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HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

Volume I:

Limitations of Einstein's Special and General Relativities, Quantum Mechanics and Quantum Chemistry

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This volume is dedicated to the memory of

Professor Jaak Lôhmus

of the Estonia Academy of Sciences, Tartu, one of the greatest experts in nonassociative algebras of the 20-th century, for nominating the author among the most illustrious applied mathematicians of all times, the only Italian name appearing in the list, for his paternity on the initiation in 1967 of research on the most general possible algebras as defined in mathematics, the Lie-admissible algebras, that are at the foundation of the covering hadronic mechanics. The nomination was done in 1990 during communist times without any advance contact with or knowledge by the author, although, after the collapse of the communist era, the author was one of the firsts to visit Tartu with his wife Carla to express his appreciation, following a rather memorable trip by train from Moscow and return, while the former USSR was in disarray. The nomination is here reported also to honor the memory of the American mathematician **A. A. Albert** *who conceived the Lie-admissible algebras in 1947, although without detailed study. It is regrettable that, following Prof. Lôhmus death in 2006, the Estonia Academy of Sciences has been under criticisms by organized interests opposing the research reported in these volumes for evident personal gains. Consequently, the International Committee on Scientific Ethics and Accountability (www.scientificethics.org) has organized a monitoring of these misconducts for appropriate treatment.*


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АКАДЕМИЯ НАУК ЭСТОНИИ

ТРУДЫ ИНСТИТУТА ФИЗИКИ

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Quasigroups and nonassociative algebras in physics

КВАЗИГРУППЫ И НЕАССОЦИАТИВНЫЕ АЛГЕБРЫ В ФИЗИКЕ

TARTU 1990 TARTU

Figure 0.1. The front page of the nomination in Russian.

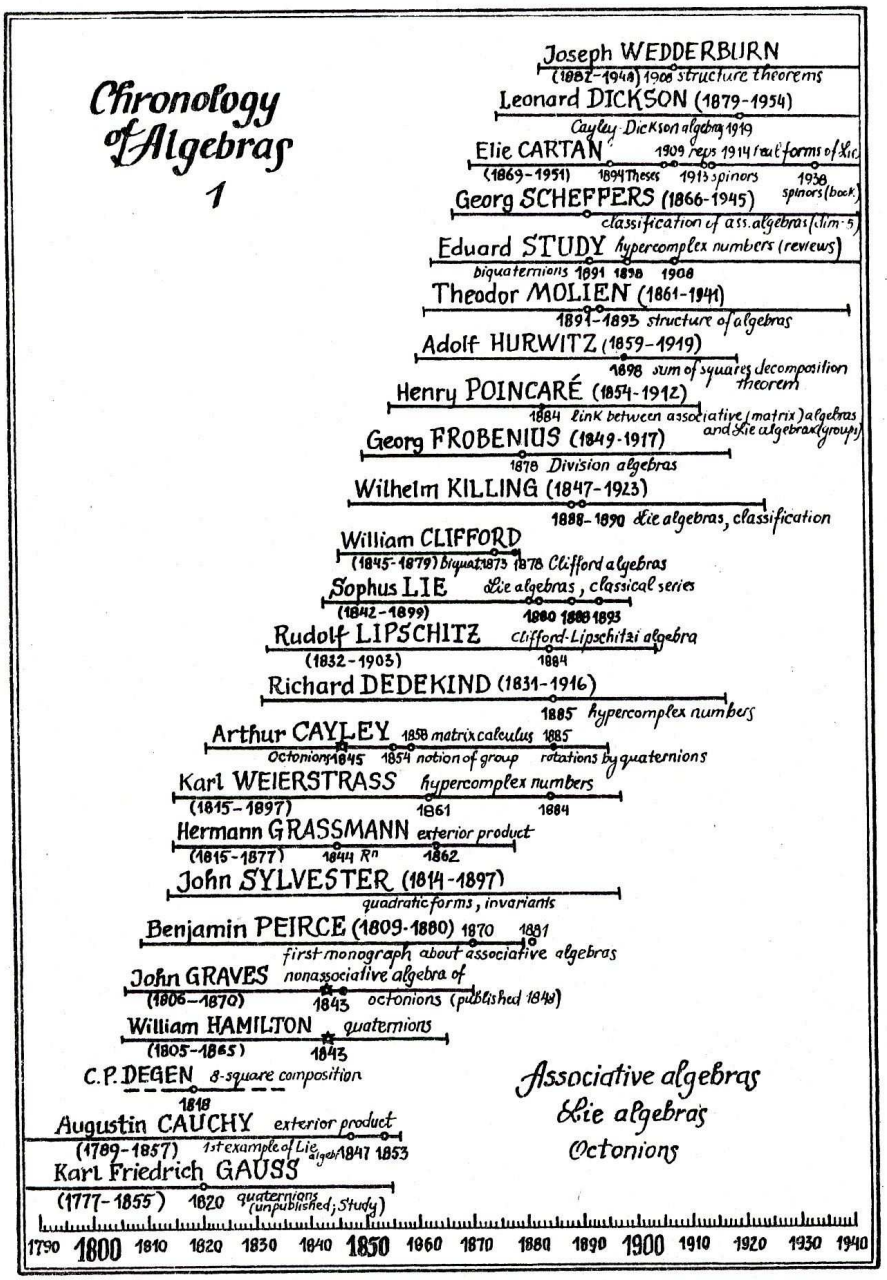

Figure 0.2. The second page of the nomination.

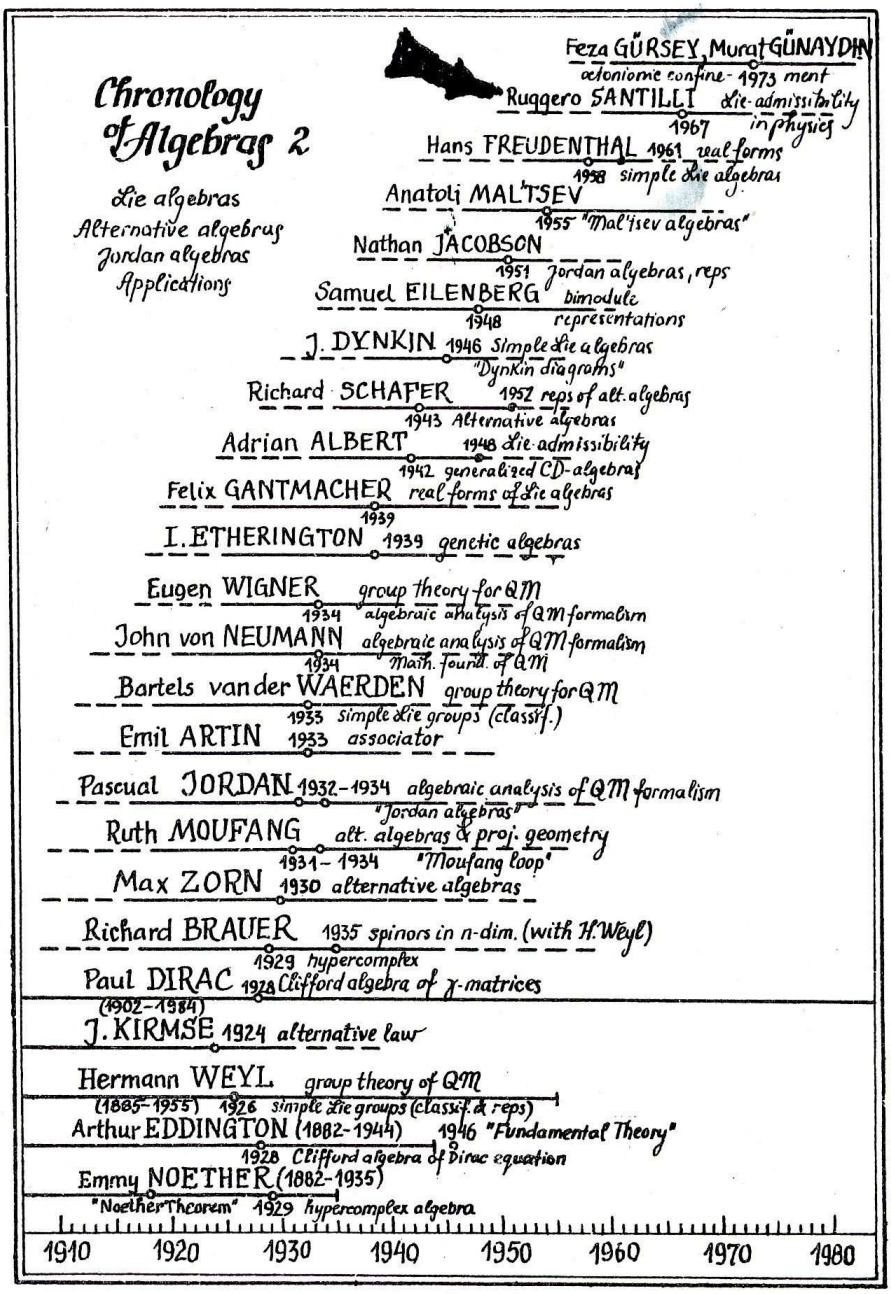

Figure 0.3. The second page of the nomination referring to a lifetime of research following the first article in the deformation of Lie algebras into Lie-admissible algebras appeared in a physics journal following only three articles in pure mathematics journals, R. M. Santilli, Nuovo Cimento **51**, 570 (1967).

Contents

Foreword

Mathematics is a subject which possibly finds itself in a unique position in academia in that it is viewed as both an Art and a Science. Indeed, in different universities, graduates in mathematics may receive Bachelor Degrees in Arts or Sciences. This probably reflects the dual nature of the subject. On the one hand, it may be studied as a subject in its own right. In this sense, its own beauty is there for all to behold; some as serene as da Vinci's "Madonna of the Rocks", other as powerful and majestic as Michelangelo's glorious ceiling of the Sistine Chapel, yet more bringing to mind the impressionist brilliance of Monet's Water Lily series. It is this latter example, with the impressionists interest in light, that links up with the alternative view of mathematics; that view which sees mathematics as the language of science, of physics in particular since physics is that area of science at the very hub of all scientific endeavour, all other branches being dependent on it to some degree. In this guise, however, mathematics is really a tool and any results obtained are of interest only if they relate to what is found in the real world; if results predict some effect, that prediction must be verified by observation and/or experiment. Again, it may be remembered that physics is really a collection of related theories. These theories are all manmade and, as such, are incomplete and imperfect. This is where the work of Ruggero Santilli enters the scientific arena.

Although "conventional wisdom" dictates otherwise, both the widely accepted theories of relativity and quantum mechanics, particularly quantum mechanics, are incomplete. The qualms surrounding both have been muted but possibly more has emerged concerning the inadequacies of quantum mechanics because of the people raising them. Notably, although it is not publicly stated too frequently, Einstein had grave doubts about various aspects of quantum mechanics. Much of the worry has revolved around the role of the observer and over the question of whether quantum mechanics is an objective theory or not. One notable contributor to the debate has been that eminent philosopher of science, Karl Popper. As discussed in my book, "Exploding a Myth", Popper preferred to refer to the experimentalist rather than observer, and expressed the view that that person played the same role in quantum mechanics as in classical mechanics. He felt, therefore, that such a person was there to test the theory. This is totally opposed to the Copenhagen Interpretation which claims that "objective reality has evaporated" and "quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness, of particles". Popper points

out that, over the years, many eminent physicists have switched allegiance from the pro-Copenhagen view. In some ways, the most important of these people was David Bohm, a greatly respected thinker on scientific matters who wrote a book presenting the Copenhagen view of quantum mechanics in minute detail. However, later, apparently under Einstein's influence, he reached the conclusion that his previous view had been in error and also declared the total falsity of the constantly repeated dogma that the quantum theory is complete. It was, of course, this very question of whether or not quantum mechanics is complete which formed the basis of the disagreement between Einstein and Bohr; Einstein stating "No", Bohr "Yes".

However, where does Popper fit into anything to do with Hadronic Mechanics? Quite simply, it was Karl Popper who first drew public attention to the thoughts and ideas of Ruggero Santilli. Popper reflected on, amongst other things, Chadwick's neutron. He noted that it could be viewed, and indeed was interpreted originally, as being composed of a proton and an electron. However, again as he notes, orthodox quantum mechanics offered no viable explanation for such a structure. Hence, in time, it became accepted as a new particle. Popper then noted that, around his (Popper's) time of writing, Santilli had produced an article in which the "first structure model of the neutron" was revived by "resolving the technical difficulties which had led, historically, to the abandonment of the model". It is noted that Santilli felt the difficulties were all associated with the assumption that quantum mechanics applied within the neutron and disappeared when a generalised mechanics is used. Later, Popper goes on to claim Santilli to belong to a new generation of scientists which seemed to him to move on a different path. Popper identifies quite clearly how, in his approach, Santilli distinguishes the region of the arena of incontrovertible applicability of quantum mechanics from nuclear mechanics and hadronics. He notes also his most fascinating arguments in support of the view that quantum mechanics should not, without new tests, be regarded as valid in nuclear and hadronic mechanics.

Ruggero Santilli has devoted his life to examining the possibility of extending the theories of quantum mechanics and relativity so that the new more general theories will apply in situations previously excluded from them. To do this, he has had to go back to the very foundations and develop new mathematics and new mathematical techniques. Only after these new tools were developed was he able to realistically examine the physical situations which originally provoked this lifetime's work. The actual science is his, and his alone, but, as with the realization of all great endeavours, he has not been alone. The support and encouragement he has received from his wife Carla cannot be exaggerated. In truth, the scientific achievements of Ruggero Santiili may be seen, in one light, as the results of a team effort; a team composed of Ruggero himself and Carla Gandiglio in Santilli. The theoretical foundations of the entire work are contained

in this volume; a volume which should be studied rigorously and with a truly open mind by the scientific community at large. This volume contains work which might be thought almost artistic in nature and is that part of the whole possessing the beauty so beloved of mathematicians and great artists. However, the scientific community should reserve its final judgement until it has had a chance to view the experimental and practical evidence which may be produced later in support of this elegant new theoretical framework.

Jeremy Dunning-Davies,
Physics Department,
University of Hull,
England.
September 8, 2007

Preface

Our planet is afflicted by increasingly cataclysmic climactic changes. The only possibility for their containment is the development of new, clean, energies and fuels. But, all possible energies and fuels that could be conceived with Einsteinian doctrines, quantum mechanics, and quantum chemistry had been discovered by the middle of the 20-th century, and they all resulted in being environmentally unacceptable either because of an excessive production of atmospheric pollutants, or because of the release of harmful waste.

Hence, the scientific community of the 21-st century is faced with the quite complex duties of, firstly, broadening conventional theories into forms permitting the prediction and quantitative study of new clean energies and fuels and, secondly, developing them up to the needed industrial maturity.

Due to organized interests in science, Einsteinian doctrines, quantum mechanics and quantum chemistry are generally depicted, particularly in Ph.D. schools in physics and chemistry, as being the final theories of nature.

In reality, it is known by experts to qualify as such, but rarely spoken, that *Einsteinian doctrines, quantum mechanics, and quantum chemistry cannot possibly be exactly valid for energy releasing processes because the said theories are invariant under time reversal, while all energy releasing processes are irreversible in time.*

As an illustration, it has been proved by graduate students in physics that, the insistent assumption of Einstein's special relativity and quantum mechanics as being exactly valid for the fusion of two nuclei into a third, $N_1 + N_2 \rightarrow N_3$ yields a finite probability of the *spontaneous* decay of the third nucleus into the original ones, $N_3 \rightarrow N_1 + n_2$, which prediction is nonscientific nonsense due to the irreversibility of the nuclear synthesis here considered. There is no need to repeat the calculations because the above inconsistency is a necessary consequence of the invariance of Einsteinian doctrines and quantum mechanics under time reversal.

Similarly, it has been proved by graduate students in chemistry that the insistent assumption of quantum chemistry as being exactly valid for an energy releasing chemical reaction, such as that of water, $2H + O \rightarrow H_2O$, yields a finite probability for the *spontaneous* disintegration of water into the original atomic constituents, $H_2O \rightarrow 2H + O$, that is also nonscientific nonsense. Again, there is no need to repeat the calculations because a given energy releasing chemical reaction and its time reversal image are statistically equivalent for quantum chemistry due to its invariance under time reversal.

The above unequivocal insufficiency of Einsteinian doctrines, quantum mechanics and quantum chemistry for energy releasing processes is amply sufficient, per se, to warrant systematic studies for their surpassing with structurally irreversible theories, namely, theories that are irreversible in time for all possible Hamiltonians, since the latter are known to be time reversal invariant. In turn, this task cannot possibly be accomplished without a broadening (called *lifting*) of the mathematics underlying orthodox theories.

The above insufficiency is also amply sufficient, per se, to denounce as scientific corruption for personal gains any objection, obstruction, or opposition to the broadening of Einsteinian doctrines, particularly when proffered by scientists at qualified institutions.

Besides the above incontrovertible limitations, orthodox theories have a number of limitations, insufficiencies or sheer inconsistencies that are well known to experts, but are not generally identified in the orthodox literature for the seemingly studious, or *de facto* implied intent of protecting organized interests on Einsteinian doctrines.

In this first volume, we shall identify in detail litany of insufficiencies, limitations or sheer inconsistencies of Einsteinian doctrines, quantum mechanics and quantum chemistry that have been generally suppressed in the technical literature by ascientific interests in science, let alone addressed and disproved. We shall also initiate the denunciation of said ascientific interests in science because, without their containment, no basic advance of human knowledge is conceivably possible, as well known by all scientists who dared to surpass Einsteinian doctrines.

The resolution of said limitations, insufficiencies or sheer inconsistencies will be presented in subsequent volumes via, firstly, the broadening of a 20-th century mathematics, then the construction of covering theories, and then their experimental verification and industrial applications.

Ruggero Maria Santilli
January 19, 2008

Legal Notice

The underwriter Ruggero Maria Santilli states the following:

1) To be the sole person responsible for the content of *Hadronic Mathematics, Mechanics and Chemistry,* Volumes I and II; to be the sole owner of the Copyrights on these two volumes; and to have recorded, beginning with 1992, the copyright ownership of a number of his main contributions in the field.

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3) There are insisting rumors that organized interests in science are waiting or the author's death to initiate premeditated and organized actions for paternity fraud via the known scheme, often used in the past, based on new papers in the field without the identification of the author's paternity, which papers are then quickly quoted as originating papers by pre-set accomplices and the fraud is then accepted by often naive or ignorant followers merely blinded by the academic credibility of the schemers. Members of these rumored rings should be aware that the industrial applications of hadronic mathematics, mechanics and chemistry have already provided sufficient wealth to set up a Paternity Protection Trust solely funded to file lawsuits against immoral academicians attempting paternity fraud, their affiliations and their funding agencies.

This legal notice has been made necessary because, as shown in Section 1.5, the author has been dubbed "the most plagiarized scientist of the 20-th century," as it is the case of the thousands of papers in deformations published without any quotation of their origination by the author in 1967. These, and other attempted paternity frauds, have forced the author to initiate legal action reported in web site [1].

In summary, honest scientists are encouraged to copy, and/or study, and/or criticize, and/or develop, and/or apply the formulations presented in these volumes in any way desired without any need of advance authorization by the copyrights owner, under the sole conditions of implementing standard ethical rules 2A, 2B, 2C. Dishonest academicians, paternity fraud dreamers, and other schemers are warned that legal actions to enforce scientific ethics are already under way [1], and will be continued after the author's death.

In faith

Ruggero Maria Santilli

U. S. Citizen acting under the protection of the First Amendment of the U. S. Constitution guaranteeing freedom of expression particularly when used to contain asocial misconducts.

Tarpon Springs, Florida, U. S. A.

October 11, 2007

[1] International Committee on Scientific Ethics and Accountability
http://www.scientificethics.org

Acknowledgments

The author expresses his deepest appreciation in memory of:

— The late Werner Heisenberg, for supporting in epistolary exchanges studies a nonlinear generalization of his celebrated equation (today known as Heisenberg-Santilli isotopic, genotopic and hyperstructural equations at the foundations of hadronic mechanics), with particular reference to their treatment via a new mathematics capable of reconstructing linearity on generalized spaces over generalized fields, since Heisenberg dedicated his last years to the evident nonlinear character of nature;;

— The late Paul A. M. Dirac, for supporting in a short but memorable meeting reviewed in Section 6.2.8, nonunitary liftings of his celebrated equation (today known as Dirac-Santilli isotopic, genotopic and hyperstructural equations) for the representation of an electron within the hyperdense medium inside the proton, with particular reference to the development of a new mathematics eliminating the vexing divergencies in particle physics, since Dirac spent his last years in attempting the elimination of divergencies amidst strong opposition by organized interests on quantum chromodynamical theologies;

— The late British philosopher Karl Popper, for his strong support in the construction of hadronic mechanics, as shown in the Preface of his last book *Quantum Theory and the Schism in Physics;*

— The late Ilya Prigogine, for pioneering the need of nonunitary broadening of quantum mechanics and statistics and his personal support for the organization of the *Hadronic Journal* since its inception;

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— The late Italian physicist Giuliano Preparata, for pioneering anisotropic departures from the geometric structure of special relativity, extended by hadronic mechanics into anisotropic and inhomogeneous media;

— The late American mathematician Robert Oehmke for pioneering work on the Lie-admissible structure of hadronic mechanics;

— The late Estonian mathematician Jaak Löhmus whose studies on nonassociative algebras, with particular reference to the octonion algebra, have been particularly inspiring for the construction of hadronic mechanics;

— and other scholars who will be remembered by the author until the end of his life.

The author expresses his appreciation for invaluable comments to all participants of: the *International Workshop on Antimatter Gravity and Anti-Hydrogen Atom Spectroscopy* held in Sepino, Molise, Italy, in May 1996; the *Conference of the International Association for Relativistic Dynamics,* held in Washington, D.C., in June 2002; the *International Congress of Mathematicians,* held in Hong Kong, in August 2002; the *International Conference on Physical Interpretation of Relativity Theories,* held in London, September 2002, and 2004; and the *XVIII Workshop on Hadronic Mechanics* held in Karlstad, Sweden, in June 2005.

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Thanks are finally due to Prof. Richard Koch of the University of Oregon for assistance in composing this monograph with TexShop, and to Dr. I. S. Gandzha for assistance in the LaTeX composition, without which help these volumes would not have been printed. Thanks are finally due to various colleagues for a technical control, including Drs. G. Mileto, M. Sacerdoti and others, and to Mrs. Dorte Zuckerman for proofreading assistance. Needless to say, the author is solely responsible for the content of this monograph due also to several additions and improvements in the final version.

Chapter 1

SCIENTIFIC IMBALANCES OF THE TWENTIETH CENTURY

1.1 THE SCIENTIFIC IMBALANCE CAUSED BY ANTIMATTER

1.1.1 Needs for a Classical Theory of Antimatter

The first large scientific imbalances of the 20-th century studied in this monograph is that caused by the treatment of *matter* at all possible levels, from Newtonian to quantum mechanics, while *antimatter* was solely treated at the level of *second quantization* [1].

Besides an evident lack of scientific democracy in the treatment of matter and antimatter, the lack of a consistent *classical* treatment of antimatter left open a number of fundamental problems, such as the inability to study whether a faraway galaxy or quasar is made up of matter or of antimatter, because such a study requires first a classical representation of the gravitational field of antimatter, as an evident pre-requisite for the quantum treatment (see Figure 1.1).

It should be indicated that classical studies of antimatter simply cannot be done by merely reversing the sign of the charge, because of inconsistencies due to the existence of only one quantization channel. In fact, the quantization of a classical antiparticle solely characterized by the reversed sign of the charge leads to a *particle* (rather than a charge conjugated antiparticle) with the wrong sign of the charge.

It then follows that the treatment of the gravitational field of suspected antimatter galaxies or quasars cannot be consistently done via the Riemannian geometry in which there is a simple change of the sign of the charge, as rather popularly done in the 20-th century, because such a treatment would be structurally inconsistent with the quantum formulation.

At any rate, the most interesting astrophysical bodies that can be made up of antimatter are *neutral.* In this case general relativity and its underlying Rieman-

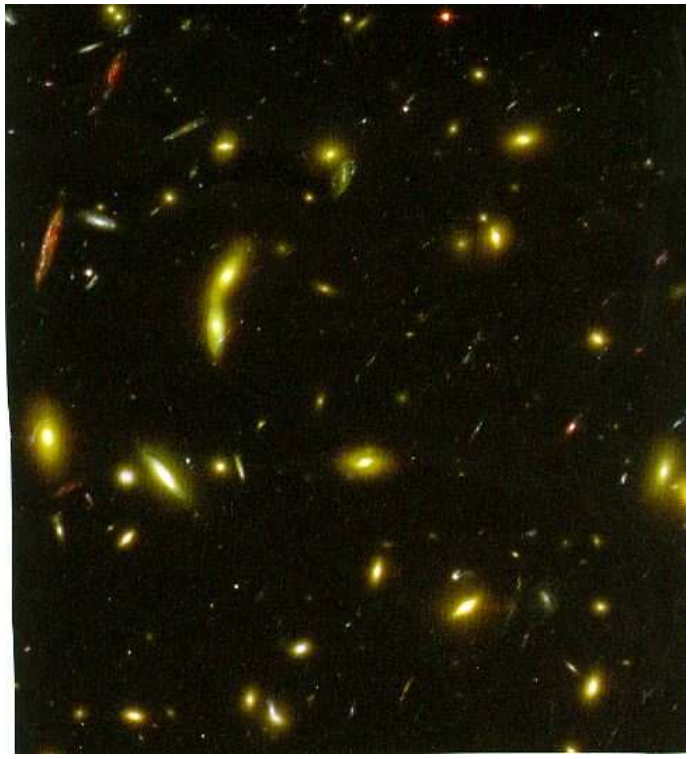

Figure 1.1. An illustration of the first major scientific imbalance of the 20-th century studied in this monograph, the inability to conduct classical quantitative studies as to whether faraway galaxies and quasars are made-up of matter or of antimatter. In-depth studies have indicated that the imbalance was not due to insufficient physical information, but instead it was due to the lack of a mathematics permitting the classical treatment of antimatter in a form compatible with charge conjugation at the quantum level.

nian geometry can provide no difference at all between matter and antimatter stars due to the null total charge. The need for a suitable new theory of antimatter then becomes beyond credible doubt.

As we shall see in Chapter 14, besides all the above insufficiencies, the biggest imbalance in the current treatment of antimatter occurs at the level of grand unifications, since all pre-existing attempts to achieve a grand unification of electromagnetic, weak and gravitational interactions are easily proved to be inconsistent under the request that the unification should hold not only for matter, as universally done until now, but also for antimatter. Hence, prior to venturing judgments on the need for a new theory of antimatter, serious scholars are suggested to inspect the entire scientific journey including the iso-grand-unification of Chapter 14.

1.1.2 The Mathematical Origin of the Imbalance

The origin of this scientific imbalance was not of physical nature, because it was due to the *lack of a mathematics suitable for the classical treatment of antimatter in such a way as to be compatible with charge conjugation at the quantum level.*

Charge conjugation is an *anti-homomorphism.* Therefore, a necessary condition for a mathematics to be suitable for the classical treatment of antimatter is that of being anti-homomorphic, or, better, anti-isomorphic to conventional mathematics.

Therefore, the classical treatment of antimatter requires *numbers, fields, functional analysis, differential calculus, topology, geometries, algebras, groups, symmetries, etc. that are anti-isomorphic to their conventional formulations for matter.*

The absence in the 20-th century of such a mathematics is soon established by the lack of a formulation of trigonometric, differential and other elementary functions, let alone complex topological structures, that are anti-isomorphic to the conventional ones.

In the early 1980s, due to the absence of the needed mathematics, the author was left with no other alternative than its construction along the general guidelines of hadronic mechanics, namely, the construction of the needed mathematics from the physical reality of antimatter, rather than adapting antimatter to pre-existing insufficient mathematics.[1]

After considerable search, the needed new mathematics for antimatter resulted in being characterized by the most elementary and, therefore, most fundamental possible assumption, that of a *negative unit,*

$$-1, \tag{1.1.1}$$

and then the reconstruction of the entire mathematics and physical theories of matter in such a way as to admit -1 as the correct left and right unit at all levels.

In fact, such a mathematics resulted in being anti-isomorphic to that representing matter, applicable at all levels of study, and resulting in being equivalent to charge conjugation after quantization.[2]

[1] In the early 1980s, when the absence of a mathematics suitable for the classical treatment of antimatter was identified, the author was (as a theoretical physicist) a member of the Department of Mathematics at Harvard University. When seeing the skepticism of colleagues toward such an absence, the author used to suggest that colleagues should go to Harvard's advanced mathematics library, select any desired volume, and open any desired page at random. The author then predicted that the mathematics presented in that page resulted in being fundamentally inapplicable to the classical treatment of antimatter, as it did indeed result to be the case without exceptions. In reality, the entire content of advanced mathematical libraries of the early 1980s did not contain the mathematics needed for a consistent classical treatment of antimatter.

[2] In 1996, the author was invited to make a 20 minutes presentation at a mathematics meeting held in Sicily. The presentation initiated with a transparency solely containing the number -1 and the statement

1.1.3 Outline of the Studies on Antimatter

Recall that "science" requires a mathematical treatment producing numerical values that can be confirmed by experiments. Along these lines, Chapter 2 is devoted, first, to the presentation of the new mathematics suggested by the author for the classical treatment of antimatter under the name of *isodual mathematics* with Eq. (1.1.1) as its fundamental *isodual left and right unit.*

The first comprehensive presentation was made by the author in monograph [94]. The first is, however, in continuous evolution, thus warranting an update.

Our study of antimatter initiates in Chapter 2 where we present the classical formalism, proposed under the name of *isodual classical mechanics* that begins with a necessary reformulation of Newton's equations and then passes to the needed analytic theory.

The operator formulation turned out to be *equivalent,* but not identical, to the quantum treatment of antiparticles, and was submitted under the name of *isodual quantum mechanics.*

Following these necessary foundational studies, Chapter 2 includes the detailed verification that the new *isodual theory of antimatter* does indeed verify *all* classical and particle experimental evidence.

In subsequent chapters we shall then study some of the predictions of the new isodual theory of antimatter, such as antigravity, a causal time machine, the isodual cosmology in which the universe has null total characteristics, and other predictions that are so far reaching as to be at the true edge of imagination.

All these aspects deal with point-like antiparticles. The study of extended, nonspherical and deformable antiparticles (such as the antiproton and the antineutron) initiates in Chapter 3 for reversible conditions and continues in the subsequent chapters for broader irreversible and multi-valued conditions.

1.2 THE SCIENTIFIC IMBALANCE CAUSED BY NONLOCAL-INTEGRAL INTERACTIONS

1.2.1 Foundations of the Imbalance

The second large scientific imbalance of the 20-th century studied in this monograph is that caused by the *reduction of contact nonlocal-integral interactions among extended particles to pre-existing action-at-a-distance local-differential interactions among point-like particles* (see Figure 1.2).

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that such a number was assumed as the basic left and right unit of the mathematics to be presented. Unfortunately, this first transparency created quite a reaction by most participants who bombarded the author with questions advancing his presentation, questions often repeated with evident waste of precious time without the author having an opportunity to provide a technical answer. This behavior continued for the remaining of the time scheduled for the talk to such an extent that the author could not present the subsequent transparencies proving that numbers with a negative unit verify all axioms of a field (see Chapter 2). The case illustrates that the conviction of absolute generality is so engraved among most mathematicians to prevent their minds from admitting the existence of *new* mathematics.

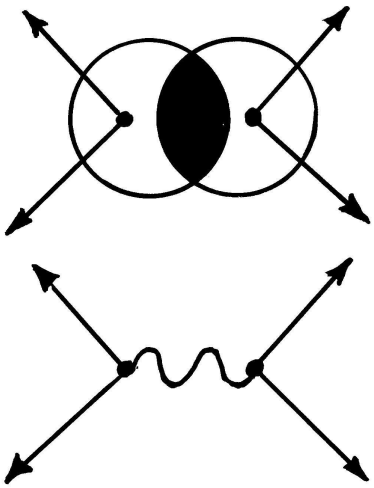

Figure 1.2. A first illustration of the second major scientific imbalance of the 20-th century studied in this monograph, the abstraction of extended hyperdense particles, such as protons and neutrons, to points, with consequential ignorance of the nonlocal and nonpotential effects caused by the deep overlapping of the hyperdense media in the interior of said particles. As we shall see, besides having major scientific implications, such as a necessary reformulation of Feynman's diagrams, the quantitative treatment of the nonlocal and nonpotential effects of this figure permits truly momentous advances, such as the conversion of divergent perturbative series into convergent forms, as well as the prediction and industrial development of basically new, clean energies and fuels.

It should be indicated that there exist numerous definitions of "nonlocality" in the literature, a number of which have been adapted to be compatible with pre-existing doctrines. The notion of nonlocality studied by hadronic mechanics is that specifically referred to *interactions of contact type not derivable from a potential and occurring in a surface, as for the case of resistive forces, or in a volume, as for the case of deep mutual penetration and overlapping of the wavepackets and/or charge distributions of particles.*

The imbalance was mandated by the fact (well known to experts to qualify as such) that *nonlocal-integral and nonpotential interactions are structurally incompatible with quantum mechanics and special relativity*, beginning with its local-differential topology, because the interactions here considered cause the catastrophic collapse of the mathematics underlying special relativity, let alone the irreconcilable inapplicability of the physical laws.

In fact, the local-differential topology, calculus, geometries, symmetries, and other mathematical methods underlying special relativity permit the sole consistent description of *a finite number of point-like particles moving in vacuum*

(empty space). Since points have no dimension and, consequently, cannot experience collisions or contact effects, the only possible interactions are at-a-distance, thus being derivable from a potential. The entire machinery of special relativity then follows. For systems of particles at large mutual distances for which the above setting is valid, such as for the structure of the hydrogen atom, special relativity is then *exactly valid.*

However, classical point-like particles do not exist; hadrons are notoriously extended; and even particles with *point-like charge,* such as the electron, do not have "point-like wavepackets". As we shall see, the representation of particles and/or their wavepackets as they really are in nature, that is, extended, generally nonspherical and deformable, cause the existence of contact effects of nonlocal-integral as well as zero-range nonpotential type that are beyond any hope of quantitative treatment via special relativity.

This is the case for all systems of particles at short mutual distances, such as the structure of hadrons, nuclei and stars, for which special relativity is *inapplicable* (rather than "violated") because not conceived or intended for the latter systems. The understanding is that the *approximate character* remains beyond scientific doubt.

Well known organized academic interests on Einsteinian doctrines then mandated the abstraction of nonlocal-integral systems to point-like, local-differential forms as a necessary condition for the validity of special relativity. This occurrence caused a scientific distortion of simply historical proportions because, while the existence of systems for which special relativity is fully valid is beyond doubt, the assumption that all conditions in the universe verify Einsteinian doctrines is a scientific deception for personal gains.

In Section 1.1 and in Chapter 2, we show the structural inability of special relativity to permit a *classical* representation of antimatter in a form compatible with charge conjugation. In this section and in Chapter 3, we show the inability of special relativity to represent extended, nonspherical and deformable particles or antiparticles and/or their wavepackets under nonlocal-integral interactions at short distances.

In Section 1.3 and in Chapter 4, we show the irreconcilable inapplicability of special relativity for all possible, classical and operator irreversible systems of particles and antiparticles. The widely ignored theorems of catastrophic inconsistencies of Einstein's gravitation are studied in Section 1.4 and in Chapter 3.

A primary purpose of this monograph is to show that the political adaptation of everything existing in nature to special relativity, rather than constructing new relativities to properly represent nature, prevents the prediction and quantitative treatment of new clean energies and fuels so much needed by mankind. In fact, new clean energies are permitted precisely by contact, nonlocal-integral

and nonpotential effects in hadrons, nuclei and stars that are beyond any dream of treatment via special relativity.

Therefore, *the identification of the limits of applicability of Einsteinian doctrines and the construction of new relativities are nowadays necessary for scientific accountability vis-a-vis society, let alone science.*

Needless to say, due to the complete symbiosis of special relativity and relativistic quantum mechanics, the inapplicability of the former implies that of the latter, and vice-versa. In fact, quantum mechanics will also emerge from our studies as being only *approximately valid* for system of particles at short mutual distances, such as for hadrons, nuclei and stars, for the same technical reasons implying the lack of exact validity of special relativity.

The resolution of the imbalance due to nonlocal interactions is studied in Chapter 3.

1.2.2 Exterior and Interior Dynamical Problems

The identification of the scientific imbalance here considered requires the knowledge of the following fundamental distinction:

DEFINITION 1.2.1: Dynamical systems can be classified into:
EXTERIOR DYNAMICAL SYSTEMS, consisting of particles at sufficiently large mutual distances to permit their point-like approximation under sole action-at-a-distance interactions, and
INTERIOR DYNAMICAL PROBLEMS, consisting of extended and deformable particles at mutual distances of the order of their size under action-at-a-distance interactions as well as contact nonpotential interactions.
Interior and exterior dynamical systems of antiparticles are defined accordingly.

Typical examples of exterior dynamical systems are given by planetary and atomic structures. Typical examples of interior dynamical systems are given by the structure of planets at the classical level and by the structure of hadrons, nuclei, and stars at the operator level.

The distinction of systems into exterior and interior forms dates back to Newton [2], but was analytically formulated by Lagrange [3], Hamilton [4], Jacobi[3][5] and others (see also Whittaker [6] and quoted references). The distinction was still assumed as fundamental at the beginning of the 20-th century, but thereafter the distinction was ignored.

[3]Contrary to popular belief, the celebrated Jacobi theorem was formulated precisely for the general analytic equations with external terms, while all reviews known to this author in treatises on mechanics of the 20-th century present the reduced version of the Jacobi theorem for the equations without external terms. Consequently, the reading of the original work by Jacobi [5] is strongly recommended over simplified versions.

For instance, Schwarzschild wrote two papers in gravitation, one of the *exterior gravitational problem* [7], and a second paper on the *interior gravitational problem* [8]. The former paper reached historical relevance and is presented in all subsequent treatises in gravitation of the 20-th century, but the same treatises generally ignore the second paper and actually ignore the distinction into gravitational exterior and interior problems.

The reasons for ignoring the above distinction are numerous, and have yet to be studied by historians. A first reason is due to the widespread abstraction of particles as being point-like, in which case all distinctions between interior and exterior systems are lost since all systems are reduced to point-particles moving in vacuum.

An additional reason for ignoring interior dynamical systems is due to the great successes of the planetary and atomic structures, thus suggesting the reduction of all structures in the universe to exterior conditions.

In the author's view, the primary reason for ignoring interior dynamical systems is that they imply the inapplicability of the virtual totality of theories constructed during the 20-th century, including classical and quantum mechanics, special and general relativities, etc., as we shall see.

The most salient distinction between exterior and interior systems is the following. Newton wrote his celebrated equations for a system of n point-particle under an arbitrary force not necessarily derivable from a potential,

$$m_a \times \frac{dv_{ak}}{dt} = F_{ak}(t, r, v), \tag{1.2.1}$$

where: $k = 1, 2, 3$; $a = 1, 2, 3, ..., n$; t is the time of the observer; r and v represent the coordinates and velocities, respectively; and the conventional associative multiplication is denoted hereon with the symbol $\times$ to avoid confusion with numerous additional inequivalent multiplications we shall identify during our study.

Exterior dynamical systems occur when Newton's force F_{ak} is entirely derivable from a potential, in which case the system is entirely described by the sole knowledge of a Lagrangian or Hamiltonian and the *truncated Lagrange and Hamilton analytic equations, those without external terms*

$$\frac{d}{dt}\frac{\partial L(t, r, v)}{\partial v_a^k} - \frac{\partial L(t, r, v)}{\partial r_a^k} = 0, \tag{1.2.2a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t, r, p)}{\partial r_a^k}, \tag{1.2.2b}$$

$$L = \frac{1}{2} \times m_a \times \mathbf{v}_a^2 - V(t, r, v), \tag{1.2.2c}$$

$$H = \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t, r, p), \tag{1.2.2d}$$

$$V = U(t, r)_{ak} \times v_a^k + U_o(t, r); \tag{1.2.2e}$$

where: $\mathbf{v}$ and $\mathbf{p}$ represent three-vectors; and the convention of the sum of repeated indices is hereon assumed.

Interior dynamical systems when Newton's force F_{ak} is partially derivable from a potential and partially of contact, zero-range, nonpotential types thus admitting additional interactions that simply cannot be represented with a Lagrangian or a Hamiltonian. For this reason, Lagrange, Hamilton, Jacobi and other founders of analytic dynamics presented their celebrated equations with *external terms representing precisely the contact, zero-range, nonpotential forces among extended particles.* Therefore, the treatment of interior systems requires the *true Lagrange and Hamilton analytic equations, those with external terms*

$$\frac{d}{dt}\frac{\partial L(t, r, v)}{\partial v_a^k} - \frac{\partial L(t, r, v)}{\partial r_a^k} = F_{ak}(t, r, v), \tag{1.2.3a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t, r, p)}{\partial r_a^k} + F_{ak}(t, r, p), \tag{1.2.3b}$$

$$L = \frac{1}{2} \times m_a \times \mathbf{v}_a^2 - V(t, r, v), \tag{1.2.3c}$$

$$H = \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t, r, p), \tag{1.2.3d}$$

$$V = U(t, r)_{ak} \times v_a^k + U_o(t, r), \tag{1.2.3e}$$

$$F(t, r, v) = F(t, r, p/m). \tag{1.2.3f}$$

Comprehensive studies were conducted by Santilli in monographs [9] (including a vast historical search) on the necessary and sufficient conditions for the existence of a Lagrangian or a Hamiltonian known as the *conditions of variational selfadjointness.* These studies permitted a rigorous separation of all acting forces into those derivable from a potential, or variationally selfadjoint (SA) forces, and those not derivable from a potential, or variationally nonselfadjoint (NSA) forces according to the expression

$$F_{ak} = F_{ak}^{SA}(t, r, v) + F_{ak}^{NSA}(t, r, v, a, ...). \tag{1.2.4}$$

In particular, the reader should keep in mind that, while selfadjoint forces are of Newtonian type, *nonselfadjoint forces are generally non-Newtonian,* in the sense

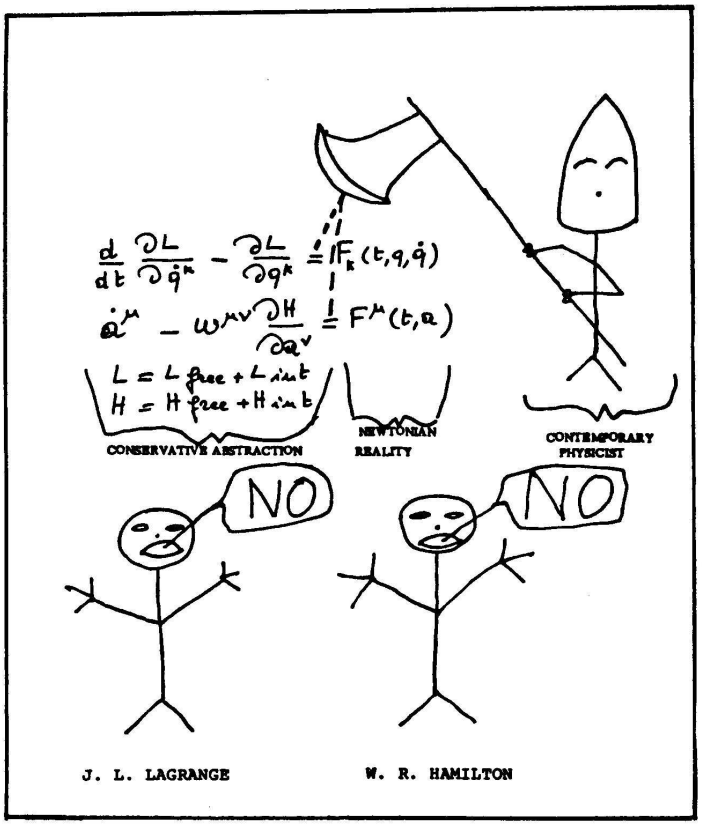

Figure 1.3. A reproduction of a "vignetta" presented by the author in 1978 to the colleagues at the Lyman Laboratory of Physics of Harvard University as part of his research under his DOE contract number DE-ACO2-80ER-10651.A001 to denounce the truncation of the external terms in Lagrange's and Hamilton's equations that was dominating physical theories of the time for the clear intent of maintaining compatibility with Einsteinian doctrines (since the latter crucially depend on the truncation depicted in this figure). The opposition by the Lyman colleagues at Harvard was so great that, in the evident attempt of tryinmg to discourage the author from continuing the research on the true Lagrange's and Hamilton's equations, the Lyman colleagues kept the author without salary for one entire academic year, even though the author was the recipient of a DOE grant and he had two children in tender age to feed and shelter. Most virulent was the opposition by the Lyman colleagues to the two technical memoirs [39,50] presented in support of the "vignetta" of this figure, for the evident reason that they dealt with a broadening of Einsteinian doctrines beginning with their title, and then continuing with a broadening of algebras, symmetries, etc.. But the author had no interest in a political chair at Harvard University, was sole interested in pursuing *new* scientific knowledge, and continued the research by dismissing the fierce opposition by his Lyman colleagues as ascientific and asocial (the episode is reported with real names in book [93] of 1984 and in the 1,132 pages of documentation available in Ref. [94]). As studied in details in these two volumes, the proper mathematical treatment of the true, historical, analytic equations, those with external terms, permits indeed the advances opposed by the Lyman colleagues, namely, the achievement of coverings of Einsteinian doctrines, that, being invariant (as shown later on), will indeed resist the test of time, while permitting the prediction and industrial development of new clean energies and fuels, thus confirming a societal, let alone scientific need for their serious study (se Footnote 1 of Volume II and subsequent footnotes for details).

of having an unrestricted functional dependence, including that on accelerations a and other non-Newtonian forms.[4]

As we shall see, nonselfadjoint forces generally have a nonlocal-integral structure that is usually reduced to a local-differential form via power series expansions in the velocities.

For instance, the contact, zero-range, resistive force experienced by a missile moving in our atmosphere is characterized by an integral over the surface of the missile and it is usually approximated by a power series in the velocities, e.g. $F^{NSA} = k_1 \times v + k_2 \times v^2 + k_3 \times v^3 + \ldots$ (see Figure 1.3).

Moreover, the studies of monographs [9] established that, for the general case in three dimensions, *Lagrange's and Hamilton's equations without external terms can only represent in the coordinates of the experimenter exterior dynamical systems, while the representation of interior dynamical systems in the given coordinates* (t, r) *of the experimenter require the necessary use of the true analytic equations with external terms.*

Whenever exposed to dynamical systems not entirely representable via the sole knowledge of a Lagrangian or a Hamiltonian, a rather general attitude is that of transforming them into an equivalent purely Lagrangian or Hamiltonian form. these transformations are indeed mathematically possible, but they are physically insidious.

It is known that, under sufficient continuity and regularity conditions and under the necessary reduction of nonlocal external terms to local approximations such as that in Eq. (1.2.4), the *Darboux's theorem* of the symplectic geometry or, equivalently, the *Lie-Koening theorem* of analytic mechanics assure the existence of coordinate transformations

$$\{r, p\} \to \{r'(r, p), p'(r, p)\}, \tag{1.2.5}$$

under which nonselfadjoint systems (1.2.2) can be turned into a selfadjoint form (1.2.1), thus eliminating the external terms.

However, coordinate transforms (1.2.5) are *necessarily nonlinear.* Consequently, the new reference frames are *necessarily noninertial.* Therefore, the elimination of the external nonselfadjoint forces via coordinate transforms cause the necessary loss of Galileo's and Einstein's relativities.

Moreover, it is evidently impossible to place measuring apparata in new coordinate systems of the type $r' = exp(k \times p)$, where k is a constant. For these reasons, the use of Darboux's theorem or of the Lie-Koening theorem was strictly prohibited in monographs [9,10,11]. Thus, to avoid misrepresentations, the following basic assumption is hereon adopted:

[4]There are serious rumors that a famous physicist from a leading institution visited NASA in 1998 to propose a treatment of the trajectory of the space shuttle during re-entry via (the truncated) Hamiltonian mechanics, and that NASA engineers kindly pushed that physicist through the door.

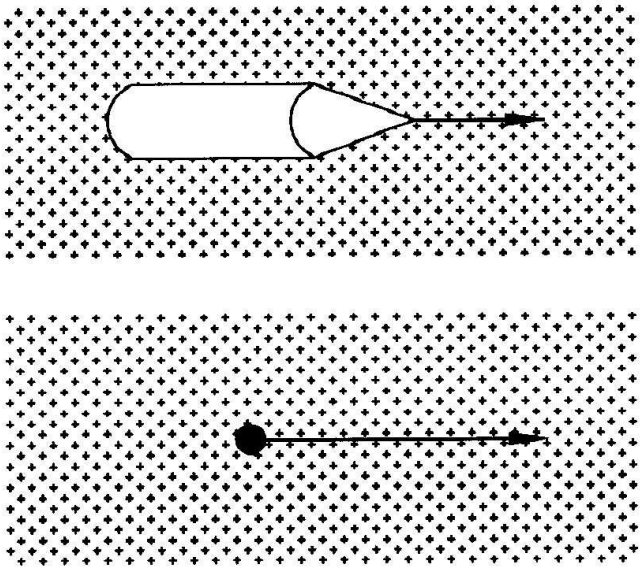

Figure 1.4. Another illustration of the major scientific imbalance studied in this monograph. The top view depicts a typical Newtonian system with nonlocal and nonpotential forces, such as a missile moving in atmosphere, while the bottom view depicts its reduction to point-like constituents conjectured throughout the 20-th century for the evident purpose of salvaging the validity of quantum mechanics and Einsteinian doctrines. However, the consistency of such a reduction has now been disproved by theorems, thus confirming the necessity of nonlocal and nonpotential interactions at the primitive elementary level of nature.

ASSUMPTION 1.2.1: The sole admitted analytic representations are those in the fixed references frame of the experimenter without the use of integrating factors, called direct analytic representations.

Only *after* direct representations have been identified, the use of the transformation theory may have physical relevance. Due to its importance, the above assumption will also be adopted throughout this monograph.

As an illustration, the admission of integrating factors within the fixed coordinates of the experimenter does indeed allow the achievement of an analytic representation without external terms of a restricted class of nonconservative systems, resulting in Hamiltonians of the type $H = e^{f(t,r,...)} \times p^2/2 \times m$. This Hamiltonian has a fully valid *canonical meaning* of representing the time evolution. However, this Hamiltonian loses its meaning as representing the energy of

the system. The quantization of such a Hamiltonian then leads to a plethora of illusions, such as the belief that the uncertainty principle for energy and time is still valid while, for the example here considered, such a belief has no sense because H does not represent the energy (see Refs. [9b] for more details).

Under the strict adoption of Assumption 1.2.1, all these ambiguities are absent because *H will always represent the energy,* irrespective of whether conserved or nonconserved, thus setting up solid foundations for correct physical interpretations.

1.2.3 General Inapplicability of Conventional Mathematical and Physical Methods for Interior Dynamical Systems

The impossibility of reducing interior dynamical systems to an exterior form within the fixed reference frame of the observer causes the loss for interior dynamical systems of all conventional mathematical and physical methods of the 20-th century.

To begin, the presence of irreducible nonselfadjoint external terms in the analytic equations causes the loss of their derivability from a variational principle. In turn, the lack of an action principle and related Hamilton-Jacobi equations causes the lack of any possible quantization, thus illustrating the reasons why the voluminous literature in quantum mechanics of the 20-th century carefully avoids the treatment of analytic equations with external terms.

By contrast, *one of the central objectives of this monograph is to review the studies that have permitted the achievement of a reformulation of Eqs. (1.2.3) fully derivable from a variational principle in conformity with Assumption 1.2.1, thus permitting a consistent operator version of Eqs. (1.2.3) as a covering of conventional quantum formulations.*

Recall that Lie algebras are at the foundations of all classical and quantum theories of the 20-th century. This is due to the fact that the brackets of the time evolution as characterized by Hamilton's equations,

$$\frac{dA}{dt} = \frac{\partial A}{\partial r_a^k} \times \frac{dr_a^k}{dt} + \frac{\partial A}{\partial p_{ak}} \times \frac{dp_{ak}}{dt} =$$

$$= \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ak}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ak}} = [A, H], \tag{1.2.6}$$

firstly, verify the conditions to characterize an *algebra* as currently understood in mathematics, that is, the brackets $[A, H]$ verify the right and left scalar and distributive laws,

$$[n \times A, H] = n \times [A, H], \tag{1.2.7a}$$

$$[A, n \times H] = [A, H] \times n, \tag{1.2.7b}$$

$$[A \times B, H] = A \times [B, H] + [A, H] \times B, \tag{1.2.7c}$$

$$[A, H \times Z] = [A, H] \times Z + H \times [A, Z], \tag{1.2.7d}$$

and, secondly, the brackets $[A, H]$ verify the *Lie algebra axioms*

$$[A, B] = -[B, A], \tag{1.2.8a}$$

$$[[A, B], C] + [[B, C], A] + [[C, A], B] = 0. \tag{1.2.8b}$$

The above properties then persist following quantization into the operator brackets $[A, B] = A \times B - B \times A$, as well known.

When adding external terms, the resulting new brackets,

$$\frac{dA}{dt} = \frac{\partial A}{\partial r_a^k} \times \frac{dr_a^k}{dt} + \frac{\partial A}{\partial p_{ak}} \times \frac{dp_{ak}}{dt} =$$

$$= \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ak}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ak}} + \frac{\partial A}{\partial r_a^k} \times F_a^k =$$

$$= (A, H, F) = [A, H] + \frac{\partial A}{\partial r_a^k} \times F_a^k, \tag{1.2.9}$$

violate the right scalar law (1.2.7b) and the right distributive law (1.2.7d) and, therefore, the brackets (A, H, F) do not constitute any algebra at all, let alone violate the basic axioms of the Lie algebras [9b].

The loss of the Lie algebras in the brackets of the time evolution of interior dynamical systems in their historical treatment by Lagrange, Hamilton, Jacobi and other founders of analytic dynamics, causes the loss of all mathematical and physical formulations built in the 20-th century.

The loss of basic methods constitutes the main reason for the abandonment of the study of interior dynamical systems. In fact, external terms in the analytic equations were essentially ignored through the 20-th century, by therefore adapting the universe to analytic equations (1.2.2) today known as the *truncated analytic equations.*

By contrast, *another central objective of this monograph is to review the studies that have permitted the achievement of a reformulation of the historical analytic equations with external terms,that is not only derivable from an action principle as indicated earlier, but also characterizes brackets in the time evolution that, firstly, constitute an algebra and, secondly, that algebra results in being a covering of Lie algebras.*

1.2.4 Inapplicability of Special Relativity for Dynamical Systems with Resistive Forces

The scientific imbalance caused by the reduction of interior dynamical systems to systems of point-like particles moving in vacuum, is indeed of historical proportion because it implied the belief of the exact applicability of special relativity

and quantum mechanics for all conditions of particles existing in the universe, thus implying their applicability under conditions for which these theories were not intended for.

A central scope of this monograph is to show that the imposition of said theories to interior dynamical systems causes the suppression of new clean energies and fuels already in industrial, let alone scientific, development, thus raising serious problems of scientific ethics and accountability.

At the classical level, the "inapplicability" (rather then the "violation") of (the Galilean and) special relativities for the description of an interior system such as a missile in atmosphere (as depicted in Figure 1.4) is beyond credible doubt, as any expert should know to qualify as such, because said relativities can only describe systems with action-at-a-distance potential forces, while the force acting on a missile in atmosphere are of contact-zero-range nonpotential type.

Despite this clear evidence, the resiliency by organized academic interests on conventional relativities knows no boundaries. As indicated earlier, when faced with the above evidence, a rather general posture is, that the resistive forces are "illusory" because, when the missile in atmosphere is reduced to its elementary point-like constituents all resistive forces "disappear."

Such a belief is easily proved to be nonscientific by the following property that can be proved by a first year graduate student in physics:

THEOREM 1.2.1 [9b]: A classical dissipative system cannot be consistently reduced to a finite number of quantum particles under sole potential forces and, vice-versa, no ensemble of a finite number of quantum particles with only potential forces can reproduce a dissipative classical system under the correspondence or other principles.

Note that the above property causes the inapplicability of conventional relativities for the description of the *individual constituents* of interior dynamical systems, let alone their description as a whole.

Rather than adapting nature to pre-existing organized interests on Einsteinian doctrines, the scope of this monograph is that of adapting the theories to nature, as requested by scientific ethics and accountability.

1.2.5 Inapplicability of Special Relativity for the Propagation of Light within Physical Media

Another case of manipulation of scientific evidence to serve organized academic interests on conventional relativities is the propagation of light within physical media, such as water.

As it is well known, light propagates in water at a speed C much smaller than the speed c in vacuum and approximately given by the value

$$C = \frac{c}{n} = \frac{2}{3} \times c << c, \quad n = \frac{3}{2} >> 1. \tag{1.2.10}$$

It is well known that electrons can propagate in water at speeds bigger than the local speed of light, and actually approaching the speed of light in vacuum. In fact, the propagation of electrons faster than the local speed of light is responsible for the blueish light, called *Cerenkov light,* that can be seen in the pools of nuclear reactors.

It is well known that *special relativity was built to describe the propagation of light IN VACUUM, and certainly not within physical media.* In fact, the setting of a massive particle traveling faster than the local speed of light is in violation of the basic axioms of special relativity.

To salvage the principle of causality it is then often assumed that *the speed of light "in vacuum" is the maximal causal speed "within water".* However, in this case there is the violation of the axiom of relativistic addition of speeds, because *the sum of two speeds of light in water does not yield the speed of light,* as required by a fundamental axiom of special relativity,

$$V_{tot} = \frac{C + C}{1 + \frac{C^2}{c^2}} = \frac{12}{13} \times c \neq C. \tag{1.2.11}$$

Vice-versa, if one assumes that *the speed of light "in water" C is the maximal causal speed "in water",* the axiom of relativistic compositions of speeds is verified,

$$V_{tot} = \frac{C + C}{1 + \frac{C^2}{C^2}} = C, \tag{1.2.12}$$

but there is the violation of the principle of causality evidently due to the fact that ordinary massive particles such as the electron (and not hypothetical *tachyons*) can travel faster than the local causal speed.

Again, the resiliency by organized interests on established relativities has no boundaries. When faced with the above evidence, a general posture is that, *when light propagating in water is reduced to photons scattering among the atoms constituting water, all axioms of special relativities are recovered in full.* In fact, according to this belief, photons propagate in vacuum, thus recovering the conventional maximal causal speed c, while the reduction of the speed of light is due to the scattering of light among the atoms constituting water.

The nonscientific character of the above view is established by the following evidence known to experts to qualify as such:

1) Photons are neutral, thus having a high capability of penetration within electrons clouds, or, more technically, the scattering of photons on atomic electron

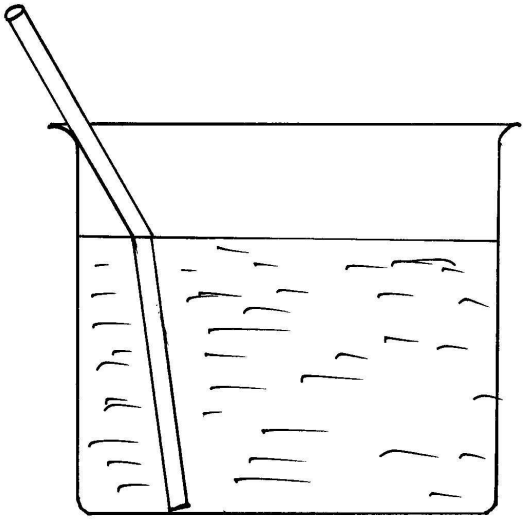

Figure 1.5. A further visual evidence of the lack of applicability of Einstein's doctrines within physical media, the refraction of light in water, due to the decrease of its speed contrary to the axiom of the "universal constancy of the speed of light". Organized academic interests on Einsteinian doctrines have claimed throughout the 20-th century that this effect is "illusory" because Einsteinian doctrines are recovered by reducing light to the scattering of photons among atoms. The political nature of the argument, particularly when proffered by experts, is established by numerous experimental evidence reviewed in the this section.

clouds (called *Compton scattering*) is rather small. Explicit calculations (that can be done by a first year graduate student in physics via quantum electrodynamics) show that, in the most optimistic of the assumptions and corrections, said scattering can account for only 3% of the reduction of the speed of light in water, thus leaving about 30% of the reduction quantitatively unexplained. Note that the deviation from physical reality is of such a magnitude that it cannot be "resolved" via the usual arbitrary parameters "to make things fit."

2) The reduction of speed occurs also for radio waves with one meter wavelength propagating within physical media, in which case the reduction to photons has no credibility due to the very large value of the wavelength compared to the size of atoms. The impossibility of a general reduction of electromagnetic waves to photon propagating within physical media is independently confirmed by the existence of vast experimental evidence on *non-Doppler's effects* reviewed in Chapter 9 indicating the existence of contributions outside the Doppler's law even when adjusted to the local speed.

3) There exist today a large volume of experimental evidence reviewed in Chapter 5 establishing that light propagates within hyperdense media, such as those in the interior of hadrons, nuclei and stars, at speed much bigger than the speed in vacuum,

$$C = \frac{c}{n} >> c, \quad n << 1. \tag{1.2.13}$$

in which case the reduction of light to photons scattering among atoms loses any physical sense (because such propagation can never reach the speed c, let alone speeds bigger than c).

In conclusion, experimental evidence beyond credible doubt has established that *the speed of light C is a local quantity dependent on the characteristics in which the propagation occurs, with speed $C = c$ in vacuum, speeds $C << c$ within physical media of low density and speeds $C >> c$ within media of very high density.*

The variable character of the speed of light then seals the lack of universal applicability of Einsteinian doctrines, since the latter are notoriously based on the philosophical assumption of "universal constancy of the speed of light".

1.2.6 Inapplicability of the Galilean and Poincaré symmetries for Interior Dynamical Systems

By remaining at the classical level, the inapplicability of Einsteinian doctrines within physical media is additionally established by the dramatic dynamical differences between the structure of a planetary system such as our Solar system, and the structure of a planet such as Jupiter.

The planetary system is a *Keplerian system,* that is, a system in which the heaviest component is at the center (actually in one of the two foci of elliptical orbits) and the other constituents orbit around it without collisions. By contrast, planets absolutely do not constitute a Keplerian system, because they do not have a Keplerian center with lighter constituents orbiting around it (see Figure 1.6).

Moreover, for a planetary system isolated from the rest of the universe, the total conservation laws for the energy, linear momentum and angular momentum are verified for each individual constituent. For instance, the conservation of the intrinsic and orbital angular momentum of Jupiter is crucial for its stability. On the contrary, for the interior dynamical problem of Jupiter, conservation laws hold only globally, while no conservation law can be formulated for individual constituents.

For instance, in Jupiter's structure we can see in a telescope the existence in Jupiter's atmosphere of *interior vortices with variable angular momentum,* yet always in such a way to verify total conservation laws. We merely have internal exchanges of energy, linear and angular momentum but always in such a way that they cancel out globally resulting in total conservation laws.

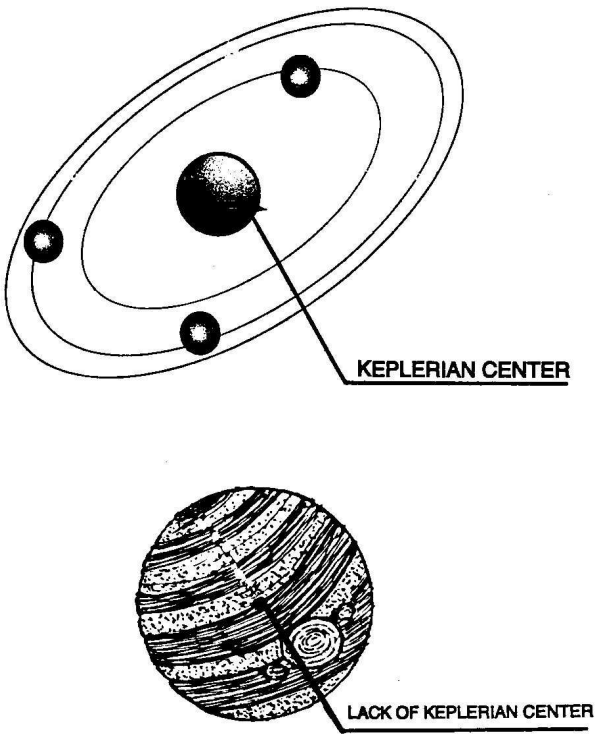

Figure 1.6. Another illustration of the second major scientific imbalance studied in this monograph, the dramatic structural differences between exterior and interior dynamical systems, here represented with the Solar system (top view) and the structure of Jupiter (bottom view). Planetary systems have a Keplerian structure with the exact validity of the Galilean and Poincaré symmetries. By contrast, interior systems such as planets (as well as hadrons, nuclei and stars) do not have a Keplerian structure because of the lack of the Keplerian center. Consequently, the Galilean and Poincaré symmetries cannot possibly be exact for interior systems in favor of covering symmetries and relativities studied in this monograph.

In the transition to particles the situation remains the same as that at the classical level. For instance, *nuclei do not have nuclei* and, therefore, nuclei are not Keplerian systems.

Similarly, the Solar system is a Keplerian system, but the Sun is not. At any rate, any reduction of the structure of the Sun to a Keplerian system directly implies the belief in the perpetual motion within a physical medium, because electrons and protons could move in the hyperdense medium in the core of a

star with conserved angular momenta, namely, a belief exiting all boundaries of credibility, let alone of science.

The above evidence establishes beyond credible doubt the following:

THEOREM 1.2.2 [10b]: Galileo's and Poincaré symmetries are inapplicable for classical and operator interior dynamical systems due to the lack of Keplerian structure, the presence of contact, zero-range, non-potential interactions, and other reasons.

Note the use of the word "inapplicable", rather than "violated" or "broken". This is due to the fact that, as clearly stated by the originators of the basic spacetime symmetries (rather than their followers of the 20-th century), Galileo's and Poincaré symmetries were not built for interior dynamical conditions.

Perhaps the biggest scientific imbalance of the 20-th century has been the abstraction of hadronic constituents to point-like particles as a necessary condition to use conventional spacetime symmetries, relativities and quantum mechanics for interior conditions. In fact, such an abstraction is at the very origin of the conjecture that the undetectable *quarks* are the physical constituents of hadrons (see Section 1.2.7 for details)..

Irrespective of whether we consider quarks or other more credible particles, all particles have a wavepacket of the order of $1\,\mathrm{F} = 10^{-13}\,\mathrm{cm}$, that is, a wavepacket of the order of the size of all hadrons. Therefore, *the hyperdense medium in the interior of hadrons is composed of particles with extended wavepackets in conditions of total mutual penetration.* Under these conditions, the belief that Galileo's and Poincaré symmetries are *exactly* valid in the interior of hadrons implies the exiting from all boundaries of credibility, let alone of science.

The inapplicability of the fundamental spacetime symmetries then implies the inapplicability of Galilean and special relativities as well as of quantum nonrelativistic and relativistic mechanics. We can therefore conclude with the following:

COROLLARY 1.2.2A [10b]: Classical Hamiltonian mechanics and related Galilean and special relativities are not exactly valid for the treatment of interior classical systems such as the structure of Jupiter, while nonrelativistic and relativistic quantum mechanics and related Galilean and special relativities are not exactly valid for interior particle systems, such as the structure of hadrons, nuclei and stars.

Another important scope of this monograph is to show that the problem of the exact spacetime symmetries applicable to interior dynamical systems is not a mere academic issue, because it carries a direct societal relevance. In fact, we shall show that broader spacetime symmetries specifically built for interior

systems predict the existence of new clean energies and fuels that are prohibited by the spacetime symmetries of the exterior systems.

As we shall see in Section 1.2.7, Chapter 6 and Chapter 12, the assumption that the undetectable quarks are physical constituents of hadrons *prohibits* possible new energy based on processes occurring in the interior of hadrons (rather than in the interior of their ensembles such as nuclei). On the contrary, the assumption of hadronic constituents that can be fully defined in our spacetime and can be produced free under suitable conditions, directly implies new clean energies.

1.2.7 The Scientific Imbalance Caused by Quark Conjectures

One of the most important objectives of this monograph, culminating in the presentation of Chapter 12, is to show that *the conjecture that quarks are physical particles existing in our spacetime constitutes one of the biggest threats to mankind because it prevents the orderly scientific process of resolving increasingly cataclysmic environmental problems.*

It should be clarified in this respect, as repeatedly stated by the author in his writings that *the unitary, Mendeleev-type, SU(3)-color classification of hadron into families can be reasonably considered as having a final character* (see e.g., Ref. [99] and papers quoted therein), in view of the historical capability of said classification to predict several new particles whose existence was subsequently verified experimentally. All doubts herein considered solely refer to the joint use of the same classification models as providing the structure of each individual element of a given hadronic family (for more details, see memoirs [100,101] and preprint [102] and Chapter 6).

Far from being alone, this author has repeatedly expressed the view that quarks cannot be physical constituents of hadrons existing in our spacetime for numerous independent reasons.

On historical grounds, the study of nuclei, atoms and molecules required *two different models,* one for the *classification* and a separate one for the *structure* of the individual elements of a given SU(3)-color family. Quark theories depart from this historical teaching because of their conception to represent with one single theory both the classification and the structure of hadrons.

As an example, the idea that the Mendeleev classification of atoms could jointly provide the structure of each individual atom of a given valence family is outside the boundary of science. The Mendeleev classification was essentially achieved via *classical theories,* while the understanding of the atomic structure required the construction of *a new theory,* quantum mechanics.

Independently from the above dichotomy classification vs structure, it is well known by specialists, but rarely admitted, that *quarks are purely mathematical quantities, being purely mathematical representations of a purely mathematical*

unitary symmetry defined in a purely mathematical complex-valued unitary space without any possibility, whether direct or implied, of being defined in our spacetime (representation technically prohibited by the O'Rafearthaigh theorem).

It should be stressed that, *as purely mathematical objects, quarks are necessary for the consistency of SU(3)-color theories.* Again, quarks are the fundamental representations of said Lie symmetry and, as such, their existence is beyond doubt. All problems emerge when said mathematical representation of a mathematical symmetry in the mathematical unitary space is assumed as characterizing physical particles existing in our spacetime.

It follows that the conjecture that quarks are physical particles is afflicted by a plethora of major problematic aspects today known to experts as *catastrophic inconsistencies of quark conjectures,* such as:

1) No particle possessing the peculiar features of quark conjectures (fraction charge, etc.), has ever been detected to date in any high energy physical laboratory around the world. Consequently, a main consistency requirement of quark conjectures is that quarks cannot be produced free and, consequently, they must be "permanently confined" in the interior of hadrons. However, it is well known to experts that, despite half a century of attempts, *no truly convincing "quark confinement" inside protons and neutrons has been achieved,* nor can it be expected on serious scientific grounds by assuming (as it is the case of quark conjectures) that quantum mechanics is identically valid inside and outside hadrons. This is due to a pillar of quantum mechanics, Heisenberg's uncertainty principle, according to which, given any manipulated theory appearing to show confinement for a given quark, a graduate student in physics can always prove the existence of a *finite probability for the same quark to be free outside the hadron, in catastrophic disagreement with physical reality.* Hence, the conjecture that quarks are physical particles is afflicted by catastrophic inconsistencies in its very conception [100].

2) It is equally well known by experts to qualify as such that *quarks cannot experience gravity* because quarks cannot be defined in our spacetime, while gravity can only be formulated in our spacetime and does not exist in mathematical complex-unitary spaces. Consequently, if protons and neutrons were indeed formed of quarks, we would have the catastrophic inconsistency that all quark believers should float in space due to the absence of gravity [101].

3) It is also well known by experts that *"quark masses" cannot possess any inertia* since they are purely mathematical parameters that cannot be defined in our spacetime. A condition for any mass to be physical, that is, to have inertia, is that it has to be the eigenvalue of a Casimir invariant of the Poincaré symmetry, while quarks cannot be defined via said symmetry because of their hypothetical fractional charges and other esoteric assumptions. This aspect alone implies numerous catastrophic inconsistencies, such as the impossibility of having the

energy equivalence $E = mc^2$ for any particle composed of quarks, against vast experimental evidence to the contrary.

4) Even assuming that, because of some twist of scientific manipulation, the above inconsistencies are resolved, it is known by experts that quark theories have failed to achieve a representation of all characteristics of protons and neutron, with catastrophic inconsistencies in the representation of spin, magnetic moment, means lives, charge radii and other basic features [102].

5) It is also known by experts that the application of quark conjectures to the structure of nuclei has multiplied the controversies in nuclear physics, while resolving none of them. As an example, the assumption that quarks are the constituents of the protons and the neutrons constituting nuclei has failed to achieve a representation of the main characteristics of the simplest possible nucleus, the deuteron. In fact, quark conjectures are afflicted by the catastrophic inconsistencies of being unable to represent the spin 1 of the deuteron (since they predict spin zero in the ground state while the deuteron has spin 1), they are unable to represent the anomalous magnetic moment of the deuteron, they are unable to represent the deuteron stability, they are unable to represent the charge radius of the deuteron, and when passing to larger nuclei, such as the zirconium, the catastrophic inconsistencies of quark conjectures can only be defined as being embarrassing [102].

In summary, while the final character of the SU(3)-color classification of hadrons into families has reached a value beyond scientific doubt, the conjecture that quarks are the actual physical constituents of hadrons existing in our spacetime is afflicted by so many and so problematic aspects to raise serious issues of scientific ethics and accountability, particularly in view of the ongoing large expenditures of public funds in the field.

On a personal note the author remembers some of the seminars delivered by the inventor of quarks, Murray Gell Mann, at Harvard University in the early 1980s, at the end of which there was the inevitable question whether Gell Mann believed or not that quarks are physical particles. Gell Mann's scientific caution (denoting a real scientific stature) is still impressed in the author's mind because he routinely responded with essentially the viewpoint outlined here, namely, Gell Mann stressed the mathematical necessity of quarks, while avoiding a firm posture on their physical reality. It is unfortunate that such a serious scientific position by Murray Gell-Manns was replaced by his followers with nonscientific positions mainly motivated by money, power and prestige.

Subsequently, quark conjectures have become a real "scientific business", as established by claim proffered by large high energy physics laboratories to have "discovered that and that quark". while in reality they had discovered a new particle predicted by SU(3)-color classification.

The decay of scientific ethics in the field is so serious, and the implications for mankind so potentially catastrophic (due to the suppression by quark conjectures as physical particles of possible new clean energies studied in Volume II) that, in the author's view, quark conjectures have been instrumental in the creation of the current scientific obscurantism of potentially historical proportions (see the *Open Denunciation of the Nobel Foundation for Heading an Organized Scientific Obscurantism* available in the web site http://www.scientificethics.org/Nobel-Foundation.htm).

1.2.8 The Scientific Imbalance Caused by Neutrino Conjectures

Another central objective of this monograph is to show that *neutrino conjectures constitute a political obstacle of potentially historical proportions against the orderly prediction and development of much needed new clean energies of "hadronic type", that is, new energies originating in the structure of individual hadrons, rather than in their collection as occurring in nuclei.*

Moreover, we shall show that *neutrino conjectures constitute an additional political obstacle also of potentially historical proportions against the study of one of the most important scientific problems in history, the interplay between matter and the universal substratum needed for the existence and propagation of electromagnetic waves and elementary particles.*

To prevent misrepresentations by vociferous (yet self-destructing) organized interests in the field, it should be stressed up-front that, as it is the case for quark conjectures, *neutrino conjectures of are necessary for the "current" treatment of weak interactions.* Therefore, a large scientific imbalance emerges only for the *political use and interpretation* of neutrino conjectures that has been dominant in the 20-th century and remains dominant to this day, namely, the use and interpretation of neutrino conjectures conceived and implemented in a capillary way for the continuation of the dominance of Einsteinian doctrines for all of physics.

Most distressing are contemporary claims of "neutrino detections" (denounced technically in Volume II) when the originator of neutrinos, Enrico Fermi, is on record by stressing that "neutrinos cannot be detected." Hence, the scientifically correct steatment would be the "detection of physical particles predicted by neutrino conjectures." As it was the case for Murray Gell-M ann, it is unfortunate that the scientific caution by Enrico Fermi was replaced by his followers with political postures essentially aiming at money, prestige and power.

In this subsections we shall show the political character of neutrino conjectures via a review the historical objections against the belief that the current plethora of neutrinos constitute actual physical particles in our spacetime. Alternative theoretical interpretations can be presented only in Chapter 6 with

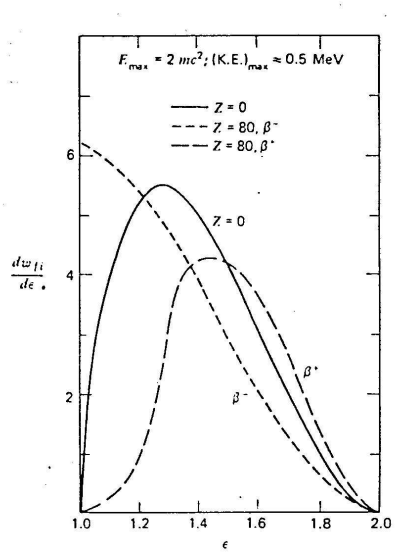

Figure 1.7. A view of the historical "bell shaped" curve representing the variation of the energy of the electron in nuclear beta decays (see, e.g., Ref. [13]). As soon as the apparent "missing energy" by the electron was detected in the early part of the 20-th century, it was claimed to be experimental evidence on the existence of a new particle with spin 1/2, charge zero and mass zero called by Fermi the "little neutron" or "neutrino".

industrial applications in Chapter 12 following the prior study and verification of *new mathematics* that is notoriously needed for true new vistas in science.

As it is well known, Rutherford [104] submitted in 1920 the conjecture that hydrogen atoms in the core of stars are compressed into a new particle he called the *neutron* according to the synthesis $(p^+, e^-) \to n$.

The existence of the neutron was subsequently confirmed experimentally in 1932 by Chadwick [105]. However, numerous objections were raised by the leading physicists of the time against Rutherford's conception of the neutron as a bound state of one proton p^+ and one electron e^-.

Pauli [106] first noted that Rutherford's synthesis violates the angular momentum conservation law because, according to quantum mechanics, a bound state of two particles with spin 1/2 (the proton and the electron) must yield a particle with integer spin and cannot yield a particle with spin 1/2 and charge zero such as the neutron. Consequently, Pauli conjectured the existence of a new neutral particle with spin 1/2 that is emitted in synthesis $(p^+, e^-) \to n.$ or in similar radioactive processes so as to verify the angular momentum conservation law.

Fermi [107] adopted Pauli's conjecture, coined the name *neutrino* (meaning in Italian a "little neutron") and presented the first comprehensive theory of the underlying interactions (called "weak"), according to which the neutron synthesis

should be written $(p^+, e^-) \to n + \nu$, where ν is the neutrino, in which case the inverse reaction (the spontaneous decay of the neutron) reads $n \to p^+ + e^- + \bar{\nu}$, where $\bar{\nu}$ is the *antineutrino.*

Despite the scientific authority of historical figures such as Pauli and Fermi, the conjecture on the existence of the neutrino and antineutrino as physical particles was never universally accepted by the entire scientific community because of: the impossibility for the neutrinos to be directly detected in laboratory; the neutrinos inability to interact with matter in any appreciable way; and the existence of alternative theories that do not need the neutrino conjecture (see Refs. [108-110] and literature quoted therein, plus the alternative theory presented in Chapter 6).

By the middle of the 20-th century there was no clear experimental evidence acceptable by the scientific community at large confirming the neutrino conjecture beyond doubt, except for experimental claims in 1959 that are known today to be basically flawed on various grounds, as we shall see below and in Chapter 6.

In the last part of the 20-th century, there was the advent of the so-called *unitary SU(3) theories* and related quark conjectures studied in the preceding subsection. In this way, neutrino conjectures became deeply linked to and their prediction intrinsically based on quark conjectures.

This event provided the first fatal blow to the credibility of the neutrino conjectures because serious physics cannot be done via the use of conjectures based on other conjectures.

In fact, the marriage of neutrino and quark conjectures within the standard model has requested the *multiplication of neutrinos,* from the neutrino and antineutrino conjectures of the early studies, to six different hypothetical particles, the so called *electron, muon and tau neutrinos and their antiparticles.* In the absence of these particles the standard model would maintain its meaning as classification of hadrons, but would lose in an irreconcilable way the joint capability of providing also the structure of each particle in a hadronic multiplet.

In turn, the multiplication of the neutrino conjectures has requested the *additional conjecture that the electron, muon and tau neutrinos have masses* and, since the latter conjecture resulted in being insufficient, there was the need for the *additional conjecture that neutrinos have different masses,* as necessary to salvage the structural features of the standard model. Still in turn, the lack of resolution of the preceding conjectures has requested the *yet additional conjecture that neutrinos oscillate,* namely, that "they change flavor" (transform among themselves back and forth).

In addition to this rather incredible litany of sequential conjectures, each conjecture being voiced in support of a preceding unverified conjecture, all conjectures being crucially dependent on the existence of quarks as physical particles despite their proved lack of gravity and physical masses, by far the biggest con-

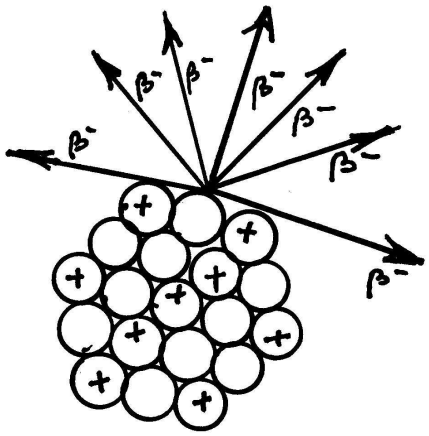

Figure 1.8. A schematic illustration of the fact that the electron in beta decays can be emitted in different directions. When the energy in the beta decay is computed with the inclusion of the Coulomb interactions between the expelled (negatively charged) electron and the (positively charged) nucleus at different expulsion directions, the nucleus acquires the "missing energy," without any energy left for the hypothetical neutrino. As we shall see in Chapter 6, rather than being a disaster, the occurrence is at the foundation of a possible basically new scientific horizon with implications sufficient to require studies over the entire third millennium.

troversies have occurred in regard to experimental claims of neutrino detection voiced by large collaborations.

To begin, both neutrinos and quarks cannot be directly detected as physical particles in our spacetime. Consequently, all claims on their existence are indirect, that is, based on the detection of actual physical particles predicted by the indicated theories. This occurrence is, per se, controversial. For instance, controversies are still raging following announcements by various laboratories to have "discovered" one or another quark, while in reality the laboratories discovered physical particles predicted by a Mendeleev-type classification of particles, the same classification being admitted by theories that require no quarks at all as physical particles, as we shall indicate in Chapter 6.

In the 1980s, a large laboratory was built deep into the Gran Sasso mountain in Italy to detect neutrinos coming from the opposite side of Earth (since the mountain was used as a shield against cosmic rays). Following the investment of large public funds and five years of tests, the Gran Sasso Laboratory released no evidence of clear detection of neutrino originated events.

Rather than passing to a scientific caution in the use of public funds, the failure of the Gran Sasso experiments to produce any neutrino evidence stimulated

massive efforts by large collaborations involving hundred of experimentalists from various countries for new tests requiring public funds in the range of hundred of millions of dollars.

The increase in experimental research was evidently due to the scientific stakes, because, as well known by experts but studiously omitted, *the lack of verification of the neutrino conjectures would imply the identification of clear limits of validity of Einsteinian doctrines and quantum mechanics.*

These more recent experiments resulted in claims that, on strict scientific grounds, should be considered "experimental beliefs" by any serious scholars for numerous reasons, such as:

1) The predictions are based on a litany of sequential conjectures none of which is experimentally established on clear ground;

2) The theory contain a plethora of unrestricted parameters that can essentially fit any pre-set data (see next subsection);

3) The "experimental results" are based on extremely few events out of hundreds of millions of events over years of tests, thus being basically insufficient in number for any serious scientific claim;

4) In various cases the "neutrino detectors" include radioactive isotopes that can themselves account for the selected events;

5) The interpretation of the experimental data via neutrino and quark conjectures is not unique, since there exist nowadays other theories representing exactly the same events without neutrino and quark conjectures (including a basically new scattering theory of nonlocal type indicated in Chapter 3 and, more extensively, in monograph [10b]).

To understand the scientific scene, the serious scholar (that is, the scholar not politically aligned to the preferred "pet theories" indicated in the Preface) should note that *neutrino and quark conjectures have requested to date the expenditure of over one billion dollars of public funds in theoretical and experimental research with the result of increasing the controversies rather than resolving any of them.*

Therefore, it is now time for a moment of reflection: scientific ethics and accountability require that serious scholars in the field exercise caution prior to venturing claims of actual physical existence of so controversial and directly unverifiable conjectures.

Such a moment of reflection requires the re-inspection of the neutrino conjecture at its foundation. In fact, it is important to disprove the neutrino conjecture as originally conceived, and then disprove the flavored extension of the conjecture as requested by quark conjectures.

As reported in nuclear physics textbooks (see, e.g., Ref. [13]), the energy experimentally measured as being carried by the electron in beta decays is a bell-shaped curve with a maximum value of 0.782 MeV, that is the difference in value between the mass of the neutron and that of the resulting proton in the

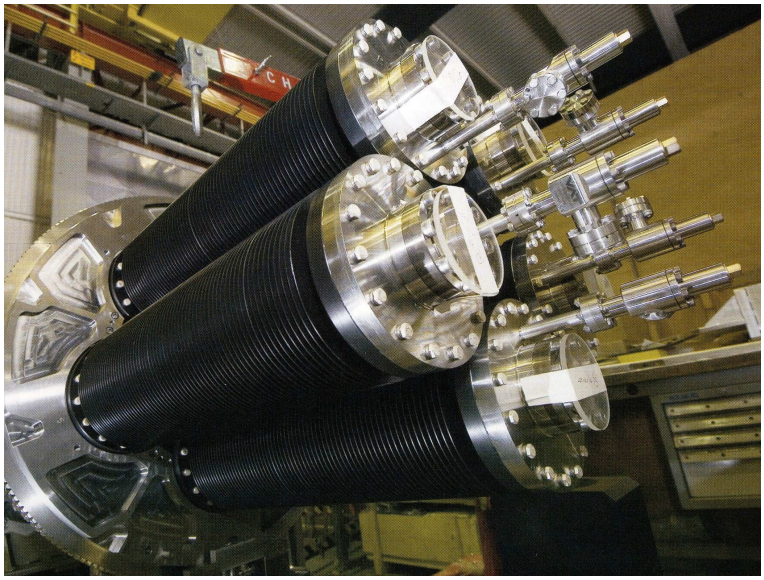

Figure 1.9. A picture of one of the "neutrino detectors" currently under construction at CERN for use to attempt "experimental measurements" of neutrinos (which one?) at the Gran Sasso Laboratory in Italy. The picture was sent to the author by a kind colleague at CERN and it is presented here to have an idea of the large funds now feverishly obtained from various governments by organized interests on Einsteinian doctrines in what can only be called their final frantic attempts at salvage the large litany of unverified and unverifiable quark, neutrino and other conjectures needed to preserve the dominance of Einstein doctrines in physics. For an understanding of the potential immense damage to mankind, we suggest the reader to study this monograph up to and including Chapter 12 on the necessity of abandoning these clearly sterile trends to achieve new clean energies.

neutron decay. As soon as the "missing energy" was identified, it was instantly used by organized interests in Einsteinian doctrines as evidence of the neutrino hypothesis for the unspoken yet transparent reasons that, in the absence of the neutrino conjectures, Einsteinian doctrines would be grossly inapplicable for the neutron decay.

As it is equally well known, the scientific community immediately accepted the neutrino interpretation of the "missing energy" mostly for academic gain, as it must be the case whenever conjectures are adopted without the traditional scientific process of critical examinations.

It is easy to see that the neutrino interpretation of the "missing energy" is fundamentally flawed. In fact, the electron in beta decays is negatively charged, while the nucleus is positively charged. Consequently, *the electron in beta decays experiences a Coulomb attraction from the original nucleus.*

Moreover, such an attraction is clearly dependent on the angle of emission of the electron by a decaying peripheral neutron. The maximal value of the energy occurs for radial emissions of the electron, the minimal value occurs for tangential emissions, and the intermediate value occur for intermediate directions of emissions, resulting in the experimentally detected bell-shaped curve of Figure 1.7.

When the calculations are done without political alignments on pre-existing doctrines, it is easy to see that the "missing energy" in beta decays is entirely absorbed by the nucleus via its Coulomb interaction with the emitted electron. Consequently, *in beta decays there is no energy at all available for the neutrino conjecture,* by reaching in this way a disproof of the conjecture itself at its historical origination.

Supporters of the neutrino conjecture are expected to present as counter-arguments various counter-arguments on the lack of experimental evidence for the nucleus to acquire said "missing energy." Before doing so, said supporters are suggested to exercise scientific caution and study the new structure models of the neutron without the neutrino conjecture (Chapter 6), as well as the resulting new structure models of nuclei (Chapter 7) and the resulting new clean energies (Chapter 12). Only then, depending on the strength of their political alignment, they may eventually realize that, in abusing academic authority to perpetrate unproved neutrino conjectures they may eventually be part of real crimes against mankind.

The predictable conclusion of this study is that theoretical and experimental research on neutrino and quark conjectures should indeed continue. However, theoretical and experimental research on theories without neutrino and quark conjectures and their new clean energies should be equally supported to prevent a clear suppression of scientific democracy on fundamental needs of mankind, evident problems of scientific accountability, and a potentially severe judgment by posterity.

For technical details on the damage caused to mankind by the current lack of serious scientific caution on neutrino conjectures, interested readers should study Volume Ii and inspect the *Open Denunciation of the Nobel Foundation for Heading an Organized Scientific Obscurantism* available in the web site http://www.-scientificethics.org/Nobel-Foundation.htm.

1.2.9 The Scientific Imbalance in Experimental Particle Physics

Another central objective of this monograph is to illustrate the *existence at the dawn of the third millennium of a scientific obscurantism of unprecedented proportions, caused by the manipulation of experimental data via the use of experimentally unverified and actually unverifiable quark conjectures, neutrino conjectures and other conjectures complemented by a variety of ad hoc parameters*

for the unspoken, but transparent and pre-meditated intent of maintaining the dominance of Einsteinian doctrines in physics.

At any rate, *experimental data are elaborated via the conventional scattering theory that, even though impeccable for electromagnetic interactions among point-like particles, is fundamentally insufficient for a serious representation of the scattering among extended, nonspherical and hyperdense hadrons* (Figure 1.2 and Chapter 3).

As a matter of fact, serious scholars and, above all, future historians, should focus their main attention on the fact that the climax of unscientific conduct by organized interests on Einsteinian doctrines occurs primarily in the *manipulation of experiments,* beginning with the control of the conditions of funding, then following with the control of the conduction of the experiments and, finally, with the control of the theoretical elaboration of the data to make sure that the orchestrated compliance with Einsteinian doctrines occurs at all levels.

Among an unreassuringly excessive number of cases existing in the literature, some of which are reviewed in Chapter 6, a representative case is that of the *Bose-Einstein correlation* in which protons and antiprotons collide at high energy by annihilating each other and forming the so-called "fireball", that, in turn, emits a large number of unstable particles whose final product is a number of correlated mesons (see, e.g., review [7] and Figure 1.7).

The simplest possible case is that of the *two-points correlation function*

$$C_2 = \frac{P(p_1, p_2)}{P(p_1) \times P(p_2)}, \tag{1.2.14}$$

where p_1 and p_2 are the linear momenta of the two mesons and the P's represent their probabilities.

By working out the calculations via unadulterated axioms of relativistic quantum mechanics one obtains expressions of the type

$$C_2 = 1 + A \times e^{-Q_{12}} - B \times e^{-Q_{12}}, \tag{1.2.15}$$

where A and B are normalization parameters and Q_{12} is the momentum transfer. This expression is dramatically far from representing experimental data, as shown in Chapter 5.

To resolve the problem, supporters of the universal validity of quantum mechanics and special relativity then introduce *four arbitrary parameters of unknown physical origin and motivation* called "chaoticity parameters" c_μ, $\mu = 1, 2, 3, 4$, and expand expression (1.2.15) into the form

$$C_2 = 1 + A \times e^{-Q_{12}/c_1} + B \times e^{-Q_{12}/c_2} + C \times e^{-Q_{12}/c_3} - D \times e^{-Q_{12}/c_4}, \tag{1.2.16}$$

which expression does indeed fit the experimental data, as we shall see. However, the claim that quantum mechanics and special relativity are exactly valid is a scientific deception particularly when proffered by experts.

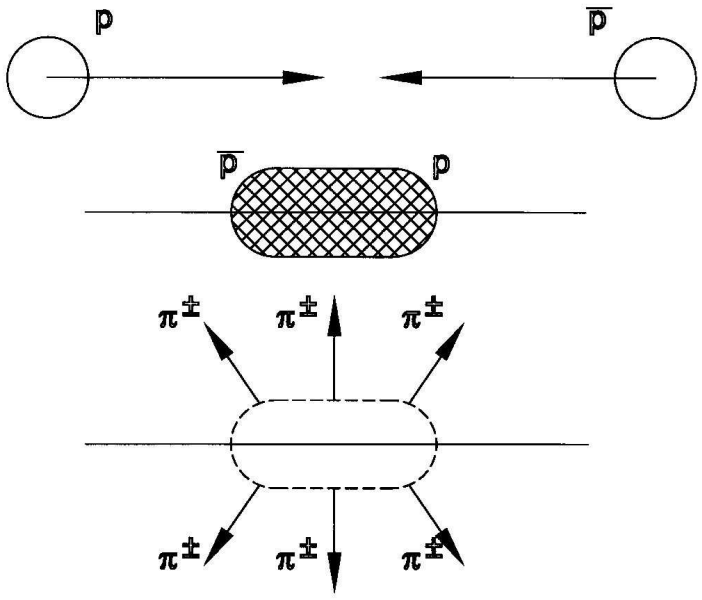

Figure 1.10. A schematic view of the Bose-Einstein correlation originating in proton-antiproton annihilations, for which the predictions of relativistic quantum mechanics are dramatically far from experimental data from unadulterated first principles. In order to salvage the theory and its underlying Einsteinian doctrines, organized interests introduce "four" ad hoc parameters deprived of any physical meaning or origin, and then claim the exact validity of said doctrines. The scientific truth is that these four arbitrary parameters are in reality a direct measurement of the deviation from the basic axioms of relativistic quantum mechanics and special relativity in particle physics.

As we shall see in technical details in Chapter 5, the quantum axiom of expectation values (needed to compute the probabilities) solely permit expression (1.2.15), since it deals with Hermitian, thus diagonalized operators of the type

$$< \psi_{\times}\psi_2| \times P \times |\psi_1 \times \psi_2 >= P_{11} + P_{22}, \tag{1.2.17}$$

while *the representation of a correlation between mesons 1 and 2 necessarily requires a structural generalization of the axiom of expectation value in such a form to admit off-diagonal elements for Hermitian operators,* for instance of the type

$$< \psi_{\times}\psi_2| \times T \times P \times T \times |\psi_1 \times \psi_2 >= P_{11} + P_{12} + P_{21} + P_{22}, \tag{1.2.18}$$

where T is a 2×2-dimensional nonsingular matrix with off-diagonal elements (and P remains diagonal).

The scientific deception occurs because quantum mechanics and special relativity are claimed to be exactly valid for the Bose-Einstein correlation when

experts, to qualify as such, know that the representation requires a structural modification of the basic axiom of expectation values as well as for numerous additional reasons, such as:

1) The Bose-Einstein correlation is necessarily due to contact, nonpotential, nonlocal-integral effects originating in the deep overlapping of the hyperdense charge distributions of protons and antiprotons inside the fireball;

2) The mathematical foundations of quantum mechanics (such as its topology), let alone its physical laws, are inapplicable for a meaningful representation of said nonlocal and nonpotential interactions as outlined in preceding sections; and

3) Special relativity is also inapplicable, e.g., because of the inapplicability of the basic Lorentz and Poincaré symmetries due to lack of a Keplerian structure, the *approximate* validity of said theories remaining beyond scientific doubt.

Admittedly, there exist a number of semiphenomenological models in the literature capable of a good agreement with the experimental data. Scientific deception occurs when these models are used to claim the exact validity of quantum mechanics and special relativity since the representation of experimental data requires necessary structural departures from basic quantum axioms.

Of course, the selection of the appropriate generalization of quantum mechanics and special relativity for an exact representation of the Bose-Einstein correlation is open to scientific debate. Scientific deception occurs when the need for such a generalization is denied for personal gains.

As we shall see, relativistic hadronic mechanics provides an *exact and invariant* representation of the experimental data of the Bose-Einstein correlation at high and low energies via unadulterated basic axioms, by providing in particular a direct representation of the shape of the $p - \bar{p}$ fireball and its density, while recovering the basic invariant under a broader realization of the Poincaré symmetry.

An in depth investigation of all applications of quantum mechanics and special relativity at large reveals that they have provided an *exact* and *invariant* representation *from unadulterated basic axioms* of *all* experimental data of the hydrogen atom, as well as of physical conditions in which the mutual distances of particles is much bigger than the size of the charge distribution (for hadrons) or of the wavepackets of particles (for the case of the electron).

1.2.10 The Scientific Imbalance in Nuclear Physics

There is no doubt that quantum mechanics and special relativity permitted historical advances in also nuclear physics during the 20-th century, as illustrated, for instance, by nuclear power plants. However, any claim that quantum mechanics and special relativity are *exactly* valid in nuclear physics is a scientific deception, particularly when proffered by experts, because of the well known inability of these theories to achieve an exact and invariant representation of numerous nu-

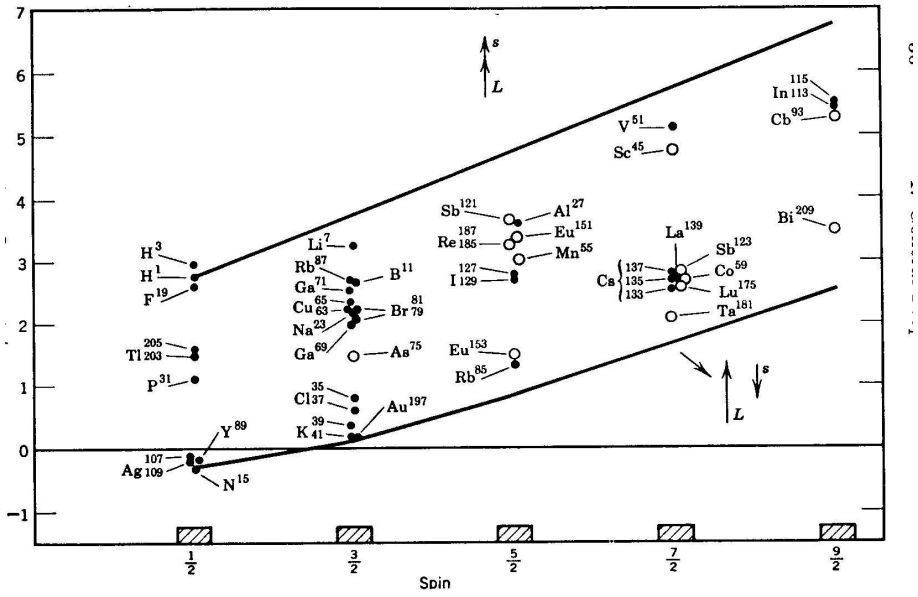

Figure 1.11. The first historical experimental evidence on the lack of exact validity of quantum mechanics in nuclear physics was given by data on nuclear magnetic moments that do not follow quantum mechanical predictions, and are instead comprised between certain minimal and maximal values, called the Schmidt Limits [13], without any possible quantum treatment. The additional suppression of the impossibility for the Galilean and Poincaré symmetries to be exact in nuclear physics due to the lack of a Keplerian center (see next figure), have essentially rendered nuclear physics a religion without a serious scientific process.

clear data despite one century of attempts and the expenditure of large public funds.

To resolve the insufficiencies, the use of arbitrary parameters of unknown physical origin and motivation was first attempted, semiphenomenological fits were reached and quantum mechanics and special relativity were again claimed to be exact in nuclear physics, while in the scientific reality the used parameters are a direct representation of *deviations* from the basic axioms of the theories as shown in detail in Chapter 5.

Subsequently, when the use of arbitrary parameters failed to achieve credible representations of nuclear data (such as nuclear magnetic moments as indicated below), organized academic interests claimed that "the deviations are resolved by deeper theories such as quark theories". At that point nuclear physics left the qualification of a true science to become a scientific religion.

Besides a plethora of intrinsic problematic aspects or sheer inconsistencies (such as the impossibility for quarks to have gravity mentioned earlier), quark

theories failed to achieve any credible representation even of the spin of individual nucleons, let alone achieve exact representations of experimental data for their bound states.

Admittedly, the deviations here considered are at times small. Nevertheless, as we shall see in Chapter 6, small deviations directly imply new clean energies that cannot be even conceived, let alone treated, via quantum mechanics. Therefore, we have a societal duty to conduct serious investigations on broader mechanics specifically conceived for nuclear physics.

The first evidence on the lack of exact character of quantum mechanics in nuclear physics dates back to the birth of nuclear physics in the 1930s where it emerged that experimental values of nuclear magnetic moments could not be explained with quantum mechanics, because, starting with small deviations for small nuclei, the deviations then increased with mass, to reach deviations for large nuclei, such as the Zirconium so big to escape any use of unknown parameters "to fix things" (see Figure 1.8).

Subsequently, it became clear that quantum mechanics and special relativity could not explain the simplest possible nucleus, the deuteron, despite vast efforts. In fact, quantum mechanics missed about 1% of the deuteron magnetic moment despite all possible relativistic corrections, as well as the questionable assumptions that the ground state of the deuteron is a mixture of various states in a way manifestly against experimental evidence.

Next, quantum mechanics and special relativity were unable to represent the spin of the deuteron, an occurrence well known to experts in the field but carefully undisclosed. The axioms of quantum mechanics require that the ground state of two particles with spin 1/2 (such as the proton and the neutron) must have spin zero (*anti-parallel or singlet coupling*), while the case with spin 1 (*parallel spin or triplet coupling*) is unstable, as a first year graduate student in physics can prove.

By contrast, the deuteron has spin 1, thus remaining fundamentally unexplained by quantum mechanics and special relativity to this day.[5] Additionally, quantum mechanics has been unable to represent the stability of the neutron, its charge radius, and numerous other data.

Perhaps the most distressing, yet generally undisclosed, insufficiency of quantum mechanics and special relativity in nuclear physics has been the failure to understand and represent nuclear forces. Recall that a necessary condition for the applicability of quantum mechanics is that *all* interactions must be derivable from a potential.

The original concept that nuclear forces were of central type soon resulted in being disproved by nuclear reality, thus requiring the addition of non-central, yet still potential forces. The insufficiency of this addition requested the introduction

[5]As we shall see in Chapter 6, the correct interpretation of the spin 1 of the deuteron has implications so deep to require a revision of the very notion of neutron.

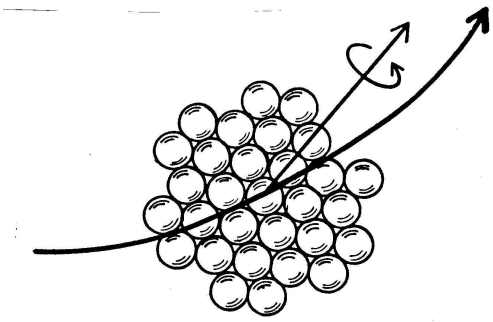

Figure 1.12. A visual evidence of the impossibility for quantum mechanics to be exactly valid in nuclear physics: the fact that "nuclei do not have nuclei." Consequently, the Galilean and Poincaré symmetries, as well as nonrelativistic and relativistic quantum mechanics, cannot possibly be exact for the nuclear structure since said symmetries demand the heaviest constituent at the center. The above occurrence establishes the validity of covering symmetries for interior systems without Keplerian centers, which symmetries are at the foundation of the covering hadronic mechanics.

of exchange, van der Waals, and numerous other potential forces. As of today, after about one century of adding new potentials to the Hamiltonian, we have reached the unreassuring representation of nuclear forces via some twenty or more different potentials in the Hamiltonian [13]

$$H = \Sigma_{k=1,2,...,n} \frac{p_k^2}{2 \times m_k} + V_1 + V_2 + V_3 + V_4 + V_5 + V_6 +$$
$$+ V_7 + V_8 + V_9 + V_{10} + V_{11} + V_{12} + V_{13} + V_{14} +$$
$$+ V_{15} + V_{16} + V_{17} + V_{18} + V_{19} + V_{20} + ... \qquad (1.2.19)$$

and we still miss a credible understanding and representation of the nuclear force!

It is evident that this process cannot be kept indefinitely without risking a major condemnation by posterity. The time has long come to stop adding potentials to nuclear Hamiltonians and seek fundamentally new approaches and vistas.

In the final analysis, an inspection of nuclear volumes establishes that nuclei are generally composed of nucleons in conditions of partial mutual penetration, as illustrated in Figure 1.9. By recalling that nucleons have the largest density measured in laboratory until now, the belief that all nuclear forces are of action-at-a-distance, potential type, as *necessary* to preserve the validity of quantum mechanics and special relativity, is pure academic politics deprived of scientific value.

As we shall see in Chapter 7, a central objective of hadronic mechanics is that of truncating the addition of potentials and re-examining instead the nuclear force from its analytic foundations, by first separating potential nonpotential forces, and then examining in details each of them.

In summary, the lack of exact character of quantum mechanics and special relativity in nuclear physics is beyond scientific doubt. The open scientific issue is the selection of the appropriate generalization, but not its need.

As we shall see in Chapter 6, the covering hadronic mechanics and isospecial relativity resolve the fundamental open problems of nuclear physics by permitting the industrial development of new clean energies based on light natural and stable elements without the emission of dangerous radiations and without the release of radioactive waste.

1.2.11 The Scientific Imbalance in Superconductivity

The condition of superconductivity in the 20-th century can be compared to that of atomic physics prior to the representation of the structure of the atom.

Recall that individual electrons cannot achieve a superconducting state because their magnetic fields interact with electromagnetic fields of atoms by creating in this way what we call *electric resistance.* Superconductivity is instead reached by deeply correlated-bonded pairs of electrons in singlet couplings, called *Cooper pairs.* In fact, these pairs have an essentially null total magnetic field (due to the opposite orientations of the two fields), resulting in a substantial decrease of electric resistance.

There is no doubt that quantum mechanics and special relativity have permitted the achievement of a good description of an "ensemble" of Cooper pairs, although each Cooper pair is necessarily abstracted as a point, the latter condition being necessary from the very structure of the theories.

However, it is equally well known that quantum mechanics and special relativity have been unable to reach a final understanding and representation of the structure of *one* Cooper pair, trivially, because electrons repel each other according to the fundamental Coulomb law.

The failure of basic axioms of quantum mechanics and special relativity to represent the *attractive* force between the two *identical* electrons of the Cooper pairs motivated the hypothesis that the attraction is caused by the exchange of a new particle called *phonon.* However, phonons certainly exist in sounds, but they have found no verification at all in particle physics, thus remaining purely conjectural to this day.

In reality, as we shall see in Chapter 7, the interactions underlying the Cooper pairs are of purely contact, nonlocal and integral character due to the mutual penetration of the wavepackets of the electrons, as depicted in Figure 1.10. As

such, they are very similar to the interactions responsible for Pauli's exclusion principle in atomic structures.

Under these conditions, the granting of a potential energy, as necessary to have phonon exchanges, is against physical evidence, as confirmed by the fact that any representation of Pauli's exclusion principle via potential interactions cause sizable deviations from spectral lines.

Therefore, the belief that quantum mechanics and special relativity provide a complete description of superconductivity is pure academic politics deprived of scientific content.

Superconductivity is yet another field in which the exact validity of quantum mechanics and special relativity has been stretched in the 20-th century well beyond its limit for known political reasons. At any rate, superconductivity has exhausted all its predictive capacities, while all advances are attempted via empirical trials and errors without a guiding theory.

As it was the case for particle and nuclear physics, the lack of exact character of quantum mechanics and special relativity in superconductivity is beyond doubt. Equally beyond doubt is the need for a deeper theory.

As we shall see in Chapter 7, the covering hadronic mechanics and isospecial relativity provide a quantitative representation of the structure of the Cooper pair in excellent agreement with experimental data, and with basically novel predictive capabilities, such as the industrial development of a new electric current, that is characterized by correlated electron pairs in single coupling, rather than electrons.

1.2.12 The Scientific Imbalance in Chemistry

There is no doubt that quantum chemistry permitted the achievement of historical discoveries in the 20-th century. However, there is equally no doubt that the widespread assumption of the exact validity of quantum chemistry caused a large scientific imbalance with vast implications, particularly for the alarming environmental problems.

After about one century of attempts, quantum chemistry still misses a historical 2% of molecular binding energies when derived from axiomatic principles without *ad hoc* adulterations (see below). Also, the deviations for electric and magnetic moments are embarrassing not only for their numerical values, but also because they are wrong even in their sign [14], not to mention numerous other insufficiencies outlined below.

It is easy to see that the reason preventing quantum chemistry from being exactly valid for molecular structures is given by contact, nonlocal-integral and nonpotential interactions due to deep wave-overlappings in valence bonds that, as such, are beyond any realistic treatment by local-differential-potential axioms, such as those of quantum chemistry (Figure 1.10).

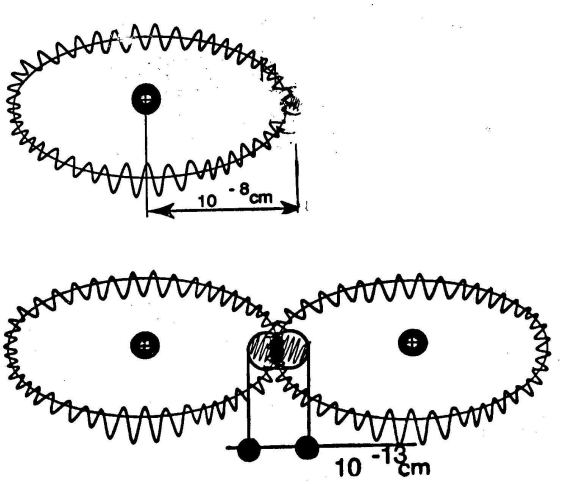

Figure 1.13. A schematic view of the fundamental conditions studied in this monograph, the deep overlapping of the extended wavepackets of electrons in valence bonds and Cooper pairs according to a singlet coupling as required by Pauli's principle. Recall that, for quantum mechanics and special relativity, electrons are points and, therefore, the conditions of this figure have no meaning at all. However, said point character can only be referred to the charge structure of the electron, since "point-like wavepackets" do not exist in nature. For the covering hadronic mechanics, superconductivity and chemistry, the point-like charge structure of the electrons remains, with the additional presence of the contact nonpotential interactions due to the overlapping of the extended wavepackets represented via a nonunitary structure. As shown in Chapters 8, 9 and 11, the treatment of the latter interactions via hadronic mechanics and chemistry has permitted the achievement, for the first time in scientific history, of an "exact and invariant" representations of molecular data from first axioms without ad hoc adulterations.

Recall that quantum mechanics achieved an exact and invariant representation of all experimental data of *one* hydrogen atom. Nevertheless, quantum mechanics and chemistry miss 2% of the binding energy of *two* hydrogen atoms coupled into the hydrogen molecule (Figure 1.11).

The only possible explanation is that in the hydrogen atom all interactions are of action-at-a-distance potential type due to the large mutual distances of the constituents with respect to the size of their wavepackets. By contrast, in the hydrogen molecule we have the mutual penetration of the wavepackets of valence electrons with the indicated contact, nonlocal-integral and nonpotential interactions at short mutual distances that are absent in the structure of the hydrogen atom.

Alternatively and equivalently, the nuclei of the two hydrogen atoms of the H_2 molecule cannot possibly be responsible for said 2% deviation. Therefore, the deviation from basic axioms can only originate in the valence bond.

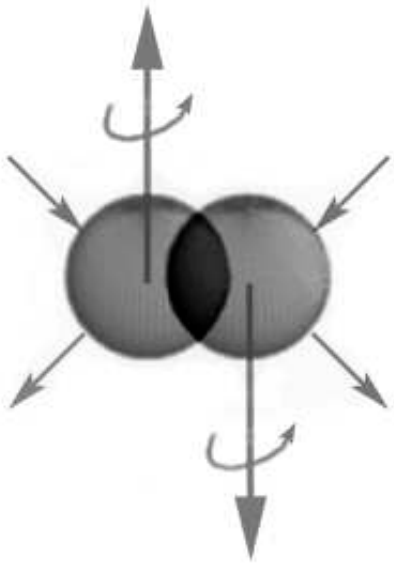

Figure 1.14. A first clear evidence of the lack of exact validity of quantum chemistry. The top view depicts one hydrogen atom for which quantum mechanics resulted in being exactly valid. The bottom view depicts two hydrogen atoms coupled into the H_2 molecule in which case quantum chemistry has historically missed a 2% of the binding energy when applied without adulteration of basic axioms "to fix things" (such as via the used of the screening of the Coulomb law and then claim that quantum chemistry is exact). Since nuclei do not participate in the molecular bond, the origin of the insufficiency of quantum mechanics and chemistry rests in the valence bond.

By no means the above insufficiencies are the only ones. Quantum chemistry is afflicted by a true litany of limitations, insufficiencies or sheer inconsistencies that constitute the best kept secret of the chemistry of the 20-th century because known to experts (since they have been published in refereed journals), but they remain generally ignored evidently for personal gains.

We outline below the insufficiencies of quantum chemistry for the simplest possible class of systems, those that are *isolated from the rest of the universe*, thus verifying conventional *conservation laws* of the total energy, total linear momentum, etc., and are *reversible* (namely, their time reversal image is as physical as the original system).

The most representative systems of the above class are given by *molecules,* here generically defined as aggregates of atoms under a valence bond. Despite undeniable achievements, quantum chemical models of molecular structures have the following fundamental insufficiencies studied in detail in monograph [11]:

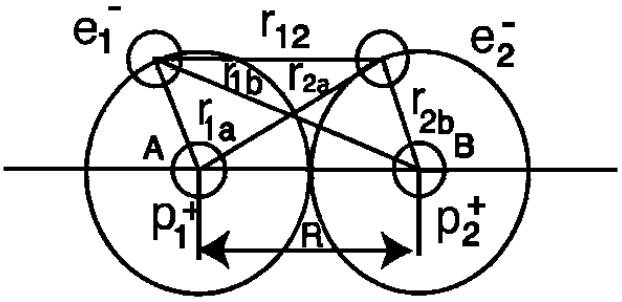

Figure 1.15. A schematic view of the fact that the total Coulomb force among the atoms of a molecular structure is identically null. As a consequence, conventional Coulomb interactions cannot provide credible grounds for molecular bonds. At the same time, existing chemical conjectures, such as the exchange and van der Waals forces, are weak, as known from nuclear physics. These facts establish that the chemistry of the 20-th century is like nuclear physics before the discovery of the strong interactions, because chemistry missed the identification of an attractive force sufficiently strong to represent molecular structure. As we shall see in Chapter 8, hadronic chemistry will indeed provide, for the first time in scientific history, the numerical identification of the missed "attractive strong attractive valence force" as being precisely of contact, nonlocal and nonpotential type. The achievement of an exact representation of molecular data is then consequential.

1: Quantum chemistry lacks a sufficiently strong molecular binding force. After 150 years of research, chemistry has failed to identify to this day the *attractive force* needed for a credible representation of valence bonds. In the absence of such an attractive force, names such as "valence" are pure nomenclatures without quantitative meaning.

To begin, the average of all Coulomb forces among the atoms constituting a molecule is identically null. As an example, the currently used Schrödinger equation for the H_2 molecule is given by the familiar expression [15],

$$(-\frac{\hbar^2}{2\mu_1}\nabla_1^2-\frac{\hbar^2}{2\mu_2}\nabla_2^2-\frac{e^2}{r_{1a}}-\frac{e^2}{r_{2a}}-\frac{e^2}{r_{1b}}-\frac{e^2}{r_{2b}}+\frac{e^2}{R}+\frac{e^2}{r_{12}})|\psi>=E|\psi>, \quad (1.2.20)$$

which equation contains the Coulomb attraction of each electron by its own nucleus, the Coulomb attraction of each electron from the nucleus of the other atom, the Coulomb repulsion of the two electrons, and the Coulomb repulsion of the two protons.

It is easy to see that, in semiclassical average, the two attractive forces of each electron from the nucleus of the other atom are compensated by the average of the two repulsive forces between the electrons themselves and those between the

protons, under which Eq. (1.2.20) reduces to two independent *neutral* hydrogen atoms *without* attractive interaction, as depicted in Fig. 1.2.12,

$$\left[\left(-\frac{\hbar^2}{2\mu_1}\nabla_1^2-\frac{e^2}{r_{1a}}\right)+\left(-\frac{\hbar^2}{2\mu_2}\nabla_2^2-\frac{e^2}{r_{2a}}\right)\right]|\psi\rangle=E|\psi\rangle. \tag{1.2.21}$$

In view of the above occurrence, quantum chemistry tries to represent molecular bonds via *exchange, van der Waals and other forces* [15]. However, the latter forces were historically introduced for *nuclear structures* in which they are known to be *very weak*, thus being insufficient to provide a true representation of molecular bonds.

It is now part of history that, due precisely to the insufficiencies of exchange, van der Waals and other forces, nuclear physicists were compelled to introduce the *strong nuclear force*. As an illustration, calculations show that, under the currently assumed molecular bonds, the molecules of a three leaf should be decomposed into individual atomic constituents by a weak wind of the order of 10 miles per hour.

To put it in a nutshell, after about one century of research, *quantum chemistry still misses in molecular structures the equivalent of the strong force in nuclear structures.*

As we shall see in Chapter 8, one of the objectives of hadronic chemistry is precisely to introduce the missing force, today known as the *strong valence force,* that is, firstly, ATTRACTIVE, secondly, sufficiently STRONG, and, thirdly, INVARIANT. The exact and invariant representation of molecular data will then be a mere consequence.

2: Quantum chemistry admits an arbitrary number of atoms in the hydrogen, water and other molecules. This inconsistency is proved beyond scientific doubt by the fact that the exchange, van der Waals, and other forces used in current molecular models were conceived in nuclear physics for the primary purpose of admitting a large number of constituents.

When the same forces are used for molecular structures, they also admit an arbitrary number of constituents. As specific examples, when applied to the structure of the hydrogen or water molecule, any graduate student in chemistry can prove that, under exchange, van der Waals and other forces of nuclear type, the hydrogen, water and other molecules admit an *arbitrary* number of hydrogen atoms (see Figure 1.13).

Rather than explaining the reason why nature has selected the molecules H_2 and H_2O as the sole possible, current molecular models admit "molecules" of the type H_5, H_{23}, H_7O, H_2O_{121}, $H_{12}O_{15}$, etc., in dramatic disagreement with experimental evidence.

3: Quantum chemistry has been unable to explain the correlation of valence electrons solely into pairs. Experimental evidence clearly establishes that the valence correlations only occur between *electron pairs* in singlet coupling. By contrast, another known insufficiency of quantum chemistry is the intrinsic inability to restrict correlations to valence pairs.

This insufficiency is then passed to orbital theories, that work well at semi-empirical levels but remain afflicted by yet unresolved problems, eventually resulting in deviations of the prediction of the theory from experimental data that generally grow with the complexity of the molecule considered.

The inability to restrict correlations to valence pairs also provides an irrefutable additional confirmation that quantum chemistry predicts an arbitrary number of constituents in molecular structures.

As we shall see in Chapter 8, thanks to the advent of the new strong valence bond, the covering quantum chemistry does indeed restrict valence bonds strictly and solely to electron pairs. The resolution of inconsistency 2 will then be a mere consequence.

4: The use in quantum chemistry of "screened Coulomb potentials" violates basic quantum principles. The inability by quantum chemistry to achieve an exact representation of binding energies stimulated the adulteration of the basic Coulomb law into the so-called *screened Coulomb law* of the type

$$F = \pm f(r) \times \frac{e^2}{r}, \tag{1.2.22}$$

that did indeed improve the representation of experimental data.

However, the Coulomb law is a fundamental invariant of quantum mechanics, namely, the law remains invariant under all possible unitary transforms

$$F = \pm\frac{e^2}{r} \to U \times (\pm\frac{e^2}{r}) \times U^\dagger = \pm\frac{e^2}{r}, \tag{1.2.23a}$$

$$U \times U^\dagger = I. \tag{1.2.23b}$$

Therefore, any structural deviation from the Coulomb law implies deviations from the basic quantum axioms.

It then follows that the only possibility of achieving screened Coulomb laws is via the use of *nonunitary transforms* of the type

$$F = \pm\frac{e^2}{r} \to W \times (\pm\frac{e^2}{r}) \times W^\dagger = \pm e^{A\times r} \times \frac{e^2}{r}, \tag{1.2.24a}$$

$$W \times W^\dagger = e^{A\times r} \neq I. \tag{1.2.24b}$$

Therefore, by their very conception, *the use of screened Coulomb laws implies the exiting from the class of equivalence of quantum chemistry.* Despite that,

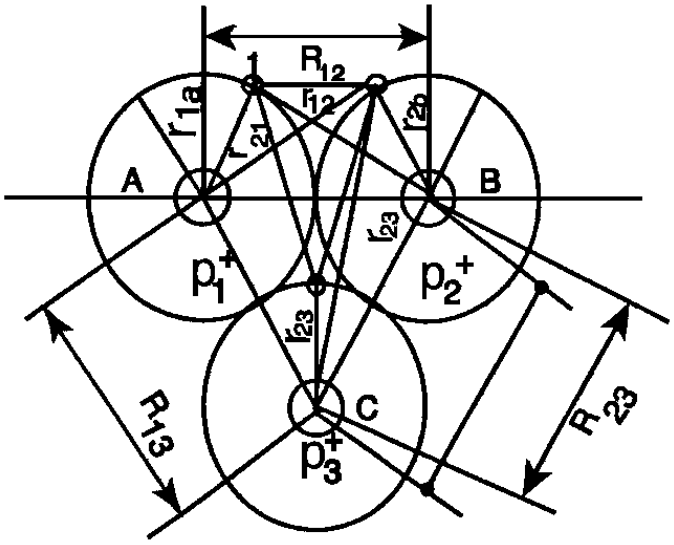

Figure 1.16. A schematic view of the fact that quantum chemistry predicts an arbitrary number of atoms in molecules because the exchange, van der Waals, and other bonding forces used in chemistry were identified in nuclear physics for an arbitrary number of constituents. Consequently, quantum chemistry is basically unable to explain the reasons nature has selected the molecules H_2, H_2O, CO_2, etc. as the sole possible molecular structures, and other structures such as H_5, H_{23}, H_7O, HO_{21}, $H_{12}O_{15}$, etc. cannot exist. As we shall see in Chapter 8, the "strong valence force" permitted by hadronic chemistry can only occur among "pairs" of valence electrons, thus resolving this historical problem in a quantitative way.

organized academic interests have continued to claim that screened Coulomb laws belong to quantum chemistry, thus exiting from the boundaries of science.

Irrespective from the above, a first year graduate student in chemistry can prove that *screened Coulomb laws cause the abandonment of the very notion of quantum in favor of the continuous emission or absorption of energy.* In fact, quantized emissions and absorptions of photons crucially depend on the existence of quantized orbits that, in turn, solely exist for unadulterated Coulomb potentials, as well known.

This insufficiency establishes the need to generalize quantum chemistry into a covering theory since the Coulomb law is indeed insufficient to represent molecular data. Rather than adapting a theory to adulterated basic axioms, it is scientifically more appropriate to build a new theory based on the needed broader axioms.

As we shall see in Chapter 8, the covering hadronic chemistry has been conceived to have a *nonunitary structure* as an evident necessary condition for novelty. In so doing, quantum chemistry naturally admits all infinitely possible

screened Coulomb laws of type (1.2.22). However, such screenings are solely admitted in the nonlocal-integral region of deep wave-overlappings of valence electrons that are of the order of $1\,\mathrm{F} = 10^{-13}\,\mathrm{cm}$, while recovering the conventional Coulomb law automatically for all distances greater that 1 F.

This conception permits the achievement of an exact representation of molecular binding energies while preserving in full the quantum structure of the individual atoms.

5: Quantum chemistry cannot provide a meaningful representation of thermodynamical reactions. The missing 2% in the representation of binding energies is misleadingly small, because it corresponds to about 1,000 Kcal/mole while an ordinary thermodynamical reaction (such as that of the water molecule) implies an average of 50 Kcal/mole. No scientific calculation can be conducted when the error is of about twenty times the quantity to be computed.[6]

As we shall see in Chapter 8, our covering hadronic chemistry does indeed permit exact thermochemical calculations because it has achieved exact representations of molecular characteristics.

6: Computer usage in quantum chemical calculations requires excessively long periods of time. This additional, well known insufficiency is notoriously due to the slow convergence of conventional quantum series, an insufficiency that persists to this day despite the availability of powerful computers.

As we shall also see in Chapter 8, our covering hadronic chemistry will also resolve this additional insufficiency because the mechanism permitting the exact representation of molecular characteristics implies a fast convergent lifting of conventional slowly convergent series.

7: Quantum chemistry predicts that all molecules are paramagnetic. This inconsistency is a consequence of the most rigorous discipline of the 20-th century, quantum electrodynamics, establishing that, under an external magnetic field, the orbits of peripheral atomic electrons must be oriented in such a way to offer a magnetic polarity opposite to that of the external field (a polarization that generally occurs via the transition from a three-dimensional to a toroidal distribution of the orbitals).

According to quantum chemistry, atoms belonging to a molecule preserve their individuality. Consequently, quantum electrodynamics predicts that the periph-

[6]The author received a request from a U. S. public company to conduct paid research on certain thermochemical calculations. When discovering that the calculations had to be based on quantum chemistry due to political needs by the company to be aligned with organized academic interests, the author refused the research contract on grounds that it would constitute a fraud of public funds, due to the excessively large error of all thermochemical calculations when based on quantum chemistry.

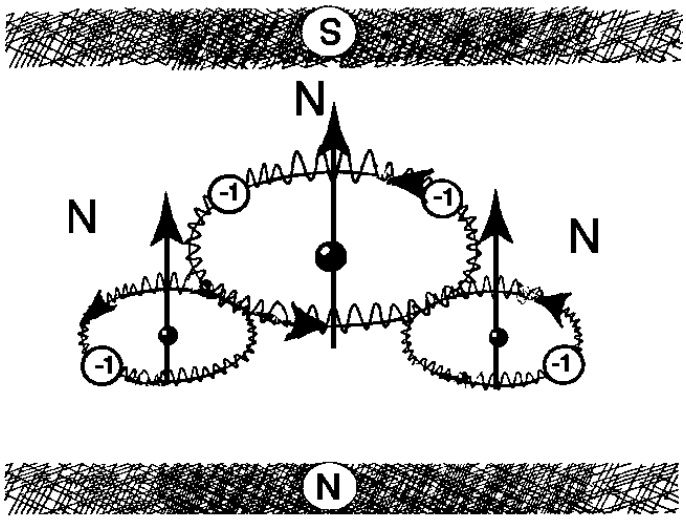

Figure 1.17. A schematic view of the prediction by quantum chemistry that water is paramagnetic, in dramatic disagreement with experimental evidence. In fact, quantum chemistry does not restrict the correlation of valence bonds to pairs. As a result, the individual valence electrons of the water molecule remain essentially independent. Quantum electrodynamics then demands the capability to polarize all valence electrons under an external magnetic field, resulting in the net magnetic polarity of this figure, and the consequential paramagnetic character of the water (as well as of all) molecules. As we shall see in Chapter 8, hadronic chemistry resolves this additional historical problem because our "strong valence force" deeply correlates valence electron pairs, thus permitting a global polarization of a molecule only in special cases, such as those with unbounded electrons.

eral atomic electrons of a molecule must acquire polarized orbits under an external magnetic field.

As a result, quantum chemistry predicts that the application of an external magnetic field, to hydrogen $H-H$, water $H-O-H$ and other molecules imply their acquisition of a net total, opposite polarity, $H_\uparrow - H_\uparrow$, $H_\uparrow - O_\uparrow - H_\uparrow$, etc., which polarization is in dramatic disagreement with experimental evidence.

The above inconsistency can also be derived from its inability to restrict the correlation solely to valence pairs. By contrast, the strong valence bond of the covering hadronic chemistry eliminates the independence of individual atoms in a molecular structure, by correctly representing the diamagnetic or paramagnetic character of substances.

No serious advance in chemistry can occur without, firstly, the admission of the above serious insufficiencies and/or inconsistencies, secondly, their detailed study, and, thirdly, their resolution via a covering theory.

Most importantly, we shall show in Chapter 10 that no resolution of the now alarming environmental problems is possible without a resolution of the above serious inconsistencies of quantum chemistry.

1.2.13 Inconsistencies of Quantum Mechanics, Superconductivity and Chemistry for Underwater Electric Arcs

Submerged electric arcs among carbon-base electrodes are known to permit the production of cost competitive and clean burning gaseous fuels via a highly efficient process since the primary source of energy is carbon combustion by the arc, the electric current used by the arc being a comparatively smaller energy. As such, submerged electric arcs have particular relevance for the main objectives of hadronic mechanics, as studied in Chapter 10 (see also monograph [11]).

An understanding of the motivations for the construction of hadronic mechanics, superconductivity and chemistry requires a knowledge of the fact that, contrary to popular beliefs, submerged electric arcs provide undeniable evidence of the following deviations from established doctrines:

1) When the liquid feedstock is distilled water and the electrodes are given by essentially pure graphite, quantum mechanics and chemistry predict that the produced gas is composed of 50% H_2 and 50% CO. However, CO is combustible in atmosphere and its exhaust is given by CO_2. Therefore, in the event said prediction was correct, the combustion exhaust of the gas should contain about 42% of CO_2. Numerous measurements conducted by an EPA accredited automotive laboratory [11] have established that the combustion exhaust contains about 4%-5% CO_2 without an appreciable percentage of unburned CO. Consequently, *the error of quantum mechanics and chemistry is of about ten times the measured value,* the error being in defect.

2) For the same type of gas produced from distilled water and carbon electrodes, quantum mechanics and chemistry predict that the thermochemical processes underlying the formation of the gas release about 2,250 British Thermal Units (BTU) per standard cubic feet (scf) (see Ref. [11]). In reality, systematic measurements have established that the heat produced is of the order of 250 BTU/scf. Therefore, *the error of quantum mechanics and chemistry is again of the order of ten times the measured quantity,* the error being this time in excess. Note that deviation 1) is fully compatible with deviation 2). In fact, the primary source of heat is the production of CO. Therefore, the production of 1/10-th of the heat predicted confirms that the CO is about 1/10-th the value predicted by quantum mechanics and chemistry.

3) Again for the case of the gas produced from distilled water and graphite electrodes, quantum mechanics and chemistry predict that no oxygen is present in the combustion exhaust, since the prediction is that, under the correct stochio-

metric ratio between atmospheric oxygen and the combustible gas, the exhaust is composed of 50% H_2O and 50% CO_2. In reality, independent measurements conducted by an EPA accredited automotive laboratory have established that, under the conditions here considered, the exhaust contains about 14% of breathable oxygen. Therefore, in this case the error of quantum mechanics and chemistry if about fourteen times the measured value.

4) Quantum mechanics and chemistry predict that the H_2 component of the above considered gas has the conventional specific weight of 2.016 atomic mass units (amu). Numerous measurements conducted in various independent laboratories have established instead that the hydrogen content of said gas has the specific weight of 14.56 amu, thus implying it a seven-fold deviation from the prediction of conventional theories.

5) Numerous additional deviations from the prediction of quantum mechanics and chemistry also exist, such as the fact that the gas has *a variable energy content, a variable specific weight, and a variable Avogadro number* as shown in Chapters 8 and 10, while conventional gases have *constant energy content, specific weight and Avogadro number,* as it is well known.

Above all *the most serious deviations in submerged electric arc occurs for Maxwell's electrodynamics,* to such an extent that any industrial or governmental research in the field based on Maxwell's electrodynamics is a misuse of corporate or public funds. At this introductory level we restrict ourselves to the indication of the axial attractive force between the electrodes and other features structurally incompatible with Maxwell's electrodynamics.

Needless to say, structural incompatibilities with Maxwell's electrodynamics automatically imply structural incompatibilities with special relativity due to the complete symbiosis of the two theories.

Note the re-emergence of the distinction between exterior and interior problems also in regard to Maxwell's electrodynamics. In fact, an arc in vacuum constitutes an exterior problem, while an arc within a liquid constitutes an interior problem. The impossibility of conducting serious industrial research via Maxwell's electrodynamics for submerged electric arcs can then be derived from the inapplicability of special relativity in the conditions considered.

The departures also extend to quantum superconductivity because the initiation of submerged electric arcs causes the collapse of the electric resistance, from very high value (as it is the case for distilled water) down to fractional Ohms. As a consequence, a submerged electric arc has features reminiscent of superconductivity. But the arc occurs at about 10,000 times the maximal temperature predicted by quantum superconductivity. The limitations of the theory is then beyond credible doubt, the only open scientific issues being the selection of the appropriate generalization.

In summary, under the above deviations, any use of quantum mechanics, superconductivity and chemistry for the study of submerged electric arcs exits the boundaries of scientific ethics and accountability. The departures of experimental evidence from old doctrines are just too big to be removed via arbitrary parameters "to fix things", thus mandating the construction of suitable covering theories.

1.3 THE SCIENTIFIC IMBALANCE CAUSED BY IRREVERSIBILITY

1.3.1 The Scientific Imbalance in the Description of Natural Processes

Numerous basic events in nature, including particle decays, such as

$$n \to p^+ + e^- + \bar{\nu}, \tag{1.3.1}$$

nuclear transmutations, such as

$$C(6,12) + H(1,2) \to N(7,14), \tag{1.3.2}$$

chemical reactions, such as

$$H_2 + \frac{1}{2}O_2 \to H_2O, \tag{1.3.3}$$

and other processes are called *irreversible* when their images under time reversal, $t \to -t$, are prohibited by causality and other laws. Systems are instead called *reversible* when their time reversal images are as causal as the original ones, as it is the case for planetary and atomic structures when considered isolated from the rest of the universe.

Yet another large scientific imbalance of the 20-th century has been the treatment of *irreversible* systems via the formulations developed for *reversible* systems, such as Lagrangians and Hamiltonian mechanics, quantum mechanics and chemistry and special relativity. In fact, all these formulations are strictly reversible, in the sense that all their basic axioms are fully reversible in time, by causing in this way limitations in virtually all branches of science.

The imbalance was compounded by use of the *truncated Lagrange and Hamilton equations* (see Section 1.2.2) based on conventional Lagrangians or Hamiltonians,

$$L = \Sigma_{k=1,2,..,n} \frac{1}{2} \times m_k \times v_k^2 - V(r), \tag{1.2.4a}$$

$$H = \Sigma_{a=1,2,..,n} \frac{\mathbf{p}_a^2}{2 \times m_a} + V(r), \tag{1.3.4b}$$

under the full awareness that *all known potentials (such as those for electric, magnetic, gravitational and other interactions), and therefore, all known Hamiltonians, are reversible.*

This additional scientific imbalance was dismissed by academicians with vested interests in reversible theories with unsubstantiated statements, such as "irreversibility is a macroscopic occurrence that disappears when all bodies are reduced to their elementary constituents".

The underlying belief is that mathematical and physical theories that are so effective for the study of one electron in a reversible orbit around a proton are tacitly believed to be equally effective for the study of the same electron when in irreversible motion in the core of a star with the local *nonconservation* of energy, angular momentum, and other characteristics.

Along these lines a vast literature grew during the 20-th century on the dream of achieving compatibility of quantum mechanics with the evident irreversibility of nature at all levels, most of which studies were of manifestly political character due to the strictly reversibility of all methods used for the analysis.

These academic beliefs have been disproved by the following:

THEOREM 1.3.1 [10b]: A classical irreversible system cannot be consistently decomposed into a finite number of elementary constituents all in reversible conditions and, vice-versa, a finite collection of elementary constituents all in reversible conditions cannot yield an irreversible macroscopic ensemble.

The property established by the above theorems dismisses all nonscientific beliefs on irreversibility, and identify the real needs, the construction of formulations that are *structurally irreversible,* that is, irreversible for all known reversible potentials, Lagrangians or Hamiltonians, and are applicable at all levels of study, from Newtonian mechanics to second quantization.

The historical origin of the above imbalance can be outlined as follows. One of the most important teaching in the history of science is that by Lagrange [2], Hamilton [3], and Jacobi [4] who pointed out that *irreversibility originates from contact nonpotential interactions not representable with a potential,* for which reason they formulated their equations with *external terms,* as in Eqs. (1.2.3).

In the planetary and atomic structures, there is no need for external terms, since all acting forces are of potential type. In fact, these systems admit an excellent approximation as being made-up of *massive points moving in vacuum without collisions* (exterior dynamical problems). In these cases, the historical analytic equations were "truncated" with the removal of the external terms.

In view of the successes of the planetary and atomic models, the main scientific development of the 20-th century was restricted to the "truncated analytic equa-

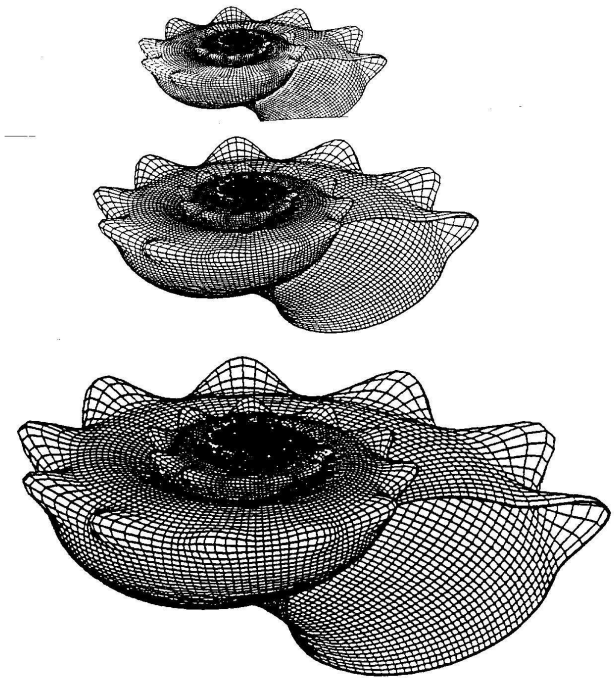

Figure 1.18. A pictorial view of the impossibility for quantum mechanics to be exactly valid in nature: the growth of a seashell. In fact, quantum mechanics is structurally irreversible, in the sense that all its axioms, geometries and symmetries, potentials, etc., are fully reversible in time, while the growth of a seashell is structurally irreversible. The need for an irreversible generalization of quantum mechanics is then beyond credible doubt, as studied in detail in Chapter 4.

tions", without any visible awareness that they are not the equations conceived by the founders of analytic mechanics.

Therefore, the origin of the scientific imbalance on irreversibility is the general dismissal by scientists of the 20-th century of the historical teaching by Lagrange, Hamilton and Jacobi, as well as academic interests on the truncated analytic equations, such as quantum mechanics and special relativity. In fact, as outlined earlier, the use of external terms in the basic analytic equations cause the inapplicability of the mathematics underlying said theories.

It then follows that no serious scientific advance on irreversible processes can be achieved without first identifying a structurally irreversible mathematics and then the compatible generalizations of conventional theories, a task studied in details in Chapter 4.

As we shall see, contrary to popular beliefs, the origin of irreversibility results in being at the ultimate level of nature, that of elementary particles in interior conditions. irreversibility then propagates all the way to the macroscopic level so as to avoid the inconsistency of Theorem 1.3.1.

1.3.2 The Scientific Imbalance in Astrophysics and Cosmology

Astrophysics and cosmology are new branches of science that saw their birth in the 20-th century with a rapid expansion and majestic achievements. Yet, these new fields soon fell pray to organized interests in established doctrines with particular reference to quantum mechanics, special relativity and gravitation, resulting in yet another scientific imbalance of large proportions.

To begin, all interior planetary or astrophysical problems are *irreversible,* as shown by the very existence of *entropy,* and known thermodynamical laws studiously ignored by supporters of Einsteinian doctrines. This feature, alone, is sufficient to cause a scientific imbalance of historical proportions because, as stressed above, irreversible systems cannot be credibly treated with reversible theories.

Also, quantum mechanics has been shown in the preceding sections to be inapplicable to all interior astrophysical and gravitational problems for reasons other than irreversibility. Any reader with an independent mind can then see the limitations of astrophysical studies for the interior of stars, galaxies and quasars based on a theory that is intrinsically inapplicable for the problems considered.

The imposition of special relativity as a condition for virtually all relativistic astrophysical studies of the 20-th century caused an additional scientific imbalance. To illustrate its dimensions and implications, it is sufficient to note that all calculations of astrophysical energies have been based on the relativistic mass-energy equivalence

$$E = m \times c^2, \tag{1.3.5}$$

namely, on the philosophical belief that the speed of light c is the same for all conditions existing in the universe (this is the well known "universal constancy of the speed of light").

As indicated earlier, this belief has been disproved by clear experimental evidence, particularly for the case of interior astrophysical media in which the maximal causal speed has resulted to be $C = c/n >> c$, $n << 1$, in which case the correct calculation of astrophysical energies is given by the equivalence principle of the isospecial relativity (see Chapter 3)

$$E = m \times C^2 = m \times c^2/n^2 >> m \times c^2,\ n << 1, \tag{1.3.6}$$

thus invalidating current view on the "missing mass", and others.

A further large scientific imbalance in astrophysics and cosmology was caused by the imposition of general relativity, namely, by one of the most controversial theories of the 20-th century because afflicted by problematic aspects and sheer inconsistencies so serious called catastrophic, as outlined in the next section.

It is hoped these preliminary comments are sufficient to illustrate the weakness of the scientific foundations of astrophysical studies of the 20-th century.

1.3.3 The Scientific Imbalance in Biology

By far one of the biggest scientific imbalances of the 20-th century occurred in biology because biological structures were treated via quantum mechanics in full awareness that the systems described by that discipline are dramatically different than biological structures.

To begin, quantum mechanics and chemistry are strictly *reversible,* while all biological structures and events are structurally *irreversible,* since biological structures such as a cell or a complete organism, admit a birth, then grow and then die.

Moreover, quantum mechanics and chemistry can only represent *perfectly rigid systems*, as well known from the fundamental rotational symmetry that can only describe "rigid bodies".

As a consequence, the representation of biological systems via quantum mechanics and chemistry implies that our body should be perfectly rigid, without any possibility of introducing deformable-elastic structures, because the latter would cause catastrophic inconsistencies with the basic axioms.

Moreover, another pillar of quantum mechanics and chemistry is the verification of total conservation laws, for which Heisenberg's equation of motion became established. In fact, the quantum time evolution of an arbitrary quantity A is given by

$$i \times \frac{dA}{dt} = [A, H] = A \times H - H \times A, \tag{1.3.7}$$

under which expression we have the conservation law of the energy and other quantities, e.g.,

$$i\ dH/dt = H \times H - H \times H \equiv 0. \tag{1.3.8}$$

A basic need for a scientific representation of biological structures is instead the representation of *the time-rate-of-variations of biological characteristics*, such as size, weight, density, etc. This identifies another structural incompatibility between quantum mechanics and biological systems.

When passing to deeper studies, the insufficiencies of quantum mechanics and chemistry emerge even more forcefully. As an example, quantum theories can well represent the *shape* of sea shells, but not their *growth in time.*

In fact, computer visualizations [16] have shown that, when the geometric axioms of quantum mechanics and chemistry (those of the Euclidean geometry)

are imposed as being *exactly* valid, sea shells first grow in a deformed way, and then crack during their growth.

Finally, the ideal systems described with full accuracy by quantum mechanics, such as an isolated hydrogen atom or a crystal, are *eternal.* Therefore, the description via quantum theories implies that biological systems are eternal.

These occurrences should not be surprising to inquisitive minds, because the birth and growth, e.g., of a seashell is strictly *irreversible and nonconservative,* while the geometric axioms of quantum theories are perfectly *reversible and conservative*, as indicated earlier, thus resulting in a structural incompatibility, this time, at the geometric level without any conceivable possibility of reconciliation, e.g., via the introduction of unknown parameters "to fix things".

Additional studies have established that the insufficiencies of quantum mechanics and chemistry in biology are much deeper than the above, and invest the *mathematics* underlying these disciplines. In fact, Illert [16] has shown that a minimally correct representation of the growth in time of sea shells requires the *doubling of the Euclidean axes.*

However, sea shells are perceived by the human mind (via our three Eustachian tubes) as growing in our *three-dimensional* Euclidean space. As we shall see in Chapter 8, the only known resolution of such a dichotomy is that via *multivalued irreversible mathematics*, that is, mathematics in which operations such as product, addition, etc., produce a *set of values*, rather than one single value as in quantum mechanics and chemistry.

At any rate, the belief that the simplistic mathematics underlying quantum mechanics and chemistry can explain the complexity of the DNA code, has no scientific credibility, the only serious scientific issue being the search for broader mathematics.

In conclusion, science will never admit "final theories". No matter how valid any given theory may appear at any point in time, its structural broadening for the description of more complex conditions is only a matter of time.

This is the fate also of quantum mechanics and chemistry, as well as special and general relativities that cannot possibility be considered as "final theories" for all infinitely possible conditions existing in the universe.

After all, following only a few centuries of research, rather than having reached a "final stage", science is only at its infancy.

1.4 THE SCIENTIFIC IMBALANCE CAUSED BY GENERAL RELATIVITY AND QUANTUM GRAVITY

1.4.1 Consistency and Limitations of Special Relativity

As it is well known, thanks to historical contributions by Lorentz, Poincaré, Einstein, Minkowski, Weyl and others, *special relativity* achieved a majestic axiomatical consistency.[7]

After one century of studies, we can safely identify the origins of this consistency in the following crucial properties:

1) Special relativity is formulated in the Minkowski spacetime over the field of real numbers;

2) All laws of special relativity are *invariant* (rather than covariant) under the fundamental *Poincaré symmetry*;

3) The Poincaré transformations and, consequently, all times evolutions of special relativity, are *canonical* at the classical level and *unitary* at the operator level with implications crucial for physical consistency.

Consequently, since canonical or unitary transforms conserve the unit by their very definition, *special relativity admits basic units and numerical predictions that are invariant in time.* After all, the quantities characterizing the dynamical equations are the *Casimir invariants* of the Poincaré symmetry.

As a result of the above features, special relativity has been and can be confidently applied to experimental measurements because the units selected by the experimenter do not change in time, and the numerical predictions of the theory can be tested at any desired time under the same conditions without fear of internal axiomatic inconsistencies.

It is well established at this writing that special relativity is indeed "compatible with experimental evidence" for the arena of its original conception, the classical and operator treatment of "point-like" particles and electromagnetic waves moving in vacuum. Despite historical results, it should be stressed that, as is the fate for all theories, *special relativity has numerous well defined limits of applicability,* whose identification is crucial for any serious study on gravitation, since

[7]It should be indicated that the name "Einstein's special relativity" is political, since a scientifically correct name should be "Lorentz-Poincaré-Einstein relativity." Also, it is appropriate to recall (as now reviewed in numerous books under testimonials by important eyewitnesses) that Einstein ended up divorcing his first wife Mileva Maric because she was instrumental in writing the celebrated paper on special relativity of 1905 and, for that reason, she had been originally listed as a co-author of that article, co-authorship that was subsequently removed when the article appeared in print. In fact, Einstein awarded his Nobel Prize money on that article to Mileva. Similarly, it should be recalled that Einstein avoided quoting Poincaré in his 1905 article following his consultation, and in documented knowledge that Poincaré had preceded him in various features of special relativity (see, e.g., the historical account by Logunov [96] or the instructive books [97,98]).

general relativity is known to be an extension of the special. Among the various limitations, we quote the following:

INAPPLICABILITY # 1: Special relativity is *inapplicable* for the *classical* treatment of antiparticles as shown in Section 1.1 and Chapter 2. This is essentially due to the existence of only one quantization channel. Therefore, the quantization of a *classical antiparticle* characterized by special relativity (essentially via the sole change of the sign of the charge) clearly leads to a quantum mechanical *particle* with the wrong sign of the charge, and definitely not to the appropriate charge conjugated antiparticle, resulting in endless inconsistencies.

INAPPLICABILITY # 2: Special relativity has also been shown to be inapplicable (rather than violated) for the treatment of both, particles and antiparticles when represented as they are in the physical reality, extended, generally nonspherical and deformable particles (such as protons or antiprotons), particularly when interacting at very short distances. In fact, these conditions imply the mutual penetration of the wavepackets and/or the hyperdense media constituting the particles, resulting in nonlocal, integro-differential and nonpotential interactions that cannot be entirely reduced to potential interactions among point-like constituents.

INAPPLICABILITY # 3: Special relativity is also afflicted by the historical inability to represent irreversible processes. This inapplicability has been identified in Section 1.3 in the reversibility of the mathematical methods used by special relativity, under which conditions the reversibility in time of its basic axioms is a mere consequence.

INAPPLICABILITY # 4: An additional field of clear inapplicability of special relativity is that for all biological entities, since the former can only represent perfectly rigid and perfectly reversible, thus eternal structures, while biological entities are notoriously deformable and irreversible, having a finite life.

INAPPLICABILITY # 5: In addition, serious scholars should keep in mind that the biggest limitation of special relativity may well result to be the forgotten universal medium needed for the characterization and propagation not only of electromagnetic waves, but also of elementary particles, since truly elementary particles such as the electron appear to be pure oscillations of said universal medium. Rather than being forgotten, the issue of the *privileged reference frame* and its relationship to reference frames of our laboratory settings appears to be more open than ever.

1.4.2 The Scientific Imbalance Caused by General Relativity on Antimatter, Interior Problems, and Grand Unifications

As indicated above, special relativity has a majestic axiomatic structure with clear verifications in the field of its original conception. By contrast, it is safe

to state that *general relativity* (see, e.g., monograph [17]) has been the most controversial theory of the 20-th century for a plethora of inconsistencies that have grown in time, rather than being addressed and resolved.

We now address some of the inconsistencies published by numerous scholars in refereed technical journals, yet generally ignored by organized interests on Einsteinian doctrines, which inconsistencies are so serious to be known nowadays as being "catastrophic". The apparent resolution of the inconsistencies will be presented in Chapters 3, 4, 5, 13, and 14.

Let us begin with the following basic requirement for any *classical* theory of gravitation to be consistent:

REQUIREMENT 1: Any consistent classical theory of antimatter must allow a consistent representation of the *gravitational field of antimatter.* General Relativity does not verify this first requirement because, in order to attempt a compatibility of classical and quantum formulations, antimatter requires negative-energies, while general relativity solely admit positive-definite energies, as well known.

Even assuming that this insufficiency is somewhat bypassed, general relativity can only represent antimatter via the reversal of the sign of the charge. But the most important astrophysical bodies expected to be made up of antimatter are neutral. This confirms the structural inability of general relativity to represent antimatter in a credible way.

REQUIREMENT 2: Any consistent classical theory of antimatter must be able to represent *interior gravitational problems.* General relativity fails to verify this second requirement for numerous reasons, such as the inability to represent the *density* of the body considered, its *irreversible* condition, e.g., due to the increase of entropy, the *locally varying speed of light,* etc.

REQUIREMENT 3: Any consistent classical theory of gravitation must permit a grand unifications with other interactions. It is safe to state that this requirement too is not met by general relativity since all attempts to achieve a grand unification have failed to date since Einstein times (see Chapter 12 for details).

REQUIREMENT 4: Any consistent classical theory of gravitation must permit a consistent operator formulation of gravity. This requirement too has not been met by general relativity, since its operator image, known as *quantum gravity* [18] is afflicted by additional independent inconsistencies mostly originating from its unitary structure as studied in the next section.

REQUIREMENT 5: Any consistent classical theory of gravitation must permit the representation of the *locally varying nature of the speed of light.* This requirement too is clearly violated by general relativity.

The above insufficiencies are not of marginal character because they caused serious imbalances in most branches of quantitative sciences.

As an illustration, the first insufficiency prevented any study whatever as to whether a far-away galaxy or quasar is made up of matter or of antimatter. The second insufficiency created a form of religion related to the so-called "black holes", since before claiming their existence, gravitational singularities must evidently come out of *interior* gravitational problems and definitely not from theoretical abstractions solely dealing with exterior gravitation. The third insufficiency has been responsible for one of the longest list of failed attempts in grand unification without addressing the origin of the failures in the gravitational theory itself. The fourth insufficiency prevented throughout the entire 20-th century a consistent quantum formulation of gravity with large implications in particle physics. The fifth insufficiency cause cosmological models that can only be qualified as scientific beliefs, rather than quantitative theories based on sound physical foundations.

It is hoped that even the most representative members of organized interests on Einsteinian doctrines will admit that any additional support for said interests is now counterproductive, since it has already passed the mark for a severe condemnation by posterity.

It is time to provide a scientific identification of the basic insufficiencies of general relativity and initiate systematic studies for their resolution.

1.4.3 Catastrophic Inconsistencies of General Relativity due to Lack of Sources

There exist subtle distinctions between "general relativity", "Einstein's Gravitation", and "Riemannian" formulation of gravity. For our needs, we here define *Einstein's gravitation* of a body with null electric and magnetic moments as the reduction of exterior gravitation in vacuum to pure geometry, namely, gravitation is solely represented via curvature in a Riemannian space $\mathcal{R}(x, g, R)$ with spacetime coordinates $x = \{x^\mu\}$, $\mu = 1, 2, 3, 0$ and nowhere singular real-valued and symmetric metric $g(x)$ over the reals R, with field equations [19,20][8]

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = 0, \tag{1.4.1}$$

[8]The dubbing of Eqs. (1.4.1) as "Einstein's field equations" is political since it is known, or it should be known by "expert" in the field to qualify as such, that Hilbert independently published the same equations, and that Einstein consulted Hilbert without quotation his work in his gravitational paper of 1916, as done by Einstein in other cases.

It is also appropriate to recall that the publication of his 1916 paper on gravitation caused Einstein the divorce from his second wife, Elsa Loewenstein, for essentially the same reason of his first divorce. In fact, unlike Einstein, Elsa was a true mathematician, had trained Einstein on the Riemannian geometry (a topic only for very few pure mathematics at that time), and was supposed to be a co-author of Einstein's 1916 paper, a co-authorship denied as it was the case for the suppression of co-authorship of his first wife Mileva for his 1905 paper on special relativity (see the instructive books [97,98]).

To avoid a scandal for the 1905 paper, Einstein donate to Mileva the proceeds of his Nobel Prize. However, he did not receive a second Nobel Prize to quite down his second wife Elsa. A scandal was then

in which, as a central condition to have Einstein's gravitation, *there are no sources for the exterior gravitational field in vacuum for a body with null total electromagnetic field (null total charge and magnetic moment).*

For our needs, we define as *general relativity* any description of gravity on a Riemannian space over the reals with Einstein-Hilbert field equations with a source due to the presence of electric and magnetic fields,

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times t_{\mu\nu}, \tag{1.4.2}$$

where k is a constant depending on the selected unit whose value is here irrelevant. For the scope of this section it is sufficient to assume that the *Riemannian description of gravity* coincides with general relativity according to the above definition.

In the following, we shall first study the inconsistencies of Einstein gravitation, that is, the inconsistencies in the entire reduction of gravity to curvature without source, and then study the inconsistency of general relativity, that is, the inconsistencies caused by curvature itself even in the presence of sources.

It should be stressed that a technical appraisal of the content of this section can only be reached following the study of the axiomatic inconsistencies of grand

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avoided for the 1916 paper via the complicity of the Princeton community, complicity that is in full force and effect to this day. Hence, Princeton can indeed be considered as being an academic community truly leading in new basic advances during Einstein's times. By contrast, Princeton is nowadays perceived as a "scientific octopus" with kilometric tentacles reaching all parts of our globe for the studious suppression, via the abuse of academic credibility, of any spark of advance over Einsteinian doctrines. In fact, no truly fundamental advance came out of Princeton since Einstein's times, thus leaving Einstein as the sole source of money, prestige and power.

The documentation of the actions by Princeton academicians to oppose, jeopardize and disrupt research beyond Einstein is vast and includes hundreds of researchers in all developed countries. It is their ethical duty, if they really care for scientific democracy and the human society, to come out and denounce publicly the serious misconducts by Princeton academicians they had to suffer (for which denunciations I am sure that the *International Committee on Scientific Ethics and Accountability* will offer its website http://www.scientificethics.org).

In regard to the author's documented experiences, it is sufficient to report here for the reader in good faith the rejection by the Princeton academic community with offensive language of *all* requests by the author (when still naive) for delivering an informal seminar on the isotopic lifting of special relativity for the intent of receiving technical criticisms. There is also documentation that, when the unfortunate session chairman of the second *World Congress in Mathematics* of the new century, the president of the Institute for Advanced Studies in Princeton prohibited presentations on Lie-isotopic and Lie-admissible algebras not only by the author, but also by the late Prof. Grigorios Tsagas, then Chairman of the Mathematics Department of Aristotle University in Thessaloniki, Greece. This volume has been dedicated to the memory of Prof. Gr. Tsagas also in view of the vexations he had to suffer for his pioneering mathematical research from decaying U. S. academia.

The climax of putrescence in the Princeton academic community is reached by the mumbo-jambo research in the so called "controlled hot fusion" under more than one billion of public funds, all spent under the condition of compatibility with Einsteinian doctrines, and under clear the technical proofs of the impossibility of its success (see Volume II for technical details).

The author spares the reader the agony of additional documented episodes of scientific misconducts because too demeaning, and expresses the view that, with a few exceptions, the Princeton academic community is nowadays an enemy of mankind.

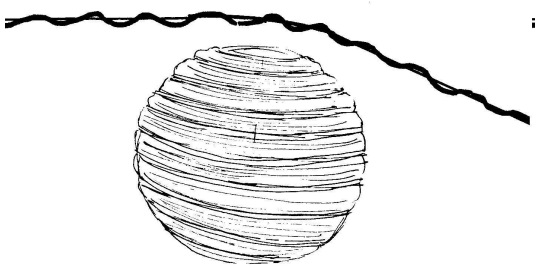

Figure 1.19. When the "bending of light" by astrophysical bodies was first measured, organized interests in Einsteinian doctrines immediately claimed such a bending to be an "experimental verification" of "Einstein's gravitation", and the scientific community accepted that claim without any critical inspection (for evident academic gains), according to an unreassuring trend that lasts to this day by being at the foundation of the current scientific obscurantism of potentially historical proportions. It can be seen by first year physics students that the measured bending of light is that predicted by the NEWTONIAN attraction. The representation of the same "bending of light" as being entirely due to curvature, as necessary in "Einstein's gravitation", implies its formulation in such a way to avoid any Newtonian contribution, with catastrophic inconsistencies in other experiments (see, e.g., next figure).

unified theories of electroweak and gravitational interactions whenever gravity is represented with curvature on a Riemannian space irrespective of whether with or without sources, as studied in Chapter 12.

THEOREM 1.4.1 [22]: Einstein's gravitation and general relativity at large are incompatible with the electromagnetic origin of mass established by quantum electrodynamics, thus being inconsistent with experimental evidence.

Proof. Quantum electrodynamics has established that the mass of all elementary particles, whether charged or neutral, has a primary electromagnetic origin, that is, all masses have a first-order origin given by the volume integral of the 00-component of the energy-momentum tensor $t_{\mu\nu}$ of electromagnetic origin,

$$m = \int d^4x \times t_{oo}^{elm}. \tag{1.4.3a}$$

$$t_{\alpha\beta} = \frac{1}{4\pi}(F_\alpha^\mu F_{\mu\beta} + \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu}), \tag{1.4.3b}$$

where $t_{\alpha\beta}$ is the *electromagnetic tensor,* and $F_{\alpha\beta}$ is the *electromagnetic field* (see Ref. [11a] for explicit forms of the latter with retarded and advanced potentials).

Therefore, quantum electrodynamics requires the presence of a *first-order source tensor* in the *exterior field equations* in vacuum as in Eqs. (1.4.2). Such a source tensor is absent in Einstein's gravitation (1.4.1) by conception. Consequently, Einstein's gravitation is incompatible with quantum electrodynamics.

The incompatibility of general relativity with quantum electrodynamics is established by the fact that the source tensor in Eqs. (1.4.2) is of *higher order in magnitude*, thus being ignorable in first approximation with respect to the gravitational field, while according to quantum electrodynamics said source tensor is of first order, thus not being ignorable in first approximation.

The inconsistency of both Einstein's gravitation and general relativity is finally established by the fact that, for the case when the total charge and magnetic moment of the body considered are null, Einstein's gravitation and general relativity allows no source at all. By contrast, as illustrated in Ref. [21], quantum electrodynamics requires a first-order source tensor even when the total charge and magnetic moments are null due to the charge structure of matter. **q.e.d.**

The first consequence of the above property can be expressed via the following:

COROLLARY 1.4.1A [21]: Einstein's reduction of gravitation in vacuum to pure curvature without source is incompatible with physical reality.

A few comments are now in order. As is well known, the mass of the electron is entirely of electromagnetic origin, as described by Eq. (3.3), therefore requiring a first-order source tensor in vacuum as in Eqs. (3.2). Therefore, Einstein's gravitation for the case of the electron is inconsistent with nature. Also, the electron has a point charge. Consequently, *the electron has no interior problem at all, in which case the gravitational and inertial masses coincide,*

$$m^{Grav.}_{Electron} \equiv m^{Iner}_{Electron}. \tag{1.4.4}$$

Next, Ref. [21] proved Theorem 1.4.1 for the case of a neutral particle by showing that the π^o meson also needs a first-order source tensor in the exterior gravitational problem in vacuum since its structure is composed of one charged particle and one charged antiparticle in high dynamical conditions.

In particular, the said source tensor has such a large value to account for the entire *gravitational mass* of the particle [21]

$$m^{Grav.}_{\pi^o} = \int d^4x \times t^{Elm}_{00}. \tag{1.4.5}$$

For the case of the interior problem of the π^o , we have the additional presence of short range weak and strong interactions representable with a new tensor $\tau_{\mu\nu}$. We, therefore, have the following:

COROLLARY 1.4.1B [22]: In order to achieve compatibility with electromagnetic, weak and strong interactions, any gravitational theory must admit two source tensors, a traceless tensor for the representation of the electromagnetic origin of mass in the exterior gravitational problem, and a second tensor to represent the contribution to interior gravitation of the short range interactions according to the field equations

$$G_{\mu\nu}^{Int.} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times (t_{\mu\nu}^{Elm} + \tau_{\mu\nu}^{ShortRange}). \tag{1.4.6}$$

A main difference of the two source tensors is that the electromagnetic tensor $t_{\mu\nu}^{Elm}$ is notoriously traceless, while the second tensor $\tau_{\mu\nu}^{ShortRange}$ is not. A more rigorous definition of these two tensors will be given shortly.

It should be indicated that, for a possible solution of Eqs. (1.4.6), various explicit forms of the electromagnetic fields as well as of the short range fields originating the electromagnetic and short range energy momentum tensors are given in Ref. [21].

Since both source tensors are positive-definite, Ref. [21] concluded that the interior gravitational problem characterizes the *inertial mass* according to the expression

$$m^{Iner} = \int d^4x \times (t_{00}^{Elm} + \tau_{00}^{ShortRange}), \tag{1.4.7}$$

with consequential general law

$$m^{Inert.} \geq m^{Grav.}, \tag{1.4.8}$$

where the equality solely applies for the electron.

Finally, Ref. [22] proved Theorem 1.4.1 for the exterior gravitational problem of a neutral massive body, such as a star, by showing that the situation is essentially the same as that for the π^o. The sole difference is that the electromagnetic field requires the sum of the contributions from *all* elementary constituents of the star,

$$m_{Star}^{Grav.} = \Sigma_{p=1,2,...} \int d^4x \times t_{p00}^{Elem.}. \tag{1.4.9}$$

In this case, Ref. [21] provided methods for the approximate evaluation of the sum that resulted in being of first-order also for stars with null total charge.

When studying a charged body, there is no need to alter equations (3.6) since that particular contribution is automatically contained in the indicated field equations.

Once the incompatibility of general relativity at large with quantum electrodynamics has been established, the interested reader can easily prove the incompatibility of general relativity with quantum field theory and quantum chromodynamics, as implicitly contained in Corollary 1.4.1B.

An important property apparently first reached in Ref. [11a] in 1974 is the following:

COROLLARY 1.4.1C [22]: The exterior gravitational field of a mass originates entirely from the total energy-momentum tensor of the electromagnetic field of all elementary constituents of said mass.

In different terms, a reason for the failure to achieve a "unification" of gravitational and electromagnetic interactions initiated by Einstein himself is that the said interactions can be "identified" with each other and, as such, they cannot be unified. In fact, in all unifications attempted until now, the gravitational and electromagnetic fields preserve their identity, and the unification is attempted via geometric and other means resulting in redundancies that eventually cause inconsistencies.

Note that conventional electromagnetism is represented with the tensor $F_{\mu\nu}$ and related Maxwell's equations. When electromagnetism is identified with exterior gravitation, it is represented with the energy-momentum tensor $t_{\mu\nu}$ and related equations (1.4.6).

In this way, *gravitation results as a mere additional manifestation of electromagnetism.* The important point is that, besides the transition from the field tensor $F_{\mu\nu}$ to the energy-momentum tensor $T_{\mu\nu}$, there is no need to introduce a new interaction to represent gravity.

Note finally the irreconcilable alternatives emerging from the studies herein considered [22]:

ALTERNATIVE I: Einstein's gravitation is assumed as being correct, in which case quantum electrodynamics must be revised in such a way to avoid the electromagnetic origin of mass; or

ALTERNATIVE II: Quantum electrodynamics is assumed as being correct, in which case Einstein's gravitation must be irreconcilably abandoned in favor of a more adequate theory.

By remembering that quantum electrodynamics is one of the most solid and experimentally verified theories in scientific history, it is evident that the rather widespread assumption of Einstein's gravitation as having final and universal character is non-scientific.

THEOREM 1.4.2 [75,110]: Einstein's gravitation is physically incompatible with the Freud identity of the Riemannian geometry for bodies with non-null electromagnetic fields, and mathematically as well as physically inconsistent with the

Freud identify for bodies with null electromagnetic fields, thus being inconsistent on geometric grounds.

Proof. the Freud identity of the Riemannian geometry requires the presence of a *first order* source tensor in the r.h.s. of the field equations, thus prohibiting Einstein attempt of reducing gravitation to sole curvature without source. In fact, the Freud identity [23] can be written in the following, as well as numerous equivalent forms:

$$R^\alpha_\beta - \frac{1}{2} \times \delta^\alpha_\beta \times R - \frac{1}{2} \times \delta^\alpha_\beta \times \Theta \; = \; U^\alpha_\beta + \partial V^{\alpha\rho}_\beta / \partial x^\rho \; = \; k \times (t^\alpha_\beta + \tau^\alpha_\beta), \quad (1.4.10)$$

where

$$\Theta \; = \; g^{\alpha\beta} g^{\gamma\delta} (\Gamma_{\rho\alpha\beta} \Gamma^\rho_{\gamma\beta} - \Gamma_{\rho\alpha\beta} \Gamma^\rho_{\gamma\delta}), \quad (1.4.11a)$$

$$U^\alpha_\beta \; = \; -\frac{1}{2} \frac{\partial \Theta}{\partial g^{\rho\alpha}_{|_\rho}} g^{\gamma\beta} \uparrow_\gamma, \quad (1.4.11b)$$

$$V^{\alpha\rho}_\beta \; = \; \frac{1}{2} [g^{\gamma\delta} (\delta^\alpha_\beta \Gamma^\rho_{\alpha\gamma\delta} - \delta^\rho_\beta \Gamma^\rho_{\alpha\delta}) +$$

$$+ (\delta^\rho_\beta g^{\alpha\gamma} - \delta^\alpha_\beta g^{\rho\gamma}) \Gamma^\delta_{\gamma\delta} + g^{\rho\gamma} \Gamma^\alpha_{\beta\gamma} - g^{\alpha\gamma} \Gamma^\rho_{\beta\gamma}]. \quad (1.4.11c)$$

Therefore, the Freud identity requires two *first order* source tensors for the exterior gravitational problems in vacuum as in Eqs. (1.4.6) reproduced from Ref. [22]. Of course, the field equations have the general form (1.4.2) in which there is indeed a source tensor in the r.h.s. Hence, for *bodies with a non-null electromagnetic fields,* there is no mathematical incompatibility between Einstein gravitation and the Fred identity.

However, it is well known that the contribution of electromagnetic fields to the gravitational field is extremely minute and definitely not of the *first order* in magnitude requested by the Freud identity, from which the first part of Theorem 1.4.2 follows.

For *bodies with null electromagnetic fields* (that is, null charge and null magnetic moments) no tensor of any type can be placed in the r.h.s. of the field equation, thus establishing the second part of Theorem 1.4 .2. **q.e.d.**

By noting that trace terms can be transferred from one tensor to the other in the r.h.s. of Eqs. (1.4.10), it is easy to prove the following:

COROLLARY 1.4.2A [75,110]: Except for possible factorization of common terms, the t- and τ-tensors of Theorem 3.2 coincide with the electromagnetic and short range tensors, respectively, of Corollary 1.4.1B.

A few historical comments regarding the Freud identity are in order. It has been popularly believed throughout the 20-th century that the Riemannian geometry possesses only *four identities* (see, e.g., Ref. [17]). In reality, Freud

[22] identified in 1939 a *fifth identity* that, unfortunately, was not aligned with Einstein's doctrines and, as such, the identity was ignored in virtually the entire literature on gravitation of the 20-th century, as it was also the case for Schwarzschild's interior solution [8].

However, as repeatedly illustrated by scientific history, structural problems simply do not disappear with their suppression, and actually grow in time. In fact, the Freud identity did not escape Pauli who quoted it in a footnote of his celebrated book of 1958 [24]. Santilli became aware of the Freud identity via an accurate reading of Pauli's book (including its important footnotes) and assumed the Freud identity as the geometric foundation of the gravitational studies presented in Ref. [10b].

Subsequently, in his capacity as Editor in Chief of *Algebras, Groups and Geometries,* Santilli requested the mathematician Hanno Rund, a known authority in Riemannian geometry [24], to inspect the Freud identity for the scope of ascertaining whether the said identity was indeed a new identity. Rund kindly accepted Santilli's invitation and released paper [26] of 1991 (the last paper prior to his departure) in which Rund confirmed indeed the character of Eqs. (3.10) as a genuine, independent, fifth identity of the Riemannian geometry.

The Freud identity was also rediscovered by Yilmaz (see Ref. [27] and papers quoted therein) who used the identity for his own broadening of Einstein's gravitation via an external *stress-energy tensor* that is essentially equivalent to the source tensor with non-null trace of Ref. [11a], Eqs. 1.4.6).

Despite these efforts, the presentation of the Freud identity to various meetings and several personal mailings to colleagues in gravitation, the Freud identity continues to remain vastly ignored, with rare exceptions (the indication by colleagues of additional studies on the Freud identity not quoted herein would be gratefully appreciated.)

Theorems 1.4.1 and 1.4.2 complete our presentation on the catastrophic inconsistencies of Einstein's gravitation due to the lack of a first-order source in the exterior gravitational problem in vacuum. These theorems, by no means, exhaust all inconsistencies of Einstein's gravitation, and numerous additional inconsistencies do indeed exist.

For instance, Yilmaz [27] has proved that Einstein's gravitation explains the 43" of the precession of Mercury, but cannot explain the basic Newtonian contribution. This result can also be seen from Ref. [21] because the lack of source implies the impossibility of importing into the theory the basic Newtonian potential. Under these conditions the representation of the Newtonian contribution is reduced to a religious belief, rather than a serious scientific statement.

For these and numerous additional inconsistencies of general relativity we refer the reader to Yilmaz [27], Wilhelm [28-30], Santilli [31], Alfvén [32,33], Fock [34], Nordensen [35], and large literature quoted therein.

1.4.4 Catastrophic Inconsistencies of General Relativity due to Curvature

We now pass to the study of the structural inconsistencies of general relativity caused by the very use of the Riemannian *curvature*, irrespective of the selected field equations, including those fully compatible with the Freud identity.

THEOREM 1.4.3 [36]: Gravitational theories on a Riemannian space over a field of real numbers do not possess time invariant basic units and numerical predictions, thus having serious mathematical and physical inconsistencies.

Proof. The map from Minkowski to Riemannian spaces is known to be *non-canonical,*

$$\eta = Diag.(1,1,1,-1) \ \rightarrow \ g(x) = U(x) \times \eta \times U(x)^\dagger, \tag{1.4.12a}$$

$$U(x) \times U(x)^\dagger \neq I. \tag{1.4.12b}$$

Thus, the time evolution of Riemannian theories is necessarily noncanonical, with consequential lack of invariance in time of the basic units of the theory, such as

$$I_{t=0} = Diag.(1cm, 1cm, 1cm, 1sec) \rightarrow I'_{t>0} = U_t \times I \times U_t^\dagger \ \neq I_{t=0}. \tag{1.4.13}$$

The lack of invariance in time of numerical predictions then follows from the known "covariance", that is, lack of time invariance of the line element. **q.e.d.**

As an illustration, suppose that an experimentalist assumes at the initial time $t = 0$ the units 1 cm and 1 sec. Then, all Riemannian formulations of gravitation, including Einstein's gravitation, predict that at the later time $t > 0$ said units have a different numerical value.

Similarly, suppose that a Riemannian theory predicts a numerical value at the initial time $t = 0$, such as the 43" for the precession of the perihelion of Mercury. One can prove that the same prediction at a later time $t > 0$ is numerically different precisely in view of the "covariance", rather than invariance as intended in special relativity, thus preventing a serious application of the theory to physical reality. We therefore have the following:

COROLLARY 1.4.3A [36]: Riemannian theories of gravitation in general, and Einstein's gravitation in particular, can at best describe physical reality at a fixed value of time, without a consistent dynamical evolution.

Interested readers can independently prove the latter occurrence from the *lack of existence of a Hamiltonian in Einstein's gravitation.* It is known in analytic mechanics (see, e.g., Refs. [17,24]) that Lagrangian theories not admitting an equivalent Hamiltonian counterpart, as is the case for Einstein's gravitation, are

inconsistent under time evolution, unless there are suitable subsidiary constraints that are absent in general relativity.

It should be indicated that the inconsistencies are much deeper than that indicated above. For consistency, the Riemannian geometry must be defined on the field of numbers $R(n, +, \times)$ that, in turn, is fundamentally dependent on the basic unit I. But the Riemannian geometry does not leave time invariant the basic unit I due to its noncanonical character. The loss in time of the basic unit I then implies the consequential loss in time of the base field R, with consequential catastrophic collapse of the entire geometry [36].

In conclusion, not only is Einstein's reduction of gravity to pure curvature inconsistent with nature because of the lack of sources, but also the ultimate origin of the inconsistencies rests in the curvature itself when assumed for the representation of gravity, due to its inherent noncanonical character at the classical level with consequential nonunitary structure at the operator level.

Serious mathematical and physical inconsistencies are then unavoidable under these premises, thus establishing the impossibility of any credible use of general relativity, for instance, as an argument against the test on antigravity predicted for antimatter in the field of matter [5], as well as establishing the need for a profound revision of our current views on gravitation.

THEOREM 1.4.4. Gravitational experimental measurements do not verify general relativity uniquely.

Proof. All claimed "experimental verifications" of Einstein's gravitation are based on the PPN "expansion" (or linearization) of the field equations (such as the post-Newtonian approximation), that, as such, is not unique. In fact, Eqs. (1.4.1) admit a variety of inequivalent expansions depending on the selected parameter, the selected expansion and the selected truncation. It is then easy to show that the selection of an expansion of the same equations (3.1) but different from the PPN approximation leads to dramatic departures from experimental values. **q.e.d.**

THEOREM 1.4.5: General relativity is incompatible with experimental evidence because it does not represent the bending of light in a consistent, unique and invariant way.

Proof. Light carries energy, thus being subjected to a bending due to the conventional Newtonian gravitational attraction, while, general relativity predicts that the bending of light is entirely due to curvature (see, e.g., Ref. [17], Section 40.3). In turn, the absence of the Newtonian contribution causes other catastrophic inconsistencies, such as the inability to represent the free fall where curvature does not exist (Theorem 1.4.6 below). Assuming that consistency is

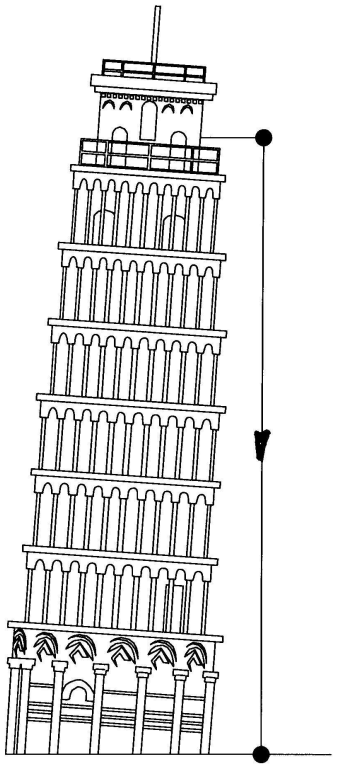

Figure 1.20. A conceptual rendering of the reason the author was unable to accept "Einstein's gravitation" as a correct theory since the time of his *high school studies,* the free fall of bodies under gravity that has to occur necessarily along a straight radial line, thus without any possible curvature. On technical terms, the free fall establishes the consistency need for any gravitational theory not only to incorporate the NEWTONIAN attraction in a clear and unambiguous way, but also in such a way that all contributions from curvature should disappear for the free fall in favor of the pure Newtonian attraction. The fact that evidence so incontrovertible continues to be denied by organized interests on Einsteinian doctrines and their vast followers, most holding chairs of high academic fame, confirm the existence of a scientific obscurantism of potentially historical proportions.

achieved with yet unknown manipulations, the representation of the bending of light is not unique because bases on a nonunique PPN approximation having different parameters for different expansions. Finally, assuming that consistency and uniqueness are somewhat achieved, the representation is not invariant in time due to the noncanonical structure of general relativity.

THEOREM 1.4.6 [110]: General relativity is incompatible with experimental evidence because of the lack of consistent representation with curvature of the free fall of test bodies along a straight radial line.

Proof. A consistent representation of the free fall of a test body along a straight radial line requires that the Newtonian attraction be represented by the field equations necessarily without curvature, thus disproving the customary belief needed to avoid Corollary 1.4.2.A that said Newtonian attraction emerges at the level of the post-Newtonian approximation. **q.e.d.**

The absence in general relativity at large, thus including Einstein's gravitation, of well defined contributions due to the Newtonian attraction and to the assumed curvature of spacetime, and the general elimination of the former in favor of the latter, causes other catastrophic inconsistencies, such as the inability to represent the base Newtonian contribution in planetary motion as shown by Yilmaz [47], and other inconsistencies [48-52].

A comparison between special and general relativities is here in order. Special relativity can be safely claimed to be "verified by experiments" because the said experiments verify numerical values uniquely and unambiguously predicted by special relativity. By contrast, no such statement can be made for general relativity since the latter does not uniquely and unambiguously predict given numerical values due, again, to the variety of possible expansions and linearization.

The origin of such a drastic difference is due to the fact that *the numerical predictions of special relativity are rigorously controlled by the basic Poincaré "invariance". By contrast, one of the several drawbacks of the "covariance" of general relativity is precisely the impossibility of predicting numerical values in a unique and unambiguous way, thus preventing serious claims of true "experimental verifications" of general relativity.*

By no means the above analysis exhausts all inconsistencies of general relativity, and numerous additional ones do indeed exist, such as that expressed by the following:

THEOREM 1.4.7 [36]: Operator images of Riemannian formulations of gravitation are inconsistent on mathematical and physical grounds.

Proof. As established by Theorem 1.4.3, classical formulations of Riemannian gravitation are noncanonical. Consequently, all their operator counterparts must be nonunitary for evident reasons of compatibility. But nonunitary theories are known to be inconsistent on both mathematical and physical grounds [36]. In fact, on mathematical grounds, nonunitary theories of quantum gravity (see, e.g., Refs. [2j,2k]) do not preserve in time the basic units, fields and spaces, while,

on physical grounds, the said theories do not possess time invariant numerical predictions, do not possess time invariant Hermiticity (thus having no acceptable observables), and violate causality. **q.e.d**

The reader should keep in mind the additional well known inconsistencies of quantum gravity, such as the historical incompatibility with quantum mechanics, the lack of a credible PCT theorem, etc.

By no means, the inconsistencies expressed by Theorems 1.4.1 through 1.4.7 constitute all inconsistencies of general relativity. In the author's opinion, additional deep inconsistencies are caused by the fact that *general relativity does not possess a well defined Minkowskian limit,* while the admission of the Minkowski space as a tangent space is basically insufficient on dynamical grounds (trivially, because on said tangent space gravitation is absent). For two additional inconsistency theorems one may consult paper [111].

As an illustration, we should recall the controversy on conservation laws that raged during the 20-th century [75]. Special relativity has rigidly defined total conservation laws because they are the Casimir invariants of the fundamental Poincaré symmetry. By contrast, there exist several definitions of total conservation laws in a Riemannian representation of gravity due to various ambiguities evidently caused by the absence of a symmetry in favor of covariance.

Moreover, none of the gravitational conservation laws yields the conservation laws of special relativity in a clear and unambiguous way, precisely because of the lack of any limit of a Riemannian into the Minkowskian space. Under these conditions, the compatibility of general relativity with the special reduces to personal beliefs outside a rigorous scientific process.

In the author view, the most serious inconsistencies of general relativity are those of *experimental* character, such as the structural impossibility for the Riemannian geometry to permit unique and unambiguous numerical predictions due to the known large degrees of freedom in all PPN expansions; the necessary *absence* of curvature to represent consistently the free fall of bodies along a straight radial line; the gravitational deflection of light measured until now being purely *Newtonian* in nature; and others.

These inconsistencies are such to prevent serious attempts in salvaging general relativity. For instance, if the deflection of the speed of light is re-interpreted as being solely due to curvature without any Newtonian contribution, then general relativity admits other catastrophic inconsistencies, such as the inability to represent the Newtonian contribution of planetary motions pointed out by Yilmaz [27] and other inconsistencies such as those identified by Wilhelm [28-30] and other researchers.

When the inconsistencies of general relativity with experimental evidence are combined with the irreconcilable incompatibility of general relativity with unified field theory and the catastrophic axiomatic inconsistencies due to lack of invari-

ance [11m], time has indeed arrived for the scientific community to admit the need for fundamentally new vistas in our representation of gravitation, without which research is turned from its intended thrilling pursue of "new" knowledge to a sterile fanatic attachment to "past" knowledge.

1.4.5 Organized Scientific Corruption on Einstein Gravitation

An indication of the greatness of Albert Einstein is that he repeatedly expressed serious doubts on his gravitational theory, with particular reference to the r.h.s. of the field equations. By comparison, organized interests on Einstein have essentially suppressed any serious scientific process in the field via the abuse of academic credibility and public funds.

On one side, authoritative criticisms on Einstein gravitation (published in serious refereed journals around the world) have increased exponentially during the 20-th century. On the other side, said organized interests have completely ignored these qualified dissident views, let alone address and disprove them also in refereed journals as required by scientific ethics and accountability.

By recalling that the latter behavior is generally perpetrated under public financial support, we see the emergence of one of the most ascientific scenario in scientific history because, jointly with the lack of serious scientific work due to lack of dismissal of catastrophic inconsistencies, we have the violation of U. S. Federal Laws due to transparent misuse of public funds.

In view of the above, Santilli suggests the conduction of a senatorial investigation on all public funds spend for research in gravitation during the past fifty years. In the event this is not possible due to potential backing by (some) politicians to their academic brothers, Santilli recommends the filing of class actions in U. S. Federal Court against federal funding agencies, such as the National Science Foundation and the Department of Energy, jointly with representative institutions abusing public funds without proper scientific process.

At any rate, the current condition of research in gravitation should not be permitted to continued by any civilized society, while such a condition will continue indefinitely in the absence of a senatorial and/or judicial intervention due to the power and capillary organization of said interests on Einstein.

To begin some indication on the gravity of the problem, it should be indicated that the rejections of dissident papers on Einstein gravitation by orthodox technical journals the world over can only be qualified as being shameful for the physics community, because perpetrated without any serious objection or disproof, thus confirming the existence of an organized corruption on Einstein gravitation (see documentations in the footnotes of Volume IV).

Some of the rejections are done with extreme studious professionalism in implementing what amount to a real scientific crime. For instance, the journals of

the America Physical Society (APS) have issued a 'final rejection" for Santilli's dissident papers. Instead, they have issued ordinary rejections that, per APS statute, allow resubmissions. The anti-scientific aspect is that such rejections are continued for years and years for the studious intent of tiring the author (see also documentations in the footnotes of Volume IV).

The rejections of dissident papers on Einstein doctrines by the journals controlled by the British Institute of Physics (IOP) are even more insidious, because perpetrated with a higher degrees of sophistication in opposing undesired physical knowledge while dreaming to portray the opposite.

The rejections of dissident papers on Einstein doctrines by orthodox journals in France, Italy, Sweden, and other countries can only be dubbed as being pathetic by comparison with the preceding ones because expressing in a transparent way their strictly political and nonscientific motivation.

The above suppression of due scientific process establishes beyond credible doubt that research in gravitation are based on academic power and political schemes all over the world, and definitely not on scientific truth. Whether intentional or not, this behavior clearly serves organized, academic, financial and ethnic interests on Einsteinian doctrines. Other views are left to naive persons or accomplices.

It is then necessary to give some indication of other forms of "dismissals" by said organized interests of the inconsistencies of Einstein's gravitation

As a general rule, said interests have no credible technical argument to oppose the catastrophic inconsistencies here considered, all published in refereed journals (of which Santilli is not an editor). Consequently, said organized interests are left with equivocal attempts to discredit Santilli, such as the dubbing of doing "fringe science" by Wikipedia (while studies ignoring the catastrophic inconsistencies are serious science in Wikipedia view).

Others, such as Dimitri Rabounski, retort to other forms of dismissal beyond credibility. In fact, following the appearance of paper [111], Rabounski released in the internet a "Review of the paper *Inconsistencies of general relativity* by R. M. Santilli" in which he claims that "Santilli is a nuclear physicist" (sic!), the evident dream being to suggest that santilli is not qualified to discuss mathematically advanced issues based on the Riemannian geometry.

In reality, far from being a nuclear physicist,Santilli is an applied mathematician who has been a *member of the Department of Mathematics at Harvard University,* as everybody can see by inspecting Santilli's CV available in the internet following an easy search at google.com. Hence, Santilli is indeed fully qualified to identify inconsistencies of Einstein gravitation.

Following these inspiring introductory lines, Rabounski passes to truly ephemeral touches of the inconsistencies, claiming misrepresentations of the theory by Santilli, yet by carefully avoiding the addressing of the main ones, such as the impossi-

bility of representing free fall with curvature, the impossibility to predict the same numbers under the same conditions at different times due to the noncanonical structure of the theory, etc. Hence, Rabounski "objections" to the catastrophic inconsistencies of Einstein gravitation identified by Santilli are purely political and without any substantive scientific content.

On the pseudo-technical side, the reader in good faith will be amused to know a seemingly technical rebuttal by Eduardo A. Notte-Cuello and Waldyr A. Rodrigues, jr., who recently released a "paper" in the arXiv with an extensive and detailed review of the derivation of the Freud identity, something well known to experts, and conclude with the statment

In this paper we proved that, contrary to the claims in [29,30] (our references [95,110) *there is no incompatibility from the mathematical point of view between the Freud identity and the Einstein-Hilbert field equations of GR.*

The "paper" then passes to claim of rebuttal of other inconsistencies of Einstein gravitation due to problems with conservation laws, that have not been addressed by Santilli due to the large dissident literature by Yilmaz and numerous other authors.

The evident dream by Notte-Cuelo and Rodrigues is that of discrediting the inconsistencies of general relativity identified by Santilli, this time, with a smoke-screen of mathematics. in fact, the above quoted "main scope" of the "paper" is scientifically vacuous because every graduate student in physics knows that, *for a body with non-null electromagnetic fields,* Einstein's field equations do have a tensor source in the r.h.s, Eq. (1.4.2), in which case there is indeed no mathematical inconsistency between the field equations and the Freud identity as stated in Theorem 1.4.2.

However, in their detailed derivation of the Freud identity by Notte-Cuelo and Rodrigues carefully avoids quoting, let alone reviewing, Rund [26] main result, namely, that the tensor in the r.h.s. of the Freud identity is of *first order in magnitude.* By comparison, the tensor in the r.h.s. of Eqs. (1.4.2) is of lilliputian value. Hence, we have the *physical* inconsistencies between Einstein gravitation and the Freud identity of Theorem 1.4.2.

The collapse of scientific value of the "paper" by Notte-Cuelo and Rodrigues is then given by Einstein's gravitation for *bodies with null charge and null magnetic moments* for which it is]it prohibited to put any tensor in the r.h.s. of the field equations, in which Notte-Cuelo and Rodrigues mimic lack of knowledge, in which case we have a *mathematical and physical* incompatibility of Einstein gravitation with the Fred identiy,

The purely political character of the "paper" by Notte-Cuelo and Rodrigues emerges when one notes that, in the event these authors did not read carefully Refs. [95,110], Santilli did notified them of the above clarifications, but, as typically the case in the field, the clarifications were ignored in the arXiv upload

(Rodrigues was then terminated as editor of Algebras, Groups and Geometries for unethical conduct in this and other cases).

The intellectual dishonestly emerges rather forcefully from the fact that Santilli studies (such as paper [110]) present *nine* different theorems of catastrophic inconsistencies of Einstein gravitation, each one being sufficient to depenn Einstein gravitation from the list of serious *physical* theories. Notte-Cuelo and Rodrigues do quote Theorem 1.4.2 based on the Freud identity, but carefully avoid the quotation of the other *eight* theorems of catastrophic inconsistencies.

Whether intentional or not, the dishonest implication that may be perceived by the naive or uneducated reader of Notte-Cuelo and Rodrigues arXiv paper on the Freud identity is that "Santilli theorem 1.4.2 is wrong and, therefore, Einstein gravitation is correct," while in the scientific reality, even assuming that Theorem 1.4.2 might be wrong (contrary to all serious evidence), Einstein gravitation remains afflicted by *eight* remaining theorems of catastrophic inconsistencies.

It is hoped the above cases illustrate the reason for Santilli suggestions to have senatorial investigations or judicial proceedings on research in gravitation. In fact, it is absolutely certain that, when under oath in front of a jury, Rabounski would have indeed documented himself before venturing that Santilli is a "nuclear physicist," and, in front of a jury for judicial proceedings, Notte-Cuelo and Rodrigues vociferous posturing would turn into anguish and positively they would state that their results confirm fully, rather than dismiss, Santilli Theorem 1.4.2.

1.5 THE SCIENTIFIC IMBALANCE CAUSED BY NONCANONICAL AND NONUNITARY THEORIES

1.5.1 Introduction

When facing the limitations of special relativity and quantum mechanics for the representation of extended, nonspherical, deformable and hyperdense particles and antiparticles under linear and nonlinear, local and nonlocal as well as potential and nonpotential forces, a rather general attitude is that of attempting their generalization via the broadening into noncanonical and nonunitary structures, while preserving the mathematics of their original formulation.

Despite the widespread publication of papers on theories with noncanonical or nonunitary structures in refereed journals, including those of major physical societies, it is not generally known that these broader theories are afflicted by inconsistencies so serious to be also known as catastrophic.

Another basic objective of this monograph is the detailed identification of these inconsistencies because their only known resolution is that presented in the next chapters, that permitted by *new mathematics* specifically constructed from the physical conditions considered.

In fact, the broadening of special relativity and quantum mechanics into noncanonical and nonunitary forms, respectively, is necessary to exit form the class of equivalence of the conventional formulations. The resolution of the catastrophic inconsistencies of these broader formulations when treated via the mathematics of canonical and unitary theories, then leaves no other possibility than that of *broadening the basic mathematics.*

To complete the presentation of the foundations of the covering hadronic mechanics, in the next two sections we shall review the inconsistencies of noncanonical and nonunitary theories. The remaining sections of this chapter are devoted to an outline of hadronic mechanics so as to allow the reader to enter in a progressive way into the advanced formulations presented in the next chapters.

1.5.2 Catastrophic Inconsistencies of Noncanonical Theories

As recalled in Section 1.2, the research in classical mechanics of the 20-th century has been dominated by *Hamiltonian systems,* that is, systems admitting their complete representation via the *truncated Hamilton equations* (1.2.2), namely, the historical equations proposed by Hamilton in which the external terms have been cut out.

For the scope of this section, it is best to rewrite Eqs. (1.2.2) in the following unified form (see monographs [9] for details)[9]

$$b = (b^\mu) = (r, p) = (r^k, p_k), \tag{1.5.1a}$$

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t, b)}{\partial b^\nu}, \tag{1.5.1b}$$

$$H = K(p) + V(t, r, p), \tag{1.5.1c}$$

$$\mu = 1, 2, 3, ..., 6, \quad k = 1, 2, 3,$$

where H is the Hamiltonian, K is the kinetic energy, V is the potential energy, $\omega^{\mu\nu}$ is the canonical Lie tensor with explicit form

$$\omega^{\mu\nu} = \begin{pmatrix} 0 & I_{3\times 3} \\ -I_{3\times 3} & 0 \end{pmatrix} \tag{1.5.2}$$

and $I_{3\times 3} = Diag(1, 1, 1)$ is the unit matrix.

[9]We continue to denote the conventional associative multiplication of numbers, vector fields, operators, etc. with the notation $A \times B$ rather than the usual form AB, because the new mathematics necessary to resolve the catastrophic inconsistencies studied in this chapter is based on various different generalizations of the multiplication. As a consequence, the clear identification of the assumed multiplication will soon be crucial for the understanding of the equations of this monograph.

In the above unified notation, the brackets of the time evolution can be written

$$\frac{dA}{dt} = [A, H] = \frac{\partial A}{\partial b^\mu} \times \omega^{\mu\nu} \times \frac{\partial H}{\partial b^\nu}, \tag{1.5.3}$$

and they characterize a *Lie algebra,* as well known.

The above equations have a *canonical structure,* namely, their time evolution characterizes a *canonical transformation*[10],

$$b^\mu \to b'^\mu(b), \tag{1.5.4a}$$

$$\omega^{\mu\nu} \to \frac{\partial b'^\mu}{\partial b^\rho} \times \omega^{\rho\sigma} \times \frac{\partial b'^\nu}{\partial b^\sigma} \equiv \omega^{\mu\nu}; \tag{1.5.4b}$$

and the theory possesses the crucial property of predicting the same numbers under the same conditions at different times, a property generically referred to as *invariance,* such as the invariance of the basic analytic equations under their own time evolution

$$\frac{db^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} = 0 \to$$
$$\to \frac{db'^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t',b')}{\partial b'^\nu} = 0. \tag{1.5.5}$$

where the invariance is expressed by the preservation of the Lie tensor $\omega^{\mu\nu}$ and of the Hamiltonian H.

It is easy to predict that future research in classical mechanics will be dominated by *non-Hamiltonian systems,* that is, systems that cannot be entirely described by the Hamiltonian and require at least a second quantity for their complete description.

Alternatively, we are referring to systems with internal forces that are partly of potential type, represented by V, and partly of nonpotential type, thus requiring new quantities for their representation.

We are also referring to the transition from *exterior dynamical systems* recalled in Section 1.3 (systems of point-like particles moving in vacuum without collisions under sole action-at-a-distance potential interactions) to *interior dynamical systems* (extended, nonspherical and deformable particles moving within a resistive medium with action-at-a-distance potential forces plus contact, nonpotential, nonlocal, and integral forces).

As also recalled in Section 1.2, exterior dynamical systems can be easily represented with the truncated Hamilton equations, while the first representation of the broader non-Hamiltonian systems is given precisely by the historical analytic

[10]For several additional different but equivalent definitions of canonical transformations one may consult Ref. [54a], pages 187-188.

equations with external terms, Eqs. (1.3.2) that we now rewrite in the unified form

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} + F^\mu(t,b,\dot{b},...), \tag{1.5.6a}$$

$$F^\mu = (0, F_k), \quad \mu = 1,2,...,6, \quad k = 1,2,3. \tag{1.5.6b}$$

Nevertheless, as also recalled in Section 1.3, the addition of the external terms creates serious structural problems since the brackets of the new time evolution

$$\frac{dA}{dt} = (A,H,F) = \frac{\partial A}{\partial b^\mu} \times \omega^{\mu\nu} \times \frac{\partial H}{\partial b^\nu} + \frac{\partial A}{\partial b^\mu} \times F^\mu, \tag{1.5.7}$$

violate the conditions to characterize an algebra (since they violate the right distributive and scalar laws), let alone violate all possible Lie algebras, thus prohibiting the studies of basic aspects, such as spacetime symmetries under nonpotential forces.

As experienced by the author, when facing the latter problems, a rather natural tendency is that of using coordinate transforms $b \to b'(b)$ to turn a systems that is non-Hamiltonian in the b-coordinates into a Hamiltonian form in the b'-coordinates,

$$\frac{db^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} - F^\mu(t,b,\dot{b},...) = 0 \to$$

$$\to \frac{db'^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H'(t,b')}{\partial b^\nu} = 0. \tag{1.5.8}$$

These transformations always exist under the necessary continuity and regularity conditions, as guaranteed by *Lie-Koening theorem* of analytic mechanics or the *Darboux Theorem* of the symplectic geometry) [9b].

This first attempt has no physical value because of excessive problems identified in Section 1.2, such as: the lack of physical meaning of quantum formulations in the b'-coordinates; the impossibility of placing a measuring apparatus in the transformed coordinates; the loss of all known relativities due to the necessarily nonlinear character of the transforms with consequential mapping of inertial into noninertial frames; and other problems.

The above problems force the restriction of analytic representations of non-Hamiltonian systems within the fixed coordinates of the experimenter, the so-called *direct analytic representations* of Assumption 1.2.1 [9].

Under the latter restriction, the second logical attempt for quantitative treatments of non-Hamiltonian systems is that of broadening conventional canonical theories into a noncanonical form at least admitting a consistent algebra in the brackets of the time evolution, even though the resulting time evolution of the broader equations cannot characterize a canonical transformation.

As an illustration of these second lines of research, in 1978 the author wrote for Springer-Verlag his first volume of *Foundations of Theoretical Mechanics* [9a]

devoted to the integrability conditions for the existence of a Hamiltonian representation (the so-called *Helmholtz's conditions of variational selfadjointness*). The evident scope was that of identifying the limits of applicability of the theory within the fixed coordinates of the experimenter.

A main result was the proof that *the truncated Hamilton equations admit a direct analytic representation in three space dimensions only of systems with potential (variationally selfadjoint) forces,*[11] thus representing only a small part of what are generally referred to as *Newtonian systems.*

In this way, monograph [9a] confirmed the need to enlarge conventional Hamiltonian mechanics within the fixed frame of the experimenter in such a way to admit a direct representation of all possible Newtonian systems verifying the needed regularity and continuity conditions.

Along the latter line of research, in 1982 the author published with Springer-Verlag his second volume of *Foundations of Theoretical Mechanics* [9b] for the specifically stated objective of broadening conventional Hamiltonian mechanics in such a way to achieve *direct universality,* that is, the capability of representing *all* Newtonian systems (universality) in the fixed frame of the experimenter (direct universality), while jointly preserving not only an algebra, but actually the *Lie algebra* in the brackets of the time evolution.

These efforts gave birth to a broader mechanics called by the author *Birkhoffian mechanics* in honor of the discoverer of the basic equations, G. D. Birkhoff [37], which equations can be written in the unified form

$$\frac{db^\mu}{dt} = \Omega^{\mu\nu}(b) \times \frac{\partial B(t,b)}{\partial b^\nu}, \tag{1.5.9}$$

where $B(t,b)$ is called the *Birkhoffian* in order to distinguish it from the Hamiltonian (since B does not generally represent the total energy), and $\Omega^{\mu\nu}$ is a *generalized Lie tensor,* in the sense that the new brackets

$$\frac{dA}{dt} = [A,B]^* = \frac{\partial A}{\partial b^\mu} \times \Omega^{\mu\nu} \times \frac{\partial B}{\partial b^\nu}, \tag{1.5.10}$$

still verify the Lie algebra axioms (see Ref. [9b] for details).

Stated in different terms, the main efforts of monograph [54b] were to show that, under the necessary continuity and regularity properties, the historical Hamilton's equations with external terms always admit a reformulation within the fixed frame of the experimenter with a consistent Lie algebra in the brackets of the time evolution,

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} + F^\mu(t,b,...) \equiv \Omega^{\mu\nu}(b) \times \frac{\partial B(t,b)}{\partial b^\nu}. \tag{1.5.11}$$

[11] The truncated Hamilton equations admit analytic representations of nonconservative systems but only *in one dimension,* which systems are essentially irrelevant for serious physical applications.

In this case, rather than being represented with H and F, non-Hamiltonian systems are represented with B and Ω.

Monograph [9b] achieved in full the intended objective with the proof that *Birkhoffian mechanics is indeed directly universal for all possible well behaved local-differential Newtonian systems,* and admits the following *generalized canonical transformations,*

$$\Omega^{\mu\nu}(b) \to \frac{\partial b'^{\mu}}{\partial b^{\rho}} \times \Omega^{\rho\sigma}(b(b')) \times \frac{\partial b'^{\nu}}{\partial b^{\sigma}} \equiv \Omega^{\mu\nu}(b'). \tag{1.5.12}$$

Monograph [9b] concluded with the indication of the apparent full equivalence of the Birkhoffian and Hamiltonian mechanics, since the latter is admitted as a particular case of the former (when the generalized Lie tensor acquires the canonical form), both theories are derivable from a variational principle, and both theories admit similar transformation properties.

Since the generalized Lie tensor $\Omega^{\mu\nu}$ and related brackets $[A, B]^*$ are antisymmetric, we evidently have conservation laws of the type

$$\frac{dB}{dt} = [B, B]^* \equiv 0, \tag{1.5.13}$$

Consequently, Birkhoffian mechanics was suggested in monograph [54b] for the representation of *closed-isolated non-Hamiltonian systems* (such as Jupiter).

The representation of *open-nonconservative non-Hamiltonian systems* required the identification of a yet broader mechanics with a consistent algebra in the brackets of the time evolution, yet such that the basic brackets are not antisymmetric. The solution was reached in monographs [38] via the *Birkhoffian-admissible mechanics* with basic analytic equations

$$\frac{db^{\mu}}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t, b)}{\partial b^{\nu}} + F^{\mu}(t, b, ...) \equiv S^{\mu\nu}(b) \times \frac{\partial B(t, b)}{\partial b^{\nu}}, \tag{1.5.14}$$

where the tensor $S^{\mu\nu}$ is *Lie-admissible* According to Santilli's [39] realization of Albert [40] abstract formulation, namely, in the sense that the generalized brackets of the time evolution

$$\frac{dA}{dt} = (A, B) = \frac{\partial A}{\partial b^{\mu}} \times S^{\mu\nu}(b) \times \frac{\partial B}{\partial b^{\nu}}, \tag{1.5.15}$$

verify all conditions to characterize an algebra, and their attached antisymmetric brackets

$$[A, B]^* = (A, B) - (B, A), \tag{1.5.16}$$

characterize a generalized Lie algebra as occurring in Birkhoffian mechanics.

The representation of the open-nonconservative character of the equations was then consequential, since the lack of antisymmetry of the brackets yields the

correct *time rate of variation of the energy* $E = B$

$$\frac{dE}{dt} = (E, E) = F_k \times v^k, \tag{1.5.17}$$

and the same occurs for all other physical quantities.

Monographs [38] then proved the direct universality of Birkhoffian-admissible mechanics for all open-nonconservative systems, identified its transformation theory and provided the following elementary, yet universal realization of the Lie-admissible tensor S for $B = H$ representing the total *nonconserved* energy

$$S^{\mu\nu} = \begin{pmatrix} 0 & I \\ -I & F/(\partial H/\partial p) \end{pmatrix}. \tag{1.5.18}$$

Note that *the Birkhoffian-admissible mechanics is structurally irreversible,* in the sense of being irreversible for all possible energies and Birkhoffian functions since the basic Lie-admissible tensor is itself irreversible, $S(t, b) \neq S(-t, b)$, thus being particularly suited to represent irreversible systems.

However, studies conducted after the publication of monographs [9,38] revealed the following seemingly innocuous feature:

LEMMA 1.5.1 [11b]: *Birkhoffian and Birkhoffian-admissible mechanics are noncanonical theories, i.e., the generalized canonical transformations, are noncanonical,*

$$\omega^{\mu\nu} \to \frac{\partial b'^{\mu}}{\partial b^{\rho}} \times \omega^{\rho\sigma} \times \frac{\partial b'^{\nu}}{\partial b^{\sigma}} \equiv \Omega^{\mu\nu}(b') \neq \omega^{\mu\nu}. \tag{1.5.19}$$

It is important to understand that *Birkhoffian and Birkhoffian-admissible mechanics are mathematically attractive, but they are not recommended for physical applications, both classically as well as foundations of operator theories.*

The canonical Lie tensor has the well known explicit form (1.5.2). Therefore, the diagonal matrix $I_{3\times 3}$ is left invariant by canonical transformations. But $I_{3\times 3}$ is the *fundamental unit of the basic Euclidean geometry.* As such, it represents in an abstract and dimensionless form the basic units of measurement, such as

$$I_{3\times 3} = Diag.(1\text{cm}, 1\text{cm}, 1\text{cm}). \tag{1.5.20}$$

By their very definition, noncanonical transformations do not preserve the basic unit, namely, they are transformations of the representative type (with arbitrary new values)

$$I_{3\times 3} = Diag.(1\text{cm}, 1\text{cm}, 1\text{cm}) \to$$

$$\to U \times I_{3\times 3} \times U^t = Diag.(3.127 \text{ cm},\ e^{-212} \text{ cm},\ \log 45 \text{ cm}), \tag{1.5.21a}$$

$$U \times U^t \neq I, \tag{1.5.21b}$$

where t stands for transposed. We, therefore, have the following important:

THEOREM 1.5.1 [53]: *Whether Lie or lie-admissible, all classical noncanonical theories are afflicted by catastrophic mathematical and physical inconsistencies.*

Proof. Noncanonical theories do not leave invariant under time evolution the basic unit. This implies the loss under the time evolution of the base field on which the theory is defined. Still in turn, the loss in time of the base field implies catastrophic mathematical inconsistencies, such as the lack of preservation in time of metric spaces, geometries, symmetries, etc., since the latter are defined over the field of real numbers.

Similarly, noncanonical theories do not leave invariant under time evolution the basic units of measurements, thus being inapplicable for consistent measurements. The same noncanonical theories also do not possess time invariant numerical predictions, thus suffering catastrophic physical inconsistencies. **q.e.d.**

In conclusion, the regaining of a consistent algebra in the brackets of the time evolution, as it is the case for Birkhoffian and Birkhoffian-admissible mechanics, is not sufficient for consistent physical applications because the theories remain noncanonical. In order to achieve a physically consistent representation of non-Hamiltonian systems, it is necessary that

1) The analytic equations must be derivable from a first-order variational principle, as necessary for quantization;

2) The brackets of the time evolution must characterize a consistent algebra admitting exponentiation to a transformation group, as necessary to formulate symmetries; and

3) The resulting theory must be invariant, that is, must admit basic units and numerical predictions that are invariant in time, as necessary for physical value.

Despite the large work done in monographs [9,38], the achievement of all the above conditions required the author to resume classical studies from their foundations.

These third efforts finally gave rise to the new *Hamilton-Santilli iso-, geno- and hypermechanics* [10b] that do verify all conditions 1), 2) and 3), thus being suitable classical foundations of hadronic mechanics, as reviewed in Chapter 3.

However, the joint achievement of conditions 1), 2) and 3) for non-Hamiltonian systems required the prior construction of *basically new mathematics,* [10a] today known as *Santilli's iso-, geno- and hyper-mathematics,* as also reviewed in Chapter 3.

This section would be grossly incomplete and potentially misleading without a study of requirement 1), with particular reference to the derivability of analytic equations from a "first-order" variational principle.

Classical studies of non-Hamiltonian systems are essential, not only to identify the basic methods for their treatment, but above all to identify quantization channels leading to unique and unambiguous operator formulations.

Conventional Hamiltonian mechanics provides a solid foundation of quantum mechanics because it is derivable from the variational principle that we write in the unified notation [9a]

$$\delta A^\circ = \delta \int [R^\circ_\mu(b) \times db^\mu - H \times dt] =$$

$$= \delta \int (p_k \times dr^k - H \times dt), \tag{1.5.22}$$

where the functions R°_μ have the canonical expression

$$(R^\circ_\mu) = (p_k, 0), \tag{1.5.23}$$

under which expression the canonical tensor assumes the realization

$$\omega_{\mu\nu} = \frac{\partial R^\circ_\nu}{\partial b^\mu} - \frac{\partial R^\circ_\mu}{\partial b^\nu}, \tag{1.5.24a}$$

$$(\omega_{\mu\nu}) = (\omega^{\alpha\beta})^{-1}. \tag{1.5.24b}$$

As it is well known, the foundations for quantization are given by the *Hamilton-Jacobi equations* here expressed in the unified notation of Ref. [9a]

$$\frac{\partial A^\circ}{\partial t} = -H, \quad \frac{\partial A^\circ}{\partial b^\mu} = R^\circ_\mu, \tag{1.5.25}$$

that can be written explicitly in the familiar forms

$$\frac{\partial A^\circ}{\partial t} + H = 0, \tag{1.4.26a}$$

$$\frac{\partial A^\circ}{\partial r^k} - p_k = 0, \tag{1.5.26b}$$

$$\frac{\partial A^\circ}{\partial p_k} = 0, \tag{1.5.26c}$$

The use of the *naive quantization*

$$A^\circ \to -i \times \hbar \times \ell n\, \psi, \tag{1.5.27}$$

yields *Schrödinger's equations* in a unique and unambiguous way

$$\frac{\partial A^\circ}{\partial t} + H = 0 \to -i \times \bar{h} \frac{\partial \psi}{\partial t} - H \times \psi = 0, \tag{1.5.28a}$$

$$\frac{\partial A^\circ}{\partial r^k} = p_k \to -i \times \hbar \times \frac{\partial \psi}{\partial r^k} - p_k \times \psi = 0, \tag{1.5.28b}$$

$$\frac{\partial A^\circ}{\partial p_k} = 0 \to \frac{\partial \psi}{\partial p_k} = 0. \tag{1.4.28c}$$

The much more rigorous *symplectic quantization* yields exactly the same results and, as such, it is not necessary for these introductory notes.

A feature crucial for quantization is Eq. (1.5.26c) from which it follows that *the canonical action A° is independent from the linear momentum*, i.e.,

$$A^\circ = A^\circ(t, r). \tag{1.5.29}$$

an occurrence generally (but not universally) referred in the literature as characterizing a *first-order action functional.*

From the naive quantization it follows that, in the configuration representation, *the wave function originating from first-order action functionals is independent from the linear momentum* (and, vice-versa, in the momentum representation it is independent from the coordinates),

$$\psi = \psi(t, r), \tag{1.5.30}$$

which property is crucial for the axiomatic structure of quantum mechanics, e.g., for the correct formulation of Heisenberg's uncertainty principle, causality, Bell's inequalities, etc.

A serious knowledge of hadronic mechanics requires the understanding of the reason *Birkhoffian mechanics cannot be assumed as a suitable foundations for quantization.* Birkhoff's equations can indeed be derived from the variational principle (see monograph [9b] for details)

$$\delta A = \delta \int [R_\mu(b) \times db^\mu - B \times dt], \tag{1.5.31}$$

where the new functions $R_\mu(b)$ have the general expression

$$(R_\mu(b)) = (A_k(t, r, p), B^k(t, r, p)), \tag{1.5.32}$$

subject to the regularity condition that $\text{Det.}\, \Omega \neq 0$, under which Birkhoff's tensor assumes the realization

$$\Omega_{\mu\nu}(b) = \frac{\partial R_\nu}{\partial b^\mu} - \frac{\partial R_\mu}{\partial b^\nu}, \tag{1.5.33a}$$

$$(\Omega_{\mu\nu}) = (\Omega)^{\alpha\beta})^{-1}, \tag{1.5.33b}$$

with *Birkhoffian Hamilton-Jacobi equations* [9b]

$$\frac{\partial A}{\partial t} = -B, \quad \frac{\partial A}{\partial b^\mu} = R_\mu. \tag{1.5.34}$$

As one can see, Birkhoffian expressions (1.5.31)–(1.5.33) appear to be greatly similar to the corresponding Hamiltonian forms (1.4.22)–(1.4.26). Nevertheless, there is a fundamental structural difference between the two equations given by the fact that *the Birkhoffian action does indeed depend on the linear momenta,*

$$A = A(t, r, p), \tag{1.5.35}$$

a feature generally referred to as characterizing a *second-order action functional.*

As a consequence, *the "wavefunction" resulting from any quantization of Birkhoffian mechanics also depends on the linear momentum,*

$$\psi = \psi(t, r, p), \tag{1.5.36}$$

by characterizing an operator mechanics that is beyond our current technical knowledge for quantitative treatment, since such a dependence would require a dramatic restructuring of all quantum axioms.

In fact, the use of a naive quantization,

$$A(t, r, p) \to -i \times \hbar \times \ell n\ \psi(t, r, p), \tag{1.5.37}$$

characterizes the following maps

$$\frac{\partial A}{\partial t} + B = 0 \to -i \times \bar{h} \frac{\partial \psi}{\partial t} - B \times \psi = 0, \tag{1.5.38a}$$

$$\frac{\partial A}{\partial b^\mu} - R_\mu = 0 \to -i \times \hbar \times \frac{\partial \psi}{\partial b^\mu} - R_\mu \times \psi = 0. \tag{1.5.38b}$$

A first problem is that the latter equations are generally nonlinear and, as such, they cannot be generally solved in the r- and p-operators. This causes the emergence of an operator mechanics in which it is impossible to define basic physical quantities, such as the linear momentum or the angular momentum, with consequential lack of currently known physical relevance at this moment.

On more technical grounds, in the lifting of Hamiltonian into Birkhoffian mechanics, there is the replacement of the r-coordinates with the R-functions. In fact, the Birkhoffian action has the explicit dependence on the R-functions, $A = A[t, R(b)] = A'(t, r, p)$. As such, the Birkhoffian action can indeed be interpreted as being of first-order, but in the R-functions, rather than in the r-coordinates.

Consequently, a correct operator image of the Birkhoffian mechanics is given by the expressions (first derived in Ref. [11b])

$$i \times \hbar \times \frac{\partial \psi[t, R(b)]}{\partial t} = B \times \psi[t, R(b)], \tag{1.5.39a}$$

$$-i \times \hbar \times \frac{\partial \psi[t, R(b)]}{\partial b^\mu} = R_\mu(b) \times \psi[t, R(b)]. \tag{1.5.39b}$$

As we shall see in Chapter 3, the above equations characterize a *covering of hadronic (rather than quantum) mechanics,* in the sense of being structurally more general, yet admitting hadronic mechanics as a particular case.

Even though mathematically impeccable, intriguing, and deserving further studies, the mechanics characterized by Eqs. (1.5.39) is excessively general for our needs, and its study will be left to the interested reader.

The above difficulties identify quite precisely the first basic problem for the achievement of a physically consistent and effective formulation of hadronic mechanics, consisting in the need of constructing a new mathematics capable of representing CLOSED (that is, isolated) non-Hamiltonian systems via a first-order variational principle (as required for consistent quantization), admitting antisymmetric brackets in the time evolution (as required by conservation laws), and possessing time invariant units and numerical predictions (as required for physical value).

The need to construct a new mathematics is evident from the fact that no pre-existing mathematics can fulfill the indicated needs. As we shall see in Chapter 3, *Santilli's isomathematics* [10a] has been constructed precisely for and does indeed solve these specific problems.

The impossibility of assuming the *Birkhoffian-admissible mechanics* as the foundation of operator formulation for OPEN (that is, nonconservative) non-Hamiltonian systems is clearly established by the fact that said mechanics *is not derivable from a variational principle.*[12]

The latter occurrence identifies a much more difficult task given by the *need to construct a yet broader mathematics capable of representing open non-Hamiltonian systems via a first-order variational principle (as required for consistent quantization), admitting non-antisymmetric brackets in the time evolution (as required by non-conservation laws), and possessing time invariant units and numerical predictions (as required by physical value).*

The lack of any pre-existing mathematics for the fulfillment of the latter tasks is beyond credible doubt. Rather than adapting nature to pre-existing mathematics, the author has constructed a yet broader mathematics, today known as *Santilli's genomathematics* [10a], that does indeed achieve all indicated objectives, as outlined in Chapter 4.

Readers interested in the depth of knowledge are suggested to meditate a moment on the implications of the above difficulties. In fact, these difficulties have caused the impossibility in the 20-th century to achieve a meaningful operator formulation of contact, nonconservative and nonpotential interactions.

A consequence has been the widespread belief that nonpotential interactions "do not exist" in the particle world, a view based on the lack of existence of their

[12]Because conventional variations δ can only characterize antisymmetric tensors of type $\omega_{\mu\nu}$ or $\Omega_{\mu\nu}$ and cannot characterize non-antisymmetric tensors such as the Lie-admissible tensor $S_{\mu\nu}$.

operator representation, with negative implications at all levels of knowledge, such as the impossibility of achieving a meaningful understanding of the origin of irreversibility.

As a consequence, the resolution of the difficulties in the quantization of nonpotential interactions achieved by hadronic mechanics implies a rather profound revision of most of the scientific views of the 20-th century, as we shall see in the subsequent chapters.

1.5.3 Catastrophic Inconsistencies of Nonunitary Theories

Once the limitations of quantum mechanics are understood (and admitted), another natural tendency is to exit from the class of equivalence of the theory via suitable generalizations, while keeping the mathematical methods used for quantum mechanics.

It is important for these studies to understand that these efforts are afflicted by catastrophic mathematical and physical inconsistencies equivalent to those suffered by classical noncanonical formulations based on the mathematics of canonical theories.

The author has dedicated his research life to the construction of axiomatically consistent and invariant generalizations of quantum mechanics for the treatment of nonlinear, nonlocal, and nonpotential effects because they are crucial for the prediction and treatment of new clean energies and fuels.

In this section we review the foundations of these studies with the identification, most importantly, of the failed attempts in the hope of assisting receptive colleagues in avoiding the waste of their time in the study of theories that are mathematically significant, yet cannot possibly have real physical value.

To begin, let us recall that a theory is said to be *equivalent to quantum mechanics* when it can be derived from the latter via any possible *unitary transform* on a conventional Hilbert space $\mathcal{H}$ over the field of complex numbers $C = C(c, +, \times)$,

$$U \times U^\dagger = U^\dagger \times U = I, \tag{1.5.40}$$

under certain conditions of topological smoothness and regularity hereon ignored for simplicity, where "$\times$" represents again the conventional associative product of numbers or matrices, $U \times U^\dagger \equiv UU^\dagger$.

As a consequence, *a necessary and sufficient condition for a theory to be inequivalent to quantum mechanics is that it must be outside its class of unitary equivalence,* that is, the new theory is connected to quantum mechanics via a *nonunitary transform*

$$U \times U^\dagger \neq I. \tag{1.5.41}$$

generally defined on a conventional Hilbert space $\mathcal{H}$ over C.

Therefore, true generalized theories must have a *nonunitary structure*, i.e., their time evolution must verify law (1.5.41), rather than (1.5.40).[13]

During his graduate studies in physics at the University of Torino, Italy, and as part of his Ph. D. thesis, Santilli [41-43] published in 1967 the following (p, q)-*parametric deformation of the Lie product* $A \times B - B \times A$, the first in scientific records

$$\begin{aligned}(A, B) = p \times A \times B - q \times B \times A = \\ = m \times (A \times B - B \times A) + n \times (A \times B + B \times A) = \\ = m \times [A, B] + n \times \{A, B\},\end{aligned} \tag{1.5.42}$$

where $p = m + n, q = n - m$ and $p \pm q$ are non-null parameters.[14]

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[13]The reader should be aware that there exist in the literature numerous claims of "generalizations of quantum mechanics" although they have a unitary time evolution and, consequently, do not constitute true generalizations. All these "generalizations" will be ignored in this monograph because they will not resist the test of time.

[14]In 1985, Biedenharn [44] and MacFairlane [45] published their papers on the simpler q-deformations

$$A \times B - q \times B \times A$$

without a quotation of the origination of the broader form by the author [41] of 1967

$$p \times A \times B - q \times B \times A.$$

Biedenharn was fully aware of origination [41] as established by the fact that Biedenharn had been part of a DOE research grant application jointly with the author and others, precisely on the latter deformations, application filed two years before the publication of paper [44] (see the full documentation in Refs. [93,94]). Unfortunately for him, Biedenharn was unable to quote origination [41] in his paper [44] for reasons explained below. Similarly, MacFairlane had been made aware of the (p, q)-deformations by the author himself years before paper [45] (see, again, the documentation in [93,94]), but was requested to abstein from proper quotation.

Ironically, by the time Biedenharn and MacFairlane published their papers, the author had already abandoned the field he initiated two decades earlier because of catastrophic inconsistencies studied in this section. The author met Biedenharn the last time prior to his departure at the *Wigner Symposium* held at Oxford University, England, in 1993. During that meeting Biedenharn confessed to the author that he had suppressed origination [41] of the q-deformations in his paper [44] because of "peer pressures from the Cantabridgean area." Biedenharn also confessed to the author that, following the publication of his paper [44], he became aware of the catastrophic inconsistencies of q-deformations, and confirmed that the "q-deformations have no physical value as treated so far."

Following the above behavior by Biedenharn and MacFairlane, the editors in the late 1980s and early 1990s of the American, British, Italian and other physical societies refused to quote paper [41] in the thousands of papers in the field, despite clear documentation of prior paternity. Because of these occurrences, the author acquired the dubbing of *the most plagiarized physicist of the 20-th century.* In reality, the author expressed his appreciation to both Biedenharn and MacFairlane because he did not want to have his name associated to thousands of papers *all* catastrophically inconsistent.

The author remembers Larry Biedenharn as a very brilliant scientist with a pleasant personality and a great potential for basic discoveries. Unfortunately, he was unable to avoid being controlled by organized interests in physics as a condition for an academic position. Consequently, he did indeed achieve a brilliant chair in physics at Duke University, but at the prize of being mainly remembered as an expert in the rotational symmetry with some ethical overtone for plagiarisms. By contrast, the author trashed out any desire for a political chair at Harvard University as a necessary condition for freedom in basic research (see book [93] and the 1132 pages of documentation [94]).

By remembering that the Lie product characterizes *Heisenberg's equations,* the above generalized product was submitted as part of the following *parametric generalization of Heisenberg's equations* in its finite and infinitesimal forms [41,42]

$$A(t) = U \times A(0) \times U^\dagger = e^{i\times H\times q\times t} \times A(0) \times e^{-i\times t\times p\times H}, \tag{1.5.43a}$$

$$i\ dA/dt = (A, H) = p\times A\times H - q\times H\times A, \tag{1.5.43b}$$

with classical counterpart studied in Ref. [43].

After an extensive research in European mathematics libraries (conducted prior to the publication of Ref. [41] with the results listed in the same publication), the brackets $(A, B) = p\times A\times B - q\times B\times A$ resulted to be *Lie-admissible* according to A. A. Albert [40], that is, the brackets are such that their attached antisymmetric product

$$[A\hat{,}B] = (A, B) - (B, A) = (p + q) \times [A, B], \tag{1.5.44}$$

characterizes a *Lie algebra.*

Jointly, brackets (A, B) are *Jordan admissible* also according to Albert, in the sense that their attached symmetric product,

$$\{A\hat{,}B\} = (A, B) + (B, A) = (p + q) \times \{A, B\}, \tag{1.5.45}$$

The following episode illustrates the above lines. In the early 1980s, the author was working at the foundation of the isotopies of the Galilei and Einstein relativities, the lifting of the rotational symmetry to represent the transition from stationary orbits with the usual *conserved* angular momentum (exact $O(3)$ symmetry), to unstable orbits with *varying* angular momentum (exact $O(3)$-admissible symmetry), discussed in details in *Elements of Hadronic Mechanics,* Volume II, with a brief review in Chapters 3 and 4 of this volume. To proceed, the author phoned the biggest U. S. expert in the rotational symmetry, Larry Biedenharn, and asked to deliver an seminar at his department to hear his critical comments. With his innate courtesy, Biedenharn quickly agreed, and set the date of the seminar. The author and his family then drove for two days, from Cambridge, Massachusetts, to Durham, North Carolina, for the meeting.

At the time of the seminar, the large lecture room at Duke University was empty (an occurrence often experienced by the author), with the sole exception of Larry Biedenharn and the chairman of the department (the author is unable to remember names of insignificant persons). Following routine presentations, the author's seminar lasted only a few seconds consisting in drawing in the blackboard a stable orbit of a satellite around Earth with exact $O(3)$ symmetry, and then drawing a decaying orbit of the same satellite during re-entry in Earth's atmosphere with "continuously decaying angular momentum and consequential breaking of the rotational symmetry." At the mere mention of this physical evidence, the department chairman went into a rage of nonscientific nonsense preventing the author from proffering any additional word for the unspoken but trivial reason that the breaking of the rotational symmetry implies the collapse of Einsteinian doctrines with consequential loss of money, prestige and power. In the middle of said rage, the author broke the chalk and left the room.

The author sensed Biedenharn's inner tragedy for, on one side, being sincerely interested in the topic while, on the other side, being forced to accept the control of his science to keep his academic job. For this reason, the author and his wife accepted the kind dinner invitation by the Biedenharns, but did run away from Duke University as fast as possible early the following morning. Had Larry Biedenharn been able to cut out the organized scientific crime at his department (where "crime" is intended in the Latin sense of damage to society for equivocal personal gains), he would have been remembered for a major structural advance in his field. The episode reinforced the soundedness of the author's decision to have trashed out Harvard University by the time of this episode as a necessary condition for freedom of scientific inquiries.

characterizes a *Jordan algebra.*

At that time (1967), only three articles on this subject had appeared in Lie- and Jordan-admissibility in the sole mathematical literature (see Ref. [41]).

In 1985, Biederharn [44] and MacFairlane [45] published their papers on the simpler q-deformations $A \times B - q \times B \times A$ without a quotation of the origination of the broader form $p \times A \times B - q \times B \times A$ by Santilli [41] in 1967.

Regrettably, Biedenharn and MacFairlane abstained from quoting Santilli's origination of twenty years earlier despite their documented knowledge of such an origination.

For instance, Biedenharn and Santilli had applied for a DOE grant precisely on the same deformations two years prior to Biedenharn's paper of 1985, and Santilli had personally informed MacFairlaine of said deformations years before his paper of 1985.

The lack of quotation of Santilli's origination of q-deformations resulted in a large number of subsequent papers by numerous other authors that also abstained from quoting said origination (see representative contributions [46-49]), for which reason Santilli has been often referred to as the "most plagiarized physicist of the 20-th century".

Ironically, at the time Biedenharn and MacFairlane published their paper on q-deformations, Santilli had already abandoned them because of their catastrophic mathematical and physical inconsistencies studied in this Section.

In 1978, when at Harvard University, Santilli proposed the following *operator deformation of the Lie product* [Ref. [50], Eqs. (4.15.34) and (4.18.11)],

$$\begin{aligned}(A\hat{,}B) = A \lhd B - B \rhd A = \\ = A \times P \times B - B \times Q \times A = \\ = (A \times T \times B - B \times T \times A) + (A \times W \times B + B \times W \times A) = \\ = [A\hat{,}B] + \{A\hat{,}B\},\end{aligned} \tag{1.5.46}$$

where $P = T + W, Q = W - T$ and $P \pm Q$ are, this time, fixed non-null matrices or operators.

Evidently, product (1.5.46) remains jointly Lie-admissible and Jordan-admissible because the attached antisymmetric and symmetric brackets,

$$[A\hat{,}B] = (A\hat{,}B) - (B\hat{,}A) = A \times T \times B - B \times T \times A, \tag{1.5.47a}$$

$$\{A\hat{,}B\} = (A\hat{,}B) + (B\hat{,}A) = A \times W \times B + B \times W \times A, \tag{1.5.47b}$$

characterizes a *Lie-Santilli and Jordan-Santilli isoalgebra* (see Chapter 4 for details).

The reader should be aware that the following alternative versions of product (1.5.46),

$$P \times A \times B - Q \times B \times A, \tag{1.5.48a}$$

$$A \times B \times P - B \times A \times Q, \tag{1.5.48b}$$

do not constitute an algebra since the former (latter) violates the left (right) distributive and scalar laws [50].

The above operator deformations of the Lie product was also submitted in the original proposal [50] of 1978 as the fundamental equations of hadronic mechanics via the following broader *operator Lie-admissible and Jordan-admissible generalization of Heisenberg's equations* in its finite and infinitesimal forms[15]

$$A(t) = U \times A(0) \times U^\dagger = e^{i \times H \times Q \times t} \times A(0) \times e^{-i \times t \times P \times H}, \tag{1.5.49a}$$

$$i\ dA/dt = (A\hat{,}H) = A \lhd H - H \rhd A =$$

$$= A \times P \times H - H \times Q \times A, \tag{1.5.49b}$$

$$P = Q^\dagger, \tag{1.5.49c}$$

which equations, as we shall see in Chapter 4, are the fundamental equations of hadronic mechanics following proper mathematical treatment.

It is an instructive exercise for the reader interested in learning the foundation of hadronic mechanics to prove that:

1) Time evolutions (1.5.43) and (1.5.49) are *nonunitary,* thus being outside the class of unitary equivalence of quantum mechanics;

2) The application of a nonunitary transform $R \times R^\dagger \neq I$ to structure (1.5.43) yields precisely the broader structure (1.5.49) by essentially transforming the parameters p and q into the operators

$$P = p \times (R \times R^\dagger)^{-1}, \quad Q = q \times (R \times R^\dagger)^{-1}; \tag{1.5.50}$$

3) The application of additional nonunitary transforms $S \times S^\dagger \neq I$ to structure (1.5.50) preserves its Lie-admissible and Jordan-admissible character, although with different expressions for the P and Q operators.

The above properties prove the following:

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[15]The author would like to be buried in Florida, the land he loved most, and have Eq. (1.5.49b) reproduced in his tombstone as follows:

Ruggero Maria Santilli
Sept. 8, 1935 - xxx, xx, xxxx

$$i\ dA/dt = A \lhd H - H \rhd A.$$

Also, the author would like his coffin to be sufficiently heavy so as to avoid floating when Florida will be submerged by the now inevitable melting of the polar ice. The author wants Eq. (1.5.49b) in his tombstone because, in view of its direct universality, it will take centuries to achieve a broader description of nature equally invariant and equally based on the axioms of a field, particularly when said equation is formulated via the multi-valued hyperstructures of Chapter 5, Eqs. (5.3).

LEMMA 1.5.2 [36]: *General Lie-admissible and Jordan-admissible laws (1.5.49) are "directly universal" in the sense of containing as particular cases all infinitely possible nonunitary generalizations of quantum mechanical equations ("universality") directly in the frame of the observer ("direct universality"), while admitting a consistent algebra in their infinitesimal form.*

The above property can be equally proven by noting that the product $(A\hat{,}B)$ is the most general possible "product" of an "algebras" as commonly understood in mathematics (namely, a vector space with a bilinear composition law verifying the right and left distributive and scalar laws).

In fact, the product $(A\hat{,}B)$ constitutes the most general possible combination of Lie and Jordan products, thus admitting as particular cases *all* known algebras, such as associative algebras, Lie algebras, Jordan algebras, alternative algebras, supersymmetric algebras, Kac-Moody algebras, *etc.*

Despite their unquestionable mathematical beauty, theories (1.5.43) and (1.5.49) possess the following catastrophic physical and mathematical inconsistencies:

THEOREM 1.5.2 [36] *(see also Refs.* [51-58]*): All theories possessing a nonunitary time evolution formulated on conventional Hilbert spaces $\mathcal{H}$ over conventional fields of complex numbers $C(c, +, \times)$ do not admit consistent physical and mathematical applications because:*

1) They do not possess invariant units of time, space, energy, etc., thus lacking physically meaningful application to measurements;

2) They do not conserve Hermiticity in time, thus lacking physically meaningful observables;

3) They do not possess unique and invariant numerical predictions;

4) They generally violate probability and causality laws; and

5) They violate the basic axioms of Galileo's and Einstein's relativities.

Nonunitary theories are also afflicted by catastrophic mathematical inconsistencies.

The proof of the above theorem is essentially identical to that of Theorem 1.5,1 (see Ref. [36] for details). Again, the basic unit is not an abstract mathematical notion, because it embodies the most fundamental quantities, such as the units of space, energy, angular momentum, etc.

The nonunitary character of the theories here considered then causes the lack of conservation of the numerical values of such units with consequential catastrophic inapplicability of nonunitary theories to measurements.

Similarly, it is easy to prove that the condition of Hermiticity at the initial time,

$$(\langle\phi| \times H^\dagger) \times |\psi\rangle \equiv \langle\phi| \times (H \times |\psi\rangle), \quad H = H^\dagger, \tag{1.5.51}$$

is violated at subsequent times for theories with nonunitary time evolution when formulated on $\mathcal{H}$ over C. This additional catastrophic inconsistency (known as *Lopez's lemma* [52,53]), can be expressed by

$$[\langle\psi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times H \times U^\dagger] \times U|\psi\rangle =$$

$$= \langle\psi| \times U^\dagger \times [(U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times U|\psi\rangle] =$$

$$= (\langle\hat{\psi} \times T \times H'^\dagger) \times |\hat{\psi}\rangle = \langle\hat{\psi}| \times (\hat{H} \times T \times |\hat{\psi}\rangle), \tag{1.5.52a}$$

$$|\hat{\psi}\rangle = U \times |\psi\rangle, \;\; T = (U \times U^\dagger)^{-1} = T^\dagger, \tag{1.5.52b}$$

$$H'^\dagger = T^{-1} \times \hat{H} \times T \neq H. \tag{1.5.52c}$$

As a result, nonunitary theories do not admit physically meaningful observables.

Assuming that the preceding inconsistencies can be by-passed with some manipulation, nonunitary theories still remain with additional catastrophic inconsistencies, such as the lack of invariance of numerical predictions.

To illustrate this additional inconsistency, suppose that the considered nonunitary theory is such that, at $t = 0$ sec, $U \times U^\dagger_{[t=0]} = 1$, at $t = 15$ sec, $U \times U^\dagger_{[t=15]} = 15$, and the theory predicts at time $t = 0$ sec, say, the eigenvalue of 2 eV,

$$H|_{t=0} \times |\psi> = 2\ eV \times |\psi> . \tag{1.5.53}$$

It is then easy to see that the same theory predicts under the same conditions the *different* eigenvalue 30 eV at $t = 15$ sec, thus having no physical value of any type. In fact, we have

$$U \times U^\dagger|_{t=0} = I, \;\; U \times U^\dagger|_{t=15} = 15, \tag{1.5.54a}$$

$$U \times H \times |\psi\rangle = (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi\rangle) =$$

$$= H' \times T \times |\hat{\psi}\rangle = U \times E \times |\psi\rangle = E \times (U \times |\psi\rangle) = E \times |\hat{\psi}\rangle, \tag{1.5.54b}$$

$$H' = U \times H \times U^\dagger, \quad T = (U \times U^\dagger)^{-1},$$

$$H' \times |\hat{\psi}\rangle\ |_{t=0} = 2\mathrm{C} \times |\hat{\psi}\rangle\ |_{t=0}, \;\; T = 1\ |_{t=0}, \tag{1.4.54c}$$

$$H' \times |\hat{\psi}\rangle\ |_{t=15} = 2\mathrm{C} \times (U \times U^\dagger) \times |\hat{\psi}\rangle\ |_{t=15} = \tag{1.5.54d}$$

$$= 30\mathrm{C} \times |\hat{\psi}\rangle\ |_{t=15}\ .$$

Probability and causality laws are notoriously based on the unitary character of the time evolution and the invariant decomposition of the unit.

Their violation for nonunitary theories is then evident. It is an instructive exercise for the reader interested in learning hadronic mechanics, superconductivity and chemistry to identify a specific example of nonunitary transforms for which the effect *precedes* the cause.

The violation by nonunitary theories of the basic axioms of Galileo's and Einstein's relativities is so evident to require no comment.

An additional, most fundamental inconsistency of the theories considered is their *noninvariance*, that can be best illustrated with the lack of invariance of the general Lie-admissible and Jordan-admissible laws (1.5.49).

In fact, under nonunitary transforms, we have, e.g., the lack of invariance of the Lie-admissible and Jordan-admissible product,

$$U \times U^\dagger \neq I \tag{1.5.55a}$$

$$\begin{aligned} U\times(A\hat{,}B)\times U^\dagger = U\times(A\lhd B - B\rhd A)\times U^\dagger = (U\times A\times U^\dagger)\times \\ \times[(U\times U^{-1})\times(U\times P\times U^\dagger)\times(U\times U^\dagger)^{-1}]\times(U\times B\times U^\dagger)- \\ -(U\times B\times U^\dagger)\times[(U\times U^{-1})\times(U\times Q\times U^\dagger)\times(U\times U^\dagger)^{-1}]\times \\ \times(U\times A\times U^\dagger) = A'\times P'\times B' - B'\times Q'\times A' = \\ = A'\lhd' B' - B'\rhd' A'. \end{aligned} \tag{1.5.55b}$$

The above rules confirm the preservation of a Lie-admissible structure under the most general possible transforms, thus confirming the direct universality of laws (1.4.49) as per Theorem 1.4.2. The point is that *the formulations are not invariant* because

$$P' = (U\times U^{-1})\times(U\times Q\times U^\dagger)\times(U\times U^\dagger)^{-1} \neq P, \tag{1.5.56a}$$

$$Q' = (U\times U^{-1})\times(U\times Q\times U^\dagger)\times(U\times U^\dagger)^{-1} \neq Q, \tag{1.5.56b}$$

that is, because *the product itself is not invariant.*

By comparison, the invariance of quantum mechanics follows from the fact that the associative product "$\times$" is not changed by unitary transforms

$$U\times U^\dagger = U^\dagger\times U = I, \tag{1.5.57a}$$

$$\begin{aligned} A\times B \to U\times(A\times B)\times U^\dagger = \\ = (U\times A\times U^\dagger)\times(U\times U^\dagger)^{-1}\times(U\times B\times U^\dagger) = A'\times B'. \end{aligned} \tag{1.5.57b}$$

Therefore, generalized Lie-admissible and Jordan-admissible theories (1.5.49) are not invariant because the generalized products "$\lhd$" and "$\rhd$" are changed by nonunitary transformations, including the time evolution of the theory itself. The same results also holds for other nonunitary theories, as the reader is encouraged to verify.

The mathematical inconsistencies of nonunitary theories are the same as those of noncanonical theories. Recall that mathematics is formulated over a given field of numbers. Whenever the theory is nonunitary, the first noninvariance is that of the basic unit of the field.

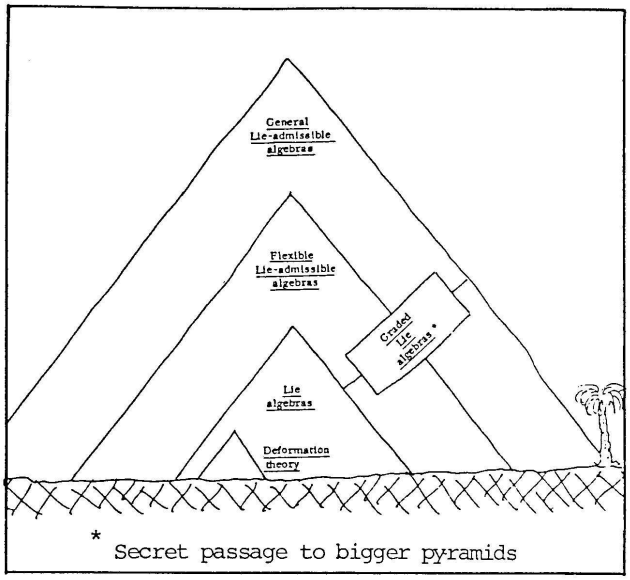

Figure 1.21. The reproduction of another "vignetta" presented by the author in 1978 to his colleagues at the Lyman Laboratory of Physics at Harvard University as part of his research under DOE (see Refs. [93,94] for details). This "vignetta" is a complement of that of Figure 1.3 on the need to maintain the external terms in the historical analytic equations because, when properly formulated, said equations yield covering, directly universal. Lie-admissible theories because Lie-admissible algebras contain as particular cases *all* algebras as defined in mathematics (universality) without the us of any transformation (direct universality). Finally, this "vignetta" was intended to illustrate that all theories preferred by the Lyman colleagues at the time, including symmetry breakings, supersymmetries, etc., were mere particular cases of the universal Lie-admissible formulations.

The lack of conservation of the unit then causes the loss of the basic field of numbers on which mathematics is constructed. It then follows that the entire axiomatic structure as formulated at the initial time, is no longer applicable at subsequent times.

For instance, the formulation of a nonunitary theory on a conventional Hilbert space has no mathematical sense because that space is defined over the field of complex numbers.

The loss of the latter property under nonunitary transforms then implies the loss of the former. The same result holds for metric spaces and other mathematics based on a field.

In short, the lack of invariance of the fundamental unit under nonunitary time evolutions causes the catastrophic collapse of the entire mathematical structure, without known exception.

The reader should be aware that the above physical and mathematical inconsistencies apply not only for Eqs. (1.5.49) but also for a large number of generalized theories, as expected from the direct universality of the former.

It is of the essence to identify in the following at least the most representative cases of physically inconsistent theories, to prevent their possible application (see Ref. [36] for details):

1) Dissipative nuclear theories [13] represented via an imaginary potential in non-Hermitian Hamiltonians,

$$H = H_0 = iV \neq H^\dagger \tag{1.5.58}$$

lose all algebras in the brackets of their time evolution (requiring a bilinear product) in favor of the triple system,

$$i \times \ dA/dt = A \times H - H^\dagger \times A = [A, H, H^\dagger]. \tag{1.5.59}$$

This causes the loss of nuclear notions such as "protons and neutrons" as conventionally understood, e.g., because the definition of their spin mandates the presence of a consistent algebra in the brackets of the time evolution.

2) Statistical theories with an external collision term C (see Ref. [59] and literature quoted therein) and equation of the density

$$i \ d\rho/dt = \rho \odot H = [\rho, H] + C, \ \ H = H^\dagger, \tag{1.5.60}$$

violate the conditions for the product $\rho \odot H$ to characterize any algebra, as well as the existence of exponentiation to a finite transform, let alone violating the conditions of unitarity.

3) The so-called "q-deformations" of the Lie product (see, e.g., [64,65,66–69] and very large literature quoted therein)

$$A \times B - q \times B \times A, \tag{1.5.61}$$

where q is a non-null scalar, that are a trivial particular case of Santilli's (p, q)-deformations (1.4.42).

4) The so-called "k-deformations" [60-63] that are a relativistic version of the q-deformations, thus also being a particular case of general structures (1.4.42).

5) The so-called "star deformations" [64] of the associative product

$$A \star B = A \times T \times B, \tag{1.5.62}$$

where T is fixed, and related generalized Lie product

$$A \star B - B \star A, \tag{1.5.63}$$

are manifestly nonunitary and *coincide* with Santilli's Lie-isotopic algebras [50].

6) Deformed creation-annihilation operators theories [65,66].

7) Nonunitary statistical theories [67].

8) Irreversible black holes dynamics [68] with Santilli's Lie-admissible structure (1.4.46) [103,104].

9) Noncanonical time theories [6971].

10) Supersymmetric theories [104] with product

$$(A, B) = [A, B] + \{A, B\} =$$

$$= (A \times B - B \times A) + (A \times B + B \times A), \tag{1.5.64}$$

are an evident particular case of Santilli's Lie-admissible product (1.4.46) with $T = W = I$.

11) String theories (see ref. [58] and literature quoted therein) generally have a noncanonical structure due to the inclusion of gravitation with additional catastrophic inconsistencies when including supersymmetries.

12) The so-called squeezed states theories [73,74] due to their manifest nonunitary character.

13) All quantum groups (see, e.g., refs. [75-77]) with a nonunitary structure.

14) Kac-Moody superalgebras [78] are also nonunitary and a particular case of Santilli's Lie-admissible algebra (1.4.46) with $T = I$ and W a phase factor.

Numerous additional theories are also afflicted by the catastrophic inconsistencies of Theorem 1.5.2, such as quantum groups, quantum gravity, and other theories the reader can easily identify from the *departures* of their time evolution from the unitary law.

All the above theories have a nonunitary structure formulated via conventional mathematics and, therefore, are afflicted by the catastrophic physical and mathematical inconsistencies of Theorem 1.5.2.

Additional generalized theories were attempted via *the relaxation of the linear character of quantum mechanics* [56]. These theories are essentially based on eigenvalue equations with the structure

$$H(t, r, p, |\psi\rangle) \times |\psi\rangle = E \times |\psi\rangle, \tag{1.5.65}$$

(i.e., H depends on the wavefunction).

Even though mathematically intriguing and possessing a seemingly unitary time evolution, these theories also possess rather serious physical drawbacks, such as: they violate the superposition principle necessary for composite systems such as a hadron; they violate the fundamental Mackay imprimitivity theorem necessary for the applicability of Galileo's and Einstein's relativities and possess other drawbacks [36] so serious to prevent consistent applications.

Yet another type of broader theory is *Weinberg's nonlinear theory* [79] with brackets of the type

$$A \odot B - B \odot A =$$

$$= \frac{\partial A}{\partial \psi} \times \frac{\partial B}{\partial \psi^\dagger} - \frac{\partial B}{\partial \psi} \times \frac{\partial A}{\partial \psi^\dagger}, \tag{1.5.66}$$

where the product $A \odot B$ is *nonassociative.*

This theory violates Okubo's No-Quantization Theorem [70], prohibiting the use of nonassociative envelopes because of catastrophic physical consequences, such as the loss of equivalence between the Schrödinger and Heisenberg representations (the former remains associative, while the latter becomes nonassociative, thus resulting in inequivalence).

Weinberg's theory also suffers from the absence of any unit at all, with consequential inability to apply the theory to measurements, the loss of exponentiation to a finite transform (lack of Poincaré-Birkhoff-Witt theorem), and other inconsistencies studied in Ref. [55].

These inconsistencies are not resolved by the adaptation of Weinberg's theory proposed by Jordan [80] as readers seriously interested in avoiding the publication of theories known to be inconsistent *ab initio* are encouraged to verify.

Several authors also attempted *the relaxation of the local-differential character of quantum mechanics* via the addition of "integral potentials" in the Hamiltonian,

$$V = \int d\tau \Gamma(\tau, \dots). \tag{1.5.67}$$

These theories are structurally flawed on both mathematical and physical grounds.

In fact, the nonlocal extension is elaborated via the conventional mathematics of quantum mechanics which, beginning with its topology, is strictly local-differential, thus implying fundamental *mathematical* inconsistencies. Nonlocal interactions are in general of contact type, for which the notion of a potential has no physical meaning, thus resulting in rather serious *physical* inconsistencies.

In conclusion, by the early 1980's Santilli had identified classical and operator generalized theories [103,104] that are directly universal in their fields, with a plethora of simpler versions by various other authors.

However, all these theories subsequently resulted in being mathematically significant, but having no physical meaning because they are noninvariant when elaborated with conventional mathematics.

As we shall see in Chapter 3 and 4, thanks to the construction of new mathematics, hadronic mechanics does indeed solve all the above inconsistencies. The clear difficulties in the solutions then illustrate the value of the result.

1.5.4 The Birth of Isomathematics, Genomathematics and their Isoduals

As it is well known, the basic equations of quantum mechanics, *Heisenberg's time evolution* of a (Hermitian) operator A ($\hbar = 1$),

$$i \times \frac{dA}{dt} = A \times H - H \times A = [A, H], \tag{1.5.68a}$$

$$H = p^2/2 \times m + V(r), \tag{1.5.68b}$$

can only represent the *conservation* of the total energy H (and other quantities) under action-at-a-distance interactions derivable from a potential $V(r)$,

$$i \times \frac{dH}{dt} = [H, H] = H \times H - H \times H \equiv 0. \tag{1.5.69}$$

Consequently, the above equations are basically insufficient to provide an operator representation of *closed non-Hamiltonian systems,* namely, systems of extended particles verifying conventional total conservation laws yet possessing internal potential; and nonpotential interactions, as it is the case for all interior problems, such as the structure of hadron, nuclei and stars.

The central requirement for a meaningful representation of closed, classical or operator interior systems of *particles* with internal contact interactions is the achievement of a *generalization of Lie's theory* in such a way to admit broader brackets, hereon denoted $[A\hat{,}B]$, verifying the following conditions:

1) The new brackets $[A\hat{,}B]$ must verify the distributive and scalars laws (3.9) in order to characterize an algebra.

2) Besides the Hamiltonian, the new brackets should admit a new Hermitian operator, hereon denoted with $\hat{T} = \hat{T}^\dagger$, and we shall write $[A\hat{,}B]_{\hat{T}}$, as a necessary condition for the representation of all non-Hamiltonian forces and effects.

3) The new brackets must be anti-symmetric in order to allow the conservation of the total energy under contact nonpotential internal interactions

$$i \times \frac{dH}{dt} = [H\hat{,}H]_{\hat{T}} \equiv 0. \tag{1.5.70}$$

For the case of *open,* classical or operator irreversible interior systems of *particles* there is the need of a *second generalization of Lie's theory* characterizing broader brackets, hereon denoted $(A\hat{,}B)$ verifying the following conditions:

1') The broader brackets (A, B) must also verify the scalar and distributive laws (3.9) to characterize an algebra;

2') The broader brackets must include *two* non-Hermitian operators, hereon denoted $\hat{P}$ and $\hat{Q}, \hat{P} = \hat{Q}^\dagger$ to represent the two directions of time, and the new brackets, denoted ${}_{\hat{P}}(A\hat{,}B)_{\hat{Q}}$, must be neither antisymmetric nor symmetric to

characterize the time rate of variation of the energy and other quantities,

$$i \times \frac{dH}{dt} \;=\; {}_{\hat{P}}(H\hat{,}H)_{\hat{Q}} \;\neq\; 0; \tag{1.5.71}$$

3') The broader brackets must admit the antisymmetric brackets $[A\hat{,}B]$ and $[A, B]$ as particular cases because conservation laws are particular cases of non-conservation laws.

For the case of closed and open interior systems of *antiparticles,* it is easy to see that the above generalizations of Lie's theory will not apply for the same reason that the conventional Lie theory cannot characterize exterior systems of point-like antiparticles at classical level studied in Section 1.1 (due to the existence of only one quantization channel, the operator image of classical treatments of antiparticles can only yield particles with the wrong sign of the charge, and certainly not their charge conjugate).

The above occurrence requires a *third generalization of Lie's theory* specifically conceived for the representation of closed or open interior systems of antiparticles at *all* levels of study, from Newton to second quantization. As we shall see, the latter generalization is provided by the isodual map.

In an attempt to resolve the scientific imbalances of the preceding section, when at the Department of Mathematics of Harvard University, Santilli [39,50] proposed in 1978 an axiom-preserving generalization of conventional mathematics verifying conditions 1), 2) and 3), that he subsequently studied in various works (see monographs [9,10,11,38] and quoted literature).

The new mathematics is today known as *Santilli's isotopic and genotopic mathematics* or *isomathematics and genomathematics* for short [81-86], where the word "isotopic" or the prefix "iso" are used in the Greek meaning of preserving the original axioms, and the word "geno" is used in the sense of inducing new axioms.

Proposal [39] for the new isomathematics was centered in the generalization (called *lifting*) of the conventional, N-dimensional unit, $I = Diag.(1, 1, ..., 1)$ into an $N \times N$-dimensional matrix $\hat{I}$ that is nowhere singular, Hermitian and positive-definite, but otherwise possesses an unrestricted functional dependence on local coordinates r, velocities v, accelerations a, dimension d, density μ, wavefunctions ψ, their derivatives $\partial\psi$ and any other needed quantity,

$$I = Diag.(1, 1, ..., 1) > 0 \to \hat{I}(r, v, a, d, \mu, \psi, \partial\psi, ...) = \hat{I}^{\dagger} = 1/\hat{T} > 0 \tag{1.5.72}$$

while jointly lifting the conventional associative product $A \times B$ among generic quantities A and B (numbers, vector fields, matrices, operators, etc.) into the form

$$A \times B \to A\hat{\times}B = A \times \hat{T} \times B, \tag{1.5.73}$$

under which $\hat{I}$, rather than I, is the correct left and right unit,

$$I \times A = A \times I \equiv A \to \hat{I}\hat{\times}A = A\hat{\times}\hat{I} \equiv A, \tag{1.5.74}$$

for all A of the set considered, in which case $\hat{I}$ is called *Santilli's isounit,* and $\hat{T}$ is called the *isotopic element.*

Eqs. (1.5.72)–(1.5.74) illustrate the isotopic character of the lifting. In fact, $\hat{I}$ preserves all topological properties of I; the isoproduct $A\hat{\times}B$ remains as associative as the original product $A \times B$; and the same holds for the preservation of the axioms for a left and right identity.

More generally, the lifting of the basic unit required, for evident reasons of consistency, a corresponding compatible lifting of *all* mathematics used by special relativity and quantum mechanics, with no exception known to this author, thus resulting in the new *isonumbers*, isospaces, isofunctional analysis, isodifferential calculus, isotopologies, isogeometries, etc. (for mathematical works see Refs. [10,11,38]).

Via the use of the above liftings, Santilli presented in the original proposal [39] a step-by-step isotopic (that is, axiom-preserving) lifting of all main branches of Lie's theory, including the isotopic generalization of universal enveloping associative algebras, Lie algebras, Lie groups and the representation theory. The new theory was then studied in various works and it is today known as the *Lie-Santilli isotheory* [81-86]. Predictably. from Eqs. (1.5.73) one can see that the new isobrackets have the form

$$[A\hat{,}B]_{\hat{T}} = A\hat{\times}B - B\hat{\times}A =$$

$$= A \times \hat{T} \times B - B \times \hat{T} \times A = [A\hat{,}B], \tag{1.5.75}$$

where the subscript $\hat{T}$ shall be dropped hereon, whose verification of conditions 1), 2), 3) is evident.

The point important for these introductory lines is that *isomathematics does allow a consistent representation of extended, nonspherical, deformable and hyperdense particles under local and nonlocal, linear and nonlinear, and potential as well as nonpotential interactions.*

In fact, all conventional linear, local and potential interactions can be represented with a conventional Hamiltonian, while the shape and density of the particles and their nonlinear, nonlocal and nonpotential interactions can be represented with Santilli's isounits via realizations of the type

$$\hat{I} = \Pi_{k=1,2,...,n} Diag(n_{k1}^2, n_{k2}^2, n_{k3}^2, n_{k4}^2) \times e^{\Gamma(\psi,\psi^\dagger)\times \int d^3r \psi^\dagger(r)_k \times \psi(r)_k}, \tag{1.5.76}$$

where: the $n_{k1}^2, n_{k2}^2, n_{k3}^2$ allow to represent, for the first time, the actual, extended, nonspherical and deformable shapes of the particles considered (normalized to the values $n_k = 1$ for the perfect sphere); n_{k4}^2 allows to represent, also for the first time, the density of the interior medium (normalized to the value $n_4 = 1$ for empty space); the function $\Gamma(\psi, \psi^\dagger)$ represents the nonlinear character of the interactions; and the integral $\int d^3r \psi^\dagger(r)_k \times \psi(r)_k$ represents nonlocal interactions due to the overlapping of particles or of their wave packets.

When the mutual distances of the particles are much greater than 10^{-13}cm = 1 F, the integral in Eq. (1.5.76) is identically null, and all nonlinear and nonlocal effects are null. When, in addition, the particles considered are reduced to points moving in vacuum, all the n-quantities are equal to 1, generalized unit (1.3.22) recovers the trivial unit, and isomathematics recovers conventional mathematics identically, uniquely and unambiguously.

In the same memoir [39], in order to represent irreversibility, Santilli proposed a broader genomathematics based on the following differentiation of the product to the right and to the left with corresponding generalized units

$$A > B = A \times \hat{P} \times B, \quad \hat{I}^{>} = 1/\hat{P}; \tag{1.5.77a}$$

$$A < B = A \times \hat{Q} \times B, \quad {}^{<}\hat{I} = 1/\hat{Q}, \tag{1.5.77b}$$

$$\hat{I}^{>} = {}^{<}\hat{I}^{\dagger}, \tag{1.5.77c}$$

where evidently the product to the right, $A > B$, represents motion forward in time and that to the left, $A < B$, represents motion backward in time. Since $A > B \neq A < B$, the latter mathematics represents irreversibility from the most elementary possible axioms.

The latter mathematics was proposed under a broader lifting called "genotopy" in the Greek meaning of inducing new axioms, and it is known today as *Santilli genotopic mathematics,* pr *genomathematics* for short [81-86].

It is evident that genoliftings (1.5.77) require a step by step generalization of all aspects of isomathematics, resulting in *genonumbers, genofields, genospaces, genoalgebras, genogeometries, genotopologies, etc.* [9b,10b,11,38a].

Via the use of the latter mathematics, Santilli proposed also in the original memoir [39] a genotopy of the main branches of Lie's theory, including a genotopic broadening of universal enveloping isoassociative algebras, Lie-Santilli isoalgebras, Lie-Santilli isogroup, isorepresentation theory, etc. and the resulting theory is today known as the *Lie-Santilli genotheory* with basic brackets

$$\hat{}_{\hat{P}}(A\hat{,}B)_{\hat{Q}} = A < B - B > A =$$

$$= A \times P \times B - B \times Q \times A = (A\hat{,}B), \tag{1.5.78}$$

where the subscripts $\hat{P}$ and $\hat{Q}$ shall be dropped from now on.

It should be noted that the main proposal of memoir [39] is genomathematics, while isomathematics is presented as a particular case for

$$(A\hat{,}B)_{\hat{P}=\hat{Q}=\hat{T}} = [A\hat{,}B]. \tag{1.5.79}$$

as we shall see in Chapters 3 and 4, the *isodual isomathematics* and *isodual genomathematics* for the treatment of antiparticles are given by the isodual image (1.1.6) of the above iso- and geno-mathematics, respectively.

1.5.5 Hadronic Mechanics

Thanks to the prior discovery of isomathematics and genomathematics, in memoir [50] also of 1978 Santilli proposed a generalization of quantum mechanics for closed and open interior systems, respectively, under the name of *hadronic mechanics,* because hyperdense hadrons, such as protons and neutrons, constitute the most representative (and most difficult) cases of interior dynamical systems.

For the case of closed interior systems of particles, hadronic mechanics is based on the following *isotopic generalization of Heisenberg's equations* (Ref. [50], Eqs. (4.15.34) and (4.18.11))

$$i \times \frac{dA}{dt} = [A\hat{,}H] = A\hat{\times}H - H\hat{\times}A. \tag{1.5.80}$$

while for the broader case of open interior systems hadronic mechanics is based on the following *genotopic generalization of Heisenberg's equations* (Ref. [50], Eqs. (4.18.16))

$$i \times \frac{dA}{dt} = (A\hat{,}H) = A < H - H > A =$$
$$= A \times P \times H - H \times Q \times A. \tag{1.5.81}$$

The isodual images of Eqs. (1.5.80) and (1.5.81) for antiparticles as well as their multivalued hyperformulations significant for biological studies, were added more recently [88].

A rather intense scientific activity followed the original proposal [50], including five *Workshops on Lie-admissible Formulations* held at Harvard University from 1978 to 1982, fifteen *Workshops on Hadronic Mechanics*, and several formal conferences held in various countries, plus a rather large number of research papers and monographs written by various mathematicians, theoreticians and experimentalists, for an estimated total of some 15,000 pages of research published refereed journals (see the *General References on Hadronic Mechanics* at the end of this volume).

It should be indicated that, following the original proposal of 1978 [50], maturity on the basic new numbers of hadronic mechanics, the *iso-, geno- and hyper-numbers and their isoduals* was reached only in 1993 [87]; a correct mathematical formulation was reached only in 1996 [88] due to problems that had remained unsolved for years; and a fully invariant physical formulation was reached only in 1997 for invariant Lie-isotopic theories [89] and invariant Lie-admissible theories [89] (see also memoir [91] for a recent review).

The lapse of time between the original proposal of 1978 and the achievement of mathematical and physical maturity illustrates the difficulties to be resolved.

As a result of all these efforts, hadronic mechanics is today a rather diversified discipline conceived and constructed for quantitative treatments of all classical

and operator systems of particles according to Definition 1.3.1 with corresponding isodual formulations for antiparticles.

It is evident that in the following chapters we can review only the most salient foundations of hadronic mechanics and have to defer the interested reader to the technical literature for brevity.

As of today, hadronic mechanics has experimental verifications and applications in particle physics, nuclear physics, atomic physics, superconductivity, chemistry, biology, astrophysics and cosmology, including numerous industrial applications outlined in monograph [92].

Hadronic mechanics can be classified into **sixteen different branches**, including: four branches of classical treatment of particles with corresponding four branches of operator treatment also of particles, and eight corresponding (classical and operator) treatments of antiparticles.

An effective classification of hadronic mechanics is that done via the main topological features of the assumed basic unit, since the latter characterizes all branches according to:

$I = 1 > 0$:
HAMILTONIAN AND QUANTUM MECHANICS
Used for the description of closed and reversible systems of point-like particles in exterior conditions in vacuum;

$I^d = -1 < 0$:
ISODUAL HAMILTONIAN AND ISODUAL QUANTUM MECHANICS
Used for the description of closed and reversible systems of point-like antiparticles in exterior conditions in vacuum;

$\hat{I}(r, v, ...) = \hat{I}^\dagger > 0$:
CLASSICAL AND OPERATOR ISOMECHANICS
Used for the description of closed and reversible systems of extended particles in interior conditions;

$\hat{I}^d(r^d, v^d, ...) = \hat{I}^{d\dagger} < 0$:
ISODUAL CLASSICAL AND OPERATOR ISOMECHANICS
Used for the description of closed and reversible systems of extended antiparticles in interior conditions;

$\hat{I}^>(r^>, v^>, ...) = (^<\hat{I})^\dagger$:
CLASSICAL AND OPERATOR GENOMECHANICS
Used for the description of open and irreversible systems of extended particles in interior conditions;

$\hat{I}^{d>}(r^{d>}, v^{d>}, ...\} = (^<\hat{I})^{d\dagger}$:
ISODUAL CLASSICAL AND OPERATOR GENOMECHANICS

HADRONIC MECHANICS

MECHANICS AND THEIR ISODUALS

Newtonian Mechanics	Isodual Newtonian Mechanics
Hamiltonian mechanics	Isodual Hamiltonian Mechanics
Quantization	Isodual Quantization
Quantum mechanics	Isodual Quantum Mechanics
Special Relativity	Isodual Special Relativity

REPRESENTATION: isolated systems of point-like particles (mechanics) and antiparticles (isodual mechanics) under local, linear and potential forces.

ISOMECHANICS AND THEIR ISODUALS

Iso-Newtonian Mechanics	Isodual iso-Newtonian Mech.
Iso-Hamiltonian mechanics	Isodual iso-Hamiltonian Mech.
Isoquantization	Isodual Isoquantization
Isohadronic mechanics	Isodual isohadronic Mech.
Isospecial Relativity	Isodual Special Relativity

REPRESENTATION: Isolated, reversible and single-valued systems of extended particles (isomechanics) and antiparticles (isodual isomechanics) under internal, local and nonlocal, linear and nonlinear, potential and nonpotential forces.

GENOMECHANICS AND THEIR ISODUALS

Geno-Newtonian Mechanics	Isodual Geno-Newtonian Mech.
Geno-Hamiltonian mechanics	Isodual Geno-Hamiltonian Mech.
Genoquantization	Isodual Genoquantization
Genohadronic mechanics	Isodual Genohadronic Mechanics
Genospecial Relativity	Isodual Genospecial Relativity

REPRESENTATION: open, irreversible and single-valued systems of extended particles (genomechanics) and antiparticles (isodual genomechanics) under external, local and nonlocal, linear and nonlinear, potential and nonpotential forces.

HYPERMECHANICS AND THEIR ISODUALS

Hyper-Newtonian Mechanics	Isodual Hyper-Newtonian Mech.
Hyper-Hamiltonian mechanics	Isodual Hyper-Hamiltonian Mech.
Hyperquantization	Isodual Hyperquantization
Hyperhadronic mechanics	Isodual Hyperhadronic Mech.
Hyperspecial Relativity	Isodual Hyperspecial Relativity

REPRESENTATION: open, irreversible and multi--valued systems of extended particles (hypermechanics) and antiparticles (isodual hypermechanics) under external, local and nonlocal, linear and nonlinear, potential and nonpotential forces.

Figure 1.22. The structure of hadronic mechanics.

Used for the description of open and irreversible systems of extended antiparticles in interior conditions;

$\hat{I}^{>} = (\hat{I}_1^{>}, \hat{I}_2^{>}, ...) = (^{<}\hat{I})^{\dagger}$:

CLASSICAL AND OPERATOR HYPERMECHANICS

Used for the description of multivalued open and irreversible systems of extended particles in interior conditions;

$\hat{I}^{d>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, ...\} = (^{<}\hat{I})^{\dagger}$:

ISODUAL CLASSICAL AND OPERATOR HYPERMECHANICS

Used for the description of multivalued open and irreversible systems of extended antiparticles in interior conditions.

In summary, a serious study of antiparticles requires its study beginning at the classical level and then following at all subsequent levels, exactly as it is the case for particles.

In so doing, the mathematical and physical treatments of antiparticles emerge as being deeply linked to that of particles since, as we shall see, the former are an anti-isomorphic image of the latter.

Above all, a serious study of antiparticles requires the admission of their existence in physical conditions of progressively increasing complexity, that consequently require mathematical and physical methods with an equally increasing complexity, resulting in the various branches depicted in Figure 5.

All in all, young minds of any age will agree that, rather than having reached a terminal character, our knowledge of nature is still at its first infancy and so much remains to be discovered.

Appendix 1.A
Crothers' Critical Analysis of General Relativity

In this appendix we reproduce *ad litteram* the independent studies on gravitation conducted by Stephen J. Crothers of the Australian Division of the Institute for basic Research, email ¡thenarmis@yahoo.com¿.

The General Theory of Relativity has now become a topic of household discussion, at least within the context of black holes, Big Bang cosmology and expansion of the Universe. These concepts have found their way into the curricula of high schools, deep into university physics courses, much research, and some pretty expensive experimental projects. Almost daily there are reports of discovery of another black hole and of physical evidence of the beginning of the Universe from the Big Bang of a cosmological singularity. So widespread now are these notions that they have taken on the mantle of verified scientific facts. Yet nothing can be further from the truth. Indeed, the evidence, both theoretical and physical, actually refutes black holes, big bangs and expansion of the Universe.

Has anyone ever found final scientific evidence on a black hole? The short answer to this question is no, not a single black hole can be claimed to have been detected beyond scientific doubt. According to the proponents of the black hole, the signatures of that bizarre object are:

1) an infinitely dense singularity, a "point-mass";

2) an event horizon.

Since nobody has ever identified in a final scientific form an infinitely dense singularity anywhere, and since nobody has ever identified an event horizon anywhere, nobody has ever identified a black hole, anywhere. Furthermore, General Relativity is claimed to be a generalisation of Special Relativity, to non-uniform motion of material bodies. However, it is very easily proved that Special Relativity forbids the existence of infinite densities, and hence it forbids singularities, i.e. infinite point-densities. So if General Relativity permits singularities (e.g. black holes), it does so in violation of Special Relativity. Indeed, according to Special Relativity, the dynamic mass m of a rest-mass m_0, moving with a speed $v < c$ along the x-axis, is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

The dynamic volume of a cuboid rest-mass m_0 is $V = x_0^3\sqrt{1-\frac{v^2}{c^2}}$, where x_0 is the length of the sides of m_0. Then the dynamic density D is

$$D = \frac{m}{V} = \frac{m_0}{x_0^3\left(1-\frac{v^2}{c^2}\right)}.$$

This is infinite when $v = c$. But according to Special Relativity no material object can acquire the speed c, of light in vacuo (equivalently, this would require an infinite amount of energy, which is impossible). Therefore, point-masses are forbidden by Special Relativity, and hence also by General Relativity if the latter is to be consistent with the former. This is sufficient to invalidate the alleged black hole singularity and the alleged Big Bang cosmological singularity.

Another simple physical argument re-affirms this result; violation of Einstein's 'Principle of Equivalence' [111]. According to this Principle [112], in a freely falling inertial frame in a sufficiently small region of Einstein's gravitational field, Special Relativity must hold. Now Einstein's field equations are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa T_{\mu\nu}\,,$$

where $G_{\mu\nu}$ is Einstein's tensor, $R_{\mu\nu}$ the Ricci tensor, κ a constant, and $T_{\mu\nu}$ the energy-momentum tensor. Einstein claimed that for the static vacuum (i.e. empty) gravitational field, $T_{\mu\nu} = 0$, so that

$$R_{\mu\nu} = 0,$$

(since in this case the Ricci curvature invariant R is also zero). It is from a solution to $R_{\mu\nu} = 0$, the so-called "Schwarzschild solution", that the black hole is alleged. Now Special Relativity permits the presence of any number of arbitrarily large (but not infinitely large) masses, which can interact. Furthermore, the very definition of an inertial frame involves the presence of mass (and in the case of Special Relativity, two masses, viz., the mass of the observer and the mass of the observed, so that relative motion of material bodies is defined). But $R_{\mu\nu} = 0$ is a statement that there are no masses permitted, by definition, in the alleged gravitational field of $R_{\mu\nu} = 0$. Therefore, Special Relativity cannot be recovered in any "freely falling" inertial frame in the spacetime of $R_{\mu\nu} = 0$ and, indeed, a "freely falling" inertial frame cannot even be present (since its very definition requires the presence of mass). Thus, Einstein's 'Principle of Equivalence' is violated by $R_{\mu\nu} = 0$, and is therefore inconsistent with the General Theory of Relativity, which is based upon the validity of his 'Principle'. Therefore, the "Schwarzschild solution" violates the 'Principle' and is consequently invalid, thereby completely invalidating the black hole, even if the latter can be deduced from the "Schwarzschild solution" by some purely formal mathematical means.

However, it has also been proved [113–131] that it is impossible to obtain the black hole from the "Schwarzschild solution" without violating the rules of differential geometry. This too is sufficient to invalidate the black hole.

It should also be noted that the concept of the black hole did not come from any observations requiring a theoretical explanation. It was generated entirely from theory (and an erroneous theory at that). It is no wonder that nobody has ever found a black hole; and there is no theory which rightly predicts them. The black hole was stillborn, and has no place in science.

The Big Bang concept and its associated expansion of the Universe is in the same boat as its cousin, the black hole. First, as shown above, the alleged cosmological singularity, an infinitely dense point-mass containing all the matter and energy of the Universe, and spacetime itself besides, is inconsistent with Special Relativity and hence also with General Relativity. Once again, if General Relativity predicted a cosmological singularity it would do so in violation of Special Relativity. Furthermore, the Big Bang and expansion of the Universe are allegedly a consequence of the Friedmann-Lemaître-Robertson-Walker (FLRW) line-element. But it has been proved that the Big Bang and associated expansion of the Universe cannot be obtained from the FLRW line-element without a gross violation of differential geometry, and so they are invalid. In actual fact, the FLRW line-element predicts an infinite, unbounded Universe, independent of time [122] – no Big Bang and no expansion.

Another interesting fact is that "Schwarzschild's solution" is not Schwarzschild's solution [7, 8, 130–134]. It is also frequently claimed that Schwarzschild deduced the black hole from his solution, with an event horizon at the "Schwarzschild radius", R_s, given by

$$R_s = \frac{2Gm}{c^2}.$$

All these claims are patently false, because Schwarzschild did not breathe a single word about black holes, never "deduced" the alleged "Schwarzschild radius", of the so-called "event horizon", and in fact obtained a solution which precludes the black hole. Here is the "Schwarzschild solution", due to David Hilbert [130, 131, 133, 134] (using $c = G = 1$),

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

wherein r is alleged to go down to zero, one way or another. But here now is Schwarzschild's real solution [7],

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{\frac{1}{3}},$$

$$0 < r < \infty,$$

wherein α is an undetermined constant, supposed a function of the mass of the source of the alleged gravitational field associated therewith. Note that when $r = 0$, Schwarzschild's line element is undefined, and there is no possibility of a black hole, which is alleged to occur in Hilbert's "Schwarzschild's solution" with infinitely dense singularity at $r = 0$ and event horizon at $r = 2m$ therein. Hilbert's "Schwarzschild's solution" violates the intrinsic geometry of the line-element, and is inconsistent with Schwarzschild's solution which does not violate the intrinsic geometry of the line-element. Also, one cannot assign a value to the constant α without introducing extraneous and *ad hoc* arguments, as Schwarzschild knew – and so he didn't. And even if Schwarzschild's solution or Hilbert's "Schwarzschild solution" were permissible, they conceive of the mass in terms of a centre of mass (i.e. a point-mass), and a centre of mass is not a physical object. There is no sense in asserting that an object and its centre of mass are identical, which is effectively what the proponents of the black hole do. In addition, the energy-momentum tensor contains all matter and energy that cause the gravitational field. Setting it to zero eliminates all causation of the gravitational field, and so causative mass cannot be introduced into the metric tensor *a posteriori* in the fashion of the proponents of black holes by their analogy with Newton's gravitational potential in the infinitely far field.

In the usual interpretation of Hilbert's "Schwarzschild's solution", the quantity r therein has never been properly identified. It has variously been called "the radius" [135, 136] of a sphere, the "coordinate radius"[137] or "radial coordinate" [17, 138] or "radial space coordinate" [139], the "areal radius" [137, 140], the "reduced circumference" [141], even "a gauge choice, which defines r" [142], but *never* for what it really is – the radius of Gaussian curvature. Being the radius of curvature it does not in fact determine the geodesic radial distance from the centre of spherical symmetry [111, 113–128, 140–145]. For a 2-D spherically symmetric geometric surface given by

$$R_c^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$R_c = R_c(r),$$

the Riemannian curvature reduces to the Gaussian curvature K, given by [143, 149–152],

$$K = \frac{R_{1212}}{g},$$

where R_{ijkm} is the Riemann tensor of the first kind and $g = g_{\theta\theta}g_{\varphi\varphi}$. Straightforward calculation gives

$$K = \frac{1}{R_c^2},$$

so that R_c is the inverse square root of the Gaussian curvature, i. e. the radius of curvature, and so r in Hilbert's "Schwarzschild's solution" is the radius of Gaussian curvature. The geodesic (or proper) radius, R_p, of Schwarzschild's solution is given by

$$R_p = \int \frac{dR}{\sqrt{1 - \frac{\alpha}{R}}},$$

and for Hilbert's black hole "Schwarzschild's solution", by

$$R_p = \int \frac{dr}{\sqrt{1 - \frac{2m}{r}}}.$$

Thus the proper radius and the radius of curvature *are not the same*; for the above, $R_p \neq R$ and $R_p \neq r$ respectively, in general [111, 113–128].

That Einstein's conception of the conservation and localisation of gravitational energy are erroneous easily follows from the fact that $R_{\mu\nu} = 0$ is inadmissible. Since the energy-momentum tensor can never be zero, Einstein's field equations can be written as

$$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0,$$

where $G_{\mu\nu}/\kappa$ gives the components of a gravitational energy tensor. Thus, when $T_{\mu\nu} = 0$, $G_{\mu\nu} = 0$, i .e. $T_{\mu\nu}$ and $G_{\mu\nu}/\kappa$, *vanish identically*. Consequently, the total energy is always zero; there is no possibility of the localisation of gravitational energy; there are no Einstein gravitational waves. The LIGO project and its counterparts around the world, such as the AIGO, are destined to detect nothing.

Einstein's pseudo-tensor is alleged to describe the localisation of gravitational energy, gravitational waves, and the flow of energy and momentum. According to the foregoing this cannot be true. This is re-affirmed by the fact that Einstein's pseudo-tensor is a meaningless collection of mathematical symbols [153]. Einstein's pseudo-tensor, $\sqrt{-g}t^{\mu}_{\nu}$, is defined as [112, 149, 153–155],

$$\sqrt{-g}t^{\mu}_{\nu} = \frac{1}{2}\left(\delta^{\mu}_{\nu}L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}}g^{\sigma\rho}_{,\nu}\right)$$

wherein L is given by

$$L = -g^{\alpha\beta}\left(\Gamma^{\gamma}_{\alpha\kappa}\Gamma^{\kappa}_{\beta\gamma} - \Gamma^{\gamma}_{\alpha\beta}\Gamma^{\kappa}_{\gamma\kappa}\right).$$

Contracting the pseudo-tensor and applying Euler's theorem yields,

$$\sqrt{-g}t^{\mu}_{\mu} = L,$$

which is a 1st-order intrinsic differential invariant that depends only upon the components of the metric tensor and its 1st derivatives. However, the mathematicians Ricci and Levi-Civita proved in 1900 that such invariants *do not exist*

[153, 156]. The invalidity of the pseudo-tensor is, of course, consistent with the invalidity of $R_{\mu\nu} = 0$. Consequently, everything built upon Einstein's pseudo-tensor is invalid. Connected with is the fact that Einstein's field equations cannot be linearised because linearisation implies the existence of a tensor that, except of the trivial case of being zero, *does not otherwise exist*, as proved by Hermann Weyl in 1944 [157].

The proponents of the Standard Model routinely ignore and attempt to suppress these facts [158, 159], because they completely invalidate their theories of black holes, big bangs and expansion of the Universe. Ironically, theoretically speaking, it is General Relativity itself which invalidates them. Observations also refute them.

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Postscript

In the history of science some basic advances in physics have been preceded by basic advances in mathematics, such as Newtons invention of calculus and general relativity relying on Riemannian geometry. In the case of quantum mechanics the scientific revolution presupposed the earlier invention of complex numbers. With new numbers and more powerful mathematics to its disposition, physics could be lifted to explain broader and more complex domains of physical reality.

The recent and ongoing revolution of physics, initiated by Prof. Ruggero Maria Santilli, lifting the discipline from quantum mechanics to hadronic mechanics, is consistent with this pattern, but in a more far-reaching and radical way than earlier liftings of physics made possible from extensions of mathematics.

Santilli realized at an early stage that basic advances in physics required invention of new classes of numbers and more adequate and powerful mathematics stemming from this. His efforts to develop such expansions of mathematics started already in 1967, and this enterprise went on for four decades. Its basic novelties, architecture and fruits are presented in the present volume. During this period a few dozen professional mathematicians world wide have made more or less significant contributions to fill in the new Santilli fields of mathematics, but the honor of discovering these vast new continents and work out their basic topology is Santillis and his alone. These new fields initiated by Santilli made possible realization of so-called Lie-admissible physics. For this achievement Santilli in 1990 received the honor from Estonia Academy of Science of being appointed as mathematician number seven after world war two considered a landmark in the history of algebra.

With regard to Sophus Lie it may be of some interest to note that the Norwegian examiners of his groundbreaking doctoral thesis in 1871 were not able to grasp his work, due to its high degree of novelty and unfamiliarity. However, due to Lie already being highly esteemed among influential contemporary mathematicians at the continent, it was not an option to dismiss his thesis. As in other disciplines, highly acknowledged after Thomas Kuhns publication of The structure of scientific revolutions in 1962, sufficiently novel mathematics implies some paradigmatic challenge. Therefore, it is not strange that some mathematicians and physicists have experienced difficulties taking the paradigmatic leap necessary to grasp the basics of hadronic mathematics or to acknowledge its far-reaching implications. Such a challenge is more demanding when scientific novelty

implies a reconfiguration of conventional basic notions in the discipline. This is, as Kuhn noted, typically easier for younger and more emergent scientific minds.

Until Santilli the number 1 was silently taken for granted as the primary unit of mathematics. However, as noted by mathematical physicist Peter Rowlands at University of Liverpool, the number 1 is already loaded with assumptions, that can be worked out from a lifted and broader mathematical framework. A partial and rough analogy might be linguistics where it is obvious that a universal science of language must be worked out from a level of abstraction that is higher than having to assume the word for mother to be the first word.

Santilli detrivialized the choice of the unit, and invented isomathematics where the crux was the lifting of the conventional multiplicative unit (i.e. conservation of its topological properties) to a matrix isounit with additional arbitrary functional dependence on other needed variables. Then the conventional unit could be described as a projection and deformation from the isounit by the link provided by the so-called isotopic element inverse of the isounit. This represented the creation of a new branch of mathematics sophisticated and flexible enough to treat systems entailing sub-systems with different units, i.e. more complex systems of nature.

Isomathematics proved necessary for the lifting of quantum mechanics to hadronic mechanics. With this new mathematics it was possible to describe extended particles and abandon the point particle simplification of quantum mechanics. This proved highly successful in explaining the strong force by leaving behind the non-linear complexities involved in quantum mechanics struggle to describe the relation between the three baryon quarks in the proton. Isomathematics also provided the mathematical means to explain the neutron as a bound state of a proton and an electron as suggested by Rutherford. By means of isomathematics Santilli was also able to discover the fifth force of nature (in cooperation with Professor Animalu), the contact force inducing total overlap between the wave packets of the two touching electrons constituting the isoelectron. This was the key to understanding hadronic superconductivity which also can take place in fluids and gases, i.e. at really high temperatures. These advances from hadronic mechanics led to a corresponding lifting of quantum chemistry to hadronic chemistry and the discovery of the new chemical species of magnecules with non-valence bounds. Powerful industrial-ecological technology exploiting these theoretical insights was invented by Santilli himself from 1998 on.

Thus, the development of hadronic mathematics by Santilli was not only motivated by making advances in mathematics per se, but also of its potential to facilitate basic advances in physics and beyond. These advances have been shown to be highly successful already. Without the preceding advances in mathematics, the new hadronic technology would not have been around. The mere existence of this technology is sufficient to demonstrate the significance of hadronic math-

ematics. It is interesting to note that the directing of creative mathematics into this path was initiated by a mathematical physicist, not by a pure mathematician. In general this may indicate the particular potential for mathematical advances by relating the mathematics to unsolved basic problems in other disciplines, as well as to real life challenges.

In the history of mathematics it is not so easy to find parallels to the achievements made by Santilli, due to hadronic mathematics representing a radical and general lifting, relegating the previous mathematics to a subclass of isomathematics, in some analogy to taking the step from the Earth to the solar system. However, the universe also includes other solar systems as well as galaxies.

In addition to isonumbers Santilli invented the new and broader class of genonumbers with the possibility of asymmetric genounits for forward vs. backward genofields, and designed to describe and explain irreversibility, characteristic for more complex systems of nature. Quantum mechanical approaches to biological systems never achieved appreciable success, mainly due to being restricted by a basic symmetry and hence reversibility in connected mathematical axioms. It represented an outstanding achievement of theoretical biology when Chris Illert in the mid-1990s was able to find the universal algorithm for growth of sea shells by applying hadronic geometry. Such an achievement was argued not to be possible for more restricted hyperdimensional geometries as for example the Riemannian. This specialist study in conchology was the first striking illustration of the potency as well as necessity of iso- and genomathematics to explain irreversible systems in biology.

Following the lifting from isomathematics to genomathematics, Santilli also established one further lifting, by inventing the new and broader class of hyperstructural numbers or Santilli hypernumbers. Such hypernumbers are multivalued and suitable to describe and explain even more complex systems of nature than possible with genonumbers. Due to its irreversible multivalued structure hypermathematics seems highly promising for specialist advances in fields such as genetics, memetics and communication theory. By the lifting to hypermathematics hadronic mathematics as a whole may be interpreted as a remarkable step forward in the history of mathematics, in the sense of providing the essential and sufficiently advanced and adequate tools for mathematics to expand into disciplines such as anthropology, psychology and sociology. In this way it is possible to imagine some significant bridging between the two cultures of science: the hard and the soft disciplines, and thus amplifying a tendency already represented to some extent by complexity science.

The conventional view of natural scientists has been to regard mathematics as a convenient bag of tools to be applied for their specific purposes. Considering the architecture of hadronic mathematics, this appears more as only half of the truth or one side of the coin. Besides representing powerful new tools to study

nature, hadronic mathematics also manifests with a more intimate and inherent connection to physics (and other disciplines), as well as to Nature itself. In this regard hadronic geometry may be of special interest as an illustration:

Isogeometry provided the new notions of a supra-Euclidean isospace as well as its anti-isomorphic isodual space, and the mathematics to describe projections and deformations of geometrical relations from isospace and its isodual into Euclidean space. However, these appear as more than mere mathematical constructs. Illert showed that the universal growth pattern of sea shells could be found only by looking for it as a trajectory in a hidden isospace, a trajectory which is projected into Euclidean space and thereby manifest as the deformed growth patterns humans observe by their senses. Further, the growth pattern of a certain class of sea shells (with bifurcations) could only be understood from the addition and recognition of four new, non-trivial time categories (predicted to be discovered by hadronic mechanics) which manifest as information jumps back and forth in Euclidean space. With regard to sea shell growth, one of this non-trivial time flows could only be explained as a projection from isodual spacetime. This result was consistent with the physics of hadronic mechanics, analyzing masses at both operator and classical level from considering matter and anti-matter (as well as positive and negative energy) to exist on an equal footing in our universe as a whole and hence with total mass (as well as energy and time) cancelling out as zero for the total universe. To establish a basic physical comprehension of Euclidean space constituted as a balanced combination of matter and antimatter, it was required to develop new mathematics with isonumbers and isodual numbers basically mirroring each other. Later, corresponding anti-isomorphies were achieved for genonumbers and hypernumbers with their respective isoduals.

Thus, there is a striking and intimate correspondence between the isodual architecture of hadronic mathematics and the isodual architecture of hadronic mechanics (as well as of hadronic chemistry and hadronic biology). Considering this, one might claim that the Santilli inventions of new number fields in mathematics represent more than mere inventions or constructs, namely discoveries and reconstructions of an ontological architecture being for real also outside the formal landscapes created by the imagination of mathematics and logic. This opens new horizons for treating profound issues in cosmology and ontology.

One might say that with the rise of hadronic mathematics the line between mathematics and other disciplines has turned more blurred or dotted. In some respect this represents a revisit to the Pythagorean and Platonic foundations of mathematics in the birth of western civilization. Hadronic mathematics has provided much new food for thought and further explorations for philosophers of science and mathematics.

If our civilization is to survive despite its current problems, it seems reasonable to expect Santilli to be honored in future history books not only as a giant in

the general history of science, but also in the specific history of mathematics. Hadronic mathematics provided the necessary fuel for rising scientific revolutions in other hadronic sciences. This is mathematics that matters for the future of our world, and hopefully Santillis extraordinary contributions to mathematics will catch fire among talented and ambitious young mathematicians for further advances to be made. The present mellowed volume ought to serve as an excellent appetizer in this regard.

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Volume II:

Isodual Theory of Antimatter Antigravity and Spacetime Machines

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This volume is dedicated to the memory of

Professor Grigorios Tsagas

in recognition of his pioneering work on
the Lie-Santilli isotheory.

Contents

Foreword

Mathematics is a subject which possibly finds itself in a unique position in academia in that it is viewed as both an Art and a Science. Indeed, in different universities, graduates in mathematics may receive Bachelor Degrees in Arts or Sciences. This probably reflects the dual nature of the subject. On the one hand, it may be studied as a subject in its own right. In this sense, its own beauty is there for all to behold; some as serene as da Vinci's "Madonna of the Rocks", other as powerful and majestic as Michelangelo's glorious ceiling of the Sistine Chapel, yet more bringing to mind the impressionist brilliance of Monet's Water Lily series. It is this latter example, with the impressionists interest in light, that links up with the alternative view of mathematics; that view which sees mathematics as the language of science, of physics in particular since physics is that area of science at the very hub of all scientific endeavour, all other branches being dependent on it to some degree. In this guise, however, mathematics is really a tool and any results obtained are of interest only if they relate to what is found in the real world; if results predict some effect, that prediction must be verified by observation and/or experiment. Again, it may be remembered that physics is really a collection of related theories. These theories are all manmade and, as such, are incomplete and imperfect. This is where the work of Ruggero Santilli enters the scientific arena.

Although "conventional wisdom" dictates otherwise, both the widely accepted theories of relativity and quantum mechanics, particularly quantum mechanics, are incomplete. The qualms surrounding both have been muted but possibly more has emerged concerning the inadequacies of quantum mechanics because of the people raising them. Notably, although it is not publicly stated too frequently, Einstein had grave doubts about various aspects of quantum mechanics. Much of the worry has revolved around the role of the observer and over the question of whether quantum mechanics is an objective theory or not. One notable contributor to the debate has been that eminent philosopher of science, Karl Popper. As discussed in my book, "Exploding a Myth", Popper preferred to refer to the experimentalist rather than observer, and expressed the view that that person played the same role in quantum mechanics as in classical mechanics. He felt, therefore, that such a person was there to test the theory. This is totally opposed to the Copenhagen Interpretation which claims that "objective reality has evaporated" and "quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness, of particles". Popper points

out that, over the years, many eminent physicists have switched allegiance from the pro-Copenhagen view. In some ways, the most important of these people was David Bohm, a greatly respected thinker on scientific matters who wrote a book presenting the Copenhagen view of quantum mechanics in minute detail. However, later, apparently under Einstein's influence, he reached the conclusion that his previous view had been in error and also declared the total falsity of the constantly repeated dogma that the quantum theory is complete. It was, of course, this very question of whether or not quantum mechanics is complete which formed the basis of the disagreement between Einstein and Bohr; Einstein stating "No", Bohr "Yes".

However, where does Popper fit into anything to do with Hadronic Mechanics? Quite simply, it was Karl Popper who first drew public attention to the thoughts and ideas of Ruggero Santilli. Popper reflected on, amongst other things, Chadwick's neutron. He noted that it could be viewed, and indeed was interpreted originally, as being composed of a proton and an electron. However, again as he notes, orthodox quantum mechanics offered no viable explanation for such a structure. Hence, in time, it became accepted as a new particle. Popper then noted that, around his (Popper's) time of writing, Santilli had produced an article in which the "first structure model of the neutron" was revived by "resolving the technical difficulties which had led, historically, to the abandonment of the model". It is noted that Santilli felt the difficulties were all associated with the assumption that quantum mechanics applied within the neutron and disappeared when a generalised mechanics is used. Later, Popper goes on to claim Santilli to belong to a new generation of scientists which seemed to him to move on a different path. Popper identifies quite clearly how, in his approach, Santilli distinguishes the region of the arena of incontrovertible applicability of quantum mechanics from nuclear mechanics and hadronics. He notes also his most fascinating arguments in support of the view that quantum mechanics should not, without new tests, be regarded as valid in nuclear and hadronic mechanics.

Ruggero Santilli has devoted his life to examining the possibility of extending the theories of quantum mechanics and relativity so that the new more general theories will apply in situations previously excluded from them. To do this, he has had to go back to the very foundations and develop new mathematics and new mathematical techniques. Only after these new tools were developed was he able to realistically examine the physical situations which originally provoked this lifetime's work. The actual science is his, and his alone, but, as with the realization of all great endeavours, he has not been alone. The support and encouragement he has received from his wife Carla cannot be exaggerated. In truth, the scientific achievements of Ruggero Santiili may be seen, in one light, as the results of a team effort; a team composed of Ruggero himself and Carla Gandiglio in Santilli. The theoretical foundations of the entire work are contained

in this volume; a volume which should be studied rigorously and with a truly open mind by the scientific community at large. This volume contains work which might be thought almost artistic in nature and is that part of the whole possessing the beauty so beloved of mathematicians and great artists. However, the scientific community should reserve its final judgement until it has had a chance to view the experimental and practical evidence which may be produced later in support of this elegant new theoretical framework.

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September 8, 2007

Preface

The author has indicated various times in his works that Albert Einstein has been the biggest scientist of the 20-th century, but also the most exploited scientist in history, because organized academic, financial and ethnic interests on Einstein have pushed the validity of his views way beyond the conditions of his original conception, by therefore turning what is supposed to be a serious scientific process into a pool of often ascientific conduits generally manipulated for personal gains.

This volume presents a solution of one of the several scientific imbalances of historical proportions caused by said ascientific interests in science, the abuse of academic authority and public funds to impose Einstein 's special and general relativity for the treatment of antimatter, while in the scientific reality Einsteinian theories have no means for a quantitative classical differentiation between neutral matter and antimatter, and even when assumed for charged classical particles, their operator image is a particle (rather than the correct charge conjugated antiparticle) with the wrong sign of the charge.

To defend the name of Albert Einstein, it must be noted that antimatter had yet to be discovered at the time of the formulation of his theories. Hence, the entire responsibility of this large scientific imbalance, and the expected severe judgment by posterity, must solely rest with said organized academic, financial and ethnic interests that extended for personal gains Einstein's views beyond the conditions of their original conception without a serious scrutiny.

The solution presented in this volume of the historical imbalance between matter and antimatter is based on the necessary development of a *new mathematics,* today called *Santilli isodual mathematics,* allowing for the first time the classical representation of antimatter as an anti-isomorphic image of that for matter. The isodual conjugation then persists under quantization, where it turns out to be equivalent to charge conjugation, thus restoring a full equivalence and scientific democracy in the treatment of matter and antimatter at all levels, from Newtonian mechanics to second quantization. The resulting *isodual theory of antimatter* then verifies, by conception and construction, all available experimental data on antimatter at the classical and operator level.

It should be stressed that, by no mans, the isodual theory of antimatter is presented as final, or complete or unique, because so many intriguing problems remain open. However, its dismissal in the absence of an alternative broadening of Einsteinian doctrines must be denounced as scientific corruption for personal

gains because the appropriate broadening of Einsteinian doctrines for antimatter is indeed open to scientific debates, but not its need.

An illustration of the damage caused to human knowledge by said ascientific interests on Einstein is given by antigravity. Said interests have dismissed, disrupted and jeopardized for over half a century professional research on the possible antigravity between matter and antimatter (here referred to gravitational repulsion) on grounds that it is not predicted by Einstein's theories.

The need to contain said ascientific interests is rendered evident by the above indicated fact that Einstein's special and general relativities have no means for a quantitative classical differentiations between a neutral matter star and its antimatter counterpart. Under these conditions, the abuse of Einstein's name must be denounced as scientific corruption for personal gains by any person who cares about human dignity, let alone scientific knowledge.

A primary objective of this volume is to show that, once ascientific interests in science are cut out, and a theory for the proper classical and operator formulation of antimatter is worked out, *gravitational repulsion between matter and antimatter is mandated by all available theoretical and experimental evidence, with no credible objection on record.*

In this volume, we also review a proposed experiment to test the gravity of positrons in horizontal flight in a vacuum tube that has been qualified by independent experimentalists in the field as being readily feasible with current technologies and, above all, resolutory.

Yet, even the consideration of this so basic an experiment has been denied by SLAC, CERN, JINR and various other laboratories throughout the world because such a consideration would imply doubts on the universal validity of Einsteinian doctrines, by illustrating in this way the alarming dimension, diversification and capillary nature of ascientific interests at physics laboratories around the world.

The final objective of this volume is to illustrate that the expected experimental verification of antigravity between matter and antimatter will imply advances in human knowledge simply beyond our imagination at this time, such as a fully causal *spacetime geometric locomotion,* here referred to motion in space and time via the alteration of the local geometry, although not for ordinary matter or antimatter, but for a particular form of matter and antimatter called isoselfdual.

It is written in history that, following the achievement of control with protracted impunity, individuals lose the understanding of the self-damaging character of their actions. It is also written in history that people have the structure they want or deserve.

In view of the above ascientific condition of science, no basic advance on antimatter, antigravity and other far reaching frontiers is possible without the joint consideration of issues pertaining to scientific ethics and accountability particu-

larly when ascientific interests in science are permitted to operate, by vile subservience or complicity, under public financial support.

To put it bluntly, the judgment expected by posterity on our contemporary scientific community will crucially depend on its capability to identify, denounce and contain ascientific and consequently asocial interests in science.

Ruggero Maria Santilli
January 19, 2008

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This legal notice has been made necessary because, as shown in Section 1.5, the author has been dubbed "the most plagiarized scientist of the 20-th century," as it is the case of the thousands of papers in deformations published without any quotation of their origination by the author in 1967. These, and other attempted paternity frauds, have forced the author to initiate legal action reported in web site [1].

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In faith

Ruggero Maria Santilli

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Tarpon Springs, Florida, U. S. A.

October 11, 2007

[1] International Committee on Scientific Ethics and Accountability
http://www.scientificethics.org

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Chapter 2

ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.1 ELEMENTS OF ISODUAL MATHEMATICS

2.1.1 Isodual Unit, Isodual Numbers and Isodual Fields

The first comprehensive study of the isodual theory for point-like antiparticles has been presented by the author in monograph [34]. However, the field is subjected to continuous developments following its first presentation in papers [1] of 1985. Hence, it is important to review the most recent formulation of the isodual mathematics in sufficient details to render this monograph self-sufficient.

In this section, we identify only those aspects of isodual mathematics that are essential for the understanding of the physical profiles presented in the subsequent sections of this chapter. We begin with a study of the most fundamental elements of all mathematical and physical formulations, units, numbers and fields, from which all remaining formulations can be uniquely and unambiguously derived via simple compatibility arguments. To avoid un-necessary repetitions, we assume the reader has a knowledge of the basic mathematics used for the classical and operator treatment of matter, including a knowledge of the fields of real, complex and quaternionic numbers. The symbol † usec in this chapter denotes conventional Hermitean conjugation, namely, transpose t plus complex conjugation ^{c}. Hence, for real numbers n we have $n^\dagger = n$, for complex numbers a we have $a^\dagger = a^c$ and for qauternions q we have $q^\dagger = q^{tc}$.

DEFINITION 2.1.1: Let $F = F(a, +, \times)$ be a field (of characteristic zero), namely a ring with elements given by real number $a = n,\ F = R(n, +, \times)$, complex numbers $A = c,\ F = C(c, +, \times)$, or quaternionic numbers $a = q,\ F = Q(q, +, \times)$,

with conventional sum $a + b$ *verifying the commutative law*

$$a + b = b + a = c \in F, \tag{2.1.1}$$

the associative law

$$(a + b) + c = a + (b + c) = d \in F, \tag{2.1.2}$$

conventional product $a \times b$ *verifying the associative law*

$$(a \times b) \times c = a \times (b \times c) = e \in F, \tag{2.1.3}$$

(but not necessarily the commutative law, $a \times b \neq b \times a$ *since the latter is violated by quaternions), and the right and left distributive laws*

$$(a + b) \times c = a \times c + b \times c = f \in F, \tag{2.1.4a}$$

$$a \times (b + c) = a \times b + a \times c = g \in F, \tag{2.1.4b}$$

left and right additive unit 0*,*

$$a + 0 = 0 + a = a \in F, \tag{2.1.5}$$

and left and right multiplicative unit I*,*

$$a \times I = I \times a = a \in F, \tag{2.1.6}$$

$\forall a,\, b,\, c \in F$*. Santilli's isodual fields (first introduced in* Refs. [1] *and then presented in details in* Ref. [2]*) are rings* $F^d = F^d(a^d, +^d, \times^d)$ *with elements given by isodual numbers*

$$a^d = -a^\dagger,\ a^d \in F, \tag{2.1.7}$$

with associative and commutative isodual sum

$$a^d +^d b^d = -(a + b)^\dagger = c^d \in F^d, \tag{2.1.8}$$

associative and distributive isodual product

$$a^d \times^d b^d = a^d \times (I^d)^{-1} \times b^d = c^d \in F^d, \tag{2.1.9}$$

additive isodual unit $0^d = 0$*,*

$$a^d +^d 0^d = 0^d +^d a^d = a^d, \tag{2.1.10}$$

and multiplicative isodual unit $I^d = -I^\dagger$*,*

$$a^d \times^d I^d = I^d \times^d a^d = a^d, \forall a^d, b^d \in F^d. \tag{2.1.11}$$

The proof of the following property is elementary.

LEMMA 2.1.1 [1,2]: *Isodual fields are fields, namely, if F is a field, its image F^d under the isodual map is also a field.*

The above lemma establishes the property (first identified in Refs. [1]) that *the axioms of a field do not require that the multiplicative unit be necessarily positive-definite, because the same axioms are also verified by negative-definite units.* The proof of the following property is equally simple.

LEMMA 2.1.2 [1,2]: *Fields F and their isodual images F^d are anti-isomorphic to each other.*

Lemmas 2.1.1 and 1.2.2 illustrate the origin of the name "isodual mathematics". In fact, to represent antimatter the needed mathematics must be a suitable "dual" of conventional mathematics, while the prefix "iso" is used in its Greek meaning of preserving the original axioms.

It is evident that for real numbers we have

$$n^d = -n, \tag{2.1.12}$$

while for complex numbers we have

$$c^d = (n_1 + i \times n_2)^d = -n_1 + i \times n_2 = -\bar{c}, \tag{2.1.13}$$

with a similar formulation for quaternions.

It is also evident that, for consistency, *all operations on numbers must be subjected to isoduality when dealing with isodual numbers.* This implies: the *isodual powers*

$$(a^d)^{n^d} = a^d \times^d a^d \times^d a^d \dots \tag{2.1.14}$$

(n times, with n an integer); the *isodual square root*

$$a^{d^{(1/2)d}} = -\sqrt{-a^\dagger}^\dagger, a^{d^{(1/2)d}} \times^d a^{d^{(1/2)d}} = a^d, \quad 1^{d^{(1/2)d}} = -i; \tag{2.1.15}$$

the *isodual quotient*

$$a^d /^d b^d = -(a^\dagger / b^\dagger) = c^d, \ b^d \times^d c^d = a^d; \tag{2.1.16}$$

etc.

An important property for the characterization of antimatter is the following:

LEMMA 2.1.3. [2]: *isodual fields have a negative-definite norm, called isodual norm,*

$$|a^d|^d = |a^\dagger| \times I^d = -(aa^\dagger)^{1/2} < 0, \tag{2.1.17}$$

where $|\ldots|$ denotes the conventional norm.

For isodual real numbers we therefore have the isodual isonorm

$$|n^d|^d \ = \ -|n| < 0, \tag{2.1.18}$$

and for isodual complex numbers we have

$$|c^d|^d \ = \ -|\bar{c}| \ = \ -(c\bar{c})^{1/2} \ = \ -(n_1^2 + n_2^2)^{1/2}. \tag{2.1.19}$$

LEMMA 2.1.4 [2]: *All quantities that are positive-definite when referred to positive units and related fields of matter (such as mass, energy, angular momentum, density, temperature, time, etc.) become negative-definite when referred to isodual units and related isodual fields of antimatter.*

As recalled Chapter 1, antiparticles have been discovered in the *negative-energy solutions* of Dirac's equation and they were originally thought to evolve *backward in time* (Stueckelberg, Feynman, and others, see Refs. [1,2] of Chapter 1). The possibility of representing antiparticles via isodual methods is therefore visible already from these introductory notions.

The main novelty is that the conventional treatment of negative-definite energy and time was (and still is) referred to the conventional unit $+1$. This leads to a number of contradictions in the physical behavior of antiparticles.

By comparison, *negative-definite physical quantities of isodual theories are referred to a negative-definite unit $I^d < 0$*. This implies a mathematical and physical equivalence between *positive-definite quantities referred to positive-definite units, characterizing matter, and negative-definite quantities referred to negative-definite units, characterizing antimatter*. These foundations then permit a novel characterization of antimatter beginning at the *Newtonian* level, and then persisting at all subsequent levels.

DEFINITION 2.1.2 [2]: *A quantity is called isoselfdual when it coincides with its isodual.*

It is easy to verify that the imaginary unit is isoselfdual because

$$i^d \ = \ -i^\dagger \ = \ -\bar{i} \ = \ -(-i) \ = \ i. \tag{2.1.20}$$

This property permits a better understanding of the isoduality of complex numbers that can be written explicitly

$$c^d \ = \ (n_1 + i \times n_2)^d \ = \ n_1^d + i^d \times^d n_2^d \ = \ -n_1 + i \times n_2 \ = \ -\bar{c}. \tag{2.1.21}$$

The above property will be important to prove the equivalence of isoduality and charge conjugation at the operator level.

As we shall see, *isoselfduality is a new fundamental view of nature* with deep physical implications, not only in classical and quantum mechanics but also in cosmology. For instance we shall see that Dirac's gamma matrices are isoselfdual, thus implying a basically new interpretation of this equation that has remained unidentified for about one century. We shall also see that, when applied to cosmology, isoselfduality implies equal distribution of matter and antimatter in the universe, with identically null total physical characteristic, such as identically null total time, identically null total mass, etc.

We should also indicate that we have assumed the isoduality of the multiplication, $\times \to \times^d = \times(-1)\times = -\times$, but *not* that of the sum, $+ \to +^d = +(-1)+ = -$. This approach may not appear entirely motivated to the mathematically inclined reader because *fields are invariant under the above defined isoduality of the sum* due to the invariance of the additive unit, $0 \to 0^d \equiv 0$ (although fields are not invariant under the isoduality of the product due to the lack of invariance of the multiplicative unit, $1 \to 1^d = -1$).

The above decision is motivated by pragmatic, rather than mathematical arguments and, more specifically, for compatibility with the more general isofields and genofields, studied in the following chapters. In fact, at the latter broader levels, we have the loss of the invariance of the axioms of a field under these broader liftings of the sum. In turn, the loss of the field axioms cause the consequential inapplicability of the theory for physical applications as currently known, that is, based on "numbers" as rings verifying the axioms of a field, thus admitting a right and left, well defined, multiplicative unit representing the selected units of measurements.

It should also be stressed that, to avoid apparent inconsistencies, the isodual conjugation must be applied to all numbers and all their multiplications (or divisions). For instance, the isodual of a real numbers $n = n \times 1$ is given by $n^d \times^d 1^d = -n \times 1 = -n$ and not by $n^d \times 1^d = n$.

We assume the reader is aware of the emergence here of *new numbers,* those with a negative unit, that have no connection with ordinary negative numbers and are the true foundations of the isodual theory of antimatter.

2.1.2 Isodual Functional Analysis

All conventional and special functions and transforms, as well as functional analysis at large, must be subjected to isoduality for consistent applications, resulting in the simple, yet unique and significant *isodual functional analysis*, studied by Kadeisvili [3], Santilli [4] and others.

We here mention the *isodual trigonometric functions*

$$\sin^d \theta^d \ = \ -\sin(-\theta), \ \ \cos^d \theta^d \ = \ -\cos(-\theta), \tag{2.1.22}$$

with related basic property

$$\cos^{d\,2d}\theta^d +^d \sin^{d\,2d}\theta^d \;=\; 1^d \;=\; -1, \tag{2.1.23}$$

the *isodual hyperbolic functions*

$$\sinh^d w^d \;=\; -\sinh(-w),\; \cosh^d w^d \;=\; -\cosh(-w), \tag{2.1.24}$$

with related basic property

$$\cosh^{d\,2d} w^d -^d \sinh^{d\,2d} w^d \;=\; 1^d \;=\; -1, \tag{2.1.25}$$

the *isodual logarithm* and the *isodual exponentiation* defined respectively by

$$\log^d n^d \;=\; -\log(-n), \tag{2.1.26a}$$

$$e_d^{X^d} = 1^d + X^d/^d 1!^d + X^{d^{2d}}/^d 2!^d + \ldots = -e^X, \tag{2.1.26b}$$

etc. Interested readers can then easily construct the isodual image of special functions, transforms, distributions, etc.

2.1.3 Isodual Differential and Integral Calculus

Contrary to a rather popular belief, the differential calculus is indeed dependent on the assumed unit. This property is not so transparent in the conventional formulation because the basic unit is the trivial number +1. However, the dependence of the unit emerges rather forcefully under its generalization.

The *isodual differential calculus*, first introduced by Santilli in Ref. [5a], is characterized by the *isodual differentials*

$$d^d x^k \;=\; I^d \times dx^k \;=\; -dx^k,\;\; d^d x_k \;=\; -dx_k, \tag{2.1.27}$$

with corresponding *isodual derivatives*

$$\partial^d/^d\partial^d x^k \;=\; -\partial/\partial x^k,\;\; \partial^d/^d\partial^d x_k \;=\; -\partial/\partial x_k, \tag{2.1.28}$$

and related isodual properties.

Note that *conventional differentials are isoselfdual*, i.e.,

$$(dx^k)^d \;=\; d^d x^{kd} \equiv d\,x^k, \tag{2.1.29}$$

but *derivatives are not isoselfdual*,

$$[\partial f/\partial x^k]^d \;=\; -\partial^d f^d/^d\partial^d x^{kd}. \tag{2.1.30}$$

The above properties explain why the isodual differential calculus remained undiscovered for centuries.

Other notions, such as the *isodual integral calculus,* can be easily derived and shall be assumed as known hereon.

2.1.4 Lie-Santilli Isodual Theory

Let $\mathbf{L}$ be an n-dimensional Lie algebra in its regular representation with universal enveloping associative algebra $\xi(\mathbf{L})$, $[\xi(\mathbf{L})]^- \approx \mathbf{L}$, n-dimensional unit $I = Diag.(1, 1, \ldots, 1)$, ordered set of Hermitian generators $X = X^\dagger = \{X_k\}$, $k = 1, 2, \ldots, n$, conventional associative product $X_i \times X_j$, and familiar Lie's Theorems over a field $F(a, +, \times)$.

The *Lie-Santilli isodual theory* was first submitted in Ref. [1] and then studied in Refs. [4-7] as well as by other authors [23-31]. *The isodual universal associative algebra* $[\xi(\mathbf{L})]^d$ is characterized by the *isodual unit* I^d, *isodual generators* $X^d = -X$, and isodual associative product

$$X_i^d \times^d X_j^d \ = \ -X_i \times X_j, \tag{2.1.31}$$

with corresponding infinite-dimensional basis characterized by the *Poincaré-Birkhoff-Witt-Santilli isodual theorem*

$$I^d, X_i^d \times^d X_j^d, \ \ i \leq j; \ \ X_i^d \times^d X_j^d \times X_k^d, \ i \leq j \leq k, \ldots \tag{2.1.32}$$

and related *isodual exponentiation* of a generic quantity A^d

$$e^{d^{A^d}} \ = \ I^d + A^d/^d 1!^d + A^d \times^d A^d/^d 2!^d + \ldots \ = \ -e^{A^\dagger}, \tag{2.1.33}$$

where e is the conventional exponentiation.

The attached *Lie-Santilli isodual algebra* $\mathbf{L}^d \approx (\xi^d)^-$ over the isodual field $F^d(a^d, +^d, \times^d)$ is characterized by the *isodual commutators* [1]

$$[X_i^d, {}^d X_j^d] \ = \ -[X_i, X_j] \ = \ C_{ij}^{k^d} \times^d X_k^d. \tag{2.1.34}$$

with classical realizations given in Section 2.2.6.

Let G be a conventional, connected, n-dimensional Lie transformation group on a metric (or pseudo-metric) space $S(x, g, F)$ admitting $\mathbf{L}$ as the Lie algebra in the neighborhood of the identity, with generators X_k and parameters $w = \{w_k\}$.

The *Lie-Santilli isodual transformation group* G^d admitting the isodual Lie algebra $\mathbf{L}^d$ in the neighborhood of the isodual identity I^d is the n-dimensional group with generators $X^d = \{-X_k\}$ and parameters $w^d = \{-w_k\}$ over the isodual field F^d with generic element [1]

$$U^d(w^d) \ = \ e^{d^{i^d \times^d {}_{w^d} \times^d X^d}} \ = \ -e^{i \times (-w) \times X} \ = \ -U(-w). \tag{2.1.35}$$

The *isodual symmetries* are then defined accordingly via the use of the isodual groups G^d and they are anti-isomorphic to the corresponding conventional symmetries, as desired. For additional details, one may consult Ref. [4,5b].

In this chapter we shall therefore use the *conventional Poincaré, internal and other symmetries* for the characterization of *matter*, and the *Poincaré-Santilli, internal and other isodual symmetries* for the characterization of *antimatter*.

2.1.5 Isodual Euclidean Geometry

Conventional (vector and) metric spaces are defined over conventional fields. It is evident that the isoduality of fields requires, for consistency, a corresponding isoduality of (vector and) metric spaces. The need for the isodualities of all quantities acting on a metric space (e.g., conventional and special functions and transforms, differential calculus, etc.) becomes then evident.

DEFINITION 2.1.3: Let $S = S(x, g, R)$ be a conventional N-dimensional metric or pseudo-metric space with local coordinates $x = \{x^k\}$, $k = 1, 2, \ldots, N$, nowhere degenerate, sufficiently smooth, real-valued and symmetric metric $g(x, \ldots)$ and related invariant

$$x^2 \ = \ (x^i \times g_{ij} \times x^j) \times I, \tag{2.1.36}$$

over the reals R. The isodual spaces, first introduced in Ref. [1] (see also Refs. [4,5] and, for a more recent account, Ref. [22]), *are the spaces $S^d(x^d, g^d, R^d)$ with isodual coordinates $x^d = x^d = -x^t$ (where t stands for transposed), isodual metric*

$$g^d(x^d, \ldots) \ = \ -g^\dagger(-x^\dagger, \ldots) \ = \ -g(-x^t, \ldots), \tag{2.1.37}$$

and isodual interval

$$\begin{aligned}(x-y)^{d^2\,d} \ &= \ [(x-y)^{id} \times^d g^d_{ij} \times^d (x-y)^{jd}] \times I^d = \\ &= \ [(x-y)^i \times g^d_{ij} \times (x-y)^j] \times I^d, \end{aligned} \tag{2.1.38}$$

defined over the isodual field $R^d \ = \ R^d(n^d, +^d, \times^d)$ with the same isodual isounit I^d.

The basic nonrelativistic space of our analysis is the three-dimensional *isodual Euclidean space* [1,9],

$$E^d(r^d, \delta^d, R^d) : r^d \ = \ \{r^{kd}\} \ = \ \{-r^k\} \ = \ \{-x, -y, -z\}, \tag{2.1.39a}$$

$$\delta^d \ = \ -\delta \ = \ Diag.(-1, -1, -1),$$

$$I^d \ = \ -I \ = \ Diag.(-1, -1, -1). \tag{2.1.39b}$$

The *isodual Euclidean geometry* is the geometry of the isodual space E^d over R^d and it is given by a step-by-step isoduality of all the various aspects of the conventional geometry (see monograph [5a] for details).

By recalling that the norm on R^d is negative-definite, the *isodual distance* among two points on an isodual line is also negative definite and it is given by

$$D^d \ = \ D \times I^d \ = \ -D, \tag{2.1.40}$$

where D is the conventional distance. Similar isodualities apply to all remaining notions, including the notions of parallel and intersecting isodual lines, the Euclidean axioms, etc.

The *isodual sphere* with radius $R^d = -R$ is the perfect sphere on E^d over R^d and, as such, it has *negative radius* (Figure 2.1),

$$R^{d2d} = (x^{d2d} + y^{d2d} + z^{d2d}) \times I^d =$$
$$= (x^2 + y^2 + z^2) \times I = R^2. \tag{2.1.41}$$

Note that the above expression coincides with that for the conventional sphere. This illustrates the reasons, following about one century of studies, the isodual rotational group and symmetry where identified for the first time in Ref. [1]. Note, however, that the latter result required the prior discovery of *new numbers,* those with a negative unit.

A similar characterization holds for other isodual shapes characterizing antimatter in our isodual theory.

LEMMA 2.1.5: The isodual Euclidean geometry on E^d over R^d is anti-isomorphic to the conventional geometry on E over R.

The group of isometries of E^d over R^d is the *isodual Euclidean group* $E^d(3) = \mathcal{R}^d(\theta^d) \times^d T^d(3)$ where $\mathcal{R}^d(\theta)$ is the isodual group of rotations first introduced in Ref. [1], and $T^d(3)$ is the isodual group of translations (see also Ref. [5a] for details).

2.1.6 Isodual Minkowskian Geometry

Let $M(x, \eta, R)$ be the conventional Minkowski spacetime with local coordinates $x = (r^k, t) = (x^\mu)$, $k = 1, 2, 3$, $\mu = 1, 2, 3, 4$, metric $\eta = Diag.(1, 1, 1, -1)$ and basic unit $I = Diag.(1, 1, 1, 1)$ on the reals $R = R(n, +, \times)$.

The *Minkowski-Santilli isodual spacetime*, first introduced in Ref. [7] and studied in details in Ref. [8], is given by

$$M^d(x^d, \eta^d, R^d) : x^d = \{x^{\mu d}\} = \{x^\mu \times I^d\} = \{-r, -c_o t\} \times I, \tag{2.1.42}$$

with isodual metric and isodual unit

$$\eta^d = -\eta = Diag.(-1, -1, -1, +1), \tag{2.1.43a}$$
$$I^d = Diag.(-1, -1, -1, -1). \tag{2.1.43b}$$

The *Minkowski-Santilli isodual geometry* [8] is the geometry of isodual spaces M^d over R^d. The new geometry is also characterized by a simple isoduality of the conventional Minkowskian geometry as studied in details in memoir.

The fundamental symmetry of this chapter is given by the group of isometries of M^d over R^d, namely, the *Poincaré-Santilli isodual symmetry* [7,8]

$$P^d(3.1) = \mathcal{L}^d(3.1) \times T^d(3.1), \tag{2.1.44}$$

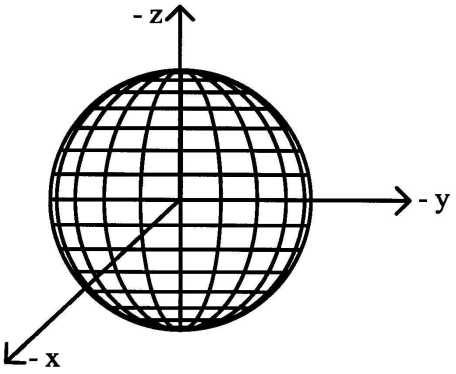

Figure 2.1. A schematic view of the isodual sphere on isodual Euclidean spaces over isodual fields. The understanding of the content of this chapter requires the knowledge that the isodual sphere and the conventional sphere coincide when inspected by an observer either in the Euclidean or in the isodual Euclidean space, due to the identity of the related expressions (2.1.36) and (2.1.38). This identity is at the foundation of the perception that antiparticles "appear" to exist in our space, while in reality they belong to a structurally different space coexisting within our own, thus setting the foundations of a "multidimensional universe" coexisting in the same space of our sensory perception. The reader should keep in mind that the isodual sphere is the idealization of the shape of an antiparticle used in this monograph.

where $\mathcal{L}^d(3.1)$ is the Lorentz-Santilli isodual group and $T^d(3.1)$ is the isodual group of translations.

2.1.7 Isodual Riemannian Geometry

Consider a Riemannian space $\Re(x, g, R)$ in $(3+1)$ dimensions [32] with basic unit $I = Diag.(1, 1, 1, 1)$, nowhere singular and symmetric metric $g(x)$ and related Riemannian geometry in local formulation (see, e.g., Ref. [27]).

The *Riemannian-Santilli isodual spaces* (first introduced in Ref. [11]) are given by

$$\begin{aligned} \Re^d(x^d, g^d, R^d): \quad x^d &= \{-x^\mu\}, \\ g^d &= -g(x), \ g \in \Re(x, g, R), \\ I^d &= Diag.(-1, -1, -1, -1) \end{aligned} \tag{2.1.45}$$

with interval

$$\begin{aligned} x^{2d} &= [x^{dt} \times^d g^d(x^d) \times^d x^d] \times I^d = \\ &= [x^t \times g^d(x^d) \times x] \times I^d \in R^d, \end{aligned} \tag{2.1.46}$$

where t stands for transposed.

The *Riemannian-Santilli isodual geometry* [8] is the geometry of spaces $\hat{\Re}^d$ over R^d, and it is also given by step-by-step isodualities of the conventional geometry, including, most importantly, the isoduality of the differential and exterior calculus.

As an example, an *isodual vector field* $X^d(x^d)$ on $\hat{\Re}^d$ is given by $X^d(x^d) = -X^t(-x^t)$. The *isodual exterior differential* of $X^d(x^d)$ is given by

$$D^d X^{kd}(x^d) \ = \ d^d X^{kd}(x^d) + \Gamma^{dk}_{i\,j} \times^d X^{id} \times^d d^d x^{jd} \ = \ DX^k(-x), \qquad (2.1.47)$$

where the Γ^d's are the components of the *isodual connection*. The *isodual covariant derivative* is then given by

$$X^{id}(x^d)_{|^{dk}} \ = \ \partial^d X^{id}(x^d)/^d \partial^d x^{kd} + \Gamma^{di}_{jk} \times^d X^{jd}(x^d) \ = \ -X^i(-x)_{|^k}. \qquad (2.1.48)$$

The interested reader can then easily derive the isoduality of the remaining notions of the conventional geometry.

It is an instructive exercise for the interested reader to work out in detail the proof of the following:

LEMMA 2.1.6 [8]: *The isodual image of a Riemannian space* $\hat{\Re}^d(x^d, g^d, R^d)$ *is characterized by the following maps*:

$$Basic \ \ Unit$$

$$I \ \to \ I^d \ = \ -I,$$

$$Metric$$

$$g \ \to \ g^d \ = \ -g, \qquad (2.1.49a)$$

$$Connection \ \ Coefficients$$

$$\Gamma_{klh} \ \to \ \Gamma^d_{klh} \ = \ -\Gamma_{klh}, \qquad (2.1.49b)$$

$$Curvature \ \ Tensor$$

$$R_{lijk} \ \to \ R^d_{lijk} \ = \ -R_{lijk}, \qquad (2.1.49c)$$

$$Ricci \ \ Tensor$$

$$R_{\mu\nu} \ \to R^d_{\mu\nu} \ = \ -R_{\mu\nu}, \qquad (2.1.49d)$$

$$Ricci \ \ Scalar$$

$$R \ \to \ R^d \ = \ R, \qquad (2.1.49e)$$

$$Einstein - Hilbert \ \ Tensor$$

$$G_{\mu\nu} \ \to \ G^d_{\mu\nu} \ = \ -G_{\mu\nu}, \qquad (2.1.49f)$$

$$Electromagnetic \quad Potentials$$

$$A_\mu \ \rightarrow \ A^d_\mu \ = \ -A_\mu, \tag{2.1.49g}$$

$$Electromagnetic \quad Field$$

$$F_{\mu\nu} \ \rightarrow \ F^d_{\mu\nu} \ = \ -F_{\mu\nu}, \tag{2.1.49h}$$

$$ElmEnergy - Momentum \quad Tensor$$

$$T_{\mu\nu} \ \rightarrow \ T^d_{\mu\nu} \ = \ -T_{\mu\nu}, \tag{2.1.49i}$$

In summary, the geometries significant for this study are: the *conventional Euclidean, Minkowskian and Riemannian geometries* used for the characterization of *matter*; and the *isodual Euclidean, Minkowskian and Riemannian geometries* used for the characterization of *antimatter*.

The reader can now begin to see the achievement of axiomatic compatibility between gravitation and electroweak interactions that is permitted by the isodual theory of antimatter. In fact, the latter is treated via negative-definite energy-momentum tensors, thus being compatible with the negative-energy solutions of electroweak interactions, therefore setting correct axiomatic foundations for a true grand unification studied in the next chapter.

2.2 CLASSICAL ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.2.1 Basic Assumptions

Thanks to the preceding study of isodual mathematics, we are now sufficiently equipped to resolve the scientific impasse caused by the absence of a classical theory of antimatter studied in Section 1.1.

As it is well known, the contemporary treatment of matter is characterized by *conventional mathematics*, here referred to ordinary numbers, fields, spaces, etc. with *positive units and norms*, thus having positive characteristics of mass, energy, time, etc.

In this chapter we study the *characterization of antimatter via isodual numbers, fields, spaces, etc., thus having negative-definite units and norms*. In particular, all characteristics of matter (and not only charge) change sign for antimatter when represented via isoduality.

The above characterization of antimatter evidently provides the correct conjugation of the charge at the desired classical level. However, by no means, the sole change of the sign of the charge is sufficient to ensure a consistent classical representation of antimatter. To achieve consistency, the theory must resolve the main problematic aspect of current classical treatments, the fact that their operator image is not the correct charge conjugate state (Section 2.1).

The above problematic aspect is indeed resolved by the isodual theory. The main reason is that, jointly with the conjugation of the charge, isoduality also conjugates *all* other physical characteristics of matter. This implies *two* channels of quantization, the conventional one for matter and a new *isodual quantization* for antimatter (see Section 2.3) in such a way that its operator image is indeed the charge conjugate of that of matter.

In this section, we study the physical consistency of the theory in its classical formulation. The novel isodual quantization, the equivalence of isoduality and charge conjugation and related operator issues are studied in the next section.

Beginning our analysis, we note that the isodual theory of antimatter resolves the traditional obstacles against negative energies and masses. In fact, *particles with negative energies and masses measured with negative units are fully equivalent to particles with positive energies and masses measured with positive units.* This result has permitted the elimination of sole use of second quantization for the characterization of antiparticles because antimatter becomes treatable at *all* levels, including second quantization.

The isodual theory of antimatter also resolves the additional, well known, problematic aspects of motion backward in time. In fact, *time moving backward measured with a negative unit is fully equivalent on grounds of causality to time moving forward measured with a positive unit.*

This confirms the plausibility of the first conception of antiparticles by Stueckelberg and others as moving backward in time (see the historical analysis in Ref. [1] of Chapter 1), and creates new possibilities for the ongoing research on the so-called "spacetime machine" studied in Chapter 5.

In this section, we construct the classical isodual theory of antimatter at the Newtonian, Lagrangian, Hamiltonian, Galilean, relativistic and gravitational levels; we prove its axiomatic consistency; and we verify its compatibility with available classical experimental evidence (that dealing with electromagnetic interactions only). Operator formulations and their experimental verifications will be studied in the next section.

2.2.2 Need for Isoduality to Represent All Time Directions

It is popularly believed that time has only two directions, the celebrated *Eddington's time arrows.* In reality, *time has four different directions* depending on whether motion is forward or backward and occurs in the future or in the past, as illustrated in Figure 2.2. In turn, the correct use of all four different directions of time is mandatory, for instance, in serious studies of bifurcations, as we shall see.

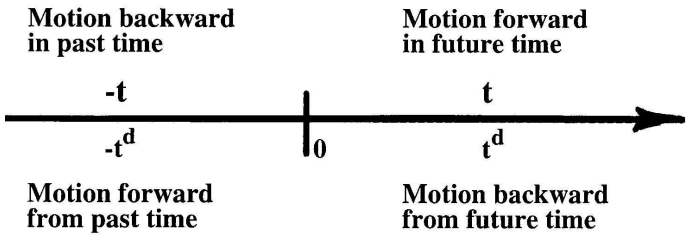

Figure 2.2. A schematic view of the "four different directions of time", depending on whether motion is forward or backward and occurs in the future or in the past. Due to the sole existence of one time conjugation, time reversal, the theoretical physics of the 20-th century missed two of the four directions of time, resulting in fundamental insufficiencies ranging from the lack of a deeper understanding of antiparticles to basic insufficiencies in biological structures and excessively insufficient cosmological views. It is evident that isoduality can indeed represent the two missing time arrows and this illustrates a basic need for the isodual theory.

It is evident that theoretical physics of the 20-th century could not explain all four directions of time, since it possessed only one conjugation, time reversal, and this explains the reason the two remaining directions of time were ignored.

It is equally evident that isoduality does indeed permit the representation of the two missing directions of time, thus illustrating its need.

We assume the reader is now familiar with the differences between time reversal and isoduality. Time reversal changes the direction of time while keeping the underlying space and units unchanged, while isoduality changes the direction of time while mapping the underlying space and units into different forms.

Unless otherwise specified, through the rest of this volume time t will be indicate *motion forward in future times,* $-t$ will indicate *motion backward in past times,* t^d will indicate motion backward from future times, and $-t^d$ will indicate *motion forward from past times.*

2.2.3 Experimental Verification of the Isodual Theory of Antimatter in Classical Physics

The experimental verification of the isodual theory of antimatter at the *classical* level is provided by the compliance of the theory with the only available experimental data, those on Coulomb interactions.

For that purpose, let us consider the Coulomb interactions under the customary notation that *positive (negative) forces represent repulsion (attraction)* when formulated in conventional Euclidean space.

Under such an assumption, the *repulsive* Coulomb force among two *particles* of negative charges $-q_1$ and $-q_2$ in Euclidean space $E(r, \delta, R)$ is given by

$$F = K \times (-q_1) \times (-q_2)/r \times r > 0, \tag{2.2.1}$$

where K is a positive constant whose explicit value (here irrelevant) depends on the selected units, the operations of multiplication $\times$ and division / are the conventional ones of the underlying field $R(n, +, \times)$.

Under isoduality to $E^d(r^d, \delta^d, R^d)$ the above law is mapped into the form

$$F^d = K^d \times^d (-q_1)^d \times^d (-q_2)^d /^d r^d \times^d r^d = -F < 0, \tag{2.2.2}$$

where $\times^d = -\times$ and $/^d = -/$ are the isodual operations of the underlying field $R^d(n^d, +, \times^d)$.

But the isodual force $F^d = -F$ occurs in the isodual Euclidean space and it is, therefore, defined with respect to the unit -1. This implies that the reversal of the sign of a repulsive force measured with a negative unit also describes repulsion. As a result, isoduality correctly represents the *repulsive* character of the Coulomb force for two *antiparticles* with *positive* charges, a result first achieved in Ref. [9].

The formulation of the cases of two particles with positive charges and their antiparticles with negative charges is left to the interested reader.

The Coulomb force between a *particle* and an *antiparticle* can only be computed by *projecting the antiparticle in the conventional space of the particle or vice-versa*. In the former case we have

$$F = K \times (-q_1) \times (-q_2)^d / r \times r < 0, \tag{2.2.3}$$

thus yielding an *attractive* force, as experimentally established. In the projection of the particle in the isodual space of the antiparticle, we have

$$F^d = K^d \times^d (-q_1) \times^d (-q_2)^d /^d r^d \times^d r^d > 0. \tag{2.2.4}$$

But this force is now measured with the unit -1, thus resulting in being again *attractive*.

The study of Coulomb interactions of magnetic poles and other classical experimental data is left to the interested reader.

In conclusion, the isodual theory of antimatter correctly represents all available classical experimental evidence in the field.

2.2.4 Isodual Newtonian Mechanics

A central objective of this section is to show that the isodual theory of antimatter resolves the scientific imbalance of the 20-th century between matter and antimatter, by permitting the study of antimatter at *all* levels as occurring for matter. Such an objective can only be achieved by first establishing the existence of a

Newtonian representation of antimatter subsequently proved to be compatible with known operator formulations.

As it is well known, the Newtonian treatment of N *point-like particles* is based on a $7N$-dimensional representation space given by the Kronecker products of the Euclidean spaces of time t, coordinates r and velocities v (for the conventional case see Refs. [33,34]),

$$S(t,\, r,\, v) = E(t,\, R_t) \times E(r,\, \delta,\, R_r) \times E(v,\, \delta,\, R_v), \tag{2.2.5}$$

where

$$r = (r_a^k) = (r_a^1,\, r_a^2,\, r_a^3) = (x_a,\, y_a,\, z_a), \tag{2.2.6a}$$

$$v = (v_{ka}) = (v_{1a},\, v_{2a},\, v_{3a}) = (v_{xa},\, v_{ua},\, v_{za}) = dr/dt, \tag{2.2.6b}$$

$$\delta = Diag.(1,\, 1,\, 1),\ k = 1,\, 2,\, 3,\ a = 1,\, 2,\, 3, \ldots,\, N, \tag{2.2.6c}$$

and the base fields are trivially identical, i.e., $R_t = R_r = R_v$, since all units are assumed to have the trivial value $+1$, resulting in the trivial total unit

$$I_{tot} = I_t \times I_r \times I_v = 1 \times 1 \times 1 = 1. \tag{2.2.7}$$

The resulting basic equations are then given by the celebrated *Newton's equations for point-like particles*

$$m_a \times dv_{ka}/dt = F_{ka}(t,\, r,\, v),\ k = 1,\, 2,\, 3,\ a = 1,\, 2,\, 3, \ldots,\, N. \tag{2.2.8}$$

The basic space for the treatment of n antiparticles is given by the $7N$-dimensional *isodual space* [9]

$$S^d(t^d, r^d, v^d) \ = \ E^d(t^d, R_t^d) \times E^d(r^d, \delta^d, R^d) \times E^d(v^d, \delta^d, R^d), \tag{2.2.9}$$

with *isodual unit* and *isodual metric*

$$I_{Tot}^d \ = \ I_t^d \times I_r^d \times I_v^d, \tag{2.2.10a}$$

$$I_t^d \ = \ -1,\ I_r^d \ = \ I_v^d \ = Diag.(-1, -1, -1), \tag{2.2.10b}$$

$$\delta^d = Diag.(1^d, 1^d, 1^d) = Diag.(-1, -1, -1). \tag{2.2.10c}$$

We reach in this way the basic equations of this chapter, today known as the *Newton-Santilli isodual equations for point-like antiparticles,* first introduced in Ref. [4],[1]

$$m_a^d \times^d d^d v_{ka}^d /^d d^d t^d \ = \ F_{ka}^d(t^d, r^d, v^d), \tag{2.2.11}$$
$$k = x,\, y,\, z,\ a = 1,\, 2, \ldots,\, n,$$

[1]Note as necessary pre-requisites of the new Newton's equations, the prior discovery of isodual numbers, spaces and differential calculus.

whose experimental verification has been provided in the preceding section.

It is easy to see that the isodual formulation is anti-isomorphic to the conventional version, as desired, to such an extent that the two formulations actually coincide at the abstract, realization-free level.

Despite this axiomatic simplicity, the physical implications of the isodual theory of antimatter are rather deep. To begin their understanding, note that throughout the 20-th century it was believed that matter and antimatter exist in the same spacetime. In fact, as recalled earlier, charge conjugation is a map of our physical spacetime into itself.

One of the first physical implications of the Newton-Santilli isodual equations is that *antimatter exists in a spacetime co-existing, yet different than our own.* In fact, the isodual Euclidean space $E^d(r^d, \delta^d, R^d)$ co-exists within, but it is physically distinct from our own Euclidean space $E(r, \delta, R)$, and the same occurs for the full representation spaces $S^d(t^d, r^d, v^d)$ and $S(t, r, v)$.

The next physical implication of the Newton-Santilli isodual equations is the confirmation that *antimatter moves backward in time in a way as causal as the motion of matter forward in time* (again, because negative time is measured with a negative unit). In fact, the *isodual time* t^d is necessarily negative whenever t is our ordinary time. Alternatively, we can say that *the Newton-Santilli isodual equations provide the only known causal description of particles moving backward in time.*

Yet another physical implication is that *antimatter is characterized by negative mass, negative energy and negative magnitudes of other physical quantities.* As we shall see, these properties have the important consequence of eliminating the necessary use of Dirac's "hole theory."

The rest of this chapter is dedicated to showing that the above novel features are necessary in order to achieve a consistent representation of antimatter at all levels of study, from Newton to second quantization.

As we shall see, the physical implications are truly at the edge of imagination, such as: the existence of antimatter in a new spacetime different from our own constitutes the first known evidence of multi-dimensional character of our universe despite our sensory perception to the contrary; the achievement of a fully equivalent treatment of matter and antimatter implies the necessary existence of antigravity for antimatter in the field of matter (and vice-versa); the motion backward in time implies the existence of a causal spacetime machine (although restricted for technical reasons only to isoselfdual states); and other far reaching advances.

2.2.5 Isodual Lagrangian Mechanics

The second level of treatment of *matter* is that via the conventional *classical Lagrangian mechanics.* It is, therefore, essential to identify the corresponding formulation for *antimatter,* a task first studied in Ref. [4] (see also Ref. [9]).

A conventional (first-order) Lagrangian $L(t,\, r,\, v) = \frac{1}{2} \times m \times v^k \times v_k + V(t,\, r,\, v)$ on configuration space (2.2.5) is mapped under isoduality into the *isodual Lagrangian*

$$L^d(t^d,\, r^d,\, v^d) = -L(-t,\, -r,\, -v), \tag{2.2.12}$$

defined on isodual space (2.2.9).

In this way we reach the basic analytic equations of this chapter, today known as *Lagrange-Santilli isodual equations,* first introduced in Ref. [4]

$$\frac{d^d}{d^d t^d} d \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d v^{kd}} d - \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d r^{kd}} d = 0, \tag{2.2.13}$$

All various aspects of the *isodual Lagrangian mechanics* can then be readily derived.

It is easy to see that isodual equations (2.3.13) provide a *direct analytic representation* (i.e., a representation without integrating factors or coordinate transforms) of the isodual equations (2.2.11),

$$\frac{d^d}{d^d t^d} d \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d v^{kd}} d - \frac{\partial^d L^d(t^d, r^d, v^d)}{\partial^d x^{kd}} d =$$

$$= m_k^d \times^d d^d v_k^d / {}^d d^d t^d - F_k^{d\,SA}(t,\, r,\, v) \;=\; 0. \tag{2.2.14}$$

The compatibility of the isodual Lagrangian mechanics with the primitive Newtonian treatment then follows.

2.2.6 Isodual Hamiltonian Mechanics

The *isodual Hamiltonian* is evidently given by [4,9]

$$H^d \;=\; p_k^d \times^d p^{dk} /{}^d 2^d \times^d m^d + V^d(t^d, r^d, v^d) \;=\; -H. \tag{2.2.15}$$

It can be derived from (nondegenerate) isodual Lagrangians via a simple isoduality of the Legendre transforms and it is defined on the $7N$-dimensional *isodual phase space (isocotangent bundle)*

$$S^d(t^d, r^d, p^d) \;=\; E^d(t^d, R_t^d) \times E^d(r^d, \delta^d, R^d) \times E^d(p^d, \delta^d, R^d). \tag{2.2.16}$$

The *isodual canonical action* is given by [4,9]

$$A^{\circ d} \;=\; \int_{t_1}^{t_2} (p_k^d \times^d d^d r^{kd} - H^d \times^d d^d t^d) \;=$$

$$= \int_{t_1}^{t_2} [R_\mu^{\circ d}(b^d) \times^d d^d b^{\mu d} - H^d \times^d d^d t^d], \tag{2.2.17a}$$

$$R^\circ \ = \ \{p, 0\}, \ b = \{x, p\}, \ \mu = 1, 2, \ldots, 6. \tag{2.2.17b}$$

Conventional variational techniques under simple isoduality then yield the fundamental canonical equations of this chapter, today known as *Hamilton-Santilli isodual equations* [4,24-31] that can be written in the disjoint r and p notation

$$\frac{d^d x^{kd}}{d^d t^d} = \frac{\partial^d H^d(t^d, x^d, p^d)}{\partial^d p_k^d}, \ \frac{d^d p_k^d}{d^d t^d} = -\frac{\partial^d H^d(t^d, x^d, p^d)}{\partial^d x^{dk}}, \tag{2.2.18}$$

or in the unified notation

$$\omega_{\mu\nu}^d \times^d \frac{d^d b^{d\nu}}{d^d t^d} \ = \ \frac{\partial^d H^d(t^d, b^d)}{\partial^d b^{d\mu}}, \tag{2.2.19}$$

where $\omega_{\mu\nu}^d$ is the *isodual canonical symplectic tensor*

$$(\omega_{\mu\nu}^d) \ = \ (\partial^d R_\nu^{\circ d} /^d \partial^d b^{d\mu} - \partial^d R_\mu^{\circ d} /^d \partial^d b^{d\nu}) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \ = \ (\omega^{\mu\nu}). \tag{2.2.20}$$

Note that isoduality maps the canonical symplectic tensor into the canonical Lie tensor, with intriguing geometric and algebraic implications.

The *Hamilton-Jacobi-Santilli isodual equations* are then given by [4,9]

$$\partial^d A^{\circ d} /^d \partial^d t^d + H^d \ = \ 0, \tag{2.2.21a}$$

$$\partial^d A^{\circ d} /^d \partial^d x_k^d - p_k^d \ = \ 0, \ \partial^d A^{\circ d} /^d \partial^d p_k^d \equiv 0. \tag{2.2.21b}$$

The *Lie-Santilli isodual brackets* among two isodual functions A^d and B^d on $S^d(t^d, x^d, p^d)$ then become

$$[A^d, {}^d B^d] \ = \ \frac{\partial^d A^d}{\partial^d b^{d\mu}} d \ \times^d \omega^{d\mu\nu} \times^d \frac{\partial^d B^d}{\partial^d b^{d\nu}} d \ = \ -[A, B], \tag{2.2.22}$$

where

$$\omega^{d\mu\nu} \ = \ (\omega_{\mu\nu}) \tag{2.2.23}$$

is the *Lie-Santilli isodual tensor* (that coincides with the conventional canonical tensor). The direct representation of isodual equations in first-order form is self-evident.

In summary, all properties of the isodual theory at the Newtonian level carry over at the level of isodual Hamiltonian mechanics.

2.2.7 Isodual Galilean Relativity

As it is well known, the Newtonian, Lagrangian and Hamiltonian treatment of matter are only the pre-requisites for the characterization of physical laws via basic relativities and their underlying symmetries. Therefore, no equivalence in the treatment of matter and antimatter can be achieved without identifying the relativities suitable for the *classical* treatment of antimatter.

To begin this study, we introduce the *Galilei-Santilli isodual symmetry* $G^d(3.1)$ [7,5,9,22-31] as the step-by-step isodual image of the conventional *Galilei symmetry* $G(3.1)$ (herein assumed to be known[2]). By using conventional symbols for the Galilean symmetry of a Keplerian system of N point particles with non-null masses $m_a,\ a = 1, 2, \ldots, n$, $G^d(3.1)$ is characterized by *isodual parameters and generators*

$$w^d \ = \ (\theta^d_k, r^{kd}_o, v^{kd}_o, t^d_o) = -w, \tag{2.2.24a}$$

$$J^d_k \ = \ \sum\nolimits_a \varepsilon_{ijk} r^d_{ja} \times^d p^k_{ja} \ = \ -J_k \tag{2.2.24b}$$

$$P^d_k \ = \ \sum\nolimits_a p^d_{ka} \ = \ -P_k, \tag{2.2.24c}$$

$$G^d_k \ = \ \sum\nolimits_a (m^d_a \times^d r^d_{ak} - t^d \times p^d_{ak}), \tag{2.2.24d}$$

$$H^d \ = \ \frac{1}{2}^d \times^d \sum\nolimits_a p^d_{ak} \times^d p^{kd}_a + V^d(r^d) \ = \ -H, \tag{2.2.24e}$$

equipped with the *isodual commutator*

$$[A^d, {}^d B^d] \ = \ \sum\nolimits_{a,k} [(\partial^d A^d /{}^d \partial^d r^{kd}_a) \times^d (\partial^d B^d /{}^d \partial^d p^d_{ak}) -$$

$$-(\partial^d B^d /{}^d \partial^d r^{kd}_a) \times^d (\partial^d A^d /{}^d \partial^d p^d_{ak})]. \tag{2.2.25}$$

In accordance with rule (2.1.34), the structure constants and Casimir invariants of the isodual algebra $G^d(3.1)$ are negative-definite. If $g(w)$ is an element of the (connected component) of the Galilei group $G(3.1)$, its isodual is characterized by

$$g^d(w^d) \ = \ e^{d^{-i^d \times^d w^d \times^d X^d}} = -e^{i \times (-w) \times X} \ = \ -g(-w) \in G^d(3.1). \tag{2.2.26}$$

The *Galilei-Santilli isodual transformations* are then given by

$$t^d \ \to \ t'^d \ = \ t^d + t^d_o \ = \ -t', \tag{2.2.27a}$$

$$r^d \ \to \ r'^d \ = \ r^d + r^d_o \ = \ -r' \tag{2.2.27b}$$

[2]The literature on the conventional Galilei and special relativities and related symmetries is so vast as to discourage discriminatory quotations.

$$r^d \ \rightarrow \ r'^d \ = \ r^d + v_o^d \times^d t_o^d \ = \ -r', \tag{2.2.27c}$$

$$r^d \ \rightarrow \ r'^d \ = \ R^d(\theta^d) \times^d r^d \ = \ -R(-\theta) \times r. \tag{2.2.27d}$$

where $R^d(\theta^d)$ is an element of the *isodual rotational symmetry* first studied in the original proposal [1].

The desired classical nonrelativistic characterization of antimatter is therefore given by imposing the $G^d(3.1)$ invariance to the considered isodual equations. This implies, in particular, that the equations admit a representation via isodual Lagrangian and Hamiltonian mechanics.

We now confirm the classical experimental verification of the above isodual representation of antimatter already treated in Section 2.2.2. Consider a conventional, classical, massive *particle* and its *antiparticle* in exterior dynamical conditions in vacuum. Suppose that the particle and antiparticle have charge $-e$ and $+e$, respectively (say, an *electron* and a *positron*), and that they enter into the gap of a magnet with constant magnetic field $\mathbf{B}$.

As it is well known, visual experimental observation establishes that particles and antiparticles under the same magnetic field have spiral trajectories of *opposite orientation*. But this behavior occurs for the *representation of both the particle and its antiparticle in the same Euclidean space*. The situation under isoduality is different, as described by the following:

LEMMA 2.2.1 [5a]: *The trajectories under the same magnetic field of a charged particle in Euclidean space and of the corresponding antiparticle in isodual Euclidean space coincide.*

Proof: Suppose that the particle has negative charge $-e$ in Euclidean space $E(r, \delta, R)$, i.e., the value $-e$ is defined with respect to the positive unit $+1$ of the underlying field of real numbers $R = R(n, +, \times)$. Suppose that the particle is under the influence of the magnetic field $\mathbf{B}$.

The characterization of the corresponding antiparticle via isoduality implies the reversal of the sign of all physical quantities, thus yielding the charge $(-e)^d = +e$ in the isodual Euclidean space $E^d(r^d, \delta^d, R^d)$, as well as the reversal of the magnetic field $B^d = -B$, although now defined with respect to the negative unit $(+1)^d = -1$.

It is then evident that the trajectory of a particle with charge $-e$ in the field B defined with respect to the unit $+1$ in Euclidean space and that for the antiparticle of charge $+e$ in the field $-B$ defined with respect to the unit -1 in isodual Euclidean space coincide (Figure 2.3). **q.e.d.**

An aspect of Lemma 2.2.1, which is particularly important for this monograph, is given by the following:

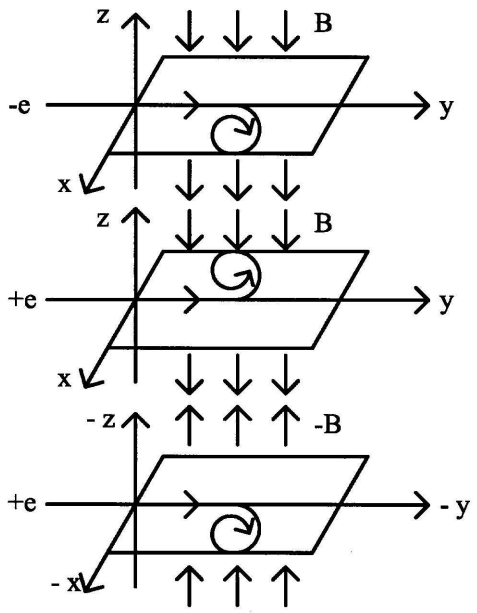

Figure 2.3. A schematic view of the trajectories of an electron and a positron with the same kinetic energy under the same magnetic field. The trajectories "appear" to be the reverse of each other when inspected by one observer, such as that in our spacetime (top and central views). However, when the two trajectories are represented in their corresponding spacetimes they coincide, as shown in the text (top and bottom views).

COROLLARY 2.2.1A: Antiparticles reverse their trajectories when projected from their own isodual space into our own space.

Lemma 2.2.1 assures that isodualities permit the representation of the correct trajectories of antiparticles as physically observed, despite their negative energy, thus providing the foundations for a consistent representation of antiparticles at the level of *first* quantization studied in the next section. Moreover, Lemma 2.2.1 tells us that the trajectories of antiparticles *appear* to exist in our space while in reality they belong to an independent space.

2.2.8 Isodual Special Relativity

We now introduce *isodual special relativity* for the classical relativistic treatment of point-like antiparticles (for the conventional case see Ref. [32]).

As it is well known, conventional special relativity is constructed on the fundamental 4-dimensional unit of the Minkowski space $I = Diag.(1, 1, 1, 1)$,

representing the dimensionless units of space, e.g., $(+1\,\text{cm}, +1\,\text{cm}, +1\,\text{cm})$, and the dimensionless unit of time, e.g., $+1$ sec, and constituting the basic unit of the conventional *Poincarè symmetry* $P(3.1)$ (hereon assumed to be known).

It then follows that *isodual special relativity* is characterized by the map

$$I \;=\; Diag.(\{1,\,1,\,1\},\,1) > 0 \rightarrow$$

$$rightarrow I^d \;=\; Diag.(\{-1,\,-1,\,-1\},\,-1) < 0. \tag{2.2.28}$$

namely, the antimatter relativity is based on *negative units of space and time*, e.g., $I^d = Diag.(-1\,\text{cm}, -1\,\text{cm}, -1\,\text{cm}, -1\,\text{sec})$. This implies the reconstruction of the entire mathematics of the special relativity with respect to the common, isodual unit I^d, including: the *isodual field* $R^d = R^d(n^d,\,+^d,\,\times^d)$ of *isodual numbers* $n^d = n \times I^d$; the *isodual Minkowski spacetime* $M^d(x^d, \eta^d, R^d)$ with isodual coordinates $x^d = x \times I^d$, isodual metric $\eta^d = -\eta$ and basic invariant over R^d

$$(x-y)^{d^{2d}} \;=\; [(x^\mu - y^\mu) \times \eta^d_{\mu\nu} \times (x^\nu - y^\nu)] \times I^d \in R^d. \tag{2.2.29}$$

This procedure yields the central symmetry of this chapter indicated in Section 2.2.6, today known as the *Poincaré-Santilli isodual symmetry* [7]

$$P^d(3.1) \;=\; \mathcal{L}^d(3.1) \times^d T^d(3.1), \tag{2.2.30}$$

where $\mathcal{L}^d(3.1)$ is the *Lorentz-Santilli isodual symmetry*, $\times^d$ is the *isodual direct product* and $T^d(3.1)$ represents the *isodual translations.*

The algebra of the connected component $P_+^{\uparrow d}(3.1)$ of $P^d(3.1)$ can be constructed in terms of the isodual parameters $w^d = \{-w_k\} = \{-\theta, -v, -a\}$ and isodual generators $X^d = -X = \{-X_k\} = \{-M_{\mu\nu}, -P_\mu\}$. The isodual commutator rules are given by [7]

$$[M^d_{\mu\nu},{}^d\, M_{\alpha\beta}]^d \;=$$

$$= i^d \times^d (\eta^d_{\nu\alpha} \times^d M^d_{\mu\beta} - \eta^d_{\mu\alpha} \times^d M^d_{\nu\beta} - \eta^d_{\nu\beta} \times^d M^d_{\mu\alpha} + \eta^d_{\mu\beta} \times^d \hat{M}^d_{\alpha\nu}), \tag{2.2.31a}$$

$$[M^d_{\mu\nu},{}^d\, p^d_\alpha] \;=\; i^d \times^d (\eta^d_{\mu\alpha} \times^d p^d_\nu - \eta^d_{\nu\alpha} \times^d p^d_\mu), \tag{2.2.31b}$$

$$[p^d_\alpha, p^d_\beta]^d \;=\; 0. \tag{2.3.31c}$$

The *Poincarè-Santilli isodual transformations* are given by[3]

$$x^{1d\prime} \;=\; x^{1d} = -x^1, \tag{2.2.32a}$$

$$x^{2d\prime} \;=\; x^{2d} = -x^2, \tag{2.2.32b}$$

[3]It should be indicated that, contrary to popular beliefs, the conventional Poincaré symmetry will be shown in Chapter 3 to be *eleven* dimensional, the 11-th dimension being given by a new invariant under change of the unit. Therefore, the isodual symmetry $P^d(3.1)$ is also 11-dimensional.

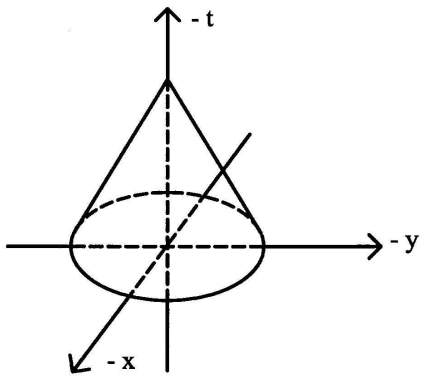

Figure 2.4. A schematic view of the "isodual backward light cone" as seen by an observer in our own spacetime with a time evolution reversed with respect to the "conventional forward light cone."

$$x^{3d\prime} \;=\; \gamma^d \times^d (x^{3d} - \beta^d \times^d x^{4d}) = -x^{3\prime}, \tag{2.2.32c}$$

$$x^{4d\prime} \;=\; \gamma^d \times^d (x^{4d} - \beta^d \times^d x^{3d}) = -x^{4\prime}, \tag{2.2.32d}$$

$$x^{d\mu\prime} \;=\; x^{d\mu} + a^{d\mu} \;=\; -x^{\mu\prime}, \tag{2.3.32e}$$

where

$$\beta^d \;=\; v^d/^d c_\circ^d \;=\; -\beta, \;\; \beta^{d2d} \;=\; -\beta^2, \;\; \gamma^d \;=\; -(1-\beta^2)^{-1/2}, \tag{2.2.33}$$

and the use of the isodual operations (quotient, square roots, etc.) is assumed.

The *isodual spinorial covering*

$$\mathcal{P}^d(3.1) = \mathcal{SL}^d(2.C^d) \times^d \mathcal{T}^d(3.1) \tag{2.2.34}$$

can then be constructed via the same methods.

The basic postulates of the isodual special relativity are also a simple isodual image of the conventional postulates [7]. For instance, the *maximal isodual causal speed in vacuum* is the speed of light in M^d, i.e.,

$$V_{max}^d \;=\; c_\circ^d \;=\; -c_\circ, \tag{2.2.35}$$

with the understanding that it is measured with a *negative-definite unit*, thus being fully equivalent to the conventional maximal speed c_o referred to a positive unit. A similar situation occurs for all other postulates.

The *isodual light cone* is evidently given by (Figure 2.4)

$$x^{d\,2\,d} \;=\; (x^{\mu d} \times^d \eta_{\mu\nu}^d \times^d x^{\nu d}) \times I^d \;=$$

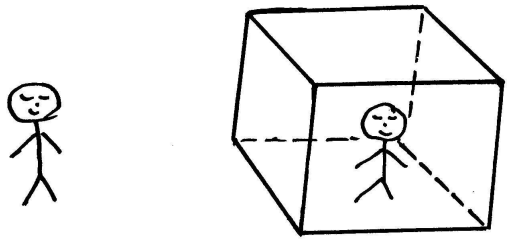

Figure 2.5. A schematic view of the "isodual cube," here defined as a conventional cube with two observers, an external observer in our spacetime and an internal observer in the isodual spacetime. The first implication of the isodual theory is that the same cube coexists in the two spacetimes and can, therefore, be detected by both observers. A most intriguing implication of the isodual theory is that each observer sees the other becoming younger. This occurrence is evident for the behavior of the internal observer with respect to the exterior one, since the former evolves according to a time opposite that of the latter. The same occurrence is less obvious for the opposite case, the behavior of the external observer with respect to the internal one, and it is due to the fact that the projection of our positive time into the isodual spacetime is indeed a motion backward in that spacetime.

$$= (-x \times x - y \times y - z \times z + t \times c_\circ^2 \times t) \times (-I) = 0. \tag{2.2.36}$$

As one can see, the above cone formally coincides with the conventional light cone, although the two cones belong to different spacetimes. The isodual light cone is used in these studies as *the cone of light emitted by antimatter in empty space (exterior problem).*

Note that the *two* Minkowskian metrics $\eta = Diag.(+1, +1, +1, -1)$ and $\eta = Diag.(-1, -1, -1, +1)$ have been popular since Minkowski's times, although both referred to the *same* unit I. We have learned here that these two popular metrics are connected by isoduality.

We finally introduce the *isodual electromagnetic waves* and related *isodual Maxwell's equations* [9]

$$F_{\mu\nu}^d = \partial^d A_\mu^d /^d \partial^d x^{\nu d} - \partial^d A_\nu^d /^d \partial^d x^{d\mu}, \tag{2.2.37a}$$

$$\partial_\lambda^d F_{\mu\nu}^d + \partial_\mu^d F_{\nu\lambda}^d + \partial_\nu^d F_{\lambda\mu}^d = 0, \tag{2.2.37b}$$

$$\partial_\mu^d F^{d\mu\nu} = -J^{d\nu}. \tag{2.2.37c}$$

As we shall see, the nontriviality of the isodual special relativity is illustrated by the fact that isodual electromagnetic waves experience gravitational repulsion when in the field of matter.

2.2.9 Inequivalence of Isodual and Spacetime Inversions

As it is well known (see, the fundamental spacetime symmetries of the 20-th century are the continuous (connected) component of the Poincaré symmetry plus discrete symmetries characterized by *space reversal* (also called *parity*) and *time reversal.*

As noted earlier, antiparticles are assumed in the above setting to exist in the same representation spacetime and to obey the same symmetries as those of particles. On the contrary, according to the isodual theory, antiparticles are represented in a spacetime and possess symmetries distinct from those of particles, although connected to the latter by the isodual transform.

The latter occurrence requires the introduction of the *isodual spacetime inversions,* that is, the isodual images of space and time inversions, first identified in Ref. [9], that can be formulated in unified coordinate form as follows

$$x^{d\mu\prime} \ = \ \pi^d \times^d x^d \ = \ -\pi \times x = $$

$$= (-r, x^4), \ \ \tau^d \times^d x^d = -\tau \times x \ = \ -(r, -x^4), \tag{2.2.38}$$

with field theoretical extension (here expressed for simplicity for a scalar field)

$$\pi^d \times^d \phi^d(x^d) \times^d \pi^{d\dagger} = \phi^d(x'^d, x'^d = (-r^d, t^d) = (r, -t), \tag{2.2.39a}$$

$$\tau^d \times^d \phi^d(x^d) \times^d \tau^{d\dagger} = \bar{\phi}^d(x''^d, x''^d = (r^d, -t^d) = (-r, t), \tag{2.2.39b}$$

where $r^d(= -r)$ is the *isodual coordinate* on space $E^d(r^d, \delta^d, R^d)$, and t^d is the *isodual time* on $E^d(t^d, 1, R_t^d)$.

LEMMA 2.2.2 [9]: *Isodual inversions and spacetime inversions are inequivalent.*

Proof. Spacetime inversions are characterized by the change of sign $x \to -x$ *by always preserving the original metric measured with positive units*, while isodual inversions imply the map $x \to x^d = -x$ *but now measured with an isodual metric* $\eta^d = -\eta$ *with negative units* $I^d = -I$, thus being inequivalent. **q.e.d.**

Despite their simplicity, isodual inversions (or isodual discrete symmetries) are not trivial (Figure 2.6). In fact, all measurements are done in our spacetime, thus implying the need to consider the *projection* of the isodual discrete symmetries into our spacetime which are manifestly different than the conventional forms.

In particular, they imply a sort of interchange, in the sense that the conventional *space* inversion $(r, t) \to (-r, t)$ emerges as belonging to the projection in our spacetime of the isodual *time* inversion, and vice-versa.

Note that the above "interchange" of parity and time reversal of isodual particles projected in our spacetime could be used for experimental verifications, but this aspect is left to interested readers.

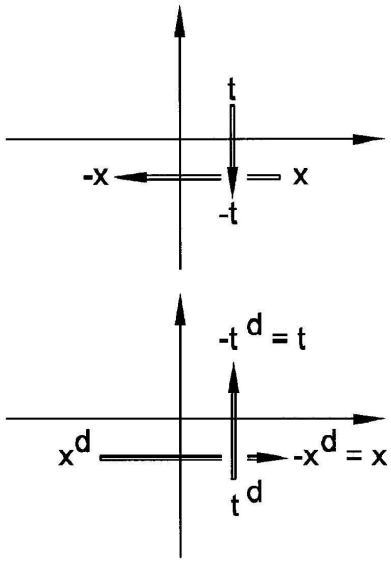

Figure 2.6. A schematic view of the additional peculiar property that the projection in our spacetime of the isodual space inversion appears as a time inversion and vice versa. In fact, a point in the isodual spacetime is given by $(x^d, t^d) = (-x, -t)$. The projection in our spacetime of the isodual space inversion $(x^d, t^d) \to (-x^d, t^d)$ is then given by $(x, -t)$, thus appearing as a time (rather than a space) inversion. Similarly, the projection in our spacetime of the isodual time inversion $(x^d, t^d) \to (x^d, -t^d)$ appears as $(-x, t)$, that is, as a space (rather than time) inversion. Despite its simplicity, the above occurrence has rather deep implications for all discrete symmetries in particle physics indicated later on.

In closing this subsection, we point out that the notion of isodual parity has intriguing connections with the parity of antiparticles in the $(j, 0) + (0, j)$ representation space more recently studied by Ahluwalia, Johnson and Goldman [10]. In fact, the latter parity results in being opposite that of particles which is fully in line with isodual space inversion (isodual parity).

2.2.10 Dunning-Davies Isodual Thermodynamics of Antimatter

An important contribution to the isodual theory has been made by J. Dunning-Davies [11] who introduced in 1999 the first, and only known consistent thermodynamics for antimatter, here called *Dunning-Davies antimatter thermodynamics* with intriguing results and implications.

As conventionally done in the field, let us represent heat with Q, internal energy with U, work with W, entropy with S, and absolute temperature with T. *Dunning-Davies isodual thermodynamics of antimatter* is evidently defined via

the isodual quantities

$$Q^d = -Q,\ U^d = -U,\ W^d = -W,\ S^d = -S,\ T^d = -T \tag{2.2.40}$$

on isodual spaces over the isodual field of real numbers $R^d = R^d(n^d, +^d, \times^d)$ with isodual unit $I^d = -1$.

Recall from Section 2.1.3 that *differentials are isoselfdual* (that is, invariant under isoduality). Dunning-Davies then has the following:

THEOREM 2.2.1 [21]: *Thermodynamical laws are isoselfdual.*

Proof. For the *First Law of thermodynamics* we have

$$dQ = dU - dW \ \equiv\ d^dQ^d = d^dU^d - d^dW^d. \tag{2.2.41}$$

Similarly, for the *Second Law of thermodynamics* we have

$$dQ = T \times dS \ \equiv\ d^dQ^d = T^d \times^d S^d, \tag{2.2.42}$$

and the same occurs for the remaining laws. **q.e.d.**

Despite their simplicity, Dunning-Davies results [21] have rather deep implications. First, the identity of thermodynamical laws, by no means, implies the identity of the thermodynamics of matter and antimatter. In fact, *in Dunning-Davies isodual thermodynamics the entropy must always decrease in time*, since the isodual entropy is always negative and is defined in a space with evolution backward in time with respect to us. However, these features are fully equivalent to the conventional increase of the entropy tacitly referred to positive units.

Also, Dunning-Davies results indicate that *antimatter galaxies and quasars cannot be distinguished from matter galaxies and quasars via the use of thermodynamics,* evidently because their laws coincide, in a way much similar to the identity of the trajectories of particles and antiparticles of Lemma 2.2.1.

This result indicates that the only possibility known at this writing to determine whether far-away galaxies and quasars are made up of matter or of antimatter is that via the predicted gravitational repulsion of the light emitted by antimatter called *isodual light* (see next section and Chapter 5).

2.2.11 Isodual General Relativity

For completeness, we now introduce the *isodual general relativity* for the classical gravitational representation of antimatter. A primary motivation for its study is the incompatibility with antimatter of the positive-definite character of the energy-momentum tensor of the conventional general relativity studied in Chapter 1.

The resolution of this incompatibility evidently requires a structural revision of general relativity [33] for a consistent treatment of antimatter. The *only* solution known to the author is that offered by isoduality.[4]

It should be stressed that this study is here presented merely for completeness, since the achievement of a consistent treatment of negative-energies, by no means, resolves the serious inconsistencies of gravitation on a Riemannian space caused by curvature, as studied in Section 1.2, thus requiring new geometric vistas beyond those permitted by the Riemannian geometry (see Chapters 3 and 4).

As studied in Section 2.1.7, the *isodual Riemannian geometry* is defined on the isodual field $R^d(n^d, +^d, \times^d)$ for which *the norm is negative-definite*, Eq. (2.1.18). As a result, *all quantities that are positive in Riemannian geometry become negative under isoduality, thus including the energy-momentum tensor.*

In fact, the energy-momentum tensor of isodual electromagnetic waves (2.2.37) is negative-definite [8,9]

$$T^d_{\mu\nu} \ = \ (4 \times \pi)^{-1d} \times^d (F^d_{\mu\alpha} \times^d F^d_{\alpha_\nu} + (1/4)^{-1d} \times^d g^d_{\mu\nu} \times^d F^d_{\alpha\beta} \times^d F^{d\alpha\beta}). \quad (2.2.43)$$

The *Einstein-Hilbert isodual equations for antimatter in the exterior conditions in vacuum* are then given by [6,9]

$$G^d_{\mu\nu} \ = \ R^d_{\mu\nu} - \frac{1}{2}^d \times^d g^d_{\mu\nu} \times^d R^d \ = \ k^d \times^d T^d_{\mu\nu}. \quad (2.2.44)$$

The rest of the theory is then given by the use of the isodual Riemannian geometry of Section 2.1.7.

The explicit study of this gravitational theory of antimatter is left to the interested reader due to the indicated inconsistencies of gravitational theories on a Riemannian space for the *conventional case of matter* (Section 1.2). These inconsistencies multiply when treating antimatter, as we shall see.

2.3 OPERATOR ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.3.1 Basic Assumptions

In this section we study the operator image of the classical isodual theory of the preceding section; we prove that the operator image of isoduality is equivalent to charge conjugation; and we show that isodual mathematics resolves all known objections against negative energies.

A main result of this section is the identification of a simple, structurally new formulation of quantum mechanics known as *isodual quantum mechanics* or, more

[4]The author would be grateful to colleagues who care to bring to his attention other "classical" gravitational theories of antimatter compatible with the negative-energy solutions needed by antimatter.

properly, as the *isodual branch of hadronic mechanics* first proposed by Santilli in Refs. [5]. Another result of this section is the fact that all numerical predictions of operator isoduality coincide with those obtained via charge conjugation on a Hilbert space, thus providing the experimental verification of the isodual theory of antimatter at the operator level.

Despite that, the isodual image of quantum mechanics is not trivial because of a number of far reaching predictions we shall study in this section and in the next chapters, such as: the prediction that antimatter emits a new light distinct from that of matter; antiparticles in the gravitational field of matter experience antigravity; bound states of particles and their antiparticles can move backward in time without violating the principle of causality; and other predictions.

Other important results of this section are a new interpretation of the conventional Dirac equation that escaped detection for about one century, as well as the indication that the isodual theory of antimatter originated from the Dirac equation itself, not so much from the negative-energy solutions, but more properly from their two-dimensional unit that is indeed negative-definite, $I_{2\times 2} = Diag.(-1,-1)$.

As we shall see, Dirac's "hole theory", with the consequential restriction of the study of antimatter to the sole second quantization and resulting scientific imbalance indicated in Section 1.1, were due to Dirac's lack of knowledge of a *mathematics based on negative units.*

Intriguingly, had Dirac identified the quantity $I_{2\times 2} = Diag.(-1,-1)$ as the unit of the *mathematics* treating the negative energy solutions of his equation, the physics of the 20-th century would have followed a different path because, despite its simplicity, the unit is indeed the most fundamental notion of all mathematical and physical theories.

2.3.2 Isodual Quantization

The isodual Hamiltonian mechanics (and its underlying *isodual symplectic geometry* [5a] not treated in this chapter for brevity) permit the identification of a new quantization channel, known as the *naive isodual quantization* [6] that can be readily formulated via the use of the Hamilton-Jacobi-Santilli isodual equations (2.2.21) as follows

$$A^{\circ d} \to -i^d \times^d \hbar^d \times^d Ln^d \psi^d(t^d, r^d), \tag{2.3.1a}$$

$$\partial^d A^{\circ d}/^d \partial^d t^d + H^d \ = \ 0 \to i^d \times^d \partial^d \psi^d/^d \partial^d t^d \ =$$

$$= \ H^d \times^d \psi^d \ = \ E^d \times^d \psi^d, \tag{2.3.1b}$$

$$\partial^d A^{\circ d}/^d \partial^d x^{dk} - \hat{p}_k \ = \ 0 \to p_k^d \times^d \psi^d \ = \ -i^d \times^d \partial_k^d \psi^d, \tag{2.3.1c}$$

$$\partial^d A^{\circ d}/^d \partial^d p_k^d = 0 \to \partial^d \psi^d/^d \partial^d p_k^d = 0. \tag{2.3.1d}$$

Recall that the fundamental unit of quantum mechanics is Planck's constant $\hbar = +1$. It then follows that the fundamental unit of the isodual operator theory is the new quantity

$$\hbar^d = -1. \tag{2.3.2}$$

It is evident that the above quantization channel identifies the new mechanics known as *isodual quantum mechanics,* or the *isodual branch of hadronic mechanics.*

2.3.3 Isodual Hilbert Spaces

Isodual quantum mechanics can be constructed via the anti-unitary transform

$$U \times U^\dagger = \hbar^d = I^d = -1, \tag{2.3.3}$$

applied, for consistency, to the *totality* of the mathematical and physical formulations of quantum mechanics. We recover in this way the isodual real and complex numbers

$$n \to n^d = U \times n \times U^\dagger = n \times (U \times U^\dagger) = n \times I^d, \tag{2.3.4}$$

isodual operators

$$A \to U \times A \times U^\dagger = A^d, \tag{2.3.5}$$

the isodual product among generic quantities A, B (numbers, operators, etc.)

$$A \times B \to U \times (A \times B) \times U^\dagger =$$

$$= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = A^d \times^d B^d, \tag{2.3.6}$$

and similar properties.

Evidently, isodual quantum mechanics is formulated in the *isodual Hilbert space* $\mathcal{H}^d$ with *isodual states* [6]

$$|\psi >^d = -|\psi >^\dagger = - < \psi|, \tag{2.3.7}$$

where $< \psi|$ is a conventional dual state on $\mathcal{H}$, and *isodual inner product*

$$< \psi|^d \times (-1) \times |\psi >^d \times I^d, \tag{2.3.8}$$

with *isodual expectation values* of an operator A^d

$$< A^d >^d = (< \psi|^d \times^d A^d \times^d |\psi >^d /^d < \psi|^d \times^d |\psi >^d), \tag{2.3.9}$$

and *isodual normalization*

$$< \psi|^d \times^d |\psi >^d = -1 \tag{2.3.10}$$

defined on the *isodual complex field* C^d with unit -1 (Section 2.1.1).

The isodual expectation values can also be reached via anti-unitary transform (2.3.3),

$$< \psi| \times A \times |\psi > \rightarrow U \times (< \psi| \times A \times |\psi >) \times U^\dagger =$$
$$= (< \psi| \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times$$
$$\times (U \times |\psi >) \times (U \times U^\dagger) =< \psi|^d \times^d A^d \times^d |\psi >^d \times I^d. \tag{2.3.11}$$

The proof of the following property is trivial.

LEMMA 2.3.1 [5b]: *The isodual image of an operator A that is Hermitian on $\mathcal{H}$ over C is also Hermitian on $\mathcal{H}^d$ over C^d (isodual Hermiticity).*

It then follows that *all quantities that are observables for particles are equally observables for antiparticles represented via isoduality.*

LEMMA 2.3.2 [5b]: *Let H be a Hermitian operator on a Hilbert space $\mathcal{H}$ over C with positive-definite eigenvalues E,*

$$H \times |\psi >= E \times |\psi >, H = H^\dagger, E => 0. \tag{2.3.12}$$

Then, the eigenvalues of the isodual operator H^d on the isodual Hilbert space $\mathcal{H}^d$ over C^d are negative-definite,

$$H^d \times^d |\psi >^d = E^d \times^d |\psi >^d, H^d = H^{d\dagger^d}, E^d < 0. \tag{2.3.13}$$

This important property establishes an evident compatibility between the classical and operator formulations of isoduality.

We also mention the *isodual unitary laws*

$$U^d \times^d U^{d^\dagger} = U^{d^\dagger} \times^d U^d = I^d, \tag{2.3.14}$$

the *isodual trace*

$$Tr^d A^d = (TrA^d) \times I^d \in C^d, \tag{2.3.15a}$$
$$Tr^d (A^d \times^d B^d) = Tr^d A^d \times^d Tr^d B^d, \tag{2.3.15b}$$

the *isodual determinant*

$$Det^d A^d = (DetA^d) \times I^d \in C^d, \tag{2.3.16a}$$
$$Det^d (A^d \times^d B^d) = Det^d \times^d Det^d B^d, \tag{2.3.16b}$$

the *isodual logarithm* of a real number n

$$Log^d n^d = -(Log\, n^d) \times I^d, \tag{2.3.17}$$

and other isodual operations.

The interested reader can then work out the remaining properties of the isodual theory of linear operators on a Hilbert space.

2.3.4 Isoselfduality of Minkowski's Line Elements and Hilbert's Inner Products

A most fundamental new property of the isodual theory, with implications as vast as the formulation of a basically new cosmology, is expressed by the following lemma whose proof is a trivial application of transform (2.3.3).

LEMMA 2.3.3 [23]: *Minkowski's line elements and Hilbert's inner products are invariant under isoduality (or they are isoselfdual according to Definition 2.1.2),*

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \equiv$$

$$\equiv (x^{d\mu} \times^d \eta^d_{\mu\nu} \times^d x^{d\nu}) \times I^d = x^{d^{2^d}}, \tag{2.3.18a}$$

$$< \psi| \times |\psi > \times I \equiv < \psi|^d \times^d |\psi >^d \times I^d. \tag{2.3.18b}$$

As a result, *all relativistic and quantum mechanical laws holding for matter also hold for antimatter under isoduality.* The equivalence of charge conjugation and isoduality then follows, as we shall see shortly.

Lemma 2.3.3 illustrates the reason why isodual special relativity and isodual Hilbert spaces have escaped detection for about one century. Note, however, that invariances (2.3.18) require the prior discovery of *new numbers,* those with negative unit.

2.3.5 Isodual Schrödinger and Heisenberg's Equations

The fundamental dynamical equations of isodual quantum mechanics are the isodual images of conventional dynamical equations. They are today known as the *Schrödinger-Santilli isodual equations* [4] (where we assume hereon $\hbar^d = -1$, thus having $\times^d \hbar^d = 1$)

$$i^d \times^d \partial |\psi >^d /^d \partial^d t^d = H^d \times^d |\psi >^d, \tag{2.3.19a}$$

$$p^d_k \times^d |\psi >^d = -i^d \times^d \partial^d |\psi >^d /^d \partial^d r^d, \tag{2.3.19b}$$

and the *Heisenberg-Santilli isodual equations*

$$i^d \times^d d^d A^d /^d d^d t^d = A^d \times^d H^d - H^d \times^d A^d = [A^d, H^d]^d, \tag{2.3.20a}$$

$$[r^d_i, p^d_j]^d = i^d \times^d \delta^{di}_j, [r^d, r^{dj}]^d = [p^d_i, p^d_j]^d = 0. \tag{2.3.20b}$$

Note that, when written explicitly, Eq. (2.3.19a) is based on an associative modular action *to the left,*

$$- < \psi| \times^d H^d = (\partial^d < \psi| \partial^d t^d) \times^d i^d. \tag{2.3.21}$$

It is an instructive exercise for readers interested in learning the new mechanics to prove the equivalence of the isodual Schrödinger and Heisenberg equations via the anti-unitary transform (2.3.3).

2.3.6 Isoselfdual Re-Interpretation of Dirac's Equation

Isoduality has permitted a novel interpretation of the conventional *Dirac equation* (we shall here used the notation of Ref. [12]) in which the negative-energy states are reinterpreted as belonging to the isodual images of positive energy states, resulting in the first known *consistent representation of antiparticles in first quantization.*

This result should be expected since the isodual theory of antimatter applies at the Newtonian level, let alone that of first quantization. Needless to say, the treatment via isodual first quantization does not exclude that via isodual second quantization. The point is that the treatment of antiparticles is no longer restricted to second quantization, as a condition to resolve the scientific imbalance between matter and antimatter indicated earlier.

Consider the conventional Dirac equation [2]

$$[\gamma^\mu \times (p_\mu - e \times A_\mu/c) + i \times m] \times \Psi(x) = 0, \tag{2.3.22}$$

with realization of Dirac's celebrated gamma matrices

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \ \gamma^4 = i \times \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{2.3.23a}$$

$$\{\gamma_\mu, \tilde{\gamma}_\nu\} \ = \ 2\times\eta_{\mu\nu}, \ \ \Psi = i \times \begin{pmatrix} \Phi \\ -\Phi^\dagger \end{pmatrix}. \tag{2.3.23b}$$

At the level of first quantization here considered, the above equation is rather universally interpreted as representing an electron under an external electromagnetic field.

The above equations are generally defined in the 6-dimensional space given by the Kronecker product of the conventional Minkowski spacetime and an internal spin space

$$M_{Tot} = M(x, \eta, R) \times S_{spin}, \tag{2.3.24}$$

with total unit

$$I_{Tot} = I_{orb} \times I_{spin} = Diag.(1,\ 1,\ 1,\ 1) \times Diag.(1,\ 1), \tag{2.3.25}$$

and total symmetry

$$P(3.1) = SL(2.C) \times T(3.1). \tag{2.3.26}$$

The proof of the following property is recommended to interested readers.

THEOREM 2.3.1 [5b]: *Pauli's sigma matrices and Dirac's gamma matrices are isoselfdual*,

$$\sigma_k \equiv \sigma_k^d, \tag{2.3.27a}$$

$$\gamma_\mu \equiv \gamma_\mu^d. \tag{2.3.27b}.$$

The above properties imply an important re-interpretation of Eq. (2.3.22), first identified in Ref. [9] and today known as the *Dirac-Santilli isoselfdual equation,* that can be written

$$[\tilde{\gamma}^{\mu} \times (p_{\mu} - e \times A_{\mu}/c) + i \times m] \times \tilde{\Psi}(x) = 0, \tag{2.3.28}$$

with re-interpretation of the gamma matrices

$$\tilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k^d \\ \sigma_k & 0 \end{pmatrix}, \ \tilde{\gamma}^4 = i \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{2.3.29a}$$

$$\{\tilde{\gamma}_{\mu}, \tilde{\gamma}_{\nu}\} = 2^d \times^d \eta_{\mu\nu}^d, \ \ \tilde{\Psi} = -\tilde{\gamma}_4 \times \Psi = i \times \begin{pmatrix} \Phi \\ \Phi^d \end{pmatrix}, \tag{2.3.29b}$$

By recalling that isodual spaces coexist with, but are different from conventional spaces, we have the following:

THEOREM 2.3.2 [9]: *The Dirac-Santilli isoselfdual equation is defined on the 12-dimensional isoselfdual representation space*

$$M_{Tot} = \{M(x, \eta, R) \times S_{spin}\} \times \{M^d(x^d, \eta^d, R^d) \times^d S_{spin}^d\}, \tag{2.3.30}$$

with isoselfdual total 12-dimensional unit

$$I_{Tot} = \{I_{orb} \times I_{spin}\} \times \{I_{orb}^d \times^d I_{spin}^d\}, \tag{2.3.31}$$

and its symmetry is given by the isoselfdual product of the Poincaré symmetry and its isodual

$$S_{Tot} = \mathcal{P}(3.1) \times \mathcal{P}^d(3.1) =$$
$$= \{SL(2.C) \times T(3.1)\} \times \{SL^d(2.C^d) \times^d T^d(3.1)\}. \tag{2.3.32}$$

A direct consequence of the isoselfdual structure can be expressed as follows.

COROLLARY 2.3.2a [9]: *The Dirac-Santilli isoselfdual equation provides a joint representation of an electron and its antiparticle (the positron) in first quantization,*

$$Dirac \ \ Equation = Electron \times Positron. \tag{2.3.33}$$

In fact, the two-dimensional component of the wave function with positive-energy solution represents the electron and that with negative-energy solutions represent the positron without any need for second quantization, due to the physical behavior of negative energies in isodual treatment established earlier.

Note the complete democracy and equivalence in treatment of the electron and the positron in equation (2.3.28), in the sense that the equation can be equally used to represent an electron or its antiparticle. By comparison, according to the

original Dirac interpretation, the equation could only be used to represent the electron [12], since the representation of the positron required the "hole theory".

It has been popularly believed throughout the 20-th century that Dirac's gamma matrices provide a "four-dimensional representation of the SU(2)-spin symmetry". This belief is disproved by the isodual theory, as expressed by the following

THEOREM 2.3.3 [5b]: *Dirac's gamma matrices characterize the direct product of an irreducible two-dimensional (regular) representation of the SU(2)-spin symmetry and its isodual,*

$$Dirac's\ \ Spin\ \ Symmetry : SU(2) \times SU^d(2). \tag{2.3.34}$$

In fact, the gamma matrices are characterized by the conventional, 2-dimensional Pauli matrices σ_k and related identity $I_{2\times 2}$ as well as other matrices that have resulted in being the exact isodual images σ_k^d with isodual unit $I_{2\times 2}^d$.

It should be recalled that the isodual theory was born precisely out of these issues and, more particularly, from the incompatibility between the popular interpretation of gamma matrices as providing a "four-dimensional" representation of the SU(2)-spin symmetry and the *lack of existence of such a representation in Lie's theory.*

The sole possibility known to the author for the reconciliation of Lie's theory for the SU(2)-spin symmetry and Dirac's gamma matrices was to assume that $-I_{2\times 2}$ is the unit of a dual-type representation. The entire theory studied in this chapter then followed.

It should also be noted that, as conventionally written, Dirac's equation *is not* isoselfdual because it is not sufficiently symmetric in the two-dimensional states and their isoduals.

In summary, Dirac's was forced to formulate the "hole theory" for antiparticles because he referred the *negative* energy states to the conventional *positive* unit, while their reformulation with respect to *negative* units yields fully physical results.

It is easy to see that the same isodual reinterpretation applies for Majorana's spinorial representations [13] (see also [14,15]) as well as Ahluwalia's broader spinorial representations (1/2, 0) + (0, 1/2) [16] (see also the subsequent paper [17]), that are reinterpreted in the isoselfdual form $(1, 2, 0) + (1, 2, 0)^d$, thus extending their physical applicability to first quantization.

In the latter reinterpretation the representation (1/2, 0) is evidently done conventional spaces over conventional fields with unit +1, while the isodual representation $(1/2, 0)^d$ is done on the corresponding isodual spaces defined on isodual fields with unit −1. As a result, all quantities of the representation (1/2, 0) change sign under isoduality.

It should be finally indicated that Ahluwalia treatment of Majorana spinors has a deep connection with isoduality because the underlying Class II spinors have a *negative norm* [16] precisely as it is the case for isoduality. As a result, the isodual reinterpretation under consideration here is quite natural and actually warranted for mathematical consistency, e.g., to have the topology characterized by a negative norm be compatible with the underlying fields.

2.3.7 Equivalence of Isoduality and charge conjugation

We come now to another fundamental point of this chapter, the proof that isoduality is equivalent to charge conjugation. This property is crucial for the experimental verification of isoduality at the particle level too. This equivalence was first identified by Santilli in Ref. [6] and can be easily expressed today via the following:

LEMMA 2.3.4 [6,5b,18]: *The isodual transform is equivalent to charge conjugation.*

Proof. Charge conjugation is characterized by the following transform of wavefunctions (see, e.g., Ref. [12], pages 109 and 176)

$$\Psi(x) \to C\Psi(x) = c \times \Psi^\dagger(x), \tag{2.3.35}$$

where

$$|c| = 1, \tag{2.3.36}$$

thus being manifestly equivalent to the isodual transform

$$\Psi(x) \to \Psi^d(x^d) = -\Psi^\dagger(-x^t), \tag{2.3.37}$$

where t denotes transpose.

A reason why the two transforms are equivalent, rather than identical, is the fact that charge conjugation maps spacetime into itself, while isoduality maps spacetime into its isodual. **q.e.d.**

Let us illustrate Lemma 2.3.4 with a few examples. As well known, the Klein-Gordon equation for a free particle

$$\partial^\mu \partial_\mu \Psi - m^2 \times \Psi = 0 \tag{2.3.38}$$

is invariant under charge conjugation, in the sense that it is turned into the form

$$c \times [\bar{\Psi} \partial^\mu \partial_\mu - \bar{\Psi} \times m^2] = 0, \;\; |c| = 1, \tag{2.3.39}$$

where the upper bar denotes complex conjugation (since $\bar{\Psi}$ is a scalar), while the Lagrangian density

$$L = -(\hbar \times \hbar/2 \times m) \times \{\partial^\mu \bar{\Psi} - i \times e \times A^\mu/\hbar \times c) \times \bar{\Psi}] \times$$

$$\times[\partial\Psi + (i \times e \times A_\mu/\hbar \times c) \times \Psi] + m \times m \times \bar{\Psi} \times \Psi \tag{2.3.40}$$

is left invariant, and the four-current

$$J_\mu = -(i \times \hbar/2 \times m) \times [\bar{\psi} \times \partial_\mu \Psi - (\partial_\mu \bar{\Psi}) \times \Psi] \tag{2.3.41}$$

changes sign

$$J_\mu \to C J_\mu = -J_\mu. \tag{2.3.42}$$

By recalling the selfduality of ordinary derivatives, Eq. (2.1.30), under isoduality the Klein-Gordon Equation becomes

$$[\partial^\mu \partial_\mu \Psi - m^2 \times \Psi]^d = \Psi^d \partial^{d\mu} \partial^d_\mu - \Psi^d \times^d m^d \times^d m^d =$$

$$= -[\bar{\Psi} \partial^\mu \partial_\mu - \bar{\Psi} \times m^2] = 0, \tag{2.3.43}$$

thus being equivalent to Eq. (2.3.39), while the Lagrangian changes sign and the four-current changes sign too,

$$J^d_\mu = -(i \times \hbar/2 \times m) \times [\bar{\Psi} \times \partial_\mu \Psi - (\partial_\mu \bar{\Psi}) \times \Psi]^d =$$

$$= (i \times \hbar/2 \times m) \times [\bar{\Psi} \times \partial_\mu \Psi - (\partial_\mu \bar{\Psi}) \times \Psi], \tag{2.3.44}$$

(where we have used the isoselfduality of the imaginary number i).

The above results confirm Lemma 2.3.4 because of the equivalent behavior of the equations of motion and the four-current, while the change of sign of the Lagrangian does not affect the numerical results.

As it is also well known, the Klein-Gordon equation for a particle under an external electromagnetic field [12]

$$[(\partial_\mu + i \times e \times A_\mu/\hbar \times c) \times$$

$$\times(\partial^\mu + i \times e \times A^\mu/\hbar \times c) - m^2] \times \Psi = 0, \tag{2.3.45}$$

is equally invariant under charge conjugation in which *either* e *or* A_μ change sign, in view of the known invariance

$$C(i \times e \times A_\mu/\hbar \times c) = i \times e \times A_\mu/\hbar \times c, \tag{2.3.46}$$

while the four-current also changes sign. By noting that the preceding invariance persists under isoduality,

$$(i \times e \times A_\mu/\hbar \times c)^d = i \times e \times A_\mu/\hbar \times c, \tag{2.3.47}$$

Eq. (2.3.45) remains invariant under isoduality, while the Lagrangian density changes sign and the four-current, again, changes sign.

Similarly, consider Dirac equation (see also Ref. [12], pp. 176-177)

$$[\gamma^\mu \times (\partial_\mu \Psi - (i \times e \times A_\mu/\hbar \times c) \times \Psi + m \times \Psi = 0, \tag{2.3.48}$$

with Lagrangian density

$$L = (\hbar \times c/2) \times \{\tilde{\Psi} \times \gamma^\mu \times [\partial_\mu \Psi + (i \times e \times A_\mu/\hbar \times c) \times \Psi] -$$
$$-(\partial^\mu \tilde{\Psi} - (i \times e \times A^\mu/\hbar \times c) \times \tilde{\Psi}] \times \gamma_\mu - m \times \tilde{\Psi} \times \Psi, \qquad (2.3.49a)$$
$$\tilde{\Psi} = \Psi^\dagger \times \gamma_4, \qquad (2.3.49b)$$

and four-current

$$J_\mu = i \times c \times \tilde{\Psi} \times \gamma_\mu \times \Psi = i \times c \times \Psi^\dagger \times \gamma_4 \times \gamma_\mu \times \Psi. \qquad (2.3.50)$$

The charge conjugation for Dirac's equations is given by the transform [12]

$$\Psi \to C\Psi = c \times S_C^{-1} \times \tilde{\Psi}^t \qquad (2.3.51)$$

where S_C is a unitary matrix such that

$$\gamma_\mu \to -\gamma_\mu^t = S_C \times \gamma_\mu \times S_C^{-1}, \qquad (2.3.52)$$

and there is the change of sign *either* of e *or* of A_μ, under which the equation is transformed into the form

$$[\partial_\mu \tilde{\Psi} - (i \times e \times A_\mu/\hbar \times c) \times \tilde{\Psi}] \times \gamma^\mu - m \times \tilde{\Psi} = 0, \qquad (2.3.53)$$

while the Lagrangian density changes sign and the four-current remains the same,

$$L \to CL = -L, \quad J_\mu \to CJ_\mu = J_\mu. \qquad (2.3.54)$$

It is easy to see that isoduality provides equivalent results. In fact, we have for Eq. (2.3.48)

$$\{[\gamma^\mu \times (\partial_\mu \Psi - i \times e \times A_\mu/\hbar \times c) \times \Psi + m \times \Psi\}^d =$$
$$= [\partial_\mu \Psi^\dagger - (i \times e \times A_\mu/\hbar \times c) \times \Psi^\dagger] \times \gamma^\mu - m \times \Psi^\dagger = 0, \qquad (2.3.55)$$

that, when multiplied by γ_4 reproduces Eq. (2.3.53) identically. Similarly, by recalling that Dirac' s gamma matrices are isoselfdual (Theorem 2.3.1), and by noting that

$$\tilde{\Psi}^d = (\Psi^\dagger \times \gamma_4)^d = \gamma_4 \times \Psi, \qquad (2.3.56)$$

we have

$$L^d = L, \qquad (2.3.57)$$

while for the four-current we have

$$J_\mu^d = -i \times c \times \Psi^\dagger \times \gamma_\mu \times \gamma_4 \times \psi. \qquad (2.3.58)$$

But the γ_μ and γ_4 anticommutate. As a consequence, the four-current does not change sign under isoduality as in the conventional case.

Note that the lack of change of sign under isoduality of Dirac's four-current J_μ confirms reinterpretation (2.3.28) since, for the latter equation, the total charge is null.

The equivalence between isoduality and charge conjugation of other equations, such as those by Weyl, Majorana, etc., follows the same lines.

2.3.8 Experimental Verification of the Isodual Theory of Antimatter in Particle Physics

In Section 2.2.3. we have established the experimental verification of the isodual theory of antimatter in classical physics that, in particle physics, requires no detailed elaboration since it is established by the equivalence of charge conjugation and isoduality (Lemma 2.3.4), and we can write:

LEMMA 2.3.5 [6,5b,18], [7]: *All experimental data currently available for antiparticles represented via charge conjugation are equally verified by the isodual theory of antimatter.*

2.3.9 Elementary Particles and their Isoduals

We assume the reader is familiar with the conventional definition of *elementary particles* as irreducible unitary representations of the spinorial covering of the Galilei symmetry $G(3.1)$ for nonrelativistic treatments and those of the Poincaré symmetry $P(3.1)$ for relativistic treatments. We therefore introduce the following:

DEFINITION 2.3.1: Elementary isodual particles (antiparticles) are given by irreducible unitary representations of the spinorial covering of the Galilei-Santilli's isodual symmetry $G^d(3.1)$ for nonrelativistic treatments and those of the Poincaré-Santilli isodual symmetry $P^d(3.1)$ for relativistic treatments.

A few comments are now in order. Firstly, one should be aware that "isodual particles" and "antiparticles" do not represent the same notion, evidently because of the negative mass, energy and time of the former compared to positive mass, energy and time of the latter. In the rest of this chapter, unless otherwise stated, the word "antiparticle" will be referred to as the "isodual particle."

For instance the word "positron" e^+ is more appropriately intended to represent the "isodual electron" with symbol e^{-d}. Similarly the, "antiproton" p^- is intended to represent the "isodual proton" p^{+d}.

Secondly, the reader should note the insistence on the *elementary* character of the antiparticles here admitted. The reason is that the antigravity studied in Chapter 4 is specifically formulated for "elementary" isodual particles, such as the isodual electron, due to a number of unsettled aspects pertaining to composite particles.

Consider, as an illustration, the case of mesons. If the π° is a bound state of a particle and its isodual, the state is isoselfdual and, as such, it *cannot* experience antigravity, as illustrated in the next section. A number of ambiguities then follow for the study of the gravity of the charged mesons $\pi^\pm$, such as the problem of ascertaining which of the two mesons is a particle and which is its isodual or,

whether the selected antiparticle is indeed the isodual image of the particle as a necessary condition for meaningful study of their gravity.

Note that essentially the same ambiguities prohibit the use of muons for a serious theoretical and experimental studies of the gravity of antiparticles, again, because of unsettled problems pertaining to the structure of the muons themselves. Since the muons are naturally unstable, they cannot be credibly believed to be elementary. Therefore, serious theoretical and experimental studies on the gravity of muons require the prior identification of their constituents with physical particles.

Finally, the reader should be aware that *Definition 2.3.1 excludes the use of quark conjectures for the gravitational studies of this monograph.* This is due to the well-known basic inconsistency of quark conjecture of not admitting any gravitation at all (see, e.g., the Appendix of Ref. [18]). In fact, gravity can only be defined in our spacetime while quarks can only be defined in their mathematical unitary internal space with no known connection with our spacetime due to the O'Rafearthaigh theorem.[5]

Also, the only "masses" that can be credibly claimed as possessing inertia are the eigenvalues of the second-order Casimir invariant of the Poincaré symmetry $p_\mu \times p^\mu = m^2$. Quarks cannot be characterized via such a fundamental symmetry, as well known. It then follows that "quark masses" are mere mathematical parameters defined in the mathematical internal complex-unitary space that cannot possibly be used as serious basis for gravitational tests.

2.3.10 Photons and their Isoduals

As it is well known, photons have no charge and, therefore, they are invariant under charge conjugation, as transparent from the simple plane-wave representation

$$\Psi(t,r) = N \times e^{i\times(k\times r - E\times t)}, \quad N \in R, \tag{2.3.59}$$

with familiar relativistic form

$$\Psi(x) = N \times e^{i\times k_\mu \times x^\mu}, \tag{2.3.60}$$

and familiar expression for the energy

$$E = h \times \nu. \tag{2.3.61}$$

As a result, matter and antimatter have been believed throughout the 20-th century to emit the same light. In turn, this belief has left fundamentally unsettled basic questions in astrophysics and cosmology, such as the lack of quantitative

[5]The possible connection between internal and spacetime symmetries offered by supersymmetric theories cannot be credibly used for gravitational tests due to their highly unsettled character and the prediction of a zoo of new particles none of which has been experimentally detected to the author's best knowledge.

studies as to whether far-away galaxies and quasars are made up of matter or of antimatter.

One of the most intriguing and far reaching implications of the isodual theory is that, while remaining evidently invariant under charge conjugation, *the photon is not invariant under isoduality,* thus admitting a conjugate particle first submitted by Santilli in Ref. [18] under the name of *isodual photon.* In particular, the isodual photon emerges as having physical characteristics that can be experimentally measured as being different from those of the photon.

Therefore, the isodual theory offers the first known possibilities of quantitative theoretical and experimental studies as to whether a far-away galaxy or quasar is made of matter or of antimatter due to detectable physical differences of their emitted light.

Note that the term "antiphoton" could be misleading because the prefix "anti" is generally assumed as referring to charge conjugation. For this reason the name of "isodual photon" appears to be preferable, also because it represents, more technically, the intended state.

In fact, the photon is mapped by isoduality into a new particle possessing all negative-definite physical characteristics, with the following simple isodual plane-wave representation

$$\Psi^d(t^d, r^d) = N^d \times^d e_d^{i^d \times^d (k^d \times^d r^d - E^d \times^d t^d)}, \quad N^d \in R^d, \tag{2.3.62}$$

with relativistic expression on isodual Minkowski space

$$\Psi^d(x^d) = N^d \times^d e_d^{i^d \times^d k_\mu^d \times^d x^{d\mu}}, \tag{2.3.63}$$

and isodual expression for the energy

$$E^d \ = \ h^d \times^d \nu^d, \tag{2.3.64}$$

where e_d is the isodual exponentiation (2.1.26b).

Note that, since i is isoselfdual, Eq. (2.1.20), *the exponent of the plane-wave representation is invariant under both charge conjugation and isoduality,* as illustrated by the following expression

$$i^d \times^d (k^d \times^d r^d - E^d \times^d t^d) \ \equiv \ i \times (k \times r - E \times t), \tag{2.3.65}$$

or its relativistic counterpart

$$i^d \times^d k_\mu^d \times^d x^{d\mu} \ \equiv \ i \times k_\mu \times x^\mu, \tag{2.3.66}$$

thus confirming the lack of contradiction between charge conjugation and isoduality.

Moreover, both the photon and the isodual photon travel in vacuum with the same (absolute) speed $|c|$, for which we have the additional identity

$$k_\mu^d \times^d k^{d\mu} \equiv k_\mu \times k^\mu = 0. \tag{2.3.67}$$

Despite the above identities, energy and time are positive-definite for the photon, while they are negative-definite for the isodual photon. As we shall see, the latter property implies that photons are attracted by the gravitational field of matter while isodual photons are repelled, thus providing a physically detectable difference.

Additional differences between light emitted by matter and that emitted by antimatter, such as those pertaining to parity and other discrete symmetries, require additional study.

All in all, the isodual theory of antimatter permits the first possibilities known to the author for future experimental measurements as to whether far-away galaxies and quasars are made up of matter or of antimatter.

2.3.11 Electrons and their Isoduals

The next truly elementary particles and antiparticles are the electron e^- and its antiparticle, the positron e^+ or the isodual electron e^{-d}. The differences between the "positron" and the "isodual electron" should be kept in mind. In fact, the former has positive rest energy and moves forward in time, while the latter has negative rest energy and moves backward in time.

Also, the electron is known to experience gravitational attraction in the field of matter, as experimentally established. As conventionally defined, the positron too is predicted to experience gravitational attraction in the field of matter (because its energy is positive).

However, as we shall see in Chapter 4, the isodual electron is predicted to experience antigravity when immersed in the field of matter, and this illustrates again the rather profound physical differences between the "positron" and the "isodual electron".

Note that, in view of their truly elementary character, isodual electrons are the ideal candidates for the measurement of the gravitational field of antiparticles.

2.3.12 Protons and their Isoduals

The next particles demanding comments are the proton p^+, the antiproton p^- and the isodual proton p^{+d}. In this case the differences between the "antiproton" and the "isodual proton" should be kept in mind to avoid major inconsistencies with the isodual theory, such as the study of the possible antigravity for antiprotons in the field of matter which antigravity cannot exist for the isodual theory (due, again, to the positive mass of the antiproton).

Note that these particles are not elementary and, as such, they are not admitted by Definition 2.3.1. moreover, as stressed earlier [18], when represented in term of quark conjectures both the proton and the antiproton cannot admit any gravity at all, let alone antigravity. As a result, extreme scientific care should be exercised before extending to all antimatter any possible gravitational measurements for antiprotons.

2.3.13 The Hydrogen Atom and its Isodual

The understanding of this chapter requires the knowledge that studies conducted on the *antihydrogen atom* (see, e.g., the various contributions in Proceedings [19]), even though evidently interesting per se, have no connection with the *isodual hydrogen atom,* because the antihydrogen atom has positive mass, for which antigravity is prohibited, and emits conventional photons. Therefore, it is important to inspect the differences between these two formulations of the simplest possible atom of antimatter.

We assume as exactly valid the conventional quantum mechanical theory of *bound states of point-like particles at large mutual distances,*[6] as available in quantum mechanical books so numerous to discourage even a partial listing.

For the case of two particles denoted with the indices 1, 2, the total state in the Hilbert space is the familiar tensorial product of the two states

$$|\psi >= |\psi_1 > \times |\psi_2 > . \tag{2.3.68}$$

The total Hamiltonian H is the sum of the kinetic terms of each state plus the familiar interaction term $V(r)$ depending on the mutual distance r,

$$H = p_1 \times p_1/2 \times m_1 + p_2 \times p_2/2 \times m_2 + V(r). \tag{2.3.69}$$

The total angular momentum is computed via the familiar expressions for angular momenta and spins

$$J = J_1 \times I + I \times J_2, \;\; S = S_1 \times I + I \times S_2, \tag{2.3.70}$$

where the I's are trivial units, with the usual rules for couplings, addition, etc. One should note that the unit for angular momenta is three-dimensional while that for spin has a generally different dimension.

A typical example of two-body bound states of particles is the *hydrogen atom* that experiences attraction in the gravitational field of matter with the well established emission of conventional photons.

[6] We are here referring to the large mutual distances as occurring in the atomic structure and exclude the short mutual distances as occurring in the structure of hadrons, nuclei and stars since a serious study of the latter is dramatically beyond the capabilities of quantum mechanics, as shown beyond scientific doubt in Chapter 3.

The study of *bound states of point-like isodual particles at large mutual distances* is an important part of isodual quantum mechanics. These bound states can be studied via an elementary isoduality of the corresponding bound states for particles, that is, via the use of the isodual Hilbert spaces $\mathcal{H}^d$ studied earlier.

The *total isodual state* is the tensorial product of the two isodual states

$$|\psi^d(r^d)>^d = |\psi_1^d(r^d)>^d \times^d |\psi_2^d(r^d)>^d = - <\psi_1(-r)| \times <\psi_2(-r)|. \quad (2.3.71)$$

The *total isodual Hamiltonian* is the sum of the isodual kinetic terms of each particle plus the isodual interaction term depending on the isodual mutual distance,

$$H^d = p_1^d \times^d p_1^d /^d 2^d \times^d m_1^d + p_2^d \times^d p_2^d /^d 2^d \times^d m_2^d + V^d(r^d). \quad (2.3.72)$$

The total *isodual angular momentum* is based on the expressions for isodual angular momenta and spin

$$J^d = J_1^d \times^d I^d + I^d \times^d J_2^d, \quad (2.3.73a)$$

$$S^d = S_1^d \times^d I^d + I^d \times^d S_2^d, \quad (2.3.73b)$$

The remaining aspects (couplings, addition theory of angular momenta, etc.) are then given by a simple isoduality of the conventional theory that is here omitted for brevity.

Note that all eigenvalues that are positive for the conventional case measured with positive units become negative under isoduality, yet measured with negative units, thus achieving full equivalence between particle and antiparticle bound states.

The simplest possible application of the above isodual theory is that for the *isodual hydrogen atom* (first worked out in Ref. [18]). The novel predictions of isoduality over that of the antihydrogen atom is that the isodual hydrogen atom is predicted to experience antigravity in the field of matter and emits isodual photons that are also repelled by the gravitational field of matter.

2.3.14 Isoselfdual Bound States

Some of the most interesting and novel bound states predicted by the isodual theory are the *isoselfdual bound states,* that is, bound states that coincide with their isodual image. The simplest case is the bound state of one elementary particle and its isodual, such as the *positronium.*

The condition of isoselfduality requires that the basic symmetry must be itself isoselfdual, e.g., for the nonrelativistic case the total symmetry must be

$$G_{Tot} = G(3.1) \times G^d(3.1), \quad (2.3.74)$$

where $\times$ is the Kronecker product (a composition of states thus being isoselfdual), with a simple relativistic extension here assumed as known from the preceding sections.

The total unit must also be isoselfdual,

$$I_{Tot} = I \times I^d, \tag{2.3.75}$$

where I represents the space, time and spin units.

The total Hilbert space and related states must also be isoselfdual,

$$\mathcal{H}_{Tot} = \mathcal{H} \times \mathcal{H}^d, \tag{2.3.76a}$$

$$|\psi >_{Tot} = |\psi > + |\psi >^d = |\psi > - < \psi|, \tag{2.3.76b}$$

and so on.

A main feature is that isoselfdual states exist in both the spacetime of particles and that of antiparticles. Therefore, the computation of the total energy must be done *either* in $\mathcal{H}$, in which case the total energy is positive, *or* in $\mathcal{H}^d$, in which case the total energy is negative.

Suppose that a system of one elementary particle and its isodual is studied in our laboratory of matter. In this case the eigenvalues for both particle and its isodual must be computed in $\mathcal{H}$, in which case we have the equation

$$\begin{aligned} i \times \partial_t |\psi >= (p \times p/2 \times m) \times |\psi > + \\ +(p^d \times^d p^d/^d 2^d \times^d m^d) \times^d |\psi > + V(r) \times |\psi > = \\ = [p \times p/2 \times m + V(r)] \times |\psi >= E \times |\psi >, \end{aligned} \tag{2.3.77}$$

under which the total energy E is evidently positive.

When the same isoselfdual state is detected in the spacetime of antimatter, it must be computed with respect to $\mathcal{H}^d$, in which case the total energy is negative, as the reader is encouraged to verify.

The total angular momentum and other physical characteristics are computed along similar lines and they also result in having positive values when computed in $\mathcal{H}$, as occurring for the conventional charge conjugation.

As we shall see shortly, the positive character of the total energy of bound states of particles and their antiparticles is crucial for the removal of the inconsistencies of theories with negative energy.

The above properties of the isoselfdual bound states have the following implications:

1) Isoselfdual bound states of elementary particles and their isoduals are predicted to be attracted in both, the gravitational field of matter and that of antimatter because their total energy is positive in our world and negative in the isodual world. This renders necessary an experimental verification of the gravitational behavior of isoselfdual bound states, independently from that of individual

antiparticles. Note that the prediction holds only for bound states of truly elementary particles and their isoduals, such as the positronium. No theoretical prediction for the muonium and the pionium is today feasible because the unsettled nature of their constituents.

2) Isoselfdual bound states are predicted to have a null internal total time $t + t^d = 0$ and therefore acquires the time of the matter or antimatter in which they are immersed, although the physical time t of the observer (i.e., of the bound state equation) is not null. This is readily understood by noting that the quantity t of Eq. (2.3.77) is our own time, i.e., we merely study the behavior of the state with respect to our own time. A clear understanding illustrated previously with the "isodual cube" of Section 2.1 is that the description of a state with our own time, by no means, implies that its intrinsic time necessarily coincides with our own. Note that a similar situation occurs for the energy because the intrinsic total energy of the positronium is identically null, $E + E^d = 0$. Yet the energy measured by us is $E_{part.} - E^d_{antipart.} = 2E > 0$. A similar situation occurs for all other physical quantities.

3) Isoselfdual bound states may result in being the microscopic image of the main characteristics of the entire universe. Isoselfduality has in fact stimulated a new cosmology, the *isoselfdual cosmology* [21] studied in Chapter 5, that is patterned precisely along the structure of the positronium or of Dirac's equation in our isoselfdual re-interpretation. In this case the universe results in having null total physical characteristics, such as null total energy, null total time, etc., thus implying no discontinuity at its creation.

2.3.15 Resolution of the Inconsistencies of Negative Energies

The treatment of antiparticles with negative energies was rejected by Dirac because of incompatibility with their physical behavior. Despite several attempts made during the 20-th century, the inconsistencies either directly or indirectly connected to negative energies have remained unresolved.

The isodual theory of antimatter resolves these inconsistencies for the reason now familiar, namely, that the inconsistencies emerge when one refers negative energies to conventional numbers with positive units, while the same inconsistencies cannot be evenly formulated when negative energies are referred to isodual numbers and their negative units.

A good illustration is given by the known objection according to which the creation of a photon from the annihilation of an electron-positron pair, with the electron having a positive energy and the positron having a negative energy, would violate the principle of conservation of the energy.

In fact, such a pair could be moved upward in our gravitational field without work and then annihilated in their new upward position. The resulting photon

would then have a blueshift in our gravitational field of Earth, thus having more energy than that of the original photon.

Presumed inconsistencies of the above type cannot be even formulated within the context of the isodual theory of antimatter because, as shown in the preceding section, the electron-positron state is isoselfdual, thus having a non-null *positive* energy when observed in our spacetime. Consequently, the lifting upward of the pair does indeed require work and no violation of the principle of conservation of the energy can be expected.

A considerable search has established that all other presumed inconsistencies of negative energy known to the author cannot even be formulated within the context of the isodual theory of antimatter. Nevertheless, the author would be particularly grateful to any colleague who brings to his attention inconsistencies of negative energies that are really applicable under negative units.

2.4 THEORETICAL PREDICTIONS OF ANTIGRAVITY

2.4.1 The Problem for Studies on Antigravity: Ethical Decay in Physics

Antigravity is one of the most ancient dreams of mankind, that has stimulated the imagination of many researchers, from various engineering fields (see, e.g., Refs. [35, 36] that also list patents), to the most advanced branches of physics (see the prediction of antigravity in supergravity theories [37, 38] and proceedings [19] for other more recent approaches).

Unfortunately, professional theoretical and experimental research in antigravity has been opposed, disrupted, or jeopardized by organized academic, financial and ethnic interests on Einsteinian doctrines on grounds that antigravity is not predicted by said doctrines. However, as we known to experts in order to qualify as such, and as established beyond credible doubt in this volume, Einsteinian doctrines do not allow a consistent classical representation of antimatter.

Hence, any opposition, obstruction, of dismissal of antigravity based on Einsteinian doctrines is sheer scientific corruption for personal gains that must be denounced by any scientist who really cares about scientific knowledge and human dignity.

A comprehensive study of antigravity was conducted by the author in monograph [34]. In this chapter we essentially present an update of the content of Ref. [34].

An experiment on the gravity of antiparticles was considered by Fairbank and Witteborn [39] via low energy positrons in vertical motion. Unfortunately, the measurements could not be completed because of claimed interferences from stray fields, excessive upward kinetic energy of the positrons, and other reasons.

There are insistent rumors that the experiment by Fairbank and Witteborn could not be completed because of disruptions by said organized interests at SLAC and elsewhere, particularly in view of a growing expectation that the experiment could indeed establish antigravity between matter and antimatter, thus cutting out of the desired dominance by Einsteinian doctrines what could amount as being half of the universe,

. The author recommends a senatorial investigation of the case to ascertain the reasons for the lack of completion at SLAC of so important an experiment, in view of its feasibility with available technologies, as well as moderate cost, while dramatically more expensive experiments fully aligned with Einsteinian doctrines were preferred at SLAC and elsewhere, and remain preferred to this day (January 19, 2008).

In the absence of a senate investigation, the author recommends the filing of a class action in the U. S. Federal Court against SLAC on grounds of misuses of public funds and other easily identifiable violations of Federal laws.

As an illustration of the need for a senatorial investigation and/or legal action, the reader in good faith should know that Burton Richter, then SLAC director, prohibited in writing in the early 1990s Santilli (a U.S. scientist) to visit SLAC (a U. S. federal laboratory) for the purpose of discussing a possible alternative of the Fairbank and Witteborn experiment via a horizontal tube (see Section 2.5.2 and references quoted therein), even though Santilli had applied for a visit fully supported by his own money and, being a theoretician, was merely looking for suggestions by experimental colleagues.

Clearly, such a denial by Burton Richter cannot be justified on scientific grounds, or on grounds of qualifications, since SLAC is bound by law as being open to visits from a variety of scientists, and Santilli qualification, honors and publications surpass most of the visitors (see Santilli CV http://www.i-b-r.org/-Ruggero-Maria-Santilli.htm www.santilli-galilei.com www.i-b-r.org and other sources).

Hence, the denial by Burton Richter for Santilli to visit SLAC, pushed to the extreme of perpetrating a clear violation of U. S. Federal Laws governing federal laboratories, had strictly nonscientific motivations. The most plausible one is the evident one, namely, the existence at SLAC and elsewhere of vociferous, organized, academic, financial and ethnic interests on Einsteinian doctrines opposing, disrupting and jeopardizing for asocial personal gains professional research on antigravity. At any rate, if the above, and much more evidence by other scientists, is not sufficient for a senatorial investigation and/or legal action, what else would be?

In view of the above, any consideration of experiments on the gravity between particle and antiparticles without a senatorial and/or judicial investigation of the past, would be a hypocritical farce acceptable by naive persons or accomplices, but definitely not in favor of scientific knowledge. At any rate, in the event said

organized academic, financial and ethnic interests on Einstein do not exist, a senatorial and/or judicial process would indeed establish their lack of existence. So, why oppose effective ways for their dismissal?

2.4.2 Outline of the Literature on Antigravity

Besides the above quoted Refs. [35, 36], [37, 38], [39], additional data on the gravity of antiparticles are those from the LEAR machine on antiprotons at CERN [40], although these data too are inconclusive because of the excessive energy of the antiprotons and other factors, including the care necessary to extend the gravity of antiprotons to all antiparticles pointed out in Chapter 2, the proved impossibility for quarks to experience gravity, let alone antigravity, and other factors.

Additional experiments on the gravity of antiparticles are based on neutron interferometry, such as the experiments by Testera [41], Poggiani [42] and others. These experiments are extremely sensitive and, as such, definite and conclusive results continue to be elusive. In particular, the latter experiments too deal with antiprotons, thus inheriting the ambiguities of quark conjectures with respect to gravity, problems in the extension to other antiparticles, and other open issues.

All further data on the gravity of antiparticles known to this author are of indirect nature, e.g., via arguments based the equivalence principle (see, e.g., Ref. [33] and papers quoted therein). Note that the latter arguments do not apply under isoduality and will not be considered further.

A review on the status of our knowledge prior to isodual theories is available in Ref. [43], that includes an outline of the arguments against antigravity, such as those by Morrison, Schiff and Good. As we shall see, the latter arguments too cannon even be formulated under isodualities, let alone be valid.

We can therefore conclude by stating that at this writing there exists no experimental or theoretical evidence known to this author that is resolutory and conclusive either against or in favor of antigravity.

One of the most intriguing predictions of isoduality is the existence of *antigravity* conceived as a reversal of the gravitational attraction, first theoretically submitted by Santilli in Ref. [44] of 1994.

The proposal consists of an experiment that is feasible with current technologies and permits a definite and final resolution on the existence or lack of the existence of the above defined antigravity.

These goals were achieved by proposing the test of the gravity of positrons in horizontal flight on a vacuum tube. The experiment is resolutory because, for the case of a 10 m long tube and very low kinetic energy of the positrons (of the order of μeV), the displacement of the positrons due to gravity is sufficiently large to be visible on a scintillator to the naked eye.

Santilli's proposal [44] was studied by the experimentalist Mills [45] to be indeed feasible with current technology, resolutory and conclusive.

The reader should be aware from these introductory lines that *the prediction of antigravity exists, specifically, for the isodual theory of antimatter and not for conventional treatment of antiparticles.*

For instance, no prediction of antigravity can be obtained from Dirac's hole theory or, more generally, for the treatment of antimatter prior to isoduality, that solely occurring in second quantization.

Consequently, antigravity can safely stated to be the ultimate test of the isodual theory of antimatter.

In this chapter, we study the prediction of antigravity under various profiles, we review the proposed resolutory experiment, and we outline some of the far reaching implications that would follow from the possible experimental verification of antigravity, such as the consequential existence of a fully *Causal Time Machine,* although not for ordinary matter, but for an isoselfdual combination of matter and antimatter.

2.4.3 Newtonian and Euclidean Prediction of Antigravity

It is important to show that the prediction of antigravity can be first formulated at the most primitive possible level, that of Newtonian mechanics and its isodual. All subsequent formulations will be merely consequential.

The current theoretical scene on antigravity is dominated by the fact that, as it is well known, the Euclidean, Minkowskian and Riemannian geometries offer no realistic possibility to reverse the sign of a gravitational mass or of the energy of the gravitational field.

Under these conditions, existing theories can at best predict a decrease of the gravitational force of antiparticles in the field of matter (see Ref. [43] for a review of these conventional studies). In any case the decreased interaction, as such, remains attractive.

Isodual mathematical and physical theories alter this scientific scene. In fact, antigravity is predicted by the interplay between the classical Euclidean geometry and its isodual. The resulting prediction of antigravity persists at all levels, that is, for flat and curved spaces and for classical or quantum formulations, in a fully consistent way without known internal contradictions.

Also, antigravity is a simple consequence of Corollary 2.3.1 according to which the observed trajectories of antiparticles under a magnetic field are the *projection* in our spacetime of inverted trajectories in isodual spacetime.

Once these aspects are understood, the prediction of antigravity becomes so simple to appear trivial. In fact, antigravity merely originates from the projection

of the gravitational field of matter in that of antimatter and vice-versa. We therefore have the following:

PREDICTION 2.4.1 [43, 5]: *The existence of antigravity, defined as a gravitational repulsion experienced by isodual elementary particles in the field of matter and vice-versa, is a necessary consequence of a consistent classical description of antimatter.*

Let us begin by studying this prediction in Euclidean and isodual Euclidean spaces. Consider the Newtonian gravitational force of two conventional (thus, positive) masses m_1 and m_2

$$F = -G \times m_1 \times m_2/r < 0, \quad G, m_1, m_2 > 0, \tag{2.4.1}$$

where G is the gravitational constant and the minus sign has been used for similarity with the Coulomb law.

Within the context of conventional theories, the masses m_1 and m_2 remain positive irrespective of whether referred to a particle or an antiparticle. This yields the well known "universal law of Newtonian attraction", namely, the prediction that the gravitational force is attractive irrespective of whether for particle-particle, antiparticle-antiparticle or particle-antiparticle.

Again, the origin of this prediction rests in the assumption that antiparticles exist in our spacetime, thus having positive masses, energy and time. Under isoduality the situation is different. For the case of antiparticle-antiparticle under isoduality we have the different law

$$F^d = -G^d \times^d m_1^d \times^d m_2^d/^d r^d > 0, \quad G^d, m_1^d, m_2^d < 0. \tag{2.4.2}$$

But this force exists in the different isodual space and is defined with respect to the negative unit -1. Therefore, isoduality correctly represents the *attractive* character of the gravitational force between two isodual particles.

The case of particle-antiparticle under isoduality requires the *projection* of the isodual particle in the space of the particle (or vice versa), and we have the law

$$F = -G \times m_1 \times m_2^d/r > 0, \tag{2.4.3}$$

that now represents a *repulsion,* because it exists in our spacetime with unit $+1$, and it is opposite to force (2.4.1). This illustrates antigravity as per Prediction 2.4.1 when treated at the primitive Newtonian level.

Similarly, if we project the particle in the spacetime of the antiparticle, we have the different law

$$F^d = -G^d \times^d m_1^d \times^d m_2/^d r^d < 0, \tag{2.4.4}$$

that also represents *repulsion* because referred to the unit -1.

We can summarize the above results by saying that *the classical representation of antiparticles via isoduality renders gravitational interactions equivalent to the electromagnetic ones, in the sense that the Newtonian gravitational law becomes equivalent to the Coulomb law, thus necessarily including both attraction and repulsions.*

The restriction in Prediction 2.4.1 to "elementary" isodual particles will soon turn out to be crucial in separating science from its political conduct, and *de facto* restricts the experimental verification of antigravity to positrons in the field of Earth.

Note also that Prediction 2.4.1 is formulated for "isodual particles" and *not* for antiparticles. This is due to the fact indicated in preceding sections that, according to current terminologies, antiparticles are defined in our spacetime and have positive masses, energy and time. As such, no antigravity of any type is possible for antiparticles as conventionally understood.

2.4.4 Minkowskian and Riemannian Predictions of Antigravity

It is important to verify the above prediction at the classical relativistic and gravitational levels.

Let $M(x, \eta, R)$ be the conventional Minkowskian spacetime with coordinates $x = (r, t)$ (as a column) and metric $\eta = Diag.(1, 1, 1, -1)$ over the field of real numbers $R(n, +, \times)$ with unit $I = Diag.(1, 1, 1, 1)$. The *Minkowski-Santilli isodual space* [8] is given by (Section 2.2.8)

$$M^d(x^d, \eta^d, R^d), \quad x^d = -x^t, \quad \eta^d = Diag.(-1, -1, -1, +1), \tag{2.4.5a}$$

$$I^d = Diag.(-1, -1, -1, -1). \tag{2.4.5b}$$

The *isodual electromagnetic field* on $M^d(x^d, \eta^d, R^d)$ is given by

$$F^d_{\mu\nu} = \partial^d_\nu A^d_\mu - \partial^d_\mu A^d_\nu = -F^d_{\nu\mu}, \quad \mu, \nu = 1, 2, 3, 4, \tag{2.4.6}$$

with *isodual energy-momentum tensor*

$$T^d_{\mu\nu} = (1^d /^d 4^d \times m^d) \times^d [F^{d\,\alpha}_\mu \times^d F^d_{\alpha\nu} +$$

$$+ (1^d /^d 4^d) \times^d g^d \times^d F^d_{\alpha\beta} \times^d F^{d\,\alpha\beta}] = -T^t_{\nu\mu}, \tag{2.4.7}$$

where g is a known constant depending on the selected unit (whose explicit value is irrelevant for this study). Most importantly, the fourth component of the isodual energy-momentum tensor is negative-definite,

$$T^d_{00} < 0. \tag{2.4.8}$$

As such, antimatter represented in isodual Minkowski geometry has negative-definite energy, and other physical characteristics, and evolves backward in time. It is an instructive exercise for the interested reader to prove that the results of the Newtonian analysis of the preceding section carry over in their entirety to the Minkowskian formulation [8].

Consider now a Riemannian space $\mathcal{R}(x, g, R)$ in (3+1)-dimensions with space-time coordinates x and metric $g(x)$ over the reals R with basic unit $I = Diag.(1, 1, 1, 1)$ and related Riemannian geometry as presented, e.g., in Refs. [33, 47]. As outlined in Section 2.1.7, the *isodual iso-Riemannian spaces* are given by

$$\mathcal{R}^d(x^d, g^d, R^d) : x^d = -x^t, \;\; g^d(x^d) = -g^t(-x^t), \tag{2.4.9a}$$

$$I^d = Diag.(-1, -1, -1, -1). \tag{2.4.9b}$$

Recall that a basic drawback in the use of the Riemannian geometry for the representation of antiparticles is the positive-definite character of its energy-momentum tensor.

In fact, this character causes unsolved inconsistencies at all subsequent levels of study of antimatter, such as lack of a consistent quantum image of antiparticles.

These inconsistencies are resolved *ab initio* under isoduality. In fact, the isodual Riemannian geometry is defined over the isodual field of real numbers R^d for which the norm is negative-definite (Section 2.2.1).

As a result, all quantities that are positive in Riemannian geometry become negative under isoduality, thus including the energy-momentum tensor. In particular, energy-momentum tensors in the Riemannian geometry are given by relativistic expression (2.1.49i) and, as such, they remain negative-definite when treated in a Riemannian space.

It then follows that in the isodual Riemannian treatment of the gravity of antimatter, all masses and other quantities are negative-definite, including the *isodual curvature tensor,* Eq. (2.1.49c).

Despite that, the gravitational force between antimatter and antimatter remain *attractive,* because said negative curvature is measured with a negative unit.

As it was the case at the preceding Euclidean and Minkowskian levels, the isodual treatment of the gravitation of matter-antimatter systems requires its projection *either* in our spacetime *or* in the isodual spacetime. This again implies a *negative curvature in our spacetime* [8] resulting in Prediction 2.4.1 of antigravity at the classical Riemannian level too.

2.4.5 Prediction of Antigravity from Isodual Einstein's Gravitation

Einstein's gravitation is generally defined (see, e.g., Ref. [33]) as the reduction of gravitation in the exterior problem in vacuum to pure curvature in a Riemannian space $\mathcal{R}(x, g, R)$ with local spacetime coordinates x and metric $g(x)$ over

the field of real numbers R *without a source,* according to the celebrated field equations

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = 0, \tag{2.4.10}$$

where $G_{\mu\nu}$ is generally referred to as the *Einstein tensor,* $R_{\mu\nu}$ is the *Ricci tensor,* and R is the *Ricci scalar.*

As it is well known, *Einstein's conception of gravitation as above identified does not permit antigravity,* and this occurrence has been a motivation for the absence of serious experimental studies in the field, as indicated in Section 1.4.1.

However, we have indicated in preceding chapters that *the problem of antigravity cannot be confidently formulated, let alone treated, in Einstein's gravitation, due to the impossibility of consistently treating antimatter.*

As indicated earlier, the only possible formulation of antimatter is that by *only* changing the sign of the charge. However, this formulation is inconsistent with quantization since it leads to particles, rather than antiparticles, with the wrong sign of the charge.

At any rate, *the most important formulation of the gravity of antimatter is that for astrophysical bodies with null total charge, as expected for an antimatter star or an antimatter neutron star.*

The impossibility for any credible treatment of antimatter is then established by the fact that *according to Einstein's conception of gravitation the gravitational fields equations for matter and antimatter stars with null total charge are identical.*

These inconsistencies are resolved by the isodual theory of antimatter because it implies the novel *isodual field equations for antimatter* defined on the isodual Riemannian space [8] $\mathcal{R}^d(x^d, g^d, R^d)$ with local isodual spacetime coordinates $x^d = -x^t$ and isodual metric $g^d(x^d) = -g^t(-x^t)$ over the isodual field of real numbers R^d

$$G^d_{\mu\nu} = R^d_{\mu\nu} - g^d_{\mu\nu} \times R^d /^d 2^d = 0. \tag{2.4.11}$$

The latter representation is based on a negative-definite energy-momentum tensor, thus having a consistent operator image, as shown in Chapter 3.

We, therefore, conclude this analysis with the following:

THEOREM 2.4.1 : Antigravity is a necessary and sufficient condition for the existence of a classical formulation of antimatter compatible with its operator counterpart.

Proof. Assume the validity of Einstein's gravitation for matter and its isodual for antimatter. Then, the former has a positive curvature tensor and the latter has a negative curvature tensor.

Therefore, the projection of the gravitational field of antimatter in the spacetime of matter implies a negative curvature tensor in our spacetime, namely,

antigravity, or, vice-versa, a positive curvature tensor in the isodual spacetime, that is also repulsive, and this proves the sufficiency. The necessity comes from the fact that the only formulation of antimatter compatible with operator counterparts is that based on negative energies and masses.

In turn, geometric formulations of negative energies and masses necessarily imply, for consistency, a negative curvature tensor. Still in turn, when projected in the space of matter, a negative curvature necessarily implies antigravity and the same occurs for the projection of matter in the field of antimatter. **q.e.d.**

2.4.6 Identification of Gravitation and Electromagnetism

In addition to the above structural inability by Einstein's equations (2.4.10) to represent antimatter, Einstein's gravitation is afflicted by a litany of inconsistencies for the treatment of *matter itself* studied in Section 1.4 whose resolution requires a number of structural revisions of general relativity.

It is important to show that the prediction of antigravity, not only persists, but it is actually reinforced for gravitational theories resolving the inconsistencies of Einstein's gravitation.

The first catastrophic inconsistency of Einstein's gravitation crucial for the problem of antigravity is that of Theorem 1.4.1 on the irreconcilable incompatibility between Einstein's lack of source in vacuum and the electromagnetic origin of mass.

As stressed in Section 1.4, this inconsistency is such that, either one assumes Einstein's gravitation as correct, in which case quantum electrodynamics must be reformulated from its foundation to prevent a first-order source in vacuum, or one assumes quantum electrodynamics to be correct, in which case Einstein's gravitation must be irreconcilably abandoned.

The second catastrophic inconsistency of Einstein's gravitation is that of Theorem 1.4.2 identifying the incompatibility of field equations (2.4.10) and the forgotten Freud identity of the Riemannian geometry,

$$R^\alpha_\beta - \frac{1}{2}\times\delta^\alpha_\beta\times R - \frac{1}{2}\times\delta^\alpha_\beta\times\Theta \;=\; U^\alpha_\beta + \partial V^{\alpha\rho}_\beta/\partial x^\rho \;=\; k\times(t^\alpha_\beta+\tau^\alpha_\beta), \quad (2.4.12)$$

where

$$\Theta \;=\; g^{\alpha\beta}g^{\gamma\delta}(\Gamma_{\rho\alpha\beta}\Gamma^\rho_{\gamma\beta} - \Gamma_{\rho\alpha\beta}\Gamma^\rho_{\gamma\delta}), \quad (2.4.13a)$$

$$U^\alpha_\beta \;=\; -\frac{1}{2}\frac{\partial\Theta}{\partial g^{\rho\alpha}_{|_\rho}}g^{\gamma\beta}\uparrow_\gamma, \quad (2.4.13b)$$

$$V^{\alpha\rho}_\beta \;=\; \frac{1}{2}[g^{\gamma\delta}(\delta^\alpha_\beta\Gamma^\rho_{\alpha\gamma\delta} - \delta^\rho_\beta\Gamma^\rho_{\alpha\delta}) +$$

$$+(\delta^\rho_\beta g^{\alpha\gamma} - \delta^\alpha_\beta g^{\rho\gamma})\Gamma^\delta_{\gamma\delta} + g^{\rho\gamma}\Gamma^\alpha_{\beta\gamma} - g^{\alpha\gamma}\Gamma^\rho_{\beta\gamma}]. \quad (2.4.13c)$$

The latter inconsistency requires the addition in the right-hand-side of Eqs. (2.4.10) of *two source tensors* for astrophysical bodies with null total charge.

As stressed in Section 1.4, the above two inconsistencies are deeply inter-related because complementary to each other, since the inconsistency of Theorem 1.4.2 is the dynamical counterpart of the inconsistency of Theorem 1.4.2 on geometric grounds.

A systematic study of the resolution of these inconsistencies was conducted by Santilli [48] in 1974.

The classical gravitational formulation of antimatter can be done in the *Riemannian-Santilli isodual space* $\mathcal{R}^d(x^d, g^d, R^d)$ studied in Sections 2.1.7 and 2.2.11.

To avoid catastrophic inconsistencies, the field equations of antimatter should be compatible with the basic geometric axioms of the isodual Riemannian geometry, including, most importantly, the *isodual Freud identity* [8], that can be written

$$R^{\alpha d}_{\beta} - \frac{1}{2}^{d} \times^{d} \delta^{\alpha d}_{\beta} \times^{d} R^{d} - \frac{1}{2}^{d} \times^{d} \delta^{\alpha d}_{\beta} \times^{d} \Theta^{d} = k^{d} \times^{d} (T^{d\alpha}_{\beta} + \Upsilon^{d\alpha}_{\beta}). \qquad (2.4.14)$$

with corresponding isodualities for Eqs. (2.4.13) here assumed as known.

These studies then leads to the following:

PREDICTION 2.4.2: [48] IDENTIFICATION OF GRAVITATION AND ELECTROMAGNETISM. In the exterior problem in vacuum, gravitation coincides with the electromagnetic interactions creating the gravitational mass with field equations

$$G^{Ext.}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times T^{Elm}_{\mu\nu}, \qquad (2.4.15)$$

where the source tensor $T^{Elm}_{\mu\nu}$ represents the contribution of all charged elementary constituents of matter with resulting gravitational mass

$$m^{Grav} = \int d^3x \times T^{Elm}_{00}, \qquad (2.4.16)$$

while in the interior problem gravitation coincides with electromagnetic interactions plus short range weak, strong and other interactions creating the inertial mass with field equations

$$G^{Int.}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times (T^{Elm}_{\mu\nu} + \Upsilon^{ShortRange}_{\mu\nu}), \qquad (2.4.17)$$

where the source tensor $\Upsilon^{ShortRange}_{\mu\nu}$ represents all possible short range interactions in the structure of matter, with inertial mass

$$m^{Inert} = \int d^3x \times (T^{Elm}_{00} + \Upsilon^{ShortRange}_{00}), \qquad (2.4.18)$$

and general law

$$m^{Inert} > m^{Grav}. \tag{2.4.19}$$

The same identification of gravitation and electromagnetism then exists for antimatter with field equations and mass expressions given by a simple isodual form of the preceding ones.

A few comments are in order. All studies on the problem of "unification" of gravitation and electromagnetism prior to Ref. [48] known to this author[7] treated the two fields as *physically distinct*, resulting in the well known historical failures to achieve a consistent *unification* dating back to Albert Einstein (see next chapter for a detailed study). An axiomatically consistent theory emerges if gravitation and electromagnetism are instead "identified", as first done by Santilli [48] in 1974.

Also, Prediction 2.4.2 implies a *theory on the origin of the gravitational field,* rather than a theory providing its "description", as available in standard treatises such as [33]. This is due to the fact that in Prediction 2.4.2 *all* mass terms are completely eliminated and replaced with the fields originating mass.

In this way, the use of any mass term in any theory is an admission of our ignorance in the structure of the considered mass.

We should indicate for completeness that the identification of exterior gravitational and electromagnetic fields appears to be disproved by the assumption that quarks are physical constituents of hadrons, owing to the known large value of their "masses".

However, as indicated in Chapter 1, gravitation solely exists in our spacetime and cannot be consistently extended to mathematical unitary symmetries. Also, the only masses that can consistently create gravitation are those defined in our spacetime, thus necessarily being the eigenvalues of the second-order Casimir invariant of the Poincaré symmetry.

Since quarks cannot be defined in our spacetime, they cannot be consistently characterized by the Poincaré symmetry and their masses are not the eigenvalues of the second-order Casimir invariant of the latter symmetry, the use of quark masses has no scientific value in any gravitational profile. This is the reason why quark "masses" have been ignored in Ref. [48] as well as in this chapter.

It is well established in quantum electrodynamics that the mass of the electron is entirely of electromagnetic origin. Therefore, a gravitational theory of the electron in which the source term solely represents the charge contribution is incompatible with quantum electrodynamics. In fact, the latter requires *the entire reduction of the electron mass to electromagnetic fields* according to Eqs. (2.4.16).

[7]Again, the author would appreciate the indication of similar contributions prior to 1974.

Note in particular that, since the electron has a point-like charge, we have no distinction between exterior and interior problems with consequential identity

$$m^{Grav}_{Electron} \equiv m^{Inert}_{Electron}. \tag{2.4.20}$$

When considering a neutral, extended and composite particle such as the π°, the absence of a source tensor of electromagnetic nature renders gravitation, again, incompatible with quantum electrodynamics, as established in Ref. [48] and reviewed in Section 1.4.

By representing the π° as a bound state of a charged elementary particle and its antiparticle in high dynamical conditions, quantum electrodynamics establishes the existence not only of a non-null total electromagnetic tensor, but one of such a magnitude to account for the entire gravitational mass of the π° according to Eq. (2.4.16) and gravitational mass

$$m^{Grav}_{\pi^\circ} = \int d^3x \times T^{Elm}_{00\,\pi^\circ}. \tag{2.4.21}$$

Unlike the case of the electron, the π° particle has a very large charge distribution for particle standards. Moreover, the structure of the π° particle implies the additional weak and strong interactions, and their energy-momentum tensor is not traceless as it is the case for the electromagnetic energy-momentum tensor.

Therefore, for the case of the π° particle, we have a well-defined difference between exterior and interior gravitational problems, the latter characterized by Eqs. (2.4.18), i.e.,

$$m^{Inert}_{\pi^\circ} = \int d^3x \times (T^{Elm}_{00} + \Upsilon^{ShortRange}_{00}) > m^{Grav}_{\pi^\circ}. \tag{2.4.22}$$

The transition from the π° particle to a massive neutral star is conceptually and technically the same as that for the π°. In fact, the star itself is composed of a large number of elementary charged constituents each in highly dynamical conditions and, therefore, each implying a contribution to the total gravitational mass of the star as well as to its gravitational field.

The separation between exterior and interior problems, the presence of only one source tensor for the exterior problem and two source tensors for the interior problems, and the fact that the inertial mass is bigger than the gravitational mass is the same for both the π° and a star with null total charge.

For the case of a star we merely have an increased number of elementary charged constituents resulting in the expression [48]

$$m^{Grav}_{Star} = \Sigma_{p=1,2,3,...} \int d^3x \times T^{Elem.Constit.}_{00}. \tag{2.4.23}$$

Note that when the star has a non-null total charge there is no need to change field equations (2.4.15) since the contribution from the total charge is automatically provided by the constituents.

As it is well known, there exist numerous other theories on the identity as well as the possible differentiation of gravitational and inertial masses (see, e.g., Ref. [33]). However, these theories deal with exterior gravitational problems while the studies here considered deal with the interior problem, by keeping in mind that inertial masses are a strictly *interior* problem, the exterior problem providing at best a geometric abstraction.

Nevertheless, one should remember that all these alternative theories are crucially based on Einstein's gravitation, while the theory presented in this section is based on quantum electrodynamics. Therefore, *none of the existing arguments on the differences between gravitational and inertial masses is applicable to the theory here considered.*

Note finally that conventional electromagnetism is represented by a *first-order tensor*, the electromagnetic tensor $F_{\mu\nu}$ of type (2.2.37a) and related first-order Maxwell's equations (2.2.37b) and (2.2.37c).

When electromagnetism is identified with exterior gravitation, it is represented with a *second-order tensor,* the energy-momentum tensor $T_{\mu\nu}$ of type (2.4.7) and related second-order field equations (2.4.15).

2.4.7 Prediction of Antigravity from the Identification of Gravitation and Electromagnetism

Another aspect important for this study is that *the identification of gravitation and electromagnetism in the exterior problem in vacuum implies the necessary existence of antigravity.*

In fact, the identification implies the necessary equivalence of the phenomenologies of gravitation and electromagnetism, both of them necessarily experiencing attraction and repulsion.

Note that this consequence is intrinsic in the identification of the two fields and does not depend on the order of the field equations (that is first order for electromagnetism and second order for gravitation as indicated earlier.

Alternatively, for the exterior problem of matter we have the field equations on $\mathcal{R}(x, g, R)$ over R

$$G_{\mu\nu}^{Ext.} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times T_{\mu\nu}^{Elm}, \tag{2.4.24}$$

in which *the curvature tensor is positive,* and for the exterior problem of antimatter we have the isodual equations on $\mathcal{R}^d(x^d, g^d, R^d)$ over R^d

$$G_{\mu\nu}^{d,Ext.} = R_{\mu\nu}^d - g_{\mu\nu}^d \times R^d/2 = k \times T_{\mu\nu}^{d,Elm}, \tag{2.4.25}$$

in which *the curvature tensor is negative.*

The prediction of antigravity, Prediction 2.4.1, follows as a trivial extension of that of the preceding sections and occurs when the gravitational field of antimatter is projected in that of matter, or vice-versa, since such a projection implies a negative curvature in a Riemannian space that, by definition, is antigravity.

The prediction of antigravity is so strong that it is possible to prove that *the lack of existence of antigravity would imply the impossibility of identifying gravitation and electromagnetism.*

In turn, the lack of such identification would necessary require the impossibility for masses to have appreciable electromagnetic origin, resulting in the need for a structural revision of the entire particle physics of the 20-th century.

2.4.8 Prediction of Gravitational Repulsion for Isodual Light Emitted by Antimatter

Another important implication of the isodual theory of antimatter is the prediction that antimatter emits a new light, the *isodual light,* that experiences repulsion when in the vicinity of the gravitational field of matter, or vice-versa [18], where the *isodual electromagnetic waves* emitted by antimatter are given by Eqs. (2.3.37), i.e.,

$$F^d_{\mu\nu} \;=\; \partial^d A^d_\mu /^d \partial^d x^{\nu d} - \partial^d A^d_\nu /^d \partial^d x^{d\mu}, \tag{2.4.26a}$$

$$\partial^d_\lambda F^d_{\mu\nu} + \partial^d_\mu F^d_{\nu\lambda} + \partial^d_\nu F^d_{\lambda\mu} = 0, \tag{2.4.26b}$$

$$\partial^d_\mu F^{d\mu\nu} = -J^{d\nu}. \tag{2.4.26c}$$

The gravitational repulsion then emerges from the negative energy of the above isodual waves when in the field of matter. Vice versa, electromagnetic waves emitted by matter are predicted to experience antigravity when in the gravitational field of antimatter because they have a positive energy.

Note that *isodual electromagnetic waves coincide with conventional waves under all known interactions except gravitation.* Alternatively, the isodual electromagnetic waves requires the existence of antigravity at a pure classical level for their proper identification.

In turn, the experimental confirmation of the gravitational repulsion of light emitted by antimatter would have momentous astrophysical and cosmological implications, since it would permit for the first time theoretical and experimental studies as to whether far away galaxies and quasars are made up of matter or of antimatter.

It is important in this connection to recall that all relativistic quantum field equations admit solutions with positive and negative energies. As it is the case for Dirac's equations, relativistic field equations are generally isoselfdual, thus admitting solutions with both positive and negative energies.

The former are used in numerical predictions, but the negative-energy states are generally discarded because they are believed to be "unphysical".

The isodual theory implies a significant revision of the interpretation of quantum field theory because *the solutions of relativistic equations with positive energy are defined in our spacetime and represent particles, while the joint solutions with*

negative energy are actually defined on the isodual spacetime and represent antiparticles.

This re-interpretation cannot be presented in this chapter for brevity. In fact, a systematic study of isodual photons requires the formulation of *isodual quantum field theory* that would render prohibitive the length of this chapter.

It is hoped that interested colleagues will indeed work out the proposed isodual quantum field theory, with particular reference to the isodual re-interpretation of advanced and retarded solutions, Green distributions, Feynman diagrams, and all that, because of various implications, such as those in conjugation of trajectories or in the transition from particles to antiparticles.

In closing, the reader should keep in mind that the isodual theory of antimatter resolves all conventional inconsistencies on negative energies as well as against antigravity (see also Section 2.3.15).

2.5 EXPERIMENTAL VERIFICATION OF ANTIGRAVITY

2.5.1 Santilli's Proposed Test of Antigravity for Positrons in Horizontal Flight

By far the most fundamental experiment that can be realized by mankind with current technologies is the measure of the gravitation of truly elementary antiparticles, such as the positron, in the field of Earth.

Irrespective of whether the outcome is positive or negative, the experiment will simply have historical implications for virtually all of physics, from particle physics to cosmology for centuries to come.

If antigravity is experimentally established, the location of the experiment is predicted to become a place of scientific pilgrimage for centuries, due to the far reaching implications, such as the consequential existence of a Causal Time Machine outlined later on in this chapter.

An inspection of the literature soon reveals that the problem of the *gravity of antiparticles in the field of Earth is fundamentally unsettled at this writing, thus requiring an experimental resolution.*

On theoretical grounds, all arguments based on the weak equivalence principle [33] are dismissed as inconclusive by the isodual theory of antimatter, since the latter predicts that bound states of particles and their isoduals experience *attraction* in the gravitational field of Earth.

At any rate, no argument against antigravity based on general relativity can be considered scientifically valid without first the resolution of the catastrophic inconsistencies of gravitation, such as those expressed by the various inconsistency theorems of Section 1.4.

Similarly, all experiments conducted to date on the test of the *gravity of antiparticles not bounded to matter* are equally inconclusive, to the author's best

knowledge.[8] A direct measurement of the gravity of positrons was considered in 1967 by Fairbanks and Witteborn [39] via electrons and positrons in a *vertical* vacuum tube.

However, the test could not be conducted because preliminary tests with electrons discouraged the use of positrons due to excessive disturbances caused by stray fields, impossibility of ascertaining the maximal height of the electrons, and other problems.

Neutron interferometric measurements of the *gravity of antiprotons* have been studied by Testera [41], Poggiani [42] and others. However, these experiments are highly sophisticated, thus implying difficulties, such as those for securing antiprotons with the desired *low energies,* magnetic trapping of the antiprotons, highly sensitive interferometric measurements of displacements, and others.

A number of important proposals to text the gravity of antimatter have been submitted to CERN and at other laboratories by T. Goldman, R. J. Hughes, M. M. Nieto, et al. [50–53], although no resolutory measurement has been conducted to date to the author best knowledge, perhaps in view of the excessive ambiguities for an accurate detection of the trajectories of antiparticles under Earth-s gravitational field in existing particle accelerators (see in this respect Figure 2.8).

Additional important references are those studying the connection between antigravity and quantum gravity [54–57], although the latter should be studied by keeping in mind Theorem 1.5.2 on the catastrophic inconsistencies of quantum gravity when realized via nonunitary structures defined on conventional Hilbert spaces and fields.[9]

In view of these unsettled aspects, an experiment that can be *resolutory* with existing technologies, that is, establishing in a final form either the existence of the lack of existence of antigravity, has been proposed by Santilli in Ref. [44] of 1994.

The experiment essentially requires a *horizontal* vacuum tube ranging from 100 meters in length and 0.5 meter in diameter to 10 m in length and 1 m in diameter depending on used energies, with axial collimators at one end and a scintillator at the other end as in Figure 2.7. The proposed test then consists in:

1) Measuring the location in the scintillator of lack of gravitational displacement via a collimated photon beam (since the gravitational displacement on photons at the considered distances is ignorable);

2) Measuring on the same scintillator the downward displacement due to Earth's gravity on an electron beam passing through the same collimators, which

[8]The author would appreciate being kept informed by experimentalist in the field.

[9]The author would like to express his sincere appreciation to T. Goldman for the courtesy of bringing to his attention the important references [50–57] that could not be reviewed here for brevity, but whose study is recommended as a necessary complement of the presentation of this monograph.

downward displacement is visible to the naked eyes for sufficiently small electron energies (for instance, we can have a downward displacement due to gravity of 5 mm, that is visible to the naked eye, for electron kinetic energies of 25 μeV along 100 m horizontal flight, or for electrons with 2 μeV along a 10 m horizontal flight); and

3) Measuring on the same scintillator the displacement due to Earth's gravity on a positron beam passing through the same collimators, which displacement is also visible to the naked eye for positron energies of the order of a few μeV.

If the displacement due to gravity of the positrons is downward, the test would establish the lack of existence of antigravity. On the contrary, the detection of an upward displacement of the positrons would establish the existence of antigravity.

An alternative proposal was submitted by Santilli [20] via the use of the so-called *particle decelerator* in the shape of a doughnut of a diameter of about 10 m and 50 cm in sectional diameter (Figure 2.8). The main idea is that low energy beams of electrons and positrons could be *decelerated* via the use of magnetic fields down to the energy needed to achieve a displacement due to gravity sufficiently larger than the dispersion to be visible to naked eye, at which point the particles are released into a scintillator.

We have stressed throughout this presentation that the only experimental verification of the theoretical prediction of antigravity recommendable at this writing, is that for *truly elementary antiparticles in the gravitational field of matter without any bound to other particles,* such as an isolated beam of positrons under the gravitation field of Earth.

Other tests of antigravity, if conducted before the above tests with positrons and used for general claims on antigravity, can likely lead to ambiguities or a proliferations of unnecessary controversies.

The reasons for this restriction are numerous. Firstly, the study of the gravity of particle-antiparticle systems, such as a bound state of one electron and one positron at large mutual distances according to quantum mechanics (QM),

$$Positronium \ = \ (e^-, \ e^+)_{QM}, \tag{2.5.1}$$

is *strongly discouraged* for a first "test of antigravity", because all theories, including the isodual theory, predict attraction of the positronium in the field of matter. Therefore, under no condition can any possible experimental verification of this prediction be used as a credible claim on the lack of existence of antigravity at large.

Second, the above restriction eliminates the use of muons for a first test of antigravity, because, in view of their instability and decay modes, and as studied in detail in the next chapter, hadronic mechanics (HM) predicts that muons are a bound state of electrons and positrons in conditions of total mutual penetrations

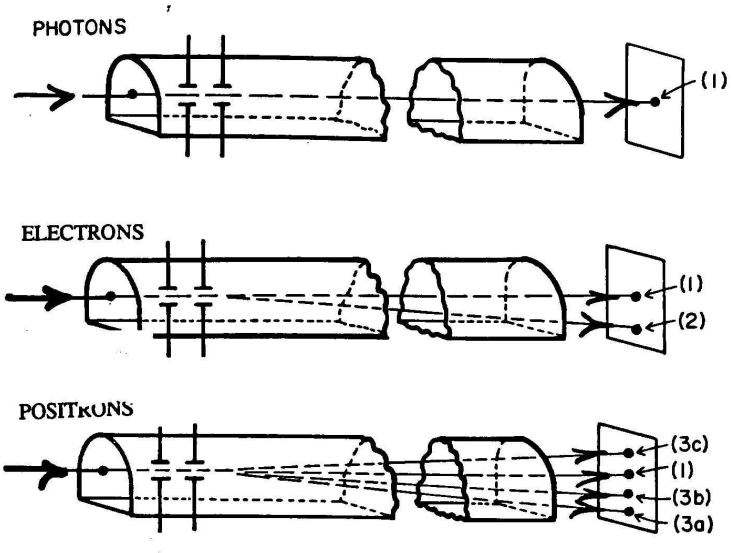

Figure 2.7. A schematic view of the proposal to test the gravity of positrons suggested by Santilli [44] in 1994 via a horizontal vacuum tube with a scintillator at the end in which a collimated beam of photons is used to identify the point in the scintillator of no displacement due to gravity, and collimated beams of very low energy electrons and, separately, positrons are used to measure displacements due to gravity. The latter are indeed visible to the naked eye for sufficiently low kinetic energy of the order of a few μeV. Santilli's proposal [44] was studied by the experimentalist J. P. Mills, jr. [45], as reviewed in the next section.

of their wavepackets at very short mutual distances,

$$\mu^{\pm} \; = \; (e^{-}, \; e^{\pm}, \; e^{+})_{HM}, \tag{2.5.2}$$

with consequential highly nonlocal effects structurally beyond any credible treatment by quantum mechanics. Under this structure, *both muons and antimuons are predicted to experience gravitational attraction only* because the possible antigravity of the positron is expected to be less than the gravity of basic electron-positron system.

A similar restriction applies against the use of mesons for first tests of antigravity because they are bound states of particles and antiparticles that, as such, are predicted not to experience antigravity in the field of matter. This is particularly the case for pions. Similarly, a first use of kaons for experiments on antigravity can only result in unnecessary controversies in view of their unsettled structure.

Serious reservation also exist for the first use of antiprotons and antineutrons due to their basically unsettled structure. As stressed earlier, the use of current quark conjecture prevents antiprotons and antineutrons to have any gravity at

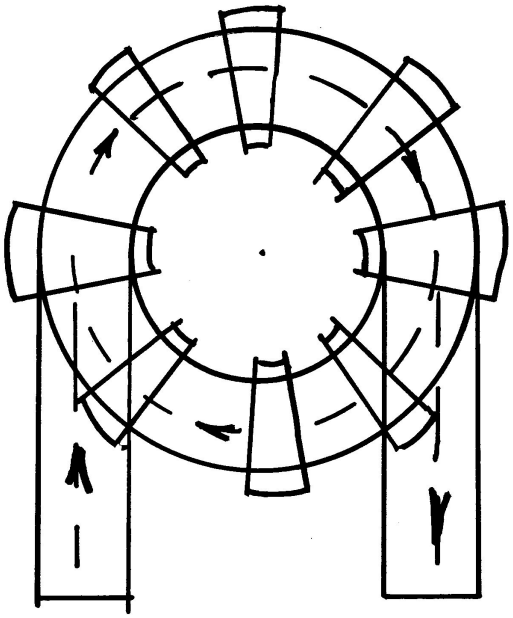

Figure 2.8. A schematic view of the alternative proposal submitted for study by the author [20] at the National High Magnetic Field Laboratory, Tallahassee, Florida, in December 1995. The main idea is to use the established techniques for "particle accelerators" for the construction of a "particle decelerator" that would slow down the initial energy of electron and positron beams down to the amounts needed to produce displacement due to gravity sufficiently bigger than the spread due to stray fields to produce a definite-resolutory answer visible to the naked eye. Suggested dimensions of the "particle decelerator" are 10 m in diameter with a sectional diameter of 0.5 m and two entrances-exits, one used for the entrance-exit of the electron beam and the other for the positron beam. The study conducted by Mills [45] for the horizontal tube indicates that the "particle decelerator" here considered is also feasible and will produce a resolutory answer.

all, let alone antigravity, as rigorously proved by the fact indicated earlier that gravity can only be defined in our physical spacetime while quarks can only be defined in their internal mathematical unitary space, as well as by the lack of credibly defines "quark masses" as inertial eigenvalues of the second order Casimir invariant of the Poincaré group (see the Appendix of Ref. [41]).

Equally equivocal can be at this stage of our knowledge the conduction of first gravitational measurements via the sole use of the antihydrogen atom for intended general results on antigravity, evidently because its nucleus, the antiproton, is believed to be a bound state of quarks for which no gravity at all can be consistently defined. Any study of antigravity under these unsettled structural

conditions can only lead to un-necessary controversies, again, if used for general results on antigravity.

It is evident that, *until baryons theories are afflicted by such fundamental problematic aspects, as the inability even to define gravity in a credible way, no gravitational measurement based on antiprotons and antineutrons can be credibly used as conclusive for all of antimatter.*

After the resolution of the gravitational behavior of unbounded positrons in the field of matter, the tests for the gravitational behavior of positronium, muons, muonium, pions, pionium, antiprotons, antineutrons, antihydrogen atom, etc. become essential to acquire an experimental background sufficiently diversified for serious advances on antimatter beyond the level of personal beliefs one way or the other.

The fundamental test of the gravity of positrons here considered was proposed by the author to the following institutions:

1) Stanford Linear Acceleration Center, Stanford, USA, during and following the Seventh Marcel Grossmann Meeting on General Relativity held at Stanford University in July 1994;

2) The Joint Institute for Nuclear Research in Dubna, Russia, during the International Conference on Selected Topics in Nuclear Physics held there in August 1994;

3) The National High Magnetic Field Laboratory in Tallahassee, Florida, during a meeting there in 1996 on magnetic levitation;

4) CERN, Geneva, Switzerland, during a presentation there of hadronic mechanics;;

5) Brookhaven National Laboratories, following the participation at the Sepino meeting on antimatter of 1996 [19];

and to other laboratories as well to universities in various countries.

It is regrettable for mankind that none of these laboratories or universities expressed interest in even considering to date such a fundamental experiment, by preferring to spend much bigger public funds for esoteric experiments manifestly lesser important than that of antigravity.

2.5.2 Santilli's Proposed Tests of Antigravity for Isodual Light

Additionally, in 1997 Santilli [18] predicted that *antimatter emits a new light, the isodual light, that is predicted to be repelled by the gravitational field of matter,* and proposed its experimental verification as the only known (or even conceivable) possibility of ascertaining whether far-away galaxies and quasars are made up of matter or of antimatter.

Measurements as to whether light emitted by the antihydrogen atoms now produced at CERN are attracted or repelled by matter is predictably more deli-

cate than the test of the gravity of the positron, evidently because gravitational displacements for photons in horizontal flight are extremely small, as well know, thus requiring very sensitive interferometric and other measurements.

The experimental detection as to whether far-away galaxies and quasars are made up of matter or of antimatter is predictably more complex and requiring longer periods of time, but with immense scientific implications whatever the outcome.

The test can be done in a variety of ways, one of which consists of measuring the deflection of light originating from far away astrophysical objects when passing near one of our planets. Comparative measurements of a sufficiently large number of galaxies and quasars should permit the detection of possible repulsions, in the event it exists.

Another test has been privately suggested by to the author by an astrophysicist and consists in reinspecting all existing astrophysical data on the deflection of light from far away galaxies and quasars when passing near-by astrophysical bodies.

In the opinion of this astrophysicist, it appears that evidence for the repulsion of light already exists in these data. Such a possible evidence has been ignored so far, and, if found, could not be admitted publicly at the moment, simply because Einstein's gravitation does not allow for any prediction of gravitational repulsion of light.

An understand is that, for these astrophysical measurements to be credible, astrophysicists must conduct the study of a vary large number of galaxies and quasars (of the order of several thousands), and the considered galaxies and quasars must be sufficiently far away to render plausible their possible antimatter structure.

2.5.3 Mills' Studies of Santilli's Proposed Tests of Antigravity

The experimentalist J. P. Mills, jr., [45] conducted a survey of all significant experiments on the gravity of antiparticles in the field of Earth, including indirect tests based on the weak equivalence principle and direct experiments with antiparticles, by concluding that the problem is basically unsettled on theoretical and experimental grounds, thus requiring an experimental resolution.

After considering all existing possible tests, Mills' conclusion is that Santilli's proposed test [44] on the measurement of the gravitational deflection of electrons and positron beams of sufficiently low energy in horizontal flight in a vacuum tube of sufficient length and shielding, is preferable over other possible tests, experimentally feasible with current technology, and providing a resolutory answer as to whether positrons experience gravity or antigravity.

As it is well known, a main technical problem in the realization of Santilli's test is the shielding of the horizontal tube from external electric and magnetic field, and then to have a tube structure in which the internal stray fields have an ignorable impact on the gravitational deflection, or electrons and positrons have such a low energy for which the gravitational deflection is much bigger than possible contributions from internal stray fields, such as the spreading of beams.

The electric field that would cancel the Earth gravitational force on an electron is given by

$$E \ = \ m_e \times g/e = 5.6 \times 10^{-11} \ \mathrm{V/m}. \tag{2.5.3}$$

As it is well known, an effective shielding from stray fields can be obtained via Cu shells. However, our current understanding of the low temperature zero electric field effect in Cu shells does not seem sufficient at this moment to guarantee perfect shielding from stray fields. Mills [45] then suggested the following conservative basic elements for shielding the horizontal tube.

Assuming that the diameter of the tube is d and the shielding enclosure is composed of randomly oriented grains of diameter λ, the statistical variation of the potential on the axis of the tube of a diameter d would then be [45]

$$\Delta V \ = \ \frac{\lambda}{d \times \sqrt{\pi}}. \tag{2.5.4}$$

As expected, *the effect of stray fields at the symmetry axis of the tube is inversely proportional to the tube diameter.* As we shall see, a tube diameter of 0.5 m is acceptable, although one with 1 m diameter would give better results.

Given a work function variation of 0.5 eV, 1 μm grains and $d = 30$ cm, we would have the following variation of the potential on the axis of the horizontal tube

$$\Delta V \ = 1 \ \mu\mathrm{eV}. \tag{2.5.5}$$

Differences in strain or composition could cause larger variations in stray fields. To obtain significant results without ambiguities for the shielding effect of low temperature Cu shells, Mills [45] suggests *the use of electrons and positrons with kinetic energies significantly bigger than* 1 μeV. As we shall see, this condition is met for tubes with minimal length of 10 m and the diameter of 1 m, although longer tubes would evidently allow bigger accuracies.

The realization of Santilli's horizontal vacuum tube proposed by Mills [45] is the following. As shown in Figure 2.9, the tube would be a long dewar tube, consisting of concentric shells of Al and Mu metals, with Pb and Nb superconducting shells and an inner surface coated with an evaporated Cu film.

There should be two superconducting shells so that they would go superconducting in sequence [Nb (9.25 K), Pb (7.196 K)], evidently for better expulsion of flux. Trim solenoids are also recommended for use within the inner shell and a multitude of connections to the Cu field for trimming electrostatic potentials.

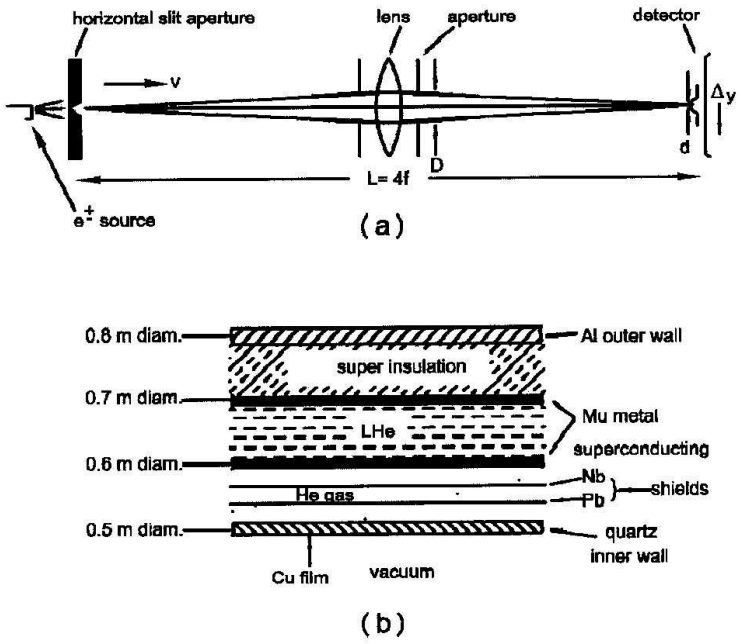

Figure 2.9. A schematic view of the realization suggested by Mills [45] of the horizontal tube proposed by Santilli [44].

As also shown in Figure 2.9, the flight tube should be configured with an electrostatic lens in its center for use of electron and positron beams in both horizontal directions, as well as to focus particles from a source at one end into a gravity deflection sensitive detector at the other end. The de Broglie wavelength of the particles results in the position resolution

$$d \;=\; 2.4 \times \pi \times \alpha_B \times \frac{c \times L}{v \times D}, \tag{2.5.6}$$

where $\alpha = 1/137$ is the fine structure constant, $a_B = 0.529\,\text{Å}$ is the Bohr radius of hydrogen, c is the velocity of light, v is the electron or positron velocity, L is the length of the horizontal path, and D is the diameter of the lens aperture in the center of the flight tube.

The *vertical gravitational deflection* is given by

$$\Delta y \;=\; g \times \frac{L^2}{2 \times v^2}. \tag{2.5.7}$$

Given $L = 100\,\text{m}$, $D = 10\,\text{cm}$, $v/c = 10^{-5}$ (i.e., for 25 μeV particles), we have

$$\Delta y = 5\,\text{mm}. \tag{2.5.8}$$

For 1 meV particles the resolution becomes

$$\Delta y = 125\,\mu\text{m}. \tag{2.5.9}$$

Therefore, one should be able to observe a meaningful deflection using particles with kinetic energies well above the expected untrimmed fluctuation in the potential.

Mills also notes that the lens diameter should be such as to minimize the effect of lens aberration. This requirement, in turn, dictates the minimum inside diameter of the flight tube to be 0.5 m.

The *electron source* should have a cooled field emission tip. A sufficient *positron source* can be provided, for example, by 0.5 ci of ^{22}Na from which we expect (extrapolating to a source five times stronger) $3 \times 10^7\, e^+/\text{s}$ in a one centimeter diameter spot, namely a positron flux sufficient for the test.

Ideal results are obtained when the positrons should be bunched into pulses of 10^4 e^+ at the rate of 10^3 bunches per second. Groups of 10^3 bunches would be collected into macrobunches containing 10^6 e^+ and 20 nsec in duration. The positrons would be removed from the magnetic field and triply brightness enhanced using a final cold Ni field remoderator to give bunches with 10^4 e^+, 10 meV energy spread, an ellipsoidal emission spot 0.1 μm high and 10 μm wide and a 1 radian divergence.

However, stray fields are notoriously weak and decrease rapidly with the distance. Therefore, there is a diameter of the vacuum tube for which stray fields are expected to have value on the axis insufficient to disrupt the test via a spreading of the beams. Consequently, the proposed tests is also expected to be resolutory via the use of very low energy positrons as available, e.g., from radioactive sources.

As a matter of fact, the detection in the scintillator of the same clear gravitational deflection due to gravity by *a few* positrons would be sufficient to achieve a final resolution, provided, of course, that these few events can be systematically reproduced.

After all, the reader should compare the above setting with the fact that new particles are nowadays claimed to be discovered at high energy laboratories via the use of extremely few events out of hundreds of millions of events on record for the same test.

The beam would then be expanded to 100 μm$\times$1 cm cross section and a 1 mrad divergence, still at 10 meV. Using a time dependent retarding potential Mills would then lower the energy spread and mean energy to 100 μeV with a 2 μs pulse width. Even assuming a factor of 1,000 loss of particles due to

imperfections in this scheme, Mills' set-up would then have pulses of about 10 positrons that could be launched into the flight tube with high probability of transmissions at energy of 0 to 100 μeV.

The determination of the gravitational force would require many systematic tests. The most significant would be the measurements of the deflection as a function of the time of flight (enhance the velocity v) $\Delta v(e\pm, \pm v)$ for both positrons and elections and for both signs of the velocity relative to the lens on the axis of the tube, $v > 0$ and $v < 0$, the vertical gravitational force on a particle of charge q is

$$F_y \ = \ -m \times g + q \times E_y + q \times v_z \times B_x/c. \tag{2.5.10}$$

The deflection is then given by

$$\Delta y = \int_0^L \int_0^{z'} q \times [E(z'') + v \times B(z'')/c] \times dz'' \times dz'/(m \times v^2) - g \times z^2/2 \times v^2. \tag{2.5.11}$$

In lowest order, Mills neglects the transverse variation in E_y and B_x and writes for the average fields

$$\epsilon \ = \ \frac{1}{L^2} \int_0^L \int_0^{z'} E_y(z'') \times dz'' \times dz', \tag{2.5.12}$$

and

$$\beta \ = \ \frac{1}{L^2} \int_0^L \int_0^{z'} B_x(z'') \times dz'' \times dz'. \tag{2.5.13}$$

Note that these are not simple averages, but the averages of the running averages. They depend on the direction of the velocity. In the approximation that there are not significantly different from simple averages, the average of the four deflection Δy for both positrons and electrons and for both signs of the velocity is independent of ϵ and β and it is given by

$$< \Delta y > = \ (g^+ + g^-) \times \frac{L^2}{v^2}. \tag{2.5.14}$$

where $g^\pm$ refers to the gravitational acceleration of $e^\pm$. Since we also have the velocity dependence of the Δy's, and can manipulate E and B by means of trim adjustments, it will be possible to unravel the gravitational effect from the electromagnetic effect in this experiment.

In summary, the main features proposed by Mills [45] for Santilli's [44] horizontal vacuum tube are that:

1) The tube should be a minimum of 10 m long and 1 m in diameter, although the length of 100 m (as proposed by Santilli [44]) and 0.5 m in diameter is preferable;

2) The tube should contain shields against internal external electric and magnetic fields and internal stray fields. According to Mills [45], this can be accomplished with concentric shells made of Al, double shells of Mu metal, double shells of superconducting Nb and Pb, and a final internal evaporated layer of fine grain of Cu;

3) Use bright pulsed sources of electrons and, separately, positrons, at low temperature by means of phase space manipulation techniques including brightness enhancement;

4) Time of flight and single particle detection should be tested to determine the displacement of a trajectory from the horizontal line as a function of the particle velocity;

5) Comparison of measurements should be done using electrons and positrons traversing the flight tube in both directions.

The *use of electrons and positrons with 25 μeV kinetic energy would yield a vertical displacement of 5 mm at the end of 100 m horizontal flight, namely, a displacement that can be distinguished from displacements caused by stray fields and be visible to the naked eye,* as insisted by Santilli [44].

Mills [45] then concludes by saying that *"... an experiment to measure the gravitational deflection of electrons and positrons in horizontal flight, as suggested by R. M. Santilli, ... is indeed feasible with current technologies.... and should provide a definite resolution to the problem of the passive gravitational field of the positron".*

2.6 SPACETIME LOCOMOTIONS

2.6.1 Introduction

In preceding sections of this monograph we have indicated the far reaching implications of a possible experimental verification of antigravity predicted for antimatter in the field of matter and vice versa, such as a necessary revision of the very theory of antimatter from its classical foundations, a structural revision of any consistent theory of gravitation, a structural revision of any operator formulation of gravitation, and others.

In this section we show that another far reaching implications of the experimental detection of antigravity is the consequential existence of a *Causal Time Machine* [46], that is the capability of moving forward or backward in time without violating the principle of causality, although, as we shall see, this capability is restricted to isoselfdual states (bound states of particles and antiparticles) and *it is not* predicted by the isodual theory to be possible for matter or, separately, for antimatter.

It should be stressed that the Causal Time Machine here considered is a *mathematical model,* rather than an actual machine. Nevertheless, science has always surpassed predictions. Therefore, we are confident that, as it has been the cases

for other predictions, one the Causal Time Machine is theoretically predicted, science may indeed permits its actual construction, of course, in due time.

As we shall see, once a Causal Time Machine has been identified, the transition to a causal SpaceTime Machine with the addition of motion in space is direct and immediate.

2.6.2 Causal Time Machine

As clear from the preceding analysis, *antigravity is only possible if antiparticles in general and the gravitational field of antimatter, in particular, evolve backward in time.* A time machine is then a mere consequence.

Causality is readily verified by the isodual theory of antimatter for various reasons. Firstly, *backward time evolution measured with a negative unit of time is as causal as forward time evolution measured with a positive unit of time.* Moreover, *isoselfdual states evolve according to the time of the gravitational field in which they are immersed.* As a result, no violation of causality is conceivably possible for isoselfdual states.

Needless to say, none of these causality conditions are possible for conventional treatments of antimatter.

The reader should be aware that we are referring here to a "Time Machine," that is, to motion forward and backward in time without space displacement (Figure 2.10). The "Space-Time Machine" (that is, including motion in space as well as in time), requires the isodualities as well as isotopies of conventional geometries studied in Chapter 3 and it will be studied in the next section.

The inability to have motion backward in time can be traced back to the very foundations of special relativity, in particular, to the basic time-like interval between two points 1 and 2 in Minkowski space as a condition to verify causality

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - (t_1 - t_2)^2 \times c^2 < 0. \tag{2.6.1}$$

defined on the field of real numbers $R(n, \times, I)$, $I = Diag.(1, 1, 1, 1)$.

The inability to achieve motion backward in time then prevents the achievement of a *closed loop* in the forward light cone, thus including motion in space and time, since said loop would necessarily require motion backward in time.

Consider now an isoselfdual state, such as the positronium or the π° meson (Section 2.3.14). Its characteristics have the sign of the unit of the observer, that is, positive time and energy for matter observers and negative times and negative energies for antimatter observers. Then a closed loop can be achieved as follows [46]:

1) With reference to Figure 2.10, expose first the isoselfdual state to a field of matter, in which case it evolved forward in time from a point at time t_1 to a point at a later time t_2 where the spacetime coordinates verify the time-like invariant (2.6.1) with $t_2 > t_1$;

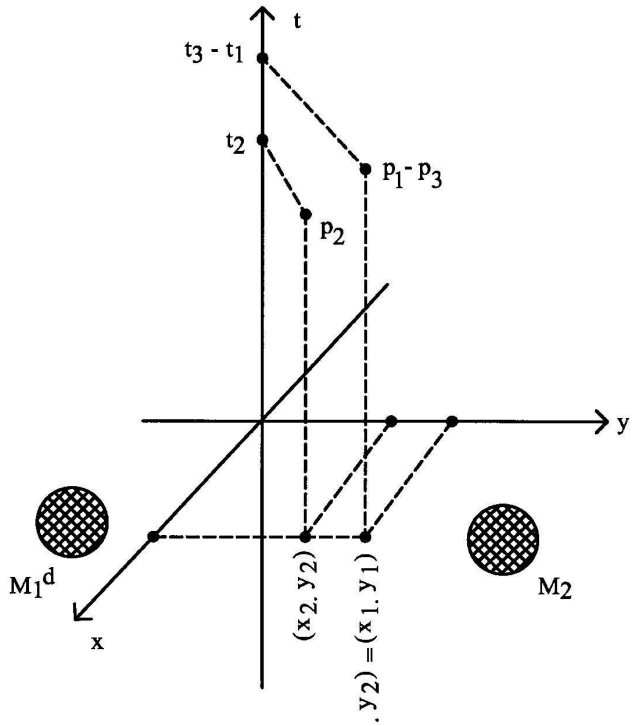

Figure 2.10. A schematic view of the simplest possible version of the "Time Machine" proposed in Ref. [46] via an isoselfdual state such as the positronium or the π° meson that are predicted to move forward (backward) in time when immersed in the gravitational field of matter (antimatter). The Time Machine then follows by a judicious immersion of the same isoselfdual state first in the fields of matter and then in that of antimatter. No causality violation is possible because of the time evolution for isoselfdual states is that of the field in which they are immersed in.

2) Subsequently, expose the same isoselfdual state to a field of antimatter in which case, with the appropriate intensity of the field and the duration of the exposure, the state moves backward in time from time t_2 to the original time t_1, where the spacetime coordinates still verify invariant (2.6.1) with $t_2 < t_1$ although in its isodual form.

We, therefore, have the following:

PREDICTION 2.6.1 [46]: Isoselfdual states can have causal motions forward and backward in time, thus performing causal closed loops in the forward light cone.

Note that the above causal Time Machine implies gravitational *attraction* for both fields of matter and antimatter, owing to the use of an isoselfdual test

particle, in which case we only have the reversal of the sign of time and related unit.

Note also that the use of a particle or, separately, of an antiparticle would violate causality.

Numerous time machines exist in the literature. However, none of them appears to verify causality and, as such, they are ignored.

Other time machines are based on exiting our spacetime, entering into a mathematical space (e.g., of complex unitary character), and then returning into our spacetime to complete the loop.

Other attempts have been based on quantum tunnelling effects and other means.

By comparison, the Causal Time Machine proposed in Ref. [46] achieves a closed loop at the classical level without exiting the forward light cone and verifying causality.[10]

2.6.3 Isogeometric Propulsion

All means of locomotion developed by mankind to date, from prehistoric times all the way to current interplanetary missions, have been based on *Newtonian propulsions*, that is, propulsions all based on *Newton's principle of action and reaction.*

As an example, human walking is permitted by the action generated by leg muscles and the reaction caused by the resistance of the feet on the grounds. The same action and reaction is also the origin of *all* other available locomotions, including contemporary automobiles or rockets used for interplanetary missions.

Following the identification of the principle of propulsion, the next central issue is the displacement that is evidently characterized by the *Euclidean distance.* We are here referring to the conventional Euclidean space $E(r, \delta, R)$ over the reals R with familiar coordinates $r = (x, y, z) \times I$, metric $\delta = Diag.(1, 1, 1)$, units for the three axes $I = I_{3\times 3} = Diag(1\ \text{cm},\ 1\ \text{cm},\ 1\ \text{cm})$ hereon used in their dimensionless form $I = Diag.(1, 1, 1)$, and Euclidean distance that we write in the isoinvariant form

$$D^2 = r^2 \times I = (x^2 + y^2 + z^2) \times I \in R. \tag{2.6.2}$$

The *geometric locomotion* can be defined as the *covering of distances via the alteration (also called deformation) of the Euclidean geometry without any use of action and reaction* . The *only* possible realization of such a geometric locomotion that avoid the theorems of catastrophic inconsistencies of Section 1.5, as well as achieves compatibility with our sensory perception (see below), is the *isogeometric*

[10]The indication by colleagues of other versions of the spacetime machine with a proved verification of causality without existing from our spacetime would be appreciated.

locomotion [5b] namely, that permitted by the *Euclid-Santilli isogeometry* and relative *isodistance.*

We are here referring to the Euclid-Santilli isospace (Section 3.2) $\hat{E}(\hat{r}, \hat{\delta}, \hat{R})$ over the isoreals $\hat{R}$ with isocoordinates $\hat{r} = (x, y, z) \times \hat{I}$, metric $\hat{\delta} = \hat{T}_{3\times 3} \times \delta$, isounits for the three isoaxes

$$\hat{I} = \hat{I}_{3\times 3} = Diag(n_1^2\,\text{cm},\ n_2^2\,\text{cm},\ n_3^2\,\text{cm}) = 1/\hat{T}_{3\times 3} > 0 \tag{2.6.3}$$

that will also be used hereon in the dimensionless form

$$\hat{I} = Diag.(n_1^2,\ n_2^2,\ n_3^2), \tag{2.6.4}$$

and *isodistance* that we write in the isoinvariant form[11]

$$\hat{D}^{\hat{2}} = \hat{r}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2) \times \hat{I} \in \hat{R}, \tag{2.6.5}$$

in which case the deformation of the geometry is called *geometric mutation.*[12]

It is evident that $\hat{D}$ can be bigger equal or smaller than D. Consequently, the isogeometric locomotion occurs when $\hat{D} < D$, as in the example below

$$\hat{I} = Diag.(n_1^2, 1, 1) \ll I = Diag.(1, 1, 1), \quad \hat{T} \gg I, \tag{2.6.6a}$$

$$\hat{D}^{\hat{2}} = (x^2/n_1^2 + y^2 + z^2) \ll D^2 = (x^2 + y^2 + z^2). \tag{2.6.6b}$$

The understanding of the above locomotion requires a knowledge of the *isobox* of Section 3.2. Consider such an isobox and assume that it is equipped with isogeometric locomotion. In this case, there is no displacement at all that can be detected by the internal observer. However, the external observer detects a displacement of the isobox the amount $x^2 - x^2/n_1^2$.

This type of locomotion is new because it is causal, invariant and occurs without any use of the principle of action and reaction and it is geometric because it occurs via the sole local mutation of the geometry.

The extension to the *Causal Spacetime Machine,* or *spacetime isogeometric locomotion* is intriguing, and can be formulated via the *Minkowski-Santilli isospace* of Section 3.2 with four-isodistance

$$\hat{D}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2 - c^2 \times t^2/n_4^2) \times \hat{I} \in \hat{R}, \tag{2.6.7}$$

[11] By "isoinvariance" we means invariance under conventional space or spacetime symmetries plus the isotopic invariance.

[12] According to the contemporary terminology, "deformations" are alterations of the original structure although referred to the original field. As such they are afflicted by the catastrophic inconsistencies of Section 1.5. The term "mutation", first introduced by Santilli in Ref. [49] of 1967, is today referred to an alteration of the original structure under the condition of preserving the original axioms, thus requiring the formulation on isospaces over isofields that avoid said theorems of catastrophic inconsistency.

Figure 2.11. An artistic rendering of the "Space Time Machine", namely, the "mathematical" prediction of traveling in space and time permitted by the isodual theory of antimatter. The main assumption is that the aether (empty space) is a universal medium characterized by a very high density of positive and negative energies that can coexist because existing in distinct, mutually isodual spacetimes. Virtually arbitrary trajectories and speeds for isoselfdual states (only) are then predicted from the capability of extracting from the aether very high densities of positive and negative energies in the needed sequence. Discontinuous trajectories do not violate the law of inertia, speeds much bigger than the speed of light in vacuum, and similarly apparently anomalous events, do not violate special relativity because the locomotion is caused by the change of the local geometry and not by conventional Newtonian motions.

where $n_4 > 0$.

The main implications in this case is the emergence of the additional *time mutation* as expected to occur jointly with any *space mutation.* In turn, this implies that the *isotime* $\hat{t} = t/n_4$ (that is, the internal time) can be bigger equal or smaller than the time t (that of the external observer).

More specifically, from the preservation of the original trace of the metric, *isorelativity predicts that the mutations of space and time are inversely promotional to each others.* Therefore, jointly with the motion ahead in space there is a motion backward in time and vice versa.

Consequently, the external observer sees the object moving with his naked eye, and believes that the object evolves in his own time, while in reality the object could evolve far in the past. Alternatively, we can say that the inspection of an astrophysical object with a telescope, by no means, implies that said object evolves with our own time because it could evolve with a time dramatically different than that after adjustments due to the travel time of light because, again, light cannot carry any information on the actual time of its source.

To further clarify this important point, *light cannot possibly carry information on the time of its source because light propagates at the speed c at which there is no time evolution.*

As a concrete example, one of the consequences of interior gravitational problems treated via Santilli's isorelativity (see Section 3.5) is that *the time of interior gravitational problems, $\hat{t} = t/n_4$, depends on the interior density n_4^2, rather than the inertial mass, thus varying for astrophysical bodies with different densities.*

This implies that if two identical watches are originally synchronized with each other on Earth, and then placed in the interior gravitational field of astrophysical bodies with different densities, they will no longer be synchronized, thus evolving with different times, even though light may continue to provide the information needed for their intercommunication.

In particular, *the time evolution of astrophysical bodies slows down with the increase of the density*,

$$\hat{t}_1 < \hat{t}_2, \quad n_{41}^2 > n_{42}^2. \tag{2.6.8}$$

It should also be noted that the above effect has no connection with similar Riemannian predictions because it is structurally dependent on the *change of the units,* rather than geometric features.

A prediction of isospecial relativity is that the bigger the density, the slower the time evolution. Thus, a watch in the interior of Jupiter is predicted to move *slower* than its twin on Earth under the assumption that the density of Jupiter (being a gaseous body) is significantly smaller than that of Earth (that can be assumed to be solid for these aspects).

As stressed in Section 2.6.1, *the above spacetime machine is a purely mathematical model*. To render it a reality, there is the need to identify the *isogeometric propulsion*, namely a source for the geometric mutations of type (2.6.5).

Needless to say, the above problem cannot be quantitatively treated on grounds of available scientific knowledge. However, to stimulate the imagination of readers with young minds of any age, a speculation on the possible mechanism of propulsion should be here voiced.

The only source of geometric mutation conceivable today is the availability of very large energies concentrated in very small regions of space, such as energies of the order of 10^{30} ergs/cm^3. Under these conditions, isorelativity does indeed predict isogeometric locomotion because these values of energy density generate very large values of isounits $\hat{I}$, with very small values of the isotopic element $\hat{T}$, resulting in isogeometric locomotions precisely of type (2.6.5).

The only possible source of energy densities of such extreme value is empty space. In fact, according to current views, space is a superposition of positive and negative energies in equal amounts each having extreme densities precisely of the magnitude needed for isogeometric locomotion.

The speculation that should not be omitted in this section is therefore that, one day in the future, the advancement of science will indeed allow to extract from space at will all needed amounts of both positive and negative energy densities.

In the event such an extraction becomes possible in a directional way, a spaceship would be able to perform all desired types of trajectories, including trajectories with sharp discontinuities (instantaneous 90 degrees turns), instantaneous accelerations, and the like without any violation of the law of inertia because, as indicated earlier, the spaceship perceives no motion at all. It is the geometry in its surroundings that has changed.

Moreover, such a spaceship would be able to cover interstellar distances in a few of our minutes, although arriving at destination way back in the time evolution of the reached system.

Science has always surpassed science fiction and always will, because there is no limit to the advancement of scientific knowledge. On this ground it is, therefore, easy to predict that, yes, one day mankind will indeed be able to reach far away stars in minutes.

It is only hoped that, when that giant step for mankind is achieved, the theory that first achieved its quantitative and invariant prediction, Santilli isorelativity, will be remembered.

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Postscript

In the history of science some basic advances in physics have been preceded by basic advances in mathematics, such as Newtons invention of calculus and general relativity relying on Riemannian geometry. In the case of quantum mechanics the scientific revolution presupposed the earlier invention of complex numbers. With new numbers and more powerful mathematics to its disposition, physics could be lifted to explain broader and more complex domains of physical reality.

The recent and ongoing revolution of physics, initiated by Prof. Ruggero Maria Santilli, lifting the discipline from quantum mechanics to hadronic mechanics, is consistent with this pattern, but in a more far-reaching and radical way than earlier liftings of physics made possible from extensions of mathematics.

Santilli realized at an early stage that basic advances in physics required invention of new classes of numbers and more adequate and powerful mathematics stemming from this. His efforts to develop such expansions of mathematics started already in 1967, and this enterprise went on for four decades. Its basic novelties, architecture and fruits are presented in the present volume. During this period a few dozen professional mathematicians world wide have made more or less significant contributions to fill in the new Santilli fields of mathematics, but the honor of discovering these vast new continents and work out their basic topology is Santillis and his alone. These new fields initiated by Santilli made possible realization of so-called Lie-admissible physics. For this achievement Santilli in 1990 received the honor from Estonia Academy of Science of being appointed as mathematician number seven after world war two considered a landmark in the history of algebra.

With regard to Sophus Lie it may be of some interest to note that the Norwegian examiners of his groundbreaking doctoral thesis in 1871 were not able to grasp his work, due to its high degree of novelty and unfamiliarity. However, due to Lie already being highly esteemed among influential contemporary mathematicians at the continent, it was not an option to dismiss his thesis. As in other disciplines, highly acknowledged after Thomas Kuhns publication of The structure of scientific revolutions in 1962, sufficiently novel mathematics implies some paradigmatic challenge. Therefore, it is not strange that some mathematicians and physicists have experienced difficulties taking the paradigmatic leap necessary to grasp the basics of hadronic mathematics or to acknowledge its far-reaching implications. Such a challenge is more demanding when scientific novelty

implies a reconfiguration of conventional basic notions in the discipline. This is, as Kuhn noted, typically easier for younger and more emergent scientific minds.

Until Santilli the number 1 was silently taken for granted as the primary unit of mathematics. However, as noted by mathematical physicist Peter Rowlands at University of Liverpool, the number 1 is already loaded with assumptions, that can be worked out from a lifted and broader mathematical framework. A partial and rough analogy might be linguistics where it is obvious that a universal science of language must be worked out from a level of abstraction that is higher than having to assume the word for mother to be the first word.

Santilli detrivialized the choice of the unit, and invented isomathematics where the crux was the lifting of the conventional multiplicative unit (i.e. conservation of its topological properties) to a matrix isounit with additional arbitrary functional dependence on other needed variables. Then the conventional unit could be described as a projection and deformation from the isounit by the link provided by the so-called isotopic element inverse of the isounit. This represented the creation of a new branch of mathematics sophisticated and flexible enough to treat systems entailing sub-systems with different units, i.e. more complex systems of nature.

Isomathematics proved necessary for the lifting of quantum mechanics to hadronic mechanics. With this new mathematics it was possible to describe extended particles and abandon the point particle simplification of quantum mechanics. This proved highly successful in explaining the strong force by leaving behind the non-linear complexities involved in quantum mechanics struggle to describe the relation between the three baryon quarks in the proton. Isomathematics also provided the mathematical means to explain the neutron as a bound state of a proton and an electron as suggested by Rutherford. By means of isomathematics Santilli was also able to discover the fifth force of nature (in cooperation with Professor Animalu), the contact force inducing total overlap between the wave packets of the two touching electrons constituting the isoelectron. This was the key to understanding hadronic superconductivity which also can take place in fluids and gases, i.e. at really high temperatures. These advances from hadronic mechanics led to a corresponding lifting of quantum chemistry to hadronic chemistry and the discovery of the new chemical species of magnecules with non-valence bounds. Powerful industrial-ecological technology exploiting these theoretical insights was invented by Santilli himself from 1998 on.

Thus, the development of hadronic mathematics by Santilli was not only motivated by making advances in mathematics per se, but also of its potential to facilitate basic advances in physics and beyond. These advances have been shown to be highly successful already. Without the preceding advances in mathematics, the new hadronic technology would not have been around. The mere existence of this technology is sufficient to demonstrate the significance of hadronic math-

ematics. It is interesting to note that the directing of creative mathematics into this path was initiated by a mathematical physicist, not by a pure mathematician. In general this may indicate the particular potential for mathematical advances by relating the mathematics to unsolved basic problems in other disciplines, as well as to real life challenges.

In the history of mathematics it is not so easy to find parallels to the achievements made by Santilli, due to hadronic mathematics representing a radical and general lifting, relegating the previous mathematics to a subclass of isomathematics, in some analogy to taking the step from the Earth to the solar system. However, the universe also includes other solar systems as well as galaxies.

In addition to isonumbers Santilli invented the new and broader class of genonumbers with the possibility of asymmetric genounits for forward vs. backward genofields, and designed to describe and explain irreversibility, characteristic for more complex systems of nature. Quantum mechanical approaches to biological systems never achieved appreciable success, mainly due to being restricted by a basic symmetry and hence reversibility in connected mathematical axioms. It represented an outstanding achievement of theoretical biology when Chris Illert in the mid-1990s was able to find the universal algorithm for growth of sea shells by applying hadronic geometry. Such an achievement was argued not to be possible for more restricted hyperdimensional geometries as for example the Riemannian. This specialist study in conchology was the first striking illustration of the potency as well as necessity of iso- and genomathematics to explain irreversible systems in biology.

Following the lifting from isomathematics to genomathematics, Santilli also established one further lifting, by inventing the new and broader class of hyperstructural numbers or Santilli hypernumbers. Such hypernumbers are multivalued and suitable to describe and explain even more complex systems of nature than possible with genonumbers. Due to its irreversible multivalued structure hypermathematics seems highly promising for specialist advances in fields such as genetics, memetics and communication theory. By the lifting to hypermathematics hadronic mathematics as a whole may be interpreted as a remarkable step forward in the history of mathematics, in the sense of providing the essential and sufficiently advanced and adequate tools for mathematics to expand into disciplines such as anthropology, psychology and sociology. In this way it is possible to imagine some significant bridging between the two cultures of science: the hard and the soft disciplines, and thus amplifying a tendency already represented to some extent by complexity science.

The conventional view of natural scientists has been to regard mathematics as a convenient bag of tools to be applied for their specific purposes. Considering the architecture of hadronic mathematics, this appears more as only half of the truth or one side of the coin. Besides representing powerful new tools to study

nature, hadronic mathematics also manifests with a more intimate and inherent connection to physics (and other disciplines), as well as to Nature itself. In this regard hadronic geometry may be of special interest as an illustration:

Isogeometry provided the new notions of a supra-Euclidean isospace as well as its anti-isomorphic isodual space, and the mathematics to describe projections and deformations of geometrical relations from isospace and its isodual into Euclidean space. However, these appear as more than mere mathematical constructs. Illert showed that the universal growth pattern of sea shells could be found only by looking for it as a trajectory in a hidden isospace, a trajectory which is projected into Euclidean space and thereby manifest as the deformed growth patterns humans observe by their senses. Further, the growth pattern of a certain class of sea shells (with bifurcations) could only be understood from the addition and recognition of four new, non-trivial time categories (predicted to be discovered by hadronic mechanics) which manifest as information jumps back and forth in Euclidean space. With regard to sea shell growth, one of this non-trivial time flows could only be explained as a projection from isodual spacetime. This result was consistent with the physics of hadronic mechanics, analyzing masses at both operator and classical level from considering matter and anti-matter (as well as positive and negative energy) to exist on an equal footing in our universe as a whole and hence with total mass (as well as energy and time) cancelling out as zero for the total universe. To establish a basic physical comprehension of Euclidean space constituted as a balanced combination of matter and antimatter, it was required to develop new mathematics with isonumbers and isodual numbers basically mirroring each other. Later, corresponding anti-isomorphies were achieved for genonumbers and hypernumbers with their respective isoduals.

Thus, there is a striking and intimate correspondence between the isodual architecture of hadronic mathematics and the isodual architecture of hadronic mechanics (as well as of hadronic chemistry and hadronic biology). Considering this, one might claim that the Santilli inventions of new number fields in mathematics represent more than mere inventions or constructs, namely discoveries and reconstructions of an ontological architecture being for real also outside the formal landscapes created by the imagination of mathematics and logic. This opens new horizons for treating profound issues in cosmology and ontology.

One might say that with the rise of hadronic mathematics the line between mathematics and other disciplines has turned more blurred or dotted. In some respect this represents a revisit to the Pythagorean and Platonic foundations of mathematics in the birth of western civilization. Hadronic mathematics has provided much new food for thought and further explorations for philosophers of science and mathematics.

If our civilization is to survive despite its current problems, it seems reasonable to expect Santilli to be honored in future history books not only as a giant in

the general history of science, but also in the specific history of mathematics. Hadronic mathematics provided the necessary fuel for rising scientific revolutions in other hadronic sciences. This is mathematics that matters for the future of our world, and hopefully Santillis extraordinary contributions to mathematics will catch fire among talented and ambitious young mathematicians for further advances to be made. The present mellowed volume ought to serve as an excellent appetizer in this regard.

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HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

Volume III:

Iso-, Geno-, Hyper-Formulations for Matter and Their Isoduals for Antimatter

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Index
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This volume is dedicated to

Professor Alvyn van der Merwe

in recognition of his Editorship of Foundations of Physics in disrespect of organized accademic, financial and ethnic interests in science that, otherwise, would have suppressed the birth of undesired advances in human knowledge.

Contents

Foreword

Mathematics is a subject which possibly finds itself in a unique position in academia in that it is viewed as both an Art and a Science. Indeed, in different universities, graduates in mathematics may receive Bachelor Degrees in Arts or Sciences. This probably reflects the dual nature of the subject. On the one hand, it may be studied as a subject in its own right. In this sense, its own beauty is there for all to behold; some as serene as da Vinci's "Madonna of the Rocks", other as powerful and majestic as Michelangelo's glorious ceiling of the Sistine Chapel, yet more bringing to mind the impressionist brilliance of Monet's Water Lily series. It is this latter example, with the impressionists interest in light, that links up with the alternative view of mathematics; that view which sees mathematics as the language of science, of physics in particular since physics is that area of science at the very hub of all scientific endeavour, all other branches being dependent on it to some degree. In this guise, however, mathematics is really a tool and any results obtained are of interest only if they relate to what is found in the real world; if results predict some effect, that prediction must be verified by observation and/or experiment. Again, it may be remembered that physics is really a collection of related theories. These theories are all manmade and, as such, are incomplete and imperfect. This is where the work of Ruggero Santilli enters the scientific arena.

Although "conventional wisdom" dictates otherwise, both the widely accepted theories of relativity and quantum mechanics, particularly quantum mechanics, are incomplete. The qualms surrounding both have been muted but possibly more has emerged concerning the inadequacies of quantum mechanics because of the people raising them. Notably, although it is not publicly stated too frequently, Einstein had grave doubts about various aspects of quantum mechanics. Much of the worry has revolved around the role of the observer and over the question of whether quantum mechanics is an objective theory or not. One notable contributor to the debate has been that eminent philosopher of science, Karl Popper. As discussed in my book, "Exploding a Myth", Popper preferred to refer to the experimentalist rather than observer, and expressed the view that that person played the same role in quantum mechanics as in classical mechanics. He felt, therefore, that such a person was there to test the theory. This is totally opposed to the Copenhagen Interpretation which claims that "objective reality has evaporated" and "quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness, of particles". Popper points

out that, over the years, many eminent physicists have switched allegiance from the pro-Copenhagen view. In some ways, the most important of these people was David Bohm, a greatly respected thinker on scientific matters who wrote a book presenting the Copenhagen view of quantum mechanics in minute detail. However, later, apparently under Einstein's influence, he reached the conclusion that his previous view had been in error and also declared the total falsity of the constantly repeated dogma that the quantum theory is complete. It was, of course, this very question of whether or not quantum mechanics is complete which formed the basis of the disagreement between Einstein and Bohr; Einstein stating "No", Bohr "Yes".

However, where does Popper fit into anything to do with Hadronic Mechanics? Quite simply, it was Karl Popper who first drew public attention to the thoughts and ideas of Ruggero Santilli. Popper reflected on, amongst other things, Chadwick's neutron. He noted that it could be viewed, and indeed was interpreted originally, as being composed of a proton and an electron. However, again as he notes, orthodox quantum mechanics offered no viable explanation for such a structure. Hence, in time, it became accepted as a new particle. Popper then noted that, around his (Popper's) time of writing, Santilli had produced an article in which the "first structure model of the neutron" was revived by "resolving the technical difficulties which had led, historically, to the abandonment of the model". It is noted that Santilli felt the difficulties were all associated with the assumption that quantum mechanics applied within the neutron and disappeared when a generalised mechanics is used. Later, Popper goes on to claim Santilli to belong to a new generation of scientists which seemed to him to move on a different path. Popper identifies quite clearly how, in his approach, Santilli distinguishes the region of the arena of incontrovertible applicability of quantum mechanics from nuclear mechanics and hadronics. He notes also his most fascinating arguments in support of the view that quantum mechanics should not, without new tests, be regarded as valid in nuclear and hadronic mechanics.

Ruggero Santilli has devoted his life to examining the possibility of extending the theories of quantum mechanics and relativity so that the new more general theories will apply in situations previously excluded from them. To do this, he has had to go back to the very foundations and develop new mathematics and new mathematical techniques. Only after these new tools were developed was he able to realistically examine the physical situations which originally provoked this lifetime's work. The actual science is his, and his alone, but, as with the realization of all great endeavours, he has not been alone. The support and encouragement he has received from his wife Carla cannot be exaggerated. In truth, the scientific achievements of Ruggero Santiili may be seen, in one light, as the results of a team effort; a team composed of Ruggero himself and Carla Gandiglio in Santilli. The theoretical foundations of the entire work are contained

in this volume; a volume which should be studied rigorously and with a truly open mind by the scientific community at large. This volume contains work which might be thought almost artistic in nature and is that part of the whole possessing the beauty so beloved of mathematicians and great artists. However, the scientific community should reserve its final judgement until it has had a chance to view the experimental and practical evidence which may be produced later in support of this elegant new theoretical framework.

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September 8, 2007

Preface

In Volume I we have identified and denounced scientific imbalance of historical proportions caused by organized academic, financial and ethnic interests on Einsteinian theories via the abuse of academic credibility and public funds to impose the validity of time reversal invariant doctrines for the treatment of irreversible events, including energy releasing processes.

A primary scope of this Volume III is the presentation of the lifelong research by the author on the generalization (called *lifting*) of Einstein's special and general relativities, quantum mechanics and quantum chemistry into forms that are structurally irreversible in time, that is, irreversible for all possible Lagrangians and Hamiltonians, since the latter are known as being all reversible.

It is evident that a task of this type cannot be achieved without the prior lifting of the entire mathematics used in the 20-th century physics, since the latter is all structurally reversible. In turn, as soon as this problem is addressed, the transition from the 20-th century reversible mathematics to its irreversible covering soon emerges as being excessive, particularly for non-mathematically oriented readers.

The latter occurrence has suggested the presentation of a progressive transition from the 20-th century mathematics to a first generalization, today known as *Santilli isomathematics,* where the prefix "iso" is intended in the Greek sense of being "axiom preserving"; the latter mathematics is then lifted into a single-valued structurally irreversible form known as *Santilli genomathematics,* where the prefix "geno" is intended in the Greek meaning of inducing new axioms; and, finally, the latter is lifted into the most general known mathematics, that of multi-valued irreversible type known as *Santilli's hypermathematics.* The corresponding mathematics for antimatter are characterized by the isodual map of Volume II.

Once the above mathematics are known, the construction of the corresponding broader relativities is elementary, yielding formulations today known as *Santilli iso-, geno-, and hyper-relativities* for matter and their isoduals for antimatter. The construction of the underlying iso-, geno-, and hyper-mechanics for matter and their isoduals for antimatter is equally elementary.

This yields a progression of formulations each one being a covering of the preceding one, for the quantitative, axiomatically consistent and invariant representation of matter and of antimatter in conditions of progressively increasing complexity.

As we shall see, besides resolving the historical imbalance on irreversibility, iso-, geno-, and hyper-formulations allow the resolution of numerous additional scientific imbalances of the 20-th century caused by adapting nature to preferred theories for evident personal gains, rather than adapting the theories to new physical reality, as done in these volumes.

Ruggero Maria Santilli
January 19, 2008

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In faith

Ruggero Maria Santilli

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Tarpon Springs, Florida, U. S. A.

October 11, 2007

[1] International Committee on Scientific Ethics and Accountability
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Chapter 3

LIE-ISOTOPIC BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

3.1 INTRODUCTION

3.1.1 Conceptual Foundations

As recalled in Chapter 1, the systems generally considered in the 20-th century are the conventional *exterior dynamical systems,* consisting of closed-isolated and reversible systems of constituents approximated as being point-like while moving in vacuum under sole action-at-a-distance potential interactions, as typically represented by planetary and atomic systems.

More technically, we can say that *exterior dynamical systems are characterized by the exact invariance of the Galilean symmetry for the nonrelativistic case and Poincaré symmetry for relativistic treatments,* with the consequential verification of the well known ten total conservation laws.

In this chapter we study the more general *interior dynamical systems of extended particles* and, separately, of *extended antiparticles,* consisting of systems that are also closed-isolated, thus verifying the same ten total conservation laws of the exterior systems, yet admit additional internal force of nonlocal-integral and nonpotential type due to actual contact and/or mutual penetration of particles, as it is the case for the structure of planets at the classical level (see Figure 3.1), and the structure of hadrons, nuclei, stars, and other systems at the operator level (see Figure 3.2).

To avoid excessive complexity, the systems considered in this chapter will be assumed to be *reversible,* that is, invariant under time reversal. The open-irreversible extension of the systems will be studied in the next chapter.

The most important methodological differences between exterior and interior systems are the following:

1) Exterior systems are completely represented with the knowledge of only *one* quantity, the Hamiltonian, while the representation of interior systems requires

the knowledge of the Hamiltonian for the potential forces, plus additional quantities for the representation of nonpotential forces, as done in the *true Lagrange and Hamilton equations,* those with external terms,

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = F_{ak}(t,r,v), \tag{3.1.1a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k} + F_{ak}(t,r,p), \tag{3.1.1b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v_{ak} \times v_a^k - V(t,r,v), \tag{3.1.1c}$$

$$H = \Sigma_a \frac{p_{ak} \times p_{ak}}{2 \times m_a} + V(t,r,p), \tag{3.1.1d}$$

$$V = \Sigma_a U(t,r)_{ak} \times v_a^k + U_o(t,r), \tag{3.1.1e}$$

$$F(t,r,v) = F(t,r,p/m), \tag{3.1.1f}$$

$$a = 1,2,3,\ldots,N; \;\; k = 1,2,3.$$

Consequently, by their very conception, interior systems are structurally beyond the representational capability of classical and quantum Hamiltonian mechanics, in favor of covering disciplines.

2) Exterior systems are of *Keplerian type*, while interior systems are not, since they do not admit a Keplerian center (see, again, Figures 3.1 and 3.2). Consequently, also by their very conception, interior systems cannot be characterized by the Galilean and Poincaré symmetries in favor of covering symmetries.

3) Exterior systems are local-differential, that is, they describe a finite set of isolated points, thus being fully treatable with the mathematics of the 20-th century, beginning with conventional local-differential topologies. By contrast, interior systems are nonlocal-integral, that is, they admit internal interactions over finite surfaces or volumes that cannot be consistently reduced to a finite set of isolated points. Consequently, interior systems cannot be consistently treated via the mathematics of classical and quantum Hamiltonian mechanics in favor of a basically new mathematics.

4) The time evolution of the Hamiltonian treatment of exterior systems characterizes a *canonical transformation* at the classical level, and a *unitary transformation* at the operator level, that we shall write in the unified form

$$U \times U^\dagger = U^\dagger \times U = I, \tag{3.1.2}$$

where $\times$ represents the usual (associative) multiplication.[1] By contrast, the time evolution of interior systems, being non-Hamiltonian, characterizes *noncanonical transformations* at the classical level and *nonunitary transformations* at the

[1]Since we shall use several types of multiplications, to avoid confusions, it is essential to identify the assumed multiplication in any mathematical treatment.

Figure 3.1. A view of Jupiter, a most representative interior dynamical system, where one can see with a telescope the dramatic differences with exterior systems, such as internal exchanges of linear and angular momentum always in such a way to verify total conservation laws. As repeatedly stated in the literature on hadronic mechanics, the structure of Jupiter has been assumed as fundamental for the construction of new structure models of hadrons, nuclei and stars, and the development of their new clean energies and fuels.

operator level, that we shall jointly write

$$U \times U^{\dagger} \neq I. \tag{3.1.3}$$

In particular, the noncanonical-nonunitary character is necessary to exit from the class of equivalence of classical and quantum Hamiltonian theories.

5) The *invariance* (rather than "covariance") of exterior systems under the Galilean or Poincaré symmetry has the fundamental implication of preserving the basic units, predicting the same numerical values under the same conditions at different times, and admitting all conditions needed for consistent applications of the theory to experimental measurements. By comparison, the loss of the Galilean and Poincaré invariance, combined with the necessary noncanonical-nonunitary structure of interior systems, activates the *theorems of catastrophic mathematical and physical inconsistencies* studied in Chapter 1 whenever treated with the mathematics of canonical-unitary theories.

In this chapter we report the rather long scientific journey that lead to a mathematically and physically consistent, classical and operator treatment of interior dynamical systems via the *isotopic branch of hadronic mechanics for matter,* and the *isodual isotopic branch for antimatter* including the resolution of all the above problems.

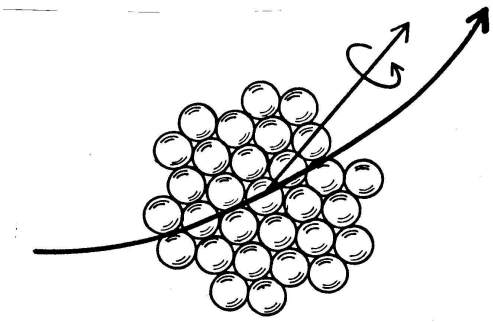

Figure 3.2. A schematic view of nuclei as they are in the physical reality, bound states of extended particles without a Keplerian center, under which conditions quantum mechanics cannot possibly be exact due to the breaking of the fundamental Galilean and Poincaré symmetries in favor of covering theories. As we shall see in this chapter, even though these breakings are small (because nucleons are in conditions of mutual penetration in nuclei of about 10^{-3} parts of their volumes), said breakings permit the prediction and industrial development of new clean energies and fuels that are prohibited by the exact validity of quantum mechanics.

Besides a number of experimental verifications reviewed in this chapter, the achievement of a consistent treatment of interior systems offers basically new structure models of hadrons, nuclei, stars, Cooper pairs, molecules and other interior structures. In turn, these new models permit quantitative studies of new clean energies and fuels already under industrial, let alone scientific development.

Stated in a nutshell, a primary aim of this chapter is to show that the assumption of a final character of quantum mechanics and special relativity beyond the conditions of their original conception (isolated point particles in vacuum) is the primary origin of the current alarming environmental problems.

The reader should be aware that, nowadays, the literature on hadronic mechanics is rather vast, having surpassed the mark of 15,000 pages of published research. As such, to avoid a prohibitive length, *the presentation in this chapter is restricted to the outline of the origination* of each topic and of the *most important developments.* Scholars interested in a comprehensive list of literature are suggested to consult the quoted references as well as those of Chapter 1.

Also to avoid a prohibitive length, the presentation of this chapter is restricted to studies of direct relevance for hadronic mechanics, namely, research fundamentally dependent on a generalization of the basic unit. The quotation of related studies not fundamentally dependent on the generalization of the basic unit cannot be reviewed for brevity.

3.1.2 Closed Non-Hamiltonian Systems

The first step in the study of hadronic mechanics is the dispelling of the belief that nonpotential forces, being nonconservative, do not permit total conservation laws, namely, that the external terms in the analytic equations (3.1.1) solely applies for open-nonconservative systems, such as an extended object moving within a resistive medium considered as external.

This belief was disproved, apparently for the first time, by Santilli in monographs [1,2]. Ref. [1] presented a comprehensive treatment of the integrability conditions for the existence of a potential or a Hamiltonian, *Helmholtz's conditions of variational selfadjointness,* according to which the total force is divided into the following two components

$$F(t,r,p,\ldots) = F^{SA}(t,r,p) + F^{NSA}(t,r,p,\ldots), \tag{3.1.4}$$

where the selfadjoint(SA) component F^{SA} admits a potential and the nonselfadjoint (NSA) component F^{NSA} does not.

We should also recall for clarity that, to be Newtonian as currently understood, a force should solely depend on time t, coordinates r and velocity $v = dr/dt$ or momenta $p = m \times v$, $F = F(t,r,v)$. Consequently, forces depending on derivatives of the coordinates of order bigger than the first, such as forces depending on the acceleration $F = F(t,r,v,a)$, $a = dv/dt$, are not generally considered Newtonian forces.

Ref. [2] then presented the broadest possible realization of the conditions of variational selfadjointness via analytic equations derivable from a variational principle, and included the first known identification of *closed non-Hamiltonian systems* (Ref. [2], pages 233–236), namely, systems that violate the integrability conditions for the existence of a Hamiltonian, yet verify all ten total conservation laws of conventional Hamiltonian systems.

Let us begin by recalling the following well known property:

THEOREM 3.1.1: Necessary and sufficient conditions for a system of N particles to be closed, that is, isolated from the rest of the universe, are that the following ten conservation laws are verified along an actual path

$$\frac{dX_i(t,r,p)}{dt} = \frac{\partial X_i}{\partial b^\mu} \times \frac{db^\mu}{dt} + \frac{\partial X_i}{\partial t} = 0, \tag{3.1.5a}$$

$$X_1 = E_{tot} = H = T + V, \tag{3.1.5b}$$

$$(X_2, X_3, X_4) = \mathbf{P}_{tot} = \Sigma_a \mathbf{p}_a, \tag{3.1.5c}$$

$$(X_5, X_6, X_7) = \mathbf{J}_{tot} = \Sigma_a \mathbf{r_a} \wedge \mathbf{p}_a, \tag{3.1.5d}$$

$$(X_8, X_9, X_{10}) = \mathbf{G}_{Tot} = \Sigma_a (m_a \times \mathbf{r}_a - t \times \mathbf{p}_a), \tag{3.1.5e}$$

$$i = 1, 2, 3, \ldots, 10; \quad k = 1, 2, 3; \quad a = 1, 2, 3, \ldots, N.$$

It is also well known that Galilean or Poincaré invariant systems do verify the above conservation laws since the X_i quantities are the generators of the indicated symmetries. However, in this case all acting forces are derivable from a potential and the systems are Hamiltonian.

Assume now the most general possible dynamical systems, those according to the true Lagrange's and Hamilton equations (3.1.1) where the selfadjoint forces are represented with the Lagrangian or the Hamiltonian and the nonselfadjoint forces are external.

DEFINITION 3.1.1 [2]: Closed-isolated non-Hamiltonian systems of particles are systems of $N \geq 2$ particles with potential and nonpotential forces characterized by the following equations of motion

$$\frac{db_a^\mu}{dt} = \begin{pmatrix} dr_a^k/dt \\ dp_{ka}/dt \end{pmatrix} = \begin{pmatrix} p_{ak}/m_a \\ F_{ka}^{SA} + F_{ka}^{NSA} \end{pmatrix}, \tag{3.1.6}$$

verifying all conditions (3.1.5), where the term "non-Hamiltonian" denotes the fact that the systems cannot be entirely represented with the Hamiltonian, thus requiring additional quantities, such as the external terms.

The case $n = 2$ is exceptional, yet it admits solutions, and closed non-Hamiltonian systems with $N = 1$ evidently cannot exist (because a single free particle is always Hamiltonian).

Closed non-Hamiltonian systems can be classified into:

CLASS α: systems for which Eqs. (3.1.5) are first integrals;

CLASS β: systems for which Eqs. (3.1.5) are invariant relations;

CLASS γ: systems for which Eqs. (3.1.5) are subsidiary constraints.

The case of closed non-Hamiltonian systems of antiparticles are defined accordingly.

The study of closed non-Hamiltonian systems of Classes β and γ is rather complex. For the limited scope of this presentation it is sufficient to see that interior systems of Class α exist.

THEOREM 3.1.2 [2]: Necessary and sufficient conditions for the existence of a closed non-Hamiltonian systems of Class α are that the nonselfadjoint forces verify the following conditions:

$$\sum_a \mathbf{F}_a^{NSA} \equiv 0, \tag{3.1.7a}$$

$$\sum_a \mathbf{p}_a \otimes \mathbf{F}_a^{NSA} \equiv 0, \tag{3.1.7b}$$

$$\sum_a \mathbf{r}_a \wedge \mathbf{F}_a^{NSA} \equiv 0. \tag{3.1.7c}$$

Proof. Consider first the case $N > 2$ and assume first for simplicity that $\mathbf{F}_a^{SA} = 0$. Then, the first nine conservation laws are verified when

$$\frac{\partial X_i}{\partial p_{ka}} \times F_{ka}^{NSA} \equiv 0, \tag{3.1.8}$$

in which case the 10-th conservation law, Eq. (3.1.5e), is automatically verified, and this proves the *necessity* of conditions (3.1.7) for $N > 2$.

The sufficiency of the conditions is established by the fact that Eqs. (3.1.7) consist of seven conditions on $3N$ unknown functions F_{ka}^{NSA}. Therefore, a solution always exists for $N \geq 3$.

The case $N = 2$ is special inasmuch as motion occurs in a plane, in which case Eqs. (3.1.7) reduce to *five* conditions on *four* functions $\mathbf{F}_{ka}^{NSA}$, and the system appears to be overdetermined. Nevertheless, solutions always exist because the verification of the first four conditions (3.1.5) automatically implies the verification of the last one, Eqs. (3.1.5e). As shown in Ref. [2], Example 6.3, pages 272–273, a first solution is given by the *non-Newtonian force*

$$\mathbf{F}_1^{NSA} = -\mathbf{F}_2^{NSA} = K \times a = K \times \frac{dv}{dt}, \tag{3.1.9}$$

where K is a constant. Another solution is given by

$$\mathbf{F}_1^{NSA} = -\mathbf{F}_2^{NSA} = M \times \frac{dr}{dt} \times \phi(M \times \dot{r} + V), \quad M = \frac{m_1 \times m_2}{m_1 + m_2}. \tag{3.1.10}$$

Other solutions can be found by the interested reader. The addition of a non-null selfadjoint force leaves the above proof unchanged. **q.e.d.**

The search for other solutions is recommended to readers interested in acquiring a technical knowledge of hadronic mechanics because such solutions are indeed useful for applications. A general solution of Eqs. (3.1.7), as well as of their operator counterpart and of their isodual images for antimatter will be identified later on in this chapter after the identification of the applicable mathematics.

It should be noted that the proof of Theorem 3.1.2 is not necessary because the existence of closed non-Hamiltonian systems is established by visual observations (Figure 3.1). At any rate, the representation of Jupiter's structure via one single function, the Lagrangian or the Hamiltonian, necessarily implies the belief in the perpetual motion within physical media, due to the necessary condition that

constituents move inside Jupiter with conserved energy, linear momentum and angular momentum.

As recalled in Chapter 1, whenever exposed to departures from closed Hamiltonian systems, a widespread posture is the claim that the non-Hamiltonian character of the systems is "illusory" (*sic*) because, when the systems are reduced to their elementary constituents, all nonpotential forces "disappear" (*sic*) and conventional Hamiltonian disciplines are recovered in full.

The political-nonscientific character of the above posture is established by the following property of easy proof by any graduate student in physics:

THEOREM 3.1.3 [3]: A classical non-Hamiltonian system cannot be consistently reduced to a finite number of quantum mechanical point-like particles and, vice-versa, a finite ensemble of quantum mechanical point-like particles cannot consistently characterize a classical non-Hamiltonian system.

The above property establishes that, rather than being "illusory," *nonpotential effect originate at the deepest and most elementary level of nature.* The property also establishes the need for the identification of methods suitable for the invariant treatment of classical and operator non-Hamiltonian systems in such a way to constitute a covering of conventional Hamiltonian treatments.

This chapter is devoted to the mathematical theoretical and experimental study of classical and operator interior system of particles and antiparticles, their experimental verifications and their novel applications.

3.1.3 Need for New Mathematics

By following the main guidelines of hadronic mechanics, we adapt the mathematics to nature, rather than adapting nature to preferred mathematics. For this purpose, we shall seek a mathematics capable of representing the following main features of interior dynamical systems:

1) Points have no dimension and, consequently can only have action-at-a-distance potential interactions. Therefore, the first need for the new mathematics is the representation of the *actual, extended, generally nonspherical shape of the wavepackets and/or of the charge distribution of the particles considered,* that we shall assume in this monograph for simplicity to have the shape of spheroidal ellipsoids with diagonal form

$$Shape_a = Diag.(n_{a1}^2, n_{a2}^2, n_{a3}^2), \quad a = 1, 2, 3, \ldots, N, \tag{3.1.11}$$

with more general non-diagonal expressions not considered for simplicity, where $n_{a1}^2, n_{a2}^2, n_{a3}^2$ represent the semiaxes of the spheroidal ellipsoids assumed as *deviation* from, or normalized with respect to the perfect sphericity

$$n_{a1}^2 = n_{a2}^2 = n_{a3}^2 = 1. \tag{3.1.12}$$

The n's are called *characteristic quantities* of the particles considered. It should be stressed that, contrary to a rather popular belief, *the n-quantities are not parameters because they represent the actual shape as derived from experimental measurements.*

To clarify this important point, by definition a "parameter" can assume any value as derived form the fit of experimental data, while this is not the case for the characteristic quantities here considered. As an example, the use for the n's of value of the order of 10^{-16} cm to represent a proton would have no physical value because the proton charge distribution is a spheroidal ellipsoid of the order of 10^{-13} cm.

2) Once particles are assumed as being extended, there is the consequential need to represent their *density.* This task can be achieved via a fourth set of quantities

$$Density_a = n_{a4}^2, \tag{3.1.13}$$

representing the *deviation* of the density of the particle considered from the density of the vacuum here assumed to be one,

$$n_{Vacuum,4}^2 = 1. \tag{3.1.14}$$

Again, n_4 is not a free parameter because its numerical value is fixed by experimental data. As an example for the case of a hadron of mass m and radius r we have the density

$$n_4^2 = \frac{m \times c^2}{\frac{4}{3} \times \pi \times r^3}, \tag{3.1.15}$$

thus establishing that n_{a4} is not a free parameter capable of assuming.

Predictably, most nonrelativistic studies can be conducted with the sole use of the space components characterizing the shape. Relativistic treatments require the additional use of the density as the forth component, resulting in the general form

$$(Shape - Density)_a = Diag.(n_{a1}^2, n_{a2}^2, n_{a3}^2, n_{a4}^2), \quad a = 1, 2, 3, \ldots, N. \tag{3.1.16}$$

3) Perfectly rigid bodies exist in academic abstractions, but not in the physical reality. Therefore, the next need is for a meaningful representation of the *deformation of shape* as well as *variation of density* that are possible under interior conditions. This is achieved via the appropriate functional dependence of the characteristic quantities on the energy E_a, linear momentum p_a, pressure P and other characteristics, and we shall write

$$n_{ak} = n_{ak}(E, p, P, \ldots), \quad k = 1, 2, 3, 4. \tag{3.1.17}$$

The reader is suggested to meditate a moment on the fact that Lagrangian or Hamiltonian theories simply cannot represent the actual shape and density

of particles. The impossibility of representing deformations of shapes and variations of density are well known, since the pillar of contemporary relativities, the rotational symmetry, is notoriously incompatible with the theory of elasticity.

4) Once particles are represented as they are in the physical reality (extended, nonspherical and deformable), there is the emergence of the following new class of interactions nonexistent for point-particles (for which reason these interactions have been generally ignored throughout the 20-th century), namely, interactions of:

I) *contact type,* that is, due to the actual physical contact of extended particle; consequently, of

II) *zero range type,* since all contacts are dimensionless; consequently of

III) *nonpotential type,* that is, not representable with any possible action-at-a-distance potential; consequently, of

IV) *non-Hamiltonian type,* that is, not representable with any Hamiltonian; consequently, of

V) *noncanonical type* at the classical level and *nonunitary type at the operator level;* as well as of

VI) *nonlinear type,* that is, represented via nonlinear differential equations, such as depending on power of the wavefunction greater than one; and, finally, of

VII) *nonlocal-integral type.* Interactions among point-particles are local-differential, that is, reducible to a finite set of isolated points, while contact interactions among extended particles and/or their wavepackets are, by conception, nonlocal-integral in the sense of being dependent on a finite surface or volume that, as such, cannot be reduced to a finite set of isolated points (see Figure 3.3).

5) Once the above new features of interior systems have been identified, there is the need not only of their mathematical representation, but above all of their *invariant representation* in order to avoid the theorem of catastrophic inconsistencies of Chapter 1.

As an illustration, Coulomb interactions have reached their towering position in the physics of the 20-th century because *the Coulomb potential is invariant under the basic symmetries of physics,* thus predicting the same numerical values under the same conditions at different times with consequentially consistent physical applications. The same occurs for other interactions derivable from a potential (except gravitation represented with curvature as shown in Section 1.4).

Along the same lines, any representation of the extended, nonspherical and deformable character of particles, their densities and their novel nonlinear, nonlocal and nonpotential interactions cannot possibly have physical value unless it is also *invariant,* and not "covariant," again, because the latter would activate the theorems of catastrophic inconsistencies of Chapter 1.

It should be indicated that an extensive search conducted by the author in 1978–1983 in the advanced libraries of Cambridge, Massachusetts, identified nu-

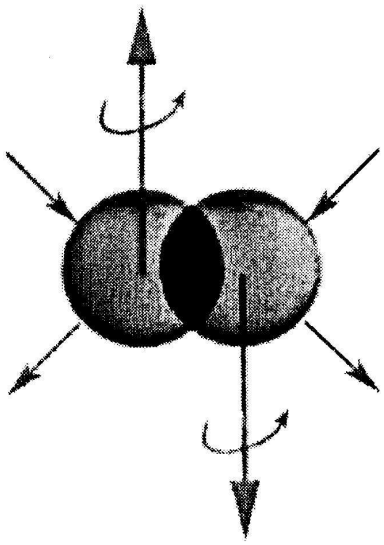

Figure 3.3. A schematic view of the fundamental interactions studied in this monograph, those originating from deep wave-overlappings of the wavepackets of particles also for the case with point-like charge as occurring in electron valence bonds, Cooper pairs in superconductivity, Pauli's exclusion principle, and other basic structures. These interactions have been ignored throughout the 20-th century, resulting in the problematic aspects or sheer inconsistencies identified in Chapter 1. As we shall see in this chapter, the representation of the new interactions here depicted with generalized units of type (3.1.19) permits the achievement of the first known, exact and invariant representation of molecular data and other data that have escaped an exact and invariant representation via quantum mechanics for about one century.

merous integral geometries and other nonlocal mathematics. However, none of them verifies all the following conditions necessary for physical consistency:

CONDITION 1: *The new nonlocal-integral mathematics must admit the conventional local-differential mathematics as a particular case under a well identified limit procedure,* because new physical advances must be a covering of preceding results. This condition alone is not verified by any integral mathematics the author could identify.

CONDITION 2: *The new nonlocal-integral mathematics must permit the clear separation of the contributions of the new nonlocal-integral interactions from those of local-differential interactions.* This second condition too was not met by any of the integral mathematics the author could identify.

CONDITION 3: *The new nonlocal-integral mathematics must permit the invariant formulation of the new interactions.* This latter condition was also vio-

lated by all integral mathematics the author could identify, thus ruling them out in a final form for consistent physical applications.

After clarifying that the mathematics needed for the correct treatment of interior systems was absent, the author was left with no other choice than that of constructing the needed mathematics. After extensive search, Santilli [4,5] suggested as the *only* possible or otherwise known solution, the invariant representation of nonlinear, nonlocal and nonpotential interactions via a generalization of the trivial unit of conventional theories. The selection was based on the fact that, whether conventional or generalized, the unit is the basic invariant of any theories. We reach in this way the following:

FUNDAMENTAL ASSUMPTION OF HADRONIC MECHANICS [4-10]: The actual, extended, nonspherical and deformable shape of particles, their variable densities and their nonlinear, nonlocal and nonpotential interactions can be invariantly represented with a generalization of the basic spacetime unit of conventional Hamiltonian theories

$$I = Diag.(1, 1, 1, 1), \tag{3.1.18}$$

into nowhere singular, sufficiently smooth, most general possible integro-differential forms, today called "Santilli isounit", of the type here expressed for simplicity for the case of two particles:

$$\hat{I} = \hat{I}^\dagger = \hat{I}_{1-2} = Diag.(n_{11}^2, n_{12}^2, n_{13}^2, n_{14}^2)\times$$
$$\times Diag.(n_{21}^2, n_{22}^2, n_{23}^2, n_{24}^2)\times$$
$$\times e^{\Gamma(t,r,\psi,\psi^\dagger,...)\times\int dr^3\times\psi^\dagger(r)\times\psi(r)} = 1/\hat{T} > 0, \tag{3.1.19}$$

with trivial generalizations to multiparticle and nondiagonal forms, where the n_{ak}^2 represents the semiaxes of the spheroidal shape of particle a, n_{a4}^2 represents its density, the expression $\Gamma(t, r, \psi, \psi, \ldots)$ represents the nonlinearity of the interaction and $\int dr^3 \times \psi^\dagger(r) \times \psi(r)$ provides a simple representation of its nonlocality. The corresponding features of antiparticles are represented by Santilli's isodual isounit

$$\hat{I}^d = -\hat{I}^\dagger = -\hat{I} < 0, \tag{3.1.20}$$

and mixed states of particles and antiparticles are represented by the tensorial product of the corresponding units and their isoduals.

Explicit examples of classical (operator) systems with nonpotential forces represented via generalized units will be given in Section 2.3 (Section 2.4).

As we shall see, the entire structure of hadronic mechanics follows uniquely and unambiguously from the assumption of the above basic unit. As a matter

of fact, some of the main features of hadronic mechanics can already be derived from the above basic assumption.

First, the maps, called in the literature *Santilli liftings*

$$I \ \rightarrow \ \hat{I}, \ I^d \ \rightarrow \ \hat{I}^d; \tag{3.1.21}$$

(where $I^d = -I$ is the isodual unit of Chapter 2 [8]) require two corresponding generalizations of the totality of the mathematical and physical formulations of conventional classical and quantum Hamiltonian theories without any exception known to this author (to avoid catastrophic inconsistencies).

As we shall see in this chapter, even basic notions such as trigonometric functions, Fourier transforms, differentials, etc. have to be lifted into two forms admitting the new quantity $\hat{I}$ and $\hat{I}^d$ as the correct left and right units.

In view of the assumed Hermiticity and positive-definiteness of $\hat{I}$, the resulting new mathematics is called in the literature *Santilli's isotopic mathematics* or *isomathematics* for short, with the corresponding *isodual isomathematics* for antimatter in interior conditions. The resulting new physical formulations are known as *Santilli isotopic mechanics* or *isomechanics* for short for the case of particles, with the *isodual isomechanics* for antiparticles.

Again in view of the fact that $\hat{I}$ is Hermitian and positive-definite, at the abstract, realization-free level there is no topological difference between I and $\hat{I}$ and, for this reason $\hat{I}$ is called *Santilli isotopic unit* or *isounit* for short.

Consequently, the new mathematical and physical formulations are expected to be *new realizations of the same axioms of conventional Hamiltonian mechanics,* and they should not be intended as characterizing "new theories" since they do not admit new abstract axioms. This illustrates the name of *isotopic mathematics* from the Greek meaning of preserving the topology.[2]

Finally, Santilli isounit $\hat{I}$ identifies in full the *covering* nature of isomechanics over conventional mechanics, as well as the type of resulting covering. This covering character is illustrated by the fact that at sufficiently large mutual distances of particles the integral in the exponent of Eq. (3.1.19) is null

$$\lim_{r \gg 1 \, \mathrm{Fm}} \int dr^3 \times \psi^\dagger(r) \times \psi(r) = 0, \tag{3.1.22}$$

in which case the actual shape of particles has no impact in the interactions and the generalized unit recovers the conventional unit[3]

$$\lim_{r \gg 1 \, \mathrm{Fm}} \hat{I} = I = Diag.(1, \, 1, \, 1, \, 1), \tag{3.1.23}$$

[2]When $\hat{I}$ is no longer Hermitian, we have the more general *genotopic mathematics* studied in Chapter 4.
[3]When the exponent of Eq. (3.1.19) is null, that is, when the mutual distances of particles are large, the characteristic quantities are constant and, consequently, terms such as $Diag.(n_{11}^{-2}, \, n_{12}^{-2}, \, n_{13}^{-2}, n_{14}^{-2})$ factor out of all equations, resulting in reduction (3.1.23).

under which limit hadronic mechanics recovers conventional quantum mechanics identically and uniquely.

The above limits also identify the important feature according to which *hadronic mechanics coincides with quantum mechanics for all mutual distances of particles sufficiently bigger than their wavepackets, while at mutual distances below that value hadronic mechanics provides a generally small corrections to quantum mechanics* (see Figure 3.3).

In this chapter we review the long and laborious scientific journey by mathematicians, theoreticians and experimentalists (see the bibliography of Chapter 1) for the achievement of maturity of formulation of the isotopic branch of hadronic mechanics, its experimental verification, its novel industrial applications, and its isodual for antimatter.

We shall begin with a review of recent developments in the construction of isomathematics that have occurred following the publication of the second edition of Vol. I of this series in 1995 [6] since these developments have important implications. We shall then identify the recent developments in physical theories occurred since the second edition of Vol. II of this series [7]. We shall then review the novel industrial applications developed since the appearance of Volumes I and II.

It should be noted that in this chapter we shall merely present recent developments. As a consequence, Volumes I and II of this series [6,7] remain useful for all detailed aspects that will not be repeated in this final volume.

A primary motivation of this volume is to present *industrial applications.* Consequently, we have selected the simplest possible mathematical treatment accessible to any experimentalists. Readers interested in utmost mathematical rigor are suggested to consult the specialized mathematical literature in the field.

Finally, the literature on the mathematics, physics and chemistry of classical and quantum Hamiltonian theories is so vast to discourage discriminatory quotations. For this reason, unless there is a contrary need, we shall abstain from quotations of works on pre-existing methods since their knowledge is a pre-requisite for the understanding of this monograph in any case.

3.2 ELEMENTS OF SANTILLI'S ISOMATHEMATICS AND ITS ISODUAL

3.2.1 Isounits, Isoproducts and their Isoduals

As indicated earlier, *Santilli isotopic mathematics*, [4–10] or *isomathematics* for short, is characterized by the map, called *lifting,* of the trivial unit $I = +1$ into a generalized unit $\hat{I}$

N-dimensional unit

$$I = +1 \to \hat{I}(t, r, p, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots), \tag{3.2.1}$$

or, more generally, by the lifting of N-dimensional units

$$I = (I_j^i) = Diag.(1, 1, 1, \ldots),\ i, j = 1, 2, \ldots, N$$

of conventional Hamiltonian theories[4] into a nowhere singular, Hermitian and positive-definite, matrix $\hat{I}$ of the same dimension N whose elements $\hat{I}_j^i$ have an arbitrary, nonlinear and integral dependence on time t, space coordinates r, momenta p, wavefunctions ψ, their derivatives $\partial\psi$, and any other needed quantity [*loc. cit.*]

$$I \ = \ (I_j^i) = Diag.(1, 1, \ldots) > 0 \rightarrow$$

$$\rightarrow \hat{I} \ = \ (\hat{I}_i^j) \ = \ \hat{I}(t, r, p, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \ldots) \ = \ 1/\hat{T} \ > \ 0. \qquad (3.2.2)$$

Isomathematics can then be defined as the lifting of all possible branches of mathematics with left and right unit I into forms admitting $\hat{I}$ as the new left and right unit.

Recall that I is the right and left unit under the conventional *associative product* $A \times B = AB$, where A, B are generic quantities (e.g., numbers, vector-fields, operators, *etc.*) for which $I \times A = A \times I = A$ for all element A of the considered set.

It is easy to see that $\hat{I}$ cannot be a unit under the same product because $\hat{I} \times A \neq A$. Therefore, for consistency, the conventional associative product $A \times B$ must be lifted into the new form first proposed by Santilli in Ref. [5] of 1978,

$$A \times B \rightarrow A\hat{\times}B = A \times \hat{T} \times B = A \times (1/\hat{I}) \times B, \qquad (3.2.3)$$

where $\hat{T}$ is fixed for the set considered, under which product $\hat{I}$ is indeed the correct left and right new unit,

$$I \times A = A \times I = A \rightarrow \hat{I}\hat{\times}A = A\hat{\times}\hat{I} = A, \qquad (3.2.4)$$

for all elements A of the considered set. In this case (only) $\hat{I}$ is called *Santilli's isotopic unit*, or *isounit* for short, and $\hat{T}$ is called *Santilli's isotopic element,* or *isoelement* for short.

Isomathematics was first submitted by Santilli in memoirs [*loc. cit.*] of 1978 and then worked out in various additional contributions by the same author, as well as by numerous mathematicians and theoreticians (see the references of Chapter 1 as well as of this section).

[4]For instance, Hamiltonian theories in 3-dimensional Euclidean space are based on the unit $I = Diag.(1, 1, 1)$ of the rotational and Euclidean symmetries, while Hamiltonian theories in Minkowski space are based on the unit $I = Diag.(1, 1, 1, 1)$ that is at the foundation of Lie's theory of the Lorentz and Poincaré symmetries.

The most salient feature of Santilli's liftings (3.2.2) and (3.2.3) is that they are *axiom preserving,* from which feature they derived their name "isotopic" [*loc. cit.*], recently contracted to the prefix "iso."

In fact, $\hat{I}$ preserves the basic topological characteristics of I. Therefore, isomathematics is expected to provide *new realizations* of the abstract axioms of the mathematics admitting I as left and right unit. In particular, the preservation of the original abstract axioms is an important guiding principle in the consistent construction of isomodels and their applications.

At this introductory stage the axiom-preserving character of generalized product (3.2.3) is easily verified by the fact that it preserves all basic axioms of the original product. In fact, the isoproduct verifies the *right and left isoscalar laws*

$$n\hat{\times}(A\hat{\times}B) = (n\hat{\times}A)\hat{\times}B, \tag{3.2.5a}$$

$$(A\hat{\times}B)\hat{\times}n = A\hat{\times}(B\hat{\times}n), \tag{3.2.5b}$$

the *right and left isodistributive laws*[5]

$$A\hat{\times}(B+C) = A\hat{\times}B + A\hat{\times}C, \tag{3.2.6a}$$

$$(A+B)\hat{\times}C = A\hat{\times}C + B\hat{\times}C, \tag{3.2.6b}$$

and the *isoassociative law*

$$A\hat{\times}(B\hat{\times}C) = (A\hat{\times}B)\hat{\times}C. \tag{3.2.7}$$

A verification of the preservation of the axioms of all subsequent constructions is crucial for a serious study and application of hadronic mechanics.

The simplest method for the construction of isomathematics as needed for various applications is given by the use of a positive-definite N-dimensional *noncanonical transform* at the classical level or a *nonunitary transform* at the operator level, here written in the unified form

$$U \times U^\dagger \neq I, \tag{3.2.8}$$

and its identification with the basic isounit of the theory

$$\hat{I} = U \times U^\dagger = 1/\hat{T} > 0, \tag{3.2.9}$$

realization first introduced by Santilli in Ref. [6,7] of 1993.

[5]The reader should keep in mind that the verification of the right and left scalar and distributive laws are necessary for any product to characterize an *algebra* as commonly understood in contemporary mathematics.

In this case, the Hermiticity of $\hat{I}$ is guaranteed because of the property,

$$(U \times U^\dagger)^\dagger = U \times U^\dagger. \tag{3.2.10}$$

Therefore, realization (3.2.9) of the isounit only requires that $U \times U^\dagger$ be a positive-definite N-dimensional matrix other than the unit matrix, from which the nowhere singularity follows, e.g., via condition

$$\text{Det}(U \times U^\dagger) > 0, \neq I. \tag{3.2.11}$$

Once the fundamental realization (3.2.9) is assumed, the construction of isomathematics follows in a simple, unique and unambiguous way. In fact, *isomathematics can be constructed by submitting conventional mathematics with left and right unit I to said noncanonical-nonunitary transform,* with very few exception, such as the isodifferential calculus that escapes construction via noncanonical-nonunitary transforms.

To begin, the isounit itself is simply given by said noncanonical-nonunitary transform of the conventional unit,

$$I \to U \times I \times U^\dagger = \hat{I}, \tag{3.2.12}$$

the isoproduct too is simply given by said noncanonical-nonunitary transform of the conventional product

$$\begin{aligned} A \times B \to U \times (A \times B) \times U^\dagger = \\ = (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) = \\ = \hat{A} \times \hat{T} \times \hat{B} = \hat{A} \hat{\times} \hat{B}, \end{aligned} \tag{3.2.13}$$

and the same simple transform holds for the construction of other aspects of isomathematics, as illustrated in this section.

As a matter of fact, the use of the above transform provides a method for the construction of isomathematics that is more rigorous than empirical liftings. For instance, by comparing Eqs. (3.2.3) and (3.2.13), we see that the lifting of the unit $I \to \hat{I} = U \times I \times U^\dagger$ implies not only the lifting of the associative product $\times \to \hat{\times} = \times (U \times U^\dagger)^{-1} \times$, but also the lifting of all elements of the set considered, $A \to \hat{A} = U \times A \times U^\dagger$.

In view of the above, the claim often expressed in the nontechnical physics literature that "the mathematics of hadronic mechanics is too difficult to comprehend" is just a case of venturing judgment without any serious knowledge of the topic.

The reader should be aware that other generalizations of the associative product, such as

$$A \bigotimes B = \hat{T} \times A \times B, \tag{3.2.14a}$$

$$A \bigodot B = A \times B \times \hat{T}, \tag{3.2.14b}$$

are unacceptable because they violate either the right or the left distributive and scalar laws, thus being unable to characterize an algebra. As such, liftings (3.2.14) are not isotopic in Santilli's sense [*loc. cit.*].

Examples of isounits have been given in Section 3.1.3. Additional examples will be provided in Sections 3.3 and 3.4. Note that, since they are Hermitian by assumption, isounits can always be diagonalized into the form of type (3.1.19).

Santilli isodual isomathematics [6–10] is the image of isomathematics under the anti-isomorphic *isodual map* of an arbitrary quantity

$$\begin{aligned} A(t, r, p, \psi, \psi^\dagger, \ldots) &\to A^d(t^d, r^d, p^d, \psi^d, \psi^{\dagger d}, \ldots) \\ &\to -A^\dagger(-t, -r^t, -p^t, -\psi^\dagger, -\psi^\dagger, \ldots), \end{aligned} \tag{3.2.15}$$

(where t denotes transposed) first submitted by Santilli in Ref. [8] of 1985 (see also Chapter 2).

The basic quantity of isodual isomathematics is then the *isodual isounit*

$$\hat{I}^d = -\hat{I}^\dagger(-t, -r^\dagger, -p^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \ldots) = 1/\hat{T}^d. \tag{3.2.16}$$

Similarly, we have the *isodual isoproduct*

$$B^\dagger \times \hat{T}^d \times A^\dagger = B^\dagger \hat{\times}^d A^\dagger, \tag{3.2.17}$$

under which $\hat{I}^d$ is indeed the right and left unit,

$$\hat{I}^d \hat{\times}^d A = A \hat{\times}^d \hat{I}^d = A, \tag{3.2.18}$$

for all A of the considered set.

Note that, *isodual map (3.2.15) must be applied for consistency to the totality of quantities of isomathematics as well as of their operations.* As an illustration, the application of the isodual map only to the quantities A, B of a product $A \times B$ and not to the product itself $\times$, leads to a host of inconsistencies.

For this and other reasons the conventional associative product is written in this monograph with the explicit notation $A \times B$ rather than the conventional notation AB. In fact, the latter would lead to gross misunderstandings and inconsistencies under the various liftings of hadronic mechanics.

Also, the construction of isomathematics is indeed recommended for physicists to be done via a noncanonical-nonunitary transform (3.2.9), while the construction of isodual isomathematics is recommended via the isodual map (3.2.15) and not via the use of an anti-isomorphic transform.

In fact, the use of anti-isomorphic transforms causes ambiguities in the very central issue, the achievement of equivalence of the isodual operator theory with

charge conjugation due to ambiguities and other technical aspects. In turn, this occurrence illustrates the significance and uniqueness of Santilli isodual map (3.2.15).

Note also that isodual isomathematics preserves the axioms, not of conventional mathematics, but of the isodual mathematics of Chapter 2, that with the simplest possible isounit unit $I^d = -I$.

Needless to say, mathematicians do not need the above elementary construction of isomathematics and its isodual since they can be formulated on abstract realization-free grounds from basic axioms.

3.2.2 Isonumbers, Isofields and their Isoduals

The first necessary isotopic lifting following that of the basic unit and product, is that of ordinary numbers. The resulting new numbers were first presented by Santilli at the 1980 meeting in Clausthal, Germany, on *Differential Geometric Methods in Mathematical Physics* and then published in a variety of papers, such as Ref. [8] of 1985, Vols. [15,16] of 1991, memoir [9] of 1993 and other works. A comprehensive presentation is available in Vol. I [6] of 1995 that also presents industrial applications of the new numbers for cryptograms and other fields. As a result of these contributions the new numbers are today known as *Santilli's isonumbers.*

The new numbers have also been studied by various authors. An important contribution has been made by E. Trell [11] in 1998 consisting in a proof of Fermat's celebrated theorem that is the simplest on record and, therefore, credibly conceivable by Fermat (as compared to other proof requiring mathematics basically unknown during Fermat's time). Unfortunately, Fermat left no record of the proof of his celebrated theorem and, therefore, there is no evidence that Fermat first studied numbers with arbitrary units. Nevertheless, Trell's proofs of Fermat's theorems remains the most plausible known to this author for being conceived during Fermat's time.

Numerous additional studies on isonumbers have been conducted by other authors. For a complete bibliography we refer interested readers to the monograph on *Santilli isonumber theory* by C.-X. Jiang [12] of 2002. Additional studies on isonumbers have occurred for their use as basis of other isostructures. Related references will be quoted in the appropriate subsequent sections.

Santilli's isonumbers have also been subjected to a generalization called *pseudo-isonumbers* identified in Ref. [9] and studies by various authors, including N. Kamiya [13] and others. However, the latter generalization violates the axioms of a field and, as such, it cannot be used for hadronic mechanics.

The reader should be aware that in this section we merely present the minimal possible properties of isonumbers sufficient for industrial applications.

Let us consider: the field $R(n,+,\times)$ of *real numbers* n with ordinary sum $+$ and product $\times$; the field $C(c,+,\times)$ of *complex numbers* $c = n_1 + i \times n_2$ where i is the imaginary unit and $n_1, n_2 \in R$; and the field $Q(q,+,\times)$ of *quaternions* $q = i_o + i_1 \times n_1 + i_2 \times n_2 + i_3 \times n_3$, where i_o is the 2-dimensional unit matrix, $i_k, k = 1,2,3$ are Pauli's matrices and $n_1, n_2, n_3 \in R$. These fields are hereon represented with the unified notation[6]

$$F(a,+,\times) : a = n,\ c,\ q, \tag{3.2.19}$$

In this section we present first the simplest possible method for the lifting of numbers via the use of a positive-definite (thus invertible) noncanonical-nonunitary transform identified with Santilli's isounit

$$I \to \hat{I} = U \times I \times U^\dagger = 1/\hat{T} > 0, \quad U \times U^\dagger \neq I. \tag{3.2.20}$$

We shall then pass to a mathematical presentation.

The isotopic lifting of ordinary numbers is easily achieved via the above map resulting in *Santilli isonumbers* for the characterization of *matter*

$$a \to \hat{a} = U \times a \times U^\dagger = a \times (U \times U^\dagger) = a \times \hat{I}, \tag{3.2.21}$$

and related *isoproduct*

$$a \times b \to U \times (a \times b) \times U^\dagger = \hat{a} \times \hat{T} \times \hat{b} = \hat{a} \hat{\times} \hat{b}, \tag{3.2.22}$$

under which $\hat{I}$ is the correct right and left isounit, Eq. (3.2.4), with the element *isozero* coinciding with the ordinary zero

$$0 \to \hat{0} = U \times 0 \times U^\dagger \equiv 0, \tag{3.2.23}$$

and, consequently, the *isosum* coinciding with the ordinary sum,

$$a + b \to U \times (a + b) \times U^\dagger = \hat{a} \hat{+} \hat{b} \equiv \hat{a} + \hat{b}. \tag{3.2.24}$$

The above liftings result in: *Santilli isofield* $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ of *isoreal isonumbers*; the isofield $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$ of *isocomplex isonumbers*; and the isofield $\hat{Q}(\hat{q}, \hat{+}, \hat{\times})$ of *isoquaternionic isonumbers*; hereon represented with the unified notation

$$\hat{F}(\hat{a}, \hat{+}, \hat{\times}), \quad \hat{a} = \hat{n}, \hat{c}, \hat{q}. \tag{3.2.25}$$

Needless to say, the liftings of the unit and of the product require a corresponding lifting of all conventional operations of numbers depending on the

[6]Octonions are not considered "numbers" because they violate the associativity property of the axioms of a field.

multiplication. By using the above noncanonical-nonunitary map, one can easily prove the *isopowers*

$$\hat{a}^{\hat{n}} = \hat{a} \,\hat{\times}\, \hat{a} \,\hat{\times}\, \ldots \,\hat{\times}\, \hat{a} \;\; (n \;\; \text{times}) = a^n \times \hat{I}. \tag{3.2.26}$$

An important particular case is the property that *isopowers of the isounits reproduce the isounit identically,*

$$\hat{I}^{\hat{n}} = \hat{I} \hat{\times} \hat{I} \hat{\times} \ldots \hat{\times} \hat{I} \equiv \hat{I}. \tag{3.2.27}$$

Similarly we have the *isosquare isoroot*

$$\hat{a}^{\hat{1/2}} = a^{1/2} \times \hat{I}^{1/2}; \tag{3.2.28}$$

the *isoquotient*

$$\hat{a} \hat{/} \hat{b} = (\hat{a}/\hat{b}) \times \hat{I} = (a/b) \times \hat{I}; \tag{3.2.29}$$

and the *isonorm*

$$\hat{|}\hat{a}\hat{|} = |a| \times \hat{I}, \tag{3.2.30}$$

where $|a|$ is the conventional norm. All these properties were first introduced by Santilli in Refs. [6–9]. The reader can now easily construct the desired isotopic image of any other operation on numbers.

Despite their simplicity, isonumbers are nontrivial. As an illustration, the assumption of the isounit $\hat{I} = 3$ implies that "2 multiplied by 3" $= 18$, while 4 becomes a prime number.

The best way to illustrate the nontriviality of the new numbers is to indicate the **industrial applications of Santilli's isonumbers,** that are a primary objective of this monograph as indicated earlier.

To begin, *all* applications of hadronic mechanics are based on isonumbers, and they will be presented later on in this chapter. In addition to that, *Santilli's isonumbers have already found a direct industrial application consisting of the isotopic lifting of cryptograms used by the industry to protect secrecy, including banks, credit cards. etc.* This industrial application was first presented by Santilli in Appendix 2.C of the second edition of Vol. I [6] of 1995, and will be reviewed later on in this chapter.

At this moment we merely mention that all cryptograms based on the multiplication depend on only one value of the unit, the quantity $+1$ dating back to biblical times. A mathematical theorem establishes that a solution of any cryptogram can be identified in a finite period of time. As a result of this occurrence, banks and other industries are forced to change continuously their cryptograms to properly protect their secrecy.

By comparison, *Santilli's isocryptograms* are based on the isoproduct and, as such, they admit an *infinite number of possible isounits,* such as, for instance, the values

$$\hat{I} = 7.2; \;\; 0.98364; \;\; 236; \;\; 1,293' \;\; 576; \;\; \text{etc.} \tag{3.2.31}$$

Consequently, it remains to be seen whether Santilli isocryptograms can be broken in a finite period of time under the availability of an infinite number of possible isounits.

Independently from that, with the use of isocryptograms banks and other industries do not have to change the entire cryptogram for security, but can merely change the value of the isounit to keep ahead of possible hackers, and even that process can be computerized for frequent automatic changes of the isounit, with clearly added safety.

Finally, another application of Santilli isocryptograms permitted by their simplicity is their use to protect the access to personal computers.

It is hoped this illustrates the industrial significance of Santilli isonumbers *per se,* that is, independently from their basic character for hadronic mechanics.

We now pass to a mathematical presentation of the new numbers.

DEFINITION 3.2.1 [9]: Let $F = F(a, +, \times)$ be a field of characteristic zero as per Definition 2.1.1. Santilli's isofields are rings $\hat{F} = \hat{F}(\hat{a}, \hat{+}, \hat{\times})$ with: elements

$$\hat{a} = a \times \hat{I}, \tag{3.2.32}$$

where $a \in F$, $\hat{I} = 1/\hat{T}$ is a positive-definite quantity generally outside F and $\times$ is the ordinary product of F; the isosum $\hat{+}$ coincides with the ordinary sum $+$,

$$\hat{a}\,\hat{+}\,\hat{b} \equiv \hat{a} + \hat{b}, \ \ \forall\, \hat{a}, \hat{b} \in \hat{F}, \tag{3.2.33}$$

consequently, the element $\hat{0} \in \hat{F}$ coincides with the ordinary $0 \in F$; and the isoproduct $\hat{\times}$ is such that $\hat{I}$ is the right and left isounit of $\hat{F}$,

$$\hat{I}\hat{\times}\hat{a} = \hat{a}\hat{\times}\hat{I} \equiv \hat{a}, \ \ \forall\, \hat{a} \in \hat{F}. \tag{3.2.34}$$

Santilli's isofields verify the following properties:

1) For each element $\hat{a} \in \hat{F}$ there is an element $\hat{a}^{-\hat{1}}$, called isoinverse, for which

$$\hat{a}\hat{\times}\hat{a}^{-\hat{1}} = \hat{I}, \ \ \forall\, \hat{a} \in \hat{F}; \tag{3.2.35}$$

2) The isosum is isocommutative

$$\hat{a}\hat{+}\hat{b} = \hat{b}\hat{+}\hat{a}, \tag{3.2.36}$$

and isoassociative

$$(\hat{a}\hat{+}\hat{b})\hat{+}\hat{c} = \hat{a}\hat{+}(\hat{b}\hat{+}\hat{c}), \ \ \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F}; \tag{3.2.37}$$

3) The isoproduct is not necessarily isocommutative

$$\hat{a}\hat{\times}b \neq \hat{b}\hat{\times}\hat{a}, \tag{3.2.38}$$

but isoassociative

$$\hat{a}\hat{\times}(\hat{b}\hat{\times}\hat{c}) = (\hat{a}\hat{\times}\hat{b})\hat{\times}\hat{c}, \quad \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F}; \tag{3.2.39}$$

4) The set $\hat{F}$ is closed under the isosum,

$$\hat{a}\hat{+}\hat{b} = \hat{c} \in \hat{F}, \tag{3.2.40}$$

the isoproduct,

$$\hat{a}\hat{\times}\hat{b} = \hat{c} \in \hat{F}, \tag{3.2.41}$$

and right and left isodistributive compositions,

$$\hat{a}\hat{\times}(\hat{b}\hat{+}\hat{c}) = \hat{d} \in \hat{F}, \tag{3.2.42a}$$

$$(\hat{a}\hat{+}\hat{b})\hat{\times}c = \hat{d} \in \hat{F}, \quad \forall \hat{a}, \hat{b}, \hat{c}, \hat{d} \in \hat{F}; \tag{3.2.42b}$$

5) The set $\hat{F}$ verifies the right and left isodistributive law

$$\hat{a}\hat{\times}(\hat{b}\hat{+}\hat{c}) = (\hat{a}\hat{+}\hat{b})\hat{\times}\hat{c} = \hat{d}, \ \ \forall \hat{a}, \hat{b}, \hat{c}, \hat{d} \in \hat{F}. \tag{3.2.43}$$

Santilli's isofields are called of the first (second) kind when $\hat{I}$ is (is not) an element of F.

The basic axiom-preserving character of the isotopies of numbers is illustrated by the following:

LEMMA 3.2.1 [9]: Isofields of first and second kind are fields (namely, they verify all axioms of a field).

Note that the isotopic lifting does indeed change the *operation* of the multiplication but not that of the sum because the isotopies here considered do change the multiplicative unit I, but not the additive unit 0, Eq. (3.2.23). This is a crucial property of hadronic mechanics best illustrated by the following property:

LEMMA 3.2. [9]: Nontrivial liftings of the additive unit 0 *and related sum violates the axioms of a field (for which reason, they are called "pseudoisofields")*

In fact, suppose that one wants to change the value of the element 0, e.g.,

$$0 \to \hat{0} = K \neq 0, \ \ K \in F. \tag{3.2.44}$$

Then, for $\hat{0}$ to remain the new additive unit, one must alter the sum into a new form admitting $\hat{0}$ as left and right additive unit, e.g.,

$$a\hat{+}b = a + (-\hat{0}) + b, \tag{3.2.45}$$

under which

$$a\hat{+}\hat{0} = \hat{0}\hat{+}a \equiv a, \ \ \forall a \in F. \tag{3.2.46}$$

However, there is no single lifting of the product such that

$$\hat{0}\hat{\times}a \neq \hat{0}, \ \ \forall a \in F, \tag{3.2.47}$$

under which there is the loss of the distributive axiom of a field, i.e.,

$$(a\hat{+}b) \times c \neq a \times c \ \hat{+} \ b \times c. \tag{3.2.48}$$

In turn, the loss of the distributive law causes very serious physical inconsistencies, such as preventing experimental applications of the theory. Therefore, *being axiom-preserving, hadronic mechanics is solely based on the isotopic lifting of the multiplicative unit and related product, but not on any lifting of the additive unit and related sum.*

Santilli's isodual isonumbers for the characterization of *antimatter* can be uniquely and unambiguously characterized via the isodual map (3.2.15). They are characterized by the *additive and multiplicative isodual isounit*

$$\hat{0} \to \hat{0}^d \equiv 0, \tag{3.2.49a}$$

$$\hat{I}^d = -\hat{I} < 0, \tag{3.2.49b}$$

where one should recall that $\hat{I}$ is real valued and positive-definite, thus Hermitian. Isodual isonumbers are then explicitly given by

$$\hat{a}^d = -\hat{a}^\dagger = -\hat{I} \times \hat{a}^\dagger. \tag{3.2.50}$$

The isodual isonumbers were first introduced by Santilli in Ref. [8] of 1985, treated mathematically in Ref. [9] of 1993 and studied extensively in Vol. I of this series [6].

The use of the same isodual map then identifies the *isodual isosum*

$$\hat{a}^d \hat{+}^d \hat{b}^d = \hat{a}^d + \hat{b}^d, \tag{3.2.51}$$

the *isodual isoproduct*

$$(\hat{a}\hat{\times}\hat{b})^d = \hat{b}^d \times^d \hat{T}^d \times^d \hat{A}^d = -\hat{b}^d \hat{\times} \hat{a}^d = -\hat{b}^\dagger \hat{\times} \hat{a}^\dagger, \tag{3.2.52}$$

and the *isodual isonorm*

$$\hat{|}\hat{a}\hat{|}^d = -\hat{|}\hat{a}\hat{|} = -|a| \times \hat{I}. \tag{3.2.53}$$

that is always *negative-definite.*

The above liftings result in: *Santilli's isodual isofield* $\hat{R}^d(\hat{n}^d, \hat{+}^d, \hat{\times}^d)$ of *isodual isoreal isonumbers*; the isodual isofield $\hat{C}^d(\hat{c}^d, \hat{+}^d, \hat{\times}^d)$ of *isodual isocomplex isonumbers*; and the isodual isofield $\hat{Q}^d(\hat{q}^d, \hat{+}^d, \hat{\times}^d)$ of *isodual isoquaternionic isonumbers*; hereon represented with the unified notation

$$\hat{F}^d(\hat{a}^d, \hat{+}^d, \hat{\times}^d), \hat{a}^d = \hat{n}^d, \hat{c}^d, \hat{q}^d. \tag{3.2.54}$$

DEFINITION 3.2.3 [9]: Let $\hat{F}(\hat{a}, \hat{+}, \hat{\times})$ be an isofield as per Definition 3.2.1. Then Santilli isodual isofields $\hat{F}^d(\hat{a}^d, \hat{+}^d, \hat{\times}^d)$ are the image of $\hat{F}$ under the isodual map (3.2.15).

LEMMA 3.2.3 [9]: Isodual isofields are fields (that is, they verify all axioms of a field).

LEMMA 3.2.4 [9]: Isodual isofields are anti-isomorphic to isofields.

As we shall see in this chapter, the latter property, jointly with the anti-isomorphic character of the isodual map, will result to be crucial for a consistent treatment of antimatter composed of extended particles with potential and non-potential internal forces.

The above properties establish the fact (first identified in Ref. [8]) that, by no means, the axioms of a field require that the multiplicative unit to be the trivial unit $+1$, because the basic unit can be a negative-definite quantity -1 as it occurs for the isodual mathematics of Chapter 2, an arbitrary positive-definite quantity $\hat{I} > 0$ as occurring in isomathematics, or an arbitrary negative-definite quantity $\hat{I}^d < 0$ as it occurs for the isodual isomathematics.

The reader should be aware that an in depth knowledge of Santilli's isonumbers and their isoduals requires an in depth study of memoir [9] or of Chapter 2 of Vol. I of this series, Ref. [6], and that an in depth knowledge of Santilli's isonumbers theory requires a study of Jiang's monograph [12].

Finally, the reader should meditate a moment on the viewpoint expressed several times in this writing to the effect that *there cannot be really new physical theories without new mathematics, and there cannot be really new mathematics without new numbers.* The basic novelty of hadronic mechanics can, therefore, be reduced to the novelty of Santilli's isonumbers.

By remembering that all "numbers" have been fully identified centuries ago, the novelty of hadronic mechanics can be reduced to the discovery that the axioms of conventional fields admit new realizations with nonsingular, but otherwise arbitrary multiplicative units.

3.2.3 Isospaces and Their Isoduals

Following the lifting of units, products and fields, the next necessary lifting is that of N-dimensional *metric or pseudo-metric spaces* with local coordinates r and Hermitian, thus diagonalized metric m over a field F, here written in the

unified notation

$$S(r,m,F):\ r=(r^k),\ m=[m_{ij}(r,\ldots)]=Diag.(m_{11},m_{22},\ldots,m_{NN}),\\ i,j,k=1,2,\ldots,N, \tag{3.2.55}$$

basic invariant

$$r^2=(r^i\times m_{ij}\times r^j)\times I=(r^t\times m\times r)\times I\ \ \in\ \ F(a,+,\times), \tag{3.2.56}$$

(where t stands for transposed) and fundamental N-dimensional unit[7]

$$I=Diag.(1,1,\ldots,1). \tag{3.2.57}$$

As now familiar, isotopies are based on the lifting of the above N-dimensional unit via a positive-definite noncanonical-nonunitary transform in the same dimension with an otherwise unrestricted functional dependence

$$I=Diag.(1,1,\ldots,1)\to\hat{I}(t,r,p,\psi,\psi^\dagger,\ldots)=U\times I\times U^\dagger=1/\hat{T}>0, \tag{3.2.58}$$

The above liftings requires that of spaces $S(r,m,R)$ into *isotopic spaces,* or *isospaces* for short, for the treatment of *matter,* hereon denoted $\hat{S}(\hat{r},\hat{M},\hat{F})$, where $\hat{r}$ denotes the *isocoordinates,* and $\hat{M}$ denotes the *isometric* defined on the isofields $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ of Section 3.2.2.

Isospaces were first proposal by Santilli in Ref. [14] of 1983 for the axiom-preserving isotopies of the Minkowskian spacetime and special relativity that are at the foundations of hadronic mechanics. Isospaces were then used by Santilli for the liftings of the various spacetime and internal symmetries (such as $SU(2)$, $SO(3)$, $SO(3.1)$, $SL(2.C)$, $G(3.1)$, $P(3.1)$, $SU(3)$, etc.) as studied later on in this chapter.

A comprehensive presentation of isospaces first appeared in monographs [15,16] of 1991 and in the first edition of Volumes I and II of this series, Ref. [6,7] of 1993 (see the second edition of 1995 for various upgradings). A mathematical study of isospaces by Santilli was presented in memoir [10] of 1996. In view of all these contributions, the new spaces are today known as *Santilli's isospaces.*

Following the appearances of these contributions, isospaces have been also studied by a number of authors for both mathematical and physical applications to be studied in subsequent sections, including the definition of isocontinuity,

[7]The basic character of the unit should be recalled here. For the case of the three-dimensional Euclidean space, $I=Diag.(1,1,1)$ is not only the basic geometric unit, but also the unit of the entire Lie theory of the rotational and Euclidean symmetries. Similarly, for the case of the Minkowski spacetime, the unit $I=Diag.(1,1,1,1)$ is at the foundations of the entire Lie theory for the Lorentz and Poincaré symmetries. We begin to see in this way the far reaching implications of isotopic generalization of the basic unit.

isotopology, isomanifolds, etc. The related literature will be presented in the appropriate subsequent sections.

In this section we identify the basic notions of Santilli isospaces. Specific types of isospaces needed for applications will be studied in subsequent sections.

The coordinates r of ordinary spaces $S(r, m, F)$ are defined on the base field $F = F(a, +, \times)$, thus being real numbers for $F = R$, complex numbers for $F = C$ and quaternionic numbers for $F = Q$.

Consequently, the *isocoordinates* $\hat{r}$ on isospaces $\hat{S}(\hat{r}, \hat{m}, \hat{F})$ must be defined on the isofields $\hat{F} = \hat{F}(\hat{a}, \hat{+}, \hat{\times})$, namely, must be *isonumbers* and, more particularly, be isoreal isonumbers for $\hat{F} = \hat{R}$, isocomplex isonumbers for $\hat{F} = \hat{C}$, and isoquaternionic isonumbers for $\hat{F} = \hat{Q}$.

Since isocoordinates are isonumbers, they can be easily constructed via the same lifting used for isonumbers, resulting in the simple definition

$$r \to \hat{r} = U \times r \times U^\dagger = r \times (U \times U^\dagger) = r \times \hat{I}. \tag{3.2.59}$$

Similarly, the metric m on $S(r, m, F)$ is an ordinary matrix in N-dimension whose elements m_{ij} are functions defined on the base field F, thus being real, complex or quaternionic functions depending on the corresponding character of F.

As we shall see shortly, a necessary condition for $\hat{S}(\hat{r}, \hat{M}, \hat{F})$ to preserve the geometric axioms of $S(r, m, F)$ (that is, for $\hat{S}$ to be an isotope of S), is that, when the unit is lifted in the amount $I \to \hat{I} = 1/\hat{T}$, the metric is lifted by the *inverse* amount $m \to \hat{m} = \hat{T} \times m$, thus yielding the transform (where the diagonal character of m is taken into account)

$$m \to U^{\dagger -1} \times m \times U^{-1} = (U \times U^\dagger)^{-1} \times m =$$
$$= \hat{T} \times m = (\hat{m}_{ij}) = (\hat{T}_i^k \times m_{kj}), \tag{3.2.60}$$

However, in this case the elements $\hat{m}_{ij}$ are not properly defined on $\hat{S}$ because they are not isonumbers on $\hat{F}$. For this purpose, the correct definition of the *isometric* is given by

$$\hat{M} = \hat{m} \times \hat{I} = (\hat{m}_{ij} \times \hat{I}) = (\hat{m}_{ij}) \times \hat{I}. \tag{3.2.61}$$

As we shall see in the next section, the above definition is independently confirmed by the isotopies of matrices. We, therefore, have the following

DEFINITION 3.2.3 [14]: Let $S(r, m, F)$ be an N-dimensional metric or pseudo-metric space with contravariant coordinates $r = (r^k)$, metric $m = (m_{ij})$ and invariant $r^2 = (r_k \times r^k) \times I = (r^i \times m_{ij} \times r^j) \times I$ over a field F with trivial unit I. Then, Santilli's isospaces are the N-dimensional isovector spaces

$$\hat{S}(\hat{r}, \hat{M}, \hat{F}) : \quad \hat{r} = (\hat{r}^k) = (r^k) \times \hat{I} \ \in \hat{F}, \tag{3.2.62a}$$

$$\hat{M} = (\hat{T} \times m) \times \hat{I} = (T_i^k \times m_{ki}) \times \hat{I} \ \in \ \hat{F}, \ \ \hat{M}^{ij} = [(\hat{M}_{pq})^{-1}]^{ij} \ \in \hat{F}, \quad (3.2.62b)$$

$$\hat{r}^k = \hat{M}^{ki} \hat{\times} \hat{r}_i = \hat{m}^{ki} \times r_i \times \hat{I}, \quad \hat{r}_k = \hat{M}_{ki} \hat{\times} \hat{r}^i = \hat{m}_{ki} \times r^i \times \hat{I}, \quad (3.2.62c)$$

$$\hat{r}^{\hat{2}} = \hat{r}^k \hat{\times} \hat{r}_k = \hat{r}^i \hat{\times} \hat{M}_{ij} \hat{\times} \hat{r}^j = (r^i \times \hat{m}_{ij} \times r^j) \times \hat{I} \ \in \ \hat{F}, \quad (3.2.62d)$$

$$i, j, k, p, q = 1, 2, \ldots, N,$$

and its projection on the original space $S(r, m, F)$, is characterized by

$$\hat{S}(r, \hat{m}, F) : \ \ r = (r^k) = (r^k) \times I \ \in \ F; \quad (3.2.63a)$$

$$\hat{m} = \hat{T} \times m = (\hat{T}_i^k \times m_{kj}) \ \in F, \ \ \hat{m}^{ij} = [(\hat{m}_{ps})^{-1}]^{ij} \ \in \ F, \quad (3.2.63b)$$

$$r^k = \hat{m}^{ki} \times r_i \ \in \ R, \ \ r_k = \hat{m}_{ki} \times r^1 \ \in \ F, \quad (3.2.63c)$$

$$r^2 = r^i \times \hat{m}_{ij} \times r^j) \times I = r^i \times (\hat{T}_i^k \times m_{kj}) \times r^j) \times I \ \in \ F. \quad (3.2.63d)$$

As one can see, expression (3.2.62) is the proper formulation of the isoinvariant on isospaces over the base isofield, and we shall write $\hat{S}(\hat{r}, \hat{M}, \hat{F})$, while expression (3.2.63) is the "projection" of the preceding space in the original space S, and we shall write $\hat{S}(r, \hat{m}, F)$, because the latter space is defined with conventional coordinates, units and products over the conventional field F by construction.

It should be stressed that *isospaces are mathematical spaces and, therefore, all physical calculations and applications will be done in the projection of isospaces over conventional spaces.* In fact, experimental measurements and events can only occur in our space time. Therefore, all physical applications of isospaces can only occur in their projection in our spacetime.

A simple visual inspection of invariants (3.2.56) and (3.2.62) establish the following

THEOREM 3.2.1 [10]: All line elements of metrics or pseudo-metric spaces with metric m and unit I, and all their isotopes possess the following invariance property

$$I \to \hat{I} = n^2 \times I, \ \ m \to \hat{m} = n^{-2} \times m, \quad (3.2.64)$$

where n is a non-null parameter.

This property too will soon acquire fundamental character, since it permits the identification, for the first time, of the property that *the Galilean and Poincaré symmetries are "eleven" dimensional,* and not ten-dimensional as believed throughout the 20-th century.

In particular, the 11-th invariance is "hidden" in conventional line elements and will permit the first and perhaps only known, axiomatically consistent grand

unification of electroweak and gravitational interactions, as studied later on in this chapter.

The nontriviality of isospaces is then expressed by the following

THEOREM 3.2.2 [14]: Even though preserving all topological properties of m (from the positive-definiteness of $\hat{I}$), the projection $\hat{m}$ of the isometric $\hat{M}$ on $\hat{S}$ over $\hat{F}$ into the original space S over F acquires an unrestricted functional dependence on any needed local variables or quantities,

$$\hat{M} \to \hat{m} = \hat{m}(t, r, p, \psi, \psi^\dagger, \ldots). \tag{3.2.65}$$

As we shall see, the above property has truly fundamental implications, since it will permit the first and only known *geometric unification of the Minkowskian and Riemannian geometries with the consequential unification of special and general relativities,* and other applications of manifestly fundamental nature.

By recalling that the basic invariant r^2 represents the square of the "distance" in S, from Eqs. (3.2.56) and (3.2.62) we derive the following additional property

THEOREM 3.2.3 [6,7,10]: The basic invariant of a metric or pseudometric space has the structure:

$$Invariant \;\; = \;\; [Length]^2 \times [Unit]^2 \tag{3.2.66}$$

The above property will soon have deep geometric implications, such as permitting different shapes, sizes and dimension for the same object under inspection by different observers, all in a way compatible with our sensory perception.

Note that invariant structure (3.2.66) is indeed new because identified for the first time by the isotopies, since the multiplication of the invariant by the unit is trivial for conventional studies and, as such, it was ignored.

It is now important to indicate the *differences between Santilli isospaces* $\hat{S}(\hat{r}, \hat{M}, \hat{F})$ *or* $\hat{S}(r, \hat{m}, F)$ *and deformed spaces* that, as well known, are given by the sole deformations of the metric, for which we use the notation $S(r, \hat{m}, F)$.

It is easy to see that *deformed spaces $S(r, \hat{m}, F)$ have a conventional noncanonical or nonunitary structure,* thus activating the theorems of catastrophic inconsistencies of Section 3.4. By comparison, Santilli isospaces have been constructed precisely to resolve these catastrophic inconsistencies via the reconstruction of canonicity or unitarity on isospaces over isofields.

Moreover, *deformed metric spaces $S(r, \hat{m}, F)$ necessarily break the symmetries of the original spaces $S(r, m, F)$, while, as we shall soon see, isospaces $\hat{S}(\hat{r}, \hat{M}, \hat{F})$ reconstruct the exact symmetries of $S(r, g, F)$.*

The implications of the latter property alone are far reaching because *all symmetries believed to be broken in the 20-th century can be proved to remain exact*

on suitable isospaces over isofields. In different terms, the "breakings of space-time and internal symmetries" studies through the 20-th century are a direct manifestation of the adaptation of new physical events to a rather limited, pre-existent mathematics because, if the underlying mathematics is suitably lifted, all believed breakings cease to exist, as already proved in Vol. II of this series [7] and updated in this volume.

Santilli's isodual isospaces for the treatment of *antimatter* are the anti-isomorphic image of isospaces under the isodual map (3.2.15) and can be written

$$\hat{S}^d(\hat{r}^d, \hat{M}^d, \hat{F}^d): \quad \hat{r}^d = -\hat{r}^\dagger, \quad \hat{M}^d = -\hat{M}, \tag{3.2.67a}$$

$$\hat{r}^{\hat{2}d} = \hat{r}^d \hat{\times}^d \hat{M}^d \hat{\times}^d \hat{r}^{t,d}. \tag{3.2.67b}$$

Isodual isospaces were introduced in Vol. I of this series [6] and then treated in various other works (see, e.g., [10,17,18]). As we shall see, they play a crucial role for the treatment of antimatter in interior conditions. The tensorial product of isospaces and their isoduals appears to be significant for basic advances in biology, e.g., to achieve a quantitative mathematical representation of bifurcations and other biological behavior.

As we shall see, all **industrial applications** of hadronic mechanics are based on isospaces to such an extent that the new isogeometries have acquired evident relevance for new patents assuredly without prior art, evidently in view of their novelty.

3.2.4 Isofunctional Analysis and its Isodual

The lifting of fields evidently requires a corresponding lifting of functional analysis into a form known as *Kadeisvili isofunctional analysis* since it was first studied by J. V. Kadeisvili [19,20] in 1992. Additional studies were done by A. K. Aringazin *et al.* [21] in 1995 and other authors.

A detailed study of isofunctional analysis was also provided in monographs [6,7] of 1995. A knowledge of these studies is necessary for any application of hadronic mechanics because all conventional functions and transforms have to be properly lifted for consistent applications, while the use of conventional (or improperly lifted) functions and transforms leads to catastrophic inconsistencies.

In essence, the consistent formulation of isofunctional analysis requires not only the preservation of the original axioms, but also the preservation of the original numerical values when formulated on isospaces over isofields, under which conditions the broadening of conventional formulations emerge in the projection of the isotopic treatment in the original space.

The latter mathematical requirement has deep physical implications, such as the preservation of the speed of light *in vacuum* as the universal invariant on *isospaces over isofield,* with consequential preservation under isotopies of all axioms of special relativity, while locally varying speeds of light within physical

media emerge in the *projection* of the isospace in our spacetime, as we shall see in subsequent sections.

The scope of this section is essentially that of providing the guidelines for the updating of Refs. [19,20,16,6,7] along the above requirements to achieve compatibility with the main lines of this presentation.

DEFINITION 3.2.4 [19,20,21, 6,7] Let $f(x)$ be an ordinary (sufficiently smooth) function on a vector space S with local variable x (such as a coordinate) over the reals R. The isotopic image of $f(x)$, called isofunctions, can be constructed via the use of a noncanonical-nonunitary transform

$$U \times f(x) \times U^\dagger = f(x) \times \hat{I} \ \in \ \hat{F}, \tag{3.2.68}$$

reformulated on isospace $\hat{S}(\hat{x}, \hat{F})$ over the isofield $\hat{F}$

$$f(x) \times \hat{I} = f(\hat{T} \times \hat{x}) \times \hat{I} = \ \hat{f}(\hat{x}) \in \hat{F}, \tag{3.2.69}$$

with projection in the original space $S(x, F)$

$$f(\hat{T} \times x) \in F. \tag{3.2.70}$$

As one can see, expression (3.2.68) coincides with the definition of isofunction in the quoted references. A feature identified since that time is the re-interpretation in such a way that the function $f(x)$ preserves its numerical value when formulated as $\hat{f}(\hat{x})$ on the isospace $\hat{S}$ over the isofield $\hat{F}$ because the variable $\hat{x}$ is multiplied by $\hat{T}$ while the unit to which such a variable is referred to is multiplied by the *inverse* amount $\hat{I} = 1/\hat{T}$. All numerical differences emerge in the *projection* of $\hat{f}(\hat{x})$ in the original space.

This is essentially the definition of isofunctions that will allow us to preserve the basic axioms of special relativity on isospaces over isofields and actually expand their applicability from motion in empty space to motion within physical media.

For the case of the simple function $f(x) = x$ we have the lifting

$$\hat{x} = U \times x \times U^\dagger = x \times (U \times U^\dagger) = x \times \hat{I} = \hat{T} \times \hat{x} \times \hat{I} \ \in \ \hat{F}, \tag{3.2.71}$$

with the projection in the original space S being simply given in this case by $\hat{T} \times x$.

More instructive is the lifting of the exponentiation into the *isoexponentiation* given by

$$e^x \to U \times e^x \times U^\dagger =$$
$$= U \times (I + x/1! + x \times x/2! + \ldots) \times U^\dagger =$$
$$= \hat{I} + \hat{x}\hat{/}\hat{1}! + \hat{x}\hat{\times}\hat{x}\hat{/}\hat{2}! + \ldots) =$$

$$= \hat{e}^{\hat{x}} = (e^{\hat{x}\times\hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T}\times\hat{x}}) \ \in \ \hat{F}, \tag{3.2.72}$$

with projection in the original space S given by

$$\hat{e}^{x} = (e^{x} \times \hat{T}) \times I = I \times (e^{\hat{T}\times x}) \ \in F, \tag{3.2.73}$$

where one should note that the function in isospace is computed over $\hat{F}$ while its projection in the original space is computed in the original field F.

The above lifting is nontrivial because of the appearance of the nonlinear integro-differential quantity $\hat{T}(t, x, \psi, \partial\psi, \ldots)$ in the exponent. As we shall see shortly, this feature permits the first known extension of the linear and local Lie theory to nonlinear and nonlocal formulations.

Let $M(x) = (M_{ij}(x))$ be an N-dimensional matrix with elements $M_{ij}(x)$ on a conventional space $S(x, F)$ with local coordinates x over a conventional field F with unit I. Then, the isotopic image of $M(x)$ or it isomatrix, is defined by

$$\hat{M}(\hat{x}) = (\hat{M}_{ij}(\hat{x})) = M(\hat{T} \times \hat{x}) \times \hat{I}, \ \ \hat{M}_{ij} \in \hat{F}, \tag{3.2.74}$$

Similarly, the *isodeterminant* of $\hat{M}$ is defined by

$$\hat{\mathrm{Det}}\hat{M} = [\mathrm{Det}(\hat{T} \times M)] \times \hat{I} \tag{3.2.75}$$

where Det represents the conventional determinant, with the preservation of the conventional axioms, e.g.,

$$\hat{\mathrm{Det}}(\hat{M}_1 \hat{\times} \hat{M}_2) = \hat{\mathrm{Det}}(\hat{M}_1) \hat{\times} \hat{\mathrm{Det}}(\hat{M}_2); \tag{3.2.76a}$$

$$\hat{\mathrm{Det}}(\hat{M}^{-\hat{I}}) = (\hat{\mathrm{Det}}\hat{M})^{-\hat{I}}, \tag{3.2.76b}$$

Note that, by construction, isomatrices and isodeterminant preserve the original values on isospaces over isofields, although show deviations when the same quantities are observed from the original space, that is, referred to the original unit.

Similarly, the *isotrace* of $\hat{M}$ is defined by[8]

$$\hat{Tr}\hat{M} = [Tr(\hat{T} \times M)] \times \hat{I}, \tag{3.2.77}$$

where Tr is the conventional trace, and it also verifies the conventional axioms, such as

$$\hat{Tr}(\hat{M}_1 \hat{\times} \hat{M}_2) = \hat{Tr}\hat{M}_1 \hat{\times} \hat{Tr}\hat{M}_2, \tag{3.2.78a}$$

[8]The isodeterminant introduced in Ref. [6], Eq. (6.3.19) is the correct form as in Eq. (3.2.77) above. However, the isotrace introduced in Eq. (6.3.20a) of Ref. [6] preserves the axioms of a trace, but not its value, as a consequence of which it is not fully invariant, the correct definition of isotrace being given by Eq. (3.2.77) above.

$$\hat{Tr}(\hat{M}^{-\hat{I}}) = (\hat{Tr}\hat{M})^{-\hat{I}}. \tag{3.2.78b}$$

The *isologarithm* is hereon defined by[9]

$$\hat{\log}_{\hat{e}}\hat{a} = \log_e a \times \hat{I}, \tag{3.2.79}$$

and admit the unique solution

$$\hat{\log}_{\hat{e}}\hat{a} = \log_e(\hat{T} \times a) \times \hat{I}, \tag{3.2.80}$$

under which the conventional axioms are preserved,

$$\hat{e}^{\hat{\log}_{\hat{e}}\hat{a}} = \hat{a}, \tag{3.2.81a}$$

$$\hat{\log}_{\hat{e}}\hat{e} = \hat{I}, \;\; \hat{\log}_{\hat{e}}\hat{I} = 0, \tag{3.2.81b}$$

$$\hat{\log}_{\hat{e}}(\hat{a}\hat{\times}\hat{b}) = \hat{\log}_{\hat{e}}\hat{a} + \hat{\log}_{\hat{e}}\hat{b}, \tag{3.2.81c}$$

$$\hat{\log}_{\hat{e}}(\hat{a}\hat{/}\hat{b}) = \hat{\log}_{\hat{e}}\hat{a} - \hat{\log}_{\hat{e}}\hat{b}, \tag{3.2.81d}$$

$$\hat{\log}_{\hat{e}}(\hat{a}^{-\hat{I}}) = -\hat{\log}_{\hat{e}}\hat{a}, \tag{3.2.81e}$$

$$\hat{b}\hat{\times}\hat{\log}_{\hat{e}}\hat{a} = \hat{\log}_{\hat{e}}(\hat{a}^{\hat{b}}). \tag{3.2.81f}$$

The lifting of trigonometric functions is intriguing and instructive (see Chapter 6 of Ref. [6] and Chapter 5 of Ref. [7] whose results in this case require no upgrading). Let $E(r, \delta, R)$ be a conventional two-dimensional Euclidean space with coordinates $r = (x, y)$ on the reals R and polar representation $x = r \times \cos\theta$ and $y = r \times \sin\theta$, $x^2 + y^2 = r^2 \times (\cos^2\theta + \sin^2\theta) = r^2$. Consider now the *iso-Euclidean space* in two dimension

$$\hat{E}(\hat{r}, \hat{\delta}, \hat{R}) : \;\; \hat{\delta} = Diag.(n_1^{-2}, n_2^{-2}), \;\; \hat{I} = Diag.(n_1^2, n_2^2), \tag{3.2.82a}$$

$$\hat{r}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2) \times \hat{I} \;\; \in \;\; \hat{R}. \tag{3.2.82b}$$

Then, the *isopolar coordinates* and related *isotrigonometric functions* on $\hat{E}$ are defined by

$$\hat{x} = \hat{r}\hat{\times}\hat{\cos}\hat{\phi}, \tag{3.2.83a}$$

$$\hat{\cos}\hat{\phi} = n_1 \times \cos(\phi/n_1 \times n_2), \tag{3.2.83b}$$

$$\hat{y} = \hat{r}\hat{\times}\hat{\sin}\hat{\phi}, \tag{3.2.83c}$$

$$\hat{\sin}\hat{\phi} = n_2 \times \sin(\phi/n_1 \times n_2), \tag{3.2.83d}$$

and they preserve the axioms of conventional trigonometric functions, such as,

$$\hat{r}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2) \times \hat{I} \;\; = r^2 \times \hat{I} \in \;\; \hat{R}. \tag{3.2.84}$$

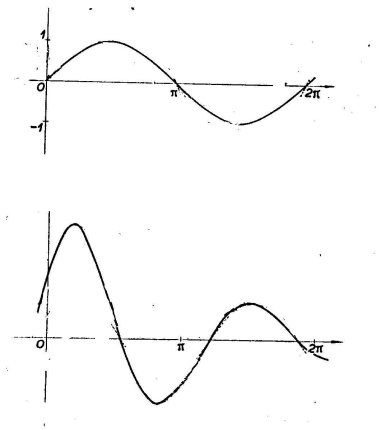

Figure 3.4. A schematic view of the conventional sinus function in Euclidean and iso-Euclidean spaces (top view) and of the projection of a possible example of the isosinus function in the conventional space.

The isotopy of spherical coordinates are treated in detail in Section 5.5 of Ref. [7]. For self-sufficiency of this volume we recall that their definition requires a three-dimensional iso-Euclidean space

$$\hat{E}(\hat{r},\hat{\delta},\hat{R}):\quad \hat{\delta}=Diag.(n_1^{-2},n_2^{-2},n_3^{-2}),\quad \hat{I}=Diag.(n_1^2,n_2^2,n_3^2), \tag{3.2.85a}$$

$$\hat{r}^{\hat{2}}=(x^2/n_1^2+y^2/n_2^2+z^2/n_3^2)\times\hat{I}\quad\in\quad\hat{R}. \tag{3.2.85b}$$

The isotopies of the conventional spherical coordinates in $E(r,\delta,R)$ then yields the following *isospherical coordinates* here presented in the projected form on $\hat{E}(r,\hat{\delta},R)$

$$x=r\times n_1\times\sin(\theta/n_3)\times\sin(\phi/n_1\times n_2), \tag{3.2.86a}$$

$$y=r\times n_2\times\sin(\theta/n_3)\times\cos(\phi/n_1\times n_2), \tag{3.2.86b}$$

$$z=r\times n_3\times\cos(\theta/n_3). \tag{3.2.86c}$$

Via the use of the above general rules, the reader can now construct all needed isofunctions.

The reader should meditate a moment on the isotrigonometric functions. In fact, they provide a *generalization of the Pythagorean theorem to curvilinear triangles.* This is due to the fact that the projection of $\hat{E}(\hat{r},\hat{\delta},\hat{R})$ into the original space $E(r,\delta,R)$ characterizes indeed curvilinear triangles, trivially, because the n-characteristic quantities are functions.

[9]Note, again, that a different definition of isologarithm was assumed in Eq. (6.7.5) of Ref. [6].

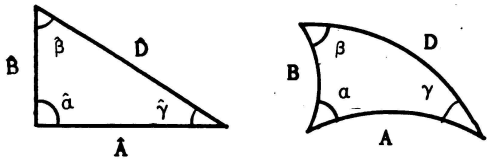

Figure 3.5. An intriguing application of isotrigonometric functions, the generalization of the conventional Pythagorean Theorem (left view) to triangles with curvilinear sides (right view). This is due to the fact that conventional triangles and the Pythagorean theorem are preserved identically on isospaces over isofields, but the projection on conventional Euclidean spaces of straight lines in isospaces over isofields are curves. Therefore in isospace we have expressions such as $\hat{A} = \hat{D}\hat{\times}\text{isosinus}(\hat{\gamma})$ with projections in the conventional space for curvilinear sides $A = D \times \text{isosinus}(\gamma)$, where A and D are now the lengths of the curvilinear sides.

However, the reader is suggested to verify that the *isotriangle*, that is, the image on $\hat{E}$ of an ordinary triangle on E coincides with the latter because the changes caused by the lifting are compensated by the inverse changes of the unit.

By noting that their value must be isonumbers, the *isointegral* can be defined by (here expressed for the simple case of isounits independent form the integration variable)

$$\hat{\int} \hat{dr} = \hat{I} \times \int \hat{T} \times d(r \times \hat{I}) = \hat{I} \times \int dr, \tag{3.2.87}$$

whose extension to the case of isounits with an explicit functional dependence on the integration variables has a complexity that goes beyond the elementary level of this presentation.

Isointegrals and isoexponentiations then permit the introduction of the following *Fourier-Kadeisvili isotransforms*, first studied in Ref. [19,20] (also represented here to avoid excessive mathematical complexities for the simpler case of isounits without an explicit dependence on the integration variables)[10]

$$\hat{f}(\hat{x}) = (\hat{1}\hat{/}2\hat{\pi})\hat{\times}\hat{\int}_{-\infty}^{+\infty} \hat{g}(\hat{k})\hat{\times}\hat{e}^{\hat{i}\hat{\times}\hat{x}}\hat{dk}, \tag{3.2.88a}$$

$$\hat{g}(\hat{x}) = (\hat{1}\hat{/}2\hat{\pi})\hat{\times}\hat{\int}_{-\infty}^{+\infty} \hat{f}(\hat{k})\hat{\times}\hat{e}^{\hat{i}\hat{\times}\hat{x}}\hat{dx}, \tag{3.2.88b}$$

with similar liftings for Laplace transforms, etc. Other transforms can be defined accordingly [6].

[10]The reader should be aware that in most applications of hadronic mechanics the isounits can be effectively approximated into constants, thus avoiding the complex mathematics needed for isointegrals and isotransforms with an explicit functional dependence on the integration variables.

We confirm in this way a major feature of isomathematics, the fact that *Hamiltonian quantities preserve not only their axioms, but also their numerical value under isotopic lifting when defined on isospaces over isofields, and all deviations occur in the projection of the lifting into the original space.*

The explicit construction of the *isodual isofunctional analysis* is also instructive and intriguing because they reveal properties that have essentially remained unknown until recently, such as the fact that *the isofourier transforms are iso-selfdual* (see also Refs. [6,7]).

3.2.5 Isodifferential Calculus and its Isodual

As indicated in Chapter 1, the delay to complete the construction of hadronic mechanics since its proposal in 1978 [5] was due to difficulties in identifying the origin of the non-invariance of its initial formulation, that is, the lack of prediction of the same numerical values for the same quantities under the same conditions, but at different times, a fundamental invariance property fully verified by quantum mechanics.

These difficulties were related to the lack of a consistent isotopic lifting of the familiar quantum mechanical momentum. More particular, *all* aspects of quantum mechanics could be consistently and easily lifted via a nonunitary transforms, except the eigenvalue equation for the linear momentum, as shown by the following lifting

$$p \times \psi(t,r) = -i \times \hbar \times \frac{\partial}{\partial r}\psi(t,r) = K \times \psi(t,r) \to$$
$$\to U \times [p \times \psi(t,r)] = (U \times p \times U^\dagger) \times (U \times U^\dagger)^{-1} \times [U \times \psi(t,r)] =$$
$$= \hat{p} \times \hat{T} \times \hat{\psi}(\hat{t},\hat{r}) = \hat{p}\hat{\times}\hat{\psi}(\hat{t},\hat{r}) =$$
$$= -i \times \hbar \times U[\frac{\partial}{\partial r}\psi(t,r)] = K \times U \times \psi(t,r) = \hat{K}\hat{\times}\hat{\psi}(\hat{t},\hat{r}), \qquad (3.2.89)$$

where $\hat{K} = K \times \hat{I}$ is an isonumber.

As one can see, the initial and final parts of the lifting are elementary. The problem rested in the impossibility of achieving a consisting lifting of the intermediate step, that based on the partial derivative.

In the absence of a consistent isotopy of the linear momentum, the early studies of hadronic mechanics lacked consistent formulations of physical quantities depending on the isomomentum, such as the isotopies of angular momentum, kinetic energy, etc.

The origin of the above problem resulted in being where expected the least, in the *ordinary differential calculus,* and this explains the delay in the resolution of the impasse.

The above problem was finally resolved by Santilli in the second edition of Refs. [6,7] of 1995 (see Section 5.4.B of Vol. I and Section 8.4.A of Vol. II) with

a mathematical presentation in memoir [10] of 1996. The resulting generalization of the ordinary differential calculus, today known as *Santilli's isodifferential calculus,* plays a fundamental role for these studies beginning with the first known structural generalization of Newton's equations in Newtonian mechanics, and then passing to the correct invariant formulation of all dynamical equations of hadronic mechanics.

For centuries, since its discovery by Newton and Leibnitz in the mid 1600, the ordinary differential calculus had been assumed to be independent from the basic unit and field, and the same assumption was kept in the earlier studies on hadronic mechanics, resulting in the lack of full invariance, inability to formulate physical models and other insufficiencies.

After exhausting all other possibilities, an inspection of the differential calculus soon revealed that, contrary to an erroneous belief kept in mathematics for about four centuries, the ordinary differential calculus is indeed dependent on the basic unit and related field.

In this section we review Santilli's isodifferential calculus in its version needed for applications and verifications of hadronic mechanics. This update is recommendable because of various presentations in which the role of $\hat{I}$ and $\hat{T}$ were interchanged, resulting in possible ambiguities that could cause loss of invariance even under the lifting of the differential calculus.

A main feature is that, *unlike all other aspects of hadronic mechanics, the isotopies of the differential calculus cannot be reached via the use of a noncanonical or nonunitary transform, and have to be built via different, yet compatible methods.*

Let $S(r, m, R)$ an N-dimensional metric or pseudo-metric space with *contravariant* coordinates $R = (r^k)$, metric $m = (m_{ij})$, $i, j, k = 1, 2, \ldots, N$, and conventional unit $I = Diag.(1, 1, \ldots, 1)$ on the reals R. Let $f(r)$ be an ordinary (sufficiently smooth) function on S, let dr^k be the differential in the local coordinates, and let $\partial f(r)/\partial r^k$ be its partial derivative.

As it is well known, the connection between covariant and contravariant coordinates is characterized by the familiar rules

$$r^k = m^{kj} \times r_j, \;\; r_i = m_{ik} \times r^k, \tag{3.2.90a}$$

$$m^{ij} = [(m_{qw})^{-1}]^{ij}. \tag{3.2.90b}$$

Let $\hat{S}(\hat{r}, \hat{M}, \hat{R})$ be an isotope of S with N-dimensional isounit $\hat{I} = (\hat{I}^i_j)$, contravariant isocoordinates $\hat{r} = (r^k) \times \hat{I}$ and isometric $\hat{M} = (\hat{M}_{ij}) = (\hat{T}^s_i \times m_{sj}) \times \hat{I}$ on the isoreals $\hat{R}$.

The connection between covariant and contravariant isocoordinates is then given by

$$\hat{r}^k = \hat{M}^{kj} \hat{\times} \hat{r}_j, \;\; \hat{r}_i = \hat{M}_{ik} \hat{\times} \hat{r}^k, \tag{3.2.91a}$$

$$\hat{M}^{ij} = [(\hat{M}_{qw})^{-1}]^{ij}. \tag{3.2.91b}$$

Therefore, on grounds of compatibility with the metric and subject to verifications later on geometric grounds, we have the following:

LEMMA 3.2.5 [10]: Whenever the isounit of contravariant coordinates $\hat{r}^k$ on an isospace $\hat{S}(\hat{r}, \hat{M}, \hat{R})$ is given by

$$\hat{I} = (\hat{I}^i_j(t, r, \ldots)) = 1/\hat{T} = (\hat{T}^j_i)^{-1}, \tag{3.2.92}$$

the isounit for the related covariant coordinates $\hat{r}_k$ is given by its inverse

$$\hat{T} = (\hat{T}^i_j(t, r, \ldots)) = 1/\hat{I} = (\hat{I}^i_j)^{-1}, \tag{3.2.93}$$

and viceversa.

The *ordinary differential* of the contravariant isocoordinates is given by $d\hat{r}^k$ with covariant counterpart $d\hat{r}_k$ and they clearly do not constitute an isotopy. The condition for the preservation of the original axioms and value for constant isounits then leads to the following

DEFINITION 3.2.5 [6,7,10]: The isodifferentials of contravariant and covariant coordinates are given respectively by[11]

$$\hat{d}\hat{r}^k = \hat{d}(r^k \times \hat{I}) = \hat{T}^k_i \times d(r^i \times \hat{I}), \tag{3.2.94a}$$

$$\hat{d}\hat{r}_k = \hat{d}(r_k \times \hat{T}) = \hat{I}^i_k \times d(r_i \times \hat{T}). \tag{3.2.94b}$$

LEMMA 3.2.6 [loc. cit.]: For one-dimensional isounits independent from the local variables, isodifferentials coincide with conventional differentials,

$$\hat{d}\hat{r}^k \equiv dr^k, \quad \hat{d}\hat{r}_k \equiv dr_k. \tag{3.2.95}$$

Note that the above property constitutes a *new invariance of the differential calculus.* Its trivial character explains the reason isodifferential calculus escaped detection for centuries. Needless to say, the above triviality is lost for isounit

[11] It should be noted that the role of $\hat{I}$ and $\hat{T}$ in this definition and that of Ref. [10] are inverted. Also, the reader should keep in mind that, since they are assumed to be Hermitian, isounits can always be diagonalized. In fact, diagonal isounits are sufficient for the verifications and applications of hadronic mechanics, while leaving to the interested reader the formulation of hadronic mechanics according to the broader isodifferential calculus of Refs. [6,7,10].

with nontrivial functional dependence from the local variables as it is generally the case for hadronic mechanics.

The *ordinary derivative* of an isofunction of contravariant coordinates is evidently given by

$$\frac{\partial \hat{f}(\hat{r}^k)}{\partial \hat{r}^k} = \lim_{\hat{d}\hat{r}^k \to 0} \frac{\hat{f}(\hat{r}^k + \hat{d}\hat{r}^k) - \hat{f}(\hat{r}^k)}{\hat{d}\hat{r}^k} \tag{3.2.96}$$

with covariant version

$$\frac{\partial \hat{f}(\hat{r}_k)}{\partial \hat{r}_k} = \lim_{\hat{d}\hat{r}_k \to 0} \frac{\hat{f}(\hat{r}_k + \hat{d}\hat{r}_k) - \hat{f}(\hat{r}_k)}{\hat{d}\hat{r}_k}. \tag{3.2.97}$$

It is then simple to reach the following

DEFINITION 3.2.4 [loc. cit.]: The isoderivative of isofunctions on contravariant and covariant isocoordinates are given respectively by

$$\frac{\hat{\partial} \hat{f}(\hat{r}^k)}{\hat{\partial} \hat{r}^k} = \hat{I}^i_k \times \frac{\partial \hat{f}(\hat{r}^i)}{\partial \hat{r}^k}, \tag{3.2.98a}$$

$$\frac{\hat{\partial} \hat{f}(\hat{r}_k)}{\hat{\partial} \hat{r}_k} = \hat{T}^k_i \times \frac{\partial \hat{f}(\hat{r}_i)}{\partial \hat{r}^k}, \tag{3.2.98b}$$

where the isoquotient is tacitly assumed.[12]

A few examples are now in order to illustrate the axiom-preserving character of the isodifferential calculus. Assume that the isounit is not dependent on r. Then, for $\hat{f}(\hat{r}^k) = \hat{r}^k$ we have

$$\frac{\hat{d}\hat{r}^i}{\hat{d}\hat{r}^j} = \hat{\delta}^i_j = \delta^i_j \times \hat{I}. \tag{3.2.99}$$

Similarly we have

$$\frac{\hat{d}(\hat{r}^i)^{\hat{n}}}{\hat{d}\hat{r}^j} = \hat{\delta}^i_j \times \hat{(}r^i)^{\hat{n}-\hat{1}}. \tag{3.2.100}$$

It is instructive for the reader interested in learning Santilli isodifferential calculus to prove that *isoderivatives in different variables "isocommute" on isospace over isofields,*

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}^i}\frac{\hat{\partial}}{\hat{\partial}\hat{r}^j} = \frac{\hat{\partial}}{\hat{\partial}\hat{r}^j}\frac{\hat{\partial}}{\hat{\partial}\hat{r}^i}, \tag{3.2.101}$$

[12]Note that the isofunction in the numerator contains an additional isounit, $\hat{f} = f \times \hat{I}$, that, however, cancels out with the isounit of the isoquotient, $\hat{/} = / \times \hat{I}$, resulting in expressions (3.2.98). Note also the lack of presence of a *factorized* isounit in the definition of the isodifferentials and isoderivatives, and this explains why the isodifferential calculus cannot be derived via noncanonical or nonunitary transforms.

but their projections on ordinary spaces over ordinary fields do not necessarily "commute".

We are now sufficiently equipped to point out the completion of the construction of hadronic mechanics. First, let us verify the axiom-preserving character of the isoderivative of the isoexponent in a contravariant coordinate for the simple case in which the isounit does not depend on the local variables. In fact, we have the expression

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}}\hat{e}^{\hat{r}} = \hat{I} \times \frac{\partial}{\partial \hat{r}}[\hat{I} \times e^{\hat{T}\times\hat{r}}] = \hat{I} \times \hat{T} \times [\hat{I} \times e^{\hat{T}\times\hat{r}}] = \hat{e}^{\hat{r}}. \tag{3.2.102}$$

Consider now the *isoplanewave* as a simply isotopy of the conventional planewave solution (again for the case in which the isounit does not depend explicitly on the local coordinates),

$$\hat{e}^{\hat{i}\hat{\times}\hat{r}\hat{\times}\hat{K}} = \hat{I} \times e^{i\times\hat{T}\times K\times\hat{r}}, \tag{3.2.103}$$

for which we have the isoderivatives

$$\frac{\hat{\partial}}{\hat{\partial}\hat{r}}\hat{e}^{\hat{i}\hat{\times}\hat{r}\hat{\times}\hat{K}} = \hat{I} \times \frac{\partial}{\partial \hat{r}}[\hat{I} \times e^{i\times\hat{T}\times K\times\hat{r}}] =$$

$$= -i \times K \times \hat{I} \times e^{i\times\hat{T}\times K\times\hat{r}} = \hat{i}\hat{\times}\hat{K}\hat{\times}\hat{e}^{\hat{i}\hat{\times}\hat{r}\hat{\times}\hat{K}}. \tag{3.2.104}$$

We reach in this way the following fundamental definition of *isomomentum*, first achieved by Santilli in Refs. [6,7] of 1995, that completed the construction of hadronic mechanics (its invariance will be proved later on in Section 3.5).

DEFINITION 3.2.7 [6,7,10]: The isolinear momentum on an iso-Hilbert space over the isofield of isocomplex numbers $\hat{C}$ (see Section 3.5 for details) is characterized by

$$\hat{p}_k\hat{\times}\hat{\psi}(\hat{t},\hat{r}) = -\hat{i}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{r}^k}\hat{\psi}(\hat{t},\hat{r}) = -i\hat{\times}\hat{I}^i_k \times \frac{\partial}{\partial \hat{r}^i}\hat{\psi}(\hat{t},\hat{r}) = \hat{K}\hat{\times}\hat{\psi}(\hat{t},\hat{r}). \tag{3.2.105}$$

Comparing the above formulation with Eq. (3.2.89), and in view of invariance (3.2.95),we reach the following

THEOREM 3.2.4 [6,7,10]: Planck's constant $\hbar$ is the fundamental unit of the differential calculus underlying quantum mechanics, i.e., quantum mechanical eigenvalue equations can be identically reformulated in terms of the isodifferential calculus with basic isounit $\hbar$,

$$p \times \psi(t,r) = -i \times \hbar \times \frac{\partial}{\partial r}\psi(t,r) \equiv -i \times \frac{\hat{\partial}}{\hat{\partial} r}\psi(t,r). \tag{3.2.106}$$

In conclusion, *Santilli's isodifferential calculus establishes that the isounit not only is the algebraic unit of hadronic mechanics, but also replaces Planck's constant with an integro-differential operator $\hat{I}$, as needed to represent contact, nonlinear, nonlocal and nonpotential effects.*

More specifically, Santilli's isodifferential calculus establishes that, while in exterior dynamical systems such as atomic structures, we have the conventional quantization of energy, in interior dynamical systems such as in the structure of hadrons, nuclei and stars, we have a superposition of quantized energy level at atomic distances plus continuous energy exchanges at hadronic distances.

Needless to say, all models of hadronic mechanics will be restricted by the condition

$$\lim_{r\to\infty} \hat{I} \equiv \hbar, \tag{3.2.107}$$

under which hadronic mechanics recovers quantum mechanics uniquely and identically.

DEFINITION 3.2.8 [6,7,17]: The isodual isodifferentials are defined by

$$\hat{d}^d\hat{r}^d = (-\hat{d}^\dagger)(-\hat{r}^\dagger) = \hat{d}\hat{r}, \tag{3.2.108}$$

while isodual isoderivatives are given by

$$\hat{\partial}^d\hat{f}^d(\hat{r}^d)\hat{/}^d\hat{d}^d\hat{r}^d = -\hat{\partial}\hat{f}(\hat{r})\hat{/}\hat{d}\hat{r}. \tag{3.2.109}$$

THEOREM 3.2.5 [6,7,17]: Isodifferentials are isoselfduals.

The latter new invariance constitutes an additional, reason why the isodual theory of antimatter escaped attention during the 20-th century.

3.2.6 Kadeisvili's Isocontinuity and its Isodual

The notion of continuity on an isospace was first studied by Kadeisvili [19] in 1992 and it is today known as *Kadeisvili's isocontinuity*. A review up to 1995 was presented in monographs [6,7]. Rigorous mathematical study of isocontinuity has been done by Tsagas and Sourlas [22–23], R. M. Falcón Ganfornina and J. Núñez Valdés [24–26] and others. For mathematical studies we refer the interested reader to the latter papers. For the limited scope of this volume we shall present the notion of isocontinuity in its most elementary possible form.

Let $\hat{f}(\hat{r}) = f(\hat{T}\times\hat{r})\times\hat{I}$ be an isofunction on an isospace $\hat{S}$ over the isofield $\hat{R}$. The *isomodulus* of said isofunction is defined by [19]

$$\hat{|}\hat{f}(\hat{r})\hat{|} = |f(\hat{T}\times\hat{r})|\times\hat{I}. \tag{3.2.110}$$

DEFINITION 3.2.9 [19,20]: An infinite sequence of isofunctions $\hat{f}_1(\hat{r})$, $\hat{f}_2(\hat{r})$, ... *is said to be "strongly isoconvergent" to the isofunction* $\hat{f}(\hat{r})$ *when*

$$\lim_{k\to\infty} \hat{|}\hat{f}_k(\hat{r}) - \hat{f}(\hat{r})\hat{|} \hat{=} \hat{0}. \tag{3.2.111}$$

while the "iso-Cauchy condition" can be defined by

$$\hat{|}\hat{f}_m(\hat{r}) - \hat{f}_n(\hat{r})\hat{|} \ < \ \hat{\delta} = \delta \times \hat{I}, \tag{3.2.112}$$

where δ *is a sufficiently small real number, and* m *and* n *are integers greater than a suitably chosen neighborhood of* δ.

The isotopies of other notions of continuity, limits, series, etc. can be easily constructed (see Refs. [6,7] for physical treatments and Refs. [22–26] for mathematical treatments).

Note that *functions that are conventionally continuous are also isocontinuous.* Similarly, *a series that is strongly convergent is also strongly isoconvergent.* However, a series that is strongly isoconvergent is not necessarily strongly convergent. We reach in this way the following important

THEOREM 3.2.6 [6,7]: Under the necessary continuity and regularity conditions, a series that is conventionally divergent can always be turned into a convergent isoform under a suitable selection of the isounit.

This mathematically trivial property has far reaching implications, e.g., the achievement, for the first time in physics, of convergent perturbative series for strong interactions, which perturbative treatments are conventionally divergent (see Section 3.4).

Similarly, the reader may be interested in knowing that, given a function which is not square-integrable in a given interval, there always exists an isotopy which turns the function into a square-integrable form [6,7]. The novelty is due to the fact that the underlying mechanism is not that of a weight function, but that of altering the underlying field.

The *isodual isocontinuity* is a simple isodual image of the preceding notions of continuity and will be hereon assumed.

3.2.7 TSSFN Isotopology and its Isodual

Topology is the ultimate foundation of quantitative sciences because it identifies on rigorous mathematical grounds the limitations of the ensuing description.

Throughout the 20-th century, all quantitative sciences, including particle physics, nuclear physics, astrophysics, superconductivity, chemistry, biology, etc.,

have been restricted to the use of mathematics based on the conventional *local-differential topology,* with the consequence that the sole admitted representations are those dealing with a finite number of isolated point-like particles.

Since points are dimensionless, they cannot have contact interactions. Therefore, an additional consequence is that the sole possible interactions are those of action-at-a-distance type representable with a potential.

In conclusion, the very assumption of the conventional local-differential topology, such as the conventional topology for the Euclidean space, or the Zeeman topology for the Minkowski space, uniquely and unambiguously restrict the admitted systems to be local, differential and Hamiltonian.

This provided an approximation of systems that proved to be excellent whenever the mutual distances of particles are much greater than their size as it is the case for planetary and atomic systems.

However, the above conditions are the exception and not the rule in nature, because all particles have a well defined extended wavepacket and/or charge distribution of the order of 10^{-13} cm. It is well known in pure and applied mathematics that the representation of the actual shape of particles is impossible with a local-differential topology.

Moreover, once particles are admitted as being extended, there is the emergence of the additional contact, zero-range nonpotential interactions that are nonlocal in the sense of occurring in a finite surface or volume that cannot be consistently reduced to a finite number of isolated points.

Consequently, it is equally know by experts that conventional local-differential topologies cannot represent extended particles at short distances and their nonlocal-nonpotential interactions, as expected in the structure of planets, strongly interacting particles, nuclei, molecules, stars and other interior dynamical systems.

The need to build a new topology, specifically conceived and constructed for hadronic mechanics was suggested since the original proposal [5] of 1978. It was not only until 1995 that the Greek mathematicians Gr. Tsagas and D. S. Sourlas [22,23] proposed the first *isotopology* on scientific record formulated on isospaces over ordinary fields. In 1996, the Italian-American physicist R. M. Santilli [10] extended the formulation to isospaces over isofields. Finally, comprehensive studies on isotopology were conducted by the Spanish Mathematicians R. M. Falcón Ganfornina and J. Núñez Valdés [24,25]. As a result, the new topology is hereon called the *Tsagas-Sourlas-Santilli-Falcón-Núñez isotopology* (or TSSFN Isotopology for short).

The author has no words to emphasize the far reaching implications of the new TSSFN isotopology because, for the first time in the history of science, mathematics can consistently represent the actual extended, generally nonspherical

and deformable shape particles, their densities as well as their nonpotential and nonlocal interactions.

As an example, Newton's equations have remained unchanged in Newtonian mechanics since the time of their conception to represent point-particles. No consistent generalization was possible due to the underlying local-differential topology and related differential calculus. As we shall see in the next section, the isodifferential calculus and underlying isotopology will permit the first known structural generalization of Newton's equations in Newtonian mechanics for the representation of extended particles.

New coverings of quantum mechanics, quantum chemistry, special relativity, and other quantitative sciences are then a mere consequence. Perhaps more importantly, the new clean energies and fuels permitted by hadronic mechanics can see their origin precisely in the TSSFN isotopology, as we shall see later on in this chapter.

In their most elementary possible form accessible to experimental physicists, the main lines of the new isotopology can be summarized as follows. Being nowhere singular, Hermitian and positive-definite, N-dimensional isounits can always be diagonalized into the form

$$\hat{I} = Diag.(n_1^2, n_2^2, \ldots, n_N^2), \quad n_k = n_k(t, r, v, \ldots) > 0, \quad k = 1, 2, \ldots, N. \quad (3.2.113)$$

Consider N isoreal isofields $\hat{R}_k(\hat{n}, \hat{+}, \hat{\times})$ each characterized by the isounit $\hat{I}_k = n_k^2$ with (ordered) Cartesian product

$$\hat{R}^N = \hat{R}_1 \times \hat{R}_2 \times \ldots \times \hat{R}_N. \quad (3.2.114)$$

Since each isofield $\hat{R}_k$ is isomorphic to the conventional field of real numbers $R(n, +, \times)$, it is evident that $\hat{R}^N$ is isomorphic to the Cartesian product of N ordinary fields

$$R^N = R \times R \times \ldots \times R. \quad (3.2.115)$$

Let

$$\tau = \{R^N, K_i\} \quad (3.2.116)$$

be the conventional *topology* on R^N (whose knowledge is here assumed for brevity), where K_i represents the subset of R^N defined by

$$K_i = \{P = (a_1, a_2, \ldots, a_N)/n_i < a_1, a_2, \ldots, a_N < m; \quad n_i, m_i, a_i \in R\}. \quad (3.2.117)$$

We therefore have the following:

DEFINITION 3.2.8 [10,22-25]: The isotopology can be defined as the simple lifting on $\hat{R}^N$ of the conventional topology on R^N, and we shall simply write

$$\hat{\tau} = \{\hat{R}^N, \hat{K}_i\}, \quad (3.2.118a)$$

$$\hat{K}_i = \{\hat{P} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N) \hat{/} \hat{n}_i < \hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N < \hat{m};\ \ \hat{n}_i, \hat{m}_i, \hat{a}_i \in \hat{R}\}. \quad (3.2.118b)$$

As one can see, the above isotopology coincides everywhere with the conventional topology of R^N except at the isounit $\hat{I}$. In particular, $\hat{\tau}$ is everywhere local-differential, except at $\hat{I}$ which can incorporate nonlocal integral terms.

It is evident that isotopology can characterize for the first time in scientific history, extended, nonspherical and deformable particles. In fact, for the case of three-dimensions in diagonal representation (3.2.113), we have the characterization of deformable spheroidal ellipsoids with variable semiaxes n_1^2, n_2^2, n_3^2 depending on local quantities, such as energy, density, pressure, etc. For the case of four-dimension the quantity n_4^2 represents, for the first time in scientific record, the density of the particle considered[13].

The reader should be aware that the above formulation of the isotopology is the simplest possible one, being restricted to the description of *one* isolated *isoparticle,* that is, an extended and nonspherical particle on isospace over isofields that, as such, has no interactions.

Consequently, numerous generalizations of the above formulations are possible and actually needed for hadronic mechanics. The first broadening is given by the case of *two* or more isoparticles in which case the basic isounit is given by the Cartesian product of two isounits of type (3.2.113). The second broadening is given by exponential factors incorporating nonlinear integral terms as in the general isounit (3.1.19). In the preceding formulation, these exponential factors have been incorporated in the n's since they are common factors.

A lesser trivial broadening of the above formulation of isotopology is given by *nondiagonal isounits* that are capable of representing nonspheroidal shapes and other complex geometric occurrences (see in Ref. [6], page 213 the case of a nondiagonal isotopy contracting the dimensions from three to one, also reviewed in the next section). The study of the latter more general formulations of isotopology is left to the interested reader.

DEFINITION 3.2.11 [22-25]: An isotopological isospace $\hat{\tau}(\hat{R}^N)$ is the isospace $\hat{R}^N$ equipped with the isotopology $\hat{\tau}$. An isocartesian isomanifold $\hat{M}(\hat{R}^N)$ is the isotopological isospace $\hat{M}(\hat{R}^N)$ equipped with a isovector structure, an isoaffine structure and the mapping

$$\hat{F} : \hat{R}^N \to \hat{R}^N; \quad \hat{a} \to \hat{f}(\hat{a}),\ \ \forall \hat{a} \in \hat{R}^N. \quad (3.2.119)$$

[13]The reader is encouraged to inspect any desired textbook in particle physics and verify the complete lack of representation of the density of the particle considered.

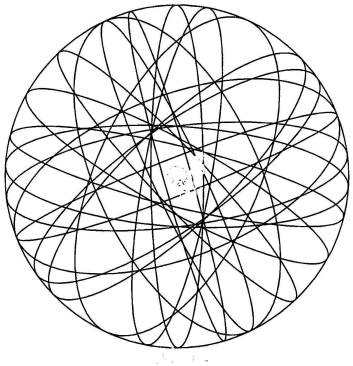

Figure 3.6. A schematic view of the "isosphere", namely, the perfect sphere on isospace over isofield represented by isoinvariant (3.2.121), that is assumed as the geometric representation of hadrons used in this monograph. The actual nonspherical and deformable shape of hadrons is obtained by projecting the isosphere in our Euclidean space, as illustrated in the last identify of Eq. (3.2.122).

An iso-Euclidean isomanifold $\hat{M}(\hat{E}(\hat{r}, \hat{\delta}, \hat{R}))$ occurs when the N-dimensional isospace $\hat{E}$ is realized as the Cartesian product (3.2.106) and equipped with isotopology (3.2.118) with basic isounit (3.2.113).

The *isodual isotopology* and related notions can be easily constructed with the isodual map (3.2.15) and its explicit study is left as an instructive exercise for the interested reader.

3.2.8 Iso-Euclidean Geometry and its Isodual

The isotopies of the Euclidean space and geometry were introduced for the first time by Santilli in Ref. [14] of 1983 as a particular case of the broader isotopies of the Minkowski space and geometry treated in the next section.

The same isotopies were then studied in various works by the same author and a comprehensive treatment was presented in Chapter 5 of Vol. I [6]. These isotopies are today known as the *Euclid-Santilli isospace and isogeometry.* The presentation of Vol. I will not be repeated here for brevity. We merely limit ourselves to outline the main aspects for minimal self-sufficiency of this monograph.

Consider the fundamental isospace for nonrelativistic hadronic mechanics, the three-dimensional *Euclid-Santilli isospace* with contravariant isocoordinates $\hat{r}$, isometric $\hat{\delta}$ over the isoreals $\hat{R} = \hat{R}(\hat{n}, \hat{+}, \hat{\times})$ (see Section 3.3)

$$\hat{E}(\hat{r}, \hat{\delta}, \hat{R}) : \quad \hat{r} = (\hat{r}^k) = (\hat{x}, \hat{y}, \hat{z}) = (r^k) \hat{\times} \hat{I} = (x, y, z) \hat{\times} \hat{I}, \quad k = 1, 2, 3; \quad (3.2.120a)$$

$$\hat{I} = Diag.(n_1^2, n_2^2, n_3^2) = 1/\hat{T} > 0, \quad n_k = n_k(t, r, v, a, \mu, \tau, \ldots) > 0, \qquad (3.2.120b)$$

$$\hat{\Delta} = \hat{\delta} \times \hat{I}; \;\; \hat{\delta} = \hat{T} \times \delta = Diag.(n_1^{-2}, n_2^{-2}, n_3^{-2}), \tag{3.2.120c}$$

with basic isoinvariant on $\hat{E}$

$$\hat{r}^{\hat{2}} = \hat{r}^i \hat{\times} \hat{\Delta}_{ij} \hat{\times} \hat{r}^j = \hat{r}^i \times \hat{\delta}_{ij} \times \hat{r}^j = \hat{r}^i \times (\hat{T}_i^k \times \delta_{kj}) \times \hat{r}^j =$$

$$= \hat{x}^{\hat{2}} + \hat{y}^{\hat{2}} + \hat{z}^{\hat{2}} = \frac{\hat{x}^2}{n_1^2} + \frac{\hat{y}^2}{n_2^2} + \frac{\hat{z}^2}{n_3^2} \; \in \; \hat{R}. \tag{3.2.121}$$

and projection on the conventional Euclidean space

$$r^2 = \frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} \; \in \; R. \tag{3.2.122}$$

where the scalar functions n_k, besides being sufficiently smooth and positive-definite, have an unrestricted functional dependence on time t, coordinates r, velocities v, acceleration a, density μ, temperature τ, and any needed local variable.

The *Euclid-Santilli isogeometry* is the geometry of the above isospaces. A knowledge of the following main features is essential for an understanding of nonrelativistic hadronic mechanics.

Since the isospaces $\hat{E}$ are all locally isomorphic to the conventional Euclidean space $E(r, \delta, R)$, it is evident that *the Euclid-Santilli isogeometry verifies all axioms of the conventional geometry,* as proved in detail in Section 5.2 of Vol. I [6]. In fact, the conventional and isotopic geometries coincide at the abstract, realization free level to such an extent that they can be expressed with the same abstract symbols, the differences between the conventional and the isotopic geometries emerging only in the selected realizations of said abstract axioms.

Note that, while the Euclidean space and geometry are unique, there exist an infinite family of different yet isomorphic Euclid-Santilli isospaces and isogeometries, evidently characterized by different isometrics in three dimension and signature $(+,+,+)$.

Recall from Section 3.2.3 that the structure of the basic invariant is given by Eq. (3.2.66). Therefore, the *isosphere*, namely, the image on $\hat{E}$ of the perfect sphere on E remains a perfect sphere. However, the projection of the isosphere on the original space E is a spheroidal ellipsoid, as clearly indicated by invariant (3.2.121). Therefore, *the isosphere on isospace over isofields unifies all possible spheroidal ellipsoids on ordinary spaces over ordinary fields.* These features are crucial to understand later on the reconstruction of the *exact* rotational symmetry for *deformed* spheres (see Fig. 3.6).

Since the functional dependence of the isometric is unrestricted except verifying the condition of positive-definiteness, it is easy to see that *the Euclid-Santilli isogeometry unifies all possible three-dimensional geometries with the signature)*

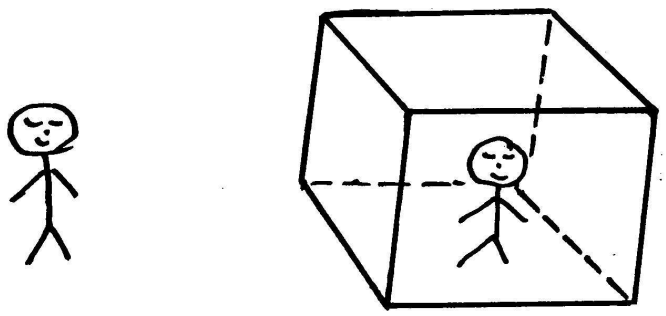

Figure 3.7. A schematic view of the "space isocube", namely, an ordinary cube inspected by two observers, an exterior observer in Euclidean space with basic units of measurements $I = Diag.(1\ \text{cm}, 1\ \text{cm}, 1\ \text{cm})$ and an interior observer on isospace with basic isounits $\hat{I} = Diag.(n_x^2\ \text{cm}, n_y^2\ \text{cm}, n_z^2\ \text{cm})$. It it then evident that, if the exterior observer measures, for instance, the sides of the cube to be $3m$, the interior observers measures different length that can be bigger or smaller than $3m$ depending on whether the isounit is smaller or bigger, respectively, than the original unit. Also, for the case of the Euclidean observer, the units in the three space directions are the same, while the corresponding isounits have different values for different directions. Therefore, the same object appears as a cube of a given size to the external observer, while having a completely different shape and size for the internal observer.

$(+,+,+)$, thus including as particular cases the Riemannian, Finslerian, non-Desarguesian and other geometries. As an example, the Riemannian metric $g_{ij}(r) = g^t$ is a trivial particular case of Santilli's isometric $\hat{\delta}_{ij}(t, r, \ldots)$. This occurrence has profound physical implications that will be pointed out in Section 3.5.

Yet another structural difference between conventional and isotopic geometries is that the former has the same unit for all three reference axes. In fact, the geometric unit $I = Diag.(1, 1, 1)$ is a dimensionless representation of the selected units, for instance, $I = diag.(1\,\text{cm}, 1\,\text{cm}, 1\,\text{cm})$. In the transition to the isospace, the units are different for different axes and we have, for instance, $\hat{I} = Diag.(n_1^2\,\text{cm}, n_2^2\,\text{cm}, n_3^2\,\text{cm})$. It then follows that *shapes detected by our sensory perception are not necessarily absolute, in the sense that they may appear basically different for an isotopic observer* (see Fig. 3.7).

Note that in the conventional space $E(r, \delta, R)$ there are two trivially different trivial units, namely, the unit $I = +1$ of the base field R and the unit $I = Diag.(1, 1, 1)$ of the space, related geometry and symmetries. The isotopies have identified for the first time the fact that *the unit of the space must coincide with the unit of the base field.*

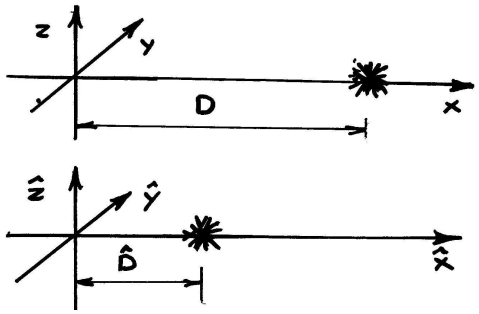

Figure 3.8. A schematic view of the geometric propulsion studied in greater details in Chapter 12, here illustrated via the contraction of distances in the transition from our coordinmates to the isotopiuc ones.

In fact, the isounit of isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{R})$ must coincide with the isounit of the isofield $\hat{R}$. It is then evident that, at the limit $\hat{I} \to I = Diag.(1, 1, 1)$ the unit matrix $I = Diag.(1, 1, 1)$ must be the unit of both the Euclidean space and of the basic field. This implies a trivial reformulation of R that is ignored hereon.

Another important notion is that of *isodistance* between two points P_1 and P_2 on $\hat{E}$ that can be defined by the expression

$$\hat{D}^2_{1-2} = (\hat{x}_1 - \hat{x}_2)^2/n_1^2 + (\hat{y}_1 - \hat{y}_2)^2/n_2^2 + (\hat{z}_1 - \hat{z}_2)^2/n_3^2. \tag{3.2.123}$$

It then follows that *local alterations of the space geometry cause a change in the distance,* an occurrence first identified in Ref. [6] as originating from a lifting of the units, and today known as *isogeometric locomotion* studied in Chapter 13. We are here referring to a new form of non-Newtonian locomotion in which objects can move without the application of a force or, equivalently, without any application of the principle of action and reaction (see Figure 3.8).

Finally, it is important to point out that *the dimensionality of the original Euclidean space is not necessarily preserved under isotopies.* This occurrence constitutes another intriguing epistemological feature because isotopies are axiom-preserving. Therefore, our senses based on the three Eustachian lobes perceive no difference in dimension between a conventional and an isotopic shape.

The epistemological question raised by the isotopies is then whether our perception of space as three-dimensional is real, in the sense of being intrinsic, or it is a mere consequence of our particular sensory perception, with different dimensions occurring for other observers.[14]

[14]As we shall see in Chapter 4, an even deeper epistemological issue emerges from our hyper-isotopies in which the unit is characterized by a *set* of values. In this case, space can be "three-dimensional" yet be "hyper-dimensional", in the sense that each dimension can be multi-valued.

The occurrence was discovered by Santilli in Ref. [6], page 213, via the following isotopic element

$$\hat{\mathbf{T}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \tag{3.2.124}$$

that is positive definite since $\text{Det}\,\hat{T} = 1$, thus being a fully acceptable isotopic element.

It is easy to see that the isoinvariant of the Euclid-Santilli isospace characterized by the above non-diagonal isotopy is given by

$$\hat{r}^{\hat{2}} = \hat{r}^i \times \hat{T}_i^k \times \delta_{kj} \times \hat{r}^j =$$

$$= \hat{x} \times \hat{z} + \hat{y} \times \hat{z} - \hat{z} \times \hat{y} = \hat{x} \times \hat{x}, \tag{3.2.125}$$

namely, in this case the *isotopic image of the three-dimensional Euclidean space is one dimensional.*

This occurrence provides another illustration of the fact that, despite their simplicity, the geometric implications of the isotopies are rather deep indeed.

The *isodual Euclid-Santilli isospace* in three dimension can be represented by the expressions

$$\hat{E}^d(\hat{r}^d, \hat{\Delta}^d, \hat{R}^d): \quad \hat{r}^d = (-\hat{x}, -\hat{y}, -\hat{z}); \tag{3.2.126a}$$

$$\hat{I}^d = Diag.(-n_1^2, -n_2^2, -n_3^2) = -1/\hat{T} > 0, \quad n_k = n_k(t, r, \ldots) > 0,$$

$$\hat{\Delta}^d = \hat{\delta}^d \times \hat{I}, \quad \hat{\delta}^d = \hat{T}^d \times^d \delta^d = Diag.(-n_1^{-2}, -n_2^{-2}, -n_3^{-2}), \tag{3.2.126b}$$

with isodual isoinvariant on $\hat{R}^d$

$$\hat{r}^{d^{\hat{2}^d}} = \hat{r}^{di} \hat{\times}^d \hat{\Delta}_{ij}^d \hat{\times}^d \hat{r}^{dj} =$$

$$= -\hat{x}^{d^{\hat{2}^d}} - \hat{y}^{d^{\hat{2}^d}} - \hat{z}^{d^{\hat{2}^d}} \quad \in \quad \hat{R}^d. \tag{3.2.127}$$

and projection on the isodual Euclidean space

$$r^{d^2} = (-x^2/n_1^2 - y^2/n_2^2 - z^2/n_3^2) \times \hat{I} \quad \in \quad R^d. \tag{3.2.128}$$

A study of the *isodual Euclid-Santilli isogeometry* from Vol. I [6] is essential for a study of antimatter in interior conditions.

3.2.9 Minkowski-Santilli Isogeometry and its Isodual

3.2.9A. Conceptual Foundations. The isotopies of the Minkowski space and geometry are the main mathematical methods of relativistic hadronic mechanics, because they are at the foundations of the Poincaré-Santilli isosymmetry, and related broadening of special relativity for relativistic interior dynamical systems.

The isotopies of the Minkowski space and geometry were first proposed by Santilli in Ref. [14] of 1983 and then studied in numerous papers (see monographs [6,7,14,15] and papers quoted therein) and are today known as *Minkowski-Santilli isospace and isogeometry.*

Due to their fundamental character, the new spaces and geometry were treated in great details in Refs. [6,7], particularly in the second edition of 1995, and that presentation is here assumed as known for brevity.

The primary purpose of this section is to identify the most salient advances occurred since the second edition of Refs. [6,7] with particular reference to the geometric treatment of gravitation.

In essence, the original efforts in the construction of relativistic hadronic mechanics were based on *two different isotopies,* the isotopies of the Minkowskian geometry for nongravitational profiles, and the isotopies of the Riemannian geometry for gravitational aspects. The presentation of Refs. [6,7] was based on this dual approach.

Subsequently, it became known that *the isotopies of the Riemannian geometry could not resolve the catastrophic inconsistencies of gravitation identified in Chapter 1 because they are inherent in the background Riemannian treatment itself, thus persisting under isotopies.*

The resolution of these catastrophic inconsistencies was finally reached by Santilli in Ref. [26] of 1998 via the *unification of the Minkowskian and Riemannian geometries into Minkowski-Santilli isogeometry.* In fact, the isometric of the latter geometry admits, as a particular cases, all possible Riemannian metrics.

Consequently, it became clear that the various methods used for the Riemannian geometry (such as covariant derivative, Christoffel symbols, etc.) are inapplicable to the conventional Minkowski space evidently because flat, but the same methods are fully applicable to the Minkowski-Santilli isogeometry.

The achievement of a geometric unification of the Minkowskian and Riemannian geometries reached in memoir [26] permitted truly momentous advances, such as the geometric unification of the special and general relativities, an axiomatically consistent grand unification of electroweak and gravitational interactions, the first known axiomatically consistent operator form of gravity, and other basic advances reviewed in Section 3.5.

3.2.9B. Minkowski-Santilli Isospaces. We now review in this subsection the foundations of the Minkowski-Santilli isospaces by referring interested readers to volumes [6,7] for details.

DEFINITION 3.2.12 [26]: Consider the conventional Minkowski space

$$M = M(x, \eta, R) : x = (x^\mu) = (r, c_o t), \tag{3.2.129a}$$

$$x^\mu = \eta^{\mu\nu} \times x_\nu, \;\; x_\mu = \eta_{\mu\nu} \times x^\nu, \tag{3.2.129b}$$

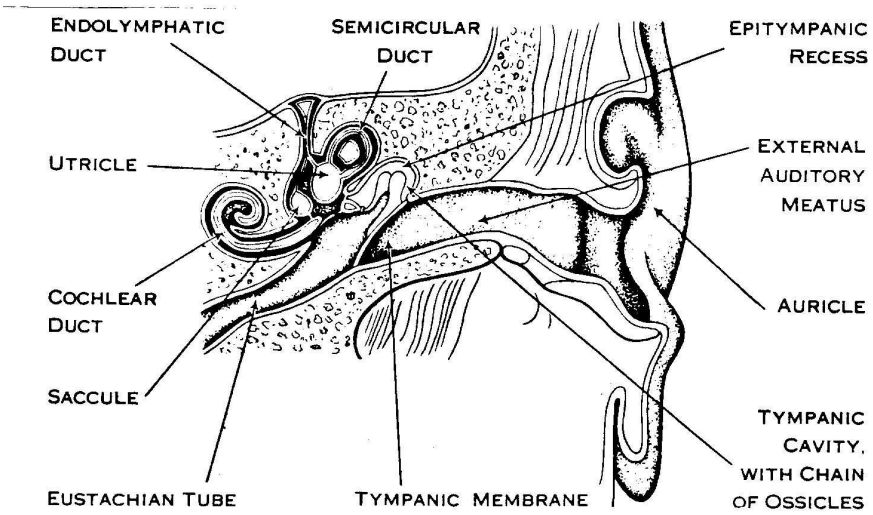

Figure 3.9. A view of the three Eustachian lobes allowing us to perceive three-dimensional shapes. The intriguing epistemological issue raised by the Euclid-Santilli isogeometry is whether living organisms with different senses perceive the same object with different shape and size than ours. As illustrated with the isobox of Figure 3.7, the same object can appear with dramatically different shapes and sizes to a conventional and an isotopic observer, as well as in dimension different than the original ones, as illustrated in the text. Another illustration of the meaning and importance of isotopies is that being axiom-preserving, different shapes, sizes and dimensions on isospaces are rendered compatible with our sensory perception.

where c_o is the speed of light in vacuum, metric

$$\eta = (\eta_{\mu\nu}) Diag.(+1, +1, +1, -1), \quad \eta^{\mu\nu} = [(\eta_{\alpha,\beta})^{-1}]^{\mu\nu}, \tag{3.2.130}$$

basic unit

$$I = Diag.(+1, +1, +1, +1), \tag{3.2.131}$$

and invariant on the reals

$$x^2 = x^\mu \times x_\mu = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \quad \in \quad R = R(n, +, \times), \tag{3.2.132}$$

$$\mu, \nu, \alpha, \beta = 1, 2, 3, 4.$$

Then, the Minkowski-Santilli isospaces can be defined by isotopies

$$\hat{M} = \hat{M}(\hat{x}, \hat{G}, \hat{R}) : \hat{x} = (\hat{x}^\mu) = (r, c_o t) \times \hat{I}, \tag{3.2.133a}$$

$$\hat{x}^\mu = \hat{G}^{\mu\nu} \hat{\times} \hat{x}_\nu, \quad \hat{x}_\mu = \hat{G}_{\mu\nu} \hat{\times} \hat{x}^\nu, \tag{3.2.133b}$$

with isometric on isospaces over isofields

$$\hat{G} = \hat{\eta} \times \hat{I} = (\hat{T}^{\rho}_{\mu} \times \eta_{\rho\nu}) \times \hat{I} =$$

$$= Diag.(\hat{T}_{11}, \hat{T}_{22}, \hat{T}_{33}, \hat{T}_{44}) \times \hat{I} \ \in \ \hat{R} = \hat{R}(\hat{n}, \hat{+}, \hat{\times}), \tag{3.2.134a}$$

$$\hat{G}^{\mu\nu} = [(\hat{G}_{\alpha,\beta})^{-1}]^{\mu\nu}, \tag{3.2.134b}$$

and isounit

$$\hat{I} = Diag.(\hat{T}^{-1}_{11}, \hat{T}^{-1}_{22}, \hat{T}^{-1}_{33}, \hat{T}^{-1}_{44}), \tag{3.2.135}$$

where $\hat{T}_{\mu\nu}$ are positive-definite functions of spacetime coordinates x, velocities v, accelerations a, densities μ, temperature τ, wavefunctions, their derivatives and their conjugates and any other needed quantity

$$\hat{T}_{\mu\nu} = \hat{T}_{\mu\nu}(x, v, a, \mu, \tau, \psi, \psi^{\dagger}, \partial\psi, \partial\psi^{\dagger}, \ldots) \ > \ 0 \tag{3.2.136}$$

isoinvariant on isospaces over the isofield of isoreal numbers

$$\hat{x}^{\hat{2}} = \hat{x}^{\mu} \hat{\times} \hat{x}_{\mu} = (\hat{x}^{\mu} \hat{\times} \hat{G}_{\mu\nu} \hat{\times} \hat{x}^{\nu}) \times I \ \in \ \hat{R} = R(\hat{n}, \hat{+}, \hat{\times}) \tag{3.2.137}$$

with projection in our spacetime

$$\hat{M}(x, \hat{\eta}, R) : x = (x^{\mu}) \times I, \tag{3.2.138a}$$

$$x^{\mu} = \hat{\eta}^{\mu\nu} \times x_{\nu}, \ \ x_{\mu} = \hat{\eta}_{\mu\nu} \times x^{\nu}, \tag{3.2.138b}$$

metric over the field of real numbers

$$\hat{\eta} = (\hat{\eta}_{\mu\nu}) = (\hat{T}^{\rho}_{\mu} \times \eta_{\rho\nu}) = Diag.(\hat{T}_{11}, \hat{T}_{22}, \hat{T}_{33}, \hat{T}_{44}) \in R = R(n, +, \times), \tag{3.2.139a}$$

$$\hat{\eta}^{\mu\nu} = [(\hat{\eta}_{\alpha,\beta})^{-1}]^{\mu\nu}, \tag{3.2.139b}$$

and invariant in our spacetime over the reals

$$x^2 = x^{\mu} \times x_{\nu} = x^{\mu} \times \hat{\eta}_{\mu\nu}(x, v, a, \mu, \tau, \psi, \psi^{\dagger}, \partial\psi, \partial\psi^{\dagger}, \ldots) \times x^{\nu} =$$

$$= T_{11} \times x_1^2 + \hat{T}_{22} \times x_2^2 + \hat{T}_{33} \times x_3^2 - \hat{T}_{44} \times x_4^2 \ \in \ R. \tag{3.2.140}$$

Note that all scalars on M must be lifted into *isoscalars* to have meaning for $\hat{M}$, i.e., they must have the structure of the isonumbers $\hat{n} = n \times \hat{I}$. This condition requires the re-definition $x \ \rightarrow \ \hat{x} = x \times \hat{I}$, $\eta_{\mu\nu} \ \rightarrow \ \hat{G}_{\mu\nu} = \hat{\eta}_{\mu\nu} \times \hat{I}$, $x^2 \rightarrow \hat{x}^{\hat{2}}$, etc.

The reader interested in learning in depth the new isogeometry should also study from the preceding sections the different realizations of the isometry whether realized in the original Minkowskian coordinates or in the isocoordinates, since the functional dependence is different in these two cases.

Note however the redundancy in practice for using the forms $\hat{x} = x \times \hat{I}$ and $\hat{G} = \hat{\eta} \times \hat{I}$ because of the identity $\hat{x}^{\hat{2}} = \hat{x}^{\mu} \hat{\times} \hat{G}_{\mu\nu} \hat{\times} \hat{x}^{\nu} \equiv (x^{\mu} \times \hat{\eta}_{\nu} \times x^{\nu}) \times \hat{I}$. For simplicity we shall often use the conventional coordinates x and the isometric will be referred to $\hat{\eta} = \hat{T} \times \eta$. The understanding is that the full isotopic formulations are needed for mathematical consistency.

A fundamental property of the infinite family of generalized spaces (3.2.133) is the lifting of the basic unit $I \ \rightarrow \ \hat{I}$ while the metric is lifted of the *inverse* amount, $\eta \ \rightarrow \ \hat{\eta} = \hat{T} \times \eta$, $\hat{I} = \hat{T}^{-1}$. This implies the preservation of all original axioms, and we have the following:

THEOREM 3.2.7 [26]: All infinitely possible isominkowski spaces $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ over the isofields $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ with a common positive-definite isounit $\hat{I}$ preserve all original axioms of the Minkowski space $M(x, \eta, R)$ over the reals $R(n, +, \times)$.

The nontriviality of the lifting is that *the Minkowskian axioms are preserved under an arbitrary functional dependence of the metric $\hat{\eta} = \hat{\eta}(x, v, a, \mu, \tau, \ldots)$* for which the sole x-dependence of the Riemannian metric $g(x)$ is only a simple *particular case*. As a matter of fact, we have the following

THEOREM 3.2.8 [26]: Minkowski-Santilli isospaces are "directly universal" in spacetime, that is, they represent all infinitely possible spacetimes with signature $(+,+,+,-)$ ("universality"), directly with the isometric and without any use of the transformation theory ("direct universality").

Note that all possible "deformations" of the Minkowski space are also particular cases of the above isospaces. However, the former are still referred to the old unit I, thus losing the isomorphic between deformed and Minkowski spaces, while the isotopies preserve the original axioms by construction.

A fundamental physical characteristic of the Minkowski-Santilli isospaces is that *it alters the units of space and time*. Recall that the unit

$$I = Diag.(\{1, 1, 1\}, 1)$$

of the Minkowski space represents in a dimensionless form the units of the three Cartesian axes and time, e.g., $I = (+1\,\text{cm}, +1\,\text{cm}, +1\,\text{cm}, +1\,\text{sec})$. Recall also that the Cartesian space-units are *equal for all axes*.

Consider now the isospaces, and recall that $\hat{I}$ is positive-definite. Consequently, we have the following lifting of the units in which the $\hat{T}_{\mu\mu}$ quantities are reinterpreted as constants

$$I = (+1\,\text{cm}, +1\,\text{cm}, +1\,\text{cm}, +1\,\text{sec}) \rightarrow$$
$$\rightarrow \hat{I} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) = 1/\hat{T}, \ \hat{I}_{\mu}^{\mu} = n_{\mu}^2, n_{\mu} > 0. \qquad (3.2.141)$$

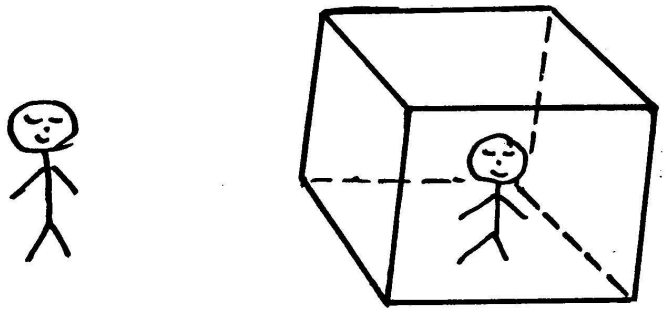

Figure 3.10. A view of the "spacetime isocube" characterized by the "space isocube" of Figure 3.7 now inspected in two spacetimes, the conventional Minkowski spacetime in the exterior and Santilli isospacetime in the interior. In addition to the variations of shape, size and dimensions indicated in Figure 3.7, the same object can be in different times for the two observers, all in a way fully compatible with our sensory perception. Consequently, seeing in a telescope a far away quasar or galaxy it does not mean that astrophysical structure is necessarily in our time, since it could be evolving far away in the future or in the past.

This means that, not only the original units are now lifted into arbitrary positive values, but the *units of different space axes generally have different values.* Jointly, the components of the metric are lifted by the *inverse* amounts n_μ^{-2}. This implies the preservation on $\hat{M}$ over $\hat{R}$ of the original *numerical* values on M over R, including the crucial preservation of the maximal causal speed c_o, as we shall see in Section 3.5.

Note also the necessary condition that *the isospace and isofield have the same isounit* $\hat{I}$. This condition is absent in the conventional Minkowski space where the unit of the space is the unit *matrix* $I = Diag.(1, 1, 1, 1)$, while that of the underlying field is the *number* $I = +1$. Nevertheless, the latter can be trivially reformulated with the common unit matrix I, by achieving in this way the form admitted as a particular case by the covering isospaces

$$M(x, \eta, R) : \; x = \{x^\mu \times I\}, \; x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \in R. \tag{3.2.142}$$

The structure of both the conventional and isotopic invariants is therefore given by Theorem 3.2.66, namely

$$\text{Basic Invariant} = (\text{Length})^2 \times (\text{Unit})^2, \tag{3.2.143}$$

which illustrates more clearly the preservation under the dual lifting $\eta \to \hat{\eta} = \hat{T} \times \eta$ and $I \to \hat{I} = 1/\hat{T}$ of the original axioms as well as numerical values.

THEOREM 3.2.9 [6,7,26]: Conventional and isotopic symmetries of spacetime are 11-dimensional.

Proof. In addition to the 10-dimensionality of the Poincaré symmetry, there is an additional 11-th dimensionality characterized by the isotransform

$$\eta \to \hat{\eta} = \eta/n^2, \;\; I \to \hat{I} = n^2 \times I, \tag{3.2.144}$$

where n is a non-null constant. **q.e.d.**

Note the crucial role of Santilli's isonumbers in the above property. This explains why the 11-th dimensionality remained undiscovered throughout the 20-th century.

A significant difference between the conventional space M and its isotopes $\hat{M}$ is that the former admit only *one* formulation, the conventional one, while the latter admit *two* formulations: that on isospace itself (i.e., expressed with respect to the isounit $\hat{I}$) and its *projection* in the original space M (i.e., expressed with respect to the conventional unit I).

Note that the projection of $\hat{M}(\hat{x}, \hat{M}, \hat{R})$ into $M(x, \eta, R)$ is not a conformal map, but an *inverse isotopic map* because it implies the transition from generalized units and fields to conventional units and fields.

The axiomatic motivation for constructing the isotopies of the Minkowskian geometry is that any modification of the Minkowski metric requires the use of *noncanonical transforms* $x \;\to\; x'(x)$,

$$\eta_{\mu\nu} \;\to\; \hat{\eta}_{\mu\nu} = \frac{\partial x'^{\alpha}}{\partial x^{\mu}} \eta_{\alpha\beta} \frac{\partial x'^{\beta}}{\partial x^{\nu}} \neq \eta_{\mu\nu}, \tag{3.2.145}$$

and this includes the case of the transition from the Minkowskian metric η to the Riemannian metric $g(x)$.

In turn, all noncanonical theories, thus including the Riemannian geometry, do not possess invariant units of space and time, thus having the catastrophic inconsistencies studied in Chapter 1. A primary axiomatic function of the isospace is that of restoring the invariance of the basic units, as established by the Poincaré-Santilli isosymmetry.

This is achieved by embedding all noncanonical content in the generalization of the unit. Invariance for noncanonical structures such as Riemannian metrics is then assured by the fact indicated earlier that, whether conventional or generalized, the unit is the basic invariant of any theory.

Stated in different terms, a primary axiomatic difference between the special and general relativities is that the time evolution of the former is a *canonical transform*, thus implying the majestic mathematical and physical consistency of special relativity recalled in Chapter 1, while the time evolution of the latter is a *noncanonical transform*, thus implying a number of unresolved problematic aspects that have been lingering throughout this century.

The reformulation of the Riemannian geometry in terms of the Minkowskian axioms is the sole possibility known to this author for achieving axiomatic consistency under a nontrivial functional dependence of the metric.

In summary, Minkowski-Santilli isospaces have the following primary applications. First, they are used for a re-interpretation of the Riemannian metrics $g(x)$ for the particular case

$$\hat{\eta} = \hat{\eta}(x) = g(x) \tag{3.2.146}$$

characterizing *exterior gravitational problems in vacuum*. Second, the same isospaces are used for the characterization of *interior gravitational problems* with isometrics of unrestricted functional dependence

$$\hat{\eta} = \hat{\eta}(x, v, a, \mu, \tau, \ldots) = g(x, v, a, \mu, \tau, \ldots) \tag{3.2.147}$$

while preserving the original Minkowskian axioms.

Since the explicit functional dependence is inessential under isotopies, our studies will be generally referred to the interior gravitational problem. Unless otherwise stated, only diagonal realizations of the isounits will be used hereon for simplicity. An example of nondiagonal isounits inherent in a structure proposed by Dirac is indicated in Section 3.5. More general liftings of the Minkowski space of the so-called *genotopic and multivalued-hyperstructural type* will be indicated in Chapter 4.

3.2.9C. Isoderivative, Isoconnection, and Isoflatness. In the preceding subsections we have presented the *Minkowskian* aspects of the new isogeometry. We are now sufficiently equipped to present the novel part of the Minkowski-Santilli isogeometry, its *Riemannian* character as first derived in Ref. [26].

Our study is strictly in local coordinates representing the *fixed* frame of the observer without any un-necessary use of the transformation theory or abstract treatments. Our presentation will be as elementary as possible without reference to advanced topological requirements, such as Kadeisvili's isocontinuity (Section 3.2.6), isomanifolds and related TSSFN isotopology (Section 3.2.7) .

Also, our presentation is made, specifically, for the (3+1)-dimensional isospacetime, with the understanding that the extension to arbitrary dimensions and signatures or signatures different than the conventional one $(+,+,+,-)$ is elementary, and will be left to interested readers.

Let $\hat{M}(\hat{x}, \hat{G}, \hat{R})$ be a Minkowski-Santilli isospace and let $\hat{M}(x, \hat{\eta}, R)$ be its projection in our spacetime as per Definition 3.2.12. To illustrate the transition from isocoordinates $\hat{x}$ to conventional spacetime coordinates x, we shall denote the projection $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, R)$. This notation emphasizes that the referral of the isospace to the conventional units and field causes the reduction of the isometric from the general form $\hat{G} = \hat{\eta} \times \hat{I}$ to $\hat{\eta} = \hat{T} \times \eta$, where, as now familiar, $\hat{I} = 1/\hat{T}$ and $\eta = Diag.(1, 1, 1, -1)$ is the familiar Minkowskian metric.

According to this notation the Riemannian content of the Minkowski-Santilli isogeometry can be unified in both its isospace formulation properly speaking and its projection in our spacetime. All differences in the interpretations whether occurring in isospace or in our spacetime are then deferred to the selection of the basic unit.

Consider now the infinitesimal version of isoinvariant (3.2.137) permitted by the isodifferential calculus

$$\hat{d}\hat{s}^{\hat{2}} = \hat{d}\hat{x}_\mu \hat{\times} \hat{d}\hat{x}^\mu \in \hat{R}. \tag{3.2.148}$$

The *isonormal coordinates* occur when the isometric $\hat{\eta}$ is reduced to the Minkowski metric η as in conventional Riemannian geometry. Consequently, isonormal coordinates coincide with the conventional normal coordinates, and the Minkowski-Santilli isogeometry verifies the *principle of equivalence* as for the conventional Riemannian geometry.

By using the isodifferential calculus, we now introduce the *isodifferential of a contravariant isovector field* on $\hat{M}$ over $\hat{R}$ [15]

$$\hat{d}\hat{X}^\beta = (\hat{\partial}_\mu \hat{X}^\beta)\hat{\times}\hat{d}\hat{x}^\mu = \hat{I}^\rho_\mu \times (\partial_\rho \hat{X}^\beta)\hat{\times}\hat{T}^\mu_\sigma \times d\hat{x}^\sigma \equiv$$

$$\equiv (\partial_\mu X^\beta) \times d\hat{x}^\mu = (\partial^\rho X^\beta) \times \hat{\eta}_{\rho\sigma} \times d\hat{x}^\sigma, \tag{3.2.149}$$

where the last expression is introduce to recall that the contractions are in isospace. The preceding expression then shows that *isodifferentials of isovector fields coincide at the abstract level with conventional differentials for all isotopies of the class here admitted* (that with $\hat{I} > 0$).

DEFINITION 3.2.13 [26]: The isocovariant isodifferential are defined by

$$\hat{D}\hat{X}^\beta = \hat{d}\hat{X}^\beta + \hat{\Gamma}^\beta_{\alpha\gamma}\hat{\times}\hat{X}^\alpha\hat{\times}\hat{d}\hat{x}^\gamma, \tag{3.2.150}$$

with corresponding isocovariant derivative

$$\hat{X}^\beta_{\hat{|}\mu} = \hat{\partial}_\mu \hat{X}^\beta + \hat{\Gamma}^\beta_{\alpha\mu}\hat{\times}\hat{X}^{\hat{\alpha}}, \tag{3.2.151}$$

where the iso-Christoffel's symbols are given by

$$\hat{\Gamma}^\beta_{\alpha\gamma}(x, v, a, \mu, \tau, \ldots) = \frac{\hat{1}}{\hat{2}}\hat{\times}(\hat{\partial}_\alpha\hat{\eta}_{\beta\gamma} + \hat{\partial}_\gamma\hat{\eta}_{\alpha\beta} - \hat{\partial}_\beta\hat{\eta}_{\alpha\gamma}) \times \hat{I} = \hat{\Gamma}_{\gamma\beta\alpha}, \tag{3.2.152a}$$

$$\hat{\Gamma}^\beta_{\alpha\gamma} = \hat{\eta}^{\beta\rho} \times \hat{\Gamma}_{\alpha\rho\gamma} = \hat{\Gamma}^\beta_{\gamma\alpha}. \tag{3.2.152b}$$

[15] We should note that the role of the isounit and of the isoelement in this presentation and in that of Ref. [26] are interchanged for general compatibility with the various applications and developments.

Note the unrestricted functional dependence of the connection which is notoriously absent in conventional treatments. Note also the abstract identity of the conventional and isotopic connections. Note finally that *local numerical values of the conventional and isotopic connections coincide when computed in their respective spaces*. This is due to the fact that in Eq.s (3.2.152) $\hat{\eta} \equiv g(x)$ for exterior problems, while the value of derivatives ∂_μ and isoderivatives $\hat{\partial}_\mu$ coincide when computed in their respective spaces.

Note however that, when projected in the conventional spacetime, the conventional and isotopic connections are different even in the exterior problem in which $\hat{\eta} = g(x)$,

$$\hat{\Gamma}_{\alpha\beta\gamma} = \frac{1}{2} \times (\hat{I}^\mu_\alpha \times \partial_\mu g_{\beta\gamma} + \hat{I}^\rho_\gamma \times \partial_\rho \hat{\eta}_{\alpha\beta} - \hat{I}^\sigma_\beta \times \partial_\sigma g_{\alpha\gamma}) \times \hat{I} \neq \Gamma_{\alpha\beta\gamma} \times \hat{I}. \quad (3.2.153)$$

The extension to covariant isovector fields and covariant or contravariant isotensor fields is consequential.

Without proof we quote the following important result from Ref. [26]:

LEMMA 3.2.7 (Iso-Ricci Lemma) [26]: Under the assumed conditions, the isocovariant derivatives of all isometrics on Minkowski-Santilli isospaces spaces are identically null,

$$\hat{\eta}_{\alpha\beta\hat{|}\gamma} \equiv 0, \quad \alpha, \beta, \gamma = 1, 2, 3, 4. \quad (3.2.154)$$

The novelty of the isogeometry is then illustrated by the fact that *the Ricci property persists under an arbitrary dependence of the metric, as well as under Minkowskian, rather than Riemannian axioms.*

The *isotorsion* on $\hat{M}$ is defined by

$$\hat{\tau}^\beta_{\alpha\gamma} = \hat{\Gamma}^\beta_{\alpha\gamma} - \hat{\Gamma}^\beta_{\gamma\alpha}, \quad (3.2.155)$$

and coincides again with the conventional torsion at the abstract level, although the two torsions have significant differences in their explicit forms when both projected in our space-time.

DEFINITION 3.2.14 [26]: The Minkowski-Santilli isogeometry is characterized by the following isotensor: the isoflatness isotensor

$$\hat{R}^\beta_{\alpha\gamma\delta} = \hat{\partial}_\delta \hat{\Gamma}^\beta_{\alpha\gamma} - \hat{\partial}_\gamma \hat{\Gamma}^\beta_{\alpha\delta} + \hat{\Gamma}^\beta_{\rho\delta} \hat{\times} \hat{\Gamma}^\rho_{\alpha\gamma} - \hat{\Gamma}^\beta_{\rho\gamma} \hat{\times} \hat{\Gamma}^\rho_{\alpha\delta}; \quad (3.2.156)$$

the iso-Ricci isotensor

$$\hat{R}_{\mu\nu} = \hat{R}^\beta_{\mu\nu\beta}; \quad (3.2.157)$$

the isoflatness isoscalar

$$\hat{R} = \hat{\eta}^{\alpha\beta} \times \hat{R}_{\alpha\beta}; \quad (3.2.158)$$

the iso-Einstein isotensor

$$\hat{G}_{\mu\nu} = \hat{R}_{\mu\nu} - \frac{\hat{1}}{2}\hat{\times}\hat{N}_{\mu\nu}\hat{\times}\hat{R},\ \hat{N}_{\mu\nu} = \hat{\eta}_{\mu\nu} \times \hat{I}; \tag{3.2.159}$$

and the isotopic isoscalar

$$\hat{\Theta} = \hat{N}^{\alpha\beta}\hat{\times}\hat{N}^{\gamma\delta}\hat{\times}(\hat{\Gamma}_{\rho\alpha\delta}\hat{\times}\hat{\Gamma}^{\rho}_{\gamma\beta} - \Gamma_{\rho\alpha\beta}\hat{\times}\hat{\Gamma}^{\rho}_{\gamma\delta}) =$$

$$= \hat{\Gamma}_{\rho\alpha\beta}\hat{\times}\hat{\Gamma}^{\rho}_{\gamma\delta}\hat{\times}(\hat{N}^{\alpha\delta}\hat{\times}\hat{N}^{\gamma\beta} - \hat{N}^{\alpha\beta}\hat{\times}\hat{N}^{\gamma\delta}); \tag{3.2.160}$$

the latter being new for the Minkowski-Santilli isogeometry.

Note the lack of use of the term "isocurvature" and the use instead of the term "isoflatness". This is due to the fact that the prefix "iso-" represents the preservation of the original axioms. The term "isocurvature" would then be inappropriate because the basic axioms of the geometry are flat.

In any case, the main problem underlying the studies herein reported is, as indicated in Chapter 1, that *curvature is the ultimate origin of the catastrophic inconsistencies of general relativity.* Consequently, all geometric efforts are here aimed at the replacement of the notion of curvature with a covering notion resolving the indicated catastrophic inconsistencies.

As we shall see better in Section 3.5, the notion of "isoflatness" does indeed achieve the desired objectives because flatness and its related invariance of gravitation under the Poincaré-Santilli isosymmetry is reconstructed on isospaces over isofields, while the ordinary curvature emerge as a mere projection in our spacetime.

3.2.9D. The Five Identities of the Minkowski-Santilli Isogeometry. By continuing our review of memoir [26], tedious but simple calculations yield the following *five basic identities of the Minkowski-Santilli isogeometry:*

Identity 1: *Antisymmetry of the last two indices of the isoflatness isotensor*

$$\hat{R}^{\beta}_{\alpha\gamma\delta} = -\hat{R}^{\beta}_{\alpha\delta\gamma}; \tag{3.2.161}$$

Identity 2: *Symmetry of the first two indices of the isoflatness isotensor*

$$\hat{R}_{\alpha\beta\gamma\delta} \equiv \hat{R}_{\beta\alpha\gamma\delta}; \tag{3.2.162}$$

Identity 3: *Vanishing of the totally antisymmetric part of the isoflatness isotensor*

$$\hat{R}^{\beta}_{\alpha\gamma\delta} + \hat{R}^{\beta}_{\gamma\delta\alpha} + \hat{R}^{\beta}_{\delta\ \alpha\gamma} \equiv 0; \tag{3.2.163}$$

Figure 3.11. Primary objectives of the Minkowski-Santilli isogeometry are the resolution of the catastrophic inconsistencies of the Riemannian formulation of exterior gravitation (Section 1.4) and a representation of interior gravitation as occurring for the Sun depicted in this figure and any other massive object. These objectives are achieved via the isotopies of the Minkowskian geometry since they are flat in isospace, thus admitting a well defined invariance for all possible gravitation, by adding sources requested by the Freud identity and other reasons, and by unifying exterior and interior gravitational problem in a single formulation in isospace that formally coincides with that for the exterior problem, the interior effects being incorporated in the isounit (see Section 3.5).

Identity 4: *Iso-Bianchi identity*

$$\hat{R}^{\beta}_{\alpha\gamma\delta\hat{|}\rho} + \hat{R}^{\beta}_{\alpha\rho\gamma\hat{|}\delta} + \hat{R}^{\beta}_{\alpha\delta\rho\hat{|}\gamma} \equiv 0; \tag{3.2.164}$$

Identity 5: *Iso-Freud identity*

$$\hat{R}^{\alpha}_{\beta} - \frac{\hat{1}}{2}\hat{\times}\hat{\delta}^{\alpha}_{\beta}\hat{\times}\hat{R} - \frac{\hat{1}}{2}\hat{\times}\hat{\delta}^{\alpha}_{\beta}\hat{\times}\hat{\Theta} = \hat{U}^{\alpha}_{\beta} + \hat{\partial}_{\rho}\hat{V}^{\alpha\rho}_{\beta}, \tag{3.2.165}$$

where $\hat{\Theta}$ is the isotopic isoscalar and

$$\hat{U}^{\alpha}_{\beta} = -\frac{1}{2}\frac{\hat{\partial}\hat{\Theta}}{\hat{\partial}\hat{\eta}^{\alpha\beta}_{\hat{|}\alpha}}\hat{\eta}^{\alpha\beta}_{\hat{|}\beta}, \tag{3.2.166a}$$

$$\hat{V}^{\alpha\rho}_{\beta} = \frac{1}{2}[\hat{\eta}^{\gamma\delta}(\delta^{\alpha}_{\beta}\hat{\Gamma}^{\rho}_{\alpha\delta} - \delta^{\rho}_{\beta}\hat{\Gamma}^{\rho}_{\gamma\delta}) + \tag{3.2.166b}$$

$$+(\delta^{\rho}_{\beta}\hat{\eta}^{\alpha\gamma} - \delta^{\alpha}_{\beta}\hat{\eta}^{\rho\gamma})\hat{\Gamma}^{\delta}_{\gamma\delta} + \hat{\eta}^{\rho\gamma}\hat{\Gamma}^{\alpha}_{\beta\gamma} - \hat{\eta}^{\alpha\gamma}\hat{\Gamma}^{\rho}_{\beta\gamma}], \tag{3.2.166c}$$

Note that the conventional Riemannian geometry is generally thought to possess only *four* identities. In fact, the *fifth* identity (3.2.165) is generally unknown in the contemporary literature in gravitation as the reader is encouraged to verify in the specialized literature in the Riemannian geometry (that is so vast to discourage discriminatory listings).

The latter identity was introduced by Freud [27] in 1939, treated in detail by Pauli in his celebrated book [28] of 1958 and then generally forgotten for a half a century, apparently because of its evident incompatibility between Einstein's conception of exterior gravitation in vacuum as pure curvature without source (see Section 3.4)

$$G^\alpha_\beta = R^\alpha_\beta - \frac{1}{2}\delta^\alpha_\beta R = 0, \tag{3.2.167}$$

and the need for a source term also in exterior gravitation in vacuum mandated by the Freud identity and other reasons

$$R^\alpha_\beta - \frac{1}{2}\delta^\alpha_\beta R - \frac{1}{2}\delta^\alpha_\beta \Theta = U^\alpha_\beta + \hat\partial_\rho V^{\alpha\rho}_\beta. \tag{3.2.168}$$

Freud's identity was rediscovered by the author during his accurate study of Pauli's historical book and studied in detail in Refs. [6,7] of 1992. Additional studies of the Freud identity were done by Yilmaz [30]. Following a suggestion by the author, the late mathematician Hanno Rund [29] studied the identity in one of his last papers and proved that:

LEMMA 3.2.8 (Rund's Lemma) [29]: Freud's identity is a bona fide identity for all Riemannian spaces irrespective of dimension and signature.

In this way, Rund confirmed the general need of a source also in vacuum (see Sections 1.4 and 3.5).

Following Ref. [26], in this paper we have presented the isotopies of the Freud identity on Minkowski-Santilli isospaces, as characterized by the isodifferential calculus. Its primary functions for this monograph is to identify the geometric structure of the *interior* gravitational problem. The persistence of the source in vacuum as per the Freud identity, electrodynamics and other needs will then be consequential, thus confirming the inconsistency of Einstein's conception of gravity in vacuum as pure curvature without source.

Note that *all conventional and isotopic identities coincide at the abstract level.*

3.2.9E. Isoparallel Transport and Isogeodesics. An isovector field $\hat X^\beta$ on $\hat M = \hat M(\hat x, \hat M, \hat R)$ is said to be transported by *isoparallel* displacement from a point $\hat m(\hat x)$ on a curve $\hat C$ on $\hat M$ to a neighboring point $\hat m'(\hat x + \hat d\hat x)$ on $\hat C$ if

$$\hat D\hat X^\beta = \hat d\hat X^\beta + \hat\Gamma^\beta_{\alpha\gamma}\hat\times\hat X^\alpha\hat\times\hat d\hat x^\gamma \equiv 0, \tag{3.2.169}$$

or in integrated form

$$\hat{X}^\beta(\hat{m}') - \hat{X}^\beta(m) = \hat{\int}_{\hat{m}}^{\hat{m}'} \frac{\hat{\partial}\hat{X}^\beta}{\hat{\partial}\hat{x}^\alpha} \frac{\hat{d}\hat{x}^\alpha}{\hat{d}\hat{s}} \hat{\times} \hat{d}\hat{s}, \tag{3.2.170}$$

where one should note the isotopic character of the integration. The isotopy of the conventional case then yields the following:

LEMMA 3.2.9 [26]: Necessary and sufficient condition for the existence of an isoparallel transport along a curve $\hat{C}$ on a $(3+1)$-dimensional Minkowski-Santilli isospace is that all the following equations are identically verified along $\hat{C}$

$$\hat{R}^\beta_{\alpha\gamma\delta} \hat{\times} \hat{X}^\alpha = 0, \quad \alpha, \beta, \gamma, \delta = 1, 2, 3, 4. \tag{3.2.171}$$

Note, again, the abstract identity of the conventional and isotopic parallel transport. However, it is easy to see that the projection of the isoparallel transport in ordinary spacetime is structurally different than the conventional parallel transport.

Consider, as an example, an extended object in gravitational fall in atmosphere (see Figure 3.12). Its trajectory is evidently irregular and depends on the actual shape of the object, as well as its weight. The understanding of the new Minkowski-Santilli isogeometry requires the knowledge of the fact that said trajectory is represented on isospace over isofields as a *straight line,* that is, via the trajectory in the absence of the resistive medium. The actual, irregular trajectory appears only in the projection of said isotrajectory in our spacetime.

If the latter treatment is represented by a rocket, one would note a twisting action as occurring in the reality of motion within physical media, which is evidently absent in the exterior case.

Along similar lines, we say that a smooth isopath $\hat{x}_\alpha$ on $\hat{M}$ with isotangent $\hat{v}_\alpha = \hat{d}\hat{x}_\alpha/\hat{d}\hat{s}$ is an *isogeodesic* when it is solution of the isodifferential equations

$$\frac{\hat{D}\hat{v}^\beta}{\hat{D}\hat{s}} = \frac{\hat{d}\hat{v}}{\hat{d}\hat{s}} + \hat{\Gamma}_{\alpha\beta\gamma} \hat{\times} \frac{\hat{d}\hat{x}^\alpha}{\hat{d}\hat{s}} \hat{\times} \frac{\hat{d}\hat{x}^\gamma}{\hat{d}\hat{s}} = 0. \tag{3.2.172}$$

It is easy to prove the following:

LEMMA 3.2.10 [26]: The isogeodesics of a Minkowski-Santilli isospace $\hat{M}$ are the isocurves verifying the isovariational principle

$$\hat{\delta} \hat{\int} [\hat{G}_{\alpha\beta}(\hat{x}, \hat{v}, \hat{a}, \mu, \tau, \ldots) \hat{\times} \hat{d}\hat{x}^\alpha \hat{\times} \hat{d}\hat{x}^\beta]^{\hat{1/2}} = 0, \tag{3.2.173}$$

where again isointegration is understood.

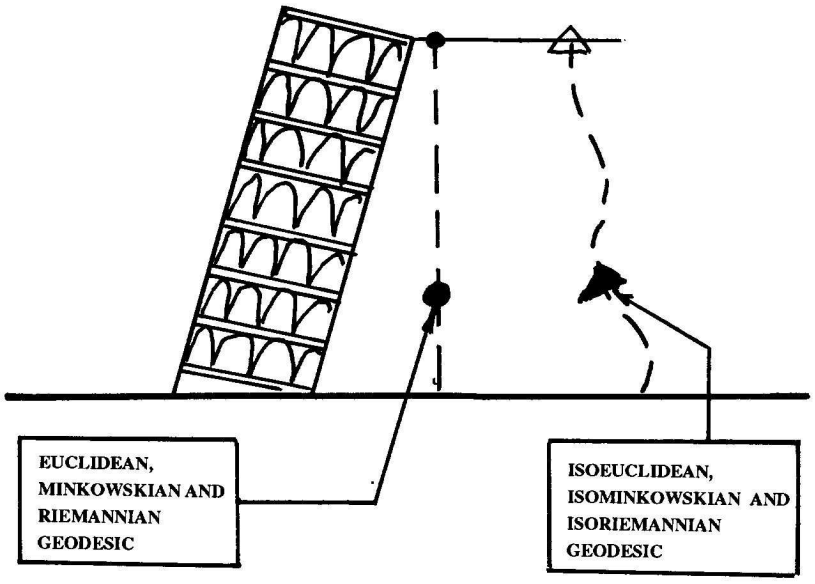

Figure 3.12. A schematic view of two objects released from the Pisa tower. The vertical trajectory represents the approximate geodesic considered by Galileo, used by Einstein and adopted until the end of the 20-th century, namely, the approximation under the lack of resistance due to our atmosphere. The Minkowski-Santilli isogeometry has been built to represent as isogeodesics actual trajectories within physical media.

Finally, we point out the property inherent in the notion of isotopies according to which

COROLLARY 3.2.10A: [26]: Trajectories in an ordinary Riemannian space coincide with the corresponding isogeodesic trajectories in Minkowski-Santilli isospace, but not with the projection of the latter in the original space.

For instance, if a circle is originally a geodesic, its image under isotopy in isospace remains the perfect circle, the *isocircle* (Section 3.2.9), even though its projection in the original space can be an ellipse. The same preservation in isospace occurs for all other curves.

The differences between a geodesic and an isogeodesic therefore emerge only when projecting the latter in the space of the former.

An empirical but conceptually effective rule is that *interior physical media "disappear" under their isogeometrization*, in the sense that actual trajectories

under resistive forces due to physical media (which are not geodesics of a Minkowski space) are turned into isogeodesics in isospace having the shape of the geodesics in the absence of resistive forces.

The simplest possible example is given by the iso-Euclidean representation of a straight stick partially immersed in water. In conventional representations the stick penetrating in water with an angle α appears as bended at the point of immersion in water with an angle $\gamma = \alpha + \beta$, where β is the angle of refraction. In iso-Euclidean representation the stick remains straight also in its immersion because the isoangle $\hat{\gamma} = \gamma \times \hat{I}_\gamma$ recovers the original angle α with $\hat{I}_\gamma = \alpha/(\alpha+\beta)$.

The situation is essentially the same for our representation of interior gravitation because the latter is represented in isospace over isofield via field equations (this time necessarily with sources) that formally coincide with conventional equations on a conventional Riemannian spacetime. Being noncanonical, all interior features are invariantly represented via generalized units.

3.2.9F. Isodual Minkowski-Santilli isospaces and isogeometry. The *isodual Minkowski-Santilli isospaces* were introduced for the first time by Santilli in Ref. [8] of 1985 and then studied in various works (see the references of Chapter 1), and can be written

$$\hat{M}^d = \hat{M}^d(\hat{x}^d, \hat{\eta}^d, \hat{R}^d) :$$

$$\hat{x}^d = \{x^{\mu d}\} \times^d \hat{I}^d = \{x^\mu\} \times (-\hat{I}) = \{r^d, c_o^d \times^d t^d\} \times^d \hat{I}^d, \tag{3.2.174a}$$

$$\hat{\eta}^d = -\hat{\eta}. \tag{3.2.174b}$$

The *isodual Minkowski-Santilli isogeometry* is the geometry of isodual isospaces M^d over R^d and was studied for the first time by Santilli in Ref. [26] of 1998.

The physically and mathematically most salient property of the latter geometry is that it is *characterized by negative units of space, time, etc., and negative norms.* Therefore, in addition to a change in the sign of the charge, we also have change of sign of masses, energies, and other quantities normally positive for matter. Similarly, we have the *isodual isospace and isotime coordinates*

$$\hat{\hat{x}}^d = \hat{x}^d \times^d \hat{I} = -\hat{x}, \;\; \hat{t}^d = t^d \times^d \hat{I}^d = -\hat{t}. \tag{3.2.175}$$

Thus, motion under isoduality is in a time direction *opposite* to the conventional motion. These features are necessary so as to have a classical representation of antimatter in interior conditions whose operator image yields indeed antiparticles (rather than particles with the wrong sign of the charge).

We also have the following important

LEMMA 3.2.12 [17]: Isodualities are independent from spacetime inversions

$$r' = \pi \times r = -r, \;\; t' = \tau \times t = -t. \tag{3.2.176}$$

Proof. Inversions occur within the same original space and keep the unit fixed, while isodualities require a map to a different space, and change the sign of the unit. Therefore, in addition to maps in different spaces, isodualities have numerical value different than the inversions. **q.e.d.**

These are the conceptual roots for the isodual theory of antimatter to predict a *new photon*, the *isodual photon* emitted by antimatter [17]. When applied to the photon, charge conjugation and, more generally, the PCT theorem, do not yield a new photon, as well known. This is not the case under isoduality because all physical characteristics change in sign and numerical value. As a result, *the isodual photon is indistinguishable from the ordinary photon under all interactions except gravitation.* In fact, as indicated in Chapter 1, the isodual photon is predicted to experience antigravity in the field of matter, thus offering, apparently for the first time, a possibility for the future study whether far away galaxies and quasars are made up of matter or of antimatter.

Another important property of isoduality is expressed by the following:

LEMMA 3.2.13 [26]: The intervals of conventional and isotopic Minkowskian spaces are invariant under the joint isodual maps $\hat{I}^d \to \hat{I}^d$ and $\hat{\eta} \to \hat{\eta}^d$,

$$\hat{x}^2 = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} \equiv [x^\mu \times (-\hat{\eta}_{\mu\nu}) \times x^\nu] \times (-\hat{I}). \tag{3.2.177}$$

As a result, *all physical laws applying in conventional Minkowskian geometry for the characterization of matter also apply to its isodual image for the characterization of antimatter.*

Note that, strictly speaking, the intervals are not isoselfdual because

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{M}_{\mu\nu} \hat{\times} \hat{x}^\mu \ \to \ \hat{x}^{d\hat{2}d} = \hat{x}^{\mu d} \times^d \hat{M}^d_{\mu\nu} \times^d \hat{x}^{\mu d} = \hat{x}^{d\hat{2}d} = -\hat{x}^{\hat{2}}. \tag{3.2.178}$$

To outline the *Riemannian* characteristics of the isodual Minkowski-Santilli isogeometry, we consider an *isodual isovector isofield* $\hat{X}^d(\hat{x}^d)$ on $\hat{M}^d$ which is explicitly given by $\hat{X}^d(\hat{x}^d) = -X^t(-x^t \times \hat{I}) \times \hat{I}$. The *isodual exterior isodifferential* of $\hat{X}^d(\hat{x}^d)$ is given by

$$\hat{D}^d \hat{X}^{\mu d}(\hat{x}^d) = \hat{d}^d \hat{X}^{\mu d}(\hat{x}^d) \hat{+}^d \hat{\Gamma}^{d\,\mu}_{\alpha\beta} \hat{\times}^d \hat{X}^{\alpha d} \hat{\times}^d \hat{d}^d \hat{x}^{\beta d} = \hat{D} \hat{X}^{t\mu}(-\hat{x}^t), \tag{3.2.179}$$

where the $\hat{\Gamma}^d$'s are the components of the *isodual isoconnection.* The *isodual isocovariant isoderivative* is then given by

$$\hat{X}^{\mu d}(\hat{x}^d)_{\hat{|}d\nu} = \hat{\partial}^d \hat{X}^{\mu d}(\hat{x}^d) \hat{/}^d \hat{\partial}^d \hat{x}^{\nu d} \hat{+} \hat{\Gamma}^{d\,\mu}_{\alpha\nu} \hat{\times}^d \hat{X}^{\alpha d}(\hat{x}^d) = -\hat{X}^{t\mu}(-\hat{x}^t)_{\hat{|}k}. \tag{3.2.180}$$

The interested reader can then easily derive the remaining notions of the new geometry. It is an instructive exercise for the interested reader to prove the

following isodualities:

$$\begin{array}{cc}
\text{Isodual isounit} & \hat{I} \to \hat{I}^d = -\hat{I}, \\
\text{Isodual isometric} & \hat{\eta} \to \hat{\eta}^d = -\eta, \\
\text{Isodual isoconnection coefficients} & \hat{\Gamma}_{\alpha\beta\gamma} \to \hat{\Gamma}^d_{\alpha\beta\gamma} = \hat{\Gamma}_{\alpha\beta\gamma}, \\
\text{Isoflatness isotensor} & R_{\alpha\beta\gamma\delta} \to R^d_{\alpha\beta\gamma\delta} = -R_{\alpha\beta\gamma\delta}, \\
\text{Isodual iso-Ricci isotensor} & \hat{R}_{\mu\nu} \to \hat{R}^d_{\mu\nu} = \hat{R}_{\mu\nu}, \\
\text{Isodual iso-Ricci isoscalar} & \hat{R} \to \hat{R}^d = \hat{R}, \\
\text{Isodual iso-Freud isoscalar} & \hat{\Theta} \to \hat{\Theta}^d = -\hat{\Theta}, \\
\text{Isodual Iso-Einstein isotensor} & \hat{G}_{\mu\nu} \to \hat{G}^d_{\mu\nu} = -\hat{G}_{\mu\nu}, \\
\text{Isodual electromagnetic potentials} & A_\mu \to A^d_\mu = -A_\mu, \\
\text{Isodual electromagnetic field} & F_{\mu\nu} \to F^d_{\mu\nu} = -F_{\mu\nu}, \\
\text{Isodual elm energy-mom. isotensor} & T_{\mu\nu} \to T^d_{\mu\nu} = -T_{\mu\nu}.
\end{array} \tag{3.2.181}$$

More detailed isogeometric studies are left to interested readers. Specific applications to gravitational treatments of matter and antimatter are presented in Section 3.5.

3.2.10 Isosymplectic Geometry and its Isodual

As it is well known, the *symplectic geometry* had an important role in the construction of quantum mechanics because it permitted the mathematically rigorous verification, known as *symplectic quantization,* that original quantization procedures, known also as *naive quantization,* were correct.

No broadening of quantum mechanics can be considered mature unless it admits fully equivalent procedures in the map from classical to operator forms known as *isoquantization* also called *hadronization* (rather than quantization).

For this purpose. Santilli [31] presented in 1988 the first known *isotopies of the symplectic geometry,* subsequently studied in various works, with a general presentation available in Vols. I, II of this series (see in particular Chapter 5 of Vol. I [6]). The new geometry is today known as *Santilli's isosymplectic geometry.*

We cannot possibly review here the isosymplectic geometry in detail and have to suggest interested readers to study Refs. [6,7]. Nevertheless, an indication of the basic lines is important for the self-sufficiency of this monograph.

Let us ignore the global (also called abstract) formulation of the symplectic geometry and consider for clarity and simplicity only its realization in a local chart (or coordinates).[16] A *topological manifold* $M(R)$ on the reals R admits the local realization as an Euclidean space $E(r, \delta R)$ with local contravariant coordinates

[16]Again, the literature on the conventional symplectic geometry is so vast to discourage discriminatory quotations.

$r = (r^i),\ i = i, 2, \ldots, N$. The *cotangent bundle* T^*M then becomes the ordinary phase space with local coordinates $(r, p) = (r^i, p_i)$, where p_i represents the tangent vectors (physically the linear momentum). The *canonical one-form* then admits the local realization

$$\theta = p_i \times dr^i. \tag{3.2.182}$$

The *fundamental (canonical) symplectic form* is then given by the exterior derivative of the preceding one form

$$\omega = d\theta = p_i \wedge dr^i, \tag{3.2.183}$$

and one can easily prove that it is closed, namely, that $d\omega \equiv 0$.

Consider now the *isotopological isomanifold* (introduced earlier) $\hat{M}(\hat{R})$ on the isoreals $\hat{R}$ with basic isounit $\hat{I}$. Its realization on local coordinates is given by the Euclid-Santilli isospace $\hat{E}(\hat{r}, \hat{\Delta}, \hat{R})$ with local contravariant isocoordinates $\hat{r} = (r^i) \times \hat{I}$. Then, the *isocotangent isobundle* $\hat{T}^*\hat{M}$ admits as local realization the *isophase isospace* with local coordinates $(\hat{r}^i, \hat{p}_i)$, where $\hat{p}$ is again a tangent isovector. The novelty is given by the fact that the unit of $\hat{p}$ is the *inverse* of that of $\hat{r}$ and we shall write

$$\hat{r} = r \times \hat{I},\ \ \hat{p} = p \times \hat{T},\ \ \hat{I} = 1/\hat{T}. \tag{3.2.184}$$

This property was identified for the first time by Santilli [31] (for a mathematical treatment see also Ref. [10]) because not identifiable in the conventional symplectic geometry due to the use of the trivial unit for which $I^{-1} \equiv I = +1$.

Consequently, we have the isodifferentials

$$\hat{d}\hat{r} = \hat{T} \times d(r \times I),\ \ \hat{d}\hat{p} = \hat{I} \times d(p \times \hat{T}). \tag{3.2.185}$$

The *isocanonical one-isoform* is then given by

$$\hat{\theta} = \hat{p}\hat{\times}\hat{d}\hat{r} = (p \times \hat{T}) \times \hat{I} \times \hat{d}(\hat{r}) = p \times \hat{T} \times d(r \times \hat{I}). \tag{3.2.186}$$

The *fundamental isocanonical two-isoform* is then given by

$$\hat{\omega} = \hat{d}\hat{\theta} = \hat{p}\hat{\wedge}\hat{d}\hat{r} = dp_i \wedge dr^i \equiv \omega, \tag{3.2.187}$$

from which the preservation of closure under isotopy, $\hat{d}\hat{\omega} \equiv \hat{0} = 0$ trivially follows.

LEMMA 3.2.14 [31,10]: The fundamental symplectic and isosymplectic two-forms coincide.

The identity of the fundamental isocanonical and canonical two-forms explains why isosymplectic geometry escaped detection by mathematicians for centuries.

It is evident that, in view of the positive-definiteness of the isounit, *the symplectic and isosymplectic geometries coincide at the global (abstract) realization-free level* to such an extent that there is not even the need of changing formulae in the literature of the symplectic geometry because the isosymplectic geometry can be expressed with the pre-existing formalism and merely subject it to a broader realization.

Despite this simplicity, the physical implications are by far non-trivial. In fact, unlike the conventional two-form, and thanks to the background TSSFN isotopology, the fundamental isocanonical two-form is universal for all possible (sufficiently smooth and regular but otherwise arbitrary) nonlocal and non-Hamiltonian systems. To illustrate this feature, let us consider a *vector field* of the cotangent bundle that must be strictly *local-differential* to avoid catastrophic inconsistencies with the underlying local-differential Euclidean topology, T^*M

$$X(r,p) = A_i(r,p) \times \frac{\partial}{\partial r^i} + B^i(r,p) \times \frac{\partial}{\partial p_i}, \tag{3.2.188}$$

or in unified notations

$$b = (b^\mu) = (r^i, p_j), \quad \mu = 1, 2, \ldots, 2N, \tag{3.2.189}$$

$$X(b) = X_\mu(b) \times \frac{\partial}{\partial b^\mu}, \tag{3.2.190}$$

is said to be a *Hamiltonian vector field* when there exists a function $H(r,p) = H(b)$ on T^*M, called the *Hamiltonian,* verifying the identity

$$A_i \times dr^i + B^i \times dp_i = -dH(r,p) \tag{3.2.191}$$

or in unified notation

$$X \rfloor \omega = dH, \tag{3.2.192}$$

that is

$$\omega_{\mu\nu} \times X^\mu \times db^\nu = -dH, \tag{3.2.193}$$

where the fundamental symplectic form has the components

$$\omega = dp_i \wedge dr^i = \frac{1}{2} \times \omega_{\mu\nu} \times db^\mu \wedge db^\nu, \tag{3.2.194}$$

$$(\omega_{\mu\nu}) = \begin{pmatrix} O_{N\times N} & -I_{N\times N} \\ I_{N\times N} & O_{N\times N} \end{pmatrix}. \tag{3.2.195}$$

Eq. (3.2.192) can hold if and only if

$$\omega_{\mu\nu} \times \frac{db^\nu}{dt} = \frac{\partial H}{\partial b^\mu}, \tag{3.2.196}$$

from which one recovers the familiar truncated Hamilton's equations

$$\frac{dr^i}{dt} = \frac{\partial H}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H}{\partial r^i}. \tag{3.2.197}$$

The main physical limitations is that *the condition for a vector field to be Hamiltonian constitutes a major restrictions because vector fields in the physical reality are generally non-Hamiltonian, besides existing from the limitations of the topology underlying the symplectic geometry.*

As we shall see in Section 3.3, the above restrictions is removed for Santilli isosymplectic geometry that acquire the character of *direct universality,* that is, the capability of representing all sufficiently smooth and regular but otherwise arbitrary vector fields (universality) in the local chart of the experimenter (direct universality).

In fact, expression (3.2.192) is lifted into the form

$$\hat{\omega}_{\mu\nu} \hat{\times} \frac{\hat{d}\hat{b}^\nu}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{b}^\mu}, \tag{3.2.198}$$

that, under the assumption for simplicity that $\hat{t} = t$, and by removing common factors, reduces to

$$\frac{dr^i}{dt} = \frac{\hat{\partial} H}{\hat{\partial} p_i} = \hat{T}^i_j(r,p) \times \frac{\partial H}{\partial p_j}; \tag{3.2.199}$$

$$\frac{dp_i}{dt} = -\frac{\hat{\partial} H}{\hat{\partial} r^i} = -\hat{I}^j_i \times \frac{\partial H}{\partial r^j}. \tag{3.2.200}$$

As we shall see better in Section 3.3, direct universality then follows from the number of free functions $\hat{T}^j_i$ as well as the arbitrariness of their functional dependence.

We shall also show that the achievement of a direct isogeometric representation of nonlocal and non-Hamiltonian vector fields representing interior dynamical problems permits their consistent map into an operator form, by therefore reaching hadronic mechanics in a mathematically rigorous, unique and unambiguous way.[17]

The construction of the *isodual isosymplectic geometry* [6] is an instructive exercise for readers interested in serious studies of antimatter in interior dynamical conditions.

[17]Note the crucial role of the isodifferential calculus for the isosymplectic geometry and its implications.

3.2.11 Isolinearity, Isolocality, Isocanonicity and Their Isodualities

In Section 3.1 we pointed out that the primary physical characteristics of particles and antiparticles in interior conditions (such as a neutron in the core of a neutron star) are nonlinear, nonlocality and noncanonicity due to the mutual penetration-overlapping of their wavepackets with those of the surrounding medium.

In the preceding subsections we have identified isotopic means for mapping linear, local and canonical systems into their most general possible nonlinear, nonlocal and noncanonical form. In this section we show how the isotopies permit the reconstruction of linearity, locality and canonicity on isospaces over isofields, called *isolinearity, isolocality and isocanonicity* for the case of particles, with their isodual counterpart for antiparticles.

The understanding of this seemingly impossible task requires the knowledge that conventional methods have only one formulation. By contrast, all isotopic methods have a dual formulation, the first in isospace over isofields, and the second when projected in ordinary spaces over ordinary fields. Deviations from conventional properties can only occur in the latter formulation because in the former all original axiomatic properties are preserved by construction.

Let $S(r, R)$ be a conventional real vector space with local coordinates r over the reals $R = R(n, +, \times)$, and let

$$r' = A(w) \times r, \;\; r'^t = r^t \times A^t(w), \;\; w \in R. \tag{3.2.201}$$

be a conventional right and left linear, local and canonical transformation on S, where t denotes transpose.

The isotopic lifting $S(r, R) \to \hat{S}(\hat{r}, \hat{R})$ requires a corresponding necessary isotopy of the transformation theory. In fact, it is instructive for the interested reader to verify that the application of conventional linear transformations to the isospace $\hat{S}(\hat{r}, \hat{R})$ causes the loss of linearity, transitivity and other basic properties.

For these and other reasons, Santilli submitted in the original proposals [4,5] of 1978 (see monographs [6,7] for comprehensive treatments and applications) the isotopy of the transformation theory, called *isotransformation theory,* which is characterized by isotransforms (where we make use of the notion of isofunction of Section 3.2.4)

$$\hat{r}' = \hat{A}(\hat{w})\hat{\times}\hat{r} = \hat{A}(\hat{w}) \times \hat{T} \times \hat{r} = [A(\hat{T} \times w) \times \hat{I}] \times \hat{T} \times (r \times \hat{I}) =$$

$$= A[\hat{T}(r, \ldots) \times w] \times \hat{r}, \tag{3.2.202a}$$

$$\hat{r}'^{\hat{t}} = \hat{r}^{\hat{t}}\hat{\times}\hat{A}^{\hat{t}}\hat{w} = \hat{r}^{\hat{t}} \times A^t[\hat{T}(r) \times w]. \tag{3.2.202b}$$

The most dominant aspect in the transition from the conventional to the isotopic transforms is that, while the former are linear, local and canonical, the latter are nonlinear in the coordinates as well as other quantities and their derivatives of arbitrary order, nonlocal-integral in all needed quantities, and noncanonical when projected in the original spaces $S(r, R)$. This is due to the unrestricted nature of the functional dependence of the isotopic element $\hat{T} = \hat{T}(r, \ldots)$.

But the conventional and isotopic transforms coincide at the abstract level where we have no distinction between the modular action $A(w) \times r$ and $\hat{A}(\hat{w})\hat{\times}\hat{r}$. Therefore, isotransforms (3.2.202) are *isolinear* when formulated on isospace $\hat{S}$ over the isofield $\hat{R}$, because they verify the conditions

$$\hat{A}\hat{\times}(\hat{n}\hat{\times}\hat{r} + \hat{m}\hat{\times}\hat{p}) = \hat{n}\hat{\times}\hat{A}\hat{\times}\hat{r} + \hat{m}\hat{\times}\hat{A}\hat{\times}\hat{p}, \quad \hat{r}, \hat{p} \in \hat{S}, \quad \hat{n}, \hat{m} \in \hat{R}. \tag{3.2.203}$$

Note that conventional transforms are characterized by a *right modular associative action* $A \times r$. Isotransforms are then characterized by the *right isomodular isoassociative action* $\hat{A}\hat{\times}\hat{r}$. Therefore, we do have the preservation of the original axiomatic structure and isotransforms are indeed an isotopy of conventional transforms.

The situation for locality and canonicity follows the same lines [4,5,6,7]. Conventional methods are local in the sense that they are defined at a finite set of isolated points. The isotopic methods are *isolocal* in the sense that they verify the condition of locality in isospaces over isofields. However, their projection on conventional space is nonlocal-integral, because that is the general characteristic of the isotopic element $\hat{T}$, as illustrated, e.g., in Eq. (3.1.202).

Similarly, conventional methods are canonical in the sense that they can be characterized via a first-order canonical action in phase space (or cotangent bundle). The isotopic methods are *isocanonical* in the sense that, as we shall see in Section 3.3, they are derivable from an *isoaction* that is first-order and canonical on isospaces over isofields, although, when projected on ordinary spaces over ordinary fields, such an isoaction is of arbitrary order.

LEMMA 3.2.15 [6,7]: All possible nonlinear, nonlocal and noncanonical transforms on a vector space $S(r, R)$

$$r' = B(w, r, \ldots) \times r, \quad r \in S, \quad w \in R, \tag{3.2.204}$$

can always be rewritten in an identical isolinear, isolocal and isocanonical form, that is, there always exists at least one isotopy of the base field, $R \to \hat{R}$*, and a corresponding isotopy of the space* $S(r, R) \to \hat{S}(\hat{r}, \hat{R})$*, such as*

$$B(w, r, \ldots) \equiv A(\hat{T} \times w), \tag{3.2.205}$$

under which

$$r' = B(w, r, \ldots) \times r \equiv A(\hat{T} \times w) \times r \equiv \hat{A}(\hat{w})\hat{\times}r, \tag{3.2.206}$$

from which the isolinear form (3.2.202) follows.

COROLLARY 3.2.15A [6,7]: Under sufficient continuity and regularity conditions, all possible ordinary differential equations that are nonlinear in ordinary spaces over ordinary fields can always be turned into an identical form that is isolinear on isospaces over isofields,[18]

$$\dot{r} - E(\dot{r}, w, \ldots) \to \hat{\dot{r}} - A[\hat{T}(\dot{r}, w, \ldots) \times \dot{r} - B[\hat{T}(\dot{r}, w, \ldots] \equiv$$
$$\equiv \hat{\dot{r}} - \hat{A}(\hat{w})\hat{\times}\hat{\dot{r}} - \hat{B}(w) = 0. \tag{3.2.207}$$

The above properties are at the foundation of the *direct universality* of isotopic methods, that is, their applicability to all possible (sufficiently smooth and regular) nonlinear, nonlocal and noncanonical systems (universality) in the frame of the experimenter (direct universality).

In order to apply isotopic methods to a nonlinear, nonlocal and noncanonical system, one has merely to identity one of its possible isolinear, isolocal and isocanonical identical reformulations in the same system of coordinates. The applicability of the methods studied in this monograph then follows.

The *isodual isotransforms* are given by the image of isotransforms (3.2.202) under isoduality, and, as such, are defined on the isodual isospace $\hat{S}^d(\hat{r}^d, \hat{R}^d)$ over the isodual isofield $\hat{R}^d$ with isodual isounit $\hat{I}^d = 1/\hat{T}^d = -\hat{I}^\dagger$. [6,7] with evident properties

$$\hat{A}^d \hat{\times}^d (\hat{n}^d \hat{\times}^d \hat{r}^d + \hat{m}^d \hat{\times}^d \hat{p}^d) =$$
$$= \hat{n}^d \hat{\times}^d \hat{A}^d \hat{\times}^d \hat{r}^d + \hat{m}^d \hat{\times}^d \hat{A}^d \hat{\times}^d \hat{p}^d, \;\; \hat{r}^d, \hat{p}^d \in \hat{S}^d, \;\; \hat{n}^d, \hat{m}^d \in \hat{R}^d. \tag{3.2.208}$$

The definition of *isodual isolinearity, isolocality and isocanonicity* then follows.

From now on, we shall use isotransforms for the study of interior dynamical systems of particles and their isodual for interior systems of antiparticles.

3.2.12 Lie-Santilli Isotheory and its Isodual

3.2.12A. Statement of the Problem. As it is well known, Lie's theory has permitted outstanding achievements in various disciplines throughout the 20-th century. Nevertheless, in its current conception and realization, Lie's theory is linear, local-differential and canonical-Hamiltonian.[19]

[18]The author has proposed for over a decade that mathematicians use the property of this Corollary 3.2.15A to identify simpler methods for the solution of nonlinear differential equations, but the request has not been met as yet, to our best knowledge.

[19]The literature on Lie's theory is also vast to discourage discriminatory listings. In any case, its knowledge is a necessary pre-requisite for the understanding of this section.

As such, Lie's theory is exactly valid for exterior dynamical systems, but possesses clear limitations for interior dynamical systems since the latter are nonlinear, nonlocal and noncanonical. This occurrence mandates a suitable revision of Lie's theory such to be exactly valid for interior dynamical systems without approximations.

Independently from that, Lie's theory in its current formulation is solely applicable to matter, evidently because there exists no antiautomorphic version of the conventional Lie's theory as necessary for the correct treatment of antimatter beginning at the classical level, as shown in Chapters 1 and 2.

Another central problem addressed in these studies is the construction of the universal *symmetry* (and not "covariance") of gravitation for matter and, independently, for antimatter, that is, a symmetry for all possible exterior and interior gravitational line elements of matter and, under antiautomorphic image, of antimatter.

Yet another need in physics is the identification of the exact symmetry that can effectively replace broken Lie symmetries, which exact symmetry cannot possibly be a conventional Lie symmetry due to the need of preserving the original dimensions so as to avoid the prediction on nonphysical effects and/or hypothetical new particles.

It is evident that Lie's theory in its current formulation is unable to solve the above identified problems. In a memoir of 1978, Santilli [4] proposed a step-by-step generalization of the conventional Lie theory specifically conceived for nonlinear, nonlocal-integral and nonpotential-noncanonical systems.

The generalized theory was subsequently studied by Santilli in a variety of papers (see monographs [1,2,6,7,14,15] and references quoted therein). The theory was also studied by a number of mathematicians and theoreticians, and it is today called the *Lie-Santilli isotheory* (see, e.g., monographs [32–37] and references quoted therein, as well as specialized papers [38–43]).

A main characteristic of the Lie-Santilli isotheory, that distinguishes it from other possible generalizations, is its isotopic character, that is, the preservation of the original Lie axioms when formulated on isospaces over isofields, despite its nonlinear, nonlocal and noncanonical structure when projected in ordinary spaces. This basic feature is evidently permitted by the reconstruction of linearity, locality and canonicity on isospaces over isofields studied in the preceding section.

To begin, let us recall that Lie's theory is centrally dependent on the basic N-dimensional unit $I = Diag.(1, 1, \ldots, 1)$ of the enveloping algebra. The main idea of the Lie-Santilli isotheory [4] is the reformulation of the entire conventional theory with respect to the most general possible isounit $\hat{I}(x, \dot{x}, \ddot{x}, \ldots)$.

One can therefore see from the very outset the richness and novelty of the isotopic theory since isounits with different topological features (such as Her-

miticity, non-Hermiticity, positive-definiteness, negative-definiteness, etc.) characterize different generalized theories.

In this section we outline the rudiments of the Lie-Santilli isotheory properly speaking, that with positive-definite isounits and its isodual with negative-definite isounits. A knowledge of Lie's theory is assumed as a pre-requisite. A true technical knowledge of the Lie-Santilli isotheory can only be acquired from the study of mathematical works such as monographs [2,6,14,36,37].

In inspecting the literature, the reader should be aware that Santilli [4] constructed the isotopies of Lie's theory as a particular case of the broader Lie-admissible theory studied in Chapter 4 occurring for non-Hermitian generalized units, and known as *Lie-Santilli genotheory.* As a matter of fact, a number of aspects of the isotheory can be better identified within the context of the broader genotheory.

The extension to non-Hermitian isounits (that was the main object of the original proposal [4]) requires the exiting of Lie's theory in favor of the covering Lie-admissible theory, and will be studied in Chapter 4.

The isotopies of Lie's theory were proposed by Santilli from first axiomatic principles without the use of any map or transform. It is today known that the isotheory cannot be entirely derived via the use of noncanonical-nonunitary transforms since some of the basic structures (such as the isodifferential calculus) are not entirely derivable via noncanonical-nonunitary transforms.

3.2.12B. Universal Enveloping Isoassociative Algebras. Let ξ be an *associative algebra* over a field $F = F(a, +, \times)$ of characteristic zero with generic elements $A, B, C, \ldots$, trivial associative product $A \times B$ and unit I. The infinitely possible isotopes $\hat{\xi}$ of ξ were first introduced in Ref. [4] under the name of *isoassociative algebras.* In the original proposal $\hat{\xi}$ coincides with ξ as vector spaces but is equipped with Santilli's isoproduct so as to admit the isounit as the correct left and right unit

$$\hat{I}(x, \dot{x}, \ddot{x}, \ldots) = 1/\hat{T} > 0, \tag{3.2.209a}$$

$$\hat{A}\hat{\times}\hat{B} = \hat{A} \times \hat{T} \times \hat{B}, \;\; \hat{A}\hat{\times}(\hat{B}\hat{\times}\hat{C}) = (\hat{A}\hat{\times}\hat{B})\hat{\times}\hat{C}, \tag{3.2.209b}$$

$$\hat{I}\hat{\times}\hat{A} = \hat{A}\hat{\times}\hat{I} \equiv \hat{A}, \;\; \forall \hat{A} \in \hat{\xi}, \tag{3.2.209c}$$

where $\hat{A}, \hat{B}, \ldots$ denote the original elements $A, B, \ldots$ formulated on isospace over isofields.

Let $\xi = \xi(L)$ be the *universal enveloping associative algebra* of an N-dimensional Lie algebra L with ordered basis X_k, $k = 1, 2, \ldots, N$, and attached antisymmetric algebra isomorphic to the Lie algebras, $[\xi(L)]^- \approx L$ over F, and let the infinite-dimensional basis $I, X_k, X_i \times X_j$, $i \leq j, \ldots$ of $\xi(L)$ be characterized by the *Poincaré-Birkhoff-Witt theorem.*

A fundamental property submitted in the original proposal [4] (see also [2], pp. 154–163) is the following

THEOREM 3.2.11 (Poincaré-Birkhoff-Witt-Santilli isotheorem): Isocosets of the isounit and the standard, isomonomials

$$\hat{I},\ \ X_k,\ \ \hat{X}_i\hat{\times}\hat{X}_j,\ i\leq j,\ \ \hat{X}_i\hat{\times}\hat{X}_j\hat{\times}\hat{X}_k,\ i\leq j\leq k,\ \ldots, \tag{3.2.210}$$

form a basis of universal enveloping isoassociative algebra $\hat{\xi}(L)$ of a Lie algebra L (also called isoenvelope for short).

The first application of the above infinite-dimensional basis is a rigorous characterization of the isoexponentiation, Eq. (3.2.72), i.e.,

$$\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}}=\hat{e}^{i\times w\times\hat{X}}=$$

$$=\hat{I}+\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}\hat{/}\hat{1}!+(\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{\times}(\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X})\hat{/}\hat{2}!+\ldots=$$

$$=\hat{I}\times(e^{i\times w\times\hat{T}\times\hat{X}})=(e^{i\times w\times\hat{X}\times\hat{T}})\times\hat{I},\ \ \hat{i}=i\times\hat{I},\hat{w}=w\times\hat{I}\in\hat{F}. \tag{3.2.211}$$

The nontriviality of the Lie-Santilli isotheory is illustrated by the emergence of the nonlinear, nonlocal and noncanonical isotopic element $\hat{T}$ directly in the exponent, thus ensuring the desired generalization.

The implications of Theorem 3.2.11 also emerge at the level of isofunctional analysis because all structures defined via the conventional exponentiation must be suitably lifted into a form compatible with Theorem 3.2.11, as illustrated by the *iso-Fourier transforms,* Eq. (3.2.88).

It is today known that the main lines of isoenvelopes can indeed be derived via the use of noncanonical-nonunitary transforms, such as

$$U\times U^\dagger\neq I, \tag{3.2.212a}$$

$$I\to\hat{I}=U\times I\times U^\dagger, \tag{3.2.212b}$$

$$X_i\times X_j\to U\times(X_i\times X_j)\times U^\dagger=\hat{X}_i\hat{\times}\hat{X}_j, \tag{3.2.212c}$$

$$X_i\times X_j\times X_k\to U\times(X_i\times X_j\times X_k)\times U^\dagger=\hat{X}_i\hat{\times}\hat{X}_j\hat{\times}\hat{X}_k,\ \text{ etc.} \tag{3.2.212d}$$

Nevertheless, the uncontrolled use of the above transforms may lead to misrepresentations. In fact, a primary objective of the Lie-Santilli isotheory is that of preserving the original generators and parameters and change instead the associative and Lie products in an axiom-preserving way to accommodate the treatment of nonlinear, nonlocal and noncanonical interactions.

The preservation of the generators is, in particular, necessary for physical consistency because they represent conserved total quantities (such as the total energy, total angular momentum, etc.). These total quantities remain unchanged in

the transition from closed Hamiltonian and non-Hamiltonian systems (see Section 3.1.2). Equivalently, the generators of Lie's theory cannot be altered by non-Hamiltonian effects.

This physical requirement can only be achieved by preserving conventional generators X_k and lifting instead their product $X_i \times X_j \to X_i \hat{\times} X_j = X_i \times \hat{T} \times X_j$, which is the original formulation of the Lie-Santilli isotheory [4] and remain the formulation needed for applications to this day. It is essentially given by the projection of the isotopic formulation on conventional spaces over conventional fields.

3.2.12C. Lie-Santilli Isoalgebras. As it is well known, Lie algebras are the antisymmetric algebras $L \approx [\xi(L)]^-$ attached to the universal enveloping algebras $\xi(L)$. This main characteristic is preserved although enlarged under isotopies (see [4,2] for details). We therefore have the following

DEFINITION 3.2.15 [4]: A finite-dimensional isospace $\hat{L}$ with generic elements $\hat{A}, \hat{B}, \ldots$, over the isofield $\hat{F}$ with isounit $\hat{I} = 1/\hat{T} > 0$ is called a "Lie-Santilli isoalgebra" over $\hat{F}$ when there is a composition $[\hat{A},\hat{B}]$ in $\hat{L}$, called "isocommutator", that is isolinear as an isovector space and such that all the following axioms are satisfied

$$[\hat{A},\hat{B}] = -[\hat{B},\hat{A}], \tag{3.2.213a}$$

$$[\hat{A},[\hat{B},\hat{C}]] + [\hat{B},[\hat{C},\hat{A}]] + [\hat{C},[\hat{A},\hat{B}]] \equiv 0, \tag{3.2.213b}$$

$$[\hat{A}\hat{\times}\hat{B},\hat{C}] = \hat{A}\hat{\times}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{\times}\hat{B}, \quad \forall \hat{A}, \hat{B}, \hat{C} \in \hat{L}. \tag{3.2.213c}$$

The isoalgebras are said to be: *isoreal, isocomplex or isoquaternionic* depending on the assumed isofield and *isoabelian* when $[\hat{A},\hat{B}] \equiv \forall \hat{A}, \hat{B} \in \hat{L}$. A subset $\hat{L}^o$ of $\hat{L}$ is said to be an *isosubalgebra* of $\hat{L}$ when $[\hat{L}^o,\hat{L}^o] \subseteq \hat{L}^o$. $\hat{L}^o$ is called an *isoideal* of $\hat{L}$ when $[\hat{L}^o,\hat{L}] \subseteq \hat{L}^o$. A *maximal isoideal* verifying the property $[\hat{L}^o,\hat{L}^o] = 0$ is called the *isocenter* of $\hat{L}$.

For the isotopies of additional conventional notions, theorems and properties of Lie algebras, one may see monograph [2,6,36,37].

We merely recall the *isotopic generalizations of the celebrated Lie's First, Second and Third Theorems* introduced in the original proposal [4], but which we do not review here for brevity. For instance, the *Lie-Santilli Second Isotheorem* reads

$$[\hat{X}_i,\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i = \tag{3.2.214a}$$

$$= \hat{X}_i \times \hat{T}(x,\dot{x},\ddot{x},\ldots) \times \hat{X}_j - \hat{X}_j \times \hat{T}(x,\dot{x},\ddot{x},\ldots) \times \hat{X}_i = \hat{C}_{ij}^k(x,\dot{x},\ddot{x},\ldots)\hat{\times}\hat{X}_k, \tag{3.2.214b}$$

where the C's, called the *structure isofunctions*, generally have an explicit dependence on the underlying isovariable (see the examples later on), and verify certain restrictions from the Isotopic Third Theorem.

It is today known that Lie-Santilli isoalgebras can be reached via a noncanonical-nonunitary transform of conventional Lie algebras. In fact, we have

$$[X_i, X_j] = C_{ij}^k \times X_k \to$$

$$U \times [X_i, X_j] \times U^\dagger = [\hat{X}_i \hat{,} \hat{X}_j] =$$

$$U \times (C_{ij}^k \times X_k) \times U^\dagger = \hat{C}_{ij}^k(x, \dot{x}, \ddot{x}, \ldots) \hat{\times} \hat{X}_k. \tag{3.2.215}$$

However, again, this type of derivation of the isotheory may be misleading in physical applications due to the need to preserve the original generators unchanged, in accordance with the original formulation [4] of 1978. In this case we shall use the following projection of the isoalgebras on the original space over the original field

$$[X_i \hat{,}\ X_j] = X_i \times \hat{T} \times X_j - X_j \times \hat{T} \times X_i = C_{ij}^k(x, \dot{x}, \ldots) \times X_k. \tag{3.2.216}$$

It has been proved (see, e.g., [2,4,6] for details) that *Lie-Santilli isoalgebras $\hat{L}$ are isomorphic to the original algebra L*. In other words, the isotopies with $\hat{I} > 0$ cannot characterize any new algebra because all possible Lie algebras are known from Cartan classification. Therefore, Lie-Santilli isoalgebras merely provide new nonlinear, nonlocal and noncanonical realizations of existing algebras. It should be stresses that the above isomorphism is lost for more general liftings as shown in the next chapter.

3.2.12D. Lie-Santilli Isogroups. Under certain integrability conditions hereon assumed, Lie algebras L can be "exponentiated" to their corresponding *Lie transformation groups* G and, vice-versa, Lie transformation groups G admit their corresponding Lie algebra L when computed in the neighborhood of the identity I.

These basic properties are preserved under isotopies although broadened to the most general possible nonlinear, nonlocal and noncanonical transformations groups.

DEFINITION 3.2.16 [4]: A right isomodular Lie-Santilli isotransformation group $\hat{G}$ on an isospace $\hat{S}(\hat{x}, \hat{F})$ over an isofield $\hat{F}$ with common isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations

$$\hat{x}' = \hat{g}(\hat{w}) \hat{\times} \hat{x}, \quad \hat{x}, \hat{x}' \in \hat{S}, \quad \hat{w} \in \hat{F}, \tag{3.2.217}$$

such that:

1) The map $\hat{g}\hat{\times}\hat{S}$ *into* $\hat{S}$ *is isodifferentiable* $\forall\hat{g}\in\hat{G}$*;*
2) $\hat{I}$ *is the left and right unit*

$$\hat{I}\hat{\times}\hat{g}=\hat{g}\hat{\times}\hat{I}\equiv\hat{g},\ \ \forall\hat{g}\in\hat{G}; \tag{3.2.218}$$

3) the isomodular action is isoassociative, i.e.,

$$\hat{g}_1\hat{\times}(\hat{g}_2\hat{\times}\hat{x})=(\hat{g}_1\hat{\times}\hat{g}_2)\hat{\times}\hat{x},\ \ \forall\hat{g}_1,\hat{g}_2\in\hat{G}; \tag{3.2.219}$$

4) in correspondence with every element $\hat{g}(\hat{w})\in\hat{G}$ *there is the inverse element* $\hat{g}^{-\hat{I}}=\hat{g}(-\hat{w})$ *such that*

$$\hat{g}(\hat{0})=\hat{g}(\hat{w})\hat{\times}\hat{g}(-\hat{w})=\hat{I}; \tag{3.2.220}$$

5) following composition laws are verified

$$\hat{g}(\hat{w})\hat{\times}\hat{g}(\hat{w}')=\hat{g}(\hat{w}')\hat{\times}\hat{g}(\hat{w})=\hat{g}(\hat{w}+\hat{w}'),\forall\hat{g}\in\hat{G},\ \hat{w}\in\hat{F}. \tag{3.2.221}$$

The I left isotransformation group is defined accordingly.

The notions of *connected or simply connected transformation groups* carry over to the isogroups in their entirety.

The most direct realization of the (connected) isotransformation groups is that via isoexponentiation,

$$\hat{g}(w)=\prod_k\hat{e}^{\hat{i}\hat{\times}\hat{w}_k\hat{X}_k}=\Big(\prod_k e^{i\times w_k\times X_k\times\hat{T}(x,\dot{x},\ddot{x},...)}\Big)\times\hat{I}, \tag{3.2.222}$$

where the X's and w's are the infinitesimal generators and parameters, respectively, of the original algebra L, with corresponding connected isotransformations

$$\hat{x}'=\hat{g}(\hat{w})\hat{\times}\hat{x}=\Big(\prod_k\hat{e}^{\hat{i}\hat{\times}\hat{w}_k\hat{X}_k}\Big)\times\hat{I}\times\hat{T}\times x\times\hat{I}=$$

$$=\Big(\prod_k e^{i\times w_k\times X_k\times\hat{T}(x,\dot{x},\ddot{x},...)}\Big)\times x\times\hat{I}. \tag{3.2.223}$$

Equations (3.2.223) hold in some open neighborhood N of the isoorigin of $\hat{L}$ and, in this way, characterize some open neighborhood of the isounit of $\hat{G}$. Consequently, under the assumed continuity and connectivity properties, Lie-Santilli isoalgebras can be obtained as infinitesimal versions of finite Lie-Santilli isogroups, as illustrated by the following finite isotransform

$$\hat{A}(\hat{w})=(\hat{e}^{\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}})\hat{\times}\hat{A}(\hat{0})\hat{\times}(\hat{e}^{-\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}})=$$

$$=(e^{i\times w\times\hat{X}\times\hat{T}})\times\hat{A}(\hat{0})\times(e^{-i\times w\times\hat{T}\times\hat{X}}) \tag{3.2.224}$$

with infinitesimal version in the neighborhood of $\hat{I}$

$$\hat{A}(\hat{d}\hat{w}) = (\hat{I} + \hat{i}\hat{\times}\hat{d}\hat{w}\hat{\times}\hat{X} + \ldots)\hat{\times}\hat{A}(0)\hat{\times}(\hat{I} - \hat{i}\hat{\times}\hat{d}\hat{w}\hat{\times}\hat{X} + \ldots) =$$

$$= \hat{A}(\hat{0}) + \hat{i}\hat{\times}\hat{d}\hat{w}\hat{\times}\hat{X}\hat{\times}\hat{A}(\hat{0}) - \hat{i}\hat{\times}\hat{d}\hat{w}\hat{\times}\hat{A}(\hat{0})\hat{\times}\hat{X}, \tag{3.2.225}$$

that can be written

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}(\hat{w})}{\hat{d}\hat{w}} = \hat{A}\hat{\times}\hat{X} - \hat{X}\hat{\times}\hat{A} = [\hat{A}\hat{,}\hat{X}]. \tag{3.2.226}$$

Note the crucial appearance of the isotopic element $\hat{T}(x, \dot{x}, \ddot{x}, \ldots)$ in the exponent of the isogroup. This ensures a structural generalization of Lie's theory of the desired nonlinear, nonlocal and noncanonical form.

Still another important property is that conventional group composition laws admit a consistent isotopic lifting, resulting in the following *Baker-Campbell-Hausdorff-Santilli Isotheorem* [4]

$$(\hat{e}^{\hat{X}_1})\hat{\times}(\hat{e}^{X_2}) = \hat{e}^{\hat{X}_3}, \tag{3.2.227a}$$

$$\hat{X}_3 = \hat{X}_1 + \hat{X}_2 + [\hat{X}_1\hat{,}\hat{X}_2]\hat{/}\hat{2} + [(\hat{X}_1 - \hat{X}_2)\hat{,}[\hat{X}_1\hat{,}\hat{X}_2]]\hat{/}\hat{12} + \ldots \ . \tag{3.2.227b}$$

Let $\hat{G}_1$ and $\hat{G}_2$ be two isogroups with respective isounits $\hat{I}_1$ and $\hat{I}_2$. The *direct isoproduct* $\hat{G}_1\hat{\times}\hat{G}_2$ is the isogroup of all ordered pairs

$$(\hat{g}_1, \hat{g}_2), \quad \hat{g}_1 \in \hat{G}_1, \hat{g}_2 \in \hat{G}_2, \tag{3.2.228}$$

with isomultiplication

$$(\hat{g}_1, \hat{g}_2)\hat{\times}(\hat{g}'_1, \hat{g}'_2) = (\hat{g}_1\hat{\times}\hat{g}'_1, \hat{g}_2\hat{\times}\hat{g}'_2), \tag{3.2.229}$$

total isounit $(\hat{I}_1, \hat{I}_2)$ and inverse $(\hat{g}_1^{-\hat{I}_1}, \hat{g}_2^{-\hat{I}_2})$.

The following particular case is important for the isotopies of inhomogeneous groups. Let $\hat{G}$ be an isogroup and $\hat{G}_{\hat{a}}$ the isogroup of all its inner isoautomorphisms. Let $\hat{G}^o_{\hat{a}}$ be a subgroup of $\hat{G}_{\hat{a}}$, and let $\Lambda(\hat{g})$ be the image of $\hat{g} \in \hat{G}$ under $\hat{G}_{\hat{a}}$. The *semidirect isoproduct* $\hat{G}\hat{\times}\hat{G}^o_{\hat{a}}$ is the isogroup of all ordered pairs

$$(\hat{g}, \hat{\Lambda})\hat{\times}(\hat{g}^o, \hat{\Lambda}^o) = (\hat{g}, \hat{\Lambda}(\hat{g}^o), (\hat{\Lambda}, \hat{\Lambda}^o), \tag{3.2.230}$$

with total isounit given by $\hat{I}_{tot} = \hat{I} \times \hat{I}^o$.

The studies of the isotopies of the remaining aspects of the structure theory of Lie groups is then consequential.

It is hoped that the reader can see from the above elements that the entire conventional Lie theory does indeed admit a consistent and nontrivial lifting into the covering Lie-Santilli formulation.

3.2.12E. Isorepresentations of Lie-Santilli Isoalgebras. Despite considerable research on the Lie-Santilli isotheory over the past 26 years, the study of the *isorepresentations* of the Lie-Santilli isoalgebras remains vastly unknown at this writing (summer 2004), with the sole exception of the *fundamental (or regular) isorepresentations* that were also identified by Santilli in the original proposal [4].

In this monograph we shall primarily use in the applications of hadronic mechanics the fundamental isorepresentations or other isorepresentations reducible to the latter.

Let L be an N-dimensional Lie algebra with N-dimensional unit $I = Diag.(1, 1, \ldots, 1)$. Let R be the fundamental, N-dimensional matrix representation of L. Let $\hat{L}$ be the isotope of L characterized by the N-dimensional isounit $\hat{I} = U \times U^\dagger > 0$. It is then evident that the *fundamental isorepresentation* of $\hat{L}$ is given by

$$\hat{R} = U \times R \times U^\dagger, \quad U \times U^\dagger = \hat{I} \neq I, \hat{I} > 0. \tag{3.2.231}$$

Interested colleagues are encouraged to study the isorepresentation theory because, as we shall see in the next sections, the fundamental notion of hadronic mechanics, that of *isoparticles,* is characterized by an irreducible isorepresentation of the Poincaré-Santilli isosymmetry.

3.2.12F. Isodual Lie-Santilli Isotheory. As indicated Chapters 1 and 2, the contemporary formulation of Lie's theory is one of the most serious obstacles for a consistent *classical* representation of antimatter, because it lacks an appropriate conjugate formulation that, after quantization, is compatible with charge conjugation.[20]

It is easy to verify that the isotheory presented above admits a consistent antiautomorphic image under isoduality, thus permitting the treatment of antimatter under nonlinearity, nonlocality and noncanonicity as occurring in interior conditions, such as for the structure of an antimatter star.

In fact, we have the *isodual universal enveloping isoassociative isoalgebra* $\hat{\xi}^d$ characterized by the *isodual Poincaré-Birkhoff-Witt-Santilli isotheorem* with infinite dimensional basis

$$\hat{I}^d, \ \ X_k^d, \ \ \hat{X}_i^d \hat{\times}^d \hat{X}_j^d, \ i \leq j, \ \ \hat{X}_i^d \hat{\times}^d \hat{X}_j^d \hat{\times}^d \hat{X}_k^d, \ i \leq j \leq k, \ \ldots. \tag{3.2.232}$$

The *isodual Lie-Santilli isoalgebra* $\hat{L}^d \approx (\hat{\xi}^d)^-$ attached to $\hat{\xi}^d$ is characterized by the *isodual Lie-Santilli Second Isotheorem*

$$[\hat{X}_i^d \hat{,} \hat{X}_j^d] = \hat{X}_i^d \hat{\times}^d \hat{X}_j^d - \hat{X}_j^d \hat{\times}^d \hat{X}_i^d = \hat{C}_{ij}^{d\ k} \hat{\times}^d \hat{X}_k^d. \tag{3.2.233}$$

[20]The reader is urged to verify that the classical treatment of antimatter via the so-called *dual Lie algebras* does not achieve antiparticles under quantization, trivially, because of the uniqueness of the quantization channel for both particles and antiparticles.

Under the needed continuity and connectivity property, the *isodual exponentiation* of $\hat{L}^d$ characterizes the *connected isodual Lie-Santilli transformation isogroup*

$$\hat{x}'^d = (\hat{g}^d(\hat{w}^d) = \hat{\prod}_k \hat{e}^{\hat{d^i}^d \hat{\times}^d \hat{w}_k^d \hat{X}_k^d}) \hat{\times}^d \hat{x}^d. \tag{3.2.234}$$

Interested readers can then easily derive any additional needed isodual property.

3.2.13 Unification of All Simple Lie Algebras into Lie-Santilli Isoalgebras

The original proposal [4] of 1978 included the *conjecture that all simple Lie algebras of dimension N can be unified into a single Lie-Santilli isoalgebra of the same dimension,* and gave an explicit example. The conjecture was subsequently proved by the late mathematicians Gr. Tsagas [42] in 1996 for all simple Lie algebras of type A, B, C and D. The premature departure of Prof. Tsagas while working at the problem prevented him to complete the proof of the conjecture for the case of all exceptional Lie algebras. As a result, the proof of the indicated conjecture remain incomplete at this writing.

For the unification here considered it is important to eliminate the restriction that the isounits are necessarily positive definite, while preserving all other characteristics, such as nowhere singularity and Hermiticity. As a result, in its simple possible form, the isounit can be diagonalized into the form whose elements can be either positive or negative,

$$\hat{I} = Diag.(\pm n_1^2, \pm n_2^2, \ldots, \pm n_N^2) = 1/\hat{T},\ n_k \in R,\ n_k \neq 0,\ k = 1, 2, \ldots, N. \tag{3.2.235}$$

The example provided in the original proposal [4], subsequently studied in detail in Refs. [8], consisted in the *classification of all possible simple Lie algebra of dimension 3.* In this case, Cartan's classification produces two non-isomorphic Lie algebras, the compact rotational algebra in three dimension $SO(3)$ and the noncompact algebra $SO(2.1)$.

The distinction between compact and noncompact algebras is lost under the class of isotopies here considered. In fact, the *classification of all possible, simple, three-dimensional Lie-Santilli isoalgebras $\hat{L}_3$ for the case of diagonal isounits* is characterized by the isounit itself and can be written

$$\hat{I} = Diag.(+1, +1, +1),\ \ \hat{L}_3 \approx SO(3), \tag{3.2.236a}$$

$$\hat{I} = Diag.(+1, +1, -1),\ \ \hat{L}_3 \approx SO(2.1), \tag{3.2.236b}$$

$$\hat{I} = Diag.(+1, -1, +1),\ \ \hat{L}_3 \approx SO(2.1), \tag{3.2.236c}$$

$$\hat{I} = Diag.(-1, +1, +1),\ \ \hat{L}_3 \approx SOI(2.1), \tag{3.2.236d}$$

$$\hat{I} = Diag.(-1,-1,-1), \quad \hat{L}_3 \approx SO(3)^d, \tag{3.2.236e}$$

$$\hat{I} = Diag.(-1,-1,+1), \quad \hat{L}_3 \approx SO(2.1)^d, \tag{3.2.236f}$$

$$\hat{I} = Diag.(-1,+1,-1), \quad \hat{L}_3 \approx SO(2.1), \tag{3.2.236g}$$

$$\hat{I} = Diag.(+1,-1,-1), \quad \hat{L}_3 \approx SO(2.1)^d, \tag{3.2.236h}$$

$$\hat{I} = Diag.(+n_1^2,+n_2^2,+n_3^2), \quad \hat{L}_3 \approx SO(3), \tag{3.2.236i}$$

$$\hat{I} = Diag.(+n_1^2,+n_2^2,-n_3^2), \quad \hat{L}_3 \approx SO(2.1), \tag{3.2.236j}$$

$$\hat{I} = Diag.(+n_1^2,-n_2^2,+n_3^2), \quad \hat{L}_3 \approx SO(2.1), \tag{3.2.236k}$$

$$\hat{I} = Diag.(-n_1^2,+n_2^2,+n_3^2), \quad \hat{L}_3 \approx SOI(2.1), \tag{3.2.236l}$$

$$\hat{I} = Diag.(-n_1^2,-n_2^2,-n_3^2), \quad \hat{L}_3 \approx SO(3)^d, \tag{3.2.236m}$$

$$\hat{I} = Diag.(-n_1^2,-n_2^2,+n_3^2), \quad \hat{L}_3 \approx SO(2.1)^d, \tag{3.2.236n}$$

$$\hat{I} = Diag.(-n_1^2,+n_2^2,-n_3^2), \quad \hat{L}_3 \approx SO(2.1), \tag{3.2.236o}$$

$$\hat{I} = Diag.(+n_1^2,-n_2^2,-n_3^2), \quad \hat{L}_3 \approx SO(2.1)^d, \tag{3.2.236p}$$

In conclusion, when studying simple algebras from the viewpoint of the covering Lie-Santilli isoalgebras, there exist *only one single isoalgebra in three dimensions,* $\hat{L}_3$ without any distinction between compact and noncompact algebras.

The *realization* of the simple isoalgebra $\hat{L}_3$ with diagonal isounits consists of 21 different Lie-Santilli isoalgebras in three dimension that can be reduced to 4 topologically different Lie algebras, namely $SO(3)$, $SO(2.1)$, $SO(3)^d$ and $SO(2.1)^d$.

All distinctions between these 21 different realizations are lost at the level of abstract Lie-Santilli isoalgebra $\hat{L}_3$.

It should be stressed that, by no means, the 21 realizations (3.2.236) exhaust all possible forms of Lie-Santilli simple isoalgebras in three dimensions because in realizations (3.2.236) we have excluded nondiagonal realizations of the isounit, as well as imposed additional restrictions on the isounit, such as single valuedness and Hermiticity.

Essentially the same results hold for the unification of the Lie Algebras of type A, B, C, and D studied by Tsagas [42].

It is hoped that interested mathematicians can complete the proof of Santilli's conjecture for the remaining exceptional algebras. In considering the problem, mathematicians are suggested to keep in mind that Hermitian and diagonal realizations of the isounit (3.2.135) are expected to be insufficient, thus implying the possible use of *nowhere singular, Hermitian, nondiagonal isounits,* or *nowhere singular, Hermitian, nondiagonal and multivalued isounits,* or *nowhere singular, non-Hermitian, nondiagonal and multivalued isounits.*

3.2.14 The Fundamental Theorem for Isosymmetries and Their Isoduals

The fundamental symmetries of the 20-th century physics deal with point-like abstractions of particles in vacuum under linear, local and potential interactions, and are the *Galilei symmetry* $G(3.1)$ for nonrelativistic treatment or the *Poincaré symmetry* for relativistic formulations.

A central objective of hadronic mechanics is the broadening of these fundamental spacetime symmetries to represent extended, nonspherical and deformable particles under linear and nonlinear, local and nonlocal and potential as well as nonpotential interactions.

In fact, as we shall see, all novel industrial applications of hadronic mechanics are crucially dependent on the admission of the extended character of particles or of their wavepackets in conditions of deep mutual penetration. In turn, the latter conditions imply new effects permitting basically new energies and fuels that are completely absent for conventional spacetime and other symmetries.

Alternatively and equivalently a central problem of hadronic mechanics is the *construction in an explicit form of the symmetries of all possible nonsingular, but otherwise arbitrary deformations of conventional spacetime and internal invariants.*

All these problems and others are resolved by the following important:

THEOREM 3.2.12 [6]: Let G be an N-dimensional Lie symmetry group of a K-dimensional metric or pseudo-metric space $S(x, m, F)$ over a field F,

$$G: \quad x' = \Lambda(w) \times x, \quad y' = \Lambda(w) \times y, \quad x, y \in \hat{S}, \tag{3.2.237a}$$

$$(x' - y')^\dagger \times \Lambda^\dagger \times m \times \Lambda \times (x - y) \equiv (x - y)^\dagger \times m \times (x - y), \tag{3.2.237b}$$

$$\Lambda^\dagger(w) \times m \times \Lambda(w) \equiv m. \tag{3.2.237c}$$

Then, all infinitely possible isotopies $\hat{G}$ of G acting on the isospace $\hat{S}(\hat{x}, \hat{M}, \hat{F})$, $\hat{M} = \hat{m} \times \hat{I} = (\hat{T}_i^k \times m_{kj}) \times \hat{I}$ characterized by the same generators and parameters of G and new isounits $\hat{I} = 1/\hat{T} > 0$ leave invariant the isocomposition on the projection $\hat{S}(x, \hat{m}, F)$ of $\hat{S}(\hat{x}, \hat{M}, \hat{F})$ on the original space $S(x, m, F)$

$$\hat{G}: \quad x' = \hat{\Lambda}(w) \times x, \quad y' = \hat{\Lambda}(w) \times y, \quad x, y \in \hat{S}, \tag{3.2.238a}$$

$$(x' - y')^\dagger \times \hat{\Lambda}^\dagger \times \hat{m} \times \hat{\Lambda} \times (x - y) \equiv (x - y)^\dagger \times \hat{m} \times (x - y), \tag{3.2.238b}$$

$$\hat{\Lambda}^\dagger(\hat{w}) \times \hat{m} \times \hat{\Lambda}(\hat{w}) \equiv \hat{m}. \tag{3.2.238c}$$

Similarly,all infinitely possible isodual isotopies $\hat{G}^d$ of $\hat{G}$ acting on the isodual isospace $\hat{S}^d(\hat{x}^d, \hat{M}^d, \hat{F}^d)$, $\hat{M}^d = (\hat{T}^d \times m^d) \times \hat{I}^d$ characterized by the isodual generators $\hat{X}_k^d$ parameters $\hat{w}^d$ and isodual isounit $\hat{I}^d = 1/\hat{T}^d < 0$ leave invariant the

isodual isocomposition on the projection $\hat{S}^d(x^d, \hat{m}^d, F^d)$

$$\hat{G}^d : \quad x'^d = \hat{\Lambda}^d \times^d x^d, \quad y'^d = \hat{\Lambda}^d \times^d y^d, \quad x^d, y^d \in \hat{S}^d, \tag{3.2.239a}$$

$$(x'-y')^{\dagger^d} \times^d \hat{\Lambda}^{\dagger^d} \times^d \hat{m}^d \times^d \hat{\Lambda}^d \times^d (x-y)^d \equiv (x-y)^{\dagger^d} \times^d \hat{m}^d \times^d (x-y)^d, \tag{3.2.239b}$$

$$\hat{\Lambda}^{\dagger^d} \times^d \hat{m}^d \times^d \hat{\Lambda}^d \equiv \hat{m}^d. \tag{3.2.239c}$$

Proof. Assume that $N = K$ and the representation Λ is the fundamental one. Recall that metrics, isometrics and isounits are diagonal. Then on $\hat{S}(x, \hat{m}, F)$ we have the identities

$$\hat{I} = U \times U^\dagger \neq I, \quad \hat{T} = (U \times U^\dagger)^{-1}, \tag{3.2.240a}$$

$$U \times (\Lambda \times m \times \Lambda) \times U^\dagger =$$

$$= (U \times \Lambda \times U^\dagger) \times (U^{\dagger^{-1}} \times m \times U^{-1}) \times (U \times \Lambda \times U^\dagger) =$$

$$= \hat{\Lambda} \times (\hat{T} \times m) \times \hat{\Lambda} = \hat{\Lambda} \times \hat{m} \times \hat{\Lambda} = \hat{m}. \tag{3.2.240b}$$

The proof of the remaining cases are equally trivial. **q.e.d.**

Note that the isotopic symmetries and their isoduals can be uniquely and explicitly constructed with the methods summarized in this section via the sole use of the original symmetry and the isounit characterizing the deformation of the original metric m.

Under our assumptions, the isosymmetries can be constructed in the needed, explicit, nonlinear, nonlocal and noncanonical forms. In fact, the existence of the original symmetry transformations plus the condition $\hat{I} > 0$ ensure the convergence of the infinite isoseries of the isoexponentiation, resulting in the needed explicit form, as we shall see in various examples in the next sections.

3.3 CLASSICAL LIE-ISOTOPIC MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

3.3.1 Introduction

One of the reasons for the majestic consistency of quantum mechanics is the existence of axiomatically consistent and invariant classical foundations, given by *classical Lagrangian and Hamiltonian mechanics,* namely, the discipline based on the *truncated analytic equations*

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = 0, \tag{3.3.1a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k}, \tag{3.3.1b}$$
$$k = 1,2,3; \quad a = 1,2,3,\ldots,N,$$

with a unique and unambiguous map into operator forms.

Following the original proposal [5] of 1978 to build hadronic mechanics, this author did not consider the new discipline sufficiently mature for experimental verifications and industrial applications until the new discipline had equally consistent and invariant classical foundations with an equally unique and unambiguous map into operator formulations.

Intriguingly, the *operator* foundations of hadronic mechanics were sufficiently identified in the original proposal [5], as we shall see in the next section. However, the identification of the *classical* counterpart turned out to be a rather complex task that required decades of research.

The objective, fully identified in 1978, was the construction of a covering of classical Lagrangian and Hamiltonian mechanics, namely, a covering of Eqs. (3.3.1), admitting a unique and unambiguous map into the already known Lie-isotopic equations of hadronic mechanics.

The mandatory starting point was the consideration of the *true Lagrange and Hamilton equations,* those with external terms

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = F_{ak}(t,r,v), \tag{3.3.2a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k} + F_{ak}(t,r,p), \tag{3.3.2b}$$

since they were conceived, specifically, for the interior dynamical systems treated by hadronic mechanics.

In fact, the legacy of Lagrange and Hamilton is that classical systems *cannot* be entirely represented with one single function today called a Lagrangian or a Hamiltonian used for the representation of forces derivable from a potential, but require additional quantities for the representation of contact nonpotential forced represented precisely by the external terms.

As such, the true Lagrange and Hamilton equations constitute excellent candidates for the classical origin of hadronic mechanics.

3.3.2 Insufficiencies of Analytic Equations with External Terms

It was indicated by Santilli [4] also in 1978 (see the review in Chapter 1 for more details) that the true analytic equations cannot be used for the construction of a consistent covering of conventional analytic equations because the new algebraic

brackets of the time evolution of a generic quantity $A(r,p)$ in phase space

$$\frac{dA}{dt} = (A,H,F) = [A,H] + \frac{\partial A}{\partial r^k} \times F_k =$$

$$= \frac{\partial A}{\partial r^k} \times \frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial r^k} \times \frac{\partial A}{\partial p_k} + \frac{\partial A}{\partial r^k} \times F_k, \tag{3.3.3}$$

violate the right distributive and scalar laws, Eqs. (3.2.5) and (3.2.6). Consequently, *the true analytic equations in their original formulation lose "all" possible algebras, let alone all possible Lie algebras.* No axiomatically consistent covering can then be build under these premises.[21]

The above insufficiency essentially established the need of rewriting the true analytic equations into a form admitting a consistent algebra in the brackets of the time evolution laws and, in addition, achieves the same invariance possessed by the truncated analytic equations.

Even though its main lines were fully identified in 1978, the achievement of the new covering mechanics resulted to require a rather long and laborious scientific journey.

This section is intended to outline the final formulation of the classical mechanics underlying hadronic mechanics in order to distinguish it from the numerous attempts that were published with the passing of time.

As a brief guide to the literature, the reader should be aware that the true analytic equations (3.3.2) are generally set for *open nonconservative systems.* These systems require the broader *Lie-admissible branch of hadronic mechanics* that will be studied in the next chapter.

Therefore, the reader should be aware that several advances in Lie-isotopies have been obtained and can be originally identified as particular cases of the broader Lie-admissible theories.

This chapter is dedicated to the study of classical and operator closed-isolated systems verifying conventional total conservation laws while having linear and nonlinear, local and nonlocal as well as potential and nonpotential internal forces.

The verification of conventional total conservation law requires classical brackets that, firstly, verify the right and left distributive and scalar laws (as a condition to characterize an algebra), and, secondly, the brackets are necessarily antisymmetric.

The brackets of conventional Hamiltonian mechanics are Lie. Therefore, a necessary condition to build a true covering of Hamiltonian mechanics is the search of brackets that are of the broader Lie-isotopic type. As a matter of

[21] For additional problematic aspects of the true analytic equations, one may consult Ref. [4] or the review in Chapter 1.

fact, this feature, fully identified in 1978 [4,5], was the very motivation for the construction of the isotopies of the Lie theory reviewed in Section 3.2.12.

In summary, the construction of a covering of the conventional Hamiltonian mechanics as the classical foundations of the Lie-isotopic branch of hadronic mechanics must be restricted to a reformulation of the true analytic equations (3.3.2) in such a way that the underlying brackets are Lie-isotopic, and the resulting mechanics is invariant.

3.3.3 Insufficiencies of Birkhoffian Mechanics

Santilli dedicated the second volume of *Foundations of Theoretical Mechanics* published by Springer-Verlag [2] in 1982 to the construction of a covering of classical Hamiltonian mechanics along the above indicated requirement. The resulting new mechanics was released under the name of *Birkhoffian mechanics* to honor G. D. Birkhoff who first discovered the underlying analytic equations in 1927.[22]

Conventional Hamiltonian mechanics is based on the *canonical action principle*

$$\delta A^o = \delta \int (p_k \times dr^k - H \times dt) = 0, \tag{3.3.4}$$

and, via the use of the unified notation

$$b = (b^\mu) = (r^i, p_j), \tag{3.3.5a}$$

$$R^o = (R^o_\mu) = (p_k, 0), \quad \mu = 1, 2, \ldots, 6, \tag{3.3.5b}$$

can be written

$$\delta A^o = \delta \int (R^o_\mu \times db^\mu - H \times dt) \equiv$$

$$\equiv \delta \int (p_k \times dr^k - H \times dt) = 0. \tag{3.3.6}$$

from which the conventional Hamilton's equations (3.3.1b) acquire the unified form

$$\omega_{\mu\nu} \times \frac{db^\nu}{dt} = \frac{\partial H}{\partial b^\mu}, \tag{3.3.7}$$

where

$$\omega_{\mu\nu} = \frac{\partial R^o_\nu}{\partial b^\mu} - \frac{\partial R^o_\mu}{\partial b^\nu} \tag{3.3.8}$$

is the fundamental (canonical) symplectic tensor (3.2.187).

[22] Interested readers should consult, for brevity, the historical notes of Ref. [2].

The fundamental (conventional Poisson) brackets of the time evolution then acquire the unified form

$$\frac{dA}{dt} = [A, H] = \frac{\partial A}{\partial b^\mu} \times \omega^{\mu\nu} \times \frac{\partial H}{\partial b^\nu}, \tag{3.3.9}$$

where

$$\omega^{\mu\nu} = [(\omega_{\alpha\beta})^{-1}]^{\mu\nu} \tag{3.3.10}$$

is the fundamental (canonical) Lie tensor.

Santilli [2] based the construction of a covering isotopic (that is, axiom-preserving) mechanics on the most general possible *Pfaffian action principle*

$$\delta A = \delta \int (R_\mu \times db^\mu - B \times dt) = 0, \tag{3.3.11}$$

where the $R_\mu(b)$ functions are now arbitrary functions in phase space, e.g., of the type

$$R(b) = (R_\mu) = (E_i(r, p), D^j(r, p)), \tag{3.3.12}$$

verifying certain regularity conditions [2].

It is easy to see that principle (3.3.11) characterizes the following analytic equations[23]

$$\Omega_{\mu\nu} \times \frac{db^\nu}{dt} = \frac{\partial B}{\partial b^\mu}, \tag{3.3.13a}$$

$$\Omega_{\mu\nu} = \frac{\partial R_\nu}{\partial b^\mu} - \frac{\partial R_\mu}{\partial b^\nu} \tag{3.3.13b}$$

is the most general possible symplectic tensor in local coordinates. Eqs. (3.3.12) were called *Birkhoff's equations* because, following a considerable research, they resulted to have been first identified by D. G. Birkhoff in 1927. The function B was called the *Birkhoffian* in order to distinguish it from the conventional Hamiltonian, since the latter represent the total energy, while the former does not.

The fundamental brackets of the time evolution then acquire the unified form

$$\frac{dA}{dt} = \frac{\partial A}{\partial b^\mu} \times \Omega^{\mu\nu} \times \frac{\partial B}{\partial b^\nu}, \tag{3.3.14a}$$

$$\Omega^{\mu\nu} = [(\Omega_{\alpha\beta})^{-1}]^{\mu\nu}. \tag{3.3.14b}$$

The covering nature of Eqs. (3.3.11)–(3.3.14) over the conventional Eqs. (3.3.4)–(3.3.10) is evident. In particular, brackets (3.3.14) are antisymmetric and verify the Lie axioms, although in the generalized Lie-Santilli isotopic form.

[23]The equations are called "analytic" in the sense of being derivable from a variational principle.

Moreover, Birkhoffian mechanics was proved in Ref. [2] to be "directly universal", that is, capable of representing "all" possible (sufficiently smooth and regular) Newtonian systems directly in the "frame of the observer" without any need for the transformation theory.

Therefore, at the time of releasing monograph [2] in 1982, the Birkhoffian mechanics appeared to have all the necessary pre-requisites to be the classical foundation of hadronic mechanics.

Unfortunately, subsequent studies established that *Birkhoffian mechanics cannot be used for consistent physical applications* because it is afflicted by the catastrophic inconsistencies studied in Section 1.4.1, with particular reference to the lack of invariance, namely, the inability to predict the same numbers for the same physical conditions at different times owing to the noncanonical character of the time evolution.

Moreover, canonical action (3.3.4) is independent from the momenta, $A^o = A^o(r)$, while this is not the case for the Pfaffian action (3.3.11) for which we have $A = A(r, p)$. Consequently, any map into an operator form implies "wavefunctions" dependent on both coordinates and momenta, $\psi(r, p)$. Therefore, the operator image of Birkhoffian mechanics is beyond our current knowledge, and its study is deferred to future generations.

The above problems requested the resumption of the search for the consistent classical counterpart of hadronic mechanics from its beginning.

Numerous additional generalized classical mechanics were identified but they still missed the achievement of the crucial invariance (for brevity, see monographs [15,16] of 1991 and the first edition of monograph [6,7] of 1993).

By looking in retrospect, the origin of all the above difficulties resulted to be where one would expect them the least, in the use of the ordinary differential calculus.

Following the discovery in 1995 (see the second edition of monographs [6,7] and Ref. [10]) of the isodifferential calculus, the identification of the final, axiomatically consistent and invariant form of the classical foundations of hadronic mechanics emerged quite rapidly.

3.3.4 Newton-Santilli Isomechanics for Matter and its Isodual for Antimatter

The fundamental character of *Newtonian Mechanics* for all scientific inquiries is due to the preservation at all subsequent levels of treatment (such as Hamiltonian mechanics, Galileo's relativity, special relativity, quantum mechanics, quantum chemistry, quantum field theory, etc.) of its main structural features, such as:

1) The underlying local-differential Euclidean topology;
2) The ordinary differential calculus; and
3) The consequential point-like approximation of particles.

Nevertheless, Newton's equations have well known notable limitations to maintain such a fundamental character for the entirely of scientific knowledge without due generalization for so many centuries.

As indicated in Chapter 1, the point-like approximation is indeed valid for very large mutual distances among particles compared to their size, as occurring for planetary and atomic systems (*exterior dynamical systems*). However, the same approximation is excessive for systems of particles at short mutual distances, as occurring for the structure of planets, hadrons, nuclei and stars (*interior dynamical systems*).

Also, dimensionless particles cannot experience any contact or resistive interactions. Consequently, dissipative or, more generally, nonconservative forces used for centuries in Newtonian mechanics are a mere *approximation* of contact nonpotential nonlocal-integral interactions among extended constituents, the approximation being generally achieved via power series expansion in the velocities.

It should be finally recalled on historical grounds that *Newton had to construct the differential calculus as a pre-requisite for the formulation of his celebrated equations.*

No genuine structural broadening of the disciplines of the 20-th century is possible without a consistent structural generalization of their foundations, Newton's equations in Newtonian mechanics.

Santilli's isomathematics has been constructed to permit *the first axiomatically consistent structural generalization of Newton's equations in Newtonian mechanics since Newton's time, for the representation of extended, nonspherical and deformable particles under linear and nonlinear, local and nonlocal and potential as well as nonpotential interactions as occurring in the physical reality of interior dynamical systems.*

By following Newton's teaching, the author has dedicated primary efforts to the isotopic lifting of the conventional differential calculus, topology and geometries [6,10] as a pre-requisite for the indicated structural generalization of Newton's equations.

To outline the needed isotopies, let us recall that Newtonian mechanics is formulated on a 7-dimensional representation space characterized by the following Kronecker products of Euclidean spaces

$$S_{tot} = E(t, R_t) \times E(r, \delta, R_r) \times E(v, \delta, R_v), \tag{3.3.15}$$

of the one dimensional space $E(t, R_t)$ representing time t, the tree dimensional Euclidean space $E(r, \delta, R_r)$ of the coordinates $r = (r_a^k)$ (where $k = 1, 2, 3$ are the Euclidean axes and $a = 1, 2, \ldots, N$ represents the number of particles), and the velocity space $E(v, \delta, R_v), v = dr/dt$.

It is generally assumed that all variables t, r, and v are defined on the same field of real numbers R. However, the unit of time is the *scalar* $I = +1$, while the

unit of the Euclidean space is the *matrix*, and the same happens for the velocities, $I_r = I_v = Diag.(1, 1, 1)$.

Therefore, on rigorous grounds, the representation space of Newtonian mechanics must be defined on the Kronecker product of the corresponding fields

$$R_{tot} = R_t \times R_r \times R_v \tag{3.3.16}$$

with total unit

$$I_{Tot} = 1_t \times Diag.(1, 1, 1)_r \times Diag.(1, 1, 1)_v. \tag{3.3.17}$$

The above total unit can be factorized into the production of seven individual units for time and the two sets of individual Euclidean axes a, y, a with corresponding factorization of the fields

$$I_{tot} = 1_t \times 1_{rx} \times 1_{ry} \times 1_{rz} \times 1_{vx} \times 1_{vy} \times 1_{vz}, \tag{3.3.18a}$$

$$R_{tot} = R_t \times R_{rx} \times R_{ry} \times R_{rz} \times R_{vx} \times R_{vy} \times R_{vz}, \tag{3.3.18b}$$

that constitute the foundations of the conventional *Euclidean topology* here assumed as known.

Via the use of Eqs. (3.1.5), *Newton's equations for closed-non-Hamiltonian systems* can then be written

$$m_a \times a_{ka} = m_a \times \frac{dv_{ka}}{dt} = F_{ka}(t, r, v) = F_{ka}^{SA} + F_{ka}^{NSA}, \tag{3.3.19a}$$

$$\sum_a \mathbf{F}_a^{NSA} = 0, \tag{3.3.19b}$$

$$\sum_a \mathbf{r}_a \bigodot \mathbf{F}_a^{NSA} = 0, \tag{3.3.19c}$$

$$\sum_a \mathbf{r}_a \bigwedge \mathbf{F}_a^{NSA} = 0, \tag{3.3.19d}$$

where SA (NSA) stands for *variational selfadjointness (variational nonselfadjointness),* namely, the verification (violation) of the integrability conditions for the existence of a potential [1], and conditions (3.3.xx), (3.3.xx) and (3.3.xx) assure the verification of conventional total conservation laws.

The isotopies of Newtonian mechanics, today known *Newton-Santilli isomechanics*, were first submitted in the second edition of monograph [5] and in the mathematical treatment [10].

They require the use of: the *isotime* $\hat{t} = t \times \hat{I}_t$ with isounit $\hat{I}_t = 1/\hat{T}_t > 0$ and related isofield $\hat{R}_t$; the *isocoordinates* $\hat{r} = (\hat{r}_a^k) = r \times \hat{I}_r$, with isounit $\hat{I}_r = 1/\hat{T}_r > 0$ and related isofield $\hat{R}_r$; and the *isovelocities* $\hat{v} = (v_{ka}) = v \times \hat{I}_v$ with isounit $\hat{I}_v = 1/\hat{T}_v > 0$ and related isofield $\hat{R}_v$.

The Newton-Santilli isomechanics is then formulated on the 7-dimensional isospace

$$\hat{S}_{tot} = \hat{E}(\hat{t}, \hat{R}_{\hat{t}}) \times \hat{E}(\hat{r}, \hat{\delta}_r, \hat{R}_{\hat{r}}) \times \hat{E}(\hat{v}, \hat{\delta}_v, \hat{R}_{\hat{v}}), \tag{3.3.20}$$

with isometrics

$$\hat{\delta}_r = \hat{T}_r \times \delta = (\hat{T}^k_{ir} \times \delta_{kj}), \quad \hat{\delta}_v = \hat{T}_v \times \delta = (\hat{T}^k_{iv} \times \delta_{kj}), \tag{3.3.21}$$

over the Kronecker product of isofields

$$\hat{R}_{tot} = \hat{R}_t \times \hat{R}_r \times \hat{R}_v, \tag{3.3.22}$$

with total isounit

$$\hat{I}_{tot} = \hat{I}_t \times \hat{I}_r \times \hat{I}_v = \\ = n_t^2 \times Diag.(n_{rx}^2, n_{ry}^2, n_{rz}^2) \times Diag.(n_{vx}^2, n_{vy}^2, n_{vz}^2). \tag{3.3.23}$$

Consequently, the isounit can also be factorized into the product of the following seven distinct isounits, with related product of seven distinct isofields

$$\hat{I}_{tot} = n_t^2 \times n_{rx}^2 \times n_{ry}^2 \times n_{rz}^2 \times n_{vx}^2 \times n_{vy}^2 \times n_{vz}^2, \tag{3.3.24a}$$

$$\hat{R}_{tot} = \hat{R}_t \times \hat{R}_{rx} \times \hat{R}_{ry} \times \hat{R}_{rz} \times \hat{R}_{vx} \times \hat{R}_{vy} \times \hat{R}_{vz}, \tag{3.3.24b}$$

and consequential applicability of the fundamental *Tsagas-Sourlas-Santilli-Falcón-Núñez isotopology* (or TSSFN Isotopology) that allows, for the first time to the author's best knowledge, a consistent representation of extended, nonspherical and deformable shapes of particles in newtonian mechanics, here represented via the semiaxes $n_\alpha^2 = n_\alpha^2(t, r, v, \ldots)$, $\alpha = t, r, v$.

Note that the isospeed is the given by

$$\hat{v} = \frac{\hat{d}\hat{r}}{\hat{d}\hat{t}} = \hat{I}_t \times \frac{d(r \times \hat{I}_r)}{dt} = v \times \hat{I}_t \times \hat{I}_r + r \times \hat{I}_t \times \frac{d\hat{I}_r}{dt} = v \times \hat{I}_v, \tag{3.3.25}$$

thus illustrating that the isounit of the isospeed cannot be the same as that for the isocoordinates, having in particular the value

$$\hat{I}_v = \hat{I}_t \times \hat{I}_r \times \left(1 + \frac{r}{v} \times \frac{1}{\hat{I}_r} \times \frac{d\hat{I}_r}{dt}\right). \tag{3.3.26}$$

The *Newton-Santilli isoequation* [6,10] can be written

$$\hat{m}_a \hat{\times} \frac{\hat{d}\hat{v}_{ka}}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{V}(\hat{r})}{\hat{\partial}\hat{r}_a^k}, \tag{3.3.27}$$

namely, *the equations are conceived in such a way to formally coincide with the conventional equations for selfadjoint forces when formulated on isospace over*

isofields, while all nonpotential forces are represented by the isounits or, equivalently, by the isodifferential calculus.

Such a conception is the only one known permitting the representation of extended particles with contact interactions that is invariant, thus avoiding the catastrophic inconsistencies of Section 1.4.1 and, in addition, achieves closure, namely, the verification of all conventional total conservation laws.

An inspection of Eqs. (3.3.27) is sufficient to see that *the Newton-Santilli isomechanics reconstructs linearity, locality and canonicity on isospaces over isofields,* as studied in Section 3.2.11. Note that this would not be the case if nonselfadjoint forces appear in the right hand side of Eqs. (3.3.27) as in Eqs. (3.3.2).

Note the truly crucial role of the isodifferential calculus for the above structural generalization of Newtonian mechanics (as well as of the subsequent mechanics), that justifies *a posteriori* its construction.

The verification of conventional total conservation laws is established by a visual inspection of Eqs. (3.3.27) since their symmetry is the *Galileo-Santilli isosymmetry* [14,15] that is isomorphic to the conventional Galilean symmetry, only formulated on isospace over isofields. By recalling that conservation laws are represented by the generators of the underlying symmetry, conventional total conservation laws then follow from the indicated invariance.

When projected in the conventional representation space S_{tot}, Eqs. (3.3.27) can be explicitly written

$$\hat{m}\hat{\times}\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}} = m \times \hat{I}_t \times \frac{d(v \times \hat{I}_v)}{dt} =$$

$$= m \times \frac{dv}{dt} \times \hat{I}_t \times \hat{I}_v + m \times v \times \hat{I}_t \times \frac{d\hat{I}_v}{dt} = -\frac{\hat{\partial}\hat{V}(\hat{r})}{\hat{\partial}\hat{r}} = -\hat{I}_r \times \frac{\partial V}{\partial r}, \qquad (3.3.28)$$

that is

$$m \times \frac{dv}{dt} = -\hat{T}_t \times \hat{T}_v \times \hat{I}_r \times \frac{\partial V}{\partial r} - m \times v \times \hat{T}_v \times \frac{d\hat{I}_v}{dt}. \qquad (3.3.29)$$

The necessary and sufficient conditions for the representation of all possible SA and NSA forces are given by

$$\hat{I}_r = \hat{T}_t \times \hat{T}_r, \qquad (3.3.30a)$$

$$m \times v \times \hat{T}_v \times \frac{d\hat{I}_v}{dt} = F^{NSA}, \qquad (3.3.30b)$$

and they always admit a solution, since they constitute a system of $6n$ *algebraic* (rather than differential) equations in the $6N + 1$ unknowns given by $\hat{I}_t$, and the diagonal $3N$-dimensional matrices $\hat{I}_r$ and $\hat{I}_v$.

Note that for $\hat{T}_t = 1$ we recover from a dynamical viewpoint the condition $\hat{I}_r = 1/\hat{I}_v$ obtained in Section 3.2.4 and 3.2.10 on geometric grounds.

As a simple illustration among unlimited possibilities, we have the following equations of motion of an *extended* particle with the ellipsoidal shape experiencing a resistive force $F^{NSA} = -\gamma \times v$ because moving within a physical medium

$$m \times \frac{dv}{dt} = \int d\sigma\, \Gamma(\sigma, \mathbf{r}, p, \dots) \approx -\gamma \times v, \tag{3.3.31a}$$

$$\hat{I}_v = Diag.(n_1^2, n_2^2, n_3^2) \times e^{\gamma \times t/m}, \tag{3.3.31b}$$

where the nonlocal-integral character with respect to a kernel Γ is emphasized. Interested readers can then construct the representation of *any* desired non-Hamiltonian Newtonian system (see also memoir [10] for other examples).

Note the natural appearance in the NSA forces of the velocity dependence, as typical of resistive forces. Note also that the representation of the extended character of particles occurs only in isospace because, when Eqs. (3.3.xx) are projected in the conventional Newtonian space, factorized isounits cancel out and the point characterization of particles is recovered.

Note finally the *direct universality* of the Newton-Santilli isoequations, namely, their capability of representing all infinitely possible Newton's equations in the frame of the observer.

As now familiar earlier, Eqs. (3.3.27) can only describe a system of *particles*. The *isodual Newton-Santilli isoequations* for the treatment of a system of *antiparticles* are given by [6,10]

$$\hat{m}_a^d \hat{\times}^d \frac{\hat{d}^d \hat{v}_{ka}^d}{\hat{d}^d \hat{t}^d} = -\frac{\hat{\partial}^d \hat{V}^d(\hat{r}^d)}{\hat{\partial}^d \hat{r}_a^{kd}}. \tag{3.3.32}$$

The explicit construction of the remaining isodualities of the above isomechanics are instructive for the reader seriously interested in a classical study of antimatter under interior dynamical conditions.

3.3.5 Hamilton-Santilli Isomechanics for Matter and its Isodual for Antimatter

3.3.5A. Isoaction Principle and its Isodual. The isotopies of classical Hamiltonian mechanics were first introduced by Santilli in various works (see monographs [6,7] and references quoted therein), and are today known as the *Hamilton-Santilli isomechanics*.

To identify its representation space, recall that the conventional Hamiltonian mechanics is represented in a 7-dimensional space of time, coordinates and momenta (rather than velocity), the latter characterizing phase space (or cotangent bundle of the symplectic geometry).

Correspondingly, the new isomechanics is formulated in the 7-dimensional isospace of isotime $\hat{t}$, isocoordinates $\hat{r}$ and isomomenta $\hat{p}$

$$\hat{S}_{tot} = \hat{E}(\hat{t}, \hat{R}_{\hat{t}}) \times \hat{E}(\hat{r}, \hat{\delta}_r, \hat{R}_{\hat{r}}) \times \hat{E}(\hat{p}, \hat{\delta}_p, \hat{R}_{\hat{p}}), \tag{3.3.33}$$

with isometrics

$$\hat{\delta}_{\hat{r}} = \hat{T}_{\hat{r}} \times \delta = (\hat{T}^k_{ir} \times \delta_{kj}), \hat{\delta}_{\hat{p}} = \hat{T}_{\hat{p}} \times \delta = (\hat{T}^k_{ip} \times \delta_{kj}), \tag{3.3.34}$$

over the Kronecker product of isofields and related isounits

$$\hat{R}_{tot} = \hat{R}_t \times \hat{R}_r \times \hat{R}_p, \tag{3.3.35a}$$

$$\hat{I}_{tot} = \hat{I}_{\hat{t}} \times \hat{I}_{\hat{r}} \times \hat{I}_{\hat{p}} =$$

$$= n_t^2 \times Diag.(n^2_{rx}, n^2_{ry}, n^2_{rz}) \times Diag.(n^2_{px}, n^2_{py}, n^2_{pz}). \tag{3.3.35b}$$

The following new feature now appears. The *isophasespace,* or, more technically, the *isocotangent bundle* of the isosymplectic geometry in local isochart $(\hat{r}, \hat{p})$ requires that the isounits of the variables $\hat{r}$ and $\hat{p}$ are *inverse* of each others (Section 3.2.3 and 3.2.10)

$$\hat{I}_{\hat{r}} = 1/\hat{T}_{\hat{r}} = \hat{I}^{-1}_{\hat{p}} = \hat{T}_{\hat{p}} > 0. \tag{3.3.36}$$

Consequently, by ignoring hereon for notational simplicity the indices for the N particles, the total isounit of the isophase space can be written

$$\hat{I}_{tot} = \hat{I}_{\hat{t}} \times \hat{I}_{\hat{r}} \times \hat{T}_{\hat{r}} = \hat{I}_{\hat{t}} \times \hat{I}_6, \tag{3.3.37a}$$

$$\hat{I}_6 = (\hat{I}^{\nu}_{\mu}) = \hat{I}_{\hat{r}} \times \hat{T}_{\hat{r}}. \tag{3.3.37b}$$

The fundamental *isoaction principle* for the classical treatment of matter in interior conditions can be written in the explicit form in the $\hat{r}$ and $\hat{p}$ isovariables

$$\hat{\delta}\hat{A}^o = \hat{\delta} \int_{t_1}^{t_2} (\hat{p}_k \hat{\times} \hat{d}\hat{r}^k - \hat{H} \hat{\times} \hat{d}\hat{t}) = \hat{\delta} \int_{t_1}^{t_2} [p_k \times \hat{T}_{\hat{r}}^{k_i(t,r,p,...)} \times \hat{d}\hat{r}^i - \hat{H} \times \hat{T}_{\hat{t}} \times d\hat{t}] = 0, \tag{3.3.38}$$

where

$$\hat{H} = \hat{p}^{\hat{2}} \hat{/} \hat{2} \hat{\times} \hat{m} - \hat{V}(\hat{r}), \tag{3.3.39}$$

is the *isohamiltonian* or simple the Hamiltonian because its projection on conventional spaces represents the orginary total energy except an inessential multiplicative factor.

By using the unified notation

$$\hat{b} = (\hat{b}^{\mu}) = (\hat{r}^i, \hat{p}_j) = (r^i, p_j) \times \hat{I}_6 = b \times \hat{I}_6, \tag{3.3.40}$$

and the isotopic image of the canonical R^o functions, Eqs. (3.3.xx),

$$\hat{R}^o = (\hat{R}^o_\mu) = (\hat{r}, \hat{0}), \tag{3.3.41}$$

the fundamental isoaction principle can be written in unified notation

$$\hat{\delta}\hat{A}^o = \hat{\delta}\int_{t_1}^{t_2}(\hat{p}_k\hat{\times}\hat{d}\hat{r}^k - \hat{H}\hat{\times}\hat{d}\hat{t}) \equiv \hat{\delta}\int_{t_1}^{t_2}(\hat{R}^o_\mu\hat{\times}\hat{d}\hat{b}^\mu - \hat{H}\hat{\times}\hat{d}\hat{t}) =$$

$$= \hat{\delta}\int_{t_1}^{t_2}(R^o_\mu \times \hat{T}^\mu_{6\nu} \times d\hat{b}^\nu - H \times \hat{T}_{\hat{t}} \times d\hat{t}) = 0. \tag{3.3.42}$$

A visual inspection of principle (3.3.38) establishes the *isocanonicity* of Hamilton-Santilli isomechanics (Section 3.2.11), namely, the reconstruction of canonicity on isospaces over isofield that is crucial for the consistency of hadronic mechanics.

In fact, the conventional action principle (3.3.4) and isoprinciple (3.3.38) coincide at the abstract, realization-free level by conception and construction.

The direct universality of classical isomechanics can be seen from the arbitrariness of the integrand of isoaction functional (3.3.38) once projected on conventional spaces over conventional fields.

An important property of the isoaction is that its functional dependence on isospaces over isofields is restricted to that on isocoordinates only, i.e., $\hat{A} = \hat{A}(\hat{r})$. However, when projected on conventional spaces, the functional dependence is arbitrary, i.e., $\hat{A}(\hat{r}) = \hat{A}(r \times \hat{I}) = \hat{A}(t, r, p, \ldots)$. This feature will soon have a crucial role for the operator image of the classical isomechanics.

It should finally be noted that isoprinciple (3.3.38) essentially eliminates the entire field of *Lagrangian and action principles of orders higher than the first,* e.g., $L = L(t, r, \dot{r}, \ddot{r}, \ldots)$ because of these higher order formulations can be easily reduced to the isotopic first-order form (3.3.38).

Recall that the action principle has the important application via the use of the *optimal control theory* of optimizing dynamical systems, However, the latter can have only been Hamiltonian until now due to the lack of a universal action functional for non-Hamiltonian systems (that constitute, by far, the system most significant for optimization). Recall also that the optimal control theory can only be applied for local-differential systems due to the underlying Euclidean topology, thus secluding from the optimization process the most important systems, those of extended, and, therefore, of nonlocal type.

Note that isoaction principle (3.3.38) occurs for all possible non-Hamiltonian as well as nonlocal-integral systems, thanks also to the underlying TSSFN isotopology (Section 3.2.7). We, therefore, have the following important:

THEOREM 3.3.1 [6,10]: Isoaction principle (3.3.38) permits the (first known) optimization of all possible nonpotential/non-Hamiltonian and nonlocal-integral systems.

The *isodual isoaction principle* [10] for the classical treatment of antimatter in interior conditions is given by

$$\hat{\delta}^d \hat{A}^d = \hat{\delta}^d \int_{t_1}^{t_2 d} (\hat{p}_k^d \hat{\times}^d d\hat{\hat{r}}^{kd} - \hat{H}^d \hat{\times}^d \hat{d}^d \hat{t}^d) =$$

$$= \hat{\delta}^d \int_{t_1}^{t_2 d} (\hat{R}_\mu^{od} \hat{\times}^d \hat{d}^d \hat{b}^\mu - \hat{H}^d \hat{\times} \hat{d}^d \hat{t}^d) = 0. \tag{3.3.43}$$

Additional isodual treatments are left to the interested reader.

3.3.5B. Hamilton-Santilli Isoequations and their Isoduals. The discovery of the isodifferential calculus in 1995 permitted Santilli [6,10] the identification of the following classical dynamical equations for the treatment of matter at the foundations of hadronic mechanics, today known as the *Hamilton-Santilli isoequations*. They are easily derived via the isovariational principle and can be written from isoprinciple (3.3.38) in disjoint notation

$$\frac{d\hat{\hat{r}}^k}{d\hat{\hat{t}}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k}, \quad \frac{\hat{\partial}\hat{p}_k}{d\hat{\hat{t}}} = -\frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{\hat{r}}^k}. \tag{3.3.44}$$

The same equations can be written in unified notation from principle (3.3.40)

$$\hat{\omega}_{\mu\nu} \hat{\times} \frac{d\hat{\hat{b}}^\mu}{d\hat{\hat{t}}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{\hat{b}}^\mu}, \tag{3.3.45}$$

where

$$\hat{\omega}_{\mu\nu} = \omega_{\mu\nu} \times \hat{I}_6 \tag{3.3.46}$$

is the *isocanonical isosymplectic tensor* that coincides with the conventional canonical symplectic tensor $\omega_{\mu\nu}$ except for the factorization of the isounit (Section 3.2.10).

To verify the latter property from an analytic viewpoint, it is instructive for the reader to verify the following identify under isounits (3.3.37)

$$\hat{\omega}_{\mu\nu} = \frac{\hat{\partial}\hat{R}_\nu^o}{\hat{\partial}\hat{\hat{b}}^\mu} - \frac{\hat{\partial}\hat{R}_\mu^o}{\hat{\partial}\hat{\hat{b}}^\nu} = \omega_{\mu\nu} \times \hat{I}_6. \tag{3.3.47}$$

A simple comparison of the above isoanalytic equations with the isotopic and conventional Newton's equations established the following:

THEOREM 3.3.2: Hamilton-Santilli isoequations (3.3.5) are "directly universal" in Newtonian mechanics, that is, capable of representing all possible, conventional or isotopic, hamiltonian and non-Hamiltonian Newtonian systems directly in the fixed coordinates of the experimenter.

It is now important to show that Eqs. (3.3.45) provide an identical reformulation of the true analytic equations (3.3.2). For this purpose, we assume the simple case in which isotime coincide with the conventional time, that is, $\hat{t} = t$, $\hat{I}_t = +1$ and we write isoequations (3.3.45) in the explicit form

$$\begin{aligned} (\omega) \times \left(\begin{array}{c} dr^k/dt \\ dp_k/dt \end{array} \right) = \left(\begin{array}{cc} 0_{3\times 3} & -I_{3\times 3} \\ I_{3\times 3} & 0_{3\times 3} \end{array} \right) \times \left(\begin{array}{c} dr^k/dt \\ dp_k/dt \end{array} \right) = \\ \left(\begin{array}{c} -dp_k/dt \\ dr^k/dt \end{array} \right) = \left(\begin{array}{c} \hat{\partial}\hat{H}/\hat{\partial}r^k \\ \hat{\partial}\hat{H}/\hat{\partial}p_k \end{array} \right) = \left(\begin{array}{c} \hat{I}^i_k \times \partial\hat{H}/\partial r^i \\ \hat{T}^k_i \times \partial\hat{H}/\partial p_i \end{array} \right). \end{aligned} \tag{3.3.48}$$

It is easy to see that Eqs. (3.3.xx) *coincide* with the true analytic equations (3.3.2) under the trivial algebraic identification

$$\hat{I}_{\hat{r}} = Diag.[I - F/(\partial H/\partial r)]. \tag{3.3.49}$$

As one can see, the main mechanism of Eqs. (3.3.45) is that of *transforming the external terms $F = F^{NSA}$ into an explicit realization of the isounit $\hat{I}_3$*. As a consequence, *reformulation (3.3.45) constitutes direct evidence on the capability to represent non-Hamiltonian forces and effects with a generalization of the unit of the theory.*

Note in particular that *the external terms are embedded in the isoderivatives.* However, when written down explicitly, Eqs. (3.3.2) and (3.3.45) coincide. Note that $\hat{I}_3$ as in rule (3.3.49) is fully symmetric, thus acceptable as the isounit of isomathematics. Note also that all nonlocal and nonhamiltonian effects are embedded in $\hat{I}$.

The reader should note the extreme simplicity in the construction of a representation of given non-Hamiltonian equations of motion, due to the algebraic character of identifications (3.3.49).

Recall that *Hamilton's equations with external terms are not derivable from a variational principle.* In turn, such an occurrence has precluded the identification of the operator counterpart of Eqs. (3.3.2) throughout the 20-th century.

We now learn that *the identical* reformulation (3.3.45) of Eqs. (3.3.2) becomes fully derivable from a variational principle. In turn, this will soon permit the identification of the unique and unambiguous operator counterpart.

It should be noted that *the Hamilton-Santilli isoequations are generally irreversible* due to the general irreversibility of the external forces,

$$F(t,\ldots) \neq F(-t,\ldots), \quad \text{or} \tag{3.3.50a}$$

$$\hat{I}(t,\ldots) = Diag.[I - F(t,\ldots)/(\partial H/\partial t)] \neq \hat{I}(-t,\ldots). \tag{3.3.50b}$$

In particular, we have irreversibility under the conservation of the total energy (see next chapter for full treatment). This feature is important to achieve compatibility with thermodynamics, e.g., to have credible analytic methods for the representation of the internal increase of the entropy for closed-isolated systems such as Jupiter.

The study of these thermodynamical aspects is left to the interested reader. In this chapter we shall solely consider *reversible closed-isolated systems* that occur for external forces not explicitly dependent on time and verify other restrictions.

An important aspect is that *the Hamilton-Santilli isoequations coincide with the Hamilton equations without external terms at the abstract level*. In fact, all differences between I and $\hat{I}$, $\times$ and $\hat{\times}$, ∂ and $\hat{\partial}$, *etc.*, disappear at the abstract level. This proves the achievement of a central objective of isomechanics, the property that the analytic equations with external terms can indeed be *identically* rewritten in a form equivalent to the analytic equations without external terms, provided, however, that the reformulation occurs via the broader isomathematics.

The *isodual Hamilton-Santilli isoequations* for the classical treatment of antimatter, also identified soon after the discovery of the isodifferential calculus, are given by

$$\hat{\omega}^d_{\mu\nu} \hat{\times}^d \frac{\hat{d}^d \hat{b}^{d\mu}}{\hat{d}^d \hat{t}^d} = \frac{\hat{\partial}^d \hat{H}^d}{\hat{\partial}^d \hat{b}^{d\mu}}, \tag{3.3.51}$$

where

$$\hat{\omega}^d_{\mu\nu} = \omega^d_{\mu\nu} \times \hat{I}_6 \tag{3.3.52}$$

is the *isodual isocanonical isosymplectic tensor*. The derivation of other isodual properties is instructed for the interested reader.

3.3.5C. Classical Lie-Santilli Brackets and their Isoduals. It is important to verify that Eqs. (3.3.44) or (3.3.45) resolve the problematic aspects of external terms indicated in Section 3.3.2 [4]. In fact, the isobrackets of the time evolution of matter are given by

$$\frac{d\hat{A}}{\hat{d}\hat{t}} = [\hat{A},\hat{H}] = \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{r}^k} \hat{\times} \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k} - \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{r}^k} \hat{\times} \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{p}_k}, \tag{3.3.53}$$

and they verify the left and right distributive and scalar laws, thus characterizing a consistent algebra. Moreover, that algebra results to be Lie-isotopic, for which reasons the above brackets are known as the *Lie-Santilli isobrackets*.

When explicitly written in our spacetime, brackets (3.3.53) recover the brackets (3.3.3) of the true analytic equations (3.3.2)

$$\frac{dH}{dt} = \frac{\partial H}{\partial r^k} \times \frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial p_k} \times \frac{\partial H}{\partial r^k} + \frac{\partial H}{\partial p_k} \times F^k = \frac{\partial H}{\partial p_k} \times F^k \equiv 0, \tag{3.3.54}$$

where the last identity holds in view of Eqs. (3.3.49). Therefore, the Hamilton-Jacobi isoequations do indeed constitute a reformulation of the true analytic equations with a consistent Lie-isotopic algebraic brackets, as needed (Section 3.3.3).

Note that, in which of their anti-isomorphic character, isobrackets (3.3.53) represent the conservation of the Hamiltonian,

$$\frac{d\hat{H}}{dt} = [\hat{H}\hat{,}\hat{H}] = \frac{\hat{\partial}\hat{H}}{\hat{\partial} r^k} \hat{\times} \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k} - \frac{\hat{\partial}\hat{H}}{\hat{\partial} r^k} \hat{\times} \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}_k} \equiv 0. \tag{3.3.55}$$

This illustrates the reason for assuming closed-isolated Newtonian systems (3.3.19) at the foundations of this chapter.

Basic isobrackets (3.3.53) can be written in unified notation

$$[\hat{A}\hat{,}\hat{B}] = \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{b}^\mu} \hat{\times} \hat{\omega}^{\mu\nu} \hat{\times} \frac{\hat{\partial}\hat{B}}{\hat{\partial}\hat{b}^\nu}, \tag{3.3.56}$$

where $\hat{\omega}_{\mu\nu}$ is the Lie-Santilli isotensor. By using the notation $\hat{\partial}^\mu = \hat{\partial}/\hat{\partial}\hat{b}^\mu$, the isobrackets can be written

$$[\hat{A}\hat{,}\hat{B}] = \hat{\partial}_\mu \hat{A} \times \hat{T}^\mu_\rho \times \omega^{\rho\nu} \hat{\partial}_\nu \hat{\partial}_\nu \hat{B}, \tag{3.3.57}$$

and, when projected in our spacetime, the isobrackets can be written

$$[A\hat{,}B] = \partial_\mu A \times \omega^{\mu\rho} \times \hat{I}^\nu_\rho \times \partial_\nu B, \tag{3.3.58}$$

where $\omega^{\mu\nu}$ is the canonical Lie tensor.

The *isodual Lie-Santilli isobrackets* for the characterization of antimatter can be written

$$[\hat{A}^d\hat{,}\hat{B}^d] = \hat{\partial}^d_\mu \hat{A}^d \hat{\times}^d \hat{\omega}^{d\rho\nu} \hat{\partial}^d_\nu \hat{\partial}^d_\nu \hat{B}, \tag{3.3.59}$$

where $\hat{\omega}^{d\mu\nu}$ is the *isodual Lie-Santilli isotensor*. Other algebraic properties can be easily derived by the interested reader.

3.3.5D. Hamilton-Jacobi-Santilli Isoequations and their isoduals. Another important consequence of isoaction principle (3.3.38) is the characterization of the following *Hamilton-Jacobi-Santilli isoequations* for matter [6,10]

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{t}} + \hat{H} = 0, \tag{3.3.60a}$$

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{r}^k} - \hat{p}_k = 0, \tag{3.3.60b}$$

$$\frac{\hat{\partial}\hat{A}^o}{\hat{\partial}\hat{p}_k} \equiv 0, \tag{3.3.60c}$$

which will soon have basic relevance for isoquantization.

Note the *independence of the isoaction $\hat{A}^o$ from the isomomenta* that will soon be crucial for consistent isoquantization.

The isodual equations for antimatter are then given by

$$\frac{\hat{\partial}^d \hat{A}^{od}}{\hat{\partial}^d \hat{t}^d} + \hat{H}^d = 0, \tag{3.3.61a}$$

$$\frac{\hat{\partial}^d \hat{A}^{od}}{\hat{\partial}^d \hat{r}^{kd}} - \hat{p}_k^d = 0, \tag{3.3.61b}$$

$$\frac{\hat{\partial}^d \hat{A}^{od}}{\hat{\partial}^d \hat{p}_k^d} \equiv 0. \tag{3.3.61c}$$

The latter equations will soon result to be essential for the achievement of a consistent operator image of the classical treatment of antimatter in interior conditions.

3.3.5E. Connection Between Isotopic and Birkhoffian Mechanics. Since the Hamilton-Santilli isoequations are directly universal, they can also represent Birkhoff's equations (3.3.13) in the fixed b-coordinates. In fact, by assuming for simplicity that the isotime is the ordinary time, we can write the identities

$$\frac{db^\mu}{dt} = \Omega^{\mu\mu}(b) \times \frac{\partial H(b)}{\partial b^\nu} \equiv \omega^{\mu\nu} \times \frac{\hat{\partial} H(b)}{\hat{\partial} b^\nu} = \omega^{\mu\rho} \times \hat{I}_{6\rho}^\nu \times \frac{\partial H}{\partial b^\nu}. \tag{3.3.62}$$

Consequently, we reach the following *decomposition of the Birkhoffian tensor*

$$\Omega^{\mu\nu}(b) = \omega^{\mu\rho} \times \hat{I}_{6\rho}^\nu(b). \tag{3.3.63}$$

Consequently, Birkhoff's equations can indeed be identically rewritten in the isotopic form, as expected. In the process, the reformulation provides additional insight in the isounit.

The reformulation also carries intriguing geometric implications since it confirms the *direct universality in symplectic geometry of the canonical two-form,* since a general symplectic two-form can always be identically rewritten in the isocanonical form via decomposition of type (3.3.xx) and then the embedding of the isounit in the isodifferential of the exterior calculus.

As an incidental note, the reader should be aware that the construction of an analytic representation via Birkhoff's equations is rather complex, inasmuch as it requires *the solution of nonlinear partial differential equations* or integral equations [2].

By comparison, the construction of the same analytic equations via Hamilton-Santilli isoequations (3.3.44) or (3.3.45) is truly elementary, and merely requires the identification of the isounit according to *algebraic* rule (3.3.49) for arbitrarily given external forces $F_k(t, r, p)$.

3.3.6 Simple Construction of Classical Isomechanics

The above classical isomechanics can be constructed via a simple method which does not need any advanced mathematics, yet it is sufficient and effective for practical applications.

In fact, *the Hamilton-Santilli isomechanics can be constructed via the systematic application of the following noncanonical transform to all quantities and operations of the conventional Hamiltonian mechanics*

$$U = \begin{pmatrix} \hat{I}_3^{1/2} & 0 \\ 0 & \hat{T}_3^{1/2} \end{pmatrix}, \tag{3.3.64a}$$

$$U \times U^t = \hat{I}_6 \neq I, \tag{3.3.64b}$$

$$\hat{I}_3 = I - \frac{F}{\partial H/\partial p} = I - \frac{F}{p/m}. \tag{3.3.64c}$$

The success of the construction depends on the application of the above noncanonical transform to the *totality* of Hamiltonian mechanics, with no exceptions. We have in this way the lifting of: the 6-dimensional unit of the conventional phase space into the isounit

$$I_6 \to \hat{I}_6 = U \times I_6 \times U^t; \tag{3.3.65}$$

numbers into the isonumbers,

$$n \to \hat{n} = U \times n \times U^t = n \times (U \times U^t) = n \times \hat{I}_6; \tag{3.3.66}$$

associative product $A \times B$ among generic quantities A, B into the isoassociative product with the correct expression and property for the isotopic element,

$$A \times B \to A \hat{\times} B = U \times (A \times B) \times U^t = A' \times \hat{T} \times B', \tag{3.3.67a}$$

$$A' = U \times A \times U^t, \quad B' = U \times B \times U^t, \quad \hat{T} = (U \times U^t)^{-1} = T^t; \tag{3.3.67b}$$

Euclidean into iso-Euclidean spaces (where we use only the space component of the transform)

$$\begin{gathered} x^2 = x^t \times \delta \times x \to \hat{x}^{\hat{2}} = U \times x^2 \times U^t = \\ = (x^t \times U^t) \times (U^{t-1} \times \delta \times U^{-1}) \times (U \times x) \times (U \times U^t) = \\ = [x'^t \times (\hat{T} \times \delta) \times x'] \times \hat{I}; \end{gathered} \tag{3.3.68}$$

and, finally, we have the following isotopic lifting of Hamilton's into Hamilton-Santilli isoequations (here derived for simplicity for the case in which the transform does not depend explicitly on the local coordinates),

$$\begin{gathered} db/dt - \omega \times \partial H/\partial b = 0 \to \\ \to U \times db/dt \times U^t - U \times \omega \times \partial H/\partial b \times U^t = \\ = db/dt \times (U \times U^t) - (U \times \omega \times U^t) \times (U^{t\ -1} \times U^{-1}) \times \\ \times (U \times \partial H/\partial b \times U^t) \times (U \times U^t) = \\ = db/dt \times \hat{I} - \omega \times (\hat{\partial} H/\hat{\partial} \hat{b}) \times \hat{I} = 0, \end{gathered} \tag{3.3.69}$$

where we have used the important property the reader is urged to verify

$$U \times \omega \times U^t \equiv \omega. \tag{3.3.70}$$

As one can see, the seemingly complex isomathematics and isomechanics are reduced to a truly elementary construction. e its universality.

3.3.7 Invariance of Classical Isomechanics

A final requirement is necessary for a physical consistency, and that is, *the invariance of isomechanics under its own time evolution*, as it occurs for conventional Hamiltonian mechanics.

Recall that a transformation $b \to b'(b)$ is called a *canonical transformation* when all the following identities hold

$$\frac{\partial b^\mu}{\partial b'^\alpha} \times \omega_{\mu\nu} \times \frac{\partial b^\nu}{\partial b'^\beta} = \omega_{\alpha\beta}. \tag{3.3.71}$$

The invariance of Hamiltonian mechanics follows from the property that its time evolution constitutes a canonical transformation, as well known.

The proof of the invariant of isomechanics is elementary. In fact, an isotransformation $\hat{b} \to \hat{b}'(\hat{b})$ constituted an *isocanonical isotransform* when all the following identities old

$$\frac{\hat{\partial} \hat{b}^\mu}{\hat{\partial} \hat{b}'^\alpha} \hat{\times} \hat{\omega}_{\mu\nu} \hat{\times} \frac{\hat{\partial} \hat{b}^\nu}{\hat{\partial} \hat{b}'^\beta} = \hat{\omega}_{\alpha\beta} = \omega_{\alpha\beta} \times \hat{I}_6. \tag{3.3.72}$$

But the above expression can be written

$$(\hat{I}_6^{\mu_\rho} \times \frac{\partial \hat{b}^\rho}{\partial \hat{b}^\alpha} \times \omega_{\mu\nu} \times \hat{I}_6^{\xi_\nu} \times \frac{\partial \hat{b}^\xi}{\partial \hat{b}'^\beta}) \times \hat{I}_6 = \omega_{\mu\nu} \times \hat{I}_6, \tag{3.3.73}$$

and they coincide with conditions (3.3.xx) in view of the identities

$$\hat{I}_6^{\mu_\rho} \times \omega_{\mu\nu} \times \hat{I}_6^{\nu_\xi} = \omega_{\rho\xi}. \tag{3.3.74}$$

Consequently, we have the following important

THEOREM 3.3.3 [6,10]: Following factorization of the isounit, isocanonical transformations are canonical.

The desired invariance of the Hamilton-Santilli isomechanics then follows.

It is an instructive exercise for the reader interested in learning isomechanics to verify that *all* catastrophic mathematical and physical inconsistencies of non-canonical theories pointed out in Chapter 1 (see Section 1.4.1 in particular) are indeed resolved by isomechanics as presented in this section.

3.4 OPERATOR LIE-ISOTOPIC MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

3.4.1 Introduction

We are finally equipped to present the foundations of *the Lie-isotopic branch of nonrelativistic hadronic mechanics* for matter and its isodual for antimatter, more simply referred to as *operator isomechanics,* and its isodual for antimatter referred to as *isodual operator isomechanics.* The new mechanics will then be used in subsequent sections for various developments, experimental verifications and industrial applications.

The extension of the results of this section to *relativistic operator isomechanics* is elementary and will be done in the following sections whenever needed for specific applications. the case of *operator genomechanics* with a Lie-admissible, rather than the Lie-isotopic structure, will be studied in the next chapter.

A knowledge of Section 3.2 is necessary for a technical understanding of operator isomechanics. For the mathematically non-inclined readers, we present in Section 3.4.8 a very elementary construction of operator isomechanics via nonunitary transforms.

Unless otherwise specified, all quantities and operations represented with conventional symbols A, H, $\times$, *etc.*, denote quantities and operations on conventional Hilbert spaces over conventional fields. All quantities and symbols of the type $\hat{A}$, $\hat{H}$, $\hat{\times}$, *etc.*, are instead defined on isohilbert spaces over isofields.

Note the use of the terms "operator" isomechanics, rather than "quantum" isomechanics, because, as indicated in Chapter 1, the notion of quantum is fully established within the arena of its conception, the transition of electrons between different stable orbits of atomic structure (exterior problem), while the assumption of the same quantum structure for the same electrons when in the core of a star (interior problems) is a scientific religion at this writing deprived of solid experimental evidence.

3.4.2 Naive Isoquantization and its Isodual

An effective way to derive the basic dynamical equations of operator isomechanics is that via the isotopies of the conventional map of the classical Hamilton-Jacobi equations into their operator counterpart, known as *naive quantization.*. More rigorous methods, such as the isotopies of symplectic quantization, essentially yields the same operator equations and will not be treated in this section for brevity (see monograph [7] for a presentation).

Recall that the *naive quantization* can be expressed via the following map of the canonical action functional

$$A^o = \int_{t_1}^{t_2} (p_k \times dr^k - H \times dt) \to -i \times \hbar \times \ln |\psi\rangle, \tag{3.4.1}$$

under which the conventional Hamilton-Jacobi equations are mapped into the Schrödinger equations,

$$-\partial_t A^o = H \to i \times \hbar \times \partial_t |\psi\rangle = H \times |\psi\rangle, \tag{3.4.2a}$$

$$p_k = \partial_k A^o \to p_k \times |\psi\rangle = -i \times \hbar \times \partial_k |\psi\rangle, \tag{3.4.2b}$$

where $|\psi\rangle$ is the *wavefunction,* or, more technically, a state in a Hilbert space $\mathcal{H}$.

Isocanonical action (3.3.38) is evidently different than the conventional canonical action, e.g., because it is of higher order derivatives. As such, the above naive quantization does not apply.

In its place we have the following *naive isoquantization* first introduced by Animalu and Santilli [44] of 1990, and here extended to the use of the isodifferential calculus

$$\hat{A}^o = \int_{t_1}^{t_2} (\hat{p}_k \hat{\times} \hat{d}\hat{x}^k - \hat{H} \hat{\times} \hat{d}\hat{t}) \to -i \times \hat{I} \times \ln |\hat{\psi}\rangle, \tag{3.4.3}$$

where $\hat{i} = i \times \hat{I}$, $|\hat{\psi}\rangle$ is the *ospwavefunction,* or, more precisely, a state of the iso-Hilbert space $\hat{\mathcal{H}}$ outlined in the next section, and we should note that $\hat{i} \hat{\times} \hat{I} \times \ln |\hat{\psi}\rangle = i \times \text{isoln} |\hat{\psi}\rangle$.

The use of Hamilton-Jacobi-Santilli isoequations (3.3.60) yields the following operator equations (here written for the simpler case in which $\hat{T}$ has no dependence on r, but admits a dependence on velocities and higher derivatives)

$$-\hat{\partial}_t \hat{A}^o = \hat{H} \to i \times \hat{\partial}_t |\hat{\psi}\rangle \ = \hat{H} \times \hat{T} \times |\hat{\psi}\rangle \ = \hat{H} \hat{\times} |\hat{\psi}\rangle, \tag{3.4.4a}$$

$$\hat{p}_k = \hat{\partial}_k \hat{A}^o \to \hat{p}_k \times \hat{T} \times |\hat{\psi}\rangle \ = \hat{p}_k \hat{\times} |\hat{\psi}\rangle \ = -\hat{i} \hat{\times} \hat{\partial}_k |\hat{\psi}\rangle, \tag{3.4.4b}$$

that constitutes the fundamental equations of operator isomechanics, as we shall see in the next section.

As it is well known, *Planck's constant $\hbar$ is the basic unit of quantum mechanics.* By comparing Eqs. (3.4.xx) and (3.4.xx) it is easy to see that $\hat{I}$ is the basic unit of operator isomechanics. Recall also that the isounits are defined at short distances as in Eqs. (3.1.xxx). We therefore have the following important

POSTULATE 3.4.1 [5]: In the transition from quantum mechanics to operator isomechanics Planck's unit $\hbar$ is replaced by the integrodifferential unit $\hat{I}$ under the condition of recovering the former at sufficiently large mutual distances,

$$\lim_{r \to \infty} \hat{I} = \hbar = 1. \tag{3.4.5}$$

Consequently, in the conditions of deep mutual penetration of the wavepackets and/or charge distributions of particles as studied by operator isomechanics there is the superposition of quantized and continuous exchanges of energy.

3.4.3 Isohilbert Spaces and their Isoduals

As it is well known, the Hilbert space $\mathcal{H}$ used in quantum mechanics is expressed in terms of states $|\psi\rangle, \ |\phi\rangle, \ldots$, with normalization

$$\langle \psi | \times | \psi \rangle = 1, \tag{3.4.6}$$

and inner product

$$\langle \phi | \times | \psi \rangle = \int dr^3 \phi^\dagger(r) \times \psi(r), \tag{3.4.7}$$

defined over the field of complex numbers $\mathcal{C} = C(c, +, \times)$.

The lifting $C(c, +, \times) \to \hat{C}(\hat{c}, \hat{+}, \hat{\times})$, requires a compatible lifting of $\mathcal{H}$ into the *isohilbert space* $\hat{\mathcal{H}}$ with *isostates* $|\hat{\psi}\rangle, |\hat{\phi}\rangle, \ldots$, *isoinner product* and *isonormalization*

$$\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \times \hat{I} = \left[\hat{\int} \hat{dr}^3 \ \hat{\psi}^\dagger(\hat{r}) \times \hat{T} \times \hat{\psi}(\hat{r}) \right] \times \hat{I} \in \hat{\mathcal{C}}, \tag{3.4.8a}$$

$$\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle = 1, \tag{3.4.8b}$$

first introduced by Myung and Santilli in 1982 [45] (see also monographs [6,7] for a comprehensive study).

It is easy to see that the isoinner product is still inner (because $\hat{T} > 0$). Thus, $\hat{\mathcal{H}}$ is still Hilbert and the lifting $\mathcal{H} \to \hat{\mathcal{H}}$ is an isotopy. Also, it is possible to prove that *iso-Hermiticity coincides with conventional Hermiticity*,

$$\langle\hat{\psi}|\hat{\times}(\hat{H}\hat{\times}|\hat{\psi}\rangle) \equiv (\langle\hat{\psi}|\hat{\times}\hat{H}^{\hat{\dagger}})\hat{\times}|\hat{\psi}\rangle, \tag{3.4.9a}$$

$$\hat{H}^{\hat{\dagger}} \equiv \hat{H}^{\dagger} = \hat{H}. \tag{3.4.9b}$$

As a result, *all quantities that are observable for quantum mechanics remain so for hadronic mechanics.*

For consistency, the conventional eigenvalue equation $H \times |\psi\rangle = E \times |\psi\rangle$ must also be lifted into the *isoeigenvalue form* [7]

$$\hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H} \times \hat{T} \times |\hat{\psi}\rangle = \hat{E}\hat{\times}|\hat{\psi}\rangle = (E \times \hat{I}) \times \hat{T} \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle, \tag{3.4.10}$$

where, as one can see, the final results are ordinary numbers.

Note the *necessity* of the isotopic action $\hat{H}\hat{\times}|\hat{\psi}\rangle$, rather than $\hat{H} \times |\hat{\psi}\rangle$. In fact, only the former admits $\hat{I}$ as the correct unit,

$$\hat{I}\hat{\times}|\hat{\psi}\rangle = \hat{T}^{-1} \times \hat{T} \times |\hat{\psi}\rangle \equiv |\hat{\psi}\rangle. \tag{3.4.11}$$

It is possible to prove that *the isoeigenvalues of isohermitian operators are isoreal*, i.e., they have the structure $\hat{E} = E \times \hat{I},\ E \in R(n, +, \times)$. As a result all real eigenvalues of quantum mechanics remain real for hadronic mechanics.

We also recall the notion of *isounitary operators* as the isooperators $\hat{U}$ on $\hat{\mathcal{H}}$ over $\hat{C}$ satisfying the isolaws

$$\hat{U}\hat{\times}\hat{U}^{\hat{\dagger}} = \hat{U}^{\hat{\dagger}}\hat{\times}\hat{U} = \hat{I}, \tag{3.4.12}$$

where we have used the identity $\hat{U}^{\hat{\dagger}} \equiv \hat{U}^{\dagger}$.

We finally indicate the notion of *isoexpectation value* of an isooperators $\hat{H}$ on $\hat{\mathcal{H}}$ over $\hat{C}$

$$\langle\hat{H}\rangle = \frac{\langle\hat{\psi}|\hat{\times}\hat{H}\hat{\times}|\hat{\psi}\rangle}{\langle\hat{\psi}|\hat{\times}|\hat{\psi}\rangle}. \tag{3.4.131}$$

It is easy to see that *the isoexpectation values of isohermitian operators coincide with the isoeigenvalues*, as in the conventional case.

Note also that *the isoexpectation value of the isounit is the isounit*,

$$\langle\hat{I}\rangle = \hat{I}, \tag{3.4.14}$$

provided, of course, that one uses the isoquotient (otherwise $\langle\hat{I}\rangle = I$).

The isotopies of quantum mechanics studied in the next sections are based on the following novel invariance property of the conventional Hilbert space [xxx], here expressed in term of a non-null scalar n independent from the integration variables,

$$\langle\hat{\phi}|\times|\hat{\psi}\rangle\times I\equiv\langle\hat{\phi}|\times n^{-2}\times|\hat{\psi}\rangle\times(n^2\times I)=\langle\phi|\hat{\times}|\psi\rangle\times\hat{I}. \tag{3.4.15}$$

Note that new invariances (3.4.15) remained undetected throughout the 20-th century because they required the prior discovery of *new numbers*, those with arbitrary units.

3.4.4 Structure of Operator Isomechanics and its Isodual

The structure of operator isomechanics is essentially given by the following main steps [47]:

1) The description of closed-isolated systems is done via *two* quantities, the Hamiltonian representing all action-at-a-distance potential interactions, plus the isounit representing all nonlinear, nonlocal and non-Hamiltonian effects,

$$H(t,r,p)=p^2/2m+V(r), \tag{3.4.16a}$$

$$\hat{I}=\hat{I}(t,r,p,\psi,\nabla\psi,\ldots). \tag{3.4.16b}$$

The explicit form of the Hamiltonian is that conventionally used in quantum mechanics although written on isospaces over isofields,

$$\hat{H}=\hat{p}\hat{\times}\hat{p}\hat{/}\hat{2}\hat{\times}\hat{m}+\hat{V}(\hat{r}). \tag{3.4.17}$$

A generic expression of the isounit for the representation of two spinning particles with point-like change (such as the electrons) in conditions of deep penetration of their wavepackets (as occurring in chemical valence bonds and many other cases) is given by

$$\hat{I}=\exp\left[\Gamma(\psi,\psi^\dagger)\times\int dv\,\psi^\dagger_\downarrow(r)\psi_\uparrow(r)\right], \tag{3.4.18}$$

where the nonlinearity is expressed by $\Gamma(\psi,\psi^\dagger)$ and the nonlocality is expressed by the volume integral of the deep wave-overlappings $\int dv\,\psi^\dagger_\downarrow(r)\psi_\uparrow(r)$. All isounits will be restricted by the conditions of being positive-definite (thus everywhere invertible) as well as of recovering the trivial unit of quantum mechanics for sufficiently big mutual distances r,

$$\lim_{r\to\infty}\int dv\,\psi^\dagger_\downarrow(r)\psi_\uparrow(r)=0. \tag{3.4.19}$$

2) The lifting of the multiplicative unit $I>0\to\hat{I}=1/\hat{T}>0$ requires the reconstruction of the entire formalism of quantum mechanics into such a form to

admit $\hat{I}$ as the correct left and right unit at all levels of study, including numbers and angles, conventional and special functions, differential and integral calculus, metric and Hilbert spaces, algebras and groups, etc., without any exception known to the authors. This reconstruction is "isotopic" in the sense of being axiom-preserving. Particularly important is the preservation of all conventional quantum laws as shown below.

3) The mathematical structure of nonrelativistic hadronic mechanics is characterized by [6]:

3a) The isofield $\hat{C} = \hat{C}(\hat{c}, +, \hat{\times})$ with *isounit* $\hat{I} = 1/\hat{T} > 0$, *isocomplex numbers* and related *isoproduct*

$$\hat{c} = c \times \hat{I} = (n_1 + i \times n_2) \times \hat{I},\ \hat{c}\hat{\times}\hat{d} = (c \times d) \times \hat{I},\ \ \hat{c}, \hat{d} \in \hat{\mathcal{C}},\ \ c, d \in C, \quad (3.4.20)$$

the isofield $\hat{R}(\hat{n}, +, \hat{\times})$ of *isoreal numbers* $\hat{n} = n \times \hat{I},\ n \in R$, being a particular case;

3b) The iso-Hilbert space $\hat{\mathcal{H}}$ with isostates $|\hat{\psi}\rangle, |\hat{\phi}\rangle, \ldots$, *isoinner product* and *isonormalization*

$$\langle\hat{\phi}|\hat{\times}|\hat{\psi}\rangle \times \hat{I}\ \in \hat{S}, \quad \langle\hat{\psi}|\hat{\times}|\hat{\psi}\rangle = 1, \quad (3.4.21)$$

and related theory of isounitary operators;

3c) The Euclid-Santilli isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{\mathrm{R}})$ with *isocoordinates, isometric and isoinvariant* respectively given by

$$\hat{r} = \{r^k\} \times \hat{I}, \quad (3.4.22a)$$

$$\hat{\delta} = \hat{T}(t, r, p, \psi, \nabla\psi, \ldots) \times \delta, \quad (3.4.22b)$$

$$\delta = Diag.(1, 1, 1), \quad (3.4.22c)$$

$$\hat{r}^{\hat{2}} = (r^i \times \hat{\delta}_{ij} \times r^j) \times \hat{I} \in \hat{\mathrm{R}}; \quad (3.4.22d)$$

3d) The isodifferential calculus and the isofunctional analysis (see Section 3.2);

3e) The Lie-Santilli isotheory with enveloping isoassociative algebra $\hat{\xi}$ of operators $\hat{A},\ \hat{B},\ \ldots$, with isounit $\hat{I}$, isoassociative product $\hat{A}\hat{\times}\hat{B} = \hat{A} \times \hat{T} \times \hat{B}$, *Lie-Santilli isoalgebra with brackets and isoexponentiation*

$$[\hat{A}, \hat{B}] = \hat{A}\hat{\times}\hat{B} - \hat{B}\hat{\times}\hat{A}, \quad (3.4.23a)$$

$$\hat{U} = \hat{e}^X = (e^{X \times \hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T} \times X}), \quad X = X^\dagger, \quad (3.4.13b)$$

and related isosymmetries characterizing groups of isounitary transforms on $\hat{\mathcal{H}}$ over $\hat{C}$,

$$\hat{U}\hat{\times}\hat{U}^\dagger = \hat{U}^\dagger\hat{\times}\hat{U} = \hat{I}. \quad (3.4.24)$$

As we shall see in Sections 3.4.8 and 3.4.9, the above entire mathematical structure can be achieved in a truly elementary way via nonunitary transforms

of quantum formalisms. Their isotopic reformulations then proves the invariance of hadronic mechanics, namely, its capability of predicting the same numbers for the same conditions at different times.

Under the above outlined structure we have the following main features:

I) Hadronic mechanics is a covering of quantum mechanics, because the latter theory is admitted uniquely and unambiguously at the limit when the isounit recovers the conventional unit, $\hat{I} \to I$;

II) Said covering is further characterized by the fact that hadronic mechanics coincides with quantum mechanics everywhere except for (as we shall see, generally small) non-Hamiltonian corrections at short mutual distances of particles caused by deep mutual overlapping of the wavepackets and/or charge distributions of particles;

III) Said covering is finally characterized by the fact that the indicated non-Hamiltonian corrections are restricted to verify all abstract axioms of quantum mechanics, with consequential preservation of is basic laws for closed non-Hamiltonian systems as a whole, as we shall see shortly.

Note that *composite hadronic systems*, such as *hadrons, nuclei, isomolecules, etc.*, are represented via the tensorial product of the above structures. This can be best done via the identification first of the *total isounit, total isofields, total isohilbert spaces*, *etc.*,

$$\hat{I}_{\rm tot} = \hat{I}_1 \times \hat{I}_2 \times \ldots, \ \hat{C}_{\rm tot} = \hat{C}_1 \times \hat{C}_2 \times \ldots, \ \hat{\mathcal{H}}_{\rm tot} = \hat{\mathcal{H}}_1 \times \hat{\mathcal{H}}_2 \times \ldots \ . \quad (3.4.25)$$

Note also that some of the units, fields and Hilbert spaces in the above tensorial products can be *conventional*, namely, the composite structure may imply *local-potential long range interactions* (e.g., those of Coulomb type), which require the necessary treatment via *conventional* quantum mechanics, and *nonlocal-nonpotential short range interactions* (e.g., those in deep wave-overlappings), which require the use of operator isomechanics.

3.4.5 Dynamical Equations of Operator Isomechanics and their Isoduals

The formulations of the preceding sections permit the identification of the following fundamental dynamical equations of the Lie-isotopic branch of hadronic mechanics, known under the name of *iso-Heisenberg equations* or *Heisenberg-Santilli isoequetions*that were identified in the original proposal of 1978 to build hadronic mechanics [5], are can be presented in their finite and infinitesimal forms,

$$\hat{A}(\hat{t}) = \hat{U} \hat{\times} \hat{A}(\hat{0}) \hat{\times} \hat{U}^{\hat{\dagger}} = \{\hat{e}^{\hat{i} \hat{\times} \hat{H} \hat{\times} \hat{t}}\} \hat{\times} \hat{A}(\hat{0}) \hat{\times} \{\hat{e}^{-\hat{i} \hat{\times} \hat{t} \hat{\times} \hat{H}}\}, \quad (3.4.26a)$$

$$\hat{i} \hat{\times} \hat{d}\hat{A} \hat{/} \hat{dt} = [\hat{A}, \hat{H}] = A \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A} = \hat{A} \times \hat{T} \times \hat{H} - \hat{H} \times \hat{T} \times \hat{A}, \quad (3.4.26b)$$

with the corresponding *fundamental hadronic isocommutation rules*

$$[\hat{b}^{\mu}, \hat{b}^{\nu}] = \hat{i} \hat{\times} \hat{\omega}^{\mu\nu} = i \times \omega^{\mu\nu} \times \hat{I}_6, \quad \hat{b} = (\hat{r}^k, \hat{p}_k), \quad (3.4.27)$$

with corresponding *iso-Schrödinger equations* for the energy, also known as *Schrödinger-Santilli isoequations* identified by Myung and Santilli [45] and Mignani [48] in 1982 over conventional fields and first formulated in an invariant way by Santilli in monograph [7] of 1995

$$\hat{i}\hat{\times}\hat{\partial}_{\hat{t}}|\hat{\psi}\rangle = \hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H}\times\hat{T}\times|\hat{\psi}\rangle = \hat{E}\hat{\times}|\hat{\psi}\rangle = E\times|\hat{\psi}\rangle, \tag{3.4.28a}$$

$$|\hat{\psi}(\hat{t})\rangle = \hat{U}\hat{\times}|\hat{\psi}(\hat{0})\rangle = \{\hat{e}^{i\hat{H}\hat{\times}\hat{t}}\}\hat{\times}|\hat{\psi}(\hat{0})\rangle, \tag{3.4.28b}$$

and *isolinear momentum* first identified by Santilli in Ref. [7] of 1995 thanks to the discovery of the isodifferential calculus

$$\hat{p}_k\hat{\times}|\hat{\psi}\rangle = \hat{p}_k\times\hat{T}\times|\hat{\psi}\rangle - \hat{i}\hat{\times}\hat{\partial}_k|\hat{\psi}\rangle = -i\times\hat{I}^i_k\times\partial_i|\hat{\psi}\rangle, \tag{3.4.29}$$

It is evident that the iso-Heisenberg equations in their infinitesimal and exponentiated forms are a realization of the Lie-Santilli isotheory of Section 3.2, which is therefore the algebraic and group theoretical structure of the isotopic branch of hadronic mechanics.

Note that Eqs. (3.4.26) and (3.4.28) automatically bring into focus the general need for a *time isounit* and related characterization of the time isodifferential and isoderivative

$$\hat{I}_t(t,r,\psi,\ldots) = \hat{T}_t > 0, \tag{3.4.30a}$$

$$d\hat{t} = \hat{I}_t\times dt, \quad \hat{\partial}_{\hat{t}} = \hat{I}_t\times\partial_t. \tag{3.4.30b}$$

Note also that $\omega^{\mu\nu}$ in Eqs. (3.4.xxx) is the *conventional* Lie tensor, namely, the same tensor appearing in the conventional canonical commutation rules, thus confirming the axiom-preserving character of isomechanics.

The limited descriptive capabilities of quantum models should be kept in mind, purely Hamiltonian and, as such, they can only represent systems which are linear, local and potential. By comparison, we can write Eq. (3.4.28a) in its explicit form

$$\begin{aligned}\hat{i}\hat{\times}\hat{\partial}_{\hat{t}}\hat{\psi} = i\times\hat{I}_t\times\partial_{\hat{t}}|\hat{\psi}\rangle = \hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H}\times\hat{T}\times|\hat{\psi}\rangle = \\ = \{\hat{p}_k\times\hat{p}_k/\hat{2}\hat{\times}\hat{m} + \hat{U}_k(\hat{t},\hat{r})\hat{\times}\hat{v}^k + \\ + \hat{U}_0(\hat{t},\hat{r})\}\times\hat{T}(\hat{t},\hat{r},\hat{p},\hat{\psi},\nabla\psi,\ldots)\times|\hat{\psi}(\hat{t},\hat{r})\rangle = \\ = \hat{E}\hat{\times}|\hat{\psi}(t,\hat{x})\rangle = E\times|\hat{\psi}(\hat{t},\hat{x})\rangle,\end{aligned} \tag{3.4.31}$$

thus proving the following

THEOREM 3.4.1 [7]: Hadronic mechanics id "directly universal" for all infinitely possible, sufficiently smooth and regular, closed non-Hamiltonian systems,

namely, it can represent in the fixed coordinates of the experimenter all infinitely possible closed-isolated systems with linear and nonlinear, local and nonlocal, and potential as well as nonpotential internal forces verifying the conservation of the total energy.

A consistent formulation of the isolinear momentum (3.4.29) escaped identification for two decades, thus delaying the completion of the construction of hadronic mechanics, as well as its practical applications. The consistent and invariant form (3.4.29) with consequential isocanonical commutation rules were first identified by Santilli in the second edition of Vol. II of this series, Ref. [7] of 1995 and memoir [10], following the discovery of the isodifferential calculus.

3.4.6 Preservation of Quantum Physical Laws

As one can see, the fundamental assumption of isoquantization is the lifting of the basic unit of quantum mechanics, Planck's constant $\hbar$, into a matrix $\hat{I}$ with nonlinear, integro-differential elements which also depend on the wavefunction and its derivatives

$$\hbar = I > 0 \to \hat{I} = \hat{I}(t, r, p, \psi, \hat{\psi}, \ldots) = \hat{I}^{\dagger} > 0. \tag{3.4.32}$$

It should be indicated that the above generalization is only *internal* in closed non-Hamiltonian because, when measured from the outside, *the isoexpectation values and isoeigenvalues of the isounit recover Planck's constant identically* [46],

$$\langle \hat{I} \rangle = \frac{\langle \hat{\psi} | \hat{\times} \hat{I} \hat{\times} | \hat{\psi} \rangle}{\langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle} = 1 = \hbar, \tag{3.4.33a}$$

$$\hat{I} \hat{\times} | \hat{\psi} \rangle = \hat{T}^{-1} \times \hat{T} \times | \hat{\psi} \rangle = 1 \times | \hat{\psi} \rangle = | \hat{\psi} \rangle. \tag{3.4.33b}$$

Moreover, the isounit is the fundamental invariant of isomechanics, thus preserving all axioms of the conventional unit $I = \hbar$, e.g.,

$$\hat{I}^{\hat{n}} = \hat{I} \hat{\times} \hat{I} \hat{\times} \ldots \hat{\times} \hat{I} \equiv \hat{I}, \tag{3.4.34a}$$

$$\hat{I}^{\hat{\frac{1}{2}}} \equiv \hat{I}, \tag{3.4.34b}$$

$$\hat{i} \hat{\times} d\hat{I} \hat{/} dt = [\hat{I}, \hat{H}] = \hat{I} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{I} \equiv 0. \tag{3.4.34c}$$

Despite their generalized structure, *Eqs. (3.4.26) and (3.4.28) preserve conventional quantum mechanical laws under nonlinear, nonlocal and nonpotential interactions* [7].

To begin an outline, the preservation of Heisenberg's uncertainties can be easily derived from isocommutation rules (3.4.27):

$$\Delta x^k \times \Delta p_k \geq \frac{1}{2} \times \langle [\hat{x}^k, \hat{p}_k] \rangle = \frac{1}{2}. \tag{3.4.35}$$

To see the preservation of Pauli's exclusion principle, recall that the regular (two-dimensional) representation of $SU(2)$ is characterized by the conventional *Pauli matrices* σ_k with familiar commutation rules and eigenvalues on $\mathcal{H}$ over C,

$$[\sigma_i, \sigma_j] = \sigma_i \times \sigma_j - \sigma_j \times \sigma_i = 2 \times i\varepsilon_{ijk} \times \sigma_k, \tag{3.4.36a}$$

$$\sigma^2 \times |\psi\rangle = \sigma_k \times \sigma^k \times |\psi\rangle = 3 \times |\psi\rangle, \tag{3.4.36b}$$

$$\sigma_3 \times |\psi\rangle = \pm 1 \times |\psi\rangle. \tag{3.4.36c}$$

The isotopic branch of hadronic mechanics requires the construction of *nonunitary images of Pauli's matrices* first constructed in Ref. [49] that, for diagonal nonunitary transforms and isounits, can be written (see also Section 3.3.6)

$$\hat{\sigma}_k = U \times \sigma_k \times U^\dagger, \quad U \times U^\dagger = \hat{I} \neq I, \tag{3.4.37a}$$

$$\begin{aligned} U = \begin{pmatrix} i\times n_1 & 0 \\ 0 & i\times n_2 \end{pmatrix}, \quad U^\dagger = \begin{pmatrix} -i\times n_1 & 0 \\ 0 & -i\times n_2 \end{pmatrix}, \\ \hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}, \end{aligned} \tag{3.4.37b}$$

where the n's are well behaved nowhere null functions, resulting in the *regular Pauli-Santilli isomatrices* [49]

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & n_1^2 \\ n_2^2 & 0 \end{pmatrix}, \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\times n_1^2 \\ i\times n_2^2 & 0 \end{pmatrix}, \ \hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}. \tag{3.4.38}$$

Another realization is given by *nondiagonal unitary transforms* [*loc. cit.*],

$$\begin{aligned} U = \begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix}, \quad U^\dagger = \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \\ \hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix}, \end{aligned} \tag{3.4.39}$$

with corresponding *regular Pauli-Santilli isomatrices*,

$$\begin{aligned} \hat{\sigma}_1 = \begin{pmatrix} 0 & n_1\times n_2 \\ n_1\times n_2 & 0 \end{pmatrix}, \ \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\times n_1\times n_2 \\ i\times n_1\times n_2 & 0 \end{pmatrix}, \\ \hat{\sigma}_3 = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}, \end{aligned} \tag{3.4.40}$$

or by more general realizations with Hermitian nondiagonal isounits $\hat{I}$ [15].

All Pauli-Santilli isomatrices of the above regular class verify the following isocommutation rules and isoeigenvalue equations on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$

$$[\hat{\sigma}_i,\hat{\sigma}_j] = \hat{\sigma}_i \times \hat{T} \times \hat{\sigma}_j - \hat{\sigma}_j \times \hat{T} \times \hat{\sigma}_i = 2 \times i \times \varepsilon_{ijk} \times \hat{\sigma}_k, \tag{3.4.41a}$$

$$\hat{\hat{\sigma^2}}\hat{\times}|\hat{\psi}\rangle = (\hat{\sigma}_1\hat{\times}\hat{\sigma}_1 + \hat{\sigma}_2\hat{\times}\hat{\sigma}_2 + \hat{\sigma}_3\hat{\times}\hat{\sigma}_3)\hat{\times}|\hat{\psi}\rangle = 3 \times |\hat{\psi}\rangle, \tag{3.4.41b}$$

$$\hat{\sigma}_3\hat{\times}|\hat{\psi}\rangle = \pm 1 \times |\hat{\psi}\rangle, \tag{3.4.41c}$$

thus preserving conventional spin 1/2, and establishing the preservation in isochemistry of the Fermi-Dirac statistics and Pauli's exclusion principle.

It should be indicated for completeness that the representation of the isotopic $S\hat{U}(2)$ also admit *irregular isorepresentations*, that no longer preserve conventional values of spin [49]. The latter structures are under study for the characterization of spin under the most extreme conditions, such as for protons and electrons in the core of collapsing stars and, as such, they have no known relevance for isomechanics.

The preservation of the superposition principle under nonlinear interactions occurs because of the reconstruction of linearity on isospace over isofields, thus regaining the applicability of the theory to composite systems.

Recall in this latter respect that conventionally nonlinear models,

$$H(t, x, p, \psi, \ldots) \times |\psi\rangle = E \times |\psi\rangle, \tag{3.4.42}$$

violate the superposition principle and have other shortcomings (see Section 1.5). As such, they cannot be applied to the study of composite systems such as molecules. All these models can be *identically* reformulated in terms of the isotopic techniques via the embedding of all nonlinear terms in the isotopic element,

$$H(t, x, p, \psi, \ldots) \times |\psi\rangle \equiv H_0(t, x, p) \times \hat{T}(\psi, \ldots) \times |\psi\rangle = E \times |\psi\rangle, \tag{3.4.43}$$

by regaining the full validity of the superposition principle in isospaces over isofields with consequential applicability to composite systems.

The preservation of causality follows from the one-dimensional isounitary group structure of the time evolution (3.4.28) (which is isomorphic to the conventional one); the preservation of probability laws follows from the preservation of the axioms of the unit and its invariant decomposition as indicated earlier; the preservation of other quantum laws then follows.

The same results can be also seen from the fact that operator isomechanics coincides at the abstract level with quantum mechanics by conception and construction. As a result, hadronic and quantum versions are *different realizations of the same abstract axioms and physical laws.*

Note that the preservation of conventional quantum laws under nonlinear, nonlocal and nonpotential interactions is crucially dependent on the capability of isomathematics to reconstruct linearity, locality and canonicity-unitarity on isospaces over isofields.

The preservation of conventional physical laws by the isotopic branch of hadronic mechanics was first identified by Santilli in report [47]. It should be indicated that the same quantum laws *are not* generally preserved by the broader

genomechanics, evidently because the latter must represent by assumption *non*-conservation laws and other *departures* from conventional quantum settings.

With the understanding that the theory does not receive the classical determinism, it is evident that isomechanics provides a variety of "completions" of quantum mechanics according to the celebrated E-P-R argument [50], such as:

1) Isomechanics "completes" quantum mechanics via the addition of nonpotential-nonhamiltonian interactions represented by nonunitary transforms.

2) Isomechanics "completes" quantum mechanics via the broadest possible (non-oriented) realization of the associative product into the isoassociative form.

3) Isomechanics "completes" quantum mechanics in its classical image.

In fact, as proved by well known procedures based on *Bell's inequalities*, quantum mechanics does not admit direct classical images on a number of counts. On the contrary, as studied in details in Refs. [51], the nonunitary images of Bell's inequalities permit indeed direct and meaningful classical limits which do not exist for the conventional formulations.

Similarly, it is evident that isomechanics constitutes a specific and concrete realization of "hidden variables" [52] λ which are explicitly realized by the isotopic element, $\lambda = \hat{T}$, and actually turned into an operator hidden variables. The "hidden" character of the realization is expressed by the fact that hidden variables are embedded in the unit and product of the theory.

In fact, we can write the iso-Schrödinger equation $\hat{H}\hat{\times}|\hat{\psi}\rangle = \hat{H} \times \lambda \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle$, $\lambda = \hat{T}$. As a result, the "variable" λ (now generalized into the operator $\hat{T}$) is "hidden" in the modular associative product of the Hamiltonian $\hat{H}$ and the state $|\hat{\psi}\rangle$.

Alternatively, we can say that hadronic mechanics provides an explicit and concrete realization of hidden variables because all distinctions between $\hat{H}\hat{\times}|\hat{\psi}\rangle$ and $H \times |\psi\rangle$ cease to exist at the abstract realization-free level.

For studies on the above and related issues, we refer the interested reader to Refs. [51] and quoted literature.

3.4.7 Isoperturbation Theory and its Isodual

We are now sufficiently equipped to illustrate the computational advantages in the use of isotopies.

THEOREM 3.4.2 [7]: Under sufficient continuity conditions, all perturbative and other series that are conventionally divergent (weakly convergent) can be turned into convergent (strongly convergent) forms via the use of isotopies with sufficiently small isotopic element (sufficiently large isounit),

$$|\hat{T}| \ll 1, \quad |\hat{I}| \gg 1. \tag{3.4.44}$$

The emerging perturbation theory was first studied by Jannussis and Mignani [53], and then studied in more detail in monograph [7] under the name of *isoperturbation theory*.

Consider a Hermitian operator on $\mathcal{H}$ over C of the type

$$H(k) = H_0 + k \times V, \quad H_0 \times |\psi\rangle = E_0 \times |\psi\rangle, \tag{3.4.45a}$$

$$H(k) \times |\psi(k)\rangle = E(k) \times |\psi(k)\rangle, \quad k \gg 1. \tag{3.4.45b}$$

Assume that H_0 has a nondegenerate discrete spectrum. Then, conventional perturbative series are *divergent*, as well known. In fact, the eigenvalue $E(k)$ of $H(k)$ up to second order is given by

$$\begin{aligned} E(k) = E_0 + k \times E_1 + k^2 \times E_2 = \\ = E_0 + k \times \langle\psi| \times V \times |\psi\rangle + k^2 \times \sum_{p \neq n} \frac{|\langle\psi_p| \times V \times |\psi_n\rangle|^2}{E_{0n} - E_{0p}}. \end{aligned} \tag{3.4.46}$$

But under isotopies we have

$$H(k) = H_0 + k \times V, \quad H_0 \times \hat{T} \times |\tilde{\psi}\rangle = \tilde{E}_0 \times |\tilde{\psi}\rangle, \quad \tilde{E}_0 \neq E_0, \tag{3.4.47a}$$

$$H(k) \times \hat{T} \times |\hat{\psi}(k)\rangle = \tilde{E}(k) \times |\hat{\psi}(k)\rangle, \quad \tilde{E} \neq E, \quad k > 1. \tag{3.4.47b}$$

A simple lifting of the conventional perturbation expansion then yields

$$\tilde{E}(k) = \tilde{E}_0 + k \times \tilde{E}_1 + k^2 \times \tilde{E}_2 + \hat{O}(k^2) =$$

$$= \tilde{E}_0 + k \times \langle\tilde{\psi}| \times \hat{T} \times V \times \hat{T} \times |\tilde{\psi}\rangle + \tag{3.4.48a}$$

$$+k^2 \times \sum_{p \neq n} \frac{|\langle\hat{\psi}_p| \times \hat{T} \times V \times \hat{T} \times |\hat{\psi}_n\rangle|^{\hat{2}}}{\tilde{E}_{0n} - \tilde{E}_{0p}}, \tag{3.4.48b}$$

whose convergence can be evidently reached via a suitable selection of the isotopic element, e.g., such that $|\hat{T}| \ll k$.

As an example, for a positive-definite constant $\hat{T} \ll k^{-1}$, expression (3.4.46) becomes

$$\begin{aligned} \tilde{E}(k) = \tilde{E}_0 + k \times \hat{T}^2 \times \langle\hat{\psi}| \times V \times |\psi*\rangle + k^2 \times T^5 \times \\ \times \sum_{p \neq n} \frac{|\langle\psi_p| \times V \times |\psi_n\rangle|^2}{\tilde{E}_{0n} - \tilde{E}_{0p}}. \end{aligned} \tag{3.4.49}$$

This shows that the original divergent coefficients $1, k, k^2, \ldots$ are now turned into the manifestly convergent coefficients $1, k \times T^2, k^2 \times T^5, \ldots$, with $k > 1$ and $\hat{T} \ll 1/k$, thus ensuring isoconvergence for a suitable selection of $\hat{T}$ for each given k and V.

A more effective reconstruction of convergence can be seen in the algebraic approach. At this introductory stage, we consider a divergent canonical series,

$$A(k) = A(0) + k \times [A, H]/1! + +k^2 \times [[A, H], H]/2! + \ldots \to \infty, \quad k > 1, \quad (3.4.50)$$

where $[A, H] = A \times H - H \times A$ is the familiar Lie product, and the operators A and H are Hermitian and sufficiently bounded. Then, under the isotopic lifting the preceding series becomes [7]

$$\hat{A}(k) = \hat{A}(0) + k \times [A\hat{,}H]/1! + k^2 \times [[A\hat{,}H]\hat{,}H]/2! + \cdots \leq |N| < \infty, \quad (3.4.51a)$$

$$[A\hat{,}H] = A \times \hat{T} \times H - H \times \hat{T} \times A, \quad (3.4.51b)$$

which holds, e.g., for the case $T = \varepsilon \times k^{-1}$, where ε is a sufficiently small positive-definite constant.

In summary, the studies on the construction of hadronic mechanics have indicated that the apparent origin of divergences (or slow convergence) in quantum mechanics and chemistry is their lack of representation of nonlinear, nonlocal, and nonpotential effects because when the latter are represented via the isounit, full convergence (much faster convergence) can be obtained.

As we shall see, all known applications of hadronic mechanics verify the crucial condition $|\hat{I}| \gg 1$, $|\hat{T}| \ll 1$, by permitting convergence of perturbative series. For instance, in the case of chemical bonds, hadronic chemistry allows computations at least one thousand times faster than those of quantum chemistry, with evident advantages, e.g., a drastic reduction of computer time (see Chapter 9). Essentially the same results are expected for hadronic mechanics and hadronic superconductivity.

The reader should meditate a moment on the evident possibility that *hadronic mechanics offers realistic possibilities of constructing a convergent perturbative theory for strong interactions.* As a matter of fact, the divergencies that have afflicted strong interactions through the 20-th century originates precisely from the excessive approximation of hadrons as points, with the consequential sole potential interactions and related divergencies.

In fact, whenever hadrons are represented as they actually are in reality, extended and hyperdense particles, with consequential potential as well as nonpotential interactions, all divergencies are removed by the isounit.

3.4.8 Simple Construction of Operator Isomechanics and its Isodual

Despite their *mathematical equivalence*, it should be indicated that quantum and hadronic mechanics are *physically inequivalent*, or, alternatively, hadronic mechanics is outside the classes of equivalence of quantum mechanics because the former is a *nonunitary image* of the latter.

As we shall see in the next chapters, the above property provides means for the explicit construction of the new model of isomechanics bonds from the conventional model. The main requirement is that of identifying the *nonhamiltonian* effects one desires to represent, which as such, are necessarily *nonunitary*. The resulting nonunitary transform is then assumed as the fundamental space isounit of the new isomechanics [46]

$$U \times U^\dagger = \hat{I} \neq I, \tag{3.4.52}$$

under which transform we have the liftings of: the quantum unit into the isounit,

$$I \to \hat{I} = U \times I \times U^\dagger; \tag{3.4.53}$$

numbers into isonumbers,

$$a \to \hat{a} = U \times a \times U^\dagger = a \times (U \times U^\dagger) = a \times \hat{I}; \quad a = n, c; \tag{3.4.54}$$

associative products $A \times B$ into the isoassociative form with the correct isotopic element,

$$A \times B \to \hat{A} \hat{\times} \hat{B} = \hat{A} \times \hat{T} \times \hat{B}, \tag{3.4.55a}$$

$$\hat{A} = U \times A \times U^\dagger, \ \hat{B} = U \times B \times U^\dagger, \ \hat{T} = (U \times U^\dagger)^{-1} = T^\dagger; \tag{3.4.55b}$$

Schrödinger's equation into the isoschrödinger's equations

$$\begin{aligned} H \times |\psi\rangle = E \times |\psi\rangle \to U(H \times |\psi\rangle) = \\ = (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi\rangle) = \\ = \hat{H} \times \hat{T} \times |\hat{\psi}\rangle = \hat{H} \hat{\times} |\hat{\psi}\rangle; \end{aligned} \tag{3.4.56}$$

Heisenberg's equations into their isoheisenberg generalization

$$\begin{aligned} i \times dA/dt - A \times H - H \times A = 0 \to \\ \to U \times (i \times dA/dt) \times U^\dagger - U(A \times H - H \times A) \times U^\dagger = \\ = \hat{i} \hat{\times} d\hat{A}/dt - \hat{A} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{A} = 0; \end{aligned} \tag{3.4.57}$$

the Hilbert product into its isoinner form

$$\begin{aligned} \langle \psi | \times | \psi \rangle \to U \times \langle \psi | \times | \psi \rangle \times U^\dagger = \\ = (\langle \psi | \times U^\dagger) \times (U \times U)^{-1} \times (U \times | \psi \rangle) \times (U \times U)^{-1} = \langle \hat{\psi} | \hat{\times} | \hat{\psi} \rangle \times \hat{I}; \end{aligned} \tag{3.4.58}$$

canonical power series expansions into their isotopic form

$$\begin{aligned} A(k) = A(0) + k \times [A, H] + k^2 \times [[A, H,], H] + \ldots \to U \times A(k) \times U^\dagger = \\ = U \times \Big[A(0) + k \times [A, H] + k^2 \times [[A, H], H] + \ldots \Big] \times U^\dagger = \\ = \hat{A}(\hat{k}) = \hat{A}(0) + \hat{k} \hat{\times} [\hat{A}, \hat{H}] + \hat{k}^{\hat{2}} \hat{\times} [[\hat{A}, \hat{H}], \hat{H}] + \ldots, \\ k > 1, \quad |\hat{T}| \ll 1; \end{aligned} \tag{3.4.59}$$

Schrödinger's perturbation expansion into its isotopic covering (where the usual summation over states $p \neq n$ is assumed)

$$\begin{aligned} E(k) = E(0) + k \times \langle\psi| \times V \times |\psi\rangle + k^2 \frac{|\langle\psi| \times V \times |\psi\rangle|^2}{E_{0n} - E_{0p}} + \ldots \to \\ \to U \times E(k) \times U^\dagger = U \times \Big[E(0) + k \times \langle\psi| \times V \times |\psi\rangle + \ldots \Big] \times U^\dagger = \\ = \hat{E}(\hat{k}) = \hat{E}(0) + \hat{k} \hat{\times} \langle\hat{\psi}| \times \hat{T} \times \hat{V} \times \hat{T} \times |\hat{\psi}\rangle + \ldots, \\ k > 1, \quad |\hat{T}| \ll 1; \end{aligned} \tag{3.4.60}$$

etc. All remaining aspects of operator isomechanics can then be derived accordingly, including the isoexponent, isologarithm, isodeterminant, isotrace, isospecial functions and transforms, *etc.* The isodual isomechanics can then be constructed via the now famil,kiar isodual map.

Note that the above construction via a nonunitary transform is the correct operator image of the derivability of the classical isohamiltonian mechanics from the conventional form via noncanonical transforms (Section 3.2.12).

The construction of hadronic mechanics via nonunitary transforms of quantum mechanics was first identified by Santilli in the original proposal [5e], and then worked out in subsequent contributions (see [12] for the latest presentation).

3.4.9 Invariance of Operator Isomechanics and of its Isodual

It is important to see that, in a way fully parallel to the classical case (Section 3.3.7), operator isomechanics is indeed invariant under the most general possible nonlinear, nonlocal and nonhamiltonian-nonunitary transforms, provided that, again, the invariance is treated via the isomathematics. In fact, any given nonunitary transform $U \times U^\dagger \neq I$ can always be decomposed into the form [12]

$$U = \hat{U} \times \hat{T}^{1/2},$$

under which nonunitary transforms on $\mathcal{H}$ over C are identically reformulated as isounitary transforms on the isohilbert space $\hat{\mathcal{H}}$ over the isofield $\hat{}$,

$$U \times U^\dagger \equiv \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}. \tag{3.4.61}$$

The form-invariance of operator isomechanics under isounitary transforms then follows,

$$\hat{I} \to \hat{I}' = \hat{U} \hat{\times} \hat{I} \hat{\times} \hat{U}^\dagger \equiv \hat{I}, \ \hat{A} \hat{\times} \hat{B} \to \hat{U} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{U}^\dagger = \hat{A}' \hat{\times} \hat{B}', \ \textit{etc.}, \tag{3.4.62a}$$

$$\begin{aligned} \hat{H} \hat{\times} |\hat{\psi}\rangle = \hat{E} \hat{\times} |\hat{\psi}\rangle \to \hat{U} \times \hat{H} \hat{\times} |\hat{\psi}\rangle = \\ = (\hat{U} \times \hat{H} \times \hat{U}^\dagger) \hat{\times} (\hat{U} \hat{\times} |\hat{\psi}\rangle) = \hat{H}' \hat{\times} |\hat{\psi}'\rangle = \\ = \hat{U} \hat{\times} \hat{E} \hat{\times} |\hat{\psi}\rangle = \hat{E} \hat{\times} \hat{U} \hat{\times} |\hat{\psi}\rangle = \hat{E} \hat{\times} |\hat{\psi}'\rangle, \end{aligned} \tag{3.4.62b}$$

where one should note the preservation of the *numerical values* of the isounit, isoproducts and isoeigenvalues, as necessary for consistent applications. The invariance of isodual isomechanics then follows rather trivially.

Note that the invariance in quantum mechanics holds only for transformations $U \times U^\dagger = I$ *with fixed* I. Similarly, the invariance of isomechanics holds only for all nonunitary transforms such $\hat{U}\hat{\times}\hat{U}^{\hat{\dagger}} = \hat{I}$ *with fixed* $\hat{I}$, and *not* for a transform $\hat{W}\hat{\times}\hat{W}^{\hat{\dagger}} = \hat{I}' \neq \hat{I}$ because the change of the isounit $\hat{I}$ implies the transition to a *different physical system.*

The form-invariance of hadronic mechanics under isounitary transforms was first studied by Santilli in memoir [46].

3.5 SANTILLI ISORELATIVITY AND ITS ISODUAL

3.5.1 Limitations of Special and General Relativities

Special and general relativities are generally presented in contemporary academia as providing final descriptions of all infinitely possible conditions existing in the universe.

The scientific reality is basically different than the above academic posture. In Section 1.1 and Chapter 2, we have shown that special and general relativities *cannot* provide a consistent classical description of antiparticles because they admit no distinction between neutral matter and antimatter and, when used for charged antiparticles, they lead to inconsistent quantum images consisting of particles (rather than charge conjugated antiparticles) with the wrong sign of the charge. Hence, *the entire antimatter content of the universe cannot be credibly treated via special and/or general relativity.*[24]

A widespread academic posture, studiously conceived for adapting nature to preferred doctrines, is the belief that the university can be effectively reduced to point-particles solely under action-at-a-distance, potential interactions. This posture is dictated by the facts that: the mathematics underlying special and general relativities, beginning with their local-differential topology, can only represent (dimensionless) point-like particles; special and genera;l relativity are notoriously incompatible with the deformation theory (that is activated whenever extended particles are admitted); and said relativities are strictly Lagrangian or Hamiltonian, thus being only able to represent potential interactions.

However, in Section 1.3 and in this chapter, we have established the "No Reduction Theorems," according to which a macroscopic extended system in nonconservative conditions (such as a satellite during re-entry in our atmosphere) cannot

[24]Particularly political is the academic posture that "antigravity does not exist because not predicted by Einstein's gravitation," when such a gravitational theory has no means for a credible representation of antimatter. As we shall see in Chapter 14, Volume II, when a credible quantitative representation of antimatter is included, antigravity (defined as gravitational repulsion) between matter and antimatter is unavoidable.

be consistently reduced to a finite number of point-particles all under potential forces and, vice versa, a finite number of quantum (that is, point-like) particles all under potential interactions cannot consistently recover a macroscopic nonconservative system. Hence, *all macroscopic systems under nonconservative forces, thus including all classical interior problems, cannot be consistently treated with special or general relativity.*[25]

Another posture in academia, also intended for adapting nature to a preferred doctrine, is that irreversibility is a macroscopic event that "disappears" (sic) when systems are reduced to their elementary constituents. This widespread academic belief is necessary because special and general relativities are *structurally reversible,* namely, their mathematical and physical axioms, as well as all known Hamiltonians are invariant under time reversal. This posture is complemented with manipulations of scientific evidence, such as the presentation of the probability of the synthesis of two nuclei into a third one, $n_1 + n_2 \to n_3$ while studiously suppressing the time reversal event that is simply unavoidable for a reversible theory, namely, the finite probability of the *spontaneous* decomposition $n_3 \to n_1 n_2$ following the synthesis. The latter probability is suppressed evidently because it would prove the inconsistency of the assumed basic doctrine. [26]

Unfortunately for mankind, the above academic postures are also used for all energy releasing processes despite the fact that they are irreversible. The vast majority of the research on energies releasing processes such as the "cold" and "hot" fusions, and the use of the vast majority of public fund,s are restricted to verify quantum mechanics and special relativity under the knowledge by experts that reversible theories cannot be exactly valid for irreversible processes/ In any case, the "No reduction theorems" prevent the consistent reduction of an irreversible macroscopic event to an ideal ensemble of point-like abstractions of

[25]Another political posture in academia is the treatment of the entire universe, thus including interior problems of stars, quasars and black-hole, with Einstein gravitation when it is well known that such a doctrine is purely "external," namely, can only represent point-like masses moving in vacuum in the gravitational field of a massive body. One can then understand the political backing needed for the credibility, e.g., of studies on black holes derived via a purely exterior theory.

[26]Serious physicists should not even redo the calculations for the probability of the spontaneous decay following the synthesis, because it is unavoidable under the assumption of the same Hilbert space for all initial and final nuclei and Heisenberg's uncertainty principle. In fact these assumptions imply that the nucleus n_1 or n_2 has a finite [probability of being outside of n_3 due to the coherence of the interior and interior Hilbert spaces. At this point, numerous additional manipulations of science are attempted to salvage preferred doctrines when inapplicable, rather than admitting their inapplicability and seeking covering theories. One of these manipulations is based on the "argument" that n_3 is extended, when extended sizes cannot be represented by quantum mechanics. Other manipulations are not worth reporting here. *The only scientific case of a rigorously proved, identically null probability of spontaneous disintegrations of a stable nucleus following its synthesis occurs when the initial and final Hilbert spaces are incoherent. This mandates the use of the conventional Hilbert space (quantum mechanics) for the initial states and the use of an incoherent iso-Hilbert space (hadronic mechanics) for the final state. This is the only possibility known to this author following half a century of studies of the problem.*

particles all in reversible conditions. Hence, *special and general relativities are inapplicable for any and all irreversible processes existing in the universe.*[27]

When restricting the arena applicability to those of the original conception (propagation of point particles and electromagnetic waves in vacuum), special relativity remains afflicted by still unresolved basic problems, such as the possibility that the relativity verifying one-way experiments on the propagation of light could be Galilean, rather than Lorentzian; the known incompatibility of special relativity with space conceived as a universal medium; and other unsettled aspects. Independently from that, we have shown in Section 1.4 that general relativity has no case of unequivocal applicability for numerous reasons, such as: curvature cannot possibly represent the free fall of a body along a straight radial line; the "bending of light" is due to Newtonian gravitation (and if curvature is assumed one gets double the bending experimentally measured); gravitation is a noncanonical theory, thus suffering of the Theorems of Catastrophic Inconsistencies of Section 1.5; etc.

In summary, on serious scientific grounds, and contrary to vastly popular political beliefs, special and general relativities have no uncontested arena of exact valid.

Far from pretending final knowledge, in this section we primarily claim the scientific honesty to have identified the above open problems and initiated quantitative studies for their resolution. Our position in regard to special relativity is pragmatic, in the sense that, under the conditions limpidly identified by Einstein, such as particles in accelerators, etc., special relativity works well. Additionally, special relativity has a majestic axiomatic structure emphasized various times by the author.

Hence, we shall assume special relativity at the foundation of this section and seek its *isotopic liftings,* namely, the most general possible formulations verifying at the abstract level the original axioms conceived by Lorentz, Poincaré, Einstein, Minkowski, Weyl and other founders. The first, and perhaps basic understanding of this section is the knowledge that *special relativity and isorelativity coincide at the abstract, realization-free level,* to such an extent that we could use the same formulae and identify the special or isotopic relativity via different meanings of the same symbols. Alternatively, to honor the memory of the founders, it is necessary to identify the widest possible applicability of their axioms before abandoning them for broader vistas.

An additional, century-old, unresolved issue is the incompatibility of special relativity with the absolute reference frame at rest with the universal substratum

[27]To honor the memory of Albert Einstein and other founders of our knowledge, it should be stressed that the use of the word "violation" would be nonscientific, since quantum mechanics and special relativity were not conceived for irreversible processes. Said disciplines have been applied to irreversible processes by Einstein's *followers* seeking money, prestige and power via the abuse of Einstein's name.

(also called *ether*) that appears to be needed for the very characterization of all visible events in the Universe [54,55[. This latter aspect is fundamental for the studies of Volume II and are treated there to avoid unnecessary repetitions.

In regard to general relativity, our position is rather rigid: no research on general relativity can be considered scientifically serious unless the nine theorems s of catastrophic inconsistencies of Ref. [75] are disproved, not in academic corridors, but in refereed technical publications. Since this task appears to be hopeless, we assume the position that general relativity is catastrophically inconsistent and seek an alternative formulation.

As we shall see, *when the memory of the founders is honored in the above sense, the broadest possible realization of their axioms include gravitation and there is no need for general relativity as a separate theory.* Thus, another basic understanding of this section is the knowledge that we shall seek a *unification of special and general relativity into one single formulation based on the axioms of special relativity, known as Santilli isorelativity.* Needless to say, such a unification required several decades of research since it required the construction of the needed new mathematics, the achievement of the unification of the Minkowskian and Riemannian geometries, and the achievement of a universal invariance for all possible spacetime line elements prior to addressing the unification itself.

A further aspect important for the understanding of this section is that, *by no means isorelativity should be believed to be the final relativity of the universe because it is structurally reversible due to the Hermiticity of the isounit and isotopic element.*[28]

This creates the need for a yet broader relativity studied in the next chapte, and known under the name of *Santilli genorelativity,* this time, based on *genotopic liftings* of special relativity or isorelativity, namely, broadening requiring a necessary departure from the abstract axioms of special relativity into a form that is *structurally irreversible,* in the sense of possessing mathematical and physical axioms that are irreversible under all possible reversible Lagrangians or Hamiltonians.

The resolution of the above indicated problems for antimatter is achieved by the isodual image of the studies of this section.

3.5.2 Minkowski-Santilli Isospaces and their Isoduals

As studied in Section 1.2, the "universal constancy of the speed of light" is a philosophical abstraction, particularly when proffered by experts without the

[28]As we shall see in the next chapter, despite its Hermiticity, the isounit can depend on time in such a way that $\hat{I}(t,...) = \hat{I}^\dagger(t,...) \neq \hat{I}(-t,...)$. In this case isotopic theories represent systems verifying total conservation laws when isolated (because of the antisymmetry of the Lie-Santilli isobrackets), yet being structurally irreversible in their interior processes, as it is the case for all interior problems when considered isolated from the rest of the universe.

additional crucial words "in vacuum", because the constancy of the speed of light has been solely proved in vacuum while, in general, experimental evidence establishes that *the speed of light is a local variable depending on the characteristics of the medium in which it propagates,* with well known expression

$$c = c_\circ/n, \tag{3.5.1}$$

where the familiar *index of refraction* n is a function of a variety of time t, coordinates r, density μ, temperature τ, frequency ω, etc., $n = n(t, r, \mu, \tau, \omega, \ldots)$.

In particular, the speed of light is generally smaller than that in vacuum when propagating within media of low density, such as atmospheres or liquids,

$$c \ll c_\circ, \quad n \gg 1, \tag{3.5.2}$$

while the speed of light is generally bigger than that in vacuum when propagating within special guides, or within media of very high density, such as the interior of stars and quasars,

$$c \gg c_\circ, \quad n \ll 1. \tag{3.5.3}$$

Academic claims of recovering the speed of light in water via photons scattering among the water molecules are afflicted by numerous inconsistencies studied in Section 1.2, and the same holds for other aspects.

Assuming that via some unknown manipulation special relativity is shown to represent consistently the propagation of light within physical media, such a representation would activate the catastrophic inconsistencies of Theorem 1.5.1.

This is due to the fact that *the transition from the speed of light in vacuum to that within physical media requires a noncanonical or nonunitary transform.*

This point can be best illustrated by using the metric originally proposed by Minkowski, which can be written

$$\eta = Diag.(1, 1, 1, -c_\circ^2). \tag{3.5.4}$$

Then, the transition from $c_\circ$ to $c = c_\circ/n$ in the metric can only be achieved via a noncanonical or nonunitary transform

$$\eta = Diag.(1, 1, 1, -c_\circ^2) \to \hat{\eta} =$$

$$= Diag.(1, 1, 1, -c_\circ/n^2) = U \times \eta \times U^\dagger, \tag{3.5.5a}$$

$$U \times U^\dagger = Diag.(1,\ 1,\ 1,\ 1/n^2) \neq I. \tag{3.5.5b}$$

An invariant resolution of the limitations of special relativity for closed and reversible systems of extended and deformable particles under Hamiltonian and non-Hamiltonian interactions has been provided by the lifting of special relativity into a new formulation today known as *Santilli isorelativity,* where: the prefix "iso" stands to indicate that relativity principles apply on isospacetime

over isofields; and the characterization of "special" or "general" is inapplicable because, as shown below, *isorelativity achieves a geometric unification of special and general relativities.*

Isorelativity was first proposed by R. M. Santilli in Ref. [58] of 1983 via the first invariant formulation of *iso-Minkowskian spaces* and related *iso-Lorentz symmetry.* The studies were then continued in: Ref. [59] of 1985 with the first isotopies of the rotational symmetry; Ref. [49] of 1993 with the first isotopies of the SU(2)-spin symmetry; Ref. [60] of 1993 with the first isotopies of the Poincaré symmetry; Ref. [51] of 1998 with the first isotopies of the SU(2)-isospin symmetries, Bell's inequalities and local realism; and Refs. [61,62] on the first isotopies of the spinorial covering of the Poincaré symmetry.

The studies were then completed with memoir [26] of 1998 presenting a comprehensive formulation of the iso-Minkowskian geometry and its capability to unify the Minkowskian and Riemannian geometries, including its formulation via the mathematics of the Riemannian geometry (such iso-Christoffel's symbols, iso-covariant derivatives, etc.). The author then dedicated various monographs to the field through the years.

Numerous independent studies on Santilli isorelativity are available in the literature, one can inspect in this respect Refs. [32–43] and papers quoted therein; Aringazin's proof [63] of the direct universality of the Lorentz-Poincaré-Santilli isosymmetry for all infinitely possible spacetimes with signature $(+, +, +, -)$; Mignani's exact representation [64] of the large difference in cosmological redshifts between quasars and galaxies when physically connected; the exact representation of the anomalous behavior of the meanlives of unstable particles with speed by Cardone et al. [65–66]; the exact representation of the experimental data on the Bose-Einstein correlation by Santilli [67] and Cardone and Mignani [68]; the invariant and exact validity of the iso-Minkowskian geometry within the hyperdense medium in the interior of hadrons by Arestov et al. [69]; the first known exact representation of molecular features by Santilli and Shillady [70,71]; and numerous other contributions.

Evidently we cannot review isorelativity in the necessary details to avoid a prohibitive length. Nevertheless, to achieve minimal self-sufficiency of this presentation, it is important to outline at least its main structural lines (see monograph [55] for detailed studies).

The central notion of isorelativity is the lifting of the basic unit of the Minkowski space and of the Poincaré symmetry, $I = Diag.(1, 1, 1, 1)$, into a 4×4-dimensional, nowhere singular and positive-definite matrix $\hat{I} = \hat{I}_{4\times 4}$ with an unrestricted functional dependence on local spacetime coordinates x, speeds v, accelerations a, frequencies ω, wavefunctions ψ, their derivative $\partial\psi$, and/or any other needed variables,

$$I = Diag.(1, 1, 1) \to \hat{I}(x, v, a, \omega, \psi, \partial\psi, \ldots) =$$

$$= 1/\hat{T}(x, v, \omega, \psi, \partial\psi, \ldots) > 0. \tag{3.5.6}$$

Isorelativity can then be constructed via the method of Section 3.4.6, namely, by assuming that the basic noncanonical or nonunitary transform coincides with the above isounit

$$U \times U^\dagger = \hat{I} = Diag.(g_{11}, g_{22}, g_{33}, g_{44}),$$

$$g_{\mu\mu} = g_{\mu\mu}(x, v, \omega, \psi, \partial\psi, \ldots) > 0, \quad \mu = 1, 2, 3, 4, \tag{3.5.7}$$

and then subjecting the *totality* of quantities and their operation of special relativity to the above transform.

This construction is, however, selected here only for simplicity in pragmatic applications, since the rigorous approach is the construction of isorelativity from its abstract axioms, a task we have to leave to interested readers for brevity (see the original derivations [7]).

This is due to the fact that the former approach evidently preserves the original eigenvalue spectra and does not allow the identification of anomalous eigenvalues emerging from the second approach, such as those of the $SU(2)$ and $SU(3)$ isosymmetries [51].

Let $M(x, \eta, R)$ be the Minkowski space with local coordinates $x = (x^\mu)$, metric $\eta = Diag.(1, 1, 1, -1)$ and invariant

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \in R. \tag{3.5.8}$$

The fundamental space of isorelativity is the *Minkowski-Santilli isospace* [58] and related topology [10,22–25], $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ characterized by the liftings

$$I = Diag.(1, 1, 1, 1) \to U \times I \times U^\dagger = \hat{I} = 1/\hat{T}, \tag{3.5.9a}$$

$$\eta = Diag.(1, 1, 1, -1) \times I \to (U^{\dagger-1} \times \eta \times U^{-1}) \times \hat{I} = \hat{\eta} =$$
$$= \hat{T} \times \eta = Diag.(g_{11}, g_{22}, g_{33}, -g_{44}) \times \hat{I}, \tag{3.5.9b}$$

with consequential isotopy of the basic invariant

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \in R \to$$

$$\to U \times x^2 \times U^\dagger = \hat{x}^{\hat{2}} = (\hat{x}^\mu \hat{\times} \hat{m}_{\mu\nu} \times x^\nu) \times I \in R, \tag{3.5.10}$$

whose projection in conventional spacetime can be written

$$\hat{x}^{\hat{2}} = [x^\mu \times \hat{\eta}_{\mu\nu}(x, v, a, \omega, \psi, \partial\psi, \ldots) \times x^\nu] \times \hat{I}. \tag{3.5.11}$$

The nontriviality of the above lifting is illustrated by the following:[29]

[29] Fabio Cardone, Roberto Mignani and Alessio Marrani have uploaded a number of papers in the section hep-th of Cornell University arXiv copying *ad litteram* the results of paper [83], including the use of the same symbols, without any quotation at all of Santilli's preceding vast literature in the field. Educators, colleagues and editors of scientific journals are warned of the existence on ongoing legal proceedings one can inspect in the web site http://www.scientificethics.org/

THEOREM 3.5.1: The Minkowski-Santilli isospaces are directly universal, in the sense of admitting as particular cases all possible spaces with the same signature $(+,+,+,-)$, such as the Minkowskian, Riemannian, Finslerian and other spaces (universality), directly in terms of the isometric within fixed local variables (direct universality).

Therefore, the correct formulation of the *Minkowski-Santilli isogeometry* requires the isotopy of all tools of the Riemannian geometry, such as the iso-Christoffel symbols, isocovariant derivative, etc. (see for brevity Ref. [15]).

Despite that, one should keep in mind that, in view of the positive-definiteness of the isounit [34,79], *the Minkowski-Santilli isogeometry coincides at the abstract level with the conventional Minkowski geometry, thus having a null isocurvature* (because of the basic mechanism of deforming the metric η by the amount $\hat{T}(x,\ldots)$ while deforming the basic unit of the inverse amount $\hat{I} = 1/\hat{T}$).

The geometric unification of the Minkowskian and Riemannian geometries achieved by the Minkowski-Santilli isogeometry constitutes the evident geometric foundation for the unification of special and general relativities studied below.

It should be also noted that, following the publication in 1983 of Ref. [58], numerous papers on "deformed Minkowski spaces" have appeared in the physical and mathematical literature (generally without a quotation of their origination in Ref. [58]).

These "deformations" are ignored in these studies because they are formulated via conventional mathematics and, consequently, they all suffer of the catastrophic inconsistencies of Theorem 1.5.1.

By comparison, isospaces are formulated via isomathematics and, therefore, they resolve the inconsistencies of Theorem 1.5.1, as shown in Section 3.5.9. This illustrates again the necessity of lifting the basic unit and related field jointly with all remaining conventional mathematical methods.

3.5.3 Poincaré-Santilli Isosymmetry and its Isodual

Let $P(3.1)$ be the conventional Poincaré symmetry with the well known ten generators $J_{\mu\nu}, P_\mu$ and related commutation rules hereon assumed to be known.

The second basic tool of isorelativity is the *Poincaré-Santilli isosymmetry* $\hat{P}(3.1)$ studied in detail in monograph [55] that can be constructed via the isotheory of Section 3.2, resulting in the isocommutation rules [58,60]

$$[J_{\mu\nu}\hat{,}J_{\alpha\beta}] = i \times (\hat{\eta}_{\nu\alpha} \times J_{\beta\mu} - \hat{\eta}_{\mu\alpha} \times J_{\beta\nu} - \hat{\eta}_{\nu\beta} \times J_{\alpha\mu} + \hat{\eta}_{\mu\beta} \times J_{\alpha\nu}), \quad (3.5.12a)$$

$$[J_{\mu\nu}\hat{,}P_\alpha] = i \times (\hat{\eta}_{\mu\alpha} \times P_\nu - \hat{\eta}_{\nu\alpha} \times P_\mu), \quad (3.5.12b)$$

$$[P_\mu\hat{,}P_\nu] = 0, \quad (3.5.12c)$$

where we have followed the general rule of the Lie-Santilli isotheory according to which isotopies leave observables unchanged (since Hermiticity coincides with iso-Hermiticity) and merely change the *operations* among them.

The *iso-Casimir invariants* of $\hat{P}(3.1)$ are given by

$$P^{\hat{2}} = P_\mu \hat{\times} P^\mu = P^\mu \times \hat{\eta}_{\mu\nu} \times P^\nu = P_k \times g_{kk} \times P_k - p_4 \times g_{44} \times P_4, \quad (3.5.13a)$$

$$W^{\hat{2}} = W_\mu \hat{\times} W^\mu, \;\; W_\mu = \hat{\epsilon}_{\mu\alpha\beta\rho} \hat{\times} J^{\alpha\beta} \hat{\times} P^\rho, \quad (3.5.13b)$$

and they are at the foundation of classical and operator *isorelativistic kinematics.*

Since $\hat{I} > 0$, it is easy to prove that *the Poincaré-Santilli isosymmetry is isomorphic to the conventional symmetry.* It then follows that *the isotopies increase dramatically the arena of applicability of the Poincaré symmetry, from the sole Minkowskian spacetime to all infinitely possible spacetimes.*

Next, the reader should be aware that *the Poincaré-Santilli isosymmetry characterizes "isoparticles" (and not particles) via its irreducible isorepresentations.*

A mere inspection of the isounit shows that the Poincaré-Santilli isosymmetry characterizes actual nonspherical and deformable shapes as well as internal densities and the most general possible nonlinear, nonlocal and nonpotential interactions.

Since any interaction implies a renormalization of physical characteristics, it is evident that *the transition from particles to isoparticles, that is, from motion in vacuum to motion within physical media, causes an alteration (called isorenormalization), in general, of all intrinsic characteristics, such as rest energy, magnetic moment, charge, etc.*

As we shall see later on, the said isorenormalization has permitted the first exact numerical representation of nuclear magnetic moments, molecular binding energies and other data whose exact representation resulted to be impossible for nonrelativistic and relativistic quantum mechanics despite all possible corrections conducted over 75 years of attempts.

The explicit form of the *Poincaré-Santilli isotransforms* leaving invariant line element (3.5.11) can be easily constructed via the Lie-Santilli isotheory and are given:

(1) The **isorotations** [11]

$$\hat{O}(3) : \hat{\mathbf{x}}' = \hat{\Re}(\hat{\theta}) \hat{\times} \hat{\mathbf{x}}, \;\; \hat{\theta} = \theta \times \hat{I}_\theta \in \hat{R}_\theta, \quad (3.5.14)$$

that, for isotransforms in the (1, 2)-isoplane, are given by

$$x^{1'} = x^1 \times \cos[\theta \times (g_{11} \times g_{22})^{1/2}] - x^2 \times g_{22} \times g_{11}^{-1} \times \sin[\theta \times (g_{11} \times g_{22})^{1/2}], \quad (3.5.15a)$$

$$x^{2'} = x^1 \times g_{11} \times g_{22}^{-1} \times \sin[\theta \times (g_{11} \times g_{22})^{1/2}] + x^2 \times \cos[\theta \times (g_{11} \times g_{22})^{1/2}]. \quad (3.5.15b)$$

For the general expression in three dimensions interested reader can inspect Ref. [7] for brevity.

Note that, since $\hat{O}(3)$ is isomorphic to $O(3)$, Ref. [59] proved, contrary to a popular belief throughout the 20-th century, that

LEMMA 3.5.1: The rotational symmetry remains exact for all possible signature-preserving $(+,+,+)$ deformations of the sphere.

The rotational symmetry was believed to be "broken" for ellipsoidal and other deformations of the sphere merely due to insufficient mathematics for the case considered because, when the appropriate mathematics is used, the rotational symmetry returns to be exact, and the same holds for virtually all "broken" symmetries.

The above reconstruction of the exact rotational symmetry can be geometrically visualized by the fact that *all possible signature-preserving deformations of the sphere are perfect spheres in isospace called isosphere.*

This is due to the fact that ellipsoidal deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the *inverse* deformation of the related unit

$$Radius \quad 1_k \to 1/n_k^2, \tag{3.5.16a}$$

$$Unit \quad 1_k \to n_k^2. \tag{3.5.16b}$$

We recover in this way the perfect sphere on isospaces over isofields

$$\hat{r}^{\hat{2}} = \hat{r}_1^2 + \hat{r}_2^2 + \hat{r}_3^2 \tag{3.5.17}$$

with exact $\hat{O}(3)$ symmetry, while its projection on the conventional Euclidean space is the ellipsoid

$$r_1^2/n_1^2 + r_2^2/n_2^2 + r_3^2/n_3^2, \tag{3.5.18}$$

with broken $O(3)$ symmetry.

(2) The **Lorentz-Santilli isotransforms** [26,29]

$$\hat{O}(3.1) : \hat{x}' = \hat{\Lambda}(\hat{v}, \ldots)\hat{\times}\hat{x}, \ \hat{v} = v \times \hat{I}_v \in \hat{R}_v, \tag{3.5.19}$$

that, for isotransforms in the (3,4)-isoplane, can be written

$$x^{1'} = x^1, \tag{3.5.20a}$$

$$x^{2'} = x^2, \tag{3.5.20b}$$

$$x^{3'} = x^3 \times \cosh[v \times (g_{33} \times g_{44})^{1/2}] -$$

$$-x^4 \times g_{44} \times (g_{33} \times g_{44})^{-1/2} \times \sinh[v \times (g_{33} \times g_{44})^{1/2}] =$$

$$= \hat{\gamma} \times (x^3 - \hat{\beta} \times \frac{g_{44}^{1/2}}{g_{33}^{1/2}} \times x^4), \tag{3.5.20c}$$

$$x^{4'} = -x^3 \times g_{33} \times (g_{33} \times g_{44})^{-1/2} \times \sinh[v(g_{33} \times g_{44})^{1/2}] +$$
$$+ x^4 \times \cosh[v \times (g_{33} \times g_{44})^{1/2}] =$$
$$= \hat{\gamma} \times (x^4 - \hat{\beta} \times \frac{g_{33}^{1/2}}{g_{44}^{1/2}} \times x^3), \tag{3.5.20b}$$

where

$$\hat{\beta}^2 = \frac{v_k \times g_{kk} \times v_k}{c_o \times g_{44} \times c_o} \hat{\gamma} = \frac{1}{(1 - \hat{\beta}^2)^{1/2}}. \tag{3.5.21}$$

For the general expression interested readers can inspect Ref. [7].

Contrary to another popular belief throughout the 20-th century, Ref. [58] proved that

LEMMA 3.5.2: The Lorentz symmetry remains exact for all possible signature preserving $(+,+,+,-)$ *deformations of the Minkowski space.*

Again, the symmetry remains exact under the use of the appropriate mathematics.

The above reconstruction of the exact Lorentz symmetry can be geometrically visualized by noting that the light cone

$$x_2^2 + x_3^2 - c_o^2 \times t^2 = 0, \tag{3.5.22}$$

can only be formulated in vacuum, while within physical media we have the *light isocone*

$$\frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - \frac{c_o^2 \times t^2}{n^2(\omega, \ldots)} = 0, \tag{3.5.23}$$

that, when formulated on isospaces over isofield, is also a perfect cone, as it is the case for the isosphere. This property then explains how the Lorentz symmetry is reconstructed as exact according to Lemma 3.5.2 or, equivalently, that $\hat{O}(3.1)$ is isomorphic to $O(3.1)$.

(3) The **isotranslations** [29]

$$\hat{\mathcal{T}}(4) : \hat{x}' = \hat{\mathcal{T}}(\hat{a}, \ldots)\hat{]} \times x = \hat{x} + \hat{A}(\hat{a}, x, \ldots), \;\; \hat{a} = a \times \hat{I}_a \in \hat{R}_a, \tag{3.5.24}$$

that can be written

$$x^{\mu'} = x^\mu + A^\mu(a, \ldots), \tag{3.5.25a}$$

$$A^\mu = a^\mu(g_{\mu\mu} + a^\alpha \times [g_{i\mu}\hat{,}P_\alpha]/1! + \ldots), \tag{3.5.25b}$$

and there is no summation on the μ indices.

We reach in this way the following important result:

LEMMA 3.5.3 [55]: Isorelativity permits an axiomatically correct extension of relativity laws to noninertial frames.

In fact, noninertial frames are transformed into frames that are inertial on isospaces over isofields, called *isoinertial,* as established by the fact that isotranslations (3.5.25) are manifestly nonlinear and, therefore, noninertial on conventional spaces while they are isolinear on isospaces, according to a process similar to the reconstruction of locality, linearity and canonicity.

The isoinertial character of the frames can also be seen from the isocommutativity of the linear momenta, Eqs. (3.5.12c), while such a commutativity is generally lost in the projection of Eqs. (3.5.12c) on ordinary spaces over ordinary fields, thus confirming the lifting of conventional noninertial frames into an isoinertial form.

This property illustrates again the origin of the name "isorelativity" to indicate that conventional relativity axioms are solely applicable in isospacetime.

(4) The novel **isotopic transformations** [60]

$$\hat{\mathcal{I}}(1) : \hat{x}' = \hat{w}^{-\hat{1}} \hat{\times} \hat{x} = w^{-1} \times \hat{x}, \ \hat{I}' = w^{-2} \times \hat{I}, \tag{3.5.26}$$

where w is a constant,

$$\hat{I} \to \hat{I}' = \hat{w}^{-2} \hat{\times} \hat{I} = w^{-2} \times \hat{I} = 1/\hat{T}', \tag{3.5.27a}$$

$$\hat{x}^{\hat{2}} = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} \equiv \hat{x}'^{\hat{2}} =$$

$$= [x^\mu \times (w^2 \times \hat{\eta}_{\mu\nu}) \times x^\nu] \times (w^2 \times \hat{I}). \tag{3.5.27b}$$

Contrary to another popular belief throughout the 20-th century, we therefore have the following

THEOREM 3.5.2: The Poincaré-Santilli isosymmetry, hereon denoted with

$$\hat{P}(3.1) = \hat{O}(3.1) \hat{\times} \hat{\mathcal{T}}(4) \hat{\times} \hat{\mathcal{I}}(1), \tag{3.5.28}$$

and, therefore, the conventional Poincaré symmetry, are eleven dimensional.

The increase of dimensionality of the fundamental spacetime symmetry as, predictably, far reaching implications, including a basically novel and axiomatically consistent grand unification of electroweak and gravitational interactions studied in Chapter 5.

The simplest possible realization of the above formalism for isorelativistic kinematics can be outlined as follows. The first application of isorelativity is that of providing *an invariant description of locally varying speeds of light propagating within physical media.* For this purpose a realization of isorelativity requires the knowledge of the *density* of the medium in which motion occurs.

The simplest possible realization of the fourth component of the isometric is then given by the function

$$g_{44} = n_4^2(x, \omega, \ldots), \tag{3.5.29}$$

normalized to the value $n_4 = 1$ for the vacuum (note that the density of the medium in which motion occurs *cannot* be described by special relativity). The above representation then follows with invariance under $\hat{P}(3.1)$.

In this case the quantities $n_k,\ k = 1, 2, 3$, represent the *inhomogeneity and anisotropy of the medium considered.* For instance, if the medium is homogeneous and isotropic (such as water), all metric elements coincide, in which case

$$\hat{I} = Diag.(g_{11}, g_{22}, g_{33}, g_{44}) = n_4^2 \times Diag.(1, 1, 1, 1), \tag{3.5.30a}$$

$$\hat{x}^{\hat{2}} = \frac{x^2}{n_4^2} \times n_4^2 \times I \equiv x^2, \tag{3.5.30b}$$

thus confirming that *isotopies are hidden in the Minkowskian axioms,* and this may be a reason why they were not been discovered until recently.

Next, isorelativity has been constructed for the invariant description of systems of extended, nonspherical and deformable particles under Hamiltonian and non-Hamiltonian interactions.

Practical applications then require the knowledge of the actual shape of the particles considered, here assumed for simplicity as being spheroidal ellipsoids with semiaxes n_1^2, n_2^2, n_3^2.

Note that the minimum number of constituents of a closed non-Hamiltonian system is two. In this case we have shapes represented with $n_{\alpha k},\ \alpha = 1, 2, , \ldots, n$.

Specific applications finally require the identification of the nonlocal interactions, e.g., whether occurring on an extended *surface* or *volume.* As an illustration, two spinning particles denoted 1 and 2 in condition of deep mutual penetration and overlapping of their wavepackets (as it is the case for valence bonds), can be described by the following Hamiltonian and total isounit

$$H = \frac{p_1 \times p_1}{2 \times m_1} + \frac{p_2 \times p_2}{2 \times m_2} + V(r), \tag{3.5.31a}$$

$$\hat{I}_{Tot} = Diag.(n_{11}^2, n_{12}^2, n_{13}^2, n_{14}^2) \times Diag.(n_{21}^2, n_{22}^2, n_{23}^2, n_{24}^2) \times$$
$$\times e^{N \times (\hat{\psi}_1/\psi_1 + \hat{\psi}_2/\psi_2) \times \int \hat{\psi}_{1\uparrow}(r)^\dagger \times \hat{\psi}_{2\downarrow}(r) \times dr^3}, \tag{3.5.31b}$$

where N is a positive constant.

The above realization of the isounit has permitted the first known *invariant and numerically exact* representation of the binding energy and other features of the hydrogen, water and other molecules [71,72] (see Chapter 9) for which a historical 2% has been missing for about one century. The above isounit has also been instrumental for a number of additional data on two-body systems whose representation had been impossible with quantum mechanics, such as the origin of the spin 1 of the ground state of the deuteron that, according to quantum axioms, should be zero.

Note in isounit (3.5.31) the nonlinearity in the wave functions, the nonlocal-integral character and the impossibility of representing any of the above features with a Hamiltonian.

From the above examples interested readers can then represent any other closed non-Hamiltonian systems.

3.5.4 Isorelativity and Its Isodual

The third important part of the new isorelativity is given by the following isotopies of conventional relativistic axioms that, for the case of motion along the third axis, can be written [29] as follows [60]:

ISOAXIOM I. The projection in our spacetime of the maximal causal invariant isospeed is given by:

$$V_{Max} = c_\circ \times \frac{g_{44}^{1/2}}{g_{33}^{1/2}} = c_\circ \frac{n_3}{n_4} = c \times n_3. \tag{3.5.32}$$

This isoaxiom resolves the inconsistencies of special relativity recalled earlier for particles and electromagnetic waves propagating within physical media such as water.

In fact, water is homogeneous and isotropic, thus requiring that

$$g_{11} = g_{22} = g_{33} = g_{44} = 1/n^2, \tag{3.5.33}$$

where n is the index of refraction.

In this case the maximal causal speed for a massive particle is $c_\circ$ as experimentally established, e.g., for electrons, while the local speed of electromagnetic waves is $c = c_\circ/n$, as also experimentally established.

Note that such a resolution requires *the abandonment of the speed of light as the maximal causal speed for motion within physical media, and its replacement with the maximal causal speed of particles.*

It happens that in vacuum these two maximal causal speeds coincide. However, even in vacuum the correct maximal causal speed remains that of particles and *not* that of light, as generally believed.

At any rate, physical media are generally opaque to light but not to particles. Therefore, the assumption of the speed of light as the maximal causal speed within media in which light cannot propagate would be evidently vacuous.

It is an instructive exercise for interested readers to prove that

LEMMA 3.5.4: The maximal causal isospeed of particles on isominkowski space over an isofield remains $c_\circ$.

In fact, on isospaces over isofields $c_\circ^2$ is deformed by the index of refraction into the form $c_\circ^2/n_4^2$, but the corresponding unit cm^2/sec^2 is deformed by the inverse amount, $n_4^2 \times \text{cm}^2/\text{sec}^2$, thus preserving the numerical value $c_\circ^2$ due to the structure of the isoinvariant studied earlier.

The understanding of isorelativity requires the knowledge that, when formulated on the Minkowski-Santilli isospace over the isoreals, Isoaxiom I coincides with the conventional axiom that is, the maximal causal speed returns to be c. The same happens for all remaining isoaxioms.

ISOAXIOM II. The projection in our spacetime of the isorelativistic addition of isospeeds within physical media is given by:

$$v_{Tot} = \frac{v_1 + v_2}{1 + \dfrac{v_1 \times g_{33} \times v_2}{c_\circ \times g_{44} \times c_\circ}} = \frac{v_1 + v_2}{1 + \dfrac{v_1 \times n_4^2 \times v_2}{c_\circ \times n_3^2 \times c_\circ}}. \tag{3.5.34}$$

We have again the correct result that *the sum of two maximal causal speeds in water,*

$$V_{max} = c_\circ \times (n_3/n_4), \tag{3.5.35}$$

yields the maximal causal speed in water, as the reader is encouraged to verify.

Note that such a result is impossible for special relativity. Note also that *the "relativistic" sum of two speeds of lights in water,* $c = c_\circ/n$, *does not* yield the speed of light in water, thus confirming that the speed of light within physical media, assuming that they are transparent to light, is not the fundamental maximal causal speed.

ISOAXIOM III. The projection in our spacetime of the isorelativistic laws of dilation of time $t_\circ$ *and contraction of length* $\ell_\circ$ *and the variation of mass* $m_\circ$ *with speed are given respectively by:*

$$t = \hat{\gamma} \times t_\circ, \tag{3.5.36a}$$

$$\ell = \hat{\gamma}^{-1} \times \ell_\circ, \tag{3.5.36b}$$

$$m = \hat{\gamma} \times m_\circ. \tag{3.5.36c}$$

$$\hat{\beta} = \frac{v_k \times g_{kk}}{c_o \times g_{44}} = \frac{v_k}{V_{Max}}, \;\; \hat{\gamma} = \frac{1}{(1 - \hat{\beta}^2)^{1/2}}, \tag{3.5.d}$$

where one should npote that, since the speed is always smaller than the maximal possible speed, $\hat{\gamma}$ cannot assume imaginary values.

Note that in water these values coincide with the relativistic ones as it should be since particles such as the electrons have in water the maximal causal speed $c_\circ$.

Note again the necessity of avoiding the interpretation of the local speed of light as the maximal local causal speed. Note also that the mass diverges at the maximal local causal speed, but *not* at the local speed of light.

ISOAXIOM IV. The projection in our spacetime of the iso-Doppler law is given by the isolaw (here formulated for simplicity for 90° *angle of aberration):*

$$\omega = \hat{\gamma} \times \omega_\circ. \tag{3.5.37}$$

This isorelativistic axioms permits an *exact, numerical and invariant representation* of the large differences in cosmological redshifts between quasars and galaxies when physically connected.

In this case light simply exits the huge quasar chromospheres already redshifted due to the decrease of the speed of light, while the speed of the quasars can remain the *same* as that of the associated galaxy. Note again as this result is impossible for special relativity.

Isoaxiom IV also permits a numerical interpretation of the internal blue- and redshift of quasars due to the dependence of the local speed of light on its frequency.

Finally, Isoaxiom IV predicts that *a component* of the predominance toward the red of sunlight at sunset is of iso-Doppler nature. This prediction is based on the different travel within atmosphere of light at sunset as compared to the zenith (evidently because of the travel within a comparatively denser atmosphere).

By contrast, the popular representation of the apparent redshift of sunlight at sunset is that via the scattering of light among the molecules composing our atmosphere. Had this interpretation be correct, the sky at the zenith should be red, while it is blue.

At any rate, the claim of representation of the apparent redshift via the scattering of light is political because of the impossibility of reaching the needed numerical value of the redshift, as serious scholars are suggested to verify.

ISOAXIOM V. The projection in our spacetime of the isorelativistic law of equivalence of mass and energy is given by:

$$E = m \times V_{Max}^2 = m \times c_\circ^2 \times \frac{g_{44}}{g_{33}} = m \times c_\circ^2 \times \frac{n_3^2}{n_4^2} = c \times n_3 \tag{3.5.38}$$

Note a crucial axiomatic difference between the conventional axiom $E = m \times c_c ir c^2$ and isoaxiom V. They coincide in vacuum, water and other media transparent to light, but are otherwise structurally different. We should note that, in early references, the conventional axiom $E = m \times c_c ir c^2$, where $c_\circ$ is the speed of light in vacuum, was lifted into the form $E = m \times c^2$ where c is the local speed of light within physical media. However, the latter form lead to inconsistencies in applications studied in Volume II (e.g., when the medium considered is opaque to light in which case both $c_\circ$ and c are meaningless) and had to be further lifted into Isoaxiom V.

Among various applications, *Isoaxiom V removes any need for the "missing mass" in the universe.* This is due to the fact that all isotopic fits of experimental data agree on values $g_{44} \gg 1$ within the hyperdense media in the interior of hadrons, nuclei and stars [7].

As a result, Isoaxiom V yields a value of the total energy of the universe dramatically bigger than that believed until now under the assumption of the universal validity of the speed of light in vacuum.

For other intriguing applications of Isoaxioms V, e.g., for the rest energy of hadronic constituents, we refer the interested reader to monographs [55,61].

The *isodual isorelativity* for the characterization of antimatter can be easily constructed via the isodual map of Chapter 2, and its explicit study is left to the interested reader for- brevity.

3.5.5 Isorelativistic Hadronic Mechanics and its Isoduals

The isorelativistic extension of relativistic hadronic mechanics is readily permitted by the Poincaré-Santilli isosymmetry. In fact, iso-invariant (3.5.13a) characterizes the following *iso-Gordon equation* on $\hat{\mathcal{H}}$ over $\hat{C}$ [55]

$$\hat{p}_\mu \hat{\times} |\hat{\psi}\rangle = -\hat{i}\hat{\times}\hat{\partial}_\mu |\hat{\psi}\rangle = -i \times \hat{I}^\nu_\mu \times \partial_\nu |\hat{\psi}\rangle, \tag{3.5.39a}$$

$$(\hat{p}_\mu \hat{\times} \hat{p}^\mu + \hat{m}_\circ^2 \hat{\times} \hat{c}^4) \hat{\times} |\hat{\psi}\rangle = (\hat{\eta}^{\alpha\beta} \times \partial_\alpha \times \partial_\beta + m_\circ^2 \times c^4) \times |\hat{\psi}\rangle = 0. \tag{3.5.39b}$$

The linearization of the above second-order equations into the *Dirac-Santilli isoequation* has been first studied in Refs. [60–62] and then by other authors (although generally without the use of isomathematics, thus losing the invariance).

By recalling the structure of Dirac's equation as the Kronecker product of a spin 1/2 massive particle and its antiparticle of Chapter 2, the Dirac-Santilli isoequation is formulated on the total isoselfadjoint isospace and related isosymmetry

$$\hat{M}^{tot} = [\hat{M}^{orb}(\hat{x}, \hat{\eta}, \hat{R}) \times \hat{S}^{spin}(2)] \times$$

$$\times [\hat{M}^{d\,orb}(\hat{x}^d, \hat{\eta}^d, \hat{R}^d) \times \hat{S}^{d\,spin}(2)] = \hat{M}^{d\,tot}, \tag{3.5.40a}$$

$$\hat{S}^{tot} = \hat{P}(3.1) \times \hat{P}^d(3.1) = \hat{S}^{d\,tot}, \tag{3.5.40b}$$

and can be written [29]

$$[\hat{\gamma}^\mu \hat{\times} (\hat{p}_\mu - \hat{e} \hat{\times} \hat{A}_\mu) + \hat{i} \hat{\times} \hat{m}] \hat{\times} |\phi(x)\rangle = 0, \tag{3.5.41a}$$

$$\hat{\gamma}^\mu = g^{\mu\mu} \times \gamma^\mu \times \hat{I}, \tag{3.5.41b}$$

where the γ's are the conventional Dirac matrices.

Note the appearance of the isometric elements directly in the structure of the isogamma matrices and their presence also when the equation is projected in the conventional spacetime.

The following generators

$$J_{\mu\nu} = (S_k,\, L_{k4}), P_\mu, \tag{3.5.42a}$$

$$S_k = (\hat{\epsilon}_{kij} \hat{\times} \hat{\gamma}_i \hat{\times} \hat{\gamma}_j)/2, \;\; L_{k4} = \hat{\gamma}_k \hat{\times} \hat{\gamma}_4/2, \;\; P_\mu = \hat{p}_\mu, \tag{3.5.42b}$$

characterize the *isospinorial covering of the Poincaré-Santilli isosymmetry.*

The notion of "isoparticle" can be best illustrated with the above realization because it implies that, *in the transition from motion in vacuum (as particles have been solely detected and studied until now) to motion within physical media, particles generally experience the alteration, called "mutation", of all intrinsic characteristics,* as illustrated by the following isoeigenvalues,

$$\hat{S}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle = \frac{g_{11} \times g_{22} + g_{22} \times g_{33} + g_{33} \times g_{11}}{4} \times |\hat{\psi}\rangle, \tag{3.5.43a}$$

$$\hat{S}_3 \hat{\times} |\hat{\psi}\rangle = \frac{(g_{11} \times g_{22})^{1/2}}{2} \times |\hat{\psi}\rangle. \tag{3.5.43b}$$

The mutation of spin then characterizes a necessary mutation of the intrinsic magnetic moment given by [29]

$$\tilde{\mu} = \Big(\frac{g_{33}}{g_{44}}\Big)^{1/2} \times \mu, \tag{3.5.44}$$

where μ is the conventional magnetic moment for the same particle when in vacuum. The mutation of the rest energy and of the remaining characteristics has been identified before via the isoaxioms.

Note that the invariance under isorotations allows the rescaling of the radius of an isosphere. Therefore, for the case of the perfect sphere we can always have $g_{11} = g_{22} = g_{33} = g_{44}$ in which case the magnetic moment is not mutated. These results recover conventional classical knowledge according to which *the alteration of the shape of a charged and spinning body implies the necessary alteration of its magnetic moment.*

The construction of the isodual isorelativistic hadronic mechanics is left to the interested reader by keeping in mind that the iso-Dirac equation is isoselfdual as the conventional equation.

To properly understand the above results, one should keep in mind that *the mutation of the intrinsic characteristics of particles is solely referred to the constituents of a hadronic bound state under conditions of mutual penetration of their wave packets (such as one hadronic constituent) under the condition of recovering conventional characteristics for the hadronic bound state as a whole (the hadron considered),* much along Newtonian subsidiary constrains on non-Hamiltonian forces, Eqs. (3.1.6).

It should be also stressed that *the above indicated mutations violate the unitary condition when formulated on conventional Hilbert spaces, with consequential catastrophic inconsistencies, Theorem 1.5.2.*

As an illustration, the violation of causality and probability law has been established for all eigenvalues of the angular momentum M different than the quantum spectrum

$$M^2 \times |\psi\rangle = \ell(\ell+1) \times |\psi\rangle, \quad \ell = 0, 1, 2, 3, \dots \ . \tag{3.5.45}$$

As a matter of fact, these inconsistencies are the very reason why the mutations of internal characteristics of particles for bound states at short distances could not be admitted within the framework of quantum mechanics.

By comparison, hadronic mechanics has been constructed to recover unitarity on iso-Hilbert spaces over isofields, thus permitting an invariant description of internal mutations of the characteristics of the constituents of hadronic bound states, while recovering conventional features for states as a whole.

Far from being mere mathematical curiosities, the above mutations permit basically new structure models of hadrons, nuclei and stars, with consequential, new clean energies and fuels (see Chapters 11, 12).

These new advances are prohibited by quantum mechanics precisely because of the preservation of the *intrinsic* characteristics of the constituents in the transition from bound states at large mutual distance, for which no mutation is possible, to the bound state of the same constituents in condition of mutual penetration, in which case mutations have to be admitted in order to avoid the replacement of a scientific process with unsubstantiated personal beliefs one way or the other (see Chapter 12 for details).

3.5.6 Isogravitation and its Isodual

As indicated in Section 1.4, there is no doubt that the classical and operator formulations of gravitation on a curved space have been the most controversial theory of the 20-th century because of an ever increasing plethora of problematic aspects remained vastly ignored. By contrast, as also reviewed in Section 1.4, special relativity in vacuum has a majestic axiomatic consistence in its *invariance* under the Poincaré symmetry.

Recent studies have shown that the formulation of gravitation on a curved space or, equivalently, the formulation of gravitation based on "covariance", is necessarily noncanonical at the classical level and nonunitary at the operator level, thus suffering of all catastrophic inconsistencies of Theorems 1.4.1 and 1.4.2.

These catastrophic inconsistencies can only be resolved via a new conception of gravity based on a *universal invariance,* rather than covariance.

Additional studies have identified profound axiomatic incompatibilities between gravitation on a curved space and electroweak interactions. These incompatibilities have resulted to be responsible for the lack of achievement of an axiomatically consistent grand unification since Einstein's times (see Chapter 14).

No knowledge of isotopies can be claimed without a knowledge that isorelativity has been constructed to resolve at least some of the controversies on gravitation. The fundamental requirement is *the abandonment of the formulation of gravity via curvature on a Riemannian space and its formulation instead on an iso-Minkowskian space* via the following steps characterizing *exterior isogravitation in vacuum,* first presented in Refs. [73,74]:

I) Factorization of any given Riemannian metric representing exterior gravitation $g^{ext}(x)$ into a nowhere singular and positive-definite 4×4-matrix $\hat{T}(x)$ times the Minkowski metric η,

$$g^{ext}(x) = \hat{T}^{ext}_{grav}(x) \times \eta; \tag{3.5.47}$$

II) Assumption of the inverse of $\hat{T}_{grav}$ as the fundamental unit of the theory,

$$\hat{I}^{ext}_{grav}(x) = 1/\hat{T}^{ext}_{grav}(x); \tag{3.5.48}$$

III) Submission of the totality of the Minkowski space and relative symmetries to the noncanonical/nonunitary transform

$$U(x) \times I^{\dagger}(x) = \hat{I}^{ext}_{grav}. \tag{3.5.49}$$

The above procedure yields the isominkowskian spaces and related geometry $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$, resulting in a new conception of gravitation, exterior isogravity, with the following main features [26]:

i) Isogravity is characterized by a universal *symmetry* (and not a covariance), the Poincaré-Santilli isosymmetry $\hat{P}(3.1)$ for the gravity of matter with isounit $\hat{I}^{ext}_{grav}(x)$, the isodual isosymmetry $\hat{P}^d(3.1)$ for the gravity of antimatter, and the isoselfdual symmetry $\hat{P}(3.1) \times \hat{P}^d(3.1)$ for the gravity of matter-antimatter systems;

ii) All conventional field equations, such as the Einstein-Hilbert and other field equations, can be formulated via the Minkowski-Santilli isogeometry since the latter preserves all the tools of the conventional Riemannian geometry, such as the Christoffel's symbols, covariant derivative, etc. [15];

iii) Isogravitation is isocanonical at the classical level and isounitarity at the operator level, thus resolving the catastrophic inconsistencies of Theorems 1.5.1 and 1.5.2;

iv) An axiomatically consistent operator version of gravity always existed and merely crept in unnoticed through the 20-th century because gravity is imbedded where nobody looked for, in the *unit* of relativistic quantum mechanics, and it is given by isorelativistic hadronic mechanics outlined in the next section.

v) The basic feature permitting the above advances is the abandonment of curvature for the characterization of gravity (namely, curvature characterized by metric $g^{ext}(x)$ referred to the unit I) and its replacement with *isoflatness,* namely, the verification of the axioms of flatness in isospacetime, while preserving conventional curvature in its projection on conventional spacetime (or, equivalently, curvature characterized by the $g(x) = \hat{T}^{ext}_{grav}(x) \times \eta$ referred to the isounit $\hat{I}_{grav}(x)$ in which case curvature becomes null due to the inter-relation $\hat{I}^{ext}_{grav}(x) = 1/\hat{T}^{ext}_{grav}(x)$) [26].

A resolution of numerous controversies on classical formulations of gravity then follows from the above main features, such as:

a) The resolution of the century old controversy on the lack of existence of consistent total conservation laws for gravitation on a Riemannian space, which controversy is resolved under the universal $\hat{P}(3.1)$ symmetry by mere visual verification that the generators of the conventional and isotopic Poincaré symmetry are the same (since they represent conserved quantities in the absence and in the presence of gravity);

b) The controversy on the fact that gravity on a Riemannian space admits a well defined "Euclidean", but not "Minkowskian" limit, which controversy is trivially resolved by isogravity via the limit

$$\hat{I}^{ext}_{grav}(x) \to I; \tag{3.5.50}$$

c) The resolution of the controversy that Einstein's gravitation predicts a value of the bending of light that is twice the experimental value, one for curvature and one for newtonian attraction, which controversy is evidently resolved by the elimination of curvature as the origin of the bending, as necessary in any case for the free fall of a body along a straight radial line in which no curvature of any type is conceivably possible or credible; and other controversies.

A resolution of the controversies on quantum gravity can be seen from the property that relativistic hadronic mechanics of the preceding section *is* a quantum formulation of gravity whenever $\hat{T} = \hat{T}_{grav}$.

Such a form of operator gravity is as axiomatically consistent as conventional relativistic quantum mechanics because the two formulations coincide, by construction, at the abstract, realization-free level.

As an illustration, whenever

$$\hat{T}^{ext}_{grav} = Diag.(g^{ext}_{11}, g^{ext}_{22}, g^{ext}_{33}, g^{ext}_{44}), \quad g_{\mu\mu} > 0, \tag{3.5.51}$$

the Dirac-Santilli isoequation (3.5.41) provides a direct representation of the conventional electromagnetic interactions experienced by an electron, represented by the vector potential A_μ, plus gravitational interactions represented by the isogamma matrices.

Once curvature is abandoned in favor of the broader isoflatness, the axiomatic incompatibilities existing between gravity and electroweak interactions are resolved because:

i) isogravity possesses, at the abstract level, the *same* Poincaré invariance of electroweak interactions;

ii) isogravity can be formulated on the *same* flat isospace of electroweak theories; and

iii) isogravity admits positive and negative energies in the *same* way as it occurs for electroweak theories.

An axiomatically consistent *iso-grand-unification* then follows, as studied in Chapter 14.

Note that the above grand-unification requires the prior *geometric unification of the special and general relativities,* that is achieved precisely by isorelativity and its underlying iso-Minkowskian geometry.

In fact, special and general relativities are merely differentiated in isospecial relativity by the explicit realization of the unit. In particular, *black holes are now characterized by the zeros of the isounit* [7]

$$\hat{I}^{ext}_{grav}(x) = 0. \tag{3.5.52}$$

The above formulation recovers all conventional results on gravitational singularities, such as the singularities of the Schwarzschild's metric, since they are all described by the gravitational content $\hat{T}_{grav}(x)$ of $g(x) = \hat{T}_{grav}(x) \times \eta$, since η is flat.

This illustrates again that *all conventional results of gravitation, including experimental verifications, can be reformulated in invariant form via isorelativity.*

Moreover, the problematic aspects of general relativity mentioned earlier refer to the *exterior gravitational problem.* Perhaps greater problematic aspects exist in gravitation on a Riemannian space for *interior gravitational problems,* e.g., because of the lack of characterization of basic features, such as the density of the interior problem, the locally varying speed of light, etc.

These additional problematic aspects are also resolved by isorelativity due to the unrestricted character of the functional dependence of the isometric that, therefore, permits a direct geometrization of the density, local variation of the speed of light, etc.

The above lines constitute only the initial aspects of isogravitation since its most important branch is *interior isogravitation* as characterized by isounit and isotopic elements of the illustrative type

$$\hat{I}^{int}_{grav} = 1/\hat{T}^{int}_{grav} > 0, \tag{3.5.53a}$$

$$\hat{T}^{int}_{grav} = Diag.(g^{int}_{11}/n_1^2, g^{int}_{22}/n_2^2, g^{int}_{33}/n_3^2, g^{int}_{44}/n_4^2), \tag{3.5.53b}$$

permitting a *geometric representation directly via the isometric of the actual shape of the body considered, in the above case an ellipsoid with semiaxes* n_1^2, n_2^2, n_3^2, *as well as the (average) interior density* n_4^2 *with consequential representation of the (average value of the) interior speed of light* $C = c/n_4$.

A most important point is that the invariance of interior isogravitation under the Poincaré-Santilli isosymmetry persists in its totality since the latter symmetry is completely independent from the explicit value of the isounit or isotopic element, and solely depends on their positive-definite character.

Needless to say, isounit (3.4.53) is merely illustrative because a more accurate interior isounit has a much more complex functional dependence with a locally varying density, light speed and other characteristics as they occur in reality.

Explicit forms of these more adequate models depends on the astrophysical body considered, e.g., whether gaseous, solid or a mixture of both, and their study is left to the interested reader.

It should also be noted that *gravitational singularities should be solely referred to interior models* evidently because exterior descriptions of type (3.5.52) are a mere approximation or a geometric abstraction.

In fact, *gravitational singularities existing for exterior models are not necessarily confirmed by the corresponding interior formulations.* Consequently, the current views on black holes could well result to be pseudo-scientific beliefs because the only scientific statement that can be proffered at this time without raising issue of scientific ethics is that *the gravitational features of large and hyperdense aggregations of matter, whether characterizing a "black" or "brown" hole, are basically unresolved at this time.*

Needless to say, exterior isogravitation is a particular case of the interior formulation. Consequently, from now on, unless otherwise specified isogravitation will be referred to the interior form.

The cosmological implications are also intriguing and will be studied in Chapter 6. It should be indicated that numerous formulations of gravitation in flat *Minkowski* space exist in the literature, such as Ref. [79] and papers quoted therein. However, these formulations have no connection with isogravity since the background space of the former is conventional, while that of the latter is a geometric unification of the Minkowskian and Riemannian spaces.

It is hoped that readers with young minds of any age admit the incontrovertible character of the limitations of special and general relativities and participate in

the laborious efforts toward new vistas because any lack of participation in new frontiers of science, whether for personal academic interest or other reason, is a gift of scientific priorities to others.

Appendix 3.A
Universal Enveloping Isoassociative Algebras

The main structural component of Lie's theory is its *universal enveloping associative algebra* $\xi(L)$ of a Lie algebra L. In fact, Lie algebras can be obtained as the attached antisymmetric part $[\xi(L)]^- \approx L$; the infinite dimensional basis of $\xi(l)$ permit the exponentiation to a finite transformation group G; and the representation theory is crucially dependent on the right and/or left modular associative action originally defined on G.

In Section 3.2.9B we have reviewed the rudiments of the *universal enveloping isoassociative algebras* $\hat{\xi}(L)$ of a Lie-Santilli isoalgebra $\hat{L}$. It is easy to see that all features occurring for $\xi(L)$ carry over to the covering isoform $\hat{\xi}(L)$.[30]

In this appendix we would like to outline a more technical definition of universal enveloping isoassociative algebras since they are at the foundations of the unification of simple Lie algebras of dimension N into a single Lie-Santilli isoalgebra of the same dimension (Section 3.2.13).

With reference to Figure **??**, the envelop $\xi(L)$ can be defined as the (ξ, τ) where ξ is an associative algebra and τ is a homomorphism of L into the antisymmetric algebra ξ^- attached to ξ such that: if ξ' is another associative algebra and τ' is another homomorphism of L into ξ'^-, a unique isomorphism γ exists between ξ and ξ' in such a way that the diagram in the l.h.s of Figure **??** is commutative. The above definition evidently expresses the uniqueness of the Lie algebra L up to local isomorphism, and illustrates the origin of the name "universal" enveloping algebra of L.

With reference to the r.h.s. diagram of Figure **??**, the universal enveloping isoassociative algebra $\hat{\xi}(L)$ of a Lie algebra L was introduced in Ref. [4] as the set $\{(\xi, \tau), i, \hat{\xi}, \hat{\tau}\}$ where: (ξ, τ) is a conventional envelope of L; i is an isotopic mapping $L \to i(L) = \hat{L} \sim L$; $\hat{\xi}$ is an associative algebra generally nonisomorphic to ξ; $\hat{\tau}$ is a homomorphism of $\hat{L}$ into $\hat{\xi}^-$; such that: if $\hat{\xi}'$ is another associative algebra and $\hat{\tau}'$ is another homomorphism of $\hat{L}$ into $\hat{\xi}'^-$, there exists a unique

[30] We use the denomination $\hat{\xi}(L)$ rather than $\hat{\xi}(\hat{L})$ to stress the fact that the generators of ξ are those of L and not of $\hat{L}$, a requirement that is essential for consistent physical applications because the generators of L represent ordinary physical quantities (such as total energy, total linear momentum, etc.) that, as such, cannot be changed by isotopies.

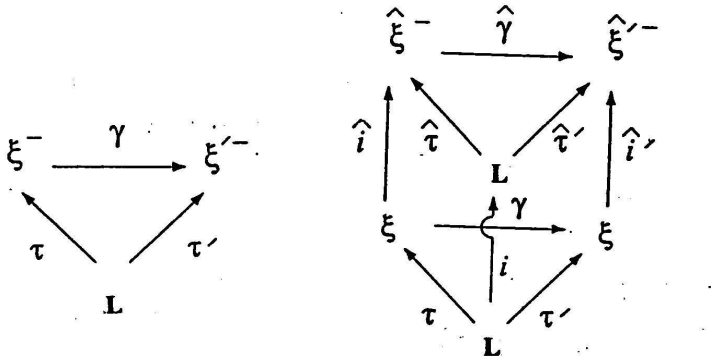

Figure 3.A.1. A schematic view of the universal enveloping associative algebra of a Lie algebra L and its lifting for the Lie-Santilli isoalgebra $\hat{L}$ according to the original proposal [4] of 1978.

isomorphism $\hat{\gamma}$ of $\hat{\xi}$ into $\hat{\xi}'$ with $\hat{\tau}' = \gamma(\hat{\tau})$ and two unique isotopies $i(\xi) = \hat{\xi}$ and $i(\hat{\xi}) = \hat{\xi}'$.

A primary objective of the above definition of isoenvelope is the *lack of uniqueness of the Lie algebra characterized by the isoenvelope* or, equivalently, the *characterization of a family of generally nonisomorphic Lie algebras via the use of only one basis*. The above definition of isoenvelope also explains in more details the variety of realization of the simple 3-dimensional Lie-Santilli isoalgebra $\hat{L}_3$ provided in Eq. (3.2.236), and may be of assistance in extending the same classification to other isoalgebras.

The above notion of isoenvelope represents the essential mathematical structure of hadronic mechanics, namely, the preservation of the conventional basis, i.e., the set of observables of quantum mechanics, and the generalization of the operations on them via an infinite number of isotopies so as to admit a new class of interactions structurally beyond the possibilities of quantum mechanics.

Appendix 3.B
Recent Advances in the TSSFN Isotopology

In Section 3.2.7 we introduced the elements of the *Tsagas-Sourlas-Santilli-Falcón-Núñez isotopology* (or TSSFN Isotopology for short). In this appendix we outline recent advances on the isotopology by the Spanish mathematicians R. M. Falcón Ganfornina and J. Núñez Valdés [24,25].

PROPOSITION 3.2.B1: Consider a mathematical structure

$$(E, +, \times, \circ, \bullet, \ldots),$$

if we construct an isotopic lifting such that:

a) Both primaries $*$, $\widehat{I}$ *and secondaries* $\star$, $\widehat{S}$ *isotopic elements are used.*

b) $(E, \star, *, \ldots)$ *is a structure of the same type as the initial, which is endowed with isounits* S, I, $\ldots$, *with respect to* $\star$, $*$, $\ldots$, *respectively.*

c) I *is an unit with respect to* $*$ *in the corresponding general set* V, *being* $T = \widehat{I}^{-I} \in V$ *the associated isotopic element.*

Then, by defining in the isotopic level the operations:

$$\widehat{a}\widehat{+}\widehat{b} = \widehat{a \star b}; \quad \widehat{a}\widehat{\times}\widehat{b} = \widehat{a * b}; \quad \ldots \tag{3.B.1}$$

And being defined in the projection level:

$$\overline{\widehat{a}} = a * \widehat{I}; \quad \alpha\overline{\widehat{+}}\beta = ((\alpha * T) \star (\beta * T)) * \widehat{I}; \quad \alpha\overline{\widehat{\times}}\beta = \alpha * T * \beta; \ \ldots \tag{3.B.2}$$

It is obtained that the isostructure $(\overline{\widehat{E}}, \overline{\widehat{+}}, \overline{\widehat{\times}}, \ldots)$ *is of the same type as the initial one.*

The study in Refs. [24,25] is made by taking into consideration both isotopic and projection levels. Equivalent results related to injective isotopies are also obtained. In the first place, Proposition 3.2.A1 is verified for topological spaces and for their elements and basic properties: isotopologies, isoclosed sets, isoopen sets, T_2, etc:

A *topological isospace* is every isospace endowed with a topological space structure. If, besides, such an isospace is an isotopic projection of a topological space, it is called *isotopological isospace*.

Similarly, they are defined concepts of *(iso)boundary isopoint, closure of a set, closed set, isointerior isopoint, interior of a set, open set, (iso)Hausdorff isospace and second countable isospace, among others.*

PROPOSITION 3.2.B2: The space from which any topological isospace in the isotopic level is obtained can be endowed with the final topology relative to the mapping $\mathbf{I}$.

The isotopic projection of a topological space is an isotopological isospace in the projection level. If such a projection is injective, then every topological isospace in such a level is, in fact, isotopological.

Similar results are obtained for the concepts of (iso)boundary isopoint, isointerior isopoint and (iso)Hausdorff isospace.

Next, Refs. [24,25] generalize Kadeisvili's isocontinuity [19]. Particularly, the basic isofield can be endowed with an isoorder, according to the following procedure.

Let $\widehat{K}$ be an isofield associated with a field K, endowed with an order $\leq$, by using an isotopology which preserves the inverse element with respect to the addition. We define the *isoorder* $\widehat{\leq}$ as $\widehat{a}\widehat{\leq}\widehat{b}$ if and only if $a \leq b$. If the isotopy is injective, the isoorder $\overline{\widehat{\leq}}$ en $\overline{\widehat{K}}$ is defined in the same way.

PROPOSITION 3.2.B3: The isoorders $\widehat{\leq}$ *and* $\overline{\widehat{\leq}}$ *are orders over* $\widehat{K}$ *and* $\overline{\widehat{K}}$*, of the same type as* $\leq$.

Let $\widehat{U}$ be a $\widehat{R}$ isovectorspace with isonorm $\widehat{||}.\widehat{||} \equiv \widehat{||.||}$ and isoorder $\widehat{\leq}$, obtained from an isotopy compatible with respect to each one of the initial operations. It will be said that an isoreal isofunction $\widehat{f}$ of $\widehat{U}$ is *isocontinuous in* $\widehat{X} \in \widehat{U}$, if for all $\widehat{\epsilon}\widehat{>}\widehat{S}$, there exists $\widehat{\delta}\widehat{>}\widehat{S}$ such that for all $\widehat{Y} \in \widehat{U}$ with $\widehat{||}\widehat{X}-\widehat{Y}\widehat{||}\widehat{<}\widehat{\delta}$, it is verified that $\widehat{|}\widehat{f}(\widehat{X})-\widehat{f}(\widehat{Y})\widehat{|}\widehat{<}\widehat{\epsilon}$. We will say that $\widehat{f}$ is *isocontinuous in* $\widehat{U}$ if it is isocontinuous in $\widehat{X}$, for all $\widehat{X} \in \widehat{U}$. Finally, when dealing with injective isotopies, the isocontinuity in the projection level is defined in a similar way.

PROPOSITION 3.2.B4: The isocontinuity in $\widehat{U}$ *is equivalent to the continuity in* U*. In the case of injective isotopies, both ones are equivalent to the one in* $\overline{\widehat{U}}$.

The isocontinuity on isotopological isospaces is also analyzed:

An *isocontinuous isomapping* in the isotopic level between two topological isospaces $\widehat{M}$ and $\widehat{N}$ is every isomapping $\widehat{f} : \widehat{M} \to \widehat{N}$ preserving closures. The definition in the projection level is given in a similar way.

PROPOSITION 3.2.B5: They are verified that:

a) $\widehat{f}$ is isocontinuous if and only if the mapping f from which comes from is continuous. That result is similar in the projection level by using injective isotopies.

b) Every isoconstant isomapping is isocontinuous.

c) Isocontinuity is preserved by both topological composition and product.

Finally, the analysis of (iso)(pseudo)metric isospaces is also concreted:

PROPOSITION 3.2.B6: Let $\widehat{M}$ be a $\widehat{K}$ isovectorspace, isotopic lifting of a vectorspace M, endowed with a (pseudo)metric d defined on an ordered field K, by using an isotopy which preserves the inverse element and compatible with respect to the addition in K. Then, the isofunction $\widehat{d}$ is an iso(pseudo)metric.

Let $(\widehat{M}, d')$ be an (iso)(pseudo)metric $\widehat{K}$ isovectorspace, endowed with an isoorder $\widehat{\leq}$. $B_{d'}(\widehat{X_0}, \widehat{\epsilon}) = \{\widehat{X} \in \widehat{M} : d'(\widehat{X}, \widehat{X_0}) \widehat{<} \widehat{\epsilon}\}$ is called *metric ball* with center $\widehat{X_0} \in \widehat{M}$ and radius $\widehat{\epsilon} \widehat{>} \widehat{S}$. If M is endowed with a (pseudo)metric d, with $\widehat{d} = d'$, then every metric ball $B_{d'} = B_{\widehat{d}} = \widehat{B_d}$ in $\widehat{M}$, which is isotopic lifting of a metric ball B_d in M, is called *metric isoball* in $\widehat{M}$.

PROPOSITION 3.2.B7: Under conditions of Proposition XXX, if $B_d(X_0, \epsilon)$ is a metric ball in M, then $\widehat{B_d(X_0, \epsilon)} = B_{\widehat{d}}(\widehat{X_0}, \widehat{\epsilon})$ is a metric ball in $\widehat{M}$.

A *metric neighborhood of an isopoint* $\widehat{X} \in \widehat{M}$ is a subset $\widehat{A} \subseteq \widehat{M}$ containing a metric ball centered in $\widehat{X}$. The set of metric neighborhoods of $\widehat{X}$ is denoted by $\widehat{\aleph}^{d'}_{\widehat{X}}$. Finally, if d' is the iso-Euclidean isodistance over $\widehat{R}^n$, the associated metric neighborhoods are called *iso-Euclidean neighborhoods.*

PROPOSITION 3.2.B8: Let d' and d'' two (iso)(pseudo)metrics over an isovectorspace $\widehat{M}$. It is verified that $\widehat{\aleph}^{d'}_{\widehat{X}} = \widehat{\aleph}^{d''}_{\widehat{X}}$ if and only if every metric ball $B_{d'}(\widehat{X}, \widehat{\epsilon})$ contains a ball $B_{d''}(\widehat{X}, \widehat{\rho})$ and every ball $B_{d''}(\widehat{X}, \widehat{\delta})$ contains a ball $B_{d'}(\widehat{X}, \widehat{\mu})$.

PROPOSITION 3.2.B9: Every isospace endowed with an (iso)(pseudo)metric is an isotopological isospace.

PROPOSITION 3.2.B10: Let $\widehat{f} : (\widehat{M}, d') \to (\widehat{N}, d'')$ *be an isomapping between* $\widehat{K}$*-isospaces endowed with (iso)(pseudo)metric and let us consider* $\widehat{X} \in \widehat{M}$*. Then,* $\widehat{f}$ *is isocontinuous in* $\widehat{X}$ *if and only if for all* $\widehat{\epsilon} \hat{>} \widehat{S}$ *there exists* $\widehat{\delta} \in \widehat{K}$ *such that* $\widehat{\delta} \hat{>} \widehat{S}$*, and if* $\widehat{Y} \in B_{d'}(\widehat{X}, \widehat{\delta})$*, then it is verified that* $\widehat{f}(\widehat{Y}) \in B_{d''}(\widehat{f}(\widehat{X}), \widehat{\epsilon})$*.*

PROPOSITION 3.2.B11: Let $\widehat{f} : \widehat{M} \to \widehat{N}$ *be an isomapping between two isotopological isospaces* $\widehat{M}$ *and* $\widehat{N}$*. If conditions of the definition of isocontinuity are satisfied, then* $\widehat{f}$ *is isocontinuous if and only if* $\widehat{f}^{-1}(\widehat{U})$ *is an isoopen of* $\widehat{M}$*, for all isoopen* $\widehat{U}$ *of* $\widehat{N}$*.*

Appendix 3.C
Recent Advances on the Lie-Santilli Isotheory

In Section 3.2.9 we have outlined the rudiments of the *Lie-Santilli isotheory.* It may be useful for the mathematically oriented reader to outline recent developments achieved by the Spanish mathematicians R. M. Falcón Ganfornina and J. Núñez Valdés [24,25,43] in the field beyond those presented in monographs [2,6,36,37].

Falcón and Núñez introduced in 2001 [37] a new construction model of isotopies which was similar to the one proposed by Santilli in 1978 although in its multivalued version presented by the same author later on [6] (see Chapter 4) because based on the use of several isolaws and isounits as operations existing in the initial mathematical structure. Such a model, which from now on will be called MCIM (*isoproduct construction model based on the multiplication*), was later generalized in Refs. [24,25,43]. In a schematic way, Santilli's isotopies can be described with the following diagram:

$$\begin{array}{ccc}
\text{Conventional Level} & \longrightarrow & \begin{array}{c}\text{General Level}\\ (V,*,\star,\ldots)\\ \cup\\ (E,\star,*,\ldots)\end{array}\\
(E,+,\times,\ldots) & & \\
\downarrow & \sharp & \downarrow \mathbf{I}\\
\text{Projection Level} & \xleftarrow{\quad\pi\quad} & \text{Isotopic level}\\
(\overline{\widehat{E}},\overline{\widehat{+}},\overline{\widehat{\times}},\ldots) & & (\widehat{E},\widehat{+},\widehat{\times},\ldots)
\end{array}$$

where, by construction:

a) The mapping $\mathbf{I}:(E,\star,*,\ldots)\to(\widehat{E},\widehat{+},\widehat{\times},\ldots):X\to\widehat{X}$ is an isomorphism.

b) The *isotopic projection* is onto:
$\pi:(\widehat{E},\widehat{+},\widehat{\times},\ldots)\to(\overline{\widehat{E}},\overline{\widehat{+}},\overline{\widehat{\times}},\ldots):\widehat{a}\to\pi(\widehat{a})=\overline{\widehat{a}}=a*\widehat{I}.$

c) $\widehat{a}\widehat{+}\widehat{b}=\widehat{a\star b};\quad \widehat{a}\widehat{\times}\widehat{b}=\widehat{a*b};\ldots.$

d) $\overline{\widehat{a}}=a*\widehat{I};\quad \alpha\overline{\widehat{+}}\beta=((\alpha*T)\star(\beta*T))*\widehat{I};\quad \alpha\overline{\widehat{\times}}\beta=\alpha*T*\beta;\ldots.$

PROPOSITION 3.2.C1: The following properties are verified:

a) The isotopic projection associated with each injective isotopic lifting is an isomorphism.

b) If the isotopic lifting used is compatible with respect to all of initial operations, then the isostructure $\widehat{E}$ is isomorphic to the initial E.

c) The relation of being isotopically equivalents is of equivalence.

d) Every isotopy $\pi \circ \mathbf{I} : (E, +, \times, \circ, \bullet, \ldots) \to (\overline{\widehat{E}}, \overline{\widehat{+}}, \overline{\widehat{\times}}, \overline{\widehat{\circ}}, \overline{\widehat{\bullet}}, \ldots)$ can be considered as an isotopic lifting which follows the MCIM, that is, every mathematical isostructure is an isostructure with respect to the multiplication.

Then, it has a perfect sense to considerer each one of the isostructures which result of applying the MCIM to conventional structures. Particularly, we can consider the construction of *Santilli's isoalgebras* (as the isotopic lifting of each algebra, which is endowed with a structure of algebra).

PROPOSITION 3.2.C2: Let U be a K-algebra and let $\widehat{U}$ be a $\widehat{K}$-isovector-space. If a $K(a, \star, *)$-algebra $(U, \diamond, \square, \cdot)$ is used in the general level, then the isotopic lifting $\widehat{U}$ corresponding to the isotopy of primary elements $\widehat{I}$ and $\square$ and secondary ones $\widehat{S}$ and $\diamond$, when MCIM is used, has a structure of isoalgebra on $\widehat{K}$, and it preserves the initial type of the algebra.

A particular type of isoalgebra is the *Lie-Santilli isoalgebra* [4]. Particularly, if $\overline{\widehat{U}}$ is the isotopic projection of a Lie-Santilli isoalgebra,

$$\widehat{I} = \widehat{I}(x, dx, d^2x, t, T, \mu, \tau, \ldots)$$

is an isounit and a basis $\overline{\widehat{U}}$, $\{\overline{\widehat{e_1}}, \ldots, \overline{\widehat{e_n}}\}$ is fixed, where $\overline{\widehat{e_i}} \,\widehat{\cdot}\, \overline{\widehat{e_j}} = \sum \overline{\widehat{c_{ij}^h}} \,\overline{\widehat{\bullet}}\, \overline{\widehat{e_h}}$, $\forall\, 1 \leq i, j \leq n$, then coefficients $\overline{\widehat{c_{ij}^h}} \in \overline{\widehat{K}}$ are the *Maurer-Cartan coefficients* of the isoalgebra, which constitute a generalization of the conventional case, since they are not constants in general, but functions dependent of the factors of $\widehat{I}$.

Another interesting isoalgebra is the *Santilli's Lie-admissible algebra* [4], that is, the isoalgebra $\widehat{U}$ such that with the commutator bracket $[.,.]_{\widehat{U}} : [\widehat{X}, \widehat{Y}]_{\widehat{U}} = (\widehat{X}\widehat{\cdot}\widehat{Y}) - (\widehat{Y}\widehat{\cdot}\widehat{X})$ is an isotopic Lie isoalgebra. The following result is satisfied:

PROPOSITION 3.2.C3: Under conditions of Proposition XXX, let us suppose that the law $\widehat{\circ}$ of the isoalgebra $\widehat{U}$ is defined according $\widehat{X}\widehat{\circ}\widehat{Y} = (X \circ Y)\square\widehat{I}$, for all $X, Y \in U$. If U is a Lie (admissible) algebra, then $\widehat{U}$ is a Lie isoadmissible isoalgebra.

In this way, Santilli's Lie-admissible isoalgebras inherit the usual properties of conventional (admissible) Lie algebras. In the same way, usual structures related with such algebras have also their analogue ones when isotopies are used.

For instance, an *isoideal* of a Lie isoalgebra $\widehat{U}$ is every isotopic lifting of an ideal $\Im$ of U, which is by itself an ideal. In particular, the *center* of a Lie isoalgebra $\widehat{U}$, $\left\{\widehat{X} \in \widehat{U} \text{ such that } \widehat{X}\widehat{\cdot}\widehat{Y} = \widehat{S},\ \forall \widehat{Y} \in \widehat{U}\right\}$, is an isoideal of $\widehat{U}$. In fact, it is verified the following result:

PROPOSITION 3.2.C4: Let $\widehat{U}$ be a Lie isoalgebra associated with a Lie algebra U and let $\Im$ be an ideal of U. Then, the corresponding isotopic lifting $\widehat{\Im}$ is an isoideal of $\widehat{U}$.

An isoideal $\widehat{\Im}$ of a Lie isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$, is called *isocommutative* if $\widehat{X}\widehat{\cdot}\widehat{Y} = \widehat{S}$, for all $\widehat{X} \in \widehat{\Im}$ and for all $\widehat{Y} \in \widehat{U}$, being $\widehat{U}$ *isocommutative* if it is so as an isoideal.

PROPOSITION 3.2.C5: $\widehat{U}$ is isocommutative if and only if U is commutative.

Lie-Santilli isoalgebras can also be introduced as follows. Given an $\widehat{K}$-isoassociative isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$, the commutator in $\widehat{U}$ associated with $\widehat{\cdot}$: $[\widehat{X}, \widehat{Y}]_S = (\widehat{X}\widehat{\cdot}\widehat{Y}) - (\widehat{Y}\widehat{\cdot}\widehat{X})$, for all $\widehat{X}, \widehat{Y} \in \widehat{U}$ is denominated *Lie-Santilli bracket product* $[.,.]_S$ with respect to $\widehat{\cdot}$. The isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, [.,.]_S)$ is then denominated *Lie-Santilli algebra.*

DEFINITION 3.2.C6: Let $\widehat{U}$ be an $\widehat{K}$-isoassociative isoalgebra associated with a K-algebra U, under conditions of Proposition XXX. Then, the Lie-Santilli algebra associated with $\widehat{U}$ is a Lie isoalgebra if the algebra U is either associative or Lie admissible.

Apart from that, a Lie-Santilli isoalgebra $\widehat{U}$ is said to be *isosimple* if, being an isotopy of a simple Lie algebra, it is not isocommutative and the only isoideals which contains are trivial. In an analogous way, $\widehat{U}$ is called *isosemisimple* if, being an isotopy of a semisimple Lie algebra, it does not contain non trivial isocommutative isoideals. Note that, this definition involves that every isosemisimple Lie isoalgebra is also isosimple. Moreover, it is verified:

PROPOSITION 3.2.C7: Under conditions of Proposition XXX, the isotopic lifting of a (semi)simple Lie algebra is an iso(semi)simple Lie isoalgebra. Particularly, every isosemisimple Lie isoalgebra is a direct sum of isosimple Lie isoalgebras.

A lie-Santilli isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$ is said to be *isosolvable* if, being an isotopy of a solvable Lie algebra, in the *isosolvability series*

$$\widehat{U}_1 = \widehat{U}, \quad \widehat{U}_2 = \widehat{U}\widehat{\cdot}\widehat{U}, \quad \widehat{U}_3 = \widehat{U}_2\widehat{\cdot}\widehat{U}_2, \ldots, \widehat{U}_i = \widehat{U}_{i-1}\widehat{\cdot}\widehat{U}_{i-1}, \ldots$$

there exists a natural integer n such that $\widehat{U}_n = \{\widehat{S}\}$. The minor of such integers is called *isosolvability index* of the isoalgebra.

PROPOSITION 3.2.C8: Under conditions of Proposition XXX, the isotopic lifting of a solvable Lie algebra is an isosolvable Lie isoalgebra.

An easy example of isosolvable Lie isoalgebras are the isocommutative isotopic Lie isoalgebras, since they verify, by definition, that $\widehat{U}\widehat{\cdot}\widehat{U} = \widehat{U}_2 = \{\widehat{S}\}$. It implies that every nonzero isocommutative Lie isoalgebra has an isosolvability index equals 2, being 1 the corresponding to the trivial isoalgebra $\{\widehat{S}\}$.

PROPOSITION 3.2.C9: Let $\widehat{U}$ be a Lie isoalgebra associated with a Lie algebra U. Under conditions of Proposition XXX, they are verified:

1) $\widehat{U}_i$ is an isoideal of $\widehat{U}$ and of $\widehat{U}_{i-1}$, for all $i \in N$.

2) If $\widehat{U}$ is isosolvable and U is solvable, then every isosubalgebra of $\widehat{U}$ is isosolvable.

3) The intersection and the product of a finite number of isosolvable isoideals of $\widehat{U}$ are isosolvable isoideals. Moreover, under conditions of Proposition XXX, the sum of a finite number of isosolvable isoideals is also an isosolvable isoideal.

By using this last result it can be deduced that the sum of all isosolvable isoideals of $\widehat{U}$ is another isosolvable isoideal, which is called *isoradical* of $\widehat{U}$. Note that it is different from the *radical* of $\widehat{U}$, which would be the sum of all solvable ideals of $\widehat{U}$. The isoradical is denoted by *isorad* $\widehat{U}$, not to be confused with *rad* $\widehat{U}$, and it will always contain $\{\widehat{S}\}$, because this last one is a trivial isosolvable isoideal of every isoalgebra. Note also that as every isosolvable isoideal of $\widehat{U}$ is a solvable ideal of $\widehat{U}$, then *isorad* $\widehat{U} \subset$ *rad* $\widehat{U}$. So, if $\widehat{U}$ is isosolvable, then $\widehat{U} =$ *isorad* $\widehat{U} =$ *rad* $\widehat{U}$, due to $\widehat{U}$ is solvable in particular.

PROPOSITION 3.2.C10: If $\widehat{U}$ is a semisimple Lie isoalgebra over a field of zero characteristic, then isorad $\widehat{U} = \{\widehat{S}\}$.

A Lie-Santilli isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$ is called *isonilpotent* if, being an isotopy of a nilpotent Lie algebra, in the series

$$\widehat{U}^1 = \widehat{U}, \quad \widehat{U}^2 = \widehat{U}\widehat{\cdot}\widehat{U}, \quad \widehat{U}^3 = \widehat{U}^2\widehat{\cdot}\widehat{U}, \ldots, \widehat{U}^i = \widehat{U}^{i-1}\widehat{\cdot}\widehat{U}, \ldots$$

(which is called *isonilpotency series*), there exists a natural integer n such that $\widehat{U}^n = \{\widehat{S}\}$. The minor of such integers is denominated *nilpotency index* of the isoalgebra.

As an immediate consequence of this definition it is deduced that every isonilpotent Lie isoalgebra is isosolvable and that every nonzero isocommutative Lie isoalgebra has an isonilpotency index equals 2, being 1 the corresponding of the isoalgebra $\{\widehat{S}\}$. Moreover, they are verified:

PROPOSITION 3.2.C11: Under conditions of Proposition XXX, the isotopic lifting of a nilpotent Lie algebra is an isonilpotent isotopic Lie isoalgebra.

PROPOSITION 3.2.C12: Let $\widehat{U}$ be a Lie isoalgebra associated with a Lie algebra U. They are verified:

1) *Under conditions of Proposition XXX, the sum of a finite number of isonilpotent isoideals of $\widehat{U}$ is another isonilpotent isoideal.*
2) *If $\widehat{U}$ is also isonilpotent and U is nilpotent, then*
   (a) *Every isosubalgebra of $\widehat{U}$ is isonilpotent.*
   (b) *Under conditions of Proposition XXX, if $\widehat{U}$ is nonzero isonilpotent, then its center is non null.*

In a similar way as the case isosolvable, the result (1) involves that the sum of all isonilpotent isoideals of $\widehat{U}$ is another isonilpotent isoideal, which is denoted by *isonihil-radical* of $\widehat{U}$, to be distinguished from the nihil-radical of $\widehat{U}$, which is the sum of the radicals ideals. It will be represented by *isonil-rad* $\widehat{U}$, which allows to distinguish it from the *nil-rad* $\widehat{U}$. It is immediate that *isonil-rad* $\widehat{U} \subset$ *nil-rad* $\widehat{U} \cap$ *isorad* $\widehat{U} \subset$ *nil-rad* $\widehat{U} \subset$ *rad* $\widehat{U}$.

Apart from that, it is possible to relate an isosolvable isotopic Lie isoalgebra with its derived Lie isoalgebra, by using the following:

PROPOSITION 3.2.C13: Under conditions of Proposition XXX, a Lie isotopic isoalgebra is isosolvable if and only if its derived Lie isoalgebra is isonilpotent.

Finally, an isonilpotent Lie isoalgebra $(\widehat{U}, \widehat{\circ}, \widehat{\bullet}, \widehat{\cdot})$ is called *isofiliform* if, being an isotopy of a filiform Lie algebra, it is verified that

$$\dim \widehat{U}^2 = n-2, \ \ \ldots, \ \ \dim \widehat{U}^i = n-i, \ \ \ldots, \ \ \dim \widehat{U}^n = 0,$$

where $\dim \widehat{U} = n$.

Note that the theory related with a filiform Lie algebra U is based on the use of a basis of such an algebra. So, starting from a basis $\{e_1, \ldots, e_n\}$ of U, which is preferably an *adapted basis*, we can deal with lots of concepts of it, such as dimensions of U and of elements of the nilpotency series, invariants i and j of U and, in general, the resting properties, starting from its structure coefficients, which are, in fact, responsible for the complete study of filiform Lie algebras.

Appendix 3.D
Relativistic Formulation of the Galilei Symmetry

As indicated in Section 3.5.1, special relativity has remained unsettled after one century of studies, even in the arena of its original conception, namely, point–particles and electromagnetic waves propagating in vacuum. A reason of the ongoing debates is connected to the alternative of Lorentz invariance for the two-ways light experiments conducted to date, and the Galilean invariance expected for one-way light experiments.

The alternative of Lorentzian vs Galilean treatments is obscured by the fact that the former applies for relativistic speeds while the latter is not perceived as such. This limitation was resolved in the early 1970s by the *relativistic formulation of the Galilean symmetry and relativity* proposed by P. Roman, J. J. Aghassi and R. M. Santilli [76-78], and known as *Galileo-Roman-Santikki-Aghassi symmetry and relativity.*[31]

In short, the alternative as to whether the ultimate relativity is of Lorentzian or Galilean type is far from being resolved. It is an easy prediction that such an alternative will not be resolved in these volumes. Consequently, in this appendix we can merely review the main ideas of the Galileo-Roman-Santilli-Aghassi (GRSA) symmetry, and leave the resolution of the alternative to future generations.

By assuming an in depth knowledge of the Galileo symmetry and its scalar extension (that we cannot possibly review here), the GRSA symmetry is based on the following assumptions:

1) The carrier space is given by the Kronecker product of the conventional Minkowski space $M(x,\eta,R)$ times a one-dimensional space $U(u)$ where u represents the proper time normalized to the dimension of length for reason clarified below,

$$S_{tot} = M(3.1) \times U(u) \tag{3.D1}$$

2) The GRSA symmetry is characterized by the following transformations

$$SO_o(3.1): \quad x^\mu \to x'^{\mu\,\prime} = \Lambda^\mu_\nu \times x^\nu, \quad \Lambda^B_\alpha eta \times \Lambda^\rho_\beta = \eta_a lpha^r ho, \tag{3.D2a}$$

$$T^a_4 : \, : x^\mu \to x'^\mu = x^\mu + a^\mu, \tag{3.D2b}$$

[31]These studies were conducted while the author was Associate Professor of Physics at Boston University under partial financial support by the USAFOSR here gratefully acknowledged.

$$T_4^b: \quad x^\mu \to x'^\mu = x^\mu + b^\mu \times u \tag{3.D2c}$$

$$T_1^\sigma: \quad u \to u' = u + \sigma, \tag{3.D2d}$$

where: Eqs. (3.D2a) are the (connected) conventional Lorentz transformations; Eqs. (3.D2b) are the conventional translations (with a^μ constants); Eqs. (3.D2c) and (3.D2d) are the new transformations with b^μ and σ non-null parameters, b^μ being dimensionless and σ having the dimension of length. Eqs. (3.D2c) were originally called *relativistic Galilean boosts,* [76] and here called *GRSA boosts,* since they are indeed a relativistic extension of the conventional nonrelativistic boosts. Eq. (3.D2d) was originally called the *relativistic Galilean time translation* [76],and it is here called the *GRSA time translation.*

3) The GRSA symmetry is then fifteen-dimensional and its connected component is written

$$GR = \{SO_o(3.1) \times T_4^b\} \times \{T_4^a \times T_1^\sigma\}, \tag{3.D3}$$

where one should note: the presence of the Poincaré group as a subgroup; the presence of the conventional Galileo group as a subgroup; and the separation of conventional translations from the Lorentz symmetry and their association to the new variable u.

Group (3.D3) admits as an invariant subgroup the group $T_4^a \times T_4^b \times T_1^\sigma$. Hence, *the GRSA group (3.D3) is an extension of the restricted Lorentz group, but not of the Poincaré group,* even though the latter is also an extension of the Lorentz group. These are central features for the understanding of the differences between the Galileo symmetry, the Poincaré symmetry and the GRSA symmetry.

The conventional Galileo group requires a scalar extension for its dynamical application, and the same occurs for the GRSA group, thus leading to the covering

$$\widetilde{GR} = T_1^\theta \times \{SL(2.C) \times T_4^b\} \times \{T_4^a \times T_1^\sigma\}, \tag{3.D4}$$

where θ is the usual phase factor.

By denoting the generators of $SL(2.C)$ with $J_{\mu\nu}$, the generators of T_4^a with $P_m u$, the generators of T_4^b with Q_μ, and the generators of T_1^σ with S, we have the following Lie algebra

$$[J_{\mu\nu}, J_{\rho\sigma}] = i \times (\eta_{\nu\rho} \times J_{\mu\sigma} - \eta_{mu\rho} \times J_{\nu\sigma} - \eta_{\mu\sigma} \times J_{\rho\nu} + \eta_{\nu\sigma} \times J_{\rho\mu}), \tag{3.D5a}$$

$$[P_\mu, J_{\rho\sigma}] = i \times (\eta_{\mu\rho} \times P_\sigma - \eta_{\mu\sigma} \times P_\rho), \tag{3.D5b}$$

$$[Q_\rho, J_{\mu\nu}] = i \times (\eta_{\mu\rho} \times Q_\nu - \eta_{\nu\rho} \times Q_\mu), \tag{3.D5c}$$

$$[P_\mu, Q_\nu] = i \times \eta_{\mu\nu} \times \ell^{-1}, \tag{3.D5d}$$

$$[S, Q_\nu] = i \times P_\nu, \tag{3.D5e}$$

$$[P_\mu, P_\nu] = [Q_\mu . Q_\nu] = [J_{\mu\nu}, S] = [P\mu, S] = 0, \tag{3.D5f}$$

where ℓ is the parameter originating from the scalar extension.

The physical interpretation is based on the following main aspects. Dynamics is assumed to verify the GRSA symmetry, with the Poincaré symmetry characterizing kinematics. Under such an assumption, the GRSA symmetry allows the introduction of a fully consistent *relativistic spacetime position operator* that is absent in relativistic quantum mechanics, with explicit expression

$$X_\mu = -\ell \times Q_\mu. \tag{3.D6}$$

In fact, the above interpretation is fully supported by commutation rules (3.D5).

Eq. (3.D6) introduces quite automatically a *universal length,* with the significant feature that *systems with different fundamental lengths are independent of each other.* The main dynamical invariant is no longer the familiar expression $P_\mu \times P^\mu = m^2$, but it is given instead by the following relativistic extension of the Galilean invariant

$$P_\mu \times P^\mu + 2 \times \ell - 1 \times S = inv. \tag{3.D7}$$

By assuming the value

$$P_\mu \times P^\mu + 2 \times \ell - 1 \times S = 0, \tag{3.D8}$$

the Galileo-Roman symmetry allows the introduction of the *relativistic mass operator*

$$\mathcal{M}^\in = \in \times \ell^{-\infty} \times \mathcal{S}. \tag{3.D9}$$

Note that the above definition is confirmed by commutation rules [3.D5) as well as from the fact that the above mass operator is invariant and a Lorentz scalar, as it should be. In particular, the eigenvalue of the above mass operator is the conventional scalar m^2 (see Ref. [76] for details). For a number of additional intriguing features of the GRSA symmetry, such as the nonlocality of the position operator "spread over" an area of radius ℓ, we have to refer the interested reader to paper [76] for brevity.

In closing with personal comments and recollections of these studies conducted some 37 years ago, there is no doubt that the GRSA group has dramatically more dynamical capabilities than the conventional Poincaré group. Also, to my best recollection, we could find no experimental data contradicting the GRSA symmetry.

Yet, the novelty of the symmetry caused a real opposition furor among colleagues, namely, a reaction that has to be distinguished from proper scientific scrutiny. Part of the opposition was due to the political attachment to Einsteinian doctrines, but part was also due to the fact that the GRSA group required technical knowledge above the average of theoretical physicists of the time. [32]

[32] See the footnotes of Volume IV.

Such huge an opposition essentially forced the author to abandon the studies in the field, a decision that he regretted later, but could not change at that time due to the need in the 1970s for the author to secure an academic position so as to feed and shelter two children in tender age and his wife.

During the 37 years that have passed since that time, the author discovered numerous theories published in the best technical journals that, in reality, did verify the GRSA symmetry, but were published as verifying the conventional Poincaré symmetry. All attempts by the author for editorial corrections turned out as being useless. That was unfortunate for the fully deserved continuation of Paul Romans name in science.

In this way, the author was exposed for to the academic rage caused by novelty and, in so doing, he acquired the necessary strength to resist academic disruptions when he proposed the construction of hadronic mechanics in 1978 [4]. Also in this way, the human experience gained by the author during his studies of the GRSA symmetry and relativity proved as being crucial for the studies on hadronic mechanics againt hardly credible obstructions, oppositions and disruptions.

Yet, the author hopes that studies on the GRSA symmetry and relativity are indeed continued by new generations of physicists, not only because of the dramatic richness of content compared to the Poincaré sub-symmetry, but also because the GRSA symmetry and its easily derivable isotopic extension appear to possess the necessary ingredients for a solution of the numerous unresolved problems of special relativity, including compatibility with the ultimate frontier of knowledge: space.[33]

[33]In the author's view, these advances may happen only when society will one future day understand the importance of scientific democracy for qualified inquiries.

Appendix 3.E
Whitney's Studies of Lorentzian vs. Galilean Relativities

3.E.1 Foreword

In this appendix we report *ad litteram* the studies on the alternative between Lorentzian vs Galilean relativities conducted by Cynthia Kolb Whitney, Editor, Galilean Electrodynamics, 141 Rhinecliff Street, Arlington, MA 02476, email dwhitney@mit.edu

3.E.2 Introduction

The art of mathematical physics lies in modeling physical processes mathematically by introducing idealizations simple enough to make the mathematics not infeasible, while at the same time complete enough to avoid rendering the physics inapplicable. It is a tough job, and we will probably never complete it. The fact is: Nature is not constrained to adhere to *any* idealizations that we introduce. History has revealed this truth over and over. But here is a brief report on progress so far.

3.E.3 Newton

The first modern mathematical physicist was Sir Isaac Newton. Important features of Newton's theory include its Universal Time, which runs the same for all observers, regardless of any absolute motion or relative motion between them. That means Newton's theory embodies Galilean Relativity. That is why I begin with him.

In Newton's *Principia* [80], the universe of discourse consisted of material bodies, whether small like apples or large like planets, reduced to point particles, with reciprocal forces between such particles, and the orbits thereby created for the particles. This universe of discourse was in total contrast to that for scientists on the European continent, which consisted of a presumed fluid 'aether', with vortices within it that carried the particles in complicated vortical orbits.

The difference in world view embodies Newton's important contribution to natural philosophy: the idea that it is right and proper to stick to describing mathematically the observable facts, without injecting any unprovable mechanical explanations. Critics forever pressed Newton for such explanations for gravity. How could it act, at a distance, without any contact? In response he included with the second edition of his *Principia* the 'General Scholium', including the remarks: "But hitherto I have not been able to discover the cause of the properties of gravity from phenomena, and I frame no hypothesis; for whatever is not deduced from the phenomena is to be called a hypothesis; and hypotheses, whether

metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy... And in us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and our sea."

Or, perhaps more memorably, "*Hypothesis non fingo.*" This statement does not mean that such an explanation is *never* to be sought; it just means that the time for such explanation can only be later on, when more facts are known. And so until that time arrives, one should just do what one can better do. If that that does finally arrive, then hypotheses need no longer be avoided; they can be embraced and tested.

In Newton's day, his purely descriptive mathematical approach was exceedingly successful. It achieved an unexpected unification between terrestrial and celestial physics. It could solve in closed form any two-body problem, with any ratio of masses involved. Given modern computers, it can handle three, or however many more, bodies. One tiny detail that it cannot do is the perihelion advance for a planet in the solar system, which is actually observable for the planet nearest the Sun, Mercury. That is, Newton's theory gets most, but not all, of that perihelion advance. For this tiny problem, Newton's theory would one day yield to Einstein's General Relativity Theory (GRT). But more comment on that development comes later.

3.E.4 Maxwell

The next batch of phenomenology for mathematical physics to deal with was revealed with the discovery and study of electromagnetic phenomena. A lot of individuals were involved, but the one who really changed things was Maxwell. He achieved an amazing unification of electricity and magnetism into electromagnetic theory [81]. It is a little unclear if he knew what he had sacrificed to get there. There was no Galilean Relativity there. Did he realize that Universal Time was gone? We do not know.

Maxwell's universe of discourse included point particles, but it put more attention onto what was between the particles: electromagnetic fields. Some particles generated the fields, while other, much smaller particles, responded to the fields, as 'test particles', unable to react back on the sources. There is an asymmetry there: Maxwell's theory is not built for a two-body problem. Indeed, because of the radiation associated with acceleration, Maxwell's theory could not handle one particularly important two-body problem: the Hydrogen atom. In part because of that problem, a totally new branch of physics, Quantum Mechanics (QM), would arise. But more comment on that development comes later.

Near the end of his Treatise on Electricity and Magnetism, Maxwell referred to a letter from Gauss to Weber expressing the opinion that the real keystone of electrodynamics would be "the deduction of the force acting between electric

particles in motion from the consideration of an action between them, not instantaneous, but propagated in time, in a similar manner to that of light." Gauss had not accomplished this, nor had Maxwell, nor had three others who had tried at the time Maxwell wrote; namely, Riemann, Clausius, and Betti. Maxwell attributed the lack of success of those three to prejudice against a hypothesis of a medium in which radiation of light and heat and electric action at a distance takes place. Maxwell was an aether man. Nevertheless, his later followers reformulated his theory without his aether, without his quaternion mathematics to represent that aether, and instead with the now-familiar field vectors. The feasibility of making the math description without requiring the aether hypothesis again illustrates Newton's point about *hypothesis non fingo*.

Later on, Lièнard and Wiechert [82, 83] did something that seems to fulfill the description that Gauss envisioned: they formulated retarded potentials, from which retarded fields follow, and with the Lorentz force law, the retarded forces follow. Their approach embodied an idea later crystallized more clearly. The idea is this: Maxwell's theory involves parameters ε_0 and μ_0 for free-space electric permittivity and magnetic permeability. They have no dependence on source or observer motion. And they imply a wave speed $c = 1/\sqrt{\varepsilon_0 \mu_0}$ that also cannot depend on source or observer motion. So the speed for potential and field retardation should also be c.

3.E.5 Einstein

Enter Einstein [84]. He elevated the idea that had emerged from Maxwell to the status of a Postulate — his famous 'Second Postulate', — which was the foundation for Special Relativity Theory (SRT). SRT does not have Galilean Relativity; it has Lorentzian relativity. Unlike Newton's theory, SRT does not have Universal Time; it has Relative Time. The idea of Relative Time is mind-boggling, and in fact leads to an extensive literature about 'paradoxes', especially about traveling twins, or trains, or clocks, or meter sticks, or buildings, *etc.*

Inasmuch as SRT is founded on Maxwell's theory, and Maxwell's theory cannot handle the Hydrogen atom, SRT is unlikely ever to be fully compatible with QM. Einstein was involved in the development of QM, through his Nobel-Prize winning work on the photoelectric effect, but he was not fond of QM, and in later years did not work so much on it. Instead, he mainly went back to SRT, embraced the Minkowski tensor formulation for it, and exploited the metric tensor therein to develop General Relativity Theory (GRT) [85].

GRT is believed to offer the explanatory hypothesis that Newton eschewed in saying "*Hypothesis non fingo*". GRT says that gravitational masses affect the metric tensor; *i.e.*, 'curve the spacetime', and responding masses travel paths that are straight in curved spacetime, or curved in flat spacetime.

Inasmuch as GRT is founded on SRT, and SRT is founded on Maxwell's theory, and Maxwell's theory cannot handle the Hydrogen atom, GRT is not likely ever to be fully compatible with QM. But scientists today do keep trying for that Holy Grail.

GRT has the same design weakness that Maxwell's theory: it is a field theory, and as such, is not designed for something so complicated as a two-body problem. Late in life, Einstein wrote about his misgivings in a letter to his friend Michel Angelo Besso: "I consider it quite possible that physics cannot be based on the field concept, *i.e.*, on continuous structures. In that case *nothing* remains of my entire castle in the air, gravitation theory included, [and the] rest of physics."

Maintaining such doubt is, I believe, the mark of a truly great scientist. Einstein's present-day followers generally do not harbor such doubts.

3.E.6 Reformulations

There have always been researchers questioning Einstein's Second Postulate, and evaluating alternatives to it. Ritz was an early [86], but not successful, example. Later, in the 1950's, began the work of P. Moon, D. Spencer, E. Moon, and many of Spencer's students [87–89]. Their work has been successful in producing a lot of very interesting results, if not in garnering all the recognition it deserves.

The key Moon-Spencer-Moon idea was a propagation process with continuing control by the source, even after the initiating 'emission' event, so that the light moves away from the source at speed c relative to that source, however arbitrarily the source itself may be moving. (This is *not* the Ritz postulate, which had the light moving at velocity $c + V$, where V was the velocity of the source at the moment of emission, and c is the velocity vector of the light if it had come from a stationary source at that moment.)

In any event, continuing control by the source implies that 'light', whatever it is, has a longitudinal extent (Of course! Light possesses wavelength, does it not?), and the longitudinal extent is expanding in time. That expansion naturally raises the question: exactly what *part* of the expanding light packet is it that moves at speed c relative to the source? The tacit hypothesis of Moon-Spencer-Moon is that the c-speed part is the leading tip of the light packet. It then follows that when a receiver is encountered, the entire longitudinal extent of the light packet must collapse instantly to the receiver. That means the trailing tail of the light packet must snap into the receiver at infinite speed. The infinite speed might be unacceptable for Einstein true believers, but maybe not for QM true believers.

3.E.7 Two Step Light

My own work [90–92] follows the Moon-Spencer-Moon lead, with one conceptual addition. My variation to the Moon-Spencer-Moon postulate is that the speed c relative to the source characterizes, not the leading tip of the light packet,

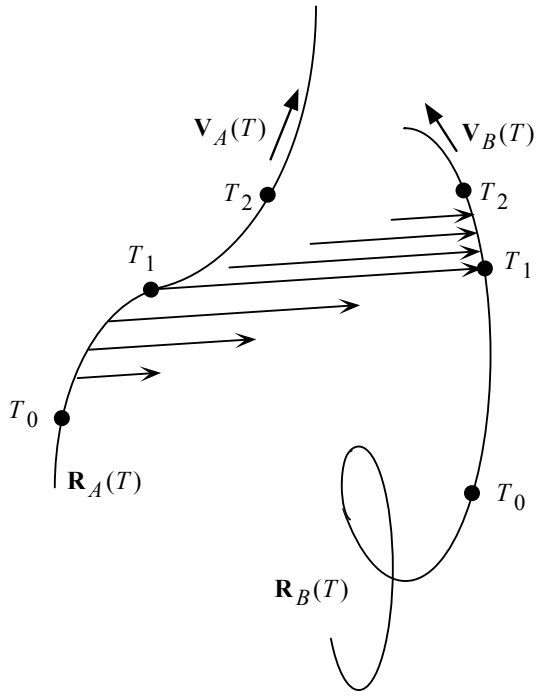

Figure 3.E.1. Illustration of Two-Step Light propagation, reprinted from [90] with permission.

but rather the mid point of the light packet. That means the leading tip must move relative to the source, not at c, but rather at $2c$. (A $2c$ anywhere is probably shocking to Einstein true-believers, but maybe not so shocking as an infinite speed would be.)

My variation on the Moon-Spencer-Moon theme introduces symmetry between light emission and absorption. The leading tip reaches the receiver in half the time for propagation at c, so there is time left for a completely symmetric absorption process, wherein the mid point of the light packet travels at speed c relative to the receiver, however arbitrarily that may move. That idea then means the tail end reels in at speed $2c$ relative to the receiver.

The fully revised light postulate is what I have called "Two-Step Light". It is illustrated in Fig. 1. The T's are Universal Times: T_0 at the beginning of the scenario, T_1 at the mid point, and T_2 at the end. Particle A is the source, and particle B is the receiver (one of possibly many candidate receivers, selected by the accidental collision with the expanding light arrow at T_1). The mid points of the light arrows may be said to conform to the Moon-Spencer-Moon favored postulate in the expansion phase of the scenario, and then with the Einstein postulate in the collapse phase of the scenario.

How can light do all that? Stay in contact with a moving source? Switch control to a moving receiver? Stay in contact with a moving receiver? At this point, I resort to saying *hypothesis non fingo*. My first job is just to work out the implications of the Two-Step Light Postulate. It is a mundane task, involving no more than high-school algebra. It has been detailed in [90] and [91]. Here I shall just summarize results.

Consider the problem of processing data consisting of successive light signals from a moving source in order to estimate the speed V of that source. If the light propagates according to the Two-Step process, but the data gets processed under the assumption of the one-step Einstein postulate, then there will be a systematic error to the estimate. In fact, the estimate turns out to be

$$v = V/\left(1 + V^2/4c^2\right). \tag{3.E.1}$$

The estimate v is always less than V, and in fact is limited to c, which value occurs at $V = 2c$. Thus v has the property of any speed in Einstein's SRT. The obvious implication is that v is an Einsteinian speed, whereas V is a Galilean speed.

One is obviously invited to look also at a related construct

$$V^{\uparrow} = V/\left(1 - V^2/4c^2\right). \tag{3.E.2}$$

The superscript $\uparrow$ is present to call attention to the fact that $V^{\uparrow}$ has a singularity, which is located at $V = 2c$, or $v = c$. That is, $V^{\uparrow}$ has the property of the so-called 'covariant' or 'proper' velocity. Interestingly, past the singularity, it changes sign. This behavior mimics the behavior that SRT practitioners attribute to 'tachyons', or 'super-luminal particles': they are said to 'travel backwards in time'. The sign change is a mathematical description, while the 'travel backwards in time' is a literary description.

The relationships expressed by (3.E.1) and (3.E.2) can be inverted, to express V in terms of v or $V^{\uparrow}$. The definition $v = V/\left(1 + V^2/4c^2\right)$ rearranges to a quadratic equation

$$\left(v/4c^2\right)V^2 - V + v = 0,$$

which has solutions

$$V = \frac{1}{v/2c^2}\left(+1 \pm \sqrt{1 - v^2/c^2}\right).$$

Multiplying numerator and denominator by $\left(+1 \mp \sqrt{1 - v^2/c^2}\right)$ converts these to the form

$$V = v \Big/ \frac{1}{2}\left(1 \mp \sqrt{1 - v^2/c^2}\right), \tag{3.E.3}$$

which makes clear that for small v, V has one value much, much larger than v, and another value essentially equal to v.

The definition $V^{\uparrow} = V/\left(1 - V^2/4c^2\right)$ rearranges to a quadratic equation

$$\left(-V^{\uparrow}/4c^2\right) V^2 - V + V^{\uparrow} = 0,$$

which has solutions

$$V = \frac{1}{-V^{\uparrow}/2c^2}\left(+1 \pm \sqrt{1 - {V^{\uparrow}}^2/c^2}\right).$$

Multiplying numerator and denominator by $\left(+1 \mp \sqrt{1 + {V^{\uparrow}}^2/c^2}\right)$ converts these to the form

$$V = V^{\uparrow} \Big/ \frac{1}{2}\left(1 \mp \sqrt{1 + {V^{\uparrow}}^2/c^2}\right), \tag{3.E.4}$$

which makes clear that for small $V^{\uparrow}$, V has one value much larger in magnitude than $V^{\uparrow}$ (which is negative there), and another value essentially equal to $V^{\uparrow}$.

To see that v and $V^{\uparrow}$ are not only qualitatively *like* Einsteinian speed and covariant speed, but in fact quantitatively *equal* to them, one can do a bit more algebra. Substitute (3.E.3) into (3.E.2) and simplify to find

$$V^{\uparrow} = \mp v \Big/ \sqrt{1 - v^2/c^2}, \tag{3.E.5}$$

which is the definition of covariant speed familiar from SRT, made slightly more precise by inclusion of the minus sign for situations beyond the singularity.

Similarly, substitute (3.E.4) into (3.E.1) and simplify to find

$$v = \mp V^{\uparrow} \Big/ \sqrt{1 + V^{\uparrow}/c^2}, \tag{3.E.6}$$

which is again a relationship familiar from SRT, made slightly more precise by inclusion of the minus sign for situations beyond the singularity.

The information content of Eqs. (3.E.1)–(3.E.6) is displayed graphically in Fig. 2. Both plot axes denote multiples of nominal light speed c. Galilean particle speed V is the independent variable. To save space, it is the absolute value of $V^{\uparrow}$ that is plotted.

Speed stands here as a proxy for many other interesting things in SRT, like momentum, relativistic mass, *etc.* SRT only offers only two speed relationships; *i.e.*, (3.E.5) and (3.E.6), whereas Two Step Light offers six relationships; *i.e.* (3.E.1) through (3.E.6). This constitutes three times the information content. That means Two Step Light offers a lot more opportunities for better explaining all the interesting things in SRT.

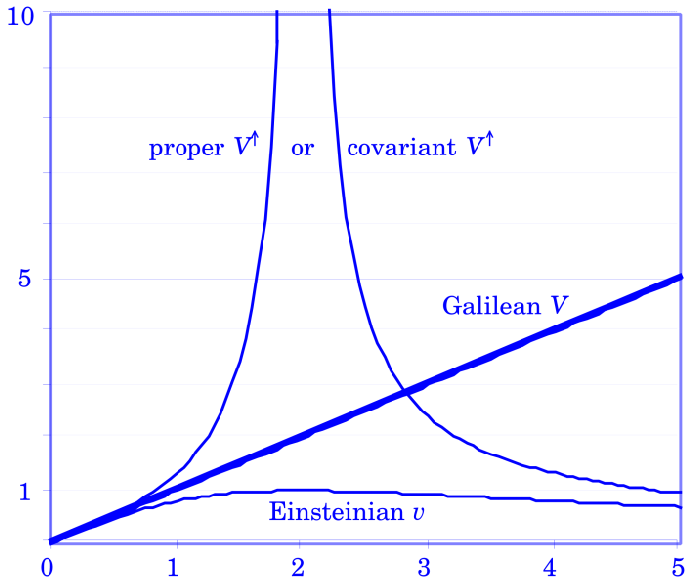

Figure 3.E.2. Numerical relationships among three speed concepts. Reprinted from [90] with permission.

3.E.8 Paradoxes Resolved

There are a great many peculiar-sounding results from SRT — that is why the word 'paradox' occurs so often in the SRT literature. But there are no paradoxes in physical reality, and there are none in Two Step Light theory. To illustrate this point, consider one rather obscure but very important case. I mentioned before the Lièнard-Wiechert retarded potentials and fields and implied forces. They have a paradoxical property.

Expressed in Gaussian units [93], the Lièнard-Wiechert scalar and vector potentials are

$$\Phi(\mathbf{x},t) = e\left[1/\kappa R\right]_{\text{retarded}} \quad \text{and} \quad A(\mathbf{x},t) = e\left[\boldsymbol{\beta}/\kappa R\right]_{\text{retarded}},$$

where $\kappa = 1 - \mathbf{n}\cdot\boldsymbol{\beta}$, with $\boldsymbol{\beta}$ being source velocity normalized by c, and $n = \mathbf{R}/R$ (a unit vector), and $\mathbf{R} = \mathbf{r}_{\text{observer}}(t) - \mathbf{r}_{\text{source}}(t - R/c)$ (an implicit definition for the terminology 'retarded'). The Lièнard-Wiechert fields expressed in Gaussian units are then

$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{(\mathbf{n}-\boldsymbol{\beta})\left(1-\beta^2\right)}{\kappa^3 R^2} + \frac{\mathbf{n}}{c\kappa^3 R}\times\left((\mathbf{n}-\boldsymbol{\beta})\times\frac{d\boldsymbol{\beta}}{dt}\right)\right]_{\text{retarded}},$$

and $\mathbf{B}(\mathbf{x},t) = \mathbf{n}_{\text{retarded}} \times \mathbf{E}(\mathbf{x},t)$. The $1/R$ fields are radiation fields, and they make a Poynting vector that lies along $\mathbf{n}_{\text{retarded}}$:

$$\mathbf{P} = \mathbf{E}_{\text{radiative}} \times \mathbf{B}_{\text{radiative}} = \mathbf{E}_{\text{radiative}} \times (\mathbf{n}_{\text{retarded}} \times \mathbf{E}_{\text{radiative}}) = E^2_{\text{radiative}} \mathbf{n}_{\text{retarded}}.$$

But the $1/R^2$ fields are Coulomb-Ampère fields, and the Coulomb field does not lie along $\mathbf{n}_{\text{retarded}}$ as one might naively expect; instead, it lies along $(\mathbf{n} - \boldsymbol{\beta})_{\text{retarded}}$.

Consider the following scenario, designed specifically for an instructive exercise in *reductio ad absurdum*. A source executes a motion comprising two components: **1)** inertial motion at constant $\boldsymbol{\beta}$, plus **2)** oscillatory motion at small amplitude and high frequency, so that there exists a small velocity $\Delta\boldsymbol{\beta}_{\text{retarded}}$ and a not-so-small acceleration $d\Delta\boldsymbol{\beta}/dt|_{\text{retarded}}$. Observe that the radiation and the Coulomb attraction/repulsion come from different directions. The radiation comes along $\mathbf{n}_{\text{retarded}}$ from the retarded source position, but the Coulomb attraction/repulsion lies along $(\mathbf{n} - \boldsymbol{\beta})_{\text{retarded}}$, which is basically $(\mathbf{n}_{\text{retarded}})_{\text{projected}}$, and lies nearly along $\mathbf{n}_{\text{present}}$. This behavior seems peculiar. Particularly from the perspective of modern Quantum Electrodynamics (QED), all electromagnetic effects are mediated by photons — real ones for radiation and virtual ones for Coulomb-Ampere forces. How can these so-similar photons come from different directions?

Two-Step Light theory resolves the directionality paradox inherent in the Linard-Wiechert fields. Because of the various $2c$'s in the mathematics, the radiation direction $\mathbf{n}_{\text{retarded}}$ changes to $\mathbf{n}_{\text{half retarded}}$, and the Coulomb attraction/repulsion direction $(\mathbf{n}_{\text{retarded}})_{\text{projected}}$ changes to $(\mathbf{n}_{\text{retarded}})_{\text{half projected}}$. These two directions are now physically the same; namely the source-to-receiver direction at the mid point of the scenario, *i.e.* $\mathbf{n}_{\text{mid point}}$. The potentials and fields become:

$$\Phi(\mathbf{x},t) = e\,[1/R]_{\text{mid point}} \quad \text{and} \quad \mathbf{A}(\mathbf{x},t) = e\,[\mathbf{V}/cR]_{\text{mid point}}\,,$$

$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{\mathbf{n}}{R^2} + \frac{\mathbf{n}}{cR} \times \left(\mathbf{n} \times \frac{d\mathbf{V}}{c\,dt}\right)\right]_{\text{mid point}} \text{and } \mathbf{B}(\mathbf{x},t) = \mathbf{n}_{\text{mid point}} \times \mathbf{E}(\mathbf{x},t)\,.$$

What is so important about the field formulations consistent with Two Step Light is the forces that they imply in a two-body system, such as the Hydrogen atom. The attractive forces are not central. They impose a torque on the system, and through that, a mechanism for energy input into the system. This can work against the energy loss due to radiation reaction. This can provide an approach for understanding atoms that is completely different from QM. One need not postulate the value of Planck's constant and the nature of its involvement in the mathematics of 'probability' waves. One can derive Planck's constant. And one can uncover a tremendous amount of previously unrecognized regularity in chemical data. Refs. [91, 92] go into all this in some detail.

3.E.9 Conclusions

About SRT: A symbol is missing from the language of SRT (namely, the Galilean speed V). As a result, Einsteinian speed v often gets conflated with Galilean speed V. Any conflation of physical concepts can cause confusion and misinterpretations of results. That is why the SRT literature has to discuss so many 'paradoxes'.

About Two Step Light: Two Step Light is a 'covering' theory; it contains all the variables and relationships familiar from SRT, but it also contains other variables and relationships as well. Users who are comfortable with the familiar need not give anything up, and users who are curious about the rest can readily make use of it.

About relativities: If one accepts Two Step Light as an explanation for SRT, then one can describe any situation of interest in terms of Galiean V and Galilean coordinate transformations. That is, one is free to use Galilean relativity rather than Lorentzian relativity if one wishes.

About QM: Like SRT, QM has required unnecessary abandonment of rationality. And there is a lot of phenomenology out there that simply is not treated by present-day QM. So it is worth re-doing QM in a different way.

About philosophy: Today's QM is rightly understood as a theory not so much of 'things', but rather of 'knowledge': what we can 'know', given our means of knowing anything about what 'is'. SRT should be understood that way too. It isn't necessarily about what 'is'; it is about what we *think*, given what data we can take, and what algorithms we allow ourselves to apply in processing that data.

Appendix 3.F
Rapoport Studies on Geometry, Torsion, Statistics, Diffusion and Isotopies

3.F.1 Introduction

In this appendix we report *ad litteram* the studies on geometry, torsion, statistics, diffusion and isotopies by Diego Lucio Rapoport of the Department of Sciences and Technology, Universidad Nacional de Quilmes, Buenos Aires, Argentina, email ¡diego.rapoport@gmail.com¿.

It is appropriate to start by quoting Prof. Santilli (see Section 6.1, Volume IV of this series): "a first meaning of the novel hadronic mechanics is that of providing the first known methods for quantitative studies of the interplay between matter and the underlying substratum. The understanding is that space is the final frontier of human knowledge, with potential outcomes beyond the most vivid science fiction of today". In this almost prophetic observation, Prof. Santilli has pointed out the essential role of the substratum, its geometrical structure and the link with consciousness. In the present appendix, which we owe to the kind invitation of Prof. Santilli, we shall present similar views, specifically in presenting both quantum and hadronic mechanics as space-time fluctuations, and we shall discuss the role of the substratum. As for the problem of human knowledge, we shall very briefly indicate on how the present approach may be related to the fundamental problem of consciousness, which is that of self-reference.

A central problem of contemporary physics is the distinct world views provided by QM and GR (short for quantum mechanics and general relativity, respectively), and more generally of gravitation. In a series of articles [94–97, 115] and references therein, we have presented an unification between space-time structures, Brownian motions, fluid dynamics and QM. The starting point is the unification of space-time geometry and classical statistical theory, which has been possible due to a complementarity of the objects characterizing the Brownian motion, i.e. the noise tensor which produces a metric, and the drift vector field which describes the average velocity of the Brownian, in jointly describing both the space-time geometry and the stochastic processes. These space-time structures can be defined starting from flat Euclidean or Minkowski space-time, and they have in addition to a metric a torsion tensor which is formed from the metric conjugate of the drift vector field. The key to this unification lies in that the Laplacian operator defined by this geometrical structure is the differen-

tial generator of the Brownian motions; stochastic analysis which deals with the transformation rules of classical observables on diffusion paths ensures that this unification is valid in both directions [116]. Thus, in this equivalence, one can choose the Brownian motions as the original structures determining a space-time structure, or conversely, the space-time structures produce a Brownian motion process. Space-time geometries with torsion have lead to an extension of the theory of gravitation which was first explored in joint work by Einstein with Cartan [98], so that the foundations for the gravitational field, for the special case in which the torsion reduces to its trace, can be found in these Brownian motions. Furthermore, in [95] we have shown that the relativistic quantum potential coincides, up to a conformal factor, with the metric scalar curvature. In this setting we are lead to conceive that there is no actual propagation of disturbances but instead an holistic modification of the whole space-time structure due to an initial perturbation which provides for the Brownian process modification of the original configuration. Furthermore, the present theory which has a kinetic Brownian motion generation of the geometries, is related to Le Sage's proposal of a Universe filled with all pervading tiny particles moving in all directions as a pushing (in contrast with Newton's pulling force) source for the gravitational field [129]. Le Sage's perspective was found to be compatible with cosmological observations by H. Arp [130]. This analysis stems from the assumption of a non-constant mass in GR which goes back to Hoyle and Narlikar, which in another perspective developed by Wu and Lin generates rotational forces [131]. These rotational forces can be ascribed to the drift trace-torsion vector field of the Brownian processes through the Hodge duality transformation [96], or still to the vorticity generated by this vector field. In our present theory, motions in space *and* time are fractal, they generate the gravitational field, and furthermore they generate rotational fields, in contrast with the pulling force of Newton's theory and the pushing force of Le Sage, or in the realm of the neutron, the Coulomb force. Furthermore, in our construction the drift has built-in terms given by the conjugate of electromagnetic-like potential 1-forms, whose associated intensity two-form generate vorticity, i.e. angular momentum; these terms include the Hertz potential which is the basis for the construction of superluminal solutions of Maxwell's equations; see [95] and references therein. So the present geometries are very different from the metric geometries of general relativity and are not in conflict with present cosmological observations.

The space-time geometrical structures of this theory can be introduced by the Einstein λ transformations on the tetrad fields [98, 95], from which the usual Weyl scale transformations on the metric can be derived, but contrarily to Weyl geometries, these structures have torsion and they are integrable in contrast with Weyl's theory; we have called these connections as RCW structures (short for Riemann-Cartan-Weyl) [94–97]. This construction is a special case of the construction of

Riemannian or Lorentzian metrics presented in Section 3.5.3, in which Santilli generalized isotopic unit takes a diagonal form with equal elements given by (the square of) a scale function, while the number field, the differential and integral calculus are the usual ones of practice in differential geometry; these restrictions will be lifted to work with a full isotopic theory for HM in extending the theory developed for QM; in distinction with HM, the usual scale transformations do not depend on anything but the space-time coordinates, thus excluding the more general non-linear non-hamiltonian case contemplated by HM. In distinction with GR which due to the lack of a source leads to inconsistencies discussed in Section 1.4, a theory based on torsion and in particular in the case of a so-called absolute parallelism in which the torsion is derived from the differential of the cotetrad field (the so-called Weitzenbock spaces), has a geometrically defined energy-momentum tensor which is built from the torsion tensor [113, 134]. Furthermore, the trace-torsion has built-in electromagnetic potential terms. We must recall that in Section 1.4 it was proved that gravitational mass has partially an electromagnetic origin. So our original setup in terms of torsion fields which can be non-null in flat Minkowski or Euclidean spaces (while in these spaces curvature is null), does not lead in principle to the inconsistencies observed before. There are other differences between the present approach and GR which we would like to discuss. In the latter theory, the space-time structure is absolute in the sense that it is defined without going through a self-referential characterization. With the introduction of torsion, and especially in the case of the trivial metric with null associated curvature tensor, we are introducing a self-referential characterization of the geometry since the definition of the manifold by the torsion, is through the concept of locus of a point (be that temporal or spatial). Indeed, space and time can only be distinguished if we can distinguish inhomogenities, and this is the intent of torsion, to measure the dislocation (in space and time) in the manifold [142]. Thus all these theories stem from a geometrical operation which has a logical background related to the concept of distinction (and more fundamentally, the concept of identity, which is prior to that of distinction) and its implementation through the operation of comparison by parallel transport with the affine connection with non-vanishing torsion.[34] In comparison, in GR there is also an operation of distinction carried out by the parallel transport of pair of vector fields with the Levi-Civita metric connection yielding a trivial difference, i.e. the torsion is null and infinitesimal parallelograms trivially close, so that it does not lead to the appearence of inhomogenities as resulting from this primitive

[34]This can be further related with multivalued logics and the appearance of time waves related to paradoxes, which in a cognitive systems approach yields the Schroedinger representation; furthermore this conception leads to the notion of reentrance of a space-time domain into itself, as a self-referential cybernetic system, and ultimately to multidimensional time; this may ultimately be linked to semiotics and its role in biology [134].

operation of distinction; these are realized through the curvature derived from the metric. But to close this discussion, we refer again to the inconsistencies that an approach based on the curvature viz a viz the present approach which places the appearence of spacetime in terms of deformations of the vacuum, and as such, has the same genesis as Isorelativity developed by Prof. Santilli and presented in Section 3.5.5.

We have shown that this approach leads to non-relativistic QM both in configuration space [96] and in the projective Hilbert state-space through the stochastic Schroedinger equation [98] (in the latter case, it was proved shown that this geometry is related to the reduction of the wave function can be described by decoherence through noise [96, 98]), and further to Maxwell's equation and its equivalence with the Dirac-Hestenes equation of relativistic QM [95, 115]. The fact that non-relativistic QM can be linked to torsion fields was unveiled recently [96]. In fact, torsion fields have been considered to be as providing deviations of GR outside the reach of present precision measurements [112]. It turns out that quantum wave-functions verifying linear or non-linear Schroedinger equations are another universal, or if wished, mundane examples of torsion fields. We shall show in the present article, that this approach extends to the strong interactions as described by HM and thus that the isotopic lift of the Schroedinger wave function is also a source for torsion, albeit one which incorporates the full non-linearity and non-hamiltonian character of the strong interactions. The quantum random ensembles which generate the quantum geometries, or which dually can be seen as generated by them, in the case of the Schroedinger equation can be associated with harmonic oscillators with disordered random phase and amplitude first proposed by Planck, which have the same energy spectrum as the one derived originally by Schroedinger [146]. The probabilities of these ensembles are classical since they are associated with classical Brownian motions in the configuration and projective Hilbert-state manifolds, in sharp contrast with the Copenhagen interpretation of QM which is constructed in terms of single system description, and they are related to the scalar amplitude of the spinor field in the case of the Dirac field, and in terms of the modulus of the complex wave function in the non-relativistic case [95, 96, 115]. We would like to recall at this stage that Khrennikov has proved that Kolmogorov's axiomatics of classical probability theory, in a contextual approach which means an a-priori consideration of a complex of physical conditions, permits the reconstruction of quantum theory [117]. Thus, Khrennikov's theory places the validity of quantum theory in ensembles, in distinction with the Copenhagen interpretation, and is known as the Vaxho interpretation of quantum mechanics. In the present approach we obtain both a geometrical characterization of the quantum domain through random ensembles performing Brownian motions which generate the space and time geometries, and additionally a characterization for single systems through the topological

Bohr-Sommerfeld invariants associated with the trace-torsion by introducing the concept of Pfaffian system developed by Kiehn in his geometro-topological theory of processes [132], specifically applied to the trace-torsion one-form [134]. Most remarkably, in our setting another relevant example of these space-time geometries is provided by viscous fluids obeying the invariant Navier-Stokes equations of fluid-dynamics, or alternatively the kinematical dynamo equation for the passive transport of magnetic fields on fluids [94, 97]. This is of importance with respect to cosmology, since cosmological observations have registered turbulent large-scale structures which are described in terms of the Navier-Stokes equations [135].

There have been numerous attempts to relate non-relativistic QM to diffusion equations; the most notable of them is Stochastic Mechanics due to Nelson [102]. Already Schroedinger proposed in 1930–32 that his equation should be related to the theory of Brownian motions (most probably as a late reaction to his previous acceptance of the single system probabilistic Copenhagen interpretation), and further proposed a scheme he was not able to achieve, the so-called interpolation problem which requires to describe the Brownian motion and the wave functions in terms of interpolating the initial and final densities in a given time-interval [102]. More recently Nagasawa presented a solution to this interpolation problem and further elucidated that the Schroedinger equation is in fact a Boltzmann equation [107], and thus the generation of the space and time structures produced by the Brownian motions has a statistical origin.[35] Neither Nagasawa nor Nelson presented these Brownian motions as space-time structures, but rather as matter fields *on* the vacuum.[36] Furthermore, Kiehn has proved that the Schroedinger equation in spatial 2D can be exactly transformed into the Navier-Stokes equation for a compressible fluid, if we further take the kinematical viscosity ν to be $\frac{\hbar}{m}$ with m the mass of the electron [105]. We have argued in [96] that the Navier-Stokes equations share with the Schroedinger equation, that both have a RCW geometry at their basis: While in the Navier-Stokes equations the trace-torsion

[35] We have discussed in [96] that the solution of the interpolation problem leads to consider time to be more than a classical parameter, but an active operational variable, as recent experiments have shown [136] which have elicited theoretical studies in [145]; other experiments that suggest an active role of time are further discussed in [96].

[36] Another developments following Nelson's approach, in terms of an initial fractal structure of space-time and the introduction of Nelson's forward and backward stochastic derivatives, was developed by Nottale in his Scale Theory of Relativity [114]. Remarkably, his approach has promoted the Schroedinger equation to be valid for large scale structures, and predicted the existence of exo-solar planets which were observationally verified to exist [106]. This may further support the idea that the RCW structures introduced in the vacuum by scale transformations, are valid independently of the scale in which the associated Brownian motions and equations of QM are posited. Nottale's covariant derivative operator turns to be a particular case of our RCW laplacian [96]. We would like to mention also the important developments of a theory of space-time with a Cantorian structure being elaborated in numerous articles by M. El Naschie [137] and a theory of fractals and stochastic processes of QM which has been elaborated by G. Ord [138].

is $\frac{-1}{2\nu}u$ with u the time-dependent velocity one-form of the viscous fluid, in the Schroedinger equation, the trace-torsion one-form incorporates the logarithmic differential of the wave function – just like in Nottale's theory [114] – and further incorporates electromagnetic potential terms in the trace-torsion one-form. This correspondence between trace-torsion one-forms is what lies at the base of Kiehn's correspondance, with an important addendum: While in the approach of the Schroedinger equation the probability density is related to the Schroedinger scale factor (in incorporating the complex phase) and the Born formula turns out to be a formula and not an hypothesis, under the transformation to the Navier-Stokes equations it turns out that the probability density of non-relativistic quantum mechanics, is the enstrophy density of the fluid, i.e. the square of the vorticity, which thus plays a *geometrical* role that substitutes the probability density. Thus, in this approach, while there exist virtual paths sustaining the random behaviour of particles (as is the case also of the Navier-Stokes equations) and interference such as in the two-slit experiments can be interpreted as a superposition of Brownian paths [107], the probability density has a purely geometrical fluid-dynamical meaning. This is of great relevance with regards to the fundamental role that the vorticity, i.e. the fluid's particles angular-momentum has as an organizing structure of the geometry of space and time. In spite that the torsion tensor in this theory is naturally restricted to its trace and thus generates a differential one-form, in the non-propagating torsion theories it is interpreted that the vanishing of the completely skew-symmetric torsion implies the absence of spin and angular momentum densities [112], it is precisely the role of the vorticity to introduce angular momentum into the present theory.

To explain the fundamental kinematical role of torsion in QM and classical mechanics of systems with Lie group symmetries, we note that if we consider as configuration space a Lie group, there is a canonical connection whose torsion tensor coefficients are non other than the coefficients of the Lie-algebra under the Lie bracket operation [128]. Thus a Lie group symmetry is characterized by the torsion tensor for the canonical connection. Thus the Lie-Santilli isotopic theory implies a deformation of the torsion tensor of the canonical connection by the generalized unit [19, 20, 22, 46, 73, 108–110].[37] With regards to another role of torsion in classical mechanics, it appears as describing friction, or more generally, non-anholonomic terms which produce additional terms in the equations of motion, which were obliterated by contemporary physics with the exception of Birkhoffian mechanics and discussed in Sections 1.2.4, 3.1, 3.3 and 4.1.2 by Prof. Santilli, which originated in the monographs [150]. In fact the attention

[37]The introduction of this generalized unit, in contrast with the basic unit of mathematics and physics, establishes a relation between these new units and physical processes which is unknown to mathematics, and is presently developed in terms of an arithmetic of forms which follows from the principle of distinction previously alluded, the multivalued logics associated to it and self-reference [134].

of this author to HM at an early stage, stemmed from his work (jointly with S. Sternberg) in classical mechanical systems with angular momentum, which could be formulated without lagrangians nor hamiltonians, and furthermore could not be reduced to the canonical form of conservative systems [155]. Further in common to HM and torsion geometries, is that the latter are associated to angular momentum densities [112], while in HM the isotopic unit incorporates spin-up spin-down couplings such as in the Rutherford-Santilli model of the neutron [108, 141, 70]. Possible relations between torsion as spin or angular momentum densities can be ventured in relation with anomalous spin interactions of the proton, and magnetic resonances [139]. Furthermore, it has been shown that completely skew-symmetric torsion can produce a spin flip of high energy fermionic matter at very high densities, and that in this situation helicity can be identified with spin [133]. An intrinsic macroscopic angular momentum would be the evidence of this phenomena.This may be of relevance when taking in consideration the time periodicity of the fine structure of histograms and its relation to macroscopic angular momentum which we have discussed in [96] and others we shall discuss in this article.

To understand the need of carrying the extensions produced by the isotopic lifts, it is based in the fact that the isotopic lift of Relativity due to Santilli (see [73]) is applicable for the electromagnetic and weak interactions but not applicable for the case of hadrons. These have a charge radius of 1 fm (10^{-13} cm) which is the radius of the strong interactions. Unlike the electromagnetic and weak interactions a necessary condition to activate the strong interaction is that hadrons enter into a condition of mutual interpenetration. In view of the developments below, we would like to stress that the modification of the symmetries of particles under conditions of possible fusion, is the first step for the usual developments of fusion theories which have been represented in terms of diffusion processes that overcome the Coulomb repulsive potential which impedes the fusion [122]; Brownian motions and other stochastic processes also appear in a phenomenological approach to the many body problem in particle and nuclear physics, but with no hint as to the possibility of an underlying space-time structure [151]. The basic idea goes back to the foundational works of Smoluchowski (independently of A. Einstein's work in the subject) in Brownian motion [123]. In the case of fusion theories, we have a gas of neutrons (which have an internal structure) and electrons, or an hadron gas; in these cases the fused particles are considered to be alike a compressible fluid with an unstable neck in its fused drops which have to be stabilized to achieve effective fusion; we can see here the figure of deformed symmetries. Thus, the situation for the application of Brownian motion to fusion is a natural extension to the subatomic scale of the original theory. We finally notice that the models for fusion in terms of diffusion do not require QM nor QCD [122]. In contrast, HM stems from symmetry group transformations that

describe the contact fusion processes that deform the neutron structure, and lead to the isotopic Schroedinger equation which in this article, together with the isotopic Heisenberg representation, will be applied to establish a link between the RCW geometries, fusion processes and diffusions. The reason for the use of the iso-Heisenberg representation, is that in Santilli's theory, the isotopic lift of the symmetries in carried out in terms of the Heisenberg-Santilli isorepresentation, where its connection with classical mechanics under the quantization rules including the isotopic lift is transparent. Similarly to QM it will turn out to be that this quantization that leads to HM can be framed in another terms, i.e. Brownian motions appear to be quantum representations with no need of a quantization of classical mechanics, which can nevertheless be achieved by taking in account the fluctuations represented by the noise tensor of these random motions.

3.F.2 Riemann-Cartan-Weyl Geometries

In this section we follow [94, 95]. In this appendix M denotes a smooth connected compact orientable n-dimensional manifold (without boundary). While in our initial works, we took for M to be space-time, there is no intrinsic reason for this limitation, in fact if can be an arbitrary configuration manifold and still a phase-space associated to a dynamical system. The paradigmatical example of the latter, is the projective space associated to a finite-dimensional Hilbert-space of a quantum mechanical system [96, 98]. We shall further provide M with an affine connection, or still by a covariant derivative operator ∇ which we assume to be compatible with a given metric g on M, i.e. $\nabla g = 0$. Here, the metric can be the Minkowski degenerate metric, or an arbitrary positive-definite (i.e. Riemannian) metric. Given a coordinate chart (x^α) $(\alpha = 1, \ldots, n)$ of M, a system of functions on M (the Christoffel symbols of ∇) are defined by $\nabla_{\frac{\partial}{\partial x^\beta}} \frac{\partial}{\partial x^\gamma} = \Gamma(x)^\alpha_{\beta\gamma} \frac{\partial}{\partial x^\alpha}$. The Christoffel coefficients of ∇ can be decomposed as:

$$\Gamma^\alpha_{\beta\gamma} = \left\{ \begin{matrix} \alpha \\ \beta\gamma \end{matrix} \right\} + \frac{1}{2} K^\alpha_{\beta\gamma}. \tag{3.F.1}$$

The first term in (3.F.1) stands for the metric Christoffel coefficients of the Levi-Civita connection ∇^g associated to g, i.e. $\left\{ \begin{matrix} \alpha \\ \beta\gamma \end{matrix} \right\} = \frac{1}{2}(\frac{\partial}{\partial x^\beta} g_{\nu\gamma} + \frac{\partial}{\partial x^\gamma} g_{\beta\nu} - \frac{\partial}{\partial x^\nu} g_{\beta\gamma}) g^{\alpha\nu}$, and

$$K^\alpha_{\beta\gamma} = T^\alpha_{\beta\gamma} + S^\alpha_{\beta\gamma} + S^\alpha_{\gamma\beta} \tag{3.F.2}$$

is the cotorsion tensor, with $S^\alpha_{\beta\gamma} = g^{\alpha\nu} g_{\beta\kappa} T^\kappa_{\nu\gamma}$, and $T^\alpha_{\beta\gamma} = (\Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta})$ is the skew-symmetric torsion tensor. We are interested in (one-half) the Laplacian operator associated to ∇, i.e. the operator acting on smooth functions on M defined as

$$H(\nabla) := 1/2 \nabla^2 = 1/2 g^{\alpha\beta} \nabla_\alpha \nabla_\beta. \tag{3.F.3}$$

A straightforward computation shows that $H(\nabla)$ only depends in the trace of the torsion tensor and g, since it is

$$H(\nabla) = 1/2\triangle_g + \hat{Q} \equiv H(g, Q), \tag{3.F.4}$$

with $Q := Q_\beta dx^\beta = T^\nu_{\nu\beta} dx^\beta$ the trace-torsion one-form and $\hat{Q}$ is the vector field associated to Q via g (the so-called g conjugate vector field to the one-form Q, i.e.

$$\hat{Q}(f) = g(Q, df), \tag{3.F.5}$$

for any smooth function f defined on M. Finally, $\triangle_g$ is the Laplace-Beltrami operator of g:

$$\triangle_g = g^{\alpha\beta} \nabla^g_{\frac{\partial}{\partial x^\alpha}} \nabla^g_{\frac{\partial}{\partial x^\beta}} = g^{\alpha\beta} \frac{\partial^2}{\partial x^\alpha \partial x^\beta} - g^{\alpha\beta} \left\{ \begin{array}{c} \gamma \\ \alpha\beta \end{array} \right\} \frac{\partial}{\partial x^\gamma}. \tag{3.F.6}$$

In this expression the partial derivatives are taken with respect to the Levi-Civita connection. Therefore, assuming that g is non-degenerate, we have defined a one-to-one mapping

$$\nabla \rightsquigarrow H(g, Q) = 1/2\triangle_g + \hat{Q}$$

between the space of g-compatible linear connections ∇ with Christoffel coefficients of the form

$$\Gamma^\alpha_{\beta\gamma} = \left\{ \begin{array}{c} \alpha \\ \beta\gamma \end{array} \right\} + \frac{2}{(n-1)} \left\{ \delta^\alpha_\beta \, Q_\gamma - g_{\beta\gamma} \, Q^\alpha \right\}, \quad n \neq 1 \tag{3.F.7}$$

and the space of elliptic second order differential operators on functions. The extensions of this laplacian to differential forms and in particular, to fluid-dynamics, has been presented in [94, 97].

3.F.3 Riemann-Cartan-Weyl Diffusions

In this section we shall recall the correspondence between RCW connections defined by (3.F.7) and diffusion processes of scalar fields having $H(g, Q)$ as its diffrential generator. Thus, naturally we have called these processes as *RCW diffusion processes*. For the extensions to describe the diffusion processes of differential forms, see [94, 97]. For the sake of generality, in the following we shall further assume that $Q = Q(\tau, x)$ is a time-dependent 1-form. In this setting τ is the universal time variable due to Stuckelberg [101]; for a very sharp account of the relation of this time to Einsten's time, t, we refer to Horwitz et al. [118]. The stochastic flow associated to the diffusion generated by $H(g, Q)$ has for sample paths the continuous curves $\tau \mapsto x(\tau) \in M$ satisfying the Itô invariant non-degenerate s.d.e. (stochastic differential equation)

$$dx(\tau) = \sigma(x(\tau)) dW(\tau) + \hat{Q}(\tau, x(\tau)) d\tau. \tag{3.F.8}$$

In this expression, $\sigma : M \times R^m \to TM$ is such that $\sigma(x) : R^m \to TM$ is linear for any $x \in M$, the noise tensor, so that we write $\sigma(x) = (\sigma_i^\alpha(x))$ $(1 \leq \alpha \leq n,$ $1 \leq i \leq m)$ which satisfies

$$\sigma_i^\alpha \sigma_i^\beta = g^{\alpha\beta}, \tag{3.F.9}$$

where $g = (g^{\alpha\beta})$ is the expression for the metric in covariant form, and $\{W(\tau),$ $\tau \geq 0\}$ is a standard Wiener process on R^m, with zero mean with respect to the standard centered Gaussian function, and covariance given by $\mathrm{diag}(\tau, \ldots, \tau)$; finally, $dW(\tau) = W(\tau + d\tau) - W(\tau)$ is an increment. Now, it is important to remark that m can be arbitrary, i.e. we can take noise tensors defined on different spaces, and obtain the essentially the same diffusion process [116]. In regards to the equivalence between the stochastic and the geometric picture, this enhances the fact that there is a freedom in the stochastic picture, which if chosen as the originator of the equivalence, points out to a more fundamental basis of the stochastic description. This is satisfactory, since it is impossible to identify all the sources for noise, and in particular those coming from the vacuum, which we take as the source for the randomness. Note that in taking the drift and the diffusion tensor as the original objects to build the geometry, the latter is derived from objects which are associated to *collective* phenomena. Note that if we start with Eq. (3.F.8), we can reconstruct the associated RCW connection by using Eq. (3.F.9) and the fact that the trace-torsion is the g-conjugate of the drift, i.e., in simple words, by lowering indexes of $\hat{Q}$ to obtain Q. We shall not go into the details of these constructions, which relies heavily on stochastic analysis on smooth manifolds [116].

Observations 1. Note that in the above construction of the s.d.e. all terms corresponding to the Levi-Civita connection $\left\{ {\alpha \atop \beta\gamma} \right\}$ have disappeared completely. In fact one can start with a Laplacian written without these terms, say

$$H := 1/2 g^{\alpha\beta} \frac{\partial^2}{\partial x^\alpha \partial x^\beta} + \hat{Q}^\alpha \partial_\alpha, \tag{3.F.10}$$

and rewrite it as

$$\frac{1}{2} \triangle_g + \tilde{b}^\alpha \partial_\alpha \tag{3.F.11}$$

with

$$\tilde{b}^\alpha = \hat{Q}^\alpha + \frac{1}{2} g^{\beta\gamma} \left\{ {\alpha \atop \beta\gamma} \right\}; \tag{3.F.12}$$

we then redefine the connection $\nabla = (\Gamma^\gamma_{\alpha\beta})$ to be compatible with g and such that $\tilde{b}^\alpha = \frac{1}{2}[g^{\beta\gamma} \left\{ {\alpha \atop \beta\gamma} \right\} - \Gamma^\alpha_{\beta\gamma}]$ so that finally our original RCW laplacian $H(\nabla)$ takes the form $H(g, \tilde{b})$ of Eq. (3.F.4) and the s.d.e. is given by (3.F.8); c.f. pages 285–289 in Ideda & Watanabe [116]. From this follows that we can write the laplacians either with the Levi-Civita covariant derivative or the usual derivative for characterizing

the diffusion processes corresponding to the Schroedinger equation; this is also valid for the iso-Schroedinger equations, starting by producing the isotopic lift of the differential operator, or further, the isotopic lift of the covariant derivative operator, the isocovariant differential introduced in Section 3.2.9.C above.

3.F.4 RCW Geometries, Brownian Motions and the Schroedinger Equation

We have shown that we can represent the space-time quantum geometries for the relativistic diffusion associated with the invariant distribution, so that $Q = \frac{1}{2}d\ln\rho$, with $\rho = \psi^2$ and $H(g, Q)$ has a self-adjoint extension for which we can construct the quantum geometry on state-space and still the stochastic extension of the Schroedinger equation defined by this operator on taking the analytical continuation on the time variable for the evolution parameter [96]. In this section which retakes the solution of the Schroedinger problem of interpolation by Nagasawa [107], we shall present the equivalence between RCW geometries, their Brownian motions and the Schroedinger equation which is a different approach to taking the analytical continuation in time, which by the way, has a very important significance in terms of considering time to be an active variable; see [96]. We shall now present the construction of non-relativistic QM with the restriction that the Hodge decomposition of the trace-torsion restricts to its exact component, excluding thus the electromagnetic potential terms of the full trace-torsion which we considered in [95, 96]. So that we take $Q = Q(t, x) = d\ln f_t(x)$ where $f(t, x) = f_t(x)$ is a function defined on the configuration manifold given by $[a, b] \times M$, where M is a 3-dimensional manifold provided with a metric, g. The construction applies as well to the general case as well, as we shall show further below. The scheme to determine f will be to manifest the time-reversal invariance of the Schroedinger representation in terms of a forward in time diffusion process and its time-reversed representation for the original equations for creation and annihilation diffusion processes produced when there is no background torsion field, whose explicit form and relation to f we shall determine in the sequel. From now onwards, the exterior differential, the divergence operator and the laplacian will act on the M manifold variables only, so that we shall write their action on fields, say $df_t(x)$, to signal that the exterior differential acts only on the x variables of M. We should remark that in this context, the time-variable t of non-relativistic theory and the evolution parameter τ, are identical [118]. Let

$$L = \frac{\partial}{\partial t} + \frac{1}{2}\triangle_g = \frac{\partial}{\partial t} + H(g, 0). \tag{3.F.13}$$

Let $p(s, x; t, y)$ be the weak fundamental solution of

$$L\phi + c\phi = 0. \tag{3.F.14}$$

The interpretation of this equation as one of creation (whenever $c > 0$) and annihilation ($c < 0$) of particles is warranted by the Feynman-Kac representation for the solution of this equation [107]. Then $\phi = \phi(t, x)$ satisfies the equation

$$\phi(s, x) = \int_M p(s, x; t, y)\phi(t, y)dy, \tag{3.F.15}$$

where for the sake of simplicity, we shall write in the sequel $dy = \mathrm{vol_g(y)} = \sqrt{\det(\mathrm{g})}dy^1 \wedge \ldots \wedge dy^3$. Note that we can start for data with a given function $\phi(a, x)$, and with the knowledge of $p(s, x; a, y)$ we define $\phi(t, x) = \int_M p(t, x; a, y)dy$. Next we define

$$q(s, x; t, y) = \frac{1}{\phi(s, x)}p(s, x; t, y)\phi(t, y), \tag{3.F.16}$$

which is a transition probability density, i.e.

$$\int_M q(s, x; t, y)dy = 1, \tag{3.F.17}$$

while

$$\int_M p(s, x; t, y)dy \neq 1. \tag{3.F.18}$$

Having chosen the function $\phi(t, x)$ in terms of which we have defined the probability density $q(s, x; t, y)$ we shall further assume that we can choose a second bounded non-negative measurable function $\breve{\phi}(a, x)$ on M such that

$$\int_M \phi(a, x)\breve{\phi}(a, x)dx = 1, \tag{3.F.19}$$

We further extend it to $[a, b] \times M$ by defining

$$\breve{\phi}(t, y) = \int \breve{\phi}(a, x)p(a, x; t, y)dx, \forall (t, y) \in [a, b] \times M, \tag{3.F.20}$$

where $p(s, x; t, y)$ is the fundamental solution of Eq. (3.F.14).

Let $\{X_t \in M, \mathcal{Q}\}$ be the time-inhomogeneous diffusion process in M with the transition probability density $q(s, x; t, y)$ and a prescribed initial distribution density

$$\mu(a, x) = \breve{\phi}(t = a, x)\phi(t = a, x) \equiv \breve{\phi}_a(x)\phi_a(x). \tag{3.F.21}$$

The finite-dimensional distribution of the process $\{X_t \in M, t \in [a, b]\}$ with probability measure on the space of paths which we denote as Q; for $a = t_0 < t_1 <$

$\ldots < t_n = b$, it is given by

$$E_Q[f(X_a, X_{t_1}, \ldots, X_{t_n-1}, X_b)] = \int_M dx_0 \mu(a, x_0) q(a, x_0; t_1, x_1) dx_1 \ldots q(t_1, x_1; t_2, x_2) dx_2 \ldots q(t_{n-1}, x_{n-1}, b, x_n) dx_n f(x_0, x_1, \ldots, x_{n-1}, x_n) := [\mu_a q >> \quad (3.F.22)$$

which is the Kolmogorov forward in time (and thus time-irreversible) representation for the diffusion process with initial distribution $\mu_a(x_0) = \mu(a, x_0)$, which using Eq. (3.F.16) can still be rewritten as

$$\int_M dx_0 \mu_a(x_0) \frac{1}{\phi_a(x_0)} p(a, x_0; t_1, x_1) \phi_{t_1}(x_1) dx_1 \frac{1}{\phi_{t_1}(x_1)} dx_1 p(t_1, x_1; t_2, x_2) \phi_{t_2}(x_2) dx_2 \ldots \frac{1}{\phi(t_{n-1}, x_{n-1})} p(t_{n-1}, x_{n-1}; b, x_n) \phi_b(x_n) dx_n f(x_0, \ldots, x_n) \quad (3.F.23)$$

which in account of $\mu_a(x_0) = \breve{\phi}_a(x_0)\phi_a(x_0)$ and Eq. (3.F.16) can be written in the time-reversible form

$$\int_M \breve{\phi}_a(x_0) dx_0 p(a, x_0; t_1, x_1) dx_1 p(t_1, x_1; t_2, x_2) dx_2 \ldots p(t_{n-1}, x_{n-1}; b, x_n) \phi_b(x_n) dx_n f(x_0, \ldots, x_n) \quad (3.F.24)$$

which we write as

$$= [\breve{\phi}_a p >><< p\phi_b]. \quad (3.F.25)$$

This is the *formally* time-symmetric Schroedinger representation with the transition (but not probability) density p. Here, the formal time symmetry is seen in the fact that this equation can be read in any direction, preserving the physical sense of transition. This representation, in distinction with the Kolmogorov representation, does *not* have the Markov property.

We define the adjoint transition probability density $\breve{q}(s, x; t, y)$ with the $\breve{\phi}$-transformation

$$\breve{q}(s, x; t, y) = \breve{\phi}(s, x) p(s, x; t, y) \frac{1}{\breve{\phi}(t, y)} \quad (3.F.26)$$

which satisfies the Chapmann-Kolmogorov equation and the time-reversed normalization

$$\int_M dx \breve{q}(s, x; t, y) = 1. \quad (3.F.27)$$

We get

$$E_{\breve{Q}}[f(X_a, X_{t_1}, \ldots, X_b)] = \int_M f(x_0, \ldots, x_n) \breve{q}(a, x_0; t_1, x_1) dx_1 \breve{q}(t_1, x_1; t_2, x_2) dx_2 \ldots \breve{q}(t_{n-1}, x_{n-1}; b, x_n) \breve{\phi}(b, x_n) \phi(b, x_n) dx_n, \quad (3.F.28)$$

which has a form non-invariant in time, i.e. reading from right to left, as

$$<< \breve{q}\hat{\phi}_b\phi_b] =<< \breve{q}\hat{\mu}_b], \tag{3.F.29}$$

which is the time-reversed representation for the final distribution $\mu_b(x) = \breve{\phi}_b(x)\phi_b(x)$. Now, starting from this last expression and rewriting it in a similar form that is in the forward process but now with $\breve{\phi}$ instead of ϕ, we get

$$\begin{aligned}\int_M dx_0\breve{\phi}_a(x_0)p(a,x_0;t_1,x_1)\frac{1}{\breve{\phi}_{t_1}(x_1))}dx_1\breve{\phi}(t_1,x_1)p(t_1,x_1;t_2,x_2)\frac{1}{\breve{\phi}_{t_2}(x_2)}dx_2 \\ \dots dx_{n-1}\breve{\phi}(t_{n-1},x_{n-1})p(t_{n-1},x_{n-1};b,x_n) \\ \frac{1}{\breve{\phi}(b,x_n)}\breve{\phi}_b(x_n)\phi(b,x_n)dx_n f(x_0,\dots,x_n)\end{aligned} \tag{3.F.30}$$

which coincides with the time-reversible Schroedinger representation

$$[\breve{\phi}_a p >><< p\phi_b].$$

We therefore have three equivalent representations for the diffusion process: the forward in time Kolmogorov representation, the backward Kolmogorov representation, which are both naturally irreversible in time, and the time-reversible Schroedinger representation, so that we can write succintly,

$$[\mu_a q >>= [\breve{\phi}_a p >><< p\phi_b] =<< \breve{q}\mu_b], \text{with } \mu_a = \phi_a\breve{\phi}_a,\ \mu_b = \phi_b\breve{\phi}_b. \tag{3.F.31}$$

In addition of this formal identity,we have to establish the relations between the equations that have led to them. We first note, that in the Schroedinger representation, which is formally time-reversible, we have an interpolation of states between the initial data $\breve{\phi}_a(x)$ and the final data, $\phi_b(x)$. The information for this interpolation is given by a filtration of interpolation $\mathcal{F}_a^r \cup \mathcal{F}_b^s$, which is given in terms of the filtration for the forward Kolmogorov representation $\mathcal{F} = \mathcal{F}_a^t, t \in [a,b]$ which is used for prediction starting with the initial density $\phi_a\breve{\phi}_a = \mu_a$ and the filtration $\mathcal{F}_t^b$ for retrodiction for the time-reversed process with initial distribution μ_b.

We observe that q and $\breve{q}$ are in time-dependent duality with respect to the measure

$$\mu_t(x)dx = \breve{\phi}_t(x)\phi_t(x)dx. \tag{3.F.32}$$

We shall now extend the state-space of the diffusion process to $[a,b] \times M$, to be able to transform the time-inhomogeneous processes into time-homogeneous processes, while the stochastic dynamics still takes place exclusively in M. This will allow us to define the duality of the processes to be with respect to $\mu_t(x)dtdx$ and to determine the form of the exact term of the trace-torsion, and ultimately,

to establish the relation between the diffusion processes and Schroedinger equations, both for potential linear and non-linear in the wave-functions. If we define time-homogeneous semigroups of the processes on $\{(t, X_t) \in [a, b] \times M\}$ by

$$P_r f(s,x) = \begin{cases} Q_{s,s+r} f(s,x), & s \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.F.33}$$

and

$$\breve{P}_r g(t,y) = \begin{cases} g Q_{t-r,t}(t,y), & r \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.F.34}$$

then

$$< g, P_r f >_{\mu_t dt dx} = < \breve{P}_r g, f >_{\mu_t dt dx}, \tag{3.F.35}$$

which is the duality of $\{(t, X_t)\}$ with respect to the $\mu_t dt dx$ density. We remark here that we have an augmented density by integrating with respect to time t. Consequently, if in our spacetime case we define for $a_t(x), \hat{a}_t(x)$ time-dependent one-forms on M (to be determined later)

$$B\alpha : = \frac{\partial \alpha}{\partial t} + H(g, a_t)\alpha_t, \tag{3.F.36}$$

$$B^0 \mu : = -\frac{\partial \mu}{\partial t} + H(g, a_t)^\dagger \mu_t, \tag{3.F.37}$$

and its adjoint operators

$$\breve{B}\beta = -\frac{\partial \beta}{\partial t} - H(g, \breve{a}_t)^\dagger \beta_t, \tag{3.F.38}$$

$$(\breve{B})^0 \mu_t = \frac{\partial \mu_t}{\partial t} - H(g, \breve{a}_t)^\dagger \mu_t, \tag{3.F.39}$$

where by $H(g, \breve{a}_t)^\dagger$ we mean the vol_g-adjoint of this operator, i.e. $H(g, \breve{a}_t)^\dagger \mu_t = \frac{1}{2} \triangle_g \mu_t - \mathrm{div_g}(\mu \breve{a}_t)$. From [96, 107] follows that the duality of space-time processes

$$< B\alpha, \beta >_{\mu_t(x) dt dx} = < \alpha, \breve{B}\beta >_{\mu_t(x) dt dx}, \tag{3.F.40}$$

is equivalent to

$$a_t(x) + \breve{a}_t(x) = d \ln \mu_t(x) \equiv d \ln (\phi_t(x) \breve{\phi}_t(x)), \tag{3.F.41}$$

$$B^0 \mu_t(x) = 0. \tag{3.F.42}$$

The latter equation being the Fokker-Planck equation for the diffusion with trace-torsion given by $a + A$, then the Fokker-Planck equation for the adjoint (time-reversed) process is valid, i.e.

$$(\breve{B})^0 \mu_t(x) = 0. \tag{3.F.43}$$

Substracting Eqs. (3.F.39) and (3.F.40) we get the final form of the duality condition

$$\frac{\partial \mu}{\partial t} + \mathrm{div}_g[(\frac{a_t - \breve{a}_t}{2})\mu_t)] \quad = \quad 0, \text{ for } \mu_t(x) = \breve{\phi}_t(x)\phi_t(x). \tag{3.F.44}$$

Therefore, we can establish that the duality conditions of the diffusion equation in the Kolmogorov representation and its time reversed diffusion lead to the following conditions on the additional elements of the drift vector fields:

$$a_t(x) + \breve{a}_t(x) = d\ \ln\ \mu_t(x) \equiv d\ \ln\ (\phi_t(x)\breve{\phi}_t(x)), \tag{3.F.45}$$

$$\frac{\partial \mu}{\partial t} + \mathrm{div}_g[(\frac{a_t(x) - \breve{a}_t(x)}{2})\mu_t(x)] = 0. \tag{3.F.46}$$

If we assume that $a_t - \hat{a}_t$ is an exact one-form, i.e., there exists a time-dependent differentiable function $S(t,x) = S_t(x)$ defined on $[a,b]\times M$ such that for $t \in [a,b]$,

$$a_t(x) - \breve{a}_t(x) = d\ \ln\ \frac{\phi_t(x)}{\breve{\phi}_t(x)} = 2dS_t(x) \tag{3.F.47}$$

which together with

$$a_t(x) + \breve{a}_t(x) = d\ \ln\ \mu_t(x), \tag{3.F.48}$$

implies that on $D(t,x)$ we have

$$a_t(x) \quad = \quad d\ \ln\ \phi_t(x), \tag{3.F.49}$$

$$\breve{a}_t(x) \quad = \quad d\ \ln\ \breve{\phi}_t(x). \tag{3.F.50}$$

Introduce now $R_t(x) = R(t,x) = \frac{1}{2}\ln\phi_t(x)\breve{\phi}_t(x)$ and $S_t(x) = S(t,x) = \frac{1}{2}\ln\frac{\phi_t(x)}{\breve{\phi}_t(x)}$, so that

$$a_t(x) \quad = \quad d(R_t(x) + S_t(x)), \tag{3.F.51}$$

$$\breve{a}_t(x) \quad = \quad d(R_t(x) - S_t(x)), \tag{3.F.52}$$

and Eq. (3.F.46) takes the form

$$\frac{\partial R}{\partial t} + \frac{1}{2}\triangle_g S_t + g(dS_t, dR_t) = 0. \tag{3.F.53}$$

Remarks. We have mentioned the fact that there is a hidden active role of time in QM [145], which in the above construction is built-in the very definition of the probability density in terms of a *final* and initial distributions. This back action of time appears to be not exclusive of QM. In the theory of growth of sea shells due to Santilli and Illert, it was shown that it cannot be explained by Minkowskian nor Euclidean geometry, but their isotopic lifts and their duals,

and this requires the introduction of time duality and four-fold time [148]; this model has been further applied to diverse problems of morphology in biology by Reverberi [149]. We further note that the time-dependent function S on the 3-space manifold, is defined by Eq. (3.F.47) up to addition of an arbitrary function of t, and when further below we shall take this function as defining the complex phase of the quantum Schroedinger wave, this will introduce the quantum-phase indetermination of the quantum evolution, as we discussed already in the setting of geometry of the quantum state-space [96, 98].

Therefore, together with the three different time-homogeneous representations $\{(t, X_t), t \in [a, b], X_t \in M\}$ of a time-inhomogeneous diffusion process $\{X_t, Q)$ on M we have three equivalent dynamical descriptions. One description, with creation and killing described by the scalar field $c(t, x)$ and the diffusion equation describing it is given by a creation-destruction potential in the trace-torsion background given by an electromagnetic potential

$$\frac{\partial p}{\partial t} + H(g, 0)(x)p + c(t, x)p = 0; \tag{3.F.54}$$

the second description has an additional trace-torsion $a(t, x)$, a 1-form on $R \times M$

$$\frac{\partial q}{\partial t} + H(g, a_t)q = 0. \tag{3.F.55}$$

while the third description is the adjoint time-reversed of the first representation given by $\breve{\phi}$ satisfying the diffusion equation on the background with no torsion, i.e.

$$-\frac{\partial \breve{\phi}}{\partial t} + H(g, 0)\breve{\phi} + c\breve{\phi} = 0. \tag{3.F.56}$$

The second representation for the full trace-torsion diffusion forward in time Kolmogorov representation, we need to adopt the description in terms of the fundamental solution q of

$$\frac{\partial q}{\partial t} + H(g, a_t)q = 0, \tag{3.F.57}$$

for which one must start with the initial distribution $\mu_a(x) = \breve{\phi}_a(x)\phi_a(x)$. This is a time t-irreversible representation in the real world, where q describes the real transition and μ_a gives the initial distribution. If in addition one traces the diffusion backwards with reversed time t, with $t \in [a, b]$ running backwards, one needs for this the final distribution $\mu_b(x) = \breve{\phi}_b(x)\phi_b(x)$ and the time t reversed probability density $\hat{q}(s, x; t, y)$ which is the fundamental solution of the equation

$$-\frac{\partial \breve{q}}{\partial t} + H(g, \breve{a}_t)\hat{q} = 0, \tag{3.F.58}$$

with additional trace-torsion one-form on $R \times M$ given by $\hat{a}$, where

$$\breve{a}_t + a_t = d\ln\mu_t(x), \text{ with } \mu_t = \phi_t\breve{\phi}_t, \tag{3.F.59}$$

where the diffusion process in the time-irreversible forward Kolmogorov representation is given by the Ito s.d.e

$$dX_t^i = \sigma_j^i(X_t)dW_t^j + a^i(t, X_t)dt, \tag{3.F.60}$$

and the backward representation for the diffusion process is given by

$$dX_t^i = \sigma_j^i(X_t)dW_t^j + \breve{a}^i(t, X_t)dt, \tag{3.F.61}$$

where $a, \breve{a}$ are given by the Eqs. (3.F.51), (3.F.52), and $(\sigma\sigma^\dagger)^{\alpha\beta} = g^{\alpha\beta}$.

We follow Schroedinger in pointing that ϕ and $\breve{\phi}$ separately satisfy the creation and killing equations, while in quantum mechanics ψ and $\bar{\psi}$ are the complex-valued counterparts of ϕ and $\hat{\phi}$, respectively, they are not arbitrary but

$$\phi\breve{\phi} = \psi\bar{\psi}. \tag{3.F.62}$$

Thus, in the following, this Born formula, once the equations for ψ are determined, will be a consequence of the constructions, and not an hypothesis on the random basis of non-relativistic mechanics.

Therefore, the equations of motion given by the Ito s.d.e.

$$dX_t^i = \text{grad}_g\phi^i(t, X_t)dt + \sigma_j^i(X_t)dW_t^j, \tag{3.F.63}$$

which are equivalent to

$$\frac{\partial u}{\partial t} + H(g, a_t)u = 0 \tag{3.F.64}$$

with $a_t(x) = d\ln\phi_t(x) = d(R_t(x) + S_t(x))$, determines the motion of the ensemble of non-relativistic particles. Note that this equivalence requires only the Laplacian for the RCW connection with the forward trace-torsion full one-form

$$Q(t, x) = d\ln\phi_t(x) = d(R_t(x) + S_t(x)). \tag{3.F.65}$$

In distinction with Stochastic Mechanics due to Nelson [102], and contemporary ellaborations of this applied to astrophysics as the theory of Scale Relativity due to Nottale [114, 106], we only need the form of the trace-torsion for the forward Kolmogorov representation, and this turns to be equivalent to the Schroedinger representation which interpolates in time-symmetric form between this forward process and its time dual with trace-torsion one-form given by $\breve{a}_t(x) = d\ln\breve{\phi}_t(x) = d(R_t(x) - S_t(x))$.

Finally, let us how this is related to the Schroedinger equation. Consider now the Schroedinger equations for the complex-valued wave function ψ and its complex conjugate $\bar{\psi}$, i.e. introducing $i = \sqrt{-1}$, we write them in the form

$$i\frac{\partial \psi}{\partial t} + H(g,0)\psi - V\psi = 0, \tag{3.F.66}$$

$$-i\frac{\partial \bar{\psi}}{\partial t} + H(g,0)\bar{\psi} - V\bar{\psi} = 0, \tag{3.F.67}$$

which are identical to the usual forms. So, we have the imaginary factor appearing in the time t, which we confront with the diffusion equations generated by the RCW connection with null trace-torsion, i.e. the system

$$\frac{\partial \phi}{\partial t} + H(g,0)\phi + c\phi = 0, \tag{3.F.68}$$

$$\frac{-\partial \breve{\phi}}{\partial t} + H(g,0)\breve{\phi} + c\breve{\phi} = 0, \tag{3.F.69}$$

and the diffusion equations determined by both the RCW connections with trace-torsion a and $\breve{a}$, i.e.

$$\frac{\partial q}{\partial t} + H(g,a_t)q = 0, \tag{3.F.70}$$

$$\frac{-\partial \breve{q}}{\partial t} + H(g,\breve{a}_t)\hat{q} = 0, \tag{3.F.71}$$

which are equivalent to the single equation

$$\frac{\partial q}{\partial t} + H(g,d\ln\phi_t)q = 0. \tag{3.F.72}$$

If we introduce a complex structure on the two-dimensional real-space with co-ordinates (R,S), i.e. we consider

$$\psi = e^{R+iS}, \quad \psi = e^{R-iS}, \tag{3.F.73}$$

viz a viz $\phi = e^{R+S}$, $\breve{\phi} = e^{R-S}$, with $\psi\bar{\psi} = \phi\breve{\phi}$, then for a wave-function differentiable in t and twice-differentiable in the space variables, then, ψ satisfies the Schroedinger equation if and only if (R,S) satisfy the difference between the Fokker-Planck equations, i.e.

$$\frac{\partial R}{\partial t} + g(dS_t, dR_t) + \frac{1}{2}\triangle_g S_t = 0, \tag{3.F.74}$$

and

$$V = -\frac{\partial S}{\partial t} + H(g,dR_t)R_t - \frac{1}{2}g(dS_t,dS_t), \tag{3.F.75}$$

which follows from substituting ψ in the Schroedinger equation and further dividing by ψ and taking the real part and imaginary parts, to obtain the former and latter equations, respectively.

Conversely, if we take the coordinate space given by $(\phi, \breve{\phi})$, both non-negative functions, and consider the domain $D = D(s,x) = \{(s,x) : 0 < \breve{\phi}(s,x)\phi(s,x)\} \subset [a,b] \times M$ and define $R = \frac{1}{2}\ln\phi\breve{\phi}, S = \frac{1}{2}\ln\frac{\phi}{\breve{\phi}}$, with R, S having the same differentiabilty properties that previously ψ, then $\phi = e^{R+S}$ satisfies in D the equation

$$\frac{\partial\phi}{\partial t} + H(g,0)\phi + c\phi = 0, \tag{3.F.76}$$

if and only if

$$\begin{aligned} -c &= [-\frac{\partial S}{\partial t} + H(g, dR_t)R_t - \frac{1}{2}g(dS_t, dS_t)] \\ &+ [\frac{\partial R}{\partial t} + H(g, dR_t)S_t] + [2\frac{\partial S}{\partial t} + g(dS_t, dS_t)], \end{aligned} \tag{3.F.77}$$

while $\breve{\phi} = e^{R-S}$ satisfies in D the equation

$$-\frac{\partial\phi}{\partial t} + H(g,0)\breve{\phi} + c\breve{\phi} = 0, \tag{3.F.78}$$

if and only if

$$\begin{aligned} -c &= [-\frac{\partial S}{\partial t} + H(g, dR_t)R_t - \frac{1}{2}g(dS_t, dS_t)] \\ &- [\frac{\partial R}{\partial t} + H(g, dR_t)S_t] + [2\frac{\partial S}{\partial t} + g(dS_t, dS_t)]. \end{aligned} \tag{3.F.79}$$

Notice that $\phi, \breve{\phi}$ can be both negative or positive. So if we define $\psi = e^{R+iS}$, it then defines in weak form the Schroedinger equation in D with

$$V = -c - 2\frac{\partial S}{\partial t} - g(dS_t, dS_t). \tag{3.F.80}$$

Remarks. We note that from Eq. (3.F.80) follows that we can choose S in a way such that either c is independent of S and thus V is a potential which is non-linear in the sense that it depends on the phase of the wave function ψ and thus the Schroedinger equation with this choice becomes non-linear dependent of ψ, or conversely, we can make the alternative choice of c depending non-linearly on S, and thus the creation-annihilation of particles in the diffusion equation is non-linear, and consequently the Schroedinger equation has a potential V which does not depend on ψ. It is important for further developments in this article that the non-linear Schroedinger equation can be turned into the iso-linear iso-Schroedinger equation by taking the non-linear terms of the potential into the

isotopic generalized unit. Indeed, the recovery of linearity in isohilbert space is achieved by the embedding of the nonlinear terms in the isounit as shown in [46]; see Eqs. (3.4.42) and (3.4.43).

3.F.4.1 Santilli-Lie Isotopies of the Differential Calculus and Metric Structures, and the Iso-Schroedinger Equation

To present the iso-Schroedinger equation, we need the Santilli-Lie-isotopic differential calculus [109, 46] and the isotopic lift of manifolds, the so-called isomanifolds, due to Tsagas and Sourlas [22]; we shall follow here the notations of Section 3.2 above. We start by considering the manifold M to be a vector space with local coordinates, which for simplicity we shall from now fix them to be a *contravariant* system, $x = (x^i), i = 1, \ldots, n$, unit given by $I = \text{diag}(1, \ldots, 1)$ and metric g which we assumed diagonalized. We shall lift this structure to a vector space $\hat{M}$ provided with isocoordinates $\hat{x}$, isometric $\hat{G}$ and defined on Santilli isonumber field $\hat{F}$, where F can be the real or complex numbers; we denote this isospace by $\hat{M}(\hat{x}, \hat{G}, \hat{F})$. The isocoordinates are introduced by the transformation $x \mapsto U \times x \times U^\dagger = x \times \hat{I} := \hat{x}$. To introduce the *contravariant* isometric $\hat{G}$ we start by considering the transformation[38]

$$g \mapsto U \times g \times U^\dagger = \hat{I} \times g := \hat{g}. \tag{3.F.81}$$

Yet from the Definition 3.2.3 follows that the isometric is more properly defined by $\hat{G} = \hat{g} \times \hat{I}$. Thus we have a transformed $M(x, g, F)$ into the isospace $\hat{M}(\hat{x}, \hat{G}, \hat{F})$. Thus the projection on $M(x, g, F)$ of the isometric in $\hat{M}(\hat{x}, \hat{G}, \hat{F})$ is defined by a contravariant tensor, $\hat{g} = (\hat{g}^{ij})$ with components

$$\hat{g}^{ij} = (\hat{I} \times g)^{ij}. \tag{3.F.82}$$

If we take $\hat{I} = \psi^2(x) \times I$ we then retrieve the Weyl scale transformations, with ψ a scale field depending only on the coordinates of M. If we start with g being the Euclidean or Minkowski metrics, we obtain the iso-Euclidean and iso-Minkowski metrics; in the case we start with a general metric as in GR, we obtain Isorelativity. We shall now proceed to identify the isotopic lift of the noise tensor σ which verifies Eq. (3.F.9), i.e. $\sigma \times \sigma^\dagger = g$. The non-unitary transform of (a diagonalized) σ is given by

$$\sigma \mapsto U \times \sigma \times U^\dagger = \sigma \times \hat{I} := \hat{\sigma}. \tag{3.F.83}$$

Then,

$$\hat{\sigma}\hat{\times}\hat{\sigma} = \sigma \times \hat{I} \times \hat{T} \times (\sigma \times \hat{I})^\dagger = (\sigma \times \sigma^\dagger) \times \hat{I} = g \times \hat{I} = \hat{g}. \tag{3.F.84}$$

[38] We shall assume, as usual, a diagonal metric.

Thus the isotopic lift of the noise tensor defined on $\hat{M}(\hat{x}, \hat{G}, \hat{R})$ is given by $\hat{\sigma} = \sigma \times \hat{I}$ which on projection to $M(\hat{x}, \hat{G}, R)$ we retrieve σ. We know follow the notations and definitions of Section 3.2.5 for the isotopic differential, and for isofunctions. We introduce the isotopic gradient operator of the isometric $\hat{G}$ (the $\hat{G}$-gradient, for short), $\widehat{\text{grad}_{\hat{G}}}$ applied to the isotopic lift $\hat{f}(\hat{x})$ of a function $f(x)$ is defined by

$$\widehat{\text{grad}_{\hat{G}}}\hat{f}(\hat{x})(\hat{v}) = \hat{G}(\hat{d}\hat{f}(\hat{x})\hat{,}\hat{v}), \tag{3.F.85}$$

for any vector field $\hat{v} \in T_{\hat{x}}(\hat{M})$, $\hat{x} \in \hat{M}$; we have denoted the inner product as $\hat{,}$ to stress that the inner product is taken with respect to the product in $\hat{F}$. Hence, the operator $\widehat{\text{grad}_{\hat{G}}}\hat{f}(\hat{x})$ can be thought as the isovector field on the tangent manifold to $\hat{M}(\hat{x}, \hat{G}, \hat{F})$ defined by

$$\hat{G}^{\alpha\beta}\hat{\times}\frac{\hat{\partial}\hat{f}(\hat{x})}{\hat{\partial}\hat{x}^{\alpha}}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\beta}} = \hat{g}^{\alpha\beta}\hat{\times}\frac{\hat{\partial}\hat{f}(\hat{x})}{\hat{\partial}\hat{x}^{\alpha}}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{i}} \times \hat{I}. \tag{3.F.86}$$

Therefore, the projection on $\hat{M}(\hat{x}, \hat{g}, F)$ of the $\hat{G}$-gradient vector field of $\hat{f}(\hat{x})$ is the vector field with components

$$\hat{g}^{\alpha\beta}\hat{\times}\frac{\hat{\partial}\hat{f}(\hat{x})}{\hat{\partial}\hat{x}^{\alpha}} = \hat{g}^{\alpha\beta}\hat{\times}\frac{\hat{\partial}\hat{f}(\hat{x})}{\hat{\partial}\hat{x}^{\alpha}}. \tag{3.F.87}$$

This will be of importance for the determination of the drift vector field of the diffusion linked with the Santilli- iso-Schroedinger equation. We finally define the isolaplacian as

$$\hat{\triangle}_{\hat{g}} = \hat{g}^{\alpha\beta}\hat{\times}\hat{D}_{\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\alpha}}}\hat{\times}\hat{D}_{\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\beta}}}. \tag{3.F.88}$$

Here $\hat{D}_{\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\alpha}}}$ is defined accordingly with Definition 3.2.13 above, by (c.f. Eq. (3.F.6) above)

$$\hat{D}_{\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\alpha}}}\hat{X}^{\beta} = \frac{\hat{\partial}\hat{X}^{\beta}}{\hat{\partial}\hat{x}^{\beta}} + \widehat{\left\{\begin{matrix}\beta\\ \gamma\alpha\end{matrix}\right\}}\hat{\times}\hat{X}^{\gamma}, \tag{3.F.89}$$

and hence it is the isocovariant differential with respect to the Levi-Civita isoconnection with isoChristoffel coefficients

$$\widehat{\left\{\begin{matrix}\alpha\\ \beta\gamma\end{matrix}\right\}} = \frac{\hat{1}}{\hat{2}}(\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\beta}}\hat{g}_{\nu\gamma} + \frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\gamma}}\hat{g}_{\beta\nu} - \frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\nu}}\hat{g}_{\beta\gamma})\hat{\times}\hat{g}^{\alpha\nu}. \tag{3.F.90}$$

We remark that from Observations 1 follows that alternatively we can define the more simpler laplacian by taking instead

$$\hat{\triangle}_{\hat{g}} = \hat{g}^{\alpha\beta}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\alpha}}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\beta}}. \tag{3.F.91}$$

In both cases we take $\hat{\sigma}$ for the corresponding isonoise term in the isodiffusion representation. The latter definition of the isolaplacian differs from the original one introduced in [22].

3.F.4.2 Diffusions and the Heisenberg Representation

Up to now we have set our theory in terms of the Schroedinger representation, since the original setting for this theory has to do with scale transformations as introduced by Einstein in his last work [100] while it was recognized previously by London that the wave function was related to the Weyl scale transformation [138], and these scale fields turned to be in the non-relativistic case, nothing else than the wave function of Schroedinger equation, both in the linear and the non-linear cases. Historically the operator theory of QM was introduced before the Schroedinger equation, who later proved the equivalence of the two. The ensuing dispute and rejection by Heisenberg of Schroedinger's equation is a dramatic chapter of the history of QM [125]. It turns out to be the case that we can connect the Brownian motion approach to QM and the operator formalism due to Heisenberg and Jordan, and its isotopic lift presented in Section 3.4.

Let us define the position operator as usual and the momentum operator by

$$q^k = x^k, \;\; p_{\mathcal{D}k} = \sigma \times \frac{\partial}{\partial x^k}, \tag{3.F.92}$$

which we call the diffusion quantization rule (the subscript $\mathcal{D}$ denotes diffusion) since we have a representation different to the usual quantization rule

$$p_k = -i \times \frac{\partial}{\partial x^k}, \tag{3.F.93}$$

with $\sigma = (\sigma_a^\alpha)$ the diffusion tensor verifying $(\sigma \times \sigma^\dagger)^{\alpha\beta} = g^{\alpha\beta}$ and substitute into the Hamiltonian function

$$H(p,q) = \frac{1}{2}\sum_{k=1}^{d}(p_k)^2 + \mathbf{v}(q), \tag{3.F.94}$$

this yields the formal generator of a diffusion semigroup in $C^2(R^d)$ or $L^2(R^d)$ which in our previous notation is written as $H(g,0)+v$. Thus, an operator algebra on $C^2(R^n)$ or $L^2(R^n)$ together with the postulate of the commutation relation (instead of the usual commutator relation of quantum mechanics $[p,q] = -i \times I$)

$$[p_{\mathcal{D}}, q] = p_{\mathcal{D}} \times q - q \times p_{\mathcal{D}} = \sigma \times I \tag{3.F.95}$$

this yields the diffusion equation

$$\frac{\partial \phi}{\partial t} \times \phi + \frac{1}{2}\sum_{k=1}^{d}(\sigma \frac{\partial}{\partial x^\alpha})^2 \times \phi + \mathbf{v} \times \phi = 0, \tag{3.F.96}$$

which coincides with the diffusion Eq. (3.F.54) provided that $c = \mathbf{v}$. Thus, in this approach, the operator formalism and the quantization postulates, allow to deduce the diffusion equation. If we start from either the diffusion process or the RCW geometry, without any quantization conditions we already have the equations of motion of the quantum system which are non other than the original diffusion equations, or equivalently, the Schroedinger equations. We stress the fact that these arguments are valid for both cases relative to the choice of the potential function V, i.e. if it depends nonlinearly on the wave function ψ, or acts linearly by multiplication on it. Further below, we shall use this modification of the Heisenberg representation of QM by the previous Heisenberg type representation for diffusion processes, to give an account of the diffusion processes that are associated with HM. This treatment differs from our original (inconsistent with respect to HM, as it turned to be proved in the later findings by Prof. Santilli) treatment of the relation between RCW geometries and diffusions presented in [119] in incorporating the isotopic lift of all structures.

Let us frame now isoquantization in terms of diffusion processes. Define isomomentum, $\hat{p}_{\mathcal{D}}$, by

$$\hat{p}_{\mathcal{D}k} = \hat{\sigma}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^k}, \text{ with } \hat{\sigma} = \sigma \times \hat{I}, \tag{3.F.97}$$

so that the kinetic term of the iso-Hamiltonian is

$$\begin{aligned} \hat{p}_{\mathcal{D}}\hat{\times}\hat{p}^{\dagger}_{\mathcal{D}} &= \hat{\sigma}\hat{\times}\hat{\sigma}^{\dagger}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}} \\ &= \hat{g}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}} = \hat{\triangle}_{\hat{g}}. \end{aligned} \tag{3.F.98}$$

We finally check the consistency of the construction by proving that it can be achieved via the non-unitary transformation

$$\begin{aligned} p_{\mathcal{D}_j} \mapsto U \times p_{\mathcal{D}_j} \times U^{\dagger} &= U \times \sigma \times \frac{\partial}{\partial x^j} \times U^{\dagger} \\ &= \sigma \times \hat{I} \times \hat{T} \times \hat{I} \times \frac{\partial}{\partial x^j} = \hat{\sigma}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^j} = \hat{p}_{\mathcal{D}j}. \end{aligned} \tag{3.F.99}$$

Note that we have achieved this isoquantization in terms of the following transformations. Firstly, we carried out the transformation

$$p = -i \times \frac{\partial}{\partial x} \to p_{\mathcal{D}} := \sigma \times \frac{\partial}{\partial x}, \tag{3.F.100}$$

to further produce its isotopic lift

$$\hat{p}_{\mathcal{D}} = \hat{\sigma}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}}. \tag{3.F.101}$$

Whenever the original diffusion tensor σ is the identity I, from Eq. (3.F.9) follows that the original metric g is Euclidean, we reach compatibility of the diffusion quantization with the Santilli-iso-Heisenberg representation given by taking the non-unitary transformation on the canonical commutation relations, which are given by

$$[\hat{q}^{i}\hat{,}\hat{p}_j] \;=\; \hat{i}\hat{\times}\hat{\delta}^{i}_{j} = i \times \delta^{i}_{j} \times \hat{I}, \tag{3.F.102}$$

together with

$$[\hat{r}^{i}\hat{,}\hat{r}^{j}] \;=\; [\hat{p}_i\hat{,}\hat{p}_j] = 0, \tag{3.F.103}$$

with the Santilli-iso-quantization rule [109, 46]

$$\hat{p}_j = -\hat{1}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{j}}. \tag{3.F.104}$$

Thus, from the quantization by the diffusion representation we retrieve the Santilli-iso-Heisenberg representation, with the difference that the diffusion noise tensor in the above construction need not be restricted to the identity.

Finally, we consider the isoHamiltonian operator

$$\hat{H} = \frac{\hat{1}}{\hat{2}\hat{\times}\hat{m}}\hat{\times}\hat{p}^{\hat{2}} + \hat{V}_0(\hat{t},\hat{x}) + \hat{V}_k(\hat{t},\hat{v})\hat{\times}\hat{v}^{k}, \tag{3.F.105}$$

where $\hat{p}$ may be taken to be given either by the Santilli isoquantization rule

$$\hat{p}_k\hat{\times}|\hat{\psi}> = -\hat{i}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{k}}\hat{\times}|\hat{\psi}>, \tag{3.F.106}$$

or by the diffusion representation $\hat{p}_{\mathcal{D}}$. $\hat{V}_0(\hat{t},\hat{x})$ and $\hat{V}_k(\hat{t},\hat{v})$ are potential isofunctions, the latter dependent on the isovelocities. Then the iso-Schroedinger equation (or Schroedinger-Santilli isoequation) [109, 46] is

$$\begin{aligned} \hat{i}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{t}}|\hat{\psi}> \;&=\; \hat{H}\hat{\times}|\psi> \\ &=\; \hat{H}(\hat{t},\hat{x},\hat{p})\times\hat{T}(\hat{t},\hat{x},\hat{\psi},\hat{\partial}\hat{\psi},\ldots)\times|\hat{\psi}>, \end{aligned} \tag{3.F.107}$$

where the wave isofunction $\hat{\psi}$ is an element in $(\hat{\mathcal{H}}, <|\hat{\times}|>, \hat{C}(\hat{c},\hat{+},\hat{\times}))$ satisfies

$$\hat{I}\hat{\times}|\hat{\psi}> = |\hat{\psi}> . \tag{3.F.108}$$

3.F.4.3 Hadronic Mechanics and Diffusion Processes

Finally, the components of drift isovector field, projected on $\hat{M}(\hat{x},\hat{g},R)$ in the isotopic lift of Eq. (3.F.63) is given by Eq. (3.F.87) with $\hat{f}=\hat{\ln}\hat{\phi}$, where $\hat{\phi}(\hat{x})=\hat{e}^{\hat{\mathcal{R}}(\hat{x})+\hat{\mathcal{S}}(\hat{x})}$ is the diffusion wave associated to the solution $\hat{\psi}(\hat{x})=\hat{e}^{\hat{\mathcal{R}}(\hat{x})+i\hat{\mathcal{S}}(\hat{x})}$ of the iso-Schroedinger equation, and its adjoint wave is $\hat{\check{\phi}}(x)=\hat{e}^{\hat{\mathcal{R}}(x)-\hat{\mathcal{S}}(x)}$. Hence, the drift isovector field has components

$$\hat{g}^{\alpha\beta}(\hat{x})\hat{\times}\frac{\hat{\partial}\hat{\ln}\hat{\phi}(\hat{x})}{\hat{\partial}\hat{x}^{\alpha}}=\hat{g}^{\alpha\beta}(\hat{x})\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\alpha}}(\hat{\mathcal{R}}_{\hat{t}}\hat{+}\hat{\mathcal{S}}_{\hat{t}})(\hat{x}). \tag{3.F.109}$$

Finally, we shall write the isotopic lift of the stochastic differential equation for the iso-Schroedinger Eq. (3.F.107). Applying the non-unitary transformation to Eq. (3.F.63), we obtain the iso-equation on $\hat{M}(\hat{x},\hat{G},\hat{R})$ for $\hat{X}_{\hat{t}}$ given by

$$d\hat{X}^{i}_{\hat{t}}=((\hat{g}^{\alpha\beta}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\alpha}}(\hat{\mathcal{R}}_{\hat{t}}\hat{+}\hat{\mathcal{S}}_{\hat{t}}))(\hat{X}_{\hat{t}})\hat{\times}d\hat{t}+\hat{\sigma}^{i}_{j}(\hat{X}_{\hat{t}})\hat{\times}d\hat{W}^{j}_{\hat{t}}, \tag{3.F.110}$$

with $d\hat{W}_{\hat{t}}=\hat{W}(\hat{t}\hat{+}d\hat{t})\hat{-}\hat{W}(\hat{t})$ the increment of a iso-Wiener process $\hat{W}_{\hat{t}}=(\hat{W}^{1}_{\hat{t}},\ldots,\hat{W}^{m}_{\hat{t}})$ with isoaverage equal to $\hat{0}$ and isocovariance given by $\hat{\delta}^{i}_{j}\hat{\times}\hat{t}$; i.e.,

$$\hat{1}\hat{/}(\hat{4}\hat{\times}\hat{\pi}\hat{\times}\hat{t})^{\hat{m}\hat{/}\hat{2}}\hat{\int}\hat{w}_{i}\hat{\times}\hat{e}^{-\hat{w}^{\hat{2}}\hat{/}\hat{4}\hat{\times}\hat{t}^{\hat{2}}}\hat{\times}\hat{d}\hat{w}=\hat{0},\quad\forall i=1,\ldots,m \tag{3.F.111}$$

and

$$\hat{1}\hat{/}(\hat{4}\hat{\times}\hat{\pi}\hat{\times}\hat{t})^{\hat{m}\hat{/}\hat{2}}\hat{\int}\hat{w}_{i}\hat{\times}\hat{w}_{j}\hat{\times}\hat{e}^{-\hat{w}^{\hat{2}}\hat{/}\hat{4}\hat{\times}\hat{t}^{\hat{2}}}\hat{\times}\hat{d}\hat{w}=\hat{\delta}^{i}_{j}\hat{\times}\hat{t},\ \ \forall i,j=1,\ldots,m \tag{3.F.112}$$

and $\hat{\int}$ denotes the isotopic integral defined by $\hat{\int}\hat{d}\hat{x}=(\int\hat{T}\times\hat{I}\times dx)\times\hat{I}=(\int dx)\times\hat{I}=\hat{x}$. Thus, formally at least, we have

$$\hat{X}_{\hat{t}}=\hat{X}_{\hat{0}}\hat{+}\hat{\int}_{\hat{0}}^{\hat{t}}(\hat{g}^{\alpha\beta}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{x}^{\alpha}}(\hat{\mathcal{R}}_{\hat{s}}\hat{+}\hat{\mathcal{S}}_{\hat{s}}))(\hat{X}_{\hat{s}})\hat{\times}\hat{d}\hat{s}+\hat{\int}_{\hat{0}}^{\hat{t}}\hat{\sigma}^{i}_{j}(\hat{X}_{\hat{s}})\hat{\times}d\hat{W}^{j}_{\hat{s}}. \tag{3.F.113}$$

The integral in the first term of Eq. (3.F.113) is an isotopic lift of the usual Riemann-Lebesgue integral [109d,19,20], while the second one is the isotopic lift of a stochastic Itô integral; we shall not present here in detail the definition of this last term, which follows from the notions of convergence in the isofunctional analysis elaborated by Kadeisvili [110] (see Section 3.2.6), and the usual definition of Itô stochastic integrals [102, 107, 116], nor the presentation of analytical conditions for their convergence which follows in principle from the isotopic lift of the usual conditions.

3.F.4.4 The Extension to the Many-Body Case

Up to know we have presented the case of the Schroedinger equation for an ensemble of one-particle systems on space-time. Of course, our previous constructions are also valid for the case of an ensemble of interacting multiparticle systems, so that the dimension of the configuration space is $3d + 1$, for indistinguishable d particles; the general case follows with minor alterations. If we start by constructing the theory as we did for an ensemble of one-particle systems (Schroedinger's "cloud of electrons"), we can still extend trivially to the general case, by considering a diffusion in the product configuration manifold with coordinates $X_t = (X_t^1, \ldots, X^d) \in M^d$, where M^d is the d Cartesian product of three dimensional space with coordinates $X_t^i = (x_t^{1,i}, x_t^{2,i}, x_t^{3,i}) \in M$, for all $i = 1, \ldots, d$. The distribution of this is $\mu_t = E_Q \circ X_t^{-1}$, which is a probability density in M^d. To obtain the distribution of the system on the three-dimensional space M, we need the distribution of the system X_t:

$$U_t^x := \frac{1}{d} \sum_{i=1}^{d} \delta_{x_i}, \tag{3.F.114}$$

which is the same as

$$U_t^x(B) = \frac{1}{d} \sum_{i=1}^{d} 1_B(X_t^i), \tag{3.F.115}$$

where $1_B(X_t^i)$ is the characteristic system for a measurable set B, equal to 1 if $X_t^i \in B$, for any $i = 1 \ldots, d$ and 0 otherwise. Then, the probability density for the interacting ensembles is given by

$$\mu_t^x(B) = E_Q[U_t^x(B)], \tag{3.F.116}$$

where E_Q is the mean taken with respect to the forward Kolmogorov representation presented above, is the probability distribution in the three-dimensional space; see [107]. Therefore, the geometrical-stochastic representation in actual space is constructable for a system of interacting ensembles of particles. Thus the criticism to the Schroedinger equation by the Copenhagen school, as to the unphysical character of the wave function since it was originally defined on a multiple-dimensional configuration space of interacting system of ensembles, is invalid [125].

3.F.5 Possible Empirical Evidence and Conclusions

We have shown that the Schroedinger and isoSchroedinger equation have an equivalent representation in terms of diffusion processes. This can be further extended to hadronic chemistry, as shown in Volume V of this series. This is an universal phenomenae since the applicability of the Schroedinger equation does

not restrict to the microcospic realm, as already shown in the astrophysical theory due to Nottale [114]; this universality is associated with the fact that the Planck constant (or equivalently, the diffusion constant) is multivalued, or still, it is context dependent, inasmuch as the velocity of light has the same feature [46]. In the case of HM this can be seen transparently in the fact that the isotopic unit plays the role, upon quantization, of the Planck constant as can seen in Eqs. (3.F.107), (3.F.108)[39], or furthermore, by its product with the noise tensor of the underlying Brownian motions. In the galactic scales, this may explain the red-shift without introducing a big-bang hypothesis [46, 73]. An identical conclusion was reached by Arp in considering as a theoretical framework the Le Sage's model of a Universe filled with a gas of particles [130], in our theory, the zero-point fluctuations described by the Brownian motions defined by the wave functions, as well as by viscous fluids, spinor fields, or electromagnetic fields [95] (and which one can speculate as related to the so-called dark energy problem). A similar view has been proposed by Santilli in which the elementary constituents are the so-called aetherinos [149], while in Sidharth's work, they appear to be elementary quantized vortices related to quantum-mechanical Kerr-Newman black holes [119]. Thus, whether we examine the domains of linear or non-linear quantum mechanics, or still of hadronic mechanics, vortices and superconductivity (which is the case of the Rutherford-Santilli model of the neutron which is derived from the previous constructions) appear as universal coherent structures; superconductivity is usually related to a non-linear Schroedinger equation with a Landau-Ginzburg potential, which is just an example of the Brownian motions related to torsion fields with further noise related to the metric. Furthermore, atoms and molecules have spin-spin interactions which will produce a contribution to the torsion field; we have seen already that the torsion geometry exists in the realm of hadronic chemistry, since we can extend the construction to the many-body case. In distinction with the usual repulsive Coulomb potential in nuclear physics, the isotopic deformations of the nuclear symmetries yield attractive potentials such as the Hulten potential, which in the range of 10^{-13} cm yields the usual potential [19, 20, 22, 46, 70, 73, 108–110, 141] without the need of introducing any sort of parameters or extra potentials. In contrast with the ad-hoc postulates of randomness in the fusion models which are considered in the usual approaches [122, 123], in the present work randomness is intrinsic to space-time or alternatively a by product of it, and in the case of HM, these geometries incorporate at a foundational level, a generalized unit which incorporates all the features of the fusion process itself: the non-canonical, non-local and non-linear overlapping of the wave functions of the ensembles which correspond to the separate ensembles under deformable collisions in which the particles lose their

[39] See Postulate 3.4.1.

pointlike structure, or in a hypercondensed plasma state, where the dynamics of the process may have a random behavior; outside of the domain of 10^{-13} cm, the hadronic fluctuations associated to the isolinear isoSchroedinger equation decay to the quantum fluctuations of the linear Schroedinger equation.

There are already empirical findings that may lead to validate the present view (see Volume IV of this series for experimental evidence in particle physics, nuclear physics, astrophysics and cosmology, and Volume V for experimental evidence in chemistry). In the last fifty years, a team of scientists at the Biophysics Institute of the Academy of Sciences of Russia, directed by S. Shnoll (and presently developed in a world net which includes Roger Nelson, Engineering Anomalies Research, Princeton University, B. Belousov, International Institute of Biophysics, Neuss (Germany), Dr. Wilker, Max-Planck Institute for Aeronomy, Lindau, and others), have carried out tens of thousands of experiments of very different nature and energy scales (α decay, biochemical reactions, gravitational waves antenna, etc.) in different points of the globe, and carried out a software analysis of the observed histograms and their fluctuations, to find out an amazing fit which is repeated with regularity of 24 hours, 27 days and the duration of a sidereal year. In these experiments the fine spectrum of their measurements reveal a non-random pattern. At points of Earth with the same local hour, these patterns are reproduced with the said periodicity. The only thing in common to these experiments is that they are occur in space-time, which has lead to conclude that they stem from space-time fluctuations, which may further be associated with cosmological fields. Furthermore, the histograms reveal a fractal structure; this structure is interpreted as appearing from an interference phenomena related to the cosmological field; we recall that diffusion processes present interference phenomena alike to, say, the two-slit experiment.[40] Measurements taken with collimators show fluctuations emerging from the rotation of the Earth around its axis or its circumsolar orbit, showing a sharp anisotropy of space. Furthermore, it is claimed that the spatial heterogeinity occurs in a scale of 10^{-13} cm, coincidently with the scale of the strong interactions [152]. Contrary to common belief, the Michelson-Morley did not provide a final dismissal of the aether, while Einstein in the course of his life supported the idea of its existence [154]. Thousands of interferometry experiments were carried out by D. Miller, Allais and others, and contemporarily very diverse setups have proved that there is a space anisotropy [153]. As a closing remark we would like to recall that Planck himself proposed the existence of ensembles of random phase oscillators having

[40]This fractal structure has been found to follow the pattern of the logarithmic Muller fractal, which is associated with the existence of a global scale for all structures in the Universe; see H. Muller, Free Energy - Global Scaling, *Raum& Zeit Special 1*, Ehlers-Verlag GmbH, ISBN 3-934-196-17-9; 2004. This leads to reinforce the thesis of time as an active field. Furthermore, the space and time Brownian motions can exist, in principle, in the different space and time scales warranted by these global scales.

the zero-point structure as the basis for quantum physics [146]. Thus, the apeiron would be related to the Brownian motions which we have presented in this work, and define the space and time geometries, or alternatively, are defined by them. So we are back to the idea due to Clifford, that there is no-thing but space and time configurations, instead of a separation between substratum and fields and particles appearing on it. Furthermore, what we perceive to be void, is the hyperdense source of actuality. The same conception has been proposed by Prof. Santilli in the main body of this volume.

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Chapter 4

LIE-ADMISSIBLE BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

NOTE; THIS CHAPTER MUST BE COMPLETED AND EDITED

4.1 INTRODUCTION

4.1.1 The Scientific Imbalance Caused by Irreversibility

As recalled in Chapter 1, physical, chemical or biological systems are called *irreversible* when their images under time reversal $t \to -t$ are prohibited by causality and/or other laws, as it is generally the case for nuclear transmutations, chemical reactions and organism growth.

Systems are called *reversible* when their time reversal images are as causal as the original ones, as it is the case for planetary and atomic structures when considered isolated from the rest of the universe, the structure of crystals, and other structures (see reprint volume [1] on irreversibility and vast literature quoted therein).

Another large scientific imbalance of the 20-th century studied in these monographs is the treatment of irreversible systems via the mathematical and physical formulations developed for reversible systems, since these formulations are themselves reversible, thus resulting in serious limitations in virtually all branches of science.

The problem is compounded by the fact that all used formulations are of Hamiltonian type, under the awareness that all known Hamiltonians are reversible over time (since all known potentials, such as the Coulomb potential $V(r)$, etc., are reversible).

This scientific imbalance was generally dismissed in the 20-th century with unsubstantiated statements, such as "irreversibility is a macroscopic occurrence that disappears when all bodies are reduced to their elementary constituents".

Figure 4.1. All energy releasing processes are irreversible over time. By contrast, all formulations of the 20th century are fully reversible over time, a limitation that is apparently responsible for the lack of industrial development of any really new form of energy for over half a century, as well as the lack of resolution of the environmental problems caused by fossil fuels combustion depicted in this figure. A primary objective of hadronic mechanics is, firstly, identify formulations that are structurally irreversible (a task addressed in this chapter), as a necessary premise for their quantitative treatment of irreversible process and the search of basically new energies (a task address in Volume II).

These academic beliefs have been disproved by Theorem 1.3.3 according to which *a classical irreversible system cannot be consistently decomposed into a finite number of elementary constituents all in reversible conditions and, vice-versa, a finite collection of elementary constituents all in reversible conditions cannot yield an irreversible macroscopic ensemble.*

The implications of the above theorem are quite profound because it establishes that, contrary to popular beliefs, *irreversibility originates at the most primitive levels of nature, that of elementary particles, and then propagates all the way to our macroscopic environment.*

In this chapter we study the contribution by the author that originated the field, as well as contributions by a number of independent authors. The presentation will mainly follow the recently published memoir [32]. Nevertheless, an in depth knowledge of the topic requires the study of (at least some of) the author's monographs [18–23,29] and those by independent authors [33–39].

The author would like to express his sincere appreciation to the *Italian Physical Society* for publishing memoir [32] in *Il Nuovo Cimento B* as a final presentation of studies in the field initiated by the author in the same Journal in paper [7] forty years earlier.

4.1.2 The Forgotten Legacy of Newton, Lagrange and Hamilton

The scientific imbalance on irreversibility was created in the early part of the 20-th century when, to achieve compatibility with quantum mechanics and special relativity, the entire universe was reduced to potential forces. Jointly, the analytic equations were "truncated" with the removal of the external terms.

In reality, Newton [2] *did not* propose his celebrated equations restricted to forces derivable from a potential $F = \partial V/\partial r$, but proposed them for the most general possible forces,

$$m_a \times \frac{dv_{ka}}{dt} = F_{ka}(t, r, v), \quad k = 1, 2, 3; \quad a = 1, 2, \ldots, N, \tag{4.1.1}$$

where the conventional associative product of numbers, matrices, operators, etc. is continued to be denoted hereon with the symbol $\times$ so as to distinguish it from numerous other products needed later on.

Similarly, to be compatible with Newton's equations, Lagrange [3] and Hamilton [4] decomposed Newton's force into a potential and a nonpotential component,they represented all potential forces with functions today known as the Lagrangian and the Hamiltonian, and proposed their celebrated equations with external terms,

$$\frac{d}{dt}\frac{\partial L(t, r, v)}{\partial v_a^k} - \frac{\partial L(t, r, v)}{\partial r_a^k} = F_{ak}(t, r, v), \tag{4.1.2a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t, r, p)}{\partial r_a^k} + F_{ak}(t, r, p), \tag{4.1.2b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v_a^2 - V(t, r, v), \quad H = \Sigma_a \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t, r, p), \tag{4.1.2c}$$

$$V = U(t, r)_{ak} \times v_a^k + U_o(t, r), \quad F(t, r, v) = F(t, r, p/m). \tag{4.1.2d}$$

More recently, Santilli [5] conducted comprehensive studies on the integrability conditions for the existence of a potential or a Lagrangian or a hamiltonian, called *conditions of variational selfadjointness*. These study permit the rigorous decomposition of Newtonian forces into a component that is variationally selfadjoint (SA) and a component that is not (NSA),

$$m_a \times \frac{dv_{ka}}{dt} = F_{ka}^{SA}(t, r, v) + F_{ka}^{NSA}(t, r, v). \tag{4.1.3}$$

Consequently, the true Lagrange and Hamilton equations can be more technically written

$$\Big[\frac{d}{dt}\frac{\partial L(t, r, v)}{\partial v_a^k} - \frac{\partial L(t, r, v)}{\partial r_a^k}\Big]^{SA} = F_{ak}^{NSA}(t, r, v), \tag{4.1.4a}$$

$$\Big[\frac{dr_a^k}{dt} - \frac{\partial H(t,r,p)}{\partial p_{ak}}\Big]^{SA} = 0, \quad \Big[\frac{dp_{ak}}{dt} + \frac{\partial H(t,r,p)}{\partial r_a^k}\Big]^{SA} = F_{ak}^{NSA}(t,r,p). \quad (4.1.4b)$$

The *forgotten legacy of Newton, Lagrange and Hamilton is that irreversibility originates precisely in the truncated NSA terms,* because all known potential-SA forces are reversible. The scientific imbalance of Section 1.3 is then due to the fact that no serious scientific study on irreversibility can be done with the truncated analytic equations and their operator counterpart, since these equations can only represent reversible systems.

Being born and educated in Italy, during his graduate studies at the University of Torino, the author had the opportunity of studying in the late 1960s the original works by Lagrange that were written precisely in Torino and most of them in Italian.

In this way, the author had the opportunity of verifying *Lagrange's analytic vision of representing irreversibility precisely via the external terms,* due to the impossibility of representing all possible physical events via the sole use of the Lagrangian, since the latter was solely conceived for the representation of reversible and potential events. As the reader can verify, Hamilton had, independently, the same vision.

Consequently, the truncation of the basic analytic equations caused the impossibility of a credible treatment of irreversibility at the purely classical level. The lack of a credible treatment of irreversibility then propagated at the subsequent operator level.

It then follows that *quantum mechanics cannot possibly be used for serious studies on irreversibility* because the discipline was constructed for the description of reversible quantized atomic orbits and not for irreversible systems.

In plain terms, while the validity of quantum mechanics for the arena of its original conception and verification is beyond scientific doubt, the assumption of quantum mechanics as the final operator theory for all conditions existing in the universe is outside the boundaries of serious science.

This establishes the need for the construction of a broadening (or generalization here called *lifting*) of quantum mechanics specifically conceived for quantitative studies of irreversibility. Since reversible systems are a *particular case* of irreversible ones, the broader mechanics must be a *covering* of quantum mechanics, that is, admitting the latter under a unique and unambiguous limit.

It is easy to see that the needed broader mechanics must also be a covering of the isotopic branch of hadronic mechanics studied in the preceding chapter, thus being a new branch of hadronic mechanics. In fact, isomechanics is itself structurally reversible due to the Hermiticity of both the Hamiltonian, $\hat{H} = \hat{H}^{\hat{\dagger}}$, and of the isotopic element, $\hat{T} = \hat{T}^{\dagger}$, while a serious study of irreversible

processes requires a *structurally irreversible mechanics,* that is, a mechanics that is irreversible for all possible reversible Hamiltonians.[1]

4.1.3 Early Representations of Irreversible Systems

As reviewed in Section 1.5.2, the brackets of the time evolution of an observable $A(r,p)$ in phase space according to the analytic equations with external terms,

$$\frac{dA}{dt} = (A,H,F) = \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}} + \frac{\partial A}{\partial r_a^k} \times F_{ka}, \tag{4.1.5}$$

violate the right associative and scalar laws.

Therefore, the presence of external terms in the analytic equations causes not only the loss of *all* Lie algebras in the study of irreversibility, but actually causes the loss of all possible algebras as commonly understood in mathematics.

To resolve this problem, the author initiated a long scientific journey beginning with his graduate studies at the University of Torino, Italy, following the reading of Lagrange's papers.

The original argument [7–9], still valid today, is to select analytic equations characterizing brackets in the time evolution verifying the following conditions:

(1) The brackets of the time evolution must verify the right and left associative and scalar laws to characterize an algebra;

(2) Said brackets must not be invariant under time reversal as a necessary condition to represent irreversibility *ab initio;*

(3) Said algebra must be a covering of Lie algebras as a necessary condition to have a covering of the truncated analytic equations, namely, as a condition for the selected representation of irreversibility to admit reversibility as a particular case.

Condition (1) requires that said brackets must be bilinear, e.g., of the form (A,B) with properties

$$(n \times A, B) = n \times (A,B), \quad (A, m \times B) = m \times (A,B); \quad n,m \in C, \tag{4.1.6a}$$

$$(A \times B, C) = A \times (B,C), \quad (A, B \times C) = (A,B) \times C. \tag{4.1.6b}$$

Condition (2) requires that brackets (A,B) should not be totally antisymmetric as the conventional Poisson brackets,

$$(A,B) \neq -(B,A), \tag{4.1.7}$$

because time reversal is realized via the use of Hermitian conjugation.

[1]An exception to this general rule we shall study later on occurs when the isotopic elements is indeed Hermitian, but explicitly dependent on time and such that $\hat{T}(t,\ldots) \neq \hat{T}(-t,\ldots)$.

Condition (3) then implies that brackets (A, B) characterize *Lie-admissible algebras* in the sense of Albert [10], namely, the brackets are such that the attached antisymmetric algebra is Lie.[2]

$$[A, B]^* = (A, B) - (B, A) = Lie. \tag{4.1.8}$$

In particular, the latter condition implies that the new brackets are formed by the superposition of totally antisymmetric and totally symmetric brackets,

$$(A, B) = [A, B]^* + \{A, B\}^*. \tag{4.1.9}$$

It should be noted that the operator realization of brackets (A, B) is also *Jordan-admissible* in the sense of Albert [10], namely, the attached symmetric brackets $\{A, B\}^*$ characterize a *Jordan algebra.* Consequently, *hadronic mechanics provides a realization of Jordan's dream, that of seeing his algebra applied to physics.*

However, the reader should be aware that, for certain technical reasons beyond the scope of this monograph, the classical realizations of brackets (A, B) are Lie-admissible but not Jordan-admissible. Therefore, Jordan-admissibility appears to emerge exclusively for operator theories.[3]

After identifying the above lines, Santilli [9] proposed in 1967 the following *generalized analytic equations*

$$\frac{dr_a^k}{dt} = \alpha \times \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\beta \times \frac{\partial H(t, r, p)}{\partial r_a^k}, \tag{4.1.10}$$

(where α and β are real non-null parameters) that are manifestly irreversible. The brackets of the time evolution are then given by

$$i \times \frac{dA}{dt} = (A, H) =$$

[2]More technically, a generally nonassociative algebra U with elements $a,\ b,\ c,\ \ldots$ and abstract product ab is said to be Lie-admissible when the attached algebra U^- characterized by the product $[a, b] = ab - ba$ verifies the *Lie axioms*

$$[a, b] = -[b, a],$$
$$[[a, b], c] + [[b, c], a] + [[c, b], a] = 0.$$

[3]More technically, a generally nonassociative algebra U with elements $a,\ b,\ c,\ \ldots$ and abstract product ab is said to be Jordan-admissible when the attached algebra U^+ characterized by the product $\{a, b\} = ab + ba$ verifies the *Jordan axioms*

$$\{a, b\} = \{b, a\},$$
$$\{\{a, b\}, a^2\} = \{a, \{b, a^2\}\}.$$

In classical realizations of the algebra U the first axiom of Jordan-admissibility is generally verified but the second is generally violated, while in operator realizations both axioms are generally verified.

$$= \alpha \times \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \beta \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}}, \tag{4.1.11}$$

whose brackets are manifestly Lie-admissible, but *not* Jordan-admissible as the interested reader is encouraged to verify.

The above analytic equations characterize the time-rate of variation of the energy

$$\frac{dH}{dt} = (\alpha - \beta) \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}}. \tag{4.1.12}$$

Also in 1967, Santilli [7,8] proposed an operator counterpart of the preceding classical setting consisting in the first known *Lie-admissible parametric generalization of Heisenberg's equation,* also called *deformed Heisenberg equations,*[4] in the following infinitesimal form

$$i \times \frac{dA}{dt} = (A, B) = p \times A \times H - q \times H \times A =$$

$$= m \times (A \times B - B \times A) + n \times (A \times B + B \times A), \tag{4.1.13a}$$

$$m = p + q, \quad n = q - p, \tag{4.1.13b}$$

where $p, q, p \pm q$ are non-null parameters, with finite counterpart

$$A(t) = e^{i \times H \times q} \times A(0) \times e^{-i \times p \times H}. \tag{4.1.14}$$

Brackets (A, B) are manifestly Lie-admissible with attached antisymmetric part

$$[A, B]^* = (A, B) - (B, A) = (p - q) \times [A, B]. \tag{4.1.15}$$

The same brackets are also Jordan-admissible in view of the property

$$\{A, B\}^* = (A, B) + (B, A) = (p + q) \times \{A, B\}, \tag{4.1.16}$$

The resulting time evolution is then manifestly irreversible (for $p \neq q$) with nonconservation of the energy

$$i \times \frac{dH}{dt} = (H, H) = (p - q) \times H \times H \neq 0, \tag{4.1.17}$$

as necessary for an open system.

Subsequently, Santilli realized that the above formulations are not invariant under their own time evolution (4.1.14) because Eqs. (4.1.11) are manifestly *nonunitary.*

[4] As we shall soon see, the term "deformed" is used for formulations that are catastrophically inconsistent because dreaming to treat new theories with the mathematics of the old ones.

The application of nonunitary transforms to brackets (4.1.12) then led to the proposal in memoir [11,12] of 1978 of the following *Lie-admissible operator generalization of Heisenberg equations* in their infinitesimal form

$$i \times \frac{dA}{dt} = A \times P \times H - H \times Q \times A = (A, H)^*, \tag{4.1.18}$$

with finite counterpart

$$A(t) = e^{i \times H \times Q} \times A(0) \times e^{-i \times P \times H}, \tag{4.1.19}$$

under the subsidiary conditions needed for consistency, as we shall see,

$$P = Q^\dagger, \tag{4.1.20}$$

where P, Q and $P \pm Q$ are now nonsingular operators (or matrices), and Eq. (4.1.16b) is a basic consistency condition explained later in this section.

Eqs. (4.1.18)–(4.1.19) are the *fundamental equations of hadronic mechanics.* Their basic brackets are manifestly Lie-admissible and Jordan admissible with structure

$$(A, B)^* = A \times P \times B - B \times Q \times A =$$

$$= (A \times T \times B - B \times T \times A) + (A \times R \times B + B \times R \times A), \tag{4.1.21a}$$

$$T = P + Q, \quad R = Q - P. \tag{4.1.21b}$$

As indicated in Section 1.5.2, it is easy to see that the application of a nonunitary transform to the parametric brackets of Eqs. (4.1.11) leads precisely to the operator brackets of Eqs. (4.1.17),

$$U \times (p \times A \times B - q \times B \times A) \times U^\dagger = \hat{A} \times P \times \hat{B} - \hat{B} \times Q \times \hat{A}, \tag{4.1.22a}$$

$$U \times U^\dagger \neq I, P = p \times (U \times (U^\dagger)^{-1}, Q = q \times (U \times U^\dagger)^{-1}, \hat{A} = U \times A \times U^\dagger. \tag{4.1.22b}$$

In particular, the application of any (nonsingular) nonunitary transforms preserves the Lie-admissible and Jordan-admissible characters. Consequently, fundamental equations (4.1.18), (4.1.19) are "directly universal" in the sense of Lemma 1.5.2.

However, the above equations *are not invariant* under their own (nonunitary) time evolution,

$$U \times (\hat{A} \times P \times \hat{B} - \hat{B} \times Q \times \hat{A},) \times U^\dagger = \hat{A}' \times P' \times \hat{B}' - \hat{B}' \times Q' \times \hat{A}', \tag{4.1.23}$$

where the lack of invariance is expressed by the lack of preservation of the numerical values of the P, Q operators because, as we shall see shortly, these operators characterize new multiplications.

By comparison, quantum mechanical brackets are indeed invariant under the class of admitted transformations, the unitary transforms

$$W \times A \times B - B \times A) \times W^\dagger = A' \times B' - B' \times A', \tag{4.1.24a}$$

$$W \times W^\dagger = W^\dagger \times W = I, A' = W \times A \times W^\dagger, B' = W \times B \times W^\dagger, \tag{4.1.24b}$$

where the invariance we are here referring to is expressed by the preservation of the associative product, namely, $A \times B$ *is not* mapped into a different product, say $A' * B'$.

As known to experts of quantum mechanics (to qualify as such), simple invariance (4.1.24) is at the foundations of the majestic axiomatic consistency of quantum mechanics, including: the prediction of the same numerical values under the same conditions at different times; the preservation of Hermiticity and, thus, of observables over time; and other basic features.

Consequently, Lie-admissible and Jordan admissible equations (4.1.18)–(4.1.19) are afflicted by the catastrophic inconsistencies of Theorem 1.5.2, as it is the fate for all nonunitary theories some of which are listed in Section 1.5. In particular, said equations do not preserve numerical predictions under the same conditions but at different times, do not preserve Hermiticity, thus do not admit observables, and have other catastrophic inconsistencies studied in detail in Section 1.5.

Moreover, in the form presented above, the dynamical equations are not derivable from a variational principle. Consequently, they admit no known unique map from classical into operator formulations.

In view of these insufficiencies, said equations cannot be assumed in the above given form as the basic equations of any consistent physical theory.

4.2 ELEMENTS OF SANTILLI GENOMATHEMATICS AND ITS ISODUAL

4.2.1 Genounits, Genoproducts and their Isoduals

The "direct universality" of Eqs. (4.1.18), (4.1.19) voids any attempt at seeking further generalizations in the hope of achieving invariance, since any nontrivial generalization would suffer the loss of any algebra in the brackets of the time evolution, with consequential inability to achieve any physically meaningful theory, e.g., because of the inability to treat the spin of a proton under irreversible conditions.

This occurrence leaves no alternative other than that of seeking a yet *new mathematics* permitting Eqs. (4.1.18), (4.1.19) to achieve the needed invariance.

After numerous attempts and a futile search in the mathematical literature of the Cantabrigian area,[5] Santilli proposed in Refs. [11,12] of 1978 the construction of a new mathematics specifically conceived for the indicated task, that

[5]Conducted in the period 1977–1978.

eventually reached mathematical maturity for numbers only in paper [13] of 1993, mathematical maturity for the new differential calculus only in memoir [14] of 1996, and, finally, an invariant formulation of Lie-admissible equations only in paper [15] of 1997.

The new Lie-admissible mathematics is today known as *Santilli genomathematics*, where the prefix "geno" suggested in the original proposal [11,12] is used in the Greek meaning of "inducting" new axioms (as compared to the prefix "iso" of the preceding chapter denoting the preservation of the axioms).

The basic idea is to lift the isounits of the preceding chapter into a form that is still nowhere singular, but *non-Hermitian*, thus implying the existence of *two* different generalized units, today called *Santilli genounits* for the description of matter, that are generally written [13]

$$\hat{I}^{>} = 1/\hat{T}^{>}, \quad {}^{<}\hat{I} = 1/{}^{<}\hat{T}, \tag{4.2.1a}$$

$$\hat{I}^{>} \neq {}^{<}\hat{I}, \quad \hat{I}^{>} = ({}^{<}\hat{I})^{\dagger}, \tag{4.2.1b}$$

with two additional *isodual genounits* for the description of antimatter [14]

$$(\hat{I}^{>})^{d} = -(\hat{I}^{>})^{\dagger} = -{}^{<}\hat{I} = -1/{}^{<}\hat{T}, \quad ({}^{<}\hat{I})^{d} = -\hat{I}^{>} = -1/\hat{T}^{>}. \tag{4.2.2}$$

Jointly, all conventional and/or isotopic products $A\hat{\times}B$ among generic quantities (numbers, vector fields, operators, etc.) are lifted in such a form admitting the genounits as the correct left and right units at all levels, i.e.,

$$A > B = A \times \hat{T}^{>} \times B, \quad A > \hat{I}^{>} = \hat{I}^{>} > A = A, \tag{4.2.3a}$$

$$A < B = A \times^{<} \hat{T} \times B, \quad A <^{<} \hat{I} =^{<} \hat{I} < A = A, \tag{4.2.3b}$$

$$A >^{d} B = A \times \hat{T}^{>d} \times B, \quad A >^{d} \hat{I}^{>d} = \hat{I}^{>d} >^{d} A = A, \tag{4.2.3c}$$

$$A <^{d} B = A \times^{<} \hat{T}^{d} \times B, \quad A <^{d} {}^{<}\hat{I}^{d} = {}^{<}\hat{I}^{d} <^{d} A = A, \tag{4.2.3d}$$

for all elements A, B of the set considered.

As we shall see in Section 4.3, the above basic assumptions permit the representation of irreversibility with the most primitive possible quantities, the basic units and related products.

In particular, as we shall see in Section 4.3 and 4.4, genounits permit an invariant representation of the external forces in Lagrange's and Hamilton's equations (4.1.2). As such, genounits are generally dependent on time, coordinates, momenta, wavefunctions and any other needed variable, e.g., $\hat{I}^{>} = \hat{I}^{>}(t^{>}, r^{>}, p^{>}, \psi^{>}, \ldots)$.

In fact, the assumption of all *ordered product to the right* $>$ represents matter systems moving forward in time, the assumption of all *ordered products to the left* $<$ represents matter systems moving backward in time, with the irreversibility being represented *ab initio* by the inequality $A > B \neq A < B$. Similar representation of irreversible antimatter systems occurs via isodualities.

4.2.2 Genonumbers, Genofunctional Analysis and Their Isoduals

Genomathematics began to reach maturity with the discovery made, apparently for the first time in paper [13] of 1993, that *the axioms of a field still hold under the ordering of all products to the right or, independently, to the left.*

This unexpected property permitted the formulation of *new numbers,* that can be best introduced as a generalization of the isonumbers [18], although they can also be independently presented as follows:

DEFINITION 4.2.1 [13]: Let $F = F(a, +, \times)$ *be a field of characteristic zero as per Definitions 2.1.1 and 3.2.1. Santilli's forward genofields are rings* $\hat{F}^> = \hat{F}(\hat{a}^>, \hat{+}^>, \hat{\times}^>)$ *with elements*

$$\hat{a}^> = a \times \hat{I}^>, \tag{4.2.4}$$

where $a \in F$, $\hat{I}^> = 1/\hat{T}^>$ *is a non singular non-Hermitian quantity (number, matrix or operator) generally outside* F *and* $\times$ *is the ordinary product of* F*; the genosum* $\hat{+}^>$ *coincides with the ordinary sum* $+$*,*

$$\hat{a}^> \hat{+}^> \hat{b}^> \equiv \hat{a}^> + \hat{b}^>, \ \ \forall \hat{a}^>, \ \hat{b}^> \in \hat{F}^>, \tag{4.2.5}$$

consequently, the additive forward genounit $\hat{0}^> \in \hat{F}$ *coincides with the ordinary* $0 \in F$*; and the forward genoproduct* $>$ *is such that* $\hat{I}^>$ *is the right and left isounit of* $\hat{F}^>$*,*

$$\hat{I}^> \hat{\times} \hat{a}^> = \hat{a}^> > \hat{I}^> \equiv \hat{a}^>, \ \ \forall \hat{a}^> \in \hat{F}^>. \tag{4.2.6}$$

Santilli's forward genofields verify the following properties:

1) For each element $\hat{a}^> \in \hat{F}^>$ *there is an element* $\hat{a}^{>-\hat{1}^>}$*, called forward genoinverse, for which*

$$\hat{a}^> > \hat{a}^{>-\hat{I}^>} = \hat{I}^>, \ \ \forall \hat{a}^> \in \hat{F}^>; \tag{4.2.7}$$

2) The genosum is commutative

$$\hat{a}^> \hat{+}^> \hat{b}^> = \hat{b}^> \hat{+}^> \hat{a}^>, \tag{4.2.8}$$

and associative

$$(\hat{a}^> \hat{+}^> \hat{b}^>) +^> \hat{c}^> = \hat{a}^> \hat{+}^> (\hat{b}^> \hat{+}^> \hat{c}^>), \ \ \forall \hat{a}, \hat{b}, \hat{c} \in \hat{F}; \tag{4.2.9}$$

3) The forward genoproduct is associative

$$\hat{a}^> > (\hat{b}^> > \hat{c}^>) = (\hat{a}^> > \hat{b}^>) > \hat{c}^>, \quad \forall \hat{a}^>, \hat{b}^>, \hat{c}^> \in \hat{F}^>, \tag{4.2.10}$$

but not necessarily commutative

$$\hat{a}^> > \hat{b}^> \neq \hat{b}^> > \hat{a}^>; \tag{4.2.11}$$

4) The set $\hat{F}^>$ is closed under the genosum,

$$\hat{a}^> \hat{+}^> \hat{b}^> = \hat{c}^> \in \hat{F}^>, \tag{4.2.12}$$

the forward genoproduct,

$$\hat{a}^> > \hat{b}^> = \hat{c}^> \in \hat{F}^>, \tag{4.2.13}$$

and right and left genodistributive compositions,

$$\hat{a}^> > (\hat{b}^> \hat{+}^> \hat{c}^>) = \hat{d}^> \in \hat{F}^>, \tag{4.2.14a}$$

$$(\hat{a}^> \hat{+}^> \hat{b}^>) > \hat{c}^> = \hat{d}^> \in \hat{F}^> \quad \forall \hat{a}^>, \hat{b}^>, \hat{c}^>, \hat{d}^> \in \hat{F}^>; \tag{4.2.14b}$$

5) The set $\hat{F}^>$ verifies the right and left genodistributive law

$$\hat{a}^> > (\hat{b}^> \hat{+}^> \hat{c}^>) = (\hat{a}^> \hat{+}^> \hat{b}^>) > \hat{c}^> = \hat{d}^>, \ \ \forall \hat{a}^>, \hat{b}^>, \hat{c}^>, \in \hat{F}^>. \tag{4.2.15}$$

In this way we have the forward genoreal numbers $\hat{R}^>$, the forward genocomplex numbers $\hat{C}^>$ and the forward genoquaternionic numbers $\hat{Q}C^>$ while the forward genooctonions $\hat{O}^>$ can indeed be formulated but they do not constitute genofields [14].

The backward genofields and the isodual forward and backward genofields are defined accordingly. Santilli's genofields are called of the first (second) kind when the genounit is (is not) an element of F.

The basic axiom-preserving character of genofields is illustrated by the following:

LEMMA 4.2.1 [13]: Genofields of first and second kind are fields (namely, they verify all axioms of a field).

Note that the conventional product "2 multiplied by 3" is not necessarily equal to 6 because, for isodual numbers with unit -1 it is given by -6 [13].

The same product "2 multiplied by 3" is not necessarily equal to $+6$ or -6 because, for the case of isonumbers, it can also be equal to an arbitrary number, or a matrix or an integrodifferential operator depending on the assumed isounit [13].

In this section we point out that "2 multiplied by 3" can be ordered to the right or to the left, and the result is not only arbitrary, but yielding different numerical results for different orderings, $2 > 3 \neq 2 < 3$, all this by continuing to verify the axioms of a field per each order [13].

Once the forward and backward genofields have been identified, the various branches of genomathematics can be constructed via simple compatibility arguments.

For specific applications to irreversible processes there is first the need to construct the *genofunctional analysis,* studied in Refs. [6,18] that we cannot review

here for brevity. The reader is however warned that any elaboration of irreversible processes via Lie-admissible formulations based on conventional or isotopic functional analysis leads to catastrophic inconsistencies because it would be the same as elaborating quantum mechanical calculations with genomathematics.

As an illustration, Theorems 1.5.1 and 1.5.2 of catastrophic inconsistencies are activated unless one uses the ordinary differential calculus lifted, for ordinary motion in time of matter, into the following *forward genodifferentials and genoderivatives*

$$\hat{d}^{>}x = \hat{T}_x^{>} \times dx, \quad \frac{\hat{\partial}^{>}}{\hat{\partial}^{>}x} = \hat{I}_x^{>} \times \frac{\partial}{\partial x}, \text{ etc}, \tag{4.2.16}$$

with corresponding backward and isodual expressions here ignored.

Similarly, all conventional functions and isofunctions, such as isosinus, isocosinus, isolog, etc., have to be lifted in the genoform

$$\hat{f}^{>}(x^{>}) = f(\hat{x}^{>}) \times \hat{I}^{>}, \tag{4.2.17}$$

where one should note the necessity of the multiplication by the genounit as a condition for the result to be in $\hat{R}^{>}$, $\hat{C}^{>}$, or $\hat{O}^{>}$.

4.2.3 Genogeometries and Their Isoduals

Particularly intriguing are the *genogeometries* [16] (see also monographs [18] for detailed treatments). They are best characterized by a simple genotopy of the isogeometries, although they can be independently defined.

As an illustration, the *Minkowski-Santilli forward genospace* $\hat{M}^{>}(\hat{x}^{>}, \hat{\eta}^{>}, \hat{R}^{>})$ over the genoreal $\hat{R}^{>}$ is characterized by the following spacetime, *genocoordinates, genometric and genoinvariant*

$$\hat{x}^{>} = x\hat{I}^{>} = \{x^\mu\} \times \hat{I}^{>}, \quad \hat{\eta}^{>} = \hat{T}^{>} \times \eta, \quad \eta = Diag.(1, 1, 1, -1), \tag{4.2.18a}$$

$$\hat{x}^{>2^{>}} = \hat{x}^{>\mu}\hat{\times}^{>}\hat{\eta}_{\mu\nu}^{>}\hat{\times}^{>}\hat{x}^{>\nu} = (x^\mu \times \hat{\eta}_{\mu\nu}^{>} \times x^\nu) \times \hat{I}^{>}, \tag{4.2.18b}$$

where the first expression of the genoinvariant is on genospaces while the second is its projection in our spacetime.

Note that the Minkowski-Santilli genospace has, in general, an explicit dependence on spacetime coordinates. Consequently, it is equipped with the entire formalism of the conventional Riemannian spaces covariant derivative, Christoffel's symbols, Bianchi identity, etc. only lifted from the isotopic form of the preceding chapter into the genotopic form.

A most important feature is that *genospaces permit, apparently for the first time in scientific history, the representation of irreversibility directly via the basic genometric.* This is due to the fact that genometrics are nonsymmetric by conception, e.g.,

$$\hat{\eta}_{\mu\nu}^{>} \quad \neq \quad \hat{\eta}_{\nu\mu}^{>}. \tag{4.2.19}$$

Consequently, *genotopies permit the lifting of conventional symmetric metrics into nonsymmetric forms,*

$$\eta^{Minkow.}_{Symm} \quad \to \quad \hat{\eta}^{>Minkow.-Sant.}_{NonSymm}. \tag{4.2.20}$$

Remarkably, *nonsymmetric metrics are indeed permitted by the axioms of conventional spaces* as illustrated by the invariance

$$(x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \equiv [x^\mu \times (\hat{T}^> \times \eta_{\mu\nu}) \times x^\nu] \times T^{>-1} \equiv$$
$$\equiv (x^\mu \times \hat{\eta}^>_{\mu\nu} \times x^\nu) \times \hat{I}^>, \tag{4.2.21}$$

where $\hat{T}^>$ is assumed in this simple illustration to be a complex number.

Interested readers can then work out backward genogeometries and the isodual forward and backward genogeometries with their underlying genofunctional analysis.

This basic geometric feature was not discovered until recently because hidden where nobody looked for, in the basic unit. However, this basic geometric advance in the representation of irreversibility required the prior discovery of basically new numbers, Santilli's genonumbers with nonsymmetric unit and ordered multiplication [14].

4.2.4 Santilli Lie-Admissible Theory and Its Isodual

Particularly important for irreversibility is the lifting of Lie's theory and Lie-Santilli's isotheories permitted by genomathematics, first identified by Ref. [11] of 1978 (and then studied in various works, e.g., [6,18–22]) via the following genotopies:

(1) The *forward and backward universal enveloping genoassociative algebra* $\hat{\xi}^>$, $^<\hat{\xi}$, with infinite-dimensional basis characterizing the *Poincaré-Birkhoff-Witt-Santilli genotheorem*

$$\hat{\xi}^> : \hat{I}^>,\ \hat{X}_i,\ \hat{X}_i > \hat{X}_j,\ \hat{X}_i > \hat{X}_j > \hat{X}_k,\ \ldots,\ i \le j \le k, \tag{4.2.22a}$$

$$^<\hat{\xi} : \hat{I},\ ^<\hat{X}_i,\ \hat{X}_i < \hat{X}_j,\ \hat{X}_i < \hat{X}_j < \hat{X}_k,\ \ldots,\ i \le j \le k; \tag{4.2.22b}$$

where the "hat" on the generators denotes their formulation on genospaces over genofields and their Hermiticity implies that $\hat{X}^> =^< \hat{X} = \hat{X}$;

(2) The *Lie-Santilli genoalgebras* characterized by the universal, jointly Lie- and Jordan-admissible brackets,

$$^<\hat{L}^> : (\hat{X}_i,\hat{X}_j) = \hat{X}_i < \hat{X}_j - \hat{X}_j > \hat{X}_i = C^k_{ij} \times \hat{X}_k, \tag{4.2.23}$$

here formulated in an invariant form (see below);

(3) The *Lie-Santilli genotransformation groups*

$$^<\hat{G}^> : \hat{A}(\hat{w}) = (\hat{e}^{\hat{i}\hat{\times}\hat{X}\hat{\times}\hat{w}})^> > \hat{A}(\hat{0}) <^< (\hat{e}^{-\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}}) =$$

$$= (e^{i\times\hat{X}\times\hat{T}^{>}\times w}) \times A(0) \times (e^{-i\times w\times{}^{<}\hat{T}\times\hat{X}}), \tag{4.2.24}$$

where $\hat{w}^{>} \in \hat{R}^{>}$ are the *genoparameters;* the *genorepresentation theory,* etc.

4.2.5 Genosymmetries and Nonconservation Laws

The implications of the Santilli Lie-admissible theory are significant mathematically and physically. On mathematical grounds, the Lie-Santilli genoalgebras are "directly universal" and include as particular cases all known algebras, such as Lie, Jordan, Flexible algebras, power associative algebras, quantum, algebras, supersymmetric algebras, Kac-Moody algebras, etc. (Section 1.5).

Moreover, when computed on the *genobimodule*

$${}^{<}\hat{B}^{>} = {}^{<}\hat{\xi}\times\hat{\xi}^{>}, \tag{4.2.25}$$

Lie-admissible algebras verify all Lie axioms, while deviations from Lie algebras emerge only in their *projection* on the conventional bimodule

$${}^{<}B^{>} = {}^{<}\xi\times\xi^{>} \tag{4.2.26}$$

of Lie's theory (see Ref. [17] for the initiation of the genorepresentation theory of Lie-admissible algebras on bimodules).

This is due to the fact that the computation of the left action $A < B = A \times^{<} \hat{T} \times B$ on ${}^{<}\hat{\xi}$ (that is, with respect to the genounit ${}^{<}\hat{I} = 1/{}^{<}\hat{T}$) yields the save value as the computation of the conventional product $A \times B$ on ${}^{<}\xi$ (that is, with respect to the trivial unit I), and the same occurs for the value of $A > B$ on $\hat{\xi}^{>}$.

The above occurrences explain the reason the structure constant and the product in the r.h.s. of Eq. (4.2.23) are those of a conventional Lie algebra.

In this way, thanks to genomathematics, *Lie algebras acquire a towering significance in view of the possibility of reducing all possible irreversible systems to primitive Lie axioms.*

The physical implications of the Lie-Santilli genotheory are equally far reaching. In fact, Noether's theorem on the reduction of reversible conservation laws to primitive Lie symmetries can be lifted to the *reduction, this time, of irreversible nonconservation laws to primitive Lie-Santilli genosymmetries.*

As a matter of fact, this reduction was the very first motivation for the construction of the genotheory in memoir [12] (see also monographs [6,18,19,20]). The reader can then foresee similar liftings of all remaining physical aspects treated via Lie algebras.

The construction of the isodual Lie-Santilli genotheory is an instructive exercise for readers interested in learning the new methods.

4.3 LIE-ADMISSIBLE CLASSICAL MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

4.3.1 Fundamental Ordering Assumption on Irreversibility

Another reason for the inability during the 20-th century for in depth studies of irreversibility is the general belief that motion in time has only two directions, forward and backward (Eddington historical time arrows). In reality, motion in time admits *four* different forms, all essential for serious studies in irreversibility, given by: 1) *motion forward to future time* characterized by the forward genotime $\hat{t}^{>}$; 2) *motion backward to past time* characterized by the backward genotime ${}^{<}\hat{t}$; 3) *motion backward from future time* characterized by the isodual forward genotime $\hat{t}^{>d}$; and 4) *motion forward from past time* characterized by the isodual backward genotime ${}^{<}\hat{t}^{d}$.

It is at this point where the *necessity* of both time reversal and isoduality appears in its full light. In fact, time reversal is only applicable to matter and, being represented with Hermitian conjugation, permits the transition from motion forward to motion backward in time, $\hat{t}^{>} \rightarrow {}^{<}\hat{t} = (\hat{t}^{>})^{\dagger}$. If used alone, time reversal cannot identify all four directions of motions. The *only* additional conjugation known to this author that is applicable at all levels of study and is equivalent to charge conjugation, is isoduality [22].

The additional discovery of two complementary orderings of the product and related units, with corresponding isoduals versions, individually preserving the abstract axioms of a field has truly fundamental implications for irreversibility, since it permits the axiomatically consistent and invariant representation of irreversibility via the most ultimate and primitive axioms, those on the product and related unit. We, therefore, have the following:

FUNDAMENTAL ORDERING ASSUMPTION ON IRREVERSIBILITY [15]: *Dynamical equations for motion forward in time of matter (antimatter) systems are characterized by genoproducts to the right and related genounits (their isoduals), while dynamical equations for the motion backward in time of matter (antimatter) are characterized by genoproducts to the left and related genounits (their isoduals) under the condition that said genoproducts and genounits are interconnected by time reversal expressible for generic quantities A, B with the relation,*

$$(A > B)^{\dagger} = (A > \hat{T}^{>} \times B)^{\dagger} = B^{\dagger} \times (\hat{T}^{>})^{\dagger} \times A^{\dagger}, \tag{4.3.1}$$

namely,

$$\hat{T}^{>} = ({}^{<}\hat{T})^{\dagger} \tag{4.3.2}$$

thus recovering the fundamental complementary conditions (4.1.17) or (4.2.2).

Unless otherwise specified, from now on physical and chemical expression for irreversible processes will have no meaning without the selection of one of the indicated two possible orderings.

4.3.2 Newton-Santilli Genoequations and Their Isoduals

Recall that, for the case of isotopies, the basic Newtonian systems are given by those admitting nonconservative internal forces restricted by certain constraints to verify total conservation laws called *closed non-Hamiltonian systems* [6b,18].

For the case of the genotopies under consideration here, the basic Newtonian systems are the conventional nonconservative systems without subsidiary constraints, known as*open non-Hamiltonian systems*, with generic expression (1.3), in which case irreversibility is entirely characterized by nonselfadjoint forces, since all conservative forces are reversible.

As it is well known, the above equations are not derivable from any variational principle in the fixed frame of the observer [6], and this is the reason all conventional attempts for consistently quantizing nonconservative forces have failed for about one century. In turn, the lack of achievement of a consistent operator counterpart of nonconservative forces lead to the belief that they are "illusory" because they "disappear" at the particle level.

The studies presented in this paper have achieved the first and only physically consistent operator formulation of nonconservative forces known to the author. This goal was achieved by rewriting Newton's equations (1.3) into an identical form derivable from a variational principle. Still in turn, the latter objective was solely permitted by the novel genomathematics.

It is appropriate to recall that Newton was forced to discover new mathematics, the differential calculus, prior to being able to formulated his celebrated equations. Therefore, readers should not be surprised at the need for the new genodifferential calculus as a condition to represent all nonconservative Newton's systems from a variational principle.

Recall also from Section 3.1 that, contrary to popular beliefs, there exist *four* inequivalent directions of time. Consequently, time reversal alone cannot represent all these possible motions, and isoduality results to be the only known additional conjugation that, when combined with time reversal, can represent all possible time evolutions of both matter and antimatter.

The above setting implies the existence of four different new mechanics first formulated by Santilli in memoir [14] of 1996, and today known as *Newton-Santilli genomechanics,* namely:

A) *Forward genomechanics* for the representation of forward motion of matter systems;

B) *Backward genomechanics* for the representation of the time reversal image of matter systems;

C) *Isodual backward genomechanics* for the representation of motion backward in time of antimatter systems, and

D) *Isodual forward genomechanics* for the representation of time reversal antimatter systems.

These new mechanics are characterized by:

1) Four different times, *forward and backward genotimes for matter systems and the backward and forward isodual genotimes for antimatter systems*

$$\hat{t}^{>} = t \times \hat{I}_t^{>}, \quad -\hat{t}^{>}, \quad \hat{t}^{>d}, \quad -\hat{t}^{>d}, \tag{4.3.3}$$

with (nowhere singular and non-Hermitian) *forward and backward time genounits and their isoduals* (Note that, to verify the condition of non-Hermiticity, the time genounits can be *complex valued.*),

$$\hat{I}_t^{>} = 1/\hat{T}_t^{>}, \quad -\hat{I}_t^{>}, \quad \hat{I}^{>d}_{\ t}, \quad -\hat{I}^{>d}_{\ t}; \tag{4.3.4}$$

2) The *forward and backward genocoordinates and their isoduals*

$$\hat{x}^{>} = x \times \hat{I}_x^{>}, \quad -\hat{x}^{>}, \quad \hat{x}^{>d}, \quad -\hat{x}^{>d}, \tag{4.3.5}$$

with (nowhere singular non-Hermitian) *coordinate genounit*

$$\hat{I}_x^{>} = 1/\hat{T}_x^{>}, \quad -\hat{I}_x^{>}, \quad \hat{I}^{>d}_{\ x}, \quad -\hat{I}^{>d}_{\ x}, \tag{4.3.6}$$

with *forward and backward coordinate genospace and their isoduals* $\hat{S}_x^{>}$, etc., and related *forward coordinate genofield and their isoduals* $\hat{R}_x^{>}$, etc.;

3) The *forward and backward genospeeds and their isoduals*

$$\hat{v}^{>} = \hat{d}^{>}\hat{x}^{>}/\hat{d}^{>}\hat{t}^{>}, \quad -\hat{v}^{>}, \quad \hat{v}^{>d}, \quad -\hat{v}^{>d}, \tag{4.3.7}$$

with (nowhere singular and non-Hermitian) *speed genounit*

$$\hat{I}_v^{>} = 1/\hat{T}_v^{>}, \quad -\hat{I}_v^{>}, \quad \hat{I}^{>d}_{\ v}, \quad -\hat{I}^{>d}_{\ v}, \tag{4.3.8}$$

with related *forward speed backward genospaces and their isoduals* $\hat{S}_v^{>}$, etc., over *forward and backward speed genofields* $\hat{R}_v^{>}$, etc.

The above formalism then leads to the *forward genospace for matter systems*

$$\hat{S}_{tot}^{>} = \hat{S}_t^{>} \times \hat{S}_x^{>} \times \hat{S}_v^{>}, \tag{4.3.9}$$

defined over the *forward genofield*

$$\hat{R}_{tot}^{>} = \hat{R}_t^{>} \times \hat{R}_x^{>} \times \hat{R}_v^{>}, \tag{4.3.10}$$

with *total forward genounit*

$$\hat{I}_{tot}^{>} = \hat{I}_t^{>} \times \hat{I}_x^{>} \times \hat{I}_v^{>}, \tag{4.3.11}$$

and corresponding expressions for the remaining three spaces obtained via time reversal and isoduality.

The basic equations are given by:

I) The *forward Newton-Santilli genoequations for matter systems* [14], formulated via the genodifferential calculus,

$$\hat{m}_a^{>} > \frac{\hat{d}^{>}\hat{v}_{ka}^{>}}{\hat{d}^{>}\hat{t}^{>}} = -\frac{\hat{\partial}^{>}\hat{V}^{>}}{\hat{\partial}^{>}\hat{x}_a^{>k}}; \tag{4.3.12}$$

II) The *backward genoequations for matter systems* that are characterized by time reversal of the preceding ones;

III) the *backward isodual genoequations for antimatter systems* that are characterized by the isodual map of the backward genoequations,

$$^{<}\hat{m}_a^d < \frac{^{<}\hat{d}^{d<}\hat{v}_{ka}^d}{<\hat{d}^{d<}\hat{t}^d} = -\frac{^{<}\hat{\partial}^{d<}\hat{V}^d}{<\hat{\partial}^{d<}\hat{x}_a^{dk}}; \tag{4.3.13}$$

IV) the *forward isodual genoequations for antimatter systems* characterized by time reversal of the preceding isodual equations.

Newton-Santilli genoequations (4.3.12) are "directly universal" for the representation of all possible (well behaved) Eqs. (1.3) in the frame of the observer because they admit a multiple infinity of solution for any given nonselfadjoint force.

A simple representation occurs under the conditions assumed for simplicity,

$$N = \hat{I}_t^{>} = \hat{I}_v^{>} = 1, \tag{4.3.14}$$

for which Eqs. (3.12) can be explicitly written

$$\hat{m}^{>} > \frac{\hat{d}^{>}\hat{v}^{>}}{\hat{d}^{>}t} = m \times \frac{d\hat{v}^{>}}{dt} =$$

$$= m \times \frac{d}{dt}\frac{d(x \times \hat{I}_x^{>})}{dt} = m \times \frac{dv}{dt} \times \hat{I}_x^{>} + m \times x \times \frac{d\hat{I}^{>}}{dt} = \hat{I}_x^{>} \times \frac{\partial V}{\partial x}, \tag{4.3.15}$$

from which we obtain the genorepresentation

$$F^{NSA} = -m \times x \times \frac{1}{\hat{I}_x^{>}} \times \frac{d\hat{I}_x^{>}}{dt}, \tag{4.3.16}$$

that always admit solutions here left to the interested reader since in the next section we shall show a much simpler, universal, *algebraic* solution.

As one can see, in Newton's equations the nonpotential forces are part of the applied force, while in the Newton-Santilli genoequations nonpotential forces are

represented by the genounits, or, equivalently, by the genodifferential calculus, in a way essentially similar to the case of isotopies.

The main difference between iso- and geno-equations is that isounits are Hermitian, thus implying the equivalence of forward and backward motions, while genounits are non-Hermitian, thus implying irreversibility.

Note also that the topology underlying Newton's equations is the conventional, Euclidean, local-differential topology which, as such, can only represent point particles.

By contrast, the topology underlying the Newton-Santilli genoequations is given by a genotopy of the isotopology studied in the preceding chapter, thus permitting the representation of extended, nonspherical and deformable particles via forward genounits, e.g., of the type

$$\hat{I}^{>} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times \Gamma^{>}(t, r, v, \ldots), \tag{4.3.17}$$

where $n_k^2,\ k = 1, 2, 3$ represents the semiaxes of an ellipsoid, n_4^2 represents the density of the medium in which motion occurs (with more general nondiagonal realizations here omitted for simplicity), and $\Gamma^{>}$ constitutes a nonsymmetric matrix representing nonselfadjoint forces, namely, the contact interactions among extended constituents occurring for the motion forward in time.

4.3.3 Hamilton-Santilli Genomechanics and Its Isodual

In this section we show that, once rewritten in their identical genoform (4.3.12), Newton's equations for nonconservative systems are indeed derivable from a variational principle, with analytic equations possessing a Lie-admissible structure and Hamilton-Jacobi equations suitable for the first known consistent and unique operator map studied in the next section.

The most effective setting to introduce real-valued non-symmetric genounits is in the $6N$-dimensional *forward genospace (genocotangent bundle)* with local genocoordinates and their conjugates

$$\hat{a}^{>\mu} = a^{\rho} \times \hat{I}_{1\,\rho}^{>\mu},\ \ (\hat{a}^{>\mu}) = \begin{pmatrix} \hat{x}_{\alpha}^{>k} \\ \hat{p}_{k\alpha}^{>} \end{pmatrix} \tag{4.3.18}$$

and

$$\hat{R}_{\mu}^{>} = R_{\rho} \times \hat{I}_{2\,\mu}^{>\rho},\ \ (\hat{R}_{\mu}^{>}) = (\hat{p}_{k\alpha}, \hat{0}), \tag{4.3.19a}$$

$$\hat{I}_1^{>} = 1/\hat{T}_1^{>} = (\hat{I}_2^{>})^T = (1/\hat{T}_2^{>})^T, \tag{4.3.19b}$$

$$k = 1, 2, 3;\ \alpha = 1, 2, \ldots, N;\ \ \mu,\ \rho = 1, 2, \ldots 6N,$$

where the superscript T stands for transposed, and nowhere singular, real-valued and non-symmetric genometric and related invariant

$$\hat{\delta}^{>} = \hat{T}_{1\,6N\times 6N}^{>} \delta_{6N\times 6N} \times \delta_{6N\times 6N}, \tag{4.3.20a}$$

$$\hat{a}^{>\mu} > \hat{R}^{>}_{\mu} = \hat{a}^{>\rho} \times \hat{T}^{>\beta}_{1\,\rho} \times \hat{R}^{>}_{\beta} = a^{\rho} \times \hat{I}^{>\beta}_{2\,\rho} \times R_{\beta}. \tag{4.3.20b}$$

In this case we have the following *genoaction principle* [14]

$$\hat{\delta}^{>}\hat{\mathcal{A}}^{>} = \hat{\delta}^{>}\hat{\int}^{>}[\hat{R}^{>}_{\mu} >_a \hat{d}^{>}\hat{a}^{>\mu} - \hat{H}^{>} >_t \hat{d}^{>}\hat{t}^{>}] =$$

$$= \delta\int[R_{\mu} \times \hat{T}^{>\mu}_{1\,\nu}(t,x,p,\ldots) \times d(a^{\beta} \times \hat{I}^{>\nu}_{1\,\beta}) - H \times dt] = 0, \tag{4.3.21}$$

where the second expression is the projection on conventional spaces over conventional fields and we have assumed for simplicity that the time genounit is 1.

It is easy to prove that the above genoprinciple characterizes the following *forward Hamilton-Santilli genoequations,* (originally proposed in Ref. [11] of 1978 with conventional mathematics and in Ref. [14] of 1996 with genomathematics (see also Refs. [18,19,20])

$$\hat{\omega}^{>}_{\mu\nu} > \frac{\hat{d}^{>}\hat{a}^{\nu>}}{\hat{d}^{>}\hat{t}^{>}} - \frac{\hat{\partial}^{>}\hat{H}^{>}(\hat{a}^{>})}{\hat{\partial}^{>}\hat{a}^{\mu>}} =$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} dr/dt \\ dp/dt \end{pmatrix} - \begin{pmatrix} 1 & K \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \partial H/\partial r \\ \partial H/\partial p \end{pmatrix} = 0, \tag{4.3.22a}$$

$$\hat{\omega}^{>} = \Big(\frac{\hat{\partial}^{>}R^{>}_{\nu}}{\hat{\partial}^{>}\hat{a}^{\mu>}} - \frac{\hat{\partial}^{>}\hat{R}^{>}_{\mu}}{\hat{\partial}^{>}\hat{a}^{\nu>}}\Big) \times \hat{I}^{>} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \hat{I}^{>}, \tag{4.3.22b}$$

$$K = F^{NSA}/(\partial H/\partial p), \tag{4.3.22c}$$

where one should note the "direct universality" of the simple algebraic solution (3.22c).

The time evolution of a quantity $\hat{A}^{>}(\hat{a}^{>})$ on the forward geno-phase-space can be written in terms of the following brackets

$$\frac{\hat{d}^{>}\hat{A}^{>}}{\hat{d}^{>}t^{>}} = (\hat{A}^{>}, \hat{H}^{>}) = \frac{\hat{\partial}^{>}\hat{A}^{>}}{\hat{\partial}^{>}\hat{a}^{>\mu}} > \hat{\omega}^{\mu\nu>} > \frac{\hat{\partial}^{>}\hat{H}^{>}}{\hat{\partial}\hat{a}^{>\nu}} =$$

$$= \frac{\partial\hat{A}^{>}}{\partial\hat{a}^{>\mu}} \times S^{\,\mu\nu} \times \frac{\partial\hat{H}^{>}}{\partial\hat{a}^{>\nu}} =$$

$$= \Big(\frac{\partial\hat{A}^{>}}{\partial\hat{r}^{>k}_{\alpha}} \times \frac{\partial\hat{H}^{>}}{\partial\hat{p}^{>}_{ka}} - \frac{\partial\hat{A}^{>}}{\partial\hat{p}^{>}_{ka}} \times \frac{\partial\hat{H}^{>}}{\partial\hat{r}^{>k}_{a}}\Big) + \frac{\partial\hat{A}^{>}}{\partial\hat{p}^{>}_{ka}} \times F^{NSA}_{ka}, \tag{4.3.23a}$$

$$S^{>\mu\nu} = \omega^{\mu\rho} \times \hat{I}^{2\mu}_{\rho}, \omega^{\mu\nu} = (||\omega_{\alpha\beta}||^{-1})^{\mu\nu}, \tag{4.3.23b}$$

where $\omega^{\mu\nu}$ is the conventional Lie tensor and, consequently, $S^{\mu\nu}$ is Lie-admissible in the sense of Albert [7].

As one can see, the important consequence of genomathematics and its genodifferential calculus is that of turning the triple system (A, H, F^{NSA}) of Eqs. (1.5) in the bilinear form $(A\hat{,}B)$, thus characterizing a consistent algebra in the brackets of the time evolution.

This is the central purpose for which genomathematics was built (note that the multiplicative factors represented by K are fixed for each given system). The invariance of such a formulation will be proved shortly.

It is an instructive exercise for interested readers to prove that the brackets $(A\hat{,}B)$ are Lie-admissible, although not Jordan-admissible.

It is easy to verify that the above identical reformulation of Hamilton's historical time evolution correctly recovers the *time rate of variations of physical quantities* in general, and that of the energy in particular,

$$\frac{dA^>}{dt} = (A^>, H^>) = [\hat{A}^>, \hat{H}^>] + \frac{\partial \hat{A}^>}{\partial \hat{p}^>_{k\alpha}} \times F^{NSA}_{k\alpha}, \tag{4.3.24a}$$

$$\frac{dH}{dt} = [\hat{H}^>, \hat{H}^>] + \frac{\partial \hat{H}^>}{\partial \hat{p}^>_{k\alpha}} \times F^{NSA}_{ka} = v^k_\alpha \times F^{NSA}_{ka}. \tag{4.3.24b}$$

It is easy to show that genoaction principle (4.3.21) characterizes the following *Hamilton-Jacobi-Santilli genoequations* [14]

$$\frac{\hat{\partial}^> \mathcal{A}^>}{\hat{\partial}^> \hat{t}^>} + \hat{H}^> = 0, \tag{4.3.25a}$$

$$\Big(\frac{\hat{\partial}^> \mathcal{A}^>}{\hat{\partial}^> \hat{a}^{>\mu}}\Big) = \Big(\frac{\hat{\partial}^> \mathcal{A}^>}{\hat{\partial}^> x^{>\,k}_a}, \frac{\hat{\partial}^> \mathcal{A}^>}{\hat{\partial}^> p^>_{ka}}\Big) = (\hat{R}^>_\mu) = (\hat{p}^>_{ka}, \hat{0}), \tag{4.3.25b}$$

which confirm the property (crucial for genoquantization as shown below) that the genoaction is indeed independent of the linear momentum.

Note the *direct universality* of the Lie-admissible equations for the representation of all infinitely possible Newton equations (1.3) (universality) directly in the fixed frame of the experimenter (direct universality).

Note also that, *at the abstract, realization-free level, Hamilton-Santilli genoequations coincide* with Hamilton's equations without external terms, yet represent those with external terms.

The latter are reformulated via genomathematics as the only known way to achieve invariance and derivability from a variational principle while admitting a consistent algebra in the brackets of the time evolution [38].

Therefore, Hamilton-Santilli genoequations (3.6.66) are indeed irreversible for all possible reversible Hamiltonians, as desired. The origin of irreversibility rests in the contact nonpotential forces F^{NSA} according to Lagrange's and Hamilton's teaching that is merely reformulated in an invariant way.

The above Lie-admissible mechanics requires, for completeness, *three* additional formulations, the *backward genomechanics* for the description of *matter moving backward in time,* and the isoduals of both the forward and backward mechanics for the description of *antimatter.*

The construction of these additional mechanics is left to the interested reader for brevity.

4.4 LIE-ADMISSIBLE OPERATOR MECHANICS FOR MATTER AND ITS ISODUAL FOR ANTIMATTER

4.4.1 Basic Dynamical Equations

A simple genotopy of the naive or symplectic quantization applied to Eqs. (3.24) yields the *Lie-admissible branch of hadronic mechanics* [18] comprising four different formulations, the *forward and backward genomechanics for matter and their isoduals for antimatter.* The forward genomechanics for matter is characterized by the following main topics:

1) The nowhere singular (thus everywhere invertible) non-Hermitian *forward genounit* for the representation of all effects causing irreversibility, such as contact nonpotential interactions among extended particles, etc. (see the subsequent chapters for various realizations)

$$\hat{I}^{>} = 1/\hat{T}^{>} \neq (\hat{I}^{>})^{\dagger}, \tag{4.4.1}$$

with corresponding ordered product and genoreal $\hat{R}^{>}$ and genocomplex $\hat{C}^{>}$ genofields;

2) The *forward genotopic Hilbert space* $\hat{\mathcal{H}}^{>}$ with *forward genostates* $|\hat{\psi}^{>}>$ and *forward genoinner product*

$$<^{<} \hat{\psi}| > |\hat{\psi}^{>} > \times \hat{I}^{>} =<^{<} \hat{\psi}| \times \hat{T}^{>} \times |\hat{\psi}^{>} > \times \hat{I}^{>} \in \hat{C}^{>}, \tag{4.4.2}$$

and fundamental property

$$\hat{I}^{>} > |\hat{\psi}^{>} >= |\hat{\psi}^{>} >, \tag{4.4.3}$$

holding under the condition that $\hat{I}^{>}$ is indeed the correct unit for motion forward in time, and *forward genounitary transforms*

$$\hat{U}^{>} > (^{<}\hat{U})^{\dagger >} = (^{<}\hat{U})^{\dagger >} > \hat{U}^{>} = \hat{I}^{>}; \tag{4.4.4}$$

3) The fundamental Lie-admissible equations, first proposed in Ref. [12] of 1974 (p. 783, Eqs. (4.18.16)) as the foundations of hadronic mechanics, formulated on conventional spaces over conventional fields, and first formulated in Refs. [14,18]

of 1996 on genospaces and genodifferential calculus on genofields, today's known as *Heisenberg-Santilli genoequations*, that can be written in the finite form

$$\hat{A}(\hat{t}) = \hat{U}^{>} > \hat{A}(0) <^{<} \hat{U} = (\hat{e}_{>}^{\hat{i}\hat{\times}\hat{H}\hat{\times}\hat{t}}) > \hat{A}(\hat{0}) < (_{<}\hat{e}^{-\hat{i}\hat{\times}\hat{t}\hat{\times}\hat{H}}) =$$

$$= (e^{i\times\hat{H}\times\hat{T}^{>}\times t}) \times A(0) \times (e^{-i\times t\times^{<}\hat{T}\times\hat{H}}), \tag{4.4.5}$$

with corresponding infinitesimal version

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = (\hat{A}\hat{,}\hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A} =$$

$$= \hat{A} \times^{<} \hat{T}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots.) \times \hat{H} - \hat{H} \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots) \times \hat{A}, \tag{4.4.6}$$

where there is no time arrow, since Heisenberg's equations are computed at a fixed time;

4) The equivalent *Schrödinger-Santilli genoequations*, first suggested in the original proposal [12] to build hadronic mechanics (see also Refs. [17,23,24]), formulated via conventional mathematics and in Refs. [14,18] via genomathematics, that can be written

$$\hat{i}^{>} > \frac{\hat{\partial}^{>}}{\hat{\partial}^{>}\hat{t}^{>}}|\hat{\psi}^{>}>= \hat{H}^{>} > |\hat{\psi}^{>}>=$$

$$= \hat{H}(\hat{r}, \hat{v}) \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \hat{\partial}\hat{\psi}\ldots) \times |\hat{\psi}^{>}>= E^{>} > |\psi^{>}>, \tag{4.4.7}$$

where the time orderings in the second term are ignored for simplicity of notation;

5) The *forward genomomentum* that escaped identification for two decades and was finally identified thanks to the genodifferential calculus in Ref. [14] of 1996

$$\hat{p}_k^{>} > |\hat{\psi}^{>}>= -\hat{i}^{>} > \hat{\partial}_k^{>}|\hat{\psi}^{>}>= -i \times \hat{I}_k^{>\,i} \times \partial_i|\hat{\psi}^{>}>; \tag{4.4.8}$$

6) The *fundamental genocommutation rules* also first identified in Ref. [14],

$$(\hat{r}^i\hat{,}\hat{p}_j) = i \times \delta_j^i \times \hat{I}^{>}, \;\; (\hat{r}^i\hat{,}\hat{r}^j) = (\hat{p}_i\hat{,}\hat{p}_j) = 0; \tag{4.4.9}$$

7) The *genoexpectation values* of an observable for the forward motion $\hat{A}^{>}$ [14,19]

$$\frac{<^{<}\hat{\psi}| > \hat{A}^{>} > |\hat{\psi}^{>}>}{<^{<}\hat{\psi}| > |\hat{\psi}^{>}>} \times \hat{I}^{>} \in \hat{C}^{>}, \tag{4.4.10}$$

under which the genoexpectation values of the genounit recovers the conventional Planck's unit as in the isotopic case,

$$\frac{<\hat{\psi}| > \hat{I}^{>} > |\hat{\psi}>}{<\hat{\psi}| > |\hat{\psi}>} = I. \tag{4.4.11}$$

The following comments are now in order. Note first in the genoaction principle the crucial independence of isoaction $\hat{\mathcal{A}}^>$ in form the linear momentum, as expressed by the Hamilton-Jacobi-Santilli genoequations (4.3.25). Such independence assures that genoquantization yields a genowavefunction solely dependent on time and coordinates, $\hat{\psi}^> = \hat{\psi}^>(t, r)$.

Other geno-Hamiltonian mechanics studied previously [7] do not verify such a condition, thus implying genowavefunctions with an explicit dependence also on linear momenta, $\hat{\psi}^> = \hat{\psi}^>(t, r, p)$ that violate the abstract identity of quantum and hadronic mechanics whose treatment in any case is beyond our operator knowledge at this writing.

Note that *forward geno-Hermiticity coincides with conventional Hermiticity.* As a result, *all quantities that are observables for quantum mechanics remain observables for the above genomechanics.*

However, unlike quantum mechanics, physical quantities are generally *nonconserved,* as it must be the case for the energy,

$$\hat{i}^> > \frac{\hat{d}^> \hat{H}^>}{\hat{d}^> \hat{t}^>} = \hat{H} \times (^<\hat{T} - \hat{T}^>) \times \hat{H} \neq 0. \tag{4.4.12}$$

Therefore, *the genotopic branch of hadronic mechanics is the only known operator formulation permitting nonconserved quantities to be Hermitian as a necessary condition to be observable.*

Other formulation attempt to represent nonconservation, e.g., by adding an "imaginary potential" to the Hamiltonian, as it is often done in nuclear physics [25]. In this case the Hamiltonian is non-Hermitian and, consequently, the nonconservation of the energy cannot be an observable.

Besides, said "nonconservative models" with non-Hermitian Hamiltonians are nonunitary and are formulated on conventional spaces over conventional fields, thus suffering all the catastrophic inconsistencies of Theorem 1.3.

We should stress the representation of irreversibility and nonconservation beginning with the most primitive quantity, the unit and related product. *Closed irreversible systems* are characterized by the Lie-isotopic subcase in which

$$\hat{i} \hat{\times} \frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = [\hat{A}, \hat{H}] = \hat{A} \times \hat{T}(t, \ldots) \times \hat{H} - \hat{H} \times \hat{T}(t, \ldots) \times \hat{A}, \tag{4.4.13a}$$

$$^<\hat{T}(t, \ldots) = \hat{T}^>(t, \ldots) = \hat{T}(t, \ldots) = \hat{T}^\dagger(t, \ldots) \neq \hat{T}(-t, \ldots), \tag{4.4.13b}$$

for which the Hamiltonian is manifestly conserved. Nevertheless the system is manifestly irreversible. Note also the first and only known observability of the Hamiltonian (due to its iso-Hermiticity) under irreversibility.

As one can see, brackets (A, B) of Eqs. (4.6) are jointly Lie- and Jordan-admissible.

Note also that finite genotransforms (4.4.5) verify the condition of genohermiticity, Eq. (4.4).

We should finally mention that, as it was the case for isotheories, *genotheories are also admitted by the abstract axioms of quantum mechanics, thus providing a broader realization.* This can be seen, e.g., from the invariance under a complex number C

$$< \psi|x|\psi > \times I =< \psi|xC^{-1} \times |\psi > \times (C \times I) =< \psi| > |\psi > \times I^{>}. \qquad (4.4.14)$$

Consequently, *genomechanics provide another explicit and concrete realization of "hidden variables" [26], thus constituting another "completion" of quantum mechanics in the E-P-R sense [27].* For the studies of these aspects we refer the interested reader to Ref. [28].

The above formulation must be completed with three additional Lie-admissible formulations, the backward formulation for matter under time reversal and the two additional isodual formulations for antimatter. Their study is left to the interested reader for brevity.

4.4.2 Simple Construction of Lie-Admissible Theories

As it was the case for the isotopies, a simple method has been identified in Ref. [44] for the construction of Lie-admissible (geno-) theories from any given conventional, classical or quantum formulation. It consists in *identifying the genounits as the product of two different nonunitary transforms,*

$$\hat{I}^{>} = (^{<}\hat{I})^{\dagger} = U \times W^{\dagger}, \quad ^{<}\hat{I} = W \times U^{\dagger}, \qquad (4.4.15a)$$

$$U \times U^{\dagger} \neq 1, \quad W \times W^{\dagger} \neq 1, \quad U \times W^{\dagger} = \hat{I}^{>}, \qquad (4.4.15b)$$

and subjecting the totality of quantities and their operations of conventional models to said dual transforms,

$$I \to \hat{I}^{>} = U \times I \times W^{\dagger}, \quad I \to^{<} \hat{I} = W \times I \times U^{\dagger}, \qquad (4.4.16a)$$

$$a \to \hat{a}^{>} = U \times a \times W^{\dagger} = a \times \hat{I}^{>}, \qquad (4.4.16b)$$

$$a \to^{<} \hat{a} = W \times a \times U^{\dagger} =^{<} \hat{I} \times a, \qquad (4.4.16c)$$

$$a \times b \to \hat{a}^{>} > \hat{b}^{>} = U \times (a \times b) \times W^{>} =$$

$$= (U \times a \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times b \times W^{\dagger}), \qquad (4.4.16d)$$

$$\partial/\partial x \to \hat{\partial}^{>}/\hat{\partial}^{>}\hat{x}^{>} = U \times (\partial/\partial x) \times W^{\dagger} = \hat{I}^{>} \times (\partial/\partial x), \qquad (4.4.16e)$$

$$< \psi| \times |\psi > \to <^{<} \psi| > |\psi^{>} >= U \times (< \psi| \times |\psi >) \times W^{\dagger}, \qquad (4.4.16f)$$

$$H \times |\psi > \to \hat{H}^{>} > |\psi^{>} >=$$

$$= (U \times H \times W^\dagger) \times (U \times W^\dagger)^{-1} \times (U \times \psi > W^\dagger), \text{ etc.} \tag{4.4.16g}$$

As a result, any given conventional, classical or quantum model can be easily lifted into the genotopic form.

Note that the above construction implies that *all conventional physical quantities acquire a well defined direction of time.* For instance, the correct genotopic formulation of energy, linear momentum, etc., is given by

$$\hat{H}^> = U \times H \times W^\dagger, \;\; \hat{p}^> = U \times p \times W^>, \text{ etc.} \tag{4.4.17}$$

In fact, under irreversibility, the value of a nonconserved energy at a given time t for motion forward in time is generally different than the corresponding value of the energy for $-t$ for motion backward in past times.

This explains the reason for having represented in this section energy, momentum and other quantities with their arrow of time $>$. Such an arrow can indeed be omitted for notational simplicity, but only after the understanding of its existence.

Note finally that a conventional, one dimensional, unitary Lie transformation group with Hermitian generator X and parameter w can be transformed into a covering Lie-admissible group via the following nonunitary transform

$$Q(w) \times Q^\dagger(w) = Q^\dagger(w) \times Q(w) = I, \; w \in R, \tag{4.4.18a}$$

$$U \times U^\dagger \neq I, \quad W \times W^\dagger \neq 1, \tag{4.4.18b}$$

$$\begin{aligned} A(w) = Q(w) \times A(0) \times Q^\dagger(w) = e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X} \rightarrow \\ \rightarrow U \times (e^{X \times w \times i} \times A(0) \times e^{-i \times w \times X}) \times U^\dagger = \\ \equiv [U \times (e^{X \times w \times i}) \times W^\dagger \times (U \times W^\dagger)^{-1} \times A \times A(0) \times \\ \times U^\dagger \times (W \times U^\dagger)^{-1} \times [W \times (e^{-i \times w \times X}) \times U^\dagger] = \\ = (e^{i \times X \times X})^> > A(0) <^< (e^{-1 \times w \times X}) = \hat{U}^> > A(0) <^< \hat{U}, \end{aligned} \tag{4.4.18c}$$

which confirm the property of Section 4.2, namely, that under the necessary mathematics *the Lie-admissible theory is indeed admitted by the abstract Lie axioms, and it is a realization of the latter broader than the isotopic form.*

4.4.3 Invariance of Lie-Admissible Theories

Recall that a fundamental axiomatic feature of quantum mechanics is the invariance under time evolution of all numerical predictions and physical laws, which invariance is due to the *unitary structure* of the theory.

However, quantum mechanics is reversible and can only represent in a scientific way beyond academic beliefs reversible systems verifying total conservation laws due to the antisymmetric character of the brackets of the time evolution.

As indicated earlier, the representation of irreversibility and nonconservation requires theories with a *nonunitary structure*. However, the latter are afflicted by the catastrophic inconsistencies of Theorem 1.3.

The only resolution of such a basic impasse known to the author has been the achievement of invariance under nonunitarity and irreversibility via the use of genomathematics, provided that such genomathematics is applied to the *totality* of the formalism to avoid evident inconsistencies caused by mixing different mathematics for the selected physical problem.

Let us note that, due to decades of protracted use it is easy to predict that physicists and mathematicians may be tempted to treat the Lie-admissible branch of hadronic mechanics with conventional mathematics, whether in part or in full. Such a posture would be equivalent, for instance, to the elaboration of the spectral emission of the hydrogen atom with the genodifferential calculus, resulting in an evident nonscientific setting.

Such an invariance was first achieved by Santilli in Ref. [15] of 1997 and can be illustrated by reformulating any given nonunitary transform in the *genounitary form*

$$U = \hat{U} \times \hat{T}^{>1/2}, W = \hat{W} \times \hat{T}^{>1/2}, \tag{4.4.19a}$$

$$U \times W^\dagger = \hat{U} > \hat{W}^\dagger = \hat{W}^\dagger > \hat{U} = \hat{I}^> = 1/\hat{T}^>, \tag{4.4.19b}$$

and then showing that genounits, genoproducts, genoexponentiation, etc., are indeed invariant under the above genounitary transform in exactly the same way as conventional units, products, exponentiations, etc. are invariant under unitary transforms,

$$\hat{I}^> \to \hat{I}^{>\prime} = \hat{U} > \hat{I}^> > \hat{W}^\dagger = \hat{I}^>, \tag{4.4.20a}$$

$$\begin{aligned} \hat{A} > \hat{B} \to \hat{U} > (A > B) > \hat{W}^\dagger = \\ = (\hat{U} \times \hat{T}^> \times A \times T^> \times \hat{W}^\dagger) \times (\hat{T}^> \times W^\dagger)^{-1} \times \hat{T}^> \times \\ \times (\hat{U} \times \hat{T}^>)^{-1} \times (\hat{U} \times T^> \times \hat{A} \times T^> \times \hat{W}^>) = \\ = \hat{A}' \times (\hat{U} \times \hat{W}^\dagger)^{-1} \times \hat{B} = \hat{A}' \times \hat{T}^> \times B' = \hat{A}' > \hat{B}', \text{ etc.}, \end{aligned} \tag{4.4.20b}$$

from which all remaining invariances follow, thus resolving the catastrophic inconsistencies of Theorem 1.3.

Note the *numerical invariances of the genounit* $\hat{I}^> \to \hat{I}^{>\prime} \equiv \hat{I}^>$, *of the genotopic element* $\hat{T}^> \to \hat{T}^{>\prime} \equiv \hat{T}^>$, *and of the genoproduct* $>\to>'\equiv>$ that are necessary to have invariant numerical predictions.

4.5 APPLICATIONS

4.5.1 Lie-admissible Treatment of Particles with Dissipative Forces

In this section we present a variety of classical and operator representations of nonconservative systems by omitting hereon for simplicity of notation all "hats"

on quantities (denoting isotopies not considered in this section), omitting the symbol $\times$ to denote the conventional (associative) multiplication, but preserving the forward (backward) symbols $>$ ($<$) denoting forward (backward) motion in time for quantities and products. The content of this section was presented for the first time by the author in memoir [32].

Let us begin with a classical and operator representation of the simplest possible dissipative system, a massive particle moving within a physical medium, and being subjected to a linear, velocity-dependent resistive force

$$m\frac{dv}{dt} = F^{NSA} = -kv, \tag{4.5.1}$$

for which we have the familiar *variation (dissipation) of the energy*

$$\frac{d}{dt}(\frac{1}{2}mv^2) = -kv^2. \tag{4.5.2}$$

Progressively more complex examples will be considered below.

The representations of system (5.1) via the *Newton-Santilli genoequations* (3.12) is given by

$$m^> > \frac{d^>v^>}{d^>t^>} = 0. \tag{4.5.3}$$

As indicated in Section 3, the representation requires the selection of *three* generally different genounits, $I_t^>, I_r^>, I_v^>$. Due to the simplicity of the case and the velocity dependence of the applied force, the simplest possible solution is given by

$$I_t^> = I_r^> = 1, \quad I_v^>(t) = e^{\frac{k\times t}{m}} = 1/T_v^>(t) > 0, \tag{4.5.4a}$$

$$m^> > \frac{d^>v^>}{d^>t^>} = m\frac{d(vI_v^>)}{dt} = m\frac{dv}{dt}I^> + kv\frac{dI_v^>}{dt} = 0. \tag{4.5.4b}$$

The representation with *Hamilton-Santilli genoequations* (3.22) is also straightforward and can be written in disjoint $r^>$ and $p^>$ notations

$$H^> = \frac{{p^>}^{2>}}{2^> > m^>} = \frac{p^2}{2m}I_p^>, \tag{4.5.5a}$$

$$v^> = \frac{\partial^> H^>}{\partial^> p^>} = \frac{p^>}{m}, \quad \frac{d^>p^>}{d^>t^>} = -\frac{\partial^> H^>}{\partial^> r^>} = 0. \tag{4.5.5b}$$

The last equation then reproduces equation of motion (5.1) identically under assumptions (5.4a).

The above case is instructive because the representation is achieved via the genoderivatives (Section 2.2). However, the representation exhibits no algebra in the time evolution. Therefore, we seek an alternative representation in which the

dissipation is characterized by the Lie-admissible algebra, rather by the differential calculus.

This alternative representation is provided by the Hamilton-Santilli genoequations (3.22) in the unified notation $a^> = (r^{>k}, p^>_k)$ that become for the case at hand

$$\frac{da^{>\mu}}{dt} = \begin{pmatrix} dr^>/dt \\ dp^>/dt \end{pmatrix} = S^{>\mu\nu}\frac{\partial^> H^>}{\partial^> a^{>\nu}} = \begin{pmatrix} 0 & -1 \\ 1 & \frac{-kv}{(\partial H/\partial p)} \end{pmatrix} \begin{pmatrix} \partial^> H^>/\partial^> r^> \\ \partial^> H^>/\partial^> p^> \end{pmatrix}, \tag{4.5.6}$$

under which we have the genoequations

$$\frac{dr^>}{dt} = \frac{\partial^> H^>}{\partial^> p^>} = \frac{p^>}{m}, \quad \frac{dp^>}{dt} = -kv, \tag{4.5.7}$$

where one should note that the derivative can be assumed to be conventional, since the system is represented by the mutation of the Lie structure.

To achieve a representation of system (5.1) suitable for operator image, we need the following *classical, finite, Lie-admissible transformation genogroup*

$$A(t) = (e^{-t\frac{\partial H}{\partial a^\mu}S^{>\mu\nu}\frac{\partial}{\partial a^\nu}})A(0)(e^{\frac{\partial}{\partial a^\nu}{}^< S^{\nu\mu}\frac{\partial H}{\partial a^\mu}t}), \tag{4.5.8}$$

defined in the 12−dimensional *bimodular genophasespace* $^<T^*M \times T^*M^>$, with *infinitesimal Lie-admissible time evolution*

$$\begin{aligned} \frac{dA}{dt} &= \frac{\partial A}{\partial a^\mu}(^< S^{\mu\nu} - S^{>\mu\nu})\frac{\partial H}{\partial a^\nu} = \\ &= (\frac{\partial A}{\partial r^k}\frac{\partial H}{\partial p_k} - \frac{\partial H}{\partial r^k}\frac{\partial A}{\partial p_k}) - (\frac{kv}{(\partial H/\partial p)})\frac{\partial H}{\partial p}\frac{\partial A}{\partial p}) = \\ &= [A, H] - kv\frac{\partial A}{\partial p}, \end{aligned} \tag{4.5.9}$$

where we have dropped the forward arrow for notational convenience, and $\omega^{\mu\nu}$ is the canonical Lie tensor, thus proving the Lie-admissibility of the S-tensors. In fact, the attached antisymmetric brackets $[A, H]$ are the conventional Poisson brackets, while $\{A, H\}$ are indeed symmetric brackets (as requested by Lie-admissibility), but they do not characterize a Jordan algebra (Section 4.1.3).

It is easy to see that the time evolution of the Hamiltonian is given by

$$\frac{dH}{dt} = -kv\frac{\partial H}{\partial p} = -kv^2, \tag{4.5.10}$$

thus correctly reproducing behavior (5.2).

The operator image of the above dissipative system is straightforward. Physically, we are also referring to a first approximation of a massive and stable elementary particle, such as an electron, penetrating within hadronic matter (such

as a nucleus). Being stable, the particle is not expected to "disappear" at the initiation of the dissipative force and be converted into "virtual states" due to the inability of represent such a force, but more realistically the particle is expected to experience a rapid dissipation of its kinetic energy and perhaps after that participate in conventional processes.

Alternatively, we can say that an electron orbiting in an atomic structure does indeed evolve in time with conserved energy, and the system is indeed Hamiltonian. By the idea that the same electron when in the core of a star also evolves with conserved energy is repugnant to reason. Rather than adapting nature to manifestly limited Hamiltonian theories, we seek their covering for the treatment of systems for which said theories were not intended for.

The problem is to identify forward and backward genounits and related genotopic elements $I^{>} = 1/T^{>}, {}^{<}I = 1/{}^{<}T$ for which the following *operator Lie-admissible genogroup* now defined on a genomodule ${}^{<}\mathcal{H} \times \mathcal{H}^{>}$

$$A(t) = (e^{iHT^{>}t})A(0)(e^{-it^{<}TH}), \tag{4.5.11}$$

and related infinitesimal form, the *Heisenberg-Santilli genoequations*

$$i\frac{dA}{dt} = A < H - H > A = A^{<}TH - HT^{>}A, \tag{4.5.12}$$

correctly represent the considered dissipative system.

By noting that the Lie-brackets in Eqs. (4.5.9) are conventional, we seek a realization of the genotopic elements for which the Lie brackets attached to the Lie-admissible brackets (5.12) are conventional and the symmetric brackets are Jordan-isotopic. A solution is then given by [32]

$$T^{>} = 1 - \Gamma, \quad {}^{<}T = 1 + \Gamma, \tag{4.5.13}$$

for which Eq. (5.12) becomes

$$i\frac{dA}{dt} = (AH - HA) - (A\Gamma H + H\Gamma A) =$$

$$= [A, H] - \{A\hat{,}H\}, \tag{4.5.14}$$

where $[A, H]$ are a conventional Lie brackets as desired, and $\{A\hat{,}H\}$ are Jordan-isotopic brackets. The desired representation then occurs for

$$I^{>} = e^{(k/m)H^{-1}} = 1/T^{>}, \quad {}^{<}I = e^{-H^{-1}(k/m)} = 1/{}^{<}T, \tag{4.5.15a}$$

$$i\frac{dH}{dt} = -\frac{kp^2}{m^2} = -kv^2. \tag{4.5.15b}$$

Note that the achievement of the above operator form of system (5.1) without the Lie-admissible structure would have been impossible, to our knowledge.

Despite its elementary character, the above illustration has deep implications. In fact, the above example constitutes the only known operator formulation of a dissipative system in which the *nonconserved* energy is represented by a *Hermitian* operator H, thus being an *observable* despite its nonconservative character. In all other cases existing in the literature the Hamiltonian is generally *non-Hermitian,* thus *non-observable.*

The latter occurrence may illustrate the reason for the absence of a consistent operator formulation of nonconservative systems throughout the 20-th century until the advent of the Lie-admissible formulations.

4.5.2 Direct Universality of Lie-Admissible Representations for Nonconservative Systems

We now show that the Lie-admissible formulations are "directly universal," namely, they provide a classical and operator representation of all infinitely possible (well behaved) nonconservative systems of N particles (universality)

$$m_n \frac{dv_{nk}}{dt} + \frac{\partial V}{\partial r_n^k} = F_{nk}^{NSA}(t, r, p, \dot{p}, ...), \quad n = 1, 2, 3, ..., N, \quad k = 1, 2, 3, \quad (4.5.16)$$

directly in the frame of the observer, i.e., without transformations of the coordinates of the experimenter to mathematical frames (direct universality).

An illustration is given by a massive object moving at high speed within a resistive medium, such as a missile moving in our atmosphere. In this case the resistive force is approximated by a power series expansion in the velocity truncated up to the 10-th power for the high speeds of contemporary missiles

$$m \frac{dv}{dt} = \Sigma_{\alpha=1,2,...,10} k_\alpha v^\alpha, \quad (4.5.17)$$

for which any dream of conventional Hamiltonian representation is beyond the boundary of science.

The direct universality of the Hamilton-Santilli genomechanics was proved in Section 3.3. The representation in geno-phase-space is characterized by the conventional Hamiltonian representing the physical total energy, and the genounit for forward motion in time representing the NSA forces, according to the equations

$$H = \Sigma_{n,k} \frac{p_{nk}^2}{2m_n} + V(r), \quad I^> = \begin{pmatrix} 1 & \frac{F^{NSA}}{(\partial H/\partial p)} \\ 1 & 0 \end{pmatrix} \quad (4.5.18)$$

under which we have the equations of motion (for $\mu, \nu = 1, 2, 3, ...6N$) [32]

$$\frac{da^{>\mu}}{dt} = \begin{pmatrix} dr_n^{>k}/dt \\ dp_{nk}^>/dt \end{pmatrix} = S^{>\mu\nu} \frac{\partial^> H^>}{\partial^> a^{>\nu}} = \begin{pmatrix} 0 & -1 \\ 1 & \frac{F^{NSA}}{(\partial H/\partial p)} \end{pmatrix} \begin{pmatrix} \partial^> H^>/\partial^> r_n^{>k} \\ \partial^> H^>/\partial^> p_{nk}^> \end{pmatrix}, \quad (4.5.19)$$

the classical, finite, Lie-admissible genosgenogroup

$$A(t) = \exp\left(-t\frac{\partial H}{\partial a^\mu} S^{>\mu\nu} \frac{\partial}{\partial a^\nu}\right) A(0) \exp\left(\frac{\partial}{\partial a^\nu}\,^{<} S^{\nu\mu} \frac{\partial H}{\partial a^\mu} t\right), \tag{4.5.20}$$

with infinitesimal time evolution

$$\frac{dA}{dt} = \frac{\partial A}{\partial a^\mu}(^{<}S^{\mu\nu} - S^{>\mu\nu})\frac{\partial H}{\partial a^\nu} =$$

$$= (\frac{\partial A}{\partial r_n^k}\frac{\partial H}{\partial p_{nk}} - \frac{\partial H}{\partial r_n^k}\frac{\partial A}{\partial p_{nk}}) - (\frac{km}{(\partial H/\partial p)})^{nk}\frac{\partial A}{\partial p_{nk}}\frac{\partial H}{\partial p_{nk}}) =$$

$$= [A, H] + \{A, H\}, \tag{4.5.21}$$

yielding the correct *nonconservation of the energy*

$$\frac{dH}{dt} = v^k F_k^{NSA}. \tag{4.5.22}$$

The operator image can be characterized by the genounits and related genotopic elements

$$I^{>} = e^{\Gamma} = 1/T^{>}, \quad {}^{<}I = e^{-\Gamma} = 1/{}^{<}T, \quad \Gamma = H^{-1}(v_n^k F_{nk}^{NSA})H^{-1}, \tag{4.5.23}$$

with finite Lie-admissible time evolution

$$A(t) = \exp(iHe^{-\Gamma}t)A(0)\exp(-ite^{+\Gamma}H) \tag{4.5.24}$$

and related Heisenberg-Santilli genoequations

$$i\frac{dA}{dt} = A < H - H > A = [A, H] + \{A\hat{,}H\} =$$

$$= (AH - HA) + (A\Gamma H + H\Gamma A), \tag{4.5.25}$$

that correctly represent the time rate of variation of the nonconserved energy,

$$i\frac{dH}{dt} = v_n^k F_{nk}^{NSA}. \tag{4.5.26}$$

The uninitiated reader should be incidentally aware that generally different genounits may be requested for different generators, as identified since Ref. [11].

In the latter operator case we are referring to an extended, massive and stable particle, such as a proton, penetrating at high energy within a nucleus, in which case the rapid decay of the kinetic energy is caused by contact, resistive, integrodifferential forces of nonlocal type, e.g., because occurring over the volume of the particle.

The advantages of the Lie-admissible formulations over pre-existing representation of nonconservative systems should be pointed out. Again, a primary advantage of the Lie-admissible treatment is the characterization of the *nonconserved* Hamiltonian with a *Hermitian,* thus *observable* quantity, a feature generally absent in other treatments.

Moreover, the "direct universality" of Lie-admissible representations requires the following comments. Recall that coordinates transformations have indeed been used in the representation of nonconservative systems because, under sufficient continuity and regularity, the Lie-Koening theorem assures the existence of coordinate transformations $(r,p) \to (r'(r,p),\ p'(r,p))$ under which a system that is non-Hamiltonian in the original coordinates becomes Hamiltonian in the new coordinates (see Ref. [6] for details). However, the needed transformations are necessarily nonlinear with serious physical consequences, such as:

1) Quantities with direct physical meaning in the coordinates of the experimenter, such as the Hamiltonian $H(r,p) = \frac{p^2}{2m} + V(r)$, are transformed into quantities that, in the new coordinates, have a purely mathematical meaning, such as $H'(r',p') = Nexp(Mr'^2/p'^3), N, M \in R$, thus preventing any physically meaningful operator treatment;

2) There is the loss of any meaningful experimental verifications, since it is impossible to place any measurement apparatus in mathematical coordinates such as $r' = KlogLr^3, p' = Pexp(Qrp), K, L, P, Q \in R$;

3) There is the loss of Galileo's and Einstein's special relativity, trivially, because the new coordinates (r', p') characterize a highly *noninertial* image of the original inertial system of the experimenter.

All the above, and other insufficiencies are resolved by the Lie-admissible treatment of nonconservative systems.

4.5.3 Pauli-Santilli Lie-Admissible Matrices

Following the study of the nonconservation of the energy, the next important topic is to study the behavior of the conventional quantum spin under contact nonconservative forces, a topic studied for the first time in memoir [32]. For this objective, it is most convenience to use the method of Suctions 4.4.2 and 4.4.3, namely, subject the conventional Pauli's matrices to two different nonunitary transforms. To avoid un-necessary complexity, we select the following two matrices

$$A = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}, \quad AA^\dagger \neq I, \ ] \ BB^\dagger \neq I, \tag{4.6.1}$$

where a and b are non-null real numbers, under which we have the following forward and backward genounits and related genotopic elements

$$I^{>} = AB^{\dagger} = \begin{pmatrix} 1 & b \\ a & 1 \end{pmatrix}, \quad T^{>} = \frac{1}{(1-ab)} \begin{pmatrix} 1 & -b \\ -a & 1 \end{pmatrix}, \tag{4.6.2a}$$

$$^{<}I = BA^{\dagger} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}, \quad ^{<}T = \frac{1}{(1-ab} \begin{pmatrix} 1 & -a \\ -b & 1 \end{pmatrix}. \tag{4.6.2b}$$

The *forward and backward Pauli-Santilli genomatrices* are then given respectively by

$$\sigma_1^{>} = A\sigma_1 B^{\dagger} = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \quad \sigma_2^{>} = A\sigma_2 B^{\dagger} = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \tag{4.6.3a}$$

$$\sigma_3^{>} = A\sigma_3 B^{\dagger} = \begin{pmatrix} 1 & b \\ a & -1 \end{pmatrix}, \quad ^{<}\sigma_1 = B\sigma_1 A^{\dagger} = \begin{pmatrix} 0 & 1 \\ 1 & (a+b) \end{pmatrix}, \tag{4.6.3b}$$

$$^{<}\sigma_2 = B\sigma_2 A^{\dagger} = \begin{pmatrix} 0 & -i \\ i & (a+b) \end{pmatrix}, \quad ^{<}\sigma_3 = A\sigma_3 B^{\dagger} = \begin{pmatrix} 1 & a \\ b & -1 \end{pmatrix}, \tag{4.6.3c}$$

in which the direction of time is embedded in the structure of the matrices.

It is an instructive exercise for the interested reader to verify that *conventional commutation rules and eigenvalues of Pauli's matrices are preserved under forward and backward genotopies,*

$$\sigma_i^{>} > \sigma_j^{>} - \sigma_j^{>} > \sigma_i^{>} = 2i\epsilon_{ijk}\sigma_k^{>}, \tag{4.6.4a}$$

$$\sigma_3^{>} > |> = \pm 1|>, \quad \sigma^{>^{2>}} > |> = 2(2+1)|>, \tag{4.6.4b}$$

$$^{<}\sigma_i >^{<} \sigma_j -^{<} \sigma_j >^{<} \sigma_i = 2i\epsilon_{ijk}^{<}\sigma_k, \tag{4.6.4c}$$

$$< | <^{<} \sigma_3 =< | \pm 1, \ ;< | <^{<} \sigma^{2>} =< |(2(2+1). \tag{4.6.4d}$$

We can, therefore, conclude by stating that *Pauli's matrices can indeed be lifted in such an irreversible form to represent the direction of time in their very structure.* However, in so doing the conventional notion of spin is lost in favor of a covering notion in which the spin becomes a locally varying quantity, as expected to a proton in the core of a star.

Consequently, the Lie-admissible formulation of Pauli matrices confirms the very title of memoir [12] proposing the construction of hadronic mechanics.

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The argument is that, while special relativity and Pauli exclusion principe are unquestionably valid for the conditions of their original conception, particles

at large mutual distances under action-at-a-distance interactions (such as for a point-like proton in a particle accelerator under long range electromagnetic interactions), by no means the same doctrines have to be necessarily valid for *one* hadronic constituent when considering all other constituents as external.[6]

The above analysis focuses the attention in an apparent fundamental structural difference between electromagnetic and strong interactions. Irrespective of whether considered part of the system (closed system) or external (open system), *electromagnetic interactions do verify Pauli principle,* as well known. The best example is given by Dirac's equation for the hydrogen atom that, as known to experts to qualify as such, represents one electron under the *external* electromagnetic field of the proton. The origin of the preservation of Pauli principle is that, whether electromagnetic interactions are closed or open, they are Hamiltonian. Lie's theory then applies with the conventional notion of spin, and Pauli principle follows.

By comparison, strong interactions are non-Hamiltonian for the numerous reasons indicated during tour analysis. Consequently, the conventional notion of spin cannot be preserved, and Pauli principle is inapplicable in favor of broader vistas. It is intriguing to note that the representation of a proton via isomechanics allows indeed a representation of its extended, nonspherical and deformable shape. Nevertheless, Pauli's principle is preserved under isotopies, as indicated in Chapter 3. Hence, the inapplicability of Pauli's principle is here referred to, specifically and solely, for open irreversible conditions at short mutual distances, exactly according to the original proposal to build hadronic mechanics [12].

The above distinction between electromagnetic and strong interactions is the conceptual foundation of monographs [40,41] suggesting the characterization of the hadronic constituents via *Lie-admissible,* rather than Lie or Lie-isotopic algebras, with the consequential inapplicability of the conventional. notion of spin. These basic issues will be studied in detail in Volume II in connection with explicit structure models of hadrons with physical constituents, that is, constituents that can be produced free in spontaneous decays while being compatible with the SU(3)-color Mendeleev-type *classification* of hadrons.

To conclude, *not only special relativity, but also Pauli principle is inapplicable (rather than violated) for a hadron under external strong interactions.* Needless to say, when a particle with the open nonconservative spin under consideration here is "completed" with the inclusion of all remaining strong interacting particles here considered as external, Pauli principle is recovered in full for the center of mass of the ensemble as a whole because the "completion" is treated via isomechanics.

[6]The reader should always keep in mind that, even though not stated in the technical literature for evident political reasons, quantum mechanics can only represent the proton as a dimensionless point.

4.5.4 Minkowski-Santilli Irreversible Genospacetime

One of the fundamental axiomatic principles of hadronic mechanics is that irreversibility can be directly represented with the background geometry and, more specifically, with the metric of the selected geometry. This requires the necessary transition from the conventional *symmetric* metrics used in the 20-th century to covering *nonsymmetric* genometric.

To show this structure, we study in this section the genotopy of the conventional Minkowskian spacetime and related geometry with the conventional metric $\eta = Diag.(1, 1, 1, -1)$ and related spacetime elements $x^2 = x^\mu \eta_{\mu\nu} x^\nu$, $x = (x^1, x^2, x^3, x^4)$, $x^4 = ct$, $c = 1$. For this purpose, we introduce the following four-dimensional non-Hermitian, nonsingular and real-valued forward and backward genounits

$$I^> = CD^\dagger = 1/T^>, \quad {}^<I = DC^\dagger = 1/{}^<T, \quad CC^\dagger \neq I, \quad DD^\dagger \neq I, \tag{4.6.5}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4.6.6}$$

where $p \neq q$ are non-null real numbers, under which we have the following forward and backward genotopy of the Minkowskian line element

$$x^2 \to x^{>2>} = Cx^2 D^\dagger = C(x^t \eta x) D^\dagger =$$
$$= (C^t x^t D^{t\dagger})(CD^\dagger)^{-1}(C\eta D^\dagger)(CD^\dagger)^{-1}(CxD^\dagger) =$$
$$= (x^t I^>) T^> \eta^> T^> (I^> x) = x^\mu \eta^>_{\mu\nu} x^\nu =$$
$$= (x^1 x^1 + x^1 q x^3 + x^2 x^2 + x^3 x^3 + x^1 p x^4 - x^4 x^4), \tag{4.6.7a}$$
$$Dx^2 C^\dagger = D(x^t \eta x) C^\dagger =$$
$$= (x^{t<} I) {}^<T {}^<\eta {}^<T ({}^<I x) = x^\mu {}^<\eta_{\mu\nu} x^\nu =$$
$$= (x^1 x^1 + x^1 p x^3 + x^2 x^2 + x^3 x^3 + x^1 q x^4 - x^4 x^4), \tag{4.6.7b}$$

resulting in the forward and backward nonsymmetric genometrics

$$\eta^> = \begin{pmatrix} 1 & 0 & q & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ q & 0 & 0 & 1 \end{pmatrix}, \quad {}^<\eta = \begin{pmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & 0 \\ q & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4.6.8}$$

exactly as desired.

Note that irreversibility selects a mutation of the line elements along a preselected direction of space and time.

Note also that the quantities p and q can be functions of the local spacetime variables, in which case the resulting *Minkowskian genogeometry* can be equipped by a suitable lifting of the machinery of the Riemannian geometry (see Ref. [16] for the isotopic case and Chapter 3).

Note finally that the above genospacetime includes, as particular case, *an irreversible formulation of the Riemannian geometry,* where irreversibility is represented at the ultimate geometric foundations, the basic unit and the metric.

It should be indicated that the above irreversible formulation of spacetime has intriguing implications for the mathematical model known as *geometric locomotion* studied in detail in monograph [73] via the isotopies of the Minkowskian geometry. In fact, a main unresolved problems is the directional deformation of the geometry as needed to permit the geometric locomotion in one preferred direction of space. An inspection of the mutated line elements (4.6.7) clearly shows that the genotopies are preferable over the isotopies for the geometric locomotion, as well as, more generally, for a more realistic geometric characterization of irreversible processes.

The construction of the *Lorentz-Santilli genotransformations* is elementary, due to their formal identify with the isotopic case of Chapter 3, and its explicit construction left as an instructive exercise for the interested reader.

4.5.5 Dirac-Santilli Irreversible Genoequation

To complete the illustrations in particle physics, we now outline the simplest possible genotopy of Dirac's equation via the genotopies of the preceding two suctions, one for the spin content of Dirac's equation and the other for its spacetime structure. Also, we shall use Dirac's equation in its isodual re-interpretation representing a direct product of one electron and one positron, the latter without any need of second quantization (see monograph [73] for detail). In turn, the latter re-interpretation requires the use of the *isodual transform* $A \to A^d = -A^\dagger$) as being distinct from Hermitian conjugation. Under the above clarifications, the *forward Dirac genoequation* here referred to can be written

$$\eta^{>\mu\nu}\gamma^>_\mu T^> p^>_\nu - im)T^>|\psi^> >= 0 \tag{4.6.9a}$$

$$p^>_\nu T^>|\psi^> >= -i\frac{\partial^>}{\partial^> x^{>\nu}}|\psi^> >= -iI^>\frac{\partial}{\partial x^>}|\psi^> >, \tag{4.6.9b}$$

with *forward genogamma matrices*

$$\gamma^>_4 = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix}\begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}\begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \begin{pmatrix} AA^d & 0 \\ 0 & -B^dB \end{pmatrix}, \tag{4.6.10a}$$

$$\gamma^>_k = \begin{pmatrix} A & 0 \\ 0 & B^d \end{pmatrix}\begin{pmatrix} 0 & \sigma_k \\ \sigma^d_k & 0 \end{pmatrix}\begin{pmatrix} A^d & 0 \\ 0 & B \end{pmatrix} = \tag{4.6.10b}$$

$$= \begin{pmatrix} 0 & A\sigma_k B^\dagger \\ B\sigma_k^d A^d & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sigma_k^> \\ <\sigma_k^d & 0 \end{pmatrix}, \tag{4.6.10c}$$

$$\{\gamma_\mu^>,\gamma_\nu^>\} = \gamma_\mu^> T^> \gamma_\nu^> + \gamma_\nu^> T^> \gamma_\mu^> = 2\eta_{\mu\nu}^>, \tag{4.6.10d}$$

where $\eta_{\mu\nu}^>$ is given by the same genotopy of Eqs. (4.6.10a).

Interested readers can then construct the backward genoequation. They will discover in this way a new fundamental symmetry of Dirac's equation that remained undiscovered throughout the 20-th century, its *isoselfduality* (invariance under isoduality.) This new symmetry is now playing an increasing role for realistic cosmologies, those inclusive of antimatter, or for serious unified theories that must also include antimatter to avoid catastrophic inconsistencies [73] (see Volume II).

It is an instructive exercise for the interested reader to verify a feature indicated earlier, the inapplicability of the conventional notion of spin and, consequently, of Pauli principle for the Dirac-Santilli genoequation. As we shall see in Volume II, the conventional Dirac equation represents the electron in the structure of the hydrogen atom. By comparison, the Dirac-Santilli genoequation represents the same electron when totally immersed in the hyperdense medium inside a proton, thus characterizing the structure of the neutron according to hadronic mechanics..

Note that, while the electron is moving forward, the positron is moving backward in time although referred to a negative unit of time, as a necessary condition to avoid the inconsistencies for negative energies that requested the conjecture of the "hole theory" (see monograph xxx for brevity).

4.5.6 Dunning-Davies Lie-Admissible Thermodynamics

A scientific imbalance of the 20-th century has been the lack of interconnections between thermodynamics, on one side, and classical and quantum mechanics, on the other side. This is due to the fact that the very notion of entropy,
indexEntropy let alone all thermodynamical laws, are centrally dependent on irreversibility, while classical and quantum Hamiltonian mechanics are structurally reversible (since all known potentials are reversible in time).

As recalled in Section 4.1, said lack of interconnection was justified in the 20-th century on the belief that the nonconservative forces responsible for irreversibility according to Lagrange and Hamilton, are "fictitious" in the sense that they only exist at the classical level and they "disappear" when passing to elementary particles, since the latter were believed to be completely reversible. In this way, thermodynamics itself was turned into a sort of "fictitious" discipline.

This imbalance has been resolved by hadronic mechanics beginning from its inception. In fact, Theorems 1.3.3 has established that, far from being "fictitious," nonconservative forces originate at the ultimate level of nature, that of elementary particles in conditions of mutual penetration causing contact nonpotential (NSA) interactions. The insufficiency rested in the inability by quantum

mechanics to represent nonconservative forces, rather than in nature. In fact, hadronic mechanics was proposed and developed precisely to reach an operator representation of the nonconservative forces originating irreversibility along the legacy of Lagrange and Hamilton.

As a result of the efforts presented in this chapter, we now possess not only classical and operator theories, but more particularly we have a *new mathematics,* the genomathematics, whose basic axioms are not invariant under time reversal beginning from the basic units, numbers and differentials.

Consequently, hadronic mechanics does indeed permit quantitative studies of the expected interplay between thermodynamics and classical as well as operator mechanics. These studies were pioneered by J. Dunning Davies [30] who introduced the first known study of thermodynamics via methods as structurally irreversible as their basic laws, resulting in a formulation we hereon call *Dunning-Davies Lie-admissible thermodynamics.* This section is dedicated to a review of Dunning-Davies studies.

Let us use conventional thermodynamical symbols, a classical form of thermodynamics, and the simple construction of irreversible formulations via two different complex valued quantities A and B. Then, the first law of thermodynamics can be lifted from its conventional formulation, that via reversible mathematics, into the form permitted by genomathematics

$$Q \to Q^> = AQB^\dagger = QI^>, \quad U \to U^> = AUB^\dagger = UI^>, \quad \text{etc.}, \tag{4.6.11a}$$

$$dQ = dU + pdV \quad \to \quad d^>Q^> = d^>U^> + p^> > d^>V^>, \tag{4.6.11b}$$

where, in the absence of operator forms, Hermitian conjugation is complex conjugation. For the second law we have

$$dQ = TdS \quad \to \quad d^>Q^> = T^> > d^>S^>, \tag{4.6.12}$$

thus implying that

$$TdS = dU + pdV \quad \to \quad T^> > d^>S^> = d^>U^> + p^> > d^>V^>. \tag{4.6.13}$$

As one can see, genomathematics permits the *first known formulation of entropy with a time arrow,* the only causal form being that forward in time. When the genounit does not depend on the local variables, the above genoformulation reduces to the conventional one identically, e.g.,

$$T^> > d^>S^> = (TI^>)I^{>-1}[I^{>-1}d(SI^>) = TdS =$$

$$= I^{>-1}d(VI^>) + (pI^>)I^{>-1}d(VI^>) = dU + pdV. \tag{4.6.14}$$

This confirms that genomathematics is indeed compatible with thermodynamical laws.

However, new vistas in thermodynamics are permitted when the genounit is dependent on local variables, in which case reduction (4.6.13) is no longer possible. An important case occurs when the genounit is explicitly dependent on the entropy. In this case the l.h.s. of Eq. (4.6.13) becomes

$$TdS + TS(I^{>-1}dI^{>}) = dU + pdV. \tag{4.6.15}$$

We then have new thermodynamical models of the type

$$I^{>} = e^{f(S)}, \;\; T^{>} > d^{>}S^{>} = T\left(1 + S\frac{\partial f(S)}{\partial S}\right) dS = dU + pdV, \tag{4.6.16}$$

permitting thermodynamical formulations of the behavior of anomalous gases (such as magnegases [21]) via a suitable selection of the $f(S)$ function and its fit to experimental data. Needless to say, equivalent models can be constructed for an explicit dependence of the genounit from the other variables. For these and other aspects we have to refer the interested reader to Volume II.

4.5.7 Ongoing Applications to New Clean Energies

A primary objective of Volume II is to study industrial applications of hadronic mechanics to new clean energies that are under development at the time of writing this first volume (2002). Hence, we close this chapter with the following preliminary remarks.

The societal, let alone scientific implications of the proper treatment of irreversibility are rather serious. Our planet is afflicted by increasingly catastrophic climactic events mandating the search for basically new, environmentally acceptable energies, for which scope the studies reported in these monographs were initiated.

All known energy sources, from the combustion of carbon dating to prehistoric times to the nuclear energy, are based on irreversible processes. By comparison, all established doctrines of the 20-th century, such as quantum mechanics and special relativity, are reversible, as recalled in Section 4.1.

It is then easy to see that *the serious search for basically new energies requires basically new theories that are as structurally irreversible as the process they are expected to describe.* At any rate, all possible energies and fuels that could be predicted by quantum mechanics and special relativity were discovered by the middle of the 20-th century. Hence, the insistence in continuing to restrict new energies to verify preferred reversible doctrines may cause a condemnation by posterity due to the environmental implications.

An effective way to illustrate the need for new irreversible theories is given by nuclear fusions. All efforts to date in the field, whether for the "cold fusion" or the "hot fusion," have been mainly restricted to verify quantum mechanics and special relativity. However, *whether "hot" or "cold," all fusion processes are irreversible, while quantum mechanics and special relativity are reversible.*

It has been shown in Ref. [31] that the failure to date by both the "cold" and the "hot" fusions to achieve industrial value is primarily due to the treatment of irreversible nuclear fusions with reversible mathematical and physical methods.

In the event of residual doubt due to protracted use of preferred theories, it is sufficient to compute the quantum mechanical probability for two nuclei to "fuse" into a third one, and then compute its time reversal image. In this way the serious scholar will see that special relativity and quantum mechanics may predict a fully causal *spontaneous disintegration of nuclei following their fusion,* namely, a prediction outside the boundary of science.

The inclusion of irreversibility in quantitative studies of new energies suggests the development, already partially achieved at the industrial level (see Chapter 8 of Ref. [20]), of the new, controlled "intermediate fusion" of light nuclei [31], that is, a fusion occurring at minimal threshold energies needed: 1) To verify conservation laws; 2) To expose nuclei as a pre-requisite for their fusion (a feature absent in the "cold fusion" due to insufficient energies), and 3) To prevent uncontrollable instabilities (as occurring at the very high energies of the "hot fusion").

It is hoped that serious scholars will participate with independent studies on the irreversible treatment of new energies, as well as on numerous other open problems, because in the final analysis lack of participation in basic advances is a gift of scientific priorities to others.

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Chapter 5

HYPERSTRUCTURAL BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

5.1 The Scientific Imbalance in Biology

In our view, the biggest scientific imbalance of the 20-th century has been the treatment of biological systems (herein denoting DNA, cells, organisms, etc.) via conventional mathematics, physics and chemistry because of various reasons studied in detail in Chapter 1.1.

We here limit ourselves to recall that biological events, such as the growth of an organism, are irreversible over time, while the mathematics of the 20-th century and related formulations are structurally reversible, that is, reversible for all possible Hamiltonians. Therefore, any treatment of biological systems via reversible mathematics, physical and chemical formulations can indeed receive temporary academic acceptance, but cannot pass the test of time.

Quantum mechanics is ideally suited for the treatment of the structure of the hydrogen atom or of crystals, namely, systems that are fully reversible. These systems are represented by quantum mechanics as being ageless. Recall also that quantum mechanics is unable to treat deformations because of incompatibilities with basic axioms, such as that of the rotational symmetry.

Therefore, *the strict application to biological systems of the mathematics underlying quantum mechanics and chemistry implies that all organisms from cells to humans are perfectly reversible, totally rigid and fully eternal.*

5.2 The Need in Biology of Irreversible Multi-Valued Formulations

It is possible to see that, despite their generality, the invariant irreversible genoformulations studied in the preceding chapter are insufficient for in depth treatments of biological systems.

In fact, recent studies conducted by Illert [1] have pointed out that the *shape* of sea shells can certainly be represented via conventional mathematics, such as the Euclidean geometry.

However, the latter conventional geometries are inapplicable for a representation of the *growth over time* of sea shells. Computer simulations have shown that the imposition to sea shell growth of conventional geometric axioms causes the lack of proper growth, such as deformations and cracks, as expected, because said geometries are strictly reversible over time, while the growth of sea shells is strictly irreversible.

The same studies by Illert [1] have indicated the need of a mathematics that is not only structurally irreversible, but also *multi-dimensional.* As an example, Illert achieved a satisfactory representation of sea shells growth via the *doubling of the Euclidean reference axes,* namely, via a geometry appearing to be six-dimensional.

A basic problem in accepting such a view is the lack of compatibility with our sensory perception. When holding sea shells in our hands, we can fully perceive their shape as well as their growth with our three Eustachian tubes. Hence, any representation of sea shells growth with more than three dimensions is incompatible with our perception of reality.

Similarly, our sensory perception can indeed detect curvature. Thus, any representation of sea shell growth with the Riemannian geometry would equally be incompatible with our sensory perception. At any rate, any attempt at the use of the Riemannian geometry for sea shell growth would be faced with fatal inconsistencies, such as the inability to represent bifurcations and other aspects since such representations would be prohibited by curvature.

These occurrences pose a rather challenging problem, the construction of yet another *new mathematics* that is

(1) Structurally irreversible over time (as that of the preceding section);

(2) Capable to represent deformations;

(3) Invariant under the time evolution in the sense of predicting the same number under the same conditions but at different times;

(4) Multi-dimensional; and, last but not least,

(5) Compatible with our sensory perception.

The only solution known to the author is that of building an irreversible *multi-valued* (rather than multi-dimensional) new mathematics, in the sense that the basic axioms of the space representation can remain three-dimensional to achieve compatibility with our sensory perception, but each axis can have more than one value, thus being multi-valued.

A search in the mathematical literature soon revealed that a mathematics verifying all the above requirements did not exist and had to be constructed.

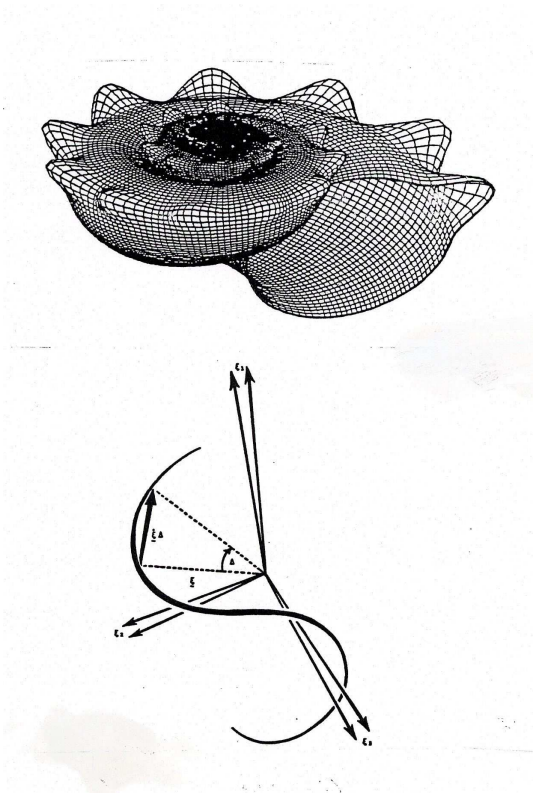

Figure 5.1. A schematic view of Illert [1] has shown that a representation of the growth over time of a seashell can be effectively done by doubling the number of reference axes. However, seashell growth is perceived by our sensory perception as occurring in three-dimensional space. The multi-valued hyperstructural branch of hadronic mechanics studied in this chapter provides a solution of these seemingly discordant requirements because, on side, it is as multi-valued as desired while, on the other side, remains three-dimensional at the abstract, realization-free level.

As an example, in their current formulations, *hyperstructures* (see, e.g., Ref. [2]) lack a well defined left and right unit thus lacking the applicability to the measurements; they do not have conventional operations, but rather the so-called *weak operations*, thus lacking applicability to experiments; they are not structurally irreversible; and they lack invariance. Consequently, conventional hyperstructures are not suitable for applications in biology.

5.3 Rudiments of Santilli Hyper-Mathematics and Hypermechanics

After a number of trials and errors, a yet broader mathematics verifying the above five conditions was identified by R. M. Santilli in monographs [3] of 1995

and in works [4,5], and subsequently studied by R. M. Santilli and the mathematician T. Vougiouklis in paper [6] of 1996 (see also mathematical study [7]). These studies resulted in a formulation today known as *Santilli hypermathematics.*

For an in depth study, including the all crucial *Lie-Santilli hypertheory,* we refer the reader to the mathematical treatments [4–7]. By assuming an in depth knowledge of genomathematics of the preceding chapter, we here limit ourselves to indicate that the selected hypermathematics is based on the assumption that the single-valued forward and backward genounits of the preceding chapter although replaced with the following *multi-valued hyperunits*

$$\hat{I}^{>}(t,x,v,\psi,\partial_x\psi,...) = Diag.(\hat{I}_1^{>},\hat{I}_2^{>},\hat{I}_3^{>}) =$$

$$= Diag.\Big[(\hat{I}_{11}^{>},\hat{I}_{12}^{>},...,\hat{I}_{1m}^{>}),(\hat{I}_{21}^{>},\hat{I}_{22}^{>},...,\hat{I}_{2m}^{>}),(\hat{I}_{31}^{>},\hat{I}_{32}^{>},...,\hat{I}_{3m}^{>})\Big], \tag{5.1a}$$

$$^{<}\hat{I}(t,x,v,\psi,...) = Diag.(^{<}\hat{I}_1,^{<}\hat{I}_2,^{<}\hat{I}_3) =$$

$$= Diag.\Big[(^{<}\hat{I}_{11},^{<}\hat{I}_{12},...,^{<}\hat{I}_{1m}),(^{<}\hat{I}_{21},^{<}\hat{I}_{22},...,^{<}\hat{I}_{2m}),$$

$$(^{<}\hat{I}_{31},^{<}\hat{I}_{32},...,^{<}\hat{I}_{3m})\Big], \tag{5.1b}$$

with corresponding *ordered hyperproducts to the right and to the left*

$$A > B = A \times \hat{T}^{>} \times B, A < B = A \times^{<} \hat{T} \times B, \tag{5.2a}$$

$$\hat{I}^{>} > A = A > \hat{I}^{>} = A, \quad ^{<}\hat{I} < AA <^{<} \hat{I} = A, \tag{5.2b}$$

$$\hat{I}^{>} = (^{<}\hat{I})^{\dagger} = 1/\hat{T}^{>}. \tag{5.2c}$$

Following the hyperlifting of the methods of the preceding chapter, we reach the following basic equations of the *multi-valued hyperstructural branch of hadronic mechanics,* first proposed by Santilli in monographs [3] of 1995 (see also the mathematical works [4–6], here written in the finite and infinitesimal forms

$$i\ dA/dt = A \lhd H - H \rhd A, \tag{5.3a}$$

$$A(t) = \hat{e}^{i\times H\times t} \lhd A(0) \rhd \hat{e}^{-i\times t\times H}, \tag{5.3b}$$

quoted in Footnote 15 of Chapter 1, where the multivalued character of all quantities and their operations is assumed.

In the above expressions the reader should recognize the diagonal elements of the genounits of the preceding chapter and then identify the multi-valued character for each diagonal element. Consequently, the above mathematics *is not* $3m$-dimensional, but rather it is 3-dimensional and m-multi-valued, namely, each axis in three-dimensional space can assume m different values.

Such a feature permits the increase of the reference axes, e.g., for $m = 2$ we have six axes as used by Illert [1], while achieving compatibility with our sensory perception because at the abstract, realization-free level hypermathematics characterized by hyperunit is indeed 3-dimensional.

It is instructive for readers interested in learning the new mathematics to prove the following

LEMMA 5.1 [3]: All rings of elements $a \times \hat{I}^{>}$ ($^{<}\hat{I} \times a$), where a is an ordinary (real, complex or quaternionic) number and $\hat{I}^{>}$ ($^{<}\hat{I}$) is the forward (backward) multivalued hyperunit, when equipped with the forward (backward) hyperproduct, verify all axioms of a field.

A good understanding of the above property can be reached by comparison with the preceding studies. The discovery of isofields [8] studied in Chapter 3 was made possible by the observation that *the axioms of a field are insensitive to the value of the unit.* As a result of which we have isoproducts of the type

$$\hat{I} = 1/3 = 1/\hat{T}, \quad 2\hat{\times}3 = 2 \times \hat{T} \times 3 = 18. \tag{5.4}$$

The discovery of genofields also in Ref. [8] was due to the observation that *the axioms of a field are additionally insensitive to the ordering of a product to the right or to the left, provided that all operations are restricted to one selected order.* This lead to *two* inequivalent multiplications, one to the right and one to the left, as necessary to represent irreversibility, such as

$$\hat{I}^{>} = 1/3 = \hat{T}, \;\; 2 > 3 = 18, {}^{<}\hat{I} = 3, \;\; 2 < 3 = 2. \tag{5.5}$$

Lemma 5.1 essentially reflects the additional property according to which *the axioms of a field are also insensitive as to whether, in addition to the selection of an ordering as per genofields, the units and (ordered) products are multivalued,* e.g.,

$$\hat{I}^{>} = \{1/3, 1/5\}, \;\; 2\hat{>}3 = \{18, 30\}, \;\; {}^{<}\hat{I} = \{3, 2\}, \;\; 2\hat{<}3 = \{2, 3\}, \tag{5.6}$$

where the results of the hypermultiplications should be interpreted as an ordered set.

Once the notion of hyperfield is understood, the construction of all remaining aspects of hypermathematics can be conducted via simple compatibility arguments, thus leading in this way to *hyperspaces, hyperfunctional analysis, hyperdifferential calculus, hyperalgebras, etc.*

Note that the resulting hyperformulations are invariant as it is the case for genomathematics. The proof of such an invariance is here omitted for brevity, but recommended to readers interested in a serious study of the field.

The above features serve to indicate that the biological world has a complexity simply beyond our imagination, and that studies of biological problems conducted

in the 20-th century, such as attempting an understanding the DNA code via numbers dating back to biblical times, are manifestly insufficient.

The above features appear to be necessary for the representation of biological systems. As an example, consider the association of two atoms in a DNA producing an organ composed by a very large number of atoms, such as a liver. A quantitative treatment of this complex event is given by representing the two atoms with α and β and by representing their association in a DNA with the hyperproduct. The resulting large number of atoms γ_k in the organ is then represented by the ordered multi-valued character of the hyperproduct, such as

$$\alpha \hat{>} \beta = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \ldots, \gamma_n, \}. \tag{5.7}$$

The above attempt at decybrings the DNA code is another illustration of our view that the complexity of biological systems is simply beyond our comprehension at this time. A mathematical representation will eventually be achieved in due time. However, any attempt at its "understanding" would face the same difficulties of attempting to understand infinite-dimensional Hilbert space in quantum mechanics, only the difficulties are exponentially increased for biological structures.

5.4 Rudiments of Santilli Isodual Hypermathematics

The *isodual hypermathematics* can be constructed via the isodual map of Chapter 2 here expressed for an arbitrary operator $\hat{A}$,

$$\hat{A}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots) \to -\hat{A}^\dagger(-\hat{t}^\dagger, -\hat{r}^\dagger, -\hat{p}^\dagger, -\hat{\psi}^\dagger, \ldots) = \hat{A}^d(\hat{t}^d, \hat{r}^d, \hat{p}^d, \hat{\psi}^d, \ldots), \tag{5.8}$$

applied to the *totality* of hypermathematics, including its operations, with no exception (to avoid inconsistencies), thus yielding *isodual hyperunits, isodual hypernumbers, isodual hyperspaces, etc.*

Consequently, the formulations here considered have *four* different hyperunits, the forward and backward hyperunits and their isoduals,

$$\hat{I}^{>}, \ {}^{<}\hat{I}, \ \hat{I}^{>d}, \ {}^{<d}\hat{I}, \tag{5.9}$$

that, in turn, have to be specialized into forward and backward *space and time hyperunits* and their isoduals.

Consequently, the formulations herein considered have *four* different *hypercoordinates*

$$\hat{x}^{>}, \ {}^{<}\hat{x}, \ \hat{x}^{>d}, \ {}^{<}\hat{x}^{d}, \tag{5.10}$$

and *four* different *hypertimes,*

$$\hat{t}^{>}, \ {}^{<}\hat{t}, \ \hat{t}^{>d}, \ {}^{<}\hat{t}^{d}. \tag{5.11}$$

In chapter 2 (see also Figure 2.2) we have studied the need for four different times. We now have the four different hypertimes for: 1) Motion forward to

future times characterized by $\hat{t}^{>}$; 2) Motion backward to past time characterized by ${}^{<}\hat{t}$; 3) Motion backward from future times characterized by $\hat{t}^{>d}$, and 4) motion forward from past times characterized by ${}^{<d}\hat{t}$. The main difference between the four times of Chapter 2 and the four hypertimes of this chapter is that the former are single-valued while the latter are multi-valued.

Note again the *necessity of the isodual map to represent all four possible time evolutions.* In fact, the conventional mathematics, such as that underlying special relativity, can only represent two our four possible time evolutions, motion forward to future time and motion backward to past time, the latter reached via the conventional time reversal operation.

The following intriguing and far reaching aspect emerges in biology. Until now we have strictly used isodual theories for the sole representation of antimatter. However, Illert [1] has shown that *the representation of the bifurcations in sea shells requires the use of all four directions of time.*

The latter aspect is an additional illustration of the complexity of biological system. In fact, the occurrence implies that the "intrinsic time" of a seashell, that is, the time perceived by a sea shells as a living organism, is so complex to be beyond our comprehension at this writing. Alternatively, we can say that the complexity of hypertimes is intended to reflect the complexity of biological systems.

In conclusion, the achievement of invariant representations of biological structures and their behavior can be one of the most productive frontiers of science, with far reaching implications for other branches, including mathematics, physics and chemistry.

As an illustration, a mathematically consistent representation of the non-Newtonian propulsion of sap in trees, all the way up to big heights, automatically provides a model of *geometric propulsion* studied in Volume II, namely propulsion caused by the alteration of the local geometry without any external applied force.

5.5 Santilli Hyperrelativity and Its Isodual

All preceding formulations can be embodied into one single axiomatic structure submitted in monographs [3,5] and today known as *Santilli hyperrelativity and its isodual,* that are characterized by:

1) The *irreversible, multi-valued, forward and backward, Minkowski-Santilli hyperspace* with the following *forward and backward spacetime hypercoordinates* and *forward and backward hyperintervals* over *forward and backward hyperfields*, and their isoduals

$$\hat{M}^{>}(\hat{x}^{>},\hat{\eta}^{>},\hat{R}^{>}),\hat{x}^{>2}=\hat{x}^{>\mu}\hat{>}\hat{\eta}^{>}_{\mu\nu}\hat{>}\hat{x}^{>\nu}\in\hat{R}^{>}, \tag{5,12a}$$

$${}^{<}\hat{M}({}^{<}\hat{x},{}^{<}\hat{\eta},{}^{<}\hat{R}),{}^{<}\hat{x}^{2}={}^{<}\hat{x}^{>\mu}\hat{<}{}^{<}\hat{\eta}_{\mu\nu}\hat{<}{}^{\hat{<}}x^{\nu}\in{}^{<}\hat{R}, \tag{5.12b}$$

$$\hat{m}^{>d}(\hat{x}^{>d},\hat{\eta}^{>d},\hat{R}^{>d},{}^{<}\hat{m}^{d}({}^{<}\hat{x}^{d},{}^{<}\hat{\eta}^{d},{}^{<}\hat{R}^{d}; \tag{5.12c}$$

2) The corresponding *irreversible, multi-valued, forward and backward Poincaré-Santilli hypersymmetry* and their isoduals here written via the Kronecker product

$$\hat{P}^{>}(3,1)time^{<}\hat{P}(3.1)\times\hat{P}^{>d}(3,1)time^{<}\hat{P}^{d}(3.1), \tag{5.13}$$

essentially given by the Poincaré-Santilli genosymmetry of the preceding chapter under a multi-valued realization of the local coordinates and their operations;

3) The corresponding *forward and backward hyperaxioms* and their isoduals:

FORWARD HYPERAXIOM I. The projection in our spacetime of the maximal causal invariant speed on forward Minkowski-Santilli hyperspace in $(3,4)$*-dimensions is given by:*

$$\hat{V}_{Max}=c_o\times\frac{b_4^{>}}{b_3^{>}}=c_o\times\frac{n_3^{>}}{n_4^{>}}=\hat{c}^{>}/\hat{b}_3^{>},\quad \hat{c}^{>}=c_o\times b_4^{>}=\frac{c_o}{\hat{n}_4^{>}}, \tag{5.14}$$

FORWARD HYPERAXIOM II. The projection in our spacetime of the hyperrelativistic addition of speeds within MULTI-VALUED physical media represented by the forward Minkowski-Santilli hyperspace is given by:

$$\hat{V}_{tot}^{>}=\frac{\hat{v}_1^{>}+\hat{v}_2^{>}}{\hat{1}^{>}+\frac{\hat{v}_1^{>}\times b_3^{>\,2}\times\hat{v}_2^{>}}{c_o\times b_4^{>\,2}\times c_o}}=\frac{\hat{v}_1^{>}+\hat{v}_2^{>}}{\hat{1}^{>}+\frac{\hat{v}_1^{>}\times n_4^{>\,2}\times\hat{v}_2^{>}}{c_o\times n_3^{>\,2}\times c_o}}. \tag{5.15}$$

FORWARD HYPERAXIOM III. The projection in our spacetime of the forward hyperdilation of forward hypertime, forward hypercontraction of forward hyperlength and the variation of forward hypermass with the forward hyperspeed are given respectively by

$$\hat{t}^{>}=\hat{\gamma}^{>}\times\hat{t}_{\circ}^{>}, \tag{5.16a}$$

$$\hat{\ell}_{\circ}^{>}=\hat{\gamma}^{>}\times\hat{\ell}, \tag{5.16b}$$

$$\hat{m}^{>}=\hat{\gamma}^{>}\times\hat{m}_{\circ}^{>}. \tag{5.16c}$$

FORWARD HYPERAXIOM IV. The projection in our spacetime of the Doppler-Santilli forward hyperlaw is given by the expression (here formulated for simplicity for 90° *angle of aberration):*

$$\hat{\omega}^{>}=\hat{\gamma}\times^{>}\hat{\omega}_{\circ}^{>}. \tag{5.17}$$

ISOAXIOM V. The projection in our spacetime of the hyperrelativistic law of equivalence of forward hypermass and the forward hyperenergy is given by:

$$\hat{E}^{>}=\hat{m}^{>}\times\hat{V}_{\max}^{2>}=\hat{m}^{>}\times c_o^2\times\frac{\hat{b}_4^{>\,2}}{\hat{b}_3^{>\,2}}=\hat{m}^{>}\times c_o^2\times\frac{\hat{n}_3^{>\,2}}{\hat{n}_4^{>\,2}}. \tag{5.18}$$

Figure 5.2. Samples of sliced seashells showing the complexity of their structure. Illert [1] has shown that a mathematical representation of their four-lobes bifurcations requires all four directions of times, namely, the knowledge by the seashell of motions forward in future and past times as well as motions backward from future and in past times. The need for multi-valued methods, plus these four different time arrows then identify our hyperstructures and their isoduals quite uniquely. Whatever the appropriate theory, it can be safely stated that the complexity of the "intrinsic time" of biological structure (that perceived by said structures rather than by us) can be safely stated to be beyond our comprehension at this writing.

In the above expressions we have used the following notations: *hypergamma and hyperbeta* are given by

$$\hat{\gamma}^{>} = (1-\hat{\beta}^{2>})^{-1/2}, \quad \hat{\beta}^{>} = \hat{v}^{2>} \times \hat{n}_4^{>\ 2}/c_o^2 \times \hat{n}_3^{>\ 2} = \hat{v}^{2>} \times \hat{b}_3^{>\ 2}/c_o^2 \times \hat{b}_4^{>\ 2}; \tag{5.19}$$

the upper symbol $^{>}$ denotes motion forward to future times; the upper symbol $\hat{x}$, etc., denotes multivalued character; and all multiplications are conventional (rather than being hyperproducts) since the hyperaxioms are expressed in their projection in our spacetime to avoid excessive complexity.

The study of the backward and isodual hyperaxioms is left to the interested reader.

A few comments are now inn order:

i) Hyperrelativity and its isodual are the most general forms of relativities known at this writing that can be formulated on numbers verifying the axioms of a field, thus admitting a well defined left and right unit with consequential applicability to measurements;

ii) Hyperrelativity and its isodual are invariant under their respective time hyperevolutions, thus predicting the same numerical results at different time, and being applicable to experiments;

iii) Hyperrelativity and its isodual are multi-valued rather than multi-dimensional, namely, they permit the representation of multi-universes in a form compatible with our sensory perception of spacetime;

iv) The speed of light in vacuum c_o has been assumed to remain unchanged under hyperlifting, thus meaning that the speed of light is the same for all vacuum foliations of spacetime.

v) Like all other quantities, hyperspeeds in general and, in particular, the hyperspeed of light must necessarily be multi-valued for consistency, namely, assume different values for different foliations of spacetime.

Note the covering character of hyperrelativity in the sense of admitting as particular cases the genorelativity of Chapter 4, the isorelativity of Chapter 3 and the conventional special relativity whenever all units return to have the value 1 dating back to biblical times.

As we shall see in Volume II, hyperrelativity and its isodual, with particular reference to the 44-multi-valued hyperdimensional hypersymmetry (5.13)[1], will allow the formulation of the most general known, thus the most complex known, cosmology that includes, for the first time, biological structure as a condition for the appropriate use of the word "cosmology" in its Greek sense.

[1]The reader should recall that the Poincaré symmetry is *eleven*-dimensional and not ten dimensional as popularly believed because of the discovery permitted by isomathematics of the additional, 11-th dimensional isoscalar isoinvariance studied in Section 3.5.

Appendix 5.A
Vougiouklis Studies on the Lie-Santilli Hyper-Theory

5.A.1 Foreword

In this appendix we report *ad litteram* the studies on the Lie-Santilli hypertheory conducted by Thomas Vougiouklis, Democritus University of Thrace, School of Education, 681 00 Alexandroupolis, Greece, email tvougiou@eled.duth.gr. These studies are fundamental for the proper mathematical formulation of the class of hyperstructures necessary for hadronic mechanics, that with a well defined left and right hyperunit.

5.A.2 Introduction

The hyperstructures were introduced by F. Marty in 1934 when he first defined the hypergoup as a set equipped with an associative and reproductive hyperoperation. The motivating example was the quotient of a group by any, not necessarily normal, subgroup. M. Koskas in 1970 was introduced the fundamental relation β^*, which it turns to be the main tool in the study of hyperstructures. T. Vougiouklis in 1990 was introduced the H_v-structures, by defining the weak axioms. The motivating example of those hyperstructures is the quotient of any group by any partition. Therefore the class of H_v-structures is the largest class of hyperstructures.

In 1996 R. M. Santilli and T. Vougiouklis, point out that in physics the most interesting hyperstructures are the one called e-hyperstructures. These hyperstructures contain a unique left ant right scalar unit, which is the most important tool in Lie-Santilli theory. In what follows we present the related hyperstructure theory mainly from the paper [6], enriched with some new results on the related hyperstructures. However one can see the books by T. Vougiouklis [27] and by P. Corsini–V. Leoreanu [13], for more definitions as well as the site: aha.eled.duth.gr, for an extensive bibliography on the concept. Moreover, in this site one can see the Vougiouklis's point of view on the birth and the history of H_v-structures in the above site: *An H_v-interview, i.e. weak, with Th. Vougiouklis,* Interviewer N. Lygeros.

5.A.3 Basic definitions

In a set H is called *hyperoperation* or *multivalued operation*, any map from $H \times H$ to the power set of H. Therefore, in a hyperoperation

$$\cdot : H \times H \to \quad \wp(H) : (x, y) \to x \cdot y \subset H$$

the result is a subset of H, instead of an element as we have in usually operations.

In a set H equipped with a hyperoperation $\cdot : H \times H \to \wp(H) - \{\varnothing\}$, we abbreviate by

WASS the *weak associativity*: $(xy)z \cap x(yz) \neq \varnothing$, $\forall x, y, z \in H$ and by

COW the *weak commutativity*: $xy \cap yx \neq \varnothing$, $\forall x, y \in H$.

The hyperstructure $(H, \cdot)$ is called H_v*-semigroup* if it is *WASS* and it is called H_v*-group* if it is reproductive H_v-semigroup, i.e. $xH = Hx = H$, $\forall x \in H$. The hyperstructure $(R, +, \cdot)$ is called H_v*-ring* if $(+)$ and $(\cdot)$ are *WASS*, the reproduction axiom is valid for $(+)$ and $(\cdot)$ is *weak distributive* with respect to $(+)$:

$$x(y+z) \cap (xy+xz) \neq \varnothing, \quad (x+y)z \cap (xz+yz) \neq \varnothing, \quad \forall x, y, z \in R.$$

An extreme class of hyperstructures is the following [23]: An H_v-structure is called *very thin* iff all hyperoperations are operations except one, which has all hyperproducts singletons except one, which is a subset of cardinality more than one.

A H_v-group is called *cyclic* [27], if there is an element, called *generator*, which the powers have union the underline set. The minimal power with the above property is called *period* of the generator. Moreover if there exist an element and a special power, the minimum one, is the underline set, then the H_v-group is called *single-power cyclic*.

The main tool to study all hyperstructures are the fundamental relations β^*, γ^* and ε^*, which are defined, in H_v-groups, H_v-rings and H_v-vector spaces, respectively, as the smallest equivalences so that the quotient would be group, ring and vector space, respectively [27]. A way to find the fundamental classes is given by analogous theorems to the following [24, 27, 28]:

THEOREM: Let $(H, \cdot)$ be a H_v-group and U be all finite products of elements of H. Define the relation β by setting $x\beta y$ iff $\{x, y\} \subset u$, $u \in U$. Then β^* is the transitive closure of β.

Analogous theorems for the relations γ^* in H_v-rings and ε^* in H_v-modules and H_v-vector spaces, are also proved.

An element is called *single* if its fundamental class is singleton.

Fundamental relations are used for general definitions. Thus, in the definition of the H_v-field the γ^* is used: A H_v-ring $(R, +, \cdot)$ is called H_v*-field* if R/γ^* is a field.

Let $(H, \cdot)$, $(H, ^*)$ be H_v-semigroups defined on the same set H. $(\cdot)$ is called *smaller* than $(^*)$, and $(^*)$ *greater* than $(\cdot)$, iff there exists an

$$f \in \mathrm{Aut}(H, ^*) \text{ such that } xy \subset f(x^*y), \ \ \forall x, y \in H.$$

Then we write $\cdot \leqslant^*$ and we say that $(H, ^*)$ *contains* $(H, \cdot)$. If $(H, \cdot)$ is a structure then it is called *basic structure* and $(H, ^*)$ is called H_b*-structure*.

THEOREM: Greater hyperoperations than the ones which are WASS or COW, are also WASS or COW, respectively.

The definition of the H_v-field introduced a new class of hyperstructures [39]:

DEFINITION: The H_v-semigroup $(H,\cdot)$ is called h/v-group if the quotient H/β^* is a group.

The h/v-groups are a generalization of the H_v-groups because in h/v-groups the reproductivity is not necessarily valid. However, sometimes a kind of *reproductivity of classes* is valid. This leads the quotient to be reproductive. In a similar way the *h/v-rings, h/v-fields, h/v-modulus, h/v-vector spaces* etc., are defined.

Definitions [33, 36, 37]. Let $(H,\cdot)$ be hypergroupoid. We *remove* h∈H, if we consider the restriction of $(\cdot)$ in the set $H-\{h\}$. $\underline{h}\in H$ *absorbs* $h\in H$ if we replace h by $\underline{h}$ and h does not appear in the structure. $\underline{h}\in H$ *merges* with $h\in H$, if we take as product of any $x\in H$ by $\underline{h}$, the union of the results of x with both h, $\underline{h}$, and consider h and $\underline{h}$ as one class with representative $\underline{h}$, therefore, h does not appear in the hyperstructure.

Hyperoperations on any type of matrices can be defined:

DEFINITION [42]: Let $A=(a_{ij})\in \mathbf{M}_{m\times n}$ be matrix and $s,t\in N$, with $1\leqslant s\leqslant m$, $1\leqslant t\leqslant n$. Then helix-projection is a map $\underline{st}$: $\mathbf{M}_{m\times n}\to\mathbf{M}_{s\times t}$: $A\to A\underline{st}=(\underline{a}_{ij})$, where $A\underline{st}$ has entries

$$\underline{a}_{ij}=\{a_{i+\kappa s,j+\lambda t}\ \big|\ 1\leqslant i\leqslant s,\ 1\leqslant j\leqslant t \text{ and } \kappa,\ \lambda\in N,\ i+\kappa s\leqslant m, j+\lambda t\leqslant n\}.$$

Let $A=(a_{ij})\in\mathbf{M}_{m\times n}$, $B=(b_{ij})\in\mathbf{M}_{u\times v}$ be matrices and $s=\min(m,u)$, $t=\min(n,v)$. We define a hyper-addition, called *helix-addition*, by

$$\begin{aligned}\oplus:\mathbf{M}_{m\times n}\times\mathbf{M}_{u\times v}&\to P(\mathbf{M}_{s\times t}):(A,B)\to A\oplus B\\ &=A\underline{st}+B\underline{st}=(\underline{a}_{ij})+(\underline{b}_{ij})\subset\mathbf{M}_{s\times t},\end{aligned}$$

where $(\underline{a}_{ij})+(\underline{b}_{ij})=\{(c_{ij})=(a_{ij}+b_{ij})\ \big|\ a_{ij}\in\underline{a}_{ij}$ and $b_{ij}\in\underline{b}_{ij})\}$. Let $A=(a_{ij})\in\mathbf{M}_{m\times n}$, $B=(b_{ij})\in\mathbf{M}_{u\times v}$ and $s=\min(n,u)$. We define the *helix-multiplication*, by

$$\begin{aligned}\otimes:\mathbf{M}_{m\times n}\times\mathbf{M}_{u\times v}&\to P(\mathbf{M}_{m\times v}):(A,B)\to A\otimes B\\ &=A\underline{ms}\cdot B\underline{sv}=(\underline{a}_{ij})\cdot(\underline{b}_{ij})\subset\mathbf{M}_{m\times v},\end{aligned}$$

where $(\underline{a}_{ij})\cdot(\underline{b}_{ij})=\{(c_{ij})=(\sum a_{it}b_{tj})\ \big|\ a_{ij}\in\underline{a}_{ij}$ and $b_{ij}\in\underline{b}_{ij})\}$.

The helix-addition is commutative, WASS but not associative. The helix-multiplication is WASS, not associative and it is not distributive, not even weak, to the helix-addition. For all matrices of the same type, the inclusion distributivity, is valid.

5.A.4 Small sets

The problem of enumeration and classification of H_v-structures, was started from the beginning [23, 22]. However, the problem becomes more complicate in

H_v-structures because we have very great numbers in this case. The partial order in H_v-structures [24], transfers and restrict the problem in finding the minimal, *up to isomorphisms*, H_v-structures. In this direction we have results by Bayon & Lygeros [10, 11]:

Let $H = \{a, b\}$ a set of two elements. There are 20 H_v-groups, up to isomorphism.

Suppose in $H = \{e, a, b\}$, a hyperoperation is defined and there exists a scalar unit, then, there are 13 *minimal* H_v-groups. The number of all H_v-groups with three elements, up to isomorphism, which have a scalar unit, is 292.

In a set with three elements there are, exactly 6.494 minimal H_v-groups. 137 are abelians and the 6.357 are non-abelians; the 6.152 are cyclic and the 342 are not cyclic.

The number of H_v-groups with three elements, up to isomorphism, is 1.026.462. More precisely, there are 7.926 abelians and 1.018.536 non-abelians; the 1.013.598 are cyclic and the 12.864 are not cyclic. The 16 of them are very thin.

The number of all H_v-groups with four elements, up to isomorphism, which have a scalar unit, is 631.609. There are 10.614.362 abelian hypergroups from which the 10.607.666 are cyclic and the 6.696 are not. There are 8.028.299.905 abelian H_v-groups from which the 7.995.884.377 are cyclic and the 32.415.528 are not.

5.A.5 Uniting elements

The *uniting elements* method was introduced by Corsini-Vougiouklis [14] in 1989. With this method one puts in the same class, two or more elements. This leads, through hyperstructures, to structures satisfying additional properties.

The *uniting elements* method is the following: Let G be algebraic structure and let d be a property, which is not valid and it is described by a set of equations; then, consider the partition in G for which it is put together, in the same partition class, every pair of elements that causes the non-validity of the property d. The quotient by this partition G/d is an H_v-structure. Then, quotient out the H_v-structure G/d by the fundamental relation β^*, a stricter structure $(G/d)\beta^*$ for which the property d is valid, is obtained.

An interesting application of the uniting elements is when more than one properties are desired. The reason for this is that some of the properties lead straighter to the classes than others. So, it is better to apply the straightforward classes followed by the more complicated ones. The commutativity and reproductivity are easy applicable properties. One can do this because the following is valid.

THEOREM [27]: Let $(G, \cdot)$ be a groupoid, and $F = \{f_1, \ldots, f_m, f_{m+1}, \ldots, f_{m+n}\}$ be a system of equations on G consisting of two subsystems $F_m = \{f_1, \ldots, f_m\}$ and $F_n = \{f_{m+1}, \ldots, f_{m+n}\}$. Let σ, σ_m be the equivalence relations defined by the uniting elements procedure using the systems F and F_m resp., and

let σ_n be the equivalence relation defined using the induced equations of F_n on the grupoid $G_m = (G/\sigma_m)/\beta^*$. Then $(G/\sigma)/\beta^* \cong (G_m/\sigma_n)/\beta^*$.

5.A.6 Theta (∂) hyperoperations

In [40] a hyperoperation, in a groupoid with a map on it, called *theta* ∂, is introduced.

DEFINITION: Let H be a set equipped with n operations (or hyperoperations) $\otimes_1, \otimes_2, \ldots, \otimes_n$ and a map (or multivalued map) $f : H \to H$ (or $f : H \to P(H)$, resp.), then n hyperoperations ∂_1, ∂_2, ..., ∂_n on H can be defined, called theta-operations by putting

$$x\partial_i y = \{f(x) \otimes_i y,\ x \otimes_i f(y)\},\ \forall x, y \in H \text{ and } i \in \{1, 2, \ldots, n\}$$

or, in case where $\otimes_i$ is hyperoperation or f is multivalued map, we have

$$x\partial_i y = (f(x) \otimes_i y) \cup (x \otimes_i f(y)),\ \forall x, y \in H \text{ and } i \in \{1, 2, \ldots, n\}.$$

If $\otimes_i$ is associative then ∂_i is WASS.

DEFINITIONS: Let $(G, \cdot)$ be a groupoid and $f_i : G \to G$, $i \in I$, be a set of maps on G. Take the map $f_\cup : G \to P(G)$ such that $f_\cup(x) = \{f_i(x) \mid i \in I\}$ and we call it the union of the $f_i(x)$. We call union theta-operation (∂), on G if we consider the map $f_\cup(x)$. A special case is to take the union of f with the identity, i.e. $\underline{f} \equiv f \cup (id)$, so $\underline{f}(x) = \{x, f(x)\}$, $\forall x \in G$, which is called b-theta-operation. We denote by ($\underline{\partial}$) the b-theta-operation, so

$$x\underline{\partial}y = \{xy, f(x) \cdot y, x \cdot f(y)\},\ \forall x, y \in G.$$

Remark that this hyperoperation is a b-operation. If $f : G \to P(G)$ then the b-theta-operation is defined by using the map $\underline{f}(x) = \{x\} \cup f(x)$, $\forall x \in G$.

Motivation for the definition of the theta-operation is the map *derivative* where only the multiplication of functions can be used. Therefore, in these terms, for two functions $s(x)$, $t(x)$, we have $s\partial t = \{s't, st'\}$, where $(')$ denotes the derivative.

Example. Taking the application on the derivative, consider all polynomials of first degree $g_i(x) = a_i x + b_i$. We have

$$g_1 \partial g_2 = \{a_1 a_2 x + a_1 b_2,\ a_1 a_2 x + b_1 a_2\},$$

so this is a hyperoperation in the set of the first degree polynomials. Moreover all polynomials x+c, where c be a constant, are units.

Properties [40, 41]. If $(G, \cdot)$ is a semigroup then:

For every f, the hyperoperation (∂) is *WASS*, and the b-theta-operation (∂) is *WASS.*

If f is homomorphism and projection, then (∂) is associative.

Reproductivity. For the reproductivity we must have

$$x\partial G = \cup_{g\in G}\{f(x)\cdot g,\, x\cdot f(g)\} = G \text{ and } G\partial x = \cup_{g\in G}\{f(g)\cdot x,\, g\cdot f(x)\} = G.$$

Thus, if $(\cdot)$ is reproductive then (∂) is also reproductive.

Commutativity. If $(\cdot)$ is commutative then (∂) is commutative. If f is into the centre of G, then (∂) is commutative. If $(\cdot)$ is COW then, (∂) is COW.

Unit elements. u is a right unit element if $x\partial u = \{f(x)\cdot u,\, x\cdot f(u)\} \ni x$. So $f(u) = e$, where e be a unit in $(G,\cdot)$. The elements of the kernel of f, are the units of (G,∂).

Inverse elements. Let $(G,\cdot)$ be a monoid with unit e and u be a unit in (G,∂), then $f(u) = e$. For given x, the element x' is an inverse with respect to u, if

$$x\partial x' = \{f(x)\cdot x',\, x\cdot f(x')\} \ni u \text{ and } x'\partial x = \{f(x')\cdot x,\, x'\cdot f(x)\} \ni u.$$

So, $x' = (f(x))^{-1}u$ and $x' = u(f(x))^{-1}$, are the right and left inverses, respectively. We have two-sided inverses iff $f(x)u = uf(x)$.

PROPOSITION: Let $(G,\cdot)$ be a group then, for all $f : G \to G$, the (G,∂) is a H_v-group.

For several results one can see [20–22, 6, 23].

In order to see a connection of the merge with the ∂-operation, consider the map f such that $f(\underline{h}) = h$ and $f(x) = x$ in the rest cases.

Example. P-hyperoperations. Let $(G,\cdot)$ be commutative semigroup and $P \subset G$. Consider the multivalued map f such that $f(x) = P\cdot x$, $\forall x \in G$. Then we have

$$x\partial y = x\cdot y\cdot P,\ \forall x,y \in G.$$

So the ∂-operation coincides with the well known class of P-hyperoperations [14].

One can define theta-operations on rings and other more complicate structures, where more than one theta-operations can be defined.

DEFINITION: Let $(R,+,\cdot)$ be a ring and $f : R \to R$, $g : R \to R$ be two maps. We define two hyperoperations (∂_+) and $(\partial\cdot)$, called both theta-operations, on R as follows

$$x\partial_+ y = \{f(x)+y,\, x+f(y)\} \text{ and } x\partial\cdot y = \{g(x)\cdot y,\, x\cdot g(y)\},\ \ \forall x,y \in G.$$

A hyperstructure $(R,+,\cdot)$, where $(+)$, $(\cdot)$ be hyperoperations which satisfy all H_v-ring axioms, except the weak distributivity, will be called *H_v-near-ring.*

PROPOSITION: Let $(R,+,\cdot)$ ring and $f : R \to R$, $g : R \to R$ maps. The hyperstructure $(R,\partial_+,\partial\cdot)$, called theta, is a H_v-near-ring. Moreover $(+)$ is commutative.

PROPOSITION: Let $(R,+,\cdot)$ ring and $f : R \to R$, $g : R \to R$ maps, then $(R, \partial_+, \partial\cdot)$, is H_v-ring.

Properties. The theta hyperstructure $(R, \partial_+, \partial\cdot)$ takes new form in special cases:

(a) If $f(x) \equiv g(x)$, $\forall x \in R$, i.e. the two maps coincide, then we have

$$x\partial \cdot (y\partial_+ z) \cap (x\partial \cdot y)\partial_+(x\partial \cdot z) = \varnothing.$$

If f is homomorphism and projection, then $(R, \partial_+, \partial\cdot)$ is H_v-ring.

(b) If $f(x) = x$, $\forall x \in R$, then $(R, +, \partial\cdot)$ becomes a multiplicative H_v-ring:

$$x\partial \cdot (y + z) \cap (x\partial \cdot y) + (x\partial \cdot z) = \{g(x)y + g(x)z\} \neq \varnothing.$$

5.A.7 The H_v-vector spaces and H_v-Lie algebras

DEFINITION [27, 34]: Let $(F,+,\cdot)$ be a H_v-field, $(V,+)$ be a COW H_v-group and there exists an external hyperoperation

$$\cdot : F \times V \to \mathcal{P}(V) : (a, x) \to ax$$

such that, for all a, b in F and x, y in V we have

$$a(x + y) \cap (ax + ay) \neq \varnothing, \quad (a + b)x \cap (ax + bx) \neq \varnothing, \quad (ab)x \cap a(bx) \neq \varnothing,$$

then V is called a H_v-vector space over F.

In the case of a H_v-ring instead of H_v-field then the H_v-modulo is defined.

In the above cases the fundamental relation ε^* is the smallest equivalence relation such that the quotient V/ε^* is a vector space over the fundamental field F/γ^*.

The general definition of a H_v-Lie algebra over a field F is the following [27, 34]:

DEFINITION. Let $(L,+)$ be a H_v-vector space over the field $(F,+,\cdot)$, $\varphi : F \to F/\gamma^*$, the canonical map and $\omega_F = \{x \in F : \varphi(x) = 0\}$, where 0 is the zero of the fundamental field F/γ^*. Similarly, let ω_L be the core of the canonical map $\varphi' : L \to L/\varepsilon^*$ and denote by the same symbol 0 the zero of L/ε^*. Consider the bracket (commutator) hyperoperation:

$$[,] : L \times L \to \mathcal{P}(L) : (x, y) \to [x, y],$$

then L is a H_v-Lie algebra over F if the following axioms are satisfied:

(L1) The bracket hyperoperation is bilinear, i.e.

$$[\lambda_1 x_1 + \lambda_2 x_2, y] \cap (\lambda_1[x_1, y] + \lambda_2[x_2, y]) \neq \varnothing,$$
$$[x, \lambda_1 y_1 + \lambda_2 y] \cap (\lambda_1[x, y_1] + \lambda_2[x, y_2]) \neq \varnothing$$

for all $x, x_1, x_2, y, y_1, y_2 \in L$ and λ_1, λ_2 in F;

(L2) $[x, x] \cap \omega_L \neq \varnothing$ for all x in L;

(L3) $([x, [y, z]] + [y, [z, x]] + [z, [x, y]]) \cap \omega_L \neq \varnothing$ for all x, y in L.

We remark that this is a very general definition therefore one can use special cases in order to face several problems in applied sciences. Moreover, from this definition we can see how the weak properties can be defined as the above weak linearity (L1), anti-commutativity (L2) and the Jacobi identity (L3).

We present here a direction to obtain results from special cases by applying ∂-operations on more complicated structures, in the sense that they have more than one operation.

THEOREM: Consider the group of integers $(Z, +)$ and let $n \neq 0$ be a natural number. Take the map f such that $f(0) = n$ and $f(x) = x$, $\forall x \in Z - \{0\}$. Then $(Z, \partial)/\beta^* \cong (Z_n, +)$.

THEOREM: Consider the ring of integers $(Z, +, \cdot)$ and let $n \neq 0$. Consider the map f such that $f(0) = n$ and $f(x) = x$, $\forall x \in Z - \{0\}$. Then $(Z, \partial_+, \partial\cdot)$ is a H_v-near-ring, with $(Z, \partial_+, \partial\cdot)/\gamma^* \cong Z_n$.

PROPOSITION: Let $(V, +, \cdot)$ be an algebra over the field $(F, +, \cdot)$ and $f : V \to V$ be a map. Consider the ∂-operation defined only on the multiplication of the vectors $(\cdot)$, then $(V, +, \partial)$ is a H_v-algebra over F, where the related properties are weak. If, moreover f is linear then we have more strong properties.

DEFINITION: Let L be a Lie algebra, defined on an algebra $(V, +, \cdot)$ over the field $(F, +, \cdot)$ where the Lie bracket $[x, y] = xy - yx$. Consider any map $f : L \to L$, then the ∂-operation is defined as follows

$$x\partial y = \{f(x)y - f(y)x,\ f(x)y - yf(x),\ xf(y) - f(y)x,\ xf(y) - yf(x)\}.$$

PROPOSITION: Let $(V, +, \cdot)$ be an algebra over the field $(F, +, \cdot)$ and $f : V \to V$ be a linear map. Consider the ∂-operation defined only on the multiplication of the vectors $(\cdot)$, then $(V, +, \partial)$ is a H_v-algebra over F, with respect to Lie bracket, where the weak anti-commutatinity and the inclusion linearity is valid.

We can see that the weak linearity is valid, more precisely, the inclusion linearity is valid: $[\lambda_1 x_1 + \lambda_2 x_2, y] \subset \lambda_1[x_1, y] + \lambda_2[x_2, y]$.

Remark that one can face the weak Jacobi identity in analogous to the above propositions as well. One can use well known maps as constants or linear.

5.A.8 Adding elements

In [33] the 'enlarged' hyperstructures were examined in the sense that an extra element, outside the underlying set, appears in one result. In both directions, enlargement or reduction, most useful in representation theory, are those H_v-structures with the same fundamental structure: Suppose we have a structure and

one element, outside of the structure, then we can attach this element in order to have a hyperstructure which becomes h/v-structure. Moreover we have the opposite problem: How one can remove at least one element of an H_v-structure or a classical structure?

The Attach Construction [33, 36, 37]. Let $(H,\cdot)$ be an H_V-semigroup and $v \notin H$. We extend the $(\cdot)$ into the set $\underline{H} = H \cup \{v\}$ as follows: $x \cdot v = v \cdot x = v$, $\forall x \in H$, and $v \cdot v = H$.

The $(\underline{H},\cdot)$ is a h/v-group where $(\underline{H},\cdot)/\beta^* \cong Z_2$ and v is a single element.

We call the hyperstructure $(\underline{H},\cdot)$ the attach h/v-group of $(H,\cdot)$.

Remarks. The core of $(\underline{H},\cdot)$ is the set H. All scalar elements of $(H,\cdot)$ are also scalars in $(\underline{H},\cdot)$ and any unit element of $(H,\cdot)$ is also a unit of $(\underline{H},\cdot)$. Finally, if $(H,\cdot)$ is *COW* (resp. commutative) then $(\underline{H},\cdot)$ is also *COW* (resp. commutative).

The motivation of the attach construction is the first kind very thin H_v-groups [23].

In the representation theory of H_v-groups by H_v-matrices one needs H_v-rings or H_v-fields which have non-degenerate fundamental structures in addition with only few of hypersums and hyperproducts to have cardinals greater than one.

THEOREM: Let $(G,\cdot)$ be semigroup and $v \notin G$ be an element appearing in a product ab, where $a,b \in G$, thus the result becomes a hyperproduct $a \otimes b = \{ab, v\}$. Then the minimal hyperoperation $(\otimes)$ extended in $G' = G \cup \{v\}$ such that $(\otimes)$ contains $(\cdot)$ in the restriction on G, and such that $(G',\otimes)$ is a minimal H_V-semigroup which has fundamental structure isomorphic to $(G,\cdot)$, is defined as follows:

$$a \otimes b = \{ab, v\},\ x \otimes y = xy,\ \forall (x,y) \in G^2 - \{(a,b)\},$$
$$v \otimes v = abab,\ x \otimes v = xab \text{ and } v \otimes x = abx,\ \forall x \in G.$$

Therefore $(G',\otimes)$ is a very thin H_V-semigroup.

If $(G,\cdot)$ is commutative then the $(G',\otimes)$ becomes strongly commutative.

5.A.9 Representations

Representations (we abbreviate here by *rep*) of H_v-groups, can be considered either by generalized permutations [25] or by H_v-matrices [26]. Here we present the matrix reps.

H_v-matrix (or *h/v-matrix*) is called a matrix with entries elements of a H_v-ring or H_v-field (or h/v-ring or h/v-field). The hyperproduct of H_v-matrices $\mathbf{A} = (a_{ij})$ and $\mathbf{B} = (b_{ij})$, of type $m \times n$ and $n \times r$, respectively, is a set of $m \times r$ H_v-matrices, defined in a usual manner:

$$\mathbf{A} \cdot \mathbf{B} = (a_{ij}) \cdot (b_{ij}) = \{\mathbf{C} = (c_{ij}) \mid c_{ij} \in \oplus \sum a_{ik} \cdot b_{kj},$$

where $(\oplus)$ denotes the *n-ary circle hyperoperation* on the hyperaddition [27]: that is the sum of products of elements of the H_v-ring is considered to be the union of

the sets obtained with all possible parentheses. However, in the case of 2×2 H_v-matrices the 2-ary circle hyperoperation which coincides with the hyperaddition in the H_v-ring. Notice that the hyperproduct of H_v-matrices does not nessesarily satisfy *WASS*.

The rep problem by H_v-matrices is the following:

H_v-matrix is called a matrix if has entries from a H_v-ring.

DEFINITION: Let $(H,\cdot)$ be H_v-group, $(R,+,\cdot)$ be H_v-ring and $\mathbf{M}_R = \{(a_{ij}) \mid a_{ij}\in R\}$*, then any*

$$\mathbf{T}: H\to \mathbf{M}_R : h\to \mathbf{T}(h) \text{ with } \mathbf{T}(h_1h_2)\cap \mathbf{T}(h_1)\mathbf{T}(h_2)\neq \varnothing,\ \forall h_1,h_2\in H,$$

is called H_v-matrix rep. If $\mathbf{T}(h_1h_2)\subset \mathbf{T}(h_1)\mathbf{T}(h_2)$, then $\mathbf{T}$ is an inclusion rep, if $\mathbf{T}(h_1h_2)=\mathbf{T}(h_1)\mathbf{T}(h_2)$, then $\mathbf{T}$ is a good rep.

In reps of H_v-groups by H_v-matrices, there are two difficulties: To find a H_v-ring and an appropriate set of H_v-matrices.

The problem of reps is very complicated mainly because the cardinality of the product of two H_v-matrices is normally very big. The problem can be simplified in several special cases such as the following:

(a) The H_v-matrices are over H_v-rings with 0 and 1 and if these are scalars. Thus the e-hyperstructures are interesting in the rep theory.

(b) The H_v-matrices are over *very thin* H_v-rings.

(c) The case of 2×2 H_v-matrices, since the 2-ary circle hyperoperation coincides with the hyperaddition in H_v-rings. This is the lowest dimensional, non degenerate, rep.

(d) The case of H_v-rings in which the strong associativity in hyperaddition is valid.

(e) The case of H_v-rings which contains singles, then these act as absorbings.

The main theorem of reps on h/v-structures, which has a completely analogous on H_v-structures [27], is the following:

THEOREM: A necessary condition in order to have an inclusion rep T of an h/v-group $(H,\cdot)$ by $n\times n$ h/v-matrices over the h/v-ring $(R,+,\cdot)$ is the following:

For all classes $\beta^*(x)$, $x\in H$ there must exist elements $a_{ij}\in H$, $i,j\in\{1,\ldots,n\}$ such that

$$T(\beta^*(a))\subset \{A=(a'_{ij}) \mid a'_{ij}\in \gamma^*(a_{ij}),\ i,j\in\{1,\ldots,n\}\}.$$

Therefore, every inclusion rep $T: H\to M_R : a\mapsto T(a)=(a_{ij})$ induces a homomorphic rep T^* of the group H/β^* over the ring R/γ^* by setting $T^*(\beta^*(a))=[\gamma^*(a_{ij})]$, $\forall\beta^*(a)\in H/\beta^*$, where the element $\gamma^*(a_{ij})\in R/\gamma^*$ is the ij entry of the matrix $T^*(\beta^*(a))$. Then T^* is called *fundamental induced rep* of T.

In analogous way other concepts of the rep theory can be transferred for h/v structures. Thus, let T be a rep of an h/v-group H by h/v-matrices over the

h/v-ring R. Denote $\mathrm{tr}_\varphi(T(x)) = \gamma^*(T(x_{ii}))$ the fundamental trace, then the mapping

$$X_T : H \to R/\gamma^* : x \mapsto X_T(x) = \mathrm{tr}_\varphi(T(x)) = \mathrm{tr}T^*(x)$$

is called *fundamental character*. There are several types of traces.

For an attached h/v-field $(\underline{H}_o, +, \cdot)$, in $\sum a_{ik} \cdot b_{kj}$ the terms $a_{ik} \cdot b_{kj}$ could be $0, v, x$ or H (where $x \in H$). But any sum is only 0 or v or H. Thus, for finite h/v-fields $(\underline{H}_o, +, \cdot)$, if the set H appears in t entries then the cardinality of the hyperproducts is $(\mathrm{card}H)^t$.

The main attached h/v-fields give to rep theory some hyperfields for reps where the cardinality of any two elements is small. The point is that 0 is absorbing.

5.A.10 The *e*-constructions

The Lie-Santilli *isotopies* born in 1970's to solve Hadronic Mechanics problems. Santilli [6], proposed a 'lifting'of the n-dimensional trivial unit matrix of a normal theory into a nowhere singular, symmetric, real-valued, positive-defined, n-dimensional new matrix. The original theory is reconstructed such as to admit the new matrix as left and right unit. The *isofields* needed in this theory correspond into the hyperstructures called *e-hyperfields*. The H_v-fields or h/v-fields can give e-hyperfields which can be used in the isotopy theory in applications as in physics or biology.

DEFINITION: Let $(\underline{H}_o, +, \cdot)$ be the attached h/v-field of the H_v-semigroup $(H, \cdot)$. If $(H, \cdot)$ has a left and right scalar unit e then $(\underline{H}_o, +, \cdot)$ is e-hyperfield, the attached h/v-field of $(H, \cdot)$.

Applications. (1) The above constructions, especially the ones in enlarging H_v-rings, or H_v-fields can be used as entries of H_v-matrices to represent H_v-groups for which the cardinality of all hypreproducts equals to 2^s, $s \in N$. This is so, since in the hyperproducts of H_v-matrices we can have one or two elements.

(2) The monomial matrix reps are based, on the ring Z_2. The enlargments of the above ring are the following hyperrings

(i) $0 \oplus 0 = 1 \oplus 1 = 1 \oplus v = v \oplus 1 = v \oplus v = 0,$
$0 \oplus 1 = 0 \oplus v = 1 \oplus 0 = v \oplus 0 = \{1, v\},$
$0 \otimes 0 = 0 \otimes 1 = 1 \otimes 0 = 0 \otimes v = v \otimes 0 = 0,$
$1 \otimes 1 = 1 \otimes v = v \otimes 1 = v \otimes v = 1,$

(ii) $0 \oplus 0 = 0 \oplus v = 1 \oplus 1 = v \oplus 0 = \{0, v\}, \;\; v \oplus v = 0,$
$0 \oplus 1 = 1 \oplus 0 = 1 \oplus v = v \oplus 1 = 1, \;\; 1 \otimes 1 = 1,$
and in the rest cases 0.

(iii) $0 \oplus 0 = 1 \oplus 1 = v \oplus v = \{0, v\}, \;\; 0 \oplus v = v \oplus 0 = 0,$

$$0 \oplus 1 = 1 \oplus 0 = 1 \oplus v = v \oplus 1 = 1, \;\; 1 \otimes 1 = 1,$$

and in the rest cases 0.

CONSTRUCTION I: Let $(H, \cdot)$ be H_v-group, then for every $(\oplus)$ such that $x \oplus y \supset \{x, y\}$, $\forall x, y \in H$, the $(H, \oplus, \cdot)$ is an H_v-ring. These H_v-rings are called *associated to* $(H, \cdot)$.

In the theory of reps of the hypergroups, in the sense of Marty, there are three types of associated hyperrings $(H, \oplus, \cdot)$ to the hypergroup $(H, \cdot)$. The hyperoperation $(\oplus)$ is defined respectively, for all x, y in H, as follows:

$$\text{type a}: \; x \oplus y = \{x, y\}, \;\; \text{type b}: \; x \oplus y = \beta^*(x) \cup \beta^*(y), \;\; \text{type c}: \; x \oplus y = H.$$

In the above types the strong associativity and strong or inclusion distributivity, is valid.

CONSTRUCTION II: Let $(H, +)$ be H_v-group. Then for every hyperoperation $(\otimes)$ such that $x \otimes y \supset \{x, y\}$, $\forall x, y \in H$, the hyperstructure $(H, +, \otimes)$ is an H_v-ring.

CONSTRUCTION III: Let $(H, +)$ be H_v-group with a scalar zero 0. Then for every $(\otimes)$ such that $x \otimes y \supset \{x, y\}$, $\forall x, y \in H - \{0\}$, $x \otimes 0 = 0 \otimes x = 0$, $\forall x \in H$, the $(H, +, \otimes)$ is an H_v-ring.

In this construction 0 is absorbing scalar but not single.

CONSTRUCTION IV: Let $(H, \cdot)$ be H_v-group. Take a $0 \notin H$ and set $H' = H \cup \{0\}$. We define the hyperoperation $(+)$ as follows: $0+0 = 0$, $0+x = H = x+0$, $x+y = 0$, $\forall x, y \in H$, and we extend $(\cdot)$ in H' by putting $0 \cdot 0 = 0$, $0 \cdot x = x \cdot 0 = 0$, $\forall x, y \in H$. Then $(H', +, \cdot)$ is a reproductive H_v-field with $H'/\gamma^* \cong Z_2$ where 0 is absorbing and single.

Appendix 5.B
Eric Trell's Hyperbiological Structures TO BE COMPLETED AND EDITED.

A new conception of biological systems providing a true advance over rather primitive prior conceptions, has been recently proposed by Erik Trell (see Ref. (164) and contributions quoted therein). It is based on representative blocks which appear in our space to be next to each other, thus forming a cell or an organism, while having in reality hypercorrelations, thus having the structure of hypernumbers, hypermathematics and hyperrelativity, with consequential descriptive capacities immensely beyond those of pre-existing, generally single-valued and reversible biological models. Regrettably, we cannot review Trell's new hyperbiological model to avoid an excessive length, and refer interested readers to the original literature (164).

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Postscript

In the history of science some basic advances in physics have been preceded by basic advances in mathematics, such as Newtons invention of calculus and general relativity relying on Riemannian geometry. In the case of quantum mechanics the scientific revolution presupposed the earlier invention of complex numbers. With new numbers and more powerful mathematics to its disposition, physics could be lifted to explain broader and more complex domains of physical reality.

The recent and ongoing revolution of physics, initiated by Prof. Ruggero Maria Santilli, lifting the discipline from quantum mechanics to hadronic mechanics, is consistent with this pattern, but in a more far-reaching and radical way than earlier liftings of physics made possible from extensions of mathematics.

Santilli realized at an early stage that basic advances in physics required invention of new classes of numbers and more adequate and powerful mathematics stemming from this. His efforts to develop such expansions of mathematics started already in 1967, and this enterprise went on for four decades. Its basic novelties, architecture and fruits are presented in the present volume. During this period a few dozen professional mathematicians world wide have made more or less significant contributions to fill in the new Santilli fields of mathematics, but the honor of discovering these vast new continents and work out their basic topology is Santillis and his alone. These new fields initiated by Santilli made possible realization of so-called Lie-admissible physics. For this achievement Santilli in 1990 received the honor from Estonia Academy of Science of being appointed as mathematician number seven after world war two considered a landmark in the history of algebra.

With regard to Sophus Lie it may be of some interest to note that the Norwegian examiners of his groundbreaking doctoral thesis in 1871 were not able to grasp his work, due to its high degree of novelty and unfamiliarity. However, due to Lie already being highly esteemed among influential contemporary mathematicians at the continent, it was not an option to dismiss his thesis. As in other disciplines, highly acknowledged after Thomas Kuhns publication of The structure of scientific revolutions in 1962, sufficiently novel mathematics implies some paradigmatic challenge. Therefore, it is not strange that some mathematicians and physicists have experienced difficulties taking the paradigmatic leap necessary to grasp the basics of hadronic mathematics or to acknowledge its far-reaching implications. Such a challenge is more demanding when scientific novelty

implies a reconfiguration of conventional basic notions in the discipline. This is, as Kuhn noted, typically easier for younger and more emergent scientific minds.

Until Santilli the number 1 was silently taken for granted as the primary unit of mathematics. However, as noted by mathematical physicist Peter Rowlands at University of Liverpool, the number 1 is already loaded with assumptions, that can be worked out from a lifted and broader mathematical framework. A partial and rough analogy might be linguistics where it is obvious that a universal science of language must be worked out from a level of abstraction that is higher than having to assume the word for mother to be the first word.

Santilli detrivialized the choice of the unit, and invented isomathematics where the crux was the lifting of the conventional multiplicative unit (i.e. conservation of its topological properties) to a matrix isounit with additional arbitrary functional dependence on other needed variables. Then the conventional unit could be described as a projection and deformation from the isounit by the link provided by the so-called isotopic element inverse of the isounit. This represented the creation of a new branch of mathematics sophisticated and flexible enough to treat systems entailing sub-systems with different units, i.e. more complex systems of nature.

Isomathematics proved necessary for the lifting of quantum mechanics to hadronic mechanics. With this new mathematics it was possible to describe extended particles and abandon the point particle simplification of quantum mechanics. This proved highly successful in explaining the strong force by leaving behind the non-linear complexities involved in quantum mechanics struggle to describe the relation between the three baryon quarks in the proton. Isomathematics also provided the mathematical means to explain the neutron as a bound state of a proton and an electron as suggested by Rutherford. By means of isomathematics Santilli was also able to discover the fifth force of nature (in cooperation with Professor Animalu), the contact force inducing total overlap between the wave packets of the two touching electrons constituting the isoelectron. This was the key to understanding hadronic superconductivity which also can take place in fluids and gases, i.e. at really high temperatures. These advances from hadronic mechanics led to a corresponding lifting of quantum chemistry to hadronic chemistry and the discovery of the new chemical species of magnecules with non-valence bounds. Powerful industrial-ecological technology exploiting these theoretical insights was invented by Santilli himself from 1998 on.

Thus, the development of hadronic mathematics by Santilli was not only motivated by making advances in mathematics per se, but also of its potential to facilitate basic advances in physics and beyond. These advances have been shown to be highly successful already. Without the preceding advances in mathematics, the new hadronic technology would not have been around. The mere existence of this technology is sufficient to demonstrate the significance of hadronic math-

ematics. It is interesting to note that the directing of creative mathematics into this path was initiated by a mathematical physicist, not by a pure mathematician. In general this may indicate the particular potential for mathematical advances by relating the mathematics to unsolved basic problems in other disciplines, as well as to real life challenges.

In the history of mathematics it is not so easy to find parallels to the achievements made by Santilli, due to hadronic mathematics representing a radical and general lifting, relegating the previous mathematics to a subclass of isomathematics, in some analogy to taking the step from the Earth to the solar system. However, the universe also includes other solar systems as well as galaxies.

In addition to isonumbers Santilli invented the new and broader class of genonumbers with the possibility of asymmetric genounits for forward vs. backward genofields, and designed to describe and explain irreversibility, characteristic for more complex systems of nature. Quantum mechanical approaches to biological systems never achieved appreciable success, mainly due to being restricted by a basic symmetry and hence reversibility in connected mathematical axioms. It represented an outstanding achievement of theoretical biology when Chris Illert in the mid-1990s was able to find the universal algorithm for growth of sea shells by applying hadronic geometry. Such an achievement was argued not to be possible for more restricted hyperdimensional geometries as for example the Riemannian. This specialist study in conchology was the first striking illustration of the potency as well as necessity of iso- and genomathematics to explain irreversible systems in biology.

Following the lifting from isomathematics to genomathematics, Santilli also established one further lifting, by inventing the new and broader class of hyperstructural numbers or Santilli hypernumbers. Such hypernumbers are multivalued and suitable to describe and explain even more complex systems of nature than possible with genonumbers. Due to its irreversible multivalued structure hypermathematics seems highly promising for specialist advances in fields such as genetics, memetics and communication theory. By the lifting to hypermathematics hadronic mathematics as a whole may be interpreted as a remarkable step forward in the history of mathematics, in the sense of providing the essential and sufficiently advanced and adequate tools for mathematics to expand into disciplines such as anthropology, psychology and sociology. In this way it is possible to imagine some significant bridging between the two cultures of science: the hard and the soft disciplines, and thus amplifying a tendency already represented to some extent by complexity science.

The conventional view of natural scientists has been to regard mathematics as a convenient bag of tools to be applied for their specific purposes. Considering the architecture of hadronic mathematics, this appears more as only half of the truth or one side of the coin. Besides representing powerful new tools to study

nature, hadronic mathematics also manifests with a more intimate and inherent connection to physics (and other disciplines), as well as to Nature itself. In this regard hadronic geometry may be of special interest as an illustration:

Isogeometry provided the new notions of a supra-Euclidean isospace as well as its anti-isomorphic isodual space, and the mathematics to describe projections and deformations of geometrical relations from isospace and its isodual into Euclidean space. However, these appear as more than mere mathematical constructs. Illert showed that the universal growth pattern of sea shells could be found only by looking for it as a trajectory in a hidden isospace, a trajectory which is projected into Euclidean space and thereby manifest as the deformed growth patterns humans observe by their senses. Further, the growth pattern of a certain class of sea shells (with bifurcations) could only be understood from the addition and recognition of four new, non-trivial time categories (predicted to be discovered by hadronic mechanics) which manifest as information jumps back and forth in Euclidean space. With regard to sea shell growth, one of this non-trivial time flows could only be explained as a projection from isodual spacetime. This result was consistent with the physics of hadronic mechanics, analyzing masses at both operator and classical level from considering matter and anti-matter (as well as positive and negative energy) to exist on an equal footing in our universe as a whole and hence with total mass (as well as energy and time) cancelling out as zero for the total universe. To establish a basic physical comprehension of Euclidean space constituted as a balanced combination of matter and antimatter, it was required to develop new mathematics with isonumbers and isodual numbers basically mirroring each other. Later, corresponding anti-isomorphies were achieved for genonumbers and hypernumbers with their respective isoduals.

Thus, there is a striking and intimate correspondence between the isodual architecture of hadronic mathematics and the isodual architecture of hadronic mechanics (as well as of hadronic chemistry and hadronic biology). Considering this, one might claim that the Santilli inventions of new number fields in mathematics represent more than mere inventions or constructs, namely discoveries and reconstructions of an ontological architecture being for real also outside the formal landscapes created by the imagination of mathematics and logic. This opens new horizons for treating profound issues in cosmology and ontology.

One might say that with the rise of hadronic mathematics the line between mathematics and other disciplines has turned more blurred or dotted. In some respect this represents a revisit to the Pythagorean and Platonic foundations of mathematics in the birth of western civilization. Hadronic mathematics has provided much new food for thought and further explorations for philosophers of science and mathematics.

If our civilization is to survive despite its current problems, it seems reasonable to expect Santilli to be honored in future history books not only as a giant in

the general history of science, but also in the specific history of mathematics. Hadronic mathematics provided the necessary fuel for rising scientific revolutions in other hadronic sciences. This is mathematics that matters for the future of our world, and hopefully Santillis extraordinary contributions to mathematics will catch fire among talented and ambitious young mathematicians for further advances to be made. The present mellowed volume ought to serve as an excellent appetizer in this regard.

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Draft dated March 10, 2008

HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

Volume IV: Experimental Verifications, Theoretical Advances and Industrial Applications in Particle Physics, Nuclear Physics and Astrophysics

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This volume is dedicated to the memory of

Don Carlo Borghi

because he found strength in his Christian faith to oppose organized academic, financial and ethnic interests disrupting experiments on the laboratory synthesis of the neutron because contrary to Einsteinian doctrines.

Contents

Foreword

These days, science is playing an ever increasing rle in the lives of each and every one of us. The public is being lectured on climate change by very authoritative sounding people; the problems of the energy requirements of the world as a whole are being discussed quite openly and widely; more and more scientific topics are being discussed openly by people in positions of authority. What is not emphasised, in fact is rarely mentioned, however, is that at the centre of all these various discussions is physics. In the world of science, physics plays a crucial, all-pervading role. If science is viewed as a bicycle wheel, physics forms the hub at the centre; all the other branches of science act as the spokes of the wheel leading outwards from this central hub. In this context, mathematics is the language of physics and must always be subservient to the physics. Chemistry is merely one branch of physics; engineering may be viewed as the practical manifestation of physical principles; physics is seen by all to be playing a bigger and bigger rle in medicine; in biology even, physics is becoming important particularly through the influence of thermodynamic principles, including that of entropy, in the examination of the theory of evolution. Hence, it is certainly not unreasonable to claim an all-pervading influence of physics in science. It must always be remembered, but frequently isnt, that physics is concerned with describing, and gaining an understanding of, the world around us. It follows that any models devised by man to achieve this are only as good as their ability to achieve this goal. Mans models will always be approximate and, therefore, always flawed. It is this which spurred Ruggero Santilli to attempt to extend the theory behind quantum mechanics and relativity when he realised that neither was, in fact, complete as a theory.

The first volume of this two volume set was devoted to the mathematical theory developed by Ruggero Santilli over a period of years in an attempt to make headway with the enormous task he had set for himself, for he had always realised that, to make any progress at all, some new mathematics would need to be developed. Mathematics as a tool of physics will always have a potential to restrict progress in physics since it is a purely manmade tool. Also, if one looks back through history to the likes of Newton and Einstein, it is apparent that each developed or introduced new mathematics in order to proceed with prodigious advances in physics. The mathematics introduced in the first volume might reasonably be considered a separate piece of work to be considered and appreciated in its own right. However, its purpose had always been to provide

a new tool to help us all in our quest to describe our universe and all that it contains. This means making the mathematics subservient to the physics; relegating the mathematics however beautiful it may be in its own right to a place on a spoke of that wheel of science referred to above. Once it takes on this rle, any results obtained theoretically are only as good as their ability to accurately portray physical phenomena. In this second volume, the link up of theory with experimental results and observation is presented. It is for the general scientific community and any other readers of this work to adjudicate on its success or failure but this judgement, which could be so crucial to us all, must be made with open minds.

The areas in which this new work may be applied are varied. At the present time, possibly the most important application might seem to be the prediction of new clean energies. This could help solve the problems of energy supply and atmospheric pollution if the predictions prove correct. Already, however, a new clean energy, magnegas, has been produced and tested independently. This fact alone must lend credence to the theory presented and should surely provide an impetus to moves to examine the other predictions in great detail on a much wider scale. This is especially important since, on the basis of our present scientific knowledge, the only realistic method of fulfilling the worlds energy needs in the not-too-distant future is via nuclear power. As well as offering possible alternatives, the new theory also offers a possible means of dealing with nuclear waste safely. This, one would have thought, would have been something governments throughout the world would have wanted to investigate as a matter of urgency. It is to be hoped that the publication of this book will refocus attention on this vitally important topic and produce the necessary reaction from around the world.

However, the new theory is not restricted in its application to matters of energy resources. For example, it also offers alternative explanations for problems in astrophysics and cosmology. One fascinating aspect of these two areas of intense scientific endeavour is that, although many observations are made, both are subject to theoretical speculation which can never be completely verified or totally disproved because the time scales involved are far too long; for example, no-one lives anywhere near long enough to truly know the full facts concerning the birth, life and death of any star the theory in that case may be beautiful, it may appear to be a reasonable explanation of all we see, but one can never be certain it is absolutely correct. This is another area where open minds are essential. However, Halton Arps observations relating to quasars caused great consternation among conventionally thinking astronomers to the extent that he has become largely ostracised by the astronomical community. It is interesting that Ruggero Santillis work leads to a possible explanation for Arps findings which should not offend those conventional astronomers too much if they view

the ideas with open minds. Again, the same body of work offers an important contribution to the debate surrounding the existence of dark matter and dark energy. This lifes work truly makes contributions to thought in diverse areas of human endeavour and should be examined far more widely than it is.

It is often said that behind every great man there is a great woman. This is true of Ruggero Santilli. It is for history, not me, to label anyone great or not but it is undoubtedly true that he has benefitted from the unswerving support and encouragement of his wife Carla. It is doubtful he would have achieved so much without this seemingly unquestioning devotion. As I wrote earlier, all Ruggero Santillis scientific achievements may be seen to be the result of tremendous teamwork; a team comprising Ruggero himself and Carla Gandiglio in Santilli.

When anyone reaches the end of these two volumes then, and only then, will they be in a position to reflect on the work as a whole and think about coming to a conclusion. As stated previously, the theoretical framework is elegant but it is here to be judged on the basis of its use in physics, since that was the reason for its genesis. View the experimental and observational evidence, as well as the basic theoretical background, with open minds before coming to any final decision. Many, probably the majority, will then regard these two volumes as representing a truly monumental piece of work which deserves dissemination to a much wider circle of people scientists, politicians, the business community, and, most of all, the general populace which ultimately pays for all scientific work, whether successful or not! The general public needs to be aware of all that is on the table for consideration, not simply those little titbits which are released for ulterior motives.

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October 8, 2007

Preface

In Volume I, we have identified the limitations of Einstein relativities and quantum mechanics; in Volume II, we have presented their resolution for antimatter; and in Volume III we have presented a sequence of covering theories for the study of matter and antimatter in conditions of progressively increasing complexity.

The main scope of this Volume IV is the presentation of experimental verifications, theoretical advances and industrial applications in particle physics, nuclear physics, astrophysics and cosmology, with particular reference to the search for new clean energies so much needed by mankind.

In preceding volumes we have initiated the denunciation of disruptions caused by organized academic, financial and ethnic interests on Einsteinian doctrines against their broadening. The understanding of this volume requires the awareness that these organized ascientific interests reach the climax of their misconduct in opposing, disrupting or jeopardizing experiments showing deviations from Einsteinian doctrines.

Such a claim has no credibility for scientists who did not conduct research beyond Einstein and, therefore, did not experience the disruption of their academic and family life by said organized interests, as it was the case for the author and so many other scientists around the world. Hence, it is necessary in this volume to initiate a documentation of what is internationally called nowadays "organized scientific crime," which documentation is presented in the footnotes of this and of the following volume.

In this way, the reader in good faith can see that the most fundamental is an experiment, the bigger the organized opposition against its conduction and that, when experimental measurements showing deviations from Einsteinian doctrines manage to escape the controlling grip by said organized interests, manipulated counter-experiments are often commissioned to reimpose the validity of Einsteinian doctrines via the abuse of academic credibility and public funds.

Since there is so much at stake for mankind, we ask the readers in good faith to defer judgment following the inspection of both, the scientific evidence and the documentation of scientific corruption. Whatever the reality, no reader with a minimum of dignity, let alone serious commitment to human knowledge, can deny that:

1) All experiments currently conducted in physics laboratories around the world are fully aligned with Einsteinian doctrines;

2) The conduction of fundamental experiments that might invalidate Einsteinian doctrines is systematically rejected at physics laboratories around the world in favor of dramatically less significant and immensely more expensive experiments strictly aligned with Einsteinian doctrines; and

3) Experiment establishing deviations from Einsteinian doctrines are discredited via the abuse of academic power rather than in the sole credible scientific way, the rerun of the measurements under an external ethical control.

To prevent becoming an accomplice, the reader in good faith should never forget that the resolution of the increasingly cataclysmic climactic changes in our planet can only be achieved via the surpassing of Einsteinian doctrines (see Volume I). Hence, in the event truly in good faith and truly committed to human dignity and scientific knowledge, the reader is expected to agree that the denunciations of scientific misconducts documented in this volume constitute a true crime against mankind.

Ruggero Maria Santilli
January 9, 2008

Legal Notice

The underwriter Ruggero Maria Santilli states the following:

1) To be the sole person responsible for the content of *Hadronic Mathematics, Mechanics and Chemistry,* Volumes I and II; to be the sole owner of the Copyrights on these two volumes; and to have recorded, beginning with 1992, the copyright ownership of a number of his main contributions in the field.

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3) There are insisting rumors that organized interests in science are waiting or the author's death to initiate premeditated and organized actions for paternity fraud via the known scheme, often used in the past, based on new papers in the field without the identification of the author's paternity, which papers are then quickly quoted as originating papers by pre-set accomplices and the fraud is then accepted by often naive or ignorant followers merely blinded by the academic credibility of the schemers. Members of these rumored rings should be aware that the industrial applications of hadronic mathematics, mechanics and chemistry have already provided sufficient wealth to set up a Paternity Protection Trust solely funded to file lawsuits against immoral academicians attempting paternity fraud, their affiliations and their funding agencies.

This legal notice has been made necessary because, as shown in Section 1.5, the author has been dubbed "the most plagiarized scientist of the 20-th century," as it is the case of the thousands of papers in deformations published without any quotation of their origination by the author in 1967. These, and other attempted paternity frauds, have forced the author to initiate legal action reported in web site [1].

In summary, honest scientists are encouraged to copy, and/or study, and/or criticize, and/or develop, and/or apply the formulations presented in these volumes in any way desired without any need of advance authorization by the copyrights owner, under the sole conditions of implementing standard ethical rules 2A, 2B, 2C. Dishonest academicians, paternity fraud dreamers, and other schemers are warned that legal actions to enforce scientific ethics are already under way [1], and will be continued after the author's death.

In faith

Ruggero Maria Santilli

U. S. Citizen acting under the protection of the First Amendment of the U. S. Constitution guaranteeing freedom of expression particularly when used to contain asocial misconducts.

Tarpon Springs, Florida, U. S. A.

October 11, 2007

[1] International Committee on Scientific Ethics and Accountability
http://www.scientificethics.org

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- The late Paul A. M. Dirac, for supporting in a short but memorable meeting reviewed in Section 6.2.8, nonunitary liftings of his celebrated equation (today known as Dirac-Santilli isotopic, genotopic and hyperstructural equations) for the representation of an electron within the hyperdense medium inside the proton, with particular reference to the development of a new mathematics eliminating the vexing divergencies in particle physics, since Dirac spent his last years in attempting the elimination of divergencies amidst strong opposition by organized interests on quantum chromodynamical theologies;

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Chapter 6

EXPERIMENTAL VERIFICATIONS AND APPLICATIONS IN PARTICLE PHYSICS, ASTROPHYSICS AND COSMOLOGY

6.1 EXPERIMENTAL VERIFICATIONS IN CLASSICAL AND PARTICLE PHYSICS

Foreword

No appraisal of the numerous experimental verifications of hadronic mechanics in all quantitative sciences can be seriously conducted without the joint addressing of the collapse of scientific ethics in the contemporary physics community due to protracted impunity caused by the studious absence of any control at any levels.

Due to this deplorable condition, the preceding version of this volume presented the experimental evidence in the main text and documentation of ethical misconducts in the footnotes, so as to provide readers with the necessary elements to unmask biased dismissals of experimental evidence for personal gains.

All footnotes on the documentation of scientific misconducts have been removed from the present version of this volume, and have been transferred to the Santilli Foundation for their editing in the proper language and uploading in the web site **http://www.santilli-foundation.org**. *No appraisal of the content of this and of the following volume can be serious without a knowledge of the documentation of ethical misconducts at all levels of the physics community presented in the indicated Foundation.*

6.1.1 Introduction

As stated in Volume I, we assume the exact validity of *special relativity,* and *quantum mechanics* for all possible *exterior dynamical problem* as conceived at the beginning of the 20-th century (and thereafter ignored), namely, physical conditions permitting the point-like abstraction of particles, and generally given by particles at large mutual distances and electromagnetic waves propagating in

vacuum hereinafter referred to a universal substratum underlying all events in the universe visible to mankind.

Typical cases of exterior dynamical problems are the propagation of light in vacuum, the structure of the hydrogen atom, particles in particle accelerators, the structure of crystals, and various other systems for which conventional theories are assumed to be exactly valid.

In this chapter, we present a number of experimental evidence in various fields establishing the exact validity of the covering *isorelativity* and *hadronic mechanics* for the more general *interior dynamical problems* as also conceived at the beginning of the 20-th century (and thereafter regrettably ignored), and generally given by physical conditions under which the point-like abstraction of particles is excessivelky approximate, thus requiring a representation of their actual size.

Interior dynamical problems generally occur for mutual distances of particles of the order of the size of their charge distributions and/or wavepackets, dynamics within physiocal media, and othrr problems such as: the propagation of light within transparent physical media; dynamics of particles within physical media opaque to light, thus lacking the central pillar of special relativity, the propagation of light; strong interactions at large, including the structure of hadrons, nuclei and stars; deep inelastic scatterings of hadrons; and other cases of extended, generally nonspherical and deformable particles at mutual distances of the order of 1 $fm = 10^{-13}$ cm or less, in which case we have the partial or total mutual penetration of the wavepackets and/or the charge distributions of particles.

In Volume I, we have established the impossibility for special relativity and quantum mechanics to be exactly valid for interior conditions due to numerous evidence, such as: the absence of a Keplerian structure in the interior of hadrons, nuclei and stars, with consequential impossibility for the Poincaré symmetry being exact; the emergence of nonlinear, nonlocal and nonpotential interactions that are dramatically beyond any possible representation by a Hamiltonian, let alone incompatible with the underlying conventional topology and related mathematics at large; and other evidence.

In reading this chapter, a clear understanding is that the *approximate* validity of special relativity and quantum mechanics for interior dynamical conditions remains beyond scientific doubt. However, as we shall see, the exact representation achieved by the covering isorelativity and hadronic mechanics have far reaching implications, such as: the lack of necessary existence in our spacetime of quarks, neutrinos, dark matter and other conjectures formulated to salvage orthodox doctrines; the conception and industrial development of much needed new clean energies and fuels simply inconceivable with conventional doctrines; and other much needed theoretical, experimental and industrial advances.

Hence, the search for suitable structural generalizations (rather than marginal touches) of special relativity and quantum mechanics is presented in these vol-

umes as a *collegial duty of the mathematical, physical and chemical communities mandated by scientific ethics and accountability in view of the huge societal implications, e.g., for the solutio of the increasingly alarming environmental problems.* Due to the evident complexity of the problems herein addressed, any rejection based on lack of total and absolutre maturity *without the joint proposal of better structural generalizions of Einsteinian doctrines and quantum mechanics,* will be considered sheer scientific corruption because, whether studious or de facto, opposes for persopnal gains advances so much needed bny mankind whose final achievement will predictably require the laborious historical process of trial and error, presentation of advan ces in the only scientifically meaning way, via publications, and their improvement also in the only scientifically possible way, via publications.

6.1.2 Space, the Final Frontier of Knowledge

As it is well known, we would not be able to hear each other's voices without Earth's atmosphere, because sound is a wave that, as such, requires a medium for its existence and propagation. In particular, sound is a *longitudinal wave,* namely, a ware whose oscillations occur in the direction of propagation, thus requiring a compressible medium, as it is the case for our gaseous atmosphere.

Similarly, we would not be able to see each other's faces without the *ether* (also called *aether,* or *space,* or *universal substratum,* or *vacuum*) conceived as a universal medium because light is also a wave, thus equally requiring a medium for its existence and propagation. In particular, light is a *transversal wave,* namely, a wave whose oscillations occur in the direction perpendicular to that of propagation, thus requiring a medium with characteristics similar to very high rigidity due to the very big value of the speed of light.[1]

The elimination of the universal substratum in the physics of the 20-th century is an excellent topic for investigation by ethically sound historians, because it is a clear illustration of how physical evidence is manipulated to fit preferred theories, and how widely manipulations are accepted because science advances by perceived credibility and/or of academic favors, and not solely because on intrinsic scientific truth.

About fifty years ago, the author decided to dedicate his life to "scientific research" intended as the unobstructed, quantitative pursue of new scientific knowledge. As such, the author never did and never will adapt evidence to preferred theories, but always did and always will adapt theories to physical reality.

[1]Contrary to a number of popular views, the transversal character of light excludes the possibility that space is compressible or that it has characteristics similar to that of a liquid. To separate science from philosophical considerations, it should be stressed that no theory on space as a universal medium can be considered scientific unless it permits a quantitative representation of the *transversal* character of light, due to its evident fundamental character.

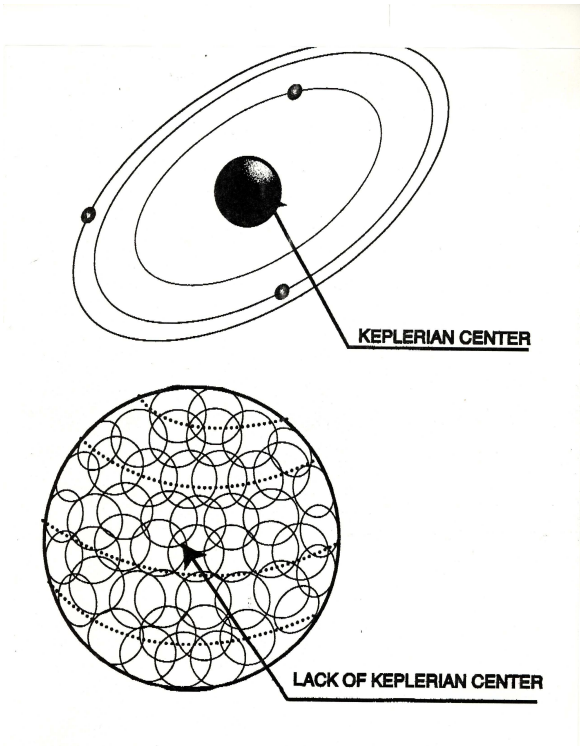

Figure 6.1. A schematic view of one of several impossibilities for special relativity and quantum mechanics as being exactly va;lid for interior dynamical problems. The figure depicts the general lack of a Keplerian structure as well as of a Keplerian center in the transition from a planetary system to the structure of one of its planets, such as Jupiter, with consequential impossibility for the central pillar of special relativity, the Poincaré symmetry, as being exact. When considering operator interior problems such as the structure of hadrons, nuclei and stars, besides the loss of the Keplerian structure, we have the additional impossibility of identifying clearly quantized orbits, thus losing the very notion of a quantum in favor of covering vistas. The theoretical studies conducted over three decades, presented in detail in EHM-I, EHM-II, Volume I and briefly summarized in this chapter, have achieved a covering of the mathematical and physical foundations of special relativity and quantum mechanics permitting an invariant formulation of interior dynamical systems without Keplerian structure and Keplerian center. This volume is dedicated to their experimental verification, theoretical advance, and industrial applications.

Einstein special relativity does not admit an absolute frame of reference, as well known. As equally well known, a universal substratum is perceived as requiring an absolute frame of reference. Consequently, the physics of the 20-th century decided that the universal substratum does not exist because not permitted by Einsteinian doctrines.

The "arguments" used to eliminate the universal substratum should be, per se, reason for investigation by ethically sound historians, because a vivid illustration

on how physics, a discipline intended as being quantitative and objective, is turned into political dogmas.

By leaving details to historians, a first argument for the elimination of the universal substratum was the reduction of light to photons that, as such, propagate like particles, thus not requiring any medium for their existence and propagation. The political character of this "argument," particularly when proffered by experts, is soon unmasked because radio waves with, say, one meter in wavelength, cannot possibly be reduced to photons in any credible way. Yet, the reduction of all electromagnetic waves to photons for the purpose of maintaining the validity of Einsteinian doctrines, was widely accepted during the 20-th century because only its serious scrutiny would case instant "disqualifications" and claims of "fringe science" by organized interests on Einsteinian doctrines.

Another argument used for adapting nature to preferred theories was the so-called *aethereal (or ethereal) wind,* namely, the evidence that Earth encounters no "wind" (that is, no resistance) during its motion through space. Therefore, the universal substratum does not exist, according to this "argument."

The first paper written by the Santilli back in 1956 [1] (when a high school student), was intended to eliminated the aethereal wind and stress the need, not only for a universal medium for the existence and motion of matter, but also for a medium with features similar to high rigidity and extreme energy density (See Ref. [2] for historical accounts).

As it is well known, the electron is a" pure" oscillation with the well known frequency of $1.236 \times 10^{20}\ Hz$,namely, without any oscillating "little mass" or other "little material entity," as proved by Schrödinger in 1935 as being the case for the variable x in Dirac's equation for the electron. This evidence mandates the need for a universal medium because in the structure of the electron, we merely have the *oscillation of a dimensionless point of the universal substratum.*

Ref. [1] indicated that, when the electron moves, it "cannot" experience any "aethereal wind" because we merely move its characteristic oscillation from one point of the aether others. Ref. [54] then suggested that the inertia (from which we compute the mass) is in actuality a tendency of the aether to oppose variations in the propagation of said oscillations.

Paper [1] then suggested that the same occurrence holds for all other elementary and, thus, composite particles. Consequently, the aether as a universal medium is necessary not only for the existence and propagation of electromagnetic waves, but also for the very existence and propagation of all elementary and composite particles and, therefore, of matter as perceived by our senses.

The main conclusion of paper [1] is that, *contrary to our sensory perception, matter is "entirely empty" and space is "entirely full,"* because matter and electromagnetic waves can be entirely reduced to pure oscillations of the aether. To be more specific in this important point, it is generally believed that matter

is "mostly empty," in the sense that, for any material substance, interatomic distances are large and then the distances between electrons and nuclei are proportionately equivalent to planetary distances. The terms "entirely empty" are referred to the fact that, following the reduction of matter to electrons, protons and neutrons, these particles too result to be empty, that is, lacking any material entity, because they are "pure oscillations" of space, that is oscillations of its point without any oscillating material entity.

When initiating his academic life in the late 1960s, the author soon discovered that any mention of the aether as a universal medium would imply instant disqualification and loss of academic jobs due to organized academic interests on Einsteinian doctrines in control of the world wide physics community. Consequently, the author had to abandon his studies of the aether and dedicate himself to other studies.

Nevertheless, *physical veritas* is not established by academic power, but by evidence. No matter how beloved and supported a given theory may be at a given time, no relativity can resist the test of time without a serious addressing of the existence of the universal substratum and its universal reference frame.

In this volume, *we assume that space is a universal medium characterized by the superposition of extremely high equal densities of positive and negative energies* that, according to the isodual theory of antimatter, can coexist because defined in physically different spaces: the conventional space over conventional numbers (with positive unit) for positive energies, and the isodual space on isodual numbers (with negative units) for negative energies (see Chapters 2 and 3 for details on the isodual theory).

It should be indicated that, when studying later on the hyperstructural branch of hadronic mechanics, the existence of matter and antimatter in separate, yet coexisting spaces is only the first example of our hyperstructures. Note that in the physics of the 20-th century, matter and antimatter were conceived as existing in the same space, but this lead to a large scientific imbalance, that matter could be treated at all levels while antimatter could be treated only at the level of second quantization. This imbalance was solved by the isodual theory of antimatter with resulting first hyperstructural character of matter and antimatter, that will be later on expanded for cosmological and other aspects.

In regard to the historical problem of compatibility of any given relativity with space conceived as a universal medium, we assume the pragmatic position that *no material system known to date can possibly identify the absolute reference frame at rest with respect to the universal medium. Hence, all issues pertaining to compatibility with the absolute reference frame are deferred to epistemological studies not contemplated in this volume.*

6.1.3 The Far Reaching Implications of Space as a Universal Medium

Far from claiming final knowledge one way or the other, the position assumed in these volumes is that *the existence of space (or ether) as a universal substratum for all events occurring in the universe is supported by sufficient evidence as being plausible, hence warranting its systematic study, because of implications simply beyond our imagination at this time,* such as:

1) As studied in detail in Section 6.2, the rest energy of the neutron is 0.78 MeV bigger than the sum of the rest energies of the proton and of the electron. As a result, the synthesis of the neutron inside stars, $p^+ + e^- \to n + \nu$, requires a minimum of 0.78 MeV (in which case there is no energy left for the neutrino). Evidently, this "missing energy" can be provided by the environment inside a star. However, due to the extreme density in the core of a star, the proton and the electron are expected to be at rest during said synthesis. It is then possible that the "missing energy" of 0.78 MeV originates from space as a universal medium with high energy density. Alternatively, the old hypothesis of continuous creation of matter in the universe could see its realization in the synthesis of the neutron inside stars, with far reaching implications. At any rate, due to the extremely high number of neutron syntheses occurring in a star every second, each one *requiring* 0.78 MeV energy, the idea of a star with *decreasing* energy is unappealing, thus mandating alternative studies. In the event the neutron is indeed a mechanism set by nature to extract energy from the ether, the possibilities for mankind are simply beyond imagination. Hence, the understanding of these volumes requires the knowledge that *hadronic mechanics is the first and only known theory permitting quantitative and invariant studies of the possible interplay between matter and the universal substratum.*

2) As we shall see, quantum mechanics is inapplicable for the neutron synthesis $p^+ + e^- \to n + \nu$ because the Schrödinger equation fails to provide physical solutions for "positive binding energies.". The non-expert reader is encouraged to verify this occurrence by attempting to solve any quantum bound state in which the usual "negative" potential is turned into a "positive" value. In fact, all physically consistent, quantum bound states (such as nuclei, atoms and molecules) have a "negative" binding energy. Hadronic mechanics was proposed by the author in memoir [14] of 1978 precisely for the achievement of a quantitative representation of the synthesis of the neutron inside stars from protons and electrons. This objective was achieved in its entirety with the numerically exact and time invariant representation of *all* characteristics of the neutron as a hadronic bound state of a proton and an electron, without any need of hypothetical quarks. In turn, the restricted of quarks as they are technically defined (purely mathematical quantities outside our spacetime for the elaboration of unitary symmetries), and the replacement of hadrons with physical constituents that can be produced free, cre-

ate far reaching possibilities for basically new *hadronic energies,* namely, energies originating from mechanisms in the interior of individual hadrons, rather than their collection. At any rate, the current quark theologies and related Quantum ChromoDynamics (QCD) imp[ly that the proton and the electron simply "disappear" at the time of the neutron synthesis to please organized interests in the field and, then, the proton and the electron "reappear" at the time of the neutron decay. These theologies have always been repugnant for Santilli, and they will always remain so, because pushing what is expected to be serious science immensely beyond any level of credibility, while opposing, disrupting and jeopardizing dissident view for personal gains.

3) When compared to interstellar distances, contemporary communications via electromagnetic waves can be compared to the communications with smoke signals during prehistoric times, evidently due to interstellar distances rendering the speed of light excessively small. Hence, serious studies on future interstellar communications require the search for *new* communications with a speed dramatically bigger than that of light, among which, the first possibility is the conception, quantitative treatment, and subsequent realization of *longitudinal waves propagating through the ether as a universal medium.* In fact, due to the very high rigidity of the universal substratum requested to represent the speed of transversal waves, longitudinal waves are predicted to propagate in space with speeds millions of times bigger than the speed of light. As well known, longitudinal waves are not predicted by Einstein special relativity (because not admitted by Maxwell's electrodynamics). However, the dismissal of the possible existence of longitudinal waves in space just because not predicted by Einsteinian doctrine is purely political and such a dismissal should itself be dismissed because nonscientific. Intriguingly, this possibility of fundamentally new form of longitudinal communications occurs if and only if neutrinos do not exist as physical particles in out spacetime, and their current "detection" is replaced precisely by longitudinal impulses. More specifically, the alternative hypothesis, called *etherino* by Santilli is that, at the time of its decay, rather than emitting a hypothetical neutrino, the neutron creates a longitudinal impulse through the ether (see Section 6.2 for details) that is currently interpreted as a particle in current experiments. The resolution of this possibility will evidently require centuries. At this point we merely indicate that the replacement of neutrinos as hypothetical physical particles with longitudinal impulses propagating though spaces witrhout any propagation of ordinary mass or energy, eliminate the current theology requested by QCD that *neutrino, nowadays assumed to have mass, can propagate throughout entire stars and galaxies without any collision at all!.* This theology has always been repugnant to Santilli and it will always remain so because, again, turning supposedly serious science dramatically beyond any level of plausibility for personal gains.

4) As indicated above, space is emerging as possessing an energy density beyond our imagination, to the extent that one cubic centimeter of space may contain more energy than that of the entire Sun. The isodual theory has established that negative energy exist in a spacetime different, yet coexisting, with that of positive energies. Hence, the isodual theory implies that space may be characterized by a a superposition of extreme equal values of positive and negative energies, with far reaching implications, such as the elimination of discontinuities at creation of the universe, the elimination of the very meaning of the search of the "age of the universe",[2] and other implications.

5) In the 20-th century, famous scientists claimed that it would be impossible for mankind to go to the ,moon and return safely. Scientific and technological advances proved them wrong. Nowadays, other scientists are on record with the claim that mankind will never travel to far away stars, and return safely to Earth, due to extreme distances. The claim is based, again, on the tacit assumption of the universal validity of Einsteinian doctrines and it is "justified" not only on ground of the time required for such a travel, but also for the need of a fuel tank as big as the entire solar system. When passing to serious science, Einstein doctrines must be assumed to have their own limitations, in which case a number of possibilities emerge as conceivable already at the current primitive stage of our scientific evolution. After all, the science fictionof a given time has been surpassed by subsequent scientific advances. With the clear understanding that serious scientific studies on interstellar travel may well require the entire third millennium, the possible existence of space as a universal medium of the above type resolves, at this time on purely *mathematical* grounds, all the above objections. In fact, the above conception of space as a universal medium of extremely high equal amount of positive and negative energies allows the *spacetime isogeometric locomotion* studied in Chapter 14 for which: a) there is no need for any "fuel tank" at all since the needed fuel could be extracted from space via mechanisms similar to that for the neutron synthesis or other yet unknown means; b) there is no limitation to speeds because the locomotion is not Newtonian, namely, without action and reaction, and occurs via a control of distances predicted by isogeometries to have unlimited speeds; and c) motion is necessarily in both space and time, since any deformation of the former requires that of the latter, and vice-versa.

It is hoped that, besides the desire of stimulating young minds of any age, the above comments illustrates a main viewpoint conveyed in these volumes: *rather than having reached final character as proffered by political interests on Einsteinian doctrines, studies on relativity laws are at their infancy, and so much remains yet to be discovered.*

[2]Because the "total age" of mater and (isodual) antimatter is zero.

6.1.4 Rudiments of Santilli Isorelativity

For minimal self-sufficiency of this volume, let us recall that special relativity and relativistic quantum mechanics are based on the "universal constancy of the speed of light" c_o that is achieved via the invariance of the line element in the *Minkowskian spacetime* $M(x, \eta, R)$ (Section I.3.5.3)

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I =$$
$$= (x^1 \times x^1 + x^2 \times x^2 + x^3 \times x^3 - x^4 \times x^4) \times I \in R,$$
$$x^4 = c_o \times t, \quad I = Diag.(1, 1, 1, 1), \tag{6.1.1}$$

under the celebrated *Lorentz symmetry* $O(3.1)$ characterized by the *Lorentz transformations* here expressed for simplicity in the $(3, 4)$ coordinates

$$x^{1'} = x^1, \quad x^{2'} = x^2, \tag{6.1.2a}$$

$$x^{3'} = \gamma \times (x^3 - \beta \times x^4), \ \ x^{4'} = \gamma \times (x^4 - \beta \times x^3), \tag{6.1.2b}$$

$$\gamma = (1 - \beta^2)^{-1/2}, \quad \beta = v^2/c_o^2, \tag{6.1.2c}$$

where: $\times$ is the conventional associative product; $+$ is the conventional sum; $I = Diag.(1, 1, 1, 1)$ is the fundamental unit of the Lorentz symmetry $O(3.1)$; for consistency, I is assumed as the unit of the base field of real numbers $R = R(n, +, \times)$; and the multiplication of the line element by I is then necessary for x^2 to be an element of the assumed base field.

However, the "universal constancy of the speed of light" is a manipulation of scientific reality, particularly when ventured by experts, whenever said statement is proffered without the crucial addition "in vacuum." In fact, the "universal constancy of the speed of light in vacuum" (namely, in exterior conditions), has been experimentally established beyond scientific or otherwise useful doubt. When this statement is contracted into "universal constancy of the speed of light" it is referred to all possible conditions existing in the universe, including interior conditions. In the latter case, not only we have no experimental evidence at all, but have robust evidence on the lack of constancy of the speed of light. Hence, when experts venture the statement of the "universal constancy of the speed of light" without the crucial specification "in vacuum," they perpetrate a manipulation of science intended to extend the validity of special relativity to all possible conditions existing in the universe.

For all cases of interior dynamical problems within a physical medium, experimental evidence establishes that *the speed of light c is a local variable* depending on the density d, temperature τ, frequency ω, and other characteristics of the medium considered, $c = c(d, \tau, \omega, ...)$, as expressed by the historical form studied in high school

$$c = c(d, \tau, \omega, ...) = \frac{c_o}{n} = \frac{c_o}{n(d, \tau, \omega, ...)}. \tag{6.1.3}$$

Organized interests on Einsteinian doctrines have attempted to dismiss the local character of the speed of light via the reduction of light to photons scattering among atoms, in which case photons propagate in vacuum, hence at the speed c_o. In Section I.1, we have shown the nonscientific character of this claim on various grounds, such as: the impossibility of reducing to photons electromagnetic waves with one meter wavelength; the inability of the reduction to photons for speeds bigger than c_o nowadays experimentally established beyond credible doubt (see Section 6.1.7); the collapse of the axioms of special relativity even for the simple case of propagation of light in water, due to either the violation of causality (because ordinary electrons can propagate in water at speeds bigger than the local speed $c = 2 \times c_o/3$) or the violation of the axiom of relativistic sums of speeds of light; and other evidence.

The only possible scientific statement is that *special relativity and, consequently, relativistic quantum mechanics, are inapplicable (rather than violated) for interior dynamical systems* because not conceived for them. To prevent exiting from the boundaries of science, the broader relativity and related mechanics can indeed be subjected to scientific debates, but not their need.

To the author's best knowledge, the first studies on the invariance of the locally varying character of the speed of light were conducted by Lorentz [3] in 1895 via Eq. (6.1.3). These studies were ignored throughout the 20-th century evidently because not aligned with organized interests on special relativity, although Lorentz studies [1] did not escape Pauli's attention who quoted them in a footnote of his book [93].

Unfortunately, Lorentz failed to achieve the invariance of $c = c_o/n(d, \tau, \omega, ...)$ and was forced to study the simpler case $c = c_o =$ constant in which case he did achieve the historical symmetry transformations (6.1.2).

The author has dedicated his research life to Lorentz's legacy [3] via decades of laborious studies reported in Volume I (as well as in the preceding volumes EHM-I and II). In essence, it emerged already at the time of the author's graduate studies in physics of the late 1960s that *Lorentz failed to achieve the invariance of the locally varying speed of light because the mathematics he used, Lie's theory, was indeed effective for the case of $c = c_o =$ constant, but basically insufficient for the broader case $c = c_o/n(d, \tau, \omega, ...)$.*

Hence, the author dedicated his efforts, firstly, to a structural generalization (called *lifting*) of Lie's theory, today known as the *Lie-Santilli iso-, geno- and hyper theory* for closed single-valued, open single-valued, and open multi-valued conditions of matter and their *isoduals* for antimatter (see Volume I for a review and EHM-I and II for detailed studies).

In particular, the author discovered that invariance was achieved if and only if any structural generalization of Lie's theory was formulated via a compatible lifting of the *totality* of the underlying mathematics, including numbers, prod-

ucts, fields, spaces, topologies, functional analysis, differential calculus, etc. The resulting new formulations are today known as *Santilli iso-, geno-, and hyper-mathematics* for matter and their *isoduals* for antimatter.

As now familiar in the field, these broader mathematics are based on the lifting of the basic unit of Lorentz symmetry, $I = Diag.(1, 1, 1, 1)$, into the most general possible units $\hat{I}, \hat{I}^{>}, \hat{I}^{\{>\}}$, called *Santilli iso-, geno- and hyper-units,* respectively, with compatible lifting of the product and of the entire conventional mathematics.

By ignoring to avoid excessive complexities the open, irreversible, single-valued case (used for the invariance of light during its absorption) and the open, irreversible, multi-valued case (used for biological processes), we here briefly outline for self-sufficiency the main lines of the isotopic lifting of the Lorentz symmetry.

The transition from the Minkowski metric for the propagation of light in vacuum, $\eta = Diag.(1, 1, 1, -c_o^2)$, to the generalized Minkowski-Santilli isometric for the propagation of light within transparent physical media, $\hat{\eta} = Diag.(1, 1, 1, -c^2)$, $c = c_o/n$ is a necessarily *noncanonical transformation* at the classical level or a *nonunitary transformation* at the operator level,

$$\eta = (1, 1, 1, -c_o^2) \to \hat{\eta} = Diag.(1, 1, 1, -c_o^2/n^2) = Z \times \eta \times Z^\dagger, \tag{6.1.4a}$$

$$Z = Diag(1, 1, 1, i/n), \quad Z \times Z^\dagger \neq I. \tag{6.1.4b}$$

The use of rotations and Lorentz transforms then yields a lifting of all remaining components of the isometric. The Lie-Santilli isotheory is constructed by applying, for reasons clarified below, the inverse of the metric transform to the totality of the mathematics underlying Lie's theory, resulting in expressions of the type

$$U \times U^\dagger = (Z \times Z^\dagger)^{-1} = Diag.(1/b_1^2, 1/b_2^2, 1/b_3^2, 1/b_4^2), =$$
$$= Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \tag{6.1.5a}$$

$$I \to \hat{I} = U \times I \times U^\dagger = Diag.(1/b_1^2, 1/b_2^2, 1/b_3^2, 1/b_4^2) = Diag.(n_1^2, n_2^2, n_3^2, n_4^2),$$
$$n_\alpha = n_\alpha(\mu, \tau, \omega, ...), n_4 = n, \tag{6.1.5b}$$

$$n \in R \to \hat{n} = U \times n \times U^\dagger = n \times (U \times U^\dagger) = n \times \hat{I} \in \hat{R}, \tag{6,1.5c}$$

$$n \times m \to \hat{n}\hat{\times}\hat{m} = U \times (n \times m) \times U^\dagger = \hat{n} \times \hat{T} \times \hat{m}, \quad \hat{T} = 1/U \times U^\dagger, \tag{6,1.5d}$$

$$[X_i, X_j] = X_i \times X_j - X_j \times X_i \to [\hat{X}_i\hat{,}\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i = U \times [X_i, X_j] \times U^\dagger, \tag{6.1.5e}$$

$$e^X \to \hat{e}^{\hat{X}} = U \times (e^X) \times U^\dagger = (e^{X \times \hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T} \times X}), \ \ etc. \tag{6.1.5f}$$

The invariance under additional nonunitary transforms is assured, provided that it is studied within the context of *isomathematics* and not that of conventional mathematics. This requires the identical reformulation of a given nonunitary transform into the *isounitary transform*,

$$W \times W^\dagger \neq I, \quad W = \hat{W} \times \hat{T}^{1/2}, \quad W \times W^\dagger \equiv \hat{W}\hat{\times}\hat{W}^{\hat{\dagger}} = \hat{W}^{\hat{\dagger}} \times \hat{W} = \hat{I}, \tag{6.1.6}$$

under which we have the invariance laws

$$\hat{I} \to \hat{W}\hat{\times}\hat{I}\hat{\times}\hat{W}^{\hat{\dagger}} \equiv \hat{I}, \tag{6.1.7a}$$

$$\hat{X}_i\hat{\times}\hat{X}_j \to \hat{W}\hat{\times}(\hat{X}_i\hat{\times}\hat{X}_j)\hat{\times}\hat{W}^{\hat{\dagger}} = \hat{X}'_i \times \hat{T} \times \hat{X}'_j = \hat{X}'_i\hat{\times}\hat{X}'_j, \quad etc. \tag{6.1.7b}$$

from which all other invariances follow. Note the *invariance of the numerical value of the isounit* $\hat{I}$ *and of the isoproduct represented by the numerical invariance of* $\hat{T}$.

The application of the above formalism to the invariance of locally varying speeds of light was achieved for the first time by R. M. Santilli in paper [4a] of 1983 at the classical level and in paper [4b] of the same year for the operator counterpart, to be studied in detail in subsequent papers [5] and additional ones. These studies achieved the invariance of the following universal *isoline isoelement* on the *Minkowski-Santilli isospace* $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$

$$\hat{x}^{\hat{2}} = \hat{x}^\mu\hat{\times}\hat{\eta}_{\mu\nu}\hat{\times}\hat{x}^\nu = [x^\mu \times \hat{\eta}_{\mu\nu}(x, d, \tau, \omega, ...) \times x^\nu] \times \hat{I} =$$

$$= [x^\mu \times g_{\mu\nu}(x, d, \tau, \omega, ...) \times x^\nu] \times \hat{I} = [x^\mu \times \hat{T}^\rho_\mu(x, d, \tau, \omega, ...) \times \eta_{\rho\nu} \times x^\nu] \times \hat{I} =$$

$$= (x^1 \times x^1/n_1^2 + x^2 \times x^2/n_2^2 + x^3 \times x^3/n_3^2 - x^4 \times x^4/n_4^2) \times \hat{I} =$$

$$= (x^1 \times x^1 \times b_1^2 + x^2 \times x^2 \times b_2^2 + x^3 \times x^3 \times b_3^2 - x^4 \times x^4 \times b_4^2) \times \hat{I} \quad \in \hat{R}, \tag{6.1.8}$$

where the n's or the b's are called the *characteristic quantities* of the medium considered and they are normalized to the corresponding values in vacuum, i.e. in vacuum we have for the index of refraction $n_4 = 1/b_4 = 1$ for which $c = c_o$, and the space components are normalized to the value of the perfect sphere (the unit of the Euclidean geometry), $n_1 = n_2 = n_3 = 1/b_1 = 1/b_2 = 1/b_3 = 1$. Note that for mathematical rigor, we should have used in Eqs. (6.1.8) the *isoquotient* $\hat{/} = / \times \hat{I}$ and all characteristic quantities should have been *isonumbers,* e.g., $\hat{n}_\alpha = n_\alpha \times \hat{I}$, resulting in the simplification used in the preceding isoelement $\hat{/}\hat{n}_\alpha = /n_\alpha$.

It should be noted that *the characteristic quantities provide a direct geometrization (that is, a geometrization via the isometric) of the deviation from the Minkowskian geometry for the vacuum caused by physical media.* Hence, the characteristic quantities $b_k = 1/n_k$, $k = 1, 2, 3$, characterize the geometric deviations from the Euclidean space for the motion of extended particle or electromagnetic waves within physical media, while the quantity $b_4 = 1/n_4$ characterizes the deviation from the Minkowskian time.

Note the *direct universality* of the isoline (6.1.8) in the sense that it includes as particular cases all possible line elements with signature $(+, +, +, -)$, thus including the Minkowskian, Riemannian, Finslerian, and any other possible line

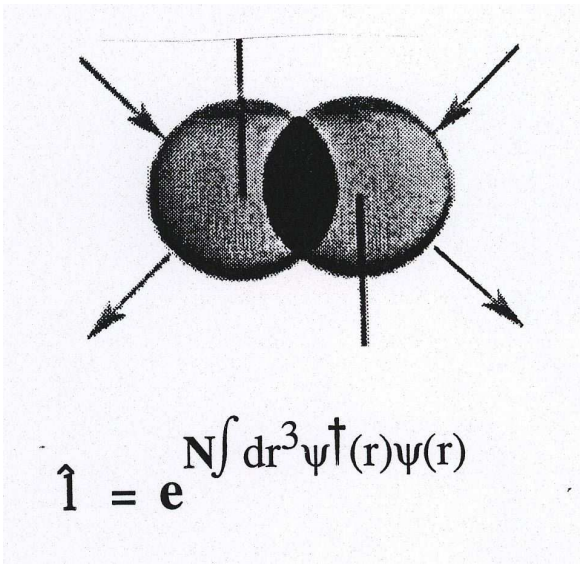

Figure 6.2. A schematic view of the *new interactions* studied in these volumes: the contact, zero-range, nonlinear, nonlocal and nonpotential interactions typical of all interior dynamical problems originating in deep mutual penetration of the wavepackets and/or charge distributions of particles as occurring in the hadronic structure, inelastic scattering, electron valence bonds, and numerous other events. Special relativity and quantum mechanics can only represent *dimensionless point-like particles,* as expected as being admitted by experts to qualify as such. Additionally, their Hamiltonians can only represent action-at-a-distance interactions derivable from a potential. Consequently, it was popularly believed throughout the 20-th century that the interactions herein considered do not exist resulting in a plethora of assumptions, insufficiencies or inconsistencies studied in details in Chapter I.1. The studies reported in these volumes required decades of research because of the difficulties, not only in representing interactions outside the capabilities of the Hamiltonian, but also achieving their *invariant representation,* i.e., a representation that would not change over time and other symmetry transformations. Following numerous trials and errors, the only consistent solution identified by the author is the representation of all non-Hamiltonian interactions and effects with a generalization of the basic unit, today known as *Santilli isounit* because the unit is the most fundamental invariant of all theories.

elements. Such a universality is said to be direct because it occurs in the space-time of the experimenter without any need for coordinate transforms. Note that, also for simplicity, we have used the *diagonal* form of the isoline isoelement. For the general nondiagonal form the interested reader may study EHM-II.

Systematic studies were conducted by Santilli on the invariance of universal line element (6.1.8), via the isotopies of: Lorentz symmetry [4a,4b]; rotational symmetry [5a,5b]; SU(2)-spin symmetry [5c,5d]; Poincaré symmetry [5e,5f]; and spinorial covering of the Poincaré symmetry [5g] (see monographs [6] for a com-

prehensive study as of 1991, and EHM, Vols. I and II, as well as Volume I of this series for details).

All preceding efforts were re-examined and further developed in paper [96] for the relativistic structure model of the neutron as a hadronic bound state of a proton and an electron studied in detail in Section 6.2.8. The most effective way to learn these advances is within the context of a specific application. Hence, we defer their treatment to Section 6.2.8 and limit ourselves here to quote the following *Lorentz-Santilliisotransformations* in the $(3, 4)$ plane (see EHM-II for the general case) that are at the foundation of these entire two volumes

$$x^{1'} = x^1, \quad x^{2'} = x^2, \tag{6.1.9a}$$

$$x^{3'} = \hat{\gamma} \times (x^3 - \frac{n_3}{n_4} \times \hat{\beta} \times x^4) = \hat{\gamma} \times (x^3 - \frac{b_4}{b_3} \times \hat{\beta} \times x^4), \tag{6.1.9b}$$

$$x^{4'} = \hat{\gamma} \times (x^4 - \frac{n_4}{n_3} \times \hat{\beta} \times x^3) = \hat{\gamma} \times (x^4 - \frac{b_3}{b_4} \times \hat{\beta} \times x^3), \tag{6.1.9c}$$

$$\hat{\gamma} = (1 - \hat{\beta}^2)^{-1/2}, \quad \hat{\beta} = v \times b_3/c_o \times b_4 = v \times n_4/c_o \times n_3, \tag{6, 1.9d}$$

$$\hat{I} = Diag.(1/b_1^2, 1/b_2^2, 1/b_3^2, 1/bs_4^2) = Diag.(n_1^2, n_2^2, n_3^2, n_4^2), \tag{6.1.9r}$$

formulated on ordinary space, rather than isospace, for simplicity.

Note that, by conception and construction, the Lorentz-Santilli isosymmetry is locally isomorphic to the conventional Lorentz symmetry, $\hat{O}(3.1) \approx O(3.1)$. Hence, the author introduced the word "isotopies" to denote, in the Greek meaning of the word, the preservation of the original axioms.

An important property, also discovered by R. M. Santilli [5], is that, contrary to popular beliefs, *the Lorentz symmetry is seven and not six dimensional.* This is due to the new *isotopic invariance* here expressed for a constant number $z \in R$

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \equiv [x^\mu \times (z^2 \times \eta_{\mu\nu}) \times x^\nu] \times (z^{-2} \times I) = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I}. \tag{6.1.10}$$

As we shall see, and as expected for any new invariance in our spacetime, the novel invariance (6.1.10) carries fundamental implications at all levels of study, from particle physics to cosmology, including far reaching advances such as the first known axiomatically consistent grand unification of electroweak and gravitational interactions studied in Chapter 14.

The fact that the new isoinvariance (6.1.10) remained un-noticed throughout the 20-th century until identified in Ref. [5] should not be surprising because its identification required the prior discovery of *new numbers,* Santilli's isonumbers with arbitrary positive-definite unit $\hat{I}$.

From now on we shall use the following terminology: the use of conventional terms, such as speed, mass, energy, etc., eill denote conventional quantities defined on the conventional Minkowski space over the conventional field of real

numbers. Terms such as *isospeed, isomass, isoenergy, etc.* will denote quantities defined on the Minkowski-Santilli isospace over the isofield of real numbers.

Santilli isorelativity (see Volume I as well as monographs [6] (as well as EHM-II and HM-I) and original references quoted therein) is based on the Poincaré-Santilli isosymmetry and the following isoaxioms (see Section I.3.5 for details):

ISOAXIOM I. The projection in our spacetime of the maximal causal invariant speed is given by:

$$V_{max} = c_o \times \frac{b_4}{b_3} = c_o \times \frac{n_3}{n_4} = \frac{c}{b_3} = c \times n_3 = c_o \times \frac{g_{44}^{1/2}}{g_{33}^{1/2}}. \tag{6.1.11}$$

ISOAXIOM II. The projection in our spacetime of the isorelativistic addition of speeds within physical media is given by:

$$v_{tot} = \frac{v_1 + v_2}{1 + \frac{v_1 \times b_3^2 \times v_2}{c_o \times b_4^2 \times c_o}} = \frac{v_1 + v_2}{1 + \frac{v_1 \times n_4^2 \times v_2}{c_o \times n_3^2 \times c_o}} = \frac{v_1 + v_2}{1 + \frac{v_1 \times g_{33} \times v_2}{c_o \times g_{44} \times c_o}}. \tag{6.1.12}$$

ISOAXIOM III. The projection in our spacetime of the isorelativistic laws of dilation of time $t_\circ$, contraction of length $\ell_\circ$ and variation of mass $m_\circ$ with speed are given respectively by:

$$t = \hat{\gamma} \times t_\circ, \tag{6.1.13a}$$

$$\ell = \hat{\gamma}^{-1} \times \ell_\circ, \tag{6.1.13b}$$

$$m = \hat{\gamma} \times m_\circ. \tag{6.1.13c}$$

ISOAXIOM IV. The projection in our spacetime of the Doppler-Santilli isolaw is given by the law (here formulated for simplicity for 90° angle of aberration):

$$\omega = \omega_o \times \frac{1 - \hat{\beta} \times \hat{cos\theta}}{\sqrt{1 - hatbeta^2}}, \tag{6.1.14}$$

ISOAXIOM V. The projection in our spacetime of the isorelativistic law of equivalence of mass and energy is given by:[3]

$$E = m \times V_{max}^2 = m \times c_o^2 \times \frac{b_4^2}{b_3^2} = m \times c_o^2 \times \frac{n_3^2}{n_4^2}. \tag{6.1.15}$$

[3]As indicated in Section 3.5, the initial formulation of Isoaxiom V was

$$E = m \times c^2 = m \times c_o^2 \times b_4^2 = \frac{m \times c_o^2}{n_4^2}.$$

However, experimental verifications of isorelativity proved this formulation to be wrong, and had to be replaced with isolaw (6.1.15). The occurrence reinforced the view that, contrary to popular beliefs in the 20-th century, *the speed of light is not, in general, the maxima. causal speed because physical media are generally opaque lo light, in which case the use of the speed of light has no mathematical or physical meaning.* It happens that in vacuum $b_3 = b_4 = 1$ and in water $b_3 = b_4$, in which case $V_{max} = c_o$, but this is a mere particular case without universal validity.

In the above isoaxioms we have

$$\hat{\beta} = v \times b_3/c_o \times b_4 = v \times n_4/c_o \times n_3 = v/V_{max}, \hat{cos\theta} = cos(\theta \times b_s) \qquad (6.1.16)$$

Since v is always smaller than or equal to the maximal causal speed V_{max}, $\hat{\beta}$ is always smaller than or equal to one and $\hat{\gamma} = (1 - \hat{\beta}^2)^{-1/2}$ cannot take imaginary values as it is the case for special relativity. For isotrigonometric functions, we refer the reader for brevity to EHM Vol. I. For detailed studies of the iso-Doppler law, one may consult EHM Vol./ II, Section 8.5.F.

Note that *the isoaxioms are not isotopies of the corresponding axioms of special relativity, because they characterize major structural departures, such as; the the maximal local speed is not, in general, the speed of light; the energy equivalence is not given by the familiar expression* $E = m \times c^2$*; etc.* These structural deviations emerge only within physical media and have major implications we shall study later on, such as the elimination of any need for dark matter..

As we shall see, these deviations are requested by experimental evidence. For instance, in the event the maximal causal speed would be the local speed of light, isorelativity would be violated by water where ordinary electrons can propagate faster than the local speed of light. On the contrary, water is homogeneous and isotropic. Consequently, for water we have $b_3 = b_4$ and the maximal causal speed *in water* is given by the speed of light *in vacuum.* In this case isorelativity verifies causality laws because ordinary electrons travels in water at speeds smaller then the local maximal causal speed. The other axioms are equally verified, such as the isorelativistic sum of speeds (see Section 6.1.7 for details).

The above structural deviations from special relativity can be understood by noting that the main meaning of the new isoaxioms is isogeometrical. Recall that the isotopies reconstruct on isospaces over isofield "all" original axioms identically. For instance, the isoimage of an hyperboloid is the perfect isosphere, the isoimage of the deformation of the light cone caused by variable speeds of light is the perfect light isocone, etc. These exact reconstructions are, evidently, at the foundations of the reconstruction of exact spacetime and internal symmetries when popularly believed as being broken due to the use of excessively elementary mathematics.

The mechanism of achieving this reconstruction is given by the lifting of any given physical quantity, say, $v^2 \to v^2 \times b_3^2$ while the corresponding unit is lifted of the *inverse* amount, $I = 1 \to \hat{I} = 1/b_3^2$. The exact reconstruction then follows from isoinvariance (6.1.10).

By the same argument, *the isotopic image of all physical media is given by the perfect isovacuum, that is, the vacuum referred to the Minkowski-Santilli isospace lover isofields.* In fact, the maximal causal speed on isospaces over isofields is the speed of light *in vacuum,* otherwise the Lorentz-Santilli isosymmetry could not be isomorphic to the conventional Lorentz symmetry.

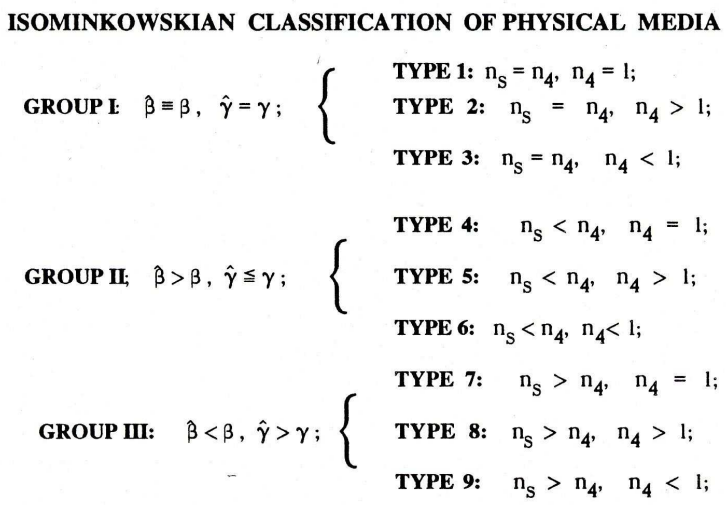

Figure 6.3. A view of the classification of physical media permitted by the Minkowski-Santilli isogeometry first proposed in Ref. [6] of 1991 (see also Figure 8.5.1 of EHM-II and Ref. [63]).

Under such isogeometrization of physical media, the projection in our space-time of the maximal causal isospeed is not the local speed of light $c = c_o \times b_4$ but instead it is given by the maximal causal speed $V_{max} = c_o \times b_4/b_3$, since isotopies preserve the axiomatic character, the speed of light being an ordinary locally variable quantity under isotopy.

The Minkowski-Santilli isogeometry permits an important classification of physical media (see Figure 6.3) under the following basic characterizations:

1) Spherical symmetry is represented which $b_k = b_s = 1/n_s = 1/n_s$, $k = 1, 2, 3$, normalized to the value $b_s = n_s = 1$ for the vacuum. Alternatively, n_s can be given in certain cases by the average of the $n_k, k = 1, 2, 3$.

2) The first direct geometric representation known to the author ("direct" because done directly with the metric) of the *density* of the medium considered is done with $b_4 = 1/n_4$ also normalized to the value $b_4 = n_4 = 1$ for the vacuum;

3) The direct geometric representation of the general *inhomogeneity* of the medium is done via a dependence of the characteristic quantities on the local radial distance r and other variables, $b_s = b_s(r, ...) = 1/n_s(r, ...)$. Such a local variations can be averaged to constants for simplicity.

4) The direct geometric representation of the general *anisotropy* is done via a difference between the space and time characteristic quantities, $b_s \neq b_4$, $n_s \neq n_4$.

5) The direct geometric representation of the *locally varying speed of light*, the maximal causal speed and the other features of isorelativity are done via Isoaxioms I to V.

The above characterizations provide the following *classical iso-Minkowskian classification of physical media* first presented in Ref. [6] of 1991, Section IV-10 (see also Ref. [63] and EHM II):

GROUP I: characterized by $n_s = n_4,\ n_4 = 1, > 1, < 1$.
These media possess the same homogeneity and isotropy of space (vacuum).

GROUP II: characterized by $n_s < n_4,\ n_4 = 1, > 1, < 1$.
These media are inhomogeneous and isotropic with low density.

GROUP III: characterized by $n_s > n_4,\ n_4 = 1, > 1, < 1$.
These media are inhomogeneous and anisotropic with high density.

GROUP I, TYPE 1: $n_s = n_4,\ n_4 = 1,\ \hat{\beta} = \beta,\ \hat{\gamma} = \gamma,\ c = c_o,\ V_{max} = c_o,\ V_{max} = c,\ V_{max} = c$.
This case represents empty space (vacuum);

GROUP I, TYPE 2: $n_s = n_4,\ n_4 > 1,\ \hat{\beta} = \beta,\ \hat{\gamma} = \gamma,\ c < c_o,\ V_{max} = c_o,\ V_{max} > c$.
These homogeneous and isotropic media originate from the isotopic invariance of the line element, Eq. (6.1.10), for $z^2 < 1$; they are transparent to light (because $V_{max} > c$); and they represent ordinary homogeneous and isotropic media such as water, or transparent liquids in general.

GROUP I, TYPE 3: $n_s = n_4,\ n_4 < 1,\ \hat{\beta} = \beta,\ \hat{\gamma} = \gamma,\ c > c_o,\ V_{max} = c_o,\ V_{max} < c$,
These homogeneous and isotropic media also originate from isotopic invariance (6.1.10) for $z^2 > 1$, and they constitute the *new media* predicted by isorelativity. A possible candidate is given by *superconductors,* as studied in Chapter 8 with electrons moving at the maximal causal speed $V_{max} = c_o$. These media can be either opaque to light (because $V_{max} < c$), or be transparent, in which case $c_{max} = V_{max}$ because the speed of light is not the maximal causal speed, but an ordinary local speed, thus being bounded by V_{max}. In case the media are opaque to light, $b_4 = 1/n_4$ preserves its meaning as a geometrization of the density with significant meaning, such as the fact that media of Type 3 are more dense than those of Type 2 (because $c_{I,3} > c_{I,2}$.

GROUP II, TYPE 4: $n_s < n_4,\ n_4 = 1,\ \hat{\beta} < \beta,\ \hat{\gamma} > \gamma,\ c = c_o,\ V_{max} < c_o,\ V_{max} < c$.
These media are the first to be non trivial, in the sense that they cannot be derived from the isotopic invariance (6.1.10). Hence, they are are inhomogeneous and anisotropic, and they are generally transparent to light, in which case $c_{max} = V_{max}$, although the case of media opaque to light (with $V_{max} < c$ should not be excluded. Expected candidates for these media are planetary atmospheres

or astrophysical chromospheres because they are of generally low density, inhomogeneous (due to the radial variation of the density) and anisotropic (due to rotations establishing a preferred direction in space). These features require a necessary departure from the Minkowskian spacetime with deep astrophysical implications, e.g., in current unfounded beliefs on cosmological redshifts. Another expected case is given by the media inside light unstable particles, such as pions, as studied in Section 6.1.7. Other expected media of this type are given by ordinary conductors.[4]

GROUP II, TYPE 5: $n_s < n_4,\ n_4 > 1,\ \hat{\beta} < \beta,\ \hat{\gamma} > \gamma,\ c < c_o,\ V_{max} < c_o,\ V_{max} \leq c.$

These are inhomogeneous and anisotropic media of generally low to moderate density (because the maximal possible speed of light is smaller than that in vacuum). As such, these media are significant for astrophysical chromospheres and other interior bodies. In fact, we shall show in Section 6.1.11 that the huge inhomogeneous and anisotropic chromospheres of quasars are media precisely of this type. Intriguoingly, the same holds for the medium inside light hadrons, as shown in Section 6.1.8.

GROUP II, TYPE 6: $n_s < n_4,\ n_4 < 1,\ \hat{\beta} < \beta,\ \hat{\gamma} > \gamma,\ c > c_o,\ V_{max} < c_o,\ V_{max} < c.$

These media too are inhomogeneous and anisotropic with expected low to moderate density. Examples are given by nuclei that are indeed, inhomogeneous and anisotropic, yet treated with the homogeneous and isotropic Minkowskian spacetime and related Poincaré symmetry, despite the fact that nuclei have no nuclei (Figure 6.1) in which case the assumption of the exact Poincaré symmetry and special relativity is mere theological, as studied in Chapter 7. The differences between media of Group II, Types 4, 5, 6 are expected to represent significant geometric differences ignored during the 20-th century because, again, nature was adapted to the homogeneous and isotropic spacetime of special relativity.

GROUP III, TYPE 7: $n_s > n_4,\ n_4 = 1,\ \hat{\beta} < \beta,\ \hat{\gamma} > \gamma,\ c = c_o,\ V_{max} > c_o,\ V_{max} > c.$

This is the first of three inhomogeneous and anisotropic media of high density that are of primary relevance for hadronic mechanics because representing the hyperdense media inside hadrons, stars, quasars and other internal astrophysical

[4]By ignoring all other arguments and experimental evidence studied in these volumes, the sole privileged space directions possessed by atmospheres or chromospheres, particles such as hadrons, and conductors at large is sufficient to prohibit the exact validity of Einsteinian doctrines due to their strict isotropic character, since anisotropy has deep geometric and dynamical implications. The appropriate broadening of Einsteinian doctrines that is applicable for basic advances in the representation of anisotropic systems, is indeed open to scientific debates, by the denial of its need is scientific corruption for personal gain in maintaining pre-established doctrines.

problems. All media of this group have $V_{max} > c_o$ and $V_{max} > c$. The first of these three media has the geometric significance that the speed of light is the same as that in vacuum, $c = c_o$.

GROUP III, TYPE 8: $n_s > n_4,\ n_4 > 1,\ \hat{\beta} < \beta,\ \hat{\gamma} > \gamma,\ c < c_o,\ V_{max} > c_o,\ V_{max} > c$.

This is a second type of inhomogeneous and anisotropic media of high density that is conceivable for extreme astrophysical conditions, such as those in the interior of black holes, in which the maximal causal speed is expected to have no limit, but the speed of light is expected to be much smaller than that in vacuum, assuming that light can even propagate in media of such extreme densities.

GROUP III, TYPE 9: $n_s > n_4,\ n_4 < 1,\ \hat{\beta} < \beta,\ \hat{\gamma} > \gamma,\ c > c_o,\ V_{max} > c_o,\ V_{max} > c,\ V_{max} > c$.

These media are experimentally verified in the interior of heavy hadrons (Section 6.1.7, 6.1.8, 6.1.9), in the interior of the fireball of the Bose-Einstein correlation (Section 6.1.10) and other hyperdense inhomogeneous and anisotropic media. As we shall see, these last media do indeed permit the prediction, quantitative development and industrial realization of basically "new" clean energies, such as energies originating from mechanism in the interior of the neutron, rather than in a nuclear structure. Due to their societal need, readers are alerted that *technical* criticisms are solicited, welcome and appreciated as part of a serious scientific process, but opposition based on tangential issues without technical relevance will be denounced as a threat to society.

Santilli isodual isorelativity for the characterization of antimatter can be easily constructed via the isodual map of Chapter I.3, and its explicit study is left to the interested reader for brevity. For recent studies on Santilli isorelativity one may consult A. K. Aringazin [7], J. F. Kadeisvili [8], K. Masuda [9], and monographs [19-24].

The reader should remember from Volume I that isorelativity unifies the special and the general into one single relativity. The unification is done beginning at the level of unification of the Minkowskian and Riemannian geometries [10] and carries over at all subsequent levels. In fact, isoelement (6.1.8) is inclusive of all possible Riemannian line elements as indicated earlier, and the Lorentz-Santilli isosymmetry $\hat{O}(3,1)$ is the universal symmetry of all possible Riemannian gravitation, first presented in Ref. [5].

However, a necessary condition for the achievement of a universal *symmetry* for all gravitational models is the abandonment of curvature since gravitation is represented in the Minkowski-Santilli isospace that is isoflat. This occurrence can also be seen from the fat that *isogravitation* [11] is characterized by

1) Factorizing any Riemannian $g(x)$ metric into a 4×4 matrix $\hat{T}(x)$ and the Minkowskian metric,

$$g_{\mu\nu} = \hat{T}^{\rho}_{\mu}(x) \times \eta_{\rho\nu}, \tag{6.1.17}$$

2) Assuming $\hat{T}(x)$ as the inverse of the new isounit,

$$\hat{I}(x) = 1/\hat{T}(x), \tag{6.1.18}$$

3) Formulating the line element with Riemannian isometric $g(x) = \hat{T}(x) \times \eta$ as an *isonumber,* that is, with respect to the isounit $\hat{I}(x) = \hat{T}(x)$,

$$\hat{x}^{\hat{2}} = [x^t \times (\hat{T} \times \eta) \times x] \times \hat{I}, \tag{6.1.19}$$

in which case the curvature represented by $\hat{T}(x)$ is essentially "cancelled out" by its inverse $\hat{I}(x)$.

The noninitiated reader should be aware that the conventional formulation of gravity, that on a curved manifold, is afflicted by numerous theorems of catastrophic mathematical and physical inconsistencies studied in details in Chapter I.1., Ref. [13], and briefly outlined in Section I.1.4. Isogravitation was formulated as the only way known to the author to bypass these inconsistency theorems, that by eliminating curvature in favor of broader geometric views [10].

A main result is the achievement in Ref. [12] of the apparently first known, axiomatically consistent grand unification of electroweak and gravitational interactions, where "axiomatically consistency" is referred to the inclusion of both matter and antimatter (the latter being rather universally ignored in grand unifications), the use of a consistent operator formulation of gravity [11], e.g., verifying the PCT theorem, and admitting compatible symmetries.

A central objective of this volume is to present a variety of experimental verifications of isorelativity for interior dynamical conditions in different fields.

6.1.5 Rudiments of Hadronic Mechanics

For minimal self-sufficiency of this volume, let us also recall that the*isotopic branch of nonrelativistic or relativistic hadronic mechanics* (first proposed in memoirs [14] of 1978) can be constructed via techniques similar to those of the preceding subsection. Any given quantum model can be lifted into the covering hadronic version via the use of a nonsingular, positive-definite, nonunitary transform on a *Hilbert space* $\mathcal{H}$ over the field of complex numbers C.

We first have the lifting of *Planck's constant* into a isounit that is positive definite (thus invertible) but otherwise possesses an unrestricted functional dependence on time t, local coordinates r, linear momentum p, wavefunctions ψ, and any other needed variable,

$$\hbar \to \hat{I}(t, r, p, E, \psi, ...) = 1/\hat{T}(t, r, p, \psi, ...) = U \times U^{\dagger} > 0, \tag{6.1.20}$$

where the dependence on energy E is trivially derived from the unrestricted dependence on the linear momentum and coordinates (see EHM-II).

The above lifting represents the impossibility of conventional quantum orbits in the hyperdense medium inside hadrons, nuclei and stars (if nothing else, due to the absence of a Keplerian structure and the consequential inapplicability of conventional Poincaré symmetry).

Lifting (6.1.20) is restricted to verify the general condition

$$Lim\ \hat{I}_{r>>1\ fm} \equiv \hbar \tag{6.1.21}$$

assuring that hadronic mechanics recovers quantum mechanics uniquely and identically at sufficiently large mutual distances of particles., thus including the recovering of conventional quantized orbits (that exist only for distances much bigger than 1 fm).

Compatibility conditions (6.1.21) will soon appear crucial for the understanding of the compatibility of our structure model of the neutron as a hadronic bound states of a proton and an electron and the conventional structure of the hydrogen atom.

We then have the lifting of $\mathcal{H}$ into the *Hilbert-Santilli isospace* $\hat{\mathcal{H}}$ expressible via the following lifting of states. inner products and expectation values of a (Hermitean) operator A

$$|\psi> \ \in \ \mathcal{H} \to |\hat{\psi}>= U \times |\psi> \in \hat{\mathcal{H}}, \tag{6.1.22a}$$

$$< \psi| \times |\psi> \times I \ \in \ R \to U \times (< \psi| \times |\psi> \times I) \times U^\dagger =$$
$$=< \psi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times |\psi> \times U \times I \times U^\dagger =$$
$$=< \hat{\psi}| \hat{\times} |\hat{\psi}> \times \hat{I} \ \in \hat{C}, \tag{6.1.22b}$$

$$< A >=< \psi| \times A \times |\psi> \times I \to U \times (< \psi| \times A \times |\psi> \times I) \times U^\dagger =$$
$$=< \hat{\psi}| \hat{\times} \hat{A} \hat{\times} |\hat{\psi}> \times \hat{I} =< \hat{A} > . \tag{6.1.22c}$$

We then have the identity

$$< \hat{I} >\equiv I = \hbar, \tag{6.1.23}$$

illustrating the fact that deviations from conventional quantization processes are *internal* and not necessarilydetectable from exterior conditions.

Similarly, we have the lifting of Heisenberg's equations into the *Heisenberg-Santilli isoequations* first proposed in Ref. [14b] of 1978 (see memoir [15] of 1996 for the first formulation via the *isodifferential calculus*)

$$i \times \frac{dA}{dt} = [A, H] \to U \times (i \times \frac{dA}{dt}) \times U^\dagger =$$

$$= \hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = i \times \hat{I}_t \times \frac{d\hat{A}}{d\hat{t}} =$$

$$= U \times [A, H] \times U^\dagger = [\hat{A}\hat{,}\hat{H}] = \hat{A} \times \hat{T}_r \times \hat{H} - \hat{H} \times \hat{T}_r \times \hat{A}, \tag{6.1.24}$$

where one should note isounits of time and space denoted with the subindeces t, r, respectively (generally ignored whenever there is no ambiguity).

Similarly, we have the lifting of canonical commutation rules into *isocanonical isocommutation rules* also introduced for the first time in memoir [14]

$$[r^i, p^j] = i \times \delta^i_j \to [\hat{r}^i\hat{,}\hat{p}_j] = \hat{i}\hat{\delta}^i_j = i \times \hat{I} \times \delta^i_j, \tag{6.1.25}$$

Similarly, we have the lifting of the Schrödinger equations into the *Schrödinger-Santilli isoequations* first formulated in an invariant form in memoir [15]

$$i \times \hbar \times \frac{\partial}{\partial t}|\psi >= H \times |\psi > \to$$

$$\to \hat{i}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{t}}|\hat{\psi}(\hat{t}, \hat{r}) >= i \times \hat{I}_t \times \frac{\partial}{\partial \hat{t}} =$$

$$= \hat{H}\hat{\times}|\hat{\psi} >= \hat{H}(\hat{r}, \hat{p}) \times \hat{T}_r(\hat{t}, \hat{r}, \hat{p}, \hat{E}, \hat{\psi}, ...) \times |\hat{\psi} > . \tag{6.1.26}$$

and the lifting of the linear momentum into *isolinear isomomentum* (reached for the first time in memoir [15] following decades of search due to the preceding absence of the isodifferential calculus

$$p_k \times |\psi >= -i \times \hbar \times \partial_k|\psi > \to U \times (p_k \times |\psi >) =$$

$$= U \times p_k \times (U \times I^\dagger)^{-1} \times U \times |\psi >= \hat{p}_k\hat{\times}|\hat{\psi} >= -U \times (i \times \hbar \times \partial_k|\psi >) =$$

$$= -\hat{i}\hat{\times}\hat{\partial}_k|\hat{\psi} >= -i \times \hat{I}^i_k \times \partial_i|\hat{\psi} >, \tag{6.1.27}$$

We should also recall the new invariance of the conventional inner product under isotopic transforms here expressed for a non-null constant $z \in R$

$$< \psi| \times |\psi > \times I \equiv < \psi| \times z^2 \times | \, psi > \times (z^-2 \times I) \equiv < \psi|\hat{\times}| \, psi > \times \hat{I}, \tag{6.1.28}$$

with extension to an arbitrary positive-definite nonunitary transform and isounit $U \times U^\dagger = \hat{I} > 0$ via the techniques of Volume I.

Note the *abstract identity of hadronic and quantum mechanics* as illustrated by the property that all relative equations and physical laws are merely differentiated by a "hat" denoting the existence of a *broader realization* of the same axioms.

The above occurrences forcefully establishes the validity of nonrelativistic and relativistic hadronic mechanics in the conditions of their applicability, evidently because of the preservation of the conventional axioms of quantum mechanics.

In turn, this forcefully establishes the validity of the Minkowski-Santilli isospaces for interior particle conditions as verified below.

Alternatively, the preservation of the abstract axioms in the transition from quantum to hadronic mechanics renders nonscientific the aprioristic selection of any of them, since the only scientific selection of the truly applicable mechanics for given conditions, that via experiments.

Note that the preceding isoequations also provide an explicit realization of *operator isogravity*, first submitted at the Marcel Grossmann meeting of 1998 [12] under the mere realization of the isounit and isotopic elements as the gravitational forms (6.1.18), (6.1.9). The consistency of operator isogravity, including the verification of the PCT theorem, is assured by the preservation of the abstract axioms of conventional relativistic quantum mechanics.

Independent reviews of hadronic mechanics are provided by monographs [19-24]. A large number of independent papers written during the bast three decades can be found in the general bibliography at the end of this volume.

6.1.6 Catastrophic Mathematical and Physical Inconsistencies of Noncanonical and Nonunitary Theories

As it is well known, classical canonical theories, or operator unitary theories, are Hamiltonian in the sense that they represent the entire system considered via the sole knowledge of a Hamiltonian. Consequently, the representation of new effects beyond the representational capabilities of a Hamiltonian, such as nonpotential interactions, has requested the use of noncanonical or nonunitary theories, e.g., theories whose time evolution verifies condition

$$U(t) \times U(t)^\dagger \neq I, \tag{6.1.29}$$

formulated on conventional mathematics.

A knowledge truly crucial for the understanding of this volume (studied in details in Section I.1.5 Theorem I.1.5.2) is that the latter theories are afflicted by the following catastrophic inconsistencies:

THEOREM 6.1 [25-32]: All noncanonical and nonunitary theories formulated via the mathematics of canonical or unitary theories (conventional numbers, spaces, functional analysis, etc.) are afflicted by catastrophic mathematical and physical inconsistencies.

On mathematical grounds, by their very definition, noncanonical and nonunitary theories do not preserve the unit,

$$I \to I' = U \times I \times U^\dagger \neq I. \tag{6.1.30}$$

Consequently, noncanonical and nonunitary theories do not preserve over time the unit I of their base fields, with consequential catastrophic collapse over time of the entire mathematical structure, including spaces, algebras, geometries, symmetries, etc. since all of them remain formulated over a base field no longer applicable at later time. An identical situation occurs under all other automorphism

On physical grounds, units of Lie symmetries represent units of measurements. For instance, the unit of the Euclidean geometry $I = Diag.1, 1, 1)$ represents in an abstract dimensionless form units actually used in tests, such as $I = Diag.(1 \ \ cm, 1 \ \ cm, 1 \ \ cm)$. Consequently, a theory with a noncanonical or nonunitary time evolution necessarily alters the numerical values of the basic units used in measurements, such as, for in stance, in the case

$$I = Diag.(1 \ \ cm, 1 \ \ cm, 1 \ \ cm) \to U \times I \times U^\dagger = I =$$

$$= Diag.(7.3 \ \ cm, 345 \ \ cm, 0.003 \ \ cm), \tag{6.1.31}$$

thus preventing any meaningful application in dynamics.

Noncanonical and nonunitary theories have additional catastrophic physical inconsistencies, such as they do not preserve over time the Hermiticity and, hence, the observability of physical quantities, namely, an operator H that is Hermitean at the initial time is not necessarily Hermitean at a subsequent time (this property is known as the *Lopez Lemma* [26,27), Eq. (I.1.5.52), i.e.

$$[< \psi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times H \times U^\dagger] \times U|\psi >=$$

$$=< \psi| \times U^\dagger \times [(U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times U|\psi >] =$$

$$= (< \hat{\psi} \times T \times H'^\dagger) \times |\hat{\psi}\rangle = \langle\hat{\psi}| \times (\hat{H} \times T \times |\hat{\psi} >), \tag{6.1.32a}$$

$$|\hat{\psi}\rangle = U \times |\psi\rangle, \ \ T = (U \times U^\dagger)^{-1} = T^\dagger, \tag{6.1.32b}$$

$$H'^\dagger = T^{-1} \times \hat{H} \times T \neq H. \tag{1.5.32c}$$

where the loss of observability follows from the general lack of commutativity of H and T. Similarly, noncanonical and nonunitary theories generally violate causality (we teach in first year graduate school of physics that the causality verified by quantum mechanics is due to its unitary structure), and other serious catastrophes.

In view of these occurrences, all papers with a noncanonical or nonunitary structure formulated with conventional mathematics, are catastrophically inconsistent and should not be considered for any serious scientific study.

Isorelativity and hadronic mechanics avoid these inconsistencies thanks to the prior discovery of new mathematics specifically constructed for the task, Santilli iso-, geno- and hyper-mathematics for matter and their isoduals for antimatter

for closed single-valued, open single-valued and open multi-valued conditions, respectively. Theorem 6.1 is bypassed because the new mathematics reconstruct canonicity or unitarity on iso-, geno- and hyper-spaces over iso-, geno-, and hyper-fields, respectively (for brevity see HM-I).

The above mathematical and [physical inconsistencies are typically suffered by the so-called *q-deformations* with *deformed Lie product* $(A, B) = A \times B - q \times B \times A$, where q is a non-null number. In fact, in this case we have the time evolution in the following infinitesimal and finite form

$$i \times \frac{dA}{dt} = A \times H - q \times H \times A, \tag{6.1.33a}$$

$$A(t) = U \times A(0) \times W^{\dagger} = (e^{i \times t \times q \times H}) \times A(0) \times (e^{-i \times t \times H}). \tag{6.1.33b}$$

directly activating Theorem 6.1.

These deformations were initiated by R. M. Santilli via paper [33] of 1967 in their broader form $(A, B) = p \times A \times B - q \times B \times A$, where p and q are non-null scalars; they were resumed in 1986 by L. C. Biedenharn [34] and A. J. Macdarlane [35] in the reduced form of the q-deformations; and they subsequently resulted in a river of papers in the field.[5] Ironically, by the time Biedenharn and Macfairlane elected to study the q-deformations, Santilli had long abandoned the field because of the catastrophic mathematical and physical inconsistencies herein considered.

Another illustration of catastrophically inconsistent theories is given by Ref. [36] of 1999 dealing with a structure dubbed by the authors "deformed Minkowski space" that is entirely identical to the Minkowski-Santilli isospace previously introduced by Santilli [3] in 1983 (including the use of exactly the same symbols!). But this "deformed space" is formulated on conventional fields and elaborated with conventional mathematics, thus being catastrophically inconsistent on mathematical and physical grounds.

In general, *all* theories departing from the conventional structure of Lie's theory (that characterized by unitary transformations on a Hilbert space over the field of complex numbers) verify Theorem 6.1, as it is the case of the *supersymmetries* [37] (see Section I.1.5 for details).

$$(A, B) = \alpha \times (A \times B - B \times A) + \beta \times (A \times B + B \times A) = \alpha \times [A, B] + \beta \times \{A, B\}, \tag{6.1.34}$$

[5]L. C. Biedenharn and A. J. Macfarlane were fully aware of the initiation of the q-deformations by Santilli [33] some twenty years earlier, as proven by the fact that in the early 1980 Biedenharn and Santilli applied for a joint DOE grant, but there was no quotation of the origination [33] in papers [34,35] because of reported ascientific pressures from the Cantabridgean academic community. As a result of this multi-faced ascientific episode, Santilli has been called *the most plagiarized physicist of the 20-th century.*

where α, β are suitable factors depending on the model at hand.[6]

The reader with a young mind of any age as well as independence from orthodox interests can now understand the reason for gravitation defined on a Riemannian space to be catastrophically inconsistent [13] at both the classical and operator levels. In fact, curvature necessarily implies that the time evolution of the theory is necessarily noncanonical at the classical level and nonunitary at the operator level, with direct activation of Theorem 6.1 (dee Ref. [13] for a total of *nine theorems of catastrophic inconsistencies of general relativity*). At any rate, general relativity admits no distinction whatever between neutral matter and antimatter. Consequently, any attempt at achieving a consistent operator theory of gravity is doomed to failure.

To avoid a mathematical treatment that may appear excessive to readers due to the applied character of this volume, in this volume we shall study experimental verifications and industrial applications formulated via the *projection* of the formulations in our conventional spacetime over conventional fields, with the clear understanding that their sole correct formulation is on iso-, geno- and hyper-spacetime over iso-, geno- and hyperfields.

6.1.7 Experimental Verifications for Arbitrary Speeds of Light

Isorelativity resolves the inconsistencies of special relativity for classical particles and electromagnetic waves propagating within physical media, including media transparent to light, such as water. In particular, isorelativity provides an *invariant* representation of locally varying speeds of light, while preserving the abstract axioms of special relativity. Since the latter is manifestly inapplicable within physical media, the physical evidence supporting the validity of isorelativity in classical mechanics over special relativity is beyond credible doubt.

Let us consider first the case of water (studied in detail in EHM-II). This medium is homogeneous and isotropic with $c < c_o$ (c in water is about 2/3 of c_o). hence, *water is an iso-Minkowskian medium of Group I, Type 2* (Figure 6.3), thus requiring that Isoaxioms I, Eqs. (6.1.11), holds for $b_3 = b_4$, as a result of which

$$V_{max} = c_o \times \frac{b_4}{b_3} = c_o \tag{6.1.35}$$

[6]Supersymmetric theories are a trivial particular case of Santilli Lie-admissible theory with product

$$(A, B) = A \times P \times B - B \times Q \times A =$$
$$= (A \times T \times B - B \times T \times A) + (A \times V \times B + B \times QV \times A) =$$
$$= [A\hat{,}B] + \{A\hat{,}B\}, \;\; P - Q = T, \;\; P - Q = V.$$

Invariance is then achieved via elaborations based on genomathematics (see [18] for brevity).

namely, *the maximal causal speed in water is the speed of light in vacuum.* This resolves the violation of causality suffered by special relativity because electrons in water can travel faster than the local speed of light, but they keep traveling at speeds smaller than the maximal causal speed.

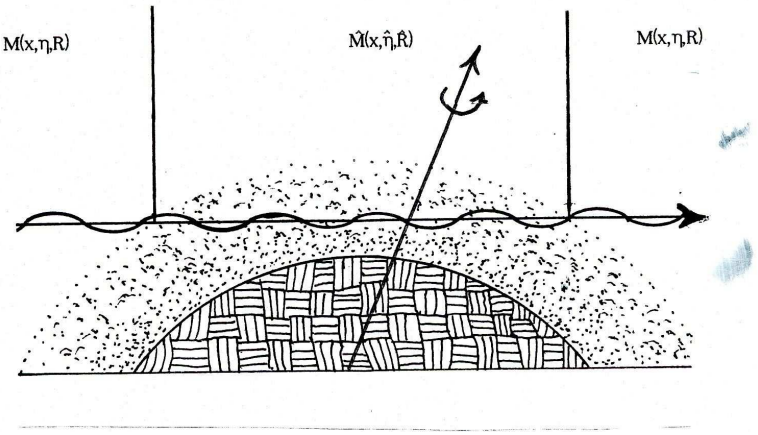

Figure 6.4. An illustration of the spacetime geometries used for the description of electromagnetic waves passing through Earth's atmosphere: the conventional Minkowski geometry is used for propagation in vacuum (exterior problem), and the Minkowski-Santilli isogeometry is used for propagation in Earth's atmosphere (interior problems). The isogeometry has been constructed for a representation of the deviations from the geometry of empty space caused by a physical medium. These deviations do not exist for special relativity because the theory abstracts all particles as idealized points for which physical media do not exist. However, the deviations emerge quite forcefully when particles are represented with their actual extended size, thus rendering inevitable contact, zero-range, nonlocal, nonlinear and nonpotential forces, e.g., of resistive type as experienced by a missile in atmosphere or, equivalently, by an electron moving within the interior of a hadron, or a proton moving in the interior of a star. In Volume I we presented No-Reduction Theorems preventing a consistent reduction of a macroscopic system with contact nonpotential interactions to a hypothetical ensemble of point-like abstractions of particles all in conservative conditions. This established that the contact nonpotential interactions existing in our physical environment originate at the ultimate level of particles, thus establishing the foundations for hadronic mechanics. In this volume we shall present numerous experimental verifications of deviations from the Minkowskian spacetime caused by physical media and then show that said deviations permit the conception and industrial development of new clean energies and fuels that are simply unthinkable for point-like abstractions of particles and their wavepackets.

Isoaxiom II, Eqs. (6.1.12), on the isorelativistic sums of speeds is also verified. For instance, the maximal causal speed verifies the isolaw

$$V_{tot} = \frac{V_{max} + V_{max}}{1 + \frac{V_{max}}{V_{max}}} \equiv V_{max}, \tag{6.1.36}$$

and this resolves the second inconsistency of special relativity in water, the fact that the sum of two maximal causal speeds in water (assumed by special relativity to be necessarily c_o to avoid violation of causality) does not yield the maximal causal speed,

$$V_{tot} = \frac{\frac{2}{3} \times c_o + \frac{2}{3} \times c_o}{1 + \frac{4 \times c_o^2/9}{c_o^2}} = \frac{12}{13} \times c_o \neq c_o. \tag{6.1.37}$$

Note that the above resolutions require *the abandonment of the speed of light as the maximal causal speed for motion within physical media, and its replacement with the maximal causal speed* (6.1.11). In fact, physical media are generally opaque to light. It happens that in vacuum these two speeds coincide. However, even in vacuum the correct maximal causal speed remains Eq. (6.1.11) and *not* that of light, as generally believed.

At any rate, to extend the applicability of special relativity beyond the conditions of its original conception, it is popularly believed that the speed of light in vacuum is the maximal causal speed also within physical media in which light cannot propagate. Such a belief has no scientific value or credibility.

The case of classical physical media opaque to light follows the same lines. Special relativity has no meaning when light cannot propagate. Isorelativity applies because physical media represented with conventional spaces over conventional fields are geometrize into a form equivalent to the vacuum when formulated on isospaces over isofields. In fact, the maximal causal speed on isospaces over isofields is c_o and not c (see Volume I for technical aspects). Alternatively, we can say that the vacuum formulated on isospaces over isofields, when projected in our space over conventional fields, characterizes physical media.

The most forceful classical verification of isorelativity is provided by the experimental evidence that electromagnetic waves can propagate within certain guides and other conditions at speeds bigger than the speed of light in vacuum [38,39] conducted at the University of Cologne, Germany, today known as the *Cologne experiment*. These experiments were confirmed via independent tests conducted in Italy (Florence), U.S.A. (Berkeley), Austria (Wien) and France (Orsay and Rennes) (see review [40] of all experimental data on $c > c_o$ up to 2000). Hence, *the existence of electromagnetic waves propagating at speeds bigger than that of light in vacuum is, nowadays, an experimental reality beyond scientific or credible doubt.*

At any rate, an entire Beethoven symphony has been transmitted at speeds $c > c_o$. Any claim of validity of special relativity for these experimental results

would be sheer corruption, for which reasons experimental evidence of speeds $c > c_o$ is often ignored in high energy physics, thus causing problems of scientific ethics and accountability of potentially historical proportions.[7]

The validity of isorelativity and relativistic hadronic mechanics for all possible speeds $c > c_o$ is established quite forcefully by the following facts:

i) Isorelativity applies for any possible local speed of light c, irrespective of whether smaller or bigger than c_o, the case $c = c_o$ being a trivial particular case;

ii) Isorelativity is the sole theory providing the invariance of arbitrary local speeds of light;

iii) Isorelativity is "directly universal," that is, including all conceivably possible (nonsingular) theories for arbitrary speeds of light (universality), directly in the spacetime of the observer without any need to use transformations of local coordinates (direct universality). This is due to the fact that, on one side, the transition from the speed of light in vacuum to locally varying speeds requires noncanonical transformations (Subsection 6.1.2) while, on the other side, isorelativity includes the most general possible noncanonical transforms.

iv) Isorelativity is the only known theory bypassing the theorems of catastrophic inconsistencies of noncanonical theories (Subsection 6.1.4) thanks to its underlying novel isomathematics;

v) Isorelativity is the sole new relativity that has permitted scientific and industrial advances on new clean energies ands fuels simply inconceivable with special relativity.

The invariant geometrization of speeds $c > c_o$ permitted by isorelativity and relativistic hadronic mechanics is elementary. With reference to experiments [38,39], in the following we outline the treatment via *the isotopic branch of hadronic mechanics,* or *isomechanics,* [8], treated via the *Minkowski-Santilli isogeometry,* although solely referred to the steady segment of the tests, that in between the guides.

The geometrization of the entire process, that starting from propagation in vacuum and then passing though guides, requires the *genotopic branch of hadronic mechanics,* or *genomechanics,* treated via the *Minkowski-Santilli genogeometry* [18] studied in detail in Volume I (see also EHM-II). The latter treatment is excessively advanced for the applied character of this volume and will be presented elsewhere.

To set up notations, let us recall the rudiments of the propagation of monochromatic electromagnetic waves in vacuum. The geometry is characterized by the

[7]The established experimental evidence on electromagnetic waves propagating in certain guides at speeds $c > c_o$ is sufficient, per se, to render equivocal the use of public funds in high energy physics experiments at Fermilab, CERN, and other laboratories all based on the assumption of the exact validity of Einsteinian doctrines within media dramatically denser than *waveguides*, such as the media inside hadrons.

conventional Minkowskian spacetime $M(x,\eta,R)$ with metric, coordinates, wavevector, and related invariants,

$$(\eta_{\mu\nu}) = (\eta^{\mu\nu}) = Diag.(1,1,1,-1), \eta_{\mu\alpha}\times\eta^{\alpha\nu} = \delta_\mu^\nu, \tag{6.1.38a}$$

$$x = (x^\mu) = (r^k, x_4) = (r^i, c_o\times t), K = (K_\mu) = (k_i, \frac{\omega}{c_o}), i = 1,2,3, \tag{6.1.38b}$$

$$x^2 = (x^\mu\times\eta_{\mu\nu}\times x^\nu)\times I = (r^i\times r^i - c_o^2\times t^2)\times I, \tag{6.1.38c}$$

$$K^2 = (K_\mu\times\eta^{\mu\nu}\times K_\nu)\times I = (k_i\times k_i - \frac{\omega^2}{c_o^2}\times I, \tag{6.1.38d}$$

where, in accordance with our formalism (Section 6.1.2), we multiply the invariants by the unit of the base field R to assure their scalar character on rigorous mathematical grounds, but such a multiplication will be ignored thereafter for notational simplicity.

An elementary electromagnetic wave propagating in empty space can be represented on a conventional Hilbert space $\mathcal{H}$ over C via the familiar wavefunction

$$\psi = e^{i\times K_\mu\times x^\mu} = e^{i\times k_i\times r^i - \omega\times t}. \tag{6.1.39}$$

We then have the linear momentum eigenvalue equation

$$p_\mu\times\psi = -i\times\partial_\mu\psi = K_\mu\times\psi, \tag{6.1.40}$$

and the well known wave equations

$$\eta^{mu\nu}\times p_\mu\times p_\nu\times\psi = \eta^{\mu\nu}\times K_\mu\times K_\nu = (k_i\times k_i - \frac{\omega^2}{c_o^2}\times\psi = 0. \tag{6.1.41}$$

The speed of electromagnetic waves in vacuum can then be represented via the known expressions

$$\frac{dr}{dt}\approx\frac{d\omega}{dk} = c_o, \tag{.6.1.42}$$

confirming that c_o is indeed the maxima;l causal speed *in vacuum,* as well known.

Recall that *isotopies are axiom-preserving.* Hence, the representation of electromagnetic waves of tests [38,39] traveling faster than c_o can be done with *exactly the same expressions* (6.1.38)-(6.1.42), only subjected to a broader realization (or interpretation). Nevertheless, for clarify, we write down the representation explicitly.

The basic space is the Minkowski-Santilli isospace $\hat{M}(\hat{x},\hat{\eta},\hat{R})$ [3] with isometric, isocoordinates, isowavevector,and related isoinvariants (see EHM-II, Volume I and the short review in Section 6.1.2)

$$(\hat{\eta}_{\mu\nu}) = (\hat{T}_\mu^\alpha\times\eta_{\alpha\nu}) = Diag.(b_1^2, b_2^2, b_3^2, b_4^2),$$

$$(\hat{\eta}^{\mu\nu}) = (\hat{I}^{\mu}_{\alpha} \times \eta^{\alpha\nu} = Diag.(b_1^{-2}, b_2^{-2}, b_3^{-2}, b_4^{-2}), \hat{\eta}_{\mu\rho} \times \hat{\eta}^{\rho\nu} = \delta^{\nu}_{\mu}, \tag{6.1.43a}$$

$$\hat{x} = (\hat{x}^{\mu}) = (\hat{r}^{i}, \hat{x}_4) = (\hat{r}^{i}, c_o \times \hat{t}), \hat{K} = (\hat{K}_{\mu}) = (\hat{k}_i, \frac{\hat{\omega}}{c_o}), \tag{6.1.43b}$$

$$\hat{x}^{\hat{2}} = (\hat{x}^{\mu} \times \hat{\eta}_{\mu\nu} \times \hat{x}^{\nu}) \times \hat{I} = (\hat{r}^{i} \times \hat{r}^{i} \times b_i^2 - c_0^2 \times \hat{t}^2 \times b_4^2) \times I, \tag{6.1.43c}$$

$$\hat{K}^{\hat{2}} = (\hat{K}_{\mu} \times \hat{\eta}^{\mu\nu} \times \hat{K}_{\nu}) \times \hat{I} = (\hat{k}_i \times \hat{k}_i \times b_i^{-2} - \frac{\hat{\omega}^2}{c_o^2} \times b_4^{-2}) \times \hat{I}, \tag{6.1.43d}$$

where the reader should keep in mind that $\hat{x}$ and $\hat{K}$ are now defined on $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$, and that the speed of light on isospace over isofields is c_o and not $c = c_o \times b_4$ (Volume I and EHM-II).

A monochromatic electromagnetic wave propagating through the guides of the Cologne experiment can be represented on a Hilbert-Santilli isospace $\hat{\mathcal{H}}$ over the isofield $\hat{C}$ via the elementary isowavefunction (the isoexponentiation (6.1.5f) and EHM-II)

$$\hat{\psi} = e^{i \times \hat{K}_{\mu} \times \hat{x}^{\mu} \times b_{\mu}^2} = e^{i \times \hat{k}_i \times r^i \times b_i^2 - \omega \times t}, \tag{6.1.44}$$

where we have ignored the multiplication by $\hat{I}$ for simplicity.

We then have the *isolinear isomomentum* equation of hadronic mechanics [15]

$$\hat{p}_{\mu} \hat{\times} \hat{\psi} = \hat{p}_{\mu} \times \hat{T} \times \hat{\psi} = -i \times \hat{\partial}_{\mu} \hat{\psi} = \hat{K}_{\mu} \times \hat{\psi}, \tag{6.1.45}$$

with isowave isoequations

$$\hat{\eta}^{mu\nu} \times p_{\mu} \times p_{\nu} \times \psi = \eta^{\mu\nu} \times K_{\mu} \times K_{\nu} = (k_i \times k_i \times b_i^{-2} - \frac{\omega^2}{c_o^2 \times b_4^{-2}} \times \times \psi = 0. \tag{6.1.46}$$

At this point we assume that the space component of the guides of tests [38.39] is isotropic, thus representable with one single space characteristic quantity, and that the symmetry axis of the tests is along the z-axis, thus allowing us to ignore the x and y components,

$$b_1 = b_2 = b_3 = b_s, r^1 = r^2 = k_1 = k_2 = 0. \tag{6.1.47}$$

We also assume that, for the steady conditions here considered, the characteristic quantities are constants or can be averaged into constants.

In correspondence of Eq. (6.1.42) we then have the expression (expressed in terms of conventional differential calculus)

$$\frac{d\hat{r}}{d\hat{t}} \approx \frac{d\hat{\omega}}{d\hat{k}} = c_o \times \frac{b_s}{b_4} = c \times b_s = V_{max}, \tag{.6.1.48}$$

namely, *the maximal causal speed of the Cologne experiment is that of isorelativity,* Eq. (6.1.11), thus providing a significant confirmation of the axiomatic structure

of isorelativity. The re-derivation of law 6.1.48) via the isodifferential calculus [15] is an instructive exercise for the reader expert on quantum mechanics, yet with insufficient knowledge of the covering hadronic mechanics.

The simp;lest possible fit of Eqs. (6.1.48) is given by assuming $b_s = 1$, as a result of which the numerical value of b_4 is trivially given by the numerical data of Refs. [38,39] for c, such as

$$b_4 = \frac{c}{c_o} = 1.5. \tag{6.1.49}$$

However, we note that *a mutation of the geometry of space requires a corresponding mutation of time and vice versa.* Hence, we exclude that we have $b_s = o$ in the Cologne experiment. Rather than being a drawback, the occurrence renders tests [38,39] quite intriguing. In fact, *depending on the assumed geometry, the Minkowski-Santilli isospace predicts that the Cologne experiment can be conducted for speeds both bigger as well as smaller than that of light in vacuum,* according to the following classification of possibilities:

$$V_{max} > c_o, I : c_o \leq c \leq V_{max}, II : c \leq c_o, \tag{6.1.50a}$$

$$V_{max} = c_o, III : c \leq c_o, \tag{6.1.50b}$$

$$V_{max} < c_o, IV : c \leq V_{max}. \tag{6.1.50c}$$

It appears that the set up of the Cologne experiment has realized only Case I of the above possibilities. The remaining cases are important, e.g., to see whether ordinary particles can travel in between the guides at speeds bigger than $c > c_o$, but smaller than V_{max}. If verified, this occurrence would constitute a superluminal reproduction of the occurrence in water in which electron travel faster than the local speed of light but slower than the maximal causal speed.

We finally mention that the *mutation*[8] of the geometry caused by the Cologne experiment is conceptually quite simple. Tests [38,39] essentially deal with the interactions at the very foundations of isorelativity and hadronic mechanics, the contact, zero-range interactions that are extended over a volume (thus being nonlocal of integral type) and not representable with a potential (thus being non-Hamiltonian hence requiring nonunitary theories), the latter condition being absolutely crucial to allow speeds $c > c_o$.[9]

[8]"mutations" are referred to invariant alterations of the spacetime geometry referred to isospaces over isofields as first introduced by Santilli [33] in 1967, while "deformations" are referred to non-invariant, thus catastrophically inconsistent alterations of the geometry referred to conventional spaces and fields.
[9]It is easy to prove that for a fully Hamiltonian theory, speeds $c > c_o$ cannot exist. In fact, orthodox physicists still deny speeds $c > c_o$ on grounds that they are not admitted by their beloved theories, a view that is both, correct, yet corrupt because based on the assumption that the old doctrines of the 20-th century, above all Einsteinian theories, are the final doctrines for all of the future history of mankind.

In turn, said non-Hamiltonian interactions cause a mutation in our terminology, namely, they change the very structure of the wavepackets, for instance, by decreasing its amplitude, with consequential decrease of the frequency $\hat{o}mega < \omega$, and increase of the speed $c > c_o$. Once the geometry of the mutation is understood, it should be possible for interested experimentalists to attempt the other cases predicted by isorelativity and hadronic mechanics, Eqs. (6.'1.50).

In conclusion, the Cologne experiments [39,40] and their numerous re-runs [40] constitute a direct experimental verification of the ultimate mathematical and physical foundations of isorelativity and relativistic hadronic mechanics with rather deep implications that will better transpire in the following analysis.
10

The serious scholar seriously interested to science should keep in mind that Albert Einstein clearly identified the limits of applicability of special relativity, "point-like particles and electromagnetic waves propagating in vacuum." The extension of the applicability of special relativity beyond the conditions limpidly identified by Einstein has been done by *Einstein's followers* for their personal gains, and not by Einstein.

6.1.8 Experimental Verifications in the Interior of Hadrons

We now study the dynamics within the hyperdense media in the interior of hadrons, nuclei and stars, hereinafter referred as *hadronic media*.

Once the evidence of the inapplicability of special relativity and its underlying Minkowskian geometry is admitted for physical media of low density such as Earth's atmosphere (Figure 6.4), the belief of their exact validity within hadronic media is nonscientific. The selection of the applicable theory is indeed open to scientific debates, by the denial of the need to surpass Einsteinian theories within hadronic media is a scientific manipulation for personal gains. This is due to numerous reasons studied in Volume I, such as:

[10]The reader should be made aware of adulterations of the above treatment existing in the literature, such as that by F. Cardone and R. Mignani, Phys. Lett. A **306**, 265 (2003). In fact, this paper: assumes $b_4 = 1$ in which case there cannot be a superluminal speed because one can prove that $c = c_o \times b_4 = c_o$ via the Lorentz-Santilli isosymmetry and the entire paper makes no sense; conventional differential equations are altered in contradiction with the rigid requirements of the Minkowski-Santilli isogeometry, as proved by the fact that they do not constitute an (axiom-preserving) isotopy; and the paper is catastrophically inconsistent because it deals with a noncanonical - nonunitary formulated via conventional mathematics (Section 6.4). As indicated in an earlier footnote, said authors call the framework "deformed Minkowski space" or "deformed special relativity" and avoid any quotation of the vast preceding literature documentedly known to them (see R. Mignani, Physics Essays **5**, 531 (1992) where the space is called "Santilli isospace"). For these and other reasons, the author filed on February 2007 at the United States Federal Court lawsuit number 8:07-CV-00308-T-23MSS available in the web site of the U. S. Federal Court or in the mirror site http://www.scientificethics.org/Lawsuit-Cardone-Mignani.htm

1) The impossibility for photons to propagate for any finite length within hyperdense hadronic media as they propagate in vacuum, with consequential collapse of the entire special relativity, including the impossibility to assume c_o as the maximal causal speed within the media considered;

2) The experimentally established absence within hadrons of a Keplerian structure with a Keplerian center, with consequential well established impossibility for the pillar of special relativity, the Poincaré symmetry, to be exact (Figure 6.1);

3) The inapplicability within hadrons of the mathematics used by special relativity, due to its strict local-differential character, with consequential sole applicability to the nonlocal-integral character of the hadronic structure; and other reasons (see Volume I for details).

The use of conjectures not directly verifiable, such as those based on the hypothetical quarks and neutrinos (see next section), is also a manipulation of science for personal gains when used in their widespread intent: preserve the exact validity of orthodox theories while opposing professional studies on alternative views.

The reader is suggested to meditate a moment on the very large amount of public money that is spent nowadays in particle physics laboratories around the world (estimated in the range of billions of dollars per year) on the assumption that special relativity and the Minkowskian geometry are exact in the interior of the hyperdense hadrons. In this way the reader has a chance of deciding whether to be part of an expected condemnation by posterity, or pursue new physical knowledge.

The epistemological, phenomenological and experimental studies on the impossibility for special relativity and the Minkowskian spacetime to be exact in the interior of hadrons can be summarized as follows. R. M. Santilli [41] submitted the hypothesis in 1982 that *the maximal causal speed in the interior of hadrons is generally bigger than that in vacuum as an intrinsic feature of strong interactions at large.*

The main argument of Ref. [41] is that the maximal possible speed under *action-at-a-distance* interactions is indeed c_o, as well-known and experimentally established, e.g.,in particle accelerators. However, under *contact zero-range* interactions, the maximal causal speed can be arbitrary because the energy balance of the latter is dramatically different than that of the former, as classically verified, e.g., in the acceleration of a balloon by Earth's atmosphere.

Strong interactions occur at mutual distances of the order of $1 fm = 10^{-13} cm$, that is also the size of all strongly interacting particles. Hence, the activation of strong interactions requires the mutual penetration and overlapping of the wavepackets and/or charge distributions of particles at short mutual distances, with ensuing contact, zero-range, nonpotential interactions. The prediction of Ref. [41] for speeds c bigger than that of light in vacuum, c_o, then applies for strong interactions at large.

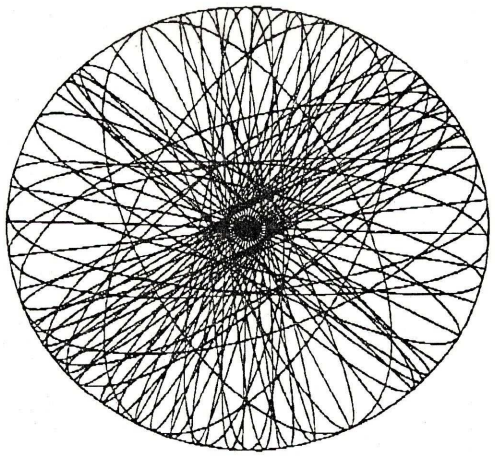

Figure 6.5. A schematic view of the *hadronic medium,* namely, the hyperdense medium inside hadrons, nuclei and stars. When combining the mathematical, theoretical and experimental evidence collected in these volumes, the belief that special relativity and quantum mechanics are "exactly" valid in the interior of hadrons is qualified as a theology without any scientific credibility. In these review lines, it is sufficient to note the impossibility for the Poincaré symmetry to be exact for the interior of hadrons due to the absence of a Keplerian structure and related Keplerian center (Figure 6.1), the impossibility for a photon to propagate in the hyperdense hadronic media in the same way as it propagates in vacuum (Figure 6.4), and numerous other evidence. Of course, when all particles and their wavepackets are abstracted as being points, the hyperdense media in the interior of hadrons disappear, although reappearing via a plethora of directly unverifiable abstractions, conjectures, beliefs and controversies, such as the belief the hadronic constituents are given by the hypothetical undetectable *point-like* quarks without any possible gravity, inertia or confinement (Chapter I.1).

Subsequently, V. de Sabbata and M. Gasperini [42] conducted the first phenomenological verification of the above hypothesis for the interior of hadrons via the use of conventional gauge theories, by obtaining maximal causal speeds up to $c = 75 \times c_o$.

More recently, various astrophysical measurements [43-46] have established the validity of the hypothesis submitted in Ref. [41] (without its quotation), by detecting masses expelled in astrophysical explosions (thus under contact interactions) at speeds $c \gg c_o$.

An additional verification of the validity of the Minkowski-Santilli isospace for the geometrization of media inside hadrons was provided in 1992 by H. B. Nielsen and I. Picek [47] [of the Niels Bohr Institute in Copenhagen, Denmark, who conducted extensive phenomenological calculations via conventional gauge

theories in the Higgs sector, and derived the following isometrics for the interior of pions and kaons,

$$\hat{\eta} = Diag.[(1-\alpha/3),(1-\alpha/3),(1-\alpha/3),-c_o\times(1-\alpha)] \equiv$$

$$\equiv\equiv Diag.(b_1^2,b_2^2,b_3^2,b_3^2,-c_o\times b_4^2) = Diag.(1/n_1^2,1/n_2^2,n_3^2,-c_o^2/n_4^2), \quad (6.1.51)$$

with numerical values for *pions*

$$b_1^2 = b_2^2 = b_3^2 = 1 + 1.2\times 10^{-3}, \quad b_4^2 = 1 - 3.79\times 10^{-3}, \quad (6.1.52)$$

and for *kaons*

$$b_1^2 = b_2^2 = b_3^2 = 1 - 2\times 10^{-4}, \quad b_4^2 = 1 + 6.1\times 10^{-4}. \quad (6.1.53)$$

As one can see, the phenomenological studies by Nielsen and Picek [47] provide a direct verification of isorelativity and relativistic hadronic mechanics, including the hypothesis [41] of speeds $c > c_o$.[11]

In fact, for pions we have $b_4 < 1$ and, consequently, speeds $c < c_o$, whereas for kaons we have $b_4 > 1$ and, therefore, $c > c_o$. Since the *charge radius* of all hadron is approximately the same, $1fm$, *the density of hadrons increases with mass.* Consequently, speeds $c > c_o$ are expected to persist for all heavier hadrons, as confirmed by subsequent data reviewed in the next sections.

The inapplicability of the conventional notions of spacetime for metrics (6.1.52)-(6.1.53), with consequential inapplicability of special relativity, are evident. The direct universality of the Minkowski-Santilli isospace and related isorelativity should equally be noted.

Intriguingly, the Minkowski-Santilli isospace requires that in the interior of hadrons we have an alteration of both space and time. Recall that the characteristic quantities characterize the isounit of the theory, Eq. (6.1.5b). Hence, from data (6.1.52), we have for pions

$$\hat{I}_{pions} = Diag(1/1.0012, 1/1.0012, 1/1.0012, 1/0.9963) =$$

$$= Diag(0.9988, 0.9988, 0.9988, 1.0037), \quad (6.1.54)$$

namely, the space isounit is smaller than 1 and the time isounit is bigger than 1. Consequently, *pions are iso-minkowskian media of Group II, Type5* (Figure 6.3).

[11]The author attempted a number of times to contact H. B. Nielsen and I. Picek at the Niels Bohr Institute in Copenhagen, to discuss the implications of their paper [47] with no replay, expectedly because such implications are in manifest conflict with organized interests on Einsteinian doctrines. The author subsequently received information that H. B. Nielsen and I. Picek had been under pressure by orthodox interests to renounce or dismiss the results of paper [47]. The Niels Bohr Institute is suggested to implement corrective measures and conduct indeed systematic studies on the inapplicability of orthodox doctrines within hadronic media so as to avoid problems of scientific ethics and accountability particularly for use of public funds.

For kaons we have the isounit

$$\hat{I}_{kaons} = Diag.(1/0.9998, 1/0.9998, 1/0.0008, 1/1.0004) =$$

$$= Diag.(1.0002, 1.0002, 1.0002, 0.9996), \tag{6.1.55}$$

namely, the space isounit is bigger than 1 and the time isounit is now smaller than 1. Consequently, *pions are iso-Minkowskian media of Group III, Type 9* (Figure 6.3).

The fundamental invariant is given by

$$x^2 = [length]^2 \times [unit]^2. \tag{6.1.56}$$

Consequently, data (6.1.52), (6.1.53) indicate that *in the interior of pion we have an isodilation of length of the order of*

$$\hat{\ell}^2 \approx .1.0012 \times \ell^2 \tag{6.1.57}$$

and an isocontraction of time of the order of

$$\hat{t}^2 \approx 0.9963 \times t^2 \tag{6.1.58}$$

while in the interior of pions we have an isocontraction of length of the order of

$$\hat{\ell} \approx 0.9998 \times \ell \tag{6.1.59}$$

and an isodilation of time of the order of

$$\hat{t} \approx 1.0004 \times t \tag{6.1.60}$$

This is a fundamental novel implication of Santilli isorelativity with vast implications at the epistemological, theoretical and experimental levels, where the novelty is given by the prediction that space and time are altered by matter as a physical medium without a direct gravitational consideration.[12]

Note that the above isodilations and isocontractions imply corresponding versions for the remaining isoactions. For instance, Isoaction V, Eq. (6.1.15) we have

$$E_{pions} = m \times V_{max} = m \times c_o \frac{b_4^2}{b_3^2} = m \times \frac{1.0037}{0.9998} =$$

$$= 1.0004 \times m \times c_o^2, \tag{6.1.61a}$$

[12]The reader should remember that the characteristic quantities do have a connection with gravitation since departures from the Minkowski metric can be interpreted as being of Riemannian character n (Section 6.1.2). However, even under such an interpretation, the prediction of alteration of space and time by isorelativity remains new, in the sense of being beyond general relativity.

$$m_{pions} = 0.9961 \times \frac{E_{pions}}{c_o^2}, \tag{6.1.61b}$$

and for kaons we have

$$E_{kaons} = m \times V_{max} = m \times c_o \frac{b_4^2}{b_3^2} = m \times \frac{0.9998}{1.0002} =$$

$$= 0.9996 \; m \times c_o^2, \tag{6.12.62a}$$

$$m_{kaons} = 1.0012 \times \frac{E_{pions}}{c_o^2} \tag{6.1.62b}$$

namely, *isorelativity predicts that the inertial mass of pions is smaller than that predicted by special relativity, while the inertial mass of kaons is bigger.* This prediction too has far reaching implications, such as the possibility of eliminating the need for the conjecture of *dark matter*, as we shall see later on in this section. The reader is encouraged to work out the remaining isoaxioms for data (6.1.61), (6.1.62).

Note that *features (6.1.61) are a consequenbce of the medium being of Group II, Type 5, and features (6.1.62) are a consequence of the medium being of Group III, Type 9.* This illustrate the profound dynamical implications of physicval media when deviating from the homogeneity and isotropy of the Minkowskian spacetime.

It should be indicated that *particles traveling in interior conditions faster than the local speed of light are not tachyons,*, or *isotachyons*, but ordinary *tardyons* or *isotardyons*. In fact, electrons traveling in water faster than the local speed of light are ordinary particles and cannot possibly be tachyons just because the speed of light is decreased. Similarly, particles traveling in the interior of kaons faster than the speed of light in vacuum, but slower than the internal maximal causal speed, are isotardyons and not tachyons or isotachyons..

In order to have true tachyons, a particle must be an isotachyon, namely, it should travel at speeds bigger than the maximal causal speed V_{max}. To the author's best knowledge, at this writing there is large experimental evidence of massive particles traveling at speeds bigger than the local speed of light, but there is no experimental evidence of true tachyons, namely, particles traveling faster than the local maximal causal speed.

6.1.9 Experimental Verifications with the Behavior of the Meanlives of Unstable Hadrons with Speed

The hyperdense character of the medium inside hadrons has been known since the discovery of protons and neutrons, and the measurement of their mass and size. In turn, dynamics within hyperdense media lead to the historical open

legacy that strong interactions have a nonlocal component due to deep wave-overlappings, namely, a condition that renders special relativity inapplicable beginning from its topology, let alone the inability to represent zero-range contact interactions extended over a volume.

Strong interactions have a range of $1lllfm$ that is essentially the size of all hadrons. It then follows that, unlike electromagnetic interactions, *a necessary condition to activate strong interactions is that hadrons enter into conditions of deep mutual overlappings* [14]. The nonlocal-integral condition of strong interactions is then beyond scientific doubt and so is the inapplicability of special relativity.

Also, to be physical, the hadronic constituents must have wavepackets of the order of the entire hadrons. This implies that, unlike the atomic constituents, *the hadronic constituents are in condition of total mutual penetration of their wavepackets, each one completely inside all others,* thus resulting, again, in a nonlocal-integral structure beyond any credible representationa capability by special relativity.

The above view *is not* in contrast with the experimental evidence that hadrons in a particle accelerator do indeed follow the laws of special relativity, because, in the high vacuum of a particle accelerator, hadrons are well approximated as being point-like particles under action-at-a-distance electromagnetic interactions, as necessary for the applicability of special relativity.

Hence, we have a dichotomy given by the exact applicability of special relativity for the center of mass behavior of hadrons in vacuum, and deviations from special relativity expected in the interior of hadrons, which dichotomy requires an experimental resolution.

Hence, the issue here addressed deals with *experimental means to detect from the outside deviations from special relativity expected in the interior of hadrons.* The answer to this question is known and it is given by *expected deviations from the prediction of special relativity on the behavior of the meanlives of unstable hadrons with speed* (or energy), i.e., deviations from the well known Einsteinian decay law

$$t = t_o \times (1 - \frac{v_k \times v_k}{c_o \times c_o})^{-1/2}. \tag{6.1.63}$$

To the authors' best knowledge, the first studies on deviations from special relativity caused by nonlocal internal effects in the structure of hadrons were conducted in 1964 by D. L. Blokhintsev and his group [48] of the JINR in Dubna, Russia. The studies were continued by L. B. Redei [49] in Italy, D. Y. Kim [50] in Canada, and others.

A rather unsettling feature of these studies was that they proposed different generalizations of the Einsteinian law (6.1.63), thus creating the problem of which law to test.

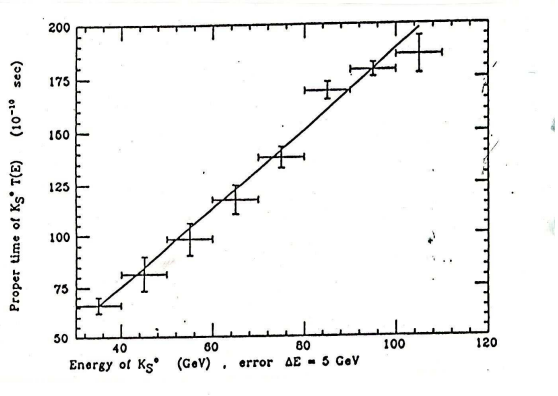

Figure 6.6. A first evidence of deviation from Einstein decay law in the meanlives of unstable hadrons is given by the linear fit of the experimental data on the K_s^o lifetime via law (6.1.63) conducted by Cardone et al [55]. The fit resulted in the value of the lifetime at rest $\tau = (0.9375 \pm 0.0021) \times 10^{-10}$ s compared to the experimental value also at rest $\tau_o = (0.8922 \pm 0.0020) \times 10^{-10}$ s with a confidence level 0.39 giving a probability of 61 % that the constant value at rest τ_o is *greater* then the actual value, namely, nonlocal internal effects are expected to *decrease* the value of the meanlife with speed. As we shall see, this behavior is connected to the *increase* of the proper time of the hadron considered compared to the proper time of an external observer. Not computed in Ref. [55] are corresponding deviations of the size of hadrons that is equally expected to deviate from Einsteinian contraction law. The reader should keep in mind that these mutations of space and time are the experimental foundation of the *isogeometric locomotion* of Chapter 13, namely, locomotion based on the control of distances via isogeometric mutations of space and time, without any Newtonian action and reaction.

In 1983 R. M. Santilli [3] proposed the iso-Minkowskian spaces $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ with *isotime dilation* as in Isoaxiom III, Eqs. (6.1.13a),

$$t = t_o \times (1 - \frac{v_k \times b_k^2 \times v_k}{c_o \times b_4^2 \times c_o})^{-1/2} = t_o \times (1 - \frac{(v_k \times v_k / n_k^2)}{(c_o \times c_o / n_4^2)})^{-1/2} \tag{6.1.64}$$

A. K. Aringazin [51] from Kazakhstan proved that the Santilli's decay isolaw is *directly universal* for all possible (signature preserving) modifications of the Minkowskian law (6.1.63) (as expected from the direct universality of isorelativity), since all generalized decay laws can be obtained as particular cases of isolaw (6.1.64) via different expansions in terms of different coefficients subjected to different truncations. Aringazin's important result is that, rather than testing a variety of seemingly different laws, the experiments can be solely conducted for isolaw (6.1.64).

The first direct experimental measurement of the behavior of the meanlife of the unstable K_os with energy was conducted in 1983 by S. H. Aronson et al. [52]

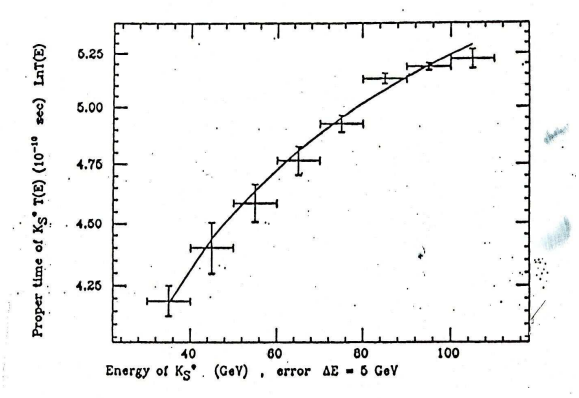

Figure 6.7. The exact fit of Santilli's iso-Minkowskian law (6.1.64) [3] provided by Cardone et al. [55] on the data of Fermilab experiment [52] from 30 to 100 GeV providing a second experimental confirmation of deviations from the Einsteinian decay law.

at Fermilab suggesting *clear deviations from the Einsteinian decat law (6.1.63) in the different energy range from* 30 *GeV to* 100 *GeV.*

Following the appearance of results [52], additional direct experimental measurements were conducted in 1987 by N. Grossman et al. [53] also at Fermilab, which tests showed *apparent verification* of the Einsteinian law (6.1.45), although in the *different* energy range from 100 to 400 GeV.

Additionally, a test of the decay law at short decay times was made by G. Alexander et al. at LEP [54], in which the events $Z^o \to \tau^+ + \tau^-$ show a clear *deviation* from the conventional law of the order of 1.1 %.

In paper [55] of 1992, F. Cardone (then of the First University in Rome, Italy) et al. proved that the Minkowski-Santilli isospace permits an exact fit of experimental data [52] (see Figure 6.6).

In the subsequent paper [56], F. Cardone et al. proved that the same Minkowski-Santilli isorepresentation unifies the seemingly discordant results of tests [52] and [53] (Figure 6.7).

In this way, Cardone et al. achieved the following numerical values of the characteristic quantities for the K^os

$$b_1^2 = b_2^2 = b_3^2 = 0.989080 \pm 0.0004, b_4^2 = 1.002 \pm 0.0007. \tag{6.1.65a}$$

$$\Delta b_k^2 = 0.007, \quad \Delta b_4^2 = 0.001. \tag{6.1.65b}$$

It is evident that the above fits constitute another experimental verification on the validity within kaons of Santilli isorelativity [3], the underlying Minkowski-Santilli isogeometry [10], and relativistic hadronic mechanics [16].

A most important feature of experimental data (6.1.65) is that theu provide an *independent confirmation f the iso-Minkowskian character of the medium within*

kaons reached in the preceding section with different procedures as being of Group III, Type 9 (Figure 6.3). Due to the general dominance of geometry over dynamics, the above independent confirmation of the iso-Minkowskian character of the medium inside kaons is the most important result of this section.

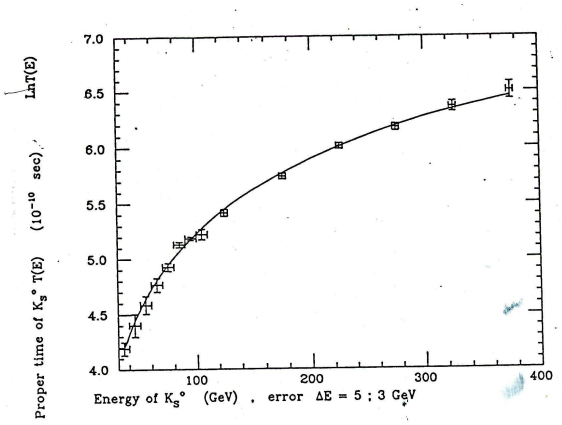

Figure 6.8. The exact fit of Santilli's iso-Minkowskian law (6.1.64) [3] provided by Cardone et al. [56] on the data of Fermilab experiment [52,53] from 30 to 400 GeV providing a third experimental confirmation of deviations from the Einsteinian decay law.

Somne of the consequences of Refs. [55,56] are the following:

1) The fits of Figure 6.6 confirm in an independent way that the maximal causal speeds in the interior of kaons is bigger than that in vacuum. In fact, values (6.1.65a) are very close to values (6.1.54) even though derived in different ways (the former via direct measurements and the latter via phenomenological calculations).

2) Results (6.1.65) confirm that the quantity $b_4 = 1/n_4$ provides a geometrization of the density of the hadron considered (again, normalized to the value $b_4 = 1/n_4 = 1$ for the vacuum), while the dependence of the characteristic quantities on the speed (or energy) is essentially in the space components $b_k = 1/n_k$,] $k = 1, 2, 3$ (also normalized to the values $b_k 1/.n_k = 1.k = 1, 2, 3$ for the vacuum).

3) Results (6.1.65) void the measurements by Grossman et al. [53] of any conclusive value, evidently because we have experimental deviations from the Minkowskian geometry even under the assumption that tests [53] are valid.

4) Results (6.1.65) establish that the rest energy of the constituents of hadrons is not given by the familiar expression $E = m \times c_o^2$, but rather by the isorenormalized Eq. (6.1.15), i.e.

$$E = m \times V_{max}^2 = m \times c_o^2 \times \frac{b_4^2}{b_3^2} = m \times c_o^2 \times \frac{n_3^2}{n_4^2}. \tag{6.1.66}$$

Since the rest energies of the particles are well known, the above isoaxiom implies that *the masses (or inertia) of the kaons are smaller than what generally assumed until now.*.

5) Results (6.1.65) establish that the frequency $\hat{\nu}$ of photons (or gluons ?) emitted in the interior of hadrons is not characterized by the traditional law $\nu = E/h$, but instead by the isorenormalized law

$$\hat{\nu} = \nu \times \frac{b_4^2}{b_3^2} = \nu \times \frac{n_3^2}{n_4^2}, \tag{6.1.67}$$

with *isoredshift* (tendency toward the red) within the physical media inside pions and *isoblueshift* (tendency toward the blue) for kaons and all other hadrons.

6) Said results establish that light emitted in the interior of hadrons is also isoredshifted or isoblueshifted, that is, it reaches the outside at a frequency smaller or bigger than that originally emitted in the interior because of mechanisms of the isospecial relativity studied later on in astrophysical verifications (essentially due to release or absorption of energy from the medium).

7) Said results establish that in the interior of kaons and all other heavier hadrons, space is contracted in the geometric sense that the Euyclidean distance becomes smaller and time flows faster than the corresppponmding quantities in the exterior. In fact, the basic units of space and time are characterized by experimental fits (6.1.65) and are given by

$$\hat{I} = (\hat{I}_{space}, \hat{I}_{times}), \tag{6.1.68a}$$

$$\hat{I}_{space} = Diag.(1.001.1.001.1.001), \quad \hat{I}_{time} = 0.9980. \tag{6.1.68b}$$

Since spacetime invariants have the structure (Sections I.3.5)

$$Invariant = (Length)^2 \times (Unit)^2 \tag{6.1.69}$$

it is evident that the increase (decrease) of a unit causes the decrease (increase) of the related length.

As an incidental note, the above features have stimulated the formulation of the so-called geometrical propulsion studied in CChapter 13, in which objects can move following a local directional change of the geometry without the application of any force visible to the outside, thus permitting, on mathematical grounds, arbitrary speeds for an outside observer.

Remarkably, features 1) to 7) are verified by all subsequent experiments, as we shall see.

A few comments are now in order. We should first indicate that the measurements by Grossman et al. [53] have been the subject of rather severe criticisms. First of all, the experimenters have made the theoretical assumption in the data elaboration of a frame in which there is no CP violation, in which case it is known

that there cannot be Minkowskian anomalies, as shown by D. Y. Kim [50] and others. Moreover, the statistics of tests [53] are insufficient for any conclusion whether in favor or against orthodox doctrines. Additional flaws of tests [53] have been identified by Yu. Arestov et al. [57]/ mThese limitatioons are discussed in detail in Appendixc 6.D.

I would like also to stress that the deviations from the Minkowskian geometry do not constitute a violation of the fundamental Lorentz symmetry. This is due to the fact that the isotopies reconstruct the Lorentz symmetry as being exact in iso-Minkowskian space, as studied in Volume I. This feature is important to disprove claims, such as that by H. B. Nielsen and I. Picek that their parameter characterizes a "violation of the Lorentz symmetry" [47]. Such a statmenmt is a mere consequence of the use for the intgerior of hadrons mathematics solely applicxable for the exterior problem in vacuum because, when the appropriate mathematics is adop[ted, the Lorentz symmetry remains fuil;lyu valid for deformation of the spacetime of type (6.1.51).

Note, however, that the Lorentz symmetry is preserved exactly at the abstract, realization free level for the nonlocal internal effects here considered. However, this is not the fate of special relativity since experimental evidence requires structural departures, such as the impossibility of assuming the speed of lighty in vacuum or inside hadrons as the maximnal causal speed in the interior of the hyperdense hadrons and other deviations represented by the Isoaxioms I-V.

The reader should be aware that the exact fits of Figures 2 and 3 were simply unavoidable, due to the direct universality of Santilli's iso-Minkowskian geometry for the representation of all infinitely possible, signature preserving deviations from the Minkowskian form. [13]

6.1.10 Experimental Verifications via the Bose-Einstein Correlation

6.1.10.A The Unavoidable Nonlocal and Non-Hamiltonian character of the Correlation

The fundamental assumption of hadronic mechanics is that strong interactions have a nonlocal component of contact, thus nonpotential type due to deep wave-overlappings at mutual distances of 1 Fermi, which component has to be represented with anything except the Hamiltonian (to prevent granting potential energy to interactions that have none, a rather common trend in the physics of the 20-th century).

[13] We should indicate the existence in the literature of several other "deformations" of the Minkowski spacetime stimulated by the isotopies [3], such as those of Refs. [58] and papers quoted therein. These deformations are formulated over conventional fields, rather than on isofields, and, as such, they verify the Theorems of catastrophic Inconsistencies of Section 6.6.

The most fundamental experimental verifications of hadronic mechanics are, therefore, those testing directly the expected nonlocality of the strong interactions. Among them, the most important tests are those on the Bose-Einstein correlation (see, e.g., Refs. [59-62]) in which:

(i) Protons and antiprotons are made to collide at very big or very small energies;

(ii) In so doing, protons and antiprotons annihilate each other in a region called the *fireball;* and

(iii) The annihilation produces various unstable hadrons whose final states are given by correlated mesons (i.e., very loosely speaking, mesons which are "in phase" with each other despite large mutual distances compared to the size of the fireball).

It is well-known in the literature that the Bose-Einstein correlation cannot be admitted by purely local theories, that is, theories dealing with a finite set of isolated point-like particles. Hence, by conception and technical realization, *the Bose-Einstein correlation is a nonlocal event.*

At this point, numerous "nonlocal theories" have been constructed for the pre-set intent of adapting physical reality to Einsteinian theories. These theories are essentially based on the attempt of reducing a nonlocal event (distributed over the finite volume of the fireball) to a finite number of isolated points, said reduction being mandatory for the applicability of the mathematics underlying Einsteinian theories, let alone their physical laws.

Since the reduction of a finite volume to a set of isolated points is a figment of academic imagination dramatically disjoint from physical reality, these "nonlocal theories" are hereon ignored.

Equally known by experts (as the author can testify), and as shown in detail below, is the fact that *the Bose-Einstein correlation is incompatible with the axiom of expectation values of quantum mechanics, thus mandating the use of a covering theory, irrespective of whether nonlocal interactions can be manipulated to verify quantum laws.*

The first exact and invariant formulation of the Bose-Einstein correlation via relativistic hadronic mechanics was done by R. M. Santilli in memoir [63] of 1962. The first of the experimental data was done by F. Cardone and R. Mignani (then at the University La Sapienza, in Rome, Italy) and provided to Santilli as a private communication. Subsequently, F. Cardone and R. Mignani provided their version of the isorelativistic treatment in paper [64] of 1996. A number of additional papers were subsequently published (such as Ref. [65]) although without structural advances.[14]

[14]It should be noted that Ref. [64] was properly written with the quotation of all originating papers and the identification of the full paternity of the various theories by Santilli. It was unfortunate that the authors subsequently elected to write a series of papers (such as those accepted by Cornell University

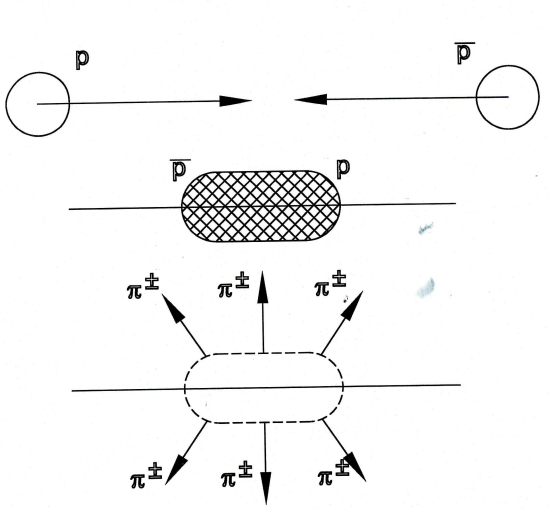

Figure 6.9. A conceptual view of the Bose-Einstein correlation in which: protons and antiprotons collide at extremely high energies; coalesce one into the other resulting into the so-called ireball (that is one of the densest media measured by mankind in laboratory until now); annihilate each other; and then result in the production of unstable particles whose final result is a large number of mesons that remain correlated at distances very large compared to the size of the fireball. Without doubt, the Bose Einstein correlation has seen the biggest scientific obscurantism in the 20-th century physics because treated under the claim that Einstein special relativity and relativistic quantum mechanics are exactly valid, while it has been known for decades that the arbitrary parameters needed for the fit of the experimental data (called "chaoticity parameters") are prohibited by the basic axioms of relativistic quantum mechanics, such as that for the vacuum expectation values (see the text). By comparison, relativistic hadronic mechanics allows an exact representation of the experimental data of the Bose-Einstein correlation while restoring the exact validity of the Lorentz and Poincaré symmetries under nonlocal and non-Hamiltonian internal effects. This episode raises the questions to be answered by the individual reader: Why do, a decreasing minority of seemingly qualified scientists continue to prefer the manipulations of the former treatment against the exact and invariant treatment of the covering theory?

In this section we shall follow the original derivation of memoir [63] due to departures from the rigorous use of relativistic hadronic mechanics of paper [64] identified below. The reader should be aware that, to avoid an excessive length, a study of the original memoir [63] is necessary for a technical knowledge of the field.

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arxiv) without any quotation whatever of Santillis originating papers. The lack of any corrective measures by both the authors and Cornell University then mandated the filing of legal action at the U./ S. federal Court one can inspect in the mirror web site http://www.scientificethics.org

6.1.10.B Conventional treatment of the Bose-Einstein correlation

We now outline the conventional treatment of the Bose-Einstein correlation via relativistic quantum mechanics by following review [59].

Consider a quantum system in 2-dimensions represented on a Hilbert space $\mathcal{H}$ with initial and final states $|a_k>$, $|b_k>$, $k = 1, 2$. The *vacuum expectation values* of an observable A are given

$$< A >=< a_k| \times A \times |b_k >= \Sigma_{k=1,2} a_k \times A_{kk} \times b_k, \tag{6.1.70}$$

which is *necessarily diagonal,* trivially, because a necessary condition for a quantity to be observable is that of being Hermitean.

The *two-points correlation function* of the Bose-Einstein correlation is defined by

$$C_2 = \frac{P(p_1, p_2)}{P(p_1) \times P(p_2)} \tag{6.1.71}$$

where $P(p_1, p_2)$ is the *two particles probability density* subjected to Bose-Einstein symmetrization, and $P(p_k)$, $k = 1, 2$, is the corresponding quantity for the k particle with 4-momentum p_k.

The two-particles density is routinely computed via the vacuum expectation value

$$P(p_1, p_2) =$$

$$= \int \psi^{\dagger}_{12}(x_1, x_2; r_1, r_2) \times \times \psi_{12}(x_1, x_2; r_1, r_2) \times F(r_1) \times F(r_2) \times d^4 r_1 \times d^4 r_2, \tag{6.1.72}$$

where ψ_{12} is the *probability amplitude* to produce two bosons at r_1 and r_2 that are detected at x_1 and x_2,

$$\psi_{12} = \frac{1}{\sqrt{2}} \times$$

$$\times (e^{i \times p_1 \times (x_1 - r_1)} \times e^{i \times p_2 \times (x_2 - r_2)} + e^{i \times p_1 \times (x_1 - p_2)} \times e^{i \times p_2 \times (x_2 - r_1)}). \tag{6.1.73}$$

Various steps (we suggest the reader to inspect in Ref. [59]) then lead to the the Gaussian form of the densities

$$F_k = \frac{1}{4 \times \pi^2 \times R^4} \times exp(-\frac{r^2}{2 \times R^2}),\ k = 1, 2, \tag{6.1.74}$$

where R is the Gaussian width and r is generally assumed to be the radius of the fireball.

Via the use of standard procedures, one reach in this way the final expression for the two-point correlation function

$$C_2 = 1 + e^{-Q^2_{12} \times R^2}, \tag{6.1.75}$$

where $Q_{12} = p_1 - p_2$ is the *momentum transfer.*

6.1.10.C Incompatibility of the Bose-Einstein correlation with Relativistic Quantum Mechanics

It is well known that the above treatment of the Bose-Einstein correlation deviates substantially from experimental data. This lead to the introduction of a first, completely unknown parameter λ, called "chaoticity parameter" and the *ad hoc* modification of law (6.1.75)

$$C_2 = 1 + \lambda \times e^{-Q_{12}^2 \times R^2}. \tag{6.1.76}$$

Note that it is impossible to derive the above parameter from any axiom of relativistic quantum mechanics. Hence, *on serious scientific grounds, the chaoticity parameter λ is the first direct evidence of the incompatibility of the Bose-Einstein correlation with quantum axioms.*

It soon turned out that adulterated expression (6.1.76) too deviates dramatically from experimental data. The problem was quickly "solved" in the conventional fashion of the 20-th century physics, via the introduction of an increasing number of completely unknown and arbitrary parameters until the desired fit of the experimental. data was achieved and then declare quantum mechanics to be exactly valid in the field.

This "solution" lead to the necessary introduction of *four completely arbitrary chaoticity parameters* and adulterated expressions of the type

$$C_2 =$$
$$= 1 + \lambda_1 \times e^{-Q_{12}^2 \times R^2} + \lambda_2 \times e^{-Q_{12}^2 \times R^2} + \lambda_3 \times e^{-Q_{12}^2 \times R^2} + \lambda_4 \times e^{-Q_{12}^2 \times R^2}, \tag{6.1.77}$$

that did eventually reach some compatibility with experimental data [59].

However, *the only scientific (that is, rigorous) way of achieving the additional terms in Eq. (6.1.77) is that via a nondiagonal formulation of the expectation values. The latter are prohibited by relativistic quantum mechanics for observable quantities as in Eq. (6.1.70).*

This establishes beyond scientific or otherwise credible doubt that *the chaoticity parameters are a direct measure of the deviation of the Bose-Einstein correlation from experimental evidence.*

Independently from that, relativistic quantum mechanics has the following insufficiencies for a serious study of the Bose-Einstein correlation:

(1) The theory can only represent the proton and the antiprotons as dimensionless points. The very existence of the fireball, let alone of the ensuring correlation, is then in question.

(ii) The above point-like abstraction of particles has a number of technical consequences, such as the factorization of the densities in Eq. (6.1.72) that, *per se,* is sufficient to prohibit correlation, as shown below;

(iii) Relativistic quantum mechanics must assume the fireball to be necessarily spherical, so as to prevent the loss of one of its central pillars, the rotational

symmetry, which feature alone is sufficient to warrant a covering theory irrespective of all other aspects, due to the dominance of spacetime symmetries over calculations.

6.1.10.D Representation of the Bose-Einstein correlation via relativistic hadronic mechanics

By falloring the first original derivation [63], we first recall that, unlike expression (6.1.70), the axiom of *isoexpectation value* for relativistic hadronic mechanics is given by

$$< \hat{A} >< \hat{a}_k| \times \hat{T} \times \hat{A} \times \hat{T} \times |\hat{b}_k >= \Sigma_{ijk=1,2}\, \hat{a}_i \times \hat{T}_i^j \times \hat{A}_{jj} \times \hat{T}_j^k \times \hat{b}_k, \quad (6.1.78)$$

where $\hat{T}$ is the isotopic element, and the "hat" denotes quantities defined on isospaces over isofields.

The main new feature is that the operator $\hat{A}$ must be Hermitean, thus diagonal, to be observable,[15] but the isotopic element does not need to be diagonal.

Santilli main contributions in memoir [63] are the proof that:

(i) *The Bose-Einstein correlation is incompatible with the axisms of relativistic quantum mechanics because of the impossibility to admit off-diagonal terms in the two-poingt correlation function from unadulterated first principles, and otehr reasons;* and

ii *The Bose-Einstein correlation is directly compatible with the axioms of the covering relativistic hadronic mechanics because of the admission of nonlocal non-Hamiltonian interactions and the appearence of off-diagonal terms from first principles.*

The rest is given by a mere application of relativistic hadronic mechanics. We assume at the foundation of the treatment Santilli isorelativity with Minkowski-Santilli isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$, isoinvariant, isometric, isotopic elementand isounit given respectively by [3]

$$\hat{x}^{\hat{2}} = (\hat{x}^\mu \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^\nu) \times \hat{I} = [x^\mu \times (\hat{T}_\mu^\nu \times \eta_{\nu\rho}) \times x^\rho] \times \hat{I} \ \in \ \hat{R}, \quad (6.1.79a)$$

$$\hat{\eta} = Diag.(b_1^2, b_2^2, b_3^2, -b_4^2) \times \Gamma = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, -1/n_4^2) \times \Gamma, \quad (6,1,79b]$$

$$\hat{T} = Diag.(b_1^2, b_2^2, b_3^2, b_4^2) \times \Gamma = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2) \times \Gamma, \quad (6.1.79c)$$

$$\hat{I} = Diag.(1/b_1^2, 1/b_2^2, 1/b_3^2, 1/b_4^2) \times \Gamma^{-1} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times \Gamma^{-1}, \quad (6.1.79d)$$

$$b_\mu = b_\mu(t, x, p, E, ...) > 0, n_\mu = n_\mu(t, x, p, E, ...) > 0, \quad (6.1.79e)$$

$$\hat{T} = \hat{T}(t, x, p, E, ...), \ \ \hat{I} = \hat{I}(t, x, p, E, ... = 1/\hat{T}), \quad (6.1.79f)$$

[15] Recall from Chapter I.3 that *iso-Hermiticity coincides with conventional Hermiticity.* Hence, all quantities that are observable for quantum mechanics remain observable for hadronic mechanics.

where: isoinvariant (6.1.79a)must be an element of the isofield $\hat{R}$ and, consequently, must have the structure of $\hat{x}^{\hat{2}} = n \times \hat{I}$, where n is a real number; the spacetime isocoordinatescoordinates must also be elements of the isofield, thus have the form $\hat{x} = x \times \hat{I},\ x = (x^\mu), \mu = 1, 2, 3, 4$; isoproducts of the isocoordinates with a generic quantity Q can be reduced for simplicity to ordinary products, $\hat{x}\hat{\times}Q = (x \times \hat{I})\hat{T} \times Q = x \times Q$ as done in isoinvariant (6.1.79a); we continue to use both notations for the characteristic quantities, $b_\mu = 1/n_\mu$ following their original formulation in [3,63] because handy in various applications; the quantity Γ is a $2x2$-matrix to be identified shortly; and one should keep in mind the explicit dependence of the characteristic quantities in time t, coordinates x, momenta p, energy E and any need additional quantity.[16]

It should be stressed that *the characteristic quantities must represent physically measurable quantities, namely,* $1/b_k^2 = n_k^2,\ k = 1, 2, 3,$ *must characterize the semiaxes of the Bose-Einstein fireball according to a proper normalization (see below), and* $1/b_4^2 = n_4^2$ *must characterize the density of the fireball in a way compatible with other experiments.*

To state this crucial point explicitly, the chaoticity parameters $\lambda_\mu,\ \mu = 1, 2, 3, 4$ are completely arbitrary and without any possible physical meaning. By constrast, the characteristic quantities $1/b_\mu^2 = n_\mu^2$ must represent concrete physical features that with experimentally verifiable numerical values as a condition for the isorepresentation to be consistent.

As a concrete illustration, in the event the fit of the experimental data yields values of the type $b_1^2 = b_2^2 = b_3^2$, the emerging isorepresentation would be inconsistent because the Bose-Ein stein fireball cannot possibly be a sphere due to the extreme energies of the collision. As a result, said fireball must be a very elongated sheroidalellipsoid, for instance, of the type $b_3^2 \gg b_1^2 = b_2^2$.

As an additional and independent condition for consistency, the numerical value of the density $b_4^2 = 1/n_4^2$ must be compatible with numerical values from different experiments on comparable densities, such as those for protons and neutrons.[17]

By continuing to follow the original derivation [63], we now represent the correlation on an iso-Hilbert space $\hat{\mathcal{H}}$ with initial and final isostates $|\hat{a}_k >,\ |\hat{b}_k >$, $k = 1, 2$, and the non-diagonal isotopic element (6.1.79c) in the explicit form

$$\hat{T} = Diag.(b_1^2, b_2^2, b_3^2, b_4^2) \times \Gamma = Diag.(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2) \times \Gamma, \qquad (6.1.80a)$$

[16]It is known since the original proposal of 1978 [14] that the isotopies restrict the topological character of the isounit but otherwise leave its functional dependence completely unrestricted. This feature is at the foundation of the representation by hadronic mechanics of features such as density, extended shape, their deformation in time,etc., that are unthinkable with quantum mechanics.

[17]As we shall see in the next section, the value of the density of the Bose-Einstein fireball allows a numerically exact representation of all characteristics of the neutron as a hadronic bound state of an (iso)proton and an (iso)electron, including the numerical value of the anomalous magnetic moment, size, meanlife and other features that cannot even be treated with quantum mechanics.

$$\Gamma =$$

$$= \begin{pmatrix} A & B \times |1 - \exp(\int dx^4 \times \psi^{\dagger}_{b2} \times \psi_{a1})| \\ C \times |1 - \exp(\int dx^4 \times \psi^{\dagger}_{a2} \times \psi_{b1})| & D \end{pmatrix} \tag{6.1.80b}$$

and the quantities $A, B, C.D$, are restricted by the condition[18]

$$Det\ \Gamma = 1. \tag{6.1.81}$$

As one can see, when used in the isoexpectation value (6.1.79), isotopic element (6.1.80):

(a) Allows indeed off-diagonal terms in the isoexpectation values;

(b) Represents the overlapping of the wavepackets of particles via the integrals in the exponents of Γ;

c) Eliminates all correlations when said overlapping is null, i.e., for the limit under condition (6.1.81)

$$Lim_{\int dx^4 \times \psi^{\dagger}_{ij} \times \psi_{jk} = 0} \Gamma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{6.1.82}$$

Next, the isorepresentation is given by a trivial isotopy of the conventional treatment [59], with the use now of the nontrivial isoexpectation values (6,1,78). We then have the *two-points isocorrelation function*

$$\hat{C}_2 = \frac{\hat{P}(p_1, p_2)}{\hat{P}(p_1) \times \hat{P}(p_2)} \tag{6.1.83}$$

where: $\hat{P}(p_1, p_2)$ is the *two-particle isoprobability density* subjected to proper symmetrization; $\hat{P}(p_k),\ k = 1, 2$, is the corresponding quantity for the k particle with 4-momentum p_k; and we ignore hereon the "hat" on variables for simplicity of notation.[19]

The two-particles isoprobability density is noa given by the isoeigenvalue expression

$$\hat{P}(p_1, p_2) =$$

$$= \int \hat{\psi}^{\dagger}_{12}(x_1, x_2; r_1, r_2) \times \hat{T} \times \hat{\psi}_{12}(x_1, x_2; r_1, r_2) \times \hat{F}(r_1, r_2) \times d^4 r_1 \times d^4 r_2, \tag{6.1.84}$$

[18]Values of determinant (6.1.81) different than 1 would merely im-ly a different renormalization of the characteristic quantities.

[19]On rigorous grounds, it should be noted that isocorrelkation function (6.1.83) is an isoscalar as it is the acse for the isoliune ele,ment (6.1.79a). This property is automatically guaranteed by the isue of an isoquotient. Fpor these mathe,matical aspects, we recommend the noninitiated reader to study Chapter I.2.

where: $\hat{\psi}_{12}$ is the *isoamplitude* for the production, as in the conventional treatment, two bosons at r_1 and r_2 that are detect ed x_1 and x_2; and the *isowavefunction* $\hat{\psi}_{ij}$ is given by a trivial isotopy of the conventional expression.

Note the crucial difference between Eq. (6.1.84) and (6.1.72) given by the isotopic lifting of all quantities and their operations and the appearance in the former of the isotopic element allowing the mixing of nondiagonal terms.

Another major difference between conventional and isotopic treatments is that *the probability densities for particles* 1 *and* 2 *are factorized in the conventional treatment (6.1.72), while they cannot be factorized in the isotopic treatment.* This is due to the fact that protons, antiprotons, and all produced mesons are point-like for relativistic quantum mechanics (as a necessary condition for a credible use of the underlying mathematics), while they are extended for the covering treatment. Hence, the separation of the densities would be equivalent to annulling all correlations.

The isotopy of the conventional treatment referred to isoexpectation values (6.1.78), including the symmetrization of the isotopic element and isowavefunctions for all possible directions, plus the assumed normalizations then leads to isodensity (9.11) of Ref. [63], i.e.,

$$\hat{F}(r_1, r_2) = \Sigma_\mu \hat{\eta}_{\mu\mu} \times \frac{b_m u^2}{4 \times \pi^2} \times e^{-\frac{1}{2} \times r^2 \times b_\mu^2} \tag{6.1.85}$$

where $r$can be interpreted as the radius of the sphere in which the correlated mesons are detected.

The continuation of calculations via a simple isotopy of the conventional treatment leads to the following expression of the two-points isocorrelation function derived for the first time in Eq. (9.12), p. 112, Ref. [63],

$$\hat{C}_2 = 1 + \Sigma_\mu \, b_\mu^2 \times e^{-\frac{\hat{Q}_{12}^{\hat{2}}}{b_\mu^2}} =$$

$$= 1 + b_1^2 \times e^{-\frac{\hat{Q}_{12}^{\hat{2}}}{b_1^2}} + b_2^2 \times e^{-\frac{\hat{Q}_{12}^{\hat{2}}}{b_2^2}} + b_3^2 \times e^{-\frac{\hat{Q}_{12}^{\hat{2}}}{b_3^2}} - b_4^2 \times e^{-\frac{\hat{Q}_{12}^{\hat{2}}}{b_4^2}}, \tag{6.1.86}$$

where, again, $Q_{12} = p_1 - p_2$.

The case of the three-points and higher isocorrelation functions is treated in Ref. [63], and it is here ignored for simplicity.

The attentive reader will have noted that, to prevent a catastrophic mixing of conventional and isotopic treatments, the *isosquare* of Eq. (6.1.86) is explicitly given by

$$\hat{Q}_{12}^{\hat{2}} = \hat{Q}_{12}^{\mu} \hat{\times} \hat{\eta}_{\mu\mu} \hat{\times} \hat{Q}_{12}^{\mu} = Q_{12}^{\mu} \times \hat{\eta}_{\mu\mu} \times Q_{12}^{\mu}, \tag{6.1.87}$$

multiplied by the isounit that is hereon ignored for simplicity.

At this point, the exponent of expression (6.1.86) ,must be reduced to quantities actually measured in the tests, the momentum transfer q_t and the characteristics values of the fireball. This reduction was also done in Ref. [63] and resulted in the following expression

$$\frac{\hat{Q_{12}^2}}{b_\mu^2} = \frac{q_t^2}{b'^2_\mu} \tag{6.1.88}$$

where $b'\mu^2$ represents renormalized expressions of the characteristic quantities. Howewer, their numerical value is unknown prior to fits of the experimental data. Hence, we assume $b'\mu^2 \equiv b_\mu^2$.

The final expression of the two-points isocorrelation function, derived for the first time in Eq. (9.25), page 119, Ref. [63] is given by one of the following equivalent expressions first achieved in Ref. [63], Eqs. (10.7), (10.8), (10.9), pages 121,122

$$\hat{C}_2 = 1 + \frac{1}{3} \times \Sigma_\mu \, b_\mu^2 \times e^{-\frac{q_t^2 \times K^2}{b_\mu^2}} =$$

$$= 1 + \frac{1}{3} \times b_1^2 \times e^{-\frac{q_t^2 \times K^2}{b_1^2}} + \frac{1}{3} \times b_2^2 \times e^{-\frac{q_t^2 \times K^2}{b_2^2}} + \frac{1}{3} \times b_3^2 \times e^{-\frac{q_t^2 \times K^2}{b_3^2}} - \frac{1}{3} \times b_4^2 \times e^{-\frac{q_t^2 \times K^2}{b_4^2}}, \tag{6.1.89a}$$

$$K^2 = b_1^2 + b_2^2 + b_3^2. \tag{6.1.89b}$$

By absorbing the k^2 term into the characteristic quantities, we have the equivalent form

$$\hat{C}_2 = 1 + \frac{K^2}{3} \times \Sigma_\mu \, b_\mu^2 \times e^{-\frac{q_t^2}{b'^2_\mu}}, \tag{6.1.90a}$$

$$b'_\mu = b_\mu / K^2. \tag{6.1.90b}$$

Another isorepresentation is given by (page 129, ref. [63])

$$\hat{C}_2 = 1 + \times \Sigma_\mu \, b_\mu^2 \times e^{-\frac{q_t^2 \times K^2}{b_\mu^2}}, \tag{6.1.91a}$$

$$K^2 = b_1^2 + b_2^2 + b_3^2 = 3. \tag{6.1.91b}$$

In the above isorepresentations, all operations are now conventional. Hence, the above expressions are the *projections* in our spacetime of the isocorrelation functions on isospace.

6.1.10.E Reconstruction of the Exact Poincaré Symmetry under Nonlocal and Non-Hamiltonian interactions of the Bose-Einstein Correlation

As indicated earlier, a crucial insufficiency of the conventional treatment of the Bose-Einstein correlation, is the inability to provide an invariant representation of

the fireball, due to its prolate character under which the conventional rotational symmetry no longer applies.

As studied in detail in Volume I, a central objective of hadronic mechanics is to restore the exact character of basic spacetime and other symmetries when popularly believed to be "broken" due to the use of excessively elementary or insufficient mathematics. It is important to show the reconstruction of the exact rotational and other spacetime symmetries for the isorelativistic treatment of the Bose-Einstein correlation as done in memoir [63]. In fact, the most important predictions of the isorepresentation characterize structural deviations from s[special relativity whose understanding, let alone rigorous derivation, can only be done at the level of isosymmetries.

With respect to Fig. 6.9, recall that the Bose-Einstein correlation creates a fireball characterized by a spheroid prolated in the direction of the proton-antiproton flight. Following its creation, the fireball expands rapidly, resulting in the correlated mesons. Consequently, the original characteristic quantities, here denoted $b'^2_k = 1/n^2_k$, have an explicit dependence on time.

By assuming that the prolateness is along the third axis, we have

$$K^2(t) = b'^2_1(t) + b'^2_2(t) + b'^2_3(t) \neq const, \;\; b'^2_3(t) \gg b'^2_1(t) = b'^2_2(t), \tag{6.1.92}$$

However, the fireball must preserve its shape during its expansion when considered as isolated from the rest of the universe. This implies that all characteristic quantities have the same factorizable time dependence, and we shall write

$$K^2(t) = k^2 \times f(t), \;\; b'^2_k(t) = f(t) \times b^2_k, \;\; k, b_k = const. \tag{6.1.93}$$

This implies the following important property

$$\frac{b'^2_k(t)}{b'^2_1(t) + b'^2_2(t) + b'^2_3(t)} = b^2_k = const. \tag{6.1.94}$$

that has been used for isorepresentation (6.1.89).

In conclusion, the fireball can be studied at the time of its formation with constant characteristic quantities $b^2_k = 1/n^2_k$ and the following isoinvariant formulated on the Euclide-Santilli isospace with isounit

$$\hat{R}^{\hat{2}} = (x^2_1 \times b^2_1 + x^2_2 \times b^2_2 + x^2_3 \times b^2_3) \times \hat{I} = (\frac{x^2_1}{n^2_1} + \frac{x^2_2}{n^2_2} + \frac{x^2_3}{n^2_3}) \times \hat{I}, \tag{6.1.95a}$$

$$\hat{I} = Diag.(1/b^2_1, 1/b^2_2, 1/b^2_3) = Diag.(n^2_1, n^2_2, n^2_3). \tag{6.1.95b}$$

As studied in Chapter 1.3, *isoinvariant (6.1.94) characterizes the perfect sphere on isosp[ace over the isofield, caleld the isosphere, and characterizes an ellipsoid only in its projection in our space.* This is due to the mechanism of the isotopies

that, in this case, must be applied to the conventional sphere in conventional space, assumed for simplicity to have radius $r = 1$. In this case the semiaxes $r_k^2 = 1$ are indeed lifted into those of the ellipsoid, $r_k^2 \to b_k^2$, but the corresponding units are lifted by the *inverse* amount, thus preserving the perfect spheridicity on isospace over isofields,

$$r_k^2 \to b_k^2, \;\; 1_k^2 \to 1/b_2^2, \tag{6.1.96}$$

Once the perfect sherical character of the fireball on isospace is understood, the reconstruction of the exact rotational symmetry for ellipsoids is trivial. In fact, we have the *Lie-Santilli isoalgebra* $\hat{O}(3)$(Ref. (63], page 115)

$$J_k = \epsilon_{ijk} \, r_i \times p_j, \tag{6.1.97a}$$

$$[J_i\hat{,}J_j] = J_i \times \hat{T} \times J_j - J_j \times \hat{T} \times J_i = b_k^2 \times J_k, \tag{6.1.97b}$$

$$J^{\hat{2}} = J \times \hat{T} \times J, \tag{6.1.97c}$$

where we have ignored for simplicity factorization of the isounit.

It is trivial to prove that the above isorotational algebra is isomorphic to the conventional algebra (due to the positive-definite character of the characteristic quantities b_k^2), $\hat{O}(3) \approx O(3)$, and this proves the reconstruction by hadronic mechanics of the exact rotational symmetry when popularly believed to be broken, a feature proved since the original proposal [14] of 1978.[20]

The reconstruction of the *exact Lorentz symmetry* $\hat{O}(3.1)$ for the Bose-Einstein correlation follows the same lines. Since the speed of light is assumed to be locally varying, we have *mutated light cones* of the type, e.g., in the (3.4)-plae

$$n\hat{2} = (x_3^2 \times b_3^2 - x_4^2 \times b_4^2) \times \hat{I} = \frac{x_3^2}{n_3^2} - \frac{x_4^2}{n_4^2} \times \hat{I}, \tag{6.1.98a}$$

$$\hat{I} = Diag.(1/b_3^2, 1/b_4^2) = Diag.(n_3^2, n_4^2). \tag{6.1.98b}$$

It is again easy to see that *the mutated light cone in our spacetime is the perfect light cone in isospace, called light isocone,* because, again, the mutation of each axis is complemented by the inverse mutation of the corresponding unit. The preservation of the original numerical values is then assumed by the structure of the isoinvariant, Eq. (6.1.69).

Once the light cone is exactly reconstructed on isospace for locally variable speeds of light, the reconstruction of the exact Lorentz symmetry became a trivial calculations (see Vol. I for brevity) and it is here left as an important exercise for the interested reader.

[20]See EHM Vol. II for realizations of the isorotational symmetry with conventional structure constants.

The same situation occurs for translations, resulting in the reconstruction of the exact Poincaré symmetry $\hat{P}(3.1)$ for all possible nonlocal and non-Hamiltonian realizations of the Bose-Einstein correlation, as first proved in Refs. [3.4].

Recall that isorelativity and special relativity coincide at the abstract, realization-free level, as confirmed by the speed of light *in vacuum* to be the constant maximal causal speed in *isospace.* Consequently, *the understanding of the isorepresentation of the Bose-Einstein correlation requires the knowledge that, rather than "violating" special relativity as at times perceived, in reality allows the maximal possible enlargement of the arena of applicability of Einsteinian axioms.*

6.1.10.F Theoretical Predictions

It is important now to identify the theoretical prediction of isorepresentation (6.1.89) so that we can compared them below with experimental data.

Prediction 1: The minimum value of the two-points isocorrelation function, first identified in Ref. [63],

$$\hat{C}_2^{Min} = 1, \tag{6.1.99}$$

evidently holding for infinite momentum transfer.

Prediction 2: The maximal value is predicted to be

$$\hat{C}_2^{Min} = 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1.67. \tag{6.1.100}$$

evidently holding for null momentum transfer. Prior to any fit, we can say that, for the isorepresentation to be valid, all data must remain between the above minimum and maximum values.

Prediction 3: Isorepresentation (6.1.89) also predicts the *maximum value of the isodensity,* occurring for $\hat{C}_2^{Max}$ (Eq. (10.27, page 127, Ref. [63]]. In fact, for $q_t = 0$ we have no correlations, in which case we have

$$b_k^2 = 1,\ k = 1, 2, 3,\ K^2 = b_1^2 + b_2^2 + b_3^2 = 3, \tag{6.1.101a}$$

$$\hat{C}_2^{Max} = 1 + \frac{K^4}{3} - \frac{K^2 \times b_4^2}{3} = 1.67, \tag{6.1.101b}$$

$$b_4^2 = 2.33,\ \ n_4^2 = 0.429,\ \ b_4 = 1.526,\ \ n_4 = 0.654. \tag{6.1.101c}$$

Prediction 4: By assuming that $K^2 = 3$ and that the fireball is very prolate, e.g., with $b_3^2 = 30 \times b_1^2 = 30 \times b_2^2$, we obtain the following prediction on the remaining characteristic quantities

$$b_1^2 = b_2^2 = 0.043,\ \ b_3^2 = 2.816,$$

$$b_1^2 = n_1^2 = n_2^2 = 10.666,\ \ n_3^2 = 0.355 \tag{6.1.102}$$

Needless to say, the above prediction is mainly referred to the *type* of isospacetime inside the fireball, rather than the numerical values per se, due to the lack of knowledge at this point of the prolateness of the fireball.

From the above predictions we then derive the following expected values[21]

$$\hat{\beta}^2 = \frac{b_3^2}{b_4^2} \times \beta > \beta^2, \tag{6.1.103a}$$

$$\hat{\gamma} = \frac{1}{(1 - \hat{\beta}^2)^{1/2}} < \gamma. \tag{6.1.103b}$$

From the isoaxioms of Section 6.3, we then have the following additional predictions:

Prediction 5: The maximal causal speed within the fireball is *bigger* than that in vacuum,

$$V_{max} = c_o \times \frac{b_4}{b_3} > c_o; \tag{6.1.104}$$

Prediction 6: Time t within the fireball flows faster than time predicted by special relativity),

$$t = \hat{\gamma} \times t_o > \gamma \times t_o; \tag{6.1.105}$$

Prediction 7: Lengths ℓ inside the fireball are smaller than lengths predicted by special relativity,

$$\ell = \hat{\gamma}^{-1} \times \ell_o < \gamma \times \ell_o; \tag{6.1.106}$$

Prediction 8: Mass behavior with speed is bigger than that predicted by special relativity,

$$m = \hat{\gamma} \times m_o > \gamma \times m_o; \tag{6.1.107}$$

Prediction 9: The energy equivalence of the fireball is bigger than that predicted by special relativity or, equivalently, for a given energy, the mass is smaller),

$$E = m \times V_{max} > E_o = m \times c_o^2; \tag{6.1.108}$$

Prediction 10: Frequencies of light emitted inside the fireball, exist the same *isoblueshifted,* namely, with an increase of frequency as compared to the corresponding behavior p[predicted by special relativity

$$\omega = \hat{\gamma} \times \omega_o. \tag{6.1.109}$$

[21]The reader may note the use of the absolute value for the definition of the $\hat{\gamma}$ in footnote 42, page 123, Ref. [63]. This was due to the lack, at the time of that memoir (1992), of experimental data on the maximal causal speed within physical media, especially those opaque to light. This information was reached subsequently with the identification of Isoaxiom I. Eq. (6.1.11), with $V_{max} = c_o \times b_4/b_3$, in which case the speed v is always smaller than or equal to V_{max}, $\hat{\beta} \leq 1$, $\hat{\gamma}$ can only assume real values, and the absolute value is no longer necessary.

Prediction 11: The speed of light within the fireball is bigger than that in vacuum,

$$c = c_o > b_4 > c_4, \tag{6.1.110}$$

by smaller than the maximal causal speed

$$c = c_o \times b_4 < V_{max} = c_o \frac{b_4}{b_3}. \tag{6.1.111}$$

As one may recall from Volume I, the isoblueshift of light is nothing mysterious because it is a mere manifestation of the high energy density of the medium in which light propagates. Isoblueshift, as the increase of frequencies as predicted by special relativity in vacuum, is then a mere consequence of the medium transfer energy to light. A similar situation occurs for all other predictions.

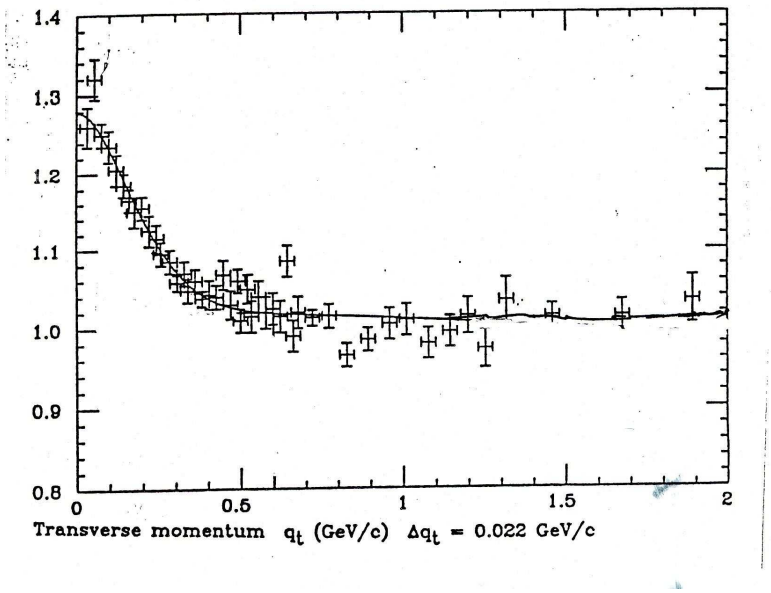

Figure 6.10. The exact fit of Santilli's two-point isocorrelation function (6.1.89) of the Bose-Einstein correlation at high energy made via the use of the experimental data from the UA1 tests at CERN [66]. The fit was done by F. Cardone and R. Mignani via a private communication to the author of 1992.

6.1.10.G Experimental verifications

It is rewarding for the author to report that *the fit of the experimental data on the Bose-Einstein correlation at high energy with the data of the UA1 experiments at CERN [66] have confirmed all the above predictions beyond the most optimistic expectations.*

The fit of Eq. (6.1.89) presented in Figure 5, page 129, Ref. [63] was conducted by F. Cardone and R. Mignani in 1992, initially reported to the author as a private communication, and then published in Ref. [64] of 1996, Table 1, page 441, resulting in the following numerical valkues of the characteristic functions for the fireball of the Bose-EWinstein correlation

$$b_1 = 0.267 \pm 0.054,\ b_2 = 0.437 \pm 0.035,\ b_3 = 1.661 \pm 0.013,\ b_4 = 1.653 \pm 0.015, \tag{6.1.112b}$$

A most important feature of the above data is that they *characterize the medium inside the fireball as being iso-Minkowskian of Group III, Type 9, thus confirming that all hadrons heavier than kaons have the same iso-Minkowskian features.* As we shall see, these geometric characterizations have primary relevance for further advances.

The fit of Figure 6.10 and the above values provide the following experimental verifications:

(1) The experimental data do indeed lie between the theoretically minimum (6.1.99) and maximal value (6.1.100);

(2) The experimental data confirm all eleven theoretical predictions (6.1.101) to (6.1.111);

(3) The experimental confirm the reconstruction of the exact character of the Poincaré symmetry for the Bose-Einstein correlation.

77).

In summary, *the fit of Figure 6.10 provides the fourth direct experimental verification of Santilli isorelativity and relativistic hadronic mechanics, this time, in their most fundamental assumption, the historical legacy of the nonlocality of strong interactions. In particular, this additional experimental verification is fully compatible with all preceding ones.*[22]

[22]The reader should be aware of the following comments on the fit of the UAI data done in Ref. [64]:

1) Fit [64] is done for *eight* parameters, the $b_\mu,\ \mu = 1, 2, 3, 4$ of the original derivation [63], plus four new parameters a_μ. This assumption turns the analysis equivalent to the conventional one, in the sense that four out of the eight parameters are equivalent to the chaoticity parameters of Eq. (6.1,77) because the Bose-Einstein correlation can only characterize *four* physical quantities, the three semiaxes of the fireball and its density. The reader shoudl be aware that the additional four parameters a_μ are inessential for the fit. Hence, relativistic hadronic mechanics requires no free parameters for the fit.

2) The redundancy of four out of eighnt parameters of fit [64] is confirmned by the fact that the b_μ and a_μ parameters are proportional to each otehr, because

$$b_1 = 0.267,\ \ b_2 = 0.0437,\ \ b_3 = 1.661, \tag{a}$$

$$a_1 = 0.053 \times 10^{-13},\ \ a_2 = 0.086 \times 10^{-13},\ \ a_3 = 0.328 \times 10^{-13} \tag{b}$$

with ratios

$$\frac{b_1}{a_1} = 5.037 \times 10^{13},\ \ \frac{b_2}{a_2} = 5.081 \times 10^{13}.\ \ \frac{b_3}{a_3} = 5.064 \times 10^{13}, \tag{c}$$

in which the very small differences of the above ratios being well within the error.

The above proportionality eliminates the need for one of the two sets of parameters because, as stressed in Ref. [63], *the characteristic quantities are always defined up to an arbitrary factor in view of the*

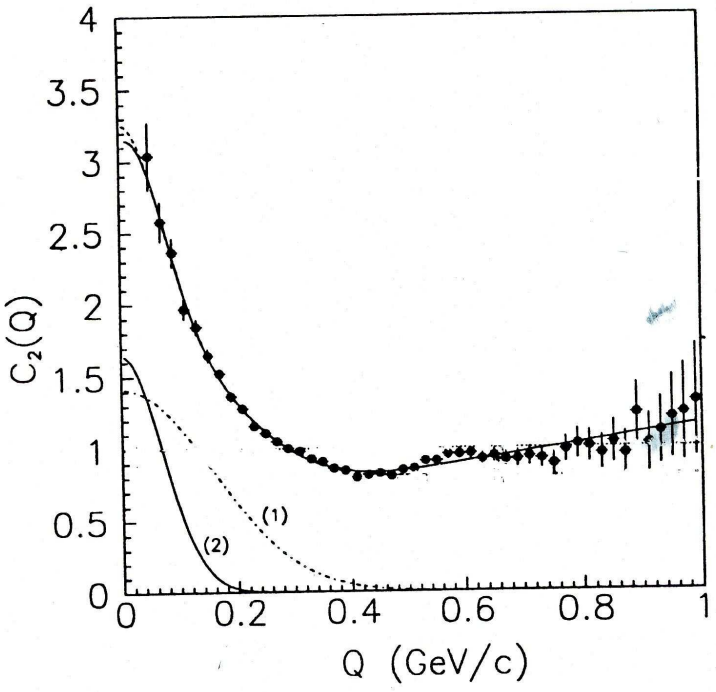

Figure 6.11. An illustration of another exact fit of the Bose-Einstein correlation from first axiomatic principles, this time of the proton-antiproton annihilation at very low energies, which can be obtained via the methods of this section. Its explicit study is ;left as an instructive exercise for the interested reader.

The repetition of the above analysis and related verification with the experimental data of the Bose-Einstein correlation at low energies (Figure 6.11) is left as an instructive exercise for the interested reader.

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isotopic invariance (6.1.69), that can be explicitly written for an arbitrary (non-null) constant C (see also Eq. (6.1.79))

$$\hat{x}^{\hat{2}} = [x^\mu \times (\hat{T}^\nu_\mu \times \eta_{\nu\rho}) \times x^\rho] \times \hat{I} = [x^\mu \times (C \times \hat{T}^\nu_\mu \times \eta_{\nu\rho}) \times x^\rho] \times (C^{-1} \times \hat{I} = \hat{x}^{\hat{2}}. \qquad (d)$$

This the reason that the characteristic quantities are normalized to the value of the vacuum, $b_\mu = 1$.

3) All formulations of Ref. [64] are based on spaces defined over conventional fields, i.e., they deal with conventional "deformations," thus dealing with conventionally nonunitary time evolutions. As a result, Ref. [64] activates the *Theorems of catastrophic Mathematical and Physical Inconsistencies* studied in detail in Section 1.5, and briefly outlined in Section 6.6.. Nevertheless these inconsistencies do not apply to values (6.1.112) since the latter are obtained by fitting expression (6.1.89) already reduced to formulations on conventional spaces over conventional fields.

Another instructive exercise for readers interested in learning hadronic mechanics is to re-derive the entire results of this section via the simple method of a nonunitary transform of the conventional treatment according to Eqs. (6.1.22) (see Section 1.3.5 for more details).

In closing, the author would like to express his sadness for the excessive abuses of the name "Einstein" through the 20-th century and continuing to this day. There is no doubt that Albert Einstein is the biggest scientist of the 20-th century, with historical contributions to mankind deserving the highest respect by all.

However, it is equally true that Albert Einstein is the scientist most abused in the history of science because mediocre academicians improperly used and abused his name for personal gains in money, prestige and power.

The use of the name "Einstein" in the "Bose-Einstein correlation" has been one of several cases of abuses of Einstein's memory because Einsteinian theories are *inapplicable* (rather than "violated") in the field, since Einstein never studied dynamical problems in the interior of hyperdense media, such as the fireball, that were inconceivable at his time.

The difference in stature between Einstein and his followers is established by the writings. Einstein has a justly deserved, towering place in the history of science because he clearly identified in his limpid writings the arena of applicability of his theories, point particles and electromagnetic waves propagating in vacuum. The comparatively lilliputian dimension of physicists abusing his name is set in history by the absence, for evident political reasons, of identification of limitations that are inherent in any physical theory.

6.1.11 Experimental Verifications in Astrophysics

One of the unsolved mysteries of contemporary astrophysics is the experimental confirmation (see Ref. [67]) of the hypothesis (see Refs. [68,69]) that certain *quasars* are physically connected to associated *galaxies,* even though they have dramatic differences in their *cosmological redshifts* (see Figure 6.12).

The Einsteinian treatment of cosmological redshift requires its interpretation via *motion in vacuum away from us,* resulting in the well known *expansion of the univ erse.* However, the evidence that quasars and galaxies with dramatically different redshifts are physically connected, thus move with the same speed, prevents any serious or otherwise scientific representation via Einsteinian theories, whether in Minkowski or Riemannian spaces.

Numerous interpretations of the above anomalous occurrence have been attempted, such as the hypothesis that the difference in cosmological redshift is due to *creation of matter* within the quasars [69]. However, none of these interpretations have acquired the necessary numerical representation for scientific credibility.

In 1991, Santilli [6] proposed the simplest possible explanation according to which the indicated difference in cosmological redshifts is merely due to *the slow-down of the speed of light in the huge quasar chromospheres* (that can be as large as entire galaxies), similar to the slow-down of the speed of light in our atmosphere (Section 6.1.7). As a result, light exits the quasar chromospheres already redshifted. A similar phenomena does not exist in the same magnitudes for a galaxy because their stars are isolated in space, and have dramatically smaller chromospheres. In this way, light from physically connected quasars and galaxies having the same expansion speed, can reach us with dramatically different redshifts.

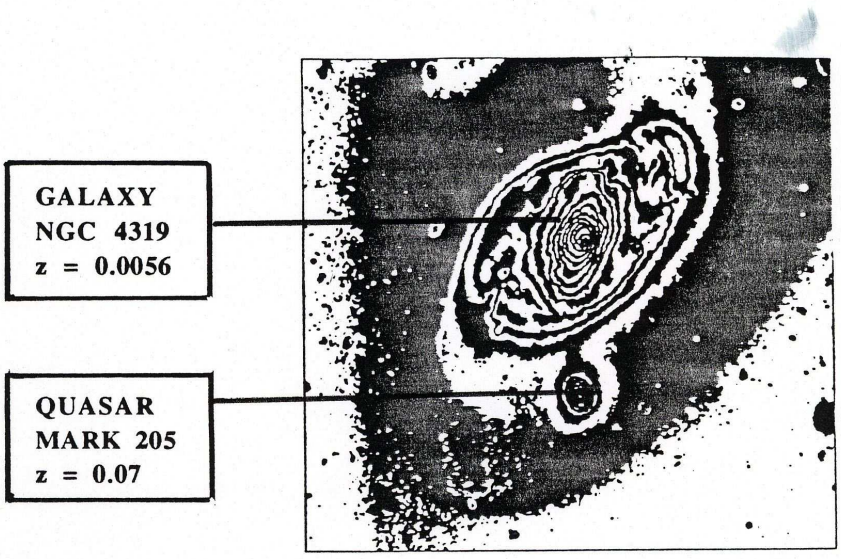

Figure 6.12. An example of clear evidence of astrophysical conditions beyond the capabilities of Einsteins special and general relativities or, equivalently, beyond the Minkowskian and Riemannian geometries: the experimentally verified (Ref. [67]) physical connection between the galaxy NGC 4319 and the quasar Mark 205, via the superposition of several gamma spectroscopic plates. By contrast, the quasar Mark 205 has a redshift with $z = 0.07$, while the associated galaxy NCG 4316 has a redshift of only $z = 0.0056$. The interpretation of this difference requires necessary departures from the Minkowskian and Riemannian geometries, because such a large difference would require that the quasar has at least 104 times the speed of the galaxy, under which conditions the quasar and its associated galaxy would have separated completely billions of years ago. Santilli's iso-Minkowskian geometry permits an exact, numerical, and invariant representation of the indicated large difference in cosmological redshifts, while restoring the abstract Minkowski and Poincaré axioms on isospaces over isofields.

The effect can first be read off in the expansion of the Minkowskian redshift (here presented for simplicity for the case of null aberration)

$$\omega = \omega_o \times (1 - v/c_o) \times \gamma \approx \omega_o \times [1 - v/c_o + \frac{1}{2} \times (v/c_o)^2 + ...] \qquad (6.1.113)$$

Since $v \ll c_o$, it is evident that a decrease of c_o will imply an increase of the redshift.

It was shown in Ref. [6], Vol. II, that the above equation is insufficient to represent astrophysical evidence, e.g., because chromospheres are anisotropic (due to their rotation) and inhomogeneous (due to the decrease of the density with the increase of the radial distance from the center), while the geometry underlying law (6.1.113) is purely isotropic and homogeneous.

By using Isopostulate IV, Eq. (6.1.14), Santilli [*loc. cit.*] suggested the following *isodoppler law* for the cosmological redshift

$$\omega = \omega_o \times [1 - (v \times b_3/c_o \times b_4) \times \hat{\gamma} \approx$$

$$\approx \omega_o \times [1 - \beta \times (b_s/b_4) + \frac{1}{2} \times \beta^2 \times (b_s/b_4)^2 + ...] \qquad (6.1.114)$$

where n_s is the space characteristic quantity in the direction of emission of light, assuming the source to be spherical for simplicity. As one can see, the above isolaw predicts an additional contribution in the redshift due to the anisotropy and inhomogeneity of quasar chromospheres.

In 1992, R. Mignani [70] provided a direct experimental verification of Santilli's Isopostulate IV and related isodoppler law for all the most important pairs of quasars and associated galazies. The verification was done via the parameter

$$B = \frac{b_s}{b_4} = \frac{(\delta\omega + 1)^2}{(\delta\omega + 1)^2 + 1} \times \frac{\delta\hat{\omega} + 1)^2 - 1}{\delta\hat{\omega} + 1)^2 + 1}, \qquad (6.1.115)$$

where $\delta\omega$ represents the measured Einsteinian redshift for galaxies, and $\delta\hat{\omega}$ represents the isotopic redshift for quasars according to Santilli's law (6.1.114).

A most important consequence of the data of Figure 6.14 is that *quasars chromospheres are iso-Minkowskian media of Group II, Type 5* (Figure 6.3). In this way, the anomalous redshift behavior here considered is reduced to the axiamatic geometric characterization of the inhomogeneity and anisotropy of astrophisical chromospheres. As we shall see, this geometric characterization will allow numerical predictions for the isoredshift expected by Sun light at sunset.

It is evident that the data of Figure 6.13 provide another experimental verification of the the very central assumption of Santilli isorelativity, the *novel modification (called mutation) of spacetime caused by physics media,* where "novelty" is intended to clarify that said modification *is not* of gravitational or any

GAL.	ω'_1	QUASAR	B	$\hat{\omega}_2$
NGC	0.018	UB1	31.91	0.91
		BSOI	20.25	1.46
NGC 470	0.009	68	87.98	1.88
		68D	67.21	1.53
NGC 1073	0.004	BSO1	198.94	1.94
		BSO2	109.98	0.60
		RSO	176.73	1.40
NGC 3842	0.020	QSO1	14.51	0.34
		QSO2	29.75	0.95
		QSO3	41.85	2.20
NGC 4319	0.0056	MARK205	12.14	0.07
NGC 3067	0.0049	3C232	82.17	0.53

Figure 6.13. A summary of Mignani's data [70] verifying Santilli's isorelativity for all major quasars that are physically associates to galaxies according to clear spectroscopic or other evidence, while having dramatically different cosmological redshifts.

previously known nature, but intrinsic in the anisotropy and/or inhomogeneity of the media.[23]

Yet another experimental verification of Santilli isorelativity is given by the exact, numerical, and invariant representation [71] of the*internal red-, and blue-shift of quasars.* We are here referring to the unexpected behavior whereby, for a given cosmological redshift, there can be relatively smaller shifts toward the read or toward the blue. This is a phenomenon that clearly confirmed Santilli's isorelativity because it is known since Newton times, although not admitetd for personal gains, that the index of refraction of lighgt has an explicit dependence on the frequency, resulting in thne beautiful separation of light into its various colors via a crystal. But the index of refraction is the characteristic quantity $1/b_4 = n_4$. Hence, thje quasars blueshifts and redshifts can be explained in a trivial way via Santilli's isorelativity, via a simple functional dependence of the

[23]Again, we use the word "mutation" suggested since the original proposal of hadronic mechanics, Re. [14]. referring to formulation defined on isospaces over isofield, so as to distinguish them from "deformations," namely, formulations defined on conventional spaces over conventional fields, because the catastrophic inconsistencies of the latter (Sections 1.3.5 and 6.1.6) were already known in 1978.

characteristic quantities on the frequency, $b_\mu = b_\mu(\omega, ...)$ (Figure 6.14). See Ref. [71] for details and fits due to their simplicity.

Note *the absolute impossibility for special and general relativities to represent the astrophysical data of this subsection.* Hence, the covering relativity that is applicable for interior astrophysical problems is open to scientific debates, but the denial of its need is outside scientific or credible doubt

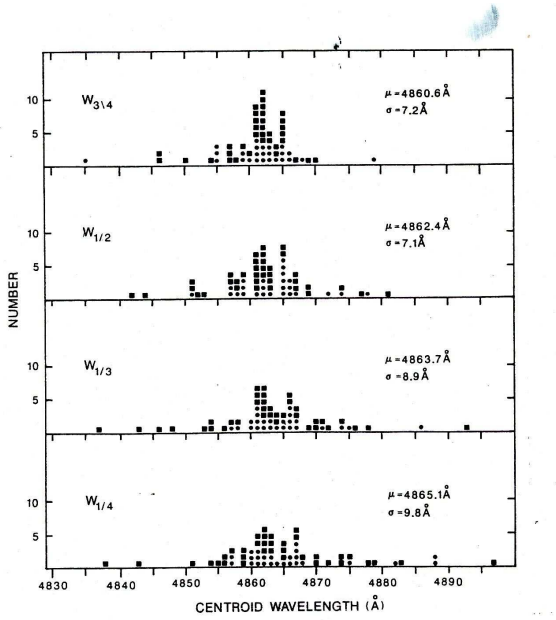

Figure 6.14. A schematic view of Sulentic's [67] discovery of the internal red- and blue-shift of quasars, that is, the decrease or increase of the cosmological redshift of quasars with corresponding variations of the light frequency. The latter occurrence is a further experimental confirmation of the validity of Santilli's iso-Minkowskian geometry for quantitative representations of cosmological redshifts. In fact, the evidence establishes a dependence of the redshift with the frequency, which is evidence of propagation of light within physical media fully known, although not admitted as of lately, since Newton's times. The iso-Minkowskian geometry then applies, e.g. because of its direct universality for interior conditions.

6.1.12 Verification via the Absence of Dark Matter and Energy

Recent astrophysical observations have established that matter in the visible universe, when computed with conventional theories, is substantially insufficient for a quantitative explanation of numerous astrophysical events, including galaxy evolutions, lensing effects, temperature distribution of hot gases, cosmic

microwave background, and other events. Specific calculations indicate that, at this writing (October, 2007) matter (or energy) in the universe as above defined can only account for 3 % of the needed mass (or energy). Consequently, 93 % of the needed mass (or energy) is missing.

The above data lead to the proposal and widespread propagation of the conjecture of *dark matter (or energy),* (see Ref. [72] for a readable account and main references) according to which the missing 93 % is carried by an unknown state of matter capable of experiencing and causing gravitation (as an evident necessary condition for a credible explanation of gravitational anomalies), yet it is "dark" in the sense of not being visible, thus not emitting or absorbing radiations, and having additional quite implausible peculiarities identified below.

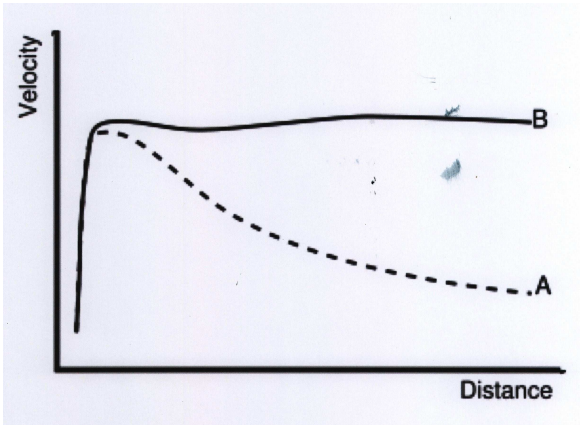

Figure 6.15. A typical illustration of the claimed need for "dark matter" [72] in a very large amount: the predicted rotation of stars in a spiral galaxy (A) and the observed behavior (B). As one can see, the observed behavior is *bigger* than that predicted by orthodox calculations. The aspect that turns the conjecture of "dark matter" into a theology is that "dark matter" should it decrease, rather than increase, the rotational motion of stars, evidently because, when assumed to constitute 93 % of the mass in the universe, "dark matter" becomes a physical medium with consequential "dark matter wind," namely, the necessary creation of a *resistance* that stars should experience while moving in the "dark matter sea." In reality, it is known by experts in the field that the conjecture of "dark matter" was submitted for the pre-meditated scope of maintaining the dominance of Einsteinian theories in conditions for which they are inapplicable. In fact, as soon as necessary deviations from Einsteinian theories are admitted in the interior of stars, quasars and black holes, there is no need at all for theological conjectures to explain the dynamics of the universe.

It is known by well informed scientist that *the conjecture of dark matter was suggested for the specific intent of salvaging the validity of Einstein special relativity in the interior of stars, quasars and black holes.* In particular, the conjecture was voiced at the time of mounting theoretical and experimental evidence of the *inapplicability* (rather than "violation") of Einsteinian doctrines in the indicated

conditions for numerous reasons, such as the emerging locally varying character of the speed of light within physical media (see Section 6.1.8 and review paper [40]). [24]

In fact, the conjecture of "dark matter" is a direct consequence of the use in astrophysical calculations of the Einsteinian energy equivalence

$$E = m \times c_o^2, \tag{6.1.116}$$

where c_o is the speed of light *in vacuum.*

Said conjecture remains implausible for several reasons indicated below, the most damaging one being the lack of a "dark matter wind" during the motion of stars. In fact, the assumption that the visible mass is only 3 % of that existing in the universe, implies that stars must move within a "dark matter sea." Under such a condition, the dynamics of stars is expected to be the *opposite* of what ventured, namely, "dark matter" should *decrease* the rotation of stars in galaxies, rather than increase them as, per primary scope of the conjecture.

The alternative is to assume that 93 % of the mass in the universe is in a state of "evanescence" so as not to cause the "dark matter wind" during the motion of stars, yet it causes gravitational effects. Such an unverifiable conjecture to support an unverifiable conjecture would cause exiting the boundaries of serious science.

As typical for all directly unverifiable conjectures ventured for the intent of salvaging Einsteinian theories, the conjecture of "dark matter" is now being complemented by additional unverifiable conjectures, such as that *"dark matter" is composed by the hypothetical neutrinos* (see next section for the basically unsettled character of the neutrino conjecture). The clear (but unspoken) intent here is to abuse academic credibility on the "evidence" for the existence of the hypothetical neutrinos as "evidence" in support of the conjecture of "dark matter," all for the pre-meditated intent of preserving Einsteinian doctrines, while studiously avoiding a mention of their possible inapplicability under extreme conditions simply unthinkable during Einstein's times.

Needless to say, studies along the above theological lines should certainly be allowed to continue.[25] However, the field would be turned into an illusory science

[24]Rather unreassuringly, the U. S. Department of Energy has recently advertised, rather widely, the availability of public funds specifically earmarked for the study of "dark energy," thus with a ‚mirror condition existing at the U. S. national Science Foundation, thus fueling rumors that U. S. Federal Agencies are controlled by organized interests on Einsteinian doctrines without a serious commitment to much needed basic advances. It is assumed that even the most unscrupulous reader will not dare to deny that strict verification of Einsteinian doctrines is a mandatory condition for securing *contemporary* federal research contracts. Documentation to the contrary would be gratefully appreciated by the author.

[25]Provided that papers written and published under public U. S. financial support explicitly quote the tentative nature of the research and the expected inapplicability of Einsteinian doctrines in the field, so as to avoid violations of U. S. Laws suitable for legal prosecution.

in the event said theological studies are not complemented with research based on *deviations* from Einsteinian doctrines within the hyperdense media inside stars, quasars and black holes.

It is hereon assumed the reader is aware of the fact that *gravitation originates from the energy of given bodies and not from their mass, the latter merely representing their inertia.* The popular misconception of assuming mass as the source of gravitation originates from Newton's equation

$$m \times \frac{dv}{dt} = F, \tag{6.1.117}$$

that was automatically extended for centuries to Newtonian gravitational attraction

$$F = G \times \frac{m_1 \times m_2}{r^2} \tag{6.1.118}$$

However, the force F in Newton's equation (6.1.117) is identically null for bodies at a constant mutual distance for which $dv/dt = 0$, while the force F in Eq. (6.1.18) is not null for the same conditions. Hence, recent studies (see EHM II and references quoted therein) have indicated that the more appropriate version of the gravitational attraction is that in terms of the energy content of the bodies,

$$F = S \times \frac{E_1 \times E_2}{r^2}, \tag{6.1.119a}$$

$$S = \frac{g}{c_o^4}. \tag{6.1.119b}$$

Needless to say, it is popularly known that formulations (6.1.18) and (6.1.19) are equivalent, since the passage from one to the other is given by a mere numerical proportionality. Such an equivalence is unquestionable for the conditions of exact applicability of Einstein's energy equivalence, namely, for *point particles moving in vacuum.*

What does not appear to be popularly known is that the equivalence between Eqs. (6.1.18) and (6.1.19) is lost when referred to *extended bodies with hyperdense interior media,* because the speed of light is no longer the maximal causal speed, assuming that light can propagate in the medium considered.

Hence, from now on, the physically important issue is the *missing energy,* in the universe, and not the missing mass.. Again, the latter merely represents the inertia as traditionally conceived through centuries, namely, as the matter tendency to oppose changes of speed. As such, inertia cannot possibly be the source of gravitation, thus leaving the energy as the only source available at this writing.

The reader accustomed to throw judgment via a quick glancing at topics in which he/she has no technical knowledge,[26] should be warned that the need to use

[26]The author remembers "distinguished" colleagues, including a Nobel Laureate in Physics, refereeing papers during the duration of time for the elevator to reach the physics department second floor. For

energy as the true gravitational source requires a serious technical knowledge of isomathematics (EHM Vol. I and Section 3.2) including the geometric unification of the Minkowskian and Riemannian geometries and a serious resolution of the *Nine Theorems of Catastrophic Inconsistency of Einstein's Gravitation* studied in Section 1.4 [73]..

At any rate, *the use of energy as the source of gravitation, rather than mass, is mandated by experimental evidence that light has no mass, yet it experiences gravitation, such as in the case of the bending of light when passing near astrophysical bodies.* In this case we evidently have the gravitational attraction

$$F = S \times \frac{E_{mass} \times E_{light}}{r^2}. \tag{6.1.120}$$

Hence, the restriction of gravitational sources to mass would solely admit *some* gravitational events in the universe, while the use of energy would admit them *all.*

As an obvious comment, the above reformulation of gravity fully represents the data of our Solar system, because the currently assumed gravitational fields of the Sun and planets are identically reformulated from mass to their isoequivalent energies. However, the reformulation is, by far, non trivial, e.g., because it may provide new insights in interior ghravitational problems, such as the speed of lighjt and maxiumal causal speed inside the Sun.

Under the above clarifications, we can quote Santilli's view [74] according to which *isorelativity eliminates the need for dark matter and energy either in full or in part.* In fact, Isoaxiom V, Eq. (6.1.15), predicts that the energy equivalence of a given mass is given by

$$\hat{E} = m \times c_o^2 \times \frac{b_4^2}{b_s^2} = m \times c_o^2 \times \frac{n_s^2}{n_4^2}, \tag{6.1.121}$$

where we have assumed for simplicity a perfect sphericity of astrophysical bodies resulting in the single value $b_s = 1/n_s$.

Santilli then pointed out that the "missing energy" ΔE can be accounted for via the value [*loc. cit.*]

$$\Delta E = m \times c_o^2 \times (\frac{b_4^2}{b_s^2} - 1) = m \times c_o^2 \times (\frac{n_s^2}{n_4^2} - 1). \tag{6.1.122}$$

Under the assumption that the mass of the universe, when computed via Einsteinian theories, is only 3 % of the needed mass, the behavior of stars and other

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the intended "review," this time is amply sufficient to identity the affiliation of the author and the compatibility of the content with Einsteinian theories.

objects in the universe can be explained via the following *average isotopic characteristics of trhe universe* applicable for the hyperdense medium inside stars, quasars and black holes

$$\frac{b_4^2}{b_s^2} = \frac{n_s^2}{n_4^2} = 94, \tag{6.1.123}$$

As an example, by using ordinary gauge theories, Ref. [42] computed the average value of the speed of light within hyperdense hadronic media to be 75-times that in vacuum, in which case we have

$$c = 75 \times c_o, \;\; b_4^2 = 75, \;\; b_s^2 = 0.079 \tag{6.1.124}$$

Needless to say, calculations [42] are merely approximate. Yet, the view that the conventional mass equivalent necessarily holds in the interior of black holes, has no scientific credibility.

Needless to say, value (6.122) and (6.1.123) are an *average* for the entire universe, under the understanding that they are based on current estimate of 93 % missing energy. Also, the values are expected to vary dramatically from stars to black holes, the latter being arguably the origin of the biggest contributions.

It should also be noted that isorelativity provides a *partial* elimination of the missing energy, because every point in space is traversed by light coming from the entire universe, thus characterizing a clear energy. Additional energy everywhere in space is provided by ordinary massive particles, such as cosmic rays, hydrogen, etc. Clearly, the latter component characterized by ordinary electromagnetic waves, particles and hydrogen has to be computed before finalizing the value of the average isotopic characteristic of the universe. Note that the latter conventional component is dramatically insufficient to account for all missing energy.

In summary, recent theological trends in astrophysics, for the pre-meditated scope of adapting nature to Einsteinian theories, have conjectured the existence of a mysterious substance existing in our spacetime, capable of experiencing and causing gravity, but unable to absorb or emit electromagnetic waves, not causing a "dark matter wind" in the motion of stars, being uniformly distributed at times to explain star rotations in Galaxies (Figure 6.15) while being entirely concentrated in a point to explain lensing effects Figure 6.16) and having other manifest basic flaws.

In this section, we have shown that the dynamics of the universe can be interpreted quantitatively by adapting the theories to the evidence, in this case, by honoring the *exact* validity of Einsteinian theories for the physical conditions limpidly expressed by Einstein ("point-particles and electromagnetic waves propagating in vacuum"), and by halting the abuse of Einstein's name and memory in pushing said validity beyond the arena of their original conception and exper-

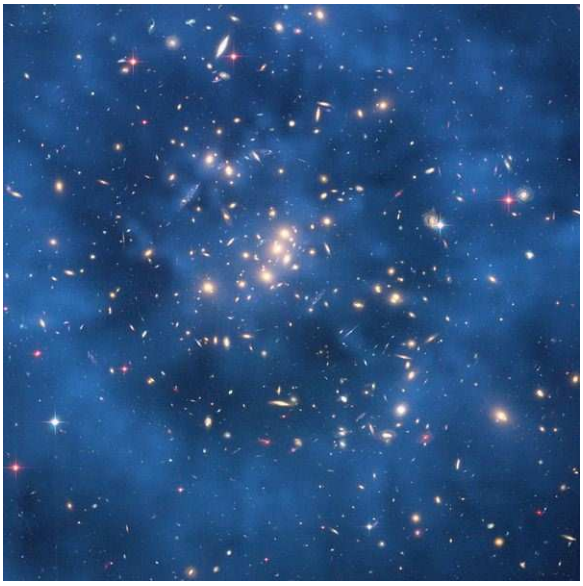

Figure 6.16. Another illustration of the widespread political preservation of Einsteinian theories under conditions for which they are inapplicable: the use of lensing effects in the universe as "evidence" in support of the conjectural "dark matter." The posturing is political because the indicated lensing effect is well known to be solely due to a highly concentrated mass in one of the foci, while the conjectural "dark matter" must be uniformly distributed in space to "interpret" the behavior of Figure 6.15. This is reminiscent of the case about one century ago of using the bending of light near an astrophysical body as "evidence" of the curvature of space, while in reality that bending is entirely due to Newtonian attraction, and, in the event curvature does indeed cause the bending of light, the prediction of Einstein's gravitation is double that measures [73]. The fact that curvature cannot possibly explain the free fall of bodies along a straight radial line, even though known to high school students, was suppressed, at times studiously, to serve a political purpose in science.

imental verifications, to the contemporary extremes of manipulation of scientific evidence that can only be euphemistically qualified as theological at best.[27]

[27]The author would like to:

1) Set a record for having received today, October 11, 2007, a visit from the Italian-British scientist, industrialist and philanthropist from London, **Dr. Francesco Fucilla**;

2) Indicate that, if properly supported by scientists, educators, politicians, economists, industrialists and historians who care about human knowledge, Dr. Fucilla can be the coordinator of a much needed *new scientific renaissance* comparable to that originated by Lorenzo de' Medici (called "Il Magnifico") in the the 1500's, not only because of Lorenzo's superior vision, but also because of the support he received by luminaries such as Andrea del Verrocchio, Leonardo da Vinci, Sandro Botticelli, Domenico Ghirlandaio, Filippino Lippi, Michelangelo Buonarroti and so many others.

3) Note with pride that Dr. Fucilla is Italian.

6.1.13 Experimental Verifications via Supernova Explosions

There is little doubt that contemporary astrophysics is one of the most theological fields of contemporary science due to the assumption of numerous fundamental aspects without serious theoretical and/or experimental evidence, and/or serious scrutiny, such as:

1) The most fundamental event in astrophysics, the *synthesis of the neutron from protons and electrons,*,

$$p^+ + e^- \to n + \nu, \tag{6.1.125}$$

is basically unknown at this writing on both theoretical and experimental grounds. On theoretical grounds the synthesis is basically unsettled because the rest energy of the neutron is 0.78 MeV *bigger* than the sum of the rest energies of the proton and the electron. Under these conditions quantum mechanics is fundamentally inapplicable due to the lack of physical meaning of Schrödinger's equations under the necessary *positive* binding energy of 0.78 MeV (in which case there is no energy available for the hypothetical neutrino). Yet, quantum mechanics is routinely applied for all calculations known to the author. On experimental grounds, the insufficiency is even greater due to the rejection by laboratories around the world of the author proposal over decades of testing synthesis (6.1.125), evidently due to its incompatibility with established doctrines (see next chapter for details). lacking fully established theoretical and experimental knowledge on the first and most fundamental synthesis (6.1.125) in a star, the rest of "astrophysics" (that is, the physics of stars") is evidently unsettle on serious scientific grounds.

2) Contemporary astrophysics is additionally based on the belief that neutrinos are physical particles in our spacetime. However, the only available quantitative representation of synthesis (6.1.125), that provided by hadronic mechanics, does not need the neutrino at all, as shown in the next section; as limpidly stated by Enrico Fermi, neutrinos cannot be experimentally detected; the indirect detections believed to be caused by neutrinos have alternative interpretations; and the neutrino conjecture remains afflicted by a number of unsettled aspects that multiply in time, rather than decrease, because unspoken. Under these conditions, any astrophysical model depending on neutrino conjectures is evidently unsettled on serious scientific grounds.

3) Astrophysical observations are interpreted via spectral analysis established on earthly experiments, namely, on the spectral emissions of essentially unperturbed atoms, while it is known that atoms subjected to extreme conditions have spectral emissions *different* than those from ideal conditions. In more explicit terms, the spectral emission, for instance, iron under the extreme densities and pressures in the core of a star is expected to be dramatically different than the spectral emission of iron as measured on in our laboratories.

[28]

At any rate, after one century of studies, the spectral emission of our Sun is still basically unknown to the authors best knowledge, e.g., because of spectral lines that should originate from orbits *smaller* than the ground state of the hydrogen, and similar unresolved anomalies. Under these conditions, the theoretical interpretation of spectral lines from a far away star, quasar or supernova via quantum mechanics can only be qualified as being unsettled at best.

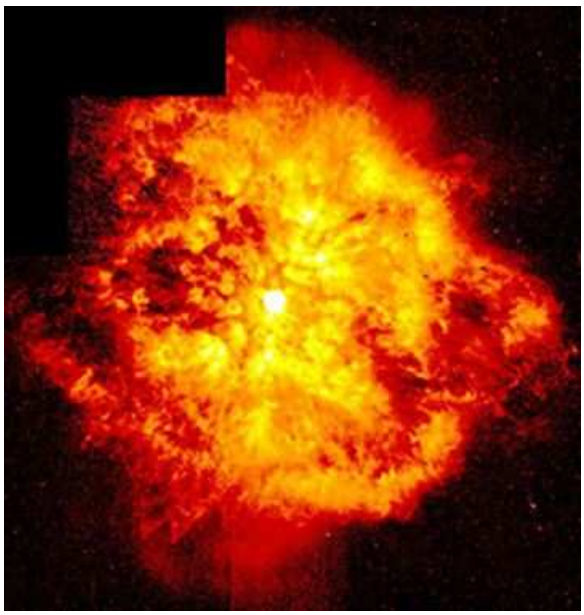

Figure 6.17. A NASA image of the nebula WR124 located 21,000 light years away showing the remnants of a supernova.

For the receptive young mind of any age interested in knowledge (rather than an academic career), the author suggest, as a pre-requisite for endless, fundamental new discoveries, to keep an open mind and study as a matter of principle *all* possibilities permitting quantitative interpretations, irrespective of wether via the use of quantum or hadronic mechanics.

[28]It is appropriate to recall here that quantum mechanics has achieved an exact representation of the spectral emission solely of the hydrogen atom and solely when in essentially unperturbed conditions, since deviations between theoretical predictions and exponential evidence begin to be significant for the helium and become embarrassing for heavy atoms such as the zirconium because of a basic reason for the proposal to construct hadronic mechanics, the emergence of nonlocal, nonpotential and non-Hamiltonian effects between atom,ic electrons that begin precisely with the helium. When all atoms, including the hydrogen, are exposed to extreme conditions of pressures as occurring in stars,m these nonlocal, nonpotential and non-Hamiltonian effects are dramatically increased, resulting in dramatic deviations between the theoretical predictions based on quantum mechanics and the experimental evidence.

Along these lines, the author suggest the conduction of quantitative studies on the *origin of the energy in supernova explosions.* As it is well known (se, e.g., Ref. [75] and large references therein), the sequence of a supernova is currently expected to be due to the exhaustion of the "nuclear fuel" in a star resulting in an expected iron core that, when reaching the Chandrasekar mass, collapses all atoms into into a neutron star, at which point contraction stops with the initiation of the explosion.

This produces one of the most violent explosions in the universe that are visible to the naked human eye on Earth as far away as tens of thousands of light years away estimated to require about 10^{50} joules of energy, namely., an amount of energy hardy comprehensible by mankind.

The issue in which the author would like to attract the attention of young mind of any age outside academic political and theological rings is that this huge energy is quickly "interpreted" as being provided by the the energy conversion of about 10 % of the original star mass. However, the *mechanism* of energy production is ignored, evidently because it is based on the synthesis of the neutron that, notoriously, cannot be treated via quantum mechanics, thus resulting in vague indications or theological feelings.

In fact, *at the time of reaching the state of a neutron star, there are no appreciable nuclear syntheses that can possibly account for the production of such un-imaginable amount of energy.* Hence, one [possibility that should be investigated, of cpourse, jointly with others, is that *the energy in a supernova may originate prior to the explosion, namely, during the formation of the neutron star.*

If the above arguments are admitted as part of others, potentially momentous advances are possible. In fact, we have recalled above that *the synthesis of the neutron does not release any energy and actually requires* 0.78 *millions electronVolts.*

Hence, the issue is addressed is: *where is the enormous amount of energy required to reach a neutron star originating from?* The issue brought to the attention of young minds of any age is the following:

SUPERNOVA HYPOTHESIS: The energy needed for a supernova explosion originates at least in part from space conceived as a universal medium of very high energy density (Section 6.1.2).

In the next section we shall study the synthesis of the neutron inside stars as solely permitted by hadronic mechanics in a quantitative, numerical. and in variant way. It is evident that, as a first possibility, the missing 0.79 MeV originates from the thermal and other energies available inside a star, are acquired by the proton and the electron during 'Rutherford's compression" of the hydrogen atom, and result in the synthesis of the neutron.

However, a scientific process cannot be claimed unless the studies include the alternative possibility that *the synthesis of the neutron inside a star is a mechanism of transfer of energy from space to matter, namely, a mechanism for continuous creation of matter in the universe.*

To conduct science as traditionally conceived, that via a quantitative and invariant ;process verifiable in laboratory, we have to halt at this point our study of supernova and defer interested reader to a study of the next section.

6.1.14 Verifications via the Bose-Einstein Condensation

The *Bose-Einstein condensation* (see, e.g., Refs. [76-78]) is one of the most mysterious events in nature that could stimulate major advances in scientific knowledge, under the condition that the memory of Satyendra Nath Bose and of Albert Einstein is indeed duly honored, but the limitations of their view is admitted as the premise the same advances, the belief in final theories being solely motivated by money, prestige and power.

There is no doubt that the initial experimental realization of the Bose-Einstein condensation can be fully treated with special relativity and quantum mechanics. However, there should be no doubt by serious scientists that its extreme realization includes *contact, zero-range, nonlocal and non-Hamiltonian interactions extended over the volume of deep wave-overlappings of the atomic electrons at short distances.*

The *approximate* character of special relativity and quantum mechanics for these novel interactions is beyond scientific doubt. By contrast, isorelativity and hadronic mechanics are the only theories known to the author that:

1) Provide an axiomatically correct representation on nonlocal;l interactions extended over a volume, beginning with the basic TSSFN isotopology specifically constructed for the interactions considered (Section 3.2.7);

2) Is "directly universal" for nonlocal and non-Hamiltonian interactions in the sense of admitting all infinitely possible interactions of the class admitted ("universality") directly in the frame of the experimenter, thus without the transformation to hypothetical frames ("direct universality"); and

3) It enjoys the same invariance of quantum mechanics, namely, admitting the same numerical predictions under the same conditions but at different times.

The conditions of applicability of relativistic hadronic mechanics to the Bose-Einstein condensation are those in which, under a sufficiently strong external magnetic field, the condensate enters into the attractive phase , shrinks beyond detection, and then explodes, by blowing off part of its atoms, the remaining parts essentially disappearing from detection.

It is known that this characteristic of BoseEinstein condensate cannot be explained with special relativity and quantum mechanics because of the evident impossibility to account for the *strongly attractive force between neutral atoms*

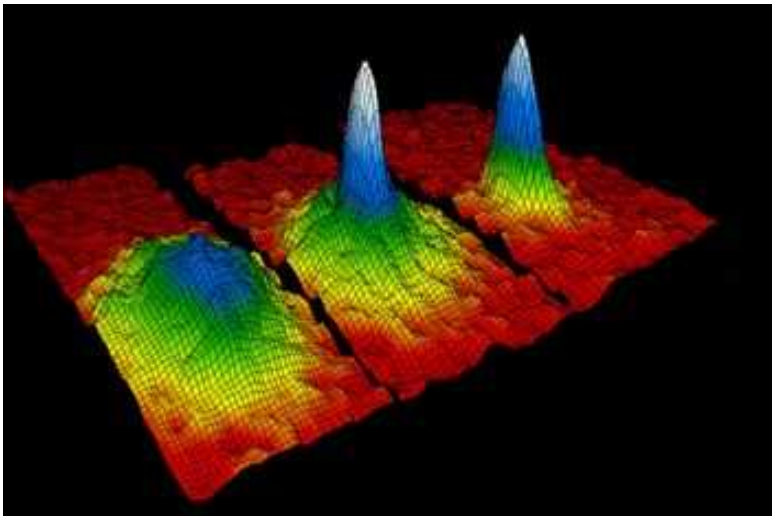

Figure 6.18. A typical illustration of the Bose-Einstein condensation from Ref. [76] showing the velocity distribution of gases: just before the appearance of the BoseEinstein condensate (left); just after the appearance of the condensate (center); and a representative sample of nearly pure condensate (right). The most intriguing event is the subsequent one with a "supernova-type behavior" that could activate an interplay between matter and space as a universal substratum of extremely high energy (Section 6.2).

that is needed for an implosions such to allow the condensate to disappear from measurements. Secondly, there is no possibility of explaining via quantum mechanics the *super-nova type behavior* of the condensate following the impletion.

Relativistic hadronic mechanics offers the possibility for a quantitative study of the above anomalous behavior. In fact, we shall study in detail in the next section that *nonlocal interactions due to wave overlappings at short distances in singlet couplings generate a strongly attractive force* that can be responsible for the synthesis of hadrons.

Additionally, relativistic hadronic mechanics offers means for quantitative studies as to whether the excessive energy needed for the super-nova phase of the condensation originates from space conceived as a universal medium of very high energy density. As we shall also see in the next section, iso-Hilbert spaces have been also conceived for a quantitative representation of the interplay between matter and the ether as a universal medium.

Stated in different terms, once the limitations of orthodox doctrines are admitted as the very premise for basic advances, the Bose-Einstein condensation could have such far reaching implications of allowing experimental means for ascertaining whether the same mechanism occurs for supernova explosions or, more

generally, whether or not there is indeed continuous creation of matter in the universe./

6.1.15 Verification in Cosmology

In preceding chapters, we have studied the various branches of hadronic mechanics consisting of methods for the representation of matter in conditions of progressively increasing complexity, such as

QUANTUM MECHANICS, representing isolated, reversible and single-valued systems of point particles under solely Hamiltonian interactions;

ISOMECHANICS, representing isolated, reversible and single-valued systems of extended, nonspherical and deformable particles under Hamiltonian and non-Hamiltonian interactions;

GENOMECHANICS, representing open, irreversible and single-valued systems of extended, nonspherical and deformable particles under Hamiltonian and non-Hamiltonian interactions;

HYPERMECHANICS, representing open, irreversible and multi-valued systems of extended, nonspherical and deformable particles under Hamiltonian and non-Hamiltonian interactions.

We have then studies the isodual images of all preceding four methods for the treatment of antimatter in conditions of corresponding, progressively increasing complexity.

These studies include the geometric unification of special and general relativity into isorelativity, their basic Poincaré-Santilli iso-, geno-, and hyper-symmetries, and the axiomatically consistent classical and operator gravity embedded in the basic unit.

For the purpose of this section we note that all distinctions between matter and antimatter are lost at the hyperstructural level, thus permitting a unification of *all* branches of hadronic mechanics into one single formulation, hypermechanics, admitting all other as particular cases. In fact, the hyperunit can be characterized by an ordered set of genounits and their isoduals,

$$\{\hat{I}\} = \{\hat{I}_1^>, \hat{I}_1^{>d}, ^< \hat{I}_1, ^< \hat{I}_1^d, I_2^>, ^< I_2^{>d}, ^< \hat{I}_2, ^< \hat{I}_2^d, ...\}, \tag{6.1.126}$$

under which the hyperproduct of two generic quantities a and b yields a corresponding ordered set of values

$$a\{\times\}b = \{c_1^>, c_1^{>d}, ^< c_1, ^< c_1^d, c_2, c_2^d, ^< c_2, ^< c_2^d, ...\} \tag{6.1.127}$$

Consequently, at this highest possible level of formulation, we have one single hyperrelativity, one single Poincaré-Santilli hypersymmetry

$$\{\hat{P}\}(3.1) = \hat{P}^>_{matter}(3.1) \times \hat{P}^{>d}_{antimatter}(3.1) \times^< \hat{P}(3.1) \times^< \hat{P}^d(3.1). \tag{6.1.128}$$

and one single hypergravity encompassing all particular cases of exterior and interior, classical and operator gravitation for both matter and antimatter.

The above defined hypermechanics have permitted the formulation of a *new cosmology*, first proposed by Santilli in Ref. [79], and now known as *hypercosmology* characterized by the following three basic assumptions:

HYPERAXIOM I: The universe is (3+1)-dimensional and multi-valued.

HYPERAXIOM II: All events in the universe verify the Poincaré-Santilli hypersymmetry.

HYPERAXIOM III: All total physical characteristics of the universe are identically null.

A few explanatory comments are in order to assist the non-initiated reader. As studied throughout Volume I, the sole possibility known to the author of achieving a consistent *classical* treatment of antimatter that verifies all experimental data and admits charge conjugate operator images, is given by the isodual theory. This theory implies that *the universe is (3.1)-dimensional but two-valued.* In fact, antimatter does indeed exist in a (3.1)-dimensional space, but the isodual conventional, iso-, geno, or hyper-Minkowski space is *different* than the conventional space of matter. This leads to a two-valued structure of the universe, namely, a structure consisting of two coexisting worlds, in which each of the (3.1) components has two values, one for matter with unit $+1$ and one for antimatter with unit -1.

The extension from the two-valued matter-antimatter spacetime to a multi-valued universe is dictated by numerous aspects, not only in astrophysics, but also in biology where multi-valuedness is necessary to attempt any minimally credible study of biological structures such as the DNA code whose complexity is beyond human imagination at this time (Chapter 5)

Note in covering structures (6.1.126)-(6.1.128) the presence of *all* possible formulations, namely: formulations for the description of matter systems moving forward and backwards in time and their isoduals for antimatter. This all encompassing generality is dictated, certainly not by elementary systems familiar to physicists, but by the complexity of the biological world that is beyond the imagination of the most educated biologist.

We have indicated in Chapter 1.5 that a credible representation of a truly elementary biological event, such as the bifurcation achieved by seashells during their growth in time, requires *all four directions of time, that is: motion forward in future time; motion forward from past time; motion backward from future time; and motion backward in past times.* Then, the most skeptic of a reader cannot deny the necessity of our isodual theories without risking a credibility collapse due to ignorance, for the evident reason that *time reversal can only achieve two out of four time directions, while the remaining two can be only achieved only via isoduality in a way compatible with classical and operator experimental*

evidence. Alternative conjugations are encouraged, provided that they are not merely epistemological, but quantitative and published in refereed journals, and with the understanding that they will remain redundant over isoduality [83].

The reader should be aware of the *dramatic differences between multi-dimensional and multi-valued theories.*. Multidimensional theories are herein defined as being characterized by a number of spacetime dimensions bigger than (3.1), such as (4.1, (3.3), etc. These theories, even though at time mathematically elegant, are herein strictly rejected on physical grounds because directly incompatible with our sensory perception that, as well known, is based on our three Eustachian lobes solely permitting a three-dimensional perception of space, and our one-dimensional perception of time.

By contrast, our multi-valued theories have been conceived and developed to achieve full compatibility with our sensory perception, while admitting a complexity of the universe beyond our imagination. As an example, when we observe a seashell in our hand, we perceive its shape in three-dimension and its evolution along our one-dimensional time. However, the same seashell can overlap a large number of spaces and their isoduals, resulting in multi-fold formulations including the four different directions of time indicated above. To state it differently, the assumption that the internal time of a seashell is necessarily the same as our time can only originate from arrogance of planetary proportions. The sole scientific statement we can venture at this writing is that the intrinsic time of a seashell is of such a complexity to be beyond our rather limited mental capabilities.

In Section 1.4., the author has shown that general theory of relativity is catastrophically inconsistent on mathematical and physical grounds for numerous diversified reasons, some of which are nowadays vexing because untreated (let alone unresolved) for about one century, while other reasons have emerged from these studies (see the *Nine Theorems of Catastrophic Inconsistencies of General Relativity* of Ref. [73]).

In the author's view, the biggest damage caused to science by general relativity has been in turning cosmology into a theology (see Appendix 6.D). One, among numerous reasons, is the structuring of gravitation on on *covariance* that leads to the impossibility of preserving the same numerical predictions under the same conditions at different times, the violation of causality, and other catastrophic inconsistencies. This is a reason for the author spending decades of his time in reformulating gravitation on a *universal symmetry* [5] as the only known way to avoid these catastrophic inconsistencies, as per historical teaching of special relativity.

The foundation of our hypercosmology on the universal hypersymmetry (6.1.128) is the single most important result of the author's lifetime of research because it governs the totality of the events in the universe, from large scale cosmological dynamics, down to the most elementary component of the universe. The clear

understanding is that we are here merely referring to a *model* that, as such, has numerous limitations, some of which are identified below, for science will never admit a "final theory."

One illustration of the theological aspect of the cosmological studies of the 20-th century is the large effort devoted to the *age of the universe* without a serious scrutiny of the limitations in the very formulation of the problem. The origin of these problems remains always the widespread studious tendency of adapting nature to Einsteinian theories.

But, as established in Chapters 1.1 and 1.2, Einsteinian theories have no *classical* mean at all to differentiate neutral matter and antimatter stars; the only differentiations available in the 20-th century being that at the level of *second quantization;* and general relativity admitting no consistent formulation at the level of *first* quantization, let alone the second. As a result of this vast scientific imbalance, in order to adapt nature to Einsteinian theories, antimatter has been intentionally ignored in the gravitational and cosmological studies of the 20-th century and the "age of the universe" has been studiously referred to matter alone, "studiously" because the antimatter component of the universe is generally *not* mentioned.

The reader who has studied seriously the content of Volume I before a quick glance at this section knows well that *the the total time of the universe is identically null for an equal distribution of matter and antimatter,* that is implicit in hyperaxiom III, of course, as a limit case.

Even by restricting the study to *the age of the matter component of the universe and, separately, the antimatter component,* there are insufficiencies in the very formulation of the "age of the universe," let alone on a possible answer, because such a question is tacitly based on the assumption of Einsteinian theories as being universally valid everywhere in the universe.

If, instead, broader vistas are admitted as possible, the problem of the age of the matter component of the universe, or its antimatter component becomes rather complex because of the *strictly local character of each of the four different times,* where the locally varying character *not* referred to gravitation, but to *the local variation of the four different time units* that include indeed gravitation, but in its *interior* formulation.

Once the initiated reader has technically understood that the total time of the universe under an equal distribution of matter and antimatter can only be identically null, then the reader can easily see that: *the total energy, the total momentum, the total angular momentum and all other characteristics of the universe are identically null* (see Volume I and Ref. [83] for technical details).

Yet another reason for the theological character of the cosmological studies of the 20-th century is the belief that the universe initiated with an *immense singularity* in the fabric of spacetime, called the "big bang" without a serious

scrutiny of its foundation. To begin, the primordial explosion is tacitly referred to solely to matter. Antimatter is studiously ignored because not treatable with Einstein's gravitation and, consequently, it is tacitly assumed not to exist.. By contrast, *the mere inclusion of antimatter would eliminate the singularity in the act of creation, evidently because the total characteristics of the universe were identically null prior to creation and so remain after creation.*

But the problem of the origin of the universe is of such a complexity to be immensely beyond our comprehension, thus demanding the only statement for true science, we do not know, and no certain answer is foreseable at this writing. To illustrate the need for serious scientific caution, we have touched in the preceding sections the possibility of *continuous creation* of matter, hence of antimatter, in the universe, and we shall enter into its quantitative studies beginning from the next section on.

It is evident that a possible continuous creation of matter in the universe renders arrogant any questions such as that on the "the age of the universe." At any rate, the author is a religious person, because the complexity of the universe is simply too enormous to be the outcome of random occurrences. The addressing of issues such as "age of the universe," the "creation of the universe," etc., indicate lilliputian intellect, because tacitly based on the unspoken, yet evident assumption of a capability to understand God's will. This is a reason for presenting our cosmological views more for the identification of the insufficiencies of existing views, rather than because actually true.

Other unreassuring cosmological studies are those on the apparent *expansion of the universe.* To begin, the views are essentially based on the Einsteinian interpretation of cosmological redshift as being due to motions away from us. However, we have shown in preceding sections that, subject to final experimental verification strongly requested in the next section,. *light can indeed slow down in the huge astrophysical chromospheres, thus being emitted already redshifted even for the case of astrophysical bodies at rest with us.* Hence, the current measurements on cosmological redshifts are indeed "actual," but their interpretation down to expansion speed are merely conjectural, again, because based on Einsteinian beliefs.

The author's view is that the "notion of expansion" of the universe appears to be supported by a number of direct or indirect astrophysical evidence, although the "numerical rate of expansion" is at this writing a mere personal belief due to the lack of experimental verifications of the Doppler-Santilli isoredshift rejected by astrophysical laboratories for decades, evidently not to question Einsteinian theories.

Additional unreassuring aspects are related to the origin of the expansion because antigravity would be a natural origin, but, according to a widespread view, "antigravity does not exist because not admitted by Einsteinian theories." Such a

view must be denounced as sheer scientific corruption for personal gains in money, prestige and power, because Einsteinian theories do not even represent antimatter, as a consequence of which any study of antigravity via Einsteinian theories is entirely vacuous. In reality, *the most plausible understanding of the expansion of the universe known to the author is that the universe is constituted by a generally homogeneous distribution of matter and antimatter galaxies experiencing mutual gravitational repulsion as studied in detail in Chapter 13.*

To be plausible, the interpretation should not only explain the *expansion per se,* but also the apparent *increase of the expansion in time.* These two occurrences cannot possibly be explained with the "big bang" since the expansion should decrease, rather than increase in time due to the "dark matter wind" that is inherent in orthodox theological beliefs). The increase of the rate of expansion can be *solely* explained via the existence of a *continuous, action-at-a-distance, repulsive force between galaxies.* In turn, the only conceivable possibility verifying these conditions is *antigravity,* and, in turn, the only known source of gravitational repulsion is *antimatter.*

We reach in this way the expectation of the existence of antimatter stars, galaxies and quasars, this time derived as the only plausible interpretation of the gravitational expansion *and* its increase in time. Unreassuringly, the author has been informed that numerous astrophysical events can only be interpreted quantitatively via antigravity, but such an interpretation cannot be voiced (or published) because antigravity is not predicted by Einsteinian doctrines.

As indicated in Volume I, the isodual theory of antimatter has been worked out because it provides, for the first time to the author's knowledge, quantitative mathematical, theoretical and experimental possibilities of ascertaining whether a far away galaxy or quasar is made up of matter or of antimatter, not only via unbiased astrophysical observations requiring antigravity, but also via unbiased interferometric studies of the light originating from galaxies and quasars to ascertain whether they it is composed by *ordinary photons,* or by *isodual photons,* The former is *attracted* by Earth's gravitational field, while the latter is *repelled,* thus rendering current interferometric techniques suitable for the detection or the denial of antimatter in the universe [83].

We should not forget that, according to unbiased reports, Earth appears to be bombarded by cosmic rays of both matter and antimatter nature, as indicated by flashes of light visible from spaceships while traversing the dark side of Earth. These flashes can be best interpreted as being due to *antimatter cosmic rays* annihilating in our atmosphere, and certainly not by *matter cosmic rays* since the latter are known to penetrate deeply into our atmosphere and definitely not to annihilate in it.

To avoid adapting nature to preferred theologies, we should not forget that the *Tunguska explosion* in Siberia in June 30, 1908, can be most plausibly explained

via an *antimatter meteorite* penetrating deep into the Earth's atmosphere and then exploding due to annihilation. This interpretation is suggested by the lack of a crater in the Tunguska event, despite a huge flattening of the local forest for over hundreds of square miles. By comparison, the hypothesis of a *matter meteorite* has no credibility since the lack of a crater would require its believed "evaporation" (sic) in atmosphere, namely, an occurrence firstly denied by all other craters caused by matter meteorites and, secondly, because the "evaporation" would have not even part of the energy needed for a scientific, that is, numerical explanation of the event.

Hence, antimatter is a most fundamental aspect of cosmology that has been forgotten during the physics of the 20-th century, to the evident detriment of researchers in the field, since their study cannot possibly pass the test of time without a full scientific democracy between matter and antimatter. It is hoped that this unreassuring trend is corrected in the 21-st century because true science cannot be done with theological beliefs or the adaptation of nature to preferred theories, but solely via the unbiased mathematical, theoretical and experimental study of *all* plausible theories, irrespective of whether compatible or not with Einsteinian doctrines

To fully understand this statement, the reader should know that, as we shall see in the final Chapter 14, all grand unification theories done throughout the 20-th century, beginning with the failed attempts by Einstein, can be proved rather easily to be catastrophically inconsistent on the sole ground that they do not include antimatter.

In closing, to keep a kilometric distance from orthodox trends, the author would like to stress that his hypercosmology has been presented as a sheer exercise of scientific curiosity without any claim of "scientific truth, and for the sole intent of showing the limitations of pre-existing cosmologies.

The author solely claims (and will defend, see the Legal Notice at the beginning of this volume) paternity of the first "cosmology" in the Greek meaning of the word, that is, including all structures in the universe, and thus *include life,* for that inclusion alone mandates all studies reported in these volumes.

6.2 HADRONIC STRUCTURE MODELS WITHOUT QUARKS AND NEUTRINOS

6.2.1 Introduction

Hadronic mechanics (hm) was proposed in memoirs [14] of 1978 for the primary purpose of achieving an exact and invariant representation of the neutron as a bound state of one proton and one electron, of course, in a generalized form (hereinafter denoted with a "hat")

$$n = (\hat{p}^+, \hat{e}^-)_{hm}. \tag{6.2.1}$$

The first rational basis for the proposal is that the proton and the electron are the only massive stable particles existing in nature. Hence, during the synthesis of the neutron in the core of stars from the hydrogen atom, the proton and the electron simply cannot "disappear" to please academicians. Consequently, the most rational assumption is that they are actual physical constituents of the neutron.

The second rational basis of the proposal is that the proton and the electron are reproduced in the *spontaneous* decay of the neutron and, as such, they simply cannot "reappear" to comply with preferred theories. Since the creation of the only known massive stable particles at the time of the neutron spontaneous decay is extremely implausible, the most rational assumption is that, again, the proton and the electron are actual physical constituents of the neutron.[29]

It is evident to undergraduate students that structure model (6.2.1) is impossible for quantum mechanics. Rather then adapting nature to preferred theories, memoirs [14] suggested adapting the theories to nature via the construction of a generalization of quantum mechanics permitting models (6.2.1).

The proposal was based on a *nonunitary transformation* of quantum mechanics (qm), as a necessary condition to exit the classes of equivalence of quantum mechanics under *unitary transforms.* The nonunitary structure was also proposed in view of the fact that, in the transition from the hydrogen atom to the neutron in the core of stars, we have the transition of the electron from the state of a point particle moving in vacuum with sole action-at-a-distance interactions, to the state of an extended wavepacket in condition of total penetration within the hyperdense medium inside the proton. The latter conditions characterize new, contact, nonlinear, nonlocal and nonpotential interactions dramatically beyond the representational capability of a Hamiltonian. The inability of representing

[29]The author experiences great difficulties in reading the particle physics literature of the 20-th century because of the presentation of particle reactions in which new particles are created without any explanation whatsoever. This posture is generally intentional to claim the validity of quantum mechanics in particle physics, since its insufficiency becomes crushing when the mechanisms creating new particles are addressed quantitatively.

the neutron synthesis with the sole knowledge of the Hamiltonian, then confirmed the need for a nonunitary theory.

Hence, memoirs [14] proposed the construction of a nonunitary image of quantum mechanics permitting a consistent map of the hydrogen atom H into the neutron exactly as occurring in the core of stars,

$$H = (p^+, e^-)_{qm} \ ; \rightarrow \ \ n = (\hat{p}^+, \hat{e}^-)_{hm} = U_n \times (p^+, e^-)_{qm} \times U_n^\dagger, \qquad (6.2.2a)$$

$$U_n \times U_n^\dagger \neq I. \qquad (6.2.2b)$$

The neutron was proposed as essentially being a new state of the hydrogen atom solely occurring at mutual distances of the order of $10^{-13} cm = 1$ fm, the hydrogen atom obeying quantum mechanics and the neutron obeying the covering hadronic mechanics. Requirement (6.2.2) then imposed *ab initio* that hadronic mechanics is solely valid at mutual distances of particles of the order of one fm, namely, for the range of strong interactions. Equivalently, map (6.2.2) requires that the excited states of the neutrons are the quantized states of the hydrogen atom, or, alternatively, that

$$Lim_{r>1fm}(U_n \times U_n^\dagger) = I. \qquad (6.2.3)$$

It was stressed in memoirs [14] that quantized orbits do exist for point particles moving in vacuum, as in the hydrogen atom, but the belief of the existence of tiny quantized orbits within the hyperdense medium inside the neutron would be pure nonscientific nonsense. This prevented in refs. [14] for ethical reasons the use of the word "quantum" for the new discipline. The name "hadronic mechanics" was selected to stress the primary intent of the new mechanics, the study of the hadronic structure or, more generally, of strong interactions.

Since quantized orbits are represented by the *basic unit of quantum mechanics,* Planck's constant $I = \hbar$, the absence of quantized orbits inside the neutron mandated a generalization, called *lifting,* of the Planck's constant into a Hermitean and positive-definite, but otherwise arbitrary, integro-differential operator $\hat{I}$. In the same way as the synthesis of the neutron is the most fundamental event in nature, its mathematical representation required the lifting of the most fundamental mathematical quantity, the basic unit, namely, synthesis (6.2.2) requires the following lifting of Planck's constant

$$U \times U^\dagger = U^\dagger \times U = I = \hbar \rightarrow U_n \times U_n^\dagger = \hat{I}_n = \hat{I}_n^\dagger = \hat{I}_n(r, p, \psi, \partial_r \psi, ...) > 0, \quad (6.2.4)$$

with the subsidiary condition following from Eq. (6.2.3)

$$Lim_{r>1fm} \hat{I}_n = I = \hbar, \qquad (6.2.5)$$

Since a mathematics based on an arbitrary (nonsingular) unit simply did not exist in 1978, all branches of mathematics had to be rewritten in such a form

admitting $\hat{I}$, rather than I, as the left and right unit at all levels. This mandated the lifting of: numbers; fields; functional analysis; topology; differential calculus; enveloping associative algebras; Lie algebras; Lie groups; Lie symmetries; Lie representation theory; Euclidean, Minkowskian, symplectic, Riemannian and other geometries; etc. These liftings illustrate the need for decades of research in pure mathematics prior to being in a position of doing serious quantitative studies on the synthesis of the neutron. The occurrence also illustrates the dimension of the resulting works (consisting of over 20,000 pages of published research by hundreds of authors outlined in the General Bibliography) of which we can regrettably touch in this section only the most salient lines.

This huge effort was motivated not only by scientific curiosity, but also by the alarming environmental problems afflicting our planet, which problems were already clear in 1978, even though irrationally dismissed. As already well known in 1978, the resolution of our environmental problems requires new clean energies and fuels. As equally known in 1978, all possible energies and fuels conceivable with quantum mechanics and special relativity had been fully discovered by that time, and all turned out to be environmentally unacceptable. The only hope for society was then the construction of suitable *generalizations* of quantum mechanics and special relativity that would at least permit the conception of new clean energies and fuels. This need provided the author the necessary strength to trash out academic putrescence and its organized opposition against the construction of hadronic mechanics denounced in the footnotes of these volumes.

In fact, if (and only if) the electron is an actual physical constituent of the neutron, then (and only then) the neutron could be stimulated to decay via resonance and/or other mechanisms, thus initiating a it new class of energies called *hadronic energies,* because different than nuclear, atomic and molecular energies and originating from mechanisms in the structure of individual hadrons, rather than in their collection. Unlike nuclear energies, the latter are expected to exist for *light nuclei,* thus being "clean" in the sense of not having sufficient energy to release harmful radiations and/or leave harmful waste, as we shall see in Chapter 11.

The extension of model (6.2.2) to some of the other baryons is elementary, e.g.,

$$\Lambda = (\hat{p}^+, \hat{\pi}^-)_{hm} \equiv (\hat{n}, \hat{\pi}^o)_{hm}, \tag{6.2.6}$$

where the reader should keep in mind the equivalence on iso-Hilbert spaces of particles that are distinct on conventional Hilbert spaces, due to internal non-Lagrangian / non-Hamiltonian exchanges and renormalizations we shall indicate in this section.

Additionally, memoirs [14] worked out in details (see Ref. [14b], Section 5) the representation of *all* characteristics of the π^o meson as a hadronic bound state of

an electron and a positron, although in their isotopic form

$$\pi^o = (\hat{e}^+, \hat{e}^-)_{hm}. \tag{6.2.7}$$

Much along lines (6.2.2) for the neutron, the above model was proposed as a nonunitary image of the positronium P

$$P = (e^+, e^-)_{qm} \ ; \rightarrow \ \pi^o = (\hat{e}^+, \hat{e}^-)_{hm} = U_{\pi^o} \times (e^+, e^-)_{qm} \times U^\dagger_{\pi^o}, \tag{6.2.8a}$$

$$U_{\pi^o} \times U^\dagger_{\pi^o} = \hat{I}_{\pi^o} > 0 \tag{6.2.8b}$$

where the reader should note from these introductory lines that quantum mechanics admits one and only one unit, Planck's constant. while hadronic mechanics admits *different isounits for different particles,* trivially, because the isounit represents contact non-Hamiltonian effects that are different for different particles.

Recall that the positronium is entirely described by one single equation, Schrödinger equation. Similarly, the nonunitary map (6.2.8) yielded *one single hadronic structure equations representing "all" features of the π^o,* including rest energy, charge radius, meanlife, charge, spin, magnetic moments, parity and spontaneous decay, the latter identifying the actual physical constituents.

Memoirs [14] then worked out the model for other mesons, resulting in "bootstrap" models of the type

$$\pi^\pm = (\hat{e}^+, e^\pm, \hat{e}^-)_{hm} \equiv (\hat{\pi}^o, \hat{e}^\pm)_{hm}, \tag{6.2.9}$$

whose spontaneous decay identifies, again, the actual physical constituents, This decay is called *hadronic tunnel effect,* in the sense that the tunneling occurs through Hamiltonian and non-Hamiltonian barriers.

The radical departures from orthodox trends of the above structure models of unstable hadrons should be noted upfront, such as:

1) The new structure models are absolutely impossible if attempted via the use of quantum mechanics for countless reasons, some of which will be identified in this section;

2) The new structure models have no need whatsoever of quark and neutrino conjectures as also shown in detail in this section; and

3) Eliminate for the structural problem the widespread tendency of looking for the "mass spectrum," a feature allowed only for classification, a point emphatically stressed in memoir [14]. In fact, nonunitary maps (6.2.2) and (6.2.8) were identified under the condition of being *spectrum suppressing,* namely, the generalized Schrödinger equation for a hadron had to characterize one state and one state only, the hadron considered, trivially, because all exited states are conventional quantum, thus atomic states, under limits (6.2.5).

The above radical departures from rather universal trends of the time (1978) require the following comments. In essence, hadron physics of the 20-th century

was dominated by the belief that the mechanics exactly valid for the description of point-like electrons moving in vacuum around atomic nuclei was also exactly valid for the description of the same particles moving within the hyperdense media inside hadrons.

Such a belief lead to scientific imbalances of historical proportions studied in Volume I and in the preceding section. The conjecture that quarks and neutrinos are physical particles in our spacetime was a mere consequence of adapting the hadronic structure to a preferred theory. The outcome was a plethora of fundamental problems that remained unresolved, because un-dressed due to the widespread illusion of achieving credibility via the academic power of the affiliations and physical societies, rather than serious scientific evidence.

To minimize misrepresentations of the intent of this section, it should be stated upfront that *we fully accepts the validity of the standard model and of the theory of weak interactions.* However, we restrict their validity to an *external,* Mendeleev-type treatments of hadrons; we deny their additional role as providing a joint representation of both, the classification and the structure of hadrons; and, by following the historical teaching for nuclei, atoms and molecules, *we seek basically new models of the hadronic structure with ordinary massive physical constituents under the condition of achieving compatibility with the established, external, Mendeleev-type theories.*

It should be noted that the new structure model of unstable hadrons did not require the addressing of the neutrino conjecture for the case of mesons and, consequently, could be worked out in its entirety already in the original memoirs [14] under the sole denial of quarks as physical particles in our spacetime. In this section, we shall review *ad litteram* the new structure model for mesons as originally conceived in 1978 by leaving additional advances to interested readers.

The explicit construction of the corresponding new structure model of unstable baryons with ordinary massive physical constituents was delayed for decades because of technical and political reasons. On technical grounds, the use of hadronic mechanics for baryons required the isotopic lifting of the $SU(2)$-spin symmetry that was unavailable at the time of the original proposal [14] (that, however, did contain the isotopies of the $O(3)$-symmetry). The first nonrelativistic structure model of the neutron as a bound state of a proton and an electron appeared in ref. [95] of 1990 following the isotopies of the spin symmetries (see Refs. [5]). Its relativistic extension appeared in papers [5f,96] only following the achievement of the the isotopies of the spinorial covering of the Poincaré symmetry in ref. [5f].

The political difficulties were caused by the fact that the belief in neutrinos as physical particles was, and remains to this day, much more entrenched in the mind of physicists than the corresponding belief for quarks. Consequently,all papers on the new structure model of the neutron and with the additional denial that neutrino as physical particles caused incredible oppositions, at times even hyster-

ical. These oppositions delayed considerably the scientific process and caused a somewhat unusual scientific situation in which, on one side, editors and reviewers mandated the maintaining of the neutrino conjecture while, on the other side, hadronic structure models did not required such a conjecture at all. This explains the presence of the neutrino conjecture in paper [95,96].

The controversies on the nature of the neutrino delayed this volume for at least ten years since it was repugnant for us to complete a lifelong research with political postures. A determining event occurred at the 2006 meeting of the *International Association for Relativistic Dynamics* (IARD) held at the University of Connecticut, in Storrs. During this meeting the participants allowed the author to express his doubts on the existence of the neutrinos as physical particles. The author has no words to express his appreciation and gratitude to all IARD members for their tolerance of dissident views, as well as his sincere respect for their commitment to true scientific democracy for qualified inquiries. Said tolerance by IARD's colleagues gave the author sufficient motivation to initiate the completion of this second volume.

The final decision to initiate the release of this volume was permitted by M. van der Merwe, Editor of Foundations of Physics for the first publication by the author [97], following four independent reviewers, with systematic doubts on the existence of the neutrinos as physical particles in our spacetime. This paper also contains considerable references of similar publications by dissident colleagues. Because of this publication, as well as numerous others by the author (such as the first paper with systematic doubts on the existence of quarks as physical particles, Ref. [88] of 1981), and numerous other pioneering works by other authors, M. van der Merwe was recently granted a Gold Medal for Scientific Merits to be granted in 2008.

As historical notes, we should recall that quantum mechanics was called "atomic mechanics" in Ref. [14], namely, a mechanics conceived and constructed for the atomic structure, in order to distinguish it from "hadronic mechanics," namely, the mechanics conceived and constructed for the hadronic structure. This terminology has been lost with the passing of time. but remains still valid as of today.

Also, electrons were said to be *mutated* when within the hyperdense medium inside hadrons, to reflect a corresponding mutation in Ref. [100] of Lie algebras into covering Lie-isotopic or Lie-admissible algebras. This dual particle and algebra meaning of the word "mutation" has remained in use and will be adopted in this section under the assumption that covering algebras are treated with new mathematics to bypass the Theorems of Catastrophic Inconsistencies of Nonunitary Theories (Section 6.1.6).

Finally, mutated electrons and positrons were called in Ref. [14] "eletons" and "antieletons," respectively. These terms have been replaced with the corre-

sponding terms used in this section, namely,. "isoelectrons" and "isopositrons", or "isodual isoelectrons" to denote the fundamental symmetry for the characterization of their mutations, the Poincaré-Santilli isosymmetry and its isodual [5].

Needless to say, due to the extreme complexity of the problem, this section includes the use of the *totality* of the preceding studies on hadronic mechanics as per classification of Figure 1.22. Readers with a vast knowledge of quantum mechanics but insufficient knowledge of the covering hadronic mechanics are discouraged from glancing at this section to prevent the illusion of its understanding.

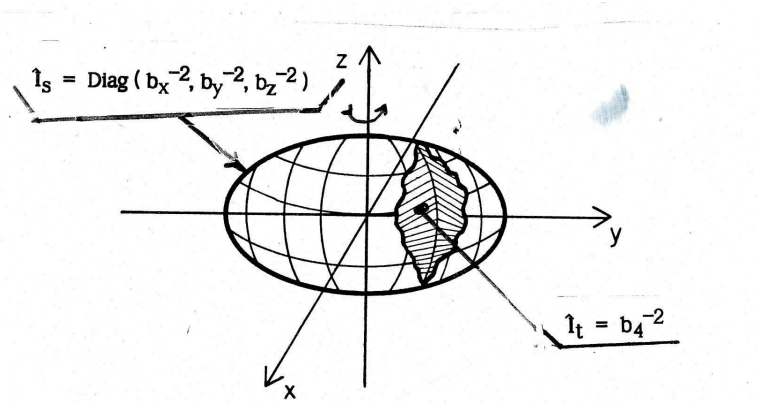

Figure 6.19. A schematic view of one of the various physical meanings of the characteristic quantities defined by isorelativity and hadronic mechanics, the representation of the actual *share* of the particle considered via the space components $b_k^2 = 1/n_k^2$, $k = x, y, z$, here depicting a spheroidal ellipsoid for simplicity (see EHM-II for other shapes represented via nondiagonal isounits), and the representation of the *density* via the fourth component $b_4^2 = 1/n_4^2$, all normalized to the values 1 for the vacuum. Note that these representations do not exist in the mathematics and physics of the 20-th century, trivially, because structurally beyond any hope of representation via a Hamiltonian. Orthodox interests claim that the characteristic quantities are "free parameters." The political nature of such a claim is unmasked by noting that its acceptance requires the belief that the size and ,mass of hadrons are also free parameters. As we shall see in the next chapter from neutron interferometric measurements, the nonspherical and deformable shape of hadrons is *measured* quite accurately and so is the density, trivially given by the ratio between the rest energy and the volume. The confirmation that the characteristic quantities are not "free parameters" will be given in this volume by showing that their numerical values for a given particle are compatible with other tests dealing with the same particle, in the same way that, after it has been measured, the mass of the neutron cannot be changed in going from one test to another.

6.2.2 Inapplicability of Quantum Mechanics for the Hadronic Structure

Rutherford [91] submitted in 1920 the hypothesis that hydrogen atoms in the core of stars are compressed into new neutral particles having the size of the proton that he called *neutrons,* according to the synthesis

$$p^+ + e^- \to n. \tag{6.2.10}$$

The existence of the neutron was confirmed in 1932 by Chadwick [92]. However, Pauli [93] noted that the spin 1/2 of the neutron cannot be represented via a quantum state of two particles each having spin 1/2. Fermi [94] adopted Pauli's objection and, for its resolution, conjectured the emission of a neutral and massless particle he called *neutrino* (meaning in Italian "little neutron") with symbol ν for the particle and $\bar{\nu}$ for the antiparticle. Fermi then developed the theory of *weak interactions* according to which the synthesis of the neutron is characterized by the reaction

$$p^+ + e^- \to n + \nu, \tag{6.2.11}$$

with or complementary reaction

$$p^+ + \bar{\nu} + e^- \to n, \tag{6.2.12}$$

and inverse reaction, the spontaneous decay of an isolated neutron,

$$n \to p^+ + e^- + \bar{\nu}. \tag{6.2.13}$$

Hence, **following Pauli's objection [93], Fermi [94] introduced the neutrino hypothesis for the specific purpose of salvaging the validity of quantum mechanics for the neutron synthesis. However, Santilli proved in 1978 [14] that quantum mechanics remains basically inapplicable (rather than violated) for the neutron synthesis for various reasons,** such as:

INAPPLICABILITY 1. Schrödinger equation does not admit physical solutions for the total energy and other physical quantities for synthesis (6.2.10) because the sum of the rest energies of the proton and of the electron,

$$m_p + m_e = 938.272\,\text{MeV} + 0.511\,\text{MeV} = 938.783\,\text{MeV}, \tag{6.2.14}$$

is *smaller* than the rest energy of the neutron,

$$m_n = 939.565\ MeV, \tag{6.2.15}$$

with "positive" energy difference

$$m_n - (m_p + m_e) = 939.565 - (938.272 + 0.511)\,\text{MeV} = 0.782\,\text{MeV}. \tag{6.2.16}$$

The above data would require a *positive binding energy,* under which Schrödinger equation becomes physically inconsistent because its indicial equation no longer admits real solutions (see Santilli [14], Shiff et al [98] and literature quoted therein). In fact, all consistent quantum bound states (such as those for nuclei, atoms and molecules) have a *negative binding energy* that results in the well known *mass defect* with familiar eigenvalue equation for the Coulomb bound state of two particles with the same mass in relative coordinates

$$(\frac{-\hbar^2}{m} \times \Delta - \frac{e^2}{r}) \times \} psi >= E \times |\psi >, \quad E \in R, \quad E < 0. \tag{6.2.17}$$

where m is the reduced mass. From data (6.2.14)-(6.2.16), the synthesis of the neutron would requires an equation with a positive binding energy of the type

$$(\frac{-\hbar^2}{m} \times \Delta + |V(r)|) \times |\psi >= E \times |\psi >, \tag{6.2.18}$$

that is physically inconsistent, as the skeptic reader is encouraged to verify.

INAPPLICABILITY 2: I n view of numerical values (6.2.14)-(6.2.16), *as written in all particle physics books of the 20-th century, synthesis (6.2.11) violates the principle of conservation of the energy* because without any specification that the l.h.s. should have the minimal kinetic energy of 0.78 MeV, in which case there is no energy left for the neutrino.

INAPPLICABILITY 3. Assuming that the proton and the electron have a relative kinetic energy of (at least) 0.78 MeV, synthesis (6.2.11) remains impossible according to quantum mechanics ,because, at that value of the kinetic energy, the proton-electron cross section is excessively small (about 10^{-20} barns).

INAPPLICABILITY 4. Assuming that the above problems are somewhat resolved via a manipulation of Schrödinger equation, it is impossible for quantum mechanics to achieve a meaningful representation of:

4.1: The meanlife of the neutron of

$$\tau_n = 15m, \tag{6.2.19}$$

, since quantum mechanics would predict a meanlife of the order of $10^{-19}s$;

4.2. The anomalous magnetic moment of the neutron

$$\mu_n = -1.913\mu_N \tag{6.2.20}$$

because, when computed from the magnetic moments of the proton

$$\mu_p = 2.792\mu_N \tag{6.2.21}$$

and of the electron

$$\mu_e = 1.001\mu_B, \tag{6.2.22}$$

would be wrong even in the sign; and of

4.3. The neutron charge radius

$$R = 10^{-13}cm, \tag{6.2.23}$$

since Bohr's radius $R = 10^{-8}cm$ is the smallest radius permitted by quantum mechanics for a "stable" bound state of a proton and an electron.

INAPPLICABILITY 5. The impossibility for quantum mechanics to reach a meaningful representation of the synthesis of the neutron is multiplied, rather than resolved, by complementary synthesis (6.2.12) because, being an antiparticle, the antineutrino carries a *negative* energy, rather than the needed positive energy and, in any case, the cross section of antineutrinos on protons and/or electrons must be assumed as being null for any serious study.

It should be noted that the above insufficiencies of quantum mechanics generally apply for the synthesis of all hadrons at large, beginning with that for the neutral pion

$$e^+ + e^- \to \pi^o, \tag{6.2.24}$$

where the "positive binding energy" is now of 133.95 MeV.

The above occurrences, presented in Ref. [14b] (see page 829, in particular) signaled the birth of hadronic mechanics. In fact, the author attempted for years to achieve a consistent solution of synthesis (6.2.11) via quantum mechanics. The confirmation by Cantabridgean colleagues that a consistent solution for Eq. (6.2.18) does not exist within the class of unitary equivalence of quantum mechanics, left no other choice than that of subjecting the conventional Schrödinger equation to a *nonunitary transform,* thus abandoning quantum mechanics for a covering theory.

6.2.3 Insufficiencies of Neutrino Conjectures

As it is well known, the neutrino hypothesis was more recently incorporated into the so-called *standard model*[30] in which the original neutrino was extended to three different particles, the *electron, muon and tau neutrinos* and their antiparticles. Neutrinos were then assumed to have masses, then to have different masses derived from the fit of experimental data, then to "oscillate" (namely, to change "flavor" or transform one type into the other), with the expectation of additional conjectures intended to bypass preceding unverifiable conjectures.

[30]The literature in the field is so vast to discourage discriminatory listings.

Despite historical advances, the neutrino hypothesis has remained afflicted by a number of basic, although generally unspoken insufficiencies addressed in Section 1.1.2.8, and outlined as follows for the self-sufficiency of this volume:

INSUFFICIENCY 1: According to the standard model, a neutral particle carrying mass and energy in our spacetime is predicted to cross very large hyperdense media, such as those inside stars, without any collision. Such a view is outside scientific reason because already questionable when the neutrinos were assumed in being massless. The recent use of massive neutrinos has rendered the view beyond the limit of plausibility because a massive particle carrying energy in our spacetime simply cannot propagate within hyperdense media inside large collections of hadrons without any collision. The general belief that this is due to the very low value of the cross section between neutrinos and other particles casts shadows on the theory, rather than resolving the inconsistency here considered.

INSUFFICIENCY 2. The fundamental reaction for the production of the (electron) neutrino, Eq. (6.2.11), generally lacks sufficient energy for the synthesis of the neutron itself, let alone the additional energy needed to characterize the hypothetical neutrino.

INSUFFICIENCY 3. As reported in nuclear physics textbooks (see Figure 1.7), the energy measured as being carried by the electron in beta decays follows a bell-shaped curve with a maximum value of the order of 0.782 MeV (depending on nuclear data). The "missing energy" (as the difference between $0.78 MeV$ and the electron energy) has been assumed throughout the 20-th century as being carried by the hypothetical neutrino. However, in view of the strongly attractive Coulomb interactions between the nucleus and the electron, the energy carried by the electron is depends on the direction of emission, with maximal value for radial emission and minimal value for tangential emission (Figure 1.8). Despite a laborious search, the author has been unable to identify in the literature much needed calculations of this aspect because if the "missing energy" is entirely absorbed by the nucleus, then, again, there is no energy left for the neutrino.

INSUFFICIENCY 4. The claims of "experimental detection" of neutrinos are perhaps more controversial than the theoretical aspects because of numerous reasons, such as:

4.1 Enrico Fermi clearly stated in his writings that "the neutrino cannot be directly detected in laboratory;"

4.2. All claims of "neutrino detections" are based on a scattering theory that is basically inapplicable for deep inelastic scatterings (Figure 1.2;

4.3. The elaboration of the data via a theory centrally dependent on the neutrino hypotheses clearly implies "experimental results" compatible with the theoretical assumptions

4.4. The claims of "neutrino detections" via the selection of extremely few events over an extremely large number of events;

4.5. The presence in recent "neutrino detectors" of radioactive sources could themselves account for the extremely few events over an enormous number of total events;

4.5. The lack of clear, physically verifiable differentiations of the various neutrinos;

4.7. The lack of uniqueness of the neutrino interpretation for the interpretation of the experimental data due to the existence of alternative interpretations without the neutrino hypothesis (see Ref. [99] and references quoted therein); and other insufficiencies.

INSUFFICIENCY 5. Numerous additional insufficiencies exist, such as the theory contains an excessive number of parameters essentially capable to achiever any desired fit, and other problems [99]. In fact, the six different "neutrino masses" are *derived* from fit of the data and, as such, could merely be arbitrary *ad hoc* parameters.

For additional studies on the insufficiencies of the neutrino hypothesis, one may consult Bagge [101] and Franklin [102] for an alternative theories without the neutrino hypothesis; Wilhelm [103] for additional problematic aspects; Mössbauer [104] for problems in neutrino oscillations; Fanchi [105] for apparent serious biases in "neutrino experiments"; and literature quoted therein.

The author would like to express his deepest appreciation to Horst E. Wilhelm because his vast physical knowledge, combined with a serious commitment to scientific inquiries, and his independence of thought were instrumental for the author to release his view on the lack of existence of neutrinos as physical particles.

6.2.4 Insufficiencies of Quark Conjectures

The view expressed by the author since the birth of quark theories (see memoir [88] of 1981) is that:

I) SU(3) color theories and more recently the standard model have provided the final Mendeleev-type, classification of particles into families;

II) Quarks are necessary for the elaboration of the theory, however,

III) On ground of strict scientific rigor, quarks should be solely defined what they are technically, purely mathematical representations of a purely mathematical internal symmetry solely definable on a purely mathematical, complex-valued unitary space.

Whenever quarks are assumed to be physiocal particles in our spacetime, numerous unresolved (and generally unspoken) insufficiencies emerge, as treated in Section 1.2.7. and outlined below for the self-sufficiency of this volume:

INSUFFICIENCY 1. According to the standard model, at the time of the synthesis of the neutron, the proton and the electron literally "disappear" from the universe to be replaced by hypothetical quarks as neutron constituents. Moreover, at the time of the neutron spontaneous decay, the proton and the electron literally "reappear" again into our spacetime. This view is beyond scientific reason, because, as pointed out in Section 6.2.1, the proton and the electron are the only *permanently stable* massive particles identified so far and, as such, they simply cannot "disappear" and then "reappear" in our spacetime just because so desired by quark supporters. The *only* plausible hypothesis is that the proton and the electron are actual physical constituents of the neutron as originally conjectured by Rutherford, although the latter view requires the adaptation of our theories to physical reality.

INSUFFICIENCY 2. When interpreted as physical particles in our spacetime, irrespective of whether we refer to mass or energy, *quarks cannot experience any gravity.* As clearly stated by Albert Einstein in his writings, gravity can only be defined in spacetime, while quarks can only be defined in the mathematical, internal, complex-valued unitary space with no known connection to our spacetime. In particular, O'Rafearthaigh's theorem prohibits quarks to be defined via our spacetime symmetries. Consequently, physicists who support the hypothesis that quarks are the physical constituents of protons and neutrons, thus of all nuclei, should see their bodies levitate due to the absence of gravity.

INSUFFICIENCY 3. When, again, interpreted as physical particles in our spacetime, *quarks cannot have any inertia.* In fact, inertia can only be rigorously admitted for the eigenvalues of the second order Casimir invariant of the Poincaré symmetry, while quarks cannot be defined via such a basic spacetime symmetry, as expected to be known by experts to qualify as such. Consequently, "quark masses" are purely mathematical parameters deprived of technical characterization as masses in our spacetime. Hence, "quark masses" are mere *ad hoc* parameters identified by pre-selected fits of data.

INSUFFICIENCY 4. Even assuming that, with unknown scientific manipulations, the above insufficiencies are resolved, it is known by experts that quark theories at the level of first quantization have failed to achieve a representation of *all* characteristics of hadrons, with catastrophic insufficiencies in the representation of spin, magnetic moment, mean lives, charge radii and other basic features of hadrons. Of course Quantum Chromodynamics (QCD) and gauge theories have provided deeper insights, but not a resolution of the controversies due to the inability to reach exact solutions of nonlinear partial differential equations.

INSUFFICIENCY 5. It is also known by experts that the application of quark conjectures to the structure of nuclei has multiplied the controversies, while re-

solving none of them. As an example, the assumption that quarks are the physical constituents of protons and neutrons in nuclei has failed to achieve a representation of the main characteristics of the simplest possible nucleus, the deuteron because:

5.1. Quark conjectures are unable to represent the spin 1 of the deuteron, since they predict spin zero in the ground state of two particles each having spin 1/2, while the deuteron has spin 1;

5.2. Quark conjectures are unable to represent the anomalous magnetic moment of the deuteron despite all possible relativistic corrections attempted for decades, because the presumed "quark orbits" are too small to fit data following polarizations or deformations;

5.3. Quark conjectures are unable to represent the stability of the neutron when a deuteron constituent;

5.4. Quark conjectures are unable to represent the charge radius of the deuteron, and

5.5. When passing to larger nuclei, such as the zirconium, the catastrophic inconsistencies of quark conjectures can only be defined as being embarrassing.

For additional references, one may consult Ref. [88] on historical reasons preventing quarks to be physical particles in our spacetime; Ref. [106] on a technical treatment of the impossibility for quarks to have gravity or inertia; Ref. [97,107] on a more detailed presentation on the topic of this section; and Wilhelm [103] for an in-depth treatment of the lack of rational priorities in quark theories.

The implications of the above insufficiencies are rather serious. In fact, they imply that *the identification of the hadronic constituents with physical particles truly existing in our spacetime is more open than ever and carries ever increasing societal implications since the assumption that quarks are physical constituents of hadrons prevents due scientific process on alternative models admitting new clean energies so much needed by mankind, as illustrated later on.*

Alternatively, we can say that *the insufficiencies of quark conjectures as physical particles in our spacetime render the current status of hadron physics essentially equivalent to our knowledge of atoms at the beginning of the 20-th century, namely, prior to the discovery of their structure. We did have at that time the Mendeleev-classification of atoms into families, but we had yet to initiate the study of the structure of individual atoms. Similarly, at this writing SU(3) color theories and the standard model have indeed provided the final classification of hadrons into family. However, on serious scientific ground the structure of individual hadrons of a given SU(3)-multiplet must be indicated as being unknown.*

As stressed in Section 6.2.1, *all alternative structure models, including those without neutrino and quark conjectures, must achieve full compatibility with the unitary models of classification, in essentially the same way according to which*

quantum structures of atoms achieved full compatibility with their Mendeleev classification.

On historical grounds, the classification of nuclei, atoms and molecules required *two different models,* one for the classification into families and a separate model for the structure of the individual elements of a given family. Quark theories depart from this historical teaching because of their original conception of attempting to represent with one single theory both, the classification and the structure of hadrons. Admittedly, in recent times quarks are differentiated whether characterizing classification and structure, but the problematic aspect persists because of the belief that one single theory can represent the totality of the phenomenology of particles. Hence, current quark theories are basically flawed in their conception. .

The view advocated by Santilli since 1978 [14] (see paper [88] of 1981 and paper [106] of 2006, all completely ignored by organized financial interests on quark conjectures to this date - November 11, 2007) is that, quite likely, history will repeat itself. The transition from the Mendeleev classification of atoms to the atomic structure required a basically new theory, quantum mechanics, due to the large differences existing in the classification and structure of atoms. Similarly, the transition from the Mendeleev-type classification of hadrons to the structure of individual hadrons will require a broadening of the basic theory, this time a generalization of quantum mechanics and special relativity due to the truly dramatic differences of the dynamics of point-particles moving in vacuum, as in the atomic structure, to the dynamics of extended wavepackets moving within hyperdense media, as in the hadronic structure.

6.2.5 Hadronic Two-Body Bound State

The *hadronic two-body bound state* was proposed and solved in the original proposal [14] (see Ref. [14b] Section 5), then used to illustrate, not only the capabilities of hadronic mechanics, but also the achievement of feature unthinkable with quantum mechanics.

The main result of the study was the achievement of a *quantitative representation of the charge independence of strong interactions,* namely, the feature known since Fermi's times that strong interactions are generally attractive irrespective of the relative signs of the charge. In turn, this is the very feature that justified the use of the name "hadronic mechanics" in the original proposal.

The above important achievement was reached by showing that *the mutual penetration of particles in singlet coupling at mutual distances of the order of the range of strong interactions (1 fm) causes a strongly attractive force independent from the sign of their charges.* There is no word to stress emphatically that this basic feature is impossible for quantum mechanics.

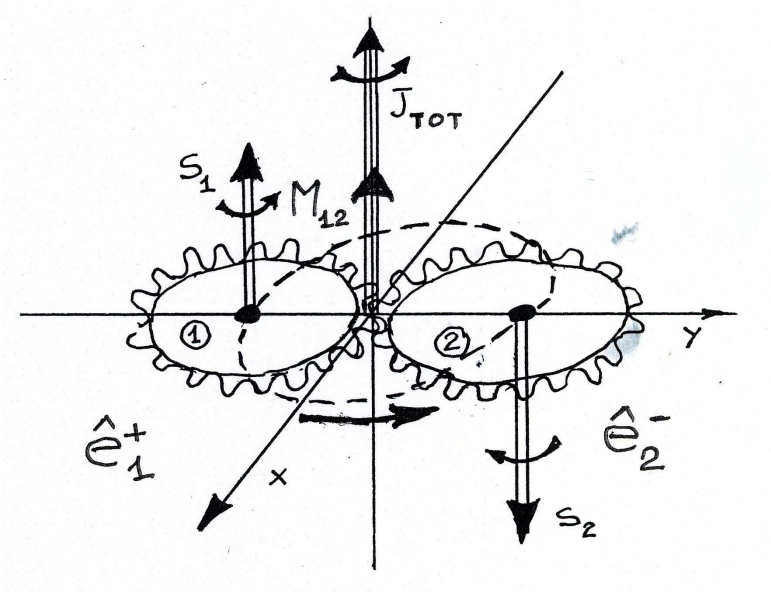

Figure 6.20. A schematic view of the *gear model* for the singlet coupling of two particles at mutual distances of the order of the range of the strong interactions pro[posed in Ref. [14b], page 852, to illustrate that stable bound states at the mutual distance here considered can only occur for singlet couplings, while triplet couplings cause strong repulsive forces, as it occurs for the coupling of ordinary gears.

This is, by far, one of the most important advances permitted by hadronic mechanics with deep implications for all structural problems, including mesons, baryons, nuclei, molecules, stars, quasars, etc. It is expected as being admitted by the mind most resilient to advances that the achievement of the first quantitative understanding of the mechanism of attraction under strong interactions is the necessary pre-requisite for basically new clean energies and fuels, thus mandating its study.

In this section we review the above features as proposed in Ref. [14b], plus the very few additional details emerged since 1978. The rest of this volume is essentially dedicated to an *application* of the content of this section. It is unfortunate that, despite the above features, proposal [14] remained ignored for decades by organized interests in quantum mechanics and special relativity, despite our bringing it to the attention of "leading" (?) physicists via letters, explanations, petitions and the like. Yet, as stressed several times, their lack of response was appreciated because a gift of scientific priority to our group. Let us begin with the following:

HADRONIC POSTULATE 1: All particles at mutual distances of the order of the strong interactions experience a strongly "attractive" force in "singlet" coupling and a "repulsive" force in "triplet" coupling.

This postulate was introduced and illustrated in proposal [14b] via the following (see Figure 6.22):

GEAR MODEL: Gears can only be coupled in singlet.

In essence, when particle wavepackets penetrate one inside the other, as in Figure 6.2, their intrinsic rotation remains allowed if and only if the coupling is with anti[parallel spin, while in the event of a coupling with parallel spins it is easy to see the emergence of a strongly *repulsive* force, exactly as it occurs for ordinary gears, trivially, because intrinsic rotations should occur for one wavepacket (one gear) moving against the other.

We assume the reader knows that this is a fundamental feature of nature. In fact, *valence electrons correlate/bond in molecular structures only in singlet pairs,* whose lack of quantitative treatment is one of the biggest century old failure of quantum chemistry. We expect the educated reader to know that a similar feature occurs in nuclear structures. It is our task to show in this section that a similar feature occurs also in the hadronic structure. Hence, from now on, unless otherwise stated, all couplings of particles pairs will be in singlet.

INSUFFICIENCY OF THE QUANTUM SCATTERING THEORY: The author has indicated for decades, to no avail, that the quantum scattering theory is fundamentally inapplicable for deep inelastic scatterings, because quantum mechanics can only represent particles as dimensionless points. Consequently, quantum mechanics has no mean to differentiate singlet and triplet couplings. Lacking such a differentiations, all "experimental results" in deep inelastic scatterings based on the conventional "quantum, scattering theory, are certainly suitable to secure large public funds, academic chairs and prizes, but they are mere "experimental beliefs" on strict scientific grounds, and they will remain so until vast theoretical and experimental studies are conducted via a covering scattering theory with a credible differentiation between singlet and triplet couplings.

A further notion needed for the understanding of this section is that of the *trigger.* In essence, experimental evidence studied later on indicates that spinning particles, such as the electrons, do not achieve a state of deep mutual penetration of their wavepackets in singlet coupling, unless there is an external intervention called "trigger." Alternatively, we can say that spinning particles have a *hadronic horizon,* given by a sphere of radius 1 fm separating the validity of quantum mechanics in the outside and that of hadronic mechanics in the inside. The "trigger" is then the external action need to cross the hadronic horizon.

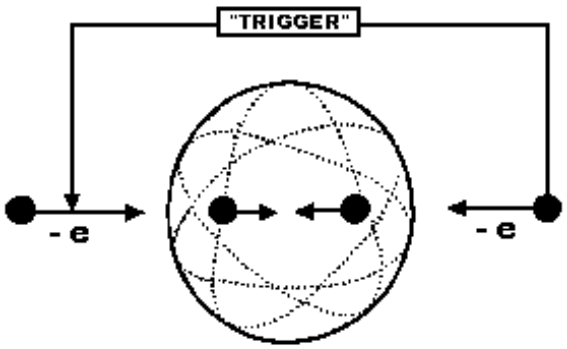

Figure 6.21. A schematic view of the *trigger,* namely, an action generally needed for two particles to cross the *hadronic horizon* and activate strong interactions. This notion will emerge better later on when we study the laboratory synthesis of the neutron from protons and electrons, and related new clean energies. At this moment we assume simple realizations of the "trigger," e.g. those merely caused by sufficient kinetic energy to achieve the deep penetration of the wavepackets needed to activate hadronic mechanics.

In nature, the best known realization of the "trigger" is the pressure in the core of stars "compressing," in Rutherford's words, the electron inside the proton to synthesize the neutron. However, we shall see in Chapter 8 that the Cooper pair in superconductivity is created thanks to a "trigger" caused by cuprates. Similarly, we shall see in Chapter 9 that electron valence bonds are triggered by nuclei.

To put it differently, isolated electrons *repel* each other due to their identical charge, and certainly cannot form any bond. An external intervention is then needed to create electron pairs in valence couplings, Cooper pairs and other structures, namely, to cross the hadronic horizon as a necessary condition to activate the charge independent, strongly attractive forces identified below.

After these background lines, we pass to a review of the two-body hadronic model proposed in Ref. [14b], Section 5. As indicated since the introductory Section 6.2.1, Eq. (6.1.2), the objective is the study of the lifting of a conventional quantum bound state under a nonunitary transform. The lifting of the center-of-mass motion is trivial and i is left to the interested reader.

Additionally, the isoeigenvalues of the *isotopic rotational symmetry* for the angular momentum component are conventional [5a,5b] and they are hereon ignored because inessential for the content of this section.Hence, we consider the important part, the nonunitary lifting of a conventional, two-body, Schrödinger's

equation in relative coordinates

$$\left(\frac{p\times p}{m}-\frac{z\times e^2}{r}\right)\times\psi(r)=E_0\times\psi(r),\ \ E_0\in R,\ \ E_0<0. \tag{6.2.25a}$$

$$p\times\psi(r)=-i\times\partial_r\psi(r), \tag{6.2.25b}$$

where r is the relative distance, m is the reduced mass and we have assumed $\hbar=1$.

As familiar to the reader who has studied the preceding parts. the desired lifting is characterized by *the same nonunitary transform applied to the totality of the quantum mechanics formalism, including the totality of their operations,* with no exception to avoid the Theorems of Catastrophic Inconsistencies of Section 6.1.6, and we shall write

$$U\times U^\dagger\neq I,\ \ U\times U^\dagger>0, \tag{6.2.26a}$$

$$I\to\hat I=U\times I\times U^\dagger=1/\hat T>0, \tag{6.2.26b}$$

$$A\to\hat A=U\times A\times U^\dagger,\ \ A=p,H,\ldots, \tag{6.2.26c}$$

$$U\times(A\times B)\times U^\dagger=\hat A\hat\times\hat B=\hat A\times\hat T\times\hat B,\hat\psi=U\times\psi\times U^\dagger, \tag{6.2.26d}$$

The fundamental invariance (intended as the preservation of the same numerical predictions under the same conditions at different times despite the nonunitary structure) is assured by the Santilli isomathematics based on the reconstruction of the totality of the conventional mathematics of quantum mechanics into a form admitting $\hat I$, rather than I, as the correct left and right generalized unit at all levels.

This requires the reformulation of the *nonunitary* transform (evidently expressed on a conventional Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$) as the *isounitary transform* on a iso-Hilbert space $\hat{\mathcal{H}}$ over the isofield of isocomplex numbers $\hat{\mathcal{C}}$, i.e.,

$$U=\hat U\times\hat T^{1/2},\ \ U\times U^\dagger=\hat U\hat\times\hat U^\dagger=\hat U^\dagger\times\hat U=\hat I, \tag{6.2.27}$$

under which we have the basic invariances

$$\hat I\to\hat I'=\hat U\hat\times\hat I\hat\times\hat U^\dagger\equiv\hat I, \tag{6.2.28a}$$

$$\hat A\hat\times\hat B\to\hat U\hat\times(\hat A\hat\times\hat B)\hat\times\hat U^\dagger=\hat A'\hat\times\hat B', \tag{6.2.28b}$$

where one should note the preservation of the numerical value of the isounit essential for measurements, from which all other invariances follow.

At this point *readers still intent in using conventional mathematics are discouraged from continuing the glancing of this section,* because it would be like

elaborating "quantum" equations with "isomathematics," resulting in a complete nonscientific nonsense. This implies the reader abandoning the use of sinus, cosinus, exponential, differential and *all* mathematics so familiar for protracted use, and the replacement with isotopic covering forms.

Under the above assumptions, the isounitary lifting of Schrödinger equations yields the Schrödinger-Santilli isoequations

$$U \times \left(\frac{p \times p}{m} - \frac{z \times e^2}{r} \right) \times \psi(r) \times U^\dagger =$$

$$= \left(\frac{1}{m} \hat{p} \times \hat{T} \times \hat{p} \times \hat{T} - \frac{z \times e^2}{r} \right) \times \hat{\psi}(r) =$$

$$= U \times [E_0 \times \psi(r)] \times U^\dagger = \hat{E} \hat{t} imes \hat{\psi} = E \times hat \psi, \tag{6.2.29a}$$

$$U \times [p \times \psi(r)] \times U^\dagger = \hat{p} \hat{\times} \hat{\psi}(r) =$$

$$= -U \times [i \times \partial_r \psi(r)] \times U^\dagger = -\hat{i} \hat{\times} \hat{\partial}_r \hat{\psi} = -i \times \hat{T} \times \partial_r \hat{\psi}(r), \tag{6.2.29b}$$

where one should note the *lifting of the numerical value of the binding energy* from E_o to E, trivially, due to the lifting of the operator from H to $H \times T$,[31] with consequential lifting of the wavefunction. One should also note that there is no isotopic element in the r.h.s of the Coulomb term because of the lifting of the fraction for which we can symbolically write

$$U \times [(/) \times \psi] \times U^\dagger = (\hat{/}) \hat{\times} \hat{\psi} = (/) \times U \times \psi \times U^\dagger = (/) \times \hat{\psi}. \tag{6.2.30}$$

Alternatively, the isounitary lifting solely generalizes operators and eigenfunctions and cannot lift scalars.

As it will soon be evident, Eqs. (6.2.29) are insufficient for the hadronic bound state because they miss the "trigger" that, being external, has to be added. The trigger here assumed is of Coulomb nature, it is represented by the addition in Eq. (6.2.29a) of the term $(e^2/r) \times \hat{T}$, and we shall write[32]

$$\left(\frac{1}{m} \hat{p} \times \hat{T} \times \hat{p} \times \hat{T} - \frac{z \times e^2}{r} + \frac{e^2}{r} \times \hat{T} \right) \times \hat{\psi}(r) = E \times \hat{\psi}(r). \tag{6.2.31a}$$

$$\hat{p} \times \hat{T} \times \hat{\psi}(r) = -i \times \hat{T} \times \partial_r \hat{\psi}(r), \tag{6.2.31b}$$

To proceed, we now assume the isounit

$$\hat{I} = Diag.(n_1^2(1), n_2^2(1), n_3^2(1), n_4^2(1)) \times Diag.(n_1^2(2), n_2^2(2), n_3^2(2), n_4^2(2)) \times$$

[31] Note that $\hat{H}$ and $\hat{T}$ do not generally commute. As a consequence, $\hat{H} \times \hat{T} \times \hat{\psi} \neq \hat{T} \times \hat{H} \times \hat{\psi}$.

[32] The sign of the trigegr will soon result to be inessential.

$$\times e^{(\psi/\hat{\psi})\times\int dr^3\hat{\psi}^\dagger(r)_{1\downarrow}\times\hat{\psi}(r)_{2\uparrow}} \tag{6.2.32}$$

where the two diagonal matrices represent the shapes (assumed to be spheroids) and the densities of the particle considered, while the last term represents the non-Hamiltonian interactions. As now familiar, the above isounit represents:

1) The nonlocality of the strong interactions expressed by the volume integral of waveoverlapping, as per historical legacy;

2) The nonlinearity of the strong interactions expressed by an explicit dependence of the isounit on the wavefunctions, also as epr historical legacy; and

3) The non-Hamiltonian character of the strong interactions, also per open historical legacy, here referred to the inability for their complete representation with a Hamiltonian and the need for a second operator, the isounit.[33]

The above isounit is excessively general for the limited scope of this section. We shall then use the approximate expression characterized by:

1) The assumption that the particles have a point-like *charge,* such as the electrons, in which case the characteristic quantities can be approximated to 1 and the two diagonal matrices in (6.2.31) be ignored in first approximation;

2) The evaluation of the volume integral into a constant; and

3) The expansion of the isoexponent terminated to the second term.

The above approximations yield the expressions

$$\hat{I} \approx e^{N\times\psi/\hat{\psi}} \approx 1 + N \times \psi/\hat{\psi}, \tag{6.2.33a}$$

$$\hat{T} \approx e^{-N\times\psi/\hat{\psi}} \approx 1 - N \times \psi/\hat{\psi}, \tag{6.2.33b}$$

$$N = \int dr^3\, \hat{\psi}^\dagger(r)_{1\downarrow} \times \hat{\psi}(r)_{2\uparrow}, \tag{6.2.32c}$$

$$|\hat{I}| \gg 1, \;\; |\hat{T}| \ll 1, \tag{6.2.33d}$$

$$Lim_{r\gg 1fm}\hat{I} = 1. \tag{6.2.33e}$$

Note that the explicit form of ψ is of the familiar Coulomb type, thus behaving like

$$\psi \approx P \times \exp(-b \times r), \tag{6.2.34}$$

[33]As we shall see in the next chapter, one of the biggest failure of the nuclear physics of the 20-th century has been the inability to understand nuclear forces, despite recent representations with a very large number of terms researchers keep adding to the Hamiltonian in the dream of finding an accurate representation. The origin of the failure is precisely the belief that the strong nuclear forces are entirely representable with a Hamiltonian while the physical reality is dramatically more complex than that. The main point here raised is that, of course, strong interactions have a Hamiltonian component, but they also have a "contact" component dramatically beyond the representational capabilities of a Hamiltonian. Such a "contact" component *cannot* be represented with a potential to prevent major physical distortions equivalent to granting a potential to resistive forces. Hadronic mechanics was built to represent such a contact, non-=Hamiltonian component in an axiomatically consistent and invariant way.

with P (approximately) constant and hadronic horizon

$$r_h = \frac{1}{b}, \tag{6.2.35}$$

while $\hat{\psi}$ behaves like (see also below)

$$\hat{\psi} \approx Q \times \left(1 - \frac{e^{-b\times r}}{r}\right), \tag{6.2.36}$$

with Q also (approximately) constant.

By introducing the *Hulthen potential*

$$V_{Hulthen} = W \frac{e^{-b\times r}}{1 - e^{-b\times r}}, \tag{6.2.37}$$

where W is Hulthen's constant, the isotopic element can be written

$$\hat{T} \approx 1 - N \times \psi/\hat{\psi} = 1 - V_0 \frac{e^{-b\times r}}{(1 - e^{-b\times r})/r}, \tag{6.2.38}$$

where we have a new Hlthen constant because it has absorbed the constant N in Eq. (6.2.38) for the Hulthen potential.

Recall that *the Hulthen potential behaves at small distances like the Coulomb potential,*

$$V_{Hulthen} \approx \frac{V_0}{b} \times \frac{1}{r}. \tag{6.2.39}$$

An understanding of the strength of the Hulthen potential is then given by the fact that the quantity b in the denominator is of the order of 10^{-13} cm, thus resulting the a multiplicative factor of the order of 10^{13}.

As a result, *inside the hadronic horizon, the Coulomb potential is absorbed by the Hulthen potential,* and we can write

$$+\frac{e^2}{r}\times\hat{T} - \frac{z\times e^2}{r} \approx +\frac{e^2}{r}\times\left(1 - \frac{V_{\rm Hulthen}}{r}\right) - \frac{z\times e^2}{r} = -V\times\frac{e^{-b\times r}}{1 - e^{-b\times r}}, \tag{6.2.40}$$

therefore resulting in the desired overall attractive force inside the hadronic horizon.

By assuming in first approximation

$$|\hat{T}| \approx \rho < 1, \tag{6.2.41}$$

and by reinstating $\hbar$ for clarity, the radial isoequation can be written

$$\left[\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}\right) + \frac{m}{\rho^2\times\hbar^2}\left(E_{hb} + V\times\frac{e^{-b\times r}}{1-e^{-b\times r}}\right)\right]\times\hat{\psi}(r) = 0, \tag{6.2.42}$$

where E_{hb} is the hadronic binding energy and,, again, we have ignored the Coulomb term because absorbed in the Hulthen potential (see Ref. [14b] for the inclusion of the Coulomb term).

The exact solution and related boundary conditions were first computed in detail in Ref. [14b], Section 5, page 837, and remain fully applicable today. By assuming the change of variable

$$x = 1 - e^{-b\times r} \tag{6.2.43}$$

Eq. (6.2.42) can be written

$$\left[x \times (1-x) \times \frac{d^2}{dx^2} - (2 \times |A|^{1/2} + 1) \times \frac{d}{dx} + \beta^2\right] \times S(x) = 0, \tag{6.2.44a}$$

$$A = \frac{m}{\hbar^2 \times \rho^2 \times b^2} \times E_{ib} < 0, \;\; \beta^2 = \frac{m \times V_0}{\hbar^2 \times \rho^2 \times b^2}, \tag{6.2.44b}$$

with boundary conditions

$$S(0) = 0, \;\; Lim_{r\to\infty} e^{-|A|^{1/2}\times b\times r} \times S(r) = 0, \tag{6.2.x45}$$

The solution of Eq. (6.2.44a) is then given by (Ref. [14b], Eq. (5.1.19), page 837)

$$G_n(x) = \Sigma_{k=1}^{k=n} \left(\begin{array}{c} n-1 \\ k-1 \end{array} \right) \times \left(\begin{array}{c} n+k+2\times|A|^{1/2}-1 \\ k \end{array} \right) \times x^k, \tag{6.2.46}$$

and can be rewritten

$$\hat{\psi}(r) =_2 F_1(2\times\gamma+1+n, 2\times\gamma, e^{-b\times r}) \times \frac{1-e^{-b\times r}}{r} \times e^{-b\times r}, \tag{6.2.47a}$$

where

$$\gamma = \frac{\beta^2 - n^2}{2\times n}, \tag{6.2.48}$$

with isorenormalized isoeigenfunctions (Ref. []14b] Eq. (5.1.29), page 839)

$$\hat{\psi}(r) = \left[\frac{\Gamma(2\times|A|^{1/2}+3)}{\Gamma(3)\times\Gamma(2\times A|^{1/2})}\right]^{1/2} \times \frac{1-e^{-b\times r}}{r} \times e^{-|A|^{1/2}\times b\times r} \tag{6.2.49}$$

The *hadronic binding energy* is then given by (Ref. [14b], Eq. (5.1.20, page 847)

$$E_{hb} = -\frac{\hbar^2\times\rho^2\times b^2}{4\times m} \times \left(\frac{m\times V_0}{\hbar^2\times\rho^2\times b^2\times n} - n\right)^2 =$$

$$= -\frac{V_0}{4 \times \beta^2} \times \left(\frac{\beta^2}{n} - n \right)^2. \tag{6.2.50}$$

The boundary conditions now demand that

$$\beta^2 = \frac{m \times V_0}{\hbar^2 \times \rho^2 \times b^2} \; > \; n^2. \tag{6.2.51}$$

The above results recovers the well known property that *the Hulthen potential has a finite spectrum of eigenvalues.* This feature begins to illustrate the hadronic bound state because the corresponding quantum state has an infinite spectrum of energy. However, as we shall see in the next section, *to be fully hadronic, the bound state must suppress the Hulthen spectrum down to only one value, the particle considered,* because, as indicated earlier, excited states would exit the hadronic horizon and be quantum mechanical.

The original derivation [14b] then proceeds to reduce the above solution to a form usable for hadronic structure models. For an isoparticle to be bounded inside the hadronic horizon b^{-1}, its isowavelength must be proportional to the horizon itself, and we shall write

$$\lambda = (k_1 \times b)^{-1}/2 \times \pi, \tag{6.2.52}$$

where k_1 is a positive quantity that must be constant for a stationary state.

Next, the *hadronic kinetic energy* E_{hk} of one constituent can be written

$$E_{hk} = \frac{\hat{p}^2}{2m} \approx \frac{\hbar^2 \times \rho^2 \times b^2}{2 \times m}, \tag{6.2.53}$$

[34]

Next, Ref. [14b] introduces the following second constant

$$k_2 = \beta^2 = \frac{m \times V_0}{\hbar^2 \times \rho^2 \times b^2} = 1 + \epsilon, \tag{6.2.54}$$

from which we have the expression

$$V_0 = k_2 \times \frac{\hbar^2 \times \rho^2 \times b^2}{m} = 2 \times k_2 \times E_{hk} \tag{6.2.55}$$

Hence, the *hadrolnic total energy* of the hadronic bound state is given by (Ref. [14b] Eq. (5.1.28), i.e.,

$$E_{ht} = 2 \times E_{hr} + 2 \times E_{hk} - E_{hb} \approx 2 \times k_1 \times [1 - (k_2 - 1)^2] \times \hbar \times b \times c_o =$$

[34]Ref. [14b], page 838, stresses the need to use the "physical" momentum $p = m \times v$, and not the "canonical" linear momentum, because, under nonpotential forces, the latter, in general, has no connection to the physical quantity.

$$= 2 \times k_1(1-\epsilon^2) \times \hbar \times b \times c_o. \tag{6.2.56}$$

where c_o is the speed of light in vacuum, and one should remember that the last approximation holds for hadronic bound states where the rest energy is ignorable with respect to the kinetic energy, as we shall see to be the case for isoelectrons.

At this point Ref. [14b] had reached an expression for the total energy of the two-body hadronic bound state that, however, depends on two unknowns, k_1, k_2. To achieve a numerical solution, Ref. [14b] introduces, as a second expression, the *meanlife* of the hadron considered, since *we solely consider unstable hadrons.* The expression selected for the meanlife is the familiar one

$$\tau^{-1} = \lambda^2 \times |\hat{\psi}(0)|^2 \times \frac{\alpha^2 \times E_{hk}}{\pi \times \hbar}. \tag{6.2.57}$$

where α is the fine structure constant, and the reader should keep in mind that the meanlife τ^{-1} is isotopic, that is, derived via isotopic methdos, like all other measurable quantities.. By using the above expressions, we can write

$$\hat{\psi}(0) \left[\frac{\frac{1}{2} \times (k_2-1) \times \Gamma[\frac{1}{2} \times (k_2-1)+2]}{3! \times \Gamma[\frac{1}{2} \times (k_2-1)]} \right]^{1/2} \times b =$$

$$= \left[\frac{\frac{1}{4} \times (k_2-1)^2 \times \Gamma[\frac{1}{2} \times (k_2--1)+1]}{6 \times \Gamma[\frac{1}{2} \times (k_2-1)]} \right]^{1/2} \times b = \frac{(k_2-1)^{3/2}}{48^{1/2}} \times b. \tag{6.2.58}$$

The meanlife of the hadronic bound state then becomes

$$\tau^{-1} = \frac{4 \times \pi}{k_1^2 \times b^2} \times \frac{(k_2-1)^3}{48} \times \frac{K_1 \times \hbar \times b \times c_o}{(137)^2 \times \hbar} =$$

$$= \frac{4 \times \pi}{48 \times (137)^2} \times \frac{(k_2-1)^3}{k_1} \times b \times c_o. \tag{6.2.59}$$

In this way, Ref. [14b], Eqs. (5.1.32), page 840, reached a *system of two equations with two unknown quantities,* k_1, k_2 expressed in terms of the total rest energy E_{tot}, the meanlife τ and the charge radius R_c of the two-body hadronic bound state, that it is reproduced identically below

$$k_1 \times [1-(k_2-1)^2] = \frac{E_{ht}}{2 \times \hbar \times b \times c_o}. \tag{6.2.60a}$$

$$\frac{(k_2-1)^3}{k_1} = \frac{48 \times (137)^2}{4 \times \pi \times b \times c_o} \times \tau^{-1} \tag{6.2.60b}$$

The most important results can be summarized as follow:

CHARGE INDEPENDENCE OF STRONG INTERACTIONS. To the best of our knowledge, hadronic mechanivs achieves the first and only known quantitative representation of the charge independence of strong interactions. As clear from the preceding analysis, this important result is achieved via the use of a force that is strongly attractive inside the hadronic horizon and such to behave like the Coulomb force, thus absorbing the latter irrespective of whether attractive or repulsive.

Alternatively, the same result can be achieved with an attractive force other than the Hulthem one not necessarily behaving like the Coulomb force inside the hadronic horizon, but sufficiently stronger than the latter as an evident condition to reach charge independence.

To fully understand the mechanism, the reader should keep ion mind that the actual representation occurs on iso-Hilbert spaces over isofields, and that the treatment presented in this section has been the *projection* of the isotopic treatment in our Euclidean space for clarity.

MASS-ENERGY ISORENORMALIZATIONS. Ref. [14b] achieved the first and only known renormalization originating from contact, non- Lagrangian / non-Hamiltonian interactions, called *isorenormalizations,* given the following liftings of quantum rest (qr) and quantum kinetic (qk) energies into the corresponding hadronic rest (hr) and hadronic kinetic (hk) energies

$$E_{qr} = m_{qr} \times c_o^2 \ \rightarrow \ \hat{E}_{hr} = m_{hr} \times c_o^2 = \frac{m_{qr}}{\rho^2} \times c_o^2, \tag{6.2.61a}$$

$$E_{qk} = \frac{1}{2 \times m_{qr}} \times p^2 \ \rightarrow \hat{E}_{hk} = \frac{1}{m_{hr}} \times p^2 = \frac{\rho^2}{m_{qr}} \times p^2, \tag{6.2.61b}$$

which are *necessary* to resolve the inconsistency of quantum mechanics under "positive" binding energies (Section 6.2.2), as we shall see in the next sections.

In fact, the resolution permitted by hadronic mechanics is that, when a quantum solution is impossible because the value of the rest energy is such to require inconsistent positive binding energies, the isorenormalized total energ becomes so large to admit a *negative* binding energy, as it is the case for the above model.

Note that *isorenormalizations are fully predicted by Santilli isorelativity.* Those considefred herein are characterized by the variation of the speed of light and maximal causal speeds within hyperdense media already established by preceding experimental verifications,

$$c = \frac{c_o}{\rho^2} = \frac{c_o}{n_4} = c_o \times b_4, \tag{6.2.62a}$$

$$V_{ma} = c_o \times \frac{n_3^2}{n_4^2} = c_o \times b_4^2$$

$$\frac{n_4^2}{n_3^2} = \frac{b_3^2}{b_4} = \rho^2, \tag{6.2.62b}$$

namely, ρ^2 is a numerical value of the geometrization of the departure of the interior of hadrons from our spacetime.

The reader with a technical knowledge of Santilli's isorelativity knows that the above isorenormalizations can be best derived from the Poincaré-Santilli isosymmetry, that causes, in general, a mutation of *all* intrinsic physical characteristics of particles.

Hence, the most insidious misrepresentation of the content of this section is the theological belief that, when immersed within the hyperdense medium inside hadrons, an ordinary particle such as the electron is the same as that in vacuum. In reality, the electron is characterized by an irreducible representation of the (spinorial covering of) the Poincaré group, while the isoelectron is characterized by a corresponding irreducible representation of the covering Poincaré-Santilli isogroup, with consequential mutations, in general, of all physical characteristics as a result of the distortions in the electron wavepacket and other features caused by the hyperdense medium.

We can say that electromagnetic interactions can only change the *kinematic* characteristics of particles while leaving their *intrinsic* characteristics (spin, parity, etc.) unchanged. By comparison, strong interactions are predicted to cause mutations of *all* characteristics, whether kinematical or intrinsic. Still alternatively, the belief that the electron has spin 1/2 when in the core of a collapsing star is pure theology proffered for personal gains without scientific credibility.

SPECTRUM SUPPRESSION. A basic assumption of hadronic mechanics is that *the excited hadronic states are quantum mechanical* [14b]. Hence, the hadronic bound state studied in this section is consistent if and only if the finite Hulthen spectrum is reduced to one, and only one energy level, that of the hadron considered. Any excitation brings the isoconstituents outside the hadronic horizon, in which the Hulthen potential is null and the state recovers the quantum form. As we shall see, the above crucial condition is indeed verified for our hadronic structure models with conventional massive particles as physical constituents.

It should be indicated that this is expected as being the case for "simple" unstable hadrons, such as light mesons and the first baryons. The possibility of exited states is not excluded for some of the baryonic resonances. Their study is rather complex since it implies the joint use of quantum and hadronic mechanics and will be left to the interested reader.

On historical notes, the most important study of the hadronic bound state following that of Ref. [14b] was done by A. O. E. Animalu [108] who applied the model for the first and only known representation of the structure of the Cooper pair and developed his isosuperconductivity theory reported in Chapter 8. An

additional study was done by Animalu and Santilli [109] that set the basis for chemical applications studied in Chapter 9. No additional study, conducted via the true use of hadronic mechanics, has occurred during the three decades since the original proposal [14b], to our best knwoeldge.

6.2.6 The π^o Meson as a Compressed Positronium

Following the detailed solution of the two-body hadronic bound state outlined in the preceding section, Ref. [14b] presented its consistent application for the *representation of all characteristics of the π^o meson as hadronic bound state of one isoelectron and one isopositron,* or as a "compressed positronium" in Rutherford's language, according to models (6.2.7), (6.2.8), Figure 6.20,

$$\pi^o = (\hat{e}^+, \hat{e}^-)_{hm}. \tag{6.2.63a}$$

$$P = (e^+, e^-)_{qm} \ \rightarrow \ \pi^o = (\hat{e}^+, \hat{e}^-)_{hm} = U_{\pi^o} \times (e^+, e^-)_{qm} \times U^\dagger_{\pi^o}, \tag{6.2.63b}$$

$$U_{\pi^o} \times U^\dagger_{\pi^o} = \hat{I}_{\pi^o} \neq I, \ \hat{I}_{\pi^o} > 0. \tag{6.2.63c}$$

The model permitted the exact and invariant representation of: rest energy E_{π^o}, meanlife τ_{π^o}, charge radius R_{π^o}, charge q_{π^o}, spin J_{π^o}, magnetic moments μ_{π^o}, space and charge parities I^G

$$E_{\pi^o} = 134.97 MeV, \ \tau_{\pi^o} = 0.84 \times 10^{-16} s, \ R_{\pi^o} = 10^{-13} cm, \tag{6.2.64a}$$

$$q_{\pi^o} = 0, \ J_{\pi^o} = 0, \ I^G = 1^-, \ \mu = 0, \tag{6.2.64b}$$

and the spontaneous decay

$$\pi^o = (\hat{e}^+, \hat{e}^-)_{hm} \ \rightarrow \ \gamma + \gamma, \quad (98.7798 \pm 0.032)\%, \tag{6.2.65}$$

representing the evident annihilation of the physical constituents, the decay

$$\pi^o = (\hat{e}^+, \hat{e}^-)_{hm} \rightarrow e^+ + e^-, \quad (7.5 \pm 2.0 \times 10^{-8})\% \tag{6.2.66}$$

representing the hadronic tunneling of the physical constituents, the remaining decays, such as

$$\pi^o = (\hat{e}^+, \hat{e}^-)_{hm} \rightarrow e^+ + e^- + \gamma \quad (1.198 \pm 0.032)\% \tag{6.2.67a}$$

$$\pi^o = (\hat{e}^+, \hat{e}^-)_{hm} \rightarrow \ (e^+, e^-)_{qm} + \gamma \quad (1.82 \pm 0.29 \times 10^8)\% \tag{6.2.67b}$$

being secondary effects.

The model is merely given by structural isoequation (9.6.42) combined with the meanlife (6.2.57) and charge radious as subsidiary constraints, merely reformulated for the π^o meson,

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \frac{m}{\rho^2 \times \hbar^2} \left(E^{\pi^o}_{hb} + V \times \frac{e^{-b \times r}}{1 - e^{-b \times r}} \right) \right] \times \hat{\psi}(r) = 0, \tag{6.2.68a}$$

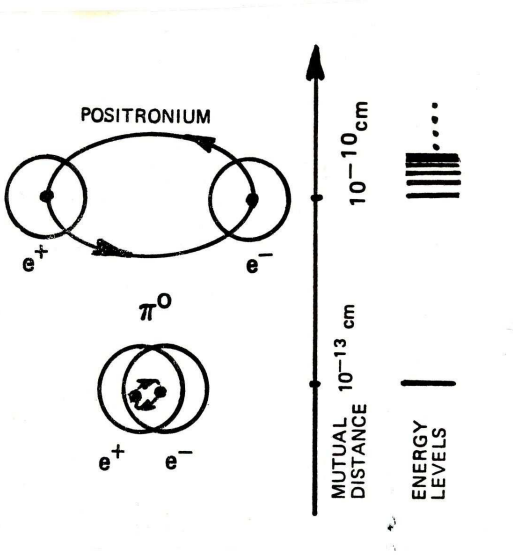

Figure 6.22. Quantum mechanics solely permits the representation of the structure of the π^o meson as a hypothetical bound state of one hypothetical quark and one hypothetical antiquark that, by conception (but not in quantitative realization) are believed as being permanently confined inside the meson, despite the extreme energies achieved in recent particles accelerators. Despite all these conjectures and shortcomings, the model can only represent *some* and definitely not all the characteristics of the particle. Hadronic mechanics allows a quantitative representation of the π^o meson as a bound state of one isoelectron and one isopositron at mutual distances of the order of the strong interactions (1 fm), as depicted in Figure 6.20. Alternatively, hadronic mechanics permits the representation of the π^o meson as a new bound state of the positronium at short distances, or, in Rutherford's words, as a "compressed positronium." Contrary to quark theologies, our hadronic model permits the exact and invariant representation of *all* characteristics of the meson, including the spontaneous decays. The model was worked out in all details in Section 5 of memoir [14b] of 1978, and has remained unchanged since that time, although ignored by organized interests in quark theologies.

$$\tau^{-1} = \lambda^2 \times |\hat{\psi}(0)|^2 \times \frac{\alpha^2 \times E_{hk}}{\pi \times \hbar}.$$

$$R_{\pi^o} = b^{-1}, \tag{6.2.68c}$$

with ensuing system (6.2.60 in the two unknown quantities $k_1 and k_2$

$$k_1 \times [1 - (k_2 - 1)^2] = \frac{135}{2 \times \hbar \times 10^{-13} \times c_o}. \tag{6.2.69a}$$

$$\frac{(k_2 - 1)^3}{k_1} = \frac{48 \times (137)^2}{4 \times \pi \times b \times c_o} \times 10^{-16}. \tag{6.2.69b}$$

By using values (6.2.65), the numerical solution is given by Eqs. (5.1.33), Ref. [14b] page 840, i.e.,

$$k_1 = 0.34. \tag{6.2.70a}$$

$$k_2 = 1 + 4.27x10^{-2}. \tag{6.2.70b}$$

The original proposal [14b] continued with the following results. Note that $\beta^2 \approx 1$. Hence, we have

$$\frac{\beta^2}{n} - n \approx 0, \quad \beta^2 = 1 + \epsilon, \quad \epsilon > 0, \quad \epsilon \approx 0, \quad n = 1. \tag{6.2.71}$$

and the hadronic binding energy, Eq. (6.2.49), is ignorable in nonrelativistic approximation,

$$E_{hb} = -\frac{V_0}{4 \times \beta^2} \times \left(\frac{\beta^2}{n} - n \right) \approx 0. \tag{6.2.72}$$

It is easy to see that the hadronic kinetic energy is also ignorable because

$$E_{hk,\pi^o} \approx k_1 \times \hbar \times b \times c_o =$$

$$= 0.34 \times (6.5 \times 10^{-22}\ MeV\ s) \times (10^{-13}\ cm) \times (3 \times 10^{12}\ cm/s) \approx$$

$$\approx 6.63 \times 10^{-23}\ MeV \tag{6.2.73}$$

Consequently, the primary contribution to the total energy of the π^o is that for the hadronic rest energy, as expected from Section 6.2.2 (Ref. [14b], Eq. (5.1.34) page 841),

$$E_{\pi^o} \approx 2 \times E_{hr,\hat{e}} = \frac{m_e \times c_o^2}{\rho^2} = 135\ MeV. \tag{6.2.74}$$

Recall from Eq. (6.2.33d) and (6.2.41) that

$$\rho^2 = |\hat{T}|^2 \ll 1, \tag{6.2.75}$$

and that the isorenormalization of the quantum rest energy (qr) into the hadronic rest energy (hr) is given by Isoaxiom V of the isospecial relativity, Eq. (6.1.15), i.e.,

$$E_{qr,e} = m_e \times c_o^2 \ \rightarrow \ E_{hr,\hat{e}} = m_e \times \frac{c_o^2}{\rho^2} = m_e \times c_o^2 \times \frac{n_3^2}{n_4^2} = m_e \times c_o^2 \times \frac{b_4^2}{b_3^2}. \tag{6.2.76}$$

Hence

$$\rho^2 = |\hat{T}|^2 = \frac{n_4^2}{n_3^2} = \frac{b_3^2}{b_4^2}. \tag{6.2.77}$$

The hadronic total energy can the n be written

$$E_{\pi^o} \approx 2 \times m_e \times c_o^2 \times \frac{b_4^2}{b_3^2} = 135\ MeV. \tag{6.2.78}$$

from which we have the numerical value

$$E_{hr,\hat{e}} = 67.5MeV \tag{6.2.79a}$$

$$\rho^2 = |\hat{T}|^2 = \frac{b_3^2}{b_4^2} \approx \ 7.5 \times 10^{-3}. \tag{6.2.79b}$$

All remaining quantities are ignorable in this first nonrelativistic approximation.

By assuming homogeneity and spherical symmetry of the π^o, we have

$$b_1 = b_2 = b_3 - 1, \tag{6.2.81a}$$

$$\rho = n_4 = 1/b_4 = 8.7 \times 10^{-1}, \tag{6.2.81b}$$

and the speed of light within the π^o is given by

$$c = 11.5 \times v_o. \tag{6.2.83}$$

This confirms that the medium insoide the π^o meson is of iso-Minkowskian Group III, type 9 (Figure 6.3, thus confirming that phenomenological calculations (6.1.51) are quite approximate, as expected. The following comments are in order:

AXIOMATIC CONSISTENCY. The above model confirms the mechanism provided by hadronic mechanics to avoid the inconsistency of quantum mechanics for the hadronic structure. Recall that a quantum treatment of model (6.2.63) would be catastrophically inconsistent since it would require a "positive" binding energy of about 134 MeV. Hadronic mechanics avoids this inconsistency via the mutation - isorenormalization of the rest energy, namely, of the maximal causal speed inside the π^o such that *the sum of the isorenormalized rest energies of the constituents is bigger than (although close to) the total energy of the π^o. As a result, the hadronic model admits a "negative" binding energy as necessary for consistency.*

Needless to say, the "negative" binding energy is that caused by the Coulomb interactions between electron and positron that has been ignored in first approximation since the latter is considerably smaller than 135 MeV. Its inclusion is left to the interested colleague.

REPRESENTATION OF ALL CHARACTERISTICS OF THE PARTICLE: Remember that quantum mechanics allows the representation of *all* characteristics of the positronium with one single equation, Schrödinger's equation. As a consequence, said equation is indeed of structural character. As stressed in Ref. [14b], Section 5, hadronic mechanics allows the same feature, this time for the π^o meson. In fact, the Schrödinger-Santilli isoequation represents all characteristics (6.2.64) of the π^o, as one can verify.

Primary decay (6.2.65) is directly represented and it is in actuality the best confirmation that the physical constituents are indeed one electron and one positron in a mutated form. Decay (6.2.66) is the hadronic tunnelling of the physical constituents and it is an additional direct confirmation that said constituents are indeed an electron and a positron. The remaining secondary decays require isorelativistic treatment that is not studied at this time. For additional comments, one may inspect Ref. [14b], pages 843, 844, with particular reference to the warning on the inability to compute these secondary decays with the conventional scattering theory due to its unitary character.

By comparison, quark conjectures *do not* represent *all* characteristics (6.2.64), but only *some* of them; they do not admit one single structural equation, but represent different characteristics with generally different procedures; and the spontaneous decays are represented via abuse of academic power, such as the claim that a quark-antiquark system can decay 98 % of the time into two photons, or the claim that the electron and the positron of decay (6.2.66) are "created" at the time of the "disappearance" of the quark-antiquark pair, all this without any explanation and without any quotation of the dissident, refereed publications such as Refs. [88, 101-105].

SUPPRESSION OF THE ATOMIC SPECTRUM. In view of subsidiary condition (6.2.51), characteristic value (6.2.70b) causes the *suppression of the atomic spectrum of energy levels down to only one state, the* π^o. In fact, the value $k_2 > 1,\ k_2 \approx 1$ implies the values $n = 1 < \beta^2 < n = 2$, by therefore suppressing in Eq. (6.2.49) all energy levels from $n = 2$ on, the only allowed level being that for $n = 1$ (see Figure 6.22)

The above atomic spectrum suppression is a most important confirmation of the validity of hadronic mechanics fully identified and emphasized in the original proposal [14b]. In fact, the π^o meson has no known excited state. Consequently, the admission of even one additional energy levels, besides that for the π^o, would be inconsistent with experimental evidence.

Besides a confirmation of validity, the suppression of the atomic spectrum has deep implications. *Model (6.2.63a) does indeed admit an infinite number of excited states, but they are those of the positronium.* Alternatively, any excitation of the energy level of the physical constituents of the π^o causes them to exit the hadronic horizon $R_c = 1$ fm, after which the Hulthen potential is null, and the hadronic model recovers the conventional Schrödinger equation of the positronium uniquely and identically.

Note that the suppression of the atomic spectrum is considered of paramount important t to avoid the illusion of studying the structure, while in reality one solely deals with the classification. Different views would require that the Schrödinger equation for the hydrogen atom must include the related Mendeleev family, which is notoriously not the case. The inability to separate the classifica-

tion from the structural problems, while at the foundations of historical studies on nuclei, atoms and molecules, has remained entranced in the minds of researchers in hadron physics due to the political condition of the field.

As we shall soon see, another basic implication of the atomic spectrum suppression is that the transition from the structure of the π^o to that of the $\pi^\pm$ requires the *increase* of the number of constituents in order to comply with physical evidence. By comparison, the classification of mesons does not require such an increase, as well known, because we have a classification via mathematical representation of a mathematical symmetry defined on a mathematical complex-valued space without any known connection to our spacetime.

IGNORABLE HADRONIC BINDING ENERGY. Another aspect, that is fundamental for the proper understanding of hadronic mechanics, but also departs dramatically from quantum settings, is that *the hadronic binding energy is so small as being ignorable in first approximation.* It is known in undergraduate studies that contact resistive forces have no potential energy. The main physical origin of structure model (6.2.63) is the *contact, zero-range, interaction* due to the complete immersion of one wavepacket within the other. Hence, any granting of energy to contact interactions responsible for structure (6.2.63) would be outside the boundary of physics.[35]

[35]When NASA initiated space missions, it became clear that classical Hamiltonian mechanics permits extreme accuracy for the orbits of satellites *in vacuum.* However, NASA engineers soon discovered that the computation of *the satellite trajectory during re-entry in atmosphere* was afflicted by serious theoretical difficulties, as well as safety concerns due to lack of accurate predictions.

When this insufficiency propagated throughout physics departments in the U.S.A., a physicist from a "leading" college visited NASA to "help" in computing re-entry trajectories. The physicist was allowed to deliver his talk as scheduled, but the affair resulted in great embarrassment because that physicist has insufficiently knowledge of the field, yet was coming from a U. S. institution crucial for NASA obtaining governmental funds.

The embarrassment by NASA engineers was due to the fact that the "physicist" from a "leading" institution had the "illusion" of treating re-entry trajectories with the only theory he knew, tcnventional Hamiltonian mechanics, that based on the truncated Hamilton equations without external terms (see Volume I). In plain language, the "physicist" was dreaming to represent re-entry trajectories with a Hamiltonian! The embarrassment by the engineers was due to the fact that, at that time, to improve the approximation of the trajectory, they had been forced to use nonpotential forces that had reached the 9-th power of the speed, e.g.,

$$F = -N_o - N_1 \times v - N_2 \times v^2 - N_3 \times v^3 - N_4 \times v^4 -$$
$$-N_5 \times v^5 - N_6 \times v^6 - N_7 \times v^7 - N_8 \times v^8 - N_9 \times v^9$$

where the Ns are positive constants. Evidently, they were dealing with a force in three dimensions immensely beyond any dream of representation with a Hamiltonian. With considerable embarrassment, NASA engineers presented great praises to the "learned" academician and gently had him return to his "leading" institution.

The episode circulated in the physics community and partially inspired the author to write two monographs on re-entry trajectories and similar non-Hamiltonian problems, under the title of *Foundations of Theoretical Mechanics,* published by Springer-Verlag, Heidelberg, Germany. Volume I, *The Inverse Problem in Newtonian Mechanics* (1978), directly relevant to the above case, presenetd a systematic study of the necessary and sufficient conditions for the existence of a potential or a Hamiltonian (the

NEARLY FREE CONSTITUENTS. Quantum bound states, such as nuclei, atoms and molecules, lead to strongly bounded constituents, as well known. By contrast. hadronic bound states lead to *nearly free constituents,* a condition reminiscent of *asymptotic freedom* in quantum electrodynamics (QCD). However, the latter theory is purely Lagrangian, thus granting a potential energy to all possible forces, under which theology, the asymptotic freedom itself becomes as quantitatively unverifiable as the quark conjectures themselves. By contrast, hadronic mechanics grants a potential only to action-at-a-distance interactions, and represents all others outside a Lagrangian or a Hamiltonian. In the latter case, the nearly free condition of the constituents has been been rigorously proved in this section for the π^o by the following evidence: 1) The lack of a potential energy by the dominant structural force, those of contact character; 2) The comparatively ignorable value of potential interactions; and 3) The virtually null value of the binding energy (see Ref. [111] for more details).

ISOSELFDUALITY PREDICTIONS. In Chapter 2, we have stressed that *iso-selfduality* (invariance under the isodual map as enjoyed by the imaginary unit i) is a new invariance of nature so fundamental that it is verified by the conventional Dirac's equation (thus leading to a basically new interpretation that escaped the physics of the 20-th century), and be assumed at the basis of our cosmology (Section 6.1.15).

Quark supporters have ignored for over a decade this new invariance, and so has been the case by the Particle Data Group who write spontaneous decay (6.2.65) without being aware that it violates this new invariance. It is easy to see

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conditions of variational selfadjointness),. Volume I then proved the impossibility for the Hamiltonian to represent nonpotential interactions in the frame of the experimenter from two dimensions on. Volume II, entitled *Birkhoffian Generalization of Hamiltonian Mechanics* (1981), to provide NASA engineers a universal variational principle (evidently necessary for optimization) applicable to all possible, sufficiently smooth re-entry trajectories with unrestricted, variationally nonselfadjoint forces much more complex than the one above.

Unfortunately, to the author's best knowledge (evidence to the contrary would be greatly appreciated for due corrections) NASA engineers were never allowed (or interested) to use the two volumes published by Springer-Verlag, because they were constrained for political reasons to continue their contacts with "leading" physicists at "leading" institutions as a condition for funding.

In turn, the absence of such a of the intended primary use of the two Springer-Verlag monographs provided additional motivation for the author being dubbed "the most plagiarized physicists of the 20-th century," because numerous other researchers subsequently published various papers in "leading" journals without any reference to the author' s two volumes in the field, publication occurred with the generally studious intent by the editors of avoiding the consultation of the author as a referee.

As a last act, the author filed in 1994 at the Massachusetts Institute of technology a request of investigations by its ethics committee to receive prophetic phone calls that the author was wasting his time, as it did turn out indeed to be the case, since academic behavior has no control whatsoever by society. As a result of all this, the huge efforts in writing the two volumes with Springer-Verlag (each volume written and rewritten several times to reach referee's acceptance, one full year being spent solely in historical search in various libraries) went into oblivion.

that *the proposed structure model of the π^o is isoselfdual,* being constituted by a particle-antiparticle system

$$\pi^o \ = \ (\hat{e}^+, \hat{e}^-)_{hm} \ \equiv \ [(\hat{e}^+, \hat{e}^-)_{hm}]^d. \tag{6.2.84}$$

Note that the same invariance is verified by quark-antiquark systems. By contrast, the r.h.s. of decay (6.2.65) is not isoselfdual,

$$\gamma + \gamma \ \rightarrow \ (\gamma + \gamma)^d \neq \gamma + \gamma. \tag{6.2.85}$$

A serious knowledge of hadronic mechanics requires the awareness of the complete democracy requested between the treatments of matter and antimatter. In turn, this leads to the prediction that *one of the two photons of decay (6.2.65) is the isodual photon γd* (Chapter 2), that is physically distinct from the conventional; photon. Contrary to what released in the Particle Data, the correct form of writing decay (6.2.65) is that verifying isoselfdual invariance

$$\pi^o \ = \ (\hat{e}^+, \hat{e}^-)_{hm} \equiv [(\hat{e}^+, \hat{e}^-)_{hm}]^d \ \rightarrow \ \gamma + \gamma^d \equiv \gamma + \gamma^d. \tag{6.2.86}$$

By recalling the need for mankind to initiate quantitative studies as to whether a far away galaxy or quasar is made up of matter or of antimatter (chapters 1 and 2), it is hoped that researchers in particle physics will eventually acknowledge basically new invariances, such as isoselfduality, particularly when they have been available for over a decade.

What is at stake, particularly for large laboratories, such as CERN, FERMILAB, DESY, RUTHERFORD, IJNR, etc. is to avoid the possible waste of truly large public sums in the laboratory fabrication of anti-hydrogen atoms for the purpose of studying their light, because the light emitted by antimatter is available in the elementary decay of the most elementary meson, the π^o.

Note that we have ignored the neutrino decays, such as

$$\pi^o = (\hat{e}^+, \hat{e}^-)_{hm} \ \rightarrow \ \nu + \bar{\nu} \ \ (8.3 \times 10^{-7}\% \ \ 90\% CL, \tag{6.2.87}$$

because purely theoretical and without any direct *experimental* evidence, since neutrinos and antineutrinos cannot be directly detected like the physical particles in the preceding decay. Neutrinos are conjectured based on the production of particles. However, the latter production admits alternative interpretation without the conjecture of the neutrinos and antineutrinos as physical particles. hence, to regain credibility, and prevent shadows of affiliations of the financial interests around the neutrino conjectures, *the Particle Data Group should restrict the data to actual physical particles directly detected in our laboratory, and remove any mention of neutrinos (or quarks) in their data, some of which listed with* 90 % *Confidence Level!.*

It is an instructive exercise for the reader interested in learning hadronic mechanics to prove that the above structure model ofc the π^o provides a realization of the *isobox* of Figure 3.7, namely, the structure presented is a mere description from an *outside observer* with our units of space and time because, for an *internal observer* with the internal units of space and time, the same structure may be dramatically different.

To set a distance from political claims, the author wants to stress that the main scope of the research herein presented is to *prove the consistency of ordinary massive physical particles as physical constituents of the* π^o without any claim of uniqueness of the model. In fact, numerous other possibilities exist along the same mechanism of regaining a positive bindingb energy under suitable isorenormalizations via forces different than the Hulthen force. Their study is left to interested researchers.

On historical comments, the only difference of the above presentation and the original one is the information gained during the three decades that passed in regard to the fact that the maximal causal speed within hadronic matter is given by $V_{max} = c_o \times (b_4/b_3)$ and not by $c = c_o \times b_4$.

6.2.7 Nonrelativistic Structure Model of the Neutron as a Hadronic Bound State of a Proton and an Electron

6.2.7.A Foreword on the Need for New Clean Energies

The neutron is one of the biggest reservoirs of clean energy available to mankind because it is naturally unstable and decays into a highly energetic electron that can be trapped with a thin metal shield, plus the hypothetical neutrino that, in the event it exists, it is innocuous. As clearly stated in the original proposal [14], hadronic mechanics was conceived and constructed for the specific purpose of providing axiomatically consistent methods for quantitative studies of the possibility of tapping the energy contained in the neutron.

Recall that *all* energies available to mankind to date, such as nuclear, atomic and molecular energies, are crucially dependent on the possibility of releasing free nuclear, atomic or molecular constituents. Hence, this historical teaching mandated the construction of a new structure model of the neutron with conventional massive physical constituents that, by central assumption, can be produced free with one mechanism or another as a condition to release the 0.78 MeV contained in the neutron structure.

In this section, we review the author's efforts [95] to achieve a nonrelativistic structure model of the neutron with physical constituents that can be produced free. The relativistic version of the model [96] will be studied in the next section. Considerable additional studies are needed prior to addressing in this volume the possible industrial utilization of the energy inside the neutron , because we are

dealing with a new class of energies, called by the author *hadronic energies,* [112] (see also the review monograph [99]) in order to distinguish them from nuclear, atomic and molecular energies, since hadronic energies originate from mechanics in the structure of individual hadrons, rather than in their collection as it is the case for nuclear energies. The need for additional studies is the reason for presenting energy related aspects in a later chapter.

The possibility of industrial applications of the structure model of this section should be compared with the impossibility of any practical application by the conjecture that the hypothetical, directly undetectable quarks are the actual constituents of the neutron. The belief that quarks are permanently confined inside the neutron, then prevents any possibility whatsoever, not even remote, of practical applications.

By no means the author suggests the termination of studies on quark conjectures and, by no means, the author claims to have resolved the historical problem of the neutron structure. However, the author insists in the ethical duty by the physics community to study alternative structure models of the neutron with actual physical constituents that can be produced free, due to the need for new clean energies to contain increasingly catastrophic climactic changes.

For this reason, the author has denounced (with real names of individuals and institutions) political obstructions against the construction of hadronic mechanics in book [89] and in the 1132 pages of documentation [90]; the author felt an ethical duty to denounced the same obstructions in the footnotes of these two volumes; and the author intends to denounce publicly any additional asocial and ascientific obstruction ventured against the efforts herein reviewed for sinister personal gains without *technical* objections published in *refereed* journals rather than verbose posturing in equivocal academic corridors.[36]

6.2.7.B Hadronic Realization of Rutherford's Conception

An exact and invariant, nonrelativistic representation of *all* characteristics of the neutron as a hadronic bound state of a proton and an electron in a mutated isotopic form, was first achieved by Santilli in Ref. [95] of 1990,

$$n = (\hat{p}^+, \hat{e}^-)_{hm}. \tag{6.2.88}$$

Equivalently, Ref. [95] achieved a representation of the neutron as a "compressed hydrogen atom" along Rutherford's historical conception [91]. Since the physical conditions of an electron compressed within the hyperdense medium inside a

[36]Serious scientists interested in contributing to the open problem of the neutron structure can be assured of appreciation, irrespective of whether their technical contributions are critical or supportive. Pseudo-scientists with a priory sinister aims, a rather frequent occurrence nowadays, are suggested to read the Legal Notice at the beginning of this volume prior to implementing their schemes.

proton are dramatically beyond a credible quantum mechanical representation, the model was achieved via a nonunitary transforms of the corresponding model for the hydrogen atom ,

$$H = (p^+, e^-)_{qm} \ \rightarrow \ n = (\hat{p}^+, \hat{e}^-)_{hm} = U_n \times (p^+, e^-)_{qm} \times U_n^\dagger, \tag{6.2.89a}$$

$$U_n \times U_n^\dagger = \hat{I}_n \neq I, \ \hat{I} > 0. \tag{6.2.89b}$$

Paper [95] did present the new structure model without any need for quark conjectures, but said paper did not address neutrino issues. The latter were addressed only recently [9797] with the outcome that *the use of hadronic mechanics does not require any neutrino conjecture for the synthesis of the neutron,* as shown below.

The model permitted the exact and invariant representation of: rest energy E_n, meanlife τ_n, charge radius R_n, charge q_n, spin J_n, magnetic moments μ_n, space and charge parities J_n^G

$$E_n = 939.56 MeV, \ \ \tau_n = 885 \ s, \ \ R_c = 10^{-13} cm, \ \ q_n = 0, \tag{6.2.90a}$$

$$J_n = \frac{1}{2}, \ \ \mu_n = -1.913 \ \mu_N, \ \ I_n^p = \frac{1}{2}^+. \tag{6.2.90b}$$

The spontaneous decay will b studied in a subsequent section since it raises fundamental openings for possible new longitudinal forms of communication.

The model solved in Ref. [95] is the particular case in which the proton has no mutation, and only the electron is mutated,

$$n = (p^+, \hat{e}^-)_{hm} \tag{6.2.91}$$

This approximation is warranted by the fact that the proton is about $2,000$ times heavier than the electron, as a result of which the isorenormalizations of the proton are very small compared to those of the electron. In any case, the study of the full model (6.2.88) requires the isorelativistic treatment not considered in this section.

6.2.7.C Representation of the Neutron Rest Energy, Meanlife and Charge Radius.

As it was the case for the π^o, the representation of all data (6.2.90a) is provided by structural isoequation (9.6.42) combined with the meanlife and charge radius as subsidiary constraints, although reformulated for the neutron [95]

$$\left[\frac{1}{r^2}\left(\frac{d}{dr} r^2 \frac{d}{dr}\right) + \frac{m}{\rho^2 \times \hbar^2}\left(E_{hb} + V \times \frac{e^{-b\times r}}{1 - e^{-b\times r}}\right)\right] \times \hat{\psi}(r) = 0, \tag{6.2.92a}$$

$$\tau^{-1} = \lambda^2 \times |\hat{\psi}(0)|^2 \times \frac{\alpha^2 \times E_{hk}}{\pi \times \hbar}. \tag{6.2.92b}$$

$$R_n = b^{-1}, \tag{6.2.92c}$$

with ensuing system (6.2.60) in the two unknown quantities k_1 and k_2

$$k_1 \times [1 - (k_2 - 1)^2] = \frac{939}{2 \times \hbar \times 10^{-13} \times c_o}. \tag{6.2.93a}$$

$$\frac{(k_2 - 1)^3}{k_1} = \frac{48 \times (137)^2}{4 \times \pi \times b \times c_o} \times 10^{-3}. \tag{6.2.93b}$$

The numerical solution is given by (Ref. [95], Eq. (2.20), page 520)

$$k_1 = 2.6. \tag{6.2.94a}$$

$$k_2 = 1 + 0.81 \times 10^{-8}. \tag{6.2.94b}$$

From the above values, we have the following features: 1) The quantity k_2 is very close to (but bigger than) 1,

$$k_2 = \beta^2 > 1, \ k_2 \approx 1; \tag{6.2.95}$$

2) The only admitted energy level is $n = 1$; 3) The hadronic binding binding energy is ignorable in first approximation,

$$E_{hb} = -\frac{V_0}{4 \times \beta^2} \times \left(\frac{\beta^2}{n} - n \right)^2 \approx 0; \tag{6.2.96}$$

4) The hadronic kinetic energy is equally ignorable as in Eq. (6.2.73); and 5) The total hadronic energy of the neutron is primarily characterized by the rest energy of the proton and the isorenormalized rest energy of the isoelectron,

$$E_n \approx E_p + E_{hr,\hat{e}} = E_p + \frac{m_e \times c_o^2}{\rho^2} = 938.272 + \frac{0.511}{\rho^2} = 939.965 \ MeV. \tag{6.2.97}$$

Hence, the isorenormalization provides the missing energy is characterized by

$$m_{\hat{e}} = \frac{0.511}{\rho^2} \ MeV = 1.294 \ MeV, \tag{6.2.98}$$

Since the proton is not mutated in this first approximation as per assumption (6.2.91), we have

$$b_1 = b_2 = b_3 = 1, \tag{6.2.99a}$$

$$\rho^2 = n_4^2 = b_4^{-2} = \frac{0.511}{1.293} = 0.395 \tag{6.2.99b}$$

$$\rho = n_4 = b^{-1} = 0.628 \tag{6.2.99c}$$

$$b_4 = n_4^{-1} = 1.592. \tag{6.2.99d}$$

Astonishingly, the above value for the characterization of the density of the neutron essentially coincides with the experimental value of the density of the fireball of the Bose-Einstein correlation, Eq. (6.1.112).[37]

[37]The reader should be aware that, due to an unfortunate clerical mistake, the published version of paper [95] is that of uncorrected galleys, rather than the final version approved by the author. This is

6.2.7.D Representation of the Neutron Spin

The representation of the spin of the neutron for structure (6.2.91) was also achieved for the first time by Santilli in Ref. [95]. Conceptually, the representation is elementary. Model (6.2.91) is possible if and only if (Figure 6.23):

A) The proton and the electron are coupled in singlet, since in triplet they would experience a strong repulsive force;

B) Following the "compression" inside the proton, the electron must acquire an orbital angular momentum equal; to the spin of the proton, otherwise the electron has to orbit within and against the hyperdense medium inside the proton, which conditions are impossible for any stable bound state;

C) Consequently, the total angular momentum of the isoelectron is identically null.

Hence, *the spin of the neutron coincides with that of the proton.*

The mathematical representation of the above structure is not trivial and delayed for years the new structure model of the neutron, since it required the previous lifting of the quantum mechanical spin that, in turn, required the prior lifting of Lie's theory and its underlying mathematics.

The isotopies of angular momentum were studied in Ref. [5a] of 1985,[38] while the isotopies of spin were first studied in Ref. [5b] of 1989. The background theory was in this way sufficiently known to allow the writing of paper [95] of 1990.[39] Following these initial studies, a number of additional papers were devoted to the isotopies of the $SU(2)$ symmetry, such as Ref. [5c] of 1993 published by the *JINR Rapid Communications*. The most comprehensive study in the field is that of paper [5d] published in 1998 by *Acta Publicanbdae Mathematicae* whose impeccable editorial review is here reported with appreciation.

Evidently, we cannot repeat here the vast literature in the isotopies of Lie's theory and are forced to outline its application to the specific problem of the neutron spin. Nevertheless, the reader should be warned that a knowledge of the Lie-Santilli isotheory is essential to prevent the illusion of having discovered "inconsistencies" (a not unfrequent occurrence),while in reality we have illiteracy of the new field. Very insidious is the rather natural expectation that the familiar notions of quantum mechanical orbital and intrinsic angular momenta for isolated particles moving in vacuum, equally apply for the same particle when immerses within the hyperdense media inside hadrons, stars or quasars.

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shown by a number of evident misprints clearly incompatible with the text. For instance, Eq. (2.45) gives the value $b_4 = 16.5$ basically, while the value for the correct rest energy of the neutron is $b_4 = 1.65$; similarly, there are evident misprints in Eqs. (2.24), (2.32) and others the reader in good faith can easily correct in any case.

[38]The publication of paper [5a] was delayed for years due to rejections by numerous journals so ascientific and political, the author felt obliged to report in the opening pages of the paper.

[39]The systematic rejections of paper [95] from all western journals without any visible scientific content have been denounced in the footnotes of Section 6.2.1.

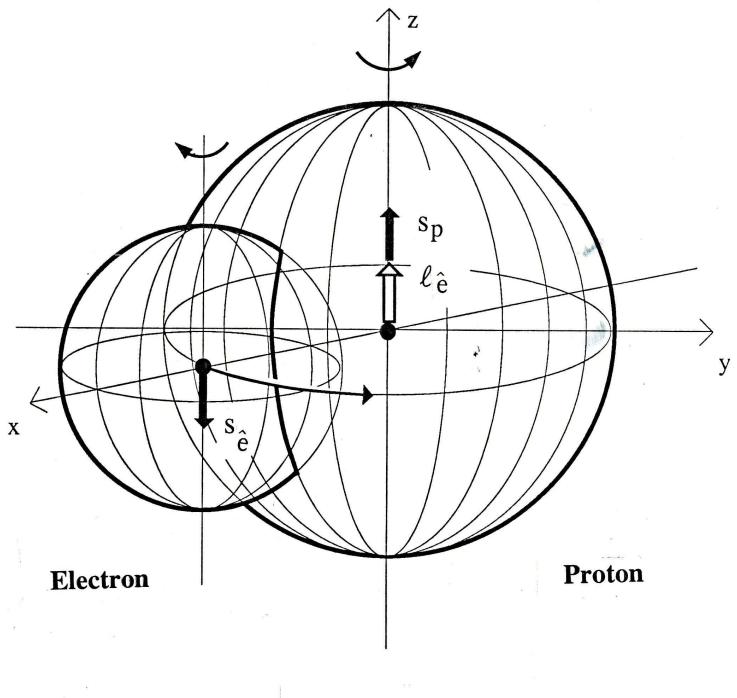

Figure 6.23. A reproduction of Figure 1, page 525, Ref. [98], providing a conceptual view of the orientations of spins and angular momenta needed to achieve a stable structure of the neutron as a hadronic bound state of a proton and an electron.

As an illustration, the third component of the spin of the electron conventionally has the value $\pm 1/2$. However, when the electron is immersed inside the proton, only one value is admitted, that for singlet coupling with the proton, while the other value characterizes strongly repulsive forces. Similarly, the idea that an electron in the core of a collapsing star still has spin $1/2$, is purely political, without any known or otherwise credible scientific support.

In view of the advances occurred since 1990, the mathematical representation of the spin of the neutron is today trivial. Recall that the proton is not mutated because 2000 times heavier than the electron, and that the coupling must be in singlet for stability. This implies that, for the case of the neutron structure, *the spin of the electron is not mutated.* The needed mutation of the quantum into the *hadronic angular momentum* (defined as the angular momentum of a particle immersed within a hadronic medium) is trivially given by the nonunitary-isounitary transforms

$$U \times U^\dagger = \hat{I} = \frac{1}{2},\ \hat{T} = 2, \tag{6.2.100a}$$

$$L_3 \times Y_{\ell,m}(\theta,\phi) = 1 \times Y_{\ell,m}(\theta,\phi) \ \rightarrow \ U \times [L_3 \times Y_{\ell,m}(\theta,\phi)] \times U^\dagger =$$

$$= \hat{L}_3 \times \hat{T} \times \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}) = \frac{1}{2} \times \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}) \qquad (6.2.100b)$$

The mutation is supported by the isotopic invariance of the Hilbert space, Eq. (6.1.28) that, in this case, reads

$$< \ell, m| \times L_3 \times |\ell, m > \times 1 \ \equiv \ U \times [< \ell, m| \times L_3 \times |\ell, m > \times 1] \times U^\dagger =$$

$$=< \hat{\ell}, \hat{m}| \times 2 \times \hat{L}_3 \times 2 \times |\hat{\ell}, \hat{m} > \times \frac{1}{2}, \qquad (6.2.101)$$

namely, *the mutation of the angular momentum from the quantum value* 1 *to the hadronic value* $\frac{1}{2}$ *is a purely internal event not detectable from the outside.*

It is instructive to review the original representation of the spin of the neutron of 1990. For this purpose, Santilli [95] used *irregular isorepresentations* of Lie-Santilli isoalgebras, namely, isorepresentations characterized by nonunitary-isounitary transforms for the generators *different* than those for the product. This difference is rather natural for the structure of the neutron, since the basic nonunitary transform for the rest energy has already been selected, Eq. (6.2.32) ,hence requiring different nonunitary - isounitary liftings for the angular momentum and for the spin.

For the representation of the hadronic angular momentum, Santilli [95] selected the following irregular isorepresentation of $\hat{SO}(3)$ based on the isodifferential calculus and isolinear momentum (6.2.29b)

$$\hat{I} = U \times I \times U^\dagger = 1/\hat{T} \neq 1, \qquad (6.2.102a)$$

$$\hat{L}_k = U \times L_k \times U^\dagger = \epsilon_{kij} \hat{r}_i \hat{\times} \hat{p}_j, \qquad (6.2.102b)$$

$$[\hat{r}_i \hat{,} \hat{r}_j] = [\hat{p}_i, p_j] = 0, \ [\hat{r}_i \hat{,} \hat{p}_j] = \hat{\delta}_{ij} = \hat{I} \times \delta_{ij} = \rho \times \delta_{ij}, \qquad (6.2.102c)$$

$$[\hat{L}_i \hat{,} \hat{L}_j] \hat{\times} \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}) =$$

$$= (\hat{L}_i \hat{\times} \hat{L}_j - \hat{L}_j \hat{\times} \hat{L}_i) \hat{\times} \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}) = \hat{i} \hat{\times} \epsilon_{ijk} \hat{L}_k \hat{\times} \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}) \qquad (6.2.102d)$$

$$\hat{L}^{\hat{2}} \hat{\times} \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}) = \Sigma_{k=1}^{k=3} \hat{L}_k \times \hat{T} \times L_k \times \hat{T} \times \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}) = \rho^2 \times \hat{\ell} \times (\hat{\ell}+1) \times \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}), \qquad (6.2.102e)$$

$$\hat{L}_3 \hat{\times} \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}) = \rho \times \hat{m} \times \hat{Y}_{\hat{\ell}\hat{m}}(\hat{\theta},\hat{\phi}), \qquad (6.2.102f)$$

$$\hat{\ell} = 1, 2, 3, ..., \quad \hat{m} = \hat{\ell}, \hat{\ell} - 1, ..., -\hat{\ell}. \qquad (6.2.102g)$$

As one can see, the isotopies lift the integer value of the angular momentum, $\hat{\ell} = 1, 2, 3, ...$, into the value $\rho \times \hat{\ell}$, where, again, $\hat{\ell} = 1, 2, 3..$, the value $\hat{\ell} = 0$ being excluded by boundary conditions, ρ being a variable depending on the local conditions.

For the study of the *hadronic spin,* (the spin of a particle when immersed within a hyperdense hadronic medium), Santilli [95], page 523, selected the following two-dimensional irregular isorepresentation of $\hat{SU}(2)$

$$\hat{I} = \begin{pmatrix} g_{11} & 0 \\ 0 & g_{22} \end{pmatrix}, \quad \hat{T} = \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & g_{22}^{-1} \end{pmatrix}, \tag{6.2.103a}$$

$$\hat{J}_1 = \frac{1}{2} \times \begin{pmatrix} 0 & g_{11}^{-1/2} \\ g_{22}^{-1/2} & 0 \end{pmatrix}, \quad \hat{J}_2 = \frac{1}{2} \times \begin{pmatrix} 0 & -i \times g_{11}^{-1/2} \\ i \times g_{22}^{-1/2} & 0 \end{pmatrix}, \tag{6.2.103b}$$

$$\hat{J}_3 = \frac{1}{2} \times \frac{\Delta^{1/2}}{2} \times \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & -g_{22}^{-1} \end{pmatrix}, \quad \Delta = Det\ \hat{I} = g_{11} \times g_{22}, \tag{6.2.103c}$$

$$[\hat{J}_1,\hat{J}_2] = i \times J_3, \quad [\hat{J}_2,\hat{J}_3] = i \times \Delta^{1/2} \times \hat{J}_1, \quad [\hat{J}_3,\hat{J}_2] = 1 \times \Delta^{1/2} \times \hat{J}_2, \tag{6.2.103d}$$

$$\hat{J}^{\hat{2}} \hat{\times} |\hat{j}, \hat{s}> = \Sigma_{k=1}^{k=3} \hat{J}_k \times \hat{T} \times \hat{J}_k \times \hat{T} \times |\hat{j}, \hat{s}> = \frac{\Delta^2}{3} \times |\hat{j}, \hat{s}>, \tag{6.2.103e}$$

$$\hat{J}_3 \hat{\times} |\hat{j}, \hat{s}> = \hat{J}_3 \times \hat{T} \times |\hat{j}, \hat{s}> = \pm \frac{\Delta}{2} \times |\hat{j}, \hat{s}>, \tag{6.2.103f}$$

Santilli [95] then computed the total angular momentum of the neutron as epr model (6.2.91)

$$J_n = J_p + \hat{L}_{\hat{e}}^{orbital} + \hat{J}_{\hat{e}}^{intrinsic} = \frac{1}{2} + \rho - \frac{\Delta}{2} = \frac{1}{2}, \tag{6.2.104}$$

resulting in the values anticipated above,

$$\rho = \frac{1}{2}, \quad \Delta = 1. \tag{6.2.105}$$

namely, the s[pin of the isoelectron is not mutated and the angular momentum is mutated in such a way that the isoelectron is merely carried out by the proton spin.

6.2.7.E Representatio of the Neutron Anomalous Magnetic Moment

The representation of the anomalous magnetic moment of the neutron also resulted in being elementary [95], *provided* that quantum views are replaced with covering vistas when dealing with dynamics within hyperdense media. The main result of paper [95] in this respect is that *a quantum representation of the anomalous magnetic moment of the neutron is impossible because quantum mechanics does not admit an orbital motion of the electron inside the proton.* By contrast,

when the hadronic orbital motion is admitted, the magnetic moment of the neutron is generated by the following *three* contributions, Ref. [*loc. cit.*], Eq. (2.40, page 526,

$$\mu_n = \mu_p - \mu_{\hat{e}}^{orbital} + \mu_{\hat{e}}^{intrinsic} \tag{6.2.106}$$

Consequently,

$$\mu_n = -1.9 \times \frac{e}{2 \times m_p \times c_o} =$$

$$= 2.7 \times \frac{e}{2 \times m_p \times c_o} - 4.6 \times \frac{e}{2 \times m_p \times c_o}, \tag{6.2.107}(6.2.107)$$

from which we derived the desired values

$$\mu_{\hat{e}}^{tot} = -4.6 \times \frac{e}{2 \times m_p \times c_o} = 2.5 \times 10^{-3} \times \mu_e, \tag{6.2.106b}$$

$$\mu_{\hat{e}}^{orbital} = (1 + 2.5 \times 10^{-3}) \times \mu_e, \tag{6.2.108}$$

where e represents the absolute value and we used: the orientation of the hadronic angular momentum and spin (Figure 6.23); the different signs of the changes of the proton to the electron; and the rescaling of Bohr's unit for the electron magnetic moment from its value in term of m_e to that in terms of m_p as needed for the neutron magnetic moment.

The plausibility of values (6.2.106c) is established by the fact that *the small value of the total magnetic moment of the isoelectron is fully compatible with the null value of its total angular momentum.*

6.2.7.F Concluding Remarks

REPRESENTATION OF ALL CHARACTERISTICS OF THE NEUTRON

It should be stressed that Ref. [95] did achieve a representation of *all* characteristics of the neutron, including rest energy, meanlife, charge radius, spin, anomalous magnetic moment, anomalous electric moment (see [95] for brevity), charge, and parities, plus a direct representation of the spontaneous decay of the neutron given by the hadronic tunneling of its physical constituents, without any theological assumption that the proton and the electron "disappear" at the time of the synthesis to protect vested interests on preferred conjectures.

It should also be stressed that the representation is invariant, due to the isounitary character of the model, namely, the numerical values remain the same under the same basic assumptions at different times. Note that the latter fundamental condition for consistency is not shared by papers using "deformations" of quantum mechanics due to their activation of the Theorems of Catastrophic Inconsistency studied earlier.

These volumes are dedicated to hadronic mechanics and not to other theories. Hence, we solely study models constructed via the full and correct use of the basic

laws of hadronic mechanics, and refer to epistemological studies all other papers, particularly when catastrophically inconsistent. The indication by colleagues of directly relevant papers in the structure of the neutron as per model (6.2.91) verifying the above crucial condition of invariance, would be greatly appreciated for due corrections.

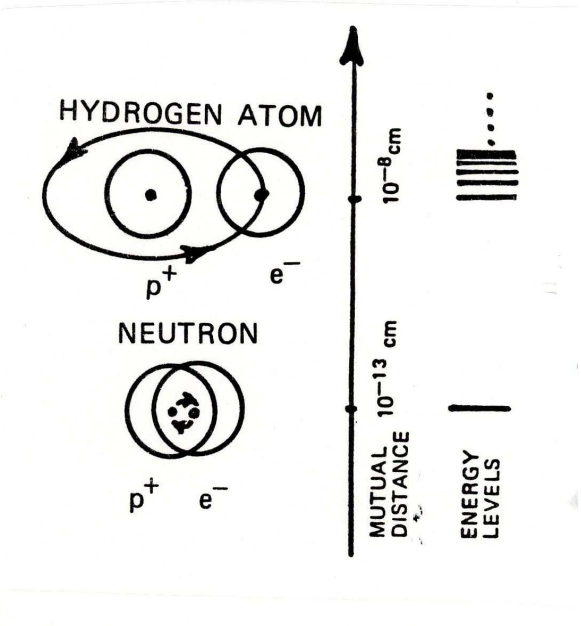

Figure 6.24. An illustration of the main objective of model (6.2.91), the representation of the neutron as a new bound state of the hydrogen at distances of one fm, along Rutherford's historical conception [91]. This conception requires that the neutron has no excited hadronic states, thus requiring the suppression of the atomic spectrum, a condition that is fully verified by hadronic mechanics. In fact, any excitation causes the neutron constituents to pass the hadronic horizon, thus recovering the conventional quantum states of the hydrogen. As we shall see, this feature has potentially fundamental relevance for new clean energies since the neutron is one of the biggest reservoirs of clean energy available to mankind.

COMPATIBILITY WITH OTHER EXPERIMENTAL DATA

As reported in Ref. [99], page 118, Santilli was astonished by value (6.2.99d) because *the numerical value of the characteristic quantity* $b_4 = 1.592$ *derived from the mere assumption that the neutron is a bound state of a proton and an electron, coincides, within the approximations herein assumed, with the numerical value of* $b_4 = 1.653$ *obtained from fit (6.1.112) of the experimental data of the Bose-Einstein correlation. Since the density of the fireball of the Bose-Einstein*

correlation is of the same order as the density of the proton, this astonishing compatiobility provides a direct experimental verification of:

1) The geometrization of the density of hadronic media via characteristic quantity $b_4 = 1/n_4$ of the Minkowski-Santilli isogeometry;

2) The structure of the neutron as a hadronic bound state of a proton and an electron; and

3) The validity of Santilli's isorelativity for the characterization of the hadronic structure, with particular reference to validity of the Poincaré-Santilli isosymmetry and related isorenormalizations (see next section).[40]

ABSENCE OF QUARK AND NEUTRINO CONJECTURES

The lack of quark conjectures was a primary motivation of model (6.2.91), since its primary intent was to reduce the constituents of the neutron to conventional,, physical, massive particles actually existing in our spacetime.

The lack of neutrino conjectures should also be noted, since a direct consequence of the spin structure of Figure 6.23. In essence, paper [95] establisghed that the historical conjecture of the neutrino originated from the inability by quantum mechanics to represent half-odd-integer angular momenta because, as soon as the latter are admitted for the electron inside the proton, the neutron does indeed originate from a compressed hydrogen atom without any need of conjecturing undetectable hypothetical particles.

Note that the lack of need for neutrino conjectures is specifically referred to the neutron *synthesis,* because the neutron *decay* is a separate problem requiring separate analysis presented later on.

SUPPRESSION OF THE ATOMIC MASS SPECTRRUM

As it was the case for the π^o, the hadronic structure model of the neutron suppressed the conventional atomic spectrum of energy down to one, and only one, energy level, that of the neutron. All excited states are, therefore, of *quantum* nature. In this way, the neutron does indeed result to be a compressed hydrogen atom according to Rutherford's historical conception [91] (see Figure 6.24).

The industrial and scientific implications of the above features are far reaching. On industrial grounds, the knowledge of the numerical value opf the isorenormalized rest energy of the electron will turn out to be crucial for the conception and development of mechanisms for the *stimulated decay of the neutron,* as one mechanism to utilize its energy.

On scientific grounds, the above features eliminate current beliefs on "neutron resonances" (see the Particle Data) and establish for hadrons too the historical teaching of nuclear, atomic and molecular physics according to which *the number*

[40]The author cannot describe the *thrill of discovery* caused by this and various other moments felt during the scientific journey presented in these volumes.

of actual constituents increases with mass, as shown in more details in a next section.

NEARLY AT REST AND NEARLY FREE CONSTITUENTS A result of Ref. ['95] particularly significant for possible industrial applications is that *the isoelectron is nearlyt at rest,* evidently in view of the very small value of the hadronic kinetic energy. Thjis implies that the "missing energy" of 0.78 MeV is embedded in the isorenormal,ization of the electrpon rest energy. In turn, such a feature is crucual to predict and test mechanisms for possible stimulated decay of the neutron [112].

Similarly, paper [95] established that *the isoelectron is nearly free,* due to the very small value of the hadronic binding energy, by confirming the similar result by the original proposal [14] for the structure of the π^o. This second result is also important for [possible industrial applications b ecause it confirms the possibility of producing free the neutron constituents with one mechanism or anotehr, as we shall see [112].

6.2.8 Relativistic Structure Model of the Neutron as a Hadronic Bound State of a Proton and an Electron

6.2.8.A Introduction

In the preceding section, we have reviewed Santilli's paper [95] of 1990 achieving the first known, nonrelativistic, exact and (time) invariant representation of *all* characteristics of the neutron as a hadronic bound state of a proton and an electron. The studies were conducted following Rutherford's legacy [91] on the synthesis of the neutron from a hydrogen atom in the core of a matter star

$$H = (p^+, e^-)_{qm} \ \rightarrow \ n = (\hat{p}^+, \hat{e}^-)_{hm}. \tag{6.2.109}$$

and were centered on the impossibility that the permanently stable proton and electron "disappear" from the universe at the time of the synthesis just to please organized interests on quantum mechanics (qm) and Einsteinian doctrines and required the prior construction of the covering hadronic mechanics (hm) and relativity specifically conceived for the problem at hand. In the preceding section we also reviewed the first achievement, also in paper [95], of a nonrelativistic, exact and invariant representation of *all* characteristics of the antineutron as a bound state of an antiproton and a positron, following the synthesis from an anti-hydrogen atom in the core of an antimatter star

$$\bar{H} = (p^-, e^+)_{qm} \ \rightarrow \ \bar{n} = (\hat{p}^-, \hat{e}^+)_{hm} \tag{6.2.110}$$

This section is devoted to a verbatim review of Santilli's paper [96] of 1996 achieving the first relativistic, exact and (time) invariant representation of *all*

characteristics of the neutron and of the antineutron according to the above syntheses.

In the hope of minimizing the predictable posturing of judging new problems with old knowledge, let us begin with the identification of the rather dramatic differences between the structure of the hydrogen atom and that of the neutron. It is hoped in this way readers will see their disqualification as serious scientists in the event they venture judgments on the extremely complex problem of the neutron structure via the use of old and decrepit mathematical and physical knowledge.[41]

As set in the history of physics, the structure of the hydrogen atom is characterized by action-at-a-distance interactions derivable from a potential between the proton and the electron assumed as being point-like, a fully acceptable abstraction in this case thanks to motion in vacuum at mutual distances much bigger than the size pof the particles. Additionally, the (absolute value of) the binding energy of the hydrogen atom is quite small compared to its total energy. In view of these and other features, *quantum mechanics did achieve an exact and invariant representation of all features of the hydrogen atom via the sole knowledge of the Hamiltonian.*

The structure of the neutron is dramatically different than the above century-old lines. To begin, the neutron is one of the densest media measured in laboratory to date; point-like wavepackets do exist in academic manipulations for preset personal gains, but do not exist in the physical reality; and the size of all wavepackets is of the same order of magnitude of the size of the neutron itself. Hence, whatever the constituents, they must be in a state of total mutual penetration of their wavepackets and/or their charge distribution.

The latter conditions cause the emergence of the old legacy that strong interactions are nonlinear (in the wavefunction), nonlocal-integral and nonpotential-nonhamiltonian, for which representation the construction of hadronic mechanics s was proposed [14]. At any rate, any attempt at reducing the conditions of total mutual penetration to point like abstractions, for the evident studious intent of preserving quantum mechanics and Einsteinian doctrines, is outside the boundary of serious science.[42]

[41] In reading this section one should keep in mind the extreme difficulties experienced by Santilli in the publication of paper [96], denounced in the footnotes of Section 6.2.1, which difficulties eventually lead Santilli to the publication of paper [96] in a remote, yet scientifically serious journal in China. In view of the huge scientific and social implications. the difficulties here denounced constitute one of the strongest evidence on the deplorable condition of physical research under public financial support currently existing, with due exceptions, in the United States of America, England, France, Germany, Sweden, Russia, and other countries.

[42] We assume the reader has some technical knowledge of the fact that quantum mechanics can solely represent particles in their point-like abstractions, and that the current attempts of adding at least one dimension via the so-called *string theories* are afflicted by catastrophic mathematical and physical inconsistencies studied in Section 6.1.6 (see Ref. [86] for specific studies). The deplorable condition of physical

Hence, the lack of exact character of quantum mechanics for the structure of the neutron is beyond credible doubt, the only debatable issue being the selection of the broader mechanics achieving an exact and (time) invariant representation of *all* characteristics of the neutron, in the same way as quantum mechanics achieved an exact and (time) invariant representation of all characteristics of the hydrogen atom.

The structure of the neutron is rendered much more complex by additional very peculiar aspects, such as the fact that synthesis (6.2.109) requires 0.78 MeV of *positive* binding energy (Section 6.2.2) under which the Schrödinger equation is no longer physically significant. Additionally, it is clear from the calculations and verifications of the preceding sections that binding energies due to potential interactions, such as those of Coulomb origin, are about 10^{-5} smaller than contributions from the strong interactions responsible for the neutron structure, thus being ignorable. Any belief of the "exact" character of Einsteinian doctrines under these conditions would be sheer scientific corruption.

In summary, the technical difficulties (whose solution required decades of laborious efforts) inherent in the problem considered are given by the facts that *any serious study of the structure of the neutron requires not only the abandonment of quantum mechanics and special relativity in favor of a suitable covering discipline, but also the achievement of an exact and invariant representation of all characteristics of the neutron without any use of any potential or Hamiltonian at all.*

The biggest mental obstacle for the understanding of this section is, therefore, due to the predictable expectation of the use of one or another potential for the representation of the neutron structurewhile, as we shall see, the exact and invariant representation of synthesis (6.2.109) has been achieved without any use nowhere of any potential or Hamiltonian. This is a necessary condition for consistency because the dominant forces are those of contact, zero-range type due to total mutual penetration of the constituents. The treatment of these interactions with any potential would then be equivalent, for instance, to representing with a potential the resistive forces experienced by a satellite during re-entry in Earths atmosphere, thus exiting all boundaries of physics.

Under the above premises, the *sole* quantitative representation of synthesis (6.2.109) known to Santilli was the construction of a *new* geometry, relativity and mechansics specifically conceived for the problem considered, while keeping a kilometric distance from the widespread opposite trend of adapting nature to pre-existing doctrines.

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research under public financial support is further documented by the fact that these inconsistencies, even though published in serious refereed journals, continue in being ignored by organized interests in the field, ratehr than being disprov ed in equally refereed journals.

As we shall see, Ref. [96] achieved several advances *prior* to addressing synthesis (6.2.109), including the construction, specifically for the neutron structure, of: the Minkowski-Santilli isogeometry; Santilli isorelativity; the Poincaré-Santilli isosymmetry; the isospinorial covering of the Poincaré-Santilli isosymmetry; the Dirac-Santilli isoequatio; and the consequential isorenormalizations of the rest energy, angular momenta, and magnetic moments of the neutron constituents.

As set in the history of science, the conventional Dirac equation for the hydrogen atom represents one electron under the external field of a proton. A fundamental result achieved for the first time by Santilli in paper [96] is that the isotopic lifting of Dirac's equation represents one electron, this time, totally immersed within the hyperdense medium inside the proton considered as external. By recalling that Dirac's equations allows the treatment of both particles and antiparticles (a feature evidently persisting under isotopies), Ref. [96] provided the first known, joint isorelativistic structure model of both the neutron and the antineutron according to syntheses (6.2.109) and (6.2.109).

On historical grounds, Santilli pointed out in Ref. [96] that, quite intriguingly, the technically most difficult problem (mutation of the total angular momentum of the electron down to the value zero) was first solved by P. A. M. Dirac in two of his last papers [13,114]. These papers remained vastly ignored by orthodox physics due to their excessive departures from preferred lines, while, by contrast, the same papers received primary attention by Santilli who quoted and reviewed them in various works (see, e.g., EHM, Volume II). Hence, an objective of this section is to establish the important historical fact that, even though without his knowledge, Dirac himself established the foundations for the quantitative treatment of the proton and the electron as actual physical constituents of the neutron.

It should be indicated that, besides Diracs papers [113,114], the literature in the field is truly large because Rutherfords legacy has stimulated countless studies since its inception of 1920. However, the greatest number of these studies have been conducted via quantum mechanics and,. as such, they are ignored here to prevent a prohibitive length. A very limited number of studies have been conducted via the use of broader mechanics other than hadronic mechanics, but they represent only some, rather than all, characteristics of the neutron and additionally suffer the catastrophic inconsistencies typical of all nonunitary theories on a conventional Hilbert space (Section 6.1.6). Consequently, inconsistent studies are equally ignored to avoid a prohibitive length of this section.

We would like to apologize to the author of these efforts for the inability of even a partial reviews to prevent discriminatory selections due to their number, and

recommend interested historians to conduct a comprehensive review of all studies conducted to date on the structure of the neutron along Rutherfords legacy.[43]

The author would grateful appreciate the indication, for proper quotation in future editions of this volume, of studies on the structure of the neutron as a bound state of a proton and an electron under the condition that: 1) they were published *prior* to 1990; 2) they are quantitative, rather than conceptual-epistemological; and 3) they achieve an exact and invariant representation of *all* characteristics of the neutron, since the representation of only some of them may bypass central issues.

6.2.8.B Poincaré-Santilli isosymmetry for the Neutron and its Isodual for the Antineutron

For the description of the dynamics of an electron orbiting in vacuum around a proton in the hydrogen atom, we assume the exact validity of the conventional Minkowski spacetime $\hat{M}(x,\eta,R)$ with local coordinates $x=(x^\mu)=(x^1,x^2,x^3,x^4)$, $x^4=c_o\times t$, where c_o is the speed of light in vacuum, with metric $\eta=Diag.(1,1,1,-1)$, unit $I=Diag.(1,1,1,1)$, field of real numbers $R(n,+,\times)$ with basic unit I, invariant $(x-y)^2=[(x^\mu-y^\mu)\times\eta_{\mu\nu}\times(x^\nu-y^\nu)]\times I\ \in\ R$, and conventional Poincaré symmetry $P(3.1)$ with generators $J_{\mu\nu},\ P_\mu$ and symmetry transformations hereinafter assumed to be known.[44]

A fundamental assumption of isorelativistic hadronic mechanics to achieve a representation of synthesis (6.2.109) without any potential or Hamiltonian, is that the transition of the electron from motion in vacuum to motion within a

[43]The author has invited Cynthia Whitney, Editor of *Galilean Electrodynamics,* to organize one or more volumes of papers on syntheses (6.2.109) and (6.2.110). Interested participants are encouraged to send their contribution directly to Whitney, under the condition that they are specifically devoted to structure models of the neutron and/or antineutrons in *with the [proton and the electron as the actual physical constituents.*

[44]Remember from Chapter 3 that the field R normally used for special relativity throughout the 20-th century is that with the trivial unit 1, in which case the invariant is given by

$$(x-y)^2=(x^\mu-y^\mu)\times\eta_{\mu\nu}\times(x^\nu-y^\nu), \tag{a}$$

and the Poincaré symmetry is believed to be 10-dimensional. By contrast, the assumption of the unit of the base field R to coincide with the 4-dimensional unit of the Poincaré symmetry, requires the invariant to have the form

$$(x-y)^2=[(x^\mu-y^\mu)\times\eta_{\mu\nu}\times(x^\nu-y^\nu)]\times I, \tag{b}$$

as a condition for said invariant to be a scalar, that is, an element of R. In turn, the latter correct way of writing the invariant allows the discovery of the 11-th dimension of the Poincaré symmetry,

$$(x-y)^2=[(x^\mu-y^\mu)\times\eta_{\mu\nu}\times(x^\nu-y^\nu)]\times I=$$
$$=\{(x^\mu-y^\mu)\times(n^2\times\eta_{\mu\nu})\times(x^\nu-y^\nu)\}\times(n^{-2}\times I),\ n\in R,\ n\neq 0. \tag{c}$$

Admittedly, at the elementary level of special relativity, alternatives (a) and (b) are of marginal. relevance. However, at the isotopic level, a number of inconsistencies emerge in the event the basic unit of the field is selected as being different than the basic unit of the symmetry. This is precisely the feature that permitted Santilli to discover the 11-th dimension of the conventional Poincar e symmetry [5e,5f].

physical medium causes an alteration of spacetime called *mutation,*. This feature is mathematically represented with the the lifting of the Minkowski metric η into a metric $\hat{\eta}$ with an arbitrary functional dependence on local coordinates x, velocities v, accelerations a, energy E, density d, temperature τ, wave function ψ, their derivatives $\partial\psi$, and any needed additional variable,

$$\eta = (\eta_{\mu\nu}) = const. \ \rightarrow \ \hat{\eta} = (\hat{\eta}_{\mu\nu}) = \hat{\eta}(x, v, a, E, d, \tau, \psi, \partial\psi, ...), \qquad (6.2.111)$$

under a number of regularity conditions identified below assuring that $\hat{\eta}$ admits η as a particular case. This condition is necessary for a quantitative representation of the neutron decay in which we have the transition from the isoelectron on a generalized spacetime with metric $\hat{\eta}$ to the ordinary electron in our spacetime with metric η.

An evident consequential condition is that the signature of $\hat{\eta}$ is the same as that of η, namely, $Sign \ \hat{\eta} = (1, 1, 1, -1)$. Hence, the generalized metric must admit the factorization into the Minkowski metric multiplied by a nonsingular 4×4-dimensional metric denoted in the field with the symbol $\hat{T}$

$$\hat{\eta} = (\hat{\eta}_{\mu\nu}) = \hat{T} \times \hat{\eta} = (\hat{T}^{\rho}_{\mu}(x, v, a, E, d, \tau, \psi, \partial\psi, ...) \times \eta_{\rho\nu}), \qquad (6.2.112a)$$

$$Det \ \hat{T} \neq 0. \qquad (6.2.112b)$$

Since the neutron is considered isolated from the rest of the universe, the above lifting must preserve *conventional* total conservation laws, namely, the total linear and angular momentum of the neutron must be conserved and the motion of its center-of-mass must be uniform.

As it is well known, a necessary and sufficient condition for the verification of these conservation laws is that *the generalized symmetry must conserve the conventional generators* $J_{\mu\nu}$, P_{μ}. The *sole* possible generalization of the Poincaré symmetry meeting the above requirement is the *Poincaré-Santilli isosymmetry* $\hat{P}(3.1)$ whose construction specifically formulated for the neutron structure (6.2.109) was done in Ref. [96] and can be outlined as follows.

The main idea of $\hat{P}(3.1)$ [5e,5f] is the reconstruction of $P(3.1)$ with respect to a generalization of its unit I assumed as being the *inverse* of the mutation of the metric,

$$\hat{I} = \hat{I}(x, v, a, E, d, \tau, \psi, \partial\psi, ...) = 1/\hat{T} > 0, \qquad (6.2.113)$$

in which case $\hat{I}$ is called the *isounit,* $\hat{T}$ is called the *isotopic element,* and the positive-definite character is assumed to preserve the topology of I. The positive-definite character is also assumed to separate the Poincaré-Santilli isosymmetry $\hat{P}(3.1)$ for the neutron from the isodual isosymmetry $\hat{P}^{d}(3, 1)$ for the antineutron, the latter requiring a negative-definite unit as assumed to be known from Volume I.

The assumption of $\hat{I}$ as the basic unit requires the reconstruction of the field R as Santilli isofield $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ (Section 3.2), with isonumbers $\hat{n} = n \times \hat{I}$, isosum trivially coinciding with the conventional sum, $\hat{+} \equiv +$, and isoproduct $\hat{n}\hat{\times}\hat{m} = \hat{n} \times \hat{T} \times \hat{m}$, under which $\hat{I}$ is the correct left and right unit.

The latter condition requires, for consistency, the isotopic lifting of the Minkowski spacetime into the Minkowski-Santilli isospacetime $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ in which the local coordinates, to be isonumbers, must have the form $\hat{x} = x \times \hat{I}$. Similarly, for the elements of the isometric being isoscalars, they must have the form $\hat{G}_{\mu\nu} = \hat{\eta}_{\mu\nu} \times \hat{I}$ herein assumed.

Ref. [96] only considered the case of a diagonal isounit and isotopic element, because fully sufficient for the structure of the neuitron, with explicit form

$$\hat{I} = Diag.(b_1^{-2}, b_2^{-2}, b_3^{-2}, b_4^{-2}) \times e^{(\psi_e/\hat{\psi}_e)\times\int dr^3\hat{\psi}^\dagger(r)_{p\downarrow}\times\hat{\psi}(r)_{e\uparrow}} =$$

$$= Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times e^{(\psi_e/\hat{\psi}_e)\times\int dr^3\hat{\psi}^\dagger(r)_{p\downarrow}\times\hat{\psi}(r)_{e\uparrow}} \qquad (6.2.114a)$$

$$hatrT = Diag.(b_1^2, b_2^2, b_3^2, b_4^2) \times e^{(\psi_e/\hat{\psi}_e)\times\int dr^3\hat{\psi}^\dagger(r)_{p\downarrow}\times\hat{\psi}(r)_{e\uparrow}} =$$

$$= Diag.(n_1^{-2}, n_2^{-2}, n_3^{-2}n_4^{-2}) \times e^{(\psi_e/\hat{\psi}_e)\times\int dr^3\hat{\psi}^\dagger(r)_{p\downarrow}\times\hat{\psi}(r)_{e\uparrow}} \qquad (6.2.114b)$$

where the $b_\mu = 1/n_\mu, \mu = 1, 2, 3, 4$ are the *characteristic quantities of the proton.* The reader is assumed to know that $b_k = 1/n_k, k = 1, 2, 3$ provide a geometrization of the *shape* of the proton, while $b_4 = 1/n_4$ provide a geometrization of its *density,* all quantities being normalized to the value 1 for the vacuum.

At this initial states of the analysis, the characteristic quantities of the proton are assumed as being local variables, $b_\mu = b_\mu(x, v, a, d, \tau, \psi, \partial\psi, ...)$, for the specific purpose of illustrating the independence of the Poincaré-Santilli isosymmetry from said local functional dependence.

Under the above assumptions, the basic isoinvariant of the neutron is then given by

$$(\hat{x} - \hat{y})^{\hat{2}} = (\hat{x}^\mu - \hat{y}^\mu)\hat{\times}\hat{G}_{\mu\nu}\hat{\times}(\hat{x}^\nu - \hat{y}^\nu) =$$

$$= [(x^\mu - y^\mu) \times \hat{\eta}_{\mu\nu} \times (x^\nu - y^\nu)] \times \hat{I} =$$

$$= [(x^1 - y^1)^2 \times b_1^2 + (x^2 - y^2)^2 \times b_2^2 + (x^3 - y^3)^2 \times b_3^2 - (x^4 - y^4)^2 \times b_4^2] \times \hat{I} =$$

$$= [(x^1 - y^1)^2/n_1^2 + (x^2 - y^2)^2/n_2^2 + (x^3 - y^3)^2/n_3^2 - (x^4 - y^4)^2/_4^2] \times \hat{I}, \quad (6.2.115)$$

where the exponent of Eqs. (6.2.114) can be considered embedded in the characteristic quantities due to their arbitrary functional dependence, or ignored at the moment, due to its cancellation by the isounit.

Note that *isoinvariant (6.2.115) contains as particular cases all infinitely possible Riemannian, Finslerian, as well as any other possible, nonsingular line element with signature* (+, +, +, -). Hence, Ref. [96] constructed the universal symmetry for all these infinitely possible line elements.

The transformations leaving invariant isoseparation (6.2.115) can be written (see Eqs. (3.4) page 183, Ref. [96])

$$\hat{x}' = \hat{\Lambda}(\hat{w})\hat{\times}\hat{x}, \quad \hat{x}' = \hat{x} + \hat{A}, \tag{6.2.116}$$

$$\hat{\Lambda}^{\dagger} \times \hat{\eta} \times \hat{\Lambda} = \Lambda \times \hat{\eta} \times \Lambda^{\dagger} = \hat{I} \times \hat{\eta} \times \hat{I}, \tag{6.2.116b}$$

$$\hat{Det}\ \hat{\Lambda} = Det\ (\Lambda \times \hat{T}) = \pm\hat{I}, \tag{6.2.116c}$$

where the quantity $\hat{A}$ is identified below and $\hat{w} = w \times \hat{I}$ represents the isoparameters.

The isoconnected component of $\hat{P}(3.1)$ is characterized by

$$\hat{Det}\ \Lambda = +\hat{I}, \tag{6.2.117a}$$

$$\hat{P}^{o}(3.1) = \hat{SO}(3.) \times \hat{A}(3.1), \tag{6.2.117b}$$

with explicit form of the *finite isotransforms* (Eqs. (3.5), page 184, Ref. [96])

$$\hat{SO}(3.1): \quad \hat{x}' = \left(\hat{e}^{i\times J_k \times w_k}\right) \hat{\times}\hat{x}\hat{\times} \left(\hat{e}^{-i\times J_k \times w_k}\right) =$$

$$= [\left(e^{i\times J_k \times \hat{T} \times w_k}\right) \times x \times \left(e^{-i\times w_k \times \hat{T} \times J_k}\right)] \times \hat{I}, \tag{6.2.118a}$$

$$\hat{A}(3.1): \quad \hat{x}' = \left(\hat{e}^{i\times P_\mu \times a_\mu}\right) \hat{\times}\hat{x}\hat{\times} \left(\hat{e}^{-i\times P_\mu \times a_\mu}\right) =$$

$$= \left[\left(e^{i\times P_\mu \times \hat{T} \times a_\mu}\right) \times x \times \left(e^{-i\times a_\mu \times \hat{T} \times P_\mu}\right)\right] \times \hat{I}, \tag{6.2.118b}$$

where $(J_k) = (J_{\mu\nu})$, P_μ, $w_k, a_\mu, k = 1,2,3,4,5,6$, $\mu,\nu = 1,2,3,4$, are conventional quantities of the Poincaré symmetry, and we have used the isoexponentiation (Section 3.2)

$$\hat{e}^{X} = (e^{X\times\hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T}\times X}), \tag{6.2.119}$$

The reformulation of finite isotransforms (6.1.118) in terms of isogenerators $\hat{J}_k = J_k \times \hat{I}$, $\hat{P}_\mu = P_\mu \times \hat{I}$ is left as an instructive exercise for the interested reader, and assumed hereon.

The computation of the *infinitesimal isotransforms* from the preceding finite forms is elementary, yielding the *Lie-Santilli isoalgebra* (Eqs. (3.6), Page 184, Ref. [96])

$$[J_{\mu\nu}\hat{,}J_{\alpha\beta}] = J_{\mu\nu} \times \hat{T} \times J_{\alpha\beta} - J_{\alpha\beta} \times \hat{T} \times J_{\mu\nu} =$$

$$i \times (\hat{\eta}_{\nu\alpha} \times J_{\beta\mu} - \hat{\eta}_{\mu\alpha} \times J_{\beta\nu} - \hat{\eta}_{\nu\beta} \times J_{\alpha\nu} + \hat{\eta}_{\mu\beta} \times J_{\alpha\nu}), \tag{6.2.120a}$$

$$[J_{\mu\nu}\hat{,}P_\alpha] = J_{\mu\nu} \times \hat{T} \times P_\alpha - P_\alpha \times \hat{T} \times J_{\mu\nu} =$$

$$= i \times (\hat{\eta}_{\mu\alpha} \times P_\nu - \hat{\eta}_{\nu\alpha} \times P_\nu), \tag{6.2.120b}$$

$$[P_\mu\hat{,}P_\nu] = P_\mu \times \hat{T} \times P_\nu - P_\nu \times \hat{T} \times P_\mu = 0. \tag{6.2.120c}$$

The initiated reader is aware of the deep meaning of the seemingly innocuous isocommutators (6.2.120c), In fact, the components of the linear momentum *do not* commute when defined over a space with an exp;licit functional dependence on the local variables. Their isocommutativity then signals the elimination of curvature for brooder vistas.

The *Casimir-Santilli isoinvariants* were also computed in Eqs. (3.7), page 184, Ref. [96], via the use of isocommutators (6.2.120) and can be written

$$\hat{C}^{(0)} = \hat{I} \tag{6.2.121a}$$

$$\hat{C}^{(1)} = \hat{P}^{\hat{2}} = \hat{P}\hat{\times}\hat{P} = (\hat{\eta}^{\mu\nu} \times \hat{P}_\mu \times \hat{T} \times \hat{P}^\nu) \times \hat{I}, \tag{6.2.121b}$$

$$\hat{C}(2) = \hat{W}\hat{\times}\hat{W}, \quad \hat{W}_m u = \epsilon_{\mu\alpha\beta\rho}\, J^{\alpha\beta} \times \hat{T} \times P_\rho. \tag{6.2.121d}$$

The explicit form of the isotransformations along the third space axis is then given by:

1)**isorotations** [5a,5b,5c,5d]

$$x^{1'} = x^1 \times \cos[\theta \times (\hat{\eta}_{11} \times \hat{\eta}_{22})^{1/2}] - x^2 \times \hat{\eta}_{22} \times \hat{\eta}_{11}^{-1} \times \sin[\theta \times (\hat{\eta}_{11} \times \hat{\eta}_{22})^{1/2}], \tag{6.2.122a}$$

$$x^{2'} = x^1 \times \hat{\eta}_{11} \times \hat{\eta}_{22}^{-1} \times \sin[\theta \times (\hat{\eta}_{11} \times \hat{\eta}_{22})^{1/2}] + x^2 \times \cos[\theta \times (\hat{\eta}_{11} \times \hat{\eta}_{22})^{1/2}]. \tag{6.2.122b}$$

2) **Lorentz-Santilli isotransforms** [5e,5f]

$$x^{1'} = x^1, eqno(6.2.123a)$$

$$x^{2'} = x^2, \tag{6.2.123b}$$

$$x^{3'} = x^3 \times \cosh[v \times (\hat{\eta}_{33} \times \hat{\eta}_{44})^{1/2}] -$$
$$-x^4 \times \hat{\eta}_{44} \times (\hat{\eta}_{33} \times \hat{\eta}_{44})^{-1/2} \times \sinh[v \times (\hat{\eta}_{33} \times \hat{\eta}_{44})^{1/2}] =$$
$$= \hat{\gamma} \times (x^3 - \hat{\beta} \times \frac{b_4}{b_3} \times x^4), \tag{6.2.123c}$$

$$x^{4'} = -x^3 \times \hat{\eta}_{33} \times (\hat{\eta}_{33} \times \hat{\eta}_{44})^{-1/2} \times \sinh[v(\hat{\eta}_{33} \times \hat{\eta}_{44})^{1/2}] +$$
$$+x^4 \times \cosh[v \times (\hat{\eta}_{33} \times \hat{\eta}_{44})^{1/2}] =$$
$$= \hat{\gamma} \times (x^4 - \hat{\beta} \times \frac{b_3}{b_4} \times x^3), \tag{6.2.123d}$$

$$\hat{\beta}^2 = \frac{v_k \times \hat{\eta}_{kk} \times v_k}{c_o \times \hat{\eta}_{44} \times c_o} = \frac{v_k \times b_k^2 \times v_k}{c_o \times b_4^2 \times c_o}, \tag{6.2.123e}$$

$$\hat{\gamma}^2 = \frac{1}{1 - \hat{\beta}^2}. \tag{6.2.123f}$$

3) **isotranslations** [96]

$$x'\mu = x_\mu + A_\mu \tag{6.2.124a}$$

$$A_\mu = A_\mu(x, v, a, E, d, \tau, \psi, \partial\psi, ...) = a_\mu \times \{\hat{\eta}_{\mu\mu} + \\ + a^\alpha \times [\hat{\eta}_{\mu\mu}, J_{\mu\alpha}]/1! + a^\alpha \times a^\beta \times [\hat{\eta}_{\mu\mu}, J_{\mu\alpha}], J_{\mu\beta}]/2! + ...\}; \tag{6.2.124b}$$

4) **space and time isoinversions** [96]

$$x' = \pi \times x = (-x^k, x^4), \tag{6.2.125a}$$

$$x' = \pi_t \times x = (x^k, -x^4); \tag{6.2.125b}$$

5) **isoselftransforms** [5e,5f,96]

$$\hat{\eta} \to \hat{\eta}' = \hat{n}\hat{\times}\hat{\eta}, \quad \hat{I} \to \hat{I}' = \hat{n}^{-1}\hat{\times}\hat{I}, \ \hat{n} \in \hat{R}, \ n \neq 0, \tag{6.2.126}$$

a property of fundamental relevance for gravitation, grand unification and other basic issues (see Chapter 14).

The following comments presented in ref. [96] should be reviewed:

A) It is easy to see the local isomorphism $\hat{P}(3.1) \approx P(3.1)$ for all positive-definite isounits. Hence, the Lorentz-Poincaré *transformations* are "inapplicable' (rather than violated) for the neutron structure (6.2.109), but *the Lorentz-Poincaré symmetry remains exact,* and only subjected to the broadest possible *realization* preserving conventional total quantities.

B) The physically most salient differences between the Poincaré symmetry and its isotopic covering is that the former solely applies for linear, local-differential and potential-Hamiltonian interactions, while the latter includes the preceding interactions and additionally teats nonlinear, nonlocal-integral and nonpotential-nonhamiltonian interactions as expected in conditions of deep mutual penetration of the wavepackets and/or charge distribution of particles.

C) The Minkowski-Santilli isospace provides a *geometric unification* of all infinitely possible spaces with signature $(+,+,+,-)$, thus including all possible Riemannian. Finslerian and other spaces (see Chapter 3 for details).

D) The Poincaré-Santilli isosymmetry is *directly universal* for all possible (non-singular) line elements with signature $(+,+,+,-)$, thus being directly universal for all possible Riemannian, Finslerian and other line elements with said signature.

E) As it is well known, no connection was considered throughout the 20-th century between strong and gravitational interactions, trivially, because strong interactions solely occur at distances of 1 fm, while gravitational models studies in the 20-th century are restricted to exterior long distance problems. However, *no distinction can be made at this stage of our studies between strong and gravitational interactions* because we are studying the interior neutron problem

within the hadronic horizon with 1 fm radius. As recalled in Section 6.1.4, Eqs. (6.1.17)-(6.1.19), all Riemannian metrics admit the factorization of the isotopic element of type (6.2.112), thus reaching line element (6.2.15).

The *Poincaré-Santilli isodual isosymmetry*

$$\hat{P}^d(3.1) = \hat{O}^d(3.1) \times \hat{A}(3.1) \tag{6.2.127}$$

for the characterization of the structure of the antineutron according to model (6.2.110) can be easily constructed from the above derivation via the *isodual map* (Chapter 2), here expressed for an arbitrary quantity

$$A(x, v, a, E, d, \psi, \partial\psi, ...) \ \rightarrow \ A^d(x^d, v^d, a^d, E^d, d^d, \psi^d, \partial\psi^d, ...) =$$
$$= -A^\dagger(-x^\dagger, -v^\dagger, -a^\dagger, -E^\dagger, -d^\dagger, -\psi^\dagger, \partial^\dagger\psi^\dagger, ...), \tag{6.2.128}$$

applied to the *totality* of the formalism, including units, numbers, fields, spaces, algebras, symmetries, etc.

6.2.8.C Santilli Isorelativity for the Neutron and its Isodual for the Antineutron

Deviations from the conventional Minkowskian spacetime causes necessary compatible deviations from special relativity. Santilli covering isorelativity [4,5] according to Isoaxioms I to V, Eqs. (6.1.11) to (6.1.16), was adopted for the interior of the neutron in Ref. [96] for synthesis (6.2.109), the isodual isorelativity being adopted for the synthesis of the antineutron (9.2.110). The same assumptions are adopted hereon. The following comments were presented in Ref. [96] and their indication may of value here:

A) The main assumption of isorelativity for the interior of the neutron is the abandonment of the speed of light as the basic invariant, and its replacement with the maximal causal speed (6.1.11). The assumption was mandarted by numerous facts, such as: the expectation of physical media opaque to light, in which case any use of the speed of light as the basic invariance is nonsensical; clar experimental evcdience in which particles move faster than the local speed of light within phyusical media, such as water, in which case the assumption of the speed of light as the basic invariant cause violation of cxausality; and otyehjr facts.

This central assumption will be derived later on from first axiomatic principles, and submitted to additional confrontation with experimental data. At this point, we merely indicxate that the assumption can be easily derived via the derivative of space with respect to time on the isocone of causal speeds in the (3,4)-plane

$$\hat{d}\hat{x}^{\hat{2}} = \hat{d}\hat{x}^3 \hat{\times} \hat{d}\hat{x}^3 - \hat{d}\hat{x}^4 \hat{\times} \hat{d}\hat{x}^4 =$$
$$= (dx^3 \times b_3^2 \times dx^3 - dx^4 \times b_4^2 \times dx^4) \times \hat{I} =$$

$$= (dx^3 \times dx^3/n_3^2 - dx^4 \times dx^4/n_4^2) \times \hat{I} = 0, \tag{6.2.129}$$

that, for b_3, b_4 independent from x, yields [5,96]

$$V_{max} = |\frac{dr}{dt}|_{max} = c_o \times \frac{b_4}{b_3} = c_o \times \frac{n_3}{n_4}. \tag{6.2.130}$$

V_{max} is essentially the maximal possible speed of the electron when a physical constituent of the neutron, that is, the maximal orbital speed of the electron when trapped within the hyperdense proton and constrained to rotate with its spin. When the neutron decays and the electron is expected, we have $b_3 = b_4 = 1$ and the conventional value c_o is recovered as maximal causal speed in vacuum.

B) The structure of the neutron is described in Ref. [96] via the use of our notions of time and length and their related units, with the understanding that the intrinsic time of the neutron, the *neutron isotime* is given by

$$\hat{t}_n = t \times \hat{I}_t, \quad \hat{I}_t = b_4^{-2} = n_4^2, \tag{6.2.131}$$

and the *neutron isolength* along the 3-axis has the expression

$$\hat{\ell}_n = \ell \times \hat{I}_\ell, \quad \hat{I}_\ell = b_3^{-2} = n_3^2, \tag{6.2.132}$$

where t and ℓ are our time and length, respectively.

It is evident that the above defined neutron proper time and proper length are *different* than our own, to such an extent that a perfectly spherical shape assumed in the outside may correspond to a different structure in the inside, trivially, due to possible different values of the space characteristic quantities.

C) Isoaxioms I to V are verified by all experimental evidence considered so far. The objective of this section is, therefore, to show that the same isoaxioms are verified also by the structure of the neutron.

D) When locally defined, that is, defined at a given value of spacetime, *isotransformations are highly nonlinear, thus mapping inertial into noninertial frames.* This is a necessary condition for the admission of unrestricted, thus generally non-Newtonian forces, such as acceleration-dependent forces. It is evident that, under such a nonlinear structure, the center of mass of an isolated neutron cannot have a uniform motion.

E) Since the objective of this section is the achievement of a *global* representation of the neutron structure, *all values of the characteristic quantities are hereon assumed as being averaged to constants, thus regaining the linearity of the isotransforms and their preservation of inertial systems* [96, page 188).

6.2.8.D The Isoselfdual Dirac-Santilli Isoequation

The next important advance presented in Ref. [96] is the construction of the isotopies of Dirac's equation in a way conform to the rules of hadronic mechanics,

today known as the *Dirac-Santilli isoequation.* The resulting isotheory is as fundamental for hadronic mechanics as the conventional Dirac equation is for quantum mechanics.

As recalled earlier, the conventional Dirac equation represents an electron moving in vacuum under the electromagnetic field of a proton, as occurring in the hydrogen atom, while the isotopic version represents the same electron when moving within hyperdense media, as occurring in the neutron structure. [45]

Recall that the Schrödinger equation represents indeed the hydrogen atom as a bound state of a proton and an electron, while Dirac;s equation *does not* because it solely represents the electron *under the field of the proton considered as external.* To avoid illusory appraisals, the reader should expect the same conceptual setting for the isotopic equation because isotopies are axiom-preserving. Hence, the Dirac-Santilli isoequation represents the dynamics of an electron immersed within the proton *considered as external.*

Additionally, we should recall that the conventional Dirac equation has been misinterpreted throughout the 20-th century as solely representing the electron, since the positron was derived via the so-called "hole theory" or other manipulations. This misinterpretation resulted to be due to the use of basically insufficient mathematical and physical insight.

Hadronic mechanics has identified fundamental flaws in this view, such as the fact that *a 4-dimensional irreducible representation of spin 1/2 does not exist.* Consequently, in the eventuality orthodox views were correct, Dirac's equation would represent the electron via a *reducible* representation of spin 1/2, thus implying that the electron is composite.

The advent of the isodual mathematics (Section 2.2) permitted the identification of the property that the conventional *gamma matrices*

$$\gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma^4 = i \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{6.2.133}$$

characterize the Kronecker product of one irreducible, two-dimensional representation of spin 1/2 time its isodual,

$$\gamma^k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k^d & 0 \end{pmatrix}, \quad \gamma^4 = i \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{6.2.134}$$

thus jointly representing an electron and a positron. In any case, this joint representation is *necessary* to achieve a full scientific democracy for particles

[45] Since the appearance of Ref.s [5,96] there have been studies on the so-called "deformation" of Dirac equation that essentially copy Santilli's result (generally without quotation of their origination) but without formulating the theory on isospaces over isofields (in the illusion of hiding the paternity fraud). These "deformations" are hereon ignored because catastrophically inconsistent, as now familiar.

and antiparticles at all levels, thus including the first quantization here considered. Alternatively, the above features are rigorously represented by the fact that *Dirac's gamma matrices are isoselfdual* (invariant under isoduality) [96].

Under the above clarifications, the construction of the Dirac-Santilli isoequation can be outlined as following. First, ref. [96] identified the *total representation space* of the conventional Dirac equations

$$S_{tot} = \{M_{orb}(x, \eta, R) \times S_{spin}(2)\} \times \{M^d_{orb}(x^d, \eta^d, R^d) \times S^d_{spin}(2)\}, \quad (6.2.135)$$

that resulted in being *twelve-dimensional,* due to the inclusion of the orbital and intrinsic spaces for both the electron and the positron.

Consequently, Ref. [96] assumed the following fundamental, twelve-dimensional, total isospace

$$\hat{S}_{tot} = \{\hat{M}_{orb}(\hat{x}, \hat{\eta}, \hat{R}) \times \hat{S}_{spin}(2)\} \times \{\hat{M}^d_{orb}(\hat{x}^d, \hat{\eta}^d, \hat{R}^d) \times \hat{S}^d_{spin}(2)\}. \quad (6.2.136)$$

The above assumption requires the use of *four different isounits and related isotopic elements,* one pair for each of the four distinct motions,

$$\hat{I}_{tot} = \{\hat{I}_{orb} \times \hat{I}_{spin}\} \times \{\hat{I}^d_{orb} \times \hat{I}^d_{spin}\}. \quad (6.23.137a)$$

$$\hat{T}^{-1}_{tot} = \{\hat{T}_{orb} \times \hat{T}_{spin}\} \times \{\hat{T}^d_{orb} \times \hat{T}^d_{spin}\}, \quad (6.2.137b)$$

with combined total orbital (to) and total spin (ts) expressions for particle and antiparticle

$$\hat{I}_{to} = \hat{I}_{orb} \times \hat{I}^d_{orb}, \quad \hat{I}_{ts} = \hat{I}_{spin} \times \hat{I}^d_{spin} \quad (6.2.138)$$

Ref. [96], Eqs. (6.1, page 189 then constructed the isotopies of Dirac's equation in the most rigorous known way, via the linearization of the second order Casimir-Santilli isoinvariant, Eq. (6.2.121b),

$$(\hat{G}^{\mu\nu} \hat{\times}_{to} \hat{P}_\mu \hat{\times}_{to} \hat{P}_\nu + \bar{m}^2_{\hat{e}}) \hat{\times}_{to} |\hat{\psi}>=$$

$$= (\hat{G}^{\mu\nu} \hat{\times}_{to} \hat{\Gamma}_\mu \hat{\times}_{to} \hat{P}_\nu + \hat{i} \hat{\times}_{to} \bar{m}_{\hat{e}}) \hat{\times}_{to} (\hat{G}^{\alpha\beta} \hat{\times}_{to} \hat{\Gamma}_\alpha \hat{\times}_{to} \hat{P}_\beta + \hat{i} \hat{\times}_{to} \bar{m}_{\hat{e}}) \hat{\times}_{to} |\hat{\psi}>= 0, \quad (6.2.139a)$$

$$\{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = \hat{\Gamma}_\mu \hat{\times}_{to} \hat{\Gamma}_\nu + \hat{\Gamma}_\nu \hat{\times}_{to} \hat{\Gamma}_\mu = \hat{2} \hat{\times}_{to} \hat{G}_{mu\nu}, \quad (6.2.139b)$$

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\} = \hat{\gamma}_\mu \times \hat{T} \times \hat{\gamma}_\nu + \hat{\gamma}_\nu \times \hat{T} \times \hat{\gamma}_\mu = 2 \times \hat{\eta}_{mu\nu}, \quad (6.2.139c)$$

$$\hat{\Gamma}_\mu = \hat{\gamma}_\mu \times \hat{I}_{to}. \quad (6.2.139d)$$

where, as shown below

$$\bar{m}_{\hat{e}} = m_e \times c_o \times \frac{b_4}{b_3}. \quad (6.2.140)$$

The above reduction is excessively general for the structure of the neutron. Hence, Ref. [96] assumed the simplified conditions

$$\hat{I}_{to} = 1/\hat{T}_{to} = \hat{I}, \quad \hat{I}_{ts} = I = Diag.(1, 1), \quad (6.2.141)$$

from which Ref. [96] derived the explicit form of the *isogamma matrices*

$$\hat{\gamma}_k = b_k \times \begin{pmatrix} 0 & \hat{\sigma}_k \\ \hat{\sigma}_k^d & 0 \end{pmatrix}, \quad \hat{\gamma}_4 = i \times b_4 \times \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{6.2.142}$$

where σ^k are the conventional Pauli matrices.

The above expressions then characterize the *Dirac-Santilli isoequation* (Eq. (6.3), p. 190, Ref. [96]),

$$(\hat{G}^{\mu\nu}\hat{\times}\hat{\Gamma}_\mu\hat{\times}\hat{P}_\nu + \hat{i}\hat{\times}\bar{m}_{\hat{e}})\hat{\times}|\hat{\psi}>= (\hat{\eta}^{\mu\nu}\times\hat{\gamma}_\mu\times\hat{P}_\nu + i\times\bar{m}_{\hat{e}})\times\hat{T}\times|\hat{\psi}>= 0. \tag{6.2.143}$$

The understanding of this section requires the knowledge that the structure of the neutron is represented via the above isoequation *without* any need to add electromagnetic potentials. The latter are crucial for the hydrogen atom but their contribution is ignorable for the neutron structure with respect to the much bigger contribution from the strong interactions 9see below). At any rate, the addition of said potential is trivial and left to the interested reader.

6.2.8.E Isospinorial Covering of the Poincaré-Santilli Isosymmetry and its Isodual

The next advance achieved in Ref. [96] is the first construction of the *isospinorial covering of the Poincaré-Santilli isosymmetry*

$$\hat{\mathcal{P}}(3.1) = \hat{SL}(2.\hat{C})\times\hat{\mathcal{A}}(3.1), \tag{6.2.144}$$

via the following realization (Eq. (6.4), page 190, ref. [96])

$$\hat{SL}(2.\hat{C}): \quad \hat{R}_k = \frac{1}{2}\times\epsilon_{kij}\Gamma_i\hat{\times}\Gamma_j, \quad \hat{S}_k = \frac{1}{2}\times\Gamma_k\hat{\times}\Gamma_4, \tag{6.2.145a}$$

$$\hat{\mathcal{A}}(3.1): \quad P_\mu. \tag{6.2.145b}$$

The verification by the above generators of commutation rules (6.2.120) is an instructive exercise for the interested reader.

The proof that the Dirac-Santilli isoequation transforms covariantly under $\hat{\mathcal{P}}(3.1)$ is instructive. Equally instructive is the proof of the isoselfduality of Eq. (6.2.143), thus eliminating the need for an isodual image. In turn, this establishes that the true symmetry of the conventional Dirac equation is the isoselfdual symmetry

$$S_{tot} = P(3.1)\times P^d(3.1). \tag{6.2.146}$$

Similarly, the total symmetry of the Dirac-Santilli isoequation is given by the isoselfdual symmetry

$$\hat{S}_{tot} = \hat{\mathcal{P}}(3.1)\hat{\times}\hat{\mathcal{P}}^d(3.1). \tag{6.2.147}$$

The reader's technical knowledge can be tested at this point via the knowledge of the reason for symmetries (6.2.146) and (6.2.147) to be *twenty two dimensional.*

6.2.8.F Isorenormalization of Spin and Angular Momentum

In order to copnduct the direct study of the hadronic structure model of the neutron as a bound state of a isoproton and an isoelectron

$$n = (\hat{p}^+, \hat{e}^-)_{hm}, \tag{6.2.148}$$

Ref. [96] studied the mutations of the intrinsic characteristics of the electron when totally immersed inside the proton, a feature called *isorenormalization* in ref. [96] for the first time, with evident isodual image for the antineutron.

Hence, Ref. [96] provided the following realization ofd the Poincaré-Santilli isosymmetry

$$\hat{O}(3.1): \quad \hat{L}_k = \epsilon_{kij}\hat{r}_i\hat{\times}\hat{P}_j, \quad \hat{S}_k = \frac{1}{2}\times\epsilon_{kij}\hat{\gamma}_i\hat{\times}\hat{\gamma}_j, \tag{6.2.149a}$$

$$[\hat{L}_i\hat{,}\hat{L}_j] = \epsilon_{ijk}b_k^{-2}\times\hat{L}_k, \tag{6.2.149b}$$

$$\hat{L}^{\hat{2}}\hat{\times}|\hat{\psi}>=(b_1^{-2}\times b_2^{-2}+b_2^{-2}\times b_3^{-2}+b_3^{-2}\times b_1^{-2})\times|\hat{\psi}>, \tag{6.2.149c}$$

$$\hat{L}_3\hat{\times}|\hat{\psi}>=\pm b_1^{-1}\times b_2^{-1}\times|\hat{\psi}>, \tag{6.2.149d}$$

$$\hat{S}^{\hat{2}}\hat{\times}|\hat{\psi}>=\frac{1}{4}\times(b_1^2\times b_2^2+b_2^2\times b_3^2+b_3^2\times b_1^2)\times|\hat{\psi}>, \tag{6.2.149e}$$

$$\hat{S}_3\hat{\times}|\hat{\psi}>=\pm b_1\times b_2\times|\hat{\psi}>, \tag{6.2.149f}$$

which realization exhibits the mutations/isorenormalizations of spin and angular momentum *necessary* for the representation of neutron structure.

6.2.8.G Isorenormalization of the Rest Energy

A direct consequence of the mutation of the speed of light,

$$c_o \ \rightarrow c = c_o\times b_4 = \frac{c_o}{n_4}, \tag{6.2.150}$$

is the isorenormalization of the rest energy of the electron in structure (6.2.148). However, the corresponding mutation

$$E_e = m_e\times c_o^2 \ \rightarrow E_{\hat{e}} = m_e\times c_o^2\times b_4^2 = m_e\times\frac{c_o^2}{n_4^2}, \tag{6.2.151}$$

would be *erroneous* because violating causality in physical media whose density is such that $b_4 > b_3$, in which case

$$c_o\times b_4 > c_o\times V_{max} = c_o\times\frac{b_4}{b_3} \tag{6.2.152}$$

.

At any rate, isorenormalization (6.2.151) would imply that. for the case of water,

$$E_{\hat{e}} \approx \frac{4 \times E_e}{9}, \tag{6.2.153}$$

since in water $b_4 \approx 2/3$. By contrast, for the correct isorenormalization (see below) we must have for an electron traveling in water $E_{\hat{e}} = E_e$ since, as indicated in Section 6.1.4, for water we have $b_4 = b_3$ due to its homnogeneity and isotropy.

In view of the above issues, Ref. [96] derived the isorenormalization of the rest energy from primitive isosymmetries. In fact, the isolinear momentum in the Lie-Santilli isoalgebra (6.2.120) has the explicit form, Eq. (5.2), p. 188, Ref. [96]

$$\hat{P}_\mu \hat{\times} |\hat{\psi}> = \hat{P}_\mu \times \hat{T} \times |\hat{\psi}> = -\hat{i}\hat{\times}\hat{\partial}_\mu |\hat{\psi}> =$$

$$= -i \times \hat{I}^\nu_\mu \times \partial_\nu |\hat{\psi}> = -i \times b_\mu^{-2} \times \partial_\mu |\hat{\psi}> = -i \times n_\mu^2 \times \partial_\mu |\hat{\psi}>, \quad nosum, \tag{6.2.154}$$

with space and time eigenvalues

$$p = (p_\mu) = (m_e \times \hat{\gamma} \times c_o \times \frac{b_4}{b_3} \times v_k, \ m_e \times \hat{\gamma} \times c_o^2 \times \frac{b_4}{b_3}), \ k = 1, 2, 3. \tag{6.2.155}$$

Consequently, the Casimir-Santilli isoinvariant (6.2.121b) assumes the explicit form

$$\hat{P}\hat{\times}\hat{P}\hat{\times}|\hat{\psi}> = \hat{G}^{\mu\nu}\hat{\times}\hat{P}_\mu\hat{\times}\hat{P}_\nu\hat{\times}|\hat{\psi}> =$$

$$= \hat{\eta}^{\mu\nu} \times \hat{P}_\mu \times \hat{T} \times \hat{P}_\nu \times \hat{T} \times |\hat{\psi}> =$$

$$= (m_e^2 \times \hat{\gamma}^2 \times c_o^2 \times \frac{b_4^2}{b_3^2} \times (v_k \times b_k^2 \times v_k) - m_e^2 \times \hat{\gamma}^2 \times c_o^4 \times b_4^2) \times |\hat{\psi}> =$$

$$= m_e^2 \times c_o^4 \times \frac{b_4^4}{b_3^4} \times \hat{\gamma}^2 \times (\hat{\beta}^2 - 1) \times |\hat{\psi}> =$$

$$= -m_e^2 \times c_o^4 \times \frac{b_4^4}{b_3^4} \times |\hat{\psi}> = -m_e^2 \times V_{max}^4 \times |\hat{\psi}>, \tag{6.2.156}$$

from which we obtain the *isorenormalization of the rest energy*

$$E_e = m_e \times c_o^2 \ \rightarrow E_{\hat{e}} = m_e \times c_o^2 \times \frac{b_4^2}{b_3^2} =$$

$$= m_e \times c_o \times \frac{n_3^2}{n_4^2} = m_e \times V_{max}^2, \tag{6.2.157}$$

that resolved the ambiguities indicated earlier.

it is easy to expect, in general, similar mutations / isorenormalizations of *all* intrinsic characteristics of the electron. This illustrates a main prediction

of hadronic mechanixcs according to which strong interactions alter all intrinsic characteristics of particles in a hadronic bound state as well as in deep inelastric scatterings of hadrons. This prediction is impossible for quantum mechanics, trivially, becauyse strog interactions are entirely represented with a Hamiltonian.

Among these predictions, it is worth recalling [96] that light emitted in the interior of of the neutron structure reaches the outside *blueshifted,* namely, with an increase of its frequency according to Isoaxiom IV via a mechanism based on the absorption of energy from the medium itself.

The fact that the rest energy of the neutron is constant establishes the impossibility for light to be created inside an isolated neutron. In turn, this confirms the impossibility of assuming the speed of light as the basic invariant for the neutron structure.

Note that no mutation of the Doppler's law is possible for light in water due to its homogeneous and isotropic character.

The isodualities of the results of this section for the antineutron are left as a useful exercise for the reader interested in new scientific vistas.

6.2.8.H Isorenormalization of Electric and Magnetic Moments

Another, well known, important role of Dirac's equation is thge characterization of the electfic and magnetic moments. The next advance of Ref. [96], Eqs. (6.5), page 190, was the repetitioon of the characterization (see, e.g., Ref. [115]), this time for isoequation (6.2.143), resulting in the *isporenormalized electric and magnetic moments,*

$$\hat{\epsilon}_{\hat{e}} = \epsilon_e \times \frac{b_3}{b_4}, \quad \hat{\mu}_{\hat{e}} = \mu_e \times \frac{b_3}{b_4}. \tag{6.2.158}$$

This derivation is also an instructive exercise for scholars interested in research intended as the pursuit of *new* knoweldge.

6.2.8.I Representation of the Neutron spin.

Following all the preceding preparatory advances, Ref. [96] specialized the results to the isorelativistic representation of the simplified structure model of the neutron and antineutron

$$n = (p^+, \hat{e}^-)_{hm}, \tag{6.2.159a}$$

$$\bar{n} = (p^-, \hat{e}^+)_{hm}, \tag{6.2.159b}$$

where the proton p^+ and the antiproton p^- are not mutated, being about $2,000$ times hgeavier than the electron and the prositron.

As now familiar from the nonrelativistic study, Figure 6.23 in particular, a necessary condition for the consistency of models (6.2.159) is that the isoelectron and the isopositron have a null total angular momentum. In turn, this is possible

if and only if

$$|\hat{L}_3| = |\hat{S}_3|, \quad |\hat{L}^{\hat{2}}| = |\hat{S}^{\hat{2}}|. \tag{6.2.160}$$

By using isorealizationb (6.2.156), the above conditions require that

$$b_1^{-1} \times b_2^{-1} = \frac{1}{2} \times b_1 \times b_2, \tag{6.2.161a}$$

$$b_1^{-2} \times b_2^{-2} + b_2^{-2} \times b_3^{-2} + b_3^{-2} \times b_1^{-2} \ = \ \frac{1}{4} \times (b_1^2 \times b_2^2 + b_2^2 \times b_3^2 + b_3^2 \times b_1^2) \tag{6.2.161b}$$

which conditions admit the unique solution (Eq. (7.3), page 192, Ref. [96])

$$b_1 = b_2 = b_3 = b_s = \sqrt{2} = 1.415, \tag{6.2.162}$$

providing the numerical value of the space characteristic quantities of the proton and antiproton as predicted by the Dirac-Santilli isoequation.

It should be noted that the important geometric result here is the sherical shape of the proton, as expected from the fact that it is assumed not to be mutated. Also, such a shape is always defined up tp scaling from the structure of the invariant (6.2.115). The actual charge radius of the neutron will be derived later on.

6.2.8.J Representation of the Neutrron Rest Energy.

From Isoaxiom V and the preceding derivation (6.2.156) we have the isorenormalization of the rest energy

$$E_e = m_e \times c_o^2 = 0.511 \ MeV \quad \rightarrow \quad E_{\hat{e}} = m_e \times c_o^2 \times \frac{b_4^2}{b_3^2} = E_n - E_p = 1.294 \ MeV, \tag{6.2.163a}$$

$$\frac{b_4^2}{b_2^2} = 2.532, \quad \frac{b_4}{b_3} = 1.592, \tag{6.2.163b}$$

where m_e is the inertial mass of the electron, and the calculations apply for both the neutron and the antineutron.[46]

The knowledge of the space characteristic quantity, Eqs. (6.2.162), then allows the computation of the numerical value of b_4

$$b_4^2 = \frac{1.293 \times 1.415}{0.511} = 3.580, \quad b_4 = 1.892, \tag{6.2.164}$$

[46]Ref. [96] used the values $E_e = 0.5 \ MeV$ and $E_n - E_p = 1.3 \ MeV$, thus resulting in the numerical value $b_4/b_3 = 1.62$. Thje difference is noted to prevent possible claims of "mistake" ventured for political objectives far from serious science.

which value, for the approximations here assumed, is fully within the corresponding nonrelativistic expression, Eqs. (6.2.99d), as well as fully within the value obtained via the fit of experimental data on the Bose-Einstein corre;lation, Eqs. (6.1.112).

It should be noted that the compairson ov values (6.2.164) and (6.1.112) indicates that the proton density (defined, again, as its rest energy divided by its volume) is *bigger* that the density of the Bose-Einstein fireball. This result, even though merely indicational at this stage of our knowledge, is correct because the fireball of the Bose-Einstein correlation is extremely elongated, thus resulting in a density lower than that of an individual proton.

6.2.8.K Representation of the Neutron Magnetic Moment.

As familiar from the analysis of the preceding section, the null value of the total angular momentum of the isoelectreon predicts that its intrinsic magnetic moment is, at best, very small. In fact, isoequation (6.2.158) permits the following numerical, exact and invariant representation of the anomalous magnetic moment of the neutron (Eqs. (7.4), page192, Ref. [96])

$$\mu_n = -1.9 \times \frac{|e|}{2 \times m_p \times c_o} = \mu_p + \hat{\mu}_{\hat{e},orb} + \hat{\mu}_{\hat{e},spin}, \tag{6.2.165a}$$

$$\mu_p = +2.7 \times \frac{|e|}{2 \times m_p \times c_o}, \tag{6.2.165c}$$

$$\mu_{e,intr} = +1.00 \times \frac{|e|}{2 \times m_e \times c_o} = 1\mu_B, \tag{6.2.165b}$$

$$\mu_{\hat{e},tot} = -4.6 \times \frac{|e|}{2 \times m_p \times m_o} = -2.4 \times 10^{-3} \times \frac{|e|}{2 \times m_e \times c_o}, \tag{6.2.165d}$$

$$\hat{\mu}_{\hat{e},intr} = +1 \times \frac{b_3}{b_4}\mu_B = \frac{1.415}{1.892}\ \mu_B = +0.747\ \mu_B, \tag{6.2.165e}$$

$$\hat{\mu}_{\hat{e},orb} = -0.744\mu_B. \tag{6.2.165f}$$

where we have used the configuration of Figure 6.23, and we should remember the change in direction of the magn etic moment caused by the chgange of the sign of the charge. The mutated electric moment of the neutron is ignored because very small in any case.

6.2.8.L Representation of the Neutron Meanlife and Charge Radius

As shown in well written treatments of the conventional Dirac equation (see, e.g., E. Corinaldesi and E. Strocchi [115], page 191 and following), the behavior of

the electron in the hydrogen atom is represented by a basic (scalar) contribution acting on each component of the wavefunction,

$$H^{(0)} \times \psi_1 = (\frac{p^2}{2 \times m_e} - e \times A_0) \times \psi_1 = E \times \psi_1, \tag{6.2.166}$$

plus an infinite series of perturbative terms, the first one of the type

$$H^{(1)} \times \psi_1 = (-\frac{(p^2)^2}{8 \times m_e^3 \times c_o^2} - \frac{e}{2 \times m_e^2 \times c_o^2 \times r} \times \frac{dA_o}{dr} \times L * s -$$

$$-\frac{e \times \hbar^2}{8 \times m_e^2 \times c_o^2} \times \Delta A_o) \times \psi_1, \tag{6.2.167}$$

where A_0 is the fourth component of the electromagnetic potential A_μ originated by the external proton, $L * S$ is the usual scalar product, and the rest is well ln known.

The repetition of the same procedure for the the case of isoequation (6.2.143) characterized by isounit (6.2.114a) has the following main implications [96]:

1) The term $-e \times A_0$ in Eq. (6.2.166) is ignorable, as for the nonrelativistic case, due to the dramatically bigger contribution from the terms $\psi_e / \hat{\psi}_{\hat{e}}$.

2) All perturbative terms are consequently ignorable, as typical for hadronic mechanics due to its capability of turning conventional weakly convergent or divergent expansions into strongly convergent isotopic forms (see later on). 3) The resulting radial equation is then identical to the non relativistic expression (6.2.92) that we rewrite in the form including the meanlife and the charge radius

$$\left[\frac{1}{r^2}\left(\frac{d}{dr} r^2 \frac{d}{dr}\right) + \frac{\bar{m}_{\hat{e}}}{\hbar^2}\left(E_{hb} + V \times \frac{e^{-b \times r}}{1 - e^{-b \times r}}\right)\right] \times \hat{\psi}(r) = 0, \tag{6.2.168a}$$

$$\tau^{-1} = \lambda^2 \times |\hat{\psi}(0)|^2 \times \frac{\alpha^2 \times E_{hk}}{\pi \times \hbar}. \tag{6.2.168b}$$

$$R_n = b^{-1}, \tag{6.2.168c}$$

in which we have replaced the quantity m/ρ^2 unknown for Eqs. (6.2.92) with the known value $m_{\hat{e}} = 1.293 MeV/c_o^2$, the representation of parity being left to the interested reader. The repetition of the same procedure as that for the nonrelativistic case then yields the desired representation. In particular, the derivation confirms that Eq. (6.2.143) predicts one and only one energy level, that of the neutron, thus suppressing again the atomic spectrum of energy.

This complete the *numerically exact and time invariant relativistic representation via the Dirac-Santilli isoequation of all characteristics of the neutron as a hadronic bound state of a proton and an electron first achieved in Ref. [96], including the representation of: rest energy; meanlife; size; spin; charge; magnetic moment; and other characteristics; the spontaneous decay being treated in the subsequent section.*

6.2.8.M Dirac's Generalization of Dirac's Equation

Santilli pointed out in Ref. [96], page 191, the important historical occurrence according to which the first mutation of the total angular momentum of the electron from half-off-integer to integer values down to the value zero, was achieved by P. A. M. Dirac in papers [113,114]

Dirac's papers remained vastly ignored by orthodox because not aligned with vested interest in old doctrines. By contrast, Santilli did study these papers in detail and presented their review in EHM, Volume II, Section 10.7, as well as in other works, including paper [96].

Due to the great historical significance of these studies by Dirac, it is important to outline here the main aspects. The reader should be aware that papers [113,114] are rather complex in conception and technical realization. Hence, by no means our brief review pays them justice, and their true understanding can only be gained by the study of the original works.

In Ref. [113] Dirac introduced the following equations called by Santilli *Dirac's generalization of Dirac's equation*

$$(a_\mu \times \partial_\mu + \beta) \times q \times \psi = 0, \tag{6.2.169a}$$

$$q = Column\ (q_1, p_1; q_2, p_2), \quad \psi = Column\ (\psi_{1+}, \psi_{1-}; \psi_{2+}, \psi_{2-}). \tag{6.2.169b}$$

By assuminmg

$$a_4 = I_{4x4}, \tag{6.2.170}$$

Dirac's a-matrices are characterized by the expression [113]

$$a_\mu \times \beta \times a_\nu + a_\nu \times \beta \times a_\mu = 2 \times \beta \times \eta_{\mu\nu}, \tag{6.2.171}$$

where $\eta_{\mu\nu}$ is the conventional Minkowski metric.

On the basis of the above structure, Dirac reaches the following realization of the a- and β-matrices

$$\beta = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad a_1 = i \times \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{6.2.172a}$$

$$a_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad a_3 = i \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{6.2.172b}$$

The angular momentum / spin is characterized by

$$S_{ij} = -(a_i \times \beta \times a_j - a_j \times \beta \times a_i) \times \frac{q \times q^t}{8}, \tag{6.2.173}$$

where t stands for transposed, and possesses the eigenvalues

$$S^2 = S_{12}^2 + S_{23}^2 + S_{31}^2 =$$

$$= \frac{1}{8} \times (q_1^2 + p_1^2 + q_2^2 + p_2^2) = J \times (J+1), \tag{6.2.174a}$$

$$J = \frac{1}{4} \times (q_1 + p_1 + q_2 + p_2) - \frac{1}{2} = \frac{1}{2} \times (n + n'), \tag{6.2.174b}$$

$$n,\ n' \ = \ 0, 1, 2, 3, ... \tag{6.2.174c}$$

thus *admitting the value $J = 0$ for the ground state.*

Dirac introduced the above theory in paper [113] for a study of two coupled oscillators with quantum numbers $q_k, p_k,\ k = 1, 2$, and then continued the analysis in paper [114]. In the author 's view, papers [113,114] are, by far, the most interesting papers ever written on oscillators.

The historical aspect particularly significant for hadronic mechanics is that, without his knowledge, *Dirac's generalization of Dirac's equation has an irreducible isotopic structure with isotopic element*

$$\hat{T} = \beta, \tag{6.2.175}$$

where irreducibility is referred to the property that papers [113,114] become inconsistent unless *entirely* elaborated with respect to the isoproducts of the type

$$A \hat{\times} B = A \times \hat{T} \times B. \tag{6.2.176}$$

In fact, Eq. (6.2.169a) can be identically written in the formalist of the Dirac-Santilli isoequation (6.2.143) according to the expressions

$$(a_\mu \times \partial_\mu + \beta) \times q \times \psi \ \equiv \ (\hat{\eta}^{\mu\nu} \times a_\mu \times \hat{T} \times p_\mu + 1) \times \hat{T} \times \hat{\psi} = 0, \tag{6.2.177a}$$

$$\hat{T} = \beta, \quad \hat{I} = \beta^{-1}, \quad \hat{\psi} = q \times \psi, \tag{6.2.177b}$$

$$p_\mu \hat{\times} \hat{\psi} = \hat{p}_\mu \times \hat{T} \times \hat{\psi} = -\hat{i} \hat{\times} \hat{\partial}_\mu \hat{\psi} = -i \times \hat{I} \times \partial_\mu \hat{\psi}, \tag{6.2.177c}$$

thus acquiring the full isotopic structure while preserving all results.

The irreducible nature of the above reformulation is established by the isoanticommutators of the a-matrices that can *only* be isotopic,, i.e. of the type

$$\{a_\mu, a\nu\} = a_\mu \hat{\times} a_\nu + a_\nu \hat{\times} a_n u =$$

$$= a_\mu \times \hat{T} \times a_\nu + a_\nu \times \hat{T} \times a_\mu = a_\mu \times \beta \times a_\nu + a_\nu \times \beta \times a_\mu = 2 \times \hat{\eta}_{mu\nu}. \tag{6.2.178}$$

The above property illustrates the reason for the name "Dirac-Santilli isoequation" suggested for structure (6.2.143) and (6.2.177) by various authors.

The necessity of the correct isotopic reformulation should be kept in mind. It is easy to prove that Dirac's original formulation is *noncanonical* at the classical level and *nonunitary* at the operator level, thus activating the now familiar inconsistencies theorems. By contrast, the isotopic reformulation reconstructs canonicity and unitarity on isospaces over isofields, thus avoiding the inconsistency theorems.

It is also interesting to note the differences between Eqs. (6.2.143) and (6.2.177a) in the representation of the total null value of the angular momentum of the electron when inside the proton. This aspect was first studied in EHM Volume II, page 498, and can be outlined as follows. The lifting of the total angular momentum

$$J_{qm} = S_{spin} + L_{orb} = \frac{1}{2} + n \ \rightarrow J_{hm} = 0, \tag{6.2.179}$$

is achieved by Eq. (6.2.143) via an isotopic lifting of the $O(3)$ and $SU(2)$ symmetries in siuch a way that

$$\hat{J}_{hm} = \hat{S}_{spin} + \hat{L}_{orb} \equiv 0. \tag{6.2.180}$$

However, the lifting occurs under a *non-null* value of the individual components,

$$|\hat{S}_3| = |\hat{L}_3| \neq 0, \tag{6.2.181a}$$

$$|\hat{S}^{\hat{2}}| = |\hat{L}^{\hat{2}}| \neq 0. \tag{6.2.181b}$$

By contrast, Dirac achieves a null value of the total angular momentum via *null* values of its components,

$$\hat{S}_{spin} \equiv \hat{L}_{orb} \equiv 0. \tag{6.2.182}$$

The latter property has deep implications, by providing additional evidence of the unique capabilities of Dirac's intuition. In fact, Santilli's solution (6.2.181) does indeed hold under the conditions it is presented, namely, that Rutherford's electron is constrained to *orbit* inside the proton along its spin (Figure 6.23). By contrast, Dirac's solution (6.2.182) holds when *Rutherford's electron is compressed all the way to the center of the proton,* since, in the latter conditions, the orbital and intrinsic angular momenta are superimpose, thus resulting in an individual null, value.

The reader should be aware that the implications of papers [113,114] are simply beyond our imagination . We limit ourselves to indicate only a few implications to prevent excessive novelty that is at times disturbing.[47]

[47]Santilli has personally experienced countless cases in which the presentation of basically novel ideas caused uncontrollable repulsions, at times with hysterical overtones, including prohibitions at the last

For structural consistency, Dirac's generalization of Dirac equation cannot be formulated on the conventional Minkowski space $M(x,\eta,R)$ and must be formulated on the Minkowski-Santilli isospace $\hat{M}(\hat{x},\hat{\eta},\hat{R})$, this time, with isometric

$$\hat{\eta} = \beta \times \eta = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \tag{6.2.183}$$

namely, *Dirac-Santilli isoequation (6.2.177) characterizes th first known nondiagonal realization of the spacetime isometric.*

Rather than being an innocuous occurrence, the implications are far reaching because the line element now reads

$$\hat{x^2} = x^\mu \times (\beta^\rho_\mu \times \eta_{\rho\nu}) \times x^\nu = x^\mu \times \hat{\eta}_{\mu\nu} =$$

$$= x^1 \times x^3 - x^2 \times x^4 - x^3 \times x^1 - x^2 \times x^4 = -2 \times x^2 \times x^4, \tag{6.2.184}$$

namely, *Dirac-Santilli isoequation (6.2.177) mutates spacetime from the conventional four-dimensions down to two-dimensions.* Moreover, the space *appears* from the outside as being 4-dimensional, while intrinsically it is only 2-dimensional, thus illustrating again the "isobox" of Chapter 3.

At this point, scientific priests solely intent in preserving old knowledge will rush to abuse their illusory academic credibility by stating that the above result is pure philosophy or a mere mathematical curiosity. Since our scientific knowledge can only be qualified as being lilliputian, having the very short life of at most two hundred years, when possible civilizations in the universe may have millions of years of scientific evolution, ascientific posturing of the above type are certainly far from reality.

Recall that, according to clear experimental evidence, the electron is a "pure oscillation" of space, namely, a structure in which there is no oscillation of a "little mass" or anything else we define as "material." In fact, Schródinger proved in 1935 (the year in which the author was born) that the variable "x" in the conventional Dirac equation for a *free* electron describes a pure oscillation, of course, of space

moment to deliver invited lectures when the novelty of the topic appeared from the abstract, and the like. This behavior by persons who are expected as being "researchers" is so abnormal to have motivated the harsh comments that *researchers who feel repulsion to novelty behave like a priest desecrating the altar or a rabbi desecrating the torah.* A person decides to become a priest to honor the altar. Similarly, a person decides to become a rabbi to honor the torah. Hence, when a person decides to become a "researcher," he/she must dedicate his/her life to the pursuit of "new" knowledge, rather than the vigil guardianship of old knowledge. Showing repulsion to the presentation of new knowledge is a violation of the very essence of research, let alone amoral and ascientific. The list of names qualifying as "scientific priests desecrating the scientific altar" would fill dozens of pages, but definitely the name of P. A. M. Dirac would not appear in that list!

as a universal substratum for all events visible to mankind (Sections 6.1.2 and 6.1.3).

Intriguingly, *Dirac-Santilli isoequation (6.2.177) establishes that, in the transition from motion in vacuum to total immersion within the hyperdense medium inside the proton, the electron performs the mutation from an oscillation in (3+1)-dimension, to one in (1+1) dimension, namely, the electron is indeed reduced to a dimensionless point in space and time.*

In turn, the above result has its own far reaching implications, such as the possible triggering of continuous creation of matter in the universe studied in the next section, at which point it appears prudent to terminate the presentation of novelties because it may be disturbing to some (but not all!) physicists, as indicated above.

6.2.8.N Dirac-Santilli Genotopic and Hyperstructural Equations

The preceding studies have been conducted by assuming the neutron as isolated from the rest of the universe, thus resulting in conventional total conservation laws that required the isotopic branch of hadronic mechanics.

However, these are ideal conditions generally not verified in reality, since the neutron is generally a member of a nuclear process, such as the synthesis of the deuteron. The latter, by conception, is irreversible, in which case the isotopic branch of hadronic mechanics does not apply, requiring the broader genotopic branch (Chapter 4). In turn, this requires a broadening of the Dirac-Santilli isoequation (6.2.143) into a structurally irreversible form.

Under the assumption of a knowledge of Chapter 4,the latter objectives can be achieved via the selection of two different, yet conjugated units for motion forward and backward in time and related genoproducts,

$$\hat{I}^{>} = (^{<}\hat{I})^{\dagger}, \quad \hat{I}^{>} = 1/\hat{T}^{>}, \quad ^{<}\hat{I} =^{<} \hat{T}, \tag{6.2.185a}$$

$$A > B = A \times \hat{T}^{>} \times B, \quad A < B = A \times^{<} \hat{T} \times B, \tag{6.2.185b}$$

the forward and backward genometrics

$$\hat{G}^{>} = \hat{\eta}^{>} \times \hat{I}^{>}, \quad ^{<}\hat{G} =^{<} \hat{I} \times^{<} \hat{\eta}, \tag{6.2.186a}$$

$$\hat{\eta}^{>} = \hat{T}^{>} \times \eta, \quad ^{<}\hat{\eta} =^{<} \hat{T} \times \eta, \tag{6.2.187b}$$

and remaining genomathematics herein assumed.

The *Dirac-Santilli forward genoequation* can then be written

$$(\hat{G}^{>\mu\nu} > \hat{\Gamma}^{>}_{\mu} > \hat{P}^{>}_{\nu} + \hat{i}^{>} > \bar{m}^{>}_{\hat{e}}) > |\hat{\psi}^{>} >=$$

$$= (\hat{\eta}^{>\mu\nu} \times \hat{\gamma}^{>}_{\mu} \times \hat{P}^{>}_{\nu} + i \times \bar{m}^{>}_{\hat{e}}) \times \hat{T}^{>} \times |\hat{\psi}^{>} >= 0. \tag{6.2.188a}$$

$$(\hat{\gamma}^{>}_{\mu}, \hat{\gamma}^{>}_{\nu}) =^{<} \hat{\gamma}_{\mu} <^{<} \hat{\gamma}_{\nu} + \hat{\gamma}^{>}_{\nu} > \hat{\gamma}^{>}_{\mu} = 2 \times \hat{\eta}^{>}_{mu\nu}, \tag{6.2.188b}$$

$$\hat{P}_\mu^> > |\hat{\psi}^> >= -i \times \hat{I}_\mu^{>\rho} \times \partial_\rho |\hat{\psi}^> >, \tag{6.2.188c}$$

with conjugate equations for the backward form.

Note that the *forward genogamma matrices* are characterized by bracket of "Jordan-admissible type." Note also that irreversibility is embedded in the most primitive possible form, in the genounits as well as in the genometrics. Note finally that genometrics are generally nondiagonal (Chapter 4).

The *Dirac-Santilli forward and backward hyperequations* are given by the preceding ones when the isounits are assumed as being nonhermitean as well as multivalued (Chapter 5).

6.2.8.O The Meeting between Dirac and Santilli

As a personal note, it may be of some value to recall that, prior to Dirac's death (occurred on October 20, 1984), Santilli had a short meeting with Dirac during a scientific conference in Florida, where Dirac had retired, during which meeting the main elements of this section were discussed. Santilli first approached Dirac by indicating interest in his papers [113,114], at which indication Dirac had one of his rare moments of visible pleasure, perhaps because extremely few physicists had been interested in the same papers.

After qualifying himself as being capable of understanding the papers (Santilli being a former member of the Department of Mathematics of Harvard University under DOE support), Santilli indicate to Dirac the extremely deep mathematical and physical implications of his work, including the surpassing of the mathematics used in the 20-th century physics, as well as the (at that time) potential representation of the synthesis of the neutron inside a star as originally conceived by Rutherford.

Santilli was aware that, in his last years, Dirac had been the victim of abuses by scientific gangsters in an illusory posture of academic power, who opposed and dubbed his late research as being "fringe science." This dubbing originated from Dirac's increasing opposition to quantum field theory due to its divergencies, thus implicitly opposing quark theologies. The highlight of the meeting occurred when Dirac instantly understood, following very few words, that the isotopies eliminate divergencies, at which point Dirac rose from his chair to sit down again and enter into a kind of "scientific trance," being clearly immerse in very deep thinking.[48]

[48]Isounit (6.2.114) verifies the properties

$$|\hat{I}_n| \gg 1, \quad |\hat{T}_n| \ll 1. \tag{a}$$

Consequently, any given *divergent* perturbative series,

$$A(w) = A(0) + w \times [A, H]/! + w^2 \times [[A, H], H]/2! + ... \to \infty,$$

$$[A, H] = A \times H - H \times A, \tag{b}$$

is turned into the strongly convergent series

$$A(w) = A(0) + w \times [A\hat{,}H]/! + w^2 \times [[A\hat{,}H]\hat{,}H]/2! + ... \to N < \infty,$$

Following a minute or so of silence, Dirac asked Santilli: "How do you manage this type of research?" at which question Santilli honestly replied "amidst huge oppositions." In fact, Santilli had just been terminated at Harvard University despite the availability of large DOE funds. In particular, Santilli met Dirac precisely while writing the book *Ethical Probe of Einstein Followers in the USA: An Insider's View* [89] and its three volumes of documentation [90] (which books were indeed publish the month and year of Dirac's death).

After an additional minute of silence, typical of his taciturn character, yet showing a deep mental activity, Dirac told Santilli something to the effect that he would help, and requested papers in the field. On his way back to the *Institute for basic Research* in Cambridge, MA, Santilli did mail to Dirac representative papers on hadronic mechanics via his address at Florida State University in Tallahassee. Unfortunately, Dirac's health deteriorated thereafter due to late age (or perhaps Santilli's papers were never released to him by FSU?), and Santilli never heard from Dirac again.

What a pity! There is no doubt that, had Dirac lived, or had Santilli visited Dirac earlier, the history of hadronic mathematics and mechanics would have been dramatically different because physical research advances on grounds of perceived relevance, and never on sole grounds of scientific content. Hence, had Dirac been able to release one single statement of interest on isomathematics and related topics, the popularity of hadronic mechanics would have been consequentially instantaneous and widespread. [49]

$$[A\hat{,}H] = A \times \hat{T} \times H - H \times \hat{T} \times A, \qquad (b)$$

under the condition (verified for the Dirac-Santilli isoequation) that $|\hat{T}| \ll w$. This is the property instantly understood by Dirac and so evident in any case. Yet, the property has been another reason for opposing, obstructing and jeopardizing the construction of hadronic mechanics by world wide organized interests, with documented prohibition since 1983 (sic) without credible technical arguments to publish papers in the American, British, French. Swedish and other Physical Societies, prohibition to present papers in various international conferences, prohibition by major particle physics laboratories to consider proposals for truly basic experiments, etc.

The political roots of the obstructions are are given by the fact that *hadronic mechanics permits a convergent perturbation theory for strong interactions,* let alone the removal of the divergencies, a feature well known to opposing interests. The point is that such a property would relegate Quantum ChromoDynamics (QCD) to pure mathematical theology, thus wiping out large public funds in the field. As indicated above, scientific gangsters dubbed Dirac's last studies as being "fringe science" because he was trying to remove the divergencies in QCD.

[49]The understanding of the meeting between Santilli and Dirac requires the knowledge that it occurred while Santilli was the victim of the organized scientific crimes perpetrated by Sidney Coleman, Steven Weinberg anmd Sheldon Glashow at Harvard University denounced in Footnote 1 of this volume.

6.2.9 The Etherino Hypothesis

6.2.10 The Etherino Hypothesis on the Neutron Synthesis

6.2.10.A The Missing Energy in the Neutron Synthesis

By no means, the advances presented in the preceding sections resolve all basic problems in the structure of the neutron. In fact, we remain, among others, with the basic problem of identifying the *the origin of the energy* $0.782\ MeV$ *missing in the reaction*

$$p^+ + e^- \ \rightarrow \ (p^+, \hat{e}^-)_{hm} = n, \tag{6.2.189a}$$

$$E_n - (E_p + E_e) = 939.565 - (938.272 + 0.511)\,\text{MeV} = 0.782\ \text{MeV}. \tag{6.2.189b}$$

with the understanding, as indicated earlier, that the reaction

$$p^+ + \bar{\nu} + e^- \ \rightarrow \ (p^+, \hat{e}^-)_{hm} = n, \tag{6.2.190}$$

has no scientific sense because the missing energy is it positive, while $\bar{\nu}$ carries a *negative* energy, being an antiparticle. Additionally, the cross section between antineutrinos and protons or positron is essentially null. Hence, in the event predictable manipulations may turn the energy of an antiparticle into a positive value (something quite possible in a field in which science is conducted via abuses of academic power rather than admission of scientific veritas), said energy cannot possibly be delivered to the proton and/or to the electron.

Note that

6.2.10.B Possible Origins of the Missing Energy in the Neutron Synthesis

The above basic question was identified and studied by Santilli in paper [97], resulting in the following possible alternatives:

HYPOTHESIS 6.2.9.I: The $0.782\ MeV$ *missing in the synthesis of the neutron originate from its environment, such as that in the interior of stars.*

Despite its seemingly plausible and rational character, the above hypothesis still remains with basic unsolved aspects. In fact, at the extreme pressures in the interior of stars, *the proton and the electron are essentially at rest at the time of the neutron synthesis.* Hence, Hypothesis I still remains with the unidentified mechanism of transferring the missing energy from the environment to the proton and/or the electron. Vague nomenclatures, such as "via the temperature," are indeed acceptable as academic parlance, but they are not adequate for the quantitative objectives of these volumes.

Independently from the above, the probability of the synthesis of the neutron is essentially null when the proton and the electron have the (relative) missing

energy of 0.782 MeV because, as indicated in Section 6.2.3, in that case their cross section becomes very small. This occurrence increases the difficulties for the transfer of energy from the environment to the proton or the electron and should not be surprising to serious scholars because it is written in history that basically new problems require basically new vistas.

In addition to the above,Hypothesis I is simply disconcerting for the author because it implies the *conception of stars as astrophysical bodies with internal mechanisms decreasing the energy in time,* while stars are one of the most majestic sources of energy in the universe. To see the differences between orthodox thinking and physical reality, physicists are suggested to multiply 0.782 $!MeV$ by the number of neutrons synthesized in a star every second, resulting in *a temperature loss* (in the sense that the heat energy is no longer usable because transferred to the neutrons) of the order of

$$E_{loss}^{star}/sec = 0.782x10^{25} MeV/sec. \tag{6.2.191}$$

Physicists should then verify that nuclear syntheses do overcome the above loss in such a way to result in a positive energy output. The above occurrences led Santilli to formulate the following alternative [97]

HYPOTHESIS 6.2.9.II: The 0.782 MeV *missing in the synthesis of the neutron originate from the ether (aether) conceived as a universal substratum for all visible events in the universe with a very high energy density.*

Needless to say, the latter hypothesis creates *more* problems than the first, as typically the case for basic advance. Yet, the serious study of unsolved basic issues requires serious scientific democracy, that is, the equal treatment of *all* possibilities, and then the selection of the correct one, after exhausting all avenues. Our interest here is merely that of "initiating" studies on the latter hypothesis, with the understanding (indicated in Section 6.1.3) that, due to their dimensions and potential outcome, the study of the ether may well require the entire third millennium.

To begin, Hypothesis II was formulated by Santilli [97] to initiate quantitative studies of the old hypothesis of *continuous creation of matter in the universe,* that has been voiced repeatedly during the 20-th century. Hence, paper [97] pointed out, apparently for the first time, that *the best possible mechanism for continuous creation in the universe is precisely the synthesis of neutrons inside stars,* via the assumption that the missing energy originates from the ether conceived as a universal medium with an extremely large energy density.

Rather than being farfetched, the hypothesis is supported by predictably insufficient, yet significant evidence, such as the fact that stars initiate their lives as being solely composed of hydrogen atoms that miss the energy needed for the

first and most fundamental nuclear synthesis, that of the neutron, after which all conventional nuclear syntheses follow.

Additionally, explicit calculations indicate that the immense energy needed for a supernova explosion, that are visible by the naked eye on Earth from very distant galaxies, simply cannot be explained via the sole use of conventional nuclear syntheses, particularly in view of the fact that supernova explosions occur at the *end of the life of stars.* This suggests again the possible existence of a mechanism extracting energy from the ether and transferring it into our spacetime.[50]

To understand Hypothesis II, one should recall from Section 6.1.2, Ref. [1], that the notion of ether as a universal substratum appears as being necessary not only for the characterization and propagation of electromagnetic waves, but also for the characterization and propagation of all elementary particles and, therefore, for the very existence of all matter in the universe.

The need of a universal medium for the characterization and propagation of electromagnetic "waves" is so strong to require no study here, e.g., for waves with 1-m wavelength for which the reduction of waves to photons (for the evident hope of eliminating the ether as a medium to preserve Einsteinian theories) loses credibility.

The same notion of ether appears necessary also for the characterization and propagation of the electron, due to its structure as a "pure oscillation," namely, an oscillation of one of the points of space in which there is no oscillation of a "little mass" as conventionally understood. Similar structures are expected for all other particles.

Once matter is *entirely* reduced to oscillations of a universal substratum [1], the transfer of energy from the substratum to our spacetime via the neutron synthesis and other events, become quite possible indeed.

It should be also recalled from Section 6.1.2. [1] that the above conception implies that, contrary to our sensory perception, *matter is totally empty, and space is totally full by a medium,* the former being mere excitations of the latter. This conception was submitted in paper [1] to illustrate the lack of existence of the "ethereal wind" [2] that delayed studies on the ether for at least one century.

In fact, under the above conception, motion of matter would merely require the transfer of the characteristic oscillations from given points of the ether to others. Mass is then characterized by the known equivalence of the energy of the characteristic oscillations, and inertia is the resistance provided by the ether against changes of motion [1].

[50]The "explanation" of supernova explosions via gravitational collapse is more controversial than the nuclear one due to known catastrophic inconsistencies of gravitational theories on a curved space studied in Section 1.4 (see paper [13]). Prior to venturing credible judgments on the structure of the universe via Einstein's gravitation, its catastrophic inconsistencies must be resolved first, not in equivocal academic corridors or via the usual silence, but via papers published in refereed journals.

6.2.10.C The Etherino Hypothesis

In order to conduct quantitative studies on the origin of the missing energy, Santilli [97] assumed that the synthesis of the neutron from protons and electrons occurs via the *absorption,* either from the environment inside stars or from the ether, of an "entity", called *etherino* (meaning in Italian "little ether") and represented with the symbol "a" (from the Latin aether), having mass 0, a minimum of 0.782 MeV energy, plus other possible features in the event necessary (see below). By unifying Hypotheses I and II, we reach in this way the following:

Etherino hypothesis on the neutron synthesis:

$$p^+ + a_n + e^- \to n, \tag{5.2.192}$$

where a_n denotes the *neutron etherino* (see below for other cases), and the energy 0.782 MeV is assumed as being "minimal" because of the presence of conventional "negative" binding energy due to the attractive Coulomb interactions between the proton and the electron at short distances, and other reasons.

The energy carried by the etherino is also assumed as being minimal in the event the neutrino exists as a physical particle, thus requiring the identification of the origin of its own energy. In fact, as now well known, the value 0.782 MeV is the minimal energy for the sole synthesis of the neutron.

It should be stressed that, in order to prevent the invention of additional hypothetical particles over an already excessive number of directly undetectable particles existing in contemporary physics, **the etherino is not a particle, but a mere mathematical symbol used to represent the transfer of the missing energy (and possibly other features) from the environment or the ether to the neutron.** The lack of characterization as a conventional physical particle will be made mathematically clear below.

Note that Hypothesis (6.2.191) was submitted [97] in lieu of (6.2.190) as a credible way to turn the negative energy of the antineutrino into the needed positive form, as well as as an attempt to resolve the excessive inconsistencies or insufficiencies of the neutrino hypothesis.

The synthesis of the antineutron in the interior of antimatter stars is evidently given by

$$p^- + \bar{a}_{\bar{n}} + e^+ \to \bar{n}. \tag{5.2.193}$$

where $\bar{a}_{\bar{n}}$ is the *antineutron antietherino,* namely an entity carrying negative energy as necessary for antimatter (Volume I). This would imply that the ether is constituted by a superposition of very large but equal densities of positive and negative energies existing in different yet coexisting spacetimes, a concept permitted by the isodual representation of antimatter with deep cosmological and epistemological implications since their total null value would avoid discontinuities at creation.

For the synthesis of the neutral pion we have the hypothesis

$$e^+ + a_{\pi^o} + e^- \to \pi^o, \tag{5.2.194}$$

where a_{π^o} is the π^o-*etherino,* namely, an entity carrying mass, charge and spin 0 and minimal energy of 133.95 MeV transferred from the ether to our spacetime. Numerous similar additional forms of etherinos can be formulated depending on the hadron synthesis at hand.

6.2.10.D Representation of the Etherino via Hadronic Mechanics

It is evident that *hadronic mechanics allows a quantitative representation of the etherino hypothesis and, more specifically, of the possible exchanges of energy between matter and the ether.* In fact, the transfer of 0.782 MeV energy to the neutron is represented via: the isotopic lifting of the unit and Hilbert spaces

$$I > 0 \ \to \ \hat{I} = 1/\hat{T} > 0 \tag{6.2.195a}$$

$$< \psi| \times |\psi > \times I \ \to < \hat{\psi}| \times \hat{T} \times |\hat{\psi} > \times \hat{I}; \tag{6.2.195b}$$

the consequential isorenormalization of the rest energy and angular moments (see the preceding sections)

$$E_e = m \times c_o^2 \ \to E_{\hat{e}} = m_e \times c_o^2 \times \frac{b_4^2}{b_3^2}; \tag{6.2.196a}$$

$$S \ \to \ \hat{S}, \ \ L \ \to \ \hat{L} \tag{6.2.196b}$$

and other isorenormalization processes.

The above representation also illustrates the purely mathematical character of the etherino as being a mere symbol to represent the *transfer* of a physical quantity to the neutron synthesis.

Once the missing energy has been transferred to the neutron constituents, evidently, it remains with the latter. and this illustrates the mechanism here considered of the continuous creation of matter in the universe.

6.2.11 Neutron Decay: Possible New Longitudinal Communications?

6.2.11.A Poincaré vs Poincaré-Santilli Symmetries

The most important implication of hadronic mechanics in the neutron synthesis (evident from the preceding two sections) is the lack of *necessary* spin 1/2 to be carried by the etherino, namely, only 0.782 MeV are needed for synthesis, since the neutron spin 1/2 is a consequence of constraining the electron to orbit within the proton with an angular momentum equal to its spin. This results in a null value of the total angular momentum of the mutated electron (isoelectron), and

the spin of the neutron coincides with that of the proton (Figure 6.23). A deeper understanding of this mechanism is now important for an initial study of the neutron decay.

At it is well known to experts, *Einstein special relativity prohibits the above representation of the neutron spin because it would require the breaking of its central pillar, the Poincaré symmetry.* Recall that the Poincaré symmetry was conceived for *Keplerian systems,* typically represented by our Solar systems, consisting of *a finite number of point-like, massive constituents without collisions in individually stable orbits around a heavier constituent, the Keplerian nucleus.*[51]

A crucial consequence is that represented via the familiar ten conservation laws of total quantities,

$$\frac{dX_i(t,r,p)}{dt} = \frac{\partial X_i}{\partial b^\mu} \times \frac{db^\mu}{dt} + \frac{\partial X_i}{\partial t} = 0, \tag{6.2.197}$$

where

$$X_1 = E_{tot} = H = T + V, \tag{6.2.198a}$$

$$(X_2, X_3, X_4) = \mathbf{P}_{Tot} = \Sigma_a \mathbf{p}_a, \tag{6/2/198b}$$

$$(X_5, X_6, X_7) = \mathbf{J}_{tot} = \Sigma_a \mathbf{r_a} \wedge \mathbf{p}_a, \tag{6.2.198c}$$

$$(X_8, X_9, X_{10}) = \mathbf{G}_{Tot} = \Sigma_a (m_a \times \mathbf{r}_a - t \times \mathbf{p}_a), \tag{6.2.198d}$$

$$i = 1, 2, 3, ..., 10; \quad k = 1, 2, 3; \quad a = 1, 2, 3, ..., N.$$

from which we have the necessary conservation, individually and separately, of the linear and angular momentum. Possible internal exchanges of linear momentum and angular momentum are prevented by the lack of collision.

There is no doubt that, as above conceived, the Poincaré symmetry is indeed exact for the above identified systems. However, the belief that Einstein's special relativity and its underlying Poincaré symmetry apply to all possible systems is sheer scientific corruption, particularly when proffered by experts with uncontrollable fanatic fervor for nonscientific aims.

In fact, the Poincaré symmetry is *inapplicable* for the neutron synthesis (rather than violated because not conceived for that) because:

1) The keplerian constituents must admit a point-like abstraction under which the neutron synthesis is impossible, e.g., because the electron would simply go through the proton without bonding;

2) The keplerian constituents must admit no collision, under which additional condition the neutron synthesis is also impossible;

[51]The lack of collision for the applicability of the Poincaré symmetry is carefully avoided in textbooks and Ph. D. courses because its admission, alone, would flair up the understanding of its limitations, with consequential unwanted search for suitable generalizations of Einsteinian doctrines.

3) The system must be time reversal invariant, namely, the time reversal event must be causal (as it is indeed the case for a Keplerian system), under which conditions the neutron synthesis is impossible because structurally irreversible and would violate the energy conservation in any case.

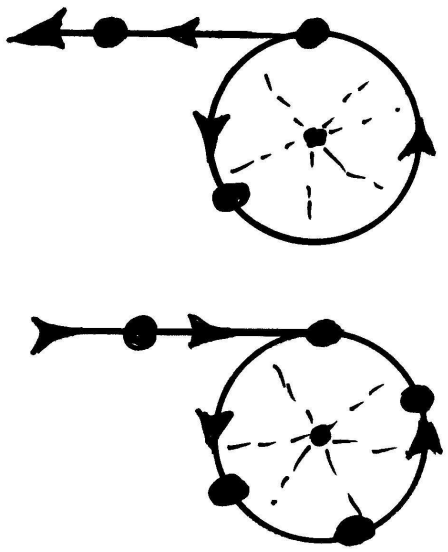

Figure 6.25. A schematic illustration of the inapplicability of Einstein's special relativity and its fundamental Poincaré symmetry for systems with constraints. The "sling shot" of this figure illustrates physical events impossible for special relativity and its Poincaré symmetry, but existing in the physical reality, such as the conversion of angular into linear momentum in the top view and the conjugate case of conversion of linear momentum into angular momentum in the lower view. When the physical reality of the neutron structure is admitted as being a constrained system, there is no need for the neutrino conjecture for both the neutron synthesis as well as for its decay. The understanding is that models without and with neutrinos do not necessarily exclude each other, the only scientific exclusion being that via unbiased experiments. The picture also illustrates the reason the author considers current "experimental results" in deep inelastic scattering as being "experimental beliefs." In fact, said results are claimed via the use of the conventional potential scattering theory under the notorious conditions of verifying Einsteinian doctrines and the Poincaré symmetry, thus (tacitly) excluding constrained conditions and exchanges of the type illustrated in this figure.

By comparison, the Poincaré-santilli isosymmetry:

1') Represents the constituents as extended, nonspherical and deformable;

2') Admits collisions between the constituents at mutual distances equal or smaller than their size; and

3') Can be extended to an irreversible formulation via the lifting of the isounit into a non-Hermitean form.

Additionally, the isosymmetry readily admits constraints on the conversion of linear into angular momentum and vice-versa (see Figure 6.25), such as the trivial constraint

$$P_k \equiv C \times \epsilon_{kij} r_i \ \wedge \ p_j. \tag{6.2.6.2.199a}$$

$$E_p \equiv E_L, \tag{6.2.199b}$$

where C is a dimensional constant that can be derived from the underlying basic conservation, that of the energy. More complex forms of constraints are left to interested colleagues.

At the limit, *the sole quantity with a certain conservation is the energy,* since all other quantities may admit one or another form of conversions among themselves and into energy. In fact, the kinetic energy carried by the linear momentum is the primitive physical quantity with ultimate conservation, since the linear momentum can transform itself into angular momentum and vice versa.

6.2.11.B Alternatives in Neutron Decay

Without doubt, *the spontaneous decay of the neutron constitutes strong evidence that the proton and the electron are its physical constituents, merely emitted via hadronic tunneling,* and we shall write

$$(p^+, \hat{e}^-)_{hm} \ \rightarrow \ (p^+ + e^- + ?)_{qm}, \tag{6.2.200}$$

where the question mark indicates the open issue of reconciling the l.h.s. treated with hadronic mechanics (hm), and the r.h.s, treated with conventional quantum mechanics (qm), the emitted particles being in vacuum.

It is also evident that the etherino hypothesis requires a reinspection of such a spontaneous decay. To conduct a true scientific analysis, rather than adopt a scientific religion, it is necessary to identify all plausible alternatives, and then reach a final selection via experiments. We reach in this way the following *three* possible alternatives [97]:

1. Neutron decay without etherino and antineutrino:

$$n = (p^+, \hat{e}^-)_{hm} = (p^+ + a_n + e^-)_{hm} \rightarrow (p^+ + e^-)_{qm}, \tag{6.2.201}$$

1. 2. Neutron decay with the emission of the etherino:

$$n = (p^+, \hat{e}^-)_{hm} = (p^+ + a_n + e^-)_{hm} \rightarrow (p^+ + e^- + a_n)_{qm}; \tag{6.2.202}$$

3. Neutron decay with the emission of the antineutrino:

$$n = (p^+, \hat{e}^-)_{hm} = (p^+ + a_n + e^-)_{hm} \rightarrow (p^+ + e^- + \bar{\nu})_{qm}. \tag{6.2.203}$$

Case 1 is fully allowed by hadronic mechanics via the transformation of the constrained angular momentum of the isoelectron into the linear momentum of the electron in vacuum as per Figure (6.2.25). This case supports the continuous creation of matter because, after having been transferred from the ether to the neutron, the originally missing energy of 0.782 MeV remain in our spacetime and are carried by the emitted particles..

Case 2 essentially implies that some or all of the originally missing energy of 0.782 MeV is returned to the ether as a universal medium. Note that this case does not necessarily imply the denial of of the continuous creation because, following its synthesis and acquisition of 0.782 MeV, the resulting neutrons generally belongs to stable nuclei.

Case 3 is very controversial and merely quoted here for completeness because the antineutrino is expected to carry *negative* energy, thus creating a number of fundamental open issues. Of course, believers in neutrinos could interchange them with antineutrinos "to fix things," but this would create a host of additional problems in the standard model.

Case 3 is primarily listed here to indicate that *the lack of existence of the neutrino for the neutron synthesis, by no means, implies that the neutrino does not exist for the neutron decay.* In different words, the neutron synthesis and its spontaneous decay are two basically different problems requiring independent treatment and. of course, separate experimental resolutions.

Needless to say, a selection between alternatives 1, 2, 3, is impossible on theoretical grounds alone, and can only be seriously achieved via experiments, such as those "requested" in the last appendix of this chapter. The problem is that the most fundamental and important a given experiment is, the bigger the organized interests against its consideration, let alone conduction.

6.2.11.C New Longitudinal Communications triggered by the Neutron?

As indicated in Section 6.1.3, when considered at interstellar distances, our current communications via electromagnetic waves can only be compared to prehistorical communications via smoke signals, due to the fact that the speed of light becomes excessively small for interstellar distances. Clearly, interstellar science will initiate the day in which quantitative research is initiated on possible new forms of communications admitting speeds millions of times bigger than that of light in vacuum. Clearly, such a scientific process can only initiate under the condition that it is beyond Einsteinian doctrines. Clearly, studies of this nature are expected to require centuries of trial and errors.

Once the problem is structured in the appropriate nonpolitical venue, systematic studies may reveal a varieties of possibilities, some of which may be already

under study experimentally, such as correlated spin effects, matter transmission, and others.

The possibility here indicated for the young mind of any age is that physical media of high rigidity, as the ether is expected to be, should indeed admit (at least) two forms of communications, the transversal ones already in use in electromagnetic communications (in which the oscillations are perpendicular to the direction of propagation), and a new, hitherto unknown communication of longitudinal character (in which the oscillations are along the direction of propagation).

Besides its intrinsic interest, the search for new communications is suggested by the possibility that *current experimental claims on "neutrino detection" are indeed real, and only in need for a more adequate interpretation.* Alternatively, we must stress that *the lack of existence of neutrinos does not necessarily invalidate available experimental data on neutrino experiments.*

The most fundamental synthesis in nature, that of the neutron, emerges again as fundamental for the above issues. In the event the missing energy of $0.782\ MeV$ in the neutron synthesis does indeed originate from the ether, its transfer to the neutron should create a form of impulse in the ether itself and its propagation cannot possibly be transversal, thus leaving as sole possibility its longitudinal form. Speeds millions of times bigger than the speed of light in vacuum are then consequential.

Consequently, *it is possible that current experiments on "neutrino detection," rather than detecting the emission of the imaginary neutrino in our spacetime,*

$$p^+e^- \to n + \nu, \tag{6.2.204}$$

detect instead a longitudinal impulse propagating through the ether, herein denoted ℓ. We reach in this way the following

Hypothesis of longitudinal impulses via the neutron synthesis:

$$p^+e^- \to n + \ell. \tag{6.2.205}$$

besides potential contributions beyond our imagination at this writing, the latter alternative would render more plausible the claims of current neutrino experiments. In fact, they are currently based on the theological belief that massive particles, such as the neutrinos in their current conception, could traverse entire stars and galaxies without any collision, a belief clearly beyond any rational basis.

the traversing of entire stars and galaxies without collision is instead fully plausible for alternative (6.2.205) since, in the latter case alone, no massive entity propagates at all, the propagation being related to a longitudinal impulse through space.

In short, interstellar communications need a new Guglielmo Marconi capable of conceiving longitudinal or other forms of very fast communications, as well as cap;able of producing them and then detecting them at a distance.

6.2.12 Laboratory Synthesis of the Neutron from Protons and Electrons

6.2.12.A The Dominant Factor in the Neutron Synthesis: Ethical Decay in Physics

Addressing the problem of the laboratory synthesis of the neutron without the joint addressing of issues pertaining to scientific ethics and accountability, would be so hypocritical to be repugnant to the author.

The synthesis of the neutron is, by far, the most fundamental synthesis in nature because the synthesis of all matter beyond the hydrogen depends on the prior synthesis of the neutron. Additionally, the neutron is an inextinguishable reservoir of energy, since it is naturally unstable (when isolated and member of certain nuclei) and decays into the proton plus a highly energetic electron that can be easily trapped with a thin metal shield.

Hence, we have a societal duty to study the possible synthesis of the neutron (done in this section) and its stimulated decay (done in the next section). Moreover, the neutron synthesis and decay offer serious possibilities to reach in due time new forms of energies via the use of *light* (rather than heavy) nuclei, thus having a realistic chance of decreasing harmful radiations and waste, whose study is mandated by scientific ethics and accountability.

Despite these transparently relevant aspects, well known to experts to qualify as such, it has been impossible for the author in about thirty years of failed attempts to locate *any* laboratory *anywhere* on Earth willing to "consider," let alone conduct, laboratory tests on the synthesis of the neutron as it occurs in stars, from protons and electrons.

At this point it is necessary to recall that experiments have their own standard of value: the more fundamental is a test, the bigger is the priority for its conduction over manifestly lesser relevant tests. Hence, the systematic refusal of laboratories in the U.S.A., England, France, Germany, Russia, Sweden, Norway, China, Japan, and other countries to conduct tests on the synthesis of the neutron seals the existence of serious problems of scientific ethics and accountability in the contemporary physics community that must be addressed by anybody with a minimum care for society.

The reason for the impossibility of even "considering" the laboratory synthesis of the neutron is that such a synthesis is impossible for Einsteinian doctrines and quantum mechanics, as reviewed in the preceding sections. Hence, the mere "consideration" of the experiment would imply laboratory directors being the target of organized scientific crime on Einsteinian doctrines, resulting in academic

and family disruptions similar to those suffered by the author for three decades (see Footnote 1 of this Volume II and the other footnotes).

The first tests on the laboratory synthesis of the neutron was done in the 1960s by a collaboration headed by Don Carlo Borghi, an Italian priest-physicist of the University of Milano, and it is today known as the *Don Borghi experiment* [116,117]. The experiment was so opposed, disrupted and jeopardized by the organized scientific crime of the time in Italy that Don Borghi was forced to leave Italy and conduct the test in Brazil.

Following the release of the claim of having successfully achieved the laboratory synthesis of the neutron, the attacks on Don Borghi's experiments in Italy and elsewhere increased in time, thus showing their true character as the outcome of organized scientific crime. In fact, all dismissals and dubbings have been based on purely theoretical considerations without first repeating the test. Finally, said character is sealed by the fact that Don Borghi experiment is quite simple and inexpensive to repeat as shown in this section, thus voiding the objections of any credibility other than the opposition to undesired human knowledge.

Due to the failure by the world wide physics community to repeat Don Borghi's experiments for over thirty years, Santilli had no other choice that repeat the experiment himself [118,119], as reported below in this section. In so doing, Santilli became the victim of additional attacks, such as that of for promoting "fringe science" (Wikipedia) and similar dubbings, again, without the prior repetition and dismissal of the new measurements.

It should be stressed that, in the event the above organized opposition was indeed convinced that the neutron synthesis is impossible in laboratory, Don Borghi and Santilli should have been *encouraged,* rather than opposed, to conduct the test since they would have been negative. In any case, it is set in history that any experiment, even when negative, is beneficial to human knowledge. Hence, evidence accumulated over some thirty years identifies clearly and incontrovertibly the existence of a world wide organized scientific crime intent in suppressing undesired human knowledge, thus perpetrating a crime against mankind.

Any person with a minimum of dignity and care for human knowledge should admit that a condition of the above type cannot continue indefinitely without inflicting severe damages to society. Somebody, somehow, somewhere has to initiate the containment of the ethical decay. For this reason, Santilli has reached the following decisions:

1) Publicly denounce for scientific corruption any scientists who criticizes experimental *measurements* on pure grounds of *theoretical* conjectures. Needless to say, experiments must indeed be subjected to theoretical elaborations and discussions. However, the use of theoretical beliefs for the dismissal of new measurements just because they are not aligned with Einsteinian doctrines is a crime against society that, if we really care, must be openly denounced as such.

Figure 6.26. A view of the Italian priest-physicist Don Carlo Borghi, originally of the University of Milano, who dedicated his research life to theoretical and experimental studies on Rutherford's legacy, namely, to the study of the neutron as a bound state of a proton and an electron. It appears that Don Borghi had indeed a fulfilling religious life, but his studies on the neutron were opposed by the organized scientific crime in Italy of the 1960s to such an extent, he had to leave Italy and conduct experiment [116,117] in Brazil. The theoretical papers by Don Borghi and his associates on the neutron structure lack the advanced mathematical knowledge needed for a consistent quantitative study of the problem. As such, these theoretical studies are inconclusive and left for inspection to interested historians. However, no matter what its final interpretation will be, Don Borghi's test [116,117] deserves indeed the historical priority of being the first experiment on the laboratory synthesis of neutrons from protons and electrons.

2) Publicly denounce as scientific corruption all rejections for the publication of *experimental* papers on sole *theoretical* machinations, because measurements can only be scientifically dismissed via counter-measurements.

3) Promote legal proceedings and/or class actions for lack of scientific ethics, absence of accountability, misuse of public funds, and other changes, against

laboratories operating under public financial support in which basic experiments on open fundamental issues, such as those on the inapplicability of Einsteinian doctrines, are dismissed in favor of other experiments generally of dramatically bigger cost, yet of comparatively insignificant (at beast) scientific relevance.

Following thirty years of documented experiences, the author's view is that the grip of the organized scientific crime on Einsteinian doctrines at laboratories such as SLAC, FERMILAB, CERN, SACLAY, RUTHERFORD, DESY, JINR and others is so strong, capillary and diversified that there is no hope for their participation in truly basic experiments until said laboratories are brought to court and the organized scientific crime currently in their control is unmasked. The author sincerely prays to be mistaken, but that requires serious basic experiments under serious ethical scrutiny.[52]

6.2.12.B Don Borghi's Experiment on the Neutron Synthesis

The first experiment on the synthesis of neutrons from protons and electrons was conducted by Don Carlo Borghi and his associates C. Giori and A. Dall'Olio in the 1960s at the CEN Laboratories in Recife, Brazil [116,117].[53]

Needless to say, Don Borghi's experiment is in need of independent reruns, either in its original form, or in one of several alternatives discussed in the next subsection. Nevertheless, Don Borghi's experiment constitutes the first historical test on Rutherford's conception of the neutron, and it is remarkable, not only because of the claimed results, but also because of its simplicity and low cost.

In essence, the experimentalists placed in the interior of a cylindrical metal chamber (called *klystron*) a hydrogen gas (at a fraction of 1 *bar* pressure) originated from the electrolytical separation of water, kept mostly ionized by an electric arc (with about 500 V and 10 mA) and traversed by microwaves (with 10^{10} s^{-1} frequency). Since protons and electrons are charged, they could not escape from the metal chamber, and remained trapped in its interior.

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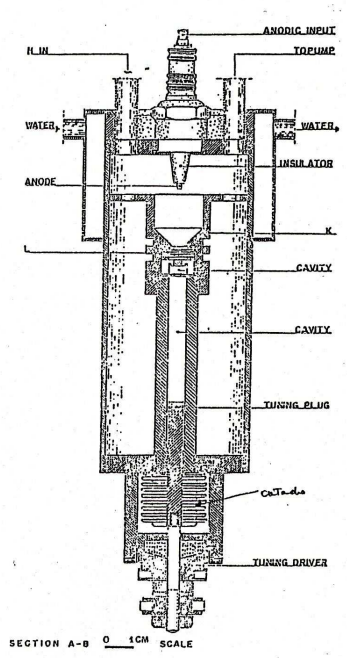
[52]The reader in good faith should be aware that, according to intelligence gathered by an investigative firm hired by the International Committee on Scientific Ethics and Accountability (www.scientificethics.org), the organized scientific crime currently controlling physics laboratories is planning indeed the repetition of the tests [116-1179], but for the premeditated intent of dismissing the tests via m,anipulations of the type illustrated in Appendix 6A, and certainly not for scientific knowledge. This is the reason for the need of continuing the conduction of the tests outside said crime and irrespective of their claims. In the final analysis, scientific truth always emerges and the bigger is the opposition against tests [116-119], the bigger will be the condemnation by posterity. **It should be final;ly noted that, at this writing (January 19, 2008), Santilli is coordinating the repetition of tests [116-119] at a number of laboratories thanks to all necessary funds provided by the industrial applications of hadronic mechanics, although Santilli is prohibiting their disclosure at this time to prevent certain disruption by the organized scientific crime.**

[53]The submission for publication [117] was done following Don Borghi's death by Santilli (without his name appearing in the paper), and publication occurred thanks to the support by N. Takybayev from Kazakhstan that is here acknowledged with most sincere gratitude.

In the cylindrical exterior of the chamber the experimentalists placed various materials suitable to be activated when exposed to a neutron flux (such as gold, silver and other substances). Following exposures of the order of weeks, the experimentalists reported nuclear transmutations due to a claimed neutron count (of up to 10^4 *cps*), apparently confirmed by beta emissions (evidently not present in the original material). For the next subsection, note that experiment [116,117] makes no claim of direct detection of neutrons, and only claims the detection of clear nuclear transmutations.

It is evident that, if confirmed, Don Borghi's experiment establishes Rutherford's conception of the neutron, as well as its treatment via hadronic mechanics, since the latter is the only mechanics permitting an exact and invariant representation of all characteristics of the neutron as a bound state of a proton and an electron.



Element -	Net mass (gr)	Date -	Max net Activity (c/10^m)
Dy (oxide)	2,580	13/02/70 06/08/70	83 107
Nb (oxide)	2,128	26/02/70 04/08/70	61 47
Pr (oxide)	5,361	26/01/70 27/07/70	69 118
Sb (oxide)	2,688	18/02/70 24/07/70	51 97
In (metallic)	0,135	21/01/70	54
Ag (metallic)	2,130	12/02/70 16/06/70	51 17
Au (metallic)	0,960	23/02/70 10/07/70	10 27
Tl (metallic)	10,260	14/02/70	41

Figure 6.27. A view of the klystron used in tests [116,117] and one of the tables of results showing nuclear transmutations that can only occur via exposure to entities originating from the interior of the klystron, which entities have to be necessarily neutral and particle size (to pass through the metal wall of the klystron) and have strong interactions (to be absorbed by nuclei).

6.2.12.C Santilli's Experiment "toward" the Neutron Synthesis.

We now report test [118-119] conducted by R. M. Santilli on the synthesis of the neutron from protons and electrons, which tests were initiated in 2006 and completed with systematic repetition and reruns in 2007. As we shall see, *Santilli experiment [1118-119] confirms Don Borghi's experiment [116-117], although in a rather suprising way identified below.* Needless to say, Santilli makes no claim of final confirmation. As a matter of fact, experimental studies in the synthesis of the neutron are at their beginning and so much remains to be done. Also, in view of this initial character, any expectation of full maturity or criticism for insufficiencies of measurements [118-119] without their rerun, shall de denounced as scientific corruption since the fields needs measurements rather then theologies.

A serious study of Dob Borghi's experiment requires: 1) The identification of the process creating the neutral, strongly interacting entities; 2) The identification of the entities themselves; and 3) The identification of the nuclear reactions they induce.

As reviewed above, tests [116-117] had two different processes in the interior of the klystron, the electric arc and the microwave. According to the original presentations [116], it *appears* that the experimentalists expected the neutral entities to originate from resonating effects caused by microwaves, and assumed the electric arc as essentially having the role of maintaining the hydrogen gas partially ionized.

Santilli conceived experiment [118] as being solely based on the use of an electric arc within a cold (i.e., at atmospheric temperature) hydrogen gas without any use of microwave at all, for the specific purpose of initiating systematic studies on the mechanism creating the neutral entities. The expectation was that, in the absence of any detection of neutral entities via the sole use of the electric arc, the addition of high frequency microwaves was expected to be necessary.

A main result of experiment [118-119] is that *the sole use of electric arcs within a hydrogen gas produces entities causing nuclear transmutations that confirm Don Borghi's experiment* [116-117]. The use of high frequency microwaves has not been studied during these measurements [118,119]. Ref. [118] contains a summary of the measurements, while Ref. [119] contains various details, including copies of the print outs of some of the numerous scans.

All tests here reported were conducted at the laboratory of the Institute for Basic Research (IBR) in Palm Harbor, Florida, with the participation of the IBR technicians Terry Allen, John T. Judy, Michael Rodriguez, Jim Alban and Ray Jones, whose professional assistance has been invaluable for the conduction of tests [118,119] as well as for their detailed rtecord and documentation.

Radiation counts were done via:

1) A photon-neutron detector model $PM1703GN$ manufactured by Polimaster, Inc., with sonic and vibration alarms as well as memory for printouts, with the photon channel activated by CsI and the neutron channel activated by LiI;

2) A photon-neutron detector SAM 935 manufactured by Berkeley Nucleonics, Inc., with the photon channel activated by NaI and the neutron channel activated by $He-3$ also equipped with sonic alarm and memory for printouts of all counts;

3) A BF^3 activated neutron detector model 12-4 manufactured by Ludlum Measurements, Inc., without counts memory for printouts;

4) An alpha, beta, gamma and X-ray detector model 907-palmRAD manufactured by Berkeley Nucleonics, Inc.; and

5) Samples of commercially available materials suitable for activation.

Electric arcs were powered by welders manufactured by Miller Electric, Inc., including a Syncrowave 300, a Dynasty 200, and a Dynasty 700 capable of delivering an arc in DC or AC mode, the latter having frequencies variable from 20 to 400 Hz.

The following three different klystrons were manufactured, tested and used for the measurements (see [119] for pictures):

Klystron I: A sealed cylindrical klystron of about 6" outside diameter (OD) and 12" height made of commercially available, transparent, PolyVinyl Chloride (PVC) housing along its symmetry axis a pair of tungsten electrodes of 0.250" OD and 1" length fastened to the tip of 0.250" OD conducting rods protruding through seals out of the top and bottom of the klystron for electrical connections. The electrodes gap was controllable by sliding the top conducting rod through the seal of the flange. The PVC was selected to be transparent so as to allow a visual detection of the arc.

Klystron II: A rectangular, transparent, PVC klystron 3" $\times$ 3" $\times$ 6" filled up with commercial grade hydrogen at atmospheric pressure and temperature traversed by a 2" long electric arc powered by a standard Whimshurst electrostatic generator.

Klystron III: A cylindrical metal klystron fabricated in schedule 80 carbon steel pipe with 12" OD, 0.5" wall thickness, 24" length and 3" thick end flanges capable of withstanding hydrogen pressure up to 500 psi with the internal arc between thoriated tungsten electrodes controlled by outside mechanisms.

A first series of measurements were initiated with Klystron I on July 28, 2006, at 2 p.m. Following flushing of air, the klystron was filled up with commercial grade hydrogen at 25 psi pressure. We first used detector $PM1703GN$ to verify that the background radiations was solely consisting of photon counts of $5-7$ $\mu R/h$ without any neutron count; we delivered a DC electric arc at 27 V and 30 A (namely with power much bigger than that of the arc used in Don

Figure 6.28. A picture of Klystron I and of detector SAM 935. Note the similarity with the klystron used in tests [116,117], but also the primary differences being given by the use of metal walls for tests [116,117], and transparent polycarbonate walls for tests [118,119]. The latter were selected so as to permit the visual identification of the creation of an interior electric arc with a gap between the electrodes, the latter being necessary to assure the creation of a discharge through the hydrogen gas.

Borghi's tests [116,117]), at about 0.125" gap for about 3 s; we waited for one hour until the electrodes had cooled down; and then placed detector $PM1703GN$ against the PVC cylinder. This resulted in the detection of photons at the rate of $10-15$ $\mu R/h$ expected from the residual excitation of the tips of the electrodes, but no neutron count at all.

However, about three hours following the test, detector $PM1703GN$ entered into sonic and vibration alarms, specifically, for neutron detections off the instrument maximum of 99 *cps* at about $5'$ distance from the klystron while no anomalous photon emission was measured. The detector was moved outside the laboratory and the neutron counts returned to zero. The detector was then returned to the laboratory and we were surprised to see it entering again into sonic and vibrational alarms at about $5'$ away from the arc chamber with the neutron count off scale without appreciable detection of photons, at which point the laboratory was evacuated for safety. After waiting for 30 m (double neutron's lifetime), we were surprised to see detector $PM1703GN$ go off scale again in neutron counts at a distance of $10'$ from the experimental set up, and the laboratory was closed for the day.

Inspection of the laboratory the following morning indicated no neutron detection in the general area, but detector $PM1703GN$ showed clear neutron counts when placed next to the PVC wall. The same detections persisted for two subsequent days until the hydrogen was flushed out of the chamber.

The test was repeated the afternoon of August 4, 2006, with the welder operating in AC mode at 30 V and 30 A plus a transformer that allowed to deliver an arc with 700 V and 1.2 A for 5 s with a gap of about 0.375". We waited again until the incandescence of the tips was extinguished and placed detector $PM1703GN$ near the cylindrical PVC wall, resulting in sonic and vibrational alarms much sooner and definitely bigger than those of the first test with DC arc requiring, again, the evacuation of the laboratory.

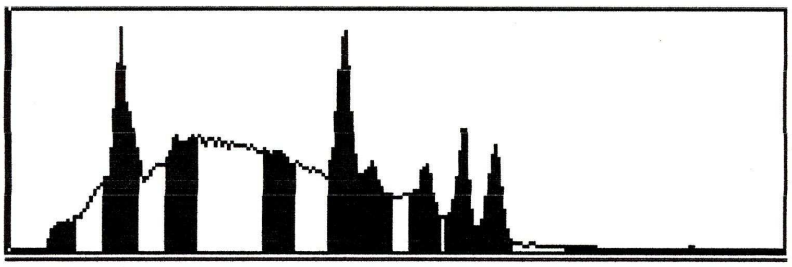
Most significantly, detector $PM1703GN$ would repeatedly go into sonic and vibrational photon alarms when placed against the cylindrical PVC wall up to three weeks following the last activation of the arc, namely, after a period of time excluding residual atomic excitations, thus confirming nuclear reactions.

During the preceding tests detector SAM 935 was used for a verification of the readings of $PM1703GN$ with rather puzzling results. In fact, detector SAM 935 did show clear detections of apparent neutrons via counts clearly above the background of 0.10 cps, but such counts had no comparison with the continuous neutron alarms of detector $PM1703GN$ (see the figures and the scans in [119]).

The settlement of this ambiguity delayed the completion of the measurements for a number of months due to the need for the proper selection and reception of a third detector. Following various theoretical studies, we selected and secured the BF^3 activated detector 12-4, namely, a neutron detector activated by nuclei *heavier* than the $He-3$ of SAM 935 and the $Li-7$ of detector $PM1703GN$. Following its arrival, confirmation of the background, and placement next to Klystron I operated as in the above reported first tests. detector 12-4 showed no neutron count at all for the entire day of the test. However, the following morning, after manually impacting the klystron, detector 12-4 showed apparent neutron counts at the rate of 50 cps for about one hour duration, namely a count much bigger than that by SAM 935 (as predicted, see below). A second impacting of the klystron produced identical results.

The traditional use of silver and gold foils placed around Klystron I was expectedly inconclusive because it showed various electron and photon emissions, but no clearly identifiable *conventional* emissions as expected from activation via the conventional use of a neutron flux.

A second series of measurements were initiated with Klystron II on August 8, 2006. Repeated tests produced no neutron detection. To simulate the "trigger" needed for the neutron synthesis, the test was repeated the following morning with an implosion due to the contamination of the chamber with air and the resulting $H-O$ combustion triggered by the arc. Despite the rudimentary nature



PEAKS FOUND

CHN	ENERGY (keV)	GROSS CPM	AMBIENT CPM	CONTINUUM CPM	NET CPM %	UNC	
16	27.0	22122	972	16148	5002 ± 4.25		
37	97.9	132198	8006	65824	58368 ± 0.89		
56	194.8	106786	7284	92488	7014 ± 6.70	U235s	
87	424.4	90276	3602	85558	1116 ± 38.5		
110	646.5	128710	2760	68672	57278 ± 0.89	Cs137	
118	739.9	67746	2180	60060	5506 ± 6.74		
137	974.9	60834	2026	40820	17988 ± 1.96	U238	
149	1143.5	65446	1142	26070	38234 ± 0.95	Co60	
160	1305.5	52112	860	16148	35104 ± 0.92	Co60	
189	1819.3	4388	122	3498	768 ± 12.3		
210	2246.2	1542	20	1352	170 ± 32.8		
224	2543.3	1706	26	550	1130 ± 5.19		

2 OF 2 LIBRARY LINES FOR Co60 FOUND Correlation = 1.00

LINE	PEAK	INTENSITY	NET CPM
1173.2	1143.5	99.90	38234
1332.5	1305.5	99.98	35104

1 OF 1 LIBRARY LINES FOR Cs137 FOUND Correlation = 0.80

LINE	PEAK	INTENSITY	NET CPM
661.7	646.5	90.00	57278

1 OF 1 LIBRARY LINES FOR U235s FOUND Correlation = 0.80

LINE	PEAK	INTENSITY	NET CPM
185.7	194.8	57.00	7014

1 OF 1 LIBRARY LINES FOR U238 FOUND Correlation = 0.80

LINE	PEAK	INTENSITY	NET CPM
1001.0	974.9	2.00	17988

NUCLIDES NOT PRESENT:

2 OF 3 LIBRARY LINES FOR U233 FOUND Correlation = 0.58
2 OF 3 LIBRARY LINES FOR Ra226 FOUND Correlation = 0.05
0 OF 1 LIBRARY LINES FOR Am241 FOUND Correlation = 0.00
0 OF 7 LIBRARY LINES FOR Eu152 FOUND Correlation = 0.00
0 OF 0 LIBRARY LINES FOR Name FOUND Correlation = 0.00

LINES NOT ASSOCIATED WITH ANY NUCLIDE:

Energy	Net CPM	Eff Corrected	
27.0	5002.0	44976.7	C
97.9	58368.0	224471.0	C

Figure 6.29. Reproduction of the print out of a typical scan of detector SAM 935 when under neutron alarm.

of the equipment, this implosion caused, by far, the biggest neutron alarms in different detectors due to off-scale *cps* without any appreciable photon detection, as confirmed and documented by the print-outs. The laboratory was evacuated again for the rest of the day, residual counts persisted for days, and the test was not repeated for safety.

A third series of tests was initiated on December 20, 2006, with Klystron III filled up with commercial grade hydrogen at 100 psi, but the tests were quickly terminated for safety due to an excessive number of counts by the various detectors as well as the virtually instantaneous disintegration of the tips of the thoriated tungsten electrodes that prohibited an electric arc for the duration of at least a few seconds.

Following completion of the tests, the detectors were returned to their manufacturers for control; they were verified to operate properly; and the printout of all readings stored in their memory was released [119] confirming the measurements reported above.

Systematic reruns of the tests in early 2007 confirmed all the above detections, including in particular their anomalous behavior. However, all tests producing protracted off scale neutron alarms, such as those with implosion or at 100 psi hydrogen gas, were not repeated for safety.

No meaningful counts were detected with the above identified klystrons in using various gases other than hydrogen, although this should not exclude possible similar effects under sufficiently more powerful arcs. No neutron, photon or other radiation was measured from electric arcs submerged within liquids. Hence, the data herein reported appear to be specific for electric arcs within a hydrogen gas under the indicated conditions.[54]

6.2.12.D Don Borghis Hypothesis of Neutroids

Recall that the synthesis of the neutron from protons and electrons has at least *two* quantum anomalies (that is, deviations from quantum mechanics), the first is given by the need of at least 0.78 MeV over the rest energies of the protons and the electron, and the second is given by the fact that the quantum spin 1/2 of the proton and the electron cannot produce the spin 1/2 of the neutron.

The usual "quick fix" to salvage Einstein special relativity and quantum mechanics, such as the assumption that the proton and the electron have 0.78 MeV relative kinetic energy, reveals a skin deep mind, because such an assumption

[54]It should be indicated that E. Conte and M. Pieralice claimed in Infinite Energy of 1999 to have synthesized neutrons from protons and electrons. However, they abstained from quoting don Borghi in their original paper, resulting in denunciations of attempting to stead the scientific priority to a dead priest; they copied ad literal Santilli paper [95] without its direct quotation, resulting in a lawsuit at the U. S. Federal Court one can inspect at www.scientificethics.org; and their neutron detection has been the subject of criticisms so severe to prevent a review.

Alarm, neutron	9/2/2006/10:50:00 AM	53 Cps
Alarm, neutron	9/2/2006/10:56:00 AM	58 Cps
Alarm, neutron	9/2/2006/10:57:00 AM	34 Cps
The device off	9/2/2006/10:58:00 AM	
The device on	9/3/2006/3:05:00 AM	
Alarm, neutron	9/3/2006/3:05:00 AM	1 Cps
Alarm, neutron	9/3/2006/3:05:00 AM	1 Cps
Calibration	9/3/2006/3:06:00 AM	
Alarm, neutron	9/3/2006/3:07:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:07:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:08:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:09:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:10:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:10:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:11:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:13:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:13:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:17:00 AM	99 Cps
Alarm, neutron	9/3/2006/3:17:00 AM	99 Cps
The device off	9/3/2006/3:17:00 AM	
The device on	9/3/2006/5:53:00 AM	
Alarm, neutron	9/3/2006/5:54:00 AM	1 Cps
Calibration	9/3/2006/5:54:00 AM	
Alarm, neutron	9/3/2006/5:54:00 AM	99 Cps
Alarm, neutron	9/3/2006/5:58:00 AM	99 Cps
Alarm, neutron	9/3/2006/5:59:00 AM	99 Cps
Alarm, neutron	9/3/2006/5:59:00 AM	99 Cps
Alarm, neutron	9/3/2006/5:59:00 AM	99 Cps
Alarm, neutron	9/3/2006/6:00:00 AM	99 Cps
Alarm, neutron	9/3/2006/6:02:00 AM	99 Cps
Alarm, neutron	9/3/2006/6:03:00 AM	99 Cps
Alarm, neutron	9/3/2006/6:04:00 AM	99 Cps
Alarm, neutron	9/3/2006/6:05:00 AM	99 Cps
The device off	9/3/2006/6:05:00 AM	
The device on	9/3/2006/7:45:00 AM	
Alarm, neutron	9/3/2006/7:46:00 AM	1 Cps
Calibration	9/3/2006/7:46:00 AM	
Alarm, neutron	9/3/2006/7:48:00 AM	99 Cps
Alarm, gamma	9/3/2006/7:58:00 AM	18 uR/h
Alarm, gamma	9/3/2006/8:03:00 AM	7 uR/h
Alarm, gamma	9/3/2006/8:03:00 AM	11 uR/h
Alarm, neutron	9/3/2006/8:04:00 AM	37 Cps
The device off	9/3/2006/8:09:00 AM	
The device on	9/3/2006/8:09:00 AM	
Alarm, neutron	9/3/2006/8:09:00 AM	1 Cps
Alarm, neutron	9/3/2006/8:09:00 AM	1 Cps

Figure 6.30. Reproduction of the print out of a typical scan of detector $PM1703GN$ when under neutron alarm. Note the separation of the background as well as of gamma detections from neutron detections.

Alarm, neutron	9/1/2006/5:57:00 AM	99 Cps
Alarm, neutron	9/1/2006/5:58:00 AM	99 Cps
Alarm, neutron	9/1/2006/5:59:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:01:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:01:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:01:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:02:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:02:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:02:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:03:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:03:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:03:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:03:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:04:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:04:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:04:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:04:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:05:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:05:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:05:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:07:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:08:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:08:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:09:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:09:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:09:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:10:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:10:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:10:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:11:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:11:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:14:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:14:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:15:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:15:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:15:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:15:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:15:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:16:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:16:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:16:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:16:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:17:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:17:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:17:00 AM	99 Cps
Alarm, neutron	9/1/2006/6:17:00 AM	99 Cps

Figure 6.31. Reproduction of the print out of a typical scan of detector $PM1703GN$ when under one of the continuous neutron alarms that required evacuation of the laboratory. Note the off-scale and duration of the alarm that disqualify as nonscientific other "interpretations" used to oppose measurements [119].

prevents the synthesis, as established by the quantitative representation of the neutron structure of the preceding sections. In fact, at the indicated relative energy, the $p-e$ cross section is of the order of 10^{-20} *barns*, at which value any synthesis is pure nonscientific nonsense.

The complexity of the neutron synthesis then emerges in its full light, jointly with the grossly insufficient character of our knowledge. These and other factors suggest caution prior to venturing claims on the full synthesis of the neutron, and this explains the reason for the title of paper [118] reporting measurements 'toward" the neutron synthesis.

Along the above lines, we can safely state that an electric discharge within a hydrogen gas at a few psi pressure and atmospheric temperature (as above described) produces "entities that:

1) Are not hydrogen atoms (because in that case no nuclear transmutation would be conceivably possible);

2) Have dimensions of the order of 1 fm as for all hadrons (otherwise the detectors would show no counts);

3) Are neutral (otherwise they would not move through walls);

4) Are stable for hadron standards (more accurate data being grossly premature at this writing);

5) Remain initially confined within the arc chamber under steady conditions, to slowly exit, except for the case of production under implosion causing rapid propagation;

6) Are generally released hours following the tests, with anomalous counts lasting for weeks;

7) Are not neutrons (otherwise we would have ordinary neutrons detections).

In fact, *all detectors systematically behaved in an anomalous way,* namely, in a way different than that for direct neutron detection as defined by the manufacturers of the equipment. This is clearly illustrated by a kind of "detectors self-activation" since detector $PM1703GN$ entered into neutron alarm with no photon count while driving miles away from the test at about $15m$ following exposure to Klystron I. The anomalous behavior was confirmed with reruns in different directions from the lab via the use of detector $PM1703GN$. The other detectors also showed similar anomalous behavior, although with different delay tines.

The most plausible interpretation is that the tests produced "entities" other than neutrons that were absorbed by nuclei of the detectors, then causing nuclear transmutations that, following a delay time, produced ordinary neutrons. In different words, the delayed detections here reported for Klystron I do indeed refer to actual neutrons, although originating from nuclear transmutations caused by the original emissions, and not from the original emission itself.

In view of the above and other anomalies, don Borghi introduced the name of *neutroids* for the entities produced inside the klystron (see below Lino Daddi historical account). We believe that the differentiation between neutrons and neutroids is quite appropriate. Consequently, to initiate quantitative studies on neutroids we we present the following:

HYPOTHESIS 6.2.12A [118]: Under steady conditions, thus excluding implosions and other impact events, a steady electric arc within a hydrogen gas at a few psi pressure and atmospheric temperature produces a new particle called neutroids and denoted with the symbol $\tilde{n}$, having the values (in nuclear units) $A = 1, Z = 0, J = 0, u = 0.008$, and we shall write

$$p^+ + e^- \ \rightarrow \ \tilde{n}(1, 0, 0, 1.008). \tag{6.2.206}$$

Note that the value $J = 0$ is assumed for the primary purpose of indicating that the total angular momentum is assumed as being conventional, thus not excluding non-null integer values requiring separate study not considered at this time. Also, the rest energy of the neutroids is assumed as being that of the proton in atomic mass units $1u = 931.49MeV$, $m_p = 938.27MeV = 1.0078 \approx 0.008u$ (see the Tables of Nuclides http://atom.kaeri.re.kr/) because the electron mass $0.511MeV = 0.0005\ u$ is ignorable for our approximation and the $p - e$ binding energy of Coulomb nature is excessively small for our approximation, being of the order of $10^{-3}MeV$).[55][56]

Our tentative interpretation at this limited level of our knowledge is that, the geometry of the electric arc is quite conducive to processes causing the synthesis of neutron-type particles. By recalling that the magnetic field created by an electric arc is directly proportional to the current and inversely proportional to the distance, in the conditions of tests [118], protons and electrons are exposed to magnetic field with an intensity of the order of $10^8\ G$ when at atomic distances from the arc.

Under so powerful a magnetic field, the geometry of the electric arc first aligns protons and electrons with their magnetic moments along the tangent to the local magnetic force (Figure 6.32). Subsequently, the same geometry is predicted to cause protons and electrons to collapse into a neutral, hadron-size particle due to

[55]It should be noted that neutroids are expected as being created also with means other than electric arcs within a hydrogen gas. The best illustration is given by numerous known reports of neutron emissions from electrolytic cells with beryllium electrodes saturated with hydrogen, which radiations could be mainly due to neutron synthesis rather than sole emissions from nuclear transmutations as interpreted so far.

[56]It should be indicated that various other names have been suggested for possible bound states of a proton and an electron below Bohrs orbit, including "mini atoms," "hydrino," and others (see Lino Daddi presentation below). Note, however, that the study herein considered under the name of neutroids refers, specifically, to hadrons, while other studies have different aims .

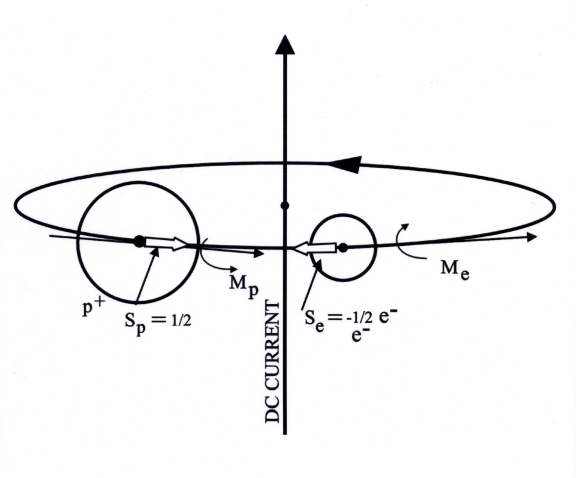


Figure 6.32. A schematic view of the alignment of protons and electrons along a magnetic force line of a DC electric arc.

the very strong Coulomb attractions at short distances of both, opposite charges and opposite magnetic polarities (Figure 6.33).

Quite intriguingly, we cannot exclude the possibility that some of the entities produced by the arc are ordinary neutrons. In fact, the behavior of all detectors for the case of the tests with implosion caused by hydrogen combustion was fully normal and, in particular, without self-activated delayed detection, thus suggesting the production of ordinary neutron.

Similarly, the off-scale neutron alarms with no photon detection were so intense for the tests with klystron III at 100 psi hydrogen gas, that the direct production of neutrons simply cannot be excluded at this writing. The point is that the joint production of a kind of "intermediate" particle between the hydrogen atom and the neutron cannot be excluded either.

Also, at this stage of our quite limited knowledge, we cannot exclude that the addition of high frequency microwaves and related resonances may have caused the production of neutrons in the original experiment by Don Borghi and his collaborators [116]. Note that detectors suitable to identify whether the transmutations originated from the actual production of neutrons, or they originated from neutroids, were not available for tests [116].

Whatever their interpretation, we can safely state that *Santilli's experiment [118,119] confirms Don Borghi's experiment [116,117] because tests [118,119] detected nuclear transmutations on various substances placed in the vicinity of the klystrons, which transmutations are the main claim of Ref. [116-117].* To be scientific, different views should provide a *quantitative* elaboration of the differences

between the two tests, namely, should *prove* that the nuclear transmutations of tests [116,117] are incompatible with those of tests [118,119].[57]

To prevent denunciations of scientific corruption, readers are suggested to avoid the use of quantum mechanical theologies for the dismissal of the above detections, such as the well known impossible existence for quantum mechanics of a bound state $p - e$ at short distances due to the uncertainty principle and other reasons (see Section 6.2.2).

In fact, quantum mechanics is exactly valid for the hydrogen atom, that is, for the dynamics of an electron moving in empty space at relative large distances from a proton. By comparison, the validity of the same discipline for the dramatically different case of the electron moving within the hyperdense medium inside the proton is a pure personal theology without any theoretical support, let above experimental. verifications. Hence, the use of quantum mechanical theologies for the dismissal of new measurements indeed scientific corruption for personal gains.

The application of quantum theologies to the neutroids would imply that the electron freely orbits within the hyperdense medium inside the proton which is nonscientific nonsense. Additionally, we have shown in preceding sections that the neutron is constituted by an electron *constrained* within the proton by said hyperdense medium, under which conditions the use of quantum mechanical theologies for the dismissal of measurements is indeed a crime against society.

6.2.12.E Again, Continuous Creation from the Synthesis of the Neutron?

As indicated in the preceding subsection, we have adopted Don Borghis hypothesis of the neutroids for scientific caution. In this subsection we would like to indicate that *any claim of production of actual neutrons in the set-up of tests [116-119] implies the admission of the continuous creation of matter according to the etherino hypothesis.* Hence, rather than disappearing for the comfort of Einsteinian interests, the hypothesis of continuous creation originated in quantitative studies on the neutron structure, reappears rather forcefully in the laboratory synthesis of the neutron, and will emerge again in the neutron decay of the subsequent section.

The above scenario originates from the following facts:

1) When the proton and the electron have a relative energy of 0.78 MeV the synthesis of the neutron is impossible for the indicated reasons, thus leaving as

[57]It should be also noted that hydrogen atoms with orbits smaller than conventional ones have been predicted in the literature (see, e.g., ref. [120] and papers quoted therein). However, these studies are aimed at processes related to *anomalous atomic orbits,* while our studies deal with *anomalous nuclear hadronic-type particles.*

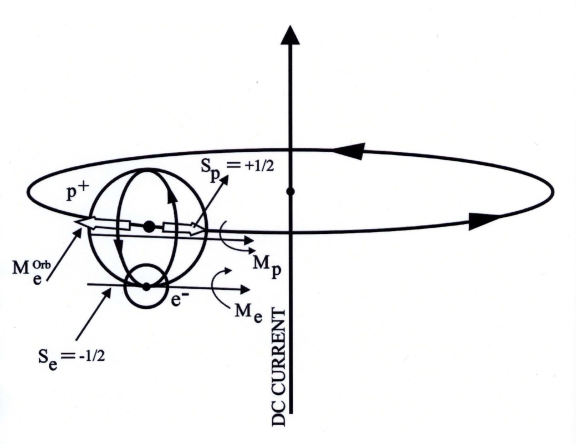


Figure 6.33. A schematic view of the neutroids expected from the collapse of the electron into the proton structure following the alignment of the preceding figure due to very strong, attractive, electric and magnetic Coulomb forces at $1F$ mutual distances due to opposing charges and magnetic polarities. Note that the coupling is in singlet, as necessary for any bond, and it is of axial character, namely, the spin of the proton and of the electron are initially aligned along a single symmetry axis. Following penetration inside the proton, the electron is expected to acquire conventional integer angular momenta, along the lines of Figure 6.23.

only possibility for the neutron synthesis that the proton and the electron are essentially at rest with respect to each other;

2) The transfer of 0.78 MeV from the physical environment to the neutron is indeed plausible for the neutron synthesis in the interior of the Sun due to its density, but the same assumption makes no sense for the comparatively insignificant hydrogen density of the tests here considered; thus, leaving

3) The only plausible explanation for the synthesis of the neutron inside the klystrons of tests [116-119] is the the origination of the missing energy of 0.78 MeV from the ether as a universal medium of very high energy density according to the etherino hypothesis.

We can then conjecture that, in the simplistic geometry of Figures 6.32 and 6.33, following penetration inside the proton, the electron is constrained to orbit with a quantum mechanical angular momentum, resulting in the neutroids, in which case there is neither an energy nor a spin anomaly. Note that in this case the initial relative kinetic energy of the proton and the electron is ignorable.

On the contrary, when the neutron synthesis occurs in the core of a star, the electron may be compressed all the way to the center of the proton, resulting in both the energy and spin anomalies. This speculation appears as being compatible with the hadronic structure model of the neutron forcefully indicating the at rest condition of the electron inside the proton, e.g., for the isorenormaliza-

tion of its mass. The same conjecture appears compatible with the cases of tests [118,119] suggesting the apparent true production of neutrons, such as those with implosion, high pressure, or other "triggers" of hadronic mechanics. Note that in the latter cases too the initial relative kinetic energy of the proton and the electron is ignorable.

As we shall see in the application of the studies herein presented well under way in the industry (and positively not in contemporary academia), the issue as to whether the electron is at rest or not when compressed inside the proton, has a primary relevance for possible new clean energies.

To understand the dichotomy here alluded to, the reader should note that, for industrialists who are currently investing substantial sums of *their* money, the proton and the electron are massive, permanently stable particles that simply cannot "disappear" at the time of the neutron synthesis. Hence, the neutron must be a kind of bound state of a proton and an electron.

By contrast, for academia abusing *public* funds, everything goes. One just needs a few accomplices at "leading" institutions and you get public funds for any desired theory. In fact, as denounced at the initiation of Section 6.2, for contemporary academia, the proton and the electron "disappear" at the time of the neutron synthesis as being replaced by hypothetical directly undetectable quarks without gravity and confinement. Additionally, at the time of the neutron decay, hypothetical quarks "disappear" while the proton and the electron "reappear," and, to complete the adaptation of nature to organized Einsteinian interests, there is the conjecture of another directly undetectable hypothetical particle, the neutrino, claimed as being "detected" via the abuse of scientific credibility and public funds.

One statement can be made with absolute certainly: public money can be easily obtained for academic games via the support of brothers at governmental agencies, but Santilli can testify that none of the industrialists he knows in three continents actively working on new clean energies would give one penny of their money to so farfetched adulterations of nature for personal gains.

In summary, all the above measurements and analyses establish beyond "credible" doubt the very motivation for the birth of hadronic mechanics in 1978 [14], namely that *the structure of the neutron, including its synthesis of this section and its stimulated decay of the next section, signal the truncation of the validity of all Einsteinian doctrines and quantum mechanics, since the usual intentional adulterations via the addition of ad hoc parameters (thanks God!) are no longer effective.*

Still in turn, this illustrates the reason seemingly reputable and indeed fully qualified scientists, such as Sidney Coleman, Steven Weinberg, Sheldon Glashow and their rapidly decreasing number of followers around the world, perpetrated such extreme gestures of scientific obstructions against new studied on the struc-

ture of the neutron as those summarized in Footnote 1 of this volume, reviewed in details in book [89] and documented in the 1,132 pages of the three volumes [90], the latter ones now being available as free download from the indicated web site. Still. in turn, this illustrate that the presentation of new experimental studies on the synthesis (and stimulated decay) of the neutron without the joint open denunciation of the organized scientific crime in the field, would be a farse serious scholars should abstain from for their own dignity, let alone for any serious pursuit of new human knowledge.

6.2.12.F Tentative Interpretation of Don Borghi and Santilli Experiments

The idea that the experimentalists of tests [116,117]. two of whom being Catholic priests, have lied in their claims is so ludricous that can only be proffered by immoral outcasts. Hence, in this work we have assumed that the claims of Refs. [116,117] are true, namely, that the various substances placed in the exterior of the klystron experienced nuclear transmutations due to a flux of *neutrons.*[58]

Needless to say, it is unknown as to whether said final neutron flux originated from the interior of the klystron, or it is due to secondary processes, such as those in the activated substances themselves or in the walls of the klystron.

In the hope, but not the certainly, of achieving an interpretation of the above measurements while avoiding continuous creation of matter in the universe, we assume that neutrons are synthesized by nuclear furnaces, namely, we assume that *neutroids are turned into neutrons when absorbed by nuclei,* and we shall write

$$\tilde{n}(1,0,0,1.008) + a \ \rightarrow \ n(1,0,1/2,1.008), \tag{6.2.207}$$

where a is the aetherino or etherino, namely, the entity permitting the energy, spin and other anomalies in the synthesis of the neutron that will not be addressed in this subsection for simplicity.

The above reaction is pure nonsense for quantum mechanics but quantitatively treated in a numerically exact and invariant way by hadronic mechanics as studied in detail in the preceding sections.

Once immersed within nuclei, there is a realistic possibility of identifying the missing 0.78 MeV from vibrational and other nuclear sources (rather than from the vacuum), and the nuclear furnace has the density sufficient to compress the electron deep inside the proton, resulting in the hadronic interpretation of the spin without any need for conjecturing hypothetical neutrinos, as per preceding studies.

[58]Beside the late Don Carlo Borghi, Don Camillo Giori is also a Catholic priest still active in a church in Parma, Italy. Unfortunately, Don Giori has been so harassed by the organized scientific crime on Einstein, that he refuses to talk to anybody about his experiment with Don Borghi.

Note that *the above assumption is sufficient alone, to represent "all" Don Borghi's data [116,117],* although, to separate quantitative science from epistemology, the reader should keep in mind that the above statement required a lifetime of research summarized in this and in the preceding three volumes of this series.

Under the above view, the substances placed by the experimentalists around the klystron of tests [116,117] absorb a large number of neutroids that, when inside nuclei, are turned into neutrons, thus creating the activation detected in the tests.

To initiate this study, we assume the usual symbol $N(A, Z, J, u)$ for ordinary nuclides as currently known, and the symbol $\tilde{N}(A, Z, J, u)$ for possible anomalous nuclides, namely, nuclides following the absorption of a nucloid not existing in available data, here called *nuclidoids*. We also assume that the binding energy of the neutroids is similar to that of an ordinary nucleon (e.g., $BE = 0.0002\ u$ for the deuteron), which assumption is a direct consequence of conversion (6.2.207). Then, for the case of gold, we have

$$Au(197, 79, 3/2.196.966) + \tilde{n}(1, 0, 0, 1.008) + a \ \rightarrow \ Au(198, 79.2, 197.972), \tag{6.2.208}$$

and, similarly, one recovers all conventional activation processes.

By comparison, the application of the above assumption to the steel casing of Don Borghi klystron yields an unknown nuclidoid

$$Fe(57, 26, 1/2, 56.935) + \tilde{n}(1, 0, 0, 1.008) + a \ \rightarrow \ \tilde{Fe}(58, 26, 1, 57.941) \tag{6.209}$$

since the tabulated nuclide is $Fe(58, 26, 0, 57.933)$.

Needless to say, the anomalous nuclide $\tilde{Fe}(58, 26, 1, 57.941)$ is expected to be highly unstable and decay in a variety of possible modes, although they do not appear to provide the source of neutrons necessary to represent Don Borghi data.

This excludes that the neutrons in Don Borghi experiment were synthesized in the walls of his klystron and confirms, quite preliminarily of course, that the neutrons were synthesized by the activating substances themselves.

Hypothesis (6.2.207) also interprets some of Santilli detections [118,119], with the understanding that the anomalous behavior of the detectors, such as the delayed neutron counts, requires special studies and perhaps the existence of some additional event not clearly manifested in Don Borghi's tests.

To initiate the study, we have the first possible reaction

$$H(1, 1, 1/2, 1.008) + \tilde{n}(1, 0, 0, 1.008) + a \ \rightarrow \ H(2, 1, 1, 2.014) \tag{6.2.210}$$

namely, we have the prediction that, under transmutation (6.2.207), the coupling of a neutroid by a proton creates the ordinary deuteron.

The understanding of this statement requires a study of the deuteron via hadronic mechanics conducted in the nuclear section of this chapter (Section 6.3).

In particular, such an understanding requires the scientific honesty to admit that quantum mechanics failed to achieve an exact representation of the simplest possible nucleus, the deuteron, following one century of attempts and a river of public money, since:

1) Quantum mechanics miserably failed to represent the deuteron spin 1 (the ground state of two particles with spin 1/2 must be 0 for quantum mechanics due to the singlet coupling mandated by stability);

2) Quantum mechanics miserably failed to achieve an exact representation of the deuteron magnetic moment despite all quarks manipulations (the hypothetical orbits of the hypothetical quarks are excessively small to make and appreciable contributions to magnetic moments)

3) Quantum mechanics miserably failed to achieve a meaningful representation of the stability of the neutron when coupled to a proton. Should we continue with embarrassing insufficiencies?

As we shall see in Section 6.3, *all* these century old problems are resolved by assuming that *the deuteron is a three-body structure composed by two protons and one electron verifying hadronic mechanics, which structure can be interpreted in first approximation as a two-body system composed by a proton and a neutron verifying quantum mechanics,* the understanding that in the latter approximation nuclear physics ceases to be an exact science.

Reaction (6.2.210) Indicates that the hydrogen content of the polycarbonate walls of Santilli's tests [118,119] do cannot possibly be considered a source of the detected neutrons.

said polycarbonate contains about 75 % carbon, in which case we have the tentative reaction

$$C(12,6,0,12.000)+\tilde{n}(1,0,0,1.008)+a \ \rightarrow$$
$$\rightarrow \ \tilde{C}(13,6,1/2,13.006)\equiv C(13,6,1/2,13.003)+\gamma, \qquad (6.211)$$

thus excluding the carbon of the polycarbonate can be a source of the detected neutrons.

Finally, said polycarbonate contains about 18.88 % oxygen for which we have the reaction yielding an unknown nuclidoid

$$O(16,8,0,16.000)+\tilde{n}(1,0,0,1.008)+a \ \rightarrow \ \tilde{O}(17,8,1/2,17.006), \qquad (6.2.212)$$

because the known nuclide is $O(17,8,5/2,16.999)$. The latter reaction too is not expected to provide the neutron counts detected by Santilli.

In conclusion, in Santilli's experiment too, it does not appear that the detected neutrons are synthesized by the walls of the klystron. This leaves as the only residual possibility that the neutrons are synthesized by the detectors themselves. To study this possibility we consider the reaction for the He_3-activated detector

$$He(3,2,1/2,3.016)+\hat{n}(1,1,0,1.008)+a \ \rightarrow$$

$$\tilde{He}(4,2,1,4.023) + EC \to He(4,2,0,4.002) + \gamma, \tag{6.2.213}$$

in which, as one can see, the detection of the neutroids is anomalous if any.

Next, we have the reaction for the Li-activated detectors

$$Li(7,3,3/2,7.016) + \hat{n}(1,1,0,1.008) + a \ \to Li(8,3,2,8.022) \ \to$$

$$\to \ Be(8,4,0,8.005) + e^- \ \to \ 2\alpha, \tag{6.2.214}$$

that do indeed behave in a way fully equivalent as to whetheher the detection refers to neutroids or neutrons.

Finally, for the base of B-actiovated detectors we have the reactions

$$B(10,5,3,10.012) + \hat{n}(1,0,0,1.008) + a \ \to \ \tilde{B}(11,5,5/2,11.018) \tag{6.2.215}$$

with the predicted spontaneous decay

$$\tilde{B}(11,5,5/2,11.018) \ \to C(11,6,3/2,11.011) + e^- + \gamma, \tag{6.2.216}$$

that do not appear to behave normally under a flux of neutroids.

From the above reactions we can see a conceivable explanation of the reason the He_3-activated detector resulted as being the least active of all in tests [118-119]. We can also see a plausible reason for the $Li-7$ activated detector as being the bests for Santilli's experiment, and that's teh reason for mandating the use of Li-activated detector for any reruns of Santilli's experiment (unless the rerun is commissioned to fake it, as indicated below, in which case care is taken to avoid its use).

Our tentative conclusion is, therefore, that *the neutrons detected in Don Borghi experiment [116,117] were synthesized by the nuclei of the activated substances, while the neutrons of Santilli experiment [118,119] were synthesized by the detectors themselves, either by their activating substance, or by their casing, the latter expected to be the origin of the delayed detection.* For instance, the explanation of the large delayed counts by the BF^3 activated detector requires a study of the absorption by its plastic casing that cannot be possibly done in this initial study.

In closing, the reader is warned against superficial conclusions, no matter how appealing. In fact, we remain with the possibility that *in Santilli's experiment, the neutroids are produced in clusters,* something reminiscent of electron clusters, but deeply connected to the new chemical species of heavy hydrogen presented in Volume V, Chapter 10 in particular. Regrettably, we cannot study this aspect at this time since it requires the prior knowledge of the entire Volume V of this series.

Yet, the latter possibility would provide a clear explanation of the large neutron counts experienced by Santilli, so intense to force various evacuations of the laboratory. In fact, it is easy to see that, if neutroids are absorbed by stable

elements in clusters such as $\tilde{n} \times \tilde{n}$, $\tilde{n} \times \tilde{n} \times \tilde{n}$, ..., following their conversion into neutrons by nuclei they would result in an excess of neutrons that can be corrected by nature with the emission of neutrons. In this case, *all* substances near the experiment, whether the walls of the klystron or the substances composing the detectors, would become a source of the detected neutrons.

A reason for suspecting the creation of neutroids in clusters in Santilli's rather than in Don Borghi's experiment is due to the much bigger power of the electric arc used by Santilli's as compared to that in Don Borghi's tests, as well as by a number of anomalous features of sufficiently powerful electric arcs, all irreconcilably incompatible with Einsteinian doctrines (see Volume I, Section 1.2.13), one of which is a radial compression of polarized structures, whether atoms or hadrons, toward the arc, thus naturally creating clusters. [59]

In conclusion, what is needed by the fundamental problem of the neutron synthesis is what it has been denied so far by academic corruption, the conduction of systematic and all encompassing measurements by a large laboratory with the necessary equipment that could not possibly be available for tests [116-119].[60]

6.2.12.G Daddi's Studies on Don Borghi's Experiment

In this section we report *ad litteram* the studies on the laboratory synthesis of the neutron conducted by Lino Daddi, former professor at the Naval Academy of Livorno, Italy, Via degli Oleandri 24, I-57128 Livorno, Italy, email linodaddi@hotmail.com

This subsection presents and comments the data deduced from two versions of the experiment carried out at "Centro de Energia Nuclear", of Pernambuco University (Brazil). Both versions [116, 117] are in English, having the same title: *EXPERIMENTAL EVIDENCE ON THE EMISSION OF NEUTRONS FROM COLD HYDROGEN PLASMA.*

Here the first version will be indicated with [A]. It is an internal report of the "Centro": Comunicação no. 25 of June 1971. It is ascribed to two Research Workers of board: Carlo Borghi (Director) and Attilio Dall'Olio (transferred from Parma University), while Camillo Giori (Parma University but in licence at "Centro") is thanked for the help in organising the experiment. This [A] version

[59] Any welder knows that, under a DC arc, electrodes are attracted to each others and they have to pull one electrode away from the other to create an arc. This feature, very well known in the industry, is carefully kept away from physics courses because it deals with a longitudinal (axial) force called *Ampere force* that is prohibited by Maxwell's equations and Einsteinian doctrines, and, consequently, it does not exist. This academic immorality is also excellent to fake Santilli's experiment. Indeed, one can rerun experiment [118,119] by ignoring the Ampere force, in which case there is a short, rather than an arc, resulting in the desired lack of production of neutroids.

[60] Widespread vulgar academic corruption on Einsteinian doctrines turns the insufficiency of the equipment used in tests [116-119] as a "motivation" for their dismissal, rather than their reruns with better equipment, for evident immoral personal gains in the immoral continuation of immoral abuses of academic power, prestige and public funds.

was presented to "First Brasilian Symposium on Radioisotopes", held at Rio de Janeiro in June 1970. Almost all measurements were performed in the six previous months.

The [B] version probably was prepared for an external publication (Authors: Borghi-Giori-Dall'Olio) and almost immediately was translated in Italian by Boscoli for a monograph of Andromeda (publisher in Bologna). After don Borghi death (happened in 1984) this [B] paper was published, without changes, in 1993 on vol. 56 of Phys. At. Nucl., page 339.

The versions are enough coincident in the description of a number of nuclear measurements. However the [A] version contains the results of some measurements which are not referred in [B] one, whereas the use of uranium is referred only in [B]. So a careful comparison between the [A] and [B] versions seems useful. My remarks will be pointed out under the [D] symbol.

In [A] the Authors are inclined to think the produced neutrons are epithermal (50–300 eV) whereas on [B] no opinion on neutron energy is asserted.

Don Borghi stated the following objet for its experiment.

[A] [B] ... in order to answer the question whether there is a proton-electron interaction different from the Coulombic one.

... there must be per unit time a very large number of electrons targeting the protons and vice-versa, since every one of such events is a chance for the hypothetical other interaction to work. These conditions are fulfilled when we have a resonant cavity filled with pure hydrogen at low pressure. mixture of free protons and free electrons called a "cold plasma".

[D] Nevertheless I think the proton-electron reactions could take place inside of the walls rather than inside of the cell. The solid walls could absorb protons as in various LENR experiments. The uncertainty of interpretation doesn't reduce the importance of the nuclear transmutations obtained in this experiment.

THE EXPERIMENT

The hydrogen

[A] About 10–20 cm^3 of hydrogen (S.T.P.) per hour flow into the klystron (highly ionized pure hydrogen at low pressure , 10^{-1} Torr). It is produced by electrolysis, (water + H_2SO_4), with a current of $\cong$ 1 A.

When the klystron contains other gas than pure hydrogen (for instance, air) no neutron flux is observed. This detail is critical, so that also a small leakage in the vacuum system reduces severely the observed flux of neutrons.

[B] The hydrogen has been always obtained by electrolysis (water + H_2SO_4), dried on silica and filtered through a vacuum tight Palladium tube (25 cm long, 0.5 cm diameter, 0.1 cm thick). The pressure of the gas in the oscillator is about 0.1 Torr.

[D] The opinion of Don Borghi the $p+e$ reaction happen in cold plasma could be wrong. The protons (but also the electrons) may be more numerous in the walls than in the cold plasma (were the pressure is very low indeed).

The $p+e$ reactions could concern the absorbed protons in the walls, according to nuclear reactions in condensed matter (LENR).

The deuterium

[A] [B] The Deuterium percentage in the input flow of hydrogen into the klystron is 1/1000. Therefore in 5 cm^3 at 0.1 Torr pressure the number of D atoms is about 10^{13}, versus 10^{16} of H.

D+D fusion

[A] [B] Possible $d-d$ reactions could actually exist in the klystron if

a) some noticeable amount of deuterium is adsorbed by the inner wall, which is of stainless steel;

b) a beam of D ions with sufficient energy falls onto these D-imbedded walls. A threshold of about 5000 V is required, whilst the klystron works at voltage of 500 V.

[D] The probability of the D+D reaction is judged very little.

Cold plasma

[A] The mechanism of ionization here used permits the permanence of positive ions together with an approximately equal number of electrons in the same volume. This mixture, possible because of the extremely high frequency, is referred to as "cold plasma" of hydrogen.

[B] The "cold plasma" is a considerable number of protons mixed and colliding with an equal number of free electrons, for a time very larger than 10^{-8} sec This limit is suggested by the known average recombination time of the ionized hydrogen atom.

Millimetric waves

[A] Metallic (stainless steel) UHF oscillator for 4 mm wavelenght for ionizing the low pressure hydrogen in the klystron-like oscillator. The peak-to-peak voltage of the microwave e.m. field does not reach to 5000 Volt (the minimum required by D+D reaction); thus is very improbable that D+D reaction contributes to the observed neutron production.

[B] The high frequency has been taken of the order 10^{10} Hertz, with amplitude large enough for ionizing the low pressure pure hydrogen contained in the resonant cavities of a klystron-like oscillator.

[D] An external probe should be necessary in order to ascertain the microwave features. If their stability will be verified, a qualitative monitoring could be committed to a nuclear counter (see item on BF_3 counter).

The cooling / moderating water

[A] The metallic klystron is cooled by a stream of water circulating onto its external wall. The cooling water shield is 1 cm thick

[B] A cooling water flow is necessary, but the cavities containing the plasma are settled outside the cooling layer, so that eventual particles outgoing from them don't cross through the water.

[D] On this argument [A] [B] are not too different, if one thinks the neutrons are produced with very low velocity.

[A] ... samples setted nearby the outer wall of the klystron, at a distance of about 15 cm from the cavities where the cold plasma is produced. The area of samples is about 4.52 cm^2, thus the solid angle is 1.6×10^{-3} sterad. The neutron emission is assumed to be spherically isotropic.

[D] The above quotation confirms between neutron source and the neutron detectors no moderator is interposed.

Indeed the solid angle is calculated by supposing no scattering of neutrons, just as when no moderator is interposed.

Neutrons or neutroids?

[A] The aim of the present work was only to show the existence of that neutron flux, whilst a more detailed analysis of both intensity and energy spectrum will be object of further works already on the way.

[B] When some proton and electron form the just said neutral bound states, then they behave possibly as neutrons and can cross throughout the walls of the oscillator.

[D] In precedent works Don Borghi named "neutroids" the hypothetical srhunk neutral particle formed by a proton and an electron (hydrogen miniatom). Such a compression is forbidden by the usually accepted interpretations of the Uncertain principle. Other Authors, by utilizing different theories, adopted the names "virtual neutron" (Chatterjee [121]), "hydrino" (Mills [122]), "hydrex" (Dufour [123]), "mass modified hydrogen atom" ($WIDOM^3$ [121]), "hydrogen miniatoms" (Daddi [124]).

Beta particles detector

[A] [B] The resulting activities have been measured by means of a Philips anticoincidence system with a steadily low background 0.9 ± 0.1 cpm.

[D] The efficiency of beta counting depends on various factors, which have to be estimated. Very important is the self-absorption factor, which depends on the thickness of the source. The activation samples used by Don Borghi are quite thick, so the percentage of the emitted beta-particles which reach the detector is often little.

Gamma rays detector

[A] [B] The gamma activity was measured by a well crystal NaI(Tl) phototube (Tracerlab 1"x1")

[D] The gamma efficiency of this well scintillator is certainly too low. Above all no spectrometric measurement is practically possible with it. So no acceptable energy determination is possible.

BF_3 counter

[A] [B] The large noise due to the microwave field of the klystron makes the signal from BF_3 detectors uncertain, also with careful electric shielding.

[D] This difficulty may turn in an advantage, by using a proportional nuclear counter (perhaps also different from BF_3 type) in order to qualitatevely monitoring the primer of the klystron. This method should be confirmed by e.m. measurements.

Activation detectors for neutrons

[A] [B] This neutron flux has been prevalently measured by the activation of samples of several elements mostly obtained from the Atomic Energy of São Paulo (see Table 6.1). From a qualitative standpoint all the samples, after a conveniently long exposure near the external wall of the klystron (for hours or several days or months) show an activity clearly many times above any reasonable fluctuation of the background.

[A] A great care having been taken in order to avoid contamination.

[B] ... as well as evident decays with short periods, whose origin cannot be other than a recent activation.

[D] The (n, γ) capture cross sections depend on the neutron velocity according to a law of the type $1/v$, so they are highest for slow neutrons (saving the presence of resonances for particular values of the energy).

Actually Don Borghi assumes in [A] version the neutrons are produced with epithermal energy, in the range 50–300 eV. This conviction derived from the verified activation of resonance detectors used in the experiment (Tl and Pr). But the non-moderation seems be unjustified, because the higher induced radioactivities would made the measurements more reliable.

They should be improved also by utilizing simple technique as the "cadmium cutting".

Table 6.1. Activation detectors

Detector (%)	Capture thermal cross section (barn)	Most important resonance cross section, barn (eV)	Produced radioisotope ($T_{\frac{1}{2}}$)	Principal activities (energy in MeV)
^{115}In (96)	157	29000 (1.4)	^{116}In* (54 m)†	β 1/0.8/06 γ 1.27
	52		^{116}In (14 s)	β 2.9
^{107}Ag (51)	45		^{108}Ag (2.4 m)	β 1.77
^{109}Ag (49)	113	12000 (5.1)	^{110}Ag (24 s)	β 2.2/2.8
^{197}Au (100)	96	30000 (4.8)	^{198}Au (2.7 d)	β 0.96 γ 0.41
^{164}Dy (28)	2600	7500 (54)	^{165}Dy* (1.26 m)	IT
	800		^{165}Dy (2.3 h)	β 0.42/0.88/1.25 γ 0.09/0.28
^{121}Sb (57)	6	1400 (6.2)	^{122}Sb (2.8 d)	β 1.4/2.0 γ 0.56
^{123}Sb (43)	2.5	1200 (2.2)	^{124}Sb (60 d)	β 0.6/2.4 γ 0.6
^{55}Mn (100)	13	1950 (337)	^{56}Mn (2.6 h)	β 2.8/1.05
^{141}Pr (100)	11	500 (23)	^{142}Pr (19 h)	β 2.17 γ 1.6
^{203}Tl (29.5)	8	450 (230)	^{204}Tl (3 y)	β 0.76
^{93}Nb (100)	6		^{94}Nb (6.2 m)	IT

†Note: transition from ^{116}In* to ^{116}In is forbidden.

Irradiation geometry

[A] samples setted nearby the outer wall of the klystron, at a distance of about 15 cm from the cavities where the cold plasma is produced. The area s of samples is about 4.52 cm^2, thus the solid angle is 0.0016 sterad.

[D] The solid angle is not 0.0016, but $s/R^2 = 0.02$ sterad. By dividing for 4π sterad (total solid angle) one obtains the geometry factor (probability) which equals, just, 0.00016.

RESULTS OF THE EXPERIMENT

Decay evaluations

[A] [B] For low level of induced radioactivity the measure has been made by an average in a time as long as possible, and when possible the half lives have been checked with the least squares method.

[D] Even if some graphs are not very clear, on the whole they are suitable for follow the activation and/or the decay, and for a qualitative control of the half-life of the produced activities.

Treatment of the statistical fluctuations

[D] The propagation of the statistical errors not seems always correctly treated. Indeed the Authors sometimes have not chosen in the better way the counting times in order to minimize the fluctuation importance.

For instance, the graphs concerning the indium and the silver can be utilized with difficulty owing to the statistical fluctuations. The standard errors in the thorium measurements (in c.p.m.) were evaluated as if the counting time was one minute (but the induced activity decays in several days!)

Activation of indium, silver and gold.

[D] Fig. 8 and Fig. 9 of [A] version present the activation and decay graphs for gold and silver. In significant Fig. 8 the gold beta activity increases and decreases at least twice. The activated gold would be measured with greater confidence and lower background in gamma-ray spectrometry thank to 411 keV photon the ^{198}Au emits. The activation of indium and silver consists in beta-emitters radioisotopes. Indium presents a strong resonance (29000 barn) at 1.44 eV; gold presents a strong resonance (34000 barn) at 4.8 eV.

These detectors, exposed bare and in a cadmium sheath, would provided, by difference, an answer to the question if the neutrons were thermal or epithermal. Don Borghi (see item on Pr, Nb and Tl) said (in [A]) convicted the energy range was put between 50 eV and 300 eV.

Activation of dysprosium and antimony

[D] The dysprosium proved to be a very good neutron detectors; that was foreseeable for the high value of the capture cross section, and for the favourable counting property of the ^{165}Dy , which half-life is 2.3 hours.

The decay curve obtained is clean and convincing. The dysprosium results could be an unquestionable proof of Don Borghi effect.

Also the antimony has shown a good activation. Its decay curve appears not much disturbed by the statistical fluctuations but the activation curve is less convincing.

The case of manganese

[A] ... a flux of epithermal neutrons, with energy $E > 10$ eV and a probable maximum between 50 and 300 eV. this upper limit is suggested by the fact that no activation is observed with Mn, which has a large resonance peak at 350 eV.

[D] This detector is not even cited in [B] version. This is the typical case in which the detector ought to be surrounded by a moderating sheath during the activation. The slowing down of the neutrons should have increased very much the measurement sensitivity. The Mn exists in nature only as the isotope 55, with a neutron thermal cross section of about 13 barn. The neutron capture forms ^{56}Mn, which emits beta particles and gamma-rays with energy of the order of MeV. Therefore the emission was observable with the used instruments.

However the complete procedure of the exposition is not referred, so the results cannot be quite acquired.

Activation of prometium, niobium and thallium

[D] Don Borghi attached importance to the prometium activation (as also of the partner niobium). The reason of such an interest is in the resonances the activation cross-section shows in the energy band of tens or hundreds eV, which could be the energy band of the emitted neutrons. The prometium decay was particularly regular.

These measures are not referred in [B], in which no hypothesis on the neutron energy is contained (in particular if they are, or not, epithermal).

^{204}Tl decays with a long half-life (3 y), so the Authors preferred don't follow its decrease. Instead an activation curve is presented, with measures which were repeated sometimes during an exposition lasted about a hundred days.

Use of thorium and uranium

[D] Nuclear reactions were also induced in naturally radioactive elements, as thorium and uranium.

Before and after exposition to neutrons, gamma measurements were made, with a little well scintillation detector (NaI). A clean modification of their radioactivity was observed, but the impossibility of energy determination prevented from deduce complete conclusions on nature of the effects.

Case of thorium

[A] [B] The activation of ^{232}Th samples, sealed in plastic tubes and in secular equilibrium with its family, has been repeatedly observed. One sample is conserved as control, and another has been irradiated for 13 days, showing an increasing gamma activity. From the initial 27750 cpm of both samples , the irradiated one reached to 43580, with a difference 16030. The activated sample has been retired and thereafter decayed toward a level 39300 cpm in 34 days.

The analysis of the decay suggests a principal contribution of ^{233}Pa ($T =$ 27 days).

[D] The effect of the exposition is very clear, so the thorium measurement should have been the main point of the work of Don Borghi, which could be acknowledged as unquestionable by anyone.

By assuming the main effect was the (n, γ) reaction of ^{232}Th, the decay chain is:

$$\underset{(22\,\mathrm{min})}{^{233}\mathrm{Th}} \Rightarrow \underset{\beta}{^{233}\mathrm{Pa}} \underset{(27\,\mathrm{d})}{\Rightarrow} \underset{\beta+\mathrm{weak}\,\gamma}{^{233}\mathrm{U}} \underset{(10^5\,\mathrm{y})}{\Rightarrow} . \tag{6.2.217}$$

In spite of the already mentioned wrong evaluation of the statistical errors, the thorium measurement seems qualitatively indisputable.

Case of uranium

[B] For U we used cylinders of high density (10 g/cm^3). Its beginning gamma activity 142700 ± 400 cpm becomes 144860 ± 400 after some 3 hours of irradiation, and thereafter may be observed to decay to the starting value with a half life of about 30 min, probably due to short lived fission product, as suggested by the secondary maximum "delayed" of about 5 min... Indeed there are almost two observable periods, namely about 2 min and 30 min.

[D] It would sufficed to protract each measure for a longer time, in order to noticeably reduce the statistical errors, which importance here are increased by the little difference between the measures.

Despite the great statistical fluctuations, the use of uranium seems supply some valid indications. A gamma activity increases, and lather decreases, as due to a secondary radioisotope produced by neutrons not directly, but from decay of initial radioisotopes (perhaps beta emitter, not observed).

The half-life should be of some minutes, therefore it seems not belonging to the chain which lead to plutonium; rather it seems due to fission of ^{235}U. The Authors supposed the formation of ^{84}Se which decays in 3 min in ^{84}Br, which decays in 32 min.

CONCLUSIONS

Evaluation of the flux

[B] The so far unknown energy spectrum of these neutrons does the values of the (n, γ) cross sections uncertain. For them no certain flux calculation is allowed.

The activity increase of about 2000 ± 500 c.p.m. of the uranium, supposed at saturation, gives a flux ... hence 10^6 n/s.

[D] In order to obtain this rough evaluation, don Borghi takes advantage from the fact the macroscopic cross section is approximately constant ($\cong 1$ cm $^{-1}$) for a large energy range. From the thorium results a very questionable calculation, present in [A] and [B] versions, takes to a similar flux value.

In spite of the many defects, not all slight, particularly in the data utilization, undoubtedly Borghi obtained clear radioactivation of thermal or resonance de-

tectors; it is therefore presumable that the cold plasma produced many neutrons (at least 10^4 n/s). The fact they produced many reactions also without having crossed any moderator suggests they were born with very low, near the thermal values, energy. This could exclude they be due to fusions between nuclei of hydrogen isotopes, for instance between the few deuterons present in the plasma.

Repetition of the experiment

[D] Those who will want repeat the Don Borghi experiment wouldn't be obliged to use all the detectors mentioned here. It will obligatory to have at disposal a low background beta counter and a good gamma spectrometer (with a large NaI crystal or solid state Ge detector).

The choice of the detectors will be made taking in account of the capture thermal cross-sections and of the half-life of the induced radioactivity. For very short times, dysprosium is preferable. For longer times indium and thorium are suitable. In gamma-ray spectrometry a good choice is a gold detector, with its half-life of 2.7 d and a lonely gamma. A further investigation should concern the walls of the cell, which may undergo alterations due to neutrons, or anyway due to nuclear reactions.

6.2.12.H Has the Don Borghi Experiment Been Secretly Redone?

As clear from the preceding analysis, Don Borghi's experiment is truly fundamental for all of physics, quite simple and rapid to rerun, and extremely inexpensive, particularly when compared to particle physics experiments preferred by academia. These aspects render implausible the idea that the experiment has never been rerun somewhere during the several decades since its original conduction.

In view of the above, there is an insistent view that the experiment has indeed been repeated by qualified laboratories. Since this view is voices by independent sources, it has sufficient plausibility for being reported here. The view here referred to is that:

1) The organized scientific crime ordered in the 1990s a U. S. nuclear physics laboratory to rerun Don Borghi experiment under the expectation that the results would be negative, hence good for immediate publication to discredit dissident views;

2) Unfortunately for said crime, the rerun confirmed Don Borghi's claim and, as such, the results could not possibly be released to the scientific community;

3) The organized scientific crime then ordered the repetition of the test to a British laboratory, and the experiment was conducted there under a fake name to cover up the real intent (the indicated fake title is "Studies of electric and magnetic polarization induced by an electric discharge in a hydrogen gas");

4) The results of the British rerun confirmed fully the U. S. tests and, therefore, the original Don Borghi's experiment;

5) Due to the enormous implications, the organized scientific crime brought the case to the attention of the Council of Foreign Relation and/or the organization known as "The New World order." A decision was reached at the top level of the crime to the effect that the results of these reruns had to be kept secret and supporters of Don Borghi experiment had to be continued to be discredited via the use of authoritative accomplices.

6.2.12.I How to Fake Don Borghi and/or Santilli Experiments

as indicated and documented in Appendix 6A, the maximal manifestation of organized scientific crimes occurs in experiments. In fact, various experiments have been commissioned for the studious intent to disprove tests indicating deviations from Einsteinian doctrines thanks to easy manipulations of data, introduction of meaningless parameters, fixing things and the like (see explicit calculations to that effect in Appendix 6A).

Due to the hardly believable ethical collapse of the physics community, experiments suggesting violations of Einsteinian doctrines suffer extreme obstructions in editorial reviews to prevent their publication (see next subsections), while counter-experiments commissioned by the organized scientific crime see their rapid publications ion Physical Review Letters, Physics Letters, Journal of Physics, and other journals of the establishment.

It is then natural to expect that the international organized scientific crime will commission reruns of Don Borghi's and/or Santilli experiments for the specific, pre-set intent of manipulating the results to disprove the claims. In view of this expectation, it may be advantageous to both, serious as well as corrupt physicists, to indicate the following easy ways to manipulate said experiments:

1) In Santilli's klystron, the electric discharge can be made under a short with no gap between the electrodes, in which case no "entities" are produced and the occurrence can be used to "disprove Santilli experiment." In fact, for the "entities" to be produced, it is necessary to have a real electric arc within a hydrogen gas with at least 15-20 Kw causing at least a minimal gap of 2-3 mm for at least 4-5 s. When there is a short without gap, the electric current propagates through the electrodes with insignificant impact in the hydrogen gas. Hence, the serious scientist will make sure to have a real gap with a real arc within the hydrogen gas, while the corrupt physicists will make sure to fake a gap in order to "disprove the results."

2) Santilli's experiment can be repeated with minimal power (say of 1 Kw), the use of a hydrogen gas with minimal pressure (say, a fraction of one psi), creating a real arc with a real gap within the hydrogen gas, resulting in no detection of any type for 2-3 days, thus claiming the "disproof of Santilli experiment."

As indicated in the preceding section, the production of the "entities" and the rapidity of their detection are proportional to the power, the pressure of the hydrogen gas and other factors (recall that at 100 psi the production of the "entities" with 30 Kw EC-AC converter was so violent to cause the evacuation of the lab and the impossibility of repeating the test for safety). Hence, it is very easy to fake Santilli's experiment by reducing the power to an minimum to have an arc, reducing the pressure of the hydrogen gas, and limiting the time of the detections.

3) Santilli experiment can be repeated with the klystron insulated from external influence such as noise, vibrations, etc., resulting in no detection for days, thus claiming the "disproof of santilli experiment." As indicated in the preceding analysis, as well as in the references quoted therein, one or two days subsequent to the arc, Santilli had to shake the klystron with a rubber hammer to finally get detections of the "entities" outside the klystron, sometime occurring one or two weeks following the arc. Hence, the more the klystron is insulated from outside influence, the better Santilli experiment can be faked.

4) Santilli experiment can be easily repeated with various neutron detectors none of which is Li-activated, then "fix things" with a sufficiently low power and gas pressure, to end up with signals clearly not of neutron type, thus "disproving Santilli claims." It has been indicated in the preceding sections that, for reasons unknown at this writing, Lithium activation is, by far, the most sensitive to the "entities." Thus, Li-activated detectors can be studiously avoided to serve interests on Einstein. Additionally, it is very easy to select detectors solely sensing to gammas, rather than the "entities, " thus reaching the preset aim of "disproving Santilli claims." The "entities" are not neutrons. Hence, it is easy to select detectors that are insensitive to the "entities;" the latter are then absorbed by the casing; resulting in sole gamma emissions. This is an excellent way to fake Santilli experiment because it allows the corrupt physicists to make the beloved claim that "no neutron has been produced in the test."

5) It is very easy to fake Santilli experiment via the mere use of the Tables of Nuclides. In fact, the transmuted nuclides caused by the absorption of the "entities" positively are not listed in the Table of Nuclides and, consequently, they do not exist. For the serious physicist we recall that the claim of production in Santilli experiment of true neutron, with consequential claims of activating conventional nuclides, is a direct admission of the continuous creation of matter in the universe. But then, the only possibility of avoiding such extreme implications is to admit that the "entities" are not neutrons, and, consequently, the activated nuclei are not listed in the Table of Nuclides.

6.2.13 Stimulated Decay of the Neutron

6.2.13.A The Dominant Factor in the Stimulated Decay of the Neutron: Ethical Decay in Physics

The neutron is *naturally unstable* (when isolated or part of certain nuclei). Moreover, depending on its nuclear environment, it exhibits a variety of meanlives from seconds all the way to thousands of years. Hence, it is quite plausible that the neutron admits one or more mechanisms called *triggers* (TR) capable of stimulating,

$$TR + n \ \rightarrow \ p^+\beta^- \tag{6.2.218}$$

where β^- can be either conventionally interpreted as an electron and a neutrino, or as an electron and an antietherino. This alternative is irrelevant for the topic of this section and, consequently, it will be ignored.

We shall merely assume the conventional nuclear interpretation of β^- as carrying a negative charge, the conventional rest energy of the electron, and null spin due to antiparallel alignment of the electron and the neutrino/etherino (when the trigger has no spin). Other conventional interpretations of β^-, e.g., when carrying spin 1 due to parallel alignment of the electron and the neutrino/etherino, will be indicated when needed.

The systematic search for the possible stimulated decay of the neutron is an ethical duty of the physics community because, in case confirmed, it creates a new type of energy called by Santilli [125] *hadronic energy,* in the sense of originating from mechanisms in the interior of hadrons, rather than in their collection as typically the case for the *nuclear energy.*

Under a possible stimulated decay, the neutron would release a highly energetic electron that can be easily trapped with a metal shield thus producing heat, plus a possible difference of electric potential between the metal shield and the original material in the event the latter is a conductor (see below).

Besides new energies, the stimulated decay of the neutron would create a new technology, called *hadronic technology,* in the sense indicated above of being based on mechanisms inside hadrons. To our best knowledge, we are referring to the first conceivable possibility of practical applications of hadron physics, since the hadron physics of the 20-th century under the control of quark believers had positively none.

The biggest problem in the study of the possibility to stimulate the decay of the neutron is not the technical profile, since all tests in the field are very simple and immensely less expensive than particle experiments currently preferred at SLAC, FERMILAB, CERN, SACLAY, DESY, RUTHERFORD, JINR and other laboratories around the world.

The biggest problem is given by the widespread ethical decay in physics because, as it is th case for the neutron synthesis, the stimulated decay of the neutron is irreconcilably incompatible with Einsteinian doctrines and quantum

mechanics[61] Consequently, organized academic, financial and ethnic interests on Einsteinian doctrines and quantum mechanics have perpetrated incredible ascientific, amoral and asocial acts of discrediting, jeopardizing and disrupting professional theoretical and experimental research in the field, as documented in this volume.

As it was the case for the laboratory synthesis of the neutron, the study of its stimulated decay without the joint addressing of issues pertaining to scientific ethics and accountability would be so hypocritical to be repugnant to this author.

Hence, the author feels an obligation of continuing the presentation of technical aspects while jointly denouncing organized scientific crimes in the field for judgment by our contemporary society, due to the pressing need for new energies, let alone a judgment by posterity.

Fortunately for society, the industry has responded to the need for new clean energies with investments in the stimulated decay of the neutron for the very reason that originates academic obstructions, *novelty.* However, industrial investments carry restrictions in disclosure because the abuse of academic authority would damage the investment, as desired by academia with due exceptions.

In fact, the author is conducting research under industrial contracts with various confidentiality restrictions. Hence, in this section we will be in a position of solely presenting information authorized for disclosure by the industry, namely, information that is old by industrial standards.

6.2.13.B The Main Hypothesis on the Neutron Stimulated Decay

The alternatives in the possible stimulated decay of the neutron are the following two:

ALTERNATIVE I: The constituents of the neutron are quarks conceived as physical particles in our spacetime.

In this case there is no possibility whatsoever, whether remove or conceivable, to stimulate the decay of the neutron, trivially, because the quarks are believed (but never rigorously proved) as being permanently confined inside the neutron. This is a reason political alignments on quark beliefs are considered a threat to society.

It should be clarified with clarity that physicists do have indeed the right to believe whatever they wish. Scientific crime occur when quark believers (currently in control pf the particle physics community) abuse their academic power to suppress, discredit and jeopardize alternative views.

[61] A variation of lifetime of the neutron would mandate the abandonment of the central pillar of special relativity, the Poincaré symmetry for broader vistas.

Ordinary crimes occur when quark believers continue to misuse public funds by ignoring, rather than disproving in refereed journals, the litany of inconsistencies for quarks to be physical particles published in refereed journals and reviewed earlier (lack of gravity due to lack of definition in our spacetime, absence of inertia, impossibility of a serious confinement due to Heisenberg uncertainty principle, etc.)[62]

ALTERNATIVE II: The constituents of the neutron are particles permitting its synthesis, the proton and the electron.

In this case, the possibility to stimulate the decay of the neutron becomes plausible and quantitatively treatable, provided that one uses the covering hadronic mechanics evidently, because Alternative II is irreconcilably incompatible with quantum mechanics.

In fact, hadronic mechanics predicts various triggers for he stimulated decay (6.2.218) depending on whether acting in the structure of the neutron or in its nuclear environment, each case having various possibilities.

The biggest contributions of hadronic mechanics to the study of the neutron has been the achievement of a consistent representation of the electron as a physical constituent of the neutron, as well as the identification of its rest energy.

Said energy cannot be conventional, due to its immersion within the hyperdense medium inside the proton with consequential deviation from the Minkowskian spacetime experimentally verified in Section 6.1. Due to these mutations, in the transition from the spacetinme of the vecuum to that within the hyperdense medium inside the proton, the electron experiences mutation (6,2,163b) of its rest energy we have called *isorenormalization,* i.e.,

$$E_e = 0.511\ MeV\ \rightarrow\ E_{\hat{e}} = 1.294\ MeV. \tag{6.2.219}$$

in which case the electron is called *isoelectron.*

An additional contribution of hadronic mechanics crucial for the task here at hand is that the isoelectron is essentially free at MeV scale, a feature called *hadronic freedom*

$$BE_{p-\hat{e}} \approx 0. \tag{6.220}$$

As assumed to be familiar by now, this is due to the fact that the force binding the un-mutated proton p and the mutated electron $\hat{e}$ is of contact non-potential

[62]As editor in chief of the *Hadronic Journal* and of other journals, Santilli has made it a point in accepting for publication papers on quark conjectures, even though he does not believe that quarks are physical particles. The understanding is that the *Hadronic Journal* has always been and will. always be available for the publication of qualified alternative or dissident views. By comparison, the journals of the American Physical Society, the British Institute of Physics, and other physical societies solely publish papers on inconsistent, yet politically aligned quark theologies while rejecting qualified alternative or dissident views with motivations whose scientific credibility can only be qualified as being pathetic.

character, thus causing no binding energy, while the binding energy due to the Coulomb attraction is too small for the approximation here considered.

Since the neutron is naturally unstable, and its meanlife varies with its environment, Santilli proposed, apparently for the first time in Ref. [125] of 1994 (see review [126] of 1996), to *test the possible stimulated decay of the neutron via the use of photons γ_r with the resonating (r) energy (or frequency) of* $1.294\ MeV$*, plus expected adjustments for binding energies in nuclei:*

$$\gamma_r + (p^+, \hat{e}^-)_{hm} = \gamma_r + n \ \rightarrow \ p^+ + \beta^-, \tag{6.2.221}$$

where the "resonating photon" has the characteristic energy or frequency

$$\gamma_r = 1.294\ MeV,\ or\ 3.129 \times 10^{20}\ Hz, \tag{6.2.222}$$

with harmonics

$$\gamma_r = (1.294 \times n)\ MeV\ or\ (3.129 \times n) \times 10^{20}\ Hz, \tag{6.2.223}$$

and subharmonics

$$\gamma = (1.294/n)\ MeV,\ or\ (3.129/n) \times 10^{20}\ Hz, \quad n = 1, 2, 3, ... \tag{6.2.224}$$

A second frequency of the resonating photon that should be taken into account is the standard characteristics frequency of the electron, that corresponding to the conventional value of its rest energy, namely,

$$\gamma_r = 0.511\ MeV,\ or\ 1.236 \times 10^{20} Hz. \tag{6.2.225}$$

Another possibility to stimulated the decay of the neutron (we can disclose here because presented in Ref. [125]) is by using photons with a wavelength equal to the neutron size. In the former case, the aim is to excite the mutated electron inside the neutron, while in the latter case the objective is to excite the entire neutron. The expulsion of its structural electron and consequential decay is then expected from the above indicated hadronic freedom.

The reader should keep in mind that isorenormalization (6.2.219) depends on the density $b_4 = 1.65$ of the hadronic medium inside the proton, which value has allowed the exact representation of the energy, spin and magnetic moment anomalies of the neutron, and has been independently verified by the experiments on the Bose-Einstein correction of Section 6.1.10.

Note also that the "density" of hadrons has been studied by hadronic mechanics for the first time and completely ignored by orthodox particle physics because incompatible with Einsteinian doctrines. Hence, expecting final knowledge in first instance would be presumptuous (or politically motivated).

Consequently, the serious reader should accept value (6.2.222)-(6.2.224) as merely preliminary and in need of a number of corrections, some of which known,

and other possible. Among the known corrections, we mention the need to adjust value (2.1.222) for nuclear binding energies when dealing with neutrons as part of a nuclear structure.

Among other conceivable corrections, we mention the possibility that deeper studies on the structure of the neutron might identify a currently unknown, potential binding force between the proton and the electron significantly stronger than their Coulomb binding force, in which case value (6.2.222) must be corrected accordingly.

Stimulated decay (6.12.221) is impossible for quantum mechanics because the cross section of a photon with the neutron is so small to be ignorable at all energies. Hadronic mechanics does recover such a behavior, with the exception of the specific value of 1.294 MeV at which the cross section is predicted to have a typical resonating peak. This behavior requires the treatment via the *isoscattering theory* not treated so far in these volumes. Interested readers may consul Chapter XII of EHM, Volume II.

The case is similar to the discovery in the 1960s of the Λ particle at CERN. A group of physicists predicted a flat cross section and no new particle. Other physicists looked for novelty, predicted a resonating peak, looked for it, and found it.

Dismissals of tests of hypothesis (6.2.221) via the use of quantum mechanics are here denounced as a blatant scientific crime because quantum mechanics was conceived to describe point particles moving in vacuum, thus being inapplicable within the hyperdense medium inside the neutron. At any rate, quantum mechanics can only represent the neutron as a dimensionless point, while the neutron in reality is an extended object.[63]

At any rate, hypothesis (6.2.221) is quantitatively predicted by hadronic mechanics from primitive axioms without adulterations and, independently from that, hypothesis (6.2.221) is plausible, thus requiring its *experimental* resolution as the only serious way to do science. Theoretical theologies in lieu of the experimental resolution are the way fanatic supporters of Einsteinian doctrines dreams of continuing to control the minds of researchers for their personal gains.[64]

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[63]A basic characteristics of hadrons that is missing in the otherwise vast information released by the Particle Data Group is that of the *size* of hadrons. This absence is shocking since said size is so fundamental that no structure model can be considered credible unless it represents quantitatively said size, as it was historically the case for the hydrogen atom. The only credible explanation is that the indication of the size, that is, of the extended character of hadrons, would attract attention on the incompatibility *ab initio* of hadrons with Einsteinian doctrines and quantum mechanics, since the latter can solely represent particles as dimensionless points.

[64]Einsteinian doctrines and quantum mechanics are assumed as being exact inside the neutron by the organized scientific crime via the abstraction of its interior to points, the selection of the constituents to forms that are compatible with the assumed theories, and the complete oblivion of catastrophic inconsistencies of such theologies identified before. Hence, the reason for the widespread fanatic belief in quark as physical particles can be understood only after the knowledge that quarks were introduced

It should be stresses that, by no means, the use of resonating photons is the only possibility to study the stimulated decay of the neutron, since numerous alternatives are possible (some of them being under industrial study and cannot be disclosed at this writing). Hence, the experimental issue is the study at large of *all* possibilities to stimulate the decay of the neutron, among which the use of resonating photons for the isoelectron and that for the neutron are only two among other possibilities.

Hence, any claim of the lack of stimulate the decay of the neutron following a possible insufficiency of numerical value (6.2.222) would be a scientific crime for personal gains.

6.2.13.C Proposed Tests for Stimulated Neutron Decays

In practical set ups, the stimulated decay of the neutron cannot possibly be tested for isolated neutrons as in reaction (6.2.221) but only when neutrons are members of a nuclear structure. Hence, the tests here considered deal with the *stimulated nuclear transmutations* of the type here written in terms of conventional nuclear specifications restricted for simplicity to atomic number A, nuclear charge Z and nuclear spin J [125]

$$\gamma_r(0,0,1) + TR + N(A,Z,J) \ \rightarrow \ N(A,Z+1,J+1) + \beta^-(0,-1,0), \quad (6.2.226)$$

under the verification of all nuclear conservation laws and superselection rules, including the conservation of the energy, charge, angular momentum, parity, etc. Additionally, the resonating frequency has to be adjusted for nuclear binding forces of proved potential origin. Finally, in certain cases, the resonating photon alone may not be sufficient, thus requiring an additional trigger.

The mechanism here considered for stimulated decay (6.2.226) is elementary. The resonating photon hitting a nucleus is expected to excite a peripheral isoelectron irrespective of whether the photon penetrates or not inside the neutron. Once excited, there is no possibility for the isoelectron other than that of leaving the neutron structure, thus causing its stimulated decay.

This is due to the fact that hadronic mechanics predicts one and only one energy level for the neutron as a generalized bound state of the proton and the electron in conditions of total mutual immersion. The range of hadronic mechanics is essentially given by the radius of the neutron (1 fm). Once excited, the isoelectron has no other possibility than that of exiting the proton and reassuring its conventional quantum features for moving in vacuum.

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as physical particles for the premeditated and studious intent of preserving the validity of Einsteinian doctrines and quantum mechanics inside hadrons.

Since in practical applications nuclei will not be hit by individual resonating photons, but by a coherent beam of the same, Santilli [125] also proposed the study of possible multiple stimulated decays of peripheral neutrons in a nucleus

$$n \times \gamma_r(0,0,1) + N(A,Z,J,0) \ \rightarrow \ N(A,Z+n,J+n) + n \times \beta^-(0,01,0), \quad (6.2.227)$$

where n = 1, 2, 3, ...

The first example suggested by Santilli for test is given by the following use of a photon with the needed resonating frequency without any trigger [125]

$$\gamma_r(0,0,-1,0) + Li(6,3,1) \ \rightarrow \ Be(6,4,0) + \beta^-(0,-1,0), \quad (6.2.228a)$$

$$Be(6,4,0) \ \rightarrow \ He(4,2,0) + 2 \times p(0,+1,1/2), \quad (6.2.228b)$$

where, by using the data in the Table of Nuclides (http://atom.kaeri.re.kr/): $Li(6,3,1)$ is naturally stable with mass 6.0151223 u; $Be(6,4,0)$ has the mass of 6.0197258 u, it is naturally unstable and decays in 5 days as shown; and $He(4,2,0)$ is stable.

Note that, for reactions (6.2.228) to take place, it is necessary that the resonating photon and the Lithium have opposite spin polarization (indicated with J = -1). Also, $Li(6,2,0)$ is *lighter* that $Be(6,3,0)$,

$$M(6,3) - M(6,4) = 6.0151223 - 6.0197258 \ u =$$

$$= -0.0046035 \ u = -4,28813276 \ MeV. \quad (6.2.229)$$

Hence, the above stimulated nuclear transmutations require a resonating frequency with the minimum energy of 6.258 MeV corresponding to values (6.2.223) for $n = 2$.

Another example of stimulated decay of the neutron suggested for tests is given by the isotope $Zn(70,30,0)$ for which we have the predicted reactions [125]

$$\gamma - r(0,0,1,0) + Zn(70,30,0) \ \rightarrow \ Ga(70,31,1) + \beta_1^-(0,-1,0), \quad (6.2.230a)$$

$$Ga(70,31,1) \ \rightarrow \ Ge(70,32,1) + \beta_2^-(0,-1,0), \quad (6.2.230b)$$

where, by using data from the Table of Nuclides, we have

1) $Zn(70,30,0)$ has the mass of 69.9253249 u and it is stable;

2) $Ga(70,31,1)$ has the mass of 69.9260277 u, is naturally unstable and decays into $Ge(70,32,1)$ with half life is of 21.14 min;

3) $Ge(70,32,0)$ has the mass of 69.9242504 u and is stable.

As one can see, the mass of $Zn(70,30,0)$ is *smaller* than that of $Ga(70,31,1)$, with negative energy difference

$$M(70,30) - M(70,31) = -0.0007028u = -0.65465MeV. \quad (6.2.231)$$

Consequently, stimulated nuclear transmutations (6.2.130) are possible if and only if the resonating photon has an energy bigger than 0.654 MeV, since all other conservation laws and superselection rules are verified.

This condition can be easily verified with resonating energy (6.2.224) with minimum value of $n = 1/4$ corresponding to 0.782 MeV (ignoring corrections due to binding energy for simplicity).

Note that the difference between the mass of the initial and final isotope is positive,

$$M(70, 30) - M(70, 32) = 0.0010745u = 1.00089033MeV. \tag{6.2.232}$$

Hence, under the minimal resonating energy of 0.782 MeV, stimulated nuclear reactions (6.2.230) have the *positive* energy output

$$M(70, 30) - M(70, 32) - E(\gamma_r) = 0.218\ MeV. \tag{6.2.233}$$

Another example suggested for tests is given by

$$\gamma_r(0, 0, 1, 0) + S(32, 16, 0) \ \rightarrow \ Cl(32, 17, 1) + \beta^-(0, -1, 0), \tag{6.2.234a}$$

$$Cl(32, 17, 1) + EC \ \rightarrow \ \alpha + Si(28). \tag{6.2.234b}$$

where: $S(32, 16, 0)$ has the mass of 31.9720707 u; $Cl(32, 17, 1)$ has mass 31.9856889 u, is naturally unstable and decays as shown in 298 ms. Again, the mass of S is smaller than that of Cl. Nevertheless, the stimulated decay of the neutron is possible by selecting a suitable resonating frequency as for the cases above.

Note that there are numerous possible selections of similar reactions, not only for the case of one but also for double or triple stimulated decays per nucleus, that cannot possibly be studied at this time, and are left to the young minds of any age.

Additionally, it should be stressed that proposal [125] is old by current industrial standards and a number of additional possibilities are under investigation by the industry. This is the reason we denounce as scientific crime any dismissal of the stimulated decay of the neutron solely based on old theoretical theologies without systematic *experimental* studies.

Final;ly, it should be stressed that *the stimulated decay of the neutron recommended for test is not for isolated neutrons, but within the context of "nuclei." But the synthesis of the neutron here considered has generated a dramatic revision of nuclear structures studied in the next section. Hence, any posturing of judgments on the proposed stimulated decay prior to an in depth study of the new vistas in nuclear structures is vulgar scientific corruption by immoral outcasts.*

6.2.13.D Classification of Hadronic Energies

We are finally equipped to introduce the main objective for which hadronic mechanics was built for, the conception and quantitative treatment of basically

new energies. They were first indicated in the original proposal [14] of 1978, treated inj detail in Ref. [125] of 1994 and further developed in monograph [99] of 1998:IndexClassification hadronic energies

DEFINITION 6.2.1: Hadronic energies are all forms of energies that cannot be predicted via quantum mechanics, but can be predicted and quantitatively treatable via the covering hadronic mechanics, and can be classified as follow:

CLASS I, when occurring at the level of individual hadrons;

CLASS II: when occurring at the level of nuclei; and

CLASS III; when occurring at the level of atoms or molecules.

In this section we shall conside one example of hadronic energies of Class I. Those of Class II will be considered in section 6.3 of this volume and those of Class III will be considered in Chapter 11 of Volume V.

6.2.13.E Examples of Hadronic Energies of Class I

Example of Hadronic Energies of Class I were submitted in Ref. [125] via the following stimulated decays,

$$n \times \gamma_r(0,0,1) + TR + N(A,Z,J) \ \rightarrow \ N(A,Z+n,J+m) + n \times \beta^-(0,-1,0), \tag{6.2.235a}$$

$$N(A,Z+n,J+m) \ \rightarrow \ N(A,Z+2,J+m) + \beta^-(0,-1,0), \tag{6.2.235b}$$

$$N(A,Z+1,J+m) \ \rightarrow \ ?, \tag{6.2.235c}$$

where: the first reaction is stimulated and the otehrs are spontaneous; and the spin of $N(A,Z+2,J+m)$ is not assumed to be necessarily $J+n$ due to possible opposing polarizations and other effects.

The original isotope should be selected in such a way to meet the following conditions:

1) The originl isotope admits the stimulated transmutation while verifying all conservation laws of the energy, angular momentum, etc.;

2) The resulting isotope admits a spontaneous β decay so that with one resonating photon we have the production of two electrons whose kinetic energy is trapped with a metal shield to produce heat;

3) The original isotope is selected to be a metal so that, following the emission of two electrons, it acquires an electric charge suitable for the production of a DC current between the metal isotope and the metal shield;

4) The energy balance is positive; and, last but not least

5) The initial and final isotopes are light, natural and stable elements so as to have a new energy that is *clean* in the sense of producing no harmful radiations (since the electrons can be easily trapped with a thin metal shield), and leave no radioactive waste.

When the above conditions are met, the original isotope is called *hadronic fuel* and the equipment used for its realization is called *hadronic reactor* [125]. It

should be stressed that, in accordance with Definition 6.2.1, the word "hadronic" is not referred to strongly interacting particles, but intended to emphasize the need of hadronic mechanics for quantitative studies.

Ref. [125] then suggested the concrete case for testing the existence of Hadronic Energies of Class I based on the isotope $Mo(100, 42, 0)$, reproduced here as originally proposed, namely, without the trigger developed subsequently, because unavailable in 1994,

$$\gamma_r(0, 0, 1) + Mo(100, 42, 0) \ \rightarrow \ Tc(100, 43, 1) + \beta^-(0, -1, 0), \qquad (6.2.236a)$$

$$Tc(100, 43, 1) \ \rightarrow \ Ru(100, 44, 0) + \beta^-(0, -1, 1), \qquad (6.2.236b)$$

where, by using again the data from the Table of Nuclides, we have:

1) $Mo(100, 42, 0)$ has the with mass of 99.9074771 u and it is stable;

2) $Tc(100, 43, 1)$ has a mass of 99.9076576 u and is naturally unstable with half life of 15.8 s and sole decaying mode into $Ru(100, 44, 0)$;

3) $Ru(100, 44, 0)$ has themass of 99.9042197 u and it is stable.

As one can see, the mass of $Mo(100, 42, 0)$ is *smaller* than that of $Tc(100, 43, 1)$,

$$M(011, 42) - M(100, 43) = 0.0001805u = 0.16803 \ MeV. \qquad (6.2.237)$$

Yet it can be verified with the minimal subharmonic resonating energy (6.2.224) for $n = 7$ (again, ignoring correctsions due to binding energies)

$$E(\gamma_r) = 0.1848 MeV. \qquad (6.2.238)$$

But the mass of the original isotope is *bigger* than that of the final isotope,

$$M(100, 42) - M(100, 44) = 0.0032574 \ u = 3.03424865 \ MeV, \qquad (6.2.239)$$

thus resulting in a positive energy output.

Hence, stimulated nuclear transmutation (6.2.236) verifies all nuclear conservation laws, while the energy output is bigger than the resonating energy, the case thus qualifying as an example of hadronic energy (HE) of Class I, with total energy output per reaction

$$HE = M(100, 42) - M(100, 44) - E(\gamma_r) - 2 \times E(e) =$$

$$= 3.034 - 0.184 - 1.022 \ MeV = 1.828 \ MeV, \qquad (6.2.240)$$

where we have subtracted the conventional rest energy of the two electrons because not usable as a source of energy in this case.

The predicted hadronic energy in this case is *two-fold,* because we first have the production of it heat acquired by the shield capturing the electrons and, jointly, we have the production of a DC *electric current* between the metal isotope $Mo(100, 42, 0)$ acquiring a positive charge due to the loss of two electrons per

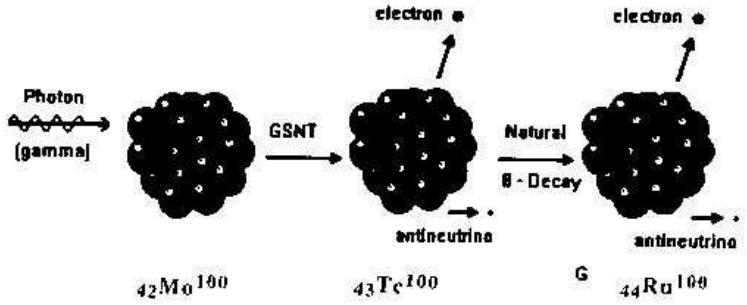

Figure 6.34. A schematic view of the proposal of Ref. [125]: use a coherent beam of resonating photons to stimulate the transmutation of the isotope $Mo(100,42)$ into $Tc(100,43)$ with the emission of a first highly energetic electron, followed by the spontaneous decay of $Tc(100,43)$ into $Ru(100,,44)$ with the emission of a second highly energetic electron, the mass of $Mo(100,42)$ being sufficiently bigger than the mass of $Ru(100,44)$ to allow a positive energy output after substracting the energy of the resonating physons, while verifying all remaining nuclear laws.

reaction, and the metal shield acquiring two negative charges, by keeping into account that each resonating photon produces two electrons.

To appraise the usable energy, let us recall the following units and their conversions

$$1\ u = 931.494\ MeV;\quad 1\ MeV = 1.602 \times 10^{-13} J = 4.45 \times 10^{-17} Wh =$$

$$= 1.511 \times 10^{-16}\ BTU;\quad 1\ Wh = 3.397\ BTU; \tag{6.2.241a}$$

$$1\ C = 6.241 \times 10^{18}\ e;\quad ,1\ A = 1C/1\ s, \tag{6.2.241b}$$

where e is the elementary charge of the electron.

Then, as an illustration, under the assumptions of using a coherent beam with resonating photons hitting a sufficient mass of $Mo(100,42,0)$ suitable to produce 10^{20} stimulated nuclear transmutations (6.2.236) per our, we have the following (see the figure):

Hadronic production of heat;

$$2 \times 10^{20}\ MeV/h = 3 \times 10^{4}\ BTU/h, \tag{6.2.242}$$

Hadronic production of electricity:

$$2 \times 10^{20} e/h = 200C/h = 55\ mA. \tag{6.2.243}$$

A few comments are now in order. We have presented an example of Hadronic Energy of Class I in its original formulation of Ref. [125] with the sole

addition of clarifications to show its plausibility, but without any addition of the industrial knowledge achieved since 1994. This has been necessary to prevent the usual non-technical attacks by academia toward novelty with evident damage to environmental research.

Evidently, the output of heat and electricity of the above proposal is moderate. Nevertheless, the use of $Mo(100, 42, 0)$ is a mere indication out of a predicted number of possibilities with heavier isotopes and expected bigger production of heat and electricity, partiularly under the use of an appropriate additional trigger.

Despite these shortcomings, *the example of hadronic energy here considered does indeed illustrate the possibility of reaching, in due time, a new form of nuclear energy without the release of harmful radiations and without leaving radioactive waste.* In fact, the electrons are not considered harmful radiations, while both the original and final isotope are light, natural, and stable elements.

In the event the objective is the production of large amounts of energy, it is necessary to consider heavy nuclei and more suitable triggers. As a working hypothesis among many, we suggest for study the possibility of using *a coherent beam of resonating photons to transmute* $U(235, 92, 7/2)$ *or other heavy nuclei into isotopes admitting spontaneous fission but without the emission of neutrons, a possibility evidently without military relevance, but of clear environmental value.* The chain reaction can then be maintained by continuing to use the beam of resonating photons as well as the trigger.

The latter possibilities are the target of academic obstructions and disruptions due to their novelty. Additionally, the equipment for their safe test is not available in most corporate laboratories. Hence, Santilli has elected to ignore them.

At this point we would like to make a comparison between the first nuclear energy, that predicted by the Italian physicist Enrico Fermi at the University of Rome, Italy, in the 1930s, and the new energy proposed by another Italian physicist, Ruggero Maria Santilli.

Fermi was forced to work with the theoretical knowledge and technologies of the 1930s essentially consisting of quantum mechanics and the use of neutrons to stimulate *nuclear fission.* This resulted in a form of energy, that was indeed historical at the time of its conception, but which is today considered environmentally insufficient due to the production of harmful radiations and the release of radioactive waste. Note that these features are inherent in the selection of *heavy nuclei.*

Santilli uses the much more advanced theoretical knowledge of the 21-st century, as well as a variety of new technologies not available during Fermi's times. These new conditions have permitted Santilli to search for new forms of nuclear energies originating from *light nuclei,* since in the latter case there is no sufficient energy to produce harmful radiation or to leave dangerous waste.

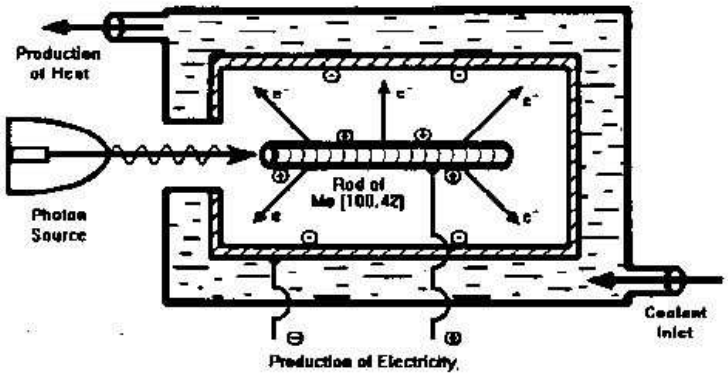

Figure 6.35. A conceptual view of the Hadronic Energy of Class I proposed for test in Ref. [125] of 1994. A coherent beam of photons with the needed resonating frequency (on the left) hits a cylindrical rod of $Mo(10,42)$ (in the center), with stimulated nuclear transmutations (6.2.236). The proposal predicts the production of two highly energetic electrons per each resonating photon that are trapped via a metal shield, thus producing heat usable via an exchanger. Since $Mo(10,42)$ is a conductor, there is the prediction of a second source of energy given by a DC current between the shield and the $Mo(100,42)$. Note that the energy of the photons not causing nuclear transmutations (6.2.236) is not lost because it becomes part of the heat produced. Note finally that the efficiency of nuclear transmutations (6.2.236) can be enhanced via suitable triggers in addition to the resonating frequency and that the use of suitably isotopes heavier than $Mo(100,42)$ would produce bigger energies.

The biggest difference between Fermi's and Santilli's times is, however, the collapse of scientific ethics in academia occurred since the 1930s. This ethical collapse is the primary origin for the lack of solution until now of our alarming environmental problems, the need of surpassing Einsteinian doctrines and quantum mechanics, e.g., via irreversible coverings to achieve a credible representation of notoriously irreversible energy releasing processes, while organized interests in academia strongly opposes the establishing of said covering theories.

In fact, Fermi's rudimentary ideas met with a very receptive, cooperative and supportive scientific environment in the USA, and the rest is well known history. By comparison, Santilli has met to this writing (February 2008) incredible oppositions, obstructions and disruptions in theoretical, let alone experimental studies of possible new energies, as documented beyond "credible" doubt in Refs. [89,90] and in the footnotes of this volume.[65]

[65] A routine posturing by a number of (but not all) academicians exposed to the need for supporting the study of "new" energies of the type presented in this section, is that "the proposal is not properly developed in a form suitable for tests." The corrupt character of the posturing is soon exposed by noting that, by applying the same rule, when exposed to therudimentary idea Enrico Fermi brought from Italy,

It is hoped receptive readers (as well as academicians) in good faith who care about science and the future of their own children understand the necessity of denouncing these obstructions as *organized scientific crimes* because clearly damaging the human society, since they manifestly damage the study of much needed new clean energies.

6.2.14 Tsagas experiment on the Stimulated Neutron Decay

The experimental verification of stimulated nuclear transmutation (6.2.236) was initiated by N. Tsagas and his group [127] at the Nuclear Engineering Department of the University of Thrace, Xanthi, Greece, with preliminary, yet positive results.

The test was conducted in the following way: 1) using a disk of $Eu(152, 63, 3)$ as the source of resonating photons (see again the Table of nuclides http://atom.kaeri.re.kr/); 2) placing said disk next to a disk of *natural* Molybdenum as target; and 3) measuring the background without any source, the emission with the Europa source alone, and the emission with the joint disks of Europa and natural Molybdenum.

Electrons originating from the Compton scattering of photons with peripheral atomic electrons can at most have 1 MeV energy, as well known. Therefore, the detection of electrons with energy over 2 MeV establishes their nuclear origin.

Since the Europa source does not emit electrons, and the Molybdenum is stable, the only possible origin of emitted electrons is due to the stimulated decay of neutrons inside the Molybdenum disk. As recalled earlier, the first reaction (6.2.236a) emits electrons with minimal energy of 2.8 MeV, while the second reaction emits electrons with energy ranging from 2.22 MeV to 3.38 MeV.

It should be indicated that Tsagas?s test [127] is limited because:

A) te tests used ordinary Molybdenum, that contains the isotope $Mo(100, 42, 0)$ only in 0.6 %, while all the remaining, rather numerous stable isotopes of the Molybdenum cannot admit the stimulated decay here considered for various reasons (see the preceding subsection).

B) The primary frequency emitted by the selected radioisotope, 1.874 MeV, *is not* the resonating frequency that should instead be 1.294 MeV less the adjustment due to the nuclear binding energy, althouth $Eu(152, 63, 3)$ does emit a number of additional photons, one of which has the energy of 0.148 MeV close to the subharmonic of the resonating energy.

C) The tests solely used detectors of the *energy* of the emitted particle, without additional detectors for the identification of their nature.

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the President of the University of Chicago should have told Fermi "You must first build a prototype of your nuclear reactor and then I will hire you."

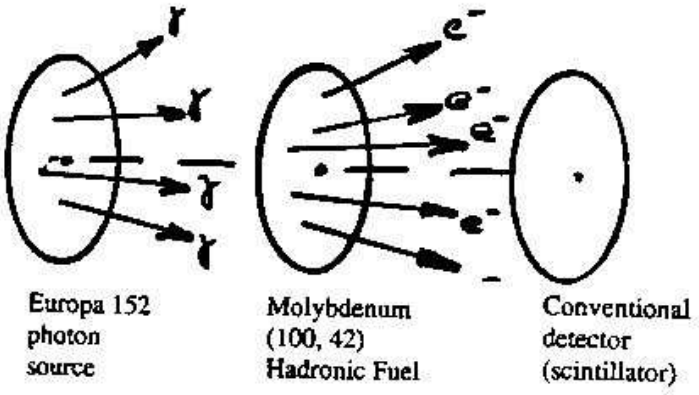

Figure 6.36. A schematic view of the first experimental verification of the stimulated decay of the neutron performed by N. Tsagas et al. [127], illustrating: the source of resonating photons (a disk of $Eu(152, 63, 3)$); the hadronic fuel selected for the test (a disk of natural Molybdenum) placed next to the Europa disk; and a scintillator capable of measuring the energy of the emitted electrons. Measurements of electrons with energy sufficiently bigger than 1 MeV establish the existence of the stimulated decay of the neutron beyond "credible" doubt because these electrons cannot be emissed by the indicated elements and cannot be of Compton origin, thus solely originating from a stimulated neutron decay.

Under these conditions, the possibilities of achieving reaction (6.2.236) are then rather limited. Yet Tsagas did indeed report the detection of emissions in the sole $Eu-Mo$ coupling in excess of 1 MeV, as shown in Figure 6.37, which unexpected occurrence renders Tsagas experiment even more interesting and meaningful, particularly in connection with the new vista on nuclear structures mandated by the structure of the neutron as a bound state of a proton and an electron of Section 6.3.

6.2.15 Santilli Experiment on the Stimulated Neutron Decay

Following Tsagas experiment [127] of 1996, Santilli proposed its repetition to numerous nuclear physics laboratories around the world, with the same results as those of the proposed experiments on the neutron synthesis, namely, obstructions and disruptions.[66]

[66] As an illustration, the editors of *Physics Review Letters* rejected a proposal to do the tests on grounds that "The author ignores the advances occurred during the past fifty years," which advances evidently refer to quark theologies without gravity inertia or confinement, neutrino beliefs with a chain of unverifiable conjectures each one in the scheme of supporting a preceding inconsistent conjectures and similar manipulation of knowledge for personal gains extremely disconnected from the quoted "knowledge."

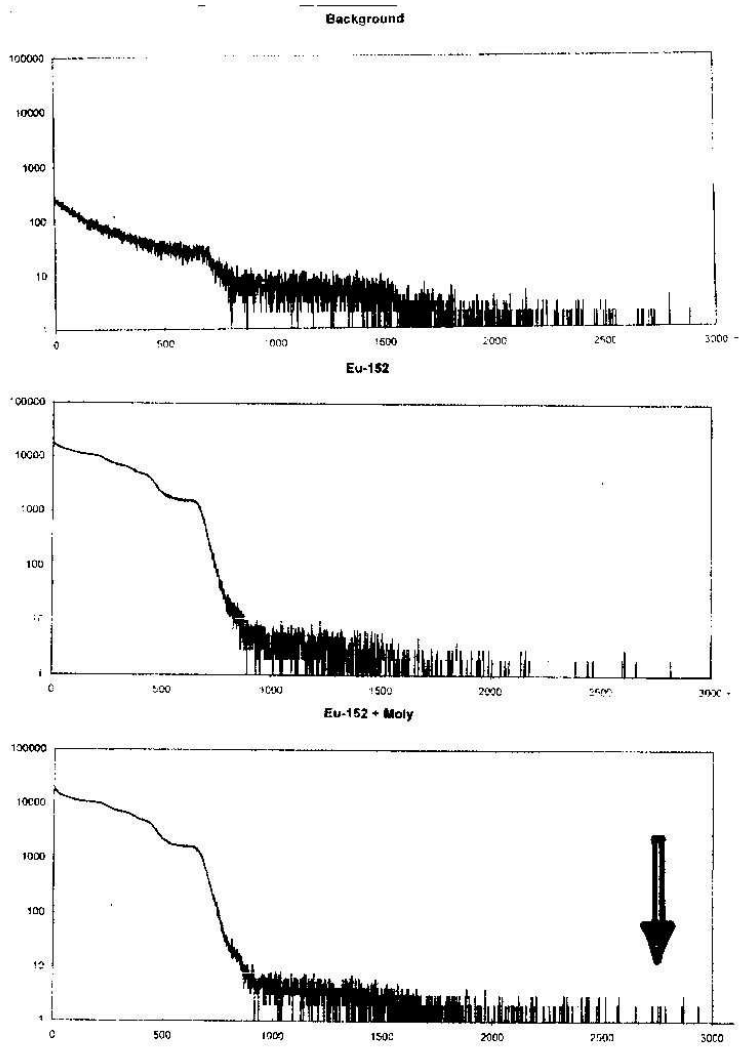

Figure 6.37. A reproduction of preliminary measurements by N. F. Tsagas and his team (private communication following ref. [127]) on the test of Santilli's hadronic energy of Class I via the model of Figure 6.36. The top view shows the background without any disk. The middle view shows the emission spectrum of Europa. The bottom view shows the emission spectrum of the coupled Europa and Molybdenum disks, with the clear presence of additional emissions with energy over 2 MeV, apparently confirming stimulated beta decay (6.2.236), opf course, in a preliminary way.

Due to the extreme conceptual, epistemological, mathematical, theoretical and experimental relevance of the needed tests, Santilli had no other alternative than that of repeating the test at the Laboratory of the Institute for Basic Research in Florida.

Santilli' s experiment was done: 1) Via the use of a pure isotope of $Mo(100, 42, 0)$; 2) the use of radioactive isotopes having the correct resonating frequency; 3) The use of energy measuring detectors; 4) The use of additional particle detectors; and 5) Conducting the test with and without additional triggers besides the resonating frequency.

Regrettably, as of today (March 10, 2008) Santilli has not received authorization by the industry providing all financial support to disclose details and results due to the ongoing collapse of ethics in academia.

6.2.16 Application to the Stimulated Decay of Radioactive Nuclear Waste

There is little doubt that the highly radioactive nuclear waste accumulating in our nuclear power plants is one of the biggest unsolved problems of contemporary society (for amn outstanding review, one should consult J. Dunning-Davies [100]).

In the U.S. alone we have over $150,000$ tons of radioactive waste currently stored at nuclear power plants. We have, therefore, surpassed the limits for safe storage in the pools of nuclear power plants. Europe has an even bigger tonnage of nuclear waste, and an unknown amount exists in other countries, all waiting for a serious addressing and a successful resolution. The yearly world production of about $30,000$ tons of additional waste then sets the premises for a truly serious environmental problem of potentially historic proportions.

As it is well known, the official position by the U.S., European and other governments has been that of transporting the highly radioactive nuclear waste to a dumping area. This "solution" has met with predictable resistance from environmental groups and residents near the proposed dumping grounds.

In fact, the transportation of the nuclear waste must occur on public streets, with evident dangers nobody can credibly deny. Assuming that local residents will permit the transit of such dangerous a material and in such a large amount, additional potentially catastrophic dangers exist in the intended storage of the nuclear waste, because it cannot be credibly predicted to be viable for tens of thousands of years.

This scenario has led to the current lack in the U. S. as well as abroad of a credible solution for the disposal of the highly radioactive nuclear waste.

After studying the problem for several years, Santilli proposed [128] 9see also Ref. [129,130]) the *conduction of systematic studies on the recycling of highly radioactive nuclear waste via its stimulated decay in the pools of the nuclear power*

plants themselves, so as to avoid its transportation altogether, as well as render nuclear power much more environmentally friendly.

In fact, the use of recent technologies allows the production of beams of coherent photons of the desired resonating energy with an equipment of moderate size, fully usable in nuclear power plants. Additionally, the new means for recycling nuclear waste are expected to create a new industry, that for the development, production and sale of the new equipment, that is expected to be needed by nuclear power plants all over the world.

Since the nuclei here considered are *very large and naturally unstable,* they are expected to admit a variety of means to stimulate their decay (rather than wait for their natural decay). Some of them are of Class II, namely, of nuclear character, that is, acting on the waste nuclei as a whole. These means will be considered in Section 6.3. In this section e consider means of Class I, namely, based on process occurring in the interior of the nuclear constituents.

The proposed recycling nuclear waste via the stimulated decay of the neutron consists of exposing the radioactive nuclear waste to an intense and coherent beam of photons with the needed resonating frequency, that can be obtained from a synchrotron of a few meters in diameter, or other equipment.

Under a certain intensity and other conditions, various peripheral neutrons of the nuclei are predicted to decay simultaneously, thus creating an*instantaneous excess of protons,* under which the stimulated decay is consequential, due to the extreme instability of these large nuclei.

On merely indicative grounds, consider the isotope $U(238, 92)$ of the waste released by nuclear power plants, that has the very long life of 4.51×10^4 years with the known harmful fission and the emission of alpha particles and other debris.

The use of the technology under consideration would imply the single stimulated transmutation (121)

$$\gamma_r(0, 1, 0) + U(238, 92, 0) \ \rightarrow \ Np(238, 93, 2) + \beta^-(0, -1, -1),$$

where Np(238, 93), which is also unstable, but with meanlife of 2.1 days , should itself be subjected to stimulated decay.

The double stimulated transmutations would yield

$$2 \times \gamma_r(0, 1, 0) + U(238, 92, 0) \ \rightarrow \ Pu(238, 94, 0) + 2 \times^- (0, -1, -1),$$

where $Pu(238, 94, 0)$, that is also unstable, should be subjected to stimulated decay until all unstable nuclei have experienced fission.

Possible tertiary artificial transmutations would yield $Am(238, 95)$ that is unstable with meanlife of 1.9 hours, spontaneous electron capture back to $Pu(238, 94)$ and emission of 2.3 MeV.

Needless to say, the objective is the recycling of nuclear waste and *not* the production of energy that, in any case, is plenty available in nuclear power plants. Hence, the possible release of energy in the above stimulated decays is irrelevant.

It should be stressed that Santilli's proposal [128] is the conduction of "comprehensive studies" in the stimulated decay of radioactive nuclear waste, because any expectation of quick solutions with little funds is either naive or corrupt, due to the dimension of the problem.

Also, the solution is not expected to originate from one single method, since it will require the combination of various methods, several of which have been already proposed and some of them patented (see Ref. [129] for brevity).

In closing it should be indicated that **Santilli has elected not to conduct any additional research in the stimulated nuclear decay because of threats not only to himself, but also to other scientists who tried to work in the field, some of them losing their job in the process.**

To give an idea of the organized scientific crime in the field, the reader in good faith should be aware that Santilli and his wife Carla organized some 18 international meetings in three continents. In 1998 they decided to organize a World Congress on Recycling Nuclear Waste to gather all the best scientists in the field and identify the needed research.

For that purpose the Santillis did set up the Scientific Committee one can see in web site [130] and attempted to organize the conference at the Nuclear Physics Department of the University of Florida in Ganeisville. Rather than assisting in the organization of a conference with such a transparent societal and environmental relevance, the outcome was such to discourage any additional attempt at organizing the conference anywhere in the USA, outcome that included the loss of a permanent job by a leading member of the Scientific Committee at a leading national laboratory.

Then, the Santilli attempted the organization of the same meeting in Europe by contacting the director of the time of the appropriate branch of the European Community in Bruxelles, C. Routti director of the EC XIi Division. Routti's behavior was so repulsive and obstructive to prevent any attempt at organizing the conference anywhere in Europe.

The announcement of the **World Congress on Recycling Nuclear waste** for the year 2000 has been left in the web site [130] as a documentation of the the fact that the lack of solution of the increasingly alarming environmental problems is due to a world wide collapse of ethics.

6.2.17 Hadronic Structure Model of Baryons with Physical Constituents

In preparation

6.2.18 Compatibility of the Hadronic Structure Models with SU(3)-Color Classifications

In preparation

6.2.19 Hadronic Structure Model of Mesons with Physical Constituents

In preparation

6.2.20 Hadronic Structure Model of Baryons with Physical Constituents

In preparation

6.2.21 Compatibility of the Hadronic Structure Models with SU(3)-Color Classifications

In preparation

6.3 EXPERIMENTAL VERIFICATIONS AND APPLICATIONS IN NUCLEAR PHYSICS

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6.3.1 Introduction

The achievement in the preceding chapter of an axiomatically consistent and time invariant representation of all characteristics of the neutron as a hadronic bound state of a proton and an electron, establishes that *nuclei are constituted by protons and electrons, the 20-th century view of being constituted by protons and neutrons being a mere first approximation.*

Serious scientists, politicians, educators and philosophers alike should never forget that stars initiate their life as being solely formed by hydrogen, that is, by protons and electrons, and that the protons and the electrons are the only known, massive, permanently stable particles. The posturing for over one century has been that the protons and the electrons "disappear" in nuclear syntheses and are replaced by neutrons for the studious adaptation of nature to preferred doctrines. Such a posturing is *de facto* scientific corruption for personal gains, because serious science requires the opposite, namely, the adaptation of the doctrines to physical evidence.

Similarly, it has been established in Section 6.2 that quantum mechanics cannot be exactly applicable to the simplest possible nuclear synthesis, that of the neutron. But then, any belief that quantum mechanics is exactly valid for more complex nuclear synthesis is sheer scientific corruption so damaging to science, particularly when proffered by experts, that must be denounced as such.

On theoretical grounds, the emerging new conception of nuclei permits the resolution of problems in nuclear physics that, having remained unsolved in over one century following the use of a river of public money, are nowadays simply embarrassing (to use an euphemism). In fact, the new conception of nuclei allows the first achievement of an invariant representation of *all* characteristics of the deuteron that, as denounced in Chapter 1, escaped resolution throughout the entire 20-th century.

On industrial grounds, the emerging new views on the structure of nuclei opens, by itself, virtually unlimited p;possibilities for new clean energies of direct social relevance, without known military applications (because occurring new light nuclei rather than heavy ones).

IN PREPARATION - NOVEMBER 1, 2005

For a summary of the content of this chapter, consult

The Physics of New Clean Energies and Fuels According to Hadronic Mechanics, R. M. Santilli, Special issue of the Journal of New Energy, 318 pages (1998).
See also www.neutronstructure.org

Foundations of Hadronic Chemistry with Applications to New Clean Energies and Fuels, R. M. santilli, Kluwer Academic Publishers, Boston-Dordrecht-London (2001).

Isodual Theory of Antimatter with Applications to Antigravity, Grand Unification and Cosmology, R. M. Santilli, Sprionger (in press)

Appendix 6.A
Ethical Problems in Particle Experiments

Only the most corrupt of a scientist can deny the existence of serious ethical problems in contemporary physics. The situation is so serious and the consequences are so grave, that *our contemporary society can be compared to the condition of the Roman empire prior to setting of Roman laws, because of basic insufficiencies of existing laws to address scientific crimes.*

Recall the Roman original definition of "crime" as damage to society. it is then evident that the manipulation by a physicist of scientific knowledge for personal gains causes damage to society dramatically bigger than ordinary crimes, such as a gun point stealing of money at a grocery store. The insufficiencies of current laws are then clearly established by the fact that the latter crime can indeed be punished with jail sentences, while the former is fully permitted despite its much more serious character.

The above view is not capricious, but based on personal experience. In fact, during the various personal attempts by the author to contain scientific corruptions, plagiarisms, frauds and other scientific crimes, judges and attorneys alike could not even understand the author's claims, let alone properly act on them in the protection of society (see http://www.scientificethics.org).

Exactly as it was the case for the Roman society over two millennia ago, our contemporary society will not enter into an era of great discoveries, capable of unthinkable advances, all the way to bring mankind to the stars (Section 6.1.3), until scientists, educators, economists, industrialists and politicians understand the need for, and implement *a new code of laws* encompassing also the control of scientific crimes.

In the Preface of this volume we indicated that the easiest manipulations of scientific knowledge occur in contemporary experiments because:

1) Manipulations of data to verify a preset theory are quite easy due to the complexity of the elaborations themselves;

2) The experimental data are generally elaborated via the very theory intended for verification, as a consequence of which, the "experimental results" must be compatible with the pre-set assumptions;

3) Very few events are often selected out of hundred of millions of events (as it is the case for the claimed "neutrino detection"), and then use of academic power to claim a pre-set result.

A dark shadow in the science history of the U. S. A. is the claim in 1995 by FERMILAB of the "discovery of the top quark" via its CDF and CO experiments with the additional claim to have "measured its mass" (174.2 GeV corresponding to the mass of a nucleus) [84]. In fact, the scientifically correct statement should have been the "detection of physical particles predicted by the unobservable top quark." At any rate, the same experimental results are admitted by other theories not assuming quarks as physical particles in our spacetime (see next chapter),

The claim to have "measured the top quark mass" passes all boundaries of serious science because quarks cannot have gravity, as well known to qualified experts (see Chapter 1 and next section), thus rendering "quark masses" mere ad hoc parameters introduced to fit a preferred theory. In any case, the unplausible high value of the "top quark mass" is a mere result of using an excessively elementary mathematics in excessively complex physical conditions because the use of isomathematics would dramatically reduce such an unreasonably high value the "quark mass" while keeping the same experimental data on physical, that is, actually observed particles.[67]

Similar dark shadows in the European history of science exist for the various claims at CERN, GRAN e and other laboratories to have "detected neutrinos" to the point of sending them across Europe from one laboratory to another, with the equal claim to have "measured neutrino masses" [85]. As limpidly stated by Enrico Fermi, "neutrinos cannot be directly detected" for the obvious reason that they are neutral./ hence, the scientifically correct statement should be the "detection of physical particles predicted by the neutrino hypothesis." Similar vast issues of scientific ethics occur in the very claim that neutrino have masses, l;et alone that they have been measured (see next chapter).

It is obvious to the educated observer in good faith that these far reaching and so objectionable claims are purely political motivations to secure money, prestige and power via the abuse of the credibility of the releasing institutions, for real science requires a dramatically more cautious language.

To illustrate the unreassuring condition of particle physics, in this appendix we show how easy is to manipulate experimental data for the pre-set objective of fitting the desired theory. The illustration is done by re-elaborating the data of Grossman's tests [53] and showing that they can be turned, from their claim of verifying Einsteinian doctrines, into a form showing deviations and full verification of Aronson's results [52].

To minimize additional scientific manipulations expected from this presentation, the author stresses that *no position is here assumed as to whether or not experiments [53[had indeed been manipulated to serve political interests, because*

[67]The reduction of current experimental beliefs on quark masses is a direct consequence of the strong convergence of divergent quantum perturbation series under isotopy (see EHM II and Chapter 3).

that position would be itself political, the only [possible scientific statement being lack of final experimental resolution at this writing one way or another.

The main objective of this section is to show the need for the conduction of contemporary particle experiments under the supervision of external Committee on Scientific Ethics and Accountability. Following fifty years of research experience, the author is forced to state again that *no basic advancement in scientific knowledge is possible without the joint consideration of scientific ethic s and accountability.*

To begin, the author wants to be on record to testify that, immediately following the appearance of Grossman's claims [53], all papers submitted to journals of the *American Physical Society* (APS) on possible deviations from the Einsteinian decay law were rejected by APS editors with written statements to the effect that "the verification of the Einsteinian decay law has been confirmed by Grossman's tests" [53].

This editorial posture must be denounced since a serious statement should have been "the validity of the Einsteinian decay law has been confirmed by tests [53] in the range from 100 to 400 GeV, but deviations have been reported by Aronson et al [52]," rather than the absolute confirmation ventured by APS editors for all values of the energy, a posture that is evidently implicit in the releases statement.

In any case, tests [53] were and remain to this day very controversial because of a number of equivocal assumptions in the data elaboration, some of which are identified below. This nonscientific posturing by APS editors confirmed (or perhaps initiated) rumors that Grossman's tests [53] had been "commissioned" by organized interests on Einsteinian doctrines following the claim of departures in Ref. [52]. Consequently, so the rumors say, the experimental data had been manipulated to meet pre-established political objectives.

As studied in the preceding and in this volume, all available conceptual, epistemological, theoretical, phenomenological and experimental evidence suggest deviations from the Minkowskian spacetime inside hadrons, with the sole exception of the Grossman tests [53]. the sole evidence that photons cannot propagate within the hyperdense medium inside hadrons is sufficient to cast serious shadows.

A re-elaboration of tests [53] was conducted in in 1998 by Yu. Arestov et al. [57] of the Institute for High Energy Physics of Protvino, Russia, by focusing the attention on the range-energy selection rule which can be applied to re-elaborate the initial data on K_s decays. In this section we shall use re-elaboration [57] and develop it further alone the lines above indicated.

Arestov et al. first obtained the raw data of tests [53] and initiated their re-elaboration via a new Monte Carlo simulation of the main features of the experiment and made new fits for K_s^o. To begin, the parameters in the full formula dN/dt for the proper time evolution are strongly correlated. This may cause a generally non-relevant regular dependence of the parameters on entities

which are not present in the formula, such as number of runs, energy, etc., apart from systematic uncertainties. Therefore, the above dependence may shadow the weak energy dependence, as can be seen from the large values of the correlation elements.[68]

Ref. [53] solved the problem of non-correlated fits by selecting the K_S^o momenta greater than $100GeV/c$, an assumption that prevents the use of the results below $100GeV/c$. By means of that energy cut, Ref. [53] obtained the data sample in which the CP violating terms contribute up to 1.6%.

A first apparent manipulation of Grossman's tests [53] occurred in looking for deviations from the Einsteinian decay law of the order of a few percentages. This is manipulatory because known by experts to be unrealistic, since all expectations are to look for deviations from the Einsteinian law of the order of 10^{-3}, as suggested by studies [48-52].

The confirmation of a possible manipulation is given by the fact that the assumption in Ref. [53] of 1.6% contribution from PC violation in the data elaboration implies looking for the energy dependence of τ_s at the level $k \times 10^{-2}$, thus rendering meaningless *ab initio* to look for more realistic deviations of the order of 10^{-3} or smaller.

Ref. [53] significantly suppressed the CP violating terms by using selection rule for the ratio R/E, where R and E represent the K_s^o range and energy. In experiment [53], R/E ranges from 2.3 to 36.1 cm/GeV. The R/E interval should be selected to make the contribution of the CP violating terms less than a desirable value, say $k \times 10^{-3}$. An effective (R, E) plot can then be calculated via Monte Carlo methods applied to the real decay volume.

Note that the above assumption caused in Ref. [57] to *lower statistics,* thus increasing the credibility of the data re-elaboration of Ref. [57] over that of the original paper [53]. In fact, under the above new assumptions, $60 - 70\%$ of the events are rejected, i.e., only $63K - 84K$ events of the total $220K$ events were used in Ref. [57]. Apart from the loss of a major part of the data, 1/3 of the decay volume in the experiment turns out to be also useless.

The large inefficiency of experiment [53] occurred because it had not been optimized for the problem. Basically, the experimental design and data selection

[68]The author jointly submitted paper [57] to *four* editors of *Physics Letters B* specifically selected because belonging to CERN, the paper essentially suggesting in due scientific language that CERN should repeat experiments [52,53] and finalize such a fundamental aspect of particle physics BEFORE spending additional public funds in the field. All four editors rejected the paper with a single signed letter stating that the paper was "excessively speculative," the same editors routinely accepting papers on neutrino and quark conjectures, evidently, as non-speculative. Following a long personal experience, it is the author's opinion that, in view of the billions of euros involved, the abuse of the laboratory credibility, the academic power of its leaders, and other factors, *no truly basic advance of physical knowledge can possibly occur at CERN without judicial injunctions for misuse of public funds and other charges initiated by European taxpayers,* the expectation that physicists at CERN may listen to scientific arguments being very naive or proffered by accomplices.

rules followed that of conventional K_s, K_l studies. A comparison of the statistics selected in re-elaboration [57] with the elaborations [53] then adds additional credibility to the rumors that Grossman's tests were commissioned.

Ref. [57] then illustrated the above arguments with two fits shown in the figure below, illustrating K_S decays at six energy values (from 125 to $375GeV$) that were generated in the decay volume with the ranges from $9.3m$ to $25.3m$. The energy dependence of the lifetime was assumed in the form

$$\tau(E) = \tau_S(1 + \epsilon E), \quad \tau_S \ = \ 0.8927, \quad \epsilon \ = \ 4 \cdot 10^{-5}. \tag{6.A.1}$$

After applying the range-energy selection rule, a sample of $64K$ events was chosen in Ref. [57] for which the contribution of the CP violating terms was less then 0.008. Namely Ref. [57] dealt with the following distribution for the proper lifetime:

$$\frac{dN}{dx} = N\{\exp(-x) \ + \ \text{CPV}\}, \tag{6.A.2}$$

where N is a normalization constant, $x = t/\tau(E)$ and CP violating terms are equal to

$$\text{CPV} \ = \ \mid \eta_{+-} \mid^2 \exp(-xy) \ + \ 2D \mid \eta_{+-}, \tag{6.A.3}$$

$$\mid \ \cos(\Delta m \ t - \phi_{+-})\exp(-x(1+y)/2) \tag{6.A.4}$$

where y stands for $\tau_s(E)/\tau_\ell$.

The values of other parameters are taken as the world average values. They are

$$\mid \eta_{+-} \mid \ = \ 2.284 \cdot 10^{-3}, \tag{6.A.5}$$

The magnitude of the CP-nonconservation parameter in the expression

$$K^o_\ell \to \pi^+\pi^-, \quad \phi_{+-} \ = \ 43.7^o, \quad \Delta m \ = \ 0.5333 \cdot 10^{10}\hbar sec^{-1} \tag{6.A.6}$$

is given by the mass difference of K^o_ℓ and K^o_s. The dilution factor D is defined as the ratio

$$\frac{N - \bar{N}}{N + \bar{N}}, \tag{6.A.7}$$

where N ($\bar{N}$) is the number of K^o ($\bar{K}^o$) produced by the proton beam on the target.

Note that Ref. [57] accepted the value $D = 0.75$ of Ref. [53]. The sequence of the mean proper lifetimes is plotted in the figure below versus E, the K^o_s laboratory energies. The dependence was obtained by simulations of K^o_s decays in the experimental volume under the conditions described above.

The figure presents two fits obtained by Arestov et al [57] with the energy-dependent formula of the type

$$\tau(E) \ = \ 0.8927(1 \ + \ p_1 E), \tag{6.A.8}$$

and the values

$$\tau(E) \ = \ c, c = 0.90 \pm 0.01, \chi^2/ndf \ = \ 0.7/5, \tag{6.A.9}$$

represented by the dashed line at top left of the figure, and

$$p_1 \ = \ (4 \ \pm \ 5), cdot10^{-5}, chi^2/ndf \ = \ 0.38/, \tag{6.A.10}$$

represented by the solid line top left.

For comparison, Ref. [57] performed also the two-parameter fit to the formula of Ref [53],

$$\tau(E) \ = \ p_2(1 \ + \ p_1 E) \tag{6.A.11a}$$

$$p_1 = (4 \ \pm \ 23) \cdot 10^{-5}, \tag{6.A.11b}$$

with $\chi^2/ndf \ = \ 0.38/4$.

There is a difference in interpretation of parameters in the two fitting formulae with the energy dependence. The parameter p_2 in the fit from paper [53] was interpreted as the zero-energy mean value of the proper lifetime. It is difficult to extrapolate the fitting formulae from the energy interval $100 - 400GeV$ to zero. Instead, Ref. [57] used the energy dependence in a limited energy interval by fit starting from a definite point. This difference in interpretation is important because, in general, various approaches in fitting procedures may lead to crucially different numerical results, thus confirming beyond credible doubt the possibility of manipulating the data elaboration to verify any pre-set beloved doctrine.

Thus, in the amount of the events selected in Ref. [57], both fits dig up well the mean value of the hidden parameter ϵ determining the energy dependence in the simulated K_S^o decays, but the error bars differ strongly. Though both results for fitting the values of p_1 are still insignificant statistically, even in the selected sample of events, the 100% error bar in fit [57] being rather promising. a pre-set goal.

An additional possibility, we note here that no firm spacetime verification of the Einsteinian decay law can be established via elaboration [53] for PC violating contributions of the order of 1.6% because possible anomalies are within the errorbars due to insufficient statistics of tests [53] and other reasons.

Arestov et al concluded their analysis in paper [57 with the statement: *The analysis of this paper establishes the insufficiencies of the tests by Grossman et al. and the need for final, more accurate measurements as the only way to resolve the now vexing fundamental problem of the spacetime geometry and physical laws holding in the interior of the hyperdense hadrons. After all, as indicated earlier, the isominkowskian fit of experiments [55-56] establishes the existence of space-time anomalies with superluminal speeds in the interior of hadrons even in the event that measurements [53] result to be correct.*

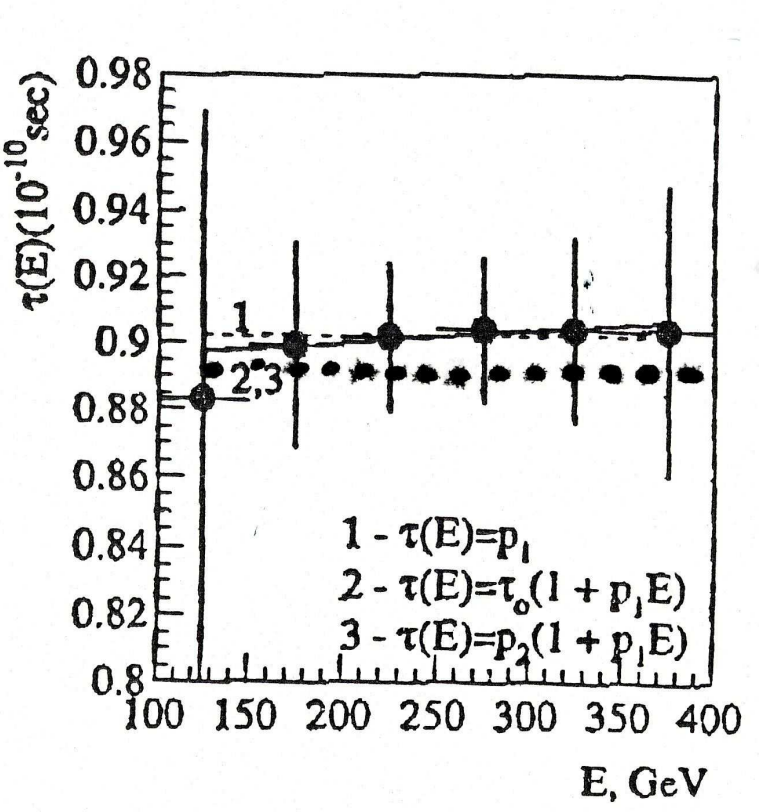

Figure 6.A.1. Re-elaboration of the experimental data of the experiments by Grossman et al [53] for the lifetime $\tau(E)$ dependence on energy. The dashed line (top left) and the continuous line (middle left) are the re-elaboration of said data as published by Arestov et al [57] to illustrate the lack of final character of the claims contained in paper [53]. The dotted line (left bottom) presents re-elaboration by the author via different fitting functions and other minor changes to illustrate how easy it is to manipulate contemporary experiments for the pre-determined intent of fitting Einsteinian doctrines. This establishes the need for the conduction of any and all particle physics experiments under the strict control of an external Committee on Scientific Ethics and Accountability. The occurrence also established the need for an in depth apolitical review of ALL recent particle experiments based on Einsteinian doctrines for conditions different then those of its original conception. Finally, the re-elaboration confirm the new for a *a new code of laws* addressing scientific manipulations, as suggested in the text

The author has re-examined the above analysis under profiles pertaining to scientific ethic s and accountability. It is evident that the "experimental verification" claimed by Grossman et al [53] could have been intentionally achieved via virtually endless manipulations of the data elaboration, all presented under

a patina of seemingly technical calculations, although solely capable of fouling readers without sufficient technical knowledge.

To mention only one among numerous possible adulterations, statistical and other conditions can be selected in such a way that the deviations from Einsteinian doctrines are of the order of magnitude of the error, and then claim verification of said doctrines. In any case, the rumors persist that this was indeed done by Grossman et al [53] and by the editors of the APS with*their* publication of the paper.

Such a visible absence of serious editorial processing is systematically implemented by APS editors for papers claiming experimental verification of Einsteinian doctrines and quantum mechanics. By contrast, the more important papers claiming experimental deviation from said doctrines are subjected by ASPS editors to brutal "reviews" intended to discourage the continued submission via a never ending sequence of criticisms on manifestly tangential issues, without issuing, in general, a formal rejection.

It is equally evident that there exist a large number of possibilities to manipulate the data to reach pre-set departures from the Einsteinian decay law. As a matter of fact, the alternatives are so many to be embarrassing. In the figure we report deviations from the Einsteinian decay law (dotted line at bottom left) obtained via a 5% change of the PC violating parameter, a different value of the fitting function and other small "adjustments."

Note that the deviations from the preceding two curves is intentionally small because it could have been as large ad desired. In particular, simple "adjustments" in the selection of the statistics, reduction of the PC parameter, suitable selection of the fitting function and others things can easily produce deviations 3-4 timers bigger than the error. Their study is left as an instructive exercise for the ethically sound scholar.

It is hoped that educators, administrators and editors seriously committed to serious science see the necessity of *a new code of laws* encompassing scientific crimes, as well as the necessity that all contemporary experiments, whether in favor or against Einsteinian doctrines, be subjected to controls by an external Committee on Scientific Ethics and Accountability prior to publication. Educators, administrators and editors should never forget that what is at stake is the ability or inability to solve increasingly alarming environmental problems in our planet. In plain language, leaving the status quo in the current conduction of basic science is not only unethical, and irresponsible but actually suicidal.

Appendix 6.B
Ethical Problems in String Theories

In preparation as of October 1, 2007.

See the catastrophic inconsistencies of string theories published in a refereed journal of which the author is not an editor

R. M. Santilli, "New problematic aspects of current string theories and their apparent isotopic resolution,"
Foundation of Physics **32**, 1111(2002)

Serious ethical probles emerge because these catastrophic inconsistencies havce remained totally ignored by organoized interests in the field.

Until physicists were playing personal games of purely mathematical in curiosity in string theories, they were tolerated. Now that laboratories are raising large public funds for experiemts on a theory proved to be catastrophically inconsistent on physical grounds, without first disproving such inconsistencies in equally refereed piublications, judicial action is necessary to prevent this unethical condition and conduction of basic scientific knowledge.

Appendix 6.C
Ethical Problems in Black Holes

In preparation as of October 1, 2007. See

Jeremy Dunning Davies, *Exploding a Myth*
Harwood, England (2007
ISBN 978-1-904275-30-5

In Santilli's view, Black holes constitute one of the most sinister episodes in the history of science because of an excessively long list of excessive ethical problems, all ignored because of the illusion of achieving credibility via the abuse of academic authority, complemented by the illusion that all physicists are naive or gullible.

As one indication, current studies of black holes, in the form appearing in publications, dishonor the memory of Schwartzchild who wrote *two* historical papers, one on the *exterior* solution and one on the *interior* problem. Even though black holes constitute the ultimate interior gravitational problem in the universe as known these days to high school students these, they are treated with the exterior solution, while being completely silent on the interior character due final incompatibility with Einsteinian doctrines, the only possible bypassing of ethical problems being an admission of scientific illiteracy.

Additional ethical problems are caused by the complete ignorance of the catastrophic mathematical and physical inconsistencies of Einstein's gravitation under the illusion that they disappear via silence, complemented by the illusion that abuses of academic authority produce certain mental slavery, while in reality setting up the illusionists for probable legal prosecution by contemporary colleagues and certain condemnation by posterity, for serious science can be solely based on a collegian addressing, rather than suppressing, of fundamental unresolved problems.

Appendix 6.D
Requested Experiments

Following a lifelong experience, the author regrets to state that physics used to be a science with an absolute standard of value, the experimental verification. Experiments themselves used to have their own standard of value, in the sense that experiments on fundamental unresolved aspects had priority over those of peripheral; relevance.

Nowadays, the standard of value is primarily set by academic power; the more fundamental a proposed experiment is, the bigger the opposition for its conduction; and, when undesired basic tests somehow manage to escape current restrictions, manipulated counter-experiments are soon commissioned to protect organized interests on Einsteinian doctrines (se Appendices 6.1.A, 6.1.B, 6.1.C).

These are the reasons for the view, repeatedly expressed by the author, that *nowadays, no basic aspects in physics can be seriously addressed without a joint consideration of issues pertaining to scientific ethics and accountability.* Hence, the author has long suggested the need for external Ethics Committees supervising basic research similar to those existing in other branches of science, particularly when the research is conducted under public financial support.

More recent events have shown that organized obstructions against undesired advances have increased with the increase of the evidence of the limitations of Einsteinian doctrines. Since the power and capillary organization of orthodox interests is beyond the imagination by outsiders, the author predicts that *no experiment on truly basic open issues is possible nowadays without legal proceedings against physics laboratories and their directors for misuse of public funds, discriminatory conduct, and other violation of federal laws.*

In this section we present n numerous basic experiments submitted by the author over three decades (see Refs. [81,6] and EHM II) to all major laboratories around the world whose list and related documentation will be disclosed at the appropriate future time in the appropriate conduit. Even the "consideration" of the experiments herein proposed by flatly rejected, let alone their "conduction."

To appraise the gravity of the situation, the "consideration" of the basic experiments reviewed below was rejected even though their costs was at times quite moderate with very large scientific implications whatever the outcome, while other experiments were preferred of immensely bigger costs to the taxpayer, without any major relevance, and often intended to test sheer theological beliefs. The

reason for this disparity documented beyond credible doubt, and continuing to this day in any case at all major physica laboratories around the world, is that the later experiments were aligned with Einsteinian doctrines while the former were not.

In view of such a deplorable condition of physics, and the expectation of its resolution via judicial proceedings, in his capacity as a U. S. taxpayer, the author has changed the original titles of "Suggested Experiments" into "Requested Experiments." Readers who interprets the content of this section as aimed at "proving Einstein wrong" and the like, are disqualified as being outside serious science because, as shown by scientific history, serious science is solely conducted via serious experiments *irrespective of whether in favor or against a preferred theory.* The endless distortions, deviations, peripherals, and the like the author has been exposed too over decades are mere schemes aimed at personal gains in money, prestige and power.

REQUESTED EXPERIMENT 1: Measure the possible isoredshift of light from a quasar before and after passing through a planetary atmosphere (such as that of Jupiter) or an astrophysical chromosphere (such as that of the Sun).

The above test was first proposed by Santilli the early 1980s when at Harvard University, and then reviewed in a number of publications such as Ref. [81] of 1988 and subsequent works (see monograph [6] in particular) and papers quoted therein.

The reason for the impossibility of astrophysical laboratories to even consider the experiment, let alone conduct it, is that, at the time of the proposals, the author was still naive, in the sense of still believing in the above quoted absolute standards of values on which the preceding history of physics was based upon. In fact, the respectful "suggestions" to consider Experiment 1 included a detailed identification of its fundamental implications. The suppression of the consideration was due to such an identification. In different terms, had the experiment been disguised by misinformation on title ands content, perhaps there would have been a chance at least for its consideration.

Had, in the United States of America, any astrophysics laboratory formally "considered" Experiment 1, that laboratory would have seen the termination of research funding by the Department of Energy, the National Science Foundation, and other governmental agencies or private foundations. Under these conditions in the U.S.A., foreign astrophysical laboratories had no other choice than align themselves with organized interests in the U.S.A. [69] It is important to identify

[69]Documentary evidence of ONE research contract existing at this writing (October 23, 2007) by the D.O.E. or the N.S.F. funding experiments that could invalidate Einsteinian doctrines would be greatly appreciated for due corrections.

the political problems that have prevented the consideration of the test so far, because useful for serious scholars seriously interested in serious science.

POLITICAL PROBLEM 1: As recalled earlier, the "universal constancy of the speed of light" is maintained within physical media via the belief that photons scatter through atoms, thus causing a believed *increase of the travel time* through the medium that appears to us as a decrease of the light speed. The important political point is that, in this way, photons continues to travel in vacuum at the "universal value" c_o. By comparison, if successful, Experiment 1 would detect a *slowdown of the speed of light itself* because it is the sole capable of causing a redshift. Admitting the possibility of detecting the local variation of the speed of light would mean terminating the dominance of Einsteinian doctrines throughtout all media in the universe, with consequential expected termination of funding perhaps in excess of one billion dollars, thus mandating the commissioning of counter-experiments, and similar scientific misconduct nowadays a routine in physics due to the total absence of any control by society.

POLITICAL PROBLEM 2: As shown below, the sole decrease of the speed of light is insufficient for serious science because the considered media are inhomogeneous and anisotropic. Experiment 1 is additionally intended to measure possible deviations from the homogeneity and isotropy of empty space, namely, something more damaging to organized interests on Einsteinian doctrines than the mere change of the speed of light, with consequential, expected, increased reactions, obstructions, schemes, manipulations, and the like.

POLITICAL PROBLEM 3: If successful, Experiment 1 would establish the exact validity within physical media of Santilli's isotopic covering of Einsteinian doctrines, including the exact validity of the iso-Minkowskian spacetime, the iso-Lorentz symmetry and related iso-axioms, namely, the proposed test would establish something expected to have truly large organized oppositions, obstructions and disruptions.

Following these necessary preliminary for outsiders to have a glimpse of the real experimental world in physic s these days, we can now pass to an outline of the scientific case to the best of our capability. To keep a kilometric distance from orthodox interests, the presentation below is submitted as tentative and conjectural, for which very reason there is the need for an experimental verification, by keeping in mind that *the orthodox interpretation is equally tentative and conjectural, again, due to the lack of direct experimental verifications.*

As well known, the conventional *Doppler's law,* for the simpler case of null angle of aberration, is given by

$$\omega = \omega_o \times \frac{1-\beta}{\sqrt{1-\beta^2}},\ \beta = \frac{v}{c_o}, \tag{6.1.129}$$

where c_o is the speed of light in vacuum, and can be written via a power series expansion

$$\omega = \omega_o[1 - \frac{v}{c_o} + \frac{1}{2} \times (\frac{v}{c_o})^2 + ...]. \tag{6.1.130}$$

As also well known, $v \ll c_o$, $v/c_o]ggv^2/c_o^2$ and, consequently, the term v/c_o dominates the expansion. We can then write

$$\omega \approx \omega_o \times (1 - \frac{v}{c_o}). \tag{6.1.131}$$

Also, $v/c_o \ll 1$. Consequently, Eq. (6.1.131) represents a *decrease* of the original frequency ω_o. Then, for $v \neq 0$, we have a *redshift* that can be defined as[70]

$$\Delta_\omega = \omega_o - \omega > 0. \tag{6.1.132}$$

It is equally evident in Eq. (6.1.131) that, in the event, for a given value of v, there is a decrease of the speed of light within the selected planetary atmosphere or astrophysical chromosphere, namely,

$$c_o \to = c = c_o \times b_4 = \frac{c_o}{n_4}, \;\; b_4 < 1, \;\; n_4 > 1, \;\; c < c_o, \tag{6.1.133}$$

Eq. (6.1.131) becomes

$$\omega \approx \omega_o \times (1 - \frac{v}{c_o} \times \frac{1}{b_4}) = \omega_o \times (1 - \frac{v}{c_o} \times n_4). \tag{6.1.134}.$$

As one can seen, *in the event, for a given v, we have a decrease of the speed of light within the medium considered, the redshift is bigger,* exactly along the Section 6.1.11.

It is equally easy to see that Eq. (6.1.134) is geometrically unbalanced and incomplete because inhomogeneity can be represented with a dependence of the index of refraction on the local coordinates, $n_4 = n_4(r, ...)$ (since n_4 represents the local density), but we lack a representation of the anisotropy of the medium considered caused by its its rotation with consequential preferred direction in space. The latter requirement leads uniquely and unambiguously to Isoaxiom IV with isotopic law

$$\omega = \omega_o \times \frac{1 - \hat{\beta}}{\sqrt{1 - \hat{\beta}^2}}, \tag{6.1.135a}$$

$$\hat{\beta} = \frac{v}{c_o} \times \frac{b_s}{b_4} = \frac{v}{c_o} \times \frac{n_4}{n_s}, \tag{6.1.135b}$$

[70]We should caution the reader that there are numerous different definitions of redshifts in astrophysics.

and final approximate expression

$$\omega \approx \omega_o(1 - \frac{v}{c_o} \times B, \qquad (6.1.136a).$$

$$B = \frac{b_s}{b_4} = \frac{n_4}{n_s} \qquad (6.1.136b)$$

where we have assumed, again, spherical symmetry for simplicity.

The following estimates of isoredshift for quasars light passing through Jupiter's atmosphere was reached in Ref. [6b], Section VII.4 and VII.5. The average value of the characteristic quantity B in the data of Fig. 6.1.13 is

$$B_{aver} = 72.58 \qquad (6.1.137)$$

from which we have the average redshift of quasars

$$\Delta^q_\omega = 1.15, \qquad (6.1.138)$$

with corresponding average redshift ofd the associated galaxies

$$\Delta^g_{aver} = 0.001. \qquad (6.1.139)$$

From astrophysical and planetary data we can assume, in first approximation, that quasar chromospheres ("q") are about 10^5 denser than Jupiter's atmosphere ("j"), and by recalling that $n_4 = 1/b_4$ represents the density d of the medium considered, we have the proportionality

$$\frac{B^q_{aver}}{B^j} \approx \frac{d^q_{aver}}{d^j}, \qquad (6.1.140)$$

with the estimate value of B for Jupiter [6b]

$$B^j_{est} = 7.3 \times 10^{-4}, \qquad (6.1.141)$$

and the corresponding estimate of the isotopic redshift for quasar light passing through Jupiter's atmosphere predicted by isorelativity

$$\Delta^q_{est} = 1.14 \times 10^{-5}. \qquad (6.1.142)$$

Individual values for b_s and b_4 can then be obtained from comparative measurements of the predicted decrease of the speed of light within Earth's atmosphere presented below, since such value would provide a good approximation of the corresponding value of b_4 for Jupiter. The value of b_s would then follow from the value of B.

"Requested" Experiment 1 suggests first to measure the quasar redshift in empty space via available instruments and techniques and then measure it again

when the same light passes through Jupiter's atmosphere. The experiment is readily feasible because it requires no new equipment, but merely the *extension* of conventionally conducted measurements only under new conditions. Also, estimate (6.1.141) is fully within current experimental feasibility.

To understand the gravity of contemporary experimental physics, noninitiated readers should know that the consideration, let alone conduction of Experiment 1 was rejected also by astrophysical laboratories that were conducting measurements of quasars redshifts, hence requiring no additional funds.

Said gravity is further illustrated by the fact that Experiment 1 requires, in reality, only a *confirmation,* since NASA planetary missions have provided apparent data showing exactly the isodoppler effect of Experiment 1 for the case of electromagnetic communications with satellites when passing though planetary atmospheres or the Sun's chromosphere. Regrettably, the author was unable to collect these data or possible references thereof, and their indication by interested colleagues would be greatly appreciated for due revisions.

The following alternative of Experiment 1 was submitted in Refs. [81,6b] but equally ignored by astrophysical laboratories:

REQUESTED EXPERIMENT 2: Measure from a satellite the possible isoredshift of light originating from a far away star or quasar when passing through Earth's atmosphere.

It is evident that possible comparative measurements of isoredshift in Jupiter's and Earth's atmospheres would yield invaluable scientific information on the geometries of physical media, particularly useful for new energies depending on spacetime anomalies, as we shall see.

The following third experiment is partially responsible for the view often expressed by the author that *the most ascientific process of contemporary physics is the scientific process.* The view is caused by a widespread dismissal of plausible dissident views, this time dealing with the *origin of the tendency toward the red of Sun light at sunset.*

REQUESTED EXPERIMENT 3: Measure at the equator the expected isoredshift of sunlight in the transition from the zenith to the horizon.

It is popularly believed that the "redness of sunsets" is caused by the absorption by our atmosphere of blue and other light resulting in the dominance of red visible by all of us. This view is not reason for debates. The problems originate when said view is assumed as the origin of the *entire* tendency toward the red at sunset, since there are *three* additional plausible contributions, all deserving experimental verification.

Conventional Doppler's effect. Earth's rotates. Hence, an observer at the equator is moved *toward* the Sun. It then follows that, at least one contribution of the "redness of sunsets," is a bona-fine, conventional redshift. In fact, said observer has the following tangential speed toward the Sun

$$v = 0.46 Kw/s \tag{6.1.143}$$

resulting in the value

$$\beta = \frac{v}{c_i} = 1.57 \times 10^{-8}. \tag{6.1.144}$$

Despite its smallness, the latter value causes a conventional Doppler's shift visible by the naked eye and given by half of the visible difference of the tendency toward the read between sunset and sunrise. In fact, the observer is moving it away from the Sun at sunrise, thus causing a *blueshift* (because in this case the negative sign in Eq. (6.1.134) is turned into a positive sign). Clearly this contribution "requires" an experimental verification or denial.

Decrease of the light speed. Light decreases in speed about 33 % in water.[71] Since the ratio of the densities of water and atmosphere is of about 10^3, in Earth's atmosphere, Sun light speed is expected to decrease in the approximate value

$$c = c_o \times b_4 = \frac{1}{3} \times c_o \times 10^{-3}. \tag{6.1.145}$$

Since effect (6.1.144) is visible to the naked eye, effect (6.1.145) "requires" an experimental verification or denial because 10^5 times bigger than the former.

Full isotopic effect. Again, law (6.1.135) for value (6.1.145) is geometrically inadequate, requiring the full isotopic law (6.1.135). The latter effect also deserves experimental verification or denial because Earth's atmosphere is expected to be a medium of Group II, Type 5, for which b_s is *smaller* than b_4, as a result of which *the anisotropy of Earth's atmosphere is expected to decrease the redshift predicted by the decrease of light speed.*.

Experiment 3 can be conducted quite easily via currently available spectrometers, by first selecting one or more spectral lines at the zenith and then following them to the horizon. Possible errors in following the Sun can be compensated with a broader selection of spectral spectral lines, with the understanding that current astrophysical equipment can follow the micrometric motion of far away stars, thus being amply sufficient for the simpler motion herein considered.

Note that Experiments 1 and 2 are intended to ascertain whether or not an *already redshifted* light can experience an additional redshift when passing through a

[71] Despite this large decrease, readers should not expect a redshift in a glass of water due to the need for a large water volume to reach a measurable effect.

medium. Experiment 3 is intended to ascertain whether or not light not originally redshifted can be redshifted by passing through a medium. Hence, Experiments 1 and 2 could be successful even in the event Experiment 3 is not.

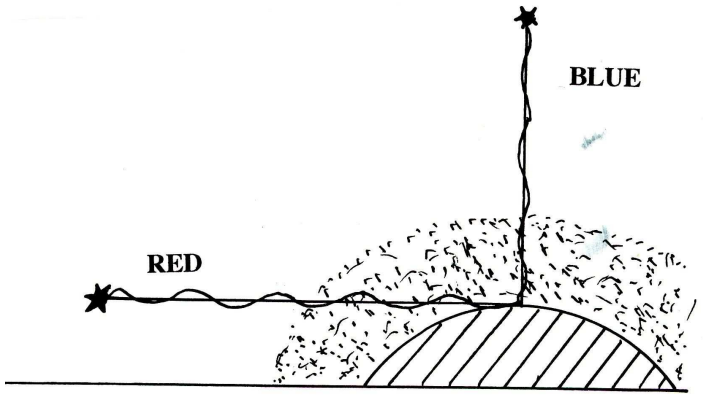

Figure 6.D.1. A schematic view of the Experiment 3 intended to ascertain whether or not,an observer at the equator following the transition from the zenith to the horizon, sunlight experiences three different contributions to the redness at sunset: 1) A conventional Doppler's redshift due to motion of the observer toward the Sun; 2) An isotopic redshift due to the predicted decrease of the speed of light within Earth's atmosphere; and 3) A blueshift reduction of the preceding redshift due to the anisotropy of the medium caused by Earth's rotation. As illustrated in the text, it should be stressed that isotopic contributions *cannot* turn blue light at the zenith into red light at the horizon. Hence, the proposed tests refer to *contributions* to the redness at sunset while keeping the conventional interpretation valid in first approximation (that Earth's atmosphere at the horizon absorbs the blue leaving the red as dominant). Despite its secondary numerical value, if confirmed, said contributions would have far reaching physical, astrophysical and cosmological implications.

We have indicated in the preceding section the current "experimental beliefs" on the expansion of the universe because redshifts measurements (that are not questioned here) are interpreted with the unverified assumption that light propagates through the immense astrophysical chromospheres at the same speed as that in vacuum. This results in an "experimental belief" because actual measurements are used to proffer personal unverified theoretical views, a rather widespread practice in contemporary physics, as we shall see.

To turn this theological condition of astrophysics into serious science, the author proposed in Ref. [6b], Sect. VII.5 the following additional test:

REQUESTED EXPERIMENT 4: Measure at one of the poles the possible isoredshift of sunlight from the zenith to the horizon.

The evident main difference between Experiments 3 and 4 is that, in the former case, we do have motion of the observer toward the Sun while, in the second case, the observer can be approximately considered to be at rest with respect to the Sun. Hence, Experiment 4 has a fundamental character for astrophysics, for which reason the author "requests" its conduction as a U. S. taxpayer. In fact, the test would permit the study:

1) Whether or not far away astrophysical bodies may exhibit a redshift while being at rest with respect to Earth;

2) Whether or not the currently believed expansion of far away astrophysical bodies should be decreased because of isotopic contributions from the slow down of the speed of light in their chromospheres; and'

3) Whether or not astrophysical bodies currently believed to be expanding from Earth are in reality moving toward Earth, trivially, because the isotopic redshift due to the chromosphere could be bigger than the blueshift due to motion.

The theology underlying the above open issue is essentially similar to that on antigravity, namely, "Einsteinian theories predict spectral shifts only under relative motion and, therefore, when there is no shift, the bodies are at rest with respect to each other." However, physical reality is definitely much more complex than this theological posturing.

The preceding experimental verifications of isorelativity have established that $n_4 = 1/b_s$ represents the local density d thus depending on the local coordinate r, $n_4(r, d, ...) = 1/b_4(r, d, ...)$. In the preceding calculations, $n_4 = 1/b_4$ has been averaged to a constant for simplicity. By contrast, the space component $n_s = 1/b_s$ depends on the speed and, trivially, from the energy E [81], $n_s(v, E, ... = i/b_s(v, E, ...)$. Hence, the isotopic law can be explicitly written

$$\omega \approx \omega_o \times [1 - \frac{v \times b_s(v, ...)}{c_o \times b_4(r, ...)}]. \tag{6.1.146}$$

Since the functional dependence of the characteristic quantity b_s on the speed is unknown at this writing, we cannot apriory assume that $\Delta_\omega = 0$ for $v = 0$ in Eq. (6.1.146). The only possible serious pursue of scientific knowledge is that via unbiased experiments, to be sure, conducted under an external Ethics Committee.

In summary the above possibilities 1), 2) and 3) may originate, not only from, a possible slowdown of the speed of light in astrophysical chromospheres, but also and independently, from, the anisotropy of the medium considered.

An illustration of one of the numerous scientific manipulations used to oppose the above proposed experiments is necessary to inform the serious scholar. The dismissal of the (at that time) "suggested" experiments was once voiced

by a seemingly senior "scientist" belonging to seemingly "leading" university on grounds that "Santilli believes that blue light at the zenith can be turned into read at the horizon via his mathematics." The following comments are then in order in the hope of at least preventing the repetition of the same "objection" against basic experiments.

The mid-blue ("b") at the zenith is characterized by the following frequency

$$\omega^b = 6.34 \times 10^1 4Hz, \tag{6.1.147}$$

while the mid red ("r") at the horizon is characterized by

$$\omega^r = 4.38 \times 10^1 4Hz, \tag{6.1.148}$$

with ratio

$$\frac{\omega^r}{\omega^b} = 0.69. \tag{6.1.149}$$

The hypothetical "redshift" from blue to red would then require

$$\omega^r = \omega^b \times [1 - \frac{v}{c_o} \times b], \tag{6.1.150a}$$

$$1 - \frac{v}{c_o} \times B = 0.69 \tag{6.1.150b}$$

$$B = 1.4 \times 10^7 \tag{6.1.150c}$$

where we have used value (6.1.143).

It is evident that *value (6.1.150c) is impossible in Earth's atmosphere.* Since it was proffered by a seemingly qualified senior "scientist" belonging to a qualified University, the statement "Santilli believes that blue light at the zenith is turned into read at the horizon via his mathematics" was an act of sheer scientific corruption intended to oppose, jeopardize or prevent undesired basic experiments for personal gains in money, power and prestige. Very unfortunately for society, physics is nowadays done via academic power. Since the abused institution was credible, the dismissal was accepted rather widely by naive followers, and the suppression of the pre-meditated experiment was fully successful.[72]

[72]Since they lack technical arguments, corrupt academicians retort to all sort of nonscientific and tangential "arguments" to prevent the conduction of undesired basic experiments. Another objection was that "the tests are not warranted because Santilli did not work then out in all the necessary experimental details." The corrupt character of the "objection" is soon identified by recalling, for instance, that the discovery of the Ω^- was done by *experiments worked out in their technological details by experimentalists* following the *purely theoretical* prediction via $SU(3)$ symmetries. The evident reason for this evident disparity is that the latter test was fuly aligned with Einsteinian theories while the former are not. Another "objection" voiced by another "physicist" is that "the experiments have no sense because Santilli believes that quasars have atmospheres." The "objection" originated from a mistake by the author in Ref. [6a] of using, in one passage, the word "atmosphere" in lieu of "chromosphere," and this was reason for the successful suppression of the tests. The author spares the reader the report of additional "objections" because their very reading is demeaning for what is supposed to be a serious scientific process.

We close this section with the following fifth fundamental tests that are mandatory for any basic advance in hadron physics, while additional tests will be reviewed and "requested" in the remaining parts of this volume.

REQUESTED EXPERIMENT 5: Achieve final experimental resolution of the behavior of the meanlives of unstable hadrons with speed.

The need to conduct this fifth experiment, and the necessity of its conduction under an external Ethics Committee, are presented and documented in Appendix 6.1.A.

It is hoped that physics laboratories will conduct the much needed *basic* tests under the strict supervision of external Ethics Committees so as to prevent their otherwise inevitable conduction under judicial injunctions due to misuse of public funds, discriminatory practices, and other violations of Federal Laws that are inherent in the current use of public funds for generally very expensive experiments based on essentially unsettled foundations.

TO BE COMPLETED WITH ADDITIONAL TRESTS

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Chapter 7

ISO-, GENO-, AND HYPER-GRAND-UNIFICATIONS AND ISO-, GENO-, HYPER-COSMOLOGIES

7.1 ISO-, GENO-, AND HYPER-GRAND-UNIFICATION

7.1.1 The Role of Antimatter in Grand Unifications

As indicated earlier, no conclusive study on antimatter can be conducted without its consistent inclusion in *grand unifications* of gravitational [1–3] and electroweak interactions [4–7]. Vice versa, no grand unification can be considered scientifically valuable without the correct inclusion of antimatter because the latter has a profound impact in the very structure of a consistent grand unification.

All studies on grand unifications conducted until now have been essentially restricted to matter. When antimatter is included, the studies have to be enlarged to *two grand unifications,* one for matter and the other for antimatter with a correct anti-automorphic (or anti-isomorphic) interconnecting map.

Consequently, the inclusion of antimatter in grand unifications introduces severe restrictions on the admissible models, which restrictions are generally absent when antimatter is ignored and grand unifications are restricted to matter alone.

We shall, therefore, avoid the review of the very large number of structurally inconsistent grand unifications published since Einstein's times and leave to the interested reader their re-examination in light of the new advances of this volume.

An in depth study of grand unifications soon reveals the need of formulating antimatter at the purely classical level, the need for abandoning curvature, and the need for a geometric unification of special and general relativities as presented in preceding chapters. It is only at the level of these broader views on grand unifications that the isodual theory of antimatter emerges as inevitable.

Even though presented at the end of this monograph, the author initiated his studies on grand unification, constructed the needed broadening or modifications

of pre-existing methods, and then achieved an invariant, axiomatically consistent grand unification.

This process requires it two decades of research before the publication of the first paper on grand unification, a lapse of time illustrating the complexity of the problem, as known in any case by the failure of the large number of preceding attempts.

The reader should be aware that, in this section, we shall exclusively study *closed-isolated systems of electroweak and gravitational interactions in vacuum* that are treatable via the Lie-isotopic branch of hadronic mechanics and its isodual. Interior problems, such as those inclusive of the origin of gravitation, require the broader Lie-admissible branch of hadronic mechanics and their treatment will be merely indicated at the end of this section for development by interested readers.

7.1.2 Axiomatic Incompatibilities of General Relativity and Electroweak Interactions

The preceding efforts for a grand unification of gauge theories of electroweak interactions and gravitation as described by general relativity are afflicted by the following axiomatic incompatibilities, first presented in Ref. [9] of 1997 (see also the related papers [10,11]):

(1) **Incompatibilities due to antimatter:** electroweak theories are *bona fide* relativistic field theories, thus characterizing antimatter via *negative-energy* solutions, while general relativity characterizes antimatter via *positive-definite* energy-momentum tensors. This first incompatibility renders manifestly inconsistent all attempts at grand unification known to this author.[1]

(2) **Incompatibilities due to curvature:** electroweak theories are essentially flat theories since they are formulated via *Minkowskian* axioms, while general relativity is centrally dependent on curvature since it is based on *Riemannian* axioms. This second incompatibility is another, independent, primary origin of the failure of the vast number of attempts at grand unification existing in the literature and carries profound implications, such as the extension to grand unification of the theorems of catastrophic inconsistencies of Section 1.4.

(3) **Incompatibilities due to spacetime symmetries:** electroweak interactions are based on the axioms of special relativity, thus verifying the fundamental *Poincaré symmetry* $P(3.1)$, while such a basic symmetry is absent in general relativity and is replaced by a generic covariance. This third incompatibility has

[1]The indication of grand unifications inclusive of antimatter would be greatly appreciated.

additional profound implications for any consistent grand unification because either one abandons the basic symmetries of electroweak interactions in favor of an unknown covariance, or one abandons general relativity for a new theory admitting a universal symmetry.

(4) **Incompatibilities due to the lack of a Minkowskian limit of general relativity:** as it is well known [1–3], general relativity admits a well defined *Euclidean* limit under PPN approximation, but one century of studies have failed to identify a corresponding well defined *Minkowskian* limit. On the other side, electroweak interactions [4–7] are formulated on a Minkowski spacetime. This fourth incompatibility of the two interactions then emerges in a number of aspects, such as irreconcilable ambiguities in the identification of total conservation laws of grand unifications when inclusive of gravitational interactions.

(5) **Incompatibilities due to the nonunitary character of quantum gravity:** as it is also well known, electroweak theories are *operator* field theories with a *unitary* structure, thus having invariant prediction of numerical values permitting meaningful experimental verifications. By comparison, all quantum formulations of general relativity (see, e.g. Ref. [8] and references quoted therein) have a *nonunitary* structure. Besides evident, additional, independent inconsistencies in attempting to combine unitary and nonunitary theories, any attempt of grand unification along contemporary views in general relativity and quantum gravity is afflicted by the theorems of catastrophic inconsistencies of Section 1.4.

It is evident that no significant advance can be achieved in grand unifications without, firstly, a serious addressing of these inconsistencies and, secondly, without their resolution.

Recall that the theory of electromagnetic interactions, when (and only when) restricted to the *vacuum*[2], has a majestic mathematical and physical consistency that eventually propagated to unified theories of electromagnetic and weak interactions.

The view adopted in this monograph, identifiable in more details only now, is that, rather than abandoning the majestic beauty of electroweak theories, we abandon instead the popular views on gravitation of the 20-th century due to their catastrophic inconsistencies and, as a condition to achieve a consistent grand unification, we reconstruct gravitational theories in such a way to have *the same abstract axioms* of electroweak theories.

[2]It is well known by expert, but rarely spoken, that Maxwell's equations have no real physical value for the treatment of electromagnetism within physical media for countless reasons, some of which have been treated in Chapter 1. As an illustration, only to locally varying character of electromagnetic waves within physical media requires a radical revision of electromagnetism in the arena considered as a condition to pass from academic politics to real science.

7.1.3 Resolution of the Incompatibilities via Isotopies and Isodualities

In this chapter we present a resolution of the above incompatibilities first achieved by Santilli in Refs. [9] of 1997 (see also Refs. [10,11] following a number of rather complex and diversified scientific journeys that can be outlined as follows:

(A) Isotopies. The scientific journey to achieve a consistent grand unification started in 1978 with memoirs [12,13] for the classical and operator isotopies. A baffling aspect in the inclusion of gravity in unified gauge theories is their geometric incompatibility.

The view that motivated Refs. [12,13] is that the difficulties experienced in achieving a consistent grand unification are primarily due to *insufficiencies in their mathematical treatment*.

Stated in plain language, the view here considered is that, due to the complexity of the problem, the achievement of an axiomatic compatibility between gravitation and electroweak interactions requires a basically new mathematics, that is, basically new numbers, new spaces, new symmetries, etc.

Following first the verification of the lack of existence in the literature of a mathematics permitting the desired consistent grand unification, and following numerous attempts, the *only* possible new mathematics resulted to be that permitted by the *isotopies* as first proposed in Refs. [12,13], namely, a generalization of the conventional trivial unit +1 of electroweak theories into the most general possible, positive-definite unit with an unrestricted functional dependence on local variables, called *Santilli's isounit*,

$$I = +1 > 0 \quad \rightarrow \quad \hat{I} = \hat{I}^\dagger = I(x, v, \psi, \partial\psi, \ldots) > 0, \tag{7.1.1}$$

and consequential compatible reconstruction of all main branches of mathematics.

The uniqueness of the isotopies is due to the fact that, *whether conventional or generalized, the unit is the basic invariant of any theory.* Therefore, the use of the unit for the generalization of pre-existing methods guarantees the preservation of the invariance so crucial for physical consistency (Sections 1.5.2 and 1.5.3).

Another aspect that illustrates the uniqueness of the isotopies for grand unifications is that *the positive-definiteness of the isounit guarantees the preservation of the abstract axioms of electroweak theories,* thus assuring axiomatic consistency of grand unification from the very beginning.

The general lines on isotopies presented in memoirs [12,13] of 1978 were then followed by laborious studies that reached mathematical and physical maturity only in memoir [14] of 1996, as outlined in Chapter 3 (see monographs [15] for a comprehensive presentation).

(B) Isodualities. The achievement of an axiomatically consistent grand unification for *matter* constitutes only *half* of the solution because, as stressed in Section 7.1.1, no grand unification can be considered physically significant without the consistent inclusion of antimatter.

The incompatibility of electroweak theories and general relativity for antimatter identified in Section 7.1.2 is only the symptom of deeper compatibility problems. As now familiar from the studies presented in this monograph, matter is treated at *all* levels, from Newtonian to electroweak theories, while antimatter is treated only at the level of *second quantization.*

Since there are serious indications that half of the universe could well be made up of antimatter (see Section 7.2), it is evident that a more effective theory of antimatter must apply at *all* levels.

Until such a scientific imbalance is resolved, any attempt at a grand unification can well prove to be futile.

Recall that charge conjugation in quantum mechanics is an *anti-automorphic map.* As a result, no classical theory of antimatter can possibly be axiomatically consistent via the mere change of the sign of the charge, because it must be an anti-automorphic (or, more generally, anti-isomorphic) image of that of matter in *all* aspects, including numbers, spaces, symmetries, etc.

The resolution of the above imbalance required a second laborious scientific journey that initiated with the proposal of the *isodual map* in memoirs [16] of 1985, here expressed for an arbitrary quantity

$$Q(x, v, \psi, \ldots) \ \ \rightarrow Q^d = -Q^\dagger(-x^\dagger, -v^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \ldots), \tag{7.1.2}$$

proposal that was followed by various studies whose mathematical and physical maturity was only reached years later in memoir [14] of 1996, as reported in Chapters 2 and 3 (see also monographs [15] for a more general presentation).

To illustrate the difficulties, it is appropriate here to note that, following the presentation in papers [16] of 1985 of the main mathematical ideas, it took the author *nine years* before publishing their application to antimatter in paper [17] of 1994.

We are here referring to the original proposal of Refs. [16,17] of mapping isounit (7.1.1) for matter into an *negative-definite* nonsingular arbitrary unit, known today as *Santilli's isodual isounits*,

$$\hat{I}(x, \psi, \partial\psi, \ldots) > 0 \quad \rightarrow \quad \hat{I}^d = -\hat{I}^\dagger(-x^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \ldots) < 0 \tag{7.1.3}$$

and its use for the characterization of antimatter at all levels, from Newtonian mechanics to second quantization.

The uniqueness of the isodual representation is given by the fact that isodualities are the *only* known liftings permitting the construction of a mathematic that is anti-isomorphic to the conventional (or isotopic) mathematics, as necessary for

a consistent representation of antimatter at all levels, while preserving the crucial invariance needed to avoid catastrophic inconsistencies.

(C) Poincaré-Santilli isosymmetry and its isoduals. The scientific journeys on isotopies and isodualities were only intended as pre-requisites for the construction of the *universal symmetry of gravitation for matter and, separately, for antimatter* in such a way to be locally isomorphic to the spacetime symmetry of electroweak interactions, the latter being an evident condition of consistency.

It is easy to see that, without the prior achievement of a new gravitation possessing an invariance, rather than the covariance of general relativity, any attempt at constructing a grand unification will prove to be futile in due time.

The complexity of the problem is illustrated by the fact that, not only gravitation for matter had to be reformulated in a form admitting a symmetry, but that symmetry had to be compatible with the basic Poincaré symmetry of electroweak theories [4–7]. Moreover, a dual compatible symmetry had to be achieved for the gravity of antimatter.

The latter problems called for a third laborious scientific journey on the *isotopies and isodualities of the Poincaré symmetry* $\hat{P}(3.1)$, today called the *Poincaré-Santilli isosymmetry and its isodual* outlined in Section 3.5 (see monographs [15] for comprehensive studies). These studies included:

1) The isotopies and isodualities of the Lorentz symmetry initiated with paper [18] of 1983 on the classical isotopies with the operator counterpart presented in paper [19] of the same year;

2) The isotopies and isodualities of the rotational symmetry first presented in papers [16][3];

3) The isotopies and isodualities of the $SU(2)$-spin symmetry, first presented in paper [20] of 1993, and related implications for local realist, hidden variables and Bell's inequalities published in Ref. [21] of 1998;

4) The isotopies and isodualities of the Poincaré symmetry including the universal invariance of gravitation, first presented in paper [22] of 1993; and

5) The isotopies and isodualities of the spinorial covering of the Poincaré symmetry first presented in papers [23,24] of 1996.[4]

[3]Papers [16] on the lifting of the rotational symmetry were evidently written before paper [19] on the lifting of the Lorentz symmetry, but appeared in print only two years following the latter due to rather unreasonable editorial processing by various journals reported in Ref. [16], which processing perhaps illustrates the conduct of some (but not all) editors when facing true scientific novelty.

[4]Ref. [24], which is the most important reference of this entire monograph (because admitting all topics as particular cases), was rejected for years by all journals of Western Physical Societies because the paper included an *industrial* application currently receiving large investments by the industry — although not by academia, — consisting in the achievement of a numerical, exact and invariant representation of *all* characteristics of the neutron as a bound state of a proton and an electron according to Rutherford. In fact, the resolution of the historical difficulties of Rutherford's conception of the neutron permits the utilization of the large clean energy contained in the neutron's structure, via its stimulated decay

We are referring here to the reconstruction of the conventional symmetries with respect to an arbitrary nonsingular positive-definite unit (7.1.1) for the isotopies, and with respect to an arbitrary nonsingular negative-definite unit (7.1.3) for the isodualities.

This reconstruction yields the most general known nonlinear, nonlocal and noncanonical or nonunitary liftings of conventional symmetries, while the locally isomorphism for isotopies) (anti-isomorphism for isodualities) with the original symmetries is guaranteed by the positive-definiteness (negative-definiteness) of the generalized units.

One should be aware that the above structures required the prior step-by-step isotopies and isodualities of Lie's theory (enveloping associative algebras, Lie algebras, Lie groups, transformation and representation theories, etc.), originally proposed by Santilli in 1978 [12], studied in numerous subsequent works and today called the *Lie-Santilli isotheory and its isodual* (see Section 3.2 for an outline and Refs. [15] for comprehensive studies).

It is evident that the Poincaré-Santilli isosymmetry and its isodual have fundamental character for these studies. One of their primary applications has been the achievement of the universal *symmetry* (rather than covariance) of all possible Riemannian line elements in their iso-Minkowskian representation [22]

$$ds'^2 = dx'^{\mu} \times g(x')_{\mu\nu} \times dx'^{\nu} \equiv dx^{\mu} \times g(x)_{\mu\nu} \times dx^{\nu} = ds^2, \tag{7.1.4}$$

Once the unit of gauge theories is lifted to represent gravitation, electroweak interactions will also obey the Poincaré-Santilli isosymmetry for matter and its isodual for antimatter, thus offering realistic hopes for the resolution of the most difficult problem of compatibility between gravitation and electroweak interactions, that for spacetime symmetries.

Perhaps unexpectedly, the fundamental spacetime symmetry of the grand unified theory of Refs. [9–11] is based on the *total symmetry of Dirac's equation*, here written with related spacetime and underlying unit (see Chapter 2 for details)

$$S_{tot} = \{SL(2.C) \times T(3.1) \times \mathcal{I}(1)\} \times \{SL^d(2.C^d) \times^d T^d(3.1) \times^d \mathcal{I}^d(1)\}, \tag{7.1.5a}$$

caused by a hard photon with a resonating frequency (numerically predicted by hadronic mechanics) that expels Rutherford's electron (the *isoelectron* with an isorenormalized mass generated by the nonlocal and non-Lagrangian interactions in the hyperdense medium inside the proton, see Chapter 6 and references quoted therein),

$$\gamma_{reson.} + n \to p^+ + e^- + \bar{\nu}.$$

Despite the undeniable mathematical consistency clear plausibility and evident large societal implications due to the need for new clean energies, Ref. [24] was rejected by all Western Physical Society without any credible scientific motivation because not aligned with organized interests in quantum mechanics and special relativity. Paper [24] was finally published in China in 1996. As a gesture of appreciation for this scientific democracy, the author organized in Beijing the 1997 International Workshop on Hadronic Mechanics (see the Proceedings of the Wiorkshops on Hadronic Mechanics listed in the General Bibliography).

$$M_{tot} = \{M(x,\eta,R) \times S_{spin}\} \times \{M^d(x^d,\eta^d,R^d) \times^d S^d_{spin}\}, \tag{7.1.5b}$$

$$I_{tot} = \{I_{orb} \times I_{spin}\} \times \{I^d_{orb} \times^d I^d_{spin}\}. \tag{7.1.5c}$$

To understand the above occurrence, the reader should be aware that isodualities imply a new symmetry called *isoselfduality* (Section 2.1), given by the invariance under the isodual map (7.1.2).

Dirac's gamma matrices verify indeed this new symmetry (from which the symmetry itself was derived in the first place), i.e.,

$$\gamma_\mu \to \gamma^d_\mu = -\gamma^\dagger_\mu = \gamma_\mu. \tag{7.1.6}$$

Consequently, contrary to a popular belief throughout the 20-th century, the Poincaré symmetry *cannot* be the total symmetry of Dirac's equations, evidently because it is not isoselfdual.

For evident reasons of consistency, the total symmetry of Dirac's equation must also be isoselfdual as the gamma matrices are. This condition identifies the total symmetry (7.1.5a) because that symmetry is indeed isoselfdual.

To understand the dimensionality of symmetry (7.1.5a) one must first recall that isodual spaces are independent from conventional spaces. The doubling of the conventionally believed ten-dimensions of the Poincaré symmetry then yields *twenty* dimensions.

But relativistic invariants possess the novel *isotopic invariance* (3.5.27), i.e.,

$$\begin{aligned}(x^\nu \times \eta_{\mu\nu} \times x^\nu) \times I &\equiv [x^\nu \times (w^{-2} \times \eta)_{\mu\nu} \times x^\nu) \times (w^2 \times I) \\ &= (x^\nu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I},\end{aligned} \tag{7.1.7}$$

with corresponding isotopic invariance of Hilbert's inner product

$$\begin{aligned}< \psi| \times |\psi > \times I &\equiv < w^{-1} \times \psi| \times |w^{-1} \times \psi > \times (w^2 \times I) \\ &= < \psi|\hat{\times}|\psi > \times \hat{I}.\end{aligned} \tag{7.1.8}$$

Consequently, the conventional Poincaré symmetry has emerged as being *eleven dimensional* at both the classical and operator levels, as first presented by Santilli in Ref. [22] of 1993 and studied in Section 3.5.3. It then follows that *the total symmetry* (7.1.5a) *of Dirac's equations is twenty-two dimensional.*

The grand unification proposed in Refs. [9–11] is based on the axiomatic structure of the conventional Dirac's equations, not as believed throughout the 20-th century, but as characterized by isotopies and isodualities.

In particular, the grand unification here studied is permitted by the new isotopic invariances (7.1.7) and (7.1.8) that are hidden in relativistic invariants [21], thus assuring the operator compatibility of the grand unification, as we shall see.

The reader should not be surprised that the two new invariances (7.1.7) and (7.1.8) remained undetected throughout the 20-th century because their identification required the prior discovery of *new numbers*, first the numbers with arbitrary positive units, and then the additional new numbers with arbitrary negative units for invariances [25].

(D) Classical and operator isogravitation. After a number of (unpublished) attempts, the resolution of numerous inconsistencies of general relativity studied in Section 1.4, plus the inconsistencies for grand unifications, requested the *isotopic reformulation of gravitation,* today known as *Santilli's isogravitation*, first presented at the VII M. Grossman Meeting on General Relativity of 1996 [26], as reviewed in Section 3.5, essentially consisting in the factorization of any given (nonsingular and symmetric) Riemannian metric $g(x)$ into the Minkowskian metric η multiplied by a 4×4-matrix $\hat{T}$,

$$g(x) = \hat{T}_{Grav}(x) \times \eta, \tag{7.1.9}$$

and the reconstruction of gravitation with respect to the isounit

$$\hat{I}_{Grav}(x) = 1/\hat{T}_{Grav}(x), \tag{7.1.10}$$

thus requiring the isotopic reformulation of the totality of the mathematical and physical methods of general relativity.

Despite its simplicity, the implications of isogravitation are far reaching, such as:

1) The isotopic reformulation permits the achievement of the universal Poincaré-Santilli isoinvariance for all possible gravitational models;

2) The isotopic reformulation eliminates curvature for the characterization of gravity, and replaces it with *isoflatness,* thus achieving compatibility with the flatness of electroweak interactions;

3) The isotopic reformulation reconstructs unitarity on iso-Hilbert spaces over isofields via the identical reformulation of nonunitary transform at the foundations of hadronic mechanics (Chapter 3)

$$U \times U^\dagger \neq I \to \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}_{Grav}, \tag{7.1.11}$$

where

$$U \times U^\dagger = \hat{I}, \quad \hat{U} = U \times \hat{T}^{1/2}_{Grav}, \tag{7.1.12}$$

thus providing the *only* known resolution of the catastrophic inconsistencies of Theorems 1.5.1 and 1.5.2.

Above all, isogravitation achieved the first and only known, axiomatically consistent operator formulation of gravitation provided by relativistic hadronic mechanics of Section 3.5, as first presented in Ref. [27] of 1997.

In fact, gravity is merely imbedded in the *unit* of relativistic operator theories. Since the gravitational isounit is positive-definite from the nonsingular and symmetric character of the metric $g(x)$ in factorization (7.1.9), the abstract axioms of operator isogravity are the conventional axioms of relativistic quantum mechanics, only subjected to a broader realization.

The preservation of conventional relativistic axioms then assures the achievement, for the first time as known by the author, of a consistent operator formulation of gravitation.[5]

(E) Geometric unification of special and general relativities. The resolution of the problems caused by lack of any Minkowskian limit of general relativity requested additional studies. After a number of (unpublished) attempts, the only possible solution resulted to be a geometric unification of special and general relativities, first presented in Ref. [28], in which the two relativities are characterized

by the same abstract axioms and are differentiated only by their realization of the basic unit. The trivial realization $I = Diag.(1, 1, 1, 1)$ characterizes special relativity, and broader realization (7.1.10) characterizes general relativity.

The latter final efforts requested the construction *ab initio* of a new geometry, today known as *Minkowski-Santilli isogeometry* [28] in which the abstract axioms are those of the Minkowskian geometry, including the abstract axiom of flatness necessary to resolve the catastrophic inconsistencies of Section 1.4, yet the new geometry admits the entire mathematical formalist of the Riemannian geometry, including covariant derivatives, Christoffel's symbols, etc. (see Section 3.2 for an outline and monographs [15] for comprehensive studies).

The important point is that at the limit

$$\lim \hat{I}_{Grav}(x) \to I, \tag{7.1.13}$$

the Minkowskian geometry and conventional special relativity are recovered identically and uniquely.

The reader should be aware that the grand unification presented in this section is centrally dependent on the Minkowski-Santilli isogeometry, the Poincaré-Santilli isosymmetry, and the isotopic formulation of gravitation. Their knowledge is a necessary pre-requisite for the technical understanding of the following sections.

[5]Note that the use of the words "quantum gravity" for operator formulation of gravitation, whether conventional or characterized by the isotopies, would be merely political. This is due to the fact that, on serious scientific grounds, the term "quantum" can only be referred to physical conditions admitting a quantized emission and absorption of energy as occurring in the structure of the hydrogen atom. By comparison, no such quantized orbits are possible for operator theories of gravity, thus rendering nonscientific its characterization as "quantum gravity". Ironically, the editor of a distinguished physics journal expressed interest in publishing a paper on "operator isogravity" under the condition of being called "quantum gravity", resulting in the necessary withdrawal of the paper by the author so as not to reduce fundamental physical inquiries to political compromises.

7.1.4 Isotopic Gauge Theories

The isotopies of gauge theories were first studied in the 1980's by Gasperini [29], followed by Nishioka [30], Karajannis and Jannussis [31] and others, and ignored thereafter for over a decade.

These studies were defined on conventional spaces over conventional fields and were expressed via the conventional differential calculus. As such, they are not invariant, as it became shown in memoirs [32], thus suffering of the catastrophic inconsistencies of Theorem 1.5.2.

Refs. [9–11] presented, apparently for the first time, the *invariant isotopies of gauge theories*, or *isogauge theories* for short, and their isoduals, those formulated on isospaces over isofields and characterized by the isodifferential calculus of memoir [14]. For completeness, let us recall that the latter theories are characterized by the following methods:

(1) Isofields [25] of isoreal numbers $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ and isocomplex numbers $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$ with: additive isounit $\hat{0} = 0$; generalized multiplicative isounit $\hat{I}$ given by Eq. (7.1.9); elements, isosum, isoproduct and related generalized operations,

$$\hat{a} = a \times \hat{I}, \;\; \hat{a}\hat{+}\hat{b} = (a + b) \times \hat{I}, \tag{7.1.14a}$$

$$\hat{a}\hat{\times}\hat{b} = \hat{a} \times \hat{T} \times \hat{b} = (a \times b) \times \hat{I}, \tag{7.1.14b}$$

$$\hat{a}^{\hat{n}} = \hat{a}\hat{\times}\hat{a}\hat{\times}...\hat{\times}\hat{a}, \tag{7.1.14c}$$

$$\hat{a}^{\hat{1/2}} = a^{1/2} \times \hat{I}^{1/2}, \;\; \hat{a}\hat{/}\hat{b} = (\hat{a}/\hat{b}) \times \hat{I}, \text{ etc.} \tag{7.1.14d}$$

(2) Isominkowski spaces [18] $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ with isocoordinates $\hat{x} = x \times \hat{I} = \{x^\mu\} \times \hat{I}$, isometric $\hat{N} = \hat{\eta} \times \hat{I} = [\hat{T}(x, \dots) \times \eta] \times \hat{I}$, and *isointerval* over the isoreals $\hat{R}$

$$\begin{aligned}(\hat{x} - \hat{y})^{\hat{2}} &= [(\hat{x} - \hat{y})^\mu \hat{\times} \hat{N}_{\mu\nu} \hat{\times} (\hat{x} - \hat{y})^\nu] \\ &= [(x - y)^\mu \times \hat{\eta}_{\mu\nu} \times (x - y)^\nu] \times \hat{I},\end{aligned} \tag{7.1.15}$$

equipped with *Kadeisvili isocontinuity* [33] and the *isotopology* developed by G. T. Tsagas and D. S. Sourlas [34], R. M. Santilli [14], R. M. Falcón Ganfornina and J. Núñez Valdés [35,36] (see also Aslander and Keles [37]). A more technical formulation of the isogauge theory can be done via the isobundle formalism on isogeometries.

(3) Isodifferential calculus [14] characterized by the following isodifferentials

$$\hat{d}\hat{x}^\mu = \hat{I}^\mu_\nu \times d\hat{x}^\nu, \tag{7.1.16a}$$

$$\hat{d}\hat{x}_\mu = \hat{T}^\nu_\mu \times d\hat{x}_\nu, \tag{7.1.16b}$$

and isoderivatives

$$\hat{\partial}_\mu \hat{f} = \hat{\partial}\hat{f}\hat{/}\hat{\partial}\hat{x}^\mu = (\hat{T}^\nu_\mu \times \partial_\nu f) \times \hat{I}, \tag{7.1.17a}$$

$$\hat{\partial}^\mu \hat{f} = (\hat{I}^\mu_\nu \times \partial_\nu f) \times \hat{I}, \;\; \hat{\partial}\hat{x}^\mu \hat{/} \hat{\partial}\hat{x}^\nu = \hat{\delta}^\mu_\nu = \delta^\mu_\nu \times \hat{I}, \text{ etc.} \tag{7.1.17b}$$

where one should note the inverted use of the isounit and isotopic element with respect to preceding formulations.

(4) Isofunctional isoanalysis [15], including the reconstruction of all conventional and special functions and transforms into a form admitting of $\hat{I}_{Grav}$ as the left and right unit. Since the iso-Minkowskian geometry preserves the Minkowskian axioms, it allows the preservation of the notions of straight and intersecting lines, thus permitting the reconstruction of trigonometric and hyperbolic functions for the Riemannian metric $g(x) = \hat{T}(x) \times \eta$.

(5) Iso-Minkowskian geometry [28], i.e., the geometry of isomanifolds $\hat{M}$ over the isoreals $\hat{R}$, that satisfies all abstract Minkowskian axioms because of the joint liftings

$$\eta \to \hat{\eta} = T(x, \ldots) \times \eta, \tag{7.1.18a}$$

$$I \to \hat{I} = T^{-1}, \tag{7.1.18b}$$

while preserving the machinery of Riemannian spaces as indicated earlier, although expressed in terms of the isodifferential calculus.

In this new geometry *Riemannian* line elements are turned into identical *Minkowskian* forms via the embedding of gravity in the deferentials, e.g., for the Schwarzschild exterior metric we have the iso-Minkowskian reformulation (Ref. [28], Eqs. (2.57)), where the spacetime coordinates are assumed to be covariant,

$$\hat{ds} = \hat{dr}^{\hat{2}} \hat{+} \hat{r}^{\hat{2}} \hat{\times} (\hat{d\theta}^{\hat{2}} \hat{+} isosin^{\hat{2}} \hat{\theta}) \hat{-} \hat{dt}^{\hat{2}}, \tag{7.1.19a}$$

$$\hat{dr} = \hat{T}_r \times dr, \hat{dt} = \hat{T}_t \times dt, \tag{7.1.19b}$$

$$\hat{T}_r = (1 - 2 \times M/r)^{-1}, \;\; \hat{T}_t = 1 - 2 \times M/r. \tag{7.1.19c}$$

(6) Relativistic hadronic mechanics [15] characterized by the *iso-Hilbert space* $\hat{\mathcal{H}}$ with *isoinner product and isonormalization* over $\hat{C}$

$$< \hat{\phi} | \hat{\times} | \hat{\psi} > \times \hat{I}, \quad < \hat{\psi} | \hat{\times} | \hat{\psi} > = \hat{I}. \tag{7.1.20}$$

Among various properties, we recall that: the *iso-Hermiticity* on $\hat{\mathcal{H}}$ coincides with the conventional Hermiticity (thus, all conventional observables remain observables under isotopies); the isoeigenvalues of iso-Hermitean operators are real and conventional (because of the identities

$$\hat{H} \hat{\times} | \hat{\psi} > = \hat{E} \hat{\times} | \hat{\psi} > = E \times | \hat{\psi} >; \tag{7.1.21}$$

the condition of *isounitarity* on $\hat{\mathcal{H}}$, over $\hat{C}$ is given by

$$\hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}, \tag{7.1.22}$$

(see memoir [27] for details).

(7) The Lie-Santilli isotheory [12] with: conventional (ordered) basis of generators $X = (X_k)$, and parameters $w = (w_k)$, $k = 1, 2, \ldots, n$, only formulated in isospaces over isofields with a common isounit; universal enveloping isoassociative algebras $\hat{\xi}$ with infinite-dimensional basis characterized by the isotopic Poincare'-Birkhoff-Witt theorem [12]

$$\hat{I},\ \hat{X}_i\hat{\times}\hat{X}_j,\ (i \leq j), \quad \hat{X}_i\hat{\times}\hat{X}_j \times \hat{X}_k,\ (i \leq j \leq k, ...) \tag{7.1.23}$$

Lie-Santilli subalgebras [12]

$$[\hat{X}_i,\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i = \hat{C}_{ij}^{k}(x, \ldots)\hat{\times}\hat{X}_k, \tag{7.1.24}$$

where the $\hat{C}$'s are the structure disfunctions; and isogroups characterized by isoexponentiation on $\hat{\xi}$ with structure [12]

$$\hat{e}^{\hat{X}} = \hat{I}\hat{+}\hat{X}\hat{/}\hat{1}!\hat{+}\hat{X}\hat{\times}\hat{X}\hat{/}\hat{2}!\hat{+}... = (e^{\hat{X}\times\hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T}\times\hat{X}}). \tag{7.1.25}$$

Despite the isomorphism between isotopic and conventional structures, the lifting of Lie's theory is nontrivial because of the appearance of the matrix $\hat{T}$ with nonlinear integrodifferential elements in the very *exponent* of the group structure, Eqs. (7.1.25).

To avoid misrepresentations, one should keep in mind that the isotopies of Lie's theory *were not* proposed to identify "new Lie algebras" (an impossible task since all simple Lie algebras are known from Cartan's classification), but to construct instead the most general possible nonlinear, nonlocal and noncanonical or nonunitary "realizations" of known Lie algebras.

(8) Isolinearity, isolocality and isocanonicity or isounitarity. Recall from lifting (7.1.25) that isosymmetries have the most general possible nonlinear, nonlocal and noncanonical or nonunitary structure. A main function of the isotopies is that of reconstructing linearity, locality and canonicity or unitarity on isospaces over isofields, properties called *isolinearity, isolocality and isocanonicity or isounitarity*. These are the properties that permit the bypassing of the theorems of catastrophic inconsistencies of Section 1.5.

As a result, the use of the conventional *linear* transformations on M over R, $X' = A(w) \times x$ violates *isolinearity* on $\hat{M}$ over $\hat{R}$.

In general, *any* use of conventional mathematics for isotopic theories leads to a number of inconsistencies which generally remain undetected by nonexperts in the field.[6]

[6]The use of conventional mathematics for isotheories would be the same as elaborating Balmer's quantum spectral lines in the hydrogen atoms with isofunctional analysis, resulting in evident major inconsistencies.

(9) Isogauge theories [9–11]. They are characterized by an n-dimensional connected and non-isoabelian isosymmetry $\hat{G}$ with: basic n-dimensional isounit (4.1.9); iso-Hermitean generators $\hat{X}$ on an iso-Hilbert space $\hat{\mathcal{H}}$ over the isofield $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$; universal enveloping associative algebra $\hat{\xi}$ with infinite isobasis (7.1.23); isocommutation rules (7.1.24); isogroup structure

$$\hat{U} = \hat{e}^{-i\times X_k \times \theta(x)_k} = (e^{-i\times X_k \times \hat{T} \times \theta(x)_k}) \times \hat{I}, \quad \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}, \tag{7.1.26}$$

where one should note the appearance of the gravitational isotopic elements in the *exponent* of the isogroup, and the parameters $\theta(x)_k$ now depend on the iso-Minkowski space; isotransforms of the isostates on $\hat{\mathcal{H}}$

$$\hat{\psi}' = \hat{U} \hat{\times} \hat{\psi} = \left(e^{-i\times X_k \times \hat{T}(x,...) \times \theta(x)_k}\right) \times \hat{\psi}; \tag{7.1.27}$$

isocovariant derivatives [28]

$$\hat{D}_\mu \hat{\psi} = (\hat{\partial}_\mu - i \hat{\times} \hat{g} \hat{\times} \hat{A}(\hat{x})^k_\mu \hat{\times} \hat{X}_k) \hat{\times} \hat{\psi}; \tag{7.1.28}$$

iso-Jacobi identity

$$[\hat{D}_\alpha \hat{,} [\hat{D}_\beta \hat{,} \hat{D}_\gamma]] \hat{+} [\hat{D}_\beta \hat{,} [\hat{D}_\gamma \hat{,} \hat{D}_\alpha]] \hat{+} [\hat{D}_\gamma \hat{,} [\hat{D}_\alpha \hat{,} \hat{D}_\beta]] = 0, \tag{7.1.29}$$

where g and $\hat{g} = g \times \hat{I}$ are the conventional and isotopic coupling constants, $A(x)^k_\mu \times X_k$ and $\hat{A}(\hat{x})^k_\mu \hat{\times} \hat{X}_k = [A(x)^k_\mu \times X_k] \times \hat{I}$ are the gauge and isogauge potentials; isocovariance

$$(\hat{D}_\mu \hat{\psi})' = (\hat{\partial}_\mu \hat{U}) \hat{\times} \hat{\psi} \hat{+} \hat{U} \hat{\times} (\hat{\partial}_\mu \hat{\psi}) \hat{-} \hat{i} \hat{\times} \hat{g} \hat{\times} \hat{A}'(\hat{x})_\mu \hat{\times} \hat{\psi} = \hat{U} \hat{\times} \hat{D}_\mu \hat{\psi}, \tag{7.1.30a}$$

$$\hat{A}(\hat{x})'_\mu = -\hat{g}^{-\hat{1}} \hat{\times} [\hat{\partial}_\mu \hat{U}(\hat{x})] \hat{\times} \hat{U}(\hat{x})^{-\hat{1}}, \tag{7.1.30b}$$

$$\hat{\delta} \hat{A}(\hat{x})^k_\mu = -\hat{g}^{-\hat{1}} \hat{\times} \hat{\partial}_\mu \hat{\theta}(\hat{x})^k \hat{+} \hat{C}^k_{ij} \hat{\times} \hat{\theta}(\hat{x})^i \hat{\times} \hat{A}(\hat{x})^j_\mu, \tag{7.1.30c}$$

$$\hat{\delta} \hat{\psi} = -\hat{i} \hat{\times} \hat{g} \hat{\times} \hat{\theta}(\hat{x})^k \hat{\times} \hat{X}_k \hat{\times} \hat{\psi}; \tag{7.1.30d}$$

non-isoabelian iso-Yang-Mills fields

$$\hat{F}_{\mu\nu} = \hat{i} \hat{\times} \hat{g}^{-\hat{1}} \hat{\times} [\hat{D}_{\hat{\mu}}, \hat{D}_\nu] \hat{\psi}, \tag{7.1.31a}$$

$$\hat{F}^k_{\mu\nu} = \hat{\partial}_\mu \hat{A}^k_\nu \hat{-} \hat{\partial}_\nu \hat{A}^k_\mu \hat{+} \hat{g} \hat{\times} \hat{C}^k_{ij} \hat{\times} \hat{A}^i_\mu \hat{\times} \hat{A}^j_\nu; \tag{7.1.31b}$$

related isocovariance properties

$$\hat{F}_{\mu\nu} \to \hat{F}'_{\mu\nu} = \hat{U} \hat{\times} \hat{F}_{\mu\nu} \hat{\times} \hat{U}^{-1}, \tag{7.1.32a}$$

$$Isotrace(\hat{F}_{\mu\nu'} \hat{\times} \hat{F}^{\mu\nu'}) = Isotrace(\hat{F}_{\mu\nu} \hat{\times} \hat{F}^{\mu\nu}), \tag{7.1.32b}$$

$$[\hat{D}_\alpha\hat{,}\hat{F}_{\beta\gamma}]\,\hat{+}\,[\hat{D}_\beta\hat{,}\hat{F}_{\gamma\alpha}]\,\hat{+}\,[\hat{D}_\gamma\hat{,}\hat{F}_{\alpha\beta}]\equiv 0; \tag{7.1.32c}$$

derivability from the isoaction

$$\hat{S}=\hat{\int}\hat{d^4}\hat{x}(-\hat{F}_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}\hat{/}\hat{4})=\hat{\int}\hat{d^4}\hat{x}(-\hat{F}^k_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}_k\hat{/}\hat{4}), \tag{7.1.33}$$

where $\hat{\int}=\int\times\hat{I}$, plus all other familiar properties in isotopic formulation.

The *isodual isogauge theory*, first proposed in Refs. [9–11], is the image of the preceding theory under the isodual map (7.1.2) when applied to the totality of quantities and their operations.

The latter theory is characterized by the isodual isogroup $\hat{G}^d$ with isodual isounit

$$\hat{I}^d_{Grav}=-\hat{I}^\dagger_{Grav}=-\hat{I}_{Grav}=-1/\hat{T}_{Grav}<0. \tag{7.1.34}$$

The elements of the base fields

$$\hat{R}^d(\hat{n}^d,\hat{+}^d,\hat{\times}^d), \tag{7.1.35}$$

are given by the isodual isoreal numbers

$$\hat{n}^d=-\hat{n}=-n\times\hat{I}, \tag{7.1.36}$$

and those of the field

$$\hat{C}^d(\hat{c}^d,\hat{+}^d,\hat{\times}^d), \tag{7.1.37}$$

are the isodual isocomplex numbers

$$\hat{c}^d=-(c\times\hat{I})^\dagger=(n_1-i\times n_2)\times\hat{I}^d=(-n_1+i\times n_2)\times\hat{I}. \tag{7.1.38}$$

The carrier spaces are the isodual iso-Minkowski spaces $\hat{M}^d(\hat{x}^d,-\hat{\eta}^d,\hat{R}^d)$ on $\hat{R}^d$ and the isodual iso-Hilbert space $\mathcal{H}^d$ on $\hat{C}^d$ with isodual isostates and isodual isoinner product

$$|\hat{\psi}>^d=-|\hat{\psi}>^\dagger=-<\psi|, \tag{7.1.39a}$$

$$<\hat{\phi}|^d\times\hat{T}^d\times|\hat{\psi}>^d\times\hat{I}^d. \tag{7.1.39b}$$

It is instructive to verify that all eigenvalues of isodual iso-Hermitean operators are *negative − definite* (when projected in our space-time),

$$\hat{H}^d\,\hat{\times}^d\,|\hat{\psi}>^d=<\psi|\times(-E). \tag{7.1.40}$$

$\hat{G}^d$ is characterized by the isodual Lie-Santilli isotheory with isodual generators $\hat{X}^d=-\hat{X}$, isodual isoassociative product

$$\hat{A}^d\hat{\times}^d\hat{B}^d=\hat{A}^d\times\hat{T}^d\times\hat{B}^d,\ \ \hat{T}^d=-\hat{T}, \tag{7.1.41}$$

and related isodual isoenveloping and Lie-Santilli isoalgebra.

The elements of $\hat{G}^d$ are the isodual isounitary isooperators

$$\hat{U}^d(\hat{\theta}^d(\hat{x}^d)) = -\hat{U}^\dagger(-\hat{\theta}(-\hat{x})). \tag{7.1.42}$$

In this way, the isodual isogauge theory is seen to be an anti-isomorphic image of the preceding theory, as desired.

It is an instructive exercise for the reader interested in learning the new techniques to study first the isodualities of the *conventional* gauge theory (rather than of their isotopies), and show that they essentially provide a mere reinterpretation of the usually discarded, advanced solutions as characterizing antiparticles.

Therefore, in the isoselfdual theory with total gauge symmetry $\hat{G}\times\hat{G}^d$, isotopic retarded solutions are associated with particles and advanced isodual solutions are associated with antiparticles.

No numerical difference is expected in the above reformulation because, as shown in Chapter 3, isotopies preserve not only the original axioms but also the original numerical value (when constructed properly).

It is also recommendable for the interested reader to verify that the isotopies are indeed equivalent to charge conjugation for all massive particles, with the exception of the photon (see Section 2.3). In fact, isodual theories predict that the antihydrogen atom emits a new photon, tentatively called by this author the *isodual photon* [38], that coincides with the conventional photon for all possible interactions, thus including electroweak interactions, *except gravitation.* This indicates that the isodual map is inclusive of charge conjugation for massive particles, but it is broader than the latter.

Isodual theories in general, thus including the proposed grand unification, predict that all *stable* isodual particles, such as the isodual photon, the isodual electron (positron), the isodual proton (antiproton) and their bound states (such as the antihydrogen atom), experience *antigravity* in the field of the Earth (defined as the reversal of the sign of the curvature tensor).

If confirmed, the prediction may offer the possibility in the future to ascertain whether far away galaxies and quasars are made-up of matter or of antimatter.

We finally note that isomathematics is a particular case of the broader *genomathematics*, also introduced for the first time in Refs. [12] of 1978 (see Chapter 4), which occurs for non-Hermitean generalized units and is used for an axiomatization of irreversibility.

In turn, genomathematics is a particular case of the *hypermathematics*, that occurs when the generalized units are given by ordered *sets* of non-Hermitean quantities and is used for the representation of multivalued complex systems (e.g. biological entities) in irreversible conditions.

Evidently both the genomathematics and hypermathematics admit an anti-isomorphic image under isoduality (see also Chapter 4).

In conclusion the methods outlined in this note permit the study of *seven* liftings of conventional gauge theories [9–11]:

(1) The *isodual gauge theories* for the treatment of antimatter without gravitation in vacuum;

(2,3) The *isogauge theories and their isoduals*, for the inclusion of gravity for matter and antimatter in reversible conditions in vacuum (exterior gravitational problem);

(4,5) The *genogauge theories and their isoduals*, for the inclusion of gravity for matter and antimatter in irreversible interior conditions (interior gravitational problems); and

(6,7) the *hypergauge theories and their isoduals*, for multivalued and irreversible generalizations.

For brevity this section is restricted to theories of type (1), (2), (3). The development of the remaining genotopies of gauge theories is left to interested readers.

7.1.5 Iso-, Geno- and Hyper-Grand-Unifications

In this section we review the *Iso-Grand-Unification* (IGU) with the inclusion of electroweak and gravitational interactions, first submitted in Refs. [9–11] via the 22-dimensional total isoselfdual isosymmetry given by isosymmetry (3.5.28) and its isodual

$$\hat{S}_{tot} = (\hat{\mathcal{P}}(3.1)\hat{\times}\hat{G}) \times (\hat{\mathcal{P}}(3.1)^d\hat{\times}^d\hat{G}^d) =$$

$$= [\hat{SL}(2,\hat{C})\hat{\times}\hat{T}(3.1)\hat{\times}\hat{\mathcal{I}}(1)] \times [\hat{SL}^d(2,\hat{C}^d)\hat{\times}^d\hat{T}^d(3.1)\hat{\times}^d\hat{\mathcal{I}}^d(1)], \qquad (7.1.43)$$

where $\hat{\mathcal{P}}$ is the Poincaré-Santilli isosymmetry [22] in its isospinorial realization [24], $\hat{G}$ is the isogauge symmetry of the preceding section and the remaining structures are the corresponding isoduals.

Without any claim of a final solution, it appears that the proposed IGU does indeed offer realistic possibilities of resolving the axiomatic incompatibilities (1)–(5) of Section 7.1.2 between gravitational and electroweak interactions.

In fact, IGU represents gravitation in a form geometrically compatible with that of the electroweak interactions, represents antimatter at all levels via negative-energy solutions, and characterizes both gravitation as well as electroweak interactions via the universal Poincaré-Santilli isosymmetry.

It should be indicated that we are referring here to the *axiomatic* consistency of IGU. In regard to the *physical* consistency we recall that isotopic liftings preserve not only the original axioms, but also the original numerical values [15].

As an example, the image in iso-Minkowskian space over the isoreals of the light cone, the isolight cone, not only is a perfect cone, but a cone with the original characteristic angle, thus preserving the speed of light in vacuum as the maximal causal speed in iso-Minkowskian space.

This peculiar property of the isotopies implies the expectation that the proposed Iso-Grand-Unification preserves the numerical results of electroweak interactions.

The reader should be aware that the methods of the recent memoir [27] permit a truly elementary, explicit construction of the proposed IGU.

As well known, the transition from the Minkowskian metric η to Riemannian metrics $g(x)$ is a *noncanonical transform* at the classical level, and, therefore, a at the operator level.

The method herein considered for turning a gauge theory into an IGU consists in the following representation of the selected gravitational model, e.g., Schwarzschild's model:

$$g(x) = T(x) \times \eta, \tag{7.1.44a}$$

$$I(x) = U \times U^\dagger = 1/\hat{T} =$$

$$Diag.[(1 - 2 \times M/r) \times Diag.(1, 1, 1),\ (1 - 2 \times M/r)^{-1}], \tag{7.1.44b}$$

and then subjecting the *totality* of the gauge theory to the nonunitary transform $U \times U^\dagger$.

The method then yields: the isounit

$$I \to \hat{I} = U \times I \times U^\dagger; \tag{7.1.45}$$

the isonumbers

$$a \to \hat{a} = U \times a \times U^\dagger = a \times (U \times U^\dagger) = a \times \hat{I},\ a = n, c; \tag{7.1.46}$$

the isoproduct with the correct expression and Hermiticity of the isotopic element,

$$A \times B \to U \times (A \times B) \times U^\dagger =$$
$$= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) =$$
$$= \hat{A} \times \hat{T} \times \hat{B} = \hat{A} \hat{\times} \hat{B}; \tag{7.1.47}$$

the correct form of the iso-Hilbert product on $\hat{C}$,

$$< \phi| \times |\psi > \to U \times < \phi| \times |\psi > \times U^\dagger =$$
$$= (< \psi| \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi >) \times (U \times U^\dagger) =$$
$$=< \hat{\phi}| \times \hat{T} \times |\hat{\psi} > \times \hat{I}; \tag{7.1.48}$$

the correct Lie-Santilli isoalgebra

$$A \times B - B \times A \to \hat{A} \hat{\times} \hat{B} - \hat{B} \hat{\times} \hat{A}; \tag{7.1.49}$$

the correct isogroup

$$U \times (e^X) \times U^\dagger = (e^{X \times \hat{T}}) \times \hat{I}, \tag{7.1.50}$$

the Poincaré-Santilli isosymmetry $\mathcal{P} \to \hat{\mathcal{P}}$, and the isogauge group $G \to \hat{G}$.

It is then easy to verify that the emerging IGU is indeed invariant under all possible additional nonunitary transforms, provided that, for evident reasons of consistency, they are written in their identical isounitary form,

$$W \times W^\dagger = \hat{I}, \tag{7.1.51a}$$

$$W = \hat{W} \times \hat{T}^{1/2}, W \times W^\dagger = \hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{I}. \tag{7.1.51b}$$

In fact, we have the invariance of the isounit

$$\hat{I} \to \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^\dagger = \hat{I}, \tag{7.1.52}$$

the invariance of the isoproduct

$$\hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^\dagger = \hat{A}' \hat{\times} \hat{B}', \text{ etc.} \tag{7.1.53}$$

Note that the isounit is *numerically* preserved under isounitary transforms, as it is the case for the conventional unit I under unitary transform, and that the selection of a nonunitary transform $W \times W^\dagger = \hat{I}'$ with value different from $\hat{I}$ evidently implies the transition to a different gravitational model.

Note that the lack of implementation of the above nonunitary-isounitary lifting to only *one* aspect of the original gauge theory (e.g., the preservation of the old numbers or of the old differential calculus) implies the loss of the invariance of the theory [32].

The assumption of the negative-definite isounit $\hat{I}^d = -(U \times U^\dagger)$ then yields the isodual component of the IGU.

Note finally that diagonal realization (7.1.44) has been assumed mainly for simplicity. In general, the isounit is positive-definite but *nondiagonal* 4×4-dimensional matrix. The Schwarzschild metric can then be more effectively represented in its isotropic coordinates as studied, e.g. in Ref. [39], pp. 196–199).

In closing, the most significant meaning of IGU is that *gravitation has always been present in unified gauge theories. It did creep-in un-noticed because embedded where nobody looked for, in the "unit" of gauge theories.*

In fact, the isogauge theory of Section 7.1.4 coincides with the conventional theory at the abstract level to such an extent that we could have presented IGU with exactly the same symbols of the conventional gauge theories without the "hats", and merely subjecting the same symbols to a more general realization.

Also, the isounit representing gravitation as per rule (7.1.9) verifies all the properties of the conventional unit I of gauge theories,

$$\hat{I}^{\hat{n}} = \hat{I}, \quad \hat{I}^{\hat{1/2}} = \hat{I}, \tag{7.1.54a}$$

$$d\hat{I}/dt = \hat{I} \hat{\times} \hat{H} - \hat{H} \hat{\times} \hat{I} = \hat{H} - \hat{H} = 0, \text{ etc.} \tag{7.1.54b}$$

The "hidden" character of gravitation in conventional gauge theories is then confirmed by the isoexpectation value of the isounit recovering the conventional unit I of gauge theories,

$$\hat{<}\hat{I}\hat{>} =< \hat{\psi}| \times \hat{T} \times \hat{I} \times \hat{T} \times |\hat{\psi} > / < \hat{\psi}| \times \hat{T} \times |\hat{\psi} >= I. \tag{7.1.55}$$

It then follows that *IGU constitutes an explicit and concrete realization of the theory of "hidden variables"* [40]

$$\lambda = T(x) = g(x)/\eta, \hat{H}\hat{\times}|\hat{\psi} >= \hat{H} \times \lambda \times |\hat{\psi} >= E_\lambda \times |\hat{\psi} >, \tag{7.1.56}$$

and the theory is correctly reconstructed with respect to the new unit

$$\hat{I} = \lambda^{-1}, \tag{7.1.57}$$

in which von Neumann's Theorem [41] and Bell's inequalities [42] do not apply, evidently because of the nonunitary character of the theory (see Ref. [21] and Vol. II of Refs. [15] for details).

In summary, the proposed inclusion of gravitation in unified gauge theories is essentially along the teaching of Einstein, Podolsky, and Rosen [43] on the "lack of completion" of quantum mechanics, only applied to gauge theories.

7.2 ISO-, GENO-, AND HYPER-SELF-DUAL COSMOLOGIES

A rather popular belief of the 20-th century was that the universe is solely composed of matter. This belief was primarily due to the scientific imbalance pertaining to antimatter as being solely studied at the level of second quantization, without any theoretical, let alone experimental, mean available for the study of antimatter.

In reality, there exists rather strong evidence that the universe is indeed composed of matter as well as antimatter and, more particularly, that some of the galaxies are made up of matter and others of antimatter.

To begin, not only the expansion of the universe, but more particularly the recently detected increase of the expansion itself, can be readily explained via an equal distribution of matter and antimatter galaxies.

In fact, antigravity experienced by matter and antimatter galaxies (studied in the preceding chapter) explains the expansion of the universe, while the continuous presence of antigravity explains the increase of the expansion.

The assumption that the universe originated from a primordial explosion, the "big bang", could have explained at least conceptually the expansion of the universe. However, the "big bang" conjecture is eliminated as scientifically possible by the increase of the expansion itself.

The "big bang" conjecture is also eliminated by the inability to explain a possible large presence of antimatter in the universe, trivially, because it would

have been annihilated at the time of the "big bang" because produced jointly with matter, as well as for other reasons.

By comparison, the only plausible interpretation at the current state of our knowledge is precisely the assumption that the universe is made up half of *matter galaxies* and half of *antimatter galaxies* due to the joint explanation of the expansion of the universe and its increase.

Independently from the above, there exists significant evidence that our Earth is indeed bombarded by antimatter particles and asteroids.

Astronauts orbiting Earth in spaceship have systematically reported that, when passing over the dark side, they see numerous flashes in the upper atmosphere that can be only interpreted as *antimatter cosmic rays*, primarily given by high energy antiprotons and/or positrons[7] originating from far away antimatter galaxies, which antiparticles, when in contact with the upper layers of our atmosphere, annihilate themselves producing the flashes seen by astronauts.

Note that the conventional *cosmic rays* detected in our atmosphere are *matter cosmic rays,* that is, high energy *particles*, such as protons and electrons, originating from a matter supernova or other matter astrophysical event.

In any case, it is evident that matter cosmic rays with sufficient energy can indeed penetrate deep into our atmosphere, while antimatter cosmic rays will be stopped by the upper layers of our atmosphere irrespective of their energy.

In addition, there exists evidence that our Earth has been hit by *antimatter meteorites* that, as such, can only originate from an astrophysical body made up of antimatter.

The best case is the very large devastation re corded in 1908 in Tunguska, Siberia, in which over one million acres of forest were completely flattened in a radial direction originating from a common center without any crater whatever, not even at the center.

The lack of a crater combined with the dimension of the devastation, exclude the origination from the explosion of a *matter asteroid*, firstly, because in this case debris would have been detected by the various expeditions in the area and, secondly, because there is no credible possibility that the mere explosion of a matter asteroid could have caused a devastation over such a large area requiring energies computed at about 100 times the atomic bomb exploded over Hiroshima, Japan.

The only plausible interpretation of the *Tunguska explosion* is that it was due to an antimatter asteroid that eventually annihilated after contact deep into our matter atmosphere.

The important point is that the numerical understanding of the Tunguska explosion requires an antimatter mass of the order of a ton, namely, an antimatter

[7]Evidently only *stable antiparticles can travel intergalactic distances without decaying.*

asteroid that, as such, can only originate from the supernova explosion of an antimatter star.

Consequently, the evidence on the existence of even one antimatter asteroid confirms the existence in the universe of antimatter stars. Since it is highly improbable that antimatter stars can exist within a matter galaxy, antimatter asteroids constitute significant evidence on the existence in the universe of antimatter galaxies.

But again, the expansion of the universe as well as the increase of the expansion itself are the strongest evidence for an essentially equal distribution of matter and antimatter galaxies in the universe, as well as for the existence of antigravity between matter and antimatter.

In any case, there exist no alternative hypothesis at all known to this author, let alone a credible hypothesis, that could explain quantitatively both the expansion of the universe and the increase of the expansion itself.

In view of the above occurrences, as well as to avoid discontinuities at creation, Santilli [44] proposed the new *Iso-Self-Dual Cosmology*, namely, a cosmology in which the universe has an exactly equal amount of matter and antimatter, much along the isoselfdual re-interpretation of Dirac's equation of Section 2.3.6.

Needless to say, such a conception of the universe dates back to the very birth of cosmology, although it was abandoned due to various reasons, including the lack of a consistent classical theory of antimatter, inconsistencies for negative energies, and other problems.

The above conception of the universe was then replaced with the "big bang" conjecture implying a huge discontinuity at creation, in which a possible antimatter component in the universe is essentially left untreated.

All the above problems are resolved by the isodual theory of antimatter, and quantitative astrophysical studies on antimatter galaxies and quasars can now be initiated at the purely classical level.

Moreover, the prediction that the *isodual light* emitted by antimatter experiences a repulsion in the gravitational field of matter [38], permits the initiation of actual measurements on the novel *antimatter astrophysics.*

Noticeably, there already exist reports that certain astrophysical events can only be explained via the repulsion experiences by light emitted by certain galaxies or quasars, although such reports could not be subjected to due scientific process since the mere existence of such a repulsion would invalidate Einstein's gravitation, as studied in Section 1.4.

Even though the assumption of an equal distribution of matter and antimatter in the universe dates back to the discovery of antimatter itself in the early 1930s, the Iso-Self-Dual Cosmology is structurally new because it is the first cosmology in scientific records based on a *symmetry,* let alone an *isoselfdual symmetry,* that

of Dirac's equation subjected to isotopies, Eqs. (7.1.43), i.e.,

$$\hat{S}_{Tot} = (\hat{\mathcal{P}}(3.1) \,\hat{\times}\, \hat{G}) \times (\hat{\mathcal{P}}(3.1)^d \,\hat{\times}^d\, \hat{G}^d) =$$

$$= [\hat{SL}(2, \hat{C}) \hat{\times} \hat{T}(3.1) \hat{\times} \hat{\mathcal{I}}(1)] \times [\hat{SL}^d(2, \hat{C}^d) \hat{\times}^d \hat{T}^d(3.1) \hat{\times}^d \hat{\mathcal{I}}^d(1)]. \tag{7.2.1}$$

In fact, virtually all pre-existing cosmologies are based on Einstein's gravitation, thus eliminating a universal symmetry *ab initio.*

Other novelties of the Iso-Self-Dual Cosmology are given by the implications, that are impossible without the isotopies and isodualities, such as:

1) The direct interpretation of the expansion of the universe, as well as the increase of the expansion itself, since antigravity is permitted by the isodualities but not in general by other theories;

2) The prediction that the universe has absolutely null total characteristics, that is, an absolutely null total time, null total mass, null total energy, null total entropy, etc., as inherent in all isoselfdual states[8];

3) The creation of the universe without any discontinuity at all, but via the joint creation of equal amounts of matter and antimatter, since all total characteristics of the universe would remain the same before and after creation.

We also mention that the isoselfdual cosmology was proposed by Santilli [44] to initiate mathematical and theoretical studies on the creation of the universe, studies that are evidently prohibited by theories with huge discontinuities at creation.

After all, we should not forget that the Bible states the creation first of light and then of the universe, while it is now known that photons can create a pair of a particle and its antiparticle.

Also, there is a mounting evidence that space (the *aether* or the *universal substratum*) is composed of a superposition of positive and negative energies, thus having all pre-requisites needed for the creation of matter and antimatter galaxies.

As one can see, a very simple property of the new number theory, the invariance under isoduality as it is the case for the imaginary unit (Section 2.1.1),

$$i \;\; \equiv i^d = -i^\dagger = -\bar{i}, \tag{7.2.2}$$

acquires a fundamental physical character for a deeper understanding of Dirac's gamma matrices (Chapter 2),

$$\gamma_\mu \equiv \gamma_\mu^d = -\gamma_\mu^\dagger, \tag{7.2.3}$$

[8]We are here referring to intrinsic characteristic of isoselfdual states, and not to the same characteristics when inspected from a matter or an antimatter observer that would be evidently impossible for the universe.

and then another fundamental character for the entire universe.

To understand the power of isodualities despite their simplicity, one should meditate a moment on the fact that the assumed main characteristics of the universe as having an equal amount of matter and antimatter, can be reduced to a primitive abstract axiom as simple as that of the new invariance (7.2.2).

Needless to say, the condition of exactly equal amounts of matter and antimatter in the universe is a *limit case,* since in reality there may exist deviations, with consequential *breaking of the isoselfdual symmetry* (7.2.1). This aspect cannot be meaningfully discussed at this time due to the abyssal lack of knowledge we now have on the antimatter component in our universe.

It should be finally indicated that, in view of the topological features assumed for the basic isounit

$$\hat{I} = \hat{I}^{\dagger} > 0, \tag{7.2.4}$$

the Iso-Self-Dual Cosmology outlined above can only represent a closed and reversible universe, thus requiring suitable broadening for more realistic theories.

Recall that, from its Greek meaning, "cosmology" denotes the entire universe. Consequently, no theory formulated until now, including the Iso-Self-Dual Theory, can be called, strictly speaking, a "cosmology" since the universe is far from being entirely composed of closed and reversible constituents.

To begin, there is first the need to represent irreversibility, since the behavior in time of all stars, galaxies and quasars in the universe is indeed irreversible.

This first need can be fulfilled with the Iso-Self-Dual Cosmology realized via isounits that are positive-definite, but explicitly time dependent,

$$\hat{I}(t, \ldots) = \hat{I}^{\dagger}(t, \ldots) \neq \hat{I}(-t, \ldots), \tag{7.2.5}$$

which feature assures irreversibility, although the universe remains closed due to the conservation of the total energy of matter and that of antimatter.

The latter model has evident limitations, e.g., in view of the possible continuous creation of matter and antimatter advocated by various researchers as an alternative to the "big bang".

The latter condition, when joint with the necessary representation of irreversibility, requires the broader *Geno-Self-Dual Cosmology*, namely, a cosmology based on the Lie-admissible lifting of symmetry (7.2.1), via the further generalization of generalized units (7.3.4) and (7.2.5) into four genounits, one per each of the four possible directions of time

$$\hat{I}^{>}, \quad -\hat{I}^{>}, \quad (\hat{I}^{>})^{d} = -{}^{<}\hat{I}, \quad -(\hat{I}^{>})^{d} = {}^{<}\hat{I}, \tag{7.2.6}$$

whose explicit construction is left to the interested reader for brevity (see Chapter 5).

Nevertheless, the latter genotopic lifting itself cannot be considered, strictly speaking, a "cosmology" because a basic component of the universe is life, for which genotopic theories are insufficient, as indicated in Section 3.7, due to their single-valuedness.

The latter need inevitably requires the formulation of cosmologies via the most general possible methods studied in this monograph, the multivalued hyperstructure of Chapter 5, resulting in the *Hyper-Self-Dual Cosmology,* namely, a cosmology based on the hyperlifting of symmetry (7.2.1) characterized by the ordered multivalued hyperunits

$$\hat{I}^{>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, \hat{I}_3^{>}, \ldots\}, \quad -\hat{I}^{>} = \{-\hat{I}_1^{>}, -\hat{I}_2^{>}, -\hat{I}_3^{>}, \ldots\}, \tag{7.2.7a}$$

$$(\hat{I}^{>})^d = \{-^{<}\hat{I}_1, -^{<}\hat{I}_2, -^{<}\hat{I}_3, \ldots\}, \quad -(\hat{I}^{>})^d = \{^{<}\hat{I}_1, ^{<}\hat{I}_2, ^{<}\hat{I}_3, \ldots\}. \tag{7.2.7b}$$

However, at this point we should remember the limitations of our mind and admit that the foundations of the Hyper-Self-Dual Cosmology, such as the multivalued hypertime encompassing all four directions of time, is simply beyond our human comprehension.

After all, we have to admit that a final scientific understanding of life will likely require thousands of years of studies.

7.3 CONCLUDING REMARKS

The analysis conducted in this monograph establishes that the isodual theory of antimatter does indeed resolve the scientific imbalance of the 20-th century caused by the treatment of matter at all levels of study, and the treatment of antimatter at the sole level of second quantization.

In fact, the isodual theory of antimatter achieves an absolute democracy of treatment of both matter and antimatter at all levels, from Newton to second quantization.

In particular, the analysis presented in this monograph establishes that the isodual theory of antimatter is verified by all known experimental data on antimatter, since the isodual theory trivially represents all available classical experimental data (Section 2.2.3), while resulting in being equivalent to charge conjugation at the operator level (Section 2.3.7), as a result of which the entire currently available experimental knowledge on antiparticles is verified by the isodual theory.

Despite its simplicity, the isodual theory of antimatter has deep implications for all quantitative sciences, including classical mechanics, particle physics, superconductivity, chemistry, biology, astrophysics and cosmology.

The most salient consequence of the isodual theory is the prediction of antigravity experienced by *elementary* antiparticles in the field of matter and vice-versa.

This prediction is a direct consequence of the very existence of a consistent classical formulation of antimatter, the electromagnetic origin of the gravitational

mass with consequential phenomenological equivalence of electromagnetism and gravitation for both attraction and repulsion, the forgotten Freud identity of the Riemannian geometry, and other aspects.

In reality, the prediction of antigravity for truly elementary antiparticles in the field of matter is rooted in so many diversified aspects that the possible experimental disproof of antigravity would likely require the reconstruction of theoretical physics from its foundations.

To minimize controversies, it should be stressed that the prediction of antigravity has been solely and specifically presented for *elementary* antiparticles, that is, for the *positron,* with the careful exclusion for first tests of any unstable or composite particles whose constituents are not seriously established as being all antiparticles.

As an illustration, we have discouraged the use in possible experiments on the gravity of the positronium as claim for final knowledge on the gravity of antimatter, because the positronium is predicted by the isodual theory to be attracted in both fields of matter and antimatter. Similarly we have discouraged the use of leptons because they may eventually result to be composite of particles and antiparticles.

Finally, we have strongly discouraged to assume experimental data on the gravity of antiprotons as final knowledge on the gravity of antiparticles, because antiprotons are today fabricated in high energy laboratories from matter components and are believed to be bound states of quarks for which no gravity at all can be consistently defined [38].

It then follows that, while all experimental data are indeed useful and should be supported, including experimental data on the gravity of antiprotons, their use for general claims on the gravity of antimatter could be deceptive.

Moreover, none of the numerous arguments against antigravity could even be properly formulated for the isodual theory, let alone have any value. As a result, the prediction of antigravity for elementary antiparticles in the field of matter is fundamentally unchallenged at this writing on theoretical grounds.

A test of the gravity of positrons in horizontal flight in a vacuum tube, that is resolutory via gravitational deflections visible to the naked eye, has been proposed by Santilli [45] and proved by the experimentalist Mills [46] to be feasible with current technology and be indeed resolutory (Section 4.2).

A comparative study of other tests has revealed that they are too delicate and require too sensitive measurements to be as resolutory as proposal [45] with current technologies.

It is hoped that the experimental community finally comes to its senses, and conducts fundamental test [45,46], rather than continuing to conduct tests of transparently less relevance at bigger public costs, because in the absence of a

final experimental resolution of the problem of antigravity, the entire theoretical physics remains essentially in a state of suspended animation.

In turn, the possible experimental verification of antigravity (as above identified) would have implications so advanced as to be at the edge of our imagination.

One of these implications has been presented in Section 13.3 with the Causal Time Machine, the novel, non-Newtonian *isolocomotion* (propulsion to unlimited speeds without any action and reaction as requested by all currently available propulsions), and other far reaching possibilities.

The experimental resolution of the existence of antigravity for *truly elementary* antiparticles is also crucial to fulfil the original scope for which the isodual theory was built, namely, to conduct quantitative studies as to whether far-away galaxies and quasars are made up of matter or antimatter.

This main scope has been achieved via the *isodual photon,* namely, the discovery that, according to the isodual theory, photons emitted by antimatter appear to have a number of physical differences with the photons emitted by matter. In particular, *the simplest possible isodual electromagnetic waves have negative energy, thus experiencing antigravity in the field of matter.*

The above prediction requires the experimental resolution as to whether light emitted by antimatter is attracted or repelled by the gravitational field of matter.

Needless to say, the current availability at CERN of the antihydrogen atom is an ideal source for such a study, with the understanding that gravitational deflections of light at short distances (as attainable in a laboratory on Earth) are extremely small, thus implying extremely sensitive measurements.

More promising is the re-inspection of available astrophysical data privately suggested to the author because said data could already include evidence of light from far-away galaxies and quasars that is repelled by astrophysical objects closer to us.

Such a repulsion could not be publicly disclosed at this time because of known opposition by organized academic interests on Einsteinian doctrines since, as well known, Einstein's gravitation prohibits the existence of antigravity (Section 4.1).

It is hoped that such organized academic interests come to their senses too, if nothing else, to avoid an easily predictable serious condemnation by posterity, in view of the well known catastrophic inconsistencies of Einstein gravitation outlined in Section 1.4.

After all, we should not forget that antiparticles were first experimentally detected in cosmic rays, thus confirming their possible origin from supernova explosions of stars made up of antimatter.

Also, there are reports of huge explosions in Earth's atmosphere before the advent of atomic bombs without any crater on the ground, such as the 1908 Tunguska explosion in Siberia, which explosions can be best interpreted as anti-

matter asteroids from far away antimatter galaxies or quasars penetrating in our atmosphere.

Therefore, it should not be surprising if light experiencing gravitational repulsion from matter is discovered first in astrophysics.

Additional tests on the possible gravitational repulsion of light emitted by antimatter can be done via the direct measurement of the deflection of light from far away galaxies and quasars when passing near one of the planets of our Solar system.

Under the assumption of using light originating from far away galaxies and quasars (to render plausible their possible antimatter nature), and for the use of a sufficient number of galaxies and quasars (to have a sufficient probability that at least one of them is made up of antimatter), these astrophysical measurements are potentially historical, and will signal the birth of the new science proposed in this monograph under the name of *antimatter astrophysics.*

The reader should be aware that, while the prediction of antigravity for *truly elementary* antiparticles is an absolute necessity for the validity of the isodual theory, the gravitational behavior of light emitted by antimatter is not that simple.

Recall from Section 13.2 that the prediction of antigravity for light emitted by antimatter is based on the negative value of its energy for the selected solution of the electromagnetic wave.

However, the photons is invariant under charge conjugation and travel at the maximal causal speed in vacuum, c. Therefore, the photon could well result to be a superposition of positive and negative energies, perhaps as a condition to travel at the speed c, in which case the photon would be an isoselfdual state, thus experiencing attraction in both fields of matter and antimatter.

As a consequence, the possible disproof of antigravity for light emitted by antimatter stars in the field of matter *would not* invalidate the isodual theory of antimatter, but merely tell us that our conception of light remains excessively simplistic to this day, since it could well be in reality a composite state of photons and their isoduals.

The issue is further complicated by the fact indicated during the analysis of this monograph that *antigravity is predicted between masses with opposite time evolutions,* as it is the case for a positron in the field of Earth. However, the photon travels at the speed of light at which speed time has no meaningful evolution.

As a result, it is not entirely clear to this author whether the sole value of negative energy for the isodual light is sufficient for the existence of a gravitational repulsion, and the issue is suggested for study by interested colleagues.

To express a personal view, it would be distressing if light solely experience gravitational attraction irrespective of whether in the field of matter or antimatter and whether originating from matter or antimatter, because this would

imply the impossibility for experimental studies as to whether far-away galaxies and quasars are made up of matter or antimatter, since all other aspects, including thermodynamics, are not detectable at large distances, thus implying the perennial inability for mankind to reach any in depth knowledge of the universe.

The author does not believe so. Advances in human knowledge have no limit, and often go beyond the most vivid imagination, as established by scientific realities that resulted in being beyond the science fiction of preceding generations.

In closing, the author hopes that the studies presented in this monograph have stimulated young minds of any age and confirmed that science will never admit final theories. No matter how precious, beloved and valid a given theory may appear to be at a given time, its surpassing with broader theories more adequate for new scientific knowledge is only a matter of time.

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Postscript

In the present second volume of his opus magnum,*Hadronic Mathematics, Mechanics and Chemistry*, Professor Ruggero Maria Santilli applies the extensive advances to pure mathematics, presented in the first volume, to a plethora of basic and far-reaching issues in the natural sciences of physics and chemistry. By these means he attains theoretical results not possible to achieve without use of these new and powerful mathematical tools or the extensions of our ontological horizon of the universe associated with the establishment of the new number fields discovered by hadronic mathematics. This second volume also presents available established experimental evidence offering crucial support to predictions from the new sciences of hadronic mechanics and chemistry, sketches of experimental design for further support and theoretical refinements (or falsifications), and emergence of new and quite spectacular technology made possible from these advances in theoretical science. Some of this technology has already been constructed and is up-and-running, and constitutes matured fruits of the quite gigantic scientific enterprise initiated by Santilli four decades ago, and with growing affiliation from co-scientists world-wide throughout the years.

In the exploring spirit of the Renaissance, one might say that the first volume offers a guiding compass and the basic skills for constructing adequate maps and ships to search for unknown continents, while this second volume presents maps as well as treasures after having successfully travelled, reached and traced unknown continents on the other side of the vast ocean of the unknown.

Scientific revolutions in the sense of Thomas Kuhn do not happen often in the history of science, and with regard to physics the last ones, quantum mechanics and Einsteins relativity theory, have now reached the age of 100 year old-timers. With the rapidly increasing number of scientists and over-all significance of scientific progress for modern society, it is not too strange from a birds-eye-view of the history of science that a new revolution has found its day.

The new theory of physics as a whole, coined hadronic mechanics by Santilli, does not question the validity of the theories of quantum mechanics and relativity theory for the physical world, given the constraints formulated by the great creators of the said theories, represented by the kind of physical objects and relations being studied by the theory, and the proper simplifications in the describing and explaining models of such objects, dependent on the nature of the objects and the available mathematics. Basically, the constraints of these theories consist in their relevance being restricted to the so-called exterior physical world, which is

the physical world outside the hadronic horizon of one femtometer. For interior relations, inside the hadronic horizon, the models and equations of these theories did not claim any immediate validity by their originators and, therefore, they are not scientifically legitimate to import inside the hadronic horizon, at least not without careful theoretical considerations on the basic problems therein involved, and without support from crucial experiments. Sad to say, this book offers much argument and evidence for a lot of such illegitimate import to have been the normal state of affairs during the second half of 20th century standard physics.

Assuming the strong interaction being adequately represented as the interaction between three point-like baryon quarks in the hadron, quantum mechanics did not succeed in establishing any good and experimentally testable model of the strong interaction, partly due to the complexities involved with the required non-linear mathematics to describe such a system. Largely because of these problems, the unification of the three other well-known forces with the strong force remained an open problem during 20th century standard physics. Equipped with the developed isomathematics, Santilli disposed the necessary tools to leave the assumption of interior point-quarks, and to describe shapes, as well as changes in shapes coined deformations, of particles with physical extension, to approach the problem of strong interactions inside the hadronic horizon. By means of isomathematics, Santilli was able to quantitatively model the neutron as a bound state of a proton and an electron, and hence to reestablish Rutherfords notion of the neutron as a compressed hydrogen atom. This achievement by Santilli was enthusiastically commented on by the great philosopher of science, Karl Popper, in his book from 1982, as a return to sanity, to that realism and objectivism for which Einstein stood.

The Rutherford-Santilli model of the neutron described the proton and the electron as a bound state with overlapping wave packets. Such a compression could only be imagined as a result of an external trigger, for example the role of pressure in the case of neutron synthesis in stars. For the neutron to stay in a bound state, the bound state had to be imagined as a singlet of a proton and an electron with opposite spins, according to the so-called gear model ruling out the possibilities of triplets or parallel spins. By 1990 Santilli had been able to publish such a model of the neutron as a mutated bound state with an exact quantitative representation of its physical characteristics: rest energy, mean life, charge radius, charge, charge parities, space, spin, and (anomalous) magnetic and electric moment.

Such a model would not have been possible by importing the quantum mechanics for exterior relations to the inside of the hadronic horizon, due to the idea of quantum quantization being contrary to the deep interpenetration of the wave packets inside the hadronic horizon and to the non-existence of exited hadronic states. Such excitation would imply tunneling through the hadronic horizon,

which by Santilli was stated as the very mechanism of the neutrons spontaneous decay. In this way Santillis model of neutron synthesis, as well as neutron decay, did not need any assumption about existence of sub- or quasi-particles as in the notion of quarks, nor was there any need to imagine said processes to rely on a somewhat mystical notion of the two stable elementary particles of the physical world, protons and electrons, being created from and resolved into intermediary states of quark assemblies. In this regard Santillis theory of the neutron offered a much simpler picture of the situation inside the hadronic horizon as well as of the relation to the exterior physical world. Elegant and adequate simplifications are what good science should be about; the question was if the theory was to become supported by experimental evidence. Such significant support was provided when the measured density of the so-called fireball in the Einstein Bose correlation of colliding proton and antiproton was shown to be very close to the hadronic calculation of the density of the neutron, as predicted by hadronic mechanics. Crucial additional support was added from the experiments headed by Prof. Tsagas in 1996 with 319 stimulated decays of the neutron, expelling the Rutherford electron when exposed to the resonance frequency of a hard photon, in accordance with the predictions from hadronic mechanics. (Sad to say, no other laboratories in the world have so far wanted to retest these results by duplicating such experiments, in spite of the great scientific, technological and ecological significance of such confirmation.)

In analogy with the neutron model, Santilli already in 1978, the birth year of hadronic mechanics, had been able to present a model of the 0 meson as a bound state of an electron and a positron with overlapping wave packets, i.e. as a compressed positronium. Also this model was able, differently from quantum mechanics, to represent all physical characteristics of the 0 meson without any additional notion of quarks, and this in one single structural equation. However, it is important to notice that the said bound state is not a bound state of the involved particles as considered outside the hadronic horizon, since physical attributes of the particles undergo some changes in this compression. Such states are, therefore, only possible to describe by means of isomathematics and from the accordingly broader concept of isoparticles.

In general, different from quantum mechanics, hadronic mechanics represents a theory of physics equipped with concepts, models and mathematics to describe and explain relations interior to the hadronic horizon. However, to be able to succeed in this, hadronic mechanics had to be developed as a lifted theory compared to quantum mechanics, thereby providing a more general theory of physics, just as valid for exterior relations as quantum mechanics, the last being a sub-field of hadronic mechanics. Therefore, it is not adequate to consider hadronic mechanics as a supplement or a competitor to quantum mechanics, but as a theory of physics with a broader explanatory power than quantum mechanics, also being

able to adequately include interior relations, as well as relations between the interior and the exterior. This broadening-from-lifting follows the general scheme of development of basic theoretical advances in physics as analyzed in David Bohms interpretation of the modern history of physics.

The theory of hadronic superconductivity, initiated by Prof. Animalu and Santilli from 1994, constitutes an important bridge between hadronic mechanics and hadronic chemistry. In superconductivity theory, as approached by quantum mechanics, it was quite a mystery how the bound state of the Cooper pair could emerge and remain, considering that two electrons are known to be repelled by the Coulomb force. However, from hadronic mechanics this became explainable with the notion of a hitherto unknown physical force becoming activated when two particles are brought into touch from an external trigger, this fifth force inducing total overlap between the two involved wave packets. Different from the four conventional forces, this was a contact force without a potential, and thus requiring a non-Hamiltonian for its mathematical description; - hence being outside the reach of quantum mechanics. Also, the force was described by hadronic mechanics not to depend on the sign of the charge of the involved particles. Thus, the Cooper pair could be explained with this force simply being stronger than the Coulomb force. Due to deep interpenetration of the wave packets, the Cooper pair, by analogy with the cases of the neutron and the compressed positronium, had to be modeled, not as conventional electrons in the exterior, but as isoelectrons.

Further, the Cooper pair in hadronic superconductivity was modeled with an 8-shaped orbit around the two nuclei involved in the superconductivity structure. This orbit shape induces an extraordinarily strong magnetic force from each nuclei, in the hydrogen atom calculated to be 1,415 times the strength of the ordinary magnetic force from the proton, and of course in opposite directions from the two nuclei. Similar superconductivity structures could then be attracted and bound together, aligning from the orientations of the extraordinarily strong magnetic forces from the nuclei, and clustering into bigger structures of atoms (as well as with the possibility to include dimers, radicals or molecules). These clusters were coined magnecules by Santilli, and were predicted from hadronic superconductivity to be discovered by experiments. This became confirmed by independent laboratories, using adequate special apparatus for such detections, from 1998 on. Santilli also invented and patented so-called plasma-arc-flow reactors, also called hadronic reactors and sometimes ecoreactors, to produce magnecules in specified types and quantities in a controlled manner. Already at Dec. 15, 1998, Santilli presented the first constructed reactor producing such new chemical species. 1998 became the take-off year of hadronic chemistry also as a scientific discipline, with a special issue of the Hadronic Journal solely dedicated to presenting the scientific foundations of this lifted and broader chemistry.

Besides Santilli the publication included among its authors Profs. Shillady and Aringazin.

The discovery of magnecules represented the first discovery of a new chemical species since the discovery of molecules in the mid-1800s. Different from molecules, magnecules have non-valence bonds and they can form much larger structures, in superfluids sometimes even visible by the naked eye. Most scientists researching superconductivity with only quantum mechanics at their disposal, believe that superconductivity is restricted to extremely low temperatures (somewhat misleading referring to temperatures far below zero as High Tc superconductivity), while hadronic chemistry has explained hadronic superconductivity to be possible also for fluids and gases, activated by the external trigger of strong and close enough magnetic fields. It is a matter of fact that hadronic reactors have been producing such magnecular gases since 1998. This is a quite bizarre situation, and also with a somewhat macabre touch, since use of magnecular gas has been proven to have highly favorable ecological applications. Compared to molecules, magnecules have many different chemical attributes, explained in detail from hadronic chemistry and experimental evidence in the present volume. For example, when used as a fuel for vehicles, exhaust from combustion of magnecular hydrogen gas has a molecular composition very different from the exhaust of molecular hydrogen gas. The first does not contain potential carcinogens of the latter, has only half the CO2 content, and adds, contrary to the latter, a significant amount (10-12

Compared to the molecular hydrogen gas, the density of the corresponding magnecular gas is about 7.5 times higher. This implies that, on the same tank volume and pressure, a car fuelled on magnecules drives 7.5 times the distance of a car fuelled on molecules. Such effective magnecular fuel is not possible to produce without hadronic reactors, which construction presupposed hadronic mechanics with related hadronic mathematics. In this way, the existing hadronic technology, and there are other examples as well, offers quite simple tests to convince any sound skeptic about the superiority of the hadronic sciences as a whole, compared to standard physics constituted inside century old paradigms.

Hadronic reactors also offer considerable advantages on the input side, because they apply either oil or water solutions as their inputs, and the degree of pollution of the inputs does not matter, insofar as they are not radioactive. In the reactor process, where the plasma reaches temperatures higher than the surface of the sun, the molecules are broken down to their constituents before being recombined as magnecules with non-valence bonds. Thereby almost all molecular polluters disappear, including for example sewage water or pharmaceutical toxins. At the output side, there is produced, along with the magnecular gas, either chemical clean water or heat that can be applied for useful purposes. Furthermore, Santilli has also succeeded in developing magnecular technology specifically

designed as an additive to coal processing in order to reduce the globally heavy load of environmental pollution from this energy technology. Also to consider among Santillis amazing inventions, is the new hadronic technology of so-called intermediate nuclear fusion.

The foundations of scientific theory behind these technological progressions, which ought to be highly welcomed in the contemporary alarming ecological situation, are not only solid, but much more extensive and by far superior to the whole disciplines of standard quantum mechanics and chemistry, as fleshed out in much detail in the present volume. It is not without good reason that Santilli in his informative mammoth article in Foundations of Physics of Sept. 2003, a journal counting eight Nobel laureates in physics in its editorial board, emphasized the discovery of magnecules as the most precious fruit of his lifelong scientific endeavor.

The radical implications of scientific revolutions are hard to overview for contemporaries, sometimes including the pioneering scientists themselves. As a prominent mathematical physicist once said to the author of this postscript: Who would have guessed, back in the 1920s, that such a bizarre theory as quantum mechanics should gain such broad applications in upcoming technology? With regard to chemistry, it appears hard to find any historic parallel to the degree of progress represented or announced by hadronic chemistry, without moving back to the discovery of the periodic table. The panorama of magnecules reveals a previously hidden landscape of a whole new chemical world. It appears nave to suppose that these landscapes are restricted to artificial creations of substances by means of human high technology. In the last sentence of his 2001 book on hadronic chemistry, Santilli predicts the discovery of hyper-magnecules in biology. Also, his hadronic theory of lightning, offering more correct calculations of its accompanied sound quantities, describes this phenomenon as basically a hadronic reaction resulting in nitrogen synthesis. This may indicate that also other phenomena in nature, including biological and physiological nature, will prove to be better understood from hadronic chemistry, especially phenomena revealing superconductivity features. Of special significance may be the research and later applications of magnecular substances for medicine and health, a field so far not systematically targeted by advanced hadronic chemistry and technology, but already with some promising accumulation of more circumstantial evidence.

From the more overarching approach of the broader hadronic chemistry Santilli, partly in cooperation with other scientists, such as Shillady and Aringazin, from the late 1990s published new models also of the much studied molecules of hydrogen and water, earlier thought to be possible to be represented exactly by means of quantum chemistry, but argued by Santilli to be given exact representation of all chemical characteristics only by means of isochemical modeling not available for quantum chemistry. In 2007 Prof. Prez-Enrquez succeeded by

using hadronic chemistry to achieve a representation of the hydrogen molecule with amazingly exact matching with experimental data (among these representing the binding energy up to the 5th digit) by further developments from the Santilli-Shillady model and the Aringazin-Kucherenko approach, an achievement the preceding quantum chemistry was quite far from realizing. Also the work by Dr. Martin Cloonan has been able to reach new insights in fields of chemistry from his Cplex-isoelectronic theory by treating highly specialized knowledge in chemistry from the theoretical framework of hadronic chemistry. These recent developments may indicate an upcoming tendency to reframe specific problems of chemistry inside the broader umbrella of hadronic chemistry and thereby propel further progressions in the fields at hand, probably a challenge most suitable for the younger among talented chemists.

For many years Santilli has emphasized growing environmental concerns as a crucial motivation for his long-lasting scientific enterprise, and in the last decade also for his more recent occupation as an inventor. In spite of the many ecologically favorable applications of magnecular technology already appearing, Santilli regards the hadronic energy connected to the beta-electron released in the neutrons spontaneous decay as the most promising source for new and clean energy, likely to become harvested by upcoming hadronic technology based on hadronic mechanics. Calculations indicate that this energy is huge, without danger of radioactive radiation, and probably capable of capture by adequate trapping and shielding devices.

Considering this promising possibility judged from the theoretical advances in hadronic mechanics, and the possibly great implications for the ecosystem, it seems strange at first glance that powerful physics institutions and laboratories around the world so far have not wanted to execute crucial experiments to support or falsify predictions and earlier experiments from hadronic mechanics regarding neutron decay. The strangeness does not shrink when considering the modest amount of resources needed to execute such experiments, compared to the gigantic budgets of CERN and the like. Hadronic mechanics has already proved to be highly successful in achieving experimental verifications of new predictions from its theoretical extensions, as well as in constructing quite amazing new and eco-friendly technology outside the reach for quantum mechanics. A nave observer from outside the world of sophisticated theoretical physics may ask why it is that hadronic mechanics is being neglected, while a stream of resources is allocated to its sub-fields of quantum mechanics and relativity theory which has only been proven valid for the physical world outside the hadronic horizon. From reading semi-popular science magazines the outside observer will gain the impression that string theory is the most advanced physics around. But if so, how come that string theory, in spite of its rich inflow of mathematical talent and money resources, backed by mighty institutions, and much activity for some 25 years,

has not been successful in creating any new and favorable technology? Could it be that much of the reason is astonishingly simple, that these mathematical models have become too detached from the physical world, somewhat similar to the epicycles of the Middle Ages, constituting a self-sufficient and well fed giraffe-like research community not needing to care about rising revolutionary physics claiming basic theoretical advances backed by direct experimental support, or about the de facto emergence of new technology from this scientific revolution?

Scientific revolutions are not a tea party, and perhaps even less so in our time when the rise of significantly more advanced scientific theory not only threatens mighty characters in huge established science institutions, prestige hierarchies and networks nourished by a priori subscription to century old paradigms, but also related established interests in energy technology, finance and politics. Santilli has often stressed the evolutionary approach to this quest, by seeking serious dialogue and mutual exploration of the issues at hand with conventional scientists and institutions. In spite of this, Santilli has to a large extent been met with a Berlin wall of ignorance or non-scientific rejection, as indicated by the amazing near non-existence of published scientific questioning of the achievements in the hadronic sciences, today piling up to at least a library of 30.000 pages of published articles and monographs. Given the seriousness of the quest, not only for the further development of science, but for the very survival of our civilization by applying new technologies made possible from hadronic mechanics and chemistry, it seems likely that a more turbulent confrontation with different establishments antagonistic to radical extensions and liftings of conventional physics, is no longer possible to avoid. Considering the grave proportions of the rising ecological crisis, it may not be exaggerated to compare the situation with that of Semmelweiss, but with the difference that Santilli also talks from theoretical science above, not below the mighty scientists not able to leave their dogmas in spite of the implied damage done for the planet. Already in his three volume work of 1986, Documentation of the Ethical Probe, Santilli presented much food for thought concerning far from optimal scientific ethics being conducted in influential scientific communities. During the last two decades the picture has turned more severe, and the footnotes in the present volume provide much further material for competent evaluation of the present situation with regard to ethical vs. non-ethical conduct in the global science ecology. It may very well be that upcoming historians of science will look at the remarkably slow post-war development of main stream physics, when comparing the amount of basic advances to the resources spent and to the amount of advances the preceding part of the century, as connected to obstructions from profound non-scientific influences, paradoxically becoming fortified and nourished inside scientific institutions themselves.

Switching the focus to the brighter side, and lifting it to the visionary horizon inspiring great minds of science and art, it is important to note that hadronic

mechanics in its very architecture involves a whole new cosmology, opening vast new territories of the cosmos for human imagination, scientific exploration and technological endeavors.

Different from Einsteins relativity theory which doesnt treat antimatter, and different from quantum mechanics which allows the existence of antimatter only at second quantization, hadronic mechanics was able to treat matter and antimatter systematically on an equal footing, corresponding to the anti-symmetric structure in hadronic mathematics between the iso-, geno- and hyperfields vs. their respective isoduals. Hadronic mechanics comprehends our physical or Euclidean universe as a combination of two distinct universes, a matter universe and an antimatter universe. These two universes have a different anchoring in supra-spacetime, respectively in isospacetime and in isodual spacetime. However, isospacetime and isodual spacetime manifest in the same 3+1D space which they share and hence is to be comprehended as double-valued. Due to the antisymmetry of the two universes, positive mass in the matter universe will be projected as negative mass when experienced in the antimatter universe, and the same the other way around, and also the same with all other physical quantities, such as time, charge and energy. For the universe as a whole combination of the matter and the antimatter universe, all these magnitudes cancel out to zero. (This is also consistent with the key notion in the ambitious theory of universal rewrite nilpotent system recently worked out by mathematical physicist Peter Rowlands.)

This implies a comprehension of space itself as a universal substratum composed of a superposition of positive and negative energies, from which matter and antimatter galaxies are continuously created. This seems to provide an elegant solution for the mystery of from where the universe, considered as a closed system, receives its energy as a whole. If the universe has a paradoxical twin structure, the puzzle may be solved from a metabolism between the two moieties from the universal substratum, where the output energy from one moiety is received with the opposite sign as input energy for the other moiety, while the energy of the total universe remains zero or nilpotent. The philosophically quite simplistic Big Bang hypothesis, popular in much 20th century physics, is an answer to a question about the origin of the universe that does not make much sense when reframed from the more sophisticated cosmology and ontology of hadronic mechanics. Regarded from hadronic cosmology, treating antimatter with scientific democracy, as Santilli likes to put it, it is not quite the same universe anymore. According to hadronic cosmology, the universe is rather comprehended as inherently and continuously re-created, as it was by the great scientist David Bohm. On this background the Big Bang (and Crunch) hypothesis may be more adequately understood as a creation myth suitable for a conflated physicalistic and entropic world view painted in scientific cosmetics.

Hadronic cosmology constitutes a platform for much more optimistic and ambitious scientific undertakings. Santillis theory of antimatter has formulated precise predictions of antigravity phenomena, and has designed experimental tests of antigravity for positrons and isodual light. Also, hadronic mechanics includes the notion of bound states of matter and antimatter, coined isoselfdual states, which opens up the possibility for time travel in the matter universe via intermediary switching onto the antimatter universe. Furthermore, Santilli describes causal spacetime machines which is the theoretical notion of way more radical space travel than the rocket technology developed half a century ago, and which applies the principle of isogeometric propulsion without Newtonian action-reaction. Hence, the realism in developing UFO technology for space travel much faster than the speed of light in vacuum, does not seem farfetched anymore from the theoretical advances of hadronic mechanics. These advances were only possible from the broadening of the theory of physics to include antimatter on an equal footing with matter, which in its turn presupposed the development of the new isonumber fields, with corresponding isogeometry, for quantitative treatments.

It is worth noticing that such space deformations are accompanied by changes in time as we ordinarily understand it. This implies a detrivialization of the conventional time concept, where the familiar time arrow reduces to just one aspect of a more complex configuration of different types of time flows. In his pioneering studies of sea shell growth from hadronic geometry Chris Illert showed in the mid-1990s that a certain class of bifurcating sea shell followed a growth path that presupposed two non-trivial kinds of time flows, perceived as jumps forward and backward in conventional time. Such discovery of non-trivial time flows in a sufficiently profound specialist study of a complex irreversible system of nature, was exactly what was expected from the new time theory of hadronic mechanics which had added four types of non-trivial categories, so-called geno-times, to the conventional notion of time. Santilli has stated that for practical purposes there is no scientific difference between the new physical principles discovered in branching sea shells and those involved in the notion of causal spacetime machines.

Throughout the last century the quest of grand unification of gravitation with the three other conventional forces of physics remained a puzzling open problem in the struggles of standard physics. Santillis theory of grand unification from hadronic mechanics presents gravitation as a macro phenomenon aggregated (with presented equations) from quantum electrodynamics de facto rooted in energy from the vacuum or universal medium. However, such a grand unification is argued by Santilli still not to be theoretically possible without acknowledging the democratic co-existence of an antimatter universe, a theory of physics not available before the development of hadronic mechanics. Accordingly, there was no mystery that grand unification became out of reach for standard physics restricted to quantum mechanics and Einstein relativity theory. From this approach

Santilli argued that grand unification was possible only as recognizing the quest as two connected grand unifications, one for the matter universe and one for the antimatter universe, to become integrated in a combined grand unification, and accordingly coined Iso-Grand-Unification, requiring isomathematics for its fulfillment.

Differently from 20th century standard physics, hadronic mechanics has provided a general scientific umbrella, sophisticated, abstract and broad enough to encompass life in its extension, at least in a much more emphatic and radical sense. This is intimately connected to the structure of the higher landscapes of hadronic mathematics, to be considered not only as tools but as structures complex enough to offer adequate maps of lifes phenomena. Due to the lack of isonumbers required to describe hadronic superconductivity, quantum mechanics was never able to catalyze much progress in chemistry, with growing disconnection between physics and chemistry as a result. For mappings of biological structures, genonumbers become crucial to grasp the fundamental irreversibility characterizing the complexity of the biological world (as well as already the behavior of stars, galaxies and quasars). After a lifting to genostructures, the whole field of isostructures, which still implied reversibility in its basic mathematical axioms, reappears only as the subfield of genostructures where reversibility constitutes a special case. The further lifting from genostructures to the much broader hyperstructures achieves not only irreversibility, but the multi-valued theory required to map even more complex structures of life. Santilli notes that when described as a multi-valued hyperstructure, the same seashell can overlap a large number of spaces and their isoduals, resulting in multi-fold formulations including the four different directions of time. The relevance of hyperstructures to describe really complex life phenomena becomes perhaps most immediately and intuitively obvious if we move to psychology and reflects on the multi-fold dynamic constellation of mind spaces and time travels involved in ordinary human thinking.

This may indicate that the top floor in the huge building of hadronic mechanics, hypermechanics, is sophisticated enough to include also mental and social phenomena. In standard physics the quest for grand unification was restricted to a unification of the four conventional physical forces, silently regarding the mental and social worlds as mystically separated from the universe or as mere epi-phenomena mirroring or emerging from the four physical forces. On this background it is highly interesting that Santilli not only presents an (iso-)grand unification of the four forces in chapter 14 of the present volume, but takes the steps all the way up to a Hyper-Grand-Unification. In the modern development of science and society, the frontier of physics has always been highly influential indirectly on other disciplines, being regarded as the most authoritative discipline concerning what is to be stated with the highest degree of scientific certainty with respect to the basic issues of our cosmos. The rise of hadronic mechanics, with

the present volume presenting a systematic overview of its most mature achievements, constitutes a much more radical scientific revolution, since the argued fruits of hypermechanics are far from being relevant only for physics, but seems directly relevant for all scientific disciplines, and this in a profound manner.

Santilli notes that all distinctions between matter and antimatter are lost at the hyperstructural level and that at this highest possible level of formulation, we have one single hyperrelativity, one single Poincar-Santilli hypersymmetry (chapter 6.1.15). In this regard the advanced science of hypermechanics is in accord with the basic notion of cosmos being a unitary whole, characterizing great natur philosophy, such as Plotinus, Kant, Hegel and Bohm. Santilli also states: The foundation of our hypercosmology on the universal hypersymmetry is the single most important result of the authors lifetime of research because it governs the totality of the events in the universe (ibid.).

Being based on symmetry, the hypercosmology of hadronic mechanics differs from Einstein gravity and other preceding cosmologies of physics. The unitary whole of the cosmos is reflected in Santilli coining this cosmology hyper-self-dual, and Santilli explicitly states the necessity of lifting the cosmology from isotopic and genotopic theories to the hypertopic level because a basic component of the universe is life (chapter 14.2) which needs multi-valued descriptions to become comprehended.

In spite of the imagined universality of the hyper-self-dual cosmology and hyper-hadronic mechanics, Santilli is careful by stating that science will never admit a final theory. This humble attitude, the complementary polarity to the visionary extreme ambition also characterizing scientific genius, differs remarkably from physicists clinging to doctrines from Einstein relativity more like religious dogma and for eternity. This was an attitude quite alien to Einstein himself who published his break-through articles without one single reference to any authority (or non-authority), and let the power of thought speak for itself.

Santilli holds Einstein in very high esteem, and declares him explicitly as the greatest scientist of the last century. However, the admiration between deeply creative and thereby related minds seems to be of another kind than that between a genius and the later followers of his established authority. One might say that Santillis admiration of Einstein is more profound, insofar as the scientific thinking of Santilli himself exposes a similar brave, original and creative line of thought. From this also follows a scientific obligation to leave home if and when the pupil reaches far enough to explore unknown higher territories in the mountains of knowledge, climbing from the shoulders of his master. Santilli is careful in the present volume, as in earlier works, to pinpoint under which constraints Einstein relativity is still to be considered valid, and at the same time to state loud and clear why the masters theories do not hold when these constraints are abandoned, and therefore was in need of a more lifted and broader theory of physics which

Santilli went out to create through forty years of hard work. Considering all the experimental evidence from the 1990s on, showing beyond serious doubt that the light speed in vacuum does not represent any ultimate barrier for velocity, explained by hadronic mechanics as a necessity inside hyper-dense hadron media, it seems quite pathetic when the authority of Einstein is mobilized as rhetoric ammunition to obstruct such theory formation and recognition.

It has been said that the real masters greatest satisfaction is when he realizes that his pupil has grown beyond the skills of himself. If allowing such an analogy for the case of clarifying proportions, Einstein ought to have every reason to evaluate his pupil Santilli with delightful satisfaction. Like Einstein, Santilli has pushed the frontier of physics far beyond earlier imagination. However, unlike Einstein, Santilli has also pushed the frontier of the whole of physics, as well as the frontiers of whole disciplines outside physics foremost chemistry and mathematics, but also theoretical biology, and with direct implications also for other disciplines, among them philosophy. So, all in all, it seems hard to doubt that history will judge Santilli as an even greater genius than Einstein.

In the history of mankind there are very few examples of scientists showing brilliance both in mathematics (whether pure or applied to physics) and in the art of invention, the Norwegian Kristian Birkeland (1867-1917) constituting one of the few worth mentioning. With his amazing patents, as well as different types of constructed hadronic reactors producing the new chemical species of magnecules, Santilli has also proven extraordinary skills as an inventor, praised by Tesla as the foremost among sciences, as well as a laboratory man. These skills, indicating intuitively precise connectedness to the rock hard and dynamic physical world, ought to give further credibility to the practical and direct relevance of the theoretical physics and chemistry of Santilli, constituting a character quite different from the more ivory tower type of mathematical physicists.

The present volume may represent a suitable closing of Santillis pioneering monographs given to the world to whom it could concern as perhaps the richest collection of scientific goodies ever presented to Mankind, whose future may depend crucially on what it does with the treasures contained in this opus magnum. With this publication, serious scientists and scholars with open and critical minds across a plethora of disciplines have been given heavy loads of precious ideas to digest and cultivate for many a year to come. In spite of Santilli often using the expression young minds of all ages, the scientific presents are doomed to primarily become appetizers to consider for the younger and most emergent upcoming among those minds, because they will become the carriers and releasers of the future, if any. Besides the thrills of discovery in absorbing the monograph itself, as well as from explorative adventures fuelled by inspiration from it, there will also be a heavy load of social responsibility and dedicated action to carry out, considering signs of rising turbulence inside as well as outside science.

At Christmas time most people appreciate Santa Claus showing up to give them exclusive presents for delight. Sad to say, this is far from always being the situation in scientific communities, nor in society at large. Considering the immense obstacles to and antagonisms, be it brute or more sophisticated, against Santilli fulfilling his mission to science and to Mankind, it is quite a mystery in itself how this man has been able to keep on track, busily creating new insights with heroic energy and steady devotion seemingly greater than life, even after entering his eighth decade on the planet. The footnotes in this volume give some indication of the emotional challenge and burden involved therein, and tells of an intellectual honesty, integrity and boldness paradigmatic for any scientist, whatever degree of intelligence or idiosyncratic inclination.

Santilli holds the dream of humanity becoming able to harvest the huge clean energy connected to the beta-electron from neutron synthesis, predicted as a realistic possibility within reach from the physics of hadronic mechanics. At the same time, hadronic mechanics points out the missing energy in this synthesis when described by conventional physics, and locates the source of this energy gap to originate from the high energy density of the universal medium, by the way a statement similar to the avant-garde Russian physicist Kozyrev arguing the stars not to be fuelled by energy from their exterior. Whatever the destiny of this dream, it must be stated beyond doubt that the life work of Santilli represents quite a neutron synthesis in itself, fuelled from beyond the stars, with the present monograph constituting a new and clean hadronic energy of parachuting fruits from the tree of advanced and matured scientific knowledge, to be picked and eaten for the delight of the world. The release of this testament of Santillis science to the world ought to be honored with the uttermost gratitude and hungry attention. Science is nothing if not living science, so I find it irresponsible not to declare the historic proportions of the Santilli legacy, as to the best of my knowledge and judgment. Hence, on the possible behalf also of some future state of the affairs of the world and its science, I take the liberty to pass a 1001 thank you to the Great Italian - and may he stay forever young.

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HADRONIC MATHEMATICS, MECHANICS AND CHEMISTRY

Volume V: Experimental Verifications, Theoretical Advances and Industrial Applications in Chemistry

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Foreword

These days, science is playing an ever increasing rle in the lives of each and every one of us. The public is being lectured on climate change by very authoritative sounding people; the problems of the energy requirements of the world as a whole are being discussed quite openly and widely; more and more scientific topics are being discussed openly by people in positions of authority. What is not emphasised, in fact is rarely mentioned, however, is that at the centre of all these various discussions is physics. In the world of science, physics plays a crucial, all-pervading role. If science is viewed as a bicycle wheel, physics forms the hub at the centre; all the other branches of science act as the spokes of the wheel leading outwards from this central hub. In this context, mathematics is the language of physics and must always be subservient to the physics. Chemistry is merely one branch of physics; engineering may be viewed as the practical manifestation of physical principles; physics is seen by all to be playing a bigger and bigger rle in medicine; in biology even, physics is becoming important particularly through the influence of thermodynamic principles, including that of entropy, in the examination of the theory of evolution. Hence, it is certainly not unreasonable to claim an all-pervading influence of physics in science. It must always be remembered, but frequently isnt, that physics is concerned with describing, and gaining an understanding of, the world around us. It follows that any models devised by man to achieve this are only as good as their ability to achieve this goal. Mans models will always be approximate and, therefore, always flawed. It is this which spurred Ruggero Santilli to attempt to extend the theory behind quantum mechanics and relativity when he realised that neither was, in fact, complete as a theory.

The first volume of this two volume set was devoted to the mathematical theory developed by Ruggero Santilli over a period of years in an attempt to make headway with the enormous task he had set for himself, for he had always realised that, to make any progress at all, some new mathematics would need to be developed. Mathematics as a tool of physics will always have a potential to restrict progress in physics since it is a purely manmade tool. Also, if one looks back through history to the likes of Newton and Einstein, it is apparent that each developed or introduced new mathematics in order to proceed with prodigious advances in physics. The mathematics introduced in the first volume might reasonably be considered a separate piece of work to be considered and appreciated in its own right. However, its purpose had always been to provide

a new tool to help us all in our quest to describe our universe and all that it contains. This means making the mathematics subservient to the physics; relegating the mathematics however beautiful it may be in its own right to a place on a spoke of that wheel of science referred to above. Once it takes on this rle, any results obtained theoretically are only as good as their ability to accurately portray physical phenomena. In this second volume, the link up of theory with experimental results and observation is presented. It is for the general scientific community and any other readers of this work to adjudicate on its success or failure but this judgement, which could be so crucial to us all, must be made with open minds.

The areas in which this new work may be applied are varied. At the present time, possibly the most important application might seem to be the prediction of new clean energies. This could help solve the problems of energy supply and atmospheric pollution if the predictions prove correct. Already, however, a new clean energy, magnegas, has been produced and tested independently. This fact alone must lend credence to the theory presented and should surely provide an impetus to moves to examine the other predictions in great detail on a much wider scale. This is especially important since, on the basis of our present scientific knowledge, the only realistic method of fulfilling the worlds energy needs in the not-too-distant future is via nuclear power. As well as offering possible alternatives, the new theory also offers a possible means of dealing with nuclear waste safely. This, one would have thought, would have been something governments throughout the world would have wanted to investigate as a matter of urgency. It is to be hoped that the publication of this book will refocus attention on this vitally important topic and produce the necessary reaction from around the world.

However, the new theory is not restricted in its application to matters of energy resources. For example, it also offers alternative explanations for problems in astrophysics and cosmology. One fascinating aspect of these two areas of intense scientific endeavour is that, although many observations are made, both are subject to theoretical speculation which can never be completely verified or totally disproved because the time scales involved are far too long; for example, no-one lives anywhere near long enough to truly know the full facts concerning the birth, life and death of any star the theory in that case may be beautiful, it may appear to be a reasonable explanation of all we see, but one can never be certain it is absolutely correct. This is another area where open minds are essential. However, Halton Arps observations relating to quasars caused great consternation among conventionally thinking astronomers to the extent that he has become largely ostracised by the astronomical community. It is interesting that Ruggero Santillis work leads to a possible explanation for Arps findings which should not offend those conventional astronomers too much if they view

the ideas with open minds. Again, the same body of work offers an important contribution to the debate surrounding the existence of dark matter and dark energy. This lifes work truly makes contributions to thought in diverse areas of human endeavour and should be examined far more widely than it is.

It is often said that behind every great man there is a great woman. This is true of Ruggero Santilli. It is for history, not me, to label anyone great or not but it is undoubtedly true that he has benefitted from the unswerving support and encouragement of his wife Carla. It is doubtful he would have achieved so much without this seemingly unquestioning devotion. As I wrote earlier, all Ruggero Santillis scientific achievements may be seen to be the result of tremendous teamwork; a team comprising Ruggero himself and Carla Gandiglio in Santilli.

When anyone reaches the end of these two volumes then, and only then, will they be in a position to reflect on the work as a whole and think about coming to a conclusion. As stated previously, the theoretical framework is elegant but it is here to be judged on the basis of its use in physics, since that was the reason for its genesis. View the experimental and observational evidence, as well as the basic theoretical background, with open minds before coming to any final decision. Many, probably the majority, will then regard these two volumes as representing a truly monumental piece of work which deserves dissemination to a much wider circle of people scientists, politicians, the business community, and, most of all, the general populace which ultimately pays for all scientific work, whether successful or not! The general public needs to be aware of all that is on the table for consideration, not simply those little titbits which are released for ulterior motives.

Jeremy Dunning-Davies,
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October 8, 2007

Preface

Following fifty years of research experience, the author can state without hesitation that quantum chemistry is the most ascientific among all quantitative sciences because of widespread denial of insufficiencies, joint with truly unbelievable obstructions against their resolution.

We have denounced in Volume I the fact that quantum chemistry is mostly based on nomenclatures, as it is the case for the valence, since it lack the identification of the actual bonding force, the proof that it is attractive, and the verification of experimental data via its use. At any rate, two identical electrons in the valence bond must repeat each other according to the basic axioms of quantum chemistry, and definitely cannot attract each other.

In Volume I, we also denounced additional insufficiencies or sheer inconsistencies of quantum chemistry, some of which are truly embarrassing, such as the prediction that all substances are paramagnetic, a prediction that is an incontrovertible consequence of the current nomenclature on valence in which, lacking a serious attractive force, individual electrons are free to acquire an orientation under an external magnetic field resulting in universal paramagnetism.

What it is still astonishing for the author is to see seemingly serious chemists being fully satisfied with views, such as that the liquid state of water is caused by "H-ridges," while denying the purely nomenclature character of such a view and opposing quantitative treatments since they would evidently require rather drastic revisions.

With the understanding that basic advances in quantitative science rarely originate from a single try, and are instead the result of a laborious process of trials and errors, the author has experienced truly incredible obstructions against any attempt of resolving the above so manifest theoretical insufficiencies.

Additionally, the author has experienced, by far, the greatest obstructions in analytic laboratories because of the virtually universal belief by analysts that novelty in chemistry cannot exist, and if it is proposed for test, it is a fraud. As a result of this conviction, analysts generally perform incredible manipulations of data and equipment for the specific intent of suppressing any hint of novelty in the measurements, and deliver the "experimental results" solely in a way fully aligned with old knowledge.

An example, among too many for comfort, is given by the "measurement" of specific weight of gaseous species that is routinely done by submitting the species to infrared scan, identifying the percentages of its molecular constituents,

and then *calculating,* rather than actually measuring, the specific weight from tabulated data, thus resulting in " experimental beliefs," and certainly not in serious experimental data.

In fact, various chemical species exposed to electric arcs show macroscopic peaks in the mass spectrum that are transparent to infrared scan, trivially, because the bond of said peaks is not that of valence for which infrared detectors where built, but much weaker. When analysts are told about the possible presence of peaks in macroscopic percentage in the mass spectrum that cannot appear in the infrared scan, thus requiring the actual measurement of the specific weight, the author has experienced the denial of the measurements, as recently the case from a large analytic laboratory in London.

The rejections by what are expected to be qualified chemical journals of any theoretical and/or experimental advance in chemistry are truly beyond belief, since the editors use the most implausible and scientifically vacuum argumentations for denying the existence of basic insufficiencies and then use them for corrupt rejections.

This ongoing theoretical and experimental conditions of chemistry is clearly unacceptable, and must be changed to prevent a historical condemnation by contemporary society, let alone by posterity, because chemistry is expected to provide the much needed new clean energies and fuels that cannot be even conceived with contemporary chemistry because, as shown in Volume I, all energy releasing processes are irreversible in time while quantum chemistry is invariant under time reversal.

By completely ignoring this ascientific condition of chemistry, this volume contains the must important experimental verification of hadronic mechanics and chemistry consisting of :

1) The achievement of a fully identified bonding force between valence electron pairs in singlet coupling originating from the deep mutual overlapping of the particle wavepackets, thus being of contact, nonlinear, nonlocal and nonpotential character, hence inconceivable with quantum chemistry due to its simplistic, purely Hamiltonian structure;

2) The proof that this new force, today called *Santilli strong valence force,* is indeed attractive; and, above all

3) The achievement for the first time of a numerically exact and (time) invariant representation of molecular characteristics for the hydrogen, water and other molecules from first axiomatic principles, without ad hoc adulterations such as the screening of the Coulomb law via its ,multiplication by an function of unknown physical or chemical origin.

The replacement of the valence nomenclature with a first, yet serious quantitative treatment then permits a number of advances yet to be explored, such as

a quantitative representation of the liquid state of water, and, subsequently, of liquid fuels, and other chemical structure still unknown at this writing.

This volume also contains one of the most rewarding results of hadronic mechanics and chemistry, the prediction, quantitative treatment and industrial realization of a new class of fuels admitting a full combustion, thus alleviating the current environmental problems, today known as *fuels with Santilli magnecular structure.*

The gravity of the ascientific condition in contemporary academic chemistry is illustrated by the fact that large investments by industries in more than one continent have been made on fuels with Santilli magnecular structure due to their proven environmental qualities while, by comparison, most academic chemists cannot even acknowledge their existence, let alone participate in their development, for fear of instant disqualification by the establishment because the use of very large public funds depends on the academic belief that all possible chemical structures in the universe can only be of molecular nature.

The clear conclusion is that academia is no longer the arena for truly basic advances because basic novelty is the ennemy for academia to kill at whatever cost. In fact, the U.S. Military decided long ago to abandon the financial support of academic research and initiated the conduction of their advanced research in secrecy. The maturity of that decision is established by a comparison between the incredible scientific advances achieved by the military during the last decades, and the absence of any truly basic advance achieved by academia in Cambridge, Massachusetts or Cambridge, England.

Being the scientific advisor of various companies, Santilli can testify that companies in various countries are nowadays following the example of the U. S. Military. In fact, some of the research contracts from corporate investments carry the prohibition of disclosing to academic advances when of basic character. and new products now reach the market without academia knowing their origination.

The author ends this last preface with the indication of the great pleasure in seeing a lifelong environmental effort being fully rewarded with industrial developments under large corporate funds, and the comparative awkward feeling of seeing his former academic colleagues essentially remaining outside the pursuit of truly basic scientific knowledge.

Ruggero Maria Santilli
January 9, 2008

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The underwriter Ruggero Maria Santilli states the following:

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This legal notice has been made necessary because, as shown in Section 1.5, the author has been dubbed "the most plagiarized scientist of the 20-th century," as it is the case of the thousands of papers in deformations published without any quotation of their origination by the author in 1967. These, and other attempted paternity frauds, have forced the author to initiate legal action reported in web site [1].

In summary, honest scientists are encouraged to copy, and/or study, and/or criticize, and/or develop, and/or apply the formulations presented in these volumes in any way desired without any need of advance authorization by the copyrights owner, under the sole conditions of implementing standard ethical rules 2A, 2B, 2C. Dishonest academicians, paternity fraud dreamers, and other schemers are warned that legal actions to enforce scientific ethics are already under way [1], and will be continued after the author's death.

In faith

Ruggero Maria Santilli

U. S. Citizen acting under the protection of the First Amendment of the U. S. Constitution guaranteeing freedom of expression particularly when used to contain asocial misconducts.

Tarpon Springs, Florida, U. S. A.

October 11, 2007

[1] International Committee on Scientific Ethics and Accountability
http://www.scientificethics.org

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Chapter 8

EXPERIMENTAL VERIFICATIONS AND APPLICATIONS IN SUPERCONDUCTIVITY

8.1 Introduction

An understanding of hadronic mechanics requires the knowledge that the new discipline and its underlying new mathematics are applicable in fields beyond particle physics, nuclear physics, and astrophysics. Another field of applicability of hadronic mechanics is superconductivity.

There is no doubt that quantum mechanics provides a good description of an *ensembles* of *Cooper (or electron) pairs* in superconductivity (see, e.g., Ref. [1]), when necessarily represented as points in oder to prevent major conflicts with the basic axioms of the theory. However, there is equally no scientific doubt that quantum mechanics cannot possibly represent the *structure of one isolated Cooper pair.*

The Cooper pair is a physical system requiring an *attractive* interaction among two *identical* electrons via the intermediate action of Cuprate ions, and the bond-correlation of the two electron is so "strong" that cooper pairs can even tunnel as a single particle according to clear experimental evidence.

But electrons repel each other according to quantum mechanics. therefore, to achieve an understanding of the bond-correlation, a conjecture was submitted according to which there is a new interaction between the two electrons mediated by a hypothetical particle called "phonon."

However, phonons represent elementary heat excitations-oscillations in a *crystal.* Consequently, it is difficult to understand how phonons can be propagated in vacuum from atom to atom in the fixed lattice of a crystal. Even assuming that this is possible, it is difficult to understand how phonons can create an *attraction* between pairs of identical electrons.

In any case, considered *ad litteram,* phonons are sound waves or at best, vibrations of the superconducting medium, in which case, again, it is evidently

difficult to understand how such vibrations could propagate in vacuum and, in case this can be explained, how could they produce a real attraction between identical electrons.

Also, the 20-th century physics has identified all possible particles. Yet, this branch of physics has no evidence of phonons, as well as of the interactions electron-phonon.

The Cooper pair (CP) is an excellent physical system to test the effectiveness of isotopic methods at large. Comprehensive studies along these lines have been conducted by A. O. E. Animalu [2] who has introduced a nonlinear, nonlocal, and non-Hamiltonian realization of hadronic mechanics for the Cooper pair known as *Animalu's isosuperconductivity* that is in remarkable agreement with experimental data, and possesses intriguing and novel predictive capacities.

8.2 Animalu's Hadronic Superconductivity and its Experimental Verification

The birth of Animalu's Hadronic Superconductivity, or *isosuperconductivity* for short can be traced back to the structure model of the π^o meson submitted by Santilli in the original proposal to build hadronic mechanics (Ref. [3], Sect. 5)

$$\pi^0 = (\hat{e}_\uparrow^+, \hat{e}_\downarrow^-)_{\rm HM}, \tag{8.1}$$

where HM stands for hadronic mechanics, and $\hat{e}^-$ represents the *isoelectron*, that is, the ordinary electron when described via the isomechanics and related Galilei-Santilli isosymmetry for nonrelativistic description or the Poincaré-santilli isosymmetry for relativistic treatments. For brevity, in this chapter we study only the nonrelativistic profile, and refer to the quoted literature for the relativistic extension.

As familiar from Chapter 6, model (8.1) is based on the property that the nonlocal-nonpotential interactions due to deep wave-overlapping results in being strongly attractive for singlet coupling (only) irrespective of whether the Coulomb interaction is attractive or repulsive.

Isosuperconductivity is based on the *isoelectron pairs* (IEP) proposed by Animalu [2] and studied in details by Animalu and Santilli [3] at the 1995 Sanibel Symposium held in Florida that can represented with the symbol

$$\text{IEP} = (\hat{e}_\uparrow^-, \hat{e}_\downarrow^-)_{\rm HM}, \tag{8.2}$$

A main property of model (8.2) is that *the attractive force caused by deep waveoverlapping of isoelectrons in singlet coupling is so strong to overcome the Coulomb interactions even when repulsive,* thus permitting the extension from model (8.1) to (8.2).

The quantitative representation of the above property can be outline as follows. Consider one electron with charge $-e$, spin up $\uparrow$ and wavefunction $\psi_\uparrow$ in

the field of another electron with the same charge, spin down $\downarrow$ and wavefunction $\psi_\downarrow$ considered as *external*. Its Schrödinger equation is given by the familiar expression

$$H_{\text{Coul.}} \times \psi(t,r) = \left(\frac{1}{2m} p_k p^k + \frac{e^2}{r}\right) \times \psi_\uparrow(t,r) = E_0 \times \psi_\uparrow(t,r), \tag{8.3a}$$

$$p_k \times \psi_\uparrow(t,r) = -i \times \partial_k \psi_\uparrow(t,r), \tag{8.3b}$$

where m is the electron rest mass. The above equation and related wavefunction $\psi_\uparrow(t,r)$ represent *repulsion*, as well known. We are interested in the physical reality in which there is *attraction* represented by a new wavefunction here denoted $\hat{\psi}_\uparrow(t,r)$.

By recalling that quantum mechanical Coulomb interactions are invariant under unitary transforms, the map $\psi_\uparrow \to \hat{\psi}_\uparrow$ is representable by a transform $\hat{\psi} = U\psi$ which is *nonunitary*, $U \times U^\dagger = U^\dagger U = \mathcal{I} \neq I$, where $\mathcal{I}$ has to be determined (see below). This activates *ab initio* the applicability of hadronic mechanics as per Sect. 1.8. The first step of the proposed model is, therefore, that of transforming system (1.28) in $\psi_\uparrow$ into a new system in $\hat{\psi}_\uparrow = U \times \psi_\uparrow$ where U is nonunitary,

$$\begin{aligned} U \times H_{\text{Coulomb}} \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times \psi_\uparrow(t,r) = \\ = \hat{H}_{\text{Coulomb}} \times T \times \hat{\psi}_\uparrow(t,r) = \\ = \left(\tfrac{1}{2m} \hat{p}_k \times T \times \hat{p}^k + \tfrac{e^2}{r}\mathcal{I}\right) \times T \times \hat{\psi}_\uparrow(t,r) = E \times \hat{\psi}_\uparrow(t,r), \end{aligned} \tag{8.4a}$$

$$\hat{p}_k \times T \times \hat{\psi}_\uparrow(t,r) = -i \times T_k^i \times \partial_i \hat{\psi}_\uparrow(t,r). \tag{8.4b}$$

System (8.4) is incomplete because it misses the interaction with the Cu^{z+} ion represented by the familiar term $-ze^2/r$ [10]. The latter are not transformed (i.e., they are conventionally quantum mechanical) and, therefore, they should be merely added to the transformed equations (1.29). The formal equations of the proposed model $\text{CP} = (e_\uparrow^-, e_\downarrow^-)_{HM}$ are therefore given by

$$\begin{aligned} \left(\frac{1}{2m} \hat{p}_k \times T \times \hat{p}^k + \frac{e^2}{r} \times \mathcal{I} - z\frac{e^2}{r}\right) \times T \times \hat{\psi}_\uparrow(t,r) = \\ = \frac{1}{2m} \hat{p}_k \times T \times \hat{p}^k \times T \times \hat{\psi}_\uparrow + \frac{e^2}{r}\hat{\psi}_\uparrow - z\frac{e^2}{r} \times T \times \hat{\psi}_\uparrow(t,r) = \\ = E \times \hat{\psi}_\uparrow(t,r), \quad \hat{p}_k \times T \times \hat{\psi}_\uparrow(t,r) = -i \times T_k^i \times \partial_i \hat{\psi}_\uparrow(t,r). \end{aligned} \tag{8.5}$$

In order to achieve a form of the model confrontable with experimental data, we need an explicit expression of the isounit $\mathcal{I}$. Among various possibilities, Animalu [10] selected the simplest possible isounit for the problem at hand, which we write

$$\begin{aligned} \mathcal{I} &= e^{-\langle \hat{\psi}_\uparrow | \hat{\psi}_\downarrow \rangle \psi_\uparrow / \hat{\psi}_\uparrow} \approx 1 - \langle \hat{\psi}_\uparrow | \hat{\psi}_\downarrow \rangle \psi_\uparrow / \hat{\psi}_\uparrow + \ldots, \\ \mathcal{T} &= e^{+\langle \hat{\psi}_\uparrow | \hat{\psi}_\downarrow \rangle \psi_\uparrow / \hat{\psi}_\uparrow} \approx 1 + \langle \hat{\psi}_\uparrow | \hat{\psi}_\downarrow \rangle \psi_\uparrow / \hat{\psi}_\uparrow + \ldots, \end{aligned} \tag{8.6}$$

under which Eqs. (8.5) can be written

$$\frac{1}{2m}\hat{p}_k T \hat{p}^k T \hat{\psi}_\uparrow - (z-1)\frac{e^2}{r}\hat{\psi}_\uparrow - \tag{8.7}$$

$$-z\frac{e^2}{r}\langle\hat{\psi}_\uparrow|\hat{\psi}_\downarrow\rangle(\psi_\uparrow/\hat{\psi}_\uparrow)\hat{\psi}_\uparrow(t,r) = E\hat{\psi}_\uparrow.$$

Now, it is well known from quantum mechanics that the radial part of $\psi_\uparrow$ in the ground state ($L=0$) behaves as

$$\psi_\uparrow(r) \approx Ae^{-r/R}, \tag{8.8}$$

where A is (approximately) constant and R is the coherence length of the pair. The radial solution for $\hat{\psi}_\uparrow$ also in the ground state is known from Eqs. (5.1.21), p. 837, Ref. [3] to behave as

$$\hat{\psi}_\uparrow \approx B\frac{1-e^{-r/R}}{r}, \tag{8.9}$$

where B is also approximately a constant. The last term in the l.h.s. of Eq. (8.9) behaves like a *Hulten potential*

$$V_0 \times \frac{e^{-r/R}}{1-e^{-r/R}}, \quad V_0 = e^2\langle\hat{\psi}_\uparrow|\hat{\psi}_\downarrow\rangle. \tag{8.10}$$

After substituting the expression for the isomomentum, the radial isoschrödinger equation can be written

$$\left(-\frac{\mathcal{I}}{2\times\hat{m}}r^2\frac{d}{dr}r^2\frac{d}{dr} - (z-1)\frac{e^2}{r} - V_0\frac{e^{-r/R}}{1-e^{-r/R}}\right)\times\hat{\psi}_\uparrow(r) = E\times\hat{\psi}_\uparrow(r), \tag{8.11}$$

where $\hat{m}$ is the isorenormalized mass of the isoelectron.

The solution of the above equation is known from Ref. [5e], Sect. 5.1. The Hulten potential behaves at small distances like the Coulomb potential,

$$V_{\text{Hulten}} = V_0 \times \frac{e^{-r/R}}{1-e^{-r/R}} \approx V_0 \times \frac{R}{r}. \tag{1.37}$$

At distances smaller than the coherent length of the pair, Eq. (1.36) can therefore be effectively reduced to the form

$$\left(-\frac{1}{2\times\hat{m}}r^2\frac{d}{dr}r^2\frac{d}{dr} - V\frac{e^{-r/R}}{1-e^{-r/R}}\right)\times\hat{\psi}_\uparrow(r) = E\times\hat{\psi}_\uparrow(r), \tag{8.12a}$$

$$V = V_0 \times R + (z-1)\times e^2, \tag{1.38b}$$

with general solution, boundary condition and related spectrum (see Ref. [3], pp. 837-838)

$$\hat{\psi}_{\uparrow}(r) = {}_2F_1(2\times\alpha+1+n, 1-\alpha, 2\times\alpha+1, e^{-r/R})e^{-\alpha\times r/R}\frac{1-e^{-r/R}}{r}, \qquad (1.39a)$$

$$\alpha = (\beta^2 - n^2)/2n > 0, \quad \beta^2 = \hat{m}\times V\times R^2/\hbar^2 > n^2, \qquad (8.12b)$$

$$E = -\frac{\hbar^2}{4\times\hat{m}\times R^2}\left(\frac{\hat{m}\times V\times R^2}{\hbar^2}\frac{1}{n} - n\right)^2, \quad n = 1, 2, 3, \ldots \qquad (8.12c)$$

where we have reinstated $\hbar$ for clarity.

Santilli [3] identified the numerical solution of Eqs. (8.12) for the hadronic model $\pi^0 = (\hat{e}^+_{\uparrow}, \hat{e}^-_{\downarrow})_{\rm HM}$ (in which there is evidently no contribution from the Cuprate ions to the constant V), by introducing the parameters

$$k_1 = \hbar/2\times\hat{m}\times R\times c_0, \quad k_2 = \hat{m}\times V\times R^2/\hbar, \qquad (8.13)$$

where c_0 is the speed of light in vacuum. Then,

$$V = 2\times k_1\times k_2^2\times\hbar\times c_0/R, \qquad (8.14)$$

and the total energy of the state $\pi^0 = (e^+_{\uparrow}, e^-_{\downarrow})_{\rm HM}$ becomes in the ground state (which occurs for $n = 1$ for the Hulten potential)

$$\begin{aligned} E_{{\rm tot},\pi^0} &= 2\times k_1\times[1-(k_2-1)^2/4]\times\hbar\times c_0/R = \\ &= 2\times k_1(1-\varepsilon^2)\times\hbar\times c_0/R. \end{aligned} \qquad (8.15)$$

The use of the total energy of the π^0 (135 MeV), its charge radius ($R\approx 10^{-13}$ cm) and its meanlife ($\tau\approx 10^{-16}$ sec), then yields the values (Eqs. (5.1.33), p. 840, Ref. [3])

$$k_1 = 0.34, \quad \varepsilon = 4.27\times 10^{-2}, \qquad (8.16a)$$

$$k_2 = 1 + 8.54\times 10^{-2} > 1. \qquad (8.16b)$$

Animalu [10a] identified the solution of Eqs. (1.39) for the Cooper pair by introducing the parameters

$$k_1 = \varepsilon\times F\times R/\hbar\times c_0, \quad k_2 = KR/\varepsilon_{\rm F}, \qquad (8.17)$$

where $\varepsilon_{\rm F}$ is the iso-Fermi energy of the isoelectron (that for hadronic mechanics).

The total energy of the Cooper pair in the ground state is then given by

$$E_{\rm Tot,\ Cooper\ pair} = 2\times k_1\times[1-(k_2-1)^2/4]\times\hbar\times c_0/R \approx k_2\times T_c/\theta_{\rm D}, \qquad (8.18)$$

where $\theta_{\rm D}$ is the Debye temperature.

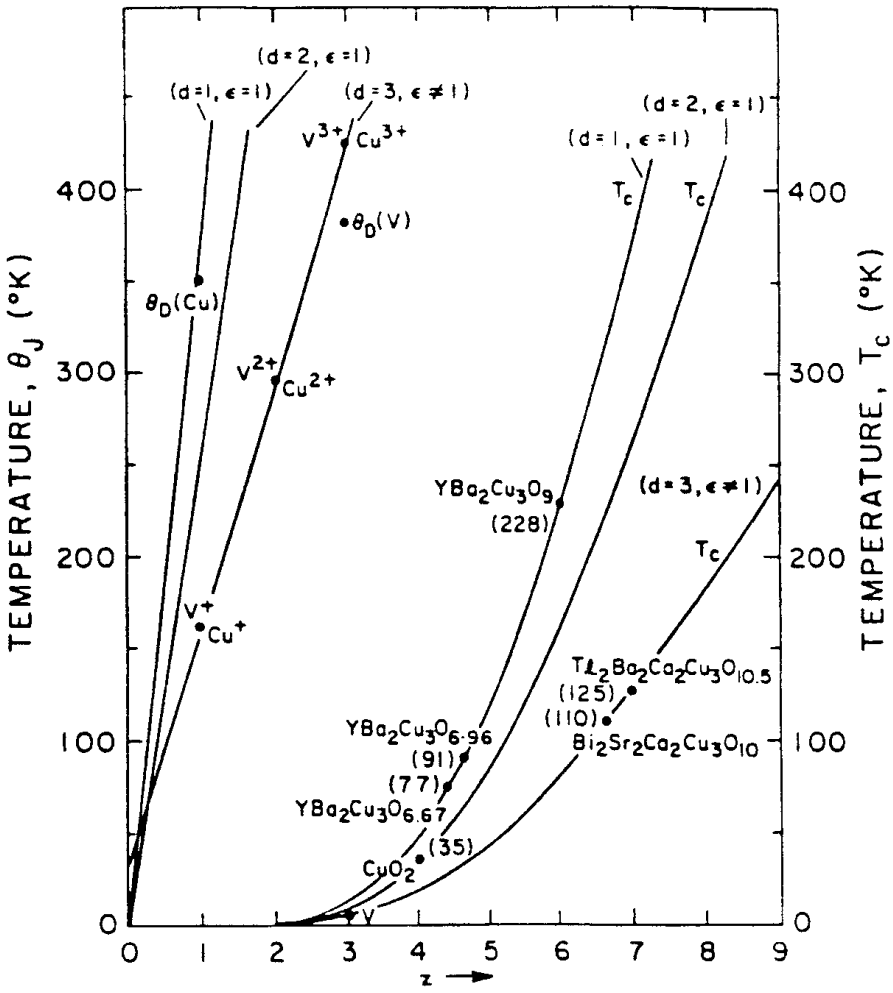

Figure 8.1. A reproduction of Fig. 10 of Ref. [10a] illustrating the remarkable agreement between the predicted dependence of T_c from the effective valence z of ions (continuous curve) and the experimental values on the "jellium temperature" for various compounds (solid dots).

Several numerical examples were considered in Refs. [2]. The use of experimental data for aluminum,

$$\theta_{\rm D} = 428^0 K, \quad \varepsilon_{\rm F} = 11.6{\rm C}, \quad T_c = 1.18^0 K, \tag{8.19}$$

yields the values

$$k_1 = 94, \quad k_2 = 1.6 \times 10^{-3} < 1. \tag{8.20}$$

For the case of $YBa_2Cu_3O_{6-\chi}$ the model yields [*loc. cit.*]

$$k_1 = 1.3z^{-1/2} \times 10^{-4}, \quad k_2 = 1.0 \times z^{1/2}, \tag{8.21}$$

where the effective valence $z = 2(7 - \chi)/3$ varies from a minimum of $z = 4.66$ for $YBa_2Cu_3O_{6.96}$ ($T_c = 91^0 K$) to a maximum of $z = 4.33$ for $YBa_2Cu_3O_{6.5}$ ($T_c = 20^0 K$). The general expression predicted by hadronic mechanics for $YBa_2Cu_3O_{6-\chi}$ is given by (Eq. (5.15), p. 373, Ref. [10a])

$$T_c = 367.3 \times z \times e^{-13.6/z}, \tag{8.22}$$

and it is in remarkable agreement with experimental data (see Figs. 1.21–1.23).

A few comments are now in order. The above Animalu-Santilli model of the Cooper pair is indeed nonlinear, nonlocal and nonpotential. In fact, the nonlinearity in $\hat{\psi}_\uparrow$ is expressed by the presence of such a quantity in Eqs. (1.31). The nonlocality is expressed by the term $\langle\hat{\psi}_\uparrow|\hat{\psi}_\downarrow\rangle$ representing the overlapping of the wavepackets of the electrons, and the nonpotentiality is expressed by the presence of interactions, those characterized by the isounit, which are outside the representational capabilities of the Hamiltonian H. This illustrates the necessity of using hadronic mechanics or other similar nonhamiltonian theories (provided that they are physically consistent), because of the strictly linear-local-potential character of quantum mechanics.

Note that, whenever the wave-overlapping is no longer appreciable, i.e., for $\langle\hat{\psi}_\uparrow|\hat{\psi}_\downarrow\rangle = 0$, $\mathcal{I} \equiv I$, quantum mechanics is recovered identically as a particular case, although without attraction.

The mechanism of the creation of the *attraction* among the *identical* electrons of the pair via the intermediate action of Cuprate ions is a general law of hadronic mechanics according to which *nonlinear-nonlocal-nonhamiltonian interactions due to wave-overlappings at short distances are always attractive in singlet couplings and such to absorb Coulomb interactions, resulting in total attractive interactions irrespective of whether the Coulomb contribution is attractive or repulsive*. As noted earlier, the Hulten potential is known to behave as the Coulomb one at small distances and, therefore, the former absorbs the latter.

Alternatively, we can say that within the coherent length of the Cooper pair, the Hulten interaction is stronger than the Coulomb force. This results in the overall attraction. Thus, the similarities between the model for the π^0 and that for the Cooper pair are remarkable. The applicability of the same model for other aspects should then be expected, such as for a deeper understanding of the valence, and will be studied in the next chapters.

Another main feature of the model is characterized also by a general law of hadronic mechanics, that *bound state of particles due to wave-overlappings at short distances in singlet states suppress the atomic spectrum of energy down to only one possible level*. The Hulten potential is known to admit a *finite* number

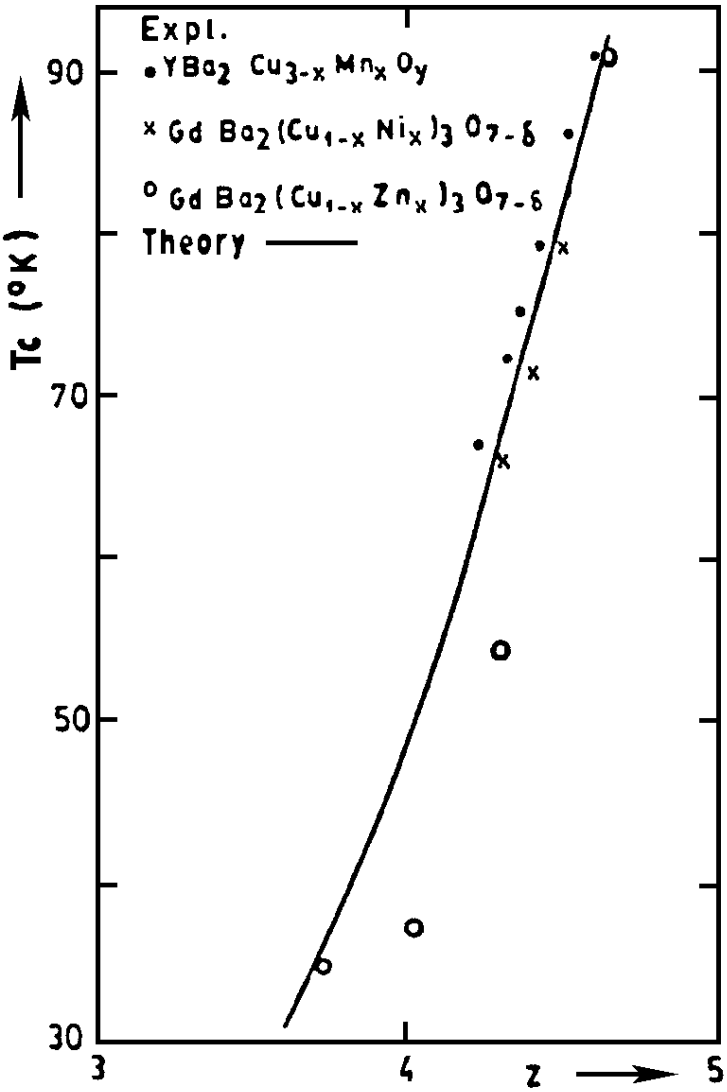

Figure 8.2. A reproduction of Fig. 5, p. 380 of Ref. [10a] showing the agreement between the prediction of isosuperconductivity for the doped 1:2:3 Cuprates and the experimental data.

of energy levels. Santilli's [5e] solution for the π^0 shows the suppression of the energy spectrum of the positronium down to only one energy level, 135 MeV of the π^0 for $k_2 > 1$. Similarly, the solutions for the Cooper pair [10] also reduce the same finite spectrum down to only one admissible level.

Table 1. $YBa_2Cu_{3-x}Mn_xO_y$
(After N.L. Saini *et al.*, Int. J. Mod. Phys. B6, 3515 (1992)

x	y	z	T_c (theory)	T_c (expt.)
0.00	6.92	4.613	88.9	91
0.03	6.88	4.541	83.5	86.6
0.09	6.87	4.447	76.7	79.0
0.15	6.91	4.387	72.6	75.0
0.21	6.92	4.312	67.6	72.0
0.30	6.95	4.212	61.3	67.0

Note: T_c (theory) $= 367.3z\exp(-13.6/z)$, where the effect of replacing Cu_3 by $Cu_{3-x}Mn_x$ is obtained by replacing 3 by $(3-x)+2x = 3+x$, which lowers the effective valence (z) on Cu^{2+} ions to $z = 2y/(3+x)$.

Table 2. $GdBa_2(Cu_{1-x}Ni_x)_3O_{7-\delta}$
(After, Chin Lin *et al.*, Phys. Rev. B42, 2554 (1990))

x	$y = 7-\delta$	z	T_c (theory)	T_c (expt.)
0.000	6.96	4.640	91.0	91
0.025	6.96	4.527	82.4	79
0.050	6.96	4.419	74.8	71
0.075	6.96	4.316	67.9	65

Note: T_c (theory) $= 367.3z\exp(-13.6/z)$, $z = 2y/3(1+x)$ as discussed in Table 1.

Table 3. $GdBa_2(Cu_{1-x}Zn_x)_3O_{7-\delta}$
(After, Chin Lin *et al.*, Phys. Rev. B42, 2554 (1990))

x	$y = 7-\delta$	z	T_c (theory)	T_c (expt.)
0.000	6.96	4.640	91.0	91
0.025	6.96	4.309	67.4	54
0.050	6.96	4.009	49.0	37
0.075	6.96	3.737	36.1	35

Figure 8.3. A reproduction of the tables of p. 379, Ref. [10a] illustrating the agreement between the predictions of the model with experimental data from other profiles.

Excited states are indeed admitted, but they imply large distances R for which nonlinear-nonlocal-nonhamiltonian interactions are ignorable. This implies that all excited states are conventionally quantum mechanical, that is, they *do not* represent the π^0 or the Cooper pair. Said excited states represent instead the discrete spectrum of the ordinary positronium, or the continuous spectrum of repulsive Coulomb interactions among the two identical electrons.

Alternatively, we can say that, in addition to the conventional, quantum mechanical, Coulomb interactions among two electrons, there is *only one additional system* of hadronic type with *only one energy level* per each couple of particles, one for $\pi^0 = (e^+_\uparrow, e^-_\downarrow)_{\rm HM}$ and the other for the Cooper pair, $\rm CP = (e^-_\uparrow, e^-_\downarrow)_{\rm HM}$.

The case of possible triplet couplings also follows a general law of hadronic mechanics. While singlets and triplets are equally admitted in quantum mechanics (read, coupling of particles at large mutual distances under their point-like approximation), this is no longer the case for hadronic mechanics (read, couplings

of particles when represented as being extended and at mutual distances smaller than their wavepackets/wavelengths). In fact, *all triplet couplings of particles under nonlinear-nonlocal-nonhamiltonian interactions are highly unstable, the only stable states being the singlets.*

This law was first derived in Ref. [5e] via the "gear model", i.e., the illustration via ordinary mechanical gears which experience a highly repulsive force in triplet couplings, while they can be coupled in a stable way only in singlets. The possibility of applying the model to a deeper understanding of Pauli's exclusion principle is then consequential, and will be studied in Chapters 4 and 5.

The connection between the proposed model and the conventional theory of the Cooper pair is intriguing. The constant in the Hulten potential can be written

$$V_0 = \hbar\omega, \tag{8.23}$$

where ω is precisely the (average) *phonon frequency.* The total energy can then be rewritten

$$E_{Tot} = 2 \times \varepsilon_{\rm F} - E \approx 2 \times k_1 \times k_2 \times \hbar \times c_0/R(e^{1/N\times V} - 1), \tag{8.24}$$

where $N \times V$ is the (dimensionless) *electron-phonon coupling constant.*

In summary, a main result of studies [2] is that *the conventional representation of the Cooper pair via a mysterious "phonon" can be reformulated without any need of such a hypothetical particle, resulting in a real, sufficiently strong attraction between the identical electrons, that is absent in the phonon theory.*

The above model of the Cooper pair see its true formulation at the relativistic level because it provides a *geometrization* of the Cooper pair, better possibilities for novel predictions and the best possible experiments tests. These profiles [10] will not be reviewed for brevity.

8.3 Novel Predictions of Animalu's Hadronic Superconductivity

As indicated in Section 1.2, besides the inability to achieve any understanding of the Cooper pair, anotehr major insufficiency of quantum mechanics is superconductivity is the well known exhaustion of all predictive capacities for the main objective of the theory, the achievement of superconductive capacity at ambient temperature.

Besides the achievement of a quantitative representation of the structure of the Cooper pair, one of the most important features of hadronic mechanics in superconductivity is precisely its capability of permitting *new* predictions.

One of them is a realistic possibility of achieving a form of superconductivity at ambient temperature that can be outlined as follows. Recall that the electric resistance originates from the interactions between the electric and magnetic fields of the electrons and those of atomic electron clouds (see Figure 8.4). Particular

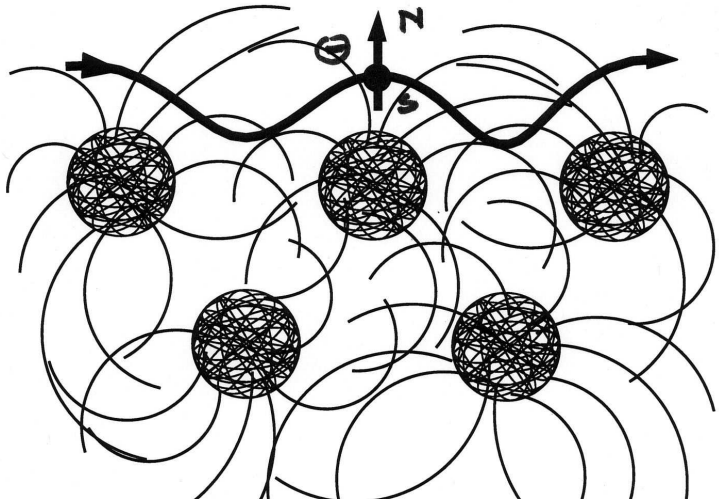

Figure 8.4. A schematic view of a conventional electric current, here represented with one electron (top view), moving in the surface of an ordinary conductor (lower view), illustrating the origin of the electric resistance due to interactions of both electric and magnetic type with the electromagnetic fields of the atoms of the conductor.

"obstructions" against the flow of electrons in conductors (thus causing resistance) originates from the interaction of the intrinsic magnetic field of electrons and the atomic electron cloud of the conductor.

The achievement of a quantitative understanding of the Cooper pair then permits the prediction and quantitative treatment of a *new electric current characterized by a flow through ordinary conductors of isoelectron pairs, rather than individual electrons,* as illustrated in Figure 8.4.

In fact, the total magnetic moment of the isoelectron pair can be considered as being null at interatomic distances, thus implying a dramatic decrease of the electric resistance, due to the reduction of the interactions between the current and the conductor to the sole Coulomb interactions.

Moreover, hadronic mechanics can assist in the creation of such a new current via the removal under sufficiently intense external electric fields of "valence pairs", rather individual electrons, from various substances (including plastic compounds and non-conducting materials), said substances being selected under the condition of having two unbonded valence electrons.

This is due to the fact that, as experimentally established in the helium, when not bonded into molecules, the electrons of a valence pair are not separated in an orbital but are generally coupled in singlet exactly along the structure of the isoelectron pair.

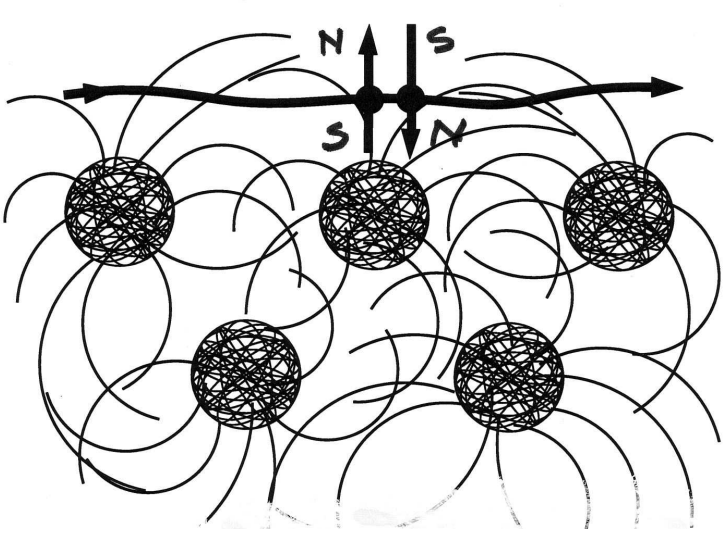

Figure 8.5. A schematic view of the new electric current predicted by hadronic superconductivity, consisting of the current of *electron pairs bonded in singlet,* in which case there is the absence of the magnetic field of the current constituents, with consequential reduction of the electric resistance.

A rather intense research to achieve superconductivity at ambient temperature is under way in corporate circles which research, unfortunately, is not generally available to academia due to its novelty, that is, the use of methods and theories generally opposed by organized interests in academia at this time. It is regrettably for scientific knowledge that this type of advanced corporate research cannot be reported in this monograph at this time.

References

[1] xxxxx

[2] A. O. E. Animalu, Hadronic J. **17**, 379 (1994).

[3] R. M. Santilli, Hadronic J. **1**, 224, 574 and 1267 (1978).

[4] A. O. E. Animalu Animalu and R. M. Santilli, Int. J. Quantum Chemistry **29**, 175 (1995).

Chapter 9

EXPERIMENTAL VERIFICATIONS AND APPLICATIONS IN CHEMISTRY

9.1 ISOCHEMICAL MODEL OF THE HYDROGEN MOLECULE

TO BE EDITED

9.1.1 Introduction

Chemistry provides some of the most important experimental verifications and applications of hadronic mechanics in its version known as *hadronic chemistry* essentially consisting of the isotopic methods of Chapter 3 for chemical processes that are invariant under time reversal (*isochemistry*), the genotopic methods of Chapter 4 for irreversible processes (*genochemistry*), the multi-valued methods of Chapter 5 for biological structures (*hyperchemistry*), and their isoduals for the first known formulation of antimatter chemistry (*isodual iso-, geno- and hyperchemistry*). A comprehensive study of these formulations is presented in monograph [67]. This chapter is essentially dedicated to a review of the experimental verifications and scientific applications of isochemistry. Industrial applications of hadronic chemistry are presented in Chapters 11 and 12.

As an indication, in Sections 1.2 and 1.3 we showed that, following one century of failed attempts, quantum chemistry was unable to represent from unadulterated basic axioms a residual amount of 2% of molecular binding energies with electric and magnetic moments being wrong even in their signs, while the improvement of the representation via the so-called "screenings of the Coulomb law" causes the loss of the quantum of energy as well as other inconsistencies.

By comparison, in this chapter we show that *hadronic chemistry has permitted the first exact and invariant representation from first principles of molecular binding energies and other molecular data without adulteration of the basic ax-*

ioms, while admitting as particular cases conventional screenings of the Coulomb law.

The studies presented in this chapter are devoted to the representation of molecular structures assumed as isolated from the rest of the universe, thus being invariant under time reversal. Consequently, unless otherwise stated, all studies of this chapter are based on *isochemistry.* various experimental verifications and applications of the broader *genochemistry* and *hyperchemistry* and their isoduals are under way by various scholars and they will be reported in their works.

As it is well known, the primary structural characteristics of *quantum chemistry* (see, e.g., Refs. [1]) are those of being:

1) *linear*, in the sense that eigenvalue equations depend on wavefunctions only to the first power;

2) *local-differential*, in the sense of acting among a finite number of isolated points; and

3) *potential*, in the sense that all acting forces are derivable from a potential energy.

Therefore, quantum chemistry is a *Hamiltonian theory*, i.e., models are completely characterized by the *sole* knowledge of the Hamiltonian operator, with a *unitary structure*, i.e., the time evolution verifies the unitarity conditions

$$U = e^{iH\times t}, \quad U \times U^\dagger = U^\dagger \times U = I, \quad H = H^\dagger, \tag{9.1.1}$$

when formulated on conventional Hilbert spaces over the conventional fields of complex numbers.

Despite outstanding achievements throughout the 20-th century, quantum chemistry cannot be considered as "final" because of numerous insufficiencies identified in Chapter 1.

A most important insufficiency is *the inability to represent deep mutual penetrations of the wavepackets of valence electrons in molecular bonds.* The latter interactions are known to be:

$\hat{1}$) *nonlinear*, i.e., dependent on powers of the wavefunctions greater than one;

$\hat{2}$) *nonlocal-integral*, i.e., dependent on integrals over the volume of overlapping, which, as such, cannot be reduced to a finite set of isolated points; and

$\hat{3}$) *nonpotential*, i.e., consisting of "contact" interactions with consequential "zero range," for which the notion of potential energy has no mathematical or physical sense.

A representation of the latter features evidently requires a *nonhamiltonian theory*, i.e., a theory which cannot be solely characterized by the Hamiltonian, and requires additional terms. It then follows that the emerging theory is *nonunitary* i.e., its time evolution verifies the law,

$$U \times U^\dagger = U^\dagger \times U \neq I, \tag{9.1.2}$$

when formulated on conventional Hilbert spaces over conventional fields.

It is evident that the above features are beyond any hope of scientific-quantitative treatment via quantum mechanics and chemistry.

In the preceding Chapter 3 we have submitted the foundations of a generalization covering of quantum chemistry under the name of *hadronic chemistry*, first submitted by Santilli and Shillady in Ref. [2], which is capable of providing an invariant representation of the above-mentioned nonlinear, nonlocal, nonpotential, nonhamiltonian, and nonunitary interactions in deep correlations of valence electrons.

In Chapter 3, we have also shown that the conventional "screenings" of the Coulomb potential (which are necessary for a better representation of experimental data) are outside the axiomatic structure of "quantum" chemistry because such screenings can only be reached via nonunitary maps of the Coulomb law, thus resulting in being particular cases of the broader hadronic chemistry.

The main purpose of this chapter is the application of hadronic chemistry to the construction of a new model of molecular bonds and its verification in the representation of experimental data of the hydrogen molecule.

Since molecular structures are considered as isolated, thus being closed, conservative, and reversible, the applicable branch of hadronic chemistry is *isochemistry*, which is characterized by the identification of the nonunitary time evolution with the *generalized unit* of the theory, called *isounit*,

$$U \times U^\dagger = \hat{I}(r, p, \psi, \partial\psi, ...) \neq I, \tag{9.1.3}$$

assumed hereon not to depend explicitly on time, and the reconstruction of the *totality* of the formalism of quantum chemistry into a new form admitting of $\hat{I}$, rather than I, as the correct right and left new unit.

The capability by the isounit to represent nonlinear, nonlocal, and nonhamiltonian interactions is evident. Its selection over other possible choices is mandated by the condition of *invariance*, that is, the prediction of the same numerical values for the same quantities under the same conditions, but at different times. In fact, whether generalized or not, the unit of any theory is the basic invariant.

A central assumption of this chapter is that *quantum mechanics and chemistry are exactly valid at all distances of the order of the Bohr radius* ($\simeq 10^{-8}$ *cm*), *and the covering hadronic chemistry only holds at distance of the order of the size of the wavepackets of valence electrons* (1 fm $= 10^{-13}$ cm).

This condition is evidently necessary, on one side, to admit the conventional quantum structure of the hydrogen atom, and, on the other side, to admit quantitative studies of the nonhamiltonian interactions of short range valence bonds.

The above condition is readily achieved by imposing that all isounits used in this chapter recover the conventional unit at distances greater than 1 fm,

$$\lim_{r \gg 1 \text{ fm}} \hat{I}(r, p, \psi, \partial\psi, \dots) = I, \tag{9.1.4a}$$

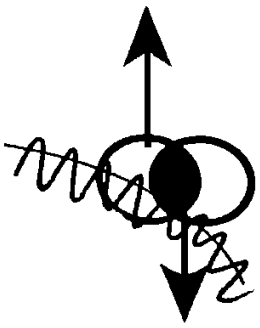

Figure 9.1. A schematic view of the central conditions studied in this chapter, the deep overlapping of the wavepackets of valence electrons in singlet coupling (to verify Pauli's exclusion principle). These conditions are known to be nonlinear, nonlocal, and nonpotential (due to the zero-range, contact character of the interactions), thus not being representable via a Hamiltonian, and, consequently, not being unitary. As a result, the ultimate nature of valence bonds is outside any credible representation via quantum chemistry. Hadronic chemistry (Chapter 3) has been built for the specific scope of representing the conditions herein considered of the bonding of valence electrons.

$$|\hat{I}| \ll 1, \quad |\hat{T}| \gg 1. \tag{9.1.4b}$$

In fact, under the above condition, hadronic chemistry recovers quantum chemistry everywhere identically. The reader should keep in mind the crucial implications of conditions (9.1.4b) which, as shown in Sect. 3.4, permit a dramatic increase of the convergence of chemical series, with corresponding decrease of computer time, as verified in the models of this chapter and of the following ones.

The reader should also note that, quite remarkably, rather than being imposed, both conditions (9.4a) and (9.4b) are naturally verified by actual chemical models.

It should be recalled that, under the assumption of representing closed-isolated systems, *isochemistry verifies all conventional laws and principles of quantum mechanics* (Chapter 3). Therefore, there is no *a priori* mean for rejecting the validity of hadronic chemistry within the small region of space of valence bonds.

It then follows that the selection of which theory is valid is referred to the capability to represent experimental data. Quantum mechanics has been capable of achieving an exact representation of all experimental data for the structure of *one individual* hydrogen atom. Therefore, quantum mechanics is here assumed as being exactly valid within such a well defined physical system, any possible improvement being redundant at best.

By comparison, quantum mechanics and chemistry have not been able to achieve an exact representation of the experimental data of the *different* conditions of molecular structures, as discussed in detail in Chapter 1. As a result,

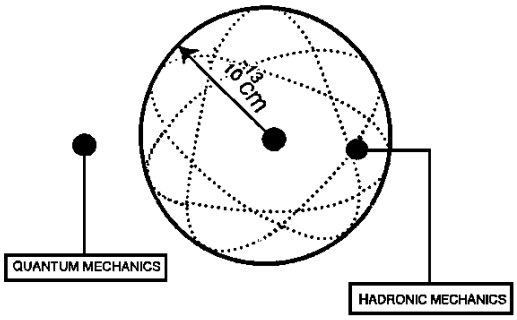

Figure 9.2. A schematic unit of the *hadronic horizon*, namely, of the sphere of radius 1 fm (= 10^{-13} cm) outside which quantum chemistry is assumed to be exactly valid, and inside which nonlinear, nonlocal, and nonpotential effects are no longer negligible, thus requesting the use of hadronic chemistry for their numerical and invariant treatment.

these theories are *not* considered as being exactly valid for the different conditions of molecular bonds (see Fig. 1.7).

As we shall see in this chapter, hadronic chemistry can indeed provide an exact representation of molecular characteristics, and, therefore, it is consider as being exactly valid for the indicated conditions of applicability.

A knowledge of *isomathematics* of Chapter 3 is essential for a technical understanding of the content of this chapter. A comprehemnsive presentation is available in monograph [67]

(see also representative papers [3, 4]).

For mathematically less inclined readers, we recall from Sect. 3.3.6 that specific applications of isochemistry can be constructed in their entirety via a simple nonunitary transform of conventional quantum chemical models. In fact such a transform adds precisely the desired short range, nonlinear, nonlocal, and nonhamiltonian effects.

9.1.2 Isochemical Model of Molecular Bonds

We now present the conceptual foundations of our *isochemical model of molecular bonds* for the simplest possible case of the H_2 molecule, which was first submitted by Santilli and Shillady in Ref. [5]. We shall then extend the model to the water and to other molecules in the subsequent chapter.

Since the nuclei of the two H-atoms remain at large mutual distances, the bond of the H_2 molecule is evidently due to the bond of the peripheral valence electrons, as generally acknowledged [1].

Our main assumption [5] is that *pairs of valence electrons from two different atoms can bond themselves at short distances into a singlet quasi-particle state*

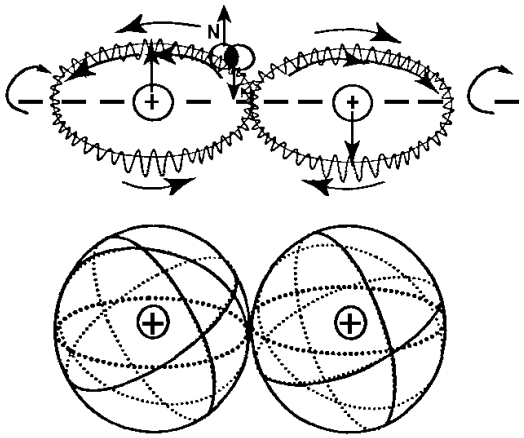

Figure 9.3. A schematic view of the proposed *isochemical model of the hydrogen molecule with fully stable isoelectronium*, where the top view refers to absolute zero degree temperature and in the absence of any motions, while the lower view includes rotations, thus recovering the conventional spherical distribution. The view is complementary to that of Fig. 9.7 for the unstable isoelectronium. The model is here depicted in terms of *orbits of the valence electrons*, rather than in terms of *orbitals, or density distributions*. The fundamental assumption is that the two valence electrons, one per each atom, correlate themselves into a bonded singlet state at short distance we have called *isoelectronium*, which is assumed in this figure to be stable. In this case the only orbit yielding a stable H-molecule is that in which the isoelectronium describes a *oo*-shaped orbit around the respective two nuclei, as it occurs for planets in certain systems of binary stars. The isoelectronium is then responsible for the *attractive force* between the two atoms. The *binding energy* is instead characterized by the *oo*-shaped orbit of the isoelectronium around the two nuclei, conceptually represented in this figure via a standing wave for a particle of spin 0, charge $-2e$, and null magnetic moment. As we shall see in this chapter, the model then permits a representation of: the reason why the H_2 and H_2O molecules have only two hydrogen atoms; the exact representation of the binding energy; the resolution of some of the inconsistencies of the conventional model; and other advances. Note finally that the model is easily extendable to dimers such as HO, HC, *etc.*, as studied in Chapter 3. The novelty in predictive character of the model can be seen from these preliminary lines. For instance, the model depicted in this figure predicts that *the hydrogen molecule becomes asymmetric, thus acquiring an infrared signature, under sufficient magnetic polarization, which removes its rotational motions.*

called "isoelectronium," which describes an oo-shaped orbit around the two nuclei similar to that of planets in binary star systems (Fig. 9.3).

It is important to note that recent studies in pure mathematics [39] have established that the *oo*-shaped orbit, called the *figure eight* solution, is one of the most stable solutions of the N-body problem.

The primary binding force of the isoelectronium is assumed to be of nonlinear, nonlocal, and nonpotential type due to contact effects in deep overlappings of the wavepackets of the valence electrons, as studied in Sect. 9.3.

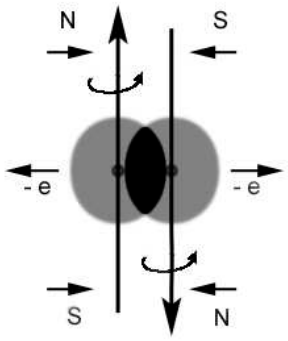

Figure 9.4. A schematic view of the conventional Coulomb forces of electrostatic and magnetostatic type in the structure of the isoelectronium. Since the charges are equal, they cause a *repulsion*. However, since the coupling is in singlet, the magnetic polarities are opposite, thus implying an *attraction*. Elementary calculations show that the magnetostatic attraction equals the electrostatic repulsion at a mutual distance of the order of 1 fm, while it becomes bigger at smaller distances, thus explaining the reason why the hadronic horizon has been set at 10^{-13} cm. This evidence establishes that the bonding force of the isoelectronium can also see its origin on purely Coulomb forces and, more particular, on the dominance of magnetic over electric effects at short distances, which is a rather general occurrence under the proper conditions (see the new chemical special of *magnecules* in Chapter 8). Despite this fully potential attractive total force, it should be stressed that the isoelectronium cannot be treated within a purely quantum mechanics context for various reasons. The first reason is that with the decrease of the distance, both electrostatic and magnetostatic effects diverge, thus preventing any serious scientific study. Hadronic mechanics and chemistry have been built precisely to remove these divergencies via the isotopies of generic products $A\hat{\times}B = A \times \hat{T} \times B$ with $|\hat{T}| \ll 1$ (Chapter 3). Therefore, the hadronic treatment of the isoelectronium permits convergent numerical predictions which would be otherwise impossible for quantum chemistry. Independently from that, the nonunitary lifting of quantum chemistry is mandated by the need to achieve an exact representation of experimental data on molecules which, as now established, requires screenings solely obtainable via nonunitary transforms of the Coulomb potential. Thus, any attempt to preserve old theories as exactly valid is doomed to failures. Despite that, the electrostatic and magnetostatic effects depicted in this figure illustrate that conventional potential effects should also be expected in the structure of the isoelectronium. In other words, rather than assuming either a purely quantum or a purely hadronic setting, we have *in media virtus*, i.e., the most plausible origin of the bonding force of the isoelectronium is that partially of potential and partially of nonpotential type. Still in turn, this implies the possibility of a significant (negative) binding energy for the isoelectronium, which is evidently that characterized by the potential component (Sect. 9.3).

However, the reader should be aware that the isoelectronium is expected to have a component of the binding force of purely potential type because, when the electrons are in singlet coupling, the magnetostatic *attraction* may be conceivably bigger than the electrostatic *repulsion* at distances of the order of one fermi or less (see Fig. 9.4 for details).

It should be stressed, however, that a purely potential origin of the isoelectronium is not expected to be exactly valid for various reasons, the most visible one being the fact that, at the very small mutual distances here considered, magnetostatic and electrostatic laws diverge, thus prohibiting reliable quantitative studies.

Hadronic chemistry has been built to resolve all divergences in the study of the isoelectronium thanks to the isomathematics with product $A\hat{\times}B = A \times \hat{T} \times B$, and the isotopic element $\hat{T}$ restricted to have absolute values much smaller than 1. In this way, the hadronic component of the isoelectronium binding force will "absorb" all divergent or otherwise repulsive effects, resulting in convergent numerical values.

The reader is also discouraged to reduce the isoelectronium to a purely quantum structure because, in this way, the theory would preserve all the insufficiencies of chemistry studied in Chapter 1, most importantly, the inability to reach an exact representation of molecular characteristics from the strict application of first quantum principles without *ad hoc* adulterations. In fact, as now well established, such an exact representation requires screenings of the Coulomb law, which can only be obtained via nonunitary transforms. The same nonunitary broadening of quantum chemistry is requested on numerous other counts independent from the isoelectronium.

Despite these limitations, the purely magnetostatic-electrostatic structure of the isoelectronium remains important *in first approximation*, because it recovers in a very simple way the hadronic horizon (Fig. 9.2), as well as the prediction by hadronic mechanics dating back to 1978 that triplet couplings are highly unstable. In fact, in the latter case, both electrostatic and magnetostatic forces would be *repulsive*, thus prohibiting any possible bound state, in beautiful agreement with Pauli's exclusion principle.

It is easy to predict that *the isoelectronium cannot be permanently stable when interpreted as a quasi-particle of about* 1 *fm charge diameter*. In fact, the mere presence of exchange forces, which remain fully admitted by isochemistry, prevents the achievement of a complete stability under the indicated small mutual distances of the electrons. As we shall see in more details in Chapter 6, there are additional technical reasons which prevent the complete stability at short distances, and actually render the isoelectronium a short lived quasi-particles when the valence electrons are assumed at mutual distances of 1 fm.

However, it is easy to see that *the isoelectronium must be fully stable when the mutual distance of the two valence electrons is permitted to be of the order of molecular size. In fact, any instability under the latter long range conditions would imply a necessary violation of the fundamental Pauli's exclusion principle.*

In different words, *the isoelectronium is one of the first known quantitative representations of Pauli's principle*, in the sense that:

1) When assumed to be of potential type, the interaction responsible for Pauli's principle implies catastrophic inconsistencies, such as shifts of experimentally established energy levels, deviations from all spectroscopic lines, etc. As a result, a quantitative representation of Pauli's principle is *impossible* for quantum mechanics, evidently due to its strictly potential character. For this reason, Pauli's principle is merely imposed in quantum mechanics without any explanations, as well known. By comparison, a quantitative representation is possible for hadronic mechanics precisely because of its admission of *nonpotential* interactions, that is, interactions which have no bearing on energy levels and spectroscopic lines.

2) Quantum mechanics admits, in general, both singlet and triplet couplings because particles are assumed to be point like as per the very topological structure of the theory. By comparison, hadronic mechanics represents particles as expended at mutual distances smaller than their wavepackets, and solely admits singlet couplings due to highly repulsive-unstable forces predicted for all triplet couplings. The latter repulsive forces originate from the drag experienced by one wavepackets when rotating within and against the rotation of the other wavepacket, as well as by the fact that in triplet couplings both magnetostatic and electrostatic effects are repulsive (Fig. 9.4); and

3) Quantum mechanics cannot provide an exact representation of an *attraction* between *identical* electrons at very short distances, as discussed earlier, in disagreement with the experimental evidence, e.g., that the two electrons of the helium are bonded most of the time, to such an extent that they are emitted in such a bonded form during photodisintegrations, and in other events. By comparison, hadronic mechanics has been built to represent precisely the *bonding* of identical electrons in *singlet* coupling under interactions *not* derivable from a potential.

The assumption of the isoelectronium as being unstable when its valence electrons are at mutual distances of molecular order, implies a violation of Pauli's principle, e.g., because of the automatic admission of triplet couplings for two electrons at the same energy level.

When assumed as being stable in the limit case of a quasi-particle of 1 fm charge radius, *the most stable trajectory of the isoelectronium is of oo-type, each o-branch occurring around each nucleus* (Fig. 9.3). As illustrated in Fig. 9.4 (see also Chapter 8), such a shape automatically prevents the inconsistent prediction of ferromagnetic character of all molecules.

When the correlation-bond is distributed over the entire molecular orbit, *the trajectory of the isoelectronium is also expected to be oo-shaped around the two nuclei with inverted direction of rotation from one o-branch to the other*. This is suggested by a variety of reasons, such as: the need of avoiding the inconsistent prediction of ferromagnetic character, the compatibility with the limit case of a

fully stable particle at short distance (which, as we shall see, can describe several *oo*-shaped orbits prior to separation), and others.

It should be indicated that the assumption of a finite lifetime of the isoelectronium irrespective of size implies the possibility of adding several H-atoms to the H_2 molecule for the duration of the unbound valence electrons, as well as other inconsistencies, such as the capability by hydrogen and water to be paramagnetic (Chapter 8).

In this chapter, we apply the above hypothesis to the construction of a new model of the hydrogen molecule and prove its capability to:

1) provide an essentially exact representation of the binding energy and other characteristics of the hydrogen molecules;

2) said representation occurs from first axiomatic principles without exiting from the underlying class of equivalence as occurring for Coulomb screenings;

3) explain for the first time to our knowledge the reason why the hydrogen molecule has only two atoms;

4) introduce an actual "strongly" attractive molecular bond;

5) achieve a much faster convergence of power series with consequential large reduction in computer times;

6) prevent inconsistencies such as the prediction that the hydrogen is ferromagnetic. In fact, whatever magnetic polarity can be acquired by the orbit around one nucleus, the corresponding polarity around the second nucleus will necessarily be opposite, due to the opposite direction of the rotations in the two *o*-branches, thus preventing the acquisition of a net total polarity North-South of the molecule.

By recalling from Chapter 3 that Gaussian screenings of the Coulomb law are a particular case of the general nonunitary structure of hadronic chemistry, one can see from these introductory lines that our first achievement on scientific records of an essentially exact representation of molecular characteristics is reduced to the proper selection of the basic nonunitary transform, because the latter will permit dramatically more restrictive screenings.

The derivability of the essentially exact representation from first axioms of hadronic chemistry without adulterations is evident.

Equally evident is the first introduction of an actual, "strongly" attractive interatomic force (where the word "strongly" does not evidently refer to strong interactions in hadron physics), which is absent in current models due to the notorious "weak" nature of exchange and other forces of current used in molecular structures (where the word "weak" does not evidently refer to the weak interactions among leptons).

The representation of the reason why the hydrogen (or water) molecule has only two H-atoms is inherent in the very conception of the isoelectronium. Once the two valence electrons of the H-atoms couple themselves into a singlet quasi-

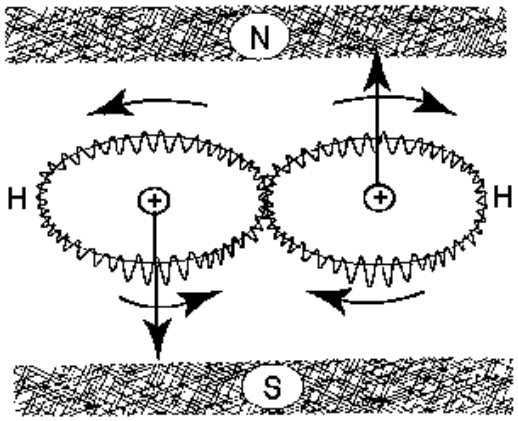

Figure 9.5. A schematic view of the impossibility for the isochemical model of the hydrogen molecule to acquire a net magnetic polarity, thus resolving a serious inconsistency of quantum chemistry. Recall from Chapter 1 that current molecular models are based on exchange, van der Waals, and other forces of nuclear origin, all implying the independence of the orbitals of the individual atoms. Under these assumptions, quantum electrodynamics demands that all molecules acquire a net total magnetic polarity North-South when exposed to an external magnetic field, in dramatic disagreement with reality. The isochemical model of molecular structure resolves this inconsistency because, as indicated in Fig. 4.3, the most stable trajectory for the isoelectronium is *oo*-shaped as it also occurs for the trajectory of planets in binary stars, with each *o*-branch around each nucleus. In this case, the rotation of the two *o*-branches are necessarily opposite to each other, thus resulting in *opposite* magnetic polarities, with the consequential impossibility to reach a *net* molecular magnetic polarity. As we shall see in Chapter 7, the above features have important industrial applications for new clean fuels and energies.

particle state, there is no possibility for a third valence electron to participate in the bound state, e.g., because we would have an impossible bound state between a fermion (the third electron) and a boson (the isoelectronium).

The achievement of a much faster convergence of the power series, or, equivalently, a dramatic reduction of computer times for the same calculations, is evident from the structure of hadronic chemistry as discussed in Chapter 3.

The avoidance of the prediction of ferromagnetic features (acquisition of a total North-South polarity under an external magnetic field) is due to the nature of the orbit of the isoelectronium, as discussed in details below and in Chapter 8.

In this chapter, we shall study two realizations of the proposed new model of the hydrogen molecule, the first model is a limiting case in which the isoelectronium is assumed to be *stable* (with an infinite lifetime) at ordinary conditions, and the second model in which the isoelectronium is assumed to be *unstable* (with a finite lifetime). The lifetime of the isoelectronium will then be computed in Chapter 6.

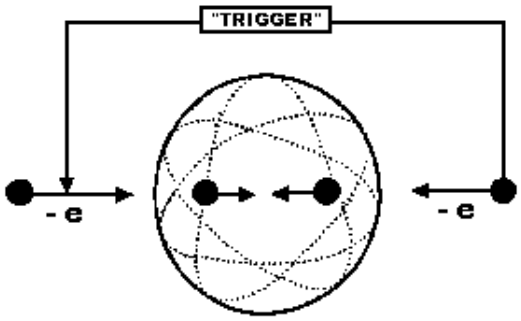

Figure 9.6. A schematic view of the *trigger*, namely, the external means suitable to force electrons with the same charge to penetrate the hadronic barrier (Fig. 9.2), in which attractive hadronic forces overcome the repulsive Coulomb barrier.

The hypothesis of the bonding of electrons at short distances was first introduced by Santilli [7a] for the structure of the π^0 meson as a hadronic bound state of one electron and one positron. Animalu [7b] and Animalu and Santilli [7c] extended the model to the Cooper pair in superconductivity as a hadronic bound state of two identical electrons.

A notion which is important for the very existence of the isoelectronium is that of a *trigger*, namely, *external (conventional) interactions, which cause the identical electrons to move one toward the other and to penetrate the hadronic horizon (Fig. 9.2) against their repulsive Coulomb interactions*. Once inside the above mentioned horizon, the attractive hadronic forces overcome the repulsive Coulomb interaction, resulting in a bound state.

In the case of the π^0 model as a bound state of an electron and a positron at short distances, there is no need for a trigger because the constituents naturally attract each other. On the contrary, the existence of the Cooper pair does indeed require a trigger, which was identified by Animalu [7b] and Animalu and Santilli [7c] as being provided by the Cuprate ions. For the case of an isolated hydrogen molecule, we conjecture that the trigger is constituted by the two H-nuclei, which do indeed attract the electrons. We essentially argue that the attraction of the electrons by the two nuclei is sufficient to cause the overlapping of the two wavepackets, thus triggering the electrons beyond the hadronic horizon.

It should be indicated that we cannot use the term "electronium" because it would imply a bound state of two identical electrons under *quantum* mechanics, which is known to be impossible. The term "electronium" would also be technically inappropriate because the constituents *are not* ordinary electrons, but rather "isoelectrons," i.e., the image of ordinary particles under *nonunitary* transforms

or, more technically, irreducible isounitary representations of the covering of the Poincarè symmetry known as the *Poincarè-Santilli isosymmetry* [3c, 3d, 4a].

We cannot close this conceptual section without a few comments regarding the possibility of treating the isoelectronium via *quantum electrodynamics* (QED), since the latter appears to be the natural discipline for a valence bond of two identical electrons at short distance. This issue is compounded by the general belief of the *unrestricted* exact validity of QED all the way to very small distances of the order of 10^{-24} cm.

It is easy to see that, as it is the case for quantum mechanics, a quantitative treatment of the isoelectronium is beyond the technical capabilities of QED for numerous conceptual and technical reasons. In fact, QED is purely linear, local and potential, while the interactions we are interested in representing are nonlinear, nonlocal and nonpotential.

In any case, it is easy to prove via the use of the Feynman diagrams that QED *cannot* represent any *attraction* between identical electrons in singlet coupling at short distance, as it occurs in the physical reality for the two electrons of the Helium, the Cooper pair, the valence electrons, and other systems. On the contrary, the *isotopies of quantum electrodynamics* (ISOQED) are expected to provide such a representation, but their study here would be vastly beyond the limited scope of this monograph.

The reconciliation between the current belief of the unrestricted exact validity of QED and the bonding of identical electrons is permitted by the fact that all experimental verifications of QED at shorter and shorter distances have been conducted via the use of *higher and higher energies*. On the contrary, the experimental verification of QED for the conditions of the isoelectronium require *smaller and smaller energies* which experimental verifications have been absent in the physics of the 20-th century due to the notorious emphasis on high energies.

As a final comment, it should be noted that the limitations of QED for the study of the isoelectronium are purely classical, and rest on *the inability of classical electrodynamics to represent the physical evidence of the attraction of identical spinning charges at sufficiently small distances, evidence which is even visible to the naked eyes, e.g., in ball lighting as created by nature, in microwave ovens or other means*.

As a matter of fact, no classical theory of electromagnetism can possibly be considered as "final" until it achieves the capability of representing the attraction of identical charges under suitable conditions. As a result, no quantum theory of electromagnetism, including QED, can be considered as "final" unless it is based on the preceding classical theory. One of the objectives of classical and operator isochemistry is precisely that of achieving such a missing representation.

9.1.3 The Limit Case of Stable Isoelectronium

We are now equipped to conduct a nonrelativistic study of the isoelectronium (Fig. 9.3) in the limit case of full stability under the assumption that the binding force is of purely hadronic type without potential contributions (Fig. 9.4). This approach is evidently done to test the effectiveness of hadronic chemistry for the numerical studies of the problem considered, since corrections due to potential effects can be easily added.

The reader should be aware upfront that *the above assumptions imply that the isoelectronium has no binding energy*, trivially, because nonpotential forces have no potential energy by conception.

The reader should be aware that the actual hadronic treatment should be conducted within the context of isomathematics, that is, on isoeuclidean and isohilbert spaces defined over isofields. To avoid excessive mathematical complexity, in this section we study the *projection* of this isotopic treatment on conventional spaces over conventional fields. However, it should be stressed that the only correct formulation remains the isotopic one.

As we shall see, the hadronic treatment of the isoelectronium yields an attraction of the type of the Hulten potential which is so strong to "absorb" at short distances all other forces, whether attractive or repulsive. However, the direct interpretation of the Hulten potential as an actual potential would be erroneous, since it solely occurs in the *projection* of the model on conventional spaces, while being completely absent in the technically appropriate treatment on isospaces over isofields. The direct interpretation of the Hulten potential as an actual potential well of quantum mechanical nature would also be in direct contradiction with the absence of binding energy.

Therefore, the assumption of the projected model as the correct one leads to insidious inconsistencies and misrepresentations, such as the possible interpretation of the isoelectronium via a potential well, which treatment is very familiar in quantum mechanics, but the same treatment has no physical meaning for the isoelectronium. This is due to the fact that, as stressed earlier, a necessary condition to avoid inconsistencies in the interpretation of Pauli's principle is that its interaction *does not* admit a potential energy, thus rendering meaningless, or at best contradictory, conventional potential wells.

Note that the emergence of a "strong" Hulten potential eliminates the issue whether the isoelectronium is due to the dominance of the attractive magnetostatic forces over the repulsive electric ones (Fig. 9.4). This is due to the fact that the Hulten potential, as we shall review shortly in detail, behaves at short distances as $constant/r$, thus absorbing all Coulomb forces, irrespective of whether attractive or not. Moreover, the unified treatment via the Hulten potential presented below eliminates the divergent character of these forces at short distances, thus permitting meaningful numerical results.

We should finally indicate, to avoid inconsistencies, that the study of this section deals with the *limit* case of *a perfectly stable isoelectronium interpreted as a quasi-particle of about 1 fm charge diameter*, while in reality such form of the isoelectronium is unstable. Moreover, in this section we shall not study the expectation that the isoelectronium persists beyond the 1 fm mutual distance of the valence electrons, as necessary to prevent violations of Pauli's principle.

We begin our quantitative analysis with the nonrelativistic quantum mechanical equation of two ordinary electrons in singlet couplings, $e^-_\downarrow$ and $e^-_\uparrow$ represented by the wavefunction $\psi_{\uparrow\downarrow}(r) = \psi(r)$,

$$\left(\frac{p \times p}{m} - \frac{e^2}{r}\right) \times \psi(r) = E \times \psi(r). \tag{9.1.5}$$

To transform this state into the isoelectronium representing the bonding of the H-electron with a valence electron of another atom of generic charge ze, we need first to submit Eq. (9.1.5) to a nonunitary transform characterizing the short range hadronic effects, and then we must add the *trigger*, namely, the Coulomb attraction by the nuclei.

This procedure yields the *isoschrödinger equation for the isoelectronium* (Chapter 1),

$$U \times U^\dagger = \hat{I} = 1/\hat{T} > 0, \tag{9.1.6a}$$

$$\hat{A} = U \times A \times U^\dagger, \quad A = p,\ H, \ldots, \tag{9.1.6b}$$

$$U \times (A \times B) \times U^\dagger = \hat{A}\hat{\times}\hat{B} = \hat{A} \times \hat{T} \times \hat{B}, \quad \hat{\psi} = U \times \psi, \tag{9.1.6c}$$

$$\left(\frac{1}{m}\hat{p} \times \hat{T} \times \hat{p} \times \hat{T} + \frac{e^2}{r} \times \hat{T} - \frac{z \times e^2}{r}\right) \times \hat{\psi}(r) = E_0 \times \hat{\psi}(r), \tag{9.1.6d}$$

$$\hat{p}\hat{\times}\hat{\psi}(r) = -i \times \hat{T} \times \nabla\hat{\psi}(r), \tag{9.1.6e}$$

where the factor $\hat{T}$ in the first Coulomb term originates from the nonunitary transform of model (9.1.5), while the same factor is absent in the second Coulomb term because the latter is long range, thus being conventional. As a result, in the model here considered the trigger is merely added to the equation.

The angular component of model (9.1.6) is conventional [3], and it is hereon ignored. For the radial component $r = |\mathbf{r}|$, we assume the isounit [7]

$$\hat{I} = e^{N \times \psi/\hat{\psi}} \approx 1 + N \times \psi/\hat{\psi}, \quad N = \int dr^3\, \hat{\psi}^\dagger(r)_{1\downarrow} \times \hat{\psi}(r)_{2\uparrow}, \tag{9.1.7a}$$

$$\hat{T} \approx 1 - N \times \psi/\hat{\psi}, \tag{9.1.7b}$$

$$|\hat{I}| \gg 1, \quad |\hat{T}| \ll 1, \tag{9.1.7c}$$

$$\lim_{r \gg 1\text{fm}} \hat{I} = 1, \tag{9.1.7d}$$

where one should note that Eqs. (9.1.7c) and (9.1.7d) are automatically verified by expressions (9.1.7a) and (9.1.7b).

Note that the explicit form of ψ is of Coulomb type, thus behaving like

$$\psi \approx N \times \exp(-b \times r), \tag{9.1.8}$$

with N approximately constant at distances near the hadronic horizon of radius

$$r_c = \frac{1}{b}, \tag{9.1.9}$$

while $\hat{\psi}$ behaves like

$$\hat{\psi} \approx M \times \left(1 - \frac{\exp(-b \times r)}{r}\right), \tag{9.1.10}$$

with M being also approximately constant under the same range [7a]. We then have

$$\hat{T} \approx 1 - \frac{V_{\text{Hulten}}}{r} = 1 - V_0 \frac{e^{-b \times r}}{(1 - e^{-b \times r})/r}, \tag{9.1.11}$$

namely, we see the appearance of a Hulten potential in this local approximation. But the Hulten potential behaves at short distances like the Coulomb one,

$$V_{\text{Hulten}} r \approx \frac{1}{b} \approx \frac{V_0}{b} \times \frac{1}{r}. \tag{9.1.12}$$

As a result, inside the hadronic horizon we can ignore the repulsive (or attractive) Coulomb forces altogether, and write

$$+\frac{e^2}{r} \times \hat{T} - \frac{e^2}{r} \approx +\frac{e^2}{r} \times \left(1 - \frac{V_{\text{Hulten}}}{r}\right) - \frac{z \times e^2}{r} = -V \times \frac{e^{-b \times r}}{1 - e^{-b \times r}}, \tag{9.1.13}$$

by therefore resulting in the desired overall *attractive* force among the identical electrons inside the hadronic horizon.

By assuming in first approximation $|\hat{T}| = \rho \approx 1$, the radial equation of model reduces to the model of π^0 meson [7a] or of the Cooper pair [7b, 7c], although with different values of V and b.

$$\left[\frac{1}{r^2}\left(\frac{d}{dr} r^2 \frac{d}{dr}\right) + \frac{m}{\rho^2 \times \hbar^2}\left(E_0 + V \times \frac{e^{-b \times r}}{1 - e^{-b \times r}}\right)\right] \times \hat{\psi}(r) = 0.. \tag{9.1.14}$$

The exact solution and related boundary conditions were first computed in Ref. [7a], Sect. 5, and remain fully applicable to the isoelectronium.

The resulting spectrum is the typical one of the Hulten potential,

$$|E_0| = \frac{\rho^2 \times \hbar^2 \times b^2}{4 \times m}\left(\frac{m \times V}{\rho^2 \times \hbar^2 \times b^2} \times \frac{1}{n} - n\right)^2, \tag{9.1.15}$$

which evidently possesses a *finite* spectrum, as well known.

To reach a numerical solution, we introduce the parametrization as in Ref. [7a],

$$k_1 = \frac{1}{\lambda \times b}, \tag{9.1.16a}$$

$$k_2 = \frac{m \times V}{\rho^2 \times \hbar^2 \times b^2}. \tag{9.1.16b}$$

We note again that, from boundary conditions, k_2 must be bigger than but close to one, $k_2 \approx 1$ [7].

We therefore assume in first nonrelativistic approximation that

$$\frac{m \times V}{\rho^2 \times \hbar^2 \times b^2} = 1. \tag{9.1.17}$$

By assuming that V is of the order of magnitude of the total energy of the isoelectrons at rest as in the preceding models [7],

$$V \approx 2 \times \hbar \times \omega \approx 2 \times 0.5\text{MeV} = 1\text{MeV}, \tag{9.1.18}$$

and by recalling that $\rho \approx 1$, we reach the following estimate for the *radius of the isoelectronium*

$$\begin{aligned} r_c = b^{-1} &\approx \left(\frac{\hbar^2}{m \times V}\right)^{1/2} = \left(\frac{\hbar}{m \times \omega_0}\right)^{1/2} = \\ &= \left(\frac{1.054 \times 10^{-27}\text{erg} \cdot \text{sec}}{1.82 \times 10^{-27}\ \text{g} \times 1.236 \times 10^{20}\ \text{Hz}}\right)^{1/2} = \\ &= 6.8432329 \times 10^{-11}\text{cm} = 0.015424288\ \text{bohrs} = 0.006843\ \text{Å}, \end{aligned} \tag{9.1.19}$$

It should be noted that: 1) the above values of r_c and V are only *upper boundary values* in the center-of-mass frame of the isoelectronium, i.e., the largest possible values under the assumptions of this section; 2) the values have been computed under the approximation of null relative kinetic energy of the isoelectrons with individual total energy equal to their rest energy; and 3) the values evidently *decrease* with the addition of the relative kinetic energy of the isoelectrons (because this implies the increase of m in the denominator).

The actual radius of the isoelectronium, when considered to be an quasiparticle as in this section, is also expected to vary with the trigger, that is, with the nuclear charges, as confirmed by the calculations presented in the next sections. This illustrates again the upper boundary character of value (9.1.19).

The value k_1 is then given by

$$k_1 = \frac{V}{2 \times k_2 \times b \times c_0} = 0.19, \quad k_2 \approx 1. \tag{9.1.20}$$

Intriguingly, the above two values for the isoelectronium are quite close to the corresponding values of the π^0 [7a] and of the Cooper pair [7b, 7c] (see also Sect. 1.9),

$$k_1 = 0.34, \quad k_2 = 1 + 8.54 \times 10^{-2}, \tag{9.1.21a}$$

$$k_1 = 1.3 \times \sqrt{z} \times 10^{-4}, \quad k_2 = 1.0 \times \sqrt{z}, \tag{9.1.21b}$$

It is important to see that, at this nonrelativistic approximation, *the binding energy of the isoelectronium is not only unique, but also identically null,*

$$|E_0| = \frac{\rho^2 \times \hbar^2 \times b^2}{4 \times m} \left(\frac{m \times V}{\rho^2 \times \hbar^2 \times b^2} - 1 \right)^2 = \frac{V}{4 \times k_2} \times (k_2 - 1)^2 = 0. \tag{9.1.22}$$

This result is crucial to prevent inconsistencies with Pauli's exclusion principle, which, as indicated earlier, *requires no potently energy between the two electrons for its interpretation in a way consistent with experimental data.*

The notion of *a bound state with only one allowed energy level* (called "hadronic suppression of the atomic spectrum" [7a]) is foreign to conventional quantum mechanics and chemistry, although it is of great importance for hadronic mechanics. In fact, any excitation of the constituents, whether the π^0, the Cooper pair or the isoelectronium, causes their exiting the hadronic horizon, by therefore re-acquiring the typical atomic spectrum. Each of the considered three hadronic states has, therefore, only one possible energy level.

The additional notion of *a bound state with null binding energy* is also foreign to quantum mechanics and chemistry, although it is another fundamental characteristic of hadronic mechanics and isochemistry. In fact, the hadronic interactions admit no potential energy, and as such, they cannot admit any appreciable binding energy, as typical for ordinary contact zero-range forces of our macroscopic Newtonian reality.

The null value of the binding energy can be confirmed from the expression of the meanlife of the isoelectronium, which can be written in this nonrelativistic approximation [7a]

$$\tau = \frac{\hbar}{4 \times \pi \times \hbar^2} |\hat{\psi}(0)| \times \alpha \times E_{\hat{e}}^{\text{Kin}} = 7.16 \times 10^4 \times \frac{k_1}{(k_2 - 1)^3 \times b \times c_0}. \tag{9.1.23}$$

The full stability of the isoelectronium, $\tau = \infty$, therefore, requires the *exact* value $k_2 \equiv 1$, which, in turn, implies $E_0 \equiv 0$.

The above derivation characterizes the *limiting assumption of a fully stable isoelectronium in nonrelativistic approximation.* By comparison, the Cooper pair under the same derivation *is not* permanently stable because its binding energy is very small, yet finite [7b], thus implying a large yet finite meanlife. Also by comparison, the π^0 *cannot* be stable, and actually has a very small meanlife, evidently because the constituents are a particle-antiparticle pair and, as such, they annihilate each other when bound at short distances.

Another important information of this section is that *the isoelectronium is sufficiently small in size to be treated as a single quasi-particle*. This property will permit rather important simplifications in the isochemical structure of molecules studied in the next sections.

By comparison, the Cooper pair has a size much bigger than that of the isoelectronium [7b, 7c]. This property is fundamental to prevent that the Cooper pair takes the role of the isoelectronium in molecular bonds, i.e., even though possessing the same constituents and similar physical origins, the isoelectronium and the Cooper pair are different, non-interchangeable, hadronic bound states.

The lack of binding energy of the isoelectronium is perhaps the most important information of this section. In fact, it transfers the representation of the binding energy of molecular bonds to the motion of the isoelectronium in a molecular structure, as studied in the next sections.

A novelty of isochemistry over quantum chemistry is that the mutual distance (charge diameter) between the two isoelectrons in the isoelectronium could, as a limited case, also be identically null, that is, the two isoelectrons could be superimposed in a singlet state. Rather than being far fetched, this limit case is intriguing because it yields the value $-2e$ for the charge of the isoelectronium, the null value of the relative kinetic energy, and an identically null magnetic field. This is a perfectly diamagnetic state, which evidently allows a better stability of the isochemical bond as compared to a quasi-particle with non-null size.

Note that, if conventionally treated (i.e., represented on conventional spaces over conventional fields), the nonunitary image of model (9.1.5) would yield *noninvariant numerical results* which, as such, are unacceptable (Sect. 1.7). This occurrence mandates the use of the covering isochemistry and related isomathematics which assures the achievement of invariant results.

Note also that the main physical idea of isounit (9.1.7) is the *representation of the overlapping of the wavepackets of the two electrons under the condition of recovering conventional quantum chemistry identically whenever such overlapping is no longer appreciable*. In fact, for sufficiently large relative distances, the volume integral of isounit (9.1.7a) is null, the exponential reduces to I, Eq. (9.1.7d), the nonunitary transform becomes conventionally unitary, and quantum chemistry is recovered identically.

It is also important to see that, under transform (9.1.7a), model (9.1.5) is implemented with interactions which are: *nonlinear*, due to the factor $\psi/\hat{\psi}$ in the exponent; *nonlocal*, because of the volume integral in (9.1.7a); and *nonpotential*, because not represented by a Hamiltonian.

We finally note that the explicit form of the isotopic element $\hat{T}$, Eq. (9.1.7b), emerges in a rather natural way as being *smaller than one* in absolute value, Eq. (9.1.7c), i.e.,

$$|\hat{T}| = |1 - N \times \psi/\hat{\psi}| \ll 1. \tag{9.1.24}$$

As pointed out in Chapter 3, this property alone is sufficient to *guarantee* that all slowly convergent series of quantum chemistry converge faster for isochemistry.

9.1.4 Isochemical Model of the Hydrogen Molecule with Stable Isoelectronium

We are now sufficiently equipped to initiate the study of the *isochemical model of the hydrogen molecule*, first submitted by Santilli and Shillady in Ref. [5] (see Figs. 9.3, 9.4 an d 9.5). In this Section we shall begin the study by identifying the equation of structure of the H-molecule under the limit assumption that the isoelectronium is perfectly stable at short distances, namely, that the two valence electrons are permanently trapped inside the hadronic horizon, resulting in the main features derived in the preceding section

$$\begin{aligned} \text{mass} \approx 1\text{MeV}, \quad \text{spin} = 0, \\ \text{charge} = 2 \times e, \quad \text{magnetic moment} \approx 0, \end{aligned} \tag{9.1.25a}$$

$$\begin{aligned} \text{radius} = r_c = b^{-1} = 6.8432329 \times 10^{-11}\text{cm} = \\ = 0.015424288 \text{ bohrs} = 0.006843 \text{ Å}. \end{aligned} \tag{9.1.25b}$$

The more realistic case when the isoelectronium is unstable at such small distances is studied later on in this chapter, where we shall also reach an essentially exact representation of the characteristics of the hydrogen molecule.

The main reason for assuming the isoelectronium to be stable at short distances with characteristics (9.1.25) is that such an approximation permits rather major structural simplifications, most notably, the transition, from the conventional hydrogen molecule (which is a *four-body system*), to the isochemical model of this section (which is a *three-body system*, Fig. 9.3). By recalling that four-body systems do not admit an exact solution, while restricted three-body systems do admit an exact analytic solution, the implications of the approximate model of this section are sufficient to warrant an inspection.

Our foundation is the conventional quantum model of H_2 molecule [1],

$$\begin{aligned} \left(\frac{1}{2\mu_1} p_1 \times p_1 + \frac{1}{2\mu_2} p_2 \times p_2 + \right. \\ \left. + \frac{e^2}{r_{12}} - \frac{e^2}{r_{1a}} - \frac{e^2}{r_{2a}} - \frac{e^2}{r_{1b}} - \frac{e^2}{r_{2b}} + \frac{e^2}{R} \right) \times |\psi\rangle = E \times |\psi\rangle. \end{aligned} \tag{9.1.26}$$

Our task is that of subjecting the above model to a transform

$$U \times U^\dagger|_{r \approx r_c} = \hat{I} = 1/\hat{T} \neq I, \tag{9.1.27}$$

which is nonunitary only at the short mutual distances

$$r_c = b^{-1} = r_{12} \approx 6.8 \times 10^{-11}\text{cm}, \tag{9.1.28}$$

and becomes unitary at bigger distances,

$$U \times U^{\dagger}|_{r \leq 10^{-10}\mathrm{cm}} \neq I, \quad I_{r \gg 10^{-10}\mathrm{cm}} = I. \tag{9.1.29}$$

This guarantees that our isochemical model coincides with the conventional model everywhere except for small contributions at small distances.

Assumption (9.1.29) also guarantees that *the conventional energy level of the individual hydrogen atoms are not altered*. In other words, assumption (9.1.29) realizes the main conception of this monograph, the exact character of quantum mechanics for the structure of *one* hydrogen atom, and its insufficiency for *two* hydrogen atoms bounded into the hydrogen molecule (Chapter 1).

The Hilbert space of systems (9.1.26) can be factorized in the familiar form (in which each term is duly symmetrized or antisymmetrized) as in Refs. [1]

$$|\psi\rangle = |\psi_{12}\rangle \times |\psi_{1a}\rangle \times |\psi_{1b}\rangle \times |\psi_{2a}\rangle \times |\psi_{2b}\rangle \times |\psi_{R}\rangle, \tag{9.1.30a}$$

$$\mathcal{H}_{\mathrm{Tot}} = \mathcal{H}_{12} \times \mathcal{H}_{1a} \times \mathcal{H}_{1b} \times \mathcal{H}_{2a} \times \mathcal{H}_{2b} \times \mathcal{H}_{R}. \tag{9.1.30b}$$

The nonunitary transform we are looking for shall act only on the r_{12} variable while leaving all others unchanged. The simplest possible solution is given by

$$U(r_{12}) \times U^{\dagger}(r_{12}) = \hat{I} = \exp\left[\frac{\psi(r_{12})}{\hat{\psi}(r_{12})} \int dr_{12} \hat{\psi}^{\dagger}(r_{12})_{1\downarrow} \times \hat{\psi}(r_{12})_{2\uparrow}\right], \tag{9.1.31}$$

where the ψ's represent conventional wavefunctions and the $\hat{\psi}$'s represent isowavefunctions.

As an alternative yielding the same results, one can transform short-range terms (isochemistry), and add un-transformed long-range terms (quantum chemistry), resulting in the radial equation

$$\left(-\frac{\hbar^2}{2 \times \mu_1} \hat{T} \times \nabla_1 \times \hat{T} \times \nabla_1 - \frac{\hbar^2}{2 \times \mu_2} \hat{T} \times \nabla_2 \times \hat{T} \times \nabla_2 + \right.$$
$$\left. + \frac{e^2}{r_{12}} - \frac{e^2}{r_{1a}} - \frac{e^2}{r_{2a}} - \frac{e^2}{r_{1b}} - \frac{e^2}{r_{2b}} + \frac{e^2}{R}\right) \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle. \tag{9.1.32}$$

By recalling that the Hulten potential behaves at small distances like the Coulomb one, Eq. (9.1.32) becomes

$$\left(-\frac{\hbar^2}{2 \times \mu_1} \times \nabla_1^2 - \frac{\hbar^2}{2 \times \mu_2} \times \nabla_2^2 - V \times \frac{e^{-r_{12} \times b}}{1 - e^{-r_{12} \times b}} - \right.$$
$$\left. - \frac{e^2}{r_{1a}} - \frac{e^2}{r_{2a}} - \frac{e^2}{r_{1b}} - \frac{e^2}{r_{2b}} + \frac{e^2}{R}\right) \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle. \tag{9.1.33}$$

The above equation does indeed achieve our objectives. In fact, it exhibits a *new explicitly attractive force between the neutral atoms of the hydrogen molecule, which force is absent in conventional quantum chemistry.* The equation also explains the reasons why the H_2 molecule admits only *two* H-atoms. As we shall see in the remaining sections, Eq. (9.1.33) also permits essentially exact representations of the binding energy and other molecular characteristics, yields much faster convergence of series with much reduced computer times, and resolves other insufficiencies of conventional models.

9.1.5 Exactly Solvable, Three-Body, Isochemical Model of the Hydrogen Molecule

Our isochemical model of the hydrogen molecule, Eqs. (9.1.33), can be subjected to an additional simplification, which is impossible for quantum chemistry. In our isotopic model, the two isoelectrons are bonded together into a single state we have called isoelectronium. In particular, the charge radius of the latter is sufficiently small to permit the values (see Fig. 9.3)

$$r_{12} \leq r_{1a}, \text{ and } r_{1b}, \quad r_{12} \approx 0, \tag{9.1.34a}$$

$$r_{1a} \approx r_{2a} = r_a, \quad r_{1b} \approx r_{2b} = r_b. \tag{9.1.34b}$$

Moreover, the H-nuclei are about 2,000 times heavier than the isoelectronium. Therefore, our model (9.1.33) can be reduced to a *restricted three body problem* similar to that possible for the conventional H_2^+ *ion* [1], but *not* for the conventional H_2 molecule.

Such a restricted model essentially consists of two H-protons at rest at a fixed mutual distance plus the isoelectronium moving around them in the *oo*-shaped orbit of Fig. 9.4, according to the structural equation

$$\left(-\frac{\hbar^2}{2\mu_1}\times\nabla_1^2-\frac{\hbar^2}{2\mu_2}\times\nabla_1^2-V\times\frac{e^{-r_{12}b}}{1-e^{-r_{12}b}}-\right.$$

$$\left.-\frac{2e^2}{r_a}-\frac{2e^2}{r_b}+\frac{e^2}{R}\right)\times|\hat{\psi}\rangle=E\times|\hat{\psi}\rangle. \tag{9.1.35}$$

Under the latter approximation, the model permits, for the first time, the achievement of an *exacts solution for the structure of the H_2 molecule*, as it is the case for the H_2^+ ion or for all restricted three-body problems. This solution will be studied in Chapter 6 via variational methods. The exact analytic solution has not been studied at this writing, and its study is here solicited by interested colleagues. At this introductory level we only limit ourselves to a few comments.

Note that *the above exact solution of the hydrogen molecule is only possible for the case of the isoelectronium fully stable at short mutual distances.* In fact,

for the case of the mutual distance of the valence electrons no longer restricted to 1 fm, the model is a full *four-body structure*, which, as such, admits no exact solution.

Note also that model (9.1.35) is the isochemical model of the H_2 molecule inside the hadronic horizon. The matching representation *outside* the hadronic horizon is presented in the next section.

Note also that the above restricted three-body model can be used for the study of the bonding of an H-atom to another generic atom, such as HO, thus permitting, again for the first time, novel exact calculations on the water as HOH, namely, as two intersecting isotopic bonds HO and OH, each admitting an exact solution, with possible extension to molecular chains, and evident extensions to other molecules.

Readers interested in studying model (9.1.35) should keep in mind that *the rest energy of the isoelectronium is unknown at this writing, thus being a free parameter suitable for fitting experimental data*. More specifically, in Eq. (9.1.35) we have *assumed* from Sect. 9.3 that

$$m_{\text{isoelectronium}} = 2 \times m_{\text{electron}}. \tag{9.1.36}$$

However, the results of Sect. 9.3 are approximate. In particular, they hold under *the assumption that the isoelectronium has no internal binding energy*. Such an assumption was made for the specific purpose of proving that nonpotential forces represented with the isounit can indeed yield a bound state. In particular, the assumption was suggested by the need to represent Pauli's exclusion principle without the introduction of a potential.

However, such a view may be solely valid at molecular distances of valence electrons, and not necessarily at short distances. As a result, the isoelectronium may indeed have an internal binding energy, that is, it can have internal forces derivable from a potential in addition to the nonpotential forces without binding energy of hadronic chemistry, as outlined in Fig. 9.4.

This is due to the fact that the structure of the isoelectronium implies *three* acting forces: *one repulsive* Coulomb force due to the same changes, plus *two attractive* forces due to the two pairs of opposite magnetic polarities in singlet configuration. The latter two attractive forces may overcome the repulsion due to the change beginning at distances of the order of one Fermi, resulting in a conceivable net attractive force derivable from a potential.

Under the latter conditions, the isoelectronium would indeed have a *negative* binding energy, resulting in the unknown value

$$m_{\text{isoelectronium}} < 2 \times m_{\text{electron}}. \tag{9.1.37}$$

The understanding is that the case $m_{\text{isoelectronium}} > 2 \times m_{\text{electron}}$ is impossible.

The unknown character of the isoelectronium mass alters considerably the perspective of restricted model (9.1.35). As we shall see in Chapter 6, it is possible

to prove via variational and other methods that *model* (9.1.35) *under assumption* (9.1.36) *does not admit exact solutions accurately representing the binding energy of the hydrogen molecule.* However, under the use of the isoelectronium mass free for fitting experimental data, the situation may be different.

Another information which should not be assumed to be exact is the *size of the isoelectronium*, Eq. (9.1.19). In fact, as stressed in Sect. 9.3, such a value too must be assumed to be an *upper boundary value*. In model (9.1.35) the isoelectronium is assumed to be point-like. However, the model can be first extended via Eq. (9.1.35) for a stable isoelectronium with a *fixed* unknown radius

$$r_c = b^{-1} \leq 6.8 \times 10^{-11}\text{cm}. \tag{9.1.38}$$

A second extension of model (9.1.35) should also be taken into consideration, that in which

$$r_c = b^{-1} \geq 6.8 \times 10^{-11}\text{cm}, \tag{9.1.39}$$

because, as stressed in Sects. 9.2 and 9.3, any assumption that the isoelectronium ceases to exist at distances bigger than 10^{-11} cm would imply a violation of Pauli's exclusion principle.

As a matter of fact, the assumed mass (9.1.36) is more in line with assumption (9.1.39), than with assumption (9.1.38), again, to prevent the existence at large mutual distances of the valence electrons of attractive internal potential forces with a binding energy which would alter conventional atomic structures.

Even though, admittedly, *the size of the isoelectronium is variable in the physical reality*, its average into a constant value may have meaning, of course, as a first approximation.

A third quantity of model (9.1.35) deserving inspection is the experimental value of the bond length, which is generally referred to the distance between the two nuclei R. In principle, such a distance is expected to be altered by a fully stable isoelectronium. Therefore, a solution of model (9.1.35) in which R is fitted from the experimental data is indeed meaningful, of course, as a first approximation.

In conclusion, in both, the four-body model (9.1.33) and the restricted three-body model (9.1.35), we have *three quantities which, in principle, can be assumed to be unknown and, therefore, should be derived from the fit of experimental data: 1) the mass of the isoelectronium; 2) the size of the isoelectronium; and 3) the bond length.*

There is no doubt that an exact analytic solution of model (9.35) suitable to represent the binding energy of the hydrogen is permitted by the above three free fits with intriguing implications for all H-bonds whose study is left to interested researchers.

9.1.6 Isochemical Model of the Hydrogen Molecule with Unstable Isoelectronium

In this section we review the study of Ref. [5] on the solution of the restricted isochemical model of the hydrogen molecule, Eq. (9.1.35) and Fig. 9.3, via conventional variational methods used in chemistry, under the assumption that the isoelectronium has characteristics (9.3.21). As we shall see, these studies have achieved an essentially exact representation of experimental data on the hydrogen molecule, including its binding energy and bond length, for the first time from exact first principles without ad hoc adulterations.

For historical papers in chemistry connected to our model, see Refs. [6]. Representative, more recent papers with technical connections to our study as outlined below are given by Refs. [8 – 38].

The possibilities that the mass of the isoelectronium be smaller than 2×mass of electron and its radius be bigger than 6.8×10^{-11} cm will not be considered in this section.

For this purpose we first note that the solution of the full model with the Hulten potential $e^{-rb}/(1-e^{-rb})$ where $r_c = b^{-1}$ is the size of the isoelectronium, implies rather considerable technical difficulties. Therefore, we shall study model (9.1.35) under an *approximation* of the Hulten potential given by one Gaussian of the type

$$\frac{e^{-rb}}{1-e^{-rb}} \approx \frac{1-Ae^{-br}}{r}, \tag{9.1.40}$$

with A a constant identified below.

It is known that a linear combination of sufficient number of Gaussians can approximate any function. Therefore, the achievement of an essentially exact representation of molecular data via approximation (9.1.40) will evidently persist under the full use of the Hulten potential.

Recall from Sect. 9.3 that the *stable* character of the isoelectronium is crucially dependent on the use of the attractive Hulten potential, which "absorbs" repulsive Coulomb forces at short distances resulting in attraction. Therefore, the weakening of the Hulten potential into the above Gaussian form has the direct consequence of turning the isoelectronium into an *unstable* state.

In this and in the following sections, we shall therefore study an isochemical model of the hydrogen molecule which is somewhat intermediary between the conventional chemical model and the isochemical model with a fully stable isoelectronium.

It should be indicated that the terms "unstable isoelectronium" should be referred as the period of time in which the two valence electrons remain within the hadronic horizon of 6.8×10^{-11} cm. The same terms *should not* be interpreted to the fact that the isoelectronium does not exist outside the hadronic horizon, because the latter view implies a number of inconsistencies, such as possible

violation of Pauli's exclusion principle, acquisition by molecules of ferromagnetic character, *etc.*

The main objective of this section is to show the achievement of the exact representation of molecular characteristics even for the case of one Gaussian approximation (9.1.40). The question whether the isoelectronium is stable or unstable evidently depends on the amount of instability and its confrontation with experimental data, e.g., on magnetic susceptibility. As such, the issue will be addressed theoretically and experimentally in a future paper.

Under the above assumption, our first step is the study of model (9.1.35) in an exemplified Coulomb form characterized by the following equation, hereon expressed in atomic units (a.u.)

$$H \times \Psi = \left(-\frac{1}{2}\nabla^2 - \frac{2}{r_a} - \frac{2}{r_b} + \frac{1}{R} \right) \times \Psi, \tag{9.1.41}$$

where the differences from the corresponding equation for the H_2^+ ion [1] are the replacement of the reduced mass $\mu = 1$ with $\mu = 2$, and the increase in the electric charge from $e = 1$ to $e = 2$.

The standard method for solving the above equation is the following. The variational calculation is set up in matrix algebra form in a nonorthogonal basis set S which has been normalized to 1. The metric of this non-orthogonal system of equations S is used to set up the orthogonal eigenvalue problem and the eigenvalues are sorted to find the lowest value. H and S are Hermitean matrices. E is a diagonal matrix with the energy eigenvalues

$$HC = ESC; \text{ define } C = S^{-\frac{1}{2}}C', \text{ then } HS^{-\frac{1}{2}}C' = ES^{-\frac{1}{2}}C', \tag{9.1.42a}$$

$$(S^{-\frac{1}{2}}HS^{+\frac{1}{2}}C') = H'C' = E(S^{-\frac{1}{2}}SS^{-\frac{1}{2}})C' = EC', \tag{9.1.42b}$$

where the last equation is obtained by multiplying the first equation from the left by $S^{-\frac{1}{2}}$, and use the unitary property that $S^{-\frac{1}{2}} = S^{+\frac{1}{2}}$ to form an orthogonal eigenvalue problem. Finally we solve for C' by diagonalizing H' and obtain $C = S^{-\frac{1}{2}}C'$.

Here the basis is formed from contracted basis sets Φ, which are fixed linear combinations of Gaussian spheres χ fitted to real shapes of spherical harmonic functions. The eigenvector column in C gives the basis coefficients of the molecular orbitals according to the expression

$$\Psi_i = \sum_{c_{i,j}} (j : \Psi_j = \sum_{a_{j,k}} \Psi_k; \ \chi = \left(\frac{2\alpha}{\pi} \right)^{3/4} \exp[-(\alpha - A)^2] = /\alpha, A). \tag{9.1.43}$$

The problem of how to form a sharp cusp on a 1s orbital is solved to a practical extent by using up to six Gaussians; here we use the very best "least-energy" 1s

orbital from Pople's group [18]. In this problem the s-, p-, d- and f-orbitals are polarization functions that merely serve to evaluate the effect of other angular components on the $1s$ orbitals which are the main terms of the $1s$-sigma bond in H_2.

Gaussian orbitals can easily be scaled to screened nuclear charge values by multiplying the Gaussian exponents by the square of the scaling factor (in effect, shrinking the space of the H-atom model) followed by renormalization of the linear combination of Gaussians. In this work the scaling constant of the $1s$ orbitals was optimized to 1.191 and the 2-, 3- and 4-shell orbitals optimized as scaled shells rather than optimizing each orbital individually.

As the orbitals were optimized using parabolic fitting to three energy values as a function of the scaling value, it became apparent that the bond length of the three-body model is much shorter than the usual value of 1.4011 Bohr (= 0.74143 Angströms). Thus, the bond length was re-optimized after optimization of the scaling for each principle shell. The scaling constants and the orbital contractions are Angströms at an energy of −7.61509174 Hartrees (= −207.2051232 eV) where the achievement of an exact representation of the binding energy is studied in detail.

Although a large basis set of $1s$, $2s$, $2p$, $3s$, $3p$, $3d$, $4s$, $4p$, and $4f$ orbitals was used, this variational energy is probably higher than the exact solution of the type used by Bates, Ledsham and Stewart [12]. However, the energy of the 6-gaussian ($6G$) Least-Energy $1s$ function [18] is −0.499946 Hartrees for the H atom so the energy quoted here should be within 0.001 Hartrees of the exact solution.

While it is expected that a collapsed isoelectronium pair would be even more unstable than a collapsed positronium quasi-particle due to the repulsive interaction of the electrons, this three-body model of H_2 predicts over 6 Hartrees added molecular stability and a substantial decrease in bond length. The $E(1)$ value of the electronium-pair of some −11.473164 Hartrees is lower than the total energy of the molecule due to the repulsion of the proton-nuclei from the $1/R$ term of the Hamiltonian.

9.1.7 Gaussian Approximation of the Isochemical Model of the Hydrogen Molecule as a Four-Body System

As indicated earlier, it is possible that the valence electrons bond themselves into the isoelectronium not in a permanent fashion, but rather in a statistical fashion, with only a percentage of their time in a bonded state, in which case the restricted three-body model is evidently insufficient. In this section we review the studies of Ref. [5] on the full four-body isochemical model of H_2, which model also permits the achievement of an exact representation of the binding energy from first principles without adulterations (see Fig. 9.7).

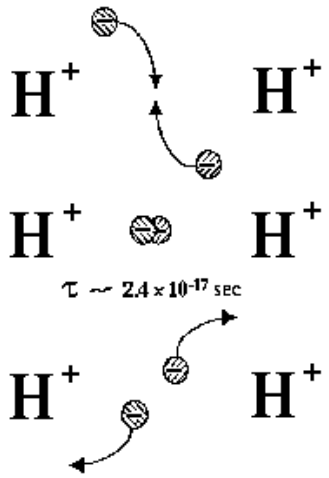

Figure 9.7. A schematic view of the *isochemical model of the hydrogen molecule with unstable isoelectronium* due to the weakening of the Hulten potential and consequential relaxation of the infinite lifetime, as a view complementary to that of Fig. 9.4 with stable isoelectronium. In this case the notion of isoelectronium essentially represents a tendency of pairs of valence electrons to correlate-bond in singlet states at short distances. The use of isochemistry, rather than conventional chemistry, is necessary because even this weaker form of the isoelectronium, as well as all screenings of the Coulomb potential at large, are nonunitary images of conventional Coulomb settings, as established in Chapter 1. Therefore, all these models require a representation on isospaces over isofields for their invariant formulation. In the text, we present the projection of such an invariant formulation on conventional spaces over conventional fields for simplicity.

A considerable effort has been made since the time of Hylleraas [20] in the 1930's to find a way to calculate the last 2% of the binding energy of molecules. Boys [22] derived a form of "configuration interaction" which offered exact variational solutions, but this proved to be very slowly convergent and only applicable to small molecules. Moller-Plesset perturbation developed by Pople *et. al.* [23] is popular today, but studies up to eighth order have been shown not to converge after huge expense in computer time. Linked-cluster diagrams by Bartlett *et. al.* [24]) multiconfiguration-self-consistent-field (MCSCF) calculations by Schaefer *et. al.* [25] and Goldstone-Bruekner-Feynman diagrammatic perturbation by Kelly [26] have all been shown to require very large computer resources, are limited to small molecules and sometimes fail to give even negative binding energies as shown by Goddard [27] for Cr_2.

All these slow and expensive methods seem to share one common feature, the use of high energy empty "virtual" orbitals from a ground state calculation, usually of Hartree-Fock-Roothaan type, to improve the representation of the ground state.

One might ask how it is possible to lower the energy by using higher energy wavefunctions. The fact that some energy lowering is found suggests electron dynamics is indeed complicated, and the rate of convergence of this method is quite slow.

The method adopted here is to use the usual Hartree-Fock-Roothaan self-consistent-field equations [1] (which also has some formal flaws such as the self-interaction terms [27]), and question the form of the Coulomb interaction of the electron.

Note that reducing the values of the Coulomb integrals will lower the energy by reducing the electron-electron repulsion while reducing the exchange terms will raise the energy, but the 1/2 factor reduces the effect of the exchange terms. Thus, a reduction of the value of the integrals will lower the energy. Note that Goddard [27] has already recommended reducing the atomic self-energy by subtracting 1.39 eV from Hartree-Fock exchange integrals in the cases of Cr_2 and Mo_2. In this way, we reach the expressions

$$FC = ESC; \quad F_{i,j} = H_{i,j} + \sum_{k,l} P_{k,l}[(i,j|k,l) - \frac{1}{2}(i,k|j,l)], \tag{9.1.44a}$$

$$(i,j|k,l) = \iint \chi_i(1)\chi_j(1)\frac{1}{r_{12}}\chi_k(2)\chi_l(2)\, d\tau_1 d\tau_2, \tag{9.1.44b}$$

$$P_{i,j} = 2\sum_n c_{n,i}c_{n,j} \quad (\text{sum } n \text{ only over occupied orbitals}). \tag{9.1.44c}$$

The 1995 paper on Cooper pairs by Santilli and Animalu [7c] invokes the non-local hadronic attractive force first identified in the π^0-meson [7a] as applied to singlet-paired electrons which form a boson particle. After using a non-local isotopic nonlinear transformation, the hadronic attraction was transformed back to real-space and modeled resulting into an attractive force which overcomes the repulsive Coulomb force. In turn, the latter occurrence constitutes the physical-chemical origin in the use of a suitably screened Coulomb potential for the binding energy.

Examination of the original 1978 paper on positronium collapse by Santilli [7a], reveals that the Hulten potential is not necessarily a unique representation of the hadronic force; since a linear combination of similar potentials could be used to represent the same hadronic bound state, provided that they characterize an attractive force among the electrons capable of overcoming their Coulomb repulsion.

This work assumes that until matrix elements of a two-Gaussian-screened-Coulomb potential can be used to approximate the real-space form of the hadronic attraction. This form has the important property that it can be merged with the general case of the four-center Coulomb or exchange integral derived by Shavitt

[16] using the famous Gaussian transform technique.

$$\frac{1}{r_{12}} = \sqrt{\frac{1}{\pi}} \int_0^\infty s^{-\frac{1}{2}} \exp[-s r_{12}^2] ds, \tag{9.1.45a}$$

$$1 + \frac{p+q}{pq} s = \frac{1}{1-t^2}. \tag{9.1.45b}$$

For future reference, note that this transform already has a pole at the lower limit where $s = 0$. This pole was removed at the last step by a change in variable, given as Eqs. (9.1.45). Shavitt was able (as a former graduate student of S.F. Boys) to show that the Gaussian transform technique reproduced the formula previously derived by Boys [6] in 1950 using electrostatic arguments. The Gaussian-lobe basis SCF programs by Shillady [8, 28] and others.

It is important to note that the formula is completely general in orientation of four Gaussian sphere lobe-orbitals as well as the distance between two electrons. As modified for description of correlation of two electrons, *such a general formula can describe angular correlation as well as distance interaction*. Thus, matrix Coulomb repulsion to model the real-space form of the hadronic attraction of two electrons.

Well-founded admiration for Shavitt's work in deriving the Coulomb interaction was rekindled as his derivation was checked. This work added the Gaussian screening as $\exp[-r^2]/r$ so that the special properties of Gaussians could be used, especially the property that polar coordinates readily separate into factorable x, y, z components. The goal is to evaluate the two-electron four-center matrix elements of the Gaussian-screened-Coulomb potential as shown below,

$$Y(r) = \frac{1 - 2\exp[-\alpha r^2]}{r}. \tag{9.1.46}$$

Intriguingly, the Gaussian exponent carried through the original derivation of the Coulomb interaction by resorting to a well known auxiliary function F_0 which has been studied by Shillady [8, 28] and others. Since both $s^{\frac{1}{2}}$ and $(s+\alpha)^{\frac{1}{2}}$ occur in the denominator of the screened-Coulomb form, two poles occur in the integral. A change of variable absorbs the term

$$1 + \frac{p+q}{pq}(s+\alpha) = \frac{1}{1-t^2}, \tag{9.1.47}$$

while the pole due to $(s+\alpha)^{-1/2}$ shifts the other pole at $s^{-\frac{1}{2}}$ to the lower limit of the integral. A smooth spike is evident at the lower value of the integration using a 70 point Simpson's Rule Integration (two ranges are used with 20 points more closely spaced near the pole and 50 points for the remaining range.)

The above work was carried out using 64 bit double precision arithmetic which provides 14 significant figures. A simple offset (δ) of 1.0×10^{-15} has provided useful results with this simple offset to avoid numerical overflow. While this pole is a problem in need of a continuous function to integrate, numerical integration seems to handle this well to 14 significant figures, particularly since the routines used for the Coulomb integrals are known to be accurate only to 12 significant figures [28].

The area under the pole-spike is estimated as a narrow triangle upon a rectangle 1.0×10^{-15} wide with the height of the triangle set at 1.79940×10^{13} times the height of the point set 1.0×10^{-15} into the range of integration (the first Simpson point).

The present code for this screened-Coulomb integral is presently slower than the corresponding function used for the Coulomb integrals due to the 70 point Simpson integration [28], but the integrand is nearly flat after the spike at $s = 0.0$ so that portion of the integrand can be evaluated more rapidly with fewer points. The simple offset of the lower limit by 1.0×10^{-15} is adequate for this monograph.

$$\left(aA(1), bB(1) \left| \frac{\exp(-\alpha(r_{12})^2)}{r_{12}} \right| cC(2), dD(2) \right) = \tag{9.1.48a}$$

$$= \frac{2\pi^{5/2}}{pq\sqrt{p+q}} e^{[ab/(a+b)]\overline{AB}^2 - [cd/(c+d)]\overline{CD}^2} \times$$

$$\times \int_{\rho}^{1} e^{-[pq/(p+q)]\overline{PQ}^2 t^2} \left(\frac{[pq/(p+q)]t^2}{[pq/(p+q)]t^2 + \alpha(t^2-1)} \right)^{1/2} dt, \tag{9.1.48b}$$

$$\rho = \delta + \frac{\alpha\sqrt{p+q}}{pq + (p+q)\alpha}, \quad p = (a+b), \quad q = c+d, \quad \delta = 1.0 \times 10^{-15}, \tag{9.1.48c}$$

$$\text{pole} = (1.79940 \times 10^{13}) e^{-[pq/(p+q)]\overline{PQ}^2 \rho^2}. \tag{9.1.48d}$$

The new integral was incorporated into the same routine used to evaluate the usual Hartree-Fock-Roothaan SCF scheme except F_0 was supplemented by the new auxiliary function (9.1.46). The H_2 molecule was treated using the same fixed-nuclei method with a bond distance of 1.4011 Bohrs. A simple basis set of just one Least-Energy $6G$-$1s$ orbital [18] centered on each H-nucleus was used to test the new program "Santilli-Animalu-Shillady-Lobe" (SASLOBE), which is set to handle up to 512 contracted orbitals.

It must be stated that the energies given are now parametrically dependent on the Gaussian-screening constant as $E(\alpha)$. The energy is variationally bound to be above the true energy in a narrow range around the optimum value. It is extremely important to note that the energy is lowered using the new attractive

hadronic term, but the optimum value is difficult to locate and "variational collapse" occurs when r_c is extended or reduced away from a shallow minimum in the energy.

In order to minimize the number of parameters in the model (only one, the Coulomb screening constant A) two equations were imposed on the Gaussian-function. First, the function was required to be equal to zero at some radial cutoff value r_c which is assumed to be the inverse of the b-variable of Sect. 9.1.5,

$$b = \frac{1}{r_c} = A\frac{\exp[-\alpha r_c^2]}{r_c}, \quad A = \exp[+\alpha r_c^2]. \tag{9.1.49}$$

Second, this radial cutoff value was used as sigma of the inverted Gaussian (radius at half-height),

$$A\exp\left[-\frac{\ln 2}{r_c^2}r_c^2\right] = \frac{A}{2}, \quad \alpha = \frac{\ln 2}{r_c}, \quad A = 2. \tag{9.1.50}$$

The upper boundary of the radius of the isoelectronium has been estimated in Sect. 9.3 to be about 0.6843×10^{-10} cm, which corresponds to 0.012931401 Bohrs. This radius does lower the Hartree-Fock-Roothaan energy noticeably for H_2, and further optimization of the pole-spike produced an SCF energy of -1.17446875 Hartrees with a cutoff radius of 0.0118447 Bohrs or 1.18447×10^{-10} cm using the minimum $1s$ basis. In conclusion, the fitted value of $b \equiv 1/r_c$ is reasonably close to the estimate value for the H_2 molecule. The minimum basis was later extended to $6G$-$1s$, $1G$-$2s$, $1G$-$2p$ for pole calibration.

Details of the exact representation of the binding energy via the above second method are presented in Appendix 9.B.

9.1.8 Summary of the Results

In order to demonstrate the advantage of the isochemical model using a Gaussian-screened-Coulomb attraction between electrons, a standard Boys-Reeves [22] calculation was carried out in Ref. [2]. This included all single- and double-excitations CISD from the ground state Hartree-Fock-Roothaan SCF orbitals for a 99×99 "codetor" [6] interaction. Only the 1s orbitals were optimized with a scaling of 1.191 for the Least-Energy $6G$-$1s$ orbitals, but the basis also included $1G$-$2s$, $2G$-$2p$, $1G$-$3s$, $1G$-$3p$, $3G$-$3d$, Thd $1G$-$4sp$ (tetrahedral array of four Gaussian spheres), and $4G$-$4f$ orbitals scaled to hydrogenic values as previously optimized [17].

The additional basis functions provide opportunity to excite electrons to higher orbitals as is the standard technique in configuration interaction, somewhat contrary to the main hypothesis of this work, which is that there is an attractive hadronic force between electron pairs inside the r_c critical radius. The results of the above calculations are summarized in Table 9.1.

Table 9.1. Summary of results for the hydrogen molecule.

Species	H_2	H_2 [a]	H_2
Basis screening			
$1s$	1.191	6.103	1.191
$2s$	0.50	24.35	0.50
$2p$	0.50	24.35	2.36
$3s$	0.34	16.23	*
$3p$	0.34	16.23	*
$3d$	0.34	-16.2[b]	*
$4sp$	0.25	12.18	*
$4f$	0.25	12.18	*
Variational energy (a.u.)	*	-7.61509174	*
SCF energy (a.u.)	-1.12822497	*	-1.13291228
CI energy (a.u.)	-1.14231305	*	*
CINO energy (a.u.)	-1.14241312	*	*
SAS energy (a.u.)	*	*	-1.174444
Exact energy (a.u.) [30]	-1.174474	*	-1.174474
Bond length (bohr)	1.4011	0.2592	1.4011
Isoelectronium radius (bohr)	*	*	0.01124995

[a] Three-body Hamiltonian (5.1).
[b] The negative $3d$ scaling indicates five equivalent three-sphere scaled to 16.20 rather than "canonical" $3d$ shapes.

The Boys-Reeves C.I. achieved an energy of -1.14241305 Hartrees based on an SCF energy of -1.12822497 Hartrees. This was followed by one additional iteration of "natural orbitals" (CINO), in which the first order density matrix is diagonalized to improve the electron pairing to first order [29]. The fact that this procedure lowered the energy only slightly to -1.14241312 Hartrees (i.e., -7.0×10^{-7} Hartrees), indicates the 99-configuration representation is close to the lower energy bound using this basis set while the isochemistry calculation produced the exact energy with a comparatively much smaller basis set.

Since SASLOBE has only a n^7 routine for the necessary integral transformation instead of the most efficient n^5 algorithm ($\simeq n$ is the number of basis functions), the SASLOBE C.I. runs are somewhat slow and required about 20 hours on a 300 MFLOPS Silicon Graphics computer.

With more efficient routines, this time can be reduced to about three hours. However, the screened-Coulomb attraction method used a smaller basis and achieved lower energies in a few seconds. It is also estimated that careful spacing of fewer quadrature points in the new integral routine can certainly reduce the SASLOBE run times by a factor of 2 at least.

Therefore it is clear that calculations in hadronic chemistry are, conservatively, at least 1,000 times faster than a C.I. calculation, an occurrence fully similar to the corresponding case in hadronic *vs.* quantum mechanics.

Another estimate is that, since the new integral corrections require a little more time than the usual Coulomb integrals (but do not take any additional storage space), the computer run-times for an isochemistry calculation should only be about three times the run-times for the corresponding Hartree-Fock-Roothaan calculation in any given basis set.

The extension of the isochemical model of the H_2 molecule to other molecules is conceptually straightforward. In particular, the notion of isoelectronium essentially restricts all possible bonds to the established ones, as it is the case for the water molecule (see next chapter).

In order to generalize the underlying quantitative treatment to molecules containing H–to–F, the pole-spike was re-optimized to obtain 100% of the correlation energy below the SCF energy in the given basis set since the SCF energy here was not quite at the Hartree-Fock limit.

Table 9.2. Isoelectronium results for selected molecules.

Species	H_2	H_2O	HF
SCF-energy (DH) (a.u.)	-1.132800^a	-76.051524	-100.057186
Hartree-Fock[d] (a.u.)			-100.07185^d
Iso-energy (a.u.)	-1.174441^c	-76.398229^c	-100.459500^c
Horizon R_c (Å)	0.00671	0.00038	0.00030
QMC energy[d,e] (a.u.)	-1.17447	-76.430020^e	-100.44296^d
Exact non-rel. (a.u.)	-1.174474^f		-100.4595^d
Corellation (%)	99.9^b	91.6^b	103.8
SCF-dipole (D)	0.0	1.996828	1.946698
Iso-dipole (D)	0.0	1.847437	1.841378
Exp. dipole (D)	0.0	1.85^g	1.82^g
Time[h] (min:s)	0:15.49	10:08.31	6:28.48

(DH^+) Dunning-Huzinaga (10S/6P), [6,2,1,1,1/4,1,1]+H_2P_1+3D1.
[a]LEAO-6G1S + optimized GLO-2S and GLO-2P.
[b]Relative to the basis set used here, not quite HF-limit.
[c]Iso-energy calibrated to give exact energy for HF.
[d]Hartree-Fock and QMC energies from Luchow and Anderson [33].
[e]QMC energies from Hammond *et al.* [30].
[f]First 7 sig. fig. from Kolos and Wolniewicz [34].
[g]Data from Chemical Rubber Handbook, 61st ed., p. E60.
[h]Run times on an O2 Silicon Graphics workstation (100 MFLOPS max.).

The energy obtained here results from the calibration of the pole-spike to the experimental value of HF, and is below the Quantum Monte Carlo (QMC) energy

of Luchow and Anderson [33], which requires hours on a much larger computer, as compared to less than 10 minutes for this work. In fact, the run times for HF were about 8 CPU minutes on a 100 MFLOP Silicon Graphics O2 workstation.

The principal value of the pole (9.1.48d) was calibrated for 100% energy of HF, H_2O has two tight sigma bonds and two diffuse lone-pairs so a single compromise value is a good test of the method. In HF the F^- is nearly spherical so an average r_c value does a better job of describing the "correlation hole" of transient isoelectronium. The computed dipole moments are in excellent agreement with the experimental values. The use of the same pole value for H_2O and HF degrades the H_2 energy slightly. The results of our studies for H–to–F based molecules are summarized in Table 9.2.

A comparison of the above date (particularly those on computer times) with corresponding data obtained via conventional approaches is instructive.

9.1.9 Concluding Remarks

The fundamental notion of the new model of molecular bonds studied in this chapter [5] is the bonding at short distances of pairs of valence electrons from two different atoms into a singlet quasi-particle state we have called *isoelectronium*, which travels as an individual particle on an oo-shaped orbit around the two respective nuclei.

The isoelectronium and related methodology are then characterized by a covering of contemporary chemistry called *isochemistry*, which is the branch of the more general *hadronic chemistry* specifically constructed to represent closed-isolated systems with linear and nonlinear, local and nonlocal, and potential as well as nonpotential internal forces.

A main assumption is that linear, local, and potential interactions are sufficient for atomic structures since the atomic distances are much bigger than the size of the wavepackets of the electrons. However, in the transition to molecular structures we have the additional presence of nonlinear, nonlocal, and nonpotential effects due to the deep penetration of the wavepackets of valence electrons, which is essentially absent in atomic structures (Fig. 1.7).

The attractive short-range interactions needed to overcome the repulsive Coulomb force in the isoelectronium structure originate precisely from nonlinear, nonlocal, and nonhamiltonian effects in deep wave-overlappings; they are described by *hadronic mechanics* [3b]; and their invariant formulation is permitted by the recently achieved broadening of conventional mathematics called *isomathematics*.

Specific experimental studies are needed to confirm the existence of the isoelectronium, by keeping in mind that the state may not be stable outside a molecule in which the nuclear attraction terms bring the electron density to some critical threshold for binding, a feature we have called the "*trigger*."

Nonrelativistic studies yield a radius of the isoelectronium of 0.69×10^{-10} cm. This "horizon" is particularly important for isochemical applications and developments because outside the horizon the electrons repel one-another while inside the horizon there is a hadronic attraction.

The same nonrelativistic studies also predict that, as a limit case, the isoelectronium is stable within a molecule, although partially stable configurations also yield acceptable results. The question of the stability vs. instability of the isoelectronium inside the hadronic horizon must therefore also be left to experimental resolutions.

The understanding is that, when the restriction to the hadronic horizon is lifted, and molecular dimensions are admitted for the inter-electron distance, the isoelectronium must be stable, otherwise violations of Pauli's exclusion principle could occur. In this sense, the isoelectronium is a direct representation of Pauli's exclusion principle.

The foundations of the isoelectronium can be seen in a paper by Santilli [7a] of 1978 on the structure of the π^0-meson as a bound state of one electron and one positron. The latter model also illustrates the capability of hadronic mechanics vs. quantum mechanics. In fact, quantum mechanics *cannot* represent the π^0 as the indicated bound state of one electron and one positron because of numerous inconsistencies, such as: the inability to represent the rest energy of the π^0, which would require a "positive" binding energy, since the sum of the rest energies of the constituents is much smaller than the rest energy of the bound state; the impossibility to represent the charge radius of the π^0, which can only be that of the positronium for quantum mechanics; the lack of representation of the meanlife of the π^0; and other insufficiencies.

By comparison, *all* the above insufficiencies are resolved by hadronic mechanics, which permits the first quantitative, numerical representation of *all* characteristics of the π^0 as a bound state of one electron and one positron at short distances, including its spontaneous decay with the lowest mode $\pi^0 \to e^- + e^+$, which results in being the hadronic tunnel effect of the constituents [7a].

In particular, the indicated model of the π^0 contains the first identification of the attractive character of nonlinear, nonlocal, and nonhamiltonian interactions due to deep wave-overlappings in singlet coupling (and their repulsive character in triplet coupling).

The isoelectronium also sees its foundations in subsequent studies by Animalu [7b] of 1994 and Animalu and Santilli [7c] of 1995 on the construction of *hadronic superconductivity* for a quantitative representation of the structure of the Cooper pair. We have in this case an occurrence similar to the preceding one for the structure of the π^0. In fact, quantum mechanics can indeed represent superconductivity, but only via an *ensemble of Cooper pairs*, all assumed to be point-like. In particular, quantum mechanics simply cannot represent the struc-

ture of *one* Cooper pair, due to the divergent character of the Coulomb repulsion between the identical electrons of the pair.

Again Animalu-Santilli hadronic superconductivity did indeed resolve this insufficiency and permitted, for the first time, the achievement of a structure model of *one* Cooper pair in remarkable agreement with experimental data. Hadronic superconductivity also shows predictive capacities simply absent in quantum mechanics, such as *the prediction of a new electric current mostly given by the motion of electron pairs, rather than the conventional electric current composed of individual electrons* (patent pending). Such a new hadronic current implies an evident reduction of the electric resistance due to the essentially null magnetic moment of the pair, as compared to the large magnetic moment of individual electrons, and its interactions with atomic electrons when moving within a conductor.

Note finally that the preceding hadronic model of the π^0 and of the Cooper pair are ultimately due to the capability of hadronic mechanics to eliminate divergencies at short distances, which is technically realized via the isotopies of the unit and related associative products of quantum mechanics

$$I \to \hat{I} = 1/\hat{T}, \tag{9.1.51a}$$

$$|\hat{I}| \gg 1, \quad |\hat{T}| \ll 1, \tag{9.1.51b}$$

$$A \times B \to A \times \hat{T} \times B, \tag{9.1.51c}$$

under which divergent or slowly convergent series can be evidently turned into rapidly convergent forms.

The tendency of identical valence electrons to bond into the isoelectronium is additionally confirmed by other evidence, such as ball lighting, which are composed by a very large number of electrons bonded together into a small region.

In summary, incontrovertible experimental evidence establishes that *electrons have the capability of bonding themselves at short distances contrary to their Coulomb repulsion*. Quantum mechanics simply cannot provide a scientific study of this physical reality. Hadronic mechanics resolved this impasse, by first identifying the conditions needed to achieve attraction, called "trigger," and then permitting quantitative numerical study of the bond.

The isoelectronium results in having deep connections with a variety of studies in chemistry conducted throughout the 20-th century [6, 8 – 38], and actually provides the physical-chemical foundations for these studies as well as their appropriate mathematical formulation for the invariance of the results.

In summary, *the isochemical model of molecular bonds submitted by Santilli and Shillady [5] is supported by the following conceptual, theoretical and experimental evidence:*

1) The isoelectronium introduces a new *attractive* force among the *neutral* atoms of a molecular structure which is absent in quantum chemistry and permits a quantitative understanding of the *strength* and *stability* of molecular bonds.

2) The isoelectronium permits an immediate interpretation of the reasons why the H_2 and H_2O molecules only admit *two* H-atoms.

3) The isoelectronium permits the achievement of a representation of the binding energy of the hydrogen molecule which is accurate to the *seventh digit*, thus allowing meaningful thermodynamical calculations.

4) The isoelectronium provides an explanation of the long known, yet little understood Pauli's exclusion principle, according to which electrons correlate themselves in singlet when on the same orbital without any exchange of energy, thus via a process essentially outside the representational capabilities of quantum mechanics and chemistry.

5) The isoelectronium is consistent with the known existence of superconducting electron-pairs which bond themselves so strongly to tunnel together through a potential barrier.

6) The isoelectronium provides a quantitative model for the explanation of electron correlations. Instead of a complicated "dance of electrons" described by positive energy excitations, the isochemistry explanation is that electrons are energetically just outside the horizon of a deep attractive potential well due to their wavefunctions overlapping beyond the critical threshold of the hadronic horizon.

7) The isoelectronium is consistent with the "Coulomb hole" studied by Boyd and Yee [35] as found from subtracting accurate explicitly-correlated wavefunctions from self-consistent-field wavefunctions. In our studies the "Coulomb hole" is re-interpreted as a "hadronic attraction".

8) The isoelectronium is also in agreement with the "bipolaron" calculated for anion vacancies in KCl by Fois, Selloni, Parinello and Car [36] and bipolaron spectra reported by Xia and Bloomfield [37].

9) The isoelectronium permits an increase of the speed in computer calculations conservatively estimated at least 1,000-fold, and prevents the inconsistent prediction that all molecules are ferromagnetic (see Chapter 7).

Moreover, *another remarkable result of this study is that the value of the radius of the isoelectronium,* 0.69×10^{-10} *cm, computed via dynamical equations in Sect. 9.3 has been fully confirmed by the independent calculations conducted in Sects. 9.1.6 and 9.1.7 via the Gaussian-lobe basis set, yielding 0.00671* Å.

We should also mention *preliminary yet direct experimental verifications of the isoelectronium offered by the ongoing experiments on photoproduction of the valence electrons in the helium indicating that electrons are emitted in pairs* [38]. The studies of this monograph warrant the systematic conduction of these experiments *specifically for the hydrogen molecule*, and the experimental finalization as

to whether electrons are emitted in an isolated form or in pairs, including relative percentages of both emissions. If conducted below the threshold of disintegration of the isoelectronium, the proposed experiments can evidently provide final proof of the existence of the isoelectronium.

We should finally note that *the representation of the binding energy and other characteristics of the hydrogen molecule exact to the seventh digit first achieved in Ref. [5] constitutes the strongest experimental evidence to date on the insufficiency of quantum mechanics and the validity of the covering hadronic mechanics for the representation of nonlinear, nonlocal, and nonpotential, thus nonhamiltonian and nonunitary effects due to deep overlappings of the "extended wavepackets" of electrons with a "point-like charge structure."*

It is evident that all the above results provide scientific credibility for the isoelectronium, the related isochemical model of molecular bonds, and the underlying hadronic chemistry, sufficient to warrant systematic theoretical and experimental studies.

As shown in Chapter 7, a significant feature of the proposed novel isochemistry is not only the capability to provide accurate representations of experimental data in shorter computer times, but also the capability to predict and quantitatively treat *new industrial applications*.

9.2 ISOCHEMICAL MODEL OF THE WATER MOLECULE

9.2.1 Introduction

Water is an extremely important compound on Planet Earth in a biological as well as geophysical sense. As a consequence, comprehensive studies on water have been conducted since the beginning of quantitative science with outstanding scientific achievements (see, e.g., Ref. [1]). Nevertheless, despite all these efforts, a number of fundamental issues on the structure of the water molecule remain still open, such as:

1) The total electrostatic force among the atomic constituents of a water molecule is null in semiclassical approximation, while the currently used forces (exchange, van der Waals and other forces [40]) are known from nuclear physics to be "weak," thus insufficient to fully explain the "strong" bond among the constituents (where the words "weak" and "strong" do not refer hereon to the corresponding interactions in particle physics). In different words, the representation of the nuclear structure required the introduction of the "strong nuclear force" because of the insufficient strength of the exchange, van der Waals and other forces. It appears that current models on the water molecule lack the equivalent of the "strong nuclear force" to achieve a full representation of molecular structures.

2) Quantum chemistry has not provided a rigorous explanation of the reason why the water molecules only has *two* hydrogen atoms. This is an evident consequence of the assumption of exchange and other nuclear-type forces which were built in nuclear physics for an *arbitrary number of constituents*, a feature which evidently persists in its entirety in molecular structures.

3) Quantum chemistry has been unable to achieve an exact representation of the binding energy of the water molecule under the rigorous implementation of its basic axioms, such as the Coulomb law. In fact, there is a historical 2% still missing despite efforts conducted throughout the 20-th century.

4) More accurate representations have been recently achieved although via the use of Gaussian screenings of the Coulomb law, which, however, are outside the class of equivalence of quantum chemistry, since they are *nonunitarily* connected to the Coulomb law.

5) Quantum chemistry cannot provide a meaningful representation of thermodynamical properties related to water. In fact, the value of 2% missing in the representation of binding energy corresponds to about 950 Kcal/mole while an ordinary thermodynamical reaction takes about 50 Kcal/mole. The use of quantum chemistry in thermodynamical calculations would, therefore, imply an error of the order of 20 times the value considered.

6) Quantum chemistry has been unable to reach an exact representation of the electric and magnetic dipole and multipole moments of the water molecule to such an extent that, sometimes, the models result in having even the *wrong sign* (see, e.g., Ref. [42a], p. 22). This insufficiency is generally assumed to be due to the incompleteness of the assumed basis, although one should not keep adding terms without deeper analysis.

7) Computer usages in quantum chemical calculations require excessively long periods of time. This occurrence, which is due to the slow convergence of conventional quantum series, has persisted to this day, despite the availability of more powerful computers.

8) Quantum chemistry has been unable to explain the "correlation energy" which is advocated for the missing percentages of the binding energies. Orbital theories work well at qualitative and semi-empirical levels, but they remain afflicted by yet unresolved problems, such as the currently used correlation among many electrons as compared to the evidence that the *correlation solely occurs for electron pairs.*

9) Quantum chemistry predicts that the water molecule is ferromagnetic, in dramatic disagreement with experimental evidence. This prediction is a consequence quantum electrodynamics, which establishes that, under an external magnetic field, the orbits of valence electrons must be polarized in such a way as to offer a magnetic polarity opposite to that of the external homogeneous field. As it is well known, the individual atoms of a water molecule preserve

their individuality in the current model of chemical bonds. As a result, quantum electrodynamics predicts that all valence electrons of the individual atoms of a water molecule acquire the same magnetic polarization under a sufficiently strong external magnetic field, resulting in a total net magnetic polarity North-South.

Particularly insidious are variational methods because they give the impression of achieving exact representations within the context of quantum chemistry, while this can be easily proved *not* to be the case. To begin, representations of 100% of the experimental data occur with the introduction of a number of *empirical parameters* which lack a physical or chemical meaning. Moreover, it is easy to prove that *variational solutions cannot be the solution of quantum chemical equations*, trivially, because the former provide 100% representations, while the latter do not. In reality, the arbitrary parameters introduced in variational and other calculations are a measure of the *deviation from the basic axioms of quantum chemistry*.

When passing from the structure of one water molecule to more general molecular structures the number of open, basic, unsolved issues increases. For instance, it is generally admitted that quantum chemistry has been unable to provide a systematic theory of the liquid state in general, let alone that of liquid water in particular [40].

Also, chemical reactions in general are *irreversible*, while the axiomatic structure of quantum chemistry is *strictly reversible* because the theory is strictly Hamiltonian and all known potential forces are reversible. This results in an irreconcilable incompatibility between the very axiomatic structure of quantum chemistry and chemical reactions in general, and those involving water in particular. In fact, an axiomatically consistent representation of irreversibility is expected to imply effects which are simply inconceivable for quantum chemistry, evidently because they are outside its structure.

When passing to water as a constituent of biological entities, the limitations of quantum chemistry reach their climax. In fact, biological structures (such as a cell) are not only irreversible (because they grow, age and die), but have such a complex structure to require multi-valued theories (also known in mathematics as hyperstructures). The expectation that quantum chemistry, with its reversible and single-valued structure, can effectively represent biological systems and their evolution is beyond the boundaries of science.

In view of the above numerous and basic limitations, in the preceding works [41] Santilli and other scientists have constructed a covering of quantum mechanics under the name of *hadronic mechanics*. By conception and construction, quantum and hadronic mechanics coincide everywhere, except inside a small sphere of radius of the order of 1 fm ($= 10^{-13}$ cm) called *hadronic horizon*, in which interior (only) the broader theory holds.

Hadronic mechanics results in being a form of "completion" of quantum mechanics much along the historical Einstein-Podolsky-Rosen argument, although achieved via the addition of contact, nonhamiltonian, nonlinear, nonlocal, and nonpotential forces due to deep overlappings of the wavepackets of particles.

On more technical grounds, hadronic mechanics is based on *new mathematics*, called *iso-, geno- and hyper-mathematics* [41c] (see Chapter 2) for the characterization of reversible, irreversible, and multivalued systems, respectively, possessing features not representable via the Hamiltonian.

These new mathematics are characterized by a progressive generalization of the trivial unit I of quantum mechanics into generalized units $\hat{I}$ of Hermitean single-valued, nonhermitean single-value, and nonhermitean multi-valued character, respectively, first proposed by Santilli in 1978 (see Refs. [41]),

$$I \to \hat{I} = \hat{I}^\dagger, \quad I \to \hat{I} \neq \hat{I}^\dagger, \quad I \to \{\hat{I}\} = \{\hat{I}_1, \hat{I}_2, \hat{I}_3, ...\} \neq \{\hat{I}\}^\dagger. \tag{9.2.1}$$

The new mathematics then emerge from the reconstruction of the conventional mathematics of quantum mechanics in such a way as to admit $\hat{I}$, rather than I, as the correct left and right unit at *all* levels.

The iso-, geno-, and hyper-mathematics characterize corresponding branches of hadronic mechanics, called *iso-, geno-, and hyper-mechanics*, which have been constructed for the corresponding representation of:

1) closed-isolated, reversible, single-valued systems with Hamiltonian and nonhamiltonian internal forces;

2) open-nonconservative, irreversible, single valued systems with unrestricted interactions with an external system; and

3) open-nonconservative, irreversible, multi-valued systems of arbitrary structure.

Subsequently, Animalu and Santilli [43] constructed *hadronic superconductivity*, with corresponding iso-, geno-, and hyper-branches (Sect. 9.1.9) for the representation of the *structure* (rather than an ensemble) of the Cooper pairs, in a way remarkably in agreement with experimental data.

In 1999, Santilli and Shillady [43a] constructed *hadronic chemistry* (Chapter 3) with corresponding branches called *iso-, geno-, and hyperchemistry*. Since molecules are considered as isolated from the rest of the universe, and are reversible in time, they are studied via *isochemistry*.

Santilli and Shillady [43a] also constructed *a new isochemical model of the hydrogen molecule* (Chapter 4) based on the assumption that *pairs of valence electrons from different atoms couple themselves into a singlet quasi-particle state called isoelectronium*.

As shown in Chapter 4, the new model was proved to resolve at least the major insufficiencies of the quantum chemical model of the hydrogen molecule, such as: explain why the molecule has only two H-atoms; represent the binding energy

to the seventh digit; achieve computer calculations which converge at least 1,000 times faster than those of quantum chemistry; and permit other advances.

The main scope of this chapter is that of studying the *new isochemical model of the water molecule* first submitted by Santilli and Shillady in Ref. [43b] via a suitable expansion of the results obtained for the hydrogen molecule.

The main assumption is that, when the valence electrons of the water molecule correlate-bond themselves into singlet pairs in accordance with Pauli's exclusion principle, there is the emergence of *new interactions structurally beyond any hope of representation by quantum mechanics and chemistry, trivially, because they are nonhamiltonian.*

In particular, the new interactions are strongly attractive, thus introducing, for the first time, a molecular bond sufficiently "strong" to represent reality. These and other features of the model, such as the sole possible correlation-bond being in pairs, will resolve all insufficiencies 1)–9) indicated earlier, as we shall see.

To provide introductory guidelines, let us recall that the main function of the isounit $\hat{I}$ (hereon assumed to be Hermitean, single-valued and positive-definite) is that of representing all interactions, characteristics and effects outside the representational capabilities of a Hamiltonian. This includes the representation of contact, nonpotential and nonhamiltonian interactions in deep overlapping of the wavepackets of valence electrons.

By recalling that, whether conventional or generalized, the unit is the fundamental invariant of any theory, the representation of the new interactions via the generalized unit assures invariance, that is, the prediction of the same numbers for the same quantities under the same conditions but at different times.

Representation of nonhamiltonian effects via quantities other than the generalized unit are encouraged, *provided that they achieve the indicated invariance*, as a necessary condition to avoid the catastrophic inconsistencies of Sect. 9.1.7.

The most fundamental mathematical, physical, and chemical notion of the new model of structure of the water molecule studied in this Chapter is, therefore, the generalization of the trivial unit +1 of current models into the isounit.

On pragmatic grounds, isochemistry can be easily constructed via a step-by-step application of the nonunitary transform

$$\hat{I} = 1/\hat{T} = U \times U^{\dagger} > 0, \tag{9.2.2}$$

to *all* aspects of quantum chemistry (Sect. 9.1.4). In particular, we shall assume that the above isounit recovers the conventional unit outside the hadronic horizon, and its average value is much bigger than 1,

$$\lim_{r \gg 1 \text{ fm}} \hat{I} = I. \tag{9.2.3a}$$

$$|\hat{I}| \gg 1. \tag{9.2.3b}$$

Assumption (9.2.3a) will assure the compatibility of a generalized discipline for the bonding of valence electrons, while preserving conventional quantum mechanics identically for the structure of the individual atoms composing the water molecule. Assumption (9.2.3b) will assure a much faster convergence of perturbative expansions, and other features.

In summary, the new isochemical model of the water molecule [43b] studied in this chapter can be constructed via the following steps:

1) Select a nonunitary transforms according to rules (9.2.2) and (9.2.3) which is representative of contact, nonlinear, nonlocal, and nonpotential effects in deep wave-overlapping, essentially similar to that used for the hydrogen molecule [43a] of the preceding Chapter;

2) Submitting to the selected nonunitary transform the totality of the notions, equations, and operations of the conventional quantum chemical model of the water molecule; and

3) Reconstructing the entire mathematics of the conventional model in such a way as to admit $\hat{I}$, rather than I, as the correct left and right unit at *all* levels, with no known exceptions. This lifting is necessary to avoid the catastrophic inconsistencies of Sect. 1.7 (e.g., to achieve invariance), thus requiring the isotopic lifting of numbers and fields, Cartesian and trigonometric functions, ordinary and partial differential equations, *etc.*

The axiomatically correct isochemical model of the water molecule is that formulated on isospaces over isofields. However, on pragmatic grounds, one can study its *projection* on ordinary spaces over ordinary numbers, *provided* that the results are interpreted with care.

For instance, in the indicated projection there is the general emergence of a *potential*, which, as such, may lead to imply that the model carries a *potential energy* and/or it can be treated via a conventional *potential well*. Such interpretations are correct if and only if the potential is well defined on isospaces over isofields. On the contrary, if said potential solely emerges in the projection, then it has a purely mathematical meaning without any associated energy.

The best illustration of the above seemingly contradictory occurrences was that for the isoelectronium of Sect. 9.3, whose structure did indeed exhibit the appearance of the *Hulten potential*, yet the quasi-particle had *no binding energy*. The reason is that binding energies are indeed well defined on isospaces over isofields via the isoschrödinger's equation and related isoeigenvalues, while the Hulten potential does not exist on isospaces, and solely occurs in the projection of the isoschrödinger's equation on ordinary Hilbert spaces.

To illustrate this important point, consider the isotopies of the conventional Schrödinger's equation via the nonunitary transform indicated above,

$$\begin{aligned} U\times(H\times|\psi\rangle) = (U\times H\times U^\dagger)\times(U\times U^\dagger)^{-1}\times(U\times|\psi\rangle) = \\ = \hat{H}\times\hat{T}\times|\hat{\psi}\rangle = \hat{H}\times|\hat{\psi}\rangle = U\times(E\times|\psi\rangle) = \\ = [E\times(U\times U^\dagger)]\times(U\times U^\dagger)^{-1}\times(U\times|\psi\rangle) = \hat{E}\times\hat{T}\times|\hat{\psi}\rangle = \\ = E\times|\hat{\psi}\rangle, \end{aligned} \tag{9.2.4}$$

with corresponding liftings of numbers and Hilbert spaces,

$$U\times n\times U^\dagger = n\times(U\times U^\dagger) = n\times\hat{I}, \quad \hat{n}\in\hat{\mathrm{R}}, \tag{9.2.5a}$$

$$U\times\langle\psi|\times|\psi\rangle\times U^\dagger = \langle\hat{\psi}|\times\hat{T}\times|\hat{\psi}\rangle\times(U\times U^\dagger) = \langle\hat{\psi}|\times|\hat{\psi}\rangle\times\hat{I}\in\hat{\mathcal{C}}. \tag{9.2.5b}$$

As one can see, binding energies $\hat{E} = E\times\hat{I}$ are fully defined on isohilbert spaces $\hat{\mathcal{H}}$ over isofields $\hat{\mathrm{R}}$, and actually acquire the conventional value E following the simplification $\hat{E}\times\hat{T}\times|\hat{\psi}\rangle = (E/\hat{T})\times\hat{T}\times|\hat{\psi}\rangle = E\times|\hat{\psi}\rangle$.

However, *the Hulten potential does not exist on isospaces over isofields*, trivially, because it does not exist in the Hamiltonian $\hat{H}$ which is fully conventional.

The Hulten potential of the isoelectronium of Sect. 9.3 emerge only when we project the real system, that on isohilbert spaces with equation $\hat{H}\times|\hat{\psi}\rangle = E\times|\hat{\psi}\rangle$, on conventional Hilbert spaces. As such, one should not expect that the Hulten potential necessarily carries an actual binding energy.

The reader should equally exercise caution for other aspects, and generally abstain from formulating opinions for hadronic chemistry essentially dependent on quantum chemical concepts and notions.

9.2.2 Main Characteristics of the Water Molecule

Water is a mixture of several different molecules in different percentages and molecular weights. In fact, we know *three* different isotopes of the hydrogen, ^{1}H, ^{2}H and ^{3}H, and *six* different isotopes of the oxygen ranging from ^{14}O to ^{19}O. In this monograph, we shall solely study the molecule $^1H_2-^{16}O$, and denote it H_2O = H-O-H where the symbol "−" is referred to the molecular bond. Such a water molecule will be studied hereon under the following conditions: 1) at absolute zero degrees °K; 2) in the absence of any rotational, vibrational, translational, or other motions; and 3) with all atoms in their ground state (see Ref. [40] for all details contained in this section).

The electrons of the individual H-atoms are assumed to be in the ground state $1s$. Of the eight electrons of the oxygen, two electrons with opposite spin orientation are in the lowest $1s$ state which is tightly bound to the nucleus; two electrons are in the next possible state $2s$; and the remaining four electrons are in the $2p$ state.

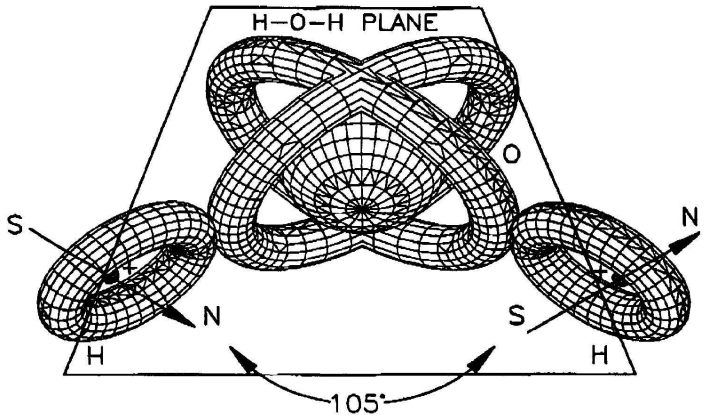

Figure 9.8. A conceptual rendering of the conventional water molecule with its typical electric polarization. Note the consequential predominance of a positive charge in the two hydrogen atoms that is responsible in part for the angle of 105° between the two HO radicals.

By using a three-dimensional reference frame with the y-z plane containing the nuclei of the H and O-atoms with origin in the latter, the $1s$ and $2s$ electrons have a spherical distribution while the $2p$ electrons are in orbitals perpendicular to the yz plane denoted $2p_x$; the remaining two electrons have orbitals perpendicular to the xz and xy planes denoted $2p_y$ and $2p_z$, respectively.

Also, the energy of formation of the water molecule from hydrogen and oxygen is -9.511 eV; the binding energy is -10.086 eV; the sum of the ground state energies of the three separate atoms is $-2,070.46$ eV; the total molecular energy at 0°K is $-2,080.55$ eV as a result of kinetic energy +2,080.6 and potential energy $-4,411.4$; the nuclear repulsion energy is +250.2 eV; the total electrostatic energy is $-2,330.8$ eV; the dissociation energy of O-H is 5.11 eV and that of H alone is 4.40 eV.

Again at 0°K and for all atoms in their ground states, the bond length of the H-O dimer is 0.95718×10^{-8} cm, while the two dimers H-O and O-H form a characteristic angle of 104.523°. Therefore, by no means the scripture H-O-H denotes that the water has a linear structure because of the indicated characteristic angles in between the two dimers H-O and O-H.

It is evident that when the individual atoms are in their excited states, the bond length and the characteristic angle change. In fact, increases of up to 8.5° have been measured for the characteristic H-O-H angle for excited states. The same characteristic angle is expected to be altered by the application of suffi-

ciently strong electric and magnetic fields, although we are unaware of accurate measurements under the indicated conditions.

The water molecule possesses an electric dipole moment of 1.83×10^{-8} e.s.u. cm and a mean quadrupole moment of -5.6×10^{-26} e.s.u. cm. It should be recalled that the very existence of a non-null value of electric dipole and quadruple moments excludes the linear structure of the water H-O-H in ordinary isolated conditions (that with a characteristic angle of 180°).

Water is a *diamagnetic* substance with a magnetic polarization (also called susceptibility) of (2.46, 0.77 and 1.42)$\times10^{-6}$ e.m.u./mole for the corresponding three space-dimension xx, yy and zz, respectively.

In first approximation, the water molecule can be represented via two individual H-O dimers with wavefunction of the molecular orbitals (m.o.'s),

$$\psi_1 = \lambda\phi(\mathrm{H}', 1s) + \mu\phi(\mathrm{O}, 2p_z), \tag{9.2.6a}$$

$$\psi_2 = \lambda\phi(\mathrm{H}'', 1s) + \mu\phi(\mathrm{O}, 2p_y), \tag{9.2.6b}$$

where λ and μ are parameters.

However, the above simple model predicts a characteristic angle of 90°. As a consequence, the model is generally modified with a mixture of electrons from the $2p$ and $2s$ states also called *hybridization*. The occurrence confirms that any model of the water with charge distributions of the valence electrons in the H-O-H plane is insufficient to represent the experimental data. In turn, this mixing creates the known two *lobes* on the side of the oxygen atom, away from the hydrogen atoms, above and below the molecular plane. This results in models of the type

$$\psi_1 = \lambda[\cos\varepsilon\psi(\mathrm{O}, 2s) + \sin\varepsilon\phi((\mathrm{O}, 2p)] + \mu\phi(\mathrm{H}', 1s), \tag{9.2.7a}$$

$$\psi_2 = \lambda[\cos\varepsilon\psi(\mathrm{O}, 2s) + \sin\varepsilon\phi((\mathrm{O}, 2p)] + \mu\phi(\mathrm{H}'', 1s), \tag{9.2.7b}$$

where ε is the hybridization parameter with generic value of the order of $\cos\varepsilon = 0.093$ confirming that the valence electrons are mainly from $2p$ states.

It should be indicated that the exact configuration, location and function of the two lone-pair electron lobes are unsettled at this writing, since they are evidently dependent on the selected theoretical model. Also, the individual electric and/or magnetic dipoles of the lobes cannot be measured (only their total values is measurable), thus implying lack of direct experimental evidence on the individual lobes.

We should also recall that the individual H-O and O-H bonds are not independent from each other, as confirmed by the different values of the dissociation energies.

Water is both an acid and a base due to dissociation of H_2O into H^+ and $O\text{-}H^-$ to the extent that the product of the concentrations $[H^+][O\text{-}H^-]$ sets up

an equilibrium whose constant value is 1.0×10^{-14}, which is the well known pH scale of the equations

$$\text{pH} = -\log_{10}[\text{H}^+], \quad \text{pOH} = 14 - \text{pH}. \tag{9.2.8}$$

In neutral water the ion concentrations are $[\text{H}^+] = [\text{O-H}^-] = 1.004 \times 10^{-7}$ mole/liter.

Water is quite polar with a dipole moment of 1.84 to 1.834 Debye and a bulk dielectric constant of 80 at 20 °C. This implies that pure water is not a good conductor, with a direct current conductivity of only 5.7×10^{-8} ohm^{-1}·cm^{-1}.

However, it is well known that small amounts of strong acids such as HCl or H_2SO_4 can make water highly conducting due to the ease with which H^+ can attach to H_2O to form H_3O^+ which then offers a *domino effect* for one H^+ to successively "bump" an H^+ off the other side of H_3O^+ and so produce a very effective conduction mechanism [44]. In fact it is well known that in aqueous solutions the transport numbers for the anions and cations are not equal, because up to 70% of the current is carried by H^+. Although OH^- typically carries much less current than H^+ in aqueous conduction of electricity (due to its larger size and lack of the domino-effect cited earlier for H^+), once a current flow is initiated additional ions are created due to collisions in solution.

An important aspect is the known existence of an equilibrium between H-O-O-H and HO^- around pH 11.63 [45] with a voltage dependence of 1.363 ± 0.0293 pH as given by M. Pourbaix for aqueous equilibria involving H^+, O-H$^-$, H$^-$, H-O-O-H and H-O$^-$. Thus, there is no doubt of the existence of small amounts of H-O-O-H in water at high pH.

In a high current process the flow of H^+ will be much greater than that of OH^- so that as H_2O is electrolyzed to $2H_2$ and $1O_2$, local concentrations/fluctuations will slightly favor higher pH (local depletion of H^+) and hence favor the existence of H-O-O-H.

We should finally mention the inability of quantum chemistry to achieve a scientific-quantitative representation (or at least an understanding) of the *different types of water when exposed to magnetic fields*, as established by the evidence, e.g., that plants grow faster when irrigated with water exposed to one type of magnetic field, while they die rapidly when exposed to a different type of magnetic field. In fact, quantum chemistry admits only *one* type of water, H_2O.

It is easy to see that this additional insufficiency of quantum chemistry is a direct consequence of the current use of exchange, van der Waals and other forces of nuclear origin under which the individual H and O atoms in the H_2O molecule preserve their individuality, thus resulting in one single configuration.

On the contrary, isochemistry introduces a real, strong bond for the valence electrons via the notion of isoelectronium. In this latter case different types of water, that is, water molecules with different physical characteristics, are indeed readily possible, as we shall see.

9.2.3 Exactly Solvable Model of the Water Molecule with Stable Isoelectronium

In the preceding Chapter 3 [43a], we have introduced the main hypothesis of the *isochemical molecular model*, according to which two electrons from two different atoms bond themselves into a singlet quasi-stable and quasi-particle state called *isoelectronium*, which describes an *oo*-shaped orbit around the nuclei, as it is the case for planets in certain binary stars (Fig. 9.3). The main characteristics of the isoelectronium in first nonrelativistic approximation are calculated in Sect. 9.3 and resulted in being:

$$\begin{gathered}\text{charge} - 2e, \quad \text{spin } 0, \quad \text{magnetic dipole moment } 0, \\ \text{mass } 1.022\text{MeV}, \quad \text{radius} = r_c = b^{-1} \approx \\ \approx (\hbar^2/m \times V)^{1/2} = (\hbar/m \times \omega)^{1/2} = 6.8432329 \times 10^{-11}\text{cm} = \\ = 0.015424288 \text{ bohrs} = 0.006843 \text{ Å}.\end{gathered} \tag{9.2.9}$$

In the above nonrelativistic approximation, the meanlife resulted in being infinite (full stability, with the understanding that relativistic corrections are expected to render such a meanlife finite (partial stability). All conventional forces of current use in chemistry (exchange, van der Waals and other forces) then hold when the valence electrons are at mutual distances bigger than the hadronic horizon.

In this Chapter, we study the *isochemical model of the water molecule* H_2O=H-O-H first introduced by Santilli and Shillady [43b], under the assumption that the molecule is considered at °C and in the absence of any rotational, oscillation or other motion. The main hypothesis is that each electron from the two H-atoms couples in singlet with one $2p$ electron from the O-atom, resulting in *two isoelectronia*, one per each H-O dimer as in Fig. 9.8.

In this Section we shall study a hadronic/isoschrödinger equation for the water molecule under the above assumptions, which equation evidently approximate, yet *exactly solvable* for the first time to our knowledge. We shall then show that the model is extendable to all other dimers comprising one hydrogen atom, such as H-C.

For this purpose, we approximate the H-O-H molecule as being composed of two intersecting identical dimers H-O with evidently only one oxygen atom. This requires a first correction due to the lack of independence of said dimers reviewed in Sect. 9.2. Moreover, in each H-O dimer we shall assume that the oxygen appears to the isoelectronium as having only one net positive charge $+e$ located in the nucleus. This evidently requires a second correction which essentially represents the screening of the various electrons of the oxygen. Additional corrections are also in order along conventional lines [40].

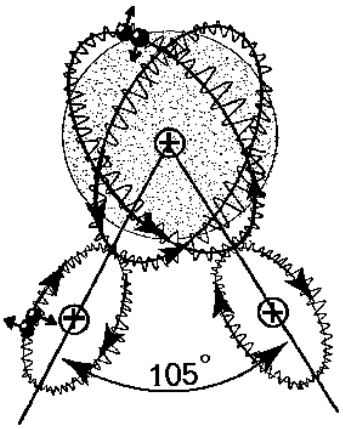

Figure 9.9. A schematic view of the proposed *isochemical model of the water molecule* here depicted at absolute zero degrees temperature and in the absence of any motion for the case of fully stable isoelectronium. It should be stressed that *at ordinary temperature rotational motions recover the conventional space distribution*, thus recovering the conventional "Mickey Mouse" configuration of the water. Also, the model is presented in terms of the *orbits* of the valence electrons (rather than in terms of *density distributions*). The fundamental assumption is that the two valence electrons, one per each pair of atoms, correlate themselves into two bonded singlet states at short distance we have called *isoelectronia*, one per each dimer H-O, which states are assumed to be mostly stable (see the text for the mostly unstable case). The water molecule is then reducible to two intersecting H-O dimers with a common O-atom. The only orbits yielding a stable water molecule are those in which each isoelectronium describes a *oo*-shaped orbit around the respective two nuclei of the H- and O-atoms. The isoelectronia are then responsible for the *attractive force* between the atoms. The *binding energy* is instead characterized by the *oo*-shaped orbits of the isoelectronia around the respective two nuclei, conceptually represented in this figure via a standing wave for a particle of spin 0 and charge $-2e$. Note that, in the absence of molecular motions, the orbits of the two isoelectronia are perpendicular to the H-O-H plane, thus confirming a characteristic of the water molecule reviewed in Sect. 9.2. Conventional exchange, van der Waals and other forces remain admitted by the model when the isoelectronia are mostly unstable. The model permits a representation of: 1) the "strong" value of the molecular bond; 2) the reason why the H_2O molecule has only two hydrogen atoms and one oxygen atom; 3) a representation of the binding energy, electric and magnetic moments accurate to several digits; and other advances studied in the text. The above model of the H-O dimer is then extendable to other H-based dimers, such as H-C.

A study of these corrections has indicated that they can all be represented via one single Gaussian screening of the Coulomb law of the type [43b]

$$\frac{+e}{r} \to \frac{+e(1 \pm e^{-\alpha r^2})}{r}, \tag{9.2.10}$$

where α is a positive parameters to be determined from experimental data, the sign "$-$" applies for the screened O-nucleus as seen from an *electron* (because of

the *repulsion* caused by the electron clouds of the oxygen), while the sign "+" applies for the screened O-nucleus as seen from the H-*nucleus* (because of the *attraction* this time caused by said electron clouds).

The resulting model is structurally equivalent to the isochemical model of the hydrogen molecule of Chapter 3 [43a], except for the modifications indicated about, and can be outlined as follows.

By denoting with the sub-indices 1 and a the hydrogen, 2 and b the oxygen, prior to the indicated screening and in the absence of all hadronic effects, the conventional Schrödinger equation of the H-O dimer with the oxygen assumed to have only one elementary charge $+e$ in the nucleus is given by

$$\left(\frac{1}{2\mu_1}p_1\times p_1+\frac{1}{2\mu_2}p_2\times p_2- \right.$$
$$\left.-\frac{e^2}{r_{1a}}-\frac{e^2}{r_{2a}}-\frac{e^2}{r_{1b}}-\frac{e^2}{r_{2b}}+\frac{e^2}{r_R}+\frac{e^2}{r_{12}}\right)\times|\psi\rangle=E\times|\psi\rangle, \tag{9.2.11}$$

As it was the case for the H_2-molecule, our task is that of subjecting the above model to a transform, which is nonunitary only at the short mutual distances $r_c=b^{-1}=r_{12}$ of the two valence electrons (here assumed to be hadronic horizon), and becomes unitary at bigger distances $\hat{I}_{r\leq 10^{-10}\text{cm}}\neq I$, $I_{r\gg 10^{-10}\text{cm}}=I$.

We assume that the state and related Hilbert space of systems (9.2.11) can be factorized in the familiar form (in which each term is duly symmetrized or antisymmetrized)

$$|\psi\rangle=|\psi_{12}\rangle\times|\psi_{1a}\rangle\times|\psi_{1b}\rangle\times|\psi_{2a}\rangle\times|\psi_{2b}\rangle\times|\psi_R\rangle, \tag{9.2.12a}$$

$$\mathcal{H}_{\text{Tot}}=\mathcal{H}_{12}\times\mathcal{H}_{1a}\times\mathcal{H}_{1b}\times\mathcal{H}_{2a}\times\mathcal{H}_{2b}\times\mathcal{H}_R. \tag{9.2.12b}$$

The nonunitary transform we are looking for shall act only on the r_{12} variable characterizing the isoelectronium while leaving all other variables unchanged. The simplest possible solution is given by

$$U(r_{12})\times U^\dagger(r_{12})=\hat{I}=e^{[\psi(r_{12})/\hat{\psi}(r_{12})]\int d\mathbf{r}_{12}\hat{\psi}^\dagger(r_{12})_{1\downarrow}\times\hat{\psi}(r_{12})_{2\uparrow}}, \tag{9.2.13}$$

where the ψ's represents conventional wavefunction and the $\hat{\psi}$'s represent isowavefunctions, for which we have, again the fundamental condition of fast convergence

$$|\hat{T}|=|(U\times U^\dagger)^{-1}|\ll 1. \tag{9.2.14}$$

We now construct the isochemical model by transforming short-range terms (isochemistry) and adding un-transformed long range ones (chemistry), thus resulting in the radial equation

$$\left(-\frac{\hbar^2}{2\times\mu_1}\times\hat{T}\times\nabla_1\times\hat{T}\times\nabla_1-\frac{\hbar^2}{2\times\mu_2}\times\hat{T}\times\nabla_2\times\hat{T}\times\nabla_2+\right.$$

$$+\frac{e^2}{r_{12}}\times\hat{T}-\frac{e^2}{r_{1a}}-\frac{e^2}{r_{2a}}-\frac{e^2}{r_{1b}}-\frac{e^2}{r_{2b}}+\frac{e^2}{R}\Big)\times|\hat{\psi}\rangle=E|\hat{\psi}\rangle. \tag{9.2.15}$$

By recalling that the Hulten potential behaves at small distances like the Coulomb one, Eq. (9.2.15) becomes

$$\Big(-\frac{\hbar^2}{2\times\mu_1}\times\nabla_1^2-\frac{\hbar^2}{2\times\mu_2}\times\nabla_2^2-V\times\frac{e^{-r_{12}\times b}}{1-e^{-r_{12}\times b}}-$$
$$-\frac{e^2}{r_{1a}}-\frac{e^2}{r_{2a}}-\frac{e^2}{r_{1b}}-\frac{e^2}{r_{2b}}+\frac{e^2}{R}\Big)\times|\hat{\psi}\rangle=E\times|\hat{\psi}\rangle. \tag{9.2.16}$$

The above model can be subjected to an important simplification. In first approximation under the assumption herein considered, the H-O dimer (9.2.16) can be reduced to a *restricted three body problem* similar to that possible for the conventional H_2^+ *ion* [41], but *not* for the conventional H_2 molecule, according to the equation

$$\Big(-\frac{\hbar^2}{2\mu_1}\times\nabla_1^2-\frac{\hbar^2}{2\mu_2}\times\nabla_1^2-V\times\frac{e^{-r_{12}b}}{1-e^{-r_{12}b}}-$$
$$-\frac{2e^2}{r_a}-\frac{2e^2}{r_b}+\frac{e^2}{R}\Big)\times|\hat{\psi}\rangle=E\times|\hat{\psi}\rangle. \tag{9.2.17}$$

The indicated corrections due to the screening of the various electrons in the oxygen and other corrections are needed in the "sensing" of the O-nucleus by the isoelectronium as well as by the H-nucleus, yielding in this way our final model

$$\Big(-\frac{\hbar^2}{2\mu_1}\times\nabla_1^2-\frac{\hbar^2}{2\mu_2}\times\nabla_2^2-V\times\frac{e^{-r_{12}b}}{1-e^{-r_{12}b}}-$$
$$-\frac{2e^2}{r_a}-\frac{2e^2(1-e^{-\alpha r_b})}{r_b}+\frac{e^2(1+e^{-\alpha R})}{R}\Big)\times|\hat{\psi}\rangle=E\times|\hat{\psi}\rangle, \tag{9.2.18}$$

where: α is a positive parameter; E is half of the binding energy of the water molecule; and, as it was the case for model (9.1.35), the mass of the isoelectronium, the internuclear distance, and the size of the isoelectronium can be fitted from the value of the binding energy and other data.

Under the latter approximation, the model admits an exact analytic solution, for the first time to our knowledge, which solution however exists only for the case of the *fully stable isoelectronium*. In fact, for the unstable isoelectronium, the model becomes a *four-body structure*, which as such admits no exact solution.

Besides being exactly solvable, model (9.2.18) exhibits a *new explicitly attractive "strong" force among the neutral atoms of the H-O dimer*, which is absent in conventional quantum chemistry; the equation also explains the reasons why the water molecule admits only *two* H-atoms.

As we shall see in the remaining sections, the model permits an essentially exact representation of the binding energy, electric and magnetic moments; the model yields much faster convergence of series with much reduced computer times, and resolves other insufficiencies of conventional models.

Finally, the model is evidently extendable with simple adjustment to an exact solution of other dimers involving the hydrogen, such as H-C. In addition, it permits the identification of electric and magnetic polarizations, which are not predictable with quantum chemistry (Chapter 8).

9.2.4 Gaussian Approximation of the Isochemical Model of the Water Molecule with Unstable Isoelectronium

The solution of the exactly solvable model (9.2.18) are not available at this writing, and its study is here encouraged. In this section, we review the studies of Ref. [43b] on a Gaussian *approximation* of the isochemical model of the water molecules, with the Hulten potential approximated to a certain Gaussian form.

It should be indicated from the onset that such an approximation implies an evident weakening of the Hulten attraction among the two isoelectrons of the isoelectronium, which, in turn, implies the instability of the isoelectronium itself, thus reaching a model which is somewhat intermediate between the full isochemical model and the conventional quantum chemical model of the water.

Despite this approximate character, the results of this section are significant because they show the capability of isochemistry to achieve an essentially exact representation of the binding energy, electric and magnetic moments and other characteristics of the water molecule.

The results of this study can be outlined as follows. Since HOOH will be slightly more allowed under the assumed conditions, collisions of HO^- with neutral H_2O and the internal repulsion within the anion could favor the release of a quasiparticle with charge $-2e$ to form OH^+. Collisions of OH^+ with OH^- will then further enhance the concentration of HOOH, and transport of $-2e$ will contribute to the current.

The question here is whether under extreme cases of forced conduction a singlet-pair of electrons (isoelectronium), can be "triggered" (Fig. 9.6) within a water molecule to form and release a $-2e$ charged isoelectronium particle which will provide an additional conduction mechanism analogous to Cooper-pairs of electrons in superconducting solids.

Since the energy depth of the V_0 parameter in the isoelectronium Hulten potential of the original 1978 derivation by Santilli [42] is not known, nor how closely the Gaussian representation fits the Hulten form, we can only match the radius of the two potentials and calculate the energy differences caused by the "sticky-electrons" model in which a transient form of isoelectronium can occur

(the Gaussian potential well may not be deep enough to ensure a permanent bound state for the isoelectronium).

The "sticky-electron" model is a parametric model which includes the magnetic dipole attraction between singlet-paired electrons, as well as the nonlocal merging of the wave-packets of each electron at short distance. The radius of the Gaussian screening is then determined empirically by fitting the calculated energy as nearly as possible to the most accurate energy values available.

As used here, it should be emphasized that the off-axis positions of the Gaussian-lobe basis sets [46-48] ensure that angular correlation is included as well as radial dependence, and can include the magnetic dipole attraction of opposite electron spins as well as merger of wavepackets.

One radial screening parameter used with off-axis basis sets parametrically covers all forms of short range attraction which may include angular dependence. Thus the present model can give us an approximate energy difference required to release an electron-pair from OH^-.

$$OH^- \to OH^+ + (2e)^{-2}. \tag{9.2.19}$$

It will be seen below that the energy difference between OH^- and OH^+ as calculated, allowing a transient form of isoelectronium, is well within the voltage accessible using capacitive discharge through water. Such a mechanism which would allow $-2e$ particles to flow through water would not be superconductivity as conventionally understood, since the freely moving molecules and ions are not constrained to lattice positions as in solids, so that resistive I^2R heating will still occur.

This is mainly due to the fact that conduction in liquids occurs by mobility of both anions and cations along with size differences, polarizability differences and special mechanisms such as the hydrogen-bonding "domino effect" for H^+ transport. In solid-state conduction, only the electrons move by ignoring in-place phonon oscillations because the atoms do not travel from one electrode to the other.

Despite the indicated lack of superconducting character, it should be indicated that yet, the essentially null magnetic moment of the $-2e$ particle would imply indeed a reduction of the resistivity.

The apparent motion of positively charges "holes" is also due to motion of electrons while the atoms merely oscillate about mean lattice positions. In solutions there is a two-way traffic with positive and negative ions travelling in opposite directions and with differing velocities, thus leading to resistive heat even up to the vaporization of the water as well as a high probability of ion collisions.

It should be noted that in recent work Ashoori *et. al.* [49] have measured migration of paired-electrons to quantum dot wells in GaAs, while Boyd and Yee [50] have observed "bipolaron" electron pairs in alkali halide lattice vacancies. Calculations leading to unexpected bipolarons in crystal lattice vacancies have

also been observed by using the method of Car and Parinello [51]. These findings in solids lend support to the concept of an electron pair as an individual particle, called by the authors isoelectronium.

The calculations given here do not prove the presence of isoelectronium particles in high current aqueous electrical conduction; they only indicate the energy threshold necessary to form the isoelectronium within the conducting solution by double-ionization of OH^-.

It is not easy to envision an experiment that would be able to analyze components of a given current, due to multiple ion species in terms of the amount of current due to $-2e$ particles, and none is proposed here. However, there may be a chemical test for such a mechanism. Once OH^- is doubly-ionized to form OH^+, collisions with $-2e$ particles would regenerate OH^- ions just as collisions of H^+ with OH^- will reform H_2O,and no new species will be evident.

However, if OH^- collides with OH^+ a new chemical species HO-OH will be formed that may last long enough in the liquid to behave as a strong oxidizing agent. Thus, organic compounds with double bonds (alkenes), which have negligible conductance, could be added to water undergoing a high current flow to cause hydroxylation of such compounds [52], (i.e., conversion of alkenes to epoxide, which are then readily hydrolyzed in the presence of H^+ to diols). Enhanced concentrations of epoxides and diols would be indirect evidence of double ionization of OH^-, according to the expressions

$$OH^+ + OH^- \rightarrow HO-OH, \tag{9.2.20}$$

$$CH_2=CHR + HO-OH \rightarrow \begin{array}{c} CH = CHR \\ \backslash\ \ / \\ O \end{array} + H_2O.$$

If the isoelectronium can be detected indirectly by a chemical method, this would in itself be an important inference on the existence of a two-electrons, spin-zero particle. More importantly, "isochemical reactions" could be driven by high conduction "liquid plasma" environments where the isoelectronium is at an enhanced concentration.

Another case of interest is that of aqueous mixtures of insoluble organic compounds forming a separate oil layer over water in an intense magnetic field of several Tesla. At normal thermal energy of room temperature $kT \cong RT$ per mole the main energy form would be random Brownian motion.

However, in the presence of a strong magnetic field HO^+ and HO^- would be constrained to favor circular motions in the magnetic field by the "cyclotron effect," but there is no obvious source of HO^+.

Since two ions of opposite sign charges would be favored to collide by both electrical attraction and by opposite path curvature in a magnetic field, there is an enhancement that when created as a normal result of H^+, OH^-,H^-, H_2O_2, HO_2^-,

H_2O equilibrium system studied by Pourbaix [45], any natural concentration of HOOH would be augmented by collision of H^+ with HOO^-.

In addition there is some slight chance that H^+ would collide with OH^- with sufficient excess energy to produce OH^+ and H^-. Thus the presence of an intense magnetic field would cause positive and negative ions to collide more easily while travelling in opposite rotational arcs in such a way as to enhance the concentration of HOOH,

$$H^+ + HOO^- \to HOOH, \tag{9.2.21a}$$

$$H^+ + OH^- \to H^- + OH^+, \tag{9.2.21b}$$

$$OH^+ + OH^- \to HOOH, \tag{9.2.21c}$$

which could then epoxidate alkenes and upon hydrolysis would lead to diols.

A direct measure of this effect would be to determine the enhanced solubility of alkenes in water. The alkenes are only slightly soluble in water ("oil and water do not mix") but alkenes converted to diols will have a measurably greater solubility in water due to the attached OH-groups. Again. If such enhanced solubility of alkenes in water can be caused by intense magnetic fields, this would be indirect evidence of the existence of an electron-pair particle with charge $-2e$.

In the description of the calculations below the key to the above possibility is that it is easy to calculate the energy of OH^- when one subtracts a small amount from the two-electron repulsion terms in the usual HFR-SCF treatment, due to the attraction of singlet-paired electrons at close range within 1.0 picometer.

In the recent *Handbook of Computational Quantum Chemistry* by Cook, Ref. [53], p. 438, it is noted that solutions to the HFR-SCF scheme may not always exist for anions. However, in the method used here convergence of the HFR-SCF method was normal for an SCF process, because the so called "self-energy" error of the Hartree-Fock method [54] (in which each electron repels all electrons including itself) is largely cancelled by the new attractive terms used here. In effect, this description of OH^- is possible because of the easy convergence of the "correlated-SCF process."

9.2.5 The Method

The model adopted in Ref. [43b] is to use the usual Hartree-Fock-Roothan self-consistent-field equations [55] (which also has some formal flaws such as the self-interaction terms [54]) and lift in a nonunitary way the form of the Coulomb interaction of the electrons.

Note that reducing the values of the Coulomb integrals will lower the energy by reducing the electron-electron repulsion while reducing the exchange terms will raise the energy, but the factor 1/2 reduces the effect of the exchange terms. Thus a reduction of the value of the integrals will lower the energy.

Note that Goodgame and Goddard [54] have already recommended reducing the atomic self-energy by subtracting 1.39 eV from Hartree-Fock exchange integrals in the cases of Cr_2 and Mo_2.

$$\text{FC} = \text{ESC}; \quad \text{F}_{i,j} = H_{i,j} + \sum_{k,l} P_{k,l}[(i,j|k,l) - 1/2(i,k|j,l)], \tag{9.2.22a}$$

$$(i,j|k,l) = \iint \chi_i(l)\chi_j(l)\frac{1}{r_{12}}\chi_k(2)\chi_l(2)d\tau_1 d\tau_2, \tag{9.2.22b}$$

$$P_{i,j} = 2\sum_n c_{n,i}c_{n,j} \quad (\text{sum } n \text{ only over occupied orbitals}). \tag{9.2.22c}$$

The 1995 paper on electron-electron pairs by Animalu and Santilli [42b] invokes the non-local hadronic attractive force evident in the π^0-meson by Santilli [42a] applied to a pair of singlet-paired electrons which form a boson quasiparticle. However, the "collapsed positronium" rapidly decays since the particle-antiparticle annihilation takes place in less than a picosecond.

In the electron-electron case it is believed that there may be a stable quasiparticle singlet bond we have called the isoelectronium. After using a non-local isotopic nonlinear transformation, the hadronic attraction was projected into real-space, and modeled with a Hulten potential.

Considerable effort was made to evaluate the matrix elements for the Hulten potential without success. Examination of the original 1978 paper on positronium collapse by Santilli [6a] revealed that the Hulten potential is not necessarily a unique representation of the hadronic force. In fact, a linear combination of similar potentials could be used to represent the Hulten potential if matrix elements of such other potentials could be evaluated.

The depth of the screened Gaussian approximation is determined by requiring that the width at half height of the Gaussian is equal to the b value of the Hulten horizon (the radius at which the Coulomb repulsion is annulled by other attractive forces). Thus, the screened Gaussian potential probably has a depth which is too shallow although the V_0 depth parameter for the Hulten potential is not known at present.

This work assumes that until matrix elements of a two-electron interaction for singlet-pairs can be found for the Hulten potential, a Gaussian-screened-Coulomb potential can be used to describe the real-space form of the hadronic attraction and as a parameter fitted to experimental energies the screening exponent probably includes other effects such as the magnetic dipole interaction of two electrons with opposite spin-magnetic-moments. This form has the important property that it can be merged with the general case of the four-center Coulomb or exchange integral derived by Shavitt [56] using the famous Gaussian transform technique.

The Gaussian transform two-electron integral for four Gaussian spheres has been used in a number of Gaussian-lobe basis SCF programs written by Shillady [57, 58] and others. It is important to note that the formula is completely general in orientation of four Gaussian sphere lobe-orbitals as well as the distance between two electrons.

As modified for a description of the correlation of two electrons, such a general formula can describe angular correlation as well as distance interaction. Thus matrix elements of a screened-Coulomb interaction were subtracted from the usual $1/r$ Coulomb repulsion to model the real-space form of the hadronic attraction of two electrons. The work outlined in this section, first presented in Ref. [43b], added the Gaussian screening as $\exp[-\alpha r^2]/r$ so that the special properties of Gaussians could be used, especially the properties that the product of two Gaussians form another Gaussian (times a re-centering factor), and that polar coordinates readily separate into factorable x, y, z components.

The goal was to evaluate the two-electron four-center matrix elements of the Gaussian-screened Coulomb potential in the expression

$$Y(r) = \frac{1 - 2\exp[-\alpha r^2]}{r}. \tag{9.2.23}$$

Amazingly, the Gaussian-Gaussian exponent and carried through the original derivation until the last step when integration over "s" is required. α is usually a very high number, this work used 0.13441885×10^7. At this point the usual Coulomb interaction resorts to a well known auxiliary function F_0 which has been studied by Shillady [57] and others.

Since both $s^{1/2}$ and $(s + \alpha)^{1/2}$ occur in the denominator of the screened-Coulomb form, two poles occur in the integral. A change of variable absorbs the pole due to $(s + \alpha)^{-1/2}$ and shifts the other pole due to $s^{-1/2}$ to the lower limit of the integral. A smooth spike is evident at the lower limit of the numerical integration using a 70 point Simpson's Rule integration (two ranges are used with 20 points more closely spaced near the pole and 50 points for the remaining range).

This work was carried out using 64 bit double precision arithmetic, which provides 14 significant figures. A simple offset δ of 1.0×10^{-15} has provided useful results with this simple offset to avoid numerical overflow.

While this pole is formally a problem in needing a continuous function to integrate, numerical integration seems to handle these Coulomb integrals are known to be accurate only to 12 significant figures. The area under the pole-spike is estimated as a narrow triangle upon a rectangle 1.0×10^{-15} wide with the height of the triangle set at 3.43207×10^8 times the height of the point set 1.0×10^{-15} into the range of integration (the first Simpson point).

The present code for this screened-Coulomb integral is presently slower that the corresponding F_0 function [56] used for the Coulomb integrals due to the

70 point Simpson integration, but the integrand is nearly flat after the spike at $s = 0.0$ so that portion of the integrand can be evaluated more rapidly with fewer points. For results presented here, the simple offset of the lower limit by 1.0×10^{-15} is adequate for this monograph. Further details on the auxiliary integral can be found in a previous paper on the H_2 molecule [59].

Work in progress indicates it may be possible to express the new auxiliary integral to an analytical expression involving the erf(x) function (see Chapter 6), but until further checks are completed this work used the Simpson integration. Note the integral is a result of a simplification of a twelve-fold integration over the volume elements of two electrons, and has been reduced to a one-dimensional integration multiplied by appropriate factors.

9.2.6 The Main Results

The geometry given for H_2O by Dunning [60] was used to carry out the usual HFR-SCF calculation after an additional $3d$ orbital mimic [58] was optimized in Ref. [43b] for the O atom and ($2s$,$2p$) orbitals were added for the H atoms. The exponent for the O$3d$ orbitals was optimized to three significant figures and the (O$3d$,H$2s$,H$2p$) exponents were (2.498, 0.500, 1.000). These polarization orbitals were added to the Dunning-Huzinaga ($10s6p$) [59] basis with the H$1s$ orbitals scaled to 1.2 which produced a lower energy than that of a 6-31G^{**} basis using the GAMESS program. The bond length of OH^+ was Angströms.

The same bond length was used for OH^- since the anion calculation using the usual HFR-SCF process was not feasible, and, in any case, the bond length is only slightly longer than that in water. The horizon cutoff value of 0.00038 Angströms optimized for H_2O was also used for OH^+ and OH^-.

The spike in the numerical integral routine was optimized by fitting the R_c cut-off value so as to obtain as near as possible the non-relativistic energy of the HF molecule as determined from Quantum Monte Carlo calculations [60]. The dipole moments for the ions are not very useful since ion dipoles are origin dependent, but they were calculated using the center-of-mass as the origin.

As we see in Table 9.3, the difference in energy between OH^- and OH^+ is 0.497621 Hartrees (13.54 eV) according to the Correlated-SCF calculations. It is clear from the standard SCF energy value for H_2O that this basis is very good, but not quite at the Hartree-Fock limit of energy. In addition, the fitting of the numerical integration spike so as to most nearly reproduce the total energy of HF is not exact.

These two artifacts introduce an energy uncertainty of about 0.0115 Hartrees, but this is less uncertainty than that of the Quantum Monte Carlo (QMC) energy of Luchow and Anderson [61]. Note that the Iso-Dipoles for H_2O and HF are very close to the experimental values which indicates that the calculated wavefunctions are of high quality.

Table 9.3. Isoelectronium results for selected molecules [43b].

	OH^+	OH^-	H_2O	HF
SCF-Energy[a]	-74.860377	-75.396624	-76.058000	-100.060379
Hartree-Fock[b]				-100.07185[b]
Iso-Energy[c]	-75.056678	-75.554299	-76.388340	-100.448029
Horizon R_c (Å)	0.00038	0.00038	0.00038	0.00030
QMC Energy[b,d]	-76.430020[d]			-100.44296[b]
Exact non-rel.				-100.4595
Iso-Dipole (D)	5.552581	8.638473	1.847437	1.8413778
Exper. Dipole			1.84	1.82

[a] Dunning-Huzinaga ($10s/6p$), (6,2,1,1,1/4,1,1)+H2s1+H2p1+3d1.
[b] Iso-Energy calibrated to give maximum correlation for HF.
[c] Hartree-Fock and QMC energies from Luchow and Anderson [61].
[d] QMC energies from Hammond, Lester and Reynolds [60].

Since the ionization energy of a neutral H atom is 13.60 eV and the energy difference of 13.54 eV would convert OH^- to OH^+, a threshold of about 13.7 eV should maintain H^+ in solution as well as transfer $(2e)^{-2}$ through an aqueous solution to or from the OH^-/OH^+ system.

These calculations indicate that there may be an enhancement of current flow with a potential above 13.7 volts across an aqueous cell and that the enhanced concentration of HOOH may be measurable above a potential of 13.7 volts. It is worth repeating that this estimate is possible largely due to the easy convergence of the Correlated-SCF process for a negative ion species; a process which is formally not defined under the usual Hartree-Fock-Roothan process [53], and most quantum chemists are familiar with the difficulty in treating negative ions using the standard Hartree-Fock-Roothaan method.

Admitting that the Correlated-SCF equations are a parametrized approximation to the Santilli derivation of the Hulten potential [42a] for a bound electron-pair, the method has the advantage of easy incorporation into an existing Hartree-Fock-Roothaan Gaussian basis program merely by subtracting a small "correlation integral" from the usual two-electron integrals.

With some thought, one should realize that fitting the single parameter (Gaussian screening exponent, α) to experimental energies, and/or Quantum Monte Carlo results will incorporate another attraction in the form of a magnetic dipole interaction between the spin moments of paired electrons. Including the magnetic dipole interaction and substituting a Gaussian form for the Hulten exponential potential leaves only a simulation of the bound electron-pair Isoelectronium. Thus, these results are for a model in which the usual HFR-SCF method

is corrected for at least two attractive interactions of electrons causing them to approach each other as if they were "sticky"; hence the term "sticky-electron-pair model."

9.2.7 Conclusions

In Chapter 3, we have presented a covering of quantum chemistry under the name of *hadronic chemistry*. In Chapter 4, we have applied the new discipline to the construction of a new model of molecular structures based on the bonding of a pair of valence electrons from different atoms into a singlet quasi-particle state called *isoelectronium*.

We have then applied the model to the structure of the hydrogen molecule, by achieving results manifestly not possible with quantum chemistry, such as: a representation of the binding energy and other features of the hydrogen molecule accurate to the *seventh digit*; an explanation of the reason why the hydrogen molecule has only *two* hydrogen atoms; a reduction of computer usage at least 1,000 fold; and other advances.

In this chapter, we have applied the isochemical model of molecular bonds to the water and other molecules with similar results. In fact, *the isochemical model of the water and other molecules is supported by the following conceptual, theoretical and experimental evidence:*

1) It introduces a new strong binding force (which is absent in current models) capable of explaining the strength and stability of molecules;

2) It explains the reason why the water molecule has only *two* hydrogen atoms and one oxygen;

3) It permits a representation of the binding energy of the water and other molecules, which are accurate to *several digits*;

4) It represents electric and dipole moments and other features of the water and other molecules, also accurate to *several digits*;

5) It permits a reduction of computer usages in calculations at least 1,000 fold; as well as it permits other achievements similar to those obtained for the hydrogen molecule.

Moreover, as it happened for the hydrogen molecule in Chapter 4, *the value of the radius of the isoelectronium, Eqs. (9.2.9) computed via dynamical equations has been fully confirmed by independent calculations for the water and other molecules conducted via the Gaussian-lobe basis set.*

The emission of electron pairs in superconductivity has been emphasized in Chapter 3. In Chapter 4 we also indicated *preliminary, yet direct experimental verifications of the isochemical model of molecular bonds offered by the ongoing experiments on photoproduction of the valence electrons in the helium indicating that electrons are emitted in pairs* [62]. The systematic repetition of these experiments *specifically for water* is here recommended. The statistical percentages

of electron pairs over the total number of emitted electrons would then establish whether the isoelectronium is fully or only partially stable.

We should finally note that *the representation of the binding energy, electric and magnetic moments and other characteristics of the water and other molecules exact to the several digits, as first achieved in Refs.* [43] *constitutes the strongest experimental evidence to date on the insufficiency of quantum mechanics and the validity of the covering hadronic mechanics for the representation of nonlinear, nonlocal and nonpotential-nonunitary effects, due to deep overlappings of the "extended wavepackets" of electrons with a point-like charge structure.*

The new isochemical model of the water molecule outlined in this chapter has a number of intriguing new applications. For instance, the correlated-SCF method is used to easily obtain an energy for the OH- anion in water, while the OH^+ ion is easily treated in either the standard or modified method. The difference in energy between the 8-electron OH^+ system and the 10-electron OH^- system is found to be 13.54 eV. This represents the energy needed to remove $(2e)^{-2}$ from OH^-. This indicates there may be a threshold for current flow in terms of $(2e)^{-2}$ as a quasi-particle in aqueous media at 13.6 eV. This voltage will also maintain H^+ in solution to some extent. Organic alkenes in solution should undergo epoxidation followed by solvolysis to diols under the conditions of abundant $(2e)^{-2}$.

Another interesting result is that the natural trace amounts of HOOH in water may be increased in water by merely placing the sample in an intense magnetic field. Positive and negative ions will traverse short arc segment paths driven by simple thermal Brownian motion in a way which will lead to an increase in collisions of oppositely charged ions. In particular, OH^- and OH^+ may undergo collisions more frequently leading to an increase in HOOH.

This additional HOOH should then be available to react with alkenes to form epoxides which will then hydrolyze in water to form diols. Such diols would be much more soluble in water than the original alkenes. This leads to the important possibility that merely exposing water-insoluble alkenes to water in a magnetic field will lead to a chemical reaction of the alkenes to form modified compounds which are more soluble in water. In other words, organic oils containing some double bonds may be made somewhat more soluble in water just by mechanical emulsification of the oils in water in an environment of a high magnetic field.

Thus, mixtures of oils and water could be mechanically agitated in a magnetic field of several Tesla to produce new oils which are chemically similar to the original oils (assuming a large organic structure) but more soluble in water after exposure to the magnetic field (see Chapter 8 for details).

Similarly, it is easy to see that, while the conventional quantum chemical model of the water molecule predicts one and only configuration, our isochemical model predicts various physically inequivalent configurations depending on the relative

orientation of the two *oo*-shaped orbits and other properties, which are under separate study.

The industrial significance of the studies outlined in this chapter will be presented in Chapters 7 and 8.

9.3 VARIATIONAL CALCULATIONS OF ISOCHEMICAL MODELS

9.3.1 Introduction

In Ref. [64a] outlined in Chapter 4, Santilli and Shillady introduced a restricted isochemical three-body model of the hydrogen molecule admitting an exact solution, and a full four-body isochemical model of the hydrogen molecule which no longer admits an exact solution.

In Ref. [64b] outlined in Chapter 5, Santilli and Shillady introduced two corresponding isochemical models of the water and other molecules, one based on a restricted three-body model of the HO dimer admitting exact solutions, and a second fully isochemical four-body model.

As also reviewed in Chapters 4 and 5, Shillady's SASLOBE variational method [64] showed the capability of the isochemical models to reach an essentially exact representation of experimental data on the hydrogen, water and other molecules, as well as resolving other shortcomings or inconsistencies of conventional quantum chemical molecular models.

A greatly detailed, independent verification of models [64a,64b] was conducted by A.K. Aringazin and M.G. Kucherenko [65a] via exact solution and by A.K. Aringazin [65b] via Ritz's variational method, by confirming all numerical results of Refs. [64].

In this chapter we outline Refs. [65] since they achieve new important insights and results on isochemistry of rather general character, and possible application to a variety of other molecules and applications of isochemistry.

9.3.2 Aringazin-Kucherenko Study of the Restricted, Three-Body Isochemical Model of the Hydrogen Molecule

In this section we outline the studies by Aringazin and Kucherenko [65a] of Santilli-Shillady exactly solvable, restricted three-body isochemical model of the H_2 molecule [64a], Eq. (9.1.35), $r_{12} \simeq 0$, i.e.,

$$-\frac{\hbar^2}{2M}\nabla^2_{ab}\psi + \left(-\frac{2e^2}{r_a} - \frac{2e^2}{r_b} + \frac{2e^2}{R}\right)\psi = E\psi. \tag{9.3.1}$$

As the reader will recall from Chapter 4, model (9.3.1) constitutes a limit case in which the two valence electrons are assumed to be permanently bonded together

into the stable singlet quasi-particle state with features (9.1.25), i.e.,

$$\text{mass} \simeq 1 \text{ MeV}, \quad \text{spin} = 0, \quad \text{charge} = 2e, \quad \text{magnetic moment} \simeq 0,$$
$$\text{radius} = r_c = b^{-1} = 6.8432329 \times 10^{-11} \text{ cm} = \tag{9.3.2}$$
$$= 0.015424288 \text{ bohrs} = 0.006843 \text{ Å},$$

which we have called *isoelectronium*.

The assumption of stationary nuclei (or, equivalently, nuclei with infinite inertia), then turns the four-body hydrogen molecule H_2 into a restricted three-body system which, as such, admits exact solution.

The reader should also recall that, the assumption of the rest energy of the isoelectronium as given by twice the electron mass is merely an upper boundary occurring when the internal forces are of purely nonpotential type. In reality, a total attractive force of purely potential type is possible because the magnetostatic attraction is bigger than the electrostatic repulsion as illustrated in Fig. 9.4. It is evident that the latter bond implies a negative binding energy resulting in a value of the isoelectronium mass

$$M_{\text{isoelectronium}} < 2m_{\text{electron}}, \tag{9.3.3}$$

which is unknown, and should be derived from fitting experimental data.

As one can see, the above restricted isochemical model of the H_2 molecule is similar to the conventional restricted three-body H_2^+ ion. To avoid confusion, we shall denote the three-body isochemical model with the "hat", $\hat{H}_2$, and the conventional (four-body) model without the "hat," H_2.

More specifically, studies [65a] were conducted under the following conditions:
1) the isoelectronium is stable;
2) the effective size of the isoelectronium is ignorable, in comparison to internuclear distance of H_2;
3) the two nuclei of H_2 are at rest;
4) the rest energy of the isoelectronium is assumed to be unknown and to be determined by the fit of the binding energy of the molecule;
5) the internuclear distance R of H_2 is also assumed to be unknown and to be fitted from the stability condition of the solution, and then compared with its experimental value.

A main result of Ref. [65a] is that the restricted three-body Santilli-Shillady model $\hat{H}_2$ is capable to fit the experimental binding energy for the following value of the isoelectronium mass,

$$M = 0.308381 m_e, \tag{9.3.4}$$

although its stability condition is reached for the following internuclear distance

$$R = 1.675828 \text{ a.u.}, \tag{9.3.5}$$

which is about 19.6% bigger than the conventional experimental value $R[H_2] = 1.4011$ a.u. $= 0.742$ Å.

These results confirm that the isochemical model (9.3.1) is indeed valid, but only in first approximation, in accordance with the intent of the original proposal [64a].

In Born-Oppenheimer approximation, i.e., at fixed nuclei, the equation for the H_2^+ ion-type system for a particle of mass M and charge q is given by

$$\nabla^2\psi + 2M\left(E + \frac{q}{r_a} + \frac{q}{r_b}\right)\psi = 0. \tag{9.3.6}$$

In spheroidal coordinates,

$$x = \frac{r_a + r_b}{R}, \quad 1 \le x \le \infty; \quad y = \frac{r_a - r_b}{R}, \quad -1 \le y \le 1; \quad 0 \le \varphi \le 2\pi, \tag{9.3.7}$$

where R is the separation distance between the two nuclei a and b, we have

$$\nabla^2 = \frac{4}{R^2(x^2-y^2)}\left(\frac{\partial}{\partial x}(x^2-1)\frac{\partial}{\partial x} + \frac{\partial}{\partial y}(1-y^2)\frac{\partial}{\partial y}\right) + \tag{9.3.8}$$

$$+\frac{1}{R^2(x^2-1)(1-y^2)}\frac{\partial^2}{\partial\varphi^2}.$$

Eq. (9.3.1) then becomes

$$\left[\frac{\partial}{\partial x}(x^2-1)\frac{\partial}{\partial x} + \frac{\partial}{\partial y}(1-y^2)\frac{\partial}{\partial y} + \frac{x^2-y^2}{4(x^2-1)(1-y^2)}\frac{\partial^2}{\partial\varphi^2} + \right. \tag{9.3.9}$$

$$\left. +\frac{MER^2}{2}(x^2-y^2) + 2MqRx\right]\psi = 0,$$

where

$$\frac{1}{r_a} + \frac{1}{r_b} = \frac{4}{R}\frac{x}{x^2-y^2}. \tag{9.3.10}$$

The use of the expression

$$\psi = f(x)g(y)e^{im\varphi}, \tag{9.3.11}$$

then allows the separation

$$\frac{d}{dx}\left((x^2-1)\frac{d}{dx}f\right) - \left(\lambda - 2MqRx - \frac{MER^2}{2}x^2 + \frac{m^2}{x^2-1}\right)f = 0,$$

$$\frac{d}{dy}\left((1-y^2)\frac{d}{dy}g\right) + \left(\lambda - \frac{MER^2}{2}y^2 - \frac{m^2}{1-y^2}\right)g = 0, \tag{9.3.12}$$

where λ is the separation constant. The exact solutions for $f(x)$ and $g(y)$ are given by the angular and radial Coulomb spheroidal functions (CSF) containing infinite recurrence relations.

Aringazin and Kucherenko [65a] calculated the energy levels via the use of recurrence relations of the type

$$Q_{k+1} = Q_k \bar{\kappa}_{N-k} - Q_{k-1} \bar{\rho}_{N-k} \bar{\bar{\delta}}_{N-k+1}, \quad Q_{-1} = 0, \quad Q_0 = 1, \tag{9.3.13}$$

where the coefficients are

$$\begin{aligned} \rho_s &= \frac{(s+2m+1)[b-2p(s+m+1)]}{2(s+m)+3}, \\ \kappa_s &= (s+m)(s+m+1) - \lambda, \\ \delta_s &= \frac{s[b+2p(s+m)]}{2(s+m)-1}. \end{aligned} \tag{9.3.14}$$

Ref. [65a] then used the value $N = 16$ for the power degree approximation of both the radial and angular components. The two polynomials have 16 roots for λ from which only one root is appropriate due for its asymptotic behavior at $R \to 0$. Numerical solution of the equation,

$$\lambda^{(x)}(p,a) = \lambda^{(y)}(p,b), \tag{9.3.15}$$

gives the list of values of the electronic ground state energy,

$$E(R) = E_{1s\sigma}(R), \tag{9.3.16}$$

which corresponds to $1s\sigma_g$ term of the H_2^+ ion-like system, as a function of the distance R between the nuclei. Note that the term "exact solution" refers to the fact that by taking greater values of N, for example $N = 50$, one can achieve higher accuracy, up to a desired one (for example, twelve decimals).

Also, the scaling method based on the Schrödinger equation has been developed which enables one to relate the final $E(R)$ dependence of different H_2^+ ion-like systems to each other.

Table 9.4 presents result of the calculations of the *minimal total energy* and the corresponding *optimal distance*, at various values of the isoelectronium mass parameter

$$M = \eta m_e, \tag{9.3.17}$$

where $M = \eta$, in atomic units.

Aringazin and Kucherenko [65a] computed some 27 tables, each with the identification of the minimum of the total energy, together with the corresponding optimal distance R. Then, they collected all the obtained energy minima and optimal distances in Table 9.4.

Table 9.4. The minimal total energy E_{min} and the optimal internuclear distance R_{opt} of Santilli-Shillady restricted three-body isochemical model $\hat{H}_2$ as functions of the mass M of the isoelectronium[a].

M, *a.u.*	E_{min}, *a.u.*	R_{opt}, *a.u.*	M, *a.u.*	E_{min}, *a.u.*	R_{opt}, *a.u.*
0.10	-0.380852	5.167928	0.32	-1.218726	1.614977
0.15	-0.571278	3.445291	0.33	-1.256811	1.566041
0.20	-0.761704	2.583964	0.34	-1.294896	1.519981
0.25	-0.952130	2.067171	0.35	-1.332982	1.476553
0.26	-0.990215	1.987664	0.40	-1.523408	1.291982
0.27	-1.028300	1.914050	0.45	-1.713834	1.148428
0.28	-1.066385	1.845688	0.50	-1.904260	1.033585
0.29	-1.104470	1.782044	0.75	-2.856390	0.689058
0.30	-1.142556	1.722645	1.00	-3.808520	0.516792
0.307	-1.169215	1.683367	1.25	-4.760650	0.413434
0.308	-1.173024	1.677899	1.50	-5.712780	0.344529
0.308381	-1.174475	1.675828	1.75	-6.664910	0.295310
0.309	-1.176832	1.672471	2.00	-7.617040	0.258396
0.31	-1.180641	1.667073			

[a] See also Figs. 9.10 and 9.11.

Table 9.5. Summary of main data and results on the ground state energy E and the internuclear distance R.

	E, *a.u.*	R, *a.u.*
H_2^+ ion, exact theory, N=16 [65a]	-0.6026346	1.9971579
H_2^+ ion, experiment [66]	-0.6017	2.00
3-body $\hat{H}_2$, M=2m_e, exact theory [65a]	-7.617041	0.258399
3-body $\hat{H}_2$, M=2m_e, var. theory [64a]	-7.61509174	0.2592
3-body $\hat{H}_2$, M=0.381m_e, exact theory [65a]	-1.174475	1.675828
4-body H_2, r_c=0.01125 a.u., V_g var. theory [64a]	-1.174474	1.4011
4-body H_2, r_c=0.01154 a.u., V_e var. theory [65b]	-1.144	1.4011
4-body H_2, r_c=0.08330 a.u., V_e var. theory [65b]	-1.173	1.3184
H_2, experiment	-1.174474	1.4011

With the fourth order interpolation/extrapolation, the graphical representations of Table 9.4 (see Figs. 9.10 and 9.11) show that the minimal total energy behaves as

$$E_{min}(M) \simeq -3.808M, \tag{9.3.18}$$

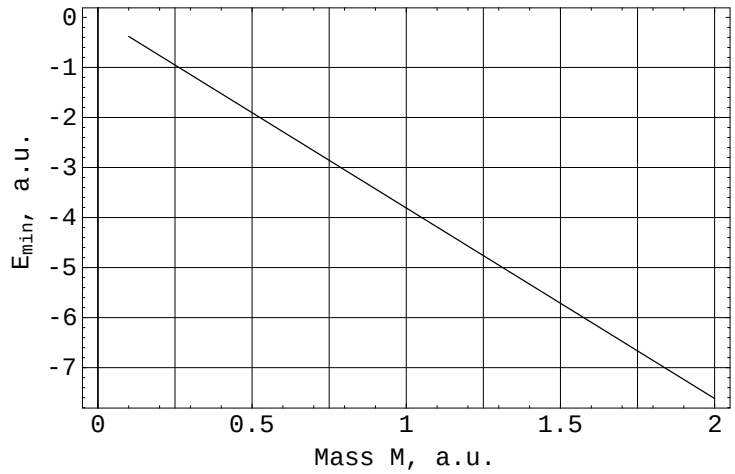

Figure 9.10. The minimal total energy $E_{min}(M)$ of the $\hat{H}_2$ system as a function of the isoelectronium mass M.

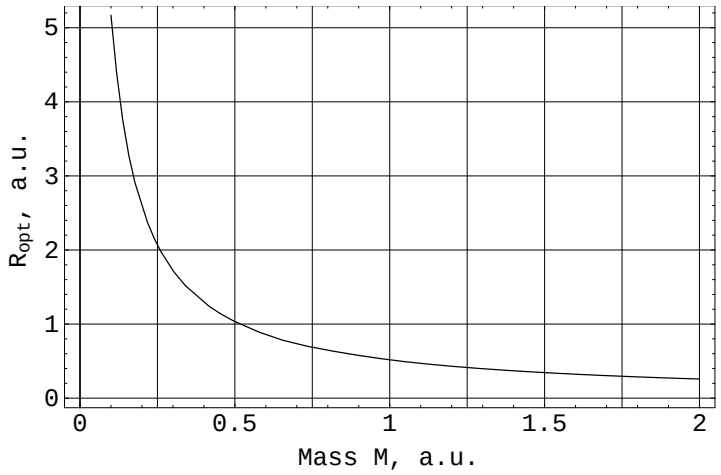

Figure 9.11. The optimal internuclear distance $R_{opt}(M)$ of the $\hat{H}_2$ system as a function of the isoelectronium mass M.

and the optimal distance behaves as

$$R_{opt}(M) \simeq 0.517/M. \tag{9.3.19}$$

One can see that at $M = 2m_e$ we have

$$E_{min}(M) = -7.617041 \text{ a.u.}, \quad R_{opt}(M) = 0.258399 \text{ a.u.}, \tag{9.3.20}$$

which recover the values obtained in Ref. [64a]

$$E_{min} = -7.61509174 \text{ a.u.}, \quad R_{opt} = 0.2592 \text{ a.u.}, \tag{9.3.21}$$

to a remarkable accuracy.

The conclusion by Aringazin and Kucherenko is that the Santilli-Shillady restricted three-body isochemical model of the hydrogen molecule is indeed valid as suggested, that is, as in first approximation. The main data and results on E_{min} and R_{opt} are collected in Table 9.5.

An important conclusion of Ref. [65a] is, therefore, that *the two valence electrons of the hydrogen molecule cannot be permanently bound inside the hadronic horizon with radius of one Fermi.*

The clear understanding, stressed in Chapter 4, is that the isoelectronium must continue to exist beyond the hadronic horizon, otherwise, in its absence, we would have a violation of Pauli's exclusion principle.

9.3.3 Aringazin Variational Study of the Four-Body Isochemical Model of the Hydrogen Molecule

In the subsequent Ref. [65b] Aringazin applied Ritz variational method to Santilli-Shillady four-body isochemical model of molecule of the hydrogen molecule (9.1.33), i.e.

$$\left(-\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 - V_0 \frac{e^{-r_{12}/r_c}}{1-e^{-r_{12}/r_c}} + \frac{e^2}{r_{12}} - \right. \tag{9.3.22}$$

$$\left. -\frac{e^2}{r_{1a}} - \frac{e^2}{r_{2a}} - \frac{e^2}{r_{1b}} - \frac{e^2}{r_{2b}} + \frac{e^2}{R} \right) |\psi\rangle = E|\psi\rangle,$$

without restriction that the isoelectronium has the permanent dimension of about one Fermi.

In particular, Aringazin's objective was to identify the ground state energy and bond length of the H_2 molecule, in Born-Oppenheimer approximation, via a Gaussian screening of the Coulomb potential, V_g, the exponential screening of the Coulomb potential,

$$V_e = -\frac{Ae^{-r_{12}/r_c}}{r_{12}}, \tag{9.3.23}$$

as well as the original Hulten potential V_h of the model (9.3.22). The resulting analysis is quite sophisticated, and cannot be reviewed herein the necessary detail. Readers seriously interested in this verification of the new isochemical model of the hydrogen molecule are suggested to study Aringazin's original memoir [65b].

The Coulomb and exchange integrals were calculated only for V_e while for V_g and V_h Aringazin achieved analytical results only for the Coulomb integrals because of the absence of Gegenbauer-type expansions for the latter potentials.

A conclusion is that *the Ritz's variational treatment of model (9.2.45) with the potential (9.3.23) is capable to provide an exact fit of the experimental data of the hydrogen molecule in confirmation of the results obtained by Santilli and Shillady [64a] via the SASLOBE variational approach to Gaussian V_g-type model.* The main data and results on the ground state energy E_{min} and internuclear distance R_{opt} are collected in Table 9.5.

Note that in the variational approach of Ref. [65b] Aringazin used a *discrete* variation of the hadronic horizon r_c and approximate exchange integral (9.3.24) that resulted in approximate fittings of the energy and distance, as shown in Table 9.5.

In addition, Ref. [65b] computed the weight of the isoelectronium phase which results to be of the order of 1% to 6% that for the case of V_e model. However, we note that this is it not the result corresponding to the original Santilli-Shillady model, which is based on the Hulten potential V_h.

An interesting result is that in order to prevent divergency of the Coulomb integral for V_h *the correlation length parameter r_c should run discrete values* due to Eq. (9.3.25). This condition has been used in the V_e model, although it is not a necessary one within the framework of this model.

As recalled earlier, Aringazin [*loc. cit.*] assumes that the isoelectronium undergoes an increase of length beyond the hadronic horizon, and the resulting two electrons are separated by sufficiently large distance. This leads us to problem of how to compute *the effective life-time of isoelectronium.*

To estimate the order of magnitude of such a life-time, Aringazin uses the ordinary formula for radioactive α-decay since the total potential $V(r)$ is of the same shape as that here considered, with very sharp decrease at $r < r_{max}$ and Coulomb repulsion at $r > r_{max}$, where r_{max} corresponds to a maximum of the potential.

This quasiclassical model is a crude approximation because in reality the electrons do not leave the molecule. Moreover, the two asymptotic regimes act simultaneously, with some distribution of probability, and it would be more justified to treat the frequency of the decay process (i.e., the tunneling outside the hadronic horizon), rather than the life-time of the isoelectronium.

However, due to the assumption of the small size of isoelectronium in comparison to the molecule size, we can study an elementary process of decay separately, and use the notion of life-time. The results of Aringazin's calculations are presented in Table 9.6.

In Ritz's variational approach, the main problem is to calculate analytically the so-called *molecular integrals.* The variational molecular energy in which we are interested, is expressed in terms of these integrals. These integrals arise when using some wave function, usually a simple hydrogen-like ground state wave function, as an infinite separation asymptotic solution, in the Schrödinger

Table 9.6. Summary of Aringazin's calculations [65b] on the lifetime of the isoelectronium, where E is relative kinetic energy of the electrons, at large distance, $r \gg r_{max}$, in the center of mass system.

Energy E, a.u.	*eV*	*Lifetime, D_0·sec*
2	54.4	$2.6 \cdot 10^{-18}$
1	27.2	$1.6 \cdot 10^{-17}$
0.5	13.6	$2.2 \cdot 10^{-16}$
0.037	1	$5.1 \cdot 10^{-6}$
0.018	0.5	4.0
0.0018	0.1	$3.1 \cdot 10^{+25}$

equation for the diatomic molecule. The main idea of Ritz's approach is to introduce parameters into the wave function, and vary them together with the separation parameter R, to achieve a minimum of the total molecular energy, which is treated as the resulting ground state energy.

In the case under study, Aringazin [*loc. cit.*] uses two parameters, γ and ρ, where γ enters hydrogen-like ground state wave function

$$\psi(r) = \sqrt{\frac{\gamma^3}{\pi}}\, e^{-\gamma r}, \tag{9.3.26}$$

and $\rho = \gamma R$ measures internuclear distance. These parameters should be varied *analytically or numerically* in the final expression of the molecular energy, after the calculation is made for the associated molecular integrals.

However, the four-body Santilli-Shillady model H_2 suggests an additional Hulten potential interaction between the electrons, which potential contains two parameters V_0 and r_c, where V_0 is a general factor, and r_c is a correlation length parameter characterizing the hadronic horizon. Thus, four parameters should be varied, γ, ρ, V_0, and r_c.

The introducing of Hulten potential leads to a modification of some molecular integrals, namely, of the Coulomb and exchange integrals. The other molecular integrals remain the same as in the case of the usual model of H_2, with well-known analytic results. Normally, the Coulomb integral, which can be computed in bispherical coordinates, is much easier to resolve than the exchange integral, which is computed in bispheroidal coordinates.

Calculations of the Coulomb integral for Hulten potential V_h appeared to be quite nontrivial [2b]. Namely, in the used bispherical coordinates, several special functions, such as polylogarithmic function, Riemann zeta-function, digamma function, and Lerch function, appeared during the calculation.

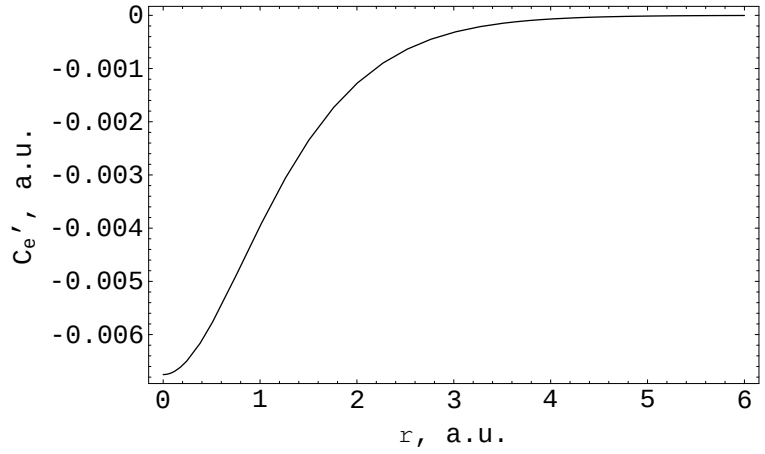

Figure 9.12. The Coulomb integral C'_e as a function of ρ, at $\lambda = 1/37$, where $\rho = \gamma R$, R is the internuclear distance, $\lambda = 2\gamma r_c$, and r_c is the hadronic horizon.

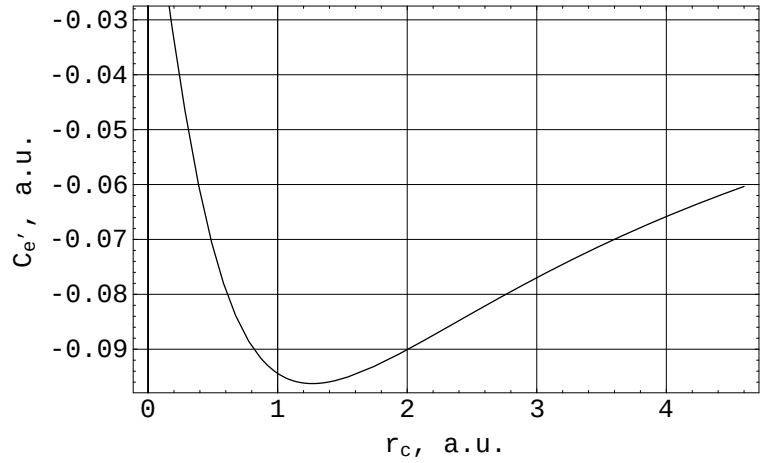

Figure 9.13. The Coulomb integral C'_e as a function of r_c, at $\rho = 1.67$. For $r_c > 0.2$ a.u., the regularized values are presented.

In order to proceed with the Santilli-Shillady approach, Aringazin [65b] invoked two different *simplified* potentials, the exponential screened Coulomb potential V_e, and the Gaussian screened Coulomb potential V_g, instead of the Hulten potential V_h. The former potentials both approximate well the Hulten potential at

short and long range asymptotics, and each contains two parameters denoted A and r_c.

In order to reproduce the short range asymptotics of the Hulten potential, the parameter A should have the value $A = V_0 r_c$, for both potentials. The Coulomb integrals for these two potentials have been calculated *exactly* owing to the fact that they are much simpler than the Hulten potential.

In particular, we note that the final exact expression of the Coulomb integral for V_g contains only one special function, the error function $\mathrm{erf}(z)$, while for V_e it contains no special functions at all. In this way, Aringazin [65b] reaches the exact expression

$$C'_e = -\frac{A\lambda^2}{8(1-\lambda^2)^4}\frac{\gamma e^{-2\rho}}{\rho}\left[-(\rho+2\rho^2+\frac{4}{3}\rho^3)+3\lambda^2(5\rho+10\rho^2+4\rho^3)-\right.$$

$$\left.-\lambda^4(15\rho+14\rho^2+4\rho^3)+\lambda^6(8+11\rho+6\rho^2+\frac{4}{3}\rho^3-8e^{2\rho-\frac{2\rho}{\lambda}})\right], \qquad (9.3.27)$$

where $\lambda = 2\gamma r_c$. This Coulomb integral is plotted in Figs. 9.12 and 9.13.

The most difficult part of calculations [65b] is the exchange integral. Usually, to calculate it one has to use bispheroidal coordinates, and needs in an expansion of the potential in some orthogonal polynomials, such as Legendre polynomials in bispheroidal coordinates. In Ref. [65b], only the exponential screened potential V_e is known to have such an expansion but it is formulated, however, in terms of bispherical coordinates (so called Gegenbauer expansion). Accordingly, the exchange integral E'_e for V_e at *null* internuclear separation, $R = 0$ (in which case one can use bispherical coordinates) was calculated exactly. After that, the R-dependence using the standard result for the exchange integral for Coulomb potential E'_C (celebrated Sugiura's result) was partially recovered,

$$E'_e \simeq \frac{A\lambda^2}{(1+\lambda)^4}\left(\frac{1}{8}+\frac{1}{2}\lambda+\frac{5}{8}\lambda^2\right)\frac{8}{5}E'_C, \qquad (9.3.28)$$

where $\lambda = 2\gamma r_c$ (see Fig. 9.14). Thus, only some approximate expression of the exchange integral for the case of V_e has been achieved. In this way, all subsequent results apply to the approximate V_e-based model.

Inserting the so-obtained V_e-based Coulomb and exchange integrals into the total molecular energy expression, the final analytical expression containing four parameters, γ, ρ, A, and r_c, was obtained. From a separate consideration of the Hulten potential case, the existence of a bound state of two electrons (which is the proper isoelectronium) leads to the following relationship between the parameters for the case of one energy level of the electron-electron system, $V_0 = \hbar^2/(2mr_c^2)$. Thus, using the relation $A = V_0 r_c$ Aringazin has $A = 1/r_c \equiv 2\gamma/\lambda$, in atomic units ($\hbar = m_e = c = 1$).

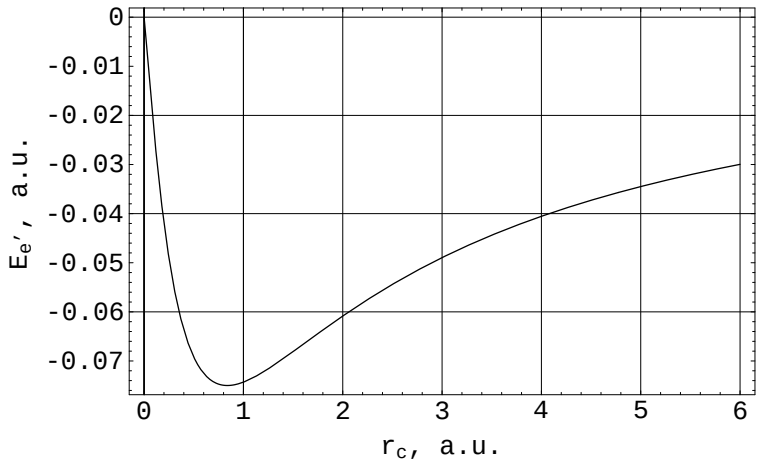

Figure 9.14. The exchange integral E'_e as a function of r_c, at $\rho = 1.67$.

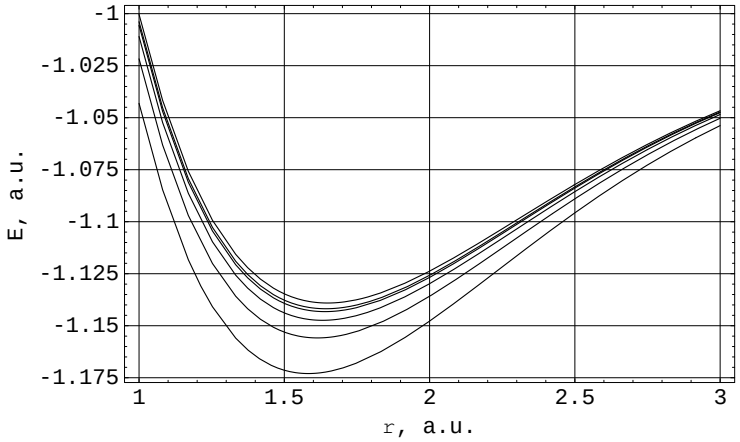

Figure 9.15. The total energy $E = E_{mol}$ as a function of ρ, at $\lambda = 1/60, 1/40, 1/20, 1/10, 1/5$. The lowest plot corresponds to $\lambda = 1/5$.

Note that Aringazin [65b] introduces the *one-level isoelectronium* characterized by the fact that the condition,

$$\lambda^{-1} = \text{ integer numbers} > 0, \tag{9.3.29}$$

follows from the analysis of the Coulomb integral for Hulten potential.

With the above set up, minimization of the total molecular energy of the V_e-based model can be made. Numerical analysis shows that the λ-dependence does

not reveal any minimum in the interval of interest,

$$4 \leq \lambda^{-1} \leq 60, \tag{9.3.30}$$

while there is a minimum of the energy for some values of γ and ρ, at fixed λ.

Therefore, 56 tables have been calculated to identify the energy minima and optimal distances for different values of λ, in the interval (9.3.30).

Table 9.7. The total minimal energy E_{min} and the optimal internuclear distance R_{opt} as functions of the correlation length r_c for the exponential screened Coulomb potential V_e.

λ^{-1}	r_c, *a.u.*	R_{opt}, *a.u.*	E_{min}, *a.u.*
4	0.10337035071618050	1.297162129235449	-1.181516949656805
5	0.08329699109108888	1.318393698326879	-1.172984902150024
6	0.06975270534273319	1.333205576478603	-1.167271240301846
7	0.05999677404817234	1.344092354783681	-1.163188554065554
8	0.05263465942162049	1.352417789644028	-1.160130284706318
9	0.04688158804756491	1.358984317233049	-1.157755960428922
10	0.04226204990365446	1.364292909163710	-1.155860292450436
11	0.03847110142927672	1.368671725082009	-1.154312372623724
12	0.03530417706681329	1.372344384866235	-1.153024886026671
13	0.03261892720535206	1.375468373051375	-1.151937408039373
14	0.03031323689615631	1.378157728092548	-1.151006817317425
15	0.02831194904031777	1.380497017045902	-1.150201529091051
16	0.02655851947236431	1.382550255552670	-1.149497886394651
17	0.02500959113834722	1.384366780045693	-1.148877823925501
18	0.02363136168905809	1.385985219224291	-1.148327310762828
19	0.02239708901865092	1.387436244558651	-1.147835285349041
20	0.02128533948435381	1.388744515712491	-1.147392910500336
21	0.02027873303335994	1.389930082626193	-1.146993041730378
22	0.01936302821907175	1.391009413196452	-1.146629840949675
23	0.01852644434336641	1.391996158084790	-1.146298491232105
24	0.01775915199935013	1.392901727808297	-1.145994983116511
25	0.01705288514774330	1.393735733699196	-1.145715952370148
26	0.01640064219648127	1.394506328745493	-1.145458555325045
27	0.01579645313764336	1.395220473843219	-1.145220372020229
28	0.01523519631632570	1.395884147817973	-1.144999330178493
29	0.01471245291356761	1.396502514589167	-1.144793644973560
30	0.01422439038752817	1.397080057337240	-1.144601770891686

Aringazin's results are collected in Tables 9.7, 9.8, and Fig. 9.15. One can see that the binding energy decreases with the increase of the parameter r_c, which corresponds to an effective radius of the isoelectronium.

In conclusion, the calculation by Aringazin [65b] reviewed in this Chapter have not identified the meanlife of the isoelectronium assumed as a quasiparticle

Table 9.8. A continuation of Table 9.8.

λ^{-1}	r_c, *a.u.*	R_{opt}, *a.u.*	E_{min}, *a.u.*
31	0.01376766836566138	1.397620687025853	-1.144422362947838
32	0.01333936209977966	1.398127830817745	-1.144254245203342
33	0.01293689977547854	1.398604504597664	-1.144096385030938
34	0.01255801083612469	1.399053372836414	-1.143947871939897
35	0.01220068312791624	1.399476798299823	-1.143807900045981
36	0.01186312715793131	1.399876883556063	-1.143675753475045
37	0.01154374612489787	1.400255505817128	-1.143550794143290
39	0.01095393745919852	1.400954915288619	-1.143320213707519
40	0.01068107105944273	1.401278573036792	-1.143213620508321
41	0.01042146833640030	1.401586548200467	-1.143112256673494
42	0.01017418516195214	1.401879953246168	-1.143015746732479
43	0.00993836493541500	1.402159797887369	-1.142923750307661
44	0.00971322867044429	1.402427000676349	-1.142835958109381
45	0.00949806639934841	1.402682399061957	-1.142752088467028
46	0.00929222969498477	1.402926758144872	-1.142671884314343
47	0.00909512514431396	1.403160778323019	-1.142595110561057
48	0.00890620863525624	1.403385101987775	-1.142521551794315
49	0.00872498034101540	1.403600319405678	-1.142451010262626
50	0.00855098030451296	1.403806973898863	-1.142383304102633
51	0.00838378454080327	1.404005566419838	-1.142318265775268
52	0.00822300158793934	1.404196559601683	-1.142255740683024
53	0.00806826944722482	1.404380381352424	-1.142195585944305
54	0.00791925286251402	1.404557428052374	-1.142137669304475
55	0.00777564089552400	1.404728067404676	-1.142081868166104
56	0.00763714476025456	1.404892640982100	-1.142028068723488
57	0.00750349588477794	1.405051466507240	-1.141976165188595
58	0.00737444417302681	1.405204839898059	-1.141926059097351
59	0.00724975644291090	1.405353037106507	-1.141877658686723
60	0.00712921502024112	1.405496315774223	-1.141830878334298

of charge radius r_c of about 1 fm. As one can see in Table 9.7, the predicted meanlife varies over a rather large range of values.

The achievement of an accurate meanlife of the isoelectronium of 1 fm charge radius can be reached only after reaching a more accurate knowledge of its rest energy. As the reader will recall from Chapter 4, the value of 1 MeV should be solely considered as an upper boundary value of the rest energy of the isoelectronium, since it holds only in the absence of internal potential forces while the latter cannot be excluded. Therefore, the actual value of the rest energy of the isoelectronium is today basically unknown.

The reader should also recall that the terms "meanlife of the isoelectronium when of charge radius of about 1 fm" are referring to the duration of time spent by two valence electrons at a mutual distance of 1 fm which is expected to be small. The understanding explained in Chapter 3 is that, *when the restriction of the charge radius to 1 fm is removed, and orbital mutual distances are admitted, the isoelectronium must have an infinite life (for the unperturbed molecule), because any finite meanlife under the latter conditions would imply the admission of two electrons with identical features in the same orbit, and a consequential violation of Pauli's exclusion principle.*

An interesting result of the Ritz variational approach to the Hulten potential studied by Aringazin [65b] is that *the charge radius of the isoelectronium r_c entering the Hulten potential and the variational energy, should run discrete set of values during the variation.*

In other words, this means that *only some fixed values of the effective radius of the one-level isoelectronium are admitted in the Santilli-Shillady model when treated via the Ritz approach.*

This result was completely unexpected and may indicate a kind of "hadronic fine structure" of the isoelectronium whose origin and meaning are unknown at this writing. It should be indicated that such a "hadronic fine structure" of the isoelectronium is solely referred to the case when r_c is restricted to be about 1 fm or less. The problem whether such a "hadronic fine structure" persists for values of r_c up to orbital distances is also unknown at this writing. It should be also indicated that this remarkable property is specific to the Hulten potential V_h, while it is *absent* in the V_e, or V_g models.

Moreover, Aringazin [65b] has achieved an estimation of *the weight of the isoelectronium phase* for the case of V_e model which appears to be of the order of 1% to 6%. This weight has been estimated from the energy contribution related to the exponentially screened potential V_e, in comparison to the contribution related to the usual Coulomb interelectron repulsive potential.

Finally, an important result of the Ritz variational four-body model studied by Aringazin [65b] is its fit to the experimental data of both the binding energy E and the bond length R of the hydrogen molecule thus providing an excellent independent confirmation of the results obtained by Santilli and Shillady [64].

Appendix 9.A
Isochemical Calculations for the Three-Body H_2 Molecule

This appendix contains a summary of the computer calculations conducted in Ref. [5] for the restricted three-body model of the hydrogen molecule according to isochemistry, Eq. (9.1.35), showing an exact representation of the binding energy. The calculations are based on the isoelectronium as per characteristics (9.1.25).

```
Gaussian-Lobe Program for Large Molecules
  set up by D. Shillady and S. Baldwin
      Richmond Virginia 1978-1997
        3 BODY H2 (Electronium)
```

ipear = 1, dt = 0.0, tk = 0.0, imd = 0, ntime = 60, mul = 1, iqd = 0, icor = 3, mdtim = 0, idb = 0.

ELECTRONIUM-PAIR CALCULATION

Atomic Core				Nuclear		Coordinates
		X		Y		Z
1.		0.000000		0.000000		0.000000
	Z1s =	6.103	Z2s =	24.350	Z2p =	24.350
	Z3s =	16.230	Z3p =	16.230	Z3d =	-16.200
	Z4p =	12.180	Z4f =	12.180		
1.		0.000000		0.000000		0.259200
	Z1s =	6.103	Z2s =	24.350	Z2p =	24.350
	Z4p =	12.180	Z4f =	12.18O	Z3d =	-16.200

Basis Size = 50 and Number of Spheres = 142 for 2 Electrons.

Distance Matrix in Angströms:

	H	H
H	0.00000	0.13716
H	0.13716	0.00000

A-B-C Arcs in Degrees for 2 Atoms.
The Center of Mass is at Xm = 0.000000, Ym = 0.000000, Zm = 0.129600.

One-Electron Energy Levels:

E(1) =	-11.473116428176	E(26) =	28.974399759209
E(2) =	-4.103304982059	E(27) =	28.974400079775
E(3) =	-1.621066945385	E(28) =	31.002613061833
E(4) =	-1.621066909587	E(29) =	31.002614578175
E(5) =	0.735166320188	E(30) =	35.201145239721
E(6) =	3.760295564718	E(31) =	38.003259639003
E(7) =	3.760295673022	E(32) =	44.948398097510
E(8) =	4.206194459198	E(33) =	44.94839B118458
E(9) =	4.813241859203	E(34) =	52.259825531212
E(10) =	11.2330B0571453	E(35) =	57.732587951875
E(11) =	15.70B645318078	E(36) =	57.732589021798
E(12) =	15.708645469273	E(37) =	68.743644612501
E(13) =	18.535761604401	E(38) =	68.743644649428
E(14) =	18.535761951543	E(39) =	73.195648957615
E(15) =	19.329445299735	E(40) =	79.303486379907
E(16) =	19.329445306194	E(41) =	85.865499885249
E(17) =	19.644048052034	E(42) =	85.865531919077
E(18) =	24.002368034839	E(43) =	127.196518644932
E(19) =	24.002368621986	E(45) =	130.602186113463
E(20) =	24.076849036707	E(46) =	130.602190550265
E(21) =	24.076853269415	E(47) =	137.484863078186
E(22) =	24.574406183060	E(48) =	158.452350229845
E(23) =	26.836031180463	E(49) =	205.158233049979
E(25) =	27.860752485358	E(50) =	446.152984041077

epair Energy = -7.615091736818.

Appendix 9.B
Isochemical Calculations for the Four-Body H_2 Molecule

In this appendix we present a summary of the computer calculations conducted in Ref. [5] for the four-body model of the hydrogen molecule, Eq. (9.1.33), according to isochemistry by using only $6G$-$1s$ orbitals for brevity. The calculations are also based on the characteristics of the isoelectronium in Eqs. (9.1.25). Note, again, the exact representation of the binding energy at -1.174447 Hartrees.

```
Gaussian-Lobe Program for Large Molecules
set up by D. Shillady and S. Baldwin
Virginia Commonwealth University
Richmond Virginia
1978-1997
Test of SASLOBE on H2
```

SANTILLI-RADIUS = 0.01184470000000.
Cutoff = (A/r)*(exp(-alp*r*r)), A = 0.20E+01, alp = 0.49405731E+04.

Atomic Core				Nuclear		Coordinates
		X		Y		Z
1.		0.000000		0.000000		0.000000
	Z1s =	1.200	Z2s =	0.000	Z2p =	0.000
	Z3s =	0.000	Z3p =	0.000	Z3d =	0.000
	Z4p =	0.000	Z4f =	0.000		
1.		0.000000		0.000000		1.401100
	Z1s =	1.200	Z2s =	0.000	Z2p =	0.000
	Z3s =	0.000	Z3p =	0.000	Z3d =	0.000
	Z4p =	0.000	Z4f =	0.000		

Basis Size = 2 and Number of Spheres = 12 for 2 Electrons.

Distance Matrix in Angströms:

	H	H
H	0.00000	0.74143
H	0.74143	0.00000

The center of Mass is at: Xm = 0.000000, Ym = 0.000000, Zm = 0.700550.

Spherical Gaussian Basis Set:

No. 1	alpha =	0.944598E+03 at X = 0.0000 Y = 0.0000 Z = 0.0000 a.u.
No. 2	alpha =	0.934768E+02 at X = 0.0000 Y = 0.0000 Z = 0.0000 a.u.
No. 3	alpha =	0.798123E+01 at X = 0.0000 Y = 0.0000 Z = 0.0000 a.u.
No. 4	alpha =	0.519961E+01 at X = 0.0000 Y = 0.0000 Z = 0.0000 a.u.
No. 5	alpha =	0.235477E+00 at X = 0.0000 Y = 0.0000 Z = 0.0000 a.u.
No. 6	alpha =	0.954756E+00 at X = 0.0000 Y = 0.0000 Z = 0.0000 a.u.
No. 7	alpha =	0.1944598E+03 at X = 0.0000 Y = 0.0000 Z = 1.4011 a.u.
No. 8	alpha =	0.7934768E+02 at X = 0.0000 Y = 0.0000 Z = 1.4011 a.u.
No. 9	alpha =	0.40798123E+01 at X = 0.0000 Y = 0.0000 Z = 1.4011 a.u.
No. 10	alpha =	0.11519961E+01 at X = 0.0000 Y = 0.0000 Z = 1.4011 a.u.
No. 11	alpha =	0.37235477E+00 at X = 0.0000 Y = 0.0000 2 = 1.4011 a.u.
No. 12	alpha =	0.12954756E+00 at X = 0.0000 Y = 0.0000 Z = 1.4011 a.u.

Contracted Orbital No. 1:
0.051420*(1), 0.094904*(2), 0.154071*(3), 0.203148*(4), 0.169063*(5), 0.045667*(6).
Contracted Orbital No.2:
0.051420*(7), 0.094904*(8), 0.154071*(9), 0.203148*(10), 0.169063*(11),
0.045667*(12).

***** Nuclear Repulsion Energy in au = 0.71372493041182. *****

Overlap Matrix:

#		at-orb	1	2
1	H	1s	1.000	0.674
2	H	1s	0.674	1.000

S(-1/2) Matrix:

#		at-orb	1	2
1	H	1s	1.263	-0.490
2	H	1s	-0.490	1.263

H-Core Matrix:

#		at-orb	1	2
1	H	1s	-1.127	-0.965
2	H	1s	-0.965	-1.127

Initial-Guess-Eigenvectors by Column:

#		at-orb	1	2
1	H	1s	0.546	1.239
2	H	1s	0.546	-1.239

One-Electron Energy Levels: E(1) = -1.249428797385, E(2) = -0.499825553916.

(1,1/1,1) =	0.75003658795676
minus (1,1/1,1) =	0.08506647783478
total (1,1/1,1) =	0.66497011012199
(1,1/1,2) =	0.44259146066210
minus (1,1/1,2) =	0.02960554295227
total (1,1/1,2) =	0.41298591770983
(1,1/2,2) =	0.55987025041920
minus (1,1/2,2) =	0.01857331166211
total (1,1/2,2) =	0.54129693875709
(1,2/1,2) =	0.30238141375547
minus (1,2/1,2) =	0.01938180841827
total (1,2/1,2) =	0.28299960533720
(1,2/2,2) =	0.44259146066210
minus (1,2/2,2) =	0.02960554295227
total (1,2/2,2) =	0.41298591770983
(2,2/2,2) =	0.75003658795676
minus (2,2/2,2) =	0.08506647783478
total (2,2/2,2) =	0.66497011012199

Block No. 1 Transferred to Disk/Memory. The Two-Electron Integrals Have Been Computed.
Electronic Energy = -1.88819368266525 a.u., Dif. = 1.8881936827,
Electronic Energy = -1.88819368266525 a.u., Dif. = 0.0000000000.
Energy Second Derivative = 0.00000000000000.

e1a =	-2.499	e1b =	-2.499		
e2a =	0.611	e2b =	0.611	e2ab =	0.611

Iteration No. = 2, alpha = 0.950000.
Electronic Energy = -1.88819368266525 a.u., Dif. = 0.0000000000.
Total Energy = -1.17446875 a.u.
One-Electron Energy Levels: E(1) = -0.638764885280, E(2) = 0.561205833046

Reference State Orbitals for 1 Filled Orbitals by Column:

#	at-orb		1	2
1	H	1s	0.546	1.239
2	H	1s	0.546	-1.239

Dipole Moment Components in Debyes:
Dx = 0.0000000, Dy = 0.0000000, Dz = 0.0000000.
Resultant Dipole Moment in Debyes = 0.0000000.
Computed Atom Charges: Q(1) = 0.000, Q(2)= 0.000.
Orbital Charges: 1.000000, 1.000000.

Milliken Overlap Populations:

#	at-orb		1	2
1	H	1s	0.597	0.403
2	H	1s	0.403	0.597

Total Overlap Populations by Atom:

	H	H
H	0.597222	0.402778
H	0.402778	0.597222

Orthogonalized Molecular Orbitals by Column:

#		at-orb	1	2
1	H	1s	0.422	2.172
2	H	1s	0.422	-2.172

Wiberg-Trindie Bond Indices:

#		at-orb	1	2
1	H	1s	0.127	0.127
2	H	1s	0.127	0.127

Wiberg-Trindie Total Bond Indices by Atoms:

	H	H
H	0.127217	0.127217
H	0.127217	0.127217

Appendix 9.C
Exact Solution of the Restricted Three-Body Hydrogen Molecule Calculated by Perez-Enriquez, Marin and Riera

9.C.1 Introduction

There exists an exact solution for the restricted three-body model of the hydrogen molecule; it was obtained by Raúl Pérez-Enríquez, José Luis Marín and Raúl Riera [68]. Their results for the three-body version of Santilli-Shillady's model [5] reproduce with high precision the values of the ground state energy as a function of the bond length as calculated by Kolos, Szalewicz and Monkhorst; these authors used a James-Coullidge type wave function with up to 259 terms (KSM curve) [69]. In the three-body approach to the hydrogen molecule, Pérez-Enríquez *et al.* assume, on the one hand, that a kind of correlated state between the electrons is present – the so-called isoelectronium state – in which these particles orbit around the nuclei in closed paths. On the other hand, they propose a hypothesis on the extension of the wave function. As a result, these authors obtain relevant results.

In an independent work, a similar kind of correlated state has been used by Pérez-Enríquez [70] to find a structural parameter that correlates linearly with the critical temperature, T_c, in the perovskite type superconductors. In this article he proposes some kind of Möbius orbital for Cooper pairs. Other contributions to the discussion about correlation between electrons have been presented. In 1993, Taut [71], by means of a pair-correlation function and density of charge for a system of two electrons in an external potential, reports that a one-particle picture may apply to systems with high charge densities.

In their approach, Pérez-Enríquez, Marín and Riera introduce as a starting point the idea that an isoelectronium is confined to a region of space; from here they work out a solution following a method similar to that of E. Ley-Koo and S. Cruz the hydrogen molecular ion inside a spheroidal box [72]; other authors have worked such a confinement for molecules under pressure [73, 74]. In this section, a brief summary of their method and its results is presented.

9.C.2 Confined Isoelectronium Approach

Taking as a point of departure Eq. (9.1.35) representing the four-body equation for the hydrogen molecule, it is possible to arrive at a couple of differential equations. In a similar way as Aringazin and Kucherenko [65], Pérez-Enríquez *et al.* find that:

$$\left\{-\frac{\hbar^2}{2M}\,\nabla_{ab}^2-\frac{\hbar^2}{2m}\nabla_{12}^2-V\frac{e^{-r_{12}/r_c}}{1-e^{-r_{12}/r_c}}+\frac{e^2}{r_{12}}\right.$$
$$\left.-\frac{2e^2}{r_a}-\frac{2e^2}{r_b}+\frac{e^2}{R}\right\}\left|\hat{\psi}\right\rangle=E\left|\hat{\psi}\right\rangle. \tag{9.C.1}$$

An equation for the iso-electrons is obtained in terms of the distance between them:

$$-\frac{\hbar^2}{2m}\nabla_{12}^2\chi+V\left(r_{12}\right)\chi=\varepsilon\chi, \tag{9.C.2}$$

with

$$V\left(r_{12}\right)=\frac{e^2}{r_{12}}-V_0\frac{e^{-r_{12}/r_c}}{1-e^{-r_{12}/r_c}}. \tag{9.C.3}$$

Similarly, for the isoelectronium in interaction with the nuclei:

$$-\frac{\hbar^2}{2M}\nabla_{ab}^2\psi+W\left(r_a,r_b,R\right)\,\psi=\left(E-\varepsilon\right)\,\psi, \tag{9.C.4}$$

with

$$W\left(r_a,r_b,R\right)=-\frac{2e^2}{r_a}-\frac{2e^2}{r_b}+\frac{e^2}{R}. \tag{9.C.5}$$

The following wave function has been used:

$$\left|\hat{\psi}\right\rangle=\chi\left(r_{12}\right)\psi\left(r_a,r_b\right). \tag{9.C.6}$$

In the following paragraphs, the solution of Eq. (9.C.4) is developed after the presentation of a condition directly related to the isoelectronium definition; a condition on the extension of the isoelectronium wave function that will provide an exact representation of the hydrogen molecule as a restricted three-body model, reproducing the behavior of the ground state KSM curve in an appreciable range of distances between nuclei in the molecule.

There are two conditions that have to be mathematically represented in order to account for the properties of isoelectronium in trying to understand the kind of movement this quasi-particle follows:

a) The formation of the quasi-particle from the two electrons involves an effective mass transformation; i.e., mass and charge of isoelectronium are $M=\nu\,m_e$ and $q=-2e$, respectively; where ν is the effective mass parameter, also called "isorenormalization of mass" and,

b) The spatial extension of the orbits of isoelectronium is limited to a defined region of space; i.e., isoelectronium orbits in a spheroidal shaped region of space.

It is worth mentioning that the latter condition, derived from the fact that the iso-electronium is a twice charged particle which surrounds both nuclei in the molecule, would imply a different behavior to that of the electron in the molecular ion. This small but heavily charged quasi-particle would have to limit its own motion to confined orbits. The hydrogen molecule with the isoelectronium orbiting two protons would appear as a confined system.

With respect to the first condition given above, it is possible to mention that a kind of scaling has been suggested in the literature before. Not just the one proposed by Aringazin and Kucherenko but from other authors as well. In particular, Svidzinsky and collaborators [75] have recently published a paper on the role of scaling while they attempt to represent the hydrogen molecule from Bohr's model. They make a dimensional scaling of the energy in this pre-quantum mechanical description. In the present approach, scaling comes from "iso-renormalization $\Rightarrow m'$", given that the central mechanism of hadronic mechanics is to transform the trivial unit 1 into a generalized quantity I^* with the consequent generalization of Plank's constant ($h \Rightarrow h'$) and its related generalized numbers, fields, spaces and so on.

Both hypotheses can be fulfilled by using an approach similar to the one used by Ley-Koo and Cruz to solve the H_2^+ molecular ion confined by a spheroidal box [72], which yields an exact solution of the differential equation using separation of variables and the condition of a vanishing wave function on the spheroidal border. The other way, whose results are reproduced here, uses a variational approach to solve Eq. (9.C.7) as it was done by Marín and Muoz [76], having the same border condition: $\psi(\xi_0, \eta, \varphi) = 0$, where ξ_0 defines the shape of the box (the inverse of eccentricity).

9.C.3 Exact Values from the Confined Isoelectronium Model

The exact solution of the three-body hydrogen molecule comes from the approach proposed by Pérez-Enríquez, Marín and Riera to the three-body Santilli-Shillady model of the hydrogen molecule (modified M3CS-S). In this context, the Schrödinger equation representing the H_2^+ has to be modified to include the above stated conditions on the mass, $M = \nu m_e$, where ν is the effective mass parameter given by the iso-renormalization, and on the charge, $q = -2e$. Thus,

$$\left\{ -\frac{\hbar^2}{2\nu m_e} \frac{4}{\rho^2 (\xi^2 - \eta^2)} \left[\frac{\partial}{\partial \xi} \left(\xi^2 - 1\right) \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \left(1 - \eta^2\right) \frac{\partial}{\partial \eta} \right. \right.$$

$$+\frac{\xi^2-\eta^2}{(\xi^2-1)(1-\eta^2)}\,\frac{\partial^2}{\partial\varphi^2}\Big]-\frac{4\,e^2}{\rho}\cdot\frac{(Z_1+Z_2)\,\xi+(Z_2-Z_1)\,\eta}{\xi^2-\eta^2}$$
$$+\,\frac{Z_1Z_2e^2}{\rho}\Big\}\psi(\xi,\eta,\varphi)=E'\,\psi(\xi,\eta,\varphi)\,. \tag{9.C.7}$$

This equation has to be solved subject to the following restriction:

$$\psi(\xi_0,\eta,\varphi)=0, \tag{9.C.8}$$

which specifies that there is a region of space of spheroidal shape where the isoelectronium moves ($\xi\le\xi_0$); at the border, the wave function vanishes. Due to the symmetry of the molecule in the ground state ($m=0$), one can suppress the azimuth variable and reduce the problem to the $z-x$ plane. Moreover, the following atomic units are used:

$$a_0=\frac{\hbar^2}{m_e e^2};\quad E'=\frac{e^2}{2a_0}E;\quad R=\frac{\rho}{a_0}.$$

Thus, the equation

$$\tilde{H}\phi=E\phi$$

is rewritten as

$$\Big\{-\frac{4}{\nu R^2(\xi^2-\eta^2)}\left[\frac{\partial}{\partial\xi}\left(\xi^2-1\right)\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\left(1-\eta^2\right)\frac{\partial}{\partial\eta}\right]\frac{8}{R}$$
$$\times\,\frac{(Z_1+Z_2)\,\xi+(Z_2-Z_1)\,\eta}{\xi^2-\eta^2}\cdot\frac{2Z_1Z_2}{R}\Big\}\phi(\xi,\eta)=E\,\phi(\xi,\eta)\,. \tag{9.C.9}$$

In order to satisfy the border conditions, a simple symmetrized variational function considering one parameter and a cut off factor can be used:

$$\phi(\alpha;\xi,\eta)=(\xi_0-\xi)\left(\exp\left[-\alpha\,(\xi+\eta)\right]+\exp\left[-\alpha(\xi-\eta)\right]\right). \tag{9.C.10}$$

The minimum energy of this modified M3CS-S molecule can be obtained by minimization of the functional of energy

$$E(\alpha)=\frac{\langle\phi|\tilde{H}|\phi\rangle}{\langle\phi\mid\phi\rangle} \tag{9.C.11}$$

subject to the following condition

$$\left.\frac{\partial E}{\partial\,\alpha}\right|_{E=E_{min}}=0. \tag{9.C.12}$$

The functional of energy depends on several parameters: the effective mass, size of the box and separation between nuclei. Then, E_{min} can be expressed as

$$E_{min}=E_{min}(\nu,\xi_0,R)\,. \tag{9.C.13}$$

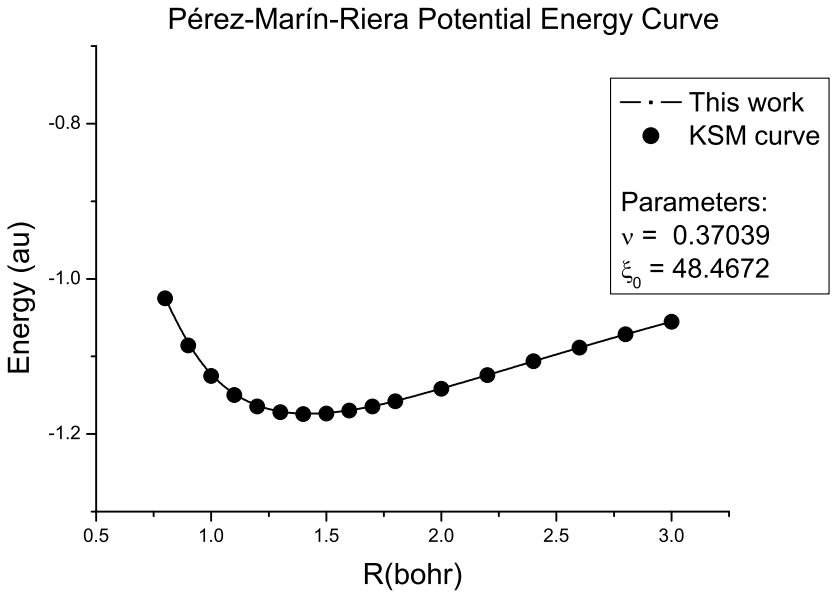

Figure 9.C.1. Comparison between Kolos data and the modified three-body Santilli-Shillady model (parameters: $\nu-$ mass; ξ_0-spheroidal shape). Figure was taken from Ref. [68].

By allowing all three parameters to change freely, a simplex optimization method, such as the Nelder-Mead method for example, yields the energy minimum ($E = -7.61509174$ au and $R = 0.2592$ bohr) as that found by Santilli-Shillady (see Table 9.1); however, one can choose the effective mass parameter and look forward to find the ground state energy of the free H_2.

Using the Nelder-Mead optimization algorithm [77] for the parameters defining the size and shape of the confining box (ξ_0 and R), Pérez-Enríquez *et al.* were able to reproduce up to the fifth significant digit the ground state energy ($E = -1.1744840$ au) at a fixed $\nu = 0.37039$. This minimum was obtained for the specific values of the box parameters ($\xi_0 = 48.46714783$; $R = 1.41847181$ bohr). It is not possible to leave this point without mentioning that the energy minimum is related to a bond length, R, that has only a 1.24% difference with respect to the experimentally observed one. The Pérez-Enríquez, Marín and Riera approach to the three-body hydrogen molecule or M3CP-M-R, may be known as the exact solution to the restricted three-body hydrogen molecule because, as shown in next subsection, it reproduces for $R \in [0.8, 3.2]$ bohr, the KSM ground state curve up to the fourth significant digit.

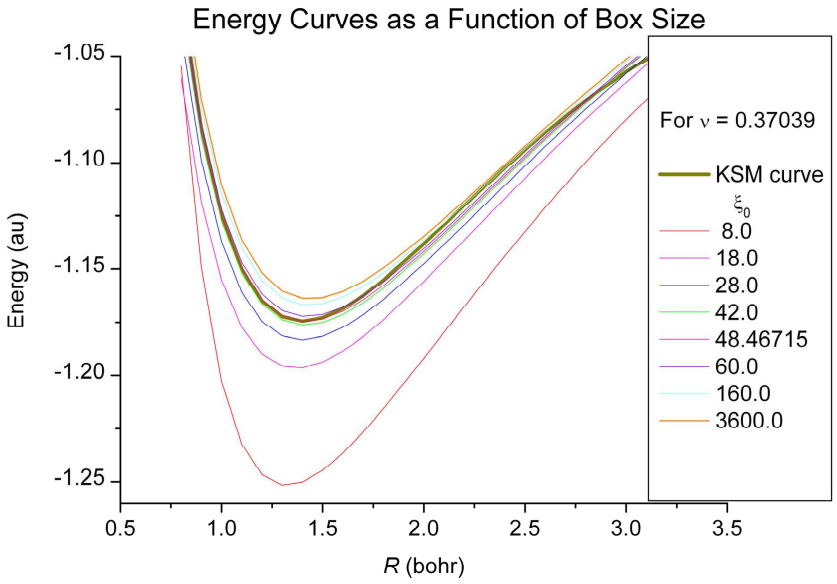

Figure 9.C.2. Energy curves showing the influence of the box size parameter.

9.C.4 Exact Values as Compared With KSM Ground State

Once the mass parameter, ν, and box size, ξ_0, were found for the optimal ground state energy, Perez-Enríquez *et al.* reproduce the KSM curve for the H_2 ground state over a range of bond lengths. In Figure 9.C.1, the graph shows the fitness of this M3CP-M-R model (line) with the values of energy as a function of R reported by Kolos *et al.* (big points). Though the curve illustrates this fact; quantitatively, a χ^2 statistical test (1.25390 with 18 degrees of freedom) confirms that with a confidence of .9999998, both data sets are identical to each other up to the 4^{th} significant digit.

Table 9.C.1 gathers both sets of energy data and compares them point to point, showing that this significant behavior lies within a defined range, $R \in [0.8, 3.2]$ bohr. As can be seen while reviewing column five the difference appears at the fourth significant digit for the worst match and up to the sixth digit for the best fit, which is located at exactly $R = 1.40$ bohr. Both characteristics – up to five digit precision on minimum energy at the experimentally observed bond length and whole curve reproduction – support the hypothesis given for the isoelectronium movement: the orbiting of isoelectronium around the nuclei limits itself to a spheroidal region of space.

The dependence of curve matching with respect to the size of the box can be analyzed using the above mentioned χ^2 statistical test. A range of box size

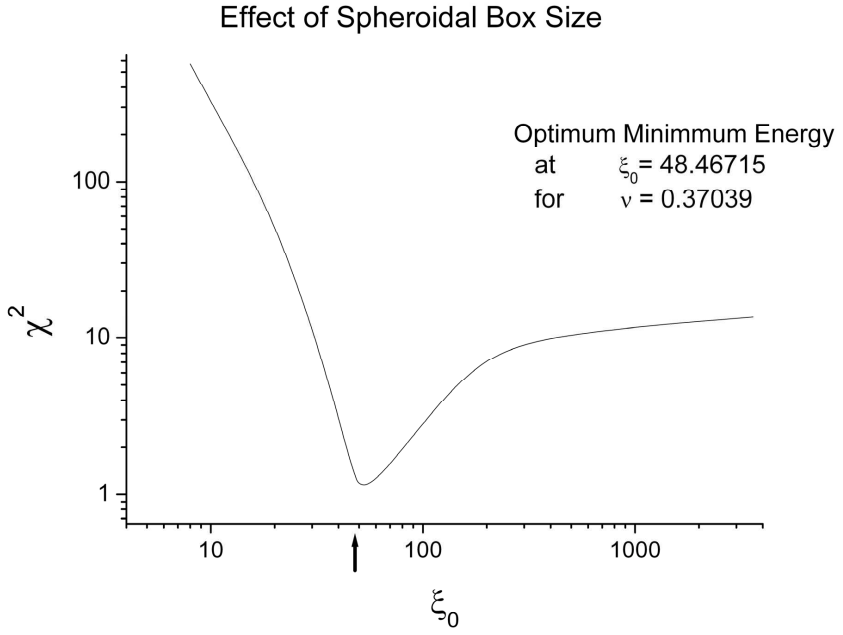

Figure 9.C.3. Importance of the box size parameter for reproducing the KSM curve.

parameter values going from 8.0 to 3600 were used by Pérez-Enríquez, Marín and Riera [78] to evaluate the sensitivity of the adjustment between data. In Figure 9.C.2, corresponding curves for some of the ξ_0 values used are shown together with the KSM curve.

In the last figure, Figure 9.C.3, the log-log graph of χ^2 vs ξ_0 is presented. As it can be seen, a point around $\xi_0 = 50$ gives a perfect match between the KSM curve and M3CP-M-R exact solution of the three-body hydrogen molecule. There is also a well defined spheroidal region for the movement of isoelectronium.

9.C.5 Final Remark

The proof given by this approach to the hydrogen molecule is a good support to the Iso-Chemical model and generates confidence on hadronic mechanics and chemistry. A final remark on the Pérez-Enríquez, Marín and Riera exact solution of the restricted three-body hydrogen molecule can be stated by saying that Santilli-Shillady's orbits, the oo-shaped orbits of isoelectronium, are a necessary condition for the extension of the electron pair; the space restriction on the free movement of this quasi particle could be present in other states of paired electrons such as the Cooper pairs in superconductivity, mainly in high Tc superconductors. The physical interpretation of the isoelectronium state, in which the interaction

Table 9.C.1. Energies from M3CP-M-R and KSM models

R[1]	α[2]	M3CP-M[3]	Kolos[4]	Diff.[5]
0.80	0.4188965	–1.024900	–1.0200565	0.0048435
0.90	0.4585059	–1.085753	–1.0836432	0.0021098
1.00	0.4964746	–1.125001	–1.1245396	0.0004614
1.10	0.5331055	–1.149680	–1.1500574	0.0003774
1.20	0.5686328	–1.164305	–1.1649352	0.0006302
1.30	0.6032813	–1.171876	–1.1723471	0.0004711
1.40	0.6371875	–1.174438	–1.1744757	0.0000377[6]
1.50	0.6705273	–1.173416	–1.1728550	0.0005610
1.60	0.7033789	–1.169826	–1.1685833	0.0012427
1.70	0.7358594	–1.164397	–1.1624586	0.0019384
1.80	0.7680469	–1.157664	–1.1550686	0.0025954
2.00	0.8319141	–1.141767	–1.1381329	0.0036341
2.20	0.8953906	–1.124237	–1.1201321	0.0041049
2.40	0.9589063	–1.106267	–1.1024226	0.0038444
2.60	1.0228130	–1.088534	–1.0857913	0.0027427
2.80	1.0871880	–1.071422	–1.0706831	0.0007389
3.00	1.1521880	–1.055136	–1.0573262	0.0021902
3.20	1.2179690	–1.039776	–1.0457995	0.0060235

Notes:

[1] Bond length (in bohr)

[2] Non linear variational parameter

[3] Data in this work with $\xi_0 = 48.467148$ and $\nu = 0.37039$

[4] Data from Kolos, Szalewicz, Monhorst [69]

[5] Absolute value of the difference

[6] Approximation up to 5^{th} significant digit at the minimum.

between electrons takes place while being inside the hadronic horizon, is restricted to a defined region around the nuclei.

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Chapter 10

INDUSTRIAL APPLICATIONS TO NEW CLEAN BURNING, COST COMPETITIVE FUELS

10.1 THE INCREASINGLY CATACLYSMIC CLIMACTIC EVENTS FACING MANKIND

10.1.1 Foreword

Some of the biggest needs of mankind to contain increasingly cataclysmic climactic events due to global warming and other large environmental problems are: 1) Remove and recycle carbon dioxide from our atmosphere; 2) Develop means for the processing of carbon dioxide in automotive exhaust; and 3) Develop new clean burning cost competitive fuels (see the content of this chapter for details).

The biggest threat to mankind in this field is the lack at this writing in all developed countries of political will to invest public funds in serious resolutions of our environmental problems. All governmental investments in the sector known to this author have been made for the *appearance* of favoring the environment while in reality favoring the myopic and self-destructing interests of the organized petroleum cartel, as it is the case for investment of public funds in hydrogen (see Section 11.1.3 for the huge environmental problems caused by current hydrogen production via the reformation of fossil fuels, while multiplying the profits of the petroleum cartel).

It should be stressed that the solution of large societal problems must be supported by *public* funds, since it is unethical to expect that individuals pay for the cost. Yet, all the research presented in this chapter has been supported by *private* funds due to the lack of public funds following solicitations by the author in the U.S.A., Continental Europe, Russia, China, Japan, Australia, and other developed countries.

One way to understand the gravity of environmental problems is to note that this chapter has been written during the month of September 2005 at the Institute for Basic Research in Florida, when the southern belt of the U.S.A. had been

Figure 10.1. A view of one of the primary responsibilities for current increasingly cataclysmic climatic events: the pollution caused by fossil fueled electric power plants.

exposed to some eighteen hurricanes and devastated by the hurricanes Kathrina and Rita, with additional hurricanes expected before the end of the 2005 season. Increasingly cataclysmic climactic events are expected in the years ahead, until the entire southern belt of the U.S.A. will become uninhabitable.[1]

Similar increasingly cataclysmic climactic events are occurring in the rest of the world. As an example, the mountain region of the Andes in Peru is experiencing an exodus of the local farmers toward the cities due to the lack of snow in winter, with expected major drought the following summer and consequential inability to grow crops.

Also, *The Economist* in England published in the fall of 2004 a report from the U. S. Pentagon releasing for the first time data on the slow down of the Gulf Stream due to the decreased density and salinity of the North Atlantic caused by the melting of the ice in the North Pole region. The complete halting of the Gulf Stream is now beyond scientific doubt, the only debatable issue remaining is that of the date, at which time England is expected to suffer from extreme cold in winter and extreme temperature in summer.

[1] At the end of the 2005 hurricane season Florida was hit by *twenty two* major climactic events, so many that the U.S. Weather Bureau exhausted all 21 letters of the English alphabet and had to name the 22-nd storm from the Greek alphabet. There is no need to wait a few years to understand that the devastating climactic events expected in the next few years are due to the lack of serious political will NOW.

The list of similar increasingly cataclysmic climactic events all over the world could now be endless.

It is at this point were the efforts for the construction of hadronic mechanics, superconductivity and chemistry acquire their full light. In fact, all possibilities of resolving our huge environmental problems via the use of conventional doctrines were long exhausted, as better illustrated in this and in the next chapter, thus establishing the need for suitable covering disciplines beyond any possible doubt.

All scientists have a direct responsibility to contribute, or at least not to oppose, serious efforts toward the solution of these increasingly cataclysmic problems via the traditional scientific process of trial and errors, by implementing genuine scientific democracy, ethics and accountability vis a vis mankind, not via a formal academic parlance, but in actual deeds, the only ones having social as well as scientific value, beginning with the admission that *the dominance of the entire universe by the rather limited Einsteinian doctrines is a purely political - nonscientific posture, and its era has now ended in favor of covering theories for physical conditions unthinkable during Einstein's times.*

10.1.2 Origin of the Increasingly cataclysmic Climactic Events

According to official data released by the U. S. Department of Energy[2], by ignoring the world-wide consumption of natural gas and coal, *we consumed in 2003 about* 74×10^6 *barrels of crude oil (petroleum) per day, corresponding to the daily consumption of about* 3×10^9 *gallons (g) or* 1.4×10^{10} *liters (L) of gasoline per day.*

When adding the world consumption of natural gas and coal, *the world consumption of fossil fuels in 2003 should be conservatively estimated to be equivalent to* 1.5×10^7 *barrels per day, corresponding to the gasoline equivalent of* 7.5×10^8 *gallon or* 2.8×10^{11} *liters per day.*

Such a disproportionate consumption is due to the average daily use in 2003 of about $1,000,000,000$ cars, $1,000,000$ trucks, $100,000$ planes plus an unidentifiable number of additional vehicles of military, agricultural, industrial and other nature, plus the large consumption of fossil fuels by electric power plants around the world.

The data for 2004 are not reported here because still debated, and estimated to be of the order of 90×10^6 barrels of crude oil (petroleum) per day. Future consumption can be best illustrated by noting that, according to official data of the Chinese government, *China is building* $500,000,000$ *(yes, five hundred million) new cars by 2015, and that the need for petroleum, in China for the year 2006 will correspond to the world consumption for 2004, including China.*

[2]See, e.g., the web site http://www.eia.doe.gov/emeu/international/energy.html

MARKET (Fuel Source)	Current U.S. Daily Market Usage (1999 Statistics)
Oil	U.S.A. - 882.8 million tons (25.5% share of world total usage) = 18,490 thousand barrels daily
Natural Gas	U.S.A. - 555.3 million tons oil equivalent (26.9% of world total usage) = 617 billion cubic meters
Coal	U.S.A. - 543.3 million tons oil equivalent (25.5% of world total usage) = 543.3 million tons oil equivalent (25.5% of world total usage)

Figure 10.2. Official data on the 2003 disproportionate consumption of fossil fuels in the U.S.A. alone.

The extremely serious environmental problems caused by the above disproportionate combustion of fossil fuels can be summarized as follows:[3]

(1) **The combustion of fossil fuels releases in our atmosphere about sixty millions metric of tons carbon dioxide CO_2 per day that are responsible for the first large environmental problem known as "global warning" or "green house effect."**[4] Of these only 30 millions metric tons are estimated to be recycled by our ever decreasing forests. This implies the release in our atmosphere of about thirty millions metric tons of unrecycled green house gases per day, which release is the cause of the "global warming" now visible to everybody through climactic episodes such as floods, tornadoes, hurricanes, etc. of increasing catastrophic nature.

[3]See for details the web site http://www.magnegas.com/technology/part6.htm. The reader should note that the calculations in this web site only treat the 2003 consumption of crude oil for automotive use. Consequently, the data therein should be multiplied by three to reach realistic values for 2003.

[4]The value of 60 million tons of CO_2 per day is easily obtained from the chemical reaction in the combustion of the indicated daily volume of fossil fuels (see for details http://www.magnegas.com/technology/part6.htm

(2) **The combustion of fossil fuels causes the permanent removal from our atmosphere of about 21 millions metric tons of breathable oxygen per day, a second, extremely serious environmental problem known as "oxygen depletion."**[5] Even though not disclosed by political circles and newsmedia, the very admission of an "excess" CO_2 in our atmosphere (that is, CO_2 no longer recycled by plants) is an admission of oxygen depletion because the "O_2 in the excess "CO_2" was originally breathable oxygen. Hence, by recalling the atomic weight of CO_2 and O_2, we have the value $\frac{32}{44} \times 30 \times 10^6 = 21.8 \times 10^8$ tons of lost oxygen per day.

It appears that, prior the introduction of oxygen depletion by the author in 2000, everybody ignored the fact that *the combustion of fossil fuels requires atmospheric oxygen.* Since only the global warming is generally considered, it appears that *newsmedia, governments and industries alike ignored the fact that we need oxygen to breath.* Only more recently, various environmental groups, unions and other concerned groups are becoming aware that *the increasing heart problems in densely populated area are indeed due to local oxygen depletion caused by excessive fossil fuel combustion.*

(3) **The combustion of fossil fuels releases in our atmosphere about fifteen millions metric tons of carcinogenic and toxic substances per day.** This third, equally serious environmental problems is euphemistically referred to by the newsmedia as "atmospheric pollution"', while in reality it refers to the primary source of the widespread increase of cancer in our societies. For instance, it has been established by various medical studies (generally suppressed by supporters of the oil cartel) that *unless of genetic origin, breast cancer is due to the inhaling of carcinogenic substances in fossil fuels exhaust.* These studies have gone so far as to establish that breast cells are very receptive to a particular carcinogenic substance in fossil fuel exhaust. After all, responsible citizens should remember and propagate (rather than myopically suppress) the fact that *the U. S. Environmental Protection Agency has formally admitted that diesel exhaust is carcinogenic.* A moment of reflection is sufficient for anybody in good faith to see that we inhale on a daily basis carcinogenic substances from gasoline exhaust in an amount that is ten thousands times bigger than carcinogenic substances ingested with food.

This is another serious environmental problem that has remained virtually ignored by all until recently due to the widespread misinformation by the newsmedia. However, the existence of this third major environmental problem caused by fossil fuel combustion has now propagated to environmental, union and other

[5]The "oxygen depletion" was first introduced by the author at the 2000 Hydrogen World Conference held in Munich, Germany (see the web site http://www.magnegas.com/technology/part6.htm).

Figure 10.3. A picture of frequent environmental disasters caused by the spill of crude oil from tankers following accidents. The replacement of crude oil with a gaseous fuel will eliminate the environmental damage, with the exception of hydrogen because, in the event the cargo of this tanker had been composed of hydrogen, its release in the atmosphere, its immediate rising to the ozone layer, and its very rapid reaction with O_3 would create a hole in the ozone layer of the size of the State of Rhode Island, with consequential increase of skin and other cancers on Earth (see Section 11.1.3 for details).

circles with predictable legal implications for the fossil fuel industry and its major users, unless suitable corrective measures are initiated, as it occurred for the tobacco industry.

It is hoped that people trapped in traffic, thus inhaling the carcinogenic fumes from the vehicle in front, will remember the above evidence and assume an active role in the support of environmentally acceptable fuels because it is written throughout history that people have the government and system they deserve.

There exist numerous additional environmental problems caused by the *global* study of fossil fuels, that is, not only the environmental problems caused by their combustion, but also those caused by their production and transportation. The latter problems are omitted here for brevity and also because the dimension of problems 1), 2) and 3) is a sufficient call for persons in good faith.

10.1.3 Serious Environmental Problems Caused by Hydrogen, Natural Gas, Ethanol, Biogases and Fuels with Molecular Structure

Whenever facing the ever increasing cataclysmic climactic events caused by fossil fuel combustion, a rather widespread belief is that the solution already exists and it is given by *hydrogen* for the large scale fuel uses of the future because hydrogen is believed to be "the cleanest fuel available to mankind."

Due to the potentially lethal implications for mankind, it is necessary to dispel this belief and indicate that, *the current production and combustion of hydrogen, whether for an internal combustion engine or for a fuel cell, causes a global pollution much greater than that caused by gasoline when compared for the same energy outputs.*

Hydrogen is indeed an environmentally acceptable fuel, but only when its production and use verify the following conditions:

CONDITION I: Hydrogen is produced via the electrolytic separation of water;

CONDITION II: The electricity used for electrolysis originates from clean and renewable energy sources, such as those of hydric, solar or wind nature; and

CONDITION III: The oxygen produced by the electrolytic process is freely released in the environment so that the subsequent hydrogen combustion leaves unchanged the existing oxygen content of our atmosphere.

However, the reality in the production and use of hydrogen is dramatically different than the above ideal conditions. In fact, hydrogen is today produced in its greatest percentage via reformation processes of fossil fuels such as methane CH_4, via the use of highly polluting electric power plants, and no oxygen is released in the atmosphere during production.

Reformation processes are preferred over electrolysis not only because of the low efficiency of the electrolytic separation of water,[6] but also due to the fact that the primary drive in the current international support for hydrogen as a fuel is to permit the petroleum cartel to multiply the profits (because the profits from the sale of the hydrogen content of fossil fuels are a multiple of the profits from the direct sale of fossil fuels, as better indicated below.)

Renewable sources of electricity, even though manifectly valuable, are so minute with respect to the enormity of the demand for fuel that cannot be taken into serious consideration. Nuclear power plants also cannot be taken into serious consideration until governments finally provide serious financial support for basic research on the stimulated decay of radioactive nuclear waste by nuclear power plants themselves, rather than the currently preferred "storage" of nuclear waste

[6]Electrolytic plants for the separation of water have an efficiency of the order of 0.8, thus yielding an efficiency for hydrogen production by volume of the order of 0.5, as compared to the efficiency in the production of magnegas discussed in the subsequent sections of this chapter that can be 10.5 in industrial recycler, that is, 21 times bigger than that of electrolysis.

in in depositories nobody wants to have near-by. These aspects begin to illustrate the reason hadronic mechanics, superconductivity and chemistry were developed, as studied in more details in the next chapter.

When inspected in real terms, the current production, transportation and use of hydrogen, if implemented in large scale such as that of fossil fuels, cause the following very serious environmental problems:

A) **Alarming oxygen depletion caused by hydrogen combustion,** namely, the permanent removal of breathable oxygen from our atmosphere and its conversion into water vapor H_2O. By remembering that oxygen is the very basis of life, we are here referring to one of the most serious environmental problems facing mankind that can become potentially lethal for large scale combustion of hydrogen irrespective of whether used as fuel or in fuel cells. When TV programs show water vapor coming out of car exhaust running on hydrogen, they are actually showing one of the most alarming environmental problems facing mankind.

It should be indicated that *gasoline combustion causes much less oxygen depletion than hydrogen combustion,* for various reasons. The first is that gasoline combustion turns atmospheric oxygen into CO_2 that is food for plants, since the chlorophyll process turns CO_2 into breathable O_2 while maintaining C for plant growth. Therefore, the oxygen depletion caused by gasoline and fossil fuels in general is only that for the *excess* of CO_2 that cannot be any longe recycled by plants due to their enormous daily releases, combined with the ongoing forest depletion.

By comparison, hydrogen turns breathable oxygen into water vapors. At this point equivocal technicians indicate that "plants also recycle water into oxygen,", which statement is correct because without water plants die, as well known. Nevertheless, if proffered by experts, the statement may be dishonest because they do not mention the fact that *our atmosphere is full of water vapor as shown by clouds and rain.* Hence, the additional water vapor originating from hydrogen combustion cannot possibly be recycled by plants. By comparison, the CO_2 content in our atmosphere was less than 1% one century ago, in which case the excess due to fossil fuel combustion was, at least initially, recycled by plants, and this is the very reason the human race is still alive today despite the current immense fossil fuel consumption the world over.

Yet another reason favoring environmentally the combustion of gasoline over hydrogen is that *the oxygen depletion caused by hydrogen combustion is a large multiple of that caused by gasoline combustion.* This additional environmental problem can be seen as follows. Gasoline combustion is based on the synthesis of CO, one of the most esoenergetic chemical reactions known to man, that releases 255 Kcal/mole, followed by the synthesis of CO_2 that releases about 85 Kcal/mole, and other reactions for a total of at least 335 Kcal/mole. By comparison, the sole chemical reaction in hydrogen combustion is the synthesis of

H_2O releasing about 57 Kcal/mole. A first year graduate student in chemistry can then compute the multiplier needed for the oxygen depletion caused by gasoline combustion to reach that of hydrogen combustion, of course, under the same energy output.

B) **Alarming environmental problems caused by current hydrogen production.** The reformation of methane and other fossil fuels for hydrogen production requires large amounts of energy because of the necessary breaking of strong molecular bonds such as CH_4. In this case, all byproducts of the reformation, such as the "green house gas" CO_2, are released into the environment. A first year graduate student in chemistry can then prove (although his/her teacher may disagree for personal academic gains) that the CO_2 released in the atmosphere for hydrogen production from CH_4 is a large multiple of the CO_2 produced in gasoline combustion.

Hence, simple calculations establish that *the current methods of hydrogen production, transportation and use release in the atmosphere carcinogenic substances, green house gases and other contaminants that are at least twenty times bigger than the contaminants releases by the gasoline production and combustion in contemporary cars with efficient catalytic converters.*

In fact, the production of hydrogen requires large amounts of energy while, by comparison, gasoline production requires considerably less energy because crude oil comes out of the group at pressure without any need of electricity, while refining processes of crude oil into gasoline are mostly chemical in nature, thus requiring minimal electric energy. The global pollution caused by gasoline is therefore essentially restricted to the pollution caused by transportation and combustion.

Being an environmentalist, the author certainly does not support gasoline as the dominant fuel. Nevertheless, scientific honesty requires the admission that *gasoline is much less polluting than hydrogen as currently produced when considered on a global scale including production, transportation and combustion.*

C) **Alarming threat to the ozone layer caused by hydrogen seepage and losses.** Another serious environmental problem caused by hydrogen is due to its *seepage,* namely, the fact that, being composed by the smallest molecule on Earth, hydrogen escapes through container walls irrespective of the used material and thickness. Consequently, the large scale use of hydrogen must take into account the inevitable release of free hydrogen that, being very light, instantly rises to the upper layer of our atmosphere all the way to the *ozone layer,* resulting in its depletion because hydrogen and ozone have one of the fastest known chemical reactions

$$H_2 + O_3 \quad \rightarrow \quad H_2O + O_2. \tag{11.1.1}$$

Again, *gasoline is preferable over hydrogen also in regard to the ozone layer.* In fact, gasoline is liquid and its vapors are heavy, thus being unable to reach the

ozone layer. Also, all byproducts of gasoline combustion are heavy and they simply cannot rise to the ozone layer. Assuming that some tornado carries byproducts of gasoline combustion all the way up to the ozone layer, they have no known reaction with the ozone that could compare with that of hydrogen, Eq. (11.1.1).

D) **Alarming environmental problems caused by the need to liquify hydrogen.** Gasoline contains about $110,000$ British Thermal Units (BTU) per gallon (g) while hydrogen contains about 300 BTU per standard cubic foot (scf). Consequently, the "Gasoline Gallon Equivalent' (GGE)' is given by 366 scf of hydrogen. Hence, the hydrogen equivalent of an average 20 gallon gasoline tank would require $7,320$ scf of hydrogen, namely, *a volume of hydrogen so big to require a trailer for its transportation in automotive uses.*

This is the reason all manufacturers testing cars running on hydrogen as a fuel, such as BMW, GM, Honda, and others, have been forced to use *liquified hydrogen.* At this point the environmental problems caused by use of hydrogen as an automotive fuel become truly serious, e.g., because hydrogen liquifies close to the absolute zero degree temperature, thus requiring large amounts of electric energy for its liquefaction, with consequential multiplication of pollution. Additional significant amounts of energy are needed to maintain the liquid state because the spontaneous transition from the liquid to the gas state is explosive without any combustion (because of the rapidity of the transition when the cooling systems ceases to operate).

At the 2000 Hydrogen World Meeting held in Munich, Germany, under BMW support, a participant from Florida stated that "If one of my neighbors in Florida purchases a car operating on liquid hydrogen, I will sell my house because in the event that neighbor leaves the car parked in his driveway to spend the weekend in Las Vegas, and the cooling systems fails to operate due to the Florida summer heat, the explosion due to the transition of state back to the gaseous form will cause a crater."

E) **Prohibitive hydrogen cost.** Commercial grade hydrogen (not the pure hydrogen needed for fuel cells) currently retails in the USA at $\$0.18/scf$. By comparison, natural gas retails at about $\$0.01/scf$. But hydrogen contains $300 BTU/scf$, while natural gas contains $1,050 BTU/scf$. Consequently, $\frac{1,050}{300} \times \$0.18 = \$0.63$, namely, *commercial grade hydrogen currently sells in the U.S.A. at* **sixty three times** *the cost of natural gas,* a very high cost that is a reflection of the low efficiency of the available processes for hydrogen production.

But, unlike magnegas and natural gas, hydrogen cannot be significantly carried in a car in a compressed form, thus requiring its liquefaction that is very expensive to achieve as well as to maintain. Consequently, simple calculations establish that *the actual cost of hydrogen in a liquified form for automotive use is at least* 200 *times the cost of fossil fuels,*

Element	MagneGas (MG)	Natural Gas	Gasoline	EPA Standards
Hydro-carbons	0.026 gm/mi	0.380 gm/mi 2460% of MG emission	0.234 gm/mi 900% of MG emission	0.41 gm/mi
Carbon Monoxide	0.262 gm/mi	5.494 gm/mi 2096% of MG emission	1.965 gm/mi 750% of MG emission	3.40 gm/mi
Nitrogen Oxides	0.281 gm/mi	.732 gm/mi 260% of MG emission	0.247 gm/mi 80% of MG emission	1.00 gm/mi
Carbon Dioxide	235 gm/mi	646.503 gm/mi 275% of MG emission	458.655 gm/mi 195% of MG emission	No EPA standard exists for Carbon Dioxide
Oxygen	9%-12%	0.5%-0.7% 0.04% of MG emission	0.5%-0.7% 0.04% of MG emission	No EPA standard exists for Oxygen

Figure 10.4. Summary of comparative measurements combustion exhaust of the new magnegas fuel (described in Section 11.3 below), natural gas and gasoline conducted at the EPA accredited automotive laboratory of Liphardt & Associated of Long Island, New York in 2000 (see for details the website http://www.magnegas.com/technology/part6.htm). As one can see, contrary to popular belief, under the same conditions (same car with same weight used with the same computerized EPA routine, for the same duration of time), natural gas exhaust contains 61% "more" hydrocarbons, about 41% "more" green house gases, and about 200% "more" nitrogen oxides than gasoline exhaust.

There is no credible or otherwise scientific doubt that, under the above generally untold large problems, hydrogen has no realistic chance of becoming a serious alternative for large use without basically *new* technologies and processes.

The above refers to the use of hydrogen as an automotive fuel for internal combustion engines. The situation for the use of hydrogen in fuel cells is essentially the same, except for different efficiencies between internal combustion engines and fuel cells that have no relevance for environmental profiles.

A possible resolution, or at least alleviation, of these problems is presented in Section 11.5.

Another widespread misrepresentation existing in alternative fuels is the belief that "the combustion of natural gas (or methane) is cleaner than that of gasoline," with particular reference to a presumed reduction of carcinogenic and green house emissions. This misrepresentation is based on the visual evidence that the flame of natural gas is indeed cleaner than that of gasoline or other liquid fuels. However, natural gas is gaseous while gasoline is liquid, with an increase of density in the trantion from the former to the latter of about $1,500$ units. Consequently, when the pollutants in the flame of natural gas are prorated to the density of gasoline, the much more polluting character of natural energes.

In any case, recent measurements reviewed later on in this chapter have disproved the above belief because, *under identical performances, natural gas is much more polluting than gasoline* (see Figure 11.3).

Further widespread misrepresentations exist for *ethanol, biogases,* and other conventional fuels, that is, fuels possessing the conventional molecular structure, because generally presented as cleaner than gasoline. In effect, ethanol combustion exhaust is the most carcinogenic among all fuels, the pollution caused by biogases is truly alarming, and the same occur for all remaining available conventional fuels.

In addition, ethanol. biogases and otehr fuels of agricultural origin leave large carbon deposits on spark plugs, piiston rings and otehr component, by decreasing considerably the life of the engines.

Hence, the mere inspection of the tailpipe exhaust is today a view of the past millennium, if not motivated by equivocal commercial, political or academic interests. The sole approach environmentally acceptable today is *the study of the global environmental profile pertaining to fuels, that including the environmental pollution causes by the production, storage, transportation, and combustion.*

In closing, equivocal commercial, political and academic interests should be made aware that, following the success of the lawsuits against the tobacco industry, environmental groups in Berlin, Washington, Tokyo and other cities are apparently preparing lawsuits for trillion dollars punitive compensation against any large scale producer or user of polluting fuel. Therefore, it appears that the best way to confront supporters of hydrogen, ethanol, biofuels and other highly polluting fuels is that via a judicial process. After all, we should never forget that the future of mankind is at stake on these issues.

10.1.4 Basic Needs for the Survival of Mankind

The most basic need for the very survival of our contemporary societies in view of the disproportionate use of fossil fuels and the increasingly cataclysmic climactic events caused by the pollutants in their combustion exhaust can be summarized as follows:

(1) **Develop "new" processes for the nonpolluting, large scale production of electricity,** that is, processes beyond the now exhausted predictive capacities of conventional doctrines. Whether for electrolysis or other uses, electricity is and will remain the basic source of energy for the synthesis of new fuels. At the same time, hydro, thermal and wind sources of energy, even though very valuable, are dramatically insufficient to fulfill the present, let alone the future needs of clean energy. Nuclear power plants have been severely damaged by governmental obstructions, both in the U.S.A., Europe and other countries, against new processes for the stimulated decay of radioactive nuclear waste by the power plants themselves, in favor of a politically motivated storage of the radioactive

waste in depositories so much opposed by local societies, thus preventing nuclear power to be a viable alternative.[7] Additionally, both the "hot fusion' and the "cold fusion" have failed to achieve industrially viable results to date, and none is in sight at this writing. The need for basically "new" clean sources of electricity is then beyond scientific doubt. This need is addressed in the next chapter because, as we shall see, the content of this chapter is a necessary pre-requisite.

(2) **Build a large number of large reactors for the large scale removal and recycling of the excess CO_2 in our atmosphere.** The containment of future production of CO_2 is basically insufficient because the existing amount in our atmosphere is sufficient to cause increasingly cataclysmic climactic events. Therefore, another major problem facing mankind is the removal of the CO_2 already existing in our atmosphere. This problem is addressed in the next subsection.

(3) **it Develop "new" fuels that are not derivable from crude oil and are capable of achieving full combustion,** that is, fuels structurally different than all known fuels due to their highly polluting character. The production of new fuels not derivable from crude oil is necessary in view of the exploding demand for fossil fuels expected from the construction in China of $500,000,000$ new cars and other factors, as well as the expected end of the petroleum reserves. This need is addressed in this chapter. the need for fuels with a new chemical structure is set by the impossibility for all available fuels, those with conventional molecular structure, to achieve full combustion. This need is addressed in this chapter.

10.1.5 Removing Carbon Dioxide from our Atmosphere and Car Exhaust

Nowadays, we have in our atmosphere a large excess CO_2 estimated to be from 100 to 300 times the CO_2 percentage existing at the beginning of the 20-th century, which excess is responsible for the "global worming" and consequential devastating climactic events.

A typical illustration is given by the Gulf of Mexico whose waters have reached in August, 2005, such a high temperature (95^oF) to kill dolphins and other marine species. This sad environmental problem is due to the fact that CO_2 is heavier than any other gas in the atmosphere, thus forming a layer on the top of the water that traps Sun light, with the resulting increase of water temperature.

All predictions establish that the current rate of CO_2 release in our atmosphere will eventually cause the water of the Gulf of Mexico to reach in the summer a

[7]For governmental politics opposing new methods for the stimulated decay of radioactive nuclear waste, one may visit the web site http://www.nuclearwasterecycling.com

steaming state, with consequential impossibility to sustain life, the only debatable aspect being the time of these lethal conditions in the absence of corrective action.

The *only* possible, rational solution of the problem is the *removal of CO_2 from our atmosphere via molecular filtration or other methods and its processing into noncontaminant gases.*

Other solutions, such as the pumping of CO_2 underground jointly with petroleum production as adopted by the petroleum company StatOil in Norway and other companies, are definitely unacceptable on environmental grounds because of the risk that the green house gas may resurface at some future time with catastrophic consequences. In fact, being a gas under very high pressure when under grounds, it is only a question of time for the CO_2 to find its way back to the surface.[8]

The technology for the molecular separation of CO_2 from our atmosphere is old and well established, thus requiring the construction of equipment in large sizes and numbers for installation in a sufficient number of location to yield appreciable results.

To understand the dimension for the sole Gulf of Mexico there is the need of a number of recyclers located in barges and/or in coastal area capable of processing at least 10 millions metric tons of air per day.

After clarifying that the technology for the removal of CO_2 from our atmosphere is fully available (only the political will is still absent at this writing in virtually all developed nations), the next issue is the selection of the appropriate processing of CO_2 into environmentally acceptable species.

According to extensive research in the problem conducted by the author and his associates, *the most efficient method for recycling CO_2 is that based on flowing the gas at high pressure through an electric arc* [5]. In fact, the arc decomposes the CO_2 molecule into carbon precipitates and breathable oxygen that can be released into the atmosphere to correct the oxygen depletion caused by fossil fuels.

Needless to say, these "CO_2 Recycling Plants can additionally remove from the environment circinogenic and other toxic pollutants via the use of the same technology of molecular sep[aration and processing.

Numerous other processes are also expected to be possible for the removal of the CO_2 excess from our atmosphere, and their indication to the author for quotation in possible future editions of this monograph would be appreciated.

Whatever environmentally acceptable solution is suggested, the main needs for serious and responsible governments is to stop the debate and discussions and initiate action *now,* when the economies of developed countries are still somewhat solid, because, later on, increasingly cataclysmic climatic events combined with

[8]In reality, petroleum companies pump CO_2 underground to increase the pressure of release of near-by crude oil, and certainly not to help the environment.

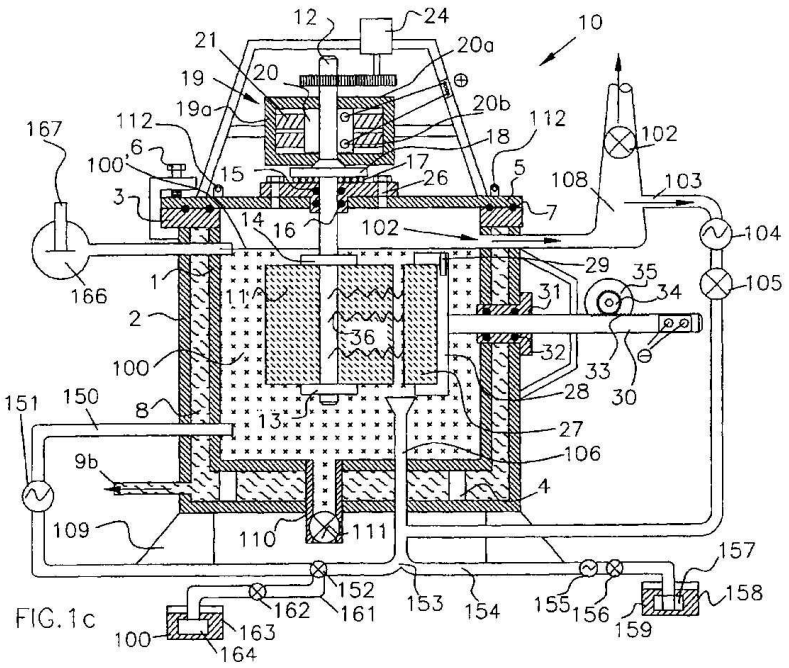


Figure 10.5. A schematic view of a preferred embodymnent for the recycling of CO_2 into C and O_2 via the use the PlasmaArcFlow technology of Refs.[5]. the main principle is that, following its separation from the atmosphere, the most efficient mean for breaking down the CO_2 bond is, by far, the electric arc..

increases in fossil fuel costs may eventually cause the collapse of said economies, at which points nations will not have the immense financial resources needed for the removal of the excess CO_2 in our planet.

IIn closing, the reader should be aware that current technologies permit the large scale production, thus at low cost, of special CO_2 absorbing cartridges that can be houses in conventional car exhaust pipes and replaced periodically. the removal of CO_2 is done via special chemicals or other means.Additional possibilities are given by passing the exhaust through a series of arcs for the recycling of CO_2 into C and breathable O_2.

In short, in this chapter we show that current technologies do indeed permit the production of environmentally acceptable fuels, while in the next chapter we shall show that other technologies permit new clean energies. Mankind is exposed to increasing cataclysmic climactic events not only because of the *lack of political will* for any action that could be considered minimally responsible at this writing (fall 2005) in any and all so-called developed countries, but also

because the political will continues to serve the oil cartel, as demonstrated by the political support of hydrogen, of course, produced from fossil fuels, despite having extremely serious environmental problems identified in the preceding subsection

10.2 THE NEW CHEMICAL SPECIES OF MAGNECULES

10.2.1 Introduction

The origin of the alarming environmental problems increasingly afflicting our planet are not due to fossil fuels per se, but rather to the strength of their conventional valence bond, since that strength has prohibited the achievement of full combustion during the past one hundred years of efforts. In fact, most of the atmospheric pollution caused by fossil fuels is due to "chunks" (such as dimers) of uncombusted fuel that are carcinogenic primarily because consisting of incomplete molecules.

In view of the above occurrence, this author proposed in Ref. [1] of 1998 a new chemical species that, by central assumption, is based on a bond much *weaker* than that of valence bonds so as to permit full combustion. For certain technical reasons indicated below the new species was submitted under the name of *magnecules* in order to distinguish the species from the conventional "molecules," and the new species is known today as *Santilli magnecules.*

In this chapter we report industrial research with the investment of several millions of dollars from private corporations that followed the proposal of Santilli magnecules [1], and resulted in the identification of three distinct new gaseous fuels with the novel magnecular structure, all achieving the original objective of full combustion without toxic substances in the exhaust. Several other substances with magnecular structure are under study and they will be reported in specialized technical journals.

This chapter is organized as follows. We shall first present the hypothesis of Santilli magnecules; we shall then study the industrial methods needed for their production, the features to be detected experimentally, and the analytic equipment needed for the detection of the new species. We shall then study three distinct gaseous fuels with magnecular structure and outline their rather vast experimental verifications. We shall finally study the experimental evidence for magnecular structures in liquids and other related aspects.

To begin, let us recall that the only chemical species with a clearly identified bond which was known prior to the advent of hadronic chemistry was that of *molecules* and related *valence bonds*, whose identification dates back to the 19-th century, thanks to the work by Avogadro (1811), Canizzaro (1858), and several others, following the achievement of scientific measurements of atomic weights.

Various candidates for possible additional chemical species are also known, such as the delocalized electron bonds. However, none of them possess a clearly identified attractive force clearly distinct from the valence.

Also, various molecular clusters have been studied in more recent times, although they either are unstable or miss a precise identification of their internal attractive bond.

An example of unstable molecular cluster occurs when the internal bond is due to an *electric polarization* of atomic structures, that is, a deformation from a spherical charge distribution without a net electric charge to an ellipsoidal distribution in which there is the predominance of one electric charge at one end and the opposite charge at the other end, thus permitting atoms to attract each other with opposite electric polarities. The instability of these clusters then follows from the known property that the smallest perturbation causes nuclei and peripheral electrons to reacquire their natural configuration, with the consequential loss of the polarization and related attractive bond.

An example of molecular clusters without a clear identification of their internal attractive bond is given by *ionic clusters*. In fact, ionized molecules have the *same positive charge* and, therefore, they *repel*, rather than attract, each other. As a result, not only the internal attractive bond of ionic clusters is basically unknown at this writing, but, when identified, it must be so strong as to overcome the repulsive force among the ions constituting the clusters.

In 1998, R. M. Santilli submitted in paper [1] (and then studied in details in monograph [2]) the hypothesis of a new type of stable clusters composed of molecules, dimers and atoms under a new, clearly identified, attractive internal bond which permits their industrial and practical use. The new clusters were called **magnecules** (patents pending) because of the dominance of magnetic effects in their formation, as well as for pragmatic needs of differentiations with the ordinary molecules, with the understanding that a technically more appropriate name would be *electromagnecules*.

The following terminology will be used herein:

1) The word *atom* is used in its conventional meaning as denoting a stable atomic structure, such as a hydrogen, carbon or oxygen, irrespective of whether the atom is ionized or not and paramagnetic or not.

2) The word *dimer* is used to denote part of a molecule under a valance bond, such as H–O, H–C, *etc.*, irrespective of whether the dimer is ionized or not, and whether it belongs to a paramagnetic molecule or not;

3) The word *molecule* is used in its internationally known meaning of denoting stable clusters of atoms under conventional, valence, electron bonds, such as H_2, H_2O, C_2H_2, *etc.*, irrespective of whether the molecule is ionized or not, and paramagnetic or not;

4) The word *magnecule* is used to denote stable clusters of two or more molecules, and/or dimers and/or atoms and any combination thereof formed by a new internal attractive bond of primarily magnetic type identified in detail in this chapter; the word *magnecular* will be used in reference to substances with the structure or features of magnecules;

5) The words *chemical species* are used to denote an essentially pure population of stable clusters with the same internal bond, thus implying the conventional chemical species of molecules as well as that of magnecules, under the condition that each species admits an ignorable presence of the other species.

In this chapter we study the theoretical prediction permitted by hadronic mechanics and chemistry of the new chemical species of magnecules and its experimental verifications, which were apparently presented for the first time by Santilli in memoir [1] of 1998.

10.2.2 The Hypothesis of Santilli Magnecules

The main hypothesis, studied in details in the rest of this Chapter, can be formulated as follows:

DEFINITION 11.2.1 [1,2] (patented and international patents pending [5]): **Santilli magnecules** *in gases, liquids, and solids consist of stable clusters composed of conventional molecules, and/or dimers, and/or individual atoms bonded together by opposing magnetic polarities of toroidal polarizations of the orbits of at least the peripheral atomic electrons when exposed to sufficiently strong external magnetic fields, as well as the polarization of the intrinsic magnetic moments of nuclei and electrons. A population of magnecules constitutes a chemical species when essentially pure, i.e., when molecules or other species are contained in very small percentages in a directly identifiable form. Santilli magnecules are characterized by, or can be identified via the following main features:*

I) Magnecules primarily exist at large atomic weights where not expected, for instance, at atomic weights which are ten times or more the maximal atomic weight of conventional molecular constituents;

II) Magnecules are characterized by large peaks in macroscopic percentages in mass spectrography, which peaks remain unidentified following a search among all existing molecules;

III) Said peaks admit no currently detectable infrared signature for gases and no ultraviolet signature for liquids other than those of the conventional molecules and/or dimers constituting the magnecule;

IV) Said infrared and ultraviolet signatures are generally altered (a feature called "mutation") with respect to the conventional versions, thus indicating an alteration (called infrared or ultraviolet mutation) of the conventional structure of dimers generally occurring with additional peaks in the infrared or ultraviolet signatures not existing in conventional configurations;

V) Magnecules have an anomalous adhesion to other substances, which results in backgrounds (blank) following spectrographic tests which are often similar to the original scans, as well as implying the clogging of small feeding lines with consequential lack of admission into analytic instruments of the most important magnecules to be detected;

VI) Magnecules can break down into fragments under sufficiently energetic collisions, with subsequent recombination with other fragments and/or conventional molecules, resulting in variations in time of spectrographic peaks (called time mutations of magnecular weights);

VII) Magnecules can accrue or lose during collision individual atoms, dimers or molecules;

VIII) Magnecules have an anomalous penetration through other substances indicating a reduction of the average size of conventional molecules as expected under magnetic polarizations;

IX) Gas magnecules have an anomalous solution in liquids due to new magnetic bonds between gas and liquid molecules caused by magnetic induction;

X) Magnecules can be formed by molecules of liquids which are not necessarily solvable in each other;

XI) Magnecules have anomalous average atomic weights in the sense that they are bigger than that of any molecular constituent and any of their combinations;

XII) A gas with magnecular structure does not follow the perfect gas law because the number of its constituents (Avogadro number), or, equivalently, its average atomic weight, varies with a sufficient variation of the pressure;

XIII) Substances with magnecular structure have anomalous physical characteristics, such as anomalous specific density, viscosity, surface tension, etc., as compared to the characteristics of the conventional molecular constituents;

XIV) Magnecules release in thermochemical reactions more energy than that released by the same reactions among unpolarized molecular constituents;

XV) All the above characteristic features disappear when the magnecules are brought to a sufficiently high temperature, which varies from species to species, called Curie Magnecular Temperature; in particular, combustion eliminates all magnetic anomalies resulting in an exhaust without magnecular features.

Magnecules are also called:

A) **elementary** *when only composed of two molecules*;

B) **magneplexes** *when entirely composed of several identical molecules*;

C) **magneclusters** *when composed of several different molecules.*

Finally, magnecules are called:

i) **isomagnecules** *when having all single-valued characteristics and being reversible in time, namely, when they are characterized by isochemistry (see Chapter 9);*

ii) **genomagnecules** *when having all single-valued characteristics and being irreversible in time, namely, when they are characterized by genochemistry;* and

iii) **hypermagnecules** *when having at least one multi-valued characteristic and being irreversible in time, namely, when they are characterized by hyperchemistry.*

The primary objective of this chapter is, first, to study the characteristic features of magnecules from a theoretical viewpoint, and then present independent experimental verifications for each feature.

All magnecules studied in this chapter are, strictly speaking, isomagnecules because single valued and reversible. The reader should be aware that all correct calculations implying single-valued irreversible chemical processes, such as chemical reactions in general, should be done with genomagnecules. Finally, all biological; structure will inevitably require the use of hypermagnecules as illustrated in Chapter 5.

The reader should keep in mind that *magnegas*, the new, clean combustible gas developed by the author [1,2,5], of Largo, Florida, has precisely a magnecular structure from which it derives its name. Nevertheless, we shall identify in this chapter other gases, liquids and solids with a magnecular structure.

By denoting the conventional valence bond with the symbol "–" and the new magnetic bond with the symbol "×", examples of *elementary magnecules* in gases and liquids are respectively given by

$$\{H-H\}\times\{H-H\},\quad \{O-O\}\times\{O-C-O\},\ etc., \tag{11.2.1a}$$

$$\{C_{15}-H_{20}-O\}\times\{C_{15}-H_{20}-O\},\ etc.; \tag{8.1b}$$

examples of *magneplexes* in gases and liquids are respectively given by

$$\{H-H\}\times\{H-H\}\times\{H-H\}\times\ldots,\ etc., \tag{11.2.2a}$$

$$\{H-O-H\}\times\{H-O-H\}\times\ldots,\ etc.; \tag{11.2.2b}$$

and examples of *magneclusters* are given by

$$\{H-H\}\times\{C-O\}\times\{O-C-O\}\times\{C=O\}\times\ldots,\ etc., \tag{11.2.3a}$$

$$\{C_{13}-H_{18}-O\}\times\{C_{14}-H_{12}-O_3\}\times\{C_{15}-H_{20}-O\}\times\ldots,\ etc. \tag{11.2.3b}$$

A generic representation of a gas magnecules requires the presence of individual atoms and dimers, such as:

$$\{H-H\}\times\{C-O\}\times H\times\{H-O-H\}\times C\times\{H-O\}\times\ldots,\ etc. \tag{11.2.4}$$

One of the most important features of magnecules is their anomalous release of energy in thermochemical reactions (Feature XIV of Definition 8.2.1), in view

of its evident importance for the industrial development of new clean fuels such as magnegas (Sects. 7.10 and 7.11).

As we shall see in detail later on, this feature is crucially dependent on the existence within the magnecules of individual atoms, such as H, C and O, and/or individual unpaired dimers, such as H–O and H–C. In fact, at the breakdown of the magnecules due to combustion, these individual atoms and dimers coupled themselves into conventional molecules via known exothermic reactions such as

$$\begin{aligned} H + H &\rightarrow H_2 + 105 \text{ Kcal/mole}, \\ C + O &\rightarrow CO + 255 \text{ Kcal/mole}, \\ H{-}O + H &\rightarrow H_2O + 28 \text{ Kcal/mole}, \ \textit{etc.}, \end{aligned} \tag{11.2.5}$$

with consequential release during combustion of a large amount of energy that does not exist in fuels with a conventional molecular structure.

In reading this chapter, the reader should keep in mind that, in view of the above important industrial, consumer and environmental implications, a primary emphasis of the presentation is the study of magnecules with the largest possible number of *unpaired atoms and dimers*, rather than molecules.

In inspecting the above representation of magnecules, the reader should also keep in mind that their linear formulation in a row is used mainly for practical purposes. In fact, the correct formulation should be via *columns*, rather than rows, since the bond occurs between one atom of a given molecule and an atom of another molecule, as we shall see in detail later on.

10.2.3 The Five Force Fields Existing in Polarized Atoms

The attractive bond responsible for the creation of magnecules originates within the structure of individual *atoms*. Therefore, it is recommendable to initiate our study via the identification of all force fields existing in a conventional atomic structure.

The sole fields in the atomic structure studied by chemists prior to Ref. [1] were the intrinsic electric and magnetic fields of electrons and nuclei (see Fig. 11.6). It was proved a century ago that these fields can only produce *valence bonds*, thus explaining the reason why molecules were the only form of atomic clustering with a clear bond admitted by chemistry until recently.

Santilli's [1] main contribution has been the identification of a *new force field in the atomic structure*, which is sufficiently strong to permit a new chemical species.

Since the inception of atomic physics, the electron of the hydrogen atom (but not necessarily peripheral electrons of more complex atoms) has been assumed to have a spherical distribution, which is indeed the case for isolated and unperturbed atomic structures (see also Fig. 811.6).

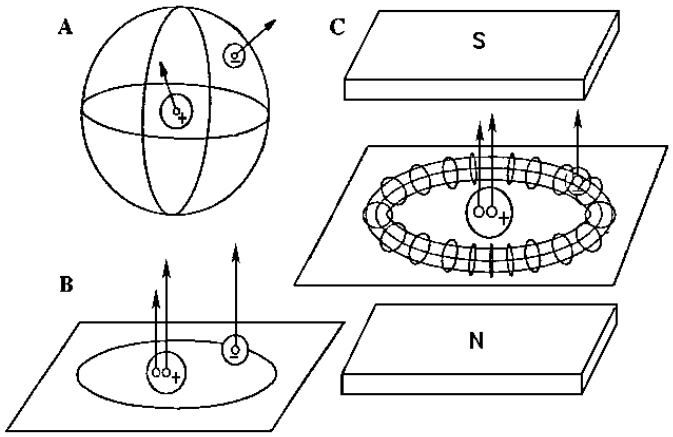


Figure 10.6. A schematic view of the force fields existing in the hydrogen atom. Fig. 11.6.A depicts an isolated hydrogen atom in its conventional spherical configuration when at absolute zero degree temperature, in which the sole force fields are given by the electric charges of the electron and of the proton, as well as by the intrinsic magnetic moments of the same particles. Fig. 11.6.B depicts the same hydrogen atom in which the orbit of the peripheral electron is polarized into a plane. In this case there is the emergence of a fifth field, the magnetic dipole moment caused by the rotation of the electron in its planar orbit. Fig. 11.6.C depicts the same hydrogen atom under an external magnetic field which causes the transition from the spherical distribution of the peripheral electron as in Fig. 11.6.A to a new distribution with the same cylindrical symmetry as that of the external field, and such to offer magnetic polarities opposite to the external ones. In the latter case, the polarization generally occurs within a toroid, and reaches the perfectly planar configuration of Fig. 11.6.B only at absolute zero degree temperature or under extremely strong magnetic fields.

However, electrons are charged particles, and all charges rotating in a planar orbit create a magnetic field in the direction perpendicular to the orbital plane, and such to exhibit the North polarity in the semi-space seeing a counter-clockwise rotation (see Fig. q11.6..B).

A main point of Ref. [1] is that the distribution in space of electron orbits is altered by sufficiently strong external magnetic fields. In particular, the latter cause the transition from the conventional spherical distribution to a new distribution with the same cylindrical symmetry of the external field, and such to exhibit magnetic polarities opposite to the external ones (Fig. 11.6.C).

Therefore, the magnetic fields of atoms *are not* solely given by the intrinsic magnetic fields of the peripheral electrons and of nuclei because, under the application of a sufficiently strong external magnetic field, atoms exhibit the additional magnetic moment caused by a polarization of the electron orbits. This third magnetic field was ignored by chemists until 1998 (although not by physicists) because nonexistent in a conventional atomic state.

As a matter of fact, it should be recalled that *orbits are naturally planar in nature, as established by planetary orbits, and they acquire a spherical distribution in atoms because of various quantum effects, e.g., uncertainties.* Therefore, in the absence of these, all atoms would naturally exhibit *five* force fields and not only the four fields currently assumed in chemistry.

On historical grounds it should be noted that theoretical and experimental studies in physics of the hydrogen atom subjected to an external (homogeneous) magnetic field date to Schrödinger's times.

10.2.4 Numerical Value of Magnecular Bonds

In the preceding section we have noted that a sufficiently strong external magnetic field polarizes the orbits of peripheral atomic electrons resulting in a magnetic field which does not exist in a conventional spherical distribution. Needless to say, the same external magnetic fields also polarize the intrinsic magnetic moments of the peripheral electrons and of nuclei, resulting into *three net magnetic polarities* available in an *atomic* structure for a new bond.

When considering molecules, the situation is different because valence electrons are bonded in singlet couplings to verify Pauli's exclusion principle, as per our hypothesis of the *isoelectronium* of Chapter 9. As a result, their net magnetic polarities can be assumed in first approximation as being null. In this case, only *two* magnetic polarities are available for new bonds, namely, the magnetic field created by the rotation of paired valence electrons in a polarized orbit plus the intrinsic magnetic field of nuclei.

It should be noted that the above results persist when the inter-electron distance of the isoelectronium assumes orbital values. In this case the total intrinsic magnetic moment of the two valence electrons is also approximately null in average due to the persistence of antiparallel spins and, therefore, antiparallel magnetic moments, in which absence there would be a violation of Pauli's exclusion principle.

The calculation of these *polarized magnetic moments at absolute zero degree temperature* is elementary [1]. By using rationalized units, the magnetic moment $M_{e\text{-orb}}$. of a polarized orbit of one atomic electron is given by the general quantum mechanical law:

$$M_{\text{e-orb.}} = \frac{q}{2m} L\mu, \tag{11.2.6}$$

where L is the angular momentum, μ is the rationalized unit of the magnetic moment of the electron, $q = -e$, and $m = m_e$.

It is easy to see that *the magnetic moment of the polarized orbit of the isoelectronium with characteristics (4.25) coincides with that of one individual electron.* This is due to the fact that, in this case, in Eq. (11.2.6) the charge in the numerator assumes a double value $q = -2e$, while the mass in the denominator also assumes a double value, $m = 2m_e$, thus leaving value (8.6) unchanged.

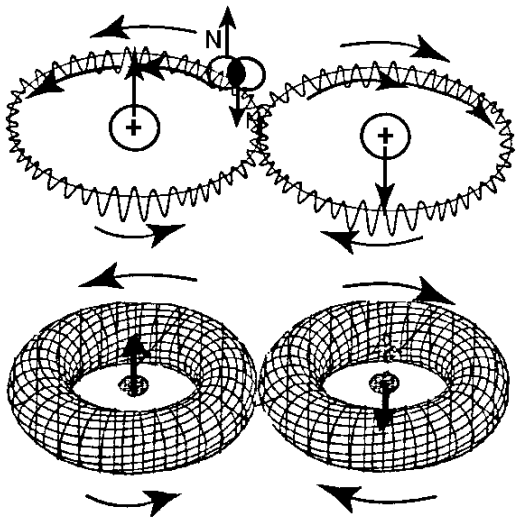


Figure 10.7. A schematic view of the magnetic fields of the isochemical model of the hydrogen molecule with isoelectronium assumed to be a stable quasi-particle. The top view represents the molecule at absolute zero degree temperature with polarization of the orbit in a plane, while the bottom view represents the molecule at ordinary temperature with a polarization of the orbit within a toroid. In both cases there is the disappearance of the *total intrinsic* magnetic moments of the electrons because they are coupled in the isoelectronium with antiparallel spin and magnetic moments due to Pauli's exclusion principle. The *lack* of contribution of the intrinsic magnetic moments of the electrons persists even when the isoelectronium has dimension much bigger than 1 fm, because the antiparallel character of the spins and magnetic moments persists, resulting in an average null total intrinsic magnetic moment of the electrons. Therefore, the biggest magnetic moment of the hydrogen molecule which can be obtained via polarizations is that of the electrons *orbits*. Note, as recalled in Sect. 9.2, the *oo*-shaped (also called figure eight) configuration has been recently proved in mathematics to be one of the most stable solutions of the N-body problem.

By plotting the various numerical values for the ground state of the hydrogen atom, one obtains:

$$M_{\text{e-orb.}} = M_{\text{isoe-orb.}} = 1,859.59\mu. \tag{11.2.7}$$

By recalling that in the assumed units the proton has the magnetic moment 1.4107 μ, we have the value [1]:

$$\frac{M_{\text{e-orb.}}}{M_{\text{p-intr.}}} = \frac{1,856.9590}{1.4107} = 1,316.33, \tag{11.2.8}$$

namely, *the magnetic moment created by the orbiting in a plane of the electron in the hydrogen atom is 1,316 times bigger than the intrinsic magnetic moment of the nucleus*, thus being sufficiently strong to create a bond.

It is evident that the *polarized magnetic moments at ordinary temperature* are smaller than those at absolute zero degrees temperature. This is due to the fact that, at ordinary temperature, the perfect polarization of the orbit in a plane is no longer possible. In this case the polarization occurs in a *toroid*, as illustrated in Fig. 8.2, whose sectional area depends on the intensity of the external field.

As an illustrative example, under an external magnetic field of 10 Tesla, an *isolated hydrogen atom* has a total magnetic field of the following order of magnitude:

$$M_{\text{H-tot.}} = M_{\text{p-intr.}} + M_{\text{e-intr.}} + M_{\text{e-orb.}} \approx 3,000\mu, \tag{11.2.9}$$

while the same hydrogen atom under the same conditions, when a component of a *hydrogen molecule* has the smaller value

$$M_{\text{H}_2\text{-tot.}} = M_{\text{p-intr.}} + M_{\text{isoe-orb.}} \approx 1,500\mu, \tag{11.2.10}$$

again, because of the absence of the rather large contribution from the intrinsic magnetic moment of the electrons, while the orbital contribution remains unchanged.

The above feature is particularly important for the study of magnecules and their applications because it establishes the theoretical foundations for the presence of isolated atoms in the structure of magnecules since *the magnetic bonds of isolated atoms can be at least twice stronger than those of the same atoms when part of a molecule*.

An accurate independent verification of the above calculations was conducted by M.G. Kucherenko and A.K. Aringazin [3], who obtained the following value via the use of alternative models,

$$\frac{M_{\text{e-orb.}}}{M_{\text{p-intr.}}} \approx 1,315\mu. \tag{11.2.11}$$

Needless to say, the quantized value of the angular momentum of the ground state of the conventional (unpolarized) hydrogen atom is null, $L = 0$, thus implying a null magnetic moment, $M = 0$. This occurrence confirms the well known feature that the magnetic moment of the orbit of the peripheral electron of a conventional (unpolarized) hydrogen atom is null.

Consequently, expressions (8.6)-(8.11) should be considered under a number of clarifications. First, said expressions refer to *the orbit of the peripheral electron under an external magnetic field* which implies an evident alteration of the value of the magnetic moment. Note that this external magnetic field can be either that of an electric discharge, as in the PlasmaArcFlow reactors, or that of another polarized hydrogen atom, as in a magnecule. This occurrence confirms a main aspect of the new chemical species of magnecules, namely, that the plane polarization of the orbits of the peripheral atomic electron is stable if and only

if said polarization is coupled to another because, if isolated, the plane polarization is instantly lost due to rotations with recover the conventional spheroidal distribution of the orbits.

Moreover, expressions (11.2.6)-(11.2.11) refer to the angular momentum of the orbit of the peripheral electron *polarized in a plane*, rather than that with a spherical distribution as in the conventional ground state of the hydrogen atom. The latter condition, alone, is sufficient to provide a non-null quantized orbital magnetic moment.

Finally, the value $L = 1$ needed for expressions (11.2.6)-(11.2.11) can be obtained via *the direct quantization of the plane polarization of a classical orbit*. These aspects have been studied in detail by Kucherenko and Aringazin [2] and Aringazin [8] (see Appendix 8.A). These studies clarify a rather intriguing property mostly ignored throughout the 20-th century according to which, contrary to popular beliefs, *the quantized angular momentum of the ground state of the hydrogen atom is not necessarily zero, because its value depends on possible external fields*.

It is important to note that the magnetic polarizations herein considered are *physical notions*, thus being best expressed and understood via *actual orbits* as treated above rather than *chemical orbitals*. This is due to the fact that *orbits are physical entities* actually existing in nature, and schematically represented in the figures with standing waves, in semiclassical approximation. By contrasts, *orbitals are purely mathematical notions* given by probability density. As a result, magnetic fields can be more clearly associated with orbits rather than with orbitals.

Despite the above differences, it should be stressed that, magnetic polarizations can also be derived via the *orbitals* of conventional use in chemistry. For example, consider the description of an isolated atom via the conventional Schrödinger equation

$$H|\psi\rangle = \left(\frac{p^2}{2m} + V\right)|\psi\rangle = E|\psi\rangle, \tag{11.2.12}$$

where $|\psi\rangle$ is a state in a Hilbert space. Orbitals are expressed in terms of the probability density $|\langle\psi| \times |\psi\rangle|$. The probability density of the electron of a hydrogen atom has a spherical distribution, namely, the electron of an isolated hydrogen atom can be found at a given distance from the nucleus with the same probability in any direction in space.

Assume now that the same hydrogen atom is exposed to a strong external homogeneous and static magnetic field B. This case requires the new Schrödinger equation,

$$\left((p - \frac{e}{c}A)^2/2m + V\right)|\psi'\rangle = E'|\psi'\rangle, \tag{11.2.13}$$

where A is vector-potential of the magnetic field B. It is easy to prove that, in this case, the new probability density $|\langle\psi'| \times |\psi'\rangle|$ possesses a *cylindrical symmetry*

precisely of the type indicated above, thus confirming the results obtained on physical grounds. A similar confirmation can be obtained via the use of Dirac's equation or other chemical methods.

An accurate recent review of the Schrödinger equation for the hydrogen atom under external magnetic fields is that by A.K. Aringazin [8], which study confirms the toroidal configuration of the electron orbits which is at the foundation of the new chemical species of magnecules. A review of Aringazin studies is presented in Appendix 11 .A. As one can see, under an external, strong, homogeneous, and constant magnetic fields of the order of 10^{13} Gauss = 10^7 Tesla, the solutions of Schrödinger equation of type (8.13) imply the restriction of the electron orbits within a single, small-size toroidal configuration, while the excited states are represented by the double-splitted toroidal configuration due to parity.

Intriguingly, the binding energy of the ground state of the H atom is much higher than that in the absence of an external magnetic field, by therefore confirming another important feature of the new chemical species of magnecules, that of permitting new means of storing energy within conventional molecules and atoms, as discussed later on in this chapter.

For magnetic fields of the order of 10^9 Gauss, spherical symmetry begins to compete with the toroidal symmetry, and for magnetic fields of the order of 10^5 Gauss or less, spherical symmetry is almost completely restored by leaving only ordinary Zeeman effects. This latter result confirms that the creation of the new chemical species of magnecules in gases as per Definition 8.2.1 requires very strong magnetic fields. The situation for liquids is different, as shown later on also in this chapter.

The magnetic polarization of atoms larger than hydrogen is easily derived from the above calculations. Consider, for example, the magnetic polarization of an isolated atom of oxygen. For simplicity, assume that an external magnetic field of 10 Tesla polarizes only the two peripheral valence electrons of the oxygen. Accordingly, its total polarized magnetic field of orbital type is of the order of twice value (8.9), *i.e.*, about 6,000 μ. However, when the same oxygen atom is bonded into the water or other molecules, the maximal polarized magnetic moment is about half the preceding value.

Note the dominance of the magnetic fields due to polarized electron *orbits* over the intrinsic *nuclear* magnetic fields. This is due not only to the fact that the former are 1,316 times the latter, but also to the fact that nuclei are at a relative great distance from peripheral electrons, thus providing a contribution to the bond even smaller than that indicated. This feature explains the essential novelty of magnecules with respect to established magnetic technologies, such as that based on *nuclear magnetic resonances.*

Note also that a main mechanism of polarization is dependent on an external magnetic field and the force actually providing the bond is of magnetic type.

Nevertheless, the ultimate origin is that of charges rotating in an atomic orbit. This illustrates that, as indicated in Sect. 11.2.1, the name "magnecules" was suggested on the basis of the predominant magnetic origin, as well as for the pragmatic differentiation with molecules without using a long sentence, although a technically more appropriate name would be "electromagnecules."

Needless to say, the polarization of the orbits is not necessarily restricted to valence electrons because the polarization does not affect the quantum numbers of any given orbit, thus applying for all atomic electrons, including those of complete inner shells, of course, under a sufficiently strong external field. As a consequence, *the intensity of the magnetic polarization generally increases with the number of atomic electrons*, namely, the bigger is the atom, the bigger is, in general, its magnetic bond in a magnecule.

Ionizations do not affect the *existence* of magnetic polarizations, and they may at best affect their *intensity*. An ionized hydrogen atom is a naked proton, which acquires a polarization of the direction of its magnetic dipole moment when exposed to an external magnetic field. Therefore, an ionized hydrogen atom can indeed bond magnetically to other polarized structures. Similarly, when oxygen is ionized by the removal of one of its peripheral electrons, its remaining electrons are unchanged. Consequently, when exposed to a strong magnetic field, such an ionized oxygen atom acquires a magnetic polarization which is similar to that of an unpolarized oxygen atom, except that it lacks the contribution from the missing electron. Ionized molecules or dimers behave along similar lines. Accordingly, the issue as to whether individual atoms, dimers or molecules are ionized or not will not be addressed hereon.

The magnetic polarizations here considered are also independent as to whether the substance considered is paramagnetic or not. This is evidently due to the fact that the polarization deals with the individual orbits of individual peripheral electrons, irrespective of whether paired or unpaired, belonging to a saturate shell or not. Therefore, the issue as to whether a given substance is paramagnetic or not will be ignored hereon.

Similarly, the polarizations here considered do not require molecules to have a net total magnetic polarity, which would be possible only for paramagnetic substances, again, because they act on individual orbits of individual atomic electrons.

We should also indicate that another verification of our isochemical model of molecular structures is the resolution of the inconsistency of the conventional model in predicting that all substances are paramagnetic, as illustrated in Figs. 1.4 and 1.5.

Recall that the atoms preserve their individualities in the conventional molecular model, thus implying the *individual* acquisition of a magnetic polarization

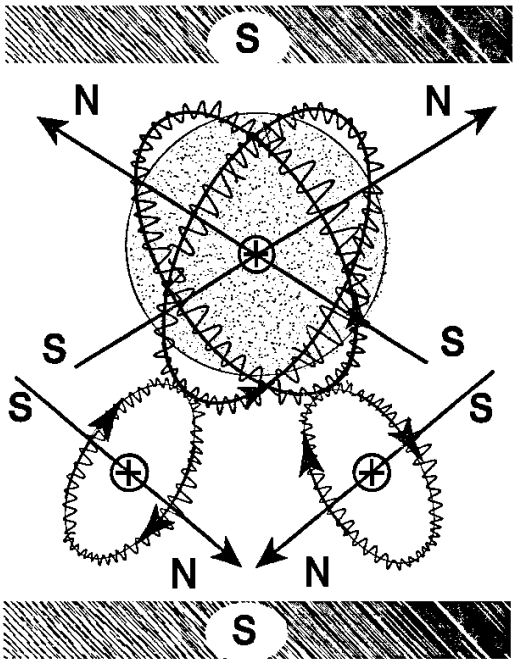


Figure 10.8. A schematic view of the resolution for the case of the water molecule of the inconsistent prediction of the conventional molecular model that water is paramagnetic (Fig. 1.14), as permitted by the Santilli-Shillady isochemical model of water molecule (Chapter 9). As one can see, the resolution is given by the impossibility for the water molecule to acquire a net magnetic polarity. Note the complexity of the geometry of the various magnetic fields which, according to ongoing research, apparently permits the first explanation on scientific record of the 105° angle between the two H–O dimers. The corresponding resolution for the case of the hydrogen is outlined in Fig. 9.5.

under an external field, with consequential net total magnetic polarities for all molecules which is in dramatic disagreement with experimental; evidence.

By comparison, in the isochemical molecular model the valence electrons are actually bonded to each other, with consequential *oo*-shaped orbit around the respective nuclei. This implies that the rotational directions of the *o*-branches are opposite to each other. In turn, this implies that magnetic polarizations are also opposite to each other, resulting in the lack of a net magnetic polarity under an external field, in agreement with nature (see Figs. 4.5 and 8.3 for more details).

10.2.5 Production of Magnecules in Gases, Liquids and Solids

At its simplest, the creation of magnecules can be understood via the old method of magnetization of a paramagnetic metal by induction. Consider a paramagnetic metal which, initially, has no magnetic field. When exposed to a constant external magnetic field, the paramagnetic metal acquires a permanent magnetic field that can only be destroyed at a sufficiently high temperature varying from metal to metal and called the *Curie Temperature*.

The mechanism of the above magnetization is well known. In its natural unperturbed state, the peripheral atomic electrons of a paramagnetic metal have a space distribution that results in the lack of a total magnetic field. However, when exposed to an external magnetic field, the orbits of one or more unpaired electrons are polarized into a toroidal shape with end polarities opposite to those of the external field.

This mechanism is called magnetic induction, and results in a stable chain of magnetically polarized orbits from the beginning of the metal to its end with polarities North-South/North-South/North-South/ ... This chain of polarizations is so stable that it can only be destroyed by high temperatures.

The creation of magnecules can be essentially understood with a similar polarization of the peripheral electron orbits, with the main differences that: no total magnetic polarization is necessary; the polarization generally apply to all electrons, and not necessarily to unpaired electrons only; and the substance need not to be paramagnetic.

To illustrate these differences, consider a diamagnetic substance, such as the hydrogen at its gaseous state at ordinary pressure and temperature. As well known, the hydrogen molecule is then a perfect sphere whose radius is equal to the diameter of a hydrogen atom, as illustrated in Fig. 11.9.A. The creation of the needed magnetic polarization requires the use of external magnetic fields capable, first, to remove the rotation of the atoms, as illustrated in Fig. 11.9.B, and then the removal of the internal rotations of the same, resulting in a planar configuration of the orbits as illustrated in Fig. 11.9 .C.

Once the above polarization is created in two or more hydrogen molecules sufficiently near each other, they attract each other via opposite magnetic polarities, resulting in the elementary magnecules of Fig. 11.10.A. Additional elementary magnecules can then also bond to each other, resulting in clusters with a number of constituents depending on the conditions considered.

A most efficient industrial production of gas and liquid magnecules is that via the *PlasmaArcFlow Reactors* [5]. As we shall see via the experimental evidence presented below, said reactors can produce an essentially pure population of gas and liquid magnecules without appreciable percentages of molecules directly detectable in the GC- or LC-MS.

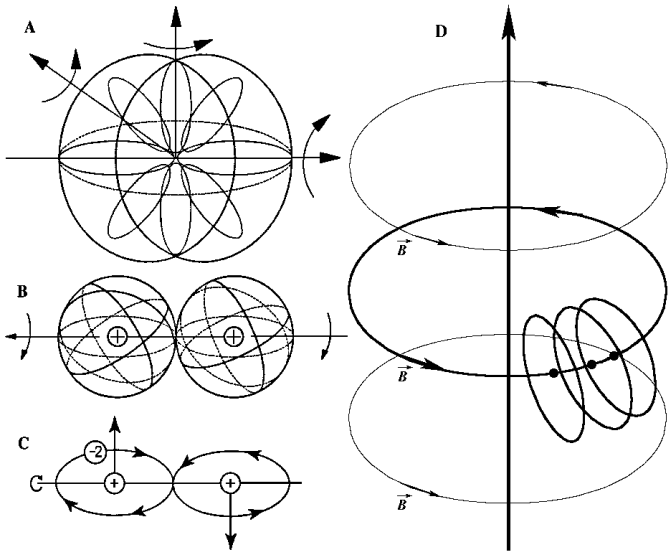


Figure 10.9. A schematic view of the main mechanism underlying the creation of magnecules, here illustrated for the case of the hydrogen molecule. It consists in the use of sufficiently strong external magnetic fields which can progressively eliminate all rotations, thus reducing the hydrogen molecule to a configuration which, at absolute zero degrees temperature, can be assumed to lie in a plane. The planar configuration of the electron orbits then implies the manifestation of their magnetic moment which would be otherwise absent. The r.h.s. of the above picture outlines the geometry of the magnetic field in the immediate vicinity of an electric arc as described in the text for the case of hadronic molecular reactors (Chapter 12). Note the *circular* configuration of the magnetic field lines around the electric discharge, the *tangential* nature of the symmetry axis of the magnetic polarization of the hydrogen atoms with respect to said circular magnetic lines, and the consideration of hydrogen atoms at *orbital distances* from the electric arc 10^{-8} cm, resulting in extremely strong magnetic fields proportional to $(10^{-8})^{-2} = 10^{16}$ Gauss, thus being ample sufficient to create the needed polarization (see Appendix 8.A for details).

The reason for these results is the intrinsic geometry of the PlasmaArcFlow itself. Recall that this technology deals with a DC electric arc submerged within a liquid waste to be recycled. The arc decomposes the molecules of the liquid into its atomic constituents; ionizes the same; and creates a plasma of mostly ionized H, C and O atoms at about $3,500°$ K. The flow of the liquid through the arc then continuously removes the plasma from the arc following its formation. Said plasma then cools down in the surrounding liquid, and a number of chemical reactions take place resulting in the formation of magnegas which bubbles to the surface of the liquid where it is collected for industrial or consumer use.

To understand the creation of a *new chemical species* defined according to Sect. 11.2.1 as an essentially pure population of *gas magnecules*, recall that mag-

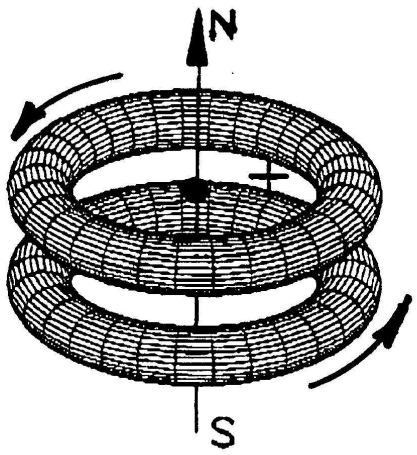


Figure 10.10. A schematic view of the simplest possible bi-atomic magnecule whose bond originates from the toroidal polarization of the orbits of peripheral atomic electrons. A first main difference with conventional molecular bonds is that, under sufficiently strong external magnetic fields, the magnecular bond may occur independently from the existence or not of valence electrons. Consequently, the two polarized atoms depicted in this figure can be arbitrarily chosen, while for conventional molecular bonds the atoms are restricted to verify known valence rules. Another major difference is that, by central conception to achieve full combustion for the case of fuels (see Section 11.1), the magnecular bond is much weaker than the molecular bond. In fact, due to its magnetic origin, the bond of this picture ceases to exist at a given temperature (the Curie Temperature) that, for the case of gaseous fuels with magnecular structure, it is usually given by the flame temperature. The main industrial as well as social result is that gaseous fuels with magnecular structure do achieve indeed total combustion without any toxic substance in its exhaust, something impossible for fuels with molecular structure, as proved by various cases studied in the subsequent sections of this chapter. Another implication also of major industrial and social relevance is that fuels with magnecular structure can be synthesized in such a way to be internally rich in oxygen (usually of liquid, rather than atmospheric origin) in order to replenish the atmospheric oxygen already depleted by fossil fuels, something equally impossible for fuels with molecular structure, as also studied later on in this chapter.

netic fields are inversely proportional to the square of the distance,

$$F_{\text{magnetic}} = \frac{m_1 m_2}{r^2}. \tag{11.2.14}$$

Therefore, an atom in the immediate vicinity of a DC electric arc with 1,000 A and 30 V, experiences a magnetic field which is inversely proportional to the square of the *orbital* distance $r = 10^{-8}$ cm, resulting in a magnetic field proportional to 10^{16} units.

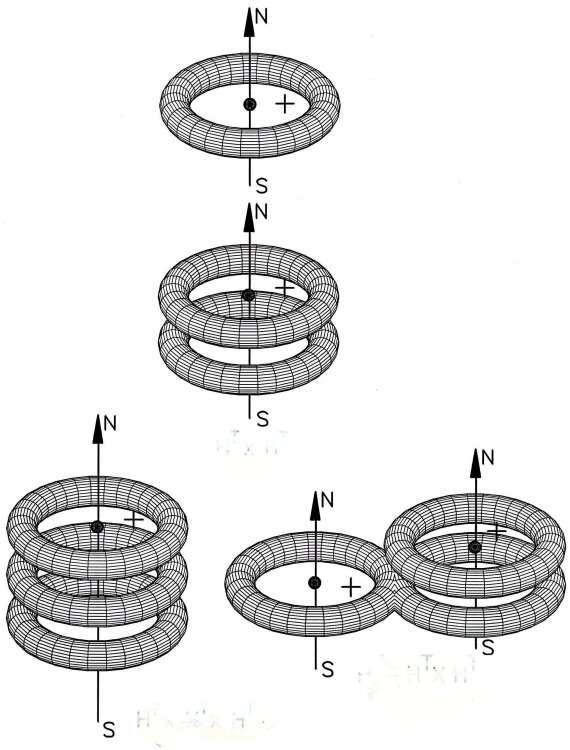


Figure 10.11. A schematic view of the simplest possible multiatomic magnecular bonds. Case A illustrates the *elementary hydrogen magnecule*. The subsequent case is that of four hydrogen atoms $(H \times H) \times (H \times H)$ (or two hydrogen molecules $H - H) \times (H - H)$) under a magnecular bond that has atomic weight very close to that of the helium. Therefore, the detection in a GC-MS scan of a peak with 4 a.m.u., by no means, necessarily identifies the helium because the peak could belong to the hydrogen magnecule. Case B illustrates a magnecule composed by a molecule and a dimer. Case C illustrates the hypothesis submitted in this monograph that the structure with 3 a.m.u. generally interpreted as a conventional "molecule" H_3 may in reality be a magnecule between a hydrogen molecule and an isolated hydrogen atom. This is due to the fact that, once the two valence electrons of the hydrogen molecule are bonded-correlated, they cannot admit the same valence bond with a third electron for numerous physical reasons, such as: the bond cannot be stable because the former is a Boson while the latter is a Fermion; the former has charge $-2e$ while the latter has charge $-e$, thus resulting in a large repulsion; *etc.*

No conventional space distribution of peripheral atomic electrons can exist under these extremely strong magnetic fields, which are such to generally cause the polarization of the orbits of *all* atomic electrons, and not only those of valence type, as well as their essential polarization in a plane, rather than a toroid.

As soon as two or more molecules near each other possessing such an extreme magnetic polarization are created, they bond to each other via opposing magnetic polarities, resulting in the elementary magnecule of Fig. 11.8.A.

Moreover, as shown earlier, isolated atoms have a magnetic field with an intensity double that of the same atom when belonging to a molecule. Therefore, as soon as created in the immediate vicinity of the electric arc, individual polarized atoms can bond to polarized molecules without any need to belong themselves to a molecule, as illustrated in Fig. 11.10.C.

Finally, recall that the PlasmaArcFlow is intended to destroy liquid molecules such as that of water. It then follows that the plasma can also contain individual highly polarized molecular fragments, such as the dimer H–O. The notion of gas magnecules as per Definition 8.2.1 then follows as referred to stable clusters of molecules, and/or dimers, and/or isolated atoms under an internal attractive bond among opposing polarities of the magnetic polarization of the orbits of peripheral electrons, nuclei and electrons when the latter are not coupled into valence bonds.

Effective means for the creation of an essentially pure population of *liquid magnecules* are given by the same PlasmaArcFlow Reactors. In fact, during its flow through the DC arc, the liquid itself is exposed to the same extreme magnetic fields as those of the electric arc indicated above. This causes the creation of an essentially pure population of liquid magnecules composed of highly polarized liquid molecules, dimers of the same liquid, and individual atoms, as established by LC-MS/UVD tests.

One way to create an essentially pure population of *solid magnecules* is given by freezing the new chemical species at the liquid level and then verifying that the latter persists after defrosting, as confirmed by various tests. Therefore, the case of solid magnecules is ignored hereon for simplicity.

By denoting with the arrow $\uparrow$ the vertical magnetic polarity North-South and with the arrow $\downarrow$ the vertical polarity South-North, and by keeping the study at the absolute zero degree temperature, when exposed to the above indicated extreme magnetic fields, the hydrogen molecule H–H can be polarized into such a form that the orbit of the isoelectronium is in a plane with resulting structure $H_\uparrow - H_\downarrow$ (Fig. 11.7).

The elementary hydrogen magnecule can then be written

$$\{H^a_\uparrow - H^b_\downarrow\} \times \{H^c_\uparrow - H^d_\downarrow\}, \tag{11.2.15}$$

where: a, b, c, d denote different atoms; the polarized hydrogen atom $H^a_\uparrow$ is bonded magnetically to the polarized atom $H^c_\uparrow$ with the South magnetic pole of atom a bonded to the North pole of atom c; and the North polarity of atom b is bonded to the South polarity of atom d (see, again, Fig. 8.5.A). This results in a strong bond due to the flat nature of the atoms, the corresponding mutual distance being very small and the magnetic force being consequently very large. Moreover, unlike the

case of the unstable clusters due to electric polarization discussed in Sect. 11.2.1, the above magnetic bonds are very stable because motions due to temperature apply to the bonded couple (11.2.15) as a whole.

For other magnecules we can then write

$$\{H_{\uparrow}-H_{\downarrow}\}\times\{C_{\uparrow}-O_{\downarrow}\}; \tag{11.2.16}$$

or, more generally

$$\{H_{\uparrow}-H_{\downarrow}\}\times H_{\downarrow}\times\{C_{\uparrow}-O_{\downarrow}\}\times\{H_{\uparrow}-O_{\downarrow}\}\times\{H_{\uparrow}-C_{\downarrow}-A-B-C\dots\}\times\dots, \tag{11.2.17}$$

where A, B, and C are generic atoms in a conventional molecular chain and the atoms without an indicated magnetic polarity may indeed be polarized but are not necessarily bonded depending on the geometric distribution in space.

Magnecules can also be formed by means other than the use of external magnetic fields. For instance, magnecules can be produced by electromagnetic fields with a distribution having a cylindrical symmetry; or by microwaves capable of removing the rotational degrees of freedom of molecules and atoms, resulting in magnetic polarizations. Similarly, magnecules can be formed by subjecting a material to a pressure that is sufficiently high to remove the orbital rotations. Magnecules can also be formed by friction or by any other means not necessarily possessing magnetic or electric fields, yet capable of removing the rotational degrees of freedom within individual atomic structures, resulting in consequential magnetic polarizations.

It is, therefore, expected that a number of substances which are today listed as of unknown chemical bond, may eventually result to have a magnecular structure.

Magnecules of type (8.15) may well have been detected in past mass spectrometric measurements, but believed to be the helium (because its molecular weight is very close to that of the helium). In fact, the same happens for the "molecule" H_3 which, in reality may be the magnecule of Fig. 11.10.C.

The destruction of magnecules is achieved by subjecting them to a temperature greater than the magnecules Curie Temperature which varies from magnecule to magnecule.

10.2.6 New Molecular Internal Bonds

As indicated in Sect. 11.2.2, and verified experimentally later on, the IR signatures of conventional molecules such as CO_2 are mutated due to the appearance of two new peaks which do not exist for the conventional molecule. By recalling that peaks in the IR signature generally represent bonds, this evidence indicates the capability by the CO_2 molecule to acquire new internal bonds in addition to those of conventional valence type.

The magnetic polarization at the foundations of magnecules predicts the existence of these new internal bonds and permits their quantitative study. Recall

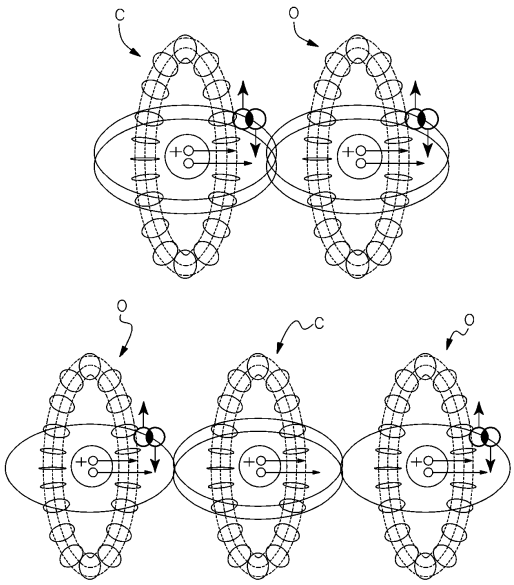


Figure 10.12. A schematic view for the cases of C=O and O–C–O of the polarization of internal atomic electrons, while preserving conventional valence bonds, and the consequential creation of new bonds in conventional molecules which are not of valence type, as later on verified experimentally via IR scans.

that external magnetic fields can polarize the orbit of valence electrons, but cannot possibly break or alter valence bonds. Recall that, consequently, sufficiently strong external magnetic fields can polarize the orbits of all atomic electrons, and not only those of the valence electrons.

Consider then a conventional molecule such as C=O. When exposed to the extreme magnetic fields as existing in the PlasmaArcFlow technology, the orbits of all internal electrons can be polarized, individually, for the carbon and the oxygen, in addition to the polarization of the two pairs of valence bonds. Note that the planes of these polarizations need not be necessarily parallel to each other, because their relative orientation dependents on the geometry at hand.

One of the various possible geometries is that in which the plane of the polarization of the internal electrons is perpendicular to that of the two pairs of valence bonds. In this case we have the birth of a *new bond of magnetic origin in the interior of a conventional molecule*, which is evidently given by the alignment of the two polarities North-South and North-South in the carbon and oxygen, and the consequential attraction of opposite polarities of different atoms, as illustrated in Fig. 11.11.A.

For the case of the O–C–O molecule we can evidently have two internal bonds of magnetic type in addition to the valence bonds, which are also given by the

alignment of the magnetic polarities, resulting in one new bond for the O–C dimer and a second one for the C–O dimer, as illustrated in Fig. 11.11.B.

As we shall see later on, the above new internal molecular bonds have major industrial and consumer implications, inasmuch as they permit the production of fuels capable of releasing under combustion anomalous amounts of energy, with consequential reduction of pollutants in the exhaust, as already proved by magnegas.

Needless to say, the creation of new internal bonds is an extreme case of IR mutation. In reality, numerous other weaker forms of mutations without the appearance of new peaks are possible and their study is left to the interested reader.

10.2.7 Main Features for the Detection of Magnecules

The experimental detection of gas magnecules requires the verification of a number of characteristic features of magnecules identified in Definition 8.2.1. In the following we focus the reader's attention on the main features of gas magnecules which must be verified via GC-MS tests. The remaining features will be considered later on.

Feature 1: Appearance of unexpected heavy MS peaks.

Gas magnecules are generally heavier than the heaviest molecule in a given gas. Peaks in the GC-MS are, therefore, expected in macroscopic percentages with atomic weights bigger than the heaviest molecule. As a concrete example, the heaviest molecule in magnegas in macroscopic percentage is CO_2 with 44 a.m.u. Therefore, GC-MS scans should only show background noise if set for over 44 a.m.u. On the contrary, peaks in macroscopic percentages have been detected in magnegas all the way to 1,000 a.m.u.

Feature 2: Unknown character of the unexpected MS heavy peaks.

To provide the initial premises for the detection of magnecules, all MS peaks of feature 1 should result in being "unknown" following the computer search among all known molecules, usually including a minimum of 150,000 molecules. Evidently, this lack of identification of the peaks, *per se*, does not guarantee the presence of a new chemical species.

Feature 3: Lack of IR signature of the unknown MS peaks.

Another necessary condition to claim the detection of magnecules is that the unknown MS peaks of feature 1 should have no IR signature other than that of the molecules and/or dimers constituents. This feature guarantees that said heavy peaks cannot possibly represent molecules, thus establishing the occurrence of a new chemical species. In fact, only very few and very light molecules can have

such a perfect spherical symmetry to avoid IR detection, while such a perfect spherical symmetry is manifestly impossible for large clusters. In regard to the constituents we are referring to IR signatures, *e.g.*, of the CO_2 at 44 a.m.u. in a cluster having 458 a.m.u.

Feature 4: Mutation of IR signatures.
The infrared signatures of conventional molecules constituting magnecules are expected to be *mutated*, in the sense that the shape of their peaks is not the conventional one. As indicated in the preceding section, the mutations most important for industrial applications are those due to the presence of *new IR peaks* representing new internal bonds. Nevertheless, various other forms of IR mutations are possible.

Feature 5: Mutation of magnecular weights.
While molecules preserve their structure and related atomic weight at conventional temperatures and pressures, this is not the case for gas magnecules, which can *mutate* in time, that is, change their atomic weight with consequential change of the shape and location of their MS peaks. Since we are referring to gases whose constituents notoriously collide with each other, magnecules can break-down during collisions into fragments which can then recombine with other fragments or other magnecules to form new clusters.

Feature 6: Accretion or emission of individual atoms, dimers or molecules.
Magnecules are expected to experience accretion or emission of individual atoms, dimer or molecules without necessarily breaking down into parts. It follows that the peaks of Feature 1 are not expected to remain the same over a sufficient period of time for the same gas under the same conditions.

Feature 7: Anomalous adhesion.
Magnetically polarized gases have anomalous adhesion to walls of disparate nature, not necessarily of paramagnetic character, as compared to the same unpolarized gas. This is due to the well-known property that magnetism can be propagated by induction, according to which a magnetically polarized molecule with a sufficiently intense magnetic moment can induce a corresponding polarization of valence and/or other electrons in the atoms constituting the wall surface. Once such a polarization is created by induction, magnecules can have strong magnetic bonds to the indicated walls. In turn, this implies that the background of GC-MS following scans and conventional flushing are often similar to the scan themselves. As a matter of fact, backgrounds following routine flushing are often used to identify the most dominant magnecules. Notice that the magnetic polar-

ization here considered does not require that the walls of the instrument are of paramagnetic type, since the polarization occurs for the orbits of arbitrary atoms.

Magnetically polarized gases additionally have mutated physical characteristics and behavior because the very notion of polarization of electron orbits implies physical alterations of a variety of characteristics, such as average size. Mutations of other characteristics are then consequential.

We should finally recall that the above features are expected to disappear at a sufficiently high temperature, evidently varying from gas to gas (Curie Temperature), while the features are expected to be enhanced at lower temperature and at higher pressure, and survive liquefaction.

10.3 THE UNAMBIGUOUS DETECTION OF MOLECULES AND MAGNECULES

10.3.1 Selection of Analytic Instruments

Current technologies offer an impressive variety of analytic instruments (see, *e.g.*, Ref. [4]), which include: Gas Chromatography (GC), Liquid Chromatography (LC), Capillary Electrophoresis Chromatography (CEC), Supercritical Chromatography (SCC), Ion Chromatography (IC), Infrared Spectroscopy (IR), Raman Spectroscopy (RS), Nuclear Magnetic Resonance Spectroscopy (NMRS), X-Ray Spectroscopy (XRS), Atomic Absorption Spectroscopy (AAS), Mass Spectrometry (MS), Laser Mass Spectrometry (LMS), Flame Ionization Spectrometry (FIS), and others.

Only some of these instruments are suitable for the detection of magnecules and, when applicable, their set-up and use are considerably different than those routinely used with great success for molecules.

Among all available chromatographic equipment, that suitable for the detection of gas magnecules is the GC with column having ID of at least 0.32 mm operated according to certain criteria outlined below. By comparison, other chromatographs do not appear to permit the entrance of large magnecules, such as the CEC, or be potentially destructive of the magnecules to be detected, such as the IC.

Among all available spectroscopic equipment, that preferable is the IR, with the understanding that such an instrument is used in a *negative* way, that is, to verify that the magnecule considered has no IR signature. The RS may also result in being preferable in various cases, while other instruments, such as the NMRS do not appear to be capable of detecting magnecules despite their magnetic nature, evidently because NMRS are most effective for the detection of microscopic magnetic environment of H-nuclei rather than large structures. Other spectroscopic instruments have not been studied at this writing.

In regard to spectrometric equipment, the most recommendable one is the low ionization MS due to the fact that other instruments seemingly destroy magnecules at the time of their detection. The study of other spectrometric equipment is left to interested researchers. Chemical analytical methods (i.e. via chemical reactions) to *detect* gas magnecules are probably not very effective since they necessarily destroy the magnecules in reaction.

As it is well known, when used individually, the above suggested instruments have considerable limitations. For instance, the GC has a great resolution of a substance into its constituent, but it has very limited capabilities to identify them. By comparison, the MS has great capabilities to identify individual species, although it lacks the ability to separate them.

For these reasons, some of the best analytic instruments are given by the combination of two different instruments. Among them, the most recommendable one is the GC combined with the MS, and denoted GC-MS. A similar occurrence holds for the IR combined to the GC-MS. As indicated since the early parts of this Chapter, the best instrument for the detection of both molecules and magnecules in gases is the GC-MS equipped with the IRD denoted GC-MS/IRD while that for liquids is the LC-MS equipped with UVD and denoted LC-MS/UVD.

Among a large variety of GC-MS instruments, only a few are truly effective for the detection of gas magnecules for certain technical reasons identified below. The instrument which has permitted the first identification of magnecules and remains the most effective at this writing (despite its considerable age for contemporary standards) is the GC Hewlett-Packard (HP) model 5890 combined with the MS HP model 5972 equipped with a large ID column and feeding line operated at the lowest temperature permitted by the instrument (about 10° C) and the longest elusion time (about 25 min).

A secondary function of the IRD is that of identifying the *dimers constituting a magnecule*, a task which can be fulfilled by various IRD. That which was used for the original discovery of magnecules and still remains effective (again, despite its age by current standards) is the IRD HP model 5965, when operated with certain criteria identified below.

A most insidious aspect in the detection of magnecule is the protracted use of any given instrument with great success in the detection of conventional molecules, and the consequential expectation that the same instrument should work equally well for the detection of magnecules, resulting in an analysis without any real scientific value because:

i) the species to be detected may not even have entered the instrument, as it is routinely the case for small syringes and feeding lines particularly for liquid magnecules (which can be so big as to be visible to the naked eye, as shown later on in this chapter);

ii) the species to be detected may have been destroyed by the measurement itself, as it is routinely the case for instruments operated at very high temperature, or flame ionization instruments which, when used for combustible gases with magnecular structure, cause the combustion of magnecules at the very time of their detection; or

iii) the detection itself may create magnecules which do not exist in the original species, as it is the case of peaks with 3 a.m.u. discussed in Fig. 11.10.

In conclusion, the separation between a true scientific measurement and a personal experimental belief requires extreme scientific caution in the selection of the analytic instrument, its use, and the interpretation of the results.

10.3.2 Unambiguous Detection of Molecules

As it is well known, a *gas molecule* is identifiable by unique and unambiguous GC-MS peaks, which are distinctly different from those of any other gas molecule. In addition, this GC-MS identification can be confirmed by IRD peaks and related resonating frequencies, which are also distinctly different for different gas molecule. Additional confirmations are possible using other analytic methods, such as those based on average molecular weight, chemical reactions and other procedures.

The advent of the new chemical species of magnecules suggests a re-examination of these analytic methods and procedures so as to separate personal opinions from actual scientific identifications. Such a re-examination is warranted by the fact that, due to extended use, claims of specific molecular identifications are nowadays generally voiced via the use of only one analytic detector.

As an illustration, most contemporary analytic laboratories conduct chemical analyses on gases via the sole use of the IRD. However, *infrared detectors do not identify complete molecules, since they can only identify the bond in their dimers*. For instance, for the case of H_2O, the IRD does not identify the complete molecule, but only its dimer H–O.

This method of identification of molecules is certainly acceptable for gases whose lack of magnetic polarization has been verified by the analysts. However, the same method is highly questionable for gases of unknown origin. In fact, we shall soon show experimental evidence of clear IR signatures for molecules which have no MS identification at all, in which case the claim of such a molecule evidently has no scientific value.

The inverse occurrence is equally questionable, namely, the claim of a given molecule from its sole identification in the MS without a confirmation of exactly the same peak in the IRD. In fact, there are several MS peaks in magnetically polarized gases which may be easily identified with one or another molecule, but which have no IR signature at all at the MS value of the atomic weight, in which case the claim of molecular identification evidently has no scientific value.

Note that the great ambiguities in the separate use of disjoint GC-MS and IRD. In fact, in this case there is no guarantee or visible evidence that exactly the same peak is jointly inspected under the MS and, separately, the IRD. In fact, a given molecule can be tentatively identified in the MS at a given a.m.u., while the same molecule may indeed appear in the IRD, although at a different value of a.m.u., in which case, again, the claim to have detected a given molecule is a personal experimental belief, rather than a scientific truth.

In conclusion, *a serious scientific identification of any given molecule requires the joint use of at least two different analytic methods, both giving exactly the same result for exactly the same peak in a unique and unambiguous way, such as the detection via MS scans with unequivocal computer identifications, confirmed by IR scans without ambiguities, thus requiring the use of GC-MS equipped with IRD.*

Additional ambiguities result from the rather widespread belief that molecules are the only possible chemical species in nature, in which case small deviations from exact identifications are generally ignored for the specific intent of adapting experimental evidence to pre-existing knowledge, rather than modifying old interpretations to fit new experimental evidence. This widespread tendency is also a reason why magnecules have not been identified until now.

As an illustration, suppose that: a GC-MS equipped with IRD detects a peak with 19 a.m.u.; said peak is identified by the MS search as the water molecule with 18 a.m.u.; and the IRD confirms the presence of the HO-dimer. Under these conditions, it is almost universally accepted in contemporary analytic laboratories that said peak with 19 a.m.u. represents the water molecule, and the spurious single a.m.u. is just an "impurity" or something to be ignored, in which case, however, we do not have a true scientific identification of the species.

In fact, it is well possible that the peak at 19 a.m.u. is constituted by a highly polarized water molecule magnetically bonded to one isolated hydrogen atom with structure

$$\{H_{\downarrow} - O_{\uparrow\uparrow} - H_{\downarrow}\} \times H_{\downarrow}. \tag{11.3.1}$$

In this case, according to our terminology, the peak at 19 a.m.u. is a magnecule and *not* a molecule, even though the MS search gives 99.99% confidence and the IR search gives 100% confidence that the species is the ordinary water molecule. After all, the magnecular bond is transparent to current IR detection, then, the latter confirms an erroneous belief.

At any rate, no claim on the peak with 19 a.m.u. can be truly scientific or otherwise credibly, unless it explains in a specific and numerical way, without vague nomenclatures, how the single a.m.u. entity is attached to the water molecule.

Recall that the valence bond requires singlet couplings to verify Pauli's exclusion principle. As a consequence, coupled pairs of valence electrons are *Bosonic states with zero spin*. Under these conditions, no nomenclature suggesting one or

another type of valence can credible explain the bonding of one single H atom to the H–O–H molecule because it would imply the bond of a *Fermion with spin* 1/2 (the valence electron of the hydrogen) with a *Boson* (the coupled valence electron pair of the water), which bond is an impossibility well known in particle physics. By comparison, the magnecular hypothesis identifies the *attractive* character of the bond in a clear and unambiguous way, and then its *numerical value* (8.9).

The detection of *liquid molecules* has problems greater than those for gas molecules, because liquid magnecules can be so big to be visible by the naked eye, in which case only their conventional molecular constituents are generally permitted to enter current instruments, resulting again in a lack of real detection.

In conclusion, the separation in the identification of molecules between a true scientific process and a personal experimental belief requires extreme care before claiming that a certain peak characterizes a molecule, since possible ambiguities exist in all cases, from small to large atomic weights. In the final analysis, as stressed above, the difference between a molecule and a magnecule may be given by what is generally considered noise, or instrument malfunction.

The most unreassuring occurrence is that all GC-MS equipped with IRD identified by this author in the USA following a laborious search belong to military, governmental, or law enforcement institutions, and none of them was identified in commercial or academic laboratories. Therefore, the great majority of analytic laboratories lack the very instrument necessary for a final and unequivocal identification of a conventional *molecules*, let alone that of magnecules.

10.3.3 Unambiguous Detection of Magnecules

Since magnecules have properties very different from those of conventional molecules, the experimental detection of magnecules requires a special care. In particular, methods which have been conceived and developed for the detection of molecules are not necessarily effective for the detection of the different chemical species of magnecules precisely in view of the indicated differences.

The first indication of a possible *gas magnecule* is given by MS peaks with large atomic weight which cannot be explained via conventional molecular hypotheses. The second indication of a gas magnecule is given by the lack of identification of said heavy peaks in the MS following a search among all known molecules. A third indication of a gas magnecule then occurs when said unknown MS peak has no IR signature, except those of its constituents with much smaller atomic weight, which occurrence establishes the lack of a valence bond. Final identification of a gas magnecule requires the knowledge of the method used in the production of the gas and other evidence.

As it is the case also for molecules, a serious spectrographic analysis of magnecules requires GC-MS detectors necessarily equipped with IRD, because only such an instrument permits the direct test of the *same peaks* under both the

MS and IR scan. Again, if the IRD operates separately from the GC-MS, the indicated joint inspection is not possible; the IRD can only detect ordinary molecular dimers; the experimental belief that the MS peak must be a molecule is then consequential.

As a concrete example verified later on with actual tests, consider the spectrographic analysis of magnegas. This is a light gas whose heaviest molecule in macroscopic percentages should be the CO_2 at 44 a.m.u. Consider now an MS peak of magnegas at 481 a.m.u. It is evident that, while small deviations could be adapted to quantum chemistry, large deviations of such an order of magnitude cannot be reconciled with established knowledge in a credible way, thus permitting the hypothesis that the MS peak in a *light* gas with 481 a.m.u. can be a magnecule. The MS scan of the peak soon establishes the impossibility for the computer to identify the peak among all existing molecules. When the GC-MS is equipped with IRD, the analyst can scan the same peak with 481 a.m.u. under the IRD and detect no signature at the 481 a.m.u. value, the only IR signature being that at 44 a.m.u. of the CO_2 as well as those of smaller molecules. The production of the gas under intense magnetic fields then confirm that the peak here considered at 481 a.m.u. is indeed a magnecule composed of a large number of ordinary light molecules, dimers and individual atoms, in accordance with Definition 8.2.1.

Note that the IRD scan in the above test has solely identified conventional molecules without any additional unknown. Yet, the conclusion that the gas considered is solely composed of molecule would be nonscientific for numerous reasons, such as: 1) magnetic bonds are transparent to IR scans with available frequencies; 2) there is no IR detection, specifically, at 481 a.m.u.; and 3) IRD do not detect molecules, but only dimers.

Therefore, even though the IRD has detected CO_2 in the above test, the actual detection was for the C–O dimer, in which case the claim of the presence of the full CO_2 molecule is a personal opinion, and not an experimental fact.

The anomalous energy content, weigh and other features of magnegas confirm the above conclusions, because the latter can only be explained by assuming that a certain percentage of IR counts is indeed due to complete molecules, while the remaining percentage is due to unpaired dimers trapped in the magnecules. The freeing of these dimers and atoms at the time of the combustion, and their recombination into molecules as in Eqs. (8.5) then explains the anomalous energy content.

In addition to the above basic requirements, numerous other precautions in the use of the GC-MS equipped with IRD are necessary for the detection of magnecules, such as:

i) the MS equipment should permit measurements of peaks at ordinary temperature, and avoid the high temperatures of the GC-MS column successfully used for molecules;

ii) the feeding lines should be cryogenically cooled;

iii) the GC-MS/IRD should be equipped with feeding lines of at least 0.5 mm ID;

iv) the GC-MS should be set to detect peaks at large atomic weights usually not expected; and

v) the ramp time should be the longest allowed by the instrument, *e.g.*, of at least 25 minutes.

It should be stressed that *the lack of verification of any one of the above conditions generally implies the impossibility to detect magnecules*. For instance, the use of a feeding line with 0.5 mm ID is un-necessarily large for a conventional light gas, while it is necessary for a gas with magnecular structure such as magnegas. This is due to the unique adhesion of the magnecules against the walls of the feeding line, resulting in occluded lines which prevent the passage of the most important magnecules to be detected, those with large atomic weight.

Similarly, it is customary for tests of conventional gases to use GC-MS with columns at high temperature to obtain readings in the shortest possible time, since conventional molecules are perfectly stable under the temperatures here considered. The use of such method would equally prevent the test of the very species to be detected, because, as indicated earlier, they have a characteristic Curie Temperature at which all magnetic features are lost. Magnecules are stable at ordinary temperatures and, consequently, they should be measured at ordinary temperatures.

Along similar lines, recall that GC-MS with a short ramp time are generally used for rapidity of results. Again, the use of such a practice, which has been proven by extensive evidence to be effective for molecules, prevents clear detection of magnecules. If the ramp time is not of the order of 25 minutes, *e.g.*, it is of the order of one minute, all the peaks of magnecules generally combine into one single large peak, as described below. In this case the analyst is generally lead to inspect an individual section of said large peak. However, in so doing, the analyst identifies conventional molecules constituting the magnecule, and not the magnecule itself.

When these detectors with short ramp times are equipped with IRD, the latter identify the infrared signatures of individual conventional molecules constituting said large unique peak, and do not identify the possible IR signature of the single large peak itself. Therefore, a GC-MS with short ramp time is basically unsuited for the detection of magnecules because it cannot separate all existing species into individual peaks.

In conclusion, the experimental evidence of the above occurrences establishes the need in the detection of gas magnecules of *avoiding, rather than using, techniques and equipment with a proved efficiency for molecules*, thus avoiding the use of GC-MS without IRD, with short ramp time, high column temperatures, microscopic feeding lines, and other techniques. On the contrary, new techniques specifically conceived for the detection of magnecules should be worked out.

The conditions for scientific measurements of *liquid magnecules* via LC-MS/UVD are more stringent than those for gases, because of the great increase, in general, of the atomic weight of liquid magnecules which are generally much larger than the IR of conventionally used feeding lines, as shown below.

This implies the possible erroneous claim that magnecules do not exist because they are not detected by the LC-MS, while in reality the magnecule to be detected could not enter into at all into the instrument.

10.3.4 Apparent Magnecular Structure of H_3 and O_3

As it is well known, chemistry has identified in GC-MS tests clusters with 3 a.m.u., which can only be constituted of three H atoms, H_3, while the familiar ozone O_3 has been known since quite some time. These structures are generally assumed to be molecules, that is, to have a valence bond according to one nomenclature or another, although this author is aware of no in depth theoretical or experimental identification of the attractive force necessary to bond the third atom to a conventional molecule.

There are serious doubts as to whether such a conventional molecular interpretation will resist the test of time as well as of scientific evidence. To begin, a fundamental property of valence bonds is that *valence electrons correlate in pairs*. Since the H_3 and O_3 structures contain the molecules H_2 and O_2 in which all available valence electrons are already bonded in pairs, the belief that an additional third valence electron could be correlated to the preceding ones violates basic chemical knowledge on valence.

Moreover, we have stressed earlier that the assumption of a third valence electron bonded to a valence pair is in violation of basic physical knowledge, because it would require the bond of a Fermion (the third electron with spin 1/2) with a Boson (the singlet valence pair with spin 0) both possessing the same negative charge. Such a hypothetical bond under molecular conditions would violate various laws in particle physics, e.g., it would imply a necessary violation of Pauli-s exclusion principle since the assumed "triplet" of electrons would have *two* identical electrons in the same structure with the same energy.

In view of the above (as well as other) inconsistencies, we here assume that *the familiar H_3 and O_3 clusters are magnecules consisting of a third H and O atom magnetically bonded to the conventional H_2 and O_2 molecules, respectively, along the structure of Fig. 11.11.C.* Note that this assumption is fully in line with

Definition 8.2.1 according to which a magnecule also occurs when one single atom is magnetically bonded to a fully conventional molecule.

The plausibility of the above structure is easily illustrated for the case of O_3. In fact, the oxygen is known to be paramagnetic, and the ozone is known to be best created under an electric discharge. These are the ideal conditions for the creation of a magnetic polarization of the orbits of (at least) the paramagnetic electrons. The attraction of opposing magnetic polarities is then consequential, and so is the magnetic bond of the third oxygen to the oxygen molecule, resulting in the magnecule $O_2 \times O$.

The above magnecular interpretation of O_3 is confirmed by various GC-MS detections of peaks with 32 a.m.u. in a magnetically treated gas originally composed of pure oxygen, in which case the sole possible interpretation is that of two magnetically bonded oxygen molecules, resulting in the magnecule $O_2 \times O_2$.

The plausibility of the magnecular interpretation is less trivial for the H_3 structure since hydrogen is diamagnetic. Nevertheless, the assumption remains equally plausible by recalling that a central feature of the new chemical species of magnecules is that *the magnetic polarization occurs at the level of each individual atom, and not at the level of a diamagnetic molecule, whose total magnetic moment remains null as illustrated in Fig. 11.7.*

In particular, the magnecular interpretation of the MS peaks at 3 a.m.u. is numerical and without ambiguities. Recall that GC equipment works by ionizing molecules. When testing a hydrogen gas, a number of H_2 molecules are separated into individual H atoms by the ionization itself. Moreover, the ionization occurs via the emission of electrons from a filament carrying current, which is very similar to that of the PlasmaArcFlow Reactors producing magnecules. Under these conditions, the filament of the GC can not only separate H-molecules but also polarize them when sufficiently close to the filament. Once such polarizations are created, their bond is a known physical law, resulting in the magnecule of Fig. 11.11, *i.e.*

$$\{H_{\downarrow} - H_{\uparrow}\} \times H_{\downarrow}. \tag{11.2.19}$$

As one can see, under the magnecular structure the bond is manifestly attractive, very strong, and numerically identified in Eq. (11.3.9). Other interpretations of the peak at 3 a.m.u. are here solicited, provided that, to be credible, they are not of valence type and the internal bond is identified in a clear, unambiguous, and numerical way.

The magnecular interpretation of H_3 is confirmed by numerous GC-MS detections of a cluster with 4 a.m.u. in a magnetically treated gas which originally was composed by pure hydrogen, under which conditions such a peak can only be constituted by two hydrogen molecules resulting in the magnecule $H_2 \times H_2$ illustrated in Fig. 11.11.

It is an easy prediction that numerous peaks detected in contemporary GC-MS or LC-MS equipment may need a magnecular re-interpretation since, as indicated earlier, the method of detection itself can create magnecules. This is typically the case when the comparison of a given MS cluster with the actual peak of a given molecules contains additional lines.

As a specific example, when the peak representing a hexanal molecule (whose heaviest constituent has 100 a.m.u.) contains additional lines at 133 a.m.u., 166 a.m.u., and 207 a.m.u., it is evident that the latter lines cannot cluster with the hexanal molecule via valence bond. The plausibility of the magnecular interpretation is then evident.

For copies of the GC-MS scans mentioned in this section, which are not reproduced here for brevity, we suggest the interested reader to contact the author.

10.3.5 Need for New Analytic Methods

In closing, we should stress that the methods for the detection and identification of magnecules are at their infancy and numerous issues remain open at this writing (spring 2001). One of the open issues relates to several detections in magnegas of IR signatures apparently belonging to complex molecules, such as light hydrocarbons, while such molecules have not been identified in the MS scans. This occurrence creates the realistic possibility that certain complex magnecules may indeed have an IR signature in view of their size. More specifically, as indicated earlier, magnecules are assumed to be transparent to currently available IRD because their inter-atomic distance is expected to be 10^4 times smaller than the inter-atomic distance in molecules, thus requiring test frequencies which simply do not exist in currently available IRD.

However, such an argument solely applies for magnecules with small atomic weight, such as the elementary magnecules of Fig. 11.10. On the contrary, magnecules with heavy atomic weight may well have an IR signature and, in any case, the issue requires specific study.

This possibility is confirmed by the fact that magnegas is created via underliquid electric arcs whose plasma can reach up to $10{,}000^o$ K. The insistence that light hydrocarbons could survive in these conditions, let alone be created, is not entirely clear. This direct observation is confirmed by the fact that no hydrocarbon has been detected in the combustion of magnegas. In fact, the cars running on magnegas (see next section) operate without catalytic converter. Direct analysis of the combustion exhaust show a *negative count* of hydrocarbons, that is, the exhaust contains less hydrocarbons than the local atmosphere which is used for basic calibration of the instrument.

In summary, we have a case in which light hydrocarbons are seemingly indicated by IR scans to exist in small percentages in magnegas, while no hydrocarbon has ever been identified in the MS scans, no hydrocarbon is expected to survive

at the extreme temperatures of the electric arcs used for their production, and no hydrocarbon has been detected in the combustion exhaust.

These occurrences illustrate again that the identification of conventional molecules via the sole use of IR scans or, equivalently, the sole use of MS scans, is, in general, a mere personal opinion without scientific foundations.

10.4 MAGNEGASES^{T}M, THE COMBUSTIBLE GASES WITH A MAGNECULAR STRUCTURE

10.4.1 PlasmaArcFlow^{TM} Reactors for Recycling Liquid Waste into the Clean Burning Cost Competitive Magnegas Fuel

In this section we summarize the results of corporate research following the investment of several millions of dollars for the conception and industrial development by the author of the first new gaseous fuels with magnecular structure, today known as *Santilli Magnegases*^{TM} (patented and international patents pending [5]), verifying the following main conditions:

1) Achievement of full combustion without any toxic content in the exhaust, thanks to its magnecular structure (Section 11.1.1);

2) Achievement of exhaust rich in breathable oxygen originating from liquids (rather than from our atmosphere) so as to replenish the oxygen depleted by fossil fuel combustion and converted into excess CO_2;

3) Have a thermodynamical equivalence with natural gas so that all equipment running on natural gas (cars, electric generators, furnaces, etc.) can also run on magnegas without structural modifications;

4) Be lighter than air and have a natural scent so that in case of leaks the gas can be easily detected and rises for safety;

5) Be synthesized via the recycling of liquid waste (such as automotive liquid wastes, city, farm or ship sewage, etc.), so as to decrease the environmental problems caused by the latter;

6) Be cost competitive with respect to fossil fuels thanks to a highly efficient and non-contaminant production process;

7) Permit the achievement of fuel independence from crude oil (petroleum) thanks to the continuous local availability as feedstock, for instance, of city sewage 24 hours per day.

The term "magnegases" is today referred to *all gaseous fuels possessing Santilli's magnecular structure.* from now on we shall study in this section a specific type of "magnegas" obtained via a new combustion of carbon obtained via a submerged electric arc. Consequently, the type of magnegas treated in this section contains carbon. other types of magnegases without carbon will be studied in the next sections.

Figure 10.13. Picture of a 250 Kw Santilli's Hadronic Reactor (also called PlasmaArcFlow Reactor) with the panels of its completely automatic and remote controls, to recycle liquid waste into magnegas usable for any fuel application, a large amount of heat and carbonaceous precipitates used to produce the electrodes. This Reactor can produce up to 5000,000 scf (140 millions liters) of magnegas per week of 24 hours work per day corresponding to 3,000 gasoline gallon equivalent (11,000 gasoline liter equivalent) of magnegas per week computed on the basis that: 1) Gasoline contains about 110,000 BTU/g (about 29,000 BTU/liter); 2) Magnegas has the low energy content of 750 BTU/scf (26 BTU/liter); and 3) the "gasoline gallon equivalent" is given by about 150 scf of magnegas ("gasoline liter equivalent" is given by about 29 liters of magnegas). PlasmaArcFlow reactors are completely self-contained. Consequently, they release no solid, liquid or gas to the environment and cause no noise or odor pollution (see website [5b] for details).

The equipment that has been developed for the production of type of magnegas here considered is given by *Santilli's hadronic reactors of molecular type* (Class III), also known as PlasmaArcFlow^{TM} Reactors (patented and international patents pending [5]), that were first built by the author in 1998 in Florida, U.S.A., , and are now in regular production and sale the world over (see the figures and web site [5b] for pictures).

PlasmaArcFlow Reactors use a submerged DC electric arc between carbon-base electrodes to achieve the complete recycling of essentially any type of (non-radioactive) liquid waste into the clean burning magnegas fuel, heat usable via exchangers, and carbonaceous precipitates used for the production of electrodes. The reactors are ideally suited to recycle antifreeze waste, oil waste, sewage, and other contaminated liquids, although they can also process ordinary fresh water. The best efficiency is achieved in these reactors for the recycling of carbon-rich liquids, such as crude oil or oil waste.

The new PlasmaArcFlow technology is essentially based on flowing liquids through a submerged DC arc with at least one consumable carbon electrode (see Figures 11.8 and 11.9). The arc decomposes the liquid molecules and the carbon electrode into a plasma at about $5,000^oC$, which plasma is composed of mostly ionized H, O and C atoms. The technology moves the plasma away from the electric arc immediately following its formation, and controls the recombination of H, O and C into magnegas, that bubbles to the surface where it is collected with various means. Other solid substances generally precipitate at the bottom of the reactor where they are periodically collected.

Since magnegas is formed under the extremely intense magnetic fields at atomic distances from the electric arc, its chemical structure is that of all possible magnecules with increasing atomic mass that can be formed from the H, C and O atoms, such as

$$\begin{gathered} H-H, \quad H\times H, \quad (H-H)\times H \quad H\times H\times H; \\ C-H, \quad C\times H, \quad (C-H)\times H, \quad C-O, \\ C\times O, \quad C\times (O-O), \quad etc. \end{gathered} \tag{11.4.1}$$

where we should note that the bond between C and O can be single, double, and triple valence type as well as of magnecular nature.

When reduced to *atomic* (rather than molecular) percentages, magnegas produced from pure water is composed of about

$$66\%H, \quad 22\%O, \quad 11\%C. \tag{11.4.2}$$

As we shall see in this section, under GC-MS/IRD detectors magnegas shows peaks from 2 amu to 1,000 amu even when produced from pure water. To understand the anomaly ands the *necessity* of a new chemical species to avoid non-scientific beliefs, the reader should be aware that the maximal molecular species predicted by quantum chemistry for magnegas produced from water is CO_2 with 44 amu.

Chemical structure (11.3.1) also explains the capability of magnegas to have combustion exhaust without any toxic components, while being rich of breathable oxygen up to 14% and more (see later on). In fact, the combustion exhaust of magnegas are given by

$$50-55\%H_2O, \quad 12-14\%O_2, \quad 5-7\%CO_2, \quad atmospheric \quad gases. \tag{11.4.3}$$

where the CO_2 content originates from the combustion of conventional carbon monoxide, that of triple bonded nature $\equiv O$ and the stochiometric ratio magnegas/atmosphere is taken into consideration.

the availability in the exhaust of a ;large percentage of breathable oxygen is primarily due to the magnecular bond of oxygen with other species that breaks

Figure 10.14. Picture of a 50 Kw Santilli's Hadronic Reactor (PlasmaArcFlow Recycler) mounted on a trailer for mobility to conduct test recycling where liquid wastes are located. This recycler can produce up to $84,000$ scf (up to $2.4M$ liters) of magnegas per week corresponding to about 560 gasoline gallon equivalent ($2,100$ gasoline liter equivalent) of magnegas per week (see website [5b] for details).

down at the *Magnegas Curie temperature* generally coinciding with the flame temperature at which all bonds of magnetic origin cease to exhaust. consequently, *magnegas exhaust is solely composed of conventional molecules without any magnecular content.*

The large percentage in the exhaust of breathable oxygen is also due to the conventional single $C-O$ and double bonded species $C=O$ that are unstable and can decompose into gaseous oxygen and carbon precipitate when the combustion is at atmospheric pressure (because high pressure $C-O$ and $C=O$ can turn into $C \equiv O$).

the efficiency of Santilli's hadronic reactor of Class III is very high because their primary source of energy is given by a new type of highly efficient and clean combustion of carbon, releasing energy that is at last 30 times the electric energy used by the arc.

In fact, we have the following *Scientific Efficiency* (SE) of PlasmaArcFlow Reactors that is evidently always *smaller* than one due to the conservation of the energy and the inevitable dispersions

$$SE = \frac{E_{tot}^{out}}{E_{tot}^{in}} = \frac{(E_{MG} + E_{heat})}{(Earc + E_{carbon} + E_{unknown})} < 1, \tag{11.4.4}$$

where E_{MG} is the combustion energy contained in magnegas, E_{heat} is the heat acquired by the liquid feedstock and the vessel,$Earc$ is the electric energy used

by the arc, E_{carbon} is the energy produced by the combustion of carbon in the plasma, and $E_{unknown}$ is an unknown source of energy due to the fact that the sum $E_{arc} + E_{carbon}$ cannot explain the total energy output (see later on in this section and the next chapter).

At the same time we have the following *Commercial Efficiency* (CE) given by the preceding one *without* the inclusion of the energy produced by the carbon combustion because the carbon content of the liquid feedstock generally brings an *income,* rather than a cost, since it is contained in the liquid waste to be, and without the unknown source of energy since it also carries no cost recycled

$$CE = \frac{E_{tot}^{out}}{E_{tot}^{in}} = \frac{(E_{MG} + E_h eat)}{Earc} >> 1, \tag{11.4.5}$$

which value is much bigger than one because, as indicated above, the energy caused by the combustion of carbon in the plasma under the electric arc and the unknown energy are a large multiple of the electric energy used by the arc.

When operated at atmospheric pressure, at 50 Kw power and at ambient temperature, the above commercial efficiency has a minimum value of about 5. However, the efficiency of Santilli's hadronic reactors increases (nonlinearly) with the increase of pressure, power and operating temperature and can assume rather high value. For instance, when operating a PlasmaArcFlow Recycler at about 150 psi (10 bars), 300 Kw and 275^oF (125^oC), the commercial efficiency can be of the order of 30, that is, per each unit of electric energy used by the arc, the reactor produces 30 times that energy in a combination of energy contained in magnegas and usable heat.

The above very high commercial efficiency of PlasmaArcFlow reactors illustrates the reason why magnegas is cost competitive with respect to all available fossil fuels.

By comparison, one should note that, in other methods based on underwater arcs, the stationary character of the plasma within the arc implies the creation of large percentages of CO_2 resulting in a CO_2 content of the exhaust much greater than that of gasoline and natural gas, measured by the author to be of the order of 18%. The resulting fuel is then environmentally unacceptable since CO_2 is responsible for the green house effect.

Recall that the primary source of the large glow created by underwater arcs is the recombination of H and O into H_2O following its separation. This recombination is the reason for the low efficiency of underwater arcs and consequential lack of industrial development until recently.

By comparison, the PlasmaArcFlow causes the removal of H and O from the arc immediately following their creation, thus preventing their recombination into H_2O, with consequential dramatic increase of the efficiency, that is, of the volume of combustible gas produced per Kwh.

Figure 10.15. A picture of a Ferrari 308 GTSi 1980 and two Honda Civic cars converted by the author to operate with the new clean burning magnegas without catalytic converter, yet surpassing all EPA exhaust requirements, having no carcinogenic or other toxic substance in the exhaust, reducing of about 50% the CO_2 emission due to gasoline combustion, reducing the operating temperature of about 25%, and emitting in the exhaust 10% to 14% breathable oxygen (see website [5b] for details).

A Ferrari 308 GTS, an SUV, two Honda and other automobiles have been converted by the author to operate on magnegas. One of these vehicles has been subjected to intensive tests at an EPA certified automotive laboratory in Long Island, New York reviewed in details in the next subsection, which tests have established that magnegas exhaust surpasses all EPA requirements *without catalytic converter*, and confirmed data (11.3.3).

In addition to the production of magnegas as a fuel, *the PlasmaArcFlow Reactors can be viewed as the most efficient means for producing a new form of hydrogen, called MagneHydrogen*^{TM}, *a carbon-free version of magnegas, with energy content and output greater than the conventional hydrogen, and at a cost smaller than that of the latter* (see next section).

Figure 10.16. A picture of a Chevrolet Suburban SUV 1992 converted by the author to operate as a bifuel gasoline/magnegas with a switch on the dashboard permitting to pass from one fuel to the other while driving. Bifuel cars are produced by numerous carmakers to operate on gasoline and natural gas. The same cars can operate on magnegas (in place of natural gas) with the sole adjustment of the pressure regulator to optimize the stochiometric ratio air/fuel, since the latter for magnegas is much smaller than that for natural gas because magnegas is very rich internally in oxygen, thus requiring a fraction of the air needed by natural gas to operate. These bifuel cars are ideally suited for the magnegas technology because, when magnegas runs out, one can still reach the magnegas refilling station on gasoline (see website [5b] for details).

10.4.2 Surpassing by Magnegas Exhaust of EPA Requirements without Catalytic Converter

As indicated above, while the chemical composition of magnegas is new, the chemical composition of magnegas combustion exhaust is fully conventional, and it has been measured with accuracy.

The tests were conducted by an EPA accredited automotive laboratory of Long Island, new York, on a Honda Civic Natural Gas Vehicle (NGV) VIN number 1HGEN1649WL000160 (the white car of Fig. 11x10), produced in 1998 to operate with Compressed Natural Gas (CNG). The car was purchased new in 1999 and converted to operate on Compressed MagneGas (CMG) in early 2000. All tests reported in this section were done with magnegas produced by recycling antifreeze waste. The conversion from CNG to CMG was done via:

1) the replacement of CNG with CMG in a 100 liter tank at 3,600 psi which contains about 1,000 cf of magnegas;

2) the disabling of the oxygen sensor because magnegas has about 20 times more oxygen in the exhaust than natural gas, thus causing erroneous readings by the computer set for natural gas; and

3) installing a multiple spark system to improve magnegas combustion.

The rest of the vehicle was left unchanged, including its computer.

Comparative tests on performance (acceleration, full load, etc.) have established that *the output power of the vehicle operating on compressed magnegas is fully equivalent to that of the same car operating on compressed natural gas.*

Comparative tests on consumption also indicate similar results. In fact, measurements of magnegas consumption per hour in ordinary city driving were conducted with the following results:

$$\begin{array}{ll} \text{Tank capacity:} & \text{1,096 cf at 3,500 psi,} \\ \text{Total duration:} & \text{about 2.5 hours,} \\ \text{Consumption:} & \text{about 7 cf/minute.} \end{array} \tag{11.4.6}$$

As one can see, a magnegas pressure tank of 1,500 cf at 5,000 psi would provide a range of about 4 hours, which is amply sufficient for all ordinary commuting and travel needs. Measurements of magnegas consumption rate per mile on highway are under way, and they are expected to yield essentially the same results holding for natural gas, namely,

$$\text{Gasoline gallon equivalent: 120 cf of magnegas.} \tag{11.4.7}$$

Preliminary measurements of magnegas combustion exhaust were conducted by the laboratory *National Technical Systems, Inc.*, of Largo, Florida, resulting in the following exhaust composition under proper combustion:

$$\begin{array}{ll} \text{Water vapor:} & \text{50\% - 60\%,} \\ \text{Oxygen:} & \text{10\% - 12\%,} \\ \text{Carbon dioxide:} & \text{6\% - 7\%,} \\ \text{Balance:} & \text{atmospheric gases,} \\ \text{Hydrocarbons, carbon monoxide,} & \\ \text{nitrogen oxides:} & \text{in parts per million (ppm).} \end{array} \tag{11.4.8}$$

Detailed magnegas exhaust measurements were then conducted at the EPA Certified, Vehicle Certification Laboratory *Liphardt & Associates* of Long Island, New York, under the Directorship of *Peter di Bernardi*, via the Varied Test Procedure (VTP) as per EPA Regulation 40-CFR, Part 86.

These EPA tests consisted of three separate and sequential tests conducted in November 2000 on a computerized dynamometer, the first and the third tests using the vehicle at its maximal possible capability to simulate an up-hill travel at 60 mph, while the second test consisted in simulating normal city driving.

Three corresponding bags with the exhaust residues were collected, jointly with a fourth bag containing atmospheric contaminants. The final measurements expressed in grams/mile are given by the average of the measurements on the three EPA test bags, less the measurements of atmospheric pollutants in the fourth bag.

The following three measurements were released by Liphardt & Associates:

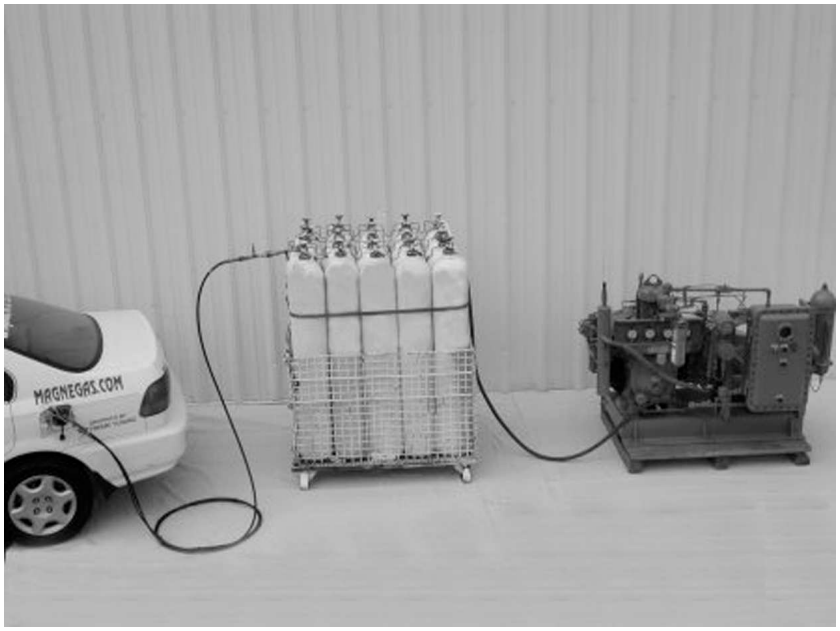


Figure 10.17. [8] A picture of the *Magnegas Refilling Station*, consisting of a standard compressor as used for natural gas and pressure bottles. The simplicity of this station should be compared with the complexity of corresponding stations for liquid hydrogen. Note that the station depicted in this figure allows current *distributors* of fuel, such as gasoline stations, to become *fuel producers*. The refill is achieved by connecting the a pressure bottle with a pressure tank in the trunk of the car; it is faster than the refill with gasoline; and it is much safer than the latter because gasoline, being liquid, spills and explodes if ignited, while magnegas, being a gas, does not spill and if ignited, burns fast in air without explosion (see website [5b] for details).

1) Magnegas exhaust measurements with catalytic converter:

HYDROCARBONS:	0.026 grams/mile, which is 0.063 of the EPA standard of 0.41 grams/mile;
CARBON MONOXIDE:	0.262 grams/mile, which is 0.077 of the EPA standard of 3.40 grams/mile;
NITROGEN OXIDES:	0.281 grams/mile, which is 0.28 of the EPA standard of 1.00 grams/mile;
CARBON DIOXIDE:	235 grams/mile, corresponding to about 6%; there is no EPA standard on CO_2 at this time;
OXYGEN:	9.5% to 10%; there is no EPA standard for oxygen at this time.

(11.4.9)

The above tests have established the important feature that *magnegas exhaust with catalytic converter imply a reduction of about 1/15 of current EPA requirement.*

2) Magnegas exhaust measurements without catalytic converter in the same car and under the same conditions as (1):

HYDROCARBONS:	0.199 grams/mile, which is 0.485 of the EPA standard of 0.41 grams/mile;
CARBON MONOXIDE:	2.750 grams/mile, which is 0.808 of the EPA standard of 3.40 grams/mile;
NITROGEN OXIDE:	0.642 grams/mile, which is 0.64 of the EPA standard of 1.00 grams/mile;
CARBON DIOXIDE:	266 grams/mile, corresponding to about 6%;
OXYGEN:	9.5% to 10%.

(11.4.10)

As a result of the latter tests, the laboratory *Liphardt & Associates* released the statement that *magnegas exhaust surpasses the EPA requirements* without *the catalytic converter*. As such, magnegas can be used in *old cars without catalytic converter while meeting, and actually surpassing EPA emission standards*.

3) Natural gas exhaust measurements without catalytic converter in the same car and under the same conditions as (1):

HYDROCARBONS:	0.380 grams/mile, which is 0.926 of the EPA standard of 0.41 grams/mile;
CARBON MONOXIDE:	5.494 gram/mile, which is 1.615 of the EPA standard of 3.40 grams/mile;
NITROGEN OXIDES:	0.732 grams/mile, which is 0.73 the EPA standard of 1.00 grams/mile;
CARBON DIOXIDE:	646.503 grams/mile, corresponding to about 9%;
OXYGEN:	0.5% to 0.7%.

(11.4.11)

The latter tests established the important property that *the combustion of natural gas emits about 2.5 times the CO_2 emitted by magnegas without catalytic converter. Note that, as well known, natural gas exhaust without catalytic converter does not meet EPA requirements*.

As an additional comparison for the above measurements, a similar Honda car running on indolene (a version of gasoline) was tested in the same laboratory with the same EPA procedure, resulting in the following data:

4) Gasoline (indolene) exhaust measurements conducted on a two liter Honda KIA:

HYDROCARBONS:	0.234 grams/mile equal to 9 times the corresponding magnegas emission;
CARBON MONOXIDE:	1.965 grams/mile equal to 7.5 times the corresponding magnegas emission;
NITROGEN OXIDES:	0.247 grams/mile equal to 0.86 times the corresponding magnegas emission;
CARBON DIOXIDE:	458.655 grams/mile equal to 1.95 times the corresponding of magnegas emission,
OXYGEN:	No measurement available.

(11.4.12)

Figure 10.18. A picture of the readings of a 4-ways exhaust analyzer testing the exhaust of the Ferrari 308 GTSi of a preceding picture operating on magnegas "without" catalytic converter. Note: the presence of 14% breathable oxygen in the exhaust; about half the CO_2 produced by the same car when running on gasoline; the very few detected hydrocarbons originate from engine oil seeping through the piston rings because magnegas "cannot" contain hydrocarbons since it is synthesized at the $5,000^oC$ of the arc at which temperature no hydrocarbon can survive; the very small content of CO in the exhaust is due to poor combustion because CO is fuel for magnegas, while it is a byproduct of the combustion for fossil fuels, as a result of which detecting CO in the exhaust of a car running on magnegas is the same as detecting gasoline in the exhaust of a car running on gasoline (see website [5b] for details).

The above data establish the environmental superiority of magnegas over natural gas and gasoline. The following comments are now in order:

1) Magnegas does not contain (heavy) hydrocarbons since it is created at $3{,}500^o$ K. Therefore, the measured hydrocarbons are expected to be due to combustion of oil, either originating from magnegas compression pumps (thus contaminating the gas), or from engine oil.

2) Carbon monoxide is fuel for magnegas (while being a combustion product for gasoline and natural gas). Therefore, any presence of CO in the exhaust is evidence of insufficient combustion.

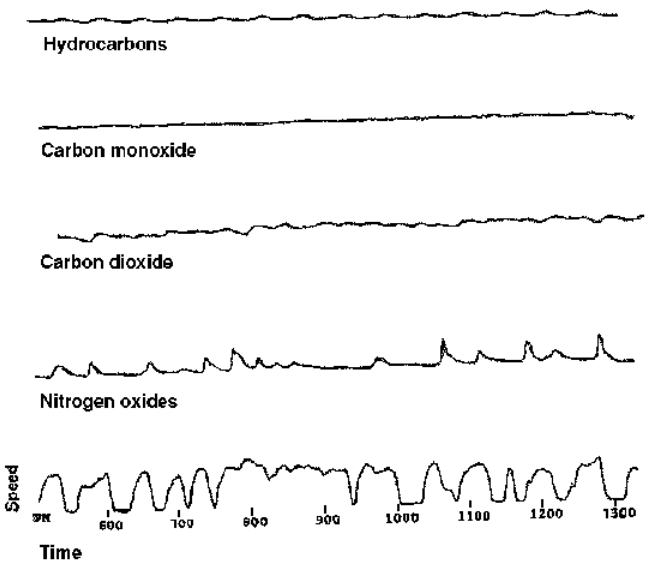


Figure 10.19. An illustration of the city part of the reported EPA test according to Regulation 40-CFR, Part 86, conducted at the Vehicle Certification Laboratory *Liphardt & Associates* of Long Island, New York on a Honda Civic Natural Gas Vehicle converted to magnegas by the author.. The first three diagrams illustrate the very low combustion emission of magnegas in city driving, by keeping in mind that most of measured emission is due to the heavy duty, hill climbing part of the EPA test. The fourth diagram on nitrogen oxides is an indication of insufficient cooling of the engine. The bottom diagram indicates the simulated speed of the car versus time, where flat tracts simulate idle portions at traffic lights.

3) The great majority of measurements originate from the first and third parts of the EPA test at extreme performance, because, during ordinary city traffic, magnegas exhaust is essentially pollutant free [5].

4) Nitrogen oxides are not due, in general, to the fuel (whether magnegas or other fuels), but to the temperature of the engine and other factors, thus being an indication of the quality of its cooling system. Therefore, for each given fuel, including magnegas, NOx's can be decreased by improving the cooling system and via other means.

5) The reported measurements of magnegas exhaust do not refer to the best possible combustion of magnegas, but only to the combustion of magnegas in a vehicle whose carburization was developed for natural gas. Alternatively, the test was primarily intended to prove that magnegas is interchangeable with natural gas without any major automotive changes, while keeping essentially the same

Figure 10.20. It is generally ignored that cruiseships leave a trail of marine death since they release in the ocean an average of 100,000 gallons of highly contaminated liquid wastes per day. The magnegas technology [5] was developed to resolve this problem via the on board recycling of all liquid waste into purified forms reusable on board without any release in the ocean.

performance and consumption. The measurements for combustion specifically conceived for magnegas are under way.

We should also indicate considerable research efforts under way to further reduce the CO_2 content of magnegas exhaust via disposable cartridges of CO_2-absorbing chemical sponges placed in the exhaust system (patent pending). Additional research is under way via *liquefied magnegas* obtained via *catalytic* (and *not* conventional) liquefaction, which liquid is expected to have an anomalous energy content with respect to other liquid fuels, and an expected consequential decrease of pollutants. As a result of these efforts, the achievement of an exhaust essentially free of pollutants and CO_2, yet rich in oxygen, appears to be within technological reach.

10.4.3 Anomalous Chemical Composition of Magnegas

As studied in the preceding section, the chemical composition of the magnegas exhaust is conventional and, therefore, can be tested with established analytic equipment and methods. However, the chemical composition of magnegas itself cannot be successfully tested with the same equipment and methods due to its novelty.

To begin, numerous tests in various analytic laboratories reviewed in below have established that magnegas results in being characterized by large peaks in macroscopic percentage all the way to 1,000 a.m.u., which peaks remain individually unidentified by the MS computer after scanning all known molecules.

By comparison, quantum chemistry predicts that the heaviest molecule in a *light* gas such as magnegas should only have 44 a.m.u., while offering no expla-

nation whatever, not even remote or indirect on the existence of detectable teaks all the way to 1,000 a.m.u.

The above differences are so drastic to provide clear experimental evidence on the fact that the magnegas structure is characterized by a *new chemical species* not predicted or considered by quantum chemistry until now.

Besides the inability to identify the clusters composing magnegas via the computer search among all known molecules, the chemical structure of magnegas is equally unidentifiable via InfraRed Detectors (IRD), because the new peaks composing magnegas have no IR signature at all, thus establishing the presence of bonds of non-valence type (because these large clusters cannot possibly be all symmetric).

Moreover, the IR signature of conventional molecules such as CO results in being *mutated* (in the language of hadronic mechanics) with the appearance of new peaks, which evidently indicate *new* internal bonds in *conventional* molecules.

In addition to all the above, dramatic differences between the prediction of quantum chemistry and reality exist for the energy content of magnegas. For instance, when produced with PlasmaArcFlow Reactors operating an electric arc between at least one consumable electrode within pure water, quantum chemistry predicts that magnegas should be a mixture of 50% H_2 and 50% CO, with traces of O_2 and CO_2.

This prediction is dramatically disproved by the fact that *both the* CO *and the* CO_2 *peaks do not appear in the MS scan in the predicted percentages, while they appear in the IR scan although in a mutated form.*

Moreover, quantum chemistry predicts that the indicated composition consisting of 50% H_2 and 50% CO should have an energy content of about 315 BTU/cf, namely, an energy content insufficient to cut metal. This prediction is also disproved by the experimental evidence that *magnegas cuts metal at least 50% faster than acetylene (which has 2,300 BTU/cf).*

Such a performance in metal cutting is more indicative of a *plasma cutting* feature, such as the metal cutting via a plasma of ionized hydrogen atoms which recombine into H_2 when cooling in the metal surface, thus releasing the energy needed for metal cutting. The problem is that magnegas is at room temperature when used for metal cutting, and it is subjected to ordinary combustion, thus requiring basically new approaches for its correct interpretation.

Nevertheless, the plasma cutting feature is indicative of the presence of isolated atoms and dimers in the magnegas structure which recombine under combustion, thus yielding a behavior and a performance similar to that of plasma cutters.

In fact, as also shown later on, GC-MS scans have indicated the presence in the anomalous peaks of *individual atoms of hydrogen, oxygen, and carbon* evidently in addition to individual molecules.

To conclude, the composition of magnegas in H, C and O *atoms* can be easily identified from the liquid used in the reactors. For instance, when magnegas is produced from water, it is composed of 50% H, 25% O, and 25% C, with corresponding percentages for other liquids such as antifreeze, crude oil, etc.

However, all attempts to reduce the chemical composition of magnegas to conventional molecules conducted by the author as well as independent chemists, have been disproved by a variety of experimental evidence.

In particular, any belief that magnegas is entirely composed by ordinary molecules, such as H_2 and CO, is disproved by experimental evidence via GC-MS and IRD detectors.

The only possible scientific conclusion at this writing is that *magnegas is composed of a new chemical species* studied below.

10.4.4 GC-MS/IRD Measurements of Magnegas at the McClellan Air Force Base

Santilli [1] had predicted that gases produced from underwater electric arcs had the new chemical structure of magnecules as clusters of molecules, dimers and individual atoms as per Definition 8.2.1, in which case conventional chemical structure (8.20) is valid only in first approximation.

Following a laborious search, Santilli [*loc. cit.*] located a GC-MS equipped with IRD suitable to measure magnecules at the *McClellan Air Force Base* in North Highland, near Sacramento, California. Thanks to the invaluable assistance and financial support by *Toups Technologies Licensing, Inc.*, of Largo, Florida, GC-MS/IRD measurements were authorized at that facility on magnegas with conventional chemical structure (8.20).

On June 19, 1998, Santilli visited the analytic laboratory of *National Technical Systems* (NTS) located at said *McClellan Air Force Base* and using instruments belonging to that base. The measurements on magnegas were conducted by analysts Louis A. Dee, Branch Manager, and Norman Wade who operated an *HP GC model 5890, an HP MS model 5972, equipped with an HP IRD model 5965.* Upon inspection at arrival, the instrument met all conditions indicated in the preceding sections then, and only then, measurements were permitted.

Thanks to a professional cooperation by the NTS analysts, the equipment was set at all the unusual conditions indicated later on. In particular, the equipment was set for the analytic method VOC IRMS.M utilizing an HP Ultra 2 column 25 m long with a 0.32 mm ID and a film thickness of 0.52 μm. It was also requested to conduct the analysis from 40 a.m.u. to the instrument limit of 500 a.m.u. This condition was necessary to avoid the expected large CO peak of magnegas at 28 a.m.u.

Moreover, the GC-MS/IRD was set at the low temperature of 10°C; the biggest possible feeding line with an ID of 0.5 mm was installed; the feeding line itself was

cryogenically cooled; the equipment was set at the longest possible ramp time of 26 minutes; and a linear flow velocity of 50 cm/sec was selected. A number of other technical requirements are available in the complete documentation of the measurements.

The analysts first secured a documentation of the *background* of the instrument prior to any injection of magnegas (also called *blank*). Following a final control that *all* requested conditions were implemented, the tests were initiated. The results, reported in part via the representative scans of Figs. 8.7 to 8.12, constitute the first direct experimental evidence of the existence of magnecules in gases.

After waiting for 26 minutes, sixteen large peaks appeared on the MS screen between 40 and 500 a.m.u. as shown in Fig. 11.21. Each of these sixteen MS peaks resulted to be "unknown," following a computer search of database on all known molecules available at *McClellan Air Force Base*, as illustrated in Fig. 11.22 No identifiable CO_2 peak was detected at all in the MS spectrum between 40 and 500 a.m.u., contrary to the presence of 9% of such a molecule in magnegas as per conventional analyses (8.20).

Upon the completion of the MS measurements, exactly the same range of 40 to 500 a.m.u. was subjected to IR detection. As expected, none of the sixteen peaks had any infrared signature at all, as shown in Fig. 11.14. Furthermore, the IR scan for these MS peaks shows only one peak, that belonging to CO_2, with additional small peaks possibly denoting traces of other substances.

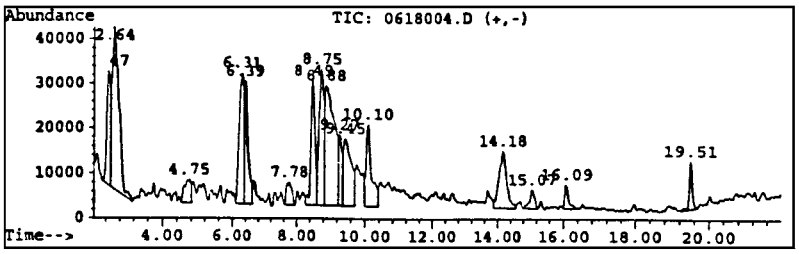
Note that the IR signature of the other components, such as CO or O_2 *cannot* be detectable in this IR test because their atomic weights are below the left margin of the scan. In addition, the IR peak of CO_2 is itself mutated from that of the unpolarized molecule, as shown in Fig. 11.24. Note that the mutation is due to the appearance of *two new peaks* which are absent in the conventional IR signature of CO_2, exactly as expected, thus confirming the hypothesis of new internal bonds as submitted in Fig. 11.12.

Note also in Fig. 24 that the computer interprets the IR signature as that belonging to CO which interpretation is evidently erroneous because CO is outside of the selected range of a.m.u.

All remaining small peaks of the IR scan resulted to be "unknown," thus being possible magnecules, following computer search in the database of IR signatures of all known molecules available at the *McClellan Air Force Base*, as illustrated in Fig. 11.25.

Following the removal of magnegas from the GC-MS/IRD, the background continued to show the same anomalous peaks of Fig. 11.21, and reached the configuration of Fig. 11.26 only after a weekend bakeout with an inert gas. Note that the latter background is itself anomalous because the slope should have been the opposite of that shown. The background finally recovered the conventional shape only after flushing the instrument with an inert gas at high temperature.

```
Information from Data File:
File       : C:\HPCHEM\1\DATA\0618004.D
Operator   : NAW
Acquired   : 18 Jun 98   3:01 pm using AcqMethod VOC_IRMS
Sample Name: TOUP'S TECH
Misc Info  : 1ML LOOP; 10C @ ULTRA COLUMN
Vial Number: 1
CurrentMeth: C:\HPCHEM\1\METHODS\DEFAULT.M
```



Retention Time	Area	Area %	Ratio %
Total Ion Chromatogram			
2.474	1753306	5.386	32.724
2.644	5091514	15.641	95.030
4.754	641528	1.971	11.974
6.307	2737749	8.411	51.098
6.390	2211258	6.793	41.272
7.782	592472	1.820	11.058
8.490	2357396	7.242	43.999
8.754	2784829	8.555	51.977
8.882	5357812	16.460	100.000
9.265	1123809	3.452	20.975
9.448	2421234	7.438	45.191
10.098	1946292	5.979	36.326
14.177	2129791	6.543	39.751
15.073	435208	1.337	8.123
16.085	389822	1.198	7.276
19.509	577433	1.774	10.777

Figure 10.21. A reproduction of the MS peaks providing the *first experimental evidence of the existence of magnecules* identified on June 19, 1998, by analysts *Louis A. Dee* and *Norman Wade* of the branch of *National Technical; Systems* (NTS) located at the *McClellan Air Force Base* in North Highland, near Sacramento, California, with support from *Toups Technologies Licensing, Inc.* (TTL) of Largo, Florida. The scan is restricted from 40 a.m.u to 500 a.m.u. The peaks refer to magnegas produced via an electric arc between consumable carbon electrodes within ordinary tap water with conventional chemical composition (8.20). Therefore, only the CO_2 peak was expected to appear in the scan with any macroscopic percentage, while no CO_2 was detected at all in the MS scan.

10.4.5 GC-MS/IRD Tests of Magnegas at Pinellas County Forensic Laboratory

Measurements on the same sample of magnegas tested at NTS were repeated on July 25, 1998, via a GC-MS/IRD located at the *Pinellas County Forensic*

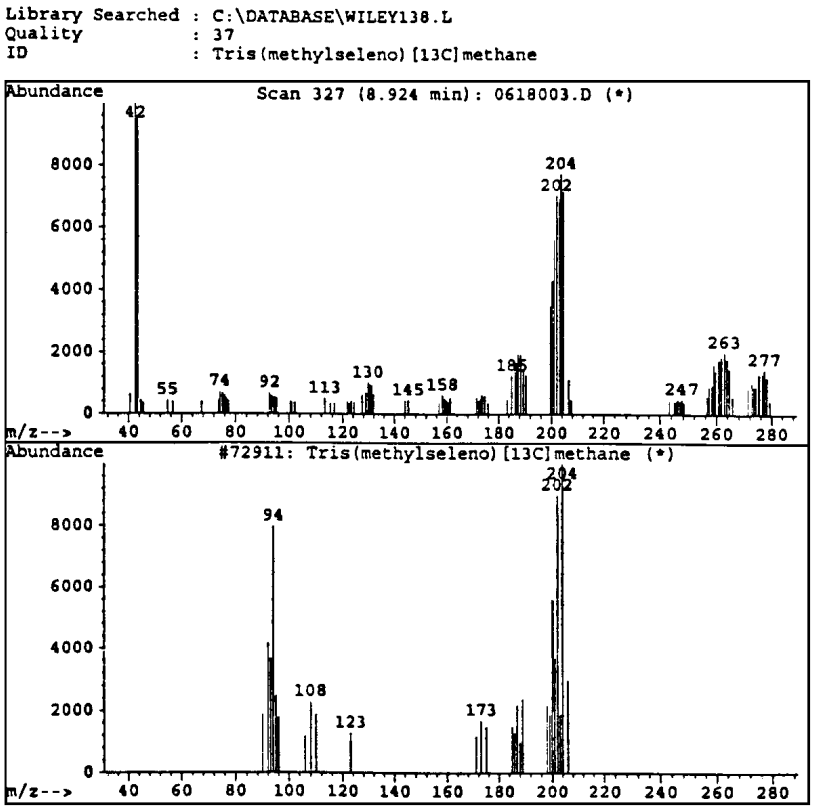


Figure 10.22. A representation of *the first experimental evidence at NTS that the peaks of Fig. 11.12 are "unknown."* The peak at the top is at 8.924 minutes, and that at the bottom shows the lack of its identification by the computer search. Note that the best fit identified by the computer does not match the peak considered. Moreover, the identified substance (methylseleno) cannot possibly exist in magnegas because of the impossible presence of the necessary elements. The same situation occurred for all remaining fifteen peaks of Fig. 11.12.

Laboratory (PCFL) of Largo, Florida, with support from *Toups Technologies Licensing, Inc.*

The equipment consisted of a *HP GC model 5890 Series II, an HP MS model 5970 and an HP IRD model 5965B.* Even though similar to the equipment used at NTS, the PCFL equipment was significantly different inasmuch as the temperature had to be increased from 10°C to 55°C and the ramp time reduced from 26 to 1 minute. The latter reduction implied the cramping of all peaks of Fig. 11.17 into one single large peak, a feature confirmed by all subsequent GC-MS tests with short ramp time.

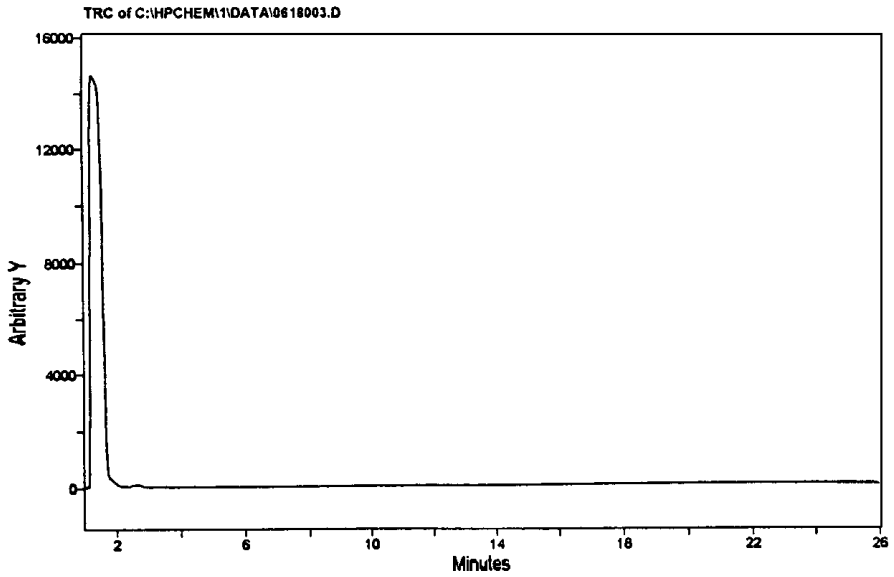


Figure 10.23. The first experimental evidence at NTS of the lack of IR signature of MS peaks. The evidence establishes the existence of large peaks in the MS that have no IR signature at all. The only identified IR signature, that for CO_2, refers to the *constituents* of the peaks of Fig. 11.12. In the above figure only the IR signature of CO_2 appears because the scan was from 40 a.m.u. to 500 a.m.u. and, as such, could not include the IR signatures of other molecules such as O_2 and CO (H_2 has no IR signature).

Despite these differences, the test at PCFL, reported in part via the representative scans of Figs. 8.13 to 8.18, confirmed *all* features of magnecules first detected at NTS. In addition, the tests provided the experimental evidence of additional features.

Following Santilli's request [1], the analysts conducted *two* MS tests of the *same* magnegas at *different times* about 30 minutes apart. As one can see in Figs. 8.13 and 8.14, *the test at PCFL provided the first experimental evidence of mutation in time of the atomic weight of magnecules.* In fact, the peak of Fig. 11.27e is macroscopically different than that of Fig. 11.28.

This difference provides evidence that, when colliding, magnecules can break down into ordinary molecules, atoms, and fragments of magneclusters, which then recombine with other molecules, atoms, and/or magnecules to form new clusters. The same scan provides first experimental evidence of the accretion or loss by magnecules of individual atoms, dimers and molecules, as discussed later on.

Figure 8.15 depicts the failure by the GC-MS/IRD to identify the peaks of Figs. 8.13 and 8.14 following a search in the database among all known molecules.

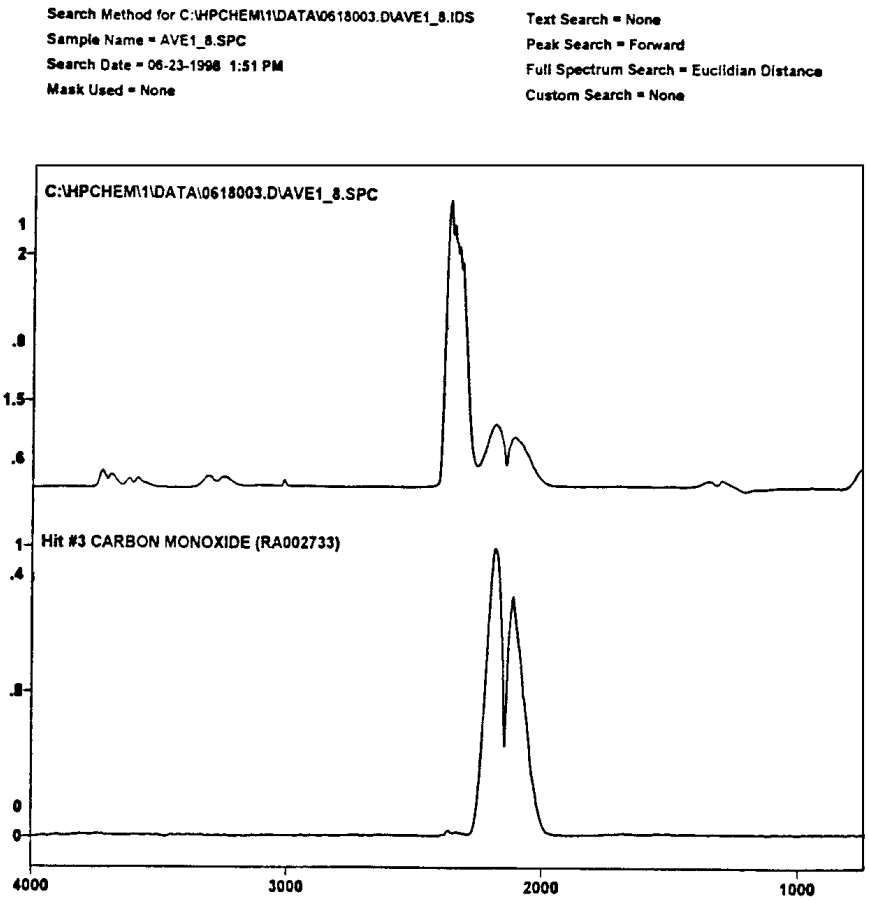


Figure 10.24. The first experimental evidence at NTS on the mutation of the IR signature of magnetically polarized conventional molecules, here referring to the CO_2 (top) compared to the result of the computer search (bottom). Note that the known, double-lobe peak of CO_2 persists in the detected peak with the correct energy, and only with decreased intensity. Jointly, there is the appearance of *two new peaks*, which are evidence of *new internal bonds within the conventional CO_2 molecule.* This evidently implies an increased energy content, thus establishing experimental foundations for the new technology of magnetically polarized fuels such as magnegas [2].*Note that the computer interprets the IR signature as that of CO, which is erroneous since CO is out of the selected range of detection.*

Figure 8.16 provides an independent confirmation that the IR scan of Fig. 11.23, namely, that the MS peaks, this time of Figs. 8.13 and 8.14, have no IR signature except for the single signature of the CO_2. However, the latter was not detected at all in said MS scans. Therefore, the CO_2 detected in said IR scan is

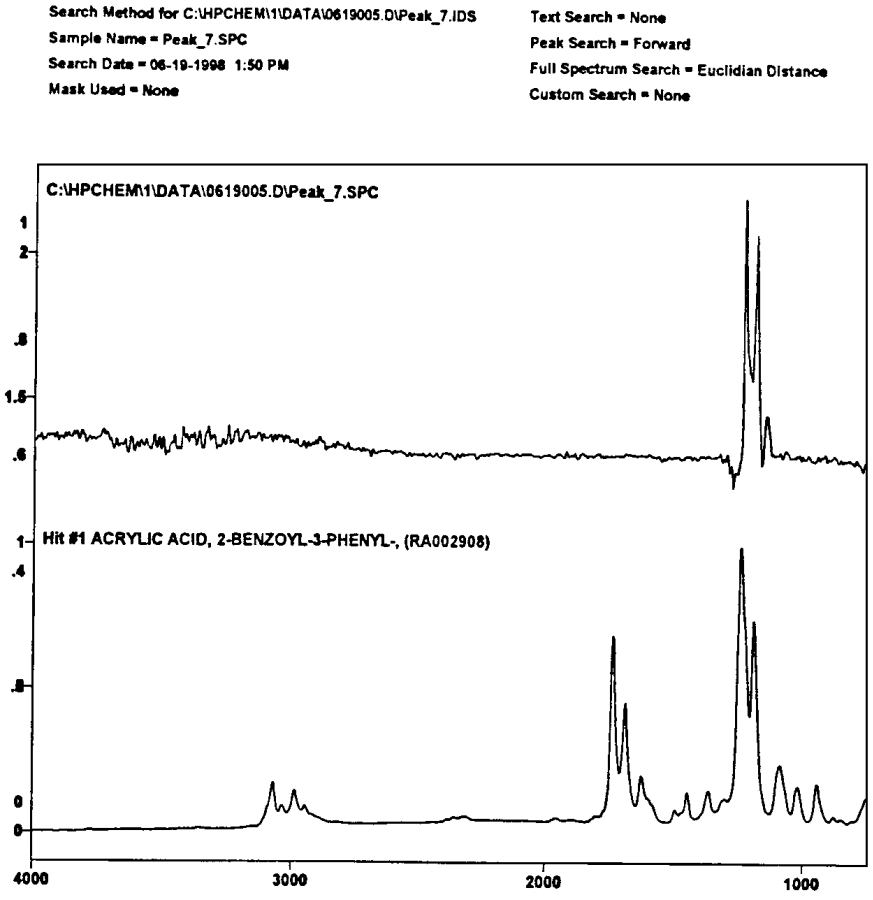


Figure 10.25. A reproduction of the lack of identification in the computer search of small peaks in the IR scan, which can therefore be additional magnecules, or IR signatures of the magnecules appearing in the MS scan.

a *constituent* of the new species detected in Figs. 8.13 and 8.14. The lack of IR signature of the MS peaks confirms that said peaks *do not* represent molecules.

Figure 8.17 confirms in full the mutated IR signature of CO_2 previously identified in Fig. 11.24, including the important presence of two new peaks, with the sole difference that, this time, the computer correctly identifies the IR signature as that of carbon dioxide.

Figure 8.18 presents the background of the instrument after routine flushing with an inert gas, which background essentially preserves the peaks of the MS scans, thus confirming the unique adhesion of magnecules to the instrument walls.

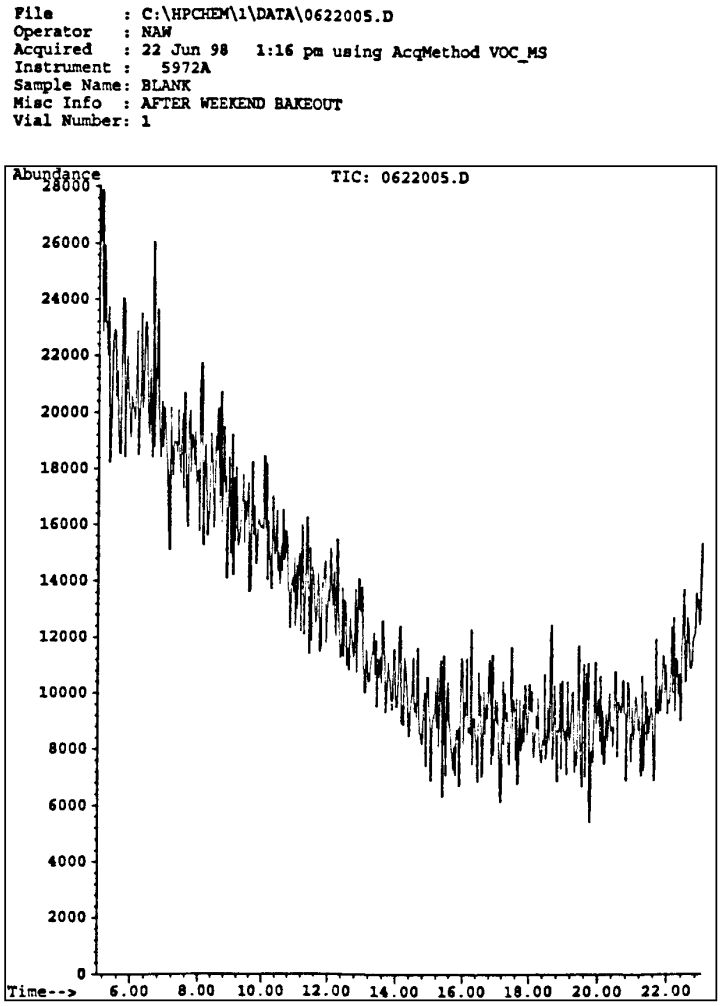


Figure 10.26. A view of the background of the preceding tests following a weekend bakeout.

10.4.6 Interpretations of the Results

A few comments are now in order for the correct interpretation of the results. First, note in the GC-MS/IRD scans that the CO_2 detected in the IRD has no counterpart in the MS scans, while none of the peaks in the MS have a counterpart in the IR scans. Alternatively, the CO_2 peak detected in the IR scans of Figs. 8.10 and 8.17 *does not* correspond to any peak in the MS scans in Figs. 8.7, 8.13 and 8.14. Therefore, said IR peak identifies a *constituent* of the MS clusters, and not an isolated molecule.

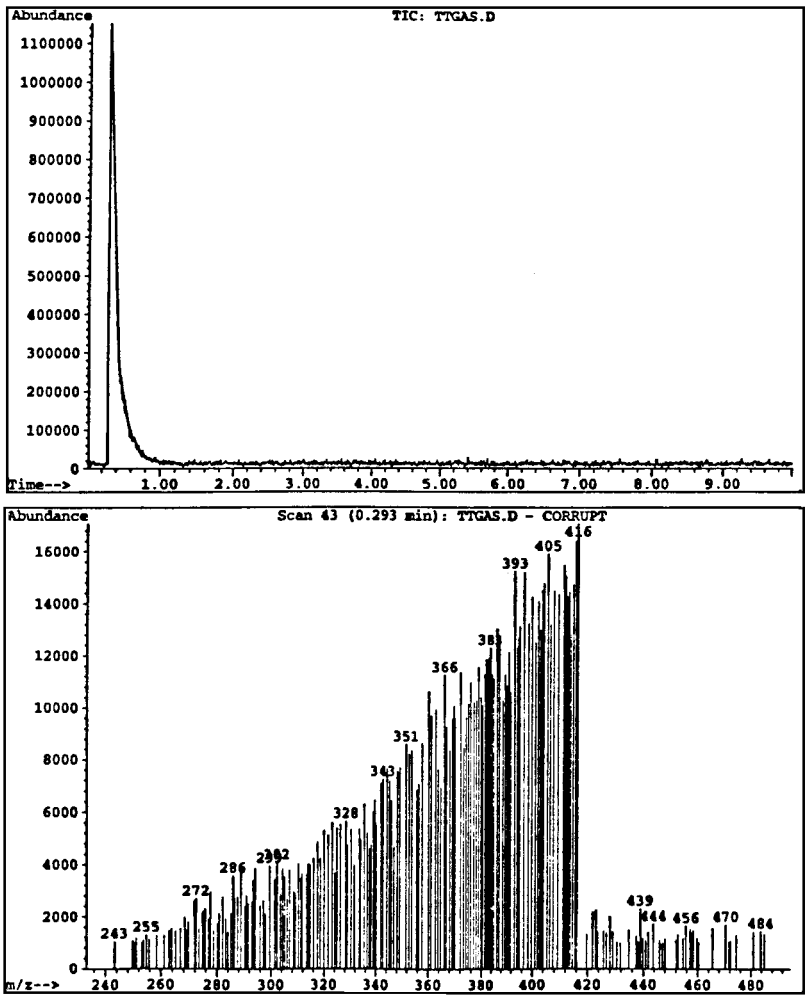


Figure 10.27. A view of the Total Ion Count (top) and MS spectrum (below) of magnegas conducted on July 25, 1998, via a HP GC-MS/IRD at the *Pinellas County Forensic Laboratory* (PCFL) of Largo, Florida, under support from *Toups Technologies Licensing, Inc.* (TTL) also of Largo, Florida. The scan is restricted to the range 40 a.m.u to 500 a.m.u. and confirm all results of the preceding NTS tests.

Moreover, the IR scan was done for the entire range of 40 to 500 a.m.u., thus establishing that said IR peak is the sole conventional constituent in macroscopic percentage in said a.m.u. range of *all* MS peaks, namely, the single constituent identified by the IRD is a constituent of all MS peaks.

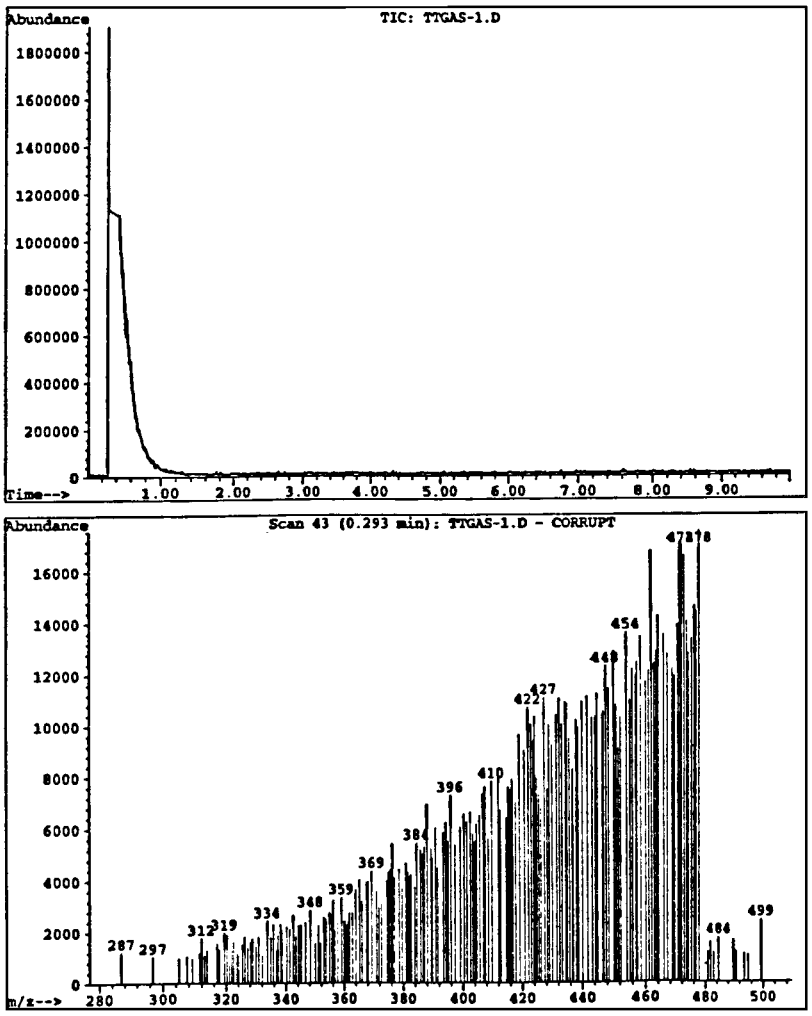


Figure 10.28. A repetition of the scan of the preceding figure conducted at PCFL in the same sample of magnegas on the same instrument and under the same conditions, but 30 minutes later. The scan provides *the first experimental evidence of the mutation of atomic weight of magnecules,* as one can see from the variation of the peaks of this figure compared with that of the preceding figure.

It should also be noted that, as recalled earlier, the IR only detects *dimers* such as C–O, H–O, *etc.*, and does not detect complete molecules. Therefore, the peak detected by the IRD *is not* sufficient to establish the presence of the complete molecule CO_2 unless the latter is independently identified in the MS. Yet the MS scan does not identify any peak for the CO_2 molecule, as indicated earlier.

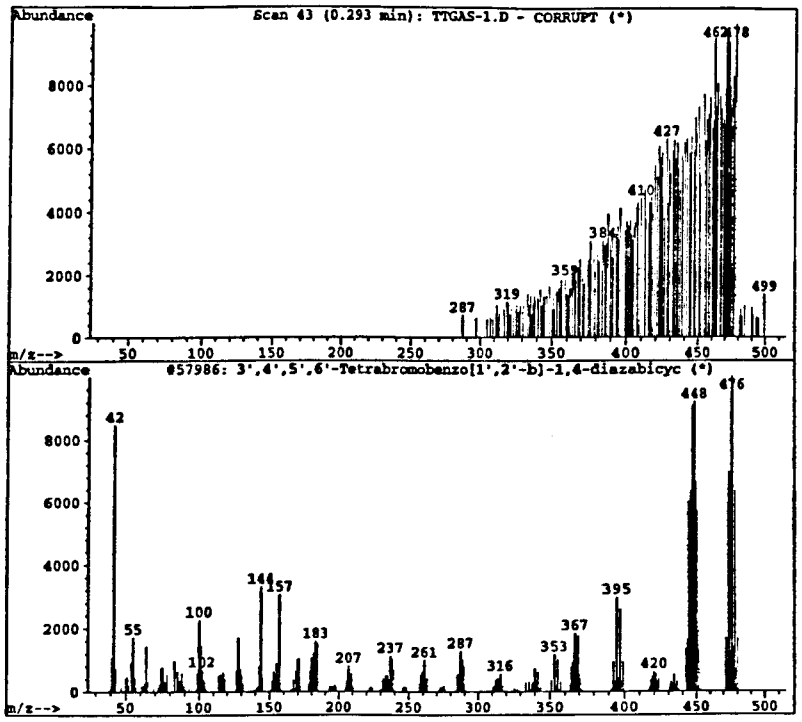


Figure 10.29. Lack of identification by the computer of the GC-MS/IRD at PCFL of the MS peaks of the preceding two scans following search among the database on all available molecules.

Despite that, the presence in the MS peaks of complete molecules CO_2 cannot be ruled out. Therefore, the most plausible conclusion is that the MS peaks represent clusters composed of a percentage of C–O dimers and another percentage of CO_2 molecules, plus other dimers, and/or molecules, and/or atoms with atomic weight smaller than 40 a.m.u., thus outside the range of the considered scans.

As indicated earlier, the presence of dimers and individual atoms in magnegas is essential for a quantitative interpretation of the large excess of energy contained in this new fuel, the order of at least three times the value predicted by quantum chemistry, which energy is released during combustion. The admission of dimers and atoms as constituents of magnecules readily explains this anomalous energy content because said dimers and atoms are released at the time of the combustion, thus being able at that time to form molecules with exothermic reactions of type (8.5). In the event magnecules would not contain dimers and atoms, their only possible constituents are conventional molecules, in which case no excess energy is possible during combustion.

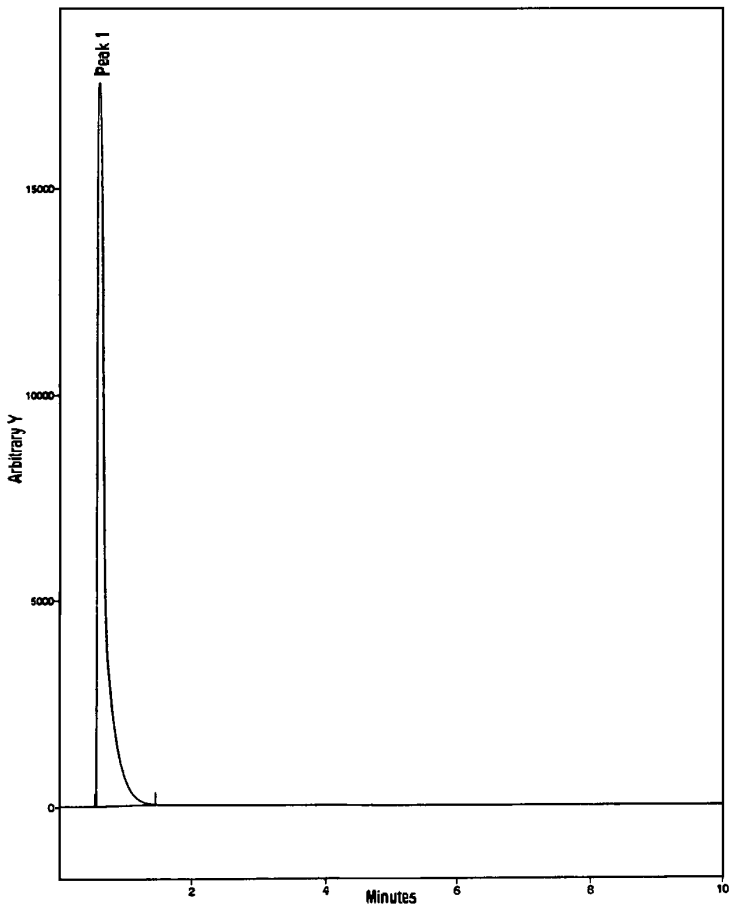


Figure 10.30. A confirmation of the lack of IR signature of the peaks of Figs. 8.13 and 8.14, as occurred for Fig. 11.14, which establishes that the MS peaks of Figs. 8.13 and 8.14 cannot have a valence bond, thus constituting a new chemical species.

The large differences of MS peaks in the two tests at NTS and at PCFL of exactly the same gas in exactly the same range from 40 to 500 a.m.u., even though done with different GC-MS/IRD equipment, illustrates the importance of having a ramp time of the order of 26 minutes. In fact, sixteen different peaks appear in the MS scan following a ramp time of 26 minutes, as illustrated by Fig. 11.12, while all these peaks collapsed into one single peak in the MS scan of Figs. 8.13 and 8.14, because the latter were done with a ramp time of about 1 minute. Therefore, *the collapse of the sixteen peaks of Fig. 11.12 into the single large peak of Figs. 8.13 and 8.14 is not a feature of magnecules, but rather it is due to the insufficient ramp time of the instrument.*

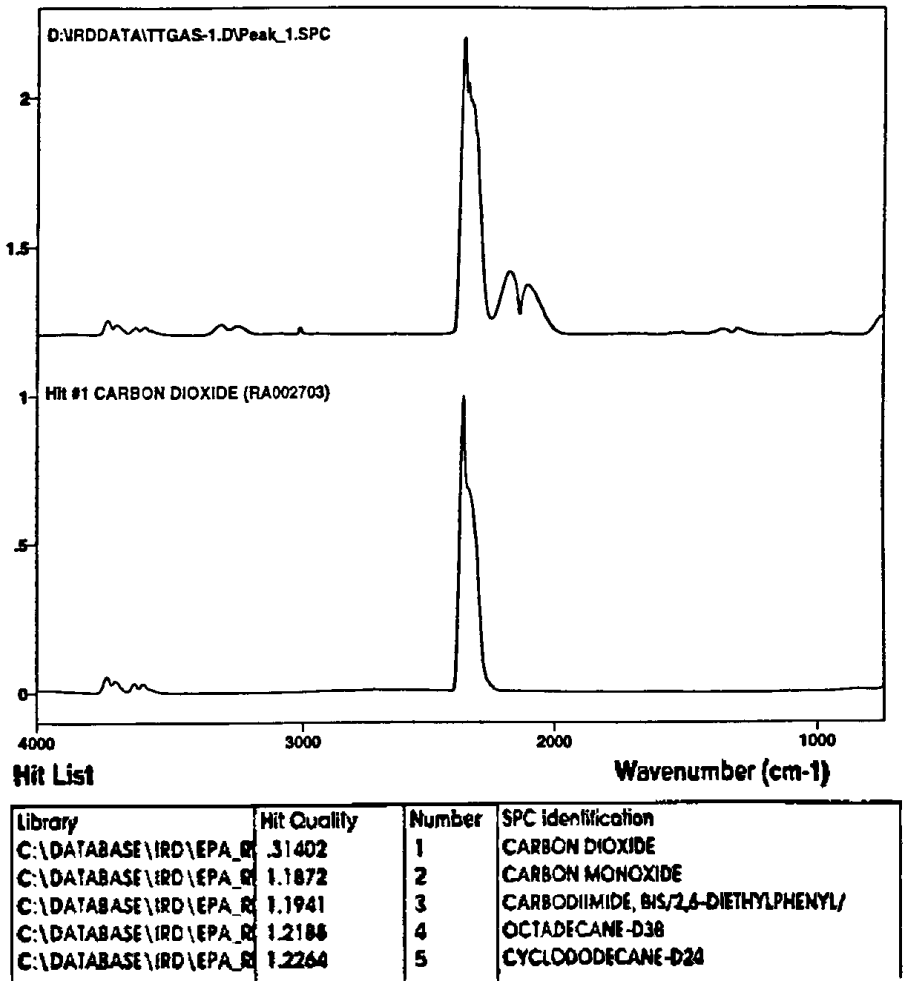


Figure 10.31. The independent confirmation at the PCFL of the NTS finding of Fig. 11.15 regarding the mutated IR signature of the CO_2 in magnegas. Note the identical shapes of the mutated IR peak in the top of the above figure, and that in Fig. 11.15 obtained via a different instrument. Note also the appearance again of two new peaks in the IR signature of CO_2, which indicate the presence of *new internal bonds* not present in the *conventional* molecule. Note finally that the instrument now correctly identifies the signature as that of the CO_2.

10.4.7 Anomalous Energy Balance of Hadronic Molecular Reactors

As is well known, the *scientific efficiency* of any equipment is *under-unity* in the sense that, from the principle of conservation of the energy and the unavoidable

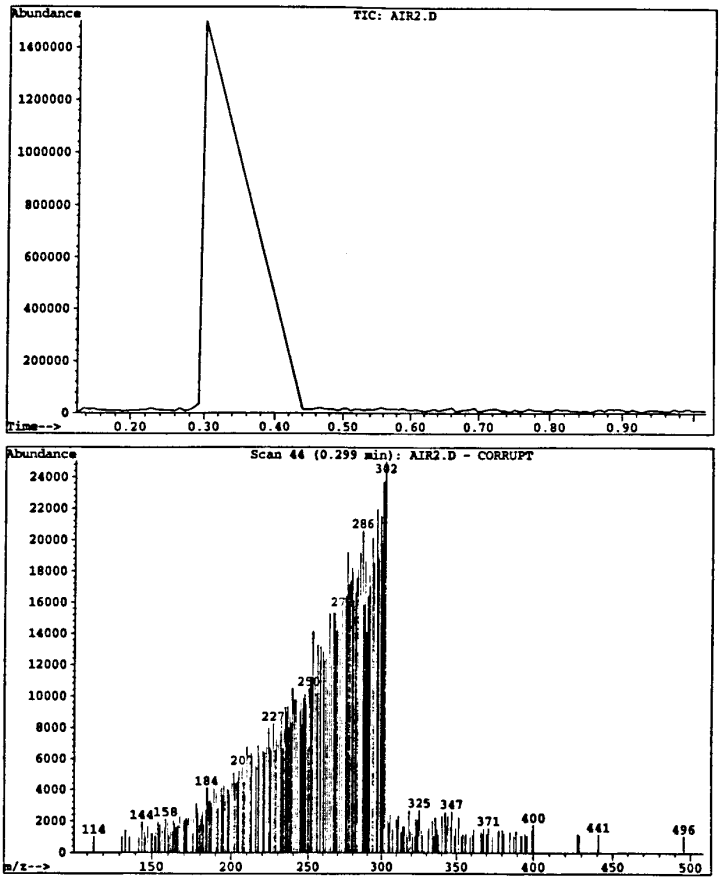


Figure 10.32. The first direct experimental verification at PCFL of the anomalous adhesion of magnecules. The figure reproduces the background of the instrument upon completion of the measurements, removal of magnegas, and conventional flushing. As one can see, the background results in being very similar to the MS scan during the tests, thus establishing that part of the gas had remained in the interior of the instrument. This behavior can only be explained via the induced magnetic polarization of the atoms in the walls of the instrument, with consequential adhesion of magnecules. It should be noted that this anomalous adhesion has been confirmed by all subsequent tests for both the gaseous and liquid states. The removal of magnecules in the instrument after tests required flushing with an inert gas at high temperature.

energy losses, *the ratio between the total energy produced and the total energy used for its production is smaller than one.*

For the case of magnegas production, the total energy produced is the sum of the energy contained in magnegas plus the heat acquired by the liquid, while the total energy available is the sum of the electric energy used for the production of

magnegas plus the energy contained in the liquid recycled. Therefore, from the principle of conservation of the energy we have the scientific energy balance

$$\frac{\text{Total energy produced}}{\text{Total energy available}} = \frac{E_{mg} + E_{heat}}{E_{electr} + E_{liq}} < 1. \tag{11.4.13}$$

An important feature of hadronic reactors is that they are *commercially over-unity*, namely, the ratio between the total energy produced and only the electric energy used for its production, is bigger than one,

$$\frac{E_{mg} + E_{liq}}{E_{electr}} > 1. \tag{11.4.14}$$

In this commercial calculation the energy contained in the liquid is not considered because liquid wastes imply an income, rather than costing money.

As a result, Santilli's hadronic molecular reactors can be viewed as reactors capable of tapping energy from liquid molecules, in much of the same way as nuclear reactors can tap energy from nuclei. An important difference is that the former reactors release no harmful radiation and leave no harmful waste, while the latter reactors do release harmful radiations and leave harmful waste.

The energy used for the production of the carbon rod, the steel of the reactors, etc. is ignored in commercial over-unity (7.34) because its numerical value per cubic foot of magnegas produced is insignificant.

The commercial over-unity of hadronic reactors is evidently important for the production of the combustible magnegas or magnetically polarized hydrogen (MagH^{TM}) at a price competitive over conventional fossil fuels.

A first certification of the commercial over-unity (7.34) was done on September 18 and 19, 1998, for the very first, manually operated prototype of hadronic reactors by the independent laboratory *Motorfuelers, Inc.*, of Largo, Florida, and included (see [8]):

1) Calibrating the cumulative wattmeter provided by *WattWatchers, Inc.*, of Manchester, New Hampshire, which was used to measure the electric energy drawn from the power lines per each cubic foot of magnegas produced;

2) The verification of all dimensions, including the volume of the column used for gas production, the volume of the liquid used in the process, etc.;

3) Repetition of numerous measurements in the production of magnegas and its energy content, calculation of the average values, identification of the errors, etc.

During the two days of tests, *Motorfuelers* technicians activated the electric DC generator and produced magnegas, which was transferred via a hose to a transparent plexyglass tower filled up with tap water, with marks indicating the displacement of one cubic foot of water due to magnegas production.

After the production of each cubic foot, the gas was pumped out of the tower, the tower was replenished with water, and another cubic foot of magnegas was

Figure 10.33. A view of metal cutting via magnegas. Independent certifications by various users have established that: 1) magnegas has a pre-heat time at least half that by acetylene (which is currently used for metal cutting and has an energy content of 2,300 BTU/cf); 2) magnegas cuts metal at least 50% faster than acetylene; 3) the cut produced by magnegas is much smoother without edges as compared to that by acetylene; 4) magnegas exhaust does not contain carcinogenic or other toxic substances, while that of acetylene is perhaps the most carcinogenic and toxic of all fuels; 5) magnegas cutting does not produce the "flash-back" (local explosion of paint over metal) typical of acetylene; 6) magnegas is dramatically safer than acetylene, which is unstable and one of the most dangerous fuels currently used; and 7) magnegas cost about 1/2 that of acetylene.

produced. The procedure was repeated several times to have sufficient statistics. The electric energy from the electric panel required to produce each cubic foot of magnegas was measured via the previously calibrated cumulative wattmeter.

As a result of several measurements, *Motorfuelers, Inc.* certified [8] that the production of one cubic foot of magnegas with the first prototype required an average electric energy of

$$E_{electr} = 122 \text{ W/cf} = 416 \text{ BTU/cf } \pm 5\%. \tag{11.4.15}$$

It should be stressed that this is the electric energy from the electric panel, thus including the internal losses of the DC rectifier. Alternatively, we can say that the arc is served by only 65% of the measured electric energy, corresponding to

$$E_{electr} = 79.3 \text{ W/cf} = 270 \text{ BTU/cf}. \tag{11.4.16}$$

The energy content of magnegas was measured on a comparative basis with the BTU content of natural gas (1,050 BTU/cf). For this purpose, technicians of *Motorfuelers, Inc.*, used two identical tanks, one of natural gas and one of magnegas, at the same initial pressure of 110 psi. Both tanks were used for 5 psi pressure decreases, under the same gas flow, to increase the temperature of the same amount of water in the same pot at the same initial temperature. The ratio of the two temperature increases is evidently proportional to the ratio of the respective BTU contents.

Following several measurements, *Motorfuelers, Inc.* certified [8] that magnegas produced from the antifreeze waste used in the reactor has about 80% of the BTU content of natural gas, corresponding to

$$E_{mg} = 871 \text{ BTU/cf } \pm 5\%. \tag{11.4.17}$$

All other more scientific tests of BTU content of magnegas conducted at various academic and industrial laboratories failed to yield meaningful results due to the energy content of magnegas for various reasons. Despite their empirical character, the measurement of BTU content done by *Motorfuelers, Inc.*, remains the most credible one.

It should be noted that the value of 871 BTU/cf is a lower bound. In fact, automotive tests reviewed in Sect. 7.9 have established that the energy output of internal combustion engines powered by magnegas is fully equivalent to that of natural gas, thus yielding a realistic value of about

$$E_{mg} = 1,000 \text{ BTU/cf}. \tag{11.4.18}$$

During the tests, it was evident that the temperature of the liquid waste in the reactor experienced a rapid increase, to such an extent that the tests had to be stopped periodically to cool down the equipment, in order to prevent the boiling of the liquid with consequential damage to the seals.

Following conservative estimates, technicians of *Motorfuelers, Inc.*, certified [8] that, jointly with the production of 1 cf of magnegas, there was the production of heat in the liquid of 285 BTU/cf plus 23 BTU/cf of heat acquired by the metal of the reactor itself, yielding

$$E_{heat} = 308 \text{ BTU/cf}. \tag{11.4.19}$$

In summary, the average electric energy of 122 W = 416 BTU calibrated from the electric panel produced one cf of magnegas with 871 BTU/cf, plus heat in the liquid conservatively estimated to be 308 BTU/cf. These independent certifications established the following *commercial over-unity* of the first, manually operated hadronic reactor within ±5% error:

$$\frac{871 \text{ BTU/cf} + 308 \text{ BTU/cf}}{416 \text{ BTU/cf}} = 2.83. \tag{11.4.20}$$

Note that, if one considers the electric energy used by the arc itself corresponding to 79.3 W/cf = 270 BTU/cf), we have the following commercial over-unity:

$$\frac{871 \text{ BTU/cf} + 285 \text{ BTU/cf}}{270 \text{ BTU/cf}} = 4.36. \tag{11.4.21}$$

In releasing the above certification, *Motorfuelers, Inc.*, noted that the arc had a poor efficiency, because it was manually operated, thus resulting in large variation of voltage, at times with complete disconnection of the process and need for its reactivation.

Motorfuelers technicians also noted that the BTU content of magnegas, Eq. (11.4.37), is a minimum value, because measured in comparison to natural gas, not with a specially built burner, but with a commercially available burner that had large carbon residues, thus showing poor combustion, while the burner of natural gas was completely clean.

Immediately after the above certification of commercial over-unity, a number of safety and health measurements were conducted on hadronic molecular reactors, including measurements on the possible emission of neutrons, hard photons, and other radiation.

David A. Hernandez, Director of the *Radiation Protection Associates*, in Dade City, Florida conducted comprehensive measurements via a number of radiation detectors placed around the reactor, with particular reference to the only radiations that can possibly escape outside the heavy gauge metal walls, low or high energy neutrons and hard photons.

Under the presence of eyewitnesses, none of the various counters placed in the immediate vicinity of the reactor showed any measurement of any radiation at all. As a result, Radiation Protection Associates released an official Certificate stating that:

> "Santilli's PlasmaArcFlow^{TM} Reactors met and exceed the regulatory regulations set forth in Florida Administrative Code, Chapter 64-E. Accordingly, the reactors are declared free of radiation leakage."

Subsequent certifications of more recent hadronic reactors operating at atmospheric pressure with 50 kW and used to recycled antifreeze waste, this time done on fully automated reactors, have produced the following measurements:

$$E_{mg} = 871 \text{ BTU/cf}, \tag{11.4.22a}$$

$$E_{liq} = 326 \text{ BTU/cf}, \tag{11.4.22b}$$

$$E_{electr} = 100 \text{ W/cf} = 342 \text{ BTU/cf}, \tag{7.42c}$$

resulting in the following commercial over-unity of automatic reactors recycling antifreeze with about 50 kW and at atmospheric pressure:

$$\frac{871 \text{ BTU/cf} + 326 \text{ BTU/cf}}{342 \text{ BTU/cf}} = 3.5. \tag{11.4.23}$$

When ordinary tap water is used in the reactors, various measurements have established a commercial over-unity of about 2.78.

It should be indicated that the commercial over-unity of the hadronic reactors increases nonlinearly with the increase of the kiloWatts, pressure and temperature. Hadronic reactors with 250 kW are under construction for operation at 250 psi and 400^o F. The latter reactors have a commercial over-unity considerably bigger than (7.43).

The origin of the commercial over-unity (7.43) is quite intriguing and not completely known at this writing. In fact, conventional chemical structures and reactions have been studied by Aringazin and Santilli [9] and shown not to be sufficient for a quantitative explanation, thus requiring a new chemistry.

Following Aringazin and Santilli [9], our first task is to compute the electric energy needed to create one cubic foot of plasma in the PlasmaArcFlow reactors as predicted by conventional quantum chemistry. Only after identifying the deviations of the experimental data from these predictions, the need for the covering hadronic chemistry can be properly appraised.

For these objectives we make the following assumptions. First, we consider PlasmaArcFlow reactor processing distilled water with the DC arc occurring between a consumable pure graphite cathode and a non-consumable tungsten anode. As indicated earlier, said reactors yield a commercial over-unity also when used with pure water. Therefore, quantum chemical predictions can be more effective studied in this setting without un-necessary ambiguities. We also assume that water and the solid graphite rod are initially at 300^o K and that the plasma created by the DC electric arc is at $3{,}300^o$ K.

The electric energy needed to create one cubic foot of plasma must perform the following transitions (see Appendix 7.A for basic units and their conversions):

1) Evaporation of water according to the known reaction

$$H_2O(\text{liquid}) \to H_2O(\text{vapor}) - 10.4 \text{ Kcal/mole}, \tag{11.4.24}$$

2) Separation of the water molecule,

$$H_2O \to H_2 + \frac{1}{2}O_2 - 57 \text{ Kcal/mole}, \tag{11.4.25}$$

3) Separation of the hydrogen molecule,

$$H_2 \to H + H - 104 \text{ Kcal/mole}, \tag{11.4.26}$$

4) Ionization of H and O, yielding a total of 1,197 Kcal.

We then have the evaporation and ionization of the carbon rod,

$$C(\text{solid}) \to C(\text{plasma}) - 437 \text{ Kcal/mole}, \tag{11.4.27}$$

resulting in the total 1,634 Kcal for 4 moles of plasma, i.e.

$$\begin{aligned} 408.5 \text{ Kcal/mol} = 0.475 \text{ kWh/mol} = 1621 \text{ BTU/mol} = \\ = 515.8 \text{ Kcal/cf} = 0.600 \text{ kWh/cf} = 2,047 \text{ BTU/cf}, \end{aligned} \tag{11.4.27}$$

to which we have to add the electric energy needed to heat up the non-consumable tungsten anode which is estimated to be 220 BTU/cf, resulting in the total of 2,267 BTU/cf. This total, however, holds at the electric arc itself without any loss for the creation of the DC current from conventional alternative current. By assuming that rectifiers, such as the welders used in PlasmaArcFlow reactors have an efficiency of 70%, we reach the total electric energy from the source needed to produce one cubic foot of plasma

$$\text{Total Electric Energy } = 3,238 \text{ BTU/cf} = 949 \text{ W/cf}. \tag{11.4.28}$$

We now compute the total energy produced by PlasmaArcFlow reactors according to quantum chemistry. For this purpose we assume that the gas produced is composed of 50% hydrogen and 50% carbon monoxide with ignorable traces of carbon dioxide. The latter is indeed essentially absent in PlasmaArcFlow reactors, as indicated earlier. In addition, CO_2 is not combustible. Therefore, the assumption of ignorable CO_2 in the gas maximizes the prediction of energy output according to quantum chemistry, as desired.

Recall that the glow of underwater arcs is mostly due to the combustion of hydrogen and oxygen back into water which is absorbed by the water surrounding the arc and it is not present in appreciable amount in the combustible gas bubbling to the surface. Therefore, any calculation of the total energy produced must make an assumption of the percentage of the original H and O which recombine into H_2O (the evidence of this recombination is established by the production of water during the recycling of any type of oil by the hadronic reactors).

In summary, the calculation of the energy produced by the PlasmaArcFlow reactors requires: the consideration of the cooling down of the plasma from $3{,}300^o$ K to 300^o K with consequential release of energy; the familiar reactions

$$H + H \to H_2 + 104 \text{ Kcal/mole}, \tag{11.4.29a}$$

$$H_2 + \frac{1}{2}O_2 \to H_2O + 57 \text{ Kcal/mole}, \tag{11.4.29b}$$

$$C + O \to CO + 255 \text{ Kcal/mole}. \tag{11.4.29c}$$

Under the assumption of 100% efficiency (that is, no recombination of water), the total energy produced is given by

$$398 \text{ Kcal/mole} = 1,994 \text{ BTU/cf}. \tag{11.4.30}$$

By assuming that the entire energy needed to heat up the non-consumable tungsten is absorbed by the liquid surrounding the electric arc in view of its continuous cooling due to the PlasmaArcFlow, we have the total heat energy of 2,254 BTU/cf.

In addition, we have the energy content of the combustible gas produced. For this purpose we recall the following known reactions:

$$\mathrm{H_2(mg)} + \frac{1}{2}\mathrm{O_2(atm)} \rightarrow \mathrm{H_2O} + 57 \text{ Kcal/mole}, \tag{11.4.31a}$$

$$\mathrm{CO(mg)} + \frac{1}{2}\mathrm{O_2(atm)} \rightarrow \mathrm{CO_2} + 68.7 \text{ Kcal/mole}. \tag{11.4.32b}$$

Consequently, the 50%-50% mixture of conventional gases $\mathrm{H_2}$ and CO has the following

$$\text{Conventional energy content of magnegas produced from water} =$$
$$= 62.8 \text{ Kcal/mole} = 249.19 \text{ BTU/mole} = 315 \text{ BTU/cf}. \tag{11.4.33}$$

Therefore, the total energy output of the PlasmaArcFlow Reactors is given by

$$E(\mathrm{mg}) + E(\mathrm{heat}) = 315 \text{ BTU/cf} + 2,254 \text{ BTU/cf} = 2,569 \text{ BTU/cf}. \tag{11.4.34}$$

It then follows that the energy efficiency of the PlasmaArcFlow reactors is underunity for the case of maximal possible efficiency,

$$\text{Energy efficiency predicted by quantum chemistry} =$$
$$= \frac{\text{Total energy out}}{\text{Electric energy in}} = \frac{E_{mg} + E_{heat}}{E_{electr}} = \frac{2,569 \text{ BTU/cf}}{3,238 \text{ BTU/cf}} = 0.79. \tag{11.4.35}$$

It is possible to show that, for the case of 50% efficiency (i.e., when 50% of the original H and O recombine into water) the total energy output evidently decreases. For detail, we refer the interested reader to Aringazin and Santilli [9].

10.4.8 Cleaning Fossil Fuel Exhaust with Magnegas Additive

Electric power plants continue to attempt the cleaning of their atmospheric pollution (see Figure 11.1) via the cleaning of their exhaust. Since the related equipment is very expensive and notoriously inefficient, these are attempts literally belonging to the past millennium. Nowadays, the exhaust of fossil fueled electric power plants can be cleaned via cost competitive improvement of the combustion.

It is known that, whether burning petroleum or coal, about 60% of the energy in the original fuel is literally thrown through the fluke, and so is the relates cost, due to the notoriously poor combustion.

It is also known in chemistry that hydrogen is the best additive to improve combustion, with consequential improvement of the environmental quality of the exhaust. In fact, hydrogen has the biggest flame temperature and speed among all known fuels. Consequently, the injection of hydrogen as an additive in the flame of fossil fuels burns the uncombusted component of the exhausts in a way proportional to the used percentage of hydrogen. A reason hydrogen as currently available has not (and cannot) be used as additive in fossil fueled electric power plants is its prohibitive cost (that in the U.S.A. is of the order of 50 times the cost of natural gas per same energy content, as recalled in Section 11.1.3).

Magnegas is the best additive for the cleaning of fossil fuel exhaust known to the author[9] because:

1) When produced from the recycling of water-base liquid wastes, magnegas contains about 65% hydrogen, thus qualifying as an effective additive to improve fossil fuel combustion;

2) The remaining components of magnegas are internally rich in oxygen, thus helping to alleviate the large oxygen depletion caused by fossil fuel combustion (Section 11.1); and

3) The cost of magnegas is competitive over that of fossil fuel, particularly when produced by the electric power plants themselves, because of the grossly reduced cost of electricity plus the possibility of producing magnegas from the recycling of city sewage, with a consequential income that covers most of the operating costs of PlasmaArcFlow Recyclers. Un der these conditions, the percentage of magnegas additive to be injected in the flame of fossil fuels becomes a corporate, rather than technical or financial decision.

Besides incontrovertible environmental advantages, the increase of profits for electric power plants in the use of magnegas additive are substantial, such as: the utilization of at least half of the fuel and related cost literally thrown through the fluke due to poor combustion; the capability of producing green electricity that notoriously brings bigger income; and the gaining of the so-called *Kyoto Credits* that, alone, bring millions of dollars of additional income.

Despite these trans[parent gains and numerous solicitations as well as the international exposure of the website [5b], no electric power plant nowhere in the world has expressed interest to this day (fall 2005) interest in at least inspecting the use of magnegas additive. This behavior was expected by the author because, as stated in the opening sentences of this chapter, profits are no longer the dominant drive in the contemporary corporate world. Politics is the dominant drive. Lack of interest for major environmental and financial gains is then another confirmation of the lack of political will toward serious environmental actions in all developed countries (for more details, visit the website [5b]).

[9]The documented indication of other additives comparable to magnegas would be sincerely appreciated.

10.4.9 Hy-Gasoline, Hy-Diesel, Hy-Ethanol, Hy-NG, Hy-Coal

Fossil fuels are sold in a disproportionate daily volumes recalled in Section 11.1, and we should expect that they will continue to be sold in ever increasing disproportionate volumes until the extinction of all petroleum reserves.

Rather than dreaming of eliminating fossil fuels from the market, scientists in general, and chemists in particular, have the ethical duty to seek additives to clean fossil fuel combustion in automotive use, namely, an usage logistically and technically different than the combustion of fossil fuels in power plants furnaces of the preceding section.

It is at this point where the irreconcilable conflict between academic interest on pre-existing theories and the societal need for new theories emerges in its full light. In fact, the best additive to clean fossil fuel combustion is, again, hydrogen (see the preceding section). However, hydrogen is a gas, while gasoline, diesel, ethanol and other fuels are liquids. Consequently, quantum chemistry provides no possibility of achieving new fuels characterized by a stable mixture of liquid fuels and gaseous hydrogen.

However, the abandonment of quantum chemistry in favor of the covering hadronic chemistry permits indeed the possible resolution of the problem. In fact. magnecular bonds are completely insensitive as to whether the constituents of a magnecular cluster partially originated from liquids and part from gases, trivially, because the bond occurs at the level of individual atoms.

The above principle has permitted the formulation of basically new *liquid* fuels known as Hy-Gasoline^{TM}, Hy-Diesel^{TM}, Hy-Ethanol^{TM}, HyCoal^{TM}, etc. (patented and international patents pending [5]), where the prefix "Hy" is used to denote a high hydrogen content. These new fuels are essentially given by ordinary fossil fuels as currently produced, subjected to a bond with magnegas or hydrogen from magnegas (see next section) via the use of special PlasmaArcFlow Reactors.

As predicted, no petroleum company has expressed to date any interest at all in even inspecting the evidence, let alone take serious initiative in these new fuels despite numerous solicitations and the transparent environmental and, therefore, financial gains, because of the origin of the current environmental problems threatening mankind: the lack of serious political will in all developed countries to this day (fall 2005), and actually the subservience of current political will to the petroleum cartel, as denounced in the opening words of this chapter (for more details, visit the website [5b]).

10.4.10 Catastrophic Inconsistencies of Quantum Mechanics, Superconductivity and Chemistry for Submerged Electric Arcs

In Chapter 9 we identified the *approximate* yet still applicable character of quantum mechanics and chemistry for molecular structures.

The analysis of this section has confirmed the content of section 1.2.11 to the effect that the divergences between submerged electric arcs and the predictions of conventional disciplines are so huge to be called "catastrophics inconsistencies" such as:

1) **Inability by quantum chemistry to identify the chemical composition of magnegas.** This occurrence is due to the fact that quantum chemistry predicts that magnegas produced via a electric arc between pure graphite electrodes submerged within distilled water is composed primarily of the molecule $H-H$ with 2 a.m.u and $C-O$ with 28 a.m.u, with traces of H_2O with 18 a.m.u. and CO_2 with 44 a.m.u. No additional species is predicted by quantum chemistry. By comparison , magnegas is composed of fully identifiable peaks in the MS ranging from 1 a.m.u to $1,000$ a.m.u. *none* of which is identifiable with the preceding molecules, resulting in catastrophic divergences in the sense that the application of quantum chemistry to magnegas would have no scientific sense, not even approximate.

2) **Inability by quantum superconductivity to represent submerged electric arcs.** Distilled water is known to be dielectric. In fact, the electric resistance between electrodes submerged within distilled water at large distance (open arc) can be of the order of 100 Ohms or so. However, when the electric arc is initiated the resistance collapses to fractional Ohms, resulting in a very high temperature kind of "superconductor" (since the arc has about $5,000^oC$). Such a collapse of electric resistance is beyond any hope of representation by quantum superconductivity. in reality, as studied in preceding chapters, the collapse is due to the basic inapplicability of Maxwell' s equations for submerged electric arcs , thus implying the basic inapplicability of the Lorentz and Poincaré symmetry, special relativity and all that in favor of covering theories.

3) **A ten-fold error in defect in the prediction of the CO_2 content of magnegas exhaust.** In fact, quantum chemistry predicts the presence of about 50% of CO in magnegas from distilled water resulting in about $40\% CO_2$ in the exhaust,, while magnegas has about 1/10-th that value;

4) **A ten fold error in excess in the prediction of heat generated by carbon combustion by the arc.** In fact, in the preceding subsection we showed that quantum chemistry predicts about $2,250$ Kcal/scf of magnegas, while the measured amount is of the order of 250 for water as feedstock. Note that the latter error confirms the preceding one.

5) **A fourteen-fold error in the prediction of oxygen in the exhaust of magnegas.** In fact, quantum chemistry predicts that, under full combustion in atmosphere, there is no oxygen in the exhaust, while magnegas shows up to 14% breathable oxygen in the exhaust.

an additional large inconsistency of quantum chemistry will be shown in the next section in regard to the hydrogen content of magnegas.

However, the most catastrophic inconsistencies are given by the fact that *magnegas has a variable energy content, a variable specific weight, and a variable Avogadro number.* The first two features are established by the fact that the energy content and density of magnegas produced from the same reactor with the same liquid feedstock increases nonlinearly with the sole increase of the operating pressure, trivially, because bigger pressures produce heavier magnecules.

The all important variation of the Avogadro number is established by the fact, verified every day in the magnegas factories around the world when compressing magnegas in high pressure bottles according to which the same increase of pressures. For instance, the transition from 20 to 120 psi requires about 40 scf, while the transition from $3,500$ psi to $3,600$ psi may require 70 scf of magnegas, an occurrence that can only be explained via the *decrease of the Avogadro number with the increase of pressure.*

The latter anomaly is necessary for gases with magnecular structure for the evident reason that the increase of pressure bonds different magnecules together, thus reducing the Avogadro number. Alternatively, the magnecular structure can be also interpreted as an unusual form of "semi-liquid" in the sense that the magnecular bond is much closer to the so called "h-bridges" of the liquid state of water. the increase of pressure evidently brings magnegas progressively closer to the liquid state, which continuous process can only occur for a variable Avogadro number.

on historical grounds, it should be recalled that *Avogadro conceived his celebrated number as being variable with physical characteristics of pressure and temperature,* a conception clearly stated on the expectation that the gaseous constituents can break down into parts due to collision and subsequent recombinations.

Subsequently, the chemistry of the time believed for decades that the Avogadro number was variable. In fact, the first measurements of the constancy of the Avogadro number maed by Canizzaro also in Italy, were initially very controversial until verified numerous times. Today we know that *the constancy of the Avogadro number for gases wit molecular structure is due to the strength of the valence bond under which no breaking of molecules is possible under increasing temperature and pressure, resulting in a constant number of constituents per mole.*

For over one century chemistry was restricted to the study of gases with molecular structure and Avogadro original conception was forgotten until resumed by the author with his gases with magnecular structure that verify all original intuition by Avogadro.

10.4.11 Concluding Remarks

The first important experimental evidence presented in this section is the independent certification of hadronic reactors of molecular type as being "commercially over-unity", that is, the ratio between the total energy produced and the electric energy needed for its production can be much bigger than one, Eq. (11.4.14).

This occurrence establishes that said hadronic reactor are based on a *a new combustion of carbon* realized via the electric arc, which combustion is much cleaner and more efficient than the combustion of carbon in a conventional furnace. In fact, the new combustion of carbon occurs in the plasma surrounding the electric arc due to the presence of oxygen originating from the liquid feedstock.

Rather than producing highly polluting exhaust, as for the combustion of carbon in a furnace, the plasma combustion produced a clean burning fuel and heat without pollution. Consequently, the plasma combustion of carbon is much more efficient than conventional combustion because pollutants in the exhaust are uncombusted fuel.

The third experimental evidence presented in this section is that establishing the existence of the new chemical species of Santilli magnecules. More specifically, said experimental evidence, plus additional tests not reported here for brevity, confirm the following features of Definition 11.4.1:

I) Magnecules have been detected in MS scans at high atomic weights where no molecules are expected for the gas considered. In fact, the biggest molecule in macroscopic percentages of the magnegas tested, that produced from tap water with conventional chemical composition (8.20), is CO_2 with 44 a.m.u., while peaks in macroscopic percentages have been detected with *ten times* such an atomic weight and more.

II) The MS peaks characterizing magnecules remain unidentified following a computer search among all known molecules. This feature has been independently verified for *each* of the sixteen peaks of Fig. 11.12, for all peaks of Figs. 8.13 and 8.14, as partially illustrated in Figs. 8.8 and 8.15, as well as for all additional MS scans not reported here for brevity.

III) The above MS peaks characterizing magnecules admit no IR signature, thus confirming that they do not have a valence bond. In fact, none of the peaks here considered had any IR signature as partially illustrated in Figs. 8.9 and 8.16, thus confirming the achievement of an essentially pure population of magnecules.

IV) The IR signature of the only molecule detected in macroscopic percentage, that of the CO_2, is mutated precisely with the appearance of two additional peaks, as shown in Fig. 11.15 and independently confirmed in Fig. 11.22. Since any peak in the IR signature represents an internal bond, the mutation here considered confirms the creation by the PlasmaArcFlow technology of new internal magnetic bonds within conventional molecules, as per Fig. 11.11.

V) The anomalous adhesion of magnecules is confirmed in both tests from the evidence that the background (blank) at the end of the tests following conventional flushing continued to show the presence of essentially the same magnecules detected during the tests, as illustrated in Figs. 8.12 and 8.18.

VI) The atomic weight of magnecules mutates in time because magnecules can break down into fragments due to collisions, and then form new magnecules with other fragments. This feature is clearly illustrated by the macroscopic differences of the two scans of Figs. 8.13 and 8.14 via the same instrument on the same gas under the same conditions, only taken 30 minutes apart.

VII) Magnecules can accrue or lose individual atoms, dimers or molecules. This additional feature is proved in the scans of Figs. 8.13 and 8.14 in which one can see that: the peak at 286 a.m.u. of the former becoming 287 a.m.u. in the latter, thus establishing the accretion of one hydrogen *atom*; the peak at 302 a.m.u. in the former becomes 319 a.m.u. in the latter, thus establishing the accretion of the H–O dimer; the peak at 328 a.m.u. in the former becomes 334 a.m.u. in the latter, thus establishing the accretion of one O_2 molecule; the peak at 299 a.m.u. in the former become 297 a.m.u. in the latter, thus exhibiting the loss of one H_2 molecule; *etc.* It should be indicated that these features have been confirmed by all subsequent GC-MS/IRD scans not reported here for brevity.

The other features of Definition 11.4.1 require measurements other than those via GC-MS/IRD and, as such, they will be discussed in the next section.

A most forceful implication of the experimental evidence presented in this section is that it excludes valence as the credible origin of the attractive force characterizing the detected clusters. This feature is forcefully established by the detection of peaks all the way to 1,000 a.m.u. in a gas solely composed of H, C and O atoms that are combined at the $10,000^oF$ of the electric arc, thus excluding hydrocarbons and other standard molecules. Even more forceful experimental evidence on new non-valence bonds will be presented in the next sections.

It is easy to predict that the emergence of "new" non-valence bonds, we have called magnecular, will inevitably imply a revision of a number of current views in chemistry, the first case coming to mind being that of the so-called *H-bridges* in the liquid state of water.

As recalled in Chapter 9, this author never accepted quantum chemistry as "the final theory" for molecular structure because *quantum chemistry lacks the explicit and numerical identification of the attractive force in valence bonds, besides the*

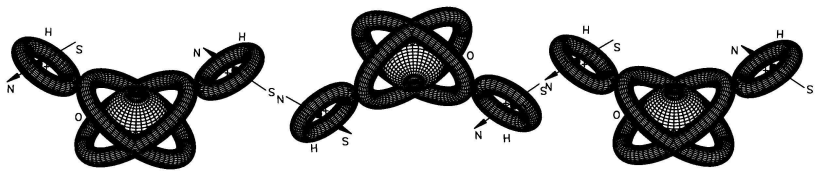


Figure 10.34. A schematic view of the magnecular interpretation of the liquid state of water. Such a state requires an ATTRACTIVE FORCE between the water molecules. But the latter are electrically neutral, diamagnetic and no unbounded electron available for valance bonds. hence, a basically new, non-valence force is needed to represent the water liquid state. Quantum chemistry suggests that such liquid state is due to the so-called "H-bridges" although the latter are pure nomenclature because they do not identify at all explicitly and numerically the attractive force among water molecules. The new chemical species of Santilli magnecules resolves the problem because the H atoms in the H_2O molecule have a polarization in a plane perpendicular to the $H-O-H$ plane, that is precisely a toroidal polarization permitting attractive forces among different H atoms identified in their attractive as well as numerical character in this section. In summary, the main hypothesis here submitted apparently for the first time is that *the liquid state of water, as well as any other liquid state, is a magnecule*, by illustrating in this way the prediction that magnecules can acquire macroscopic dimensions (see additional studies on magnecules in liquids later on in this chapter).

fact that, according to quantum mechanics, two identical electrons should repel, rather than attract each other, as a consequence of which the name "valence" is a mere nomenclature without sufficient scientific content.

This was essentially the situation in molecular structures such as the water molecule H_2O. Hadronic mechanics and chemistry have resolved this insufficiency by identifying explicitly and numerically the ATTRACTIVE force between IDENTICAL ELECTRONS that is responsible for the water and other molecules.

In this section we have learned that molecules admit non-valence bonds originating from toroidal polarizations of the orbitals. The magnecular origin of the H-bridges in the liquid state of water is then inevitable because, as recalled in Chapter 9, the orbitals of the H atoms in H_2O *do not* have a spherical distribution, but a distribution that is perpendicular to the $H-O-H$ plane, thus being precisely of the toroidal type underlying magnecules (see Fig. 11.34 for details).

In closing the author would like to stress that the above findings, even though independently confirmed numerous times, should be considered preliminary and in need of additional independent verifications, which are here solicited under the suggestion that:

1) Only peaks in macroscopic percentages should be initially considered to avoid shifting issues of primary relevance into others of comparatively marginal importance at this time;

2) The internal *attractive* force necessary for the very existence of cluster is identified in clear numerical terms without vague nomenclatures deprived of an actual physical reality, or prohibited by physical laws; and

3) The adopted terminology is identified with care. The word "magnecule" is a mere name intended to denote a chemical species possessing the specifically identified characteristics I) to XV) of Definition 11.4.1 which are distinctly different than the corresponding characteristics of molecules. Therefore, the new species can not be correctly called molecules. The important features are these distinctly new characteristics, and not the name selected for their referral.

10.5 THE NEW MAGNECULAR SPECIES OF HYDROGEN AND OXYGEN WITH INCREASED SPECIFIC WEIGHT

10.5.1 Resolution of Hyrogen Problems Permitted by the Magnegas Technology

In Section 11.1.3 we have pointed out serious problematic aspects caused by large scale use of hydrogen, including its large oxygen depletion, vast pollution caused by its current production methods, threat to the ozone layer, seepage and excessive costs due to the need of liquefaction. It is important to note that the new chemical species of magnecules permits the resolution, or at least the alleviation of these problems.

As indicated in Section 11.3, magnegas is synthesized from liquids that are very rich in hydrogen, such as water-base or oil-base liquid wastes. Consequently, magnegas generally contains about 66% hydrogen, not in a valence bond with other atoms as it is the case for CH_4, but *in a magnecular mixture* with other gases, thus permitting simple methods of molecular or other separations without the need of large energies to break valence bonds. Hence, the Magnegas Technology [5b] offers the following possibilities:

A) **Reduction of oxygen depletion caused by hydrogen combustion.** As indicated in the preceding sections, the new magnecular bond has been developed to achieve full combustion as well as to permit the inclusion of oxygen when prohibited by valence bonds, resulting in a fuel that is internally rich in oxygen originating not from the atmosphere, but from liquid wastes. in fact, magnegas exhaust routinely contains up to 14% of breathable oxygen, and such a percentage can be increased following suitable development. It then follows that *the combustion of hydrogen produced via its separation from magnegas causes dramatically less oxygen depletion than that of hydrogen originating from reformation or electrolysis,* since none of the latter processes release oxygen in the atmosphere.

B) **Reduction of environmental pollution in hydrogen production.** Admittedly, the production of magnegas currently requires the use of commer-

cially available electricity that is polluting because of generally fossil origin. However, PlasmaArcFlow Recyclers release no solid, liquid or gaseous contaminant in the environment; the electric energy used by the arc is about 1/20-th the operating energy (since the rest is given by a very clean carbon combustion via the arc); and the efficiency of PlasmaArcFlow Recyclers can be up to twenty times that of electrolysis. Consequently, the production of hydrogen from magnegas is dramatically less polluting then conventional methods, with the understanding that, when the new clean energies presented in the next chapter achieve industrial maturity, hydrogen production from magnegas will release zero environmental pollutants.

C) **Reduction of the threat to the ozone layer caused by hydrogen seepage and leaks.** Besides a basically new production method, a necessary condition for hydrogen to be a really viable fuel for large scale use is that of achieving a *new magnecular form of hydrogen* consisting of clusters sufficiently large to avoid seepage, as well as to prevent that, in case of leaks, hydrogen quickly rises to the ozone layer. This new species in studied in the next subsections.

D **Elimination of the need for liquefaction of hydrogen.** This objective is related to the preceding one. In fact, the achievement of a magnecular form of hydrogen automatically implies an increase of the specific weight over the standard value of 2.016 a.m.u. that, in turn, automatically implies the reduction of container volumes, with consequential possibility of using hydrogen in a compressed form without any need for its liquefaction. Note that, lacking such heavier form, hydrogen has no realistic possibility of large scale use due to the extreme costs and dangers of changes of state from liquid to gas.

E) **Dramatic reduction of hydrogen cost.** Magnegas produced in volumes is cost competitive with respect to fossil fuels such as natural gas. Consequently, the biggest contribution of the Magnegas Technology to the hydrogen industry is the dramatic reduction of current hydrogen production costs down to values compatible with fossil fuels costs, as shown in more details in the next subsection. Additional advantages over conventional hydrogen are permitted by its magnecular structure as shown below.

10.5.2 The Hypothesis of the New Chemical Species of MagneHydrogen^{TM} and MagneOxygen^{TM}

In paper [18] the author submitted, apparently for the first time, the hypothesis that conventional hydrogen H_2 and oxygen O_2 gases can be turned into a new species with magnecular structure here called *MagneHydrogen*^{TM} and *MagneOxygen*^{TM} 9as well as of other gases), with suggested chemical symbols MH and MO, respectively (patented and international patents pending).

The foundations of the above hypothesis are essentially those given in preceding sections. As recalled earlier, the hydrogen molecule is diamagnetic and, therefore,

it *cannot* acquire a total net magnetic polarity. Nevertheless, the orbits of the *individual H atoms* can acquire a toroidal polarization under a sufficiently strong external magnetic field. The opposite magnetic moments of the two H atoms then explain the diamagnetic character of the hydrogen molecule as illustrated in Figure 11.7.

The aspect important for the hypothesis of MH and MO is that the toroidal polarization of the orbits of the electrons of the individual H atoms, plus the polarization of the intrinsic magnetic moments of nuclei and electrons in the H_2 molecule is sufficient for the creation of the desired new chemical species with bigger specific weight, because the new bonds can occur between pairs of individual H atoms, as illustrated in Figures 11.10 and 11.11.

The creation of MO is expected to be considerably simpler than that of MH because oxygen is paramagnetic, thus having electrons free to acquire an overall magnetic polarity which is absent for the case of MH. Nevertheless, the achievement of a significant increase of the specific weight of the oxygen will require the toroidal polarization of at least some of the peripheral atomic electrons, in addition to a total magnetic polarization.

The primary technological objective is, therefore, that of achieving physical conditions and geometries suitable for the joint polarization of *atoms*, rather than molecules, in such a way to favor their coupling into chains of opposing magnetic polarities. In the final analysis, the underlying principle here is similar to the magnetization of a ferromagnet, that is also based on the polarization of the orbits of unbounded electrons. The main difference (as well as increased difficulty) is that the creation of MH requires the application of the same principle to a *gaseous*, rather than a solid substance.

Under the assumption that the original gases are essentially pure, MH can be schematically represented

$$(H_\uparrow - H_\downarrow) \times H_\uparrow, \tag{11.5.1a}$$

$$(H_\uparrow - H_\downarrow) \times (H_\uparrow - H_\downarrow), \tag{11.5.1b}$$

$$(H_\uparrow - H_\downarrow) \times (H_\uparrow - H_\downarrow) \times H_\uparrow, etc. \tag{11.5.1c}$$

while MagneO can be schematically represented

$$(O_\uparrow - O_\downarrow) \times O_\uparrow, \tag{11.5.2a}$$

$$(O_\uparrow - O_\downarrow) \times (O_\uparrow - O_\downarrow), \tag{11.5.2b}$$

$$(O_\uparrow - O_\downarrow) \times (O_\uparrow - O_\downarrow) \times O_\uparrow .etc. \tag{11.5.3c}$$

where the arrows now indicate possible polarizations of more than one electron orbit.

By keeping in mind the content of the preceding sections, the achievement of the above magnecular structure does imply that MH and MO have specific

weight and energy content greater than the corresponding values for unpolarized gases. The numerical values of these expected increases depend on a variety of factors discussed in the next subsections, including the intensity of the external magnetic field, the pressure of the gas, the time of exposure of the gas to the external field, and other factors.

A first important feature to be subjected to experimental verification (reviewed below) is the expected increase of specific weight. By recalling that the *gasoline gallon equivalent for hydrogen* is about 366 scf, the achievement of a form of MH with five times the specific weight of conventional hydrogen would reduce the prohibitive volume of $7,660$ scf equivalent to 20 g of gasoline to about $1,500$ scf. This is a volume of MH that can be easily stored at the pressure of $4,500$ psi in carbon fiber tanks essentially similar in volume and composition to that of a natural gas tank. As a result, the achievement of MH with sufficiently high specific weight can indeed eliminate the expensive liquefaction of hydrogen in automotive use, with consequential reductions of costs.

Another basic feature to be subjected to experimental verification (reviewed below) is that the combustion of MH and MO releases more energy than the combustion of conventional H and O gases. It then follows that

I) The use for internal combustion engines of MH with a sufficiently high specific weight is expected to eliminate liquefaction, yield essentially the same power as that produced with gasoline, and permit a dramatic decrease of operating costs;

II) The use of MH and MO in fuel cells is expected to yield a significant increase of voltage, power and efficiency; and

III) The use of liquefied MH and MO as fuels for rocket propulsion is expected to permit an increase of the payload, or a decrease of the boosters weight with the same payload.

Moreover, recent studies scheduled for a separate presentation have indicated that the *liquefaction of MH and MO appears to occur at temperatures bigger than those for conventional gases,* thus implying an additional reduction of costs. This expectation is due to the fact that magnecules tend to aggregate into bigger clusters with the increase of the pressure, evidently due to their magnetic polarizations, which feature evidently favors liquefaction.

It is evident that the same principles outlined above also apply for other gases, and not necessarily to H and O gases alone. In fact, the processing of any gaseous fossil fuel via the principles here considered permits the increase of its specific weight as well as of its energy output, thus permitting a consequential decrease of storage volume, increase of performance and decrease of costs.

Note that the hypothesis of MH and MO is an extension of H_3 and O_3 to arbitrary values H_n and O_m as permitted by local values of pressure and temperature. Alternatively, the experimental evidence on MH and MO reviewed later

on in this section confirms the magnecular structure of H_3 and O_3 presented in Section 11.3.4.

10.5.3 Industrial Production of MagneHydrogen^{TM} and MagneOxygen^{TM}.

As indicated earlier, the magnetic polarization of the orbits of peripheral atomic electrons requires extremely strong magnetic fields of the order of billions of Gauss. These values are of simply impossible realization in our laboratories with current technologies, that is, at distances of the order of inches or centimeters. These magnetic fields cannot be realized today even with the best possible superconducting solenoids cooled with the best available cryogenic technology.

The only possible, industrially useful method of achieving magnetic fields of the needed very high intensity is that based on direct current (DC) electric arcs with currents of the order of thousands of Amperes (A) when considered at atomic distances, i.e., of the order of 10^{-8} cm. As illustrated in Fig. 11.9, the magnetic field created by a rectilinear conductor with current I at a radial distance r is given by the well known law

$$B = kI/r, \tag{11.5.4}$$

where k = 1 in absolute electromagnetic units. It then follows that, for currents in the range of 10^3 A and distances of the order of the size of atoms r = 10^{-8} cm, the intensity of the magnetic field B is of the order of 10^{13} Oersted, thus being fully sufficient to cause the magnetic polarization of the orbits of peripheral atomic electrons.

Under the above conditions schematically represented in Fig. 11.9, atoms with the toroidal polarization of their orbits find themselves aligned one next to the other with opposing polarities which attract each other, thus forming magnecules. The electric arc decomposes the original molecule, thus permitting the presence of isolated atoms or radicals in the magnecular structure as needed to increase the energy output (Section 3).

In this way, the process transforms the original gas with its conventional molecular structure into a new chemical species consisting of individual atoms, radicals and complete molecules all bonded together by attractive forces among opposite magnetic polarities of the toroidal polarization of the orbits of peripheral atomic electrons.

In the event the original gas has a simple diatomic molecular structure, such as H_2, the magnecular clusters are composed of individual polarized H atom and ordinary polarized molecules H_2 as in Fig. 11.11. In the event the original gas has the more complex diatomic structure of O_2, the magnecular clusters are composed of individual polarized O atoms, O-O single bonds, and O_2 molecules with additional internal bonds as in Fig. 11.12. In the event the original gas has the more complex diatomic structure CO with triple valence bonds, the magnecular

clusters are more complex and are generally composed of individual C and O atoms, single bonds C-O, double bond C=O, conventional molecules CO and O_2 with internal new bonds as in Fig. 11.12, plus possible C-complexes. Original gases with more complex conventional molecular structure evidently imply more complex magnecular clusters with all possible internal atomic arrangements.

It is evident that the resulting new species is not composed of all identical magnecules, and it is composed instead of a variety of magnecules from a minimum to a maximum number of atomic components, which have been measured to reach 1,000 a.m.u. and even more. The specific weight of the magnecular gas is then given by the average weight of all different magnecules, as indicated earlier.

Needless to say, a number of alternative methods for the industrial production of MH and MO are possible as identified in the existing patent applications. An alternative method worth mentioning here is the use of solenoids. The reader should however be aware that the latter cannot decompose molecules. Therefore, the MagneGases produced via the use of electric discharges and solenoids are different.

Another type of MH important for this study is that obtained from MagneGas [5]. When MagneGas is produced from a hydrogen rich liquid feedstock (such as water or liquids of fossil origin), it may contain up to 60% hydrogen in a form already polarized by the electric arc used for its production. Therefore, the hydrogen content of MagneGas is indeed a particular form of MH which can be separated via a variety of available technologies, such as filtration, cryogenic cooling and other processes.

This particular form of MH (whose features are identified in the next subsection) is particularly suited as fuel for internal combustion engines, rather than for fuel cells. This is due to the expected presence of very small C and O impurities which do not permit their use in fuel cells.

This particular type of MH derived from MagneGas has already been tested for automotive usage and proved to have a performance essentially similar to that of gasoline without any need of liquefaction, as needed instead by hydrogen vehicles currently tested by BMW, GM and other automakers. The tests were conducted via the conversion of two Honda and one Ferrari cars to operate on the new fuels (see [5] for brevity).

Above all, this particular type of MH has resulted to be cost competitive with respect to fossil fuels, of course, when produced in sufficiently large volumes. This cost competitiveness is due to a variety of factors, including (see [5] for detail):

1) the use of hydrogen rich wastes as liquid feedstock, such as city and farm sewage, antifreeze and or oil waste, etc., which implies an *income*, rather than a cost;

2) the possible utilization of steam at 400^o produced by the cooling of the highly esoenergetic processes of the reactors, which steam can be used for other *income*

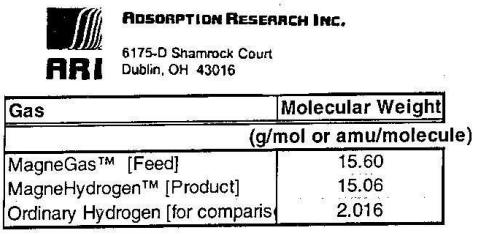


Figure 10.35. A view of the main results on the measurement of specific weight on a specific form of MH produced from magnegas released by Adsorptions Research Laboratory, in Ohio, under signature by its laboratory director. It should be stressed that the high value of specific weight was due to a specific treatment not expected to be possible on a industrial basis. Therefore, the specific weight ofg MH industrially production from magnegas is expected to have about three times the specific weight of H_2, thus sufficient to render MH equivalent to natural gas as far as energy content is concerned (because natural gas contains about $1,000$ BTU/scf, $H2$ contains about 300 BTU/scf, consequently $MH = 3H_2$ would contain BTU/scf close to those of natural gas)

producing applications, such as desalting seawater via evaporation, production of electricity via turbines, heating of buildings, and other income producing uses; and

3) the unusually high efficiency of Santilli Hadronic Reactors of molecular types used for the process which brings the cost of electricity down to $0.005/scf$.

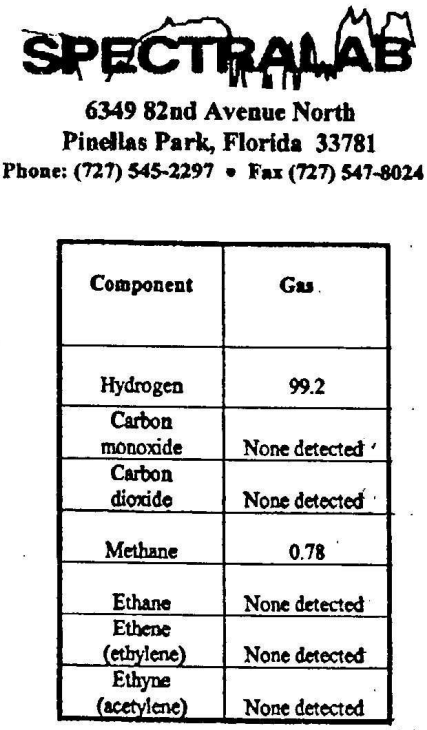
Specific equipment and designs for the industrial production of MH, MO, and other magnetically polarized gases are available on request.

10.5.4 Experimental Evidence on MagneHydrogen^{TM} and MagneOxygen^{TM}

It is now important to review the experimental evidence supporting the existence of MH and MO.

The first tests were conducted with MH produced from MagneGas as indicated in the preceding subsection. MagneGas was first produced by using antifreeze waste as liquid feedstock. The combustible gas was then passed through 5 Armstrong zeolite filters, which essentially consist of a microporous molecular sieve selecting a gas via the so-called "molecular sieving," or molecular size exclusion.. The filtered gas was then subjected to the following three measurements:

1) This type of MH was first subjected to analytic measurements by a laboratory via Gas Chromatography (CG) and independent tests for confirmation were conducted via Fourier Transform Infrared Spectroscopy (FTIS). All measurements were normalized, air contamination was removed, and the lower detection limit was identified as being 0.01%. The results are reported in Fig. 11.35. As



SPECTRALAB

6349 82nd Avenue North
Pinellas Park, Florida 33781
Phone: (727) 545-2297 • Fax (727) 547-8024

Component	Gas
Hydrogen	99.2
Carbon monoxide	None detected
Carbon dioxide	None detected
Methane	0.78
Ethane	None detected
Ethene (ethylene)	None detected
Ethyne (acetylene)	None detected

Figure 10.36. A summary view of the spectroscopic analyses conducted by Spectra lab of largo, Florida, showing 99.2% hydrogen in the species of MH here considered. Note the "experimental belief" that the species here considered contains 0.78% methane. MH produced from magnegas cannot possibly contain methane since magnegas is formed at about $10,000^oF$ of the electric arc at which methane cannot possibly survive. in reality, the analytic instrument has detected a magnecular species with 16 a.m.u and identified that species with methane due to lack of info in the computer data banks.

one can see, these measurements indicate that this particular type of MagneH is composed of 99.2% hydrogen and 0.78% methane, while no carbon monoxide was detected.

2) The average specific weight of this type of MH was measured by two independent laboratories as being 15.06 a.m.u., while conventional pure hydrogen has the specific weight of 2.016 a.m.u., thus implying a 7.47 fold increase of the specific weight of conventional hydrogen.

3) The same type of MH used in the preceding tests was submitted to CG-MS scans via the use of a HP GC 5890 and a HP MS 5972 with operating conditions specifically set for the detection of magnecules (Section 5 and Ref. [5]). The results of these third tests are reproduced in Fig. 11.37. As one can see, by

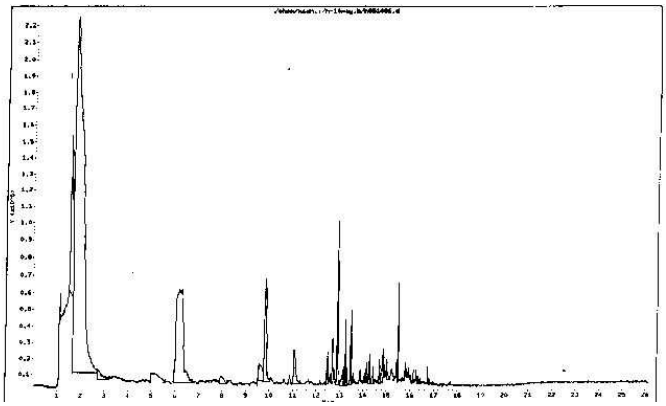


Figure 10.37. A view of one of the numerous GC-MS scans conducted at the Toxic Analytic Laboratory in California establishing in a final form the magnecular character of the species here studied. In fact, the molecular seeving process used by Adsorption Research Laboratory could only allow the separation of hydrogen and definitely not the numerous heavy clusters identified in this scan. Since hydrogen has only one electron and, consequently can only form under valence bond H_2, the heavy species of hydrogen here considered establishes the existence of a *non-valence* bond beyond any possible or otherwise credible doubt.

keeping in mind the results of GC-FTIS of Fig. 11.36, the GC-MS measurements should have shown only two peaks, that for hydrogen H_2 at about 2 a.m.u., and that for methane CH_4 at about 16 a.m.u. On the contrary, these GC-MS tests confirm the existence of a large peak at about 2 a.m.u. evidently representing hydrogen, but do not show any peak at 16 a.m.u. proportional to the 0.78% of methane, and exhibit instead the presence of a considerable number of additional peaks in macroscopic percentages all the way to 18 a.m.u. This GC-MS scan establishes the existence beyond credible doubt of a magnecular structure in the type of MH here studied. Note, in particular, *the existence of well identified peaks in macroscopic percentage with atomic weight of 3, 4, 5, 6, 7, 8 and higher values which peaks, for the gas under consideration here, can only be explained as magnecules composed of individual H atoms as well as H molecules in increasing numbers.*

The above measurements 1), 2) and 3) confirm the capability to produce hydrogen with a multiple value of their standard specific weight, and consequential increased energy content.

Next, to test MO in fuel cells, the author had constructed by technicians in Florida a rudimentary apparatus based on the use of automotive sparks powered by an ordinary car battery, the system operating at about 15 psi. Two types of MO, denoted by MO_1 and MO_2, were produced from pure oxygen for comparative purposes.

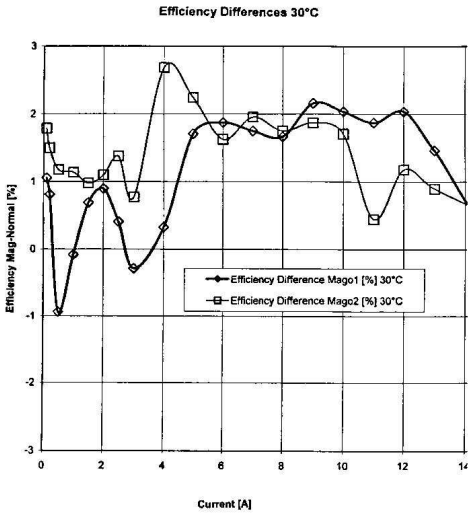


Figure 10.38. A schematic view of the voltage increase in a test fuel cell operated with ordinary pure hydrogen and the two samples of MO produced by rudimentary equipment.

This type of MO was tested in lieu of ordinary oxygen in a 2-cell Proton Exchange Membrane (PEM) fuel cell operated with conventional high purity hydrogen. The membrane material was Nafion 112; the catalyst in the electrodes was platinum acting on carbon; the plates for heat transfer were given by two nickel/gold plated material; the temperature of the fuel cell was kept constant via ordinary cooling means; the current was measured via a HP 6050A electronic load with a 600 W load module; a flow rate for oxygen and hydrogen was assigned for each current measurement; both oxygen and hydrogen were humidified before entering the cell; the measurements reported herein were conducted at 30 degrees C.

The results of the measurements are summarized in Figs. 11.38, 11.39 and 11.40 that report relative measurements compared to the same conditions of the cell when operated with ordinary pure oxygen. As one can see, these measurements show a clear increase of the voltage, power and efficiency of the order of 5% when the cell was operated with MO_1 and MO_2. The increase was consistent for both samples except differences within statistical errors.

To appraise these results, one should note that the types of MO used in the test were produced via rudimentary equipment based on intermittent sparks operated with an ordinary automotive battery, and with the pressure limited to 15 psi. By comparison, the industrial production of MO should be done with an array of arcs each operated with continuous currents of thousands of Amperes, and at

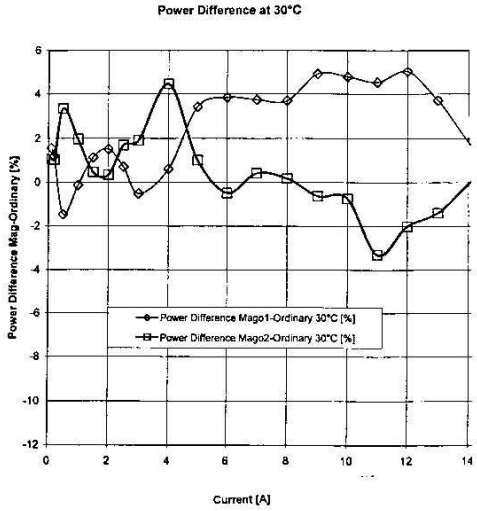


Figure 10.39. A schematic view of the power increase in a test fuel cell operated as in Fig. 11.38 confirming the results of the latter.

pressures of thousands of psi. It is evident that the latter conditions are expected to imply a significant increase of the performance of the fuel cells when operated with MO. Still bigger increases in voltage, power and efficiency occur when the fuel cells are operated with both MO and MH for the reasons discussed in Section 3. These latter tests are under way and are contemplated for reporting in a future research..

In summary, the systematic character of the experimental results, combined with the limited capabilities of the used equipment, appear to confirm the hypothesis of new forms of hydrogen and oxygen with magnecular structure capable of producing an industrially significant increase in voltage, power and efficiency of fuel cells. Independent measurements are here solicited for the finalization of these issues.

10.5.5 Conclusions

Despite the known uneasiness created by novelty, the rather vast experimental evidence, only partially reproduced in this section to avoid a prohibitive length, supports the following results:

1) The *existence of a new chemical species whose bonding force is not of valence type* (from the absence of infrared signature and various other evidence as in Figs. 6 and 7), which has been interpreted by this author as being due to the only fields available in a molecular structure, the electric and magnetic fields, and called

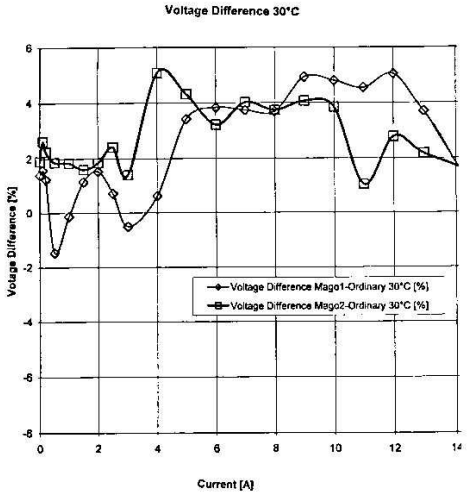


Figure 10.40. A schematic view of the efficiency increase in a test fuel cell operated as in Fig. 11.38, which provide additional confirmation of the latter results.

electromagnecules in general, the name *Santilli magnecules* being used to denote the dominance of magnetic over electric effects (Section 2). Other researchers may prefer different nomenclatures and search for esoteric fields other than the electric and magnetic fields, with the understanding that the *non-valence novelty* of the new species is outside scientific debate.

2) The *existence of a form of hydrogen with about seven times the atomic weight of molecular hydrogen which eliminates the need for liquefaction in automotive use, while having a power output essentially similar to that of gasoline, and being cost competitive with respect to fossil fuel when produced in large scale.* This is the new species of hydrogen, called by this author MagneH^{TM} (patented and international patents pending) which is derived via filtering, cryogenic separation or other means from the new combustible fuel called *Santilli MagneGas*^{TM} (international patents pending). The latter gas s is produced via DC electric arcs between carbon-base consumable electrodes submerged within a hydrogen rich liquid feedstock, such as fresh or salt water, antifreeze or oil waste, city or farm sewage, crude oil, etc.

3) The *industrial capability of turning conventional hydrogen and oxygen into new species with bigger atomic weight and energy content for use in fuel cells with increased voltage, power and efficiency.*

4) The *existence of new forms of liquid hydrogen and oxygen for rocket propulsion with increased trust, and consequential increased payload or decreased boosters' weight with the same payload.*

5) The *experimental evidence of dramatic departures from quantum chemistry in support of the covering hadronic chemistry* [5].

Evidently, these studies are in their infancy and much remains to be done, both scientifically and industrially. Among the existing intriguing open problems we mention:

A) The identification of *new analytic equipment specifically conceived for the detection of magnecules*. In fact, researchers in the field know well the dramatic insufficiency for tests on magnecular substances of currently available analytic equipment specifically conceived for molecular substances.

B) The *identification of the possible frequency at which magnecules may have an infrared signature.* For instance, the detection of methane in the MH tests of Fig. 11.36 has a mere indicative value, rather than being an actual experimental fact. In any case, the detection of methane is not confirmed by at least one second independent test to achieve final scientific character. Also, a peak at 16 a.m.u. which is necessary in the GC-MS scans of Fig. 11.27 to confirm the presence of methane (CH_4), is missing. Finally, the original MagneGas is created in the $10{,}000^o$ F of electric arcs at which temperature no methane can survive. In view of the above, a more plausible possibility is that the "methane" detected by the analyses of Fig. 11.37 is, in reality, the infrared signature of a magnecule.

C) The *study of the liquefaction of MagneGases* on a comparative basis with the liquefaction of the same gases with conventional molecular structure. This study is recommended particularly for rocket propulsion, due to the expected new species of *liquid magnecules* [5], the liquefaction itself at a temperature bigger than the conventional ones, the increase in trust and the reduction in liquefaction costs.

D) The *study of the possible storage of energy in inert gases* via the mechanism of internal magnetic polarization and resulting new molecular bonds illustrated in Figs. 5 and 8. In fact there exist patents as well as reported test engines operating on inert gases which are generally dismissed by academia because of the believed "inert" character of these cases. Perhaps, a more open mind is recommendable for truly basic advances.

E) The *study of nonlinear deviations from the perfect gas law and the Avogadro number* which are inherent in magnecular clustering since they can break down into fragments due to collision and then have different recombinations, resulting in a population with generally varying number of constituents, while keeping constant statistical averages.

Needless to say, the author solicits the independent verification of all results presented in this section without which no real scientific advance is possible.

10.6 HHO, THE NEW GASEOUS AND COMBUSTIBLE FORM OF WATER WITH MAGNECULAR STRUCTURE

10.6.1 Introduction

Studies on the electrolytic separation of water into hydrogen and oxygen date back to the 19-th century (for a textbook on the water molecule see, e.g., Ref. [20a] and for an account on its electrolytic separation see, e.g., Ref. [20b]). More recently, there has been considerable research in the separation of water into a mixture of hydrogen and oxygen gases. These studies were initiated by Yull Brown in 1977 via equipment generally referred to as electrolyzers and the resulting gas is known as "Brown gas" (see patents [21]).

In accordance with these patents as well as the subsequent rather vast literature in the field, the Brown gas is defined as a combustible gas composed of conventional hydrogen and conventional oxygen gases having the exact stochiometric ratio of 2/3 (or 66.66% by volume) of hydrogen and 1/3 (or 33.33% by volume) of oxygen.

In this section the author (a physicist) presents to the chemistry community for its independent verification various measurements on an apparently new mixture of hydrogen and oxygen hereon referred to as the HHO gas (international patent pending) developed by Hydrogen Technology Applications, Inc., of Clearwater, Florida (www.hytechapps.com). The new HHO gas is regularly produced via a new type of electrolyzer and has resulted to be distinctly different in chemical composition than the Brown gas, even though both gases share a number of common features.

The main scope of this section is to report, apparently for the first time, new clusters of hydrogen and oxygen atoms contained in the HHO gas, which clusters appear to escape the traditional valence interpretation and constitute one of the novelties of the HHO gas over the Brown gas.

Another objective of this sec tion is to initiate quantitative studies on the rather unique features of the HHO gas that do not appear to be representable via the conventional quantum chemistry of hydrogen and oxygen gases.

Yet another objective of this section is to present a working hypothesis to initiate the understanding of the capability by the HHO electrolyzers to perform the transition of water from the liquid to a gaseous state via a process structurally different than evaporation or separation, due to the use of energy dramatically less than that required by said evaporation or separation.

The final objective of this section is the submission, apparently for the first time, of it a new form of the water molecule created by the removal of its natural electric polarization and consequential collapse of the two HO dimers, from their conventional configuration with 105^o to a new configuration in which the two

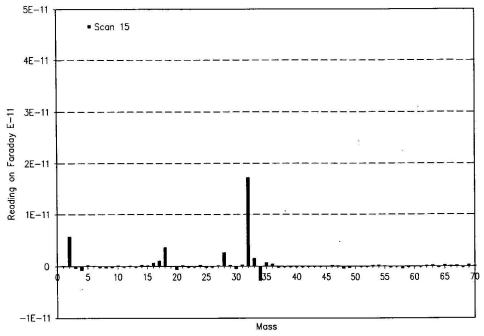


Figure 10.41. A view of one of the GC scans on the HHO gas conducted by Adsorption Research Laboratories showing conventional as well as anomalous peaks.

dimers are collapsed one against the other due to strongly attractive opposing magnetic polarizations (see below for details and pictures).

Due to the loss of electric polarization, polymerization and other features, the above new form of the water molecule permits a plausible representation of the creation of the HHO gas from liquid water without the evaporation energy. Its unstable character also permits a plausible interpretation on the experimental measurements of all anomalous features of the HHO gas.

Independent verification by interested chemists of the various measurements reported in this section are solicited, jointly with the conduction of additional much needed tests. Samples of the HHO gas can be obtained at any time by contacting Hydrogen Technology Applications, Inc. at their website www.hytechapps.com.

10.6.2 Experimental Measurements on the New HHO Gas

Under visual inspection, both the HHO gas results to be odorless, colorless and lighter than air, as it is also the case for the Brown gas. Their first remarkable feature is the efficiency E of the electrolyzer for the production of the gas, here simply defined as the ratio between the volume of HHO gas produced and the number of Watts needed for its production. In fact, the electrolyzers rapidly convert water into 55 standard cubic feet (scf) of HHO gas at 35 pounds per square inch (psi) via the use of 5 Kwh, namely, an efficiency that is at least ten times the corresponding efficiency of conventional water evaporation, thus permitting low production costs.

The above efficiency establishes the existence of a transition of water from the liquid to the gaseous state that is not caused by evaporation. By keeping in mind the combustible character of the HHO gas compared to the noncombustible char-

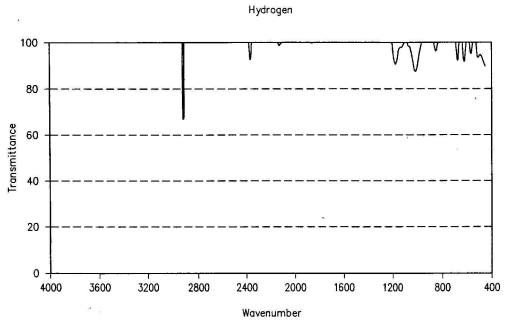


Figure 10.42. The IR signature of a conventional H_2 gas made by the PdMA laboratory.

acter of water vapor, the above efficiency suggests the existence of new chemical processes in the production of the gas that deserve quantitative studies.

A second important feature is that the HHO gas does not require oxygen for combustion since the gas contains in its interior all oxygen needed for that scope, as it is also the case for the Brown gas. By recalling that other fuels (including hydrogen) require atmospheric oxygen for their combustion, thus causing a serious environmental problem known as oxygen depletion, the capability to combust without any oxygen depletion (jointly with its low production cost) render the gas particularly important on environmental grounds.

A third feature of the gas is that it does not follow the PVT of gases with conventional molecular structure, since the gas reacquires the liquid water state at a pressure of the order of 150 psi, while conventional gases acquire the liquid state a dramatically bigger pressures. This feature suggests that the gas here considered does not possess a conventional molecular structure, namely, a structure in which the bond is of entire valence type.

A fourth feature of the gas is its anomalous adhesion (adsorption) to gases, liquids and solids, as verified experimentally below, thus rendering its use particularly effective as an additive to improve the environmental quality of other fuels, or other applications. This feature is manifestly impossible for conventional gases H_2 and O_2, thus confirming again a novel chemical structure.

A fifth feature of the gas is that it exhibits a widely varying thermal content, ranging from a relatively cold flame in open air at about 150^oC, to large releases of thermal energy depending on the substance to which the flame is applied to, such as the instantaneous melting of bricks requiring up to $9,000^oC$.

The measurements conducted by the author at various independent laboratories on the HHO gas can be summarized as follows.

On June 30, 2003, Adsorption Research Laboratory of Dublin, Ohio, measured the specific weight of the HHO gas and released a signed statement on the result-

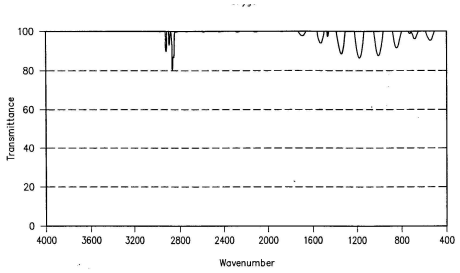


Figure 10.43. The IR signature of a conventional O_2 gas made by the PdMA laboratory.

ing value of 12.3 grams/mole. The same laboratory repeated the measurement on a different sample of the gas and confirmed the result.

The released value of 12.3 grams/mole is anomalous. In fact, the conventional separation of water into H_2 and P_2 produces a mixture of 2/3 HBN_2 and 1/3 O_2 that has the specific weight $(2 + 2 + 32)/3 = 11.3$ grams/mole.

Therefore, we have the anomaly of 12.3 - 11.2 = 1 gram/mole, corresponding to 8.8% anomalous increase in the value of the specific weight. Rather than the predicted 66.66% of H_2 the gas contains only 60.79% of the species with 2 atomic mass units (amu), and rather than having 33.33% of O_2 the gas contains only 30.39% of the species with 32 amu.

These measurements provide direct experimental evidence that the HHO gas is not composed of a sole mixture of H_2 and O_2, but has additional *heavier* species.

Moreover, the HHO gas used in the tests was produced from distilled water. Therefore, there cannot be an excess of O_2 over H_2 to explain the increased specific weight. The above measurement establishes the presence in HHO of 5.87% of hydrogen and 2.94% oxygen bonded together into species heavier than water, as identified below via mass spectroscopy and other analytic measurements.

Adsorption Research Laboratory also conducted scans of the HHO gas via a Gas Chromatographer (GC) reproduced in Fig. 11.41 establishing the presence in the HHO gas of the following species here presented in order of their decreasing percentages:

1) A first major species with 2 amu expectedly representing gaseous hydrogen;

2) A second major species with 32 amu expectedly representing gaseous oxygen;

3) A large peak at 18 amu expectedly representing water vapor;

4) A significant peak with 33 amu expectedly representing a new species expectedly of non-molecular nature;

5) A smaller yet clearly identified peak at 16 amu expectedly representing atomic oxygen;

6) Another small yet fully identified peaks at 17 amu expectedly representing the radical OH whose presence in a gas is also anomalous;

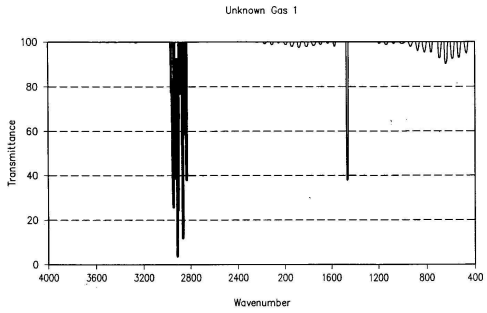


Figure 10.44. The IR signature of the HHO gas made by the PdMA laboratory. When compared to the IR scans of Figures 2 and 3, this scan shows that the HHO gas is not a mixture of H_2 and O_2 gases.

7) A small yet fully identified peak at 34 amu expectedly representing the bond of two dimers HO that is also anomalous for a gas;

8) A smaller yet fully identified peak at 35 amu that cannot be identified in any known molecule;

9) Additional small peaks expected to be in parts per million.

It should be added that the operation of the GC detector was halted a few seconds following the injection of the HHO gas, while the same instrument was operating normally with other gases. This anomalous behavior can be best interpreted via an anomalous adhesion of the gas to the walls of the feeding line as well as of the column and other parts of the instruments, an anomalous adhesion confirmed by additional tests reviewed below.

On July 22, 2003, the PdMA Corporation in Tampa, Florida, conducted InfraRed (IR) scans reported in Figures 2, 3 and 4 via the use of a Perkin-Elmer IR scanner model 1600 with fixed point/single beam. The reported scans refer to a conventional H_2 gas (Fig. 11.42), a conventional O_2 gas (Fig. 11.43), and the HHO gas Fig. 11.44).

Inspection of these scans shows a substantial differences between HHO gas and H_2 and O_2 gases. In fact, the latter gases are symmetric molecules, thus having very low IR peaks, as confirmed by scans 2 and 3. The first anomaly of HHO is that of showing comparatively much stronger resonating peaks. Therefore, the indicated IR scans establish that the HHO gas has an asymmetric structure, which is remarkable since the same feature is absent for the conventional mixture if H_2 and O_2 gases.

Moreover, H_2 and O_2 gases can have at most two resonating frequencies each, one for the vibrations and the other for rotations. Spherical distributions of orbitals and other features imply that H_2 has essentially only one IR signature as

confirmed by the scan of Fig. 11.42, while O_2 has one vibrational IR frequency and three rotational ones, as also confirmed by the scans of Fig. 11.43.

Inspection of the IR scans for the HHO gas in Fig. 11.44 reveals additional novelties. First, the HHO scan show the presence of at least nine different IR frequencies grouped around wavenumber 3000, plus a separate distinct frequency at around wavenumber 1500.

These measurements provide experimental evidence that the species with 18 amu detected in the GC scans of Fig. 11.41 is not water vapor, but a yet unknown bond of two hydrogen and one oxygen atoms.

In fact, water vapor has IR frequencies with wavelengths 3756, 3657, 1595, their combination and their harmonics (here ignored for simplicity). The scan for the HHO gas in Fig. 11.44 confirms the presence of an IR signature near 1595, thus confirming the molecular bond HO, but the scan shows no presence of the additional very strong signatures of the water molecules at 3756 and 3657, thus establishing the fact that the peak at 18 amu is not water as conventionally understood in chemistry.

On July 22, 2003, the laboratory of the PdMA Corporation in Tampa, Florida measured the flash point, first on commercially available diesel fuel, detecting a flash point of 75^oC, and then of the same fuel following the bubbling in its interior of the HHO gas, detecting the flash point of 79^oC.

The latter measurement too is anomalous because it is known that the addition of a gas to a liquid fuel *reduces* its flash point generally by half, rather than *increasing* it as in the above measurement, thus implying the expected flash value of about 37^C for the mixture of diesel and HHO gas. Therefore, the anomalous increase of the flash point is not of 4^oC, but of about 42^oC.

Such an increase cannot be explained via the assumption that HHO is contained in the diesel in the form of a gas (otherwise the flash point would decrease), and requires the occurrence of some type of anomalous bond between the gas and the liquid that cannot possibly be of valence type.

An experimental confirmation of the latter bond was provided on August 1, 2003, by the Southwest Research Institute of Texas, that conducted mass spectrographic measurements on one sample of ordinary diesel as used for the above flash point measurements, here reported in Fig. 11.45, and another sample of the same diesel with HHO gas bubbled in its interior, here reported in Fig. 11.46.

The measurements were conducted via a Total Ion Chromatogram (TIC) and Gas Chromatography Mass Spectrometry GC-MS manufactured by Hewlett Packard with GC model 5890 series II and MS model 5972. The TIC was obtained via a Simulated Distillation by Gas Chromatography (SDGC).

The column was a HP 5MS $30 \times 0,25$ mm; the carrier flow was provided by helium at 50^oC and 5 psi; the initial temperature of the injection was 50^oC with a temperature increase of 15^oC per minute and the final temperature of 275^oC.

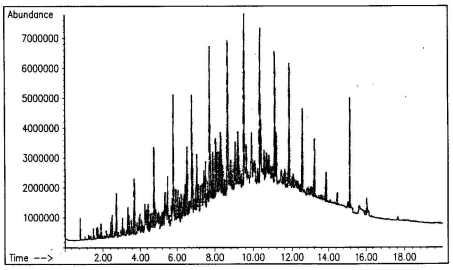


Figure 10.45. A TIC of the GC-MS scans of conventionally sold diesel fuel made by Southwest Research Institute.

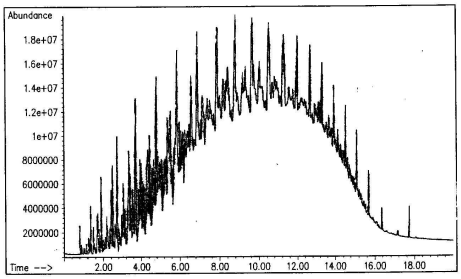


Figure 10.46. A TIC of the GC-MS scans made by Southwest Research Institute on the same diesel fuel of Figure 5 in which the HHO gas had been bubbled through, showing the alteration of the TIC both in shape as well as increased mass, thus indicating a new bond between diesel and HHO that cannot be of valence type (since HHO is gaseous and diesel is liquid. In any case, all valence electrons in both the gas and the liquid are used by conventional molecular bonds.

The chromatogram of Fig. 11.45 confirmed the typical pattern, elusion time and other feature of commercially available diesel. However, the chromatograph of the same diesel with the HHO gas bubbled in its interior of Fig. 11.46 shows large structural differences with the preceding scan, including a much stronger response, a bigger elusion time and, above all, a shift of the peaks toward bigger amu values.

Therefore, the latter measurements provide additional confirmation of the existence of an anomalous bond between the diesel and the HHO gas, precisely as predicted by the anomalous value of the flash point and the clogging up of GC feeding lines. In turn such a bond between a gas and a liquid cannot possibly be

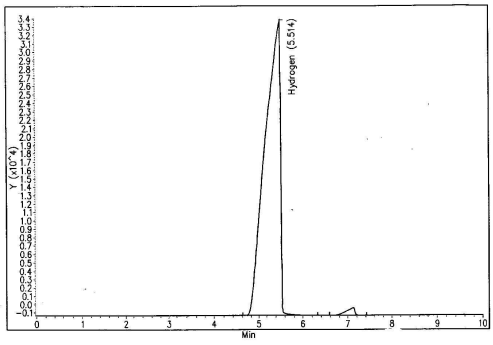


Figure 10.47. A TIC of the GC-MS scans on the HHO gas made by Toxic LTD Laboratories showing the H_2 content of the HHO gas.

of valence type, since all valence electrons are expected to be coupled in both the liquid and the gas.

Further mass spectrographic measurements on the HHO gas were done on September 10, 2003, at SunLabs, of the University of Tampa, Florida, via the use of a very recent GC-MS Clarus 500 by Perkin Elmer, one of the most sensitive instruments currently available to detect hydrogen.

Even though the column available at the time of the test was not ideally suited for the separation of all species constituting the HHO gas, the latter measurements confirmed the preceding results.

In fact, the scan of Fig. 11.50 confirms the presence in the HHO gas of a basic species with 2 amu representing hydrogen, plus a species with 5 amu that cannot admit any valence or molecular interpretation for the HHO gas even if the species is formed by the spectrometer.

In conclusion, the experimental measurements of the flash point and of the scans of Figs. 5 and 6 establish beyond doubt the capability by the HHO gas to have an anomalous bond with liquid fuels, that is, a bond that is not of valence type.

Additional analyses on the chemical composition of the HHO gas were done by Air Toxic LTD of Folsom, California, via the scans reproduced in Figs. 7, 8 and 9. These scans confirmed that H_2 and O_2 are the primary constituents of the HHO gas. However, the same measurements identify the following anomalous peaks:

a) A peak in the H_2 scan at 7.2 minutes elusion times (Fig. 11.47);

b) A large peak in the O_2 scan at 4 minutes elusion time (Fig. 11.48); and

c) An anomalous blank following the removal of the HHO gas (Fig. 11.49), because said blank shows the preservation of the peaks of the preceding scans,

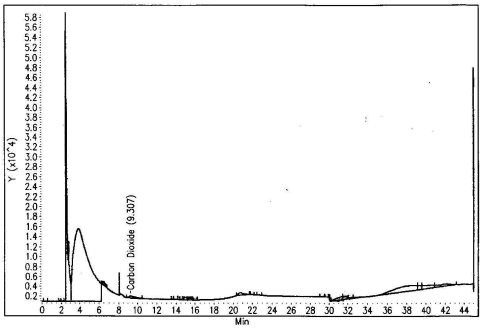


Figure 10.48. A TIC of the GC-MS scans on the HHO gas made by Toxic LTD Laboratories showing the peaks belonging to H_2 and O_2 , plus anomalous peaks.

an occurrence solely explained via anomalous adhesion of the HHO gas to the interior walls of the instrument.

The scan of Fig. 11.51 provides evidence of a species with mass 16 amu that can only be interpreted as atomic oxygen, thus providing additional indication of the presence in the HHO gas of atomic hydrogen as expected from its capabilities, although the species, again, could be separated by the spectrometer due to the expected weak nature of the bond. The latter could not be detected in the preceding scan due to the impossibility of the instrument here considered to detect a species with 1 amu. The same scan of Fig. 11.51 confirms the presence in the HHO gas of a species with 17 amu and a species with 18 amu detected in earlier tests.

The scan of Fig. 11.52 establishes the presence in the HHO gas of species with 33 and 34 amu, while the species with 35 amu detected in preceding measurements was confirmed in other scans here not reported for brevity.

The tests also confirmed the blank anomaly, namely, the fact that the blank of the instrument following the removal of the gas continues to detect the basic species constituting the gas, which blank is not reproduced here for brevity, thus confirming the anomalous adhesion of the HHO gas to the interior walls of the instrument.

In summary, the above analytic measurements establish the following properties of the HHO gas:

I) An anomalous increase in specific weight of 1 gram/mole (or 8.8% in volume) establishing the presence in the HHO gas of species heavier than the predicted mixture of H_2 and O_2, thus establishing the presence in the HHO gas of new species composed of H and O atoms that cannot possibly have valence bonds.

II) The GC scans done by Adsorption Research (Fig. 11.41) confirm the presence of chemical species in the HHO gas that cannot have a valence interpre-

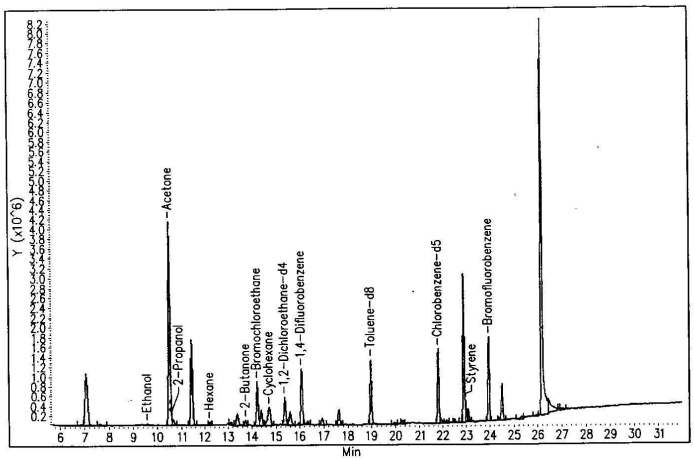


Figure 10.49. One of the anomalous blanks of the GC-MS scans made by Toxic LTD Laboratories following the tests of the HHO. The blank is firstly anomalous because only the background should have been detected, thus indicating a bond between the HHO gas and the walls of the instrument, whose most plausible explanation is the magnetic polarization by induction of said walls by a form of magnetic polarization of the species composing the HHO gas. the second reasons for the anomalous nature of the blank is that the substances detected cannot possibly exist in the HHO gas produced from distilled water, thus showing an accretion of bonds to the instrument walls.

tation, such as the species with 17 amu, 33 amu, 34 amu, and 35 amu, besides conventional species with 2 amu, 16 amu and 18 amu, all species independently confirmed by other tests, such as the scans of Figs. 10, 11 and 12

III) The halting of the GC instrument in the scans of Fig. 11.41 after a few seconds following the injection of the HHO gas, while the same instrument works normally for conventional gases, is experimental evidence for an anomalous adhesion by the HHO gas to the internal walls of the instrument, to such a level of occluding the column and causing the shut down of the scan;

IV) The large increase of the flash point of diesel fuel following inclusion of the HHO gas also constitutes experimental evidence of anomalous adhesion by the HHO gas, this time, to a liquid fuel that cannot also be of valence type since all valence electrons available in both the liquid and the gas are expected to be paired;

V) The mass spectrometric measurements on the mixture of diesel and HHO (Figs. 5 and 6) provide additional experimental confirmation of an anomalous bond between the HHO gas and diesel;

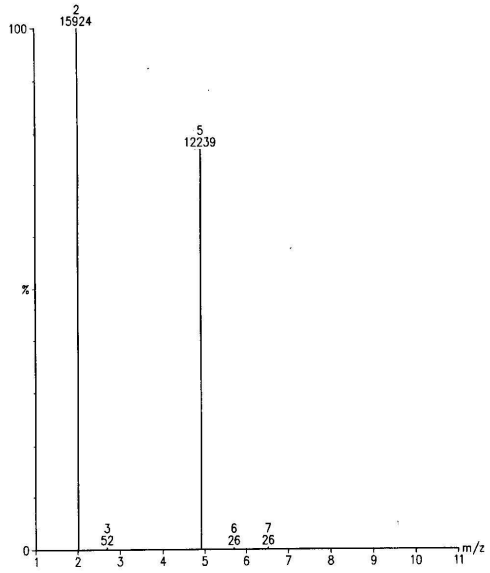


Figure 10.50. The scan conducted by SunLabs on the HHO gas confirming the presence of H_2, plus additional anomalous peaks, such as the species at 5 amu, that cannot possibly admit a molecular interpretation.

VI) The additional scans of Figs. 7, 8 and 9 confirm all the preceding results, including the anomalous blank following the removal of the HHO gas, thus confirming the anomalous adhesion of the HHO gas to the internal walls of the instrument;

VII) The capability by the HHO gas to melt instantaneously tungsten and bricks is the strongest evidence on the existence in the HHO gas of basically new chemical species that cannot possibly have a valance bond, since a mixture of $2/3$ H_2 and $1/3$ O_2 cannot melt instantly tungsten and bricks , as any interested chemist is encouraged to verify.

It should be indicated that a number of species in the HHO gas, particularly those with higher specific weight, are expected to be unstable and, as such, decomposed by the analytic instrument itself, In different terms, by no means GC, IR and other scans should be expected to detect *all* constituents of the HHO gas, since a number of them are expected to be decomposed or altered by the ionization and other processes connected to the scans themselves.

10.6.3 Magnecular Interpretation of the Measurements

The first experimental evidence supporting the magnecular structure of the HHO gas is its capability of instantly melting tungsten and bricks. In fact, such

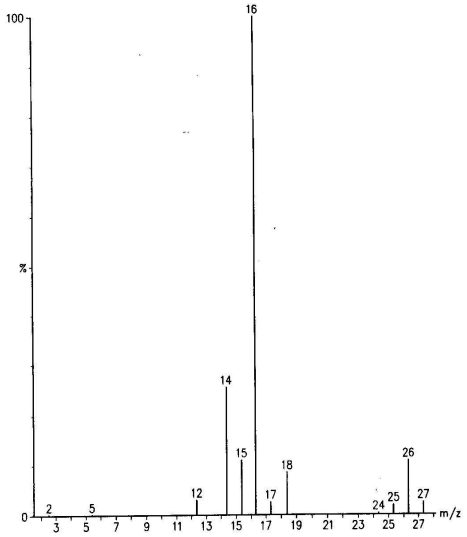


Figure 10.51. The scan conducted by SunLabs on the HHO gas detecting a peak at 16 amu that confirms the presence of atomic oxygen in the HHO gas of Fig. 11.51, plus a peak at 17 amu indicating the presence of traces of the radical $O-H$, a peak at 18 amu indicating the presence of water vapor all three species also detected in the scan of Fig. 11.51, as well as additional anomalous peaks at 12, 14, 25, 26, 27 amu that, for the case of the HHO gas produced from distilled water cannot admit a molecular interpretation.

a capability can only be explained via the presence in the HHO gas, not only of atomic (that is, unbounded) hydrogen as depicted in the top of Fig. 11.9, but also of atomic hydrogen with the toroidal polarization of their orbitals as depicted in the bottom of Fig. 11.10.

In fact, no instantaneous melting of bricks is possible without the hydrogen contained in the HHO gas rapidly penetrating within deeper layers of the brick structure. Such a rapid penetration cannot be explained with atomic hydrogen, although it can be readily explained via the polarized hydrogen atom of the bottom of Fig. 11.10.

Besides having a smaller sectional area that favors fast penetration, polarized H-atoms cause an induced polarization of the orbitals of the atoms of the brick, their consequential attraction to the polarized H atoms, and the latter rapid penetration within deep layers of the brick structure. In turn, faster penetration within the lattice of solids implies a bigger reactivity that, in turn, causes a bigger melting temperature.

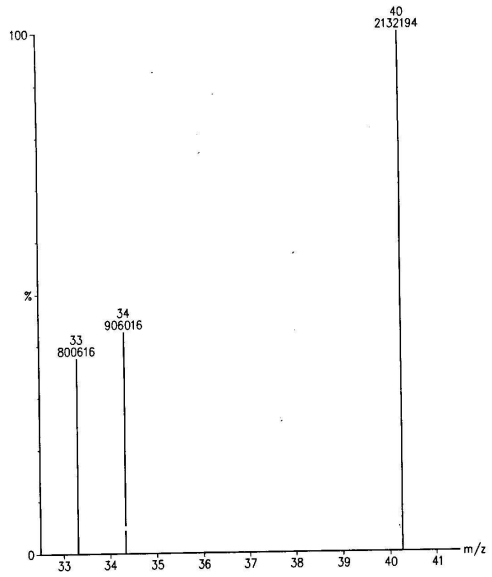


Figure 10.52. A sample of various additional scans conducted by SunLabs on the HHO gas detecting anomalous peaks at 33, 34 and 40 amu that cannot possibly have a consistent molecular interpretation. Intriguingly, the peak at 35 amu detected in other tests did not appear sequentially in this scan, and appeared instead in other scans here not reported for brevity, thus indicating that the peaks of this and of the preceding scans conducted by SunLabs are, in actuality, the constituents of the clusters composing the HHO gas, and not the actual constituents themselves.

Moreover, polarized atomic hydrogen as well as oxygen are needed to explain the anomalous adhesion of the HHO gas to internal walls of detection instruments as well as to other substances.

Note that the studies of the Brown gas [2] have indicated the need for *atomic hydrogen.* Therefore, the presence of *atomic and polarized hydrogen* is a novelty of the HHO gas.

Evidently, individual hydrogen atoms cannot maintain their polarization as in Fig. 11.9 in view of motions caused by temperature, as well known. The only known possibility for maintaining said polarization is that polarized H atoms bond themselves with opposing magnetic polarities as depicted in Fig. 11.11. In fact, rotations and vibrations due to temperature occur for such bonded H atoms as a whole, while individually preserving said polarization.

In turn, bonds of polarized atomic hydrogen constitute the very basic bond of magnecules, thus supporting the hypothesis of the magnecular structure of the HHO gas.

Note that a conventional hydrogen gas cannot acquire any magnetic polarization because the conventional hydrogen molecules is diamagnetic. However, as established in Refs. [21], the diamagnetic character refers to the hydrogen *molecule* as a whole, because quantum mechanics establishes that each individual hydrogen *atom* of a hydrogen molecule can indeed acquire a magnetic polarization under sufficiently strong external magnetic fields.

The diamagnetic character of the hydrogen molecules, as depicted in Fig. 11.10, is due to the fact that the individual magnetic polarizations of its H atoms are opposite to each other, and are at such a close mutual distances to cancel each other when inspected at sufficiently large distances.

Needless to say, the above hypothesis on the polarization of atomic hydrogen also applies to oxygen, the latter being known to be paramagnetic, resulting in atomic oxygen with the spherical distribution of orbitals, polarized atomic oxygen with the polarization of at least the valance electrons, and pairs of bonded polarized oxygen atoms as depicted in Fig. 11.11.

The first prediction of the magnecular structure of the HHO gas is that the species at 2 amu and 32 amu detected by mass spectroscopy could, in actuality, be constituted by a mixture of the conventional molecules H_2 and O_2 and a percentage of the same atoms although with the magnecular bond, since the latter are expected to have essentially the same atomic weight than the former.

The separation of hydrogen molecules and magnecules is possible via instruments based on magnetic resonance techniques because the conventional hydrogen molecule is diamagnetic (Fig. 11.9) while the hydrogen magnecule has a distinct magnetic polarity (Fig. 11.11).

It is easy to see that the magnecular hypothesis on the chemical structure of the HHO gas permits a quantitative interpretation of all anomalous species reported in the preceding section.

As now familiar, let us denote the conventional valence bond with the usual symbol "-" and the magnecular bond with the symbol "$\times$". According to this notation, $H_2 = H - H$ represents the molecule of Fig. 11.10 while $H \times H$ represents the magnecule of Fig. 11.11. Molecular bonds are notoriously restricted to valence pairing, in the sense that no additional atom can be bonded when all available valence pairs are coupled. By contrast, magnecular bonds do not have such a restriction, in the sense that atoms can indeed be added to a magnecule under the sole condition of the availability of opposite magnetic polarizations.

Needless to say, for the HHO gas at ambient temperature and pressure, the stability of the magnecular clusters is inversely proportional to the number of their constituents. As a result, magnecular clusters with relatively low atomic weight are expected to exist in significant percentages, while those with large atomic weight are expected to be present at best in parts per millions.

The magnecular hypothesis permits the following interpretations of the species composing the HHO gas: the species with 3 amu is interpreted as a combination of the magnecules $H \times H \times H$ or $(H-H) \times H$; the species with 4 amu is interpreted as a combination of $(H-H) \times (H-H$, $(H-H) \times H \times H$, or $H \times H \times H \times H$, heavier magnecular bonds solely of hydrogen atoms being unstable due to collisions; the species with 17 amu is interpreted as a combination of the traditional dimer $H - O$ and the magnecular bond $H \times O$; the species with 33 amu is interpreted as a mixture of $(O - O) \times H$, $(H - O) \times O$ and $O \times O \times H$; the species with 34 amu is interpreted as a mixture of $(H - H) \times (O - O) \times (H - H) \times H$ and similar configurations; the species with 35 amu is interpreted as a mixture of $(O-O) \times (H-H) \times (H-H) \times H$ and equivalent configurations (see Fig. 11.11); and other magnecular species in progressively smaller percentages.

10.6.4 the New Gaseous and Combustible Form of Water

Besides a quantitative interpretation of the chemical structure of all species contained in the HHO gas, as well as of its anomalous thermal content and adhesion, perhaps the biggest contribution of the magnecular hypothesis is a quantitative interpretation of the formation of the HHO gas despite the lack of evaporation or separation energy.

Recall that nature has set the water molecule $H_2O = H - O - H$ in such a way that its H atoms do not have the spherical distribution, and have instead precisely the polarized distribution of Fig. 11.10 along a toroid whose symmetry plane is perpendicular to that of the $H - O - H$ plane, as depicted in Fig. 153, and established in the technical literature (see, e.g., Ref. [1=20a]).

It is also known that the H-O-H molecule at ambient temperature and pressure, even though with a null total charge, has a high electric polarization (namely, a deformation of electric charge distributions) with the predominance of the negative charge density localized in the O atom and the complementary predominant positive charge density localized in the H atoms [1a]. This feature causes a repulsion of the H atoms due to their predominantly positive charges, resulting in the characteristic angle of (about) 105^o between the $H - O$ and $O - H$ dimers as depicted in Fig. 11.54.

It is well established in quantum mechanics that toroidal polarizations of the orbitals of the hydrogen atom as in the configuration of Fig. 11.11 create very strong magnetic fields with a symmetry axis perpendicular to the plane of the toroid, and with a value of said magnetic field sufficient for the creation of the new chemical species of magnecules [3].

It then follows that, in the natural configuration of the $H - O - H$ molecule, the strong electric polarization caused by the oxygen is such to weaken the mag-

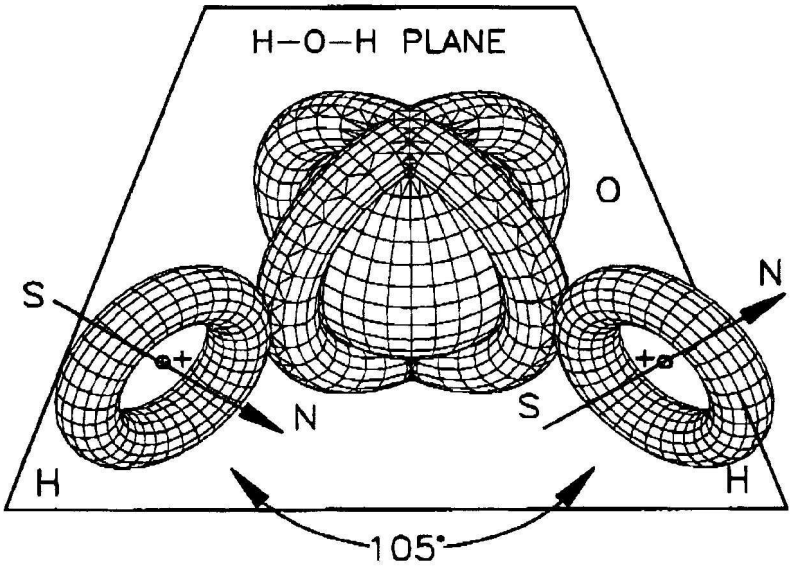


Figure 10.53. A conceptual rendering of the conventional water molecule without any electric polarization. This rendering is primarily intended to illustrate the experimentally established feature that the orbitals of the two hydrogen atoms do not have a spherical distribution as in Fig. 11.413, but have instead a distribution essentially perpendicular to the $H-O-H$ plane (see Refs. [20] for details) here conceptually represented with a toroid. The strong valence bond needed to achieve the first known exact representation of the experimental data of the water molecule achieved in Ref. [21] requires that the corresponding orbitals of the valence electrons of the oxygen have a corresponding polarized distribution here also conceptually depicted with toroids perpendicular to the $H-O-H$ plane around the spherical core of the remaining electrons of the oxygen atom.

netic field of the toroidal polarization of the H-orbital resulting in the indicated repulsion of the two H-atoms in the $H-O-H$ structure.

However, as soon as the strong electric polarization of the molecule $H-O-H$ is removed, the strong attraction between opposite polarities of the magnetic fields of the polarized H atoms become dominant over the Coulomb repulsion of the charges, resulting in a new configuration of the water molecule depicted in Figs. 19 and 20 apparently presented in this section for the first time.

Therefore, a central hypothesis of this section is that the electrolyzer developed by Hydrogen Technology Applications, Inc., is such to permit the transformation

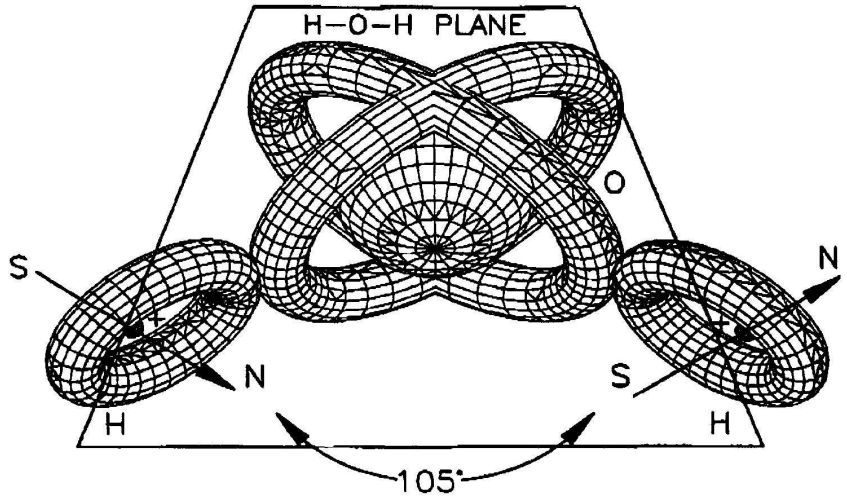


Figure 10.54. A conceptual rendering of the conventional water molecule of Fig. 16, this time with the electric polarization as occurring in nature. Note the consequential the predominance of a positive charge in the two hydrogen atoms that is responsible in part for the angle of 105^o between the two $H-O$ radicals.

of the water molecule from the conventional $H-O-H$ configuration of Fig. 11.54 to the basically novel configuration of Fig. 11.55.

By using the above identified symbols for molecules and magnecules, the conventional water molecule is represented by $H-O-H$ while the new configuration of Fig. 11.55 is represented by $(H \times H)-O$, where the symbol "-" evidently denotes double valence bond.

The plausibility of the new form of water is supported by the fact that, when $H-O-H$ is liquid, the new species $(H \times H)-O$ is expected to be gaseous. This is due to various reasons, such as the fact that the hydrogen is much lighter than the oxygen in the ratio 1 to 16 amu. As a result, the new species $(H \times H)-O$ is essentially equivalent to ordinary gaseous oxygen in conformity with conventional thermodynamical laws, since the transition from the liquid to the gas state implies the increase of the entropy, as well known.

Alternatively, the loss of electric polarization in the transition from $H-O-H$ to $(H \times H)-O$ is expected to cause the loss of the processes permitting the very existence of the water molecule, such as the hydrogen bridges between dimers $O-H$ of different molecules. Transition to a gaseous form is then consequential,

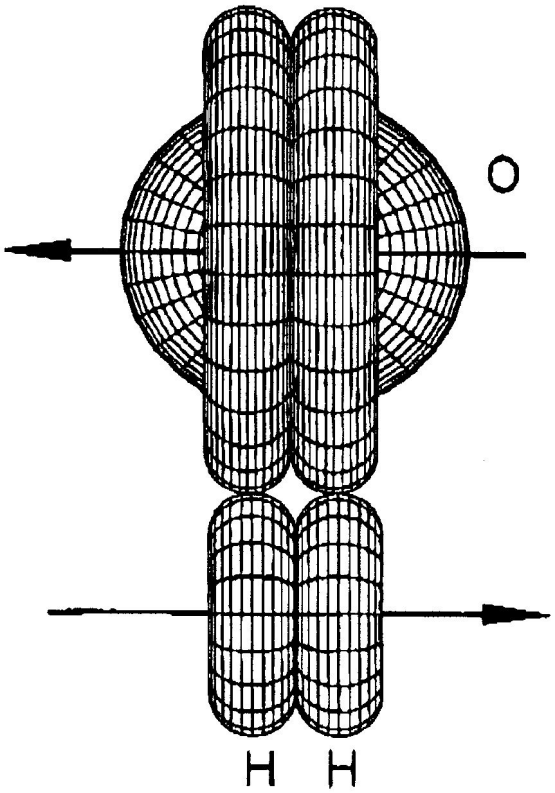


Figure 10.55. A conceptual rendering of the central hypothesis submitted for the first time in this section, namely, the $H - O - H$ molecule in which all electric polarizations have been removed, with the consequential collapse of the two polarized H-atoms one into the other due to their neutral charge and strongly attractive opposing magnetic polarities. This hypothesis permits a quantitative interpretation of the transition of state from liquid to gas achieved by the HHO electrolyzers via processes structurally different than evaporation energy. In fact, unlike the configuration of Fig. 11.11, that of this figure can only exist at the gaseous state due to the loss of the processes permitting the liquid state, such as hydrogen bridges between pairs of water molecules. It should be noted that the configuration here depicted is unstable and decomposes into atomic oxygen, as detected in the HHO gas, plus the new magnecular species $H \times H$ that has indeed been detected but it is generally interpreted as $H - H$.

thus confirm the plausibility of the new form of water $(H \times H) - O$ proposed in this section.

However, it can also be seen that the new form of water $(H \times H) = O$ is *unstable,* and decomposes in $H \times H$ and O. This decomposition is supported by the clear evidence in the HHO gas of atomic oxygen, as well as of the species

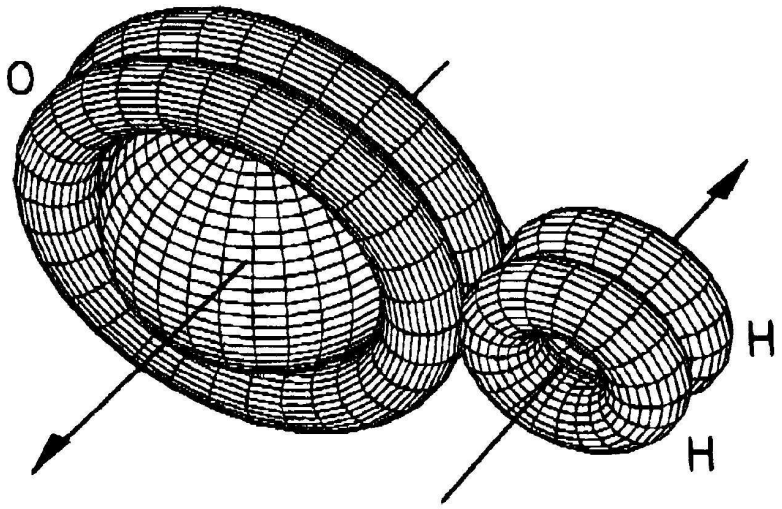


Figure 10.56. A conceptual rendering of a perspective view of the central hypothesis submitted for the first time in this section via Fig. 11.55, better illustrating the bond via opposing magnetic polarities of the two H-atoms, as well as the unstable character of the configuration due to collision with other species and intrinsic instabilities to be studied in a forthcoming section.

with 2 amu that is normally interpreted as being $H - H$, while we suggest the additional possibility that such a species is, at least in part, $H \times H$.

10.6.5 Contributions of Hadronic Chemistry Toward the Future Understanding of the Complexities of Water

There is no doubt that, being the foundation of life, water is by far the most complex chemical structure in nature. Any chemist who believes to have achieved a final understanding of water via quantum chemistry should be removed from the scientific community because of either mental or ethical problems, and the same holds for chemists using hadronic chemistry.

It is merely hoped that the efforts presented in this section have achieved another step in the study of water beyond those permitted by quantum chemistry, with the understanding that a serious understanding of water may well require efforts throughout this third millennium.

Recall that quantum chemistry was unable to achieve an exact and invariant representation of the main characteristics of the water molecule from unadulterated first principles despite efforts over the past century. In fact, a historical 2% has been missing in the representation of the water binding energy, while the

representation of its electric and magnetic moments was embarrassingly wrong even in the signs.

An improvement of the numerical representation was achieved via the so-called "screening of the Coulomb law", that is, the multiplication of the Coulomb potential by an arbitrary function of unknown physical or chemical origin, $\frac{q_1 \times q_2}{r} \rightarrow f(r) \times \frac{q_1 \times q_2}{r}$. However, as indicated since Chapter 1, this type of screening implies the abandonment of the notion of "quantum" of energy, trivially, due to the loss of all quantized orbits, as well as the exiting from the basic axioms of quantum mechanics, because the transition from the Coulomb potential to its screened form requires nonunitary transforms.

Independently from these basic shortcomings, the fundamental problem of quantum chemistry, whether with or without screening processes, remains the fact that the name "valence" is a pure nomenclature, since it does not identify in explicit and numerical terms the *attractive force* needed for two hydrogen atoms to be bounded to the oxygen atom in the structure $H - O - H$, and electrons repel each other in any case for quantum mechanics and chemistry.

Besides fundamental insufficiencies in a numerically exact and invariant representation of the main characteristics of the water molecules, additional vast insufficiencies exist for the liquid and solid state of water. As an example, the use of the "H-bridges" to represent the liquid state of water is another case of basically ascientific nomenclature because, again, of the lack of any identification of the *attractive force* needed to explain the bond of neutral and diamagnetic water molecules in their liquid state.

When water becomes part of biological organisms, the open problems became so great to be beyond our imagination at this writing (also because most chemists believe that the water molecule remains the same).

As shown in Chapter 9, the *isotopic branch of hadronic chemistry,* or *isochemistry* for short, was first and most fundamentally focused in the identification of the *attractive force* in the singlet coupling of two valence electrons, which identification required a necessary nonunitary theory since the valence force resulted to have a contact, thus non-Hamiltonian character.

Thanks to this basic advance, the isochemistry permitted, for the first time in scientific history, the numerically exact and invariant representation not only of the binding energy but also of the electric and magnetic moments of the water molecules (Section 9.3).

Subsequently, in Section 11.5 we indicated that the liquid state of water appears to be of magnecular character since the H-atoms in the H_2O structure have by nature a toroidal polarization in a plane perpendicular to the $H - O - H$ plane, thus permitting the magnecular bond between two H atoms of different water molecules $H_\uparrow \times H \downarrow$ that is referred to as "H-bridges" (see Figure 11.34.

In this section, we have shown that water admits a previously unknown gaseous and combustible state achievable from the liquid state *without* the evaporation energy believed to be necessary by quantum chemistry. In turn, such a feature indicates our basic lack of understanding of the conventional water evaporation itself, trivially, because of the lack of conventional identification of the force responsible for the liquid state.

To understand the limited character of the advances permitted by isochemistry, it is important to recall that they have been achieved via a lifting of quantum chemistry that is strictly reversible in time as the original theory.

Consequently, *isochemistry is strictly inapplicable (rather than violated) for any irreversible process involving the water molecule,* such as the very creation of the molecule itself

$$2 \times H + \frac{O_2}{2} \quad \rightarrow \quad H_2O + 57Kcal/mole. \tag{11.6.1}$$

Any insistence in the use of a reversible theory, whether quantum chemistry of its isotopic covering, for the above irreversible process may imply severe scientific drawbacks. As one example among many, it is generally believed in chemistry that the above process is unique and immutable.

On the contrary, the use of the *genotopic branch of hadronic chemistry,* or *genochemistry* for short, establishes that the rate of the above process depends on the distribution of the orbitals.

Recall that the H atoms in the H_2 molecule have a spherical distribution. Consequently, to achieve reaction (11.6.1), nature has to first break down the H_2 molecule and then polarize the orbitals of the individual H atoms from their spherical to the above indicated toroidal polarization.

Genochemistry then predicts that basically new advances over reaction (1.6.xx) can be achieved with the combustion of H and O, firstly, if we start from *atomic* H and/or O atoms (because in this case there is no need to separate atoms prior to their new bond) and, secondly, via the use of *atomic and polarized* H and O atoms, that is, by preparing them in the form as appearing in the $H-O-H$ molecule. Similar basic advances can be obtained in various other chemical reactions.

Despite these possibilities, genochemistry remains basically insufficient for further advances in the study of the water molecule because the theory is indeed irreversible but single-valued. It is an easy prediction that further advances in the study of water, particularly when a member of a biological structure, will require the *hyper-structural branch of hadronic chemistry,* or *hyperchemistry* for short, due to its multi-valued character. In turn, the latter broadening will inevitably require the notion of *hypermagnecule* (Definition 11.2.1).

At that point the complexities of water and its role as the basis of life appear in their full light, e.g., because of the joint need of all four directions of time (Section 2.1), each time being multivalued (Chapter 5).

In summary, water is perhaps the best illustration of the fact that the human adventure in science will never end.

10.7 EXPERIMENTAL EVIDENCE OF MAGNECULES IN LIQUIDS AND SOLIDS

10.7.1 Preparation of Liquid Magnecules used in the Tests

In early 1998 Santilli [1] obtained a number of samples of *fragrance oils* from *Givaudan-Roure Corporation* (GR) with headquarters in Teaneck, New Jersey. About 50 cc of various samples of perfectly transparent fragrance oils were placed in individual glass containers. One polarity of an alnico permanent magnet with 12,000 G and dimension $1/2'' \times 1'' \times 2''$ was immersed within said oils.

Starting with a perfect transparency, after a few days a darkening of the oils became visible, jointly with a visible increase of the viscosity, with changes evidently varying from oil to oil. Subsequently, there was the appearance of granules of dark complexes in the interior of the oil which were visible to the naked eye. Both the darkening and the viscosity increased progressively in subsequent days, to reach in certain cases a dark brown color completely opaque to light. The viscosity increased to such an extent that the oil lost all its fluidity.

It should be stressed that the above visible effects are of pure magnetic origin because of the lack of any other contribution, *e.g.*, the complete absence of any additives. After the immersion of the permanent magnets, all samples were left undisturbed at ordinary room conditions. The indicated effects remain unchanged to this day, thus showing that the changes were stable at ordinary conditions of temperature and pressure.

Santilli's [1] main hypothesis on the darkening of the oils is that their molecules acquire a magnetic polarization in the orbits of at least some of their atomic electrons (called in chemistry *cyclotron resonance orbits*), by therefore bonding to each other according to Definition 11.2.1 in a way similar to the corresponding occurrence for gases.

It should also be indicate that the immersion of one polarity of a permanent magnet in fragrance oils is, evidently, a rudimentary way to create magnecules in detectable percentage although not an essentially pure population of magnecules as requested for a new chemical species (see Sect. 11.2). A number of more sophisticated magnetic polarization techniques are now available with rather complex geometries. Also, as indicated in Sect. 11.6, an essentially pure population of liquid magnecules can be reached via the PlasmaArcFlow reactors described in Section 11.4.

10.7.2 Photographic Evidence of Magnecules in Liquids

The above alteration of the structure of fragrance oils was confirmed by photographs taken by the GR Research Laboratory in Dubendorf, Switzerland, via a microscope with minimal magnification, as illustrated in the pictures of Figs. 8.19 and 8.20.

The pictures of Fig. 11.57 refer to the GR fragrance oil received under the code "ING258AIN, Text 2" subjected to the rudimentary magnetic polarization indicated in the preceding section under the respective magnification 10X and 100X.

As one can see, these photographs establish that, under the indicated magnetic treatment, the oil has acquired a structure of the type of "brick layering" which is visible under only 10X magnification, and is per se highly anomalous for a liquid that was originally fully transparent. Note that the magnecules are not constituted by the individual "bricks," but rather by the dark substance which interlock said "bricks." This point is important to understand the size of the magnecule here considered which covers the entire 50 cc of the liquid.

The photographs in Figs. 8.20 were taken at the University of South Florida in St. Petersburg via a microscope with the same magnifications 10X and 100X, but refer to a different GR fragrance oil received under the code "Mixture 2" and magnetically treated to such a point of completely losing transparency and fluidity. As one can see, the latter picture provides confirmation that, following exposure to a 12,000 G magnetic field, fragrance oil molecules bond together into rather large clusters estimated to be well in excess of 10,000 a.m.u., that is, with an atomic weight which is dramatically bigger than that of the largest molecule composing the oil, as per Feature I) of Definition 11.2.1.

Inspection of the various photographs shows a variety of sizes of magnecules, thus establishing their lack of unique characteristics for any given liquid. This evidently confirms the *lack* of a valence bond. Inspection of the samples also show the magnecules capability of increasing their size via the accretion of further oil molecules.

Other photographic documentations of various magnecules in liquids were done, by confirming the findings of Figs. 8.19 and 8.20.

10.7.3 Spectroscopic Evidence of Liquid Magnecules at the Tekmar-Dohrmann Corporation

The first experimental evidence of magnecules in liquids was established on May 5, 1998, by analysts *Brian Wallace* and *Mia Burnett* at *Tekmar-Dohrmann Corporation* (TDC) in Cincinnati, Ohio, operating a *Tekmar 7000 HT Static Headspacer Autosampler* equipped with a Flame Ionization Detector (FID). The tests were repeated on May 8 and 11, 1998, by confirming the preceding results. It should be noted that the Tekmar equipment lacks the computer search as

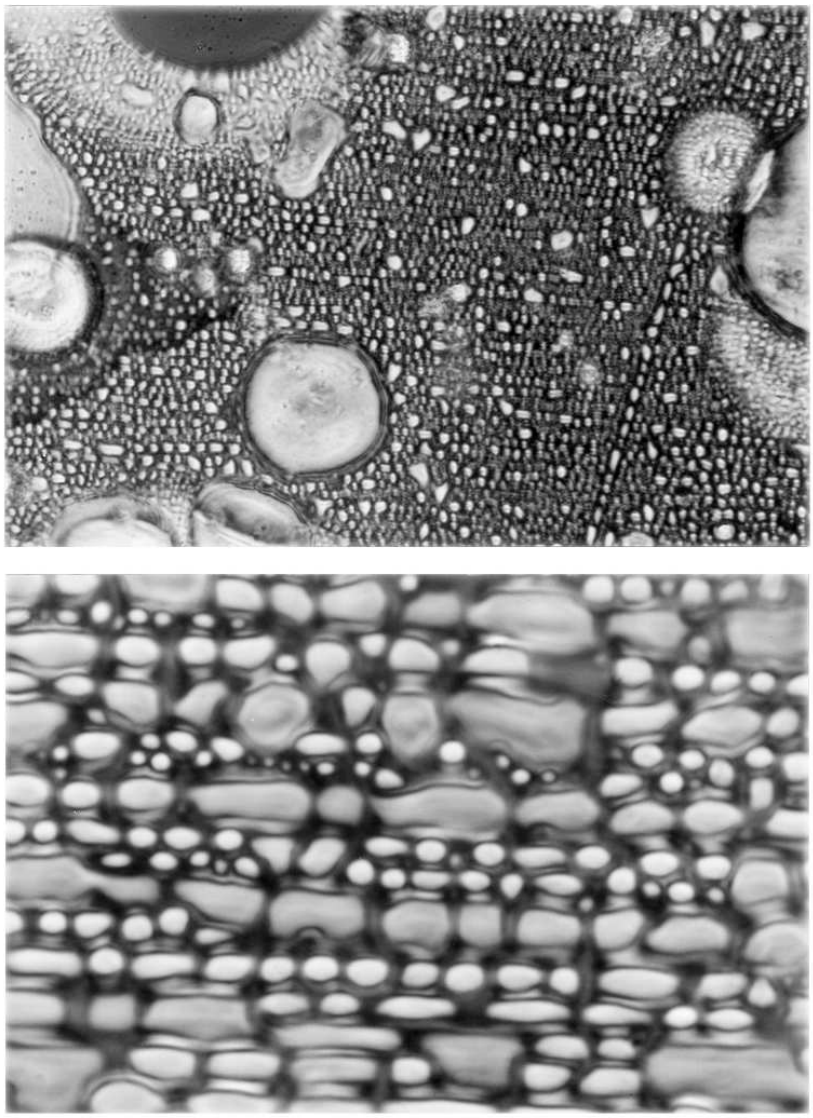


Figure 10.57. A photographic evidence of magnecules in liquids obtained at the *Givaudan-Roure Research Laboratory* in Dubendorf, Switzerland, in the GR fragrance oil "ING258IN Test 2" under magnifications 10X and 100X [1].

well as the UV scan. Also, the instrument had limited capability in atomic weight. Finally, the FID was permitted in this case because the liquids were not combustible.

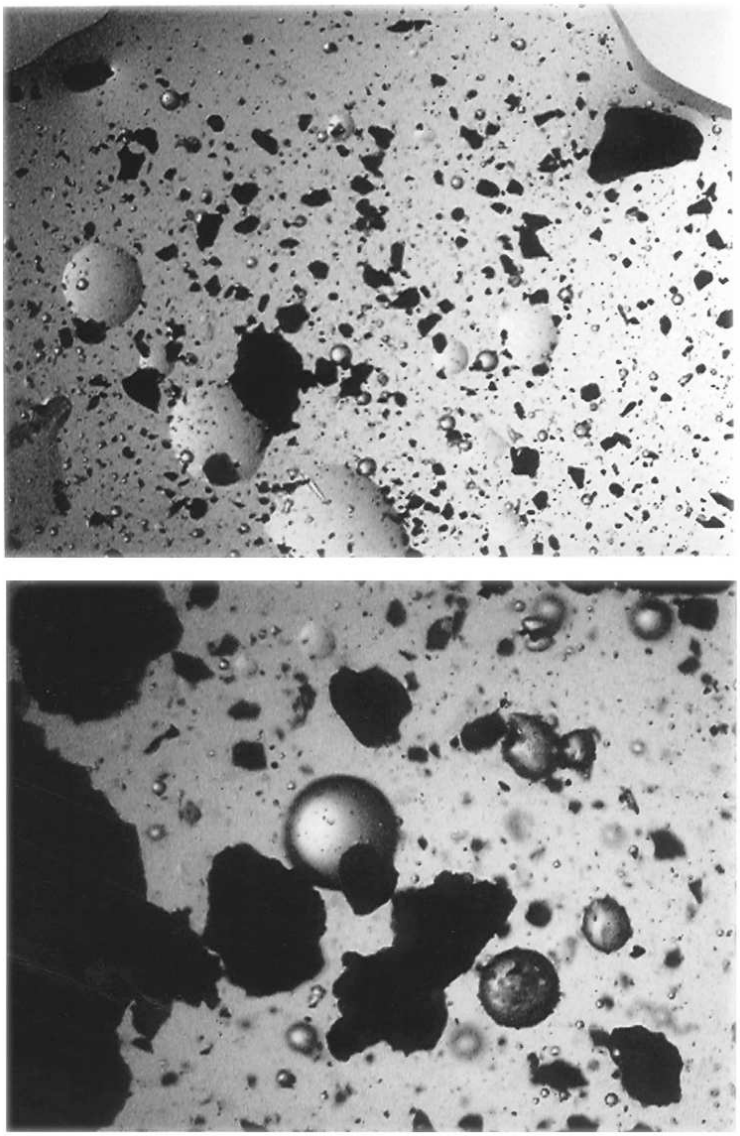
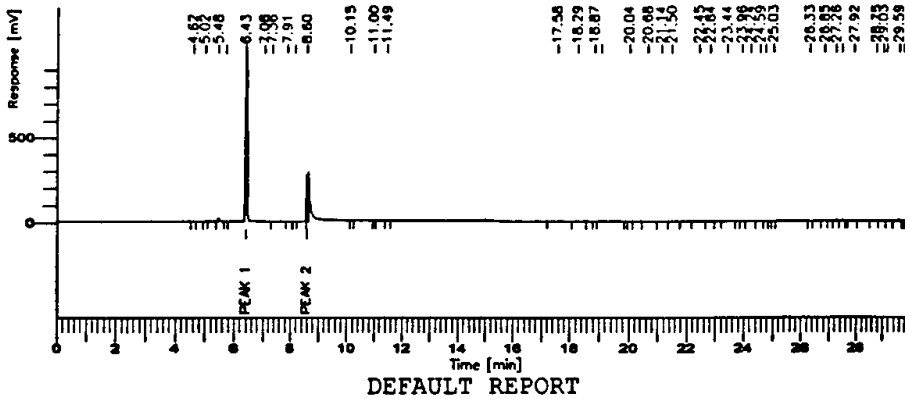


Figure 10.58. Confirmation of magnecules in GR fragrance oil "Mixture 2" under 10X and 100X obtained at the University of South Florida in St. Petersburg. Note the difference in sizes of the magnecules and their difference with those of Fig. 11.57 [1].

The measurements were done on: Sample 1, pure (magnetically untreated) GR "Fragrance Oil 2"; Sample 2, magnetically untreated tap water; and Sample 3, a magnetically treated mixture of the two.

Software Version: 4.0<4J28>
Date: 5/5/98 08:18 AM
Sample Name : 500ul Perfume Oil ID#1
Data File : C:\TC4\HP210\MY04003.RAW Date: 5/4/98 04:44 PM
Sequence File: C:\TC4OLD\HP210\MY04.SEQ Cycle: 3 Channel : B
Instrument : 772_-_2 Rack/Vial: 0/0 Operator: mb
Sample Amount : 1.0000 Dilution Factor : 1.00



DEFAULT REPORT

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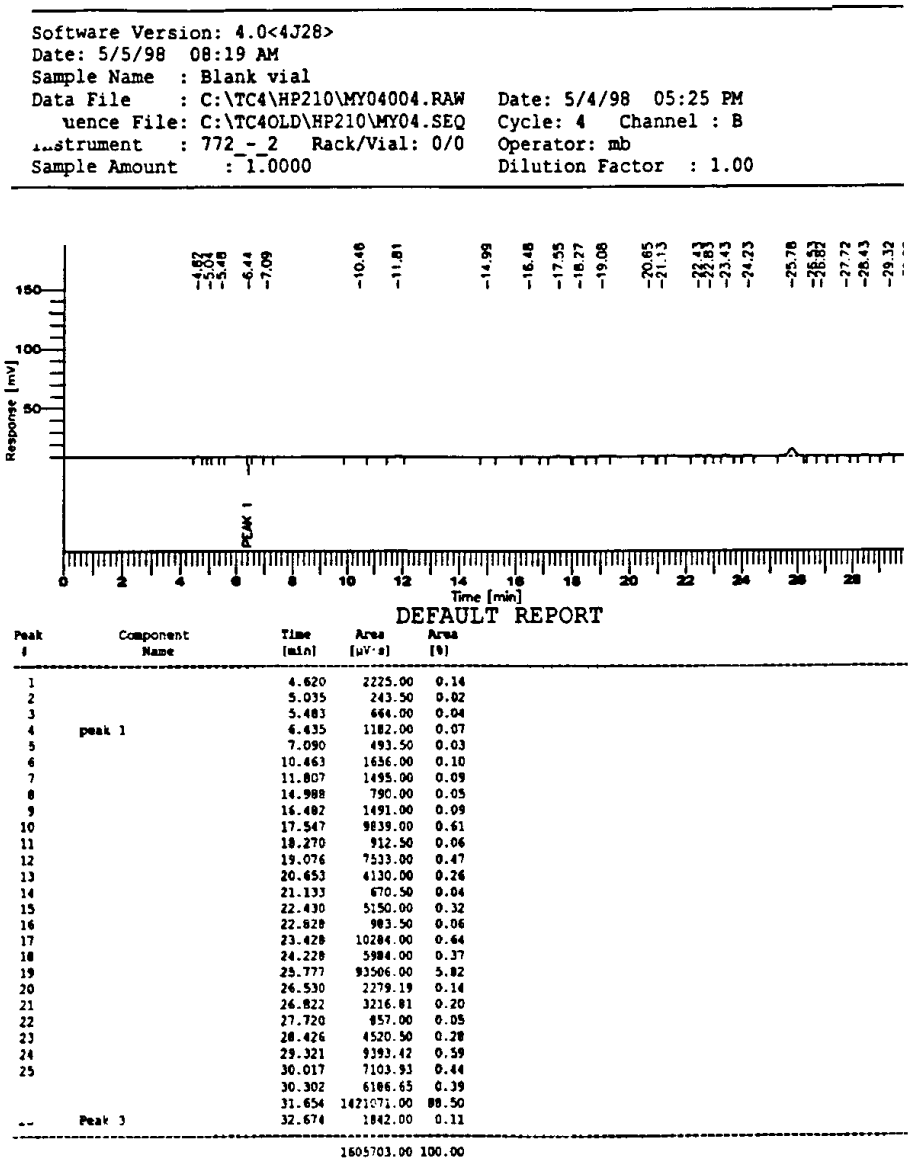
2		5.022	432.00	0.00	25		23.958	1292.73	0.01
3		5.479	68060.38	0.43			24.241	5968.77	0.04
4		5.731	1120.12	0.01			24.587	980.85	0.01
5	peak 1	6.430	3894142.72	24.38			24.764	700.15	0.00
6		7.077	23106.00	0.14	29		25.034	221.00	0.00
7		7.355	8426.66	0.05	30		26.330	189.00	0.00
8		7.911	1549.62	0.01	31		26.849	7912.00	0.05
9		8.163	190.00	0.00	32		27.264	19913.14	0.12
10	peak 2	8.604	509716.00	3.19	33		27.461	10844.36	0.07
11		10.146	829.00	0.01	34		27.918	1332.00	0.01
12		10.999	338.00	0.00	35		28.753	751.69	0.00
13		11.485	798.00	0.00	36		29.026	3094.14	0.02
14		17.582	9646.00	0.06			29.163	2312.68	0.01
15		18.294	814.50	0.01			29.589	30846.00	0.19
16		18.871	748.51	0.00	39		29.750	383.00	0.00
17		19.082	12390.99	0.08	40		30.320	6254.00	0.04
18		20.043	582.50	0.00	41		31.370	3617.37	0.02
19		20.679	4000.46	0.03	42		31.720	51605.63	0.32
20		21.139	654.15	0.00	43		32.296	268.00	0.00
21		21.500	716.38	0.00	44		32.519	87913.29	0.55
22		22.452	4196.12	0.03	45	Peak 3	32.742	11181133.21	70.00
23		22.837	1128.08	0.01					
24		23.437	10546.00	0.07				15973772.00	100.00

Figure 10.59. A first scan done on May 5, 1998, 8.18 a.m. at *Tekmar Dohrmann Company* (TDC) in Cincinnati, Ohio, via a Tekmar 7000 HT Static Headspacer Autosampler with a Flame Ionization Detector (FID).

Despite these limitations, *the results of the Tekmar tests provided the first direct spectroscopic evidence of the existence of magnecules in liquids, including the first direct experimental evidence of water magneplexes* as per Definition 11.2.1. In particular, these tests established that magnecules in liquids have the same main features of the magnecules in gases.



DEFAULT REPORT

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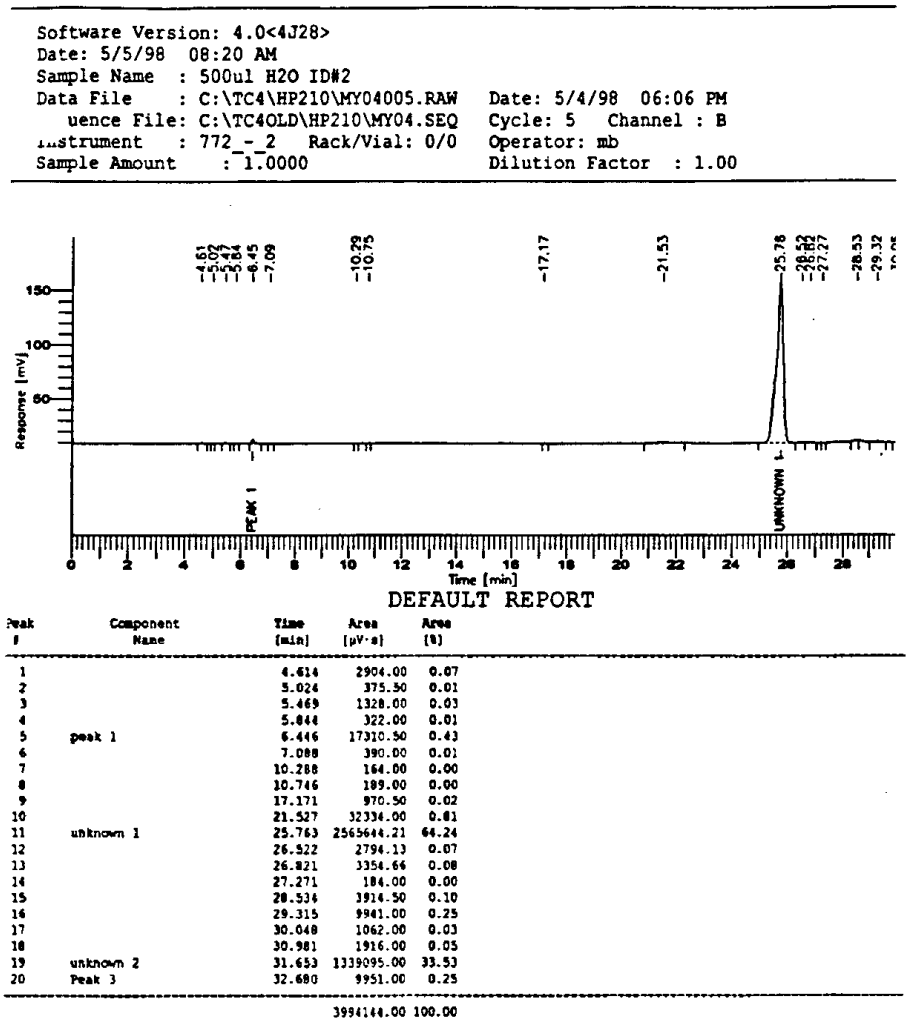
2		5.035	243.50	0.02
3		5.483	664.00	0.04
4	peak 1	6.435	1182.00	0.07
5		7.090	493.50	0.03
6		10.463	1636.00	0.10
7		11.807	1495.00	0.09
8		14.988	790.00	0.05
9		16.482	1491.00	0.09
10		17.547	9839.00	0.61
11		18.270	912.50	0.06
12		19.076	7533.00	0.47
13		20.653	4130.00	0.26
14		21.133	670.50	0.04
15		22.430	5150.00	0.32
16		22.828	903.50	0.06
17		23.428	10284.00	0.64
18		24.228	5984.00	0.37
19		25.777	93506.00	5.82
20		26.530	2279.19	0.14
21		26.822	3216.81	0.20
22		27.720	857.00	0.05
23		28.426	4520.50	0.28
24		29.321	9393.42	0.59
25		30.017	7103.93	0.44
		30.302	6186.65	0.39
		31.654	1421071.00	88.50
--	Peak 3	32.674	1842.00	0.11
			1605703.00	100.00

Figure 10.60. The scan at TDC on 5/5/98 at 8.19 a.m. to check that the background is correct.

To avoid a prohibitive length we reproduce only a few representative scans in Figs. 8.21 to 8.25 [1]. Figure 8.21 reproduces the origin test of the fragrance oil without magnetic treatment. Note the dominance of three molecules denoted "Peak 1" with 24.28%, "Peak 2" with 3.19% and "Peak 3" with 70.00%. Fig-

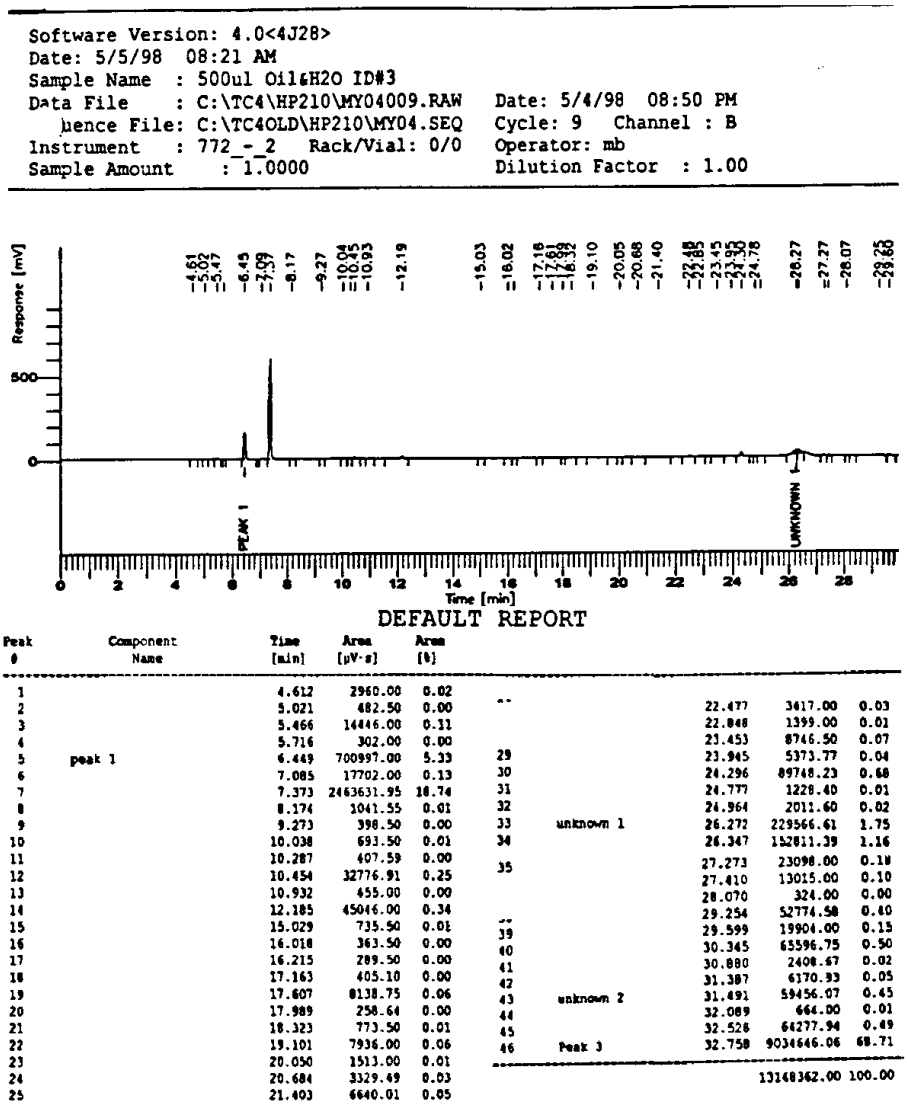


DEFAULT REPORT

Peak #	Component Name	Time [min]	Area [µV·s]	Area [%]
1		4.614	2904.00	0.07
2		5.024	375.50	0.01
3		5.469	1328.00	0.03
4		5.844	322.00	0.01
5	peak 1	6.446	17310.50	0.43
6		7.088	390.00	0.01
7		10.288	164.00	0.00
8		10.746	189.00	0.00
9		17.171	970.50	0.02
10		21.527	32334.00	0.81
11	unknown 1	25.763	2565644.21	64.24
12		26.522	2794.13	0.07
13		26.821	3354.66	0.08
14		27.271	184.00	0.00
15		28.534	3914.50	0.10
16		29.315	9941.00	0.25
17		30.048	1062.00	0.03
18		30.981	1916.00	0.05
19	unknown 2	31.653	1339095.00	33.53
20	Peak 3	32.680	9951.00	0.25
			3994144.00	100.00

Figure 10.61. The scan at TDC on 5/5/98 at 8.19 a.m. on the magnetically treated water which constitutes experimental evidence of magnecules in water given by the large unknown peak.

ure 8.22 depicts the background which is shown to be correct. Figure 8.23 represent the scan of magnetically treated water with a large "unknown 1" with 64.24% and "unknown 2" with 33.53% totaling 97.78%. This is evidence of the creation of magnecules in water, also called magneplexes according to Definition 11.2.1. Figure 8.24 represents a scan of the magnetically treated combination of water and fragrance oil with "unknown 1" 1.75% and "unknown 2" with 0.45%. An important information of this scan is that the original Peak 1 of Fig. 11.59



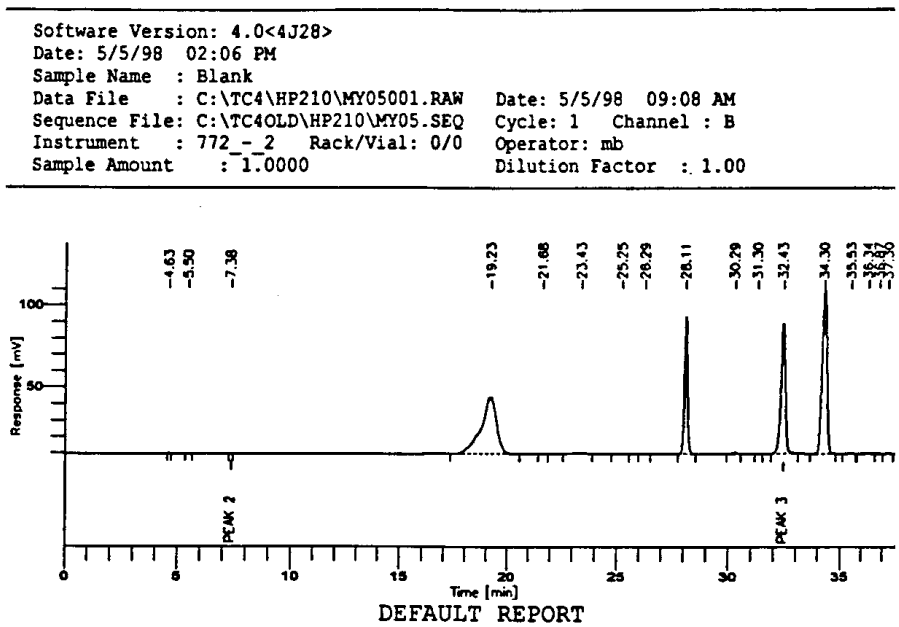
Software Version: 4.0<4J28>
Date: 5/5/98 08:21 AM
Sample Name : 500ul Oil&H2O ID#3
Data File : C:\TC4\HP210\MY04009.RAW Date: 5/4/98 08:50 PM
Sequence File: C:\TC4OLD\HP210\MY04.SEQ Cycle: 9 Channel : B
Instrument : 772_-_2 Rack/Vial: 0/0 Operator: mb
Sample Amount : 1.0000 Dilution Factor : 1.00

DEFAULT REPORT

Peak #	Component Name	Time [min]	Area [µV·s]	Area [%]
1		4.612	2960.00	0.02
2		5.021	482.50	0.00
3		5.466	14446.00	0.11
4		5.716	302.00	0.00
5	peak 1	6.449	700997.00	5.33
6		7.085	17702.00	0.13
7		7.373	2463631.95	18.74
8		8.174	1041.55	0.01
9		9.273	398.50	0.00
10		10.038	693.50	0.01
11		10.287	407.59	0.00
12		10.454	32776.91	0.25
13		10.932	455.00	0.00
14		12.185	45046.00	0.34
15		15.029	735.50	0.01
16		16.018	363.50	0.00
17		16.215	289.50	0.00
18		17.163	405.10	0.00
19		17.607	8138.75	0.06
20		17.989	258.64	0.00
21		18.323	773.50	0.01
22		19.101	7936.00	0.06
23		20.050	1513.00	0.01
24		20.684	3329.69	0.03
25		21.403	6640.01	0.05
--		22.477	3617.00	0.03
		22.848	1399.00	0.01
		23.453	8746.50	0.07
29		23.945	5373.77	0.04
30		24.296	89748.23	0.68
31		24.777	1228.40	0.01
32		24.964	2011.60	0.02
33	unknown 1	26.272	229566.61	1.75
34		26.347	152811.39	1.16
35		27.273	23098.00	0.18
		27.410	13015.00	0.10
		28.070	324.00	0.00
--		29.254	52774.58	0.40
39		29.599	19904.00	0.15
40		30.345	65596.75	0.50
41		30.880	2408.67	0.02
42		31.387	6170.93	0.05
43	unknown 2	31.491	59456.07	0.45
44		32.089	664.00	0.01
45		32.526	64277.94	0.49
46	Peak 3	32.758	9034646.06	68.71
			13148362.00	100.00

Figure 10.62. The scan on 5/5/98 at 8.21 a.m. on the magnetically treated mixture of water and fragrance oil of scan 8.21 which constitutes evidence of magnecules given by two unknown peaks.

with 24.28% and Peak 3 with 70.00% have been decreased to the values 5.33% and 68.71%, respectively. This is evidence that the missing percentages of these molecules have been used in the formation of magnecules. Figure 8.25 reproduces the background following the tests and routine flushing. As one can see, the scan



DEFAULT REPORT

Peak #	Component Name	Time [min]	Area [μV·s]	Area [%]
1		4.633	2621.00	0.05
2		5.496	695.50	0.01
3	peak 2	7.379	293.00	0.01
4		19.227	1795761.00	33.57
5		21.679	1071.00	0.02
6		23.432	18393.00	0.34
7		25.247	6824.50	0.13
8		26.290	2393.00	0.04
9		28.110	835986.00	15.63
10		30.291	12877.00	0.24
11		31.296	1435.00	0.03
12	Peak 3	32.425	1135236.50	21.22
13		34.296	1508602.50	28.20
14		35.532	6202.50	0.12
15		36.344	9033.76	0.17
16		36.866	1934.19	0.04
17		37.297	7005.45	0.13
18		37.914	2731.10	0.05
			5349096.00	100.00

Figure 10.63. The scan at TDC on 5/5/98 at 2.26 p.m. on the background with anomalous adhesion confirming the corresponding anomalous background for gas magnecules.

preserves macroscopic percentages of the preceding scans, thus confirming the anomalous adhesion also existing in gas magnecules.

10.7.4 Spectroscopic Evidence of Liquid Magnecules at Florida International University

Additional comprehensive tests via a modern equipment for LC-MS equipped with UVD were conducted on the GR fragrance oil "ING258IN Test 2" of Figs. 8.19 on December 1, 1998, at the chemistry laboratory of *Florida International Uni-*

versity (FIU) in Miami, Florida. The tests were then repeated on December 17 and 18 by confirming the preceding results.

The tests were conducted under a number of technical characterizations specifically selected to detect magnecules, among which include:

1) Total Ion Chromatogram (TIC) collected under the positive ion atmospheric pressure electrospray ionization (ESI+) mode;

2) Integrated TIC with retention times and areas for the most abundant peaks;

3) Raw mass spectra for all peaks identified in item 2;

4) HP LC chromatograms collected at fixed wavelength of 254 cm; and

5) UV-visible spectra form the HPLC diode array detector with 230–700 mm.

The tests were conducted on the following samples:

I) Sample GR331, the magnetically untreated, fully transparent GR fragrance oil "ING258IN Test 2";

II) Sample GR332, magnetically treated "ING258IN Test 2" with 10% Dipropylene Glycol (DPG);

III) Sample GR332S, bottom layer of the preceding sample;

IV) Sample GR335, magnetically treated mixture 4% GR fragrance oil "ING258IN Test 2", 0.4% DPG and 95% tap water; and

V) Sample GR335O, visible dark clusters in the preceding sample.

To avoid a prohibitive length of this presentation, only representative scans are reproduced in Figs. 8.26 to 8.30 [1]. As one can see, these scans provide a second experimental evidence of magnecules in liquids as evident in comparing the peaks of the untreated liquid with those of the treated one.

A few comments are in order. To understand the FIU measurements the reader should keep in mind that the liquid is that of Fig. 11.57. Consequently, *the magnecules to be tested are visible to the naked eye. Therefore, only minute fragments entered the capillary feeding lines of the LC-MS/UVD instrument.*

Finally, the reader should keep in mind that the magnetic polarization of the test has been minimal, and *the liquid does not constitute a pure population of liquid magnecules.* The latter case is available from the PlasmaArcFlow reactors of Section 11.4 whose study is here omitted.

10.7.5 Experimental Verification of Mutated Physical Characteristics

In addition to the preceding *chemical* features, the existence of magnecules implies the mutation of *physical* characteristics, such as increase of the specific density and viscosity. This is due to the fact that magnetic bonds among ordinary molecules imply an evident reduction of intermolecular distances, thus resulting in more molecules per unit volume, as compared to the magnetically untreated substance. The increases in density and viscosity are then consequential.

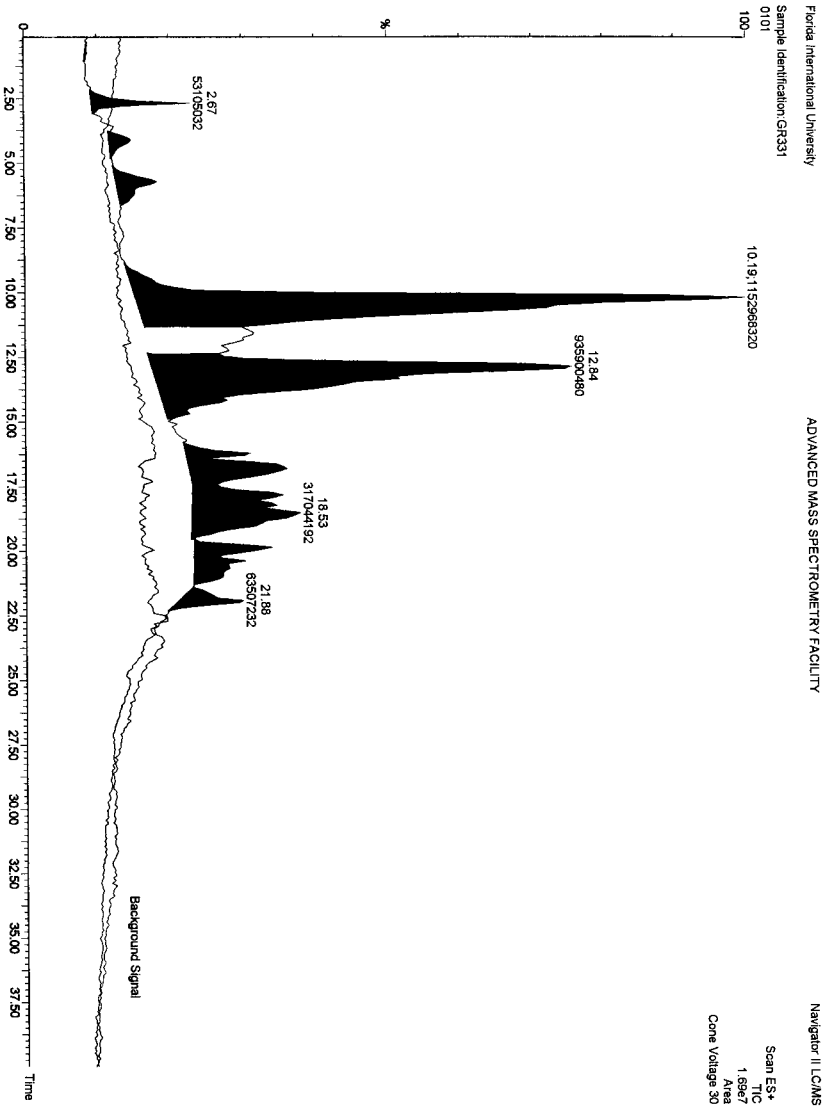


Figure 10.64. Scan on the untreated GR oil "ING258IN Test 2" of Fig. 11.56 (GR331 of the text) conducted at *Florida International University* (FIU).

A most intriguing feature of gas magnecules with important scientific and industrial implications is that *the Avogadro number of a gas with magnecular structure is not constant, or, equivalently, the so-called "gas constant" R of a gas with magnecular structure is an (expectedly nonlinear) function of P, V, T,*

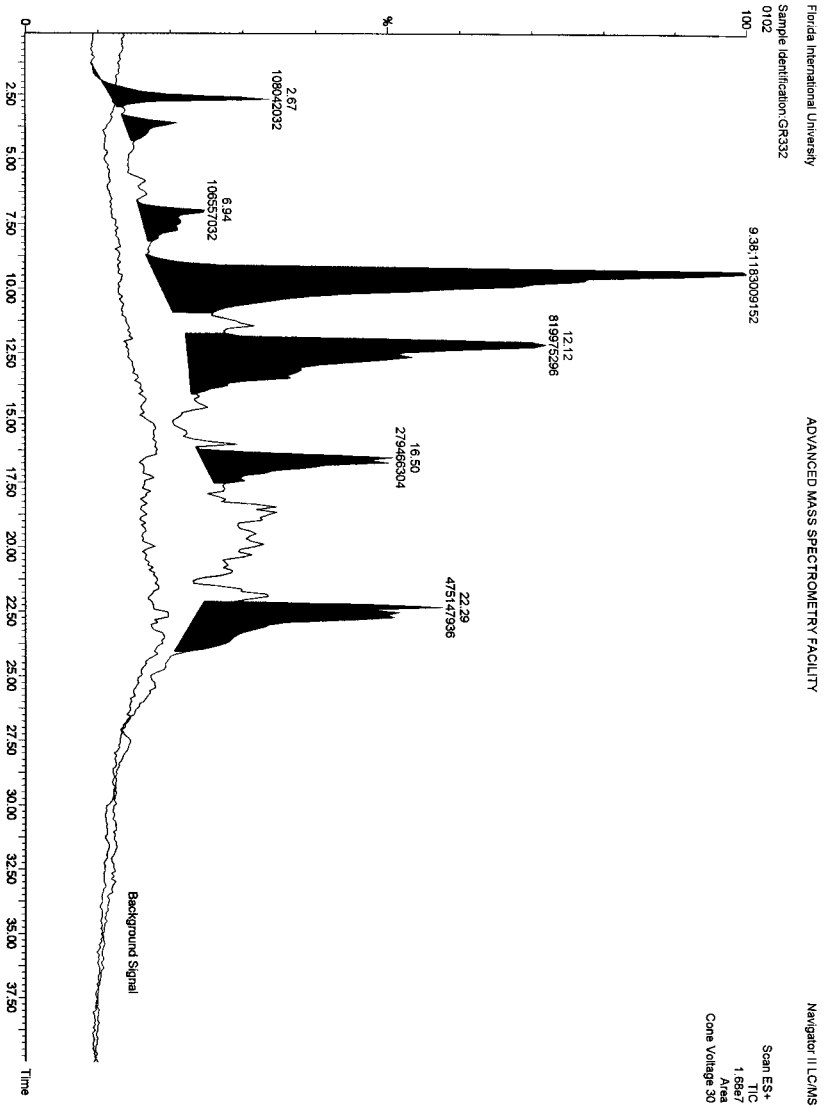


Figure 10.65. Scan at FIU of Sample GR332.

$R = R(P, V, T)$, resulting in the generalized gas law

$$\frac{PV}{T} = nR(P, V, T), \tag{8.21}$$

where the explicit dependence of R on P, V, and T depends on the magnecular gas considered.

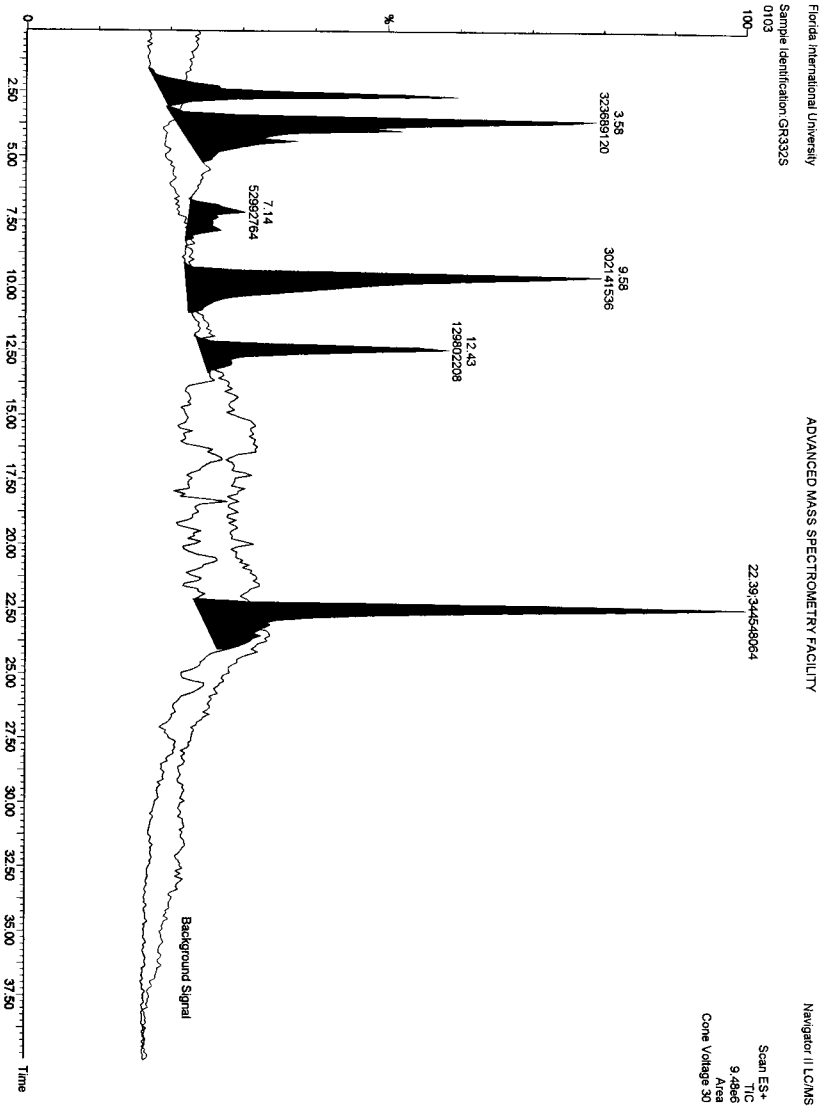


Figure 10.66. Scan at FIU of Sample GR332S.

The variation of the Avogadro number for gas with magnecular structure has been proved by routine tests at *USMagnegas, Inc.,* Largo, Florida, establishing that:

1) The number of constituents of a gas with magnecular structure decreases with a sufficient increase of the pressure;

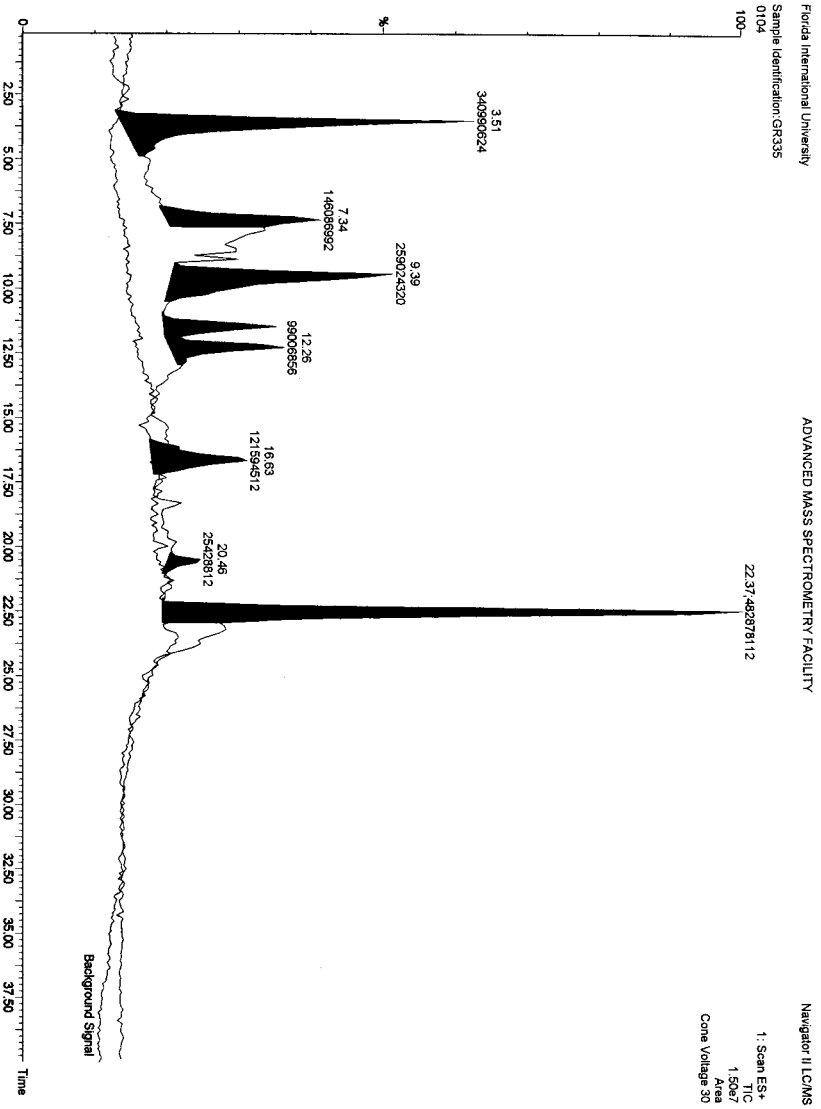


Figure 10.67. Scan at FIU of Sample GR335.

2) Given a fixed and sealed tank with volume V of a gas with magnecular structure at given pressure P and temperature T, after bringing this tank to a sufficiently higher temperature $T' > T$, and then returning it to the original temperature T, the pressure of the tank is not the original pressure P but a generally bigger pressure $P' > P$;

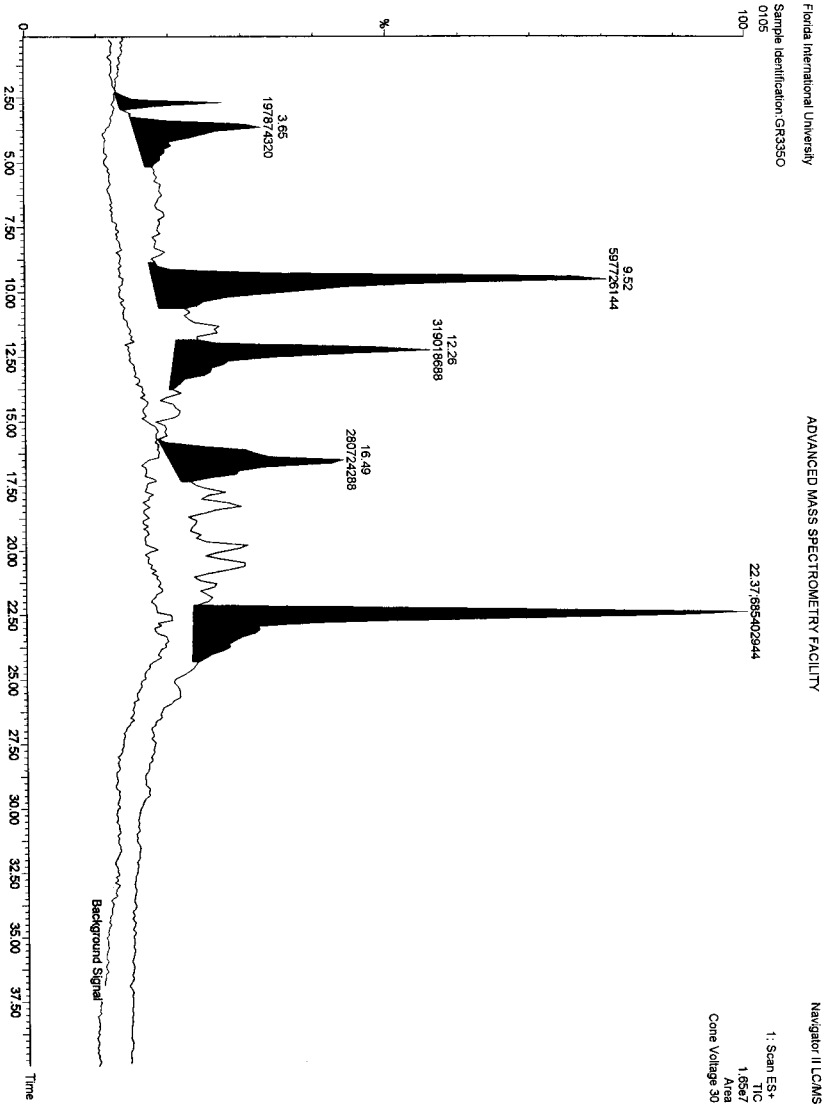


Figure 10.68. Scan at FIU of Sample GR335O.

3) The increase of pressure of a gas with magnecular structure requires a volume which generally increases with the pressure itself, that is, if the increase of pressure in a given tank from 100 psi to 200 psi requires V cf of magnecular gas, the same increase of pressure in the same tank via the same gas, this time from 4,000 psi to 4,100 psi at the same temperature does not require the same vol-

ume V but a volume V' of the magnecular gas bigger than the original volume, $V' > V$.

The above deviations from the conventional gas law are easily explained by the fact that *the increase of pressure in a gas with magnecular structure generally implies the aggregation of magnecules into bigger clusters, with consequential decrease of the number of constituents.* Similarly, *the increase of temperature generally implies the breaking down of magnecules into smaller clusters, with consequential increase of the number of constituents and resulting anomalous increase of pressure.* It then follows that, if the increase of temperature of a given fixed volume is beyond the Curie Magnecular Point (Definition 8.2.1), all magnetic polarizations are terminated with consequential increase of the number of constituents due to the reduction of magnecules to molecules. This implies that the return of the gas to the original temperature does not restore the original magnecules, and, consequently, the return to the original temperature generally occurs at an increased pressure due to the increased number of constituents.

We now report measurements of specific density, viscosity and other characteristics of fluids with magnecular structure which confirm the above GC-MS/IRD and LC-MS/UVD tests, by providing final evidence on the existence of magnecules as per Definition 8.2.1.

All tests were done via the use of ordinary tap water and a number of GR fragrance oils. All samples here considered were prepared by conventionally mixing tap water and one fragrant oil, and then submitting that mixture to rather weak permanent magnets of 200 G (much weaker than those used for the fragrance oils of Figs. 8.18 and 8.19). All samples resulted in being very stable without any measurable change over a period of about one year, and survived freezing followed by defrosting. The various samples were numbered from 1 to 25.

The measurements of the specific density were conducted on March 9, 1998 by the *U.S. Testing Company, Inc.* (USTC) of Fairfield, New Jersey. The results of the tests are presented in Figs. 8.31 and 8.32.

Sample 1 is ordinary untreated tap water. Sample 2 is ordinary tap water magnetically treated for about 5 minutes. Samples 3 and 4 were tap water treated with other magnetic equipment. Sample 5 was ordinary untreated GR fragrance oil "APC Fragrance." Sample 6 was a mixture of fragrance oil 5 with tap water magnetically treated for about 5 minutes. Mixtures 7 and 8 were the same mixture 5 although treated with other equipment. Sample 17 was a magnetically treated GR oil "Air Freshener 1." Mixture 19 was Fragrance 17 with tap water 16 magnetically treated for 5 minutes. Note that all measurements were done to an accuracy of the fourth digit. Therefore, numerical results up to the third digit can be considered accurate.

In the transition from Sample 1 (untreated water) to Sample 2 (magnetically treated water) there is an increase in the specific density in the macroscopic

REPORT OF TEST

SGS U.S. Testing Company Inc.

291 Fairfield Avenue
Fairfield, NJ 07004-3833
Tel: 973-575-5252
Fax: 973-244-1694

Report Number: 103947
Date: 03/09/98
Page: 1 of 1

Millennium Results

Density of	g/mL	% Change Density vs Ordinary Water
Sample #1	0.9805	0
Sample #2	0.9889	+0.86
Sample #3	0.9804	0
Sample #4	0.9853	+0.49
Fragrant #5	0.9720	NA
Mixture #6	0.9967	+1.65
Mixture #7	0.9982	+1.80
Mixture #8	0.9902	0.99
Treated Water #16	0.9893	0.89
Frag Treated # 17	0.9453	NA
Mixture #18	0.9902	0.99
Mixture #19	0.9929	1.26

Samples were transferred to a separatory funnel. The layers were allowed to separate. The water layer was withdrawn into a funnel with Whatman #4 filter paper. The filtrate was transferred to a preweighed 10 mL volumetric flask. The sample was weighed to 0.0001 grams and the density calculated.

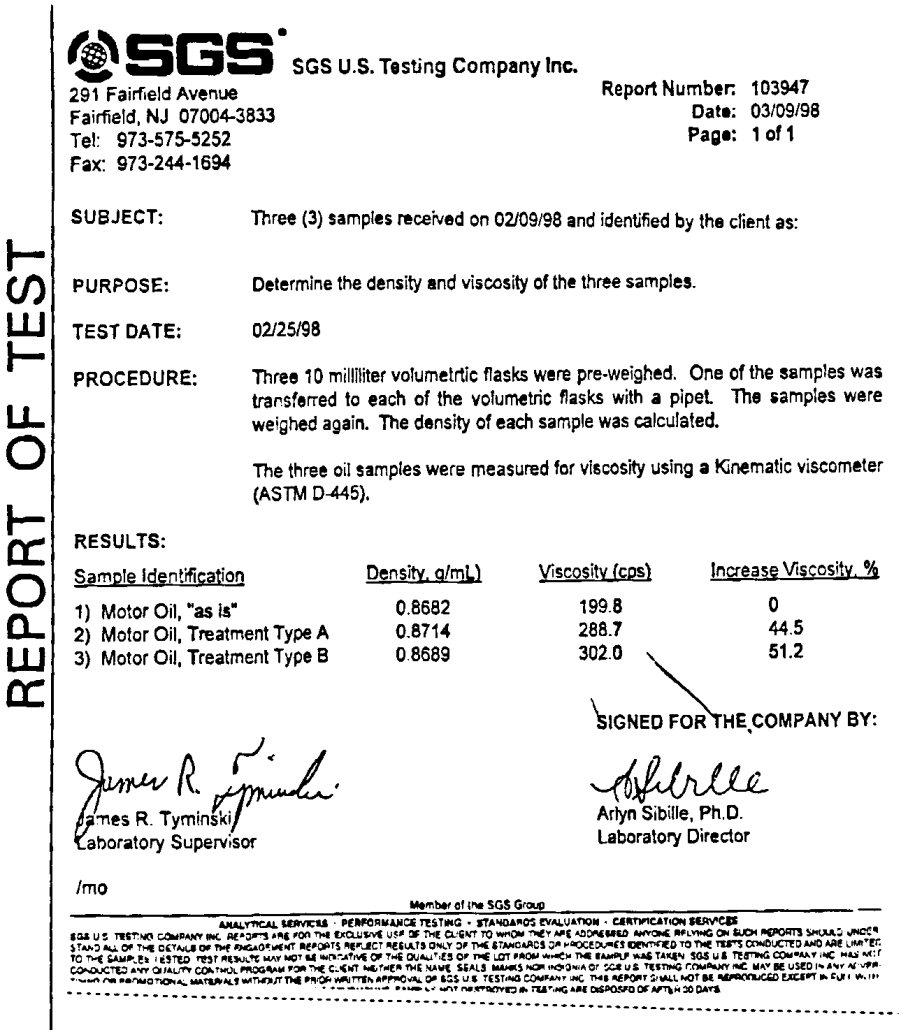
When the samples were pure substances, they were transferred directly to preweighed 10 mL volumetric flasks.

Calculations:

Weight flash with sample - weight flask ÷ volume of flask = g/ mL

Arlyn Sibille, Ph.D.

Figure 10.69. USTC measurements of specific density on magnetically treated liquids.



REPORT OF TEST

SGS U.S. Testing Company Inc.
291 Fairfield Avenue
Fairfield, NJ 07004-3833
Tel: 973-575-5252
Fax: 973-244-1694

Report Number: 103947
Date: 03/09/98
Page: 1 of 1

SUBJECT: Three (3) samples received on 02/09/98 and identified by the client as:

PURPOSE: Determine the density and viscosity of the three samples.

TEST DATE: 02/25/98

PROCEDURE: Three 10 milliliter volumetrtic flasks were pre-weighed. One of the samples was transferred to each of the volumetric flasks with a pipet. The samples were weighed again. The density of each sample was calculated.

The three oil samples were measured for viscosity using a Kinematic viscometer (ASTM D-445).

RESULTS:

Sample Identification	Density, g/mL)	Viscosity (cps)	Increase Viscosity, %
1) Motor Oil, "as is"	0.8682	199.8	0
2) Motor Oil, Treatment Type A	0.8714	288.7	44.5
3) Motor Oil, Treatment Type B	0.8689	302.0	51.2

SIGNED FOR THE COMPANY BY:

James R. Tyminski
Laboratory Supervisor

Arlyn Sibille, Ph.D.
Laboratory Director

/mo

Member of the SGS Group

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Figure 10.70. USTC measurements of viscosity on magnetically treated liquids.

amount of 0.86%, thus confirming the indicated mutation of the specific density of water under a magnetic treatment. In turn, the increase in density supports the existence of magneplexes in magnetically treated water as per the scan of Fig. 11.65.

As well known, fragrance oils are (generally) *lighter* than water, *i.e.*, the specific density of the untreated fragrance in Sample 5 is *smaller* than that of the untreated water in Sample 1. According to quantum chemistry, the specific den-

sity of any mixture of the above two liquids, whether solution, suspension or dispersion, should be *in between* the lighter and heavier specific densities.

On the contrary, as one can see, *the specific density of the magnetically treated mixture of GR fragrance with tap water, Sample 6, resulted in being bigger than that of the densest liquid, the water.* This measurement constitutes additional, rather strong, direct experimental verification of the mutation of physical characteristics in liquids under magnetic fields.

A remarkable point is that the *magnetic mutations of density are macroscopically large.* In fact, they were called by an analyst "UPS-type anomalies", meaning that the shipment via UPS of a given volume of a magnetically treated liquid may require an increase of the shipping cost of the same volume of untreated liquid due to the macroscopic increase in the weight.

A further prediction of magnetically polarized liquids is the increase of its viscosity. This is evidently due to the arbitrary size of an individual magnecule, as well as the tendency of the same to bond to near-by molecules, resulting in accretions, not to mention the anomalous adhesion to the walls of the container, which has been systematically detected for all magnetically polarized liquids.

As indicated earlier, in certain cases the increase of viscosity is so large as to be first visible to the naked eye, and, when the treatment is sufficiently protracted, the increase in viscosity is such as to lose the customary liquid mobility.

Ordinary engine oils are particularly suited for magnetic treatment because, when properly treated, their increase in viscosity is so dramatic as to be visible to the naked eye jointly with a visible change in visual appearance (color, texture, opacity, *etc.*).

The measurements on viscosity are reported in Fig. 11.70. The selected engine oil was an ordinarily available 30-40 Castrol Motor Oil subjected to a particular type of magnetic treatments via two different kinds of equipment called of Type A and B. All treatments were done at ordinary conditions without any additive or change of any type. As one can see, *measurement 2 shows a dramatic increase in the viscosity in the magnetically treated oil of 44.5%.*

The above experimental results evidently provide additional support for the existence of magnecules.

The tests also provide evidence of the anomalous adhesion of liquids with magnecules, which is established in this case by a dramatic, macroscopic increase of adhesion of the oil to the walls of the glass container.

The same macroscopic anomaly is confirmed at the microscopic level. During the measurement of viscosity there was such an anomalous adhesion of the magnetically treated oils to the walls of the instrument that said oil could not be removed via routine cleaning with acetone and required the use of strong acids.

This anomalous adhesion is further experimental evidence of the existence of magnecules, because of their predicted capability to induce the polarization of

the orbits of the valence electrons of the atoms in the walls of the container, thus resulting in anomalous adhesion via magnetic bonds due to induction.

It is evident that the mutations of density and viscosity implies the expected mutation of *all* other physical characteristics of the liquid considered. These measurements are left to the interested researchers.

The existence of mutation of *physical* characteristics then implies the mutation of *chemical* features. At this moment, we can only indicate the visual evidence reported by the analysts of USTC according to whom the reaction of magnetically treated oils with acetone is dramatically different from that with untreated oil, including mutations in color, texture and other appearances.

10.7.6 Concluding Remarks

The theoretical and experimental evidence presented in this Chapter establishes that the chemical species of molecules, defined as stable clusters of atoms under a valance bond, does not exhaust all possible chemical species existing in nature.

This conclusion is proved beyond scientific doubt, for instance, by macroscopic percentage of stable clusters, with atomic weight of several hundreds a.m.u., in light gases without an infrared signature where heaviest possible detected molecule is the CO_2 with 44 a.m.u.; the mutation of transparent oils into a completely opaque substance without fluidity; the joint increase of the specific density for both gaseous and liquid cases; and other evidence.

Needless to say, the final *characterization* and *detection* of the new chemical species submitted in Refs. [1,2] and reviewed in this chapter will require a considerable collegial effort, since the methods presented in this chapter are manifestly preliminary, with the understanding that, again, the *existence* of the new chemical species is outside scientific doubts.

As a matter of fact, the proposed new chemical species of magnecules, which, according to Definition 11.2.1 includes that of molecules, cannot be considered itself as the final chemical species in nature as it is the fate proved by history for all scientific discoveries.

As an example, the reformulation of magnecules via the hyperstructural branch of hadronic chemistry implies the prediction of the broader chemical species of *hypermagnecules* which is apparently more suitable to represent living organisms due to its inherent irreversibility, multidimensional structure compatible with our three-dimensional sensory perception, and other features needed for a more adequate representation of the complexities of living organisms. The *novelty* of this possible species is then an evident consequence of its novel features. Its *need* is established by the fact that current attempts to decipher the DNA code via the numbers used for molecules and magnecules dating back to biblical times have

little chance of success, thus mandating the use of broader numbers, such as the hypernumbers and related multi-dimensional structures.

All in all, we can safely conclude that science is a discipline that will never admit final theories.

Appendix 10.A
Aringazin's Studies on Toroidal Orbits of the Hydrogen Atom under an External Magnetic Field

In the main text of this chapter we have presented the theoretical and experimental foundations of the new chemical species of magnecules which is centrally dependent on individual atoms acquiring a generally toroidal configuration of the orbits of at least the peripheral electrons when exposed to sufficiently intense external magnetic fields, as originally proposed by Santilli [1] and reviewed in the main text of this Chapter.

In this Appendix we outline the studies by Aringazin [8] on the Schrödinger equation of the hydrogen atom under a strong, external, static and uniform magnetic field which studies have confirmed the toroidal configuration of the electron orbits so crucial for the existence of the new chemical species of magnecules.

It should be stressed that when considered at orbital distances (i.e., of the order of 10^{-8} cm), atoms and molecules near the electric arc of hadronic reactors (Section 11.4), and in the plasma region, are exposed to a strong magnetic field, whose intensity may be high enough to cause the needed magnetic polarization (see Fig. 11.4.D).

A weak, external, static, and uniform magnetic field B causes an anomalous Zeeman splitting of the energy levels of the hydrogen atom, with ignorably small effects on the electron charge distribution. In the case of a more intense magnetic field which is strong enough to cause decoupling of a spin-orbital interaction (in atoms), $e\hbar B/2mc > \Delta E_{jj'} \simeq 10^{-3}$ eV, i.e., for $B \simeq 10^5$ Gauss, a normal Zeeman effect is observed, again, with ignorably small deformation of the electron orbits.

More particularly, in the case of a *weak* external magnetic field B, one can ignore the quadratic term in the field B because its contribution is small in comparison with that of the other terms in Schrödinger equation, so that the *linear* approximation in the field B can be used. In such a linear approximation, the wave function of electron remains unperturbed, with the only effect being the well known Zeeman splitting of the energy levels of the H atom. In both Zeeman effects, the interaction energy of the electron with the the magnetic field is assumed to be much smaller than the binding energy of the hydrogen atom, $e\hbar B/2mc \ll me^4/2\hbar^2 = 13.6$ eV, i.e., the intensity of the magnetic field is much smaller than some characteristic value, $B \ll B_0 = 2.4 \cdot 10^9$ Gauss $= 240000$ Tesla

(recall that 1 Tesla = 10^4 Gauss). Thus, the action of a weak magnetic field can be treated as a small perturbation of the hydrogen atom.

In the case of a *very strong* magnetic field, $B \gg B_0$, the quadratic term in the field B makes a great contribution and cannot be ignored. Calculations show that, in this case, a considerable deformation of the electron charge distribution in the hydrogen atom occurs. More specifically, under the influence of a very strong external magnetic field a magnetic confinement takes place, i.e., in the plane perpendicular to the direction of magnetic field (see Fig. 8.4.D), the electron dynamics is determined mainly by the action of the magnetic field, while the Coulomb interaction of the electron with the nucleus can be viewed as a small perturbation. This adiabatic approximation allows one to separate variables in the associated Schrödinger equation [9]. At the same time, in the direction of the magnetic field the motion of electron is governed both by the magnetic field and the Coulomb interaction of the electron with the nucleus.

The highest intensities of magnetic fields maintained macroscopically at large distances in modern magnet laboratories are of the order of $10^5 - 10^6$ Gauss (~ 50 Tesla), i.e., they are much below $B_0 = 2.4 \cdot 10^9$ Gauss ($\sim 10^5$ Tesla). Extremely intense external magnetic fields, $B \geq B_c = B_0/\alpha^2 = 4.4 \cdot 10^{13}$ Gauss, correspond to the interaction energy of the order of the mass of electron, $mc^2 = 0.5$ MeV, where $\alpha = e^2/\hbar c$ is the fine structure constant. In this case, despite the fact that the extremely strong magnetic field does characterize a stable vacuum in respect to creation of electron-positron pairs, one should account for relativistic and quantum electrodynamics (QED) effects, and invoke Dirac or Bethe-Salpeter equation. These contributions are of interest in astrophysics, for example, in studying the atmosphere of neutron stars and white dwarfs which are characterized by $B \simeq 10^9 \ldots 10^{13}$ Gauss.

Aringazin [8] has focused his studies on magnetic fields with intensities of the order of $2.4 \cdot 10^{10} \leq B \leq 2.4 \cdot 10^{13}$ Gauss, at which value nonrelativistic studies via the Schrödinger equation can be used to a very good accuracy, and the adiabatic approximations can be made.

Relativistic and QED effects (loop contributions), as well as effects related to finite mass, size, and magnetic moment of the nucleus, and the finite electromagnetic radius of electron, reveal themselves even at low magnetic field intensities, and can be accounted for as very small perturbations. Additional effects are related to the apparent deviation from QED of strongly correlated valence bonds as studies in Chapter 4. These effects are beyond the scope of the presented study, while being important for high precision studies, such as those on stringent tests of the Lamb shift.

It should be noted that locally high-intensity magnetic fields may arise in plasma as the result of nonlinear effects, which can lead to the creation of stable self-confined structures having nontrivial topology with knots [10]. More

particularly, Faddeev and Niemi [10] recently argued that the static equilibrium configurations within the plasma are topologically stable solitons describing knotted and linked fluxtubes of helical magnetic fields. In the region close to such fluxtubes, we suppose the magnetic field intensity may be as high as B_0. In view of this, a study of the action of strong magnetic field and the fluxtubes of magnetic fields on atoms and molecules becomes of great interest in theoretical and applicational *plasmachemistry*. Possible applications are conceivable for the new chemical species of magnecules.

As a result of the action of a very strong magnetic field, atoms attain a great binding energy as compared to the case of zero magnetic field. Even at intermediate $B \simeq B_0$, the binding energy of atoms greatly deviates from that of the zero-field case, and even lower field intensities may essentially affect chemical properties of molecules of heavy atoms. This occurrence permits the creation of various other bound states in molecules, clusters and bulk matter [9, 11, 12].

The paper by Lai [12] is focused on very strong magnetic fields, $B \gg B_0$, motivated by astrophysical applications, and provides a good survey of the early and recent studies in the field, including studies on the intermediate range, $B \simeq B_0$, multi-electron atoms, and H_2 molecule. Several papers using variational/numerical and/or analytical approaches to the problem of light and heavy atoms, ions, and H_2 molecule in strong magnetic field, have been published within the last years (see, e.g., references in [12]). However, highly magnetized molecules of heavy atoms have not been systematically investigated until Santilli's proposal for the new species of magnecules [1]. One of the surprising implications is that for some diatomic molecules of heavy atoms, the molecular binding energy is predicted to be several times bigger than the ground state energy of individual atom [13].

To estimate the intensity of the magnetic field which causes considerable deformation of the ground state electron orbit of the H atom, one can formally compare Bohr radius of the H atom in the ground state, in zero external magnetic field, $a_0 = \hbar^2/me^2 \simeq 0.53 \cdot 10^{-8}$ cm $= 1$ a.u., with the radius of orbit of a single electron moving in the external static uniform magnetic field $\vec{B}$.

The mean radius of the orbital of a single electron moving in a static uniform magnetic field can be calculated exactly by using Schrödinger's equation, and it is given by

$$R_n = \sqrt{\frac{n + 1/2}{\gamma}}, \tag{11.A.1}$$

where $\gamma = eB/2\hbar c$, B is intensity of the magnetic field pointed along the z axis, $\vec{B} = (0, 0, B)$, $\vec{r} = (r, \varphi, z)$ in cylindrical coordinates, and $n = 0, 1, \ldots$ is the principal quantum number. Thus, the radius of the orbit takes *discrete* set of values (10.A), and is referred to as Landau radius. This is in contrast to well

known *classical* motion of electrons in an external magnetic field, with the radius of the orbit being of a continuous set of values.

The energy levels E_n of a single electron moving in said external magnetic field are referred to as Landau energy levels,

$$E_n = E_n^{\perp} + E_{k_z}^{\parallel} = \hbar\Omega(n + \frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m}, \qquad (11.A.2)$$

where $\Omega = eB/mc$ is so called cyclotron frequency, and $\hbar k_z$ is a projection of the electron momentum $\hbar\vec{k}$ on the direction of the magnetic field, $-\infty < k_z < \infty$, m is mass of electron, and $-e$ is charge of electron.

Landau's energy levels $E_n^{\perp}$ correspond to a discrete set of round orbits of the electron which are projected to the transverse plane. The energy $E_{k_z}^{\parallel}$ corresponds to a free motion of the electron in parallel to the magnetic field (*continuous* spectrum), with a conserved momentum $\hbar k_z$ along the magnetic field.

In regard to the above review of Landau's results, we recall that in the general case of a *uniform* external magnetic field the coordinate and spin components of the total wave function of the electron can always be separated.

The corresponding coordinate component of the total wave function of the electron, obtained as an exact solution of Schrödinger equation for a single electron moving in the external magnetic field with vector-potential chosen as $A_r = A_z = 0$, $A_\varphi = rB/2$,

$$-\frac{\hbar^2}{2m}\left(\partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\varphi^2 + \partial_z^2 - \gamma^2 r^2 + 2i\gamma\partial_\varphi\right)\psi = E\psi, \qquad (11.A.3)$$

is of the following form [9]:

$$\psi_{n,s,k_z}(r,\varphi,z) = \sqrt{2\gamma}I_{ns}(\gamma r^2)\frac{e^{il\varphi}}{\sqrt{2\pi}}\frac{e^{ik_z z}}{\sqrt{L}}, \qquad (II.A.4)$$

where $I_{ns}(\rho)$ is Laguerre function,

$$I_{ns}(\rho) = \frac{1}{\sqrt{n!s!}}e^{-\rho/2}\rho^{(n-s)/2}Q_s^{n-s}(\rho); \qquad (II.A.5)$$

Q_s^{n-s} is Laguerre polynomial, L is normalization constant, $l = 0, \pm1, \pm2, \ldots$ is azimuthal quantum number, $s = n - l$ is radial quantum number, and $\rho = \gamma r^2$.

Spin components of the total wave function are trivially given by

$$\psi(\frac{1}{2}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \psi(-\frac{1}{2}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad (II.A.6)$$

with the corresponding energies $E_{spin} = \pm\mu_0 B$, to be added to the energy (10.A); $\mu_0 = e\hbar/2mc$ is Bohr magneton.

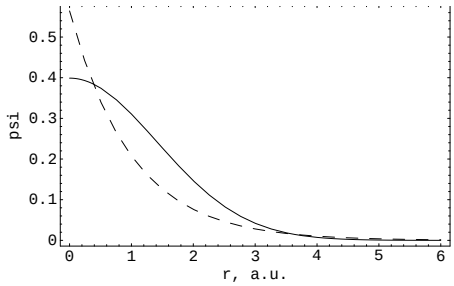

Figure 10.A.1. Landau's ground state wave function of a single electron, ψ_{000} (solid curve), Eq. (10.A), in a strong external magnetic field $B = B_0 = 2.4 \cdot 10^9$ Gauss, as function of the distance r in cylindrical coordinates, and (for a comparison) the hydrogen ground state wave function (at *zero* external magnetic field), $(1/\sqrt{\pi})e^{-r/a_0}$ (dashed curve), as function of the distance r in spherical coordinates. The associated probability densities are shown in Fig. **??**; 1 a.u. $= a_0 = 0.53 \cdot 10^{-8}$ cm.

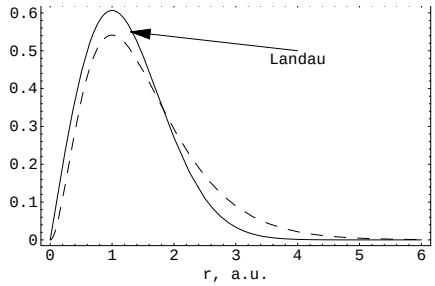

Figure 10.A.2. Probability density for the case of Landau's ground state of a single electron, $2\pi r|\psi_{000}|^2$ (solid curve), Eq. (10.A), in a strong external magnetic field $B = B_0 = 2.4 \cdot 10^9$ Gauss, as a function of the distance r in cylindrical coordinates, and (for a comparison) the probability density of the hydrogen atom ground state (at *zero* external magnetic field), $4\pi r^2|(1/\sqrt{\pi})e^{-r/a_0}|^2$ (dashed curve), as function of the distance r in spherical coordinates. The associated wave functions are shown in Fig. 10.A.1; 1 a.u. $= 0.53 \cdot 10^{-8}$ cm.

For the *ground* Landau level, i.e. at $n = 0$ and $s = 0$, and zero momentum of electron in the z-direction, i.e. $\hbar k_z = 0$, we have from (10.A)

$$E_0^{\perp} = \frac{e\hbar B}{2mc}, \tag{$II.A.7$}$$

and due to Eq. (10.A) the corresponding normalized ground state wave function is

$$\psi_{000}(r, \varphi, z) = \psi_{000}(r) = \sqrt{\frac{\gamma}{\pi}}\, e^{-\gamma r^2/2}, \tag{$II.A.8$}$$

$\int_0^{\infty}\int_0^{2\pi} r dr d\varphi\ |\psi_{000}|^2 = 1.$

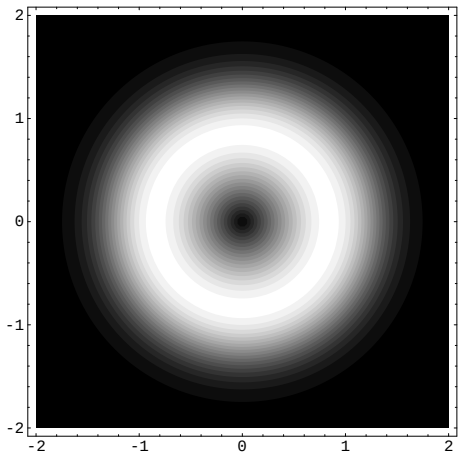

Figure 10.A.3. Contour plot of the (r, φ) probability density for the case of Landau's ground state of a single electron, $2\pi r|\psi_{000}|^2$, Eq. (10.A), in strong external magnetic field $B = B_0 = 2.4 \cdot 10^9$ Gauss, as a function of the distance in a.u. (1 a.u. $= 0.53 \cdot 10^{-8}$ cm). The lighter area corresponds to a bigger probability of finding the electron. The set of maximal values of the probability density is referred to as an "orbit".

The corresponding (smallest) Landau's radius of the orbit of electron is

$$R_0 = \sqrt{\frac{\hbar c}{eB}} \equiv \sqrt{\frac{1}{2\gamma}}, \qquad (II.A.9)$$

in terms of which ψ_{000} reads

$$\psi_{000} = \sqrt{\frac{1}{2\pi R_0^2}}\; e^{-\frac{r^2}{4R_0^2}}. \qquad (II.A.10)$$

Figure 10.A.1 depicts Landau's ground state wave function of a single electron, ψ_{000}, in the strong external magnetic field $B = B_0 = 2.4 \cdot 10^9$ Gauss ($R_0 = 1$ a.u.), and (for a comparison) of the hydrogen ground state wave function, at *zero* external magnetic field, $(1/\sqrt{\pi})e^{-r/a_0}$. Figures 10.A.2 and 10.A.3 display the associated probability density of the electron as a function of the distance r from the center of the orbit, the radius of which is about 1 a.u.

The condition that Landau's radius is smaller than Bohr's radius, $R_0 < a_0$ (which is adopted here as the condition of a considerable "deformation" of the electron orbit of the H atom) then implies

$$B > B_0 = \frac{m^2 c e^3}{\hbar^3} = 2.351 \cdot 10^9 \text{ Gauss}, \qquad (II.A.11)$$

where m is mass of electron. Equivalently, this deformation condition corresponds to the case when the binding energy of the H atom, $|E_0^{Bohr}| = |-me^4/2\hbar^2| = 0.5$ a.u. = 13.6 eV, is smaller than the ground Landau energy $E_0^{\perp}$.

The above critical value of the magnetic field, B_0, is naturally taken as an *atomic unit* for the strength of the magnetic field, and corresponds to the case when the pure Coulomb interaction energy of the electron with nucleus is equal to the interaction energy of the single electron with the external magnetic field, $|E_0^{Bohr}| = E_0^{\perp} = 13.6$ eV, or equivalently, when Bohr radius is equal to Landau radius, $a_0 = R_0 = 0.53 \cdot 10^{-8}$ cm.

It should be stressed here that the characteristic parameters, Bohr's energy $|E_0^{Bohr}|$ and Bohr's radius a_0, of the H atom have the purpose to establish a criterium for the critical strength of the external magnetic field of the hydrogen atom under the conditions here considered. For other atoms the critical value of the magnetic field may be evidently different.

After outlining the quantum dynamics of a single electron in an external magnetic field, Aringazin [8] turns to the consideration of the H atom under an external static uniform magnetic field.

In the cylindrical coordinate system (r, φ, z), in which the external magnetic field is $\vec{B} = (0, 0, B)$, i.e., the magnetic field is directed along the z-axis, Schrödinger's equation for an electron moving around a fixed proton (Born-Oppenheimer approximation) in the presence of the external magnetic field is given by

$$-\frac{\hbar^2}{2m}\left(\partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\varphi^2 + \partial_z^2 + \frac{2me^2}{\hbar^2\sqrt{r^2+z^2}} - \gamma^2 r^2 + 2i\gamma\partial_\varphi\right)\psi = E\psi, \tag{II.A.12}$$

where $\gamma = eB/2\hbar c$.

The main problem in the nonrelativistic study of the hydrogen atom in an external magnetic field is to solve the above Schrödinger equation and find the energy spectrum. This equation is not analytically tractable so that one is led to use approximations.

In the approximation of a *very* strong magnetic field, $B \gg B_0 = 2.4 \cdot 10^9$ Gauss, Coulomb interaction of the electron with the nucleus is not important, in the transverse plane, in comparison to the interaction of the electron with external magnetic field. Therefore, in accord to the exact solution (10.A) for a single electron, one can look for an approximate ground state solution of Eq. (10.A.3) in the form of factorized transverse and longitudinal parts,

$$\psi = e^{-\gamma r^2/2}\chi(z), \tag{II.A.13}$$

where $\chi(z)$ is the longitudinal wave function to be found. This is so called *adiabatic approximation*. In general, the adiabatic approximation corresponds to the case when the transverse motion of electron is totally determined by the

intense magnetic field, which makes the electron "dance" at its cyclotron frequency. Specifically, the radius of the orbit is then *much smaller* than Bohr radius, $R_0 \ll a_0$. The remaining problem is thus to find longitudinal energy spectrum, in the z direction.

Inserting tehe wave function (10.A.3) into the Schrödinger equation (10.A.3), multiplying it by ψ^*, and integrating over variables r and φ in cylindrical coordinate system, one gets the following equation characterizing the z dependence of the wave function:

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + \frac{\hbar^2\gamma}{m} + C(z)\right)\chi(z) = E\chi(z), \qquad (II.A.14)$$

where

$$C(z) = -\sqrt{\gamma}\, e^2 \int_0^\infty \frac{e^{-\rho}}{\sqrt{\rho + \gamma z^2}}\, d\rho = -e^2\sqrt{\pi\gamma}\, e^{\gamma z^2}[1 - \mathrm{erf}(\sqrt{\gamma}|z|)], \qquad (II.A.15)$$

where $\mathrm{erf}(x)$ is the error function.

The arising effective potential $C(z)$ is of a nontrivial form, which does not allow to solve Eq. (10.A.3) analytically, so one can approximate it by simple potentials, to make an estimation on the ground state energy and wave function of the H atom.

At high intensity of the magnetic field, $\gamma \gg 1$ so that under the condition $\gamma\langle z^2\rangle \gg 1$ one can ignore ρ in the square root in the integrand in Eq. (10.A.3). Then, one can perform the simplified integral and obtain the result

$$C(z) \simeq V(z) = -\frac{e^2}{|z|}, \quad \text{at } \gamma\langle z^2\rangle \gg 1, \qquad (II.A.16)$$

which appears to be a pure Coulomb interaction of electron with the nucleus, in the z direction. Due to the exact result (10.A.3), $C(z)$ tends to zero as $z \to \infty$. However, a remarkable implication of the exact result is that $C(z)$ is *finite* at $z = 0$, namely, $C(0) = -\sqrt{\pi\gamma}\, e^2$, so that the effective potential $C(z)$ can *not* be well approximated by the Coulomb potential.

The exact potential $C(z)$ can be well approximated by the *modified* Coulomb potential,

$$C(z) \simeq V(z) = -\frac{e^2}{|z| + z_0}, \qquad (II.A.17)$$

where z_0 is a parameter, $z_0 \neq 0$, which depends on the field intensity B due to

$$z_0 = -\frac{e^2}{C(0)} = \frac{1}{\sqrt{\pi\gamma}} = \sqrt{\frac{2\hbar c}{\pi e B}}. \qquad (II.A.18)$$

The analytic advantage of this approximation is that $V(z)$ is *finite* at $z = 0$, being of Coulomb-type form. Therefore, Eq. (10.A.3) reduces to *one-dimensional* Schrödinger equation for the Coulomb-like potential,

$$\left(\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + \frac{e^2}{|z| + z_0} + \frac{\hbar^2\gamma}{m} + E\right)\chi(z) = 0. \qquad (II.A.19)$$

In the atomic units ($e = \hbar = m = 1$), using the notation

$$E' = \frac{\hbar^2\gamma}{m} + E, \quad n^2 = \frac{1}{-2E'}, \qquad (II.A.20)$$

introducing the new variable $x = 2z/n$, and dropping $x_0 = 2z_0/n$, to simplify representation, the above equation can be rewritten as

$$\left[\frac{d}{dx^2} + \left(-\frac{1}{4} + \frac{n}{x}\right)\right]\chi(x) = 0, \qquad (II.A.21)$$

where $x > 0$ is assumed. Introducing new function $v(x)$ defined as $\chi(x) = xe^{-x/2}v(x)$, one gets the final form of the equation,

$$xv'' + (2 - x)v' - (1 - n)v = 0. \qquad (II.A.22)$$

Noting that it is a particular case of Cummer's equation,

$$xv'' + (b - x)v' - av = 0, \qquad (II.A.23)$$

the general solution is given by

$$v(x) = C_1\,{}_1F_1(a, b, x) + C_2U(a, b, x), \qquad (II.A.24)$$

where

$${}_1F_1(a, b, x) = \frac{\Gamma(b)}{\Gamma(b - a)\Gamma(a)}\int_0^1 e^{xt}t^{a-1}(1 - t)^{b-a-1}\,dt \qquad (II.A.25)$$

and

$$U(a, b, x) = \frac{1}{\Gamma(a)}\int_0^\infty e^{-xt}t^{a-1}(1 + t)^{b-a-1}\,dt \qquad (II.A.26)$$

are the confluent hypergeometric functions, and $C_{1,2}$ are constants; $a = 1 - n$ and $b = 2$. Hence, for $\chi(x)$ one has

$$\chi(x) = (|x|+x_0)e^{-(|x|+x_0)/2}\left[C_1^{\pm}\,{}_1F_1(1 - n, 2, |x| + x_0) + C_2^{\pm}U(1 - n, 2, |x| + x_0)\right], \qquad (II.A.27)$$

where the parameter x_0 has been restored, and the "$\pm$" sign in $C_{1,2}^{\pm}$ corresponds to the positive and negative values of x, respectively (the modulus sign is used for brevity).

Let us consider first the $x_0 = 0$ case. The first hypergeometric function ${}_1F_1(1-n, 2, x)$ is finite at $x = 0$ for any n. At big x, it diverges exponentially, unless n is an integer number, $n = 1, 2, \ldots$, at which case it diverges polynomially. The second hypergeometric function $U(1-n, 2, x)$ behaves differently, somewhat as a mirror image of the first one. In the limit $x \to 0$, it is finite for integer $n = 1, 2, 3, \ldots$, and diverges as $1/x$ for noninteger $n > 1$ and for $0 \leq n < 1$. In the limit $x \to \infty$, it diverges polynomially for integer n, tends to zero for noninteger $n > 1$ and for $n = 0$, and diverges for noninteger $0 < n < 1$.

In general, because of the prefactor $xe^{-x/2}$ in the solution (10.A.3) which cancels some of the divergencies arising from the hypergeometric functions, we should take into account *both* of the two linearly independent solutions, to get the most general form of normalizable wave functions.

As a consequence, for $x_0 \neq 0$ the eigenvalues may *differ* from those corresponding to $n = 1, 2, \ldots$ (which is a counterpart of the principal quantum number in the ordinary hydrogen atom problem) so that n is allowed to take some *non-integer* values from 0 to ∞, provided that the wave function is normalizable.

For even states, in accord to the symmetry of wave function under the inversion $z \to -z$, one has

$$C_1^+ = C_1^-, \quad C_2^+ = C_2^-, \quad \chi'(0) = 0. \tag{$II.A.28$}$$

Also, since $n = 1$ gives $E' = -1/(2n^2) = -1/2$ a.u., one should seek normalizable wave function for n in the interval $0 < n < 1$, in order to achieve lower energy value. If successful, $n = 1$ indeed does not characterize the ground state. Instead, it may correspond to some excited state.

Analysis shows that *normalizable* wave functions, as a combination of *two* linearly independent solutions, for the modified Coulomb potential *does exist* for various *non-integer* n. Focusing on the ground state solution, Aringazin considers values of n ranging from 0 to 1. Remind that $E' = -1/2n^2$ so that for $n < 1$ the energy lower than $E' = -0.5$ a.u.

For $n < 1$, the first hypergeometric function is not suppressed by the prefactor $xe^{-x/2}$ in the solution (10.A.3) at large x so we are led to discard it as an unphysical solution by putting $C_1 = 0$. A normalizable ground state wave function for $n < 1$ is thus may be given by the second term in the solution (10.A.3). Indeed, the condition $\chi'(x)_{|x=0} = 0$ implies

$$\begin{aligned} &\tfrac{1}{2}e^{-(x+x_0)/2}C_2[(2-x-x_0)U(1-n,2,x+x_0)- \\ &\qquad -2(1-n)(x+x_0)U(2-n,3,x+x_0))]_{|x=0} = 0. \end{aligned} \tag{$II.A.29$}$$

The l.h.s of this equation depends on n and x_0, so one can select some field intensity B, calculate associated $x_0 = x_0(B)$ and find n, from which one obtains the ground state energy E'. On the other hand, for the ground state this condition can be viewed, *vice versa*, as an equation to find x_0 at some selected n.

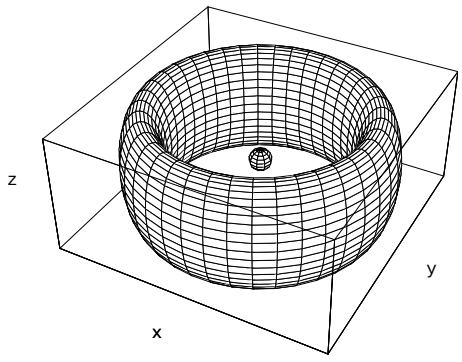

Figure 10.A.4. A schematic view on the H atom in the ground state under a very strong external magnetic field $\vec{B} = (0, 0, B)$, $B \gg B_0 = 2.4 \cdot 10^9$ Gauss, due to the *modified* Coulomb approximation studied in the text. The electron moves on the Landau orbit of small radius $R_0 \ll 0.53 \cdot 10^{-8}$ cm resulting in the toroidal structure used for the new chemical species of magnecules. The vertical size of the atom is comparable to R_0. The spin of the electron is antiparallel to the magnetic field.

For example, taking the noninteger value $n = 1/\sqrt{15.58} \simeq 0.253 < 1$ Aringazin found $x_0 = 0.140841$. This value is in confirmation with the result $x_0 = 0.141$ obtained by Heyl and Hernquist [14]. On the other hand, x_0 is related in accord to Eq. (10.A.3) to the intensity of the magnetic field, $x_0 = 2z_0/n$, from which one obtains $B \simeq 4.7 \cdot 10^{12}$ Gauss. Hence, at this field intensity the ground state energy of the hydrogen atom is determined by $n = 1/\sqrt{15.58}$.

The total ground state wave function is given by

$$\psi(r, \varphi, x) \simeq \sqrt{\frac{1}{2\pi R_0^2}} e^{-\frac{r^2}{4R_0^2}} (|x| + x_0) e^{(|x|+x_0)/2} U(1 - n, 2, |x| + x_0), \quad (II.A.30)$$

where n is determined due the above procedure, and the associated three-dimensional probability density is schematically depicted in Fig. 10.A.4.

One can see that the problem remarkably difference than the ordinary three-dimensional problem of the hydrogen atom, for which the principal quantum number n must be integer to get normalizable wave functions, and the value $n = 1$ corresponds to the lowest energy.

The modified Coulomb potential approach provides qualitatively correct behavior, and suggests a *single* Landau-type orbit shown in Fig. 10.A.4 for the *ground* state charge distribution of the hydrogen atom. This is in full agreement with Santilli's study [1, 11] of the hydrogen atom in a strong magnetic field.

Accurate analytic calculation of the ground and excited hydrogen wave functions made by Heyl and Hernquist [14] in the adiabatic approximation leads to

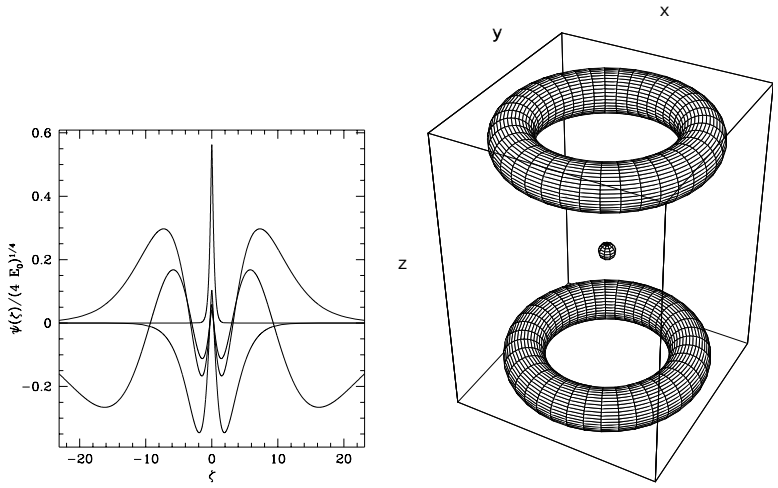

Figure 10.A.5. The axial wavefunctions of hydrogen in an intense magnetic field (analytic calculation) for $B = 4.7 \cdot 10^{12}$ Gauss. The first four even states with axial excitations, $|000\rangle$ (ground state), $|002\rangle$, $|004\rangle$, and $|006\rangle$ (left panel), and odd states $|001\rangle$ and $|003\rangle$ (right panel) are depicted; $n = 1/\sqrt{15.58}$, $\zeta = 2z/n$ corresponds to x in the used notation; z in a.u., 1 a.u. $= 0.53 \cdot 10^{-8}$ cm (reproduction of Figure 3 by Heyl and Hernquist [14]).

the longitudinal parts of the wave functions shown in Fig. 10.A.5, which reproduces the original Fig. 3 of their work; $\zeta = 2\pi\alpha z/\lambda_e$; $B = 4.7 \cdot 10^{12}$ Gauss. They used the modified Coulomb potential of the type (10.A.3), and the additional set of linearly independent solutions of the one-dimensional modified Coulomb problem in the form

$$(|x| + x_m)e^{-(|x|+x_m)/2}{}_1F_1(1-n, 2, |x| + x_m) \int^{|x|+x_m} \frac{e^t}{(t\,{}_1F_1(1-n, 2, t))^2}\, dt, \qquad (II.A.31)$$

where $m = 0$ corresponds to the ground state. For the ground state with $n = 1/\sqrt{15.58}$, they found $x_0 = 0.141$, which corresponds to $B = 4.7 \cdot 10^{12}$ Gauss. This result is in agreement with the study made above.

One can see from Fig. 10.A.5 that the peak of the ground state wave function $|000\rangle$ is at the point $z = 0$, while the largest peaks of the excited wave functions are away from the point $z = 0$ (as it was expected to be). Consequently, the associated longitudinal probability distributions (square modules of the wave functions multiplied by the volume factor of the chosen coordinate system) are symmetric with respect to $z \to -z$, and their maxima are placed in the center $z = 0$ for the ground state, and away from the center for the excited states. The computed ground state $|000\rangle$ binding energy of the hydrogen atom for different field intensities are [14]:

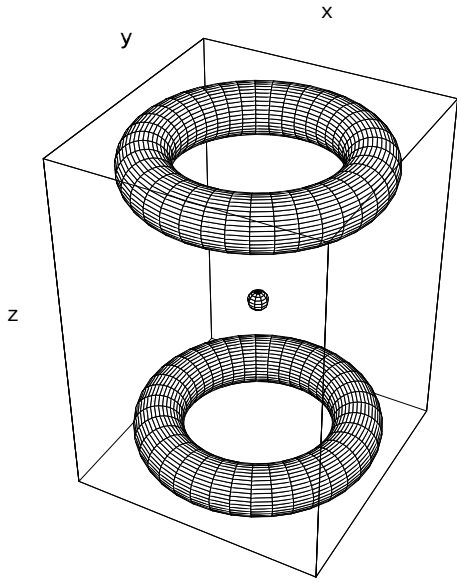

Figure 10.A.6. A schematic view on the H atom in an excited state under a very strong external magnetic field $\vec{B} = (0, 0, B)$, $B \gg B_0 = 2.4 \cdot 10^9$ Gauss. One electron moves simultaneously on two toroidal orbits of radius R_0 which are shown schematically as torii in the different $(x,\ y)$ planes, one torus at the level $z = -L_z$ and the other at the level $z = +L_z$, with the nucleus shown in the center at $z = 0$. Each torus represents the $(x,\ y)$ probability distribution as shown in Fig. 10.A.3 but with small Landau radius, $R_0 \ll a_0$. The spin of electron is aligned antiparallel to the magnetic field.

Magnetic field B (Gauss)	Binding energy, $\|000\rangle$ state (Rydberg)
4.7×10^{12}	15.58
9.4×10^{12}	18.80
23.5×10^{12}	23.81
4.7×10^{13}	28.22
9.4×10^{13}	33.21
23.5×10^{13}	40.75
4.7×10^{14}	47.20

Heyl and Hernquist calculated the first-order perturbative corrections to the above energies and obtained the values, which are in a good agreement with the results by Ruder *et al.* [9] and Lai [12].

The associated probability density of the above *excited* states is evidently of a cylindrical (axial) symmetry and can be described as *two* Landau orbits of radius R_0 in different (r, φ) planes, one at the level $z = -L_z$, and the other at the level $z = +L_z$, with the nucleus at $z = 0$, as schematically depicted in Fig. 10.A.6.

Presence of two Landau orbits occurs in accord to the excited wave functions, which is symmetrical with respect to the inversion, $z \to -z$, and the largest peaks of which are away from the center $x = 0$. The electron moves simultaneously on these two Landau orbits.

A review of approximate, variational, and numerical solutions can be found in the paper by Lai [12]. The accuracy of numerical solutions is about 3%, for the external magnetic field in the range from 10^{11} to 10^{15} Gauss. Particularly, due to the variational results [12], the z-size of the hydrogen atom in the ground state is well approximated by the formula $L_z \simeq [\ln(B/B_0)]^{-1}$ a.u.; the transverse (Landau) size is $L_\perp \simeq (B/B_0)^{-1/2}$ a.u.; and the ground state energy $E \simeq -0.16[\ln(B/B_0)]^2$ a.u., with the accuracy of few percents, for $b \equiv B/B_0$ in the range from 10^2 to 10^6. One can see for $B = 100B_0$, that the variational study predicts the ground state energy $E = -3.4$ a.u. $= -92.5$ eV, the transverse size $L_\perp$ of about 0.1 a.u. $= 0.53 \cdot 10^{-9}$ cm, and the z-size L_z of about 0.22 a.u. This confirms the result of the modified Coulomb analytic approach.

Since a zero-field ground state case is characterized by perfect spherically symmetric electron charge distribution in the H atom, intermediate intensities of the magnetic field are naturally expected to imply a distorted spherical distribution. However, a deeper analysis is required for the intermediate magnetic field intensities because the adiabatic approximation is not longer valid in this case.

As to the multi-electron atoms, an interesting problem is to study action of very strong external magnetic field on He atom (see. e.g., Refs. [12, 14]) and on the multi-electrons heavy atoms, with outer electrons characterized by a *nonspherical* charge distribution, such as the p-electrons in Carbon atom, orbitals of which penetrate the orbitals of inner electrons. In fact, a very intense magnetic field would force such outer electrons to follow *small round* toroidal orbits. In addition to the effect of a direct action of the magnetic field on the inner electrons, a series of essential rearrangements of the whole electron structure of the atom seems to occur with the variation of the field strength. The magnetic field competes with the Coulomb energy, which is different for different states of electrons, and with the electron-electron interactions, including spin pairings. However, it is evident that at sufficiently strong fields, all the electron spins are aligned antiparallel to the magnetic field — fully spin polarized configuration — while at lower field intensities various partial spin polarized configurations are possible.

In accord to the numerical calculations based on the density matrix theory by Johnsen and Yngvason [13], which is in good agreement with the Hartree-Fock treatment of a very strong magnetic field, the inner domain in iron atom (26 electrons) is characterized by a slightly distorted spherically symmetric distribution, even at the intensities as high as $B = 100B_0 \ldots 1000B_0$. The outer domain appears to be of specific, highly elongated distribution along the direction of the magnetic field as shown in Fig. 10.A.7. The possible interpretation that the inner

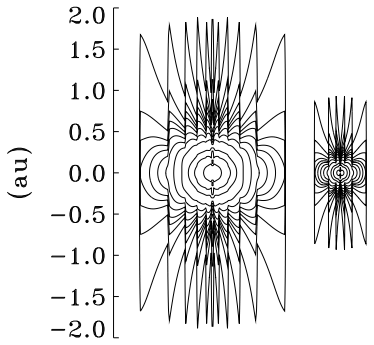

Figure 10.A.7. Contour plots of the (r, z) plane electronic density of iron atom according to the density matrix theory at two different magnetic field strengths, 10^{11} Gauss (left) and 10^{12} Gauss (right). The outermost contour encloses 99% of the negative charge, the next 90%, then 80% etc., and the two innermost 5% and 1% respectively (reproduction of Fig. 5 by Johnsen and Yngvason [13]).

electrons remain to have a spherical distribution while outer electrons undergo the squeeze seems to be not correct unless the spin state of the iron atom is verified to be partially polarized. So, we can conclude that all the electrons are in the highly magnetically polarized state (Landau state mixed a little by Coulomb interaction), and the electronic structure is a kind of *Landau multi-electron cylindrical shell*, with the spins of all the electrons being aligned antiparallel to the magnetic field (fully spin polarized configuration).

Another remark regarding Fig. 10.A.7 is that the contours indicating a nearly spherical distribution will always appear since the Coulomb center (nucleus) is not totally eliminated from the consideration (non-adiabatic approximation), and it forces a spherical distribution to some degree, which evidently depends on the distance from the center (closer to the center, more sphericity). We note that outer contours in Fig. 10.A.7 is in qualitative agreement with Fig. 10.A.6 in the sense that the predicted charge distribution reveals symmetry under the inversion $z \to -z$, with the characteristic z-elongated Landau-type orbits.

An interesting problem is to study H_2 molecule under the action of a strong external static uniform magnetic field using Schrödinger's equation. However, prior to that study, it would be useful to investigate the simpler two-center H_2^+ ion, since it can give valuable information on the features of the full hydrogen molecule under the action of a strong magnetic field. We refer the interested reader to Refs. [12, 14, 15] for studies on H_2^+ ion and H_2 molecule in strong magnetic field. Figure 10.A.8 displays the ground and first excited state wave functions of H_2^+ [14].

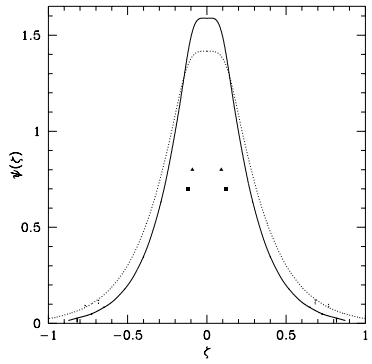

Figure 10.A.8. A schematic view of the ground and first-excited state of H_2^+ ion. The solid line traces $|000\rangle$, and the dashed line follows $|0\text{-}1\,0\rangle$. The triangles give the positions of the protons for the ground state and the squares for the excited state. The magnetic field $B = 4.7 \cdot 10^{12}$ Gauss is pointed along the internuclear axis; $\zeta = 2\pi\alpha z/\lambda_e$ denotes z in a.u.; 1 a.u. $= 0.53 \cdot 10^{-8}$ cm (reproduction of Figure 5 by Heyl and Hernquist [14]).

APPENDIX 10.A

Table 10.A.1. Basic units and their conversions.

1 kWh	860 Kcal = 3413 BTU	1 cf	28.3 liters
1 Kcal	3.97 BTU	1 cf[a]	1.263 mol
1 eV	3.83×10^{-23} Kcal	N_A	6.022×10^{23} mol^{-1}
1 cal	4.18 J	$N_Ak/2$	1 cal/(mol·K)
1 mole[a]	22.4 liters = 0.792 cf	R	8.314 J/(mol·K) = 1.986 cal/(mol·K)

[a] An ideal gas, at normal conditions.

Table 10.A.2. Specific heat capacities. $p = 1$ atm, $T = 25^o$ C.

H_2(gas)	29.83 J/(mol·K)	7 cal/(mol·K)	
H_2O (liquid)	4.18 J/(gram·K)	1 cal/(gram·K)	18 cal/(mol·K)
Graphite (solid)	0.71 J/(gram·K)	0.17 cal/(gram·K)	2 cal/(mol·K)
O_2 (gas)	29.36 J/(gram·K)	7 cal/(gram·K)	
H (gas)	14.3 J/(gram·K)	3.42 cal/(gram·K)	
O (gas)	0.92 J/(gram·K)	0.22 cal/(gram·K)	
Fe (solid)	0.45 J/(gram·K)	0.11 cal/(gram·K)	6 cal/(mol·K).

Table 10.A.3. Average binding energies, at T=25^o C.

	Kcal/mol		*Kcal/mol*		*Kcal/mol*
H–H	104.2[a]	C=O	192.0[d]	O=O	119.1[b]
C–C	82.6	O–H	110.6	C=C	145.8
C–O	85.5	C≡C	199.6	C=O	255.8[c]

[a] in H_2; [b] in O_2; [c] in carbon monoxide; [d] in carbon dioxide.

Table 10.A.4. Evaporation heats and first ionization potentials.

	Kcal/mol	*Atoms*	*eV*
Water	10.4	H	13.6
Graphite	171.7	C	11.26
		O	13.6

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Chapter 11

INDUSTRIAL APPLICATIONS TO NEW CLEAN ENERGIES

11.1 IN PREPARATION - NOVEMBER 1, 2005

For a summary of the content of this chapter, consult

The Physics of New Clean Energies and Fuels According to Hadronic Mechanics, R. M. Santilli, Special issue of the Journal of New Energy, 318 pages (1998).

Foundations of Hadronic Chemistry with Applications to New Clean Energies and Fuels, R. M. santilli, Kluwer Academic Publishers, Boston-Dordrecht-London (2001).

Isodual Theory of Antimatter with Applications to Antigravity, Grand Unification and Cosmology, R. M. Santilli, Sprionger (in press)

Postscript

In the present second volume of his opus magnum,*Hadronic Mathematics, Mechanics and Chemistry*, Professor Ruggero Maria Santilli applies the extensive advances to pure mathematics, presented in the first volume, to a plethora of basic and far-reaching issues in the natural sciences of physics and chemistry. By these means he attains theoretical results not possible to achieve without use of these new and powerful mathematical tools or the extensions of our ontological horizon of the universe associated with the establishment of the new number fields discovered by hadronic mathematics. This second volume also presents available established experimental evidence offering crucial support to predictions from the new sciences of hadronic mechanics and chemistry, sketches of experimental design for further support and theoretical refinements (or falsifications), and emergence of new and quite spectacular technology made possible from these advances in theoretical science. Some of this technology has already been constructed and is up-and-running, and constitutes matured fruits of the quite gigantic scientific enterprise initiated by Santilli four decades ago, and with growing affiliation from co-scientists world-wide throughout the years.

In the exploring spirit of the Renaissance, one might say that the first volume offers a guiding compass and the basic skills for constructing adequate maps and ships to search for unknown continents, while this second volume presents maps as well as treasures after having successfully travelled, reached and traced unknown continents on the other side of the vast ocean of the unknown.

Scientific revolutions in the sense of Thomas Kuhn do not happen often in the history of science, and with regard to physics the last ones, quantum mechanics and Einsteins relativity theory, have now reached the age of 100 year old-timers. With the rapidly increasing number of scientists and over-all significance of scientific progress for modern society, it is not too strange from a birds-eye-view of the history of science that a new revolution has found its day.

The new theory of physics as a whole, coined hadronic mechanics by Santilli, does not question the validity of the theories of quantum mechanics and relativity theory for the physical world, given the constraints formulated by the great creators of the said theories, represented by the kind of physical objects and relations being studied by the theory, and the proper simplifications in the describing and explaining models of such objects, dependent on the nature of the objects and the available mathematics. Basically, the constraints of these theories consist in their relevance being restricted to the so-called exterior physical world, which is

the physical world outside the hadronic horizon of one femtometer. For interior relations, inside the hadronic horizon, the models and equations of these theories did not claim any immediate validity by their originators and, therefore, they are not scientifically legitimate to import inside the hadronic horizon, at least not without careful theoretical considerations on the basic problems therein involved, and without support from crucial experiments. Sad to say, this book offers much argument and evidence for a lot of such illegitimate import to have been the normal state of affairs during the second half of 20th century standard physics.

Assuming the strong interaction being adequately represented as the interaction between three point-like baryon quarks in the hadron, quantum mechanics did not succeed in establishing any good and experimentally testable model of the strong interaction, partly due to the complexities involved with the required non-linear mathematics to describe such a system. Largely because of these problems, the unification of the three other well-known forces with the strong force remained an open problem during 20th century standard physics. Equipped with the developed isomathematics, Santilli disposed the necessary tools to leave the assumption of interior point-quarks, and to describe shapes, as well as changes in shapes coined deformations, of particles with physical extension, to approach the problem of strong interactions inside the hadronic horizon. By means of isomathematics, Santilli was able to quantitatively model the neutron as a bound state of a proton and an electron, and hence to reestablish Rutherfords notion of the neutron as a compressed hydrogen atom. This achievement by Santilli was enthusiastically commented on by the great philosopher of science, Karl Popper, in his book from 1982, as a return to sanity, to that realism and objectivism for which Einstein stood.

The Rutherford-Santilli model of the neutron described the proton and the electron as a bound state with overlapping wave packets. Such a compression could only be imagined as a result of an external trigger, for example the role of pressure in the case of neutron synthesis in stars. For the neutron to stay in a bound state, the bound state had to be imagined as a singlet of a proton and an electron with opposite spins, according to the so-called gear model ruling out the possibilities of triplets or parallel spins. By 1990 Santilli had been able to publish such a model of the neutron as a mutated bound state with an exact quantitative representation of its physical characteristics: rest energy, mean life, charge radius, charge, charge parities, space, spin, and (anomalous) magnetic and electric moment.

Such a model would not have been possible by importing the quantum mechanics for exterior relations to the inside of the hadronic horizon, due to the idea of quantum quantization being contrary to the deep interpenetration of the wave packets inside the hadronic horizon and to the non-existence of exited hadronic states. Such excitation would imply tunneling through the hadronic horizon,

which by Santilli was stated as the very mechanism of the neutrons spontaneous decay. In this way Santillis model of neutron synthesis, as well as neutron decay, did not need any assumption about existence of sub- or quasi-particles as in the notion of quarks, nor was there any need to imagine said processes to rely on a somewhat mystical notion of the two stable elementary particles of the physical world, protons and electrons, being created from and resolved into intermediary states of quark assemblies. In this regard Santillis theory of the neutron offered a much simpler picture of the situation inside the hadronic horizon as well as of the relation to the exterior physical world. Elegant and adequate simplifications are what good science should be about; the question was if the theory was to become supported by experimental evidence. Such significant support was provided when the measured density of the so-called fireball in the Einstein Bose correlation of colliding proton and antiproton was shown to be very close to the hadronic calculation of the density of the neutron, as predicted by hadronic mechanics. Crucial additional support was added from the experiments headed by Prof. Tsagas in 1996 with 319 stimulated decays of the neutron, expelling the Rutherford electron when exposed to the resonance frequency of a hard photon, in accordance with the predictions from hadronic mechanics. (Sad to say, no other laboratories in the world have so far wanted to retest these results by duplicating such experiments, in spite of the great scientific, technological and ecological significance of such confirmation.)

In analogy with the neutron model, Santilli already in 1978, the birth year of hadronic mechanics, had been able to present a model of the 0 meson as a bound state of an electron and a positron with overlapping wave packets, i.e. as a compressed positronium. Also this model was able, differently from quantum mechanics, to represent all physical characteristics of the 0 meson without any additional notion of quarks, and this in one single structural equation. However, it is important to notice that the said bound state is not a bound state of the involved particles as considered outside the hadronic horizon, since physical attributes of the particles undergo some changes in this compression. Such states are, therefore, only possible to describe by means of isomathematics and from the accordingly broader concept of isoparticles.

In general, different from quantum mechanics, hadronic mechanics represents a theory of physics equipped with concepts, models and mathematics to describe and explain relations interior to the hadronic horizon. However, to be able to succeed in this, hadronic mechanics had to be developed as a lifted theory compared to quantum mechanics, thereby providing a more general theory of physics, just as valid for exterior relations as quantum mechanics, the last being a sub-field of hadronic mechanics. Therefore, it is not adequate to consider hadronic mechanics as a supplement or a competitor to quantum mechanics, but as a theory of physics with a broader explanatory power than quantum mechanics, also being

able to adequately include interior relations, as well as relations between the interior and the exterior. This broadening-from-lifting follows the general scheme of development of basic theoretical advances in physics as analyzed in David Bohms interpretation of the modern history of physics.

The theory of hadronic superconductivity, initiated by Prof. Animalu and Santilli from 1994, constitutes an important bridge between hadronic mechanics and hadronic chemistry. In superconductivity theory, as approached by quantum mechanics, it was quite a mystery how the bound state of the Cooper pair could emerge and remain, considering that two electrons are known to be repelled by the Coulomb force. However, from hadronic mechanics this became explainable with the notion of a hitherto unknown physical force becoming activated when two particles are brought into touch from an external trigger, this fifth force inducing total overlap between the two involved wave packets. Different from the four conventional forces, this was a contact force without a potential, and thus requiring a non-Hamiltonian for its mathematical description; - hence being outside the reach of quantum mechanics. Also, the force was described by hadronic mechanics not to depend on the sign of the charge of the involved particles. Thus, the Cooper pair could be explained with this force simply being stronger than the Coulomb force. Due to deep interpenetration of the wave packets, the Cooper pair, by analogy with the cases of the neutron and the compressed positronium, had to be modeled, not as conventional electrons in the exterior, but as isoelectrons.

Further, the Cooper pair in hadronic superconductivity was modeled with an 8-shaped orbit around the two nuclei involved in the superconductivity structure. This orbit shape induces an extraordinarily strong magnetic force from each nuclei, in the hydrogen atom calculated to be 1,415 times the strength of the ordinary magnetic force from the proton, and of course in opposite directions from the two nuclei. Similar superconductivity structures could then be attracted and bound together, aligning from the orientations of the extraordinarily strong magnetic forces from the nuclei, and clustering into bigger structures of atoms (as well as with the possibility to include dimers, radicals or molecules). These clusters were coined magnecules by Santilli, and were predicted from hadronic superconductivity to be discovered by experiments. This became confirmed by independent laboratories, using adequate special apparatus for such detections, from 1998 on. Santilli also invented and patented so-called plasma-arc-flow reactors, also called hadronic reactors and sometimes ecoreactors, to produce magnecules in specified types and quantities in a controlled manner. Already at Dec. 15, 1998, Santilli presented the first constructed reactor producing such new chemical species. 1998 became the take-off year of hadronic chemistry also as a scientific discipline, with a special issue of the Hadronic Journal solely dedicated to presenting the scientific foundations of this lifted and broader chemistry.

Besides Santilli the publication included among its authors Profs. Shillady and Aringazin.

The discovery of magnecules represented the first discovery of a new chemical species since the discovery of molecules in the mid-1800s. Different from molecules, magnecules have non-valence bonds and they can form much larger structures, in superfluids sometimes even visible by the naked eye. Most scientists researching superconductivity with only quantum mechanics at their disposal, believe that superconductivity is restricted to extremely low temperatures (somewhat misleading referring to temperatures far below zero as High Tc superconductivity), while hadronic chemistry has explained hadronic superconductivity to be possible also for fluids and gases, activated by the external trigger of strong and close enough magnetic fields. It is a matter of fact that hadronic reactors have been producing such magnecular gases since 1998. This is a quite bizarre situation, and also with a somewhat macabre touch, since use of magnecular gas has been proven to have highly favorable ecological applications. Compared to molecules, magnecules have many different chemical attributes, explained in detail from hadronic chemistry and experimental evidence in the present volume. For example, when used as a fuel for vehicles, exhaust from combustion of magnecular hydrogen gas has a molecular composition very different from the exhaust of molecular hydrogen gas. The first does not contain potential carcinogens of the latter, has only half the CO2 content, and adds, contrary to the latter, a significant amount (10-12

Compared to the molecular hydrogen gas, the density of the corresponding magnecular gas is about 7.5 times higher. This implies that, on the same tank volume and pressure, a car fuelled on magnecules drives 7.5 times the distance of a car fuelled on molecules. Such effective magnecular fuel is not possible to produce without hadronic reactors, which construction presupposed hadronic mechanics with related hadronic mathematics. In this way, the existing hadronic technology, and there are other examples as well, offers quite simple tests to convince any sound skeptic about the superiority of the hadronic sciences as a whole, compared to standard physics constituted inside century old paradigms.

Hadronic reactors also offer considerable advantages on the input side, because they apply either oil or water solutions as their inputs, and the degree of pollution of the inputs does not matter, insofar as they are not radioactive. In the reactor process, where the plasma reaches temperatures higher than the surface of the sun, the molecules are broken down to their constituents before being recombined as magnecules with non-valence bonds. Thereby almost all molecular polluters disappear, including for example sewage water or pharmaceutical toxins. At the output side, there is produced, along with the magnecular gas, either chemical clean water or heat that can be applied for useful purposes. Furthermore, Santilli has also succeeded in developing magnecular technology specifically

designed as an additive to coal processing in order to reduce the globally heavy load of environmental pollution from this energy technology. Also to consider among Santillis amazing inventions, is the new hadronic technology of so-called intermediate nuclear fusion.

The foundations of scientific theory behind these technological progressions, which ought to be highly welcomed in the contemporary alarming ecological situation, are not only solid, but much more extensive and by far superior to the whole disciplines of standard quantum mechanics and chemistry, as fleshed out in much detail in the present volume. It is not without good reason that Santilli in his informative mammoth article in Foundations of Physics of Sept. 2003, a journal counting eight Nobel laureates in physics in its editorial board, emphasized the discovery of magnecules as the most precious fruit of his lifelong scientific endeavor.

The radical implications of scientific revolutions are hard to overview for contemporaries, sometimes including the pioneering scientists themselves. As a prominent mathematical physicist once said to the author of this postscript: Who would have guessed, back in the 1920s, that such a bizarre theory as quantum mechanics should gain such broad applications in upcoming technology? With regard to chemistry, it appears hard to find any historic parallel to the degree of progress represented or announced by hadronic chemistry, without moving back to the discovery of the periodic table. The panorama of magnecules reveals a previously hidden landscape of a whole new chemical world. It appears nave to suppose that these landscapes are restricted to artificial creations of substances by means of human high technology. In the last sentence of his 2001 book on hadronic chemistry, Santilli predicts the discovery of hyper-magnecules in biology. Also, his hadronic theory of lightning, offering more correct calculations of its accompanied sound quantities, describes this phenomenon as basically a hadronic reaction resulting in nitrogen synthesis. This may indicate that also other phenomena in nature, including biological and physiological nature, will prove to be better understood from hadronic chemistry, especially phenomena revealing superconductivity features. Of special significance may be the research and later applications of magnecular substances for medicine and health, a field so far not systematically targeted by advanced hadronic chemistry and technology, but already with some promising accumulation of more circumstantial evidence.

From the more overarching approach of the broader hadronic chemistry Santilli, partly in cooperation with other scientists, such as Shillady and Aringazin, from the late 1990s published new models also of the much studied molecules of hydrogen and water, earlier thought to be possible to be represented exactly by means of quantum chemistry, but argued by Santilli to be given exact representation of all chemical characteristics only by means of isochemical modeling not available for quantum chemistry. In 2007 Prof. Prez-Enrquez succeeded by

using hadronic chemistry to achieve a representation of the hydrogen molecule with amazingly exact matching with experimental data (among these representing the binding energy up to the 5th digit) by further developments from the Santilli-Shillady model and the Aringazin-Kucherenko approach, an achievement the preceding quantum chemistry was quite far from realizing. Also the work by Dr. Martin Cloonan has been able to reach new insights in fields of chemistry from his Cplex-isoelectronic theory by treating highly specialized knowledge in chemistry from the theoretical framework of hadronic chemistry. These recent developments may indicate an upcoming tendency to reframe specific problems of chemistry inside the broader umbrella of hadronic chemistry and thereby propel further progressions in the fields at hand, probably a challenge most suitable for the younger among talented chemists.

For many years Santilli has emphasized growing environmental concerns as a crucial motivation for his long-lasting scientific enterprise, and in the last decade also for his more recent occupation as an inventor. In spite of the many ecologically favorable applications of magnecular technology already appearing, Santilli regards the hadronic energy connected to the beta-electron released in the neutrons spontaneous decay as the most promising source for new and clean energy, likely to become harvested by upcoming hadronic technology based on hadronic mechanics. Calculations indicate that this energy is huge, without danger of radioactive radiation, and probably capable of capture by adequate trapping and shielding devices.

Considering this promising possibility judged from the theoretical advances in hadronic mechanics, and the possibly great implications for the ecosystem, it seems strange at first glance that powerful physics institutions and laboratories around the world so far have not wanted to execute crucial experiments to support or falsify predictions and earlier experiments from hadronic mechanics regarding neutron decay. The strangeness does not shrink when considering the modest amount of resources needed to execute such experiments, compared to the gigantic budgets of CERN and the like. Hadronic mechanics has already proved to be highly successful in achieving experimental verifications of new predictions from its theoretical extensions, as well as in constructing quite amazing new and eco-friendly technology outside the reach for quantum mechanics. A nave observer from outside the world of sophisticated theoretical physics may ask why it is that hadronic mechanics is being neglected, while a stream of resources is allocated to its sub-fields of quantum mechanics and relativity theory which has only been proven valid for the physical world outside the hadronic horizon. From reading semi-popular science magazines the outside observer will gain the impression that string theory is the most advanced physics around. But if so, how come that string theory, in spite of its rich inflow of mathematical talent and money resources, backed by mighty institutions, and much activity for some 25 years,

has not been successful in creating any new and favorable technology? Could it be that much of the reason is astonishingly simple, that these mathematical models have become too detached from the physical world, somewhat similar to the epicycles of the Middle Ages, constituting a self-sufficient and well fed giraffe-like research community not needing to care about rising revolutionary physics claiming basic theoretical advances backed by direct experimental support, or about the de facto emergence of new technology from this scientific revolution?

Scientific revolutions are not a tea party, and perhaps even less so in our time when the rise of significantly more advanced scientific theory not only threatens mighty characters in huge established science institutions, prestige hierarchies and networks nourished by a priori subscription to century old paradigms, but also related established interests in energy technology, finance and politics. Santilli has often stressed the evolutionary approach to this quest, by seeking serious dialogue and mutual exploration of the issues at hand with conventional scientists and institutions. In spite of this, Santilli has to a large extent been met with a Berlin wall of ignorance or non-scientific rejection, as indicated by the amazing near non-existence of published scientific questioning of the achievements in the hadronic sciences, today piling up to at least a library of 30.000 pages of published articles and monographs. Given the seriousness of the quest, not only for the further development of science, but for the very survival of our civilization by applying new technologies made possible from hadronic mechanics and chemistry, it seems likely that a more turbulent confrontation with different establishments antagonistic to radical extensions and liftings of conventional physics, is no longer possible to avoid. Considering the grave proportions of the rising ecological crisis, it may not be exaggerated to compare the situation with that of Semmelweiss, but with the difference that Santilli also talks from theoretical science above, not below the mighty scientists not able to leave their dogmas in spite of the implied damage done for the planet. Already in his three volume work of 1986, Documentation of the Ethical Probe, Santilli presented much food for thought concerning far from optimal scientific ethics being conducted in influential scientific communities. During the last two decades the picture has turned more severe, and the footnotes in the present volume provide much further material for competent evaluation of the present situation with regard to ethical vs. non-ethical conduct in the global science ecology. It may very well be that upcoming historians of science will look at the remarkably slow post-war development of main stream physics, when comparing the amount of basic advances to the resources spent and to the amount of advances the preceding part of the century, as connected to obstructions from profound non-scientific influences, paradoxically becoming fortified and nourished inside scientific institutions themselves.

Switching the focus to the brighter side, and lifting it to the visionary horizon inspiring great minds of science and art, it is important to note that hadronic

mechanics in its very architecture involves a whole new cosmology, opening vast new territories of the cosmos for human imagination, scientific exploration and technological endeavors.

Different from Einsteins relativity theory which doesnt treat antimatter, and different from quantum mechanics which allows the existence of antimatter only at second quantization, hadronic mechanics was able to treat matter and antimatter systematically on an equal footing, corresponding to the anti-symmetric structure in hadronic mathematics between the iso-, geno- and hyperfields vs. their respective isoduals. Hadronic mechanics comprehends our physical or Euclidean universe as a combination of two distinct universes, a matter universe and an antimatter universe. These two universes have a different anchoring in supra-spacetime, respectively in isospacetime and in isodual spacetime. However, isospacetime and isodual spacetime manifest in the same 3+1D space which they share and hence is to be comprehended as double-valued. Due to the antisymmetry of the two universes, positive mass in the matter universe will be projected as negative mass when experienced in the antimatter universe, and the same the other way around, and also the same with all other physical quantities, such as time, charge and energy. For the universe as a whole combination of the matter and the antimatter universe, all these magnitudes cancel out to zero. (This is also consistent with the key notion in the ambitious theory of universal rewrite nilpotent system recently worked out by mathematical physicist Peter Rowlands.)

This implies a comprehension of space itself as a universal substratum composed of a superposition of positive and negative energies, from which matter and antimatter galaxies are continuously created. This seems to provide an elegant solution for the mystery of from where the universe, considered as a closed system, receives its energy as a whole. If the universe has a paradoxical twin structure, the puzzle may be solved from a metabolism between the two moieties from the universal substratum, where the output energy from one moiety is received with the opposite sign as input energy for the other moiety, while the energy of the total universe remains zero or nilpotent. The philosophically quite simplistic Big Bang hypothesis, popular in much 20th century physics, is an answer to a question about the origin of the universe that does not make much sense when reframed from the more sophisticated cosmology and ontology of hadronic mechanics. Regarded from hadronic cosmology, treating antimatter with scientific democracy, as Santilli likes to put it, it is not quite the same universe anymore. According to hadronic cosmology, the universe is rather comprehended as inherently and continuously re-created, as it was by the great scientist David Bohm. On this background the Big Bang (and Crunch) hypothesis may be more adequately understood as a creation myth suitable for a conflated physicalistic and entropic world view painted in scientific cosmetics.

Hadronic cosmology constitutes a platform for much more optimistic and ambitious scientific undertakings. Santillis theory of antimatter has formulated precise predictions of antigravity phenomena, and has designed experimental tests of antigravity for positrons and isodual light. Also, hadronic mechanics includes the notion of bound states of matter and antimatter, coined isoselfdual states, which opens up the possibility for time travel in the matter universe via intermediary switching onto the antimatter universe. Furthermore, Santilli describes causal spacetime machines which is the theoretical notion of way more radical space travel than the rocket technology developed half a century ago, and which applies the principle of isogeometric propulsion without Newtonian action-reaction. Hence, the realism in developing UFO technology for space travel much faster than the speed of light in vacuum, does not seem farfetched anymore from the theoretical advances of hadronic mechanics. These advances were only possible from the broadening of the theory of physics to include antimatter on an equal footing with matter, which in its turn presupposed the development of the new isonumber fields, with corresponding isogeometry, for quantitative treatments.

It is worth noticing that such space deformations are accompanied by changes in time as we ordinarily understand it. This implies a detrivialization of the conventional time concept, where the familiar time arrow reduces to just one aspect of a more complex configuration of different types of time flows. In his pioneering studies of sea shell growth from hadronic geometry Chris Illert showed in the mid-1990s that a certain class of bifurcating sea shell followed a growth path that presupposed two non-trivial kinds of time flows, perceived as jumps forward and backward in conventional time. Such discovery of non-trivial time flows in a sufficiently profound specialist study of a complex irreversible system of nature, was exactly what was expected from the new time theory of hadronic mechanics which had added four types of non-trivial categories, so-called geno-times, to the conventional notion of time. Santilli has stated that for practical purposes there is no scientific difference between the new physical principles discovered in branching sea shells and those involved in the notion of causal spacetime machines.

Throughout the last century the quest of grand unification of gravitation with the three other conventional forces of physics remained a puzzling open problem in the struggles of standard physics. Santillis theory of grand unification from hadronic mechanics presents gravitation as a macro phenomenon aggregated (with presented equations) from quantum electrodynamics de facto rooted in energy from the vacuum or universal medium. However, such a grand unification is argued by Santilli still not to be theoretically possible without acknowledging the democratic co-existence of an antimatter universe, a theory of physics not available before the development of hadronic mechanics. Accordingly, there was no mystery that grand unification became out of reach for standard physics restricted to quantum mechanics and Einstein relativity theory. From this approach

Santilli argued that grand unification was possible only as recognizing the quest as two connected grand unifications, one for the matter universe and one for the antimatter universe, to become integrated in a combined grand unification, and accordingly coined Iso-Grand-Unification, requiring isomathematics for its fulfillment.

Differently from 20th century standard physics, hadronic mechanics has provided a general scientific umbrella, sophisticated, abstract and broad enough to encompass life in its extension, at least in a much more emphatic and radical sense. This is intimately connected to the structure of the higher landscapes of hadronic mathematics, to be considered not only as tools but as structures complex enough to offer adequate maps of lifes phenomena. Due to the lack of isonumbers required to describe hadronic superconductivity, quantum mechanics was never able to catalyze much progress in chemistry, with growing disconnection between physics and chemistry as a result. For mappings of biological structures, genonumbers become crucial to grasp the fundamental irreversibility characterizing the complexity of the biological world (as well as already the behavior of stars, galaxies and quasars). After a lifting to genostructures, the whole field of isostructures, which still implied reversibility in its basic mathematical axioms, reappears only as the subfield of genostructures where reversibility constitutes a special case. The further lifting from genostructures to the much broader hyperstructures achieves not only irreversibility, but the multi-valued theory required to map even more complex structures of life. Santilli notes that when described as a multi-valued hyperstructure, the same seashell can overlap a large number of spaces and their isoduals, resulting in multi-fold formulations including the four different directions of time. The relevance of hyperstructures to describe really complex life phenomena becomes perhaps most immediately and intuitively obvious if we move to psychology and reflects on the multi-fold dynamic constellation of mind spaces and time travels involved in ordinary human thinking.

This may indicate that the top floor in the huge building of hadronic mechanics, hypermechanics, is sophisticated enough to include also mental and social phenomena. In standard physics the quest for grand unification was restricted to a unification of the four conventional physical forces, silently regarding the mental and social worlds as mystically separated from the universe or as mere epi-phenomena mirroring or emerging from the four physical forces. On this background it is highly interesting that Santilli not only presents an (iso-)grand unification of the four forces in chapter 14 of the present volume, but takes the steps all the way up to a Hyper-Grand-Unification. In the modern development of science and society, the frontier of physics has always been highly influential indirectly on other disciplines, being regarded as the most authoritative discipline concerning what is to be stated with the highest degree of scientific certainty with respect to the basic issues of our cosmos. The rise of hadronic mechanics, with

the present volume presenting a systematic overview of its most mature achievements, constitutes a much more radical scientific revolution, since the argued fruits of hypermechanics are far from being relevant only for physics, but seems directly relevant for all scientific disciplines, and this in a profound manner.

Santilli notes that all distinctions between matter and antimatter are lost at the hyperstructural level and that at this highest possible level of formulation, we have one single hyperrelativity, one single Poincar-Santilli hypersymmetry (chapter 6.1.15). In this regard the advanced science of hypermechanics is in accord with the basic notion of cosmos being a unitary whole, characterizing great natur philosophy, such as Plotinus, Kant, Hegel and Bohm. Santilli also states: The foundation of our hypercosmology on the universal hypersymmetry is the single most important result of the authors lifetime of research because it governs the totality of the events in the universe (ibid.).

Being based on symmetry, the hypercosmology of hadronic mechanics differs from Einstein gravity and other preceding cosmologies of physics. The unitary whole of the cosmos is reflected in Santilli coining this cosmology hyper-self-dual, and Santilli explicitly states the necessity of lifting the cosmology from isotopic and genotopic theories to the hypertopic level because a basic component of the universe is life (chapter 14.2) which needs multi-valued descriptions to become comprehended.

In spite of the imagined universality of the hyper-self-dual cosmology and hyper-hadronic mechanics, Santilli is careful by stating that science will never admit a final theory. This humble attitude, the complementary polarity to the visionary extreme ambition also characterizing scientific genius, differs remarkably from physicists clinging to doctrines from Einstein relativity more like religious dogma and for eternity. This was an attitude quite alien to Einstein himself who published his break-through articles without one single reference to any authority (or non-authority), and let the power of thought speak for itself.

Santilli holds Einstein in very high esteem, and declares him explicitly as the greatest scientist of the last century. However, the admiration between deeply creative and thereby related minds seems to be of another kind than that between a genius and the later followers of his established authority. One might say that Santillis admiration of Einstein is more profound, insofar as the scientific thinking of Santilli himself exposes a similar brave, original and creative line of thought. From this also follows a scientific obligation to leave home if and when the pupil reaches far enough to explore unknown higher territories in the mountains of knowledge, climbing from the shoulders of his master. Santilli is careful in the present volume, as in earlier works, to pinpoint under which constraints Einstein relativity is still to be considered valid, and at the same time to state loud and clear why the masters theories do not hold when these constraints are abandoned, and therefore was in need of a more lifted and broader theory of physics which

Santilli went out to create through forty years of hard work. Considering all the experimental evidence from the 1990s on, showing beyond serious doubt that the light speed in vacuum does not represent any ultimate barrier for velocity, explained by hadronic mechanics as a necessity inside hyper-dense hadron media, it seems quite pathetic when the authority of Einstein is mobilized as rhetoric ammunition to obstruct such theory formation and recognition.

It has been said that the real masters greatest satisfaction is when he realizes that his pupil has grown beyond the skills of himself. If allowing such an analogy for the case of clarifying proportions, Einstein ought to have every reason to evaluate his pupil Santilli with delightful satisfaction. Like Einstein, Santilli has pushed the frontier of physics far beyond earlier imagination. However, unlike Einstein, Santilli has also pushed the frontier of the whole of physics, as well as the frontiers of whole disciplines outside physics foremost chemistry and mathematics, but also theoretical biology, and with direct implications also for other disciplines, among them philosophy. So, all in all, it seems hard to doubt that history will judge Santilli as an even greater genius than Einstein.

In the history of mankind there are very few examples of scientists showing brilliance both in mathematics (whether pure or applied to physics) and in the art of invention, the Norwegian Kristian Birkeland (1867-1917) constituting one of the few worth mentioning. With his amazing patents, as well as different types of constructed hadronic reactors producing the new chemical species of magnecules, Santilli has also proven extraordinary skills as an inventor, praised by Tesla as the foremost among sciences, as well as a laboratory man. These skills, indicating intuitively precise connectedness to the rock hard and dynamic physical world, ought to give further credibility to the practical and direct relevance of the theoretical physics and chemistry of Santilli, constituting a character quite different from the more ivory tower type of mathematical physicists.

The present volume may represent a suitable closing of Santillis pioneering monographs given to the world to whom it could concern as perhaps the richest collection of scientific goodies ever presented to Mankind, whose future may depend crucially on what it does with the treasures contained in this opus magnum. With this publication, serious scientists and scholars with open and critical minds across a plethora of disciplines have been given heavy loads of precious ideas to digest and cultivate for many a year to come. In spite of Santilli often using the expression young minds of all ages, the scientific presents are doomed to primarily become appetizers to consider for the younger and most emergent upcoming among those minds, because they will become the carriers and releasers of the future, if any. Besides the thrills of discovery in absorbing the monograph itself, as well as from explorative adventures fuelled by inspiration from it, there will also be a heavy load of social responsibility and dedicated action to carry out, considering signs of rising turbulence inside as well as outside science.

At Christmas time most people appreciate Santa Claus showing up to give them exclusive presents for delight. Sad to say, this is far from always being the situation in scientific communities, nor in society at large. Considering the immense obstacles to and antagonisms, be it brute or more sophisticated, against Santilli fulfilling his mission to science and to Mankind, it is quite a mystery in itself how this man has been able to keep on track, busily creating new insights with heroic energy and steady devotion seemingly greater than life, even after entering his eighth decade on the planet. The footnotes in this volume give some indication of the emotional challenge and burden involved therein, and tells of an intellectual honesty, integrity and boldness paradigmatic for any scientist, whatever degree of intelligence or idiosyncratic inclination.

Santilli holds the dream of humanity becoming able to harvest the huge clean energy connected to the beta-electron from neutron synthesis, predicted as a realistic possibility within reach from the physics of hadronic mechanics. At the same time, hadronic mechanics points out the missing energy in this synthesis when described by conventional physics, and locates the source of this energy gap to originate from the high energy density of the universal medium, by the way a statement similar to the avant-garde Russian physicist Kozyrev arguing the stars not to be fuelled by energy from their exterior. Whatever the destiny of this dream, it must be stated beyond doubt that the life work of Santilli represents quite a neutron synthesis in itself, fuelled from beyond the stars, with the present monograph constituting a new and clean hadronic energy of parachuting fruits from the tree of advanced and matured scientific knowledge, to be picked and eaten for the delight of the world. The release of this testament of Santillis science to the world ought to be honored with the uttermost gratitude and hungry attention. Science is nothing if not living science, so I find it irresponsible not to declare the historic proportions of the Santilli legacy, as to the best of my knowledge and judgment. Hence, on the possible behalf also of some future state of the affairs of the world and its science, I take the liberty to pass a 1001 thank you to the Great Italian - and may he stay forever young.

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