Hull Design

Hull Shape

Approach

My approach to defining the hull geometry starts with the overall characteristics, like the length, displacement, center of buoyancy, and prismatic coefficient. These are the most important parameters in defining the hull's carrying capacity and its wave resistance.

The underwater cross section shape is defined by dimensionless shape parameters that control the beam/depth ratio, the angle of the sides near the waterline, and the slackness of the bilge. I vary these parameters smoothly along the length of the hull to produce a fair shape. The wetted area is the biggest influence on the hull's low speed resistance, and this is largely determined by the cross sectional shape. The wave drag is also influenced by the cross sectional shape, but this is not as strong an influence as the manner in which the area of each cross section changes along the length of the hull - the cross sectional area distribution.

The underwater hull shape is determined by combining the section shape with the cross sectional area distribution. I have used Fourier series to define how the area changes along the length of the hull. Once I have the cross sectional area distribution, I size the cross section shapes at each station to match the desired cross sectional area at that station.

This approach has the advantage that I can change the section shapes without affecting the overall hull characteristics. It also makes it easy to shrink or stretch the hull while maintaining a similar shape.

Cross Sectional Area Distribution

I've chosen to use some simple Fourier series for defining the cross sectional area distribution because the Fourier series produce a smooth, pleasing variation of the areas, and because they can be integrated analytically to get the overall characteristics. Similar functions have been used for tank test models that are reported in the literature, so I can use this information to help estimate the hull's resistance. It also meets the criteria above, since the free parameters are the length of the waterline, displacement, center of buoyancy, and prismatic coefficient. The relationships are:

$$X=\left[\frac{-1}{2}\cdot\cos(\xi)-l_{cb}\cdot(1-\cos(2\cdot\xi))\right]\cdot L_{wl}$$

$$0\le\xi\le\pi$$

$$S=\frac{D}{\rho \cdot g \cdot L_{wl}} \cdot \left[\frac{4}{\pi} \cdot \sin(\xi) + \left[\frac{2}{\pi} - \frac{5}{(8 \cdot Cp)}\right] \cdot \sin(3 \cdot \xi) + \left[\frac{-2}{\pi} + \frac{3}{(8 \cdot Cp)}\right] \cdot \sin(5 \cdot \xi)\right]$$

X is the longitudinal distance along the hull, and S is the cross sectional area distribution. D is the design displacement, Lwl the design length of the waterline, and Cp the prismatic coefficient.

As x varies from p to 0, it sweeps the sections from bow to to stern. When x = p/2, the cross sectional area is at its maximum. This section will be located twice as far aft of midships as is the center of buoyancy

This constant-length, constant-volume approach also leads to a different interpretation for the prismatic coefficient, Cp. Traditionally, Cp has been considered a measure of how full the ends of the hull are. Cp is defined as:

$$Cp=\frac{V}{S_x \cdot L}$$

But when volume and length are kept constant, the choice of Cp controls the size of the maximum cross sectional area:

$$S_x=\frac{V}{Cp \cdot L}$$

Volume and length are largely determined by the purpose of the boat and its load carrying capacity. The choice of Cp, because of its effect on the maximum cross sectional area, largely determines the maximum beam.

This is why there is an optimum Cp for different design speeds (Froude numbers). A larger Cp will result in blunter ends, which would be expected to cause more wave drag. However, it also narrows the beam, improving the length-to-beam ratio, which lowers wave drag. For Cp's below the optimum, the hull is too beamy, and for Cp's above the optimum, the ends are not fine enough.

Values of optimum Cp for monohulls can be found in "Principles of Yacht Design". The values should generally lie between 0.5 and 0.6, with higher values used for higher design speeds. For these functions, values of Cp less than p/7 (0.45) will result in non-physical shapes, with negative areas near the ends. Setting Cp greaer than 15p/64 (0.73) will result in necking in the waist of the hull, forming two bulges instead of a single maximum beam.

Here's a bit of trivia from the strange world department: if you set Cp to 3p/16 (0.589), lcb to zero, and use circular cross sections, the resulting shape is known as a Sears-Haack body. This is the shape that produces the minimum wave drag in supersonic flight. Coincidentally, this Cp also produces the minimum drag for a boat that is operated at hull speed (Froude number = 0.40)!

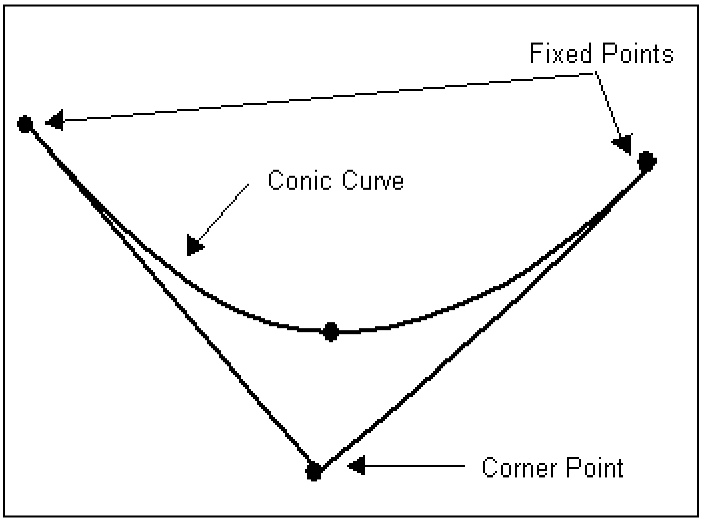
Conic Section Lofting

Now that I have the cross sectional area distribution, the next job is to define the shapes of the cross sections. One way to do this is by piecing together curves that are parts of ellipses, parabolas, and hyperbolas. If one has two intersecting line segments forming a "V", one can find a conic section that has these properties:

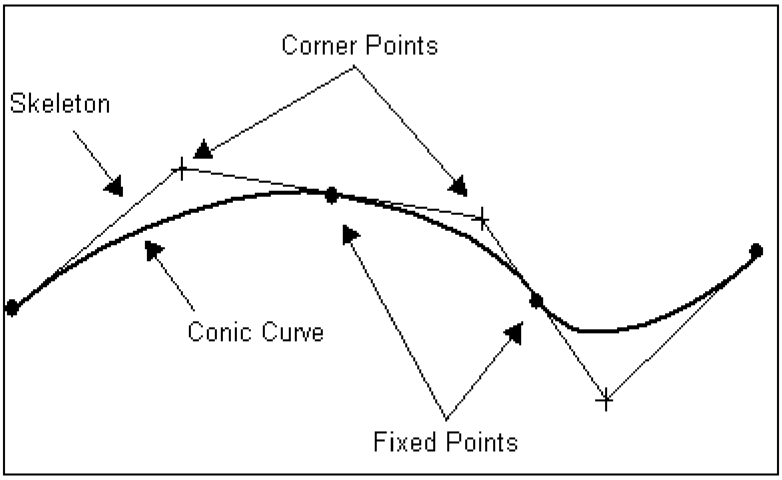
a) It passes through the end points of the V.

b) It is tangent to the lines at the end points.

c) It can be made to pass through any third point which is located inside the area enclosed by the "V".



By joining together a number of such curves, one can approximate nearly any shape. The points where the curves join together are fixed in space, and straight lines are drawn through these points to form an outer skeleton. The points where the lines through the fixed points intersect are called corner points. The degree of curvature between the fixed points is controlled by either defining a third fixed point inside the V, or by a parameter that represents how much the curve is attracted to the corner point.



This approach to lofting is very intuitive, since the shape can be sketched out using the straight line skeleton. The curves are smooth and continuous, and it is easy to see what the effect will be of changing any of the defining points. Although I will be presenting the algebraic equations that define these curves, one doesn't need a computer to draw them. They can be quickly created using just a compass and straight edge. For more details, see Dan Raymer's "Aircraft Design: A Conceptual Approach".

More aeronautical trivia: the legendary P-51 Mustang was one of the first aircraft to be designed using conic lofting. Part of its remarkable performance has been attributed to the fairness of the resulting lines.

Analytically, the conic curve is defined by six coefficients. The parameter, t, is used to sweep the curve from one end to the other. When t = -1 or +1, the curve is at the end points. When t = 0, the curve is at the fixed interior point. This point is defined by the parameter, r, which is the relative distance from the corner point to the midpoint between the fixed ends. When r = 0, the curve goes through the corner point, and has a sharp break. When r = 1, the curve is a straight line between the end points. Separate curves are used for all three coordinate directions (X, Y, Z) so that there is no problem with multiple values. The basic relationships are:

Equation for the curve:

$$0 = c1 \cdot x^2 + c2 \cdot x \cdot t + c3 \cdot t^2 + c4 \cdot x + c5 \cdot t + c6$$

Midpoint between the ends:

$$x4 = \frac{(x1 + x3)}{2}$$

The third fixed point in the "throat":

$$x5 = \rho \cdot (x4 - x2) + x2$$

$$x5 = \frac{\rho}{2} \cdot (x1 + x3) + (1 - \rho) \cdot x2$$

Taking the derivative with respect to t:

$$0 = c1\cdot\left(2\cdot x\cdot\frac{dx}{dt}\right) + c2\cdot\left(x + t\cdot\frac{dx}{dt}\right) + c3\cdot 2\cdot t + c4\cdot\left(\frac{dx}{dt}\right) + c5$$

By applying the conditions at t = -1, 0, 1, five equations result that can be used to solve for the coefficients:

@ t = -1, x = x1, x' = x2-x1

@ t = 0, x = x5

@ t = 1, x = x3, x' = x3-x2

$$0 = c1\cdot x1^2 - c2\cdot x1 + c3 + c4\cdot x1 - c5 + c6$$

$$0 = c1\cdot x5^2 + c4\cdot x5 + c6$$

$$0 = c1\cdot x3^2 + c2\cdot x3 + c3 + c4\cdot x3 + c5 + c6$$

$$0 = 2\cdot c1\cdot x1\cdot(x2 - x1) + c2\cdot(2\cdot x1 - x2) - 2\cdot c3 + c4\cdot(x2 - x1) + c5$$

$$0 = 2\cdot c1\cdot x3\cdot(x3 - x2) + c2\cdot(2\cdot x3 - x2) + 2\cdot c3 + c4\cdot(x3 - x2) + c5$$

Some special cases have to be considered. If x5 = 0 then c6 = 0, and c5 = 1. Otherwise, c6 = 1.

Putting the equations in matrix form:

$$\begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x1^2 & -x1 & 1 & x1 & -1 \\ x5^2 & 0 & 0 & x5 & 0 \\ x3^2 & x3 & 1 & x3 & 1 \\ 2\cdot x1\cdot(x2 - x1) & 2\cdot x1 - x2 & -2 & x2 - x1 & 1 \\ 2\cdot x3\cdot(x3 - x2) & 2\cdot x3 - x2 & 2 & x3 - x2 & 1 \end{bmatrix} \cdot \begin{bmatrix} c1 \\ c2 \\ c3 \\ c4 \\ c5 \end{bmatrix}$$

$$c6 = 1$$

If x5 = 0:

$$\begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} x1^2 & -x1 & 1 & x1 \\ x3^2 & x3 & 1 & x3 \\ 2 \cdot x1 \cdot (x2 - x1) & 2 \cdot x1 - x2 & -2 & x2 - x1 \\ 2 \cdot x3 \cdot (x3 - x2) & 2 \cdot x3 - x2 & 2 & x3 - x2 \end{bmatrix} \cdot \begin{bmatrix} c1 \\ c2 \\ c3 \\ c4 \end{bmatrix}$$

$c5 = 1$

$c6 = 0$

These can be solved using standard numerical methods, such as Gauss-Jordan elimination.

Once the coefficients are in hand, other points on the curve can be obtained, given values for t. If c1 = 0, then the solution is straightforward:

$$x = \frac{-\left(-c3 \cdot t^2 - c5 \cdot t - c6\right)}{(-c2 \cdot t - c4)}$$

If, in addition to c1 = 0, (c2)t = -c4, there is no solution.

If c1 is not zero, there are two solutions:

$$x = \begin{bmatrix} \frac{-1}{(2 \cdot c1)} \cdot \left[c2 \cdot t + c4 + \sqrt{(c2 \cdot t + c4)^2 + -4 \cdot c1 \cdot \left(t^2 \cdot c3 + t \cdot c5 + c6\right)}\right] \\ \frac{-1}{(2 \cdot c1)} \cdot \left[c2 \cdot t + c4 - \sqrt{(c2 \cdot t + c4)^2 + -4 \cdot c1 \cdot \left(t^2 \cdot c3 + t \cdot c5 + c6\right)}\right] \end{bmatrix}$$

Which solution is correct is somewhat problematic. I compute both, and pick the one that lies inside the "V" formed by the defining points. This means that the coefficients alone are not sufficient to evaluate the curve - one must also have the original defining points. This means that I have to store nine or ten quantities for each segment: the two endpoints, corner point, six coefficients, and (optionally) the slackness parameter, r. And this has to be repeated for both of the two spatial dimensions, Y and Z. This bulkiness is probably the biggest drawback to the method.

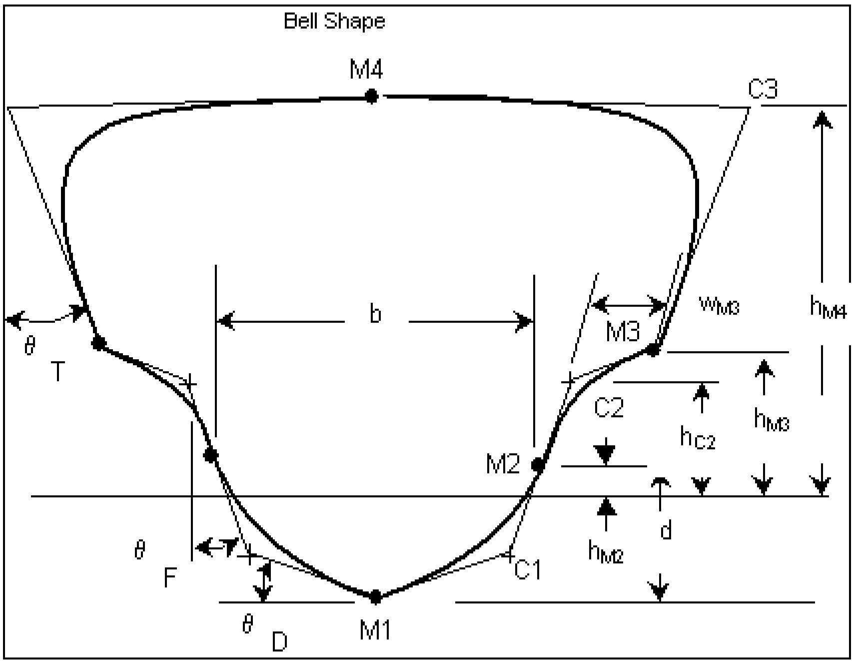
In addition to the case of c1 = 0, one also has to be careful of the case where

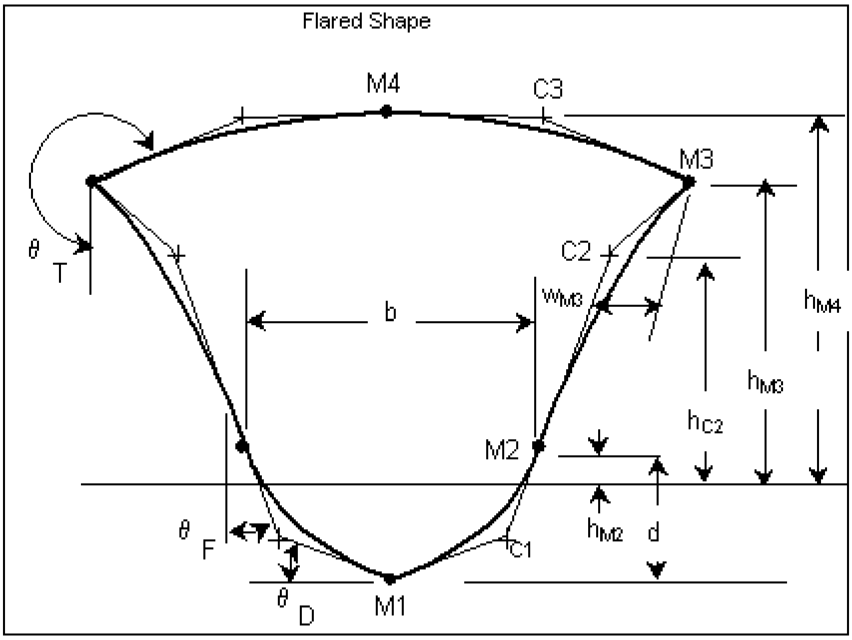
$$(c2 \cdot t + c4)^2 + -4 \cdot c1 \cdot \left(t^2 \cdot c3 + t \cdot c5 + c6\right) < 0$$

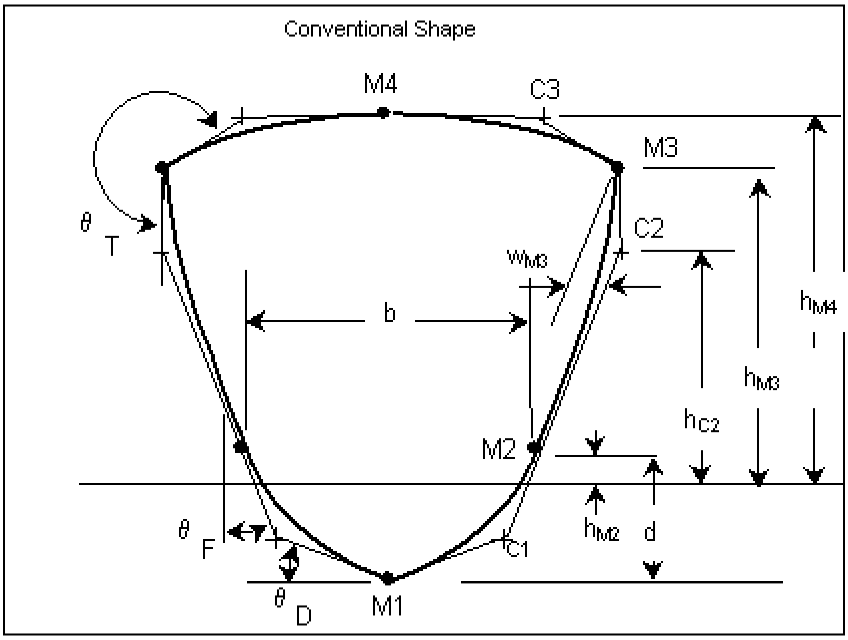
This also means there is no solution, and the coefficients are probably invalid.

Cross Section Shapes

The actual hull form is finally determined through the choice of section shape. I have used the conic section lofting to define a generic shape that can be adapted to different hull forms by simply varying the parameters. One segment is used to define the underwater shape, and three more segments define the shape of the topsides and deck. Dimensionless parameters are used to define the underwater portion, which makes it easier to size it to the cross sectional area distribution. The topsides are sized relative to the underwater portion, but the vertical distances are kept fixed.







Twelve parameters define the section shape. The first five, along with the cross sectional area, define the underwater portion:

BDR beam/depth ratio = b/d

tan(qD) tangent of deadrise angle

r1 bilge slackness parameter

hM2 height of moldline M2 above the design waterline

tan(qF) slope of the hull near the waterline

BDR is the most important parameter, as it controls the depth of the hull and has the most effect on the wetted area. A semicircle has a beam/depth ratio of 2.0, so BDR should be approximately two if minimum wetted area is the object. Decreasing BDR toward the bow will reduce the amount of rocker that would otherwise result from the shape of the cross sectional area distribution, and will make the bow finer. Increasing BDR toward the stern will give broader, flatter sections, but will also steepen the curve of the buttlines.

The deadrise angle controls how much "V" there is to the bottom. Setting the deadrise to zero results in a round bottom. The parameter r1 determines how hard the bilge is. A value of zero results in a sharp chine at C1, and a value of 1 results in a straight line between M1 and M2. Values in the neighborhood of 0.3 to 0.5 result in smoothly rounded shapes.

The mold line M2 is intended to shape the design waterline. Raising it above the waterline maybe necessary to improve the numerical characteristics at the ends, where the cross sectional area goes to zero. It may also be desireable to raise the M2 moldline for hulls with overhangs. The slope, tan(qF), at M2 is defined relative to vertical. A zero value results in a vertical exit of the topsides from the water.

I originally structured the shape of the topsides to form a bell shape, similar to that used in Shuttleworth catamaran designs. However, this same structure can represent the flared shapes of many Newick designs, as well as a conventional hull shape, with or without tumblehome. The parameters defining the topsides are:

hC2 height of C2 above design waterline

r2 curvature parameter for segment M2-M3

hM3 height of M3 above design waterline

r3 curvature parameter for segment M3-M4

hM4 height of M4 above design waterline

wM3 width of hull flare, relative to the extended slope at M2

tan(qT) slope of topsides from vertical

wM3 controls how much the topsides below M3 deviate from a straight line. Positive values will push M3 outboard, forming a knuckle or flare. Negative values will produce tumblehome. A zero value will result in a straight line between M2 and M3, regardless of the slope or the position of C2. hC2 will generally be less than or equal to hM3. If they are equal, the slope of the hull at the knuckle will be parallel to the waterplane.

The moldline M3 forms a sharp chine. In the bell shaped section, with its rounded topsides, M3 locates the knuckle. On the other shapes, M3 is the shear. How this line varies along the hull has a major influence on the appearance of the boat. The final mold line, M4, determines the height of the cabin, and must be designed in concert with M3.

A zero value for tan(qT) will result in a vertical topside starting at the knuckle in the bell shape. A comparatively large negative value is required for the more conventional shapes. In the latter two cases, this parameter will control the slope of the deck at the hull/deck joint. Setting tan(qT) equal to tan(qF) and wM3 equal to 0 will result in a smooth join at M3, with no crease.

The degree of curvature in the topsides is controlled by the parameters r2 and r3. In most cases, the shape is not very sensitive to these parameters, due to the shallow angles of the skeleton at the corner points. For the bell shape, however, r3 has a major effect on the shape. A small value will result in drawing the hull toward C3, and this can be used to create a straighter topsides and a more conventional, sharp edged, shear line.

I have chosen to define symmetrical hull shapes, largely for ease of manufacture and because I don't have good tools for analyzing the hydrodynamics of asymmetrical shapes. For asymmetrical hulls, the parameters above would have to be doubled, with a set for each side, and an additional parameter defined that contolled the lateral position of the section according to the camber in the hull centerline.

Adjusting Section Shapes to the Cross Sectional Area

Finally, I put all this together and determine the width and remaining dimensions of the section shape. This is an iterative process, but it converges very rapidly. It uses the numerical integration described in the next chapter, Hydrostatics. So I will skip over the mechanics of calculating the underwater cross sectional area numerically, and cover how the shape is adjusted to match.

The first step is to get an approximate value for the section dimensions. The skeleton of the underwater shape is very useful here, since it is easy to calculate the area inside the skeleton, and this allows me to calculate the beam as a function of the area. C1 is located at the intersection of the skeleton lines through M1 and M2, and the coordinates of M2 can also be written in terms of the hull depth:

$$Y_{C1}=\frac{1}{2}\cdot d\cdot\frac{\left(-BDR+2\cdot\tan\left(\theta_F\right)\right)}{\left(-1+\tan\left(\theta_F\right)\cdot\tan\left(\theta_D\right)\right)}$$

$$Z_{C1}=\left[1-\frac{1}{2}\cdot\tan\left(\theta_D\right)\cdot\frac{\left(-BDR+2\cdot\tan\left(\theta_F\right)\right)}{\left(-1+\tan\left(\theta_F\right)\cdot\tan\left(\theta_D\right)\right)}\right]\cdot d$$

$$Y_{M2}=bdr\cdot\frac{d}{2}$$

$$Z_{M2}=-h_{M2}$$

I use the area of the skeleton to get the first approximation of the cross sectional area, S0:

$$S=\frac{1}{8}\cdot\frac{\left(-\tan\left(\theta_D\right)\cdot BDR^2-4\cdot\tan\left(\theta_F\right)+4\cdot BDR\right)}{\left(1-\tan\left(\theta_F\right)\cdot\tan\left(\theta_D\right)\right)}\cdot d_0^{\,2}\ \ldots$$
$$+-\frac{1}{2}\cdot BDR\cdot h_{M2}\cdot d_0+\frac{1}{2}\cdot h_{M2}\cdot\tan\left(\theta_F\right)$$

This is solved for the first guess at the hull depth, d0 using the quadratic formula, setting S equal to the desired area. Once a definite value for the depth is known, all the other dimensions of the section can be calculated, including the offsets.

A simpler first guess, especially good for rounded sections, is to base the depth on the area of an ellipse:

$$S=\frac{\pi}{2}\cdot d_0\cdot\left(\frac{BDR}{2}\cdot d_0\right)$$

$$d_0 = \sqrt{\pi \cdot \left(\frac{4}{BDR} \cdot \frac{S}{\pi^2} \right)}$$

The next step is to numerically integrate the underwater area of the section, S0, using the first guess at the offsets, using the techniques described under Hydrostatics. This will probably be a little smaller than the design area, S. The area scales as d2 if hM2 is small, so the final step is to obtain the revised estimate for d, d1:

$$d_1 = d_0 \cdot \sqrt{\frac{S}{S_0}}$$

The section dimensions are recalculated based on d1. These last two steps may be repeated until the design and actual areas match as closely as desired.

Tables of Offsets

The preceding relationships are sufficient for completely defining the shape of the hull's canoe body. However, if x is varied in a regular manner, the spacing of the sections will not be uniform. The stations will actually be very well placed for numerical purposes, with more stations in locations, such as the bow and stern, where the lines are changing rapidly. However, the typical convention is to have stations distributed evenly, forming a multiple of ten intervals between the perpendiculars at the ends of the design waterline.

If the longitudinal locations of the stations are known, the following relationships can be used to find x so that the cross sectional area can be computed for that location.

If lcb is zero,

$$\xi = \pi - \operatorname{acos}\left(2 \cdot \frac{X}{L_{wl}} \right)$$

Otherwise,

$$\xi = \pi - \operatorname{acos}\left[\frac{1}{\left[8 \cdot \left(l_{cb} \right) \right]} \cdot \left[-1 + \sqrt{\left(1 + \frac{32}{L_{wl}} \cdot l_{cb} \cdot X + 64 \cdot l_{cb}^{2} \right)} \right] \right]$$

A similar situation exists with respect to creating waterlines and buttlines at regular intervals. Computing the value for t, given an x, is similar to solving for x, given t. The same equations can be used, if t and x are exchanged, along with the corresponding coefficients (c1 for c3 and c4 for c5).

I have chosen to evaluate the shape at even values of t, however, to create an internal table of offsets. I generate waterlines, buttlines, and diagonals by interpolating this internal table's intersection with an arbitrary cutting plane. This procedure is necessary in any event to compute the waterline plane for arbitrary pitch and roll attitudes, so it is convenient to use it to generate the hull's lines.

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Hull Design

Hydrostatics

Approach

Hydrostatics is all about the forces and moments generated by the buoyancy of the boat at zero speed and in flat water. The trends in these static forces and moments are indicative of the load carrying ability and the stability of the yacht under sail. I use the hydrostatic calculations to determine:

displacement - determines load carried by the hull

wetted surface area - important for hull resistance calculations

transverse center of buoyancy - a measure of the heeling trim and stability

longitudinal center of buoyancy - a measure of the pitching trim and stability

pitching moment - longitudinal trim and stability at large rotations

rolling moment - transverse trim and stability at large rotations

waterplane area - the sensitivity of the displacement to changes in immersion

moments of inertia of the waterplane area - stability of the hull for small rotations

A key prerequisite to doing these calculations for any degree of immersion or rotation is the ability to interpolate the internal table of offsets to determine where the waterplane is, and what parts of the hull are above and below the waterline. I do this by running through the points for each section, testing to see if they are above or below the waterline. This is easy when the points are transformed to the earth axis system - positive Ze values indicate that the point is below the water, and negatvie Ze values indicate that it is above the water. When successive points are above and below the waterplane, the segment can be interpolated to find the point at the waterplane.

The points below the waterline are integrated to calculate the cross section area and the girth. The centroid of the sectional area is also calculated. Integrating the wetted girth along the length yields the wetted area. Integrating the cross sectional area gives the displacement, and integrating the area times its distance from the reference center yields the pitching and rolling moments. Dividing the moment by the displacement gives the location of the center of buoyancy. Computation of the waterplane area and moment of inertia is similar.

As mentioned a the previous section, once the hulls are rotated, their coordinates are no longer arranged in convenient stations, perpendicular to the waterplane. This makes the calculation of hydrostatics more complicated. Three general approaches suggest themselves:

- rotate the coordinates into the earth axis system, and integrate them as they are, which requires a complex integration scheme,

- interpolate the hull coordinates in the earth axis system to produce "nice" stations, which requires a great deal of computation as the inerpolation is redone for every rotation,

  - compute the hydrostatics in the hull coordinate system, taking into account that gravity no longer acts along the hull's Z axis. These forces and moments are then transformed back into the boat or earth coordinates, as desired.

Forces and Moments

I've chosen to use the third method, and will be computing all the forces and moments in the hull's reference system, and about the individual hull's reference center. These are then transformed to the boat coordinate system. All the quantities in this section are in the body axis system of the original hull, unless otherwise stated. The gravity vector, relative to the hull, is in the direction:

$$\mathrm{Vertical\_down}=\begin{bmatrix} \begin{array}{l} -\sin(\Theta)\cdot\cos(\tau)\cdot\cos(\iota)\;\ldots \\ +\sin(\Phi)\cdot\cos(\Theta)\cdot\sin(\tau)\cdot\cos(\iota)\;\ldots \\ +-\cos(\Phi)\cdot\cos(\Theta)\cdot\sin(\iota) \end{array} \\ \begin{array}{l} \sin(\Theta)\cdot\sin(\tau)\cdot\cos(\kappa)\;\ldots \\ +-\sin(\Theta)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\;\ldots \\ +\sin(\Phi)\cdot\cos(\Theta)\cdot\cos(\tau)\cdot\cos(\kappa)\;\ldots \\ +\sin(\Phi)\cdot\cos(\Theta)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\;\ldots \\ +\cos(\Phi)\cdot\cos(\Theta)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} \\ \begin{array}{l} -\sin(\Theta)\cdot\sin(\tau)\cdot\sin(\kappa)\;\ldots \\ +-\sin(\Theta)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\;\ldots \\ +-\sin(\Phi)\cdot\cos(\Theta)\cdot\cos(\tau)\cdot\sin(\kappa)\;\ldots \\ +\sin(\Phi)\cdot\cos(\Theta)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\;\ldots \\ +\cos(\Phi)\cdot\cos(\Theta)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array} \end{bmatrix}=\begin{bmatrix} \mathrm{Vert}_x \\ \mathrm{Vert}_y \\ \mathrm{Vert}_z \end{bmatrix}$$

The points around the stations are unevenly distributed, but there are a lot of them, so I've chosen to use simple, low order methods for approximating the integrals needed to compute sectional properties. In order to get the properties for the whole hull, these sectional properties need to be integrated along the length of the hull. Here I can use the fact that the stations are evenly spaced for an even number of intervals, and use Simpson's rule.

The cross section area is computed by a discrete approximation derived from Green's Theorem:

$$S=\int Y\,dZ=-\left(\int Z\,d{-Y}\right)=\frac{1}{2}\cdot\left(\int Y\,dZ-\int Z\,dY\right)$$

$$S=\frac{1}{4}\cdot\left[\sum_{i=2}^{n}\left((Y(i)+Y(i))\cdot(ZY(i)-Z(i-1))-(Y(i)-Y(i-1))\cdot(Z(i)+Z(i+1))\right)\right]$$

The Y's and Z's in this case are only those points which lie at or below the waterplane. The moments of the cross section area about the Y and Z axes are given by:

$$Mz=\frac{1}{2}\cdot\left(\int Y^2\,dZ\right)$$

$$My=\int Y\cdot Z\,dZ$$

$$M_z=\frac{1}{6}\cdot\sum_{i=2}^{n}\left(Y(i)^2+Y(i)\cdot Y(i-1)+Y(i-1)^2\right)\cdot(Z(i)-Z(i-1))$$

$$M_y=-\frac{1}{6}\cdot\sum_{i=2}^{n}(2\cdot Z(i)\cdot Y(i)+2\cdot Z(i-1)\cdot Y(i-1)+Z(i-1)\cdot Y(i)+Z(i)\cdot Y(i-1)\cdot(Z(i)-Z(i-1))$$

And the centroid of the section is found by dividing the moments by the area:

$$Y_c=\frac{M_z}{S}$$

$$Z_c=\frac{M_y}{S}$$

The displacment is found by integrating the cross section areas using Simpson's rule (m must be odd and the stations evenly spaced DX apart):

$$D=\rho\cdot g\cdot\int_{\frac{-Lwl}{2}}^{\frac{Lwl}{2}} S\,dX$$

$$D=\left(\frac{\rho\cdot g\cdot\Delta X}{3}\right)\cdot\left[\sum_{j=2}^{m-1}\left[3+(-1)^j\right]\cdot S(j)\ +S(1)+S(m)\right]$$

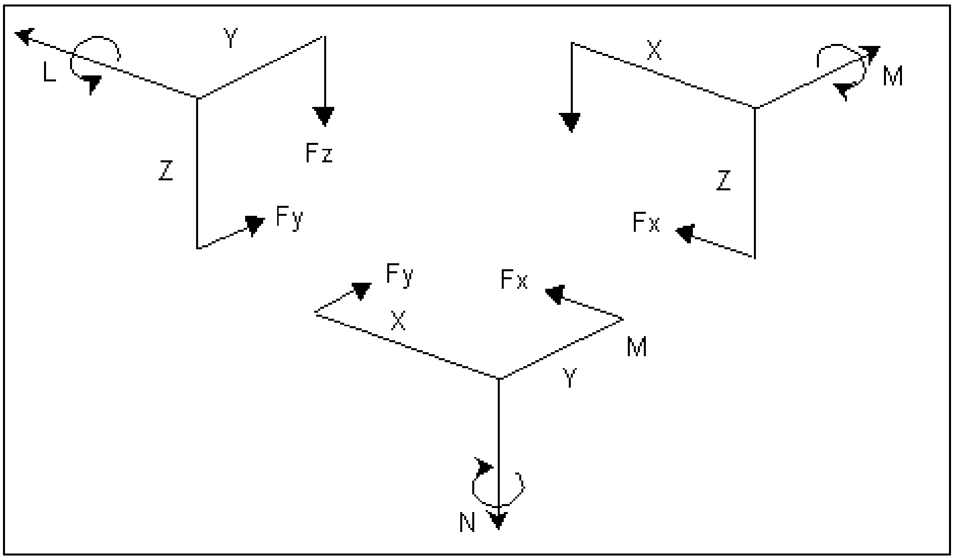
The force on the hull is equal to the displacement, in the opposite direction to vertical down:

$$\begin{bmatrix} F_{xh} \\ F_{yh} \\ F_{zh} \end{bmatrix} = -\begin{bmatrix} Vert_x \cdot D \\ Vert_y \cdot D \\ Vert_z \cdot D \end{bmatrix}$$

The moments are found by computing the center of buoyancy in each axis, and applying the total forces at that location:

$$\begin{bmatrix} X_{cb} \\ Y_{cb} \\ Z_{cb} \end{bmatrix} = \frac{\rho \cdot g \cdot \Delta X}{3 \cdot D} \cdot \left[\left[\sum_{j=2}^{m-1} \left[3 + (-1)^j\right] \cdot S(j) \cdot \begin{bmatrix} X_c(j) \\ Y_c(j) \\ Z_c(j) \end{bmatrix} \right] \ldots \right.$$
$$\left. + S(1) \cdot \begin{bmatrix} X_c(1) \\ Y_c(1) \\ Z_c(1) \end{bmatrix} + S(m) \cdot \begin{bmatrix} X_c(m) \\ Y_c(m) \\ Z_c(m) \end{bmatrix} \right]$$

$$\begin{bmatrix} L_h \\ M_h \\ N_h \end{bmatrix} = \begin{bmatrix} 0 & Fz & -Fy \\ -Fz & 0 & Fx \\ Fy & -Fx & 0 \end{bmatrix} \cdot \begin{bmatrix} X_{cb} \\ Y_{cb} \\ Z_{cb} \end{bmatrix} = \begin{bmatrix} Fz \cdot Y_{cb} - Fy \cdot Z_{cb} \\ -Fz \cdot X_{cb} + Fx \cdot Z_{cb} \\ Fy \cdot X_{cb} - Fx \cdot Y_{cb} \end{bmatrix}$$



The hull's contribution to the boat's forces and moments are computed by rotating to the boat axis system and transfering the moments to the boat reference center:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \cos(\tau)\cdot\cos(\iota) & -\sin(\tau)\cdot\cos(\kappa) \ldots + \cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa) & \sin(\tau)\cdot\sin(\kappa) \ldots + \cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa) \\ \sin(\tau)\cdot\cos(\iota) & \cos(\tau)\cdot\cos(\kappa) \ldots + \sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa) & -\cos(\tau)\cdot\sin(\kappa) \ldots + \sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa) \\ -\sin(\iota) & \cos(\iota)\cdot\sin(\kappa) & \cos(\iota)\cdot\cos(\kappa) \end{bmatrix} \cdot \begin{bmatrix} F_{xh} \\ F_{yh} \\ F_{zh} \end{bmatrix}$$

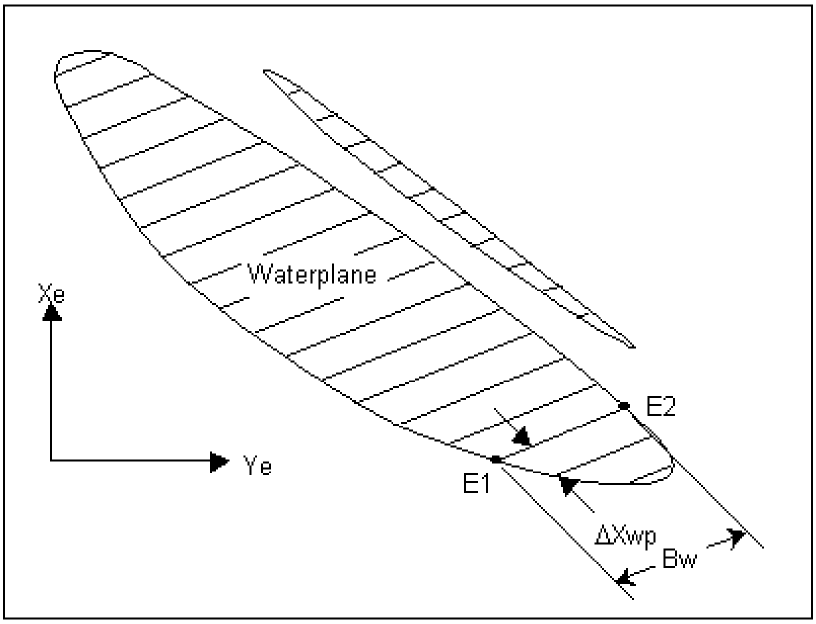
$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} \cos(\tau)\cdot\cos(\iota) & -\sin(\tau)\cdot\cos(\kappa) \ldots + \cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa) & \sin(\tau)\cdot\sin(\kappa) \ldots + \cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa) \\ \sin(\tau)\cdot\cos(\iota) & \cos(\tau)\cdot\cos(\kappa) \ldots + \sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa) & -\cos(\tau)\cdot\sin(\kappa) \ldots + \sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa) \\ -\sin(\iota) & \cos(\iota)\cdot\sin(\kappa) & \cos(\iota)\cdot\cos(\kappa) \end{bmatrix} \cdot \begin{bmatrix} L_h \\ M_h \\ N_h \end{bmatrix} \ldots + \begin{bmatrix} Yo\cdot F_z - Zo\cdot F_y \\ Zo\cdot F_x - Xo\cdot F_z \\ Xo\cdot F_y - Yo\cdot F_x \end{bmatrix}$$

Once the forces and moments for an individual hull are computed in the body axis system, they can be summed with the forces and moments for the other hulls to obtain the forces and moments acting on the whole boat.

Waterplane Parameters

The area and moments of inertia of the waterplane are useful because they indicate the sensitivity of the forces and moments. These parameters are computed in a similar manner to the hull forces and moments. However, in this case, it isn't necessary to do all the rotations back and forth between the earth and hull axis systems, since the orientation of the waterplane and the forces acting on it are understood. The forces and moments can be found in the earth axis system, and then transformed to the boat axis system, if desired.

The cross sections intersect the waterplane in straight lines. Because of the hollow in some of the cross section shapes, it is possible that there will be multiple segments where the waterplane intersects the cross section. Each segment has the endpoints E1 and E2, and the length of the segment is Bwp . These points around the waterplane can be transformed to the waterplane system and the waterplane quantities computed there. Note that the segments will not be parallel to the Ywp axis if the hull is toed in or out.



$$\begin{bmatrix} Xwp \\ Ywp \end{bmatrix} = \begin{bmatrix} \begin{array}{l} \cos(\Theta)\cdot\cos(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +-\cos(\Phi)\cdot\sin(\Theta)\cdot\sin(\iota) \end{array} & \begin{array}{l} -\cos(\Theta)\cdot\sin(\tau)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Theta)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\cos(\tau)\cdot\cos(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\sin(\Theta)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} & \begin{array}{l} \cos(\Theta)\cdot\sin(\tau)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Theta)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Phi)\cdot\sin(\Theta)\cdot\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\sin(\Theta)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array} \\ \begin{array}{l} \cos(\Phi)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi)\cdot\sin(\iota) \end{array} & \begin{array}{l} \cos(\Phi)\cdot\cos(\tau)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +-\sin(\Phi)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} & \begin{array}{l} -\cos(\Phi)\cdot\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Phi)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array} \end{bmatrix} \cdot \begin{bmatrix} Xh_{wp} \\ Yh_{wp} \\ Zh_{wp} \end{bmatrix} \ldots$$

$$+ \begin{bmatrix} \cos(\Theta) & \sin(\Phi)\cdot\sin(\Theta) & \cos(\Phi)\cdot\sin(\Theta) \\ 0 & \cos(\Phi) & -\sin(\Phi) \end{bmatrix} \cdot \begin{bmatrix} Xo \\ Yo \\ Zo \end{bmatrix}$$

$$B_{wp} = \sqrt{\left(X2_{wp} - X1_{wp}\right)^2 + \left(Y2_{wp} - Y1_{wp}\right)^2}$$

The lines where the section plane intersects the waterplane are equally spaced, but compared to the spacing of the sections in the hull axis system, the spacing will be closer. There will be nj such segments (typically one or two) for each station, according to the number of patches of the waterplane, with a zero distance used for sections that are entirely above or below the waterplane.

$$\Delta X_{swp} = \sqrt{\left(\begin{array}{l} \cos(\Theta)\cdot\cos(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +-\cos(\Phi)\cdot\sin(\Theta)\cdot\sin(\iota) \end{array}\right)^2 + \left(\begin{array}{l} \cos(\Phi)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi)\cdot\sin(\iota) \end{array}\right)^2}$$

Let the independent parameters t and u represent the directions along the station segments and perpendicular to them, such that t = 0 at point E1, t = 1 at point E2, and u=1 at one end of the waterplane patch and nj at the other end. An elemental area of the waterplane is then:

$$X_{wp} = X1(u) + (X2(u) - X1(u))\cdot t$$

$$Y_{wp} = Y1(u) + (Y2(u) - Y1(u)) \cdot t$$

$$dA_{wp} = \sqrt{(X2(u) - X1(u))^2 + (Y2(u) - Y1(u))^2} \cdot \Delta X_{swp} \cdot dt \cdot du$$

And the waterplane areas can be found by integrating the area along the t and u directions:

$$A_{wp} = \int_1^{n_j} \int_0^1 \frac{dA_{wp}}{dt \cdot du} dt\, du = \int_1^{n_j} B_{wp} \cdot \Delta X_{swp}\, du$$

The area of the waterplane is found by integrating the waterplane beams using Simpson's rule:

$$A_{wp} = \left(\frac{\Delta X_{swp}}{3}\right) \cdot \left[\sum_{j=2}^{m-1} \left[3 + (-1)^j\right] \cdot \sum_{i=1}^{n_j} B_{wp}(i,j) + \sum_{i=1}^{n_1} B_{wp}(i,1) + \sum_{i=1}^{n_m} B_{wp}(i,m) \right]$$

The centroid of the waterplane is found by taking the moments of the waterplane beams, and dividing by the area, just as was done for the section areas. Once the centroid is found in waterplane axes, it can be transformed back into boat axes if desired:

$$Xc_{wp} = \frac{1}{A_{wp}} \cdot \int_1^{n_j} \int_0^1 Y_{wp} \cdot \frac{dA_{wp}}{dt \cdot du} dt\, du$$

$$Xc_{wp} = \frac{1}{A_{wp}} \cdot \int_1^{nj} B_{wp}(u) \cdot \Delta X_{swp} \cdot \frac{(Y1(u) + Y2(u))}{2} du$$

$$Yc_{wp} = \frac{1}{A_{wp}} \cdot \int_1^{n_j} \int_0^1 X_{wp} \cdot \frac{dA_{wp}}{dt \cdot du} dt\, du$$

$$Yc_{wp} = \frac{1}{A_{wp}} \cdot \int_1^{nj} B_{wp}(u) \cdot \Delta X_{swp} \cdot \frac{(X1(u) + X2(u))}{2} du$$

$$Xc_{wp} = \left(\frac{\Delta X_{swp}}{2 \cdot A_{wp}}\right) \cdot \left[\sum_{j=2}^{m-1} \left[3 + (-1)^j\right] \cdot \sum_{i=1}^{n_j} B_{wp}(i,j) \cdot (Y1(i,j) + Y2(i,j)) \; \ldots \\ + \sum_{i=1}^{n_1} B_{wp}(i,1) \cdot (Y1(i,1) + Y2(i,1)) \; \ldots \\ + \sum_{i=1}^{n_m} B_{wp}(i,m) \cdot (Y1(i,m) + Y2(i,m))\right]$$

$$Yc_{wp} = \left(\frac{\Delta X_{swp}}{2 \cdot A_{wp}}\right) \cdot \left[\sum_{j=2}^{m-1} \left[3 + (-1)^j\right] \cdot \sum_{i=1}^{n_j} B_{wp}(i,j) \cdot (X1(i,j) + X2(i,j)) \; \ldots \\ + \sum_{i=1}^{n_1} B_{wp}(i,1) \cdot (X1(i,1) + X2(i,1)) \; \ldots \\ + \sum_{i=1}^{n_m} B_{wp}(i,m) \cdot (X1(i,m) + X2(i,m))\right]$$

The centroid of the waterplane can be transformed back to the boat body axis system, remembering that, by definition, the vertical location of the waterplane in the earth axes is zero:

$$\begin{bmatrix} Xc_b \\ Yc_b \\ Zc_b \end{bmatrix} = \begin{bmatrix} \cos(\Theta) \cdot \cos(\Psi) & \cos(\Theta) \cdot \sin(\Psi) \\ \sin(\Phi) \cdot \sin(\Theta) \cdot \cos(\Psi) \; \ldots + -\cos(\Phi) \cdot \sin(\Psi) & \sin(\Phi) \cdot \sin(\Theta) \cdot \sin(\Psi) \; \ldots + \cos(\Phi) \cdot \cos(\Psi) \\ \cos(\Phi) \cdot \sin(\Theta) \cdot \cos(\Psi) \; \ldots + \sin(\Phi) \cdot \sin(\Psi) & \cos(\Phi) \cdot \sin(\Theta) \cdot \sin(\Psi) \; \ldots + -\sin(\Phi) \cdot \cos(\Psi) \end{bmatrix} \cdot \begin{bmatrix} Xc_{wp} \\ Yc_{wp} \end{bmatrix}$$

The moments of inertia, or second moments, of the waterplane area is computed similarly. These are used to calculate the stability for small motions.

$$Ix_{wp} = \int_1^{n_j} \int_0^1 Y_{wp}^2 \cdot \frac{dA_{wp}}{dt \cdot du} \, dt \, du$$

$$Ix_{wp} = \int_1^{n_j} B_{wp}(u) \cdot \Delta X_{swp} \cdot \frac{\left(Y1(u)^2 + Y1(u) \cdot Y2(u) + Y2(u)^2\right)}{3} \, du$$

$$Iy_{wp} = \int_1^{n_j} \int_0^1 Y_{wp}^2 \cdot \frac{dA_{wp}}{dt \cdot du} \, dt \, du$$

$$Iy_{wp} = \int_{1}^{n_j} B_{wp}(u)\cdot\Delta X_{swp}\cdot\frac{\left(X1(u)^2 + X1(u)\cdot X2(u) + X2(u)^2\right)}{3}\,du$$

$$Ix_{wp} = \left(\frac{\Delta X_{wp}}{3}\right)\cdot\left[\sum_{j=2}^{m-1}\left[3+(-1)^j\right]\cdot\sum_{i=1}^{n_j} B_{wp}(i,j)\cdot\Delta X_{swp}\cdot\left(Y1(i,j)^2 + Y1(i,j)\cdot Y2(i,j) + Y2(i,j)^2\right) + \sum_{i=1}^{n_1} B_{wp}(i,j)\cdot\Delta X_{swp}\cdot\left(Y1(i,1)^2 + Y1(i,1)\cdot Y2(i,1) + Y2(i,1)^2\right) + \sum_{i=1}^{n_m} B_{wp}(i,j)\cdot\Delta X_{swp}\cdot\left(Y1(i,m)^2 + Y1(i,m)\cdot Y2(i,m) + Y2(i,m)^2\right)\right]$$

$$Iy_{wp} = \left(\frac{\Delta X_{wp}}{3}\right)\cdot\left[\sum_{j=2}^{m-1}\left[3+(-1)^j\right]\cdot\sum_{i=1}^{n_j} B_{wp}(i,j)\cdot\Delta X_{swp}\cdot\left(X1(i,j)^2 + X1(i,j)\cdot X2(i,j) + X2(i,j)^2\right) + \sum_{i=1}^{n_1} B_{wp}(i,1)\cdot\Delta X_{swp}\cdot\left(X1(i,1)^2 + X1(i,1)\cdot X2(i,1) + X2(i,1)^2\right) + \sum_{i=1}^{n_m} B_{wp}(i,m)\cdot\Delta X_{swp}\cdot\left(X1(i,m)^2 + X1(i,m)\cdot X2(i,m) + X2(i,m)^2\right)\right]$$

Wetted Surface Area

The area of the wetted surface is important for calculating the resistance of the boat. Since the wetted area is independent of the choice of axis system, I've chosen to avoid transformations and use coordinates in the hull axis system. I've chosen to use the method of computing the wetted girths at each section, and adjusting them to account for the slope of the hull by the secant method. These adjusted girths are then integrated along the length using Simpson's rule.

As with the displacement calculation the portion of the hull below the waterplane is found by testing each point around a section to see if it is above or below the waterplane. The distance between points at or below the waterplane are added into the girth, and points above the waterplane are ignored. The girth is computed by taking the straight-line distance between successive points, weighted by the secant of the longitudinal surface slope:

$$G_{wet}(j) = \sum_{i=2}^{n} \mathrm{Sec}(i,j)\cdot\sqrt{(Y(i,j) - Y(i-1,j))^2 + (Z(i,j) - Z(i-1,j))^2}$$

The secant weighting is a correction for the fact that the girths will be integrated along the length of the hull to get the wetted area, but the actual distance along the surface is greater than the station spacing, depending on how steeply the hull tapers at a given section. The secant is taken by projecting the vector normal to the surface onto the plane of the section, and the normal vector is found by taking the cross product of the line segment and a vector connecting the corresponding points on the adjacent sections:

$$\begin{bmatrix} Xn(i,j) \\ Yn(i,j) \\ Zn(i,j) \end{bmatrix} = \begin{bmatrix} X(i,j+1) - X(i,j-1) \\ Y(i,j+1) - Y(i,j-1) \\ Y(i,j+1) - Y(i,j-1) \end{bmatrix} \times \begin{bmatrix} X(i,j) \\ Y(i,j) - Y(i-1,j) \\ Z(i,j) - Z(i-1,j) \end{bmatrix}$$

$$\begin{bmatrix} Xn(i,j) \\ Yn(i,j) \\ Zn(i,j) \end{bmatrix} = \begin{bmatrix} (Y(i,j+1) - Y(i,j-1)) \cdot (Z(i,j) - Z(i-1,j)) \ \ldots \\ + -(Y(i,j+1) - Y(i,j-1)) \cdot (Y(i,j) - Y(i-1,j)) \\ (Y(i,j+1) - Y(i,j-1)) \cdot X(i,j) \ \ldots \\ + -(X(i,j+1) - X(i,j-1)) \cdot (Z(i,j) - Z(i-1,j)) \\ (X(i,j+1) - X(i,j-1)) \cdot (Y(i,j) - Y(i-1,j)) \ \ldots \\ + -(Y(i,j+1) - Y(i,j-1)) \cdot X(i,j) \end{bmatrix}$$

$$\begin{bmatrix} Xn(i,j) \\ Yn(i,j) \\ Zn(i,j) \end{bmatrix} = \begin{bmatrix} (Y(i,j+1) - Y(i,j-1)) \cdot (Z(i,j) - Z(i-1,j)) \ \ldots \\ + -(Y(i,j+1) - Y(i,j-1)) \cdot (Y(i,j) - Y(i-1,j)) \\ (Y(i,j+1) - Y(i,j-1)) \cdot X(i,j) \ \ldots \\ + -2 \cdot \Delta X \cdot (Z(i,j) - Z(i-1,j)) \\ 2 \cdot \Delta X \cdot (Y(i,j) - Y(i-1,j)) \ \ldots \\ + -(Y(i,j+1) - Y(i,j-1)) \cdot X(i,j) \end{bmatrix}$$

$$Sec(i,j) = \frac{\sqrt{Xn(i,j)^2 + Yn(i,j)^2 + Zn(i,j)^2}}{\sqrt{Yn(i,j)^2 + Zn(i,j)^2}} = \sqrt{1 + \frac{Xn(i,j)^2}{Yn(i,j)^2 + Zn(i,j)^2}}$$

In the case of the end sections, only one sided differences are used to compute the normals:

$$\begin{bmatrix} Xn(i,1) \\ Yn(i,1) \\ Zn(i,1) \end{bmatrix} = \begin{bmatrix} (Y(i,2) - Y(i,1)) \cdot (Z(i,1) - Z(i-1,1)) - (Y(i,2) - Y(i,1)) \cdot (Y(i,1) - Y(i-1,1)) \\ (Y(i,2) - Y(i,1)) \cdot X(i,1) - \Delta X \cdot (Z(i,1) - Z(i-1,1)) \\ \Delta X \cdot (Y(i,1) - Y(i-1,1)) - (Y(i,2) - Y(i,1)) \cdot X(i,1) \end{bmatrix}$$

$$\begin{bmatrix} Xn(i,m) \\ Yn(i,m) \\ Zn(i,m) \end{bmatrix} = \begin{bmatrix} (Y(i,m) - Y(i,m-1)) \cdot (Z(i,m) - Z(i-1,m)) \ \ldots \\ + -(Y(i,m) - Y(i,m-1)) \cdot (Y(i,m) - Y(i-1,m)) \\ (Y(i,m) - Y(i,m-1)) \cdot X(i,m) - \Delta X \cdot (Z(i,m) - Z(i-1,m)) \\ \Delta X \cdot (Y(i,m) - Y(i-1,m)) - (Y(i,m) - Y(i,m-1)) \cdot X(i,m) \end{bmatrix}$$

Finally, the wetted girths are integrated along the hull to get the wetted area of the hull:

$$S_{wet} = \left(\frac{\Delta X}{3}\right) \cdot \left[\sum_{j=2}^{m-1} \left[3 + (-1)^j\right] \cdot G_{wet}(j) + G_{wet}(1) + G_{wet}(m) \right]$$

Hydrostatics of the Complete Multihull

Once the hydrostatic quantities are found for each hull, it only remains to ocmbine them together to get the characteristics of the whole boat. The displacement and wetted area can simply be summed up over the hulls to get the displacement and wetted area for the whole boat.

The forces and moments were already computed in the boat axis system, and so X, Y, Z components of the forces and the moments (L, M, N) can also be summed for all the hulls. Once the forces and moments are in hand, they can be transformed from the boat axis system to any other desired axis system.

The waterplane areas were computed in the waterplane axis system, as were the moments of inertia of the waterplane area, so these, too, can be summed over the hulls. The longitudinal and transverse centers of buoyancy are perhaps less of interest for the complete multihull, since the forces and moments are already known. But if desired, the location of the complete center of buoyancy can be found by transforming the moments to the waterplane axis system and dividing by the displacement.

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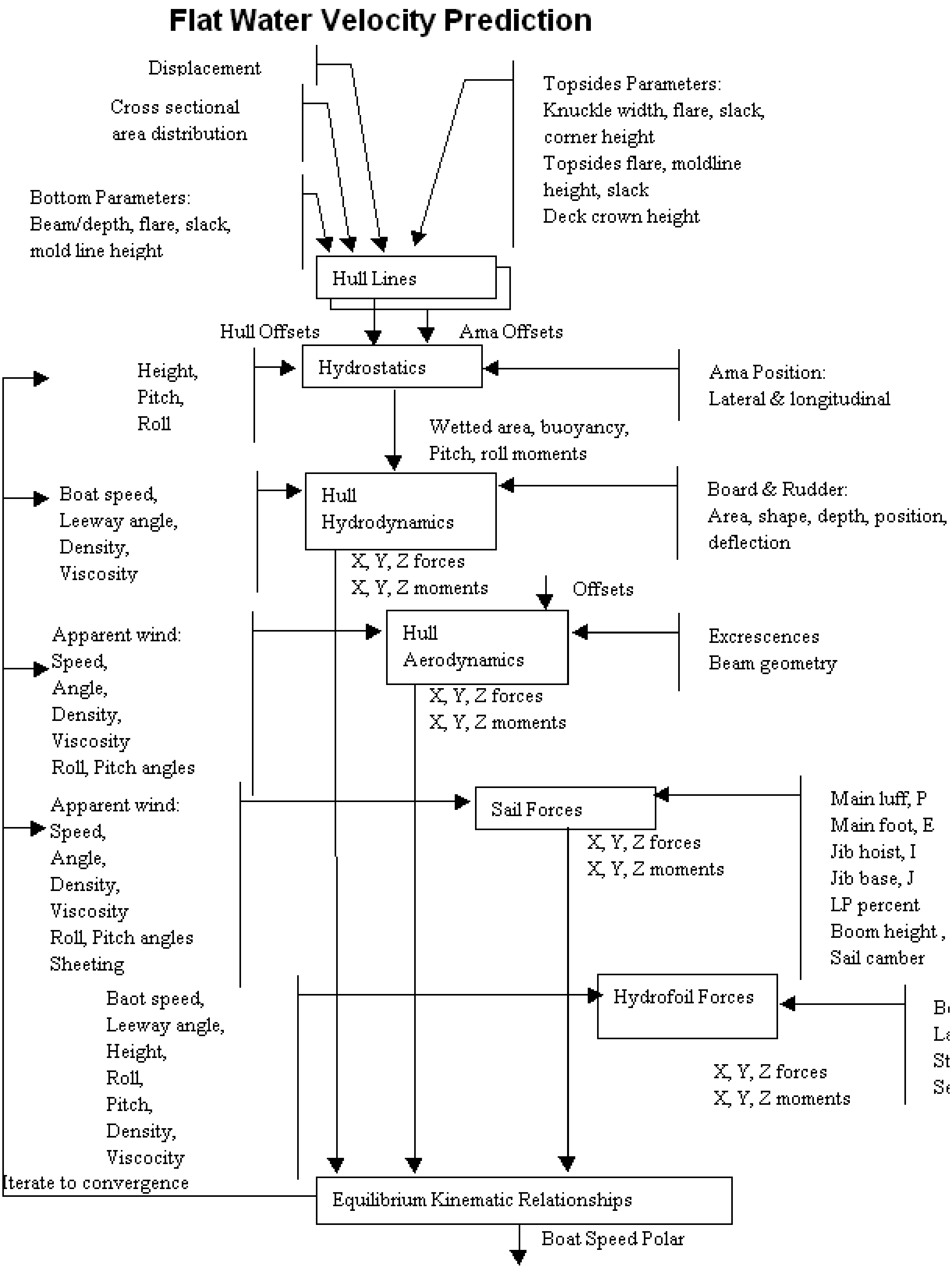
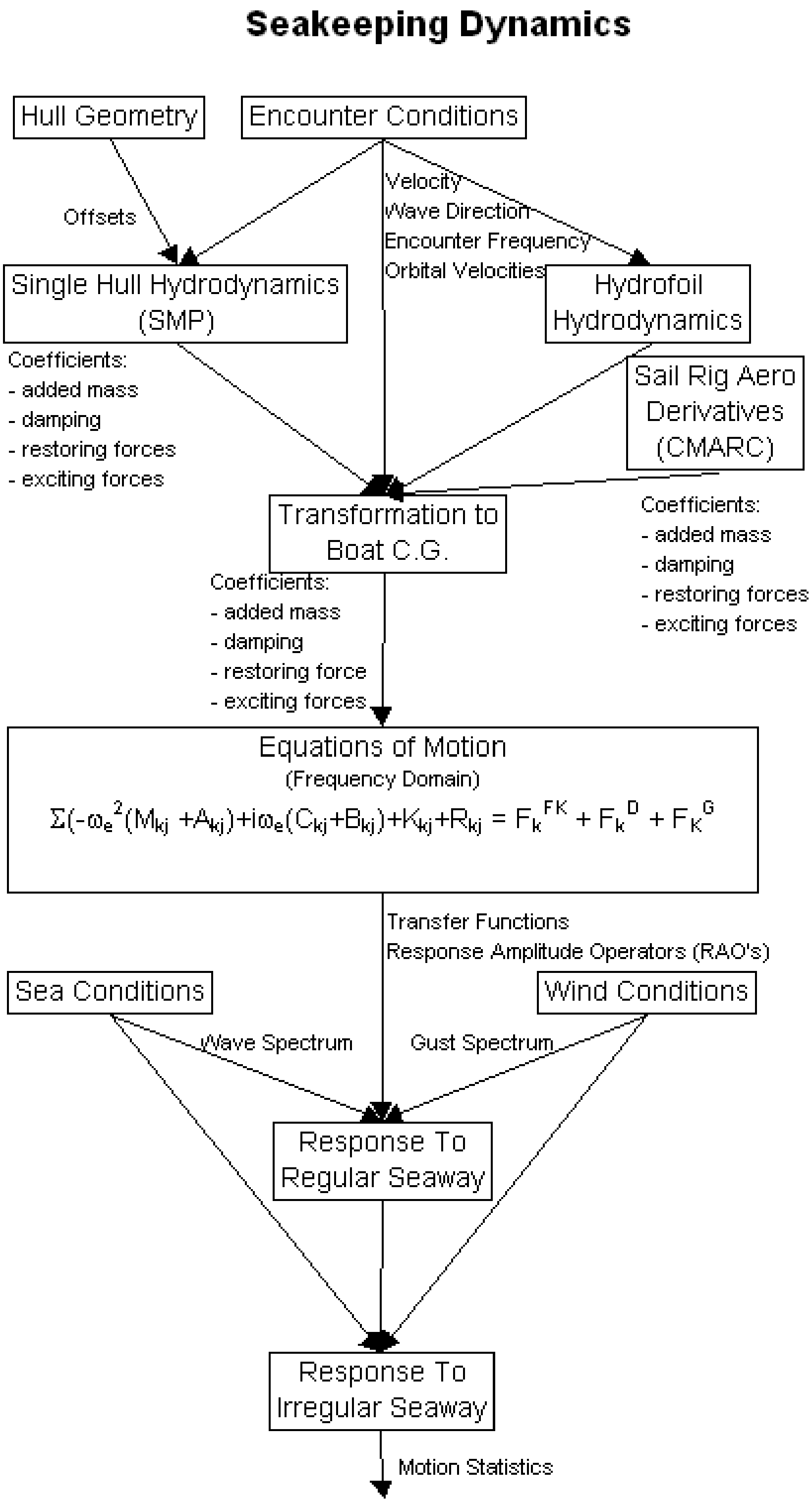



Seakeeping Dynamics
Hull Geometry
Encounter Conditions
Offsets
Velocity
Wave Direction
Encounter Frequency
Orbital Velocities
Single Hull Hydrodynamics (SMP)
Hydrofoil Hydrodynamics
Sail Rig Aero Derivatives (CMARC)
Coefficients:
- added mass
- damping
- restoring forces
- exciting forces
Transformation to Boat C.G.
Coefficients:
- added mass
- damping
- restoring forces
- exciting forces
Coefficients:
- added mass
- damping
- restoring force
- exciting forces
Equations of Motion
(Frequency Domain)
$\Sigma(-\omega_e^2(M_{kj}+A_{kj})+i\omega_e(C_{kj}+B_{kj})+K_{kj}+R_{kj} = F_k^{FK} + F_k^{D} + F_K^{G}$
Transfer Functions
Response Amplitude Operators (RAO's)
Sea Conditions
Wind Conditions
Wave Spectrum
Gust Spectrum
Response To Regular Seaway
Response To Irregular Seaway
Motion Statistics

Hull Design

Axis System Conventions

Approach

My first step in designing the hull is to define some axis systems I can use throughout the design process, including defining the shape of the hull, calculating its performance, and analyzing the dynamics of the boat's motion.

Body axis systems are always fixed to the body - they move and rotate with it. The reference point for the hull body axis system is the midpoint of the hull, at the design waterline. The X, Y, and Z axes form a right-handed coordinate system. The X axis is measured positive forward of midships, and negative aft. The Y axis is measured positive to starboard, and negative to port. The Z axis is measured positive down, and negative up. The position of a hull is given by the position of the midpoint of the hull relative to the reference point for the whole boat. For defining the geometry of monohulls, trimarans, and proas, this is the reference point of the main hull. For catamarans, this is the midships point, mid way between the two hulls, at the waterline. The body axis system will be relocated to the boat's center of gravity for dynamic analyses.

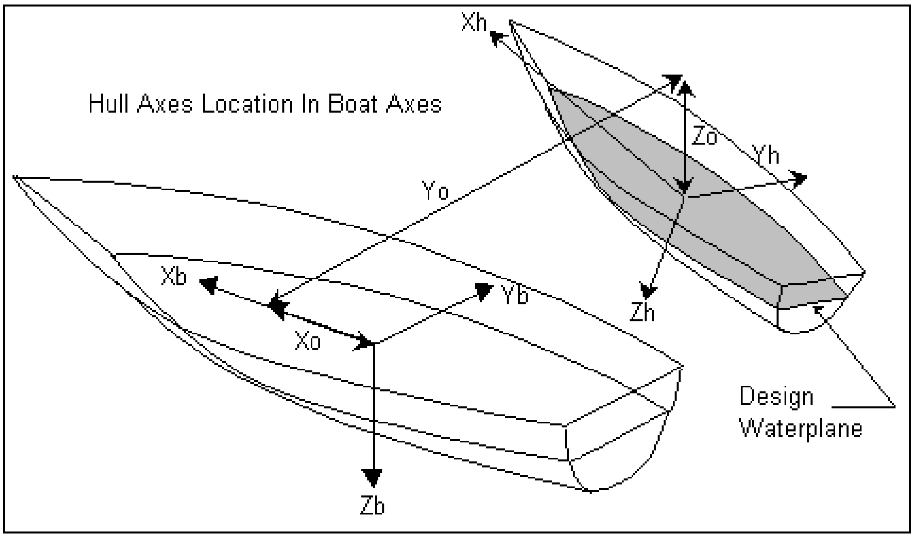
I've chosen the midpoint as the origin for the body axis system because it is near the center of gravity and center of buoyancy, and it avoids having all the X coordinates be negative numbers. An aft-starboard-up coordinate system starting at the forward end of the design waterline would be just as good for the geometry, but the boat's velocity would be negative, which is also awkward.

Forces applied to the hull are also positive when acting forward, starboard, and down, as are the velocity components, U (surge), V (sway), W (heave). The moments about the X, Y, and Z axes, L, M, and N, and the angular rates, P, Q, and R, are positive rolling to starboard, pitching bow up, and yawing to starboard.

The earth axis system, used to orient the boat, is positive to North, East, and down. The boat's location is measured from an earth axis system that doesn't move, and a moving earth axis system that is attached to the boat but doesn't rotate.

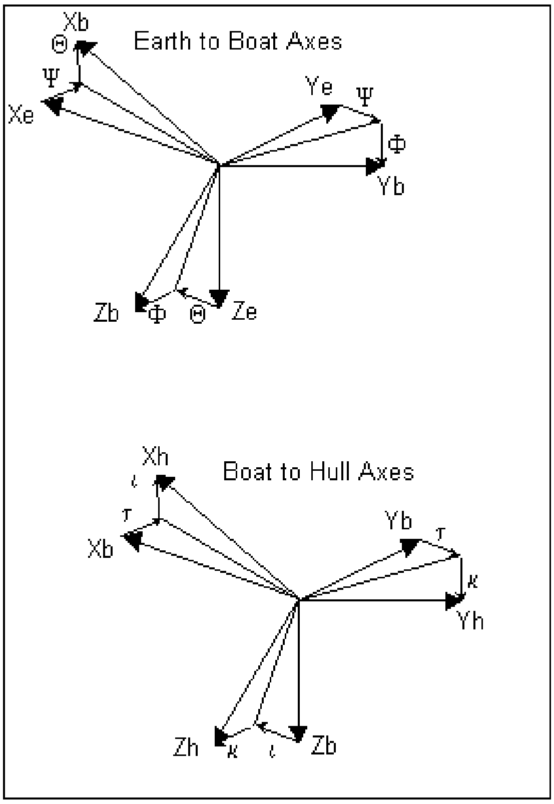
Axes Rotations

To compute the hydrostatics and motion for a complete multihull vessel, it is necessary to assemble the hulls in relation to each other, as they will be assembled in the complete boat. This will involve translation - lateral, longitudinal, and vertical - to position the hulls, and rotation. Rotation is necessary to consider hulls that are canted so as to be vertical when the boat is heeled, but also for amas on a trimaran that may be pitched up or hulls that are toed in or out, as with flip-tackers. Rotation of the coordinates is also necessary to compute the hydrostatics when the boat is in a heeled condition, or trimmed up or down by the bow.



Rotating the points is done by multiplying the X-Y-Z vector of coordinates by a transformation matrix. There is a transformation matrix for rotations about each axis, and these can be combined into a single transformation matrix. These transformations are necessary to change from one axis system to another, say from the body axis system (forward, starboard, keelward) to the earth axis system (North, East, down). The order of the rotations is important, since each successive rotation is done about an axis in an intermediate axis system. These transformation matrices turn out to have a special mathematical property - the inverse of the matrix is equal to its transpose. This makes it easy to change from going from earth to body axes to going from body to earth axes - just swap elements across the diagonal and reverse the order of the matrix multiplications.

I have chosen to use the same convention as is used in aeronautical practice in order to make it easy to use existing derivations for the equations of motion when flying on hydrofoils. Going from the earth to the body axis one rotates first in yaw about the Z axis- a change in the boat's heading. Then one rotates in pitch about the new Y axis, and finally one rotates in roll about the final X axis. There is some evidence that boats tend to pitch in the heeled plane of symmetry, rather than in the vertical plane, which would make a yaw-roll-pitch order more natural. However, I will be sticking with the yaw-pitch-roll convention.



The individual rotations are: Earth to intermediate system 1

$$\begin{bmatrix} X1 \\ Y1 \\ Z1 \end{bmatrix} = \begin{bmatrix} \cos(\Psi) & \sin(\Psi) & 0 \\ -\sin(\Psi) & \cos(\Psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Xe \\ Ye \\ Ze \end{bmatrix}$$

Intermediate system 1 to intermediate system 2

$$\begin{bmatrix} X2 \\ Y2 \\ Z2 \end{bmatrix} = \begin{bmatrix} \cos(\Theta) & 0 & -\sin(\Theta) \\ 0 & 1 & 0 \\ \sin(\Theta) & 0 & \cos(\Theta) \end{bmatrix} \cdot \begin{bmatrix} X1 \\ Y1 \\ Z1 \end{bmatrix}$$

Intermediate system 2 to body axes

$$\begin{bmatrix} Xb \\ Yb \\ Zb \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Phi) & \sin(\Phi) \\ 0 & -\sin(\Phi) & \cos(\Phi) \end{bmatrix} \cdot \begin{bmatrix} X2 \\ Y2 \\ Z2 \end{bmatrix}$$

These can be combined together into one single rotation matrix (here's where the order comes in):

$$\begin{bmatrix} Xb \\ Yb \\ Zb \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Phi) & \sin(\Phi) \\ 0 & -\sin(\Phi) & \cos(\Phi) \end{bmatrix} \cdot \begin{bmatrix} \cos(\Theta) & 0 & -\sin(\Theta) \\ 0 & 1 & 0 \\ \sin(\Theta) & 0 & \cos(\Theta) \end{bmatrix} \cdot \begin{bmatrix} \cos(\Psi) & \sin(\Psi) & 0 \\ -\sin(\Psi) & \cos(\Psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Xe \\ Ye \\ Ze \end{bmatrix}$$

$$\begin{bmatrix} Xb \\ Yb \\ Zb \end{bmatrix} = \begin{bmatrix} \cos(\Theta)\cdot\cos(\Psi) & \cos(\Theta)\cdot\sin(\Psi) & -\sin(\Theta) \\ \sin(\Phi)\cdot\sin(\Theta)\cdot\cos(\Psi)\ \ldots \\ + -\cos(\Phi)\cdot\sin(\Psi) & \sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\Psi)\ \ldots \\ + \cos(\Phi)\cdot\cos(\Psi) & \sin(\Phi)\cdot\cos(\Theta) \\ \cos(\Phi)\cdot\sin(\Theta)\cdot\cos(\Psi)\ \ldots \\ + \sin(\Phi)\cdot\sin(\Psi) & \cos(\Phi)\cdot\sin(\Theta)\cdot\sin(\Psi)\ \ldots \\ + -\sin(\Phi)\cdot\cos(\Psi) & \cos(\Phi)\cdot\cos(\Theta) \end{bmatrix} \cdot \begin{bmatrix} Xe \\ Ye \\ Ze \end{bmatrix}$$

Going from body to earth axes:

$$\begin{bmatrix} Xe \\ Ye \\ Ze \end{bmatrix} = \begin{bmatrix} \cos(\Theta)\cdot\cos(\Psi) & \sin(\Phi)\cdot\sin(\Theta)\cdot\cos(\Psi)\ \ldots \\ + -\cos(\Phi)\cdot\sin(\Psi) & \cos(\Phi)\cdot\sin(\Theta)\cdot\cos(\Psi)\ \ldots \\ + (\sin(\Phi)\cdot\sin(\Psi)) \\ \cos(\Theta)\cdot\sin(\Psi) & \sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\Psi)\ \ldots \\ + \cos(\Phi)\cdot\cos(\Psi) & \cos(\Phi)\cdot\sin(\Theta)\cdot\sin(\Psi)\ \ldots \\ + -\sin(\Phi)\cdot\cos(\Psi) \\ -\sin(\Theta) & \sin(\Phi)\cdot\cos(\Theta) & \cos(\Phi)\cdot\cos(\Theta) \end{bmatrix} \cdot \begin{bmatrix} Xb \\ Yb \\ Zb \end{bmatrix}$$

The procedure for assembling the hulls in an arbitrary arrangement is to first rotate the hulls in toe (t, positive bow to starboard), incidence (i, positive bow up), and cant (k, positive rolled to starboard), using the expression above. Then position the hull by adding to all the coordinates the position of the reference center relative to the boat's reference center.

Note that this can play havoc with the station definitions. If toe in/out or incidence is used, the points for a given station will no longer be all at the same longitudinal station for the whole boat (X coordinates will not be the same for a given hull cross section). This doesn't cause any fundmental difficulty, as long as one carries along the X coordinate as well as Y and Z for each point.

$$\begin{bmatrix} Xb \\ Yb \\ Zb \end{bmatrix} = \begin{bmatrix} \cos(\tau) & -\sin(\tau) & 0 \\ \sin(\tau) & \cos(\tau) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\iota) & 0 & \sin(\iota) \\ 0 & 1 & 0 \\ -\sin(\iota) & 0 & \cos(\iota) \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\kappa) & -\sin(\kappa) \\ 0 & \sin(\kappa) & \cos(\kappa) \end{bmatrix} \cdot \begin{bmatrix} Xh \\ Yh \\ Zh \end{bmatrix} + \begin{bmatrix} Xo \\ Yo \\ Zo \end{bmatrix}$$

$$\begin{bmatrix} Xb \\ Yb \\ Zb \end{bmatrix} = \begin{bmatrix} \cos(\tau)\cdot\cos(\iota) & -\sin(\tau)\cdot\cos(\kappa)\ \ldots \\ + \cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa) & \sin(\tau)\cdot\sin(\kappa)\ \ldots \\ + \cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa) \\ \sin(\tau)\cdot\cos(\iota) & \cos(\tau)\cdot\cos(\kappa)\ \ldots \\ + \sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa) & -\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ + \sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa) \\ -\sin(\iota) & \cos(\iota)\cdot\sin(\kappa) & \cos(\iota)\cdot\cos(\kappa) \end{bmatrix} \cdot \begin{bmatrix} Xh \\ Yh \\ Zh \end{bmatrix} \ldots + \begin{bmatrix} Xo \\ Yo \\ Zo \end{bmatrix}$$

I've found it useful to have a special case of the earth axis system, which is aligned with the waterplane and the boat's X axis. Thus, the heading angle, Y, is zero, but the pitch and roll angles are the same as for the earth axis system. The origin of the waterplane axis system is also at the mean water surface, so after rotating from body to waterplane orientation, the coordinates are translated vertically to account for the height of the boat's reference center relative to the waterplane.

$$
\begin{bmatrix} Xwp \\ Ywp \\ Zwp \end{bmatrix} =
\begin{bmatrix}
\begin{array}{l} \cos(\Theta)\cdot\cos(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +-\cos(\Phi)\cdot\sin(\Theta)\cdot\sin(\iota) \end{array} &
\begin{array}{l} -\cos(\Theta)\cdot\sin(\tau)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Theta)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\cos(\tau)\cdot\cos(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\sin(\Theta)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} &
\begin{array}{l} \cos(\Theta)\cdot\sin(\tau)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Theta)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Phi)\cdot\sin(\Theta)\cdot\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\sin(\Theta)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\sin(\Theta)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array} \\
\begin{array}{l} \cos(\Phi)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi)\cdot\sin(\iota) \end{array} &
\begin{array}{l} \cos(\Phi)\cdot\cos(\tau)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +-\sin(\Phi)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} &
\begin{array}{l} -\cos(\Phi)\cdot\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Phi)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array} \\
\begin{array}{l} -\sin(\Theta)\cdot\cos(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi)\cdot\cos(\Theta)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +-\cos(\Phi)\cdot\cos(\Theta)\cdot\sin(\iota) \end{array} &
\begin{array}{l} \sin(\Theta)\cdot\sin(\tau)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Theta)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\cos(\Theta)\cdot\cos(\tau)\cdot\cos(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\cos(\Theta)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\cos(\Theta)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} &
\begin{array}{l} -\sin(\Theta)\cdot\sin(\tau)\cdot\sin(\kappa)\ \ldots \\ +-\sin(\Theta)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Phi)\cdot\cos(\Theta)\cdot\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi)\cdot\cos(\Theta)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Phi)\cdot\cos(\Theta)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array}
\end{bmatrix}
\cdot \begin{bmatrix} Xh \\ Yh \\ Zh \end{bmatrix} \ldots
$$

$$
+ \begin{bmatrix} \cos(\Theta) & \sin(\Phi)\cdot\sin(\Theta) & \cos(\Phi)\cdot\sin(\Theta) \\ 0 & \cos(\Phi) & -\sin(\Phi) \\ -\sin(\Theta) & \sin(\Phi)\cdot\cos(\Theta) & \cos(\Phi)\cdot\cos(\Theta) \end{bmatrix} \cdot \begin{bmatrix} Xo \\ Yo \\ Zo \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ Zref \end{bmatrix}
$$

A similar axis system is the stability axis system. The stability axis system will be used to linearize the equations of motion to analyze the boat's dynamics. The stability axis system is a body axis system, fixed to the body at the boat's reference point, and moving and rotating with the body. However, at some time, typically then the boat is in steady trimmed motion, the orientation is frozen to be parallel to the waterplane axis system:

$$
\begin{bmatrix} Xs \\ Ys \\ Zs \end{bmatrix} =
\begin{bmatrix}
\begin{array}{l} \cos(\Theta_0)\cdot\cos(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi_0)\cdot\sin(\Theta_0)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +-\cos(\Phi_0)\cdot\sin(\Theta_0)\cdot\sin(\iota) \end{array} &
\begin{array}{l} -\cos(\Theta_0)\cdot\sin(\tau)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Theta_0)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi_0)\cdot\sin(\Theta_0)\cdot\cos(\tau)\cdot\cos(\kappa)\ \ldots \\ +\sin(\Phi_0)\cdot\sin(\Theta_0)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Phi_0)\cdot\sin(\Theta_0)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} &
\begin{array}{l} \cos(\Theta_0)\cdot\sin(\tau)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Theta_0)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Phi_0)\cdot\sin(\Theta_0)\cdot\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi_0)\cdot\sin(\Theta_0)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Phi_0)\cdot\sin(\Theta_0)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array} \\
\begin{array}{l} \cos(\Phi_0)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi_0)\cdot\sin(\iota) \end{array} &
\begin{array}{l} \cos(\Phi_0)\cdot\cos(\tau)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Phi_0)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +-\sin(\Phi_0)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} &
\begin{array}{l} -\cos(\Phi_0)\cdot\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Phi_0)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Phi_0)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array} \\
\begin{array}{l} -\sin(\Theta_0)\cdot\cos(\tau)\cdot\cos(\iota)\ \ldots \\ +\sin(\Phi_0)\cdot\cos(\Theta_0)\cdot\sin(\tau)\cdot\cos(\iota)\ \ldots \\ +-\cos(\Phi_0)\cdot\cos(\Theta_0)\cdot\sin(\iota) \end{array} &
\begin{array}{l} \sin(\Theta_0)\cdot\sin(\tau)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Theta_0)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi_0)\cdot\cos(\Theta_0)\cdot\cos(\tau)\cdot\cos(\kappa)\ \ldots \\ +\sin(\Phi_0)\cdot\cos(\Theta_0)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\sin(\kappa)\ \ldots \\ +\cos(\Phi_0)\cdot\cos(\Theta_0)\cdot\cos(\iota)\cdot\sin(\kappa) \end{array} &
\begin{array}{l} -\sin(\Theta_0)\cdot\sin(\tau)\cdot\sin(\kappa)\ \ldots \\ +-\sin(\Theta_0)\cdot\cos(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +-\sin(\Phi_0)\cdot\cos(\Theta_0)\cdot\cos(\tau)\cdot\sin(\kappa)\ \ldots \\ +\sin(\Phi_0)\cdot\cos(\Theta_0)\cdot\sin(\tau)\cdot\sin(\iota)\cdot\cos(\kappa)\ \ldots \\ +\cos(\Phi_0)\cdot\cos(\Theta_0)\cdot\cos(\iota)\cdot\cos(\kappa) \end{array}
\end{bmatrix}
\cdot \begin{bmatrix} Xh \\ Yh \\ Zh \end{bmatrix} \ldots
$$

$$
+ \begin{bmatrix} \cos(\Theta_0) & \sin(\Phi_0)\cdot\sin(\Theta_0) & \cos(\Phi_0)\cdot\sin(\Theta_0) \\ 0 & \cos(\Phi_0) & -\sin(\Phi_0) \\ -\sin(\Theta_0) & \sin(\Phi_0)\cdot\cos(\Theta_0) & \cos(\Phi_0)\cdot\cos(\Theta_0) \end{bmatrix} \cdot \begin{bmatrix} Xo \\ Yo \\ Zo \end{bmatrix}
$$

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