.0051	.0068	.0085	.0102	.0119	.0136	.0153	.0170
	.2	0	.0028	.0055	.0083	.0111	.0139	.0166	.0194	.0222	.0250	.0277
	.3	0	.0033	.0066	.0099	.0133	.0166	.0199	.0232	.0265	.2098	.0331
	.4	0	.0034	.0068	.0103	.0137	.0171	.0205	.0239	.0273	.0308	.0342
	.5	0	.0032	.0063	.0095	.0127	.0159	.0190	.0222	.0254	.0286	.0317
	.6	0	.0027	.0054	.0080	.0107	.0134	.0161	.0187	.0214	.0241	.0268
	.7	0	.0020	.0040	.0061	.0081	.0101	.0121	.0141	.0161	.0182	.0202
	.8	0	.0013	.0026	.0039	.0052	.0065	.0077	.0090	.0103	.0116	.0129
	.9	0	.0006	.0012	.0018	.0023	.0029	.0035	.0041	.0047	.0053	.0059
	D	0	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	0	-.0003	-.0006	-.0009	-.0012	-.0015	-.0018	-.0021	-.0024	-.0027	-.0030
	.2	0	-.0005	-.0010	-.0015	-.0020	-.0026	-.0031	-.0036	-.0041	-.0046	-.0051
	.3	0	-.0006	-.0013	-.0019	-.0025	-.0032	-.0038	-.0044	-.0051	-.0057	-.0063
	.4	0	-.0007	-.0014	-.0020	-.0027	-.0034	-.0041	-.0048	-.0055	-.0061	-.0068
	.5	0	-.0007	-.0013	-.0020	-.0027	-.0033	-.0040	-.0047	-.0053	-.0060	-.0067
	.6	0	-.0006	-.0012	-.0018	-.0024	-.0030	-.0036	-.0042	-.0048	-.0054	-.0060
	.7	0	-.0005	-.0010	-.0015	-.0019	-.0024	-.0029	-.0034	-.0039	-.0044	-.0049
	.8	0	-.0003	-.0007	-.0010	-.0014	-.0017	-.0020	-.0024	-.0027	-.0031	-.0034
	.9	0	-.0002	-.0004	-.0005	-.0007	-.0009	-.0011	-.0012	-.0014	-.0016	-.0018
	E	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0423	.0746	.0969	.1092	.1116	.1039	.0862	.0585	.0340	.0298
− Area		0	-.0118	-.0235	-.0353	-.0471	-.0589	-.0707	-.0824	-.0942	-.1191	-.1742
Total Area		0	.0305	.0511	.0616	.0621	.0527	.0332	.0038	-.0357	-.0851	-.1446

MOMENTS/PL — SPAN 2

	Unit load at	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0
	.1	-.0195	-.0167	-.0139	-.0110	-.0082	-.0053	-.0025	.0004	.0032	.0060
	.2	-.0379	-.0324	-.0269	-.0214	-.0159	-.0103	-.0048	.0007	.0062	.0117
	.3	-.0539	-.0461	-.0382	-.0304	-.0225	-.0147	-.0069	.0010	.0088	.0166
	.4	-.0663	-.0567	-.0470	-.0374	-.0277	-.0181	-.0085	.0012	.0108	.0205
	.5	-.0740	-.0633	-.0525	-.0417	-.0310	-.0202	-.0094	.0013	.0121	.0229
	.6	-.0758	-.0648	-.0538	-.0427	-.0317	-.0207	-.0097	.0014	.0124	.0234
	.7	-.0705	-.0602	-.0500	-.0397	-.0295	-.0192	-.0090	.0013	.0115	.0218
	.8	-.0569	-.0486	-.0403	-.0321	-.0238	-.0155	-.0072	.0010	.0093	.0176
	.9	-.0338	-.0288	-.0239	-.0190	-.0141	-.0092	-.0043	.0006	.0055	.0104
	B	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	.0664	.0568	.0472	.0376	.0280	.0183	.0087	-.0009	-.0105	-.0201
	.2	.0134	.1314	.1094	.0875	.0655	.0436	.0216	-.0003	-.0223	-.0443
	.3	-.0220	.0817	.1853	.1490	.1126	.0762	.0399	.0035	-.0328	-.0692
	.4	-.0422	.0456	.1334	.2213	.1691	.1169	.0647	.0126	-.0396	-.0918
	.5	-.0501	.0211	.0924	.1636	.2349	.1661	.0974	.0287	-.0401	-.1088
	.6	-.0484	.0061	.0607	.1153	.1699	.2245	.1391	.0536	-.0318	-.1172
	.7	-.0399	-.0014	.0371	.0755	.1140	.1525	.1909	.0894	-.0121	-.1137
	.8	-.0271	-.0036	.0200	.0436	.0671	.0907	.1142	.1378	.0214	-.0951
	.9	-.0129	-.0024	.0081	.0186	.0292	.0397	.0502	.0607	.0712	-.0582
	C	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	.0095	.0019	-.0056	-.0131	-.0206	-.0281	-.0357	-.0432	-.0507	-.0582
	.2	.0154	.0032	-.0091	-.0214	-.0337	-.0459	-.0582	-.0705	-.0828	-.0951
	.3	.0185	.0038	-.0109	-.0256	-.0403	-.0549	-.0696	-.0843	-.0990	-.1137
	.4	.0190	.0039	-.0112	-.0264	-.0415	-.0566	-.0718	-.0869	-.1021	-.1172
	.5	.0177	.0036	-.0104	-.0245	-.0385	-.0526	-.0667	-.0807	-.0948	-.1088
	.6	.0149	.0031	-.0088	-.0207	-.0325	-.0444	-.0562	-.0681	-.0799	-.0918
	.7	.0112	.0023	-.0066	-.0156	-.0245	-.0334	-.0424	-.0513	-.0603	-.0692
	.8	.0072	.0015	-.0042	-.0100	-.0157	-.0214	-.0271	-.0328	-.0385	-.0443
	.9	.0033	.0007	-.0019	-.0045	-.0071	-.0097	-.0123	-.0149	-.0175	-.0201
	D	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	-.0017	-.0003	.0010	.0023	.0037	.0050	.0064	.0077	.0091	.0104
	.2	-.0029	-.0006	.0017	.0040	.0062	.0085	.0108	.0130	.0153	.0176
	.3	-.0035	-.0007	.0021	.0049	.0077	.0105	.0133	.0161	.0190	.0218
	.4	-.0038	-.0008	.0022	.0053	.0083	.0113	.0143	.0174	.0204	.0234
	.5	-.0037	-.0008	.0022	.0051	.0081	.0111	.0140	.0170	.0199	.0229
	.6	-.0033	-.0007	.0020	.0046	.0073	.0099	.0125	.0152	.0178	.0205
	.7	-.0027	-.0006	.0016	.0037	.0059	.0080	.0102	.0123	.0145	.0166
	.8	-.0019	-.0004	.0011	.0026	.0041	.0057	.0072	.0087	.0102	.0117
	.9	-.0010	-.0002	.0006	.0014	.0021	.0028	.0037	.0045	.0053	.0060
	E	0	0	0	0	0	0	0	0	0	0
+ Area		.0266	.0503	.0975	.1300	.1430	.1363	.1100	.0652	.0332	.0305
− Area		-.0858	-.0437	-.0447	-.0507	-.0566	-.0626	-.0685	-.0755	-.1149	-.2032
Total Area		-.0592	.0066	.0528	.0793	.0864	.0737	.0415	-.0103	-.0817	-.1727

REACTIONS/P and SHEARS/P

	Unit load at	R_A	R_B	R_C	R_D	R_E	V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}	V_{DE}	V_{ED}
SPAN 1	A	1.0	0	0	0	0	1.0	0	0	0	0	0	0	0
	.1	.8776	.1427	-.0256	.0073	-.0017	.8776	-.1224	.0203	.0203	-.0058	-.0058	.0017	.0017
	.2	.7566	.2828	-.0503	.0142	-.0034	.7566	-.2434	.0394	.0394	-.0108	-.0108	.0034	.0034
	.3	.6383	.4177	-.0713	.0202	-.0049	.6383	-.3617	.0560	.0560	-.0154	-.0154	.0049	.0049
	.4	.5240	.5449	-.0878	.0249	-.0060	.5240	-.4760	.0689	.0689	-.0189	-.0189	.0060	.0060
	.5	.4152	.6617	-.0980	.0278	-.0067	.4152	-.5848	.0769	.0769	-.0211	-.0211	.0067	.0067
	.6	.3132	.7654	-.1003	.0284	-.0068	.3132	-.6868	.0787	.0787	-.0216	-.0216	.0068	.0068
	.7	.2193	.8539	-.0933	.0264	-.0063	.2193	-.7807	.0732	.0732	-.0201	-.0201	.0063	.0063
	.8	.1349	.9242	-.0753	.0213	-.0051	.1349	-.8651	.0591	.0591	-.0162	-.0162	.0051	.0051
	.9	.0613	.9737	-.0447	.0127	-.0030	.0613	-.9387	.0351	.0351	-.0096	-.0096	.0030	.0030
	B	0	1.0	0	0	0	0	-1.0/0	0/1.0	0	0	0	0	0
SPAN 2	.1	-.0640	.9853	.0273	-.0244	.0059	-.0640	-.0640	.9313	-.0687	.0186	.0186	-.0059	-.0059
	.2	-.1047	.9479	.1977	-.0537	.0129	-.1047	-.1047	.8432	-.1568	.0408	.0408	-.0129	-.0129
	.3	-.1256	.8658	.3236	-.0840	.0202	-.1256	-.1256	.7403	-.2597	.0638	.0638	-.0202	-.0202
	.4	-.1300	.7373	.4574	-.1115	.0268	-.1300	-.1300	.6273	-.3727	.0847	.0847	-.0268	-.0268
	.5	-.1214	.6303	.5915	-.1322	.0317	-.1214	-.1214	.5090	-.4910	.1004	.1004	-.0317	-.0317
	.6	-.1030	.4929	.7182	-.1423	.0342	-.1030	-.1030	.3899	-.6101	.1081	.1081	-.0342	-.0342
	.7	-.0783	.3531	.8301	-.1360	.0331	-.0783	-.0783	.2748	-.7252	.1049	.1049	-.0331	-.0331
	.8	-.0507	.2180	.9194	-.1154	.0277	-.0507	-.0507	.1683	-.8317	.0877	.0877	-.0277	-.0277
	.9	-.0234	.0984	.9786	-.0707	.0170	-.0234	-.0234	.0751	-.9249	.0537	.0537	-.0170	-.0170
	C	0	0	1.0	0	0	0	0	0	-1.0/0	0/1.0	0	0	0
SPAN 3	.1	.0170	-.0707	.9786	.0986	-.0234	.0170	.0170	-.0537	-.0537	.9249	-.0751	.0234	.0234
	.2	.0277	-.1154	.9194	.2190	-.0507	.0277	.0277	-.0877	-.0877	.8317	-.1683	.0507	.0507
	.3	.0331	-.1360	.8301	.3531	-.0783	.0331	.0331	-.1049	-.1049	.7252	-.2748	.0783	.0783
	.4	.0342	-.1423	.7182	.4929	-.1030	.0342	.0342	-.1081	-.1081	.6101	-.3899	.1030	.1030
	.5	.0317	-.1322	.5915	.6303	-.1214	.0317	.0317	-.1004	-.1004	.4910	-.5090	.1214	.1214
	.6	.0268	-.1115	.4574	.7573	-.1300	.0268	.0268	-.0847	-.0847	.3727	-.6273	.1300	.1300
	.7	.0202	-.0840	.3236	.8659	-.1256	.0202	.0202	-.0638	-.0638	.2597	-.7403	.1256	.1256
	.8	.0129	-.0537	.1977	.9479	-.1047	.0129	.0129	-.0408	-.0408	.1568	-.8432	.1047	.1047
	.9	.0059	-.0244	.0873	.9853	-.0640	.0059	.0059	-.0186	-.0186	.0687	-.9313	.0640	.0640
	D	0	0	0	1.0	0	0	0	0	0	0	-1.0/0	0/1.0	0
SPAN 4	.1	-.0030	.0127	-.0447	.9737	.0613	-.0030	-.0030	.0096	.0096	-.0351	-.0351	.9387	-.0613
	.2	-.0051	.0213	-.0753	.9242	.1349	-.0051	-.0051	.0162	.0162	-.0581	-.0581	.8651	-.1349
	.3	-.0063	.0264	-.0933	.8539	.2193	-.0063	-.0063	.0201	.0201	-.0732	-.0732	.7807	-.2193
	.4	-.0068	.0284	-.1003	.7656	.3132	-.0068	-.0068	.0216	.0216	-.0787	-.0787	.6868	-.3132
	.5	-.0067	.0278	-.0980	.6817	.4152	-.0067	-.0067	.0211	.0211	-.0769	-.0769	.5848	-.4152
	.6	-.0060	.0249	-.0878	.5449	.5240	-.0060	-.0060	.0189	.0189	-.0689	-.0689	.4760	-.5240
	.7	-.0049	.0202	-.0713	.4177	.6383	-.0049	-.0049	.0154	.0154	-.0560	-.0560	.3617	-.6383
	.8	-.0034	.0142	-.0502	.2828	.7566	-.0034	-.0034	.0108	.0108	-.0394	-.0394	.2434	-.7566
	.9	-.0017	.0073	-.0259	.1427	.8776	-.0017	-.0017	.0056	.0056	-.0203	-.0203	.1224	-.8776
	E	0	0	0	0	1.0	0	0	0	0	0	0	0	-1.0
+ Area		.4731	1.4460	1.5707	1.4460	.4731	.4731	.0296	.7737	.0653	.7854	.0937	.6743	.1177
− Area		-.1177	-.1234	-.1306	-.1234	-.1177	-.1177	-.6743	-.0937	-.7854	-.0653	-.7737	-.0296	-.4731
Total Area		.3554	1.3246	1.4401	1.3246	.3554	.3554	-.6447	.6800	-.7201	.7201	-.6800	.6447	-.3554

A .1 .2 .3 .4 .5 .6 .7 .8 .9 B .1 .2 .3 .4 .5 .6 .7 .8 .9 C .1 .2 .3 .4 .5 .6 .7 .8 .9 D .1 .2 .3 .4 .5 .6 .7 .8 .9 E

Span 1 — Span 2 — Span 3 — Span 4

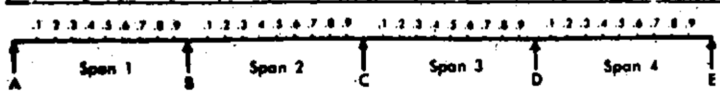
Influence coefficients — Four continuous spans.

L = Length of EXTERIOR spans; length of interior spans = NL

N = 1.4

Unit load at		MOMENTS/PL — SPAN 1: A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	SPAN 2: .1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0876	.0757	.0638	.0514	.0392	.0271	.0149	.0028	-.0094	-.0215	-.0188	-.0161	-.0133	-.0106	-.0079	-.0051	-.0024	.0003	.0031	.0058
	.2	0	.0758	.1516	.1275	.1033	.0791	.0549	.0307	.0066	-.0176	-.0418	-.0365	-.0312	-.0259	-.0206	-.0152	-.0099	-.0046	.0007	.0060	.0113
	.3	0	.0641	.1281	.1922	.1562	.1203	.0843	.0484	.0125	-.0235	-.0594	-.0519	-.0443	-.0368	-.0292	-.0217	-.0141	-.0066	.0010	.0085	.0161
	.4	0	.0527	.1054	.1581	.2107	.1634	.1161	.0688	.0215	-.0258	-.0731	-.0638	-.0546	-.0453	-.0360	-.0267	-.0174	-.0081	.0012	.0105	.0198
	.5	0	.0418	.0837	.1255	.1674	.2092	.1510	.0929	.0347	-.0235	-.0816	-.0713	-.0609	-.0505	-.0401	-.0298	-.0194	-.0090	.0013	.0117	.0221
	.6	0	.0316	.0633	.0949	.1266	.1582	.1898	.1215	.0531	-.0152	-.0836	-.0730	-.0623	-.0517	-.0411	-.0305	-.0199	-.0093	.0014	.0120	.0226
	.7	0	.0222	.0445	.0667	.0889	.1112	.1334	.1556	.0778	.0001	-.0777	-.0678	-.0580	-.0481	-.0383	-.0284	-.0185	-.0088	.0015	.0111	.0210
	.8	0	.0137	.0275	.0412	.0549	.0687	.0824	.0961	.1099	.0236	-.0627	-.0547	-.0468	-.0388	-.0308	-.0229	-.0149	-.0069	.0010	.0090	.0169
	.9	0	.0063	.0126	.0188	.0251	.0314	.0377	.0439	.0502	.0565	-.0372	-.0325	-.0278	-.0230	-.0183	-.0136	-.0089	-.0041	.0006	.0053	.0101
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0071	-.0141	-.0212	-.0283	-.0353	-.0424	-.0494	-.0565	-.0636	-.0706	.0693	.0593	.0492	.0392	.0292	.0191	.0091	-.0010	-.0110	-.0210
	.2	0	-.0116	-.0231	-.0347	-.0462	-.0578	-.0694	-.0809	-.0925	-.1041	-.1156	.0113	.1382	.1151	.0920	.0689	.0458	.0227	-.0004	-.0235	-.0466
	.3	0	-.0139	-.0277	-.0416	-.0555	-.0693	-.0832	-.0971	-.1110	-.1249	-.1387	-.0271	.0844	.1960	.1575	.1191	.0806	.0422	.0037	-.0347	-.0732
	.4	0	-.0144	-.0287	-.0431	-.0574	-.0718	-.0861	-.1005	-.1149	-.1292	-.1436	-.0490	.0457	.1403	.2349	.1795	.1241	.0687	.0134	-.0420	-.0974
	.5	0	-.0134	-.0268	-.0402	-.0536	-.0670	-.0804	-.0938	-.1072	-.1206	-.1340	-.0572	.0196	.0965	.1733	.2501	.1769	.1037	.0306	-.0426	-.1158
	.6	0	-.0114	-.0227	-.0341	-.0455	-.0568	-.0682	-.0796	-.0910	-.1023	-.1137	-.0548	.0040	.0629	.1218	.1807	.2395	.1484	.0573	-.0338	-.1249
	.7	0	-.0086	-.0173	-.0259	-.0346	-.0432	-.0518	-.0605	-.0692	-.0778	-.0864	-.0449	-.0034	.0381	.0796	.1211	.1626	.2041	.0956	-.0129	-.1214
	.8	0	-.0056	-.0112	-.0168	-.0224	-.0280	-.0335	-.0391	-.0447	-.0503	-.0559	-.0303	-.0051	.0204	.0458	.0712	.0966	.1221	.1475	.0229	-.1016
	.9	0	-.0026	-.0052	-.0078	-.0103	-.0129	-.0155	-.0181	-.0207	-.0233	-.0258	-.0143	-.0031	.0082	.0196	.0309	.0423	.0536	.0650	.0763	-.0623
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0019	.0037	.0056	.0075	.0094	.0112	.0131	.0150	.0168	.0187	.0106	.0025	-.0056	-.0137	-.0218	-.0299	-.0380	-.0461	-.0542	-.0623
	.2	0	.0030	.0061	.0091	.0122	.0152	.0183	.0213	.0244	.0274	.0305	.0173	.0041	-.0091	-.0224	-.0356	-.0488	-.0620	-.0752	-.0884	-.1016
	.3	0	.0036	.0073	.0109	.0146	.0182	.0218	.0255	.0291	.0328	.0364	.0206	.0047	-.0109	-.0267	-.0425	-.0583	-.0740	-.0898	-.1056	-.1214
	.4	0	.0037	.0075	.0112	.0150	.0187	.0225	.0262	.0300	.0337	.0375	.0212	.0050	-.0112	-.0275	-.0437	-.0600	-.0762	-.0925	-.1087	-.1249
	.5	0	.0035	.0069	.0104	.0139	.0174	.0208	.0243	.0278	.0313	.0347	.0197	.0046	-.0104	-.0255	-.0405	-.0556	-.0706	-.0857	-.1008	-.1158
	.6	0	.0029	.0058	.0088	.0117	.0146	.0175	.0205	.0234	.0263	.0292	.0166	.0039	-.0088	-.0214	-.0341	-.0468	-.0594	-.0721	-.0847	-.0974
	.7	0	.0022	.0044	.0066	.0088	.0110	.0132	.0154	.0176	.0198	.0220	.0124	.0029	-.0066	-.0161	-.0256	-.0351	-.0446	-.0542	-.0637	-.0732
	.8	0	.0014	.0028	.0042	.0056	.0070	.0084	.0098	.0112	.0126	.0140	.0079	.0019	-.0042	-.0102	-.0163	-.0224	-.0284	-.0345	-.0405	-.0466
	.9	0	.0006	.0013	.0019	.0025	.0032	.0038	.0044	.0051	.0057	.0063	.0036	.0008	-.0019	-.0046	-.0074	-.0101	-.0128	-.0156	-.0183	-.0210
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	0	-.0003	-.0006	-.0009	-.0012	-.0015	-.0018	-.0021	-.0024	-.0027	-.0030	-.0017	-.0004	.0009	.0022	.0035	.0048	.0061	.0074	.0088	.0101
	.2	0	-.0005	-.0010	-.0015	-.0020	-.0025	-.0030	-.0036	-.0041	-.0046	-.0051	-.0029	-.0007	.0015	.0037	.0059	.0081	.0103	.0125	.0147	.0169
	.3	0	-.0006	-.0013	-.0019	-.0025	-.0032	-.0038	-.0044	-.0050	-.0057	-.0063	-.0036	-.0008	.0019	.0046	.0074	.0101	.0128	.0155	.0183	.0210
	.4	0	-.0007	-.0014	-.0020	-.0027	-.0034	-.0041	-.0047	-.0054	-.0061	-.0068	-.0038	-.0009	.0020	.0050	.0079	.0108	.0138	.0167	.0197	.0226
	.5	0	-.0007	-.0013	-.0020	-.0026	-.0033	-.0040	-.0046	-.0053	-.0060	-.0066	-.0037	-.0009	.0020	.0049	.0077	.0106	.0135	.0163	.0192	.0221
	.6	0	-.0006	-.0012	-.0018	-.0024	-.0030	-.0036	-.0042	-.0047	-.0053	-.0059	-.0034	-.0008	.0018	.0043	.0069	.0095	.0121	.0146	.0172	.0198
	.7	0	-.0005	-.0010	-.0014	-.0019	-.0024	-.0029	-.0034	-.0039	-.0043	-.0048	-.0027	-.0006	.0014	.0035	.0056	.0077	.0098	.0119	.0140	.0161
	.8	0	-.0003	-.0007	-.0010	-.0014	-.0017	-.0020	-.0024	-.0027	-.0030	-.0034	-.0019	-.0005	.0010	.0025	.0040	.0054	.0069	.0084	.0098	.0113
	.9	0	-.0002	-.0003	-.0005	-.0007	-.0009	-.0010	-.0012	-.0014	-.0016	-.0017	-.0010	-.0002	.0005	.0013	.0020	.0028	.0036	.0043	.0051	.0058
	E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0430	.0761	.0991	.1121	.1152	.1082	.0912	.0643	.0390	.0347	.0306	.0560	.1090	.1465	.1615	.1540	.1240	.0725	.0347	.0294
− Area		0	-.0138	-.0277	-.0415	-.0554	-.0692	-.0831	-.0969	-.1108	-.1383	-.1928	-.0918	-.0429	-.0441	-.0522	-.0604	-.0685	-.0767	-.0859	-.1308	-.2316
Total Area		0	.0292	.0484	.0576	.0567	.0460	.0251	-.0057	-.0465	-.0993	-.1581	-.0612	.0131	.0649	.0943	.1011	.0855	.0473	-.0134	-.0966	-.2022

Unit load at		REACTIONS/P: R_A	R_B	R_C	R_D	R_E	SHEARS/P: V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}	V_{DE}	V_{ED}
SPAN 1	A	1.0	0	0	0	0	1.0	0	0	0	0	0	0	0
	.1	.8785	.1366	-.0233	.0066	-.0017	.8785	-.1215	.0182	.0182	-.0050	-.0050	.0017	.0017
	.2	.7583	.2772	-.0452	.0131	-.0034	.7583	-.2416	.0364	.0364	-.0098	-.0098	.0034	.0034
	.3	.6406	.4087	-.0643	.0187	-.0048	.6406	-.3594	.0503	.0503	-.0139	-.0139	.0048	.0048
	.4	.5269	.5331	-.0781	.0230	-.0059	.5269	-.4731	.0619	.0619	-.0171	-.0171	.0059	.0059
	.5	.4184	.6607	-.0882	.0257	-.0066	.4184	-.5816	.0691	.0691	-.0191	-.0191	.0066	.0066
	.6	.3164	.7844	-.0903	.0263	-.0068	.3164	-.6836	.0708	.0708	-.0196	-.0196	.0068	.0068
	.7	.2223	.8435	-.0840	.0245	-.0063	.2223	-.7777	.0658	.0658	-.0182	-.0182	.0063	.0063
	.8	.1373	.9158	-.0678	.0198	-.0051	.1373	-.8627	.0531	.0531	-.0147	-.0147	.0051	.0051
	.9	.0628	.9667	-.0402	.0117	-.0030	.0628	-.9372	.0316	.0316	-.0087	-.0087	.0030	.0030
	B	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0	0	0
SPAN 2	.1	-.0706	1.0037	.0852	-.0246	.0063	-.0706	-.0706	.9331	-.0669	.0182	.0182	-.0063	-.0063
	.2	-.1156	.9616	.1944	-.0544	.0140	-.1156	-.1156	.8460	-.1540	.0404	.0404	-.0140	-.0140
	.3	-.1387	.8724	.3196	-.0854	.0220	-.1387	-.1387	.7437	-.2563	.0634	.0634	-.0220	-.0220
	.4	-.1436	.7744	.4538	-.1138	.0292	-.1436	-.1436	.6308	-.3692	.0844	.0844	-.0292	-.0292
	.5	-.1340	.6481	.5882	-.1351	.0347	-.1340	-.1340	.5121	-.4879	.1004	.1004	-.0347	-.0347
	.6	-.1137	.5062	.7138	-.1458	.0375	-.1137	-.1137	.3925	-.6075	.1063	.1063	-.0375	-.0375
	.7	-.0864	.3838	.8388	-.1416	.0364	-.0864	-.0864	.2767	-.7233	.1052	.1052	-.0364	-.0364
	.8	-.0559	.2254	.9188	-.1188	.0305	-.0559	-.0559	.1695	-.8305	.0881	.0881	-.0308	-.0305
	.9	-.0258	.1015	.9784	-.0727	.0187	-.0258	-.0258	.0757	-.9243	.0640	.0640	-.0187	-.0187
	C	0	0	1.0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 3	.1	.0187	-.0727	.9784	.1015	-.0258	.0187	.0187	-.0640	-.0640	.9243	-.0757	.0258	.0258
	.2	.0308	-.1188	.9188	.2254	-.0559	.0305	.0308	-.0881	-.0881	.8305	-.1695	.0559	.0559
	.3	.0364	-.1416	.8388	.3632	-.0864	.0364	.0364	-.1052	-.1052	.7233	-.2767	.0864	.0864
	.4	.0375	-.1458	.7138	.5062	-.1137	.0375	.0375	-.1063	-.1063	.6075	-.3925	.1137	.1137
	.5	.0347	-.1351	.5882	.6481	-.1340	.0347	.0347	-.1004	-.1004	.4879	-.5121	.1340	.1340
	.6	.0292	-.1138	.4538	.7744	-.1436	.0292	.0292	-.0844	-.0844	.3692	-.6308	.1436	.1436
	.7	.0220	-.0854	.3196	.8324	-.1387	.0220	.0220	-.0634	-.0634	.2563	-.7437	.1387	.1387
	.8	.0140	-.0544	.1944	.9616	-.1156	.0140	.0140	-.0404	-.0404	.1540	-.8460	.1156	.1156
	.9	.0063	-.0246	.0852	1.0037	-.0706	.0063	.0063	-.0182	-.0182	.0669	-.9331	.0706	.0706
	D	0	0	0	1.0	0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 4	.1	-.0030	.0117	-.0402	.9687	.0628	-.[illegible]	-.0030	.0087	.0087	-.0315	-.0315	.9372	-.0628
	.2	-.0051	.0198	-.0678	.9159	.1373	-.0051	-.0051	.0147	.0147	-.0531	-.0531	.8627	-.1373
	.3	-.0063	.0245	-.0840	.8435	.2223	-.0063	-.0063	.0182	.0182	-.0658	-.0658	.7777	-.2223
	.4	-.0068	.0263	-.0903	.7544	.3164	-.0068	-.0068	.0196	.0196	-.0708	-.0708	.6836	-.3164
	.5	-.0066	.0257	-.0882	.6607	.4184	-.0066	-.0066	.0191	.0191	-.0691	-.0691	.5816	-.4184
	.6	-.0059	.0230	-.0781	.5331	.5269	-.0059	-.0059	.0171	.0171	-.0619	-.0619	.4731	-.5269
	.7	-.0048	.0187	-.0643	.4087	.6406	-.0048	-.0048	.0139	.0139	-.0503	-.0503	.3594	-.6406
	.8	-.0034	.0132	-.0452	.2772	.7583	-.0034	-.0034	.0098	.0098	-.0354	-.0354	.2416	-.7583
	.9	-.0017	.0068	-.0233	.1368	.8785	-.0017	-.0017	.0050	.0050	-.0182	-.0182	.1215	-.8785
	E	0	0	0	0	1.0	0	0	0	0	0	0	0	-1.0
+ Area		.4803	1.5138	1.6765	1.5138	.4803	.4803	.0347	.8210	.0588	.8382	.1004	.6936	.1384
− Area		-.1384	-.1351	-.1176	-.1351	-.1384	-.1384	-.6928	-.1004	-.8382	-.0588	-.8210	-.0347	-.4803
Total Area		.3419	1.3787	1.5589	1.3787	.3419	.3419	-.6581	.7206	-.7794	.7794	-.7206	.6581	-.3419



Influence coefficients — Four continuous spans.

L = Length of EXTERIOR spans; length of interior spans = NL

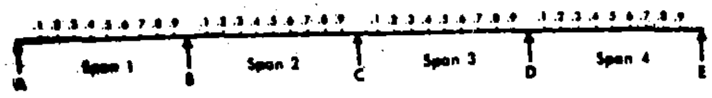
N = 1.5

Table 20.43

MOMENTS/PL

Unit load at		A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	.1	.2	.3	.4	.5	.6	.7	.8	.9	C
		SPAN 1											**SPAN 2**									
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0879	.0758	.0636	.0517	.0396	.0275	.0155	.0034	-.0087	-.0208	-.0181	-.0155	-.0129	-.0102	-.0076	-.0049	-.0023	.0003	.0030	.0056
	.2	0	.0760	.1519	.1279	.1039	.0799	.0558	.0318	.0078	-.0163	-.0403	-.0353	-.0300	-.0248	-.0198	-.0147	-.0096	-.0044	.0007	.0058	.0109
	.3	0	.0643	.1285	.1928	.1571	.1214	.0856	.0499	.0142	-.0215	-.0573	-.0500	-.0427	-.0354	-.0282	-.0209	-.0136	-.0063	.0010	.0082	.0155
	.4	0	.0530	.1059	.1589	.2119	.1648	.1177	.0707	.0236	-.0234	-.0705	-.0615	-.0526	-.0436	-.0347	-.0257	-.0167	-.0078	.0012	.0101	.0191
	.5	0	.0421	.0843	.1264	.1685	.2107	.1528	.0949	.0371	-.0208	-.0787	-.0687	-.0587	-.0487	-.0387	-.0287	-.0187	-.0087	.0013	.0113	.0213
	.6	0	.0319	.0639	.0958	.1278	.1597	.1917	.1236	.0556	-.0125	-.0806	-.0703	-.0601	-.0498	-.0396	-.0294	-.0191	-.0089	.0013	.0116	.0218
	.7	0	.0225	.0450	.0675	.0900	.1126	.1351	.1576	.0801	.0026	-.0749	-.0654	-.0559	-.0463	-.0368	-.0273	-.0178	-.0083	.0012	.0108	.0203
	.8	0	.0140	.0279	.0419	.0558	.0698	.0837	.0977	.1117	.0256	-.0606	-.0527	-.0451	-.0374	-.0297	-.0220	-.0144	-.0067	.0010	.0087	.0164
	.9	0	.0064	.0128	.0192	.0257	.0321	.0385	.0449	.0513	.0577	-.0359	-.0313	-.0268	-.0222	-.0176	-.0131	-.0085	-.0040	.0006	.0052	.0097
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0077	-.0155	-.0232	-.0310	-.0387	-.0465	-.0542	-.0620	-.0697	-.0774	.0721	.0617	.0512	.0408	.0303	.0199	.0094	-.0010	-.0115	-.0219
	.2	0	-.0127	-.0253	-.0380	-.0507	-.0634	-.0760	-.0887	-.1014	-.1141	-.1267	.0090	.1448	.1206	.0964	.0722	.0480	.0238	-.0004	-.0247	-.0489
	.3	0	-.0152	-.0304	-.0456	-.0608	-.0760	-.0912	-.1064	-.1216	-.1368	-.1520	-.0325	.0870	.2064	.1659	.1254	.0849	.0444	.0039	-.0366	-.0771
	.4	0	-.0157	-.0315	-.0472	-.0629	-.0787	-.0944	-.1102	-.1259	-.1416	-.1574	-.0559	.0455	.1470	.2484	.1898	.1313	.0727	.0141	-.0444	-.1030
	.5	0	-.0147	-.0294	-.0441	-.0587	-.0734	-.0881	-.1028	-.1175	-.1322	-.1469	-.0644	.0180	.1004	.1828	.2652	.1876	.1100	.0324	-.0451	-.1227
	.6	0	-.0125	-.0249	-.0374	-.0498	-.0623	-.0748	-.0872	-.0997	-.1121	-.1246	-.0614	.0018	.0650	.1282	.1914	.2546	.1578	.0610	-.0358	-.1327
	.7	0	-.0095	-.0189	-.0284	-.0379	-.0473	-.0568	-.0663	-.0758	-.0852	-.0947	-.0501	-.0056	.0390	.0836	.1281	.1727	.2173	.1018	-.0136	-.1291
	.8	0	-.0061	-.0122	-.0184	-.0245	-.0306	-.0367	-.0429	-.0490	-.0551	-.0612	-.0339	-.0066	.0207	.0480	.0753	.1026	.1299	.1572	.0245	-.1082
	.9	0	-.0028	-.0057	-.0085	-.0113	-.0141	-.0170	-.0198	-.0226	-.0255	-.0283	-.0161	-.0039	.0083	.0204	.0326	.0448	.0570	.0692	.0814	-.0664
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0020	.0041	.0061	.0082	.0102	.0123	.0143	.0164	.0184	.0204	.0118	.0031	-.0056	-.0143	-.0230	-.0317	-.0404	-.0491	-.0577	-.0664
	.2	0	.0033	.0067	.0100	.0133	.0166	.0200	.0233	.0266	.0300	.0333	.0191	.0050	-.0092	-.0233	-.0375	-.0516	-.0658	-.0799	-.0941	-.1082
	.3	0	.0040	.0079	.0119	.0159	.0199	.0238	.0278	.0318	.0357	.0397	.0225	.0060	-.0109	-.0279	-.0447	-.0615	-.0784	-.0953	-.1122	-.1291
	.4	0	.0041	.0082	.0122	.0163	.0204	.0245	.0286	.0327	.0367	.0408	.0235	.0061	-.0112	-.0286	-.0459	-.0633	-.0806	-.0980	-.1153	-.1327
	.5	0	.0038	.0076	.0113	.0151	.0189	.0227	.0264	.0302	.0340	.0378	.0217	.0057	-.0104	-.0264	-.0425	-.0585	-.0746	-.0906	-.1067	-.1227
	.6	0	.0032	.0063	.0095	.0127	.0158	.0190	.0222	.0253	.0285	.0317	.0182	.0046	-.0087	-.0222	-.0356	-.0491	-.0626	-.0760	-.0895	-.1030
	.7	0	.0024	.0047	.0071	.0095	.0119	.0142	.0166	.0190	.0214	.0237	.0136	.0034	-.0063	-.0166	-.0267	-.0368	-.0469	-.0570	-.0670	-.0771
	.8	0	.0015	.0030	.0045	.0060	.0075	.0090	.0105	.0120	.0135	.0150	.0086	.0023	-.0041	-.0105	-.0169	-.0233	-.0297	-.0361	-.0425	-.0489
	.9	0	.0007	.0013	.0020	.0027	.0034	.0040	.0047	.0054	.0061	.0067	.0039	.0010	-.0019	-.0047	-.0076	-.0105	-.0133	-.0162	-.0191	-.0219
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	0	-.0003	-.0006	-.0009	-.0012	-.0015	-.0018	-.0021	-.0024	-.0027	-.0030	-.0017	-.0004	.0008	.0021	.0034	.0046	.0059	.0072	.0084	.0097
	.2	0	-.0005	-.0010	-.0015	-.0020	-.0025	-.0030	-.0035	-.0040	-.0045	-.0050	-.0029	-.0008	.0014	.0035	.0057	.0078	.0099	.0121	.0142	.0164
	.3	0	-.0006	-.0012	-.0019	-.0025	-.0031	-.0037	-.0044	-.0050	-.0056	-.0062	-.0036	-.0009	.0017	.0044	.0070	.0097	.0123	.0150	.0176	.0203
	.4	0	-.0007	-.0013	-.0020	-.0027	-.0034	-.0040	-.0047	-.0054	-.0060	-.0067	-.0039	-.0010	.0018	.0047	.0076	.0104	.0133	.0161	.0190	.0218
	.5	0	-.0007	-.0013	-.0020	-.0026	-.0033	-.0039	-.0046	-.0052	-.0059	-.0066	-.0038	-.0010	.0018	.0046	.0074	.0102	.0129	.0157	.0185	.0213
	.6	0	-.0006	-.0012	-.0018	-.0023	-.0029	-.0035	-.0041	-.0047	-.0053	-.0059	-.0034	-.0009	.0016	.0041	.0066	.0091	.0116	.0141	.0166	.0191
	.7	0	-.0005	-.0010	-.0014	-.0019	-.0024	-.0029	-.0033	-.0038	-.0043	-.0048	-.0027	-.0007	.0013	.0033	.0054	.0074	.0094	.0115	.0135	.0155
	.8	0	-.0003	-.0007	-.0010	-.0013	-.0017	-.0020	-.0023	-.0027	-.0030	-.0034	-.0019	-.0005	.0009	.0024	.0038	.0052	.0066	.0081	.0095	.0109
	.9	0	-.0002	-.0003	-.0005	-.0007	-.0009	-.0010	-.0012	-.0014	-.0016	-.0017	-.0010	-.0003	.0005	.0012	.0019	.0027	.0034	.0042	.0049	.0056
	E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0438	.0778	.1012	.1151	.1189	.1127	.0965	.0703	.0445	.0403	.0349	.0620	.1211	.1639	.1811	.1728	.1398	.0803	.0354	.0284
− Area		0	-.0161	-.0322	-.0483	-.0644	-.0805	-.0966	-.1127	-.1288	-.1554	-.2135	-.0989	-.0424	-.0435	-.0540	-.0644	-.0749	-.0863	-.0969	-.1476	-.2618
Total Area		0	.0277	.0456	.0530	.0507	.0384	.0161	-.0162	-.0585	-.1109	-.1732	-.0640	.0196	.0776	.1099	.1167	.0979	.0535	-.0166	-.1122	-.2334

Unit load at		R_A	R_B	R_C	R_D	R_E	V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}	V_{DE}	V_{ED}
		REACTIONS/P					**SHEARS/P**							
SPAN 1	A	1.0	0	0	0	0	1.0	0	0	0	0	0	0	0
	.1	.8792	.1373	-.0211	.0063	-.0017	.8792	-.1208	.0165	.0165	-.0044	-.0044	.0017	.0017
	.2	.7597	.2723	-.0406	.0123	-.0034	.7597	-.2403	.0320	.0320	-.0089	-.0089	.0034	.0034
	.3	.6427	.4028	-.0583	.0175	-.0048	.6427	-.3573	.0455	.0455	-.0127	-.0127	.0048	.0048
	.4	.5295	.5265	-.0716	.0215	-.0059	.5295	-.4705	.0560	.0560	-.0156	-.0156	.0059	.0059
	.5	.4213	.6412	-.0799	.0240	-.0066	.4213	-.5787	.0625	.0625	-.0174	-.0174	.0066	.0066
	.6	.3194	.7465	-.0818	.0245	-.0067	.3194	-.6806	.0640	.0640	-.0178	-.0178	.0067	.0067
	.7	.2251	.8344	-.0761	.0228	-.0062	.2251	-.7749	.0595	.0595	-.0166	-.0166	.0062	.0062
	.8	.1396	.9084	-.0614	.0184	-.0050	.1396	-.8604	.0480	.0480	-.0134	-.0134	.0050	.0050
	.9	.0641	.9644	-.0364	.0109	-.0030	.0641	-.9359	.0285	.0285	-.0079	-.0079	.0030	.0030
	B	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0	0	0
SPAN 2	.1	-.0774	1.0121	.0832	-.0247	.0067	-.0774	-.0774	.9347	-.0653	.0179	.0179	-.0067	-.0067
	.2	-.1267	.9754	.1913	-.0550	.0150	-.1267	-.1267	.8487	-.1513	.0399	.0399	-.0150	-.0150
	.3	-.1520	.8988	.3162	-.0868	.0237	-.1520	-.1520	.7468	-.2532	.0630	.0630	-.0237	-.0237
	.4	-.1574	.7913	.4502	-.1159	.0317	-.1574	-.1574	.6340	-.3660	.0842	.0842	-.0317	-.0317
	.5	-.1469	.6619	.5852	-.1381	.0378	-.1469	-.1469	.5151	-.4849	.1003	.1003	-.0378	-.0378
	.6	-.1246	.5196	.7135	-.1492	.0408	-.1246	-.1246	.3950	-.6050	.1084	.1084	-.0408	-.0408
	.7	-.0947	.3732	.8270	-.1452	.0397	-.0947	-.0947	.2785	-.7215	.1055	.1055	-.0397	-.0397
	.8	-.0612	.2319	.9178	-.1217	.0333	-.0612	-.0612	.1706	-.8294	.0884	.0884	-.0333	-.0333
	.9	-.0283	.1045	.9781	-.0747	.0204	-.0283	-.0283	.0762	-.9238	.0543	.0543	-.0204	-.0204
	C	0	0	1.0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 3	.1	.0204	-.0747	.9781	.1045	-.0283	.0204	.0204	-.0543	-.0543	.9283	-.0762	.0283	.0283
	.2	.0333	-.1217	.9178	.2319	-.0612	.0333	.0333	-.0884	-.0884	.8294	-.1706	.0612	.0612
	.3	.0397	-.1452	.8270	.3732	-.0947	.0397	.0397	-.1055	-.1055	.7215	-.2785	.0947	.0947
	.4	.0408	-.1492	.7135	.5196	-.1246	.0408	.0408	-.1084	-.1084	.6050	-.3950	.1246	.1246
	.5	.0378	-.1381	.5852	.6619	-.1469	.0378	.0378	-.1003	-.1003	.4849	-.5151	.1469	.1469
	.6	.0317	-.1159	.4502	.7913	-.1574	.0317	.0317	-.0842	-.0842	.3660	-.6340	.1574	.1574
	.7	.0237	-.0868	.3162	.8988	-.1520	.0237	.0237	-.0630	-.0630	.2532	-.7468	.1520	.1520
	.8	.0150	-.0550	.1913	.9754	-.1267	.0150	.0150	-.0399	-.0399	.1513	-.8487	.1267	.1267
	.9	.0067	-.0247	.0832	1.0121	-.0774	.0067	.0067	-.0179	-.0179	.0653	-.9347	.0774	.0774
	D	0	0	0	1.0	0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 4	.1	-.0030	.0109	-.0364	.9644	.0641	-.0030	-.0030	.0079	.0079	-.0285	-.0285	.9359	-.0641
	.2	-.0050	.0184	-.0614	.9084	.1396	-.0050	-.0050	.0134	.0134	-.0480	-.0480	.8604	-.1396
	.3	-.0062	.0228	-.0761	.8344	.2251	-.0062	-.0062	.0166	.0166	-.0595	-.0595	.7749	-.2251
	.4	-.0067	.0245	-.0818	.7465	.3194	-.0067	-.0067	.0178	.0178	-.0640	-.0640	.6806	-.3194
	.5	-.0066	.0240	-.0799	.6412	.4213	-.0066	-.0066	.0174	.0174	-.0625	-.0625	.5787	-.4213
	.6	-.0059	.0215	-.0716	.5265	.5295	-.0059	-.0059	.0156	.0156	-.0560	-.0560	.4705	-.5295
	.7	-.0048	.0175	-.0583	.4028	.6427	-.0048	-.0048	.0127	.0127	-.0455	-.0455	.3573	-.6427
	.8	-.0034	.0123	-.0406	.2723	.7597	-.0034	-.0034	.0089	.0089	-.0320	-.0320	.2403	-.7597
	.9	-.0017	.0063	-.0211	.1373	.8792	-.0017	-.0017	.0046	.0046	-.0165	-.0165	.1208	-.8792
	E	0	0	0	0	1.0	0	0	0	0	0	0	0	-1.0
+ Area		.4878	1.5826	1.7818	1.5826	.4878	.4878	.0403	.8694	.0533	.8909	.1070	.7135	.1610
− Area		-.1610	-.1473	-.1065	-.1473	-.1610	-.1610	-.7135	-.1070	-.8909	-.0533	-.8694	-.0403	-.4878
Total Area		.3268	1.4355	1.6753	1.4355	.3268	.3268	-.6732	.7624	-.8376	.8376	-.7624	.6732	-.3268



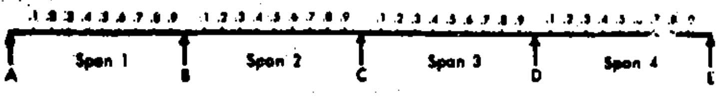
Influence coefficients — Four continuous spans.

L = Length of EXTERIOR spans; length of interior spans = NL

N = 1.6

Unit load at		MOMENTS/PL Span 1 A	.1	.2	.3	.4	.5	.6	.7	.8	.9	B	Span 2 .1	.2	.3	.4	.5	.6	.7	.8	.9	C
SPAN 1	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	.1	0	.0880	.0760	.0640	.0520	.0400	.0280	.0160	.0040	-.0080	-.0200	-.0175	-.0149	-.0124	-.0099	-.0073	-.0048	-.0022	.0003	.0029	.0054
	.2	0	.0761	.1522	.1283	.1044	.0806	.0567	.0328	.0089	-.0150	-.0389	-.0339	-.0290	-.0240	-.0191	-.0142	-.0092	-.0043	.0007	.0056	.0106
	.3	0	.0645	.1289	.1934	.1579	.1224	.0868	.0513	.0158	-.0198	-.0553	-.0483	-.0412	-.0342	-.0272	-.0201	-.0131	-.0061	.0008	.0080	.0150
	.4	0	.0532	.1064	.1596	.2128	.1660	.1192	.0724	.0256	-.0212	-.0680	-.0594	-.0507	-.0421	-.0334	-.0248	-.0161	-.0075	.0012	.0098	.0185
	.5	0	.0424	.0848	.1272	.1696	.2120	.1544	.0968	.0393	-.0183	-.0759	-.0663	-.0566	-.0470	-.0373	-.0277	-.0180	-.0084	.0013	.0110	.0206
	.6	0	.0322	.0644	.0967	.1289	.1611	.1933	.1256	.0578	-.0100	-.0778	-.0679	-.0580	-.0481	-.0382	-.0283	-.0184	-.0086	.0013	.0112	.0211
	.7	0	.0228	.0455	.0683	.0911	.1139	.1366	.1594	.0822	.0049	-.0723	-.0631	-.0539	-.0447	-.0355	-.0263	-.0171	-.0080	.0012	.0104	.0196
	.8	0	.0142	.0283	.0425	.0567	.0708	.0850	.0992	.1133	.0275	-.0583	-.0509	-.0435	-.0361	-.0287	-.0212	-.0138	-.0064	.0010	.0084	.0158
	.9	0	.0066	.0131	.0196	.0262	.0327	.0392	.0458	.0523	.0588	-.0346	-.0302	-.0258	-.0214	-.0170	-.0126	-.0082	-.0038	.0006	.0050	.0094
	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 2	.1	0	-.0084	-.0169	-.0253	-.0337	-.0422	-.0506	-.0590	-.0675	-.0759	-.0843	.0748	.0640	.0531	.0423	.0314	.0206	.0098	-.0011	-.0119	-.0228
	.2	0	-.0138	-.0276	-.0414	-.0552	-.0690	-.0828	-.0966	-.1104	-.1242	-.1380	.0067	.1513	.1260	.1007	.0754	.0501	.0248	-.0005	-.0258	-.0511
	.3	0	-.0166	-.0331	-.0497	-.0662	-.0828	-.0993	-.1159	-.1324	-.1490	-.1656	-.0381	.0894	.2168	.1743	.1317	.0892	.0466	.0041	-.0385	-.0810
	.4	0	-.0171	-.0343	-.0514	-.0685	-.0857	-.1028	-.1199	-.1371	-.1542	-.1714	-.0631	.0452	.1535	.2618	.2001	.1384	.0766	.0149	-.0468	-.1085
	.5	0	-.0160	-.0320	-.0480	-.0640	-.0799	-.0959	-.1119	-.1279	-.1439	-.1599	-.0719	.0162	.1042	.1922	.2803	.1983	.1163	.0343	-.0476	-.1296
	.6	0	-.0136	-.0271	-.0407	-.0543	-.0678	-.0814	-.0950	-.1085	-.1221	-.1356	-.0681	-.0006	.0669	.1345	.2020	.2695	.1671	.0646	-.0379	-.1403
	.7	0	-.0103	-.0206	-.0309	-.0412	-.0515	-.0618	-.0721	-.0825	-.0928	-.1031	-.0554	-.0078	.0398	.0875	.1351	.1827	.2304	.1080	-.0144	-.1367
	.8	0	-.0067	-.0133	-.0200	-.0266	-.0333	-.0400	-.0466	-.0533	-.0600	-.0666	-.0374	-.0083	.0209	.0501	.0793	.1085	.1377	.1669	.0260	-.1148
	.9	0	-.0031	-.0062	-.0092	-.0123	-.0154	-.0185	-.0215	-.0246	-.0277	-.0308	-.0178	-.0047	.0083	.0213	.0343	.0474	.0604	.0734	.0864	-.0705
	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 3	.1	0	.0022	.0044	.0067	.0089	.0111	.0133	.0155	.0178	.0200	.0222	.0129	.0037	-.0056	-.0149	-.0242	-.0334	-.0427	-.0520	-.0613	-.0705
	.2	0	.0036	.0072	.0108	.0145	.0181	.0217	.0253	.0289	.0325	.0361	.0210	.0060	-.0091	-.0242	-.0393	-.0544	-.0695	-.0846	-.0997	-.1148
	.3	0	.0043	.0086	.0129	.0172	.0215	.0258	.0301	.0344	.0387	.0430	.0251	.0071	-.0109	-.0288	-.0468	-.0648	-.0828	-.1008	-.1187	-.1367
	.4	0	.0044	.0088	.0133	.0177	.0221	.0265	.0309	.0353	.0398	.0442	.0257	.0073	-.0112	-.0296	-.0481	-.0665	-.0850	-.1034	-.1219	-.1403
	.5	0	.0041	.0082	.0122	.0163	.0204	.0245	.0286	.0326	.0367	.0408	.0238	.0067	-.0103	-.0274	-.0444	-.0614	-.0785	-.0955	-.1126	-.1296
	.6	0	.0034	.0068	.0103	.0137	.0171	.0205	.0239	.0273	.0307	.0342	.0199	.0056	-.0086	-.0229	-.0372	-.0514	-.0657	-.0800	-.0942	-.1085
	.7	0	.0026	.0051	.0077	.0102	.0128	.0153	.0179	.0204	.0230	.0255	.0149	.0042	-.0065	-.0171	-.0278	-.0384	-.0491	-.0597	-.0704	-.0810
	.8	0	.0016	.0032	.0048	.0064	.0080	.0097	.0113	.0129	.0145	.0161	.0094	.0026	-.0041	-.0109	-.0175	-.0242	-.0310	-.0377	-.0444	-.0511
	.9	0	.0007	.0014	.0022	.0029	.0036	.0043	.0050	.0057	.0065	.0072	.0042	.0012	-.0018	-.0048	-.0078	-.0108	-.0138	-.0168	-.0198	-.0228
	D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPAN 4	.1	0	-.0003	-.0006	-.0009	-.0012	-.0015	-.0018	-.0021	-.0024	-.0027	-.0030	-.0017	-.0005	.0007	.0020	.0032	.0045	.0057	.0069	.0082	.0094
	.2	0	-.0005	-.0010	-.0015	-.0020	-.0025	-.0030	-.0035	-.0040	-.0045	-.0050	-.0029	-.0008	.0013	.0033	.0054	.0075	.0096	.0117	.0137	.0158
	.3	0	-.0006	-.0012	-.0019	-.0025	-.0031	-.0037	-.0043	-.0049	-.0056	-.0062	-.0036	-.0010	.0016	.0041	.0067	.0093	.0119	.0145	.0170	.0196
	.4	0	-.0007	-.0013	-.0020	-.0027	-.0033	-.0040	-.0046	-.0053	-.0060	-.0066	-.0039	-.0011	.0017	.0045	.0072	.0100	.0128	.0156	.0183	.0211
	.5	0	-.0006	-.0013	-.0019	-.0026	-.0032	-.0039	-.0045	-.0052	-.0058	-.0065	-.0038	-.0011	.0016	.0044	.0071	.0098	.0125	.0152	.0179	.0206
	.6	0	-.0006	-.0012	-.0017	-.0023	-.0029	-.0035	-.0041	-.0046	-.0052	-.0058	-.0034	-.0010	.0015	.0039	.0063	.0088	.0112	.0136	.0160	.0185
	.7	0	-.0005	-.0009	-.0014	-.0019	-.0024	-.0028	-.0033	-.0038	-.0042	-.0047	-.0027	-.0008	.0012	.0032	.0051	.0071	.0091	.0111	.0130	.0150
	.8	0	-.0003	-.0007	-.0010	-.0013	-.0017	-.0020	-.0023	-.0027	-.0030	-.0033	-.0019	-.0005	.0008	.0022	.0036	.0050	.0064	.0078	.0092	.0106
	.9	0	-.0002	-.0003	-.0005	-.0007	-.0009	-.0010	-.0012	-.0014	-.0015	-.0017	-.0010	-.0003	.0004	.0011	.0019	.0026	.0033	.0040	.0047	.0054
	E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+ Area		0	.0446	.0791	.1037	.1182	.1228	.1174	.1019	.0765	.0503	.0462	.0395	.0683	.1336	.1822	.2019	.1927	.1546	.0887	.0367	.0275
− Area		0	-.0186	-.0371	-.0557	-.0742	-.0928	-.1113	-.1299	-.1484	-.1762	-.2362	-.1070	-.0423	-.0430	-.0559	-.0688	-.0816	-.0945	-.1085	-.1653	-.2938
Total Area		0	.0260	.0420	.0480	.0440	.0300	.0061	-.0280	-.0719	-.1259	-.1900	-.0675	.0260	.0906	.1263	.1331	.1111	.0601	-.0198	-.1286	-.2663

Unit load at		REACTIONS/P R_A	R_B	R_C	R_D	R_E	SHEARS/P V_{AB}	V_{BA}	V_{BC}	V_{CB}	V_{CD}	V_{DC}	V_{DE}	V_{ED}
SPAN 1	A	1.0	0	0	0	0	1.0	0	0	0	0	0	0	0
	.1	.8800	.1350	-.0192	.0059	-.0017	.8800	-.1200	.0150	.0150	-.0042	-.0042	.0017	.0017
	.2	.7611	.2680	-.0372	.0115	-.0033	.7611	-.2389	.0291	.0291	-.0082	-.0082	.0033	.0033
	.3	.6447	.3986	-.0529	.0163	-.0047	.6447	-.3553	.0413	.0413	-.0116	-.0116	.0047	.0047
	.4	.5320	.5189	-.0652	.0201	-.0058	.5320	-.4680	.0509	.0509	-.0143	-.0143	.0058	.0058
	.5	.4241	.6327	-.0727	.0224	-.0065	.4241	-.5759	.0568	.0568	-.0159	-.0159	.0065	.0065
	.6	.3222	.7359	-.0745	.0230	-.0066	.3222	-.6778	.0581	.0581	-.0163	-.0163	.0066	.0066
	.7	.2277	.8263	-.0692	.0213	-.0062	.2277	-.7723	.0541	.0541	-.0152	-.0152	.0062	.0062
	.8	.1417	.9019	-.0558	.0172	-.0050	.1417	-.8583	.0436	.0436	-.0122	-.0122	.0050	.0050
	.9	.0654	.9605	-.0332	.0102	-.0030	.0654	-.9346	.0259	.0259	-.0073	-.0073	.0030	.0030
	B	0	1.0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0	0	0
SPAN 2	.1	-.0843	1.0206	.0814	-.0249	.0072	-.0843	-.0843	.9362	-.0638	.0176	.0176	-.0072	-.0072
	.2	-.1380	.9892	.1884	-.0556	.0161	-.1380	-.1380	.8511	-.1489	.0395	.0395	-.0161	-.0161
	.3	-.1656	.9153	.3129	-.0882	.0255	-.1656	-.1656	.7497	-.2503	.0627	.0627	-.0255	-.0255
	.4	-.1714	.8083	.4469	-.1181	.0342	-.1714	-.1714	.6370	-.3630	.0839	.0839	-.0342	-.0342
	.5	-.1599	.6777	.5624	-.1410	.0408	-.1599	-.1599	.5178	-.4822	.1002	.1002	-.0408	-.0408
	.6	-.1356	.5329	.7113	-.1527	.0442	-.1356	-.1356	.3972	-.6028	.1085	.1085	-.0442	-.0442
	.7	-.1031	.3833	.8255	-.1488	.0430	-.1031	-.1031	.2802	-.7198	.1087	.1087	-.0430	-.0430
	.8	-.0666	.2383	.9171	-.1249	.0361	-.0666	-.0666	.1717	-.8283	.0888	.0888	-.0361	-.0361
	.9	-.0308	.1074	.9779	-.0768	.0222	-.0308	-.0308	.0766	-.9234	.0546	.0546	-.0222	-.0222
	C	0	0	1.0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0	0	0
SPAN 3	.1	.0222	-.0768	.9779	.1074	-.0308	.0222	.0222	-.0546	-.0546	.9234	-.0766	.0308	.0308
	.2	.0361	-.1249	.9171	.2383	-.0666	.0361	.0361	-.0888	-.0888	.8283	-.1717	.0666	.0666
	.3	.0430	-.1488	.8255	.3833	-.1031	.0430	.0430	-.1057	-.1057	.7198	-.2802	.1031	.1031
	.4	.0442	-.1527	.7113	.5329	-.1356	.0442	.0442	-.1085	-.1085	.6028	-.3972	.1356	.1356
	.5	.0408	-.1410	.5624	.6777	-.1599	.0408	.0408	-.1002	-.1002	.4822	-.5178	.1599	.1599
	.6	.0342	-.1181	.4469	.8083	-.1714	.0342	.0342	-.0839	-.0839	.3630	-.6370	.1714	.1714
	.7	.0255	-.0882	.3129	.9153	-.1656	.0255	.0255	-.0627	-.0627	.2503	-.7497	.1656	.1656
	.8	.0161	-.0556	.1884	.9892	-.1380	.0161	.0161	-.0395	-.0395	.1489	-.8511	.1380	.1380
	.9	.0072	-.0249	.0814	1.0206	-.0843	.0072	.0072	-.0176	-.0176	.0638	-.9362	.0843	.0843
	D	0	0	0	1.0	0	0	0	0	0	0	-1.0 / 0	0 / 1.0	0
SPAN 4	.1	-.0030	.0102	-.0332	.9605	.0654	-.0030	-.0030	.0073	.0073	-.0259	-.0259	.9346	-.0654
	.2	-.0050	.0172	-.0558	.9019	.1417	-.0050	-.0050	.0122	.0122	-.0436	-.0436	.8583	-.1417
	.3	-.0062	.0213	-.0692	.8263	.2277	-.0062	-.0062	.0152	.0152	-.0541	-.0541	.7723	-.2277
	.4	-.0066	.0230	-.0745	.7359	.3222	-.0066	-.0066	.0163	.0163	-.0581	-.0581	.6778	-.3222
	.5	-.0065	.0224	-.0727	.6327	.4241	-.0065	-.0065	.0159	.0159	-.0568	-.0568	.5759	-.4241
	.6	-.0058	.0201	-.0652	.5189	.5320	-.0058	-.0058	.0143	.0143	-.0509	-.0509	.4680	-.5320
	.7	-.0047	.0163	-.0529	.3986	.6447	-.0047	-.0047	.0116	.0116	-.0413	-.0413	.3553	-.6447
	.8	-.0033	.0115	-.0372	.2680	.7611	-.0033	-.0033	.0082	.0082	-.0291	-.0291	.2389	-.7611
	.9	-.0017	.0059	-.0192	.1350	.8800	-.0017	-.0017	.0042	.0042	-.0150	-.0150	.1200	-.8800
	E	0	0	0	0	1.0	0	0	0	0	0	0	0	-1.0
+ Area		.4956	1.6548	1.8868	1.6548	.4956	.4956	.0462	.9187	.0485	.9434	.1136	.7361	.1855
− Area		-.1855	-.1598	-.0970	-.1598	-.1855	-.1855	-.7361	-.1136	-.9434	-.0485	-.9187	-.0462	-.4956
Total Area		.3101	1.4950	1.7898	1.4950	.3101	.3101	-.6899	.8051	-.8949	.8949	-.8051	.6899	-.3101



Influence coefficients — Four continuous spans.

L = Length of EXTERIOR spans; length of interior spans = NL.

N = 1.7

CHAPTER 21

Baker's Method for Ultimate Load Analysis of Indeterminate Concrete Structures

Use may be made of the flexibility method of structural analysis extended to loading beyond elastic behaviour of the structure but allowing plastic rotations at the hinges to within certain empirically predetermined permissible values. Concrete, being far less ductile as compared to steel, makes the estimation of allowable plastic rotations at hinges and of the average effective flexural rigidity of the cracked members over their lengths very difficult. Consequently, the solution to the problem, being only as accurate as the empirical evaluation of the above items, may not always be significantly close to truth. This is an established fact in the case of concrete.

For an n times statically indeterminate structure loaded to beyond its elastic behavioural capacity, assuming all n hinges are formed, the n compatibility equations of inelastic stability just prior to collapse of the structure are

$$
\begin{aligned}
V_{10} + V_{11}p_1 + V_{12}p_2 + \ldots + V_{1n}p_n &= -\theta_{p_1} \\
V_{20} + V_{21}p_1 + V_{22}p_2 + \ldots + V_{2n}p_n &= -\theta_{p_2} \\
V_{n0} + V_{n1}p_1 + V_{n2}p_2 + \ldots + V_{nn}p_n &= -\theta_{p_n}
\end{aligned}
$$

which may also be simply written as

$$V_{r0} + V_{rs}\,p_s = -\theta_{pr}$$

r ranging from 1 to n

s ranging from 1 to n

where

$V_{r0} = \int \dfrac{m_r m_0}{EI}\,ds$ = the movement at release no: r due to applied loading (designated with suffix zero)

$V_{rs} = \int \dfrac{m_r m_s}{EI}\,ds$ = the movement at release no: r due to unit value of plastic movement (bi-action) p applied at (release) hinge no: s

θ_{pr} = permissible plastic rotation at release (or hinge) no: r. $p_1, p_2, p_3, \ldots, p_n$, $m_1, m_2, m_3, \ldots, m_n$ and m_0 have already been defined in Ch. 20 (flexibility method of structural analysis).

EI is the effective modulus of rigidity (average of section to section) in a member assumed cracked as the structure is considered near a state of collapse.

Procedure

Step 1 Calculate the number of statical indeterminacies in the structure. Introduce as many releases into the structure, seeing that it remains statically stable. Draw $m_1, m_2, \ldots, m_n$ diagrams, the bending moment diagrams due to unit values of unknown biactions $p_1, p_2, \ldots, p_n$, respectively, the latter representing the plastic moments at hinges $1, 2, \ldots, n$, respectively, in this case on the statically made determinate structure. Draw m_0 diagram, the bending moment diagram due to applied loading, on the statically made determinate structure. (For all this follow the same procedure as in steps 1–4 in Ch. 20.)

Step 2 Work out θ_p values (i.e., permissible plastic rotations) for each (release) hinge from the empirical formulae suggested by Baker.[1]

Step 3 Work out EI values for each beam-member and column-member from the empirical formulae suggested by Baker.[1]

Step 4 Work out the influence coefficients V_{r0} and V_{rs} (r = 1 to n, s = 1 to n).

Step 5 Work out the first-trial values of the plastic moments $p_1, p_2, \ldots, p_n$, from the empirical formulae suggested by Baker.[1]

Step 6 Substituting the values worked out in steps 4 and 5, evaluate the left hand sides of the n compatibility equations mentioned earlier above. If these fall within their respective permissible values established in step 2 above, it is OK, if not, change one or some or (if necessary) all the assumed first trial values of $p_1, p_2, \ldots, p_n$, until the values worked out from the left hand sides of the said n compatibility equations fall within their respective θ_p values (established in step 2). The compatible $p_1, p_2, \ldots, p_n$ values are thus established (representing one possible compatible solution).

Step 7 Total ultimate moment at any section (immediately prior to collapse of the structure) can now be readily estimated from

$$m_t = m_0 + (m_1 p_1 + m_2 p_2 + \cdots + m_n p_n)$$

where $m_0, m_1, m_2, \ldots, m_n$ are the ordinates of these diagrams at that particular section.

REFERENCES

1. Baker, ALL, *Ultimate Load Design*. Concrete Publications, London.
2. Raina, VK,'Analysis of a Multistorey Multibay Reinforced Concrete Frame by Baker's Ultimate Load Method''. Internal Teach-In, Unpublished, Imperial College, London.

CHAPTER 22

Effect of Differential Settlement of Supports in a Statically Indeterminate Structure

22.1 EFFECTS OF (AN ASSUMED) PIER SETTLEMENT ON THE MOMENTS IN THE SUPERSTRUCTURE

In the simple case shown in Fig. 22.1 where a continuous beam of constant depth with a large number of identical spans is subjected to the settlement of one pier by a given amount δ, one may easily derive the effect in terms of moments and stresses in the superstructure. Taking the fixed end moment $M' = 6EI\delta/l^2$, the moments over the piers, at midspan and at quarter-span sections are

Over the pier subjected to settlement	$+0.732M'$
Over the adjacent piers	$-0.464M'$
Midspan moment	$+0.134M'$
Quarter-span amount	$+0.433M'$

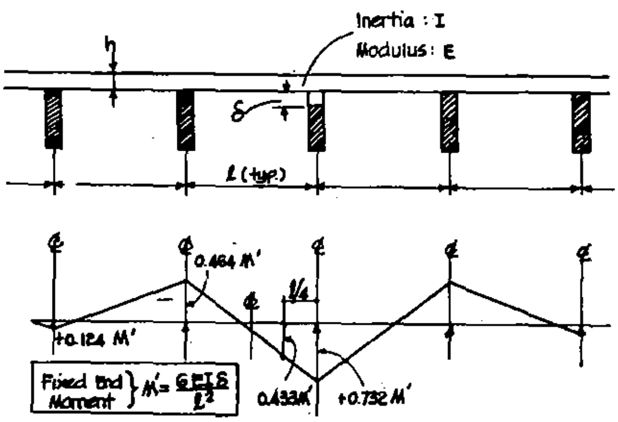


Fig. 22.1 *Effect of differential settlement on a continuous beam with equal spans and constant depth*

The stress produced at an extreme fibre at a section in the superstructure where moment is M, is $f = Mc/I$, where c is the distance between the centroid and the extreme fibre. If the moment is expressed as $M = kM'$, the stress becomes,

$$f = k\frac{6Ec\delta}{l^2}$$

which can be rewritten as follows

$$f = 6kE\frac{c}{h}\frac{h}{l}\frac{\delta}{l}$$

The value of c/h varies between 0.4 and 0.6 and that of h/l between $\frac{1}{18}$ and $\frac{1}{22}$.

Considering the quarter-span section close to the pier where settlement occurred, the stress in the bottom fibre in the superstructure will be, with $k = 0.433$ and E for concrete as 300,000 kips/ft^2 (for longterm loading), and assuming $\frac{c}{h} = 0.55$ and $\frac{h}{l} = \frac{1}{18.32}$,

$$f = 23,400\frac{\delta}{l} \qquad \text{(approx.)}$$

For a settlement $\delta = \frac{l}{1000}$, the stress is equal to 23.4 kips/ft^2 (11.4 kg/cm^2) at the bottom fiber, a very nominal value!

For a 100 ft span, the corresponding settlement is $\delta = 0.1$ ft = 1.2 inches (3 cm approx.).

The amount of settlement to be considered actually is only the part taking place after continuity is achieved in the deck.

In conclusion it can be readily seen that for the same stress effect, a longer span can take bigger differential settlement and conversely even a relatively small differential settlement can produce a significant stress in the deck in a relatively short span. In practice, aggregate additive moments (and shears) should be considered at each critical section due to possible settlement of each support individually.

22.2 CALCULATING THE EFFECT OF DIFFERENTIAL SETTLEMENT OF SUPPORTS IN A STATICALLY INDETERMINATE STRUCTURE BY THE FLEXIBILITY METHOD

If the structure is statically determinate the support settlements do not set up any moments.

Let an n times statically indeterminate structure be called S_n and the statically made determinate structure be called S_o. Consider a 2-span continuous beam ABC in which support B has sunk by δ vertically, [Fig. 22.2(a)] causing an angular movement (at B) of $\theta = \theta_1 + \theta_2 = \frac{2\delta}{l}$. There are two ways of finding the bending moments caused by support sinking.

First method Imagine a shear-type biaction p_1 [Fig. 22.2(b)] is introduced at B as shown, to make $\delta = 0$. Thus, (movement at B due to p_1) + $\delta = 0$, or $V_{11}p_1 + \delta = 0$, where $V_{11} = \int \frac{m_1 m_1}{EI} ds$, m_1 being the BMD due to $p_1 = 1$ on S_o.

$\therefore p_1$ can be known and total moment caused at any section (by the sinking of the support) found out from the product $m_1 p_1$ at that section.

Second method Imagine, instead, a moment-type biaction p_1 [Fig. 22.2(c)] is introduced at B.

For compatibility of deformations:

$$V_{11}\, p_1 + \theta = 0,$$

where

$$V_{11} = \int \frac{m_1 m_1}{EI} ds, m_1 \text{ being BMD on } S_o \text{ due to } p_1 = 1.$$

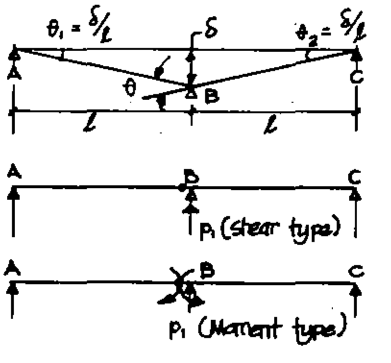


Fig. 22.2

$\therefore p_1$ can be known, and total moment caused at any section (by the sinking of support) found out from the product $m_1 p_1$ at that section.

Now, let us consider a 3-span continuous beam $ABCD$ where supports B and C sink by amounts δ_1 and δ_2 as shown in Fig. 22.3 causing angular movements θ_1 and θ_2 at B and C.

$$\left.\begin{aligned} \theta_1 &= \alpha - \beta = \frac{\delta_1}{l_1} - \frac{\delta_2 - \delta_1}{l_2} \\ \theta_2 &= \gamma + \beta = \frac{\delta_2}{l_3} + \frac{\delta_2 - \delta_1}{l_2} \end{aligned}\right\}, \text{ hence known.}$$

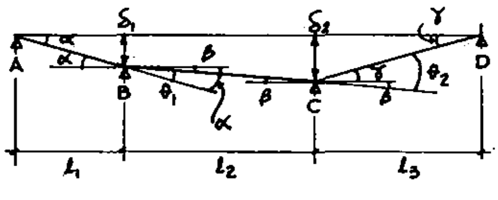


Fig. 22.3

Here again, if we introduce moment-biactions p_1 and p_2 at B and C, causing angular movements of $(V_{11}p_1 + V_{12}p_2)$ at B and $(V_{21}p_1 + V_{22}p_2)$ at C

where

$$V_{11} = \int \frac{m_1 m_1}{EI} ds, \quad V_{22} = \int \frac{m_2 m_2}{EI} ds,$$

$$V_{12} = V_{21} = \int \frac{m_1 m_2}{EI} ds,$$

m_1 being BMD on S_o due to unit p_1, etc.

Then, for total movements at B and C individually to be zero for compatibility of deformations, we have

at B: $(V_{11}p_1 + V_{12}p_2) + \theta_1 = 0$

at C: $(V_{21}p_1 + V_{22}p_2) + \theta_2 = 0$

whence p_1 and p_2 can be found, and total moment (due to support-sinking) at any section evaluated from

$$(m_1 p_1 + m_2 p_2)$$

Numerical Example

EI constant, 2 span continuous beam, $n = 1$ (Fig. 22.4) due to p_1, angular movement at B is

$$V_{11}p_1 = \left(\int \frac{m_1 m_1}{EI} ds\right) p_1$$

where

$$\int \frac{m_1 m_1}{EI} ds = \frac{2}{EI}\left[\frac{20}{6}\left(0 + 4 \times \frac{1}{2} \times \frac{1}{2} + 1\right)\right]$$

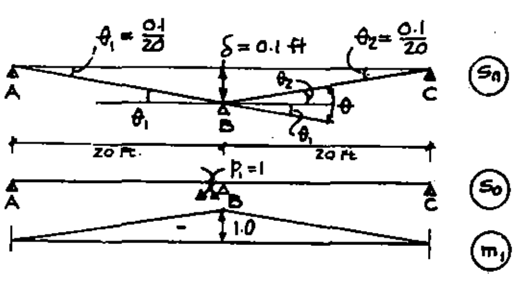


Fig. 22.4 *Numerical example*

$$= \frac{2}{3} \times \frac{20}{EI}$$

$$\theta \text{ (due to } \delta) = \frac{0.1}{20} + \frac{0.1}{20} = 0.01 \text{ radian}$$

from $(V_{11}p_1 + \theta) = 0$, we get $\frac{2}{3} \times \frac{20}{EI}p_1 + 0.01 = 0$

$$p_1 = -\frac{3EI}{4000}$$

Hence total moment (due to support sinking), m^t, at different sections:

$$m_A^t = m_1p_1 = 0$$

$$m_B^t = m_1p_1 = (-1) \times \left(-\frac{3EI}{4000}\right) = \frac{3EI}{4000}$$

$$m_C^t = m_1p_1 = 0$$

CHAPTER 23

Reinforced Concrete Design

23.1 GENERAL BACKGROUND AND PRINCIPLE OF REINFORCED CONCRETE DESIGN

As explained earlier, the load-deformation curve for reinforced concrete is linear during the early stages of its loading history — which is the elastic phase — and thereafter, the deformation increases much more rapidly through the elastoplastic phase to the plastic phase.

The design of reinforced concrete based on the elastic behaviour (i.e., corresponding only to the early elastic phase) is based on five basic assumptions, viz., the stress strain curve is a straight line, plane sections remain plane even after bending (i.e., strains at various fibre levels down any section remain linearly compatible with each other), and the material is homogeneous, isotropic and crackfree. The first assumption is not really true if the total load-deformation curve is kept in view (in case of any unforeseen distress, the deformational response may well go past the linear elastic phase). The third, the fourth and the fifth assumptions are unrealistic in the context of structural concrete, more so in reinforced concrete in which cracking (within limits) is incumbent by virtue of the neutral axis concept. Micro-cracking in concrete commences right from the instant of its initial setting, and as the applied load effect increases, visible cracking sets in even in prestressed concrete, invalidating the rationality of the perpetuality of the above assumptions. This being so, it is very important to ponder whether using the formulae which are derived on the basis of the aforementioned assumptions can hence yield results which may be irrational vis-a-vis the actual stresses in the structure. It is, therefore, more realistic to accept the fact that structural concrete is partially cracked and design it for a loading that is higher than the working load, even if higher material stresses have to be allowed. This higher load could be a certain factored value of the working load, but still lower than the ultimate load that would cause the physical collapse of the structure.

This approach is particularly necessary for designing against those of the load effects under which the collapse could be sudden (for example shear and torsion) because these effects are associated with very little redistribution. This is why there is no increase in allowable stress under such load effects. Unlike shear (and torsion) fortunately the bending moment undergoes redistribution, and unless the sections are over-reinforced (which is unusual), the structure will accept enough visible deformation in high moment zones (Which behave like hinges) and thereby, give enough warning of impending failure. This, however, will depend upon the ductility of the structure in these high moment zones. Otherwise these zones will not 'rotate' enough, thus leading to shortfall in moment redistribution and an earlier collapse without the other hinging zones in the structure maturing fully. (Full maturity of a hinge means that its critical section develops its full ultimate moment capacity).

'Elastic' Versus 'Load-Factor' Approach

(i) In *elastic* (i.e., working load) analysis of reinforced concrete, flexural compressive stress in concrete is always assumed to be a certain fraction of its crushing strength and tensile stress in steel is assumed to be a certain fraction of its yield strength. The saftey factor is then defined as the ratio between failure stress and working stress. This, however, does not mean that if stress in concrete is slightly exceeded, the concrete will crush. Nor does it mean that concrete will not crush until load is increased by the ratio between the crushing strength of concrete and this stress. The former is true since reasonable over-stressing is found to be possible. The latter is true, since, beyond elastic range, the stress-strain relation is non-linear, whereby ultimate compressive strain in concrete develops while the compressive strain is still less than its crushing strength. Thus it is apparent that elastic analysis is conservative in its outlook and does not consider the structural performance beyond the elastic range.

(ii) In *plastic* (i.e., ultimate load) analysis it is assumed that the member is behaving plastically or non-linearly (past the elastic range), and that it is loaded to its ultimate such that it is about to collapse and excessive deformation has set in certain critical zones. This excessive deformation constitutes yield lines in slabs and plastic hinges in columns and beams. Thus, it is apparent that plastic (i.e., ultimate load) analysis is perhaps too broad and risky an outlook.

(iii) In the *load factor approach* the assumed applied load, while still a certain fraction of the ultimate load, is assumed to be a certain factor times the working load. This type of analysis, in which the permissible stresses in concrete and steel are also 'load-factored' values of the elastic stresses permitted in them hitherto, is an analysis intermediate between elastic and ultimate concepts. This type of design analysis, called Load Factor Analysis is, therefore, far more logical and reasonable to apply.

(iv) The basic difference between the elastic and load-factor methods of analysis is that with the elastic method we ensure that the structure will behave satisfactorily at working load but only assume that hopefully there will be a satisfactory factor of safety against failure, whereas with the load factor method we ensure that we have a satisfactory factor against failure but only assume that the structure will behave satisfactorily at working load.

(v) It follows then, that if a Load Factor approach is followed (when the section is designed for a load effect equal to suitably factored working load value, adopting load-factored material stresses, and the two sets of load factors may be different), at least certain serviceability criteria must additionally be ensured in order to guard against any unsightly deformations like large crack-widths and deflections.

(vi) This load factor method, intermediate between working load and ultimate load, (sometimes loosely referred to as ultimate strength method by some) should be adopted as much for its consequence after construction as for its refinements in logic and practical precision during design.

Acceptance of the Concepts

It took decades to accept the load factor approach in the design of reinforced concrete building elements. It is taking longer to accept this approach for the design of bridge elements, possibly because the bridge school feels, that unless the values for load factors on various loads and material stresses are established beyond any shadow of doubt, any such refinement in design method—without a matching assurance on quality control and possible load-variation, may lead to collapse of some bridges, and that this can be much more catastrophic than the partial collapse of a building. This may, however, be only part of the reason for resistance to acceptance, the other reasons being the average designer's limited understanding and design capability coupled with lack of inertial impulse needed to get out of the beaten-track. Fortunately, at least for shear (and torsion), the load factor approach has now been accepted by many a bridge-code, perhaps more because shear (and torsion) failures can be ruthlessly sudden, so much so that the design authorities cannot afford to be complacent much longer.

Having thus briefly introduced the subject of philosophy of design of reinforced concrete, the remainder of the current chapter will now very briefly attempt a quick refresher for practical design of reinforced concrete elements by the two stated methods. It will be presumed that the reader is reasonably conversant with the basic subject-matter and has himself already carried out some reinforced concrete design.

23.2 ELASTIC DESIGN METHOD

This method of design is also referred to as *working load method of design* or *modular-ratio method of design*. Let us consider this in two sections, one being *beams and slabs* and the other *columns*.

Beams and Slabs

• ***Bending***

(See Fig. 23.1)

(i) M = applied moment, MR = moment of resistance of section as singly reinforced section

A_s = total tensile steel A'_s = compression steel

E_s and E_c = modulii of elasticity of steel and concrete

f_s = tensile stress in steel, f_c = maximum flexural compressive stress in concrete (permissible values).

Now MR for Type A cross-section = (area × stress) × (lever arm) = $(bndf_c/2)(d - nd/3)$

$$= \left[n\frac{f_c}{2}(1 - n/3)\right] bd^2 = Rbd^2,$$

where $R = n\dfrac{f_c}{2}(1 - n/3)$

or $n \cdot \dfrac{f_c}{2} \cdot j$,

where $j = (1 - n/3)$, and n from strain compatibility, is obtained from:

$$\frac{f_c}{E_c}/nd = \frac{f_s}{E_s}/(d - nd),$$

i.e., $(mf_c/f_s) = n/1 - n,$

so that $n = \dfrac{1}{1 + \dfrac{f_s}{mf_c}}$, where $m = E_s/E_c$ (modular ratio)

and MR for Type B cross-section singly reinforced ≈ $b \cdot h_t \cdot \dfrac{f_c}{2} \cdot \left(d - \dfrac{h_t}{2}\right)$

(ii) If the cross-section is of Type A or Type B (see Fig. 23.1)

(a) if $M <$ the respective MR then the section is O.K. as singly reinforced; then reinforcement is as follows:

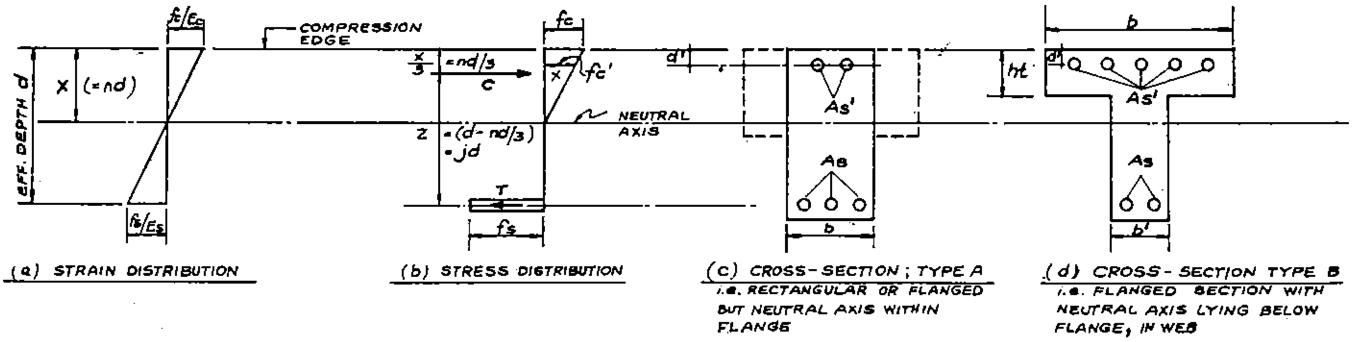


Fig. 23.1 *Bending*

- In Type A cross-section

$$A_s = \frac{M}{f_s jd} \text{ and } As' \text{ is not required.}$$

- In Type B cross-section: Ignoring the web-portion in compression

$$A_s = \frac{M}{f_s(d - h_t/2)} \text{and } As' \text{ is not required.}$$

(b) if $M >$ the respective MR then the section requires to be reinforced in compression also (i.e. doubly reinforced) with additional tension reinforcement to balance the compression steel, then the reinforcement is as follows:

- In Type A cross-section

$$A_s = \frac{(MR)}{f_s jd} + \frac{M - (MR)}{f_s(d - d')} \text{ and } A_s' = \frac{M - (MR)}{mf_c'(d - d')}$$

where $f_c' = f_c \dfrac{(nd - d')}{nd}$

- In Type B cross-section ignoring the web-portion in compression:

$$A_s = \frac{(MR)}{f_s(d - h_t/2)} + \frac{M - (MR)}{f_s(d - d')}, \text{ and}$$

$$A_s' = \frac{M - (MR)}{mf_c'(d - d')}$$

where $f_c' = f_c \dfrac{(nd - d')}{nd}$

(iii) If the cross-section is (a) other than of types A or B, or (b) even if it is of type A or type B but it is required to do an exact elastic design (e.g., either not ignore the part of the web in compression or, alternatively, assume values for A_s and the section dimensions by trials and then check stresses in concrete and steel, until stresses are found satisfactory), or (c) the section and reinforcement are known and exact stresses in concrete and steel are required to be found under a given M, then in these cases:

- locate the neutral axis by equating about it the moments of 'effective areas' in compression and tension (i.e., equate the 'moments of areas' of 'concrete and steel in the compression zone' and that of 'steel in tension zone') about the unknown location

- ***Shear***

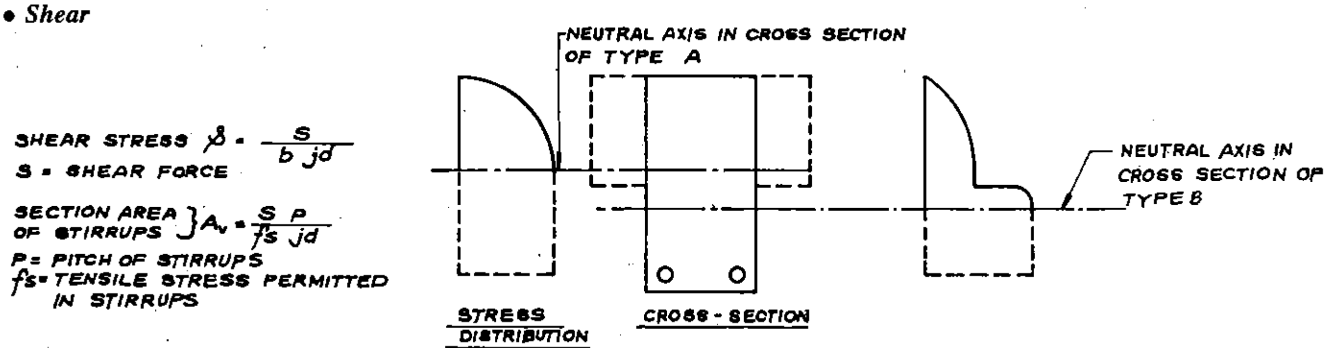


Fig. 23.2 *Shear*

NOTE: **For design against SHEAR (and TORSION) refer to Chapter 24 for details** of the neutral axis. (Note that in reinforced concrete the location of the neutral axis depends on f_c, f_s and m, and, therefore, not directly on the applied moment M. Although the actual values of f_c and f_c obviously depend on M their 'ratio'—whether of their actual values (which can be known only after the design) or of their limiting values—that enters the formula for n, does not affect the value of n very much).

- estimate the moment of inertia I (second moment of area) of the 'effective section' about the neutral axis.
- then compute the maximum flexural compressive stress in concrete and maximum tensile stress in steel from $\left(\frac{M}{I}y_c\right)$ and $\left(\frac{M}{I}y_t m\right)$ respectively, y_c and y_t being the distances of neutral axis from the extreme compression and tension fibres.

- ***Local Bond***

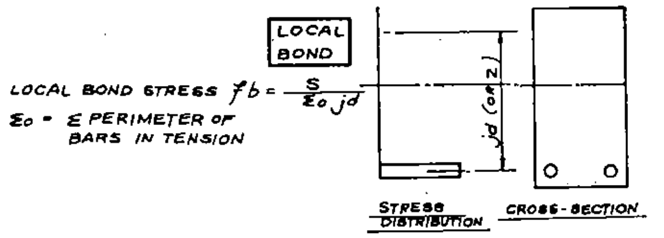


Fig. 23.3 *Local bond*

- ***Crack Control***

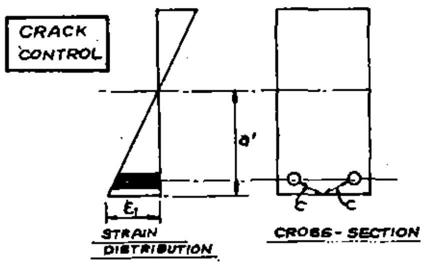


Fig. 23.4 *Crack-control*

For Crack Control, refer to "Serviceability Limit State of Cracking" given ahead in this chapter.

Columns

These may be either 'concentrically loaded' or 'eccentrically loaded' in service.

Concentrically Loaded Columns

In these the longitudinal (i.e., the main) reinforcement is bound together either by independent links or by continuous (helical) binders. The design and detailing are fairly simple in each case and reference may be made to any standard code for this purpose. Reference may also be made to ch. 10 in this book so as to first establish whether the column is short or long. If short, then design the section for the load-value by the pertinent formula in the code. However, if the column is 'long* then first establish the enhanced value of its design load and then design its section by the pertinent codal formula. In case of really long* columns, alternatively establish the design load and possible moments by the second order theory taking into account the buckling effects and then design the section for this load by the pertinent codal formula.

Eccentrically Loaded Columns (Bending about any axis)

As in the case of concentrically loaded columns, here too it is necessary to first establish whether the column is short or long. If short, the load and moment values remain as established from the first order theory, and sections can be designated for these by the regular method.**. However, if the column is long*, then first establish the design values of the enhanced load and moment as for a long column and then design its sections by the regular method**. In case of really long* columns, alternatively establish the design values of the load and the moment by the second order theory taking into account the buckling effects, and then, for the so established design values of load and moment at various sections, design these sections by the regular method**

23.3 LOAD-FACTOR DESIGN METHOD

This method of design is also referred to as the *ultimate strength design method,* and, some even go as far as (wrongly) calling it 'ultimate load design method'. We shall consider this subject in three sections, one being the background information in the run up to the second section that deals with beams and slabs and the third that deals with the columns.

Background Information

Significance of the Load-Factor and its Assessment

When a structure is designed on an elasto-plastic load-characteristic basis, it is analysed for a load equal to the working load multiplied by appropriate load factors. Since a great deal of care is usually taken when analysing the structure, it is only reasonable to assume that a similar

* *Long Column*: Details are given in Ch. 10 under reference.

** *Regular method*: Details of this elastic design method are given in Ch. 25 and it covers biaxial and indeed any axis bending.

Table 23.1 *Elastic Design—permissible stresses in concrete and steel bars as per BS CP 110*

	Concrete (f_{cu} = 28-day standard cube strength) (N/mm²)				
	Concrete → (f_{cu})	30.0	37.5	45.0	
Permissible	Bending stress	10.0	12.5	12.5	
	Direct stress	7.6	9.5	9.5	
	Shear : beams	0.87	0.87	0.87	
	: slabs	Related to percentage of reinforcement			†plus 25% for deformed bars
	Average bond†	1.00	1.00	1.00	
	Local bond†	1.47	1.47	1.47	
	Modulus of elasticity	28kN/mm²	30.25kN/mm²	35.5kN/mm²	
	Reinforcement (N/mm²)				
		Mild Steel ≯ 40mm dia.	**Mild Steel** > 40mm dia.	**High-yield** steel	
Permissible	Tension	140	125	230	Subject to crack-width control
	Compression	125	110	175	
	Range of variation	265	235	325	
	Shear reinforcement	140	125	175	
	Yield stress	200	200	330	
	Characteristic stress	250	250	425	
	Modulus of elasticity	200kN/mm²	200kN/mm²	200kN/mm²	

degree of care should also be taken in assessing the load factor.

The term load factor tends to imply that it is only a factor against possible overload, but, this is not strictly true. In order to see what this factor covers, it is convenient to trace the steps an engineer treads before obtaining his final design.

His first task is to assess his working load, and this implies that this working load is the one associated with the length of life of the structure. He, therefore, has to view, not necessarily the working load today, but also that which will occur in the next fifty years or so. It is only reasonable to suppose that this estimate is subject to a certain amount of doubt, and the degree of doubt will clearly depend on his source of information. His best assessment will be obtained from information based on a careful statistical study over a large number of years; his worst assessment will be when he has to purely guess. Part of the load factor should, therefore, bear a relation to the accuracy of the assessment of the maximum working load.

The next step in the design is to analyse the structure under a given loading configuration. Any analysis is based on assumptions which rarely are a representation of all the characteristics of the structure in the field. Certain aspects are usually neglected because the effects are small, but they still exist and, therefore, the analysis is not usually a complete picture of the true behaviour. Coupled with the analysis are further assumptions concerning the dimensions, and with each successive assumption the structure behaves in a slightly different way from its supposed replica. Having obtained from the analysis the numerical values of the forces and moments in the structure, the beams, slabs and columns may then be designed to 'fail' at these moments. The design equations are again based on assumptions, usually neglecting creep, neglecting thermal stresses and neglecting foundation settlements. It is, therefore, realistic that some part of the load factor should also include an item which represents the expected inaccuracy of the particular analysis that has been used.

In the ultimate strength equations, values for the strength of concrete, the yield stress of steel. The section dimensions, and the value for the steel percentage are involved. Engineers should also remember the days when concrete is poured in the rain or in hot weather or at near freezing temperatures, the days shutters slip, and the days when there is no one to supervise or inspect the work in progress. Certainly, part of the load factor must include a provision against all these, because these things do happen!

The value of the load factor should also include an item which tries to measure the seriousness of collapse, since it is clearly more important to prevent failure if hundreds of people will be killed and the mishap will cause an economic crisis! Consequently, the load factor must be modified and increased if this is likely to happen (degree of seriousness of the possible mishap).

In a philosophical approach to structural design, mainly due to the work of the international committees, the CEB published its *Recommendations for an International Code of Practice for Reinforced Concrete* in 1963, generally known as the *Blue Book*, and later in conjunction with the FIP, a complementary report dealing with prestressed concrete. Further to these, there was published in 1970 *The Interna-*

tional Recommendations for the Design and Construction of Concrete Structures, giving the principles and recommendations, and generally known as the *Red Book*.

In design the following points have to be taken into consideration

(i) Variations in materials in the structure and in the test specimens
(ii) Variations in loading
(iii) Constructional inaccuracies
(iv) Accuracy of design calculations
(v) Safety and serviceability of the structure

The various criteria required to define the serviceability or usefulness of any structure can be described keeping in mind the following headings as being 'unfit for use':

(i) *Collapse* Failure of one or more critical sections; overturning or buckling.
(ii) *Deflection* The deflection of the structure or any part of the structure adversely affecting the apearance or efficiency of the structure.
(iii) *Cracking* Cracking of the concrete which may adversely affect the appearance or efficiency of the structure.
(iv) *Vibration* Vibration, from forces due to wind or earthquake or machinery, may cause discomfort or alarm, damage to the structure, or may interfere with its proper function.
(v) *Fatigue* Where loading is predominantly cyclic in character the fatigue effects have to be considered.
(vi) *Durability* Porosity of concrete, leading to ingress of deleterious materials and
(vii) *Fire Resistance* Insufficient resistant to fire leading to (i), (ii) and (iii) above.

When any structure is rendered unfit for use for its designed function by one or more of the above causes, it is said to have entered a *limit state*.

(a) *Ultimate limit state* Ultimate limit state is preferred to collapse.
(b) *Serviceability limit states* Deflection, cracking, vibration.
(c) *Other limit states* Special requirements for unusual or special functions of a structure.
(d) *Other considerations* Fatigue, durability, fire resistance, lightning, etc.

The purpose of design then, is to ensure that the structure being designed will not become unfit for the use for which it is required, i.e., that it will not cross a particular limit state. The essential basis of the design method, therefore, is to consider each limit state and to provide a suitable margin of safety correspondingly.

Accepting the fact that the strengths of constructional materials vary, as also do the loads on the structure, two partial safety factors may be used. One will be for mateials and is designated γ_m, and the other, for loading, is termed γ_f. These factors will vary for the various limit states. As new knowledge on either materials or loading becomes available, the factors can be amended quite easily without the complicated procedure to amend one overall factor as in the earlier days.

Characteristic Strength of Materials

For both concrete and reinforcement, the codes use the term characteristic strength instead of the 28-day works cube (or cylinder) strength and yield stress, although it is still related to these. The characteristic strength for all materials has the notation f_k and is defined as the value of the strength of concrete, and of the yield or proof-stress of reinforcement, below which not more than 5% of the test results may be expected to fall.

The value, therefore, is

$$f_k = (f_m - 1.64\ s)$$

where f_m is the mean strength of actual test results determined in accordance with a standard procedure, s is the standard deviation, 1.64 is the value of the constant required to comply with not more than 5% of the test results falling below the characteristic.

(a) Concrete The strength of the concrete is based on tests made on cubes (UK practice) at an age of 28 days unless there is satisfactory evidence that a particular testing regime is capable of predicting the 28 days strength at an earlier age. For concrete the characteristic strength has the notation f_{cu}. The quantity 1.64 s is called the 'current margin' and the concrete mix should be designed to have a mean strength greater than the required characteristic strength by at least this current margin. If there is insufficient data from cube tests to enable a value for the standard deviation to be used, then the margin for the initial mix design must be given an actual numerical value depending on the required characteristic strength.

Concrete is classified into grades, where the grade number is the chaacteristic cube strength. This is shown in Table 23.2 which also indicates the lowest grade for compliance with appropriate use.

From this table it will be seen that the lowest grade of concrete for reinforced concrete with dense aggregate is 20.

So, assuming that this grade of concrete is being used as a designed mix and, from test data the standard deviation is taken as 7.5 N/mm^2 then, a mean strength of (20 + 1.64 × 7.5) = 32.3 N/mm^2 would be the target strength. With sufficient data the current margin for this grade of concrete is 15 N/mm^2 and the mean strength would be (20 + 15) = 35 N/mm^2.

Table 23.2 *Grades of Concrete (BS CP 110)*

Grade	Characteristic strength N/mm^2	Lowest grade for compliance with appropriate use
7	7.0	—Plain concrete
10	10.0	
15	15.0	—Reinforced concrete with light weight aggregate
20	20.0	—Reinforced concrete with dense aggregate
25	25.0	
30	30.0	—Concrete with post-tensioned tendons
40	40.0	—Concrete with pre-tensioned tendons
50	50.0	
60	60.0	

Strength compliance is judged by test results from a suitable testing regime where the rate of sampling and testing depends on the nature of the work and the volume of concrete at risk. For example, a higher rate of sampling would be required for highly stressed structural members. Also, it would be appropriate to have higher rates of sampling and testing at the start of the work to establish the level of quality quickly. The actual rate of sampling will fluctuate, but at least one set of samples should be taken from each days' concrete of each particular grade, subject to a specified minimum number per batch or per 50 m^3 of poured concrete. Compliance with the specified characteristic strength may be assumed if,

(i) The average strength from any group of four consecutive test cubes exceeds the specified characteristic by not less then 0.5 times the current margin,

(ii) Each individual test result is greater than 85% of the specified characteristic strength

If only one cube result fails to meet the second requirement, then only that particular batch from which the cube was taken does not comply. But if the average strength of any group of four consecutive cubes fails to meet the first requirement then all the batches between the including those from which the first and last samples were taken do not comply. In this case, the mix proportions shall be modified to increase the strength and the engineer will then determine the action to be taken. (These 'criteria' specifications can vary depending on which code governs. In this connection, reference may also be made to the relevant chapter in the author's other book, *Concrete for Construction—Facts and Practice*.)

(b) Reinforcement The reinforcement may comply with BS 4449, BS 4461 or BS 4483, all of which specify the tests for compliance to obtain the characteristic strength which has the notation f_y. The designation of the reinforcement with its specified characteristic strength is shown in Table 23.3.

From the table it will be seen that the characteristic strength of high yield bars depends on whether they are

Table 23.3

Designation	Nominal sizes. mm.	Specified characteristic strength (f_y) N/mm^2
Hot rolled mild steel (BS4449)	All sizes	250
Hot rolled high yield (BS4449)	All sizes	410
Cold worked high yield (BS4461)	Up to and including 16	460
	Over 16	425
Hard drawn steel wire	Up to and including 12	485

'hot rolled' or 'cold rolled' worked. A further sub-division is made in BS CP 110 to determine the bond characteristics.

Characteristic Loads.

For loading we use the 'characteristic' load (F_k) as the basis. Ideally, this should be determined from the mean load and its standard deviation from the mean, and using the same probability as for the materials we should say that $F_k = F_m + 1.64s$. The characteristic load would be that value of loading so that not more than 5% of the spectrum of loading throughout the life of the structure will lie above the value of the characteristic load.

Design Strength of Materials

The design strengths of the materials can be obtained by dividing the characteristic strengths by the partial safety factor γ_m i.e. design strength = f_k/γ_m.

γ_m takes account of possible differences between the material in the actual structure and the strength derived from test specimens.

In concrete, this would cover such items as insufficient compaction, differences in curing, etc. For reinforcement, such items as the difference between the assumed and the actual cross-sectional areas (caused by rolling tolerances, corrosion, etc.)

The values of γ_m for each material will be different for the different limit states by virtue of the different probabilities that can be accepted for each individually.

Table 23.4 sets out these values, as per BS CP 110.

Table 23.4 *Values as per BS CP 110*

Limit state	Values of γ_m	
	Concrete	Steel
Ultimate	1.5	1.15
Deflection	1.0	1.0
Cracking	1.3	1.0

Considering the values for both materials, the factor for the ultimate limit state is higher than the others, because, not only must the probability of failure be decreased, but failure could be localised. So the γ_m factor also contains an allowance for this and as a compressive failure in concrete is sudden and without warning the factor for concrete is higher than for reinforcement.

Deflection is related to the whole member and the factor for both materials is 1.0.

For cracking, only parts of the member are affected and a factor in between 1.0 and 1.5 for concrete has been selected, but kept at 1.0 for reinforcement.

When analysing any cross-section within the structure, the properties of the materials should be assumed to be those associated with their design strength appropriate to the limit state being considered.

The short-term design stress-strain curve for concrete is shown in Fig. 23.5 and by putting-in the relevant value of γ_m, depending on the limit state being considered, the appropriate design stress-strain curve can be obtained, as shown in Fig. 23.6.

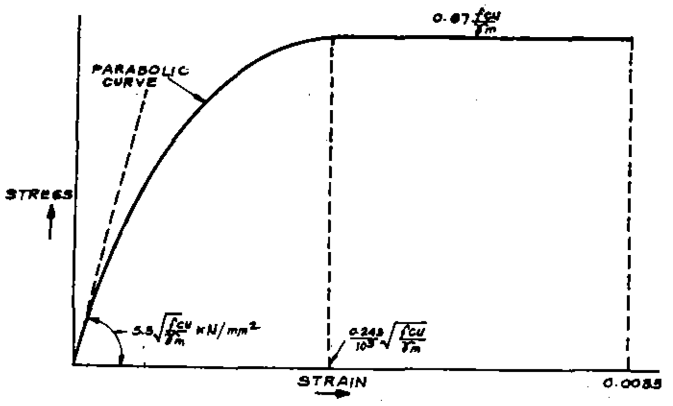


Fig. 23.5 *Short-term design stress-strain relation for normal weight concrete (f_{cu} in N/mm²) (BS CP 110)*

The design strength has been defined as characteristic strength devided by γ_m and yet the maximum stress value is given as 0.67 $\frac{f_{cu}}{\gamma_m}$ (BS CP 110). The reason for this is that the characteristic strength has been derived from tests on cubes. It is well established from tests that the maximum compressive stress at failure in a member of the same concrete as a cube has a value in the region of 0.8 f_{cu}. This is a peak value, and as an additional safety factor against compressive failure this value has been reduced to 0.67 f_{cu}, which agrees with the present design methods using ultimate load. If a cylinder was to be used in determining the characteristic strength, the factor would be of the order of 0.85 as the cylinder strength is nearer the actual behaviour and is approximately 0.8 × cube strength.

For the serviceability limit states Poisson's Ratio may be taken as 0.2.

For reinforcement the short term stress strain relationship is shown in Fig. 23.7.

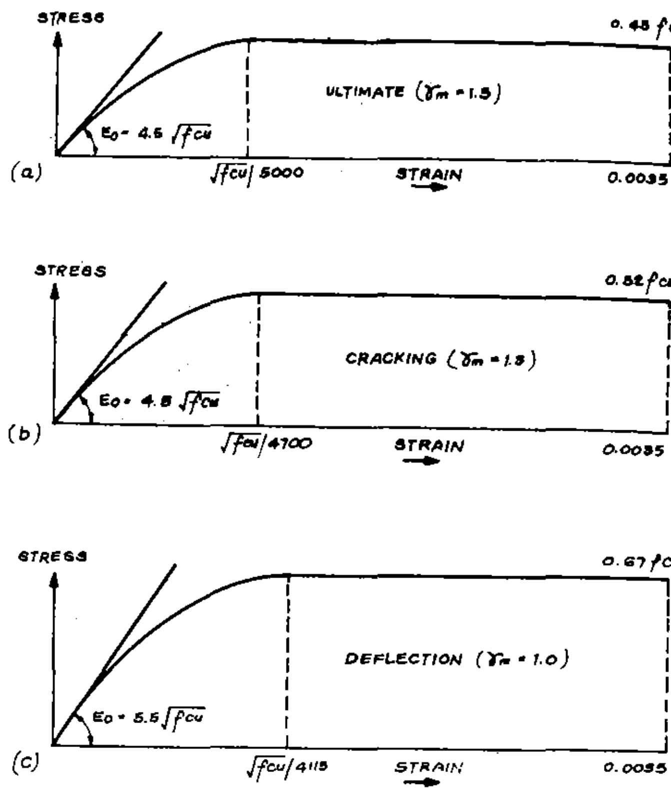


Fig. 23.6 *Short-term design stress-strain curves for concrete ultimate and serviceability limit states (BS CP110)*

NOTE: (1) Ultimate Limit State — Alternative Mean Stress 0.4 f_{cu}
(2) Serviceability Limit States — Alternative Linear Stress-Strain Relationship with Specified Value for E_0 Dependent on f_{cu}
(3) f_{cu} in N/mm² and E in kN/mm²

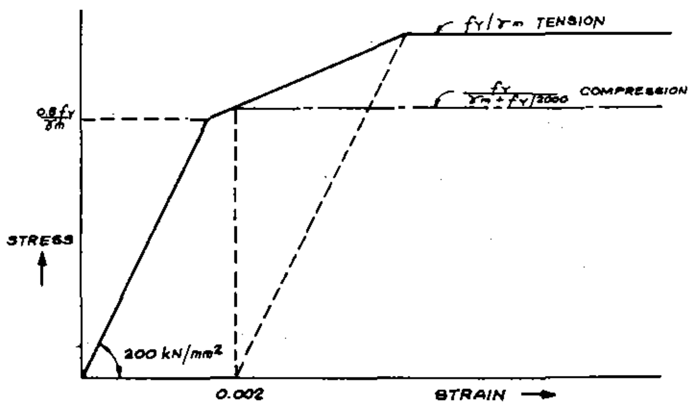


Fig. 23.7 *Short-term design stress-strain relation for reinforcement (f_y in N/mm²)*

The relationship, which is trilinear, is for all grades of reinforcement, and again, by putting-in the relevant factors for γ_m, the appropriate curves for the different limit states can be obtained and these are shown in Fig. 23.8. The elastic modulus for all types of loading may be taken as 2000 kN/mm^2.

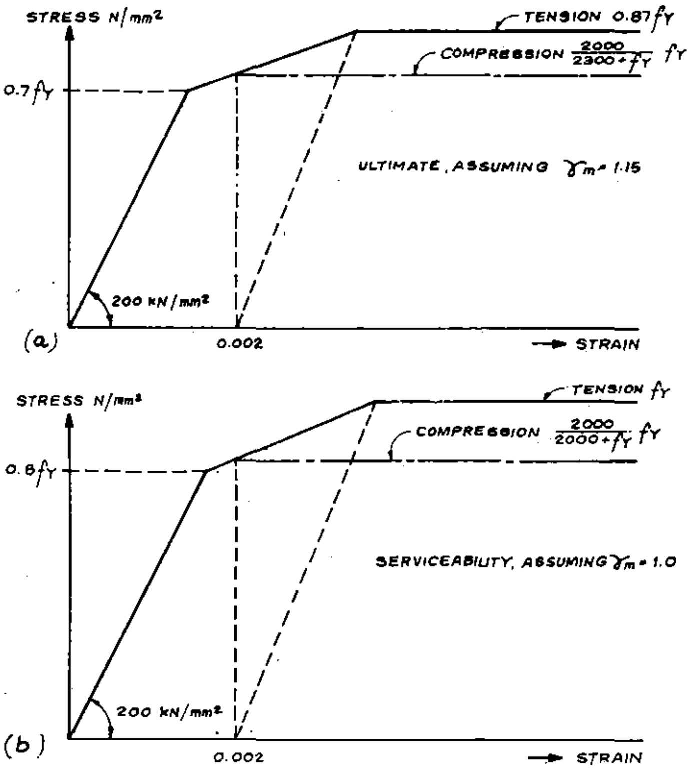


Fig. 23.8 *Short-term design stress-strain curves for reinforcement (ultimate and serviceability limit states)*

Design Loads

The design load is obtained by multiplying the characteristic load by the other partial safety factor γ_f. This factor γ_f is introduced to take account of,

(i) Possible unusual increases in the load beyond those in deriving the characteristic load
(ii) Inaccurate assessment of effects of loading
(iii) Variations in dimensional accuracy achieved in construction,
(iv) The importance of the limit-states being considered. γ_f varies for the different limit-states and values as per CP110 are set out in Tables 23.5 and 23.6.

The criteria to be complied with for the various limit states are broadly as follows.

(i) *Ultimate limit state* The strength of the structure should be sufficient to withstand the design loads.
(ii) *Serviceability limit states*
  (a) *Deflection* The engineer must be satisfied that deflections are not excessive having regard to the particular structure, but reasonable limits may be set by the pertinent code of practice.
  (b) *Cracking* The assessed surface width of cracks should not, in general, exceed 0.3 mm and, for particularly aggressive environments, the assessed surface crack widths at points nearest the main reinforcement should not, in general, exceed 0.004 times the nominal cover. The BS CP 110 points out that it is not possible to predict an absolute maximum crack width and the possibility of some cracks being wider than the above must be accepted unless special precautions are taken.
  (c) *Vibration* Limits for this are generally not given in the codes. Reference may be made to a separate chapter in this book on this subject.
  (d) *Other limit states* Special structures will comply with additional limit states, as considered necessary by the engineer.

Table 23.5 *Values of γ_f: Ultimate Limit State*

Load combination	Dead load		Imposed load		Wind load
	max.	min.	max.	min.	
1. Dead and imposed load	1.4	1.0	1.6	0	0
2. Dead and wind load	1.4	0.9	0		1.4
3. Dead and imposed and wind load	1.2		1.2		1.2

Table 23.6 *Values of γ_f: Serviceability Limit State*

Load combination	Dead load	Imposed load	Wind load
1. Dead and imposed load	1.0	1.0	0
2. Dead and wind load	1.0	0	1.0
3. Dead and imposed and wind load	1.0	0.8	0.8

Beams and Slabs

• In estimating the ultimate (capacity) moment of resistance of a reinforced concrete section, the following assumptions can, therefore, be made. (BS CP 110):

In Concrete in Flexural Compression

1. The strain distribution in the concrete in compression is derived from the assumption that plane sections remain plane, and

2. The stress distribution in the concrete in compression is derived from the stress strain curve in Fig. 23.5 with γ_m = 1.5 (refer to Fig. 23.6(a) and Fig. 23.10). Or, it may be taken as a rectangle with a stress value of $0.4 f_{cu}$ over the whole compression zone (Fig. 23.9).

In both cases, the strain at the outermost compression

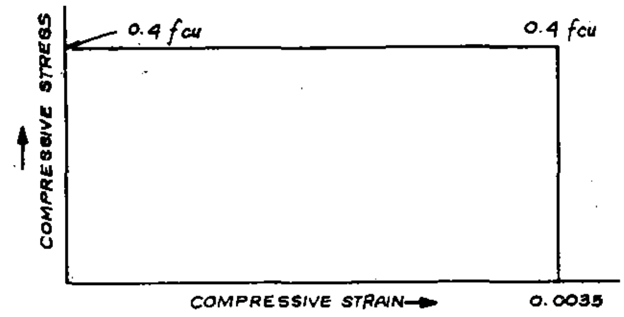


Fig. 23.9

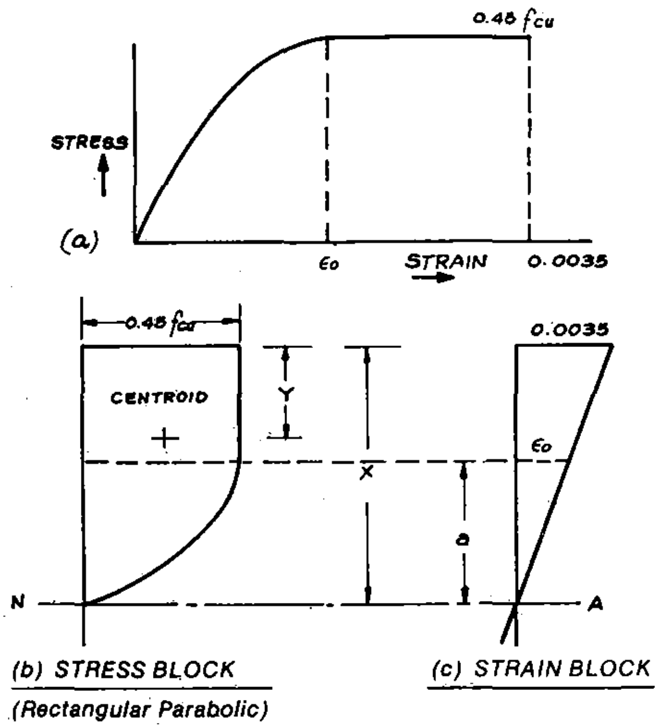


Fig. 23.10

fibre at failure is taken as 0.0035. Where beams are reinforced in tension only, the depth of the concrete in compression is limited to half the effective depth, i.e., $x \leq 0.5d$. The first diagram is usually referred to as the rectangular-parabolic stress block. From the co-ordinates given, the properties of this stress block for a particular concrete grade can be worked out. A chart relating the properties is shown in Fig. 23.11 and it can be seen that the constant stress value of 0.4 f_{cu} for the equivalent rectangle case is a fairly good approximation in the lower grades, but decreases with the higher grades of concrete (see line for coefficient K_1).

The values in Fig. 23.11 can be obtained basically by setting up two equations, one for the area of the stress block, and the other by equating the moments of the total stress-block area and of its component areas about (say) the extreme compression edge. This is done as follows:

From strain block $\dfrac{a}{x} = \dfrac{\varepsilon_0}{0.0035}$

$$\begin{aligned}
\text{Area of stress block} &= 0.45 f_{cu} x - \frac{0.45 f_{cu} a}{3} \\
&= 0.45 f_{cu} x \left(1 - \frac{\varepsilon_0}{3 \times 0.0035}\right) \\
&= \frac{0.45 f_{cu}}{0.0035} \left(0.0035 - \frac{\varepsilon_0}{3}\right) x \\
&= f_m x = K_1 f_{cu} x
\end{aligned}$$

Taking moments about compressive face and if y is the distance of the centroid of the total rectangular-parabolic stress block from the compressed face, then

$$\begin{aligned}
y &= \frac{0.45 f_{cu} x \frac{x}{2} - \frac{0.45}{3} f_{cu} a \left(x - \frac{a}{4}\right)}{0.45 f_{cu} \left(x - \frac{a}{3}\right)} \\
&= \frac{6x^2 - a(4x - a)}{4(3x - a)} \\
&= \left\{ \frac{\left(2 - \frac{a}{x}\right)^2 + 2}{4\left(3 - \frac{a}{x}\right)} \right\} x \\
&= \left\{ \frac{\left(2 - \frac{\varepsilon_0}{0.0035}\right)^2 + 2}{4\left(3 - \frac{\varepsilon_0}{0.0035}\right)} \right\} x \\
&= K_2 x
\end{aligned}$$

In deriving values for f_m, K_1 and K_2 substitute $\dfrac{\sqrt{f_{cu}}}{5000}$ for ε_0 (with f_{cu} in N/mm^2)

3. The tensile strength of the concrete is ignored.

In Reinforcement

In reinforcement,

1. The strain in the reinforcement is derived from the assumption that plane sections remain plane, and
2. The stress in the reinforcement is derived from the stress-strain curve in Fig. 23.7 with $\gamma_m = 1.15$, i.e., Fig. 23.8(a). There is no simplified version of this diagram. A chart giving the actual values for the various types of reinforcement is shown in Fig. 23.12.

• The above are the basic assumptions and, in the actual design to find the amount of reinforcement required, we can use,

(a) Design charts, or
(b) Design formulae, or

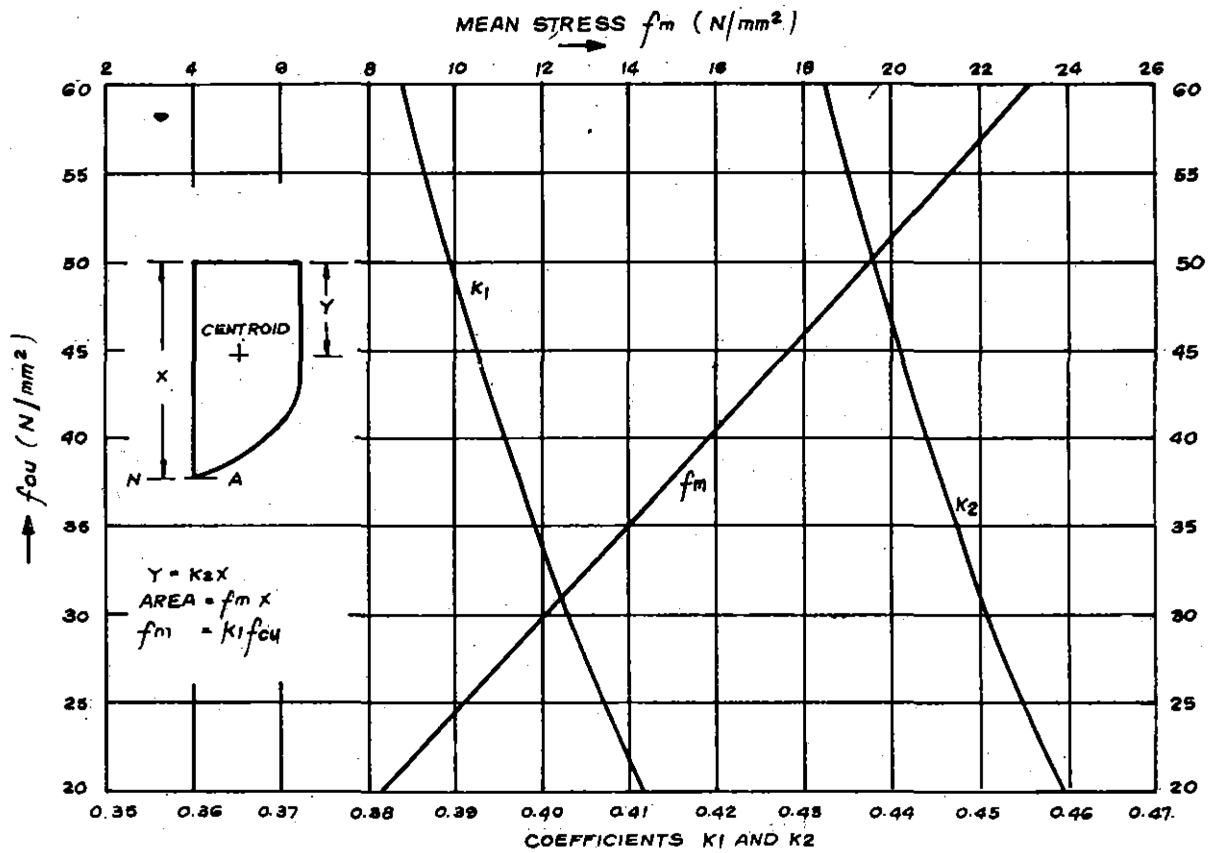


Fig. 23.11 *Properties of rectangular-parabolic stress block*

(c) Strain compatibility

(a) Using Design Charts These have been prepared using the *rectangular-parabolic* stress block for concrete, and the stress-strain curves for the reinforcement. The charts for rectangular sections, reinforced in tension only, or in tension and compression, are given in Part 2 of BS CP 110, which also gives the derivation. Each chart is for a particular grade of concrete, a particular strength of reinforcement, and, in the case of beams reinforced in compression too, for particular value of d'/d.

(b) Using Design Formulae (listed in Table 23.7) There are seven equations as listed ahead in Table 23.7 which can be derived quite simply. The first five deal with rectangular sections or flanged sections where the neutral axis lies within the compression flange, and the remaining two deal with flanged sections where the neutral axis is below the compression flange. In deriving the formulae, a rectangular stress block of maximum depth $0.5d$ and a uniform compressive stress of $0.4f_{cu}$ are assumed. There is a formula for the ultimate resistance moment based on the strength of the concrete and singly reinforced section, above which compression reinforcement is required.

Tensile stress in reinforcement is assumed as $0.87f_y$ as per Fig. 23.8(a). Compressive stress in reinforcement $\left[\dfrac{2000}{2300+f_y}f_y \text{ in Fig. 23.8(a)}\right]$ is assumed as $0.72f_y$. A formula is given for the lever arm z which again depends on the amount of tension reinforcement, but unlike in elastic design as the lever arm is no longer required for shear, etc. in the load factor method, it is not important.

In using the formulae for rectangular sections, it is most important to note that they only apply for (a maximum redistribution of about 10%) the neutral axis in the limit at half the effective depth. With $x/d = 0.5$ (i.e., $n_u = 0.5$), the lever arm, $\left(d - \dfrac{x}{2}\right)$ or $\left(d - \dfrac{n_u d}{2}\right)$ i.e. z, must be 0.75 d. Then, based on the concrete in compression (i.e., based on concrete side), $M_u = 0.4f_{cu}(b(0.5d))(0.75d) = 0.15f_{cu}bd^2$, which is Eq. 23.2 in Table 23.7. For this particular value, sometimes referred to as the balance point, M_u, based on the reinforcement in tension, $= (0.87f_y)A_s z$, and substituting 0.75 d for z we get (by equation the two M_u values),

$$A_s = 0.23bdf_{cu}/f_y$$

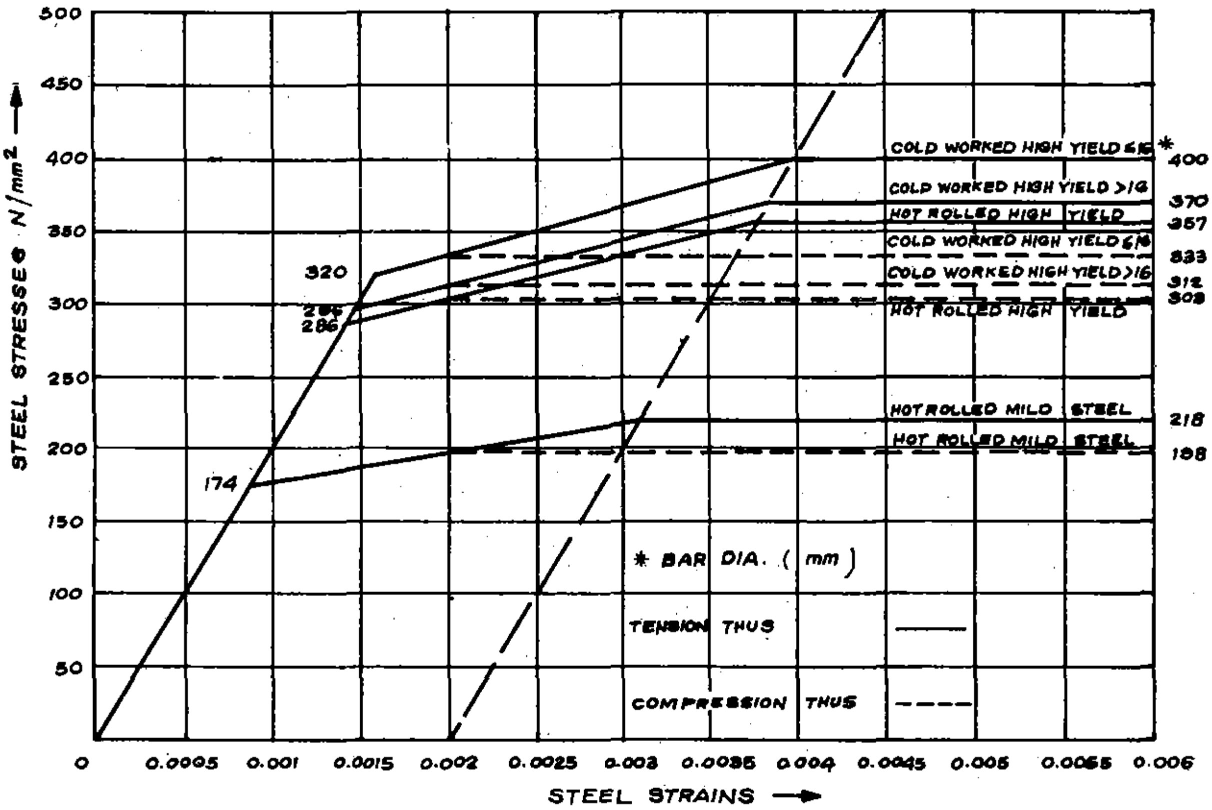


Fig. 23.12 *Design stress-strain curves for ultimate limit state*

If we have more than 10% redistribution, $\frac{x}{d}$ must be less than 0.5, and then although z is greater than $0.75d$ the value of $\frac{M_u}{bd^2}$, based on the concrete side, will get reduced. With 20% redistribution: $\frac{x}{d} \simeq 0.4$, $z \simeq 0.8\ d$, so $M_u \simeq 0.128 f_{cu} bd^2$. With 30% redistribution: $\frac{x}{d} \simeq 0.3$, $z \simeq 0.85\ d$, so $M_u \simeq 0.102\ f_{cu} bd^2$.

So as with the design charts, the more the redistribution, the sooner the compression steel is required.

There is also the condition that the lever arm shall not exceed 0.95 d, which means that for small steel percentages (near the minimum percentages) the value of the lever arm obtained from Eq. (23.5) in Table 23.7 cannot be used. The restriction is not great in any case since the maximum value of z is approximately 0.96 d (using the minimum steel percentage and Grade 20 concrete).

When compression steel is required, Eqs. (23.3) and (23.4) in Table 23.7 should be used. In these equations, the design stress in the compression steel is taken as 0.72 f_y, which is a simplification of $\frac{2000 f_y}{2300 + f_y}$ [Fig. 23.8(a)]. An important condition in the use of Eq. (23.3) is that d'/d is not greater than 0.2. This is to ensure that with a neutral axis depth of 0.5 d, the strain in the compression steel has reached 0.002, i.e., the design stress in compression has been reached. If more than 10% redistribution has been carried out, the values for the resistance moment and force in the concrete should be modified, as explained above, for sections reinforced in tension only. The values for the resistance moment in Eq. (23.3) will be as noted earlier, and the force in Eq. (23.4) will become $(0.16 f_{cu} bd)$ and $(0.12 f_{cu} bd)$ for 20% and 30% redistributions, respectively. The maximum values of d'/d will be 0.16 and 0.12, respectively.

When we have flanged beams with the neutral axis below the flange, the ultimate resistance moment is given by Eqs. (23.6) and (23.7) in Table 23.7. From these equations, it can be seen that any concrete below the flange is ignored. Where redistribution has been carried out, reducing the moment in the span, we have the requirement concerning the neutral axis depth. If this is greater than the depth of the flange, we can still use Eqs. (23.6) and (23.7) but if it is less than or equal to the depth of the flange, we can substitute the actual-depth for the flange-depth in these equations.

If we need the resistance of the concrete in compression to be greater than that given by Eq. (23.7), we have to revert

Table 23.7 *(based on BS CP110): Formulae for Ultimate Resistance Against Moment: Beams and Slabs*

Design formulae: Provided that the amount of redistribution of the elastic ultimate moments has been less than 10%, the following formulae may be used to calculate the ultimate moment of resistance of a solid slab, or of a rectangular beam or of a flanged beam, a ribbed slab or a voided slab section, when the neutral axis lies within the compression flange depth.

- For sections without compression reinforcement the ultimate momennt of resistance may be taken as the lesser of the values obtained from Eqs. (23.1) and (23.2) below.
- Eqs. (23.3) and (23.4) may be used for sections with compression reinforcement.

A rectangular stress block of maximum depth 0.5d and a uniform compressive stress of $0.4f_{cu}$ has been assumed.

$$M_u = (0.87f_y)A_s z \quad \text{(based on tension failure)} \tag{23.1}$$

$$M_u = (0.15f_{cu}bd^2) \quad \text{(based on compression failure)} \tag{23.2}$$

$$M_u = (0.15f_{cu}bd^2) + (0.72f_y)A'_s(d - d') \quad \text{(adding the effect of compression steel)} \tag{23.3}$$

$$(0.87f_y)A_s = 0.2f_{cu}bd + 0.72f_yA'_s \quad \text{(equating tension to total compression)} \tag{23.4}$$

where M_u is the ultimate resistance moment
A_s is the area of tension reinforcement
$A_{s'}$ is the area of compression reinforcement
b is the width of the section.
d is the effective depth to the tension reinforcement (from compression edge)
d' is the depth to the compression reinforcement (from compression edge)
f_y is the characteristic strength of the reinforcement
z is the lever arm (*i.e.*, $\left(d - \frac{n_u d}{2}\right)$ where n_u is the neutral axis factor at ultimate and $n_u \ngtr 0.50$, so that $z = 0.75d$ and the section remains under-reinforced.
f_{cu} is the characteristic strength of the concrete

- when d'/d is greater than 0.2 Eq. (23.3) should not be used and the resistance moment should be calculated with the aid of strain compatibility approach (explained earlier in the text).
- The term $0.72f_y$ in Eqs. (23.3) and (23.4) is a simplification of expression $\dfrac{f_y}{(r_m + f_y/2000)}$ shown in Fig. 23.8(a)
- The lever arm, z in Eq. (23.1) may be calculated from the equation

$$z = \left(1 - \frac{1.1f_yA_s}{f_{cu}bd}\right)d \tag{23.5}$$

(solving this equation in the limit when $z = 0.75d$ corresponds to tension = compression)

- The value of z shall not be taken as greater than $0.95d$
- The ultimate resistance moment of a flanged beam may be taken as the lesser of the values given by Eqs. (23.6) and (23.7) where h_t is the thickness of the flange.

$$M_u = (0.87f_y)A_s\left(d - \frac{h_t}{2}\right) \quad \text{(based on tension failure)} \tag{23.6}$$

$$M_u = 0.4f_{cu}bh_t\left(d - \frac{h_t}{2}\right) \quad \text{(based on compression failure)} \tag{23.7}$$

- Where it is necessary for the resistance moment to exceed the value given by Eq. (23.7) for instance if neutral axis lies in the web and/or the applied factored moment is higher, the section should be analysed with the aid of strain-compatibility approach (explained earlier in the text).

to the basic assumptions and use strain compatibility.

(c) Using Strain-Compatibility Approach In a section which is non-rectangular in the compression zone (call it above neutral axis), we cannot use design charts or formulae directly, and so we cannot quickly find the area of reinforcement required for a given ultimate moment or vice versa. We, therefore, have to revert to first principles and use the 'strain-compatibility' principle. But even with this method, it is only possible to find the ultimate resistance moment based on an assumed steel area. The basic principle of strain-compatibility method is that for a given section (including the assumed reinforcement), we can assume the neutral axis depth, by trial and error, and work out the total compression force C and the total tension force T for each trial unit the two forces are equal, then we can work out the ultimate moment of resistance of the assumed section by taking moments of these C and T forces about any fibre level in the section and check whether this M_u is adequate.

As for flexural compressive stress block in concrete, one can use a rectangular-parabolic stress block and proceed by dividing the section into segments. But at this would be a long and tedious process, we may simply assume a rectangular stress block with a uniform compression stress of $0.4f_{cu}$.

As for steel, the stress-strain curve for reinforcement is as shown in Fig. 23.8(a).

Using a linear strain-profile, determine the strains in the reinforcements and, from the stress-strain profile for the particular reinforcement, find the stress in the reinforcement and hence the force in it.

We only have to assume a trial neutral axis depth and draw the strain block with the maximum compression strain in the concrete as 0.0035 at the top edge and work out the strains in various reinforcement bars, by proportion. Then work out tension force in each bar (= its area × stress corresponding to strain in it) and hence the total T. Also work out the compression force C in concrete in (flexural) compression (= its area $\times 0.4f_{cu}$). Neutral axis location is OK when T equals C, otherwise repeat the procedure with a fresh trial for neutral axis depth until T equals C. Then take moments of C and T forces; as explained above, in order to find M_u.

- ***Interesting Comparison between the British and the ACI/AASHTO Approaches***

(i) In Appendix 6 (given at the end of this book) are quoted with courtesy relevant extracts on this subject from the ACI and AASHTO stipulations as a matter of comparative companion reading. This American practice is based on 'cylinder-crushing-strength of concrete', a 'rectangular-flexural-compressive stress-block' and certain reduction (i.e. ϕ) factors applied on the limit state strength computation.

"(ii) • The B.S. flexural compressive stress block is assumed rectangular all the way in Neutral axis, the stress value is taken a constant of 0.4 of cube strength of concrete ($0.4f_{cu}$), and the distance to neutral axis from the compression edge is taken equal to $d/2$ in the limit but actually is estimated from $C \equiv T$, i.e. from: $(b.n_u d)(0.4f_{cu}) \equiv (A_s 0.87 f_y)$ for a rectangular section as an example, which gives $n_u = \dfrac{2.175A_s \cdot f_y}{bd \cdot f_{cu}}$, sothat the lever arm between C and T is: $(d-n_u d/2)$, i.e. $(1-n_u/2)d$, i.e. $(1 - \dfrac{1.1A_s \cdot f_y}{bd \cdot f_{cu}})d$.

And, then, $M_u = (C \text{ or } T)\cdot(d - n_u d/2)$.

- Although the ACI/AASHTO stress block, after Whitney, is also assumed rectangular, but its depth is taken to range from 0.85 to 0.65 of $n_u d$ (not full $n_u d$), depending on if the concrete cylinder strength is upto 4000 p.s.2 or more. The stress value is taken a constant of 0.85 of Cylinder Strength of Concrete (i.e. $0.85f_{cyl.}$, which is nearly $0.71f_{cu}$). The depth of the stress block, 'a', is then estimated from $C \equiv T$, i.e. from: $(b.a)(0.85f_{cyl.}) \equiv (A_s \cdot f_y)$, which gives: $a = \dfrac{A_s \cdot f_y}{b.(0.85f_{cyl.})}$, sothat the lever arm between C and T is then:

$$(d-a/2) = [d-\frac{0.59A_s \cdot f_y}{b \cdot f_{cyl.}}], \text{ i.e. } (1-\frac{0.71A_s \cdot f_y}{bd \cdot f_{cu}})d$$

And, then $M_u = 0.9[(C \text{ or } T\cdot(d-a/2)]$, 0.9 factor is as stated in the ACI/AASHTO stipulations

- It is clear therefore that the lever arm is bigger in the ACI/AASHTO approach than in the B.S. approach and is likely to give slightly larger M_u relatively. However, the compression force C in the two approaches may not show much difference because the stress block depth relatively is more in the British approach while the stress value is relatively more in the American approach ($0.7f_{cu}$ vs. $0.4f_{cu}$)"

The above procedure for estimating M_u from compatibility of strains has also been explained in rigorous detail in Ch. 27 in this book, using a 'rectangular-parabolic' stress block and higher values for the maximum flexural compressive stress and strain in concrete. But these higher values correspond to the collapse limit state, not an intermediate load-factored limit state, and therefore, the M_u so calculated is the limiting capacity value. [These higher values of stress and strain in concrete are not codified and are based on earlier research done by the author at Imperial College, London, under Prof. ALL Baker (1964).] It will be noted in the said chapter that the averaging stress factor β for the part-parabolic stress block and its centroid-locating factor η have been worked out in terms of concrete strain at the top of the part-parabola in a manner similar to that explained earlier. These β and η factors allow general applicability of that method even if the section is of any shape, not merely rectangular.

- ***Comparison Between Various Flexural Compressive Stress Blocks***

(a) With a rigorous limit state analysis, the resistance moment from the concrete side obtained when a rectangular stress block is assumed, is 0.4 $f_{cu}bx$, and thus ranges from $8bx$ when f_{cu} is equal to 20 N/mm^2 to $16bx$ when f_{cu} equals 40 N/mm^2. These values compare with resistances of $8.14bx$ and $15.66bx$ respectively when a parabolic-rectangular stress block is assumed. It can in fact be shown that for values of f_{cu} of less than 28.14 N/mm^2, the choice of a parabolic-rectangular stress block gives a greater moment resistance, while for higher values of f_{cu} the resistance given by a rectangular stress block is greater. Also, the depth to the centroid of a parabolic-rectangular stress block varies between $0.458x$ and $0.444x$ as f_{cu} increases from 20 to 40 N/mm^2 compared with the constant value of $0.5x$ for a rectangular stress block. The relationship between the moments of resistance provided by the alternative assumptions depends on the ratio of x/d but typical comparative figures are as given in Table 23.8.

The values in Table 23.8 indicate that, while normally showing slight advantage over a rectangular distribution of stress, the choice of parabolic-rectangular stress distribution in the concrete is most advantageous for low values of f_{cu} and high ratios of x/d.

For other than simple rectangular sections the calculations with a parabolic-rectangular stress block are often complex and the choice of a rectangular stress block here is most desirable.

For sections reinforced in tension only, it is sometimes slightly advantageous and never disadvantageous (except when x/d is less than 0.1) to use the CP 110 simplified expressions rather than a rigorous analysis with a rectangular stress block although a slight advantage, in terms of achieving an increased resistance moment and a slight reduction in reinforcement, may be obtained by employing a parabolic-rectangular stress block, especially with low values of f_{cu}.

Table 23.8

Concrete strength f_{cu}	Neutral axis-depth factor x/d(i.e., $n_u d/d$)	Resistance moment M_u			Percentage increase in M_u provided by (i)	
		(i) Parabolic rectangular stress-block	(ii) CP 110 rectangular stress-block	(iii) Modified* rectangular stress-block	over (ii)	over (iii)
20 N/mm^2	0.3	2.105 bd^2	2.040 bd^2	2.076 bd^2	+3.2%	+ 1.4%
	0.6†	3.536 bd^2	3.360 bd^2	3.504 bd^2	+5.2%	+ 0.9%
40 N/mm^2	0.3	4.071 bd^2	4.080 bd^2	4.152 bd^2	−0.2%	− 2.0%
	0.6†	6.890 bd^2	6.720 bd^2	7.008 bd^2	+2.5%	− 1.7%

* See (b) above.

† Such value of x/d can only be adopted if compression reinforcement is provided.

(b) The assumption of a uniform rectangular stress block is slightly disadvantageous when considering sections subjected to combined bending and thrust where the latter predominates. This is because the assumed shape of the parabolic-rectangular stress block provides some resistance to bending, whereas in such a condition, a uniform rectangular stress block does not. The purpose of the uniform rectangular stress block is to provide a simple, yet fairly accurate representation of the parabolic-rectangular distribution for use in calculations which would otherwise be unnecessarily complex. One simple means of improving this correspondence is to employ a uniform stress of 0.45 f_{cu} over a depth of 0.85 x (instead of 0.40 f_{cu} over a depth of x). The resulting resistance moments given by the latter assumption are also set out in Table 23.8 and show clearly the improved correspondence obtained.

- ***Serviceability Limit State of Cracking***

Following the reasonable conditions for this purpose:

(i) The assessed surface crack width shall not in general exceed 0.3 mm.

(ii) When members are exposed to a particularly aggressive environment, such as the 'very severe' category (i.e., exposed to sea water and with abrasion), the assessed surface crack widths at points nearest the main reinforcement should not exceed 0.004 times the nominal cover to the main reinforcement.

In a 'normal environment' condition (i) will suffice, while in the 'very severe category' the more critical of conditions (i) and (ii) will govern. For this category, the minimum grade of concrete is 40 N/mm^2 and the nominal cover is 60 mm. Or we can have 50 mm cover using grade 50 concrete. Using the latter, we should have the requirement that at points nearest the main reinforcement the crack width should not exceed 0.2 mm whilst it must not exceed 0.3 mm at any other position.

With large covers to the main bars it will generally be found that unless the bars are at fairly close centres it will be the 0.3 mm criteria that will govern the design.

The widths of flexural cracks at a particular point on the surface of a member depend primarily on three factors:

(i) The proximity to the point under consideration, of reinforcing bars perpendicular to the cracks,

(ii) The proximity of the neutral axis to the point under consideration,

(iii) The average surface strain at the point under consideration.

Referring to Fig. 23.4 the surface crack width in reinforced concrete under flexure may be taken as equal to ($3.3c\epsilon_1$) for deformed bars or ($3.8c\epsilon_1$) for plain bars. If $a' < c$, then substitute a' for c.

The formulae give acceptably accurate results in most normal design circumstances, but it should be emphasized that cracking is a random phenomenon and that an absolute maximum crack width cannot be predicted. The formula is designed to give a width of crack which has an acceptably small chance of being exceeded. Thus an occasional crack slightly larger than the predicted width should not be considered as cause for concern. But if a significant number of the cracks in a structure exceed the calculated width, reasons other than the structural nature of the phenomenon should be sought to explain their presence. (In certain cases, the occurrence of cracks due to plastic shrinkage of setting concrete, much before any load comes on the structure, could be a possible confusing reason.)

Columns

The following is applicable for columns which could be under 'only direct load' or 'direct load and bending moment'.

After estimating the load-factored design values* of

* The 'design values' are obtained after taking into account whether the

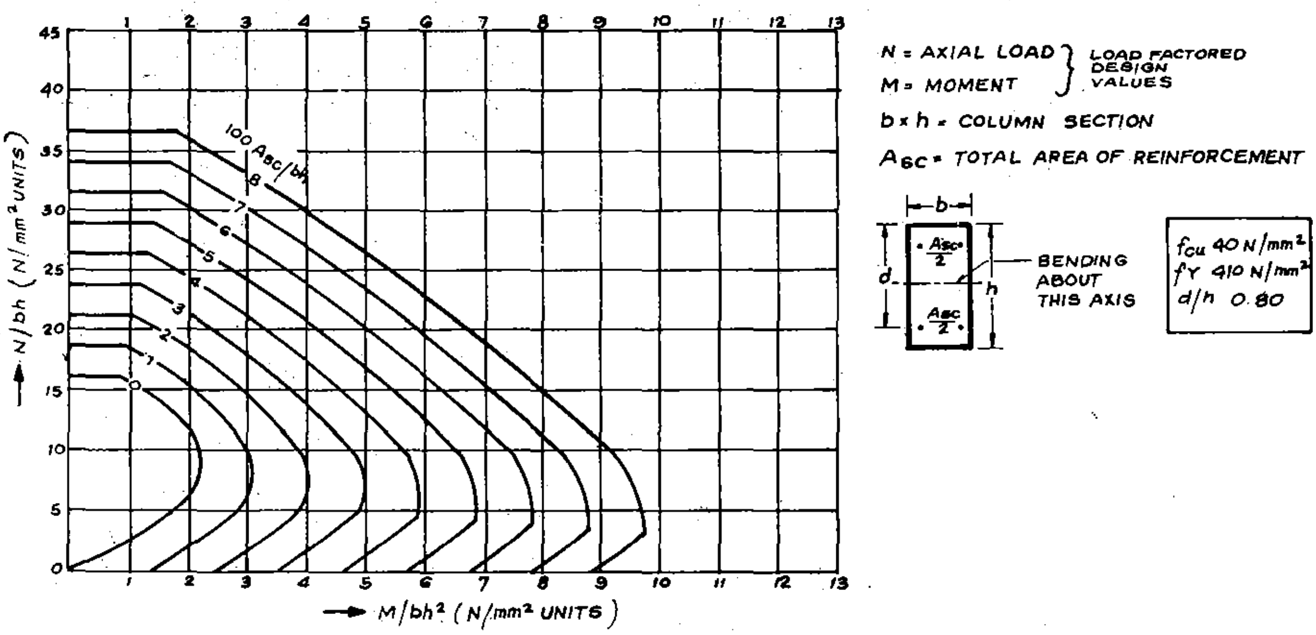


Fig. 23.13 *Rectangular columns*

the axial load and the appropriate moment, the necessary column reinforcement can be calculated by using:

(a) *design charts* for symmetrically reinforced rectangular or circular columns, or
(b) *formulae* for symmetrically or unsymmetrically reinforced rectangular columns, or
(c) *Strain-compatibility* for non-rectangular columns with symmetrical or unsymmetrical reinforcement.

(a) By Using Design Charts These have been prepared using the rectangular parabolic stress block for concrete and the trilinear stress-strain curves for reinforcement as for beams. The charts for rectangular columns are given in Part 2 of BS CP 110 and a typical chart is shown in Fig. 23.13. Charts for circular columns are given in Part 3 of BS CP 110. Each chart is for a particular grade of concrete, a particular characteristic strength of reinforcement, and a particular d/h ratio (i.e., the positioning of the reinforcement).

Knowing N/bh and M/bh^2, the area of reinforcement can be found from the appropriate chart.

It should be noted that A_{sc} is the total area of reinforcement and this is divided equally between the faces parallel to the axis of bending. Any reinforcement in the depth faces of the section is not taken into account.

column is short or long as referred to earlier and as explained in Ch. 10 of this book. If the column is long, then the load and moment values obtained from the first order theory are either directly enhanced by dividing them by the appropriate reduction factor or are modified by doing the buckling analysis (the theory of second order). All this has been very clearly explained in Ch. 10.

On the design chart itself it will be seen that the lines for the percentages of reinforcement flatten off when they approach N/bh axis. This cut off line is to ensure that the accidental moment of 0.05 Nh is automatically allowed for. The 'kinks' in the chart correspond to those in the stress-strain curve for steel.

Above the point B (Fig. 23.14) in the design chart the section is controlled by compression in the concrete and below point B by tension in the reinforcement.

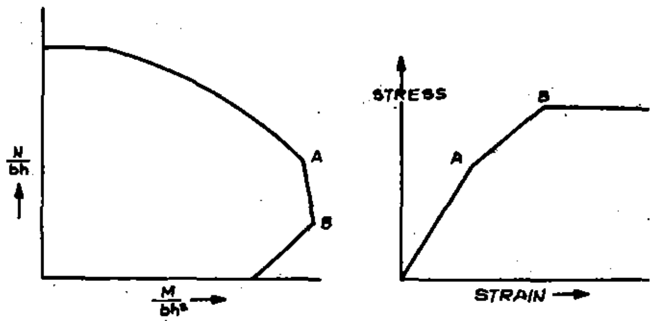


Fig. 23.14 *Relationship of "kinks" on design chart to stress-strain curve of steel*

Note Even for biaxial bending the said design charts can be used with certain additional effort and understanding. For more details see reference 2 and CP 110.

(b) By Using Formulae (listed in Table 23.9) CP 110 gives design formulae (listed in Table 23.9 ahead) which can be derived quite simply and used for rectangular columns with symmetrical or asymmetrical reinforcement in the faces parallel to the axis of bending.

With symmetrical reinforcement, a design chart would generally be used, but with asymmetrical reinforcement the charts mentioned earlier cannot be used. The formulae give a trial and error method (which is a modified form of using 'strain compatibility method', described ahead).

If the ultimate axial load on a column does not exceed the value given by Eq. (1) in Table 23.9

$$N = 0.4F_{cu}b(h - 2e)$$

then, only nominal reinforcement is required, provided the resultant eccentricity $e = M/N$ does not exceed $(h/2-d')$, where h is the depth of the section in the plane of bending, d' is the depth from the surface to the reinforcement in the more highly compressed face.

When the ultimate axial load is greater than that given by Eq. (1), the strength of the section may be assessed using Eqs. (2) and (3) in Table 23.9. The diagrams for the different conditions are shown in Fig. 23.15.

The procedure is to calculate e, assess which of the five cases is appropriate, use Eq. (2) to calculate areas of reinforcement in each face to give the required value of N, and then use this area of reinforcement in Eq. (3) to find the value of M, which should not be less than the ultimate design moment. If the values are not satisfactory, then try a different case, and the results obtained in the first trial should indicate which case to try next (see Examples 1 and 2 ahead).

Use of these Design Formulae

Example 1 Rectangular column 400 × 300 mm carrying a load of 1200 kN and a moment of 120 kNm about the major axis (load factored). Concrete is grade 30(f_{cu} = 30 N/mm²) and the cover to the main reinforcement is 30 mm. Reinforcement has a characteristic strength of 410 N/mm² $h = 400$ mm, $b = 300$ mm, $N = 1200$ kN, $M = 120$ kNm

$$e = \frac{120 \times 10^6}{1200 \times 10^3} = 100 \text{ mm}$$

Assuming $d' = 40$, $\frac{h}{2} - d' = 200 - 40 = 160$

So, $e < \frac{h}{2} - d'$

From Eq. (1)

$$N = 0.4 \times 30 \times 300(400 - 200) \times 10^{-3}$$
$$= 720 \text{ kN, i.e., < axial load}$$

Table 23.9 *(reference: BS CP 110) Formulae for ultimate resistance against axial load and moment—Rectangular Columns*

Following formulae may be used as appropriate, for the design of a rectangular column section having its longitudinal reinforcement in the two faces parallel to the axis of bending whether that reinforcement is symmetrical or not.

(i) In a column where the ultimate axial load does not exceed the value N given by Eq. (1) only nominal reinforcement is required

$$N = 0.4f_{cu}b(h - 2e) \quad (1)$$

provided the resultant eccentricity $e = M/N$ does not exceed $(h/2 - d')$.

where f_{cu} is the characteristic strength of the concrete

M is the maximum moment due to ultimate loads about the axis considered

b is the breadth of the section

h is the depth of the section in plane of bending,

d' is the depth from the surface to the reinforcement in the more highly compressed face

(ii) When the ultimate axial load N is greater than that given by Eq. (1) the strength of the section may be assessed using Eqs. (2) and (3).

$$N = 0.4f_{cu}bd_c + 0.72f_yA'_{s1} + f_{s2}A_{s2} \quad (2)$$
$$M = 0.2f_{cu}bd_c(h - d_c) + 0.72f_yA'_{s1}(h/2 - d') - f_{s2}A_{s2}(h/2 - d_2) \quad (3)$$

where f_{cu}, M, b, h and d' are as given above.

A'_{s1} is the area of compression reinforcement in the more highly compressed face.

A_{s2} is the area of reinforcement in the other face which may be considered as being either in compression, inactive, or in tension as the resultant eccentricity of load increases and d_c decreases from h to $2d'$.

f_{s2} is the stress in this reinforcement which should not exceed $0.72f_y$ when compressive or $-0.87f_y$ when tensile.

d_2 is the depth to this reinforcement from this face

d_c is the depth of concrete in compression which may be chosen as appropriate, but shall not be taken as less than $2d'$ (the equivalent of neutral axis, which, here, because of direct load and moment, is not the same as effective centroidal axis)

(iii) As an alternative to (ii) when the resultant eccentricity e is not less than $(h/2 - d_2)$, the axial load may be ignored and the column section may be designed to resist purely an increased moment M_a,

$$M_a = M + N(h/2 - d_2) \quad (4)$$

The area of tension reinforcement necessary to provide resistance to this increased moment may be reduced by the amount $N/0.87F_y$.

Note: As in the case of beams and slabs (Table 23.7), here also.

- Maximum compressive strain in concrete = 0.0035
- Maximum compressive stress in concrete = $0.4f_{cu}$ and its distribution is assumed to be rectangular within depth d_c
- Maximum compressive stress in reinforcement: $0.72f_y$
- Maximum Tensile stress in reinforcement: $0.87f_y$

Try Case $(C)d_c = h$ (Fig. 23.15)

$$N = (0.4 \times 30 \times 300 \times 400 + 0.72 \times 410 \times A'_{s1}) \times 10^{-3}$$
$$= 1440 + 0.3A'_{s1}$$

This is greater than the actual load even without taking A'_{s1} into account. But we must have minimum of 1% longitudinal reinforcement, i.e., 1200 mm², and minimum bar size of 12 mm,

Try $A'_{s1} = 2/25(982 \text{ mm}^2)$, $A_{s2} = 2/12(226 \text{ mm}^2)$

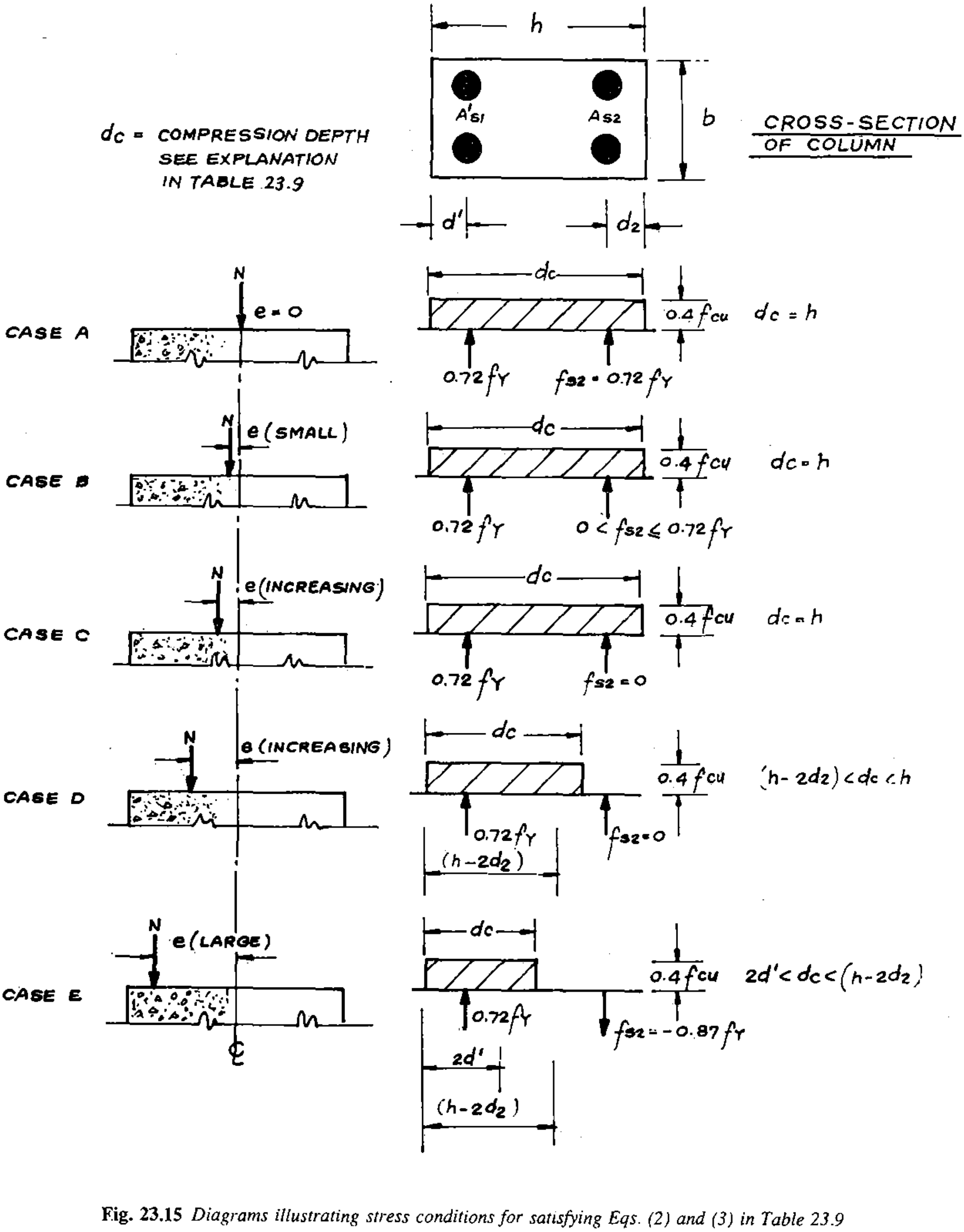


Fig. 23.15 *Diagrams illustrating stress conditions for satisfying Eqs. (2) and (3) in Table 23.9*

$$M = 0 + 0.72 \times 410 \times 982(200 - 40) \times 10^{-6}$$
$$= 46 \text{ kNm}$$

N is too large and M is too small.
Try Case (D) with $d_c = h - 2d_2 = 320$ mm
$f_{s2} = 0$, $A'_{s1} = 982$ mm^2

$$N = (0.4 \times 30 \times 300 \times 320 + 0.72 \times 410 \times 982) \times 10^{-3}$$
$$= 1152 + 290 = 1442 \text{ kN}, > \text{actual}$$
and $M = 0.2 \times 30 \times 300 \times 320(400 - 320) + 0.72 \times 410 \times 982(200 - 40) \times 10^{-6}$
$$= 46 + 46 = 92 \text{ kNm}$$

As N is too large and M is too small we can go to case (E) and we should have to take f_{s2} as $-0.87f_y$ which would reduce N and increase M,
Try Case (E) with $d_c < (h - 2d_2)$ say 300 mm
$f_{s2} = -0.87 \times 410$, $A_{s1} = 982$ mm^2, $A_{s2} = 226$ mm^2

$$N = (0.4 \times 30 \times 300 \times 300 + 0.72 \times 410 \times 982 - 0.87 \times 410 \times 226) \times 10^{-3}$$
$$= 1080 + 290 - 80 = 1290 \text{ kN}$$
and $M = 0.2 \times 30 \times 300 \times 300(100) + 0.72 \times 410 \times 982(160) + 0.87 \times 410 \times 226 \times 160 \times 10^{-6}$
$$= 54 + 46 + 13 = 113 \text{ kN}$$

If we now increase A_{s2} to 2/16 (402 mm^2)

$$N = 1080 + 290 - 143 = 1207 \text{ kN}$$
and $$M = 54 + 46 + 23 = 123 \text{ kNm}$$

This is satisfactory, and the assumed reinforcement OK.

Example 2 (See Fig. 23.16)

LOAD FACTORED VALUES:
ULT Design Load = 144 kN
ULT Design Moment = 160 kNm
Assume $f_y = 425$ N/mm^2 $f_{cu} = 30$ N/mm^2
$e = \dfrac{160}{144} = 1.11$ m. This is obviously large for the assumed section dimensions and certainly the A_{s2} will be in tension, so assume a value of d_c less than $(h - 2d_2)$ but more than $2d'$ (i.e., Case E in Fig. 23.15),
$(h - 2d_2) = 500 - 85 = 415$ mm
$2d' = 2 \times 38 = 76$ mm
Try $d_c = 200$ mm and A_{s1} and A_{s2} as shown in Fig. 23.16, hence;

$$N = \{0.4 \times 30 \times 200 \times 200 + 0.72 \times 425 \times 402 + (-0.87 \times 425 \times 981)\} \times 10^{-3}$$
$$= 480 + 123 - 364 = 239 \text{ kN}$$

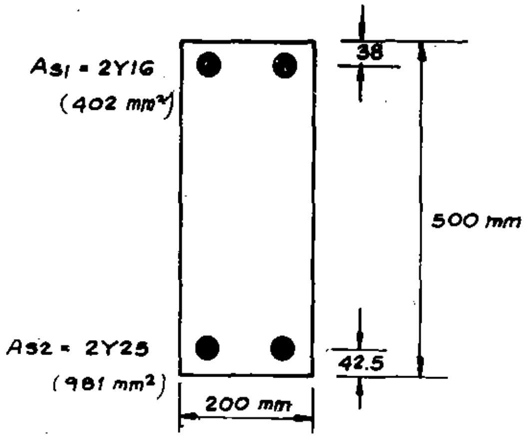


Fig. 23.16

This is then greater than design N of 144 so reduce value of d_c to

$$\frac{385}{0.4 \times 30 \times 200} = 161 \text{ mm}$$

This now gives value for N as 144 kN, hence

$$M = \{0.2 \times 30 \times 200 \times 161(500 - 161) + 0.72 \times 425 \times 402(250 - 38) -(-0.87) \times 425(250 - 42.5)981\} \times 10^{-6}$$
$$= 65.5 + 26.1 + 75.5 = 167.1 \text{ kNm, satisfactory.}$$

Hence assumed section and reinforcement OK.

(c) By Using Strain Compatibility

As with the beams and slabs, if the column has an irregular section, the best method is that of strain-compatibility, using the rectangular stress block for the concrete in compression (value = $0.4F_{cu}$) and maximum compressive strain equal to 0.0035. Again, this can only be done if a first-trial area of reinforcement is assumed first.

Assume a trial neutral axis depth, draw the strain block (as in the case of a beam or a slab, as explained earlier), work out the strain values in the reinforcements (from the strain block, by proportion) and hence tension forces in them and hence the Total Tension, T. Also work out the total compression in concrete and compression steel (if any). If T, C and the externally applied axial load N are in equilibrium them trial is OK, otherwise try again with a more suitable depth of neutral axis, until finally $(T + N) = C$. Then, as in the case of beams and slabs, the ultimate moment of resistance of the assumed section can be found by taking moments of T and C forces about any fibre-level in the section, and its magnitude should be ensured against the applied (load-factored) moment.

- ***Crack Control in Columns***

As per CP110 cracks due to bending in a column designed for an ultimate axial load greater than $0.2 f_{cu}.A_c$ are unlikely to occur and, therefore, no check is required. A more lightly loaded column subject to bending should be considered as a beam for the purpose of crack control.

- ***Concentrated Loads on Slabs***

The critical section is taken on a perimeter 1.5 h from the boundary of the loaded area as shown in Fig. 23.17.

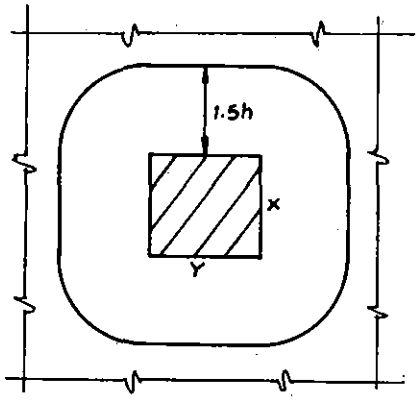


Fig. 23.17 *Perimeter for shear stress in solid slabs (plan)*

The critical perimeter has a length l and if the dimensions of the load contact area are x and y, then:

$$l = 2(x + y) + 2\pi(1.5h) = 2(x + y) + 3\pi h$$

where h is the overall slab thickness.

It is assumed that the shear stress, v, has a constant value throughout the effective depth and length of this critical perimeter. No shear reinforcement is required when this shear stress is within the permissible value stipulated in clause 3.4.5 of BS CP110 to which reference may be made. Generally, shear reinforcement is required in slabs only in special circumstances (e.g., in flat-slabs).

23.4 DETAILING

The UK Concrete Society's report on *Standard Details* and their joint report with the Institution of Structural Engineers on *The Detailing of Reinforced Concrete* may be referred to for workmanlike detailing. The detailing of reinforcement is no less important than designing the sections. For this purpose, due note should be taken of the relevant local stipulations and workmanlike details. Some relevant information is also given in a separate chapter in this book which may be referred to. Practical detailing is learnt only by actually working on the 'drawing board' in a practising office, not by merely computing the stresses, much less by merely proof reading others' calculations and drawings in the name of checking.

REFERENCES

1. Classical works by authorities like KW Johansen, LL Jones, RH Wood, ALL Baker, Hognestad, Hansen, McHenry, Hajnal Konyi, FN Pannell, and Laupa, Seiss and Newmark.
2. "RC Design to CP 110 Simply explained", by AH Allen (from which some material has been taken with grateful thanks).
3. UK Concrete Society Report on 'Standard Details'.
4. 'The Detailing of Reinforced Concrete', joint Report by the UK. Concrete Society and the UK Institution of Structural Engineers, London.

CHAPTER 24

Practical Design against Shear and Torsion and Design of Short-cantilevers and Deep-beams

24.1 PRINCIPLE OF DESIGN AGAINST SHEAR

Concrete subjected to (bending and) shear will fail in tension when the principal tensile stress (diagonal tension) exceeds its modulus of rupture. This would suggest that principal tensile stress should obviously be guarded against. This stress t at a particular fibre level is calculatable from the following expression derived on the basis of classical elastic theory assuming that the material is homogeneous, crack-free, isotropic and elastic,

$$t = -\frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + s^2} \text{ (–ve sign for tensile stress)}$$

where f = the magnitude of the bending tensile stress
s = the magnitude of the transverse shear stress

at the particular fibre level under consideration. Greatest t will occur where a greatest s and greatest f coexist. In reinforced concrete, greatest permissible f being zero, t then equals s and this is why in reinforced concrete, traditionally only shear stress s used to be checked in the elastic analysis. However, in prestressed concrete, at the fibre where s is greatest, f could be compressive, zero or tensile since some flexural tension may be permitted. This would suggest to calculate t value at various critical fibre levels (at each critical section) in such a case, and this is what used to be done traditionally (and the greatest t was restricted to a permissible fraction of the modulus of rupture).

However, since shear does not accept any significant redistribution (unlike flexure) and, therefore, its failure can be rather sudden (which is why seldom did any code permit any increase in shear stress, unlike in bending stress), we have to be more careful in handling shear. Since the aforementioned elastic formula is based on the stated assumptions none of which are really accurately true in case of concrete (because concrete at least has micro cracks in it and is not truly elastic, homogeneous or isotropic) it is, therefore, wholly irrational to design for shear by trying to calculate and control principal tension in this manner. It would be more rational in case of shear to somehow directly consider the concrete in its cracked state (the load-factor approach), even if one took into account the shear which the unreinforced concrete could take till it cracked (reinforcement taking only the balance). This would be more realistic than using the purely elastic approach for shear under the realistic conditions that prevail. (In structural engineering it is more rational to attempt an approximate solution to an exact problem than a solution to an approximated problem, for obvious reasons.)

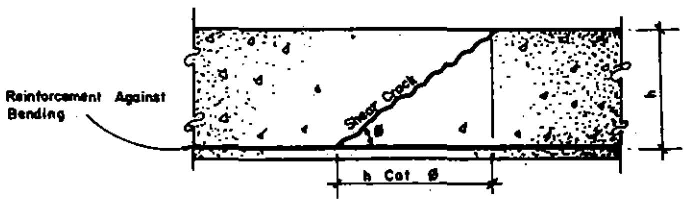


Fig. 24.1

Explained in simple words, if vertical shear reinforcement is A_v at pitch p, then the shear force V it can take within the spread of the shear crack in a concrete member shown in Fig. 24.1 is,

$$V = \frac{A_v}{p} h \cot\phi f_s$$

where f_s is the tensile stress in the shear reinforcement.

In the limit, ultimate shear on reinforcement would be,

= (Ultimate shear on the section) – (shear capacity of unreinforced concrete)

= $V_u - V_c$

with f_s reaching the value of tensile yield stress in reinforcement, f_{sy}

And assuming approximately that $\phi = 45°$ and h is the lever arm d between longitudinal tension and compression forces in the section, then,

$$V_u - V_c = \frac{A_v}{p} d f_{sy}$$

so that shear reinforcement required would simply be,

$$A_v = \frac{(V_u - V_c)p}{f_{sy}d}$$

In this formula,

- V_c, in kg, may be assumed equal to $(10bd)$, where b (the breadth of rib) and d (as defined below) are in cm
- d = 0.8 of effective depth in case of reinforced concrete
  = effective depth or 0.8 of overall depth, whichever greater, in case of prestressed concrete,
- V_u = load factored dead and live load shears, less any relief in shear due to prestress and sloping soffit (assuming unit load factor on prestress)
- f_{sy} = yield stress in vertical stirrups whose cross sectional area is A_v placed at a horizontal pitch p.

However, if $\frac{V_u}{bd}$ exceeds 0.15 of the standard 28-day concrete cylinder crushing strength, the section should be increased in order not to exceed this limit and only then A_v calculated. (This condition is from an overall philosophical standpoint to avoid shear-compression failure and to cover the effects of approximations made in converting the structure into the approximated mathematical model.)

Also note that it is preferable to provide vertical stirrups for shear rather than crank the flexural reinforcement since in the latter detail the resultant vectorial outward force at the bend in the cranked reinforcement has the tendency to wedge-out the concrete lying within the bend.

24.2 DESIGN OF SHORT-CANTILEVERS, CORBELS AND BRACKETS

The design procedure for short cantilevers, brackets and corbels recognizes the deep beam or simple truss action of these short shear-span members, as illustrated in Fig. 24.2. Four possible failure modes must be controlled: (a) Direct shear failure at the interface between bracket or corbel and supporting member; (b) Yielding of the tension tie due to moment and direct tension; (c) Crushing of the internal compression strut; and (d) Localized bearing or shear failure under the loaded area.

The design provisions given here apply only to members having a shear span-to-depth ratio of unity or less ($a/d' \leq 1$) since, for longer spans, diagonal tension cracks may form and the use of 'horizontal shear reinforcement' may not suffice. Furthermore, the method has not been validated by tests beyond $a/d = 1$. (For cases with $a > d'$, the usual design procedures for flexure and for shear should be applied.)

The critical section is designed to resist simultaneously a

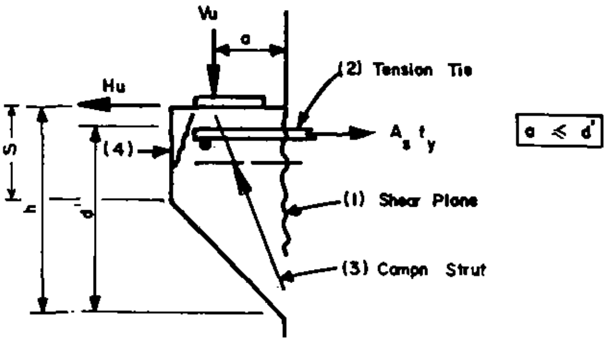


Fig. 24.2 *Structural action of short cantilevers*

shear V_u, a moment $[V_u a + H_u(h - d')]$, and a horizontal tensile force H_u; latter being caused either due to friction at the bearing area (and) or due to restrained shrinkage, creep and thermal deformation.

For design purposes the total reinforcement required is divided into three parts, with each determined separately—(A_{vf}) area of shear-friction reinforcement to resist direct shear V_u; (A_f) area of flexural reinforcement to resist moment $V_u a + H_u$, $(h - d')$; and (A_t) area of tensile reinforcement to resist direct tensile force H_u.

Once the separate areas of reinforcement A_{vf}, A_f and A_t have been determined, the actual reinforcement to be provided, A_s and A_h, may be sized, where A_s will act as the primary tension reinforcement and A_h will act as shear reinforcement (placed as horizontal stirrups, one below the other, below A_s).

Design Steps

(See Fig. 24.3)

Condition 'Shear-span to effective depth ratio'; a/d', is ≤ 1. (If $a > d'$, design for flexure and shear as per usual procedures.)

Step 1 Ensure $S/d' \geq 0.5$

Step 2 Ensure $\frac{V_u}{bd} \leq 0.15 f'_c$, otherwise revise section dimensions

V_u = ultimate shear value
b = width of cantilever/bracket/corbel
f'_c = 28-day standard cylinder strength of concrete used.
d = 0.8 of effective depth $d'(d = 0.8d')$

Step 3 Calculate shear-friction reinforcement A_{vf}:

$$A_{vf} = \frac{V_u}{0.85 f_{sy} \mu}$$

f_{sy} = yield stress value of the reinforcement used
μ = 1.4 for concrete placed monolithically across interface

1.0 for concrete placed against hardened concrete but with roughened surface

0.7 for concrete anchored to structural steel

0.6 for concrete placed against hardened concrete but with surface not roughened.

NOTE Only monolithic construction recommended

Step 4 Calculate direct-tension reinforcement A_t:

$$A_t = \frac{H_u}{0.85 f_{sy}}$$

H_u = 1.7 × actual horizontal force in working load condition if clearly defined but $\nless 0.2V_u$

Step 5 Calculate flexural-tension reinforcement A_f:

$$A_f = \frac{[V_u a + H_u(h - d')]}{0.85 f_{sy} d}$$

$d = 0.8d'$ (as already defined), all other symbols as already defined or shown in Figs. 24.2 and 24.3.

Step 6 Compute total primary tensile reinforcement A_s

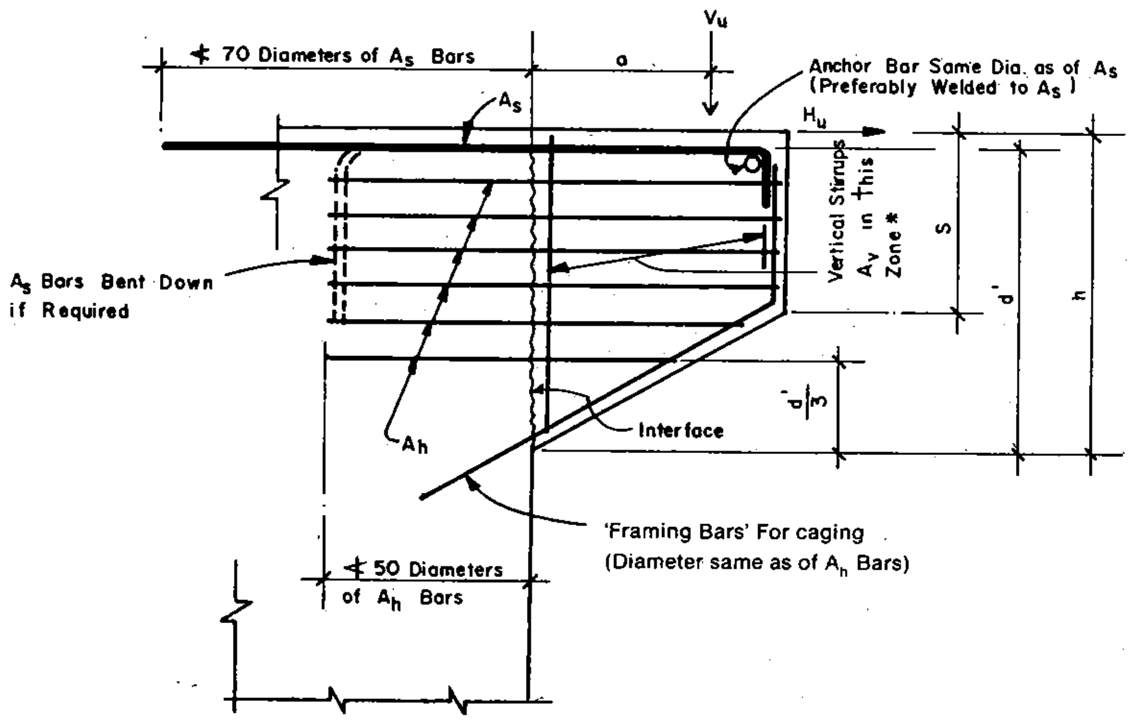
$$A_s \left.\begin{matrix} \nless (A_f + A_t) \\ \nless \left(\frac{2}{3}A_{vf} + A_t\right) \\ \nless (0.04 f_c'/f_{sy}) bd' \end{matrix}\right\} \text{Provide largest of these three magnitudes as } A_s$$

Step 7 Calculate total section area A_h of stirrups (closed ties) to be provided horizontally, one below other, below and next to A_s:

$A_h \nless 0.5A_f$ and $\nless 0.333A_{vf}$, provide larger of the two. These stirrups shall be provided below A_s and within a depth of $2/3d'$ below A_s, as indicated in Fig. 24.3.

24.3 DESIGN OF AN ARTICULATION (i.e., A HALVING JOINT)

A cut-out 'recessed' seating arrangement whereby one deck sits on another, may be referred to as an 'articulation' or a 'halving joint'. An articulation is obviously a very important part of the whole and it needs special attention



$$* \; A_v = 0.50 \frac{(V_u - V_c)p}{f_{sy} d}$$

symbols as explained earlier.

Fig. 24.3 *Detailing of A_s and A_h reinforcements in a short cantilever (or bracket or corbel) (any other reinforcement not shown).*

in design, construction and maintenance. Heavy shear and bending stresses on account of abrupt reduction of depth of section occur at the articulations. The design of an ordinary reinforced concrete articulation is generally done by superimposing the bending stresses due to vertical and horizontal reactions at the bearing and the shear stresses along a plane emanating from the end of the articulation block. The assumption is that the concrete in the articulation will crack in an undesirable manner. These cracks should be sealed with a rigid filler (rich non-shrink cement grout or, preferably, a suitable epoxy resin formulation) soon after their appearance and ceasing to widen further.

Design Steps

(See Fig. 24.4)

Condition Shear span to effective depth ratio, a/d', is ≤ 0.6. (If $a/d' > 0.6$, then redimension to suit.)

Step 1 Ensure that $S \nless 0.4$ of overall depth of main part of beam.

Step 2 Ensure that $\frac{V_u}{bd} \leq 0.15 f_c'$.

(otherwise revise section dimensions)

V_u = ultimate shear value

b = width of articulation section

f_c' = 28-day standard cylinder strength of concrete used

d = 0.8 of effective depth $d'(d = 0.8d')$

Step 3 Calculate total horizontal steel A_s:

$$A_s = A_{vf} + A_t$$

$$\text{where } A_{vf} = \frac{V_u}{0.85 f_{sy} \mu}$$

$$A_t = \frac{H_u}{0.85 f_{sy}}$$

A_{vf} = shear-friction reinforcement

A_t = direct-tension reinforcement

f_{sy} = yield stress value of reinforcement used

μ = 1.4 for concrete placed monolithically across interface, and only monolithic concreting recommended

H_u = 1.7 × actual horizontal force in working load condition, if clearly defined, but $\nless 0.2V_u$ for this calculation

Step 4 Calculate total vertical stirrups A_v:

$A_v = A_s/\mu$

Step 5 Calculate total horizontal stirrups A_h:

$A_h = 0.5A_s$

Step 6 Calculate total inclined stirrups A_i and provide these inclined stirrups at angle θ to horizontal (preferably 45°) such that they intersect the line of action of V_u, going well past outer edge of the bearing.

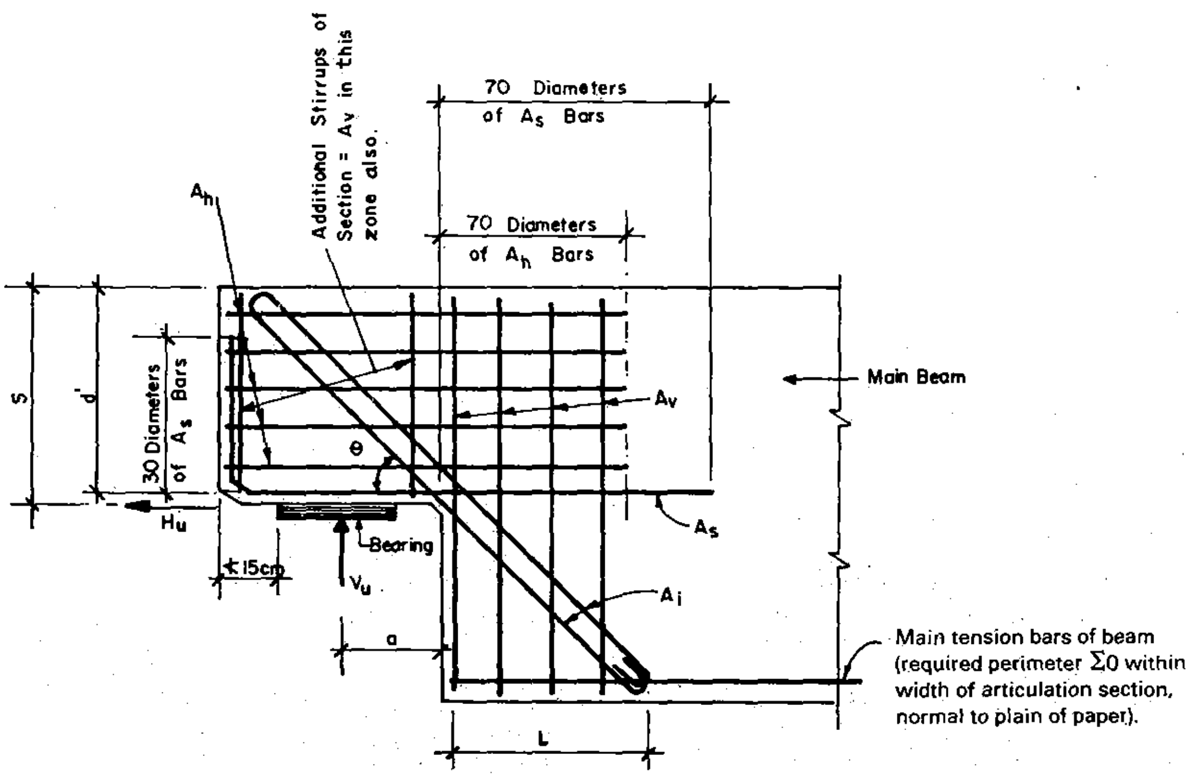


Fig. 24.4 *Detailing of A_i, A_s, A_v and A_h reinforcements in an articulation (other reinforcements not shown, main tension bars of beam only indicated).*

A_i shall be larger of the two values obtained from the following two equations:

(i) $A_i f_{sy} \sin\theta = \dfrac{V_u}{0.85}$ (for resisting V_u)

(ii) $A_i \cos\theta = \dfrac{[V_u a + H_u(S - d')]}{0.85 f_{sy} d}$ (for resisting moments)

Step 7 Ensure that $l \nless \dfrac{0.5V_u}{\Sigma 0 f_{bu}}$

$\Sigma 0$ = perimeter of the main tension reinforcement of the main beam at the articulation (see Fig. 24.4)

f_{bu} = ultimate anchorage bond stress between the aforementioned main tension reinforcement and the concrete (ranges from 14–19 kg/cm^2 for plain bars and 19–28 kg/cm^2 deformed bars, for concrete grades 200–350 kg/cm^2 standard concrete cylinder strengths, respectively).

24.4 DESIGN OF DEEP BEAMS

Design of deep beams (depth > about half the clear span) is a special subject. The design proposals produced by Kong, Sharp and others are based on the results of several hundred tests and, unlike most other procedures, are also applicable to deep beams with web openings. Details of the method are presented ahead with slight modifications on coefficients k_1 and k_2 to suit partial load factors and the assumed load factor approach. As the depth of a beam becomes greater in proportion to its span, the distribution of stress differs from that assumed for a normal beam. In addition, the particular arrangement of the applied loads and of the supports has an increasing influence on this stress distribution. Thus if the ratio of clear span to depth is less than 2 to 3 for a freely-supported beam, or $2^1/_2$ to 4 for a continuous system, it should be designed as a deep beam.

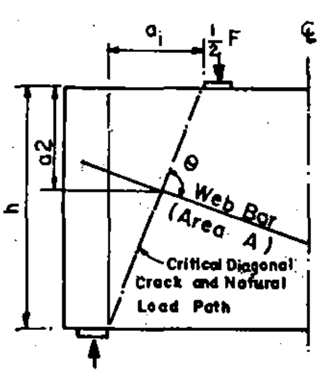


Fig. 24.5

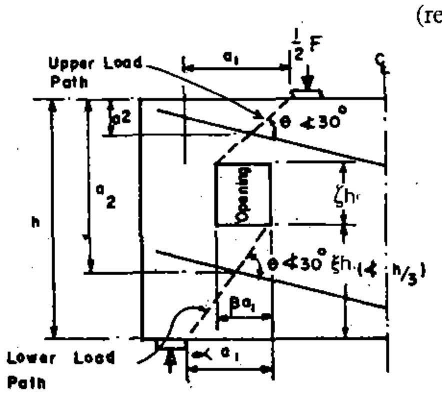


Fig. 24.6

Notation (Figs. 24.5 and 24.6)

A area of individual web bar

$A_{s_{req.}}, A_{s_{prov.}}$ minimum area of main steel required and actual area provided

a_1 clear distance from edge of load to face of support

αa_1 distance from inner edge of opening to face of support

βa_1 width of opening

a_2 depth at which web bar intersects critical diagonal crack

b breadth of beam

d effective depth to main steel

f_t cylinder splitting tensile strength of concrete (see Table 24.1 ahead)

f_{sy} yield strength of reinforcement

h overall depth of beam

k_1, k_2 empirical coefficients for concrete and reinforcement. Take k_1 as 0.7 for normal-weight concrete and 0.5 for light-weight concrete: take k_2 as 100 for plain round bars and 225 for deformed bars.

l span of beam between centres of supports

M ultimate moment

V ultimate shearing force

V_1 shearing force resisted by concrete and main reinforcement only

θ angle between bar being considered and critical diagonal crack

ξ distance of bottom of opening from beam soffit expressed as proportion of total depth of beam

ζ depth of opening expressed as proportion of total depth of beam

Design Procedure (Steps)

(refer to table ahead)

(i) Calculate ultimate bending moment M acting on beam

(ii) Calculate area of main reinforcement required from formula (A)

(iii) Calculate ultimate shearing force V acting on beam

(iv) Calculate suitable minimum breadth of beam (or check, if breadth is specified) from formula (B)

(v) Sketch elevation of beam and compute angle θ for main steel

(vi) Calculate shearing resistance V_1 for beam with main reinforcement only from formula (C); thus determine shearing resistance $(V - V_1)$ to be provided by web reinforcement

(vii) From sketch of beam, measure values of θ and a_2 for each individual web bar

(viii) Calculate area of web bars required from formula (D).

Design formula	Without openings in beam	With openings in beam
(A)	$A_{s_{req}} \nless \dfrac{1.9M}{f_{sy}l}$ or $\dfrac{1.9M}{f_{sy}h}$	$A_{s_{req}} \nless \dfrac{1.9M}{f_{sy}l}$ or $\dfrac{1.55M}{f_{sy}\xi h}$
(B)	$b \simeq \dfrac{0.65V}{k_1(h-0.35a_1)f_1}$	$b \simeq \dfrac{0.55V}{k_1(\xi h - 0.35\alpha a_1)f_t}$
(C)	$V_1 = k_1(h-0.35a_1)f_t b + k_2 A_{s_{prov}} d \sin^2\theta/h$	$V_1 = k_1(\xi h - 0.35\alpha a_1)f_t b + k_2 A_{s_{prov}} d \sin^2\theta/h$
(D)	$V - V_1 = k_2 \Sigma A a_2 \sin^2\theta/h$	$V - V_1 = 1.5k_2 \Sigma A a_2 \sin^2\theta/h$

NOTES

(i) The formulae are only known to be applicable if the following conditions apply: $l/h \ngtr 2$. Static loads only occur and these are applied to top of beam only a_1/h is not greatly outside range of 0.23 to 0.70. Positive anchorage is provided to main reinforcement.

(ii) Restrictions to θ and ξh shown in diagrams only apply when opening intersects line of critical diagonal crack. If opening is reasonably clear of this line, the effect of the opening may be disregarded completely when considering shearing resistance.

(iii) For distributed loads, substitute statically-equivalent twin concentrated loads (i.e., replace uniform load F by two concentrated loads of $\frac{1}{2}F$ at distances of $\frac{1}{4}l$ from supports.

(iv) The more nearly perpendicular a web bar is to the principal diagonal crack, the more effective it is in resisting shearing and limiting cracking: its effectiveness also increases with increasing depth a_2. However, inclined web reinforcement may be more expensive to bend and fix.

(v) If openings are present, web reinforcement must pass both above and below them.

Table 24.1

If cylinder splitting tensile strength is not known, estimate as follows:

Cube strength f_{cu} (N/mm^2)	Cylinder splitting tensile Strength f_t(N/mm^2)
20	2.24
25	2.50
30	2.74
40	3.16
50	3.54

Example Design the reinforcement for the deep beam shown in Fig. 24.7 which supports an ultimate load that (including self-weight) can be represented by the twin concentrated loads shown of 625 kN, using 25 grade concrete and mild steel bars (cube strength = 25 N/mm^2). The concentrated loads exert a bending moment of $625 \times 10^3 \times 400 = 250 \times 10^6$ N-mm on the beam. Thus the area of main reinforcement required is the greater of either

$$\frac{1.9M}{f_{sy}l} = \frac{1.9 \times 250 \times 10^6}{250 \times 1{,}650} = 1{,}152 \text{ mm}^2$$

or

$$\frac{1.55M}{f_{sy}\xi h} = \frac{1.55 \times 250 \times 10^6}{250 \times 800} = 1{,}938 \text{ mm}^2$$

Provide four 25 mm bars ($A_s = 1{,}963$ mm^2) against bending. Since $f_{cu} = 25$ N/mm^2, take $f_t = 2.5$ N/mm^2. Then, since $V = 625 \times 10^3$,

breadth of section required $\simeq \dfrac{0.55V}{k_1(\xi h - 0.35\alpha a_1)f_t} = \dfrac{0.55 \times 625 \times 10^3}{0.7(800 - 0.35 \times 300)2.5} = 283$ mm;

say $b = 300$ mm. Thus the shearing resistance provided by the concrete together with the main reinforcement only is

$$\begin{aligned} V_1 &= k_1(\xi h - 0.35\alpha a_1)f_t b + k_2 A_{s_{prov}} d \sin^2\theta/h \\ &= 0.7(800 - 0.35 \times 300)2.5 \times 300 + \\ &\quad 100 \times 1{,}963 \times 1{,}425 \times 0.877/1{,}500 \\ &= 528 \times 10^3 \text{ N}, \end{aligned}$$

(note that $\tan\theta = 800/300$, hence $\sin^2\theta = 0.877$)

Thus the balance of $625 \times 10^3 - 528 \times 10^3 = 97 \times 10^3$ must be provided by the web reinforcement. If horizontal links are provided at the depths shown in Fig. 24.7 $\sin^2\theta = 0.877$ for each link and since $\Sigma A a_2 \sin^2\theta/h = (200 + 350 + 750 + 900 + 1{,}050 + 1{,}200 + 1{,}350)A \times 0.877/1{,}500 = 3.39A$, thus

$A = (V - V_1)/(1.5k_2 \times 3.39) = 97 \times 10^3/(3.39 \times 100 \times 1.5) = 191$ mm^2 (2 legs of a link). Provide 12 mm dia. 2 legged links horizontally and vertically as shown.

NOTE The Europeon Concrete Committee recommends that the section areas of vertical bars and horizontal bars should not be less than 0.25% in case of mild steel/0.20% in case of deformed bars, of horizontal and vertical cross-sectional areas of the beam, respectively.

24.5 DESIGN AGAINST COMBINED SHEAR AND TORSION

The basic philosophy explained earlier remains valid here also. Since transverse shear (vertical or horizontal) as well as torsion, both cause shear stresses, it is only rational to consider them simultaneously and suggest a combined approach for total shear-torsion design. Where torsion is absent, the terms corresponding to torsion can be ignored without affecting the combined approach.

Behaviour Under Torsion, Flexure and Associated Shear

When a section is subjected to pure torsion, the torsional shear stress thus caused can have different values at different fibres in the section depending on the shape of the section. But, as the torque approaches its ultimate value, a certain

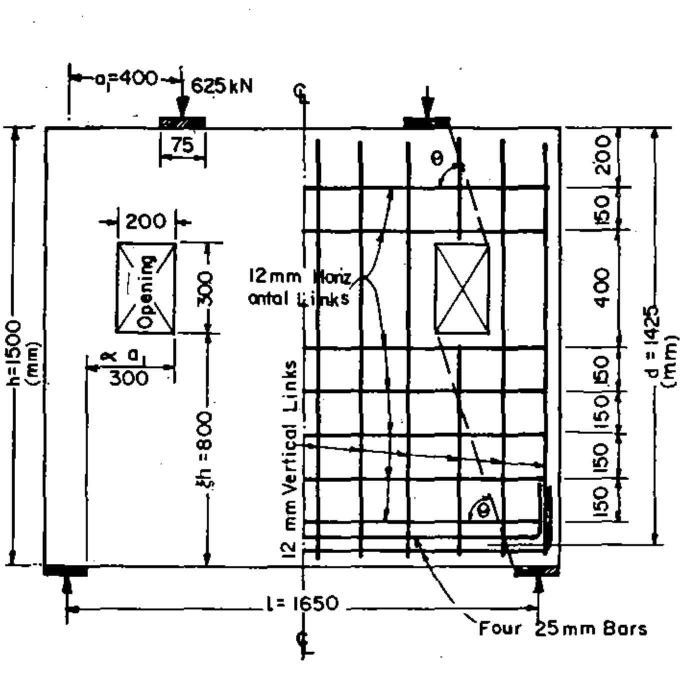


Fig. 24.7 *(Based on Fig. 24.6)*

amount of redistribution of stress takes place among the fibres in that section with the result that, ultimately a more or less unifrom stress exists over a large part of the section. For practical purposes, the ultimate torsional shear stress is, therefore, assumed constant throughout the section.

Flexure creates internal longitudinal compression (and tension) in a member. This compression helps in halting the propagation of spiral-cracking caused by torsion. For this reason, the presence of bending moment does not appear to decrease pure torsional strength of a member except when the applied bending moment is greater that about 80% of the ultimate moment of resistance of the section.[1,2]

Even when a section, which is under combined torsion, shear and moment, has cracked, it still has torsional strength owing to the presence of internal longitudinal compression which locks the torsional cleavage-fracture in the compression zone. This allows the unreinforced torsion capacity (T_c) of a concrete section to be calculated by simply limiting the principal tensile stress at the centroidal level to a suitable value.

Torsion failure, unlike most flexural failures, is sudden (as is a transverse shear failure), and is particularly violent in prestressed concrete owing to unleashing of stored energy in tendons. Torsional moment is associated with very little redistribution, as is the case with shear.

For pure or predominant torsion, cracks first appear on the longer faces of the section and at about 45° to the axis of member. But in pure or predominant bending, the cracks usually appear first on the face stretched by bending, then spread to the side faces, and finally to the compression face, by when concrete in the compression zone may also show distress and disintegration owing to crushing.

Under combined bending and torsion, the cracking is intermediate between about 45° spiral-form and emanating inclined form, and greater the effect of bending moment the steeper is the direction of cracks.

Effect of Reinforcement

Provision of (closed) hoops and longitudinal steel crossing the lines of potential cracks increases the torsional strength of a concrete member, delays the collapse, and lessens the violence of failure. Torsion causes diagonal tension on all faces of a member and hence needs closed hoops and longitudinal bars all round. Transverse vertical shear causes shear stress which varies from zero at top and bottom fibres to a maximum in between, correspondingly causes diagonal tension only on vertical faces and, therefore, needs only vertical stirrups and/or bentup bars.

Transfer of Shear after Cracking

In reinforced concrete, where sections are cracked even under working load, shear is transferred from one (cracked) section to another as a moment caused by the lever arm between the differentials in longitudinal (flexural) compression and tension across the section. The shear, of course, can be the transverse shear or the torsional shear, or both. Transfer of this shear will be accomplished so long as there is bending moment to create the aforesaid differentials. In prestressed concrete, during the elastic phase the sections are theoretically assumed crackfree (by allowing no or little tension) and as such, the shear is assumed to be transferred from section to section as in a homogeneous body. In the ulitmate stage, when cracks have occurred and, therefore, the flexural tensile stress in concrete has been relieved, the situation becomes akin to that in cracked *RC*, the only addition being the presence of a direct stress. Consequently, the shear-transfer from one (cracked) section to another is accomplished in no way less than it is in reinforced concrete.

Design Approach

The upper limit to the usefulness of torsional reinforcement is reached when principal tensile stress approaches a ceiling.[2] Although longitudinal and hoop steel help limit the spreading of diagonal cracks, a stage comes when the concrete between the diagonal cracks gives-in as a compression strut. Any additional reinforcement will obviously be useless. Hence the limitation on the sum of the transverse and torsional shear stresses (see step 3 ahead).

The diagonal tension due to torsion may be taken by the vectors of the tensions in the hoops and the longitudinals, while the diagonal compression due to torsion may be resisted by concrete. The area of hoop steel may be obtained by equating the strain energy stored in hoop steel and in the concrete in principal compression to the work done by the torsion in rotating the beam.[3] Bach and Graf at Stuttgart have experimentally shown that hoops without longitudinal reinforcement are ineffective against torsion and that an equal volume of longitudinal steel should be provided.

For the same bending stress, the principal tensile stress is a function of shear stress. Thus torsion is a primary design factor if torsional shear stress exceeds transverse shear stress. However, if the latter exceeds the former, then transverse shear is a primary design factor.

It has been suggested[2] that if torsion be the primary design factor, reinforcement be designed for it after taking due account of the torsion-taking-capacity of unreinforced section, while entire Transverse shear be taken on additional reinforcement. But if torsion is only a secondary design factor, reinforcement be designed for transverse shear after taking due account of the transverse-shear-taking-capacity of the unreinforced section, while entire torsion be taken on additional reinforcement.

Suggested 'Step by Step' Procedure

Step 1 Estimate V_u (load-factored dead and live load shears less any relief in shear due to prestress and sloping soffit, assuming unit load factor on prestress) and T_u (load factored torsional moment, due to dead and live load and any other applied load) at the section.

Step 2 Evaluate equivalent ultimate transverse shear stress (v_u) and the ultimate torsional shear stress (σ_u)

v_u may be estimated from $v_u = \dfrac{V_u}{bd}$

where b = breadth of rib

d = '0.8 of effective depth' in case of reinforced concrete, and

= '0.8 of overall depth or the effective depth, whichever greater' in case of prestressed concrete

and σ_u may be found using the appropriate formula given in colume 3 in Table 24.2 of formulae for shear stress due to torsion (elastic and ultimate) for different shapes of sections.

It may be noted that σ_u is constant throughout the section, but v_u (the greatest ultimate transverse shear stress) may occur at about the mid-height fibre or the mid-width fibre depending on whether it is vertical transverse shear or horizontal transverse shear. In case both these shears (V_{uv} and V_{uh}) coexist, then $v_{uv} = \dfrac{V_{uv}}{bd}$ and $v_{uh} = \dfrac{V_{uh}}{DB}$ (where D = overall depth, B = 0.8 of breadth of rib) and use bigger of v_{uv} and v_{uh}

Step 3 Check that $(v_u + \sigma_u) \not> 0.15$ of standard 28-day concrete cylinder crushing strength

If this is not so, then increase section, recalculate v_u and σ_u, and ensure the above stipulation before going to Step 4 ahead.

Step 4 Now it is either $v_u > \sigma_u$ or $\sigma_u > v_u$; proceed correspondingly,

... if $v_u > \sigma_u$ (i.e., transverse shear governs, then design for full T_u and only for part of V_u)

then $A_t = \dfrac{T_u p_t}{f_{sy} 0.8 x_1 y_1}$

and $A_v = \dfrac{(V_u - V_c)}{f_{sy} d} p_v$

where V_c = (10 bd) in kg, b and d in cm.

(V_c = shear capacity of concrete alone)

... if $\sigma_u > v_u$ (i.e., torsion governs, then design for full V_u and only for part of T_u)

then $A_t = \dfrac{(T_u - T_c)}{f_{sy} 0.8 x_1 y_1} p_t$

where T_c (the torsion capacity of concrete alone) may be found using the appropriate formula given in column 3 in Table 24.2 of formulae for shear stress due to Torsion on different shapes of sections, assuming the limiting torsional shear stress $\sigma \neq 0.1$ of standard cylinder strength,

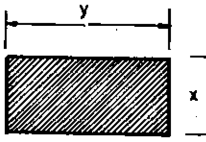
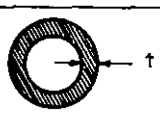
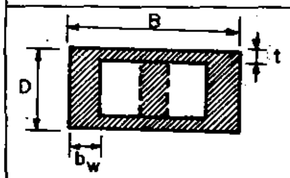
and $A_v = \dfrac{V_u p_v}{f_{sy} d}$

where
p_t = pitch of torsion hoops (effectively closed stirrups)
p_v = pitch of vertical stirrups (for vertical transverse shear)
A_t = twice the section area of torsion hoop leg, hoop placed at pitch p_t ($1/2 A_t$ is section area of one horizontal or vertical leg of hoop),
A_v = section area of all vertical shear stirrup legs at the section, placed at pitch p_v,
f_{sy} = yield stress in hoop and stirrup reinforcements,
x_1, y_1 = the lengths of the short and long sides of the torsion hoop,
d = as defined above in Step 2.

Step 5 Provide longitudinal reinforcement perpendicular to the hoops, inside them, and proportionately distributed along the hoop perimeter); section area of this longitudinal reinforcement is,

$$A_l = A_t \frac{x_1 + y_1}{p_t}$$

Table 24.2 *Relation between torsional shear stress σ and torsional moment T*

Column 1	Column 2	Column 3
Section	in *ELASTIC phase* T and σ are elastic (working-load) values	*At ULTIMATE* T and σ are the ultimate values
 	$\sigma = \dfrac{16T}{\pi D^3}$ at surface at any point	$\sigma = \dfrac{12T}{\pi D^3}$ at every point in section
 x = small side y = long side	at middle of long side, on surface, $\sigma = \dfrac{T}{\alpha_1 x^2 y}$ at middle of short side, on surface, $\sigma = \dfrac{T}{\alpha_2 x y^2}$ α_1 and α_2 from Table 2.1 in Reference 2 $\sigma = 0$ at corners and cg (ALSO SEE NOTE* below)	$\sigma = \dfrac{2T}{x^2(y - x/3)}$ at every point in section
 	$\sigma = T/2At$ at any point in section, A = mean of the areas enclosed by inner and outer boundaries of the tube of thickness t	Same formula as for Elastic phase (col. 2)
 Neglect Any Intermediate Webs	at middle of long side: $\sigma = \dfrac{T}{2t(D-t)(B-b_w)}$ at middle of short side: $\sigma = \dfrac{T}{2b_w(D-t)(B-b_w)}$	$\sigma = \dfrac{T}{(b_w + t)(D-t)(B-b_w)}$ at every point in the section
T or L section	See Reference 2 (ALSO SEE NOTE* below)	See Reference 2

**NOTES*

(i) *Rectangular section (Fig. 24.8)*

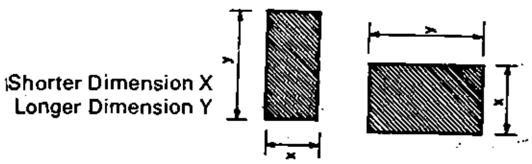


Fig. 24.8

Torsional constant is,

$J = \gamma x^3 y$, where γ depends on the ratio of $\dfrac{y}{x}$

This was dicovered by St. Venant and the γ values are tabulated below

y/x	1	1.5	2	3	5	>5
γ	0.14	0.20	0.23	0.26	0.29	0.33

A formula which will give values within 4% of these is

$$\gamma = 0.33 - 0.21\frac{x}{y}\left(1 - \frac{x^4}{12y^4}\right)$$

(ii) *T, I or L sections (Fig. 24.9)*

Torsional constant is

$J = \gamma_1 b_w^3 h + \gamma_2 h_t^3 (b_t - b_w) + \gamma_3 h_b^3 (b_t - b_w)$

The γ's depend on the individual y/x ratio (i.e., ratio of longer side to smaller side) for each rectangle where it should be remembered that y and x are the longer and shorter dimensions respectively, e.g., for the top flange y is $(b_t - b_w)$ and x is h_t.

A different value will be found if the section is divided up as follows (Fig. 24.10):

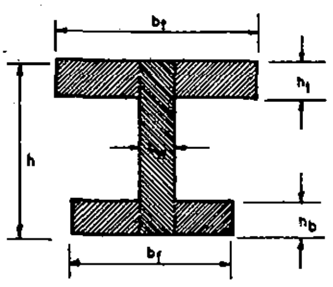


Fig. 24.9

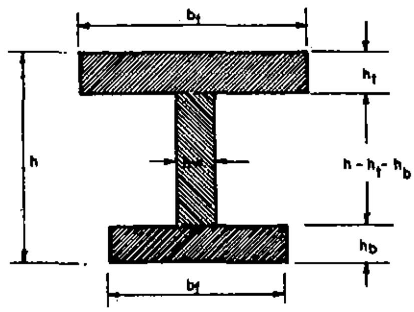


Fig. 24.10

$$J = \gamma_1 h_t^3 b_t + \gamma_2 h_b^3 b_t + \gamma_2 b_w^3 (h - h_t - h_b)$$

Where torsion clauses are based on the work of Prof. Cowan, it is the latter method of dividing up the section which is suggested. Where proportioning the torsional moment between individual rectangles, the widest rectangle should be taken as long as possible. To be consistent therefore, if b_w is greater than h_t and h_b use the first method of dividing up the section. If not, then the use the second method.

REFERENCES

1. Swami, N, Behaviour and Ultimate Strength of Prestressed Concrete Hollow Beams under Combined Bending and Torsion—(Magazine of *Concrete Research*, vol. 14, no. 40, C and CA., London), March 1962.
2. Cowan and Lyalin, *Reinforced and Prestressed Concrete in Torsion*, Edward Arnold, London.
3. Cowan, "Torsion of Rectangular Elastic Isotropic Beam Reinforced with Helics of another Material", *Appl. Scientific Research*, (Delft) section A, 3, 1952.
4. Raina, VK, "Elastic and Ultimate Design Against Diagonal Cracking in Prestressed Concrete Members Under Combined Torsion, Flexure and Shear", Seminar on Problems in Prestressing, ING of IABSE Madras, February 1970, IRC, New Delhi.
5. ACI and AASHTO Design Specifications.

CHAPTER 25

Design of a Reinforced Concrete Section Subject to Combined Axial Thrust and Any-Axis Bending—Simplified Practical Method

Synopsis

Most of the methods available for designing reinforced concrete sections subjected to axial thrust and bending about any axis—orthogonal or oblique, are at best terribly theoretical and at worst inaccurate. This leaves a practising designer to the mercy of 'established' graphs and charts whose scope is very limited, in that, they are not available for all shapes of sections. This chapter very simply derives the general expression for stresses for the general cases of combined axial thrust and oblique bending in a section of *any* shape. The design procedure is then illustrated, step by step, through five typical examples.

NOTES 1. The method outlined in this presentation is applicable even if the section is prestressed. In such a case the prestress force and moment may be added respectively to the axial thrust and bending moment (and the concept becomes akin to that in partial —prestressing where reinforcement is provided to carry certain amount of tension).

2. The method outlined in this presentation is 'exact' and yet very 'simplified'.

3. The author acknowledges with grateful thanks Engr. S.G. Athavale who initially evolved the basic content of the theoretical derivation presented here.

25.1 INTRODUCTION

Under a purely axial load (zero eccentricity), an RC section suffers only direct compression. As the eccentricity is introduced and increased, additional bending stresses (tensile and compressive) are created and so long as the resultant stresses (calculated using full section properties) are within permissible limits, cracking of concrete is insignificant. However, if the resultant maximum tensile stress in concrete exceeds its permissible value, the section is then assumed to have cracked and full section properties cannot be assumed effective any more. The effective section then is less than the full physical section as the concrete below the neutral axis (i.e., in tension) is considered ineffective. The thrust P, applied at an eccentricity e from the physical centroid of the overall section (e = resultant applied bending moment/P) is withstood only by the effective section whose centroid is no more the physical centroid of the overall section. If e' is the shift between these two centroids, then the thrust P in fact acts at an eccentricity of only $(e - e')$ from the centroid of the effective section.

In a section subjected to a thrust combined with two orthogonal moments (causing bending about an oblique axis), in working-load design one of the approaches (though erroneous) is to first assume a monoaxial bending with thrust, and on the section properties of this monoaxially cracked section then superimpose the effect of the other orthogonal moment and later finally apply some stress correction—if at all. Obviously, such a method is less powerful because it is an indirect procedure in attempting to arrive at the final bending behaviour of the section—not to speak of ignoring the e' effect in an oblique direction as well as the effect of rotation of neutral axis.

It is better to have approximate solution to an exact problem instead of an exact solution to an approximate problem.

25.2 THEORY

Consider the section as shown in Fig. 25.1, subjected to a thrust P and a resultant bending moment M_R (vectorial resultant), such that this combination is equivalent to P applied at a distance $e = \dfrac{M_R}{P}$ from the physical centroid of the overall section, as shown. Let f be the extreme compressive stress, n the distance of the neutral axis below the extreme compressed fibre, p the stress at the level of the centroid of the effective section, x, y, and $\bar{y}$ the various distances as shown, and dA the transformed area of the element shown shaded (Fig. 25.1).

Force in the element = stress × area

$$= \frac{f}{n}(x + \bar{y})dA \qquad (25.1)$$

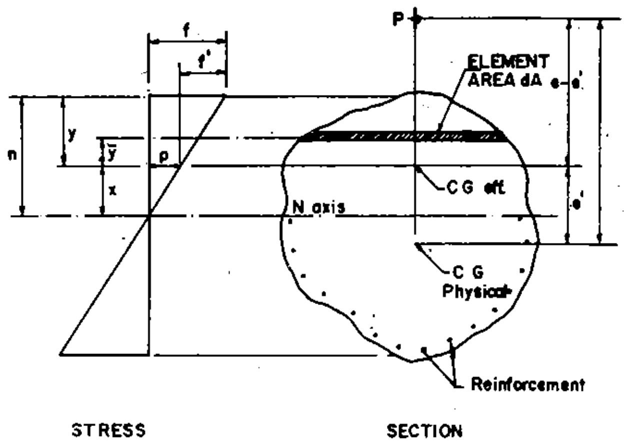


Fig. 25.1

and also $$= \left(p + \frac{f'}{y}\bar{y}\right) dA \tag{25.2}$$

Its moment about centroid of effective section

$$= \text{force} \times \text{arm}$$

$$= \left(p + \frac{f'}{y}\bar{y}\right) dA \cdot \bar{y} \tag{25.3}$$

Equating external and internal forces,

$$P = \Sigma \frac{f}{n}(x + \bar{y})dA$$

$$= \frac{f}{n}(\Sigma x dA + \Sigma dA \cdot \bar{y}) \tag{25.4}$$

But $\Sigma dA \cdot \bar{y}$, the moment of area about its own centroid, is equal to zero, so that,

$$P = \frac{f}{n}(\Sigma x dA + 0)$$

$$= \frac{f}{n} x \Sigma dA$$

$= p \cdot A_{\text{eff}}$ ($\because \frac{f}{n}x = p$ and ΣdA = total effective area A_{eff}.)

Thus $$p = \frac{P}{A_{\text{eff}}} \tag{25.5}$$

Equating moments due to external and internal forces bout the centroid of effective section,

$$P(e - e') = \Sigma \left(p + \frac{f'}{y}\bar{y}\right) dA \cdot \bar{y}$$

$$= p \Sigma dA \cdot \bar{y} + \frac{f'}{y} \Sigma dA \cdot (\bar{y})^2 \tag{25.6}$$

but $\Sigma dA \cdot \bar{y} = 0$, and $\Sigma dA \cdot (\bar{y})^2$ = second moment of effective area about its own centroid i.e., I_{eff}, thus,

$$P(e - e') = 0 + \frac{f'}{y} I_{\text{eff}}$$

so that $$f' = \frac{P(e - e')}{I_{\text{eff}}} y \tag{25.7}$$

Since total stress f at any fibre distant $\pm y$ from centroid of effective section comprises of direct compression p and bending stress $\pm f'$, thus,

$$f = p \pm f'$$

substituting from Eqs. (25.5) and (25.7) we get;

$$f = \frac{P}{A_{\text{eff}}} \pm \frac{P(e - e')}{I_{\text{eff}}} y \tag{25.8}$$

However, it should be noted that this formula is applicable when the cracked section is symmetrical about the load line. (Load line is the prependicular from load point P to the axis of resultant moment.) In other words, this formula is applicable when the neutral axis is parallel to the axis of the resultant moment so that load line would be perpendicular to neutral axis itself. This is so in the case of circular sections, solid or annular, under monoaxial, biaxial or multiaxial bending. This is so also in the case of rectangular, square and triangular (equilateral and isosceles) sections, solid or box, so long as the resultant moment acts about an axis which falls perpendicular to the axis of symmetry of the effective section.

When the cracked section is not symmetrical about the load line, i.e., the load line is not a principal axis, then the neutral axis is not parallel to the axis of the resultant moment. The simple superimposition of bending stresses calculated individually with respect to the two orthogonal axes is obviously invalid if these axes are not principal axes, because then it is the case of the effective section being asymmetrical about these axes. In such a case, the effective section is subjected to bending about two orthogonal axes—neither of which is a principal axis—and the superimposition of stresses has then to be on the following lines:

If $T - T$ and $L - L$ are the two orthogonal axes through the physical centroid of the overall section $b \times d$, M_T and M_L are moments about these axes and P the axial thrust, then the extreme fibre stresses in the cracked section are given by,

$$\frac{P}{A_{\text{eff}}} \pm \left[\frac{P(e_L - e'_L) - P(e_T - e'_T)\left(\dfrac{I_{LT_{\text{eff}}}}{I_{\text{eff}_T}}\right)}{I_{\text{eff}_L} - \left(\dfrac{I^2_{LT_{\text{eff}}}}{I_{\text{eff}_T}}\right)}\right] \times$$

$$\left\{\left(\frac{b}{2} - e'_L\right) \text{ or } \left(\frac{b}{2} + e'_L - \text{cover to reinforcement}\right)\right.$$

$$\left.\text{depending on which fibre}\right\}$$

$$\pm \left[\frac{P(e_T - e'_T) - P(e_L - e'_L)\left(\dfrac{I_{LT_{\text{eff}}}}{I_{\text{eff}_T}}\right)}{I_{\text{eff}_T} - \left(\dfrac{I^2_{LT_{\text{eff}}}}{I_{\text{eff}_L}}\right)}\right] \times$$

$$\left\{\left(\frac{d}{2} - e'_T\right) \text{ or } \left(\frac{b}{2} + e'_T - \text{ cover to reinforcement}\right),\right.$$

$$\left.\text{depending on which fibre}\right\}$$

where e_T and e_L are the load eccentricities from $T-T$ and $L-L$ axes, respectively $\left(e_T = \dfrac{M_T}{F}\right.$ and is measured along $L-L$ axis, $e_L = \dfrac{M_L}{F}$ and is measured along $T-T$ axis$\Big)$,

e'_T and e'_L are the distances from the T and L axes to the cg of the effective section.

I_{eff_T} and I_{eff_L} are the second moments of area (moments of inertia) of the effective section about axes parallel to T and L axes passing through its centroid.

$I_{LT_{\text{eff}}}$ is the product of inertia of the effective section about axes parallel to L and T axes passing through its centroid. The inclination ϕ of the axis of the resultant moment to $L-L$ axis is given by,

$$\tan\phi = \frac{M_T}{M_L}$$

The inclination ϕ' of the N-axis is given by,

$$\tan\phi' = \frac{M_T \cdot I_L - M_L \cdot I_{LT}}{M_L \cdot I_T - M_T \cdot I_{LT}}$$

(I_{LT} represents product of inertia)

$\phi = \phi'$ only when $I_L = I_T$ and $I_{LT} = 0$

To locate the inclination ϕ in a cracked section it is sufficient to initially adopt the properties of uncracked section in the above formula, taking $I_{LT} = 0$. However, as has been shown in Examples 4 and 5 ahead, the value of ϕ can be finally verified by the following formula,

$$\tan\phi = \frac{P(e_T - e'_T)I_{\text{eff}_L} - P(e_L - e'_L)I_{LT_{\text{eff}}}}{P(e_L - e'_L)I_{\text{eff}_T} - P(e_T - e'_T)I_{LT_{\text{eff}}}}$$

or $$\tan\phi = \frac{(e_T - e'_T)I_{\text{eff}_L} - (e_L - e'_L)I_{LT_{\text{eff}}}}{(e_L - e'_L)I_{\text{eff}_T} - (e_T - e'_T)I_{LT_{\text{eff}}}}$$

25.3 EXAMPLES

Example 1

Case of a solid rectangular RC section subjected to axial thrust and monoaxial bending about a side. Consider a solid rectangular RC section (Fig. 25.2) of size 120 cm × 60 cm. Modular ratio $m = 14$. Axial thrust $P = 98$ tonnes and bending moment $M = 83.5$ tm about axial parallel to its breadth.

Step 1 Assume 40 cm^2 of mild steel in compression as well as in tension flanges, as shown.

Step 2 Assume neutral axis at 55 cm from extreme compression fibre.

Step 3 $A_{\text{eff}} = (60 \times 55) + (14 - 1)40 + (14 \times 40)$

$$= 3300 + 520 + 560 = 4380 \text{ cm}^2$$

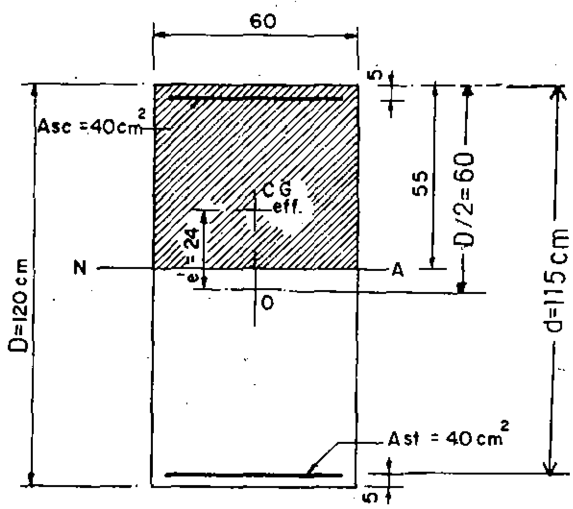


Fig. 25.2

Step 4 Distance of CG_{eff} from physical centroid (take moments of effective areas about physical centroid)

$$e' = \frac{(3300 \times 32.5) + (520 \times 55) - (560 \times 55)}{4380} = 24 \text{ cm.}$$

$$e - e' = \frac{83.5 \times 100}{98} - 24.0 = 85.2 - 24.0 = 61.2 \text{ cm}$$

Step 5 $I'_{\text{eff}} = 60 \times \dfrac{55^3}{12} + 60 \times 55 \times 8.5^2 + 520 \times 31^2$

$$+560 \times 79^2$$

$$= 50{,}70{,}000 \text{ cm}^4$$

Step 6 Distance of neutral axis below CG_{eff}

$$\text{At } y \text{ below } CG_{\text{eff}}, \text{ stress} = \frac{P}{A_{\text{eff}}} - \frac{P(e-e')y}{I_{\text{eff}}}$$

This stress is zero at neutral axis, so that diatance of neutral axis below CG_{eff} will thus be given by:

$$y = \frac{I_{\text{eff}}}{A_{\text{eff}}(e-e')}$$

i.e., $\dfrac{50,70,000}{4380 \times 61.2} = 18.0$ cm compared to (24–5)

$= 19$ cm, assumed, hence OK

Step 7 (i) Max. compressive stress in concrete,

$$= \frac{P}{A_{\text{eff}}} + \frac{P(e-e')\left(\frac{D}{2} - e'\right)}{I_{\text{eff}}}$$

$$= \frac{98,000}{4380} + \frac{98,000 \times 61.2}{50,70,000}\left(\frac{120}{2} - 24\right)$$

$= 22.4 + 42.5 = 64.9$ kg/cm²

OK for the concrete chosen.

(ii) Max. tensile stress in steel,

$$= m\left\{\frac{P}{A_{\text{eff}}} - \frac{P(e-e')\left(d - \frac{D}{2} + e'\right)}{I_{\text{eff}}}\right\}$$

$$= 14\left\{22.4 - \frac{98,000 \times 61.2(115 - 60 + 24)}{50,70,000}\right\}$$

$= 14(22.4 - 93.4) = -994$ kg/cm²

OK for steel chosen.

Example 2

Case of a solid circular RC section subjected to axial thrust and bending.

Consider a solid circular RC section (Fig. 25.3) of 120 cm dia. Modular ratio $m = 18$. Axial thrust $P = 32$ tonnes and bending moment $M = 30.5$ tm.

Step 1 Assume 35 cm² of mild steel uniformly distributed along the periphery.

Step 2 Assume neutral axis at 36 cm from extreme compression fibre.

Step 3 Reading the values of constants from Fig. 25.4 for section properties of circular segment above neutral axis,

for $\dfrac{x}{d} = \dfrac{36}{120} = 0.3$, $A = 0.65$, $B = 0.20$

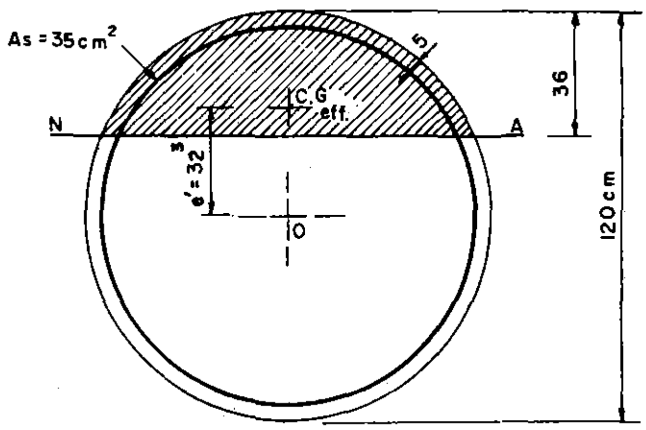


Fig. 25.3

and $C = 0.02$

$\therefore \bar{x} = 0.65 \times 60 = 39$ cm (distance of cg of segment from centre of circle)

$A = 0.20 \times 120^2 = 2880$ cm² (section area of segment)

and $I = 0.02 \times 60^4 = 2{,}59{,}200$ cm⁴ (second moment of area of segment about its own centroid).

Step 4 For convenience taking transformed area of steel as $(m-1)A_s$, both in tensile as well as in compressive zones, the effective area is,

$A_{\text{eff}} = 2880 + (18-1)35$

$= 2880 + 595 = 3475$ cm²

Step 5 Distance of CG_{eff} from physical centroid of whole section is

$e' = \dfrac{2880 \times 39}{3475} = 32.3$ cm (by taking moments of effective areas about physical centroid)

$\therefore e - e' = \dfrac{30.5 \times 100}{32} - 32.3 = 95.3 - 32.3$

$= 63$ cm

Step 6 $I_{\text{eff}} = 2,59,200 + 2880 \times 6.7^2 + \left(595 \times \dfrac{55^2}{2} + 595 \times 32.3^2\right)$

$= 19{,}08{,}000$ cm⁴

Step 7 Distance of neutral axis below CG_{eff} (as in Example 1) is,

$$\frac{I_{\text{eff}}}{A_{\text{eff}}(e - e')} = \frac{19,08,000}{3475 \times 63} = 8.7 \text{ cm}$$

compared to $(32.3 - 24.00) = 8.3$ cm assumed, OK

Step 8 (i) Max. compressive stress in concrete,

$$= \frac{P}{A_{\text{eff}}} + \frac{P(e - e')(D/2 - e')}{I_{\text{eff}}}$$

$$= \frac{32000}{3475} + \frac{32000}{19,08,000} \times 63 \left(\frac{120}{2} - 32.3\right)$$

$$= 9.2 + 29.2 = 38.4 \text{ kg/cm}^2, \text{ OK}$$

(ii) Max. tensile stress in steel,

$$= m\left(\frac{P}{A_{\text{eff}}} - \frac{P(e - e')(d - D/2 + e')}{I_{\text{eff}}}\right)$$

$$= 18\left(9.2 - \frac{32000 \times 63(115 - 60 + 32.3)}{19,08,000}\right)$$

$$= 18(9.2 - 92.0) = -1490 \text{ kg/cm}^2, \text{ OK}$$

Example 3

Case of a hollow circular RC section subjected to axial thrust and bending.

Consider a hollow circular RC section (Fig. 25.5) of outer dia. 600 cm and inner dia. 360 cm. Modular ratio $m = 18$. Axial thrust $P = 630$ tonnes and bending moment $M = 1850$ tm.

Step 1 Assume 100 cm^2 of mild steel at outer as well as inner faces of hollow section, uniformly distributed along the peripheries.

Step 2 Assume neutral axis at 150 cm from extreme compression fibre.

Step 3 Taking constants from Fig. 25.4 for the two circular segments (bounded by outer and inner circles above neutral axis,

Outer circle

$$\frac{x}{d} = \frac{150}{600} = 0.25$$

$\therefore A = 0.70;\quad B = 0.15;\quad C = 0.01$

Inner circle

$$\frac{x}{d} = \frac{30}{360} = 0.083$$

$\therefore A = 0.905;\quad B = 0.03;\quad C = 0$ (approx.)

so that $\bar{x} = 0.70 \times 300 = 210$ cm (for bigger segment)

$\bar{x} = 0.905 \times 180 = 163$ cm (for smaller segment)

Area = $0.15 \times 600^2 = 54{,}000$ cm^2 (for bigger segment)

Area = $0.03 \times 360^2 = 3890$ cm^2 (for smaller segment)

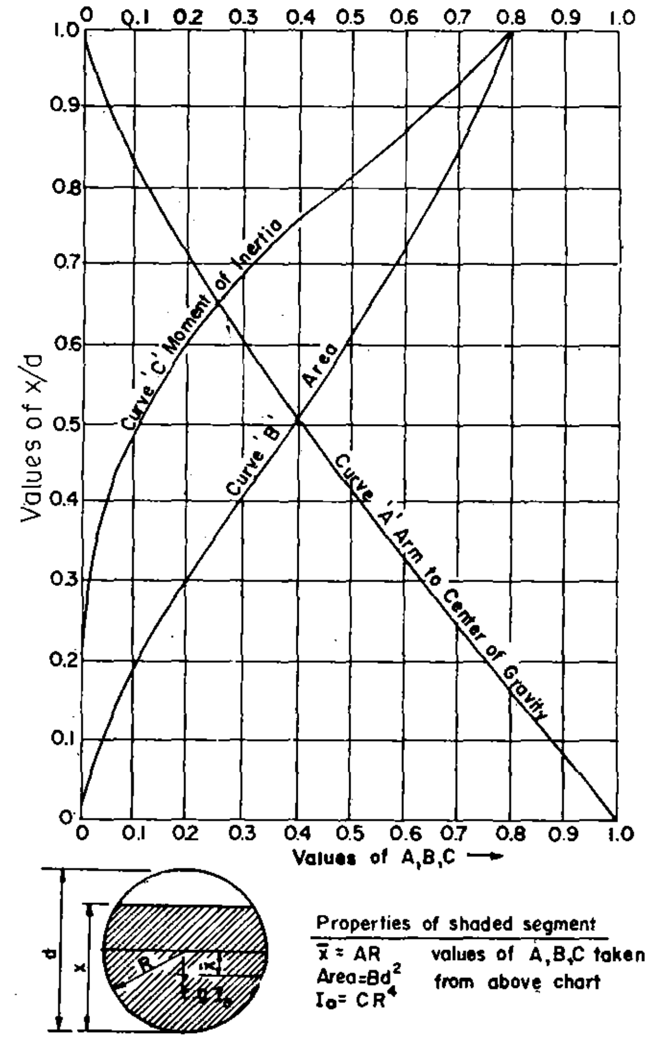


Fig. 25.4 *Constants for Properties of Circular Segments (Ref. page 703 of RC Fundamentals by Phil M Ferguson, 2nd Edition, Wiley Eastern.)*

$I = 0.01 \times 300^4 = 810{,}00{,}000$ cm^4 (for bigger segment)

$I = 0$ (approx.) (for smaller segment)

Step 4 For convenience, taking transformed area of steel as $(m-1)A_s$, both in tensile and compressive zones,

$$A_{\text{eff}} = 54,000 - 3890 + (18 - 1)(100 + 100) = 53510 \text{ cm}^2$$

Step 5 Distance of CG_{eff} from physical centroid:

$$e' = \frac{(54,000 \times 210) - (3890 \times 163)}{53510} = 200 \text{ cm}$$

$$\therefore (e - e') = \frac{1850 \times 100}{630} - 200 = 294 - 200 = 94 \text{ cm}$$

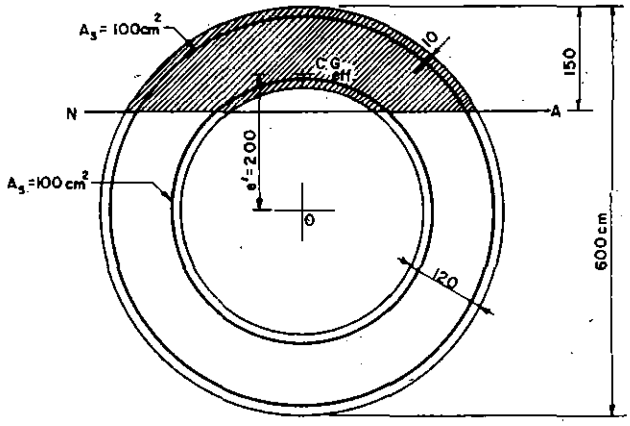


Fig. 25.5

Step 6 $I_{eff} = 810,00,000 + 54,000 \times 10^2 - 3890 \times 37^2$

$$+(18-1)100\left(\frac{290^2+190^2}{2}\right)+(18-1)$$
$$(100+100)200^2$$

$= 25,79,00,000 \text{ cm}^4$

Step 7 Distance of neutral axis below CG_{eff} is given by,

$$\frac{I_{eff}}{A_{eff}(e-e')} = \frac{25,79,00,000}{53510 \times 94}$$

$= 51.2$ cm compared to $200 - 150 = 50$ cm assumed, OK

Step 8 (i) Max. compressive stress in concrete

$$= \frac{P}{A_{eff}} + \frac{P(e-e')(D/2-e')}{I_{eff}}$$

$$= \frac{630,000}{53,510} + \frac{630,000 \times 94(600/2-200)}{25,79,00,000}$$

$= 11.8 + 23.0 = 34.8$ kg/cm², OK

(ii) Max. tensile stress in steel

$$= m\left(\frac{P}{A_{eff}} - \frac{P(e-e')(d-D/2+e')}{I_{eff}}\right)$$

$$= 18\left(11.8 - \frac{6,30,000 \times 94(590-300+200)}{25,79,00,000}\right)$$

$= 18(11.8 - 112.4) = -1810$ kg/cm², OK

Example 4

Case of a solid rectangular section subjected to axial thrust and biaxial bending.

Consider a solid rectangular RC section (Fig. 25.6) of size 75 cm× 200 cm Modular ratio m = 14. Axial thrust P = 160 tonnes. Bending moments M_T = 180 mt. and M_L

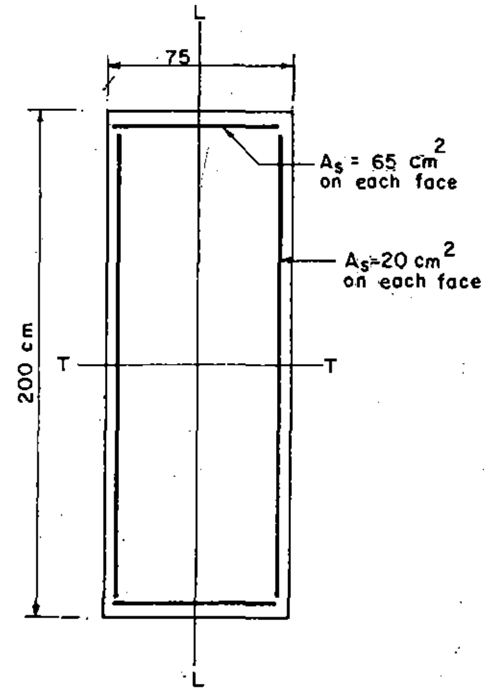


Fig. 25.6

= 50 mt. TT and LL axes are orthogonal axes parallel to breadth and length of section, respectively.

Step 1 Assume 65 cm² of mild steel along each breadth face and 20 cm² of mild steel along each length face as shown.

Step 2 Properties of uncracked section

$$I_{TT} = \frac{75 \times 200^3}{12} + (14-1)$$
$$\left\{65 \times 95^2 + \frac{20 \times 190^2}{12}\right\} \times 2$$
$$= 500,00,000 + 168,20,000$$
$$= 668,20,000 \text{ cm}^4$$

$$I_{LL} = 200 \times \frac{75^3}{12} + (14-1)$$
$$\left\{20 \times 32.5^2 + 65 \times \frac{65^2}{12}\right\} \times 2$$
$$= 70,40,000 + 11,44,000$$
$$= 81,84,000 \text{ cm}^4$$

$$I_{TL} = 0$$

Step 3 *Location of neutral axis:* For the initial trial, using the properties of uncracked section,

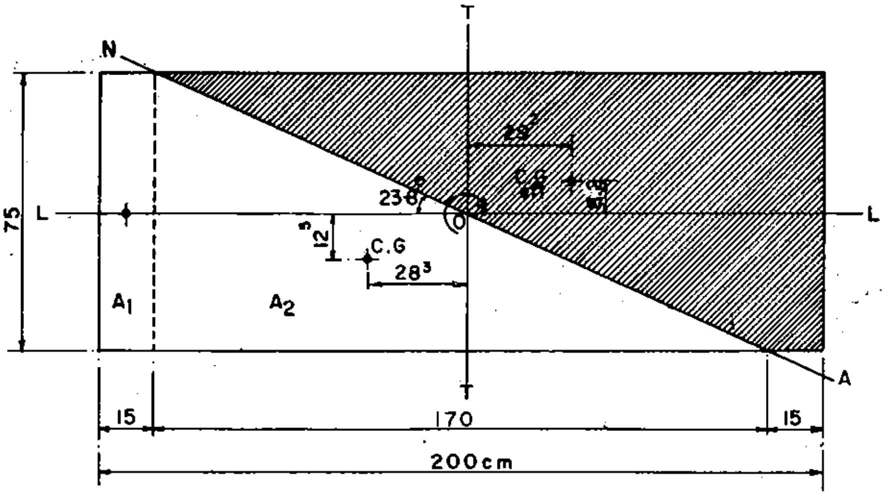


Fig. 25.7

$$\tan\phi = \frac{M_T I_{LL}}{M_L I_{TT}} = \frac{180 \times 81,84,000}{50 \times 668,20,000} = 0.44$$

$\therefore \phi = 23.8°$

Assume neutral axis to pass through physical centroid (point O) of uncracked section at an inclination of 23.8° to LL axis (Fig 25.7).

Step 4 Taking transformed area of steel as $(m-1)A_s$, both in tensile and compressive zones,

$$A_{\text{eff}} = (15 \times 75) + \left(75 \times \frac{170}{2}\right) + (14-1)(65+20)2$$

$$= 1125 + 6375 + 2210 = 9710 \text{ cm}^2$$

Step 5 Ordinates of CG_{eff} *wrt* the physical centroid: (take moments of effective area about TT and LL axes respectively).

$$e'_T = \frac{(1125 \times 92.5) + (6375 \times 28.3)}{9710} = 29.2 \text{ cm}$$

$$e'_L = \frac{6375 \times 12.5}{9710} = 8.2 \text{ cm}$$

Step 6 $\therefore e_T - e'_T = \dfrac{180 \times 100}{160} - 29.2 = 83.3$ cm

$$e_L - e'_L = \frac{50 \times 100}{160} - 8.2 = 23.05 \text{ cm}$$

Step 7 See Fig. 25.7.

$$I_{\text{eff}_T} = \frac{500,00,000}{2} + 168,20,000 - 9710 \times 29.2^2$$

$$= 335.4 \times 10^5 \text{ cm}^4$$

$$I_{\text{eff}_L} = \frac{70,40,000}{2} + 11,44,000 - 9710 \times 8.2^2$$

$$= 40.12 \times 10^5 \text{ cm}^4$$

$$I_{LT_{\text{eff}}} = 1125(63.3)(-8.2) + 6375(4.3)(-0.9) + 2210(-29.2)(-8.2)$$

$$= -0.80 \times 10^5 \text{ cm}^4$$

Step 8 Check for $\tan\phi'$

$$\tan\phi' = \frac{(83.3 \times 40.12) - 23.05(-0.8)}{(23.05 \times 335.4) - 83.3(-0.8)}$$

$= 0.43$ compared to 0.44 assumed, OK

Step 9 Check for position of neutral axis:
Stress anywhere along the neutral axis must be zero.

As a check, stress at point O (Fig. 25.7) must be zero as it lies on the assumed neutral axis in this trial. Let us find it out:

$$\text{Stress at point O} = \left(\frac{160,000}{9710}\right) - \left(\frac{160,000\left\{23.05 - 83.3 \times \dfrac{(-0.80)}{335.4}\right\}}{\left(40.12 - \dfrac{0.8^2}{335.4}\right) \times 10^5}\right) 8.2 - \frac{160,000\left\{83.3 - 23.05 \times \dfrac{(-0.8)}{40.12}\right\}}{\left(335.4 - \dfrac{0.8^2}{40.12}\right) \times 10^5} \times 29.2$$

= -2.0 kg/cm^2 compared to zero if neutral axis really passed through point O. This means a fresh trial is necessary. The new neutral axis will be parallel to the assumed one but bodily slightly displaced.

For the purposes of this illustration if the assumed neutral axis is taken as final, then the extreme stresses are as follows:

Step 10 (i) Max. compressive stress in concrete,

$$= \frac{160,000}{9710} +$$

$$\frac{160,000\left\{23.05 - 83.3 \times \frac{(-0.8)}{335.4}\right\}}{\left(40.12 - \frac{0.8^2}{335.4}\right) \times 10^5}(37.5 - 8.2) +$$

$$\frac{160,000\left\{83.3 - \frac{23.05(-0.8)}{40.12}\right\}}{\left(335.4 - \frac{0.8^2}{40.12}\right) \times 10^5}(100 - 29.2)$$

$$= 16.5 + 27.2 + 28.3 = 72 \text{ kg/cm}^2$$

(ii) Max. tensile stress in steel,

$$= 14\left\{16.5 - \frac{27.2(37.5 + 8.2 - 5)}{29.3} - \frac{28.3(100 + 29.2 - 5)}{70.8}\right\}$$

$$= 14\{16.5 - 39.6 - 49.8\}$$

$$= -1020 \text{ kg/cm}^2$$

Example 5

Case of a hollow rectangular section subjected to axial thrust and biaxial bending. Consider a hollow rectangular RC section (Fig. 25.8) of outer dimensions 140 cm × 295 cm with 42 cm thick walls. Modular ratio $m = 11$. Axial thrust $P = 345$ tonnes. Bending moments $M_L = 280$ mt and $M_T = 225$ mt. TT and LL axes are orthogonal axes parallel to breadth and length of section, respectively.

Step 1 Assume reinforcement as shown in Fig. 25.8.

Step 2 Properties of uncracked section,

$$I_{LL} = \left(295 \times \frac{140^3}{12}\right) - \left(211 \times \frac{56^3}{12}\right) + (11 - 1)\left\{55 \times 65^2 + 20 \times 33^2 + 45\times\frac{130^2}{12} + \frac{15 \times 66^2}{12}\right\} \times 2$$

$$= 676,00,000 - 31,00,000 + 64,60,000$$

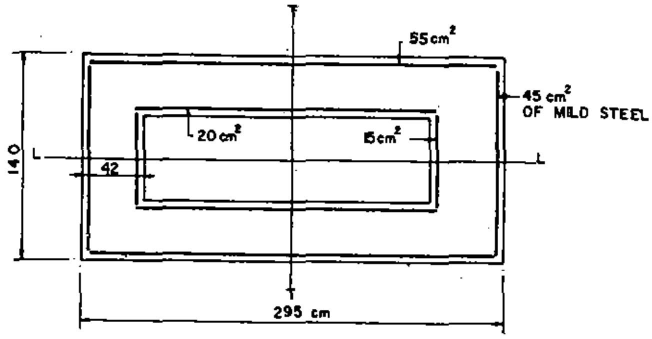


Fig. 25.8

$$= 709.6 \times 10^5 \text{cm}^4$$

$$I_{TT} = \left(140 \times \frac{295^3}{12}\right) - \left(56 \times \frac{211^3}{12}\right) + (11 - 1)\left\{45 \times 142.5^2 + 15 \times 110.5^2 + 55 \times \frac{285^2}{12} + 20 \times \frac{221^2}{12}\right\} \times 2$$

$$= 29,84,00,000 - 437,50,000 + 309,60,000$$

$$= 2856.1 \times 10^5 \text{ cm}^4$$

$$I_{TL} = 0$$

Step 3 Location of neutral axis.

For the initial trail, using the properties of uncracked section,

$$\tan\phi = \frac{M_T \cdot I_{LL}}{M_L \cdot I_{TT}} = \frac{225 \times 709.6}{280 \times 2856.1} = 0.20$$

$$\therefore \phi = 11°$$

Assume neutral axis inclined at 11° to LL axis and intersecting TT axis at O′ to 20 cm above physical centroid, O.

Step 4 Taking transformed area of steel as $(m - 1)A_s$, both in tensile and compressive zones,

$$A_{\text{eff}} = (20.5 \times 295) + \left(59 \times \frac{295}{5}\right) - \left(29.1 \times \frac{145.5}{2}\right) + (11 - 1)(55 + 20 + 45 + 15) \times 2$$

$$= 6050 + 8700 - 2110 + 2700$$

$$= 15340 \text{ cm}^2$$

Step 5 Ordinates of CG_{eff} *wrt* physical centroid:

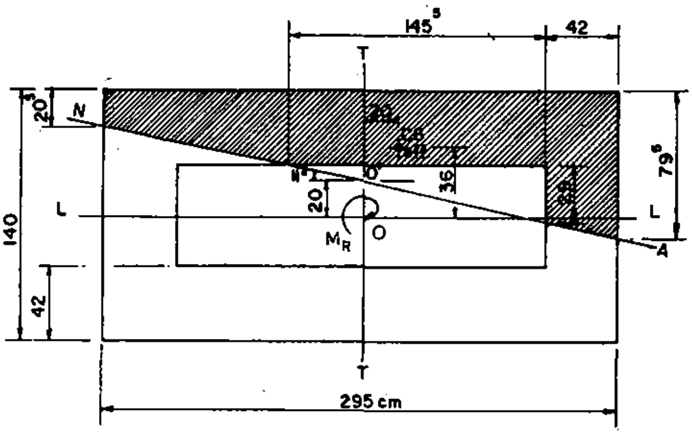


Fig. 25.9

Take moments of effective area about LL and TT axes, respectively

$$e'_L = \frac{(6050 \times 59.75) + (8700 \times 29.83) - (2110 \times 18.3)}{15340}$$

$= 38.0$ cm.

$$e'_T = \frac{(8700 \times 49.17) - (2110 \times 57)}{15340}$$

$= 20.0$ cm.

Step 6 $e_L - e'_L = \dfrac{280 \times 100}{345} - 38.0 = 43.2$ cm

$$e_T - e'_T = \frac{225 \times 100}{345} - 20.0 = 45.2 \text{ cm}$$

Step 7 See Fig. 25.9.

$$I_{\text{eff}_L} = \left(295 \times \frac{20.5^3}{12} + 6050 \times 21.75^2\right) + \left(295 \times \frac{59^3}{36} + 8700 \times 8.17^2\right) - \left(145.5 \times \frac{29.1^3}{12} + 2110 \times 19.7^2\right) + (64,60,000 + 2700 \times 38^2)$$

$= 148.22 \times 10^5$ cm^4

$$I_{\text{eff}_T} = \left(20.5 \times \frac{295^3}{12} + 6050 \times 20^2\right) + \left(59 \times \frac{295^3}{36} + 8700 \times 29.17^2\right) - \left(29.1 \times \frac{145.5^3}{36} + 2110 \times 37^2\right) + (309,60,000 + 2700 \times 20^2)$$

$= 1221.5 \times 10^5$ cm^4

$$I_{TL_{\text{eff}}} = 6050\,(21.75)(-20) + 8700(-8.17)\times(29.17) - 2110(-19.7)(37) + 2700(-38.0)(20.0)$$

$= -9.80 \times 10^5$ cm^4

Step 8 Check for $\tan \phi'$:

$$\tan \phi' = \frac{45.2(148.22) - 43.2(-9.80)}{43.2(1221.5) - 45.2(-9.80)} = 0.135$$

$\therefore \phi' = 8°$ compared to 11° assumed,

OK good enough.

Step 9 Check for position of neutral axis.
Stress anywhere along neutral axis should be zero.
Check at O'.

$$\text{Stress at O}' = \frac{345,000}{15340} - \left(\frac{345,000\left\{45.2 - 43.2\dfrac{(-9.80)}{148.22}\right\}}{\left(1221.5 - \dfrac{9.8^2}{148.22}\right) \times 10^5}\right) \times 20.0 - \left(\frac{345,000\left\{43.2 - \dfrac{45.2(-9.80)}{1221.5}\right\}}{\left(148.22 - \dfrac{9.8^2}{1221.5}\right) \times 10^5}\right)(-20.0 + 38.0)$$

$= 22.5 - 2.7 - 18.3 = 1.5$ kg/cm^2, compared to zero

if the neutral axis really passed through point O'. This means a fresh trial is necessary. The new neutral axis will be almost parallel to the assumed one but bodily slightly displaced.

Step 10 For the purposes of this illustration if the assumed neutral axis is taken as final then the extreme stresses are as follows:

(i) Max. compressive stress in concrete:

$$= \frac{345,000}{15340} + \frac{345,000\left\{145.2 - 43.2\dfrac{(-9.80)}{148.22}\right\}}{\left(1221.5 - \dfrac{9.8^2}{148.22}\right) \times 10^5}(147.5 - 20.0) + \frac{345,000\left\{143.2 - 45.2\dfrac{(-9.80)}{1221.5}\right\}}{\left(148.22 - \dfrac{9.8^2}{1221.5}\right) \times 10^5}(70.0 - 38.0)$$

$= 22.5 + 17.5 + 32.5 = 72.5 \text{ kg/cm}^2$

(ii) Max. tensile stress in steel:

$$\doteq 11 \left\{ 22.5 - 17.5 \frac{(147.5 + 20.0 - 5)}{127.5} - 32.5 \frac{(70 + 38 - 5)}{32.0} \right\} \doteq 1150 \text{ kg/cm}^2$$

CONCLUSION

The work presented here is of immense practical application which only practising designers can best appreciate. It is, of course, based on elastic analysis.

CHAPTER 26

Post-tensioned Prestressing of Concrete

26.1 GENERAL PRINCIPLES OF PRESTRESSED CONCRETE

One of the best definitions of prestressed concrete is given by the ACI Committee on Prestressed Concrete:

> "Prestressed concrete: Concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from given external loadings are countered to a desired degree. In reinforced-concrete members the prestress is commonly introduced by tensioning the steel reinforcement."

The basic principle of prestressing is not limited to structures in concrete; it has been applied to steel construction as well. When two plates are joined together by hot-driven rivets or high-tensile bolts, the connectors are highly prestressed in tension and the plates in compression, thus enabling the plates to carry tensile loads between them.

Whether prestressing is applied to steel or concrete, its ultimate purpose is twofold—first, to induce desirable strains and stresses in the structure; second, to counterbalance undesirable strains and stresses. In prestressed concrete, the steel is pre-elongated so as to avoid excessive lengthening under service load, while the concrete is precompressesd so as to prevent cracks under tensile stress. Thus an ideal combination of the two materials is achieved.

26.2 STAGES OF LOADING

One of the considerations peculiar to prestressed concrete is the plurality of stages of loading to which a member or structure is often subjected. Some of these stages of loading occur also in non-prestressed structures, but others exist only because of prestressing. For a cast-in-place structure, prestressed concrete has to be designed for at least two stages: the initial stage during prestressing and the final stage under external loadings. For precast members, a third stage, that of handling and transportation, has to be investigated. During each of these three stages, there are again different periods when the member or structure may be under different loading conditions. These will be analyzed below.

Initial Stage

The member or structure is under prestress but not subjected to any superimposed external loads. This can be further subdivided into the following periods, some of which may not be important and hence may be neglected in certain designs.

Before Prestressing: Before the concrete is prestressed, it is quite weak in carrying load; hence the yielding of its supports must be prevented. Provision must be made for the shrinkage of concrete if it might occur. This is often significant because any shrinkage cracks will destroy the capacity of the concrete to carry tensile stresses.

During Prestressing: This is a critical test for the strength of the tendons. Oftentimes, the maximum stress to which the tendons will be subjected throughout their life occurs at that period. It occasionally happens that an individual wire may be broken during prestressing, owing to defects in its manufacture. But this is seldom significant, since there are often many wires in a member. If a bar is broken in a member with only a few bars, it should be properly replaced. For concrete, the prestressing operations impose a severe test on the bearing strength at the anchorages. Since the concrete is not aged at this period while the prestress is at its maximum, crushing of the concrete at the anchorages is possible if its quality is inferior. Again, unsymmetrical and concentrated prestress from the tendons may produce overstresses in the concrete. Hence the order of prestressing the various tendons must often be studied before hand.

At Transfer of Prestress: For pre-tensioned members, the transfer of prestress is accomplished in one operation and within a short period. For post-tensioned members, the transfer is often gradual, the prestress in the tendons being transferred to the concrete one by one. In both cases, there is no external load on the member except its own weight. Thus, the initial prestress, with little loss as yet taking place, imposes a serious condition on the concrete and often controls the design of the member. For economic reasons the design of a prestressed member often takes into account the weight of the member itself in holding down the cambering effect of prestressing. This is done on the assumption of a given condition of support for the member. If that condition is not realized in practice, failure of the member might result. For example, the weight of a simply supported prestressed girder is expected to exert a maximum positive moment at midspan which counteracts the negative moment due to prestressing. If the girder is cast and prestressed on soft ground without suitable pedestals at the ends, the expected positive moment may be absent and the prestressing may produce excessive tensile stresses on top fibres of the girder, resulting in its failure. Likewise, if the supporting staging is very flexible (not massive and rigid), the cambering effect due to prestress may still not allow the self weight of the structure to act freely since the precompressed staging may, despite its decompression, still keep touching the soffit of structure, thereby allowing only a part of dead load to act. This may cause severe tension in top fibre.

Decentering and Retensioning: If a member is cast and prestressed in place, it generally becomes self-supporting during or after prestressing. Thus the falsework can be removed after prestressing, and no new condition of loading is imposed upon the structure. Some concrete structures are retensioned, i.e., prestressed in two or more stages. Then the stresses at various stages of tensioning must be studied.

Intermediate Stage

This is the stage during transportation and erection. It occurs only for precast members when they are transported to the site and erected in position. It is highly important to ensure that the members are properly supported and handled at all times. For example, a simple beam designed to be supported at the ends will easily break if lifted farther away from its ends.

Not only during the erection of the member itself, but also when adding the superimposed dead loads, attention must be paid to the conditions of support and loading. This is especially true for a cantilever layout, when partial loading may result in more serious bending than a full loading.

Final Stage

This is the stage when the actual working loads come on the structure. As for other types of construction, the designer must consider various combinations of live loads on different portions of the structure with lateral loads such as wind and earthquake forces, and with strain loads such as those produced by settlement of supports and temperature effects. For prestressed concrete structures, especially those of unconventional types, it is often necessary to investigate their cracking and ultimate loads, in addition to the working load.

These will be discussed as follows:

Working Load: To design for the working load is a check on excessive stresses and strains. It is not necessarily a guarantee of sufficient strength to carry overloads. However, an engineer familiar with the strength of prestressed-concrete structures may often design conventional types and proportions solely on the basis of working-load computations.

Cracking Load: Cracking in a prestressed-concrete member signifies a sudden change in the bond and shearing stresses. It is sometimes a measure of the fatigue strength. For certain structures, such as tanks and pipes, the commencement of cracks presents a critical situation. For structures subject to corrosive influences, for unbonded tendons where cracks are more objectionable, or for structures where cracking may result in excessive deflections, an investigation of the cracking load seems important.

Ultimate Load: Structures designed on the basis of working stresses may not always possess a sufficient margin for overloads. This is true, for example, of prestressed-concrete members under direct tensile loads. Since it is desirable that a structure does possess a certain minimum overload capacity, it is often necessary to determine its ultimate strength. In general, the ultimate strength of a structure is defined by the maximum load it can carry before collapsing. However, before this load is reached, permanent yielding of some parts of the structure may already have developed. Although any strength beyond the point of permanent yielding may serve as additional guarantee against total collapse, some engineers consider such strength as not usable and prefer to design on the basis of usable strength rather than the ultimate strength. However ultimate strength is more easily computed and is more commonly accepted as a criterion for design.

In addition to the above normal loading conditions, some structures may be subject to repeated loads of appreciable magnitude which might result in fatigue failures. Some structures may be under heavy loads of long duration, resulting in excessive deformations due to creep, while others may be under such light external loads that the camber produced by prestressing may become too pronounced as time goes on. Still others may be subject to undesirable vibrations under dynamic loads. These are special conditions which the engineer must consider for his individual case.

26.3 PRESTRESSED VESRSUS REINFORCED CONCRETE

As it is assumed that readers are already acquainted with reinforced concrete, it will be interesting to compare prestressed concrete with it. The most outstanding difference between the two is the employment of materials of higher strength for prestressed concrete. In order to utilize the full strength of the high-tensile steel, it is necessary to resort to prestressing to prestretch it. Pretressing the steel and anchoring it against the concrete produces desirable strains and stresses which serves to reduce or eliminate cracks in concrete. Thus the entire section of the concrete becomes effective in prestressed concrete, whereas only the portion of section above the neutral axis is supposed to act in the case of reinforced concrete.

The use of curved tendons will help to carry some of the shear in a member. In addition, precompression in the concrete tends to reduce the diagonal tension. Thus it is possible to use a smaller section in prestressed concrete to carry the same amount of external shear in a beam.

High-strength concrete, which cannot be economically utilized in reinforced-concrete construction, is found to be desirable and even necessary with prestressed concrete. In reinforced concrete, using concrete of high strength will result in a smaller section calling for more reinforcement and will end with more costly design. In prestressed concrete, high-strength concrete is required to match with high-strength steel in order to yield economical proportions. Stronger concrete is also necessary to resist high stresses at the anchorages and to give strength to the thinner sections so frequently employed for prestressed concrete.

Each material or method of construction has its own field of application. When welding was first developed in the 1930s, some engineers were over enthusiastic and believed that it would replace riveting altogether, which it has not done even yet. Prestressed concrete is likely to have a similar course of development. Not for a long time will it be used in as great quantity as reinforced concrete. But a new type of construction, basically sound in its strength and economy, is likely to have a rapid rate of growth and to be adaptable to new and unprecedented situations and requirements.

The advantages and disadvantages of prestressed concrete as compared with reinforced concrete will now be discussed in respect to their serviceability, safety and economy.

Serviceability: Prestressed-concrete design is more suitable for structures of long spans and those carrying heavy loads, principally because of the higher strengths of materials employed. Prestressed structures are more slender and hence more adaptable to artistic treatment. They yield more clearance where it is needed. They do not crack under working loads, and whatever cracks may be developed under overloads will be closed up as soon as the load is removed, unless the load is excessive. Under dead load, the deflection is reduced, owing to the cambering effect of prestress. This becomes an important consideration for structures such as long cantilevers. Under live load, the deflection is also smaller because of the effectiveness of the entire uncracked concrete section, which has a moment of inertia 2 to 3 times that of the cracked section. Prestressed elements are more adaptable to precasting because of the lighter weight.

So far as serviceability is concerned, the only shortcoming of prestressed concrete is its lack of weight. Although seldom encountered in practice, there are situations where weight and mass are desired instead of strength. For these, plain or reinforced concrete could serve just as well and at lower cost.

Safety: It is difficult to say that one type of structure is safer than another. The safety of a structure depends more upon its design and construction than upon its type. However, certain inherent safety features in prestressed concrete may be mentioned. There is partial testing of both the steel and the concrete during prestressing operations. For many structures, during prestressing, both the steel and the concrete are subject to the highest stresses that will exist in them during their life of service. Hence, if the materials can stand prestressing, they are likely to possess sufficient strength for the design service loads.

When properly designed by the present conventional methods, prestressed-concrete structures have overload capacities similar to and perhaps slightly higher than those of reinforced concrete. For the usual designs, they deflect appreciably before ultimate failure, thus giving ample warning before impending collapse. The ability to resist shock and impact loads and repeated working loads has been shown to be as good in prestressed as in reinforced concrete. The resistance to corrosion is better than that of reinforced concrete for the same amount of cover, owing to denser concrete and the non-existence of cracks. If cracks should occur, corrosion can be more serious in prestressed concrete. Regarding fire resistance, high-tensile steel is more sensitive to high temperatures, but, for the same amount of minimum cover, prestressed tendons can have a greater average cover because of the spread and curvature of the individual tendons.

Prestressed-concrete members do require more care in design, construction, and erection than those of ordinary concrete, because of the higher strength, smaller section, and sometimes delicate design features involved. Although prestressed-concrete construction has been practised only since the late 1940s, it is possible to conclude from experience that the life of such structures can be as long if not longer than that of reinforced concrete.

Economics

From an economic point of view, it is at once evident that smaller quantities of materials, both steel and concrete, are required to carry the same loads, since the materials are of higher strength. There is also a definite saving in stirrups, since shear in prestressed concrete is reduced by the inclination of the tendons, and the diagonal tension is further minimized by the presence of prestress. The reduced weight of the member will help in economizing the sections; the smaller dead load and depth of members will result in saving materials from other portions of the structure. In precast members, a reduction of weight saves handling and transportation costs.

In spite of the above economies possible with prestressed concrete, its use cannot be advocated for all conditions. First of all, the stronger materials will have a higher unit cost. More auxiliary materials are required for prestressing, such as end anchorages, and grouts. More formwork is also needed, since non-rectangular shapes are often necessary for prestressed concrete. More labour is required to place steel in prestressed concrete, especially when the amount of work involved is small. More attention to design is involved, and more supervision is necessary; the amount of additional work will depend upon the experience of the engineer and the construction crew, but it will not be serious if the same typical design is repeated many times.

From the above discussion, it can be concluded that prestressed concrete design is more likely to be economical when the same unit is repeated many times or when heavy loads and long spans are encountered. It should also find suitable application when combined with precasting or semi-precasting such as composite or lift-slab construction. Each structure must be considered individually. The availability of good designers, of experienced crews, of pre-tensioning factories and construction gear and of competitive bidding often helps to tip the balance in favour of prestressed concrete.

For more details on the subject of relative economics, see Chs. 18 and 42 of this book.

26.4 SYSTEMS

Linear or Circular Prestressing: Circular prestressing is a term applied to prestressed circular structures, such as round tanks, silos, and pipes, where the prestressing tendons are wound around in circles. As distinguished from circular prestressing, the term linear prestressing is often employed to include all other structures such as beams and slabs. The prestressing tendons in linearly prestressed structures are not necessarily straight; they can be either bent or curved, but they do not go round and round in circles as in circular prestressing.

Pre-Tensioning and Post-Tensioning: The term pre-tensioning is used to describe any method of prestressing in which the tendons are tensioned before the concrete is placed. It is evident that the tendons must be temporarily anchored against some abutments or stressing beds when tensioned and the prestress transferred to the concrete after it has set. This procedure is employed in precasting plants or laboratories when permanent beds are provided for such tensioning; it is also applied in the field where abutments can be economically constructed. In contrast to pre-tensioning, post-tensioning is a method of prestressing in which the tendon is tensioned after the concrete has hardened. Thus, the prestressing is almost always performed against the hardened concrete, and the tendons are anchored against it immediately after prestressing. This method can be applied to members either precast or cast in place.

End-Anchored or Non-End-Anchored Tendons: When post-tensioned, the tendons are anchored at their ends by means of mechanical devices to transmit the prestress to the concrete. Such a member is termed end-anchored. Occasionally, though rarely, a post-tensioned member may have its tendons held by grout with no mechanical end anchorage. In pre-tensioning, the tendons generally have their prestress transmitted to the concrete simply by their bond action near the ends. The effectiveness of such stress transmission is limited to wires and strands of small size. Anchorages have been developed for pre-tensioning so as to permit the use of tendons of larger diameter.

Bonded or Unbonded Tendons: Bonded tendons denote those that are bonded throughout their length to the surrounding concrete. Non-end-anchored tendons are necessarily bonded ones; end-anchored tendons may be either bonded or unbonded to the concrete. In general, the bonding of post-tensioned tendons is accomplished by subsequent grouting; if unbonded, protection of the tendons from corrosion must be provided by galvanizing, greasing, or some other means. Sometimes, bonded tendons may be purposely unbonded along certain portions of their length (generally in pre-tensioning).

Precast, Cast-in-Place, Composite Construction: Precasting involves the placing of concrete away from its final position, the members being cast either in a permanent plant or somewhere near the site of the structure, and eventually erected at the final location. Precasting permits better control in mass production and then is often economical. Cast-in-place concrete requires more form and falsework per unit of product but saves the cost of transportation and erection, and it is a necessity for large and heavy members. In between these two methods of construction, there are tilt-up wall panels and lift slabs which are constructed at places near or within the structure and then erected to their final position; no transportation is involved for these. Oftentimes, it is economical to precast part of a member, erect it, and then cast the remaining portion in place. This procedure is called composite construction. The precast elements in a structure of composite construction can be more easily joined together than those in a totally precast structure. By composite construction, it is possible to save much of the form and falsework required for total cast-in-place construction. However, the suitability of each type must be studied with respect to the particular conditions of a given structure.

Partial or Full Prestressing: A further distinction between the types of prestressing is sometimes made depending upon the degree of prestressing to which a concrete member is subject. When a member is designed so that under the working load there are no tensile stresses in it, then the concrete is said to be fully prestressed. If some tensile stresses will be produced in the member under working load, then it is termed partially prestresed. For partial prestressing, additional rebars are frequently provided to reinforce the portion under tension. In practice, it is often difficult to classify a structure as being partially or fully prestressed since much will depend upon the magnitude of the working load used in design. For example, highway bridges may be designed for full prestressing, though actually they are subject to tensile stresses during the passage of heavier vehicles. On the other hand, roof beams designed for partial prestressing may never be subjected to tensile stresses since the assumed live loads may never act on them.

Protection of Tendons

A fundamental requirement for prestressing tendons is that they must be protected against corrosive attack in order to maintain the integrity of the structure. Where the cables are contained within ducts inside a concrete section this protection is provided by pumping cement grout into the duct after prestressing has taken place. Where the tendons are external to the section other options become available, including providing an added casing of concrete directly around the tendons, or using a polyvinylchloride coating to enclose the cable which has been prepacked with grease.

Tendon Profiling

Because of the changes in stress induced by the application of loading at different sections along a beam, the optimum use of prestressing requires that the position of the tendons should change within the section from one point to another along the length. In post-tensioning this is achieved without difficulty by fixing the cable ducts to the required profile before concreting. Where pre-tensioning is used this obviously cannot be done, since the tendon is stressed prior to concreting. The tendon can be pulled or deflected out of a straight line to approximate to the ideal profile but this harping adds complexity to the stressing procedure and is only reckoned as appropriate for large units. An alternative technique for simulating the effect of a profile is to break the bond between selected tendons and the surrounding concrete near the ends of the unit. This 'debonding' is achieved by placing sleeves over the selected tendons and thus rendering them ineffective in those zones. A plastic hosepipe is commonly used for this purpose.

Parasitic Forces

In statically-indeterminate structures, parasitic forces are set up by the prestressing force in addition to the moments induced by the eccentricity of the prestressing cable (measured from the centroid of the section) along the member. An exception to this occurs where the shape of the cable profile is such that it would not modify the support reactions — a so-called 'concordant' profile — which is rarely achieved in practice. This coincides with the resultant line of thrust.

Parasitic forces develop because the structure is not free to deform as the prestressing forces alone would dictate. The deformation of the structure is constrained by the supports, and constraining forces therefore develop at these supports, modifying the reactions to external loading and inducing a pattern of moments in the structure: i.e. parasitic moments, or secondary moments.

For design purposes it is necessary to assess the effective eccentricity of the cable at points along the structure, this eccentricity not necessarily being equal to the physical eccentricity of the cable measured from the centroid of the section. The effective eccentricity can be assessed by treating the forces exerted on the structure due to the prestressing force as a loading case, thus producing bending-moment and shearing-force diagrams. The effective bending moment at any section, divided by the prestressing force, then gives an effective eccentricity at that point. Details of analysis are given ahead.

Sections at which Stresses should be Checked

Although consideration of the stresses at known critical cross-sections (e.g., at midspan and near the supports) may show satisfactory stresses at all stages, where prestressing cables are curved it is necessary to check the stresses at various sections throughout the span, and this should be done even otherwise.

Shear-Lag Effect on Section Properties

Difficulties can arise in calculating the stresses produced by prestressing forces and applied loading on a voided slab of box deck near the supports. Section properties are well defined within the span. Near the supports, however, the situation is complicated by shear-lag and by the presence of a diaphragm which is solid.

The effect of shear lag is to remove the side cantilevers from the effective section resisting the bending moments. For prestressing calculations the gross area — i.e., including the cantilevering slabs — still nearly applies to the 'average' stress induced by prestressing forces.

At the face of the diaphragm member, the section is again well defined. Any tapering of the voids that is adopted to assist in providing shearing resistance or a more gradual distribution of stress between the fully voided section and the solid section at the centre-line must, of course, be reflected in the section properties assumed at this point.

At the centre-line of the pier the real section is solid, but the uniform stress due to the prestressing force requires a certain distance to redistribute itself over the solid section in place of the voided section for the rest of deck. It is therefore questionable whether stress calculations that are based on the solid section give valid answers. As the size of void increases it becomes clearer that stresses evaluated at the face of the diaphragm are those which are most relevant for design purposes.

An additional effect of the diaphragm is to modify the longitudinal bending moments because the support reactions are distributed through the width of the diaphragm, rather than acting at a point, as is usually assumed when calculating moments and shearing forces on the deck. This means that any stress calculations that are based on peak values of the bending moments tend to be academic, and do not reflect the real stress situation in the bridge. This shortcoming is generally accepted as a contribution towards conservatism in design, but where calculations for severe temperature stresses must also be taken into account, demanding substantial residual stresses under applied loads it seems appropriate to consider this width of support.

Also, see Ch. 19 in this book for Shear-Lag.

Loss of Prestress

Although the force transmitted to a structure at the anchorage points may be well defined at the time stressing of the tendons takes place, these stresses will be modified by the subsequent behaviour of the structure. The prestressing tendons and the concrete are sustaining each other in an equilibrium state of stress, arising from the strains locked in by the stressing operations. Steel subjected to locked-in strain experiences some loss of stress with time, this phenomenon being known as relaxation. Concrete is subject to shrinkage and to minor changes in the stress/strain relationship with time, these effects being known as creep. In the process of stressing, the loads imposed by successive tendons as they are stressed cause the structure to deform elastically, thereby modifying the locked-in strain in those tendons that have previously been tensioned. All of these effects reduce the locked-in forces, thus resulting in a loss of prestress. An additional loss of prestress in post-tensioned tendons arises from the friction between the cable and the duct, which leads to a progressive reduction in the actual force in the cables as the distance from the jacking point increases.

The total loss of prestress is usually substantially greater with pre-tensioning than with post-tensioning because the whole prestressing force is applied to the member simultaneously in pre-tensioning, and the demand for rapid fabrication usually leads to the stress being transferred into the member while the concrete is still relatively 'green' and thus more susceptible to losses due to creep. Also, pre-tensioning is affected by the entire shrinkage of the concrete, whereas in the case of post-tensioning some of this shrinkage takes place prior to the commencement of prestressing, and this part does not, therefore, affect the prestressing force.

Fuller details are given ahead.

Anchor Blocks (End Blocks)

At the point where a prestressing cable is anchored the full prestressing force in that cable is transmitted to the structure. This represents a substantial force concentrated on a very small area, and early prestressed concrete design and construction produced examples of failure in the region of anchorages, due to the splitting of the concrete. Similar problems can still overtake the unwary designer. The effect of these large anchorage forces is not unlike that of driving a wedge into the structure, and the shape of many prestressing anchorages encourages the analogy.

Several theories were developed as a basis for the design of anchorage zones (Freysinnet, Guyon, Magnel, Borsch, Bleich, Siever, Morsch, etc.), producing significant differences in the results. The Cement and Concrete Associsation (London) undertook a substantial series of tests, presenting its findings in two research reports, the first dealing with single anchorages and the second with group effects.

The prime requirement resulting from the calculations derived from these research reports is the need for heavy reinforcement perpendicular to the axis of prestressing cable and immediately behind the anchorage, to withstand the splitting forces arising from the wedge-like action of the anchorage.

An additional requirement in the region of prestressing anchorages is the need to examine the way in which the forces disperse into the full section of the deck. For example, in a box-section deck, groups of anchorages are usually concentrated in concrete 'anchor blocks' formed by an enlargement of the web. The dispersal of these large concentrated forces into the full cross-section gives rise to shearing and tensile stresses around the blocks. Each case requires individual assessment, assumptions being made as to the flow of the forces, and reinforcement being provided to cater for the shearing and tensile stresses that result. Within the anchor blocks themselves the design approach can be based on deep-beam theory.

The substantial amounts of reinforcement often necessary in the region of anchorages call for particular care in detailing. It is all too easy to adopt an arrangement of reinforcement which fits well on a drawing but proves almost impossible to assemble on site. If there is a conflict between the need to provide the theoretical amount of reinforcement required and the resulting production of an arrangement so congested as to prevent good concreting in that region, it is obviously even more important that well-compacted concrete should support an anchorage than that the calculated area of reinforcement should be present.

The fixing of closely-spaced reinforcement can usually be facilitated by detailing the steel in such a way that the reinforcement required in the bottom of the deck to deal with slab or diaphragm action can be fixed first. Anchorage-block reinforcement is then assembled on top of this mat, followed by the top surface reinforcement for the diphragm or slab, which is assembled on top of the bursting steel. The system of stirrups adopted must permit this sequence.

For bursting steel itself many designers favour the use of a spiral immediately around the anchorage. This follows from the development of an early form of anchorage for small prestressing systems, which was constructed with fine aggregate concrete inside a contiguously wound helix. The attraction of the spiral is its unquestionable effectiveness in all directions within the planes of the splitting force, and the fact that it unites a significant lump of concrete with the anchorage. Only in smaller stressingg systems is this spiral adequate on its own; however, a system of links, U-bars and straight bars will often be needed to supplement the spiral where conventional links are used, care should be taken to detail reinforcement to a shape which avoids a double thickness of steel at the overlap closing the link. A close spacing of reinforcement is needed in an anchorage zone.

Details of designing reinforcement in anchorage zones are given ahead.

Shear

The capacity of a prestressed concrete member to resist shear depends on the amount of bending moment acting in conjunction with shearing force at the section. This is best appreciated by considering the case of a simply-supported beam subject to concentrated loads at third points.

Adjacent to the supports, the limiting shear capacity of a beam depends on its ability to sustain the diagonal compressive force arising from the shearing force, without causing buckling of the section.

Moving away from the supports into the area where the pattern of shearing stresses is fully developed (clear of the influence of the support reaction), but where bending stresses are modest, any distress arising from excessive shear will be shown by the formation of diagonal cracks near the neutral axis of the member —where the shearing stresses are a maximum—and extending towards the boundaries of the section as the load increases; until failure occurs either by the shear stirrups yielding or the web buckling in compression.

In the vicinity of the two applied loads is a region of shearing stress associated with high bending stresses. In this area failure of the section will be heralded by the formation of cracks beginning at the extreme tensile fibre and extending upward into the member as the load increases. The propagation of these cracks impinges on the cross-sectional area available to provide effective shearing resistance, and thus reducing the shear capacity. Once a critical point is reached, the shearing and bending effects combine to extend the crack to the point where failure occurs. In the case of a continuous beam it is normal to find areas of high shearing stresses and high bending stresses together, so that this mode of failure becomes critical in such cases.

Between the two applied loads mentioned above, nominal shearing stress will be combined with high bending stresses. Failure in this zone would occur essentially as a result of pure bending.

The approach necessary to calculate the shear capacity differs according to the zone concerned. Inclined prestressing cables have a marked effect on the principal stresses in a web subjected to shear and are therefore significant in assessing the shear capacity where bending does not produce flexural cracks. The moment at which flexural cracks commence to form is unaffected by the inclination of the stressing cables so that, where the shear capacity is limited by flexural cracking, inclined tendons do not help.

For a detailed discussion on design against 'shear' (and 'Shear and Torsion') refer to Ch. 24 in this book.

Ultimate Moment of Resistance

The basic aim in prestressed concrete design is to maintain the section in compression under all conditions of loading. However, where overload does occur, tensile stresses can develop to such an extent that the section begins to crack, and the whole basis of evaluating the capacity of the section to resist load changes.

There are fundamental differences between the behaviour of bonded and unbonded tendons in ultimate load conditions. Since a bonded tendon is united with the concrete section at all points along its length, the concrete and the prestressing tendon will continue to act together, responding to the same strain-distribution curve at each section, until failure takes place. With unbonded tendons, whereas the concrete section will be responding to the varying systems of force and distributions of strain at individual sections, the tendon is stressed by a strain that is locked into it by being held at points far removed from each other along the length of the member. It is, therefore, insensitive to particular distributions of strain of individual sections so that, although the concrete at a particular section may be strained to a point, approaching its ultimate capacity, the prestressing tendon crossing the section may only be stressed to a level approximating to its normal working load.

The general approach to assessing the ultimate resistance of a prestressed concrete member is to seek, by trial and adjustment, a balance between the internal and external forces based on an assumed distribution of strain.

For details about calculating the absolute ultimate moment of resistance of a section refer to Ch. 27 in this book, where details are also given about how to estimate the actually attainable moments of resistance in keeping with compatibility of deformations and consequent moment-redistribution.

Description of the Construction Phases

Where a construction is not entirely cast *in situ* on a single support system, the consulting engineer must specify to the contractor, in a detailed manner, the different construction phases in order that there is no ambiguity on the job site. This specification shall necessarily include those phases which may influence the stability of the structure, either during construction or in service. It is not always essential to define those which are part of the concreting sequences, form stripping and tensioning ... which, considered in relation to any one phase, have little or no effect on the calculations.

Particular attention must be paid to the clamping and unclamping of temporary bearings and blocking pieces.

Order of Tensioning

The order in which cables are tensioned forms an integral part of the design. It will be drawn up after verification of the provisional phases of construction. Particular attention shall be given to the necessary concrete strength with respect to the more highly stressed sections of the structure, and the anchorage areas.

When the structure is simple, the tensioning sequence will figure on the cable layout. In the case of large structures comprising several cable layouts, or for structures to be built in several phases, the tensioning sequence will be included in the specification for the construction stages.

NOTE 1. For some details about prestressing steel (wires, strands and bars) and prestressing anchorages, see Ch. 6 in this book.

2. While some design details are given in their relevant contexts in other chapters in this book, details as per ACI practice are available in APPENDIX 6 to this book.

Losses in Prestress

The prestress imparted to 'post-tensioned' prestressed concrete structure suffers a reduction owing to various causes. These causes are:

A. *Frictional resistance* Felt by the extending tendon at the time the tendon is tensioned, *and the 'slip'* of the tensioned tendon at the anchorages at the instant it is locked in the anchorages to release the stressi[ng] jacks.

B. *Shrinkage of concrete* over an extended peri[od] of time.

C. *Creep of concrete* over an extended period [of] time.

D. *Elastic shortening of concrete* each time tendon is stressed.

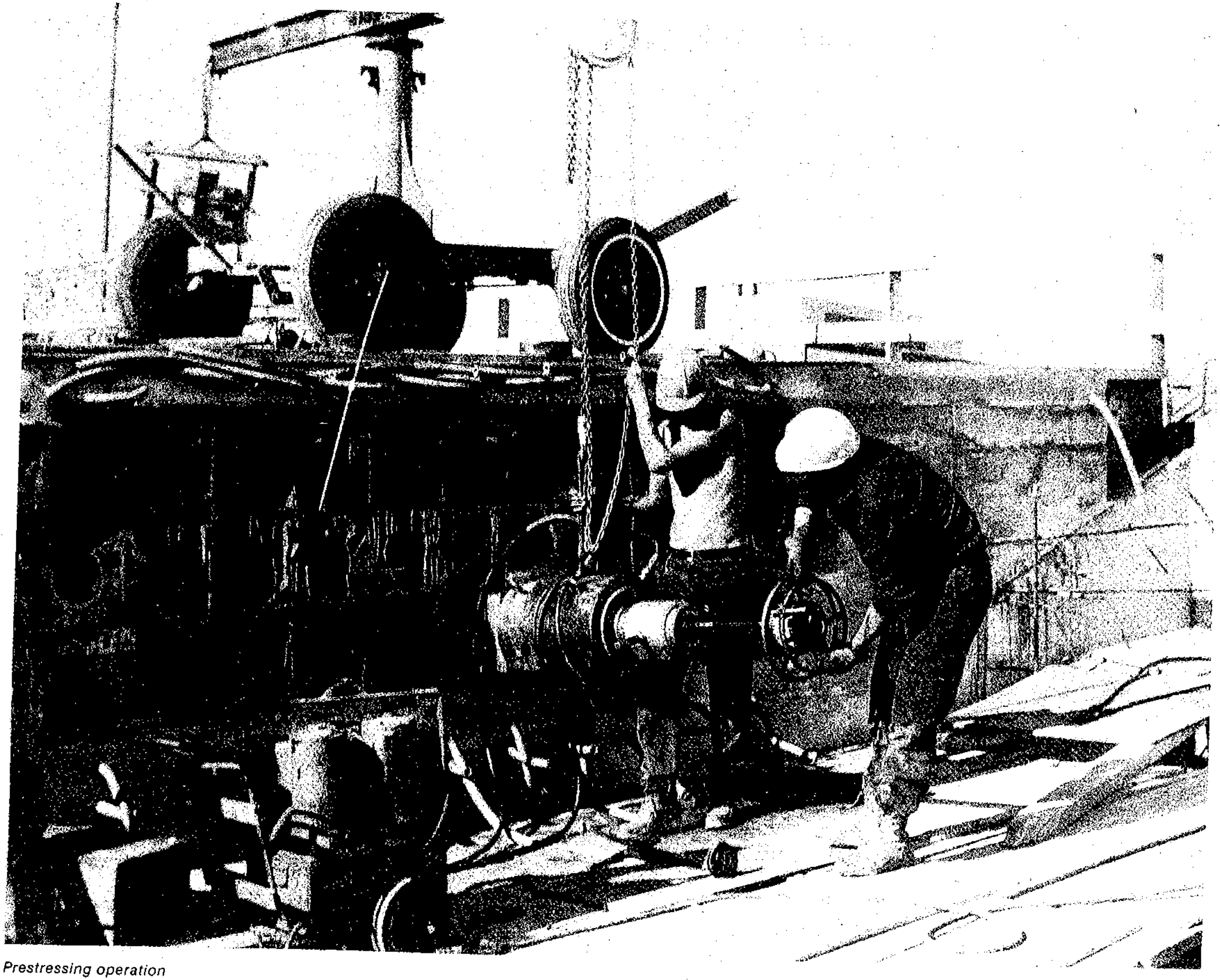
E. *Relaxation of the tendon steel* over a period [of] time.

In pre-tensioned prestressed concrete: Loss 'A' i[s] obviously absent, loss 'B' is relatively more (sinc[e] even the shrinkage cracks that have occurred prio[r] to transfer, absorb some of the already applie[d] prestress unlike in post-tensioning where such cracks get closed during tensioning process and only then the pressure gauge registers the dormant prestress), loss 'D' is almost double of that in the case of post-tensioning (since shortening caused by the transfer of each tendon is felt by all tendons as all tendons are transferred simultaneously, unlike in post-tensioning where shortening caused by prestress in any particular tendon is felt only by those of the tendons already stressed, so that the last stressed tendon suffers nothing, the first stressed tendon suffers due to all the subsequent tendons and variation in-between is almost linear), and loss 'E' is almost identical to that in post-tensioning.

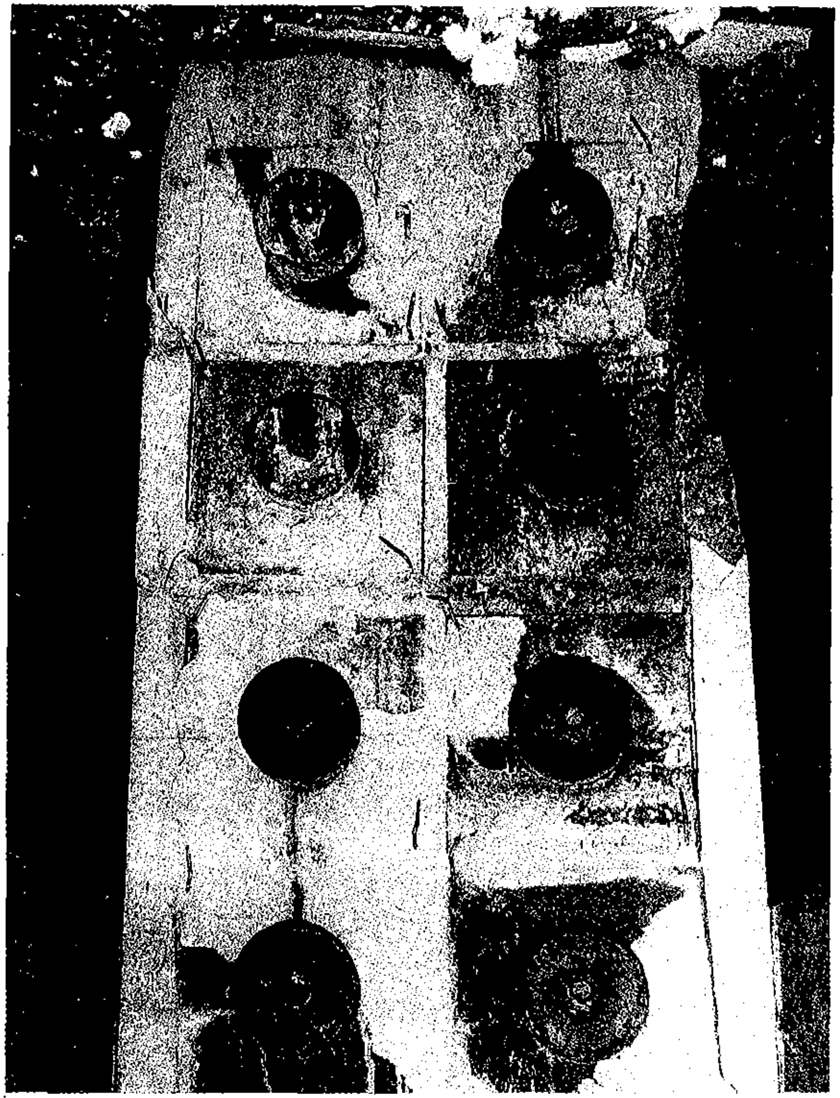
A. Loss due to Friction and Slip

By doing the necessary friction calculation and the slip-effect calculation in advance of the design of the structure, the actual initial prestress forces in each tendon, at each section after friction and slip effects, are calculated and only these values used in design. This takes care of friction and slip loss.

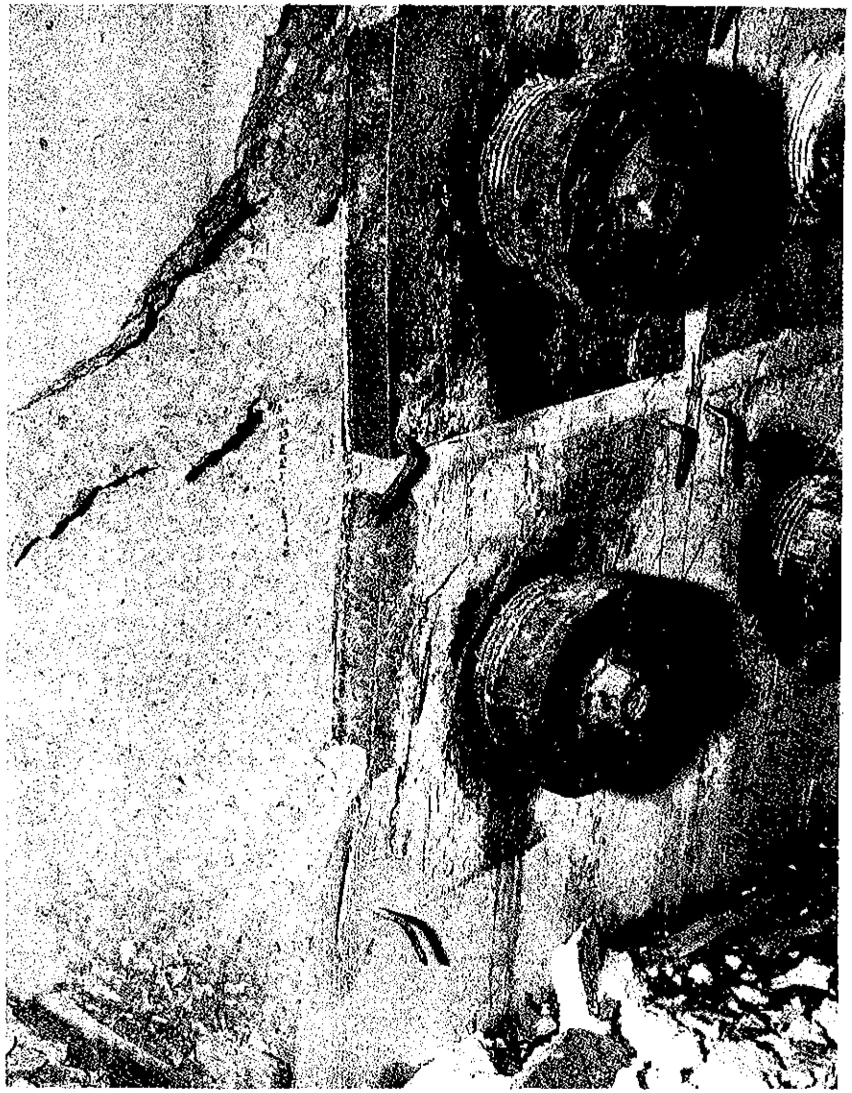
Details of these calculations are given ahead. For the purpose of only a general 'feel', for instance in a symmetrically parabolically profiled tendon (with a central straight stretch) in a simple span of about 35 m, stressed from both ends equally and simultaneously, the friction will reduce the force in the tendon from $0.75F_u$ at jacking ends to about $0.64F_u$ at midspan; also that the slip effect could travel up to 10 m, or more from jacking-ends towards mid-span (depending on tendon profile), reducing the force at the jacking-end from $0.75F_u$ before slip to about $0.65F_u$ (or lower) after slip (F_u = ultimate tensile force possible in the tendon). Also, the extension in a tendon is usually of the order of about 4.5-5.0 mm per meter of extending length between null point to jacking point (assuming average initial stress in cable to be about 0.65 of ultimate and $E = 2 \times 10^6 \text{kg/cm}^2$). This again is only a rough figure for a



Prestressing operation



Distress in anchorage zone noticed while prestressing (caused by crushing of improperly vibrated concrete)



Another view showing distress in anchorage zone concrete general idea, exact figure for each stressing-end depending on actual prestress force distribution along tendon length (taking due account of actual friction losses), actual location of the null point (the point in the cable where forces from the two jacking-ends are equal and hence does not 'move') and the correct E value of the tendon-steel.

It is preferable to stress each tendon from both its ends (simultaneously and equally) unless for instance the method of construction prevents it, e.g. lack of jacking space, or use of coupler, etc. For one thing, this permits fuller use of cable steel. One-end stressing in a structure symmetrical about its mid-span, will require checking stresses even at the mirror-image sections owing to unequal prestress forces at such sections, which only adds to the design labour. However, the biggest advantage of both-end stressing is the much better chance of salvaging the structure should an inadvertent mortar or grout leak block the tendon for some portion within its length. In such a case with live anchorages at both ends, most of the tendon length can still be tensed and, despite the rock-anchor type behaviour of cable within the blockage, changes remain for the anchor force distributions from the opposing sides within the blockage, to intersect. The overall stresses at this critical section within the blockage in such a case, can then be rechecked backwards using the actual (even though) reduced value of the cable force at the section. Again, two-end stressing is advantegeous in short cables where the magnitude of slip — which is independent of tendon length — can be a significant proportion of the total elongation. Here, anchoring at only one end is done first and then slip loss is taken up by compensatory (further) stressing at the other end and only then is that end anchored. (See ahead for discussion on 'cable blockage'.)

B. Loss due to Shrinkage of Concrete

This loss in a tendon, stressed at a particular age of concrete, is the product of the Residual Shrinkage Strain in concrete from that day onwards and E the modulus of elasticity of cable steel, and this product, a stress, can then be expressed as a percentage of average initial stress in the tendon. As an example, if the tendon is stressed when concrete is 7 days old, it is possible to estimate the loss in this tendon due to shrinkage of concrete during any required time interval therebeyond (e.g., between 7 to 14 days, 14 to 28 days, etc.) knowing residual shrinkage strains of concrete on those days.

Values for these are given for example in the IRC Specifications, but their exactitude is only academic. Total residual shrinkage strain value beyond, say 7 days (when usually most of the prestress is applied in many practical cases), is more important. Its product with E of cable steel, expressed as a percentage of the average initial stress in the tendon (usually 0.64 of ultimate) is of the order of 6 whether the construction is wholly cast *in situ* or partly precast. Half of this may be assumed to occur within the first month and balance thereafter (with a fifth in the second month, another like amount in the third month and the balance thereafter). For cables stressed at concrete age 28 days or more, the above figure of 6 may be taken as 3, remembering that hair-splitting exactitude in estimating these figures is immaterial so long as total of losses 'B', 'C', 'D', and 'E', referred to earlier, is taken as stated in the Table 26.1 given ahead. (Loss due to shrinkage of concrete reduces if humidity increases.)

C. Loss due to Creep of Concrete

This loss in a tendon, stressed at a particular age of concrete, is the product of the Residual Creep Strain in concrete from that day onwards and E of the cable steel. However, since creep is caused if either the loading is long term or the rate of loading is slow, its residual strain is always expressed per certain weighted average compressive stress (f_{av}) in concrete at the level of centroid of the appropriate cables under the appropriate loading condition, usually 100 kg/cm^2. Therefore the above product has to be multiplied accordingly by 100 and divided by the actual f_{av} (kg/cm^2) as prevalent. Noting that f_{av}, being an intermediate fibre level stress, can therefore generally not be more than about 80% of maximum permissible (extreme fibre) compressive stress, it may be taken as ranging between 95 to 115 kg/cm^2, cast in situ to precast construction. Consequently it will only be a slight over estimation of the loss if, on paper, f_{av} is directly taken as 100 kg/cm^2. This will only ease the computations without any unnecessary (and at any rate a pointless) hair-splitting. (Anyone who has time to waste and has lost track of the aim of this exercise within its practical meaning, may indulge in mensurational paper work here, but he will not be doing anything worthwhile by hair splitting.)

Here also, it is possible to estimate the loss in a tendon (due to creep of concrete) during any required time interval after stressing the tendon if only the residual creep strains of concrete on those days are known. Values for these are given for example in the IRC Specifications (in terms of concrete strengths, which at any rate can be approximated to the age of concrete and the strain values related to concrete age). Again, the total residual creep strain value beyond say 7 days (when usually most of the prestress is applied in many practical cases) is more important. Its product with E of cable steel (after adjustment of $100/f_{av}$ explained earlier) expressed as a percentage of the average initial stress in the tendon (usually 0.64 of ultimate) is of the order of 10, whether the construction is wholly cast in situ or partly precast.

A fifth of this may be assumed to occur within the first month and the balance thereafter (with a fifth in the second month, another like amount in the third month and balance two-fifths thereafter). For cables stressed at concrete age 28 days or more, the above figure of 10 may be taken as 8. (Loss in prestress due to creep of concrete initially decreases if humidity increases and subsequently increases if the humidity continues to increase.)

D. Loss due to Elastic Shortening of Concrete

As explained in the beginning, in post-tensioned concrete this loss may be assumed to vary linearly from the maximum in the cable stressed first to zero in the cable stressed last. Hence, may be taken as half of the maximum, latter being the product of 'strain in concrete due to a weighted average compressive stress in concrete at the level of centroid of all tendons under all dead loads and all initial prestress' and E of the cable steel.

i.e., $$\frac{1}{2}\left(\frac{f'_{av}}{E_{concrete}}\right) E_{steel}$$

i.e. $$\frac{1}{2} m f'_{av}$$

Value of f'_{av}, as against that of f_{av} mentioned earlier in reference to loss due to creep of concrete, is obviously higher because of the stated loading condition. This value may safely be taken as 120 kg/cm^2 whether the construction is cast in situ or precast.

The above loss is assumed to occur at the very time an individual cable is stressed.

The above stress loss of $(1/2\, m f'_{av})$, when expressed as a percentage of the average initial stress in the tendon (usually 0.64 of ultimate), is of the order of 3.5, assumed to occur at time of stressing the tendons.

E. Loss due to Relaxation of Tendon Steel

Cable steel under constant tension tends to relax in as much as its stretched fibres creep away from each other. This reduces the prestressing force in the tendon and the magnitude of reduction depends on how much stress-relieving has gone into the steel in its manufacturing process. It ranges between 4 to 7 per cent of the (average) initial prestressing force (or stress) in the tendon and most of it is assumed to occur in the first 1000 hours after tensioning the tendon, with majority of it occurring in its earlier stages.

Suggested values of loss of prestressing force in stage-I tendons (PSI) stressed at concrete age 7 days and in stage-II tendons (PSII) stressed at concrete age 28 days, expressed as percentage of the (average) initial prestressing force in the cables: (Post-tensioned concrete) are given in Table 26.1.

Reference may also be made for another simple approach given for post-tensioned as well as pre-tensioned concrete in the AASHTO Design Specifications, which, in principle, are reproduced below with courtesy and acknowledgement: "Loss of prestress due to all causes, excluding friction, may be determined by the following method. The method is based on normal weight concrete and one of the following types of prestressing steel: 250- or 270-ksi, seven-wire, stress-relieved strand; 240-ksi stress-relieved wires; or 145- to 160-ksi smooth or deformed bars. For data regarding the properties and effects of lightweight aggregate concrete and low-relaxation tendons, refer to documented tests or see authorized suppliers.

TOTAL LOSS

$$\Delta f_s = SH + ES + CR_c + CR_s$$

where Δf_s = total loss excluding friction in pounds per square inch

SH = loss due to concrete shrinkage in pounds per square inch

ES = loss due to elastic shortening in pounds per square inch

CR_c = loss due to creep of concrete in pounds per square inch

Table 26.1

Prestress	Loss between (% of av. P_i)	Due to shrinkage of concrete (% of av. P_i)	Due to creep of concrete (% of av. P_i)	Due to elastic shortening of concrete (% of av. P_i)	Due to relaxation of steel (% of av. P_i)	Total loss (% of av. P_i) P_i = initial prestressing force after friction and slip
Stage I (applied at 7 days)	7—28 days	3	2	3.5	6	14.5
	after 28 days	3	8	—	—	11.0
	Total	6	10	3.5	6	25.5
Stage II (applied at 28 days)	after 28 days	3	8	3.5	6	20.5

NOTE: Loss of prestressing force on account of 'curvature-friction and wobble-friction' and 'slip at anchorages at the time of releasing the tensioning-jacks' is accounted for through the friction-effect calculations, and only the 'after friction and slip' prestressing force values used as P_i values.

CR_s = loss due to relaxation of prestressing steel in pounds per square inch

Shrinkage of Concrete: Pre-tensioned members

$$SH = 17{,}000 - 150\ RH$$

Post-tensioned members ~

$$SH = 0.80(17{,}000 - 150\ RH)$$

where RH = mean annual ambient relative humidity in per cent

Elastic shortening:

Pre-tensioned members

$$ES = \frac{E_s}{E_{ci}} f_{cir}$$

Post-tensioned members

$$ES = 0.5\frac{E_s}{E_{ci}} f_{cir}$$

where E_s = modulus of elasticity of prestressing steel strand, which can be assumed to be 28×10^6 psi.

E_{ci} = modulus of elasticity of concrete in psi at transfer of stress, which can be calculated from:

$$E_{ci} = 33w^{3/2}\sqrt{f'_{ci}}$$

in which w is the concrete unit weight in pounds per cubic foot and f'_{ci} is in pounds per square inch (cylinder crushing strength).

f_{cir} = concrete stress at the centre of gravity of the prestressing steel due to prestressing force and dead load of beam immediately after transfer; f_{cir} shall be computed at the section or sections of maximum moment. (At this stage, the initial stress in the tendon has been reduced by elastic shortening of the concrete and tendon relaxation during placing and curing the concrete for pre-tensioned members, or by elastic shortening of the concrete and tendon friction for post-tensioned members. The reductions to initial tendon stress due to these factors can be estimated, or the reduced tendon stress can be taken as $0.63\ f'_s$ for typical pretensioned members.)

Creep of Concrete: Pre-tensioned and post-tensioned members.

$$CR_c = 12f_{cir} - 7f_{cds}$$

where f_{cds} = concrete stress at the center of gravity of the prestressing steel due to all dead loads except the dead load present at the time the prestressing force is applied.

Relaxation of Prestressing Steel:

Pretensioned members:

250- to 270-ksi Strand

$$CR_s = 20{,}000 - 0.4ES - 0.2(SH + CR_c)$$

Post-tensioned members:

250- to 270-ksi Strand

$CR_s = 20{,}000 - 0.3FR - 0.4ES - 0.2(SH + CR_c)$

240- ksi Wire

$CR_s = 18{,}000 - 0.3FR - 0.4ES - 0.2(SH + CR_c)$

145- to 160-ksi Bars

$CR_s = 3{,}000$

where FR = friction loss stress reduction in psi below the level of $0.70\ f'_s$ at the point under consideration.

$\left.\begin{array}{l} ES,\ SH \\ \text{and } CR_c \end{array}\right\}$ = appropriate values as determined for either per-tensioned or post-tensioned members.

In lieu of the preceding method, the following estimates of total losses may be used for prestressed members or structures of usual design. These loss values are based on use of normal weight concrete, normal prestress levels, and average exposure conditions. For exceptionally long spans, or for unusual designs, the method given above must be used.

Estimate of Prestress Losses

	Total loss	
	$f'_c = 4{,}000$ psi	$f'_c = 5{,}000$ psi
Pretensioning (Strand)	—	45,000 psi
Post-Tensioning*		
Wire or Strand	32,000 psi	33,000 psi
Bars	22,000 psi	23,000 psi

* *friction losses not included.*

f'_c = 28-day standard cylinder crushing strength of concrete

f'_s = ultimate tensile stress of prestressing steel.

26.5 CABLE FRICTION CALCULATIONS

1. If the force in the cable at its stressing-end (i.e., at its jacking-end) is P_0 (generally 0.75 of ultimate, but never more than 0.80 of ultimate), then allowing for loss in this force on account of frictions due to curvature of cable profile

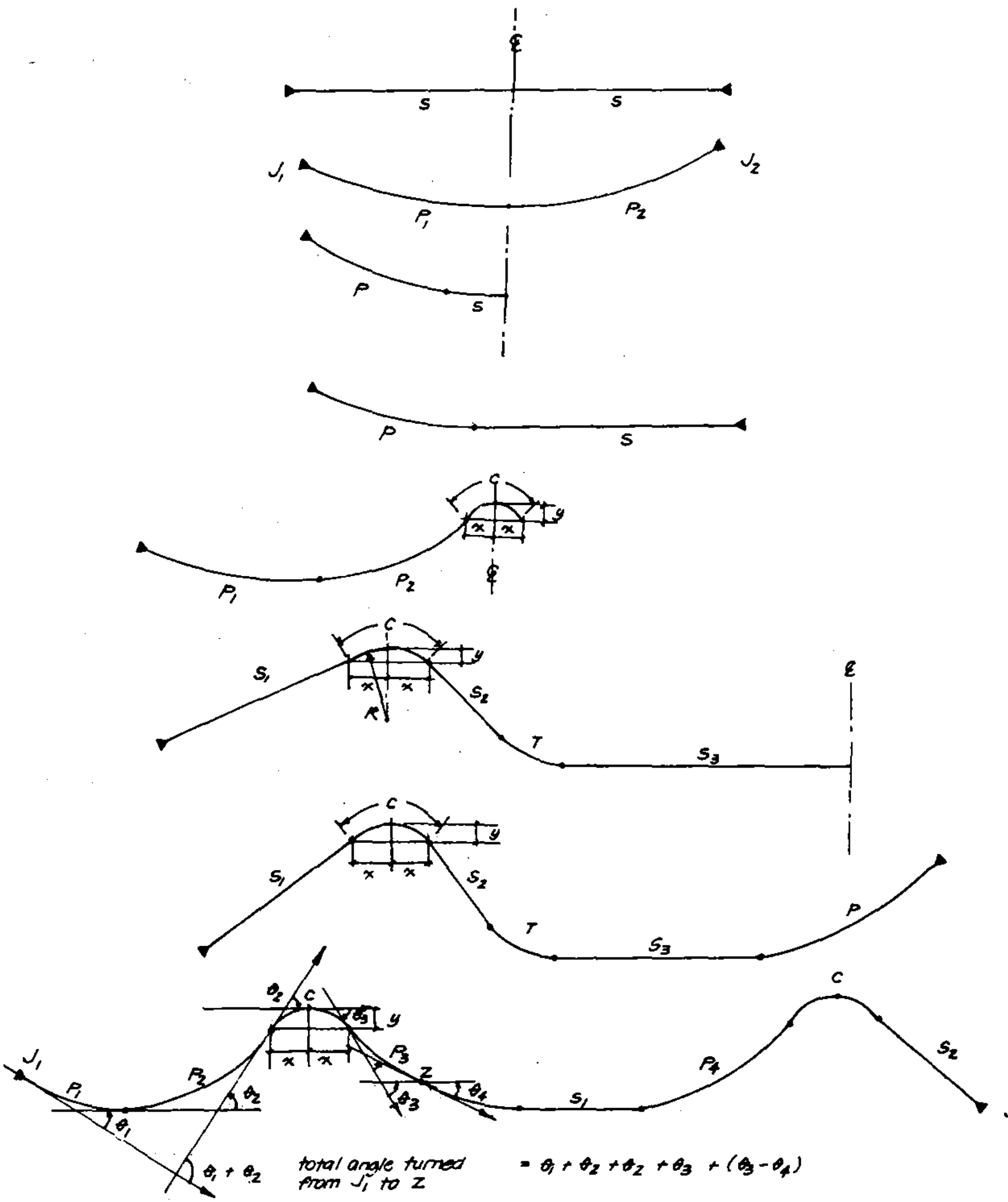


Fig. 26.1(a) ▶ = jacking point
S = straight
P = parabolic
C = circular
T = curved 'transition'

x and y to be so fixed as to give R the radius of the circular curve as not less than the permitted minimum bending radius for the cable and duct adopted.

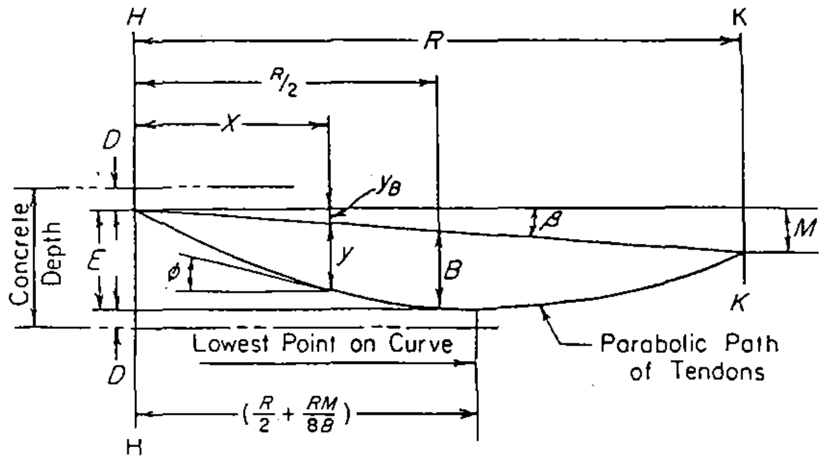


Fig. 26.1(b) *Geometry of parabolic path of tendons where ends of parabola are at different elevations*

$Y = 4B\dfrac{X}{R}\left(1-\dfrac{X}{R}\right)$

$Y_B = M\dfrac{X}{R}$

$\tan\phi = \dfrac{4B}{R}\left(1-\dfrac{2\lambda}{R}\right)$

L = Length of tendon along parabolic curve

$= R\left(\sec.\beta + \dfrac{2.67n^2}{\sec.^3\phi}\right)$

$n = \dfrac{B}{R}$

w_T = uniform upward load applied to concrete by tendons (Force F)

$= \dfrac{8BF}{R^2}$

V_H and V_K = concentrated downward loads applied at supports by tendons = $F\tan\phi$

At $H, X = O$ and $\tan\phi = \dfrac{4B+M}{R}$

At $K, \bar{X} = R$ and $\tan\phi = \dfrac{M-4B}{R}$

and wobble effect up to a point distant x from the jacking point, **the force in the cable at that point** will be:

$$P_x = P_0 \cdot e^{-(\mu\theta + kx)}$$

where θ = total angle turned, in elevation and plan, between the two points

x = distance between the two points, measured along the cable profile (but since rise of cable is small compared to its projected length in plan, x may easily be taken as distance in plan between the two points)

μ = curvature friction coefficient

k = wobble friction coefficient

Values of μ and k depend on type of duct housing the cable (e.g. plane, corrugated, galvanized, flexible, rigid, etc.) and, to some extent, even on the amount of care exercised in handling the duct and how symmetrically and untustled the tendon elements stay inside the duct. As an example, for a rigid, corrugated and unrusted duct, housing high tensile steel wires assumed placed along a circle in cross-section, with the duct remaining unsquashed all along its length, μ may be taken as 0.25 per radian and k as 0.003 per meter length. Duct manufacturers give μ and k values for different types of ducts while the codes usually specify the upper bound values.

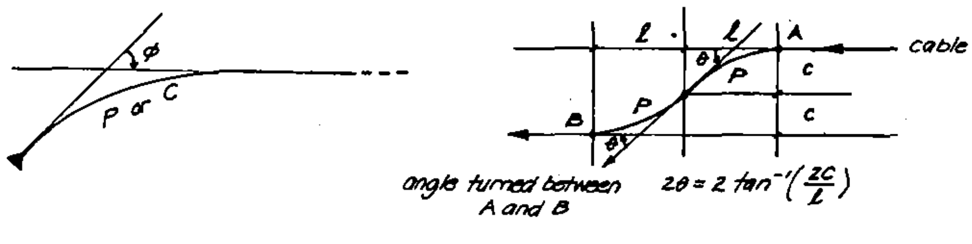


Fig. 26.2

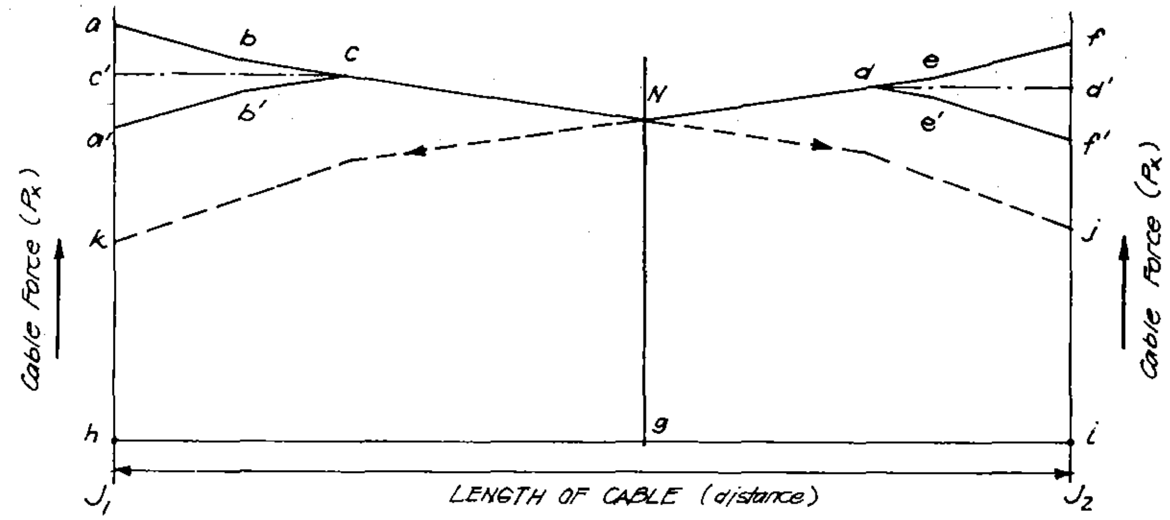


Fig. 26.3

2. **Angles turned through in elevation and plan** between the jacking point and any point in the cable length can be found out from geometry of the cable profile in elevation and plan. Some of the usual cable profiles in elevation, adopted for various cables in various situations are indicated ahead~ (not drawn to same horizontal and vertical scales). Fig. 26.1.

In plan the cable may turn in profile, depending on the situation, and some of these profiles are indicated below (drawn to different scales in length and breadth directions) ~ Fig. 26.2.

3. **Evaluating 'change in cable length'** [either the 'cable elongation' owing to stressing or the 'cable slip' owing to small reverse movement at the instant the stressing jack is released and the cable is locked in its anchorage]:

If δ_x be the movement (i.e., the change of length) in a small cable length dx, then

$$\frac{\delta x}{dx} = \frac{\text{change in length}}{\text{original length}} = \text{strain} = \frac{F_x}{AE}$$

where F_x is the cable force at that section, and A and E the section area and modulus of elasticity of cable. Then,

$$\delta_x = \frac{F_x dx}{AE}$$

Summing up on both sides,

$$\Sigma\delta_x = \Sigma\frac{F_x dx}{AE} = \frac{1}{AE}\Sigma(F_x dx)$$

LHS represents the total movement, and $\Sigma(F_x dx)$ represents the area of (Force × length) diagram, causing the movement.

Hence, in Fig. 26.3 (for explanation see ahead):

(i) Cable elongation at jacking end J_1

$= \frac{1}{AE}$ (area *abc Ngha)*

(ii) Cable elongation at jacking end J_2

$= \frac{1}{AE}$ (area *fedNgif)*

(iii) Cable slip at end $J_1 = \frac{1}{AE}$ (area *abcb'a'a)*

(iv) Cable slip at end $J_2 = \frac{1}{AE}$ (area *fede'f'f)*

4. **Estimation of**

- Initial force P_i in the cable at any section (after friction and slip)
- Null point N (-location of)
- Cable elongation at each stressing-end
- Jack pressure at each stressing-end
- Slip at each stressing-end

For each chosen trial cable profile, assumed stressed from both ends J_1 and J_2 simultaneously and equally, proceed step-wise as follows:

Step 1 Work out inclination of tangent to cable profile with horizontal at various points in the cable profile from its geometry. Also work out the angle turned through in plan by the cable between various points.

Step 2 Assume J_1 as the stressing-end and J_2 as the dummy-end, work out total angle turned through as well as distance from J_1 up to *each* point in the cable (moving towards J_2), thus work out value of $(\mu\theta + kx)$ up to each point with respect to J_1 and hence the P_x value $\left(= P_0 e^{-(\mu\theta+kx)}\right)$ at each of these points. This can best be done in a tabular format. Plot these P_x values at the respective sections along the cable length, *line abc Nj* in Fig. 26.3.

Step 3 Assume J_2 as the stressing-end and J_1 as the dummy-end, repeat Step 2, and we plot the line *fedNk* in Fig. 26.3.

Step 4 Point of intersection of the above two lines, N, is then the *NULL POINT*.

Step 5 Cable extensions (as explained earlier):

- Cable elongation at $J_1 = \dfrac{1}{AE}$ (area *abcNgha*)
- Cable elongation at $J_2 = \dfrac{1}{AE}$ (area *fedNgif*)

Step 6 Jack-pressures (assuming 96% jack efficiency, usual):

at J_1 : $\dfrac{\text{force } ha}{\text{ram area of Jack} \times 0.96}$

at J_2 : $\dfrac{\text{force } if}{\text{ram area of Jack} \times 0.96}$

Step 7 (i) Unless the prestressing system adopted is free of slip (e.g. in a 'nut' or 'button head' system), a certain amount of slip is bound to occur when locking the cable into its anchorage (generally 6 to 10 mm). Normally, initial prestressing force in a cable should not exceed 0.7 of its ultimate at any section, but a Jacking Force P_0 of up to 0.8* of ultimate is allowed so long as after slip-effect the prestressing force along the cable at no section exceeds 0.7 of ultimate, and this slip-effect is brought on (by releasing the jack and locking the cable) within about 2 minutes after reaching 0.8 of ultimate force in the cable at the jacking point.

(ii) Assuming that the slip effects from the two jacking ends die out at sections c and d, then, as explained earlier:

$$\text{slip at } J_1 = \frac{1}{AE} \text{ (area } abcb'a'a)$$

$$\text{slip at } J_2 = \frac{1}{AE} \text{ (area } fede'f'f)$$

It has to be understood that the effect of slip is to lower the lines abc and fed to their mirror-images $a'b'c$ and $f'e'd$, shown in Fig. 26.3.

Reason: On account of slip up to c (consider stressing-end J_1 for example) all points in the cable from a up to c move towards c and, therefore, relative to c carry no force i.e. have the same force as at c. This lowers line abc to horizontal $c'c$. In addition, the physical slip movement from a to c attracts opposing friction, which, in magnitude is same as between c and a, only reversed this time. This lowers $c'c$ to $a'b'c$ (mirror image of abc wrt $c'c$).

(iii) So, for the first trial, locate points c and d where the vertical ordinates equal 0.7 of ultimate force in cable and work out the magnitudes of the slips at the two ends. If these are equal to or more than the normal slips associated with the prestressing system, then OK, and specify the same as required slips in the drawing. Otherwise, re-select locations of points c and d (at ordinates less than 0.7 of ultimate force value) till the estimated slip values equal the above referred normal slip values. This line $a'b'cNde'f'$ is established.

Step 8 Vertical ordinates below line $a'b'cNde'f'$ at various sections may be read off and these then represent the P_i (initial prestress force) in the cable at these sections, and their products with $\cos\theta$ and $\sin\theta$ at the respective sections represent the horizontal and the vertical components of the initial prestress force in the cable at those sections (θ being inclination of tangent to cable profile in elevation with the horizontal at the section). "$P_i \cos\theta e$" represents the primary initial prestress moment due to the cable at any section where e is eccentricity of the cable wrt centroid of the section.

NOTE: Repeat above steps for each cable and then sum up "$P_i \cos\theta e$" values and "$P_i \sin\theta$" values of all cables in each stage of prestress at each section for (first calculating secondary prestress values in indeterminate structures and then for) stress calculations.

5. If the **cable is stressed only from one end**, say J_1, then in Fig. 26.3 the prestressing force distribution before slip will be the ordinates below $abcj$ and the Null point will shift to j itself. Slip at the jacking end will be $\dfrac{1}{AE}$ (area $abcb'a'a$), and ordinates below line $a'b'cj$ will represent the P_i values after friction and slip. Extension at stressing end will be $\dfrac{1}{AE}$ (area $abcjiha$).

6. If the **cable is short or friction is too large,** the slip effects from the two stressing ends may even overshoot N (Fig. 26.3). In such a case, the mirror images of lines abN and feN (drawn wrt horizontal through N) will then bodily lower down further by constant ordinates X and Y, respectively, such that

$\dfrac{1}{AE}(X \cdot gh) + \dfrac{1}{AE}$ (area between abN and its mirror image line) = magnitude of normal slip (from where X can be evaluated and the ordinates P_i read off)

and $\dfrac{1}{AE}(Y \cdot gi) + \dfrac{1}{AE}$ (area between feN and its mirror image line) = magnitude of normal slip (from where Y can be evaluated and the ordinates P_i and off).

In such a case if the cable was stressed from only one

* Preferably not more than 0.75 of ultimate force in the cable.

end, say from end J_1, and $\frac{1}{AE}$ (area between abj and its mirror image line) was less than the magnitude of the normal slip, then mirror image line of abj (wrt horizontal through j) would bodily lower further by a constant ordinate Z, such that:

$\frac{1}{AE}(Z \cdot hi) + \frac{1}{AE}$ (area between abj and its mirror image line) = magnitude of normal slip (from where Z can be evaluated and then the ordinates P_i read off).

26.6 EFFECT OF CABLE-BLOCKAGE

Sometimes a cable in a short zone of its length may get blocked due to accidental leakage of slurry from concrete at the time of casting or even from leakage of grout from a neighbouring cable.

This blockage might become a solid lock which might be realised only at the time of attempting to stress this particular cable. (Refer to explanation given in Sec. 26.4 under 'Losses in Prestress' of this chapter as to how 'both-end stressing' can be particularly advantageous in such an emergency.)

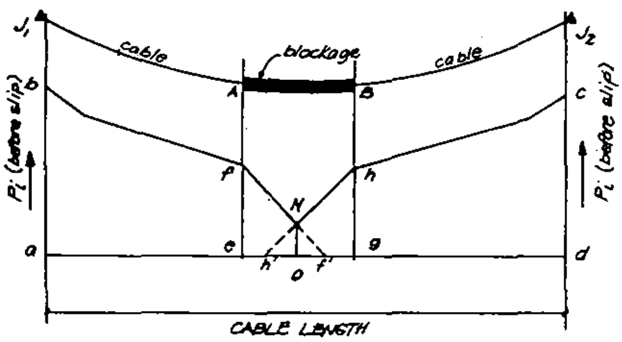


Fig. 26.4

In such a case proceed as follows:

Step 1 Try 'see-saw' stressing, i.e., carefully 'pull' the cable from one end at a time (keeping the other end 'locked' in the jack at that time) and release, taking care not to increase the pulling force beyond 0.75 to 0.77 of ultimate force possible in the cable. By this alternate see-saw stressing and releasing process try to break the grout-block. The process can take considerable time and, unless the blockage is not too hard and or is reasonably small in length, it may not be possible to break it. At any rate, in the process, the approximate locations of points A and B (limits of the blockage) can be reasonably predicted since the extending cable length up to its jacking point can be roughly estimated from the available extension, knowing that the rate of extension is usually about 4.5–5 mm per meter of the extending length for an average cable force of 0.6 of ultimate. If the blockage is released, carry out the scheduled stressing, but if blockage remains intact, then proceed to Step 2 ahead.

Step 2 Stress the cable from each end, gradually and simultaneously (taking precaution the blockage does not suddenly break with a jerk), raising the pulling force to 0.8 of the ultimate force. Note down the extension at each end, lock the cable at each end and note down the slips, as usual. Thus ordinates ab and dc (before slips), each equal to 0.8 of ultimate, are plotted (see Fig. 26.4).

- Knowing the approximate locations of blockage points A and B (which, where micro-drilling tools and expertise are available, can be located reasonably accurately) and hence the values of $(\mu\theta + Kx)$ from J_1 to A and from J_2 to B, the ordinates ef and gh Fig. 26.4 (being the cable forces at A and B can be quickly evaluated and plotted.
- The blockage acts like a rock anchor for the embedded cable from A towards B and from B towards A. If the force ef dies down to zero at f' and gh dies down to zero at h', then force $ef = f_b \times p_m \times$ (length ef') and force $gh = f_b \times p_m \times$ (length gh')
  where: f_b = average bond stress afforded by the grout in the blockage and p_m = effective perimeter of cable section gripped.

  Thus the lengths ef' and gh' can be estimated and luckily if lines hh' and ff' intersect, the minimum prestressing force ON (at the null point N) can be worked out and this value of force in this particular cable used in checking the overall stresses at this section. Chances are the stresses will turn out acceptable. Even if the extreme fibre stress turns out to be slightly tensile, the non-prestressed reinforcement within the tensile stress block can be checked to carry this tension force (at a reasonable over-stress as acceptable). Thus, the structure may be salvaged.

26.7 DESIGN OF CABLE-ANCHORAGE ZONES (END BLOCKS) IN POST-TENSIONED PRESTRESSED CONCRETE

Owing to extremely high bursting and spalling stresses set up in concrete immediately in front of a prestressing anchorage at the (very) time of tensioning the prestressing cable, design of reinforcement in the anchorage zone is of paramount importance. Actually, the stresses far exceed any elastically permissible values and this complicates the analysis. The stress situation is somewhat akin to that in concrete above (or below) a bearing plate or those in the immediate neighbourhood of a concrete hinge bearing (on either side of its throat). The prestressing force emanating from the anchorage travels into the concrete member initially in the shape of trajectories that set up strees-eddies in three dimensions within a certain distance — generally taken equal to three-quarters to full depth of beam at anchoring face. The zone within this distance is sometimes referred to as the 'lead-in' zone. The prestress force is assumed to come on the full 'effective section' only from the end of the lead-in zone.

If the section of the beam at the end of the lead-in zone is assumed rectangular (Case I), then the centroid of prestress force diagram coincides with that of the prestress stress diagram at that section. However, if the section is not rectangular (Case II), then the prestress stress diagram and the section area diagram may be multiplied together to draw the prestress force diagram at that section. Generally it is enough to assume the situation as Case I without any significant effect on the final detailing. Then the following step by step procedure may be followed for designing the anchorage zone reinforcement:

Step 1 Draw the prestress 'stress' diagram at the end of the lead-in zone.

Step 2 Divide the area fo this diagram into n equal area compartments, n being the number of cables at the end of the lead-in zone, stressed equally. Then if the cables at the end of the lead-in zone fall in line with the centroids of their respective strees-compartments, it is a case of Linear Force Distribution, otherwise it is a case of Non-linear Force Distribution. If it is the former, then go directly to each individual anchorage and draw for each its surrounding prism whose side-dimension $2a$ may be taken as the leasser of (a) twice the distance of the anchor centreline from the nearest 'free-edge' and (b) twice its distance from the nearest boundary of its 'neighbouring prism'. (Thus the prism side-dimension $2a$ for each individual anchor in this case is established.) However, if the case is one of 'Non-linear Force Distribution', then proceed by the method of 'successive resultants' { i.e., locate the resultants R_1, R_2, etc., of every two successive anchors down the elevation, treating the odd left out anchor (if any) as its own resultant (Fig. 26.5); then locate the resultants R_1', R_2',... of every two successive resultants already established (R_1' being the resultant of R_1 and R_2, R_2 being the resultant of R_3 and R_4 and so on) down the elevation; repeat the same process to locate the still bigger resultants R_1'', R_2'',... of every 2 successive earlier resultants R_1' and R_2', R_3' and R_4' and so on. Continue this all the way till one final total resultant R is located. Draw its surrounding prism whose side dimension is twice its distance from the nearest free-edge; then, within this prism, draw the prisms for each of its two immediate sub-resultants, noting that the prism side-dimension of a sub-resultant is the lesser of twice its distance from the nearest free-edge or twice its distance from the nearest boundary of its parent-prism'; proceed in this way backward to the sub-resultants of each of the just dealt sub-resultants, until, in the end, the prism side-dimension $2a$ for each individual anchor is established} until the prism side-dimension $2a$ is established for each anchor.

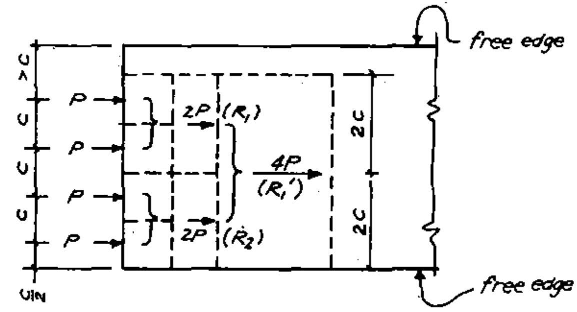


Fig. 26.5

Step 3 Establish the value of the concentration ratio $\frac{2a'}{2a}$ for each 'anchor'. ($2a'$ is the width of the loaded area of an anchorage, which for circular anchors may be taken as the 'side-dimension' of 'equivalent-area square'.)

Step 4 Estimate the bursting force T behind each anchor from the following formula:

$$T = 0.30P\left(1 - \frac{2a'}{2a}\right)$$

where P is the jacking force in the anchorage.

Step 5 Area of reinforcement steel required against bursting behind an individual anchor, is:

$$A_s = T/f_s$$

where f_s = working (elastic) tensile stress permitted in the reinforcement.

This A_s is to be provided at the centroid of the bursting force distribution behind the anchor, which may be assumed at $\frac{2a}{4}$ from the face of anchor, along its axis. This A_s is generally provided partly in the form of (a) a helix (of length $2 \times \frac{2a}{4}$ i.e.

of length a) placed touching the anchor face, and (b) 2, 3, or 4 grill meshes as close to the anchor as detailing would permit duly spaced between each other. If diameter of helix bar is d, its pitch is p, then area of steel provided by it against bursting is,

$$A_h = \left(\frac{\pi}{4}d^2\right) 2\frac{a}{p}$$

If the overall detailing of each grill mesh is such that behind each anchor we get 2 horizontal bars above and 2 below it and 2 vertical bars on either side of it (i.e., 4 bars in each direction, per mesh), and if say 4 grill meshes are provided one after another as described above, then section area of each bar in each mesh is,

$$= \frac{(A_s - A_h)}{4} \times \frac{1}{4}$$

Small bar diameters must be adopted, preferably 6 mm for helix and 10 mm for mesh. Also low tensile strength reinforcement is preferable. All this helps limit crack width.

Step 6 In addition to the above detailed reinforcement against bursting, anti-spalling reinforcement needs to be provided within the distance between the concrete face and the far end of the anchor. Normally 2 successively placed grill meshes, made of 10 mm diameter reinforcement bar, so as to give at least 4 horizontal and 4 vertical bars about each anchor, per grill, will suffice. This may be suitably increased proportionately if jacking force per anchor exceeds about 100 T.

The C and CA method of design of reinforcement in the anchorage Zone (after establishing the correct $2a$ value by Guyon's method explained in Step 2 above) is summarised below (for each anchor):

- See Fig. 26.6
- Prestressing force at anchoring = P
- Crushing stress $f_c = \frac{P}{2a^2}$ (kg/cm^2)
- Tensile stress in concrete

$$fy = f_c \left[0.4625 \left(\frac{a'}{a}\right)^2 - 1.3\frac{a'}{a} + 1.1\right] \text{ kg/cm}^2$$

- Bursting force $T = P\left(0.5 - 0.4\frac{a'}{a}\right)$, shown shaded.
- Permitted tensile stress in concrete $f_t = 0.39\sqrt{u_t}$ kg/cm^2
- u_t = standard cube crushing strength of concrete at transfer in kg/cm^2
- Bursting reinforcement $A_{st} = \frac{T}{f_{st}}\left[1 - \left(\frac{Kf_t}{f_y}\right)^2\right]$

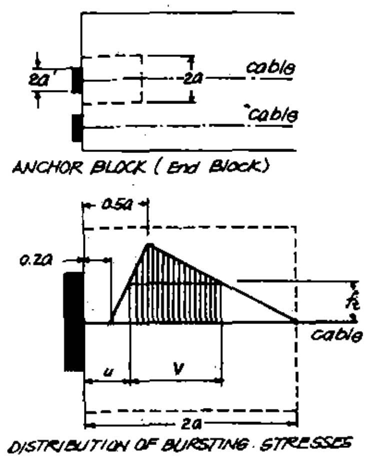


Fig. 26.6

where $K = 0.5\frac{a'}{a} + 1.12$

- Location of bursting steel ~ within dimension v:

$$u = \left(0.2 + \frac{0.3Kf_t}{f_y}\right) a \text{ and } v = 1.8a\left(1 - \frac{Kf_t}{f_y}\right)$$

However, additional reinforcement, the anti-spalling reinforcement, should be provided as explained in Step 6 above.

Typical Detailing at Certain Anchorages ~

(a) Fully embedded (dead) anchorages (Fig. 26.7): When a passive (dead) anchorage is embedded in the concrete it develops deformations beneath anchorage due to compression at tensioning and tensile stresses behind the anchorage.

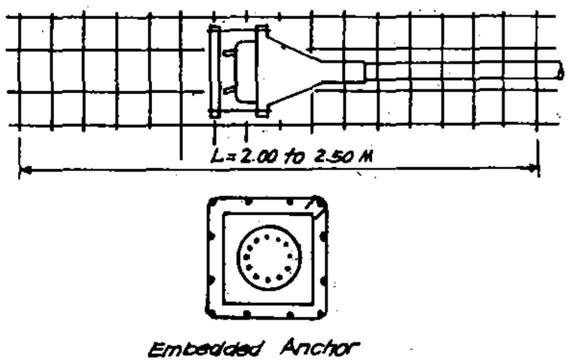


Fig. 26.7

To avoid cracking, it is necessary to include bars parallel to the cables to hold the concrete.

To avoid problems it appears that the steel should take up 20 to 30% of the anchorage force.

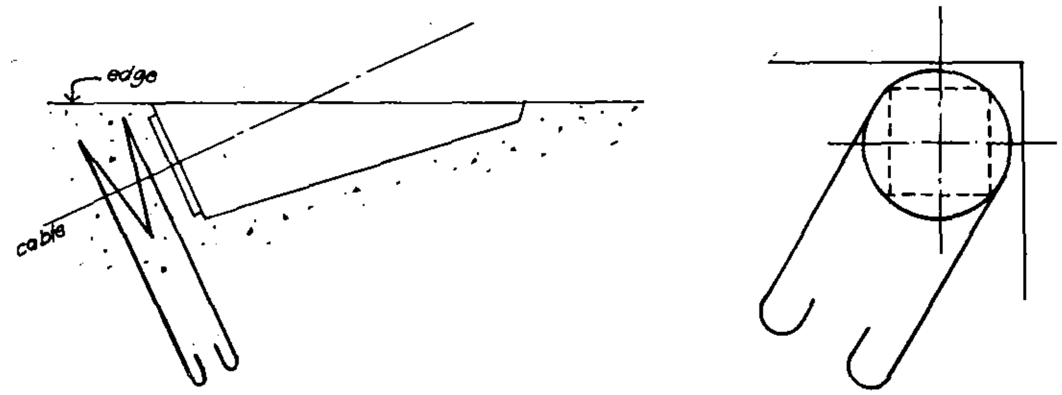


Fig. 26.8

(b) Anchorages at the 'edges' of structures (Fig. 26.8): It is necessary to provide special reinforcement for anchorages which are close to the surface of the concrete, by stirrups, tying the anchorage into the body of the structure.

(c) Anchorages in the thickening of webs (Fig. 26.9): This type of anchorage involves the following stress:
- a tensile stress in that part of the web situated beyond the anchorage. This is the same tensile stress as that of the embedded anchorage.
- moments in the web due to the eccentricity of the tendon.
- a shear force between the web and the thickening.

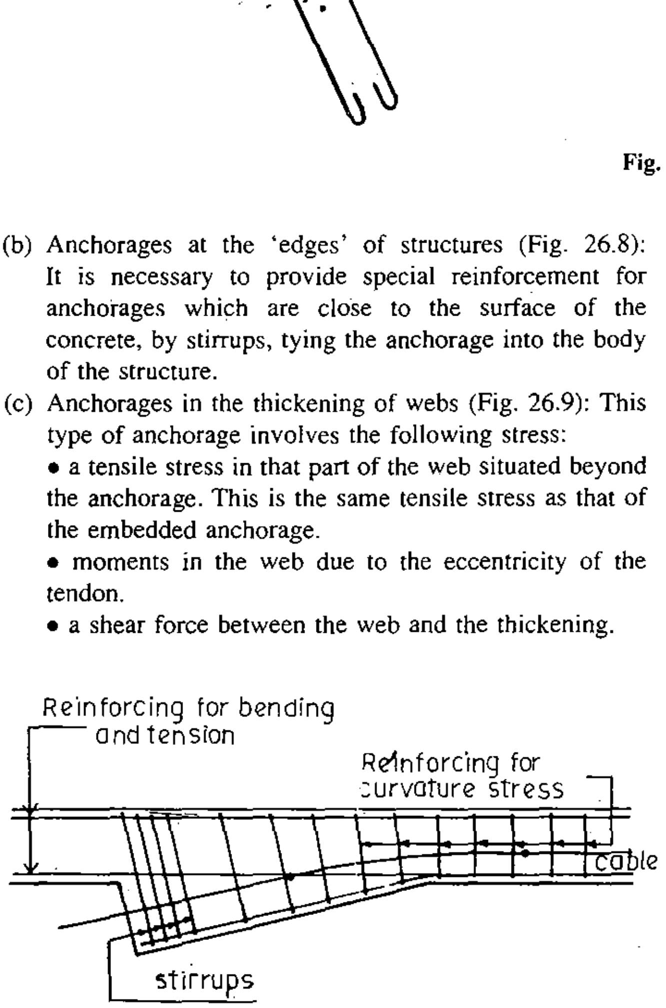


Fig. 26.9

To counter these stresses it is necessary to provide longitudinal reinforcement in the web together with stirrups in front of the anchorage to resist splitting on the web at the thickening.

Finally, in the cable's curved portion it is necessary to resist the side thrust of the cable. These forces are taken up by stirrups which should continue for some distance beyond the zone of curvature (shown in Fig. 26.9). This is because of inaccuracies at the site which may considerably alter the position of the zone of deviation.

26.8 CONCORDANT CABLE PROFILES IN CONTINUOUS PRESTRESSED CONCRETE BEAMS

A concordant cable is one which coincides with the resultant line of thrust at every section. (Resultant line of thrust is the locus of the centroids of the prestress force diagrams, section to section, and its position at any section is given by the 'total' prestressing moment divided by the prestressing force at that section, measured w.r.t.c.g. of the section.) Position of resultant line of thrust is given by $\frac{m_t^p}{P}$ with respect to the centroid of section. Position of cable is given by $\frac{m_o^p}{P}$ with respect to the centroid of section, m_o^p being the primary moment, $P \cdot e^*$, at the section. If the two have to be the same, then:

$$\frac{m_o^p}{P} = \frac{m_t^p}{P}$$

But m_t^p (total prestress moment) $= m_o^p + \sum_{s=1 \text{ to } n} m_s p_s$

where : $m_o^p =$ primary prestress moment and

$\sum_{s=1 \text{ to } n} m_s p_s =$ secondary prestress moment

Hence $\frac{\Sigma m_s p_s}{P} = 0$

Since $P \neq 0$, and m_s is a function of p_s (moment diagram due to unit p value), thus p_s must be zero. Then, from the general equation of compatibility ($V_{ro} + V_{rs}p_s = 0$) it follows that: $V_{ro} = 0$.

* "$P \cos\theta \cdot e$" in fact, as explained earlier.

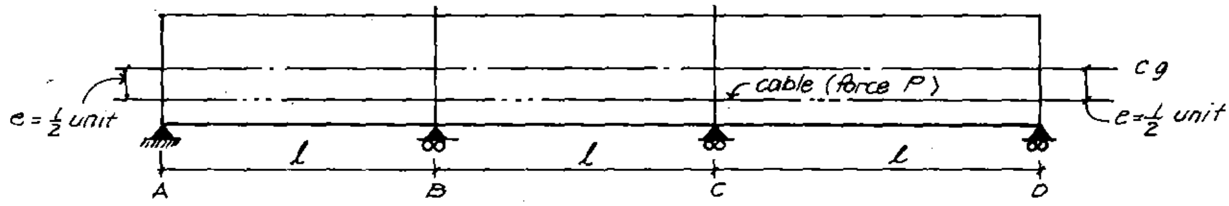


Fig. 26.10

This means:

$$\int \frac{m_r m_o^p}{EI} ds + \int \frac{s_r s_o^p}{GA} ds + \int \frac{n_r n_o^p}{EA} ds = 0,$$

$r = 1$ to n

$n =$ no. of statical indeterminacies.

This then is the general condition of concordance. In beams, where shear and thrust energies are particularly insignificant, the second and third terms may be ignored, so that in continuous prestressed concrete beams the general condition of cable-concordance reduces to:

$$\int \frac{m_r m_o^p}{EI} ds = 0$$

i.e., 'that cable profile is concordant whose primary prestress moment m_o^p diagram when mated with each of the $m_1, m_2, \ldots, m_n$ diagrams of the continuous beam, with $1/EI$ diagram, adds up to zero'.

Example on Plotting Resultant Line of Thrust

The above 3-span continuous beam (Fig. 26.10), of constant cross-section, is prestressed by means of a straight cable as shown. Locate the resultant line of thrust (a purely academic case, only to explain the method):

Procedure:

(i) Calculate n (statical indeterminacies),

(ii) Introduce biactions at the 'n' releases,

(iii) Draw m_o^p diagram (the BMD on S_o due to applied load of prestress),

(iv) Draw $m_1, m_2, m_3 \ldots$ diagrams (due to unit biactions, on S_o)

(v) Find $V_{10}\left(= \int \frac{m_1 m_0}{EI} ds\right)$,

$V_{20}\left(= \int \frac{m_2 m_0}{EI} ds\right)$,

$V_{11}\left(= \int \frac{m_1 m_1}{EI} ds\right)$, and

$V_{12}\left(= \int \frac{m_1 m_2}{EI} ds\right)$

(vi) Find p_s from $V_{ro} + V_{rs} p_s = 0$

(vii) Find $m_t^p (= m_0^p + m_1 p_1 + m_2 p_2)$ at each section.

(viii) Find $\frac{m_t^p}{P}$ at these sections, which gives ordinates of the Resultant Line of Thrust at those sections, wrt centroid of sections.

Solution: $n = 3(M - N + 1) - R = 3(6 - 4 + 1) - 7 = 2$, introduce 2 releases and, therefore, 2 biactions p_1 and p_2. Let the releases be moment releases by the sides of the intermediate supports, as shown in Fig. 26.11. EI is constant.

$$EI\, V_{10} = \Sigma m_1 m_o^p = 2\left\{\frac{l/2}{3}\left(0 + \frac{P}{2}1 + 4\frac{P}{2}\frac{1}{2}\right)\right\} = \frac{Pl}{2}$$

$$EI\, V_{20} = \Sigma m_2 m_o^p = \frac{Pl}{2}$$

$$EI\, V_{11} = \Sigma m_1 m_1 = 2\left\{\frac{l/2}{3}\left(0 + 1 + 4\frac{1}{2}\frac{1}{2}\right)\right\} = \frac{2l}{3}$$

$$EI\, V_{12} = \Sigma m_1 m_2 = \frac{l/2}{3}\left(0 + 0 + 4\frac{1}{2}\frac{1}{2}\right) = \frac{l}{6}$$

$$EI\, V_{22} \equiv EI\, V_{11} = \frac{2l}{3}$$

$$EI\, V_{21} \equiv EI\, V_{12} = \frac{l}{6}.$$

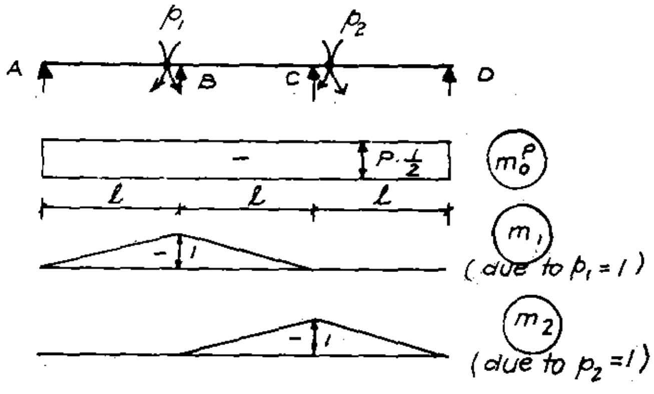


Fig. 26.11

- Substituting in the compatibility equations

$$V_{10} + V_{11} p_1 + V_{12} p_2 = 0$$

and $\quad V_{20} + V_{21}p_1 + V_{22}p_2 = 0$

We have: $\quad \frac{Pl}{2} + \frac{2l}{3}p_1 + \frac{l}{6}p_2 = 0$

and $\quad \frac{Pl}{2} + \frac{l}{6}p_1 + \frac{2l}{3}p_2 = 0$

giving: $\quad p_1 = p_2 = -\frac{3}{5}P$

- Thus total prestressing moments m_t^p are:

$$m_{t_A}^p = (m_0^p + m_1 p_1 + m_2 p_2)_A = -P/2 + 0 + 0 = -P/2$$

$$m_{t_B}^p = (m_0^p + m_1 p_1 + m_2 p_2)_B = -P/2 + (-1)\left(-\frac{3}{5}P\right) = P/10$$

$$m_{t_C}^p = P/10 \text{ and } m_{t_D}^p = -P/2$$

- This 'total' prestressing moment diagram is shown in Fig. 26.12, and dividing its ordinates by the prestressing force at each section (P, in this case) we obtain the profile and location of the resultant line of thrust as shown in the second part of Fig. 26.12.

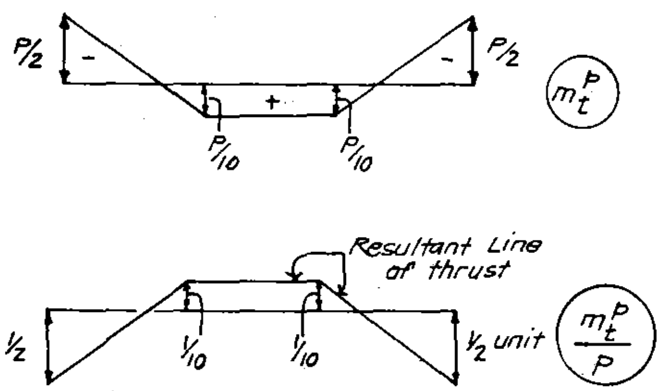


Fig. 26.12

26.9 NON-CONCORDANCE AND ESTIMATION OF SECONDARY PRESTRESS MOMENTS, SHEARS AND REACTIONS IN CONTINUOUS PRESTRESSED CONCRETE BEAMS

In practice when carrying out a workman-like practical design of a continuous prestressed concrete beam-type bridge-deck, the continuous cable profile is fixed from practical considerations of detailing and ease rather than purely from theoretical considerations (of plotting cable zones with their theoretical upper and lower limits, then 'fishing' for a concordant profile and then linearly transforming the same, should the former lie outside the physical depth of the beam!). Consequently, in most practical designs, the continuous cable profile is not concordant, i.e., does not coincide with the resultant line of thrust (see description given earlier in 26.8 in which the concordant cable has been explained in some detail). The eccentricity between the two at any section sets up another moment (and its associated shears and consequent support reactions). This is generally referred to as 'parasitic' (parasite-like) or 'secondary' prestress moment, causing parasitic shears and reactions. These would be zero if the tendon profile only coincided with the resultant line of thrust. ***It must be remembered that while the simple prestress moment resulting from the mere product of 'horizontal prestressing force and its eccentricity with respect to the centroid of the concrete section' (called 'primary' prestress moment, m_o^p) is a purely internal entity—and, if inclined, has its primary prestress shear, but never any support reactions, the secondary prestress moment and shear are associated with their balancing support reactions (i.e., external reactions) and are thus an external entity like any external load.*** Since these secondary moments cause vertical reactions at supports, the total overall beam length may be imagined as a 'simple' long beam between its two ends, subjected to upward or downward vertical loads at the intermediate support locations (equal to the support reactions due to secondary prestress moments). ***Hence, the secondary prestress moment diagram between two successive supports is always linear.*** At those supports where secondary prestress reaction opposes the self dead load reaction, it should be ensured that the latter is more so that there is no 'lifting up' of the beam at such supports (as otherwise this would change the support conditions for various loadings). Generally, the self dead load reactions are higher anyway. ***It must be understood that since these secondary prestress moments, shears and reactions, are directly proportional to EI (like the fixity moments due to any applied loading in an indeterminate structure), their magnitude reduces when cracking sets-in and EI reduces. Beyond elastic limit, with each additional hinge forming, as the structure cracks and tends to come closer and closer to a statically determinate form, the magnitude of these parasitic values keeps reducing till they disappear when the structure becomes determinate.*** In a simply supported prestressed concrete beam (or in any statically determinate prestressed concrete structure) there is only primary prestress moment (and shear and thrust), causing no support reactions (primary prestress being an internal entity) and no secondary prestress effect. Having understood that the secondary prestress effect is like any other externally applied loading on the indeterminate structure, it can easily be evaluated, say by the Flexibility Method of influence coefficients (explained earlier in Ch. 20) taking the primary prestress moment

• EXAMPLE

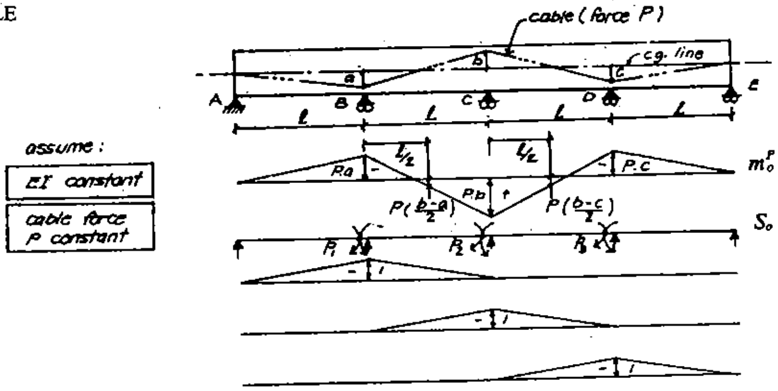


Fig. 26.13

diagram, m_o^p, as the m_o (statical) moment diagram due to the 'applied loading'.

Hence, in a practical case of a continuous prestressed concrete beam, in order to work out the total prestress moments: $m_t^p(= m_o^p + \sum_{r=1}^{n} m_r p_r^p)$, total prestress shears: $s_t^p(= s_o^p + \sum_{r=1}^{n} s_r p_r^p)$ and the support reactions due to secondary prestress, exactly the same procedure as explained in the earlier referred Flexibility Method may be followed, taking the primary prestress moment diagram m_o^p (the diagram showing the products of horizontal prestress force and its eccentricity wrt to centroid of the concrete section, at various sections) directly as being the statical moment diagram m_o due to applied loading (prestress in this case) on the statically made determinate structure. Secondary prestress moment and shear 'at any section' would then be simply: $\sum_{r=1}^{n} m_r p_r^p$(i.e., $m_1 p_1 + m_2 p_2 + \ldots + m_n p_n$) and $\sum_{r=1}^{n} s_r p_r^p$(i.e., $s_1 p_1 + s_2 p_2 + \ldots + s_n p_n$), respectively, with $m_1, m_2, \ldots$ and $s_1, s_2, \ldots$ being the ordinates of these diagrams 'at the section' and p_r^p(i.e., $p_1, p_2, \ldots, p_n$) being the biaction p-values (under prestress case) solved out of the n compatibility equations.

The corresponding support reactions can then readily be worked out knowing that the secondary prestress moment diagram is linear in between successive supports. The same procedure holds good for prestressed concrete frames.

It is of interest to remember that in a 2-span practical prestressed concrete continuous beam, the secondary prestress moment at the central support is of the same sign as the primary prestress moment there (i.e., sagging type, causing compression at top fibre and tension at bottom fibre) and can be almost equal in magnitude. Hence its usefulness should not be underestimated. In terms of effect, it is not just secondary. At span points, it can of course be opposite in effect to the primary prestress moment. Again, in a 3-span case too it assists the primary moment at the intermediate supports but opposes it in the span regions. Parasitic prestress moments, shears, and reactions (and thrusts in frames) must be thoroughly accounted for.

Interesting Special (Academic) Case (Fig. 26.13)

In a continuous psc beam if the cable profile is 'linear between successive supports' and of 'zero eccentricity at ends', then the total-moment due to prestress at any section is zero ($m_t^p = 0$), or, in other words, the resulting line of thrust coincides with the centroidal axis of beam, or that it is axial. Thus, in such a case the prestress causes pure compression P/A and no bending and not even shear.

It should also be noted that one of these two cable-profiles is a linear transformation of the other and the two have same resultant line of thrust. (See ahead for the 'principle of Linear Transformation' and its effect of not altering the total prestress moment.)

Refer to Fig. 26.13:

• $n = 3(8 - 5 + 1) - 9 = 3$, so the beam of 4 spans needs 3 releases to make it statically determinate (S_o). Let us introduce 3 moment releases as shown in Fig. 26.13: p_1, p_2 and p_3, m_o^p, m_1, m_2 and m_3 are shown in Fig. 26.13.

$$\bullet\ EI\,V_{10} = \Sigma m_1 m_0 = \frac{l/2}{3}\left\{0 + Pa \times 1 + 4 \times \frac{1}{2} \times \frac{Pa}{2}\right\} + \frac{l/2}{3}\left\{Pa \times 1 + 0 + 4 \times (-\tfrac{1}{2}) \times P(\frac{b-a}{2})\right\} = \frac{Pl}{6}(4a - b)$$

$$EI\,V_{20} = \Sigma m_2 m_0 = \frac{l/2}{3}\{0 + Pb \times (-1) + 4 \times \left(-\frac{1}{2}\right) \times P\left(\frac{b-a}{2}\right)\} + \frac{l/2}{3}\{-Pb + 0 + 4 \times \left(-\frac{1}{2}\right) \times P\left(\frac{b-c}{2}\right)\} = \frac{Pl}{6}(a - 4b + c)$$

$$EI\,V_{30} = \Sigma m_3 m_0 = \frac{l/2}{3}\{0 + Pc + 4 \times \left(-\frac{1}{2}\right)$$

$\times P\left(\frac{b-c}{2}\right)\} + \frac{l/2}{3}\{Pc + 0 + 4 \times \frac{1}{2} \times \frac{Pc}{2}\}$

$= \frac{Pl}{6}(4c - b)$

$EI\ V_{11} = \Sigma m_1 m_1 = 2 \times \frac{l/2}{3}\left\{0 + 1 + 4 \times \frac{1}{2} \times \frac{1}{2}\right\}$

$= \frac{2l}{3}, \equiv EI\ V_{22} \equiv EI\ V_{33}$

$EI\ V_{12} = \Sigma m_1 m_2 = \frac{l/2}{3}\{0 + 0 + 4 \times \frac{1}{2} \times \frac{1}{2}\}$

$= \frac{l}{6}, \equiv EI\ V_{21} \equiv EI\ V_{23} \equiv EI\ V_{32}$

$EI\ V_{13} = \Sigma m_1 m_3 = 0, \equiv EI\ V_{31}$

- Substituting in the 3 compatibility equations:

$$V_{10} + V_{11}p_1 + V_{12}p_2 + V_{13}p_3 = 0$$
$$V_{20} + V_{21}p_1 + V_{22}p_2 + V_{23}p_3 = 0$$
$$V_{30} + V_{31}p_1 + V_{32}p_2 + V_{33}p_3 = 0$$

We get: $p_1 = -Pa,\ p_2 = Pb,\ p_3 = -Pc$

- Hence : , $m_{t_A}^{P} = (m_0 + m_1 p_1 + m_2 p_2 + m_3 p_3)_A$
  $= 0 + 0 + 0 + 0 = 0$

Similarly $m_{t_B}^{P} = -Pa + (-1)(-Pa) + 0 + 0 = 0,$

$m_{t_C}^{P} = 0,\ m_{t_D}^{P} = 0,\ m_{t_E}^{P} = 0,$

and infact m_t^P at any section is found to be equal to zero.

- Hence the position of resultant line of thrust ($= m_t^P/P$ at any section wrt cg of the section) is also equal to zero wrt cg of each section, i.e. coincides with cg line.
- ***This means that the resultant line of thrust is coinciding with the section cg line all along, i.e., is axial, which is the same as if the cable itself was placed at zero eccentricity wrt cg of section at each section.***
- ***It should also be noted that even the total shear due to prestress ($s_t^P = s_o + \sum s_r p_r$) in the present type of cable is zero at each section, eventhough the cable looks inclined in each span.***

26.10 LINEAR TRANSFORMATION OF CONTINUOUS PRESTRESSING TENDON PROFILES IN CONTINUOUS BEAMS

In continuous prestressed concrete beams designed theoretically, if the 'theoretically established' tendon profile lies outside the beam in certain zones, it can be brought back into the beam by the principle of linear transformation without changing the resultant line of thrust (i.e., without changing the total prestressing effect).

The principle of linear transformation of tendon profile in continuous beams states that:

"The resultant line of thrust remains unaltered, i.e., the total prestress moment, etc., remain unchanged even if the position of tendon is shifted up or down, vertically, at any or all the intermediate supports, by any amount, so long as the position of the tendon at the two ends is kept unchanged and the new profile in each span is obtained by pure linear transformation of the previous profile" (Fig. 26.14).

Proof

Consider, for example, a 3-span continuous prestressed concrete beam with a given cable profile as shown in Fig. 26.15(a). If this cable profile is linearly transformed as shown in Fig. 26.15(b) (downward by $2a$ at one intermediate support and by $2b$ upward at the other) let us calculate and see if the total prestressing moments, etc., remain unchanged, the transformation having been done according to the above-stated principle (Fig. 26.14).

NOTE For ease, assume EI is constant, horizontal prestressing force P constant throughout, and spans equal.

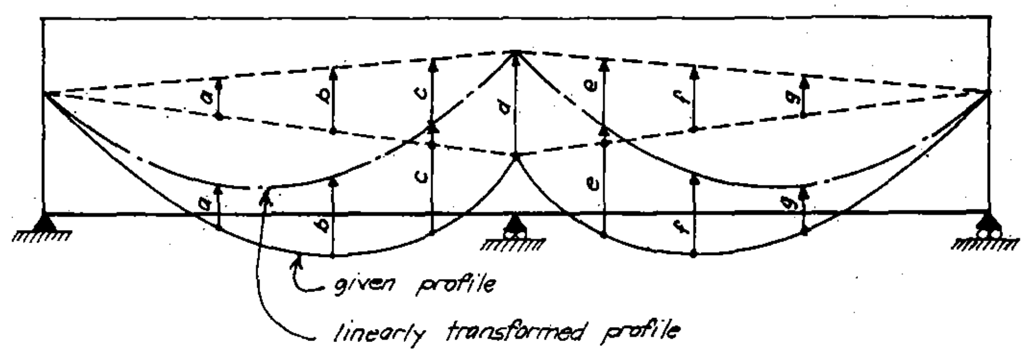


Fig. 26.14

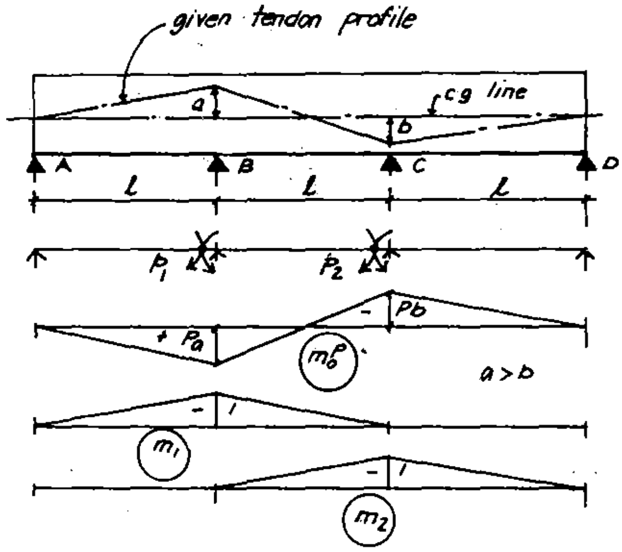


Fig. 26.15(a)

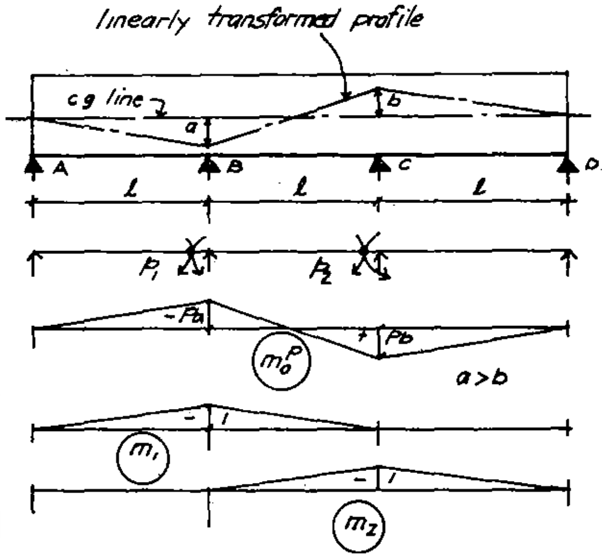


Fig. 26.15(b)

$V_{10} + V_{11}p_1 + V_{12}p_2 = 0$

$V_{20} + V_{21}p_1 + V_{22}p_2 = 0$

$$V_{10} = \int \frac{m_1 m_o^p}{EI} = \frac{1}{6EI}\left(0 - Pa - 4\frac{Pa}{2}\frac{l}{2}\right)$$

$$+\frac{l}{6EI}\left(-Pa - 4\frac{1}{2}P\frac{(a-b)}{2}\right)$$

$$= \frac{Pl}{6EI}(b - 4a)$$

$$V_{20} = \int \frac{m_2 m_o^p}{EI} = \frac{Pl}{6EI}(4b - a)$$

$$V_{12} = V_{21} = \int \frac{m_1 m_2}{EI} = \frac{l}{6EI}$$

$$V_{11} = V_{22} = \int \frac{m_1 m_1}{EI} = \frac{2l}{3EI}$$

Substituting in the above two equations, and solving these, we get:

$$p_1 = Pa, \; p_2 = -Pb$$

so that total p.s. moment at, say B, is

$m_{t_B}^p = m_0^p + m_1 p_1 + m_2 p_2 = Pa - 1Pa + 0 = 0$

likewise $m_{t_{B-C}}^p = m_0^p + m_1 p_1 + m_2 p_2$

$$= 1P\frac{(a-b)}{2} + \left(-\frac{1}{2}\right) Pa + \left(-\frac{1}{2}\right)(-Pb) = 0$$

and $m_{t_c}^p = -Pb + 0 + (-1)(-Pb) = 0$

also, p.s. shear, say at just to left of B, is $s_{t_{B_L}}^p = s_0^p + s_1 p_1 + s_2 p_2$

$$= \frac{Pa}{l} + \left(-\frac{1}{l}\right) Pa + 0 = 0$$

likewise, say at middle of span $A - B$

is $s_{t_{A-B}}^p = \dfrac{Pa}{l} + \left(-\dfrac{1}{l}\right) Pa = 0$

and at A, reaction = shear = $Pa/l + \left(-\frac{1}{l}\right) Pa = 0$

$V_{10} + V_{11}p_1 + V_{12}p_2 = 0$

$V_{20} + V_{21}p_1 + V_{22}p_2 = 0$

$$V_{10} = \int \frac{m_1 m_o^p}{EI} = \frac{Pl}{6EI}(4a - b)$$

$$V_{20} = \int \frac{m_2 m_o^p}{EI} = \frac{Pl}{6EI}(a - 4b)$$

$V_{12} = V_{21} = l/6EI$

$V_{11} = V_{22} = 2l/3EI$

Substituting in the above two equations, and solving these, we get:

$$p_1 = -Pa, \quad p_2 = Pb$$

so that $m_{t_B}^p = -Pa - 1(-Pa) + 0 = 0$

$$m_{t_{B-C}}^p = -P\frac{(a-b)}{2} + (-1/2)(-Pa) + (-1/2)Pb$$
$$= 0$$

and $m_{t_c}^p = Pb + 0 + (-1)Pb = 0$

also p.s. shear, say at just to left to B,

is $\quad s_{t_{B_L}}^p = s_o^p + s_1 p_1 + s_2 p_2$

$$= -\frac{Pa}{l} + (-1/l)(-Pa) + 0 = 0$$

at A, shear = reaction = $-\dfrac{Pa}{l} + (-1/l)(-Pa) = 0$

likewise, say at middle of span $A - B$

Shear: $s_{t_{A-B}}^p = -\dfrac{Pa}{l} + (-1/l)(-Pa) + 0 = 0$

Hence, it can be seen that total moments, shears, and reactions etc., due to prestress remain same despite the linear transformation.

NOTE ***Regarding linear transformation of cable profiles in Prestressed Concrete Frames:*** There being no intermediate support in the transom of a single span frame, the entire tendon profile in the transom is bodily lifted or lowered in its linear transformation by the same amount e. Consequently this P_e moment diagram for the transom (i.e., the effect of linear transformation alone) is rectangular. As for the leg cable profile, it may be only 'bodily rotated' for its linear transformation, thereby giving a triangular P_e moment diagram as a result of this transformation. The effect of the above moment diagrams (rectangular in transom and triangular in the legs) can be readily worked by the flexibility method and it is seen that:

(i) ***In case of vertical legs, only additional thrust is caused in transom alone, and no other change***

(ii) ***In case of inclined legs, additional thrust is caused in transom as well as in the legs, no other change.***

However, all this is only of academic interest. ***In any practical design the cable profiles are selected from convenience of detailing and simplicity and then for such prefixed tendon profiles first the friction calculations are done so that exact initial prestress force normal to section is known at each section. Its product with cable eccentricity at each section gives the m_o^p (primary prestress moment) at various sections. This diagram is used as the 'applied load' statical moment m_o^p diagram and the frame analysed for this applied loading by the Flexibility Method, and the secondary prestress: moments $(m_1p_1 + m_2p_2 + \ldots)$, shears $(s_1p_1 + s_2p_2 + \ldots)$ and thrusts $(n_1p_1 + n_2p_2 + \ldots)$ calculated after working out the influence coefficients and $p_1, p_2, \ldots$ values from the compatibility equations.***

26.11 LIMITS OF LINEAR TRANSFORMATION OF CABLE PROFILES IN CONTINUOUS BEAMS

Effect of tendon transformation on ultimate strength. Contrary to general belief, it has been shown[1] that linear transformation of cable profiles in continuous prestressed concrete beams can, in some cases, lead to reduction in the ultimate load carrying capacity of the beam. This is possible when the transformation results in bringing the tendon close to the compression edge (at the first forming hinge sections), thereby reducing the effective depth. As the steel area remains the same, the proportion of steel-area to effective concrete area increases, making the section 'over reinforced', which in turn reduces the rotation capacity of the hinge. This can lead to a premature collapse of the hinge, resulting in an early failure of the beam (see also Ch. 28).

Limits of linear transformation

However, it has been shown[1] that the ultimate strength may not be seriously affected so long as it is ensured that linear transformation does not reduce the effective depth at any critical section to less than a third of the overall depth of section (see also Chs. 27 and 28).

REFERENCE

1. Raina, V.K., "Ultimate Moment Distribution in Continuous Prestressed Concrete Beams", *Ph.D. Thesis*, University of London, 1966.

CHAPTER 27

Behaviour of a Structure during the Loading History all the Way up to Collapse, and Estimation of Moments Attained at 'Ultimate' taking 'Redistribution' into Account

Synopsis

After elastic design, simply ensuring the ultimate moment of resistance of a critical section equals a certain load factor times its working load moment, does not necessarily guarantee that the structure will sustain the desired ultimate load, unless the structure accepts 100% redistribution of moments at 'ultimate'. Therefore, it is important to investigate the actually attainable ultimate moments at critical sections taking into account the compatibility of deformations and check that the (thus calculated) actually mobilisable ultimate strength of every critical section is not less than the required (codified) applied ultimate moment in a given statically indeterminate structure.

Notation

$M_2(M_u)$ moment of resistance at limit L_2
M_1 moment of resistance at limit L_1
M_0 applied bending moment at which critical section cracks (limit L_0)
M_{ps} prestressing moment
P prestressing force
e_s eccentricity of P, wrt centroid of section
w dead load per unit run
M_{DL} dead load bending moment
EI_{un} uncracked flexural rigidity
θ rotation (θ_p, α: plastic rotation)
A cross-section area
Z section modulus
z distance between hinge section and the adjacent point of contraflexure
d effective depth
D overall depth
l span
l_p equivalent plastified length
W_u applied ultimate load
f_{cu} cube strength of concrete at 28 days (crushing)
f'_c cylinder strength of concrete at 28 days (crushing)
f''_c maximum flexural compressive stress in concrete
e_p effective prestrain in tendon
e_{cp} strain in concrete at level of tendon due to stress under prestress and dead load
e_{csu} strain in concrete at level of tendon due to applied load at limit L_2
e_{cse} strain in concrete at level of tendon due to applied load at limit L_1
e_c strain in concrete at any fibre in flexural compression
n neutral axis factor
p'' $\dfrac{\text{volume of binders per unit legth of beam}}{\text{volume of concrete in unit length of beam}} \times 100$
ϕ curvature (ϕ_2, ϕ_1, ϕ_0, ϕ_{ps}: due to M_2, M_1, M_0, M_{ps}, respectively)

Suffixes 2 (or u) and 1 (or e) refer to stages at limits L_2 and L_1 respectively.

Suffixes l and r refer to 'left' and 'right' of hinge.

Suffixes c, ps, and ms refer to concrete, prestress and mild steel respectively.

27.1 DEFORMATION CHARACTERISTICS

The early attempts to formulate a design procedure for reinforced concrete recognised the inelastic non-homogeneous characteristics of the composite material. This early emphasis on the inelastic properties was discontinued when the elastic straight line theory was introduced in the early part of this century. There are only a few papers in the period 1910 to 1930 which discuss the inelastic properties of the material, but since then there has been an ever increasing interest in this subject. Evans and Hognestad have published excellent reviews of this work. There have been many alternative stress blocks suggested for the flexural compressive stress distribution in concrete, such as, a cubic parabola by Mensch, a quarter ellipse by Kempton-Dyson, a fifth degree parabola by Lyse, a combination of parabola and rectangle by Rüsch, and a combination of parabola and trapezium by Hognestad.

Structural analysis is the evaluation of the distribution and magnitude of the load-effects (viz., bending moments, shearing forces and twisting moments) due to the applied loads, while structural 'design' embodies the calculation of section sizes and properties of the members for resisting the load-effects acting on them. M. Guyon has phrased it as follows: "The first object is to 'know', within the limits of our ability to do so; and the second is to 'apply', without going beyond what we know."

The flexural compressive stress distribution in concrete alters greatly between the working load and ultimate load owing to its inelastic behaviour beyond the elastic limit. When a structure is designed on the basis of elastic theory such that at working load the maximum concrete stress is a third of the cube crushing stress, it does not infer that the structure can carry thrice the working load before it fails even if it is statically determinate, nor is it possible to predict the collapse load by elastic analysis because the elastic theory embraces only the initial portion of the total load-deformation curve. In a statically indeterminate structure loaded to collapse, the bending moment at deformed sections will not necessarily be proportional to the applied load. When a member is designed according to the elastic theory, the code of practice requires that its ultimate moment of resistance is not less than a certain load factor times the applied working load moment. After cracking sets in and some sections start yielding, redistribution of moments commences. At the collapse of a structure it is possible that some critical sections may be unable to contribute their full (i.e., absolute) ultimate moment of resistance owing to the lack of redistribution and the obligation to compatibility. In such a case, the structure will have collapsed when the applied moment at these sections was still less than that required by the code. Therefore, it is preferable in a structure, designed on an elastic basis, to estimate the actually 'feasible' ultimate moment distribution, and check that the (thus calculated) actually mobilisable ultimate moment at every critical section (after redistribution) is not less than the anticipated applied ultimate moment required by the code. (In determinate structures, however, 100% redistribution can be assumed directly, but what has been stated above still holds.)

27.2 COLLAPSE BEHAVIOUR OF CONTINUOUS BEAMS

Consider a continuous beam being loaded proportionally from zero load to collapse load. During the loading history, a stage is reached when some critical sections being to yield (limit L_1). With total load on the 'beam as a whole' increasing, the burden of resisting this additional load falls more and more upon other sections that have not yet yielded. The yielding sections act like hinges, in that their stiffness is much reduced and they visibly rotate. The hinging regions are commonly idealised as being effective at the maximum-moment section and are called *plastic hinges*.

The bending moments at the sections of reduced stiffness do not increase proportionally with load and hence an increase in load must be taken by a rapid increase in moment at other less highly stressed sections. This adjustment in the distribution of bending moment is commonly called the 'redistribution' of moments.

Each time a plastic hinge is formed, the number of statical indeterminacies reduces by one, until eventually the beam becomes statically determinate in respect of such loading as remains to be applied to it. Then the development of yet another plastic hinge will make the beam unstable—a 'mechanism' causing its collapse! However, a premature collapse (i.e., when the number of hinges formed is less than $n+1$, n being the number of statical indeterminacies) is possible when any hinge reaches its defomational limit earlier and/or any individual span becomes unstable. A complete or full redistribution (adaptation) refers to the formation of $n+1$ hinges at failure with each hinged section attaining its full (absolute) ultimate moment of resistance through the phenomenon of redistribution.

The essential difference between steel and concrete structures is that in the former, the stress increases at high strains (i.e., strain hardening), while in the latter, after a distinct maximum stress, the stress decreases with increasing strain (i.e., strain softening). As a result of this, steel beams generally do not have a distinct collapse but exhibit large changes in geometry. On the other hand, owing to strain softening in concrete, the moment at a section does not remain constant once the ultimate value is reached, but diminishes with increasing deformation. This causes a distinct collapse in case of concrete. When the moment at a particular section is decreasing while the applied load on the whole is increasing, the yoke of resisting this load falls more and more on less deformed sections. The sharper the falling of moment at one section, the quicker the increase at others, and hence greater the redistribution.

27.3 PROPOSED METHOD FOR ESTIMATING TRUE ULTIMATE BENDING MOMENT DISTRIBUTION

In reinforced and prestressed concrete members, a stress-strain curve cannot be generalised into a basic relationship, simply because concrete is a heterogeneous composite material, in which, both the quality as well as the quantity of the constituent materials can vary. Instead a 'moment-deformation' curve is more fundamental[1] in such a material, the deformation being either curvature or rotation. Again, a moment-curvature relationship is less generalisable as compared to a moment-rotation relationship because, as tests have shown,[2] the former seems to vary from section to section in a member even when the sections are exactly identical in composition, orientation and loading. The same tests have confirmed that the latter appears to be much less variable, probably because it refers to an entire hinging-region rather than to a section in it.

The necessity to idealise a *moment-deformation* characteristic is obvious because the actual experimental curve cannot be available every time for every member. The simplest idealisation is a bilinear idealisation, which is assumed to consist of one straight line from the origin to the point representing moment and deformation at yield (Limit L_1), and a second straight line joining the points representing moment and deformation at yield and at ultimate (Limit L_2). However, in prestressed concrete cases, since cracking commences quite late in comparison with that in reinforced concrete, a closer approximation to the actual curve is a trilinear idealisation which takes account of the uncracked phase (Limit L_0) prior to yeild.

The proposed[1,2] method assumes a moment-curvature idealisation in the elastic phase and a moment-rotation idealisation in the plastic phase and takes account of both elastic as well as plastic deformations. The compatibility requirement at ultimate is considered under the cumulative effects of elastic and plastic conditions. The attainable plastic rotation at a hinge is split up into parts occurring to its left and right.

For the elastic deformations, two idealisations (a bilinear and a trilinear) are suggested, any one of which may be adopted. The bilinear moment curvature curve assumes a constant cracked EI (intermediate between the uncracked EI and the constant cracked EI of Baker's) equal to $\left[\dfrac{M_1 + M_0}{\phi_1 + \phi_0}\right]$ (slope OL_1 in Fig. 27.1), and neglects the initial rotation due to prestress. The trilinear idealisation, a more sophisticated approach, assumes uncracked EI for moments up to M_0 throughout and then a constant cracked EI (much smaller than that in the bilinear case) given by $\left[\dfrac{M_1 - M_0}{\phi_1 - \phi_0 + \phi_{ps}}\right]$ (slope L_0L_1 in Fig. 27.3) for moments between M_0 and M_1. The effect of initial rotation due to prestress is taken into account as well.

Based on these assumptions, the curvature distribution at the limit L_1 is represented by the area aa_1dbgc_1ca in Fig. 27.2. The moment-cuvature relationship is then represented by $aL_0L_1L_2$ in Fig. 27.3.

The plastic rotation in each of the proposed idealisations is estimated from the empirical formula recommended by Baker and Amarkone[3] viz.,

$$\theta_p = 0.4(e_{cu} - e_{ce})z/d$$

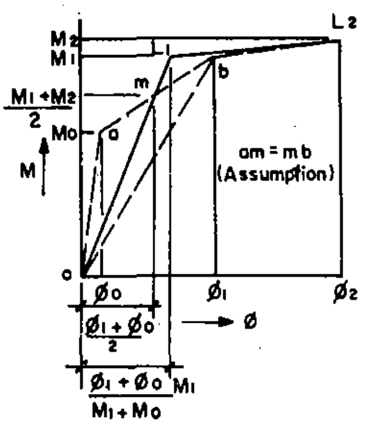


Fig. 27.1

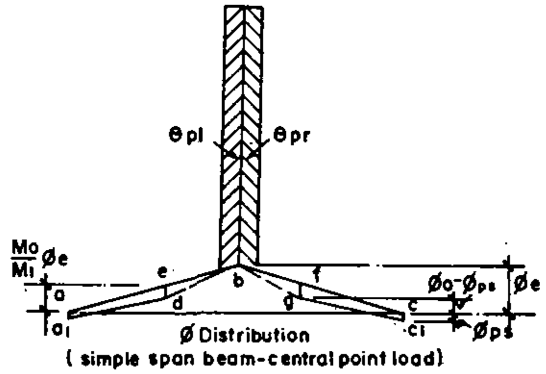


Fig. 27.2

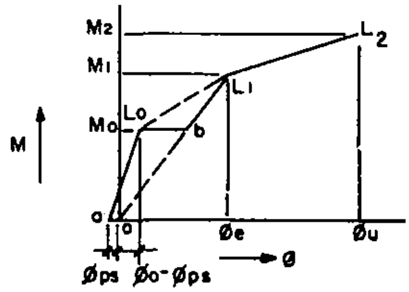


Fig. 27.3

The validity of this assumption was investigated[2] and found to involve a theoretical error of between 2–7%. The above formula is derived from its more general form

$$\theta_p = 0.8(e_{cu} - e_{ce})K_1K_3z/d$$

where K_1 is assumed to vary from 0.7 to 0.9 depending on type of steel and K_3 from 0.6 to 0.9 depending on grade of concrete. If the product of K_1 and K_3 is taken as 0.5

on an average, which is no less arbitrary than the choice from the above-mentioned ranges of K_1 and K_3, there is no justification for making a modification of 2–7% in the plastic rotation formula to suit the proposed idealisations.

Summarising, to estimate the ultimate moment distribution (and hence the ultimate load) in a given continuous beam, first the necessary calculations at the limits L_0, L_1 and L_2 may be done for each critical section (see sec. 27.6) and the appropriate EI values for each hinge region calculated. A suitable first-trial ultimate moment distribution may then be assumed, the cracked and uncracked parts marked off and the m_i diagrams drawn. The assumed ultimate moment diagram is correct provided the n compatibility equations are satisfied, otherwise a new trial is necessary. (The prestress moment diagram is considered in the case of a trilinear idealisation, as explained.)

In order to explain the compatibility calcutions based on this method, Examples 1 and 2 are given below. The former is based on the proposed bilinear idealisation and latter on the proposed trilinear idealisation. The same procedure applies to reinforced concrete beams also.

EXAMPLE 1 The 2-span beam is shown in Fig. 27.4.

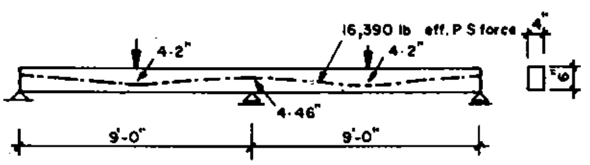


Fig. 27.4

From calculations at limits L_0, L_1 and L_2 for the 'mid-span' and 'support' sections the results are shown in Table 27.1.

At the collapse of the beam, assume that at the central support section 100% of its M_2 (i.e., 1.00 × 143,700 = 143,700 lb in.) is attained, while at the mid-span sections only 93% of their M_2 (i.e., 0.93 × 132,000 = 122,800 lb in.) is attained. (This in fact is the final of three trials which almost completely satisfies the involved compatibility equation, as will be seen ahead.)

Because of symmetry, only one symmetrical half need be considered Fig. 27.5(a) shows the assumed ultimate moment distribution, in which by marking off the ordinates pq and $p'q'$ = (M_1 values of the respective hinges) the M elastic diagram is curved out. Figure 27.5(b) represents the m_1 diagram in the span.

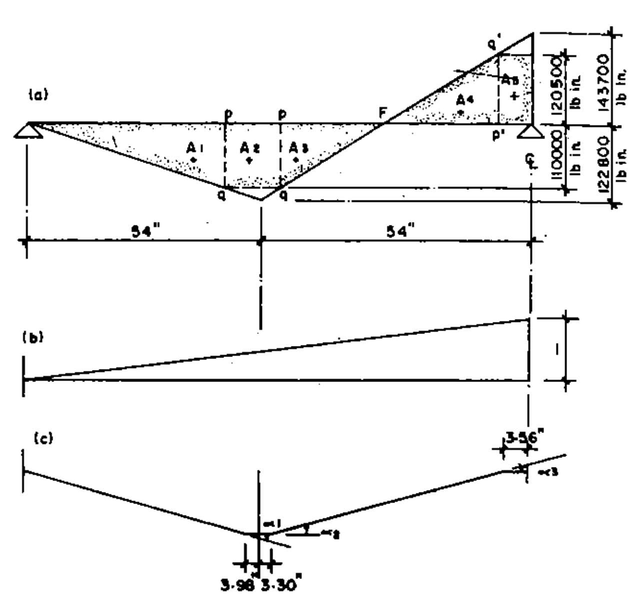


Fig. 27.5

The position of the point of contraflexure F is worked out from geometry. Similarly the areas A_1 to A_5, positions of their centroids, and the ordinates of the m_i diagram are worked out from geometry.

Attainable θ_{pl} and θ_{pr} at the hinges and their positions are worked out in Table 27.2.

Knowing the appropriate EI value in the span and support hinge regions (126 × 10^6 and 137 × 10^6 lb in.^{2}, respectively), values are substituted algebraically in the compatibility condition:

$$\int \left(\frac{M}{EI}\right)_{\text{elastic}} \cdot m_i\, dx + \Sigma \alpha m_i = 0$$

or

$$\Sigma \frac{Am_i}{EI} + \Sigma \alpha m_i = 0$$

The left-hand side equals 0.0014 or 1.4/1000, which is negligible, so that lhs. equals rhs. Hence compatibility may be assumed as achieved under the assumed ultimate

Table 27.1 *(lb and in. units)*

Section	M_0	M_1	M_2	e_{ce}	e_{cu}	ϕ_0 ($\times 10^{-3}$)	ϕ_1 ($\times 10^{-3}$)	$\frac{M_1+M_0}{\phi_1+\phi_0}$ ($\times 10^6$)	Max. θ_{pl} ($\times 10^{-3}$ radian)	Max. θ_{pr} ($\times 10^{-3}$ radian)
Mid-span	50,600	110,000	132,000	.002	.0038	0.120	1.160	126.0	0.1695 z_l	0.1697 z_r
Support	54,200	120,500	143,700	.002	.0039	0.130	1.150	137.0	0.1705 z_l	0.1705 z_r

Table 27.2 *Calculation of l_{pl}, l_{pr}, θ_{pl} and θ_{pr}*

	Midspan hinge	Central support hinge
z_l	54″	28.7″
z_r	25.3″	—
$l_{pl} = 0.5d\left(\frac{z_l}{d}\right)^{1/4}$	3.98″	3.56″
$l_{pr} = 0.5d\left(\frac{z_r}{d}\right)^{1/4}$	3.30″	—
Attainable θ_{pl}	$0.1695 \times 54 \times 10^{-3} \times \frac{122{,}800 - 110{,}000}{132{,}000 - 110{,}000}$ = 0.0053 rad. = α_1 in Fig. 27.5(c)	$0.1705 \times 28.7 \times 10^{-3} \times \frac{143{,}700 - 120{,}500}{143{,}700 - 120{,}500}$ = 0.0049 rad. = α_3 in Fig. 27.5(c)
Attainable θ_{pr}	$0.1695 \times 25.3 \times 10^{-3} \times \frac{122{,}800 - 110{,}000}{132{,}000 - 110{,}000}$ = 0.0024 rad. = α_2 in Fig. 27.5(c)	— —

moment distribution diagram, latter representing the actually mobilisable, i.e., 'attainable' moments at 'ultimate' (100% at the central support and only 93% of the absolute full value at the midspan sections).

Ultimate load W_u, applied at each load point, may now be calculated from the statical condition:

$$\left[\frac{W_u l}{4} + \frac{wl^2}{8}\right] - \left(\begin{array}{c}\text{fixed-end moment at}\\ \text{mid-span from ultimate BMD}\end{array}\right) = \left(\begin{array}{c}\text{actual attained ultimate}\\ \text{moment at mid-span}\end{array}\right)$$

so that, W_u = 7080 lb

EXAMPLE 2 The 3-span beam shown in Fig. 27.6.

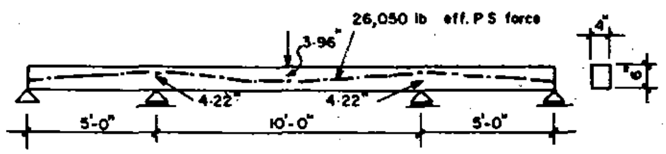


Fig. 27.6

From calculations at limits L_2, L_1 and L_0 for the loaded section (called *midspan* section hereafter) and the (intermediate) support sections, values are as in Table 27.3.

At the collapse of the beam, assume that at the midspan section 100% of its M_2 (i.e., 1.00 × 129,000 = 129,000 lb in.) is attained, while at each of the support sections only 92.4% of their M_2 (i.e., 0.924 × 161,840 = 149240 lb in.) is attained. (This again is the final of the various trials.)

Because of symmetry, only one m_i diagram need be worked with. Figure 27.7(b) shows the assumed ultimate moment distribution, in which, by marking off the respective M_1 ordinates in various hinge regions, the M elastic diagram is carved out. By further marking off the respective M_0 ordinates, the uncracked and cracked parts are marked. Figure 27.7(d) represents one m_i diagram. Figure 27.7(a) represents the prestress moment diagram.

The position of the points of contraflexure are worked out by geometry. Similarly, the areas A_1 to A_{10}, positions of their centroids, and the ordinates of the m_i diagram under the latter, are found by geometry.

Attainable θ_{pl} and θ_{pr} at the hinges are worked out in Table 27.4.

Knowing the appropriate EI values in each hinge region, values are substituted in the compatibility condition:

$$\Sigma\frac{Am_i}{EI} + \Sigma\alpha m_i = 0$$

Table 27.3 *(lb and in. units)*

Section	M_0	M_1	M_2	e_{ce}	e_{cu}	ϕ_0 (×10⁻³)	ϕ_{ps} (×10⁻³)	ϕ_1 (×10⁻³)	EI_{un} (×10⁶)	$\frac{M_1 - M_0}{\phi_1 - \phi_0 + \phi_{ps}}$ (×10⁶)	Max. θ_{pl} (×10⁻³ radian)	Max. θ_{pr} (×10⁻³ radian)
Mid-span	64,500	117,900	129,000	.002	.0058	.138	.020	1.290	467	45.6	.452 z_l	.452 zr
Supports	80,920	149,240	161,840	.0019	.0064	.173	.068	1.120	467	67.7	.432 z_e	.432 zr

Table 27.4 *Calculation of θ_{pl} and θ_{pr}*

	Midspan hinge	Support hinge
z_l	27.8″	60″
z_r	27.8″	32.2″
Attainable θ_{pl}	$0.452 \times 27.8 \times 10^{-3} \times \dfrac{129{,}000 - 117{,}900}{129{,}000 - 117{,}900}$	$0.432 \times 60 \times 10^{-3} \times \dfrac{149{,}240 - 149{,}240}{161{,}840 - 149{,}240}$
	= .0126 rad	= 0
	= α_3 in Fig. 27.7(c)	= α_1 in Fig. 27.7(c)
Attainable θ_{pr}	$0.452 \times 27.8 \times 10^{-3} \times \dfrac{129{,}000 - 117{,}900}{129{,}000 - 117{,}900}$	$0.432 \times 32.2 \times 10^{-3} \times \dfrac{149{,}240 - 149{,}240}{161{,}840 - 149{,}240}$
	= .0126 rad.	= 0
	= α_4 in Fig. 27.7(c)	= α_2 in Fig. 27.7(c)
	($\alpha_3 = \alpha_4$ by symmetry)	($\alpha_2 = \alpha_5$ by symmetry)

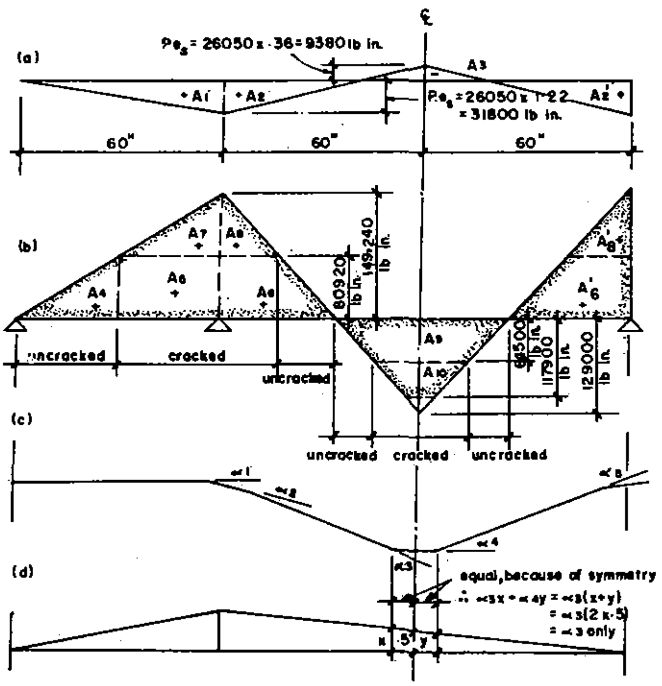


Fig. 27.7

and its lhs, algebraically, works out to :

$$\left\{\frac{A_1 m_i}{EI} + \left(\frac{A_2 m_i}{EI} + \frac{A_2' m_i}{EI}\right) + \frac{A_3 m_i}{EI} + \frac{A_4 m_i}{EI} + \frac{A_5 m_i}{EI} + \left(\frac{A_6 m_i}{EI} + \frac{A_6' m_i}{EI}\right) + \frac{A_7 m_i}{EI} + \left(\frac{A_8 m_i}{EI} + \frac{A_8' m_i}{EI}\right) + \frac{A_9 m_i}{EI} + \frac{A_{10} m_i}{EI} + (\alpha_3 x + \alpha_4 y)\right\} = 0.0003$$

which is negligible, so that lhs = rhs.

Hence compatibility may be assumed as achieved under the assumed ultimate moment distribution, latter indicating that only 92.4% of the absolute ultimate moment of resistance of the support sections is actually mobilisable at collapse of the structure. Ultimate load W_u, applied at the loaded section, may now be calculated, = 9150 lb.

Degree of accuracy A large number of 2 and 3 span prestressed concrete beams have been tested and also analysed.[2]

The following summary shows the degree of accuracy with which the proposed method predicted the ultimate load carrying capacity of these beams, the degree of accuracy being expressed as,

$$\frac{\text{Theoretically calculated ultimate load}}{\text{Experimentally measured ultimate load}}\,\% \pm \begin{array}{c}\text{Standard}\\ \text{deviation }\%\end{array}$$

Idealisation	2-span beams	3-span beams
Bilinear	89.6% ± 7.9%	97.4% ± 5.7%
Trilinear	92.6% ± 5.9%	99.4% ± 7.6%

These results indicate that the degree of error involved, in general, is smaller in 3-span beams than in 2-span beams. This implies that more redistribution occurs in 2-span beams than can be predicted.

27.4 EFFECT OF TENDON TRANSFORMATION ON ULTIMATE STRENGTH

Contrary to the general belief, held by some, it has been shown[2] that linear transformation of cable profiles in continuous prestressed concrete beams can, in some cases, lead to reduction in the ultimate load carrying capacity of the beam. This is possible when the transformation results in bringing the tendon close to the compression edge at the first forming hinge section, thereby reducing the effective depth. As the steel area remains the same, the proportion of 'steel-area' to 'effective concrete area' increases, making the section over reinforced, which in turn reduces the rotation capacity of the hinge. This can lead to a premature collapse of the hinge, resulting in an early failure of the beam. See Ch. 28 for details.

However, it has been shown[2] that the ultimate strength may not be seriously affected so long as it is ensured that linear transformation does not reduce the effective depth at any critical section to less than a third of the overall depth of section.

Concluding Remarks

It is essential to investigate the truly attainable (mobilisable) ultimate bending moment distribution in an elastically designed structure in order to know its true ultimate load carrying capacity and the truly mobilisable moments of resistance (as against their 'absolute' values) at various critical sections.

The suggested method, based on compatibility of deformations, shows significant accuracy with which the suggested idealisations predict the actual measured ultimate load of 2- and 3-span continuous prestressed beams.

Though it is preferable to adopt a trilinear idealisation in all prestressed concrete cases, the adoption of a bilinear one does not lead to any significant degree of error.

NOTE A similar procedure can be applied to reinforced concrete.

27.5 IMPORTANCE OF WORKMAN-LIKE 'DETAILING'

As explained, at collapse-load condition, it is quite possible due to inability of certain hinges to continue to rotate and due to obligation to compatibility of deformations, that full redistribution of moments may not take place, i.e., all necessary hinges may not get the chance to fully mature. Hence, all the critical sections may not attain moment values equal to their full ultimate moment capacities. Either this should be investigated* and it should be ensured that these actually attainable moments at collapse stage (not the full ultimate moment capacities) are not less than the factored applied moments at each critical section, or at least very thorough attention should be given to the detailing of section dimensions and amount and flow of reinforcement and prestressing force, guarding against any abrupt changes, cut-offs, cut-outs, etc., that can severely impair the ultimate rotational capacity of a hinging region which can lead to incomplete redistribution and early ultimate failure.

* This can be done by various methods given for instance by Guyon, Macchi, Cooke, Raina (see method outlined in section 27.4), etc., but many designers may not be up-to-date on this. Hence, the need for a resolute and intensely workman-like detailing cannot be over emphasised.

27.6 ESTIMATION OF VARIOUS LIMITING MOMENTS AND STRESS-STRAIN RELATIONSHIP

1. The absolute value of the ultimate moment of resistance (Limit L_2) of a prestressed concrete section.
2. The moment of resistance of a prestressed concrete section at elastic limit (Limit L_1)
3. The moment of resistance of a prestressed concrete section at cracking limit (Limit L_0)
4. The stress-strain blocks for concrete in flexural compression.

1. Calculation at limit L_2 for finding M_2, e_{cu} and $n_u d$ at the 'collapse limit state' (see also Ch. 23 for load-factored type limit state allowed in the regular codes of practice).

(i) Assume a suitable first-trial $n_u d$.

(ii) Estimate e_{cu} and f''_c from the empirical formulae (a) or (b), and (c) or (d), respectively

(a) $$e_{cu} = .0015\left(1.45 + 1.5\rho'' + \frac{0.7 - .1\rho''}{n_u} - f'_e \times 10^{-4}\right) \text{ but } \not> .01$$

(b) $$e_{cu} = .0042\left(\frac{d}{D} + 0.4p''\right)\frac{A_c \times 10^{-2}}{A_{ps} + 0.05A_{ms}} \text{ but } \not> 0.01 \text{ and } \not< 0.0035$$

(c) $f''_c = f'_c(.8 + .1/n_u)$, but $\not> f'_c$ where f'_c may be taken as .85 f_{cu}

(d) $f''_c = 0.85 f_{cu}$

Formulae (a) and (c) are after Baker and Amarkone[3] while formulae (b) and (d) are after author[2] of this book. It may be seen that formulae (a) and (c) involve n_u and are, therefore, dependent on the trial and error calculations, while the proposed formulae have the advantage of involving only known parameters. It has been shown[2] that the proposed formulae are just as accurate,

$$\text{average } \frac{(b)}{(a)} = 1.04 \pm \text{ standard deviation of } 0.103$$

$$\text{average } \frac{(d)}{(c)} = 1.002 \pm \text{ standard deviation } 0.012.$$

(iii) Draw the strain-block in relation to the effective section, work out the location of compressive strain of 0.002, and draw the flexural compressive stress-block [see item 4 ahead].

(iv) Find out total tension T and total compression C in the effective section: if $T = C$, trial alright, otherwise, repeat assuming a fresth suitable value for $n_u d$ until $T = C$ finally.

(v) Calculate M_2, the (absolute) ultimate moment of resistance of the section, by taking moments of T and C forces about any fibre-level in the section.

2. Calculation at limit L_1 (for finding $M_1 \cdot e_{ce}$ and $n_e \cdot d$)

(a) (i) Assume a suitable first trail $n_e \cdot d$.

(ii) Assume total strain in the extreme tendon (or at c.g. of tendons, if bunched together) equal to its 0.1% proof strain value, so that $e_{cse} = 0.1\%$ proof strain value $-e_p - e_{cp}$), then if e_{ce} works out to 0.002 or less, then draw the corresponding stress-block and find out T and C. If $T = C$, then calculate M_1 by taking their moments about any fibre level in the section. If $T \neq C$, then may be by merely trying a fresh value of $n_e d$ (with e_{ce} still working out to less than or equal to 0.002), T may equal C, in which case the section is 'under reinforced' unless $e_{ce} = 0.002$ in which case reinforced' unless $e_{ce} = 0.002$ in which case the section is 'balanced' and M_1 can be found.

If instead it is impossible to get $T = C$ in the above trial without e_{ce} exceeding 0.002, it means that the section is 'over reinforced', in which case proceed as follows:

(b) (i) Assume a revised suitable first trial $n_e \cdot d$

(ii) Assume $e_{ce} = 0.002$, work out e_{cse} and check that the total strain $(e_p + e_{cp} + e_{cse})$ in the extreme tendon (or at cg of tendons, if bunched together) is less than 0.1% proof strain value. Draw the corresponding stress-block and work out T and C. If $T \neq C$, repeat with a fresh value for $n_e d$ until $T = C$ with $(e_p + e_{cp} + e_{cse})$ still $< 0.1\%$ proof strain value. Then calculate M_1 as explained above.

3. Calculation at limit L_0 ***(for finding*** M_0***)***

Where parasitic moments are absent, M_0 may be calculated from the limiting condition that cracking will commence when the resultant tensile stress equals the modulus of rupture (f_{m0d}):

$$\frac{M_0 + M_{DI}}{Z} - \left(\frac{P}{A} + \frac{P \cdot e_s}{Z}\right) = f_{m0d}$$

Where parasitic moments are present, by virtue of the fact that they start diminishing right from the instant of microcracking so that their amount actually present at the time of visible cracking is uncertain, the term Pe_s in the above formula (which now represents primary as well as secondary moments) becomes indeterminate. Hence, in the author's opinion, in such cases, M_0 may not be evaluated from the above formula. On the basis of test results, it is suggested in such cases to assume $M_0 = 0.55\ M_2$ if untensioned steel is not considered in calculating M_2 and $M_0 = 0.50M_2$ if untensioned steel is considered.

4. Stress strain blocks for concrete in flexural compression

These are assumed as shown in Fig. 27.8. It is convenient to divide the effective concrete section into rectangles, so that when taking moments of their compressive forces, the lever arms are the same as of their stress-blocks.

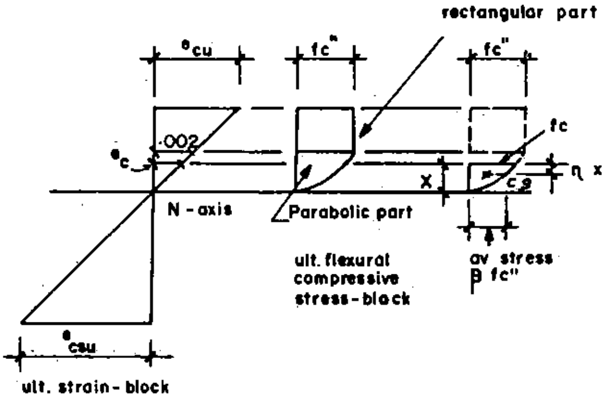


Fig. 27.8

For the rectangular part of the stress-block, or part thereof, 'average stress' equals $f''c$ and 'location of its centroid' lies at the middle of its depth.

For the parabolic part of the stress-block, or part thereof (i.e., $e_c \leq 0.002$), the 'average stress equals $\beta f_c''$ and 'location of its centroid below its top equals $\eta \cdot x$. The values of the coefficients β and η depend on the value of concrete strain e_c at the level of the top of the considered part of the parabolic stress-block. It may be shown[2] that,

$$\left.\begin{aligned} \beta &= \frac{e_c}{0.002}\left(1 - \frac{e_c}{0.006}\right) \\ \text{and } \eta &= \frac{4 - (e_c/.002)}{12[1 - (e_c/0.006)]} \end{aligned}\right\} \text{ for } 0 < e_c \leq 0.002$$

REFERENCES

1. Raina, V.K., "Estimation of Ultimate Load Carrying Capacity of Continuous Prestressed Concrete Beams in Flexure" *Consulting Engineer*, London, July, 1967.
2. Raina, V.K., "Ultimate Moment Distribution in Continuous Prestressed Concrete Beams", PhD Thesis, University of London, 1966.
3. Baker, A.L.L., and A.M.N. Amarkone, "Inelastic Hyperstatical Frames—Analysis and Application of the International Correlated Tests". CEB 9th Full Meeting, Ankara, September, 1964.

CHAPTER 28

Effect of Tendon Transformation on Ultimate Strength of Continuous Prestressed Concrete Beams

It is common knowledge in prestressed concrete that even if the position of the tendon in a continuous beam is shifted at any or all the intermediate supports (by any amount, vertically, up or down), he resultant line of thrust remains unchanged (i.e., the total prestress moment, etc. remains unaltered) so long as the position of the cable at the end support sections is unchanged and the new profile of the tendon in each span is obtained by pure 'linear transformation' of its parent profile. This is established even mathematically (see Linear Transformation in Ch. 26 of this book), but the fact remains that this principle holds so long as the beam is behaving purely elastically. After cracking sets in, and if the load is still increased, plastic rotations are created in the beam in its hinging regions, moments get redistributed, flexural rigidity (EI) of cracked sections reduces, and as a consequence, the prestress 'parasitic' moments begin to reduce until they disappear completely when all the n hinges are formed and fully developed (n being the number of statical indeterminacies of the structure). Should this full redistribution occur, then obviously the load taking capacity of the structure would be different in the two profile cases since the prestress primary moments (which are then the prestress total moments, parasitic ones having disappeared) are different for the two profiles. At least theoretically this could be the case, provided full redistribution takes place. (The latter is generally not true and hence some parasitic moments still exist at the instant of collapse, the hence the above stated principle of linear transformation of tendon profiles may continue to hold, at least nearly.)

An attempt has been made here to show theoretically that the ultimate carrying capacity of a continuous prestressed concrete beam does not remain unaffected with tendon transformation. Basing the analysis on the assumptions that, at ultimate of the beam, each critical section attains its full ultimate moment of resistance and that the sections are not over reinforced, a formula has been derived for the percentage change in the ultimate load carrying capacity caused by linear transformation of the parent cable. The derivation is based purely on an equilibrium concept. However, the above-mentioned assumptions may not hold good in some cases, which limits the applicability of this equilibrium method formula.

28.1 ANALYTICAL DERIVATION OF THE PROPOSED FORMULA

First, an expression is derived for the ultimate moment of resistance of a prestressed concrete section. The term representing force in the tendon is replaced by a mathematical equation expressing *force* in terms of *strain* in the tendon. Later, the variable part e_{csu} (in the strain $e_p + e_{cp} + e_{csu}$), called e in short, is expressed by a transcendental equation linking it with the location of the tendon in the section in order to make the cable force sensitive to linear transformation. Assuming the each critical section attains its full ultimate moment of resistance, an expression is derived for the maximum free BM that a span of a continuous beam can carry with parent cable as well as with transformed cable in terms of ultimate moments of resistance of the sections concerned. Formulae derived earlier for the latter are then substituted and the percentage change in the 'maximum free BMs' in the two cases worked out. Hence a percentage change in the ultimate loads that can be carried in the two cases, is obtained.

Though the analysis is carried out assuming a rectangular section of the beam, it is still applicable in the case of other sections so long as the effective section at ultimate is rectangular.

Derivation

Consider an intermediate span of any prestressed concrete continuous beam, as shown in Fig. 28.1. The cable profile in the chainline represents the parent cable, while that in the full-line represents a linearly transformed cable — transformations being $+\delta_1$ at (left hand support) section 1 and δ_3 at (right hand support) section 3. Consequently, the effective depths at sections 1 and 3 increase, while that at an intermediate section 2, distant βl from 1, decreases by δ_2 such that,

$$\delta_2 = (1 - \beta)\delta_1 + \beta\delta_3.$$

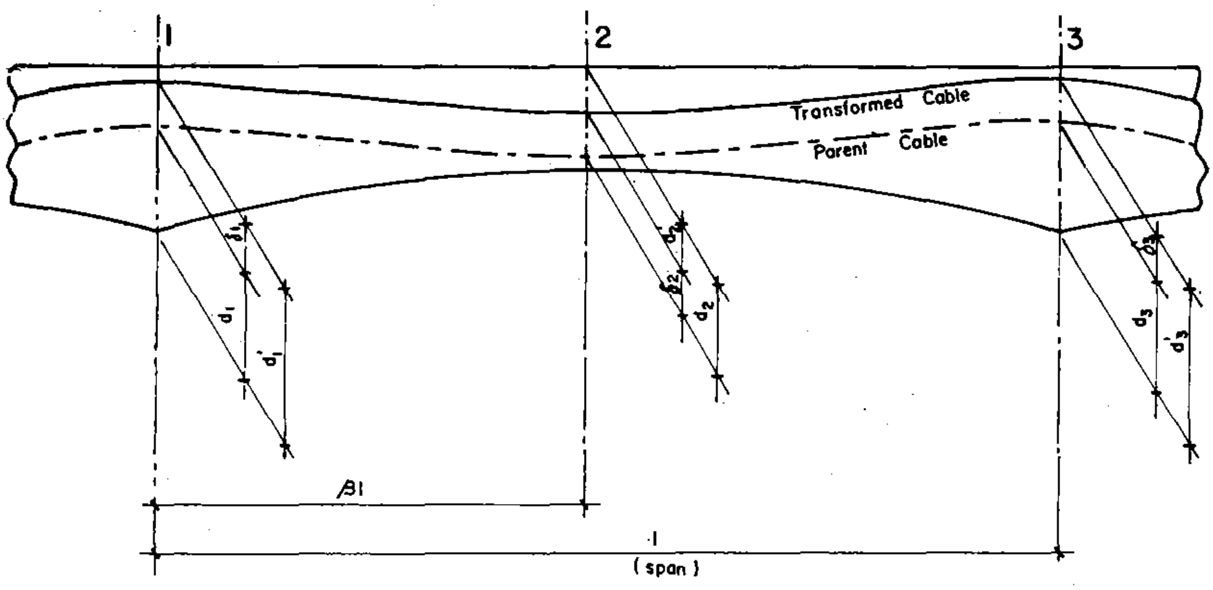


Fig. 28.1

Let

B = width of concrete section

A_s = area of cable steel in the section

A = area of untensioned steel in the section

D = eff. depth from compression edge to cg of untensioned steel

α = ratio of average flexural compressive stress in concrete at ultimate of f_c'', where $f_c'' = 0.85 f_{cu}$ (f_{cu} = 28 days cube strength of concrete)

γ = ratio of depth up to centre of compression at ultimate of depth up to neutral axis at ultimate. (NB : α and γ refer to 'total' flexural compression stress block at ultimate)

t_r = stress in cable steel in the section at ultimate of the section

t = stress in untensioned steel in the section at ultimate of the section

βl = distance of 'maximum span-moment section' from (left hand support) section 1

l = span

d_1, d_2, d_3 = eff. depths to tendon under parent profile case, at sections 1, 2 and 3

d_1', d_2', d_3' = eff. depths to tendon under transformed profile case, at sections 1, 2 and 3

M_1, M_2, M_3 = ultimate moments of resistance under parent profile case of sections 1, 2 and 3

M_1', M_2', M_3' = ultimate moments of resistance under transformed profile case of sections 1, 2 and 3

$e_{cu1}, e_{cu2}, e_{cu3}$ = e_{cu} values under parent profile case at sections 1, 2 and 3

$e_{cu1}', e_{cu2}', e_{cu3}'$ = e_{cu} values under transformed profile case at sections 1, 2 and 3

$e_1, e_2, e_3,$ = e_{csu} (or e) values under parent profile case at sections 1, 2 and 3

$e_1', e_2', e_3',$ = e_{csu} (or e) values under transformed profile case at sections 1, 2 and 3

d_0 = overall depth (general symbol)

NOTE See Ch. 27 for explanation of notation e_{cu}, e_p, e_{cp}, e_{csu}, f_c'', d, p'', A_c, A_{ps}, A_{ms}, $n_u d$, C and T.

$a = d_2/d_1$, $c = d_3/d_1$, $r = \delta_1/d_1$, $q = \delta_3/\delta_1$, $p = \delta_2/\delta_1$

Evidently, $\delta_1 = d_1' - d_1$, $\delta_2 = d_2 - d_2'$, $\delta_3 = d_3' - d_3$,

$$\left.\begin{aligned} \text{and} \quad & d_1' = (1+r)d_1 \\ & d_2' = (a - pr)d_1 \\ \text{and} \quad & d_3' = (c + qr)d_1 \end{aligned}\right\} \qquad (A)$$

Values of e_{cu} and f_c'' may be found from the following proposed empirical formulae,

$$e_{cu} = 0.0042(d/d_0 + 0.4p'')\frac{A_c \times 10^{-2}}{A_{ps} + 0.05A_{m.s.}}$$

but $\nless 0.0035$ and $\ngtr 0.001$

$$f_c'' = 0.85 f_{cu}$$

These formulae have an advantage over those recommended by Baker and Amarkone, in that they are independent of trial and error calculations. Validity of the former in comparison with the latter has already been established by the author and some details and given in Ch. 27.

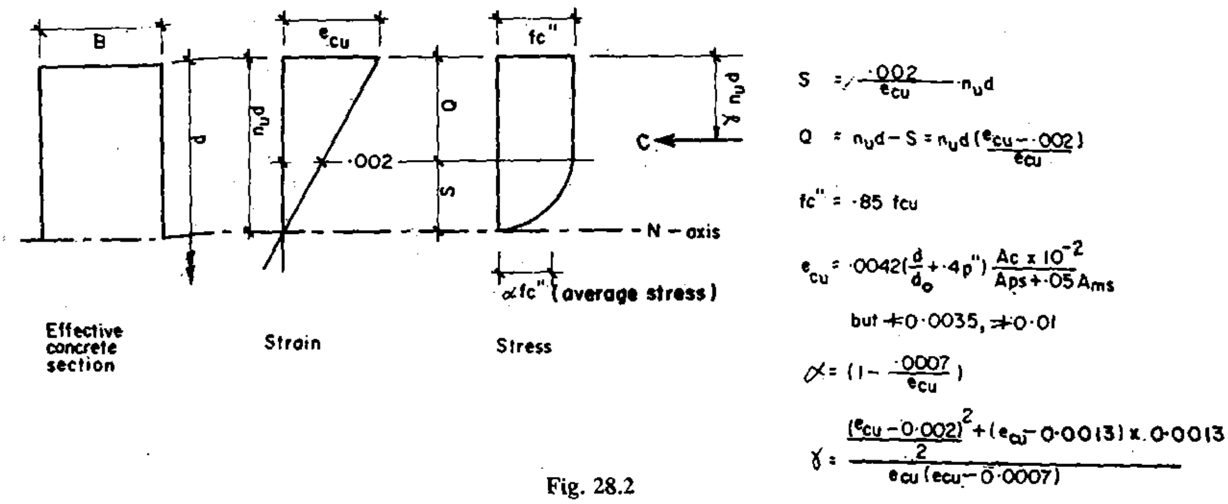


Fig. 28.2

The stress-strain block for concrete under ultimate flexural compression is assumed to be as shown in Fig. 28.2. Value of α may be obtained by equating the areas of the 'average' (equivalent) stress block and the assumed stress block:

$$n_u d \cdot \alpha f_c'' = (Qf_c'' + 2/3Sf_c'')$$

substituting for Q and S (Fig. 28.2) and simplifying, we get:

$$\alpha = \left(1 - \frac{0.0007}{e_{cu}}\right)$$

Value of γ may be obtained by equating the moments of areas of the 'average' (equivalent) stress block and the assumed stress block, say, about the compression edge:

$$(Qf_c'' + 2/3Sf_c'')\gamma n_u d = (Qf_c'')Q/2 + (2/3Sf_c'')(Q + 3/8S)$$

substituting for Q and S and simplifying, we get:

$$\gamma = \frac{\dfrac{(e_{cu} - 0.002)^2}{2} + (e_{cu} - 0.0013)0.0013}{e_{cu}(e_{cu} - 0.007)}$$

From the internal equilibrium condition $C = T$, at ultimate of a section,

$$(Bn_u d)\alpha f_c'' = A_s t_r + \Sigma At$$

$$\therefore \quad n_u = \frac{A_s t_r + \Sigma At}{B\alpha f_c'' d} \quad (28.1)$$

Taking moment of internal forces, say about centre of compression, we get an expression for ultimate moment of resistance of the section, viz.,

$$M_u = A_s t_r(d - \gamma n_u d) + \Sigma At(D - \gamma n_u d) \quad (28.2$$

At the ultimate of the section, the untensioned stee invariably yields and therefore carries its yield force. Th shift in the position of ultimate centre of compression fron that in the parent cable case to that in the transformed cable case may be assumed to be small in comparison to the lever arm of the untensioned steel whose contribution to M_u and T is small enough. Therefore, the contribution of the untensioned steel to the 'change' in M_u under the two cases may be neglected. Accordingly, for the present purpose, the Eqs. (28.2) and (28.1) may be simply written as:

$$M_u = A_s t_r \cdot (d - \gamma n_u d) \quad (28.3)$$

and

$$n_u = \frac{A_s t_r}{B\alpha f_c'' d} \quad (28.4)$$

substituting Eq. (28.4) in (28.3) we get:

$$M_u = (A_s t_r)d - (A_s t_r)^2 \frac{\gamma}{B\alpha f_c''} \quad (28.5)$$

$(A_s t_r)$, the tensile force in the tendons, depends on total strain $(e_p + e_{cp} + e_{csu})$ in them, of which e_{csu} depends on the location of the tendons in the section.

The load-strain curve of a high tensile steel tendon resembles a half range sine wave, and may be expressed in the form:

$$y = K\psi(x) \quad (28.6)$$

making use of Fourier series. Since usually the normal prestressed concrete continuous beams are not over-reinforced, the total strain in cable steel at the ultimate of a hinge section may be assumed to be about 0.0095.

For one 0.276″ dia. HTS wire (that was used in the continuous beams tested by the author), a reasonably accurate relationship between the load y and the strain x, for strain values of about 0.0095, may be expressed as follows:

$$y = 14400 \sin \frac{\pi x}{0.03} \text{ (lb units)} \qquad (28.7)$$

where 0.03 represents its ultimate strain (the half-range). Hence in general the load-strain relation for tendons may be written as:

$$y = b \sin(K_3 x) \qquad (28.8)$$

where y = load of force $(A_s f_r)$
x = total strain in the tendon
b and K_3 = coefficients, depending on type of HTS, former also depending on the number of tendons.
y and b having same units.
For the particular 0.276″ dia. HTS wires, therefore:
$b = (14,400N)$ lbs, N being number of wires
$K_3 = \pi/0.03$
Substituting into Eq. (28.5), we get:

$$M_u = yd - y^2 \frac{\gamma}{B\alpha f_c''}$$
$$= bd \sin(K_3 x) - \frac{b^2\gamma}{B\alpha f_c''} \sin^2(K_3 x) \qquad (28.9)$$

But $x = e_p + e_{cp} + e_{csu}$ (total strain in tendon), calling $(e_p + e_{cp})$ as K_1, and e_{csu}, as e, we have:

$$x = K_1 + e$$

$\therefore\ M_u = bd \sin\,(K_3K_1 + K_3 e) - \frac{b^2\gamma}{B\alpha f_c''} \sin^2(K_3K_1 + K_3 e)$ calling K_3K_1 as K_2 and $\frac{b^2\gamma}{B\alpha f_c''}$ as K_4, we have:

$$M_u = bd \sin(K_2 + K_3 e) - K_4 \sin^2(K_2 + K_3 e) \qquad (28.10)$$

Thus:

$$\left.\begin{aligned}
M_1 &= bd_1 \sin(K_2 + K_3 e_1) - K_{4_1} \sin^2(K_2 + K_3 e_1)\\
M_2 &= bd_2 \sin(K_2 + K_3 e_2) - K_{4_2} \sin^2(K_2 + K_3 e_2)\\
M_3 &= bd_3 \sin(K_2 + K_3 e_3) - K_{4_3} \sin^2(K_2 + K_3 e_3)\\
M_1' &= bd_1' \sin(K_2 + K_3 e_1') - K_{4_1}' \sin^2(K_2 + K_3 e_1')\\
M_2' &= bd_2' \sin(K_2 + K_3 e_2') - K_{4_2}' \sin^2(K_2 + K_3 e_2')\\
M_3' &= bd_3' \sin(K_2 + K_3 e_3') - K_{4_3}' \sin^2(K_2 + K_3 e_3')
\end{aligned}\right\} \qquad (28.10a)$$

From strain-compatibility across a section, based on linear strain distribution from compression-edge to tendon, we have,

$$e = e_{cu} \frac{d - n_u d}{n_u d}$$
$$= e_{cu} \left(\frac{1}{n_u} - 1\right)$$

Substituting Eq. (28.8) into (28.4) and then this into the above, we get:

$$e = e_{cu} \left(\frac{B\alpha f_c'' d}{b \sin(K_3 x)} - 1\right)$$

Calling $\frac{B\alpha f_c''}{b}$ as H and substituting for x as before, we get:

$$e = e_{cu} \left(\frac{Hd}{\sin(K_3K_1 + K_3 e)} - 1\right)$$

and $\because\ K_3K_1 = K_2$

$$\therefore\ e = e_{cu} \left(\frac{Hd}{\sin(K_2 + K_3 e)} - 1\right)$$

so that $\sin(K_2 + K_3 e) = \dfrac{Hde_{cu}}{(e_{cu} + e)}$ (28.11)

which is a *transcendental equation*.
Thus

$$\left.\begin{aligned}
\sin(K_2 + K_3 e_1) &= \frac{H_1 d_1 e_{cu1}}{(e_{cu1} + e_1)}\\
\sin(K_2 + K_3 e_2) &= \frac{H_2 d_2 e_{cu2}}{(e_{cu2} + e_2)}\\
\sin(K_2 + K_3 e_3) &= \frac{H_3 d_3 e_{cu3}}{(e_{cu3} + e_3)}\\
\sin(K_2 + K_3 e_1') &= \frac{H_1' d_1' e_{cu1}'}{(e_{cu1}' + e_1')}\\
\sin(K_2 + K_3 e_2') &= \frac{H_2' d_2' e_{cu2}'}{(e_{cu2}' + e_2')}\\
\sin(K_2 + K_3 e_3') &= \frac{H_3' d_3' e_{cu3}'}{(e_{cu3}' + e_3')}
\end{aligned}\right\} \qquad (28.11a)$$

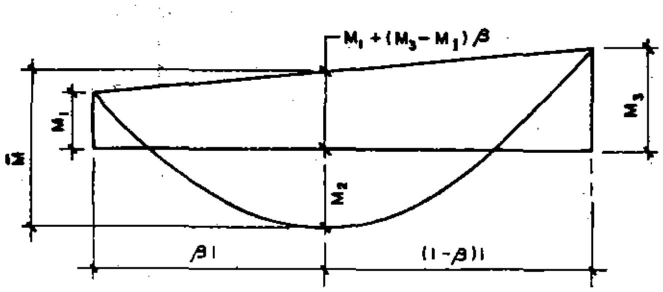
Assuming full redistribution of bending-moments at ultimate of the structure so that each critical section attains a BM equal to its M_u, and referring to Fig. 28.3, we have:
In parent cable case

$$\bar{M} = (1 - \beta)M_1 + M_2 + \beta M_3 \qquad (28.12)$$

In transformed cable case

$$\bar{M}' = (1 - \beta)M_1' + M_2' + \beta M_3' \qquad (28.13)$$

where $\bar{M}$ = maximum free BM due to total loading on span in parent cable case.



Moment diagram at failure
(parent cable case)

Fig. 28.3

$\bar{M}'$ = maximum free BM due to total loading on span in transformed cable case.

The 'change' in the maximum free BMs that the span is, therefore, able to carry, relative to parent cable case, is:

$$\frac{\bar{M}-\bar{M}'}{\bar{M}} = 1 - \frac{(1-\beta)M_1' + M_2' + \beta M_3'}{(1-\beta)M_1 + M_2 + \beta M_3}$$

$$= 1 - X \qquad (28.14)$$

where
$$X = \frac{(1-\beta)M_1' + M_2' + \beta M_3'}{(1-\beta)M_1 + M_2 + \beta M_3} \qquad (28.15)$$

Substituting into Eq. (28.15) the Eq. (28.10a) and then into this new equation substituting Eq. (28.11a) and then into this still new equation substituting equation (A), and noting in general that

$K_4 H \left(= \dfrac{b^2\gamma}{B\alpha f_c''} \times \dfrac{B\alpha f_c''}{b}\right)$ is simply equal to $b\gamma$, we get after a very laborious simplification:

then
$$\bar{M} = \bar{W}(1-\beta)\beta l + \frac{wl^2}{2}\beta(1-\beta) \qquad (28.17)$$

and
$$\bar{M}' = \bar{W}'(1-\beta)\beta l + \frac{wl^2}{2}\beta(1-\beta) \qquad (28.18)$$

so that
$$\frac{\bar{M}-\bar{M}'}{\bar{M}} = \frac{(1-\beta)\beta l(\bar{W}-\bar{W}')}{\bar{W}(1-\beta)\beta l + \dfrac{wl^2}{2}\beta(1-\beta)} \qquad (28.19)$$

Special case: Where the term $\dfrac{wl^2}{2}\beta(1-\beta)$ (own weight BM) is relatively negligible:

$$\frac{\bar{M}-\bar{M}'}{\bar{M}'} = \frac{\bar{W}-\bar{W}'}{\bar{W}} \qquad (28.20)$$

Connecting Eqs. (28.20) and (28.14), we get the percentage change in ultimate load carrying capacity of the span, below that of the parent-profile case viz. $\left(\dfrac{\bar{W}-\bar{W}'}{\bar{W}}\right)\cdot 100$, equal to $(1-X)\cdot 100$ where X is given by Eq. (28.16).

Procedure Thus the procedure for evaluating the percentage change in ultimate load carrying capacity, from this equilibrium method formula, may be summarised as follows:

Step 1 Find the load-strain $(y : x)$ relationship for a tendon in the from: $y = b\sin\dfrac{\pi x}{U}$, where b is as explained earlier, and U = ultimate tensile strain.

$$X = \frac{H_1'(1-\beta)\left[\dfrac{(1+r)^2(e_{cu1}')}{(e_{cu1}'+e_1')} - \dfrac{\gamma_1' e_{cu1}'^2(1+r)^2}{(e_{cu1}'+e_1')^2}\right] + H_2'\left[\dfrac{(a-pr)^2(e_{cu2}')}{(e_{cu2}'+e_2')} - \dfrac{\gamma_2' e_{cu2}'^2(a-pr)^2}{(e_{cu2}'+e_2')^2}\right] + H_3'\beta\left[\dfrac{(c+qr)^2 e_{cu3}'}{(e_{cu3}'+e_3')} - \dfrac{\gamma_3' e_{cu3}'^2(c+qr)^2}{(e_{cu3}'+e_3')^2}\right]}{H_1(1-\beta)\left[\dfrac{(e_{cu1})}{(e_{cu1}+e_1)} - \dfrac{\gamma_1 e_{cu1}^2}{(e_{cu1}+e_1)^2}\right] + H_2\left[\dfrac{a^2 e_{cu2}}{(e_{cu2}+e_2} - \dfrac{\gamma_2 e_{cu2}^2 a^2}{(e_{cu2}+e_2)^2}\right] + H_3\beta\left[\dfrac{c^2 e_{cu3}}{e_{cu3}+e_3} - \dfrac{\gamma_3 e_{cu3}^2 c^2}{(e_{cu3}+e_3)^2}\right]} \qquad (28.16)$$

NOTE In the above analysis b has been assumed constant at all sections.

However, should it vary for different sections, then its account can be taken in Eq. (28.10a) and a modified formula for X worked out.

Now if

w = equivalent uniformly distributed own weight of the beam per unit length

$\bar{W}$ = applied (equivalent point) load at section 2, corresponding to $\bar{M}$.

and $\bar{W}'$ = applied (equivalent point) load at section 2, corresponding to $\bar{M}'$

Step 2 Estimate e_{cu} and f_c'' at each critical section, in the parent cable case as well as in the transformed cable case, using the proposed empirical formulae:

$$e_{cu} = 0.0042(d/d_0 + 0.4p'')\frac{A_c \times 10^{-2}}{A_{ps} + 0.05A_{ms}},$$

but $\nless .0035$ and $\ngtr .01$

and $f_c'' = 0.85 f_{cu}$

Step 3 Estimate α, γ and H at each critical section, in the parent cable case as well as in the transformed cable case, using the following formulae:

$\alpha = (1 - 0.0007/e_{cu})$

$$\gamma = \frac{(e_{cu} - 0.002)^2/2 + (e_{cu} - 0.0013)0.0013}{e_{cu}(e_{cu} - 0.0007)}$$

and $H = B\alpha f_c''/b(b = N$ times 'b for one tendon' found in Step 1 above, N being the number of tendons at the section).

Step 4 Plot 'Hd versus e' relationship from

$$Hd = (1 + e/e_{cu})\sin(K_2 + K_3 e) \quad \text{(Eq. 28.11)}$$

where $K_2 = \frac{\pi}{U}(e_p + e_{cp})$

[take $e_{cp} = 0.0002$, approximately]

$$K_3 = \frac{\pi}{U}$$

and e_{cu} = average of the e_{cu} values found in step 2 above (an approximation)

Step 5 (i) Referring to the left hand support section, the right hand support section and the maximum-bending-moment span section by suffixes 1, 3 and 2 in the parent cable case, respectively, and by suffixes $1'$, $3'$ and $2'$ in the transformed cable case, respectively, then H_1, H_2, H_3, H_1', H_2' and H_3' stand known from step 3. Read off e from the Hd versus e plot, corresponding to $Hd = H_1d_1, H_2d_2, \ldots$ and $H_3'd_3'$. Thus e_1, e_2, e_3, e_1', e_2' and e_3' are known.

(ii) Find the value of the terms $(1 + r)^2$, $(a - pr)^2$, $(c + qr)^2$, a^2 and c^2 from the relations:
$(1 + r) = d_1'/d_1$, $(a - pr) = d_2'/d_1$,
$(c + qr) = d_3'/d_1$
$a = d_2/d_1$, and $c = d_3/d_1$ respectively

Step 6 Evaluate X from Eq. (28.16).

Step 7 Then $(1 - X)\cdot 100$ represents the percentage change in "ultimate load carrying capacity below that of the parent cable case". [If $(1 - X)\cdot 100$ turns out to be positive, it means a decrease in carrying capacity compared with parent cable case.] This is so if self weight is negligible. Otherwise $\left(\frac{\bar{W} - \bar{W}'}{\bar{W}}\right)$ may be evaluated from Eq. (28.19) noting that in it $(1 - X)$ represents $\left(\frac{\bar{M} - \bar{M}'}{\bar{M}}\right)$ (Eq. 28.14).

It has to be noted that in case of an ordinary two-span beam, only two terms in the numerator and in the denominator in the above expression for X exist. This is so because, for instance if the span under consideration be the right hand one, then $M_3 = 0$ in Fig. 28.3, and so, therefore in Eqs. (28.12) and (28.13) too. Similarly if the span under consideration be the left hand one, then $M_1 = 0$ in Fig. 28.3, and so, therefore, in Eqs. (28.12) and (28.13) too.

CHAPTER 29

Simplified Method for the Analysis of Torsional Moment as an Effect of a Horizontally Curved Multispan Continuous Deck*

Synopsis

A simplified method of calculation for torsional effects on a bridge structure is presented.[1] Analysis and calculations are shown for torsional moments as an effect of loading on a horizontally curved, multispan, continuous bridge. Special attention is given to the analysis for torsional moment resulting from applied longitudinal bending moments (which include parasitic† prestressing moments but not primary prestress moments). A method of application is given in a step-by-step form with full interpretation of the problem, and a numerical example is solved in the same step-by-step form to illustrate the method.[1]

29.1 INTRODUCTION

Structures for highway interchanges of the type shown in Fig. 29.1 (see art plate) are usually curved in plan. The superelevation often varies, and in most cases the vertical alignment is curved. An important requirement for such structures is that the construction depth of the deck should be as shallow as possible. A number of roadways cross one another at points in close proximity and the vertical distances between them must include the construction depth for each structure. Deep deck systems result in increased excavation or fill and increase the overall size of the interchange.

Further requirements are that piers should be as few and as small as possible, and that they should be placed to suit the roadways beneath the structure rather than to conform to any simple structural requirements.

Cast-in-place post-tensioned concrete structures can be made to meet all of the foregoing requirements very economically and with results that are aesthetically pleasing. To develop the full potential of the material, however, it is necessary to use a more sophisticated structural configuration and to carry out a more complete analysis. The concept involved is one of *total design*, since advantage is taken of the latent structural strength which is not normally utilized.

Cross-Sections

Figure 29.2 shows a number of different types of cross-sections which have been used. The sections using round voids are cast in one operation whereas sections similar to Fig. 29.2(d) usually require construction joints and are more liable to shrinkage cracks. This factor and the more complex formwork go against the use of box sections except for long spans.

Section (a) has been used for spans from 125–150 ft (38–46 m). Sections (b) and (c) have been found useful for a wide range of spans from 70–150 ft (21–46 m), and section (d) is suitable for spans in excess of 150 ft. The width of the cross-sections does not often exceed 50 ft (15 m). Roadway widths greater than 50 ft are usually carried on two or more separate structures.

Structural Capacity

It is common practice to utilize sections such as those described only as a means of spanning in the longitudinal direction. Some of the additional structural capacities may, however, be identified as follows:

(a) *Transverse flexural strength:* All of the sections shown have the ability to resist transverse bending due to vertical loads. With the section depths and widths commonly used, a single column at the center of the section produces transverse bending moments which are within the capacity of the deck section using normal reinforcement.

(b) *Lateral flexural strength:* All sections are able to resist very large bending moments due to lateral forces. For bridges of normal length it is usually not necessary to resist lateral forces at piers. The abutments can be designed to provide adequate lateral restraint at negligible expense.

(c) *Torsional strength about longitudinal axis:* The sections shown in Fig. 29.2 have a very large resistance to twisting. Except for very long bridges, it is sufficient to resist twisting only at the abutments. Wide piers, piers

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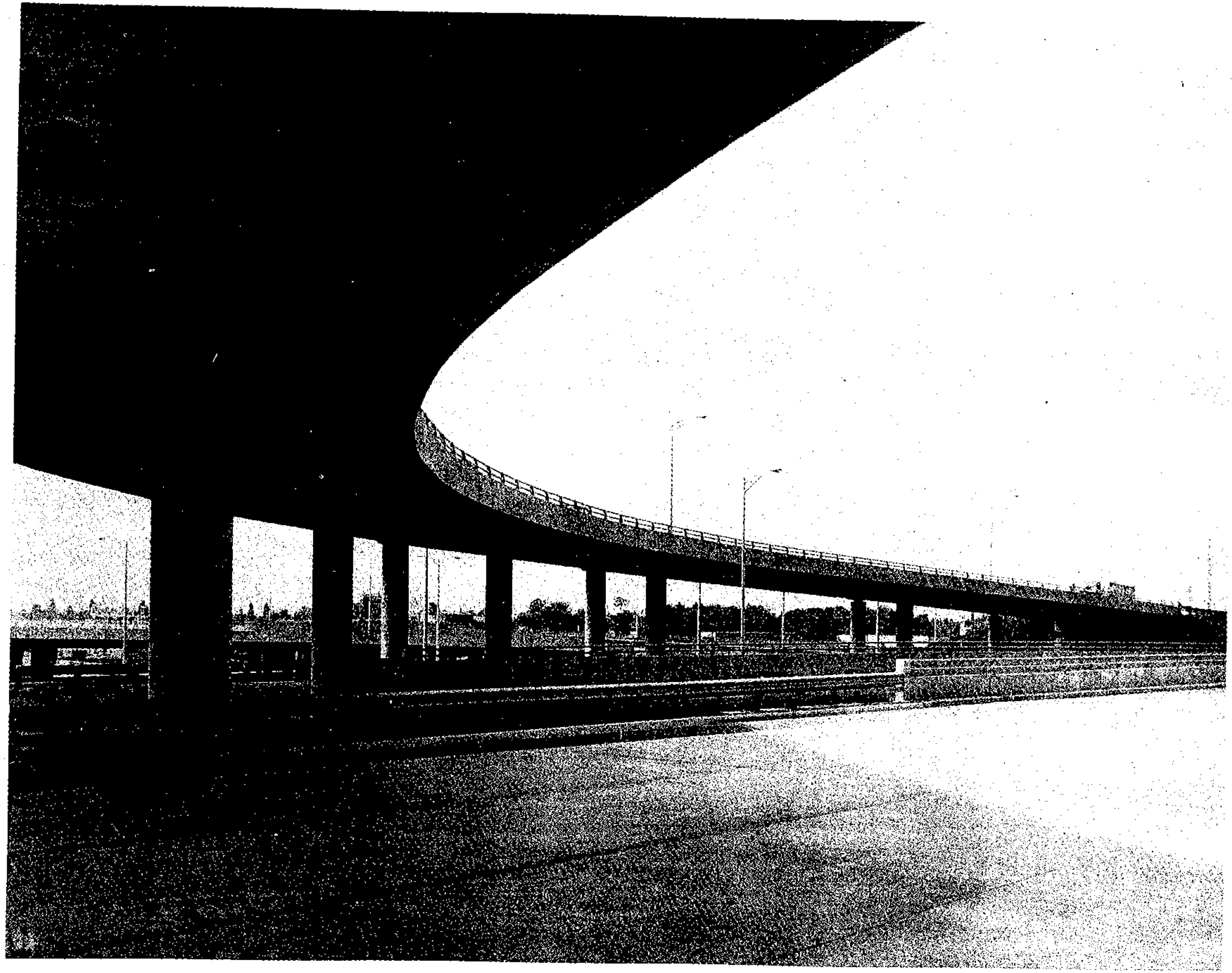
* The author wishes to thank Witecki, Bassi, Lin and Richardson, and acknowledges with grateful thanks the material taken from their papers as enlisted in the references 1 and 2 of this chapter.

† Only the parasitic prestress effect is akin to an (externally) applied load, not the primary prestress effect.



A curved continuous prestressed concrete bridge at the Spadina-Interchange, Toronto, Canada

Fig. 29.1 *Curved bridges in highway interchange*

integral with the deck, or piers made up of many columns are seldom necessary.

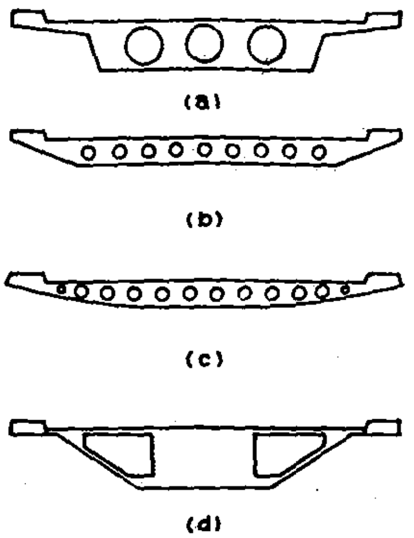


Fig. 29.2 *Types of bridge cross-sections used in Ontario*

Utilization of the structural capacities described above can lead to considerable economy, particularly in the reduction of the number and size of the columns. Initially this was done only when space limitations left no alternative, but the improved appearance and reduced costs that resulted have led to a general policy of reducing substructures to a minimum.

Large transverse bending moments occur particularly when only one column is used at each pier. Approximate calculations of transverse flexural deflection however are usually sufficient to demonstrate that the distribution of longitudinal bending stress is not significantly affected by transverse bending. Nearly all structures of the type considered can be designed longitudinally as beams, and transverse bending can be considered separately.

Twisting of the deck of a straight bridge can affect the transverse distribution of longitudinal bending stresses, but the loading required to produce such twisting is seldom critical for longitudinal flexure, and torsion can be considered separately. For curved bridges, however, longitudinal bending does affect the magnitude of torsional stresses very significantly and, if the curvature is sharp, torsion can increase longitudinal bending stresses to a limited extent.

29.2 LONGITUDINAL ANALYSIS

The bridge shown in Fig. 29.3 was built with a curvature of 30 degrees. The effect of the curvature on the longitudinal bending moments was investigated analytically, and the results for curvature from 0 deg. to 60 deg. are shown in Fig. 29.4. It can be seen that the bending moment at the center of the bridge is not appreciably affected by curvature and that the bending moment at the pier for 60 deg. curvature, radius 96 ft (30 m), is only 3% greater than for an equivalent straight bridge. The curves shown are for deal load. This is encouraging for the present analysis.

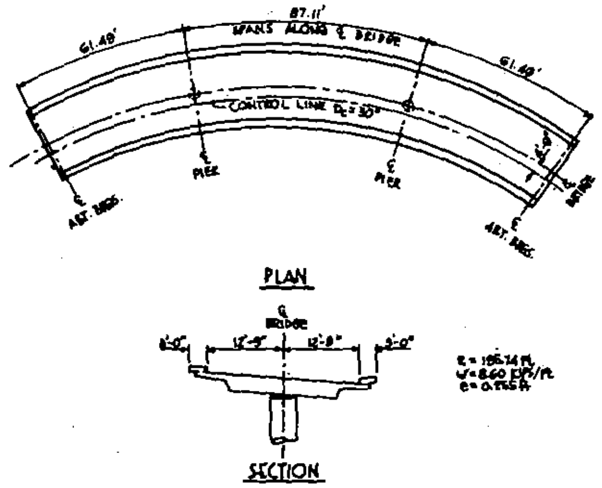


Fig. 29.3 *Dimensions of 30 degrees curvature bridge* (* 'Degree' of a curve is the angle subtended at the center by a 100 ft. length of arc.)*

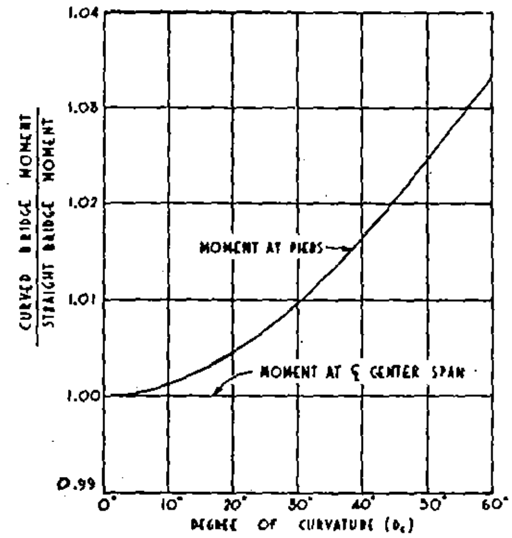


Fig. 29.4 *Effect of bridge curvature on longitudinal dead load moments*

A curvature of 60 deg. is far beyond the limit of normal highway design. Other variables are involved in the above comparison such as type of loading, width-to-span ratio, and elastic properties; but investigation of a large number of cases has shown that for normal design purposes it may be assumed that the longitudinal bending moments, shears, and reactions are not significantly affected by curvature, and above about 300 ft. radius it can be safely assumed that in a curved-in-plan superstructure the values of longitudinal bending moments, shears and reactions are almost same as in a straight-in-plan case whose spans arc equal to arc spans of the former.

29.3 TORQUE ANALYSIS

An approximate method is available for the calculation of torque in curved bridges.[1] It is restricted to curved beams which have constant or nearly constant elastic properties. It is felt that the method is quite adequate for all normal bridge-design problems provided there are no drastic changes in cross-section properties. The procedure obtains torison values by calculating the shear due to a hypothetical distributed torque loading.

Consider a small element of length ds of a curved beam as shown in Fig. 29.5.

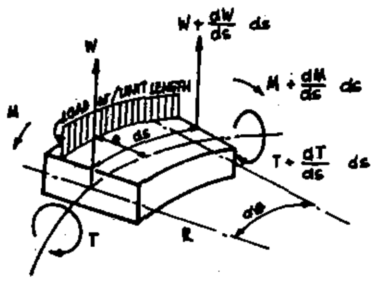


Fig. 29.5 *Torque analysis element for curved beam*

where t = applied torque per unit length at a section, = (w.e. + t')

R = radius of element ds at center-line of section

t' = any locally applied torque at a section (distributed over a dispersion width or uniformly distributed)

T = torsional moment created as a result of applied M, w and t'.

M = moment (externally applied, including parasitic moment due to non-concordant prestress)

W = shear

w = uniformly distributed load per unit length (self dead load)

$e = \dfrac{\gamma I_{yy}}{R{\cdot}w}$ eccentricity of the center of gravity of the uniformly distributed self dead load, measured from the center-line of the section

γ = density of material

I_{yy} = moment of inertia about vertical axis through centroid of section

a = distance from center-line of torque span to point under consideration

torque span = length of structure between points of torsional restraints (single bearing support has no torsional restraint)

Assuming $\cos d\theta = 1$ and $\sin d\theta = d\theta = ds/R$, the equilibrium equations for the element ds are,

$$\frac{dW}{ds} = -w \text{ or } W = \int -w ds \tag{29.1}$$

$$\frac{dT}{ds} = \frac{M}{R} + t \text{ or } T = \int \left(\frac{M}{R} + t\right) ds \tag{29.2}$$

$$\frac{dM}{ds} = W - \frac{T}{R} \text{ or } M = \int \left(W - \frac{T}{R}\right) ds \tag{29.3}$$

For long torque spans and/or sharply curved beams, Eq. (29.2) can be modified as follows to improve the accuracy of the approximate method (see Fig. 29.6):

$$T = \int \left[\frac{M}{R} + t\cos(a/R)\right] ds \tag{29.4}$$

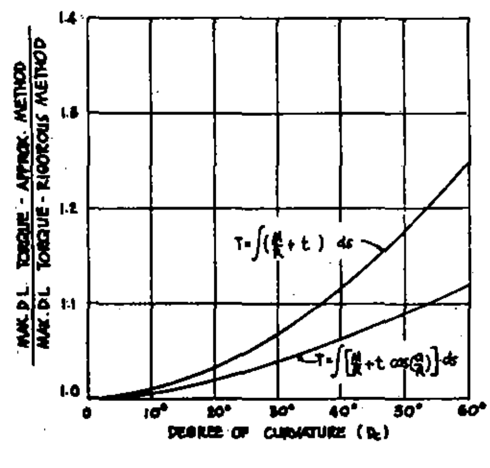


Fig. 29.6 *Effect of bridge curvature on accuracy of approximate method of torque analysis*

From Eq. (29.4) it is evident that the torque at any point along a curved beam is numerically equal to the shear (computed for the *torque span*) due to the application of a hypothetical load of $[(M/R + t\cos(a/R))]$ along the beam.

The magnitude of the torsional stiffness does not appear in the method. It is obvious that the torsional stiffness could affect the actual torque values. Using the rigorous method, the effect of torsional stiffness on torque was investigated for a wide range of values. Figure 29.7 is a graphic presentation of the results and shows that, for a very wide range of torsional-stiffness values, the maximum torque (dead load is again used as an example) at the abutment is not appreciably affected. For low values of torsional stiffness, the torque decreases rapidly with stiffness to zero. Cross-sections of the type shown in Fig. 29.2 have torsional-stiffness values in the range in which torque is almost independent of stiffness.

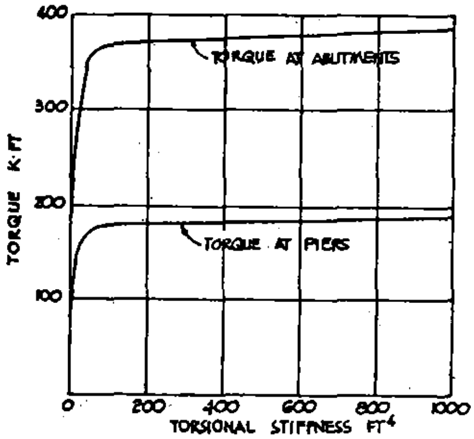


Fig. 29.7 *Effect of torsional stiffness on dead-load torques*

Application

The formula mentioned in Eq. (29.2) can now be applied to calculate the torsional moments as an effect of a horizontally curved multispan continuous bridge.

Step 1 Determine the geometry of the bridge.

Step 2 Establish the deck cross-section and compute the dead load, superimposed dead load, live load and any other anticipated load.

Step 3 Define uniformly distributed torque loading, t (it may have non-uniform values at certain sections).

Step 4 'Straighten out' the bridge deck to its 'full developed length' and compute bending moments for applied loadings, including prestress parasitic moments, not prestress primary moments. ('Straight' bridge means one with spans in length equal to 'arc spans' of curved bridge.)

Step 5 Add all of the computed bending moments and divide the result by radius R to obtain distributed torque loading M/R along the centerline of the bridge.

Step 6 Add the results of step 3 and step 5, at corresponding sections, to obtain the total distributed torque loading, $M/R+t$, along the centerline of the bridge (geometrical center line, not centroid of mass).

Step 7 Establish supports capable of restraining the torsional moment and consider the developed length of deck between these supports as a series of simply supported beams — torque spans.

Step 8 Apply the result of step 6 as a distributed load on the previously defined simply supported beams and compute simple beam reactions and shear diagram for each of the beams. These reactions are the concentrated torsional moments developed at the ends of the beams, whereas the ordinates of the shear diagram represent the torsional moment at any given section.

The total effect of torsion on support restraining the torque is the algebraic sum of the torsional moment at the two adjoining ends of the assumed simple beams, above that support.

Step 9 Compute stresses due to torsional moment at the desired sections along the deck; design the necessary reinforcement for (combined shear and) torsion.

NOTE Only stresses due to the torsional moment have been considered here. Stresses due to longitudinal bending moment, transverse bending moment, shear (principal tension) have to be considered separately.

Also see Annexure at chapter-end for more information.

Numerical Example

The structure considered here is a curved, three-span, continuous bridge with prestressed, post-tensioned, concrete superstructure as shown in Figs. 29.8, 29.9 and 29.10. The superstructure is of solid concrete and has a constant cross-section throughout (Fig. 29.10). It is supported by two abutments and two intermediate single bearing columns (Fig. 29.9). Bearing at the columns are rubber 'pot' bearings allowing free horizontal movement and rotation of the superstructure. Bearings at the abutments are considered to be capable of restraining the torsional moment.

In order to shorten the calculations in the example that follows, only the torsional moment as an effect of dead load and live load is considered, omitting the effect of prestress.

Step 1 The geometry of the bridge is shown in plan, in Fig. 29.8. The bridge is on a 13-deg. curve and the radius of curvature at the centerline of the bridge is R = 440.737 ft.

Step 2 Establish the deck cross-section and compute the dead load, superimposed dead load, live load, etc. (Fig. 29.11).

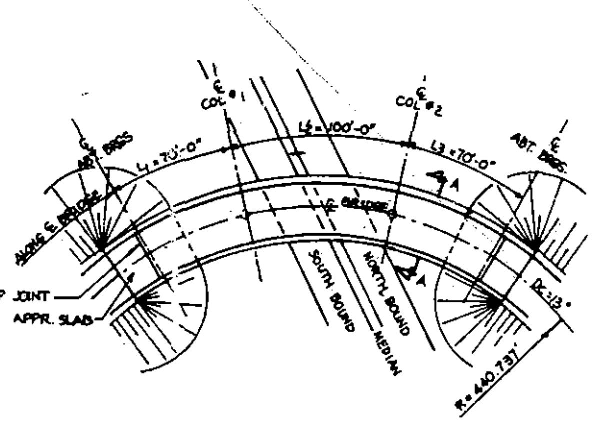


Fig. 29.8 *Plan*

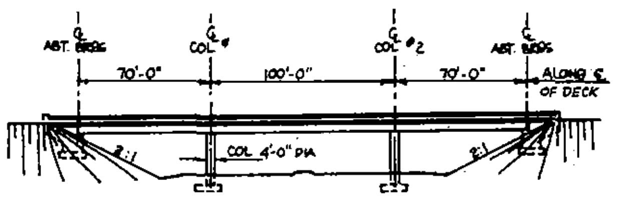


Fig. 29.9 *Elevation*

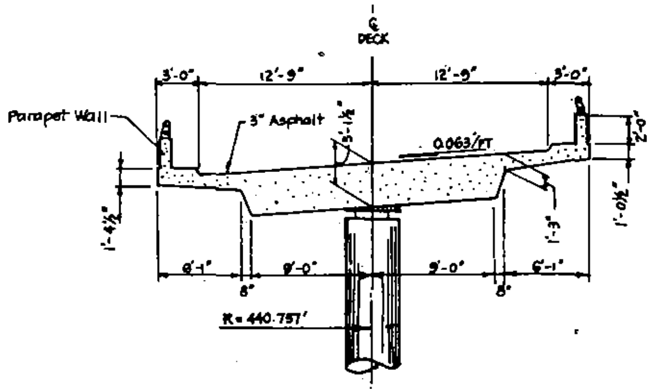


Fig. 29.10 *Cross-section A – A*

Location of CG of deck

Distance: CG from top of deck $Y_t = 1.28$ ft

CG from bottom of deck $Y_b = 1.84$ ft

Dead load

Area of deck cross-section = 73.796 sq. ft

Weight of deck,

including curbs, ω = 73.796× 0.150

= 11.07 k/ft

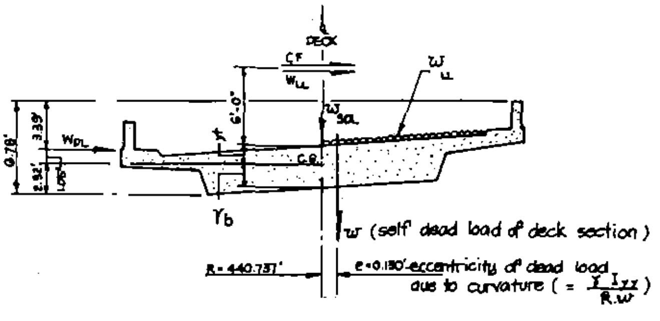


Fig. 29.11 *Cross-section showing all applied loading*

Superimposed dead load

Parapet walls = 0.833 × 2.000 × 0.150 × 2 = 0.50 k/ft

Asphalt (3-in.) = 25.50 × 0.250 × 0.150 = 0.96 k/ft

Total ω_{SDL} = 1.46 k/ft

Live load

Loading as per AASHTO HS20-44 including impact taken as max 30%.

(a) *Uniform vertical load*

Uniformly distributed load applied as shown in Fig. 29.11 to obtain maximum torsional effect. Concentrated load as per AASHTO specifications omitted in this example to simplify calculations.

$$\omega_{LL} = 0.064 \times 12.75 \times 1.30 = 1.06 \text{ k/ft}$$

(b) *Centrifugal force*

Maximum design speed (ramp) = 40 mph, as per AASHTO specifications.

$$C = 0.00117 \times 40^2 \times 13 = 24.34\% \text{ say, } 25\%.$$
$$CF = 0.25 \times 1.06 = 0.27 \text{ k/ft}$$

Wind

Loading as per AASHTO specifications:

(a) *Load on structure* acting at center of gravity of exposed area:

$$W_{DL} = 6.78 \times 0.050 = 0.34 \text{ k/ft}$$

(b) *Wind on live load* acting at point 6 ft 0-in. above deck:

$$W_{LL} = 0.100 \text{ k/ft}$$

Step 3 Determine distributed torque loading, t.

Loading	*Moment*		
Self weight of deck	11.07 × 0.13	= 1.34	(= ωe)
Live load	1.06 × 12.75× 0.50	= 6.26	(= t')
Centrifugal force	0.27 × (6.00 + 1.28)	= 1.57	
Wind on structure	0.340× 1.05	= 0.36	
Wind on live load	0.100× (6.00 + 1.28)	= 0.73	
		Total: 10.26 kft/ft	

Step 4 'Straighten' out the bridge deck to its full developed length and compute dead load bending moment, superimposed dead load bending moment, live load bending moment, etc.

Steps 5 and 6

Tabulate the bending moments calculated in step 4 and compute the (total) distributed torque load, $M/R + t$, along the centerline of the bridge at corresponding sections (Table 29.1). Since the bridge is symmetrical, the calculations are limited to the point of symmetry at the centerline of the middle span (Fig. 29.12). These calculations are for every tenth section of each span.

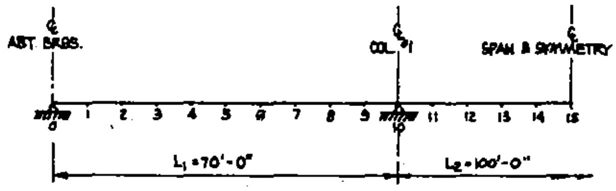


Fig. 29.12 *Developed length of bridge deck (half shown)*

Step 7 As mentioned before, the abutments, in the present case will be the only supports 'capable of restraining the torsional moment'. Therefore the full developed length of the deck between the abutments must be regarded as a single 'torque span'.

Step 8 Using the values of $M/R + t$ from Table 29.1 construct a distributed torque loading diagram as in Fig. 29.13.

Applying the above 'conjugate' loading (Fig. 29.13) on the previously defined 'torque span'

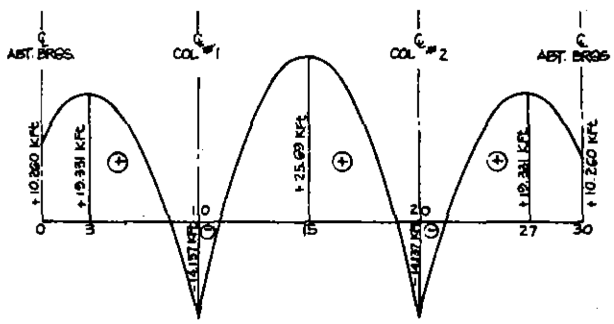


Fig. 29.13 *Distributed torque loading diagram*

(step 7), construct the shear diagram as for a simply supported beam (Fig. 29.14). This represents the final torsional moment diagram; where the ordinates are the final torsional at any given section.

The diagram, Fig. 29.14 shows ordinates at every tenth section of the 'torque span'.

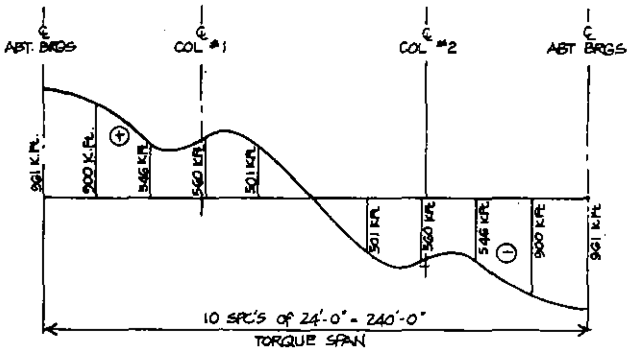


Fig. 29.14 *Final torsional moment diagram*

Table 29.1 *Computation of total distributed torque loading*

SECTION	Longitudinal bending moment $M(k \cdot ft)$					R	$\frac{M}{R}$	t	$\frac{M}{R} + t(kft/ft)$
	DL	SDL	LL	OTHER	TOTAL				
0	0	0	0		0		0		10.260
1	1646	217	156		2019	440.737 ft	4.580	10.260 kft/ft	14.840
2	2732	360	262		3354	440.737 ft	7.610	10.260 kft/ft	17.870
3	3257	429	312		3998	440.737 ft	9.071	10.260 kft/ft	19.331
4	3222	425	309		3956	440.737 ft	8.976	10.260 kft/ft	19.236
5	2626	346	251		3223	440.737 ft	7.313	10.260 kft/ft	17.573
6	1470	193	140		1803	440.737 ft	4.091	10.260 kft/ft	14.351
7	− 247	− 33	− 24		− 304	440.737 ft	− 0.690	10.260 kft/ft	9.570
8	−2524	− 333	−242		− 3099	440.737 ft	− 7.031	10.260 kft/ft	3.229
9	−5361	− 707	−513		− 6581	440.737 ft	−14.932	10.260 kft/ft	− 4.672
10	−8759	−1155	−839		−10753	440.737 ft	−24.397	10.260 kft/ft	−14.137
11	−3612	− 476	−346		− 4434	440.737 ft	−10.060	10.260 kft/ft	0.200
12	392	52	37		481	440.737 ft	1.091	10.260 kft/ft	11.351
13	3252	428	311		3991	440.737 ft	9.055	10.260 kft/ft	19.315
14	4967	655	476		6098	440.737 ft	13.8355	10.260 kft/ft	24.095
15	5540	730	530		6800	440.737 ft	15.428	10.260 kft/ft	25.688

Step 9 Compute stresses due to the torsional moment at the desired sections along the deck and design the necessary reinforcement for (combined shear and) torsion. (This portion is outside the scope of the present chapter but has been dealt with separately in Ch. 24 in this book to which reference may be made for the reinforcement design.)

REFERENCES

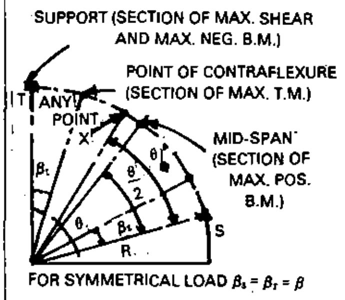
1. Witecki, A., "Simplified Method for the Analysis of Torsional Moment as an Effect of a Horizontally Curved, Multispan Continuous Bridge," *1st International Symposium on Concrete Bridge Design*, SP-23 American Concrete Institute, Detroit, 1968.
2. Bassi, K.G., WL Lin and BS Richardson, "Continuous Post-tensioned Torsionally Stiff Concrete Bridges", *1st International Symposium on Concrete Bridge Design*, SP-23, American Concrete Institute, Detroit, 1968.

ANNEXURE

Rapid Analysis of Circularly-Curved-in-Plan Girders*

BOW GIRDERS (CIRCULAR ARCS)

APPLICABLE TO (i) Beams forming a complete circular system with supports spaced uniformly, and with identically load on each span.
(ii) Beam of single-span with ends rigidly fixed.



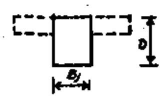
Section	Bending Moment	Twisting Moment	Shear Force
Support	UK_wR or $\frac{C_2G + C}{C_1G + 1} K_wR$	SK_wR or $\frac{C_4G + C_5}{C_1G + 1} K_wR$	QK_w
Point of Contra-flexure	ZERO	VK_wR or $\frac{C_6G + C_7}{C_1G + 1} K_wR$	NK_w
Mid-span	YK_wR or $\frac{C_8G + C_9}{C_1G+1} K_wR$	ZERO	ZERO

		Uniformly Distributed Load ($K_w = wR$) w lb. per Foot							Central Point Load P.($K_w = P$)	
ANGLE θ		30°	45°	60°	72°	90°	120°	180°	90°	180°
NO. OF SUPPORTS		12	8	6	5	4	3	(2)	4	(2)
C_1		—	—	10.4	7.5	4.5	2.4	—	4.5	—
SUPPORT	U	0.02 Approx.	0.053	—	—	—	—	1.00	—	0.5
	C_2	—	—	0.99	1.00	1.00	1.11	—	0.97	—
	C	—	—	0.12	0.14	0.24	0.45	—	0.25	—
	S	NEGLIGIBLE		—	—	—	—	0.30	—	0.18
	C_4	NEGLIGIBLE		0.025	0.03	0.03	0.27	—	0.04	—
	C_5	NEGLIGIBLE		0.02	0.02	0.03	0.09	—	0.04	—
	V	0.001 Approx.	0.004	—	—	—	—	0.12	—	0.09
	C_6	—	—	0.08	0.10	0.13	0.16	—	0.17	—
	C_7	—	—	ZERO	0.01	0.02	0.03	—	0.26	—
MID-SPAN	Y	0.01 Approx.	0.023	—	—	—	—	ZERO	—	0.318
	C_8	—	—	0.40	0.38	0.32	0.11	—	0.19	—
	C_9	—	—	0.10	0.08	0.05	0.002	—	ZERO	—
SHEAR	Q	0.28	0.39	0.53	0.63	0.79	1.05	1.57	0.5	0.5
	N	0.20	0.22	0.26	0.35	0.40	0.49	0.66	0.22	0.18
Angle β		6°	10°	15°	18°	22°	36°	52°	$25\,^1/_2°$	$57\,^1/_2°$

$G = \frac{\text{Torsional Rigidity}}{\text{Flexural Rigidity}}$

G is approximately equal for rectangular, TEE, and ELL Sections with the same ratio of D:B.

$\frac{D}{B}$	1.0	1.5	2.0	2.5	3.0
G	1.12	0.70	0.47	0.33	0.22



* Refer to: Schulz M., and M. Chedraui, "Tables for circularly curved horizontal beams with symmetrical uniform loads", *Journal of the American Concrete Institute*, vol. 28, no. 11, May 1957, pp. 1033–1040 and Spyropoulos, P.J., "Circularly curved beams transversely loaded", *Journal of the American Concrete Institute*, vol. 60, no. 10, October 1963, pp. 1457–1469.

Numerical Example

A bow girder in 18 in. deep and 18 in. wide, has a radius of 12 ft, and subtends an angle of 90 deg. The ends are rigidly fixed and the total load is 45,000 lb uniformly distributed. Find the maximum negative bending moment, the maximum positive bending moment, and the maximum twisting moment. Thus θ = 90 deg; R = 12 ft;

$$G = 1.12; w = \frac{45,000}{\pi \times 2 \times 12 \times 0.25} = 2,390 \text{ lb per ft;}$$

K_w = 2,390 × 12 =28,700 lb

According to the table given above:

The maximum negative bending moment occurs at the support, and the coefficients are $C_1 = 4.5$; $C_2 = 1.00$; and $C = 0.24$. The maximum negative bending moment

$$= \frac{(1.00 \times 1.12) + 0.24}{(4.5 \times 1.12) + 1} \times 28{,}700 \times 12 = 77,400 \text{ ft-lb}$$

The maximum positive bending moment occurs at midspan; $C_8 = 0.32$; $C_9 = 0.05$. The maximum positive bending moment

$$= \frac{(0.32 \times 1.12) + 0.05}{6.04} \times 28{,}700 \times 12 = 23,200 \text{ ft-lb}$$

The maximum twisting moment occurs at the point of contraflexure (β = 22 deg.) $C_6 = 0.13$; $C_7 = 0.02$. The maximum twisting moment is

$$= \frac{(0.13 \times 1.12) + 0.02}{6.04} \times 28{,}700 \times 12 = 9{,}360 \text{ ft-lb}$$

CHAPTER 30

Temperature Stresses in Concrete Bridge Decks —Simple Design Method

Synopsis

After briefly reviewing the background of the problem and clearly pinpointing the three causes that build up the thermal stresses, the phenomenon of distribution of temperature through the deck-depth is examined and idealisation of the non-linear differential thermal gradient suggested. The possible thermal crackpatterns are discussed. Some useful observations in relation to thermal stresses, including their effect on ultimate load carrying capacity of the member, their cyclic nature as distinct from the phenomenon of fatigue and their possible order of magnitude, are reported. Finally, suitable recommendations are made and typical numerical examples are presented clearly in a manner that would suit hand computation in a practising design office.

30.1 INTRODUCTION

Thermal effects have come in for close scrutiny in recent years. In the past the effect of non-linearity of temperature distribution across the deck-depth was not well understood so that, where thermal effect was considered (e.g., in an 'arch' or in a 'portal'), it was essentially the effect of rise or fall in the overall mean-temperature of the body over a long period of time.

As structures became more complex and the distribution of temperature through the deck-depth, particularly its nonlinear profile, began to be understood better, need for a closer look into thermal effects arose.

It is now realised that, in certain types of structures, the combined effects of 'change in the body-temperature', the 'nonlinear distribution of thermal gradient through the deck-depth' (and in case of a portal, across the abutment leg-thickness) and the 'continuity effect' owing to the constraints offered to free hogging (or sagging) desire of the structure (under unequal extreme fibre temperatures) by the intermediate and fixed supports (if any), can create flexural stress that can match the dead and live load stresses.

30.2 THE THREE CAUSES

(i) Effect of Change (Rise or Fall) in the 'Mean' Temperature of the Body of the Deck

For the purpose of this effect, it is assumed that the temperature of the entire body of deck has one 'mean' value at any instant of time and that this 'body-mean-temperature' rises or falls over a long period of time, thereby wanting the structure to 'heave'. If the structure is free to permit this 'heave' i.e., is free to expand or contract (e.g., simply supported beam or a continuous beam), this causes no thermal stresses. However, if the structure is unable to permit such a heave (e.g., arch, frame, fixed beam), i.e., offers constraint to its desire to heave, moments etc. are then caused, which create stresses (**thermal stress Type 1**). These moments can be evaluated by the usual methods of theory of elasticity. (See Annexure at chapter-end.)

(ii) The Effect of 'Nonlinear' Distribution of Temperature across the Deck-Depth

If the top surface of the concrete deck is hotter than its soffit surface (temperatures T_1 and T_2 in Fig. 30.1), the ordinate T of the thermal gradient at any intermediate depth, as measured from the base line $a - b$, has been discovered to follow a nonlinear variations as shown in Fig. 30.1.

Figure 30.2 indicates the build-up of the total thermal gradient. Its uniform part ⓐ, at the instant of consideration, is akin to the 'body-mean-temperature', the effect of change in which over a long period of time, is already taken care of earlier. However, the variable part ⓑ, better called the 'differential thermal gradient', would heat each fibre to a different degree, the variation being nonlinear. If the fibres were free to each other (i.e., unrestrained) then they could happily accept the corresponding non-linear thermal strains αt (α being the coefficient of expansion/contraction). But since their deformations must follow a linear law (plane sections must remain plane), they will not accept these non-linearly related strains, and the difference between the final 'linear' strain gradient ⓓ and the 'unrestrained' strain gradient ⓒ (i.e., $\epsilon_0 - y\theta - \alpha t$) will represent the

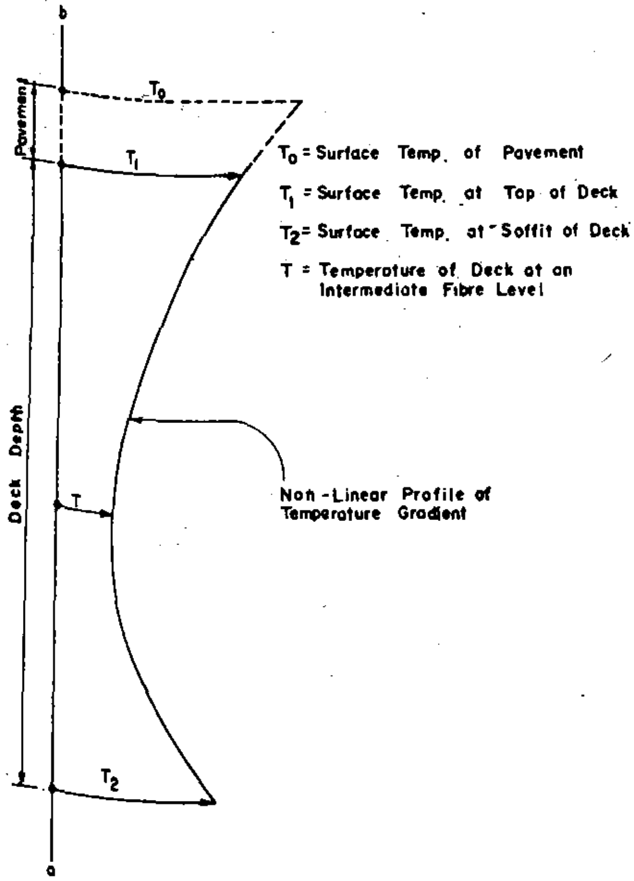


Fig. 30.1 *Total thermal gradient*

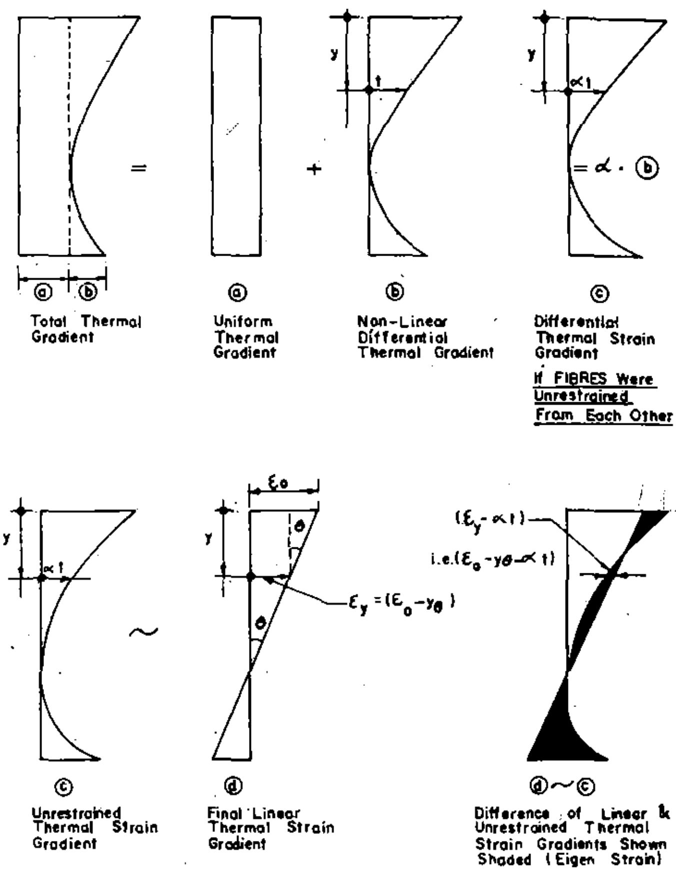


Fig. 30.2

uneven 'internal disturbance'. Its strain effect may be called the Eigenstrain and its stress effect may be called the Eigenstress, both of which would be zero if only the thermal gradient were linear (which it is not). This Eigenstress (and the Eigenstrain), as can be seen, is purely an internal entity, not associated with any support reactions.

Both its 'force' (at any section) and its 'moment' about any fibre in that section must sum up to zero (i.e., self-equilibriating).

As shown in Fig. 30.2, the ordinate of the shaded diagram ⓓ-ⓒ, at y below top, represents the Eigenstrain at the fibre, i.e.,

$$\text{eigenstrain}: \epsilon_{ei} = \epsilon_0 - y\theta - \alpha t \qquad (30.1)$$

and $\therefore$

$$\text{eigenstress}: f_{ei} = E_C \cdot \epsilon_{ei} = E_C(\epsilon_0 - y\theta - \alpha t) \qquad (30.2)$$

where E_C is the modulus of elasticity of concrete. If A be the section area of this fibre, then:

since $\Sigma A f_{ei} = 0 \ \therefore E_C \Sigma A(\epsilon_0 - y\theta - \alpha t) = 0$

and since $\Sigma[(A f_{ei}) y] = 0 \ \therefore E_C \Sigma A(\epsilon_0 - y\theta - \alpha t) y = 0$

From these equations it follows that:

$$\epsilon_0 \Sigma A - \theta \Sigma A y = \alpha \Sigma A t \qquad (30.3)$$

and

$$\epsilon_0 \Sigma A y - \theta \Sigma A y^2 = \alpha \Sigma A y t \qquad (30.4)$$

Thus, in any given deck section, subjected to a known differential temperature gradient, the section can be broken into rectangular elements, the terms ΣA, ΣAy, ΣAy^2, ΣAt and ΣAyt quickly evaluated and a mere substitution in Eqs. (30.3) and (30.4) will yield the values of extreme fibre strain ϵ_0 (at $y = 0$) and the strain gradient θ, whereafter Eigenstress at any fibre can be readily evaluated from Eq. (30.2) **(thermal stress Type 2).**

Eigenstress, on its own, may be small or significant, depending on,

- depth of section
- thickness and colour of pavement
- wind speed
- orientation of bridge and incidence of sun rays
- ambient temperature
- material properties,

— thermal conductivity
— specific heat
— thermal diffusivity
— coefficient of thermal expansion and contraction
— coefficient of absorptivity
— coefficient of surface-heat-transfer

- surface temperature
- shape of thermal gradient.

The distribution of Eigenstress, not being linear, when added to the thermal 'Continuity' stress (see ahead), may show significant stresses not only at extreme fibres but also at intermediate fibres (e.g., mid height portion of webs) which are heavily loaded under shear. This can produce longitudinal cracks in webs.

(iii) Effect of Intermediate-Support Restraint on the Free Hogging (or Sagging) Desire of the Structure caused by Unequal Extreme Fibre Temperatures—the 'Continuity Effect'

In a beam-type deck, the difference of temperature between the extreme surfaces causes hogging (or sagging) of the beam.

If the beam is simply supported, it merely hogs (or sags) as its supports do not prevent rotation. This free deformation is not a 'moment-induced' deformation, but merely a 'strain-induced' deformation, and no moment is caused, Fig. 30.3.

However, if the beam is continuous, its aforementioned desire to freely hog (or freely sag) will be 'constrained' at the intermediate supports (presence of dead load reactions will prevent it from lifting up and presence of supports will prevent it from going down at these supports). This 'continuity' effect sets up moments that cause additional stresses (**thermal stress Type 3**) called 'continuity' stresses.

These 'continuity moments may simply be computed by superposition of 'free curvature moments M' and the 'distributed moments' due to equal and opposite moments M applied at the two ends of the continuous beam necessary to restore zero moment condition at the two ends. This is shown diagrammatically in Fig. 30.4 and solutions of continuity moments and reactions in some typical continuous beams are indicated in Fig. 30.5.

While here the discussion is restricted only to beams the equivalent of the aforementioned 'continuity moments' in the longitudinal frames (i.e., single or multi-bay portals) would be the 'frame moments' set up due to constraints to free hogging (or sagging) because of rigid joints, and can be analysed by the usual classic methods of elastic analysis. (See Annexure at chapter-end.)

Review

A review of three causes under subhead 30.2 above clearly indicates that the thermal stresses are caused:

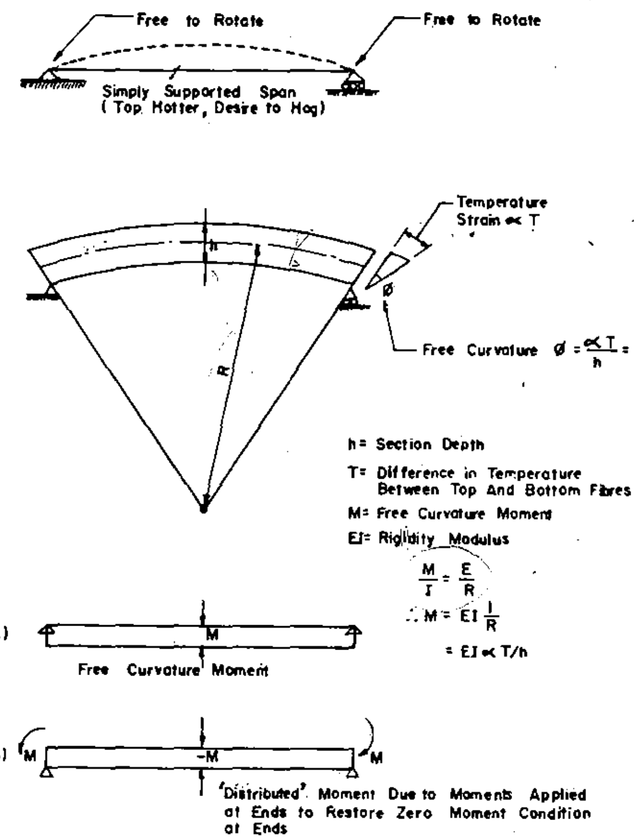


Fig. 30.3 *Net moment = (a) + (b) = zero, in the simple span*

— in a simply supported deck ... only by one cause i.e., 'Eigenstress'.
— in a continuous beam deck ... by two causes i.e., 'Eigenstress' and 'Continuity effect'.
— in a frame deck ... by all three causes i.e., 'Eigenstress', 'Continuity effect' and 'Change in body-mean-temperature'.

30.3 PREDICTING TEMPERATURE DISTRIBUTION THROUGH DECK-DEPTH (FOR EVALUATING EIGENSTRESS)

Theoretical Background

The thermal response of an isotropic solid with a discrete boundary in contract with air, Fig. 30.6, subjected to variations in ambient temperature θ_a and radiation q, is governed by the general Fourier Conduction Equation (Reference 5 at the chapter-end):

$$\frac{k}{\rho c}\left[\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right]=\frac{\partial\theta}{\partial t} \tag{30.5}$$

with the boundary condition

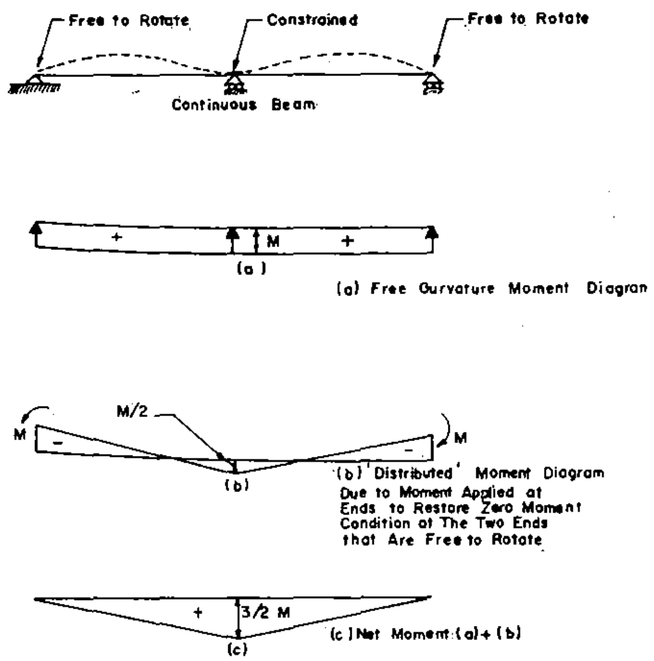


Fig. 30.4

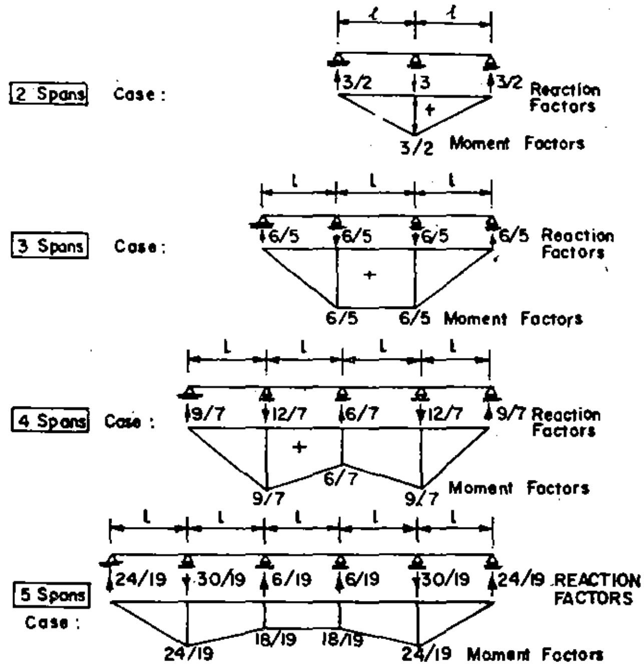
$$k\frac{\partial\theta}{\partial_n} + q + H(\theta_a - \theta_0) = 0$$

where k = conductivity of the solid
ρ = its density
c = its specific heat
θ_0 = boundary temperature of the solid
H = boundary heat transfer coefficient (depending mainly on wind speed)
n = the direction normal to the boundary

q, θ_a, and H are complex functions of time, and therefore a direct mathematical solution of Eq. (30.5) is almost impossible. Use of 3-dimensional (x, v, and z directions) finite element technique is appropriate. However, for Bridge decks, the problem can easily be reduced to a 1-dimensional heat flow problem since in most bridge decks the temperature variation along longitudinal (x) and breadth (z) axes is of little significance—only in the depth (y) direction it matters most. Hence Eq. (30.5) can be reduced to

$$\frac{k}{\rho c}\frac{\partial^2\theta}{\partial y^2} = \frac{\partial\theta}{\partial t} \tag{30.7}$$

which may be solved along with Eq. (30.6) by finite difference method. To analyse the thermal response by such method in a given deck section, it may be necessary to



REACTION = (Reaction Factor)$\cdot M/l$
MOMENT = (Moment Factor)$\cdot M$
where M = free-curvature moment
$= EI \propto T/h$ (see Fig. 30.3)

Fig. 30.5 *Examples of continuity moments and reactions for various cases of prismatic continuous beams of equal spans due to temperature difference 'T' between top and bottom (top hotter).*

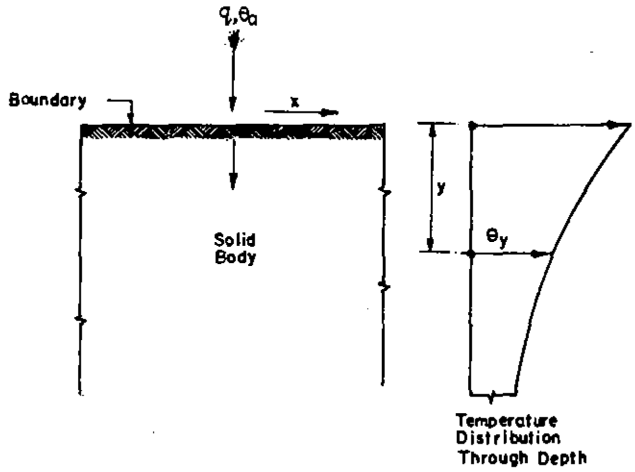


Fig. 30.6

analyse a few representative locations within the section and then combine the total results for the section. Finally a critical temperature distribution through the deck-depth may be established.

Computer Analysis

The Research Report 23–3F (Project 3–5–74–23) by the 'Center for Highway Research' at the University of Texas at Austin, USA, reports of the TSAP (Temperature and Stress Analysis Program) computer program developed there sometime back (1977–78). This program first predict the temperatures at various 'points' at 'various sections', based on certain relevant input data and a one-or-two-dimensional heat flow model, and then uses it and predicts thermally induced stress assuming the 'structure' as a one dimensional (i.e., linear elastic) beam.

Subsequently they have also developed SHELL-8 program which basically uses the temperature input from TSAP and then analyses thermal stresses assuming the deck to be a 2-dimensional grillage with 6 degrees of freedom at each 'node'.

Such programs are obviously powerful tools but unfortunately are beyond the reach of most designers, who therefore need a more straight-forward approach, a reasonably idealised temperature distribution (depicted in Fig. 30.2b) and a workman-like solution to the problem. (See Recommendations and Numerical Examples ahead.)

Idealisation of the (Differential) Thermal Design Gradient

It has been observed that out of the 'differential thermal gradient', shown in Fig. 30.2b, majority of the top surface temperature cools off in about top 25 cm depth of deck. The rate of cooling thereafter slows down considerably. The bottom surface temperature, up the depth, cools relatively much slower than the top surface temperature.

Owing to complex variations possible among the various dependant factors listed earlier, it is very difficult to generalise the shape of this Design Gradient.

However, until more refinement and accuracy is possible, the (differential) thermal Design Gradient (shown in Fig. 30.2b) may be assumed to be of the idealised shape put forth in the Recommendations under 30.6 ahead.

30.4 THERMAL CRACK PATTERN

Although temperature differential is reversible (though not with same intensity) so that there is the tendency to alternately hog and sag, the worst effect in a continuous beam deck generally occurs when top is hotter. This causes a tendency to hog, which invites downward reactions (restraints) at intermediate supports and consequent upward reactions at abutments, and hence a sagging type Bending Moment.

This, in turn, can lead to (thermal) cracking in the bottom of the central span in a 3-span continuous beam and around central pier in 2-span continuous beam and increase in reactions at Abutments.

Cracks, parallel to length of the bridge, may also occur in the bottom to top slab due to sagging moments set up in it owing to downward restraints exerted by its intermediate web supports against its desire to hog up under higher surface temperature.

Horizontal cracks may also occur in the webs when significant horizontal thermal gradient exists through webs. This happens in thick webs (thickness generally greater than 30 cm).

Unless sections are thin, considerable temperature differentials can be set up and cracks will be induced by thermal stresses. It is a good idea then to permit partial prestressing (rather than go for full prestress) i.e., permit tensile crack under working load in combination with sufficient small diameter reinforcement bars at close pitch in tension zones, and rely on the reduction in flexural rigidity due to thermal cracking to reduce the 'continuity' moments. (Unfortunately for various other but valid reasons, partial prestressing is not yet allowed in many codes.) It is true that upon thermal cracking and consequent reduction in stiffness (EI), the 'continuity' stresses (which are caused in the first place because of restraint to deformation at supports as the member tries to hog or sag) reduce.

Provision of additional small diameter bars at close spacing in crack-critical zones does not avoid thermal cracks, it only distributes them better and reduces their widths. But one of the simplest means to avoid thermal cracking is to avoid thick webs and slabs in hollow boxes.

Owing to Poisson's effect (that 'compression in a given direction causes tension perpendicular to it') there is already some tension in the transverse direction because of longitudinal compressive stresses, hence cracks may easily occur in combination with temperature in thick webs and slabs.

It has to be remembered that in certain hot places the surface of a box-deck on a hot day can heat up to about 70 to 80°C with the air inside the box reaching about 40 to 50°C. During the following night, while the temperature outside the box can fall rapidly, the warm air inside can maintain its temperature for a relatively longer period. The resulting temperature gradients between outside and inside create bending moments in the transverse direction, with a tendency for longitudinal cracks in webs and slabs. These moments can be large where webs and bottom slabs are thicker (e.g., near supports) than where they are thinner (e.g., in span area). Figure 30.7 (after **Leonhardt**) shows transverse bending moments in two box sections with different web and soffit-slab thicknesses under a 20°C temper-

2·5 to 3·5 MPa

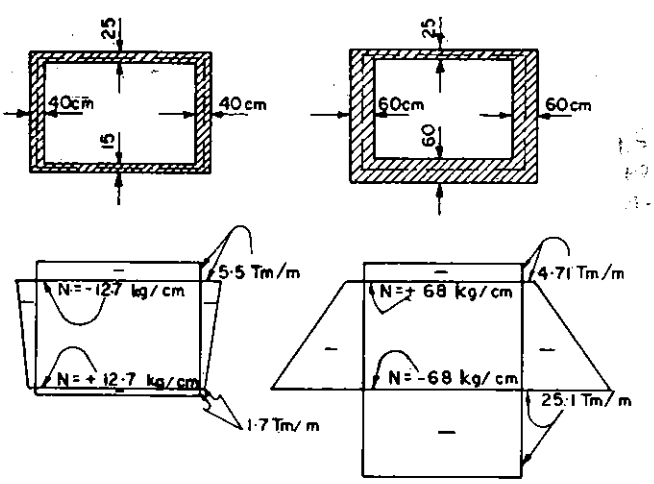


Fig. 30.7 *Transverse bending moments in hollow box sections due to temperature gradient of 20° Centigrades between inside and outside of box*
Left: Typical section in span
Right: Typical section near intermediate supports

ature difference between inside and outside (calculated by the usual methods of theory of elasticity; same as the continuity moments shown earlier; only the frame is in transverse direction). The results are revealing and show how high the thermal moments due to 'Continuity effect' can be.

30.5 THERMAL STRESSES

Possible Order of Magnitude

In the various Research Reports of the 'Center for Highway Research' at the University of Texas at Austin, USA (also see under Computer Analysis earlier) it has been shown that the total 'temperature induced tensile stresses in prestressed concrete slab type decks' and in composite decks comprised of 'prestressed beams and reinforced concrete slab', can be of the order of up to 60 and 80% respectively, of the modulus of rupture of concrete.

On the other hand, in composite decks consisting of 'steel beams and reinforced concrete slab', the total 'temperature induced tensile stresses' can reach about 10% of the total stress due to dead and live loads—but this 10% stress is well within the extra permitted under combination with temperature.

In certain box section prestressed concrete decks (45 cm webs), well detailed otherwise but poorly detailed against the effect of possible difference in temperature between inside and outside (sun beating on the outside while the atmosphere inside the box still thawing from winter conditions), the resultant tensile stresses far exceeded the modulus of rupture of concrete and longitudinal unsightly cracks developed in the webs.

'Eigenstresses' of 25 to 35 kg/cm^2 compression at top and bottom with 15 to 25kg/cm^2 tension in mid height region are not uncommon. In addition, reversible 'continuity stress' (resulting from the 'continuity effect' in continuous beams and frames) of 10 to 15 kg/cm^2 (compressive or tensile) at top and bottom, varying linearly in between it also not uncommon.

Hence it is important to estimate these stresses and ensure that the resultant stresses, in combination with those from other loads and forces, are safe.

Character

As pointed out in the earlier referred Research Reports, in general, Highway Bridges are subject to 'cyclic temperature induced stresses' at the rate of 30 cycles per month (365 cycles a year) and the frequency of one cycle is 24 hours. Furthermore the daily fluctuations of the thermal stresses are not necessarily at their maximum. Hence this cyclic character should not be compared to the classical fatigue phenomenon in which several hundred cycles per minute, are normally applied.

Effect on Ultimate Load Carrying Capacity

At working load, thermal effect may be provided for as described earlier but its effect on ultimate load carrying capacity of the structure is usually small (Reference 5). This can be readily realised from Fig. 30.8. After considering the factored values of Dead and Live Loads, for a given thermal effect T, it is clear that its effect is relatively small owing to the drastic flattening of the moment-deformation curve in the yield zone. Temperature effect can even be disregarded in studying the ultimate strength of the structure.

Some Useful Observations

- Largest eigenstress and continuity stress can be induced in a concrete bridge deck, of given geometry, during summer time (when the solar radiation is maximum), particularly if the outer webs are protected by the shade of the overhangs.
- Eigenstresses increase with increase in depth of section, and are least in thin slabs.
- Continuity stresses reduce considerably due to thermal cracking.
- Unfavourable stresses will result also when the daily range of the ambient temperature is large and wind speed minimum.
- Higher wind speed reduces thermal inequalities in general and consequently reduces both the eigenstresses as well as the 'continuity stresses'. Hence bridges in exposed locations, as opposed to the bridges in sheltered locations, have lesser thermal stress problems.

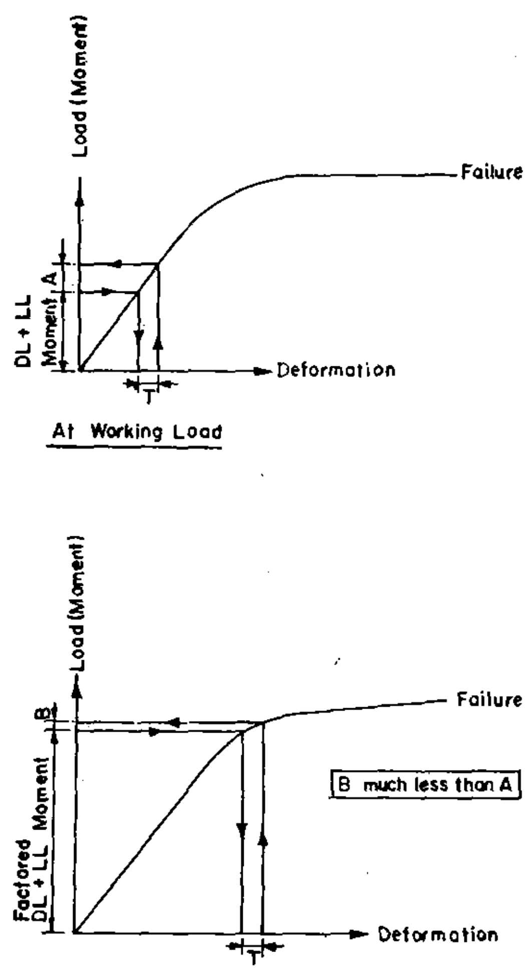


Fig. 30.8

• High 'tensile' stresses due to temperature cannot practically be avoided unless the section thicknesses are reduced (and) or expensive prestress is introduced. Otherwise, provision of small diameter reinforcement bars at close spacing (10–12 cm) in crack-critical zones is advisable in order to avoid wide cracks and instead create finer cracks, better distributed.

• Thermal effect is not very significant in assessing the ultimate strength (load capacity) of a structure since it does not reduce the ultimate carrying capacity very much.

• Asphaltic wearing course, i.e., a 'black' pavement, accentuates thermal stresses in the deck since a black cover absorbs more heat.

• An increase in pavement thickness reduces thermal stresses.

• 'Asphalt' has higher absorptivity and emissivity coefficients (0.90 and 0.92, respectively) compared to grey 'concrete' surfaces (0.50 and 0.88, respectively). Hence Asphaltic wearing course will transmit higher temperature to the deck. However, after the concrete wearing course has slightly aged and become dirty and black, it too will transmit higher temperature to the deck.

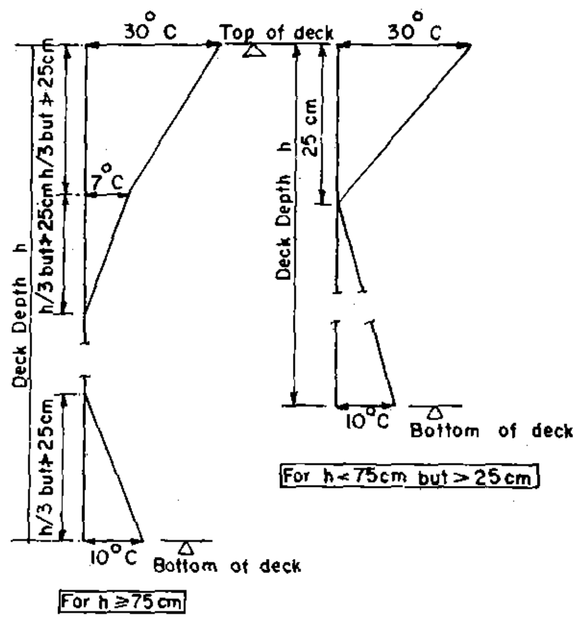
• Surface Condition of a steel girder in a composite deck can create very high (even more than 50°C) difference of temperature between steel and concrete if its surface is grey black oxidised (absorptivity 0.95) compared with a mere 14°C if its surface is white (absorptivity 0.15).

30.6 RECOMMENDATIONS

Until more dependable data are collected, the bridges in essentially hot climates, where shade temperatures reach 50 to 55°C, may be designed for the following thermal effects:

a) 'The effect of a 50°C overall rise (or overall fall) in body-mean-temperature (i.e. ± 25°C range)', *using half the instantaneous modulus of elasticity of concrete*. (Its effect is felt in arches, frames and fixed beams, not in simply supported and continuous beams which are not restrained from movement. However, this effect is required in them for estimating movements at bearings and expansion-joints.)

b) 'Eigenstresses resulting from the non-linearity of the temperature distribution across the deck depth' as shown in Fig. 30.9, *adopting the instantaneous modulus of elasticity of concrete*. The effect of re-radiation (soffit hotter than top) may be ignored owing to the predominant nature of temperature.



Note Eigenstress may be ignored when h < 25 cm

Fig. 30.9 *Recommended non-linear differential thermal gradients*

Same as above but across the thickness of abutment-leg in a frame, with front hotter.

c) 'Continuity stresses' resulting from the 'continuity moments' caused by the restraint to free hogging (or sagging) desire of the structure under a reversible 'lump' temperature difference of $T = \pm 12°C$ between the extreme fibres, *using the instantaneous modulus of elasticity of concrete*.

NOTE: Field observations of actual long term variations in the body-mean-temperatures and of the distribution of temperature across the deck-depth of different types of deck-sections, should be undertaken in order to refine the above recommendations. In this connection Reference 2 is useful. This work may be undertaken by National Road Research Institutes in collaboration with the relevant Ministry Authorities.

30.7 NUMERICAL EXAMPLES

A) Simply Supported Concrete Deck

Estimate the thermal stresses in a simply supported beam-and-slab concrete deck (instantaneous $E_c = 2.4 \times 10^5$ kg/cm^2). Representative 'beam' section is as shown in Fig. 30.10. Follow the Recommendations stated earlier in 30.6.

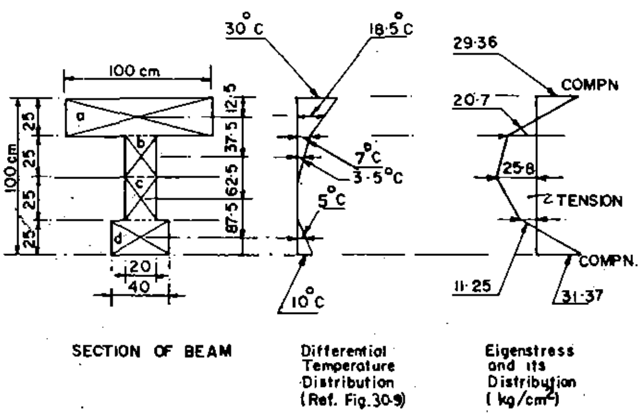


Fig. 30.10

Solution

Step 1 Stresses due to variation in body-mean-temperature are zero, since the deck is free to move longitudinally.

Step 2 Eigenstresses will exist since distribution of temperature across deck-dept is non-linear. These are estimated as shown in Table 30.1.

Refer to Eqs. (30.3) and (30.4) and Fig. 30.10.

Substituting in Eqs. (30.3) and (30.4) and taking $\alpha = 1.17 \times 10^{-5}/°C$:

$$\epsilon_0 \cdot (0.45) - \theta(0.16875) = 1.17 \times 10^{-5}(5.3)$$
$$\epsilon_0(0.16875) - \theta(0.10703) = 1.17 \times 10^{-5}(1.08125)$$

solving these, we get $\epsilon_0 = 2.2868 \times 10^{-4}$ and $\theta = 2.4235 \times 10^{-4}$ rad./m.

Hence eigenstress can be evaluated from Eq. (30.2) tabularly as shown in Table 30.2, taking:

$E_c = 2.4 \times 10^5$ kg./cm^2 $\quad \theta = 2.4235 \times 10^{-4}$ rad./m,
$\varepsilon_0 = 2.2868 \times 10^{-4}$, and $\alpha = 1.17 \times 10^{-5}/°C$

The above calculated eigenstress values, f_{ei}, have been plotted in Fig. 30.10 for a pictorial view.

Step 3 Stresses due 'continuity effect' are zero, deck being simply supported and hence free to hog or sag.

Hence, total of the thermal stresses in this case (a simple span deck) is only the eigenstress as calculated in Step 2 above, shown in Fig. 30.10.

B) Continuous-Beam Concrete Deck

Estimate the thermal stresses in a prismatic 2-span continuous beam-and-slab concrete deck (instantaneous $E_c = 2.4 \times 10^5$ kg/cm^2). Representative 'beam' section is same as used in the previous simply supported deck illustration, shown in Fig. 30.10. Follow the Recommendations stated earlier.

Solution

Step 1 Stresses due to variation in body-mean-temperature are zero since the deck is free to move longitudinally.

Table 30.1

ZONE	A (m^2)	y (m)	Ay (m^3)	Ay^2 (m^4)	t $(°C)$	At	Aty
a	$1.0 \times 0.25 = 0.25$	0.125	0.03125	0.00391	18.5	4.625	0.57812
b	$0.20 \times 0.25 = 0.05$	0.375	0.01875	0.00703	3.5	0.175	0.06563
c	$0.20 \times 0.25 = 0.05$	0.625	0.03125	0.01953	0	0	0
d	$0.40 \times 0.25 = 0.10$	0.875	0.08750	0.07656	5.0	0.500	0.43750
sum	0.45		0.16875	0.10703		5.30	1.08125
	ΣA		ΣAy	ΣAy^2		ΣAt	ΣAty

Table 30.2

y (m)	$y\theta$	t (°C)	αt	$f_{ci} = E_c(\epsilon_0 - y\theta - \alpha t)$ kg/cm²
0	0	30	3.51×10^{-4}	−29.36 (compr.)
0.25	0.606×10^{-4}	7	0.819×10^{-4}	20.70 (tension)
0.50	1.212×10^{-4}	0	0	25.80 (tension)
0.75	1.818×10^{-4}	0	0	11.25 (tension)
1.00	2.424×10^{-4}	10	1.17×10^{-4}	−31.37 (compr.)

Step 2 Eigenstresses will exist since distribution of temperature across deck-depth is non-linear. These are estimated exactly as illustrated in the previous example and since section assumed is the same, the eigenstresses are also the same. These then are as shown in Fig. 30.10.

Step 3 Stresses due to 'continuity effect' will exist in the present case since a continuous beam, despite desiring to hog up (or sag down) freely between its abutments because of difference in extreme fibre temperatures, is prevented from doing so because of the presence of intermediate supports, which compel the beam to hog (or sag) only in between the consecutive supports.

First the moments due to this 'continuity effect' may be worked out as explained earlier and in Fig. 30.4. This moment diagram, in the present case, is then as shown in Fig. 30.11, and the 'continuity stress' at any fibre at any section will be obtained by dividing the 'moment' at that section by the appropriate section modulus to the concerned fibre.

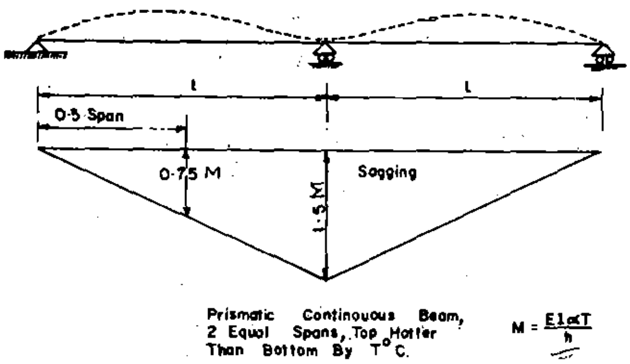


Fig. 30.11 *'Continuity' moment diagram*

As an example, let us find these stresses at the central support section (where second moment of area of the section is $I = 46 \times 10^5$ cm⁴ and $y_t = 37.5$ cm, $y_b = 62.5$ cm)

Continuity moment = $1.5EI\alpha T/h$

$$= \frac{1.5 \times (2.4 \times 10^5) \times (46 \times 10^5) \times (1.7 \times 10^{-5}) \times (\pm 12)}{100}$$

$= 23.25 \times 10^5$ kg.cm (reversible)

Continuity stresses—

$$\text{at top: } \pm \frac{23.25 \times 10^5}{46 \times 10^5} \times 37.5 = \pm 18.95 \text{ kg/cm}^2$$

$$\text{at bottom: } \pm \frac{23.25 \times 10^5}{46 \times 10^5} \times 62.5 = \pm 31.59 \text{ kg/cm}^2$$

(varying linearly in between)

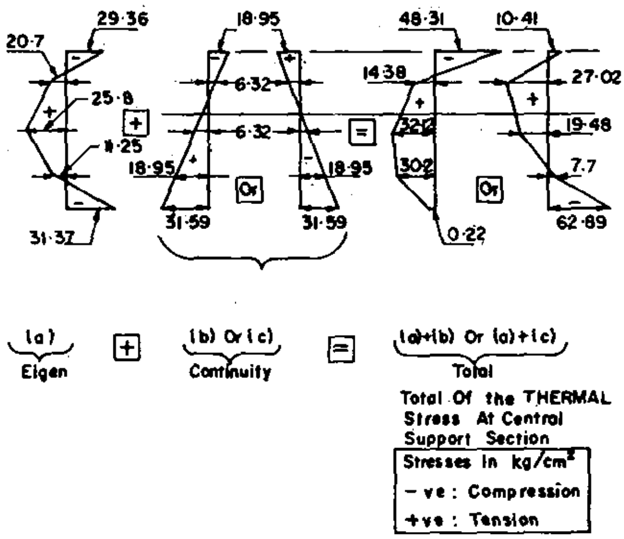


Fig. 30.12

Hence the total thermal stress configuration at the central support section, being the superposition of the above calculated eigenstresses and the continuity stresses, is as shown in Fig. 30.12. This may be checked at other critical sections too.

REFERENCES

1. Priestly, MJN, 'Thermal Gradients in Bridges—Some Considerations', *New Zealand Eng.*, Wellington, Vol. 27, No: 7, July, 1972.
2. Emerson, M., 'The calculation of distribution of temperature in Bridges', *UK Transport and Road Research Laboratory*, Report LR. 561, 1973.
3. Hunt, Bruce and Cook, 'Thermal calculations for Bridge Design', Proc., ASCE, Vol. 101, ST9, Sept. 1975
4. Hambly, EC 'Temperature distributions and stresses in Concrete Bridges', *The Structural Engineer*, Vol 56A, No: 5, May 1978.

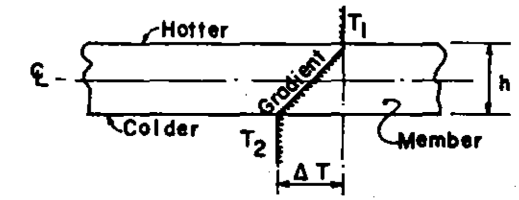
5. Priestly, MJN, 'Design of Concrete Bridges for temperature gradients', *ACI Journal*, May 1978.
6. Leonhardt, *Ribschaden an Betonbrucken—Ursachen und Abhilfe* (i.e. Crack damages at Concrete Bridges—cause and remedy), Beton und Stahlbetonbau, 2/1979.
7. El Badri and Amin Ghali, 'Nonlinear temperature distribution and its effects on bridges', IABSE Zurich, *Periodica* 3/1983, Aug. 1983.

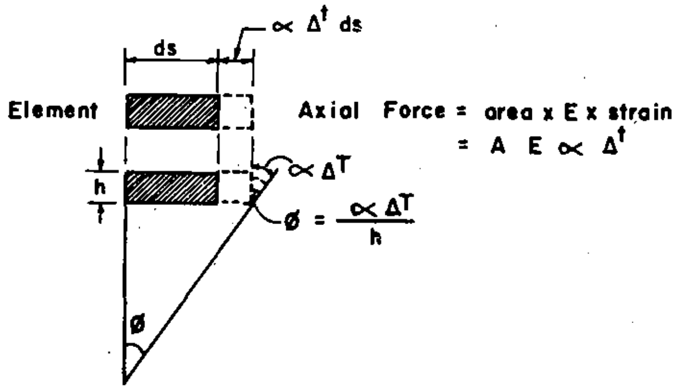
ANNEXURE

Estimation of the Effects of "Change in Body Mean Temperature" and "Unequal Extreme Fibre Temperatures" (by the Flexibility Method)

Assumptions:

- temperature varies linearly (note that the effect of non-linearity of temperature variation through section-depth is considered separately in Eigenstress calculation, described in the body of the chapter).
- no temperature 'discontinuity' between member surface and air.





$$\phi = \text{curvature} = \frac{1}{R}$$

$$\frac{M}{I} = \frac{E}{R}, \quad M = EI\phi$$

$$= EI\frac{\alpha \Delta T}{h}$$

$T_1, T_2 \ldots$ extreme fibre temperatures

$\Delta T = T_1 - T_2$ = difference in extreme fibre temperatures

$T_0 \ldots$ initial 'mean' temperature

$T_1 = T_0$ + maximum rise in temperature

$T_2 = T_0$ − maximum fall in temperature

$\Delta t = \left(\frac{T_1 + T_2}{2} - T_0\right)$ = change in body mean temperature

α = coefficient of thermal expansion/contraction of the material

h = section − depth

- axial force (thrust, tensile/compressive) = section area $A \times E \times \alpha\Delta t$
- bending moment $M = EI\alpha\Delta T/h$

These 'bending moment' $(EI\alpha\Delta T/h)$ and 'thrust' $(AE\alpha\Delta t)$ diagrams, when drawn along the structure, represent the 'applied load' moment and thrust diagrams on the statically made determinate structure. These are 'akin' to m_0 and n_0 diagrams explained in the Flexibility Method given earlier, but do not physically exist if the structure is made determinate (just as there is no m_0 diagram in case of differential settlement of supports in a continuous beam). Rest of the procedure is same as in the regular Flexibility Method. The following numerical example explains the process in a simple workman-like manner. However, thrust energy must be considered in evaluating the influence coefficients, viz.

$$V_{rs} = \int \frac{m_r m_s}{EI}\, ds + \int \frac{n_r n_s}{EA}\, ds$$

and $$V_{ro} = \int \frac{m_r m_o}{EI}\, ds + \int \frac{n_r n_o}{EA}\, ds$$

The general compatibility equations of elastic stability remaining:

$$V_{ro} + V_{rs} p_s = 0 \left\{ \begin{array}{l} r \text{ ranging from 1 to } n, \\ s \text{ ranging from 1 to } n, \\ n \text{ being the number of statical-indeter-} \\ \quad \text{minancies in the structure.} \end{array} \right.$$

Here, consequently,

$$V_{ro} = \int \frac{m_r (EI\alpha\Delta T/h)}{EI}\, ds + \int \frac{n_r (AE\alpha\Delta t)}{EA}\, ds$$

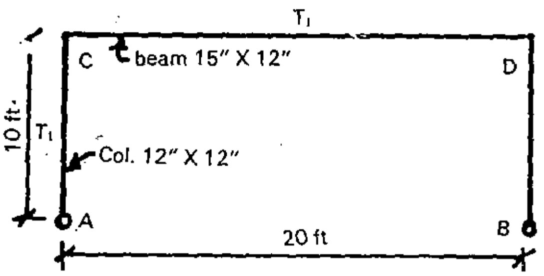
or simply, $V_{ro} = \alpha \int m_r (\Delta T/h)\, ds + \alpha \int n_r (\Delta t)\, ds$

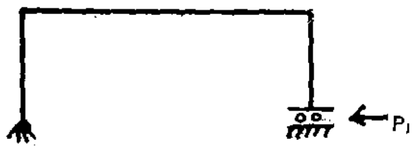
Numerical Example

A two hinged portal in reinforced concrete,* 10ft. high, 20ft. span, T_1 on outsides and T_2 on insides, columns 12″ × 12″, transome 15″ × 12″,

$$T_1 = T_0 + 35°\text{F}, \quad T_2 = T_0 - 15°\text{F},$$
$$\Delta T = T_1 - T_2 = 50°\text{F}$$

* Actually in the case of structural concrete the Flexibility Analysis shown in this Example should infact be done separately for the $(\Delta T/h)$ case using instantaneous E (this being the case of "Temperature difference between extreme fibres") and then separately for the (Δt) case taking E as half of the instantaneous E value (this being the case of "change in body-mean-temperature"—a long term effect).





$$\Delta t = \frac{T_1 + T_2}{2} - T_0 = 10°\text{F}, \quad \alpha = \frac{1}{180,000}\ \text{per°F}$$

$h_{\text{beam}} = h_b = 1.25\text{ft.}, \quad h_{\text{col.}} = h_c = 1.0ft,$

breadth $= 1.0$ft.

$E = 2 \times 10^6$ psi or 288×10^6 psf

(assumed constant for this example),

$$I_{\text{beam}} = I_b = \frac{1}{12} \times 1(1.25)^3 = 0.1628\ \text{ft}^4$$

$$I_{\text{col.}} = I_c = \frac{1}{12} \times 1(1)^3 = 0.0833\ \text{ft}^4$$

$\therefore I_b = 1.95 I_c$

$A_{\text{beam}} = A_b = 1.25 \times 1 = 1.25\ \text{ft}^2$

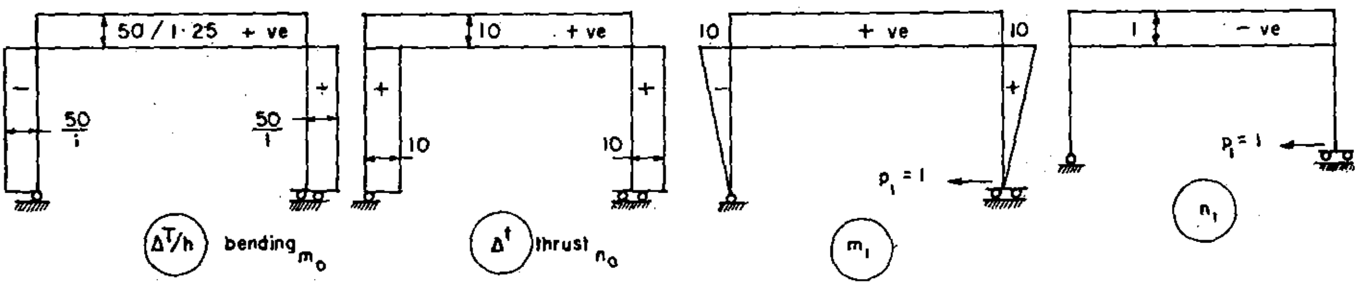
$A_{\text{col.}} = A_c = 1 \times 1 = 1.00\ \text{ft}^2$

$\therefore A_b = 1.25 A_c$

$\alpha_s = 3(M - N + 1) - R = 3(4 - 4 + 1) - 2 = 1 \ldots$

once statically indeterminate. (i.e. $n = 1$)

Introduce one release, a shear biaction p_1, at (say) right side foot:



$\Delta T/h$ and Δt diagrams are as shown. Due to unit p_1 the m_1 and n_1 diagrams are as shown.

$$V_{10} + V_{11} p_1 = 0, \quad \therefore p_1 = \frac{-V_{10}}{V_{11}}$$

$$V_{10} = \int \frac{m_1 \cdot (EI \propto \Delta T/h)}{EI}\, ds + \int \frac{n_1 \cdot (EA \propto \Delta t)}{EA} \cdot ds$$

$$= \alpha \left[\int m_1 \cdot \Delta^T/h \cdot ds + \int n_1 \cdot \Delta^t \cdot ds \right]$$

$$= \alpha \left[\left\{ 2 \times \frac{10}{6} \left(0 + 4 \times \frac{-10}{2} \times \frac{-50}{1} + (-10) \times \frac{-50}{1} \right) \right\} + \left\{ \frac{20}{6} \left(\frac{50}{1.25} \times 10 + 4 \times \frac{50}{1.25} \times 10 + \frac{50}{1.25} \times 10 \right) \right\} + \left\{ \frac{20}{6} (10 \times (-1) + 4 \times 10 \times (-1) + 10 \times (-1)) \right\} \right]$$

$$= +0.07111$$

$$V_{11} = \int \frac{m_1 m_1}{EI}\, ds + \int \frac{n_1 n_1}{EA}\, ds$$

$$= \left[\left\{ 2 \times \frac{10}{6EI_c} \left(0 + 4 \times \frac{-10}{2} \times \frac{-10}{2} + (-10)(-10) \right) + \frac{20}{6EI_b} (10 \times 10 + 4 \times 10 \times 10 + 10 \times 10) \right\} + \left\{ \frac{20}{6EA_b} (-1 \times -1 + 4 \times (-1)(-1) + (-1)(-1)) \right\} \right]$$

$$= \left(\frac{1692.3}{EI_c} + \frac{16}{EA_c} \right)$$

Substituting the values for E, I_c, A_c and finally the values of V_{10} and V_{11}, we finally get $p_1 = -1008$ lb

Hence, as a result of the above noted 'change in body mean temperature' and 'the temperature difference between extreme fibres':

moment at $A = m_1 p_1 = 0$, at $B = m_1 p_1 = 0$, at C in $AC = m_1 p_1 = (-10)(-1008) = 10,080$ lb.ft

at C and D in $CD = m_1 p_1 = 10 \times (-1008) = -10,080$ lb.ft.

at D in $BD = m_1 p_1 = 10 \times (-1008) = -10,080$ lb.ft.

thrust at $A = n_1 p_1 = 0$, at $B = n_1 p_1 = 0$, at C in AC and D in $BD = n_1 p_1 = 0$

at C and D in $CD = n_1 p_1 = (-1)(-1008) = 1008$ lbs

shear at any section in $AC = s_1 p_1 = \left(-\frac{10}{10} \right)(-1008) =$ 1008 lbs

at any section in $CD = s_1 p_1 = 0 \times (-1008) = 0$

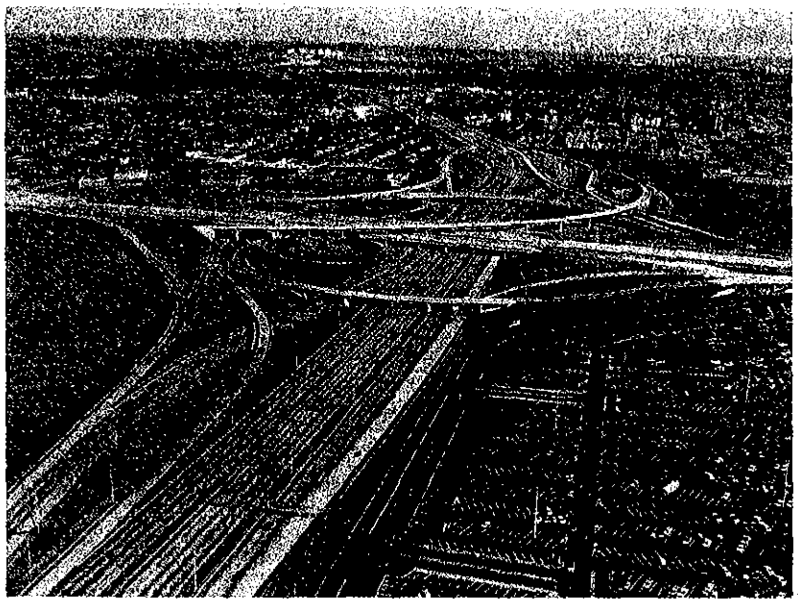


Fig. 31.1 *The Spadina interchange on Highway 401, Toronto*

Fig. 31.2 *Intersection of Highway 401 and Avenue Road, Toronto*

Fig. 31.3 *The interchange at Highways 400 and 401, Toronto*

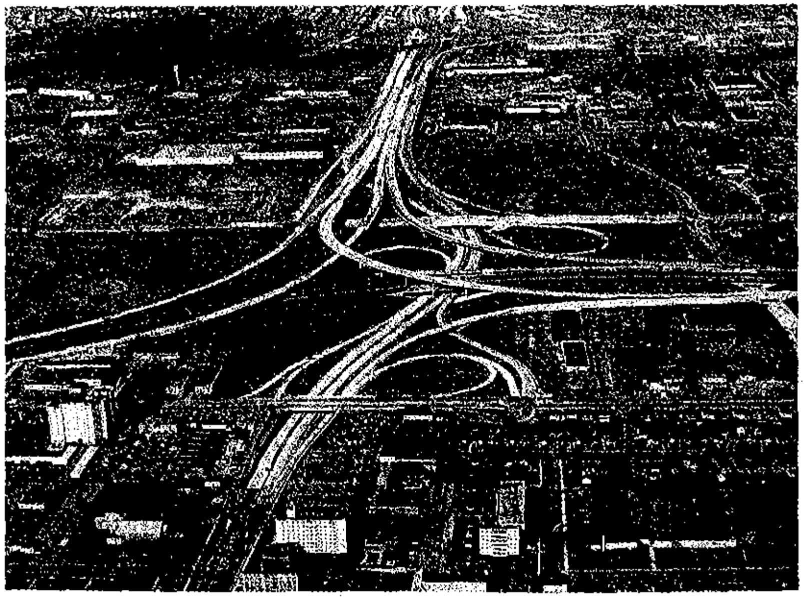


Fig. 31.4 *Highway 27 and Queen Elizabeth way interchange, Toronto*

(Photographs courtesy : Dept. Transptn. Commncn., Ontario)

Fig. 31.1 *The Spadina interchange on Highway 401, Toronto*

Fig. 31.2 *Intersection of Highway 401 and Avenue Road, Toronto*

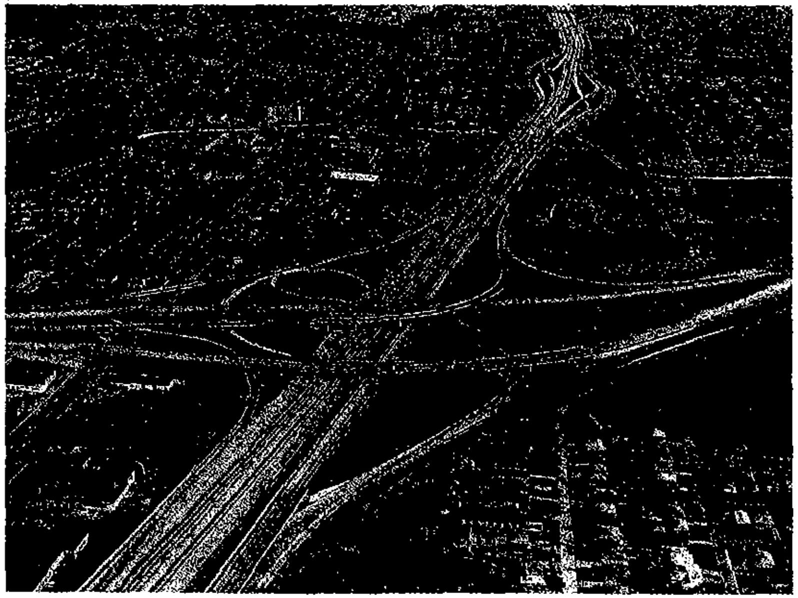


Fig. 31.3 *The interchange at Highways 400 and 401, Toronto*

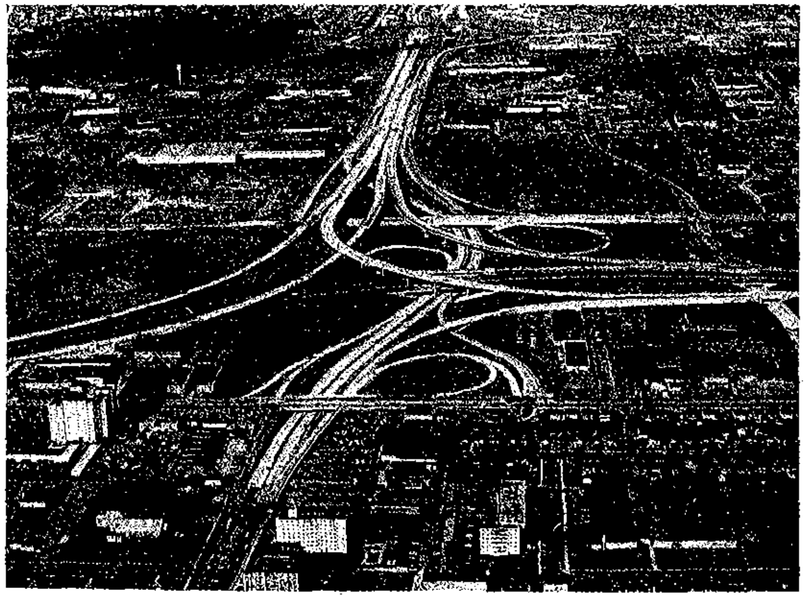


Fig. 31.4 *Highway 27 and Queen Elizabeth way interchange, Toronto*

(Photographs courtesy : Dept. Transptn. Commncn., Ontario)

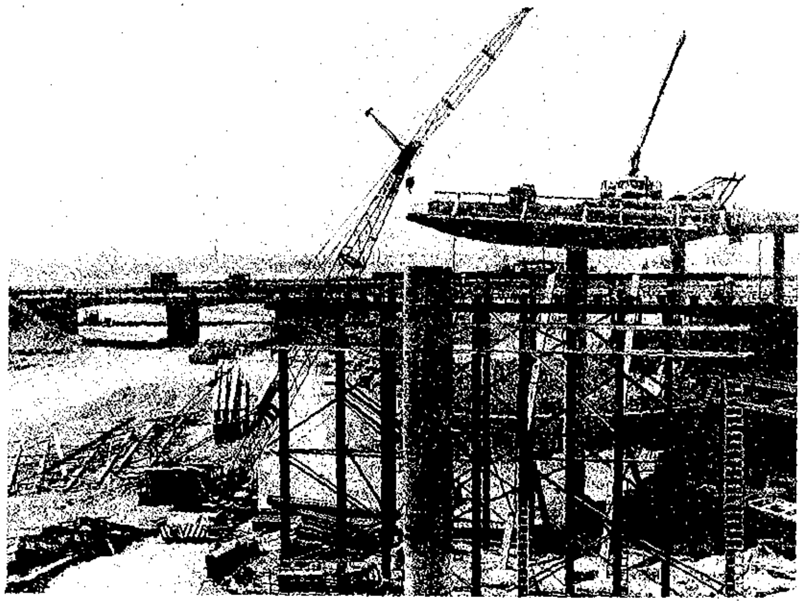


Fig. 31.5 *A bridge at Highway 27 and Queen Elizabeth way interchange, Toronto (under construction)*

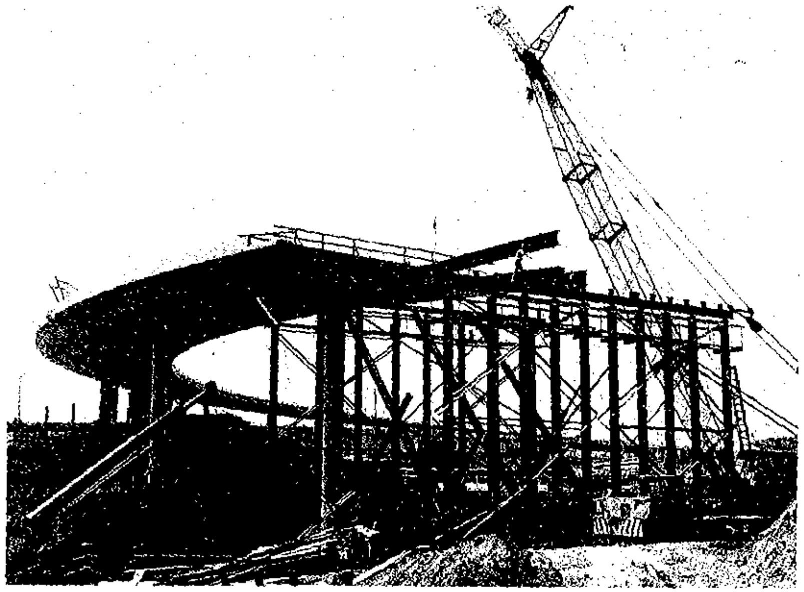


Fig. 31.6 *The steel frame of the bridge at Highway 27 and Queen Elizabeth way interchange, Toronto (temporary structure)*

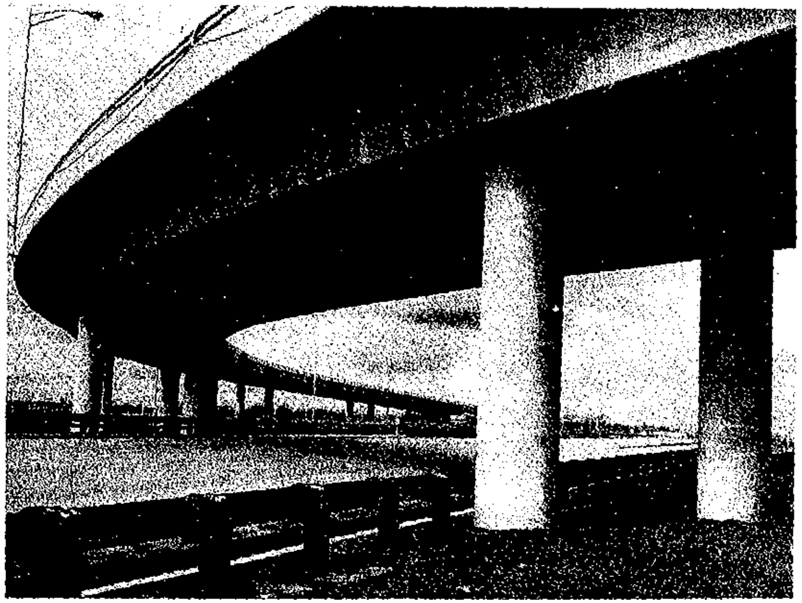


Fig. 31.7 *A bridge at junction of Highways 400 and 401, Toronto*

Fig. 31.8 *A bridge at the Spadina interchange of Highway 401, Toronto*

(Photographs courtesy : Dept. Transptn. Commncn., Ontario)

CHAPTER 31

Transverse Analysis of Some Typical Concrete Deck-Sections, i.e. Analysing a Deck Cross-section

31.1 BACKGROUND

With the development of modern highway networks, involving complex interchanges at their crossings, the bridge designer today is frequently confronted with the problem of bridging skewed and sloping gaps, which may, more often than not, have sweeping curves in plan. This factor, coupled with the stringent requirements for sight distance and clearance, often compels the designer to consider newer deck sections for meeting the conditions both aesthetically and economically. Needless to say, the designer can tackle the problem with the aid of the digital computer but for which, it would have been impossible to speedily analyse the combination of continuity, curve and skew.

The days are past when a highway would be realigned at a crossing in order to build a bridge according to the designer's conventional methods of analysis. Now the designer has to explore more refined techniques of calculation to fit in with the complicated geometry of the bridge alignment and the complex behaviour of the new types of deck sections. It would be almost unusual these days to think of bridging at highway intersection gap by the good old girder and slab section, except at right bridge crossings with almost no restrictions on the section depth, etc.

The present-day trend in Europe and in the American continent is to use box-sections. These sections enable hogging and sagging moments to be resisted efficiently, the spans being almost always continuous. Moreover, such sections are also sturdy enough to withstand torsion, and have more efficient load distribution characteristics. Figures 31.1 to 31.8 (art plates) illustrate some modern highway inter-section crossings using the latest types of prestressed concrete box-section bridges.

The current methods for the analysis of the transverse slab moments in bridge decks vary from the straight forward 'continuous beam on rigid supports' analysis to the 'plate' and 'grid' harmonic analyses, depending on the type of deck. For reasonably realistic answers by some of these methods, the use of a digital computer becomes essential, and this either leaves an average designer helpless, or he has to indulge in glorified guesswork.

It is the purpose here to try and derive the transverse slab moments by relatively easily applied and yet reasonably accurate procedures. The designer, nevertheless, will have to exercise his own judgement in particular cases. The results can only be as accurate as the validity of the assumptions that are made in deriving them. Given here are the methods of transverse analysis of five deck sections. Each of these methods of analysis is supplemented by an illustrative numerical example of an actual bridge.

Some Typical Deck Sections

Some of the more popular types of prestressed concrete deck sections are shown in Figs. 31.9 to 31.13. Figure 31.9 shows a twin-box-section bridge deck, interconnection being provided only by a relatively flexible top slap. This section looks slender and lends itself very efficiently to precast segmental construction. Figure 31.10 shows a multicell box-section with webs of equal thickness. Such a section is eminently suitable for in-situ construction, and has inherent strength against torsion and moment variation. It is generally made solid at the supports. Figure 31.11 represents a multicell box-section with a stub central web. It has all the advantages of the multicell box-section with webs of equal thickness and is particularly suited for bridges involving single column-pier supports. Figure 31.12 illustrates a two-cell box-section with solid central spine. This too has all the advantages of the multicell box-section with stub central web, and is suited for a bridge carrying less number of traffic lanes. Figure 31.13 shows a voided deck section. This type of bridge deck is particularly suitable where the depth of construction is severely restricted. This deck section may also be referred to as a voided slab deck. All voids need not be of the same size and even the same void need not be of a constant diameter along the length of the bridge. Void formers in hard cardboard, or thin sheet-metal are used for forming the voids in in-situ construction. Invariably such bridge decks are made solid at the piers and the abutments.

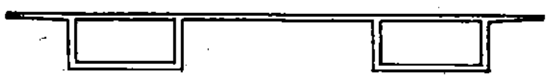


Fig. 31.9 *Twin-box-section bridge deck with interconnecting top slab*

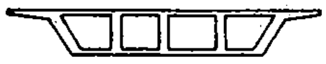


Fig. 31.10 *Multicell box-section with webs of equal thickness*

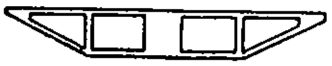


Fig. 31.11 *Multicell box-section with a stub central web*

Fig. 31.12 *Two-cell box-section with solid central spine*

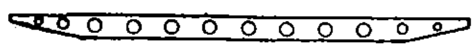


Fig. 31.13 *Voided deck-section*

CASE (A): Twin-box-section with an interconnecting top slab

Figure 31.14 is illustrative of a twin-box-section deck in which the boxes are interconnected only by a top slab. The two boxes can be loaded in different proportions by the applied live load, which itself may be variable along the length and width of the bridge. This can lead to heavy reversal of deformations in the deck slab, whereby the support and the span sections C and D may in fact be subjected to sagging and hogging bending moments, respectively. The cantilever section A, and the span section B may be analysed by conventional methods since they behave more like similar sections in a normal continuous beam on rigid supports, by virtue of the geometry of the deck section. For the critical sections C and D under applied live load, a more elaborate analysis is necessary because of the relative flexibility of the slab interconnecting the two boxes. A method is presented here for this analysis, in which the live load is placed for the maximum bending effect at section C. The procedure may be repeated for any other load position.

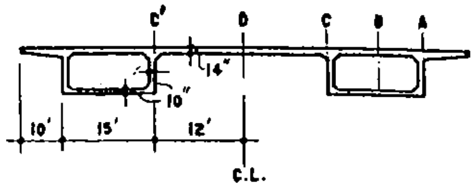


Fig. 31.14 *Twin-box-section deck with the boxes interconnected by the top slab*

• ***Analysis*** : Any particular distributed load may at any section be resolved into two loads, one of which is symmetrical and the other asymmetrical in the cross-section of the deck (Fig. 31.15). Under the symmetrical load, the system will deform symmetrically about the axis of symmetry of the cross-section, so that the shear there will be zero. Under the asymmetrical load, the system will deform asymmetrically about the axis of symmetry of the cross-section, so that the bending moment there is zero. If the system is 'cut' through the axis of symmetry, all along its length, it can be rendered statically determinate by introducing two redundant bi-actions q_x and M_x, the vertical shear and bending moment respectively. The third stress resultant, namely, the thrust bi-action, may be neglected here since the two boxes will not roll inwards or outwards laterally after introducing the 'cut'. The slab spanning in the transverse direction is assumed to be replaced by transverse beams of width dx, whose torsional rigidity is disregarded. The redundants q_x and M_x are functions of the longitudinal ordinate x and may be expressed as compound sine-curves as given below:

$$q_x = q_1 + q_2\left(\alpha \sin\frac{\pi x}{L} + \beta \sin\frac{3\pi x}{L}\right)$$

where $\alpha + \beta = 1$, L is the longitudinal span.

$$M_x = M_1 + M_2\left(\gamma \sin\frac{\pi x}{L} + \delta \sin\frac{3\pi x}{L}\right)$$

where $\gamma + \delta = 1$.

The two redundants may be evaluated from the two compatibility equations for continuity at the cut. In the case of symmetric loading, the slope at midspan of the slab is zero, while in the asymmetric case, the deflection at midspan of the slab is zero. In the symmetric case of loading, the rotations caused at the cut by the load and by M_x will sum up to zero, and in the asymmetric case of loading, the deflections caused at the cut by the load and by q_x will sum up to zero.

It should be noted that since the expressions for q_x and M_x themselves involve three unknowns each, there will in

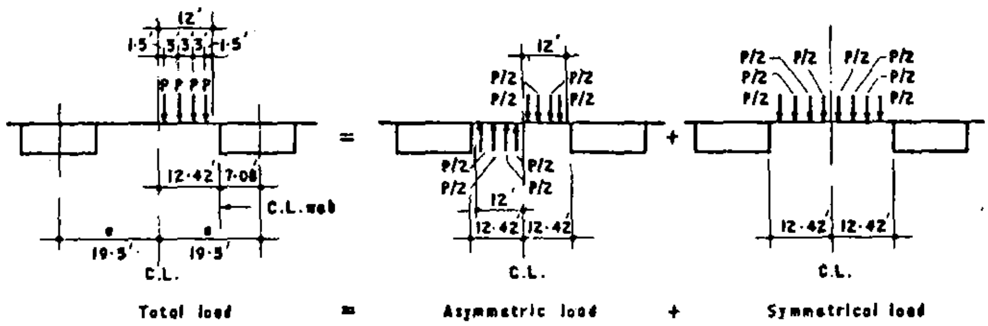


Fig. 31.15 *Total distributed load at a section resolved into asymmetrical and symmetrical loads*

fact be three unknowns in each of the two compatibility equations. These may, however, be solved by substituting for three values of x $\left(\text{say, } x = 0, x = \dfrac{L}{6} \text{ and } x = \dfrac{L}{2}\right)$ in each compatibility equation, thus getting three simultaneous equations for q_x and three for M_x. Solving these will completely define the two redundants.

• ***Wheel load as a mathematical function***: The live load under consideration here is the one according to the British highway bridge loading. This is placed on the bridge longitudinally as shown in Fig. 31.16. Transversely it is so placed on the bridge-deck as to create the maximum gap at the cut. It is assumed that a 45-ton HB axle of four wheels is dispersed transversely over a width of 12 ft (3.66 m). The applied live load may be expressed as a smooth continuous loading function in terms of Fourier series as

$p(x) = \dfrac{2P}{L}\left(3.7398 \sin\dfrac{\pi x}{L} - 1.8942 \sin\dfrac{3\pi x}{L}\right)$ on two boxes; details are given in Annexure I.

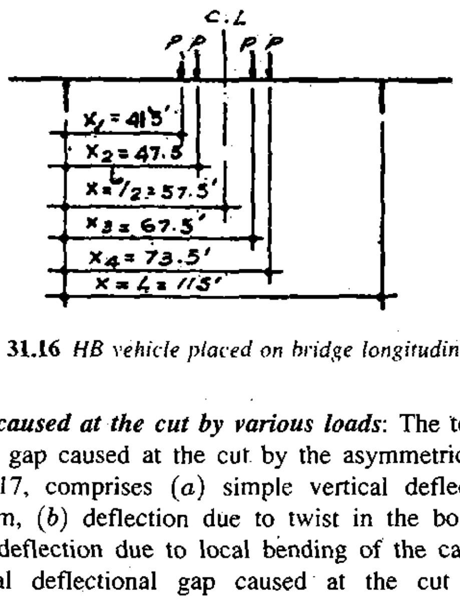


Fig. 31.16 *HB vehicle placed on bridge longitudinally*

• ***Gaps caused at the cut by various loads***: The total deflectional gap caused at the cut by the asymmetrical load, Fig. 31.17, comprises (a) simple vertical deflection of box beam, (b) deflection due to twist in the box beam, and (c) deflection due to local bending of the cantilever. The total deflectional gap caused at the cut by q_x, Fig. 31.18, consists of (d) simple vertical deflection of box beam, (e) deflection due to twist in the box beam and (f) deflection due to local bending of the cantilever. Hence one compatibility equation is formed as given below:

$$(a) + (b) + (c) = (d) + (e) + (f) \qquad (31.1)$$

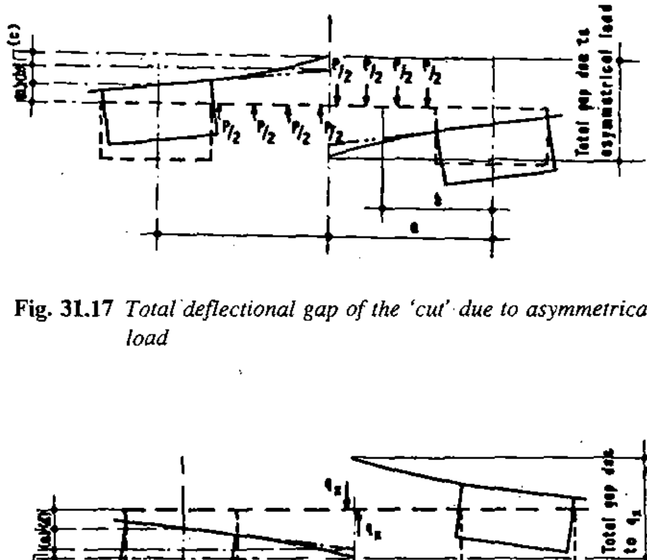


Fig. 31.17 *Total deflectional gap of the 'cut' due to asymmetrical load*

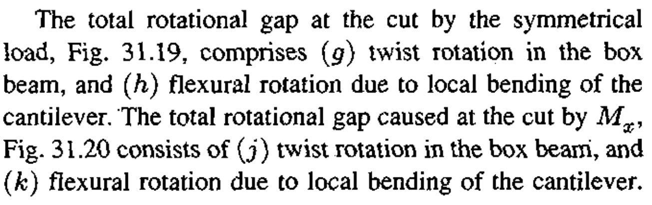


Fig. 31.18 *Total deflectional gap at the 'cut' due to q_x*

The total rotational gap at the cut by the symmetrical load, Fig. 31.19, comprises (g) twist rotation in the box beam, and (h) flexural rotation due to local bending of the cantilever. The total rotational gap caused at the cut by M_x, Fig. 31.20 consists of (j) twist rotation in the box beam, and (k) flexural rotation due to local bending of the cantilever.

A second compatibility equation is formed as follows:

$$(g) + (h) = (j) + (k) \tag{31.2}$$

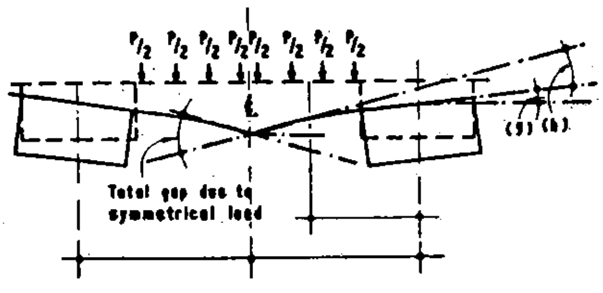


Fig. 31.19 *Total rotational gap at the 'cut' due to symmetrical load*

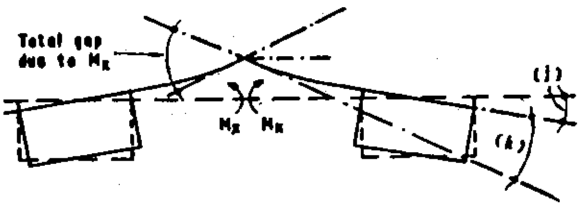


Fig. 31.20 *Total rotational gap at the 'cut' due to M_x*

• ***Formulae for various deflections and rotations***: Expressions for the various deflections and rotations, referred to as (a), (b), (c), (d), (e) and (f), and (g), (h), (j) and (k) respectively, are worked out in Annexure 2, and their final values are given as follows:

$$(a) = \frac{PL^3}{81\pi^4 EI_b}\phi_1 + \frac{PL}{15.39 EI_b}\phi_2$$

$$(b) = \frac{PabL}{9\pi^2 GK}\phi_3$$

$$(c) = \frac{Pl^4}{96LEI_c}\phi_4$$

$$(d) = \frac{q_1}{67.13EI_b}\phi_6 + \frac{q_2 L^4}{81\pi^4 EI_b}\phi_5 + \frac{(2\alpha - 1)L^2 q_2}{62.25EI_b}\phi_7$$

$$(e) = \frac{a^2 q_1}{2GK}\phi_8 + \frac{a^2 q_2 L^2}{9\pi^2 GK}\phi_9$$

$$(f) = \frac{q_1 l^3}{3EI_c} + \frac{q_2 l^3}{3EI_c}\phi_{10}$$

$$(g) = \frac{PbL}{9\pi^2 GK}\phi^3$$

$$(h) = \frac{Pl^3}{72LEI_c}\phi_4$$

$$(j) = \frac{M_1}{2GK}\phi_8 + \frac{M_2 L^2}{9\pi^2 GK}\phi_{11}$$

$$(k) = \frac{M_1 l}{EI_c} + \frac{M_2 l}{EI_c}\phi_{12}$$

where
$$\phi_1 = 302.8\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}$$
$$\phi_2 = x(x - L)$$
$$\phi_3 = 33.62\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}$$
$$\phi_4 = 3.7398\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}$$
$$\phi_5 = 81\alpha\sin\frac{\pi x}{L} + (1-\alpha)\sin\frac{3\pi x}{L}$$
$$\phi_6 = x(2.8x^3 - 5.6Lx^2 + 1.8L^2 x + L^3)$$
$$\phi_7 = x(x - L)$$
$$\phi_8 = x(L - x)$$
$$\phi_9 = 9\alpha\sin\frac{\pi x}{L} + (1-\alpha)\sin\frac{3\pi x}{L}$$
$$\phi_{10} = \alpha\sin\frac{\pi x}{L} + (1-\alpha)\sin\frac{3\pi x}{L}$$
$$\phi_{11} = 9\gamma\sin\frac{\pi x}{L} + (1-\gamma)\sin\frac{3\pi x}{L}$$
$$\phi_{12} = \gamma\sin\frac{\pi x}{L} + (1-\gamma)\sin\frac{3\pi x}{L}$$

a = distance from centre-line of box to centre-line of cut
b = distance from centre-line of box to centre-line of applied load
l = half of the clear distance between two boxes
L = longitudinal span of the deck
EI = flexural rigidity (suffix b for box beam and c for cantilever)
GK = torsional rigidity of a box beam

For the example under illustration

l = 12 ft
L = 115 ft
a = 19.5 ft
b = 13.5 ft
p = 45 tons
EI_b = 142000 × 10^6 lb ft^2 (per box beam)
EI_c = 57.15 × 10^6 lb ft^2/ft width (per cantilever)
GK = 144500 × 10^6 lb ft^2 per box beam, Annexure 3.

• ***Evaluation of various deflections and rotations***: Substituting for P, a, b, L, l, EI_b, EI_c, and GK, in the formulae derived earlier for various deflections and rotations,

$(a) = 10^{-10}$ $(1367767.2957\phi_1 + 53061.9718\phi_2)$
$(b) = 10^{-10}$ $(2377022.2420\phi_3)$
$(c) = 10^{-10}$ $(33128380.6900\phi_4)$
$(d) = 10^{-10}$ $[0.0010q_1\phi_6 + 1560.4488\, q_2\phi_5 + 14.9612$
$(2\alpha - 1)q_2\phi_7]$

$(e) = 10^{-10}\ (13.1522 q_1\phi_8 + 0.2961\ q_2\phi_9)$
$(f) = 10^{-10}\ [100787.4015(q_1 + q_2\phi_{10})]$
$(g) = 10^{-10}\ (121898.00\ \phi_3)$
$(h) = 10^{-10}\ (3680000.00\ \phi_4)$
$(j) = 10^{-10}\ (0.0346\ M_1\phi_8 + 10.3015\ M_2\phi_{11})$
$(k) = 10^{-10}\ (2099.7375 M_1 + 2099.7375 M_2\phi_{12})$

Substituting these values in compatibility Eqs. (31.1) and (31.2)

$$q_1\Delta_1 + q_2\Delta_2 = 53061.9718\ \Delta_3 \qquad (31.3)$$
$$M_1\Delta_4 + M_2\Delta_5 = 121898.00\ \Delta_6 \qquad (31.4)$$

where $\Delta_1 = (0.001\phi_6 + 13.1522\phi_8 + 100787.4015)$
$\Delta_2 = [1560.4488\ \phi_5 + 14.9612\ (2\alpha - 1)\ \phi_7 + 0.296\ \phi_9 + 100787.4015\ \phi_{10}]$
$\Delta_3 = (25.776\ \phi_1 + \phi_2 + 44.797\phi_3 + 624.33\ \phi_4)$
$\Delta_4 = (0.0346\ \phi_8 + 2099.7375)$
$\Delta_5 = (10.3015\ \phi_{11} + 2099.7375\ \phi_{12})$
and $\Delta_6 = (\phi_3 + 30.21\ \phi_4)$

Equation (31.3) contains three unknowns, namely q_1, q_2 and α, and, therefore, it is essential to develop three simultaneous equations in order to solve for the three unknowns. These three simultaneous equations may be obtained by substituting for Δ_1, Δ_2 and Δ_3, in it at three locations of x, say at $x = 0$, $\frac{L}{6}$, and $\frac{L}{2}$. Equation (31.4) also contains three unknowns, namely, M_1, M_2 and γ, for evaluating which it is again essential to generate three simultaneous equations. This may be done by substituting for Δ_4, Δ_5 and Δ_6 at the same three locations of x as in the case of Eq. (31.3) as explained above.

• ***Evaluation of the various ϕ and Δ functions***: Values of various ϕ functions and Δ functions at $x = 0$, $x = \frac{L}{6}$ and $x = \frac{L}{2}$, as computed from their relevant formulae derived earlier, are calculated and are given in Table 31.1.

Solving simultaneously the three equations generated from Eq. (31.3) by substituting for Δ_1, Δ_2, and Δ_3 in it, at $x = 0$, $x = L/6$, and $x = L/2$:

$$q_1 = 0$$
$$q_2 = 1558.51 \text{ lb/ft width}$$
$$\alpha = 0.8905$$

Hence the redundant q_x is defined and may be expressed as $q_x = 1558.51\left(0.8905 \sin\frac{\pi x}{L} + 0.1095 \sin\frac{3\pi x}{L}\right)$

Similarly solving simultaneously the three equations generated from Eq. (31.4) by substituting in it for Δ_4, Δ_5, and Δ_6, at $x = 0$, $x = \frac{L}{6}$, and $x = \frac{L}{2}$:

$$M_1 = 0$$
$$M_2 = 4735.40 \text{ lb ft/ft width}$$
$$\gamma = 1.7212$$

Table 31.1 *Values of various ϕ and Δ functions*

Function	At $x = 0$	At $x = L/6$	At $x = L/2$
ϕ_1	0	149.5058	304.6942
ϕ_2	0	−1836.8055	−3306.2500
ϕ_3	0	14.9158	35.5142
ϕ_4	0	−0.0243	5.6340
ϕ_5	0	$39.5\alpha + 1$	$82\alpha - 1$
ϕ_6	0	33739133.2320	74332765.6250
ϕ_7	0	−1836.8055	−3306.2500
ϕ_8	0	1836.8055	3306.2500
ϕ_9	0	$3.5\alpha + 1$	$10\alpha - 1$
ϕ_{10}	0	$1 - 0.5\alpha$	$2\alpha - 1$
ϕ_{11}	0	$3.5\gamma + 1$	$10\gamma - 1$
ϕ_{12}	0	$1 - 0.5\gamma$	$2\gamma - 1$
Δ_1	100787.4015	not required	not required
Δ_2	0	$(-43716.5657\alpha + 129828.961)$	$(428465.5\alpha - 52882.679)$
Δ_3	0	2669.8679	9655.9525
Δ_4	2099.7375	not required	not required
Δ_5	0	$(2110.039 - 1013.814\gamma)$	$(-2110.039 + 4302.49\gamma)$
Δ_6	0	14.1817	205.7173

Hence the redundant M_x is defined and may be expressed as

$$M_x = 4735.40\left(1.7212 \sin\frac{\pi x}{L} - 0.7212 \sin\frac{3\pi x}{L}\right)$$

Now, with q_x and M_x determined, and the applied live load $p(x)$ already defined, the bending moment and shear at the critical section C at any location along the span can be worked out. For example, at the midspan location, *i.e.*, at $x = L/2$:

$$p(x) = \frac{2P}{L}\left(3.7398 \sin\frac{\pi x}{L} - 1.8942 \sin\frac{3\pi x}{L}\right)$$
$$= \frac{2 \times 45 \times 2240}{115}(3.7398 + 1.8942)$$
$$= 9865 \text{ lb per ft width}$$

i.e., as a uniformly distributed load $w = \frac{9865}{12}$

$$= 822 \text{ lb per ft run per ft width}$$
$$q_x = 1558.51\left(0.8905 \sin\frac{\pi x}{L} + 0.1095 \sin\frac{3\pi x}{L}\right)$$
$$= 1558.51(0.8905 - 0.1095)$$
$$= 1218 \text{ lb/ft width}$$
$$M_x = 4735.40\left(1.7212 \sin\frac{\pi x}{L} - 0.7212 \sin\frac{3\pi x}{L}\right)$$

$$= 4735.40(1.7212 + 0.7212)$$
$$= 11550 \text{ lbft./ft.width}$$

Hence total bending moment at C, Fig. 31.21

$$= -\frac{wl^2}{2} + q_x l + M_x$$
$$= -\left(\frac{822 \times 12^2}{2}\right) + (1218 \times 12) + 11550$$
$$= -59200 + 14600 + 11550$$
$$= -33050 \text{ lb ft/ft width (hogging)}$$

Bending moment at C', Fig. 31.21,

$$= -q_x l + M_x$$
$$= -14600 + 11550$$
$$= -3050 \text{ lb ft/ft width, i.e., hogging}$$

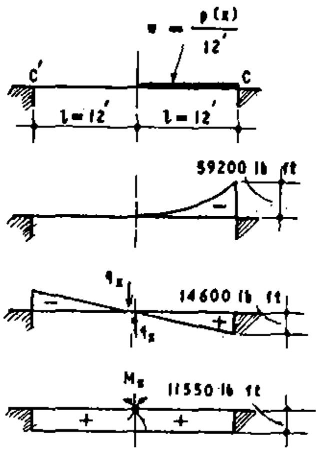


Fig. 31.21 *Bending moment distribution in-between boxes*

However, as the vehicle can be placed towards C' also, sections C and C' should both be designed for the bigger moment.

It is, however, necessary to be careful of the true bending moment at the critical section C, because for the vehicle placed in the longitudinal direction as shown in Fig. 31.16, physically there is no axle sitting at $x = L/2$ so that, in fact there are bending moment at C has very little effective contribution from $p(x)$; in other words, the bending moment at C

$$= q_x l + M_x$$
$$= 14600 + 11550$$
$$= 26150 \text{ lb ft/ft width, i.e., sagging}$$

Hence the critical sections C and C' in fact should be designed for a reversal of moments ranging from a hogging moment of 33,050 lb ft. to a sagging moment of 26,150 lb ft per ft width.

• ***Torsion in a box beam***: According to the general equation for torsion

$$\frac{T}{K} = \frac{G\theta}{L} \equiv G\frac{d\theta}{dx} \text{ so that } GK\frac{d\theta}{dx} = T$$

where G is modulus of rigidity, K the polar moment of inertia of the section, T the torsion, and θ the twist. Torsion in a box beam due to applied symmetric or asymmetric live load is given by:

$$T = GK\frac{d\theta}{dx}$$
$$= -\frac{Pb}{L}\left(-3.7398\frac{L}{\pi}\cos\frac{\pi x}{L} + 1.8942\frac{L}{3\pi}\cos\frac{3\pi x}{L}\right)$$

Details are given in Annexure 2. The maximum value of torsion occurs at $x = 0$ and at $x = L$, and is given by

$$-\frac{Pb}{L}\left(-3.7398\frac{L}{\pi} \sim 1.8942\frac{L}{3\pi}\right)$$

i.e., $-\frac{45 \times 2240 \times 13.5}{3\pi}(11.2194 \sim 1.8942)$
$$= 1343000 \text{ lb ft}$$

Torsion in a box beam due to qx is given by

$$T = GK\frac{d\theta}{dx} = aq_2\left[\frac{\alpha L}{\pi}\cos\frac{\pi x}{L} + \frac{(1-\alpha)L}{3\pi} \times \cos\frac{3\pi x}{L}\right]$$

Details are given in Annexure 2. Its maximum value occurs at $x = 0$ and $x = L$, and is given by:

$$\frac{aq_2 L}{3\pi}[3\alpha + (1 - \alpha)]$$

i.e., $\frac{19.5 \times 1558.5087 \times 115}{3\pi}\,[(2 \times 0.8905) + 1]$
$$= 1,031,000 \text{ lb ft}$$

Torsion in a box beam due to M_x is given by

$$T = GK\frac{d\theta}{dx} = M_2\left[\frac{\gamma L}{\pi}\cos\frac{\pi x}{L} + \frac{(1-\gamma)L}{3\pi} \times \cos\frac{3\pi x}{L}\right]$$

This equation is similar to the one above; only aq_2 is replaced by M_2, and α by γ. Maximum value occurs at $x = 0$ and at $x = L$, and is given by

$$\frac{M_2 L}{3\pi}[3\gamma + (1 - \gamma)]$$

i.e., $\frac{4735.40 \times 115}{3\pi}\,[(2 \times 1.7212) + 1]$
$$= 256,000 \text{ lb ft}$$

At a support in a box beam, combining the torsional moments resulting from the symmetric load, the asymmetric load, q_x and M_x, the aggregate torsional reaction:

= 1,343,000 ± 1,343,000 ± 1,031,000 + 256,000
= 1,399,000 lb ft, and −775,000 lb ft respectively, in the two boxes.

• ***Transverse load distribution between the two boxes***: For determining the severest loading that a box has to take, it is sufficient to assume that the slab distributes the live load into the two boxes in accordance with the principle of simple statics, without considering the compatibility of deformations of the section as a whole. From the point of view of safety there can be no objection to this method of transverse load distribution. Such an approach can, however, result in the overdesign of the boxes, even if local overloading of the deck slab has been taken care of by analysing the moments by the compatibility of deformations method referred to earlier.

In order to estimate the reaction factors for the two boxes under a particular live load on the deck, it is sufficient to determine the total vertical deflection of each box, say, e_1 and e_2; then the corresponding reaction factors are $e_1/(e_1+e_2)$ and $e_2/(e_1+e_2)$. It is of interest to observe that, under HB loading, the reaction factor for the load-side box is found to be 1.195 on the basis of simple statics between the boxes as against 0.833 found on the basis of compatibility of deformations.

• ***Procedure for determining*** e_1 ***and*** e_2: The live load is placed in the most eccentric position on the deck section, and is split into symmetric and asymmetric parts. A 'cut' is introduced in the interconnecting slab, releasing the two redundants q_x and M_x, the former valid in the case of asymmetric loading and the latter in the case of symmetric loading. The compatibility equation is framed in the case of asymmetric load (which involves q_x), and q_x is determined as explained earlier. Since M_x does not contribute to the vertical deflection of the box beams, there is no need to frame the second compatibility equation.

With q_x known, the simple vertical deflection of each of the two boxes is determined, say at $x = L/2$, due to the asymmetric load, the symmetric load, and q_x, using the formulae similar to those given under (a) and (d) earlier. Then the algebraic addition of the relevant deflections gives e_1 and e_2.

Also refer to Chapter 37 for transverse distribution of Live Load in two-box girder deck where the boxes are essentially interconnected only by the common topslab and support and endcross girders.

CASE (B): Multicell box-section with equal-thickness webs

The analysis takes into account differential deflection of the constituent beams under the applied load, and derives the transverse slab moments on the basis of the strain energy dissipated in flexing the transverse slab resting on elastically sinking supports. Considering a typical continuous concrete box-section bridge as shown in Fig. 31.22, having spans L_m, m being = 1, 2, 3, ..., n, it is assumed that the bridge deck has n vertical or nearly vertical webs, as shown in Fig. 31.23(a) and (b). For the design of the transverse (top) slab the governing live load is assumed to be a wheeled truck load, because the transverse slab spans are short. Assuming that the deck is composed of n 'slab-web-slab' elements, Fig. 31.23(c), interconnected and sinking elastically, the moment of inertia (second moment of area) of the entire effective section is given by:

$$I = \phi \sum_{w=1}^{n} I_w \tag{31.5}$$

where I_w = the moment of inertia of the w^{th} 'slab-web-slab' component element, and ϕ = the reduction factor, because

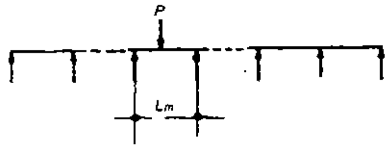


Fig. 31.22 *Typical continuous concrete box-section bridge*

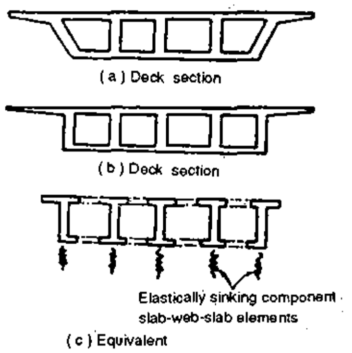


Fig. 31.23 *Deck section with vertical or nearly vertical webs, and equivalent slab-web, slab elements, interconnected and elastically sinking*

the moment of inertia of the whole may not be equal to the arithmetic sum of the moments of inertia of the component elements. In the case of bridge decks consisting of similar beams interconnected by a nominal number of diaphragms and a top slab, ϕ is very close to unity, and I is equal to nI_w.

Let i be the sum of the moments of inertia of the top and bottom transverse slabs over the effective dispersion width. The truck, with weight P, representing the live load, is placed on the span L_m of the continuous bridge, Fig. 31.22 so as to cause almost the greatest span deflection under it. This deflection is given by

$$\Delta = \frac{PL_m^3}{KEI} \tag{31.6}$$

Where K is a constant determined from the boundary conditions of the particular structural system, and EI is the sectional flexural rigidity. If R_w is the reaction caused in the w^{th} component element, and the relative shear-distortion of these component elements is assumed on the basis of I_w being the section property equivalent to that afforded by the transverse interconnection by means of the slabs, then using Eq. (31.5), the w^{th} element deflects by

$$\Delta_w = \frac{R_w L_m^3}{KEI_w}, \quad w \text{ being } = 1, 2, \ldots, n \tag{31.7}$$

If U be the strain energy stored in the system, then its partial derivative with respect to web reaction R_w gives the deflection in the w^{th} element in the direction of R_w. In other words,

$$\frac{\partial U}{\partial R_w} = \Delta_w = \frac{R_w L_m^3}{KEI_w} \tag{31.8}$$

Releasing the strain energy stored in the system and equating the work done in so doing:

$$\frac{\partial U}{\partial R_w} = \frac{\partial}{\partial R_w} \int_O^S \frac{M^2}{2Ei}\,dx \tag{31.9}$$

$$= \int_O^S \frac{M}{Ei}\frac{\partial M}{\partial R_w}\,dx \tag{31.10}$$

Combining Eqs. (31.10) and (31.8)

$$\int_O^S \frac{M}{Ei}\frac{\partial M}{\partial R_w}\,dx = \frac{R_w L_m^3}{KEI_w} \tag{31.11}$$

Simplification yields

$$\int_O^S \frac{M}{i}\frac{\partial M}{\partial R_w}\,dx = \frac{1}{KI_w} L_m^3 R_w \tag{31.12}$$

where $w = 1, 2, 3, \ldots, n$.

The n unknowns, $R_1, R_2, \ldots, R_n$, can readily be evaluated by solving the n simultaneous Eqs. (31.12). Knowing these reactions in the elasticity sunk component support elements, the transverse bending moments in the slab can be easily written down.

• ***Application of the Method***: A two-span continuous bridge deck of four-cell box-section, as shown in Figs. 31.24 and 31.25 is considered. It is required to analyse the design moments in the transverse slab. Loads to be considered are:

live load — AASHTO H20-S16-44 truck
dead load — self and superimposed

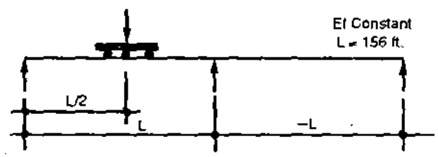


Fig. 31.24 *Two-span continuous bridge of four-cell box-section*

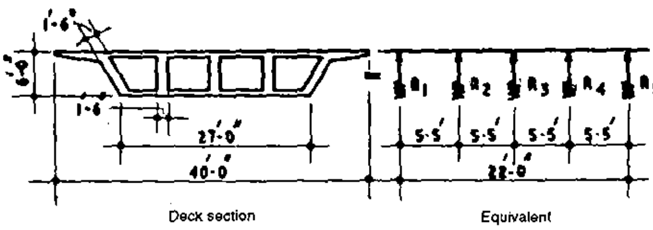


Fig. 31.25 *Deck section and equivalent elements*

Analysis for the case of live load is given below, and the analysis for the cases of dead load can be carried out in a similar manner.

From the relevant bridge calculations:

$EI =$ constant throughout
$I = 395.109$ ft^4
$i = 0.037$ ft^4
$L = 156$ ft
$n = 5$ (four cells)

Placing the truck as shown in Fig. 31.24, the maximum deck deflection works out to $\Delta = \frac{3}{200}\frac{PL^3}{EI}$. This may be compared with Eq. (31.6).

All the webs being of equal thickness, and because of the overall symmetry of the section, it may be assumed that ϕ is equal to unity in Eq. (31.5), so that each

$I_w = \frac{I}{n} = \frac{I}{5}$. This may be compared with Eq. (31.5)

Hence, $\Delta_w = \frac{3}{200}\frac{R_w L^3}{EI_w}$

where w = 1, 2, 3, 4, and 5. This may be compared with Eq. (31.7).

Now, for example, if the truck is placed symmetrically on the deck cross-section, as shown in Fig. 31.26, then R_1 will equal R_5, R_2 will equal R_4, leaving only three unknowns, namely, R_1, R_2 and R_3, to be found. In fact, only R_1 and R_2 need to be found, because R_3 will then follow from the equilibrium condition between the applied load and the reactions $R_1 R_2$ and R_3. Using Eq. (31.12) the required two simultaneous equations for R_1 and R_2 are:

$$\int_A^E \frac{M}{i}\frac{\partial M}{\partial R_1}dx = \frac{3}{200} \times \frac{5}{I} L^3 R_1 \qquad (31.13)$$

and
$$\int_A^E \frac{M}{i}\frac{\partial M}{\partial R_2}dx = \frac{3}{200} \times \frac{5}{I} L^3 R_2 \qquad (31.14)$$

Owing to the symmetry about the centre, Fig. 31.26, the integration need be performed only from A to C, and the result doubled. Equations (31.13) and (31.14) may then be reduced as given below:

$$2\int_A^C \frac{M}{i}\frac{\partial M}{\partial R_1}dx = \frac{3}{200} \times \frac{5}{I} L^3 R_1 \qquad (31.15)$$

and
$$2\int_A^C \frac{M}{i}\frac{\partial M}{\partial R_2}dx = \frac{3}{200} \times \frac{5}{I} L^3 R_2 \qquad (31.16)$$

Noting that i is constant, further simplification yields

$$\int_A^C M\frac{\partial M}{\partial R_1}dx = 0.0375\frac{i}{I} L^3 R_1 \qquad (31.17)$$

and
$$\int_A^C M\frac{\partial M}{\partial R_2}dx = 0.0375\frac{i}{I} L^3 R_2 \qquad (31.18)$$

From Fig. 31.26, it is clear that the limit A to C comprises the sub-limits A to B, B to Y, Y to Z and Z to C. Considering these separately:

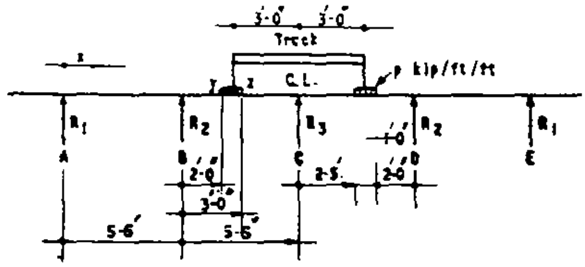


Fig. 31.26 *AASHTO truck placed symmetrically on deck section*

(i) From A to B
$M = R_1 x$, where x varies from 0 to 5.5 ft.

$$\frac{\partial M}{\partial R_1} = x, \text{ and } \frac{\partial M}{\partial R_2} = 0$$

(ii) From B to Y
$M = R_1(5.5 + x) + R_2 x$, where x varies from 0 to 2 ft.

$$\frac{\partial M}{\partial R_1} = 5.5 + x, \text{ and } \frac{\partial M}{\partial R_2} = x$$

(iii) From Y to Z
$M = R_1(5.5 + x) + R_2 x - \frac{p(x-2)^2}{2}$
where x varies from 2-3 ft

$$\frac{\partial M}{\partial R_1} = 5.5 + x, \text{ and } \frac{\partial M}{\partial R_2} = x$$

(iv) From Z to C
$M = R_1(5.5 + x) + R_2 x - p(x - 2.5)$, where x varies from 3–5.5 ft.

$$\frac{\partial M}{\partial R_1} = 5.5 + x, \text{ and } \frac{\partial M}{\partial R_2} = x$$

Substituting these values in Eqs. (31.17) and (31.18), integrating, and finally solving for R_1 and R_2, it is found that

$$R_1 = 49p \text{ lb/ft.}$$
$$R_2 = 483p \text{ lb/ft.}$$

The pressure intensity p under the heaviest wheel load of the truck may be found by dividing the load including impact factor by the dimensions of its dispersion, parallel and perpendicular to the bridge. In the present case these dimensions may be assumed as follows:

(i) dispersion dimension parallel to bridge: based on the longitudinal dispersion in plan, this may be taken to be equal to the span of the transverse slab when the load is placed at mid-span i.e., 5.5 ft; according to the AASHTO specifications, the upper limit is 7 ft.

(ii) dispersion dimension perpendicular to bridge: based on the wheel contact width plus 45° dispersion up to the upper layer of reinforcement in the slab; this may be taken as equal to 1.0ft in the present case.

Then $p = \dfrac{16,000 \times 1.30 \text{ impact factor}}{1000 \times (5.5 \text{ ft} \times 1.0 \text{ ft})} = 3.782$ kip/ft./ft

Therefore $R_1 = -49 \times 3.782 = -185$ kip/ft

and $R_2 = 483 \times 3.782 = 1{,}825$ kip/ft

The bending moments in the slab can be easily obtained from Fig. 31.26, as follows:

At middle of span AB:

$$\text{Bending moment } = R_1 \times \frac{5.5}{2} = -185 \times 2.75 \times 1000 = -509 \text{ lb ft/ft}$$

At support B:

$$\text{Bending moment } = R_1 \times 5.5 = 185 \times 5.5 \times 1000 = -\ 1{,}018 \text{ lb ft/ft}$$

At middle of span BC:

Bending moment

$$= R_1\left(5.5 + \frac{5.5}{2}\right) + R_2 \times \frac{5.5}{2} - (0.75)^2\,\frac{p}{2}$$
$$= -185 \times 8.25 + 1{,}825 \times 2.75 - \frac{3782}{2}(0.75)^2$$
$$= 2{,}429 \text{ lb ft/ft}$$

At support C:

Bending moment

$$= R_1(5.5 + 5.5) + R_2 \times 5.5 - (p \times 1) \times 3$$
$$= -185 \times 11.0 + 1{,}825 \times 5.5 - 3{,}782 \times 3$$
$$= -3{,}340 \text{ lb ft/ft.}$$

CASE (C): Multicell box-section with stub central web

Figure 31.27 represents a multicell box-section with stub central web. For its transverse analysis it is assumed that the Vierendeel type frames on either side of the centre-line of the deck are rigidly attached to the stub central web. The statically indeterminate system with various loads (self-weights and live load trucks together with any cantilever bending moment) is shown in Fig. 31.28. $I_{12}, I_{13}, \ldots$ and I_{56} represent second moments of area (moments of inertia) of members 1-2, 2-3, ..., and 5-6, respectively. The frame being six times statically indeterminate, it is rendered

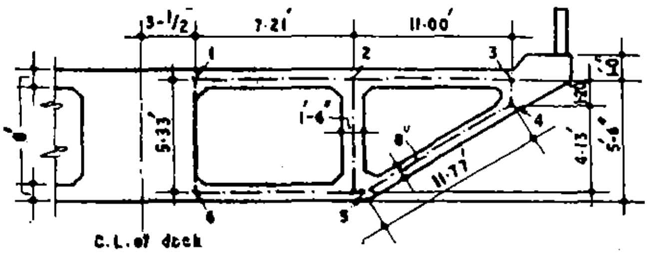


Fig. 31.27 *Multicell box-section with stub central web*

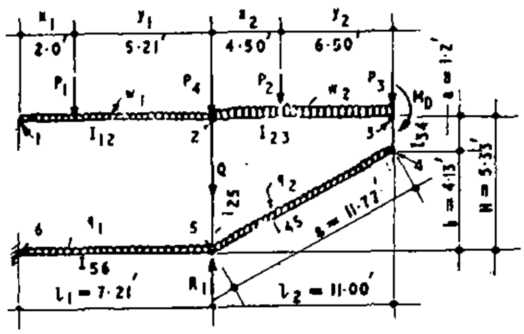


Fig. 31.28 *Statically indeterminate system with various loads*

statically determinate by introducing two cuts in it as shown in Fig. 31.29. The six stress resultants released are p_1, p_2, p_3, p_4, p_5, and p_6. These can be evaluated from the following compatibility equations once the influence coefficients δ_{rs} and δ_{ro} are evaluated (r = 1 to 6, s = 1 to 6):

$$\delta_{10} + \delta_{11}p_1 + \delta_{12}p_2 + \delta_{13}p_3 + \delta_{14}p_4 + \delta_{15}p_5 + \delta_{16}p_6 = 0 \tag{31.19}$$
$$\delta_{20} + \delta_{21}p_1 + \delta_{22}p_2 + \delta_{23}p_3 + \delta_{24}p_4 + \delta_{25}p_5 + \delta_{26}p_6 = 0 \tag{31.20}$$
$$\delta_{30} + \delta_{31}p_1 + \delta_{32}p_2 + \delta_{33}p_3 + \delta_{34}p_4 + \delta_{35}p_5 + \delta_{36}p_6 = 0 \tag{31.21}$$
$$\delta_{40} + \delta_{41}p_1 + \delta_{42}p_2 + \delta_{43}p_3 + \delta_{44}p_4 + \delta_{45}p_5 + \delta_{46}p_6 = 0 \tag{31.22}$$
$$\delta_{50} + \delta_{51}p_1 + \delta_{52}p_2 + \delta_{53}p_3 + \delta_{54}p_4 + \delta_{55}p_5 + \delta_{56}p_6 = 0 \tag{31.23}$$
$$\delta_{60} + \delta_{61}p_1 + \delta_{62}p_2 + \delta_{63}p_3 + \delta_{64}p_4 + \delta_{65}p_5 + \delta_{66}p_6 = 0 \tag{31.24}$$

where $\delta_{10} = (\delta_1 P_1 + \delta_1 P_2 + \delta_1 P_3 + \delta_1 P_4) + (\delta_1 w_1 + \delta_1 w_2) + (\delta_1 q_1 + \delta_1 q_2) + \delta_1 Q + \delta_1 M_D + \delta_1 R_1$

It may be noted that $\delta_{rs} = \int \dfrac{m_r m_s}{EI}\, ds$

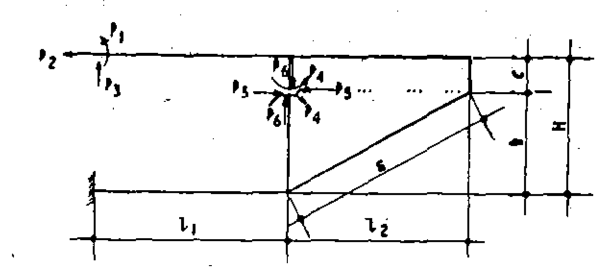


Fig. 31.29 *System made statically determinate by introducing two cuts*

$$= \int \frac{m_r m_s k}{E}\, ds,$$

where $k = \dfrac{1}{I}$

$$\delta_{ro} = \int \frac{m_r m_o}{EI}\, ds$$

$$= \int \frac{m_r m_o k}{E}\, ds,$$

where $k = \dfrac{1}{I}$

m_o = bending moment diagram on the determinate structure due to applied loads P_1, P_2, P_3, P_4, w_1, w_2, q_1, q_2, Q, M_D and R_1.

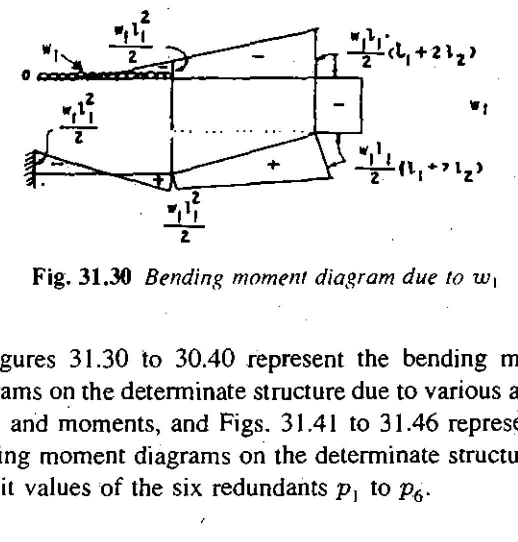


Fig. 31.30 *Bending moment diagram due to* w_1

Figures 31.30 to 30.40 represent the bending moment diagrams on the determinate structure due to various applied loads and moments, and Figs. 31.41 to 31.46 represent the bending moment diagrams on the determinate structure due to unit values of the six redundants p_1 to p_6.

- **Calculation of the various influence coefficients**

The influence coefficients involved in Eq. (31.19) are:

$$\delta_{11} = k_{12}l_1 + k_{23}l_2 + k_{34}a + k_{45}S + k_{56}l_1$$

$$\delta_{12} = 0.50\ k_{34}a^2 + 0.50\ k_{45}S(H + a) + k_{56}l_1 H$$

$$\delta_{13} = -0.50\ k_{12}l_1^2 - 0.50\ k_{23}l_2(2l_1 + l_2) - k_{34}a \times (l_1 + l_2) - 0.50\ k_{45}S(2l_1 + l_2) - 0.50\ k_{56}l_1^2$$

$$\delta_{14} = k_{23}l_2 + k_{34}\ a + k_{45}\ S$$

$$\delta_{15} = -k_{23}l_2 a - 0.50\ k_{34}a^2 + 0.50\ k_{45}\ Sb$$

$$\delta_{16} = 0.50\ k_{23}l_2^2 + k_{34}al_2 + 0.50\ k_{45}\ Sl_2$$

$$\delta_1 w_1 = 0.166\ k_{12}w_1 l_1^3 + 0.50\ k_{23}l_2 w_1\ (l_1^2 + l_1 l_2) + 0.50\ k_{34}w_1 l_1 a(l_1 + 2l_2) + 0.50\ k_{45}\ Sw_1 \times (l_1^2 + l_1 l_2)$$

$$\delta_1 w_2 = 0.166\ k_{23}w_2 l_2^2 + 0.50\ k_{34}w_2 l_2^2 a - 0.50\ k_{56}l_1 w_2(l_2^2 + l_1 l_2)$$

$$\delta_1 q_1 = 0.166\ k_{56}q_1 l_1^3$$

$$\delta_1 q_2 = -0.166\ k_{45}q_2 l_2^2 S - 0.50\ k_{56}l_1 q_2(l_2^2 + l_1 l_2)$$

$$\delta_1 Q = 0.50\ \frac{Qa}{H}l_2(k_{23}l_2 + 2k_{34}a + k_{45}S) - 0.50\ k_{56}Ql_1^2$$

$$\delta_1 P_1 = 0.50\ k_{12}P_1 y_1 + 0.50\ k_{23}P_1 l_2(2y_1 + l_2) + k_{34}P_1 a(y_1 + l_2) + 0.50\ k_{45}P_1 S(2y_1 + l_2) + 0.50\ k_{56}P_1 y_1^2 - 0.50\ k_{56}P_1 x_1^2$$

$$\delta_1 P_2 = 0.50\ k_{23}P_2 y_2^2 + k_{34}P_2 y_2 a + 0.50\ k_{45}P_2 \times y_2^2\frac{S}{l_2} - 0.50\ k_{45}P_2 x_2^2\frac{S}{l_2} - 0.50\ k_{56}P_2 l_1 \times (2x_2 + l_1)$$

$$\delta_1 P_3 = -0.50\ k_{45}P_3 l_2 S - 0.50\ k_{56}P_3 l_1(l_1 + 2l_2)$$

$$\delta_1 P_4 = 0.50\ P_4 l_2(k_{23}l_2 + 2k_{34}a + k_{45}S) - 0.50\ k_{56}P_4 l_1^2$$

$$\delta_1 R_1 = 0.50\ k_{56}R_1 l_1^2$$

$$\delta_1 M_D = -k_{34}M_D a - k_{45}M_D S - k_{56}M_d l_1$$

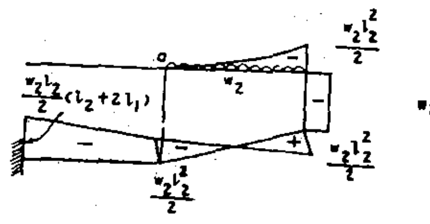


Fig. 31.31 *Bending moment diagram due to* w_2

The influence coefficients involved in Eq. (31.20):

$$\delta_{21} = 0.50\ k_{34}a^2 + 0.50\ k_{45}S(H + a) + k_{56}l_1 H$$

$$\delta_{22} = 0.333\ k_{34}a^3 + 0.333\ k_{45}(H^2 + Ha + a^2)S$$
$$+k_{56}H^2 l_1$$

$$\delta_{23} = -0.50\ k_{34}a^2(l_1 + l_2) - 0.166\ k_{45}S$$
$$[H(3l_1 + l_2) + a(3l_1 + 2l_2)] - 0.50\ k_{56}Hl_1^2$$

$$\delta_{24} = 0.50\ k_{34}a^2 + 0.50\ k_{45}S(H + a)$$

$$\delta_{25} = -0.166\ k_{34}a^3 + 0.166\ k_{45}bS(2H + a)$$

$$\delta_{26} = 0.50\ k_{34}a^2 l_2 + 0.166\ k_{45}l_2 S(H + 2a)$$

$$\delta_2 w_1 = 0.25\ k_{34}a^2 w_1 l_1(l_1 + 2l_2) + 0.083\ k_{45}w_1 l_1$$
$$\times S[3l_1(a + H) + 2l_2(2a + H)]$$

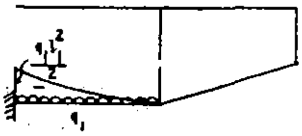


Fig. 31.32 *Bending moment diagram due to* q_1

$$\delta_2 w_2 = 0.25\ k_{34}w_2 l_2^2 a_2 - 0.083\ k_{45}w_2 l_2^2\ S(H - a)$$
$$-0.50\ k_{56}w_2 l_1 l_2 H(l_1 + l_2)$$

$$\delta_2 q_1 = -0.166\ k_{56}q_1 l_1^3 H$$

$$\delta_2 q_2 = -0.042\ k_{45}q_2 l_2^2 S(3H + a) - 0.50\ k_{56}q_2 l_2$$
$$\times(l_1 + l_2)Hl_1$$

$$\delta_2 Q = 0.50\ k_{34}Qa^3\frac{l_2}{H} + 0.166\ k_{45}QaS\frac{l_2}{H}$$
$$\times(2a + H) - 0.50\ k_{56}Ql_1^2 H$$

$$\delta_2 P_1 = 0.50\ k_{34}P_1 a^2(y_1 + l_2) + 0.166\ k_{45}P_1 S$$
$$\times[3y_1(H + a) + l_2(H + 2a)] + 0.50\ k_{56}$$
$$\times P_1 y_1^2 H - 0.50\ k_{56}P_1 x_1^2 H$$

$$\delta_2 P_2 = 0.50\ k_{34}P_2 a^2 y_2 - 0.166\ k_{45}P_2 x_2^2$$
$$\times\left[3H - \frac{x_2}{l_2}(H - a)\right]\frac{S}{l_2} + 0.166\ k_{45}P_2 y_2^2$$
$$\times\left[3a + \frac{y_2}{l_2}(H - a)\right]\frac{S}{l_2} - 0.50\ k_{56}P_2 l_1 H$$
$$\times(2x_2 + l_1)$$

$$\delta_2 P_3 = -0.166\ k_{45}P_3 l_2 S(2H + a)$$
$$-0.50\ k_{56}P_3 l_1 H(l_1 + 2l_2)$$

$$\delta_2 P_4 = 0.50\ k_{34}P_4 a^2 l_2 + 0.166\ k_{45}P_4 Sl_2$$
$$\times(2a + H) - 0.50\ k_{56}P_4 l_1^2\ H$$

$$\delta_2 R_1 = 0.50\ k_{56}R_1 l_1^2 H$$

$$\delta_2 M_D = -0.50\ k_{45}M_D S(H + a) - k_{56}M_D Hl_1$$
$$-0.50\ k_{34}M_D a^2$$

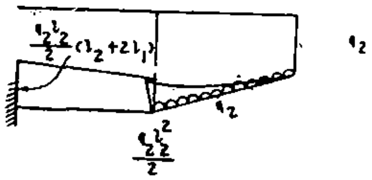


Fig. 31.33 *Bending moment diagram due to* q_2

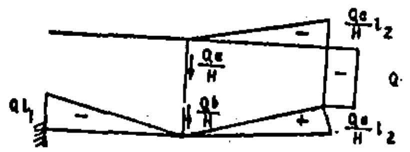
The influence coefficients involved in Eq. (31.21):

$$\delta_{31} = -0.50\ k_{12}l_1^2 - 0.50\ k_{23}l_2(2l_1 + l_2)$$
$$-k_{34}a(l_1 + l_2) - 0.50\ k_{45}S(2l_1 + l_2)$$
$$-0.50\ k_{56}l_1^2$$

$$\delta_{32} = -0.50\ k_{34}a^2(l_1 + l_2) - 0.166\ k_{45}S[H(3l_1 + l_2)$$
$$+\ a(3l_1 + 2l_2)] - 0.50\ k_{56}Hl_1^2$$

$$\delta_{33} = 0.333\ k_{12}l_1^3 + 0.333\ k_{23}l_2(3l_1^2 + 3l_1 l_2 + l_2^2)$$
$$+\ k_{34}a(l_1 + l_2)^2 + 0.333\ k_{45}S$$
$$\times(3l_2 + 3l_1 l_2 + l_2^2) + 0.333\ k_{56}l_1^3$$

$$\delta_{34} = -0.50\ k_{23}l_2(2l_1 + l_2) - k_{34}a(l_1 + l_2)$$
$$-0.50\ k_{45}S(2l_1 + l_2))$$



Portion of Q force above cut $= \dfrac{Qa}{H}$

Portion of Q force above cut $= \dfrac{Qb}{H}$

Total $= Q$

Fig. 31.34 *Bending moment diagram due to* Q

$$\delta_{35} = 0.50\ k_{23}l_2 a(2l_1 + l_2) + 0.50\ k_{34}a^2$$
$$\times(l_1 + l_2) - 0.166\ k_{45}bS(3l_1 + l_2)$$

$$\delta_{36} = -0.166\ k_{23}l_2^2(3l_1 + 2l_2) - k_{34}l_2 a(l_1 + l_2)$$

$$-0.166\ k_{45} l_2 S(3l_1 + 2l_2)$$

$$\begin{aligned}
\delta_3 w_1 = {} & -0.125 k_{12} w_1 l_1^4 - 0.166\ k_{23} w_1 l_1 l_2 \\
& \times (3l_1^2 + 4.5 l_1 l_2 + 2l_2^2) - 0.50\ k_{34} w_1 l_1 a \\
& \times (l_1^2 + 3l_1 l_2 + 2l_2^2) - 0.166\ k_{45} w_1 l_1 S \\
& \times (3l_1^2 + 4.5 l_1 l_2 + 2l_2^2) - 0.083\ k_{56} w_1 l_1^4
\end{aligned}$$

$$\begin{aligned}
\delta_3 w_2 = {} & -0.042\ k_{23} w_2 l_2^3 (4l_1 + 3l_2) \\
& -0.50\ k_{34} w_2 l_2^2 a(l_1 + l_2) \\
& -0.083\ k_{45} w_2 l_2^3 S \\
& +0.166\ k_{56} w_2 l_1^2 l_2 (1.5 l_2 + l_1)
\end{aligned}$$

$$\delta_3 q_1 = 0.042\ k_{56} q_1 l_1^4$$

$$\begin{aligned}
\delta_3 q_2 = {} & 0.042\ k_{45} q_2 l_2^3 S(4l_1 + l_2) \\
& +0.166\ k_{56} q_2 l_1^2 l_2 (1.5 l_2 + l_1)
\end{aligned}$$

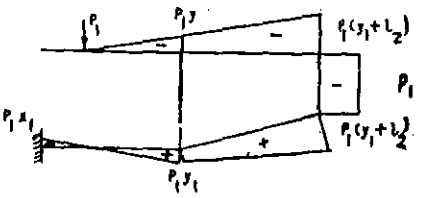


Fig. 31.35 *Bending moment diagram due to* P_1

$$\begin{aligned}
\delta_3 Q = {} & -0.166\ k_{23}\ Qa\frac{l_2^2}{H}(3l_1 + 2l_2) - k_{34} Qa^2 \\
& \times \frac{l_2}{H}(l_1 + l_2) - 0.166\ k_{45} Qa l_2 \frac{S}{H} \times \\
& (3l_1 + 2l_2) + 0.166\ k_{56}\ Q l_1^3
\end{aligned}$$

$$\begin{aligned}
\delta_3 P_1 = {} & -0.166\ k_{12} P_1 y_1^2 (3l_1 - y_1) - 0.166 \\
& \times k_{23} P_1 l_2 [3y_1(2l_1 + l_2) + l_2(3l_1 + 2l_2)] \\
& -k_{34} P_1 a(y_1 + l_2)(l_1 + l_2) - 0.166\ k_{45} \\
& \times P_1 S[3y_1(2l_1 + l_2) + l_2(3l_1 + 2l_2)] \\
& -0.166\ k_{56} P_1 l_1^2 (2y_1 - x_1)
\end{aligned}$$

$$\begin{aligned}
\delta_3 P_2 = {} & -0.166\ k_{23} P_2 y_2^2 (3l_1 + 2l_2 + x_2) \\
& -k_{34} P_2 y_2 a(l_1 + l_2) + 0.166\ k_{45} P_2 x_2^2 \\
& \times (3l_1 + x_2)\frac{S}{l_2} - 0.166\ k_{45} P_2 y_2^2\ (3l_1 + 2l_2 \\
& + x_2)\frac{S}{l_2} + 0.166\ k_{56} P_2 l_1^2\ (3x_2 + l_1)
\end{aligned}$$

$$\begin{aligned}
\delta_3 P_3 = {} & 0.166\ k_{45} P_3 l_2 S(3l_1 + l_2) + 0.166\ k_{56} \\
& \times P_3 l_1^2 (l_1 + 3l_2)
\end{aligned}$$

$$\begin{aligned}
\delta_3 P_4 = {} & -0.166\ k_{23} P_4 l_2^2 (3l_1 + 2l_2) \\
& -k_{34} P_4 a l_2 (l_1 + l_2) - 0.166\ k_{45} P_4 l_2 S \\
& \times (3l_1 + 2l_2) + 0.166\ k_{56} P_4 l_1^3
\end{aligned}$$

$$\delta_3 R_1 = -0.166\ k_{56} R_1 l_1^3$$

$$\begin{aligned}
\delta_2 M_D = {} & k_{34} M_D a(l_1 + l_2) + 0.50\ k_{45} M_D S \\
& \times (2l_1 + l_2) + 0.50\ k_{56} M_D l_1^2
\end{aligned}$$

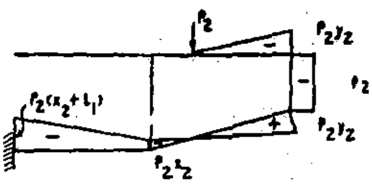


Fig. 31.36 *Bending moment diagram due to* P_2

The influence coefficients involved in Eq. (31.22):

$$\delta_{41} = k_{23} l_2 + K_{34} a + k_{45} S$$

$$\delta_{42} = 0.50 k_{34} a^2 + 0.50\ k_{45} S(H + a)$$

$$\begin{aligned}
\delta_{43} = {} & -0.50\ k_{23} l_2 (2l_1 + l_2) - k_{34} a(l_1 + l_2) \\
& -0.50\ k_{45} S(2l_1 + l_2)
\end{aligned}$$

$$\delta_{44} = k_{33} l_2 + k_{34} a + k_{45} S + k_{25} H$$

$$\begin{aligned}
\delta_{45} = {} & -k_{23} l_2 a - 0.50\ k_{34} a^2 + 0.50\ k_{45} bS \\
& -0.50\ k_{25} a^2 + 0.50\ k_{25} b^2
\end{aligned}$$

$$\delta_{46} = 0.50\ k_{23} l_2^2 + k_{34} l_2 a + 0.50 k_{45} l_2 S$$

$$\begin{aligned}
\delta_4 w_1 = {} & 0.50\ k_{23} l_2 w_1 (l_1^2 + l_1 l_2) + 0.50\ k_{34} w_1 l_1 a \\
& \times (l_1 2 l_2) + 0.50\ k_{45} S w_1 (l_1^2 + l_1 l_2)
\end{aligned}$$

$$\delta_4 w_2 = 0.166\ k_{23} w_2 l_2^2 + 0.50\ k_{34} w_2 l_2^2 a$$

$$\delta_4 q_1 = 0$$

$$\delta_4 q_2 = -0.166\ k_{45} q_2 l_2^2 S$$

$$\delta_4 Q = 0.50\ Qa\frac{l_2}{H}(k_{23} l_2 + 2k_{34} a + k_{45} S)$$

$$\begin{aligned}
\delta_4 P_1 = {} & 0.50\ k_{23} P_1 l_2 (2y_1 + l_2) + k_{34} P_1 a(y_1 + l_2) \\
& +0.50\ k_{45} P_1 S(2y_1 + l_2)
\end{aligned}$$

$$\begin{aligned}
\delta_4 P_2 = {} & 0.50\ k_{23} P_2 y_2^2 + k_{34} P_2 y_2 a + 0.50\ k_{45} P_2 \\
& \times y_2^2\ \frac{S}{l_2} - 0.50\ k_{45} P_2 x_2^2 \frac{S}{l_2}
\end{aligned}$$

$$\delta_4 P_3 = -0.50\ k_{45} P_3 l_2 S$$

$$\delta_4 P_4 = 0.50\ P_4 l_2 (k_{23} l_2 + 2k_{34} a + k_{45} S)$$

$$\delta_4 R_1 = 0$$

$$\delta_4 M_D = -k_{34} M_D a - k_{45} M_D S$$

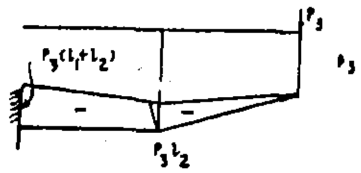


Fig. 31.37 *Bending moment diagram due to P_3*

The influence coefficients involved in Eq. (31.23):

$$\delta_{51} = -k_{23} l_2 a - 0.50\ k_{34} a^2 + 0.50\ k_{45} Sb$$

$$\delta_{52} = -0.166\ k_{34} a^3 + 0.166\ k_{45} bS(2H + a)$$

$$\delta_{53} = 0.50\ k_{23} l_2 a (2l_1 + l_2) + 0.50\ k_{34} a^2 (l_1 + l_2)$$
$$-0.166\ k_{45} bS(3l_1 + l_2)$$

$$\delta_{54} = k_{23} l_1 a - 0.50\ k_{34} a^2 + 0.50\ k_{45} bS$$
$$-0.50\ k_{25} a^2 + 0.50\ k_{25} b^2$$

$$\delta_{55} = k_{23} l_2 a^2 + 0.333\ k_{34} a^3 + 0.333\ k_{45} b^2 S$$
$$+0.333\ k_{25} a^3 + 0.333\ k_{25} b^3$$

$$\delta_{56} = -0.50\ k_{23} l_2^2 a - 0.50\ k_{34} l_2 a^2 + 0.166$$
$$\times k_{45} l_2 bS$$

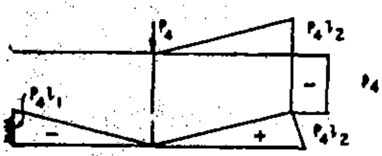


Fig. 31.38 *Bending moment diagram due to P_4*

$$\delta_5 w_1 = -0.50\ k_{23} l_2 a w_1 l_1 (l_1 + l_2) - 0.25\ k_{34} w_1 l_1$$
$$\times a^2 (l_1 + 2l_2) + 0.166\ k_{45} w_1 l_1 bS(1.5l_1 + l_2)$$

$$\delta_5 w_2 = -0.166\ k_{23} w_2 w_2 l_2^3 a - 0.25\ k_{34} w_2 l_2^2 a^2$$
$$-0.083\ k_{45} w_2 l_2^2 bS$$

$$\delta_5 q_1 = 0$$

$$\delta_5 q_2 = -0.125\ k_{45} q_2 l_2^2 bS$$

$$\delta_5 Q = -0.50\ k_{23} Q\ a^2 \frac{l_2^2}{H} - 0.50\ k_{34}\ Qa^3 \frac{l_2}{H} + 0.166$$
$$k_{45}\ Qabs\ \frac{l_2}{H}$$

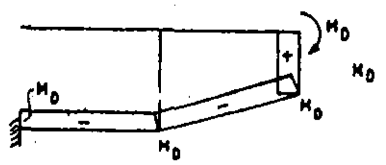


Fig. 31.39 *Bending moment diagram due to M_D*

$$\delta_5 P_1 = -0.50\ k_{23} P_1 l_2 a (2y_1 + l_2)$$
$$-0.50\ k_{34} P_1 a^2 (y_1 + l_2) + 0.166\ k_{45}$$
$$\times P_1 bS(3y_1 + l_2)$$

$$\delta_5 P_2 = -0.50\ k_{23} P_2 y_2^2 a - 0.50\ k_{34} P_2 y_2 a^2$$
$$+0.166\ k_{45} P_2 bS(y_2 - 2x_2)$$

$$\delta_5 P_3 = -0.333\ k_{45} P_3 l_2 bS$$

$$\delta_5 P_4 = -0.50\ k_{23} P_4 a l_2^2 - 0.50\ k_{34} P_4 a^2 l_2$$
$$+0.166\ k_{45} P_4 bSl_2$$

$$\delta_5 R_1 = 0$$

$$\delta_5 M_D = 0.50\ k_{34} M_D a^2 - 0.50\ k_{45} M_D bS$$

The influence coefficients involved in Eq. (31.24):

$$\delta_{61} = 0.50\ k_{23} l_2^2 + k_{34} a l_2 + 0.50\ k_{45} Sl_2$$

$$\delta_{62} = 0.50\ k_{34}\ a^2\ l_2 + 0.166\ k_{45}\ l_2\ S(H + 2a)$$

$$\delta_{63} = -0.166\ k_{23} l_2^2\ (3l_1 + 2l_2) - k_{34} l_2 a \times (l_1 + l_2)$$
$$-0.166\ k_{45}\ l_2 S(3l_1 + 2l_2)$$

$$\delta_{64} = 0.50\ k_{23}\ l_2^2 + k_{34} l_2 a + 0.50\ k_{45}\ l_2 S$$

$$\delta_{65} = -0.50\ k_{23}\ l_2^2\ a - 0.50\ k_{34}\ l_2 a^2 + 0.166\ k_{45}\ l_2 bS$$

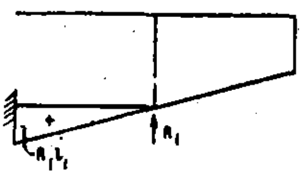


Fig. 31.40 *Bending moment diagram due to R_1*

$$\delta_{66} = 0.333\ k_{23} l_2^3 + k_{34} l_2^2 a + 0.333\ k_{45} l_2^2 S$$

$$\delta_6 w_1 = 0.166\ k_{23} w_1 l_1 l_2^2\ (1.5l_1 + 2l_2) + 0.50\ k_{34}\ w_1 l_1 l_2\ a(l_1 + 2l_2) + 0.166\ k_{45} \times w_1 l_1 l_2 S(1.5l_1 + 2l_2)$$

$$\delta_6 w_2 = 0.125\ k_{23}\ w_2\ l_2 + 0.50\ k_{34}\ w_2 l_2^3 a + 0.083 k_{45} w_2\ l_2^3 S$$

$$\delta_6 q_1 = 0$$

$$\delta_6 q_2 = -0.042\ k_{45}\ q_2 l_2^3 S$$

$$\delta_6 Q = 0.333\ k_{23} Qa \frac{l_2^3}{H} + k_{34}\ Qa^2 \frac{l_2^2}{H} + 0.333\ k_{45}\ QaS \frac{l_2^2}{H}$$

$$\delta_6 P_1 = 0.166\ k_{23}\ P_1 l_2^2\ (3y_1 + 2l_2) + k_{34} P_1 l_2 a \times (y_1 + l_2) + 0.166\ k_{45}\ P_1 l_2 S(3y_1 + 2l_2)$$

$$\delta_6 P_2 = 0.166\ k_{23}\ P_2 y_2^2 (2l_2 + x_2) + k_{34} P_2 l_2 y_2 a + 0.166\ k_{45}\ P_2 l_2 S(2y_2 - x_2)$$

$$\delta_6 P_3 = -0.166\ k_{45}\ P_3\ l_2^2 S$$

$$\delta_6 P_4 = 0.333\ k_{23}\ P_4\ l_2^3 + k_{34}\ P_4 a l_2^2 + 0.333\ k_{45}\ P_4\ S l_2^2$$

$$\delta_6 R_1 = 0$$

$$\delta_6 M_D = -k_{34}\ M_D l_2 a - 0.50\ k_{45}\ M_D\ l_2 S$$

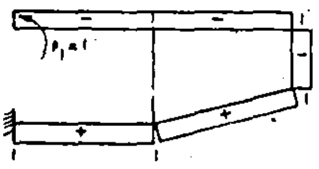


Fig. 31.41 *Bending moment diagram due to m_1*

In the design example under illustration, on one-foot width basis:

$w_1 = w_2 =$	138 lb/ft	
$q_1 =$	100 lb/ft	
$q_2 =$	108 lb/ft	
$P_1 = P_2 =$	3,400 lb	
$P_3 =$	4,680 lb	
$P_4 =$	3,460 lb	
$Q =$	1,350 lb	
$R_1 =$	3,640 lb	(upward force due to differential cable push in transverse section)
$M_D =$	2,030 lbft./ft.	

$$I_{12} = \frac{1}{12} \% \left(\frac{8}{12}\right)^2 = 0.025\ \text{ft}^4$$

so that $k_{12} = \dfrac{l}{I_{12}} = 40.0\ \text{ft}^{-4}$

Similarly $k_{23} = 40\ \text{ft}^{-4}$

$k_{34} = 0.77\ \text{ft}^{-4}$

$k_{45} = 40.0\ \text{ft}^{-4}$

$k_{56} = 40.0\ \text{ft}^{-4}$

$k_{25} = 3.57\ \text{ft}^{-4}$

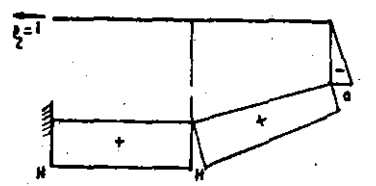


Fig. 31.42 *Bending moment diagram due to m_2*

Substituting the above in kip and ft. units in the formulae for the various coefficients determined earlier, the values obtained are given below for Eq. (31.19)

$\delta_{11} =$	1488.524,	$\delta_1 q_2 =$	− 4140.758,
$\delta_{12} =$	3074.886,	$\delta_1 Q =$	122.075,
$\delta_{13} =$	− 13672.45 ,	$\delta_1 P_1 =$	36636.36,
$\delta_{14} =$	911.724,	$\delta_1 P_2 =$	− 3453.30,
$\delta_{15} =$	443.648,	$\delta_1 P_3 =$	− 31830.936,
$\delta_{16} =$	5019.564,	$\delta_1 P_4 =$	13770.39,
$\delta_1 w_1 =$	8608.044,	$\delta_1 R_1 =$	− 3794.84,
$\delta_1 w_2 =$	− 2758.75,	$\delta M_D =$	− 1543.05,
$\delta_1 q_1 =$	− 248.85,		

Therefore, $\delta_{10} = 11366.38$

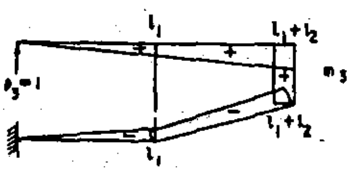


Fig. 31.43 *Bending moment diagram due to m_3*

For Eq. (31.20)

$\delta_{21} = \delta_{12}$		$\delta_2 q_2 =$	− 21069.11,
$\delta_{22} =$	13876.125,	$\delta_2 Q =$	− 5459.396,
$\delta_{23} =$	− 23235.55,	$\delta_2 P_1 =$	58135.289,
$\delta_{24} =$	1537.72,	$\delta_2 P_2 =$	− 43178.00,
$\delta_{25} =$	3827.84,	$\delta_2 P_3 =$	− 152784.27,
$\delta_{26} =$	6651.419,	$\delta_2 P_4 =$	3840.344,
$\delta_2 w_1 =$	12111.63,	$\delta_2 R_1 =$	− 20226.49,

$\delta_2 w_2 = -23936.04, \quad \delta M_D = -6242.03,$

$\delta_2 q_1 = -1326.48,$

Therefore, $\delta_{20} = 200134.56$

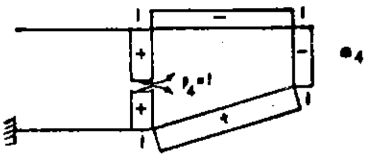


Fig. 31.44 *Bending moment diagram due to m_4*

For Eq. (31.21)

$\delta_{31} =$	δ_{13},	$\delta_3 q_2 =$	20017.40,
$\delta_{32} =$	δ_{23},	$\delta_3 Q =$	$-$ 18750.979,
$\delta_{33} =$	166453.06,	$\delta_3 P_1 =$	$-$ 471823.37,
$\delta_{34} =$	$-$ 11593.086,	$\delta_3 P_2 =$	$-$ 58314.67,
$\delta_{35} =$	$-$ 3811.04,	$\delta_3 P_3 =$	196236.37,
$\delta_{36} =$	$-$ 72747.04,	$\delta_3 P_4 =$	$-$ 243093.85,
$\delta_3 w_1 =$	$-$ 116901.28,	$\delta_3 R_1 =$	9083.78
$\delta_3 w_2 =$	$-$ 13977.04,	$\delta_3 M_D =$	14291.951,
$\delta_3 q_1 =$	453.88,		

Therefore, $\delta_{30} = -682777.81$

For Eq. (31.22):

$\delta_{41} = \delta_{14}, \quad \delta_{42} = \delta_{24}, \quad \delta_{43} = \delta_{34},$

$\delta_{44} = 930.75, \quad \delta_{45} = 471.52, \quad \delta_{46} = 5019.56,$

$\delta_4 w_1 = 8264.83, \quad \delta_4 w_2 = 1227.33, \quad \delta_4 q_1 = 0$

$\delta_4 q_2 = -1021.13, \delta_4 Q = 1525.65, \quad \delta_4 q_1 = 33216.80$

$\delta_4 P_2 = 4494.14, \quad \delta_4 P_3 = 12118.39, \quad \delta_4 P_4 = 17367.69,$

$\delta_4 R_1 = 0, \quad \delta_4 M_D = -957.60,$

Therefore, $\delta_{40} = 51992.32$

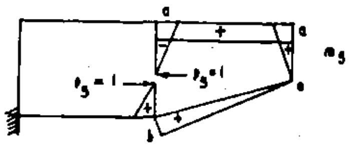


Fig. 31.45 *Bending moment diagram due to m_5*

For Eq. (31.23)

$\delta_{51} = \delta_{15}, \quad \delta_{52} = \delta_{25}, \quad \delta_{53} = \delta_{35}, \delta_{54} = \delta_{45},$

$\delta_{55} = 3394.00, \quad \delta_{56} = 640.38, \quad \delta_5 w_1 = 2214.55.$

$\delta_5 w_2 = -4163.00, \delta_5 q_1 = 0, \delta_5 q_2 = -3176.18,$

$\delta_5 Q = 194.639, \quad \delta_5 P_1 = 9967.185,$

$\delta_5 P_2 = -6203.40, \quad \delta_5 P_3 = -33332.82,$

$\delta_5 P_4 = -6518.459, \quad \delta_5 R_1 = 0, \quad \delta_5 M_D = -1972.44,$

Therefore, $\delta_{50} = -42989.93$

For Eq. (31.24)

$\delta_{61} = \delta_{16}, \delta_{62} = \delta_{26}, \delta_{63} = \delta_{36}, \quad \delta_{64} = \delta_{46},$

$\delta_{65} = \delta_{56}, \delta_{66} = 36810.66, \delta_6 w_1 = 54449.06,$

$\delta_6 w_2 = 17361.91, \quad \delta_6 q_1 = 0, \quad \delta_6 q_2 = -2835.52,$

$\delta_6 Q = 11188.233, \quad \delta_6 P_1 = 213343.16,$

$\delta_6 P_2 = 50402.21, \quad \delta_6 P_3 = -44256.37,$

$\delta_6 P_4 = 127364.912, \quad \delta_6 R_1 = 0, \quad \delta_6 M_D = -5277.14,$

Therefore, $\delta_{60} = -421740.45$

Substituting these values of influence coefficients in compatibility Eqs. (31.19) to (31.24) gives

$$1488.52\, p_1 + 3074.89\, p_2 - 13672.45\, p_3 + 911.72\, p_4 + 433.65\, p_5 + 5019.56\, p_6 + 11366.4 = 0$$

$$3074.89\, p_1 + 13876.13\, p_2 - 23235.55\, p_3 + 1537.72\, p_4 + 3827.84\, p_5 + 6651.42\, p_6 - 200134.6 = 0$$

$$-13672.45\, p_1 - 23235.55\, p_2 + 166453.06\, p_3 - 11593.08\, p_4 - 3811.04\, p_5 - 72747.04\, p_6 - 682777.8 = 0$$

$$911.72\, p_1 + 1537.72\, p_2 - 11593.08\, p_3 + 930.75\, p_4 + 471.52\, p_5 + 5019.56\, p_6 + 51992.3 = 0$$

$$443.65\, p_1 + 3827.84\, p_2 - 3811.04\, p_3 + 471.52 p_4 + 3394.00\, p_5 + 640.38\, p_6 - 42989.9 = 0$$

$$5019.56\, p_1 + 6651.42\, p_2 - 72747.04\, p_3 + 5019.56\, p_4 + 640.38\, p_5 36810.66\, p_6 + 421740.5 = 0$$

Solving these six equations simultaneously gives

$$\left.\begin{aligned} p_1 &= 42.68 \\ p_2 &= 26.84 \\ p_3 &= 12.15 \\ p_4 &= 14.95 \\ p_5 &= -11.62 \\ p_6 &= 0.046 \end{aligned}\right\} \text{(kip and ft. units)}$$

With the six redundants p_1, p_2, p_3, p_4, p_5, and p_6 thus determined, the bending moment at any point can be readily calculated from the following formula:

Bending moment at any section

$= m_0 + m_1 p_1 + m_2 p_2 + m_3 p_3 + m_4 p_4 + m_5 p_5 + m_6 p_6$

where m_0 = sum of the ordinates of the bending moment diagrams due to applied load and moments

Thus the bending moment, say at point 1, Fig. 31.12:

$= 0 + (-1.00) p_1 + 0$

$= (-1.00) \times 42.68 = 42.680$ lb ft/ft width (hogging)

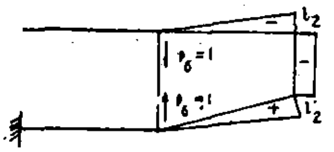


Fig. 31.46 *Bending moment diagram due to m_6*

Similarly, thrust n and shear s can also be readily calculated at any point from the formulae:

$n = n_0 + n_1p_1 + n_2p_2 + n_3p_3 + n_4p_4 + n_5p_5 + n_6p_6$

$s = s_0 + s_1p_1 + s_2p_2 + s_3p_3 + s_4p_4 + s_5p_5 + s_6p_6$

Thus thrust, say at point 1, Fig. 31.27:

$= 0 + (-1.00)p_2 + 0$

$= (-1.00) \times 26.84 = -26{,}840$ lb ft/ft width (tension)

and shear at point 1, Fig. 31.27:

$= 0 + (1.00)p_3 + 0$

$= (1.00) \times 12.15 = 12{,}150$ lb/ft width

Similarly, the bending moment, thrust and shear force can be determined right round the frame, and the analysis completed.

CASE (D): Two-cell box-section with solid central spine

Figure 31.47 represents a two-cell box-section with a solid central spine. For its transverse analysis it is assumed that the triangulated frame on either side of the centreline of the deck is rigidly attached to the solid central spine. The statically indeterminate system with various loads (self-weights and live load trucks, together with any cantilever bending moment), is shown in Fig. 31.48. I_1, I_2 and I_3 represent second moments of area (moments of inertia) of the three members of the frame in Fig. 31.48. The frame being three times statically indeterminate, is rendered determinate by introducing one cut in it as shown in Fig. 31.49. The three stress resultants released are p_1, p_2, and p_3. These can be evaluated from the following compatibility equations once the influence coefficients δ_{rs} and δ_{ro} are evaluated ($r = 1$ to 3, $s = 1$ to 3):

$$\delta_{10} + \delta_{11}\, p_1 + \delta_{12}\, p_2 + \delta_{13}\, p_3 = 0 \qquad (31.25)$$

$$\delta_{20} + \delta_{21}\, p_1 + \delta_{22}\, p_2 + \delta_{23}\, p_3 = 0 \qquad (31.26)$$

$$\delta_{30} + \delta_{31}\, p_1 + \delta_{32}\, p_2 + \delta_{33}\, p_3 = 0 \qquad (31.27)$$

where $\delta_{10} = \delta_1 w + \delta_1 q + (\delta_1 P_1 + \delta_1 P_2) + \delta_1 M_D$, and so on. w, q, P_1 and P_2 are the applied loads, and M_D the applied moment.

It may be noted that

$$\delta_{rs} = \int \frac{m_r m_o}{EI}\, ds = \int \frac{m_r m_s k}{E}\, ds, \text{ where } k = \frac{1}{I}$$

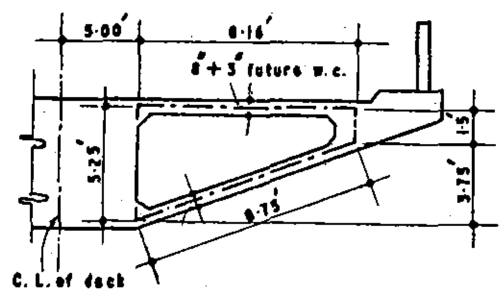


Fig. 31.47 *Two-cell box-section with solid central spine*

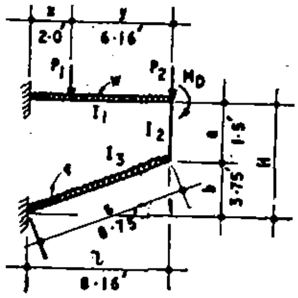


Fig. 31.48 *Statically indeterminate system with various loads*

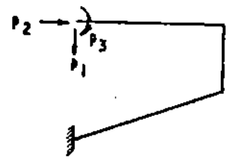


Fig. 31.49 *Statically determinate system*

$$\delta_{ro} = \int \frac{m_r m_o}{EI}\, ds = \int \frac{m_r m_o k}{E}\, ds, \text{ where } k = \frac{1}{I}$$

m_o = bending moment diagram on the determinate structure due to applied loads and moments.

Figures 31.50 to 31.54 represent the bending moment diagrams on the determinate structure due to various applied loads and moments, and Figs. 31.55 to 31.57 represent the bending moment diagrams on the determinate structure due to unit values of the three redudants p_1, p_2 and p_3.

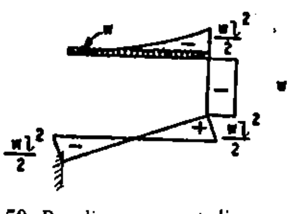


Fig. 31.50 *Bending moment diagram due to w*

• Calculation of various influence coefficients

~ *Influence coefficients involved in Eq (31.25)*

$$\delta_{11} = 0.333\ k_1 l^3 + k_2\ al^2 + 0.333\ k_3 l^2 s$$
$$\delta_{12} = -0.500\ k_2 a^2 l - 0.166\ k_3\ ls(H+2a)$$
$$\delta_{13} = -0.500\ k_1 l^2 + k_2 la - 0.500\ k_3 ls$$
$$\delta_1 w = 0.125\ k_1\ wl^4 + 0.500\ k_2\ wl^3 a + 0.083\ k_3 wl^3 s$$
$$\delta_1 q = -0.042\ k_3\ ql^3 s$$
$$\delta_1 P_1 = 0.166\ k_1\ P_1 ly^2\left(3 - \frac{y}{l}\right) + k_2 P_1\ lya + 0.166\ k_3\ P_1 y^2 s\left(3 - \frac{y}{l}\right) + k_3 P_1 s\frac{x^3}{l}$$
$$\delta_1 P_2 = -0.166\ k_3\ P_2 l^2 s$$
$$\delta_1 M_D = -0.500\ M_D l(2\ k_2 a + k_3 s)$$

Fig. 31.51 *Bending moment diagram due to q*

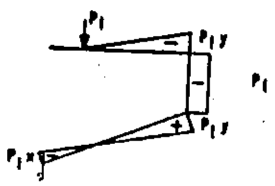


Fig. 31.52 *Bending moment diagram due to p_1*

Influence coefficients involved in Eq. (31.26)

$$\delta_{21} = -0.50\ k_2 a^2 l - 0.166\ k_3 ls(H+2a)$$
$$\delta_{22} = 0.333\ k_2 a^3 + 0.333\ k_3 s(H^2 + Ha + a^2)$$
$$\delta_{23} = 0.500\ k_2 a^2 + 0.500\ k_3 s(H+a)$$
$$\delta_2 w = -0.250\ k_2 wl^2 a^2 + 0.083\ k_3 wl^2 s(H-a)$$
$$\delta_2 q = 0.043\ k_3 ql^2 s(3H+a)$$
$$\delta_2 P_1 = -0.500\ k_2 P_1 a^2 y + 0.166\ k_3 P_1 x^2 \times \left[3H - \frac{x}{l}(H-a)\right]\frac{s}{l} - 0.166\ k_3 P_1 y^2 \times \left[3a + \frac{y}{l}(H-a)\right]\frac{s}{l}$$
$$\delta_2 P_2 = 0.166\ k_3 P_2 ls(2H+a)$$
$$\delta_2 M_D = 0.500\ k_2 M_D a^2 + 0.500\ k_3\ M_D s\ (H+a)$$

~ *Influence coefficients involved in Eq. 31.27*

$$\delta_{31} = -0.500\ k_1 l^2 - k_2 la - 0.500\ k_3 ls$$
$$\delta_{32} = 0.500\ k_2 a^2 + 0.500\ k_3 s(H+a)$$

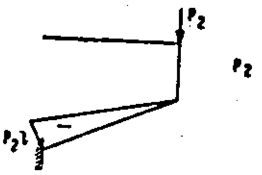


Fig. 31.53 *Bending moment diagram due to p_2*

$$\delta_{33} = k_1 l + k_2 a + k_3 s$$
$$\delta_3 w = -0.166\ k_1\ wl^3 - 0.500\ k_2\ wl^2 a$$
$$\delta_3 q = 0.166\ k_3 ql^2 s$$
$$\delta_3 P_1 = -0.500\ P_1 y(k_1 y + 2k_2 a) - 0.500 k_3 P_1 \times (y^2 - x^2)\frac{s}{l}$$
$$\delta_3 P_2 = 0.500\ k_3 P_2 ls$$
$$\delta_3 M_D = k_2 M_D a + k_3 M_D s$$

In the design example under illustration on 1-ft width basis:

w= 137 lb/ft
q= 110 lb/ft
P_1= 2,670 lb
P_2= 4,440 lb
M_D= 2,390 lb ft

$$I_1 = \left(\frac{1}{12}\right) \times 1 \times \left(\frac{8}{12}\right)^3 = 0.025\ \text{ft}^4$$

Hence, $k_1 = \dfrac{1}{I_1} = 40\ \text{ft}^{-4}$

Similarly, $k_2 = 0.633\ \text{ft}^{-4}$
$k_3 = 40.0\ \text{ft}^{-4}$

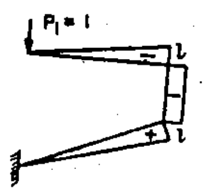


Fig. 31.54 *Bending moment diagram due to M_D*

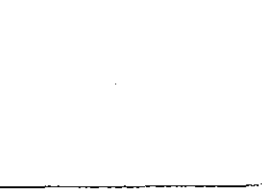


Fig. 31.55 *Bending moment diagram due to m*

Substituting the above in kip and ft. units in the formulae for the various influence coefficients derived earlier, the values obtained are given below:

For Eq. (31.25):

$\delta_{11} = 15061.04$, $\quad \delta_1 q = -\ 879.80$,
$\delta_{12} = -\ 3917.10$, $\quad \delta_1 P_1 = 24750.07$,
$\delta_{12} = -\ 2767.46$, $\quad \delta_1 P_2 = -\ 17176.92$,
$\delta_1 w = 5234.85$, $\quad \delta_1 M_D = -\ 3431.37$,

therefore, $\delta_{10} = 8496.33$

For Eq. (31.26):

$\delta_{21} = \delta_{12}$, $\quad \delta_2 q = 1901.51$,
$\delta_{22} = 4389.70$, $\quad \delta_2 P_1 = -\ 4172.31$,
$\delta_{23} = 1181.97$, $\quad \delta_2 P_2 = 25259.84$,
$\delta_2 w = 990.85$, $\quad \delta_2 M_D = 2824.92$,
therefore, $\delta_{20} = 26804.81$

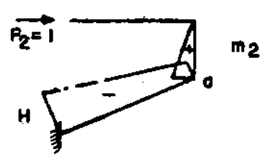


Fig. 31.56 *Bending moment diagram due to m_2*

For Eq. (31.27):

$\delta_{31} = \delta_{13}$, $\quad \delta_3 q = 425.38$,
$\delta_{32} = \delta_{23}$, $\quad \delta_3 P_1 = -\ 3985.67$,
$\delta_{33} = 677.35$, $\quad \delta_3 P_2 = 6340.32$,
$\delta_3 w = -\ 498.76$, $\quad \delta_3 M_D = 838.77$,
therefore, $\delta_{30} = 3119.99$

Substituting these values of the influence coefficients in the three compatibility Eqs. (31.25), (31.26) and (31.27):

$$15061.04p_1 - 3917.10p_2 - 2767.46p_3 + 8496.33 = 0$$
$$-3917.10p_1 + 4389.70p_2 + 1181.97p_3 + 26804.81 = 0$$
$$-2767.46p_1 + 1181.97p_2 + 677.35p_3 + 3119.99 = 0$$

Solving these three equations simultaneously:

$$\left.\begin{aligned} p_1 &= -3.848 \\ p_2 &= -7.696 \\ p_3 &= -6.630 \end{aligned}\right\} \quad \text{(kip and ft. units)}$$

With the three redundants p_1, p_2 and p_3 determined, the bending moment at any point may now be readily calculated from the following:

Bending moment at any section $= m_0 + m_1 p_1 + m_2 p_2 + m_3\, p_3$ where m_0 = sum of the ordinates of the bending moment diagrams due to applied loads and moments.

Thus the bending moment, say at the junction between the top slab and the central spine,

$= 0 + (1.00)p_3 = 1.00 \times (-6.630)$
$= 6{,}630$ lb ft per ft width (hogging)

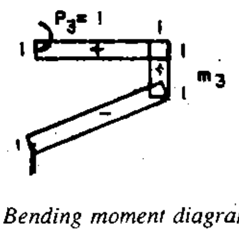


Fig. 31.57 *Bending moment diagram due to m_3*

Similarly, the thrust at this section:

$= n_0 + n_1 p_1 + n_2 p_2 + n_3 p_3$
$= 0 + (1.00)p_2 = 1.00 \times (-7.696)$
$= 7{,}696$ lb ft per ft width (tension)

The shear at this section

$= s_0 + s_1 p_1 + s_2 p_2 + s_3 p_3$
$= 0 + (-1.00)p_1 = (-1.00) \times (-3.8484)$
$= 3{,}848$ lb per ft width.

Similarly, the bending moment, thrust and shear can be calculated at any other point in the frame and the analysis completed.

CASE (E) Voided deck section

Figure 31.58 represents a section through a voided slab deck. For the transverse analysis, such a deck section may be assumed as comparising of two elastic cantilevers protruding on either side of centre-line of the deck, on rigidly holding the other.

Considering one of these cantilevers, if M and S represent the total bending moment and shearing force at the root of this cantilever per foot of its width, and if w and l represent the equivalent uniformly distributed load and the equivalent cantilever span respectively, such that

$$\frac{wl^2}{2} \equiv M$$

and $$wl \equiv S$$

then, rearrangement of terms gives

$$S\frac{l}{2} = M$$

or $$l = \frac{2M}{S} \qquad (31.28)$$

and $$w = \frac{S^2}{2M} \qquad (31.29)$$

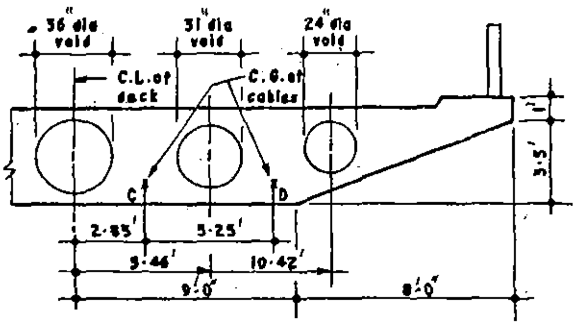


Fig. 31.58 *Voided slab deck*

It is further assumed that the longitudinal prestressing cables support the cross-section of the deck all along their length in a manner somewhat akin to the cables in a suspension bridge. The longitudinal prestressing cables are assumed to provide the supporting reactions in proportion to the product of their number and drape in a particular span. The bending moment and shearing force at any point in the transverse section of the deck can then be calculated from statics.

In the design example under illustration, per foot-width basis, the bending moment M and shearing force S at the root of the cantilever are as follows:

Load	M	S
Self and superimposed deadload	58,800 lb ft	7,315 lb
Live Load	31,500 lb ft	4,930 lb
TOTAL	90,300 lb ft	12,245 lb

Therefore, for the equivalent loaded cantilever

$$l = \frac{2M}{S} = \frac{2 \times 90.3}{12.245} = 14.72 \text{ ft}$$

$$w = \frac{S^2}{2M} = \frac{12.245^2}{2 \times 90.3} = 832 \text{ lb/ft}$$

Figure 31.59 represents the transverse section of the deck in terms of the equivalent loaded cantilevers, described above, in which A, B, C and D are the concealed supports provided by the longitudinal prestresssing cables. In the present case there are four 24/0.6 in Freyssinet multistrands at A, B, C and D, each. All the four cable groups are similarly profiled so that they all have the same drapes. Thus each of the four supports provides a reaction of $\frac{w(2l)}{4}$ i.e. $\frac{0.832(2 \times 14.72)}{4} = 6{,}120$ lb

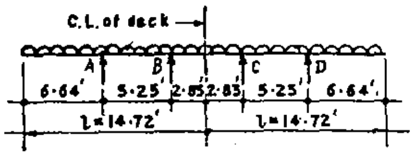


Fig. 31.59 *Equivalent elastic cantilevers of length l*

Hence the bending moment, say at the centre-line of the deck

$$= 6.12(2.83 + 5.25 + 2.83) - 0.832 \times \frac{(14.72)^2}{2}$$
$$= -23{,}200 \text{ lb ft per ft width (hogging).}$$

Thus the bending moment may be determined at any point in the transverse section of the deck.

CASE (F): Analysis of 'Deck-slab' in ordinary 'Beam-&-slab' type of deck and ordinary 'Slab' type of deck

This may be done by various simplified/emperical methods, e.g., those given in the AASHTO (and A.C.I.) bridge design specifications (which are summarised below for a ready reference), those suggested by Westergaard as early as 1930 (!), those suggested in the Indian Roads Congress Specifications for bridge design (which calculate the effective-width of wheel load dispersion by yet other emperical formulae (!) and require the use of 'influence-line', etc. Rüsch and Hergenroader have produced 'influence-surfaces, for analysing moments in skew slabs. Subsequently, Püchher also has produced 'influence surfaces' for computing moments in deck slabs under wheel loads. Each approach has its own limitations. However, the below described AASHTO-ACI approach is good enough for most cases, and is extremely quick to apply:

Design of Deck Slab—Empirical Method

- **Limitations**

The method described here applies to monolithic concrete deck slabs carrying highway loads. It should not be used where the skew angle exceeds about 20 degrees. The skew angle is defined as the horizontal angle between the abutment and the perpendicular to the bridge centre line.

For skews greater than 20 degrees, the slab design should be based on model tests, studies of highway skew slab bridges done by other agencies, a rational analysis based on principles of the theory of plates, or other acceptable methods (e.g. plane-grid finite element).

The reinforcement at the obtuse corners should receive special consideration.

- **One-way Slab**

The design of one-way slabs should be based on the analysis of a strip of unit width at right angles to the supports considered as a rectangular beam. Bending moment per unit width of slab due to a standard truck load, should be calculated according to empirical formulae given below, unless more exact methods are used. In the following

M_l = Live load moment per foot width of slab, in foot-pounds
l_n = Clear span length of slab, in feet
E = Effective width of slab resisting a wheel or other concentrated load, in feet
P_l = Load on one rear wheel of truck, in pounds

(a) Main reinforcement perpendicular to traffic (Spans 2-24 feet, inclusive). The live load moment for simple spans should be determined by the following formula (impact not included):

$$M_l = \frac{l_n + 2}{32} P_l$$

In slabs continuous over three or more supports, a continuity factor of 0.8 should be applied to the above formula for both positive and negative moments.

(b) Main reinforcement parallel to traffic: Longitudinally reinforced slabs should be designed for the appropriate truck or lane loading, which ever causes larger design moment. The effective width of slab resisting a wheel load should be estimated to be

$$E = 4 + 0.06\ l_n \text{ (in feet)}$$

but not to exceed 7 ft. The effective slab width resisting lane load should be taken as $2E$.

For simple spans, the maximum live load moment per foot width of slab, without impact, may be approximated by the following formulae:

Loading: In case of AASHTO Truck
Spans up to and including 50 feet:
$M_l = 900 l_n$ foot-pounds
spans 50–100 ft
$M_l = 1000(1.30 l_n - 20.0)$ foot-pounds.

Moments in continuous spans should be determined by a suitable analysis using the truck or appropriate lane loading.

Edge beams should be provided for all slabs having main reinforcement parallel to traffic. The beam may consist of a slab section additionally reinforced, a beam integral with and deeper than the slab, or an integral reinforced section of slab and curb. The edge beam should be designed to resist a live load moment of $0.10\ P_{l n}$ for simple spans. For continuous spans, 80% of the above calculated value should be used for both positive and negative moments, unless a greater reduction results from a more exact analysis.

• Two-way Slab

Two-way slabs are supported on all four sides by beams, girders or walls and reinforced in both directions. For rectangular slabs simply supported on all four sides, the proportion of the load carried by the short span of the slab may be estimated by the following equations:

For load uniformly distributed, $p = \dfrac{l_2^4}{l_1^4 + l_2^4}$

For load concentrated at centre, $p = \dfrac{l_2^3}{l_1^3 + l_2^3}$

where: p = proportion of load carried by short span
l_1 = length of short span of slab
l_2 = length of long span of slab.

where l_2 exceeds $1\frac{1}{2}$ times l_1, the entire load should be assumed to be carried by the short span.

For concentrated load, the effective slab width, E, for the load carried in either direction should be determined in accordance with (a) and (b) above.

Moments obtained should be used in designing the center half of the short and long spans. The reinforcement steel in the outer quarters of both short and long spans may be reduced 50 per cent.

For other supporting conditions at the edges, the formulae for p can be adjusted to account for the restraining effects the edges. At the ends of the bridge and at the intermediate points where the continuity of the slab is broken, the edges should be supported by diaphragms or other suitable means.

• Ribbed Slabs

A two-way system consisting of a slab with equally closely spaced ribs of similar size in two directions may be analyzed by the empirical method as an equivalent two-way slab. For two-way systems in which the ribs are not equally spaced or are of different size, an elastic analysis may be used in which the system is treated as an equivalent orthotropic plate. A model analysis may also be used.

• Cantilever Slabs

(a) *Truck loads:* Cantilever slabs may be designed to support truck wheel load independent of edge support along the end of the cantilever. The provisions given below include the effect of wheels on parallel elements.

(1) *Main reinforcement perpendicular to traffic:* The effective slab width, perpendicular to traffic, for each wheel load should be

$$E = 0.8\chi_1 + 3.75 \text{ (in feet)}$$

where χ_1 = distance in feet from load to point of support.

(2) *Main reinforcement parallel to traffic:* The effective slab width, parallel to traffic, for each wheel load, should be

$$E = 0.35\chi_1 + 3.2 \text{ (in feet) but not to exceed 7.0 feet}$$

(b) *Railing loads:* The effective width of slab resisting railing post loadings should be estimated as $E = 0.8\chi_2 + 3.75$ if no parapet is used and $E = 0.8\chi_2 + 5.0$ if a parapet is used. In the above expressions. χ_2 is the distance in feet from the center of the post to the point under investigation.

REFERENCES

1. "Models for Concrete Structures," *ACI Publication SP-24*; American Concrete Institute, Detroit, 1970.
2. Kennedy, J.B., and K.G. Tamber, "Problems of Skew in Concrete Bridge Design, Department of Highway, Ontario, Report No. 144.
3. Scordelis, A.C., R.E. Davis and K.S. Lo, "Load Distribution in Concrete Box Girder Bridges," International Symposium on Concrete Bridge Design, Toronto, April 1967.
4. Tung, D.H., "Torsional Analysis of Single Thin-walled Trapezoidal Concrete Box Girder Bridges," International Symposium on Concrete Bridge Design, Toronto, April, 1967.
5. Van De Pittee, D., "Prestressed Concrete Suspension Bridges," *Proceedings on World Conference of Prestressed Concrete*, 1967.
6. Dudra, J., "Design and Construction of Hudson Hope Bridge," *Journal of the Prestressed Concrete Institute, April*, 1966.
7. Scordelis, A.C., and C. Meyer, "Wheel Load Distribution in Concrete Box Girder Bridges," College of Engineering, University of California, January, 1969.
8. Podolny, Jr. Walter, "Cable Stayed Bridges of Prestressed Concrete," *PCI Journal*, Vol. 18, No. 1, January-February 1973, pp. 68–79; Closure-*PCI Journal*, Vol. 18, No. 5 September-October 1973, pp. 115–116.
9. Westergaard, H.M., "Computation of Stresses in Bridge Slabs due to Wheel Loads," *Public Roads*, March, 1930.
10. Mehmel, A., and H. Weise, "Model Investigation of Shew Slabs on Elastically Yielding Point Supports," Cement and Concrete Association, London, 1965.
11. Rusch, H., and A. Hergenroder, "Influence Surfaces for Moments in Shew Slabs," Cement and Concrete Association, London.
12. "Recommended Practice for Segmented Construction in Prestressed Concrete," *PCI Journal*, March-April, 1975.
13. "Analysis of Continuous Skewed Slab Bridge Structures," Ohio Department of Highways, 1961.
14. Schleicher, C., and Wegener, Borwin, "Continuous Skew Slabs," Veb Verlag Lur Bauwesen, 2nd Ed. Berlin.
15. Hool, G.A., and H.E. Pulver, "Reinforced Concrete Construction," Vol. 1, 1973, McGraw-Hill Book Co., Inc.

ANNEXURES

ANNEXURE 1

Expressing the wheel-load as a mathematical function: In order to transform the HB loading into a smooth continuous loading function, the Fourier series is made use of. The equation for a point load P at $x = x_1$ in the span may be expressed as

$$p(x_1) = 2\frac{P}{L} \sum_{n=1,2,3,\ldots} \sin\frac{n\pi x_1}{L} \sin\frac{n\pi x}{L}$$

Considering only the first three terms in the series:

$$p(x_1) = \frac{2P}{L}\left(\sin\frac{\pi x_1}{L}\sin\frac{\pi x}{L} + \sin\frac{2\pi x_1}{L}\sin\frac{2\pi x}{L} + \sin\frac{3\pi x_1}{L}\sin\frac{3\pi x}{L}\right)$$

For all four point loads, the equation of equivalent continuous loading curve becomes

$$\begin{aligned} p(x) &= p(x_1) + p(x_2) + p(x_3) + p(x_4) \\ &= \frac{2P}{L}\left[\sin\frac{\pi x}{L}\left(\sin\frac{\pi x_1}{L} + \sin\frac{\pi x_2}{L} + \sin\frac{\pi x_3}{L} + \sin\frac{\pi x_4}{L}\right) + \sin\frac{2\pi x}{L}\left(\sin\frac{2\pi x_1}{L} + \sin\frac{2\pi x_2}{L} + \sin\frac{2\pi x_3}{L} + \sin\frac{2\pi x_4}{L}\right)\right. \\ &\quad \left. + \sin\frac{3\pi x}{L}\left(\sin\frac{3\pi x_1}{L} + \sin\frac{3\pi x_2}{L} + \sin\frac{3\pi x_3}{L} + \sin\frac{3\pi x_4}{L}\right)\right] \end{aligned}$$

Referring to Fig. 31.16 it is noted that $x_1 = 41.5$ ft, $x_2 = 47.5$ ft, $x_3 = 67.5$ ft, and $x_4 = 73.5$ ft; substituting these values in the above equation:

$$p(x) = \frac{2P}{L}\left(3.7398\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}\right)$$

It may be noted that this is the load on the two boxes.

ANNEXURE 2

Derivation of formulae for various deflections and rotations

(a) Simple vertical deflection of a box beam due to asymmetric load: The intensity of loading on one box is $\left(\frac{1}{2}\right) p(x)$, Fig. 31.60 and is given by

$$\frac{P}{L}\left(3.7398\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}\right)$$

From the general equation of flexure

$$EI_b\frac{d^2y}{dx^2} = -M_x, \; (M_x \text{ being sagging moment})$$

Therefore, $EI_b\dfrac{d^4y}{dx^4} = -\dfrac{d^2M_x}{dx^2}$

The second differential of moment represents the first differential of shear, which represents the load intensity in magnitude.

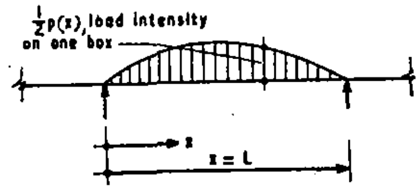


Fig. 31.60 *Variable downward loading along span*

Hence, $EI_b\dfrac{d^4y}{dx^4} = \dfrac{P}{L}\left(3.7398\sin\dfrac{\pi x}{L} - 1.8942\sin\dfrac{3\pi x}{L}\right)$

Integrating both sides four times in succession, and evaluating the four constants of integration from the four boundary conditions, at $x = 0$ and at $x = L$, each,

$EI_b\dfrac{d^2y}{dx^2} = 0.13PL$, (see Annexure 4)

and, $y = 0$

Finally it is found that

$$y = \frac{PL^3}{81\pi^4 EI_b}\phi_1 + \frac{PL}{15.39EI_b}\phi_2$$

where $y =$ deflection of box beam

$$\phi_1 = 302.8\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}$$

$$\phi_2 = x(x - L)$$

(b) Deflection at the 'cut' due to twist in the box-beam caused by asymmetric load, and (g) twist rotation in a box beam caused by symmetric load: The magnitude of twist rotation caused in a box beam by asymmetric as well as symmetric load is the same. The intensity of torque-loading on a box (Fig. 31.61),

$$\begin{aligned} &= \text{load} \times \text{arm} \\ &= \frac{1}{2}p(x)b \\ &= \frac{Pb}{L}\left(3.7398\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}\right) \end{aligned}$$

This represents $-\frac{dT}{dx}$ (just as first differential of shear represents the loading in magnitude). Assuming GK as constant and noting that

$$GK\frac{d\theta}{dx} = T, \text{ (as explained in the text)},$$

$$GK\frac{d^2\theta}{dx^2} = \frac{dT}{dx}$$

$$= -\frac{Pb}{L}\left(3.7398\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}\right)$$

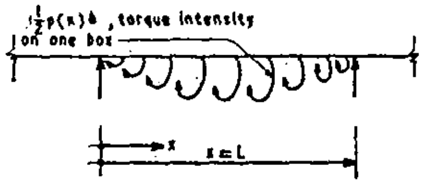


Fig. 31.61 *Variable torque loading along span*

Integrating both sides twice in succession, and evaluating the two constants of integration from the two boundary conditions, at $x = 0$ and at $x = L$, each, $\theta = 0$, finally it is seen that,

$$\theta = \frac{PbL}{9\pi^2 GK}\theta_3 \quad (= (\text{g}))$$

where θ = twist rotation in the box beam under symmetric or asymmetric loading

$$\phi_3 = \left(33.62\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}\right)$$

Hence the deflection at the 'cut' due to twist in the box beam caused by asymmetric load

$$= \text{twist rotation} \times \text{arm}$$

$$= \theta a$$

$$= \frac{PabL}{9\pi^2 GK}\phi_3 \quad (= (\mathbf{b}))$$

(c) Deflection at the cut due to local bending of cantilever under asymmetric load: This is the case of a cantilever subjected to a uniformly distributed total load of $\frac{1}{2}p(x)$. Its intensity, based on 12-ft dispersion (explained in the text), is $\frac{1}{24}p(x)$ lb/ft/ft. Deflection at the free end of cantilever under this uniformly distributed load

$$= \frac{\frac{1}{24}p(x)l^4}{8EI_c}$$

where $p(x) = \frac{2P}{L}\left(3.7398\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}\right)$

Hence deflection

$$= \frac{Pl^4}{96EI_c L}\phi_4$$

where $\phi_4 = \left(3.7398\sin\frac{\pi x}{L} - 1.8942\sin\frac{3\pi x}{L}\right)$

(d) Simple vertical deflection of a box beam due to q_x: The intensity of loading (Fig. 31.62):

$$q_x = q_1 + q_2\left[\alpha\sin\frac{\pi x}{L} + (1-\alpha)\sin\frac{3\pi x}{L}\right]$$

Now $EI_b\frac{d^4y}{dx^4} = q_x$ (the loading)

$$= q_1 + q_2\left[\alpha\sin\frac{\pi x}{L} + (1-\alpha)\sin\frac{3\pi x}{L}\right]$$

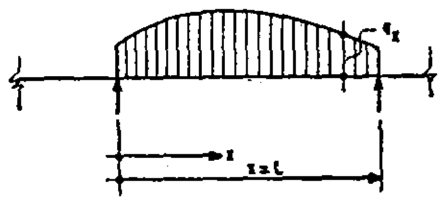


Fig. 31.62 *Variable load q_x along span*

Integrating both sides four times in succession, and evaluating the four constants of integration from the four boundary conditions, at $x = 0$ and at $x = L$, each,

$EI_b\frac{d^2y}{dx^2} = [q_1 + 0.6\ q_2(2\alpha - 1)] \times 0.0536\ L^2$ (see Annexure 4)

and $y = 0$.

Finally it is seen that

$$y = \frac{q_1}{67.13EI_b}\phi_6 + \frac{q_2L^4}{81\pi^4 EI_b}\phi_5 + \frac{(2\alpha-1)L^2q_2}{62.25EI_b}\phi_7$$

where $\phi_6 = x(2.8x^3 - 5.6Lx^2 + 1.8L^2x + L^3)$

$$\phi_5 = \left[81\alpha\sin\frac{\pi x}{L} + (1-\alpha)\sin\frac{3\pi x}{L}\right]$$

$$\phi_7 = x(x - L)$$

(e) Deflection at the cut due to twist in the box beam caused by q_x: The intensity of torque-loading on a box

$$= \text{load} \times \text{arm} = q_x \cdot a$$

$$= aq_1 + aq_2\left[\alpha\sin\frac{\pi x}{L} + (1-\alpha)\sin\frac{3\pi x}{L}\right]$$

Following the same explanation as given under (*b*) earlier, for the present case

$$GK\frac{d^2\theta}{dx^2} = \frac{dT}{dx}$$
$$\equiv -q_x a$$

Substituting for q_x, integrating both sides of the above equation twice in succession, and then evaluating the two constants of integration from the two boundary conditions, at $x = 0$ and at $x = L$, $\theta = 0$, it is finally found that

$$\theta = \frac{aq_1}{2GK}\phi_8 + \frac{aq_2L^2}{9\pi^2GK}\phi_9$$

where θ = twist rotation in the box beam due to q_x,

$$\phi_8 = x(L - x)$$
$$\phi_9 = x\left[9\alpha\sin\frac{\pi x}{L} + (1 - \alpha)\sin\frac{3\pi x}{L}\right]$$

Hence deflection at the 'cut' due to twist in the box beam caused by q_x

$$= \text{twist rotation} \times \text{arm}$$
$$= \theta_a$$
$$= \frac{a^2q_1}{2GK}\phi_8 + \frac{a^2q_2L^2}{9\pi^3GK}\phi_9$$

(f) Deflection at the 'cut' due to local bending of cantilever under q_x: This is the case of a cantilever subjected to a point load q_x at its tip. Deflection at the tip is equal to $\dfrac{q_xl^3}{3EI_c}$

$$i.e.\ \frac{q_1l^3}{3EI_c} + \frac{q_2l^3}{3EI_c}\left[\alpha\sin\frac{\pi x}{L} + (1 - \alpha)\sin\frac{3\pi x}{L}\right]$$
$$= \frac{q_1l^3}{3EI_c} + \frac{q_2l^3}{3EI_c}\phi_{10}$$

where $\phi_{10} = \alpha\sin\dfrac{\pi x}{L} + (1 - \alpha)\sin\dfrac{3\pi x}{L}$

(h) Flexural rotation at the cut due to local bending of cantilever under symmetric load: This is the case of a cantilever subjected to a uniformly distributed total load of $\frac{1}{2}p(x)$, whose intensity is $\frac{1}{24}p(x)$ lb/ft/ft, as explained under (c) earlier. Flexural rotation at the free end of the cantilever under this uniformly distributed load

$$= \frac{\frac{1}{24}p(x)l^3}{6EI_c}$$
$$= \frac{Pl^3}{72EI_cL}\phi_4\ \{p(x) \text{ and } \phi_4 \text{ have been defined already}\}.$$

(j) Twist rotation in a box-beam due to M_x.

$$M_x = M_1 + M_2\left[\gamma\sin\frac{\pi x}{L} + (1 - \gamma)\sin\frac{3\pi x}{L}\right]$$

M_x being analogous to the product q_xa, in the formula for θ in (*e*) earlier, q_xa is replaced by M_x thus:

$$\theta = \frac{M_1}{2GK}\phi_8 + \frac{M_2L^2}{9\pi^2GK}\phi_{11}$$

where θ = twist rotation in box-beam due to M_x

$$\phi_8 = x(L - x)$$
$$\phi_{11} = 9\gamma\sin\frac{\pi x}{L} + (1 - \gamma)\sin\frac{3\pi x}{L}$$

(k) Flexural rotation at the 'cut' due to local bending of cantilever under M_x: This is the case of a cantilever subjected to a constant moment M_x along its length. Flexural rotation caused at the tip of the cantilever is given by

$$\frac{M_1l}{EI_c}$$
$$i.e.\ \frac{M_1l}{EI_c} + \frac{M_2l}{EI_c}\left[\gamma\sin\frac{\pi x}{L} + (1 - \gamma)\sin\frac{3\pi x}{L}\right]$$
$$\text{or } \frac{M_1l}{EI_c} + \frac{M_2l}{EI_c}\phi_{12}$$

where $\phi_{12} = \gamma\sin\dfrac{\pi x}{L} + (1 - \gamma)\sin\dfrac{3\pi x}{L}$

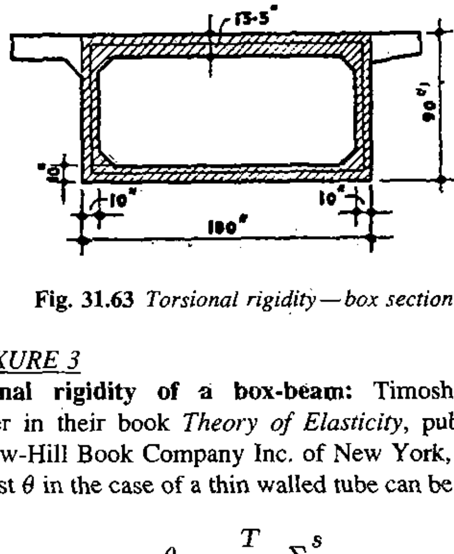


Fig. 31.63 *Torsional rigidity—box section*

<u>*ANNEXURE 3*</u>

Torsional rigidity of a box-beam: Timoshenko and Goodier in their book *Theory of Elasticity*, published by McGraw-Hill Book Company Inc. of New York, show that the twist θ in the case of a thin walled tube can be expressed as

$$\theta = \frac{T}{4A^2G}\Sigma\frac{s}{t}$$

where A is the mean of the areas enclosed by the outer and the inner boundaries of the cross-section of the tube, s is the length, and t the thickness of the elements composing the cross-section of the tube.

In the present instance (Fig. 31.63)

$$A = \frac{(180 \times 90) + (160 \times 66.5)}{2}$$
$$= 13420 \text{ in}^2 \text{ or } 93.3 \text{ ft}^2$$
$$\Sigma\frac{s}{t} = \frac{170}{10} + \frac{170}{13.5} + \frac{78.25}{10} \times 2$$
$$= 45.25$$
$$G = \frac{E}{2(1 + \text{Poisson's ratio})}$$
$$= \frac{3 \times 10^6}{2(1 + 0.15)}$$
$$= 1.304 \times 10^6 \text{ lb/in}^2$$
$$\text{or } 187.8 \times 10^6 \text{ lb/ft}^2$$

$$\text{Therefore, } \theta = \frac{T \times 45.25}{4 \times 93.3^2 \times 187.8 \times 10^6}$$
$$= \frac{T}{14.45 \times 10^{10}}$$

so that, $\frac{T}{\theta} = 14.45 \times 10^{10}$ lb/ft^2

per unit length, $GK = \frac{T}{\theta}\left(\text{from} \frac{T}{K} = \frac{G\theta}{L}\right)$

Hence, $GK = 14.45 \times 10^{10}$ lb/ft^2

ANNEXURE 4

Bending moment in the box-beam at support sections due to asymmetric load (placed as in Fig. 31.16 and due to redundant q_x:

(i) due to asymmetric load placed longitudinally as in Fig. 31.16. This load, on one box-beam, comprises 4 axles of halved HB vehicle, Fig. 31.15. By reading under the load the ordinates of the bending moment influence diagram for an intermediate support ($x = 0$ or $x = L$) of a multispan continuous beam (assuming constant EI), the bending moment, $\left(i.e.\ EI \cdot \frac{d^2y}{dx^2}\right)$, is obtained. This comes to 0.13 PL (of the hogging type). This approach is approximate.

(ii) due to q_x:

$$q_x = q_1 + q_2\left[\alpha \sin\frac{\pi x}{L} + (1 - \alpha)\sin\frac{3\pi x}{L}\right]$$

Therefore, at $x = 0$ and at $x = L$, $qx = q_1$, and at $x = \frac{L}{2}$, $q_x = q_1 + q_2(2\alpha - 1)$. Note that q_1 distribution is rectangular, *i.e.*, it is a uniformly distributed load, q_2 distribution is sinusoidal, *i.e.*, it lies between a convex parabola and a triangle, which may, therefore, be approximated into an equivalent rectangular distribution whose average ordinate is 0.6 of the maximum, *i.e.* $0.6q_2(2\alpha - 1)$.

Thus, for the present consideration only, q_x may be approximated into a uniformly distributed load whose intensity is given by $q_1 + 0.6q_2(2\alpha - 1)$. Placing this load on the bending moment influence diagram for an intermediate support ($x = 0$ or $x = L$) of a multispan continuous beam (assuming constant EI), the bending moment $\left(i.e. EI \cdot \frac{d^2y}{dx^2}\right)$ is obtained. This comes to:

0.0536 $L^2\{q_1 + 0.6q_2(2\alpha - 1)\}$ and is of hogging type.

CHAPTER 32

Expansion Joints for Bridge Decks

A deck movement joint is defined as a structural discontinuity between two elements, at least one of which is a deck element, designed to permit relative translation and/or rotation between the two. This definition includes all joints in the deck or between the deck and the abutments at which 'movement' can take place, but excludes construction joints. An expansion joint implicitly also refers to a contraction joint and hence it is more rational to call it a movement joint. However, in general, it is called an expansion joint merely because of colloquial usage of the name.

The width of the gap, the maximum linear expansion and contraction and rotation of the bridge structure at the gap are the predominant factors in the design. The design of the joint should allow free rotation and deflection or shifting of the structure at the edges without damage or inconvenience to the traffic. Structurally, the expansion joint has to be strong enough to withstand the knocking of the wheels of vehicles passing over the bridge, and the latter is the main cause for gradual damage and disintegration of joints.

The basic philosophy sometimes adopted is that 'minimum number of joints be used consistent with the need for articulation of the structure'. The approach to eliminate as many expansion joints as possible is taken because of the generally poor performance of deck joints. There are considerable benefits to be derived from eliminating joints entirely, where possible, as might be the case for curved structures or those which have flexibility in the support system, such as, with the slender or hinged columns. The structure acts like a bow with the superstructure fixed at both abutments. Movement at intermediate supports is accommodated by sliding bearings.

Where deck joints are required, the following two types should be generally excluded because of unsatisfactory service records: any joint which has been found prone to failure within a few years of installation, and joints placed directly in concrete without provision for armour at its edges with the roadway surface whereby they fail because of spalling of the concrete. This type of failure was common when it was the normal practice to install compression seals (see ahead) in direct contact with a concrete deck slab.

Deck joints divide into those which are designed to be sealed (waterproof) and those that are of open type (designed to permit the free flow of water through the joint). All joints must be designed so that none of the components fail at serviceability limit states and the sealed joints should remain watertight. Since deck joints do not directly affect the safety of a structure, some damage can be tolerated from conditions exceeding serviceability limit states since it would be uneconomical to design joints for ultimate limit states! Acceptable damage is considered to be that which can be repaired while the deck remains in service, even though one or more lanes may have to be closed to traffic to permit the repairs to be carried out.

32.1 SOURCES OF MOVEMENTS AND LOADS AT EXPANSION JOINTS

Movements in a structure result from either 'loading conditions' or a 'change in volume' of the components of the structure. The magnitude of the movement must be calculated for each structure since it depends upon the nature of the structure and its response to static and dynamic forces. The most common sources of loads on and movements in deck joints are direct impact of wheels, temperature, and those caused by soil conditions which result in the movement of supports. Conversely, some joints, particularly compression seals or those which accommodate movement by the shear deformation of an elastomer, can impose some forces on the structure which may have to be included in the design of the structure. This characteristic of compression seals and certain proprietary joints may prevent substitution of these joints for those which do not exert forces on the structure—unless the forces are not significant.

A checklist, which has been found to be a convenient method of ensuring that all factors affecting the magnitude of movements and loads have been considered in design, is as follows:

(a) Properties of the materials in the structure, including coefficient of thermal expansion, modulus of elasticity, Poisson's ratio, creep and shrinkage strains
(b) Effective temperature variation in the structure
(c) Sizes and properties of the structural members
(d) Method and sequence of construction

(e) Tilt, settlement and movement of supports
(f) Construction tolerances
(g) Static and dynamic response of the structure
(h) Interaction of the forces to which the structure may be subjected, including dead and live loading; wind, earthquake and earth pressures
(i) Structural restraints, if any

Joints which are considered 'fixed' may be fixed against translation, but not rotation. Flexural deflection of a member results in rotation at the joint, and failure to accommodate this rotation has been a common cause of leakage and deterioration at 'fixed' joints.

The 'plan-geometry' of a bridge superstructure is an important factor in determining the movements which take place at different points in the deck structure. Bridges which are straight and narrow in plan will generally exhibit movements along their centreline in response to changes in temperature. Bridges which are curved in plan or with skew joints have two components of movement at deck joints, a longitudinal component along the centreline of the superstructure and a transverse component perpendicular to the longitudinal centreline as shown in Figs. 32.1 and 32.2. The transverse component can cause serious problems with some types of expansion devices, for example, 'finger' joints (sometimes called 'tooth' or 'comb' joints). Binding up of the joint can result in large forces on the joints, bearings and abutments with corresponding reactions at intermediate piers. (Skewing the fingers in the finger joints can be costly and cumbersome.)

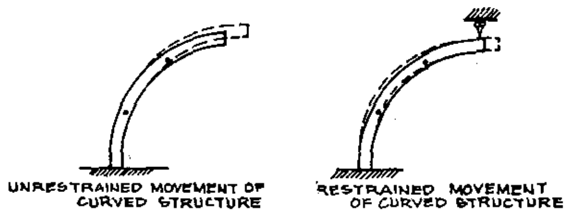


Fig. 32.1 *Response of curved structures*

To determine the actual movement of a bridge super-structure, at all the points of attachment of the superstructure, the position of the zero movement point must be established. The position of this point depends upon the geometry of the superstructure, the location of the piers and bearings and their relative shear stiffnesses. For straight structures, the zero movement point is generally located on the longitudinal centreline of symmetry as shown in Fig. 32.3. In curved structures the zero movement point will generally not be located on the longitudinal centreline and may even fall outside the plan of the superstructure. Movements are always towards and away from the zero

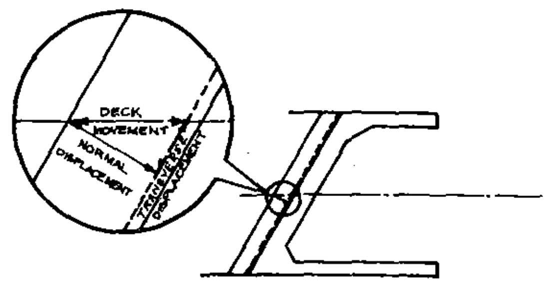


Fig. 32.2 *Normal and transverse displacements across a skewed joint*

movement point as shown in Fig. 32.4. The calculation of the location of the zero movement point can be quite involved for complex structures. Detailed explanations for determining the location of this point have been presented in two separate chapters in this book.

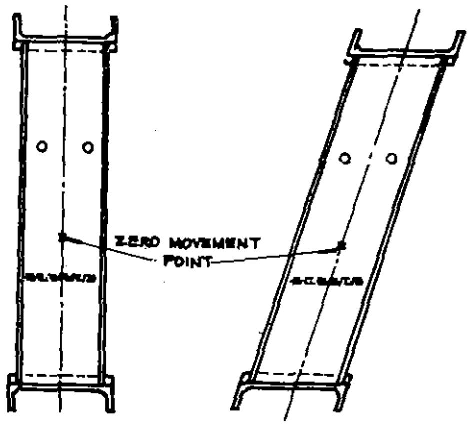


Fig. 32.3 *(a) 'Right' superstructure (b) 'Skewed' superstructure. Zero movement point in straight and skewed superstructures*

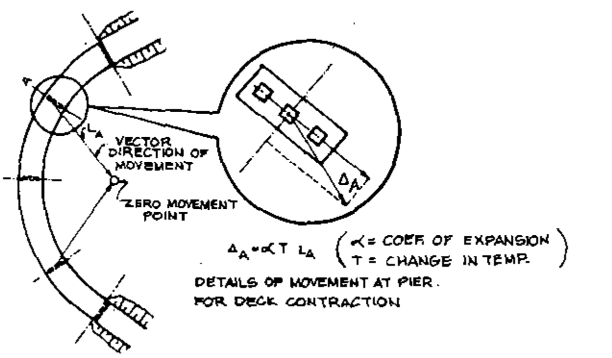


Fig. 32.4 *Zero movement point in curved structures*

32.2 FUNCTIONAL REQUIREMENTS OF EXPANSION JOINTS

The first step towards a performance specification is a statement of the required results. Four such requirements for deck joints are:

(a) The components of the joint must accommodate the translation and rotation of the structure at the joint.
(b) The joint must provide for the passage of traffic across the joint without significantly impairing the riding quality of the pavement surface. It must not be a safety hazard to road users, including pedestrians and cyclists where they are permitted on the bridge.
(c) The joint shall be almost maintenance-free for the design-life of the structure.
(d) Deck joints must be constructed and detailed such that they prevent damage to the deck, bearings, piers and abutments from water, deicing chemicals and other foreign materials, either by preventing water from passing through the joint or by permitting water to drain through the joint in such a manner that damage to the deck, bearings, pier or abutment, does not occur. The selection of an 'open' type joint (which allows water to percolate through) will normally require particular attention to the details of components of the structure other than the joint itself.

The maximum permissible gap in a joint at the serviceability limit states is generally limited to 100 mm for deck joints with a 'single' opening, and to 80 mm for each opening in deck joints with 'multiple' openings. The gap is measured perpendicular to the length of the joint at the deck surface. These limits are based on experience and have been found to be the maximum values which can be permitted, consistent with maintaining an adequate quality of ride across the joint for user comfort and safety of vehicle loads. While improvements in ride-quality result from keeping the maximum gap as small as possible, this requires the use of more multiple opening joints which cost considerably more than joints with single openings. Since 'incompressibles' do get trapped in many types of sealed joint, the minimum gap should be not less than 20 mm in order to prevent the transfer of forces through the debris in the joint. Consequently, limits on the minimum gap also increase the maximum gap for the same total movement capacity.

Open Type Joints

Under certain conditions, as open type of joint can provide a good enough, effective and an economic solution. Satisfactory performance depends upon the provision of an effective deck drainage system, control of flow of surface drainage through the joint and disposal of the run-off from the site. The success of open joints in the past has been limited by the damage (to the portion of the structure immediately beneath the joint) from debris and run-off, especially in areas where deicing salts are used. Wet dirt often accumulates on the flanges of steel beams or around bearings, and leads to corrosion. Chlorides penetrate concrete surfaces, causing the spalling of concrete and the corrosion of embedded reinforcement. The amount of deterioration can be minimized by removal of the accumulated debris by routine washing (followed by spot repainting in steel structures) but this has rarely been done in practice.

Sealed Type Joints

Sealed type of joints are required to seal the surface of the deck, including curbs, sidewalks, medians and, where necessary, even the barrier or parapet walls, against the seepage of moisture and other contaminants through the joint. It is also a requirement that the geometry of the joint be such as to prevent the accumulation of water and roadway debris in the joint. Metal to metal sliding surfaces have not performed satisfactorily, largely because of the effects of vibration, the accumulation of debris and the fact that they cannot be sealed effectively. The presence of debris and, in the winter months, possibly ice, has been found to restrict the movement capabilities of joints, displacing thin membrane seals. Where the movement of the joint is accommodated through the change in geometry of a thin 'gland' or 'membrane strip', these should not be in direct contact with vehicles in order to prevent their failures from wear and puncture by the trapped incompressibles.

Some of the proprietary joints, claimed to be 'self-cleansing', have been found to be no less vulnerable to damage by accumulated debris than other types of joints.

Deck joints having elastomeric components in direct contact with the deck surface, do not provide adequate support for the adjacent edges of the deck wearing surface. Nosings of epoxy mortar or similar material are sometimes used between the edges of the deck surface and the joint to try and prevent spalling of the concrete edge of the deck.

The chief factors in determining whether to use an open or a sealed joint are the cost and the location of the site. The total cost of each system must be compared and, for open joints, this includes the cost of disposing of surface run-off. The total costs of sealed joints may include the cost of providing for additional forces on the structure, and an increase in maintenance costs and frequency of inspection.

In general, open joints are well suited to secondary highways where little sand and salt is supplied during winter

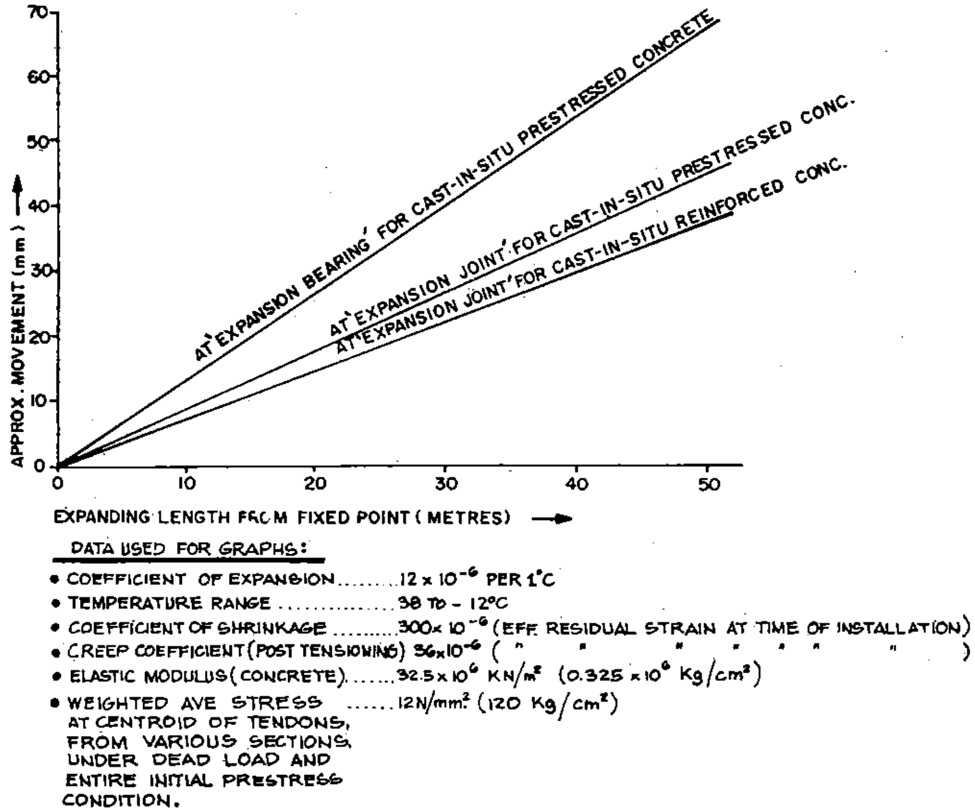


Fig. 32.5

maintenance operations. Sealed joints tend to be more suited to urban areas where the costs of providing for the surface run-off can be high.

Movements at Joints—A Relative Comparison

As explained earlier, the movements to be catered for at (bearings and) expansion joints arise from the following causes:

(i) Thermal expansion and contraction of the superstructure and in certain cases even of substructure
(ii) Shrinkage of concrete
(iii) Creep in the concrete
(iv) Elastic shortening under prestress
(v) Displacement of the structure under load or any other action.

Because expansion joints are installed at a late stage in construction, therefore some of these movements will already at least partly have taken place. Hence, less total movement has to be catered for at expansion joints than in bearings at the same location.

Elastic shortening under prestress is normally assessed on the mean stress induced in the deck section by the prestress and dead loads.

In most bridges, displacements of the structure under load, produce very minor movements.

Figure 32.5 shows an approximate comparison of the total movements at an expansion joint in cases of cast-in-situ reinforced concrete deck and prestressed concrete deck, for various expanding lengths. (Various assumptions made are given in the figure itself.) Also shown is the movement at the expansion bearing (for various expanding lengths) in case of a cast-in-situ prestressed concrete deck. (Note that the movement at the bearing is more than at the expansion joint for the same deck since the latter is installed much after the deck is cast on top of the bearing and, therefore, suffers movements only due to temperature and parts of concrete shrinkage and creep).

Early Types of Expansion Joints

The structural steel 'angles and plate' type, shown in Fig. 32.6, is one of the oldest types of expansion joints. It has never performed satisfactorily, primarily because of its inefficient (even though cheaper) detail that lends itself as an easy prey both to poor installation as well as to poor response to wheel-knocking and rotational deformation.

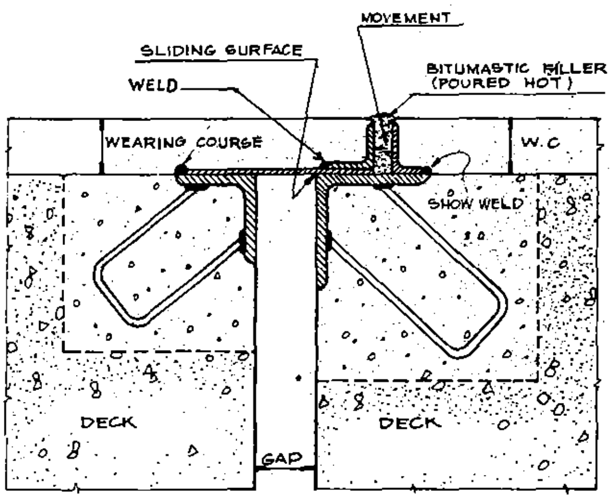


Fig. 32.6 *Angle and plate joint (Ancient)*

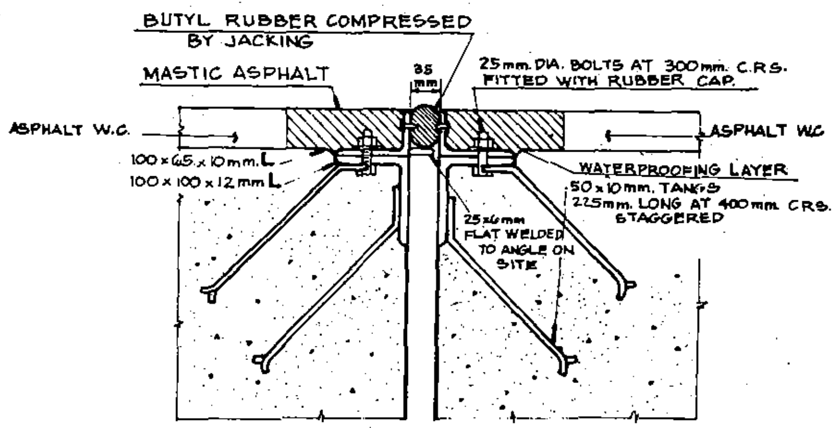


Fig. 32.7 *Angle joint sealed with butyl rubber strip (Ancient)*

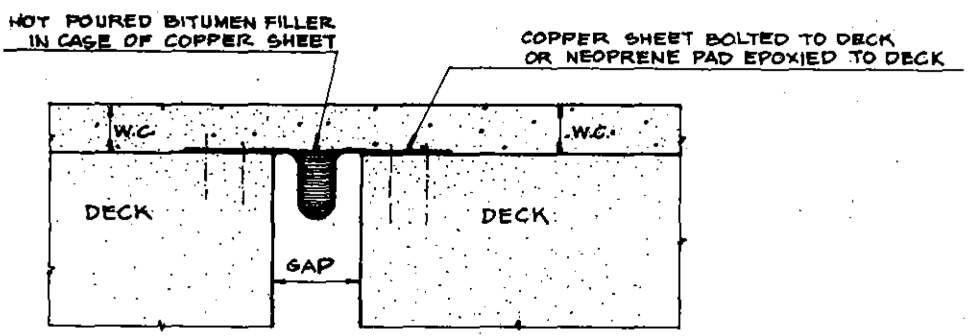


Fig. 32.8 *Bellow-type joint*

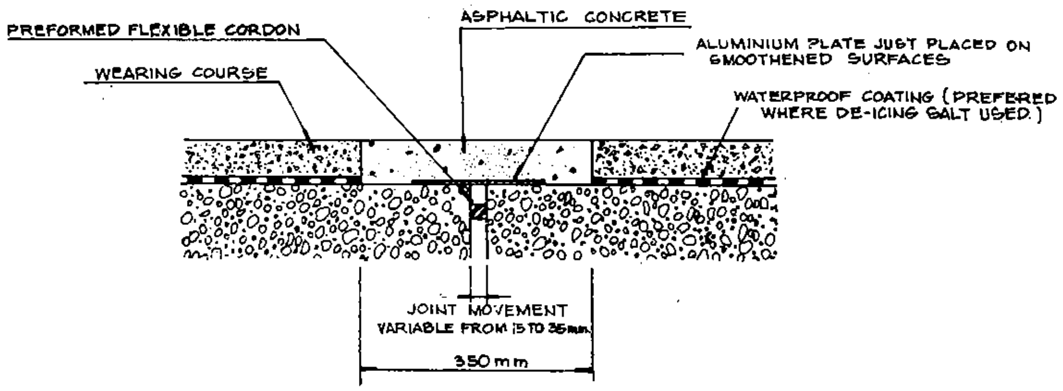


Fig. 32.9 *Buried plate type joint*

Figure 32.7 shows a slightly less inefficient cousin of the same type of joint. Figures 32.8 and 32.9 respectively show the Bellow-type and the Buried-plate-type joints, which permit a continuous wearing surface, thus permitting a better ride.

All these joints can basically cater only for small to medium amounts of movements (say 15-50 mm).

Elastomeric (Neoprene) Compressed Seals

There are two basic types of extruded elastomeric seals. One is the 'open-cell compression-seal' dependent upon its ability to maintain pressure on the joint side-walls, with varying degrees of stress. The other type consists of 'strip-seals' and 'box-configuration seals' using locking lugs or ears placed in compression within the cavity of a retainer (Figs. 32.10 and 32.11).

The former seals may be designated 'compression seals' and the latter addressed as either 'strip seals' or 'box seals' for whatever case applies, as will be seen in the figures ahead.

These types of seals are necessarily dependent upon the material they are made of. This material is what makes 'preformed' elastomeric seals superior to all other types of seals and sealants.

Generically, the basic material is polychloroprene, otherwise known as Neoprene. Polychloroprene was selected because of its agreeable physical attributes.

Since the first uses of these seals were oriented to the transportation field, they had to resist deterioration from exposure to weather, sunlight, oils, chemicals, heat, abrasion, and impact, while still remaining elastic during their expected life cycle.

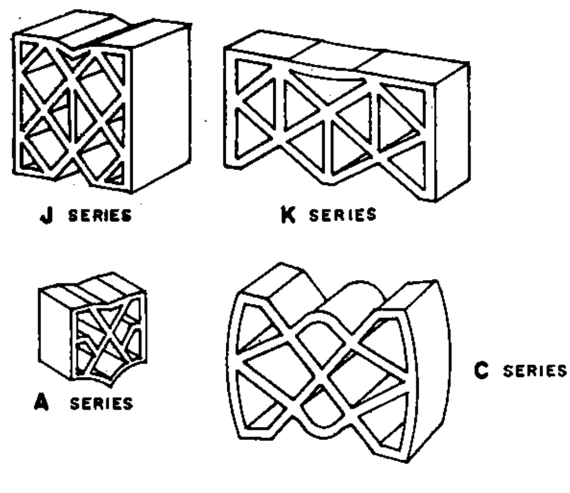


Fig. 32.10 *Standard compression seals (Glands)*

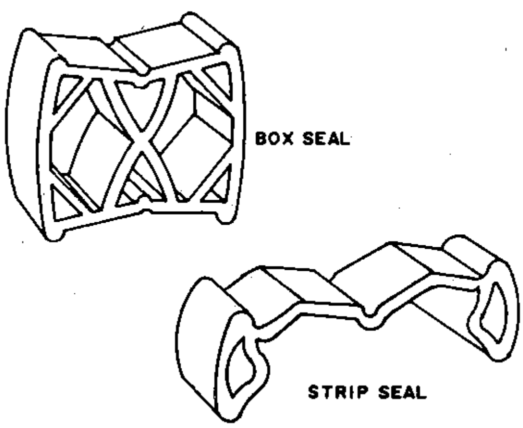


Fig. 32.11 *Lock in compression seals (Glands)*

32.3 COMPRESSION SEALS

Compression seals derive their name from the fact that they

Table 32.1 *Typical Compression Seal Data*

	Style No.	*W*	*H*	*W* min	*MIW*	*W* max	*M*	*CH*	*G* min
SERIES — J	J-175	1.3/4 (44.45)	2 (50.80)	0.76 (19.30)	1.02 (25.91)	1.48 (37.59)	0.72 (18.27)	2.3/16 (55.56)	0.56 (14.22)
	J-200	2 (50.80)	2.1/16 (52.39)	0.92 (23.37)	1.19 (30.27)	1.70 (43.18)	0.78 (19.81)	2.15/32 (62.71)	0.52 (13.21)
	J-225	2.1/4 (57.15)	2.5/8 (66.68)	1.06 (26.92)	1.36 (34.54)	1.91 (48.51)	0.85 (21.59)	2.7/8 (73.03)	0.56 (14.72)
	J-250	2.1/2 (63.50)	2.3/4 (69.85)	1.15 (29.21)	1.49 (37.85)	2.12 (53.85)	0.97 (24.64)	3.1/8 (79.38)	0.65 (16.51)
	J-300	3 (76.20)	3 (76.20)	1.30 (33.02)	1.74 (44.20)	2.55 (64.77)	1.25 (31.75)	3.11/16 (93.66)	0.80 (20.32)
	J-325	3.1/4 (82.55)	3.1/8 (79.38)	1.43 (36.32)	1.90 (48.26)	2.76 (70.10)	1.33 (33.78)	3.13/16 (96.84)	0.81 (20.57)
	J-350	3.1/2 (88.90)	3.1/2 (88.90)	1.47 (37.34)	2.01 (51.05)	2.97 (75.44)	1.50 (38.10)	4.1/8 (104.78)	0.85 (21.59)
	J-400	4 (101.60)	4.23/32 (119.86)	1.77 (44.96)	2.35 (59.69)	3.40 (86.36)	1.63 (41.40)	5.1/4 (133.35)	1.02 (25.91)
	J-450	4.1/2 (114.30)	4.1/2 (114.30)	1.82 (46.23)	2.52 (64.01)	3.82 (97.03)	2.00 (50.80)	5.11/32 (135.73)	1.07 (27.18)
	J-500	5 (127.00)	5.5/16 (134.94)	1.75 (44.45)	2.64 (67.06)	4.25 (107.95)	2.50 (63.50)	5.7/8 (149.22)	1.00 (25.4)
	J-600	6 (152.40)	5.3/4 (146.05)	2.03 (51.56)	3.12 (79.25)	5.10 (129.54)	3.07 (77.98)	6.5/8 (168.27)	1.28 (32.51)
SERIES — K	K-300	3 (76.20)	2.19/32 (65.88)	1.25 (31.75)	1.71 (43.43)	2.55 (64.77)	1.30 (33.02)	3.02 (76.71)	0.62 (15.75)
	K-350	3.1/2 (88.90)	2.1/2 (63.50)	1.25 (31.75)	1.86 (47.24)	2.97 (75.44)	1.72 (43.69)	3.0 (76.20)	0.82 (15.75)
	K-400	4 (101.60)	2.1/2 (63.50)	1.63 (41.40)	2.26 (57.40)	3.40 (86.36)	1.77 (44.96)	3.10 (78.74)	0.88 (22.35)
	K-500	5 (127.00)	3.1/4 (82.55)	2.00 (50.80)	2.80 (71.12)	4.25 (107.95)	2.25 (57.15)	3.62 (91.95)	1.25 (31.75)
	K-600	6 (152.40)	4 (101.60)	2.25 (57.15)	3.27 (83.06)	5.10 (129.54)	2.85 (72.39)	4.37 (111.0)	1.50 (38.10)

NOTE JW–The recommended 'joint width at the time of installation' is considered the same as MIW or larger. When ambient temperatures are over 85° F the compression seal becomes difficult to install.

KEY:
- W — Seal Width—Nominal Uncompressed
- H — Seal Height—Nominal Uncompressed
- W min — Seal Width—Compressed at the Highest Anticipated Temp.
- MIW — Minimum Installation Width
- W max — Seal Width—at Lowest Anticipated Temp.
- M — Total Seal Movement Capability (W max minus W min)
- JD — Minimum Recommended Joint Depth for Seal
- CH — Compressed Height of Seal
- G min — Clear gap between 'nitches' or 'support ears' supporting the Compn. Seal.

Dimensions in inches (mm)

are compressed when inserted into the joint-opening, and remain in the state of compression during all movement phases of the joint.

They are generally designated by size according to their width. A movement rating is usually established, for each width seal, along with other pertinent data, by the manufacturer. These critical dimensions and data are available in table form published by the various manufacturers (Table 32.1. typical).

Naturally, the seal to be used, in any particular case, must have a nominal width which is greater than the largest expected gap opening. Therefore, the 'working dimensions' of compression seals vary generally from 85% of nominal width—when the joint is fully open, to approximately 40-50% of nominal width—when the joint-gap is at its minimum dimension, (Fig. 32.12). Figure 32.13 shows a compression seal with armoured structural steel edges.

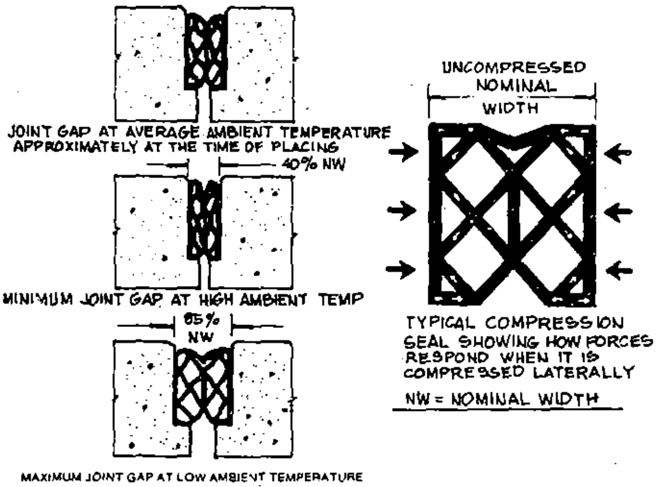


Fig. 32.12 *Compression seals are always in stressed state. They maintain watertightness and keep out incompressibles by maintaining sidewall pressure on the joint interfaces*

In order to design an expansion joint, the engineer would first determine the joint movements that will take place due to temperature, creep, shrinkage, elastic shortening, etc. After this anticipated movement is ascertained, the joint geometry can easily be determined. Simply stated, the joint geometry/configuration for the entire movement cycle should accommodate the physical characteristics of the compression seal selected (i.e., physical properties of a seal exhibiting a movement rating equal to or greater than the expected joint movement).

After the designer has obtained the joint movement, he can utilize the manufacturers charts for getting the maximum and minimum gap openings as well as other data for the seal selected. Knowing the ambient temperature range, he can then proceed with calculations for the joint geometry.

Satisfactory performance of compression seals is judged upon by their ability to maintain sidewall-contact with the joint-gap interfaces at all times, thereby creating a watertight seal and preventing incompressibles from accumulating in the joint or penetrating downward along the sidewalls.

Therefore, several physical properties of a compression seal have been identified which serve to provide data to ascertain the expected performance capability of the elastomer and, hence, of the seal itself. These various parameters have been incorporated into specifications, and certain test procedures have been developed for quality assurance.

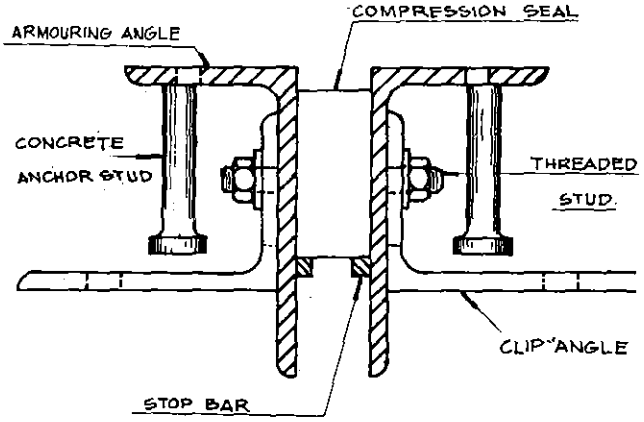


Fig. 32.13 *Cross-section of armoured joint with compression seal*

Physical Properties of Compression Seal (see Table 32.2)

- **Tensile Strength and Elongation**

After it has been established that a rubber composition has desirable characteristics and the product has been placed in production, tensile strength and elongation measurements are excellent control tools. They are sensitive to manufacturing mishaps and tell the rubber technologists whether or not the product has been properly vulcanized and the ingredients thoroughly blended. However, it is good to keep in mind that tensile strength and elongation cannot be used in design calculations, and they bear little relation to a rubber product's ability to perform its function.

It is often argued that these factors, while not in themselves significant, are nonetheless an index to general all round quality. This simply is not so, with exceptions at the extremes of the tensile strength range. For example, compounds below 1,000 psi

Table 32.2 *Physical Properties of Preformed Elastomeric Joint Seals*

Property		Requirements	ASTM Method
• Tensile Strength, min. psi		2000	D 412
• Elongation at break, min. per cent		250	D 412
• Hardness, Type A Durometer:			D 2240
—Lock in Seals		60 ± 7	(modified)
—Stdd. Compression Seals		55 ± 5	
• Compression set, 70 hr. @ 212° F, max.		40%	D 395 Method B (modified)
• Oven aging, 70 hr. @ 212° F:			
—Tensile strength loss max		20%	
—Elongation loss max.		20%	D 573
—Harness, Type A Durometer (count 'points' can easily vary, see explanation in text ahead)		0 to + 10	
• Ozone resistance: 20% strain, 300 pphm, in air 70 hr at 104° F (wiped with toluence to remove surface contamination)		No cracks	D 1149
• Low temperature recovery, 72 hr at 14° F, 50% deflection, min.	Used for Standard Compression Seals Only	88%	D-2628-69 SECTION 7
• Low temperature recovery, 22 hr at −20° F, min. 50% deflection		83%	D-2628-69 SECTION 7
• High temperature recovery, 70 hr. at 212° F, min. 50% deflection		85%	D-2628-69 SECTION 7

(70 kg/cm^2) in tensile strength are usually rather poor in most mechanical properties while those with tensile strength over 3,000 psi (210 kg/cm^2) usually have good properties. On the other hand, for those in the middle range say from 1,500 to 3,000 psi (105 to 210 kg/cm^2), where the great majority of rubber products fall, a correlation is at best haphazard between tensile strength and such properties as resilience, abrasion resistance, compression set, and flexlife.

- **Hardness**

Hardness, as applied to rubber products, is defined as the relative resistance of the surface to indentation usually taken by a device called a durometer. There are two problems encountered with the validity of this test:

(i) The measured values are unreliable because of mechanical considerations. It is very common to obtain a five-point variation in readings made by people using different durometer gauges on the same place of rubber; in some cases, these differences may be as high as 10 points.

(ii) The characteristic that is measured rarely bears any relation to the ability of a rubber product to function properly.

The point of this discussion is to alert those who are responsible for quality assurance that minor deviations of this specification are not criteria enough for rejection.

- **Compression Set**

The compression set values are helpful in that a rubber composition can be eliminated from consideration for sealing applications if it falls in the distinctly high range of set. This is one more test to examine quality control of the rubber matrix.

- **Oven Aging and Ozone Resistance**

The oven aging test was designed for predicting natural aging in an accelerated time frame. Ozone attack is evident by the formation of a few deep cracks or myriad small cracks, usually occurring at right angles to the direction of applied stress. It is a common phenomenon since most compression seals are under stress and ozone is universally present in the atmosphere. Ozone is generated from oxygen in the air and by Ultra-violet light from the sun. (Higher altitudes have higher ozone content.)

However, ozone attack has nearly ceased to be a problem since all neoprene compounds, used for premolded elastomeric sealers, have additives to arrest this problem. The performance test for ozone resistance is normally not required although the specification must state this requirement to ensure its continued observance.

- **Recovery Test**

The recovery test is a limiting measure of 'performance predictability'.

The test at 14°F will determine crystallization tendencies of an elastomer, and the high and low temperature recovery values will provide data in predicting field performance under similar environmental conditions. The high temperature test also suffices for an accelerated aging test in determining long range performance.

The seal's ability to 'recover', after compression, to its nominal width, under all ambient temperature conditions in the field, is probably the single most important factor affecting its performance.

Web-Configuration of Compression Seals

One of the factors in the seal design is the internal web configuration of the seal which is often ignored by users and always neglected in specifications.

Certain geometric web configurations can produce excellent laboratory results, but are often misleading. For example, a configuration utilizing webs only in the horizontal and vertical planes (series of squares) will create a seal with high initial pressure and lower ultimate pressures, a trait most desirable in a compression seal. But, in the field, where the seal must be installed by forcing it into the joint-crevice (not by merely placing it between a set of plattens), this box-like configuration becomes 'unstable'. This phenomenon parallels that of a truss designed similarly with squares only, which would more easily permit lateral forces to cause a failure in the direction of force (Fig. 32.14).

In the case of a compression seal similarly designed, where a force is applied to the top surface in order to squeeze the seal in the joint, lateral collapse takes place causing the seal to roll or twist in the joint, resulting in poor sidewall contact, thereby allowing the incompressibles and moisture to penetrate and eventually migrate downward, ultimately leading to failure (Fig. 32.14).

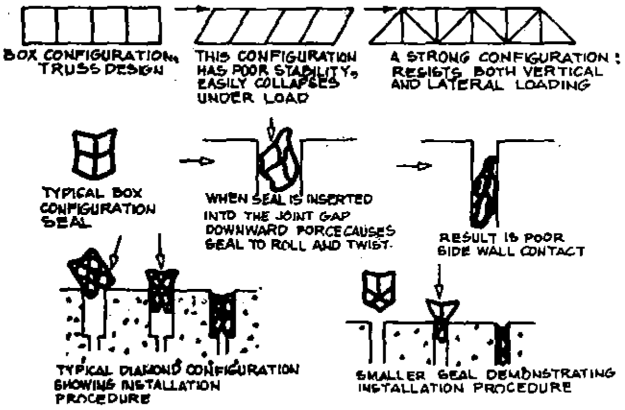


Fig. 32.14

Compression Seal Installation

It is normal practice to include, in tabular form (Table 32.2 and Fig. 32.12), the temperature range on the contract plans. Factors, such as creep, shrinkage, elastic-shortening and the sequence of construction, need to be taken into account in calculating the installation settings. The effective or installation temperature may be taken as the mean shade air temperature for the 48 hours prior to joint installation for concrete structures, and for the previous 24 hours for composite structures with a concrete deck slab and steel beams. The difference reflects the fact that concrete is a poor thermal conductor and changes in the effective temperature occur more slowly in concrete structures than in steel structures. More sophisticated methods are rarely necessary since the temperature will change even during the time taken to install the joint. There is also a danger that more complex procedures will not be understood by the contractor's personnel and the inspectors. Where the joints are to be installed in concrete, temporary connections (if any) must be released as soon as the concrete is placed to prevent distress in the concrete from the movement of the structure.

Field observations have shown a lower incidence of damage in joints where concrete end-dams are at least 400 mm wide since a wider concrete surface appears to reduce the dynamic load on the joint. A detail which has been found satisfactory is illustrated in Fig. 32.15 for the case where a compression seal is used. A similar end-dam detail can be used with other types of joint. The steel angles between the end-dam and the bituminous concrete are necessary to prevent spalling of the concrete. Vent-holes, approximately 5 mm diameter at 250 mm centres, are required in the top of all angles to prevent air entrapment when placing the concrete. A minimum angle thickness of 12 mm is desirable. The (minimum) 200 mm gap beneath the joint facilitates inspection and improves ventilation so that the ballast wall and the ends of the beams are less susceptible to damage if leakage should occur.

Installation of compression seal is normally a relatively easy task if the correct seal for given joint opening is specified and other simple procedures are adhered to (Figs. 32.14, 32.16 and 32.17).

The proper joint gap-opening is, without question, important to the ease for installing compression seal. Therefore, manufacturers normally indicate the minimum gap-openings permissible for field installation, in their product tables. It is preferred to design the joint geometry so that at the time of installation the joint interstice is greater than the recommended minimum.

Even so, it would be difficult to insert a typical compression seal into a joint-gap without the aid of a lubricant. Elastomeric material has a high coefficient of

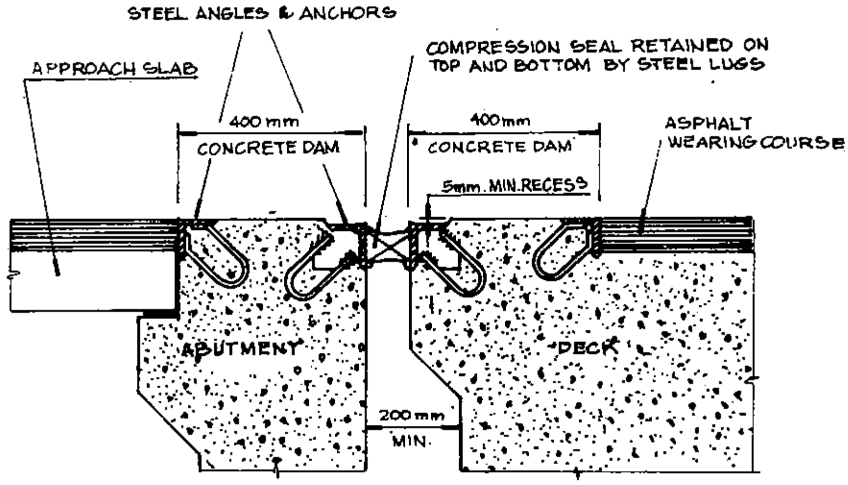


Fig. 32.15 *Typical detail for installation of a deck joint assembly*

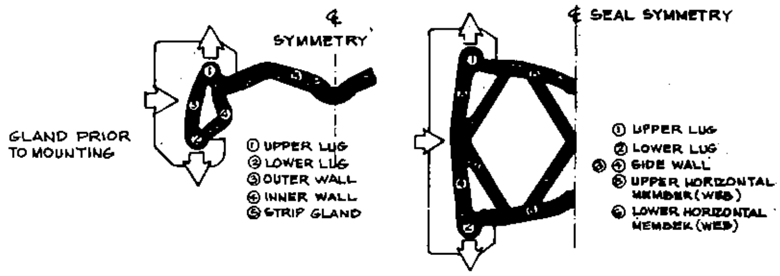


Fig. 32.16

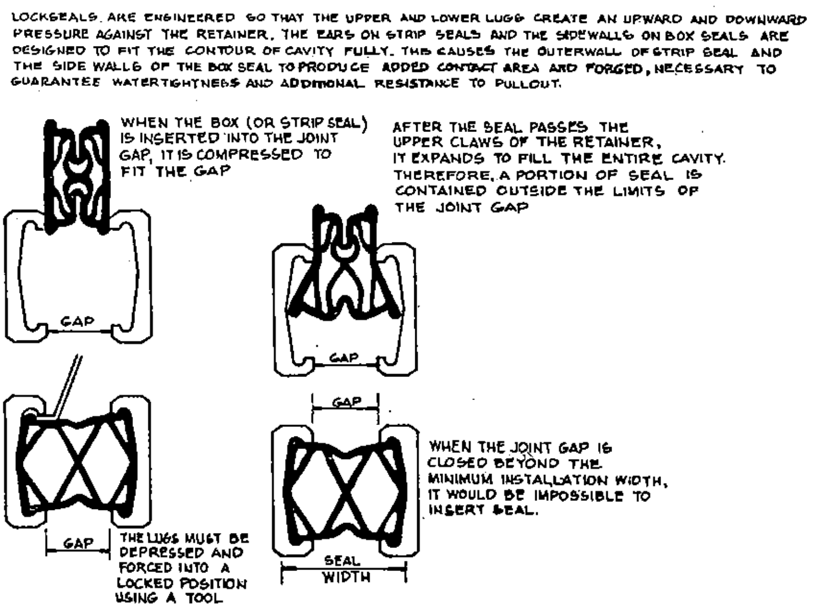


Fig. 32.17

friction, and under field conditions, where the joint-gap is predetermined by design, it would sometimes be impossible to insert the seal into it unless a lubricant is utilized.

Oil soap was the first lubricant to be used on the earliest installations. Since that time, it has been determined that an adhesive, in a single component, system, containing a high degree of lubricity when applied to the joint interfaces would give better performance.

There are, basically, two types of lubricant-adhesives currently in use today. One is used where compression seals are inserted directly into joints with concrete faces and the other is used for installing seals in structures having armoured (metal faced) joints.

The first type is used primarily for contraction joints and small expansion joints in pavements and slabs. It consists of a one component polychloroprene compound having a solids content of 24% ± 3% by weight. The low solids content permits the use of a spray applicator which is a necessity in cases where miles of seal are placed. This adhesive primes the joint faces and seals small holes and imperfections in concrete surfaces.

Another type, used for larger expansion joints and in conjunction with steel and other metal faced joints, consists of a one part moisture curing polyurethane, an aromatic hydrocarbon solvent mixture, with a solids content of approximately 72% ± 3% by weight.

The larger the seal the larger the cell openings or cavities. In order for the rubber to be able to span these greater

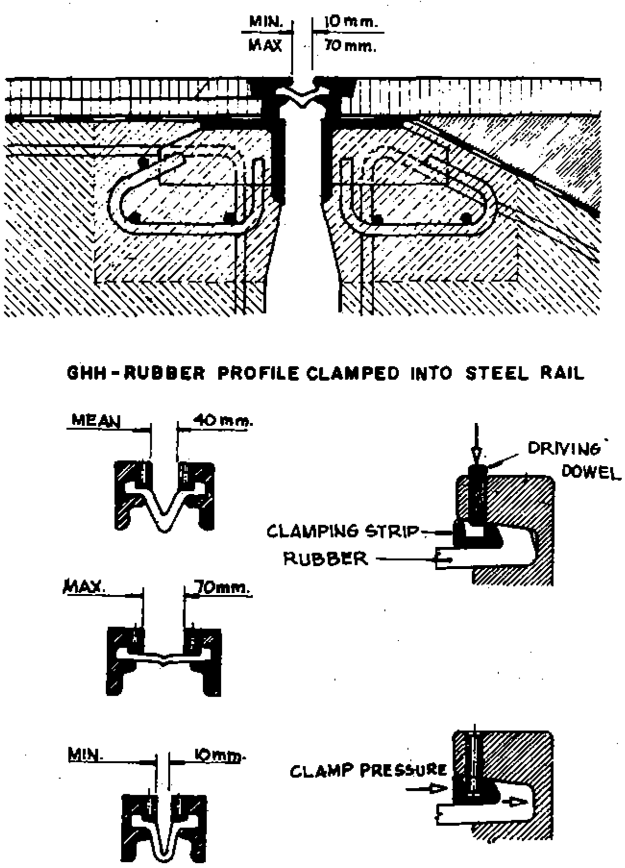


Fig. 32.18

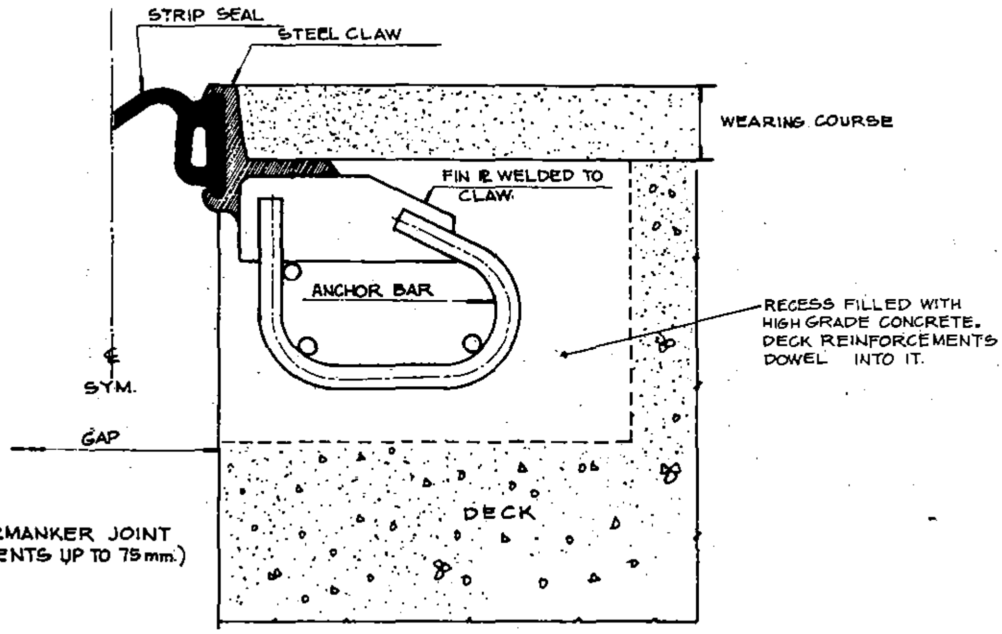


Fig. 32.19

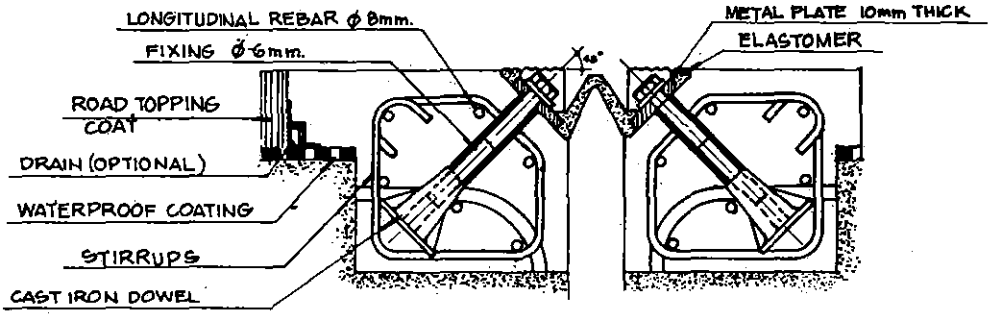


Fig. 32.20

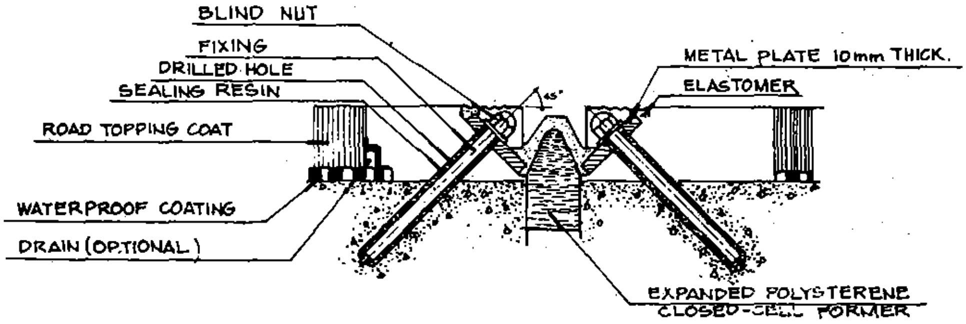


Fig. 32.21

open dimensions, the webs must be constructed thicker and heavier for increased strength. Due to the physical characteristics of rubber, this increase in mass is not strictly an arithmetical ratio to size of seal.

Therefore, because of these factors, and the maximum allowable joint-gap openings, other alternatives were needed for movements larger than about 50 mm and for joints on skews which tended to magnify these limiting features.

Among these alternatives are the 'mechanically secured' elastomeric seals which are a form of compression seal. (Figs. 32.18 to 32.38). These seals derive their waterproofing characteristics from the fact they are placed in a state of compression at the point where moisture or the incompressibles can enter. This stress is contained within the locking device in the case of 'strip-seals'.

The uniqueness of compression type strip-seals is the complete elimination of ancillary component parts such as plates or clamps. They are self-contained with no components to rust or wear, that could cause failure.

A derivation of the strip-seal is the box-type 'lock-seal' which works on the same principles as the former. Their main advantage is that they have secondary webs providing additional strength, and their ability for self-cleansing of the incompressibles is superior to that of strip-seals. (This claim is not entirely true, though.)

Another desirable aspect of these types of compression seals is that they afford resistance to tension forces. Strip seals can be stretched up to 100% of their width before pulling out and box seals stretch approximately 25% beyond their normal movement range. While the stretch force will permit the expansion joint to overtravel in the open direction, it should never be used in the operating range.

The Freyssinet joints shown in Figs. 32.20 and 32.21, composed of an elastomeric membrane stiffened by two lateral metallic plates, can take movements up to 65 mm.

The joints may be installed in the traditional manner, in rebates (recesses) in a rapid economical manner, within the thickness of the road wearing course.

Wabo Strip Seal (Figs 32.22 and 32.23)

This joint system consists of a clawlocked neoprene strip between two steel shapes forming a sealed joint. A preformed 'hinge' controls folding movements of the seal element.

Versatile in design, this system can be prefabricated ready for placement, or field assembled. There are several series of seal elements to offer solutions to a wide range of field applications.

The S series seals (Fig. 32.23), which mate with the Type A steel extrusion (Fig. 32.22), meet typical job requirements with movement ratings up to 5" (125 mm).

The SE-series, also capable to movements up to 5" (125 mm) has a typical usage in rehab (rehabilitative) work where a low height replacement system is required. The type

M, E and W steel extrusions (Fig. 32.22) are used with this series of seals.

For applications where an even traffic surface must be maintained, the L (LE) and EF (EFE) series seals (Fig. 32.23) provide the answer for movements up to 4" (100 mm).

Due to the mechanically-locked feature of the Strip Seal products, this system is excellent in applications where differential slab movements are anticipated.

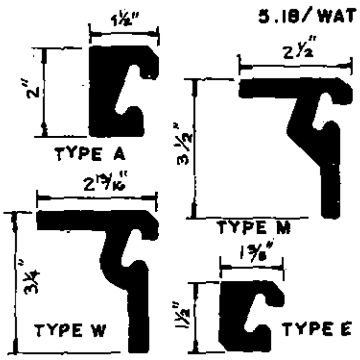
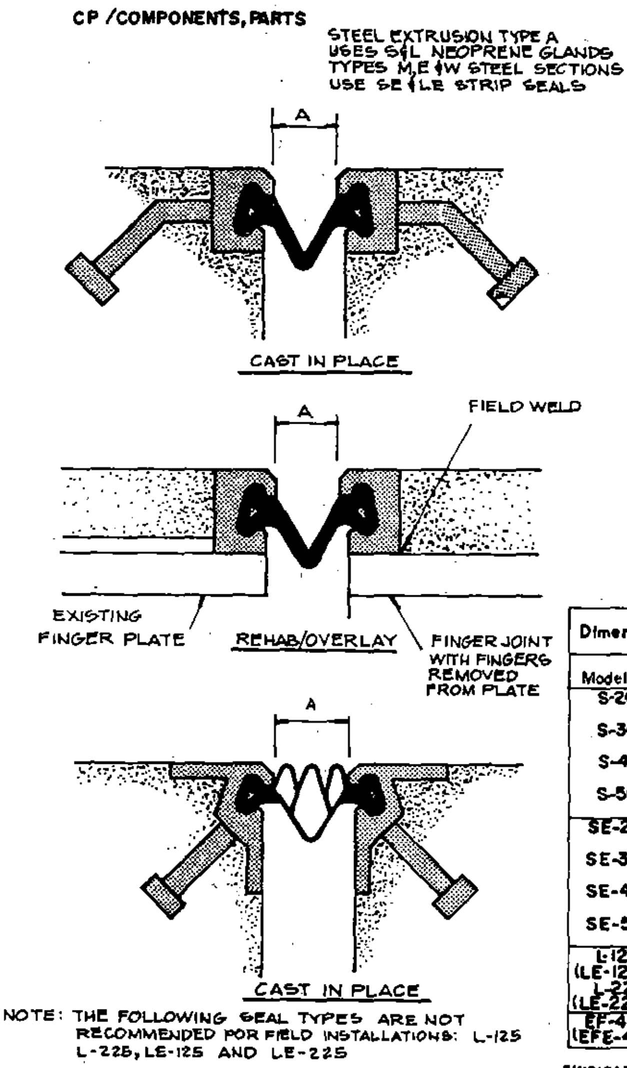
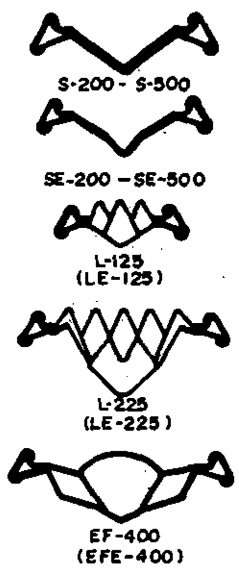


Fig. 32.22

32.4 LARGE MOVEMENT EXPANSION JOINTS

Basically the "ELASTOMER-BASED" joints exploit the





Dimension Table · Bold numbers indicate inches; Light numbers indicate millimeters

Model No	Movement	Min. Joint, Width A	Max. Joint Width A	Installation Width A *
S-200	2 50.80	0 0	2 50.80	1 1/2 38.10
S-300	3 76.20	0 0	3 76.20	1 1/2 38.10
S-400	4 101.60	0 0	4 101.60	1 1/2 38.10
S-500	5 127.00	0 0	5 127.00	2 50.80
SE-200	2 50.80	0 0	2 50.80	1 1/2 38.10
SE-300	3 76.20	0 0	3 76.20	1 1/2 38.10
SE-400	4 101.60	0 0	4 101.60	1 1/2 38.10
SE-500	5 127.00	0 0	5 127.00	1 1/2 38.10
L-125 (LE-125)	1 1/4 31.75	1 25.40	2 1/4 57.15	1 3/4 44.45
L-225 (LE-225)	2 1/4 57.15	1 3/4 44.45	4 101.60	2 3/4 69.85
EF-400 (EFE-400)	4 101.60	1/2 12.70	4 1/2 114.30	2 1/4 57.15

*INDICATES MINIMUM WIDTH AT WHICH SEAL CAN BE INSTALLED

Fig. 32.23

elastic deformation of an elastomeric 'gland' in order to allow relative movements at the joint gap.

They can cover a wide range of movements by 'parallel coupling' of several basic units (distribution of the movement) (Figs. 32.24 to 32.38)

By the late 1960s several devices were introduced on the market which utilized compression seals 'in parallel', enabling engineers to use two or more seals, simultaneously, to obtain the desired larger movement rating. This system is referred to as a 'Modular Joint'

The modular expansion joint was a breakthrough in engineering design of movement joints, providing an alternative to the articulated joints, finger joints, and slider-plate assemblies.

Fassbinder proposed a watertight modular joint consisting of a deformable closing element made up of flat steel plates and foamed plastic intermediate layers forming a composite unit. Later, a tubular preformed elastomeric seal was utilized in place of the foamed plastic (Fig. 32.24).

Subsequent to this modular expansion joint, 'heavy duty' compression seals were placed in parallel between structural shapes resting on support bars. This particular concept, with modifications, served well for almost a decade (Fig. 32.25).

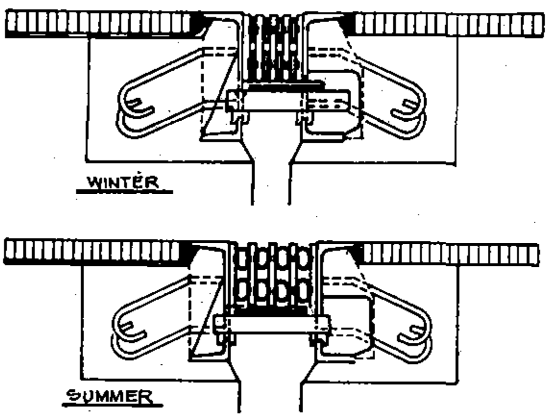


Fig. 32.24 *The Fassbinder modular expansion joint*

The latest models of modular expansion joint systems are using the 'locked in seal' principles simply because of their advantages, including their greater operating range. These newer models have been made heavier, stronger, more durable, and are practically maintenance free.

The modular expansion joints have gone through a period of growth much the same as any new product might. There were problems that had to be overcome (seal's performance and the mechanical aspects).

The early joints were noisy due to flexural rebound, they leaked because of pressure decay, some seals opened more

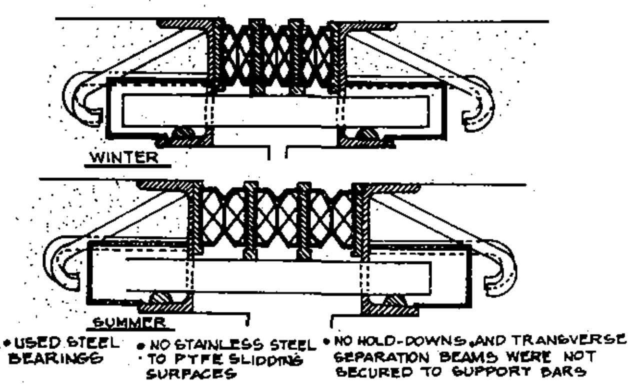


Fig. 32.25 *Early American modular expansion joints*

than others causing seal failures, and surfaces in contact with one another induced unwanted forces. Happily the present state of art has overcome many of these problems.

In case of very large movements the modular joints require supporting by a rather complex mechanical system (Figs. 32.25 to 32.34).

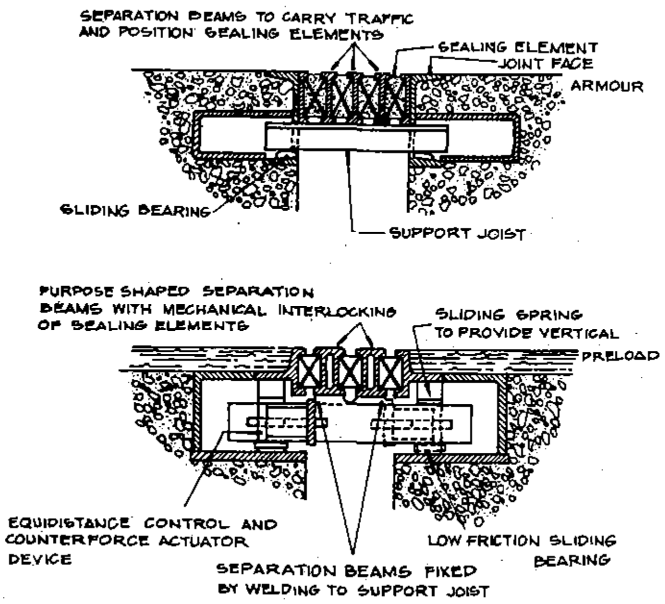


Fig. 32.26

All newer types of modular joints, with good performance records, have transverse components attached to the support bars, to prevent flexural rebound; the support bars are held down with some form of pressure device; contact surfaces are usually stainless steel to PTFE (Teflon); bearings for support bars are polyurethane; and some form of equalization device or even a spring is utilized for maintaining equidistant spacing of the elastomeric elements.

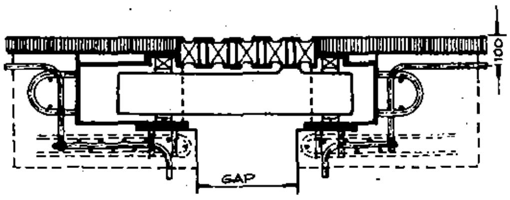


Fig. 32.27

The basic principles of modular joints have not changed radically over the years. They achieve a sealed waterproofed expansion joint, with large movement capability and low maintenance characteristics.

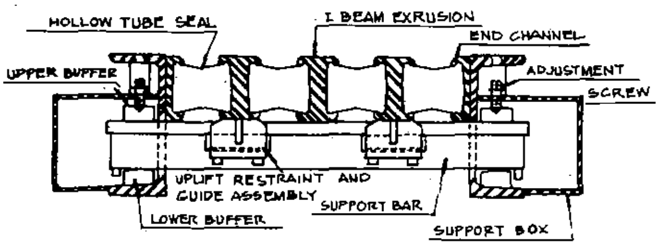


Fig. 32.28 *Cross-section of ACME-BETA joint*

The new 'locked in compression seals' have proved extremely reliable with respect to performance in both waterproofing and material endurance over the years.

The technologically improved materials and designs of appurtenant components have increased their maintenance-free service-life, providing quiet, functional and durable

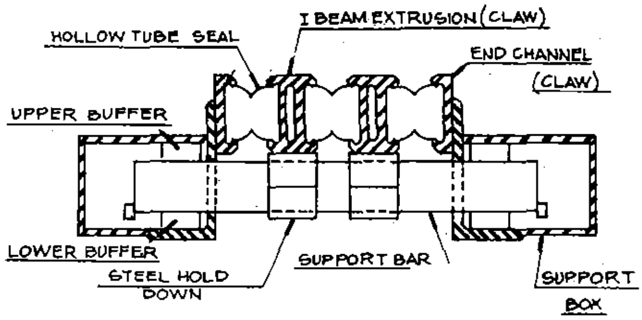


Fig. 32.30 *Cross-section of WABO-BETA joint*

(modular) expansion joints.

Wabolum Joint (Fig. 32.32) (by Watson-Bowman Group)

This is a modern aluminium modular joint system, providing a watertight seal through use of continuous neoprene sealing strips. Joint movements to 32" can be accommodated—through the addition of aluminium and neoprene standard components (Fig. 32.32).

Aluminium grade 6061-T6(ASTM) is employed to meet structural requirements necessary for heavy traffic loadings and to facilitate ease in field assembly due to the metal's lightweight characteristics.

Standard sections can be made and welded to handle a variety of skew, parapet and curb conditions.

This is manufactured by the Watson-Bowman-Acme in the US.

'Wabo-Maurer' Systems (by Watson-Bowman-Maurer Combine)

See Fig. 32.34 and its table.

For heavily-trafficked bridge structures requiring large

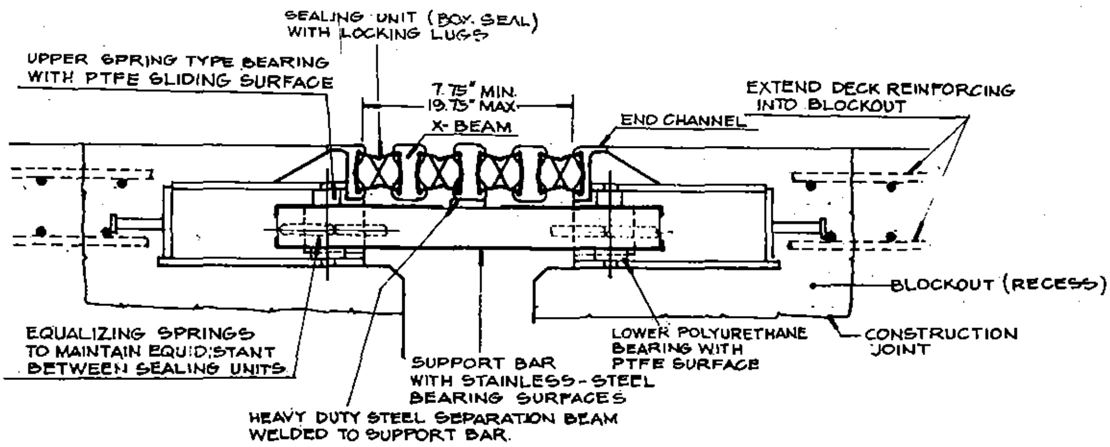


Fig. 32.29 *Newer type modular joint system with many improvements*

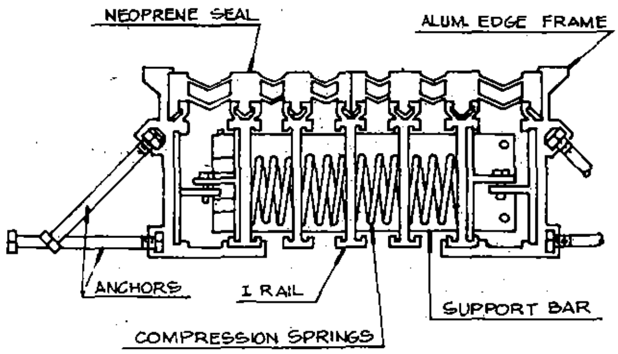


Fig. 32.31 *Cross-section of delastiflex D.L. joint*

movement joints, this system provides a sealed joint through use of low-stress neoprene tubes mechanically locked between rigid extruded steel shapes. With movement capabilities to 26", this field proven, system guarantees a long and maintenance-free expansion-joint life.

Other features of this system include:

- A 'unitized' construction as each extruded steel Support Beam is welded to an individual Support Bar, ensuring rigidity in resisting horizontal forces imparted from traffic.
- Urethane Bearings at the top and at the bottom of the Support Bar absorb impact loading with noise free action while securing against lifting forces.
- Teflon PTFE to Stainless Steel sliding surfaces for nearly friction-free support bar sliding action.
- Counterforce Mechanisms which neutralize internal forces, thus minimizing undesirable compressive forces exerted on deck edges.
- Heavy-duty Edge Member and Anchor Unit, which minimize edge member deflection while transmitting the forces into structural concrete.
- Wabo-Maurer Systems described here are manufactured according to US Patents 3604322 and 3626822 and Canadian Patents 852961 and 882450.

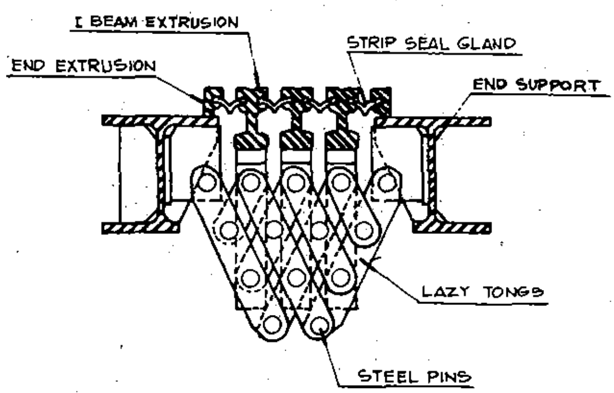


Fig. 32.33 *Cross-section of rheinstal "Lazy-tongs" type joint*

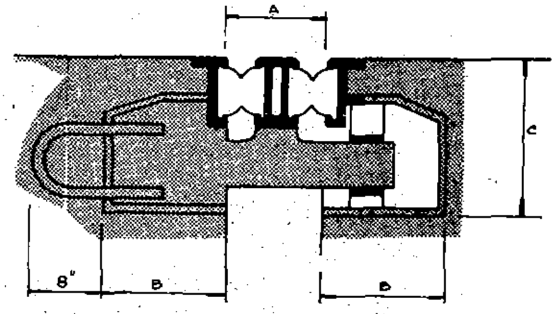


Fig. 32.34

NOTE : See Dimension Table on next page.

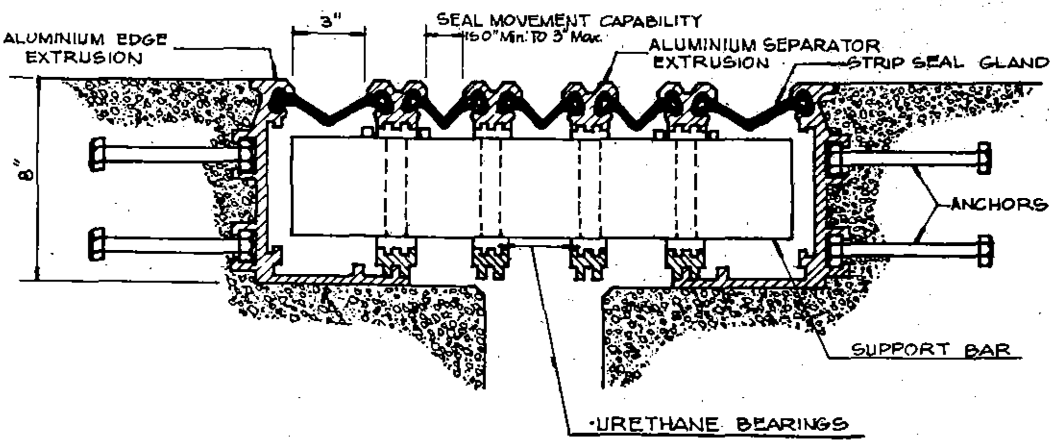


Fig. 32.32

Dimension Table (Fig. 32.34) Unbracketed numbers indicate inches, bracketed numbers indicate millimetres

			Dimensions			
			A		B	C
Model No.	Movement	Weight	Min.	Max.		
D-300	3.0	30 plf	0	3.0		
(One Seal)	(76.20)		(0)	(76.20)	—	—
D-600	6.0	75 plf	(3.0)	9.0	8.5	10.5
(Two Seals)	(152.40)		(76.20)	(228.60)	(215.90)	(266.75)
D-900	9.0	115 plf	6.0	15.0	11.5	10.5
(Three Seals)	(228.60)		(152.40)	(381.0)	(292.10)	(266.70)
D-1200	12.0	175 plf	9.0	21.0	14.5	10.5
(Four Seals)	(304.80)		(228.60)	(533.40)	(368.30)	(266.70)
D-1500	15.0	240 plf	12.0	27.0	17.5	10.5
(Five Seals)	(381.00)		(304.80)	(685.80)	(444.50)	(266.70)
D-1800	18.0	335 plf	15.0	33.0	20.5	10.5
(Six Seals)	(457.20)		(381.00)	(838.20)	(520.70)	(266.70)
D-2100	21.0	445 plf	18.0	39.0	23.5	10.5
(Seven Seals)	(533.40)		(457.20)	(990.60)	(596.90)	(266.70)
D-2400	24.0	590 plf	21.0	45.0	26.5	10.5
(Eight Seals)	(609.60)		(533.40)	(1143.00)	(673.10)	(266.70)
D-2700	27.0	765 plf	24.0	51.0	29.5	10.5
(Nine Seals)	(685.80)		(609.60)	(1295.40)	(749.30)	(266.70)
D-3000	30.0	880 plf	27.0	57.0	32.5	10.5
(Ten Seals)	(762.00)		(685.80)	(1447.80)	(825.50)	(266.70)

NOTE The above dimensions refer to systems using foreign-made steel extrusions. The dimensions vary slightly for those using American-made extrusions.

Waboflex-SR Joint (by Watson-Bowman Group)

See Figs. 32.35 to 32.38.

A moulded neoprene monolithic unit, Waboflex-SR is a rubber cushion expansion joint. Eight sizes enables the SR series to handle movements to 13". Moulded in standard 6 ft. lengths (except SR 13 which is 4 ft. long), this system requires a minimum of field labour to install in bridge decks, ramps, and roadways.

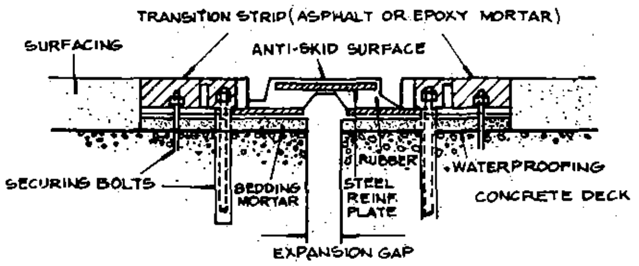


Fig. 32.35

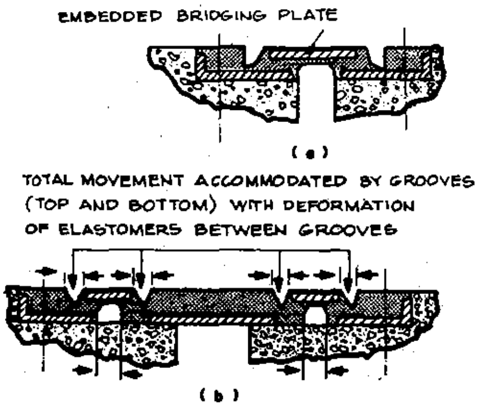


Fig. 32.36

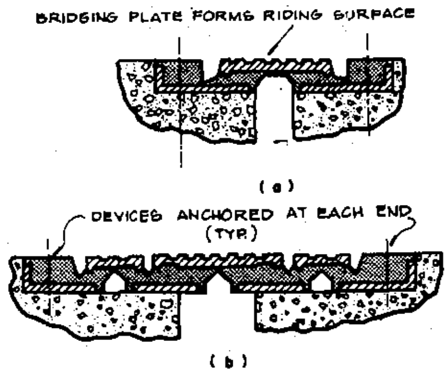
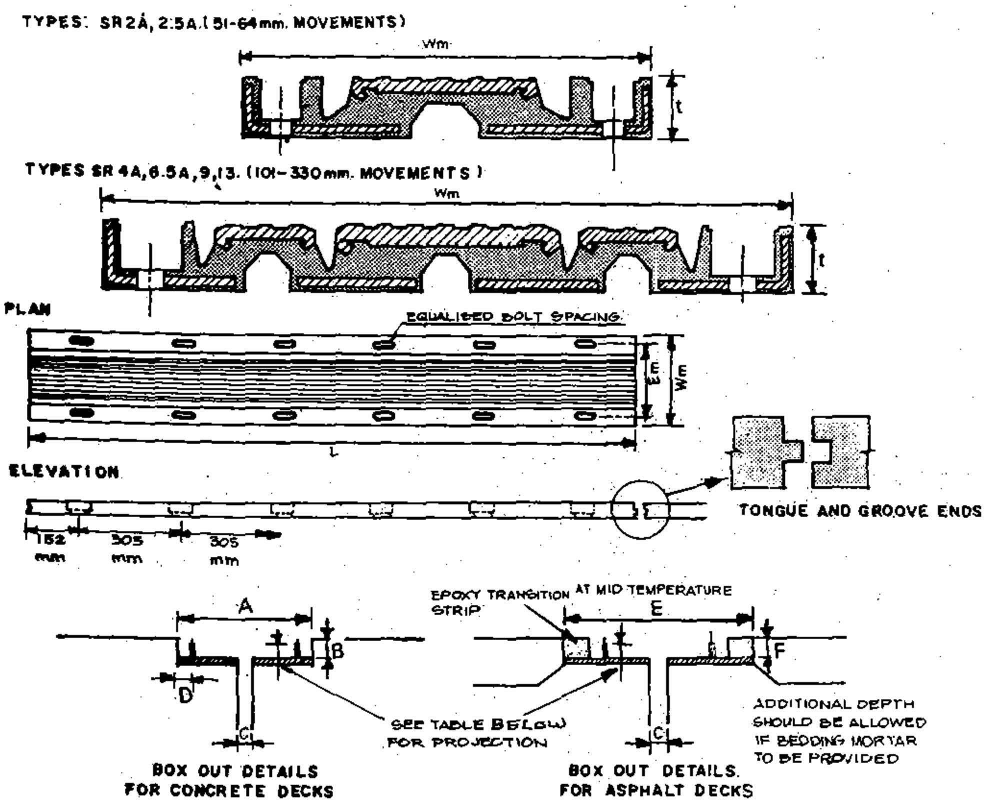


Fig. 32.37

Typical cross-section (SR2A-SR13) consists of steel angles and plates embedded in the neoprene moulded section, arranged so as to meet structural requirements. High strength extruded aluminium sections, affixed to the upper surface in the moulding process, are grooved to provide skid resistance as well as long life performance under heavy traffic loadings.

Further features include unitized, welded curb sections



WABOFLEX DIMENSIONS							BOX OUT DIMENSIONS							
							CONCRETE DECK						ASPHALT DECK	
Model No.	Total Movement	*L* Length	Wm Width	*t* Thickness	Em Bolt Centres	Wt of Unit	A Min.	A Max	B Depth	C Min.	C Max.	D Bolt ℄	E Width	F Depth
	mm	mm	mm	mm	mm	kg	mm	mm	mm	mm	mm	mm	mm	mm
SR 2A	51	1829	270	40	213	40.83	251	302	44	25	76	32	370	44
SR 2.5A	64	1829	343	46	280	55.35	324	387	50	35	98	38	543	50
SR 4A	101	1829	578	54	499	104.80	540	641	58	25	127	46	778	58
SR 6.5A	165	1829	711	76	616	183.76	635	800	80	38	203	51	911	80
SR 9	229	1829	950	95	851	303.99	841	1070	99	44	273	52	1200	99
SR 13	330	1219	1397	127	1270	492.22	1238	1568	131	51	380	67	1697	131

NOTE "m" Suffix denotes dimensions at mean temp. when Waboflex in relaxed state

Fig. 32.38 *Physical properties of Waboflex SR expansion joints*

Table 32.3 *Mechanical Properties of Waboflex SR Expansion Joints*

Waboflex Type	SR2A	SR2.5A	SR 4A	SR 6.5A	SR 9	SR 13
Design Movement mm	51 kg/m	64 kg/m	101 kg/m	165 kg/m	229 kg/m	330 kg/m
Compressive force required in moving joint sections through the recommended movement range	2118	1328	738	1210	1845	2214
Tensile force required in moving joint sections through the recommended movement range	2214	1107	789	1299	2037	2362

Refer to Servicised Ltd., UK, for forces required in case of Skew Deck Joints.

Table 32.4 *Waboflex SR Fixing Specification*

Fixing Specification		WABOFLEX TYPE SR 2A	SR 2.5A	SR 4A	SR 6.5A	SR 9	SR 13
Length of each unit mm		1,829	1,829	1,829	1,829	1,829	1,219
No. of fixings per unit		12	12	12	12	12	8
Studding*	Diameter mm	12	16	20	24	24	30
	Length* mm	140	170	200	230	235	245
Hole in Structural	Diameter mm	14	18	22	28	28	35
Concrete	Depth mm	110	135	160	185	185	185
Stud projection above bedding mm		25	30	35	40	45	55
Resin Capsule Diameter mm		12	16	20	24	24	30
Flat Washers	Outside Diameter mm	27	30	38	44	44	57
	Thickness mm	3.0	2.8	2.8	4.5	4.5	4.5
Spring lock washer thickness mm		3.0	3.5	4.0	4.0	4.0	6.0
Hex.Nut	Thickness mm	10.0	13.0	15.5	19.0	19.0	24.0
	Width across Flats mm	19.0	24.0	30.0	35.5	35.5	45.0
Torque Setting	kg m.	5.5	9.0	12.0	14.0	14.0	21.0
	ft lbs.	40	65	85	100	100	150
Bitu-Thene Mastic** Litres		0.92	0.92	0.92	0.92	0.92	0.61
Two-part synthetic rubber for tongue & groove joints		50 joints	34 joints	17 joints	10 joints	6 joints	3 joints
Uncured Rubber Strip	width mm	270	343	578	711	950	1397
For Butt Joints	Depth mm	40	46	54	76	95	127
Bolt hole sealant black verti seal † lb per unit		2.79	4.32	9.92	15.34	21.73	26.4
• Ancillary materials per metre length of joint							
PML Screed Primer kg		0.12	0.17	0.23	0.25	0.32	0.45
PML Epoxy Mortar QS †† litres (kg)		1.85 (3.7)	2.39 (4.78)	3.51 (7.02)	3.96 (7.92)	5.21 (10.42)	7.41 (14.82)
PML NS/FG Transition Mortar litres (kg)		4.0 (10.29)	9.2 (23.66)	10.8 (27.77)	15.2 (39.09)	23.75 (61.07)	38.1 (97.97)

- All studs, washers and hex nuts are supplied in mild steel to BS970 Part 1 220 MO7 having a tensile strength of 432–494 MN/m^2 and zinc plated to BS 3382 Part 2 1961 5–10 microns thick.

* Standard stud lengths assuming maximum 5 mm epoxy bedding mortar.

** Bitu-thene Mastic supplied in 3-litre units.

† Bolt hole sealant, black vertiseal supplied in 5 lb. packs.

†† Assuming 5 mm thickness.

for leakproof sealing and moulded 'tongue and groove' connections between standard sections.

Waboflex-SR is an advanced design of elastomeric expansion jointing system developed for bridges, ramps and roadways. It is designed for long-life performance under heavy traffic. Latest improvements incorporated in Waboflex-SR provide stress reduction and "armoured" skid resistant wearing surfaces.

A skid resistant system is achieved with ribbed surface high strength aluminium 6061-T6 extrusions specially grooved for maximum friction. There are no large surface areas of rubber exposed to traffic and a continuous drainage groove assures rapid surface water run-off.

The armoured aluminium surface, ribs and impact edges protect the neoprene rubber from tyre abrasion.

The aluminium extrusions are 'T' locked, preventing delamination and resisting the buckling forces encountered in cycling. Steel angles and plates in A36 steel are embedded in the neoprene moulding to provide structural reinforcement.

These joints are compounded from	Du Pont neoprene to ASTM specifications

Tensile strength	D-412
Elongation	D-412
Hardness	D-2240
Compression set	D-395
Ozone resistance	D-1149
Over ageing	D-573
Low/High temperature	
Recovery	D-2628-67T

There is a significant reduction in the stress required to stretch or compress Waboflex SR Systems to the correct temperature-related setting.

It is recommended that Waboflex SR systems should be set 3-4 mm below wearing surface. Experience has shown that there is migration, erosion or flow of hot mix asphalt away from expansion joint systems leaving it exposed to traffic and general attrition.

Pre-Setting of Waboflex SR Joints—Calculations of Movements

The extent of movements in bridge deck expression joints should be based on the local highway authority's design requirements for the accumulative effects of temperature, prestress, creep and shrinkage, deflection of deck and rotation at the supports.

Figure 32.39 shows the pre-tension or precompression dimensions required for the Waboflex SR range to suit the Effective Bridge Deck Temperature at the time of installation and, therefore, the centreline to bolt offset. The theoretical Mean Temperature, at which the Waboflex SR is in a relaxed state, should be provided by the design engineer as part of the design specification.

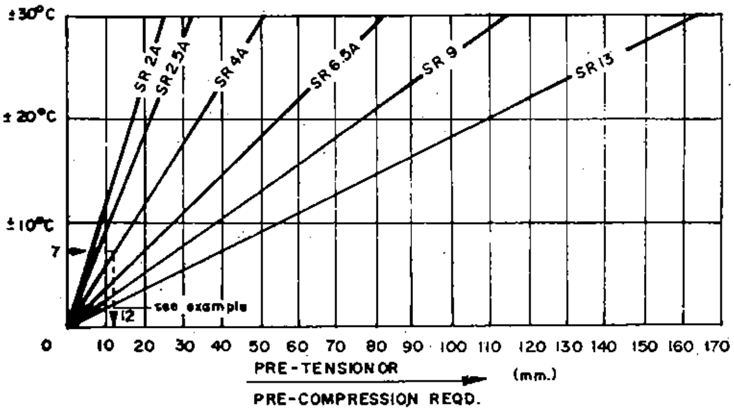


Fig. 32.39

Preset of Waboflex SR Range

It has been proved by extensive field testing that an approximate method of establishing the "Effective Bridge Deck Temperature", within a ± 3°C tolerance, without resorting to complex calculations and elaborate temperature gauges, is by measuring the 'Mean Shade Temperature' beneath the deck at 8.00 hours ± 1 hour and increasing this temperature by 3°C. (Refer to UK Department of the Environment TRRL 696 by Mary Emerson). Another way of establishing the movement of the bridge is by monitoring the structure over a period prior to installing the bridge expansion joint.

- EXAMPLE

Waboflex:

Theoretical Mean Deck Temperature given by Engineer = 20°C

Mean Shade Temperature at 08.00 hours = 10°C

Effective Bridge Deck Temperature = 10°C + 3°C = 13°C

Change from Theoretical to Effective-Bridge Deck Temperature: 20°–13°C = 7°C

From Fig. 32.39 it would be necessary to extend the Waboflex SR 4A by 12 mm (by using 'Companders' i.e. compression of extension clamps).

- Therefore, to obtain correct bolt offset from mean centreline for drilling studs:

Bolt centres when relaxed (Em, from Table in Fig. 32.38)	= 499 mm
plus pre-tension	= 12 mm
	511 mm
	÷2
∴ offset	= 255.5 mm

32.5 INSTALLATION OF WABOFLEX SR SYSTEMS

These instructions should be read in conjunction with working drawings prepared by the suppliers. Waboflex SR Expansion Joints can be easily installed by the main contractor in 'cutout' sections of surfacing in existing decks and specially prepared concrete strips of new decks without resorting to specialist subcontractors. The application procedure consists of drilling fixing holes at predetermined positions, jacking the sections together to ensure water-tightness and bolting them down. Ideally, a site survey should be carried out after installation of the road surfacing and pavement lines to obtain an accurate profile of the bridge expansion joints on which the working drawings are based:

Tools Required

Heavy Duty Disc Cutter
Concrete Scabbler
Electric Wire Brush
Drill-Hammer/Rotary type with pillar drilling rig
Air Compressor for cleaning holes
Drive-Unit for installing studding
Small Jack
Ratchet, Wrench and Sockets, speed brace with 100 short extension
Torque Wrench
'Companders'—width adjusting clamps to extend or compress Waboflex to suit ambient temperature (available on hire from suppliers)
'T' Paddle and Mastic Gun with Follower Plate
Remaining ancillaries are found as normal site equipment and materials.

Materials Required:

* Screed Primer
* Bedding Mortar
Stud anchors and Resin capsules
Bitu-thene Mastic
Prefabricated Kerb Sections
Straight Waboflex Sections
Heavy Duty Flat Washers
Spring Lock Washers
Nuts
Cleaning and Priming Solutions
Two-part Liquid Cold Rubber Polymer
* Transition Strip Mortar
Bolt Hole Sealant (Black Vertiseal)
Uncured Rubber Strips for butt joints
Tool Cleaner

* These materials are available from the suppliers.

- **Procedure: (Steps 1 to 10, Figs. 32.40 to 32.49):** Determine mean centreline of expansion joint by establishing setting-out points at extremity of carriageway, taking into account any irregularities produced during concreting of the deck. Using chart shown in Fig. 32.38 mark width of box-out symmetrically about centre line with a chalked string line.

Remove the road surfacing materials over the specified

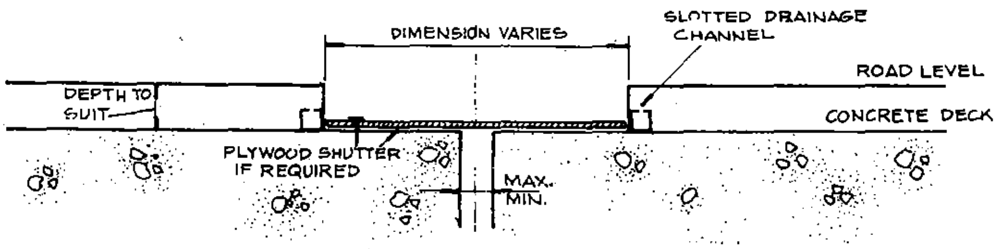


Fig. 32.40 *(Step 1)*

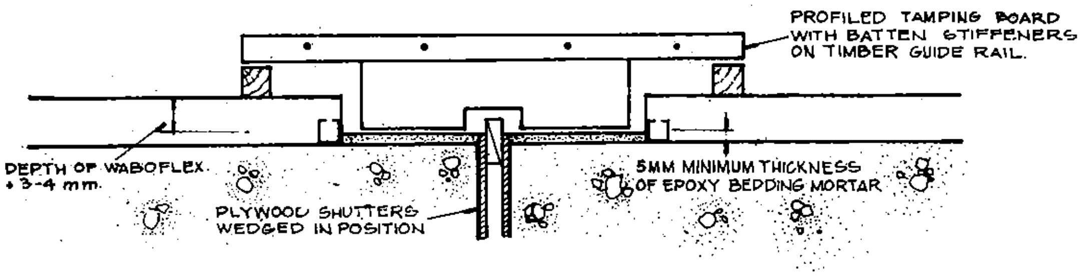


Fig. 32.41 *(Step 2)*

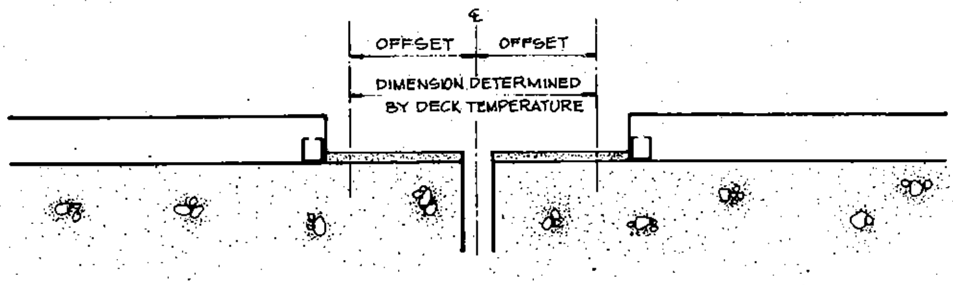


Fig. 32.42 *(Step 3)*

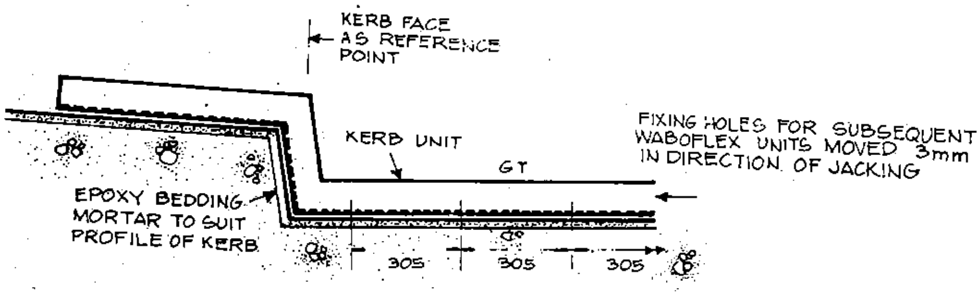


Fig. 32.43 *(Step 4)*

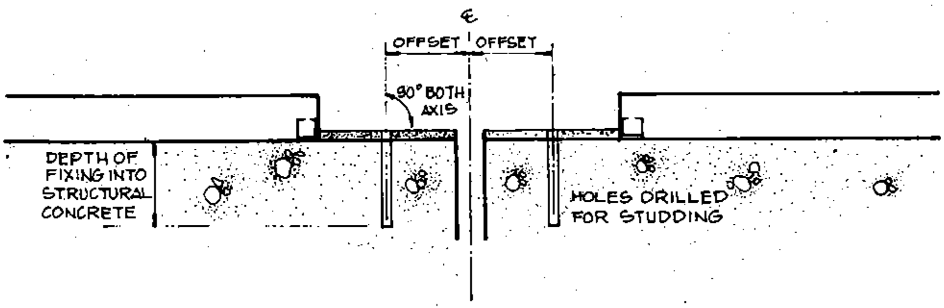


Fig. 32.44 *(Step 5)*

width, measured symmetrically about the centre line of the expansion joint gap. If the road surfacing has not been laid, a piece of plywood can be fixed to the concrete deck to allow easy removal of the surfacing after laying has been completed. A heavy duty disc cutter should be used to cut through the surfacing materials to the level of the concrete deck and the material between the saw cuts broken out and cleared away.

The concrete surface within this area should be mechanically scabbled, in order to remove surface laitence and bituminous materials that may still be adhering to the concrete. Following this operation all lose material should be removed by brush and air line.

When required, the level of the concrete surface should

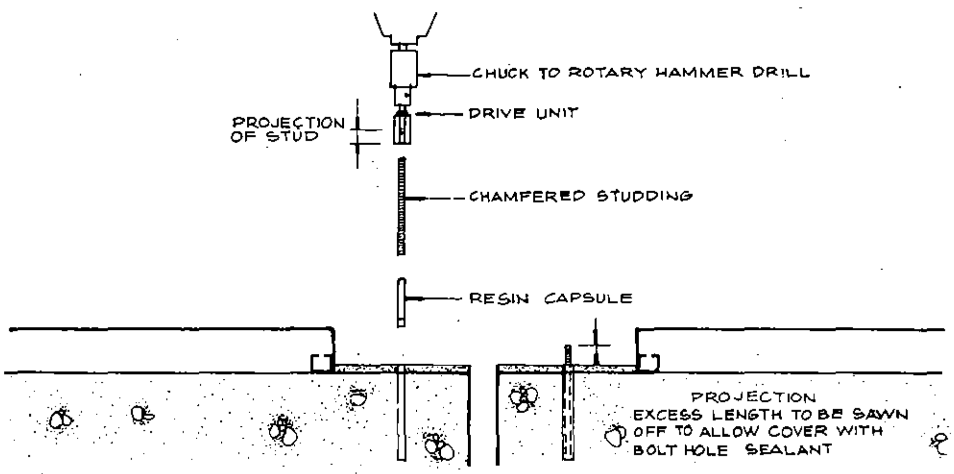


Fig. 32.45 (Step 6)

be made-up, using a bedding mortar, mixed and applied as specified by the manufacturer to ensure a true and even surface on which to bed the Waboflex to the correct level in relation to the existing road surfacing. This is achieved by having timber battens on each side of the joint acting as guide rails and using a profiled tamping board with stabilising batten. A steel float can be used to smooth off any tamping marks.

A longitudinal check should be made with a straight edge to ensure that the Waboflex will have an even seating.

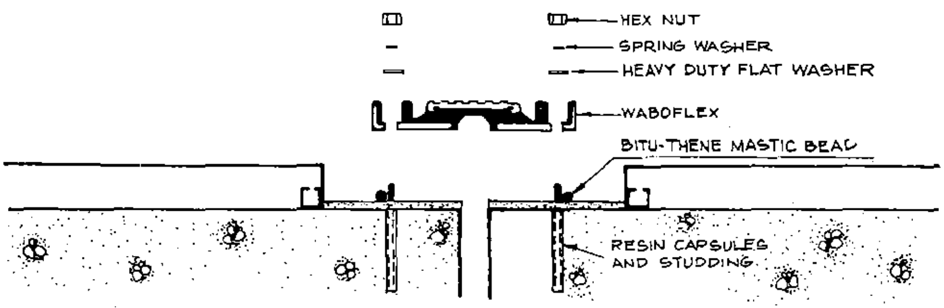


Fig. 32.46 (Step 7)

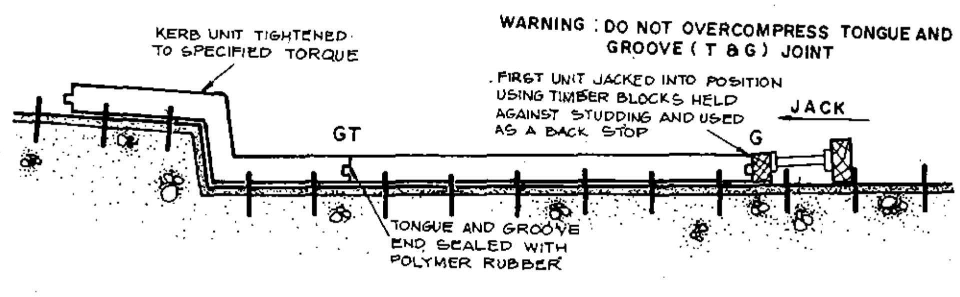


Fig. 32.47 (Step 8)

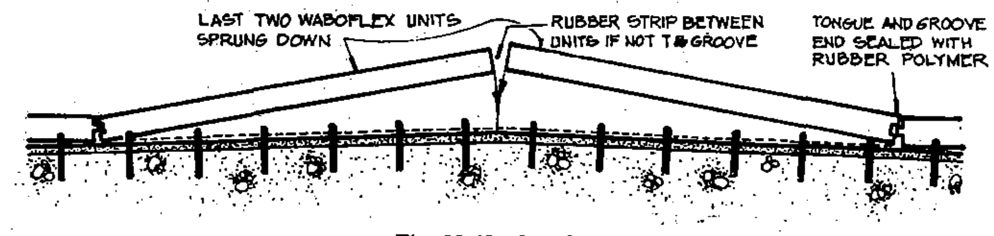


Fig. 32.48 *(Step 9)*

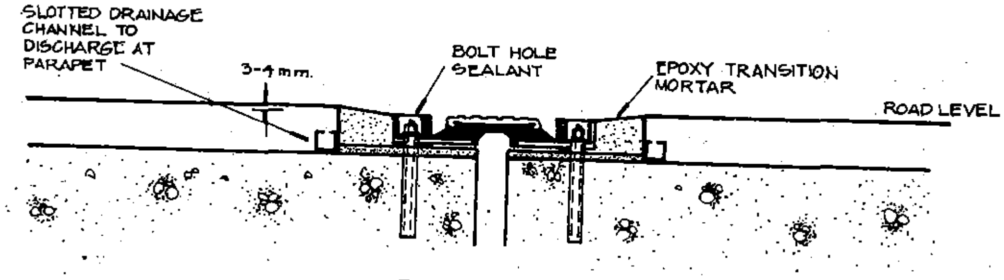


Fig. 32.49 *(Step 10)*

It is essential to use the kerb unit at the lowest level of the cross falls to act as a reference point for establishing the transverse setting out of the holes for the main deck.

'Longitudinal' means along Waboflex, parallel to joint. Transverse means 90° across the joint. The transverse centre line of the fixing studs should be set out symmetrically about the centre line of the expansion joint by marking the bedding mortar and projecting the setting-out across the carriageway, using a string line or theodolite when a string line is impractical or inaccurate.

The distance between fixing stud centre lines will be dependent on the relationship between actual bridge deck temperatures at the time of construction and at the time of fixing. An approximate method of determining the bridge deck temperature and stud hole centres is shown in the Example under Fig. 32.39. These dimensions should be calculated in conjunction with a representative of the Design Authority. In the majority of cases Waboflex will be installed in a 'relaxed state' without resorting to precompression or tension.

The longitudinal setting-out of the studs should commence by laying the lowest kerb unit in line with the kerb face. The sections of Waboflex can be used as a template to mark out the longitudinal positions of the fixing studs.

A rotary hammer-drill, with pillar drilling rig for accurate positioning and square alignment to the bedding surface, should be used to form holes to the diameter and depth specified in Table 32.4. In the absence of a rig, use a steel template as a guide to prevent the drill bit from wandering.

Where steel reinforcement is encountered, it will be necessary to employ a hollow core, water cooled diamond tipped drilling bit. Following the use of a diamond drill, the hole should be roughened using the rotary hammer drill.

Holes to be thoroughly blown free of loose materials and free water and the specified anchor capsule inserted with the filler cap at the bottom.

A drive unit adaptor is fitted to the rotary hammer drill to accommodate the studding and act as a depth check. The chamfered end is driven to the bottom of the hole, mixing the resin and hardener together. Care should be taken not to over mix, as this will cause the compound to extrude from the hole. The fixed studs should be left undisturbed for a period as specified. Should oversize holes be drilled in error, they should be filled with polyester grout with studs aligned vertically on both axes.

When the studding is firmly set, a continuous bead of Bitu-thene Mastic should be gunned or trowelled along the outside of the Waboflex stud line on a clean and dry surface.

Installation of the Waboflex should commence from the kerb, upstand or end-unit at the lowest level of the roadway. The width of the first unit may be 'compressed' or 'extended' to suit the joint-width, using companders, and fitted over the studding. One heavy duty flat washer, one lock washer, and a nut are tightened to the specified torque.

The adjacent unit is adjusted to width clamps as before and placed over the studding, ensuring that the tongue and grooved end faces 'correspond'. Before bringing these faces together the tongue and grooved ends should be treated with cleaning and priming solutions followed by a coating of cold-applied polymer-rubber into the groove. This unit is then jacked into position to close the units and form a tight fitting joint. The fixings are tightened down to the specified torque. Any excess compound extruded when units are brought together should be removed. This procedure is continued across the width of the bridge until the last two units are ready to position. This would occur at the crown of the road or one unit out from the highest kerb. When sawn ends abut, inserts of rubber strips should be included prior to jacking the units together.

In order to achieve a tightly-fitting joint when the final piece is inserted, it will normally be necessary to measure the length of the closing piece required and cut to suit on site and spring the last two units down into position in compression. Careful cutting of these sections can be carried out with the equipment used for cutting the road surfacing. The temperature of the Waboflex should not be allowed to rise excessively (to avoid damage to the rubber).

The dimension for the closing pieces should not be taken until all the remaining units are in position and firmly fixed.

After the installation has been completed a minimum period of four hours should be allowed to elapse before checking the torque on the locating studs and, if required, these should be re-torqued to the specified figure.

After re-torquing, the stud hole recesses should be cleaned (without using solvents), and filled with bolt-hole sealant. The gap between the edges of the Waboflex and the adjacent road surfacing should then be filled with flexibilised epoxy mortar mixed and applied as specified. Ensure that this material is fully compacted and 'steel-float finished'.

- **Finger-type Expansion Joint**

The most silent expansion joint is still the 'finger type', in steel, which can be used for expansions of up to ± 400 mm (Fig. 32.50). The fingers should have a trapezoidal cross section so that no stones etc., can remain sticking between the fingers. Of course this joint is not watertight and it is necessary to drain the water which runs through between the fingers. It can be collected in a drain gutter which should be made of a

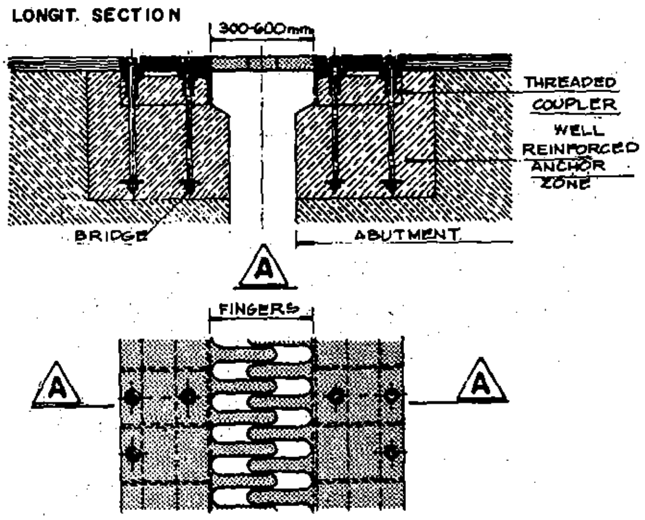


Fig. 32.50 *Finger type expansion joint*

fibre reinforced plastic membrane and should easily be approachable for inspection and cleaning (Fig. 32.51).

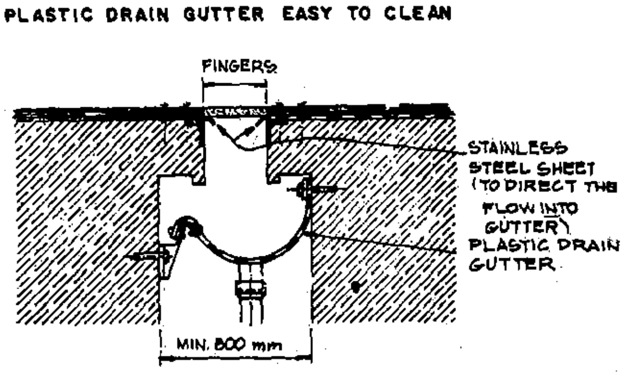


Fig. 32.51 *Gutter with plastic membrane*

- **Freyssinet's 'FT' and 'Monobloc' Expansion Joints**

See Fig. 32.52.

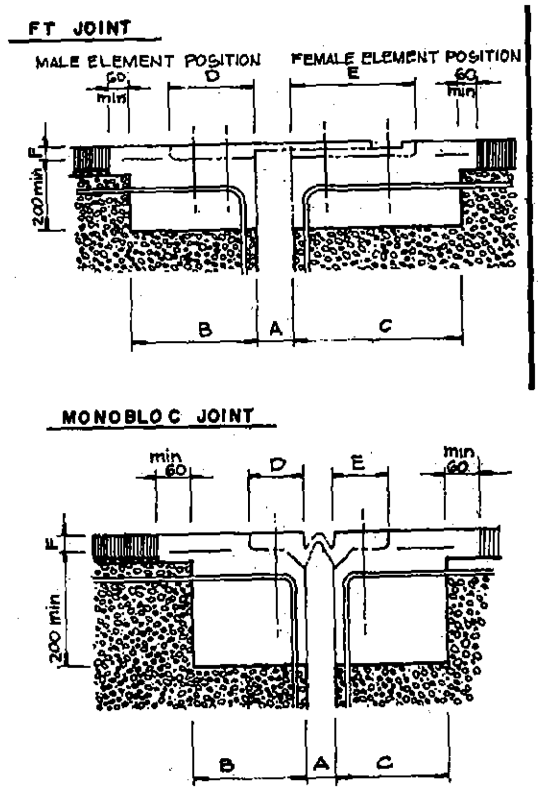


Fig. 32.52

(a) *THE FT JOINT:* The FT joint (0-600 mm movement) consists of two basic components: a male element and a female element, both made of a combination of STEEL and NEOPRENE. The front part of the male element slides back and forth over the female element (contraction and expansion). The front part of the male element generally has 'fingers' that move in the receiving grooves of the female element.

(b) *THE MONOBLOC JOINT*: The MONOBLOC joint (0–50 mm movement) consists of a moulded elastomer profile vulcanized between two identical steel or aluminium elements.

The advantages of these neat joints (as indeed in certain other proprietary types too) are:

- No thrust on the structure (unlike compressing an elastic section)
- No bracket fixings
- Flexibility
- Compact size
- Self-cleansing
- Easily adjusted to road level
- Retightenable
- Movement up to 600 mm, in 10 standard models, and
- Models for skew bridges up to 50 degrees (with or without fingers, as convenient).

(d) *KERB AND FOOTWAY JOINTS BY FREYSSINET*

- See Fig. 32.53.
- A complementary range of kerb and footway joints for each type of FT and MONOBLOC joint is available from FREYSSINET INTERNATIONAL. These sliding joints are also made of combination of STEEL and NEOPRENE. The kerb joint is always adapted to the kerb profile used for a particular structure.
- Footway joints are manufactured in 1 metre lengths with 10 fixing studs per linear metre.

(e) *INSTALLATION OF FREYSSINET EXPANSION JOINTS*

This is shown in Fig. 32.54 step and sequence wise:

Step 1 REBATES (recesses) IN THE STRUCTURAL CONCRETE

Rebates are formed across the entire width of the deck during the construction of the structure.

Step 2 PROTECTION OF REBATES AND SURFACING

The rebates are filled with loose gravel and covered with a thick steel plate. The waterproofing and black top are then laid continuous across the rebate.

Step 3 CUTTING OUT THE SURFACING AND CLEANING OF REBATES

The black top and waterproofing are sawn through on either side of the rebate. The surfacing, water-

(c) *RANGE AND DETAILS OF FREYSSINET 'FT' AND MONOBLOCK' JOINTS (see Fig. 32.52)**

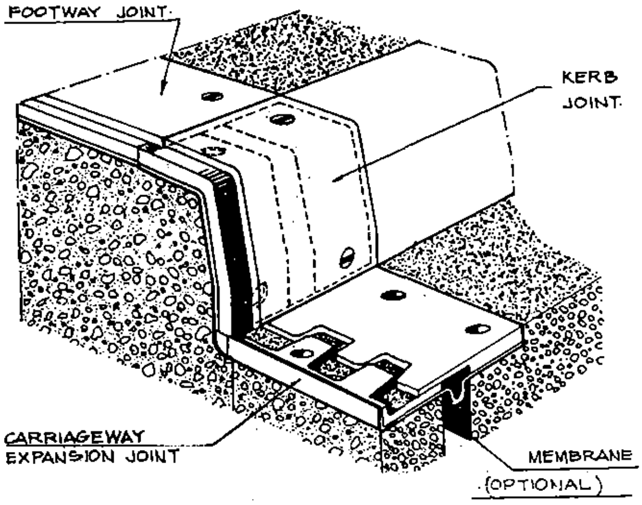
Type	Total movement	A		B	C	D	E	F	Length of each element	Fixing Device			Weight
		min	max							ϕ	Number of fixings		
											male el.	female el.	kg.
MONOBLOC 15	15	10	25	150	150	60	60	26	997	14	5	5	13
FT 25	25	15	40	200	200	100	100	27	747	14	5	5	30
FT 30 D*	30	10	40	200	200	100	130	22	997	16	5	5	22
MONOBLOC 30	30	15	45	200	200	105	105	28	997	16	5	5	23
FT 50	50	20	70	250	250	115	145	33	997	16	4	4	41
FT 50 D*	50	20	70	250	250	120	155	33	747	14	5	5	52
MONOBLOC 50	50	20	70	200	200	100	100	30	997	16	4	4	26
FT 75	75	20	95	300	300	135	190	39	997	16	5	3	53
FT 75 D*	75	20	95	300	300	135	190	39	997	16	5	3	50
FT 100	100	20	120	350	350	160	250	47	747	16	5	3	84
FT 150	150	20	170	350	450	220	360	50	747	16	8	8	151
FT 200	200	30	230	450	600	300	450	50	497	16	6	6	190
FT 250	250	50	300	500	700	330	560	57	497	16	7	6	250
FT 300	300	50	350	650	750	370	550	90	497	20	6	4	430
FT 400	400	50	450	650	850	370	650	90	497	20	6	4	470
FT 500	500	50	550	650	950	370	750	90	497	20	6	4	505
FT 600	600	50	650	650	1050	370	850	90	497	20	6	4	560

— All dimensions in mm.

— D* indicates absence of teeth (fingers)

TYPE	opening	A		B	C	D	E	Weight per 1 m
		min	max					kg.
JTFT 25	25	20	45	200	200	100	100	25
JTFT 30	30	20	50	200	200	100	100	25
JTFT 50	50	20	70	200	200	100	100	25
JTFT 75	75	20	95	200	200	120	100	28
JTFT 100	100	20	120	250	200	150	100	32
JTFT 150	150	20	170	300	200	200	100	39
JTFT 200	200	30	230	350	200	260	100	64
JTFT 250	250	50	300	400	250	310	150	86

All dimensions in mm (JTFT 300 to 600 also available from Freyssinet international).



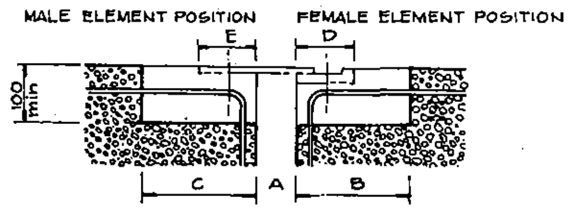


Fig. 32.53

proofing, and gravel are removed and the rebate is thoroughly cleaned.

Step 4 PLACING OF DRAIN (option) AND REINFORCING

A drain may be placed along the edge of the waterproofing cutaway. The rebate is reinforced with mild steel stirrups and continuity bars. A polystyrene and plywood void-former is then positioned.

Step 5 ADJUSTMENT AND CONCRETING

The joint, complete with fixings, placed in position, element by element. The embedding concrete which must be thoroughly vibrated, is poured into the rebate on either side of the void-former.

Step 6 PLACING THE WATERTIGHT MEMBRANE (option) AND FINAL TIGHTENING

When the concrete has set, a watertight membrane may be placed between the joint and it's seating. The final tightening is then carried out with a torque wrench.

Anchorage of Expansion Joints

Each side of the joint must be anchored to the deck (and abutment, as the case may be) so as to safety transfer all static and dynamic loads from the joint assembly to the structure. It is normal practice to design anchorages by neglecting any contribution from concrete under the joint. Field experience and laboratory testing (References 1 and 2) have shown that at least the following minimum anchorage requirements should be satisfied:

(a) Where the deck joint assembly is attached by reinforcing bars cast into concrete, the total cross-sectional area of such bars on each side of the joint should not be less than 1600 mm^2/m and the bar spacing be not more than 250 mm. Perforated plates giving an equivalent anchorage capacity may also be used. The anchorage system should be fixed to the main structural reinforcement. In the case of perforated plates or loops, this can be done by passing the deck transverse bars through these loops.

(b) Where the joint assembly is attached directly to the steel superstructure by welding or bolting, the welds or bolts should have an ultimate capacity of not less than 600 kn/m. The spacing between the bolts or between welds should not exceed 250 mm.

(c) Any other method of anchorage should have a capacity at least equal to (a) or (b) above.

Anchor bolts, set in concrete, have not been found terribly satisfactory as they tend to work loose under dynamic loading (Ref. 3). Loosening of the bolts has been observed with both friction type anchors and with bolts set in grout. Anchor bolts installed nearer the face of the joint than the first transverse reinforcing bar have also caused spalling of the concrete at the joint.

The steel structure of expansion joints must be strongly anchored to the deck slab, especially in case of concrete decks where securing by welding is generally not possible. There must be sufficient overlapping of the anchor bars

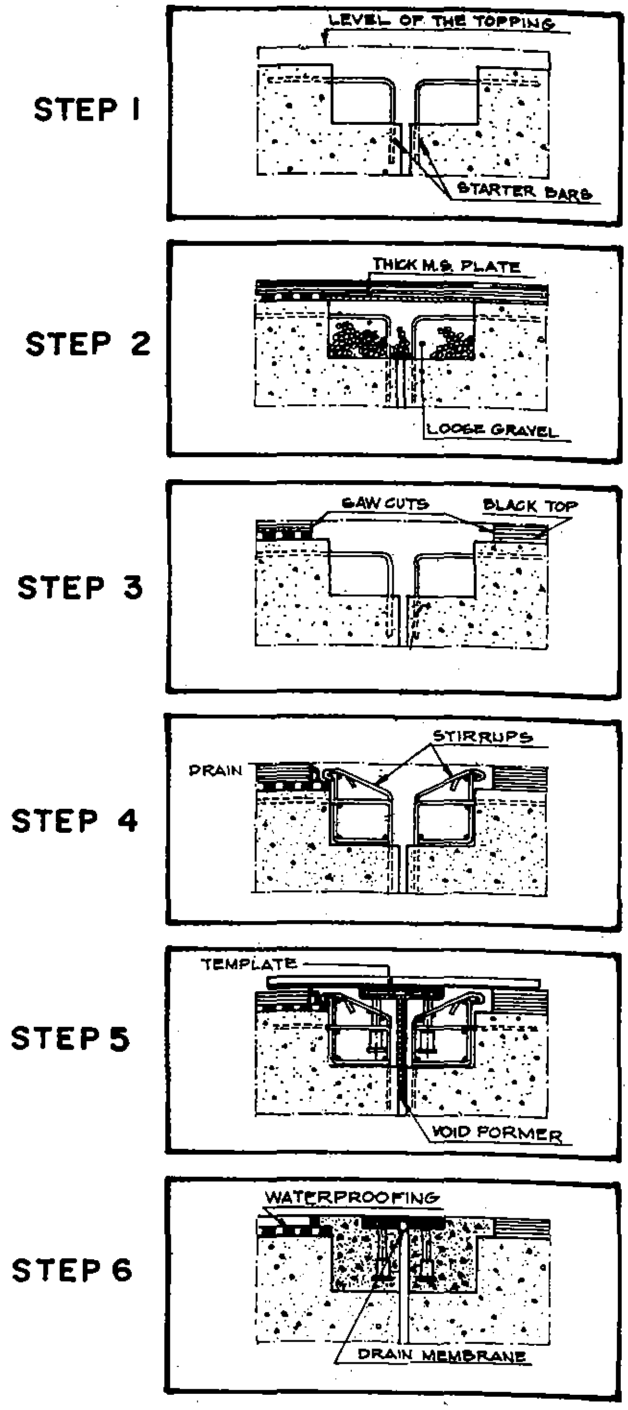


Fig. 32.54

with the reinforcement of the slab. Round ribbed reinforcing bars should be used and safely welded to the steel profiles (angles, fins, etc.).

The detailing of such reinforcement must be planned with attention to the 'placing' process. The gap must be well cleaned and wetted before the high strength concrete is placed and compacted in the recesses.

A word of general caution: Too often a proprietary joint is selected from a manufacturer's catalogue at the last moment, improperly installed by unskilled labour, and then not maintained well. Where a competing system is actually installed under an 'or approved equal' provision in the contract documents, the joint may bear little resemblance to that shown on the plans. Appropriate modifications in anchorage or blockout details may not always be attended to in advance. Consequently, it is not surprising that the performance of deck joints has often not been satisfactory.

Proprietary Joints

There are many organisations which manufacture these joints. While the products of many of them differ only slightly in actual details, one may not be able to say the same for their qualities. In selecting an appropriate joint the Engineer must carefully weigh the manufacturer's claims on performance against the actually required functions of rotation and translation and the required directions of the incumbent movements, the fixing details, the first-cost and the maintenance problems.

Some of the well known brands are: Maurer, Freyssinet-International, Thormajoint, Transflex, Waboflex,* Gutehoffnungshutte (GHH), and Demag. Strict adherance to quality controls and actual performance-record remain the governing yard-sticks for impartial selection of the actual product.

REFERENCES

1. Lee. D.J., "The Theory and Practice of Bearings and Expansion Joints for Bridges," Cement and Concrete Association, UK, 1971.
2. Black, W.P.M., "Performance of Expansion Joints in Great Britain", Laboratory Note No. LN/900/WPMB, B.B. 309, Transport and Road Research Laboratory, U.K., 1965.
3. "Technical Memorandum (Bridges) No. BE 3/72-Expansion Joints for Use in Highway Bridge Decks," Department of Transport, London, England, 1972.
4. Emerson, M., "Bridge Temperatures for Setting Bearings and Expansion Joints," Report No. SR 479, Transportation and Road Research Laboratory, UK, 1979.
5. Howard, Needless, Tammen and Bergendoff, "Bridge Deck Joint-Sealing Systems, Evaluation and Performance Specifications," NCHRP Report 204, Transportation Research Board, USA, 1979.
6. Koster, W., "Expansion Joints in Bridges and Concrete Roads," Transatlantic Arts Inc., New York, 1969.

* Servicised Ltd.

CHAPTER 33

Parapets and Railings for Highway Bridges

The requirement for a parapet to provide a safeguard against a vehicle which is out of control, plunging over the edge of a bridge, cannot be specified in terms of a static loading condition. The ability to absorb or redirect the energy of an errant vehicle is a function of the flexibility and constructional details of a parapet as much as on the nature and speed of the vehicle. Design regulations have, therefore, been based on the 'containment' requirements in terms of a specified weight of a vehicle and its approach angle and speed. The assessment of suitable parapet designs has become a matter of tests rather than design calculations.

It would be impracticable to stipulate that a parapet should be capable of containing 'any vehicle' travelling at 'any speed'. Requirements must be rationalized, and very few incidents have arisen in which vehicles have plunged through parapets, although there is inevitably much publicity in instances where this does occur with a consequent loss of life.

The selection of the type of parapet for a bridge is of fundamental importance to its appearance. In fact, for traffic users crossing a bridge, the parapet is likely to be the only indication that they are on a bridge structure. The fundamental choice is between a 'solid concrete parapet' usually surmounted by a single rail, and a more-open 'metal parapet'. Each can have visual merits depending on the general configuration of the bridge structure.

Other things being equal, if a three-span or four-span bridge is required over a motorway to carry a local road, with consequent light loading, it would seem inappropriate to introduce heavy concrete parapets onto a structure which would otherwise be slender.

Because it is very important to the finished appearance of a bridge, the parapet and its supporting upstand merit particular attention during detailing. The main potential hazard is weathering as a result of water staining. Even where the parapet is non-corrosive, such as where it is of aluminum, if water running off the parapet is allowed to run over the front face of the supporting upstand, this will lead to severe staining in time, which will have a disfiguring effect. The width of the supporting upstand, therefore, needs to be ample to accommodate the parapet post fixings and base plate, with a sufficient marging of width to ensure that the water drains into the bridge rather than over the front face.

Where a metal parapet is to be used, a choice must be made between steel, which will then require painting (not only in the course of construction but as a regular item of maintenance), and aluminium, which has gained widespread favour. Its colour is complementary to that of concrete, and the absence of any need for routine maintenance in the form of painting is a significant advantage.

33.1 DEFINITIONS

(a) **Safety Fence/Guard Rail**
A continuous metal beam intended to redirect an errant vehicel along the line of the fence. Guard Rail generally has wire ropes instead. These fences may be used where the road-embankment is less than 4 m high and is steeper than 1V to 4H.

(b) **Parapet**
A protective fence or wall at the edge of a bridge or similar structure.

(c) **Vehicle-Parapet**
A parapet designed to 'contain' vehicles on a structure from which pedestrians, animals and cyclists are excluded by Order.

(d) **Pedestrian-Parapet**
A parapet designed to safeguard pedestrians, but not intended to contain vehicles.

(e) **Vehicle-Pedestrian Parapet**
A parapet designed to contain vehicles and to safeguard pedestrians.

33.2 CLASSIFICATION OF HIGHWAY BRIDGE PARAPETS

While different countries may have separate designations and classifications for these, the British Deptt. of Environment, previously 'The UK Deptt. of Transport' (reference its Technical Memorandum No.: BE5) divided these parapets into six categories, P1 to P6. Of these, the P3 category has since been discontinued and the remaining five are briefly described in Table 33.1 ahead.

Table 33.1 *Application and Containment Requirements for Parapets*

Parapet Group Designation	Application	Containment for which Designed
P1	Vehicle-parapets for bridges carrying motorways or roads to motorway standards* (excluding motorway bridges over railways and high risk locations).	1.5 t vehicle at 113 km/hr. and 20° angle of impact
P2	Vehicle-pedestrian parapets for bridges carrying all purpose roads and for accommodation bridges (excluding bridges over railways and high risk locations).	1.5 t vehicle at 80 km/hr. and 20° angle of impact
P3	Parapet of this designation is no more in vogue.	
P4	*Pedestrian-parapets* for use on *footbridges* and bridges carrying bridleways (excluding bridges over railways).	
P5	*Parapets for use over railways* (excluding use on bridges at high risk railway locations):	
	i. On bridges carrying motorways or roads to motorway standards*	As for Group P1
	ii. On bridges carrying all purpose roads	As for Group P1
	iii. On footbridges	As for Group P4
P6	High Containment vehicle-parapets and vehicle-pedestrian parapets at high risk locations (excluding accommodation bridges).	30 t vehicle at 64 km/hr. and 20° angle of impact

* Roads from which pedestrians, animals, pedal-cycles and vehicles drawn by animals are excluded by order.

NOTE: In general. P2 type parapets are intended to have half the strength of P1 parapets (see Table 33.2 ahead). These strengths are approximately proportional to the square of the speed of the vehicle to be contained.

33.3 VARIOUS DETAILS

High Containment Parapets (Type P6)

High containment parapets are provided at certain high risk locations where the likelihood of parapet impact and consequent dangers resulting from parapet penetration are judged to be more than the hazards resulting from containment of errant vehicles and their redirection back into the traffic stream.

At certain locations, the nature of the area below the bridge may justify the High Containment parapet. Such a location could be when:

- the bridge is above a high speed railway line or motorway
- the bridge is above a busy railway line or motorway
- the bridge is close to an area occupied by people or by valuable installations or used for storage of hazardous materials
- the bridge is over any type of railway line or motorway but has 'inferior' horizontal or vertical alignment
- the bridge is part of a complex interchange where driver's error is more likely
- where road junctions are very close to the bridge or its approaches
- where the road arrangement and actual traffic have a record of accidents.

Materials

All parapets may be constructed of metal or reinforced concrete.

Height

The height of parapets is measured above the 'adjoining paved surface' and should not be less than the following:

1000 mm	...	for vehicle-parapets and vehicle-pedestrian parapets except as given ahead.
1500 mm	...	for accommodation bridges
1150 mm	...	for footbridges except over railways
1800 mm	...	for bridleways
1250 mm	or	
1500 mm	...	for all bridges over railways (and the height could be more)
1500 mm	...	for High Containment applications.

Shape/Form of Various Parapets (Figs. 33.1 to 33.4)

Reinforced concrete parapets or metal parapets with shaped plinths may be used for all vehicle-parapets and vehicle-pedestrian applications. P5 has same containment standard as for P1. (Reference may be made to the stated BE5 (or its latest version) for more details, as also of P4 and P5.)

Parapet type P6 may have the form same as that shown in Fig. 33.1 or Fig. 33.2 but should be of adequate section strength (see Table 33.3 ahead) and 1500 mm tall. (The minimum height of shaped reinforced concrete parapets is kept between 1250 and 1500 mm above the adjoining paved surface.) Special conditions at particular sites may require taller parapets; these cases should be considered on their individual merits and as a departure from the standards.

Standard Bridge Parapet and Metal Handrail for Expressway Traffic Conditions

See Fig. 33.5 and Fig. 33.6.

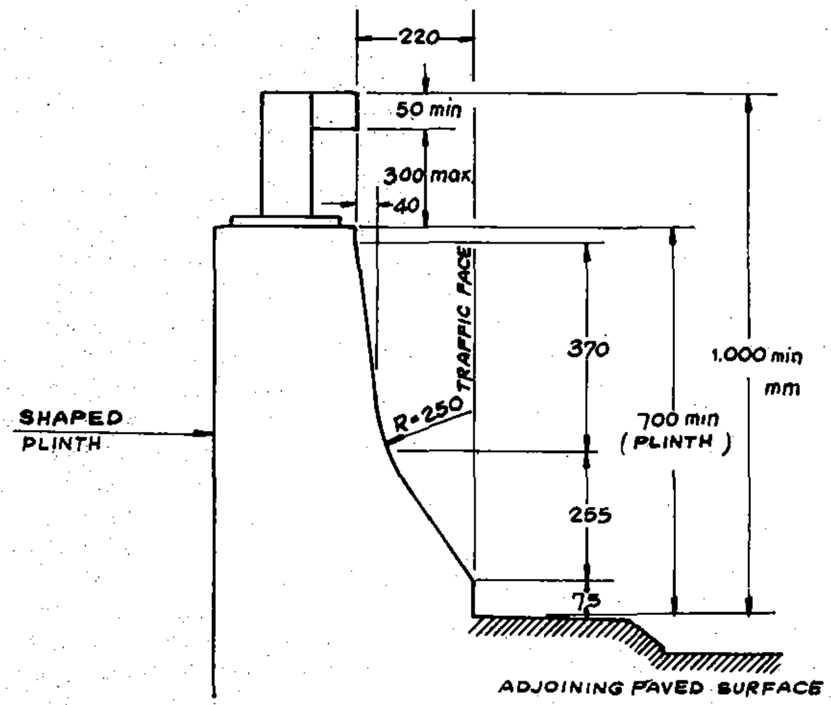


Fig. 33.1 *P1 'Vehicle-Parapet' with plinth 700 mm high or more for use in motorway under bridges*

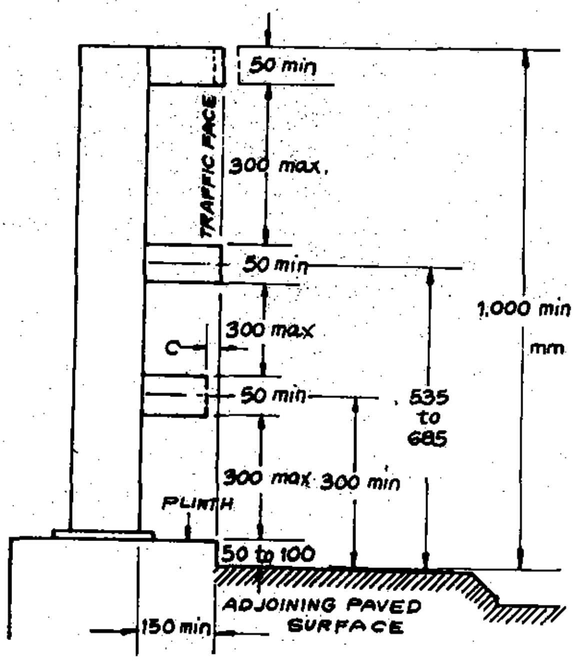


Fig. 33.2 *P1 'Vehicle-Parapet' with plinth less than 700 mm high for use on motorway under-bridges*

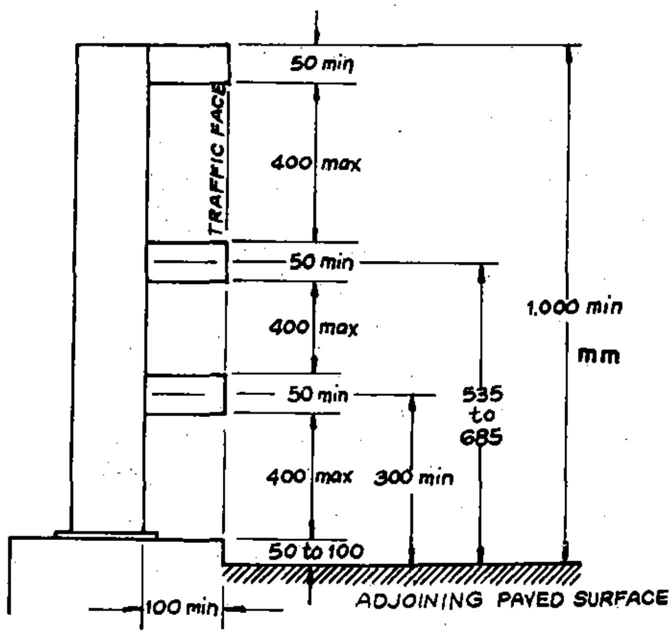


Fig. 33.3 *P2 'Vehicle-Pedestrian Parapet' for use on all purpose road bridges. The design speed being stated*

Notes for Parapet and Railing:

(i) All concrete shall be $f_c' = 250$ kg/cm^2 (28 days cylinderstrength)

(ii) All dimensions are in cm unless stated otherwise.

(iii) Reinforcing steel shall be grade 60 ($F_y = 4200$ kg/cm^2)

(iv) Concrete cover shall be 4 cm, unless otherwise shown.

(v) All exposed concrete edges shall provide 2 cm, chamfer.

(vi) Metal hand railing shall be approved by the Engineer.

(vii) Parapet shall be cast-in-situ.

(viii) All structural steel shall be galvanized and shall conform to AASHTO M-183.

(ix) All the exposed edges of structural steel shall be rounded.

(x) All connections of structural steel shall provide 6 mm fillet weld on all sides around.

NOTE: These details have demonstrated superior safety aspects for high-containment of Expressway Traffic.

Parapet used on Many US Highways

Figure 33.7 shows typical dimensions of a standard Parapet Kerb with metal hand rail as adopted by many authorities in the USA. It has demonstrated very satisfactory safety aspects for Expressway Traffic Conditions.

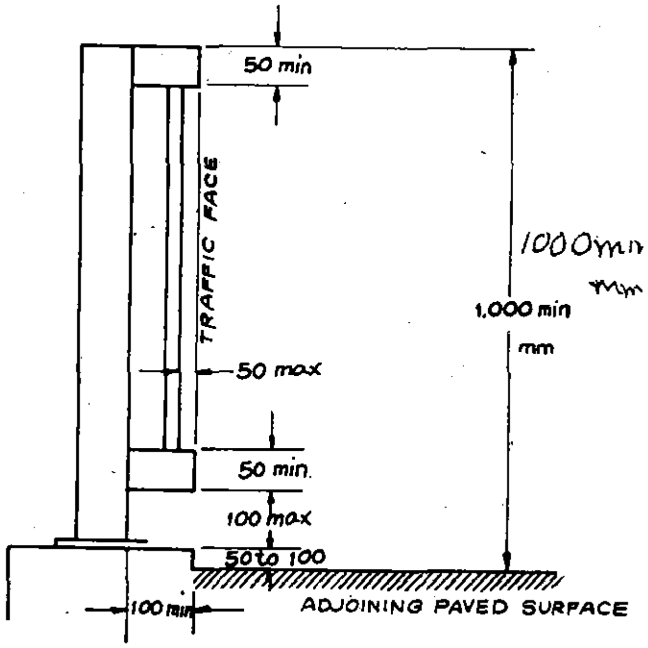


Fig. 33.4 *P2 'Vehicle-Pedestrian Parapet' for use in road bridges where speed is restricted to 48 km/hr*

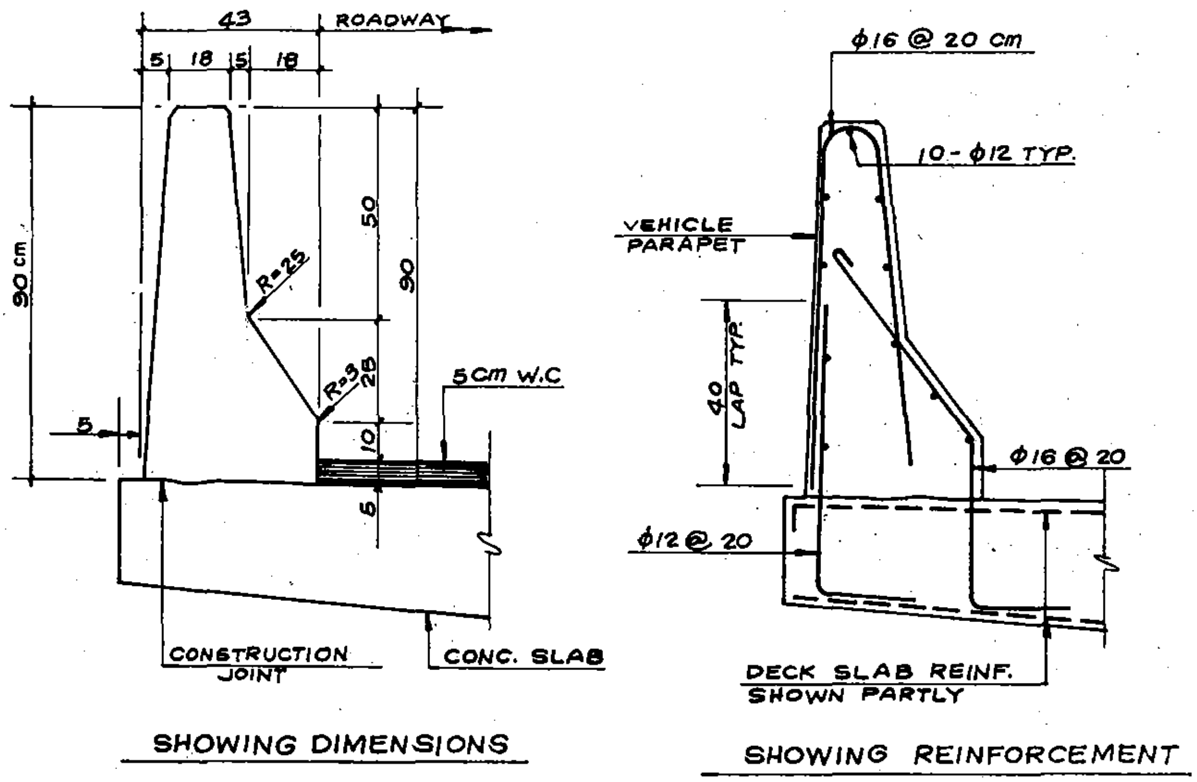


Fig. 33.5 *Vehicle-Parapet for bridge without sidewalk*

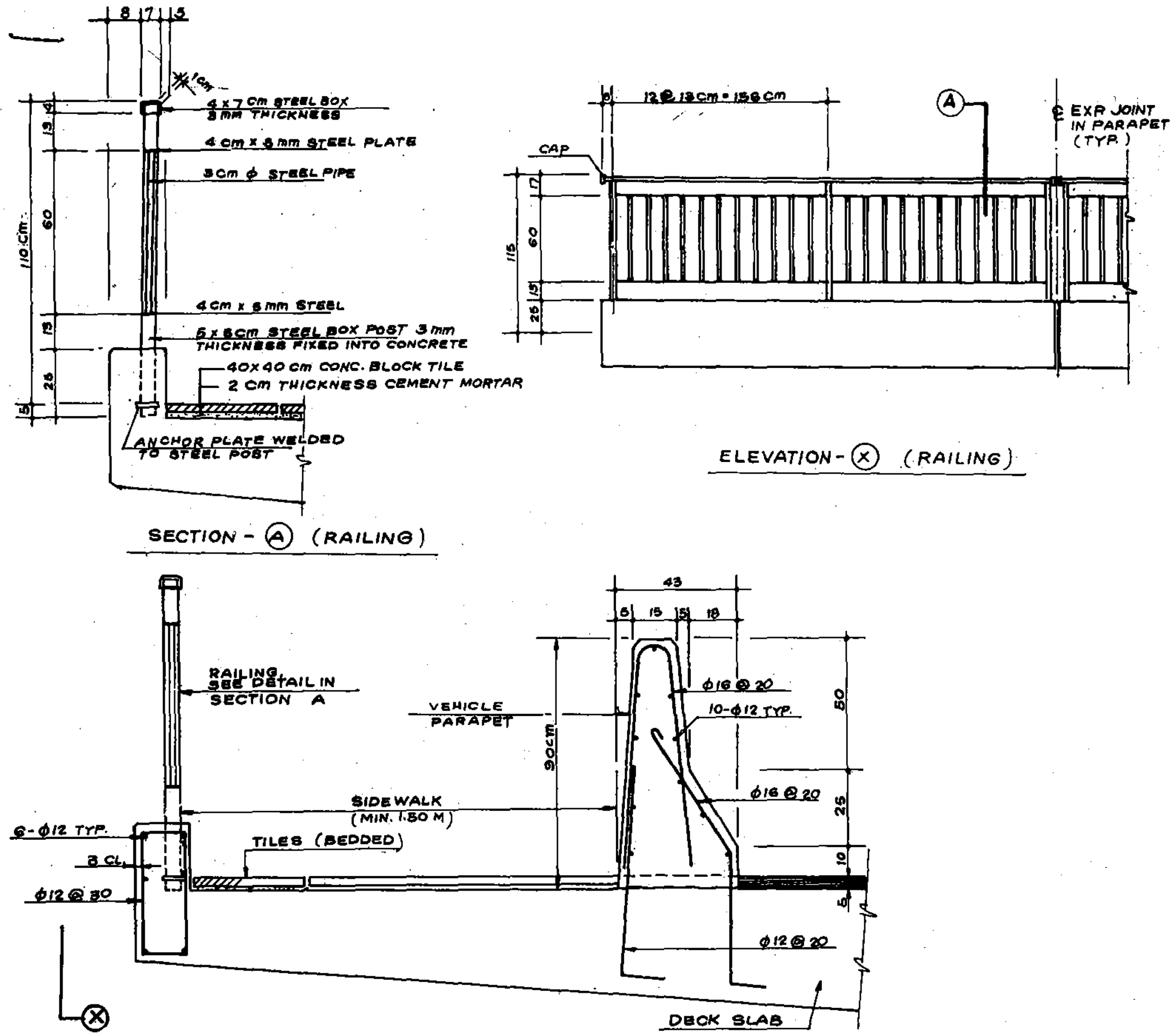


Fig. 33.6 *Vehicle-Parapet and railing for bridge with side-walk (slab reinforcement not shown.)*

An Almost Fool-proof Parapet for Expressway Traffic

There is no such thing as a fool-proof parapet. However, for an 'almost' fool-proof parapet system for Expressway traffic, it is suggested that a 300 mm unmountable RC kerb in place of the usual 230 mm standard kerb be provided along with the Fig. 33.8 arrangement.

It has been confirmed by the Road Research Laboratory in the UK (who have supervised dynamic testing of parapet details) that even a 230 mm kerb does not throw the errant vehicle up because the shock absorbers of the vehicle absorb the impact.

Requirements for Metal Members of Metallic Parapets

Reference may be made to the UK DOE Technical Memorandum BE5 (or its latest version) regarding the following requirements:

- elongation
- minimum thickness of metal
- protection against corrosion
- maximum distance between posts supporting the longitudinal members
- strength of the effective longitudinal members

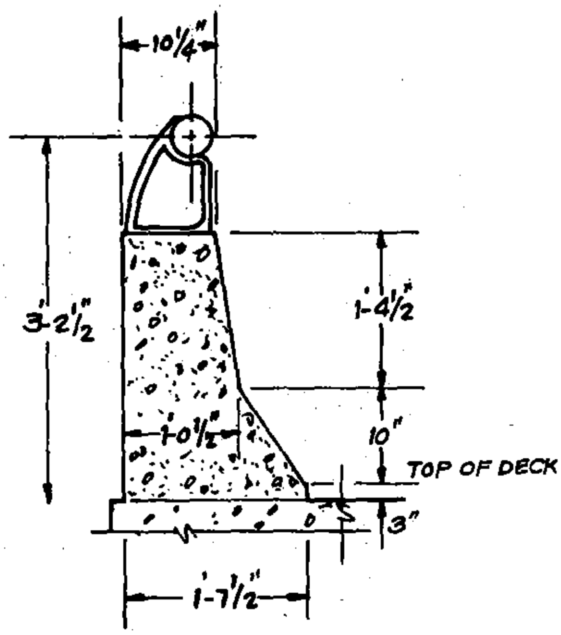


Fig. 33.7 *Vehicle-Parapet Kerb (reinforcement not shown) AASHTO/ACI*

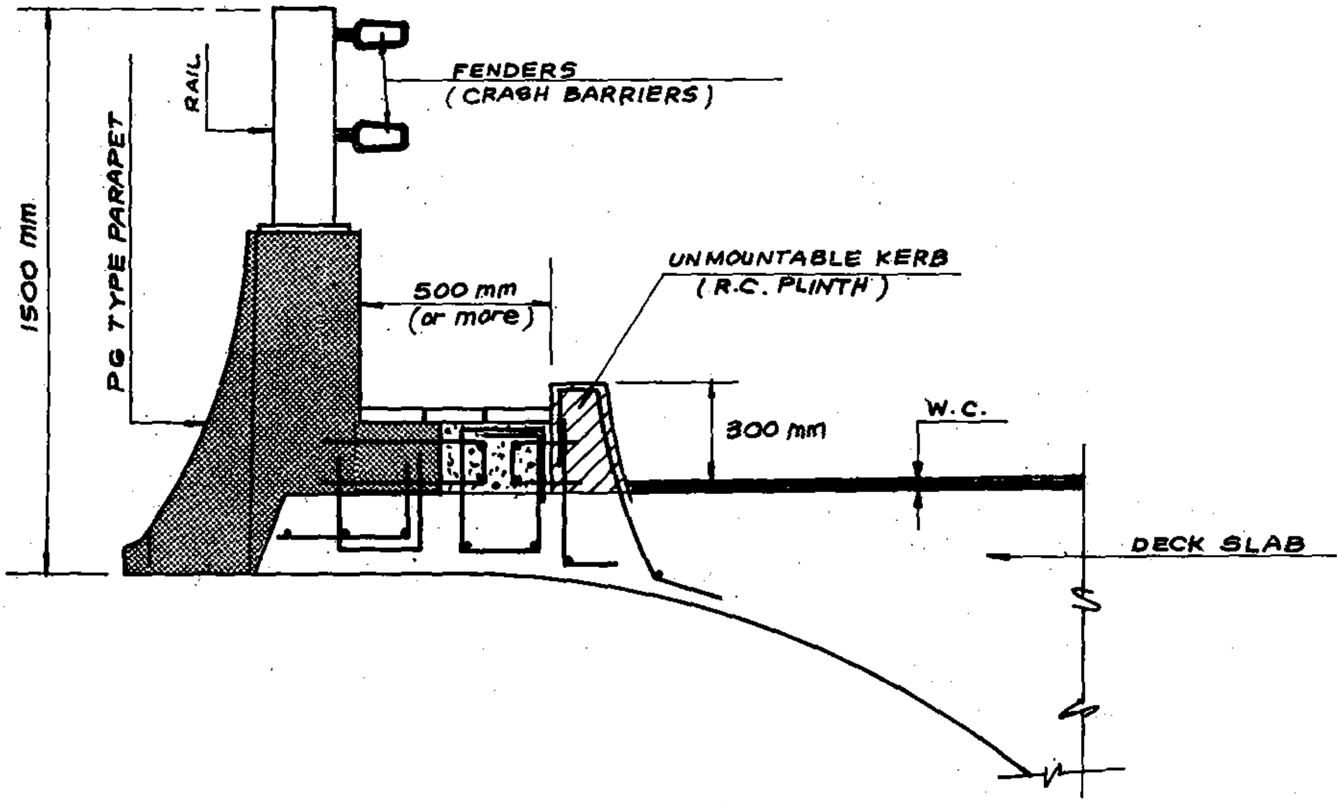


Fig. 33.8

- structural continuity of the longitudinal members
- strength of the supporting posts under transverse loading
- strength of supporting posts under longitudinal loading
- strength and descriptions of post-fixings
- details of fixing longitudinal members to supporting posts (strength criteria)

The steel design is essentially based on plastic approach.

Requirements for Reinforced Concrete Parapets and Plinths

Here too reference may be made to the above referred BE5 (or its latest version) on which the following are based:

(a) **Minimum Concrete Strength**

The concrete should have a minimum 28-day works 'cube' strength of 30 N/mm^2

(b) **Design Requirements for Reinforced Concrete In-situ and Precast Parapets (Groups P1, P2, P4 and P5)**

The 'vehicle' and 'vehicle-pedestrian' parapets shall be of the form shown in Fig. 33.9, except that where the permitted traffic speed does not exceed 48 km/hr the front face of the parapet may be vertical. For P4 parapets the front face of the parapets shall always be vertical.

Generally the shaping of the top of the parapet should be as shown in Fig. 33.9 to prevent anybody walking on top of these walls, but for parapets on bridges carrying motorways, or roads to motorway standards, from which pedestrians are excluded by order, such shaping is not essential.

For 'vehicle' and 'vehicle-pedestrian' parapets the minimum length of each of the 'precast' reinforced concrete panels shall be 1200 mm.

For the design requirements for these parapets see Table 33.2.

(c) **Design Requirements for Reinforced Concrete In-situ and Precast Plinths (Groups P1, P2, P4 and P5)**

The requirements given in (iii) and (iv) below apply only to 'plinths' for 'vehicle' and 'vehicle-pedestrian' parapets. Reference should be made to BE5 for the Group P4 plinth requirements.

(i) **Height**

A reinforced concrete plinth shall always be provided under a metal parapet. The plinth height for 'vehicle' and 'vehicle-pedestrian' parapets, measured above the adjoining paved surface, shall either be (1) not less than 50 mm and not greater than 100 mm, or (2) shall be at least 800 mm. Where the permitted traffic speed does not exceed 80 km/hr the above limitation (2) on plinth height may be reduced to 700 mm [see (iii)].

(ii) **Strength**

The strength of the plinth shall be sufficient to withstand the moment and shear developed by the fixing of the parapet posts.

(iii) **Plinth as an Effective Longitudinal Member**

A plinth may be considered as an effective longitudinal member provided its height is at least 700 mm above the adjoining paved surface, and that it can support a horizontal transverse force of "F/n" kN, applied as a point load to its top edge, in addition to the forces induced by the posts (F being the appropriate force given in Table II in BE5 and n the number of effective longitudinal members). If the derived ultimate moments of resistance at any point for the plinth are less than the

Table 33.2 *Design Requirements for Reinforced Concrete Parapets (P1, P2, P4 and P5)*

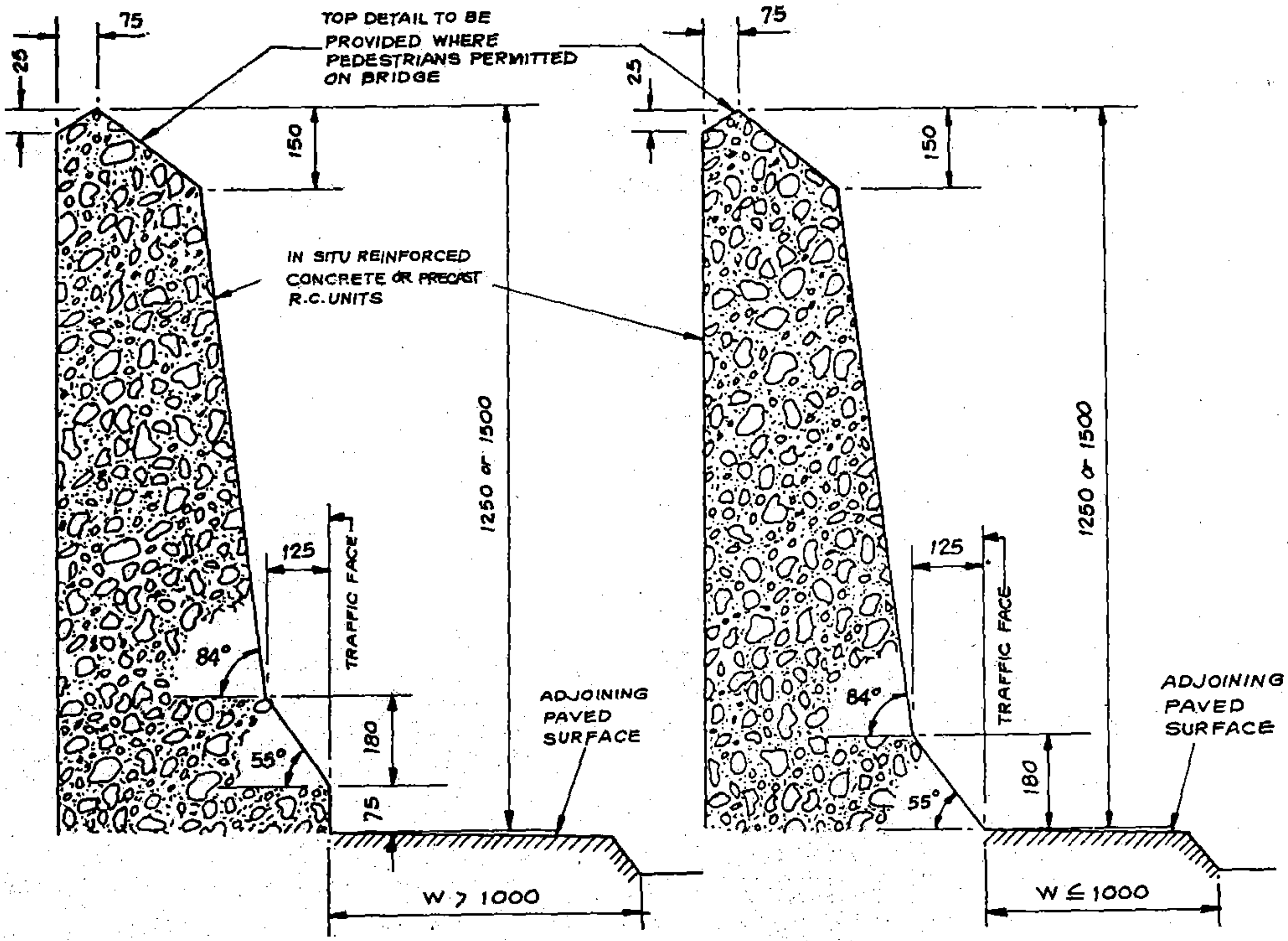
Item No.	Requirement	Parapet Group [see Note (iii)] P1	P2
1.	Minimum ultimate moment of resistance, at the base of wall [see Note (i)] for bending in the vertical plane, with reinforcement adjacent to the traffic face [see Note (ii)].	25 kNm/m	12.5 kNm/m
2.	Minimum ultimate moment of resistance for bending in the horizontal plane, with reinforcement adjacent to outer face [see Note (ii)].	12.5 kNm/m	6.25 kNm/m
3.	Minimum ultimate moment of resistance of anchorages at the base of a precast reinforced concrete panel.	37.5 kNm/m	18.7 kNm/m
4.	Minimum ultimate transverse shear resistance at vertical joints between precast panels, or at vertical joints made between lengths of in-situ parapet.	65.5 kN/m of joint	32.5 kN/m of joint
5.	Minimum thickness of RC wall.	180 mm	150 mm

NOTES:

(i) The base of wall refers to horizontal sections of the parapet within 300 mm above the adjoining paved surface level. The minimum ultimate moment of resistance shall reduce linearly from the 'base of wall" value to zero at top of the parapet.

(ii) In addition to the main reinforcement, in items 1 and 2 above, distribution steel equal to 50% of the main reinforcement shall be provided in the respective faces.

(iii) For Group P4 (pedestrian parapets) and Group P5 (parapets for bridges over railways) see BE5 for details.



PROFILE WHEN W > 1000

PROFILE WHEN W ≤ 1000

NOTES:

1. W is the width of adjoining paved surface between Kerb and Traffic-face of Parapet.
2. Dimensions are in mm.
3. Sketches not to scale.

Fig. 33.9 *Shaped Concrete Parapets (see item 5 in Tables 33.2 and 33.3 ahead for wall thickness).*

Table 33.3 *Design Requirements for High-Containment In-situ Reinforced Concrete Parapets (P6)*

Item No.	Requirement	
1.	Minimum ultimate moment of resistance at base of wall [see Note (i)] for bending in the vertical plane with reinforcement adjacent to traffic-face [see Note (ii)]	
	• For end sections [see Note (iii)]	165 kNm/m
	• For intermediate sections [see Note (iii)].	125 kNm/m
2.	Minimum ultimate moment of resistance for bending in the horizontal plane, with reinforcement adjacent to outer face [see Note (ii)]	62.5 kNm/m
3.	Minimum ultimate horizontal transverse shear resistance	220 kN/m
4.	Minimum ultimate transverse shear load to be transferred at vertical joints made between lengths of in-situ parapets [see Note (iv)]	165 kN
5.	*Minimum thickness of RC wall*: at top	250 mm
	: at adjoining paved surface level	430 mm

NOTES:

(i) The base of wall refers to horizontal sections of the parapet within 300 mm above or below the adjoining paved surface level. The minimum ultimate moment of resistance shall reduce linearly from the 'base of wall' value to zero at top of the parapet.

(ii) In addition to the main reinforcement in items 1 and 2 above, distribution steel equal to 50% of the main reinforcement shall be provided in the respective faces.

(iii) For design purposes the parapet shall be divided into 'end-sections' extending a distance not greater than 3.0 m from ends of the parapet and 'intermediate-sections' extending along the remainder of the parapet.

(iv) The minimum ultimate transverse shear load may be reduced to 50 kN if the 3.0 m longitudinal lengths adjacent to and on either side of the joint are designed as 'end-sections'. The shear load shall be assumed to be uniformly distributed over the joint length.

appropriate moments given in Table 33.2 the values as specified in the table shall be used.

(iv) **Precast Sections**

Where a plinth is built of precast concrete sections, they shall be not less than 1200 mm long. The fixing moment of the section of the plinth to the deck shall be not less than the holding down moment of the post fixing, plus 1-1/2 times the moment induced by F/n where appropriate. If the derived ultimate fixing moment is less than the moment given in item 3 of Table 33.2, the value as specified in the table shall be used.

When a precast concrete plinth acts as an effective longitudinal member, the vertical joints shall be

Table 33.4 *Additional Design Requirements for High-Containment In-situ Reinforced Concrete Parapets (P6)*

Item No.	Additional Requirements	
1.(a)	Minimum ultimate moment of resistance, at level *d* (metres) below the adjoining paved surface level, for bending in the vertical plane with reinforcement adjacent to traffic face—	
	• For end sections } [see Note (i)]	$165 + 48.(d - 0.3)$ kNm/m
	• For intermediate sections } [see Note (i)]	$125 + 12.(d - 0.3)$ kNm/m

NOTE:

(i) For design purposes the parapet shall be divided into 'end-sections' extending a distance not greater than $3.0 + 3 \cdot (d - 0.3)$ metres from ends of the parapet and 'intermediate-sections' extending along the remainder of the parapet.

Table 33.5 *Design Requirements for High-Containment Precast Reinforced Concrete Parapets (P6)*

Loading No.	Requirement
	The minimum ultimate moment of resistance and horizontal transverse shear resistance of the precast panels [see Note (i)] shall be sufficient to withstand the more severe of the following ultimate design forces:
1.	A transverse force of 330 kN applied to each panel over a horizontal length of 1.5 m at a height of 0.85 m above the adjoining paved surface.
2.	A transverse force of 330 kN applied over a horizontal length of 3.0 m at the top of the parapet.

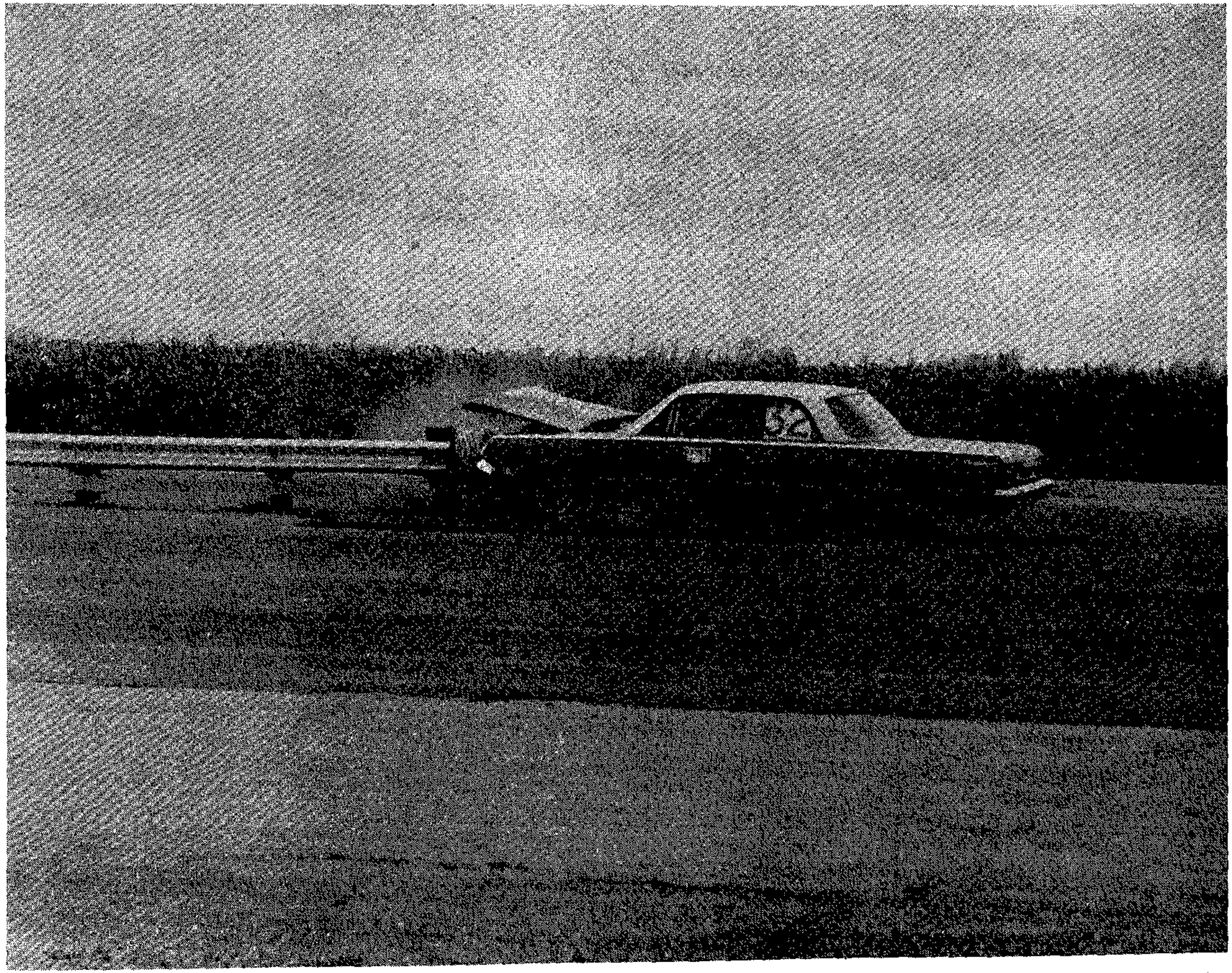
NOTES:

(i) If the ultimate moment and shear resistances derived from Table 33.5 are less than the values given in Tables 33.3 and 33.4 [items 1 and 1(a) end-sections, and item 2 and item 3], Tables 33.3 and 33.4 values shall be used.

(ii) The minimum ultimate moment of resistance and horizontal transverse shear resistance of the precast panel anchorages shall be 1.5 times the values calculated at the base of the precast panels.

(iii) The minimum ultimate transverse shear resistance at vertical joints between precast panels shall be 110 kN. The effect of shear transfer between adjacent panels shal' be ignored when applying the above requirements.

(iv) The requirement for minimum thickness given under item 5 of Table 33.3 shall apply to precast panels also. The minimum length of panels shall be 1500 mm.



Vehicle parapet (crash-barrier) impact test provided with shear keys or dowels which will transfer the appropriate load given in item 4 of Table 33.2.

(d) Design Requirements for Reinforced Concrete High-containment In-situ and Precast Parapets (Group P6)

(a) **In-situ Reinforced Concrete Parapets (P6)**
The design requirements for these parapets are given in Table 33.3. Parapets which are joined to the supporting structure at a level deeper than 300 mm below the adjoining paved surface level, shall, in addition, comply with the requirements of Table 33.4.

(b) **Precast Reinforced Concrete Parapets (P6)**
The design requirements for these parapets are given in Table 33.5.

Requirements for Dynamic Testing of Prototype Vehicle-Parapet (Reference BE5)

1. **Testing Arrangements**
The testing facilities and method shall be capable of demonstrating that the prototype parapet meets the appropriate 'containment standard' and other criteria given below.

The prototype parapet shall be not less than 30 metres long. Unless otherwise agreed, it shall be identical in every detail, including the capacity of its post fixings, to the production parapet which it represents. The impact vehicle shall be a roadworthy production model of approved type. The steering should be in good condition and the bodywork reasonably free from corrosion. Howsoever the vehicle is guided to the impact point, it shall be unrestrained at and after the time of impact.

The angle of impact shall always be 20° to the line of the parapet.

2. **Containment Standards**

P1 (application as described in Table 33.1)	1.5 t at 113 km/hr.
P2 (application as described in Table 33.1)	1.5 t at 80 km/hr.

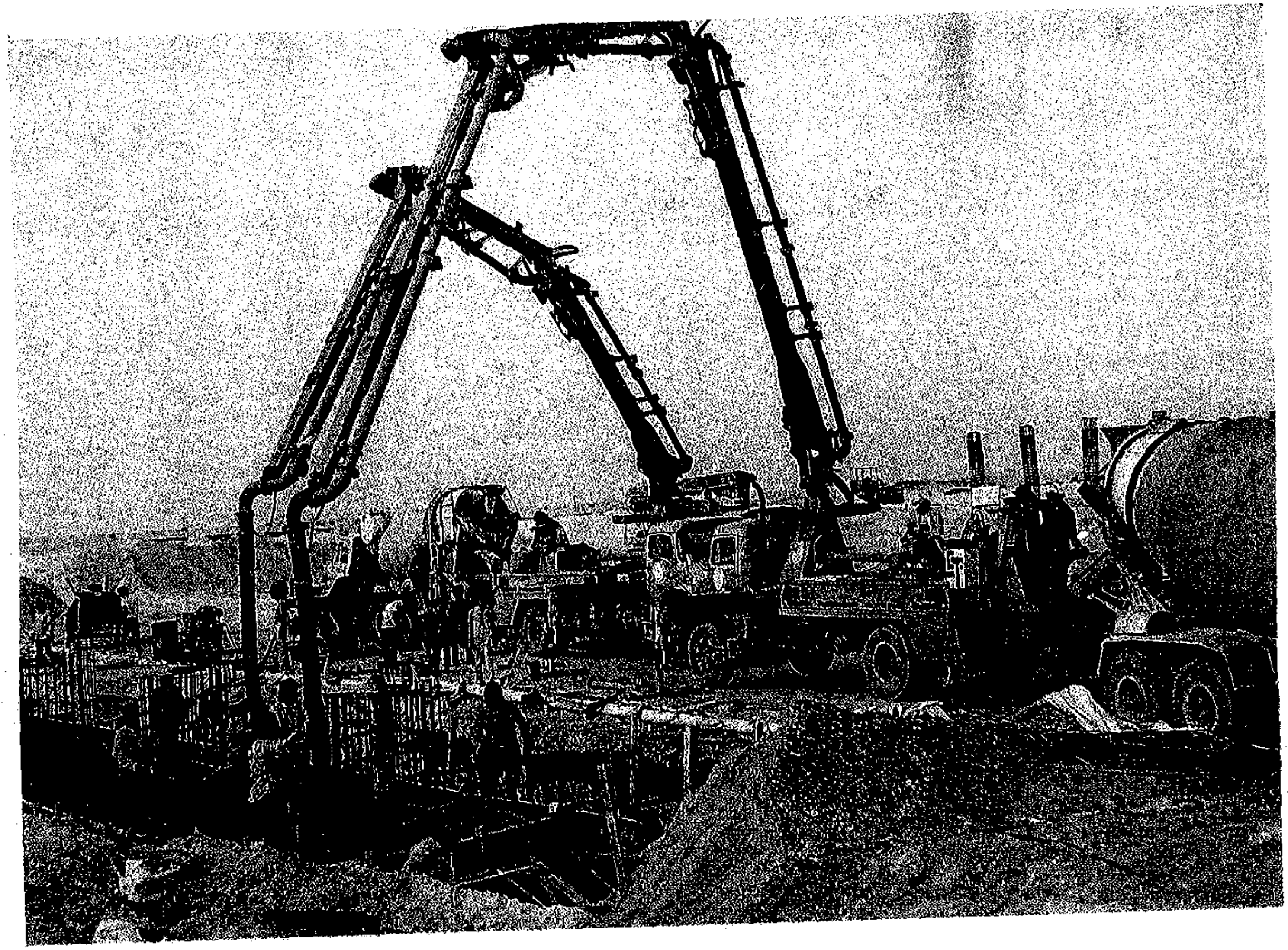
3. **Criteria for Acceptance**
(i) The vehicle shall be contained.
(ii) No part of the parapet shall become detached.
(iii) The transverse displacement of the parapet shall not exceed 800 mm.
(iv) Examination of the concrete base under the baseplate of a metal parapet after testing should show no visible signs of damage or cracking.
(v) No part of the vehicle shall be arrested abruptly by impact with a post.
(vi) The test vehicle shall not roll over.
(vii) The car shall preferably be steered back to the line of the parapet but if steerback is not achieved, it shall be redirected at not more than 12° to the line of the parapet.

CHAPTER 34

Construction Techniques

This subject has already been dealt with in quite some detail in the author's other book: "*Concrete Bridge Practice: Construction, Maintenance and Rehabilitation*" to which reference may be made. Given below for ready reference are the heads under which this subject has been discussed in it.

- **CONSTRUCTION TECHNIQUES**
  - **Techniques for Constructing 'Substructure':**
    - — for footings, piles and caissons
    - — stability of floating caissons
    - — floating the well curb from shore
    - — cofferdams (false steining)
    - — floating pile-driving plant
    - — support by ground-anchors
    - — site preparation
    - — excavation methods
    - — stability of slopes (in different strata)
    - — some unusual pier construction techniques (examples of actual bridges)
  - **Construction of Reinforced-Earth Walls and Abutments**
  - **Techniques for Constructing 'Superstructure':**
    - — General
      - Crane erection, Jacking or counter-weighing, Cantilever erection, metal erection — trusses, false-work piers, cable erection.
    - — mobile crane (floating-type, gantry-type, crawler-type)
    - — use of service platform, temporary trestle, erection span with launching-nose,
    - — casting beams vertically and then pivotting down
    - — casting beams along embankment and then swivelling them into position
    - — free cantilever method for segmental Construction:
      - various details
      - cast in-situ vs. precast segmental methods
    - — span by span construction method
    - — progressive placement method
    - — incremental-launching (push-out) method
    - — loads applied to segmentally constructed deck during construction.
    - — two additional techniques for constructing continuous p.s.c. decks on tall piers.
    - — temporary structures (i.e. Construction Apparatus):
      - general
      - casting yards (various details)
      - launching girders and trusses (various methods)
      - movement of precast elements (various phases)
      - movement of erection equipment
      - working of a typical Launching Girder for launching precast p.s.c. beams (stage-wise).
      - crane systems (Derrick Trolleys, Guy Derrick Cranes, Tower Cranes, Portal Gantries, etc.)
    - — manufacture of precast segments:
      - long-line casting,
      - short-line casting,
      - handling, temporary assembly,
    - — placing precast segments:
      - independent lifting equipment
      - beam and winch method
      - launching girder method
    - — notable construction details of some major bridges:
      - two p.s.c. bowstring decks (U.K. and France)
      - open spandrel arches for a bridge cast vertically (South Africa)
      - Vijle Fjord Bridge (Segmental)
      - Houston Ship Channel Bridge
      - Medway Bridge (U.K.)
      - Paris Belt-way Bridge (upstream)
      - Oleron Viaduct (France)
      - Linn Cove Viaduct (U.S.A.)
      - Eastern Scheldt Bridge (Rotterdam)
      - Pasco-Kennewick Bridge (U.S.A)
      - Gladsville Arch Bridge (Australia)
      - Pontchartrain Bridge
      - Caracas Arches (Venezuela)



Laying foundation using pumped concrete

CHAPTER 35

Construction Considerations

This subject has already been dealt with in quite some detail in the author's other book: *Concrete Bridge Practice: Construction, Maintenance and Rehabilitation* to which reference may be made. Given below for ready reference are the heads under which this subject has been discussed in it.

CONSTRUCTION CONSIDERATIONS

- General
- Excavation and backfill:
  - — general
  - — preparation of channel
  - — depth of footings
  - — preparation of foundations (in various strata)
  - — sheeting, cofferdams, open cribs
  - — Caisson (well) foundations:
    - various types
    - important details
    - problems involved
    - temporary islands
    - sinking through various strata
    - sand blowing
    - artesian conditions
    - counteracting tilts and shifts
    - tackling obstacles met with in well-sinking
  - — backfilling, placing, compacting
  - — drainage
- Foundation Piles:
  - — general
  - — equipment
  - — construction and driving procedures (various details)
- Concrete . . . production and control:
  - — various details regarding the Aggregates, cement, water, admixtures, proportioning and finalising a mix and its workability; Batching, mixing, delivering, placing, vibrating and finishing of concrete.
- Control of Surface Evaporation and its Importance
- Joints (Construction, Expansion, and Cold)
- Deposition of concrete under water:
  - — various details pertinent to Tremie concreting
- Inspection and Testing 'for' Concrete:
  - — for batching, mixing, delivery, etc.
  - — for sampling and testing of materials
- Curing of Concrete:
  - — various details about membrane-curing, watercuring and steam-curing.
- Concreting in adverse weather conditions:
  - — in cold weather
  - — in hot weather
- Concrete exposed to sea water
- Falsework
- Forms (Shutters)
- Reinforcing Steel (various considerations)
- Falsework for structural steel work
- Bearings and Anchors under steel deck (relevant to composite construction)
- Structural Steel Expansion Joints and Roller Bearings
- Assembly and alignment of steel decks (relevant to composite construction)
- Connections in Steel Decks (relevant to composite construction)—various details.
- Painting of Structural Steel (relevant to composite construction).
- Inspection of Structural Steel (relevant to composite construction).
- Prestressed Concrete:
  - — various details more pertinent to prestressed concrete in addition to those given earlier for concrete.
- Prestressing:
  - — proprietary systems
  - — pretensioning components (considerations and various details)
  - — post-tensioning components (considerations and various details)
  - — Prestressing reinforcement (various considerations)
- Placement of reinforcement, inserts and bearings prior to casting prestressed concrete
  - — various details and considerations

- Prestressing the Structure:
  - some important considerations and precautions
- Precast-Segment Manufacture and Erection:
  - manufacture
  - erection of segments
- Epoxy bonding agents for precast segments
- Inspection of precast segment-joining procedure
- Epoxy bonding agent TESTS·
  - details of 7 recommended tests
- Grouting of cable ducts:
  - various considerations and details
- Load testing a Bridge Superstructure:
  - purpose
  - 'Test Load' and Computation of deflections (theoretically)
  - Load application and Testing
  - Assessment of Results
  - Precautions
  - Some Important Considerations
- Prestressed Rock Anchors:
  - various considerations and requirements
- Application of Sprayed Concrete and Mortar
  - various considerations
- Common problems associated with concrete:
  - workability
  - excessive salts in aggregates
  - cracking in finished concrete
  - settlement at abutments
  - settlement around culverts
  - settlement of falsework
- Considerations for making good concrete in Hot/Windy/Marine atmospheric conditions.
- Care for Structural Concrete:
  - general
  - corrosion of reinforcement
  - reaction between aggregate and cement
  - resistance to various chemicals/substances
  - resistance to sulphate attack
  - resistance to soft moorland and peaty waters
  - resistance to sewage
  - resistance to erosion and abrasion
  - resistance to fire
- Bridge Construction Execution: General preparation, mobilisation, and stores.
  - various items based on construction considerations.
- SEQUENCE of Road Construction, Typical EQUIPMENTS used, and their RATED CAPACITIES

CHAPTER 36

Cantilever Construction of Bridges

36.1 A MODERN CONSTRUCTION TECHNIQUE

It is no longer necessary to emphasize the advantages of prestressed concrete, the birth of which is linked with the name of Freyssinet. A modern bridge is a prestressed concrete bridge and among prestressed bridges those built by the cantilever method demonstrate the latest refinements of this construction technique.

Deep valleys are crossed without high and expensive scaffolding and rivers without the risk of having the scaffolding washed away by a flood.

36.2 VARIOUS DETAILS*

The Two Ways of Building in Cantilever

Cast In-situ Construction

The bridge is cast in-situ with sections 3 to 6 m long cantilevering symmetrically on both sides of the pier. Formwork is supported by a steel truss attached to the completed part of the bridge and which moves from one completed section to the next.

The sequence of operations consists of:

- Setting up and adjusting the steel truss and shuttering
- Placing reinforcement and prestressing ducts
- Concreting
- Threading the cables to be anchored in the section and stressing
- Releasing the shuttering and moving on to the next section

Normal construction time is one section nearly per week per formwork unit. This time can be reduced by using heat curing.

Construction with Precast Segments

Bridge segments are precast in a yard with special forms using the immediately preceding unit as end boxing. A perfectly matching joint is thus obtained.

Precast segments are placed by means of a mobile launching girder or, when access under the bridge is possible with barges or trucks by means of a crane or a mobile hoist located at the extremity of the cantilevers.

Joints are glued with epoxy resin. Segments are held temporarily by ties and then prestressed by cables threaded through empty ducts. The speed of placing is 4 to 8 segments per day.

The Bridge Type

Usually the bridges have a continuous superstructure which has now practically replaced the midspan hinge system used earlier. Hinges are complicated and, because of creep, the riding surfaces often develop a severe break of the profile at this point.

In continuous structures, expansion joints may be spaced very far apart. Continuous bridges up to 1000 m long have been carried out without intermediate joints and this is perhaps not a limit provided the bearing and pier design is correctly integrated into the total design of the structure. Free longitudinal movement of the bridge due to creep and temperature change is allowed for by placing the structure on elastomer or sliding (teflon) bearings. Pier flexibility can also be used to accommodate these movements and the superstructure fixed to the piers. In this case pier flexibility is obtained either by height or by using single or double thin plate walls thus reducing the flexural resistance.

Single span bridges with counterweight abutments have also been built.

A typical cross-section would be a 'box' girder of constant or variable depth.

Transverse prestressing of the top slab is not uncommon.

The Choice of Spans and the Construction System

The use of cast in-situ construction or precast segments depends on the size of the bridge and on the span lengths.

The field of application for bridges built in cantilever is very wide and covers mainly spans from 50 to 200 m. At present a new trend consists in using precast segments for smaller spans, say of about 35 m, for elevated roads or flyovers, when scaffolding must be avoided and existing road facilities maintained beneath the structure.

The cantilever method of construction is virtually the only one to be considered today for spans over 70 m, a field

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* also see Chapter 18

in which it competes successfully with steel construction. For smaller spans it competes with other erection methods, in particular casting in-situ on falsework which is still economical if not too high.

The most economical span length when a free choice is possible depends on the cost of foundations and on the height of the piers. Experience shows that economical spans range from 70 to 110 m for average conditions of profile, piers and foundations. Superstructures of variable, depth are often used in this range, whereas constant depth is maintained for smaller spans.

For very large spans (over 110 m), since precast segments would be too heavy and, therefore, the equipment for handling them too expensive, the cast in-situ method of construction is generally used—unless the number of spans is large.

For conventional spans, the choice between casting in-situ and using precast segments depends essentially on the bridge surface area, due to the elevated cost of the precasting plant and placing facilities, as well as construction time. A figure of approximately 5000 m^2 can be given as a minimum for justifying precast construction. It must be noted that this cost is reduced when placing of segments can be carried out using standard available equipment (floating or truck canes) and when the weight of segments is small (shorter spans). Thus, this method has been used successfully for small spans of urban viaducts and flyovers.

The Desirable Prestressing System

For good detailing and easy construction, the prestressing system must present certain specific properties:

- Cables must be easily threaded into the ducts after completion of each pair of sections.
- Tendon force must not be too small, in order to avoid cable congestion at the top of the cross sections near the piers and not too large to avoid excessive stress concentrations near the anchorages.
- The void left in the duct around the cable must be large enough for good grouting.
- The most convenient final cable force depends on the size of the structure and should be chosen to respect the above criteria.

The frequently used cables are the 12 T 13 (12/0.5") strand cable and the 12 ϕ 8 mm wire cable (both Freyssinet) and similar sized cables of other prestressing systems like BBR, VSL, etc.

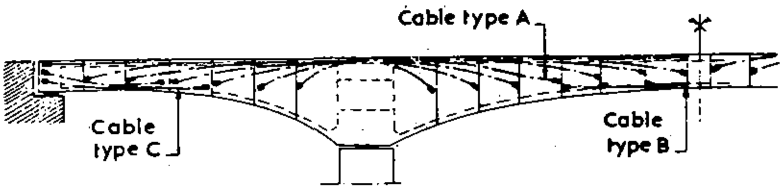
The Cost Advantage

The cost advantages of the cantilever construction technique have been demonstrated in successful tenders where this technique was in competition with other methods and other materials, in particular steel. When comparing with steel, maintenance costs have to be taken into account.

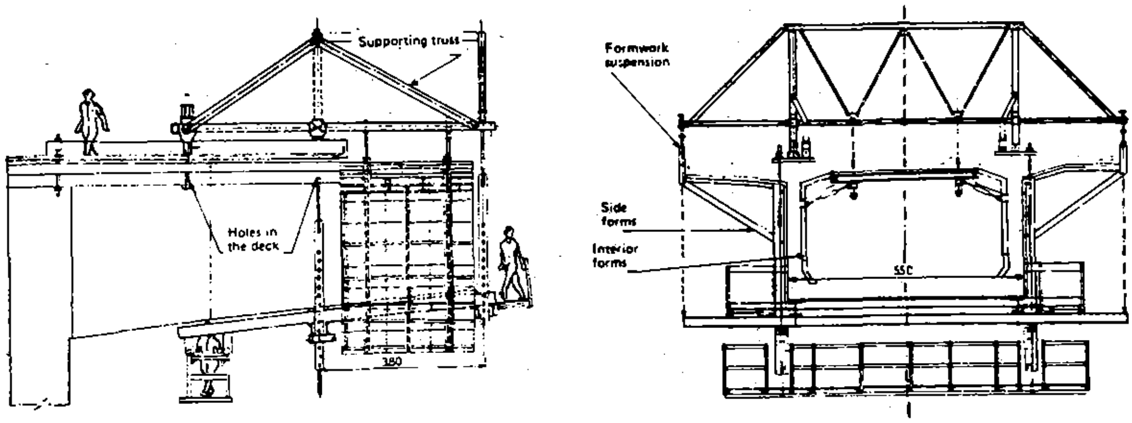
The rapid growth of this technique is the best proof of its value from the economic point of view.

Services Provided by Various Prestressing Agencies

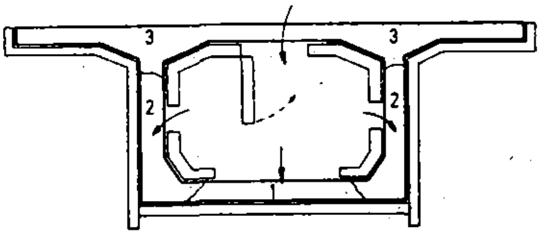
- Preliminary feasibility studies
- Technical assistance
- Supervision of design and construction
- Technical advice during design
- Computation using specially developed computer programs
- Supply of suitable prestressing material and equipment on a sale or hire basis
- Design of special equipment for cantilever construction: mobile falsework for cast-in-situ projects; precasting—cells, launching girders, etc ... for precast segmental structures
- Supply of the above equipment including its commissioning
- Technical assistance on site including complete subcontracting of prestressing operations



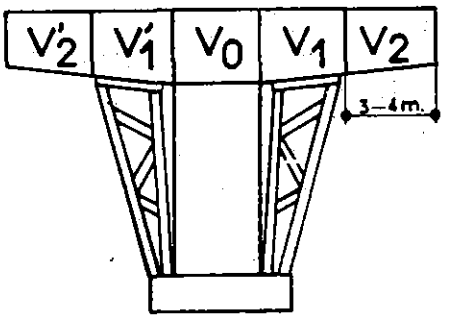
TYPICAL CABLE LAYOUT FOR CANTILEVER CONSTRUCTION



MOBILE FALSEWORK FOR CANTILEVER CONSTRUCTION



CONCRETING SEQUENCES FOR A TYPICAL SEGMENT



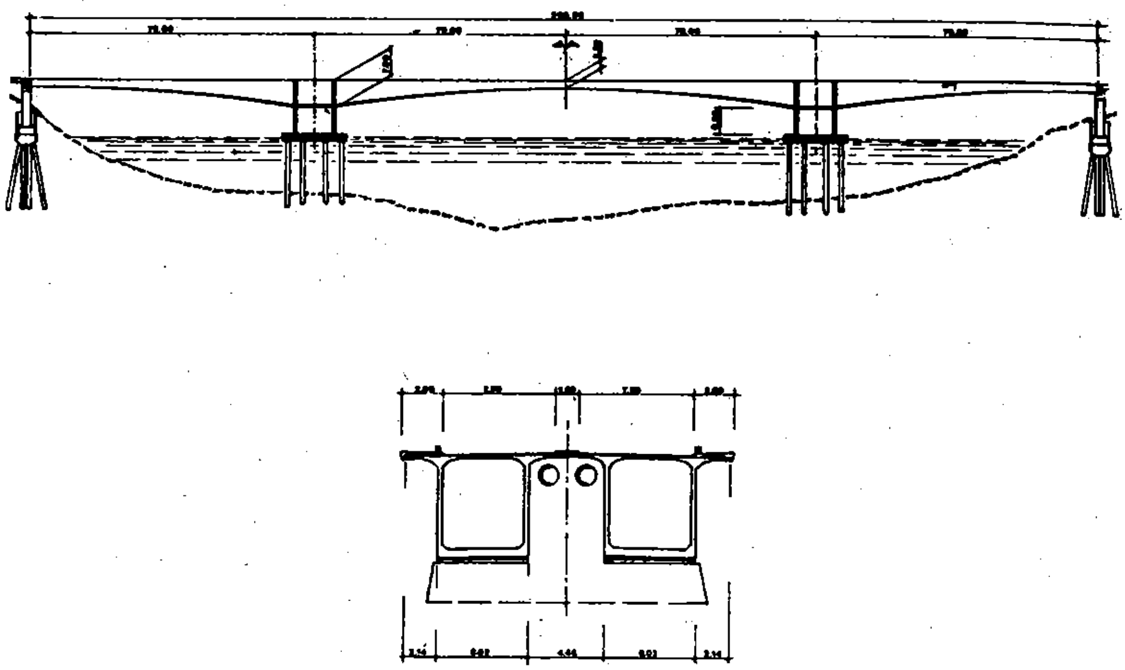
CASTING OF THE SEGMENTS NEAR A PIER

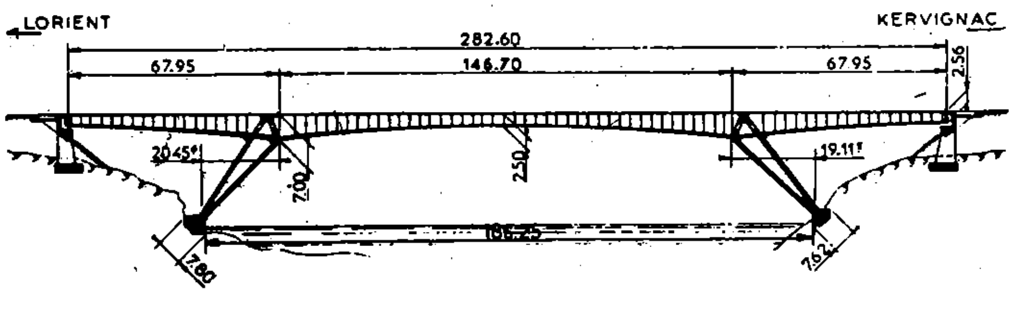
BRIDGE OVER THE SETUBAL LAGOON ARGENTINA 1970

140 m main span fixed on twin wall piers.

Design Europe Etudes

Contractor Christiani and Nielsen

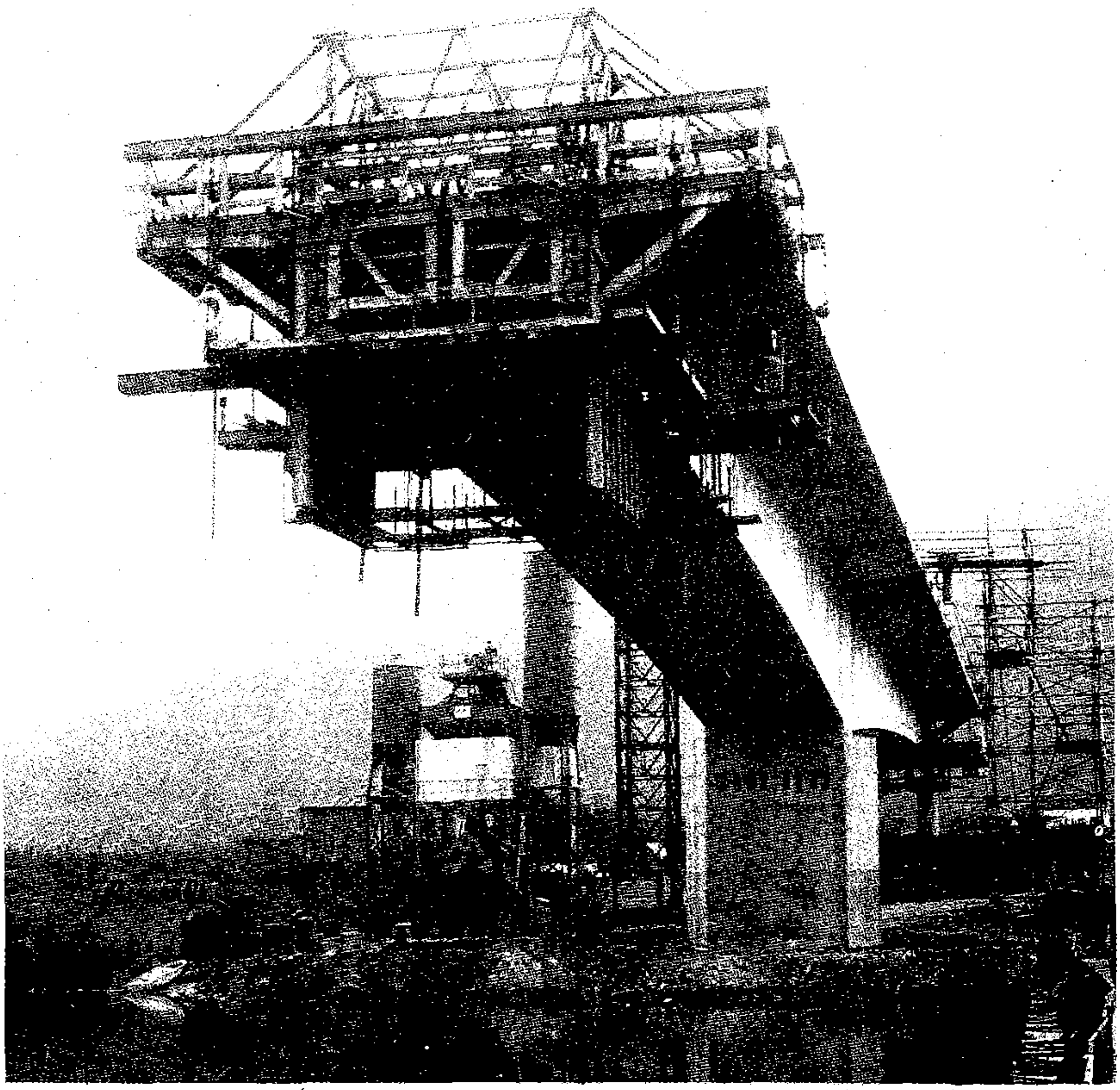




"BONHOMME" BRIDGE OVER THE BLAVET RIVER, MORBIHAN FRANCE 1974

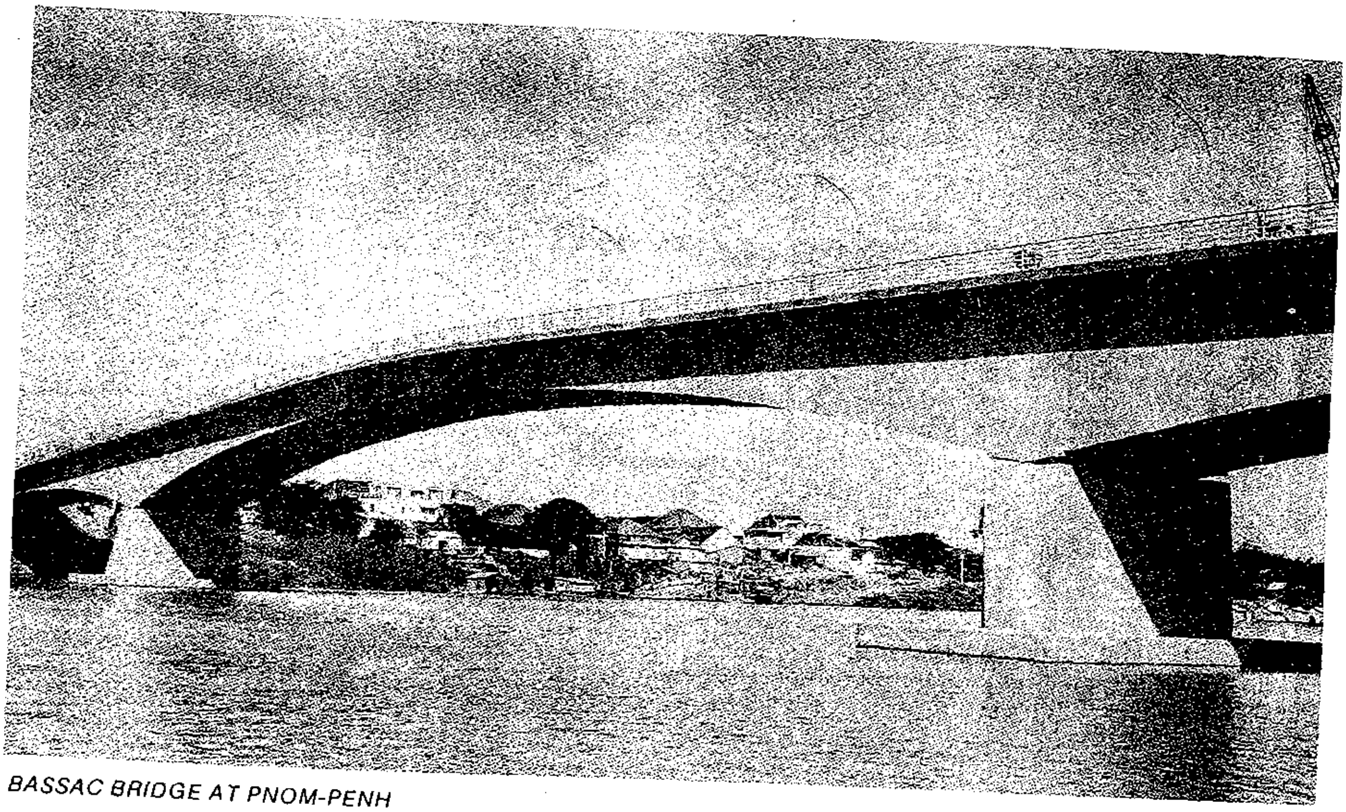
Racked legs of this 186m span portal frame bridge were built using steel tube falsework which served as temporary support during the cantilever construction. For adjusting the geometry of the bridge, flat jacks were placed under the legs and at mid span.

Design and Construction: Campenon Bernard



CAST IN-SITU CANTILEVER

BOUSSENS BRIDGE OVER THE GARONNE RIVER — *FRANCE 1969*
Spans : 49 m. 96 m. 49 m.
Design : Europe Etudes
Contractor : J.C. Stribick et Fils

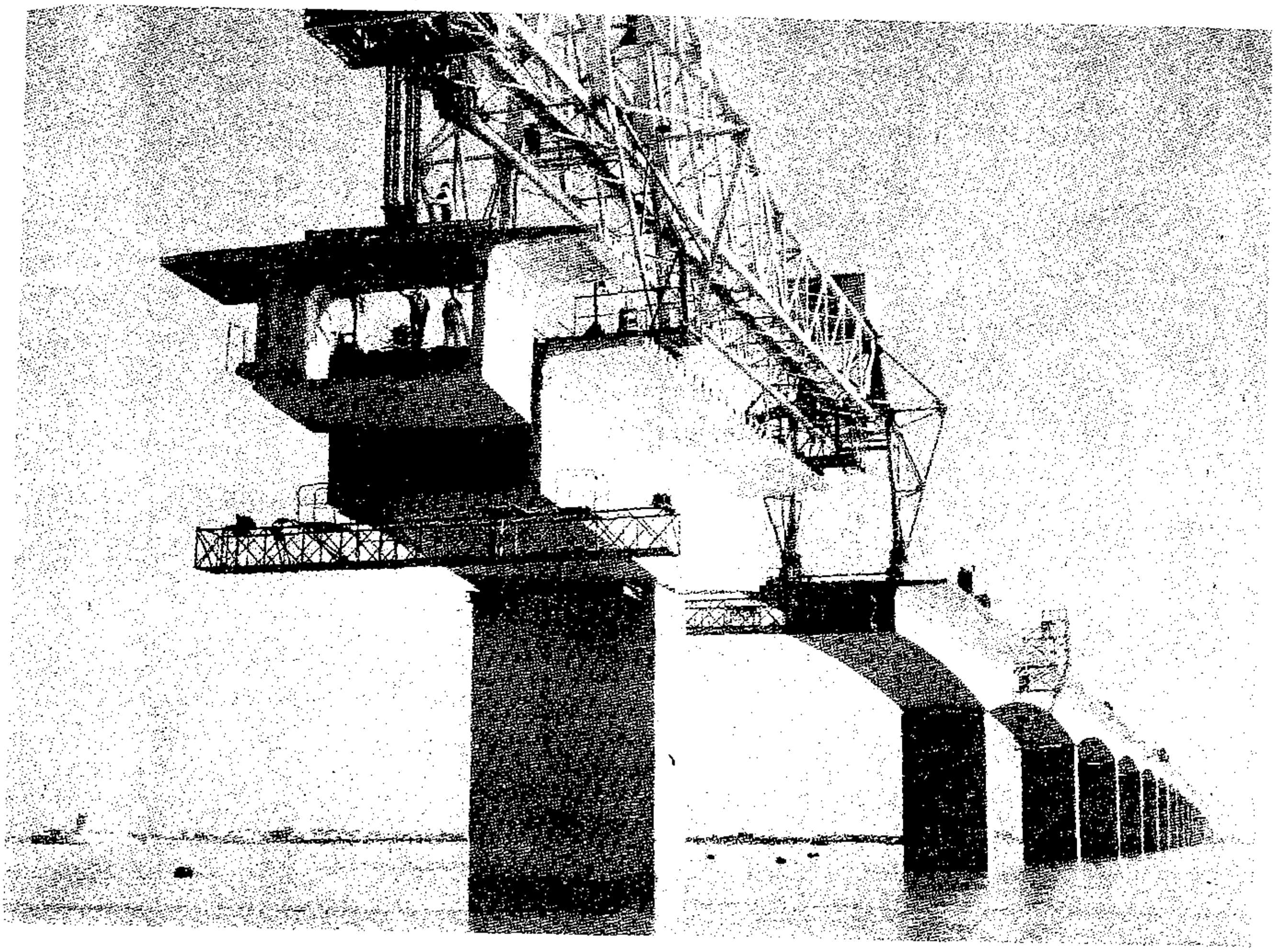


BASSAC BRIDGE AT PNOM-PENH

Spans 75 m-120 m-75 m. Continuous single box section with depth varying from 2.55 m to 6.90 m. *CAMBODIA 1966*
Design and Construction : Compagnie Francaise de Dragages et de Travaux Publics.

CHANGHUNG BRIDGE OVER THE SHIUKULUANCHI RIVER *TAIWAN 1968*

Single 120 m span hinged at the centre and fixed on counterweight type abutments.
Design : Japan Bridge and Structure Institute
Contractor : Kung Sing Eng. Corp.



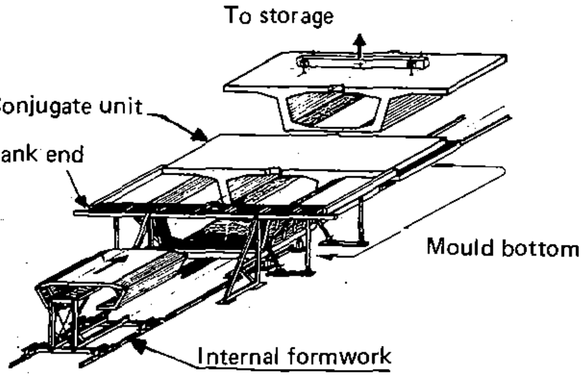
PRECAST SEGMENTAL CANTILEVER

BRIDGE BETWEEN THE OLERON ISLAND AND THE CONTINENT

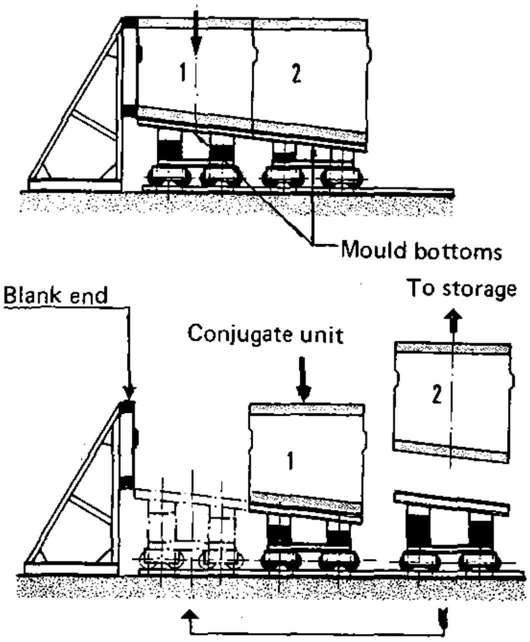
FRANCE 1966

Longest bridge in France (2.862 m) 26 main spans of 79 m.
Design and Construction : Campenon Bernard

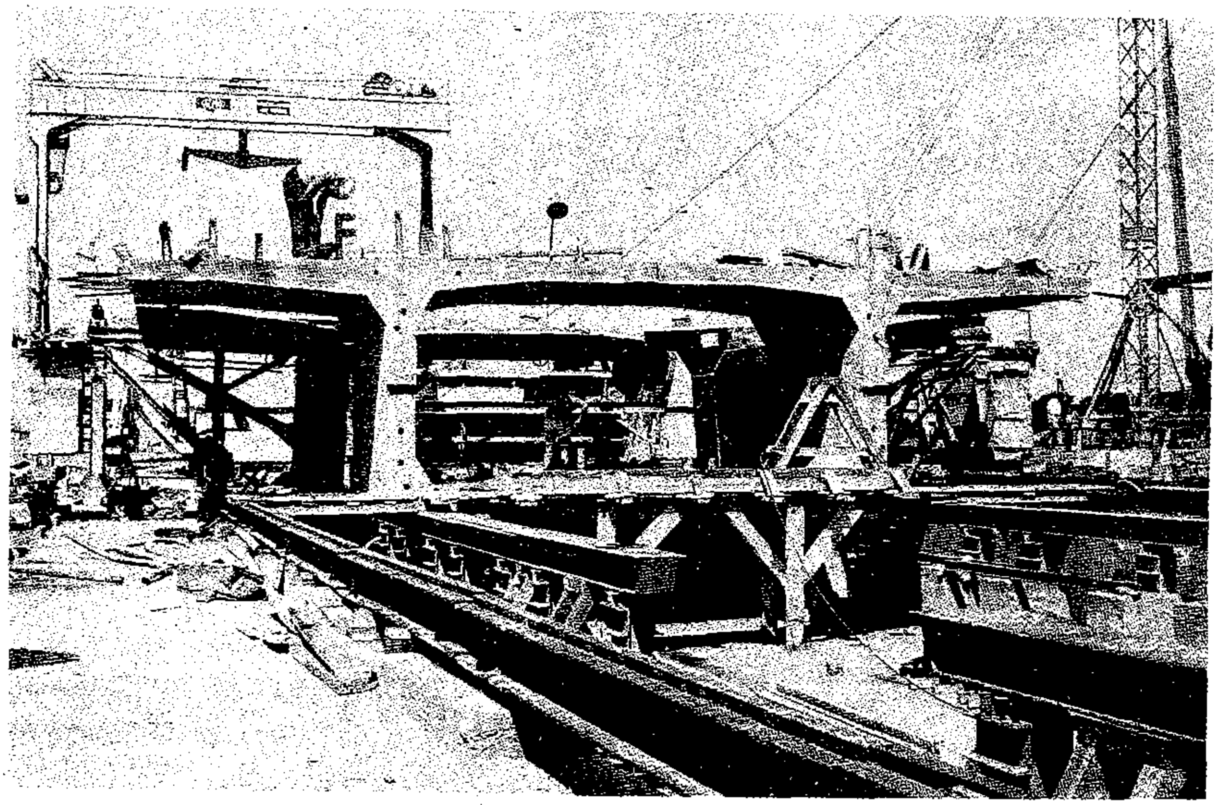
SCHEME OF A PRECASTING CELL

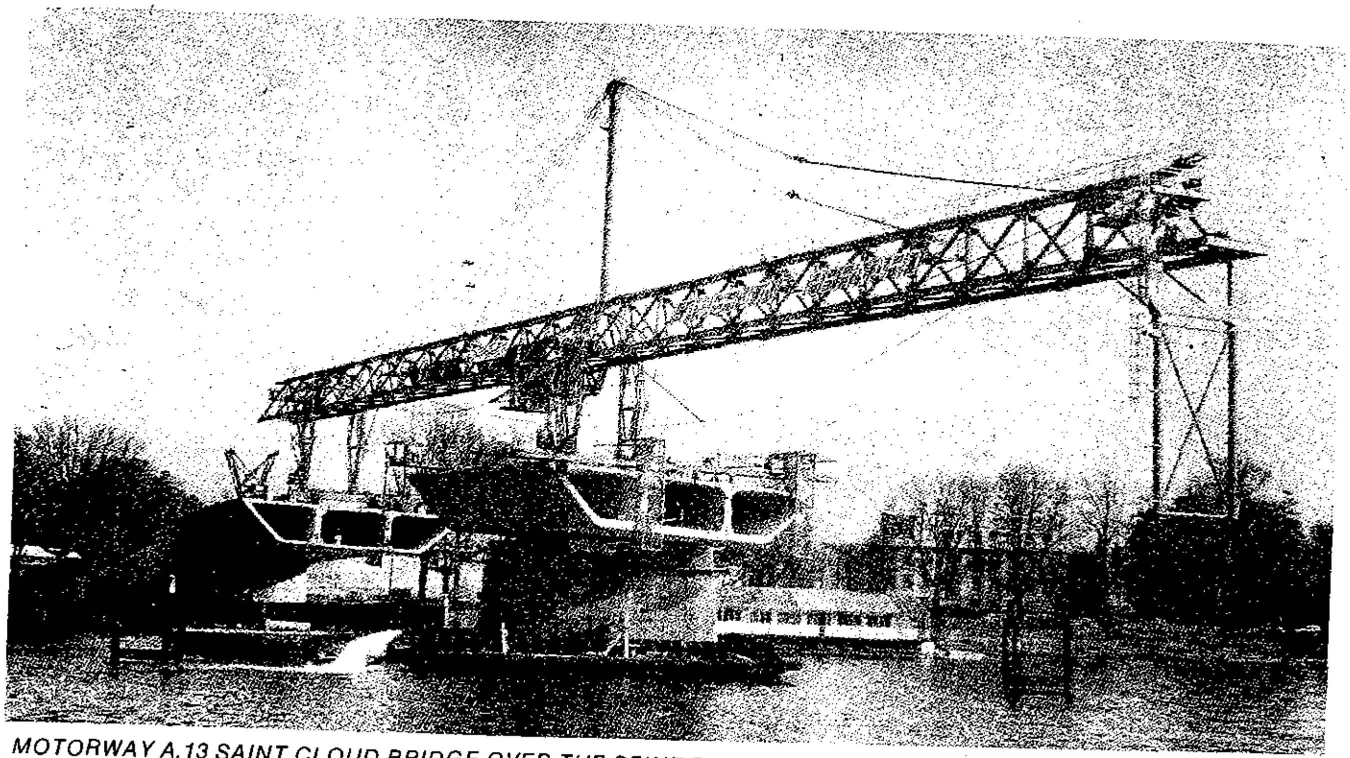


PERSPECTIVE VIEW



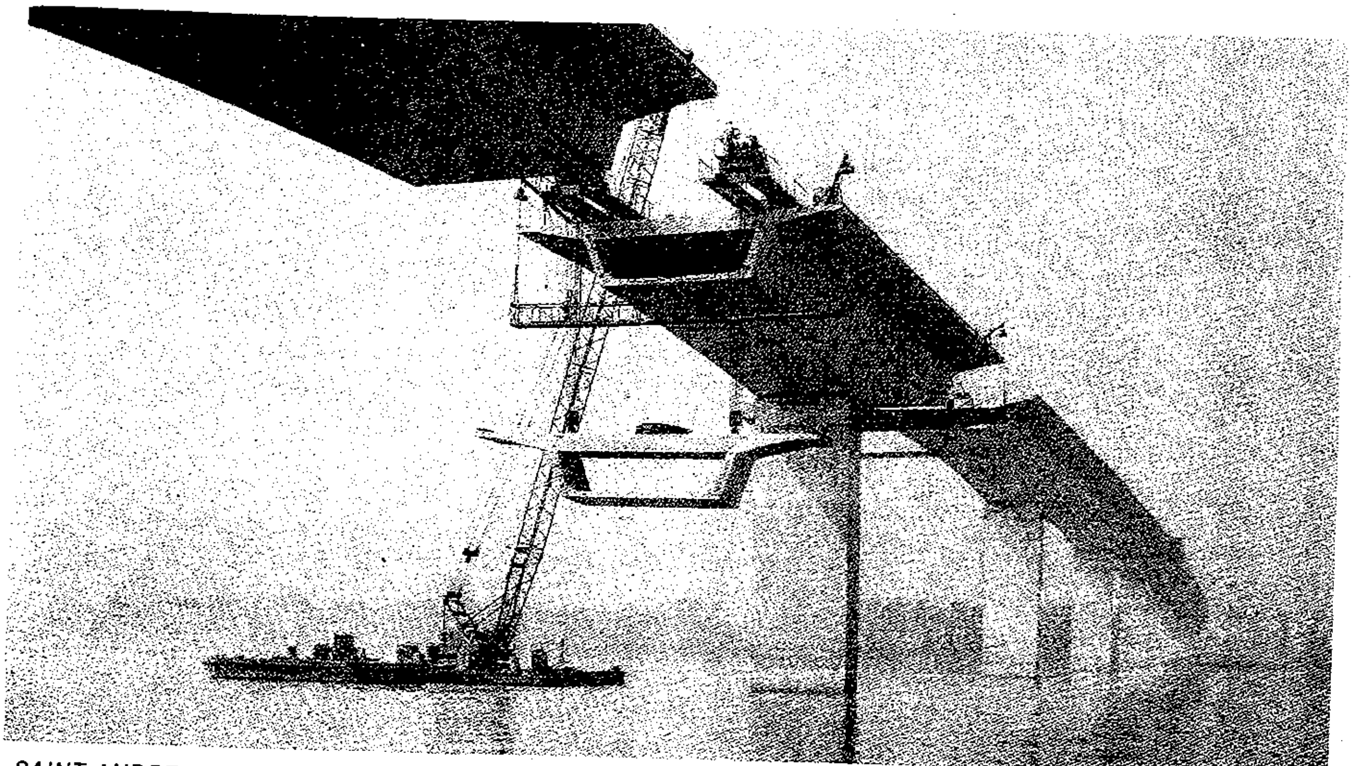
PRINCIPLE OF OPERATION





MOTORWAY A.13 SAINT CLOUD BRIDGE OVER THE SEINE RIVER — FRANCE 1973

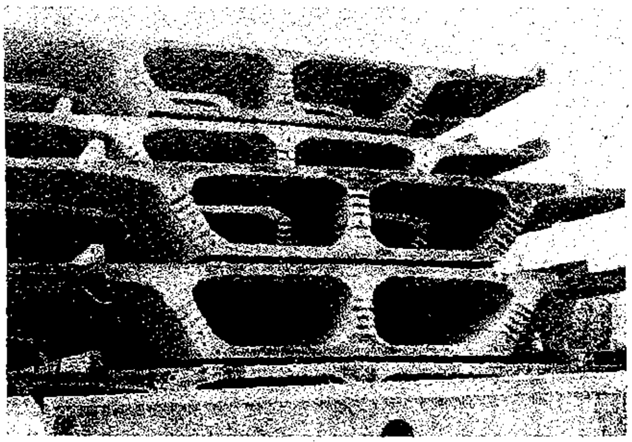
1,103 m long curved bridge and straight viaduct with spans of 46 m to 102 m. Exceptionally heavy precast segments (up to 130 tonnes), 20 m wide, placed by launching girder.
Design and Construction : Campenon Bernard



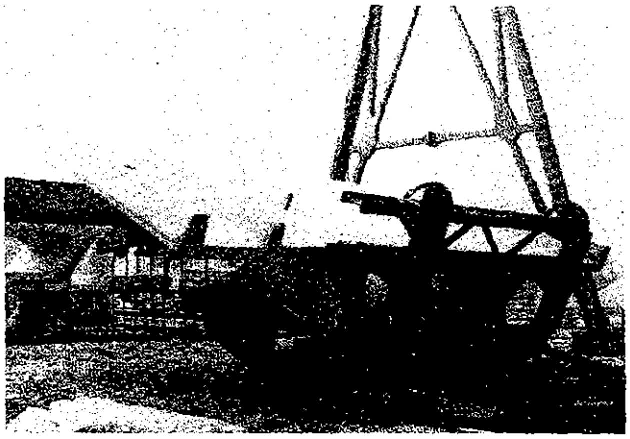
SAINT-ANDRE-DE-CUBZAC BRIDGE OVER THE DORDOGNE RIVER — FRANCE 1974

1,278 m long structure with 37 m to 95 m spans. Segments placed by beam and winch lifting devices.
Design and Construction : Campenon Bernard

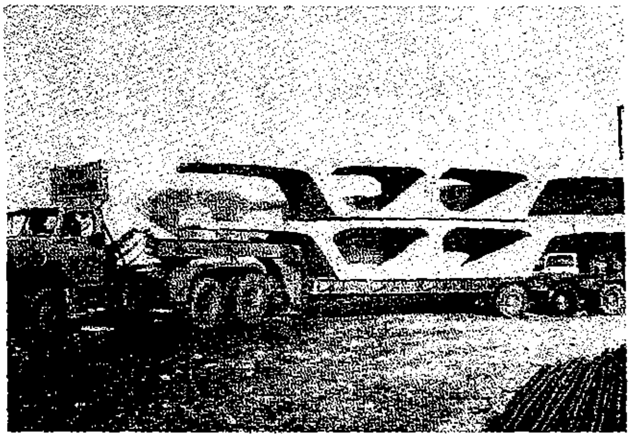
The construction of 146 overbridges for the Alpine Motorways project is an example of cantilever construction of small bridges using segments cast in a central plant, delivered on trucks, placed by crane and temporarily assembled by bars before threading the tendons. Total construction time is two weeks per bridge, foundations included !



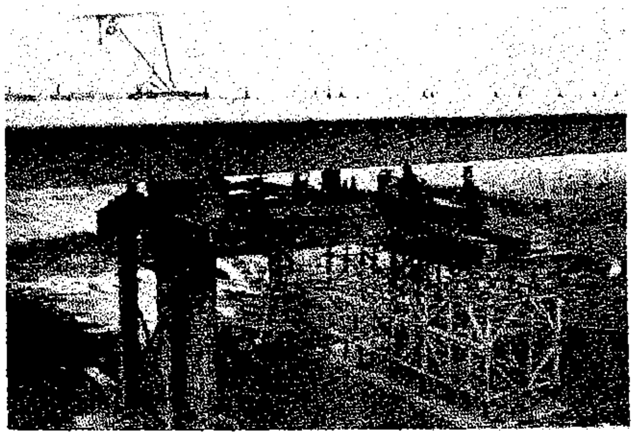
Storage of segments



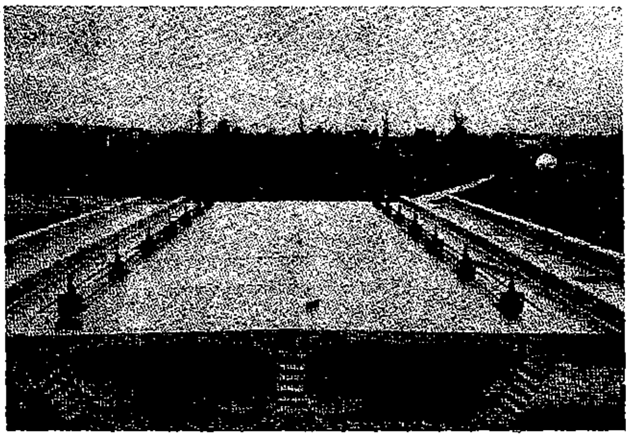
Turning a segment



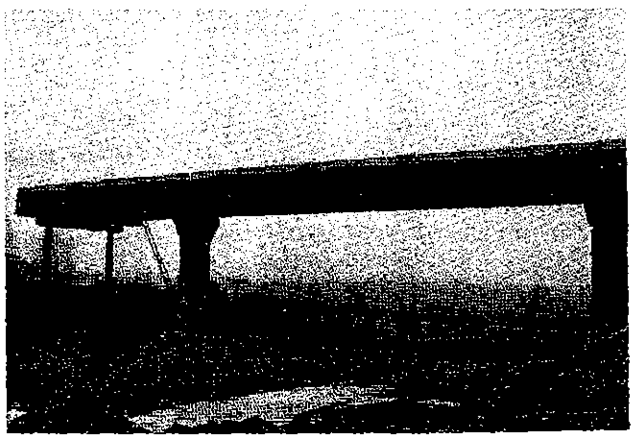
Transportation of segments



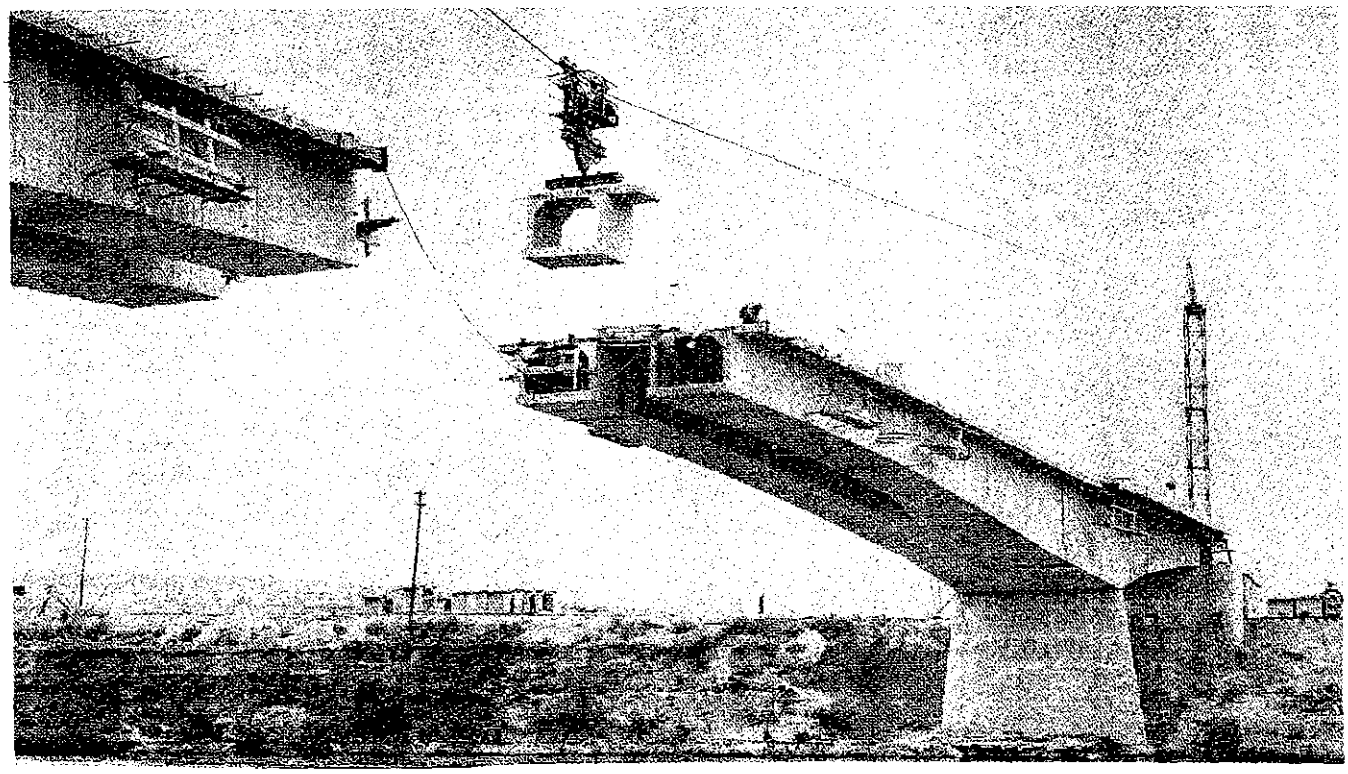
Cantilever support



Temporary cantilever prestress using bars

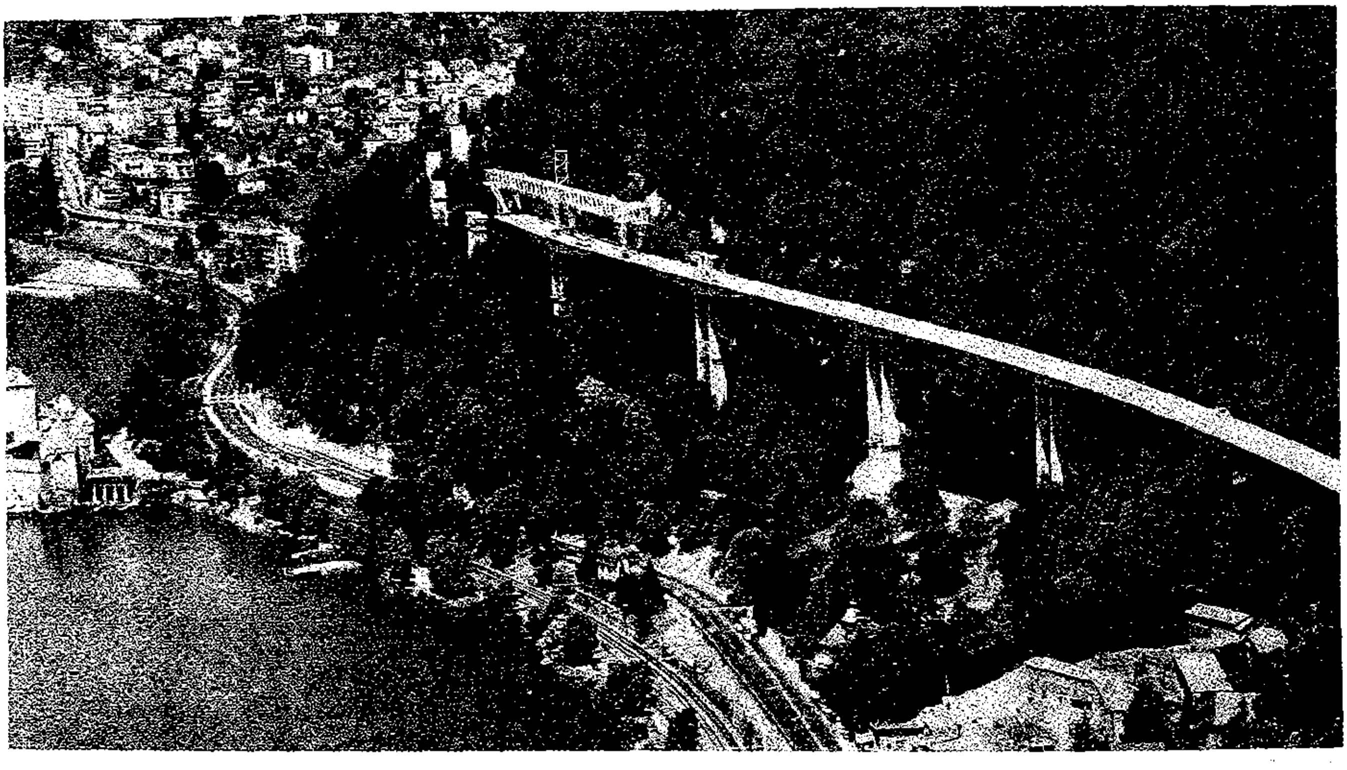


Completed overbridge



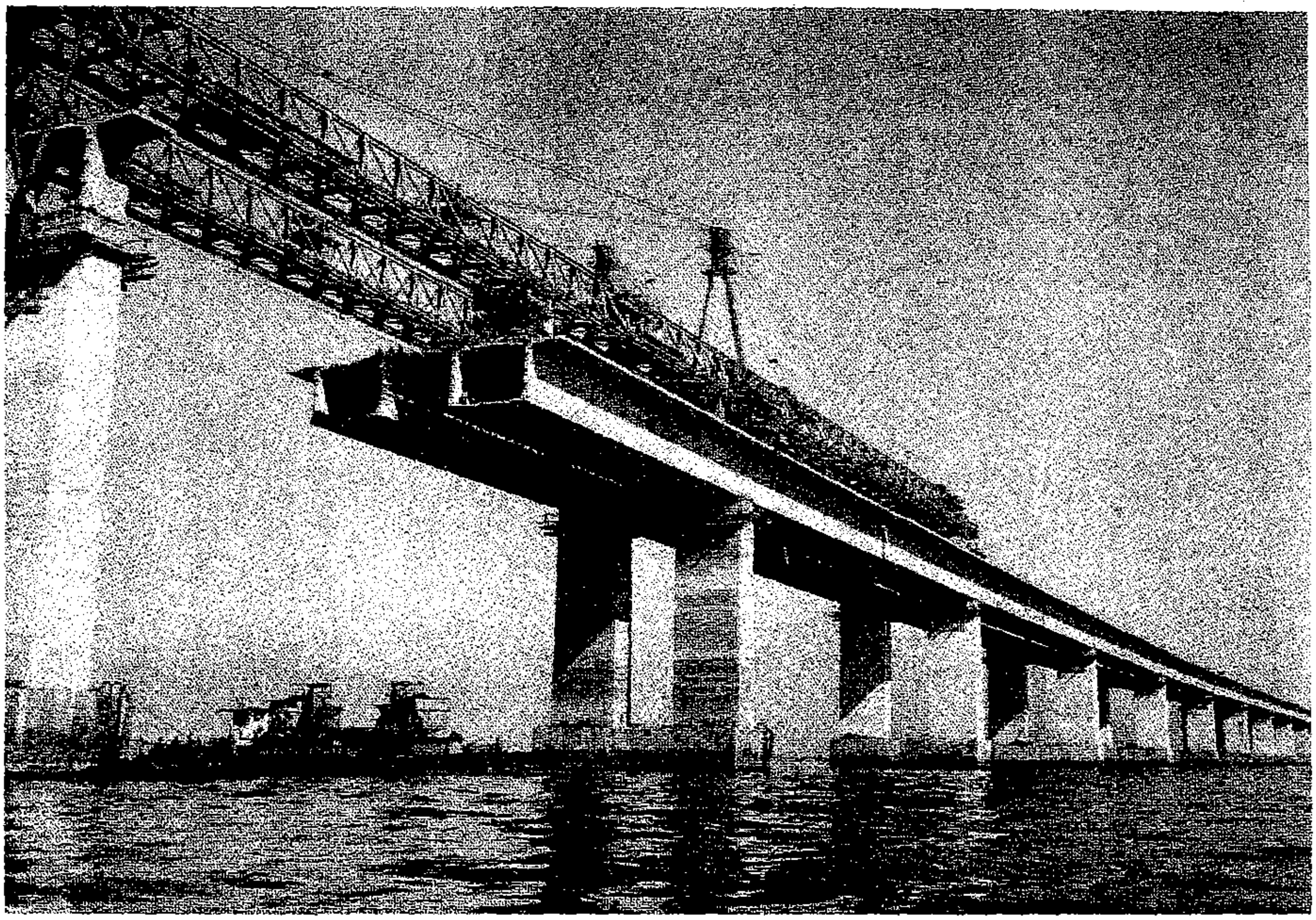
CASTEJON BRIDGE OVER THE RIO EBRO SPAIN 1968

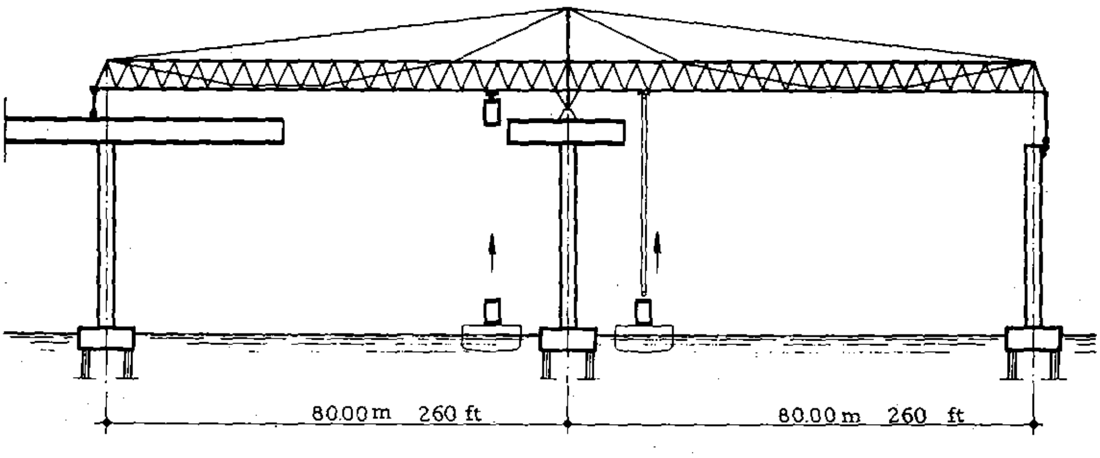
Spans 25 m-101 m-50 m-3 × 28 m. 8 to 10 ton segments placed by cableway.
Design : Pr. C.F. Casado
Contractor : Agroman S.A.



CHILLON VIADUCT ON THE GENEVA LAKE MOTORWAY SWITZERLAND 1969

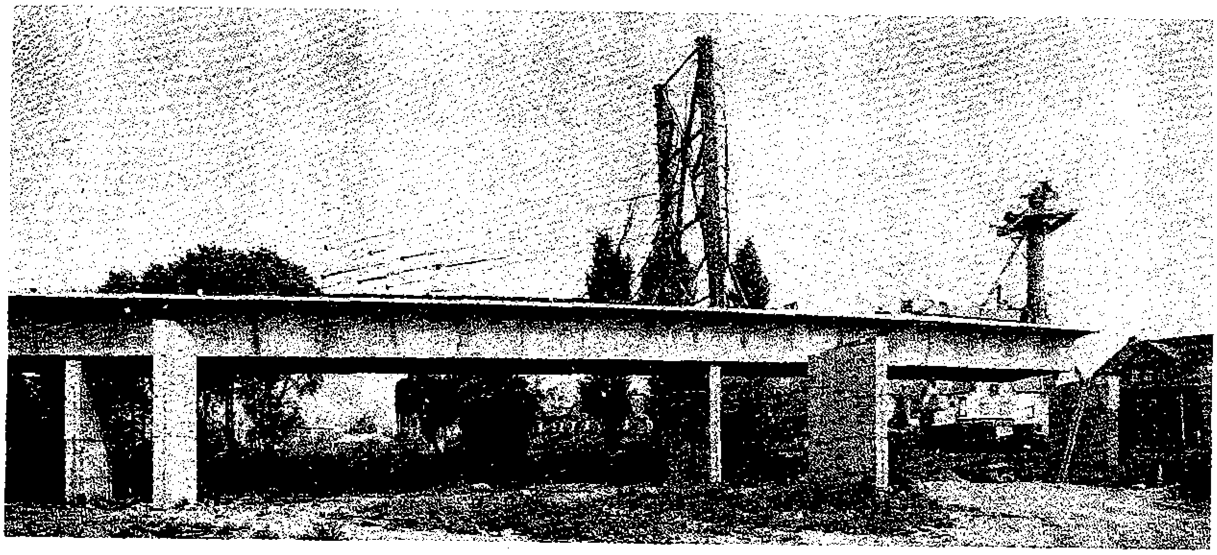
2.210 m long structure with two separate decks. 92 m, 98 m and 104 m spans with superelevated curves.
Design : J.C. Piguet
Contractor : P. Chapuisat and Dantan Frères S.A.

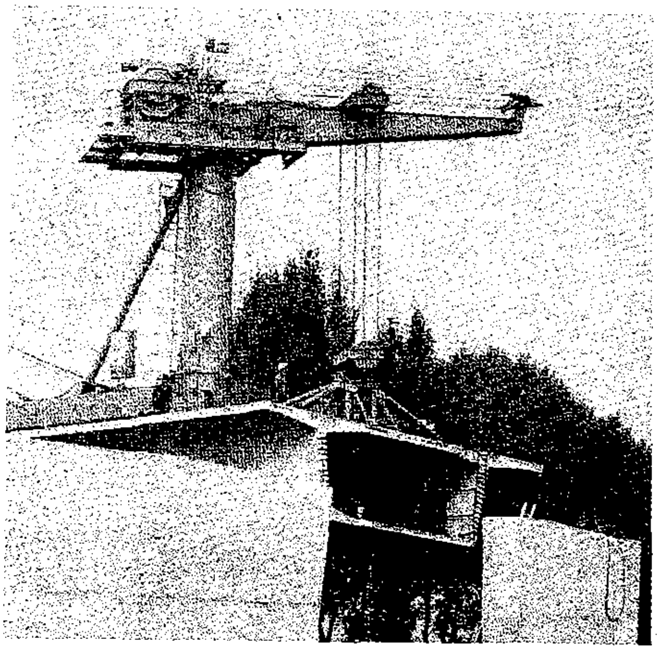




BRIDGE BETWEEN RIO AND NITEROI ACROSS THE GUANABARA BAY *BRAZIL 1973*

Longest cantilever built bridge in the world (7,884 m). 5 continuous sections in 80 m long spans. Segments placed by several launching girders.
Design : A.A. Naronha. Servicos de Engenharia S.A.
Contractor : Consorcio Constructor Guanabara Ltda
Equipment design and technical assistance : Campenon Bernard



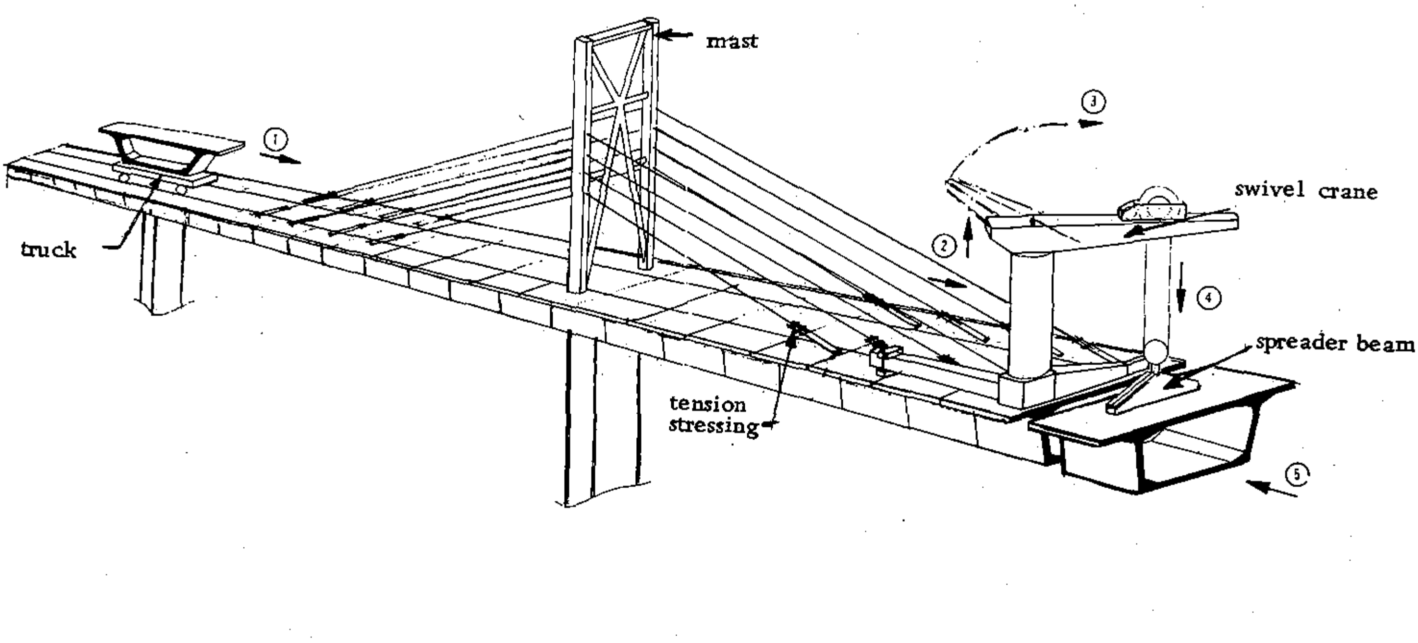


ROMBAS VIADUCT FRANCE 1974

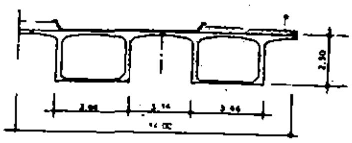
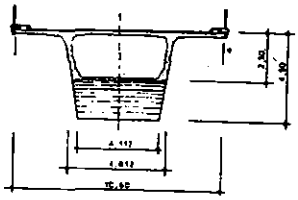
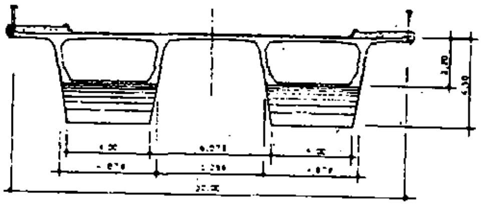
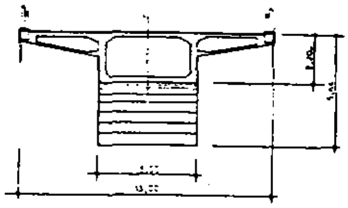
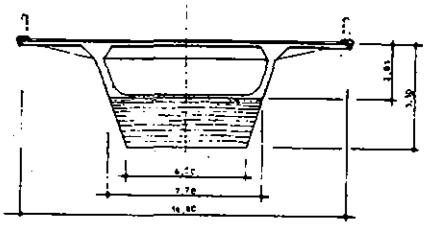
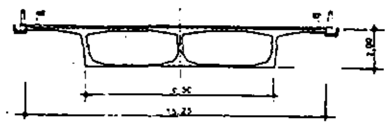
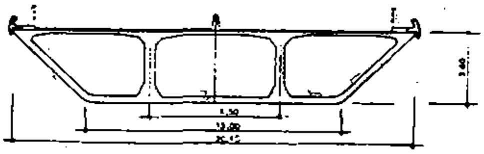
9 spans from 25 m to 45 m.

Design and Construction : Campenon Bernard

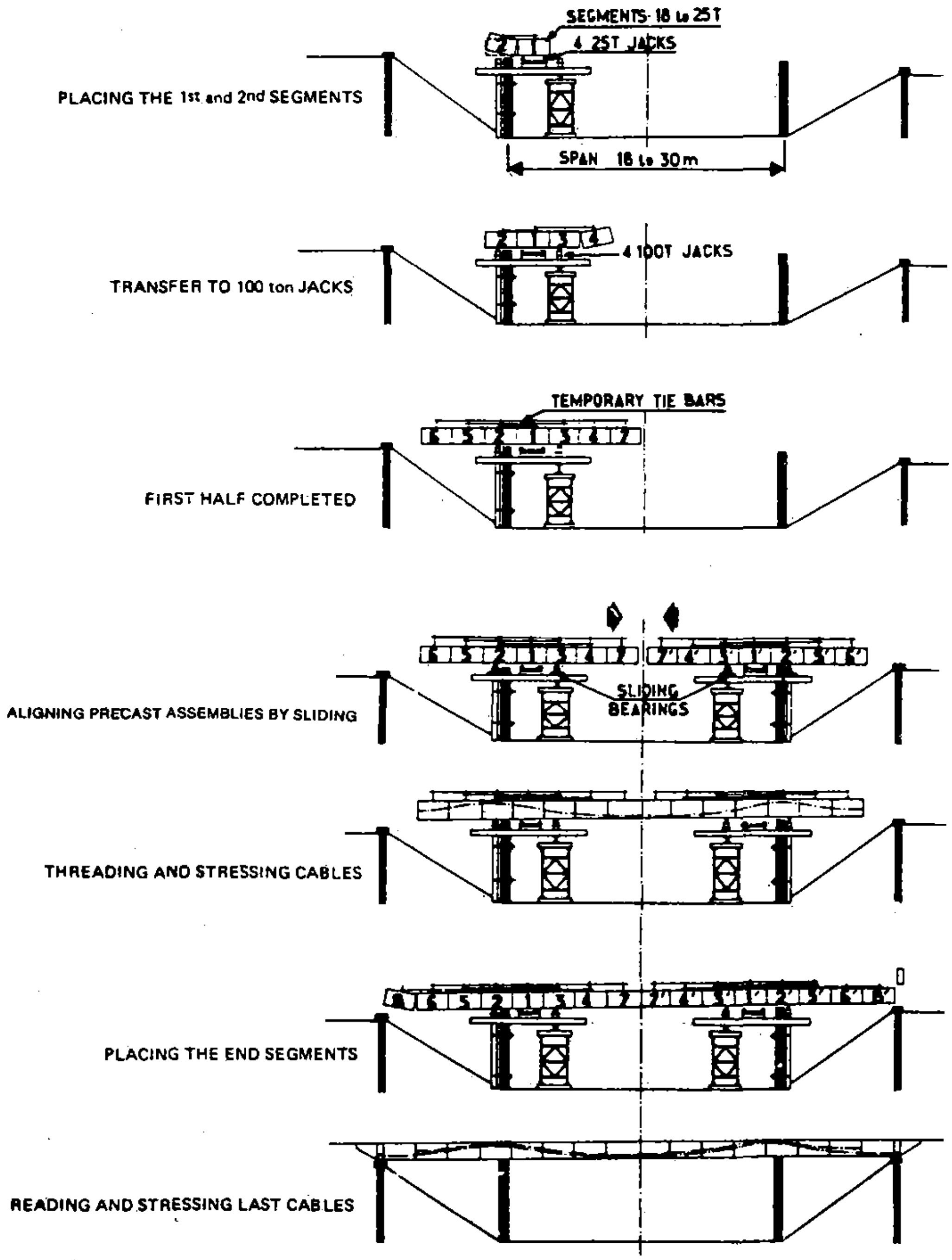
According to this new technique for bridges spanning up to 40-45 m, the cantilever construction proceeds directly from one pier to the next. The first part of each span is cantilevered and the second part is supported by a mast and stays system. A pair of stays is placed for each new segment. Prestressing cables are threaded during cantilevering and stressed from the inside of the box section. Segments are placed by a swivel crane which is moved progressively along the deck.



EVOLUTION OF SEGMENT CROSS-SECTION AND WEIGHTS

BRIDGE	CROSS-SECTION (Dimensions in meters)	SEGMENT LENGTH	MAXIMUM SEGMENT WEIGHT (Tonnes)
CHOISY-LE-ROI		2.50 m 8.20 ft	25
SEUDRE		3.30 m 10.80 ft	75
BLOIS		3.50 m 11.50 ft	75
CHILLON		3.20 m 10.50 ft	80
SAINT-ANDRE-DE-CUBZAC		3.40 m 11.20 ft	80
B3 SOUTH		2.50 m - 3.40 m 8.20 ft - 11.20 ft	50
SAINT-CLOUD		2.25 m 7.40 ft	130

ALPINE MOTORWAY OVERBRIDGES: ERECTION SCHEME ~



CHAPTER 37

Considerations in the Design of Prestressed Concrete Box-Girder Decks with Special Reference to Cantilever-Construction

The design of concrete box girders is a complex task requiring extensive technical skill and construction experience. This subject is described, below in eight specific subheads 37.1 to 37.8.

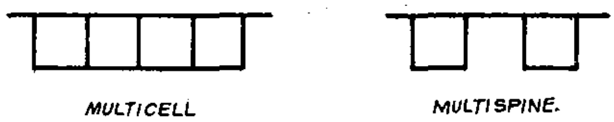


Fig. 37.1

37.1 INITIAL DESIGN

Why a Box Girder?

The popularity of the concrete box girder has risen steadily over the years to the point where it is now almost commonplace.

The reasons for this advance have been well documented by several authors in the past, and are listed briefly below:

(a) The box section is a highly efficient structural form which allows optimum use of materials on long-span bridges where reduction of dead weight is all-important.

The shape of the cross-section may be varied along the span to suit the loading envelope for the bridge, and the high lateral stability and torsional rigidity of the concrete box make it particularly suitable for curved bridges.

(b) In many cases it allows a finer pier design and thus a more attractive structure.

(c) Although its construction is more complex than other sections, recent advances in construction technology have overcome this disadvantage to the point where it is now competitive over a wide range of spans.

Types of Box Girder

Among the more important factors which must be considered are:

(a) *Forms of Cross-section*
  - (i) Haunched or constant section longitudinally
  - (ii) Multicell or multispine transversely (Fig. 37.1)
  - (iii) Position of diaphragms

(b) *Method of Erection*
  - (i) Cantilever, either symmetric [Figs. 37.2 (i)] or eccentric [Fig. 37.2 (ii)].
  - (ii) Span-by-span on temporary staging (Fig. 37.3).

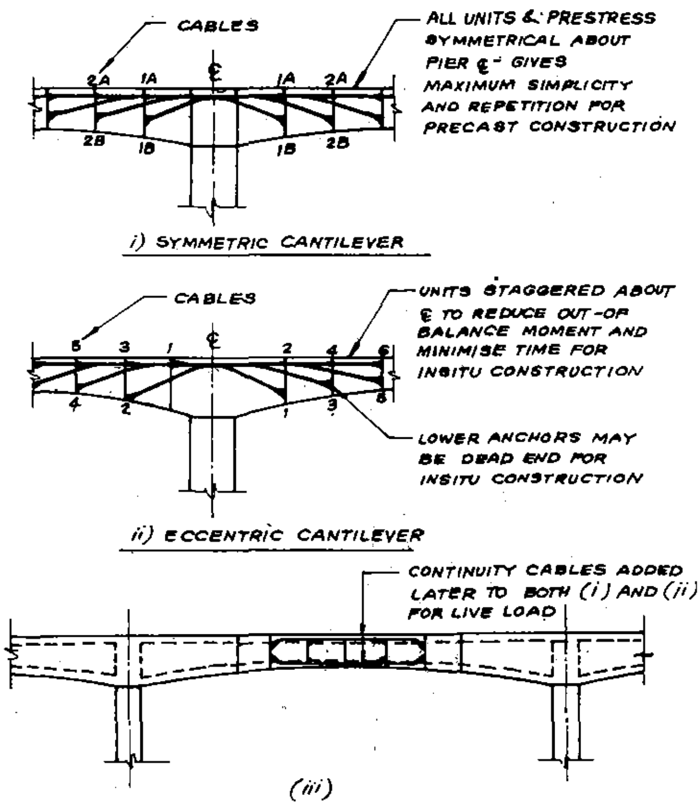


Fig. 37.2

(c) *Type of Construction*
  - (i) In-situ.

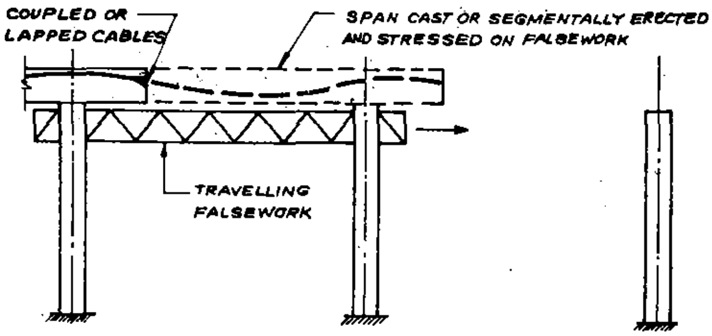


Fig. 37.3

(ii) Precast, including the method of jointing for this

(d) *Type of Prestress*

(i) Longitudinal [dependent on (a) above]

(ii) Transverse (for deck slab bending essentially)

(iii) Vertical (for web shear).

To assist the designer in this task the Cement and Concrete Association (London) has published a comprehensive survey of some major concrete box girders designed (1), with comparisons between them designed to indicate the most efficient ranges for the parameters involved.

Because of problems in detailing the thin sections used in these structures it is imperative that all initial sizes be reviewed in terms of anchorage location, duct trajectories, concrete placing, etc., before work on the final design is begun.

37.2 ANALYSIS

The accurate analysis of box girder bridges is a complex problem which has received considerable attention in recent years.

Dead Load Stresses

(a) *Straight Bridges*

For straight bridges, simple beam theory is generally sufficient to estimate dead load stresses. The dead load analysis must, of course, take full account of the various stages of construction.

Two secondary effects which should be evaluated for each structure are:

(i) *Shear lag*

In regions of high shear (i.e., at and near the supports) the 'stiffness' of the cross-section is effectively 'reduced' by the flexibility of the flanges in shear. For sections with large flanges the effect can be considerable. In prestressed structures, however, the effect may not be very important as a large percentage of the dead load is balanced by the prestress.

(ii) *Creep redistribution of moments* Wherever span-by-span construction is employed or wherever continuity between a number of spans is made some time after construction, some creep redistribution of the dead load moments will occur (Fig. 37.4)

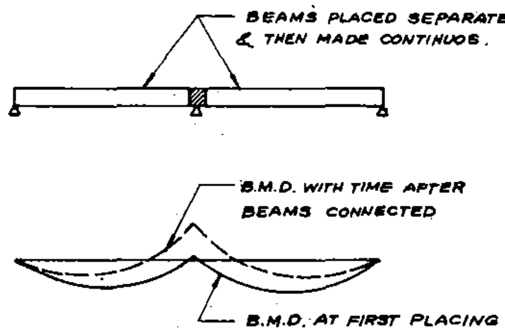


Fig. 37.4

(b) *Curved Bridges*

In the case of curved bridges the dead load moments are accompanied by torques for equilibrium.

For box structures of moderate curvature the method given in reference (2) and shown here in Fig. 37.5 is generally sufficient. The longitudinal bending moments are computed as for a straight beam and the expression for the incremental torque given in Fig. 37.5 is integrated along the beam from points of known zero torque to give the final torque at any section. (Also see separate chapter in this book, on this subject.)

Live Load Stresses

Methods of Analysis

The five methods generally pointed to (for the present purpose) are outlined below in principle.

(a) *Simple Beam Theory*

In its basic form, this theory assumes only simple bending theory and torsion of thin-walled members. Warping and distortion (Fig. 37.6) of the cross-section are ignored. Local bending moments in the deck slab are found from influence surfaces etc., and transverse moments, around the box by moment distribution etc., as shown in Fig. 37.7.

The structures may be analysed by a simple computer grillage analysis.

Representative (i.e., distributed) forces are first determined at each node of the grillage.

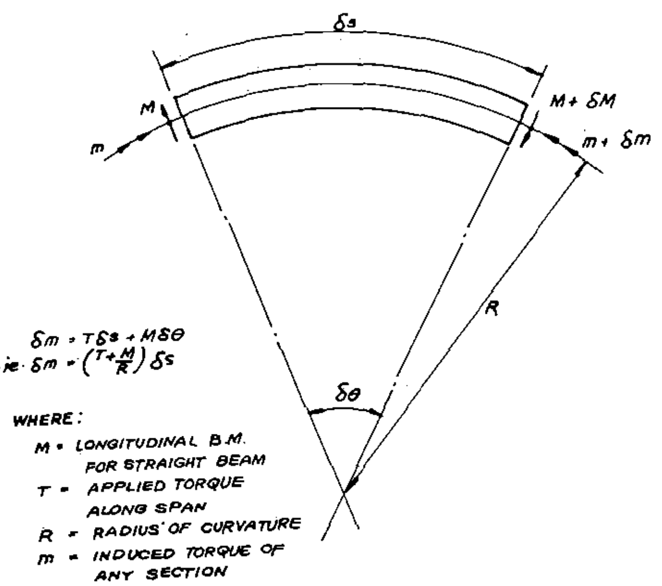


Fig. 37.5

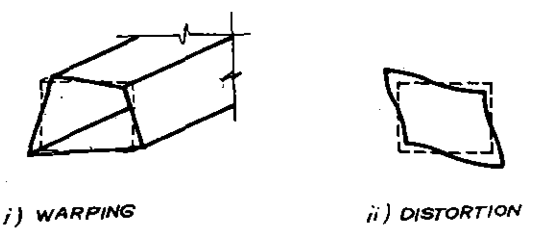


Fig. 37.6

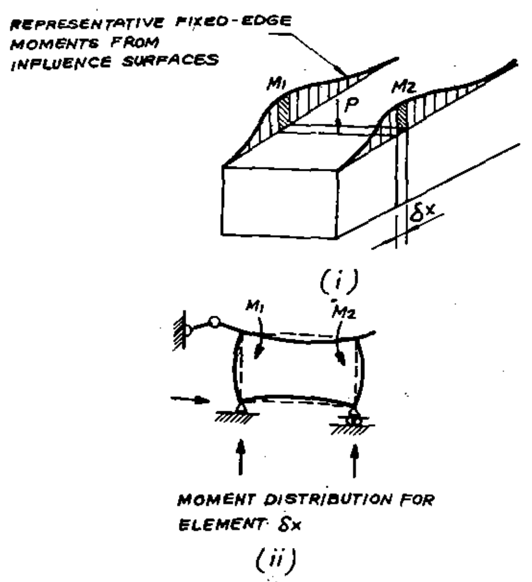


Fig. 37.7

These loads and the structure may then be separated into symmetric and antisymmetric components.

The structure may thus be analysed in each condition of symmetry and the final results obtained by superposition. Variation of section properties along the span (including arbitrary reductions in flange width for shearlag near the supports) can be handled in this way, and the method can be extended to curved structures also.

The accuracy of the method is acceptable only when the distortional 'stiffness' of the box is high and when the span length is such that warping stresses are small. If this is not the case, then one of the methods in (b) below must be used.

(b) *Approximate Allowances for Warping and Distortion*

A number of methods exist.

Some of these methods, e.g., that of Richmond (3) can be extended to grillage analyses of the form discussed in (a) above, and they can, therefore, be expected to give improved accuracy where the methods of (a) become inadequate.

(c) *Folded Plate Analysis (References 4 and 5)*

The basis of this method is shown in Fig. 37.8. The structure is represented as an assembly of plates joined rigidly along their longitudinal edges. All points on these edges have the four degrees of freedom shown in Fig. 37.8(i). Individual plate stiffness matrices for these displacements are derived from classical plate theory and the overall stiffness matrix for edge loading of the structure assembled from these. Wherever loads are applied away from the edges of a plate the fixed edge moments and forces must first be determined as in Fig. 37.8(ii). These are then applied to the structures as in Fig. 37.8(iii). In all cases, solution is by Fourier series and the final results are obtained by superposition of the results for each harmonic of loading.

The method is suitable for straight or curved, simply supported, structures of constant cross-section. Continuous structures can be handled by first treating the entire structure as simply supported and then eliminating displacements at the interior supports as a second stage in the analysis. Variation of overall depth or plate thickness in the longitudinal direction cannot be handled. However, the method is academically good, but is seldom used in practice.

(d) *Finite Strip (Reference 6)*

This method is essentially an approximate and more versatile version of the folded plate method. The loaded surfaces are subdivided into more plates, as shown in Fig. 37.9, so that variations in transverse thickness can be accommodated and the first stage in the folded plate analysis of (c) above can be eliminated by applying loads directly to the longitudinal edges between plates. The degrees of freedom chosen are the

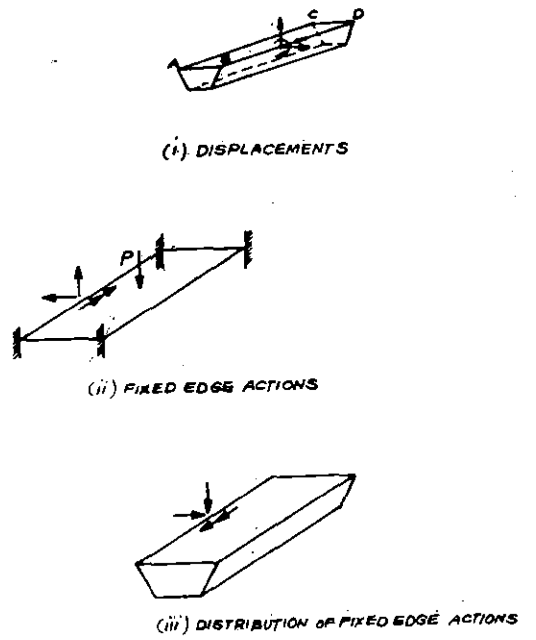


Fig. 37.8

same but in this case the variation of these displacements across a strip is approximated by polynomials. Solution is again by superposition of loading harmonics. The remarks made in (c) above concerning curved structures, continuous spans and variable depth structures apply equally to this method. Accuracy of the method is almost as good as for folded plate case (c). The method is seldom used in practice.

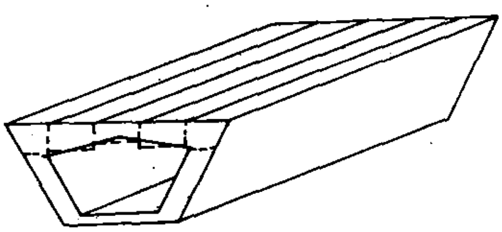


Fig. 37.9

(e) *Finite Element*

The basis of the method is shown in Fig. 37.10. The entire structure is divided into small elements and the stiffness of the structure is assembled from the membrane and plate-bending stiffness of each element.

The method is the only one that is truly general and powerful and can handle all types of structure. It has even been extended to the analysis of structures subject to cracking (7). Its accuracy depends on the nature and number of elements used. True compatibility of deformation

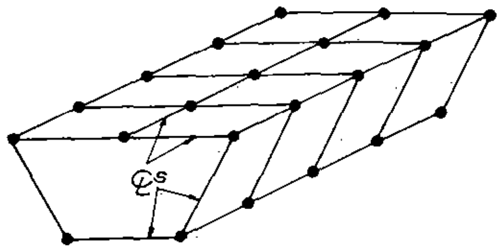


Fig. 37.10

is achieved only at the nodes, and the accuracy of the representation between nodes depends on the suitability of the assumed displacement functions for the strain field in question and upon the size of the grid element chosen. Considerable research has been directed recently towards finding the most suitable types of element for box girder work (8 and 9) and good agreement between experiment and theory can be obtained. The disadvantage of the method is the large computer storage required and the time consuming input-synthesis and output-analysis.

Comparison of Methods

In general it can be said that for single or multiple cell boxes, straight-in-plan, method (a) is good enough, while for curved-in-plan decks that are 'wide' and or 'shallow', method (e) is preferable.

As for "method versatility":

(i) Of all the methods discussed only the finite element method is truly versatile and accurate.

(ii) The approximate methods (a) and (b) are also versatile but their accuracy is inadequate for more complex structures.

(iii) The folded-plate and finite strip methods have limitations which make them unattractive to the design engineer.

Thus the approximate methods (a) and (b) are recommended for straightforward structures and the finite element method for more complex structures. (Also see Ch. 19 in this book on the associated subject of the distribution of the applied load transversely among the longitudinals of the deck.)

The approximate methods have been used successfully for the design of many major structures in the past and there is no reason why they should not continue to be used where appropriate in the future.

The finite element method is likely to be adopted ultimately as the general method for all this work. Simplified commercial programs for box girder structures are already available and it will not be long before even more sophisticated programs are in use.

Effect of Diaphragms

The influence of the number and position of diaphragms is discussed in reference (10). It is shown that the spacing of diaphragms must be selected carefully if the warping stresses in the box are not to be exaggerated. For the 'average' straight concrete box girder, it has been shown by several authors (e.g., References 11 and 12) that there is no need for transverse diaphragms except at the supports, and from the construction viewpoint also this is certainly the best solution.

Differential Temperature

This is an important effect which has been recognised only comparatively recently. It is now well documented, and a clear method of analysis is given in Ch. 30 of this book.

Prestress

(a) *Straight Bridges*

In general, simple beam theory is adequate for the prestress analysis of straight bridges. The more complex 'Finite Element' method may be used if the structural complexity warrants it.

(b) *Curved Bridges*

The comments in (a) above apply here with the additional point that care must be taken in the evaluation of torques induced by post-tensioning. For the usual case where the prestress is distributed symmetrically about the transverse centreline (Fig. 37.11) with cables moving essentially in the vertical plane, it can be shown (13) that it is only the secondary or parasitic prestress moments which induce torques in the structure. These torques may be evaluated from the secondary moments by the method referred to in Fig. 37.5 (as also the separate chapter on this subject in this book).

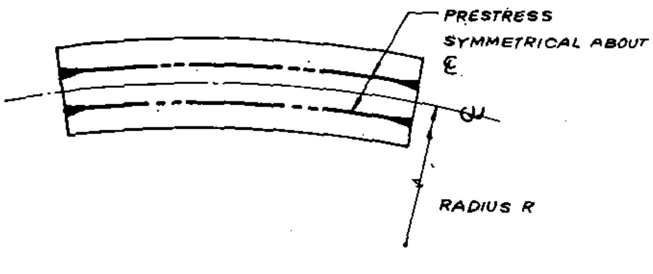


Fig. 37.11

Distortional Resonance and Instability

The minimum dimensions of concrete boxes are usually dictated by detailing requirements and the rigidity of the cross-section is thus usually more than adequate for the above effects.

Reference is made in (10) however to the case of a large thin-walled box where the natural frequency of distortional vibration was found to be in the normal range of traffic vibration.

Creep Redistribution within the Cross-section

In thin-walled and closed-cell structures like box girders there can be considerable differences in the rate of curing of the various components of the cross-section and consequent differences in the rate of shrinkage and creep.

This in turn, can give rise to variations in the distribution of the stresses within the cross-section, to the extent where the final distribution may be different from that prevailing initially.

37.3 FINAL DESIGN

Determination of the final concrete dimensions and reinforcement for concrete box girder bridges is a lengthy process requiring constant review of the initial member sizes and stresses.

The essential steps in this process may be summarised broadly as follows:

(A) Working Stress Design

i.e. (i) Determination of bending and shear reinforcement to control stresses and limiting the deflections at working loads.

(ii) A check on such miscellaneous effects as the influence of construction variables on design assumptions, and the fatigue limit of deck reinforcement.

(B) Overload Design (Check)

i.e., overload capacity

(C) Ultimate Load (Moment) Check

for both bending and shear.

(A) Working Stress Design

(a) Longitudinal Bending

Post-tensioning must be provided to control longitudinal stress. The determination of suitable prestress layouts for this purpose is a relatively straightforward problem and is not discussed here. The following points, more peculiar to box girder design are, however, important:

(i) The thin sections involved in box girder design can make detailing of the prestressing difficult. This should be given careful attention early in the design.

(ii) High stresses in design can give rise to large creep strains, particularly in in-situ construction, and consequently to large creep deflections if the prestress design is significantly different from a load balance.

The main cables may also be used for relief of vertical shear near the supports.

(b) Transverse Bending

Design of the (transverse) reinforcement for the bending within deck cross-section is also straightforward and is not discussed further here. Where, as is generally the case, the only resistance to deformation of the cross-section between support diaphragms is the bending strength of the cross-section itself, care should be taken to give the corners of the box adequate strength in transverse bending, e.g., as shown in Fig. 37.12.

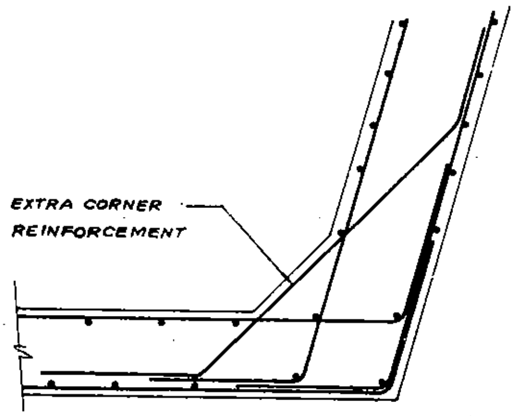


Fig. 37.12

Extra transverse reinforcement should also be provided in the flanges for the following effects (where appropriate):

(i) Splitting forces adjacent to any flange-anchored longitudinal prestressing cables,
(ii) Transverse forces due to inclination of the webs (from frame action of transverse section).

(c) Shear (Transverse Shear at Various Sections Longitudinally)

The behaviour of 'large' prestressed concrete beams in shear is still not clearly understood. Recommendations for the shear design of these members are correspondingly lacking. While the ultimate strength in shear may be estimated with reasonable confidence, it is not clear yet just how the various resisting mechanisms contribute to this strength throughout the loading range.

Current opinion (14, 15) on working stress design for shear is that it can be omitted if a number of requirements for ultimate load design are met. (See separate chapter in this book on the philosophy of design against shear.)

Principal tension is the webs may be controlled in any of the following ways:

(i) Increased web thickness (though not advisable beyond about 45 cm, from thermal stresses stand point).
(ii) Vertical prestressing in the webs [Fig. 37.13(i)]. This method is effective but it can give unacceptable increases in the principal compressions if used excessively. (Construction-wise cumbersome.)
(iii) Short cap cables for shear relief [Fig. 37.13(ii)]. (Note that this is in addition to the normal shear relief afforded by draped cables of the type shown dotted.)
(iv) Inclination of the longitudinal cables in cantilever construction [Fig. 37.13(iii)].
(v) Haunching of the member so that vertical component of the longitudinal flange force counteracts the shear [see Fig. 37.13(iv) and Reference 16]. For box girders with substantial bottom flanges, the above reduction may be approximated by the vertical component of the force in the bottom flange.

Extra reinforcement should also be included in the webs locally wherever longitudinal cables are anchored in them.

(d) Torsion

Torsion is usually of concern only in curved bridges.

Care should be taken with thin-flanged sections that the principal tensions in the flanges due to combined shear and torsion are limited [see (c) above]. The cross-sectional hoop reinforcement should be detailed to provide 'continuity' for torsional shear flow. The transverse reinforcement should be closely spaced to confine the longitudinal corner bars which should themselves be substantial. (See separate chapter on this subject in this book.)

(e) Miscellaneous Checks

(i) Construction Variables

In some cases assumptions may be made in design which are not fully realised in practice, e.g., the formwork for a continuous span may be assumed rigid when in fact it can never be fully so. An effort should always be made to establish the upper and lower bounds for these effects, in the same way as it is normal to allow for a small (5%) variation in the final prestressing force achieved.

(ii) Fatigue of Deck Reinforcement

In the thin flexible deck slabs of box girders there is a risk that continual overstressing of the transverse deck reinforcement can lead to premature fatigue failures. This is especially true in partially prestressed designs where the deck steel may be cycling anywhere between 300 kg/cm^2 compression and 1700 kg/cm^2 tension. It should be remembered that the design of the deck slab is

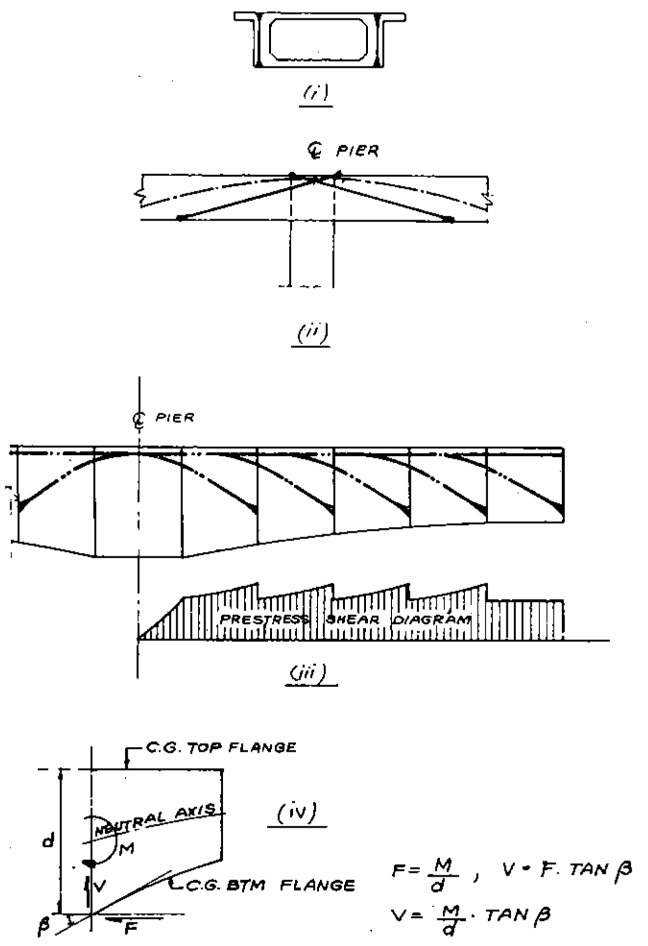


Fig. 37.13

frequently governed more by individual wheel loading than by overall truck loading, and that the fatigue life of the deck slab should, therefore, be appreciably higher than that of the girder.

The fatigue life of the deck slab reinforcement should thus always be checked by limiting the stress variation range in the steel and a reduced upper limit of its stress.

(B) Overload

The overload capacity check of prestressed concrete bridges involves a method that limits crack widths under overload according to the nature of the structure and the environment. The crack widths may be determined either by analysis from first principles of the cracked and partially prestressed section, or by the use of hypothetical allowable tensions which have been found to give comparable results for most common girder sections.

Extra non-prestressed reinforcement may thus be added to bring a structure up to a desired overload grading.

The longitudinal reinforcement should be closely spaced in the cross-section so as to minimise flexural crack widths. If the reinforcement is concentrated mainly in the webs, the crack spacing in the flanges will be governed mainly by the rate of shear transfer from the webs and will, therefore, be comparatively large and the cracks will be wide. If, however, the reinforcement (including where possible the bonded prestressed reinforcement) is distributed evenly over the cross-section, then the crack spacing will be governed mainly by the ultimate bond stress along the reinforcement in the normal fashion and the cracks will be correspondingly smaller in width and closer together. All post-tensioning ducts should be rigid, corrugated and well grouted to ensure maximum efficiency in this respect.

(C) Ultimate Load

(a) Bending

The ultimate strength of the box girder in flexure can be adjusted in the normal fashion by the use of non-prestressed reinforcement which is discussed in a separate chapter in this book.

Care should be taken in assessing the amount of plastic redistribution available from the girder, as discussed in the said separate chapter.

(b) Shear

See details in a separate chapter in this book.

37.4 DETAILING

The importance of detailing has been stressed constantly throughout this book. The following are some of the more critical factors which must be resolved as early as possible in the design stage.

Cable Layout

The prestressing layout adopted should be as simple as possible and should be directed towards the following objective:

(i) Optimum concrete flow access to webs
(ii) Minimum interference of cables with reinforcing steel
(iii) Maximum repetition and standardisation
(iv) Optimum speed of construction

Thus the arrangement of Fig. 37.14(i) for cantilever construction, for example, is preferable to that of Fig. 37.14(ii).

Duct Curvature Problems

Care must be taken wherever ducts change direction that there is sufficient concrete against the inside curve of

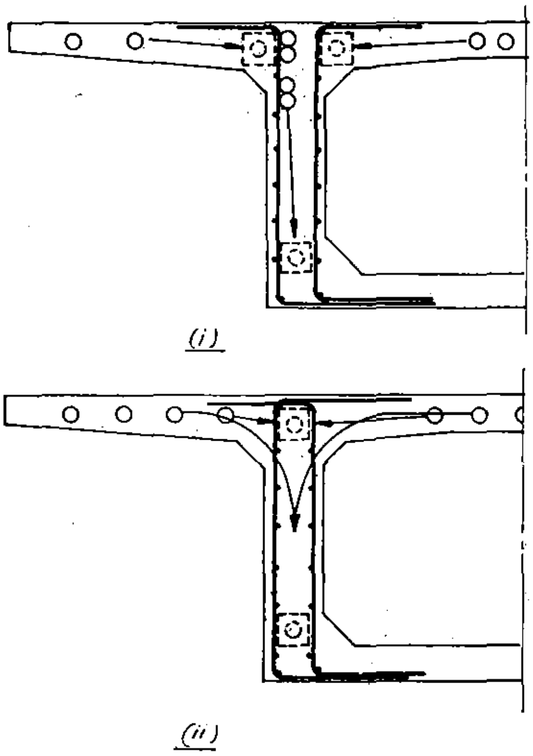


Fig. 37.14

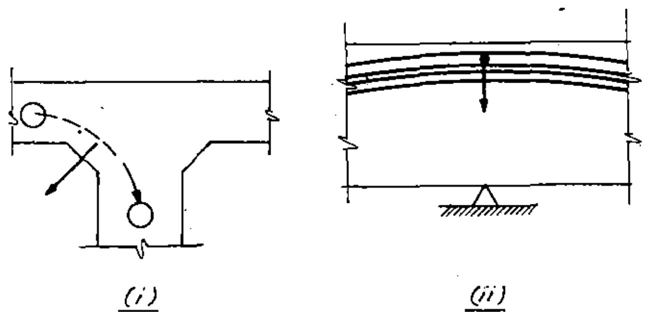


Fig. 37.15

the duct to resist the radial pressures from the tendon. Figure 37.15 shows two examples of this.

Where there is curvature over supports [Fig. 37.15(ii)], the ducts should be separated vertically by at least one and preferably two duct diameters.

Where the ducts run straight for some distance the opinion of the codes is divided but it seems acceptable to bundle the ducts at least in pairs either horizontally or vertically.

Care must also be taken during segmental construction that each new segment of duct is correctly aligned with the previous segment. [See reference (17) for example, where a special coupler had to be designed to keep friction losses down to the design limits.]

Construction Joints

Deliberate construction joints are generally of concern only in precast designs. In this case the designer may choose from the following:

(i) Normal (30–45 cm) in-situ concrete joints with reinforcement continuous through the joint.
(ii) Fine concrete (5–10 cm) in-situ joints with no reinforcement
(iii) Zero-tolerance epoxy joints with shear keys (Fig. 37.16)

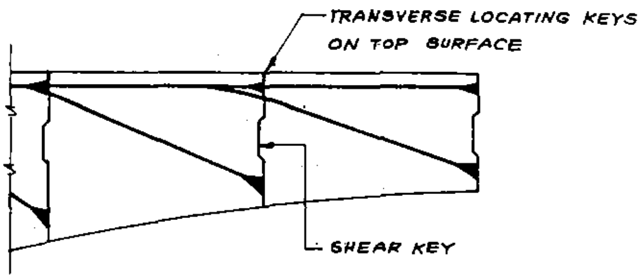


Fig. 37.16

The delays in construction inherent in (i) and (ii) must be weighed against the stringent precasting requirements for (iii).

Wherever possible it is generally recommended that the cross-section should be poured in one operation, namely:

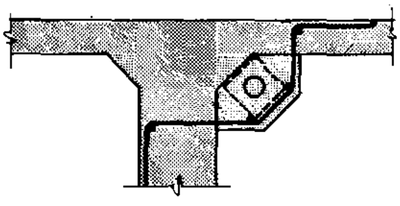
(i) Pour bottom slab, [re-trowel the wet concrete in-slab... see (iv) ahead]
(ii) Pour webs, ensuring good compaction of the concrete in the critical fillet regions at the base of the webs. (Access holes for vibrators should be provided in the formwork above the fillets to facilitate this.)
(iii) Pour top slab.
(iv) Lightly revibrate the top slab as required in the vicinity of the webs to minimise the effect of concrete settlement (plastic settlement cracks). Also re-trowel the wet slab-concrete against plastic shrinkage cracks (as needed, depending on if surface evaporation is considerable).

Particular care must be taken to ensure good compaction around anchorage locations.

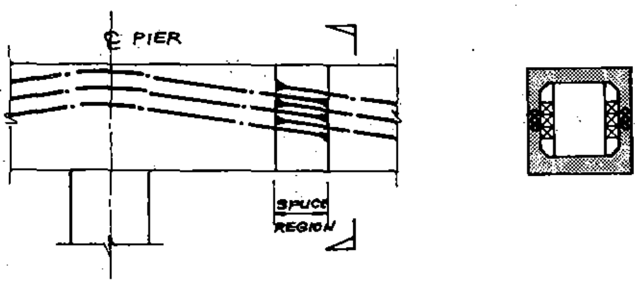
If construction joints in the cross-section cannot be avoided they should be placed away from anchorage locations at points of low shear and should be well 'prepared' before further concreting to ensure a degree of roughness at least equivalent to a fractured concrete surface.

Anchor Blocks (End Blocks, Anchorage Zones)

In any form of in-situ construction where travelling formwork is used, there is usually a conflict between design and construction requirements for the internal anchor blocks construction, whether it be for the continuity cables in cantilever construction [Fig. 37.17(i)] or the main cables in span-by-span construction [Fig. 37.17(ii)], poses a severe problem for the contractor.



(i)



(ii)

Fig. 37.17

37.5 CONSTRUCTION

The paramount requirements for box girder construction are the strictest supervision and quality control. Supervision should be directed by experienced engineers acquainted with aspects of the design, and all developments during construction should be examined closely.

The following items should be given special attention:

Formwork and Falsework

Formwork and falsework for concrete box girder bridges must be of the highest standard.

The entire falsework must be adequately rigid to ensure sound construction throughout and the formwork for the cross-section must be rigid and almost watertight. Access for inspection and concrete compaction is particularly important.

Specialist services for falsework design are now available and the use of these should be encouraged wherever possible.

Concrete

(a) Mix Design

The concrete for the superstructure must be durable and of the desired high strength, and satisfy a number of other conflicting criteria such as good workability and low creep and shrinkage characteristics.

Concrete mix designs and placing procedures must, therefore, be reviewed carefully beforehand, and wherever possible some form of trial run should be arranged to test these procedures.

(b) Curing

Careful curing is essential if shrinkage and thermal cracking of the cross-section is to be minimised.

The internal formwork should be removed as soon as possible to prevent undue 'propping' of the cross-section, and differential thermal expansion between formwork and concrete should also be limited. Steam curing procedures may need special review in this respect.

Erection

(a) Epoxy Jointing (Precast Only)

Where epoxy jointing is used in precast work the preparation of the mating concrete surfaces must be closely supervised.

Some preparation of the mating surfaces (e.g., by light sand-blasting) is generally required.

(b) General

All movements of the superstructure during erection should be constantly reviewed to ensure that they fall within anticipated limits and that the final state of stress throughout the structure will be acceptable.

The overall alignment of the superstructure must also be checked in cantilever construction to ensure successful span closure. The design should incorporate adequate facilities for the control of this alignment.

Grouting

Careful supervision of all grouting operations is essential if the desired durability and ultimate load capacity are to be achieved.

All joints between ducts should be carefully sealed and all cables in any one zone of the cross-section should preferably be grouted simultaneously to minimise the possibility of grout leakage between ducts.

Grouting should be performed only by skilled operators and facilities for emergency flushing with fresh water should be provided wherever possible.

37.6 DEFLECTION OF CANTILEVER BRIDGES AND CAMBER DESIGN

Each cantilever arm consists of several segments, fabricated, installed, and loaded at different points in time. It is important, therefore, to predict accurately the deflection curves of the various cantilevers so as to provide adequate camber either in the fabrication plant for precast segmental construction or for adequate adjustment of the form-travellers for cast-in-situ construction.

While the structure is statically determinate, the cantilever arm deflections are due to:

(a) The concrete girder weight
(b) The weight of the travellers or the segment-placing equipment
(c) The cantilever prestress

If the structure is finally a continuous type, then after continuity between individual cantilevers is achieved, the structure becomes statically indeterminate and continues to undergo additional deflections for the following reasons:

(d) Continuity prestress
(e) Removal of travellers or segment-placing equipment (reverse loading, but on continuous structure)
(f) Removal of 'provisional supports' and release of 'deck to pier connections' if any (again a reverse loading)
(g) Placing of superimposed loads

Even if the structure is non-continuous, items (e), (f) and (g) still cause deflection.

Subsequent long-term deflections due to concrete-creep and prestress-losses will also take place. Compensation for the deflections resulting in the following three states must be provided-for by adequate camber or adjustment:

1. Cantilever arms (during construction)
2. Short term in-service deck } Statically determinate or
3. Long term in service deck } indeterminate

The concrete modulus of elasticity varies both with the 'age at the time of first loading' and with the 'duration of the load'. Deflections of states 2 and 3* above are easily accommodated by changing the theoretical longitudinal profile by the corresponding reverse amount in each section to offset the anticipated future deflections. A more delicate problem is to further accurately predict, allow for, and adequately follow the deflections of the individual cantilever arms during construction (at each stage there is a new cantilever). For this it is necessary to analyse each construction stage and to determine the deflection curve of each successive cantilever arm (as construction proceeds segment by segment). A simple case with a five-segment cantilever is shown in Fig. 37.18. The broken line represents the envelope of the various deflection curves or the 'space-trajectory', followed by the cantilever-tip at each construction stage (i.e., at the end of each segment addition). For more practical details see 37.7 ahead.

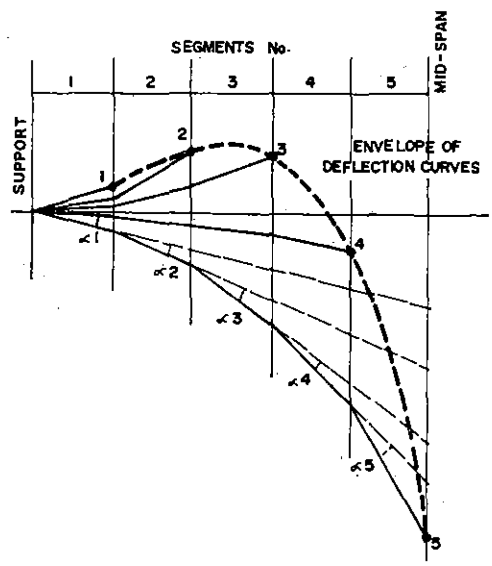


Fig. 37.18 *Deflections of a typical cantilever*

By changing the relative angular positions of the various segments by small angles, such as α_1, α_2, and so on, the cantilever should be assembled to its final length with a satisfactory longitudinal profile as shown in Fig. 37.19, for the simple case considered.

It is interesting to compare the relative importance of deflections and camber for cast-in-situ and precast construction. Figure 37.20 shows values for an actual structure, where computations have been made for the two different methods. The calculational assumptions given in Fig. 37.20 indicate that in most cases the difference would be even more significant if a cast-in-situ cycle of less than one week were employed and if precast segments were stored for more than two weeks.

37.7 PRACTICAL PROBLEMS IN CAST-IN-SITU CONSTRUCTION CAMBER CONTROL

Before proceeding with the cantilever construction proper,

* Deflection, at various sections, due to permanent loads (dead load and final prestress) as enhanced by creep of concrete over a long period of time (stabilising period is generally about 3 years), should be pre-determined at each section and allowed—for during construction in the form of precamber in reverse direction on the actually required longitudinal profile of the deck. A good enough estimate of this deflection may be based on assuming the effective modulus of elasticity of concrete as 0.45 of its instantaneous value generally given in the codes. This will nearly take care of the changed concrete modulus on account of long term creep effect and the variations in humidity, temperature, water-cement ratio, etc., from section to section, during the period of construction. However, this simplification will apply more for atmospheric conditions not far from moderate, with atmospheric shade-temperatures of about (25 ± 20)°C. In a different environment, the designer may use more appropriate assumptions and the actual feed back from the construction site.

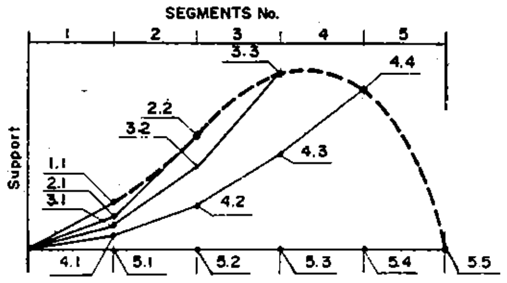


Fig. 37.19 *Choice and control of camber*

LEGEND

1.1 Means deflected position of the tip of segment 1 due to itself and its prestress.

2.1 Deflected position of the tip of segment 1 under the Wt. and prestress of segment 1 and 2.

2.2 Deflected position of the tip of segment 2 under Wt. and prestress of segment 1 and 2.

3.1 Deflected position of the tip of segment 1 under the Wt. and prestress of segments 1, 2 and 3.

3.2 Deflected position of the tip of segment 2 under the Wt. and prestress of segments 1, 2 and 3 and so on.

a starting base must first be completed on the various piers. This first special segment, called a 'pier-segment' or a 'pier-table', is generally constructed on a temporary platform anchored by prestressing the pier top, Fig. 37.21.

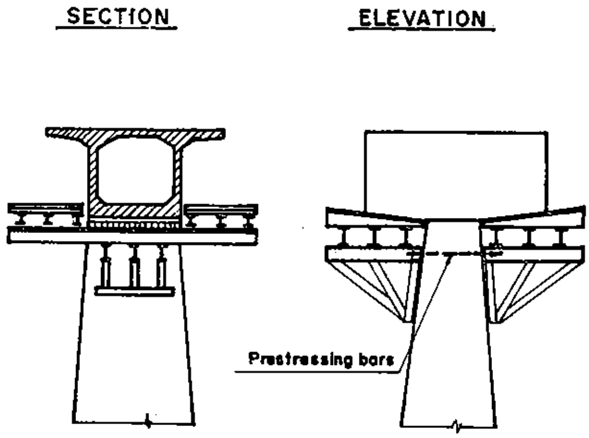


Fig. 37.21 *Construction of the pier-segment for a cast-in-situ cantilever deck*

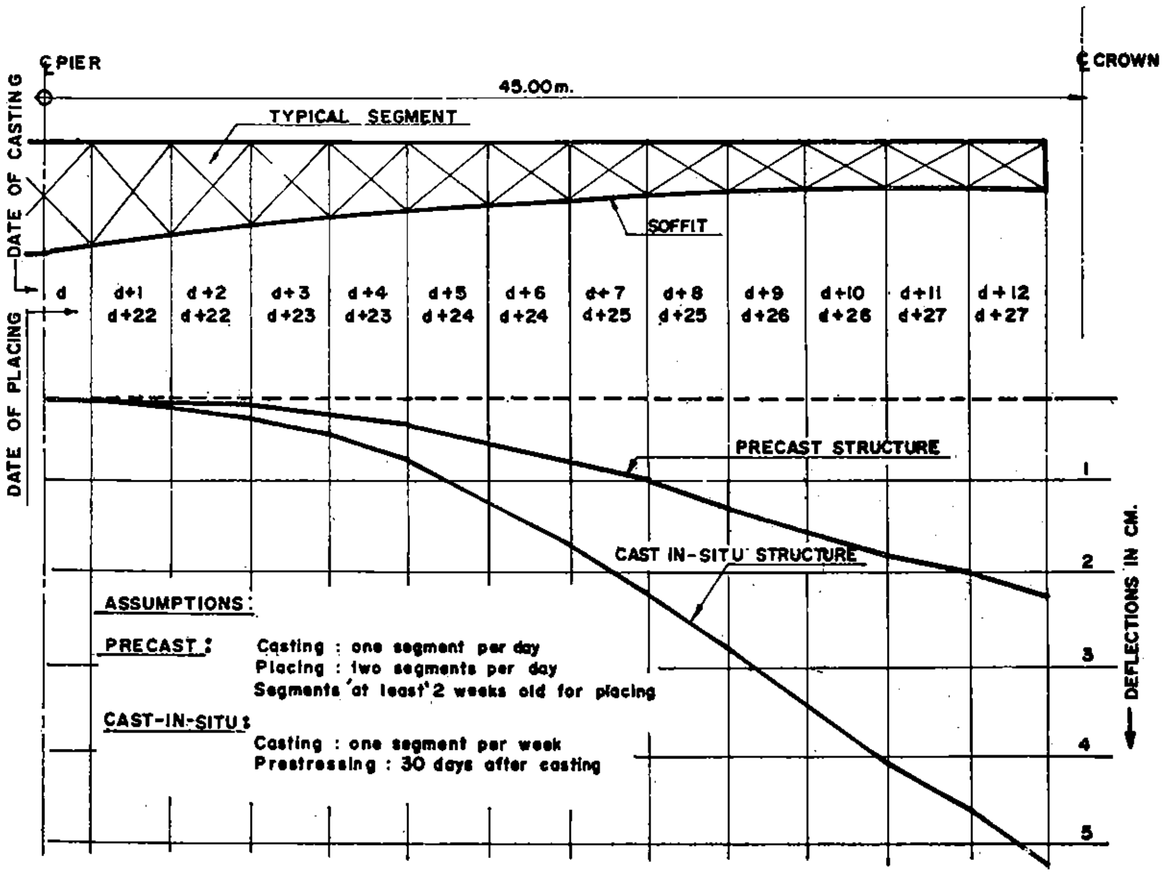


Fig. 37.20 *Comparison of soffit deflections between precast and cast-in-situ structures.*

This special segment may either be given the minimum length to insure adequate connection to the pier for the stability of the future cantilever or else be of such length as to allow both travellers to be installed simultaneously (Fig. 37.22).

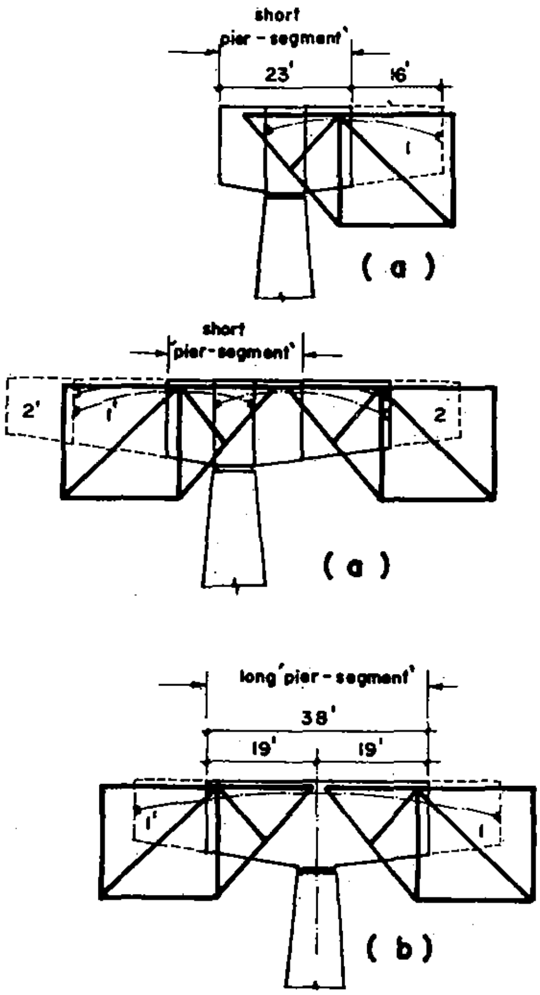


Fig. 37.22 *Start of cantilever construction from the pier-segment. (a) Short pier-segment—successive installation of travellers. (b) Long 'pier-segment'—simultaneous installation of travellers*

Another important problem relates to the safety of the travellers during construction. Apart from the difficulties of ensuring pier safety in the event a form-traveller fell during transfer from one position to the next, the difficulties would be even greater in the event of an accident during the casting operation. Consequently, all precautions must be taken both at the design stage and during construction to eliminate this potential hazard. The load-carrying members of the traveller must be carefully inspected and load-tested before use so as to practically eliminate the danger of structural failure.

The most critical areas are in the safety of the suspension rods and the transfer of the traveller-reactions to the concrete. Preferably all suspension rods and anchor bars should be doubled. Also, the prestressing tendons must have an adequate margin of safety. Use of only a single strand or a single bar in each web of the box should be avoided. Rather one or more multistrand-tendons with individual anchors for each strand or at least two prestress-bars should be used.

Worldwide use of cast-in-situ cantilever construction has established an extremely good safety record; no less than that for cast-in-situ construction on false work. Accidents are very few and far between; however, designers and constructions must always be safety conscious.

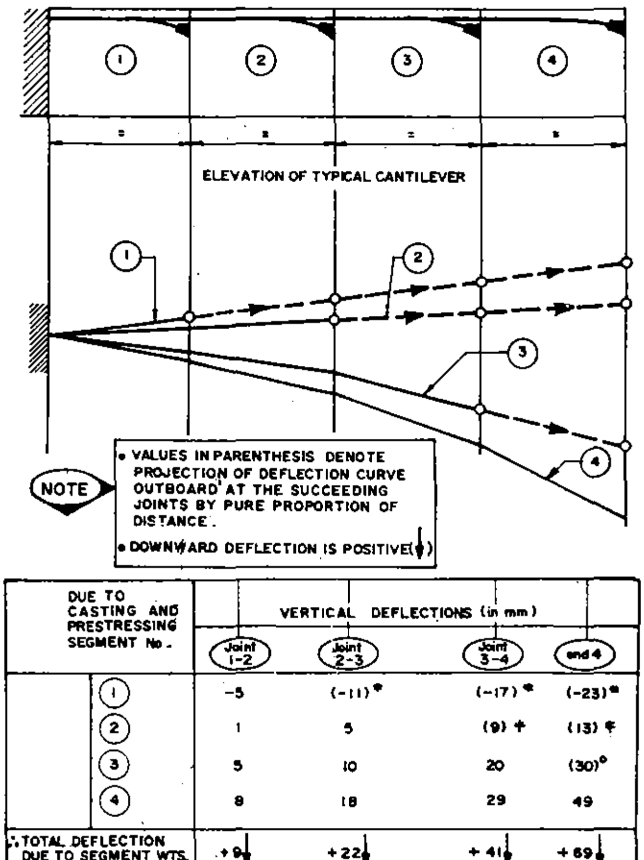
The most critical 'practical' problem of cast-in-situ construction is 'deflection control', particularly for long-span structures. There are five categories of deflections (or space geometrical movements of the structure) during construction and after completion:

1. Deflection of the travellers themselves under the weight of the concrete segment. This value is given by the manufacturer or may be computed and checked at the site during the first operations.
2. Deflection of the concrete cantilever arms 'during' construction. For 'each' casting of a pair of segments, the weight of the concrete segments and the corresponding cantilever prestress forces impose upon the cantilever a new deflection curve.
3. Deflections of the various cantilever arms 'after' construction and after removal of the travellers (but before continuity is achieved with the other parts of the deck, if continuous).
4. Short- and long-term deflections of the completed structure, including the effect of superimposed dead loads (kerbs, railings, pavements, utilities) and live loads.
5. Short- and long-term pier shortenings and foundation settlements, if any.

Using the data available on concrete properties and foundation conditions, the designer should compute the various deflections mentioned under items 3, 4, and 5 above, assuming the bridge unloaded for foundation settlements. and for long-term concrete deck deflections, and half the design live load for computation of the short-term concrete deck deflections.

The sum of the various deflection values obtained in the successive sections of the deck allows the construction of a 'camber diagram', which should be added algebraically to the 'theoretical' longitudinal profile of the bridge deck to determine, for each cantilever arm, an adequate 'casting-curve'. (As pointed out earlier, this 'theoretical longitudinal deck profile' is the 'actually required' deck profile with 'reverse' deflections due to all dead loads and final prestress added to it, taking into account, reduction in concrete E value due to long term creep.) This casting curve is the goal towards which construction proceeds during cantilever casting. The essential difficulty is that no absolute coordinates are available in a system where everything changes at each construction stage (transfer of traveller, concrete casting, or cantilever prestressing).

A very simple example may illustrate the solution of the problem of accommodating the deflections described under item 2 above. For simplicity, assume only a four-segment cantilever arm, for which a 'horizontal' longitudinal profile is the required 'theoretical longitudinal deck profile'. See Fig. 37.23.



DUE TO CASTING AND PRESTRESSING SEGMENT No.	VERTICAL DEFLECTIONS (in mm) Joint 1-2	Joint 2-3	Joint 3-4	end 4
1	-5	(-11)*	(-17)*	(-23)*
2	1	5	(9)†	(13)†
3	5	10	20	(30)°
4	8	18	29	49
∴ TOTAL DEFLECTION DUE TO SEGMENT WTS. AND THEIR PRESTRESS.	+9	+22 x	+41 y	+69 z

Fig. 37.23 *'Partial' deflections due to 'girder weight' and 'prestressing' at each construction stage ('partial', because the 'Theoretical longitudinal deck profile, referred to in the text, already includes the creep causing long-term deflections from permanent loads)*

* by proportion of -5 mm at 1–2
† by proportion of 1 mm at 1–2 and 5 mm at 2–3
° by proportion of 10 mm at 2–3 and 20 mm at 3–4

The designer analyses the various deflection curves for each construction step (casting segment or precasting). The typical results are shown in Fig. 37.23. The cumulative deflection curve is immediately obtained together with the camber diagram, Fig. 37.24. The use of the camber diagram for determining the adequate deflection at each construction stage is simple; however, it is much less simple to use in a proper manner in the field, and experienced surveyors have often made mistakes.

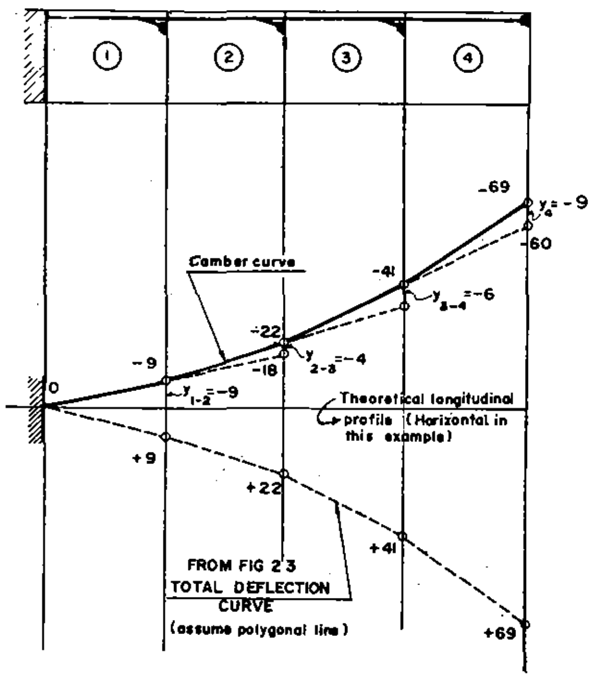
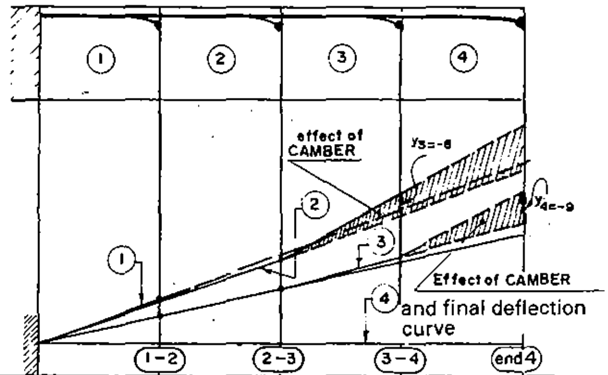


Fig. 37.24 *Cumulative deflection curve and choice of camber (i.e. pre-camber)*

When properly used, the camber diagram allows the determination, at each joint, of the offset values such as y_{1-2}, y_{2-3} and y_{3-4} at each point, which will bring the traveller in the proper position to realize the desired final geometry. The sketch and table in Fig. 37.25 show how to use the camber diagram properly. It is very important to realize that at no construction stage does the profile of the cantilever coincide with either the 'final deflection curve' or the 'camber diagram'. (Profile refers to the soffit-profile.)

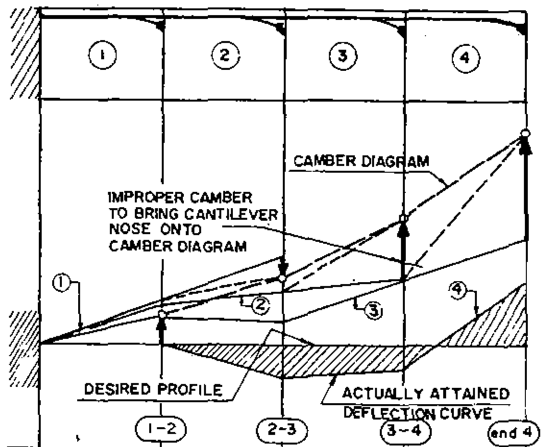
The natural tendency would be to build-up the traveller to the required offset to make its nose fall exactly on the camber diagram. The results of this improper procedure are shown in detail in Fig. 37.26. The bridge is built with an undesired double curvature, particularly undesirable toward the end of the cantilever. When the mistake is discovered, it is usually too late to put into effect any remedial measures,



		1–2	2–3	3–4	end 4
SEGMENT 1	CAMBER	−9	(−18)*	(−27)*	(−36)*
	LOADS **	−5	(−11)	(−17)	(−23)
	TOTAL	−14	(−29)	(−44)	(−59)
SEGMENT 2	CAMBER	0	−4	(−8)∓	(−12)∓
	LOADS **	+1	+5	(+9)	(+13)
	TOTAL	−13	−28	(−43)	(−58)
SEGMENT 3	CAMBER	0	0	−6	(−12)°
	LOADS **	5	10	20	(+30)
	TOTAL	−8	−18	−29	(−40)
SEGMENT 4	CAMBER	0	0	0	−9
	LOADS **	+8	+18	+29	+49
	TOTAL	0	0	0	0

Fig. 37.25 *Follow-up of Deflections with proper use of camber diagram*

* by proportion of −9mm at 1–2
∓ by proportion of −4mm at 2–3
o by proportion of −6mm at 3–4
*... from Fig. 37.23 for respective segment.



		1–2	2–3	3–4	end 4
SEGMENT 1	CAMBER	−9	(−18)	(−27)	(−36)
	LOADS	−5	(−11)	(−17)	(−23)
	TOTAL	−14	(−29) x_1	(−44)	(−59)
SEGMENT 2	CAMBER	0	$\frac{+29-22}{x_1-x}$ = +7	(14)	(21)
	LOADS	1	5	(9)	(13)
	TOTAL	−13	−17	(−21) y_1	(−25)
SEGMENT 3	CAMBER	0	0	$\frac{+21-41}{y_1-y}$ = −20	(−40)
	LOADS	5	10	20	30
	TOTAL	−8	−7	−21	−35 z_1
SEGMENT 4	CAMBER	0	0	0	$\frac{+35-69}{z_1-z}$ = −34
	LOADS	8	18	29	49
	TOTAL	0.	+11	+8	−20

Fig. 37.26 *Follow-up of deflections with improper use of camber diagram (x, y, z from Fig. 37.23)*

because the final shape of a cantilever depends essentially upon the accuracy of the geometry near the piers, where the deck is subjected to the highest moments and where its deflections have the greatest effect at midspan.

37.8 CHARACTERISTICS OF PRECAST SEGMENTS AND MATCH-CAST EPOXY JOINTS

Developed originally to allow a rapid and safe assembly of precast segments at the construction site, the technique of 'match-casting' was progressively refined as experience was gained.

The epoxy resin plays several important roles:

1. During assembly but before hardening

(a) To lubricate the mating surfaces while final positioning takes place.
(b) To compensate for minor imperfections in the match-cast surfaces.

2. In the finished structure after hardening

(a) To ensure the watertightness of the joints, especially in the top slab.
(b) To participate in the structural resistance by transmitting compression and shear forces. However, before hardening of the epoxy resin, the joints present no shear resistance whatsoever, because in this state, the epoxy behaves like a perfect lubricant. It is, therefore, necessary to provide shear keys in each web in order to ensure the shear-force transfer between segments. These keys, as well as those situated in the top slab, also allow a very accurate assembly of one segment with the another.

During assembly of the deck, some sort of temporary fixation, either mechanical or by means of prestress bars, allows the placing-equipment (launching girder or crane) to be quickly unloaded without waiting for the cantilever tendons to be stressed.

Figure 37.27 shows how a typical segment can be assembled into the existing structure using a temporary apparatus located on the top and bottom slabs, which is used to create forces F_1 and F_2 which ensure the equilibrium of

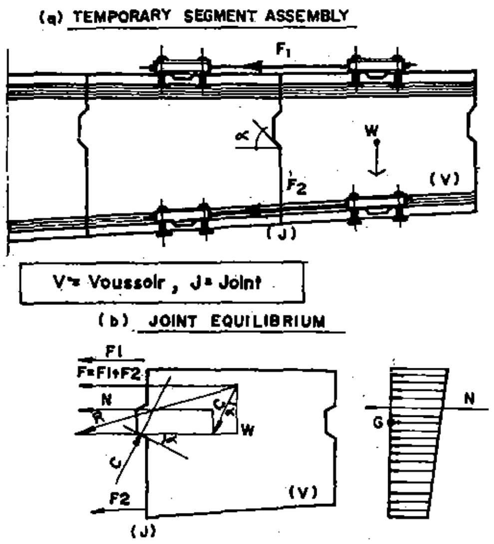


Fig. 37.27 *Temporary assembly of precast segments*

the new segment at the joint.

These two forces F_1 and F_2, combined with the weight W of the segment, give the resultant force R, which is inclined with respect to the joint. Because of the very small coefficient of friction of the epoxy, the vertical shearing component of R can be balanced only by the vertical component of the reaction C which exists normal to the inclined bottom face of the web shear keys. The resultant R is composed, therefore, of the oblique reaction C supported by the shear keys, and a horizontal reaction N, which is responsible for securing the joint.

The axial stress distribution at the joint cross-section differs in this case from what would be obtained by ordinary calculations. It is obvious that V is smaller than F (the sum of forces F_1 and F_2). Let α be the angle of the key support faces with respect to the horizontal: then $F-N = W \cdot \tan\alpha$ and for a typical case of $\tan\alpha = 0.50$, $F - N = W/2$. Consider a segment weighing $W = 50$ tons, temporarily assembled by a prestress force of 100 tons located in the slabs, the axial force reduction is $(F - N) = \frac{W}{2} = \frac{50}{2} =$ 25 tons—that is, 25% of the total applied prestress force.

If the rate of erection of the precast segments is sufficient to ensure the positioning of say four segments before the resin in the first joint has set, then the reduction in the effective axial force in this joint will be $4 \times 25 = 100$ T, which more or less corresponds to loosing one tendon of twelve 1/2 inch diameter strands. The same conclusion would be valid when the permanent prestressing was used to ensure the temporary stability of the cantilever.

This reduction of the effective prestress force should be taken into account while verifying the cantilever resistance and stability. Failure to do so may result in temporary joint opening, which is undesirable although not dangerous for stability.

It is also preferable to choose the intensity and the point of application of the forces F_1 and F_2 in such a manner as to allow the axial force N to be as close as possible to the section centroidal axis, thus ensuring a nearly uniform axial stress distribution over the total depth (and hence a resin-film of constant thickness) during construction.

Structural Importance of Epoxy Resins

The resistance or strength of different cantilevers is ensured by groups of tendons, which may be referred to as 'cantilever-tendons'. These may be straight or curved in profile, and anchored on the various segment faces. The stressing operations remain in the critical path of construction because no new pair of segments can be placed before the previous pair has been stressed to the existing cantilever.

The final group of tendons in continuous structures joins together the different cantilevers and makes the structure continuous. They are anchored either in block-outs in the bottom slab or in the 'fillets' at the 'junction between the top-slab and the webs', after upward deviation towards top slab level.

A bad choice, or improper use, of the epoxy resin can be a critical factor concerning the shear resistance of the joints, and for this reason joints of this type require strict quality control.

In general, the different types of epoxy resins available have final strengths substantially exceeding that of concrete, so they do not constitute a weak point in themselves. Several conditions must be satisfied, however, in order that the resin cures properly:

1. Mixing the constituents in their correct proportions.
2. Eliminating any solvents that have a fatal effect on the properties of the resin.
3. Avoiding any flexibility additives, such as thiokol, that greatly increase the deformability of the epoxy.
4. Mixing and applying the epoxy carefully.

With respect to the last point, the surfaces to be joined must be specially treated if the best results are to be obtained. Comparative tests have shown that sand blasting the concrete surfaces gives the most satisfactory results, the surfaces being kept clean, dry, and free from grease, etc. during placing. In damp or rainy weather alcohol is burnt on the joint surfaces to eliminate surface moisture. The water present in the concrete itself has no detrimental effects on the performance of the resin.

It has also been established, that rapid placing of successive segments has a favorable effect on the properties of the resin. The additional compressive stress applied to an epoxy joint while still not fully set (i.e., while still under polymerization) when the next segment is prestressed, improves the resin's ultimate mechanical properties.

In addition to the precautions taken to ensure correct curing, one may provide against the risk of bad results by including shear keys in the bottom slab also.

It is preferable that the precast segments are equipped with interlocking keys in the top and bottom slabs and in most of the height of the webs. This configuration of shear keys at regular intervals, improves the behaviour of joints by relieving the epoxy of its structural role.

37.9 FATIGUE IN PRESTRESSED CONCRETE BRIDGES

Basically, prestressed concrete resists dynamic and cyclic loadings very well. Eugene Freyssinet demonstrated this fact sixty years ago. He tested two identical telegraph poles under dynamic loading. One was of reinforced concrete and the other of prestressed concrete; both were designed for the same loading conditions. The reinforced concrete member failed earlier than the prestressed concrete member which sustained the dynamic load for several million cycles.

Fatigue in concrete itself has never been a problem in any known practical structure, because a variation of compressive stress in concrete may be supported indefinitely—so long as it never turns tensile and either the applied load never exceeds about 0.6 of ultimate load or the maximum compressive stress does not exceed about 0.55 of cylinder crushing strength. When reference is made to fatigue in prestressed concrete, it is always inferred that fatigue problems arise in the prestressing steel or conventional reinforcing steel as a result of ingress after the cracking of concrete due either to bending or to shear. If concrete cracking could be avoided in prestressed concrete structures, the fatigue problem would be almost completely eliminated.

Figure 37.28 shows fatigue behaviour of prestressing strands currently used in prestressed concrete structures. The diagram shows the limit of 'stress variation' causing fatigue-failure versus the 'mean stress' in the prestressing steel. For convenience, both values are expressed as a ratio with respect to the ultimate tensile strength. For a steel stress of 60% of the ultimate, the acceptable range of variation is $\pm$ 8% of the ultimate for a number of cycles between 10^6 and 10^7. Using for example 270 ksi quality strand, this variation is therefore about $\pm$ 22,000 psi, or a 'total' range of about 44,000 psi.

Because dynamic loading on a bridge is of a short-term nature, the concrete modulus is high and the ratio between steel and concrete moduli is of the order of 6. Consequently, the maximum concrete stress in an uncracked

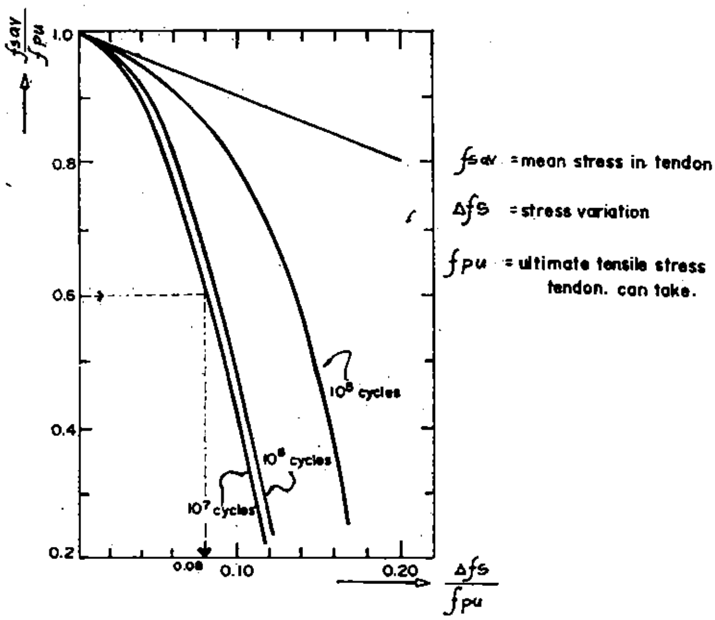


Fig. 37.28 *Fatigue behaviour of prestressing strands*

section that would cause a fatigue failure would have to be about 44,000/6 = 7330 psi (516 kg/cm^2)—a value which is probably nine times the 'stress variation' under design live loads in highway box girder bridges. An uncracked practical prestressed concrete structure is, therefore, completely safe with respect to fatigue, regardless of the magnitude of live loads. A limited amount of cracking, although considered inadvisable from the corrosion point of view, is not critical if kept under control.

REFERENCES

1. Swann R.A., "A Feature Survey of Concrete Box Spine Beam Bridges," Cement and Concrete Association, London, June, 1972.
2. Witecki A.A., "Torsional Moments in Horizontally Curved Bridges", ACI Publication SP23 on Concrete Bridge Design, 1969.
3. Richmond B., "Twisting of Thin Walled Box Girders," Proc. ICE Papers 6868, April, 1966, 7031, January 1968, 7174 and 7189, August, 1969.
4. Scordelis A.C., et al. "Load Distribution in Concrete Box Girder Bridges," ACI Publication SP23 on Concrete Bridge Design, 1969.
5. Chu k-h, E., and Dudnik "Concrete Box Girder Bridges as Folded Plates," ACI Publication SP23 on Concrete Bridge Design, 1969.
6. Cheung Y.K., "Analysis of Box Girder Bridges by the Finite Strip Method," ACI Publication SP26 on Concrete Bridge Design, 1971.
7. Trikha, D.N. and A.D. Edwards, "Analysis of Concrete Box Girder Before and After Cracking." Proceedings, Institution Civil Engineers, Paper 7571, 1972.
8. Sisodiya R.G., et al., "New Finite Elements with Application to Box Girder Bridges," Proceedings Institution Civil Engineers, Paper 7479S, 1972.
9. Lim, P.K.T., et al, "Finite Element Analysis of Curved Box Girder Bridges," Conference, Development in Bridge Design and Construction, Cardiff, 1971.
10. Kristek V., "Theory and Research on Thin-Walled Prestressed

Concrete Beams," Session IV, 6th FIP Conference, 1970.
11. Campbell-Allen D. and R.J.L., Wedgwood "Need for Diaphragms in Concrete Box Girders," *Journal of Structural Division* ASCE, March 1971.
12. Sisodiya R.G., et al, "Diaphragms in Single and Double Cell Box Girder Bridges with Varying Angles of Skew," *ACI Journal*, July 1972.
13. Garrett R.J., and R.A. Cochrane, "The Analysis of Prestressed Beams Curved in Plan with Torsional Restraint at the Supports." Institution of Structural Engineers, March, 1970.
14. FIP-CEB, "International Recommendations for the Design and Construction of Concrete Structures," Prague, 1970.
15. Leonhardt F., "Shear and Torsion in Prestressed Concrete," Proceedings, 6th FIP Congress, Prague, 1970.
16. Discussion, "The Design and Construction of Kingston Bridge and Elevated Approach Roads, Glasgow," *Journal of the Institution of Structural Engineers*, August, 1971.
17. Nundy F.S., "Problems in the Construction of Western Avenue Extension," Proceedings of Institution of Civil Engineers, February, 1972.
18. Maisel B.I., "Review of Literature Related to the Analysis and Design of Thin-walled Beams," Cement and Concrete Association, July, 1970.
19. Lacey G.C. et al., "State of the Art for Long Span Prestressed Concrete Bridges of Segmental Construction," *PCI Journal*, Sept.-Oct., 1971.
20. Cusens A.R., "Box and Cellular Girder Bridges—A State of the Art Survey," ACI Publication SP26 on Concrete Bridge Design, 1971.
21. Wittfoht H., "Considerations of the Cross Sectional Design of Prestressed Concrete Bridges with Reference to Bridges built in 1960," Cement and Concrete Association, Translation No. 124, 1966.
22. Muller J., "Long Span Precast Prestressed Concrete Bridges Built in Cantilever," ACI Publication SP23 on Concrete Bridge Design, 1969.
23. Wittfoht H., "Prestressed Concrete Bridge Construction with Stepping Formwrok Equipment," ACI Publication SP23 on Concrete Bridge Design, 1969.
24. Somerville G., "Tests on a 1/12 Model of the Mancunian Way," Technical Report TRA/394, Cement and Concrete Association, London.
25. Homberg/Ropers, "Fahrbahnplatten mit Veranderlicher Dicke," Springer-Verlag, 1965.
26. Maugh L.C., "Statically Indeterminate Structures," 2nd edn, Wiley, pp. 412–422.
27. Report on Urato Bay Bridge, Japan, Engineering News Record, July 13, 1972.
28. Podolny W. and J.M. Muller, "Construction and Design of Prestressed Concrete Segmental Bridges," John Wiley and Sons. (... from which some material has been included with grateful acknowledgment.)

CHAPTER 38

Design and Construction of Cable-stayed Bridge Decks—Some Information*

Present day construction techniques use cable-stays for many types of structures, such as stadia, aircraft hangars, telecommunication towers, large chimneys and cooling towers, electricity pylons and, perhaps the most important, long-span stayed bridges, which are undergoing an impressive and fast development.

38.1 BRIEF HISTORY

The construction of cable-stayed bridges did not really start until the very early years of the 18th century. Though initially expected to develop very rapidly, its development was halted by two serious accidents:

Pryburg bridge, Great Britain, collapsed in 1818 due to oscillations caused by the wind; and the Saale river bridge, Germany, collapsed in 1824 due to overloading by a crowd watching a regatta.

These accidents were no doubt caused by the insufficient technology of the 'stays' and their 'connections'. Composed of bars and chains, their mechanical properties were poor and their cross-section not large enough. During construction, there were no proven methods of applying the correct tension to the stays which were, therefore, ineffective, causing the deck to attain dangerous deformations.

The French scientist Navier, who studied these accidents, recommended suspension bridges rather than stayed, and for about 100 years, only a few stayed bridges were built, of which, Whealing bridge over the Ohio river and the rail bridge over the Niagara, both built by Roebling in the US in 1846 and 1855, may be cited. The aerial ferries built in Marseille and Nantes (France) by Arnodin, in 1903 and 1905, are early examples of pure cable stayed bridges.

In the 1950s, the necessity to rebuild the bridges destroyed during World War II, specially in Germany, together with the progress made in steel construction technology (steel and cables of high mechanical properties, orthotropic slab, etc.) were factors which influenced the revival and development of cable-stayed bridges. Engineers like Dischinger, and others proved the superiority of stayed bridges over suspended bridges for spans of some hundreds of metres.

It is generally admitted that the first modern cable-stayed bridge is the Stromsund bridge, built in Sweden in 1955.

More than 70 other major bridges of this type, with steel or concrete decks, have since then been built or are under design or construction.

38.2 THE EVOLUTION OF THE STAYS

The stays of the first modern bridges consisted of single or multiple arrangements of the following main basic elements: locked-coil wire ropes, twisted strand ropes, bars, strand of twisted wires, and compacted parallel wires. The ultimate tensile strength of these elements was in the range of 1100 to 1500 N/mm^2.

The 'stress variation' in cable-stays may reach high values and the fatigue properties are, therefore, of the utmost importance. Fatigue here is so important because of large stress variation caused in the stays even by the passing of live load on the bridge. Approximately, it can be assumed that the deck is very flexible so that the stress in the stays is nearly directly proportional to the applied load. Moreover, the stays, essentially the direct tension carriers, also suffer bending stress from the bending moment caused by the combined effect of axial tension and self weight 'sag' in them. If this is not 'subdued' (by using damping guides near anchorages), the resulting stresses and, therefore, the stress variation can be even higher. This can enhance the fatigue effect further.

Although some of the above-mentioned elements, specially parallel wire ropes, possessed relatively good fatigue properties, their anchorages were unable to withstand high 'stress amplitudes', and it became necessary to find new types of stays.

Through development and research undertaken basically in Germany, mainly by Prof. F. Leonhardt, a type of stay has been developed which uses parallel wires with ultimate strength of about 1700 N/mm^2, of the same type as those

* The author acknowledges with grateful thanks Engr. P. Xercavins for the information reproduced here.

used for prestressed concrete.

During the 1970s 7-wire strand to a very large extent superseded the wire in post-tensioning technology, and following this, research was directed towards the use of prestressing strand for cable-stays.

Practice has proven the satisfactory dynamic properties of good quality strand with ultimate static strength up to 1900 N/mm^2.

Parallel strand stays have been used for long span bridges, among which two landmarks in this new technology are worth noting, Brotonne Bridge over the Seine river, France, and Rande Bridge near Vigo, Spain.

Numerous other applications of this technology may be envisaged, such as the anchorage of semi-submersible off-shore platforms, and cable stayed roofs similar to the type used for the Olympic Stadium in Munich, designed by German Engineers and for which strand was adopted.

Economy

It is worth pointing out that cable stayed 'concrete-deck' bridges have the potential to outbid the concrete box girders in such 'low' span ranges as 150 to 250 m. The large concrete box girder alternatives have some inherent problems of durability and serviceability which are likely to be magnified with growth in span-length. The large free cantilevers suffer a big time dependent deformation which may take as long as five or seven years to stabilise. Errors in their prediction at design stage lead to a 'break' in profile at the central hinge (commonly preferred by some) and result in serviceability defects which are difficult to rectify. Precast segmental construction also needs additional attention. The first generation voussoirs in use rely on the epoxy adhesive for transfer of shear and impose severely low tolerance on joint geometry, field mixing, curing and application of epoxy compound. Errors can pose grave durability risks. Discontinuity of reinforcement across the joint face leaves little room for control of cracking caused by non-linear temperature distribution which has led to distress of many box girder bridges abroad in the recent past.

Many of these problems could perhaps be cut down with the adoption of cable stayed concrete decks. The biggest advantages are the simplicity of the form and accessibility of the deck and stays to inspection. The low depth simple beam and slab deck of cable stayed bridge does not require the large volume of prestressing steel consumed in concrete box girders, interiors of which are not easily accessible to inspection. In case of damages, repair and restoration can be a costly experience. The cable-stays on the other hand are replaceable by design.

Aerodynamic Stability

It is common knowledge that a cable stayed deck has inherent superiority over suspension bridges in dynamic behaviour. The multiple stay system provides additional system-damping of larger number of cables, 'all with different natural frequencies' and a shallower depth of deck! The resulting improved aerodynamic behaviour of multiple stay cable deck has influenced new thinking on efficient deck geometry. It may be recalled, that streamlined aerofoil profiling of deck geometry was found necessary for aerodynamic stability of suspension bridges. The flat sharp nosed closed box section of Severn, Bosphorous and Humber suspension bridges are typical examples of application of this principle enunciated by Leonhardt in 1952. Later, it was found that an open section like the one adopted by Leonhardt for the cable stayed Knie bridge (1969) at Dusseldorf with two triangular boxes at edges was equally aerodynamically stable. However, for the design of twin rail-cum-road cable stayed bridges over Parana-Palmas and Guazu (1972-77), a dynamic model of the whole bridge was tested. It proved that the multiple stays provided such heavy damping as to wipe out any probability of large amplitude resonance oscillations. The flutter theory and the sectional wind tunnel tests have little applicability in checking the aerodynamic stability of the full multiple stay system deck configuration. This new knowledge has led to a striking simplification of the deck geometry of modern bridges.

Concrete Deck

The multiple stay system lends itself to the use of pre-fabricated concrete decks with ease. The all-reinforced concrete Dnepr bridge in USSR built in 1963, is a pioneering example. The modest 144 m span and 6.3 m wide deck was erected by suspending precast reinforced concrete deck elements from cable stays. A nominal amount of steel was used in deck except at cable connections. Leonhardt's more modern Pasco bridge made full use of the advantages of the multistay system in deck erection. The pre-fabricated deck elements were 24.33 m wide and 8.33 m long. These were erected by a pair of stay-cables located in the segment. An overview of the Pasco bridge design shows that dead load bending moments can be largely neutralised. Normal forces can be significant. Live loads and temperature differential between the steel stays and the deck are the major sources of bending moment in deck. 'Partially prestressed concrete' is the obvious solution for the normal cable stayed concrete deck. The Pasco deck was essentially proportioned by ultimate limit state requirements. Tensile stresses were permitted for full live load and temperature combination. But these were kept low.

Unlike steel bridges, shrinkage and creep effects comprise an important design consideration in concrete decks. Improved knowledge and advanced mathematical modelling tools have aided modern design greatly. In practice, initial forces introduced in the stays during erection, and allowing for subsequent redistribution of differential moments by the end of erection and in service, makes liberal room for economical design. To avoid creep problems, 'composite deck' with concrete slab on steel girders is rarely used in long span cable-stayed bridges. The Hooghly bridge is an isolated exception. Very long span bridges of all types (beams, cable-stayed and suspension), in modern times, invariably use an all steel orthotropic plate deck. Considerable saving in dead load and steel consumption makes its use almost mandatory. For Hooghly, lack of infrastructure facilities and fabrication experience posed problems and were the main reasons for the isolated exception.

"The construction of spans of up to 400 m with concrete decks (even longer spans with steel decks) using cable-stay techniques, derived from prestressing that enable simple installation and the eventual replacement of cable-stays, have nearly pushed beyond the 500 m limit the field previously reserved exclusively for suspended bridges."

The specifications relative to cable-stayed bridges are numerous since this technique is still relatively recent.

38.3 ANALYSIS (*Brief Note*)

A cable-stayed bridge is a statically indeterminate structure with high degree of redundancy. Analysis of cable stayed bridges considering three dimensional space action is a relatively complex problem but if suitable assumptions are made, the problem may be solved by two dimensional analysis and results can be obtained with reliable accuracy for practical design. Several investigators have used two dimensional approach and many bridges have been designed on that basis and constructed. Undoubtedly the cable-stays display a non-linear behaviour and the non-linearity is due to large displacements, 'catenary-action' of cables (which, therefore, are non-linear) and bending moment-axial force interaction in stays, girders and towers. Hence in strict terms, the principle of superposition is not applicable and consequently stresses and displacements cannot accurately be determined by use of simple influence lines. However, various investigators have studied the non-linearity effect on cable stayed bridges and found that non-linearity is not significant and linear analysis gives satisfactory results particularly if damping guides are used near anchorages. Nevertheless, for very large spans and important structures non-linear analysis must be carried out and its effect studied.

The modern approach to structural analysis can be broadly classified as:

(i) Stiffness (displacement) approach,
(ii) Flexibility (force) approach.

Any computerised analysis procedure generally uses one of the above approaches, depending upon the problem in question and convenience of the analyst. A mixed method of analysis has also been by Podolony and Scalzi. The finite element technique is another method used for the analysis of cable-stayed bridges. The well known 'stiffness' approach seems to be the most commonly employed technique of analysing a cable-stayed bridge. In the stiffness method, following assumptions may be made:

(i) The structural system is idealised as interconnected system of linear elements joined to one another at nodal points only.
(ii) The towers are idealized by beam-elements capable of taking axial deformation as well as bending.
(iii) The stays are assumed as uniaxial truss elements capable of resisting axial force only, not bending. (This, infact is wrong, unless damping moments in the stays caused due to axial tension and 'sag' in them).
(iv) The nodes may be selected at the points of interaction of cables and girders.
(v) Foundations may not be considered in this analysis.

38.4 DEFLECTION CRITERIA

A task committee of the American Society of Civil Engineers has prepared tentative recommendations for cable-stayed bridge structures according to which the static deflection in highway bridges under live load should not exceed 1/500 of the span length between piers i.e. central span, unless static and dynamic analyses indicate that serviceability requirements are met. This static live load deflection of 1/500th of the span length is excess of that allowed by American Association of State Highway and Transportation Officials (AASHTO) specifications. The guideline is generally for short length bridges (for spans of 150 m or less) and if the same specification is adhered to for long span bridges of a flexible nature, it would result in an unduly conservative design. In Germany, bridges are being designed with deflection limit ranging from 1/500 to 1/225 of the span length.

AASHTO recommends that until further experience is gained, the 1/500 limit should be adopted for design.

The deck superstructure can be of steel or concrete. Many of the bridges built around the world have used steel decks but many others have been built using concrete decking and generally a prestressed box girder system. A concrete girder will have more weight per unit length (and hence will need relatively more cable material to support the girder) and heavier section of towers and substructure. Steel deck will undoubtedly be a lighter construction.

Proportions

For an economical design of a cable-stayed bridge the ideal side to central span ratio may be about 0.40 and the tower height in such bridges should be about 1/6 of main span for minimum tension in the cables (and hence for less area of cables), minimum girder moments and minimum base moment in the tower. However, economy can be achieved within a range of 0.35 to 0.45 of side to central span ratio under different span lengths and with different types of cable arrangements within a tower height of 1/4 to 1/8 of central span.

With the increase in allowable deflection limit the total cable area required decreases. Effect of tower height on cost of superstructure is almost linear for a central span range of 200–300 m.

The economical range of main span for a cable-stayed bridge of prestressed concrete box girder type appears to be 200–300 m, and if deflection criterion is relaxed then a cable-stayed bridge with central span of up to 500 m could become competitive, particularly if lighter steel deck is adopted. Steel deck, involving high grade steel and special welding electrodes and technology, may be costlier in certain countries, but this has to be offset against reduced cable steel and propagation of technology.

38.5 OPERATION OF A CABLE STAY

By its structure and its installation, the cable stay is sometimes confused with a prestressing cable.

The comparison between a prestressed concrete bridge built in cantilever and a cable-stayed bridge will enable the fundamental differences to be distinguished (Fig. 38.1).

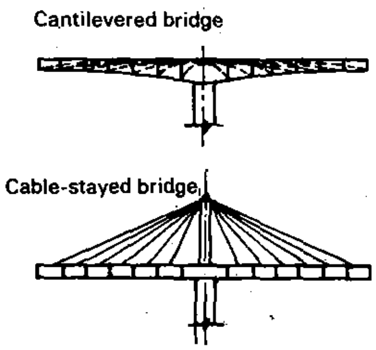


Fig. 38.1 *Segmental bridges*

Both types of bridge use the same construction procedure that consists of building the deck with successive segments from a pier, but the basic difference may be realised from the following considerations:

(a) In the case of a cantilever bridge, the prestressing cable acts as an 'active' force on the deck, latter being very rigid compared to the actual deformation of the cable. Because of this rigidity (of the deck) the cable can be stressed to a high proportion, even independently of the applied dead and live loads. The subsequent stress variations in the cables will result from those undergone by the concrete, in proportion to the moduluses of the two materials, steel and concrete. Under the action of the variable loads, a stress variation of the order of 50 Mpa in the steel and 10 Mpa in the concrete may occur.

(b) In a cable stayed bridge, the cables are the 'stressed elements' of the cross-linked framework formed by the deck, the pylon and the cable stays. They are *passive* suspenders (but adjustable) and should not be stressed to 'values' other than the tensions resulting from the loads applied at the nodes (the deck-stay combined-effect points) as otherwise this will cause deformations that are incompatible with the actual response of the deck (Fig. 38.2).

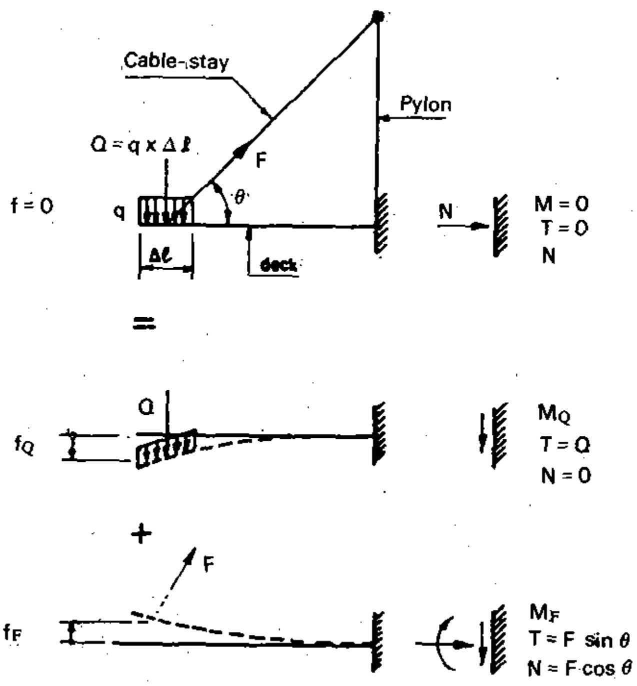


Fig. 38.2 *Cross-linked framework formed by the deck, the pylon and the cable-stay*

In this model, the deck is a 'continuous beam on multiple elastic bearings' latter being the suspenders, i.e., the cable-stays. The aim of the adjustments is to result in the lowest possible moments in the deck, in keeping with the spacing of the stays. Any 'variation of applied load' is immediately expressed in the stays by a 'stress variation' which, in a preliminary approximation of an infinitely flexible deck, is proportional to variation in the applied load. The 'order of magnitude' may be easily estimated.

The dead load of a concrete-deck cable stay is in the region of about 1500 kg per m^2 of bridge (overall width), or 1,800 kg per m^2 when reduced to the width of the carriageway, whereas the live load may be taken as about 400 kg per m^2 for spans of some 200 metres or so.

Hence, the ratio between the live load and the overall loads is about:

$$\frac{400}{400+1800} = 0.182$$

If the maximum stay tension is fixed at 0.45 RG, then, with the characteristic ultimate stress RG = 1800 Mpa (N/mm^2), the stress variation in the stays is about: $0.182 \times 0.45 \times 1800 = 147$ Mpa(N/mm^2) (i.e. 1470 kg/cm^2), which is large.

It can be seen from this calculation that the steel used for stays is subjected to large stress variations. Hence a detailed analysis of the strand (and cable stay) 'fatigue' performance is thus necessary.

As a secondary consequence, the assumption of a stay behaving as a linear 'tension rod' introduces a theoretical non-linearity since the self weight of the tension rod causes a slight deflection (sag) that varies with the tension (Fig. 38.3). The result is a non-linearity of elongation with tension.

In practice, for the analysis of the structures, it is enough to consider a constant apparent modulus owing to very slight variation in this modulus in service.

$$\frac{E}{E_0} = \frac{1}{1 + \frac{\gamma^2 \, l^2 \, E_0}{12\,\sigma^3}}$$

E = modulus of elasticity
Eo = modulus of elasticity of the straight cable
γ = specific weight of the cable (total weight ; steel volume)
l = horizontal span of cable
σ = Tensile stress in the cable

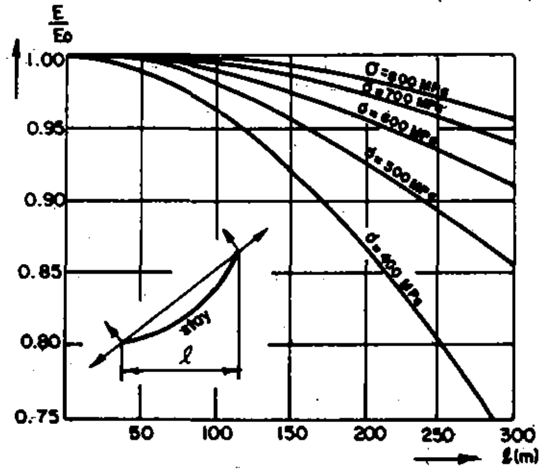


Fig. 38.3 *Effective modulus of elasticity E in terms of horizontal span and tensile stress in the stay cable. (Ernst's formula)*

38.6 ALTERNATE STRESSES IN CABLE-STAYS

Two complementary aspects must be taken into consideration in the analysis of alternate stresses:

(a) The first concerns the working loads to be taken into account for the large number of loading cycles.

(b) The second concerns the bending stresses that superimpose on the simple axial stresses when the cable-stays are subjected to load variations, bending or vibrations (due to wind effects, etc.).

Aspect (a)

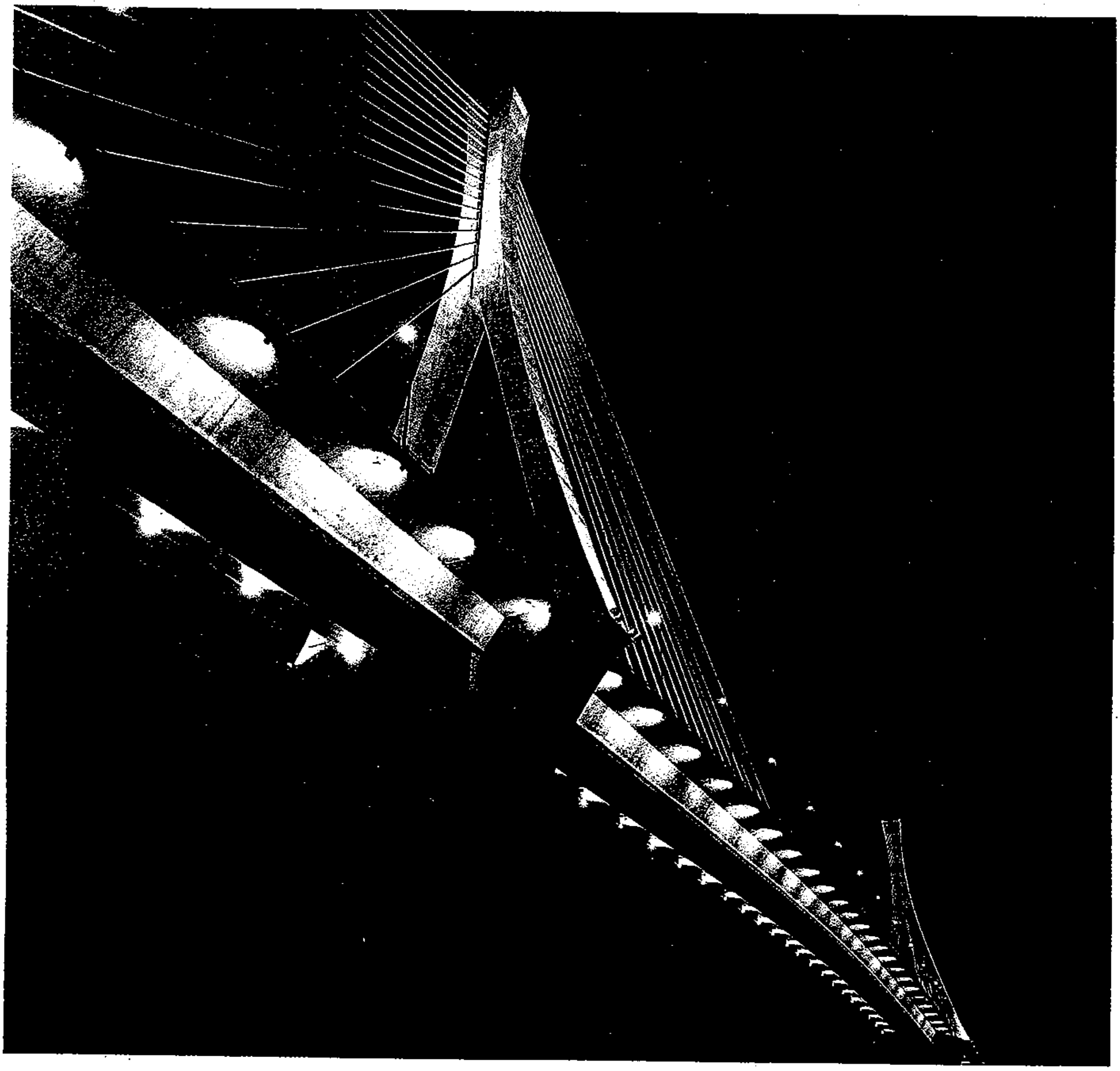
The daily live loads that are applied frequently enough and cause fatigue are much lower than the conventional design live loads. Fatigue is caused only by tens of thousands of loading cycles. The usual value of 2 million cycles in terms of fatigue would require a daily frequency of 54 passages of live load for 100 years. The live loads corresponding to this frequency are far much lower than the design loads, and very often too low to be taken into consideration with regard to fatigue. However, the design loads must be taken into account: for example, if the deck is very light (steel), or for particularly high operating loads, or in the case of short cable-stayed spans, or for railway bridges where the probability of heavy loads is much higher.

Therefore, the intensity of loads to be taken into account depends on the one hand on the number of envisaged cycles and, on the other, on the type of load.

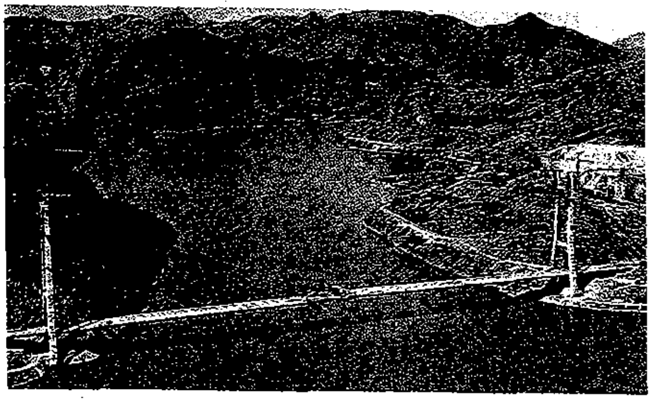
This intensity, related to the log of number of cycles, is expressed by a curve known as 'loading distribution spectrum'. The aspect of this curve may vary considerably, depending on whether it concerns an action related to a natural phenomenon (Fig. 38.4a) or a "determinist" action, for example the loading spectrum of a railway bridge where the loads and frequencies are more clearly defined (Fig. 38.4b). Generally speaking, for road bridges, the spectrum is unknown as it depends upon numerous parameters: its aspect may, however, be taken as that of the spectrum of a natural phenomenon.

In the absence of precise information, assumptions are made that often result in considering a nominal rated load (that tries to take into consideration the different parameters mentioned) and a probability of loading which may have repetition of 2.10^6 cycles.

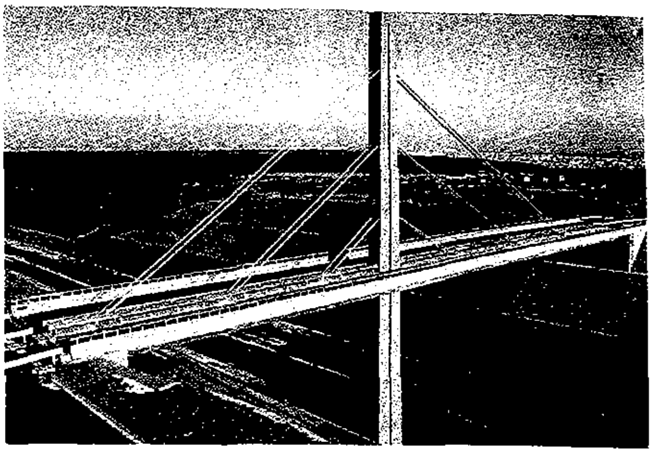
The French working loads are usually rated with a coefficient of 0.5, whereas the less intense AASHTO live loads can be rated with a coefficient of 0.7. It should be



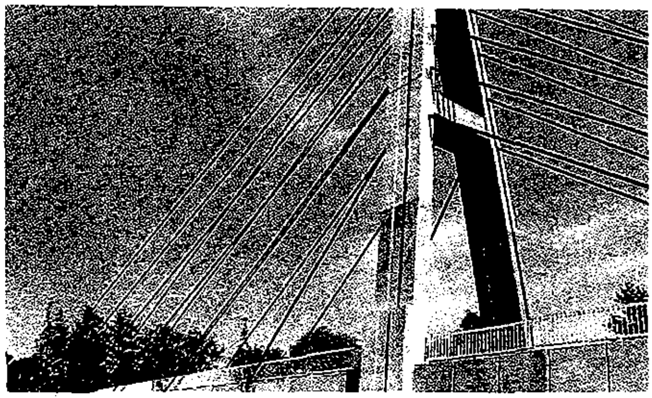
A cable-stayed bridge deck



Barrios de Luna Bridge (Spain)

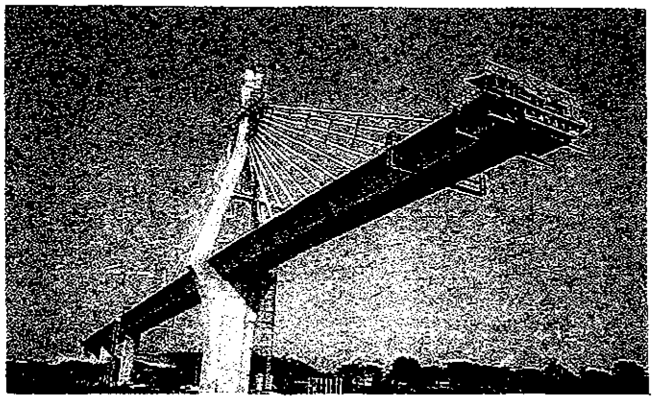


Nishikigaoka Bridge (Japan)

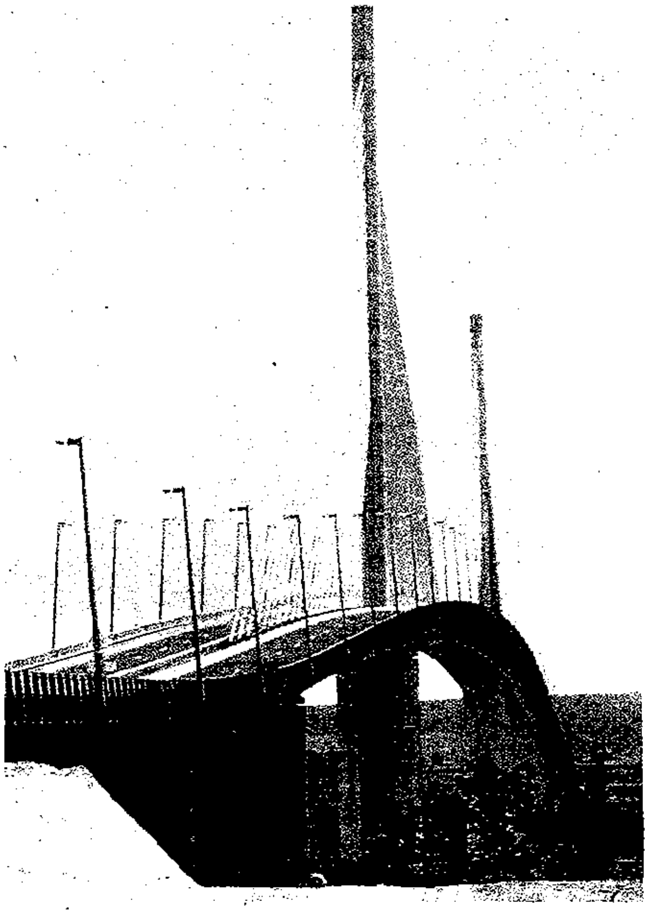


Illhof Footbridge (France)

Meylan Footbridge (France)



Coatzacoalcos Bridge (Mexico)



Brotonne Bridge (France)

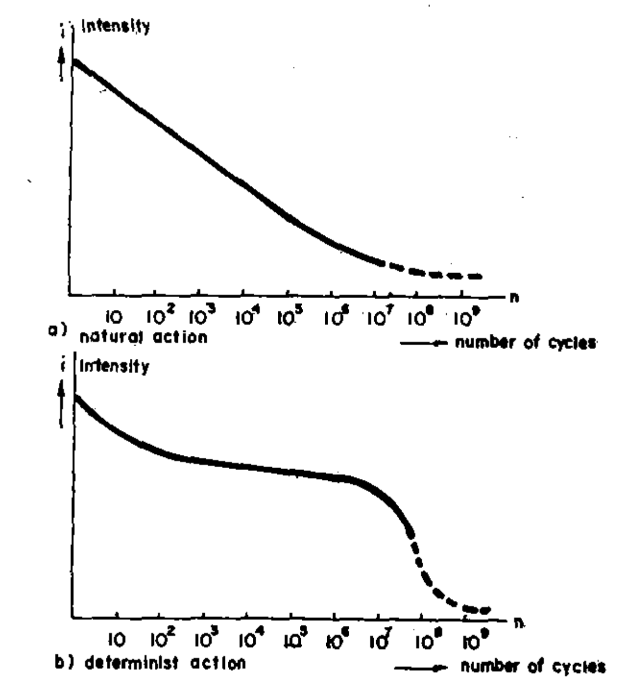


Fig. 38.4 *Spectra of intensity of loading i in terms of the number of cycles n*

noted that the West German DIN (Deutsches Institut fur Normung) standards define a variable coefficient, according to the type of their live load:

- 1.0 for railway loading
- 0.8 for category 30 traffic loading
- 0.5 for category 60 traffic loading

Aspect (b)

This aspect has often been under-estimated and even neglected if reference is made to the specifications of numerous tenders that insist only on the axial stress variations without a mention of bending stresses. The latter are produced:

- Either by the variations of stay tension and distortion, impeded at the fixed anchorages (Fig. 38.5),
- Or by stay vibrations caused by wind effects.

An 'articulation' at the anchorage could theoretically eliminate these stresses, but the relatively intricate and costly arrangement for this, renders this solution hardly applicable in practice.

Even the oscillation due to wind, and the deflection of the deck under applied loads, generate bending stresses which, if added to those previously mentioned and with the axial stress, involve making special arrangements in order to resist fatigue effects.

By analysing the induced bending stresses, the value of the bending moment in the stay may be obtained at any point

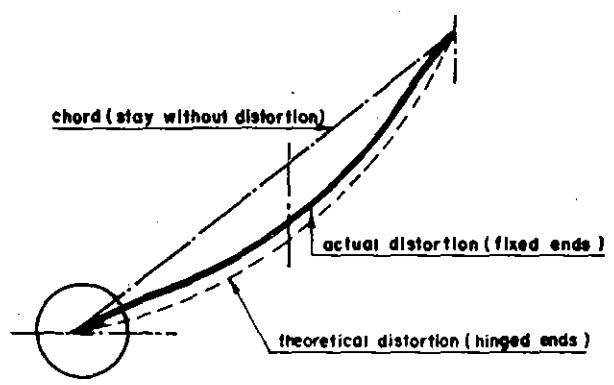


Fig. 38.5 *Distortion of a cable-stay*

x distant from the anchorage, as a function of the angle α between the tangent at the fixed end and the tangent at the distortion (Fig. 38.6).

$$M = \alpha\sqrt{EI \cdot F} \cdot e^{-kx} = \sqrt{EI \cdot \sigma_t A} \cdot e^{-kx}$$

and

$$k = \sqrt{\frac{F}{EI}}$$

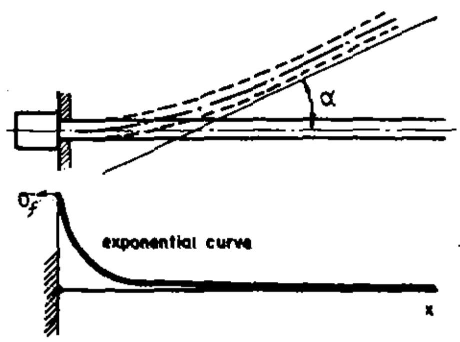


Fig. 38.6 *Distorted stay cable at its extremity and corresponding bending stress in terms of α*

where F: Force in the stay cable
E: modulus of elasticity of steel
I: inertia of the stay cable.*

* The inertia I to be considered is:

— Either n times the inertia of one of the n constituent wires of the strands, when they are independent of each other, which is the case with strands grouted with a flexible product inside a sheath, or individually protected strands.

— Or the inertia of the entire group of constituent wires when they are bounded together by a stiff, adhesive product, e.g., cement grout.

The corresponding 'bending stress' is equal to: $\sigma_f = \dfrac{M}{I} \cdot \dfrac{d}{2}$

giving:

$$\sigma_f = 2\alpha \frac{d}{4i} \sqrt{E\sigma_t} e^{-kx}$$

where

i = radius of gyration corresponding to 'd' ($i = \sqrt{\frac{I}{A}}$)

σ_t = 'axial stress' in the stay

$$k = \sqrt{\frac{F}{EI}} \text{ or } k = \frac{1}{i}\sqrt{\frac{\sigma_t}{E}}$$

$$\left(\because \sqrt{\frac{F}{I}} = \sqrt{\frac{\sigma_t \cdot A}{I}}\right)$$

d = diameter of reference**

** The diameter d to be considered is:

- either, the diameter of one constituent wire when the wires of the strands and independent of each other e.g., strands with flexible injected protection.
- or, the diameter of the 'group' of constituent wires when they are bonded together by a stiff, adhesive product, e.g. cement grout.

If one assumes a uniform circular cross-section for the stay, then: $i = d/4$

so that:

$\sigma_f = 2\alpha\sqrt{E\alpha_t} \cdot e^{-kx}$

From its maximum value $(\sigma_f)_{\max}$ at the fixed end ($x = 0$) σ_f decreases exponentially:

$\sigma_f = (\sigma_f)_{\max} e^{-kx}$

where $(\sigma_f)_{\max} = 2\alpha\sqrt{E\alpha_t}$ = 'reference bending stress'.

For angles of deflection of the order of 10^{-2} to 10^{-3} radians and

$$E = 200\ 000 \text{ MPa and}$$

$$\sigma_t = 500 \text{ MPa, we get}$$

$$(\sigma_f)_{\max} = 20 \text{ MPa for } \alpha = 10^{-3} \text{ radian, and}$$

$$(\sigma_f)_{\max} = 200 \text{ MPa for } \alpha = 10^{-2} \text{ radian.}$$

Thus the bending stress 'variation' for restrained rotations at the fixed end may be of the same order of magnitude as the axial stress 'variations'. Taking this observation into account, it is recommended that adequate steps be taken to reduce to a minimum the bending stresses at the anchorage position.

Local reinforcement of the stay by means of a tube of appropriate thickness, bonded to the wires, is one of several solutions. Another method, both economical and effective, which is used on Freyssinet stay cables, consists in passing the cable through a guide situated at a distance l from the anchorage. The reduction in bending stress is quite considerable, once the distance is sufficiently great, as compared with the design diameter d. Figure 38.7 values of reduction coefficients β_A (A = anchorage) and β_G (G = guide).

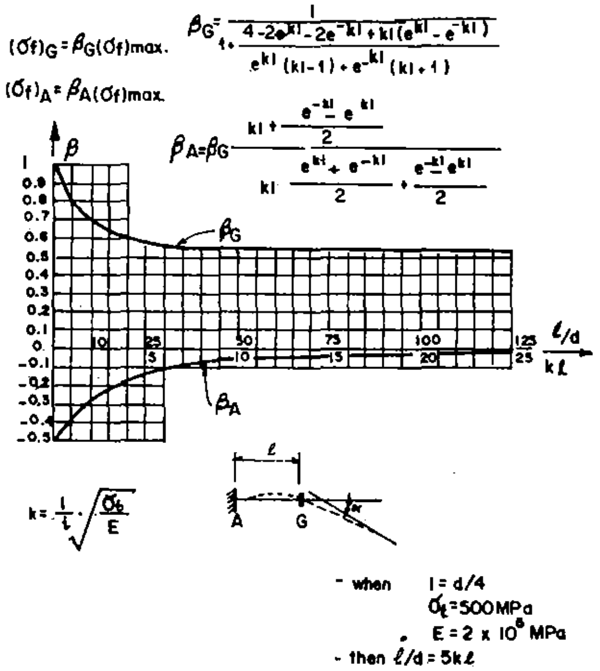


Fig. 38.7 *Reduction coefficients β_A and β_G for bending stress σ_f*

For a ratio $l/d = 30$, the bending stress at the anchorage position becomes only 10% of the reference bending stress $(\sigma_f)_{\max}$ and the reduction of stress at the guide position is approx. 50%.

Figure 38.8 shows two cases of bending stress distribution:

(i) The first is that of a stay composed of wires bonded together (e.g. by rigid grout) where d may be taken as being equal to the external dia. 0.20 m and where a guide is placed at 2.00 m from the anchorage. In this case $l/d = 10$ and the residual value of the bending stress at the guide is about 69% of the reference bending stress. This value could be further reduced by local strengthening of the protector tube.

(ii) The second case is that of a stay composed of independent strands (i.e. with flexible protection), where d may be taken as being equal to the diameter of a constituent wire (0.005 m for a 15 mm dia. strand), and where the guide is placed at 2.00 m from the anchorage.

In this case $l/d = 400$, and the bending stress at the guide position is 51% of the reference bending stress, which may be considerably reduced by using a guide of adequate length and nature: the peak of the curve is flattened out and the residual values is very low.

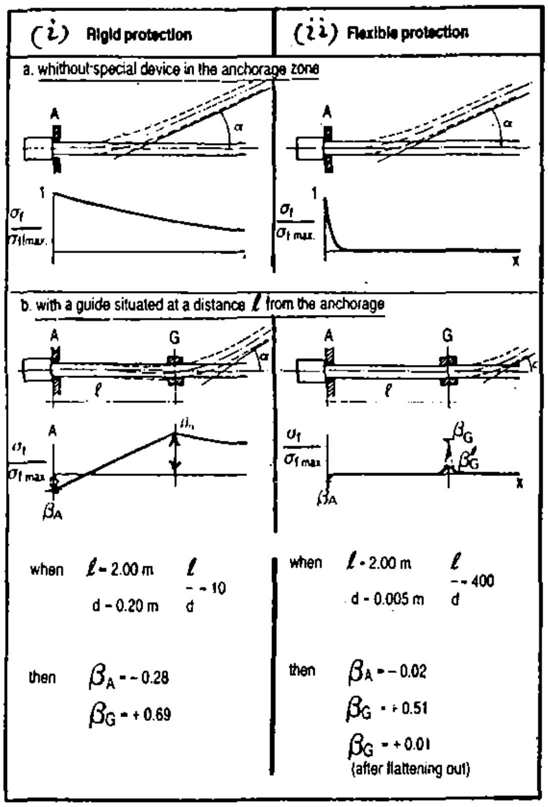


Fig. 38.8 *Bending stress distribution according to the protection type*

This brief analysis shows that, without special precautions, the bending stresses are far from negligible and that it is necessary to provide devices capable of reducing them considerably

38.7 CABLE STAY RELIABILITY

Cable-stays, the principal elements of cable stayed bridges, must present a total operational guarantee through their design and type. It is for this reason that the protection against corrosion must be particularly well designed and carefully executed.

Furthermore, to be in a position to easily and rapidly repair a cable stay damaged by a traffic accident that is never absolutely unavoidable, it is strongly advised to have cable-stays that can be replaced without impairing the structure of either the pylon or the deck.

38.8 GENERAL CABLE STAY ARRANGEMENT

The technique of modern cable-stays, derived from that of prestressing cables, also used construction procedures inherited from prestressed concrete and particularly the cantilever construction method, as indicated earlier.

In this method of construction, the distribution of the cable-stays is consistent with the progression, connected with the length of segments, and this determines the strength required of the cable stay tendons. The length of deck supported by a cable stay system is in the order of 5 to 10 metres for concrete decks and 10 to 20 metres for steel decks. The load capacity of each cable stay can be between 200 tonnes and 1000 tonnes, or even more, during the operational phase. A cable stay generally supports between 50 m^2 and 250 m^2 of concrete deck and 100 m^2 to 500 m^2 of steel deck.

The 'arrangement' of the cable-stays determines the design of both the pylon and the deck.

In the transverse section of the bridge the staying may be formed by a single axial layer or two lateral layers, depending upon the carriageway layout and the width of the bridge (Figs. 38.9 and 38.10, respectively):

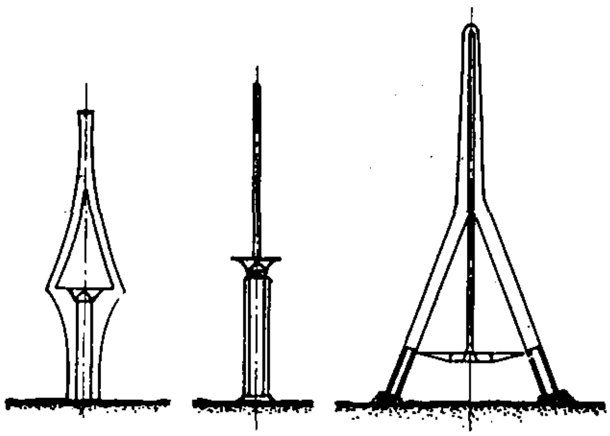


Fig. 38.9 *Transverse arrangement of pylons... One axial layer of stays*

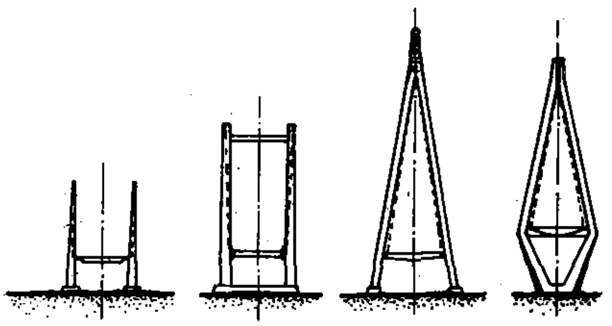


Fig. 38.10 *Transverse arrangement of pylons... Two lateral layers of stays*

- Two layers (or two planes) of stays are recommended:

— when the deck has only two lanes, so as not to separate them by the otherwise single axial set of stays, hence avoiding traffic jams in the event of an accident
— when the deck carries and odd number of lanes
— when the bridge is very wide (so that two-plane pick-up is better).

- In the other cases, and particularly for carriageways with two lanes in each direction, the solution consisting of an axial layer is possible:
  — A central, i.e. single (axial) layer implies a box-girder deck (Fig. 38.11) in order to withstand twisting effects due to asymmetrical load variations. In this case, the pylon may be either a single mast (BROTONNE Bridge) or an inverted-Y (COATZACOALCOS Bridge) with the cable-stays anchored at the top, the splayed legs absorbing transverse forces when imposed by local conditions (wind, earthquake, etc.).

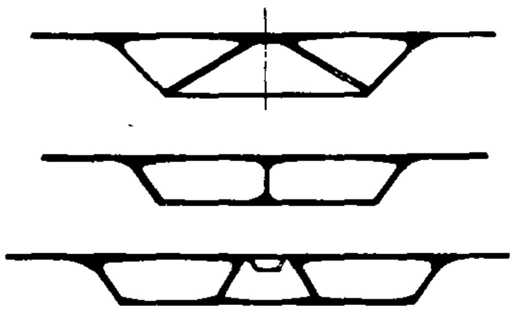


Fig. 38.11 *Cross-section of the deck for one axial layer of stays*

- Two lateral layers result in a layout with two longitudinal bearing systems at right angles to the layers in the form of lattice or plate girders for steel bridges or ribs in the case of a concrete deck, the two bearing systems being connected together (Fig. 38.12).

The pylon in this case has:
— either two independent masts, that may or may not be braced at or near the top, depending upon transverse stresses
— or the form of an inverted-Y or A, the cable-stays in this case being anchored in the vertical portion of the Y or along the inclined portions of the A.

Longitudinally, three typical layouts are encountered (Fig. 38.13):
- fan type
- harp type
- mixed type

The choice of the system is derived from a compromise between a purely economical pursuit and an aesthetic pursuit.

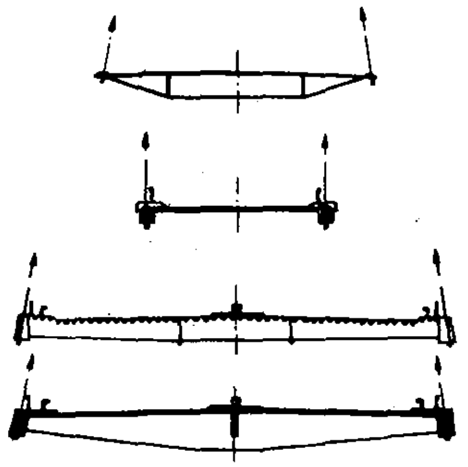


Fig. 38.12 *Cross-section of the deck. Two lateral layers*

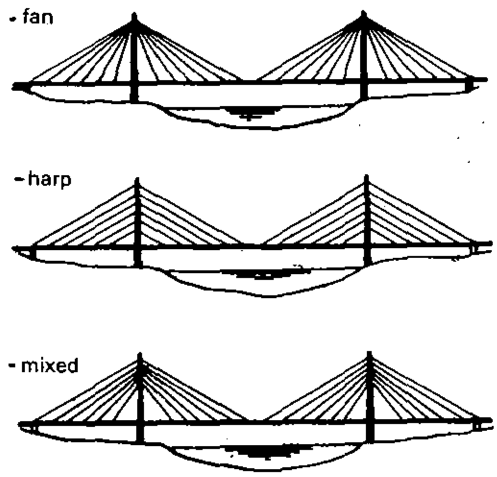


Fig. 38.13 *Longitudinal arrangements*

The fan type solution is more aesthetic and is, as a rule, the most economical for a pylon slenderness ratio $h/l \leq 0.30$ (Fig. 38.14), because for an equal pylon height, the average inclination of the cable-stays is obviously greater in fan system, and the average stress in the cable-stays is, therefore, lower. They are, however, longer but with a more favourable anchorage cost repercussion. But, as the cable-stays converge towards a single point at the pylon top, the arrangement becomes very complex and any subsequent stay replacement is difficult.

It is for this reason that a 'mixed' solution is often adopted, by spreading the anchorages along the mast according to their dimensions, closer to the top.

The structural response of the pylon varies, depending on the chosen solution: the fan-type solution increasing buckling problems obviously (greater effective strut length),

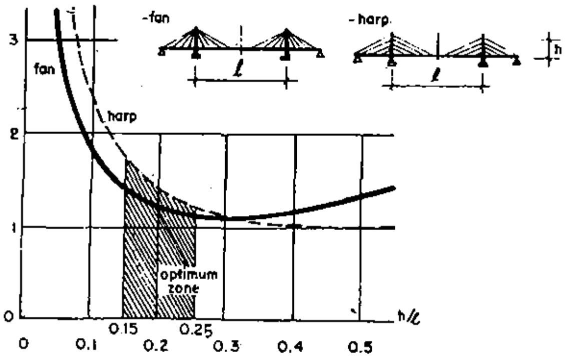


Fig. 38.14 *Compared weight of stay cables in terms of slenderness*

the harp-solution increases the bending moments. This has to be understood very clearly.

Arrangement at the Pylon

The transfer of forces from the cable-stays to the pylon may be achieved by three different arrangements:

(a) A 'saddle', permitting the continuity of the cable stay (Fig. 38.15)

(b) The crossing of rectilinear cable-stays 'inside' the pylon (Fig. 38.16)

(c) 'Relay' devices incorporated into the pylon, a relay connecting the upper anchorages of associated cable-stays (Figs. 38.17 and 38.18).

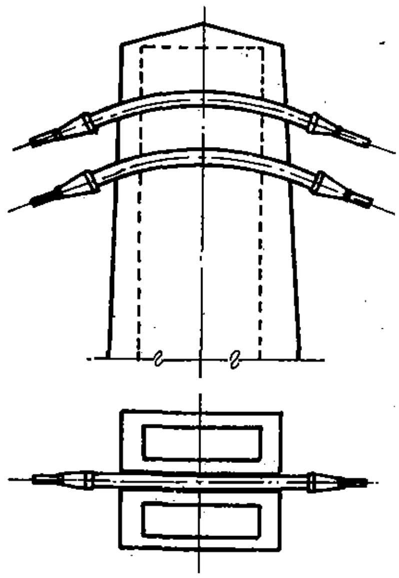


Fig. 38.15 *Saddle*

(a) The merits of the saddle are continuity and the elimination of upper anchorage but the delicate sequencing of the strands at installation results in preference being given in certain cases to systems with upper and lower anchorage that ensure improved control to fatigue endurance uniformity. The eventual 'replacement' of cable-stays is more difficult with saddle arrangement than in the other solutions. A longitudinal retaining system must be incorporated with the saddle because of the asymmetry of forces (Fig. 38.15).

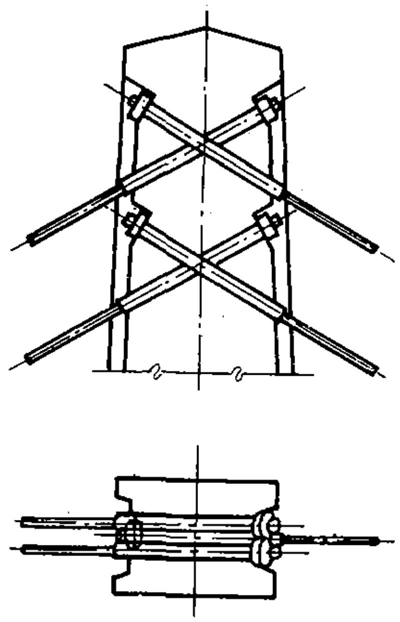


Fig. 38.16 *Crossing of stay cables*

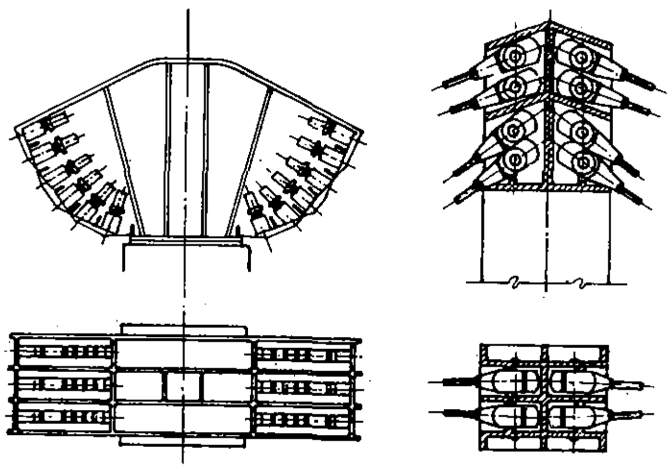


Fig. 38.17 *Incorporated relay device. Grouped anchorages (FAN system)*

(b) The crossing of rectilinear cable-stays inside the pylon is a simple solution for the installation and eventual replacement of a cable stay. It does not

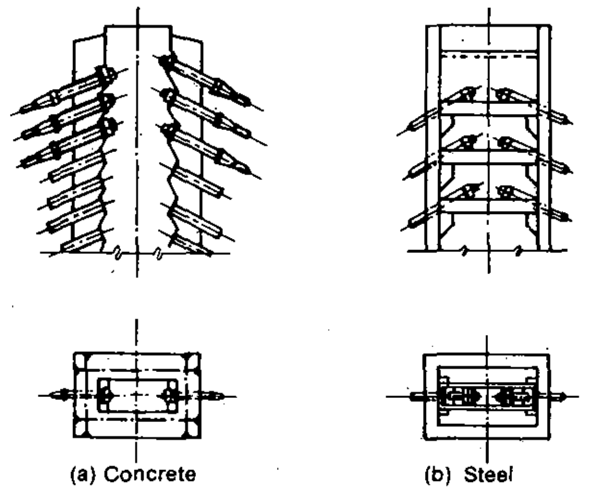


Fig. 38.18 *Incorporated relay device. Distributed anchorages*

permit a symmetrical arrangement of the cable-stays on either side of the pylon (a simple layer on one side and a divided layer on the other), and it also causes twisting moments in the pylon by couple alternation (two opposite cable-stays cannot cross over in the same plane), if an asymmetric solution is adopted (Fig. 38.16).

(c) There are many types of 'relays' that can be incorporated into the pylon, as for example:
- the concrete of the pylon itself compressed by cables or short prestressed bars [Fig. 38.18(a)].
- a coupling (connecting the two anchorages) consisting of steel sections or prestressed concrete [Fig. 38.18(b)], a multiple connecting box.
- a special saddle connected to the upper ends of the cable-stays by 'couplers'.

Arrangement of Stays at Deck Level

The first cable stayed bridges comprised relatively of 'few and rather widely spaced apart stays'. The cantilever construction method, the desire to limit cable stay tendon capacity, and also the possibility of 'rapidly' being able to replace a cable-stay without causing excessive stresses in the deck, resulted in bridge designers adopting a 'closely spaced' cable stay arrangement.

Depending on the type of cable stay chosen (a central plane layer of stays or two lateral planes or layers of stays) access to the anchorage will be either from inside the box-girder or externally by means of mobile platforms.

Furthermore, certain special arrangements must be made depending on the case:

- special protection of the cable stay over a certain height from the carriageway
- incorporation of special vibration-damping system

38.9 DIFFERENT TYPES OF CABLE-STAYS

The first cable-stays used the same cables as for suspension bridges, particularly locked-coil cables. The latter consists of stranded cables with an outer layer of Z-shaped wires that are interlaced to form a locked unit. These cables were factory made including the anchorages which are formed by pouring metal into a socket between the flared ends of wires. The yield point fluctuating between 1100 MPa and 1500 MPa, depending on the elements employed. The modulus of elasticity is in the neighbourhood of 150,000 MPa (1.5×10^6 kg/cm^2) Fig. 38.19.

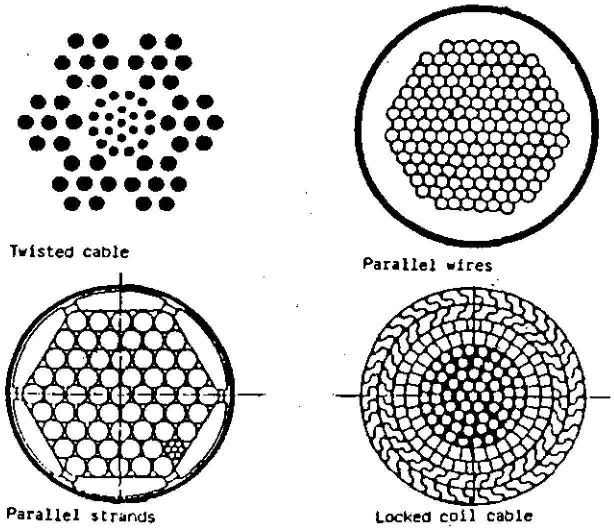


Fig. 38.19 *Different types of cable-stays*

The first cable-stayed bridges were mainly of steel, the stress 'variations'—as is the case with these decks—were high, and the locked-coil cables were not entirely satisfactory.

Parallel wire cables, evolved from prestressing techniques, were then proposed and used after having successfully undergone fatigue tests.

During the end of the sixties and throughout the seventies 7-wire strands were widely developed for prestressing and for this reason the research work undertaken on cable-stays resulted in the use of strands with a very high yield point (1860 MPa) and excellent endurance qualities and a modulus of elasticity similar to that of the wires.

BROTONNE Bridge was one of the first large cables stay bridges to use strands. It was followed by RANDE Bridge in Spain, and COATZACOALCOS Bridge in Mexico.

38.10 THE FREYSSINET CABLE STAY

Freyssinet stays consist of a bundle of parallel strands of 15 mm (0.6″) dia., of the same type as currently used for prestressed concrete. For this application, however, the strand must be properly selected for its fatigue strength. Generally, good quality steel will have a fatigue performance which makes it suitable for this purpose.

Over the free length, the strands are enclosed in a tube, generally of polyethylene, which allows the threading and grouting of the stay. A spiral of steel wire inside the polyethylene sheath ensures that proper grout cover is provided around the bundle of strands (Fig. 38.20).

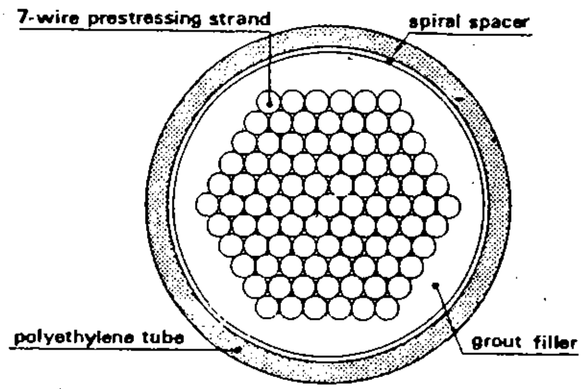


Fig. 38.20 *Section of a Freyssinet cable stay*

The cable-ends are locked into anchorage devices which have been specially designed for this purpose, incorporating technological features which have proven their security and economy within the limits of prestressed concrete techniques.

The strands are individually anchored in the blocks by means of jaws or swaged sleeves, depending on stressing and installation procedures.

Other components of the anchorage devices are:

- a trumpet, which guides the strands to the anchor block with the appropriate angle and spacing.
- a trumpet extension (heavy steel pipe), the length and gauge of which have been selected in order to reduce the range of stresses transmitted to the anchorage.
- a light pipe used to extend the stay anchorage device for structural reasons.
- a neoprene damper is generally inserted between the light steel pipe and the structure in order to restrict wind-induced oscillation in the stays.

The stay anchor exists in two versions: fixed and adjustable. The adjustable type is provided with an outer thread which allows fine adjustment of the stay tension by turning a collar. Both versions can be used at the upper or lower end of the cable stay.

The Cable Stay Comprises of 3 Types of Zones (Fig. 38.21):

(a) The free length zone.
(b) The transition zones (between the anchorages and the free length zone, particularly concerned by the problems of flexural fatigue).
(c) The anchorages themselves.

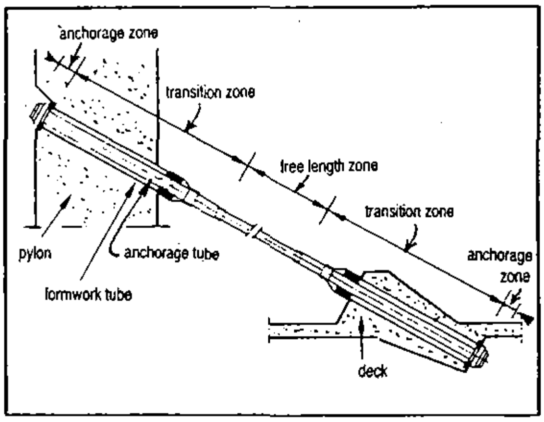


Fig. 38.21 *General layout of the Freyssinet cable-stay*

(a) *The free length zone* consists of a bundle of 15 mm (nominal) diameter parallel strands, number of strands in the bundle depending on capacity required. (See ahead for 'Protection of Stay' for purpose of protecting the stay.)

(b) *The transition zone* comprises essentially of:
- an 'anchorage-tube' connected to the anchorage, of equal or greater length than the formwork-tube, permitting sliding (and hence the complete stay-structure independence) to allow for re-stressing, adjusting or destressing if necessary, both during construction and during the service life of the bridge.
- a 'guide' placed at the end of the anchorage tube. Its functions are:
  - o to reduce bending stresses in the cable at the anchorage outlet and at the guide outlet.
  - o to reduce the dimension of the cable as it issues from the structure.
  - o finally, to act as a barrier between the transition zone and the free length zone in the case where the types of protection are different in the two zones (for example: flexible in the transition zone, rigid in the free length zone).

The type and the dimensions of the guide are such that bending stress variations are rendered negligible compared to the axial tensile stresses in a cable-stay with flexible protection, and to within controlled limits (by means of appropriate dimensioning of the anchorage-tube) in the case of rigid protection (as explained earlier in some detail).

(c) *The anchorage itself* is the essential element, being naturally the most sensitive part with regard to fatigue endurance in the absence of special precautions.

The design of anchorages for cable-stays is based on the following ideas:

- use of prestressing techniques by anchoring the strands with jaws specially designed for their fatigue endurance,
- arranging the anchorage and the transition zone so as to obtain the perfect distribution of strands, eliminating 'contact' between the strands. (These 'contacts' are responsible for the group effect that reduces the fatigue strength of a bundle compared to that of its components.) This is made possible:
  - o by the use of strands, each having high yield point and fatigue endurance qualities, hence a limited number of strands to obtain a given force and the possibility to sequence these strands,
  - o by the miniaturization of jaws (made possible by using high-performance materials to manufacture the anchorage), allowing a reduction of the centre to centre distance of strands, and, therefore, of the deviations.

Protection of Stay

This is an essential operation to guarantee a dependable service life. In most of the existing modern cable stay bridges, the protection consists of injecting cement grout, a technique derived directly from prestressing.

Through experiences on numerous nuclear containment vessels and cable-stay bridge sites, special equipment complying to the requirements of international specifications related to grouting have been developed. This has not prevented the continuation of research work to reduce grouting operations on site to a strict minimum. This is in fact the case for cable stays with 'flexible' protection, where the free length zone consists of self-protected sheathed strands and where grouting is reduced to the transition and anchorage zones.

A 'flexible' product of the epoxy pitch, grease, wax, etc., ... type is then used.

For the protective elements a choice between the different protection systems (Fig. 38.22) may be classified as follows:

- rigid type protection, for example:
  * bare strands in a steel sheath injected with cement grout.
  * same arrangement, but in a high density polyethylene sheath (PEHD)
- flexible type protection, for example:
  * bare strands in a PEHD sheath grouted with epoxy pitch, grease or wax
  * individually sheathed strands (in PEHD) grouped together either by hose clamps and placed in an eventual outer sheath, or by a spirally wound band.

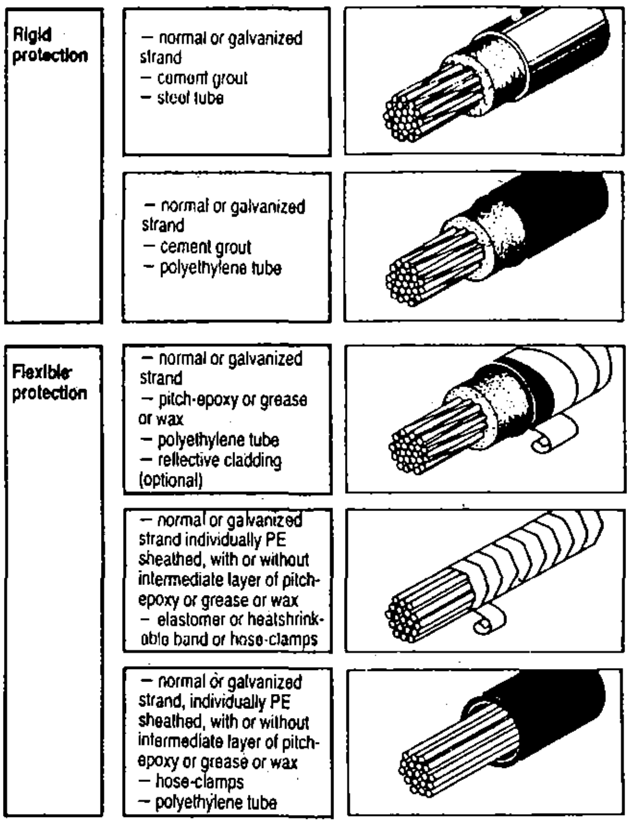


Fig. 38.22 *Various types of protection*

Depending on the environment, a 'reflective coating' may also be applied to the protecting surface to avoid radiation problems.

Durability of the cables is a major concern for the stay cables exposed to ambient atmosphere. Extensive use of highly stressed steel tendons all over the world in bridges even at marine locations discounts inhibition, but leaves little room for complacence.. Bare ropes, protected by mere painting or by mere galvanising may be vulnerable to corrosion. In the Kohlbrandt cable stayed bridge in Germany, all stay cables of locked coil ropes had to be replaced in 1979 after about five years of service due to corrosion damages.

A more effective protective system for parallel wire cables comprises two successive barriers—the first one is the PE tube sheathing and the second is cement grout injected into the annular space round the wire bundle.

Grout continuity and no damage to the duct is ensured by rigid controls on grouting pressure and grouting sequence. An epoxy enriched cement is used. In some cases like Brotonne, France, or Mainbruck, Hochst, West Germany, steel tubes have been used for sheathing. Rigid steel tubes have some advantages in contributing to stiffness. PE tubes, however, offer comparative ease in handling and are generally preferred. The typical thickness of a 160 mm PE tube is 8.6 mm and the duct is not sensitive to handling, flexing and lateral pressures. The corrosion protection effectiveness of the cement grout is reported to have been tested by BBR for the Schiller-Strasse bridge, after 12 years of service. A high pH value of pore water in dense grout, rust free cable-surface, and no degradation in mechanical properties of the PE tubes were confirmed.

Going by the above evidence, the durability life of the stay cables can be predicted as 'high'. However, a modern system of deck and tower anchoring does provide for easy replacement of stays to cater for accidental damages.

38.11 BBR STAY AND ANCHORAGE

BBR-HIAM (high amplitude) parallel wire cables, *with their own anchorage system*, have been developed to perfection by the combined efforts of the famous Swiss firm BBR (well known for its prestressing system) and the German consulting firm Leonhardt and Andra. The individual wires forming a parallel wire stay bundle are no different from the familiar 7 mm wires used for prestressing. The wires are bundled into a hexagonal section shape and enclosed by a PE (polyethylene) duct. A strand, 10 mm in diameter, is wound round the wires as a spacer. The wires are provided with a variant of BBR buttonhead anchorage. The buttonheads can provide adequate static strength. But, in order to statisfy high fatigue strength demands, the anchorage system has some special features. The wires in the bundle are flared out inside a steel casing before being held at a steel plate by usual BBR buttonheads. The conical space inside the casing is filled with a mixture of small steel pellets of 1.5–2.0 mm diameter and epoxy resin.

The cold cured mass in epoxy matrix, which carries the tendon load by arching action developed in the pellets, is claimed to ensure anchorage efficiency with superior fatigue. In the molten-zinc-filled-sockets of ropes, used earlier, the steel was damaged due to high temperature. A degradation of elastic modulus and a stress range as low as 100 MPa resulted. The Freyssinet and the HiAm BBR anchorages, in sharp contrast, do not reduce either the static or fatigue strength of the wires. Conclusive evidence have been derived from static and dynamic tests in which failure has invariably occurred in free length of wires. The static and dynamic properties of the stay cables are, therefore, determined by the mechanical properties of the constituent wires.

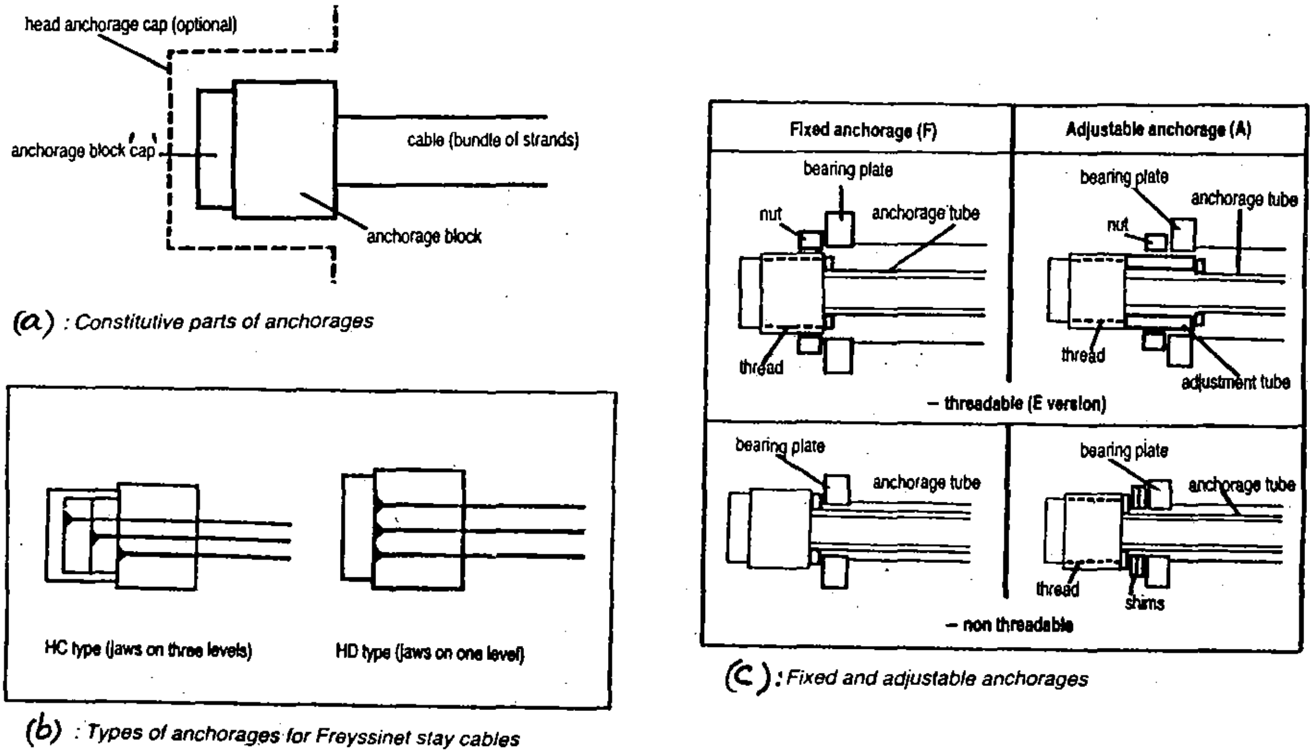


Fig. 38.23 *Constitutive parts of the anchorage*

38.12 FREYSSINET STAY ANCHORAGE

Two ranges of anchorages are proposed by the *Freyssinet System:* the HC anchorage and the HD anchorage (Fig. 38.23).

- The HC anchorage features a staggered arrangement of the jaws in 3 levels, resulting in the virtually complete suppression of deviations. The resistance of the anchorage to fatigue is practically that obtained with a mono-strand anchored in the same conditions. (The HC anchorage was used for the COATZACOALCOS Bridge in Mexico.)
- The HD anchorage, which is based on the same principles, comprises of a single level of jaws. It has a slightly large diameter than the HC anchorage, but is more rapidly installed.

For both types of anchorage, a cap covers the rear-end to ensure impermeability when grouting. A second cap filled with grease may be added to protect the emergent extremity of the anchorage.

Finally, to allow for re-stressing operations, two adjustable load transfer systems are proposed:

- by means of screwing a threaded collar on a threaded adjustment tube, placed in front of the anchorage
- by means of inserting shims, in a simplified version.

In both systems, the anchorage block at the extremity of the cable stay where re-stressing is carried out is itself threaded, enabling the whole anchorage assembly to be pulled i.e. stressed without direct re-stressing of the previously stressed and eventually grouted strands.

The E version (slip-on) allows the front of the anchorage without its threaded collar to pass through the bearing plate for the installation of prefabricated cable-stays.

38.13 STRENGTH OF THE STAYS

In typical cable stayed structures the stays are subjected to large load variations so that their static strength as well as their fatigue strength become critical, as indicated earlier.

Stays may be assembled from strand conforming to current standards for prestressed concrete strand, but additional fatigue requirements have to be considered.

Static Strength

The stays and their anchorage devices have been so conceived that their ultimate static strength can be considered equal to the sum of the guaranteed ultimate loads of the strands of which they are composed. The ultimate strength for instance of FREYSSINET cable-stays is obtained through bond and wedge action.

Table 38.1 gives some physical characteristics and ultimate loads for the FREYSSINET range of stays. It has been prepared for use with strand conforming to various standards and corresponds to cables containing the full number of strands which the anchor can hold. It is obvious that a stay may contain a lesser number of strands than its anchors allow for.

Operating Stress in the Cable Stay

The 'operating stresses' of cable-stays are traditionally limited to a fraction of their characteristic ultimate tensile stress, the normally imposed values vary between 40–45% of the characteristic ultimate tensile stress.

This percentage is lower than that considered in standard structural steel work because of the enhanced fatigue problems here, mentioned previously. Also, higher stresses would give rise to ductile cable-stays (increased deflection), ***and it is necessary to have a sufficient margin for the transfer of cable stay reactions to the neigbouring cable-stays in the event of changing a stay.***

A closer analysis of deformability and fatigue would enable in many cases, particularly for road bridges with concrete decks, to increase the maximum stress. It is not correct to go beyond 50% of the characteristic ultimate tensile stress, as long as this value has not been ratified by

Table 38.1 *Some characteristics and ultimate load for the Freyssinet stay size**

Type of Strand		Single Strand	Freyssinet Stay Size*: 19H15	27H15	**37H15**	48H15	**61H15**	75H5	**91H15**
15.24 mm (0.6″).	A	140	2660	3780	5180	6720	8540	10500	12740
grade 270. to	Afpu	260.7	4953	7039	9646	12514	15902	19552	23724
ASTM A-416	g	1.102	20.94	29.75	40.77	52.90	67.22	82.65	100.28
15.7 mm super	A	150	2050	4050	5550	7200	9150	11250	13650
strand to	Afpu	265	5035	7155	9805	12720	16165	19875	24115
Euronorm EU 138	g	1.19	22.42	31.86	43.66	56.64	71.98	88.50	107.38
15.2 mm. class III	A	139	2641	3753	5143	6672	8479	10425	12649
to French Circular	Afpu	252.1	4790	6807	9328	12100	15378	18907	22941
N° 73–175	g	1.091	20.73	29.46	40.37	52.37	66.55	81.82	99.28

A—nominal cross section in mm^2; Afpu—ultimate force of the element in kN; g—nominal weight in kg/m.

* Recommended standard sizes are indicated in bold characters. The other sizes can be envisaged when quantities justify.

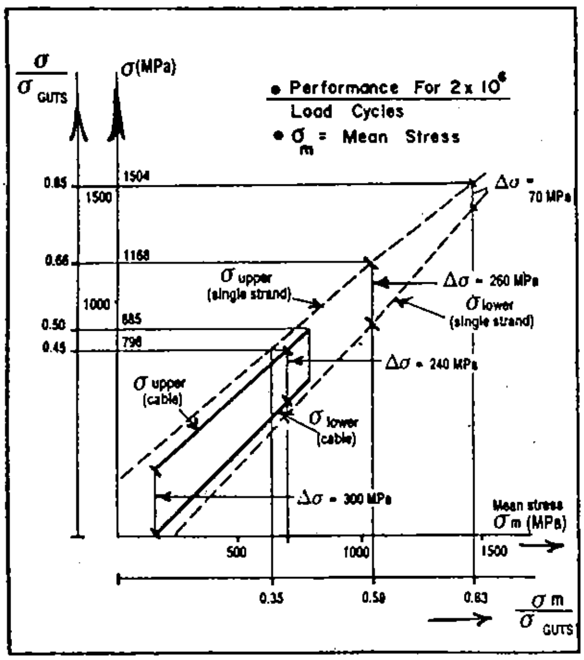
fatigue experience. (For strands, the characteristic ultimate tensile stress is sometimes converted to their 'breaking load' — this stress times the actual steel section area of the strand, and referred to as such.)

'Upper Stress' Limited by Fatigue Endurance Performance

The endurance of the constituent strands should be close to that of the total cable stay itself and the 'group effect' should be relatively low.

The 'anchorage performance' also depends on the 'strand performance', and the latter itself is the subject of specifications with relation to fatigue.

Generally, the steel suppliers draw-up 'WÖHLER curves' and 'SMITH diagrams', latter deduced from a group of WÖHLER curves (Fig. 38.24).



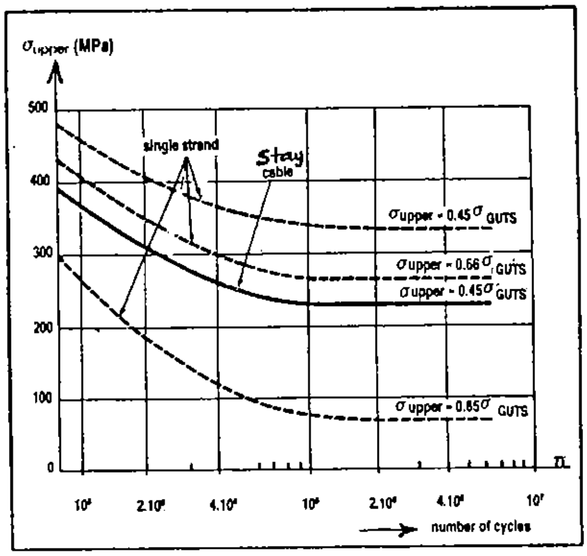
(a) Compared Smith's Diagrams for a Single Strand Dia. 15mm & a Stay Cable. Number of Cycles 2 x 10⁶

Fig. 38.24 *(a) Smith's diagrams*

The performance of stay-anchorages is also presented in similar forms from unit tests and tests on complete anchorages.

An interesting point concerning anchorage performances is their ability to support a stress variation $\Delta\sigma$ of 240 Mpa for 2 million cycles with a maximum stress of 0.45 σ_{GUTS}, i.e. 0.45 RG. (σ_{GUTS} or RG, as defined earlier, is the characteristic ultimate tensile stress.)

From this experimental data, the upper stress (expressed as a fraction of σ_{GUTS}) may be plotted against the ratio 'q' of extreme stresses (Fig. 38.25) where



(b) Wöhler's Curves for a Single Strand Dia. 15mm and a Stay Cable with Freyssinet anchorages

- σ_{GUTS} = RG = *Characteristic or Guranteed ultimate tensile strength*
- σ = *stress in tendon*
- $\Delta\sigma$ = *stress variation*
- $\sigma_{upper} = \sigma_{maximum}$ (or $\sigma_{max.}$)
- $\sigma_{lower} = \sigma_{minimum}$ (or $\sigma_{min.}$)

Fig. 38.24 *(b) Wöhler's curves*

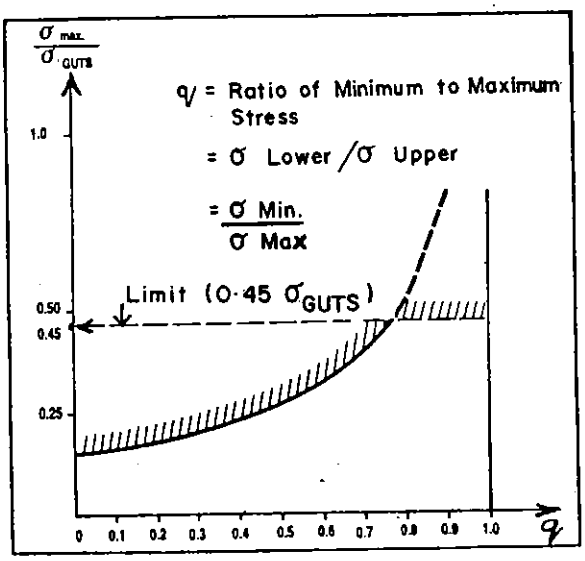


Fig. 38.25 *(Maximum stress in terms of σ_{GUTS}) vs.'q'*

$$q = \frac{\sigma_{min}(\text{or } \sigma_{lower})}{\sigma_{max}(\text{or } \sigma_{upper})}$$

and also against the stress variation $\Delta\sigma$ (Fig. 38.26) where $\Delta\sigma = (\sigma_{max} - \sigma_{min})$ or $(\sigma_{upper} - \sigma_{lower})$.

These diagrams have been limited to the maximum stress (σ_{upper} or $\sigma_{max.}$) of 0.45 RG (i.e. $0.45\sigma_{GUTS}$). They do not include a safety factor, this being left to the designer and state of art applicable to the concerned site and country.

The margin for this safety factor may be obtained either by reducing the 'amplitude' of allowable stress (for a realistic amplitude of variable loads causing fatigue), or by increasing the variable loads, or partially on both.

38.14 SPECIFICATION OF STRANDS FOR CABLE-STAYS

The strand used for stay cables must comply with the requirements of one of the following standards:

- ASTM A 416-80
- EURONORM 138-79
- B.S. 5896: 1980
- any other equivalent standard

They must in all events comply with clause relative to fatigue in B.S. 5896: 1980, or in EURONORM 138-79, that is to say ability to withstand 2.10^6 cycles with:
$\sigma_{upper} = 0.80$ of actual breaking stress and $\Delta\sigma = 195$ MPa.

Standard	ASTM A 416-80	EURONORM 138-79	BS 5896: 1980
Strand designation	• 600 strand Grade 270	Strand Fe-7S. 1770-15.7-Relax 2 EU 138/6	BS 5896/3 Super strand-1770 15.7-relax 2
Nominal diameter (mm)	15.24	15.70	15.70
Nominal tensile strength (MPa)	1862	1770	1770
Nominal steel area (mm^2)	140	150	150
Nominal weight (kg/m)	1.102	1.180	1.180
Specified characteristic breaking load (kN)	260.7	265.0	265.0
Specified characteristic 0.1% proof load (kN)		225.0	225.0
Load at 1% elongation (kN)	221.5	233.0	233.0
Minimum elongation at max. load (%)	3.5	3.5	3.5
for L = (mm)	610	500	500

Dimensions and properties of strands used for Freyssinet stay cables (L = length of test piece).

38.15 SPECIFICATION OF ANCHORAGES FOR CABLE-STAYS

All the component elements of for instance Freyssinet anchorages are designed to ultimate strength requirements.

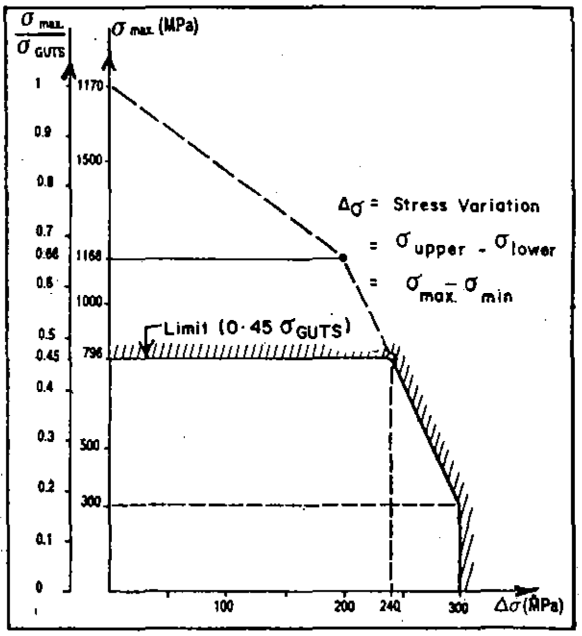


Fig. 38.26 *(Maximum stress in terms of σ_{GUTS}) vs. ($\Delta\sigma$)*

High strength alloy steels are used and are treated in order to give the following mechanical characteristics:

—*Nut and adjusting tube*—steel complying with French standard NF A 35-552

- Breaking strength : 600 MPa min.
- 0.2% proof stress : 400 MPa
- Elongation at rupture : 17% min.
- Charpy U Notch (KCU) : 50 J/sq. cm. min.

—*Anchorage blocks*—steel complying with French standard NF A 35-552

- Breaking strength : 800 MPa min.
- 0.2% proof stress : 600 MPa min.
- Charpy U Notch (KCU) : 60 J/sq. cm. min.

—*Bearing plate*—steel complying with French standard NF A 35-501

- Breaking strength : 490 MPa min.
- 0.2% proof stress : 305 MPa
- Charpy V Notch (KCV) at 0°C : 35 J/sq. cm. min.

38.16 SPECIFICATION OF PROTECTIVE MATERIALS FOR CABLE-STAYS

Stay cables are protected by means of either rigid or flexible materials depending on the design.

The rigid materials are:

—Steel tube complying for example with French standard NF A 49-112 or equivalent. The steel employed has the following characteristics:

- Breaking strength : 540 MPa min.

- 0.2% Proof stress : 300 MPa
- Elongation at rupture : 22% min.

—cement grout for injection of the stays, of the same type as that used for prestressing tendons.

The grout must therefore comply with the specifications and grouting procedures of the latter.

The flexible materials are:

—high density polyethylene tube or coating of the same nature on individually protected strands.

The polyethylene (HDPE) complies for instance with French standard NF T 54–072 class 5.3, or with the equivalent ISO specifications 161/1 and 3607, which require:

- Yield stress : 19 MPa min.
- Elongation at rupture : 350% min.
- Carbon black content : 2.3% ± 0.3%

The internal parts of for instance the Freyssinet anchorage are protected by injecting flexible epoxy resin having the following characteristics:

- Viscosity at 20°C : 1800 mPas (cPs)
- Pot life (for 500 g) : 5 1/2 hr, at 20°C
  : 1 1/2 hr, at 50°C

38.17 SUPPLY OF 'STAY' STEEL

The stay cables may be supplied to site fully fabricated from the factory. ***The sequence of operations for instance for a BBR stay comprises the following:*** 'Individual wires are cut to exact length required, on a special cutting bench. The wires are coated with oil, pulled through a die to form the bundle, wrapped by the spacer strands, and then the anchor head is fitted at one end by button heading. The PE duct is drawn over the bundle and the duct joints are welded. The second anchor head is fitted. Epoxy resin and steel pellets are filled in the end cone held vertically and cured with programmed tempering. The cables are then reeled on bobbins, for shipping to site. Even long hauls from factory to site are not uncommon. For the Novi Sad bridge, the cables were delivered from Switzerland (Stahlton AG) to the Yugoslavian site, by rail, on 48 steel drums, the largest one weighing 18 tonnes.

The wires used for stay cables are of the familiar prestressing steel grade of S 1500–1600 for instance. An equivalent elastic modulus of 1.9×10^6 kg/cm^2 normally ensures adequate stiffness and good dynamic behaviour. ***In view of the critical importance of the mechanical properties, elaborate testing programme, especially for 'stress range' $\Delta\sigma$ has been a significant feature of the cable stayed bridges. A few salient results for two bridges, Novi Sad bridge, Yugoslavia, and Pasco, USA, are given below:***

"... From the 916 coils of wires used for the bridge over the Danube, Novi Sad, 210 specimens were tested. They have been randomly chosen over the whole lot and tested at 5 ranges of stress amplitudes, each having 42 samples. The number of cycles of failure was checked up to a maximum of 2×10^6 cycles. From the results, arranged according to statistical methods, the necessary inference (Wöhler field) of the lot could be drawn".

"... Design criteria are in general taken from the values related to 2×10^6 load cycles assuming that this level represents the material's infinite fatigue resistance. In the case of Novi Sad bridge the intrinsic fatigue strength (variation) of the lot was found to be 378 MPa taking into account a rupture probability of 5% on an upper stress level of 735 MPa. The fatigue strength of the cables can be calculated as 240 MPa with a factor of safety of 1.6. The fatigue damage of these cables subjected to 2×10^6 load cycles in this range will not be more than 5%, even in the case of accidental-overloading of the cables".

"... For the Pasco-Kennewick Bridge two cables with 83 wires each have been tested (with excellent results) at the University of Texas at Austin. The cables were subjected to an elevated temperature creep test, a room temperature creep test, a fatigue test (after 2 million cycles between 0.25 and 0.45 of breaking load, fatigue (stress variation) 160 MPa, no wire broke) and an ultimate strength test upon completion of the fatigue test: a maximum load of 4636 KN was developed corresponding to 1.04 of σ_{GUTS}. Only the first wire broken indicated a fatigue crack, no other wires showed any evidence of fatigue damage. The distribution of wire failures along the cable represents the statistical distribution very closely".

38.18 INSTALLATION OF A CABLE STAY

These may be installed in their final position, by individual threading of strands or by threading the whole bundle of strands, raising the 'cable' which may be prepared on the deck or fabricated/assembled in the factory.

However, in the case of factor prefabrication, the use of a flexible sheath is absolutely necessary because of transportation, and special precautions must be taken to limit the consequences of winding-stresses on a large diameter sheath.

The ease with which certain types of stays (e.g., FREYSSINET cable-stays) are installed renders their factory prefabrication unprofitable, as transportation is more expensive due to the special transport and handling equipment required.

The stay cables may be *installed in many different ways* while in each case providing the same performance guarantees.

They may be installed in their final position equally well:

- either by threading the strands one by one or in a complete bundle;
- or by raising the cable complete with anchorages, prefabricated on site or in the factory.

In fact, the ease with which the Freyssinet stay cables are installed renders it unnecessary, in most cases, to have recourse to prefabrication in the factory — a costly operation due to the special handling and transport equipment which it requires, particularly for the larger range of stays.

The choice between complete in-situ assembly or on-site prefabrication depends on the method and the speed at which the deck is constructed.

The different construction sites in which Freyssinet International has participated provide evidence of the flexibility of the techniques proposed.

'On-site' Fabrication and Installation (Fig. 38.27)

This method has proved to be the most economical due to the limited amount of special equipment required. For example, in the case of a stay with polyethylene sheath, the general scheme of operations is as follows:

1. Installing of the anchorages and anchorage tubes in their final position.
2. Welding together 12 m lengths of polyethylene tube on the deck or in a prefabrication area.
3. Moving into position and lifting the upper end of the pre-assembled sheath by means of a lifting system placed on the pylon.
4. Bringing the lower end as near as possible to the lower anchorage.
5. Threading the first strands through the upper anchorage block, the sheath and the lower anchorage using a single-strand pushing machine.
6. Stressing the first strand to a predetermined value using a monostrand jack. This operation can be carried out either at the adjustable or the fixed anchorage position.
7. Successive threading and stressing of all the strands, as for the first, while checking forces and displacements.
8. Control and adjustment stressing of the complete stay.
9. Protecting the anchorage zones (grouting).
10. Corrosion protecting the free length (grouting, for example).

This method applies to straight stay cables, without deviations, having upper and lower anchorages. In the case of a stay with a deviation saddle the procedure is a little different although based on the same principles of threading with a pushing machine as for Coatzacoalcos bridge.

Prefabrication and Installation of a Cable Stay

Prefabrication theoretically offers the advantages of a more easily controlled production and a reduction of work-time on site.

On the other hand, factory prefabrication is virtually limited to PE sheathed low capacity cables with a small-diameter flexible-sheath so as to limit the transport equipment to reasonable dimensions and weight to comply with drum diameters (latter preferably not greater than 20-25 times the diameter of the cable stay sheath of PE).

For pre-assembled cable stays all the components of the stay, excluding the permanent corrosion protection of the free length, are assembled and controlled in the plant. The main operations are:

Stay assembly

The prestressing steel is cut to length, with temperature corrections as required, within very close tolerances. The polyethylene sheath is pulled through the bundle and welded, and the anchors and other accessories assembled at each end. The anchorage resin compound is poured into the anchorages. The complete stays are finally coiled in reels, reusable or disposable.

Shipment

Loaded reels containing one or more cables may be shipped by truck, rail or ocean freight. Dependent upon the cable sizes and lengths, individual reels may weigh up to about 20 tons.

Erection

At the jobsite the reels are installed in 'dispensers' with automatic breaks. An erection module installed on the top of the pylon, which includes a winch and a 'messenger cable' provided with hangers, are used to bring the cable stay in place. Specially designed tensioning equipment is used to place the anchorages in position and further to adjust to the desired tension.

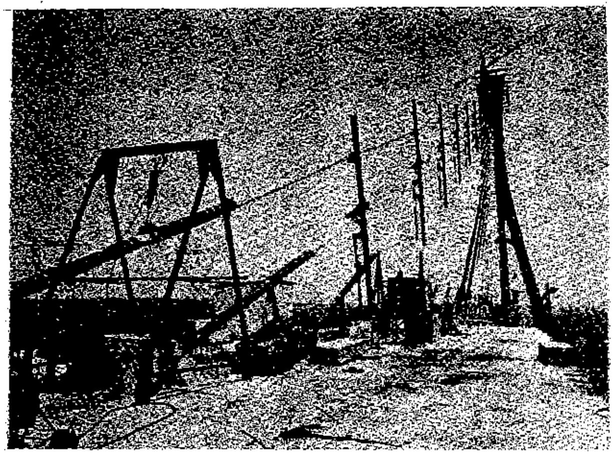
Corrosion protection of the free length

This operation is carried out, as in the case of assembled in-situ stays, by injecting cement grout in the sheath.

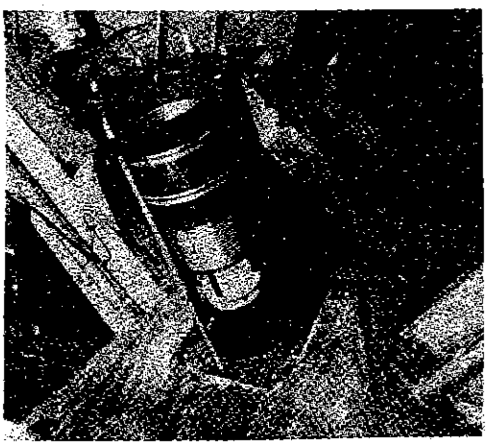
Stressing; Adjusting; Detensioning of Stays

The design of Freyssinet stay cable anchorages offers the same advantages for stressing as those used for prestressed concrete. The cable passes through a centre hole jack and the slack is taken up without the necessity of cutting to a precise length (Fig. 38.28).

Moreover, in parallel-strand cable stays, due to the sequencing of the parallel strands, stressing may be



Installation of stay cables



Installation of a lower anchorage (Freyssinet)

Threading and classifying the strands through the tube (Freyssinet)

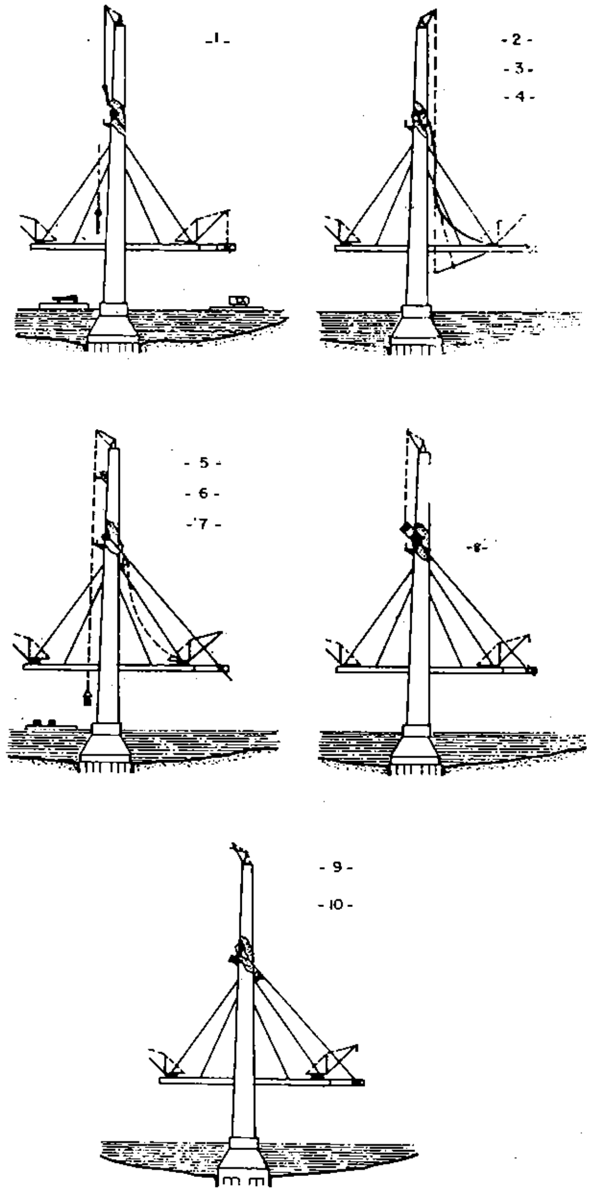


Fig. 38.27 *On site installation (typical scheme of operations)*

performed advantageously on site, strand by strand, with a lightweight easily handled jack. For a semi-prefabricated cable stay (i.e., the one with a simple pre-blocked anchorage) again the stressing may be carried out strand by strand, whereas for a completely prefabricated cable stay (in which both anchorages are pre-blocked) stressing is performed with a jack drawing all the strands simultaneously. Table 38.2 indicates the jacks to be used according to the tendon to be stressed.

Furthermore, an adjusting or a control jack may be used. This jack may remain available to the client for the

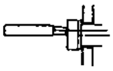
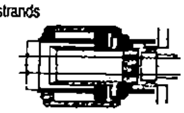
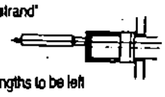
Type of operation and equipment	Type of protection		Installation		
	Rigid protection	Flexible protection	Installation on site	Semi-prefabricated stay cable (one isostatic anchorage)	Prefabricated stay cable (two isostatic anchorages)
• Stressing strand by strand 	X	X	X	X	
• Stressing of all the strands 	X	X	X	X	X
• Adjusting the tension using a special jack 	X	X	X	X	X
• Adjusting the tension using a standard jack with special accessories 	X	X	X	X	X
• Detensioning strand by strand*  adequate jacking lengths to be left		X	X	X	X
• Detensioning of all the strands*  adequate jacking lengths to be left	X	X	X	X	X

Fig. 38.28 *Stressing, adjusting and detensioning of a Freyssinet stay cable*

Table 38.2 *Principal Characteristics of Freyssinet Tensioning Jacks*

Number of strands	Type of jack	Length* (mm)	External diameter (mm)	Weight (kg)	Internal diameter (centre hole jack) (mm)	Tensioning area (cm^2) (ram)	Max. tensioning pressure (bar) kg/cm^2
19	K 350	502	440	430	156	490	625
31 to 37	K 500	718	515	780	203	766	600
55 to 61	K 700	727	640	1160	250	980	625
73 to 91	K 1000	783	770	1450	310	1431	625

* Dimensions without accessories.

supervision of cable stays. It is screwed to the anchorage block and enables restressing. Considering its function (adjustment), the stroke may be limited to reduce the weight.

The stressing procedure is confined to the compliance with the regulations in force in the country concerned.

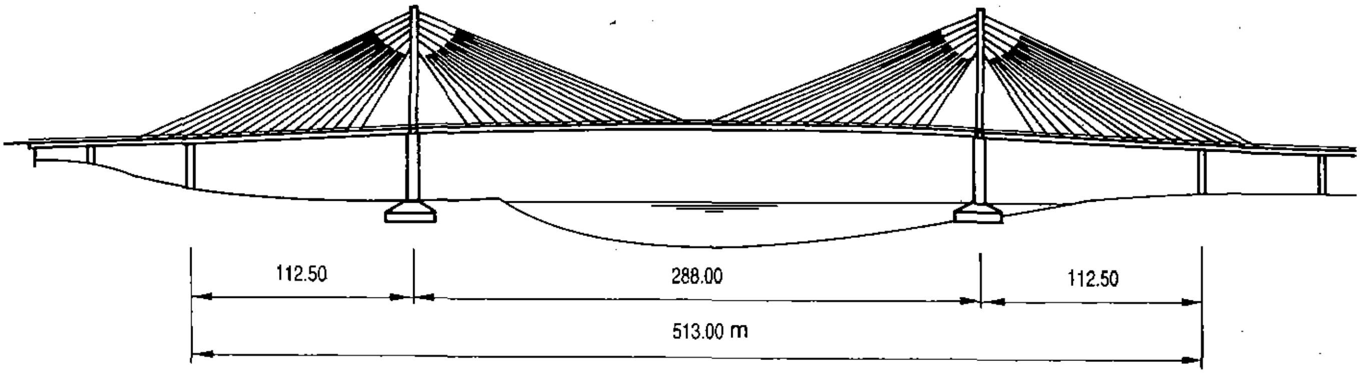
The cable stay may be completely destressed, either strand by strand (in the case of a flexible protection), providing that the necessary extra length has been provided for, or, the 'whole stay' (by the clamping of a special 'friction-piece' taken up by the stressing jack, adapted for destressing, whatever the type of protection).

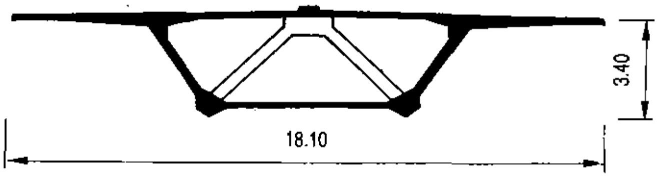
Salient Details of the Cable-stayed Hooghly Bridge (Calcutta, India):

~ **Main Bridge:**

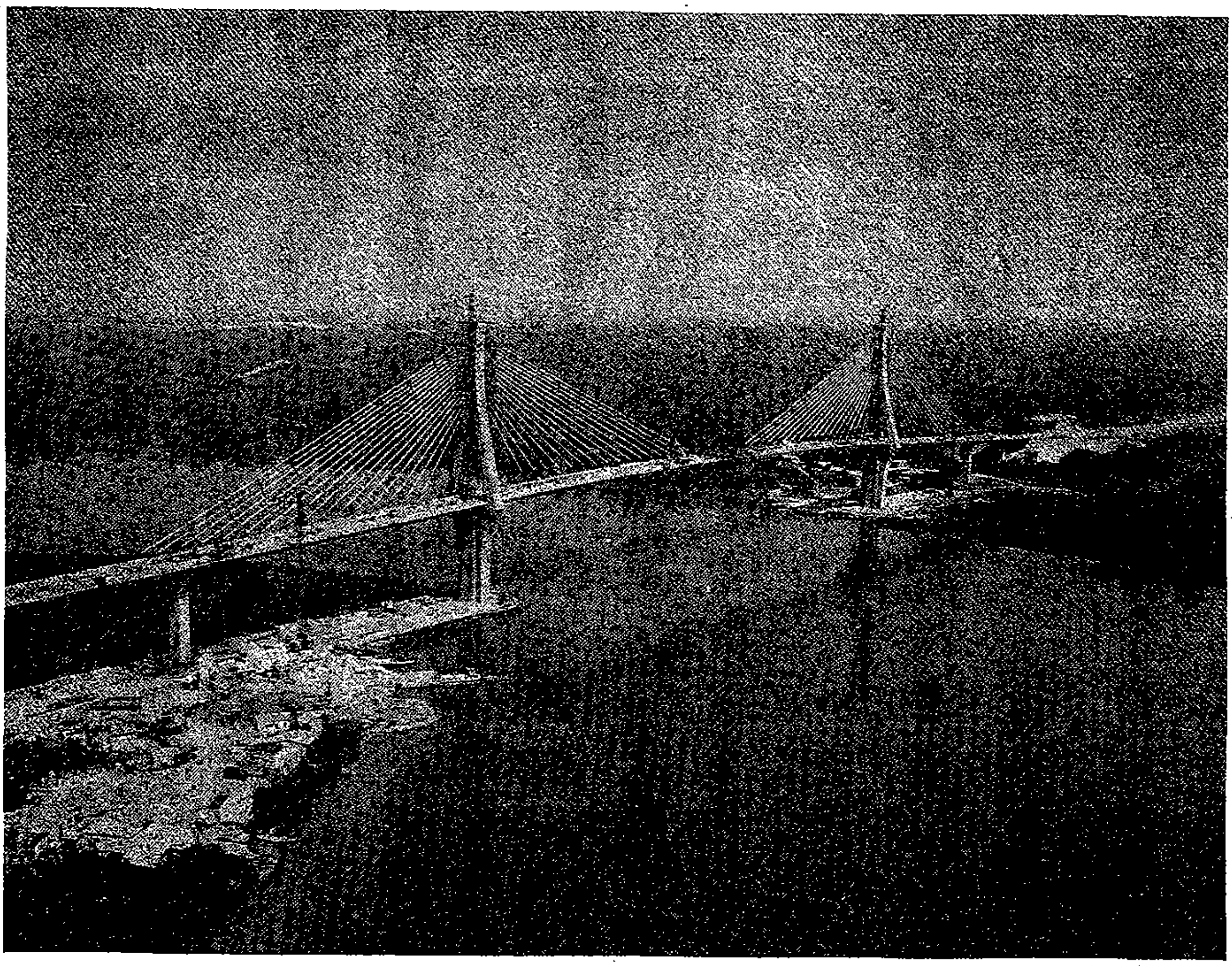
—Type of structure	Cable stayed 'composite deck'—reinforced concrete carriageway slab on steel I-beams.
—Length:	822.96 m.
—Main spans	457.20 m.
—Side spans (2)	182.80 m., each
—Navigational clearance	33.87 m/34.38 m.
—Roadway	Two 3-lane carriageways, 12.3 m each, 1.7 m. median, 2.5 m. footways
—Deck:	Concrete slab 230 mm thick, two outer steel I-girders 28.10 m apart and a central I-girder.
—Cable:	Parallel wire cable, σ_{GUTS} = 1600 MPa, BBR (HiA) (HiAm anchorage). (σ_{GUTS} = characteristic ult. strength).
—Towers:	Two steel towers. Height 122.30 m from pier top.
—Piers	Concrete piers. Height 12.39 m/16.05 m from foundation top, for main towers.
—Foundations:	Well foundation—twin wells, Main foundations—well diameter: 20.60 m/21.35 m Concrete Steining thickness—1.50 m/2.25 m (9 cells) Depth—21.96 m/29.95 m. Cellular space unfilled, watertight and accessible to maintenance inspection.
—Contractor	M/S. Bhagirathi Bridge Construction Co., India (A consortium of Gammon India Ltd. and BBJ).
—Consultant (Prime)	Leonhardt and Andrä, West Germany.
—Advisory Consultant	FFP, UK
—Approx. Cost (Bridge only)	Rs. 600 million*
—Executing Agency:	Hooghly River Bridge Commission

* The cost figure is only indicative of the project volume.



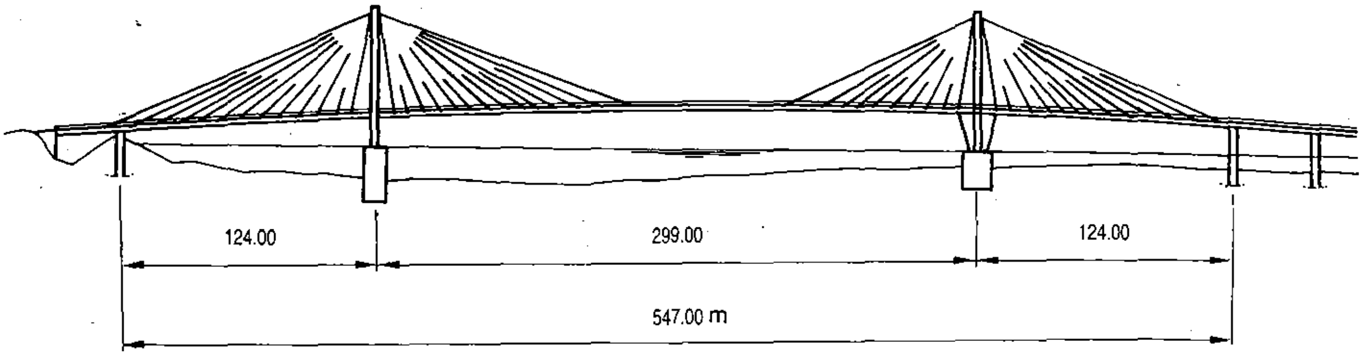


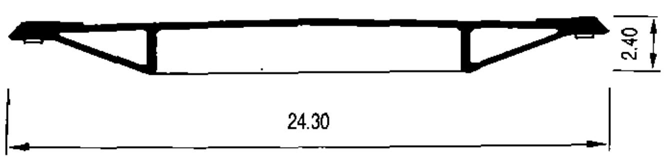
Owner :Secretaria de Communicaciones y Transportes
Designer : SOGELERG (France)
Contractor : ICA (Mexico)
Sub-contractor for stays : Freyssinet de Mexico SA (Mexico)
Period of construction :1980-1984
Type of stay-cables: 37 and 61 HC 15 Freyssinet stay-cables.



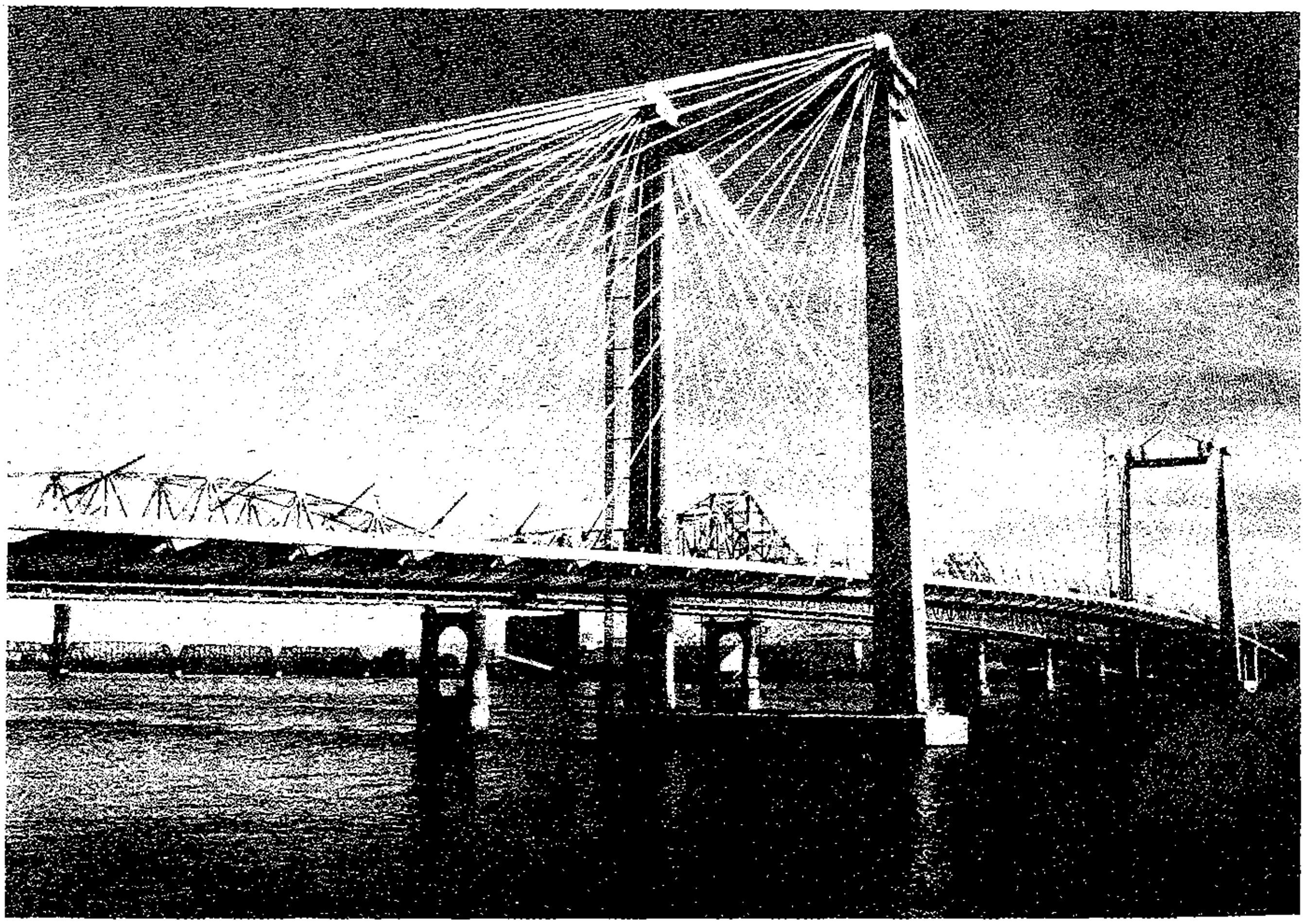
COATZACOALCOS BRIDGE

MEXICO

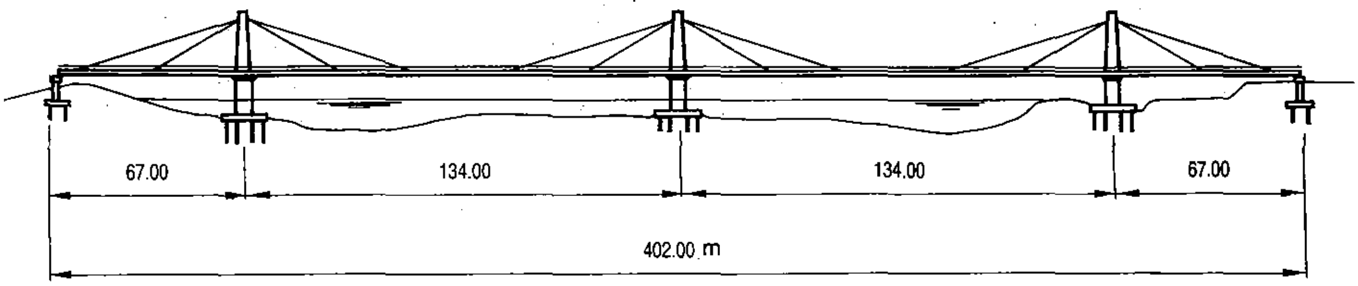


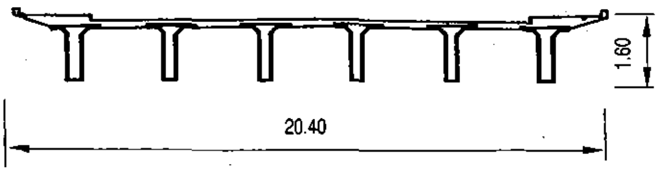


Owner : Federal Highway Administration and Cities of Pasco and Kennewick (Washington)
Designer : A. Grant and Associates (USA) in collaboration with Profs. Leonhardt and Andrä (Germany)
Contractor : Peter Kiewit Sons Co (USA)
Sub-contractor for stays : The Prescon Corporation (USA)
Period of construction : 1976-1978
Type of stay cables : HIAM cables with wires dia. 1/4"



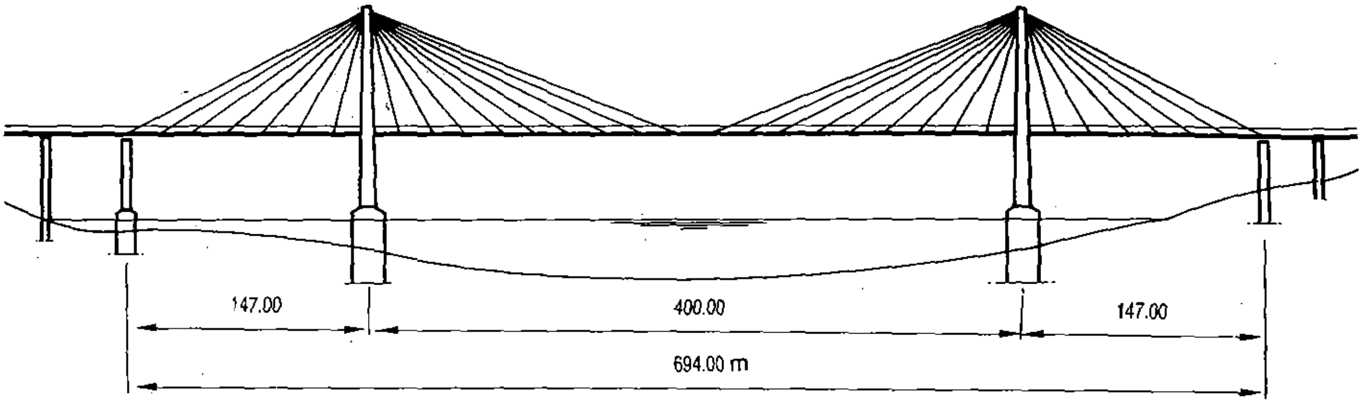
PASCO AND KENNEWICK BRIDGE *UNITED STATES OF AMERICA*

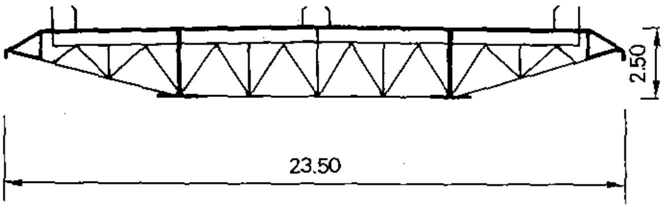




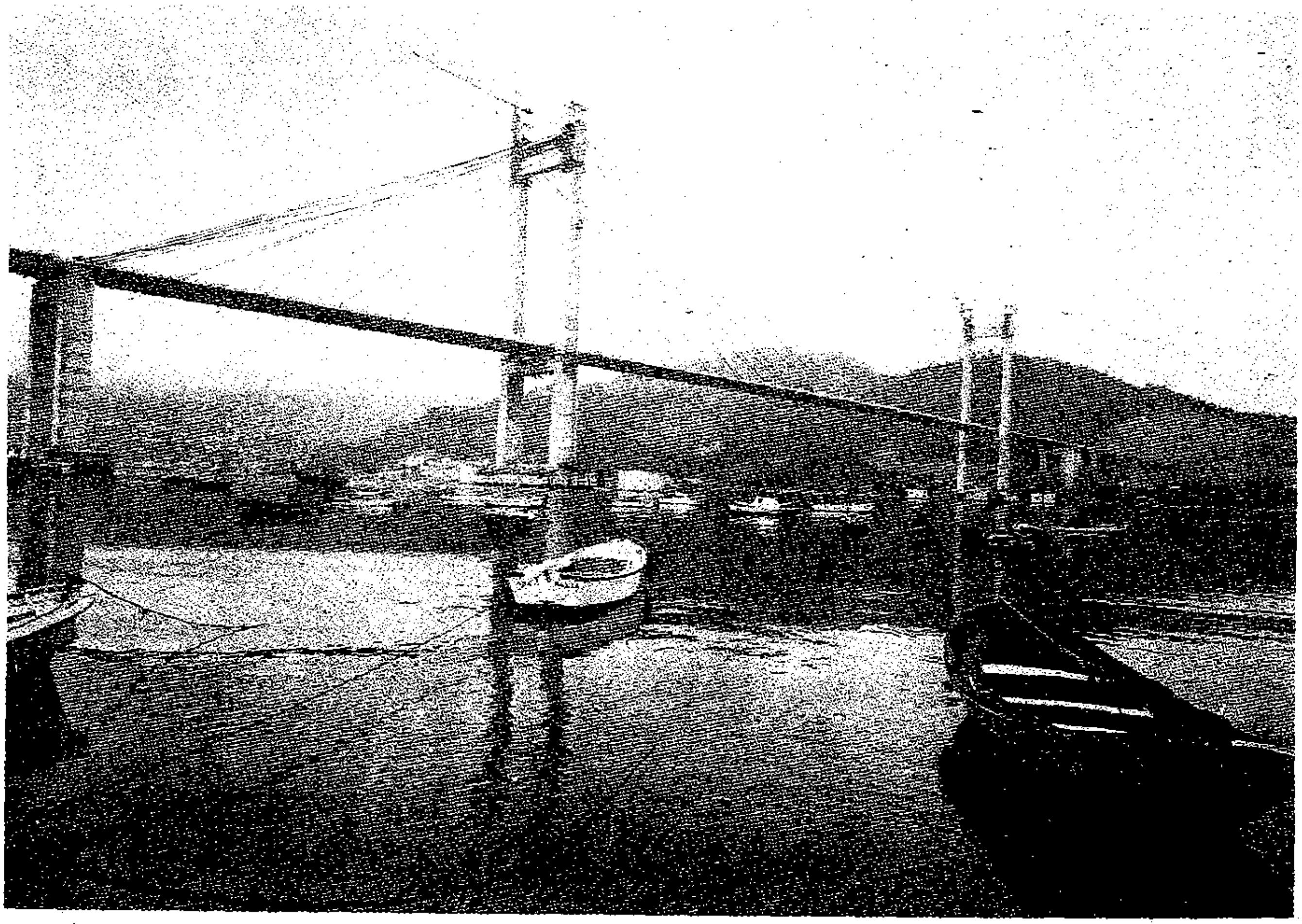
Owner : Taiwan Highway Bureau
Designer : TY Lin International (USA)
Contractor : Kung Sin Engineering Co (Taiwan)
Sub-contractor for stays : FKK (Japan)
Period of construction : 1975-1977
Type of stay cables : bundles of 14 (12 T 13) cables

KWANG FU BRIDGE *TAIWAN*

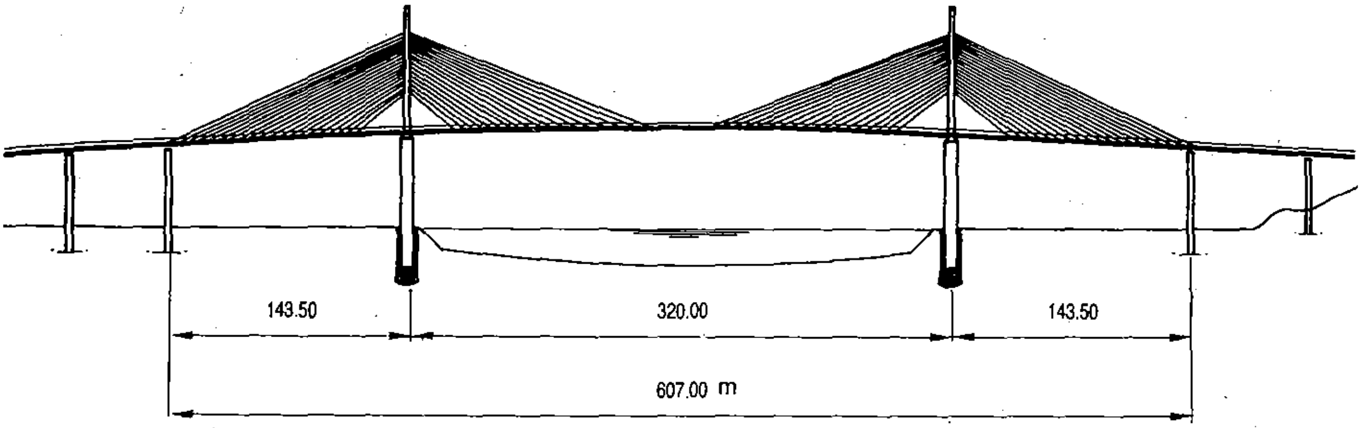


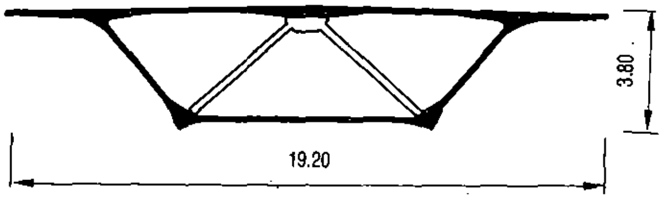


Owner : Audasa
Designer : *Profs del Pozo, de Miranda and Passaro (Spain)*
Contractor : C.Y.T.-M.Z.O.V. (Spain)
Sub-contractor for stays : Freyssinet SA (Spain)
Period of construction : 1975-1978
Type of stay cables : 37 H 15, 61 H 15 and 91 H 15 Freyssinet stay cables

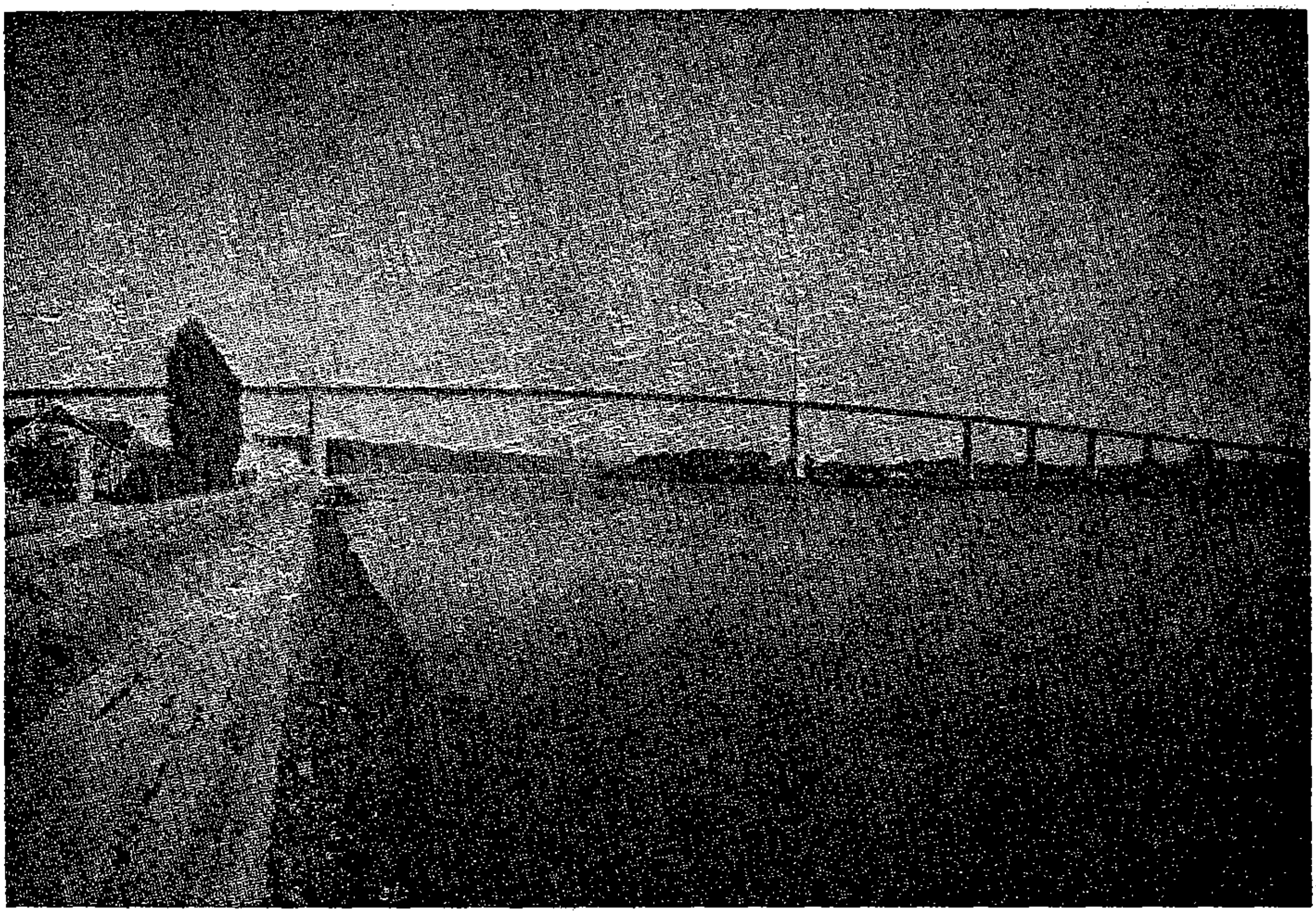


RANDE BRIDGE *SPAIN*





Owner : Departement de la Seine-Maritine
Designer : Campenon Bernard (France)
Contractor : Campenon Bernard (France)
Sub-contractor for stay anchorages : Freyssinet International (France)
Period of construction : 1974-1977
Type of stay-cables: 39 to 60 T15 continuous stays with lower adjustable anchorages.



BROTONNE BRIDGE *FRANCE*

Some Noteworthy Cable Stayed Bridges

Name	Country	Type of Structure	Main Span (m)	Deck	Inauguration	Freyssinet Group Member
Danube Canal bridge	Austria	Road bridge	119.00	Concrete	1975	Vorspann Technik (Austria)
Brotonne bridge	France	Road bridge	320.00	Concrete	1977	Freyssinet International (France)
Pasco-Kennewick bridge	USA	Road bridge	299.00	Concrete	1977	Prescon Corporation (USA)
Kwang Fu bridge	Taiwan	Road bridge	134.00	Concrete	1978	FKK (Japan)
Rande bridge	Spain	Road bridge	400.00	Steel	1978	Freyssinet SA (Spain)
Meylan footbridge	France	Footbridge	79.00	Concrete	1979	Freyssinet International (France)
Ilhof footbridge	France	Footbridge	63.50	Concrete	1980	Freyssinet International (France)
Cotton Tree Drive footbridge	Hong Kong	Footbridge	60.00	Concrete	1980	Freyssinet Hong Kong Ltd (Hong Kong)
Nishikigaoka bridge	Japan	Road bridge	52.00	Concrete	1982	FKK (Japan)
Luling bridge	USA	Road bridge	362.00	Steel	1983	The Prescon Corporation (USA)
Barrios de Luna bridge	Spain	Road bridge	440.00	Concrete	1983	Freyssinet SA (Spain)
Coatzacoalcos bridge	Mexico	Road bridge	288.00	Concrete	1984	Freyssinet de Maxico (Mexico)
Abidjan Cathedral	Ivory Coast	Cable stayed roof	90.00	Concrete	1985	Freyssinet International (France)
Sambre bridge	Belgium	Railway bridge	108.00	Steel	under construction	Laboratoire de Cinematique (Belgium)
Wandre bridge	Belgium	Road bridge	183.00	Concrete	under construction	Laboratoire de Cinematique (Belgium)

REFERENCES

1. Tentative Recommendation for Cable-Stayed Bridge Structures by Task Committee on Cable Suspended Structures of the Committee on Special Structures of the Committee on Metals of the Structural Division Journal ASCE, Structural Division, May, 1977, pp-929.
2. Podolony, Walter and J.B. Scalzi, "Construction and Design of Cable Stayed Bridges", John Wiley and Sons, New York, 1976.
3. Smith, B.S. "The Single Plane Cable Stayed Girder Bridge: A Method of Analysis Suitable for Computer Use", Proceedings of Institution of Civil Engineers, London, 1967.
4. Tang, M.C., "Analysis of Cable Stayed Girder Bridge", *Journal of Structures Division*, ASCE, August, 1971.
5. Chauvin, A., Developments in Technology of Bridge Stays, 10th Congress FIP, New Delhi, February, 1986.
6. Podolny, Jr. W. and J.F. Fleming, Historical Development of Cable-stayed Bridges, *Journal of the Structural Division*, Proceedings of the American Society of Civil Engineers, September, 1972.
7. Bibliography and Data on Cable-stayed Bridges, *Journal of the Structural Division*, October, 1977.
8. Troitsky, M.S., Cable-stayed Bridges: Theory and Design, Crosby Lockwood Staples, London, 1977.
9. Leonhardt, F., Latest Developments of Cable stayed Bridges for Long Spans, Bygningsstatiske Meddelelser, Copenhagen, 1974.
10. Leonhardt, F. and W. Zellner, Cable-stayed Bridges, IABSE Surveys-S-13/80. IABSE, *Periodica* 2/1980, May, 1980.
11. Podolny Jr, W., The Evolution of Concrete Cable-stayed Bridges, Concrete International, August, 1981.
12. Birkenmaier, M., Fatigue Resistant Tendons for Cable-stayed Construction, IABSE Proceedings P-30/80, IABSE, *Periodica* 2/1980, May, 1980.
13. Birkenmaier, M., Recent Developments and Trends in the Design and Construction of Cable-stayed Bridges, Proceedings of the 9th Congress FIP, Stockholm, June, 1982.
14. Sanchez-Sanchez, A., L. Paulik, F. Paredes and C. De La Fuente, The Cable-stayed Bridge of Coatzacoalcos II (Mexico), Proceedings of the 10th Congress FIP, New Delhi, February, 1986.
15. Sanchez-Sanchez, A., and L. Paulik, The Cable-supported Bridge of Coatzacoalcos II (Mexico), 12th Congress IABSE, Vancouver, September, 1984.
16. Matt, P., H.R. Muller and U. Morf, Cables for Cable-stayed Structures, FIP Notes, 1985/1.
17. Sanchez-Sanchez, A., L. Paulik, P. Jartoux and A. Chauvin, La Construction du pont a haubans de Coatzacoalcos II, Conference du 21 mai 1984, Annales de l'ITBTP, Janvier, 1986.
18. Walther, R., B. Houriet, W. Isler, and P. Moïa, Ponts haubanès. Presses polytechniques romandes, 1985.
19. Combault, J., K. Duviard, R. Bunzli, E. Maennel J.-L. Salomon, M. Virlogeux and M. Falempin: La Construction de la passerelle de l'Ilhof (France). Travaux, January, 1985.
20. Mathivat, J., and B. Lenoir, Evolution des ponts à haubans en biton journie d'etude AFPC-ITBTP, du 25 November, 1982, Annales de l'ITBTP Janvier, 1984.
21. Podolny, W. Jr, and J. Muller, Construction and Design of Prestressed Concrete Segmental Bridges, John Wiley and Sons, New York, 1982.
22. Combault, J., F. Conversy and P. Thivans, Structures haubanées. Contrainte de flexion locale dens les haubans. Conference FIP, Stockholm, June, 1982.
23. Mathivat, J. and B. Lenoir, Evolution des ponts a haubans multiples repartis en beton. La Technique Francaise du Béton Précontraint, AFB special issue for the 9th Congress, FIP, Stockholm, June, 1982.
24. Mondorf, P.E., and C. De La Fuente., Development des haubans pour grands ponts, La Technique Francaise du Beton Precontraint AFB special issue for the 9th congress, FIP, Stockholm, June 1982.
25. De La Fuente., C., and A.A. Dinis: Prestressing Steel used for Cable-stayed Bridges, *Freyssinet Magazine*, 1982/4.
26. Freyssinet International, Cable Stays, Ref, FI 2061 A, 1981.
27. Xercavins, P., and P.E. Mondorf, Parallel Strand Cable Stays, Static and Fatigue Strength, 11th Congress IABSE, Vienna, September 1980.
28. Moreau, P., M. Placidi and M. Virlogeux, Meylan and Illhof Pedestrian Bridges, Design and Erection, 11th Congress, IABSE, Vienna, September 1980.
29. Mathivat J., Construction par encorbellment des ponts en beton precontaint, Eyrolles, Paris, 1979.
30. The Great Belt Bridge, Fatigue Tests with Cable Models, 1st series of tests, Ministry of Publics Works, Copenhagen, August, 1979.
31. Le Franc, M., and J. Roche, Pontsà haubans, rapport général. Jounees AFPC, Juin 1978.
32. Muller J., J. Mathivat and J. Combault, Le pont de Brotonne, La Technique Francaise du Béton Precontraint, AFB special issue for the 8th congress FIP, London, 1978.
33. Andrä, W. and R. Saul, Varsuche mit Bundeln aus parallelen Drähten und Litzen für die Nordbrücke Mannheim-Ludwigshafen und das Zelfdach in München. Die Bautechnik, Sept., Oct. and Nov. 1974.

CHAPTER 39

Vibration of Bridge Decks

39.1 INTRODUCTION

The passage of any load over a bridge causes the span to deflect from its equilibrium position relatively quickly. Thereafter, the inherent elasticity of the structure tends to restore the bridge deck to its equilibrium position, causing a series of oscillations. This phenomenon continues till either the structure comes back to its equilibrium position or is again activated by the passage of another load. The effects of bridge vibrations are two-fold, causing (i) structural damage in extreme cases, and (ii) unpleasant physiological and psychological reactions on humans.

The effect of vibration is to cause additional stresses in the structure over and above the static effects. The earlier studies conducted in the field aimed at the evaluation of those additional effects leading to the provision of suitable impact factors in the Codes of Practice. This step contemplated to account and provide for the extra stresses over and above the static effects but did not qualitatively analyse the exact role played by vibrations in rendering a structure unsafe. Certain thumb rule provisions incorporated in the Codes of Practices, such as limiting the ratio of deflection to span and restricting the span-depth ratios, tend indirectly to make the structure more rigid and thus less prone to vibrations. But even these provisions are not based on the evaluations of the frequency and amplitude of vibration likely to occur, and hence cannot be taken as a guarantee against the occurrence of undue vibrations even under normal loads.

As regards wind, it can affect either directly or indirectly. The direct action is envisaged by the wind force acting on vehicles, pedestrians and bridges, which could be a serious problem on exposed bridges like suspension bridges or bridges who approaches are shielded by surrounding objects acting like wind breakers. To meet this contingency, the design could provide for protective parapets, etc. The indirect effect relates to the effect on the user of the bridge due to the wind force 'exciting' the structure, tending to create a feeling of instability. The solution of this problem lies in making the structure more stiff with as high a value of structural damping as possible. Studies conducted do not tend to result in specific recommendations/guidelines for the designer.

Vehicles provide the major force causing excitation on bridges. Studies conducted indicate that the vibration set-up is influenced by the following parameters:

(i) rigidity and first-mode natural frequency of vibration of the bridge deck,
(ii) natural frequencies of the vehicle system and the suspension systems,
(iii) vehicle speed,
(iv) ratio of vehicle weight(s) to bridge weight,
(v) irregularities in bridge deck and approaches,
(vi) ill functioning of expansion joints,
(vii) frequency of live load application due to the passage of multiple axles,
(viii) motion induced in the bridge before application of the live load (particularly important for continuous spans), and
(ix) the damping characteristics of the bridge and the vehicle.

Actual observations on bridges have shown that the foregoing factors have wide variations in their relative influence in bridges, making it practically impossible to evolve a generalised solution applicable to all bridge cases. Whatever studies have been conducted, the results are applicable to only specific cases and probably only some of the broad conclusions reached may apply to other cases as well. Further, due to the large number of interacting parameters between the bridge and the vehicle, often attempts are based on quite a few simplifying assumptions which are far from reality, tending to make the conclusions all the more limited to a particular case. This only emphasizes the fact that each case will need to be treated on its own merit rather than have a solution based on the results of other cases.

From the point of view of pedestrian force, the effect of marching troops seems to be the only force worth considering, causing excitation to bridges. But the troops should break step while crossing a bridge and thus not affect the bridge response materially. The effect of pedestrian force may be of relevance only in case of footbridges.

The vehicular force constitutes the main exciting force causing vibrations in bridges. This is also reflected by the large number of studies conducted to evaluate the effect of moving loads.

39.2 STUDIES

The studies conducted can be divided basically into two parts i.e. (1) theoretical analyses and model studies, and (2) field observations. In some cases where observation of dynamic response was made on actual bridges, the results were also substantiated by theoretical studies. Sometimes the two differed, making a conclusion difficult, the primary aim of all the tests conducted being to evaluate the free response and forced response of a bridge so as to determine the increase in static deflection, moment and shear. Based on the studies conducted, some of the conclusions that have been reached are as follows:

(i) For predicting the motion of a loaded simple 'beam and slab' span at points directly under the path of the load, the initial conditions need to be completely defined.
(ii) Simple span bridges, when loaded, vibrate with their first mode of natural frequency, or a forced frequency depending upon the loading vehicle.
(iii) The observed rigidity of simple span bridges is more than that arrived at by usual design computations (due partly to wearing course and parapets—and even the kerbs—contributing to the deck section properties).
(iv) Pavement irregularity, causing fluctuations in the magnitude of the applied axle load, is the key factor influencing the amplitude of motion of simple span bridges.
(v) The amplitude of vibration for single span bridges is not generally well related to vehicle speed even for a particular vehicle.
(vi) Vibration intensity measured in terms of human reaction to sustained vibration, indicated that "unpleasant" levels of vibration are not uncommon.

The results of the studies indicated that designers should consider the inter-relationship between vehicle loads, pavement irregularity and bridge vibration. The irregularity in the bridge approaches and at the expansion joints tend to introduce significant motion in the vehicle system increasing vibratory motion.

The above findings are generally true for continuous spans also. Although continuity increases the strength and stiffness of each span in a continuous bridge, natural vibration occurs at the same frequency as if the span were simply supported.

Since continuous spans are usually lighter than simply supported spans, it is evident that the continuous spans in general will vibrate at lower frequencies than simply supported spans of the same strength.

Compared to 'frequency' of vibrations the results for 'amplitude' of vibration were far less consistent. Amplitude is significantly affected by the condition of bridge deck and approaches and the characteristics of the bridge and vehicle. Correlation of data of vibratory amplitude, obtained with various parameters and design factors, showed a significant relationship between amplitude factor and stiffness, indicating thereby the bridges of less stiffness showed a clear tendency for much larger values of vibratory amplitude.

Oechler compared simply supported, continuous and cantilever type bridges and concluded that cantilever bridges have the following characteristics:

(i) greatest susceptibility to vibration,
(ii) greatest maximum duration of vibration,
(iii) lowest natural frequency,
(iv) highest amplitude of vibration,
(v) greatest psychological discomfort, and
(vi) those designed with composite action have larger amplitudes of vibration.

The findings reached are of a very general type, leading to only broad conclusions and dependent on the bridge and vehicle parameters, emphasizing that individual problems have to be tackled independently to arrive at desired solutions. For the purpose of a designer's need, more detailed tests seem to be necessary which will clearly prescribe the upper limit to vibrations likely to occur and suggest ways and means of restricting them within acceptable limits, so that the deflection, bending moments and shear forces, as accounted for in the design, are not exceeded in actual practice by the passage of moving loads.

39.3 CODAL PROVISIONS

The various Codal provisions which correlate indirectly with the dynamic response of bridges are those pertaining to impact factors, limitation in the ratio of deflection to span and restriction in the span-depth ratios. These provisions have been evolved with time, based on a quantitative analysis of the bridge behaviour when subjected to live loads, i.e., after accounting for the increase in the static values due to the dynamic effect. Even these provisions vary from one Code of Practice to the other, indicating a lack of uniformity in approach as well as actual values. Whatever studies have been conducted, they do not help in arriving at a generalised qualitative solution, so that the provisions made in the Codes of Practices could be linked up with vibrations, amplitude, etc., likely to occur. However, a larger number of these studies generally seem to conclude that the additional safety factors incorporated in the design by way of impact considerations or limiting the deflection and depth of member with respect to span, are sufficient to account for the overstresses caused due to the dynamic effect of live loads, thus indicating that the overall structural safety is not impaired on account of the vibration effect.

Some schools of thought recommend separate "amplification factors" to be applied for calculation of deflections, moments and shears. The deflection amplification factor being higher than the others because of the severe restrictions placed on dynamic deflections considering human susceptibility to vibration. However, so far there is lack of research and lack of conclusive findings to substantiate the above concept for being included as Codal requirements, though the basic reasoning seems to be appropriate.

Studies that have been conducted with idealised axle systems to seek information with respect to the frequencies of vibration of the bridge when loaded and unloaded, are not conclusive enough for incorporation in the Codes. The mathematical equations that have been evolved are based on a number of assumptions, and may not be obtainable in and applicable to other cases. This thus indicates that the prediction of frequencies whether natural or forced, for any bridge, needs to be evaluated individually by obtaining actual test values rather than by any theoretical equation. Impact is a dynamic feature related mainly to the surface characteristics of the bridge and the spring constant of the vehicle. No doubt these will not be the same in all countries, and, because of this, there may be some differences between the impact values in different countries. But it is difficult to appreciate the wide variation even in the basic approach of different countries in allowing for impact, as brought out below:

(i) The British Standard specifies an impact allowance of 25 per cent to be added to the axle load, or the pair of adjacent wheels which produces the greatest bending moment or shear as the case may be. The stipulations in the Norwegian standard are similar: the only difference is that instead of 25 per cent, 38.5 per cent impact is added to the heaviest axle load.

(ii) In the majority of the remaining countries, impact is related to the loaded length (span length) although the exact relationship varies considerably from country to country.

(iii) Austria, India and Japan specify different impact for concrete and steel bridges, the factor for steel being more than that for concrete. (This apparently is based on the principle that a lighter structure will be subjected to a more dynamic effect.) Finland too has a similar approach, but it distinguishes only timber bridges from the others by specifying a lower impact factor for them, presumably on account of the damping effect of timber.

(iv) The British standard and the loading standards of West Germany. and Italy ignore impact when the span length exceeds 30m, 50m, and 100m, respectively. But even for longer spans, countries like Australia and India specify certain minimum values of impact.

(v) All the standards give an upper limit for the impact allowance either with respect to the type of vehicle (tracked or wheeled) or in relation to the type of bridge (concrete, steel or timber) and the value of this varies from 25 per cent to 65 per cent in different standards.

(vi) Unlike other countries, Belgium and France relate the impact factor to the dead load of the bridge structure. The principle behind this is, however, implicit in the impact formulae being used by many other countries which relate impact to the type of bridge and length of span. The impact formula of Belgium is further complicated by including the speed of the vehicle in it.

(vii) Austria specifies different impact factors for the directly loaded and indirectly loaded main girders on concrete bridges, the factor for the former being more than that for the later. Again in the case of steel bridges, it distinguishes between the first and second lanes of steel bridges, specifying higher impact factors for the first than the second and allowing no impact for lanes in excess of two.

The formulae for impact allowance specified by some countries are unnecessarily complicated and not fully justified, particularly since the effect of live load on the bridge is comparatively less than that of dead load for span lengths of approximately 25 m and above. These clearly show that the present knowledge about the dynamic behaviour of live load on bridge structures is inadequate and there is need for more research in this field.

39.4 HUMAN ASPECT

Bridge response to a moving load is dependent on mass, stiffness, damping properties of the bridge and dynamic properties of the moving loads, resulting in vibrations either at the natural frequency or at the frequency of the applied exciting force. The normal range of fundamental frequency of bridges varies between 1 and 20 cps. This may coincide with the range of frequencies of vehicles, resulting in the possibility of resonance.

The human reactions to vibrations are peculiar and need to be examined from two points: (i) physiological effects on the body and (ii) psychological effect on the person. The human body has its own standards for feeling the effects of natural frequency, and is capable of detecting small levels of vibration. The human reaction to these vibrations results in the physiological effects. The psychological effects are more difficult to define, varying from one human being to the other. This aspect is more linked with the human feelings and is a problem of educating and familiarising the user in such a situation.

39.5 FREQUENCIES CAUSING PHYSIOLOGICAL AND PSYCHOLOGICAL EFFECTS

The assessment of the nature of human reactions, as related to the large variations in the stimulus causing the vibration, is rather complex. However, studies conducted to evaluate physiological response of humans have been successful in laying down the following frequencies at which feeling of discomfort or resonance may occur in the human body:

1/4-1 cps — associated with motion sickness,
2 cps — head resonance (for horizontal movement); also associated with motion sickness,
4–6 cps — major resonance of the whole body,
7–9 cps — abdominal resonance,
10–12 cps — unspecified trunk resonance.

The predominance of the above frequencies will result in adverse physiological effects like discomfort, annoyance, etc., specially when subjected to such conditions for a long period.

However, laying down similar limiting frequencies from the point of view of adverse psychological effects is rather difficult, as no standardised scale can be evolved due to the wide variation in human reactions. For instance, one person may react unfavourably with heights and even a small degree of vibration on a bridge whereas another person may not feel this at all. As such this is a problem wherein the response varies largely from user to user, depending upon familiarity and usage.

39.6 FATIGUE AND TOLERANCE LEVELS

Apart from the physiological and psychological effects, is the fatigue effect and the difficulty the user feels in performing normal duties when subjected to long duration of vibrations. To analyse this problem, researchers have considered the reaction of the pedestrian as the deciding factor in comparison to the passengers in the vehicle, as the latter remain on the bridge relatively for a very short duration. Studies conducted have revealed that there is an appreciable reduction in 'tolerance level' if a person is standing still rather than walking on a bridge. This was observed due to the flexing of the knees in the process of walking creating a hinge effect reducing the amount of excitation transmitted to the body! To study the levels of human tolerance to vibration, work has been done by Reiher and Meister, Dieckman, Goldman, Janeway and Bryce. However, these works do not seem to cover vibration effects on people when walking, or the tolerance of people to transient vibrations. In 1965, the Road Research Laboratory, U.K. conducted some tests to determine the tolerance limits for pedestrians walking and standing on a bridge. The amplitude and frequencies were varied and human reactions ascertained. The acceptable and non-acceptable levels of vibration for all subjects based on the above tests for walking and standing were reproduced as an amplitude frequency plot and in the form of acceleration against frequency.

Tests indicated that cantilever bridges were of the livelier type, needing more care in design. However, the R.R.L., U.K., concludes that test data available are insufficient to enable comparisons with other types of bridges.

39.7 WORK ALREADY DONE AND ITS RELATION TO THE PRESENT PROBLEM

Extensive studies have been carried out by the University of Illinois, U.S.A., Queen's University, Ontario, Canada, and Technical University of Prague, Czechoslovakia, to study the dynamic response of highway bridges. On many occasions, the experimental data on models or prototypes, have been supplemented by theoretical analyses involving simplified assumptions and idealisations. These tend to limit the application of the results on similar problems existing in case of other highway bridges. Based on past studies, it is felt that the main input parameters which influence the dynamic response of a continuous bridge can be summarised as below:

(i) Assumptions and idealisations made for simplification of the study,
(ii) Aspect ratio of the bridge (span/width), and other relevant geometrical and structural details,
(iii) Boundary conditions at the ends,
(iv) Nature of moving vehicle system,
(v) Speed parameter, generally defined as $VT/2L$, where V is the velocity of the moving vehicle system, T is the period of the bridge and L is the characteristic length of the bridge.

In addition to the above parameters, the following four factors will also influence the validity of the results obtained:

(i) How the response data are obtained,
(ii) Method adopted for calculating the natural frequencies and the numerical results obtained,
(iii) Method adopted for analysing the forced response of the bridge and the results obtained, and
(iv) General conclusions drawn with respect to the dynamic increments for deflections and stresses.

The tests consist of two parts: (1) dynamic load-tests, and (2) static load-tests. In the dynamic load-tests, different types of loadings are employed for the purpose, with differing numbers of trucks and convoys moving in one or both directions, maintaining certain speeds and spacings.

The accelerations are measured by an instrument-probe and acceleration vs. time plots are recorded automatically through an instrumentation set up on a photosensitive paper. From these measured accelerations for the various points of the bridge, obtained for different types of test loadings, different speed parameters, and for different directions of movement of trucks, the required amplitudes of dynamic displacements are calculated from which the corresponding dynamic increments are derived. From these data it has been possible to summarise what are commonly called as 'impact factors', both for deflections and for loads.

The static tests are planned in order to obtain the deflection factors for salient points.

39.8 PRACTICAL FACTS

Prestressed or reinforced concrete bridges usually do not have vibration problems because the amplitudes of these vibrations in concrete bridges are very small and there is really no adverse effect on traffic or on human beings. Only a certain range of frequencies may disturb the feelings of some human beings, but this disturbance usually is in a tolerable range unless it is a light pedestrian bridge where people are not quickly passing across.

Again, it is usually not so strong that it could not be accepted by pedestrians. If you sit in a car, you even do not feel all these vibrations. Therefore, we do not really need to go into any 'limitations' of such vibrations. Also, the stresses which come to these bridges by vibrations or by frequent loadings are such that there is no reason for being worried for the structural safety, if the codal impact factors on live load are included and the road surface is smooth.

The impact factors which have been measured in many concrete bridges under traffic in Germany show that the impact is so small that the stresses caused by the impact are small and never reach the maximum stresses by extreme loads which are specified for ultimate strength. It is different in steel bridges and specially different with steel bridges for railroads. There we have to consider, of course, the impact especially for the welded structures which are sensitive against frequently repeated loadings leading to the reduced fatigue strength. Impact factor actually reduces with higher speed, speeds higher than about 50–60 kilometres per hour. The deformations become smaller because the structure does not have sufficient time to react and get the large deflection which we calculate for statical loading. Therefore, high speeds of vehicles do not really affect the highway bridges.

The quantitative value and mode of vibration of the vehicle, which causes various physical discomforts and structural criticality, depends mainly on the type of vehicle, vehicle-suspension, surface characteristics and the structural system. Generally, for concrete structures, vibration, apart from causing a certain amount of unpleasant physical feeling to pedestrians, is not a significant factor. The vibrations are normally more pronounced in prestressed concrete structures as compared to reinforced concrete structures in view of their smaller mass and hence that of the damping system. Again, a continuous bridge generally manifests a higher level of vibration as compared to what is experienced with simply supported spans, on account of possible superimposition of the vibration modes of multiple vehicles traversing the deck, and lighter mass, other things being equal.

39.9 PRACTICAL APPROACH FOR VIBRATION ANALYSIS

Descriptions given above are all very well but not very helpful to the regular designer who finally still needs a guideline. For this purpose reference may be made to B.S. C.P. 117-Part 2—1967 and Lenzen's Criteria (Fig. 39.1). The following procedure may be adopted for a quick and workman-like solution of the problem.

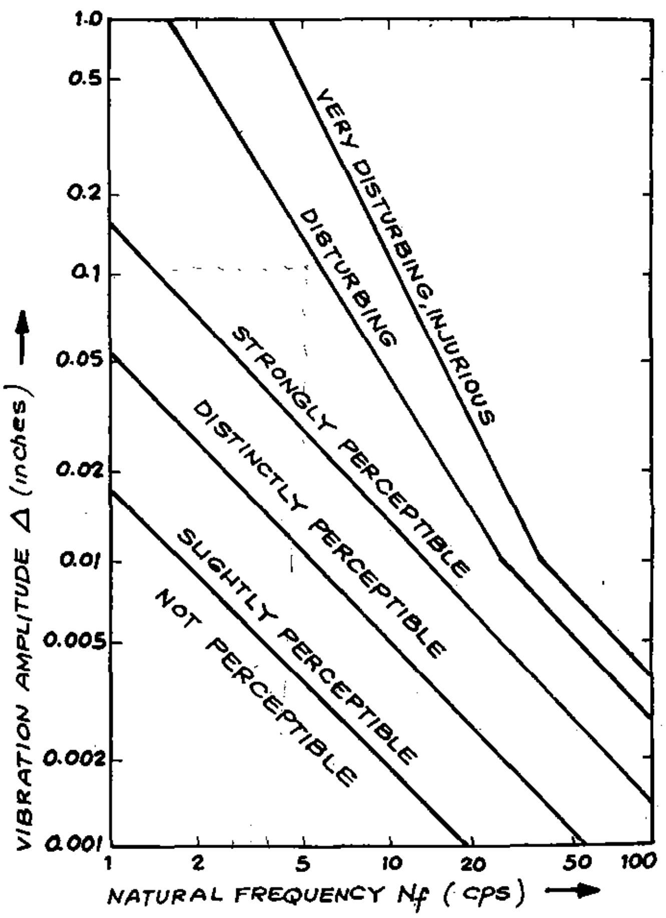


Fig. 39.1 *Lenzen's criteria*

Step 1 Estimate the maximum span deflection δ (in inches) under a single 20-ton hypothetical point-load placed in the centre of span and cross-section, using fully composite (average) flexural rigidity of the deck section of full-width (EI).

Step 2 Estimate the fundamental natural frequency N_f of the span from:

$$N_f = \frac{2}{L^2}\sqrt{\frac{EIg}{w_d}} \text{ cycles per second}$$

where w_d = dead weight, including finishes, in tons per foot of bridge span,

g = 32.2 ft./sec^2

EI = Flexural rigidity, in foot and ton units, of the full-width section of the bridge deck,

and L = span (ft)

Step 3 Take $\Delta = 0.40\delta$ if $N_f > 4$ cps
and $\Delta = 0.75\delta$ if $N_f > 4$ cps

where Δ is the maximum amplitude of vibration, in inches (δ also in inches).

Step 4 Estimate the maximum acceleration A from:

$$A = 40\Delta N_f^2 \text{in/sec}^2$$

Step 5 Ensure that the product A $\Delta \not> 5$ in^2/sec^2 and that the vibration characteristic from Fig. 39.1 (from known values of N_f and Δ) is as desirable.

CHAPTER 40

Use of Freyssinet Flat Jacks

40.1 INTRODUCTION

The Freyssinet flat jack is a thin hydraulic jacking device capable of exerting large forces simply and economically.

Devised by M. Freyssinet and used by him as long ago as 1938 for strengthening dams, it soon became evident that its field of application was extremely varied and it has progressively extended to all spheres of construction, either to improve the quality of foundations, to simplify the building of structures and to increase their resistance due to the prestress thus applied, or for carrying out repairs, reinforcing and controls, and ensuring a maximum of operating safety.

Over the past thirty years, Freyssinet International has acquired considerable experience in a wide variety of possible applications, using flat jacks of varied shapes and forces.

The Freyssinet flat jack (Fig. 40.1) is a hydraulic capsule in the form of a flat double saucer, made of two steel plates welded together around the exterior of the pressformed peripheral beading. Two nozzles, one of which acts as a vent, enable the flat jack to be inflated. The upper and lower plates are forced part, exerting a uniform pressure on the surfaces of both parts of the structure between which the flat jack is placed. The pressure inside the jack may reach 15N/mm^2 and the opening (or stroke), that is to say the final clearance between the two plates, corresponds to the diameter of the beading (normally 25 mm).

40.2 ADAPTATION AND COMBINATIONS

- The table below indicates the sizes of standard circular jacks. Jacks of different sizes and shapes, rectangular, triangular or oblong, for special application, can be supplied by Freyssinet International.

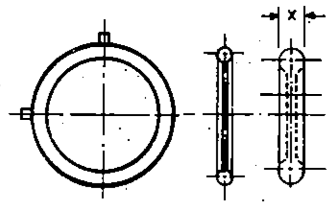


Fig. 40.1

- Jacks with openings in excess of those shown can also be supplied on condition that the bearing capacity is reduced, the permissible pressure in the beading decreasing when the diameter increases.
- To increase the movement, however, it is preferable to compound the jacks in layers (Fig. 40.2).

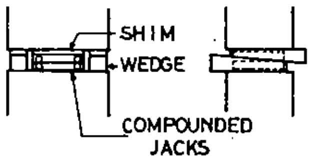


Fig. 40.2

- If a load has to be distributed over a large area, it is advisable to use a series of flat jacks interconnected and inflated simultaneously using a common manifold (Fig. 40.3).

40.3 FORCE EXERTED ACCORDING TO THE OPENING

The force exerted by a Freyssinet flat jack varies slightly

Standard Sizes

Max-force in Tonnes	2	7	15	40	50	60	80	110	170	240	370	600	830	900
Outside diameter in mm	70	120	150	220	250	270	300	350	420	480	600	750	870	920
Thickness in mm (x)	15	25	25	25	25	25	25	25	25	25	35	35	35	35
Minimum access* gap in mm	25	35	35	35	35	35	35	35	35	35	45	45	45	45
Opening in mm (x)	15	25	25	25	25	25	25	25	25	25	35	35	35	35

* Minimum gap required to place flat jack and steel thrust plates.

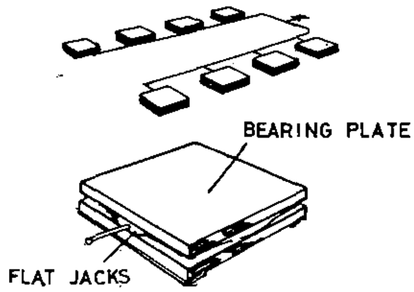


Fig. 40.3

according to the opening (or stroke) and the pressure. The active surface (quotient of the force divided by the hydraulic pressure) decreases when the opening increases.

It should be noted, however, that the force exerted is, in all events, mainly dependent on the reaction acting against the jack.

40.4 METHODS OF 'INSTALLATION'

The double-saucer shape of the flat jack requires the use of intermediate thrust membranes between the jack and the structure. A number of methods are used:

Steel Plate

The method is used when the jack is to be recovered on completion of duty (Fig. 40.4).

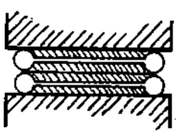


Fig. 40.4

A steel saucer plate is placed in each of the saucer recesses. The steel plates may be designed with handling lugs for ease of manipulations and for extraction after inflation.

Epoxy Resin

This method is used when the jack is to remain permanently in position. The dished surfaces are filled and levelled with a hard setting epoxy resin which has the advantage of being light, easy to handle and non-corrodible (Fig. 40.5).

Pre-Cast Concrete Blocks

Used as a permanent installation where the concrete block is to remain part of the visible structure, or where a number of jacks are compounded in series (Fig. 40.6).

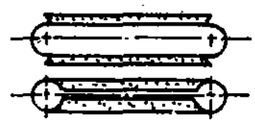


Fig. 40.5

The concrete blocks are reinforced with one or several layers of reinforcing mesh depending on their thickness.

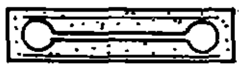


Fig. 40.6

40.5 'INFLATION' TECHNIQUE

(a) Temporary Installation

When the jack is to be used for a limited period and subsequently removed, inflation is normally done with hydraulic fluid using normal pumping techniques and a circuit designed according to the structural balance or 'fail-safe' requirements. Experience in circuit design is necessary, and discussion is advised on all but the simple applications.

(b) Permanent Installation

Where Freyssi Flat Jacks are to remain permanently installed, inflation by chemical hard setting resin is normal. Often in these cases, temporary oil inflation is required for periods of up to six months for structural stability to be achieved or to allow for the effects of a continuing construction programme.

This can be done quite simply where each jack in turn can temporarily be depressurised for replacement of the oil by the chemical resin: where this cannot be permitted, a more advanced technique, involving a double-banked jack circuit, must be used—developed by Freyssinet International.

Freyssi Flat Jacks may equally be inflated with cement grout, but, in these cases, the working force should be reduced to 50% of normal to ensure correct inflation free of blockages at a pressure not exceeding 7.5 N/mm^2.

40.6 RECOVERY AND RE-USE OF JACKS

Freyssi Flat Jacks remain in their inflated shape on release of hydraulic pressure and, on recovery, may be returned to their closed position in a press. The number of re-uses that can be obtained is dependent on the extent of the opening used and the working pressure, and, as a guide, from 5–10 uses may be expected under average conditions.

However, if the conditions of use demand reliability or calibrated accuracy from the jacks, *it is recommended that they be used only once* as the cost of replacement is comparatively low.

40.7 APPLICATIONS

Freyssi Flat Jacks find wide application in the civil and structural engineering industry wherever application or control of large forces are required or where structural or foundation strains have to be induced. Essentially simple and compact, they are so often used in situations for which their use was not foreseen, such as remedial measures or structural additions, as they are used in new constructions in which they form part of the structural concept.

Amongst the many applications can be listed:

* Control of thrust forces
* Pre-stressing between abutments
* Adjustment of support reactions
* Structural pre-loading
* Structural lifting
* Underpinning
* Measurement of forces
* Thrust maintenance

Five Specific examples of actual use are described ahead to illustrate the adaptability of Freyssinet Flat Jacks.

• Maintaining Constant Thrust

The present 'Minster' at York (England) dates back to the eleventh century with later additions to the tower up to the fifteenth century and some rebuilding following a fire in the nineteenth century. Due to gradual foundation settlement causing severe cracking, extensive structural repair has had to be undertaken.

The Consulting Engineers wished to apply a steady thrust to the East Wall, which was, at the start of operations, more than 2 ft. out of vertical. A series of steel-framed flying shores were erected and double-banked Freyssi Flat Jacks inserted at their bases to provide in each an axial thrust of 160 tons (Fig. 40.7)

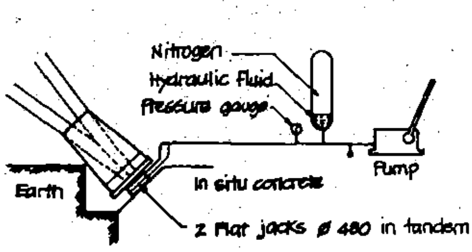


Fig. 40.7

However, due to the considerable rigidity of the 14 ft. thick masonry walls and of the foundations of the shores, temperature effects would have caused the thrust to vary from zero to twice the force required; this was not acceptable structurally, but, in any case, would have doubled the cost of the steel frames.

A hydraulic circuit was, therefore, fitted with nitrogen-filled compensating accumulators designed to maintain the actual thrust to within 5% of the given mean under the most adverse temperature conditions. Control records show that this was achieved.

• Prestressing between Abutments

A length of 1 3/4 miles of the road tunnel section, constructed through Mont Blanc to link France and Switzerland, was pre-stressed longitudinally with Freyssi Flat Jacks (Fig. 40.8).

The end abutment sections were each designed to withstand a maximum thrust of 7700 tons and, between them, the concrete section was stressed such that a minimum of 225 lb/in^2 was maintained under the most adverse ambient conditions.

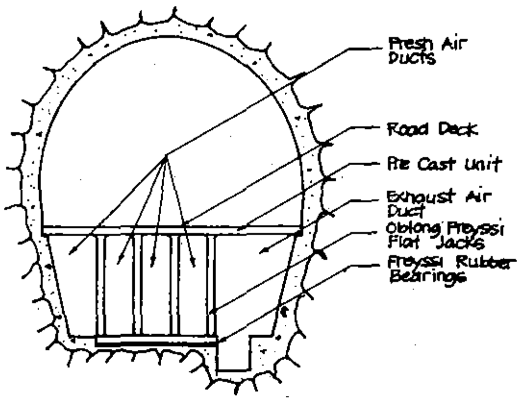


Fig. 40.8

The concrete sections, containing, in the lower chambers, the air ducts and, on the top, the road surface, were pre-case in 32 ft. lengths and jointed in-situ. Each sixteenth joint was 'active' and Freyssi jacks were inserted across almost the whole section. These were case in pairs into lightly reinforced concrete block.

In order to provide adequate strain capacity at each joint, which had to compensate for concrete shrinkage, elastic shortening and creep under pre-stress and for temperature movement, itself estimated at ±1.5″ per joint, the Freyssi Flat Jacks were arranged in seven layers giving a maximum capacity of 7″ movement. 'Active' joints had to be provided at frequent intervals in order to localise the restraints to longitudinal force caused by the rubber bearings on which the road section was supported.

The Freyssi Flat Jacks were inflated by oil in layers, each layer provided 1 in. of movement; they were later cement-grouted. The prestress was readjusted through a period of nine months as shrinkage and creep took place and observations continued so that further correction may be carried out as necessary. More than 2,000 oblong shaped Freyssi Flat Jacks were used in this project.

• Control of Arch Thrust

The Gladesville Bridge, constructed over the Parramotta River just east of Sydney, is a concrete arch bridge with a clear span of just over 1000 ft. It is the largest of its type in the world.

Constructed as four independent arches, later made monolithic, it is entirely made of pre-cast voussoirs erected on steel centring. A battery of Freyssi Flat Jacks was inserted at both third points of the arch, by means of which the whole span has lifted clear of the support frame for de-centring; this created the arch thrust which was then adjusted to the correct point of application (Fig. 40.9).

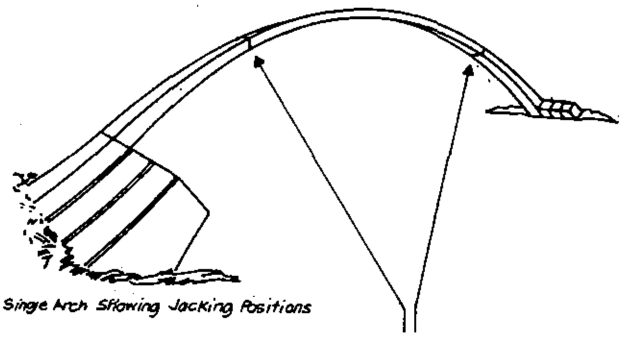


Fig. 40.9

As each of the arches, in its independent condition, was extremely sensitive to horizontal curvature, due to differential drying and the thermal effects of sun, a sophisticated circuit, including trimmer controls, was provided.

At each section of each arch, 56 Freyssi Flat Jacks, capable of exerting a maximum total thrust of 6500 tons, were installed; these were banked in layers of four jacks to provide a maximum of 4″ movement (Fig. 40.10).

• Transfer of Forces and Lifting

(a) Freyssi Flat Jacks were used to help preserve a matchless example of cultural history. The temple at Abu Simbel was due to be inundated by the rising waters of the Nile behind the newly constructed Aswan High Dam. A scheme to raise the temple just over 200 ft. was financed through UNESCO

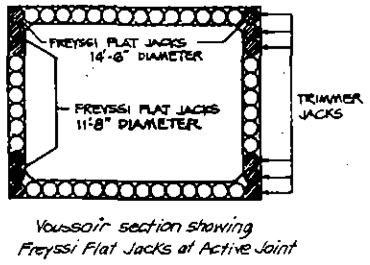


Fig. 40.10

just in time to carry out the work which, in principle, involved cutting away the surrounding rock and sawing the temple into blocks of 20–30 tons weight for re-erection at the higher level.

To take the load of the roof as the vault-effect was removed, a series of vertical columns were erected, into which strut forced were gradually induced by Freyssi Flat Jacks as the dismantling proceeded (Fig. 40.11).

The magnitude of the operation can be judged from the fact that 540 Flat Jacks of 220 mm dia. were used.

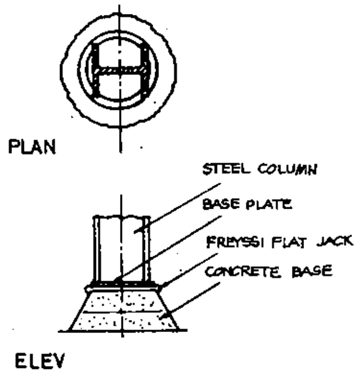


Fig. 40.11

(b) These jacks can also be used conveniently to temporarily lift bridge-girders (momentarily) in order to 'relieve' the neoprene bearings under them so that these bearings need to be designed only for the subsequent deformation permanently. This leads to economy in their design. These jacks can also be used to lift the super structures momentarily in order to reset the steel bearing plates and rollers to save on their design travels.

• Structural Levelling

The spectacular project, called Habitat'67, is a new concept of modern urban living. It was erected as a part of the Montreal Expo'67.

Habitat has 158 apartments each made of individual precast units so arranged that each forms the roof garden for the one above. The structure rises to 120 ft., contains six elevator shafts and has a system of longitudinal 'streets' for horizontal communication. All the elements were of prestressed concrete.

The 'street' girders were each made of six pre-cast elements individually erected with a special crane; these elements were supported temporarily on scaffolding and adjusted to correct line and level using Freyssi Flat Jacks of 18.9″ diameter with a maximum thrust of 222 short tons. 12/0.5″ Freyssinet pre-stressing cables were then inserted and stressed, after which the flat jacks were recovered and subsequently re-used.

CHAPTER 41

Fire Resistance of Structural Concrete

41.1 GENERAL

Fire resistance of structural concrete is assumed to have reached its limit when:

(1) The service load cannot be supported any longer.
(2) The structural member ceases to prevent the passage of flames and combustion gases.
(3) An excessive transfer of heat to the unexposed side occurs.

The latter two criteria are applicable to floors only, which perform a separating function. Although they are the criteria for failure in fire, in a structural sense they could be compared with the serviceability limit since the structure can be repaired after the fire provided the deformations are not excessive.

The conditions for simply supported beams are much more unfavourable than those for continuous beams or for beams which are built-in at the ends or capable of having thermal restraint. In various countries specific regulations have been made which do not quite agree with each other and are often of an arbitrary nature, except in the United States where more practical considerations apply from the point of view of fire insurance and the requirement of possible repair. There is a great difference between theoretical tests and practical experience in the case of actual fire.

In a newly built bungalow in North Carolina at which a fire occurred following a collision involving an oil drum and lasted for 1 1/2 hr. (during which temperature of 1400°F (750°C) must have been reached since glass melted), the walls were badly damaged and had to be rebuilt, but the prestressed lightweight concrete floor and roof consisting of double tee-beams without special insulation retained their full carrying capacity. This was ascertained from a subsequent loading test.

• *The different methods of dimensioning prestressed concrete members for a definite fire resistance rating*:

There are three possibilities of dimensioning prestressed concrete members to resist fire for a definite time.

(1) Test of full-sized members giving specific minimum dimensions of standard constructions.
(2) Tables showing minimum dimensions in which the cover to the steel (in fact the distance from the centroid of the steel) and the minimum dimensions are specified. These tables vary for different countries and are mostly based on conclusions made from tests. Obviously method (1) which is based on test results should have preference.
(3) Computation method. When the individual conditions are known, or can be reasonably assumed, fire resistance may be established from computation. In this case, the effect of continuity and/or actual restraint can be taken into account and thermal expansion and restraint, if any, ought to be considered.

The excellent performance of the prestressed concrete construction in Avianca Building, Bogota, was, no doubt, largely due to good detailing of the individual precast members as well as the entire structure.

41.2 SIMPLY SUPPORTED (UNRESTRAINED) SLABS AND BEAMS

If the underside of a slab is exposed to fire, the bottom of the slab will expand more than the top, resulting in a deflection of the slab. The tensile strength of the concrete and steel near the bottom of the slab will decrease as the temperature increases. When the strength of the steel at elevated temperature reduces to that of the stress *in the steel, flexural* collapse will occur.

41.3 CONTINUOUS SLABS AND BEAMS

Structures that are continuous or otherwise statically indeterminate undergo changes in stresses when subjected to fire. Such changes in stress result from temperature gradients within structural members, or changes in strength of structural materials at high temperatures, or both.

If the underside is exposed to fire, the bottom of the beam becomes hotter than the top and tends to expand more than the top. This differential heating causes the ends of a continuous beam to tend to lift from their supports thus increasing the reaction at the interior support. This action results in a redistribution of moments, i.e., the negative moment at the interior support increases while the positive moments decrease.

During the course of a fire, the negative moment reinforcement (at top) remains cooler than the positive moment reinforcement because it is farther and hence better protected from the fire. Thus, the increase in negative moment can be accommodated. Generally, the redistribution that occurs is sufficient to cause yielding of the negative moment reinforcement. The resulting decrease in positive moment means that the positive moment reinforcement can afford to be heated to a higher temperature before failure will occur. Thus, it is apparent that the fire endurance of a continuous reinforced concrete beam is generally significantly longer than that of a similar simply supported beam loaded to the same moment intensity.

ASTM E119-79 specifies the test methods and procedures for determining the fire resistive properties of building components, and is the common standard.

41.4 FIRE ENDURANCE OF FLOORS AND ROOFS WHICH HAVE RESTRAINT TO THERMAL EXPANSION

If a fire occurs beneath a small interior portion of a large reinforced concrete slab, the heated portion will tend to expand and push against the surrounding part of the slab. In turn, the unheated part of the slab exerts restraining compressive forces on the heated portion. The compressive force, or thrust, acts near the bottom of the slab when the fire first occurs, but as the fire progresses the line of action of the thrust rises. If the surrounding slab is thick and heavily reinforced, the thrust forces that occur can be quite large, but considerably less than those calculated by use of elastic properties of concrete and steel together with appropriate coefficients of expansion. At high temperatures, creep and stress relaxation play an important role. Nevertheless, the thrust is generally great enough to increase the fire endurance significantly. In most fire tests of restrained assemblies, the fire endurance is determined by temperature rise of the unexposed surface rather than by structural considerations, even though the steel temperatures often exceed 815°C (1500°F).

41.5 HEAT TRANSMISSION

In addition to structural integrity, ASTM E119 limits the average temperature rise of the unexposed (top) surface of floors or roofs to 139°C (250°F) during standard fire tests. For concrete slabs, the temperature rise of the top surface is dependent mainly upon the thickness, unit weight, moisture content, and aggregate type. Other factors that affect temperature rise, but to a lesser extent, include air content, aggregate moisture content at the time of mixing, maximum size of aggregate, water-cement ratio, cement content, and slump.

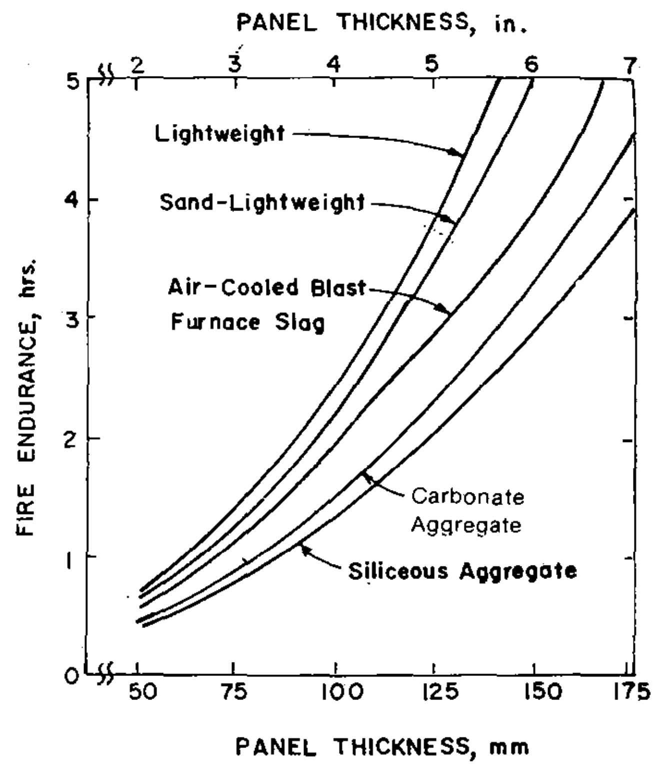


Fig. 41.1 *Effect of slab thickness and aggregate type on fire endurance of concrete slabs. [Based on 139°C (250°F) rise in temperature of unexposed surface.]*

• *Effect of Slab Thickness and Aggregate Type*

Figure 41.1 shows the relationship between slab thickness and fire endurance for structural concretes made with a wide range of aggregates. The curves are for air-entrained concretes fire tested when the concrete was at the standard moisture condition (75 per cent RH at middepth), made with air-dry aggregates having a nominal maximum size of 19 mm (3/4 in.). On the graph, lightweight aggregates include expanded clay, shale, slate, and fly ash that make concrete having a unit weight of about 1520 to 1680 kg/m^3 (95 to 105 pcf) without sand replacement. The unit weight of air cooled blast-furnace slag aggregate has little effect on the resulting fire endurance of the normal weight concretes in which it is used.

• *Effect of Unit Weight*

Fire endurance generally increases with a decrease in unit weight. For structural concretes, the influence of aggregate type may overshadow the effect of unit weight. For low density concretes, a relationship exists between unit weight (oven-dry) and fire endurance, as shown in Fig. 41.2. The

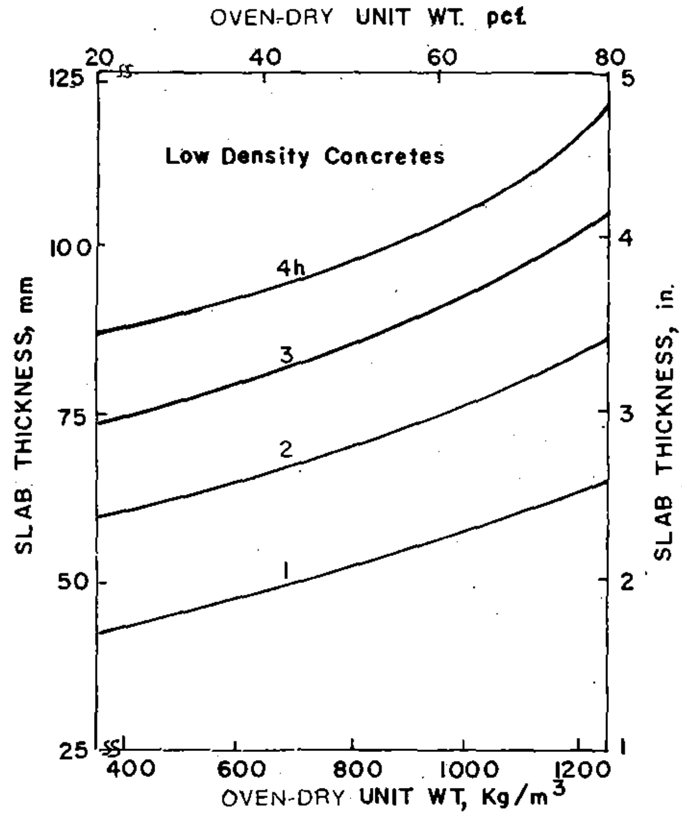


Fig. 41.2 *Effect of dry unit weight and slab thickness on fire endurance of low density concretes. [Based on 139°C (250°F) rise in temperature of unexposed surface]*

curves in Fig. 41.2 represent average values for concretes made with dry vermiculite or perlite, or with foam (cellular concrete), with or without masonry sand.

• *Effect of Moisture Condition*

The moisture content of the concrete at the time of test and the manner in which the concrete is dried affect fire endurance. Generally, a lower moisture content or drying at elevated temperature of 50 to 95°C (120 to 200°F) reduces the fire endurance. A method is available for adjusting fire endurance of concrete slabs for moisture level and drying environment (Appendix 5, ASTM E119-79).

• *Effect of Air Content*

The fire endurance of a concrete slab increases with an increase in air content, particularly for air contents above 10 per cent. Also, the improvement is more pronounced for lightweight concrete.

• *Effect of Sand Replacement in Lightweight Concrete*

As indicated in Fig. 41.1, replacement of lightweight aggregate fines with sand results in somewhat shorter fire endurance periods.

• *Effect of Aggregate Moisture*

The influence on fire endurance of absorbed moisture in aggregates at the time of mixing is insignificant for normal weight aggregates but may be significant for lightweight aggregates. An increase in aggregate moisture increases the fire endurance. Thus, the fire endurances obtained from Fig. 41.1 represent minimum values.

• *Effect of Water-cement Ratio, Cement Content and Slump*

Results of a few fire tests indicate that these factors, per se, within the normal range for structural concretes, have almost no influence on fire endurance.

• *Effect of Maximum Aggregate Size*

For normal weight concretes, fire endurance is improved by decreasing the maximum aggregate size.

41.6 FIRE ENDURANCE OF WALLS

In fire test of walls consisting of plain concrete, reinforced concrete and concrete masonry units, the fire endurance is generally governed by heat transmission rather than structural consideration assuming that the structural requirement of the design code had been satisfied.

41.7 REINFORCED CONCRETE COLUMNS

Reinforced concrete columns have performed well during exposure to fire throughout the history of concrete construction.

Table 41.1 *Fire Endurances Proposed by Hull and Ingberg*

Aggregate type	Minimum area of round or square cross section, cm^2 (sq in.)	Concrete cover, mm(in.)	Fire endurance classification, hr.
Siliceous	710 (110)	38 (1-1/2)	1-1/2
Siliceous	1,290 (200)	38 (1-1/2)	2-1/2
Siliceous	1,290 (200)[1]	38 (1-1/2)	3-1/2
Siliceous	1,613 (250)	64 (2-1/2)	3
Siliceous	1,613 (250)[1]	64 (2-1/2)	6
Traprock & slag[2]	1,290 (200)	38 (1-1/2)	4
Carbonate	1,290 (200)	38 (1-1/2)	6

1. Mesh in cover
2. Air-cooled slag

Columns larger than 305 mm (12 in.) in diameter or

305 mm (12 in.) square are assigned 3 hr and 4 hr fire resistance classifications in most building codes in America.

It is suggested that the information in Table 41.1 be used for designing reinforced concrete columns for exposure to fire. This information is based on the results of a comprehensive series of fire tests on concrete columns reported in 1925. Only those types of columns which are commonly used today are included in the table.

Columns designed in accordance with the requirements of Table 41.1 have been used in concrete buildings for about 50 years. These ratings combined with requirements for structural adequacy have given economical column sizes that have performed well.

Recently analytical procedures have been developed for estimating temperature distributions in concrete columns during exposure to fire and for designing concrete columns for specific fire endurances and loads.

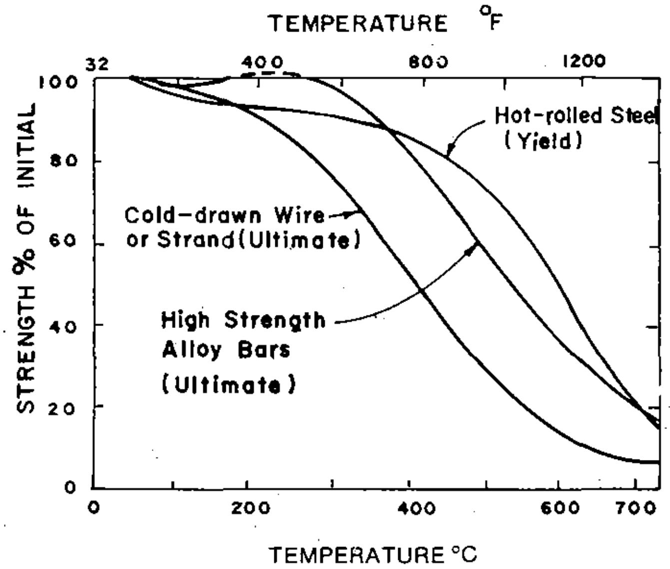


Fig. 41.3 *Strength of certain steels at high temperatures*

41.8 PROPERTIES OF STEEL AT HIGH TEMPERATURES

• *Strength*

Figure 41.3 shows the influence of temperature on the strength of certain steels. Included are data on the yield stress of structural steels and ultimate strengths of cold-drawn steel and high strength alloy steel bars used in prestressed concrete. Generally, the strengths of steels decrease with increasing temperature but ultimate strengths of hot-rolled steels are often slightly higher at temperatures up to about 260°C (500°F) than they are at room temperature.

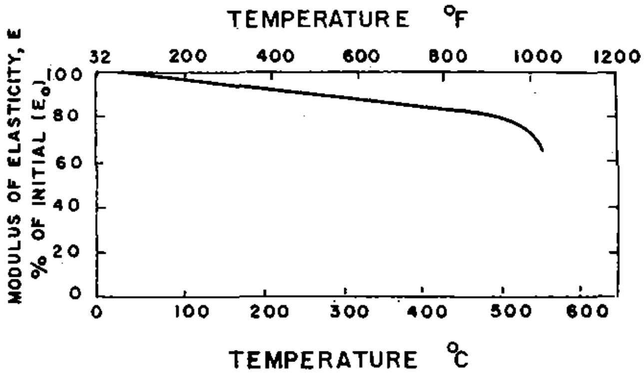


Fig. 41.4 *Modulus of elasticity of steel at high temperatures*

• *Modulus of Elasticity*

The modulus of elasticity of steel decreases with increasing temperature as shown in Fig. 41.4. Young's modulus for ferritic steels decreases linearly up to about 480°C (900°F). Above 480°C (900°F) the modulus decreases at a higher rate. Most investigators indicate that modulus of elasticity is not dependent upon microstructure of the steel. Consequently, the curve in Fig. 41.4 is representative of the types of steel used in concrete construction. However, there is a greater decrease in Young's modulus for prestressing steel than that shown in Fig. 41.4.

• *Thermal Expansion*

The average linear thermal expansion of ferritic steels over a temperature range of 200 to 650° (400 to 1200°F) is shown in Fig. 41.5. The coefficient of thermal expansion is not constant over this temperature region but increases as temperature increases. The temperature dependence of the coefficient of thermal expansion, α, is approximately by the formula:

$$\alpha = (11 + 0.0036\theta_1) \times 10^{-6}/°\text{C or}$$
$$\alpha = (6.1 + 0.002\theta_2) \times 10^{-6}/°\text{F}$$

in which $\theta_1 (\theta_2)$ is temperature in degrees Celsius (degrees Fahrenheit)

• **Stress-Strain relationships**

Stress-strain relationships for several types of steel have been reported by Harmathy and Stanzak.

Such curves for an ASTM A-36 steel are shown in Fig. 41.6. Figure 41.7 shows a family of stress-strain curves for cold-drawn prestressing steel, ASTM A421.

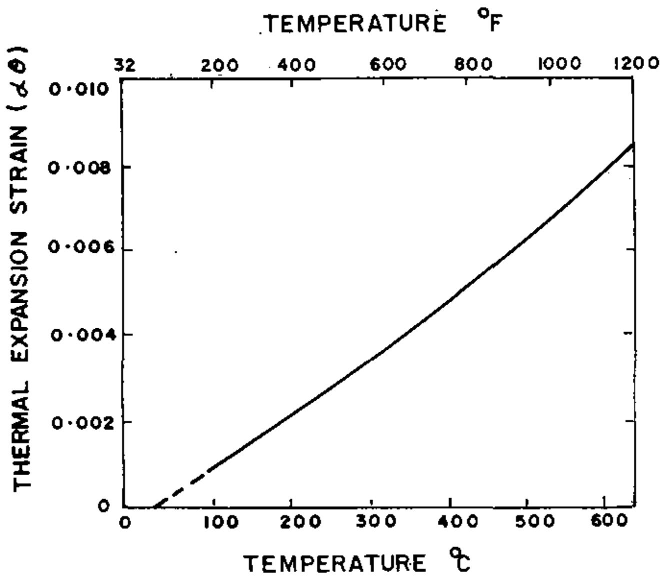


Fig. 41.5 *Thermal expansion of ferritic steels at high temperatures*

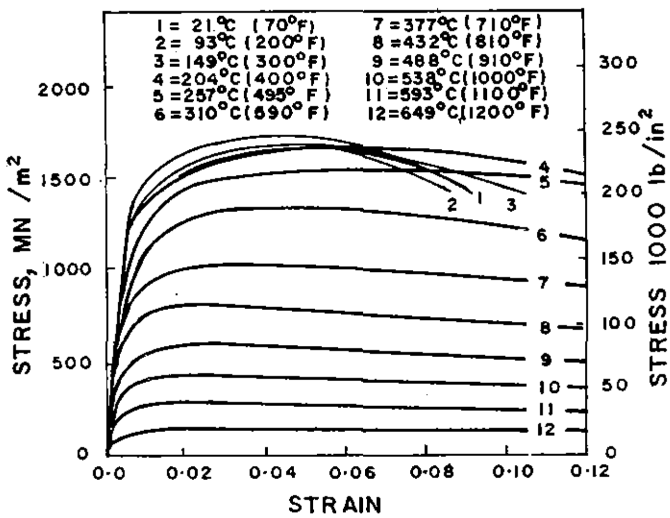


Fig. 41.7 *Stress-strain curves for prestressing steel (ASTM A-421) at various high temperatures*

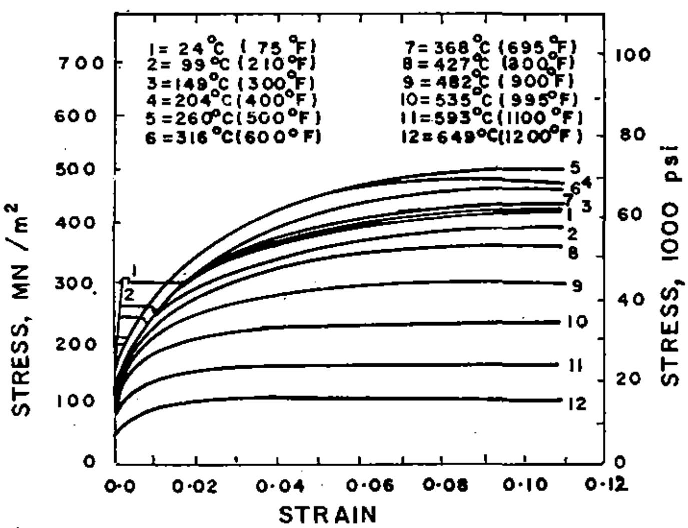


Fig. 41.6 *Stress-strain curves for structural steels (ASTM A-36) at various high temperatures*

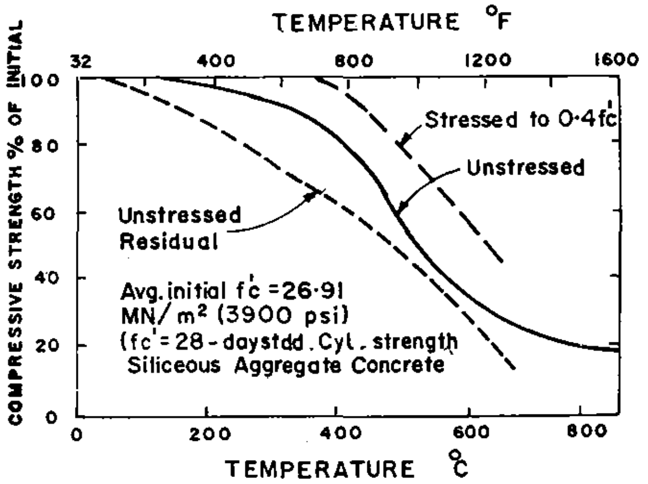


Fig. 41.8 *Compressive strength of siliceous aggregate concrete at high temperature and after cooling*

41.9 PROPERTIES OF CONCRETE AT HIGH TEMPERATURES

• ***Compressive Strength***

Compressive strengths of concretes made with different types of aggregates are shown in Figs. 41.8, 41.9, 41.10. Curves designated "unstressed" are for specimens heated to test temperature with no superimposed load and tested hot. Strengths of specimens heated while stressed to 0.4 f_c' (f_c' = 28-day cylinder compressive strength of concrete) and then tested hot are designated "stressed to $0.4f_c'$". The "unstressed residual" strengths were determined from specimens heated to test temperature, cooled to room temperature, stored in air at 75 per cent RH for 6 days and then tested in compression. Note that the "stressed" strengths are higher than the "unstressed" strengths. Abrams found that stress levels of 0.25 to 0.55 f_c' had little effect on the strength obtained. The "unstressed residual" strengths were in all cases lower than the strengths determined by the other two procedures. Abrams also noted that original concrete strengths between 27.6 and 44.8 MPa

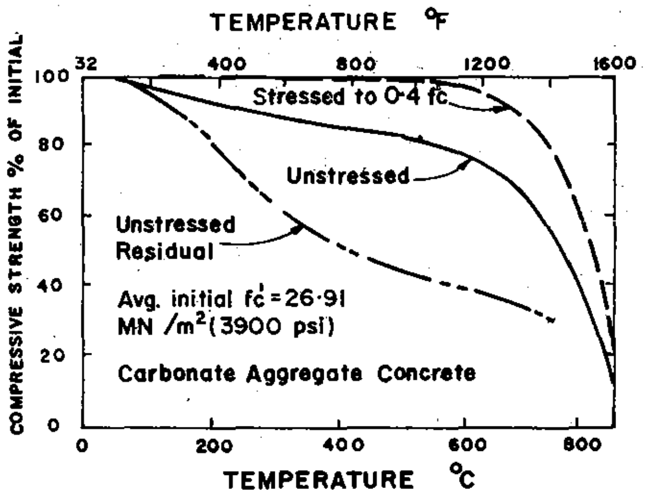


Fig. 41.9 *Compressive strength of carbonate aggregate concrete at high temperature and after cooling*

(4000 and 6500 psi) had little effect on the percentage of strength retained at test temperature. In Fig. 41.10 the "sanded" specimens were made with sand replacing 60 per cent of the lightweight fines, by volume.

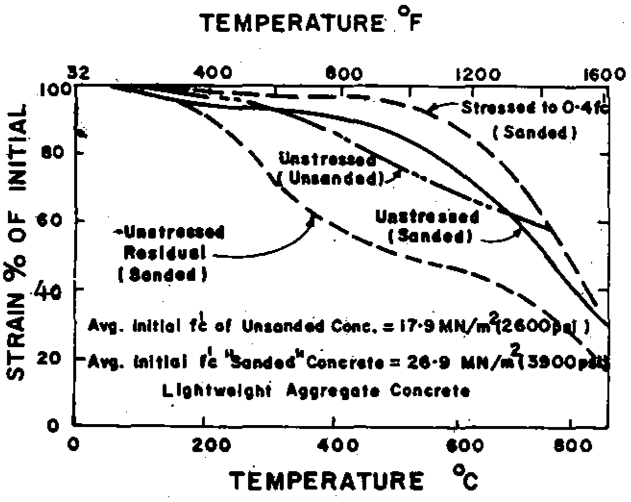


Fig. 41.10 *Compressive strength of lightweight concrete at high temperature and after cooling*

The "unsanded" concrete was the kind used in masonry block manufacture. Harmathy and Berndt reported data on the compressive strength of cement paste and a lightweight concrete from tests performed on specimens held at the target temperature in no-load condition for a period of 1 to 24 hr.

Further data on the strength of concrete at high temperatures have been reported by Zoldners, Malhotra, Saeman and Washa, Binner, et al., and Weigler and Fischer.

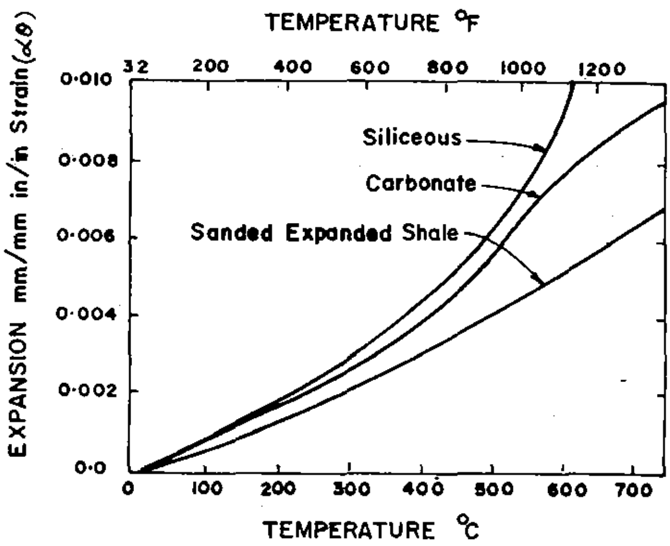


Fig. 41.11 *Thermal expansion of concrete at high temperatures*

• ***Thermal Expansion***

Figure 41.11 shows data on linear thermal expansion of concretes made with different aggregates. The data were obtained by Cruz.

Harmathy and Allen studied the thermal expansion of 16 different concretes used in masonry units. Among these, pumice concretes were found to exhibit considerable shrinkage at temperatures above 315°C (600°F). Dettling pointed out that thermal expansion of concrete is influenced by aggregate type, cement content, water content, and age. Philleo performed tests on a carbonate aggregate concrete using a different technique. He obtained somewhat higher values than those obtained by Cruz at temperatures above 370°C (700°F)

• ***Modulus of Elasticity and Shear Modulus***

Figures 41.12 and 41.13 show the effect of high temperatures on the moduli of elasticity and shear of concretes made with three types of aggregate. The data were obtained by Cruz.

From Cruz's data, it appears that aggregate type and concrete strength do not significantly affect moduli at high temperature.

Philleo obtained values for modulus of elasticity of a carbonate aggregate concrete using a dynamic method. His results agree closely with those obtained by Cruz up to about 370°C (700°F). From 370 to 650°C (700 to 1200°F),

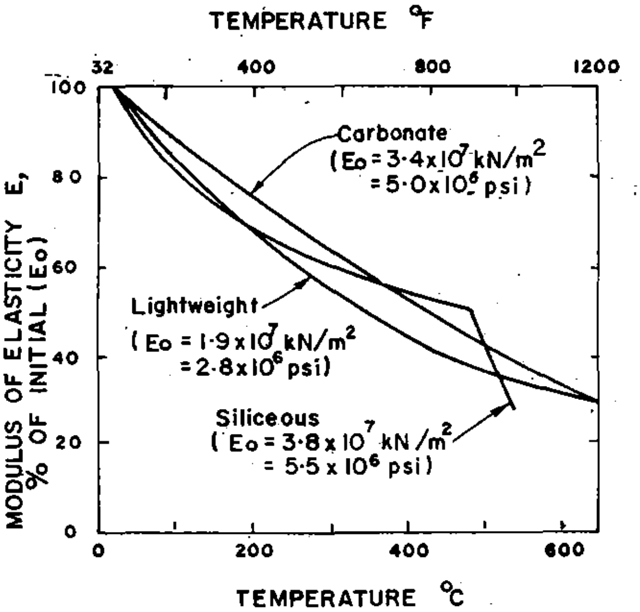


Fig. 41.12 *Modulus of elasticity of concrete at high temperatures*

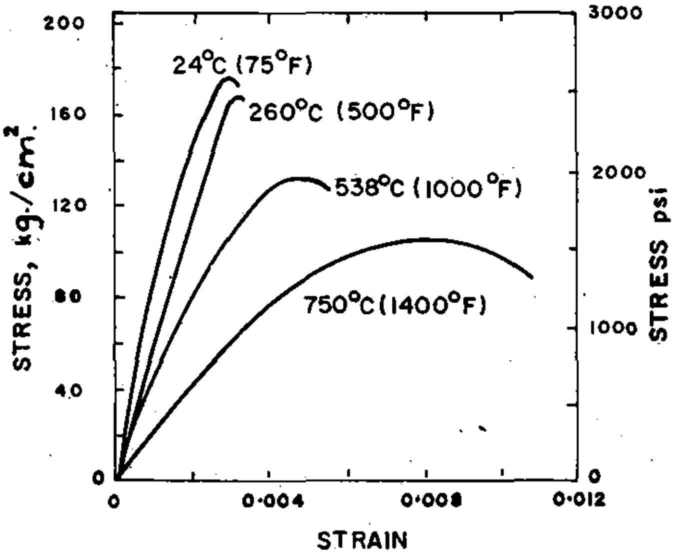


Fig. 41.14 *Stress-strain curves for a lightweight masonry concrete at various high temperatures*

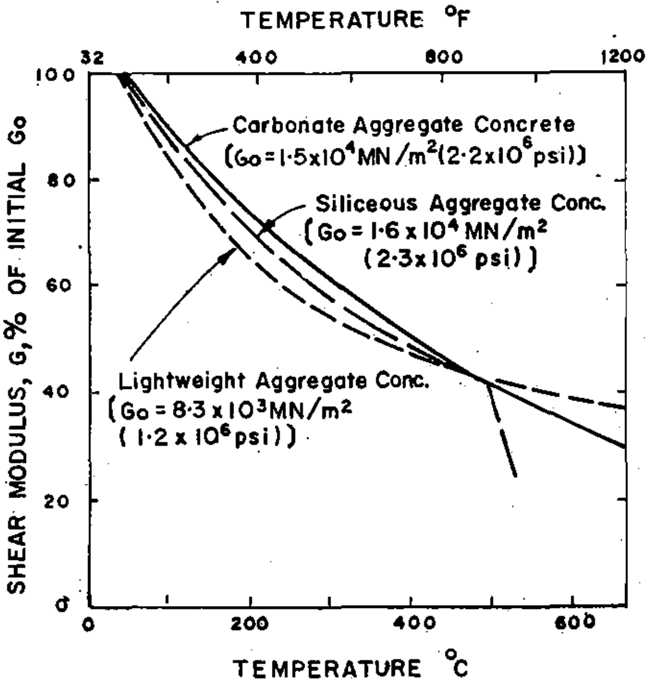


Fig. 41.13 *Shear modulus of concrete at high temperatures*

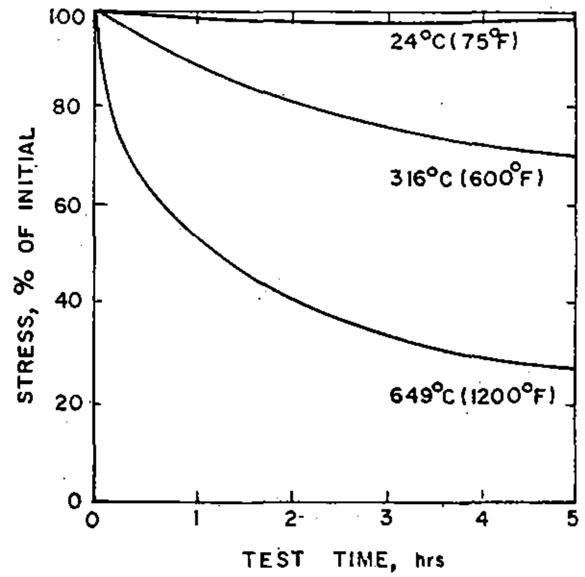


Fig. 41.15 *Stress relaxation of a carbonate aggregate concrete*

Philleo obtained higher values. Harmathy, and Saeman and Washa determined the modulus of elasticity in compression and found little change up to about 200°C (400°F).

- ***Poisson's Ratio***

Philleo and Cruz reported data on Poisson's ratio of concrete at high temperatures. Even though Philleo indicated a decrease in Poisson's ratio, both he and Cruz pointed out that results were erratic and no general trend of the effect of temperature was clearly evident.

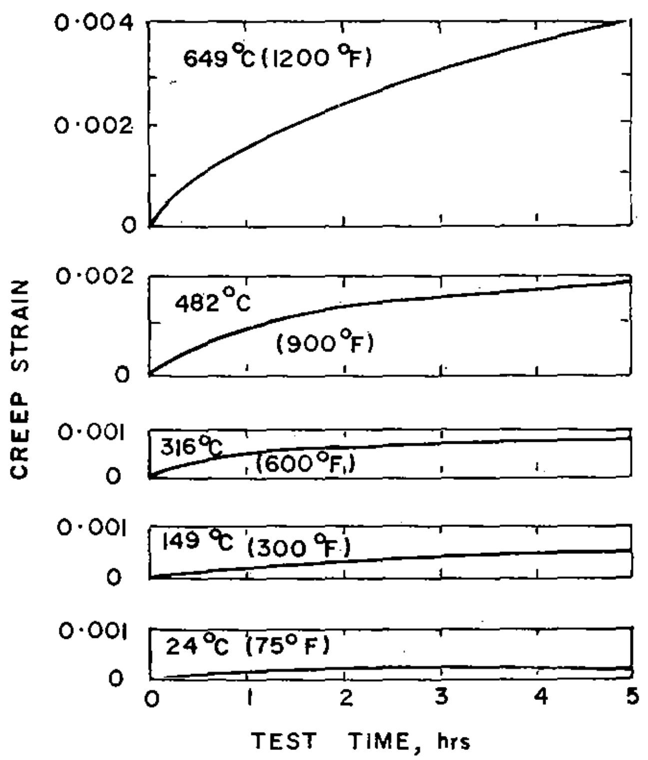


Fig. 41.16 *Creep of a carbonate aggregate concrete at various temperatures (applied stress = 12.42 MN/m^2 (1800 psi), f_c' = 27.6 MN/m^2 (4000 psi))*

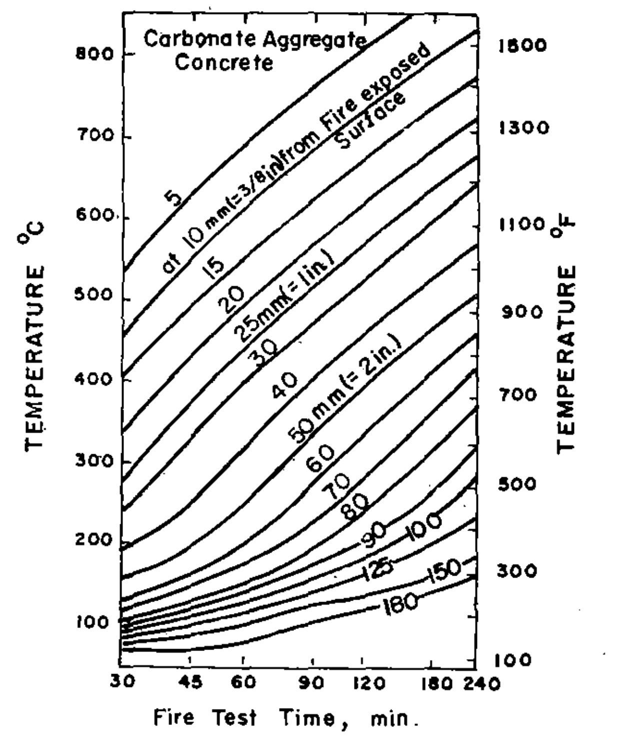


Fig. 41.17 *Temperatures within slabs during fire tests — carbonate aggregate concrete*

• ***Stress-strain Relationships***

Stress-strain relationships (in compression) of a lightweight masonry concrete (expanded shale aggregate) were reported by Harmathy. Figure 41.14 shows some of the data. Kordina and Schneider studied the stress-strain response of normal-weight concretes at variable temperatures under a number of loading conditions.

• ***Stress Relaxation and Creep***

Some data on stress relaxation and creep at high temperatures of a carbonate aggregate concrete were reported by Cruz. Figures 41.15 and 41.16 show the data graphically for a 5-hr test period. Nasser and Neville reported that age, moisture-condition, type and strength of concrete, and stress-strength ratio affect creep of concrete at high temperatures. Mukaddam and Bresler conducted studies on the creep of concrete at variable temperatures.

41.10 TEMPERATURE DISTRIBUTION WITHIN CONCRETE MEMBERS EXPOSED TO FIRE

• ***Slabs***

Figures 41.17, 41.18 and 41.19 show temperatures within concrete slabs during fire tests. Total slab thickness did not significantly affect the temperatures except for very thin slabs or when the temperatures were less than about 204°C (400°F). Temperatures in slabs were obtained from specimens 3 × 3 ft in plan with protected edges.

• ***Rectangular and Tapered Joists***

Computed and measured temperatures within rectangular beams made with quartzitic gravel have been reported by Ehm. The beam sizes he tested ranged in size from 64 × 305 mm (2.5 × 12 in.) to 279 × 559 mm (11 × 22 in.).

Figures 41.20, 41.21 and 41.22 show temperature distributions along the "centre-line" at various distances from the bottom of the beam and for widths up to 254 mm (10 in.) for normal-weight carbonate aggregate concrete for fire endurance periods of 1, 2, and 3 hr. The width b is the beam width for rectangular members and the width at a distance u from the bottom for the tapered member. These charts were generated from test data obtained from tests of rectangular and tapered members. Tests were carried out in Underwriters' Laboratories Floor Furnace, Northbrook, Illinois, and Portland Cement Association's Beam Furnace, Skokie, Illinois. Temperature distributions obtained in other furnaces may differ from those shown due to differences

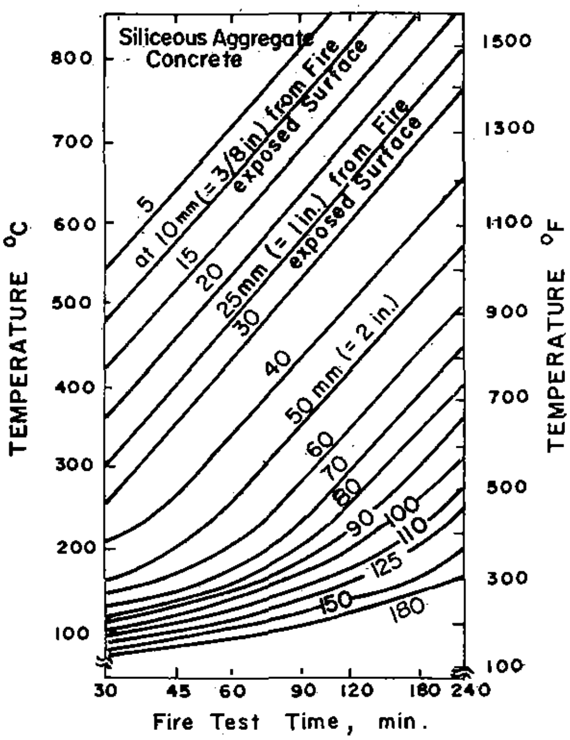


Fig. 41.18 *Temperatures within slabs during fire tests—siliceous aggregate concrete*

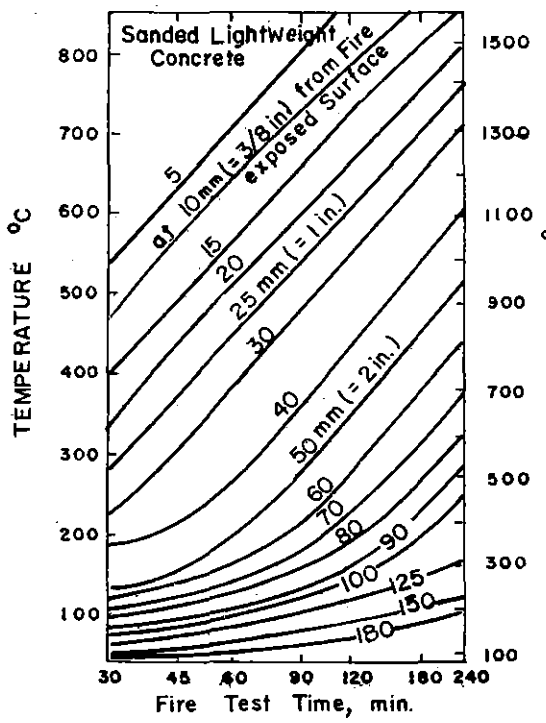


Fig. 41.19 *Temperatures within slabs during fire tests—sanded lightweight concrete*

in furnace size and design, furnace wall construction, and flame type.

Some National Codes of Practice specify certain "cover" and/or "minimum dimensions" to achieve a specific period of fire rating. These vary to a great extent in different countries. In the USA the P.C.I. Committee on Fire Resistance Rating adopted a Code provision in December 1971 which, however, has not been published on the consideration that some values are rather conservative.

In Table 41.2 the British Code CP 110 requirements, and the P.C.I. proposals (1971) have been incorporated.

In the British Code the end-restraint or continuity has not been directly taken into consideration. Advice has been given in such a case to use the values for the next lower rating-time.

In the American proposal a continuous beam or slab is defined as that which has been designed for continuity at least for superimposed dead load and full live load, and has at least 20 per cent of the maximum negative moment reinforcement made continuous "throughout the span".

The American proposal also includes restrained beams and slabs which are framed into a building in such a manner that restraints to thermal expansion occur although structurally the support condition could be simply supported.

By comparison it appears that the main difference between the American proposal and the British Code lies in the fact that the former recognizes the different behaviour of continuous beam "spaced up to" 4 ft (1.2 m) on centres and beams "speed over" 4 ft (1.2 m) on centres. Closely spaced beams act like an anisotropic slab showing better transverse distribution of load. The capacity of load distribution to adjacent ribs significantly improves fire performance since it appreciably reduces the risk, generally present in primary beams, of sudden collapse due to serious spalling. Obviously there is no clear-cut demarcation (beams having slightly more spacing than 4 ft (1.2 m) may not act very differently) but the demarcation line has been drawn on the basis of practical experience, especially with standard double-T units having rib centres on 4 ft (1.2 m).

For continuous beams spaced up to 4 ft (1.2 m) centres the dimensions (width and cover) are considerably higher in the British Code (except the cover for 1 hr rating). It should be noted that for a fire rating period of over 2 hr. the dimensions in the American proposal are almost halved for this category of beam. Even for beams spaced over 4 ft (1.2 m) centres the dimensions in the American proposal are appreciably less for fire rating periods of 2 to 4 hr. whereas the dimensions for 1 to 1-1/2 hr. period are slightly higher.

Regarding simply-supported *beams*, the British and American results are comparable although the American figures are slightly on the high side.

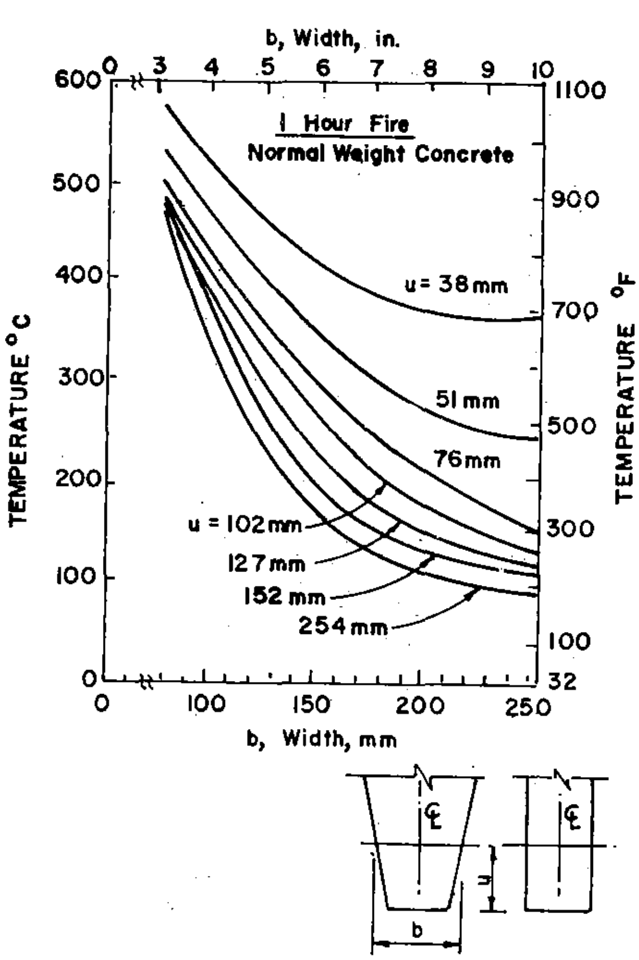


Fig. 41.20 *Temperatures in normal weight concrete rectangular and tapered units at 1 hr of fire exposure*

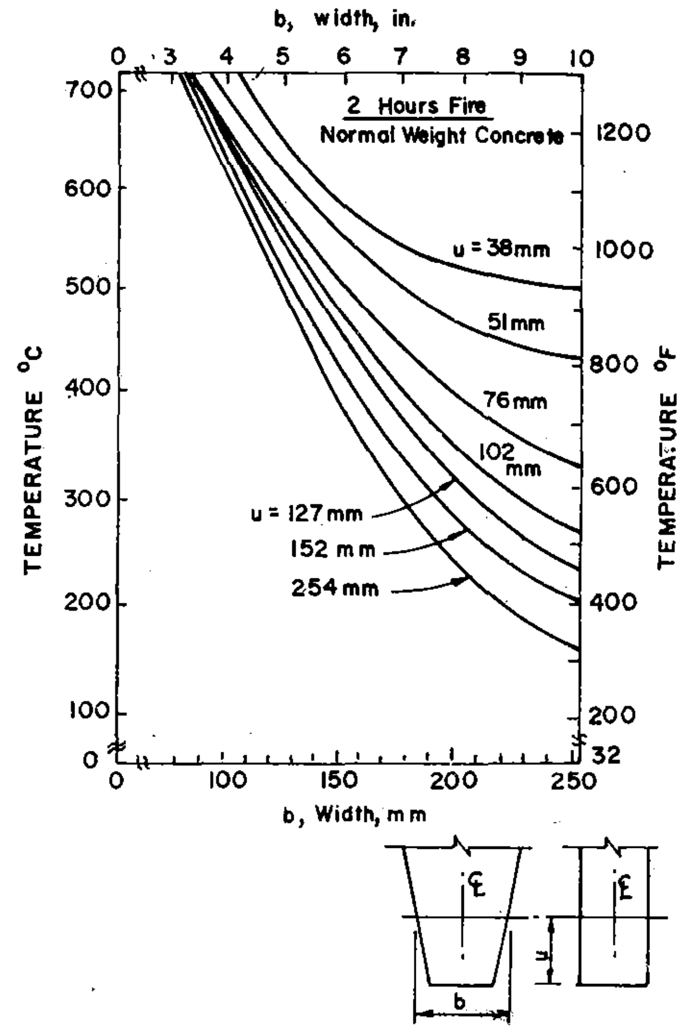


Fig. 41.21 *Temperatures in normal weight concrete rectangular and tapered units at 2 hr of fire exposure*

With regard to simply supported *slabs* the American proposals for 2 to 4 hr. range are less favourable, but below 2 hr. the difference is very small. As far as continuous slabs are concerned, the cover in the American proposal is considerably less than in the British Code expect for the 1 hr. rating. The thickness, on the contrary, is slightly higher in the American proposal.

Fire rating dimensions below 1 hr. have not been included in the Table, as the British Code gives dimensions relating to a 1/2 hr. period whereas the American proposal gives a 3/4 hr. period.

With regard to slabs the British Code also includes specifications for cored slab, hollow ribbed floor, upright T-slabs and inverted channel or U-slab. It is apparent that the P.C.I. results are quite progressive in many respects.

For prestressed lightweight concrete members Table 41.3 shows the dimensioning as proposed in the C.E.B. Manual on Lightweight Concrete. The definition of continuous and restrained members in this case is the same as in the American proposal.

In the case of a tapering beam the width refers to that at the centroid of steel. Where the cover exceeds 40 mm, supplementary reinforcement in the form of welded mesh should be provided to avoid spalling.

In the thickness of a floor, non-combustible floor finishes may be included.

If the end-span of a continuous beam or slab is so designed that the end-support can take the same moment as the intermediate support, the end-bay can be considered as continuous.

P.C.I. published a report on the Fire Resistance of Post-Tensioned Structures. The specification for cover corresponding to different periods of fire rating is virtually identical to that in the American proposal shown in Table 41.2.

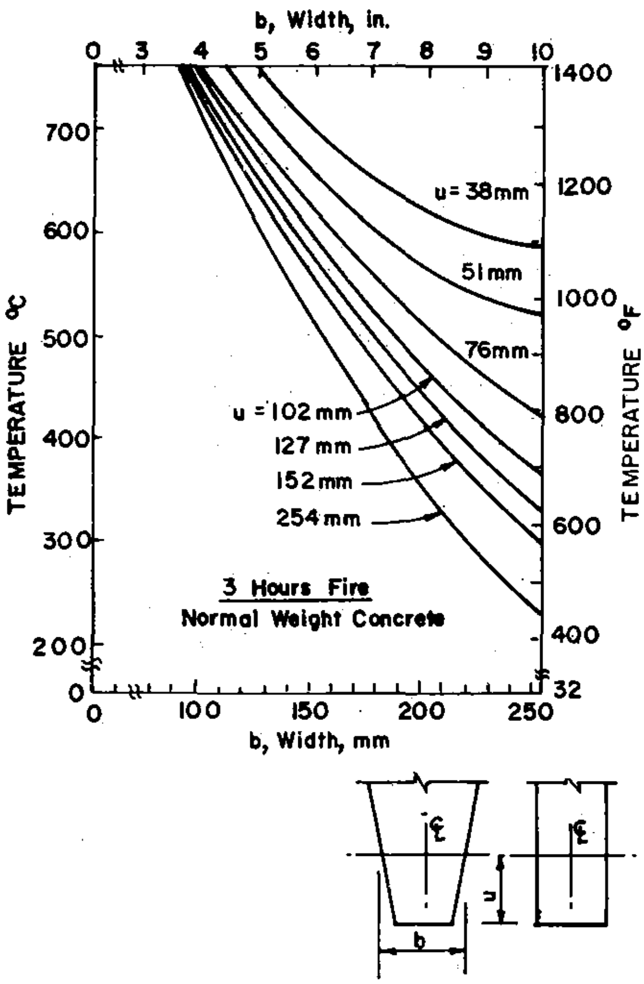


Fig. 41.22 *Temperatures in normal weight concrete rectangular and tapered units at 3 hr of fire exposure*

Table 41.2 *Minimum Dimensions in Millimetres for General Concrete*

	End condition				4 hours	3 hours	2 hours	1 1/2 hours	1 hour
BEAM	Simply supported	British code		Width	280	240	180	140	110
				Cover	100	85	65	50	40
		American proposal		Width	305	255	195	155	115
				Cover	95	80	65	50	45
	Continuous beams	British code		Width	240	180	140	110	80
				cover	85	65	50	40	25
		American proposal	1.2 m or less on centres	Width	110	98	79	65	65
				Cover	45	38	33	33	33
			Over 1.2 m on centres	Width	195	155	115	115	115
				Cover	65	50	45	45	45
SLAB	Simply supported	British code		Thickness	150	150	125	125	100
				Cover	65	50	40	30	25
		American proposal		Thickness	178	158	127	110	90
				Cover	71	59	46	38	28
	Continuous beams	British code		Thickness	150	125	125	100	90
				Cover	50	40	30	25	15
		American proposal		Thickness	178	158	127	110	90
				Cover	25	25	20	20	20

Table 41.3 *Minimum Dimensions in Millimetres for Lightweight Concrete*

	Member type and support condition	Minimum dimension	Minimum dimensions for fire resistance rating indicated in millimetres				
			4 hours	3 hours	2 hours	1 1/2 hours	1 hour
1	Beams, simply supported	Width	250	200	160	130	100
		Cover	100 (80)	90 (65)	75 (50)	65 (40)	50 (30)
2	Beams, composite with slab and continuous, spaced 1200 mm or less on centres	Width	100	90	70	65	65
		Cover	35	35	35	30	30
3	Beams, composite with slab and continuous, spaced over 1200 mm on centres	Width	200	180	130	100	90 (80)
		Cover	70 (65)	60 (50)	50 (40)	45 (30)	40 (20)
4	Simply supported slab	Thickness	135	115	95	80	65
		Cover	60	50	35	30	25
5	Continuous solid slab	Thickness	135	115	95	80	65
		Cover	30	25	20	20	20

NOTE In this table the figures in brackets indicate the appropriate values as specified in the British Code of Practice CP 110.
In CP 110 no provision has been made for continuous beams spaced up to 1200 mm on centres. Therefore, the approximate values for continuous beams in the British Code have been shown.
For prestressed lightweight concrete slab the British Code (CP 110) suggests the same dimensions as the normal-weight aggregate concrete.

SELECTED BIBLIOGRAPHY

1. *Reinforced Concrete Fire Resistance*, Concrete Reinforcing Steel Institute, Chicago, 1980.
2. *FIP/CEB Report on Methods of Assessment of Fire Resistance of Concrete Structural Members*, Cement and Concrete Association, London, 1978, p.91.
3. Harmathy, T.Z., "Determining the Temperature History of Concrete Construction Following Fire Exposure," ACI Journal, *Proceedings* V.65, No. 11, Nov. 1968, pp. 959–964.
4. Prestressed Concrete Institute, Prestressed Concrete Resists Fire, Chicago, 1968, p. 12 R202-68 (Containing research results by T.Y. Lin and Associates Inc., Chicago).
5. Bobrowski, J., Presentation of the Report of the Joint Committee of the Institution of Structural Engineers and the Concrete Society on the *Fire resistance of concrete structures*, Institution of Structural Engineers and Concrete Society, Birmingham, 1975.
6. Gustaferro, A.H., "Design of Prestressed Concrete for Fire Resistance," *Journal*, Prestressed Concrete Institute, V. 18, No. 6, Nov.-Dec. 1973, pp. 102–116.
7. Iding, R. et al., *Behavior of Reinforced Concrete Under Variable Elevated Temperatures*, Department of Civil Engineering, University of California, Berkeley, 1975.
8. Janney, W. and R. Elstner, *Fire Damage to the Avianca Building*, Bogota: FIP Congress, New York, 1974.
9. Harmathy, T.Z. and W.W. Stanzack, "Elevated-Temperature Tensile and Creep Properties of Some Structural and Prestressing Steels," *Fire Test Performance*, STP-464, American Society for Testing and Materials, Philadelphia, 1970, pp. 186–208.
10. "Steels for Elevated Temperature Service," U.S. Steel Corporation, Pittsburgh, 1965, p. 90.
11. Garofalo, E., P.R. Malenock, and G.V. Smith, 'The Influence of Temperature on the Elastic Constants of Some Commercial Steels," *Symposium on Determination of Elastic Constants*, STP-129, American Society for Testing and Materials, Philadelphia, 1952.
12. Dannenberg, Deutschniann, and Melchior, "Effect of Heat on Tensile Strength in Tests of Prestressing Steel (Warmzerreissversuche mit Spannstahlen)," *Bulletin* No. 122, Deutscher Ausschuss fur Stahlbeton, Berlin, 1957, pp. 11–35. Also, English Translation, *Foreign Literature Study* No. 483, Portland Cement Association.
13. Dorn, J.E., "Some Fundamental Experiments on High-Temperature Creep," *Journal of Mechanics, Physics and Solids*, V.3, 1954, p. 85.
14. Gustaferro, A.H., "Fire Resistance of Post-Tensioned Structures," *Journal*, Prestressed Concrete Institute, V. 18, No. 2, Mar-Apr. 1973, pp. 38–63.
15. *Symposium on Fire Resistance of Concrete*, SP-5, American Concrete Institute, Detroit, 1962, p.88.
16. Bryson, J.O., and D. Gross, "Techniques for the Survey and Evaluation of Live Floor Loads and Fire Loads in Modern Office Buildings," *Building Science Series* No. 16, National Bureau of Standards, Washington D.C., 1967, p. 30.
17. Gustaferro, A.H., M.S. Abrams, and E.A.B. Salse, "Fire Resistance of Prestressed Concrete Beams, Study C, Structural Behaviour During Fire Tests," *Research and Development Department Bulletin* No. RD009.01B, Portland Cement Association, Skokie, 1971, p.29.
18. "Fire Test of a Simple, Statically Indeterminant Beam," *Report* No. B1-59-22, TNO Institute for Structural Materials and Building Structures, Delft. Also, English Translation, SLA Translation Center, John Crerar Library, Chicago.
19. Ehm, H., and R. von Postel, "Tests of Continuous Reinforced Beams and Slabs Under Fire," *Proceedings*, Symposium on Fire Resistance of Prestressed Concrete, English Translation, SLA Translation Center, John Crearar Library, Chicago.
20. Abrams, M.S., et al., "Fire Endurance of Continuous Reinforced Concrete Beams," *Preliminary Report* of the Tenth Congress of the International Association for Bridge and Structural Engineering, Portland Cement Association, Skekie, 1976.
21. Gustaferro, A.H., "Temperature Criteria at Failure," *Fire Test Performance*, STP 464, American Society for Testing and Materials, Philadelphia, 1970, pp.68–84.
22. Abrams, M.S., and A.H. Gustaferro, "Fire Endurance of Concrete Slabs as Influenced by Thickness, Aggregate Type, and Moisture," *Journal*, PCA Research and Development Laboratories, V. 10, No. 2, May 1968, pp.9–24. Also, *Research Department Bulletin* No. 223, Portland Cement Association.
23. Abrams, M.S., and A.H. Gustaferro, "Fire Endurance of Two-Course Floors and Roofs," ACI Journal, *Proceedings*, V. 66, No. 2, Feb. 1969, pp. 92–102.

24. Gustaferro, A.H., M.S. Abrams, and Litvin Albert, "Fire Resistance of Lightweight Insulating Concretes," *Lightweight Concrete,* SP-29, American Concrete Institute, Detroit, 1971, pp. 161–180.
25. Lie. T.T., "Calculation of Fire Resistance of Composite Floor and Roof Slabs," *Fire Technology,* Feb. 1978.
26. Harmathy, T.Z., "Thermal Properties of Concrete at Elevated Temperatures," *Journal of Materials,* V. 5, No. 1, Mar. 1970, pp.47–74.
27. Abrams, M.S., "Compressive Strength of Concrete at Temperatures to 1600F," *Temperature and Concrete,* SP-25, American Concrete Institute, Detroit, 1971, pp. 33–58.
28. Malhotra, H.L., "The Effect of Temperature on the Compressive Strength of Concrete," *Magazine of Concrete Research* (London), V. 8, No. 23, Aug. 1956, pp. 85–94.
29. Harmathy, T.Z., and J.E. Berndt, "Hydrated Portland Cement and Lightweight Concrete at Elevated Temperatures," ACI Journal, *Proceedings,* V. 63, No. 1, Jan. 1966, pp. 93–112.
30. Cruz, Carlos R., "Elastic Properties of Concrete at High Temperatures," *Journal,* PCA Research and Development Laboratories, V. 8, No. 1, Jan. 1966, pp. 37–45. Also, *Research Department Bulletin* No. 191, Portland Cement Association.
31. Pettersson, O., "Theoretical Design of Fire Exposed Structures." *Bulletin* No. 51, Division of Structural Mechanics and Concrete Construction, Lund Institute of Technology.
32. Gustaferro, A.H., "How to Design Prestressed Concrete for a Specific Fire Endurance," *Introductory Report,* Tenth IABSE Congress (Tokyo, 1976), International Association for Bridge and Structural Engineering, Zurich, 1976.
33. Gustaferro, A.H., and L.D. Martin, *Design for Fire Resistance of Precast Prestressed Concrete,* Prestressed Concrete Institute, Chicago, 1977, p.81.
34. Gustaferro, A.H., and C.C. Carlson, "An Interpretation of Results of Fire Tests of Prestressed Concrete Building Components," *Journal,* Prestressed Concrete Institute, V. 7. No. 5, Oct. 1962, pp. 14–43.
35. Lie, T.T., and D.E. Allen, "Calculation of the Fire Resistance of Reinforced Concrete Columns," *Technical Paper* No. 378, Division of Building Research, National Research Council of Canada, Ottawa.
36. Thompson, J.P., "Fire Resistance of Reinforced Concrete Floors," *Publication* No. EB065B, Portland Cement Association, Skokie, 1963, p.32.
37. Becker, J.M., H. Bizri, and B. Bresler, "A Computer Program for the Fire Response of Structures—Thermal," Report No. UCB FRG 71-1, Department of Civil Engineering, University of California, Berkeley, Jan. 1974, Walls.
38. Abrams, M.S., and A.H. Gustaferro, "Fire Endurance of Prestressed Concrete Units Coated with Spray-Applied Insulation," *Journal,* Prestressed Concrete Institute, V. 17, No. 1, Jan-Feb. 1972, pp.82–103.
39. Abrams, M.S., and B. Erlin, "Estimating Post-Fire Strength and Exposure Temperature of Prestressing Steel by a Metallographic Method," *Journal,* PCA Research and Development Laboratories, V. 9, No. 3, Sept. 1967, pp. 23–33. Also, *Research Department Bulletin* No. 219, Portland Cement Association.
40. Harmathy, T.Z., "Deflection and Failure of Steel-Supported Floors and Beams in Fire," *Symposium on Fire Test Methods—Restraint and Smoke,* STP-422, American Society for Testing and Materials, Philadelphia, 1967, pp.40–62.
41. Lie, T.T., and T.Z. Harmathy, "Fire Endurance of Concrete-Protected Steel Columns," ACI Journal, *Proceedings,* V. 71, No. 1, Jan.1974, pp.29–32.
42. Dettling, H., "The Thermal Expansion of Hardened Cement Paste, Aggregates, and Concretes (Die Warmedehnung des Zementsteines, der Gesteine und der Betone)," *Bulletin* No. 164, Deutscher Ausschuss fur Stahlbeton, Berlin, 1964, Part 2, pp. 1–64. Also, English Translation, *Foreign Literature Study* No. 458, Portland Cement Association.
43. Allen, L.W., and T.Z. Harmathy, "Fire Endurance of Selected Concrete Masonry Walls," ACI Journal, *Proceedings,* V.69, No. 9, Sept.1972, pp. 562–568.
44. Harmathy, T.Z., "Design of Concrete Masonry Walls for Fire Endurance," *Behavior of Concrete Under Temperature Extremes,* SP-39, American Concrete Institute, Detroit, 1973, pp. 179–203.
45. Nasser, K.W., and A.M. Neville, "Creep of Old Concrete at Normal and Elevated Temperatures," ACI Journal, *Proceedings,* V. 64, No. 2, Feb.1967, pp. 97–102.
46. Mukaddam, M.A., and B. Bresler, "Behavior of Concrete under Variable Temperature and Loading," *Concrete for Nuclear Reactors,* SP-34, American Concrete Institute, Detroit, 1972, pp. 771–797.
47. Mukaddam, M., "Creep *Analysis* of Concrete at Elevated Temperatures," ACI Journal, *Proceedings,* V.71, No. 2, Feb.1974, pp.72–78.
48. Carman, A.D., and R.A. Nelson, "The Thermal Conductivity and Diffusivity of Concrete," *Bulletin* No. 122, Engineering Experimental Station, University of Illinois, Urbana, Apr. 1921, p.32.
49. Zoldners, N.G., "Effect of High Temperatures on Concrete Incorporating Different Aggregates," *Mines Branch Research Report,* No. 64, Department of Mines and Technical Surveys, Ottawa, May 1960, p.48.
50. Raina, V.K., "Concrete for Construction—Facts and Practice," Tata McGraw-Hill, New Delhi.

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CHAPTER 42

Economics and Quantity-Trends in Alternative Bridge Structure Schemes

42.1 SYNOPSIS

Talking alone never pulled out any stumps. Only first-hand, first-rate, commercially biased, result-oriented practising professional experience, gained in cold-blooded competition, gives a workman-like understanding of the true mechanics of what actually constitutes 'economics' in the study of alternative bridge structure schemes in given situations. This, tempered with scientific unbiased analysis of the results, and sifted with clinically inquisitive curiosity of a practising professional, can unravel the 'quantity figures' on which the economics is based.

Such 'quantity figures' represent a fund of practically useful information which is not found in straight textbooks, class-room lectures or in laboratory exercises and research reports. (Real professionals are generally too busy to dwell in mundane publications.)

Such distilled 'quantity figures', when presented in a tool-kit manner, reflect the usefulness of hundreds of competitive engineering man-months of top quality computer aided analysis and design of myriads of alternative actual bridge structures, with many of which the author was personally involved first hand, more as a doer than as just a checker.

With this as the background, the author presents here first the facts of bridge economics and then the results of certain alternative case-studies that show the relative economics of different structure schemes. This is followed by various practical inferences and finally are presented the actual all-important 'quantity trends' in various alternative bridge structure schemes. These represent an invaluable amalgam of high strung perspiration and inspired slog.

42.2 INTRODUCTION

There is no one form of design which would be always most economical. It is only by comparing a few designs that the economic design can be found in a particular set of conditions.

However, sometimes the quantities of concrete and steel expressed per square metre of deck area are quoted as indicative of economy although these figures are not the only ones which govern the cost of bridge. Other considerations which influence the cost are: the length of the individual spans, the type of deck cross-section, the number of longitudinals in cross-section, the width of the bridge, the depth and type of foundation, excavation and fill involved, the cost of formwork, the cost of materials and labour, whether reinforced concrete or prestressed concrete, whether precast or cast-in-situ, the method of erection, the size of contract, etc.

Difficult foundations alone may justify increase of span lengths to reduce the number of foundations, in spite of increase in the cost of the deck owing to longer spans.

For smaller spans, i.e. below 30 m, the comparison is even more difficult and depends entirely on local circumstances.

Figure 42.1 shows the usual relationship between the length of span and the cost of the bridge per unit of plan area of the deck. It is comparatively high for short spans, reduces for medium spans with repetitive elements, and rises for longer spans.

Figure 42.2 indicates average relative quantities of concrete and steel per square unit of deck plan area for various types of decks for a given and design criteria. It appears that for spans smaller than about 35 m, simply-supported beams are probably the cheapest type of construction from purely materials point of view.

For spans larger than about 45 m, the cost of simply supported beams increases rapidly and becomes practically prohibitive at more than about 60 m.

Where the deck is less than about 8 m above the ground and foundations are simple and shallow, shorter spans of the order of about 18 m to 25 m can be more economical. Still shorter spans, using about 10 m span, 3 to 4 span continuous modules, reinforced concrete solid slab deck, may be more convenient if not more economical but this might adversely affect the aesthetics and also require more subsequent maintenance, particularly if protection against scour at its numerous piers is called for at a later date.

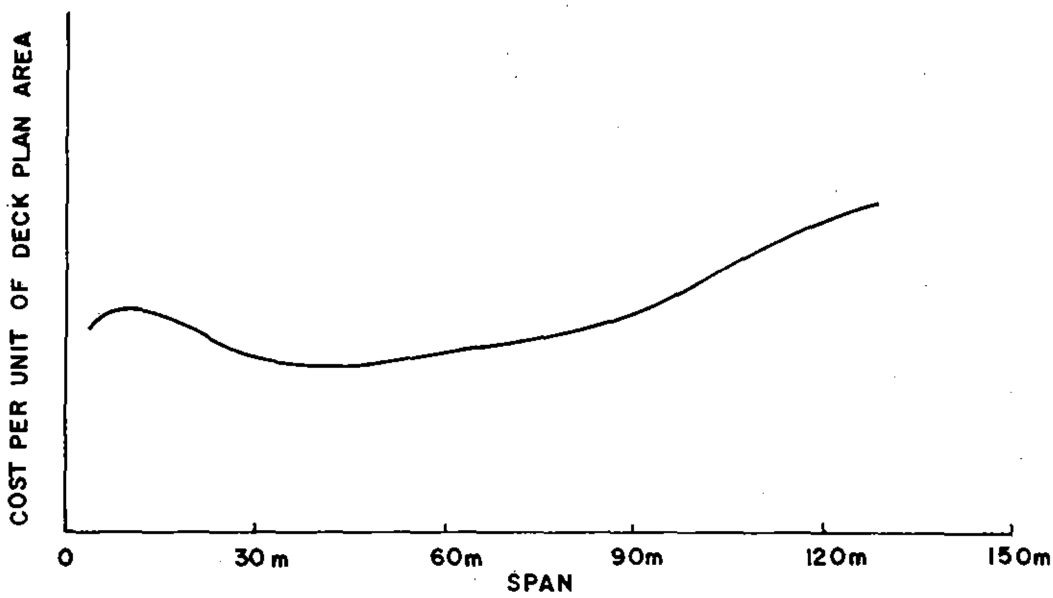

Fig. 42.1

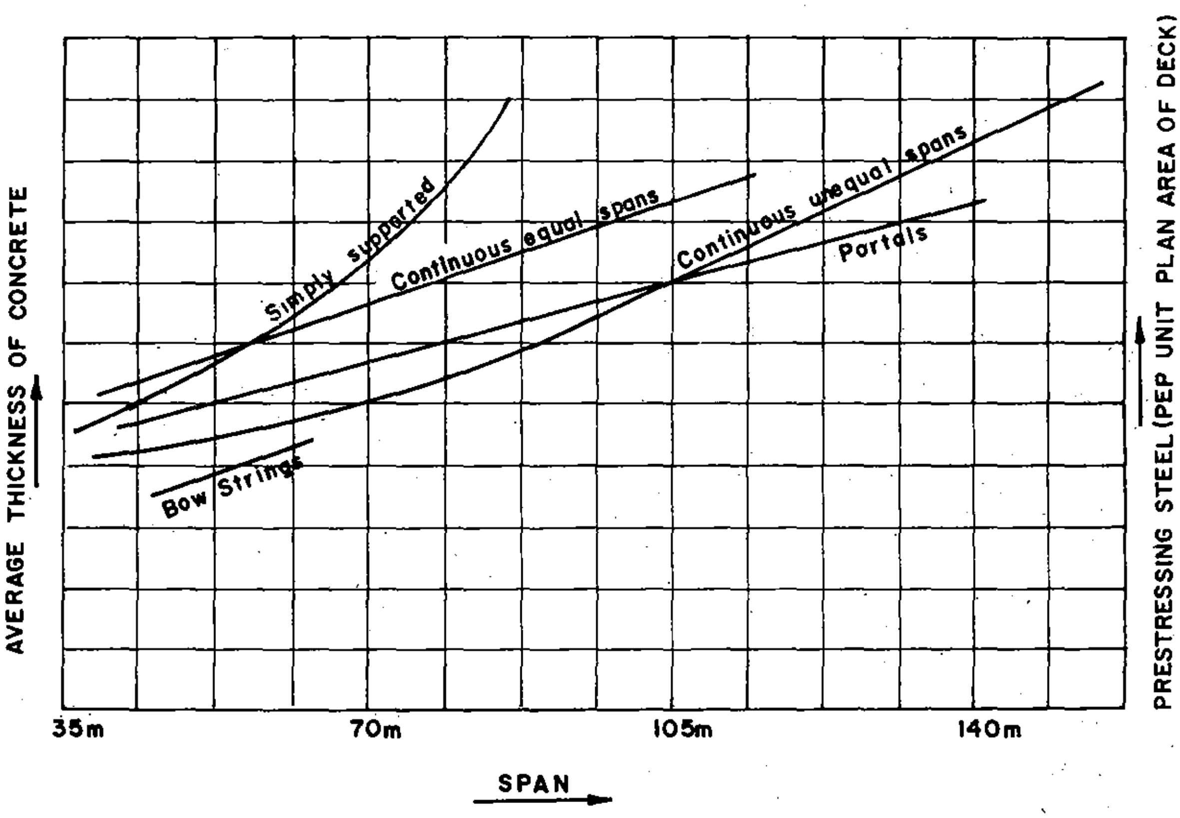

Fig. 42.2 *Approximate quantities of concrete and steel in various types of bridge decks*

NOTE Actual values will depend on the loading and the design specifications. The above curves are only indicative and relative.

In the 18 m to 25 m span range, under the circumstances, the most economical deck for a multi-span bridge could be:

— precast prestressed concrete simply supported girders which may be made continuous for finishings and live load by the in-situ reinforced concrete top slab (assuming that the effect of differential settlement of foundations is small).
— Precast reinforced concrete girders simply supported for self weight but temporarily propped at one-third points till the cast in place top slab becomes composite with the beams. (May be made continuous for finishings and live load.)
— voided prestressed concrete slab, continuous if differential settlement of foundations is not a problem.

Continuous beams of equal spans do not seem to bring significant savings up to about 45 m spans, but become more economical above that span length, particularly if the cost of foundations is not, in general, different from what it would be in case of simply supported beams.

Continuous beams with unequal spans offer noticeable economics. The range of applicability of such solutions depends on the ratio of short span to long span. With this ratio approaching 0.3, the bridge deck may be considered 95% fixed and the longer span may be increased even to about 90 m. For these very high degrees of fixity obtained by alternating short spans, foundations may become expensive because of the extreme piers needing to resist the uplift. Counter-weights or anchorages may become necessary which can add considerably to cost.

Portal frames lead to quantities somewhat similar to those in continuous beams with unequal spans. Generally, portals would be slightly more expensive than continuous beams with unequal spans for spans less than about 90 m. The cost of foundations, however, may tilt the balance.

It appears that for portals the quantities of steel increase slowly with the span. This is because of the thrust, although this thrust must be paid for in the foundations, depending on the nature of the ground.

Concrete arches are seldom prestressed because prestressing adds very little to the natural advantage of thrust in arch design.

Bowstring girders are cheaper solutions from the point of view of quantities of materials, but the cost of formwork is high and may effect the overall economy. Their better aesthetic appearance, however, cannot be debated out in terms of economy alone.

A tenderer's policy may involve putting large sums of money against certain items of the work because those particular items would be completed earlier in the game, and the money will thus be recouped at an early stage in the contract. All such factors can distort the data collected from tender bids as a basis for building up cost and quantity information data bank. However, scathing exposure to and a workman-like analysis of a very large number of fiercely competitively designed and actually constructed bridges, of various types, first hand, in cold commercial competition (which is far different from class-room lecturing, pure academic noise, or restrictive staid office experience, where one does not actually evolve, conceive and design the alternatives—and build first hand, but essentially only proof reads and oversees someone else's work) can unveil the treasure of 'quantity-trends' and unfold the 'economics of alternative bridge structure schemes'.

42.3 ECONOMICS—VARIOUS CONSIDERATIONS

This chapter is intended to provide a guide for the preliminary planning basically of reinforced and prestressed concrete bridges. Discussion and typical geometries are presented for several commonly used types of bridge structures. The list is not intended to be complete. Suggestions made should be regarded as general guidelines only.

Preliminary design and dimensional proportioning are necessary for preliminary cost estimates and comparative economics of all bridge structures, and also for the analysis for all indeterminate bridge structures. Through advance layout and approximate dimensional proportioning, based on theory, practice, experience, and judgement, one may establish probable geometries and dimensions which will permit the designer to analyze and design the structure. With further revisions, the engineer can then produce a complete set of contract drawings for the structure.

Before moving on to the more specific considerations, it would be appropriate to review several factors which will have a direct bearing on the decision-making process. First of all, it should be recognized that the high cost of construction-labour has forced changes toward industrialized construction methods. Technological progress in mass production of high performance precast and prestressed concrete with compressive strengths of 450 to 600 kg/cm^2 are already available. Methods and equipment for transporting and erecting concrete elements weighing over 100 tonnes are now available.

In longer span bridges, more attention will be paid to reduction in superstructure dead loads. As a result, the utilization of thinner sections and lower density concretes will receive more consideration in future.

Factors to be Considered

In selecting the type of structure for a particular site, the following factors should be debated, decided and established:

Geometry

1. Horizontal and vertical clearances
2. Permissible structure-depth and support-widths
3. Detour or traffic accommodation during construction
4. Utilities: those to be crossed and those to be carried.

First-cost and Ease of Maintenance

1. Sub-surface conditions
2. Total time of construction
3. Construction details — repetitive elements
4. Total cost including approach fills
5. Requirements for any future extensions.

Safety

1. During construction — to adjacent property, utilities, construction-crew, and highway/channel traffic
2. Minimizing traffic hazard in completed structure.

Appearance

1. Similarity to and harmony with adjacent structures
2. Congruence with the environment
3. Use of attractive shapes and surface treatments.

Waterway Crossing — Special Requirements

1. Bridge length, waterway, and pier spacing, to accommodate flood discharge
2. Backwater effects
3. Maintenance of waterway-traffic during construction
4. Navigational requirements and approvals.

Railway Crossings — Special Requirements

1. Overhead clearances
2. Loadings, impact, and vibration
3. Operational restrictions and approvals.

Additional Factors Particularly Associated with Precast Prestressed Construction

1. Cost of tooling (formwork, cranes, etc.)
2. Length of production run
3. Plant production versus on-site production
4. Weights of elements
5. Transportation and erection costs
6. Material production, transportation and erection strategy.

In general, the items to be finalized as early as practical are as follows:

(a) Typical sections and alignment (vertical and horizontal)
(b) Span lengths and arrangement
(c) Structure type and span to depth ratios — including type of materials
(d) Other considerations.

Typical Sections and Alignment

Many bridges have had to be redesigned because of modifications of roadway alignment, indecisions on the number of lanes, change of waterway clearances, width of shoulders, type of curb and railing, and size of pipelines to be carried. It is necessary that these items be established as early as possible to avoid expensive and time-consuming redesign.

Span Lengths and Arrangement

For grade separation bridges the total length is established by the facility crossing under the structure. Location and type of supports are dictated by foundation conditions, required clearances, safety, and aesthetics. Long spans with few supports make the most graceful composition, constitute the least obstruction and, aesthetically, create sensations that are very satisfying.

For river crossings the span lengths may be dictated by any combination of the following:

(a) Foundation requirements
(b) Height of piers
(c) Waterway area and other hydraulic considerations
(d) Navigation requirements
(e) Type of debris expected during flood stages
(f) Facility of construction
(g) Economy

Structure Type, Span to Depth Ratio and Various Other Considerations

In the following subsections the general characteristics of the common types of concrete bridge structures are indicated for reference in preliminary studies. In addition to structural aspects, various other considerations are listed (Fig. 42.3).

Reinforced Concrete Slab Bridge

Structural: Span to depth ratios should be about 15 for simple spans and 19 to 24 for continuous spans; solid slabs are used for spans from approx. 5 to 14 m; cored or voided slabs are used for spans from 14 to 25 m.

Appearance: Neat and simple; desirable for low short spans.

Construction: Details and formwork simplest.

Traffic: May be impeded by falsework if cast-in-place, due to reduced clearances. Guide-rail should protect falsework openings for traffic lanes.

Construction time: Shortest of any cast-in-place construction.

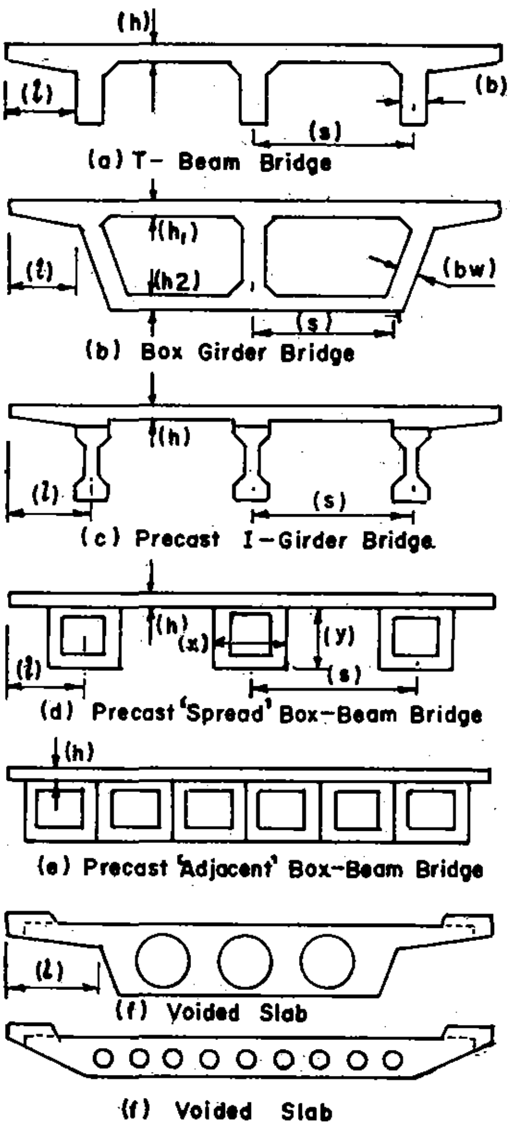

Fig. 42.3 *Concrete bridge deck configurations*

Maintenance: Very little except at hinges. Future widening may be difficult.

Reinforced Concrete T-Beam Bridge

Structural: Span-depth ratio is generally 14 ± for simple spans; 16 ± for continuous spans; higher ratios are possible, but riding qualities are affected by creep characteristics of concrete; span range: 10 to 25 m.

Appearance: Cluttered in view from bottom; elevation is neat and simple.

Construction: Requires good finish on all surfaces; formwork is complicated.

Traffic: May be impeded by falsework if cast-in-place, due to reduced clearances. Guide-rail should protect falsework openings for traffic lanes.

Construction time: More than for slabs due to forming, but not excessively longer.

Maintenance: Low, except that bearing and hinge details may require attention.

Reinforced Concrete Box Girder Bridge

Structural: Span-depth ratio is generally 16 ± for simple spans; 18 ± for continuous spans; higher ratios are possible, but riding qualities are affected by creep characteristics of concrete; high torsional resistance makes it suitable on curved alignment; span range of 25 to 70 m.

Appearance: Neat and clean lines from all views; utilities, pipes and conduits can be concealed.

Construction: Rough form finish is satisfactory on inside surfaces; formwork is complicated.

Traffic: May be impeded by falsework due to reduced clearances. Guide-rail should protect falsework openings for traffic lanes.

Construction time: More than for slabs or T-beams due to staging of concrete placement, but still not excessively long.

Maintenance: Low, except that bearing and hinge details may give some trouble. Future widening may be difficult.

Prestressed Concrete Slab Bridges

Cast-in-place, Post-Tensioned

Structural: Voided slabs up to about 40 m. where a high span-depth ratio is required; either simple or continuous spans; span-depth ratios as high as 40 ± have been used (generally 30); high torsional resistance, making it very suitable for curved alignment, especially on single columns.

Appearance: Neat and simple; desirable for low short spans.

Construction: More complicated than conventional reinforced concrete. Sequence of stressing and grouting should be supervised by specialists.

Traffic: May be impeded by falsework due to reduced clearances. Guide-rail should protect falsework openings for traffic lanes.

Construction time: About same as conventional reinforced slabs.

Maintenance: Very little except at hinges.

Precast, Pretensioned or Post-Tensioned

Structural: Voided slabs for spans 10 to 25 m.; span-depth ratio: 25 to 30.

Appearance: Neat and simple; desirable for low short spans.

Construction: Details and formwork very simple; plant fabrication methods are favourable and applicable; field erection may be fast.

Traffic: No falsework required; units placed by cranes; no prolonged impediment to traffic.

Construction time: Time for erection of precast elements at site a minimum.

Maintenance: Very little except at hinges.

Prestressed Concrete Girder Bridges

Cast-in-place, Post-Tensioned

Structural: Generally applicable to spans 25 to 200 m., either T or box girder form; either simple, cantilever, or continuous; span-depth ratio is usually 20 ± for simple spans and 25 ± for continuous spans; as high as 33 ± have been used; haunched box girders, to 200 m, with span-depth ratio at mid-span 50 ± and at supports 33 ±, both cast-in-place and precast combinations.

Appearance: Beams may be cluttered in view from bottom; elevation neat and simple.

Construction: More complicated than conventional reinforced girders.

Traffic: May be impeded by falsework unless using cantilever construction which is practical for major structures only.

Construction time: Slightly longer than for conventionally reinforced T-beam or box girders.

Maintenance: Low except that bearing and hinge details may require attention.

Precast T, I, U and Box Shaped Beams, with in-situ Slab

Structural: Applicable to spans 10 to 50 m.; the span-depth ratio for I-beam is 18 ± for simple spans and 20 ± for continuous spans; for spread box beams is 18 to 22; and for adjacent box beams is 25 to 30.

Appearance: I-beam is similar to T-beam, except for bulbous bottom of prestressed girder sections; spread box-beam is similar to T-beam; adjacent box beam is similar to cast-in-place box girder.

Construction: Fabrication is more complicated than in conventional reinforced concrete girders; longer span girders require very careful handling after fabrication in plant and at job site; standard girders are preferred; special sections are more expensive; standard forms are stocked by fabricators; can be used as simple spans, but preferably used as continuous spans under live load; precast girders with deck slab cast-in-situ.

Traffic: No falsework is required except for long spans with precast segments to be post-tensioned unless stage stressed by a cantilever method.

Construction time: Time for erection of precast element at site a minimum, but may require slab falsework.

Maintenance: Low, except that bearing and hinge details may require attention.

Rigid-Frame Bridges

Structural: Integral rigid negative-moment knees greatly reduce the positive span moment and (overturning) moment at foundation level; single rigid portal frames will adapt to narrow water channels, railways, subways, and divided or undivided highways underneath; double-span rigid frames suitable for divided multi-lane highways underneath. Rigid frames (with or without side spans) can cross multi-lane divided highways with a wide centre median. The horizontal member may be of any of the construction forms described earlier. The advantage of a variable moment of inertia can be easily incorporated; preliminary proportioning can start with a thickness at the knee equal to approximately twice that at the crown.

Appearance: Graceful and clean—well adjusted to stone facing.

Construction: Usually requires curved form-work for variable depth.

Traffic: May be impeded by falsework due to reduced clearances. Guide-rail should protect falsework openings for traffic lanes.

Construction time: Similar to that of other types.

Maintenance: Low, except for potential back-fill settlement, limited flexibility for future widening.

Arch Bridges

- *Fixed arch (Filled or open spandrel)*

Structural: Horizontal reactions created an arch greatly reduce the otherwise large positive moment in the centre; constant depth for small spans and variable moment of inertia for medium and long spans; spans as long as 300 m have been built; rise-to-span ratio varies with topography; thickness at springing lines usually is slightly more than twice that at the crown; filled spandrels are used only with short spans; for medium and long deck spans, open spandrels with roadways carried by columns are the rule; in a through arch, the centre deck usually is carried by hangers and side decks by columns; use long single spans over deep waterways and shorter multiple spans over wide shallow waters with rocky bed.

Appearance: Graceful and attractive, especially over deep gorges, ravines, or a large waterway.

Construction: Either using falsework or cantilever method.

Traffic: When traffic cannot be diverted, the cantilever method may be used in lieu of falsework.

Construction time: Usually longer than other structures.

Maintenance: Low.

- *Two-Hinged Rib Arch*

All statements made for the fixed arch apply, except the following.

Structural: The two hinges usually are placed at springing lines, either at the same level or not, depending on topography; sections at the hinges, being moment-free, take thrusts, constant sections are used more with two-hinged rib arches than with fixed arches; design of hinges becomes complicated for heavy long-span arches where fixed arch should prevail; roadways usually are supported on columns carried by the deck arch; in a through arrangement, only the side decks are carried by columns while the centre deck is supported by hangers; multiple two-hinged arch spans are uncommon.

- *Tied Arch Bridge*

All statements made for the two hinged arch rib apply, except for the following structural aspects

(a) Tied-arch bridges are used where the supporting rock foundation cannot resist the arch thrust.
(b) The horizontal thrust of the arch is entirely taken by the tie.
(c) The tied arch is always used as a through span.
(d) The deck floor of a tied-arch bridge is always carried by hangers.
(e) Multiple tied arch spans are a succession of individual tied arches.

Truss Bridge

Applicable to spans 35 to 80 m.

Structural: A truss bridge may be supported as a simple, cantilever, or continuous structure. It may have parallel chords, inclined upper chord in a through type, or inclined lower chord in a deck type. It may be prestressed, precast, and field jointed; or precast, and post-tensioned. For spans longer than usual for girders it may prove competitive. A span-depth ratio of 10 or less should be sued for a deck truss. For about equal quantity of materials, a truss has less deflection than its girder counterpart. However, concrete trusses are uncommon.

The Vierendeel truss has not at all been widely used. A rational analysis is available to solve the three unknowns in each member with computer programmes.

Appearance: Probably the least attractive of any bridge type.

Construction: Complicated forming makes the construction slow.

Traffic: Requires extensive falsework, makes it difficult to continue traffic under the bridge during entire construction.

Construction time: Usually quite long.

Maintenance: Moderate because of complexity of trusses. Future widening is a real problem.

Suspension and Cable Stayed Bridges

Structural: Cable stayed girder bridges have been used for spans of 400 to 500 m; with cellular piers, to carry towers and cable stays. Suspension type bridge serves well for spans exceeding about 300 m.

Appearance: Graceful and delicate; well adapted to both urban and rustic environment.

Construction: At difficult crossings the suspension bridge is simpler to construct than the other common bridge types; stayed-type bridges are especially adapted to long spans.

Traffic: Falsework usually is not necessary; deck may be constructed by the cantilever method or suspended from cables.

Construction time: Considerable.

Maintenance: Moderate, because of complexity of suspension system.

Initial Geometric Proportioning of Deck

The deck slab overhang, designated as in Fig. 42.3 should be provided as required to produce desirable aesthetic effect and reduce the span moment.

Girder spacing S may vary as justified by comparing the cost of more girders against that of additional slab thickness. It is generally more economical to use greater depth of girder (or stem) and wider spacing. Usual ranges are 1.6 to 2.5 m for T-beam bridges, 2 to 3.5 m. for box-girder bridges, 2 to 4.5 m for precast prestressed I-beam bridges, and 2 to 4 m for precast, prestressed box-beam bridges.

Stem width, b, shall be a minimum of 27 cm for up to 8 longitudinal bars. It is often necessary to increase this thickness over continuous supports to accommodate compressive forces.

For prestressed girders the stem width should preferably be 30 cm minimum to facilitate provision of prestressing tendons. The bottom slab thickness should be approximately 1/20 of clear span between webs, but should not be less than 15 cm.

The thickness of the bottom slab, h_2, may be increased near continuous supports to accommodate compressive forces.

The webs of box girder superstructures should have a minimum thickness b of 20 cm. It is often useful to increase the thickness of webs near supports to provide adequate concrete shear resistance.

Precast box beams ordinarily have a width of 100 to 120 cm and heights in the range of 60 to 120 cm.

Abutments

There are two basic types of abutments: open type or spill-through type and closed type or non-spill through type.

Open type abutments are more economical than closed type since they allow the backfill to spill out in between their columns thereby reducing the active earth pressure. But, due to this very reason, the backfill may settle down, should it be washed through under heavy rains. In such a case a deep curtain wall should be provided extending downward from the abutment cap, for at least 1.5 m below the spill-line.

Piers or Bents

The term "pier" pertains to an intermediate substructure unit between the abutments. A "bent" is a pier consisting of one or more columns in the substructure unit, with or without cap.

Pile Bents Type are economical but are suitable for stream crossings where floating debris is minimal. They may be competitive in placid waters and in dry beds where soil is poor and footings laid at shallower depths might settle. The preferable minimum width of cap is 75 cm.

Pile bents are unsightly for bridges 6 m or more in height and unsuitable if scour is significant.

Solid Piers They are located in streams where floating debris and rolling boulders accompany the fast current. They are preferable for long spans, and may be supported on spread footings, pile foundations or caissons, as appropriate. A pier wall should have a low thickness/height ratio, but should not be less than 30 cm thick. This is the minimum thickness that should be used with two planes of reinforcing steel.

Multi-Column Bents and Multi-Columns They generally support dry land structures. They normally replace pile bents when spread footings are recommended. They may be supported on either spread footings or pile foundations. Columns may be circular, rectangular or variable in section to provide architectural effect.

The thickness/height ratio of normal height uniform section columns should be in the range of 1/12 to 1/15 for appearance as well as structural reasons. Unnecessarily thick columns can invite high temperature and shrinkage stresses and very slender ones will require buckling analysis.

Single-Column Bents and Single Columns They can sometimes be used to avoid skewed piers. They are adapted to viaducts over city streets where the location of the column may be restricted.

Foundations

Footings are appropriate where scour is low and soil bearing capacity is enough not to create settlement problems, or if rock is available at a shallow depth. In any case, if subsoil water is encountered footings are feasible only if dewatering is possible. Otherwise piles are preferable, which enables transferring the loads directly to lower and firmer substrata, without requiring dewatering. These are costlier than footings, in general. However, in case of deep scour, rolling boulders, floating debris and strong current, the (generally costlier) caissons become necessary. Caissons are relatively massive and generally suit spans not less than 25 m., preferably longer.

Economic Evaluation

If the form of construction chosen for each component of a bridge is functional and well proportioned, for the conditions imposed by the layout and location of the structure, it is likely that the overall solution is an economic one. ***The appropriateness of various forms of construction to differing situations and ranges of span is a judgement usually attributed to professional experience. Involvement in the first hand design and construction of varying types of structures is the best means of developing a keen sense of their suitability in differing situations.***

It is certainly a waste of effort to carry our lengthy calculations on optimization for component parts within a solution which is itself inappropriate. In most engineering design situations there are a myriad practical constraints on the proportions and options open.

Some of the most commonly quoted factors relating to economy can yield disappointing results. It is often stated that contractors prefer methods of construction which are speedy to erect. Once committed to a project the contractor's preferences are likely to be very much orientated in that direction, but this is not to say that he would give the best tender price for the solution which offers the most rapid construction. ***Within a highway contract there may well be individual structures which are critical in terms of allowing access through a site, but differences in the overall cost on the project which might result from the delayed or early completion of such a structure are unlikely to be reflected in the costs attributed to it in the overall bills of quantities.***

There are no permanent rules about the comparative costs of differing solutions for construction, because changes take place in the relative cost and availability of labour and materials from one time to another, and developments in constructional techniques.

There are inevitable difficulties in forecasting the cost of civil engineering works, and these are aggravated during times of economic uncertainty or rapid inflation. Quite apart from the general financial climate of the country, the rates adopted by contractors are obviously influenced by their current workload, so that low pricing

cannot be expected during a period when the construction industry is heavily committed.

Costs in tenders submitted by several contractors frequently show marked disparities in the sums of money assigned to individual portions of the work, even where the total tender prices may be similar. Bridge designers have always been sensitive to this problem because the cost of the bridges is commonly only part of the total cost of a highway project. There may therefore be wide discrepancies between the prices placed against bridgeworks by different competitors, within overall tender prices which may be close. There can also be widely differing rates against indentical items in comparable works within separate contracts.

Difficulty arises in making comparisons between works that are not similar. A comparison of the difference between deep spread footings in difficult ground and the use of piling would be sensitive to certain rates which therefore require detailed examination to confirm their reliability. Similar items may demand significantly different rates within alternative solutions. An obvious example is the formwork required to abutment-wall-construction if the options are between a plain surface for a cantilevered wall and the shaping required to form counterforts. The rates for the formwork in such cases will be markedly different.

It is also important to identify an appropriate unit for making cost comparisons. It can be misleading to assume idealized conditions when making comparisons between alternatives. Most choices of this kind bring, in their wake, a number of peripheral issues, some of which may be significant in terms of cost. There can be other valuable guidelines to the economy of a form of construction, apart from direct calculations of cost. For example, the contents of materials — concrete, reinforcing or prestressing steel — expressed per unit area of bridge deck, etc. may be useful guidelines to economy; indicating the quantity trends.

42.4 SOME USEFUL INFERENCES

Given below, in a workman-like summary form, are certain inferences relevant to quantities in the more usual types of bridges. These are essentially in the vein of a guideline, indicating the relative trend in the quantities in well designed alternatives, all designed for same loads using the same set of design specifications.

For same deck-depth, a box section deck has less concrete than that in a voided deck, and ultimately is cheaper. However, practical considerations regarding access for formwork eliminate box decks of less than 1.2 m depth. In such cases, voided decks are preferable (Fig. 42.4).

There is an 'optimum depth' for a 'given' span for 'minimum prestress'. Very shallow decks require high prestress. Box decks require lesser prestress than

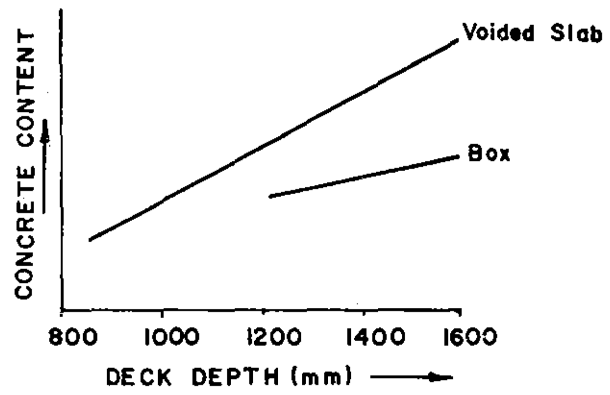

Fig. 42.4

corresponding voided decks (Fig. 42.5).

A box with thin walls may require heavy reinforcement to resist local bending, but there is little change in it with increase in deck depth, and overall quantity may still be less in the case of box deck when compared with a well detailed voided deck ***(where void formers require to be held down against floatation)*** (Fig. 42.6)

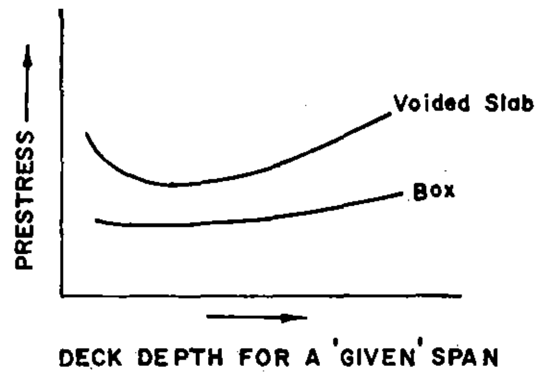

Fig. 42.5

Where deck depth enables a box construction, its overall cost can be cheaper than that of the voided slab. But the cost comparison is sensitive to the slenderness of the deck in relation to the span. An actual comparison is therefore necessary for the span under consideration (Fig. 42.7).

Whatever form of construction is adopted the unit cost per square metre of a bridge deck tends to increase with the span (Fig. 42.8).

If the cost of an individual pier and its foundation is fairly constant, it is apparent that the total cost of the pier substructures will reduce with increase in the span. However, for a given bridge length the cost of bridge finishings is constant, regardless of the span lengths adopted,

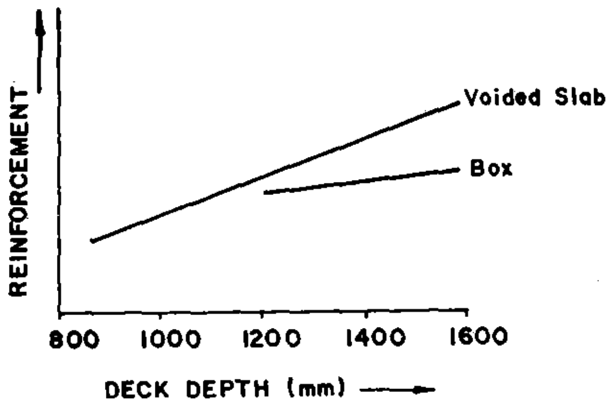

Fig. 42.6

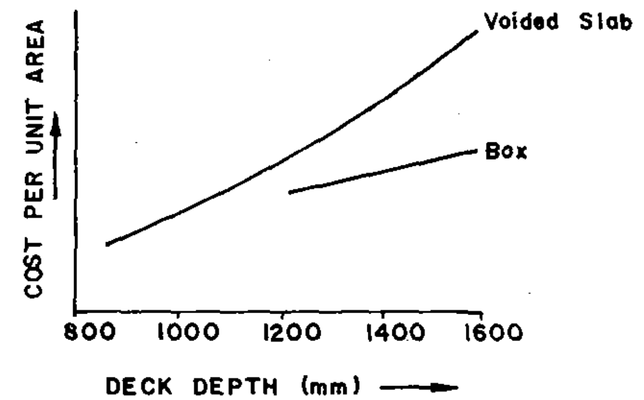

Fig. 42.7

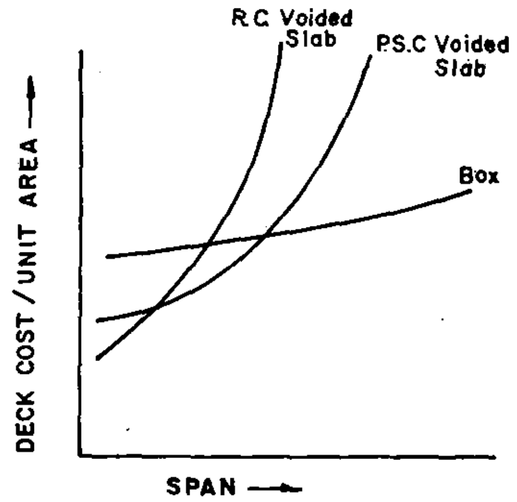

Fig. 42.8

and nearly so is the cost of its abutments (Fig. 42.9).

As the span increases, cost of deck becomes greater. Since pier costs are primarily related to height, longer spans

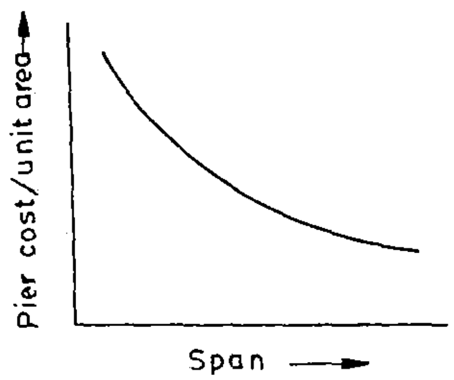

Fig. 42.9

will be economical where piers are tall and foundations deep.

For a given span length, a particular deck depth is optimum; prestress will increase for lesser or more depth. If a very slender deck is adopted, high stresses develop which demand a high prestress. The shallowest deck is not, therefore, the most economical (Fig. 42.10).

For a given span, the quantity of reinforcement increases *with increase in deck-depth in prestressed concrete but* reduces in reinforced concrete. However, some details, such as diaphragms, require less steel with greater depths (Fig. 42.11).

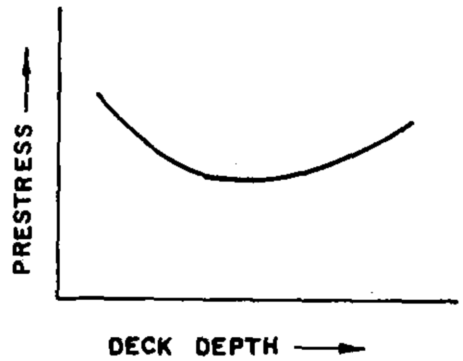

Fig. 42.10

Formwork for the soffit is only marginally affected by increases in depth. However, side shutters and void forms obviously increase with increasing depth (Fig. 42.12).

Even with the lower grade of concrete called for in mass abutments and consequent lower unit rates for such concrete, *mass abutments may be cheaper only when either other rates* are high and/or seismic activity is very low (Fig. 42.13).

For high walls the reinforcement needed in cantilever abutments can become very heavy, particularly if there is

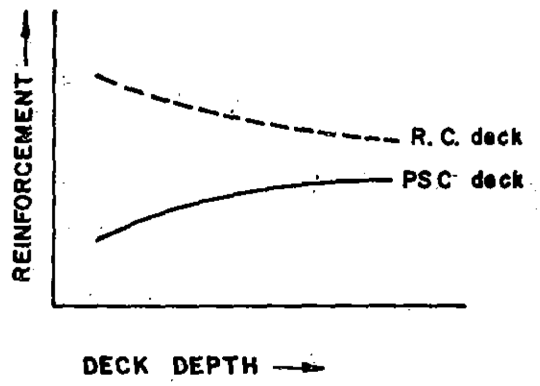

Fig. 42.11

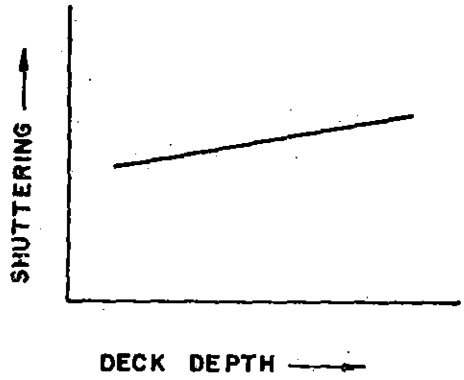

Fig. 42.12

difference in the rates between large and small diameter bars (Fig. 42.14).

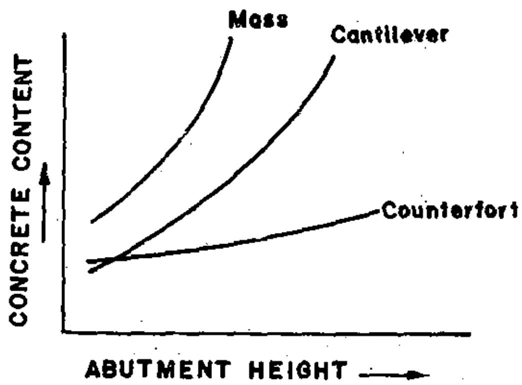

Fig. 42.13

Complicated shuttering and details needed in counterfort abutment construction attract higher rates (Fig. 42.15)

Apart from theoretical economic considerations in abutments, the choice between wall types may be influenced by the speed of construction and practical considerations

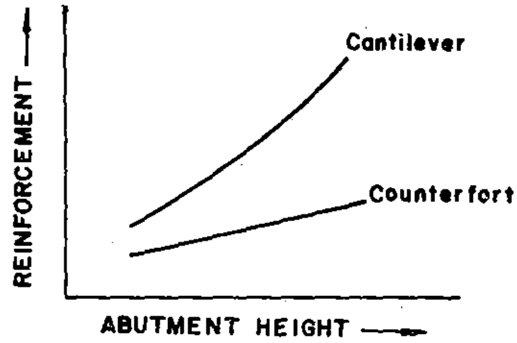

Fig. 42.14

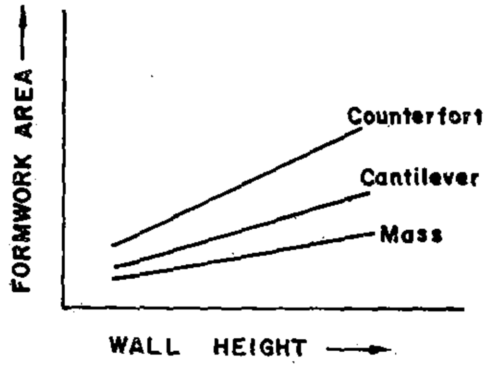

Fig. 42.15

relating to the concreting of wall sections at a given height.

42.5 ADDITIONAL FACTORS

Some of the additional factors that have a marked effect on the overall cost and hence on the relative economy of the bridge structure are:

(a) current material costs in the market or as supplied,
(b) current cost of transportation of materials to site including all handling and storage,
(c) economic climate in the country at the time of costing the bid,
(d) whether there is abundance or scarcity of civil engineering works in the current market,
(e) current labour wages and labour availability,
(f) possibility of increase in material costs and labour wages during the currency of contract, i.e. price escalation incident to inflation,
(g) location of the site of work with respect to the nearest rail and road head (effect on hauling and movement costs),
(h) length of working season per year,
(i) quantity of work involved,
(j) difficulties in construction (e.g. extents of detours required, including their maintenance; whether construction is on a new alignment which may be undisturbed by

traffic or whether the construction is in an urban area and therefore handicapped by heavy traffic volumes, etc.),

(k) whether any major equipments and apparatus are required to be bought afresh,

(l) contractor's overhead expenses, profit margin and promotional expenses (whether any special provisions to be built in!) and

(m) whether a low 'first-cost' is the over-riding constraint, which may result in higher ultimate cost (partly because of possible underestimation in design and costing at bid time, and partly because of higher subsequent maintenance cost), or a more sedate costing (including provision for better looks) would be encountered.

The present chapter deals basically with 'economy and quantity-trends' as a result of alternative structure schemes. Details about 'costing', i.e. 'pricing the costs' of 'constructing' a particular item of work, the 'consultancy fee' and the 'overheads', have been discussed separately in the author's other books.

42.6 TYPICAL COMPARISON—CASE STUDY 1

Problem

To design in detail suitable structural alternatives for a right-bridge, crossing a six lane divided highway, and to compare their strategic dimensions, quantities and relative costs.
Deck width: about 10.4 m (number of design lanes: 3)
Soffit of Deck: about 5.5 m above the highway.
Abutments: as dwarf walls resting on approach embankments. *Piers:* 2-column frame with a cap, on 45T or more working load capacity cast-in-situ piles (each 12 m or more in length). Minimum length of bridge (end to end) 46.4 m (to clear the dual-carriageway highway including its median and hard shoulders). Length may be suitably increased for aesthetics and to offer better proportioned end spans. *AASHTO Live Load:* HS 20–44 (3-axle 32.66 T Truck per lane). AASHTO Design Specifications. Seismic activity at site: G/20. Wind intensity: normal, Neoprene bearings.

Span-layout Selection

Four arrangements for 'span layout' were chosen:

(i) 8 m-18.3 m-18.3 m-8 m ('4-span' arrangement)
(ii) 14.5 m-36.5-14.5 m... ('3-span' arrangement)
(iii) 22.9 m-22.9 m...... ('2-span' arrangement)
(iv) 45.7 m......... ('single-span' arrangement)

These four span-arrangements were considered to be the upper and lower limiting arrangements by which the same highway could be crossed, some span arrangements affording a 'lighter' look owing to additional end spans, and others creating a blunter look. (Some clients prefer the lighter look for aesthetics, even if the first-cost is higher.) Arrangements (iii) and (iv) can also permit a future widening of the highway at minimal cost.

Structure-Schemes

Scheme A: cast-in-situ R.C. beam-and-slab fully continuous deck.
Scheme B: cast-in-situ R.C. box-girder fully continuous deck.
Scheme C: precast p.s.c. I-girders with cast-in-situ diaphragms and deck slab, simply supported for these dead-loads, continuous for finishings and live load.
Scheme D: welded structural steel girders and cross beams with cast-in-situ R.C. slab, made composite for finishings and live load; each span simply supported (continuity would have meant either no shear-connectors in hogging moment zones, or, if shear connectors were provided there, then additional longitudinal reinforcement in such zones to take bending tension in order to ensure integrity of concrete in tension zone of continued composite action. In balance, only simple spans were therefore adopted).
Scheme E: cast-in-situ p.s.c. box girder fully continuous deck.
Scheme F: cast-in-situ p.s.c. box girder, simple span.

(i) in the '4-span' arrangement: Schemes A, B, C and D.
(ii) in the '3-span' arrangement: Schemes B, E, and D.
(iii) in the '2-span' arrangement: Schemes B, E, C and D.
(iv) in the 'single-span' arrangement: Scheme F only.

Main Quantities

The main quantities for various structure schemes for each of the four span-layouts (sub- and super-structure) are indicated in Table 42.1. In its last row are also shown the 'comparative' overall costs in proportion to that of the most economical alternative assumed equal to 1.00. (For unit prices, etc., see the next sub-section.)

Unit Prices, Comparative-Cost Picture, and Deck-Depths

Suffice it to say that prevalent all inclusive unit prices (as were relevant to the site at the time the investigation was made) were considered. Their individual values are of little significance in the discussion here. What is of significance here are the relative overall costs of the various alternative structural schemes expressed 'as proportions of the lowest-cost alternative'. The latter is shown as 100 per cent in Table 42.1A.

Conclusions

As can be seen from above, the most economical scheme here appears to be the '2-span' arrangement in R.C. box girder, fully continuous deck (i.e. in Scheme B), designated as 100 per cent. The '4-span' arrangement in cast-in-situ R.C. beam and slab fully continuous deck (i.e. in Scheme A) follows very closely with 100.9 per cent. This is followed by the '2-span' arrangement in the cast-in-situ p.s.c. box girder fully continuous deck (i.e. in Scheme E) with 102.7 per cent, a figure which is of some general interest since it shows that the depth of a structure can be reduced at not much additional cost. The variation in the relative position of the precast prestressed "I" girder (scheme C) in the 4-span and 2-span alternatives arises from the fact that the shorter prestressed units are relatively more costly due to the essentially certain fixed costs of prestressing operations. The composite welded steel girder type bridge (scheme D) shows a consistent differential greater than 20 per cent, a result which issues from the high cost of structural steel at the concerned site at the considered time.

Table 42.1

	4-span alternative				3-span alternative			2-span alternative				1-span alternative
Item	SCHEME A R.C. BEAM AND SLAB CONT-INUOUS	SCHEME B R.C. BOX GIRDER CONTI-NUOUS	SCHEME C PRE-CAST PRES-TRES-SED 'I' GIR-DERS MADE SEMI-CON-TINU-OUS	SCHEME D COM-POSITE WEL-DED STRUC-TURAL S.G. SIMPLE SPANS	SCHEME B R.C. BOX GIRDER CON-TINU-OUS	SCHEME E PRES-TRES-SED CAST IN SITU BOX GIRDER CON-TINU-OUS	SCHEME D COMP-OSITE WEL-DED STRUC-TURAL STEEL GIRDER SIMPLE SPANS	SCHEME B R.C. BOX GIRDER CON-TINU-OUS	SCHEME E PRES-TRES-SED CAST IN SITU BOX GIRDER CON-TINU-OUS	SCHEME C PRE-CAST PRES-TRES-SED 'I' GIR-DERS MADE SEMI-CON-TINU-OUS	SCHEME D COM-POSITE WEL-DED STRUC-TURAL STEEL GIRDER SIMPLE SPANS	SCHEME F PRE-STRESSED CAST IN SITU BOX GIRDER SIMPLE SPAN
Structure excavation (m^3)	142	147	147	142	178	178	171	82	81	70	70	33
Structure backfill (m^2)	77	71	71	77	92	90	86	77	73	62	62	47
Prestressed girder (No.)			10							10		
Prestressing Steel and operations (kg.)						2417			4456			10318
Class 'A' concrete (m^3)	317	314	268	222	468	152	236	298	105	201	197	83
Cast in place prestressed concrete (m^3)						314			191			274
Reinforcing steel (T)	45.5	53.5	39.7	22.8	79.4	36.8	25.0	43.6	20.9	24.9	20.5	27.0
Structural Steel including paint (T)				45.5			79.4				46.3	
Furnishing concrete piling (m)	408	408	408	384	658	658	610	518	518	449	449	375
Driving Piles (No.)	34	34	34	32	50	50	46	36	36	32	32	22
Barrier Railing (m)	122	122	122	122	150	150	150	114	114	114	114	144
Comparative overall 'costs'	1.009	1.041	1.175	1.255	1.491	1.524	1.741	1.000	1.027	1.076	1.232	1.220

Table 42.1A

	4-span alternative				3-span alternative			2-span alternative				single span alternative
Scheme	A	B	C	D	B	E	D	B	E	C	D	F
Girder depth (m)	1.22	1.07*	1.16	1.07	1.98	1.68	2.13	1.37	0.99	1.30	1.30	1.98
Comparative overall costs	100.9 %	104.1 %	117.5 %	125.5 %	149.1 %	152.4 %	174.1 %	100.0 %	102.7 %	107.6 %	123.2 %	122.0 %

*Practically it will be very difficult to construct a box section deck less than about 1.20 m deep, so, to that extent, Scheme B in this 4-span alternative is more of academic interest than of any practical value.

The results of the study also point out the cost effect of significantly increased span lengths. In the 3-span alternative, the main span was increased from 18.3 m to 36.5 m and costs increased by about 50%. With a problem of the kind under study, the effect of the removal of one support is lost in the much greater increased cost of the superstructure.

It is also apparent in the above case that, for example, an increase in pile lengths to 24 m would have increased the cost of all bridges, but the cost of the steel bridges would have increased at a slower rate since fewer piles are required in their foundations owing to lesser dead load. Conversely, shorter piles tend to decrease overall costs and favour the concrete bridges here. From this, it should be apparent that the precise cost relationships among the various types of bridges can be determined only by the kind of detailed study which gives due consideration to all such variations.

42.7 TYPICAL COMPARISON—CASE STUDY 2

Problem

To design in detail suitable structural alternatives for a right-bridge, crossing a wide and almost flat-bedded stream, and to compare their strategic dimensions, quantities and 'relative' costs per unit plan-area of deck. The stream is subject to minor seasonal floods, scour depth is small, open foundations on sandy strata possible. Deck width: 15.85 m. Clear roadway (Number of design lanes: 4). Soffit of deck: about 10 m above soffit of footing. Abutments: non-spill through type. Piers: either 5-column frame with a cap, or a slender shaft with a hammer head, or a wall type, resting on spread footing. Minimum length of bridge: about 51 m, but could even be about 55, 60, 75 or even about 90 m, end to end. Could even be much longer, in which case the above modules could be filled in various combinations as required and as feasible. Aesthetics: not any significant criterion as the bridge is located out in wilderness, away from any urban area. Low cost main consideration. Live Load : Saudi Ministry of Communications new 3-axle 600 KN Truck, per lane. AASHTO design specifications, Seismic activity nil. Wind intensity normal. Neoprene bearings.

Alternative 'Span-layouts' and 'Structure-schemes'

Seven arrangements were chosen.

Scheme A: R.C. fully continuous cast-in-situ constant depth slab spans, with column-frame piers, spans being:
7.75 m-10 m-10 m-10 m-10 m-7.75 m = 55.50 m

Scheme B: R.C. fully continuous cast-in-situ haunched slab spans, with hammer headed shaft piers, spans being:
15.25 m-20 m-15.25 m = 50.5 m.

Scheme C: p.s.c. fully continuous cast-in-situ voided slab-spans, with wall type piers, spans being:
15.25 m-20 m-15.25 m = 50.5 m

Scheme D: Similar to Scheme C, but spans being:
22.75 m-30 m-22.75 m = 75.5 m

Scheme E: p.s.c. fully continuous cast-in-situ box girder with wall type piers, spans being:
22.75 m - 30 m - 22.75 m = 75.5 m

Scheme F: 20 m precast p.s.c. girders with cast-in-situ R.C. diaphragms and deck slab, simply supported for these dead loads, and rendered continuous for finishings and live load (i.e. semi-continuous deck), with hammer headed shaft piers, spans being:
20.25 m - 20 m - 20.25 m = 90.50 m

Scheme G: Similar to Scheme *F* but with 30 m precast p.s.c. girders, spans being:
30.25 m-30 m-30.25 m = 90.50 m

Main Quantities

The main quantities for these seven structure-schemes are indicated in following Table 42.2. In it are also shown the 'comparative' overall costs per unit plan-area of the respective decks in proportion to that of Scheme G~ the most economical alternative (assumed as 100 per cent). Rather than local unit prices, the 'relative' costs per unit plan area of deck are of significance here: These are indicated in Table 42.2 and the inference is rather interesting. Also shown in the last row in Table 42.2 are the structural deck depths as a matter of interest to practitioners.

Conclusions

As can be seen from Table 42.2 the most economical scheme here appears to be Scheme G, consisting of 30 m precast p.s.c. girders simply supported for self-weight and that of cast-in-situ R.C. diaphragms and deck slab but rendered continuous over piers (by longitudinally appropriately reinforced deck slab) for finishings and live load, designated as 100%. This is followed by a similar deck arrangement, Scheme F (107%), which utilises shorter spans and a slightly smaller deck depth. Then follow, Scheme E (fully continuous cast-in-situ p.s.c. box girder, 117.6%) and Scheme A (fully continuous cast-in-situ R.C. slab spans of constant depth, 123%), However, the latter involves too many more foundations (apart from the crowding of columns) which is not helpful for subsequent maintenance activity. Schemes D and C (fully continuous cast-in-situ p.s.c. voided slabs of longer and shorter span modules, respectively) relative costs: 127.6% and 130.4%, and Scheme B (fully continuous cast-in-situ R.C. haunched slab-spans): 130%, appear to be the costliest in the present

Table 42.2

QUANTITY ESTIMATES								
DESCRIPTION	Unit	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E	Scheme F	Scheme G
Unclassified structural excavation	m^3	2480	2090	2015	2050	2050	2015	2050
Concrete in R.C. Substructure	m^3	1008	960	1025	1171	1171	966	977
Concrete in R.C. Superstructure	m^3	557	600	—	—	—	—	—
Concrete in p.s.c. Superstructure	m^3	—	—	532	978	664	439	742
Concrete in retaining walls	m^3	—	—	—	—	—	—	—
Elastomeric bearings	1000 cm^3	90	130	150	200	200	160	227
Metal hand railing	m	120	110	110	160	160	130	190
Reinforcement bars Grade 60	T	166	146	86	119	163	114	152
Prestressing steel (including anchorages, cost of ducts, cable making, stressing and grouting)	T	—	—	9.5	20.5	20.5	9.5	20.5
R.C. Slope protection	m^3	38	38	38	38	38	38	38
Expansion joint	m	33.6	33.6	33.6	33.6	33.6	33.6	33.6
Structural steel (minor)	T	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Cost in Saudi Riyals per m^2 of deck-plan area (*Riyadh—1984*)		1919	2028	2036	1992	1836	1670	1561
Comparative costs per unit deck-plan area compared to that in the lowest cost alternative (Scheme G)		123%	130%	130.4%	127.6%	117.6%	107%	100%
Structural deck depth (m)		0.56	0.60 increasing to 1.0 at piers	0.80	1.20	1.20	1.40 (beam : 1.15)	1.60 (beam : 1.38)

case. The structural scheme that was cheaper in case study 1 (i.e. fully continuous 2-span cast-in-situ R.C. box girder, each span 22.9 m) was in fact tried here also with span arrangements of (i) 15.25 m-20- m-15.25 m (= 50.50 m), and (ii) 25.25 m-25.25 m (= 50.50 m). The latter arrangement came very close (105%) to Scheme G, while the former was nor far (110%). However, precast p.s.c. girders had an edge in the present case owing to the relatively low price assumed for these girders, based on the centralised location of the precasting yard relative to the total length of the highway project and the location of various bridges in it where precast girders could be used.

42.8 QUANTITY TRENDS

42.8.1 Appreciation

Since prices of materials, wages of labour, operational costs of plant and equipment and even the margins for overheads and profits can vary, depending on incidence of inflation, availability of materials and personnel, logistics of site, availability of works, and scores of other factors (some of which have been outlined earlier), it is futile to talk in terms of any lasting 'cost' trends. Instead, 'quantity' trends have a better utility in practice. These too of course will depend on the design specifications (codes of practice) and road live load adopted and the incumbent magnitudes of seismicity, wind, flood velocity, scour depth at, foundations, safe bearing capacity of substrata at founding levels, properties of soil below design scour level and of the backfill, etc., which again can vary from site to site.

Given this back-drop, certain quantity-trends mentioned in the succeeding subsections (although they are based on a vast number of actually professionally designed bridges in commercial practice), must still be regarded essentially to be 'indicative' only, and that too valid for the particular loading and design specifications mentioned against them.

Having said this, ***it must however be clearly understood that such 'quantity figures' nevertheless represent a fund of practical information that is not found in straight textbooks, class-room lectures or in any research reports. Talking alone never pulled out any stumps!*** These 'figures are a treasure, and represent the essence of hundreds of engineering man-months of competitive hard-hitting, practice-oriented, computer-aided, commercially-biased professional analysis and the design of myriads of alternative bridge structure schemes, with many of which the author was personally involved first-hand (more as a doer, than as just a checker).

Summarised in the Succeeding Paragraphs are Certain "Quality-Trends" Arranged as Follows

... in subsection 42.8.2—For HS-20-44 (32.5^T *3-axle) AASHTO Truck per lane* ~

Item A. Continuous beam-and-slab R.C. decks:

... average concrete thickness and span to depth ratios for particular beam spacings, for various spans.

Item B. Continuous beam-and-slab R.C. decks:

... reinforcement in deck for different span to depth ratios for particular beam-spacings, for various spans.

Item C. Continuous box-girder R.C. decks:
... average concrete thickness and span-to-depth ratios for particular web spacings, for various spans.

Item D. Continuous box-girder R.C. decks:
... reinforcement in deck for different span-to-depth ratios for particular beam-spacings, for various spans.

Item E. Simply supported precast p.s.c. girders with cast-in-situ R.C. diaphragms and deck slab:
... girder depth, and quantities of concrete, reinforcement, and prestressing steel per girder, for various spans.

Item F. Simply supported cast-in-situ p.s.c. box section deck:
... prestressing steel per unit deck plan area for different span-to-depth ratios with particular web spacings, for various spans.

Item G. 2-equal spans continuous cast-in-situ p.s.c. box section deck:
... prestressing steel per unit deck plan area for different span-to-depth ratios with particular web spacings, for various spans.

Item H. 3-equal spans continuous cast-in-situ p.s.c. box section deck:
... prestressing steel per unit deck plan area for different span to depth ratios with particular web spacings for various spans.

Item I. 3-span continuous (0.75-1-0.75) cast-in-situ p.s.c. box section deck:
... prestressing steel per unit deck plan area for different span-to-depth ratios with particular web spacings, for various spans.

Item J. 4-equal spans continuous cast-in-situ p.s.c. box section deck:
... prestressing steel per unit deck plan area for different span-to-depth ratios with particular web spacings, for various spans.

Item K. 4-span continuous (0.75-1-1-0.75) cast-in-situ p.s.c. box section deck:
... prestressing steel per unit deck plan area for different span-to-depth ratios with particular web spacings, for various spans.

Item L. Simply supported composite steel girders with R.C. deck slab:
... structural steel per unit deck plan area for particular girder spacings, for various span to depth ratios, for various spans.

Item M. Concrete and reinforcement in non-spill through type abutments on spread footings or piles, for spans up to about 40 m (piles excluded):
... in cantilever type, counterfort type, hinged-to-deck type and rigid-frame type cases.

... in subsection 42.8.3 —for 3-axle 600KN Truck per lane (new Saudi loading), three design lanes roadways ~

Item N. Simply supported precast p.s.c. girders with cast-in-situ R.C. slab and only (half-depth) end cross beams (no intermediate cross beams, so that, where needed, the slab shutters move on trolleys that move on girder bottom bulbs, and hence a much speedier construction is possible), assuming static transverse load distribution among girders: 3 design-lanes carriageway:
... quantities of concrete, reinforcing steel and prestressing steel, in 15-, 20-, 25- and 35-m spans, respectively.

Item O. Concrete and reinforcement in cantilever type non-spill through abutment of different heights, (on footings or piles) for spans up to about 40 m (piles excluded):
... quantities of concrete and reinforcement per unit length of abutment for skew angles of 0° and 35°, and quantity of reinforcement per cubic content of concrete, for overall abutment heights of 5.5, 6.5, 7.5, 8.5, 9.5 and 10.5 m, respectively.

Item P. Concrete and reinforcement in cantilever type spill-through (2-columnar abutment of different heights, on footings or piles, for spans up to about 40 m (piles excluded):
... quantities of concrete and reinforcement in the abutment for skew angles of 0° and 35°, and quantity of reinforcement per cubic content of concrete, for overall abutment heights of 9.5, 11.5, 13.5, 15.5, and 16.5 m, respectively.

Item Q. Concrete and reinforcement in 2-columns pier of different heights, on footings or piles, for spans up to about 40 m (piles excluded):
... quantities of concrete and reinforcement in the pier for skew angles of 0° and 35°, and quantity of reinforcement per cubic content of concrete, for overall pier heights of 8, 10, 12, 14, 16, 18, 20 and 22 m, respectively.

Item R. Cantilever type retaining wall, retaining the roadway:
... quantities of concrete and reinforcement per unit length of wall and quantity of reinforcement per cubic content of concrete, for overall wall heights of 5, 7, 9, 11, and 13 m, respectively.

Item S. Cantilever type retaining wall, retaining sloped

embankment:
...quantities of concrete and reinforcement per unit length of wall and quantity of reinforcement per cubic content of concrete, for overall wall heights of 8, 10 and 12 m, respectively.

... in subsection 42.8.4 ~

Item T. p.s.c. butterfly deck, constructed by free cantilever method:
...some typical section sizes and quantities for a 2 design-lanes wide deck with footpaths, comprising 54.5 m free cantilever arms supporting 6.0 m riding (floating) spans, designed for British HA and 45 units of HB loading and Indian Class A and Class 70R loadings.

... in subsection 42.8.5 ~

Item U. Approximate quantities in certain important items of work in a regular riverbridge designed to I.R.C. Specifications (e.g. average thickness of concrete deck in R.C. and p.s.c.; reinforcement in R.C. deck slab, in p.s.c. and in R.C. decks, in pier caps, in pier shafts, in well-caps, in well-steinings, prestress-steel; steel structurals for well cutting-edges and strake panels.

... in subsection 42.8.6 ~

Item V. Approximate quantities in shutter form-work, in staging falsework and platform in trusses supporting cast-in-situ p.s.c. girders of up to about 35-m span, and quantities in travelling gantries used in cantilever construction of superstructure.

42.8.2 Items A to M, below, are based generally on following Design Parameters

1. design live load: AASHTO HS-20-44 3-axle truck of 32.5 T (or the specified lane load) per lane, No. of design lanes: 3.
2. design based on: AASHTO Specifications.
3. reinforcement bars: 2800 kg/cm^2 yield.
4. concrete: 210 and 400 kg/cm^2 in R.C. and p.s.c., respectively (28-day cylinder).
5. prestressing steel: 16600 kg/cm^2 minimum U.T.S.
6. structural steel: 2800 kg/cm^2 yield.
7. spacing between webs or girders: 2.1 to 3.2 m in the limit, but generally 2.1 to 2.6 m.
8. web width: 35 cm in R.C. beam-and-slab
   20 cm in R.C. box girder
   30 cm in p.s.c. box girder
   (web width flared near supports to twice the thickness).
9. wobble and curvature friction coefficients: 0.003/m and 0.25/rad. respectively.
10. bottom slab thickness in box girders: 14 cm generally, but flared to 30 cm between 0.9 span and 0.95 span and constant beyond.
11. Soil: • safe bearing capacity: 3 kg/cm^2 at founding level
    • angle of internal friction: 30° (non-cohesive)
    • angle of wall friction: 18°.
12. Seismicity: G/20 horizontal, ± G/40 vertical or wind of normal intensity.

NOTE : (i) Quantities of parapets, kerbs and wearing course excluded.
(ii) Span/depth ratio is associated with sum total of web width.

Item A. Continuous beam-and-slab R.C. decks (cast-in-situ)

Span lengths (m)	15	23	30
Span/depth	12.5	13.5	14
Av. thickness of concrete in deck (cm)	36.2	44.5	55.2

• Item B. Continuous beam-and-slab R.C. decks (Cast-in-situ)

Span lengths (m)	15				23				30			
Span/depth	15	14	13.5	12.5	15	14	13.5	12.5	15	14	13.5	12.5
Reinforcement bars in kg/m^3 of concrete in deck	178	172	165	156	159	154	150	145	150	145	—	—

• Item C. Continuous box girder R.C. decks (Cast-in-situ)

Span lengths (m)	15	23		30		38	
Span/depth	16.6	18.75	15	20	16.6	20.8	17.8
Av. thickness of concrete in deck (cm)	39.7	42.8	45.2	46.5	49	49.4	53

• Item D. Continuous box girder R.C. decks (Cast-in-situ)

Span lengths (m)	15		23			30				38		
Span/depth	15	16	15	17	20	15	17	18	20	17.5	18.8	20
Reinforcement bars in kg/m^3 of concrete in deck	174	184	182	195	221	184	198	212	228	203	217	224

• Item E. Simply-supported precast prestressed girders with Cast-in-situ R.C. diaphragms and deck-slab

Span (m)	12			18			24			30			36	
Girder* depth (m)	0.92	1.22	1.53	1.07	1.38	1.68	1.22	1.53	1.68	1.38	1.53	1.68	1.53	1.68
Concrete/girder (m^3)	4	4.6	5.9	6.3	7.6	9.1	8.9	10.7	11.8	12	13.1	14.3	15.6	16.8
Wt. per girder (T)	10	11.5	14.75	15.75	19	22.75	22.25	26.75	29.5	30	32.75	35.75	39	42
Reinforcement/girder (kg)	274	320	364	400	482	568	568	704	750	772	872	954	1067	1136
Reinforcement in kg/m^3 of girder concrete	68.5	69.6	61.7	63.5	63.4	62.4	63.8	65.8	63.6	64.3	66.6	66.7	68.4	67.6
Prestressing steel/girder (kg)	182	160	138	378	287	251	600	464	428	909	800	709	1250	1081
Prestressing steel in kg/m^3 of girder concrete	45.5	34.8	23.4	60	37.8	27.6	67.4	43.4	36.3	75.8	61.1	49.6	80.2	64.4

*Girder spacing = 2.10 m.

N.B: For reinforcement in R.C. deck slab, see ahead.

• Item F. Simply-supported P.S.C. box girder (Cast-in-situ)

Span (m)	30				38				45				53			
Span/depth	20	22	25	28.5	20	22	25	28.5	20	22	25	28.5	20	22	25	28.5
Prestressing steel in kg/m^2 of deck-plan area	12.5	13.5	15.7	18.1	14.7	15.9	18.1	20.3	17.6	19.1	21	23.5	21.5	23	25	27

• Item G. 2-equal spans continuous P.S.C. box deck (Cast-in-situ)

Span (m)	30					38					45					53					60				
Span/depth	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33
Prestressing steel in kg/m^2 of deck-plan area	10.9	12	13.8	15.9	22.2	13.4	14.4	16.3	18.8	24.2	15.9	17.3	19.2	21.7	26.6	18.8	20.3	22.7	25.7	29.5	21.7	23.1	26.1	29.1	33.4

• Item H. 3-equal spans continuous P.S.C. box deck (Cast-in-situ)

Span (m)	30					38					45					53					60				
Span/depth	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33
Prestressing steel in kg/m^2 of deck-plan area	11.4	12.8	13.8	15.8	22.6	13.3	14.4	16.3	18	23.6	15.3	16.7	18.2	20.7	25.4	18	19.2	21.2	23.1	27.6	21	22.7	24.2	26.6	31.5

• Item I. 3-span continuous (0.75-1-0.75) P.S.C. box deck (Cast-in-situ)

Central Span (m)	30					38					45					53					60				
Span/depth*	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33
Prestressing steel in kg/m^2 of deck-plan area	7.9	8.1	10.4	11.9	—	9.4	10.4	11.9	13.8	—	11.9	12.8	13.8	16.3	19	13.9	15.4	16.3	18.3	21.7	16.8	17.8	18.8	20.7	24.7

*for central span; same depth in outer spans.

• Item J. 4-equal spans continuous P.S.C. box deck (Cast-in-situ)

Span (m)	30					38					45					53					60				
Span/depth	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33
Prestressing steel in kg/m^2 of deck-plan area	10.6	11.9	13.3	15.8	—	12.8	14.4	15.8	18.8	—	15.4	16.8	18.8	21.5	25.6	18.2	19.8	21.7	24.6	19.3	21.2	23.2	25.1	28.1	33

• Item K. 4-span continuous (0.75-1-1-0.75) P.S.C. box deck (Cast-in-situ)

Interior Spans (m)	30					38					45					53					60				
Span/depth*	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33	20	22	25	28	33
Prestressing steel in kg/m^2 of deck-plan area	9.4	10.5	11.5	12.9	—	12.5	13	14.4	15.8	—	15.4	16.3	17.3	19.3	21.2	18.5	19.3	20.7	22.7	25	22.2	23.2	24.2	26.6	29.5

*for interior spans; same depth in outer spans.

- Item L. Simply supported structural steel girders composite with R.C. slab

Span (m)	18	21	24	27	30	36	42
Span/overall depth	14	14	14	16.5	16.5	20	20
Structural steel in kg/m^2 of deck-plan area	113	123	133	142	152	182	226

- Item M. Non-spill-through R.C. abutments on footings or piles, for spans up to about 40 m (piles excluded)

Overall Height (m)	Cantilever type		Counterfort type		Hinged to deck type		Rigid frame type (abutment leg + footing)			
	Reinfmt. *	Concrete **	Reinfmt.	Concrete	Reinfmt.	Concrete	Reinfmt.		Concrete	
							18 m span span/depth = 14.7	30 m span span/depth = 20	18 m span span/depth = 14.7	30 m span span/depth = 20
3.0	239 (58.3)	4.1	—	—	209 (83.6)	2.5	—	—	—	—
4.5	417 (78.7)	5.3	551 (62.6)	8.8	298 (82.8)	3.6	—	—	—	—
6.0	655 (80.9)	8.1	640 (65.3)	9.8	417 (74.5)	5.6	—	—	—	—
7.5	953 (75)	12.7	760 (65)	11.7	640 (70.3)	9.1	566 (72.6)	625 (60.7)	7.8	10.3
9.0	—	—	953 (67.6)	14.1	—	—	633 (68.1)	715 (55.5)	9.3	12.9
10.5	—	—	1132 (69)	16.4	—	—	700 (63.1)	790 (52.3)	11.1	15.1

NOTE 1. *'*Reinforcement*' is in kg/metre length of abutment.
2. **'*Concrete*' is in m^3/metre length of abutment.
3. *Figures within brackets* are reinforcement in kg/m^3 of concrete.
4. Cantilever returns and footings (pile caps) included in averaging.

42.8.3 Items N, O, P, Q, R and S below, are based generally on the following Design Parameters

1. design live load: 3-axle 600 KN truck, or the specified lane load, per lane, (the new loading of the Saudi Ministry of communications), 3 design lanes carriageway of 12.6 m. betweer kerbs.
2. design based on AASHTO specifications.
3. reinforcement bars: 4200 kg/cm^2 yield.
4. concrete: 210 and 400 kg/cm^2 in R.C. and p.s.c. respectively (28-day cylinder).
5. prestressing steel: 16600 kg/cm^2 minimum U.T.S.
6. spacing between precast p.s.c beams: 2.11 m for 15-, 20- and 25-m spans, 1.91 m for 35 m span.
7. precast p.s.c. beams and c.i.s. R.C. slab (as per table below).
8. wobble and curvature friction coefficients: 0.003/m and 0.25/rad., respectively.
9. Soil: • safe bearing capacity: 3 kg/cm^2 at founding level.
   • angle of internal friction: 30° (non cohesive).
   • angle of wall friction: 18°.
10. Seismicity: G/20 horizontal, ± G/40 vertical or wind of normal intensity.
11. Abutments and piers on R.C. footings or piles.

NOTE (i) Quantities of parapets, kerbs and wearing course excluded.
(ii) In piers and abutments, quantities of R.C. footings or pile caps are included in the averaging, but piles are excluded. Quantities of R.C. cantilever returns are included in the abutments.

Simple span (m)	Precast p.s.c. beams			R.C. slab c.i.s. over beam flanges : thickness (cm)
	depth (m)	btm. bulb width	top flange width	
15	1.10	60	2.10	22
20	1.30	60	2.10	22
25	1.50	60	2.10	22
35	2.40	70	1.00	22

• Item N. Simply supported precast p.s.c. girders with cast-in-situ R.C. slab and only (half-depth) end cross beams (no intermediate cross beams, so that where needed, the slab shutters move on trolleys that move on girder bottom bulbs, and hence a much speedier construction is possible), assuming static transverse load distribution among Girder; 3-design lanes carriageway of 12.60 m clear roadway width between kerbs:

Span (m)	Concrete		Reinforcement bars			Prestressing Steel		
	Qtty. (m^3) (one span)	m^3/m^2 of deck plan area i.e. av. thickness of deck (m)	Qtty. (T) (one span)	kg/m^3 of deck concrete	kg/m^2 of deck-plan area	Qtty. (T) (one span)	kg/m^3 of deck concrete	kg/m^2 of deck-plan area
15	117	0.619	18.8	161	100	3.0	25.7	16
20	166	0.659	27.2	164	108	5.5	33.2	22
25	215	0.683	34.8	162	111	7.8	36.3	25
35	344	0.780	54.5	159	124	15.0	43.6	34

• Item O. Concrete and reinforcement in cantilever type non-spill-through abutment of different heights (on footings or piles), for spans up to about 40 m (piles excluded):

Overall ht. of abutment above foundation level (m)	Concrete m^3/m length of abutment	Reinforcement bars	
		kg/m length of abutment	kg/m^3 of concrete
'1'	'2'	'3'	'4'
5.5	8.2	615	75
6.5	10.1	788	78
7.5	11.8	968	82
8.5	14.3	1230	86
9.5	16.7	1503	90
10.5	20.1	1870	93

• Item P. Concrete and reinforcement in cantilever type spill-through (2-columnar) abutment of different heights, on footings or piles, for spans up to about 40 m, (piles excluded); 12.6 m clear roadway width between kerbs:

Overall height of abutment above foundation level (m)	Concrete (m^3)	Reinforcement bars	
		Qtty. (kg)	kg/m^3 of concrete
'1'	'2'	'3'	'4'
9.5	191	13,500	71
11.5	240	18,200	76
13.5	298	24,138	81
15.5	364	30,940	85
16.5	399	34,314	86

NOTE (i) Table shows quantities in one Abutment for 0° skew case.
(ii) For 35° skew, quantities in columns '2' and '3' increase by about 20%.

• Item Q. Concrete and reinforcement in a 2-columnar pier of different heights, on footings or piles, for spans up to about 40 m (piles excluded); 12.6 m clear roadway width between kerbs:

Overall height of pier above foundation level (m)	Concrete (m^3)	Reinforcement bars	
		Qtty. (kg)	kg/m^3 of concrete
'1'	'2'	'3'	'4'
8	125	10,000	80
10	153	12,200	80
12	194	15,500	80
14	229	18,300	80
16	267	21,400	80
18	306	24,500	80.
20	349	28,270	81
22	397	32,158	81

NOTE (i) Table shows quantities in one pier for 0° skew.
(ii) For 35° skew, quantities in columns '2' and '3' increase by about 20%.

• Item R. Cantilever type retaining wall, retaining the roadway (surcharge effect of live load considered):

Overall height of retaining wall above foundation level (m)	Concrete m^3/m length of wall	Reinforcement bars	
		kg/m length of wall	kg/m^3 of concrete
5	3.1	216	70
7	5.9	510	87
9	9.3	890	96
11	13.8	1370	99
13	18.9	1926	102

• Item S. Cantilever type retaining wall, retaining 3H : 2V sloped embankment:

Overall height of retaining wall above foundation level (m)	Concrete m^3/m length of wall	Reinforcement bars	
		kg/m length of wall	kg/m^3 of concrete
8	10.3	1070	104
10	15.1	1650	109
12	21	2350	112

42.8.4 Item T: Prestressed Concrete 'Butterfly' Deck, i.e. Double Cantilever Deck, Constructed by Free Cantilever Method, Segment-by-Segment:

Some typical sections and quantities for a 2-lane bridge deck (7.50 m carriageway width between kerbs), with two footpaths (1.50 m each), comprising 54.5 m free cantilever arms supporting 6.0 m riding (floating) spans, designed for Indian Roads Congress Specifications and I.R.C. Class A and Class 70R loading and British (HA and 45 units of HB) loadings designed to U.K. D.O.E. Specifications. Concrete grade: 425 kg/cm^2 (28 day *cube*). Reinforcement bars: Grade 60 (4200 kg/cm^2 yield) and Grade 40 (2800 kg/cm^2 yield). Prestressing steel: 16300 kg/cm^2 minimum U.T.S., wobble and curvature friction coefficients 0.003/m and 0.25/rad., respectively. Quantities of parapets, kerbs and wearing course excluded:

I. Typical Deck Section Sizes Single cell 4.50 m overall width structural box depth: approx. 8.0 m deep at cantilever root and 2.0 m at tip. Web thickness: 250 mm at cantilever tip increasing to 350 mm at cantilever root in two steps of 50 mm at suitable intermediate sections (thickening affected on inner side). Soffit slab thickness: 200 mm at cantilever tip increasing to 600 mm at cantilever root, thickness changing linearly within a segment in whichever segments required, constant in others. Thickness details of deck slab (top slab) retained unchanged throughout.

II. Quantities of Reinforcement Bars

(i) *From consideration of transverse bending (box section)*
- *in webs:*
  Grade 60 vertical bars ~
  80 kg/m^3 of concrete in webs.
  Grade 40 longitudinal bars ~
  20 kg/m^3 of concrete in webs.
- *in top slab (including in the cantilevering footpaths):*

Grade 60 transverse bars — 235 kg/m run of bridge length.
Grade 40 longitudinals — 110 kg/m run of bridge length.
— *in soffit slab:*
Grade 60 transverse bars — 40 kg/m^3 of soffit slab concrete.
Grade 40 longitudinals — 20 kg/m^3 of soffit slab concrete.

(ii) *Additional reinforcement against shear and torsion (longitudinal design):*
Grade 60 — approx. 50 kg/m^3 of concrete in webs.

(iii) *Overall, total reinforcement bars in terms of Grade 60 steel*
120 kg/m^3 of total structural deck concrete, for preliminary estimate.

III. For the same grade of concrete and same average deck thickness, the initial prestressing force required at root of cantilever

(i) 7500 T if designed to Indian (IRC) loadings stated above (62 Nos. 12/0.5 inch Freyssi multistrands), and
(ii) 9200 T if designed to British loadings stated above (56 Nos. 12/0.6 inch Freyssi multistrands).
(iii) Corresponding *quantities of prestressing strand steel are 55 kg/m^3 of concrete in one double cantilever butterfly deck for the Indian loading and 74 kg/m^3 for the British loading.*

IV. Total dead weight of one butterfly (i.e. double cantilever) deck; including wearing course, parapets, kerbs, and lighting posts

2500 T approx.

V. Quantity of structural concrete (425 kg/cm^2 28-day cube strength):

— in one butterfly (i.e. double-arm) deck including footpaths: 765 m^3 approx.
— in its soffit slabs alone 235 m^3
— in its webs alone 250 m^3

VI. Average thickness of deck i.e. quantity of deck concrete in m^3/m^2 of deck-plan area:

— based on 7.50 m 'clear roadway width' as the plan width, ... 94 cm.
— based on approximately 10.8-m 'overall plan-width of deck', ... 64 cm.

42.8.5 Item U: A Normal River Crossing Bridge with Caisson Foundations, Designed to Indian Roads Congress Specifications:

Particulars

- 2-lane 7.5-m clear carriageway, with 1.5-m wide footpaths on either side.
- design live load: (2 lanes of) Class A or (1 lane of) Class 70-R Indian Roads Congress (I.R.C) Loading, and 400 kg/m^2 footpath loading (footpath itself capable of withstanding an isolated point load of 4T on 30 cm diameter contact area).
- structural design to the I.R.C. Specifications.
- soffit of deck: about 6 m above average river-bed level.
- seismicity: G/20 horizontal coefficient, and
  $\pm$ G/40 vertical coefficient.
- wind: of normal intensity.
- flood velocity: 4 m/sec. (max. mean).
- flood depth: 16 m (to maximum scour level).
- abutments and piers on caisson foundations.
- caisson (under pier) taken to about 7 m below max. scour level.
- safe bearing capacity of soil at founding level: 40 T/m^2.
- angle of internal friction of soil below maximum scour level: 30° (and skin friction ignored).
- reinforcement bars: 4200 kg/cm^2 (yield).
- prestressing steel: 16,000 kg/cm^2 ultimate tensile stress.
- wobble and curvature friction coefficients: 0.003/m and 0.25/rad., respectively.
- 28-day cube strength of concrete:
  — in deck 'longitudinally' or 'longitudinally and transversely' prestressed ... 350 kg/cm2.
  — in deck entirely in R.C. 210 kg/cm^2
  — in R.C. piers 250 kg/cm^2.
  — in caissons ... ranges from a minimum of 110 kg/cm^2 in plain concrete case (still having a certain minimum amount of reinforcement) to 160 kg/cm^2 in R.C. case.

Approximate Quantity Trends

I. Average 'thickness' of concrete in deck (based on 'overall plan width of deck, including footpaths)

1. 3-girder beam-and-slab, cast-in-situ, longitudinally in p.s.c., transversely in R.C. ... 48 to 60 cm for 30 to 45 m simple spans.
2. 3-webbed cast-in-situ R.C. box girder ... 45 cm for 20 to 25 m simple spans, and 48 to 56 cm for balanced cantilever spans and continuous spans of 30 to 40 m.

II. Reinforcement bars

1. in R.C. deck slab of longitudinally prestressed deck	180 kg/m^3 of deck slab concrete
2. in R.C. deck slab of longitudinally only reinforced deck (average thickness of concrete in deck is more than in p.s.c case.)	120 kg/m^3 of deck slab concrete
3. if deck is prestressed longitudinally as well as transversely	55kg/m^3 of total concrete in deck
4. in R.C. box section superstructure- -for simple spans 20 to 25m	220 kg/m^3 of total concrete in deck
—for balanced cantilever and continuous spans of about 30 to 40 m range	260 kg/m^3 of total concrete in deck
5. in parapet beams (spanning between the tips of cross beams)	400 kg/m^3 of parapet beam concrete
6. in R.C. pier caps and abutment caps	230 kg/m^3 of respective cap concrete
7. in R.C. pier shafts and abutment shafts: • solid type 100 • cellular type 120	kg/m^3 of pier/ abutment shaft concrete
8. in R.C. caps of caissons	90 kg/m^3 of cap concrete but varies with diameter of caisson.
9. in caisson steinings: • in thicker plain concrete steinings...	13 kg/m^3 of steining concrete (minimum)
• in thinner steining designed in R.C....	depends on design
10. in caisson-curbs (well-curbs)	80 kg/m^3 of curb concrete (minimum)

III. Prestressing Steel in Deck

1. For 30 to 45 m simple spans, 3-girder section, prestressed only longitudinally ... 40 to 50 kg/m^3 of total concrete in deck.
2. Prestressing steel for transversely prestressing the deck slab and cross beams ... 50 kg/m^3 of concrete in deck slab and cross beams.

IV. Structural Steel (2800 kg/cm^2 yield) in Caissons

1. For cutting edge of caisson
   - in Angle-and-Plate type assembly ... 50 to 85 kg/m length of cutting edge depending on type of substrata.
   - in V-shaped-plate assembly ... 160 kg/m length of cutting edge (used in hard-sub-strata).
2. For strake plates including angle stiffeners, etc. (armour plates on inner side of curb and for up to 2m on inner side of steining above curb, and for part-height on outer side of curb):
   - on inner surface of well-curb ... 80 kg/m^2 of that surface
   - on inner surface of well-steining ... 45 kg/m^2 of that surface
   - on outer surface of well-curb ... 50 kg/m^2 of that surface

42.8.6 Item V. Quantities of Structural Steel in Certain General Construction Items

1. For shutter form-work: • p.s.c. girder webs	107 kg/m^2 of surface to be shuttered.
• of deck slab	45 kg/m^2 of soffit surface area.
2. For building up staging falsework from bed-level.	11 to 14 kg/m^3 of volume of air-space under deck-casting platform.
3. For making the deck-casting platform	42 to 45 kg/m^2 of its surface area.
4. For trusses for supporting the staging falsework for casting the decking in situ, over a span of about 35m.	90 T for a 3-girder decking (2-lane deck), or about 290 kg/m^2 casting-platform plan-area.
5. For 2 travelling gantries needed for casting the box deck of a 2-lane deck by free cantilever method.	2 @ 40 = 80T approx.

CHAPTER 43

Guidelines for Professionally Preparing and Submitting 'Detailed Structural Analysis and Design Calculations' and 'Construction Drawings' for Client's Counter-checking and Record

1. After preparing the preliminary design and outline drawings and completing the necessary investigations, it is essential to carry out detailed design calculations and prepare detailed construction-drawings, both for cross-checking as well as official records prior to taking up the actual construction work.
2. These calculations must be prepared in a step-by-step workmanlike manner and submitted along with fully detailed working drawings after through checking at the designer's end. The following guidelines may be followed:

A. Analysis and Design 'Calculations'

A.1 *Give a summary write-up about:*
- — the type of bridge,
- — span arrangement and spans,
- — cross-sections of foundations and deck,
- — method of 'construction' and 'loading-sequence',
- — types of bearings used and their locations,
- ... giving simple explanatory sketches.

A.2 ***State the design 'loading' and maximum number of 'design lanes' possible to be loaded. Also mention the following:***
- — the design criteria adopted,
- — the materials assumed,
- — the codes of practice followed,
- — the 'normal permissible' stresses adopted in various materials,
- — the soil characteristics assumed for back-fill as also at the founding-level as well as of the soil mass between the maximum scour level and founding-level for each foundation (as applicable),
- — differential settlement of supports considered (if any),
- — wind and seismic intensities assumed,
- — temperature range considered for global variation,
- — differential temperature gradient assumed across deck depth (and across abutment-wall in case of framed structure), etc.

A.3 *Give details about:*
- — 'maximum' scour levels at piers and abutments (as applicable),
- — normal bed (ground) levels,
- — 'low water' and 'high flood' levels and afflux (LWL, HFL, AHFL),
- — 'normal safe bearing capacity' of substrata at various founding levels under 100% or 50% buoyancy conditions (depending on if substrata below foundations is soil or good rock),
- — In case of prestressing, state complete assumptions. e.g. type of tendon*, μ and K values (curvature and wobble friction coefficients), minimum bending radius, ultimate tensile stress, section area and ultimate tensile force per tendon, E_s value, location of cables in section at various sections, cable-profiles in plan and in elevation, etc. together with the calculations for friction, position of 'null-point', jacking pressures and or stressing-end forces in cables, extensions, slip effect, and 'initial primary prestress'' horizontal force and its moment and shear values at each section. Also give the assumptions made in the calculation of losses in prestress; what portion of which loss acts on what section and on what span-arrangement must be clearly enumerated?

'Precamber', necessary to be given (in reverse direction) at various sections, must be calculated and presented, and clearly shown on the drawings and as to with respect to what profile? This precamber information is necessary in order to counter the expected long-term deflection at various sections

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* It is advisable to assume a suitable but a definite system of prestressing and prepare the "execution" drawings in full detail, rather than not assume any definite prestressing system and give only values of initial prestressing force at various sections (after slip and friction) and its profile. The contractor can still be given the choice for his "alternative" by saying "or equivalent".

under all permanent loads and final prestress and should be computed on E_c value as reduced by creep, etc.

A.4 ***If any Computer Programs are used, describe***

- their brief details and salient features including the assumptions made in the mathematical modelling,
- the input data required and the details of its printout (output),
- what idealisation necessary; for instance if a 'plane-grid' is adopted then give:
  - a representative sketch showing this grid, the coordinate axes and the coordinates to each joint,
  - show 'joint' and 'member' numbers in it,
  - work out and state the section properties allotted to various members,
  - clearly show various loading 'cases' and loads 'apportioned' to each member/joint under each loading case... this is most important!

A.5 ***Presenting all the calculations***

This should be done in a methodical step-by-step, coherent and indexed manner, giving computations for each element; e.g. deck (longitudinals, cross beams, deck slab, end-blocks, etc.), bearings, abutments (stability and design), piers (stability and design), etc. for all incumbent loading cases as also for each construction stage, including for 'one-span dislodged' condition if such dislodging is possible.

A.6 ***At the end of 'each' analysis, the values of various stress resultants***

i.e., shears, moments, thrusts, torques, reactions, etc. and in case of prestressing, the initial and final values of primary and secondary prestress moments, shears, thrusts and reactions *must be clearly tabulated before proceeding with 'design'.*

B. Detailed Drawings

Show 'general arrangement' of the 'complete structure' as well as 'individual details' of 'each' structural component in the same serial as the actual construction will require! Drawings should show various details including the following.

- the materials and types of cement and steel used,
- the concrete strengths and their values at various stages of construction,
- the proposed founding levels and caution about actual founding levels,
- 'normal safe soil bearing capacity' at the proposed founding levels, stating buoyancy conditions as applicable,
- 'cover' to reinforcement in different members,
- details of structural-welding, shear-connectors, stiffeners, etc.,
- 'type and 'location' of each bearing and values of 'vertical load' on it under various load combinations together with the 'associated horizontal forces, amount of movements and rotations in orthogonal directions;—
- 'type' and 'details' of prestressing tendons (including: profiles, E_s and ultimate tensile stress/force values, required jacking-pressures and or jacking forces in cables at their stressing-ends, extensions, slips and their 'limiting' values; whether stressing from both ends or one end only; 'sequence of prestressing' and minimum concrete strength at each stage of construction; precautions while stressing the cables (details given in the chapter on 'construction considerations' in the author's book *Concrete Bridge Practice: Construction, Maintenance and Rehabilitation*) bar-bending schedules. In detailing, take care about: shape and 'development length' of shear stirr-ups and torsion hoops, bar cut-off locations and length of laps in primary reinforcement, 'bundling' of primary reinforcement in and near webs, spacing of bars from considerations of crack control, minimum reinforcement details, special reinforcements at bearing locations and cut-outs and around prestressing anchorages, etc.),
- explain the system adopted for reinforcement-nomenclaturing,
- profile-ordinates for precamber to deck soffit,
- Design live load and codes of practice followed,
- reference to 'Special Specifications' and 'General Notes for construction'.
- any other relevant detail.

CHAPTER 44

Engineering Drawings and Working Drawings

44.1 ENGINEERING DRAWINGS

General

Engineering drawings are those prepared by the Engineer for the Owner. The engineering drawings and the structural specifications form a part of Contract Documents. Engineering drawings should contain an adequate set of notes and all other essential information in a form that can be quickly and correctly interpreted. These drawings should convey definite instructions and show reinforcing bars and welded wire fabric. Engineering and working drawings may be combined.

The responsibility of the Engineer is to furnish a clear statement of design requirements; the responsibility of the Detailer is to carry out these requirements. The Engineer, in either his specifications or drawings should not merely refer the Detailer to an applicable building code for information to use in preparing placing drawings. Instead, this information should be interpreted by the Engineer and shown by him in the form of specific design details or notes for the Detailer to follow. The Engineer should require in his specification that working drawings be submitted for his approval.

Materials

The minimum standard media for production of engineering drawings shall be pencil on tracing paper. Other media providing improved reproducibility or durability such as ink, tracing cloth, or polyester film may be utilized.

Sizes

Drawings should be made in standard sizes. All sheets in any one set of drawings should be the same size. There are well-recognized sets of standard sizes, approximating to ~

18 × 24 in.	(45 × 60 cm.)
24 × 36 in.	(60 × 90 cm.)
27 × 36 in.	(70 × 90 cm.)
34 × 44 in.	(85 × 110 cm.)

All dimensions are to cutting line outside of margin. Border lines are inside of this.

Direction

An arrow indicating the direction of north should be placed beside every plan that concerns layout.

Scales

The scales used should be indicated on all engineering drawings, preferably under the title of each view. Drawings that may be enlarged or reduced in reproduction should show a graphic scale as well as a descriptive one to aid the user.

Lettering

All lettering shall be clear, legible and for normal full-size prints, not less than 1/8 in. high. If small-scale photographic prints are to be made for field use, lettering must be correspondingly larger, and meet microfilming standards.

Additional Details

Engineering drawings and/or specifications for elements like beams, girders, columns, walls, and foundations shall show type and grade of steel, service live load, concrete strength, concrete dimensions, class of tension splice or lap lengths, concrete cover for the reinforcement, required joints, and any other information needed for the preparation of reinforcement working drawings.

Dimensions

Since the engineering drawings for highway structures usually are a combination of engineering and working drawings from which the structure will be built, all dimensions should be clearly shown. Drawings must show the exact dimensions of concrete protection for all reinforcement. Where separate working drawings are prepared, structural dimensions may be omitted following the same practice as for buildings.

Reinforcement

Combination 'engineering-cum-working' drawings must show the size, spacing, and location of the bars and welded wire fabric in the structure. The list of bars must show the number of pieces, size, length, mark of bars, and bending details of all bend bars.

Reinforcement of larger structures is sometimes detailed, fabricated, and delivered by units, for example, footings, abutments, piers, and girders. The bar list may be similarly subdivided. If the structure is sufficiently large, a separate drawing and bar list is usually made for each unit.

Reinforcing bars for foundations, piers, abutments, wing-walls, and slabs are usually shown on the plan, section, or elevation. Cross-sections must be provided for clarification where necessary. The bar list must be a complete summary of materials required.

Stirrup Anchorage

The Engineer must show or specify by notes the sizes, spacings, location, and types of all stirrups. These types include open stirrups and closed stirrups (or stirrup-ties).

There are various permissible methods of anchorage, but the most common is to be use one of the standard stirrup-tie types.

Where stirrup support bars are required, they must be specified, by the Engineer. In designing the anchorage, allowance must be made to ensure that the ends of the stirrup hook are fully encased in concrete, as when hooks turn outward into shallow slabs.

Where the design requires closed stirrup-ties for vertical shear, then the closure may consist of overlapped standard 90-deg end hooks of one-or two-piece stirrups, or properly spliced pairs of U-stirrups. Where design requires closed ties for torsion, the closure may consist of overlapped standard 135-deg hooks of one- or two-piece ties enclosing a longitudinal bar. At least one longitudinal bar must be located at each interior corner of the stirrups or ties; the size of this bar should be equal to at least the diameter of the stirrup (12 mm. minimum). These details should be indicated by the engineer. He must also show the general arrangement of all such bars and stirrups.

Bundled Bars

When bars are placed in contact with each other in groups of two, three, or four—known as "bundledbars"—the minimum clear space provided between bundles must be equal to 1.5 times the diameter of a single round bar having an area equivalent to the area of the bundle.

Splices

General

In beams or girders that require bars longer than can be carried in stock, splices must be specified. The Engineer must show or specify by notes how the splicing is to be realized, viz. lapping, welding, or mechanical connections.

The Engineer must also show by details on engineering drawings the location of all splices. In beams or girders, splices should preferably be made where the stress in the bar is minimum, i.e., at the point of inflection. Splices where the critical design stress is tensile should be avoided by the Engineer wherever possible. Lapped bars may be either in contact or separated. The Engineer should show or note on the drawings whether splices are to be staggered or made at the same location. Bars to be spliced by noncontact lapped splices in flexural members shall not be spaced transversely more than 1/5 the length of lap nor 6 in.

Lap Splices

Since the strength of a lap splice does not increase directly with the length of lap, and varies with bar diameter, concrete strength, position of the bar, distance from other bars, and type of stress (compressive or tensile), it is necessary for the engineer to show location of all splices, and to indicate by "C" or "T" whether tension or compression controls. If tension controls, he should indicate class of splice required and whether it is "top" or "other." Preferably he should dimension each splice. Where bars of two sizes are lap spliced under the design criteria of the preceding sentence, the Detailer will use appropriate tensile lap splice for the smaller bar, unless otherwise noted.

At column bar splice points, sufficient bars (or dowels) from the lower column must extend into the upper column to provide not less than the sectional area of the bars in the upper column. These bars must extend the minimum distance required for column splices. At least four bars should be so extended. The Engineer should note that unless otherwise specified or shown on engineering drawings, the Detailer will detail the remaining bars in the lower column extending to within 3 in of the top of the floor or other member transmitting the additional load to the column. Where the tops of column bars are less than 2 m above the top of footings or pedestals, the bars should extend into the footing or pedestals. Normally, dowels will be used only if specifically noted on engineering drawings.

Dowels for lap splices at column offsets should have a cross-sectional area at least equal to that of the bars above and they must extend both above and below the splice points, as shown or specified by the Engineer.

The Engineer should also be aware that it is a standard practice in the industry when detailing column verticals to use the appropriate lap splice length for the bars in the column above. This applies regardless of differences in sizes.

For column bars at a lapped splice it should be noted that the amount of offset of the bars is greater for rectangular columns than for round columns. Column verticals to be lap spliced in square or rectangular columns, where column size does not change, are usually shop offset bent into the column above, unless otherwise shown by the Engineer. Column verticals in round columns where column sizes do not change must be offset bent only if a maximum number of lap spliced bars is desired in the column above, but the Engineer must indicate which arrangement is desired.

Where the depth of the footing, or footing and pedestal combined, is less than the minimum length of embedment required for dowels of a certain size, the size of dowel should be decreased and the number of dowels should be increased to give equivalent area and shown on the engineering drawings. Hooks at the bottom of the bars may be desirable to resist tension, but the hook may not be considered in determining the embedment provided for compression.

Butt Splices

Full welded splices or mechanical connections may be specified or, for compression only, end bearing splices may be specified as butt splices for vertical column bars. For 40 mm (and above) bars, butt splices must be used. Special preparation of the ends of the vertical bars is usually required for butt splices. Where bars are welded, the most common practice is to provide a square-cut end at the top of the lower bar and a double-beveled end on the bottom of the upper bar. Field preparation of ends by flame cutting is satisfactory. Where a mechanical splice is used, both ends of the bar may be either square-cut or standard-shear cut, depending on the type of connection used. Since the points of splices are usually staggered between alternate vertical bars and the splice location depends upon the design requirements, the Engineer must indicate the types of splices permissible, their location, and end preparation required.

Connection Details

Rigid Frame Corners

In designing the corner connection of a rigid frame, care must be taken, particularly in providing full continuity around as large a uniform radius as possible, keeping the centre within the concrete. This point is important in splicing the top bars from the girder to the outside bars in the column. The engineer must provide complete information, showing radius of bend and location, and dimensions of lap splices. If welding or a mechanical splice is to be used, a full description must be provided.

Wall Intersections and Corners

When horizontal wall reinforcement is required by design, all the horizontal bars in one, or sometimes both, faces of a wall should be sufficiently extended past a corner or intersection to be fully developed.

The engineer must indicate which, if any, horizontal reinforcement shall be extended, how far it shall be extended, and how it shall be anchored at intersections and corners of walls and footings. In areas where the applicable building code requires earthquake-resistant design, standard practice requires anchorage of all horizontal bars.

Walls with loads that open corner intersections must be reinforced differently from walls with loads that close such intersections.

Closed Stirrups

Where the engineering drawings show closed stirrups, these stirrups may be closed by two-piece stirrups using overlapping standard 90 deg end hooks enclosing a longitudinal bar, or by properly spliced pairs of U-stirrups or a standard one-piece stirrup-tie. At least one longitudinal bar must be located at each corner of the section, the size of this bar to be at least equal to the diameter of the stirrup but no less than 12 mm. These details must be shown by the Engineer. It should be noted that the use of 90-deg hooks and lapped splices in closed stirrups is not considered effective in situations where the member is subjected to high torsional stress. Tests have shown premature failure due to spalling of the concrete cover and consequent loss of anchorage in the 90-deg hooks and lapped splices in these situations.

Avoiding Bar Congestion at Joints

It is important for the Engineer to study bar layouts carefully and give the detailer the proper information. This study will point up congestion at beam-column joints of beam bars, column bars, and joint hoops. A large-scale drawing or a model or mock-up of the joint details may be worthwhile to assure that a design can be assembled and concrete can be placed.

44.2 WORKING DRAWINGS

Definition

Working drawings are working drawings for the fabrication and placing of reinforcing steel. These drawings may comprise bar lists, schedules, bending details, placing details, and placing plans or elevations. They may be prepared entirely manually or include computer printout.

Scope

Working drawings are intended to convey the Engineer's intent as covered in the Contract Documents. The Contract Documents plus any additions thereto, such as "Addenda" issued by the Engineer (per terms agreed upon in the contract if issued after the contract is made), constitute the sole authority for information in working drawings. The working drawings must include all information necessary for complete fabrication and placing of all reinforcing steel and bar supports.

Procedure

Working drawings are prepared by a detailer in accordance with the Engineer's instructions contained in the Contract Documents. Any necessary additional information must be supplied by the contractor concerning field conditions, field measurements, construction joints, and sequence of placing concrete. After approval by the Engineer, including necessary revisions, the drawings may be used by the fabricator and placer.

Drawing Standards

Working drawings are prepared to the same general standards as engineering drawings.

Layout

Drawings usually show a plan, elevations, sections, and details of a structure, accompanied by schedules for footings, columns, beams, and slabs. The plan should be drawn in the upper left corner of the sheet, with the elevations and details below and to the right of the plan. Schedules (and bending details) should be placed in the upper right corner of the drawing.

Unless otherwise specified, standard industry practice is followed—drawings are prepared by pencil on tracing paper.

Other media providing equal or improved reproducibility or durability may be utilized, such as ink, computer or mechanically generated graphics and printout, tracing cloth, or polyester film.

An arrow indicating the direction of north should be placed beside every plan view concerning layout.

Symbols and Notation

Standard symbols and abbreviations for working drawings are shown below:

# or Φ	To indicate size of deformed bar (High yield, Grade 60)
ϕ	Plain rounds (mild steel, Grade 40)
@	Spacing centre to centre
$\rightleftharpoons$	Direction in which bars extend
$\leftrightarrow$	Limits of area covered by bars

Pl	Plain Bar	OF	Outside Face
Bt	Bent	NF	Near Face
Str	Straight	FF	Far Face
Stir	Stirrup	EF	Each Face
Sp	Spiral	Bott	Bottom
IF	Inside Face	EW	Each Way
Cl	Clear	T	Top

Where unusual details or conditions require use of other (special) symbols or abbreviations, the drawings must provide an explanation of the notation applied.

Schedules

The reinforcement of elements and many other parts of structures can best be shown in tabular form commonly referred to as a schedule. The schedule is a compact summary of all the bars complete with the number of pieces, shape and size, lengths, marks, and bending details from which shop orders can be easily and readily written. While these schedules usually include the bending details for bent bars, separate bending detail schedules may be used.

General Requirements

Upon receipt of the engineering drawings, the fabricator takes the following steps:

1. Prepares working or shop drawings (including bending details).
2. Obtains engineer's approval, if required.
3. Prepares bar lists (shop lists) and fabricates the reinforcing bars.
4. Tags, bundles, and delivers the fabricated reinforcing bars to the job site.

Sometimes the working drawings but certainly the shop drawings must show the size, shape, and location of bars in the structure, including bar supports, if supplied by the fabricator. They also serve as the basis for preparing bar lists.

To assure proper interpretation of the engineering drawings and the contractor's requirements, the fabricator's shop drawings are usually submitted for approval to the contractor before shop fabrication is begun.

For the convenience of both contractor and fabricator, reinforcement is detailed, fabricated, and delivered by units which generally consist of footings, walls, columns, etc. A separate shop drawing and bar lists is usually made for each unit. For small structures the entire requirement may be handled as one unit. For large projects, the contractor may desire a unit, to be divided to correspond with his construction schedule. Such arrangements, between the contractor and fabricator, with the Engineer's approval, are made *before the detailing is begun.* All sections should be kept as large as practicable since it is more economical to detail and fabricate for large units, especially where there is apt to be a duplication of bars.

Unlike the customary practice in the field of reinforced concrete buildings, many highway departments prepare a combination 'engineering and working' drawing. The combination drawing includes a list of reinforcing materials from which the bar fabricator prepares his bar lists. The reinforcing bar placer uses the combination drawing to place the reinforcing bars.

Schedules

Highway structure engineering drawings most often show details of the various elements directly on the plan or elevation. Schedules are sometimes used for piers, small structures, and even retaining walls. Highway engineering drawings usually include, when completely detailed, a type of schedule that is really a bill of material, sometimes segregated by elements of a structure. These drawings are used by the reinforcing bar fabricator to prepare shop bar lists.

Dimensions

When the drawings for highway structures are combination engineering and working drawings from which the structure must be built, all dimensions must be clearly shown. The builder of a bridge should not have to compute any needed dimensions. Drawings must show the exact dimensions of concrete protection for all reinforcement. For example, they must plainly show whether the cover dimension specified on a girder is the clear distance from the main reinforcement or the clear distance from the stirrups. Where separate shop drawings are prepared, structural dimensions may be omitted.

Reinforcement

Drawings must show the size, spacing, and location of the bars in the structure. The bar schedule (combined drawing) must show the number of pieces, size, length, mark of bars, and bending details of all bent bars.

Reinforcement for larger structures is usually detailed, fabricated, and delivered by units for the convenience of both contractor and fabricator, for example, footings, abutments, piers, and girders. The bar list is then similarly subdivided. If the structure is sufficiently large, a separate drawing and bar schedule is made for each unit.

Reinforcing bars for foundations, piers, abutments, wingwalls, and slabs are usually shown on the plan, section, or elevation. Reinforcement may be shown in the simplest and clearest manner; however, the bar schedule must be a complete summary.

To be certain that all of the reinforcement is properly placed or positioned in a unit, a cross-section is frequently required in addition to the plan and elevation of the unit whereon the bars are shown.

Bar Supports

Plain metal supports are widely used as a means of securely holding reinforcement in proper position while concrete is being placed. Plastic coated or stainless legs can be specified to avoid possible rusting at points of exposure. Precast mortar/concrete blocks are also used.

Other types of proprietary supports are available and may be suitable.

Class of bar supports required should be specified in the Contract Documents.

Reinforcing steel must be accurately located in the forms, and firmly held in place before and during the placing of concrete. Adequate supports are necessary to prevent displacement during construction and to keep the steel at a proper distance from the forms. Bar supports are sometimes specified to be "sufficient in number and strength to carry properly the reinforcing steel they support." However, the detailer shall show bar supports as required.

Bar supports are not intended to and shall not be used to support runways for concrete buggies or similar loads.

Side-Form Spacers

All steel reinforcement must be firmly held in place before and during casting of concrete by means of built-in concrete blocks, metallic or plastic supports, spacer bars, wires, or other devices adequate to insure against displacement during construction and to keep the steel at the proper distance from the forms. Selection of the type of spacer has traditionally been the responsibility of the constructor. Detailing of side form spacers is not a standard requirement and is performed only when specifically required by the contract documents. The reinforcing bar shop drawings need only show, and the reinforcing bar fabricator will only be responsible to supply, those side form spacers that are equal to the standard wire bar supports.

44.3 CAUTION

It has to be clearly understood that one cannot become a good practising engineer unless one goes through the 'drawing-board', as it were, making detailed working drawings for at least a year or two before attempting a meaningful design assignment. As pointed out in another chapter, good and practical design has to go hand in hand with workman-like drawings, and it is one of those facts — perhaps unpalatable to the young engineers — which cannot be short-circuited or side-tracked if the designer desires to become workman-like and useful. Only a good hard exposure to 'detailing' on the drawing-board, in a practising design office, gives that all too important 'touch of class' that does not come by the mere ability to calculate the stresses — howsoever impressively! (Also see section 23.4 in Ch. 23 on 'Detailing'.)

CHAPTER 45

Pre-Tender Data-Questionnaire—Design-cum-Construct Bridge-Tender

- Tender for Bridge at Station ... (km ...) on ... Road near ... across River ... (as applicable).
- Collect information about the Project from the Department (i.e. the client) as well as from the site and its neighbourhood concerning various aspects of technical requirements, site-conditions, availability of various materials for construction including the leads involved, restrictions imposed and competition expected.
- The following design-data, as pertinent to the bridge site, are required in order to take up the design of the bridge.

Roadway Requirements

1. Number of traffic lanes and overall carriageway width.
2. Any hard shoulders, cycle tracks, and footpaths, if so, then their widths.
3. Road formation levels at crown of wearing course along the bridge.
4. Whether wearing course is in asphaltic or reinforced concrete.
5. Arrangement of lighting, drainage, and parapets on the bridge.

Structural Requirements

6. Live load details (including of that on hard shoulders, cycle tracks and footpaths).
7. Type of bridge generally anticipated (i.e. type of foundations and superstructure) and any particular construction method specially suitable to site.
8. Wind velocity and pressure for design (based on height above mean retarding surface).
9. Horizontal and vertical seismic coefficients (applicable on full gravity weight/reaction of mass above embedment level for working out seismic forces), if the area is in a seismic zone.
10. Any skew, plan curvature and vertical curve.
11. Whether abutments should be spill-through or non-spill-through type; and any particular type of abutment arrangement preferred.
12. Minimum overall length of the bridge (if a condition).
13. If pneumatic sinking of caisson foundations is anticipated, then type of steel-strakes and cutting-edge required.

Hydrological Requirements

14. Maximum design flood discharge expected to pass under the bridge.
15. 'Maximum mean' or 'maximum' velocity of flood flow.
16. Effective linear waterway required under the bridge (after allowing for 'average thickness' of each pier and its foundation, between high flood level and normal scour level, ignoring the earth fills in front of the abutments).
17. High flood level, afflux and low water level.
18. Vertical clearance required between the affluxed high flood level and soffit of deck from the considerations of unobstructed flow of floating debris with the flood discharge.
19. Vertical and horizontal clearances required under the bridge from any navigational requirements (if a condition).
20. Size (weight) of the floating debris expected to hit a pier and distance in which it is expected to stop after hitting the pier.
21. Size (weight) of any barge expected to hit a pier and any preferences for fender arrangement.
22. Average river-bed level and cross-section of river (or of the bridge crossing) under the bridge.
23. 'Normal' and 'design (i.e. maximum)' scour levels at piers and abutments.
24. 'Minimum' founding levels at piers and abutments from consideration of maximum scour, etc.
25. If foundations are on rock, then minimum penetration required in rock (depending on type of rock).
26. Whether approaches to the two abutments are easily

possible even during floods (from construction standpoint).

27. Whether there are any perennial river channels and whether they can be diverted during working season.
28. Any temporary islands required (in the river channels) for construction of foundations.
29. Any tidal variations at site, if so, how much and timings during various seasons.

Subsurface Soil Parameters

30. Normal safe bearing capacity of substrata at the 'minimum founding levels' under flood/saturated condition, and rate of its increase per metre depth there-below.
31. Whether the sub-soil below the founding level can be assumed 100% buoyant (50% in case of ordinary rock, lesser in case of hard sound rock) for checking the overall stability of the foundation (i.e. the floatation condition).
32. Average 'particle size' of 'soil' and the corresponding 'silt factor' of the eroding soil (required for estimating the 'normal scour depth').
33. Properties of the predominating type of soil around piers and abutments below their respective maximum scour levels, as also of the 'earth fill' (above maximum scour level) around/behind abutments. These are essentially: angles of internal friction and wall friction, cohesion value, and submerged and dry densities.
34. Any particular method 'specifically' recommended for estimating relief due to passive less active earth pressures below maximum scour level in case of caisson foundations.
35. If the subsoil below maximum scour level is saturated and cohesionless, then any increase in minimum depth of foundations as a precaution against soil liquefaction under design seismic shake.
36. Relevant details of any existing bridge in the vicinity of the proposed bridge and details of previous flood marks, discharges, surface velocities and flood-spreads in the area.
37. Type of coarse-aggregates and fine-aggregates available at site or near it. Whether the coarse aggregates are natural gravel or crushed type; and whether they are alkali-reactive (certain aggregates have some silicates and carbonates in them which react with the alkali in the cement and this leads to extensive cracking of hardened concrete later). Whether the sand needs crushed component to bring it up to acceptable fineness modulus, and whether it contains unacceptable amounts of deleterious materials (e.g. clay, silt, chlorides, sulphates, etc.) which may require extensive washing prior to actual use. Whether the aggregates are highly absorptive and contain more than acceptable amount of flaky particles, are dirty and require washing.
38. Environmental conditions, saline/marine atmosphere, prevalence of windy and hot climatic conditions (or cold weather conditions), etc.
39. Availability of good potable water that can be used for making and curing concrete.
40. Rainfall per year, and 'which and how many are the wet months' per year, its effect on working season.

CHAPTER 46

'Maintenance Management System' for Highway Bridges

46.1 INTRODUCTION

Although a vast revenue resource is consumed in building a highway network (including its structures), managing its maintenance and then actually executing the maintenance work can prove even more exacting and costly if what has been built must remain operational for the intended long-term safe use. In the past the maintenance work has traditionally been a mere ongoing process conducted many a time by those who long ceased to be technical—or almost! But never before has the need for maintenance been more pressing. This calls for a scientific assessment of the problem for a workman-like understanding and execution. This requires:

(i) a thorough examination of the detailed inventories;
(ii) carrying out detailed condition surveys and visual and hands-on inspections;
(iii) analysing the observations and structures in order to unfold the causes of structural distresses;
(iv) carrying out the structural investigative computations and, where called for, the appropriate *in situ* tests on material-samples and existing structures; and
(v) ultimately writing the prescriptions for rehabilitation and repair or outright demolition and replacement.

It can readily be realised that first Maintenance Management (i.e. a rationalised methodology for assessment of the condition of what exists and how to repair and rehabilitate it to retain its operational status ... in order to enjoy its revenue return if not financial-return), and then the execution of Maintenance (i.e. the physical execution of the maintenance decisions taken), form a whole new dimension of the society's rightful demand on the Engineer's Services if an existing highway network system has to remain there for the purpose it was constructed in the first place! The effective aggregate of the aforementioned two exercises is what is referred to as the Maintenance Management System (MMS) in the present-day technical jargon.

It is only natural that in each country the appropriate Ministry responsible for the country's highway system should take up this MMS work on priority basis, either departmentally (which, as the experience shows, ultimately does not generally work) or by engaging the services of appropriately qualified consulting engineering firms.

46.2 ELEMENTS OF THE AGREEMENT

In general, the Maintenance Consultancy Agreement* should consist of the following elements:

I. Instructions to Bidders.
II. Submission of Proposals.
III. Information and Documents relating to the Companies submitting Proposals.
IV. Evaluation of Submissions.
V. Consulting Services Agreement 'Form'.
VI. Terms of Reference for the Highways Maintenance Management System (TOR for the HMMS).
VII. Survey Activities A, B and C for Bridges, Culverts and Tunnels, and Schedule of Unit-Rates. (See ahead for details of Activities A, B and C.)
VIII. Outline of Bridge Surveys ... Objective, Survey Activities A, B and C, and estimation of Numbers and Types of Crew and Equipment required.
IX. Conditions of Employment of the Consultant.
X. Appendices (various).

NOTE: In subsequent discussion 'Ministry' refers to the authority that owns the Highway and Bridge Network.

46.3 INCEPTION REPORT

The Highways Maintenance Management System comprises a suite of sub-systems, some of which are independent but all of which are inter-related. Before proceeding with the inventory and detailed sub-systems development, it is necessary to prepare and submit the Inception Report which includes annotated flow charts, proposed computer files and record layouts illustrating the system as a whole and the inter-relationships of the sub-systems together with a brief commentary on the contents and objectives of each sub-system. The Inception Report shall include a description of proposed reporting formats for the sub-systems, the overall capability of the system in relation to updating and maintenance and, where relevant, proposals for testing and validation of the system and its sub-systems. It shall also

* See Reference 1.

include a general outline of the proposals for producing users' handbooks and manuals, and training of the staff to be assigned by the Ministry. After review of the inception report by the Ministry and completion of required relevant revisions prepare the revised report. On receipt of written approval by the Ministry, commence work according to the approved work plans.

The Inception Report shall include,

(i) Proposals for executing the inventory and condition survey, traffic counts and vehicle weighings; proposals for measurement of pavement quality and sampling frequencies; the classification system proposed for the condition of each highway element; the type and number of equipments proposed; and a work program and manning schedule.

(ii) Proposals for the organization, specifications, and flow charts for the data based management system, its flexibility for interaction with other systems and sub-systems and future expansion.

(iii) Description of the Pavement Management System (PMS), including:
 (a) mathematical models proposed for use with PMS;
 (b) the capability of the PMS model for addressing the various issues; and
 (c) a work plan for developing and testing the prediction sub-model.

(iv) Description of the maintenance management sub-system proposed for bridges and other structures, including:
 (a) procedures to be used for visual examinations and subsequent structural and engineering investigations; and
 (b) methodology to be used in establishing priorities for maintenance, rehabilitation or replacement of bridges and other structures, where and as applicable.

(v) Description of the methodology to be used for establishing quality standards and costs for the maintenance of non-pavement related elements.

46.4 INVENTORY AND CONDITION SURVEY OF THE ENTIRE HIGHWAY NETWORK

It is necessary to develop procedures for recording the type, location and condition of all highway elements and features required for each maintenance management sub-system. Locations shall be listed to the nearest 1/100th of a kilometre when not otherwise coincident with kilometre markers. It will be required to prepare a complete classification of features and elements that would be covered by the inventory. A partial list of such elements and features is given below:

(a) Pavements including ramps, frontage roads, acceleration and deceleration lanes and shoulders
(b) Bridges: vehicular and pedestrian
(c) Culverts
(d) Tunnels
(e) Retaining walls
(f) Slope protections
(g) Guardrails
(h) Medians
(i) Pavement markings
(j) Highway signs
(k) Intersections
(l) Fences
(m) Rest areas
(n) Sidewalks
(o) Landscaping
(p) Lighting
(q) Traffic signals

It is necessary to carry out an inventory and condition survey for the entire network through the collection of information relevant to type, location and condition of each of the elements. All such inventory data shall be entered on special forms to be designed to suit the Ministry. The Ministry may already have developed special data collection forms for related applications.

Review these forms and suggest appropriate modifications, identifying code numbers while maintaining compatibility with the existing forms in the Ministry.

In collecting data on the condition of bridges and other structures, one may concern oneself initially with detailed hands-on visual examinations of the structural elements of their substructures and superstructures to insure the discovery of any critical weaknesses in these elements.

Where weaknesses are identified, then it is necessary to undertake a detailed structural investigation to determine the cause and the appropriate corrective action, the requirements of which are detailed ahead.

As part of the visual examination one has to also identify those conditions which will require routine maintenance (e.g. painting and repair of railings, repair of bridge-approaches and guardrails).

NOTE: The discussion hereafter shall be restricted only to *'structures'* since the present discussion deals only with bridges.

46.5 MAINTENANCE AND REHABILITATION OF BRIDGES AND OTHER STRUCTURES

In developing maintenance and rehabilitation requirements for bridges and other structures and their replacement, where necessary, develop a methodology for (a) the evaluation of the structural capabilities and overall performance of these features (including the substructures and superstructures); and (b) the determination of the optimum course of action for each bridge, culvert, tunnel, etc. Then undertake a complete detailed 'visual survey' and 'hands-on inspection' of all superstructures and substructures of every bridge, culvert, tunnel, etc. in the network, to identify and systematically record any signs of distress or deterioration. Develop a methodology whereby the condition of each structure could be rated to establish:

(i) Those elements which are in good condition without any signs of deterioration and requiring no immediate action;
(ii) Items which show signs of distress or deterioration but which, under the operation conditions of the total structure, may not advance to a serious defect and which can be easily rectified through routine maintenance;
(iii) Items which show signs of deterioration and which due to distress under operation conditions would affect the structural integrity of the elements (cracking, corrosion, etc.);
(iv) Item which show signs of progressive deterioration which could lead to their failure due to excessive or repetitive loading, etc. This may necessitate restrictions in loads or should the load category be upgraded, to replacement or strengthening of the structure; and
(v) Elements which have been underdesigned or poorly constructed which show structural cracks or deterioration. Further checks by the consultant may require complete replacement or recommendations to limit the use of such structure.

46.6 BRIDGE MAINTENANCE TEAM (BMT)

Brief guidelines are presented here in order to assist the Maintenance Department to achieve a *result-oriented performance* from its Bridge Maintenance Team (BMT) in a *workman-like manner*.

• Composition of BMT (Bridge Maintenance Team):

The BMT shall, in addition to suitable number of engineers have atleast two design engineers and one Data-Entry cum Programmer attached to it. The former will assist the BMT in carrying out the *Structural Investigative Computations* in the on-going assignments, and the latter will put on tape the details of all the highway structures for an effective and rapidly-retrievable Data-base.

• Training of BMT:

A Structures Expert should impart practice-oriented professional-type training to the BMT, covering the following aspects:

A. Bridge Features:

1. *Classification* of Bridges
2. The *Substructure*
3. The *Superstructure*
4. Bridge *Bearings* and their possible Problems
5. Bridge *Expansion-Joints* and their possible Problems
6. *Economics* and *Quantity-Trends* in Alternative Reinforced and Pre-stressed Concrete Bridges

B. Bridge Maintenance:

8. *Deterioration* and *Rehabilitation* of Concrete Bridges
9. *Maintenance* of Bridges
10. *Distresses* in various Bridge-Elements and their *Reporting* (Questionnaire)
11. Some *Construction Considerations*

C. Concrete Technology and Field Practice:

12. The Story of Concrete ~
    - o Historical
    - o Coarse Aggregate
    - o Fine Aggregate
    - o Cement
    - o Water
    - o Admixtures
    - o Control of Surface Evaporation
    - o Underwater Concreting
    - o Joints ... Construction/Movement/Cold
    - o Mixing Considerations
    - o Transporting the Mix
    - o Finishing the laid Concrete
    - o Curing
    - o Sampling and Testing Concrete
    - o Acceptance Criteria:
        - — Cylinders
        - — Cores (in case of dispute)
    - o Dense Concrete
13. Concrete *Mix Design*
14. Practical Facts about *Properties of Concrete*
15. Practical Facts about *Durability of Concrete*
16. Concrete in *HOT* and/or *MARINE Environment*

Training the Engineers of Various Road Districts:

(i) The Structures Expert of the Ministry, together with the head of BMT, shall also train the designated engineers from the various Road Districts, in different aspects of Bridge Maintenance, familiarizing them appropriately enough with the associated design and construction features as well.
(ii) This sharply *focused* training would cover:
    — *Deterioration* and *Rehabilitation* of Concrete
    — Bridge *Bearings* and their possible Problems

- Bridge *Expansion-Joints* and their possible Problems
- *Maintenance* of Bridges
- *Distresses* in various Bridge-Elements and their *Reporting* (Questionnaire)
- *Inspect-and-Explain* Field-Visits

• Working Methodology

The BMT should have access to all the "as-built' drawings and associated information available with the Ministry. This shall include even the individual Maintenance Contract Documents. (This is necessary since going through longer bureaucratic channels can cause time-wasting bottlenecks. However, all such borrowed Documents shall be duly returned to the concerned for preserving the records as they should be.)

The BMT, with the active assistance of their Data-Entry-cum-Programmer, shall prepare an inventory of all Bridges (indeed of all the highway structures), clearly recording the updated distresses in and the corresponding maintenance actions taken for each. The details in this inventory shall be updated annually.

The BMT shall draw up a working-strategy in such a manner as to automatically double-check through spot-reinspections as to whether the suggested maintenance actions were really and actually carried out in the field. The feed-back must then be entered on the data-base tape for the appropriate structure for future reference.

The BMT must keep an updated record of the salient features of the exact scope of work of each Maintenance Contractor (e.g. whether routine maintenance or preventive maintenance, or both, and their itemized details).

The BMT should utilize the *simplified Distress-Reporting-System* as described in Chapter 47 ahead. (This is in the format of a "Questionnaire" and can be strengthened by introducing into it a simplified Condition-Rating-System suggested in the present chapter.)

One engineer from the BMT must always participate in the "Handing Over" exercise. This will make the subsequent maintenance activity more convenient and meaningful in evaluation. Last but not least, the BMT engineers should be involved in the formulation of the Maintenance Contract Document. Their experience through direct involvement with the maintenance work can be of great help in drafting the Maintenance contract documents.

• Scope of Work for BMT

~ *BMT's Obligations and Interaction:*

(i) BMT engineers *shall be obligated* to be familiar with "Design" and "Construction" *features* of Bridges and shall be responsible for:
  - Thoroughness of field inspection.
  - Analysis of findings and observations.
  - Monitoring of distresses.
  - Recommendations for correction of defects (in consultation with Experts, where necessary).
  - Locating areas of structures with incipient problems, and
  - Programming preventive and routine maintenance actions.

(ii) The BMT engineers *shall interact with* various Experts in the Ministry in the following fields, as and when the need arises:
  - Design
  - Construction
  - Materials
  - Soils
  - New Techniques
  - Plant and Equipment, and
  - Emergency Repairs

~ *Objective:*

(i) To ensure "serviceability of structures" (for safe and uninterrupted traffic flow) and for "gainful protection of investment", the BMT *'has to be able to identify the need'* for Structure-Maintenance-Rehabilitation-Replacement (as necessary), *take the appropriate 'action-decisions'* and then *ensure their timely execution in the field.*

(ii) Each component of each structure must be inspected for deficiencies-deterioration-distress, e.g.
  - *Concrete components* warrant inspecting for various cracking patterns (longitudinal, diagonal, mesh pattern, etc.), deck spalling, joint spalling, and other signs of distress.
  - *Steel structural components* warrant inspecting for rust and loss of section due to similar corrosive action, damage due to stress, buckles, kinks, and collision and other indications of strength-reduction and distress.
  - etc.

~ *Summary of BMT's Activities:*

- Timely *'inspection'* of structures.
- *'Condition-survey'* of structures and their *'rating'*.
- *'Maintenance Action-decisions'* and their *'execution'*.

46.7 STRUCTURE—INSPECTIONS

• Basis

- Original completion reports must be available for all bridges.

— These form the basis for periodic inspections.
— Data thus collected from inspection to inspection should be assessed to determine need for any remedial measures (Maintenance Actions).

• Inspections shall be Carried Out Under 3 Categories, viz:

(i) Routine Inspections . . . at monthly intervals:

— Not very skilled operation, but is conducted quickly every month.
— *Purpose*: to report fairly obvious deficiencies/developments.

(ii) Detailed Inspections . . . at annual intervals:

— *General Type*—one year.
— *Major Type*—the next year.

In the General Type:

- o Cover all elements against a CHECKLIST.
- o Visual and hands-on inspection, supplemented by Standard Instruments and Aids.
- o Produce a written report for each part/element.

In the Major Type:

- o Inspect more intensively than in the "General Type", and this may require setting up special access facilities.

(iii) Special Inspections . . . after special events:

— This type of inspection is needed after special events, e.g.:
  - o Heavy Flood,
  - o Heavy Earthquake,
  - o Passage of high intensity live load.

— May need the services of an experienced Designer as it may involve certain:
  - o Design Checks, and
  - o Tests.

46.8 GUIDANCE FOR BMT (... OUTLINES)

Establishing the Condition-Rating System

Normally there are *five categories of "condition"* to be applied, *as reiterated below:*

a) Structures containing elements which are in good (includes new) condition without any signs of deterioration and requiring no immediate action.
b) Structures containing elements which show signs of distress or deterioration, but which, under the operating conditions of the total structure, may not advance to a serious defect, and which can be easily rectified through routine maintenance.
c) Structures containing elements which show signs of deterioration and which, due to distress under operation conditions, would affect structural integrity of the elements (cracking, corrosion, deflection, unusual vibration, any other deformation).
d) Structures containing elements which show signs of progressive failure due to excessive loading, possibly necessitating load restriction, strengthening, or replacement.
e) Structures containing elements which have been underdesigned or poorly constructed or are overloaded, and show structural cracks or deterioration, possibly resulting in use-limitations or replacement.

The elements examined in the condition survey (see Activities A, B and C ahead) should be categorized and sorted *to result in a Condition—Rating*, following the above described general guideline, which can be used numerically from 5 to 0, as enumerated below, to determine the course of action:

5.. No Action
4.. Routine Maintenance
3.. Special Maintenance/Rehabilitation/Repair
2.. Reduced allowable load after special repair
1.. Further investigation, but in the interim reduce load and speed and use speed breakers and rumble strips in the approaches to the bridge.
0.. Replacement.

The Condition Rating can also be made on a scale of 9 to 0 (Equivalent Rating Conditions of the US Federal Highway Authority) as depicted in Table 46.1.

Table 46.1 *Equivalent Rating Conditions*

Description	Rating
o Not applicable	N
o New condition	9
o Good condition—no repairs needed	8
o Generally good condition—potential exists for minor maintenance	7
o Fair condition—potential exists for major maintenance	6
o Generally fair condition—potential exists for minor rehabilitation	5
o Marginal condition—potential exists for major rehabilitation	4
o Poor condition—repair or rehabilitation required immediately	3
o Critical condition—need for repair or rehabilitation urgent. Facility should be closed until the indicated repair is completed	2
o Critical condition—facility is closed. Study should determine the feasibility for repair	1
o Critical condition—facility is closed and is beyond repair.	0

Inspection Guidance

~ ***"Inventory" and "Condition Survey"***

Each structure has to be:

— Inspected,
— Evaluated, *and*
— Classified

in relation to various parameters affecting its performance and structural integrity.

~ ***Methodology***

I. Give identity No.: to each bridge based on the *'type of structure'*.
II. Define *'elements/components for inspection'*.
III. Give *'guidelines for inspection and the remedies/repairs'*.

Type of Structure

Once the preliminary identification of the structure and crossing-type have been established, the *identification of the type of structure* should be made. With the utilization of computer analysis in mind, coupled with an awareness of the many possible combinations of structure types, *a coding system for identification of the structure-type* is advisable. The three digit code for identification of "type of structure" which is used by FHWA, US DOT, is recommended.

An example of the utilization of the three digit code for identification of "type of structure" is given in Table 46.2.

Table 46.2 *Type of Structure*

1st Digit		2nd and 3rd Digits	
1	Reinforced concrete, determinate	01	Slab
2	Reinforced concrete, indeterminate	02	Stringer/Multi-beam of girder
3	Steel, determinate	03	Girder & floor-beam system
4	Steel, indeterminate	04	Tee beam
5	Prestressed concrete, determinate	05	Box beam or Girder-multi
6	Prestressed concrete, indeterminate	06	Box beam or Girder-single
7	Timber	07	Frame
8	Stone or masonry	08	Orthotropic
9	Aluminium	09	Truss deck-type
		10	Truss through-type
		11	Arch deck-type
		12	Arch through-type
		13	Suspension
		14	Stayed girder
		15	Movable lift-type
		16	Movable bascule-type
		17	Movable swing-type
		18	Tunnel
		19	Culverts
		20	Other

Elements to be Inspected

o *Bridge 'Superstructure' Components*

1. Deck Components:
   a. Deck slab
   b. Kerbs/walkways
   c. Bridge railing
   d. Expansion joints/expansion devices
   e. Drainage devices
2. Beams/Girders (longitudinals)
3. Diaphragms or Cross Frames
4. Truss Components:
   a. Floor beams
   b. Connections
   c. End posts
   d. Top chord
   e. Lower chord
   f. Lateral bracing
   g. Sway bracing
   h. Portals
   i. Lacing
5. Bearing devices
6. Arch Components:
   a. Arch
   b. Arch columns
   c. Supports
7. Other Components (if applicable):
   a. Spandrel walls
   b. Suspension system
   c. Hangers
   d. Suspenders
   e. Anchorage
   f. Towers
8. Paint/snowcem
9. Connections:
   a. Welds
   b. Rivets
   c. Bolts
   d. Hinges/Articulations
10. Bridge machinery (if applicable)
11. Collision damage preventers/reducers (if applicable)

o *Bridge 'Substructure' Components:*

1. Abutments:
   a. General condition
   b. Beam seats and shaft/wall/columns
   c. Backwalls/Dirt walls
   d. Wingwalls
   e. Footings/Piles, etc.
   f. Settlement

g. Scour

2. Piers
   a. General condition
   b. Beam seats and shaft/wall/columns
   c. Footings/piles, etc.
   d. Fenders and dolphins
   e. Scour
   f. Settlement
3. Paint (if applicable)
4. Debris on beam seats and bearings

o *Culverts (if applicable):*
   a. General condition
   b. Alignment
   c. Adequacy
   d. Debris accumulation
   e. Headwall or endwall condition
   f. Condition at inlet and outlet

o *Channel Condition (if applicable):*
   a. Alignment
   b. Protection (vegetation, riprap, etc.)
   c. Waterway adequacy
   d. Scour
   e. Deflection of water-course and the resulting ponding and oblique attack on some foundations

o *Approaches to Bridge or Culvert:*
   a. Alignment
   b. Grade
   c. Approach slabs
   d. Guard rail
   e. Embankment condition
   f. Relief joints

o *General Components:*
   a. Warning signs
   b. Navigational lights
   c. Utilities on bridge (pipes, cables, etc.)
   d. General appraisal and operating status.

NOTE: In addition to the "serviceability" and "functionability" of bridge components, each component must be analyzed for structural deficiencies and/or deterioration. *Concrete structural components warrant inspection for:~* various cracking patterns (longitudinal, diagonal, mesh pattern, etc.), deck spalling, joint spalling, and other signs of distress or deterioration. *Steel structural components warrant inspection for:~* rust and loss of section due to similar corrosive action, stress cracks, buckles, kinks, collision damage, and other indications of strength reduction and distress. *Timber structural components should be inspected for:~* structural decay or damage due to fungus, insects, dry rot, weather attack, chemical attack, life cycle wear, collision, and other causes of loss of structural integrity. *Stone and masonry components warrant inspection for:~* damage due to weathering, spalling, plant growth (in joints and cracks) abrasion due to wind-blown particles, seasonal expansion/contraction, and other strength reducing factors.

(c) Inspection — Guidance

The Maintenance Engineer should be familiar with:

— Terminology and elements of bridge structure.
— Tools, devices of specialized equipment needed for inspection and testing.
— Methodology of inspection,
— What to look for,
— Types of "materials" used in the bridge and, hence, be ready for the "types of distress/deterioration" expected.
— Possible "effects" of failure of one or more component elements due to overload or collision (assuming design and construction were satisfactory).
— Results of foundation movement, e.g. from:
  o Lateral movement settlements
  o Vertical movement settlements
  o Pile settlement
  o Rotational or tipping movement which could be caused by:
    * Slope slides
    * Bearing failure
    * Consolidation of soil
    * Seepage/frost-action
    * Drag-down of piles (negative skin-friction)
— Results of damage in Waterway.

• ***What to Look for***

o *Approaches:*
— Unevenness
— Settlement
— Cracks in approach slab may indicate hollows settlement beneath.
— Examine its joint with abutment wall.
— Conditions of shoulders, slopes, drainage approach guard rail.

o *Waterway*†

o *Piers and Abutments, including foundations/piles*
— Foundations† scour, movements, concrete condition, reinforcement condition, etc.

o *Stringers:*
— *Timber Stringers:*
  * Splitting
  * Cracking/damage
  * Excessive deflection
  * Insect attack/decay

† For possible problems in 'Waterways', possible problems due to various types of 'Foundation Movements', various types of cracks and deteriorations possible in concrete, corrosion problems, etc., (and their Rehabilitation) — See References 2, 3 and 4.

* Crushing
* Connections: clean, tight
* Bearings/seatings

Steel Stringers:

* Cracking
* Corrosion
* Bearings/seatings
* Connections
* Misalignment: flanges, webs
* Damage

Concrete Stringers: (R.C./p.s.c.)

* Cracking†
* Corrosion†
* Damage: spalling, cold joints

o *Steel Girders:*

* Cracking and corrosion
* Corrosion particularly at contact surfaces where moisture may ingress (e.g. flange plates, gussets, splices, rivets and bolts, weldlines)
* Re-entrant corners
* Sections of section/size change
* Flanges and webs: misalignment, damage/buckling
* Web stiffeners (any evidence of buckling)
* Any unusual deflections under passing loads
* Examine hinges
* Hanger bars w.r.t. ambient temperature.

o *R.C. Members:*

* 'Spotting' by rust
* Leaching (dissolution and baring of matrix)
* Spalling — in critical zones of structure: (e.g. splash zones, water line at low tide levels, portions exposed to atmospheric attack, particularly on the windward side).
* Give special attention to: girder webs, soffit slabs (in box decks), articulations, deck slabs from soffit side, bearing areas.

o *Pre-stressed Concrete Members:*

In addition to the examinations recommended for R.C. members, examine against:

* Loss of camber
* Excessive deflection
* Distress in anchorage zones, at junctions of diaphragms, cracks in webs (sometimes due to radial cable thrust)

NOTE: Wherever any distress is found, immediately:

* Indicate it on the structure by *Highlighter*, marking its extent, width, etc., and *date it* at each inspection (for continuous monitoring).
* Note it down on paper also and report it.
* Monitor it at regular intervals.

o *Bearings:*

Examine that they are:

* Functioning properly
* Clean (and not stuck/locked-up against rotation/sliding, as required).
* Anchor bolts — that they are secured.
* In neoprene bearings — no abnormal bulging, radial cracking, horizontal cracking, etc.
* In pot bearings — seal is intact/no leakage.
* In spherical bearings and cylindrical bearings — the Teflon sheet is intact.
* Examine pedestals.

o *Expansion Joints:*

* Is space for thermal movements adequate?
* Is the joint clear of debris/surfacing material?
* Is the Neoprene uncracked/uncut? (in those of the joints that incorporate Neoprene either as a seal or in modular manner)
* Are the fingers and plates freely working (in finger-type joints)?
* Is the joint assembly securely in place? (Otherwise, if loose, they can become traffic hazard).
* Is the concrete adjacent to the joint sound and free of laminations (sound by a 2 kg. hammer)?

o *Decks:*

Concrete decks, examine for:

* Cracks — various types
* Leaching
* Scaling
* Pot-holing
* Spalling
* Other signs of deterioration
* Evaluate each item for its effect on the structure and the rehabilitative work required to restore structural integrity.
* Rusting of reinforcement
* Asphaltic and other type of w.c. may hide the deterioration in deck slab, etc. until after the damage has become excessive.
* Where "permanent forms" have been used in the deck construction, some such panels may require to be removed (if possible) to closely examine portions on them.
* etc.

Steel Decks, examine for:

* Corrosion — particularly at bearing surface
* Unsound welds

† For possible problems in 'Waterways', possible problems due to various types of 'Foundation Movements', various types of cracks and deteriorations possible in concrete, corrosion problems, etc., (and their Rehabilitation)··· See References 2 and 3.

* Rivets, bolts, stiffeners
* Splices
* etc.

Timber Decks, examine for:

* Decay at contact surfaces where the surfaces bear (on stringers, between layers of planking, etc.)
* Any looseness developed in nailing/bolting
* Any abnormal deflections under passing loads (due to looseness of connections or damage to member-sections, etc.)
* etc.

NOTE: *All decks should be examined for:*

* Slipperiness (hazard)
* Drainage (no ponding on deck or near it)
* Drains/scuppers...... open and clear
* Ensure draining water does not harm other areas of bridge and those around it.

o *Road Kerbs:*

* *Concrete Kerbs:* check as for concrete, steel.
* *Timber Kerbs:* splits and decay should be checked.
* Note the condition of any painting (visibility-purpose painting).
* Examine against any loss of height from building-up of surfacing on road surface.

o *Side-walks (foot-paths):*

Concrete sidewalks:

* Cracks
* Scaling
* Spalling
* Pot holing
* Other deteriorations (e.g. rusting of steel)
* Condition at joints

Timber sidewalks:

* Soundness of timber
* Adequacy of floor planks
* Nails/bolts... secured or working loose?
* Slipperiness

Steel sidewalks:

* Corrosion/perforations
* Connections (secured or working loose)

All sidewalks should also be examined for:

* Drainage
* Deflection

o *Parapets/Railings:*

Concrete Railings:

* Cracks
* Spalls
* Scaling
* Other deteriorations (e.g. rusting of reinforcement).
* Member connections

Metal Railings:

* Condition of paint and corrosion
* Bolts
* Rust stains where posts are set in pockets (may be necessary to remove some of the mortar/grouting from around the postbase in order to examine the severity of corrosion).

Timber Railings:

* Any insect attack and decay
* Securedness of all connections
* Breakages in timber

All handrails should also be examined for:

* Damage from traffic—pedestrian or vehicular
* Vertical and horizontal alignments (foundation settlement and deficiencies in bearings may show up through these)
* Joints—ensure that they are free and working (to accommodate deck movements)
* Free of hazardous projections

o *Crash Barriers:*

Examine for:

* Traffic damage—structural
* Traffic damage—alignment
* Traffic damage—hazardous projections
* The damages in steel—as stated elsewhere.
* The damages in concrete—as stated elsewhere.
* Anchor bolts/nuts... for securedness.

o *Wearing Course:*

Examine for:

* Surface condition—cracks, spalls, pot holes, disintegration, rideability, etc.

o *Trusses in Steel:*

* Sight along roadway railing and kerbs and along chord members for:

o Any misalignment—vertical, horizontal.

—Any deviations?

* Check each member:
  — Compn. members for: kinks, bows, washers connections.
  — Counter braces: in these the "counters" should be just optimally tight (against rattling and vibrating), more would cause abnormal stresses and less would render them useless!

NOTE: Any "looped" rod-type tension members should be checked against cracks in the loop.

* Damage from passing traffic (particularly in the "portal bracings")
* All lateral bracing members against: damage, vibration movement.
* Condition of paint/corrosion.
* Condition of "connection details": bolts, rivets, welds, gussets, packing Plates, pins.

(Particularly where any rubbish/contaminants can collect as the surface drainwater washes the deck.).

* Look for rust-generation at "bearing surface", particularly in multi-plated sections — howsoever difficult or inconvenient! This is where the trouble may begin!!!
* Cracks/breaks, etc. as stated earlier.

o *Trusses in Timber:*

Examine against:

* Some of the items pointed out above for "Steel Trusses" as are relevant here.
* Alignment against sag — may indicate partial failure in joints or improper adjustment of steel vertical rods at connections.
* Splits/decay from insect attack:
Particularly where moisture may collect, e.g. at contact surfaces and around holes through which rod bolts are fitted.
Also: look out for *decay* at the joints, particularly where prone to collection of dirt and debris.
* Accumulation of fire-hazards around the bridge (if any).

— General Cautions:

* Observe/inspect the bridge during passage of high flood, heavy load, and combination thereof against any excessive deflection and vibration.
* Watch out against accumulation of fire hazard. (Collection of drift-wood, debris, floating logs and trees, illegal constructions nearby or below, storage of inflammable materials nearby or below, etc.)
* Try to observe the movements and rotations at the Expansion — Joints and Bearings at extreme temperatures.
* Condition of p/c slabs and of their supporting arrangement where such slab elements exist (... any missing panels, rattling panels, broken panels... traffic hazard).
* Examine behaviour of construction joints and cold joints under passing heavy loads.

46.9 OUTLINE OF BRIDGE SURVEYS

Objective, Survey Activities A, B and C, and Estimation of Numbers and Types of Crew and Equipment Required (See References 1 and 2):

The impetus for evaluation and prediction of deterioration and defects in highway structures covers a spectrum of reasons from safety of the travelling public to the integrity of the structures itself to development of a data bank information as a base of an overall highway maintenance management program. Thus, the basic maintenance considerations and objectives of highway structures (bridges, culverts and tunnels) management system are:

(i) To preserve the investment made in existing highway facilities;
(ii) To provide continuing adequate levels of safety and convenience to the highway users;
(iii) To upgrade and improve existing facilities as may be economically practical through minor maintenance;
(iv) To conserve and protect the aesthetic and ecological features of the environment; and
(v) To ensure effective and economical utilization of resources in the accomplishment of maintenance programs.

These objectives will be accomplished through the effective management of maintenance operations and resources. In fact, an up-to-date inventory data of all bridges, culverts and tunnels provides the basis for maintenance management systems.

In developing the bridge model, the recorded information available may not be sufficient to fulfil the needs of the model.

In appreciation of above, the description of Activities A, B and C and the inspection procedures to be followed to obtain the data required are as follows:

• ***Activity A***

Activity A includes the detailed visual and hands-on inspection of all bridges and highway structures and the creation of permanent information files designed to serve the future information requirements.

The purpose of the detailed inspection procedure is to provide basic data that will satisfy the following information needs:

(i) Provide the required information for the proposed mathematical model of the highway structure system. The development of the model and the input parameters will necessarily govern the type and level of information that must be obtained and assembled during the detailed inspection. Other mathematical indicators, such as 'sufficiency' type of ratings, may also be calculated in whole or in part from the basic data collected during the detailed inspection phase.
(ii) Provide information necessary to identify minor and major items of deterioration, the identification of which will require Activities B and C, which address such deteriorated items. Activity A will provide all or part of the information necessary to evaluate these items under Activities B and C.
(iii) Provide an accurate, permanent record file for all

highway structures, including the following permanent reference files:

(a) A *standardized inventory sheet*, stored electronically, that will contain enhanced FHWA* type information and act as the data base for the mathematical model. This form must be designed to provide the necessary information for the above tasks and will, to a large extent, define the detailed inspection effort. Obtaining information in a form directly usable in succeeding evaluation steps is considered extremely important. Several highway agencies, including the State of Pennsylvania in the U.S.A., have developed enhanced FHWA-type inventory sheets which may prove useful in developing a form that is specific here and serve the information needs of the mathematical model.

(b) A *supplementary file* will be established to store information relevant to a given structure, but beyond the scope of the inventory form. Specific information that may be included in such a file includes sketches of deteriorated elements, measurements, notations and other information that is beyond the scope of the inventory form, but may prove useful in future investigations. This file would be a "hard copy" file, but electronically referenced from the inventory file.

(c) A *photographic file* showing the general features of each structure, cataloging deteriorated features, should be established. Photographs are a valuable tool that can quickly provide information not easily obtained and stored by other methods. Standard views of each structure should be photographed, along with any specific deteriorated items, or items of general interest. Photographs should be adequately labelled and stored in loose leaf, 3-ring bound, hard covers, suitable for updating data.

(d) A *permanent reference file of 'as-built' drawings* should be established for all highway structures as soon as possible. As-built drawings, or design-drawings if as-built were not completed, contain information of the as-built structures.

A concerted effort should be made to obtain, catalog and maintain a file of all available as-built drawings. Availability of as-built drawings may prove invaluable if major rehabilitations or modifications are necessary in the future.

The search for as-built drawings may have to extend to the original design consultants if drawings are not presently available. However, further passage of time will only make the search more difficult when a need for the drawings arises.

* Federal Highways Authority, Washington D.C., U.S.A.

- ***Activity B***

Activity B includes the evaluation of structural components identified in Activity A that exhibit minor deterioration, the preparation of a report and recommendations addressing such deterioration, and the preparation of sufficient details and specifications for repair. Activity B will be initiated as a result of the findings of the detailed field inspection of Activity A, if found necessary. In fact, 'bridge and other highway structures' maintenance work is not as clearly defined as highway maintenance, since the structure work is generally reported as either 'major' structure maintenance or 'minor' structure maintenance. A realistic approach to a bridge maintenance program will evaluate the deterioration and defects against appropriate codes to identify work activities under major or minor structure maintenance.

In general, defects and deteriorations in non-structural elements and in 'structural elements due to non-structural causes' could be defined as minor structural distress (except where plastic shrinkage and plastic settlement cracks are severe). Typical types of defects and deteriorations in non-structural elements are given below:

(i) Clogged drainage — scupper.
(ii) Pavement migration.
(iii) Defects in concrete posts and handrails.
(iv) Failure in expansion joints.
(v) Cracking or movement in the approach slab.
(vi) Parapet geometry.
(vii) Scour at footings.
(viii) Slope failure.

Typical defects and deteriorations in structural elements due to non-structural causes are given below:

(i) Plastic and drying shrinkage cracks.
(ii) Scaling and spalling.
(iii) Corrosion of reinforcement.
(iv) Inadequate concrete cover.
(v) Plastic settlement cracks.

In addition, minor repairs may call for routine-type bridge maintenance, such as:

(i) Regular cleaning of bridges, including hand sweeping, cleaning of pipe drains, scuppers and bridge seats, and removing debris from expansion joints.
(ii) Repair or replacement of expansion joints to restore smooth safe riding surfaces.
(iii) Replacement of expansion joint seal material of bridge expansion joints to insure water-tight seals.
(iv) Repair or replace bridge handrails
(v) Sealing of bridge decks to prevent damage from chemical attack.
(vi) Applying a coat of oil and mineral mixture as

corrosion protection membrane to concrete bridge decks and curbs.

Data will be provided during Activity B on a separate form outlining the maintenance and rehabilitation needs of the structure. A work-order should be prepared for repair actions by a contractor. The said work-order will include sufficient sketches, specifications and cost estimates to define the scope of work in order to execute it.

• *Activity C*

Activity C includes the evaluation of structural components identified in Activity A that exhibit major defects and deterioration with possible safety-related implications, the preparation of a report and recommendations addressing the situation. Activity C will be initiated as a result of the findings of the detailed field inspection of Activity A. Deteriorated or damaged bridge components that may pose an immediate safety hazard will, of course, be addressed in an appropriate time frame.

Activity C will be accomplished through the following specific steps:

(i) Obtain sufficient information during the field and inventory survey to evaluate the need for major rehabilitation of the deteriorated bridge or culvert elements. The data collected from field inspection such as problem type, location, deterioration rate, safety relation, dimensions and measurement, sketches, and photos as well as inventory data will provide sufficient information to conduct a preliminary investigation and evaluation. It will also relate the problem against the coding system as it is related to the safety of the travelling public, integrity of the structure and the anticipated amount of repair required to restore the elements to good condition. Thus, the analysis of this investigation with accurate detailing of the problem will be reported in appropriate form to initiate the Activity C operation. Typical defects and deteriorations which may initiate Activity C are given below:
   (a) Physical distress or loss of patches of concrete because of corrosion of reinforcement, scaling, spalling and structural cracks.
   (b) Repair or replacement of bridge expansion joints.
   (c) Foundation settlement.
   (d) Cracks and crushing of concrete members due to foundation movement.
   (e) Pier cap spalls.
   (f) Fractured abutment back wall.
   (g) Horizontal and vertical cracks at the end blocks of prestressed concrete girder.
   (h) Damage to strands of prestressed concrete girders.
   (i) Fatigue cracking in steel members.
   (j) Corrosion in steel structures.
   (k) Brittle fracture in steel members.

(ii) Submit request to the Ministry to initiate Activity C.

(iii) Upon notice to proceed, obtain as-built plans, design computations, standard design criteria and any other pertinent data relating to the design and construction of the structures from the authorities, as available.

(iv) Conduct an in-depth inspection of the deteriorated structural elements. The said inspection shall include sufficient measurements to analyze the condition of the elements.

In case a concrete deck is found to have one or more of the following during field condition survey:
   (a) Visible concrete spalls which have occurred in the deck riding surface and/or evidence of unsound concrete in the exposed surface of the deck slab which may indicate structural distress,
   (b) Extensive deterioration of asphalt overlay logically due to underlying concrete deterioration,
   (c) Evidence of delaminations (horizontal fracture planes) in the concrete deck and structural cracking and /or deformation,
   (d) Evidence of reinforcing steel corrosion, and
   (e) Evidence of inadequate concrete cover over the reinforcing steel, etc., and the Preliminary investigation raises questions about the structural adequacy of the concrete deck to carry current traffic loads, a request to the authorities to initiate Activity C should be submitted. Upon receiving notice to proceed, a detailed field inspection will be made to further define the inadequacies. In this case some of the following recommended test components may have to be conducted as an evaluation system:
      — Delamination detection with appropriate equipment to determine extent of internal fractures of the concrete.
      — Determination of the extent of reinforcing steel corrosion.
      — Determination of the compressive strength of the concrete.
      — Chemical analysis to determine extent of chloride contamination at the levels of principal reinforcements.

      In other cases such as steel bridges whose as-built plans are not available, field measurements shall be taken to locate and size all structural components for the purpose of completing the inventory data as well as carrying out the necessary structural computation.

(v) Prepare a report, outlining the analysis and findings of the in-depth inspection which is concerned with the evaluation of the load carrying capacity of the structure. The report will indicate the structural investigative computation* and its basis, degree of accuracy, methods of analysis and factor of safety. If the following cases arise from the structural analysis computations, then conducting field load tests† may be needed:

(a) The findings indicate a low degree of certainty in the load carrying capacity calculations due to existence of different types of deteriorations in many structural components.

(b) The evaluated factor of safety does not meet the code specifications.

(c) The collected measured data and results of the conducted tests as well as inventory information are not enough to calculate the load carrying capacity of the structure with different degree of accuracy.

However, field load tests will not be made unless it is absolutely necessary and the only solution to determine the load carrying capacity of the structure.

(vi) If required, and upon approval of the Ministry, perform necessary load tests and undertake analysis of the structure of determine load capacity (structural investigative computations*).

(vii) Prepare a report outlining all the results of the structural investigation. The report will include the recommendations to restore, if possible, the structure to its original loading capabilities. If rehabilitation is required the report will include work-orders consisting of plans, methods and procedures of repair, required personnel and manhours, types and number of equipments, and required quantity and types of materials. This information will be provided in sufficient detail to enable a contractor to undertake the work. If the structure cannot be rehabilitated, a recommendation will be made to demolish the structure, detailing the method to be used to accomplish the work in a safe manner, retaining elements that can be reused in the new construction.

A detailed list of all possible and anticipated defects and deterioration for all structural and non-structural elements for each group of bridges will be defined and classified. These defects and deterioration will be as identified in the guidelines earlier but should be detailed out in the inspection and procedures manual which should be developed specifically for the purpose. Some of the types of defects and deterioration in concrete bridges may be re-summarised as follows:

• ***In Structural Elements***

— Corrosion of reinforcement
— Structural fracture of concrete
— Structural cracking
— Low concrete cover
— Distress in shear
— Moisture trapped in box beams
— Spalling of concrete cover
— Fractured back wall
— Deterioration of abutment
— Failed slope wall
— Pier cap spalls
— Corrosion of roller or fixed bearing
— Movement of bearing system
— Undercutting at footing, scouring at foundations
— Movement or settlement of piers and abutments

• ***In Non-structural Elements***

— Failure of expansion joints
— Leaking of expansion joints
— Clogged drainage — scrupper
— Determination of concrete in non-structural elements
— Parapet geometry
— Horizontal or vertical movement in the approach slab
— Cracking in non-structural elements
— Rough riding surface
— Pavement migration
— Failed drainage system
— Erosion of soil slope wall
— Slope failure

In general and in most cases, the deterioration and defects in non-structural elements are minor and trigger Activity B; defects in the structural elements can be major and can trigger Activity C. However, in some cases defects in non-structural elements may be a major problem, such as serious failure of expansion joints. Also, in some other cases defects and deterioration in structural elements could be considered as a minor problem, such as low concrete cover or minor cracking due to non-structural causes. Criteria will be formally established for determining the required and

* The structural *investigative computations* refer to all structural analysis needed to determine the governing bending moments, shears, torsions, reactions, etc. at critical sections of the existing structure, in order to establish the structural stability, material stresses, and other serviceability criteria (e.g. crack widths, deflections, vibrations, etc.) for ascertaining the acceptability of the structure. All investigative computations are to be done for both substructure and superstructure, taking into account the *current* design live loads as well as other forces (e.g. floods, wind, etc.). It should also be noted that these are only investigative computations and are not the same as preparing a fresh detailed design of a new structure.

† e.g. concrete cores, reinforcement tensile strength, various load tests (against bending, shear etc.,) of slab/longitudinals, etc.

necessary activity.

Two steps may have to be established for this criteria. The first step is to develop a 'coding system' to rate the condition of each element (structural and non-structural). The inspector should use the Procedures Manual and his own judgement in rating the condition of each element and its relationship to the entire structure. The second step is to establish 'consideration criteria' which are directly related to the degree and level of deterioration or defects. Among the consideration criteria the following are considered important:

(i) The safety of the travelling public.
(ii) The integrity of the structure.
(iii) The amount of repair necessary to restore the member to good condition.

A trained engineer will analyze and evaluate the rating elements which will be provided in Activity A in their relationship to the safety of the travelling public, the integrity of the structure, the other items or components of the bridge, and the amount of repair required to restore the member to good condition. Using this procedure, the engineer will be able to decide whether Activity B or C is required for the existing condition.

46.10 INSPECTION PROCEDURES

(a) Inspection Forms

Three different forms may be developed to list information obtained in the field. The first form will be filled out for every structure inspection and will entail the condition of each element of the structure. A numerical system will be used to indicate the amount of deterioration each element exhibits. For multi-span bridges, each span's elements will be rated individually. An overall rating for the entire structure will be computed based on the individual ratings of its components. This form will correspond to the requirements of Activity A.

The second form, which will provide information as part of Activity B, will entail the listing of those structural items which require repairs. The inspectors will take sufficient measurements to get an estimate of the quantities so that a contractor will have an idea of the scope of work. This form, in addition to some drawn field sketches and cost estimates, will be detailed enough to serve as a work-order for a contractor.

For structural conditions encountered in the field which warrant major rehabilitation or replacement (Activity C), a form listing all the pertinent items should be made available for the inspectors. This complete form should provide necessary information for the initiation of an analysis of the structure.

All three forms should be simple in nature, requiring a minimum of effort to complete. Most of the input will be noted by either the checking of boxes or the information can be easily key punched, resulting in a computerized readout of data.

(b) Inspection and Procedures Manual

Prior to the initiation of the inspection operations a manual should be developed which will be used as a guide by all parties involved in the inspection operations. The said manual will contain the following features:

(i) *Procedures to be followed for the inspection of bridges, tunnels and culverts* The information will be specific, indicating what to look for when inspecting each individual element. In addition, instructions will be provided on how to rate the bridge and its elements.
(ii) *Maintenance of traffic stipulations* Details will be provided on how to safely divert traffic around an inspection site, including the placement of traffic control devices and the effective use of flagmen.
(iii) *Use of inspection form* Instructions on how to fill out the inspection forms will be provided.
(iv) *List of emergency contacts* A list will be prepared of individuals to be contacted within each district in case of emergency. Local officials, police, fire departments, medical facilities, etc. will be indicated so that if the need arises, the concerned individuals can be readily notified.
(v) *Organisation chart* A chart showing the chain of command for the entire inspection operation will be prepared so that everyone is aware of who is their immediate supervisor. This chart will also show the makeup of each inspection team.
(vi) *Form processing* This section will provide instructions on how the various inspection forms are to be processed. Information such as where the forms are to be transmitted how often, to whom, etc. will be clearly outlined.

46.11 MEANS OF ACCESS AND THE REQUIRED EQUIPMENT

Means of Access for Inspection:

1. The simplest mode of inspection, visual inspection, is not free from problems. The most vulnerable areas in concrete bridges (whether reinforced or prestressed) are obviously the deck and beam soffits. Reinforcements in main load carrying members is concentrated near the soffit surfaces mostly, and signs of cracking, corrosion, spalling and exposure are to be found there. But generally, soffit is also the most inaccessible. Without proper and safe access, no inspection is possible. The problem of accessibility gets further aggravated in long span, high rise bridges, and those crossing deep perennial rivers or across tidal estuaries. The traditional access through platforms erected on barges

or scaffolding, even from dry river beds, are time consuming, costly, and can be ineffective. These are not viable means of inspection.

2. The accessibility problem has long been successfully solved through the use of *snooper crawlers*, which are movable vehicle-mounted crane-arms, tailored to the needs of horizontal and vertical accessibility all along the span, substructure and bearings. These mobile inspection units have been in wide use in many countries. Besides inspection units have been in wide use in many countries. Besides inspection, this also provides a working platform for minor repairs which would otherwise take a long time. The major advantage of this kind of equipment is its versatility. Though the equipment is costly, yet its cost-effectiveness needs little explaining. If inspection is considered inescapable, such a mobile inspection unit is the only means available, and once procured, it can be used for a large number of bridges very effectively.
3. If the professionals mean business and are prepared to discharge the responsibility of maintenance of bridges placed upon them, they must insist on the purchase of proper tools of inspection. Even when damage has taken place, the lack of means of access stand in the way of proper assessment and the timely corrective repairs. Should we rebuild at an enormous cost, rather than spend a comparatively nominal sum in providing proper means of inspection and timely remedies?
4. For very long span and/or wide bridges with large navigational clearances, like the cable-stayed suspension system or steel girder type, permanently installed travelling gantries and lifts are a must if inspection is to be carried out meaningfully, and timely maintenance assured.
5. Access to the piers and pier-frames of large height and bearings must be provided from the deck, and must be provided for at the design stage itself, so that even in routine inspections, the inspecting engineer has easy and safe access, which is essential if inspection is not to be avoided. However, no single equipment may be able to serve all the needs of maintenance inspection. It is essential that during the design stage itself the question of accessibility should receive adequate attention, and the means of access built-in as a part of the structure, to enable easy inspection and maintenance of maximum parts of the bridge.

- ***'Equipment' for Field-assessments during 'Inspection':***

(i) Rebound hammer for in-situ assessment of the strength of concrete (Schmidt hammer).
(ii) Magnetic detector for "locating reinforcement bars",
(iii) Mechanical extensometer and "transparent templates" for reading of crack-widths from surface contact.
(iv) Hydraulic devices, pressure transducers or load cells for measurement of forces or pressures, etc.
(v) — "Platforms" erected from "ground" or on
— "Barges" ... not very speedy or convenient.
— *"Ladders"* ... limited utility and range
— *"Cherry-Pickers" moving on bed (vehicle-mounted)*: ... limited to working from ground level but not on water and its reach is only about 10 m, ... not useful for taller heights.
— *"Snooper-Crawler", moving on deck itself*: ... costly equipment, but most versatile and elegantly suited for tall and long spans (together with an EXPLORER which can go down vertically along a pier/abutment from top down):
  * It is basically a *Vehicle-mounted multiple elbowed crane-arm* which can crawl and permit snooping of otherwise inaccessible bridge elements, e.g. underside of decks, beams, cross beams, expansion joints, bearings, pier and abutment caps, etc.
  * It also has a working platform, from which repairs can be attended to.
  * Its EXPLORER permits the inspectors to go down (and up) the tall substructure vertically and carry out the inspections elegantly.

- ***Equipment and Personnel***

(a) Based on type and numbers of structures to be surveyed, and within what required time frame, it is contemplated that three different categories of structural inspection crews will need to be established. Pertinent data regarding each is as follows:

Survey-Crew 'Type'	*Equipment Proposed*	*Items Inspected*	*No: Involved*
(A)	Ladders, cherry-pickers	'(1A) Bridges' less than 10.7* metres high	x_1 no. of bridges
		'(2A) Box culverts' more than 1.5 metres high	y_1 no. of box culverts
		'(3A) Tunnels'	z no. of tunnels
(B)	Snoopers	'(1B) Bridges' more than 10.7* metres high	x_2 no. of bridges + those bridges in (1A) unreachable from below
(C)	Video cameras	'(1C) Box culverts' less than 1.5 metres high	y_2 no. of box culverts
		'(2C) Pipe culverts'	y_3 no. of pipe culverts

*10.7 m is the maximum height reach of a cherry-picker vehicle if inspection has to be done from below the bridge.

The estimate of inspection time for each of the items is as follows:

Item (1A): (x_1 bridges) $\times$ ('b' spans/bridge) $\times$ 1/5 day/span = n_1 days

Item (2A): (y_1 box culverts) $\times$ ('c' barrels/culvert) $\times$ ('d' metres/barrel) $\times$ (1/200 day per m per barrel) = n_2 days.

Item (3A): (z tunnels) $\times$ ('e' metres/tunnel) $\times$ (1/100 metres/day) = n_3 days

Total days of type (A) crew = $n_1 + n_2 + n_3$ = **'L' days**

Item (1B): (x_2 bridges $\times \left\{ \begin{array}{c} \text{'}f\text{' spans} \\ \text{per bridge} \end{array} \right\} \times$ (1/2 day per span) = **'M' days**

This is the total days of Type (B) crew

Item (1C): (y_2 culverts) $\times$ ('g' barrels/culvert) $\times$ (15 metres/barrel) $\times$ (1/250 day/m per barrel) = **'N_1' days**

Item (2C): (y_3 culverts) $\times$ ('h' metres/culvert) $\times$ 1/250 day/m ...... = **'N_2' days**

Total days of culvert survey crew = $N_1 + N_2$ = **'N' days**

- The amount of inspection time available as per project—Precedence chart = P months/crew $\times$ 4.33 wks/mo. $\times$ 6 days/week = Q 'crew-days
- *Number of crews required for each crew-type*

$$\text{Crew Type (A)} = \frac{L \text{ days}}{Q \text{ crew-days}} = \text{'}H\text{' (nearest higher whole number)}$$

$$\text{Crew Type (B)} = \frac{\text{M days}}{\text{Q}} = \text{'J'}$$

$$\text{Crew Type (C)} = \frac{N \text{ days}}{Q} = \text{'K'}$$

$$\text{Total} = (\text{H} + \text{J} + \text{K} =) \text{ 'V' no. of crews}$$

The typical inspection crew will be composed of four full-time members and a part-time administrator with skill levels as follows:

Grade C: Trained Structural Engineer, field crew leader in charge of Activities A and B and preliminary assessment for disposition of a structure into Activity C.

Grade D: Trained Junior Civil Engineer responsible for inventory and condition measurements and filling-in of forms.

Grade E: Assistant, equipment operator if applicable, or traffic controller.

Administrator: Part-time worker, responsible for logistics support and coordination with local authorities.

Inspection crews will be supervised by a Senior Structural Engineer (Grade B) who will be responsible for three field teams at a time, preparation of reports, and will have the governing opinion for disposition of a structure into Activity C.

(b) *A partial list of required equipment for the 'V' no. of crews is given below:*

				Not required
• For bridges...	(1B)... 'Bronto' 4-lane Skylift (Snooper) (See Note 1 ahead)	—		decide
• For bridges...	(1A)... Cherry-picker 4-WD	—		decide
• For bridges, culverts, tunnels (traffic control)	Two cabin pickup 4-WD with flashing arrow panel	—		decide
• For culverts	(1C) & (2C) Culvert video Inspection	—		's' (See Note 2 ahead)
• Non-destructive testing pundit		—		—
• Destructive and redesign check	1. Concrete core drilling 2. Full-scale load testing 3. Bearing capacity check	—		Lease as needed
• Bridge deck testing	Ground Penetrating Radar (GPR) (See Note 3 ahead)	—	—	
• Traffic warning signs	Survey crew	—	—	
	Right/left lane closed-1/2 mile	—	—	
	Pavement width transition	—	—	
	End of work zone	—	—	decide
	Traffic cones	—	—	
• Miscellaneous	35 mm camera/flash	—	—	
	Portable computers	—	—	
	30′ ladder	—	—	
	15′ ladder	—	—	
	50′ pocket tape	—	—	
	6′ folding ladder		—	
	100′ tape		—	
	Chipping hammer		—	
	Calipers		—	
	Scraper		—	
	Small magnifying glass		—	
	Flash lights		—	
	Hard hats		—	
	Marking tools		—	
	Safety belts		—	
	Crack-template gauges		—	
	Drag chain		—	decide
	Swiss hammer		—	
	Pachometer*		—	
	Red flags		—	
	Rubber boots		—	
	Rain suits		—	
	Wire brushes		—	
	First aid kits		—	
	Steel toe shoes		—	
	Goggles		—	
	Gloves		—	
	Walkie-Talkie		—	

*For detecting reinforcement (and rust).

Notes

1. The ('Bronto' Skylift) Snooper provides access to the underside of the superstructure from the existing roadway surface for those structures which are excessively high or which due to the terrain cannot be viewed from the ground below via ladders or cherry-pickers. The only alternative to the use of this equipment is to construct a scaffolding system hanging from the underside of the bridge. This system, which would have to be constructed and dismantled in each span would be costly and very time consuming. In addition, the bridge condition may not permit hanging scaffolding system from the structures point of view.
2. The videos provide a method of inspecting those box culverts which could not be inspected by reasonably easily walking through them. Assume there exist approximately T_1 culverts which are 1.5 metres of less in height (which, by virtue of their span of 6 metres—distance between outside walls—will need to be inspected as part of the contract) and T_2 no. of pipe culverts. The amount of effort required to inspect these structures has to be determined. Assume that the T_1 box culverts and T_2 pipe culverts add up to approximately q metres length. In an available inspection time of Q days/crew, at an inspection rate of 250 metres/day, the total crews required will be $(q/Q = 250) = s$, and thus the number of video cameras necessary is likewise s.
3. For bridges having a concrete deck with an asphalt wearing surface, the Ground Penetrating Radar (GPR) is the best device and is recommended for purchase to detect deterioration, defects and any delamination problem in the concrete deck under the asphaltic cover. This device will help in determining the deck condition without removing the asphalt cover and in estimating the amount and type of rehabilitation required especially in Activity C. It will provide the necessary data required to help the decision maker to define the amount and type of action to restore the concrete deck to good condition. For instance the Virginia Department of Highways and Transportation in the U.S.A. has adopted the GPR technique for bridge inspection and has prepared a manual to assist engineers in the use of the radar technique.

46.12 SCHEDULE OF UNIT RATES FOR INVESTIGATIVE STRUCTURAL COMPUTATIONS FOR BRIDGES AND CULVERTS (... to be sought from the consultant if job let out on contract)

Type of Structure	Per structure Statically Determinate	 Indeterminate
I. Bridge Superstructure[1]		
(a) Slab type Solid, voided, reinf. conc./prest. conc.	...	...
(b) Stringer/multi-beam girder*	...	...
(c) Tee beam*	...	...
(d) Box beam/girder–multiple*	...	...
(e) Box beam/girder–single spread*	...	...
(f) Girder and floor beam system Reinf. conc. slab supported on transversely placed floor beams which rest on main longitudinal steel girders.	...	...
(g) Frame		
(i) Single bay	NA	...
(ii) Multiple bays	NA	...
(h) Orthotropic deck	...	...
(i) Truss-deck type	...	...
(j) Truss-through type	...	...
(k) Arch-deck type	...	...
(l) Arch-through type	...	...
(m) Suspension	...	...
(n) Stayed girder	....	...
(o) Movable—lift type	...	...
(p) Movable—bascule type	...	...
(q) Movable—swing type	...	...
II. Culverts[1]		
(a) Single span single cell, reinf. conc. box section	...	...
(b) Multi-span multiple cell, reinf. conc. box section		

NOTE: Consultants should note that in a group of structures whose superstructures (at least) are essentially identical, the fee for the investigative structural computation work will be paid only for one such structure out of the group owing to the advantage of repeat usage of the said computational work.

46.13 SCHEDULE OF UNIT RATES FOR 'STATIC LOAD' AND 'MATERIAL' TESTS (... to be sought from the consultant if job let out on contract)

I. Conducting equivalent static load test as per relevant standards in order to ascertain the allowable service load capacity.

   A. *On deck-slab (in beam- and slab-type or box-type superstructure)*

      (i) against punching shear at one critical section in a span ...

[1]May be skewed.

**Section:* beam and slab type (beams may be individual solid webs or individual boxes or a single/multi cell box); *Construction:* wholly cast *in situ* or partly precast and partly cast *in situ: Material:* reinf. conc. or prest. and reinf. conc. or steel beams with reinf. conc. slab (composite/non-composite).

(ii) each repeat of A (i) above in the same span or in any other span of the same structure ...
(iii) against flexure at one critical section in a span ...
(iv) each repeat of A (iii) above in same span or in any other span of the same structure ...

B. On solid-slab superstructure

(i) against flexure at a critical section in a span ...
(ii) each repeat of B (i) above in the same span or in any other span of the same bridge ...

C. On a longitudinal 'beam-member' of the superstructure

(i) against flexure at one critical section in a span ...
(ii) each repeat of C(i) above in the same span or in any other span of the same bridge ...
(iii) against transverse shear at one critical section in a span ...
(iv) each repeat of C (iii) above in the some span or in any other span of the same bridge ...

II. Conducting as per the relevant A.S.T.M. the compressive strength tests on drilled-out concrete cores (including the cost of drilling, etc., all inclusive) and tensile strength tests on test lengths of reinforcement bars/structural steel elements taken out as representative samples from the structures in question (including cost of obtaining the sample, making good the structure. etc., all inclusive):

(i) per acceptable concrete core test ...
(ii) per acceptable reinforcement sample test ...
(iii) per acceptable structural steel sample test ...

III. Conducting non-destructive testing of concrete using ultrasonic technique: per complete set of tests to ascertain the concrete quality and strength in the entire superstructure per span of approximately overall plan dimensions 30 m × 17 m. (This rate shall be applied pro rata for different plan dimensions.) ...

46.14 STRUCTURE-MAINTENANCE 'REPORTS' AND 'RECORDS'

Function

One of the important functions of bridge maintenance is to prepare a complete, accurate and current record of each bridge on the highway system. Much of the usefulness of the information obtained from field investigations depends upon its availability in a permanent and concise record. The records must be maintained in an orderly system and be readily available to any interested authorized person. Records should provide a full history of the structure including all recommendations for strengthening and repair along with the actions which have been taken on these recommendations. The file will provide data on the capacity of the structure, including the calculations substantiating reduced-load-limits. Complete record information in a good usable form is vital to the function of providing a safe, smooth riding highway.

Reports

- The basic records shall contain the general description, history, dimensions, and condition of the structure, as well as any recommendations which may have been necessary at and since the time of the first investigation. In making a report, keep in mind that money may be allocated or repairs designed, based on this information. Furthermore, it is a legal record which may form an important element in some future litigation. *The language used in reports should be clear and concise and, in the interest of uniformity, the same phraseology should be used as far as possible to avoid ambiguity of meaning.* The information contained in reports is obtained from field investigations, supplemented by reference to "as-built" or "field checked" plans. The source of all information contained in a report should be clearly evident.
- A report should be made for each bridge-investigation, even though it may only be a cursory investigation. Such short inspections are made many times for the purpose of checking some specific item where a problem or change may be anticipated. Even though no changes are evident in this inspection and the condition seems relatively unimportant, documenting this information would be valuable in the future.

  At least two photographs of each bridge investigated, one showing a roadway view and one a side elevation view, should be included *as part of the original bridge report.* Other photos necessary to show major defects or other important special features, should also be included. A photo showing utilities on the structure is desirable.
- Necessary stress calculations to determine the safe load limit based on field measurements or checked plans, should be available for quick and easy reference.
- A 'paint record' for each steel structure should be a part of the bridge report file. Subsequent paint inspections and painting work performed will be a part of the

record, thus keeping it current and providing information which will be valuable in programming future painting requirements.

Original Bridge Report (OBR)

Original bridge reports should show a minimum of the following information:

(i) *Bridge Number:* The official number assigned to structure.

(ii) *Date of Investigation:* Date on which the field investigation was made.

(iii) *Name:* The full name of the bridge. Other common names by which it is known may be placed in parentheses following the official name.

(iv) *Location:* Location of the bridge must be sufficiently described so that it can be readily spotted on a map or found in the field. It will normally be located by route number, district, and log-kilometre.

(v) *Description:* Briefly give all pertinent data concerning the type of structure. The type of superstructure will generally be given first followed by the type of piers and type of abutments along with their foundation. If the bridge is on piles, the type of piles should be stated. If data is available, indicate the type of soil upon which footings are founded, maximum bearing pressures, pile capacities, etc.

(vi) *Skew:* Normally, the skew angle will be taken from the plans and it is to be recorded to the nearest degree. If no plans are available, the angle is to be measured. If the skew angle is 0 degree, it should be so stated. The skew angle is the angle between the centre-line of a pier and a line normal to the roadway centre-line.

(vii) *Spans:* The number of spans and the span lengths are to be listed. These shall be listed in the same direction as the log-kilometre. Span lengths shall be recorded centre to centre (c/c) or clear open distance (clr) between piers, bents, or abutments. Measurements shall be along centre-line of the roadway.

(viii) *Total Length:* This shall be the overall length and shall be the length of the roadway which is supported on the bridge structure. This will normally be the length from paving notch to paving notch, or between front faces of Abutment-backwalls (dirt walls) measured along centre-line.

(ix) *Roadway Width:* This shall be the minimum clear width between kerbs. On divided roadways such as found on freeways under overcrossings, the roadway width will be taken as the travelled way between shoulders. Also, the shoulders and median width will be given.

(x) *Surfacing:* Type of wearing surface and its thickness.

(xi) *Sidewalks:* If only one is present, the side shall be noted thus: '1 at the rate of 1.5 m (east)'. Sidewalks on both sides are noted thus: '2 at the rate of 1.5 m'. If there are no sidewalks, note 'none'.

(xii) *Railing:* The type and material of the railing is to be listed.

(xiii) *Alignment:* Note whether the bridge is tangent or on a curve. If the bridge is on a curve, state the radius of the curve if plans are available for this information. On the older roads and bridges, a comparison of the alignment with the general alignment of the road should be made.

(xiv) *Traffic Lanes:* State the number of 'actual' traffic-lanes and number of 'design' traffic-lanes.

(xv) *Design Live Loading:* The live loading for which the bridge was designed will be stated if it is known. A structure widened or otherwise altered so that different portions have different live load designs, is to have each live loading specified.

(xvi) *Waterway:* May be classed as: 'not a factor', 'excessive', 'sufficient', 'barely sufficient', or 'insufficient'. The velocity of the stream should be classed with reference to its scouring probabilities, such as: 'normally high velocity', 'normally medium velocity'. A statement also should be made describing the material making up the stream bed. In cases where the cross-section is uniform, a statement may be made of the average clear height, and the profile usually submitted may be dispensed with.

(xvii) *Other Features Crossed:* List facilities over which the structure crosses in addition to the main obstacle.

(xviii) *Clearances:* A clearance diagram should be made for each structure which restricts the vertical clearance under the deck, such as overcrossings, underpasses, through-truss bridges, channels, etc. The minimum number of vertical measurements shown on the diagram will be at each edge of the travelled way and the minimum vertical clearance within the travelled way.

(xix) *Date Built:* Date of the completion. If the bridge has been widened since the time of the original construction, the widening date should also be shown thus: ''widened-(date)'. A similar notation should be made for any major reconstruction.

(xx) *Plans:* State what plans are available, where they are filed, and if they are ''as-built''.

(xxi) *Plans and Dimensions:* When plans are not on file, sufficient data shall be included under this heading to permit an adequate stress analysis of the entire structure, when needed.

(xxii) *Condition of Structure:* All signs of distress, failure, or defects worthy of mention, as well as descriptions of condition and appraisal, shall be noted with sufficient accuracy so that another man at a future date can easily make a comparison of condition or rate of disintegration/distress.

Photographs and sketches should be used freely as needed to illustrate and clarify conditions of structural elements. Good diagrams are very helpful at future investigations in determining progression of cracks, spalls, and various other defects, and to help determine changes and their magnitude.

(xxiii) *Restrictions:* Note any load, speed or traffic restrictions in force on the bridge and if known, record date of establishment and identification of agency who put the restriction or restrictions in force.

(xxiv) *Miscellaneous:* Include information on high water marks, unusual loadings or conditions, and such general statements as cannot be readily incorporated into the other headings.

(xxv) *Stress Analysis:* Some structures will require a stress analysis to determine their operating and inventory ratings or, where necessary, their load limits for posting. A general statement of the results of the stress analysis with a note on which members were found to be weak, what stresses were used, and any other modifying factors which were assumed in the analysis, should be given.

(xxvi) *Recommendations:* Under this heading, all directions for repair and maintenance, posting, rescinding of posting, or other instructions regarding the bridge, are included.

(xxvii) *Signature:* Reports are to be signed and dated by the person conducting the investigation.

(xxviii) *Channel Profile:* A sheet showing the channel profile at the upstream side of a bridge over a waterway should be a part of the original bridge report. The sketch should show the foundation of the structure and where available, description of material upon which footings are founded, elevation of the pile tips and/or of the footings of piers and abutments. This information is valuable for reference in anticipating possible scour problems through yearly observations and especially useful to detect serious conditions during period of heavy flow. Vertical measurements should be made or referenced to a part of the structure such as the top of kerb or top of railing which is readily accessible during high water.

Soundings, in addition to the single line channel profile are necessary at some river piers to provide adequate information on scour conditions and how the piers may be affected. Such requirements will vary with stream velocity and general channel stability. The necessity of additional soundings must be determined. These soundings will normally be limited to an area within a radius of 30 m from a pier.

(xxix) *Encroachment:* An encroachment sheet should be submitted with the original sheet of the original bridge report when there is one or more utility on the structure. A utility in the immediate area, though not fastened to the bridge, should also be included, such as a sewer line, etc.

(xxx) *Environmental Conditions:* Any unusual environment which may have an effect on the structure, such as salt spray, industrial gases, etc., should be noted in the report.

(xxxi) *Average Daily Traffic:* State the ADT along with the date or record. This information should be updated at intervals of approximately 5 years.

Revised Original Bridge Report (ROBR)

When a bridge is significantly altered by widening, lengthening or by some other manner which extensively modifies the structure, a new original bridge report should be written. The new report should be filled out completely using the new dimensions and description as necessary. Many of the items will be unchanged and remain the same as in the old original report.

The revised original report should clearly indicate that it supersedes the existing report. However, the old original report is still an important record for the structures and should be retained in the file.

Supplementary Bridge Report (SBR)

Reports on a structure are to be made for investigations made subsequent to the date of the original bridge report. Following are items that should be included in the *supplementary report:*

(i) *Bridge Number:*
(ii) *Date:* Date upon which the supplementary field investigation was made.
(iii) *Bridge Name and Location:* Same as for the original report.
(iv) *Work Done:* All work that has been done to the bridge since the last inspection should be listed.
(v) *Revised Dimensions:* This information is to be given when some maintenance or improvement work has

altered the dimensions of the structure. New bed profiles for unstable channels should be mentioned and plotted on the profile record sheet.

(vi) *Condition of Structure:* Similar as for original report.

(vii) *Restrictions:* Note any changes in load, speed, or traffic restrictions in force on the bridge from that recorded in the original bridge report or in a previous supplementary bridge report. Record the date of establishment and identification of agency who put the restriction or restrictions in force, if known.

(viii) *Revised Stress Analysis:* This information is to be given when maintenance or improvement work or reduced strength of members has altered the condition of the structure, thus making another stress analysis necessary.

(ix) *Recommendations:* All instructions for work, postings, or other items requiring acting from others will be included here in the same manner as was done in the original report.

(x) *Signature:* The report must be signed and dated by the person responsible for the report.

Records — Check List

— ***Function***

— ***Reports:***

1. *General*
2. *Original Bridge Report:*
   - o Bridge Number
   - o Date of Investigation
   - o Name
   - o Location
   - o Description
   - o Skew
   - o Spans
   - o Total Length
   - o Roadway Width
   - o Surfacing
   - o Sidewalks
   - o Railing
   - o Alignment
   - o Traffic Lanes
   - o Design Live Loading
   - o Waterway
   - o Other Features Crossed
   - o Clearances
   - o Date Built
   - o Plans
   - o Plans and Dimensions
   - o Condition of Structure
   - o Restrictions
   - o Miscellaneous
   - o Stress Analysis
   - o Recommendations
   - o *Signature*
   - o Channel Profile
   - o Encroachments
   - o Environmental Conditions
   - o Average Daily Traffic
3. *Revised Original Bridge Report*
4. *Supplementary Bridge Report:*
   - o Bridge Number
   - o Date
   - o Bridge Name and Location
   - o Work Done
   - o Revised Dimensions
   - o Conditions of Structure
   - o Restrictions
   - o Revised Stress Analysis
   - o Recommendations
   - o *Signature*

46.15 REHABILITATION WORK

This will include guidelines for repairing various types of distresses. (See References 2 and 3.)

REFERENCES

1. Raina, V.K., *"Consultancy and Construction Agreements for Bridges, including Field Investigations,"* Tata McGraw-Hill Publishers, New Delhi.
2. Raina, V.K., *"Concrete Bridge Practice—Construction, Maintenance, and Rehabilitation*, Tata McGraw-Hill Publishers, New Delhi.
3. Raina, V.K., *"Concrete for Construction—Facts and Practice*, Tata McGraw-Hill Publishers, New Delhi.
4. Raina, V.K., ***"Inspection, Repair, Strengthening, Testing and Load Capacity Evaluation of Concrete Bridges,"*** Tata McGraw-Hill, New Delhi.

CHAPTER 47

'Bridge-Distress' Reporting—A Workman-like Approach

> **NOTE: Also refer to Chapter 5 in the author's other book: *Inspection, Repair, Strengthening, Testing and Load Capacity Evaluation of Concrete Bridges* for a more detailed presentation of the subject.**

47.1 INTRODUCTION

- In order to receive 'accurate-enough' feedback from the field-engineers regarding any distress in various component-elements of bridges, it is a good idea to train them in filling a simplified 'Report-Form' and ensure that they understand its details (remembering that their levels of technical understanding may be rather humble, for various reasons).
- For this purpose a simplified Questionnaire-type Report-Form is presented in 47.2 below, which should be read in accompaniment with its ATTACHMENT 'A'. An 'example' of how to fill this Report-Form for a particular bridge (hypothetical) is then presented in ATTACHMENT 'B' in order to orchestrate the Bridge-Distress Reporting methodology.
- Sometimes extra vigilance is required in respect of 'Special Bridges'. In such a case it becomes necessary to identify the Special Bridges. A bridge may be simple rather than complicated in terms of its structural details but yet it may well qualify as a Special Bridge simply because it may lie in an urban (congested) area where detouring may be very difficult, or that it may lie in an important artery which cannot be afforded to be closed, or that it may be so high that detouring through the bed so much below it may not be possible. A bridge may also qualify as a Special Bridge owing to possibilities of its becoming unpassable because of chances of:
  - — failure by instability,
  - — failure due to fatigue,
  - — failure of its components,
  - — failure of its systems,
  - — partial failure due to severe exposure conditions
  - and — difficulty in maintenance owing to complicated structural arrangements or features.

For this purpose 'Definition of a Special Bridge' has been briefly indicated in ATTACHMENT 'C'. This may be of assistance while deciding as to which bridges are 'special'.

47.2 BRIDGE-DISTRESS REPORTING (IN A QUESTIONNAIRE FORMAT). (FOR APPORTIONING APPROPRIATE CONDITION-RATINGS ... SEE THE DISCUSSION ON THIS SUBJECT IN CHAPTER 46.)

<u>QUESTIONNAIRE:</u>

- NOTES *for the field-engineer reporting the distress:*

1. For definition of "Special Bridge", see enclosed Attachment 'C'.
2. From your records and information about each "special bridge" in your jurisdiction, please indicate what problems, if any, were encountered (or are likely to be encountered) during its service-life* that required (are likely to require) special efforts for Maintenance and Repair.

   The problems could be in Foundations, Abutments, Piers, Bearings, Expansion Joints, Superstructure, etc.

   Please also indicate any of the Construction-Stage problems, information about which may be important during Maintenance phase.
3. For each Special Bridge with problematic Maintenance, the necessary information should be given briefly in the format given below. Please also describe any relevant information that may not have been covered in this Questionnaire.
4. *Assistance in pin-pointing some of the items that may call for special vigilance and Maintenance:*

   Purely for your assistance, some possible Problems that can call for Special Maintenance-effort have already been pointed out and 'serial numbered' in the enclosed Attachment 'A'. Please go through these carefully.
5. Where you find that some details and Problems of your Bridges can be appropriately expressed by the descriptions given in the Attachment 'A', there the 'corresponding Serial Numbers' should be inserted in the relevant boxes in this Questionnaire. This will save time and retain uniformity of expression and adequacy of detail. This will help in the overall evaluation task ultimately. (Attachment 'B' shows an Example of how to do the distress Reporting.)

• REPORTING:

(a) Bridge's Name: ....
(b) At Stn. ....
(c) Location .....
(d) Type ....
(e) Longitudinal Spans and Arrangement (Sketch) · ·
....
....
(f) Cross-section of Superstructure (Sketch) ...
....
....
(g) Date Constructed..................................
(h) Construction Contractor's Name ....
(i) Construction Supervised by ....
(j) Design Consultant ....
(k) Maintenance Contractor's Name ....
(l) Why do you think it is a Special Bridge (if it is a Special Bridge): ....

1. FOUNDATIONS:

Type: [ ], ...

Problems: [ ], [ ], ...

*Condition Rating** [ ], [ ], [ ], ...

• (element by element)

• • *Hence decide if Bridge useable 'globally', subject to what Maintenance-Actions,* [or] *'Close Bridge till remedial measure taken',* [or] *'Replacement (and temporary auxiliary supports) necessary, meanwhile close the Bridge.*

Brief descriptions with explanatory sketches:
....
....

2. ABUTMENTS & PIERS:

Type: [ ], [ ], [ ], ...

Problems: [ ], [ ], ...

Condition Rating: [ ], [ ], [ ], ...

• (element by element)

• • *Hence decide if Bridge useable 'globally', subject to what Maintenance Actions,* [or] *'Close Bridge till remedial measures taken,'* [or] *the Replacement (and temporary auxilliary supports) necessary, but meanwhile close the Bridge.*

Brief descriptions with explanatory sketches:
....
....

* For explanation, see discussion in Chapter 46.

3. BEARINGS:

Type: [ ], ...

Problems: [ ], [ ], ...

Condition Rating: [ ], [ ], ...
(each)

Brief descriptions with explanatory sketches:
....
....

4. EXPANSION JOINTS:

Type: [ ], ...

Problems: [ ], [ ], ...

Condition Rating: [ ], [ ],
(each)

Brief descriptions with explanatory sketches:

5. SUPERSTRUCTURE:

Type: [ ], ...

Problems: [ ], [ ], ...

Condition Rating: [ ], [ ], [ ], ...

• (element by element)

• • *Hence decide if Superstructure useable 'globally' subject to what Maintenance-Actions,* [or] *'Close Bridge till remedial measures taken',* [or] *Superstructure must be replaced immediately (meanwhile close the bridge.)*

Brief descriptions with explanatory sketches:
....
....

6. DRAINAGE:

Problem: [ ]; ....

Condition Rating: [ ], [ ], ...

Brief descriptions with explanatory sketches:
....
....

7. CORROSION IN STRUCTURAL-STEEL

Type: [ ], ...

Problem: [ ], [ ], ...

Condition Rating: [ ], [ ], ...
(each element)

Brief descriptions with explanatory sketches:
....
....

ATTACHMENT 'A' to BRIDGE 'QUESTIONNAIRE'

1 - FOUNDATIONS (symbol F)

- *Types:*
  - # **F**.1 footings (open foundations)
  - # **F**.2 piles
  - # **F**.3 caissons
  - # **F**.4 other (explain)
- *Some Problems may arise from:*
  - # **F**.a - undermining due to scour,
  - # **F**.b - dislodging of Gabions/protection works,
  - # **F**.c - exposure of piles,
  - # **F**.d - concrete cracking (from numerous causes*)
  - # **F**.e - loosening of anchor-holds,
  - # **F**.f - settlement (differential, overall),
  - # **F**.g - movement/tilting,
  - # **F**.h - overload/under-design**
  - # **F**.i - other (explain)

---

* *these could be:*

- plastic shrinkage of concrete,
- plastic settlement of concrete,
- early thermal and shrinkage movements and lack of adequately 'spaced' and 'functioning' movement-joints,
- drying shrinkage of concrete (long term),
- rusting of steel under moist conditions, due to:
  - — oxidation of steel,
  - — carbonation of concrete,
  - — chloride-ion attack on steel (chloride salts from surrounding soil and water or from aggregates themselves if contaminated),
- sulphate attack on cement in concrete (sulphate salts may ingress from surrounding soil and water or may also be present as impurities in the aggregates),
- alkali-aggregate reaction (where aggregates contain reactive silicates and carbonates),
- Temperature:
  - — variation
  - — gradients
- weathering and disintegration of concrete,
- crazing in concrete,
- fire and/or explosion

etc.

(see REFERENCES 1, 2 and 3.)

** *resulting in distress due to:*

- excessive flexure, shear, and/or torsion
- excessive direct force (axial compression/tension)
- excessive bearing pressure
- restraint against designed deformation
- inadequate soil-bearing conditions, etc.

2 - ABUTMENTS & PIERS: (Symbols **A** *and* **P***)*

- *Types:*
  - # **A**.1 —open-type abutments
  - # **A**.2 —closed-type abutments
  - # **A**.3 —other (explain)
  - # **P**.1 —solid wall type pier
  - # **P**.2 —frame type pier
  - # **P**.3 —single shaft type pier, solid
  - # **P**.4 —single shaft type pier, hollow
  - # **P**.5 —other (explain)
- *Some Problems may arise from:*
  - # **AP**.a — tendency to move/slide/rotate, which may also result in change in plumb and line and consequent misalignment/displacement in the Bearings (with all the consequent effects),
  - # **AP**.b — consequences of:
    - —foundation scour
    - —foundation settlements
  - # **AP**.c — impact hits from passing vehicles (and from barges and floating debris in case of bridges in water bodies),
  - # **AP**.d — height more than about 10 m, so that unusual and special equipment may be needed for inspecting and carrying out the Maintenance Work in inaccessible zones,
  - # **AP**.e — Concrete cracking (from numerous causes*)
  - # **AP**.f — overload/under-design
  - # **AP**.g — other (explain)

3 - BEARINGS (Symbol **B***)*

- *Types:*
  - # **B**.1 — 'Elastomeric' type:
    - generally used for relatively short-span and medium-span bridges.
  - # **B**.2 — 'Pot' type:
    - generally used for medium-span bridges.
  - # **B**.3 — 'Spherical' and 'Cylindrical' (teflon coated) type:
    - generally used where deck-rotation and/or loads are large.
  - # **B**.4 — other (explain)
- *Some Problems may arise from:*
  - # **B**.a — In the **Elastomeric Bearings** (which may be provided with a bonded PTFE layer and a stainless steel sliding plate when large movements are to be catered for), problems may arise from:
    (i) compression bulging of sides,

(ii) unequal vertical deformation among a row of bearings in one line, owing to failure of one or more bearings due to excessive load,

(iii) unequal vertical deformation owing to improper seating surfaces between the bearing and the deck-soffit and/or the pedestal underneath,

(iv) 'surface' cracking,

(v) 'radial' cracking in the bulges,

(vi) excessive shear-deformation (due either to under-design or manufacturing defect).

B.b — In the **Pot Bearings** (of 'fixed', 'free to slide in any direction' or 'guided to slide in a particular direction' types), problems may arise from:

(i) excessive rotation causing excessive deformation in the potted elastomeric disc and in the surrounding seal. (The temporary clamping bolts should either have been removed after installation or should have sheared subsequently.)

(ii) In the case of sliding types, if the PTFE bonded to the top of the middle-plate is outstretchingly visible, then the top plate is not long enough in the direction of principal-movement. Such a bearing may need to be replaced (since such naked PTFE, open to ingress of dirt, may soon cease to function as designed).

B.c — In the **Spherical and Cylindrical Steel Bearings** (former capable of rotation in any direction and the latter capable of rotation in only one direction), with mating convex and concave (coated) steel surfaces, problems may arise that are somewhat similar to those in the case of Pot Bearings described earlier.

B.d — In the case of **crude forms of bearings** (such as felt or tar-paper layers), problems, may arise in the deck above and in the concrete under the bearings from cracking due to 'constraint against free rotation and sliding'.

B.e — Other (explain)

4 - EXPANSION JOINTS (Symbol **EJ***)*

- *Types:*

EJ.1 — Buried-type (These are ancient type. In these a joint-filler is filled in the movement-accommodating-gap of the joint, and the wearing course is either carried across or butts with it from either side),

EJ.2 — Elastomeric-type (These are most commonly used in the modern-day bridges. They are waterproofed and modular in build-up, making transporting, handling and installing easy. Can take large movements of superstructure.)

EJ.3 — Finger-type (Generally these are steel plates cut out into intermeshing finger arrangement, bolted to the edges of the adjacent decks.)

EJ.4 — Other (explain)

- *Some Problems may arise from:*

EJ.a — In the **Buried-type** joints:

(i) swelling-up of wearing course,

(ii) weeping out of the filler,

(iii) consequent thudding by the passing wheels and riding-discomfort,

(iv) cracking at edges.

EJ.b — In the **Elastomeric-type** joints:

(i) Tearing of elastomer (and consequent exposure of their reinforcing steel plates where such plates exist). This may be due to manufacturing defect or due to breaking of concrete in the supporting deck-edges,

(ii) loosening of anchor bolts (and obvious consequences).

EJ.c — In the **Finger-Type** joints:

(i) loosening of anchor bolts,

(ii) fracture of fingers (owing to combined effect of high impact pounding from passing wheels and fatigue from highly repetitive nature of such loading, the effect becoming severer as the anchor bolts loosen.)

EJ.d — Other (explain)

5 - SUPERSTRUCTURE (Symbol S*)*

- *Types:*
  - *The* **span-arrangement** *may be:*

S.1 · Simply supported spans

S.2 · Balanced-Cantilever and segmentally built Free-Cantilever Spans
S.3 · Semicontinuous spans (i.e. simply supported precast p.s.c. girders with continuous r.c.deck slab over piers, making them continuous for subsequent loads)
S.4 · Continuous spans (either cast as such, or built by stage-construction, or built by segmental cantilever method)
S.5 · Frame (single or multi-span)
S.6 · Cable-Stayed spans
S.7 · Other (explain)

- *The* **construction** *may be in:*
  - reinforced concrete
  - reinforced and prestressed concrete
  - composite or non-composite structural steel and reinforced concrete
  - structural steel (plate girder or Truss)
  - Other (explain)
- *The* **superstructure cross-section** *may be:*
  - (i) · 'r.c. solid slab'-type (cast-in-situ, c.i.s.),
  - (ii) · 'r.c. voided slab'-type (c.i.s.),
  - (iii) · 'r.c. box section'-type (c.i.s.),
  - (iv) · 'r.c. beams-and-slab'-type (c.i.s.),
  - (v) · 'precast r.c. beams and cast-in-situ slab'-type,
  - (vi) · 'precast p.s.c. beams and cast-in-situ slab'-type,
  - (vii) · 'p.s.c. voided slab'-type (c.i.s.),
  - (viii) · 'p.s.c. box'-type (c.i.s. or precast segmental),
  - (ix) · 'precast box segments prestressed with precast concrete web, diaphragm and slab elements'-type,
  - (x) · 'steel beams and concrete slab' (composite or non-composite)-type,
  - (xi) · other (explain)
- *Some Problems may arise from:*

S.a · concrete cracking (from numerous causes*)
S.b · settlement of foundations (differential, overall)
S.c · movement/tilting of foundations
S.d · overload/under-design**
S.e · accidental hit from passing traffic
S.f · accidental fire
S.g · distress in Articulations (Halving Joints)
S.h · Shear lag effect (particularly during the construction stages) whereby the applied prestress is absorbed by only part of the assumed deck-section, leading to cracking in unprestressed regions.
S.i · inaccessibility for inspection (because ... piers taller than about 10 m. so that crane and ladder arrangement not enough; etc.)
S.j · other (explain)

*, ** : See footnotes on page 649.

6 - DRAINAGE (Symbol D)

D. Inefficient drainage of water from and around the bridge can be hazardous for the traffic and can also lead to unsightly staining of concrete surfaces. When such ponded water gets mixed with any wind-blown sand and surrounding soil containing sulphate and chloride salts, the resulting attacks on concrete and steel can lead to harmful cracking of concrete and rusting of steel and continuous disintegration of the effected reinforced (and prestressed) concrete components of the bridge. Footings, piles and pile-caps, buried and boxed portions of substructure, caps and seating beams, abutment-walls, boxed-decks, etc., can all fall prey to such distress and damage.

7 - CORROSION IN STRUCTURAL-STEEL (Symbol C)

C. Steel beams, boxes and trusses, used in superstructures for example, are susceptible to rusting and corrosion-distress due to oxidation and chloride-attack under moist conditions. Hence the anti-rust painting must always be ensured as a rust-protection means. Detection of paint-failure requires close hands-on examination. Where rust-inhibiting steel (CORTEN) is used without any protective-coating, the effectiveness of the rust-inhibiting iron-oxide coating should be checked for distress (e.g. signs of unevenness and flaking of the surface protecting layer). The most likely areas of rust-formation are: external faces of outer girders, all welds/rivets/bolts, site joints, and gusset connections. Old steel bearings also need a special examination to ensure whether rust has not incapacitated them from performing the intended rocking/rolling/sliding, as the case may be. (See References 1 and 2.)

REFERENCES

1. Raina, V.K., *"Concrete for Construction—Facts and Practice,"* Tata McGraw-Hill, New Delhi.
2. Raina, V.K., *"Concrete Bridge Practice—Construction, Maintenance and Rehabilitation,"* Tata McGraw-Hill, New Delhi.
3. Raina, V.K., *"Concrete Bridges—Inspection, Repair, Strengthening, Testing and Load Capacity Evaluation,"* Tata McGraw-Hill, New Delhi.

ATTACHMENT 'B' (EXAMPLE)

An 'EXAMPLE' of how to fill the BRIDGE 'QUESTIONNAIRE' if the information given in the ATTACHMENT 'B' has to be conveyed:

NOTE : **Condition Ratings have not been indicated here but in an actual case the appropriate RATINGS should be shown, based on the Ratings discussed in Chapter 46.**

PROCEED as follows:

— items (a) to (l) ... fill-in as relevant to the particular bridge in question.

— ***FOUNDATIONS:***

- If FOUNDATION is, say, of 'footing' type, then insert $\boxed{\text{F.1}}$ in the box in front of 'Type'.
- If the Problems are, say, 'undermining due to scour' and 'settlement', then insert $\boxed{\text{F.a}}$ and $\boxed{\text{F.f}}$ in the boxes in front of 'Problems'.
- Also, give brief descriptions with simple free-hand sketches as to 'which foundations', 'extents of scour and settlement', etc.

— ***ABUTMENTS & PIERS:***

- If ABUTMENTS are, say, of 'open' type and PIERS are 'solid wall' type, insert $\boxed{\text{A.1}}$ and $\boxed{\text{P.1}}$ in the boxes in front of 'Type'.
- If the Problem is, say, 'foundation scour' only in a particular Pier, then insert $\boxed{\text{P.b (scour)}}$ in one box in front of 'Problems'.
- Also, give a brief description with simple free-hand sketches as to 'which pier', 'problem caused', etc.

— ***BEARINGS:***

- If BEARINGS are, say, of 'Elastomeric' type, then insert $\boxed{\text{B.1}}$ in the box in front of 'Type'.
- If the problem is, say, 'compression bulging of sides', then insert $\boxed{\text{B.a. (i)}}$ in one box in front of 'Problems'.
- Also, give a brief description as to 'which Bearing', 'extent of bulging/damage', etc.

— ***EXPANSION JOINTS:***

If it is, say, an 'Elastomeric' type Joint and the elastomer has torn off, and 'breaking of the supporting concrete edge of deck' has been the cause, then insert $\boxed{\text{EJ.2}}$ & $\boxed{\text{EJ.b.(i) (due to breaking of the supporting concrete edge)}}$ in the boxes in front of 'Type' and 'Problems', respectively.

Also, give necessary descriptions with explanatory sketches.

— ***SUPERSTRUCTURE:***

- If it is, say, 'p.s.c. continuous spans' of 'box' section, built using 'precast segments', and the problem is, say, damage due to an 'accidental hit from an over-height speeding truck', then insert $\boxed{\text{S.4 (viii), precast segmental}}$ and $\boxed{\text{S.e}}$ in the boxes in front of 'Type' and 'Problems', respectively.
- Also give brief description as to 'location' and 'extent', and explanatory sketches.

NOTE:

No other problems in this bridge.

ATTACHMENT 'C' to BRIDGE QUESTIONNAIRE

Definition of a 'Special' Bridge Structure

1. General:

Highway structures may be considered Special when their failure, partial or total, causes disfunction of the highway system. The more serious the disruption, the more Special the Structure in terms of maintenance requirements. The gravity of a failure may be assessed in terms of potential loss of life and property, capital cost of rehabilitation work or the disruption to traffic with resultant loss in trade, communication and strategic planning. Special Bridge Structures can thus be categorised in different ways. A system using failure mechanism is used to describe Special Structures here.

2. Failure by Instability:

Structural failure by instability is sudden and total, and often involves loss of life and property. Such Special Structures are:

a) Bridges in escarpments whose foundations may be undermined by precipitation run off during the rainy season.
b) Wadi-Bridges whose foundations may be undermined by scour.
c) Bridges in unstable geological formation which may be damaged by landslides.

3. Fatigue Failure:

Like failure by instability, fatigue failures are sudden and often catastrophic. Some examples of fatigue failure are:

a) Failure of tie-pins in suspended spans of steel bridges.
b) Unbonded external prestressing strands.
c) Hangers of suspension bridges.
d) Stay anchorages of stay bridges.

4. Component Failure:

Component failure is generally not catastrophic but unless corrected, will lead to structural failure. The most common types of such failures are:

a) Bridge bearings such as elastomeric, pot, spherical and steel bearings of various types.
b) Bridge expansion joints, including nosings.

5. Systems Failure:

Failure of electro-mechanical systems may make structures inoperable. Examples of such systems are:

a) Electro-mechanical units of moveable bridges **e.g.** swing, bascule and lift bridges.
b) Ventilation of long tunnels.
c) Drainage and Pumping Systems of Underpasses.

6. Partial Failure From Severe Exposure Conditions:

Bridges in severe environment such as marine and coastal, are particularly susceptible to partial failure from material deterioration. To this category must be added Structures where the ground contains high concentration of salts such as in 'Sabkha' areas.

7. Special Structures by Scale, Design and Construction:

In addition to the Special Structures categorised above by failure types, there are Structures which are complex both by design and construction technique. There are also Structures which are Special by virtue of their scale. The maintenance requirements of such Structures must take into account the special features of design, construction and scale.

a— Such a bridge could even be a short single span type if, for instance, it is a frame or has a curved or skewed deck. On the contrary, a Special Bridge does not necessarily have to be long. For example, a multi-span bridge, comprising of a number of relatively short simply supported spans with superstructure comprising of say cast-in-situ reinforced concrete 'slab' or 'beam-and-slab' arrangement, resting through Neoprene Pads on a mundane substructure of not too tall piers on shallow open-foundations (not subjected to any significant scour), may qualify as a Special Bridge only if it is strategically located.

b— On the other hand, if a bridge has significantly tall piers (in excess of about 10 m.), involves deep foundations (that are also likely to be subjected to significant scour), and has a superstructure that is longitudinally continuous with or without halving-joints (possibly also curved and skewed in plan), involves prestressing in one or both directions, has special (modular or finger type) expansion joints to cater for large deck movements, etc., then, such a bridge is a Special Structure. It calls for a precise understanding of the function and performance of its foundations, the superstructure, and of the special expansion joints and bearings. It also calls for special equipment for carrying out inspection and attending to the maintenance work.

c— In addition to these special features, following are some examples of Bridges whose design as well as construction are Special in themselves, and they consequently call for Special effort both for understanding their structural mechanics as well as in their monitoring and maintenance:

—Frame, single or multi-span
—Orthotropic deck
—Continuous Truss—deck type
—through type
—Cable-Stayed bridge
—Suspension bridge
—Movable Superstructure—LIFT type
—SWING type
—BASCULE type

8. Examples of Some of the SPECIAL Features of Bridges (which on their own may or may not qualify the Bridge to be a Special Bridge):

— Abutments and Piers being taller than about 10m, (requiring Special Plant and Equipment for inspecting and repair work).
— Foundations subjected to significant Scour and possible undermining.
— Special Bearings (of Elastomeric type, Pot type, Spherical type etc.) subjected to high loads and large movements and rotations.
— **Superstructures** of the following types:
  - o Statically determinate superstructures (simply supported spans, or balanced cantilevers), involving spans in excess of about 25 to 30m. These may comprise of 'p/c psc beams and in-situ top slab' or 'box-sections', **etc.**
  - o Statically indeterminate superstructures, involving

individual spans in excess of about 10 to 15 m. (It could be a voided slab, or 'simple-span beams rendered semicontinuous by in-situ continuous r.c.slab' or 'box-section'.)

— The 'Halving-Joints' (i.e. the 'Articulations') in the Superstructures.

— The Expansion Joints, particularly those that cater for movements of an 'aggregate superstructure-length in excess of about 30 to 40m' from the Fixed Bearing point (or Zero-Movement-Point).

CHAPTER 48

Bridge Engineering — Some Topical Reflections

48.1 INTRODUCTION

Bridge design and construction have come a long way during the past few centuries and 'much water has flowed down' particularly in the past hundred years or so during which more bridges have been built than in all the previous centuries. In fact large number of today's existing bridges have been built only during the past fifty years or so and the rate of design and construction has been the highest essentially during the past twentyfive years.

This rather recent rapid rate of growth in the comprehension of the science of analysis and design and the art of building bridges has led to numerous developmental experiences. Secret of success being constancy of purpose, a workman like introspection leads to certain 'reflections', some of which perhaps are topical. The topics chosen here are:

- the 'bridge culture' (a brief historical note),
- rational approach to structural design,
- 'new' codes of practice (split load-factors),
- design-education,
- cure against cracking of concrete — not by calculations alone!
- life care of road bridges,
- concrete structure... deterioration and rehabilitation.
- appropriate technology for developing countries.
- blue-print for guiding technical development in the field of civil engineering public works in developing countries.
- inspection check-list (... field work).

While conservatism may have a place in some fields of activity, a robustly practical engineer has to be constantly on the run, merely to keep up. This would not be so bad if the engineering activity had only a short life span before it went obsolete (e.g. in electronics, space technology and weaponry). Unfortunately for the poor bridge engineer, whose bridges must last upwards of half a century at least, long range forecasting of the changes in social organization, which influence the quality and quantity of traffic, becomes a predicament. This calls for an amalgam of intuition, introspection, and reflection, by the initiated restless minds.

48.2 THE 'BRIDGE CULTURE' (A brief historical note)

1. The primitive bridge building must have started with a trial and error process. In some misty morning of prehistoric past, a human trying to cross a stream, probably saw a fallen tree across it. When he tried to clamber over, it might have broken and dropped him in the drink. So he thought real hard and felled a bigger tree and this took him across. The first primitive single span wooden beam bridge was thus designed and built that day. In reality the modern bridge hardly conceal their ancestry. Vine ropes secured on two towering trees on the opposite banks of a deep gorge, with a bamboo or brushwood deck or without one, were the earliest forerunners of the suspension bridges. The first arch bridge might have similarly been built about four thousand years back in the Euphrates valley.
2. ***The man who creates technology, the engineer with the aid of the scientist, is the society's most radical revolutionary. He is the fundamental agent of all social changes.***
3. To trace this glorious history of bridges from the beginning, we should perhaps divide it into two periods, as it is customary to talk about history in terms of periods. The first period would be before concrete — the stone to steel ages and second in which we are living, after concrete. Stone and timber were the traditional building materials since the start of civilisation. Stone arches used mostly in early bridges date back to the days of Babylon. Romans learned the art of building arches from the Ettruscans. These arches were semi-circular. Flat circular arches appeared in full glory during the renaissancé period. Although the first theory of arches came to be established as late as 1695 and was used in practical design only in 1729, the optimum profile of the stone arch had been found very early by artist builders intuitively and has changed very little since. The proven durability of material and the long experience in intuitive proportioning made masonry arch bridges the most popular form of construction even in the early days of railways until iron bridges took over. Arch did continue to be built in iron, steel and concrete

because of its unmatched aesthetical qualities, besides structural efficiency.

4. While stone and timber remained the common building materials for bridges, the mid-nineteenth century demanded stronger and bigger bridges over large rivers for railways. So around 1840 the transition period from timber to steel began. In this period cast iron (and later wrought iron) was tried out by bridge builders. The first recorded Western use of iron in bridges was a chain bridge built in 1734 by the German army across the Oder river in Prussia. However, cast iron was not found very suitable for building large bridges. A combination of cast iron for compression members and wrought iron for tension members was first used in truss bridges from 1840 onwards, specially for railway bridges.
5. In 1856, Bessemer patented a process for making large quantities of steel economically. In 1861 Siemens and Martin introduced the open hearth process. In the last part of the 19th century the new material steel caught the imagination of bridge builders. By 1890 the advantages of the steel were undisputed and the use of structural steel spread with speed to all countries. Wrought iron was completely expelled from all new bridges and by the turn of the century steel brought in spans unlimited, built for the heavy loads of railway traffic. The Firth of Forth cantilever bridge of 520m span or Brooklyn bridge, Roebling's suspension bridge of 490m span, were a few of the famous achievements of the 19th century to mark the beginning of modern era of bridge engineering. By the turn of the century, the growing use and availability of structural steel and greater skills in design analysis paved the way for larger span bridges. Multiple span girder bridges, arch bridges and cantilever bridges in steel reached up to maximum spans of 500m or more with comparative ease. Howrah bridge (1943), a steel cantilever bridge with a main span of 150 m, is a typical example of the large crop of outstanding bridges built in steel in the earlier years of this century.
6. A very substantial advance in the understanding and theoretical analysis of the load response of the structural system of suspension bridges was made by the twenties of this century and many elegant bridges were built, like Lindenthal's Manhattan bridge (1909) with 450m span, followed by Steinman's 340m span Florianopolis bridge and the Delaware river bridge of 530m span in 1926, to name a few at random. The big leap forward came in 1931 with the construction of George Washington bridge, which has been acclaimed by Stussi as a "great and most important step in the evolution of the art of bridge engineering". Le Corbusier was enchanted with this bridge and in his 1937 publication "When Cathedrals Were White" said: "The George Washington Bridge is the most beautiful bridge in the world... It is blessed, the seat of grace." The bridge broke the 1000m span barrier and its span length of 1060m was double that of Delaware bridge, the then longest span in the world. It also proved, by the successful construction of an eight-lane major roadway without stiffening girders, an important fact sensed by Navier and intuitively made use of by Roebling. The bridge was built by Amman, a Swiss engineer who migrated to America in 1904. Three decades later, this 'foremost bridge builder in the world' contributed his last great masterpiece to the land of his adoption. Amman's Verrazano Narrows Bridge in New York was opened in 1964, ten months before he died. Standing with a main span of 1300m, the longest realised until now in U.S.A., the bridge is the highest landmark in the history of long span suspensions bridge building, which has flourished since the thirties, leading to the construction of economical, gracefully slender and ambitious structures in large numbers in all parts of the world. The lessons in ensuring aerodynamic stability were learnt in a hard way by the tragedy of the 855m Tacoma Narrows Bridge which fluttered and perished in 1940 in a 64 km/h wind. The progress has gone unabated with innovations, like the stiffened deck of Severn bridge, to replace the deep stiffening trusses and the future may reveal yet newer directions of development.
7. In 1820, Navier in opting for the classical suspension system, condemned the cable stayed solution as unsuitable. It remained so until in 1938 Dischinger developed a suspension system which was actually a combination of cable stayed and classical suspension bridge types. The actual development of the modern cable stayed bridge started with the construction of Stromsund bridge in Sweden around 1953 by DEMAG. Then in an unprecedented spurt of development followed the famous Rhine family of cable stayed bridges, which at present number about a dozen with spans up to 300m and more.
8. Leonhardt hypothecated in his 1972 IABSE paper that for spans of 750m to 1500m the cable stayed system was technically and economically superior to the classical suspension bridge especially with regard to aerodynamic stability. The use of A-form pylons with corresponding inclination of planes of cables, fan shaped smaller diameter cables at closer spacing etc., are the new trends in the design to realise the large spans. High cable stresses to ensure favourable stiffness ratios and flat aerodynamically shaped beam sections with sharp wind noses are being used. The cable-stayed system with a steel or concrete deck will surely find

increasing use in many future bridges, from the modest foot-bridge to the mighty vehicular bridges, not only for the slim attractive looks which are no mean virtues, but also as the economically viable alternative to other conventional bridge types.

9. The demands of the bridge builder and his reliance on scientific knowledge led to the birth of structural mechanics. Timoshenko in his 'History of Strength of materials' points out: "The construction of the first railroads greatly affected the development of strength of materials by presenting a series of new problems especially in bridge engineering which had to be solved". So the midpoint of nineteenth century marked a turning point in the history of bridge building by a change to scientific method of design of bridges and consequently for all structural design. Stussi remarked in the IABSE Symposium on 'Concepts of Safety of Structures' (London, 1969) that the scientific period of design "was introduced by Louis Navier (1785-1836) who provided the transition to scientifically based construction with his principal work; "Resume des Lecons" (1826) and thereby fundamentally created the applied science of statics". Navier gave these lessons on 'the application of mechanics in the establishment of construction and of machines' in the Royal School of Bridges and Roads (L'Ecole Royale des Ponts et Chaussees), Paris, and some of his students were to emerge as eminent engineers in future years. After Navier, the development of engineering design was fostered by very many pioneers like Carl Culmann, Otto Mohr, Friedrich Engesser, to name only a few of the giants.

   This complete break from the past attitudes, and the zeal infused by the scientific thinking, was directly responsible for the scale of achievements of the engineer in the nineteenth century which appears to be massive indeed when compared to those of the past centuries all taken together. Even the great advances made by the Romans in bridge building had led to maximum span of a mere 35m. From the time of the Romans till the end of the eighteenth century advances made were irregular until the bridge engineer of the nineteenth century reached for the sky with his new scientific thinking. An environment of progress unlimited, where the engineer walks hand in hand with the researcher-scientist has been the keynote of the engineer's work since. Bridge building has thus become a highly intellectualised activity.

10. The use of concrete, an artificial conglomerate of gravel or broken stone, sand, and lime or cement, is of great antiquity. Concrete made with 'surkhi' has been used in India traditionally. And the glorious history of Roman concrete, used to build such famous structures as the Pantheon, is more well-known. But the art almost went into oblivion until its grand revival in the recent times marked by the entry of portland cement into the scene.

11. With the progress of portland cement concrete, it came to be used in bridges of substitute stone. Concrete was first used for a bridge of 13.5m span across a canal in France in 1840.

12. However, concrete never emerged as the 'many splendoured thing' until the art of reinforcing, fully developed by the turn of this century, made it so. This bold engineering concept of 'reinforcing the weak tensile zone of concrete by stronger iron inserts' was really established in the mid-nineteenth century itself. Of course, like many inventions in civil engineering, the story can always be traced to hoary past. The use of iron reinforced masonry may date back to first century B.C. by the Romans or even earlier in the Indian subcontinent. But we have to come back to more recent times which heralded its general structural applications. The Frenchman Lambot's famous rowboat 3.30m long 'made by plastering Roquefort cement on a skeleton network of iron and wire' was followed by his English patent in 1855. Coigent, another Frenchman, took out a patent at about the same time on the structural use of reinforced concrete. In 1855, Wilkinson secured in England a patent for a concrete arch floor, reinforced with tie bars, which had been discussed by Fairbirn in his book (1864). Moiner's first patent was taken three years later. Many others took about this time patents on reinforced concrete in one form or other, in various lands. Thus in the nineteenth century, reinforced concrete was still in the empirical period of patents and specialists.

13. The early twentieth century saw a dramatic change in attitudes to reinforced concrete. Improved concrete materials, mass production of good quality cement and better knowledge of proportioning, made attainable quality standards of concrete much higher than before. What threw the doors of progress wide open was the development of a design theory to interpret adequately the 'mechanics of the structural behaviour of reinforced concrete'. The search for a basis of rational concrete 'structure' design began in the last part of the nineteenth century itself. Thaddeus Hyatt is credited to be the first to establish the basis of analysis of stress in reinforced concrete by stating 'the principle of bond' and that 'the reinforcement must be able to resist sufficient tensile stresses to balance the compressive stresses in concrete'. Hyatt was an American lawyer by education but inventor by nature and was later to take patents on deformed bars. He published his 28-page book on 'the

use of portland cement concrete combined with iron' in 1877 which may have antedated the design principles to emerge by good two decades. Koennen, a government architect in Berlin, was commissioned to deduce the methods of computation of reinforced concrete sections and he published his design rules in 1886. Coignet's ideas on elastic design were printed two years later. Inelastic theories of design, which were to be rejected rather irrationally later, also appeared in 1898.

14. So in the early years of the 20th century, the stage was set for reinforced concrete to play the role destined for it. Professional societies flourished and research results were eagerly devoured for translation into applications. The familiar infrastructure of a national and generalised design and construction practice was born.
15. The days of the patents and specialists were ended. Specifications of reinforced concrete materials and design rules were published by professional societies in all advanced countries. The Prussian regulations of 1907 were reported to be most complete set of design rules at that time. In 1909, the joint code of ACI, ASCE, and other professional societies of America, interested in reinforced concrete, made its grand entry. The French Commission on reinforced concrete had formulated its design rules in 1906.
16. Reinforced concrete won the hearts of bridge builders so readily because of economy. So a 'comparison of cost with iron/steel or masonry' was invariably reported while talking about a reinforced concrete bridge in the early years of this century. Here is an example quoted from the 1904 proceedings of Institution of Civil Engineers (extract of a paper from La Genie Civil, Paris) about a bridge just constructed, spanning the river Aisne at Soissons made throughout in reinforced concrete' costing £ 7,700. "For a similar bridge in masonry, £ 1,875 would have to be added (to the cost figure), while if the bridge were in steel, an extra cost of £ 1,460 would be entailed." These were very telling figures to guide one's options. But direct saving in construction cost was not all that mattered. The outstanding virtue of reinforced concrete that made the difference was 'its durability and minimal need for maintenance care'. Doubts about corrosion of reinforcement embedded in the cracked tensile zone and exposed to the elements could have thrown cold water on the bridge engineers' enthusiasm for reinforced concrete and stunted its progress. Fortunately they were too pragmatic to be so inhibited.
17. Post-war years saw the fast emergence of reinforced concrete in its full glory. The massive rebuilding of bridges and buildings in war ravaged Europe brought new pressures on the pace of development of reinforced concrete. Higher strengths with higher quality of materials were the obvious avenues open. The history of a century of development of reinforced concrete materials and methods of making good and better concrete is itself a fascinating 'subject'. The fundamentals of making good concrete were established firmly by the twenties. Abrams, giving his well-known correlation of water-cement ratio with concrete strength, wrote in 1919; 'use the smallest quantity of mixing water that will produce a plastic or workable concrete'. With careful control of water and proportioning, attention was focused on 'workability, placing, and compaction'. Freyssinet proposed 'compaction by vibration in 1917. Alongside came the improvement in composition and fineness of grinding of cement, and the strength of concrete increased.
18. For the bridge builder, a companion development of reinforcing steel to higher strengths, specially in post-war years in Europe, made a big impact. Mild steel bars became the universal form of reinforcement since the 1900s. In Europe, they used it plain; Americans preferred it deformed. The designers had chained themselves universally to a low permissible stress of 125 MPa and the bogey of cracking and corrosion made higher strength reinforcement a taboo. The visions of economy, possible with high strength reinforcement, made Europeans conquer these irrational apprehensions, egged on by the pressure of post-war years.
19. The big break in the art of reinforcing the concrete came with the introduction of 'prestressing'. The idea of prestressing, conceived and put to practice in the 20th century in the single happening of most significance in the entire history of the art and technology of construction. Man must have practised prestressing for centuries. In some distant past, an unknown genius might have used iron rings in wooden barrels or iron bands on wooden cart's wheels, exploiting the concept of 'precompression'. The man who first gave the form and content to this concept, the originator of prestressed concrete, was none other than Eugene Freyssinet, a great builder of our times.
20. A lesser man would have been content with the fame and fortune he had in 1930. But Freyssinet was a man with a mission in his heart. His life carries an obvious message. His bitter struggles for seven long years for the cause of prestressed concrete made history. If history has to have its heroes, here was one for the history of bridges.
21. Freyssinet took patents in 1928. He was out to sell the history's most exciting building material, but with no buyers around. He had reportedly told his friend Le Corbusier: "I reached my goal. Now I am looking

around to see what I can use this discovery for. And in my opinion modern society needs housing parks and highways." Le Corbusier was so touched by the sentiments of his friend as to conclude: "into that one short sentence he has crammed a vast wealth of poetry, lyricism, of solidarity, of concern for mankind and the hearts of men".

22. The big boom in prestressed concrete came only after the war. Although France and Belgium led initially, the development and use of prestressed concrete flourished in all countries in Europe and soon spread to all parts of the world. Prestressed concrete came to be used most in bridge construction. Of about the 500 bridges built in Germany during 1949-53, seventy per cent were in prestressed concrete. Prestressed concrete was introduced in USA in 1949 by Magnel's Walnut Bridge and at about the same time in India by the Coleroon bridge, using Freyssinet system. Bridge builders revelled in prestressed concrete as it helped them to realise larger and yet larger spans in concrete. As more liked to use steel where it was possible to use concrete, prestressed concrete had to face demands of increasingly larger spans, with improved techniques of design, construction and materials.
23. In 1960 beam bridges had reached spans up to 160m and Morandi's bridge across the Lake of Maracaibo was under construction with spans of 235m by the help of stay cables. In 1970, the longest span of beams reached 230m in Japan, and for cable-stayed concrete bridges, designs had been made with about 300m spans.
24. The achievement in building long-span bridges in prestressed concrete are too close to out times to afford a historical perspective. The development of new landmarks in span, form, and construction technology, is growing at a dizzy pace. The panorama is so vast as to prohibit any attempt to survey all its features in this limited space.
25. Hamana bridge in Japan also claims attention as one of the best among a growing family of long-span prestressed concrete bridges erected by cantilever construction method. The Hamana bridge has a 240m main span with a central hinge. The originator of this type of construction, double cantilever box beam superstructures built without the use of falsework, which has revolutionised the building of prestressed concrete bridges, is Ulrish Finsterwalder—one of the greatest bridge builders of our times. A later modification of this method is the cantilever construction with precast segments introduced in the 1963 construction of Choisy-le-Roi bridge in Paris by Enterprises Compenon Bernard. Since then the precast segmental construction in cantilever has gained worldwide acceptance, mainly for the fast erection. In recent European projects, a fantastic rate of erection of urban viaducts of two or three spans of 35m to 45m in a single week has been reported. The span range of economical application of box beam bridges erected by cantilever construction is believed to lie normally between 50 to 200m. Hamana bridge seems to extend this limit by a good 40m. Urado Bay bridge in Japan is a 270m span cantilever type bridge. Use of lightweight aggregates in future may increase the span range of these elegant bridge types further.
26. In brief, the present development of prestressed concrete proved to the hilt that Freyssinet's seven years of ordeal did not go in vain. Continuing efforts to expand its scope of application has led to growingly imaginative forms of construction. These structures carry a central message that the man-made environment of structures in these last two decades of the century can be built not only economically, but also with elegance and dignity. Amongst these prestressed concrete structures, bridges occupy pride of place. Of course, prestressed concrete has been used and developed in all kinds of structures. Yet the idea of prestressing arose out of bridges and in bridges do appear the most impressive forms of engineering applications of prestressed concrete.
27. ***It is an undeniable professional responsibility on the part of a bridge engineer to have the cultural depth of knowing the history of his profession.***

48.3 RATIONAL APPROACH TO STRUCTURAL DESIGN

1. The basic principles underlying the philosophy of structural design are that (i) the structure should have a predetermined useful life depending on whether it is to be a monumental structure, permanent structure or just a temporary structure, (ii) the load, causing failure of a structure, should be sufficiently greater than the working loads, so that the probability of the failure of a structure during its life-time is less than a limit specified and the required safety is in-built into it, (iii) the deformations, vibrations and deflections of the structure under the applied working loads are not large enough to impair the safety or serviceability of the structure during its life time, and (iv) economic considerations with regard to design, construction and maintenance are duly recognized. In other words, the aim is to design a structure with maximum safety at a minimum cost. In so doing, methodologies like the elastic analysis followed by the more recent plastic or limit state analyses are in vogue currently, the 'limit state approach' being the latest trend.
2. The traditional method of design was based on

the concept of 'allowable stresses' and associated conventionally with theory of elasticity. This required a comparison between maximum stress under the working loads and the fracture stress of the material (or the stress level at which the material failed) to respond elastically. The factor of safety determining the allowable stresses, catered for a margin of safety. The factor of safety, therefore, revealed a subjective striving on the part of the designer to have a measure of safety, fully accounting for limitations of knowledge and arbitrariness of assumptions involved. Subjective ignorance regarding the resistance of materials at the limiting state of failure was thus added to the objective uncertainty associated with imperfection of human observation.

The safety factor being fixed rather arbitrarily based on subjective arguments, the most refined design by elastic theory and allowable stress method could have doubtful merits. Its very validity remained to be ascertained in absence of identification of objective conditions obtaining at failure. Perfection of design method demanded removal of the element of subjective ignorance. The expedient assumption of linear elasticity at all load levels was not true. 'Linearity of the stress-strain diagram of a material is a desirable property, only so far as it creates the theoretical conditions for the application of the relatively simple methods of classical elastic theory. It is the very slight deviation from elasticity, being the expression of the ability of the material to relieve by limited yielding and localised excessive stresses, that creates the practical conditions for the application of the elastic theory in design of engineering structures. Recognition of inelastic behaviour of materials under certain conditions led to the use of theory of design for plasticity to be adopted in structural design.

3. The plastic theory is based on the failure concept and recognition of the formation of requisite number of plastic hinges necessary for making a structure geometrically and compatibly deformable. The moment of resistance at the plastic hinge is assumed to be constant and independent of angular changes. Further, the plastic moment of resistance is proportional to the yield stress of the material; the latter, for purposes of design, being assumed to be constant. However, researchers have found that for complete sections of beams, the yield stress varies, thus making the assessment of moments at plastic hinges uncertain. The load factors built in the plastic design are more to account for the unforeseen contingencies such as weaknesses and deterioration of material, defects of fabrication and construction, errors in design and detailing, rather than only load variations. The plastic behaviour of a beam is simple but that of a beam-column complicated and uncertain, the latter being affected by angular rotations of the ends of the columns. Aspect like lateral instability, a complex phenomenon in the elastic range itself, becomes all the more complicated in the plastic stage, making its assessment possible only by approximations. The applicability of the plastic theory has been practised mostly for continuously acting loads and there are deficiencies in the approach when intermittent live loads also act. Thus, even the plastic approach suffers from uncertainty with regard to material properties, residual stress and yield stress, lack of adequate knowledge in respect of plastic behaviour of beam-column and empiricism and inadequacy of provisions relating to lateral instability and action of live loads.
4. With the design method correlated to failure state, it is necessary to introduce basic modifications to the method of application of factor of safety. The conventional definition of a factor of safety as the ratio between the two stress levels now losses its meaning, as the basic assumption of stress ratio being identical to load ratio is no longer held valid. Safety can, therefore, be rationally defined in terms of load ratios only. So factor of safety is introduced by way of comparison between increased value of load or load effects with relevant resistance of structure. The revised method of design by this rational way of introducing the factor of safety is commonly known as the limit state design method. Of course, every rational design is necessarily a limit design. It is the specification of the 'limiting conditions' that may differ for different approaches to design. The limit state design method compels the designer to make an objective assessment of the mechanics of resistance of structure relevant to the limit state investigated. The design method is no longer chained to elastic theory. The element of subjective ignorance associated with allowable stress design is removed with the perfections in design analysis, obtained by continual theoretical and experimental investigation. The redefined factor of safety reflects the measure of inescapable uncertainty due to inherent imperfections of human observations. For a rationalised application of the method, a unique factor of safety is often substituted by 'different and partial safety factors' valid for different limiting states of ultimate failure and serviceability limit states of deformation, local damage, etc. Apart from load increment factors, reduction factors to cater for uncertainties in quality of materials are also specified separately.
5. The recent trends in development of rational concepts of structural safety have been marked by the introduction of a new philosophy of design. The structural engineer today generally uses a deterministic analysis of safety

problems, presuming a complete knowledge of load and structural response, both considered as a function of non-random parameters. So he would strive for absolute safety which is no more than convenient fiction. The design approach, to be rational, has to be a probabilistic one, where basic parameters are considered as random. In the words of Freudenthal, one of the pioneers of probabilistic design philosophy, the laws of structural design must be considered as a combination of functional and statistical relationships, functional so far as the laws of theory of structures are concerned and statistical to the extent that real physical properties appear as parameters of functional relations'. The probabilistic approach to safety, the first attempts towards which could be traced back to the late thirties, has been discussed by many devotees and sceptics alike for a long time. The principles of the probabilistic design philosophy have today found general acceptance. The problem now being investigated and sought to be solved relates to the direct applicability of principles to practical design and computation. A few major difficulties faced in the application of the probabilistic approach to practical design rules are: (1) choice of appropriate probability model, (2) introduction of subjective elements into probabilistic structure (no amount of observable data, it is apprehended, may be sufficient to make entirely objective decisions in a civil engineering design), and (3) interpretation of probabilistic information in a form which leads to rational decision.

6. In view of the above, the need for performance studies of structures and a continual monitoring of data to improve upon the various parameters involved in design cannot be over-emphasised..
7. Structural failures are often on account of bad workmanship, human error, incorrect approach, etc., and to cover such contingencies, often the design loads are arbitrarily increased, resulting in the present design loads being hypothetical. The factors attributing to failure cannot be clubbed under one common factor like safety or increase in loads or reduction in structural strength.

48.4 NEW CODES OF PRACTICE (SPLIT 'LOAD-FACTORS'!)

1. The good old working stress design philosophy, sometimes supplemented by the so-called ultimate strength or load factor design, is abandoned. The new format adopted is, of course, limit state design combined with a semi-probabilistic approach of safety. Looking back, the death of the conventional code format has been a historical process spreading over quite a few decades, with a slow but steady progress from the East to the West before the twain could meet. It was the Russians who set the ball rolling. As far back as 1938, they chose to break away from working stress design though it continued to enjoy wide patronage in the rest of the world. The limit state design, method was developed in U.S.S.R. in the period 1947–49 and approved in 1955. The Russian Standard on reinforced concrete NITU 123–55 was re-elaborated in 1962 through SNIP II-B. 1.62 and the design method was adopted by virtue of the recommendations of COMECON in East Europe also, in 1963. In 1964, CEB brought out its first edition of a model code adopting semi-probabilistic limit state design method followed by the joint FIP/CEB second edition in 1970. The design principles or the code format adopted by CEB and FIP are now commonly advocated by other international organisations such as ISO (International Standards Organisation), CIB (Conseil International du Baitment) and CECM (Convention Europeene de la Construction Metallique). A new era of international agreement and unification in "structure" design philosophy has dawned. Many national codes have, as a result, changed directly or indirectly reflecting the influence of CEB Code format. The British Unified Code appearing in late 1972 has a new limit state design format, following CEB model code closely. So, the replacement of the traditional code format by the new one is almost the inevitable natural process like shedding of old clothes and taking on new ones.
2. The single distinguishing feature of the new design philosophy or code format has been claimed to be the 'rationalisation of the concepts of safety'. The new philosophy is meant to open the eyes of the engineers and designers to the non-deterministic character of safety. Structural safety is not a physical measurable property. It is a demand of the society for guarding against any danger to life and property, and, therefore, depends on the social attitudes and environment. One cannot help recalling that oft-quoted passage from the Code of Hammurabi, King of Babylon (2500 B.C.) — "If a builder builds a house for a man and the house which he has built collapses and causes the death of the owner of the house, the builder shall be put to death". The Babylonian builder had of course aimed for absolute safety. He could hardly afford to think or talk of any probability or risk of failure. Has the pre-historic tradition changed? The talk of finite probabilities of failure remains as much a taboo among most of the lay public and even of the engineering profession, though heads may not literally roll today! This irrational attitude has also shaped the 'structure' design codes. Traditional code format identifies loads, strengths of materials, structural response and dimensions by nominal deterministic values. It effectively hides implicit uncertainties involved

and any inkling about the probability distribution of loads, structural characteristics, consequences of failures, or errors in formulae used in analysis and design, etc. This dogmatic approach, implying the non-existent, absolute, risk-free decision is considered highly unethical and unhealthy in the scientific environment of today. It has to be explicitly recognised that associated with every human socio-economic activity is 'a risk which depends on various losses possible and the probability with which they are incurred'. The object of structural design is to reduce the risk of failure to an 'optimum' level. As money for building a structure that the society can afford to pay is 'not unlimited', the safety level decision can only be obtained in terms of 'optimisation'. The main purpose of a code should be maximisation of utility from the standpoint of society. So, the question 'how safe is safe enough' involves interdisciplinary socio-economic decision-making.

3. The last decade or two has seen an explosion of rational thinking on structural safety. IABSE, in close cooperation with other international agencies, has played a laudable role in projecting these developments through various publications and symposia on the subject. The London Symposium (1969) on "Concepts of Safety of Structures and Methods of Design" will, perhaps, be readily recalled. In the ASCE-IABSE Conference on Tall Buildings (Lehigh University, 1972) also, the state-of-art reports of various technical committees like "Structural Safety and probabilistic Methods" (TC 10), "Load Factor (Limit States) Design" (TC 19). "Commentary on Structural Standards" (TC 20) and "Limit State Design" (TC 26), provided a most interesting exposition of the subject viewed from various angles of principles and practice. Literature on Structural Safety linked to probabilistic and optimisation approaches has been steadily piling up. The results of theoretical research now promise development of a safety theory 'based on probability and economic considerations. . .to replace previous obscure fancies'. As a result of this continuing and intensive scientific discussion, there is general agreement that time has come for writing code specifications that handle the non-deterministic character of safety and uncertainty involved in an explicit manner.

   Divergent opinions have, however, been expressed as to the form and complexity and scope of such code formats. The ideal form of specifying safety, it has been pointed out, is through 'reliability optimisation'. But even the impatient do not find it rational to implement it immediately!

4. Dr. Blakey, presiding over the discussions on limit state design philosophy (Proceedings: Australian and New Zealand Conference on Tall Buildings, Sydney, 1973) narrated the following interesting anecdote in his opening remarks: "I am remembering one of my colleagues who was enthused with the limit state concept to the extent that he circularised many interested parties with the intention of gaining their support for the inclusion of these techniques in building codes. He received a variety of replies and he summed them up as saying—"*This is a great idea. P.S. what does it mean.*" The little postscript, though looking rather naive, asks a very relevant question which deserves to be answered fully by the code makers. Paraphrased a bit, it raises the vital issue whether *the split safety factors* of the new code format, the definition of limit states, etc., represent any significant departure from or improvements upon the deterministic conventional load factors and the design approach. It is not difficult to sell the 'great idea' of the new principle as the first step on the road to full freedom from deterministic approach. But has the first step been taken at all in reality? If the 'characteristic' loads are nothing but the old 'nominal' values, and safety factors are just intuitively fished out, the end result of the design process can hardly be expected to be improved merely *by the magic jargons of limit states, characteristic values, etc.* When new code format goes little beyond split-safety-factors, progress towards improvement in design process, or the end result, depends entirely on how rationally they are calibrated. So, if the calibration is not done on the basis of scientific theories, and experimental data not statistically interpreted, the difference between the traditional and the new method is completely drowned and only the high sounding jargon sticks out. Dr. Neville, explaining the background of fixing limit state load factors of the British Unified Code relating to the primary combination of dead load and live load concedes the following in a frank discussion: "The values of these partial (safety) factors were chosen so as to produce designs which are essentially the same as the existing designs. I remember in the Committee we asked the Cement and Concrete Association in England to run a number of designs with various figures and the argument was that if one designs this sort of a room for this sort of loading, the new British Code method should produce the same size of slabs, the same size of beams, the same size of columns because we know that these things work and nobody was prepared to stick his neck out and sign his name to a document which will allow us to design something which is significantly smaller in size. I might just as well let the cat out of the bag and say that the factors 1.4 and 1.6 are there by the grace of God. There was a period of 5 months when the values were 1.5 for dead and 1.5 for live. The committee agreed to that in response to the argument from the designers who said it

is far too much work for us to use 1.4 on dead and 1.6 on live. We agreed to it until about five meetings later, when two most outspoken consultants were away and the others managed to regain their position." (Neville: Discussion Session 4, Limit State Philosophy Loads, *Proceedings Australian New Zealand Conference on Tall Buildings*, 1973). The semi-probabilistic basis of new split-safety factors thus comes down in a heap, like a house of cards! Some food for thought—this.

48.5 DESIGN EDUCATION

1. What makes 'structure' design-education a special concern is its creative design content. An education for creative design may be the professed aim of any course in structure design, but achieving it by a systemised process to yield the best results is not that easy. Creativity is something which cannot perhaps be taught. It is an experience one must live through to learn. So it is futile to imagine that textbooks or classroom lectures, however scholarly they may be, can make a designer. "Can design be taught?" The question has been examined rather thoroughly in a paper bearing the same title by Harris (Procs. Instn. Civ. Engrs., Part 1, 1980, Aug.). Practising engineers are sometimes prone to look down upon the entire proposition as fictive. According to these pessimists, the educator and the practitioner live in two separate worlds, with communication almost shut off. They simply do not talk the same language at all. So the educator can teach design theory alone and nothing more. ***Any attempt to teach the students some design by learned professors who may themselves lack their moorings in 'actual competitive' design practice, could do more harm than good.*** Unlearning the wrong lessons may take longer than learning first hand closer to the scene of action.
2. Such pessimism about design-education seems to be an over-reaction. But with a little introspection, it is quite feasible to modify it for the right kind of design-education. Complete absence of design-education course content, and exposure to theory alone, could injure the young minds. Real-life design practice lies far from the crowd of design theories. Lack of appreciation of this basic fact of life can ail the system.

   In examining the question what can be taught and how much can be taught, Harris defines the elements of a design process, and weighs off each separately. His conclusions, quoted below, are interesting:

   "If the various operations be examined as to which 'can be taught' and which 'can be taught about' (those in which the student can be exercised effectively and those about which he can be warned and informed, ready for closer acquaintance in practice), one would have a list like this:
   (a) Appreciation: Can be exercised, even in the absence of a real client.
   (b) Conception: Exercise is possible and desirable.
   (c) Appraisal: Self-criticism can be developed.
   (d) Decision: No replacement for reality.
   (e) Checking and Elaboration: Present education concentrates on the former; as for the latter, to sense its importance is about all that can be contrived."
3. A systematic attempt towards developing a design-education model, which concentrates on every element of the design process, has been described by Cowan in a recent paper (Procs. Instn. of Civ. Engrs., Part 1, 1981, Nov.). On the basis of the findings on the application of the model, Cowan agrees with Harris fully that design 'can be and should be taught'. His recommendations are quoted below:

   "A programme of design-education, which systematically explores each element in an expressed statement of the design process, is both feasible and worthwhile and should be included as a part of the overall curriculum in engineering education."
4. The problem 'how' design should be taught has engaged attention of other authors. For example, William et al.* has identified five types of basic approach to design education. These are:
   (1) ***Classical classroom approach***: A concept is explained in a lecture, and a single answer problem that involves the concept is assigned as homework.
   (2) ***Socratic approach***: By his questions, the instructor leads the students through the problem-solving situation.
   (3) ***Guided design approach***: A small group of students is given an open-ended problem. They are guided through the solution of the problem by printed instructions that ask them to perform a step in decision-making process and then provide feedback on what would have been accomplished. The student must learn new material to solve the problem.
   (4) ***Case studies approach***: The instructor chooses an open-ended problem that requires the student to work at or a little above his level of development. The student may be required to learn some new material before he can solve the problem. His solution is compared with actual solution.
   (5) ***Professional approach or authentic involvement***: Here the problem is directly encountered. There are no clues to the method of solution, none of the tools required in the solution are identified and there is no certainty that an accepted solution exists".

* (Present and future directions in design-education, Engineering issues, ASCE, January 1975)

5. Obviously, the efficacy of the approaches increases as we go down the list. The preferences of a professional approach over the classical classroom or Socratic approach would hardly need a comment. However, the problem that stares us in face is how to translate these preferences into action within the framework of the engineering education system that prevails.
6. The impact of computers in structure design is another aspect of design-education which may be of critical concern to the profession. Emphasis on mathematical computation has to be reviewed critically in this context. While the analyst-designer must have adequate knowledge of the mathematics of 'structure' analysis, ***he need not be a deft expert in computing moments, shears, and stresses in structure elements. The job may be taken over and done better by computer people. It would be more important than ever for the student to understand almost instinctively how the structure really works, how to proportioning of its various elements affects its load response, and what detailing it takes to make the structure practical and tick. The good designer can and should be taught to 'think' more than 'compute'.***
7. The young graduate may be well versed in all sophisticated analytical techniques, but his pride and self-confidence may get a rude jolt when he is unable to read a plan, let alone really design a bridge or a structure in all its 'practical detail'.
8. ***The fault lies with the system which forgot to teach him the fundamental fact that design is mostly 'detailing' which is made on drawing boards, not on the calculation sheets, nor in the computer. The only way out of this impasse is to open the doors of universities wide enough to bring more of the design-office into the classroom.*** The mechanics of a stable, effective, planned, and rightly motivated 'educator-practitioner' interaction must be evolved with a national priority.

48.6 CURE AGAINST CRACKING OF CONCRETE—NOT BY CALCULATIONS ALONE!

1. Cracking and crackwidth-prediction-and-control have been much in the limelight in recent years. The problem has engaged growing attention of researchers, code-makers, designers and constructors, to such an extent that it almost selects itself as one of the most discussed topics of the last decade, among people dealing with concrete.
2. All this talk about crackwidth-computation may lead one to believe that the panacea of all ills lies in accurate prediction of flexural crackwidth or shear crackwidth. Nothing could be farther from truth. Computing crackwidth is an important design task but a designer with overreliance on crackwidth-computation for crack control may soon find it disappointing.
3. Cracking of concrete occurs due to tensile stresses. But the occurrence of such tensile stresses may be traced to various sources other than flexural tension. Had flexural stresses been the one and only reason for tensile stresses in concrete, 'full' prestressing would have easily solved the problem!
4. There are other kinds of tensile stresses. It is, therefore, to be expected that the concrete structure remains unimpressed by the most sophisticated prediction of flexural crackwidth and may crack due to these other tensile stresses.
5. One of the commonest causes of cracking in structures is the lack of reinforcing at locations where significant tensile stresses occur under predictable or unpredictable combinations of effects. Another kind comprises avoidable 'deficiencies in construction quality of concrete resulting in so-called plastic cracking'.
6. In concrete, seeds of most cracking distress are sown before hardening and application of loading. Deficiencies and consequent damages that have been done to concrete at birth or at a very young age cannot be undone after it matures and is put into service. If visible cracking has not appearing already in the concrete when poorly made, it remains very vulnerable due to unhealed scars. Reinforcement does not help, and computations go haywire!
7. To guard against such cracking, one must understand one's concrete better. It is a multiphase, inherently heterogeneous, complex material. In engineering use, we interpret the behaviour of concrete at phenomenon level, which can be deemed as little better than prescientific. The status of knowledge on microstructure of concrete and associated theories of crack propagation and fracture of concrete have grown fast in recent years. It is now clear that microcracks, lying hidden from view, hold the key to the cracking behaviour of concrete like all other properties pertaining to strength and deformation.
8. Microcracks are minute flaws or discontinuities inherent to heterogeneous nature of concrete. They appear in concrete, not yet loaded, or lightly loaded while lying in the forms, waiting to gain its mechanical strength. The cracks are too narrow to falsify the use of elastic theory or the assumption of a homogeneous isotropic material by the structural designer. (Some microcracks may also get healed partly or completely with the crystals precipitating during the continued hydration of cement. But this autogeneous healing does not solve all problems.) They nevertheless create, through their

geometry, zones of potential fracture. So it is but expected that any excessive micro-cracking that occurs before hardening of concrete may prove to be the major source of cracking distress in the service life of the structure.

9. Restraint to free deformation while concrete hardens (plastic shrinkage, plastic settlement, etc.) is the primary cause of random non-structural cracking. Cracking can occur in otherwise sound concrete when still fresh due to the tendency of concrete to contract, caused by loss of moisture or change in temperature. Other reasons are differential settlement over obstructions, such as large pieces of aggregates or steel, separation of aggregates and matrix due to settlement of latter with local bleeding under coarse aggregate. Segregation and faulty compaction result in voids and honeycombs.
10. Even when deficient quality of construction induces excessive microcracking which soon develops into visible cracks, these may lie undetected for long despite their early appearance. These are usually obscured at early age of concrete by water bleeding initially to the surface of concrete. All looks well until the curing is over or the concrete is put into service. Unfavourable ambient conditions soon magnify the damages for all to see and ponder over. But post-mortem analysis often ignores the original sin.
11. The tensile strength of concrete develops very slowly and the concrete obviously remains very vulnerable to cracking at a young age. Deficiencies in composition of concrete, placing and curing, etc., and more significantly the adverse environmental conditions (wind, heat and cold) are agents which magnify the distress. Even if visible cracking is not very evident, due to severe microcracking, the tensile strength of the hardened concrete may be much smaller than anticipated and severe cracking distress under subsequent load-induced tensile stresses can occur.
12. As mentioned earlier, cracking in concrete yet to harden, can be attributed basically to (1) temperature change, and (2) drying shrinkage. The mechanism of cracking due to temperature change is simple enough to appreciate. Cement liberates heat during hydration. While temperature rises, the plastic concrete hardens as it cools. A temperature differential can thus be set up between the outer and inner portions of the concrete mass during this temperature change. This can be aggravated by environmental conditions. Restraint to free deformation results in tensile stress (not necessarily flexural).
13. Similarly, drying shrinkage generates tensile stresses due to restraint of deformation. Drying is largest at the exposed surface which tends to shrink more than the interior, yielding a restraint to the deformation of interior layers.
14. Conscious efforts to reduce these restraint stresses and the consequent risks of cracking in plastic concrete may involve some elementary care in making good concrete. For reducing the cracking caused by temperature change, the temperature in the interior of concrete must be kept down and there are many ways of doing it. The most obvious one is to strike at the grass-roots. Cement is the source of heat of hydration. So the less the cement, less is the temperature change. Of course, cement is also the primary source of strength. So an optimum quantity, just enough to gain the desired strength, should be used. Non-optimal use of cement may make the concrete costlier or even stronger than desired but never better! Merely richer mix for the same target strength can in the limit be poorer in quality. It has been reported from a survey of a number of mass concrete structures in USA that the only concretes free from cracking were the 'leanest'. Perhaps temperature differences alone cannot explain the observed better cracking resistance of leaner mixes. Lower modulus of elasticity and lower drying shrinkage characteristics may also contribute equally.
15. The above elementary rule of making better concrete with less cement may be too well-known to be restated. Yet it remains a fact of life that misconceived reliance on liberal addition to cement content to compensate for lack of controls on construction quality continues. Accent should be on durability and bond more than on extra strength alone. This calls for coarser gradation, cleaner dust-free aggregates of good quality, tight water-cement ratio, casting in low-wind low-heat low-cold weather, with adequate curing. It is worse when people add extra cement hoping it will 'take care' of strength when the worker quietly adds more water arbitrarily to permit him an easier workability locally. This in fact leads later to all sorts of non-load-induced random cracks that subsequently accentuate, which ultimately destroys the integrity of the structure.
16. Technical Report No: 22 of the Concrete Society indicates various non-load induced cracks that can plague the concrete in a structure. It shows their most common locations, their primary and secondary causes, remedies and approximate times of appearance, and is therefore a good general guide. More details may be found in the report itself.
17. A designer doing voluminous computations of crack-width should perhaps first ensure that the mix of the concrete going into the structure is properly designed and followed because excess of cement alone can lead to loss of quality in respect of cracking behaviour.
18. Caution against using too finely ground cement for

not so high grades of concrete is also called for. Modern high-strength cements are ground much finer than earlier. They liberate heat of hydration much faster and increase the risk of setting up a thermal gradient in a thick member. So, where cracking behaviour is a critical consideration, use of a very finely ground high-strength cement may not be favoured.

19. For preventing excessive drying shrinkage of concrete, it is important to protect the concrete surface by continued curing. Members with large thickness, or those cast in adverse ambient condition, will require greater care. Premature exposure by removal of forms also can aggravate the risk. In control of drying shrinkage, the effect of water content and mix proportions is most important. Choosing the lowest water-cement ratio practicable is necessary from the viewpoint of cracking resistance also.
20. 'Settlement cracking' also appears in concrete before hardening. This type of cracking can be seen in poorly laid concrete on upper surface of slab following approximately the top reinforcing pattern. The obstruction to the downward movement of solid particles by the reinforcement grid is the obvious cause. The incompatibility of the concrete mix, placing, and compacting techniques with the reinforcement arrangement contribute to the distress. This can be controlled by slightly revibrating the concrete before its initial setting. Plastic shrinkage cracks (which start at top and descend downwards, sometimes following the obstruction-creating path of the reinforcement grid and sometimes appearing normal to wind direction) can be controlled by robust timely trowelling. They are caused when the rate of surface evaporation is faster than the rate of internal bleeding (rising upward of water in fresh concrete).
21. The assumption of uniform stress distribution over the total 'effective' compression flange width only brushes aside a considerable deviation of stresses at the flange-edges. But such inaccuracies do not spell any trouble nor do they signify any gross error. Gross errors occur when the designer fails to subjectively account for the possible significant impact of the inherent inaccuracies of his analysis. *In spite of the large sophistication in the tools of mathematical analysis of a structure, a 'structure' design can hardly live by computation alone. Unless design decisions are supplemented by subjective engineering judgement of the actual structural behaviour, the finest efforts may end in disaster. This lesson has been learnt the hard way many times in this century. All the brilliant mathematics of the first Tacoma Narrows bridge design did not save it from the collapse due to aerodynamic instability. The small geometric inaccuracies ignored in analysis brought about startling disasters of mighty steel bridges. Added to all this are the unavoidable differential temperature stresses even whose prediction is inaccurate.*
22. In some of the recent concrete bridges some unsightly cracking has appeared due to tensile stresses ignored in common design practice. The principal agent in these cases has been identified as temperature variation. Leonhardt has reported instances of severe unanticipated cracking of beam webs due to non-uniform temperature distribution and consequent stresses. Priestley has talked about thermal-stress-induced cracking of prestressed concrete box girders of a major urban motorway viaduct in New Zealand. All this cracking was found to be strongly correlated to the ambient temperature and solar radiation. Ignoring these stresses, as we have been doing in earlier design practice, cannot but be termed as a gross error. *While these stresses may be pre-calculated, cracking of concrete due to them cannot be avoided unless sections are thin. Best bet is to provide closely spaced reinforcement bars of small diameter along the skin in crack-critical zones.* This will allow more cracks but of narrower width. Here again quality control exercised in making the concrete will greatly help in limiting the total cracking.

48.7 LIFE-CARE OF ROAD BRIDGES

1. We cannot afford to forget that there is a hidden price on every investment made today—maintenance and repair costs of a much longer tomorrow. The bill is usually delayed but inevitably becomes due and must be paid. The price for neglect of maintenance care can be staggering, and costly national investment may be imperilled.
2. The learned designer and the expert builder may turn back and retort that all this is too much ado about nothing. The designer today is better equipped and so is the builder. Sophisticated structural analysis and rational considerations of safety have resulted in more economic and safer structures. Materials and construction techniques have improved. Bridges are designed better, built better, to last their lifetime.
3. There can be no disputing these claims. But the fact remains that engineer's job hardly ends with the bridge being built and put into operation. It only starts a new phase of activities and this is no less challenging even if less glamourous. The benefits promised to the society by the planner and designer are now directly measured by the real life performance of the bridge.
4. A long true vigil of the engineer starts. He must care for the minutest details to ensure peak performance of

the facility for the longest period possible. Interest and involvement of the engineer in maintenance flow from the central philosophy of the profession as serving the use and convenience of man.

5. An associated technical activity of equal importance is the 'performance measurement' to serve as feedback to future planning, designing and construction activities. No human ever did anything perfect. Bridge builders have always a lot to learn from past mistakes, if correctly monitored. Continuing improvement in engineering judgement could result from such performance measurement and thus gain tangible benefits.
6. Maintenance measures for road bridges suffer as the inventory of bridges in service grows in years. Older bridges do not receive the attention they need until too late when irreparable damages may occur. Bridges constructed and put into operation get left to themselves. Even routine maintenance inspection and reporting go by default. 'Pockets' that do not drain, expansion joints that rattle, or rollers clogged in dirt that do not roll, remain mute witnesses to lack of even cursory care. Records of bridge details and their current condition may not always be available. Even systematic documentation of design, construction data measurements and completion plans may be missing. However, the overall picture is not as gloomy. Most of the bridge structures are generally in good enough shape. But dark patches like these can blemish the general scene.
7. Signs of complacency can creep in. In bridge maintenance practice what may prevail is an unplanned maintenance. A damage detected by design or accident calls for corrective operations. If the results are satisfactory, these operations end with a finality. There may be little feedback for future planned preventive care for either the affected structure or for similar vulnerable ones. Adequate maintenance-planning and organisation are the obvious needs.
8. Planned maintenance is the obvious option. This may be defined as 'work organised with forethought, control and records.' It comprises two essential elements:
   (a) Planned preventive maintenance
   (b) Planned corrective maintenance

   Both of these must be organised with 'forethought, control and records.' They must be mutually balanced and go hand in hand.
9. The main objectives of maintenance planning of bridges may be restated as:
   (a) To preserve the investment;
   (b) To provide adequate level of safety of user;
   (c) To provide adequate level of comfort and convenience of vehicular travel;
   (d) To ensure economy in expenditure and resources.
10. These general statements apply to overall highway maintenance as well. However, the special requirements and considerations of bridge maintenance are easily discernible. The technical priorities often become confused because of the intrinsic difference in user reaction. The travelling public may be quite vocal about the inconvenience caused by pavement potholes, but serious signs of distress of a bridge structure may escape notice and criticism.

    If priorities in maintenance schedule are motivated by public reaction, urgent bridge maintenance operations may get deferred due to constraints of money and manpower limitation. Such deferments may prove costlier and more catastrophic than imagined. Planned maintenance can correct these pitfalls.
11. Bridge maintenance has a special character and requires special skills. The surface defects may be cosmetic and easily restored. But the more important factors to which the objectives are principally oriented are prevention of the 'loss of property'. Hazards to human lives which cannot be measured in economic terms are to be guarded against. Maintenance operations are basically aimed to prevent premature loss of facility and furnish possible forewarning.
12. Unfortunately if often happens that attention to maintenance requirements of bridges is aroused by an accident. The example readily recalled is the Silver Bridge collapse in U.S.A. on December 15, 1967. The tragedy took the toll of 16 lives of men, women and children. In its wake, deep public concern and apprehension were created over maintenance responsibilities and performances. Committees deliberated; U.S. Congress held hearings; Federal and State legislations were enacted. AASHTO Bridge Committee in cooperation with Bureau of Public Roads undertook the development of a comprehensive Manual for Bridge Maintenance Inspections. The concluding remarks of the 'President's Task Force on Bridge Safety Committee 3 — National Study to Assure Bridge Safety', quoted below, indicate the extent of feverish activities that followed:

    "Federal, state, and local government agencies having responsibility for bridge inspection and maintenance, are either presently intensifying existing programs or developing new improved programs... The concern for the safety of the bridge user has been greatly aroused. New inspection equipment is being purchased, sophisticated devices are being employed for inspection of bridge members and determination of scour at bridge piers, bridges are being reappraised as to load carrying capacities, hundreds of consultant engineering contracts are being let to inspection services. . . This concern by the public is reflected in the legislation passed by the 90th Congress which makes bridge inspection on Federal-Aid highway systems 'mandatory' and further provides for the establishment of Inspection Standards and a training program for bridge inspectors".

13. A bridge built is to fail as surely as a man born dies. It is designed and built for a finite life and finite risk of failure. An engineer, therefore, needs to be motivated only by the basic ethical objective of maximisation of utility. Incident-generated responses often vitiate rational decision-making and ethical optimisation. In some countries one has seen design seismic coefficients shoot up immediately following a strong tremor and perhaps change again with time! Maintenance is a long-term activity. It is important to avoid these arbitrary lows and highs, sudden spurts in awareness, followed by the spells of complacency in planning bridge maintenance.
14. Monitoring the quality of performance of the bridge and assessing whether the specified level is being maintained is the controlling function of maintenance management. The monitoring that may be decided upon must be a method of regular observation and testing. Routine maintenance inspection by fully qualified and trained bridge inspectors is an essential part of the monitoring system. But the scope of such inspection as well as supplemental procedures necessary must vary to suit special requirements. Observation and testing should be done with instrumentation as and where needed.
15. "The approach of an engineer to a problem of maintenance should be identical to the approach of a doctor to his patient. It involves 'detection', 'diagnosis' and 'cure' with an overriding requirement for 'prevention'. In order to proceed, the engineer must have the knowledge of the various forms of deterioration which are known to occur (the symptoms), the cause or causes of various types of decay (the disease) and how to correct the condition (cure)." 'Expertise' needed for maintenance is therefore too important to be ignored.
16. Last but not least is the 'performance measurement'. Of course, this is not an activity completely isolated from maintenance. It is, however, identified by the more specialised purpose of feedback and is, therefore, more selective in operation.
17. Organised scientific enquiries into performance characteristics of bridges in service are needed to find the right answers. For example, possible large deflections due to concrete creep, likely to stabilise after say 3 to 5 years of service, effects of restraint stresses due to temperature gradients, vibration, and the serviceability/distortion problems of large span thin-walled box bridge beams, etc. Monitoring 'performance data' by planned instrumentation could serve as an important aid to the solution of these problems.
18. Acquisition of 'loading history data' of bridges is another basic area of performance measurement. Studies on intensity, frequency of truck traffic loading and trends are associated with basic parameters of bridge design and linked to the optimisation of the entire transportation system cost.

48.8 CONCRETE STRUCTURE—DETERIORATION AND REHABILITATION (... for details refer to the author's books referred to on p: 652

(A) Conceptual Understanding

Structure	Element	Causes of damage	Investigations	Types of damage	Rehabilitation requirements	Methods of rehabilitation
Super-structure	1. Asphalt layer 2. Concrete slab 3. Parapet 4. R.C. girders 5. P.S.C. girders 6. Box type structures 7. Articulations	See (B) ahead	See (C) ahead	See (D) ahead	After due inspection, decide between: * Replacement * Repair * Strengthening and * Protection	See (E) & (F) ahead
Substructure	1. Pier cap 2. Piers, abutmnts 3. Pile caps, footings 4. Piles					
Others	1. Retaining walls 2. Approach slabs 3. Bearings 4. Expansion joints					

(B) Causes of Damage

1. *Quality of Constituents of Concrete Itself*:
   1. Cement
   2. Aggregates
   3. Water
   4. Additives
2. *Environmental Effects*:
   1. Corrosion/chloride attack/sulphate attack
   2. Alkali-aggregate reaction
   3. Marine conditions/sea environment
   4. Weathering.
      * Thermal variations
      * Sand abrasion
      * Waves
      * Rain/humidity
      * Wind
3. *Foundation Problems*:
   1. Settlements/Differential settlements
   2. Variation in subsoil water level
   3. Ground movements
4. *Construction Errors*:
   1. Poor quality concrete/construction practice.
   2. Form work/false work problems
   3. Inadequate compaction
   4. Incorrect detailing of reinforcement
   5. Inadequate and delayed curing
   6. Excessive construction loads
5. *Design Errors*:
   1. Design load underestimated
   2. Under-design/strength overestimated
   3. Omission of appropriate joints
      * Construction joints
      * Movement joints
   4. Unacceptable cold joints
   5. Unaccounted internal stresses:
      * Thermal
      * Shrinkage
      * Creep
      * Unforeseen load conditions
6. *Unexpected or Unprecedented Load Conditions*:
   1. Illegal overloading/speeding
   2. Unusual floods
   3. Unusual earthquakes

(C) Investigations

	Type	Analysis
(A)	*Routine Examinations*:	
	1. Compressive strength	'Schmidt' Hammer REBOUND
	2. Crack measurements	Crack microscope or equivalent
	3. Crack monitoring	Mechanical gauge/tell-tales
	4. Concrete cover	Magnetic cover meter
	5. Carbonation	Phenolphthalein test
	6. Corrosion detection	Colebrand pathfinder
	7. Corrosion determination	Corrosion test
	8. Moisture contents	Moisture detector
	9. 'Joint' monitoring	Crack monitor
	10. Chloride concentration	Drilling cores/powder analysis to plot the chloride penetration profile
	11. 'Mix' composition	Valuation of components for analysis of internal faults.
(B)	*Special Examination*:	
	1. Concrete strain	Load testing, use of strain gauges
	2. Structural deformation	Levelling instruments/survey
	3. Soil pressure	Capsule load cells
	4. Load measurements	Load cells
(C)	*Special Equipments*:	
	1. Electric Power supply	Stand-by generator, etc.
	2. Core drilling	Rotary cutting tool
	3. Pick and knock devices	Hammers (ordinary/electrical)
	4. Temperature measurement	Thermometers/Thermocouples
	5. Humidity measurement	Hygrometers
	6. Tests on fresh concrete	Properties, according to standards
	7. Load application rigs	Hydraulic jacks, frames, anchors, etc.
	8. Laboratory testing	Various laboratory facilities

(D) Types of Damage

1. *Spalling*
2. *Swelling/bulging*
3. *Crumbling*
4. *Cracking and spalling due to corrosion of Reinforcement*
   * Corrosion of steel can be due to its exposure to moisture and atmospheric oxygen and corrosive chemicals and fumes.
   * Corrosion of steel can also be due to ingress or presence of chlorides in combination with moisture. This is an electro-chemical process where electric currents are set up between points in reinforcements located in weaker and stronger chloride solutions, leading to eating-up of reinforcement due to formation of iron oxide (rust).
   * Corrosion of steel can also be caused by Carbonation (increase in acidity in the moisture in concrete owing to reduction in its pH value because of formation of calcium carbonate from calcium hydroxide (released by hydration of cement) and the atmospheric CO_2)
   * Corrosion in each of the above cases leads to the 'formation of Iron-Oxide' (rust) which occupies higher volume and hence causes radial expansion of the bars and consequent bursting—pressure that causes cracking along the line of reinforcement. Soon rust-stains weep out of these cracks, rust increases, cracks widen, and ultimately the surrounding concrete spalls!
5. *Cracking due to sulphate attack*:
   * Ingress of sulphates from surrounding soil, water, or atmosphere, or sulphates that may be present in the aggregates themselves and/or in the mix-water or curing-water, chemically react with C_3 A in cement in the presence of moisture, forming the sulpho-aluminate crystals which occupy more volume and hence exert expansive pressures that cause cracking.
6. *Cracking due to alkali-aggregate reaction*
   * Alkali in cement reacts with certain reactive carbonates and silicates found in certain aggregates, causing the formation of silicate-gel which exerts expansive pressures causing extensive cracking.
7. Surface erosion/abrasion.
8. Crazing (surface cracking generally within laitance thickness).
9. Plastic shrinkage and plastic settlement cracks (within the first 3 to 4 hrs.)
10. Early thermal movement cracks (after 1 day but generally within 3 to 4 weeks of casting; causing by:
    — lack of movement joints
    — presence of constraints to free movement
11. Drying shrinkage cracks
12. Cracks due to overstressing of steel and concrete.

NOTE Cracks can be 'live' or 'dormant', depending on whether they continue to open or do not open further.

(E) Methods of Rehabilitation

(1) *Outright Replacement*
— Demolish and reconstruct

(2) *Repair* First take necessary precautions of closing traffic if required, and temporarily supporting the structure carefully and then:

(a) *Remove unsound concrete, using*:
- Hammers
- Pneumatically operated tools/compressor
- Very high pressure water jet

(b) *Clean reinforcement steel all around, using*:
- Wire brushes
- Sand blasting
- High pressure water jet

(c) *Restore removed concrete, by*:
- Shotcreting (by layers)

or • Guniting (by layers)
or • Epoxy-mortar
or • Epoxy concrete
or • Normal (Higher strength) concrete

(d) *Seal cracks*: (after suitably cleaning the cracks/pressure cleaning):
* *for dormant cracks*:

— Seal by pressure injecting a suitable epoxy formulation (unless low-pressure cement grouting can dependably do the job).
* for live cracks:

— These are like "movement joints". Cut a chase or a groove (a recess) along the crack (dimensions depending on type of crack), then seal with a suitable Flexible filler (mastics, thermo-plastics, elastomers) and, where called for, use an additional covering strip of flexible pre-formed rubberised or tar-bonded strip.

3. *Strengthening*

(a) Bolts, anchors, additional structural steel
* Consult Trade Representatives (e.g. HILTI, CIBA-GEIGI, etc.)

(b) Externally bonded steel plates
* Consult Trade Representatives (e.g. HILTI, CIBA-GEIGI, etc.)

(c) External prestressing
* Consult Specialist Agencies (e.g. V.S.L., Freyssinet, B.B.R., etc.)

4. *Protection*:
   (a) *Protection Against Corrosion of Steel (Chloride Attack and Carbonation Effect)*:
   * Although many Trade-products may be available from the Trade (e.g. CIBA-GEIGI) which can be used even subsequently, but their long term efficacy is doubtful. It is best to use dense concrete:
   * Increase minimum cover suitably (by about 25%)
   * Increase minimum cement content (to about 350 to 450 kg/m^3, depending on the conditions)
   * Reduce w/c ratio to about 0.37 to 0.40
   * Use plasticiser (additive) to gain on workability
   * Compact concrete by thorough vibration
   * Adopt slightly more fines in the total aggregate content.
   * Adopt early and adequate curing.
   * Paint concrete surface by high-pressure hot bitumen, if not objected to.
   * Preferably use Portland Blast Furnace slag cement containing about 65 to 70% of the slag in the cement (by weight). This gives a very dense paste which inhibits diffusion of chloride ions (and even alkali ions), thus inhibiting attacks from chlorides and sulphates and even the alkali-aggregate reaction.

   (b) *Protection Against Sulphate Attack*:
   * Use sulphate-resisting cement (Type V cement: ASTM) which has low C_3A content, and dense concrete as in (a) above.

   (c) *Protection Against Alkali-aggregate Reaction*:
   • Use low alkali cement
   • If such aggregates must be used, then at least replace part by non-reactive aggregates even if long haulage is involved.
   • Use Portland Blast Furnace slag cement as explained above.

5. *Special methods*:
   (a) Replacement of expansion-joints/bearings
   (b) Replacement Operations involving heavy-lifting and massive sliding/moving.
   (c) Ground and rock anchoring to stabilise unstable abutments, piers, walls.
   (d) Strengthening of foundations (various methods)
   (e) Strengthening of articulations locally

NOTE For all these five items it is advisable to consult specialist agencies like: V.S.L., Freyssinet, B.B.R., etc.

(F) Crack Repair Illustrations

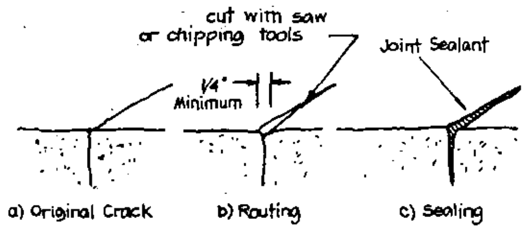


Fig. 48.1 *Repair of crack by routing and sealing is a method suitable for cracks that are dormant and not structurally significant. Routing and cleaning before installing the sealant add significantly to life of the repair.*

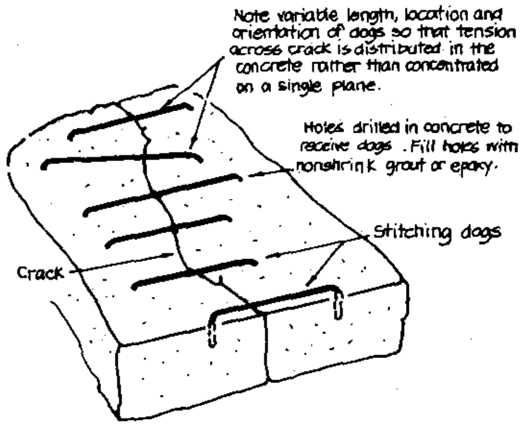


Fig. 48.2 *Crack repair by stitching restores tensile strength across major cracks. Where there is a water problem, the crack should be made water tight first to protect the stitching dogs from corrosion.*

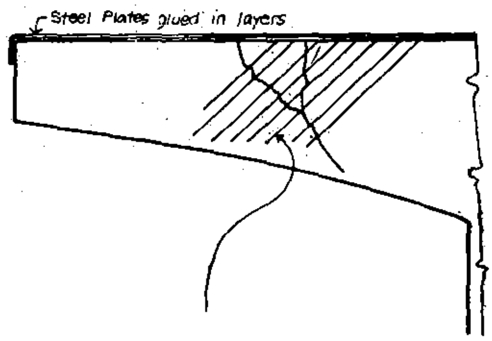


Fig. 48.3 *Added reinforcement installed to strengthen repair. Holes are drilled at right angles to the crack then filled with epoxy before bars are inserted.*

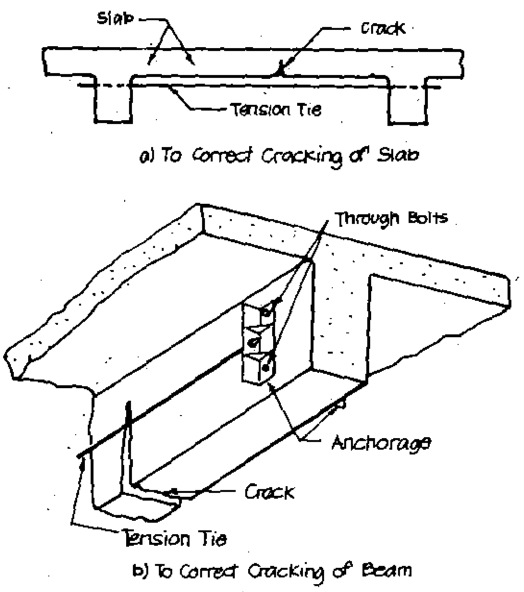


Fig. 48.4 *External prestressing can close cracks and restore structural strength. Careful analysis of the effects of the tensioning force must be made or the crack may migrate to another position.*

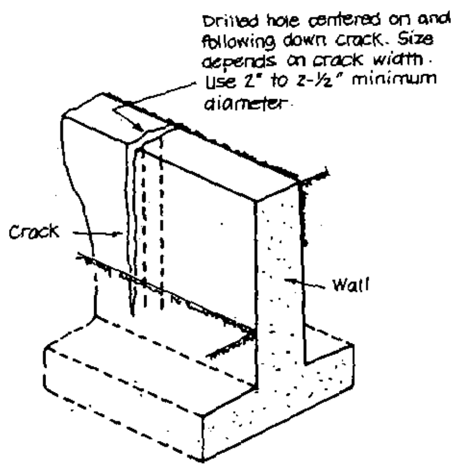


Fig. 48.5 *Drilling and plugging is a repair method well suited to vertical cracks in retaining walls. The repair material becomes a structural key to resist loads and prevent leakage through the crack.*

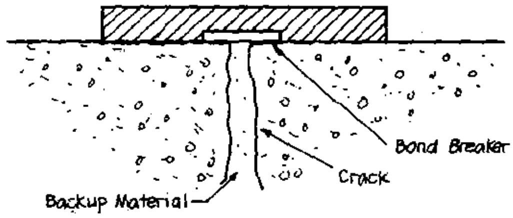


Fig. 48.6 *Flexible surface sealant can be used over narrow cracks subject to movement, if appearance is not a consideration. Note bond breaker over the crack itself.*

48.9 APPROPRIATE TECHNOLOGY FOR DEVELOPING COUNTRIES

Introduction

A lesser developed country (LDC) would obviously like to catch up with the modern technological advancement on the one hand and create more employment on the other. LDC technology needs are enormous compared to their relatively meagre resources and therefore most of the linkage problems (e.g. industry-university links) should better be matters of urgent policy design rather than just matters of occasional reminding.

Assessment of the 'appropriateness' of technology is not a new idea. Workers revolt against machines during the industrial revolution, the clinical testing of pharmaceutical products, or even the cost-benefit analysis of development projects, all involve technology assessment in one form or another.

It requires people with talent, tact, discretion and imaginative analysis to assess the appropriateness of technology for a given LDC keeping in view the limitations of prevailing standards of education, experience and society. Suppose in a poor country with dense population a low cost housing scheme has to be executed. It should obviously be a very simple type of building, fully made of locally available materials using local skill, and all the work should be done by the local hand. Water, sewerage and electricity are of course a must. Now suppose such a housing scheme has also set up in an oil rich country where materials may be unavailable locally (even the water may be brackish) and there is little skilled labour available. Therefore a good part of the materials and staff and labour have to be imported. These imported people have to live and that means providing for them temporary housing, transportation, recreation, schools for their children, medical care and leave arrangement. Also, materials have to be transported, which might mean waiting in congested harbours, traffic jams, overloads, etc. There may also be insufficient communication due to possible lack of telex and telephone facilities. Consequence of these considerations should obviously be a design which asks for minimum manpower and short construction time. The lesser the man-hours, the lesser are the problems. Production method may well have to be extremely advanced and sophisticated, equipment heavy and expensive, materials imported and of first class quality — and not necessarily cheap. In the above two cases obviously the technology which is appropriate for one is not so for the other — and yet both technologies are appropriate for the individual LDCs.

All developing countries lack to some extent the economic infrastructure, the social infrastructure and the productive assets. This unfortunate phenomenon was probably accentuated due to the dominating powers using them as sources of labour and raw materials and as markets for finished goods.

With objectives like international free trade, infant industry protection, and employment creation, the expression 'appropriate technology' becomes difficult to use. International trade between developed and less developed countries may cause dependence through ownership of technology.

In many instances where the shift from very labour intensive to intermediate class of technology appears an economically attractive proposition, the non-availability of the required capital can act as dominant constraint.

Phenomena

A technology is appropriate if it involves simplicity of production, use and maintenance, and cheapness and independence, without being wholly mystified by experts. It has a much closer relationship to life and can be a means of raising consciousness.

The cities exploit the countryside and themselves are sacrificed to the requirements of international commerce. No area lives for itself alone. Self-sufficiency and self-reliance are most important so that:

- the economy can proceed at stable pace,
- the people can define their own goals,
- the system is not too dependent on alien processes,
- the people are reasonably protected against commercial exploitation.

Intermediate technology, appropriate technology, or convenient technology — which one? We want the correct technology for the environment prevailing at a particular time in the particular part of the world under question. This correct technology is best described as appropriate technology. Appropriate technology must be subject to the prevailing local conditions such as the indigenously available raw materials, manpower, plant, power, etc.

Developmental process transforms the traditional into modern. It is the technology-gap that really creates the income-gap. After three decades of development efforts, however, unemployment and the compromise between labour intensive versus mechanised techniques have emerged as the most critical problems of today in many developing countries.

The value of manpower training programmes for professional practice and workman like production has been underestimated, whereas the importance of research has been overestimated in the past. It is more important to extend research into practice than only make more research which is not put to use. Manpower training is capital saving process while research is capital using process, and though the latter should never be stopped altogether, the former definitely deserves more attention in the present situation.

It is essential to improve the knowledge and skills in the developing countries for better utilisation of technology. This demands building the necessary infrastructure in the LDCs which includes educational, scientific and technical disciplines. There is no shame in first copying and then creating one's own.

Different types of alternative technologies are best identifiable in terms of the communities that have to practise them. An attempt should be made to cast the goals into a set of rules which the technologists might follow. The latter should not be left to judge intuitively as to whether a given line of development meets the goals. Some typical goals are:

- Regional self-sufficiency
- Resources conservation
- Direct control by consumers
- Absence of exploitation
- Ecological equilibrium

The LDC's first problem generally is to find employment for large reserves of manpower by investing in industries which may even be of somewhat doubtful profitability. Sometimes the engineering firms commissioned to make the techno-economic studies are connected with plant manufacturers or process holders and therefore may give a slant in their report towards the choice of certain types of plants and processes. The choice is between less advanced technology in order to absorb available manpower or more advanced technology which could be export-oriented. LDCs many times are not properly equipped to face the choice of technology. The engineering firms justifiably keep their secrets jealously and are not always prepared to pass them on the others. ***The 'tied' aid is not the ideal background for choosing the right technology.***

Development

Economists can elegantly make projections and the scientists, being no less, may matchingly project the consequences of the developments in science. It may be desirable to subordinate science to economy in the early

stages since the R&D facilities and resources may be scarce.

The economy of LDCs requires certain capital intensive industries which require a great deal of investment. The examples are steel, cement, aluminium, paper, certain essential chemicals and fertilizers and, most of all, the road and rail building as also the ports. Many such industries can give birth to ancillary small-scale industries and in the process create employment necessitating local technology. Many of these, however, require rather long periods of gestation that cannot be helped.

The success of construction investment depends on the suitability of the overall concept of a project. Designs must not only be suitable for local conditions but appropriate use of indigenous materials, local skills and manpower should be tied in. The total cost of the project including maintenance should be in proportion to the financial resources of the state.

Development can be categorised into three heads: ***Economic***, ***Social*** and ***Industrial***:

(i) ***Economic*** For developing agricultural and industrial production the major bottleneck is transportation. Owing to lack of it, a commodity may be sold at a certain time at a very high cost in big cities while farmers in far flung rural areas may not find a buyer for it because they may be unable to transport it owing to bad roads. Another type of bottleneck in the transport sector can be the lack of port facilities and there are instances when the waiting time for loading and unloading ships reached as high as three to five months at certain times! Hence the development of roads and ports is of basic importance for improving the economic infrastructure. The urgent need in transport sector is for more roads of all types, ranging from national highways to feeder rural roads, coupled with coordinated development of port facilities. The main problem here is the choice of optimum standards to meet the present and the projected future needs, the choice of the appropriate construction methods and the balance between labour intensive and the capital intensive techniques.

(ii) ***Social*** This requires developing low cost housing, schools, hospitals, administrative offices, community recreation centres, water supply, sewerage, rural electrification, etc.

(iii) ***Industrial*** This includes industries, power stations, irrigation structures, canal networks, etc.

When the financial inputs are scarce and labour and skills are relatively easily available, then whether labour intensive methods should be used (even when they may be more costly) in order to relieve unemployment, is a question for debate. Political environment has a lot to do with this debate and it is difficult to generalise on this as there are so many shades of it. There are dictatorships of left and right, powerful monarchies, various brands of democracy, influential aristocracies and straight military government. Any study which fails to take careful account of the political pressure and constraints is liable to prove a waste of time.

Feasibility

To handle the heavy work-load including the preliminary and detailed engineering, substantial staff is needed and it is here that the locally available engineers, technicians, draughtsmen and foremen can be trained in order that the host country strengthens her own infrastructure. The design standards, for example in the case of roads, particularly with regard to the road widths, radii and drainage, should be very carefully considered. Requirement of the width of the carriageway should be extremely carefully studied as any extra over the optimum will lead to a considerable loss of resources and any shortfall may lead to traffic congestion and bottlenecks. Where necessary, the minimum radius of a curve and the gradient might have to be suitably altered in order to avoid long and deep cuts and fills so as to reduce the danger of slides during heavy rains. This is particularly true for tropical countries and obviously detailed soil investigation is of paramount importance in the design stage. The roads may be generally paved with lighter pavement (such as double surface treatment) and in certain cases the roads are even left as simply gravel roads. This, sometimes, can lead to increased cost of maintenance, not to talk of the danger of accidents due to development of corrugations. Similarly in the design of structures, an account has to be taken of the availability of the locally available materials and skills. For example in some LDCs whether the bridge superstructure should be in steel or in concrete will obviously depend on what is easily available and the skill required for the same. By and large, many LDCs now possess cement works and produce steel reinforcing bars and light steel sections and therefore a concrete superstructure might well be more appropriate.

A typical road might begin as an earth track and then gradually be surfaced and re-constructed. In LDCs where the financial resources are scarce but labour and skills are available cheaply, obviously the choice is for labour intensive techniques and on the face of it there would be nothing wrong if materials for fill work are taken from the nearby pits along the route, loaded in the cane baskets and head-carried to the site and dumped, without calling for much earth moving machinery. But inherent fault may be the lack of proper compaction, which, in due course, may call for heavy maintenance cost for repairs and running. The question of using primitive construction methods compared to at least partially mechanised operations should therefore not be made purely a matter of 'minimum first cost'. A road has to be built to a certain standard and does require a certain minimum of mechanised operations which should not be substituted for by manual labour alone. The appropriateness of the technology will lie in the indepth study of the variables and preferably should not be sacrificed for the available technology alone.

Assessment of feasibility is a comparison between two or more alternative future situations, a systematic comparison of relative merits and demerits from technical, financial, economical, social, environmental, administrative and political points of view.

Financially the feasibility assessment would mean: 'what is the return of revenue for a certain financial input'. Economical feasibility assessment would mean to go further and study not just the revenue return for the financial input but also investigate the good it might do to the general public at large. Some projects that are not bright from the financial assessment angle, may still be economically desirable. Projects like railways, roads and ports will generally be profitable even from the financial angle but projects like hospitals, motorways, flood protection works, public health projects and community development centres may be economically viable even if not very much financially.

Most engineering projects have a long life and are undertaken more in the interest of future generations. Since the final decisions generally are not taken by those who conduct the study but by politicians, it is incumbent that political implications will at some stage influence the decision. In some LDCs the political importance of a new road/port/airport/bridge or an industrial estate may be more overpowering than any of the other criteria in the total feasibility analysis. The aim of a feasibility report should not be so much as to recommend a decision but to provide information that can lead to it.

Methodology

LDCs must endeavour to develop their own engineering capabilities in order to break their dependence on DC markets. Even technical assistance, unless restricted, pruned and evaluated regularly, reinforces relations of dependence.

Technology aid should be conceived not merely for filling LDC resource gaps, but as an attempt at developing the capability of LDC people. Unfortunately, many a time, technological issues in LDCs are discussed by non-technologist, and despite their pious intentions, this does not always help.

The DCs should make information available to LDCs and the U.N. should play a big role in this field. But this does not mean it would be possible for the U.N. to establish a data bank to give information on all available technical designs and processes. Every LDC should establish a technical information service to provide appropriate technical information. Advice and funds may be obtained through multilateral and bilateral aid sources. A considerable part of this information may be obtained for a small fee. However, know how and detailed information can still not be available without high cost.

DCs should be invited to aid the LDCs and to make an effort to translate their techniques into the ones that match the economic and labour conditions and the in-situ environments of the latter.

Detailed information about appropriate technologies should be collected, subjected to field trials and documented in suitable form for communication to the developing countries.

Environmental care is of importance not only to DCs but also to LDCs though the latter may not always afford its priority. The damage to environment from insufficiently planned development activities can in fact result in decrease rather than increase in the happiness. Mistakes made in the earlier years of the development of the DCs need not be repeated all over again in the LDCs.

After the projects are constructed, many times the element of maintenance is given secondary importance and soon the lack of it take its toll. The maintenance is of paramount importance and this requires training of manpower where not available.

Suggestions

All human beings, and engineers are no exception, are strongly linked to their own limits of experience. Extrapolation concerning change of size, type of loading, climate, construction materials, the levels of efficiency, feed-back, feed-forward and above all the way of life, should be taken up very carefully.

Planning and construction of a country's road network is generally placed under the jurisdiction of the PWDs working under the jurisdiction of its Ministry of Transport. These PWDs, particularly in LDCs, unfortunately may not be fully conversant with the latest and appropriate technical developments in modern methods of design and construction, and, for certain reasons cannot keep pace with the furious standards of workmanship and efficiency of the consultants and constractors. It is therefore necessary in such a case that the PWD concentrates on the overall planning and maintenance, leaving the engineering and construction to the consultants and contractors (of course typing them up with necessary contractual obligations).

Security lies in a technology that follows the natural-balance cycles. It is of great significance to bend technologies to suit the renewable sources of energy like wind, sun, water, vegetable-fuels, plant and animal products, soil, stone and above all the human labour.

It is not a good solution to use second-hand machinery, because, even if it may require less investment, it may well lead to an increase in production-cost and thereby widen the already existing technological gap between the DCs and LDCs. ***While some LDCs may have the finance (like the oil wealth) and therefore may be capable of calling for the most advanced technological expertise, they generally are handicapped in receiving technology. They may even suffer on account of the non-availability of their own skilled staff for taking over and, in the process, may spend far too much without fully utilising it. On the other hand the poorer LDCs may have most of the manpower already available but the lack of finance may compel them to search for self-reliance and compromise technology.*** A technology service should be established at the international level which could help the developing countries to set up technology cells and train skilled personnel.

Some capital intensive and high technology industries will still be needed even when the general policy may be to encourage small-scale industries. A certain measure of foreign investment and technical collaboration may still be required. It is necessary not to confuse self-reliance here and to accept the foreign investment and collaboration in such zones not from a sense of inadequacy but from a sense confidence essential to such sectoral development.

So far as possible, the whole of design should be carried out locally in the LDC even though in some cases it may even increase the cost. This kind of cost increment should be treated as a welcome investment. To get the entire package from a foreign country will only make the concerned LDC more dependent in future.

Practical difficulties arise in LDCs because the level of dependable information is usually low and sometimes statistical information available may be unreliable and therefore one should not incorporate more detail in the analysis than the data permits. The poorer the data, the simpler and cruder will be the analysis. Therefore in some of the LDCs it may be better to simplify the methodology since data may not be good enough. Their priorities may be different. This may require lower standards to be accepted for items like transport, electricity and water, in order to realise resources for basic housing, sewerage, health and education.

The continued and timely supply of various modern farm equipments and inputs such as factors, power-tillers, fertilisers, pesticides, etc., is very important for the agricultural base and the latter forms the back-bone of the infrastructure needed for technology development. To ensure this, indirect taxes should be lowered on these farm equipments and inputs. Agriculture is a high-risk industry, where, in case of a failure, the farmer loses all. Therefore it might be prudent to tax the farmer (as opposed to those owning farm holdings on big scale) on the output he sells than on the input he has to buy.

Certain industries suffer from lack of adequate demand to what they are capable of producing even within the existing limitations. Others face not so much the lack of demand as the inability to produce enough owing to a variety of reasons as for example: power shortage and transportation bottle-necks. These difficulties are magnified sometimes owing to their tie up with agricultural production. These problems require improvement in efficiency, better management and re-scheduling the priorities. Support and assistance through economic and fiscal policies is called for.

Between the two extremes of 'computers for people' approach on the one hand and 'spreading information as to how to block the operation of computer systems' on the other, lies the approach of compromise technology which is essentially labour based.

Although the case of each individual LDC has to be considered on its own merits, a very careful study will require to be made regarding whether labour intensive techniques, capital intensive techniques or a partial combination of the two should be adopted. Certain LDCs have some skilled and ample unskilled manpower available but not enough capital, and some other have the necessary capital but not enough manpower. ***If a time bound programme has to be followed it is essential to take up manpower training on a massive scale and this is where the U.N.'s D.E.S.D. comes in.***

48.10 BLUE-PRINT FOR GUIDING TECHNICAL DEVELOPMENT IN THE FIELD OF CIVIL ENGINEERING PUBLIC WORKS IN DEVELOPING COUNTRIES

(A) Synopsis

Even knowledge, unless indexed properly, cannot be quickly tapped when faced with a time-bound problem. By the same token it is essential that any technical development programme be at least guided, if not regimented, for a time-bound progress. This calls for first understanding the problem, then gearing up the means to tackle it and then marshalling a blue-print approach for solving it.

The presentation here indentifies some of the relevant civil engineering public works together with some of their details and possible problems, and reviews the working system of the Public Works Departments.

Then, in an attempt to rectify the rut and guide the technical development process, the author suggests a more relevant system of manpower-training and technical education.

After describing various phases that are incumbent in a civil engineering public work, the possible U.N. Technical Advisory assistance is recommended. The importance of applied research against pure research and the element of competition are stressed.

(B) Introduction

1. ***'The first object is to 'know'—within limits of our ability to do so, and the second is to 'apply'—without going beyond what we know.''***
2. If a time bound progress has to be achieved in technical development, it is essential to evaluate the problem and decide upon the solution philosophy in a workman-like manner.
3. Talented people have to tactfully use their discretion in preparing, appraising, planning and executing the technical development programmes, although actual implementation is not always easy. The political influence in any country, more so in a developing country, can thwart such activity unless tact and discretion are used.
4. Even after three-and-a-half decades of development efforts, the end-product less than matches the needs of the multiplying multitude of masses. Technical development can be achieved by massive manpower training as relevant to indigenous conditions and conversion of techniques to suit the local availability of materials, skills and environment.
5. This subject assumes all the more importance as the developing countries focus more attention on the mechanisms for achieving collective self-reliance. There is no alternative for the developing countries but essentially to search from within to strengthen their own infrastructures as the major burden of their development must finally be borne by them only. Even though many of these countries in the past have been physically conquered, politically subjugated, culturally aggressed and economically exploited, there is no point in wasting further time in this rhetoric as otherwise history will only repeat itself. We, as today's well-wishers of humankind, have an urgent job to do and must remember that the only thing dark about these countries is our own ignorance about them.
6. Whether it is the 'wealthy poor', the 'well-to-do poor', the 'less poor' or the 'rock-bottom' developing countries, they constitute a world of their own.
7. The aim basically has to be to bend the methods and technologies, to suit the indigenous availability of manpower, materials and equipment and simultaneously strengthen the infrastructure by a massive programme of technical training. The present paper attempts at putting forth a workman-like plan for achieving just this.

(C) Some Relevant Civil Engineering Public Works

1. Let us first identify the more relevant civil engineering public works and then identify some of their details briefly and possible problems.
2. ***The more relevant civil engineering public works are:***
   (a) Roads and Railways,
   (b) Bridges,
   (c) Buildings, e.g. Low cost housing, Administrative Offices, Schools, Hospitals, Community Development Centres, etc.
   (d) Water supply, drainage and sewerage works,
   (e) Canal networks and flood-protection schemes, and
   (f) Ports and Harbours.
3. ***Some details given very briefly***

(a) *Roads and Railways* The need of the project and terms of reference; reconnaissance survey, route location and alignment; rate of progress for laying out the alignment; minimum vertical and horizontal curves, gradients and radii; drainage; possibility of aerial photography for locating stretches in difficult locations: topographical and geological features; future development of areas fed by the road/rail; 'formation width' in relation to the contemplated future traffic; lands to be appropriated and acquired; structural works involved (e.g. bridges, culverts, causeways, drainage arrangements, retaining walls, rest houses, etc.); sources of materials (and leads involved), labour and plant available locally; how much labour-intensive and how much mechanised techniques to be adopted; plant and equipment requirement; quality of structure, possible causes of disintegration of road surface; subsoil drainage and control, method of rolling; size and grading of metal; design loads, traffic and speed; maintenance; preparation of preliminary estimates; agency for execution; project report.

(b) *Bridges* Need of the bridge; approaches to the bridge; appropriate location of the bridge; effective linear waterway requirement in keeping with the catchment area, drainage run-off, afflux, bed scour, etc.; vertical clearance requirement; maximum scour depth at foundations; minimum founding depths, right or skew bridge crossing; bridge width (i.e. single-lane, double-lane or multi-lane); requirement of footpaths—if any; design loadings; type of foundation (wells, piles, open foundations, etc.); spill-through or non-spill-through type of abutments; subsoil investigations and soil parameters; high flood level and low water level; wind intensity; earthquake intensity; maximum mean flood velocity; temperature variation; type of bridge, method of construction and optimum span length; possibility of ground movements and unequal settlement of foundations; whether concrete or steel superstructure; type of bearings and expansion joints; permissible stresses during construction and during service conditions, considering the appropriate load combinations; river training works (wired boulder crates and mattresses; guidebunds, spurs, etc.); preliminary design, drawings, quantities and estimate;

agency for execution; project report.

(c) *Buildings, e.g. low cost housing, administrative offices, schools, hospitals, community development centres, etc.* Need of the particular project at a particular location and terms of reference; functional requirements, i.e. estimation of the type and volume of service to cater for and correspondingly designation of the size, number of floors and the allied services involved; architectural requirements; structural requirements; preparation of preliminary architectural plans; preliminary structural designs, drawings, bills of quantities and estimates; agency for execution; project report.

(d) *Water supply, drainage and sewerage projects*

(i) ***Water supply***: Reasons necessitating the scheme/additional water supply; quality and quantity of existing supply and its sources; possible sources of additional supply and arrangements for its intake; discharge from wells, tests of yield from wells and construction of tube-wells and open wells; well linings, number of people, number of animals and the volume of industry to be catered for; estimated daily requirement and how calculated; purification, distribution and pumping; impurities in water, hard and soft waters and particular chemical processes required; storage and service reservoirs; settling basins and sedimentation tanks; coagulation, sterilisation and chlorination; colour and taste removal; domestic storage tanks; elevated tanks; pressurised supply; leakage and wastage of water; different methods of distribution and design of the system; design of mains and supply pressures at different points and taps; economical velocities in distribution pipes; whether supply will be metered; piping-material and jointing systems; type of pumps and power for working them; the total cost of the scheme, the annual recurring cost and the depreciation of the plant and equipment; cost estimates; executing agency; project report and terms of reference.

(ii) ***Drainage and sewerage***: Statement of problem; surface drainage; rainfall runoff from catchment, time of concentration and flood drainage, permeability of soil; combined and separate systems of drainage; open drains; underground sewers; size of sewers for different systems; volume of sewage for design, storm overflow; self-cleansing velocity in sewer lines; flushing of drains and sewers; section shape of sewers; choice of materials for making sewers; problems in setting out sewer-lines, and excavation; laying and jointing pipes and sewer lines, sewerage crossing; manholes, ventilation and maintenance; drainage traps and inspection chambers; testing against leakage; plumbing and fixtures; standards for public sanitary convenience; house disposal systems; disposal of sullage from towns; location of disposal works; preliminary treatment of sewerage; chemical precipitation of sewerage, filtration; sludge digestion and activated process; treatment of industrial wastes; methods of sewage disposal; cost estimates; executing agency; project report and terms of reference.

(e) *Canal networks and flood protection schemes* Need for a particular project; suitability of the soil for construction of embankments; stabilising side-slopes against slip failure; alignment of river and flood embankments; hydraulic gradient; closing of breaches; pitching and revetting materials for core-wall; cut-off trenches; construction of spurs; design and layout of distributories; water courses; service roads; top widths and boundary ditches; lining of canals; canal cross-sections; seepage and evaporation losses in canals and reservoirs; suitability of water for irrigation and of soil for reclamation; water logging; soil erosion; silting up in canals and reservoirs; storage of rain water run off; desilting and maintenance of canals; eradiction of water weeds; cross drainage works; aquaducts and syphons; problems in the design of weir floors; aprons and cut-off walls; non-silting velocity in canals; flood escape channels; cost estimates; executing agency; project report and terms of references.

(f) *Ports and harbours* Need for a particular project and terms of reference; cost-benefit forecast; effect of the project on the hinterland and its development; functional requirements; structural requirements; collection of data on wind, waves, currents, seismic intensity and sea bed contours; soundings; subsoil characteristics, scour patterns; alternative locations and alignments; recommended location and alignment and type of berthing structure with reasons; the overall integrated scheme; marine movements and road transportation of passengers and cargo; particulars of the soil investigation to be conducted and its possible effect on the overall cost; location of suitable entrance approach channel and availability of the turning radius for the ships; fixing the levels of the top of the berths and of the dredged floor on the basis of the estimated astronomical highest and lowest tides; extent of dredging and the requirements of the tug boats/turning dolphins; requirement of accessories on the berthing structures; buoys and entrance channel marks; types of fenders and recommendations; type of piles and foundation to be adopted; design criteria, e.g. loading, current forces, wave force, wind force, seismic force, scour of sea bed, impact force from docking of ships and hydrodynamic effect during docking; mooring-line forces, etc.; type of mooring dolphins and considerations for designs; preparation of preliminary designs, drawings, quantities and cost estimates; expected time required for construction; executing agency; project report.

4. ***Possible problems***

Problems can also raise from inadequacy and vague wording of the contract clauses and therefore it is necessary that the contract documents are prepared thoroughly and handled by those who are conversant with this subject and its legality. Examples of possible troublesome clauses are clauses pertaining to the amount of bond, the amount of third party insurance, time for completion of the works, amount of liquidated damage, length of maintenance period after handing over, the percentage and limit of retention money, the period within which the payment is to be made after the certificates have been issued, upward revision in cost owing to escalation in labour wages and material prices (incident to inflation), any change in scope of work necessitated either due to change in specifications and/or physical site conditions, accident or injury to workmen and the consequent compensation, interference with traffic and adjoining properties, access to site, expected risks and agreement about nomination of arbitrator, etc.

(D) Public Works System of Working

1. Before we proceed further, it will be appropriate to briefly touch upon the ***public works system of working***.
2. The ***Ministry in charge of Public Works***, under its Secretary (who is answerable to the Hon'ble Minister) has generally a Director General, the Additional Directors General (one for each discipline) and the Zonal Chief Engineers working under the Secretary. Each Chief Engineer has his compliment of Superintending Engineers, Executive Engineers, Assistant Engineers and technicians together with the supporting Administrative and Accounts staff.
3. While the Hon'ble Minister and the political party in power may change, the rest of the staff generally remains. ***Unfortunately the pulls and pushes of bureaucracy and obligation to hierarchy and political influence in most developing countries sometimes frustrate some of these engineers into becoming mere bureaucrats which prevents them from dabbling in the delights of bold, decisive and independent planning, design and construction.*** This has to be taken explicit note of in understanding the system of the state run public works departments (P.W.D.s). These departmental engineers may therefore generally not be fully conversant with the latest and with the appropriate technical developments in modern methods of design, construction and management, and for understandable reasons, cannot keep pace with the furious standards of result and profit oriented workmanship and efficiency of the independent professional consultants and contractors. It is therefore necessary in such a case that the P.W.D. concentrates on overall general planning and maintenance leaving the engineering and construction to the acknowledged Consultants and Contractors, of course tying them to necessary contractual obligations.
4. Most engineering projects have a long life and are undertaken more in the interest of future generations. Since the final decisions generally are not taken by those who conduct the feasibility and cost-benefit studies but by politicians, it is incumbent that ***political implications will at some stage influence the decision.*** In some developing countries the political importance of a new project may be more overpowering than any of the other criteria in its feasibility analysis.
5. ***Cost of works***

   (a) The total cost of a works is not only the final cost of its construction but also includes the expenditure to be incurred for its maintenance and the interest lost on the capital investment.

   (b) Cost of civil engineering public works are sometimes paid in instalments. Equal periodical payments which are allowed to accumulate, each earning its own interest, are called annuities. This sum of money is set aside annually to accumulate compound interest and form what is called a sinking fund in order to clear a debt.

   (c) In estimating operating expenses of engineering works, an allowance is made for depreciation. In calculating this, a certain life-span is first assumed for the work/plant, and then annuity, at an assumed rate of compound interest, for the assumed life span, is estimated so as to equal the cost of the work/plant and this provides a fund by means of which the work/plant may be replaced/replenished when needed.
6. If a particular job has to be done and delivered by a privately run contracting agency, its total cost will not only include the working cost of materials, labour, plant, engineering time and overheads, but also profits and provision for promotional expenses. The cost of engineering time will have to include not just the gross salaries but also certain employee benefits (e.g. provident fund, gratuity, leave travel, medical benefits, insurance and compensation, leave salary, etc.) and a bonus as an incentive for straining to stay alive in the competitive cut-throat market. Against this, the P.W.D. system hardly provokes competition in-house, and the result can be an out-of-focus design and costing and less than workman-like manner of working. (If a private firm worked with laxity, it would soon fold up owing to incisive competition from rivals.) Their overheads and costs mount owing to an excessively hierarchical manner of working and lack of independence and practising professional touch.

agency for executis not uncommon to see more engineers
D. overseeing a job than the number of
(c) Buildings, e.g. ie Consultant/Contractor actually doing
schools, hospitals art of doing a work with ones own
Need of the part thus become a casualty. This endemic
and terms of reting of many of the state-run P.W.D.s
estimation of the e drastically altered for a meaningful
for and correspon opment programme to flourish.
of floors and the
requirements; str **Technical Education**
preliminary archit
designs, drawings, indeed necessary, to tilt the technical
for execution; projeto an applied direction than leave it
mical.

(d) Water supply, &
(i) **Water supply:** students successfully clearing the 12-
ditional water chooling, a large majority should be
supply and its dustrial Training Institutes for shoplevel
supply and ap 3 year vocational training courses in
from wells, tesical fields. This cadre would form the
of tube-wells astructure at the grass-root level.
of people, nuthe interested school leaving students,
industry to be ter intellect and better bent of mind,
and how calced to the standard 5-year engineering
pumping; imput the Institutes of Technology. Here
and particular could be a rigorous blend of academic
service reservching. These would-be engineers should
tanks; coagulatith exposure to the views of practising
and taste removited from industry) by forging the
tanks; pressuriustry-university link-ups. In course of
water; different engineers who will percolate into
of the system; of the infrastructure, providing pivotal
at different poiat may be termed as battalion and
distribution pipme of them of course will have to man
piping-material later in their lives.
power for work of these graduate engineers should
the annual recu 'specialisation' on the basis of their
plant and equipt. This specialisation has to be really
project report ang over a period of about 3 years: about
(ii) **Drainage and s**is postgraduate study in the particular
drainage; rainfaon at the University level, followed by
concentration analf years' professional training, partly
combined and ily in construction, by rotation. Some
drains; undergrong engineers may of course go in for
systems; volume without the practical training back-
self-cleansing ve
and sewers; sectivity in the field of civil engineering
for making seweves not only design and construction
and excavation; rative and budgetary control, it is
lines, sewerage subjects of public-administration and
maintenance; draluded in the under-graduate studies.
testing against lethe technicians (who have come out of
for public sanitaring Institutes) and the engineers (who
have come out of the Institutes of Technology) are put through refresher courses at periodic intervals during the course of their service in order to keep them in touch and updated.

(F) Relating Experience to Level of Responsibility

1. This relationship should be interpreted very carefully, particularly in regard to minimum qualifications, experience, duties required to be performed, and the requirements of supervision, decision-making and commitment. The level of responsibility pertains more to a position than to person. Promotions should not be based purely on 'age' and 'number' of years of experience, because some people may show considerable promise at a relatively early age and unless they are picked up and nurtured, the resulting frustration can be degenerative. An individual's position will have a lot to do with the merit of his background and experience, his efficiency in performance and of course the prevailing market value.
2. However, it is always a good idea to make lateral recruitment at higher levels from time to time for injecting fresh blood into the system. Some experts in individual fields should also be brought in on a term-appointment basis in order to get the benefit of their expertise for obvious advantage.

(G) Phases

1. Construction of a civil engineering public work project should be looked at in *five phases*, namely feasibility investigation, engineering investigation, design, construction, and maintenance.

(a) In the ***feasibility investigation phase*** broad studies are undertaken including the questions of viability, and consequently experts of many disciplines may have to be involved. Net product is a report upon which the Clients, together with their financial advisors, will decide if the scheme has to progress further.

(b) In the ***engineering investigation phase*** the study enters more deeply into the engineering aspects and detailed site surveys and soil investigations, etc. Special investigations may be necessary in order to facilitate the study of alternative engineering solutions.

(c) In the ***design phase*** detailed engineering design and drawings are prepared together with the contract documents, specifications and Bills of Quantities, in order to enable the Client to invite tenders for evaluation, acceptance and taking up construction.

(d) In the ***construction phase*** the authorities have to see that the construction proceeds in accordance with the drawings and specifications under appropriate supervision on site. The appointment of specialist Inspectors and Expeditors may be called for. The authorities have to check the progress and the quality of the work in relation to the programme and terms of the contract.

(e) In the *maintenance phase* the authorities may have to devise the maintenance procedures and effect their implementation lest lack of maintenance should nullify the entire effort.

(H) U.N. Assistance

1. ***The U.N. and its affiliates should strengthen their technical Advisory Teams which can assist on the following lines:***

   (a) Review the need for different public works projects in individual countries.

   (b) Assess the techno-economic feasibility of the assistance-requests from the individual countries for various public work projects.

   (c) Detail out the terms for field inspections and project reporting.

   (d) Assist in evaluating the appropriateness of the relevant experts who may be drafted in for individual assignments.

   (e) Review the write-ups of specifications and contract documents.

   (f) Assist in scrutinising the technical proposals and bids, and in making recommendations for the award of contracts.

   (g) Oversee and analyse the field reports and review the implementation of projects periodically.

   (h) Assist in-site supervision and quality control checks, maintenance checks, and manpower-training programmes through table-top discussions and refresher courses, clearly distinguishing between 'theory' and 'professional-practice'.

   (i) Study the existing work methods of the various Public Works Departments by working from within them for short periods of time, and then suggest ways of improving the same in an attempt to make them competitively self-reliant.
2. However, the U.N. cannot establish a data-bank to give information on all available technical designs and processes. It is the developed countries that should establish technical information services to provide appropriate technical information. Advice and funds may be obtained through multilateral and bilateral aid sources.
3. An international appropriate technology service should be established at the U.N. level which could help the developing countries to set up technology cells and train the skilled personnel.

(I) R and D

1. Much depends on how result-oriented is the technical training imparted to its recipients. ***Manpower training is a capital-saving process while research is capital-using process, and though the latter should never be stopped altogether, the former definitely deserves more attention in the present context.***
2. The less developed countries many times are not properly equipped to face the choice of technology. ***It may be desirable to subordinate science to economy, at least in the early stages***, since the R and D facilities and resources may be scarce and the results desired faster. ***It has to be remembered that for the purpose of development, time cannot be stilled and, therefore, good use has to be made of local skills, manpower and equipment even as we go along the developmental process transforming the traditional into the modern, without waiting too much for research.***

(J) Competition

Healthy but stiff competition alone leads to improvements. Therefore while some designs and techniques must be standardised for the obvious dual aim of simplifying procedures and cutting out avoidable multiplication of effort and time, the element of competition for better and bolder designs and construction methods should be religiously encouraged. ***This will lead to better designers and constructors which finally are the barometer of the quality of technical development.***

48.11 INSPECTION CHECKLIST (... field work)

An inspection checklist should be developed by the safety inspector or safety engineer with the help of executives, superintendents, and foremen, tailored to the requirements of the individual operations of the work. General checklists which apply to most of the problems, likely to be encountered will be easily developed. In addition, 'special checklists' for each individual job will be necessary, as peculiar problems are bound to occur. These will include, but will not be limited to, soil conditions, access roads, storage areas, power and utility lines, special equipment, camp housing, and other features.

The following list is offered only as a general guide, and is not intended to be comprehensive, nor should it be followed in this form if job requirements would be better served otherwise.

General Checklist

1. Accident Prevention Organization

a. Schedule for posting safety material.

b. Hard hat requirements.

c. Safety meetings scheduled and posted.

2. ***Housekeeping and Sanitation***
   a. General neatness of working areas.
   b. Regular disposal of waste and trash.
   c. Passageways and walkways clear.
   d. Adequate lighting.
   e. Projecting nails removed.
   f. Oil and grease removed.
   g. Waste containers provided and used.
   h. Sanitary facilities adequate and clean.
   i. Drinking water tested and approved.
   j. Adequate supply of water.
   k. Salt tablets.
   l. Drinking cups or sterilized bubblers.
3. ***First Aid***
   a. First aid station.
   b. First aid supplies.
   c. First aid instruction on the job.
   d. Telephone numbers and locations of nearby physicians.
   e. Telephone number and location of nearest hospital.
   f. Injuries reported promptly to proper persons.
4. ***Fire Prevention***
   a. First instructions to personnel.
   b. Fire extinguishers identified, checked, lighted.
   c. Phone number of fire department posted.
   d. Hydrants clear, access to public thoroughfare open.
   e. Good housekeeping.
   f. NO SMOKING posted and enforced where needed.
5. ***Electrical Installations***
   a. Adequate wiring, well insulated.
   b. Fuses provided.
   c. fire hazards checked.
   d. Electrical dangers posted.
   e. Proper fire extinguishers provided.
6. ***Hand Tools***
   a. Proper tool being used for each job.
   b. Neat storage, safe carrying.
   c. Inspection and maintenance.
   d. Damaged tools repaired or replaced promptly.
7. ***Power Tools***
   a. Good housekeeping where tools are used.
   b. Tools and chords in good condition.
   c. Proper grounding.
   d. Proper instruction in use.
   e. All mechanical safeguards in use.
   f. Tools neatly stored when not in use.
   g. Right tool being used for the job at hand.
   h. Wiring properly installed.
   i. Enough men used to handle material.
8. ***Powder Actuated Tools***
   a. Local laws and ordinances complied with.
   b. All operators qualified.
   c. Tools and charges protected from unauthorized use.
   d. Competent instruction and supervision.
   e. Tools checked and in good working order.
   f. Tools not used on any but recommended materials.
   g. Safety goggles or face shields.
   h. Flying hazard checked by backing up, removal of personnel, or use of captive stud tool.
9. ***Ladders***
   a. Stock ladders inspected and in good condition.
   b. Stock ladders not spliced.
   c. Properly secured, top and bottom.
   d. Siderails on fixed ladders extend above top landing.
   e. Built-up ladders constructed of sound materials.
   f. Rungs not over 12 inches on centre (preferably)
   g. Stepladders fully open when in use.
   h. Metal ladders not used around electrical hazards.
   i. Proper maintenance and storage.
10. ***Scaffolding***
    a. Erection under proper supervision.
    b. All structural members adequate for use.
    c. All connections adequate.
    d. Safe tie-in to structure.
    e. Ladders and working areas free of debris, grease.
    f. Proper footings provided.
    g. Passersby protected from falling objects.
    h. Supports plumb, adequate crossbracing provided.
    i. Guard rails and toeboards in place.
    j. Scaffold machines in working order.
    k. Ropes and cables in good condition.
    l. Frequent inspection.
11. ***Hoists, Cranes and Derricks***
    a. Inspect cables and sheaves.
    b. Check slings and chains, hooks and eyes.
    c. Equipment firmly supported.
    d. Outriggers used if needed (stability).
    e. Power lines inactivated, removed or at safe distance.
    f. Proper loading for capacity at lifting radius.
    g. All equipment properly lubricated and maintained.
    h. Signalmen where needed.
    i. Signals understood and observed.
12. ***Heavy Equipment***
    a. Regular inspection and maintenance.
    b. Lubrication and repair of moving parts.
    c. Lights, brakes, warning signals operative.
    d. Wheels checked when necessary, tyre pressures checked.
    e. Haul roads well maintained and laid out properly.
    f. Protection when equipment is not in use.
13. ***Motor Vehicles***
    a. Regular inspection and maintenance.
    b. Qualified operators.

c. Local and state vehicle laws and regulations observed.
d. Brakes, lights, warning devices operative.
e. Weight limits and load sizes controlled.
f. Personnel carried in a safe manner.

14. ***Conveyers and Cableways***
a. Proper inspection and maintenance.
b. Good housekeeping.
c. Screens and other protection where needed.
d. Adequate inspection, maintenance ladders, and walkways.
e. Lighting.

15. ***Marine Equipment***
a. Waterways rules and regulations observed.
b. Life jackets.
c. Life preservers.
d. Fire protection.
e. Engine and machinery room safety.
f. Floating pipelines, walkways, and handrails.
g. Handrails aboard vessels.
h. Decks clear, free of oil and grease.
i. Lights.
j. Cable.

16. ***Equipment Maintenance***
a. Planned maintenance and inspection programme.
b. Adequate equipment records.
c. Proper oils, fuels, lubricants used.

17. ***Garages and Repair Shops***
a. Fire hazards.
b. Dispensing of fuels and lubricants.
c. Good housekeeping.
d. Lighting.
e. Carbon monoxide dangers.

18. ***Barricades***
a. Floor openings planked over or barricaded.
b. Roadways and sidewalks effectively protected.
c. Adequate lighting provided.
d. Traffic controlled.

19. ***Handling and Storage of Materials***
a. Neat storage area, clear passageways.
b. Materials neatly stacked.
c. Stacks on firm footings, not too high.
d. Proper number of men for each operation.
e. Men picking up loads correctly.
f. Materials protected from heat and moisture.
g. Protection against falling into hoppers and bins.
h. Dust protection observed.
i. Extinguishers and other fire protection.
j. Traffic routing and control.

20. ***Excavation and Shoring***
a. Shoring of adjacent structures.
b. Shoring and sheathing as needed for soil and depth.
c. Public roads and sidewalks supported and protected.
d. Materials not too close to edge of excavations.
e. Lighting at night.
f. 'Water' controlled.
g. Equipment at safe distance from edge.
h. Ladders or stairs provided where needed.
i. Equipment ramps adequate, slope not too great.
j. Frequent inspection.

21. ***Pipelines***
a. Shoring and bracing as needed.
b. Equipment in working order.
c. Proper access to deep trenches.

22. ***Demolition***
a. Operations planned ahead.
b. Shoring of adjacent structures.
c. Material chutes.
d. Sidewalk and other public protection.
e. Clear operating space for trucks and other vehicles.
f. Adequate access ladders or stairs.
g. Regular supervision.

23. ***Pile Driving***
a. Proper storage procedures.
b. Unloading only by properly instructed and experienced workmen.
c. Steam lines, slings, etc., in operating condition.
d. Piledriving rigs properly supported.
e. Ladders on frames.
f. Care observed by 'top' man.
g. Cofferdams maintained and inspected.
h. Adequate pumping available.
i. Proper bracing and ties in current.

24. ***Tunnels***
a. Adequate ventilation.
b. Adequate lighting.
c. Good housekeeping.
d. Tunnel supports.
e. Electrical lines.
f. Operation of hauling equipment.
g. Proper personal protection.
h. Gas detectors used.
i. Proper transportation of personnel.
j. Control of blasting operations.
k. Dust protection.
l. Drilling safety observed.

25. ***Shafts***
a. Ladders, stairs adequate and safe.
b. Top of shaft barricaded, toeboards provided.
c. Adequate lighting.
d. Ventilation.
e. Inspection and maintenance of elevators and hoists.
f. Signals being used.
g. Shoring and bracing.

26. ***Compressed Air Work***
   a. Time limits for pressure and exertion observed.
   b. Decompression chamber provided.
   c. Gauges operating.
   d. Exhaust valves.
   e. Signals and telephone connection.
   f. Ventilation.
   g. Sanitary facilities.
   h. Medical Doctor at site.
27. ***Boilers***
   a. State and local regulations and codes observed.
   b. Pressure gauges checked and operating.
   c. Equipment in good working order.
28. ***Pressure Vessels***
   a. Gauges operating and checked regularly.
   b. Equipment in sound working condition.
   c. Connections properly made.
29. ***Explosives***
   a. Qualified operators.
   b. Proper transport vehicles.
   c. Local laws and regulations observed.
   d. Storage magazines constructed per regulations or as recommended.
   e. Experienced personnel handling explosives at all times.
   f. Cases opened with wooden tools only.
   g. NO SMOKING posted and observed where appropriate.
   h. Detonators tested before each shot.
   i. All personnel familiar with signals, and signals properly used at all times.
   j. Inspection after each shot.
   k. Proper protection and accounting for all explosives at all times.
   l. Proper disposition of wrappings, waste, and scrap.
   m. Advise residents nearby of blasting cap danger.
   n. Check radio frequency hazards.
30. ***Inflammable Gases and Liquids***
   a. All containers clearly identified.
   b. Proper storage practices observed.
   c. Fire hazards checked.
   d. Proper storage temperatures and protection.
   e. Proper types and number of extinguishers nearby.
   f. Carts for moving cylinders.
31. ***Welding and Cutting***
   a. Qualified operators.
   b. Screens and shields.
   c. Goggles, gloves, clothing.
   d. Equipment in operating condition.
   e. Electrical equipment 'grounded'.
   f. Power cables protected and in good repair.
   g. Fire extinguishers of proper type nearby.
   h. Inspection for fire hazards.
   i. Inflammable materials protected.
   j. Gas cylinders chained upright.
   k. Gas lines protected and in good condition.
32. ***Steel Erection***
   a. Safety nets.
   b. Hard hats, safety shoes, gloves.
   c. Taglines for tools.
   d. Fire hazards at rivet forge and welding operations.
   e. Floor opening covered and barricaded.
   f. Ladders, stairs, or other access provided.
   g. Hoisting apparatus checked.
33. ***Concrete Construction***
   a. Forms properly installed and braced.
   b. Adequate shoring, plumbed and crossbraced.
   c. Shoring remains in place until strength is attained.
   d. Proper curing period and procedures.
   e. Check heating devices/cooling devices (adverse weather)
   f. Mixing and transport equipment supported, and traffic planned and routed.
   g. Adequate runways.
   h. Protection from cement dust.
   i. Hardhats, safety shoes, shirts covering skin.
   j. Nails and stripped-form material removed from area.
34. ***Masonry***
   a. Proper scaffolding.
   b. Masonry saws properly equipped, dust protection provided.
   c. Safe hoisting equipment.
35. ***Highway Construction***
   a. Laws and ordinances observed.
   b. Competent flagmen properly dressed, instructed, posted.
   c. Adequate warning signs and markers.
   d. Equipment not blocking right-of-way.
   e. Traffic control through construction site.
   f. Adequate marking and maintenance of detours.
   g. Dust control.
   h. Adequate lighting.
36. ***Quarries and Gravel Pits***
   a. Proper signals and safety procedures for blasting.
   b. Screening plants operated safely.
   c. Traffic routed and controlled.
37. ***Prestressing***
   a. Approved execution drawings and details of anchorages, cables, tendons, ducting, groutpipes, tendon profiles, type of stressing jacks, limiting values of Jacking Forces, tendon extensions, expected slip at anchors (pull-in), adjustment of extensions with variation in Modulus of Elasticity value, required concrete strengths at various stages

of prestressing etc.

b. Instructions: 'what to do' if acceptable extensions or jack pressures not attained at appropriate conjoint values.
c. Check on long term slips (24 hr)
d. Grouting instructions (cleaning, mix, pressure procedure).
e. Protection against heating, rusting, pitting, oiling, etc.
f. Adequately experienced crew, conversant with all procedures and cautions in prestress application.
g. Maintenance of jacks, oil baths, gauges, pressure hoses, joints and connections in lines, master wedges, including periodical calibration of pressure-gauges.
h. Proper storage of tendons, ducts, anchorages and equipment.

NOTE: Also see the relevant details in the chapter on "Construction considerations" in the author's other book: *"Concrete Bridge Practice ... Construction, Maintenance and Rehabilitation"*.

SOME USEFUL REFERENCES

1. Carison, R.W., D.L. Houghton, and M. Polivka "Causes and Control of Cracking in Unreinforced Mass Concretes, ACI Journal, *Proceedings*, V.76. No.7, July 1979, pp. 821–837.
2. Verbeck, G.J. "Mechanisms of Corrosion of Steel in Concrete". *Corrosion of Metals in Concrete*, SP-49, American Concrete Institute, Detroit, 1975, pp. 21–38.
3. Beeby, A.W., "Cracking, Cover and Corrosion of Reinforcement", *Concrete International: Design & Construction*, V.5, No.2, Feb. 1983, pp. 35–40.
4. "Durability of concrete Bridge Decks", *NCHRP Synthesis of Highway Practice* No.57, Transportation Research Board, Washington, D.C., May 1979, p. 61.
5. Verbeck, G. "Carbonation of Hydrated Portland Cement." *Cement and Concrete.*, STP-205. American Society for Testing and Materials, Philadelphia. 1958. pp. 17–36. Also, *Research Department Bulletin* No. 87, Portland Cement Association.
6. Kaminetzky, D. "Failures During and After Construction". *Concrete Construction*. V. 26. No. 8, Aug. 1981, pp. 641–649.
7. Mast, R.F., "Roof Collapse at Antioch High School," *Journal Prestressed Concrete Institutes*, V. 26, No. 3, May-June 1981, pp. 29–53.
8. Gergely, P. and L. A. Lutz, " Maximum Crack Width in Reinforced Concrete Flexural Members," *Causes, Mechanism, and Control of Cracking in Concrete*, SP-20, American Concrete Institute, Detroit, 1968, pp. 87–117.
9. Broms, B.B., "Crack Width and Crack Spacing in Reinforced Concrete Members." ACI Journal, *Proceedings*, V. 62. No. 10, Oct. 1965, pp. 1237–1256.
10. Broms, B.B., and L.A. Lutz, "Effects of Arrangement of Reinforcement on Crack Width and Spacing of Reinforced Concrete Members," ACI Journal, *Proceedings*, V. 62, No. 11. Nov. 1965, pp. 1395–1410.
11. Iliston, J.M., and R.F. Stevens, "Long-Term Cracking in Reinforced Concrete Beams," *Proceedings*, Institution of Civil Engineers (London), Part 2. V. 53. Dec. 1972, pp. 445–459.
12. Stratton, F. W., R. Alexander, and W. Nolting, "Cracked Structural Concrete Repair through Epoxy Injection and Rebar Insertion." *Report* No. FHWA-KS-RD. 78-3, Kansas Department of Transportation, Topeka, Nov. 1978, p. 56.
13. "Forensic Investigations—Part 1, some Tools," *Building Failure Forum*, V. I. No. 6, June 1981, pp. 11–12.
14. Clear, K.C. and R.E. Hay, "Time-to-Corrosion of Reinforcing Steel in Concrete Slabs. V.I. Effects of Mix Design and Construction Parameters," *Report* No. FHWA-RD-73-32, Federal Highway Administration, Washington, D.C. 1973, 103 pp.
15. Malhotra, V.M., *Testing Hardened Concrete: Nondestructive Methods*, ACI Monograph No. 9, American Concrete Institute/Iowa State University Press, Detroit, 1976, p. 118.
16. "Concrete Repair Restoration," *ACI Compilation* No. 5. American Concrete Institute, Detroit, 1980, p. 118. (compiled from *Concrete International: Design & Construction*, V. 2. No. 9, Sept. 1980).
17. Stratton, F. Wayne, McCollon, and F. Bruce, "Repair of Hollow or Softened Areas in Bridge Decks by Rebonding with Injected Epoxy Resin or Other Polymers," *Report* No. K-F-72-5, State Highway Commission of Kansas, Topeka, July 1974, p. 104.
18. Johnson, M. Sydney, *Deterioration, Maintenance, and Repair of Structures*, McGraw-Hill Book Co., New York, 1965, p. 373.
19. Stratton, F. Wayne. "Custom Concrete Drill Helps Repair Shear Cracks in Bridge Girders," *Concrete International Design & Construction*, V. 2, No. 9. Sept. 1980, pp. 118–119.
20. Stratton, F. Wayne; Alexander, Roger; and Nolting, William, "Development and Implementation of concrete Girder Repair by Post-Reinforcement," *Report* No. FHWA-KS-82-1, Kansas Department of Transportation, Topeka, May 1982, p. 31.
21. ***American Concrete Institute***

201.1R-68	(Reaffirmed 1979) Guide for Making a Condition Survey of Concrete in Service.
201.2R-77	(Reaffirmed 1982) Guide to Durable Concrete
207.1R-70	(Reaffirmed 1980) Mass Concrete for Dams and Other Massive Structures
207.2R-73	(Reaffirmed 1980) Effect of Restraint, Volume Change, and Reinforcement on Cracking of Massive Concrete
224R-80	Control of Cracking in Concrete Structures
302.1R-80	Guide to Concrete Floor and Slab Construction
304-73	(Reaffirmed 1983) Recommended Practice for Measuring, Mixing, Transporting and Placing Concrete
305R-77	(Revised 1982) Hot Weather Concreting
308-81	Standard Practice for Curing Concrete
309-72	(Revised 1982) Standard Practice for Consolidation of Concrete
309.2R-82	Identification and Control of Consolidation-Related Surface Defects in Formed Concrete
318-83	Building Code Requirements for Reinforced Concrete
345-82	Standard Practice for Concrete Highway Bridge Deck Construction
347-78	(Reaffirmed 1984) Recommended Practice for Concrete Formwork
350R-83	Concrete Sanitary Engineering Structures
503R-80	Use of Epoxy Compounds with Concrete
503.3-79	Standard Specification for Producing a Skid-Resistant Surface on Concrete by the Use of Multi-Component Epoxy System
504R-77	Guide to Joint Sealants for Concrete Structures
517.2R-80	Accelerated Curing of Concrete at Atmospheric Pressure-State of the Art
546.1R-80	Guide for Repair of Concrete Bridge Superstructures
548R-77	(Reaffirmed 1981) Polymers in Concrete

22. ***American Society for Testing and Materials***

C 595-82	Standard Specification for Blended Hydraulic Cements

C 597-71	(Reapproved 1979) Standard Test Method for Pulse Velocity through Concrete
C 876-80	Standard Test Method for Half Cell Potentials of Reinforcing Steel in Concrete
C 881-78	Standard Specifications for Epoxy-Resin-Base Bonding Systems for Concrete

23. ***American Association of State Highway and Transportation Officials (AASHTO)***
    ... Various Design Specifications

APPENDIX 1

Metric Units and Conversion Factors

Subject	Imperial Unit	S.I. Unit	Symbol	Conversion factor
Length	mile	kilometre	km	1 mile = 1.609 km
	yard	metre	m	1 yd = 0.914 m
	foot	metre or millimetre	m or mm	1 ft = 0.305 m = 304.8 mm
	inch	millimetre	mm	1 in = 25.400 mm
Area	square mile	square kilometre	km^2	1 $mile^2$ = 2.590 km^2
	acre	square kilometre or hectare	km^2 or ha	1 acre = 0.004 km^2 = 0.405 ha
	square yard	*square metre*	m^2	1 yd^2 = 0.836 m^2
	square foot	square metre	m^2	1 ft^2 = 0.093 m^2
	square inch	square millimetre	mm^2	1 in^2 = 645.16 mm^2
Volume	cubic yard	cubic metre	m	1 yd^3 = 0.765 m^3
	cubic foot	cubic metre	m^3	1 ft^3 = 0.0283 m^3
	cubic inch	cubic millimetre	mm^3	1 in^3 = 16.387.1 mm^3
Capacity	gallon	litre	litre	1 gal = 4.546 litre
Mass of materials	*ton*	*tonne*	*tonne*	*1 ton = 1.016 tonne*
	hundred weight	kilogramme	kg	1 cwt = 50.802 kg
	pound	kilogramme	kg	1 lb = 0.454 kg
	ounce	gramme	g	1 oz = 28.350 g
Density	pounce per cubic foot	kilogramme per cubic metre	kg/m	1 lb/ft^3 = 16.019 kg/m^3
	pound per cubic yard			1 lb/yd^3 = 0.593 kg/m^3
Force	pound force	newton	N	1 lbf = 4.448 N
	ton force	kilonewton	kN	1 tonf = 9.964 kN
	pound force per foot	newton per metre	N/m	1 lb/ft = 14.591 N/m
	ton force per foot	kilonewton per metre	kN/m	1 ton/ft = 32.690 kN/m

Subject	Imperial Unit	S.I. Unit	Symbol	Conversion factor
Pressure	pound force per square foot	newton per square metre	N/m^2	$1\ lbf/ft^2 = 47.880\ N/m^2$
	pound force per square inch	newton per square millimetre	N/mm^2	$1\ lbf/in^2 = 0.00689\ N/mm^2$
	ton force per square foot	kilonewton per square metre	kN/m^2	$1\ ton/ft^2 = 107.250\ kN/m^2$
	ton force per square inch	kilonewton per square millimetre	kN/mm^2	$1\ tonf/in^2 = 0.0154\ kN/mm^2$
Stress	pound force per square inch	newton per square millimetre	N/mm^2	$1\ lbf/in^2 = 0.00689\ N/mm^2$
	ton force per square inch	newton per square millimetre	N/mm^2	$1\ tonf/in^2 = 15.444\ N/mm^2$
	ton force per square foot	kilonewton per square metre	kN/m^2	$1\ tonf/ft^2 = 107.250\ kN/m^2$
Modulus of elasticity	pound force per square inch	newton per square millimetre	N/mm^2	$1\ lbf/in^2 = 0.006.89\ N/mm^2$
Bending	pound force inch	newton millimetre or newton metre	N mm or N m	1 lbf in = 112.985 N mm = 0.1113 N m
	pound force foot	newton metre	N m	1 lbf ft = 1.356 N m
	ton force foot	kilonewton metre	kN m	1 tonf ft = 3 037 kN m
Section modulus	in^3	mm^3	mm^3	$1\ in^3 = 16\ 386\ mm^3$
Second moment of area	in^4	mm^4	mm^4	$1\ in^4 = 416\ 210\ mm^4$

DECIMALS OF AN INCH
For each 64th of an inch
With Millimetre Equivalents

Fraction	1/64ths	Decimals	Millimetres (Approx.)	Fraction	1/64ths	Decimals	Millimetres (Approx.)
...	1	.015625	0.397	...	33	.515625	13.097
1/32	2	.03125	0.794	17/32	34	.53125	13.494
...	3	.046875	1.191	...	35	.546875	13.891
1/16	4	.0625	1.588	9/16	36	.5625	14.288
...	5	.078125	1.984	...	37	.578125	14.684
3/32	6	.09375	2.381	19/32	38	.59375	15.081
...	7	.109375	2.778	...	39	.609375	15.478
1/8	8	.125	3.175	5/8	40	.625	15.875
...	9	.140625	3.572	...	41	.640625	16.272
5/32	10	.15625	3.969	21/32	42	.65625	16.669
...	11	.171875	4.366	...	43	.671875	17.066
3/16	12	.1875	4.763	11/16	44	.6875	17.463
...	13	.203125	5.159	...	45	.703125	17.859
7/32	14	.21875	5.556	23/32	46	.71875	18.256
...	15	.234375	5.953	...	47	.734375	18.653
1/4	16	.250	6.350	3/4	48	.750	19.050
...	17	.265625	6.747	...	49	.765625	19.447
9/32	18	.28125	7.144	25/32	50	.78125	19.844
...	19	.296875	7.541	...	51	.796875	20.241
5/16	20	.3215	7.938	13/16	52	.8125	20.638
...	21	.328125	8.334	...	53	.828125	21.034
11/32	22	.34375	8.731	27/32	54	.84375	21.431
...	23	.359375	9.128	...	55	.859375	21.828
3/8	24	.375	9.525	7/8	56	.875	22.225
...	25	.390625	9.922	...	57	.890625	22.622
13/32	26	.40625	10.319	29/32	58	.90625	23.019
...	27	.421875	10.716	...	59	.921875	23.416
7/16	28	.4375	11.113	15/16	60	.9375	23.813
...	29	.453125	11.509	...	61	.953125	24.209
15/32	30	.46875	11.906	31/32	62	.96875	24.606
...	31	.484375	12.303	...	63	.984375	25.003
1/2	32	.500	12.700	1	64	1.000	25.400

APPENDIX 2

Some General Data

SI PREFIXES

Multiplication Factor			Prefix	Symbol
1 000 000 000 000 000 000	=	10^{18}	exa	E
1 000 000 000 000 000	=	10^{15}	peta	P
1 000 000 000 000	=	10^{12}	tera	T
1 000 000 000	=	10^{9}	giga	G
1 000 000	=	10^{6}	mega	M
1 000	=	10^{3}	kilo	k
100	=	10^{2}	hecto	h
10	=	10^{1}	deka	da
0.1	=	10^{-1}	deci	d
0.01	=	10^{-2}	centi	c
0.001	=	10^{-3}	milli	m
0.000 001	=	10^{-6}	micro	μ
0.000 000 001	=	10^{-9}	nano	n
0.000 000 000 001	=	10^{-12}	pico	p
0.000 000 000 000 001	=	10^{-15}	femto	f
0.000 000 000 000 000 001	=	10^{-18}	atto	a

Temperature Conversion Tables

$$\text{deg C} = \frac{\text{deg F} - 32}{1.8}, \quad \text{deg F} = 1.8 \text{ deg C} + 32$$

deg C	deg F		deg F	deg C
−40	−40.0		−40	−40.0
−35	−31.0		−30	−34.4
−30	−22.0		−20	−28.8
−25	−13.0		−10	−23.3
−20	−4.0		0	−17.7
−15	+5.0		+10	−12.2
−10	14.0		20	−6.6
−5	23.0		30	−1.1
0	32.0		40	+4.4
+5	41.0		50	10.0
10	50.0		60	15.5
15	59.0		70	21.1
20	68.0		80	26.6
25	77.0		90	32.2
30	86.0		100	37.7
35	95.0		110	43.2
40	104.0		120	48.8
45	113.0		130	54.4
50	122.0		140	60.0

(Contd.)

deg C	deg F		deg F	deg C
55	131.0		150	65.5
60	140.0		160	71.1
65	149.0		170	76.6
70	158.0		180	82.2
75	167.0		190	87.7
80	176.0		200	93.3
			210	98.8
90	194.0		220	104.4
100	212.0			
120	248.0		240	115.5
140	284.0		260	126.6
160	320.0		280	137.7
180	356.0		300	148.9
200	392.0		400	204.4
300	572.0		500	260.0
400	752.0		600	315.6
500	932.0		800	426.7

An *approximate* method of converting temperatures which is correct to within 5 deg F, between 0 and 100 deg F, is—

$$\text{deg C} = \frac{\text{deg F} - 30}{2}$$

Shrinkage of Concrete

The shrinkage of concrete made with quartz aggregate due to ageing is approximately—

Time	*Shrinkage (per cent)*
After 28 days	0.025
After 3 months	0.035
After 12 months	0.050

Conversion Factors

Note that it is strict continental practice to use lower case letters always for unit unless the unit derives from a name, e.g. Watt—W, Fahrenheit—F, and kilowatt—kW.

To convert	Into	Multiply by	Reciprocal
Linear			
Inches	centimetres	2.54	0.3972
Inches	metres	0.0254	39.3701
Feet	centimetres	30.4799	0.0328
Yards	metres	0.9144	1.0936
Miles (5,280 ft)	kilometres	1.6093	0.62137
Chains	miles	0.0215	80.0
Furlongs	miles	0.125	8.0
Area			
Acres	square chains	10.0	0.1
Acres	hectares	0.4047	2.471
Acres	square miles	0.0016	640.0
Acres	square yards	4,840.0	0.0002
Square inches	square centimetres	6.4516	0.155
Square feet	square decimetres	9.2903	0.1076
Square yards	square metres	0.8361	1.196
Square chains	square metres	404.7	0.0025
Square chains	square yards	484.0	0.0021

(Contd.)

To convert	Into	Multiply by	Reciprocal
Square feet	square metres	0.0929	10.764
Square yards	hectares	0.00008	11,960.0
Square inches	square metres	0.0000645	1,550.0
Volume			
Cubic inches	cubic centimetres	16.387	0.061
Cubic feet	cubic metres	0.02832	35.32
Cubic feet	litres	28.317	0.0353
Cubic yards	cubic metres	0.7645	1.308
Cubic inches	cubic decimetres	0.01639	61.024
Miscellaneous			
British Thermal Units	calories	252.0	0.004
British Thermal Units	kilocalories	0.252	3.968
Btu per square foot	kilocalories per square metre	2.713	0.369
Atmospheres	pounds per square inch	14.7	0.0679
Feet of water	pounds per square inch	0.4335	2.3067
Metres of water	pounds per square inch	1.418	0.7031
Inches of mercury	pounds per square inch	0.4912	2.036
Millimetres of mercury	pounds per square inch	0.0193	51.71
Kilowatts	horsepower	1.34	0.746
Miles per hour	feet per second	1.4667	0.6818
Weight			
Ounce (Avoirdupois)	drams	16	0.0625
Ounce (Av.)	grammes	28.35	0.0353
Ounce (Troy)	grammes	31.1035	0.03215
Pound (16 oz)	kilogrammes	0.45359	2.2046
Hundred weights (112 lb)	kilogrammes	50.8	0.0197
Tons (2,240 lb)	tonnes (= 1,000 kg)	1.016	0.9842
Hundred weights	quintal (= 100 kg)	0.508	1.968
Pounds	tons	0.0004	2,240.0
Fluid			
Cubic feet	litres	28.317	0.0353
Gallons (Imp.)	litres	4.546	0.22
Gallons	cubic feet	0.1605	6.232
Litres	cubic centimetres	1,000.0	0.001
Pints	litres	1.7598	0.568
Gallons (Imp.)	gallons (U.S.)	1.2	0.8327
Circular			
Degrees	radians	0.0175	57.29
Compound			
lb/lineal foot	kg/lineal metre	1.488	0.6711
ton/lineal foot	kg/lineal metre	3,333.0	0.0003
lb/in^2	kg/cm^2	0.0703	14.223
lb/in^2	kg/mm^2	1.575	0.635
lb/ft^2	kg/m^2	4.883	0.2048
lb/in^2	ton/ft^2	0.0643	15.55
lb/ft^2	kg/m^3	16.019	0.9625
lb/yd^3	kg/m^3	0.5917	1.686
lb/gal	kg/1	0.0998	10.0166
ft-lb	kg-$\bar{1}$	0.1382	7.233
ft-ton	tonne-m	0.3096	3.229

(Contd.)

To convert	Into	Multiply by	Reciprocal
Horsepower	Force de Cheval	1.0139	0.9863
ft^3/sec	m^3/hr	101.94	0.098
ft^3/hr	l/sec	0.0787	127.2
lb/hp	kg/Force de Cheval	0.447	2.237
Grain/gal	g/l	0.0143	70.15

Figures Relating to Water

1 cubic foot of fresh water weighs 62.4 lb
1 cubic foot of sea water weighs 64.0 lb
1 British gallon of fresh water weighs 10.0 lb
1 United States gallon = 0.83 British gallon
1 litre of fresh water = 0.22 British gallon
1 cubic foot of fresh water = 6.24 British gallons
Head of fresh water in feet × 0.433 = pressure (lb/in.^{2})
Head of sea water in feet × 0.444 = pressure (lb/in.^{2})

Weights of Constructional Materials

Miscellaneous materials					N/m^2	lb/ft^2
				Clay floor tiles	575	12
				Pavement lights	1,200	25
		kN/m^3	lb/ft^3	Damp-proof course	48	1
	Tarmacadam	22.6	144		N/m^2 per mm thickness	lb/ft^2 per in thickness
	Macadam (waterbound)	25.1	160			
	Snow: compact	2.4 to 8.0	15 to 50	Felt (insulating)	1.9	1
	loose	0.8 to 1.9	5 to 12			
				Paving slabs (stone)	26.4	14
	Vermiculite (aggregate)	0.8	5	Granite sets	28.3	15
	Terracotta	20.8	132	Asphalt	22.6	12
	Glass	26.7	170	Rubber paving	15.1	8
	Cork: granular	1.2	$7\frac{1}{2}$	Polyvinylchloride	19 (av.)	10 (av.)
	compressed	3.8	24	Glass-fibre (forms)	1.9	1
Timber	General	7.9(av.)	50(av.)	Wooden boarding & blocks	N/m^2 per mm	lb/ft^2 per in
	Douglas fir	4.7	30	Softwood	4.7	$2\frac{1}{2}$
	Yellow pine, spruce	4.7	30	Hardwood	7.5	4
	Pitch pine	6.6	42	Hardboard	10.4	$5\frac{1}{2}$
	Larch, elm	5.5	35	Chipboard	7.5	4
	Oak (English)	7.1 to 9.4	45 to 60	Plywood	6.1	$3\frac{1}{4}$
	Teak	6.3 to 8.6	40 to 55	Blockboard	4.7	$2\frac{1}{2}$
	Jarrah	9.4	60	Fibreboard	2.8	$1\frac{1}{2}$
	Greenheart	10.2 to 11.8	65 to 75	Wood-wool	5.7	3
	Quebracho	12.6	80	Plasterboard	9.4	5
				Weather boarding	3.8	2
Stone and other materials	Natural stone (solid)				kN/m^3	lb/ft^3
	Granite	25.1 to 28.7	160 to 183	Stone rubble (packed)	22.0	140
	Limestone: Bath stone	20.4	130	Quarry waste	14.1	90
	Marble	26.7	170	Hardcore (consolidated)	18.9	120
	Portland stone	22.0	140	All-in aggregate	19.6	125
	Sandstone	22.0 to 23.6	140 to 150			
	Slate	28.3	180			

(Contd.)

Metals, steel constructions, etc.		kN/m³	lb/ft³			
	Iron: cast	70.7	450	Structural steelwork	Net weight of member	
	wrought	75.4	480	rivetted	+ 10% for cleats, rivets,	
	ore: general	23.6	150		bolts, etc.	
	(crushed) Swedish	36.1	230	welded	+ $1\frac{1}{4}$% to $2\frac{1}{2}$% for welds, etc.	
	Steel (see also below)	77.0	490	Rolled sections: beams	+ $2\frac{1}{2}$%	
	Copper: cast	85.6	545	stanchions	+ 5% (extra for caps & bases)	
	wrought	87.7	558	Plate-webgirders	+ 10% for rivets or welds;	
	Brass	83.3	530		stiffeners, etc.	
	Bronze	87.7	558			
	Aluminium	27.2	173			
	Lead	111.0	707			
	Zinc (rolled)	70.0	446			
				Steel stairs: industrial	N/m	lb/ft
		g/mm²	lb/in²	type 1m or 3ft wide	820	56
		per metre	per foot	Steel tubes:		
				50mm or 2 in bore	45 to 60	3 to 4
	Steel bars	7.85		Gas piping:		
		(7.85T/m³)	3.4	20mm or $\frac{3}{4}$ in	18	$1\frac{1}{4}$
	Rail tracks				kN/m of track	lb/ft of track
	standard gauge	Bull-head rails, chairs, transverse timber (softwood)			2.4	165
	main lines	sleepers, etc.				
		Flat-bottom rails, transverse prestressed concrete sleepers, etc.			4.1	280
		Add for electric third rail			0.5	35
		Add for crushed stone ballast			25.5	1,750
					kN/m²	lb/ft²
		Overall average weight: rails, connections, sleepers, ballast, etc.			7.2	150
					kN/m of rail	lb/ft of rail
		Bridge rails, longitudinal timber sleepers, etc.			1.1	75

To convert values in kN to values in kg multiply by 102.

Normal Concrete:	kN/m³	kg/m³
—plain	22	2250
—reinforced	24	2450
—prestressed reinforced	25	2550

COEFFICIENTS OF EXPANSION (α per 100°)

The coefficient of linear expansion (α) is the change in length, per unit of length, for a change of one degree of temperature. The coefficient of surface expansion is approximately two times the linear coefficient, and the coefficient of volume expansion, for solids, is approximately three times the linear coefficient.

A bar, free to move, will increase in length with an increase in temperature and will decrease in length with a decrease in temperature. The change in length will be αtl where α is the coefficient of linear expansion, t the change in temperature, and l the length. If the ends of a bar are fixed, a change in the temperature (t) will cause a change in the unit stress of $E\alpha t$, and in the total stress of $AE\alpha t$, where A is the cross-sectional area of the bar and E the modulus of elasticity.

The following table gives the coefficient of linear expansion for *100°*, or *100 times the value indicated above.*

EXAMPLE: A piece of medium steel is exactly 40 feet long at 60°F. Find the length at 90°F assuming the ends free to move.

$$\text{Change of length} = \alpha tl = \frac{.00065 \times 30 \times 40}{100} = .0078 \text{ ft}$$

The length at 90°F is 40.0078 ft.

EXAMPLE: A piece of medium steel is exactly 40 feet long and the ends are fixed. If the temperature increases 30°F. What is the resulting change in the unit stress?

Change in unit stress = $E\alpha t$ =

$$\frac{29,000,000 \times .00065 \times 30}{100} = \begin{array}{c}\text{5655 lb. per sq. in.}\\ \text{(397.6 kg./cm}^2\text{)}\end{array}$$

COEFFICIENTS OF EXPANSION FOR 100 DEGREES = 100α

Materials	Linear expansion		Materials	Linear expansion	
	Centigrade	Fahrenheit		Centigrade	Fahrenheit
METALS AND ALLOYS			STONE AND MASONRY		
Aluminium, wrought	.00231	.00128	Ashlar masonry	.00063	.00035
Brass	.00188	.00104	Brick masonry	.00061	.00034
Bronze	.00181	.00101	Cement, Portland	.00126	.00070
Copper	.00168	.00093	Concrete	.00099	.00055
Iron, cast, gray	.00106	.00059	Granite	.00080	.00044
Iron, wrought	.00120	.00067	Limestone	.00076	.00042
Iron, wire	.00124	.00069	Marble	.00081	.00045
Lead	.00286	.00159	Plaster	.00166	.00092
Magnesium, various alloys	.0029	.0016	Rubble masonry	.00063	.00035
Nickel	.00126	.00070	Sandstone	.00097	.00054
Steel mild	.00117	.00065	Slate	.00080	.00044
Steel, stainless, 18–8	.00178	.00099			
Zinc, rolled	.00311	.00173			
TIMBER			TIMBER		
Fir (Parallel to fibre)	.00037	.00021	Fir (perpendicular to fibre)	.0058	.00032
Maple (Parallel to fibre)	.00064	.00036	Maple (perpendicular to fibre)	.0048	.00027
Oak (Parallel to fibre)	.00049	.00027	Oak (perpendicular to fibre)	.0054	.00030
Pine (Parallel to fibre)	.00054	.00030	Pine (perpendicular to fibre)	.0034	.00019

EXPANSION OF WATER

If Maximum Density = 1 unit at 4°C, then

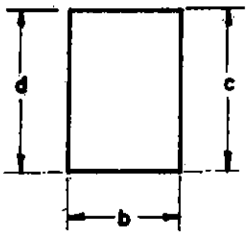
C°	Volume	C°	Volume	C°	Volume	C°	Volume	C°	Volume	C°	Volume
0	1.000126	10	1.000257	30	1.004234	50	1.011877	70	1.022384	90	1.035829
4	1.000000	20	1.001732	40	1.007627	60	1.016954	80	1.029003	100	1.043116

APPENDIX 3

Properties of Geometric Sections and Shapes

PROPERTIES OF GEOMETRIC SECTIONS

RECTANGLE
Axis of moments on base



$$A = bd$$

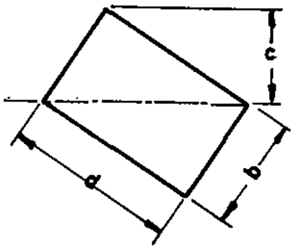
$$c = d$$

$$I = \frac{bd^3}{3}$$

$$S = \frac{bd^2}{3}$$

$$r = \frac{d}{\sqrt{3}} = .577350\ d$$

RECTANGLE
Axis of moments on diagonal



$$A = bd$$

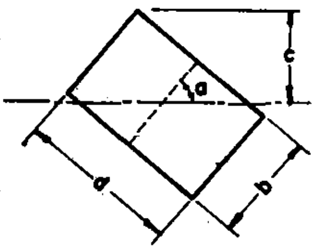
$$c = \frac{bd}{\sqrt{b^2 + d^2}}$$

$$I = \frac{b^3 d^3}{6(b^2 + d^2)}$$

$$S = \frac{b^2 d^2}{6\sqrt{b^2 + d^2}}$$

$$r = \frac{bd}{\sqrt{6(b^2 + d^2)}}$$

RECTANGLE
Axis of moments any line through centre of gravity



$$A = bd$$

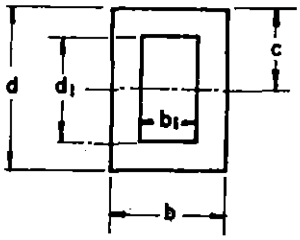
$$c = \frac{b \sin a + d \cos a}{2}$$

$$I = \frac{bd(b^2 \sin^2 a + d^2 \cos^2 a)}{12}$$

$$S = \frac{bd(b^2 \sin^2 a + d^2 \cos^2 a)}{6(b \sin a + d \cos a)}$$

$$r = \sqrt{\frac{b^2 \sin^2 a + d^2 \cos^2 a}{12}}$$

HOLLOW RECTANGLE
Axis of moments through centre



$$A = bd - b_1 d_1$$

$$c = \frac{d}{2}$$

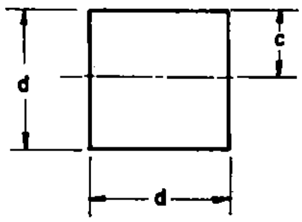
$$I = \frac{bd^3 - b_1 d_1^3}{12}$$

$$S = \frac{bd^3 - b_1 d_1^3}{6d}$$

$$r = \sqrt{\frac{bd^3 - b_1 d_1^3}{12A}}$$

$$Z = \frac{bd^2}{4} - \frac{b_1 d_1^2}{4}$$

SQUARE
Axis of moments through centre



$$A = d^2$$

$$c = \frac{d}{2}$$

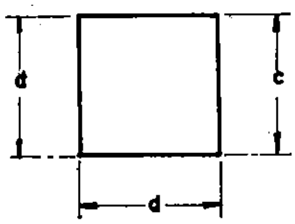
$$I = \frac{d^4}{12}$$

$$S = \frac{d^3}{6}$$

$$r = \frac{d}{\sqrt{12}} = .288675\ d$$

$$Z = \frac{d^3}{4}$$

SQUARE
Axis of moments on base



$$A = d^2$$

$$c = d$$

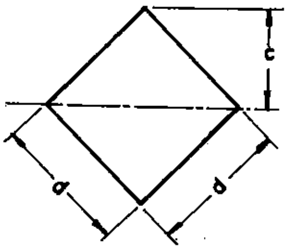
$$I = \frac{d^4}{3}$$

$$S = \frac{d^3}{3}$$

$$r = \frac{d}{\sqrt{3}} = .577350\ d$$

PROPERTIES OF GEOMETRIC SECTIONS *(Contd.)*

SQUARE
Axis of moments on diagonal



$$A = d^2$$

$$c = \frac{d}{\sqrt{2}} = .707107\ d$$

$$I = \frac{d^4}{12}$$

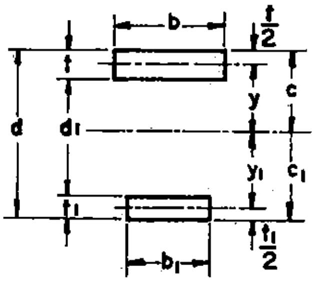
$$S = \frac{d^3}{6\sqrt{2}} = .117851\ d^3$$

$$r = \frac{d}{\sqrt{12}} = .288675\ d$$

$$Z = \frac{2c^3}{3} = \frac{d^3}{3\sqrt{2}}$$

$$= .235702\ d^3$$

UNEQUAL RECTANGLES
Axis of moments through centre of gravity



$$A = bt + b_1t_1$$

$$c = \frac{1/2bt^2 + b_1t_1(d - 1/2t_1)}{A}$$

$$I = \frac{bt^3}{12} + bty^2 + \frac{b_1t_1^3}{12}$$

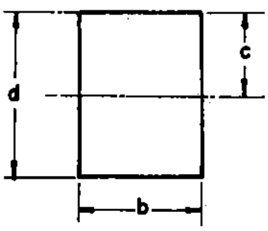
$$+ b_1t_1y_1^2$$

$$S = \frac{I}{c} \quad S_1 = \frac{I}{c_1}$$

$$r = \sqrt{\frac{I}{A}}$$

$$Z = \frac{A}{2}\left[d - \left(\frac{t + t_1}{2}\right)\right]$$

RECTANGLE
Axis of moments through centre



$$A = bd$$

$$c = \frac{d}{2}$$

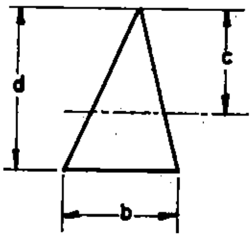
$$I = \frac{bd^3}{12}$$

$$S = \frac{bd^2}{6}$$

$$r = \frac{d}{\sqrt{12}} = .288675\ d$$

$$Z = \frac{bd^2}{4}$$

TRIANGLE
Axis of moments through centre of gravity



$$A = \frac{bd}{2}$$

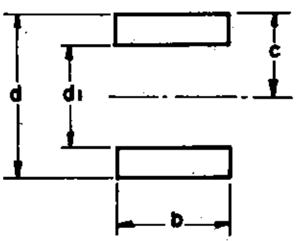
$$c = \frac{2d}{3}$$

$$I = \frac{bd^3}{36}$$

$$S = \frac{bd^2}{24}$$

$$r = \frac{d}{\sqrt{18}} = .235702\ d$$

EQUAL RECTANGLES
Axis of moments through centre of gravity



$$A = b(d - d_1)$$

$$c = \frac{d}{2}$$

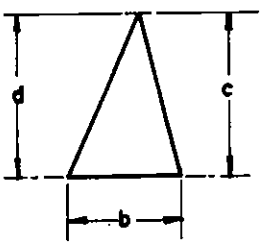
$$I = \frac{b(d^3 - d_1^3)}{12}$$

$$S = \frac{b(d^3 - d_1^3)}{6d}$$

$$r = \sqrt{\frac{d^3 - d_1^3}{12(d - d_1)}}$$

$$Z = \frac{b}{4}(d^2 - d_1^2)$$

TRIANGLE
Axis of moments on base



$$A = \frac{bd}{2}$$

$$c = d$$

$$I = \frac{bd^3}{12}$$

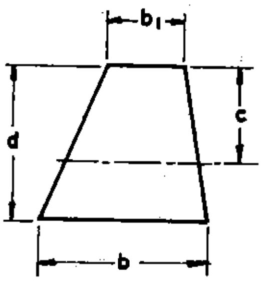
$$S = \frac{bd^2}{12}$$

$$r = \frac{d}{\sqrt{6}} = .408248\ d$$

PROPERTIES OF GEOMETRIC SECTIONS (*Contd.*)

TRAPEZOID

Axis of moments through centre of gravity



$$A = \frac{d(b + b_1)}{2}$$

$$c = \frac{d(2b + b_1)}{3(b + b_1)}$$

$$I = \frac{d^3(b^2 + 4bb_1 + b_1^2)}{36(b + b_1)}$$

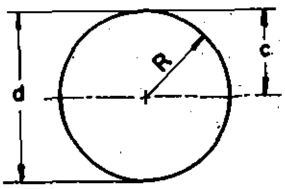
$$S = \frac{d^2(b^2 + 4bb_1 + b_1^2)}{12(2b + b_1)}$$

$$r = \frac{d}{6(b + b_1)}$$

$$\sqrt{2(b^2 + 4bb_1 + b_1^2)}$$

CIRCLE

Axis of moments through centre



$$A = \frac{\pi d^2}{4} = \pi R^2$$

$$= .785398\ d^2$$

$$= 3.141593 R^2$$

$$c = \frac{d}{2} = R$$

$$I = \frac{\pi d^4}{64} = \frac{\pi R^4}{4}$$

$$= .049087\ d^4$$

$$= .785398\ R^4$$

$$S = \frac{\pi d^3}{32} = \frac{\pi R^3}{4}$$

$$= .098175\ d^3$$

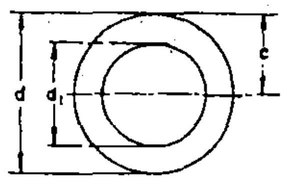
$$= .785398\ R^3$$

$$r = \frac{d}{4} = \frac{R}{2}$$

$$Z = \frac{d^3}{6}$$

HOLLOW CIRCLE

Axis of moments through centre



$$A = \frac{\pi(d^2 - d_1^2)}{4} = .785398\ (d^2 - d_1^2)$$

$$c = \frac{d}{2}$$

$$I = \frac{\pi(d^4 - d_1^4)}{64} = .049087\ (d^4 - d_1^4)$$

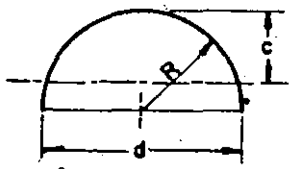
$$S = \frac{\pi(d^4 - d_1^4)}{32d} = .098175\ \frac{d^4 - d_1^4}{d}$$

$$r = \frac{\sqrt{d^2 + d_1^2}}{4}$$

$$Z = \frac{d^3}{6} - \frac{d_1^3}{6}$$

HALF CIRCLE

Axis of moments through centre of gravity



$$A = \frac{\pi R^2}{2} = .1.570796\ R^2$$

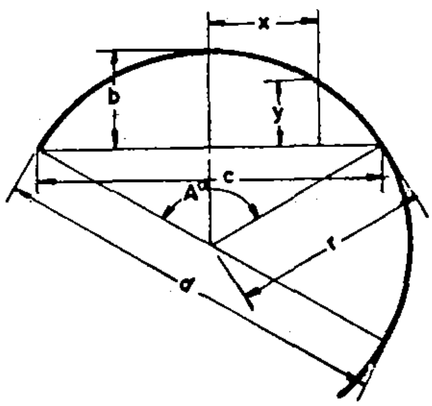
$$c = R\left(1 - \frac{4}{3\pi}\right) = .575587\ R$$

$$I = R^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) = .109757\ R^4$$

$$S = \frac{R^3}{24}\frac{(9\pi^2 - 64)}{(3\pi - 4)} = .190687\ R^3$$

$$r = R\ \frac{\sqrt{9\pi^2 - 64}}{6\pi} = .264336\ R$$

PROPERTIES OF THE CIRCLE



PROPERTIES OF THE CIRCLE (*Contd.*)

Circumference $= 6.28318\ r = 3.14159$ d

Diameter $= 0.31831$ circumference

Area $= 3.14159\ r^2$

Arc $\quad a = \dfrac{\pi r A^\circ}{180^\circ} = 0.017453\ r A^\circ$

Angle $A^\circ = \dfrac{180^\circ a}{\pi r} = 57.29578 \dfrac{a}{r}$

Radius $\quad r = \dfrac{4b^2 + c^2}{8b}$

Chord $\quad c = 2\sqrt{2br - b^2} = 2r \sin \dfrac{A}{2}$

Rise $\quad b = r - 1/2\sqrt{4r^2 - c^2} = \dfrac{c}{2} \tan \dfrac{A}{4}$

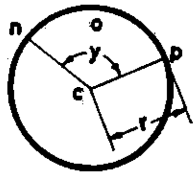
$$= 2r \sin^2 \frac{A}{4} = r + y - \sqrt{r^2 - x^2}$$

$$y = b - r + \sqrt{r^2 - x^2}$$

$$x = \sqrt{r^2 - (r + y - b)^2}$$

Diameter of circle of equal periphery as square	= 1.27324 side of square
Side of square of equal periphery as circle	= 0.78540 diameter of circle
Diameter of circle circumscribed about square	= 1.41421 side of square
Side of square inscribed in circle	= 0.70711 diameter of circle

CIRCULAR SECTOR



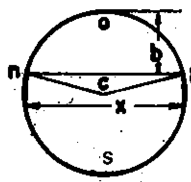
r = radius of circle, y = angle ncp in degrees

Area of Sector $ncpo$ = 1/2 (length of arc $nop \times r$)

$$= \text{Area of circle} \times \frac{y}{360}$$

$$= 0.0087266 \times r^2 \times y$$

CIRCULAR SEGMENT



r = radius of circle $\quad x$ = chord $\quad b$ = rise

Area of segment nop = Area of sector $ncpo$ − Area of triangle ncp

$$= \frac{(\text{Length of arc } nop \times r) - x(r - b)}{2}$$

Area of Segment nsp = Area of Circle − Area of segment nop

VALUES FOR FUNCTIONS OF π

$\pi = 3.14159265359$, log = 0.4971499

$\pi^2 = 9.8696044$, log = 0.9942997

$\dfrac{1}{\pi} = 0.3183099$, log = $\bar{1}.5028501$

$\sqrt{\dfrac{1}{\pi}} = 0.5641896$, log = $\bar{1}.7514251$

$\pi^3 = 31.0062767$, log = 1.4914496

$\dfrac{1}{\pi^2} = 0.1013212$, log = $\bar{1}.0057003$

$\dfrac{\pi}{180} = 0.0174533$, log = $\bar{2}.2418774$

$\sqrt{\pi} = 1.7724539$, log = 0.2485749

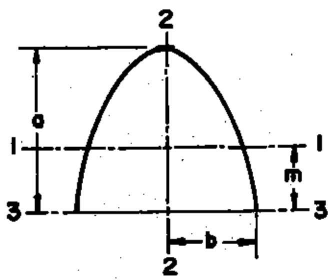
$\dfrac{1}{\pi^3} = 0.0322515$, log = $\bar{2}.5085504$

$\dfrac{180}{\pi} = 57.2957795$, log = 1.7581226

NOTE: Logs of fractions such as $\bar{1}.5028501$ and $\bar{2}.5085500$ may also be written 9.5028501–10 and 8.5085500–10 respectively.

PROPERTIES OF GEOMETRIC SECTIONS (*Contd.*)

PARABOLA



$$A = \frac{4}{3}ab$$

$$m = \frac{2}{5}a$$

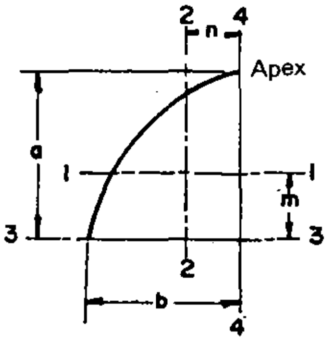
$$I_1 = \frac{16}{175} \times a^3 b$$

$$I_2 = \frac{4}{15} \times ab^3$$

$$I_3 = \frac{32}{105} \times a^3 b$$

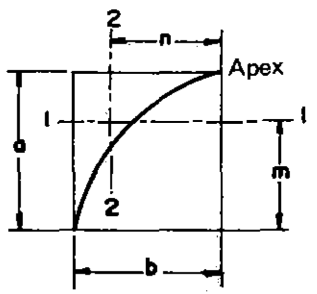
PROPERTIES OF GEOMETRIC SECTIONS (*Contd.*)

HALF PARABOLA



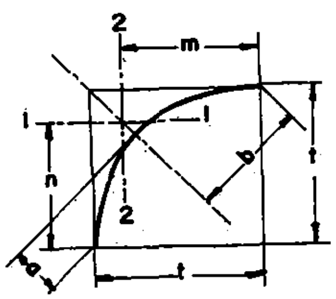
$$A = \frac{2}{3}ab$$
$$m = \frac{2}{5}a$$
$$n = \frac{3}{8}b$$
$$I_1 = \frac{8}{175} \times a^3 b$$
$$I_2 = \frac{19}{480} \times a^3 b$$
$$I_3 = \frac{16}{105} \times a^3 b$$
$$I_4 = \frac{2}{15} \times ab^3$$

COMPLEMENT OF HALF PARABOLA



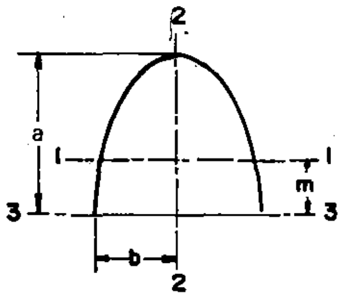
$$A = \frac{1}{3}ab$$
$$m = \frac{7}{10}a$$
$$n = \frac{3}{4}b$$
$$I_1 = \frac{37}{2100}a^3 b$$
$$I_2 = \frac{1}{80}ab^3$$

PARABOLIC FILLET IN RIGHT ANGLE



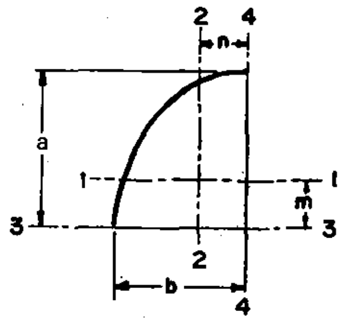
$$a = \frac{t}{2\sqrt{2}}$$
$$b = \frac{t}{\sqrt{2}}$$
$$A = \frac{1}{6}t^2$$
$$m = n = \frac{4}{5}t$$
$$I_1 = I_2 = \frac{11}{2100}t^4$$

* HALF ELLIPSE



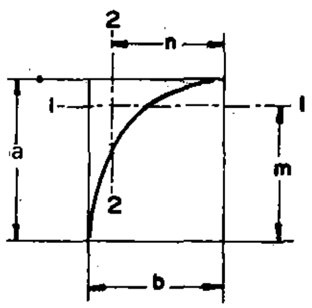
$$A = \frac{1}{2}\pi ab$$
$$m = \frac{4a}{3\pi}$$
$$I_1 = a^3 b\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$$
$$I_2 = \frac{1}{8}\pi ab^3$$
$$I_3 = \frac{1}{8}\pi a^3 b$$

* QUARTER ELLIPSE



$$A = \frac{1}{4}\pi ab$$
$$m = \frac{4a}{3\pi}$$
$$n = \frac{4b}{3\pi}$$
$$I_1 = a^3 b\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$$
$$I_2 = ab^3\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$$
$$I_3 = \frac{1}{16}\pi a^3 b$$
$$I_4 = \frac{1}{16}\pi ab^3$$

* ELLIPTIC COMPLEMENT



$$A = ab\left(1 - \frac{\pi}{4}\right)$$
$$m = \frac{a}{6\left(1 - \frac{\pi}{4}\right)}$$
$$n = \frac{b}{6\left(1 - \frac{\pi}{4}\right)}$$

(*Contd.*)

* To obtain properties of half circle, quarter circle and circular complement substitute $a = b = R$.

PROPERTIES OF GEOMETRIC SECTIONS (*Contd.*)

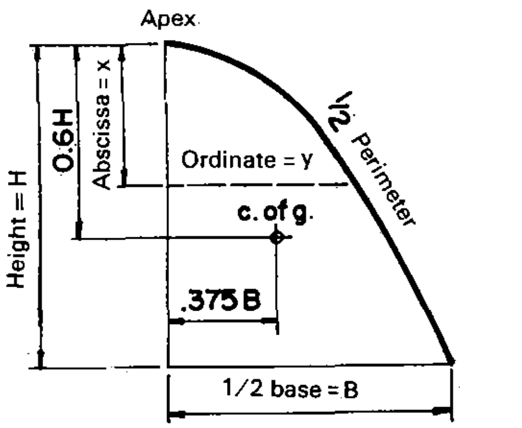
$$I_1 = a^3b\left(\frac{1}{3} - \frac{\pi}{16} - \frac{1}{36(1-\frac{\pi}{4})}\right)$$

$$I_2 = ab^3\left(\frac{1}{3} - \frac{\pi}{16} - \frac{1}{36(1-\frac{\pi}{4})}\right)$$

PROPERTIES OF PARABOLA AND ELLIPSE

PARABOLA

When $H \div B = 0.1$ or less, approximate 1/2 perimeter $= \sqrt{B^2 + \frac{4}{3}\cdot H^2}$ or use formulae for circular arcs.

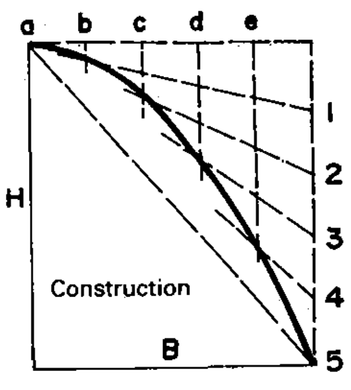


Area = 2/3HB

Perimeter $P = B^2 \div H$

$x = y^2 \div P$

$y = \sqrt{x P}$

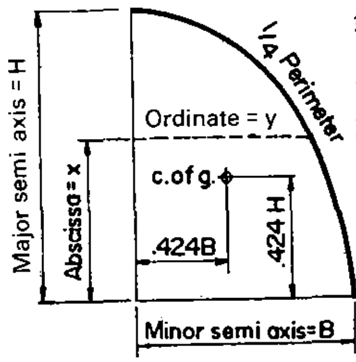


PROPERTIES OF PARABOLA AND ELLIPSE (*Contd.*)

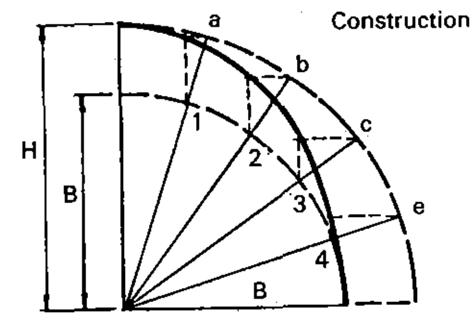
ELLIPSE

$(x^2 \div H^2) + (y^2 \div B^2) = 1 \quad x = (H \div B)\sqrt{B^2 - y^2}$

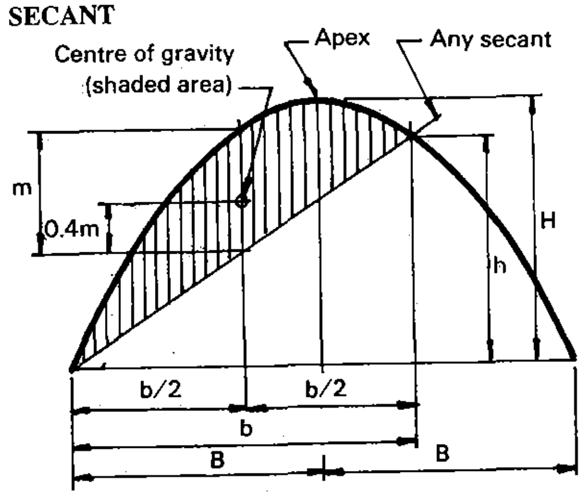
$y = (B \div H)\sqrt{H^2 - x^2}$



Approximate 1/4 perimeter $= \frac{\pi}{4}\sqrt{2(H^2+B^2)}$



AREA BETWEEN PARABOLIC CURVE AND SECANT



Length *b* may vary from O to 2B.

(Contd).

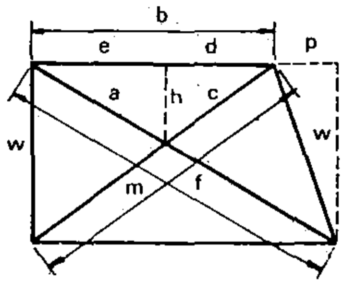
$$h = Hb\left(\frac{2B-b}{B^2}\right)$$

$$m = \frac{Hb^2}{4B^2}$$

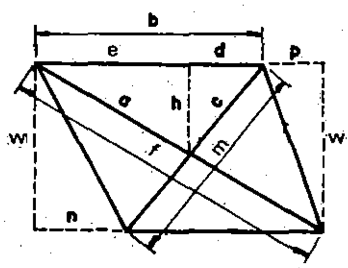
$$\text{Shaded area} = \frac{2}{3}bm$$

$$= \frac{Hb^3}{6B^2}$$

BRACING FORMULAS



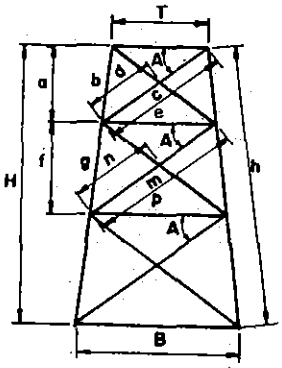
Given	To Find	Formula
bpw	f	$\sqrt{(b+p)^2+w^2}$
bw	m	$\sqrt{b^2+w^2}$
bp	d	$b^2 \div (2b+p)$
bp	e	$b(b+p) \div (2b+p)$
bfp	a	$bf \div (2b+p)$
bmp	c	$bm \div (2b+p)$
bpw	h	$bw \div (2b+p)$
afw	h	$aw \div f$
cmw	h	$cw \div m$



Given	To Find	Formula
bpw	f	$\sqrt{(b+p)^2+w^2}$
bnw	m	$\sqrt{(b-n)^2+w^2}$
bnp	d	$b(b-n) \div (2b+p-n)$
bnp	e	$b(b+p) \div (2b+p-n)$
$bfnp$	a	$bf \div (2b+p-n)$
$bmnp$	c	$bm \div (2b+p-n)$
$bnpw$	h	$bw \div (2b+p-n)$
afw	h	$aw \div f$
cmw	h	$cw \div m$

Given	To Find	Formula
bpw	f	$\sqrt{(b+p)^2+w^2}$
bkv	m	$\sqrt{(b+k)^2+v^2}$
$bkpvw$	d	$bw(b+k) \div [v(b+p)+w(b+k)]$
$bkpvw$	e	$bv(b+p) \div [v(b+p)+w(b+k)]$
$bfkpvw$	a	$fbv \div [v(b+p)+w(b+k)]$
$bkmpvw$	c	$bmw \div [v(b+p)+w(b+k)]$
$bkpvw$	h	$bvw \div [v(b+p)+w(b+k)]$
afw	h	$aw \div f$
cmv	h	$cv \div m$

PARALLEL BRACING



$k = (\log B - \log T) \div$ no. of panels. Constant k plus the logarithm of any line equals the log of the corresponding line in the next panel below.

$a = TH \div (T+e+p)$

$b = Th \div (T+e+p)$

$c = \sqrt{(1/2T+1/2e)^2+a^2}$

$d = ce \div (T+e)$

$\log e = k + \log T$

$\log f = k + \log a$

$\log g = k + \log b$

$\log m = k + \log c$

$\log n = k + \log d$

$\log p = k + \log e$

The above method can be used for any number of panels.

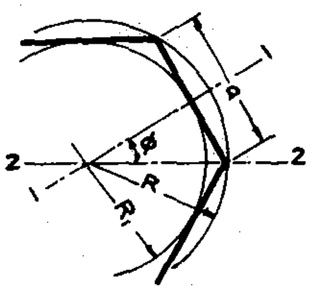
(Contd.)

In the formulas for "a" and "b" the sum in parentheses, which in the case shown is $(T+e+p)$, is always composed of all the horizontal distances except the base.

PROPERTIES OF GEOMETRIC SECTIONS AND STRUCTURAL SHAPES

REGULAR POLYGON

Axis of moments through centre



n = Number of sides

$$\phi = \frac{180^\circ}{n}$$

$$a = 2\sqrt{R^2 - R_1^2}$$

$$R = \frac{a}{2\sin\phi}$$

$$R_1 = \frac{a}{2\tan\phi}$$

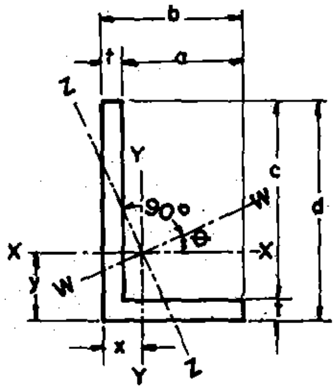
$$A = \frac{1}{4}na^2\cot\phi = \frac{1}{2}nR^2\sin 2\phi = nR_1^2\tan\phi$$

$$I_1 = I_2 = \frac{A(6R^2 - a^2)}{24} = \frac{A(12R_1^2 + a^2)}{48}$$

$$r_1 = r_2 = \sqrt{\frac{6R^2 - a^2}{24}} = \sqrt{\frac{12R_1^2 + a^2}{48}}$$

ANGLE

Axis of moments through centre of gravity



Z-Z is axis of minimum I

$$\tan 2\theta = \frac{2K}{I_y - I_x}$$

$$A = t(b+c),\ x = \frac{b^2 + ct}{2(b+c)},\ y = \frac{d^2 + at}{2(b+c)}$$

K = Product of Inertia about X–X and Y–Y

$$= \mp\frac{abcdt}{4(b+c)}$$

$$I_x = \frac{1}{3}[t(d-y)^3 + by^3 - a(y-t)^3]$$

$$I_y = \frac{1}{3}[t(b-x)^3 + dx^3 - c(x-t)^3]$$

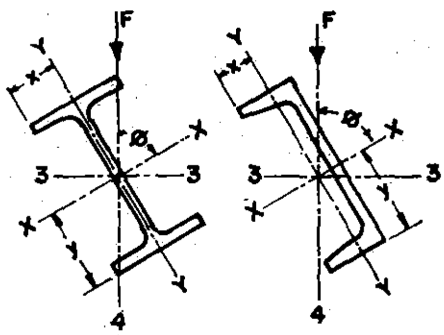
$$I_z = I_x\sin^2\theta + I_y\cos^2\theta + K\sin 2\theta$$

$$I_w = I_x\cos^2\theta + I_y\sin^2\theta - K\sin 2\theta$$

K is negative when heel of angle, with respect to c.g., is in 1st or 3rd quadrant, positive when in 2nd or 4th quadrant.

BEAMS AND CHANNELS

Transverse force oblique through centre of gravity



$$I_3 = I_x\sin^2\phi + I_y\cos^2\phi$$

$$I_4 = I_x\cos^2\phi + I_y\sin^2\phi$$

$$f_b = M\left(\frac{y}{I_x}\sin\phi + \frac{x}{I_y}\cos\phi\right)$$

where M is bending moment due to force F.

APPENDIX 4

Mathematical Data

$\pi = \frac{355}{113}$ (approx.) $= \frac{22}{7}$ (approx.) $= 3.141592654$ (approx.)

One radian $= \frac{180°}{\pi} = 57.3°$ (approx.) $=$ $57.2957795°$ (approx.)

Length of arc subtended by an angle of one radian = radius of arc.

One degree Fahrenheit = 5/9 degree Centigrade or Celsius.
Temperature of t°F $= 5/9(t - 32)$°C.
Temperature of t°C $= (1.8t + 32)$°F.

Base of Naperian logarithms, $e = \frac{193}{71}$ (approx.) $=$

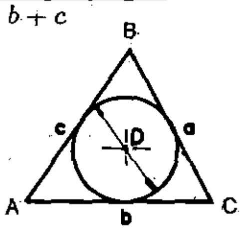
$\frac{2721}{1001}$ (approx.) $= 2.718281828$ (approx.)

To convert common into Naperian logarithms, multiply by $\frac{76}{33}$ (approx.) $= \frac{3919}{1702}$ (approx.) $= 2.302585093$ (approx.).
Nominal value of $g = 9.80665$ m./sec^2 $= 32.174$ ft/sec^2.
Diameter of inscribed circle of a triangle:

$$D = \frac{2b\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}}{a + b + c}$$

For isosceles triangle, $a = c$:

$$D = \frac{b\sqrt{4a^2 - b^2}}{2a + b}$$



Solution of Triangles

Applicable to any triangle ABC in which $AB = c$;

$BC = a$; $AC = b$; $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Area $= \frac{bc\sin A}{2} = \frac{ac\sin B}{2} = \frac{ab\sin C}{2} =$

$\sqrt{s(s-a)(s-b)(s-c)}$, where $s = 1/2(a + b + c)$.

$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ $\quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

THE GREEK ALPHABETS

Letters from the Greek alphabet are frequently used as symbols in engineering problems.

Alpha	A	α	Iota	I	ι	Rho	P	ρ
Beta	B	β	Kappa	K	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	T	τ
Delta	Δ	δ	Mu	M	μ	Upsilon	Υ	υ
Epsilon	E	ϵ	Nu	N	ν	Phi	Φ	ϕ
Zeta	Z	ζ	Xi	Ξ	ξ	Chi	X	χ
Eta	H	η	Omicron	O	o	Psi	Ψ	ψ
Theta	Θ	θ	Pi	Π	π	Omega	Ω	ω

ALGEBRAICAL FORMULAE

Products and Factors

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$
$$(x + a)(x - a) = x^2 - a^2$$
$$(x \pm a)(x^2 \pm ax + a^2) = x^3 \pm a^3$$
$$(x \pm a)^2 = x^2 \pm 2ax + a^2$$
$$(x \pm a)^3 = x^3 \pm 3x^2a + 3xa^2 \pm a^3$$

General Solution to a Quadratic Equation

If $\quad ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

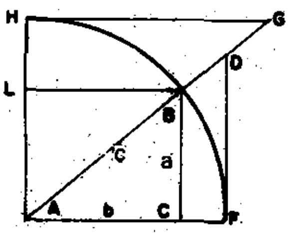
Indices

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$\log(ab) = \log a + \log b$$
$$\log(a/b) = \log a - \log b$$
$$\log(a^n) = n\log a$$
$$\log\sqrt[n]{a} = \log a \times 1/n$$

STANDARD FORMS OF DIFFERENTIALS AND INTEGRALS

Differential $\frac{dy}{dx}$	Function $\longleftarrow y \longrightarrow$	Integral $\int y\,dx$
1	x	$\frac{1}{2}x^2 + C$
0	a	$ax + C$
1	$x \pm a$	$\frac{1}{2}x^2 \pm ax + C$
$2x$	x^2	$\frac{1}{3}x^3 + C$
nx^{n-1}	x^n	$\frac{1}{n+1}x^{n+1} + C$ (except when $n = -1$)
$-x^{-2}$	x^{-1}	$\log_e x + C$
$\frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$	$u \pm v \pm w$	$\int u\,dx \pm \int v\,dx \pm \int w\,dx$
$u\frac{dv}{dx} + v\frac{du}{dx}$	uv	No general form
$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	u/v	No general form
$\frac{du}{dx}$	u	$ux - \int x\,du + C$
e^x	e^x	$e^x + C$
x^{-1}	$\log_e x$	$x(\log_e x - 1) + C$
$0.4343x^{-1}$	$\log_{10} x$	$0.4343x(\log_e x - 1) + C$
$a^x \log_e a$	a^x	$\frac{a^x}{\log_e a} + C$
$\cos x$	$\sin x$	$-\cos x + C$
$-\sin x$	$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x$	$-\log_e \cos x + C$
$\cosh x$	$\sinh x$	$\cosh x + C$
$\sinh x$	$\cosh x$	$\sinh x + C$
$\operatorname{sech}^2 x$	$\tanh x$	$\log_e \cosh x + C$
$-\frac{1}{(x+a)^2}$	$\frac{1}{x+a}$	$\log_e(x + a) + C$

TRIGONOMETRIC FUNCTIONS



Radius $AF = 1$

$$= \sin^2 A + \cos^2 A = \sin A \operatorname{cosec} A$$
$$= \cos A \sec A = \tan A \cot A$$

$$\text{Sine } A = \frac{\cos A}{\cot A} = \frac{1}{\operatorname{cosec} A} = \cos A \tan A = \sqrt{1 - \cos^2 A} = BC$$

$$\text{Cosine } A = \frac{\sin A}{\tan A} = \frac{1}{\sec A} = \sin A \cot A = \sqrt{1 - \sin^2 A} = AC$$

$$\text{Tangent } A = \frac{\sin A}{\cos A} = \frac{1}{\cot A} = \sin A \sec A = FD$$

$$\text{Cotangent } A = \frac{\cos A}{\sin A} = \frac{1}{\tan A} = \cos A \operatorname{cosec} A = HG$$

$$\text{Secant } A = \frac{\tan A}{\sin A} = \frac{1}{\cos A} = AD$$

$$\text{Cosecant } A = \frac{\cot A}{\cos A} = \frac{1}{\sin A} = AG$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sec^2\theta - \tan^2\theta = 1$$
$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} \qquad \tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$

$$\sin\theta + \sin\phi = 2\sin 1/2(\theta + \phi)\cos 1/2(\theta - \phi)$$
$$\sin\theta - \sin\phi = 2\cos 1/2(\theta + \phi)\sin 1/2(\theta - \phi)$$
$$\cos\theta + \cos\phi = 2\cos 1/2(\theta + \phi)\cos 1/2(\theta - \phi)$$
$$\cos\theta - \cos\phi = -2\sin 1/2(\theta + \phi)\sin 1/2(\theta - \phi)$$
$$\sin\theta\sin\phi = 1/2[\cos(\theta - \phi) - \cos(\theta + \phi)]$$
$$\sin\theta\cos\phi = 1/2[\sin(\theta + \phi) + \sin(\theta - \phi)]$$
$$\cos\theta\cos\phi = 1/2[\cos(\theta + \phi) + \cos(\theta - \phi)]$$

$$\text{Sin } A = \cos A, \quad \tan A = 2\sin\frac{A}{2}\cdot\cos\frac{A}{2}$$
$$= \sqrt{1/2(1 - \cos 2A)} = \sqrt{1 - \cos^2 A}$$

$$\text{Cos } A = \frac{\sin A}{\tan A} = 2\cos^2\frac{A}{2} - 1 = 1 - 2\sin^2\frac{A}{2}$$
$$= \sqrt{1 - \sin^2 A}$$

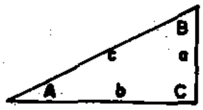
$$\text{Tan } A = \frac{\sin A}{\cos A} = \frac{\sin 2A}{1 + \cos 2A} = \sqrt{\frac{1}{\cos^2 A} - 1}$$

$$\text{Sin }\frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}} \qquad \text{Sin } 2A = 2\sin A\cdot\cos A$$

$$\text{Cos }\frac{A}{2} = \sqrt{\frac{1 + \cos A}{2}} \qquad \text{Cos } 2A = 2\cos^2 A - 1$$

$$\text{Tan }\frac{A}{2} = \frac{1 - \cos A}{\sin A} \qquad \text{Tan } 2A = \frac{2\tan A}{1 - \tan^2 A}$$

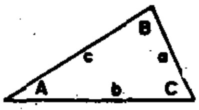
RIGHT ANGLED TRIANGLES



$$a^2 = c^2 - b^2$$
$$b^2 = c^2 - a^2$$
$$c^2 = a^2 + b^2$$

Known	Required					
	A	B	a	b	c	Area
a, b	$\tan A = \frac{a}{b}$	$\tan B = \frac{b}{a}$			$\sqrt{a^2 + b^2}$	$\frac{ab}{2}$
a, c	$\sin A = \frac{a}{c}$	$\cos B = \frac{a}{c}$		$\sqrt{c^2 - a^2}$		$\frac{a\sqrt{c^2 - a^2}}{2}$
A, a		$90° - A$		$a \cot A$	$\frac{a}{\sin A}$	$\frac{a^2 \cot A}{2}$
A, b		$90° - A$	$b \tan A$		$\frac{b}{\cos A}$	$\frac{b^2 \tan A}{2}$
A, c		$90° - A$	$c \sin A$	$c \cos A$		$\frac{c^2 \sin 2A}{4}$

OBLIQUE ANGLED TRIANGLES



$$s = \frac{a + b + c}{2}$$

$$K = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Known	Required					
	A	B	C	b	c	Area
a, b, c	$\tan \frac{1}{2}A = \frac{K}{s-a}$	$\tan \frac{1}{2}B = \frac{K}{s-b}$	$\tan \frac{1}{2}C = \frac{K}{s-c}$			$\sqrt{s(s-a)(s-b)(s-c)}$
a, A, B			$180° - (A + B)$	$\frac{a \sin B}{\sin A}$	$\frac{a \sin C}{\sin A}$	
a, b, A		$\sin B = \frac{b \sin A}{a}$			$\frac{b \sin C}{\sin B}$	
a, b, C	$\tan A = \frac{a \sin C}{b - a \cos C}$				$\sqrt{a^2 + b^2 - 2ab \cos C}$	$\frac{ab \sin C}{2}$

TRIGONOMETRICAL RATIOS

Angle Degrees	Sine	Tangent	Cotangent	Cosine	—
1	0.0175	0.0175	57.2900	0.9998	89
2	0.0349	0.0349	28.6363	0.9994	88
3	0.0523	0.0524	19.0811	0.9986	87
4	0.0698	0.0699	14.3007	0.9976	86
5	0.0872	0.0875	11.4301	0.9962	85
6	0.1045	0.1051	9.5144	0.9945	84
7	0.1219	0.1228	8.1443	0.9925	83
8	0.1392	0.1405	7.1154	0.9903	82
9	0.1564	0.1584	6.3138	0.9877	81
10	0.1736	0.1763	5.6713	0.9848	80
11	0.1908	0.1944	5.1446	0.9816	79
12	0.2079	0.2126	4.7046	0.9781	78
13	0.2250	0.2309	4.3315	0.9744	77
14	0.2419	0.2493	4.0108	0.9703	76
15	0.2588	0.2679	3.7321	0.9659	75
16	0.2756	0.2867	3.4874	0.9613	74
17	0.2924	0.3057	3.2709	0.9563	73
18	0.3090	0.3249	3.0777	0.9511	72
19	0.3256	0.3443	2.9042	0.9455	71
20	0.3420	0.3640	2.7475	0.9397	70
21	0.3584	0.3839	2.6051	0.9336	69
22	0.3746	0.4040	2.4751	0.9272	68
23	0.3907	0.4245	2.3559	0.9205	67
24	0.4067	0.4452	2.2460	0.9135	66
25	0.4226	0.4663	2.1445	0.9063	65
26	0.4384	0.4877	2.0503	0.8988	64
27	0.4540	0.5095	1.9626	0.8910	63
28	0.4695	0.5317	1.8807	0.8829	62
29	0.4848	0.5543	1.8040	0.8746	61
30	0.5000	0.5774	1.7321	0.8660	60
31	0.5150	0.6009	1.6643	0.8572	59
32	0.5299	0.6249	1.6003	0.8480	58
33	0.5446	0.6494	1.5399	0.8387	57
34	0.5592	0.6745	1.4826	0.8290	56
35	0.5736	0.7002	1.4281	0.8192	55
36	0.5878	0.7265	1.3764	0.8090	54
37	0.6018	0.7536	1.3270	0.7986	53
38	0.6157	0.7813	1.2799	0.7880	52
39	0.6293	0.8098	1.2349	0.7771	51
40	0.6428	0.8391	1.1918	0.7660	50
41	0.6561	0.8693	1.1504	0.7547	49
42	0.6691	0.9004	1.1106	0.7431	48
43	0.6820	0.9325	1.0724	0.7314	47
44	0.6947	0.9657	1.0355	0.7193	46
45	0.7071	1.0000	1.0000	0.7071	45
—	COSINE	COTANGENT	TANGENT	SINE	ANGLE DEGREES

APPENDIX 5

Estimation of Bursting Tensile Stress in 'Caisson-Steining' under 'Pneumatic-Sinking' Conditions

Where it is anticipated to sink the caisson by means of 'pneumatic-sinking', it is necessary that the steining (and the well-curb) are made with concrete of not less than 160 to 200 kg/cm^2 28 days cube strength. This concrete should be easily workable and well compacted so as to make it adequately impermeable, lest the compressed air should leak out. Obviously any injury to the concrete, resulting from explosive charges used in the process of well-sinking, should be avoided. Generally no special hoop reinforcement is required either in the steining or in the curb (the caisson-shoe) for 'sinking-depths' to about 30m. (This corresponds to about 30 T/m^2 or 3 kg/cm^2, i.e., 3 atmospheres pressure, and is the upper limit of human endurance.) Unless the steining is academically thin and the nominal hoop reinforcement in the steining and the total reinforcement in the well-curb are less than the minima stipulated in the appropriate codes, for instance the Indian Roads Congress-'Bridge Design Stipulations' (for more details see the chapter on 'substructure' in this book), no special reinforcement is required. Nevertheless a resolute and quick workman-like check is necessary at the time the quantities have to be decided and when design and drawings are to be finalised.

The following simplified assumptions and analysis may be adopted:

1. Assume that the corbel (i.e. the air-lock stool) will be laid at the 'top' of the total anticipated caisson-height.
2. Ignore the relief against bursting from the surrounding soil mass (for various practical reasons).
3. Assume low-water-level condition exists, and that the caisson is at the stage of 'sinking completed'.
4. Internal pneumatic air pressure is at least 15% higher than the maximum water pressure from outside (for obvious reasons).
5. Check the greatest bursting tensile stress in the steining (this will be in the top unit-meter depth of the steining) on its inner as well as outer faces and, only if this exceeds the section-capacity, check this stress at lower depths too in order to detail the steining section down the depth. It is advisable that the maximum tensile stress be limited to less than the permissible tensile stress in concrete (uncracked section basis), since, otherwise, should the concrete have to crack in order to transfer the hoop-tension to the hoop reinforcement, the resulting cracking can cause leakage of air pressure.
   *See explanatory note ahead**.
6. Work out the 'average' tensile stress in the concrete in the curb-section (= average bursting pressure within its height × mean radius of curb × curb-height, divided by section area of curb) and limit it to the permissible tensile stress in concrete, ignoring the nominal reinforcement in the curb.

***EXPLANATORY NOTE ON ITEM 5**

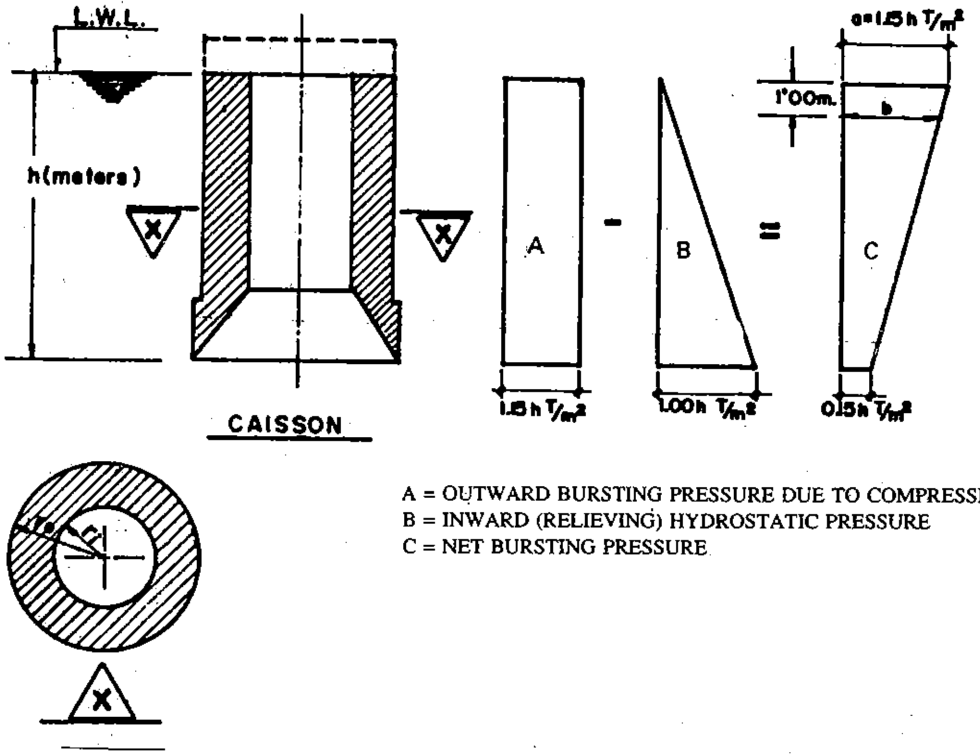
—Refer to Fig. A5.1 on p: 709, which is self explanatory

—Steining is a thick-shell (of known inner and outer radii r_i and r_0), hence Lame's formula can be used to estimate f_{t_i} and f_{t_0}, the inner and outer peripheral hoop stresses, from

$$f_t = \frac{p \cdot r_i^2}{(r_0^2 - r_i^2)} \left\{1 + \frac{r_0^2}{r^2}\right\} \qquad \text{(A 5.1)}$$

where $r = r_i$ for f_t on inner surface,
and
$r = r_0$ for f_t on outer surface,
p being the 'average' hoop stress.

Hence f_{t_i} and f_{t_0} can be found from Eq. (A 5.1) in terms of p. 'Average' hoop tension T in the top 1-metre depth of steining of thickness $(r_0 - r_i)$, is $T = \left\{\frac{f_{t_0} + f_{t_i}}{2} \times 1.00 \times (r_0 - r_t)\right\}$ in tonne units, with f_{t_0} and f_{t_i} (and p) in T/m^2 units, and, r_0 and r_i in metre units. Equating the T to the maximum bursting force which exists in the top 1-metre depth of steining (which is = $\left(\frac{a+b}{2}\right) r_i \times 1.00$ tonnes, a and b as in the figure), obtain the value for p. Hence f_{t_i} and f_{t_o} can be quickly evaluated, and the greater of these, i.e. f_{t_i}, ensured against the permissible tensile strength of concrete and either hoop reinforcement designed as needed, or alternatively, concrete section and/or strength modified to suit.



A = OUTWARD BURSTING PRESSURE DUE TO COMPRESSED AIR.
B = INWARD (RELIEVING) HYDROSTATIC PRESSURE
C = NET BURSTING PRESSURE

Fig. A 5.1

APPENDIX 6

Extracts from the A.C.I. Committee Report 343 and the AASHTO Design Specifications

Some extracts are reproduced below with acknowledgement and courtesy purely as a ready reference 'companion reading' (However, reference must be made to the original texts for details).

... 'NOTATIONS' explained at the end of this Appendix.

NOTE: For Load Factor Design (R.C.), See Sections 7.3, etc., ahead.

(A) SERVICE-LOAD DESIGN

8.7 Permissible Stresses for Prestressed Flexural Members at Service Conditions

8.7.1 Flexural stresses in concrete immediately after transfer, before losses due to creep and shrinkage, should not exceed the following:

(a) Compression: Pre-tensioned $0.60f'_{ci}$
Post-tensioned $0.55f'_{ci}$

(b) Tension in member without bonded auxiliary reinforcement (unprestressed or prestressed) in the tension zone $3\sqrt{f'_{ci}}$

Where the calculated tensile stress exceeds this value, bonded reinforcement should be provided to resist the total tensile force in the concrete computed on the assumption of an uncracked section.

8.7.2 Flexural stresses in concrete at service loads, after allowance for all prestress losses, should not exceed the following:

(a) Compression $0.40f'_c$

(b) Tension in precompressed tension zone:
Members with bonded auxiliary reinforcement to control cracking $6\sqrt{f'_c}$
Members with bonded auxiliary reinforcement to control cracking, but exposed to corrosive environments or severe exposure conditions $3\sqrt{f'_c}$
Members without bonded auxiliary reinforcement 0

8.7.3 The permissible stresses recommended in Sections 8.7.1 and 8.7.2 may be exceeded if it is shown by tests or analysis that performance will not be impaired.

8.7.4 Stress in prestressing steel should not exceed the following:

(a) Due to temporary jacking force, but not greater than the maximum value recommended by the manufacturer of the steel or of the anchorages $0.80f_{pu}$

(b) Pre-tensioning tendons immediately after transfer or post-tensioning tendons immediately after anchoring $0.70f_{pu}$

8.8 Service Load Designing (R.C.)

8.8.1 *General*

Nonprestressed members may be designed with reference to service loads and allowable stresses. The stresses in concrete should not exceed the values given in Table A6–8.8.1 the tensile stress in reinforcement, f_s, should not exceed the following:

Grade 40 or Grade 50 reinforcement 20,000 psi
Grade 60 reinforcement 24,000 psi

8.8.2 *Development of Reinforcement*

The calculated tension or compression of the reinforcement at each section must be developed on each side of that section by proper embedment length, end anchorage or, for tension only, hooks. Development of reinforcement should be computed by procedures recommended in Chapter 13.

8.8.3 *Flexure*

For investigation of service load stresses the straight-line theory of stress and strain in flexure should be used and the assumptions of Section 8.1 should apply.

8.8.4 *Compression Members*

The combined axial load and moment capacity of compression members should be taken as 35 per cent of that computed in accordance with the recommendations of Section 7.3.5. Slenderness effects should be included according to the recommendations of Section 7.3.6. The term P_u in Eq. (7.20) should be replaced by 2.85 times the design axial load.

8.8.5 *Shear*

8.8.5.1 The design shear stress should be computed by:

$$v = V/b_w d$$

where b_w should be taken as the width of web and d should be taken as the distance from the extreme compression fibre to the centroid of the longitudinal tension reinforcement. For a circular section, b_w should be taken as the diameter and d need not be taken less than the distance from the extreme compression fibre to the centroid of the longitudinal reinforcement in the opposite half of the member.

8.8.5.2 When the reaction in the direction of the applied shear, introduces compression into the end region of the member, sections located less than distance d from the face of the support may be designed for the same v as that at a distance d.

8.8.5.3 The shear stress carried by the concrete, v_c, should not exceed $0.95\sqrt{f_c'}$ when v exceeds v_c, shear reinforcement should be provided.

8.8.5.4 Shear reinforcement should conform to the general requirements of Section 7.3.7.1. When shear reinforcement perpendicular to the axis of the member is used, the required area should be computed by:

$$A_v = \frac{(v - v_c)b_w s}{f_s}$$

8.8.5.5 When $(v - v_c)$ exceeds $2f_c'$, the maximum spacings given in Section 7.3.7.1.6 should be reduced by one-half. The maximum shear carried by concrete plus shear reinforcement should not exceed $5\sqrt{f_c'}$.

8.8.6 *Shear Stress in Slabs and Footings*

8.8.6.1 The shear capacity of slabs and footings in the vicinity of concentrated loads or reactions should be governed by the more severe of two conditions:

(a) The slab or footing acting as a wide beam, with a critical section extending in a plane across the entire width

Table A6-8.8.1 *Allowable Stresses*

Description		Basic value	Allowable Stresses: For strength of concrete shown, $f_c' =$ 3000 psi	$f_c' =$ 4000 psi	$f_c' =$ 5000 psi
Modular ratio, $n = \dfrac{E_s}{E_c}$ For normal weight concrete	n	$\dfrac{29,000,000}{57,000\sqrt{f_c'}}$	9	8	7
Flexure					
Extreme fibre stress in compression	f_c	$0.4f_c'$	1200	1600	2000
Extreme fibre stress in tension (plain concrete)*	f_t	$1.6\sqrt{f_c'}$	88	102	113
Modulus of rupture*	f_r	$7.5\sqrt{f_c'}$	410	475	530
Shear					
Beams:					
Shear carried by concrete*	v_c	$0.95\sqrt{f_c'}$	52	60	67
Maximum shear carried by concrete plus shear reinforcement	v	$5\sqrt{f_c'}$	274	316	354
Slabs and footings:					
Shear carried by concrete*	v_c	$1.8\sqrt{f_c'}$	99	114	127
Maximum shear carried by concrete plus shear reinforcement	v	$3\sqrt{f_c'}$	165	190	212
Bearing on loaded area†	f_b	$0.3f_c'$	900	1200	1500
Bearing on loaded area subjected to high edge stresses due to deflection or eccentric loading†	f_b	$0.225f_c'$	675	900	1125

* When lightweight aggregate concretes are used, the allowable stresses should be multiplied by 0.75 for "all-lightweight" concrete, and 0.35 for "sand-lightweight" concrete. Linear interpolation may be used when partial sand replacement is used.

† When the supporting surface is wider on all sides than the loaded area, the allowable bearing stress on the loaded area may be increased by $\sqrt{A_2/A_1}$, but not more than 2. When the supporting surface is sloped or stepped, A_2 may be taken as the area of the lower base of the largest frustum of a right pyramid or cone contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal.

and located at a distance d from the face of the concentrated load or reaction area. For this condition the slab or footing should be designed in accordance with Section 8.8.5.

(b) Two-way action for the slab or footing, with a critical section perpendicular to the plane of the slab and located so that its periphery is a minimum and approaches no closer than $d/2$ to the periphery of the concentrated load or reaction area. For this condition the slab or footing should be designed in accordance with Sections 8.8.6.2 and 8.8.6.3.

8.8.6.2 The peripheral shear stress should be computed by:

$$v = V/b_0 d$$

in which V and b_0 are taken at the critical section defined in Section 8.8.6.1 (b). The peripheral shear stress should not exceed the shear stress carried by the concrete $v_r = 1.8\sqrt{f'_c}$ unless shear reinforcement is provided in accordance with Section 8.8.6.3, in which case v_u should not exceed $3\sqrt{f'_c}$.

8.8.6.3 Shear reinforcement consisting of bars or wires may be provided. For design of such shear reinforcement, shear stresses should be investigated at the critical section defined in Section 8.8.6.1(b) and at successive sections more distant from the support; and the shear stress v_c, carried by the concrete at any section should not exceed $0.9\sqrt{f'_c}$, where v exceeds v_c, shear reinforcement should be provided according to Section 8.8.5.4.

8.4 Distribution of Reinforcement in Flexural Members

8.4.1 Only deformed reinforcement should be considered effective for principal reinforcement, except that plain bars or smooth wire may be used as spirals or reinforcement ties for confinement. All tension reinforcement should be well distributed in zones of maximum tension.

When the design yield strength, f_y, for tension reinforcement exceeds 40,000 psi, cross-sections of maximum tension stress should be so proportioned that the calculated stress in the reinforcement at service load, f_s, in kips per square inch, does not exceed the value computed by:

$$f_s = \frac{z}{\sqrt[3]{d_c A}} \qquad (8.1)$$

but f_s should not be greater than $0.6f_y$,

where A = effective tension area of concrete surrounding the main tension reinforcing bars and having the same centroid as the reinforcement, divided by the number of bars, sq. in. When the main reinforcement consists of several bar sizes, the number of bars should be computed as the total steel area divided by the area of the largest bar used.

d_c = thickness of concrete cover measured from the extreme tension fibre to the centre of the bar located closest thereto.

8.4.2 The quantity z in Eq. (8.1) should not exceed 170 kips per in. for members in moderate exposure conditions and 130 kips per in. for members in severe exposure conditions. Where members are exposed to very aggressive exposure or corrosive environments, such as deicer chemicals, the denseness and nonporosity of the protecting concrete should be considered, or other protection, such as a waterproof protecting system, should be provided in addition to satisfying Eq. (8.1).

8.4.3 Where girder flanges are in tension, the tension reinforcement should be distributed over an effective width as defined in Section 7.2.11.2 or 7.2.12.2, or a width equal to one-tenth of the girder span, whichever is smaller. If the effective slab width exceeds one-tenth of the span, additional longitudinal reinforcement should be provided in the outer portions of the slab.

8.4.4 If the depth of the side face of a flexural member exceeds 2 ft. longitudinal reinforcement having a total area not less than 10 per cent of the principal tension reinforcement should be placed, one-half near each side face of the member, and distributed in the zone of flexural tension. The spacing of such reinforcement should not exceed 12 in. or the width of the web, whichever is less. Such reinforcement may be included in computing the flexural capacity only if a stress and strain compatibility analysis is made to determine stresses in the individual bars.

(B) LOAD-FACTOR DESIGN (R.C.)

7.3 STRENGTH REQUIREMENTS

7.3.1 *Required Strength*

7.3.1.1 Bridge structures and structural members should be designed to have strength at all sections at least equal to the structural effects of the load groups which represent various combinations of loads and forces to which the structure may be subjected. Each part of such structure should be proportioned for the group loads that are applicable, and the maximum design required should be used.

7.3.1.2 The serviceability requirements of Chapter 8 should also be satisfied to insure adequate performance at service load levels.

7.3.2 *Strength*

The strength of a member or cross-section in terms of load, moment, shear, or stress should be taken as the strength computed in accordance with the recommendations and assumptions of this Section 7.3 modified by a capacity reduction factor, ϕ.*. The following values of ϕ should be used:

For flexure	$\phi = 0.90$
For shear and torsion	$\phi = 0.85$
For spirally reinforced compression members	$\phi = 0.75$
For tied compression members	$\phi = 0.70$
For bearing on concrete	$\phi = 0.70$

The value of ϕ may be increased linearly from the value for compression members to the value for flexure as the axial design load, P_u, decreases from $0.10f'_cA_g$ or P_b, whichever is smaller, to zero.

For precast prestressed units produced in plants having an acceptable quality control program, the capacity reduction factors in Section 9.2 may be applied. Development lengths specified in Chapter 13, do not require a ϕ factor.

7.3.3 *Design Assumptions*

The strength design of members for flexure and axial loads should be based on the assumptions given in this section, and on satisfaction of the applicable conditions of equilibrium and compatibility of strains.

(a) Strain in the reinforcing steel and concrete should be assumed directly proportional to the distance from the neutral axis.

(b) The maximum usable strain at the extreme concrete compression fibre should be assumed equal to 0.003.

(c) Stress in reinforcement below the specified yield strength, f_y, for the grade of steel used should be taken as E_s times the steel strain. For strains greater than that corresponding to f_y, the stress in the reinforcement should be considered independent of strain and equal to f_y.

(d) Tensile strength of the concrete should be neglected in flexural calculations of reinforced concrete.

(e) The relationship between the concrete compressive stress distribution and the concrete strain may be assumed to be a rectangle, trapezoid, parabola, or any shape which results in prediction of strength in substantial agreement with the results of comprehensive tests.

(f) The recommendations of Section 7.3.3(e) may be considered satisfied by an equivalent rectangular concrete stress distribution for concrete having a compressive strength f'_c of 8000 psi or less. This stress block is defined as follows: A concrete stress of 0.85 f'_c should be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fibre of maximum compressive strain. The distance c from the fibre of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The factor β_1 should be taken as 0.85 for strengths, up to and including 4,000 psi. For strengths above 4,000 psi β_1 should be reduced continuously at a rate of 0.05 for each 1,000 psi of strength in excess of 4,000 psi, but β_1 should not be taken less than 0.65.

7.3.4 *Flexure*

7.3.4.1 *Minimum Reinforcement of Nonprestressed Flexural Members*

7.3.4.1.1 At any section of a flexural member where tension reinforcement is required by analysis, the reinforcement provided should be adequate to develop a moment strength at least 1.2 times the cracking moment calculated on the basis of the modulus of rupture of the concrete. The minimum reinforcement ratio, $\rho_{\min}$ required for this condition may be obtained from the following approximate expressions.

Rectangular Sections:

$$\rho_{\min} = \frac{2\sqrt{f'_c}}{f_y}$$

T-Sections: $\rho_{\min} = 0.4\left(1 + \frac{b_w}{b}\right)\frac{\sqrt{f'_c}}{f_y}$

I-Sections: $\rho_{\min} = 2\left[1 - (1 - b_w/b)(1 - 2h_f/h)^3\right]\frac{\sqrt{f'_c}}{f_y}$ (7.1)

7.3.4.1.2 The minimum reinforcement recommendations may be waived if the area of reinforcement provided at a section is at least one-third greater than that required by analysis.

7.3.4.2 *Maximum Reinforcement of Nonprestressed Flexural Members*

7.3.4.2.1 For flexural members, the reinforcement ratio provided should not exceed 0.75 of that ratio ρ_b, which would produce balanced conditions for the section under flexure.

7.3.4.2.2 Balanced conditions exist at a cross-section when the tension reinforcement reaches its yield strength, f_y, just as the concrete in compression reaches its assumed ultimate strain of 0.003.

7.3.4.3 *Rectangular Sections with Nonprestressed Tension Reinforcement Only*

For rectangular sections, the design moment strength may be computed by:

$$M_u = \phi\left[A_s f_y d\left(1 - 0.6\rho\frac{f_y}{f'_c}\right)\right] \tag{7.2}$$

* The coefficient provides for the possibility that small adverse variations in material strengths, workmanship, and dimensions, while individually within acceptable tolerances and limits of good practice, may combine to result in under capacity. The capacity reduction factors given here are intended to apply to reinforced concrete, excluding prestress concrete. All ϕ factors given in this chapter agree with those given in ACI318-71. Those given in Chapter 9 for prestressed concrete differ with those given for prestressed concrete in AASHTO Standard Specifications for Highway Bridges and in ACI 318-71.

$$= \phi\left[A_s f_y\left(d - \frac{a}{2}\right)\right] \tag{7.3}$$

where

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

The balanced reinforcement ratio, ρ_b, for rectangular sections with tension reinforcement only is given by:

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y}\frac{87,000}{87,000 + f_y} \tag{7.4}$$

7.3.4.4 *I- and T-Sections with Tension Reinforcement Only*

7.3.4.4.1 When the compression flange thickness is equal to or greater than the depth to the neutral axis, a/β_1, the design moment strength may be computed by the equations given in Section 7.3.4.3.

7.3.4.4.2 When the compression flange thickness is less than a/β_1, the design moment strength may be computed by:

$$M_u = \phi\left[(A_s - A_{sf})\, f_y\left(d - \frac{a}{2}\right) + A_{sf} f_y \,(d - 0.5h_f)\right] \tag{7.5}$$

where

$$A_{sf} = \frac{0.85 f_c'(b - b_w)h_f}{f_y}$$

$$a = \frac{(A_s - A_{sf})f_y}{0.85 f_c' b_w}$$

The balanced reinforcement ratio, ρb, for I- and T-sections with tension reinforcement only is given by:

$$\rho b = \left[\frac{b_w}{b}\frac{0.85\beta_1 f_c'}{f_y}\frac{87,000}{87,000 + f_y} + \rho_f\right] \tag{7.6}$$

where

$$\rho f = \frac{A_{sf}}{b_w d}$$

7.3.4.5 *Rectangular Sections with Compression Reinforcement*

For rectangular sections and flanged sections in which the neutral axis lies within the flange, the design moment strength may be computed by:

$$M_u = \phi\left[(A_s - A_s')\, f_y\left(d - \frac{a}{2}\right) + A_s' f_y\,(d - d')\right] \tag{7.7}$$

where

$$a = \frac{(A_s - A_s')f_y}{0.85 f_c' b}$$

and the following condition should be satisfied:

$$\frac{(A_s - A_s')}{bd} \geq 0.85\beta_1 \frac{f_c' d'}{f_y d}\frac{87,000}{87,000 - f_y} \tag{7.8}$$

When the value of $(A_s - A_s')/bd$ is less than the value given by Eq. (7.8), so that the stress in the compression reinforcement is less than the yield strength, or when effects of compression reinforcement are neglected, the moment strength may be computed by the equations in Section 7.3.4.3 except when a general analysis is made based on stress and strain compatibility using the assumptions given in Section 7.3.3.

The balanced reinforcement ratio, ρb, for rectangular sections with compression reinforcement is given by:

$$\rho b = \frac{0.85\beta_1 f_c'}{f_y}\frac{87,000}{87,000 + f_y} + \rho' \frac{f_{sb}'}{f_y} \tag{7.9}$$

where

f_{sb}' = stress in compression reinforcement at balanced conditions

$= 87,000\left(1 - \frac{d'}{d}\frac{87,000 + f_y}{87,000}\right) \leq f_y$

7.3.4.6 *Other Nonprestressed Cross-sections*

For other cross-sections and for conditions of nonsymmetrical bending, the design moment strength, $M_u = \phi M_t$, should be computed by a general analysis based on stress and strain compatibility using the assumptions given in Section 7.3.3. The requirements of Section 7.3.4.2 should also be satisfied.

7.3.4.7 *Prestressed Concrete Members*

7.3.4.7.1 The design moment strength for prestressed concrete members may be computed by the same strength design procedures and equations recommended for non-prestressed members. For prestressing steel, f_{ps} should be substituted for f_y.

7.3.4.7.2 In lieu of a more precise determination of f_{ps} based on strain compatibility, and provided that f_{se} is not less than 0.5 f_{pu}, the following approximate values should be used:

Bonded members

$$f_{ps} = f_{pu}\left(1 - 0.5\rho_p \frac{f_{pu}}{f_c'}\right) \tag{7.10}$$

Unbonded members

$$f_{ps} = f_{se} + 10,000 + \frac{f'_c}{100\rho_p} \qquad (7.11)$$

but f_{ps} should not be taken more than $f_{se} + 60,000$, nor the yield strength of the prestressing tendons.

7.3.4.7.3 Nonprestressed reinforcement, when used in combination with prestressed steel, may be considered to contribute to the tensile force in a member at design moment strength an amount equal to its area times its yield strength.

7.3.4.7.4 The amount of prestressed and nonprestressed steel used for calculation of the design moment strength should be such that

$$A_{ps}\frac{f_{ps}}{bdf'_c}$$

or

$$A_s\frac{f_y}{bdf'_c} + A_{ps}\frac{f_{ps}}{bdf'_c} - A'_s\frac{f_y}{bdf'_c}$$

is not greater than 0.30 For flanged sections, b should be taken as the web width and the steel area should be that required to develop the compressive strength of the webs only.

7.3.4.7.5 When a steel ratio in excess of 0.30 is used, the design moment strength should be based on the compression portion of the internal resisting moment couple. The following expressions satisfy the intent of this recommendation:

For rectangular sections, or flanged sections in which the neutral axis is within the flange:

$$M_u = \phi(0.25f'_c bd^2) \qquad (7.12)$$

For flanged sections in which the neutral axis falls outside the flange:

$$M_u = \phi[0.25f'_c b_w d^2 + 0.85f_c(b - b_w)] \\ h_f(d - 0.5h_f) \qquad (7.13)$$

7.3.4.7.6 The total amount of prestressed and nonprestressed reinforcement provided should be adequate to develop a moment strength at least 1.2 times the cracking moment calculated on the basis of the modulus of rupture of the concrete.

7.3.5 *Nonprestressed Compression Members with or without Flexure*

7.3.5.1 *General Requirements*

7.3.5.1.1 The design of cross-sections subject to combined flexure and axial load should be based on stress and strain compatibility using the assumptions given in Section 7.3.3. For prestressed members refer to Chapter 9. Slenderness effects should be included according to the recommendations of Section 7.3.6.

7.3.5.1.2 All members subjected to a compression load should be designed for an eccentricity equal to the greatest of

(a) that corresponding to the maximum design moment which accompanies this compression load, or
(b) 0.05 h for spirally reinforced compression members, or 0.10 h for tied compression members, about either axis, or
(c) 1 in. about either axis.

7.3.5.2 *Minimum Reinforcement for Compression Members*

7.3.5.2.1 Longitudinal reinforcement for compression members should be not less than 0.01 nor more than 0.08 times the gross area of the section. The minimum number of longitudinal reinforcing bars should be six for bars in a circular arrangement and four for bars in a rectangular arrangement. For other shapes, one bar should be placed at each corner. The minimum size of bar should be # 5.

7.3.5.2.2 In a compression member which has a larger cross-section than required by considerations of loading, a reduced effective area may be used for determining minimum longitudinal reinforcement, provided that in no case should less longitudinal reinforcement be used than that required by the minimum section designed with 1% of longitudinal reinforcement.

7.3.5.3 *Compression Member Strength*

The following provisions may be used as a guide to define the range of the load-moment interaction relationship for members subjected to combined flexure and axial load.

7.3.5.3.1 Pure compression. The axial design load strength of a cross-section in pure compression, P_0, may be computed by:

$$P_0 = \phi[0.85f'_c(A_g - A_{si}) + A_{st}f_y] \qquad (7.14)$$

Concentric loading is a hypothetical loading condition since all members subjected to a compression load should be designed for eccentricities not less than the values recommended in Section 7.3.5.1.2.

7.3.5.3.2 Pure flexure. The assumptions given in Section 7.3.3 or the applicable equations for flexure given in Section 7.3.4 may be used to compute the design moment strength, M_u, for the condition of pure flexure.

7.3.5.3.3 Balanced conditions. Balanced conditions for a cross-section are defined in Section 7.3.4.2. For a rectangular section with reinforcement in one or two faces and located at approximately the same distance from the axis of bending, the balanced load, P_b, and balanced moment, M_b, may be computed by:

$$P_b = \phi[0.85f'_c ba_b + A'_s f'_s - A_s f_y] \qquad (7.15)$$

and

$$M_b = P_b e_b = \phi\left[0.85 f_c' b a_b \left(d - d'' - \frac{a_b}{2}\right) + A_s' f_s' (d - d' - d'') + A_s f_y d''\right] \quad (7.16)$$

where

$$a_b = \left(\frac{87,000}{87,000 + f_y}\right)\beta_1 d$$

$$f_s' = 87,000\left(1 - \frac{d'}{d}\frac{87,000 + f_y}{87,000}\right) \le f_y$$

7.3.5.3.4 Combined flexure and axial load. The design strength under combined flexure and axial load should be based on stress and strain compatibility using the assumptions given in Section 7.3.3. The strength of a cross-section is controlled by tension when the axial design load strength, P_w, is less than P_b (or e is greater than e_b). The strength of a cross-section is controlled by compression when the axial design load strength, P_w is greater than P_b (or e is less than e_b).

The combined axial load and moment strength should be multiplied by the appropriate capacity reduction factor, ϕ, for spirally reinforced or tied compression members. The value of ϕ may be increased linearly from the value for compression members to the value for flexure as the axial design load strength, P_w, decreases from $0.10\, f_c' A_g$ or P_b, whichever is smaller, to zero.

7.3.5.4 *Biaxial Loading*

In lieu of a general section analysis based on stress and strain compatibility for a loading condition of biaxial bending, the design strength of noncircular members subjected to biaxial bending may be computed by the following approximate expressions:

$$P_{uxy} = \frac{1}{(1/P_{ux}) + (1/P_{uy}) - (1/P_0)} \quad (7.17)$$

when the applied axial design load,

$$P_u \ge 0.1 f_c' A_g$$

or

$$\frac{M_x}{M_{ux}} + \frac{M_y}{M_{uy}} \le 1 \quad (7.18)$$

when the applied axial design load,

$$P_u < 0.1 f_c' A_g$$

For disproportionate rectangular sections, Eq. (7.17) may yield unconservative results. In such cases, an analysis should be made based on stress and strain compatibility using the assumptions given in Section 7.3.3.

7.3.6 *Strength Reduction for Length of Compression Members*

7.3.6.1 Wherever possible the design of compression members should be based on a comprehensive analysis of the structure. Such analysis should be a second-order (Ref. 7.1) analysis, taking into account the deformation of the structure and the duration of the loads. When axial loads are of sufficient magnitude to reduce stiffness or increase fixed-end moments, such effects should be included. When a second-order analysis is made, additional correction of moments for length effects are not required.

In lieu of a second-order analysis, the design of compression members may be based on the approximate procedure recommended in Section 7.3.6.2.

7.3.6.2 *Approximate Method for Length Effects for Compression Members*

7.3.6.2.1 For purposes of determining the limiting dimensions of compression members, the unsupported length l_u should be taken as the clear distance between lateral supports, except as recommended in Subsections (a), (b), (c), and (d) below.

(a) In pile bent construction, l_u for the pile should be the clear distance between the lowest lateral support described in Section 7.3.6.2.1 (c) and the lower extremity at which the pile may be assumed to be fixed, depending on the soil conditions.

(b) For compression members supported on spread or pile footings, l_u should be the clear distance between the footing and the underside of the deeper flexural member framing into the compression member in the direction of potential translation at the next higher level.

(c) For members restrained laterally by struts, ties or beams, l_u, should be the clear distance between consecutive struts in each vertical plane: provided that, to be an adequate support two such struts should meet the compression member at approximately the same level, and the angle between vertical planes through the struts should not vary more than 15 degrees from a right angle. Such struts should be of adequate dimensions and have sufficient anchorage to restrain the member against lateral deflection.

(d) For compression members restrained laterally by struts or beams with brackets used at the junction, l_u should be the clear distance between the lower support or point of fixity of the base, and the lower edge of the bracket, provided that the bracket width equals that of the beam or strut and is at least half that of the compression member.

7.3.6.2.2 The length which produces the greatest ratio of length to radius of gyration of the section compatible with the loading and support conditions should be considered.

7.3.6.2.3 The radius of gyration may be taken equal to 0.30 times the overall depth in the direction of bending for rectangular members, and 0.25 times the diameter for circular members. For other shapes, the radius of gyration may be computed for the gross concrete section.

7.3.6.2.4 The effective length of compression members should be taken as kl_u, where the effective length factor, k, should not be less than that recommended in Subsections (a) and (b) below, Ref. 7.2.

(a) Lateral or sidesway instability should be considered in defining the effective length, kl_u, for all structures in which the total translational stiffness of the bracing elements is less than six times the sum of the combined translational stiffnesses of all of the compression members in the level under consideration. All other structures may be assumed to be restrained against lateral instability. For structures subject to sidesway, the value of kl_u should be determined with due consideration of cracking and reinforcement on relative stiffness. When determining the stiffness ratio of beams and columns to obtain k, the beam stiffness should be that of the cracked section, and column stiffness should be obtained using Eq. (7.22) with β_d taken as zero. In no case should k be taken as less than 1.0.

(b) In structures restrained against translation, the value of k should be taken equal to 1.0 unless an analysis shows that a smaller value may be used.

7.3.6.2.5 For compression members braced against sidesway, the effects of slenderness may be neglected when kl_u/r is less than 34–12 M_1/M_2. For compression members not braced against sidesway, the effects of slenderness may be neglected when kl_u/r is less than 22. For all compression members with kl_u/r greater than 100, a second-order analysis should be made. M_1 is the smaller end moment on the compression member calculated from a conventional elastic analysis, positive if member is bent in single curvature, negative if bent in double curvature. M_2 is the larger end moment on the compression member calculated from a conventional elastic analysis, always positive.

7.3.6.2.6 Compression members should be designed using the applied design axial load, P_u, from a conventional elastic analysis and a magnified moment M_c defined by

$$M_c = \delta M_2 \tag{7.19}$$

where

$$\delta = \frac{C_m}{1 - P_u/\phi P_c} \geq 1.0 \tag{7.20}$$

and

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \tag{7.21}$$

In lieu of a more precise calculation, EI, in Eq. (7.21) may be taken either as

$$EI = \frac{(E_c I_g/5) + E_s I_s}{1 + \beta_d} \tag{7.22}$$

or alternatively

$$EI = \frac{E_c I_g/2.5}{1 + \beta_d} \tag{7.23}$$

where β_d is the ratio of maximum design dead load moment to maximum design total load moment, always positive.

For members braced against sidesway and without transverse loads between supports C_m may be taken as

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \tag{7.24}$$

but not less than 0.4.

For members braced against sidesway and with transverse loads between supports in the plane of loading, C_m may be determined by analysis. For all other cases, C_m should be taken as 1.0.

7.3.6.2.7 When design of compression members is governed by the minimum eccentricities recommended in 7.3.5.1.2 M_2 should be based on the minimum eccentricity with conditions of curvature determined by either of the following:

(a) When the actual computed eccentricities are less than the minimum, the computed end moments may be used to evaluate the conditions of curvature.

(b) If computations show that there is no eccentricity at both ends of the member, conditions of curvature should be based on a ratio of M_1/M_2 equal to one.

7.3.6.2.8 When compression members are subject to bending about both principal axes, the moment about each axis should be amplified by δ, computed from the corresponding conditions of restraint about that axis.

7.3.6.2.9 In structures which are not braced against sidesway, the flexural members should be designed for the total magnified end moments of the compression members at the joint.

7.3.6.2.10 When a group of compression members on one level comprise a bent, or when they are connected integrally to the same superstructure, and collectively resist the sidesway of the structure, the value of δ should be computed for the member group, P_u and P_c should be taken as the ΣP_u and ΣP_c for all members in the group. In designing each member in the group, δ should be taken as the larger of (a) the value computed for the group as a whole, or (b) the value computed for the individual compression member assuming its ends to be braced against sidesway.

7.3.7 *Shear*

7.3.7.1 *Shear Reinforcement—General Requirements*

7.3.7.1.1 A minimum area of shear reinforcement should be provided in all flexural members, except slabs and footings, where the design shear stress is greater than one-half the shear stress carried by the concrete.

7.3.7.1.2 Where shear reinforcement is required the area provided should be not less than

$$A_v = \frac{50b_w s}{f_y} \tag{7.25}$$

where b_w and s are in inches.

7.3.7.1.3 Shear reinforcement may consist of:

(a) Stirrups perpendicular to the axis of the member or making an angle of 45 degree or more with the longitudinal tension reinforcement.

(b) Welded wire fabric with wires located perpendicular to the axis of the member.

(c) Longitudinal bars with a bent portion making an angle of 30 degrees or more with the longitudinal tension bars.

(d) Combinations of stirrups and bent bars.

(e) Spirals.

7.3.7.1.4 Web reinforcement should extend to a distance d from the extreme compression fibre and should be anchored at both ends to develop the design yield strength of the reinforcement.

7.3.7.1.5 Spacing of web reinforcement. Where web reinforcement is required and is placed perpendicular to the axis of the member, it should be spaced not further apart than $0.50d$ in nonprestressed concrete and $0.75h$ in prestressed concrete, but not more than 24 in. Inclined stirrups and bent bars should be so spaced that every 45 degree line, extending toward the reaction from the middepth of the member, $0.50d$, to the longitudinal tension bars, should be crossed by at least one line of shear reinforcement.

7.3.7.1.6 The design yield strength of nonprestressed shear and torsion reinforcement should not exceed 60,000 psi.

7.3.7.2 *Shear Strength*

7.3.7.2.1 The design shear stress v_u should be computed by:

$$v_u = \frac{V_u}{\phi b_w d} \tag{7.26}$$

where b_w should be taken as the width of web and d should be taken as the distance from the extreme compression fibre to the centroid of the longitudinal tension reinforcement. For prestressed concrete members d need not be taken less than $0.80h$. For a circular section, d need not be taken less than the distance from the extreme compression fibre to the centroid of the longitudinal reinforcement in the opposite half of the member.

7.3.7.2.2 When the reaction in the direction of the applied shear introduces compression into the end region of the member, sections located less than a distance d from the face of the support may be designed for the same v_u as that computed at a distance d. For prestressed concrete members, sections located at a distance less than $h/2$ may be designed for the shear computed at $h/2$.

7.3.7.2.3 When the design shear stress exceeds the shear stress carried by the concrete, shear reinforcement should be provided. Whenever applicable, the effects of torsion according to the recommendations of Section 7.3.8 should be added.

7.3.7.3 *Shear Stress Carried by Concrete for Nonprestressed Members*

7.3.7.3.1 The shear stress carried by the concrete should not exceed:

$$v_c = 2\sqrt{f_c'} \tag{7.27}$$

unless a more detailed analysis is made in accordance with Sections 7.3.7.3.2 or 7.3.7.3.3.

For members subjected to axial tension or torsion, the shear stress carried by the concrete should not exceed the values given in Sections 7.3.7.3.4 or 7.3.7.3.5.

7.3.7.3.2 The shear stress carried by the concrete may be computed by:

$$v_c = 1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u} \tag{7.28}$$

but v_c should not exceed $3.5\sqrt{f_c'}$. The quantity $V_u d/M_u$ should not be taken greater than 1.0, where M_u is the bending moment occurring simultaneously with V_u at the section considered.

7.3.7.3.3 For members subjected to axial compression, v_c may be computed by:

$$v_c = 2\left(1 + 0.0005\frac{N_u}{A_g}\right)\sqrt{f_c'} \tag{7.29}$$

The quantity N_u/A_g should be expressed in pounds per square inch.

7.3.7.3.4 For members subjected to significant axial tension, shear reinforcement should be designed to carry the total shear, unless a more detailed analysis is made using

$$v_c = 2\left(1 + 0.002\frac{N_u}{A_g}\right)\sqrt{f_c'} \tag{7.30}$$

where N_u is negative for tension, the quantity N_u/A_g should be expressed in pounds per square inch.

7.3.7.3.5 At cross-sections subject to a design torsion stress exceeding $1.5f_c'$, in accordance with the recommendations of Section 7.3.8, the shear stress carried by the concrete should not exceed:

$$v_c = \frac{2\sqrt{f_c'}}{\sqrt{\left(1+\frac{v_{tu}}{1.2v_u}\right)^2}} \tag{7.31}$$

7.3.7.4 *Shear Stress Carried by Concrete for Prestressed Members*

7.3.7.4.1 For members having an effective prestress force at least equal to 40% of the tensile strength of the flexural reinforcement, the shear stress carried by the concrete should not exceed:

$$v_c = 0.6\sqrt{f_c'} + 700\frac{V_u d}{M_u} \tag{7.32}$$

but v_c need not be taken less than $2f_c'$ nor should v_c be greater than $5f_c'$ unless a more detailed analysis is made in accordance with Section 7.3.7.4.2. When applying Eq. (7.32), M_u is the bending moment occurring simultaneously with V_u at the section considered and d should be the distance from the extreme compression fibre to the centroid of the prestressing tendons.

7.3.7.4.2 The shear stress carried by the concrete may be computed as the lesser of v_{ci} or v_{cw}.

$$v_{ci} = 0.6\sqrt{f_c'} + \frac{\left(V_d + \frac{V_i M_{cr}}{M_{max}}\right)}{b_w d} \tag{7.33}$$

but need not be taken less than $1.7\sqrt{f_c'}$, where

$$M_{cr} = (I/y_t)(6\sqrt{f_c'} + f_{pc} - f_d)$$

$$v_{cw} = 3.5\sqrt{f_c'} + 0.3f_{pc}' + V_p/b_d w \tag{7.34}$$

Alternatively, v_{cw} may be taken as the shear stress corresponding to a multiple of deadload plus live load which results in a computed principal tensile stress of $4\sqrt{f_c'}$ at the centroidal axis of the member, or at the intersection of the flange and the web when the centroidal axis is in the flange. In a composite member, the principal tensile stress should be computed using the cross-section which resists live load.

When applying Eqs (7.33) and (7.34), d should be the distance from the extreme compression fibre to the centroid of the prestressing tendons or $0.8h$, whichever is greater. The values of M_{max} and V_i in Eq. (7.33) should be computed from the load distribution causing maximum moment to occur at a section.

In a pretensioned member in which the section at a distance $h/2$ from the face of the support is closer to the end of the beam than the transfer length of the tendons, the reduced prestress should be considered when calculating v_{cw}. This value of v_{cw} should also be taken as the maximum limit for Eq. (7.32). The prestress force may be assumed to vary linearly from zero at the end of the tendon to a maximum at a distance from the end of the tendon equal to the transfer length, assumed to be 70 diameters for strand and 100 diameters for single wires.

7.3.7.5 The recommendations for shear stress carried by the concrete apply to normal weight concrete. When lightweight aggregate concretes are used, one of the following modifications should apply:

(a) When f_{ct} is determined, the stress, v_c or v_{tc} should be modified by substituting $f_{ct}/6.7$ for $\sqrt{f_c'}$ but the value of $f_{ct}/6.7$ used should not exceed $\sqrt{f_c'}$.

(b) When f_{ct} is not determined, all values of $\sqrt{f_c'}$ affecting v_c, v_{tc}, or M_{cr} should be multiplied by 0.75 for "all-lightweight" concrete, and 0.85 for "sand-lightweight" concrete. Linear interpolation may be used when partial sand replacement is used.

7.3.7.6 *Design of Shear Reinforcement*

7.3.7.6.1 Shear reinforcement should conform to the general recommendations of Section 7.3.7.1. When shear reinforcement perpendicular to the axis of the member is used, the required area should be computed by:

$$A_v = \frac{(v_u - v_c)b_w s}{f_y} \tag{7.35}$$

7.3.7.6.2 When inclined stirrups or bent bars are used as shear reinforcement, the following applies:

(a) When inclined stirrups are used, the required area should be computed by:

$$A_v = \frac{(v_u - v_c)b_w s}{f_y(\sin\alpha + \cos\alpha)} \tag{7.36}$$

(b) When shear reinforcement consists of a single bar or a single group of parallel bars, all bent up at the same distance from the support, the required area should be computed by:

$$A_v = \frac{(v_u - v_c)b_w d}{f_y \sin\alpha} \tag{7.37}$$

in which $(v_u - v_c)$ should not exceed $3\sqrt{f_c'}$.

(c) When shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support, the required area should be computed by Eq. (7.36).

(d) Only the centre three-fourths of the inclined portion of any longitudinal bar that is bent should be considered effective for shear reinforcement.

7.3.7.6.3 Where more than one type of shear reinforcement is used to reinforce the same portion of the member, the required area should be computed as the sum for the various types separately.

7.3.7.6.4 When $(v_u - v_c)$ exceeds $4\sqrt{f_c'}$, the maximum spacings recommended in Section 7.3.7.1 should be reduced by one-half.

7.3.7.6.5 The value of $(v_u - v_c)$ should not exceed $8\sqrt{f_c'}$.

7.3.8 *Combined Torsion and Shear for Nonprestressed Members*

7.3.8.1 Torsion effects should be included for shear and bending whenever the design torsion stress exceeds $1.5\sqrt{f_c'}$. For members with rectangular or flanged sections, the design torsion stress should be computed by:

$$v_{tu} = \frac{3T_u}{\phi \Sigma x^2 y} \tag{7.38}$$

where $\Sigma x^2 y$ should be taken for the component rectangles of the section, but the overhanging flange width used in design should not exceed three times the thickness of the flange. The rectangles may be so formed as to provide the largest value of $\Sigma x^2 y$.

7.3.8.2 Rectangular box sections may be taken as solid sections, provided that the wall thickness, h, is at least $x/4$. For box sections with a wall thickness less than $x/4$, but greater than $x/10, \Sigma x^2 y$ should be multiplied by $4h/x$. When h is less than $x/10$, the stiffness of the wall should be considered.

7.3.8.3 Hollow box sections with a wall thickness less than $x/4$, in which the longitudinal torsion reinforcement consists of less than eight bars distributed around the section perimeter, should have at each interior corner a fillet with minimum leg size of $x/6$. When the longitudinal torsion reinforcement consists of eight or more bars distributed around the section perimeter, fillets should be provided at each interior corner, with a minimum leg size of $x/12$ but preferably not more than 4 in.

7.3.8.4 Sections located less than a distance d from the face of the support may be designed for the same torsion as that computed at a distance d.

7.3.8.5 The torsion stress carried by the concrete should not exceed:

$$v_{tc} = \frac{2.4\sqrt{f_c'}}{\sqrt{1 + \left(1.2\dfrac{v_u}{v_{tu}}\right)^2}} \tag{7.39}$$

7.3.8.6 When the design torsion stress v_{tu} exceeds the torsion stress carried by the concrete v_{tc}, torsion reinforcement should be provided.

7.3.8.7 For members subjected to significant axial tension, torsion reinforcement should be designed to carry the total torque, unless a more detailed analysis is made in which v_{tc} given by Eq. (7.39), and v_c given by Eq. (7.31) are multiplied by $(1 + 0.002N_u/A_g)$, where N_u is negative for tension.

7.3.8.8 For noncompact open cross-sections, warping torsional resistance may be taken into account based on an analysis (7–3).

7.3.8.9 *Design of Torsion Reinforcement*

7.3.8.9.1 Torsion reinforcement, where required, should be provided in addition to reinforcement required to resist shear, flexure, and axial forces. The reinforcement required for torsion may be combined with that required for other forces, provided the area furnished is the sum of the individually required areas and the most restrictive requirements for spacing and placement are met.

7.3.8.9.2 The required area of closed stirrups should be computed by:

$$A_t = \frac{(v_{tu} - v_{tc})s\Sigma x^2 y}{3\alpha_t x_1 y_1 (f_y)} \tag{7.40}$$

where $\alpha_t = [0.66 + 0.33(y_1/x_1)]$, but not more than 1.50.

7.3.8.9.3 The spacing of closed stirrups should not exceed $(x_1 + y_1)/4$, or 12 in., whichever is the smaller.

7.3.8.9.4 The required area of longitudinal bars should be computed by:

$$A_l = 2A_t \frac{x_1 + y_1}{s} \tag{7.41}$$

or by

$$A_l = \left[\frac{400xs}{f_y}\left(\frac{v_{tu}}{v_{tu} + v_u}\right) - 2A_t\right]\left(\frac{x_1 + y_1}{s}\right) \tag{7.42}$$

whichever is the greater. The value of A_l computed by Eq. (7.42) need not exceed that obtained by substituting

$$\frac{50b_w s}{f_y} \text{ for } 2A_t$$

7.3.8.9.5 The spacing of longitudinal bars, distributed around the perimeter of the stirrups, should not exceed 12 in. At least one longitudinal bar should be placed in each corner of the stirrups.

7.3.8.9.6 Torsion reinforcement should be provided at least a distance $(d + b)$ beyond the point theoretically required.

7.3.8.9.7 For T-, I-, or L-sections, the arrangement of the torsion reinforcement should reflect the choice of the component rectangles.

7.3.8.9.8 The design torsion stress, v_{tu}, should not exceed:

$$v_{tu} = \frac{12\sqrt{f_c'}}{\sqrt{1 + \left(1.2\frac{v_u}{v_{tu}}\right)^2}} \quad (7.43)$$

7.3.9 *Shear Friction*

7.3.9.1 These provisions apply where it is inappropriate to consider shear as a measure of diagonal tension, and particularly in design of reinforcing details for precast concrete structural elements.

7.3.9.2 A crack should be assumed to occur along the shear path. Relative displacement should be considered resisted by friction maintained by shear-friction reinforcement across the crack. This reinforcement should be approximately perpendicular to the assumed crack.

7.3.9.3 The shear stress v_u should not exceed $0.2f_c'$ nor 800 psi.

7.3.9.4 The required area of reinforcement snould be computed by:

$$A_{vf} = \frac{V_u}{\phi f_y \mu} \quad (7.44)$$

The coefficient of friction, μ, should be 1.4 for concrete cast monolithically, 1.0 for concrete placed against hardened concrete, and 0.7 for concrete placed against as-rolled structural steel.

7.3.9.5 Direct tension across the assumed crack should be provided for by additional reinforcement.

7.3.9.6 The shear-friction reinforcement should be well distributed across the assumed crack and should be adequately anchored on both sides by embedment, hooks, or welding to special devices.

7.3.9.7 When shear is transferred between concrete placed against hardened concrete, the interface should be rough, clean, and free of laitance with a full amplitude of approximately 1/4 in. When shear is transferred between as-rolled steel and concrete, the steel should be clean and without paint.

7.3.10 *Horizontal Shear Design for Composite Concrete Flexural Members*

7.3.10.1 In a composite member, full transfer of the shear forces should be assured at the interfaces of the separate components.

7.3.10.2 Full transfer of horizontal shear forces may be assumed when all of the following are satisfied:

(a) the contact surfaces are clean and intentionally roughened

(b) minimum ties are provided in accordance with Section 7.3.10.6

(c) web members are designed to resist the entire vertical shear, and

(d) all shear reinforcement is anchored into all intersecting components. When all of the above are not satisfied, horizontal shear should be fully investigated.

7.3.10.3 The design horizontal shear stress, v_{dh}, may be computed at any cross-section as

$$v_{dh} = \frac{V_u}{\phi b_v d}$$

in which d is for the entire composite section. Alternatively, in any segment not exceeding one-tenth of the span, the actual change in compressive or tensile force to be transferred may be computed, and provisions made to transfer that force as horizontal shear to the supporting element.

7.3.10.4 The horizontal shear may be transferred at contact surfaces using the permissible horizontal shear stress, v_h, stated below.

(a) When ties are not provided, but the contact surfaces are clean and intentionally roughened, permissible v_h = 80 psi.

(b) When the minimum tie requirements of Section 7.3.10.6 are provided and the contact surfaces are clean but not intentionally roughened, permissible v_h = 80 psi.

(c) When the minimum tie requirements of Section 7.3.10.6 are provided and the contact surfaces are clean and intentionally roughened, permissible v_h = 350 psi.

(d) When v_{dh} exceeds 350 psi, design for horizontal shear should be made in accordance with Section 7.3.9.

7.3.10.5 When tension exists perpendicular to any surface, shear transfer by contact should be assumed only when the minimum tie requirements of Section 7.3.10.6 are satisfied.

7.3.10.6 *Ties for Horizontal Shear*

7.3.10.6.1 When vertical bars or extended stirrups are used to transfer horizontal shear, the tie area should not be less than that required by Section 7.3.7.1.1 and the spacing should not exceed four times the least dimension of the supported element nor 24 in.

7.3.10.6.2 Ties for horizontal shear may consist of single bars, multiple leg stirrups, or the vertical legs of welded wire fabric. All ties should be adequately anchored into the components by embedment or hooks.

7.3.10.7 *Measure of Roughness*

Internal roughness should be assumed only when the contact surface is roughened, clean, and free of laitance. Roughness should have a full amplitude of approximately 1/4 in.

7.3.11 *Special Recommendations for Brackets and Short Cantilevers*

7.3.11.1 These recommendations apply to brackets and short cantilevers having a shear-span-to-depth ratio, a/d, of unity or less. The distance d should be measured at a section adjacent to the face of the support, but should not be taken greater than twice the depth of the member at the outside edge of the bearing area.

7.3.11.2 When special provisions are made to avoid tension due to restrained shrinkage and creep, so that the member is subject to shear and moment only, the design shear stress should not exceed:

$$v_u = 6.5\left(1 - 0.5\frac{\alpha}{d}\right)(1 + 64\rho_v)\sqrt{f'_c} \tag{7.45}$$

where

$$\rho_v = \frac{A_s + A_h}{bd}$$

but not greater than

$$0.20\frac{f'_c}{f_y}$$

and A_h should not exceed A_s.

7.3.11.3 For members subject to direct tension, the design shear stress should not exceed:

$$v_u = \left[6.5 - 5.1\sqrt{\frac{N_u}{V_u}}\right]\left[1 - 0.5\frac{\alpha}{d}x\right]$$
$$\left\{1 + \left[64 + 160\sqrt{\frac{N_u^3}{V_u}}\right]\rho\right\}\sqrt{f'_c} \tag{7.46}$$

where ρ should not exceed $0.13\ f'_c/f_y$ and N_u/V_u should not be taken less than 0.20. The tensile force N_u should be regarded as a live load even when it results from creep, shrinkage, or temperature change.

7.3.11.4 Closed stirrups or ties parallel to the main tension reinforcement having a total cross-sectional area A_h not less than 0.50 A_s should be uniformly distributed within two-thirds of the effective depth adjacent to the main tension reinforcement.

7.3.11.5 The ratio $\rho = A_s/bd$ should not be less than $0.04\ (f'_c/f_y)$.

7.3.12 *Special Recommendations for Deep Flexural Members*

7.3.12.1 These recommendations apply to members with a span-to-depth ratio, l_v/d, less than five and the members are loaded at the top or compression face.

7.3.12.2 The shear stress carried by the concrete should not exceed:

$$v_c = 2\sqrt{f'_v} \tag{7.47}$$

unless a more detailed analysis is made using

$$v_c = \left(3.5 - 2.5\frac{M_u}{V_u d}\right)\left(1.9\sqrt{f'_c} + 2500\rho_w\frac{V_u d}{M_u}\right) \tag{7.48}$$

where the term $\left(3.5 - 2.5\frac{M_u}{V_u d}\right)$ should not exceed 2.5, and v_c should not exceed $6\sqrt{f'_c}$ · M_u and V_u are the bending moment and shear occurring simultaneously at the critical section defined by Section 7.3.12.3.

7.3.12.3 The critical section for shear measured from the face of the support should be taken at 0.15 l_n for uniformly loaded flexural members and 0.50 for members with concentrated loads, but not greater than d. Shear reinforcement required at the critical section should be used throughout the span.

7.3.12.4 The design shear stress should not exceed $8\sqrt{f'_c}$ when l_n/d is less than 2. When l_n/d is between 2 and 5, the design shear stress should not exceed:

$$v_u = \frac{2}{3}\left(10 + \frac{l_n}{d}\right)\sqrt{f'_c} \tag{7.49}$$

7.3.12.5 The area of shear reinforcement should be computed from:

$$\frac{A_v}{s}\frac{\left(1 + \frac{l_n}{d}\right)}{12} + \frac{A_{vh}}{s_2}\frac{\left(11 - \frac{l_n}{d}\right)}{12} = \frac{(v_u - v_c)b_w}{f_y} \tag{7.50}$$

7.3.12.6 The area of shear reinforcement A_v perpendicular to the main reinforcement should not be less than 0.0015 bs, and s should not exceed $d/5$ or 18 in. The area of shear reinforcement A_{vh} parallel to the main reinforcement should not be less than 0.0025 bs_2 and s_2 should not exceed $d/3$ or 18 in.

7.3.13 *Special Recommendations for Slabs and Footings*

7.3.13.1 The shear strength of slabs and footings in the vicinity of concentrated loads or reactions should be governed by the more severe of two conditions:

(a) The slab or footing acting as a wide beam, with a critical section extending in a plane across the entire width and located at a distance d from the face of the concentrated load or reaction area. For this condition the slab or footing should be designed in accordance with the recommendations in Sections 7.3.7.1 through 7.3.7.6.

(b) Two-way action for the slab or footing, with a critical section perpendicular to the plane of the slab and located so that its periphery is a minimum and approaches no closer than $d/2$ to the periphery of the concentrated load or reaction area. For this condition the slab or footing should be designed in accordance with Sections 7.3.13.2 and 7.3.13.3.

7.3.13.2 The peripheral shear stress should be computed by:

$$v_u = \frac{V_u}{\phi b_0 d} \tag{7.51}$$

in which V_u and b_0 are taken at the critical section defined in Section 7.3.13.1(b). The peripheral shear stress should not exceed the shear stress carried by the concrete $v_c = 4\sqrt{f_c'}$ unless shear reinforcement is provided. If the ratio of the short side to the long side of the concentrated load or reaction area, B_c, is less than 0.5, v_u should not exceed the larger of $v_c = (2 + 4B_c)\sqrt{f_c'}$ or $2\sqrt{f_c'}$, unless shear reinforcement is provided. If shear reinforcement is provided in accordance with Section 7.3.13.3, v_u should not exceed $6\sqrt{f_c'}$.

7.3.13.3 Shear reinforcement consisting of bars or wires may be provided. For design of such shear reinforcement, shear stresses should be investigated at the critical section defined in Section 7.3.13.1(b) and at successive sections more distant from the support; and the shear stress carried by the concrete at any section should not exceed $2\sqrt{f_c'}$. Where the peripheral shear stress exceeds the shear stress carried by the concrete, shear reinforcement should be provided.

7.3.13.4 Shear reinforcement consisting of steel I- or channel shapes should be designed in accordance with the following provisions, which do not apply where shear is transferred to a column from an edge of a slab.*

7.3.13.4.1 Each shearhead should consist of steel shapes so fabricated by welding that each pair of the four arms is continuous through the column. The ends of shearheads may be cut at angles up to 30 degrees with the horizontal, provided that the plastic moment capacity of the remaining tapered section is adequate to resist the shear force attributed to that arm of the shearhead. The ratio α_v, between the EI-value for each shearhead arm and that for the surrounding composite cracked slab section of width $(c_2 + d)$ should not be less than 0.15. All compressive flanges of the steel shapes should be located within $0.3d$ of the compressive surface of the concrete slab. The steel shapes should not be deeper than 70 times their web thickness.

7.3.13.4.2 The full plastic moment of resistance, M_p, required for each arm of the shearhead should be computed by:

$$M_p = \frac{V_u}{\phi 8}\left[h_v + \alpha_v\left(l_v \frac{c_1}{2}\right)\right] \tag{7.52}$$

where ϕ is the capacity reduction factor for flexure and l_v is the minimum length of each shearhead arm required to comply with the requirements of Sections 7.3.13.4.3 and 7.3.13.4.4.

7.3.13.4.3 The critical slab section should be perpendicular to the plane of the slab. The section should cross each shearhead arm at points three-quarters of the distance

$$\left(l_v - \frac{c_1}{2}\right)$$

from the column face to the end of the shearhead, and it should be so located that its periphery is a minimum. However, the critical section need not approach closer than $d/2$ to the periphery of the column.

7.3.13.4.4 The shear stress, v_u, should not exceed $4\sqrt{f_c'}$ on the critical section defined in Section 7.3.13.4.3.

7.3.13.4.5 The shearhead may be assumed to contribute a resisting moment, M_v, to each column strip of the slab computed by:

$$M_v = \frac{\phi\alpha_v V_u}{8}\left(l_v - \frac{c_1}{2}\right) \tag{7.53}$$

where ϕ is the capacity reduction factor for flexure, and l_v is the length of each shearhead arm actually provided. However, M_v should exceed neither 30% of the total moment resistance required for each column strip of the slab, nor the change in column strip moment over the length l_v, nor the value of M_p given by Eq. (7.52).

7.3.14 *Transfer of Moments to Compression Members*

7.3.14.1 Except where a compression member is restrained on three or four sides by flexural members of approximately equal depth, the shear force, V_u, acting on the compression member within the depth of the larger flexural member should be taken as the ultimate force exerted by the tensile reinforcement framing directly into the compression member from that flexural member. The shear strength should be calculated in accordance with Sections 7.3.7.2 through 7.3.7.6. Minimum shear reinforcement should be provided in accordance with Section 7.3.7.1.1.

7.3.14.2 When unbalanced gravity load, wind, or earthquake causes transfer of bending moment between compression and flexural member, the per cent of the moment given by

$$\frac{100}{\left[1 + \frac{2}{3}\sqrt{\frac{c_1}{c_2}}\right]}$$

should be considered transferred by flexure across the periphery of the critical section defined in Section 7.3.13.1 (b), and the remainder by eccentricity of the shear about the centroid of the critical section. Shear stresses should be taken as varying linearly about the centroid of the critical section and the maximum shear stress should not exceed $4\sqrt{f_c'}$.

7.3.15 *Bearing*

7.3.15.1 Bearing stresses should not exceed $0.85\,\phi\, f_c$, except as provided below.

7.3.15.2 When the supporting surface is wider on all sides than the loaded area, the permissible bearing stress on the loaded area may be multiplied by $\sqrt{A_2/A_1}$, but not more than 2.

* Tests indicate that, due to torsional effects and other peculiarities, the behaviour of shearheads located at a slab edge differs substantially from that at other locations.

7.3.15.3 When the supporting surface is sloped or stepped, A_2 may be taken as the area of the lower base of the largest frustrum of a right pyramid or cone contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal.

7.3.15.4 When the loaded area is subjected to high edge stresses due to deflection or eccentric loading, the permissible bearing stress on the loaded area should be multiplied by a factor of 0.75.

REFERENCES

7.1 Anderson, A.R., and S.E. Moustafa, "Ultimate Strength of Prestressed Concrete Piles and Columns," ACI Journal, *Proceedings V. 67*, No. 8, August 1970, pp. 620–635.

7.2 MacGregor, J.G., J.E. Breen, and E.O. Pfrang, "Design of Slender Concrete Columns," ACI Journal, *Proceedings V. 67*, No. 1, January 1970, pp. 6–28.

7.3 Kollbrunner, C.F., and F. Basier, *Torsion in Structures*. Springer Verlag, New York, Inc., New York, 1969.

7.4 *CEB Information Bulletin # 83*, pp. 65–66.

DETAILING OF REINFORCEMENT

13.2 Development and Splices of Reinforcement

13.2.1 *Development Requirements, General*

13.2.1.1 The calculated tension or compression in the reinforcement at each section should be developed on each side of that section by embedment length or end anchorage or a combination thereof. For reinforcement bars in tension, hooks may be used in developing the reinforcement bars.

13.2.1.2 Tension reinforcement may be anchored by bending it across the web and making it continuous with the reinforcement on the opposite face of the member, or anchoring it there.

13.2.1.3 The critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates, or is bent.

13.2.1.4 Reinforcement should extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of the member, 12 bar diameters, or one-sixteenth of the clear span, whichever is greater, except at supports of simple spans and at the free end of cantilevers.

13.2.1.5 Continuing reinforcement should have an embedment length not less than the development length l_d beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure.

13.2.1.6 Flexural reinforcement should not be terminated in a tension zone unless one of the following conditions is satisfied:

(a) The shear at the cutoff point does not exceed two-thirds that permitted, including the shear strength of furnished web reinforcement.

(b) Stirrup area in excess of that required for shear is provided along the terminated reinforcement over a distance from the termination point equal to three-fourths the effective depth of the member. The excess stirrup-area should not be less than 60 $b_w s/f_y$. The resulting spacing should not exceed $(d/8)\beta_b$ where β_b is the ratio of the area cut off to the total area of reinforcement at the section.

(c) For # 11 and smaller bars, the continuing bars provide double the area required for flexure at the cut-off point and the shear does not exceed three-fourths that permitted.

13.2.2 *Positive Moment Reinforcement*

13.2.2.1 At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members should extend along the same face of the member into the support. In beams, the length of this extension should be at least 6 in. When a flexural member is part of the lateral load resisting system, the positive reinforcement required to be extended into the support should be anchored to develop the full f_y in tension at the face of the support.

13.2.2.2 At simple supports and at points of inflection, positive moment tension reinforcement should be limited to a diameter such that l_d computed for f_y does not exceed

$$M/V + l_a$$

where M is the computed moment capacity assuming all positive moment tension reinforcement at the section to be fully stressed. V is the maximum applied design shear at the section. l_a at a support should be the sum of the embedment length beyond the centre of the support and the equivalent embedment length of any furnished hook or mechanical anchorage, l_a at a point of inflection should be limited to the effective depth of the member or $12d_b$, whichever is greater. The value M/V in the development length limitation may be increased 30% when the ends of the reinforcement are confined by a compressive reaction.

13.2.3 *Negative Moment Reinforcement*

13.2.3.1 Tension reinforcement in a continuous, restrained, or cantilever member, or in any member of a rigid frame, should be anchored in or through the supporting member by embedment length, hooks, or mechanical anchorage.

13.2.3.2 At least one-third the total reinforcement provided for negative moment at the support should have an embedment length beyond the point of inflection not less than the effective depth of the member, 12 bar diameters, or one-sixteenth of the clear span, whichever is greater.

13.2.4 *Special Members*

Adequate end anchorage snould be provided for tension reinforcement in flexural members where reinforcement stress is not directly proportional to moment, such as: sloped, stepped, or tapered footing; brackets; deep flexural members, or members in which the tension reinforcement is not parallel to the compression face.

13.2.5 *Development Length of Deformed Bars and Deformed Wire in Tension*

The development length l_d, in inches, of deformed bars and deformed wire in tension should be computed as the product of the basic development length of (a) and the applicable modification factor or factors of (b), (c) and (d), but l_d should be not less than recommended in (e).

(a) The basic development length should be
For # 11 or smaller bars
(Note 1) .... $0.04A_b f_y/\sqrt{f'_c}$
but not less than
(Note 2) .... $0.004d_b f_y$
For # 14 bars
(Note 3) .... $0.085f_y/\sqrt{f'_c}$
For # 18 bars
(Note 3) .... $0.11f_y/\sqrt{f'_c}$
For deformed
wire .... $0.03d_b f_y/\sqrt{f'_c}$

(b) The basic development length should be multiplied by the applicable factor or factors for:
Top reinforcement (Note 4) 1.4
Reinforcement bars with f_y greater than 60,000 psi .... $2–60,000/f_y$

(c) When lightweight aggregate concrete is used, the basic development lengths should be multiplied by 1.33 for "all lightweight" concrete and 1.18 for "sand-lightweight" concrete with linear interpolation when partial sand replacement is used, or the basic development length may be multiplied by 6.7 $\sqrt{f'_c/f_{ct}}$, but not less than 1.0, when f_{ct} is determined.

(d) The basic development length may be multiplied by the applicable factor or factors for:
Reinforcement being developed in the length under consideration and spaced laterally at least 6 in. on centre and at least 3 in. from the side face of the member to the edge bar, measured in the direction of the spacing ...... 0.8. Where anchorage or development for f_y is not specifically required, reinforcement in flexural members in excess of that required ...... (A_s required)/(A_s provided) Reinforcement bars enclosed within a spiral which is no less than 1/4 in. diameter and no more than 4 in. pitch ...... 0.75.

(e) The development length, l_d, should be taken as not less than 12 in. except in the computation of lap splices and anchorage of web reinforcement.

NOTES:

1. The constant carries the unit of 1/in.
2. The constant carries the unit of in. 2/lb.
3. The constant carries the unit of in.
4. Top reinforcement is horizontal reinforcement so placed that more than 12 in. of concrete is cast in the member below the bar as part of the same concrete placement that encases the bar.

13.2.6 *Development Length of Deformed Bars in Compression*

The development length l_d for bars in compression should be computed as $0.02f_y d_b/\sqrt{f'_c}$ but should not be less than $0.0003f_y d_b$ or 8 in. Where excess bar area is provided, the l_d length may be reduced by the ratio of area required to area provided. The development length may be reduced 25% when the reinforcement is enclosed by spirals not less than 1/4 in. in diameter and not more than 4 in. pitch.

13.2.11 *Development Length of Prestressing Strand*

13.2.11.1 Three or seven-wire pretensioning strand should be bonded beyond the critical section for a development length, in inches, not less than

$$(f_{ps} - 2/3f_{se})d_b$$

when d_b is the nominal diameter in inches, f_{ps} and f_{se} are expressed in kips per square inch and the expression in the parentheses is used as a constant without units. Investigation may be limited to those cross-sections nearest each end of the member which are required to develop their full strength under the specified design load.

13.2.11.2 Where bonding of the strand does not extend to the end of the member the bonded development length recommended in Section 13.2.11.1 should be doubled.

13.2.12 *Mechanical Anchorage*

Any mechanical device capable of developing the strength of the reinforcement without damage to the concrete may be used as anchorage.

13.2.13 *Anchorage of Web Reinforcement*

13.2.13.1 Web reinforcement should be carried as close to the compression and tension surfaces of the member as cover requirements and the proximity of other steel will permit, and in any case the ends of single leg, single U-, or multiple U-stirrup should be anchored by one of the following means:

(a) A standard hook plus an effective embedment of 0.5 l_d. The effective embedment of a stirrup leg should be taken as the distance between the mid-depth of the member $d/2$ and the start of the hook (point of tangency).

(b) Embedment above or below the middepth, $d/2$, of the beam on the compression side for a full development length l_d but not less than 24 bar diameters or, for deformed bars or deformed wire, 12 in.

(c) For # 5 or smaller bars bending around the longitudinal reinforcement through at least 135 degrees, the effective embedment of a stirrup degree should be taken as the distance between the mid-depth of the member $d/2$ and the start of the hook (point of tangency).

NOTATION

a = depth of equivalent rectangular stress block, in. [Article 7.3.3 (f)]
a_b = depth of equivalent rectangular stress block for balanced conditions, in.
a_v = ratio of stiffness of shearhead arm to surrounding composite slab section
A = effective tension area of concrete surrounding the main tension reinforcing bars and having the same centroid as that reinforcement, divided by the number of bars, sq. in. When the main reinforcement consists of several bar sizes the number of bars shall be computed as the total steel area divided by the area of the largest bar used.
A = axial load deformation (Chapter 5)
A = maximum expected acceleration at bedrock at the site expressed as a fraction of gravity (Section 5.5.3)
A_b = area of an individual bar, sq. in.
A_c = area of core of spirally reinforced compression measured to the outside diameter of the spiral, sq. in.
A_g = gross area of section, sq. in.
A_h = area of shear reinforcement parallel to main tension reinforcement, sq. in.
A_{ps} = area of prestressed reinforcement in tension zone, sq. in.
A_{pst} = total area of prestressing steel (Chapter 11)
A_s = area of tension reinforcement, sq. in.
A'_s = area of compression reinforcement, sq. in.
A_{sa} = area of bonded reinforcement in tension zone, sq. in.
A_{se} = area of stirrups transverse to potential bursting crack and within a distance s, in.
A_{sf} = area of reinforcement to develop compressive strength of over-hanging flanges of I- and T-sections, sq. in.
A_{st} = total area of longitudinal reinforcement (in compression members), sq. in.
A_t = area of one leg of a closed stirrup resisting torsion within a distance s, sq. in.
A_l = total area of longitudinal reinforcement to resist torsion sq. in.
A_v = area of shear reinforcement within a distance, s, or area of shear reinforcement perpendicular to main reinforcement within a distance, s, for deep beam, sq. in.
A_{vf} = area of shear-friction reinforcement, sq. in.
A_{vh} = area of shear reinforcement parallel to the main reinforcement within a distance s, sq. in.
A_w = area of an individual wire, sq. in.
A_1 = loaded area, sq. in (Sections 7.3.15 and 9.11.3.1)
A_2 = maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the loaded area, sq. in., (Sections 7.3.15 and 9.11.3.1)
b = width of compressive face of member, in.
b_a = width of bearing plate measured in the same direction as d_a
b_e = width of concrete section in plane of potential bursting crack, in.
b_0 = periphery of critical section for slabs and footings, in.
b_v = the width of the cross-section being investigated for horizontal shear, in.
b_w = web width, or diameter of circular section, in.
B = buoyancy (Chapter 5)
B = band width (Section 11.5.4)
c = distance from extreme compressive fibre to neutral axis, in.
c_w = shape factor used to change basic wind pressure
c_1 = size of rectangular or equivalent rectangular column, capital or bracket measured in the direction in which moments are being determined.
c_2 = size of rectangular or equivalent rectangular column, capital or bracket measured transverse to the direction in which moments are being determined.
C = construction, handling and erection loads (Section 5)
C = a coefficient equal to A.R.S/Z (Section 5.5.3)
C = a stiffness parameter (Section 5.7.5.3)
C_m = a factor relating the actual moment diagram to an equivalent uniform moment diagram
CF = centrifugal force (Chapter 5)
d = distance from extreme compressive fibre to centroid of tension reinforcement, in.
d = size of pile (Chapter 11)
d' = distance from extreme compressive fibre to centroid of compression reinforcement, in.
d'' = distance from centroid of gross section, neglecting

the reinforcement, to centroid of tensile reinforcement, in.

d_a = distance from edge of member to centre of anchor plate, in.

d_b = nominal diameter of bar, wire, or prestressing strand, in.

d_c = thickness of concrete cover measured from the extreme tensile fibre to the centre of the bar located closest thereto, in.

D = dead load

D = distribution factor (Table 5.7.5.3)

DS = displacement of supports

e = eccentricity of design load parallel to axis measured from the centroid of the section. It may be calculated by conventional methods of frame analysis, in.

e = base of Napierian logarithms

e_b = M_b/P_b = eccentricity of the balanced condition load-moment relationship, in.

E = effective width of concrete slab resisting wheel or other concentrated load, ft.

E = earth pressure (Chapter 5)

E = efficiency or the decimal fraction of the single pile value to be used for each pile in the group (Chapter 11)

E_c = modulus of elasticity of concrete, psi (Section 7.2.4, Section 8.1.1 (d), and Section 9.3)

E_{ci} = modulus of elasticity of concrete at transfer of stress, psi

E_s = modulus of elasticity of steel, psi

EI = flexural stiffness of compression members

EQ = earthquake force

f = sag of suspension cables

f_a = axial stress, psi (Chapter 11)

f_b = bending stress, psi (Chapter 11)

f_b = average bearing stress in concrete on loaded area

f_c = extreme fibre compressive stress in concrete at service loads

f'_c = specified compressive strength of concrete, psi

f'_{ci} = compressive strength of concrete at time of initial prestress, psi

$\sqrt{f'_c}$ = square root of specified compressive strength of concrete, psi

f_{cds} = change in concrete stress at centre of gravity of prestressing steel due to all dead loads except the dead load acting at the time the prestressing force is applied, psi

f_{cp} = concrete bearing stress under anchor plate of post-tensioning tendon, psi

f_{cir} = concrete stress immediately after transfer at centre of gravity of prestressing steel, psi

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi

f_d = stress due to dead loads at the extreme fibre of a section at which tensile stresses are caused by applied load, psi

f_f = stress range ksi

f_h = tensile stress developed by a standard hook, psi (Section 13.2.8.1)

f_l = loss in effective prestress, psi

f_{lc} = loss in prestressing steel stress due to creep, psi

f_{le} = loss in prestressing steel stress due to elastic shortening, psi

f_{lf} = loss in prestressing steel stress due to friction, psi

f_{lp} = total loss in prestressing steel stress, psi

f_{lr} = loss in prestressing steel stress due to relaxation, psi

f_{ls} = loss in prestressing steel stress due to shrinkage, psi

f_{min} = minimum stress level where tension is positive and compression is negative, psi

f_0 = fundamental natural frequency of the guideway, Hz.

f_{pc} = maximum allowable compressive stress exclusive of effective prestress, psi (Chapter 11)

f_{pc} = compressive stress in the concrete, after all prestress losses have occurred, at the centroid of the cross-section resisting the applied loads or at the junction of the web and flange when the centroid lies in the flange, psi. (In a composite member, f_{pc} will be the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange when the centroid lies within the flange, due to both prestress and to bending moments resisted by the precast member acting alone.)

f_{pe} = compressive stress in concrete due to prestress only after all losses, at the extreme fibre of a section at which tensile stresses are caused by applied loads, psi

f_{po} = steel stress at jacking end of post-tensioning tendon, psi

f_{ps} = calculated stress in prestressing steel at design load, psi

f_{pu} = ultimate strength of prestressing steel, psi

f_r = modulus of rupture of concrete, psi

f_s = tensile stress in reinforcement at service loads, psi

f_{se} = effective stress in prestressing steel, after losses, psi

f'_s = stress in compressive reinforcement, psi

f'_{sb} = stress in compressive reinforcement at balanced conditions

f_t = extreme fibre tensile stress in concrete at service loads

x_1 = shorter centre-to-centre dimension of a closed rectangular stirrup
y = longer overall dimension of a rectangular part of a cross-section
y_1 = longer centre-to-centre dimension of a closed rectangular stirrup
y_t = distance from the centroidal axis of cross-section, neglecting the reinforcement, to the extreme fibre in tension
z = a quantity limiting distribution of flexural reinforcement
Z = reduction for ductility and risk assessment (Section 5.5.3)
α = total angular change of prestressing steel profile in radians from jacking end to any point x from that end
α = angle between inclined shear reinforcement and longitudinal axis of member
α_v = ratio of stiffness of shearhead arm to surrounding composite slab section
β_b = ratio of area of bars cut off to total area of bars at the section
β_c = ratio of the short side to the long side of the concentrated load or reaction area in Section 7.3.13.2
β_d = the ratio of maximum design load moment to maximum design total load moment, always positive
β_1 = a factor defined in Section 7.3.3
σ = moment magnification factor for compression members
μ = coefficient of friction (Section 7.3.9.4)
μ = curvature friction coefficient (Table 9.4.3)
ρ = constant for standard hook
ρ = ratio of nonprestressed reinforcement = A_s/bd
ρ = density of air
ρ' = compression reinforcement ratio = A'_s/bd
ρ_b = reinforcement ratio producing balanced conditions (Section 7.3.4.3)
ρ_f = $A_{sf}/b_w d$
$\rho_{\min}$ = minimum tension reinforcement ratio = A_s/bd in Section 7.3.4.1
ρ_p = ratio of prestressed reinforcement, A_{ps}/bd
ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced compression member
ρ_v = $A_s + A_h/bd$
ρ_w = reinforcement ratio $A_s/b_w d$
φ = capacity reduction factor
ϕ = $\tan^{-1}(d/s)$ expressed in degrees (Chapter 11)
χ_1 = distance from load to point of support, ft. (Section 10.4.5)
χ_2 = distance from centre of rail post section under investigation (Section 10.4.5)
ψ = curvature at section due to shrinkage
ω = $\rho f_y/f'_c$
ω' = $\rho' f_t/f'_c = A'_s f_y/bdf'_c$
ω_p = $\rho_p f_{ps}/f'_c = A_{ps} f_{ps}/bdf'_c$
$\omega_w, \omega_{pw}, \omega_w$ = reinforcement indices for flanged sections computed as for ω, ω_p, and ω' except that b is the web width and the steel area is that required to develop the compressive strength of the web only.

APPENDIX 7

Some of the U.K. (DTp.) Department of Transport Bridge Engineering Technical Memoranda (now DTp., previously DoE. — Dept. of Environment):

NUMBER	TITLE	DATE
H2/74	Suite of road design and analysis programs Program HECB/R/12 (FREEWAY)	11/3/74
H5/74	Suite of road design and analysis programs Program system (HOPS)	15/5/74
H6/74	Design flows for motorways and rural all-purpose roads (Corrigendum: 1/4/75)	12/8/74
H9/74	System of drainage and analysis programs Program HECB/R/7/ (DAPHNE) and HECB/R/11 (SAFRON)	3/9/74
H6/75	Design of rural motorway-to-motorway interchanges General guidelines	25/4/75
H9/75	Grass cutting and hedgerow treatment on trunk roads and motorways	11/7/75
H10/75	Clearance to bridges on dual two-lane roads	25/7/75
H13/75	Highways optimisation program system (HOPS) Hops user guide	11/9/75
H15/75	Suite of road design and analysis programs Program HECB R/18 (SETTLE)	30/9/75
H17/75	Design of rural motorway-to-motorway interchanges Single lane links	1/12/75
H18/75	Design of rural motorway-to-motorway interchanges Merging and diverging lanes	1/12/75

NUMBER	TITLE	DATE
H3/76	Model contract document for site investigation	Feb 1976
H7/76	Standardisation in format and preparation of drawings	4/6/76
H15/76	Introduction to specification for road and bridge work 1976 and notes for guidance on specification	17/12/76
H3/77	Suite of highway maintenance analysis programs Program suite HECB/R16 (CHART)	15/6/77
H4/77	Standard terms of appointment for testing firms employed on roadworks	21/6/77
H5/77	Method of measurement for road and bridge works 1977 and notes for guidance and library of standard item descriptions	29/6/77
H6/77	Suite of slope stability programs Program HECB/R/22 (CIRCA 1.0)	9/12/77
H2/78	Highway cost model system program HECB/R/20 (COSMOS)	11/1/78
H3/78	Notes for guidance and library of standard item descriptions for use with the method of measurement for road and bridge works, 1977 (Corrigendum, 1/3/78)	12/1/78
H5/78	Model contract document for topographical surveys	Feb 1978

NUMBER	TITLE	DATE
IM 4	Pulverised fuel ash Backfilling to structures	19/12/69
IM 5	Formation of continuity joints in bridge decks	21/1/70
IM 7	Use of stainless steel in bridge works	27/7/70
IM 9	Line and level on log-span structures	14/8/70
IM 11	PTFE in bridge bearings	23/10/70
IM 12	New proprietary products	30/4/71
BE 5	The design of highway bridge parapets (Third revision 16/11/73)	30/9/70
BE 11	Lightweight aggregate concrete for use in highway structures	20/4/69
BE 13	Fatigue risk in Bailey bridges	8/4/68
BE 14	Headroom standards (Addendum 1; 29/1/71)	11/12/68
BE 16	Provisional fatigue requirements for steel bridges	Jan. 69
BE 18	Suite of bridge design and analysis programs	10/7/69
BE 23	Shear key decks	27/11/70
BE 27	Waterproofing and surfacing of bridge decks	17/6/70
BE 28	Unauthorised access to motorway bridges	11/6/71
BE 29	MOT/C & CA standard bridge beams	2/7/71
BE1/72	Department of the Environment suite of bridge design and analysis programs	28/3/72
BE3/72	Expansion joints for use in highway bridge decks	31/10/72
BE7/72	Appointment of testing firms employed on bridgeworks	15/8/72
BE1/73	Reinforced concrete for highway structures (1st Revision; 9/8/73)	30/1/73
BE2/73	Prestressed concrete for highway structures	30/1/73
BE3/73	The assessment of highway bridges for construction and use vehicles	28/2/73

NUMBER	TITLE	DATE
BE6/73	Application of the Merrison Committee's interim design and workmanship rules for steel box-girder bridges	9/10/73
BE7/73	Rules for the design and construction of preflexed beams in highway bridges	9/10/73
BE8/73	Approval in principle and calibration of computer programs for use in DoE highway structures on trunk roads and motorways	12/11/73
BE1/74	The independent checking of erection proposals and temporary work details for major highway structures on trunk roads and motorways	28/2/74
BE2/74	Suite of bridge design and analysis programs Program HECB/B/12 (COLDES)	1/5/74
BE3/74	Suite of bridge design and analysis programs Program HECB/B/11 (MUPDI)	21/8/74
BE4/74	Suite of bridge design and analysis programs Program HECB/B/13 (STRAND)	21/8/74
BE5/74	High-alumina cement concrete	16/8/74
BE6/74	Suite of bridge design and analysis programs Program HECB/B/15 (ORTHOP) and design charts HECB/B1/5	15/10/74
BE7/74	Lateral loading on piled foundations	17/12/74
BE1/75	List of computer programs examined by the Highway Engineering Computer Branch (Corrigendum, 7/2/75)	21/1/75
BE3/75	Suite of bridge design and analysis programs Program HECB/B/14 (QUEST)	19/2/75
BE4/75	Suite of bridge design and analysis programs	20/2/75

NUMBER	TITLE	DATE
	Program HECB/B/16 (CASKET)	
BE5/75	Rules for the design and use of Freyssinet concrete hinges in highway structures	7/3/75
BE1/76	Design requirements for elastomeric bridge bearings	13/2/76
BE2/76	List of computer programs examined by the Highway Engineering Computer Branch (Department of the Environment)	5/3/76
BE3/76	Interim rules for design and construction of plate girders and rolled section beams in bridges	31/3/76
BE4/76	Suite of bridge design and analysis programs Program HECB/B/13 (STRAND 2)	25/3/76
BE5/76	Evaluation of highway structures	20/8/76
BE6/76	Suite of bridge design and analysis programs Program HECB/B/20 (PREBEM)	5/11/76
BE7/76	Suite of bridge design and analysis programs Program HECB/B/8 (RETWAL)	20/12/76
BE1/77	Standard highway loadings	

NUMBER	TITLE	DATE
BE2/77	List of computer programs examined by the Highways Engineering Computer Branch DTp	18/2/77
BE3/77	Suite of bridge design and analysis programs Program HECB/B/9 (GRIDS)	22/3/77
BE4/77	The inspection of highway structures	1/4/77
BE5/77	Suite of bridge design and analysis programs Program HECB/B/7 (PGROUP)	1/4/77
BE6/77	Amendments to the 5th edition "Specification for Road & Bridgeworks"	18/4/77
BE7/77	Departmental standard (Interim) motorway sign/signal gantries	25/4/77
BE9/77	Suite of bridge design and analysis programs Program HECB/B/17 (MINIPONT)	16/11/77
BE1/78	Design criteria for footbridges and sign/signal gantries	17/2/78
BE2/78	List of computer programs examined by the highway engineering computer branch DTp	29/3/78
BE3/78	Reinforced earth retaining wall and bridge abutments for embankments	27/4/78

Subject Index